SATPREP

Applications of Sine and Cosine Rule

1. The following diagram shows a triangle with sides 5 cm, 7 cm, 8 cm.

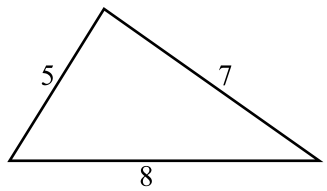


Diagram not to scale

Find

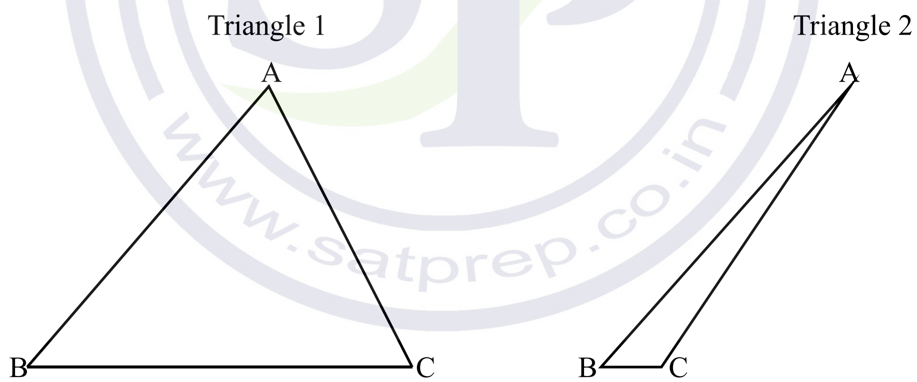
(a) the size of the smallest angle, in degrees;

(b) the area of the triangle.

2. The diagrams below show two triangles both satisfying the conditions

$$AB = 20 \text{ cm}, AC = 17 \text{ cm}, A\hat{B}C = 50°.$$

Diagrams not to scale



(a) Calculate the size of $A\hat{C}B$ in **Triangle 2.**

(b) Calculate the area of **Triangle 1**.

3. In the triangle PQR, PR = 5 cm, QR = 4 cm and PQ = 6 cm.

Calculate

(a) the size of $P\hat{Q}R$;

(b) the area of triangle PQR.

4. A triangle has sides of length 4, 5, 7 units. Find, to the nearest tenth of a degree, the size of the largest angle.

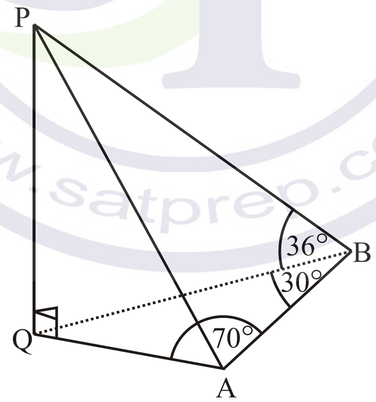
5. Two boats A and B start moving from the same point P. Boat A moves in a straight line at 20 km h^{-1} and boat B moves in a straight line at 32 km h^{-1}. The angle between their paths is 70°.

Find the distance between the boats after 2.5 hours.

6. In a triangle ABC, AB = 4 cm, AC = 3 cm and the area of the triangle is 4.5 cm^2.

Find the **two** possible values of the angle $B\hat{A}C$.

7. The diagram shows a vertical pole PQ, which is supported by two wires fixed to the horizontal ground at A and B.



$BQ = 40$ m

$P\hat{B}Q = 36°$

$B\hat{A}Q = 70°$

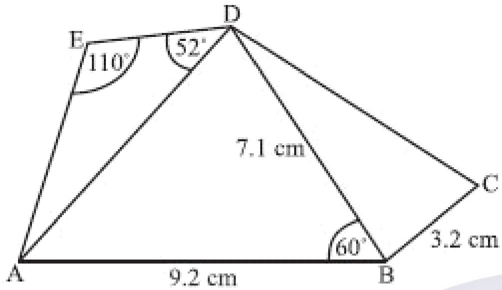
$A\hat{B}Q = 30°$

Find

(a) the height of the pole, PQ;

(b) the distance between A and B.

8. The following diagram shows a pentagon ABCDE, with AB = 9.2 cm, BC = 3.2 cm, BD = 7.1 cm, $A\hat{E}D = 110°$, $A\hat{D}E = 52°$ and $A\hat{B}D = 60°$.



(a) Find AD.

(b) Find DE.

(c) The area of triangle BCD is 5.68 cm^2. Find $D\hat{B}C$.

(d) Find AC.

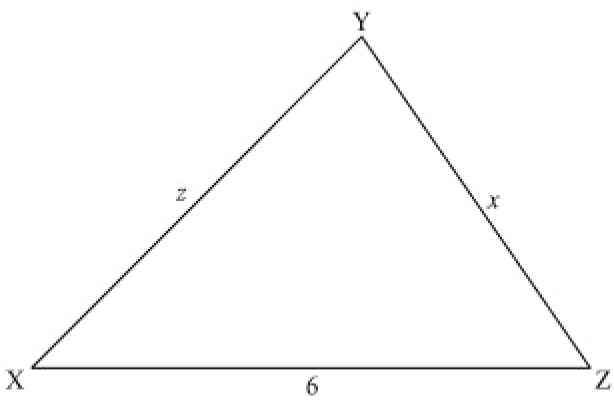
(e) Find the area of quadrilateral ABCD.

9. (a) Let $y = -16x^2 + 160x - 256$. Given that y has a maximum value, find

(i) the value of x giving the maximum value of y;

(ii) this maximum value of y.

The triangle XYZ has XZ = 6, YZ = x, XY = z as shown below. The perimeter of triangle XYZ is 16.



(b) (i) Express z in terms of x.

(ii) Using the cosine rule, express z^2 in terms of x and $\cos Z$.

(iii) Hence, show that $\cos Z = \dfrac{5x-16}{3x}$.

Let the area of triangle XYZ be A.

(c) Show that $A^2 = 9x^2 \sin^2 Z$.

(d) Hence, show that $A^2 = -16x^2 + 160x - 256$.

(e) (i) Hence, write down the maximum area for triangle XYZ.

(ii) What type of triangle is the triangle with maximum area?

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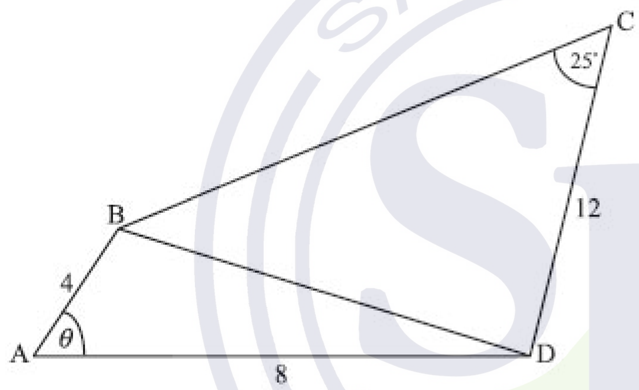
10. The diagram below shows a quadrilateral ABCD. AB = 4, AD = 8, CD =12, $B\hat{C}D = 25°$, $B\hat{A}D = \theta$.



(a) Use the cosine rule to show that $BD = 4\sqrt{5-4\cos\theta}$.

Let $\theta = 40°$.

(b) (i) Find the value of $\sin C\hat{B}D$.

(ii) Find the two possible values for the size of $C\hat{B}D$.

(iii) Given that $C\hat{B}D$ is an acute angle, find the perimeter of ABCD.

(c) Find the area of triangle ABD.

Answers :

1. (a) The smallest angle is opposite the smallest side.

$$\cos\theta = \frac{8^2 + 7^2 - 5^2}{2\times 8\times 7}$$

$$= \frac{88}{112} = \frac{11}{14} = 0.7857$$

Therefore, $\theta = 38.2°$

(b) Area $= \frac{1}{2} \times 8 \times 7 \times \sin 38.2°$

$= 17.3 \text{ cm}^2$

2. (a) $\frac{\sin(\mathrm{A\hat{C}B})}{20} = \frac{\sin 50°}{17}$

$\Rightarrow \sin(\mathrm{A\hat{C}B}) = \frac{20\sin 50°}{17} = 0.901$

$\mathrm{A\hat{C}B} > 90° \Rightarrow \mathrm{A\hat{C}B} = 180° - 64.3° = 115.7°$

$\mathrm{A\hat{C}B} = 116$ (3 sf)

(b) In Triangle 1, $\mathrm{A\hat{C}B} = 64.3°$

$\Rightarrow \mathrm{B\hat{A}C} = 180° - (64.3° + 50°)$

$= 65.7°$

Area $= \frac{1}{2}(20)(17)\sin 65.7° = 155\ (\text{cm}^2)$ (3 sf)

3. (a) Evidence of using the cosine rule

eg $\cos\ \mathrm{P\hat{Q}R} = \frac{p^2 + r^2 - q^2}{2\,pr}$, $q^2 = p^2 + r^2 - 2pr\ \cos\ \mathrm{P\hat{Q}R}$

Correct substitution

eg $\frac{4^2 + 6^2 - 5^2}{2\times 4\times 6}$, $5^2 = 4^2 + 6^2 - 2\times 4\times 6\ \cos Q$

$$\cos\ \mathrm{P\hat{Q}R} = \frac{27}{48} = 0.5625$$

$$\mathrm{P\hat{Q}R} = 55.8°\ (0.973 \text{ radians})$$

(b) Area = $\frac{1}{2}pr\sin\ \text{P}\hat{\text{Q}}\text{R}$

For substituting correctly $\frac{1}{2}\times 4\times 6\ \sin 55.8$

$= 9.92\ (\text{cm}^2)$

4. ***Note:*** *Award (M1) for identifying the largest angle.*

$\cos\alpha = \dfrac{4^2+5^2-7^2}{2\times 4\times 5}$

$= -\dfrac{1}{5}$

$\Rightarrow \alpha = 101.5°$

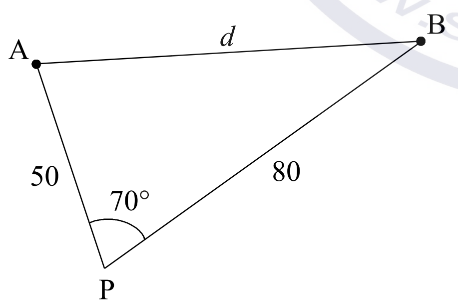
OR Find other angles first

$\beta = 44.4°$ $\gamma = 34.0°$

$\Rightarrow \alpha = 101.6°$

Note: *Award (C3) if not given to the correct accuracy.*

5.



OR
2.5 × 20 = 50
2.5 × 32 = 80

$d^2 = 50^2 + 80^2 - 2\times 50\times 80\times\cos 70°$
$d = 78.5$ km

6. Area of a triangle = $\frac{1}{2} \times 3 \times 4 \sin A$

$\frac{1}{2} \times 3 \times 4 \sin A = 4.5$

$\sin A = 0.75$

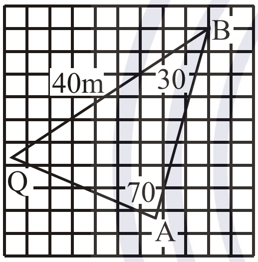
$A = 48.6°$ and $A = 131°$ (or 0.848, 2.29 radians)

Note: *Award (C4) for 48.6° only, (C5) for 131° only.*

7. (a) $\frac{PQ}{40} = \tan 36°$

$\Rightarrow PQ \approx 29.1$ m (3 sf)

(b)



$A\hat{Q}B = 80°$

$\frac{AB}{\sin 80°} = \frac{40}{\sin 70°}$

Note: *Award (M1) for correctly substituting.*

$\Rightarrow AB = 41\ 9.$ m (3 sf)

8. (a) Evidence of choosing cosine rule

eg $a^2 = b^2 + c^2 - 2bc \cos A$

Correct substitution

eg $(AD)^2 = 7.1^2 + 9.2^2 - 2(7.1)(9.2) \cos 60°$

$(AD)^2 = 69.73$

$AD = 8.35$ (cm)

(b) $180° - 162° = 18°$

Evidence of choosing sine rule

Correct substitution

eg $\dfrac{\text{DE}}{\sin 18°} = \dfrac{8.35}{\sin 110°}$

DE = 2.75 (cm)

(c) Setting up equation

eg $\frac{1}{2}\,ab\sin C = 5.68$, $\frac{1}{2}\,bh = 5.68$

Correct substitution

eg $5.68 = \frac{1}{2}(3.2)\,(7.1)\sin \text{D}\hat{\text{B}}\text{C}$, $\frac{1}{2}\times 3.2\times h = 5.68$, $(h = 3.55)$

$\sin \text{D}\hat{\text{B}}\text{C} = 0.5$

$\text{D}\hat{\text{B}}\text{C}$ 30° and/or 150°

(d) Finding $\text{A}\hat{\text{B}}\text{C}$ $(60° + \text{D}\hat{\text{B}}\text{C})$

Using appropriate formula

eg $(\text{AC})^2 = (\text{AB})^2 + (\text{BC})^2$, $(\text{AC})^2 = (\text{AB})^2 + (\text{BC})^2 - 2\,(\text{AB})$
$(\text{BC})\cos \text{ABC}$

Correct substitution (allow ***FT*** on **their** seen $\text{A}\hat{\text{B}}\text{C}$)

eg $(\text{AC})^2 = 9.2^2 + 3.2^2$

AC = 9.74 (cm)

(e) For finding area of triangle ABD

Correct substitution Area = $\frac{1}{2}\times 9.2\times 7.1\sin 60°$

$= 28.28...$

Area of ABCD = 28.28... + 5.68

$= 34.0\ (cm^2)$

9. (a) **METHOD 1**

***Note**: There are many valid algebraic approaches to this problem (eg completing the square, using $x = \frac{-b}{2a}$). Use the following mark allocation as a guide.*

(i) Using $\frac{dy}{dx} = 0$

$-32x + 160 = 0$

$x = 5$

(ii) $y_{max} = -16(5^2) + 160(5) - 256$

$y_{max} = 144$

METHOD 2

(i) Sketch of the correct parabola (may be seen in part (ii))

$x = 5$

(ii) $y_{max} = 144$

(b) (i) $z = 10 - x$ (accept $x + z = 10$)

(ii) $z^2 = x^2 + 6^2 - 2 \times x \times 6 \times \cos Z$

(iii) Substituting for z into the expression in part (ii)

Expanding $100 - 20x + x^2 = x^2 + 36 - 12x \cos Z$

Simplifying $12x \cos Z = 20x - 64$

Isolating $\cos Z = \frac{20x - 64}{12x}$

$\cos Z = \frac{5x - 16}{3x}$

***Note**: Expanding, simplifying and isolating may be done in any order, with the final A1 being awarded for an expression that clearly leads to the required answer.*

(c) Evidence of using the formula for area of a triangle

$$\left(A = \frac{1}{2} \times 6 \times x \times \sin Z\right)$$

$$A = 3x \sin Z \left(A^2 = \frac{1}{4} \times 36x^2 \times \sin^2 Z\right)$$

$$A^2 = 9x^2 \sin^2 Z$$

(d) Using $\sin^2 Z = 1 - \cos^2 Z$

Substituting $\frac{5x-16}{3x}$ for $\cos Z$

for expanding $\left(\frac{5x-16}{3x}\right)^2$ to $\left(\frac{25x^2 - 160x + 256}{9x^2}\right)$

for simplifying to an expression that clearly leads to the required answer

eg $A^2 = 9x^2 - (25x^2 - 160x + 256)$

$A^2 = -16x^2 + 160x - 256$

(e) (i) 144 (is maximum value of A^2, from part (a))

$A_{max} = 12$

(ii) Isosceles

10. (a) For **correct** substitution into cosine rule

$$\text{BD} = \sqrt{4^2 + 8^2 - 2 \times 4 \times 8 \cos\theta}$$

For factorizing 16, $\text{BD} = \sqrt{16(5 - 4\cos\theta)}$

$$= 4\sqrt{5 - 4\cos\theta}$$

(b) (i) BD = 5.5653 ...

$$\frac{\sin \text{C}\hat{\text{B}}\text{D}}{12} = \frac{\sin 25}{5.5653}$$

$\sin \text{C}\hat{\text{B}}\text{D} = 0.911$ (accept 0.910, subject to ***AP***)

(ii) $\text{C}\hat{\text{B}}\text{D} = 65.7°$

Or $C\hat{B}D = 180 -$ their acute angle

$= 114°$

(iii) $B\hat{D}C = 89.3°$

$$\frac{BC}{\sin 89.3} = \frac{5.5653}{\sin 25} \text{ or } \frac{BC}{\sin 89.3} = \frac{12}{\sin 65.7} \text{ (or cosine rule)}$$

$BC = 13.2$ (accept 13.17…)

Perimeter $= 4 + 8 + 12 + 13.2$

$= 37.2$

(c) Area $= \frac{1}{2} \times 4 \times 8 \times \sin 40°$

$= 10.3$