

## Double angle trigonometric Identity

Verify each identity.

$$1) 2\sin x \cos x - \cos 2x = 2\sin^2 x - 1 + \sin 2x$$

$$2) \cos^2 x + \tan^2 x \sin^2 x = \sec^2 x \sin^2 x + \cos 2x$$

$$3) \frac{2\sin^2 x}{\tan x} = \sin 2x$$

$$4) 1 + \cos 2x - \tan 4x \cos 4x = 2\cos^2 x - \sin 4x$$

$$5) \frac{\tan x}{\sin 2x} = \frac{\sec^2 x}{2}$$

$$6) \csc^2 x \tan x = \frac{2}{\sin 2x}$$

$$7) \frac{1 + \tan^2 x}{\tan^2 x} = \frac{2}{1 - \cos 2x}$$

$$8) \tan^2 x (1 + \cos 2x) = \frac{2}{\csc^2 x}$$

$$9) \frac{\tan x}{1 - \cos 2x} = \frac{1}{\sin 2x}$$

$$10) 3\cos^2 x = \cos 2x + 2 - \sin^2 x$$

## Answers to Double angle trigonometric Identity

1)  $2\sin x \cos x - \cos 2x$       Use  $\cos 2x = 1 - 2\sin^2 x$

$2\sin x \cos x - 1 + 2\sin^2 x$       Use  $\sin 2x = 2\sin x \cos x$

$2\sin^2 x - 1 + \sin 2x$       ■

2)  $\cos^2 x + \tan^2 x \sin^2 x$       Use  $\cos 2x = \cos^2 x - \sin^2 x$

$\sin^2 x \tan^2 x + \sin^2 x + \cos 2x$       Use  $\tan^2 x + 1 = \sec^2 x$

$\sec^2 x \sin^2 x + \cos 2x$       ■

3)  $\frac{2\sin^2 x}{\tan x}$       Use  $\tan x = \frac{\sin x}{\cos x}$

$\frac{2\sin^2 x \cos x}{\sin x}$       Cancel common factors

$2\cos x \sin x$       Use  $\sin 2x = 2\sin x \cos x$

$\sin 2x$       ■

4)  $1 + \cos 2x - \tan 4x \cos 4x$       Use  $\cos 2x = 2\cos^2 x - 1$

$2\cos^2 x - \tan 4x \cos 4x$       Decompose into sine and cosine

$2\cos^2 x - \frac{\sin 4x}{\cos 4x} \cdot \cos 4x$       Simplify

$2\cos^2 x - \sin 4x$       ■

5)  $\frac{\tan x}{\sin 2x}$       Use  $\sin 2x = 2\sin x \cos x$

$\frac{\tan x}{2\sin x \cos x}$       Use  $\tan x = \frac{\sin x}{\cos x}$

$\frac{\sin x}{2\cos^2 x \sin x}$       Cancel common factors

$\frac{1}{2\cos^2 x}$       Use  $\sec x = \frac{1}{\cos x}$

$\frac{\sec^2 x}{2}$       ■

6)  $\csc^2 x \tan x$       Decompose into sine and cosine

$\left(\frac{1}{\sin x}\right)^2 \cdot \frac{\sin x}{\cos x}$       Simplify

$\frac{1}{\sin x \cos x}$       Create a common factor

$\frac{2}{2\sin x \cos x}$       Use  $\sin 2x = 2\sin x \cos x$

$\frac{2}{\sin 2x}$       ■

$$7) \frac{1 + \tan^2 x}{\tan^2 x} \quad \text{Use } \tan^2 x + 1 = \sec^2 x$$

$$\frac{\sec^2 x}{\tan^2 x} \quad \text{Decompose into sine and cosine}$$

$$\frac{\left(\frac{1}{\cos x}\right)^2}{\left(\frac{\sin x}{\cos x}\right)^2} \quad \text{Simplify}$$

$$\frac{1}{\sin^2 x} \quad \text{Use } \sin^2 x = \frac{1 - \cos 2x}{2}$$

$$\frac{2}{1 - \cos 2x} \quad \blacksquare$$

$$8) \tan^2 x(1 + \cos 2x) \quad \text{Use } \tan^2 x = \frac{1 - \cos 2x}{1 + \cos 2x}$$

$$\frac{(1 + \cos 2x)(1 - \cos 2x)}{1 + \cos 2x} \quad \text{Cancel common factors}$$

$$1 - \cos 2x \quad \text{Use } \cos 2x = 1 - 2\sin^2 x$$

$$2\sin^2 x \quad \text{Use } \csc x = \frac{1}{\sin x}$$

$$\frac{2}{\csc^2 x} \quad \blacksquare$$

$$9) \frac{\tan x}{1 - \cos 2x} \quad \text{Use } \cos 2x = 1 - 2\sin^2 x$$

$$\frac{\tan x}{2\sin^2 x} \quad \text{Use } \tan x = \frac{\sin x}{\cos x}$$

$$\frac{\sin x}{2\cos x \sin^2 x} \quad \text{Cancel common factors}$$

$$\frac{1}{2\sin x \cos x} \quad \text{Use } \sin 2x = 2\sin x \cos x$$

$$\frac{1}{\sin 2x} \quad \blacksquare$$

$$10) 3\cos^2 x \quad \text{Use } \cos^2 x = \frac{1 + \cos 2x}{2}$$

$$\frac{3(1 + \cos 2x)}{2} \quad \text{Use } \cos 2x = 1 - 2\sin^2 x$$

$$\frac{6(1 - \sin^2 x)}{2} \quad \text{Cancel common factors}$$

$$3(1 - \sin^2 x) \quad \text{Use } \cos 2x = 1 - 2\sin^2 x$$

$$\cos 2x + 2 - \sin^2 x \quad \blacksquare$$