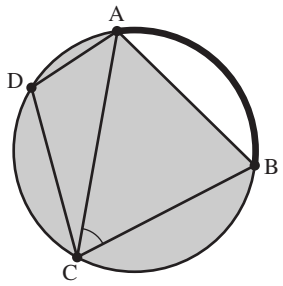
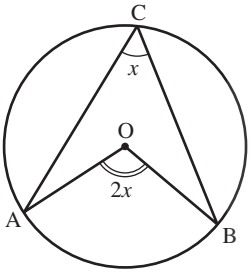
Circle theorems

There are four theorems on angles in circles that you should know. First you need to learn some words.

- The straight line AB is a **chord**.
- The curved line AB (in bold) is an **arc**.
- The chord AB divides the circle into two **segments** – the major segment (shaded) and the minor segment (unshaded).
- $A\hat{C}B$ is the angle **subtended** by AB at C.
- ABCD is a **cyclic quadrilateral**.

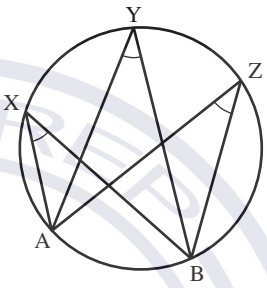


Theorem 1
The angle subtended at the centre of a circle is twice the angle subtended at the circumference.



$A\hat{O}B = 2 \times A\hat{C}B$

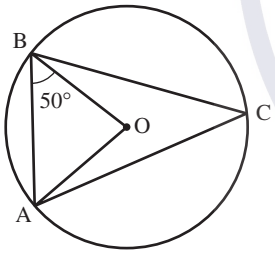
Theorem 2
Angles subtended by an arc in the same segment of a circle are equal.



$A\hat{X}B = A\hat{Y}B = A\hat{Z}B$

The proof of this theorem is given in the section on proof.

(a) Given $A\hat{B}O = 50°$, find $B\hat{C}A$.



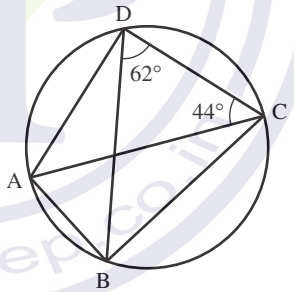
Triangle OBA is isosceles (OA = OB).

$\therefore$ $O\hat{A}B = 50°$

$\therefore$ $B\hat{O}A = 80°$ (angle sum of a triangle)

$\therefore$ $B\hat{C}A = 40°$ (angle at the centre)

(b) Given $B\hat{D}C = 62°$ and $D\hat{C}A = 44°$ find $B\hat{A}C$ and $A\hat{B}D$.



$B\hat{D}C = B\hat{A}C$ (both subtended by arc BC)

$\therefore$ $B\hat{A}C = 62°$

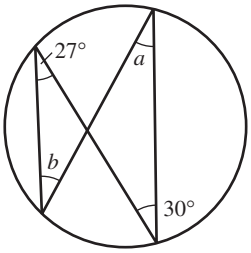
$D\hat{C}A = A\hat{B}D$ (both subtended by arc DA)

$\therefore$ $A\hat{B}D = 44°$

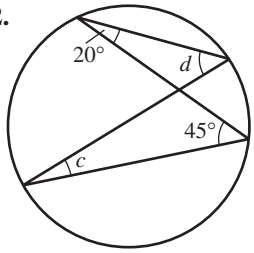
Exercise 1

Find the angles marked with letters. A line passes through the centre only when point O is shown.

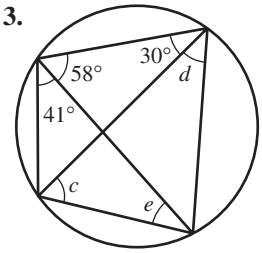
1.



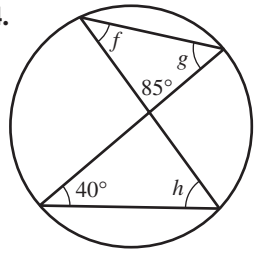
2.



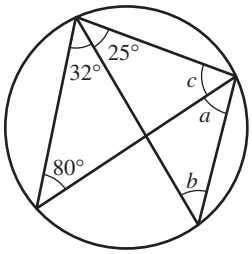
3.



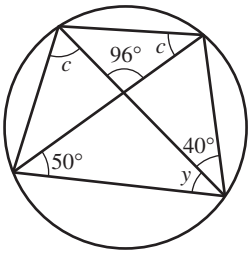
4.



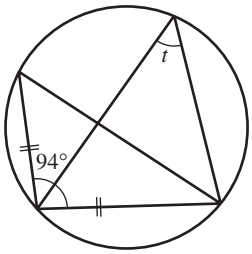
5.



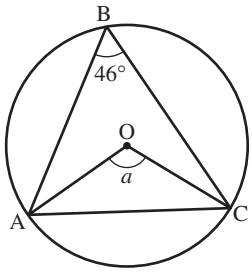
6.



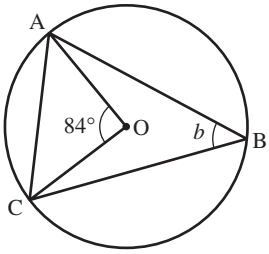
7.



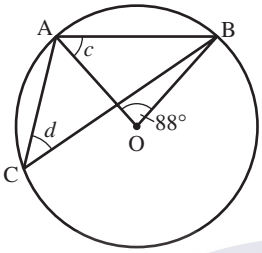
8.



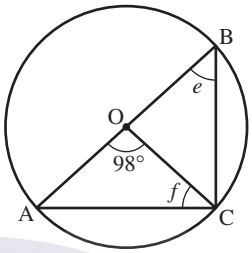
9.



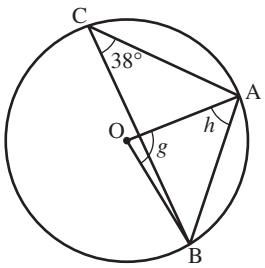
10.



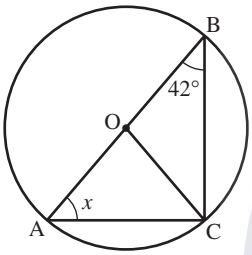
11.



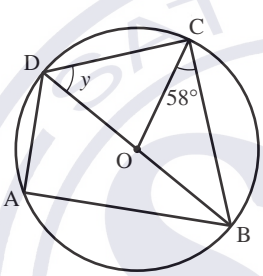
12.



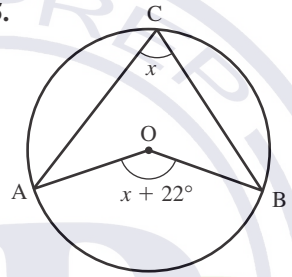
13.



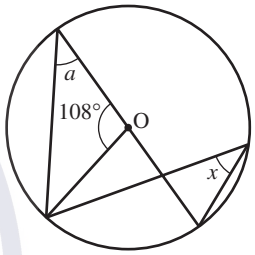
14.



15.

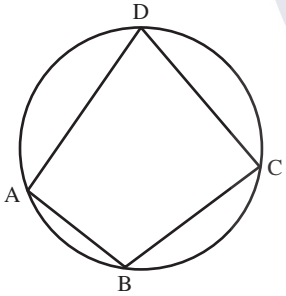


16.



Theorem 3

The opposite angles in a cyclic quadrilateral add up to 180° (the angles are supplementary).



$\widehat{A} + \widehat{C} = 180^\circ$

$\widehat{B} + \widehat{D} = 180^\circ$

Find a and x.

$a = 180^\circ - 81^\circ$

(opposite angles of a cyclic quadrilateral)

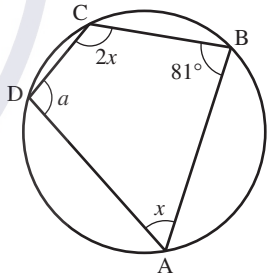
$\therefore \quad a = 99^\circ$

$x + 2x = 180^\circ$

(opposite angles of a cyclic quadrilateral)

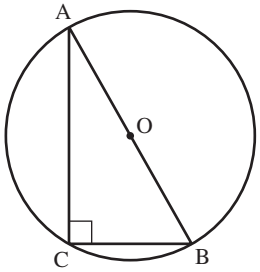
$3x = 180^\circ$

$\therefore \quad x = 60^\circ$



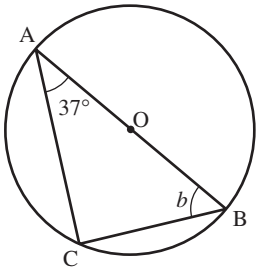
Theorem 4

The angle in a semicircle is a right angle.



In the diagram,
AB is a diameter.
$\widehat{ACB} = 90^\circ$.

Find b given that AOB is a diameter.



$\widehat{ACB} = 90^\circ$ (angle in a semicircle)

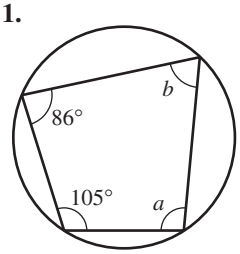
$\therefore \quad b = 180^\circ - (90 + 37)^\circ$

$= 53^\circ$

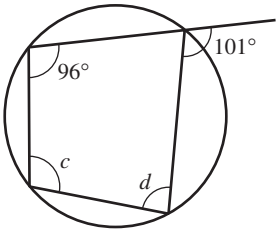
Exercise 2

Find the angles marked with a letter.

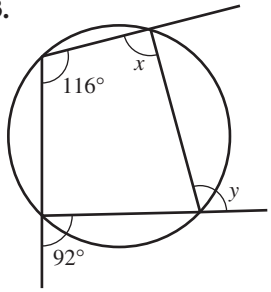
1.



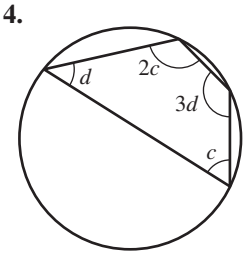
2.



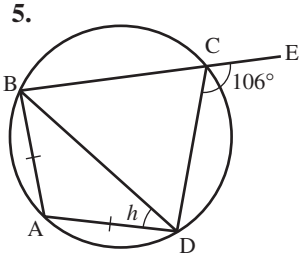
3.



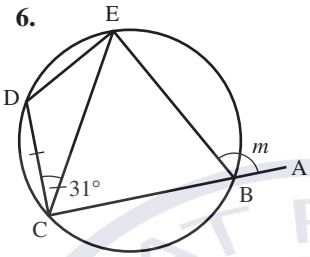
4.



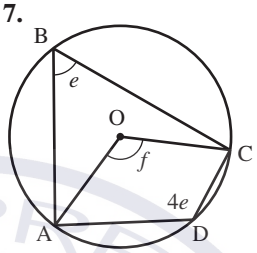
5.



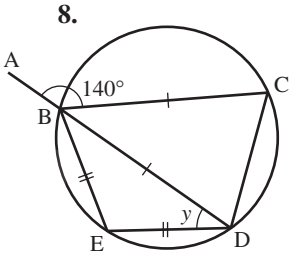
6.



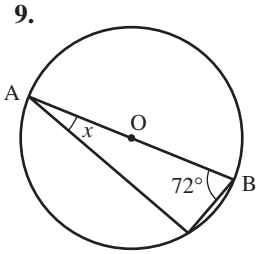
7.



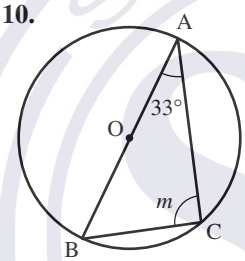
8.



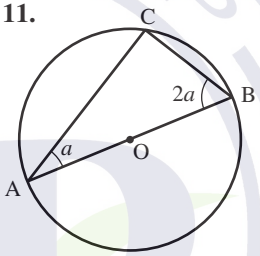
9.



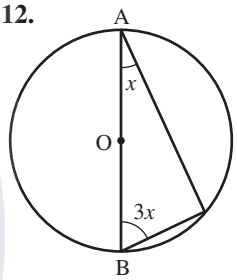
10.



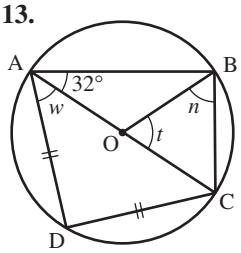
11.



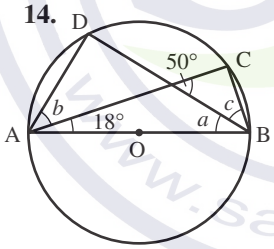
12.



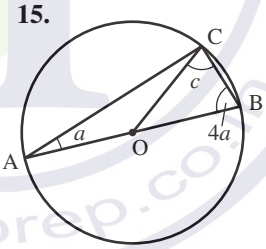
13.



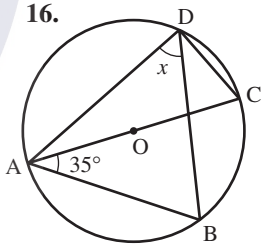
14.



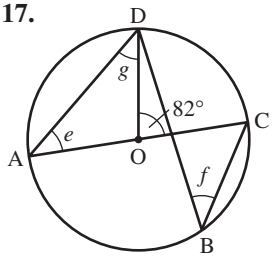
15.



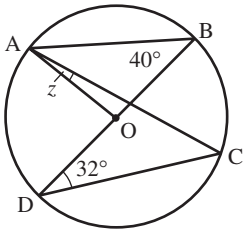
16.



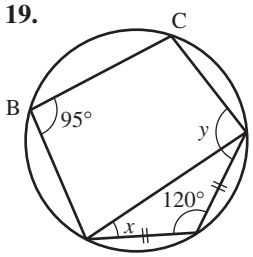
17.



18.



19.



20.