SATPREP
DERIVATIVES

On problems 1 – 4, find the critical points of each function, and determine whether they are relative maximums or relative minimums by using the Second Derivative Test whenever possible.

1. $f(x)=x^3-3x^2+3$

2. $f(x)=x+\dfrac{4}{x}$

3. $f(x)=\sin x-\cos x,\ 0\le x\le 2\pi$

4. $f(x)=2\sin x+\cos(2x),\ 0\le x\le 2\pi$

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5. Suppose that the function f has a continuous second derivative for all x and that $f(-1)=2,\ f'(-1)=-3,\ f''(-1)=5$. Let g be a function whose derivative is given by $g'(x)=\left(x^4-6x^3\right)\left(3f(x)+2f'(x)\right)$ for all x.

(a) Write an equation of the line tangent to the graph of f at the point where $x=-1$.

(b) Does g have a local maximum or a local minimum at $x=-1$? Justify your answer.

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6. Conside the curve given by $x^2+4y^2=7+3xy$.

(a) Show that $\dfrac{dy}{dx}=\dfrac{3y-2x}{8y-3x}$.

(b) Show that there is a point P with x-coordinate 3 at which the line tangent to the curve at P is horizontal. Find the y-coordinate of P.

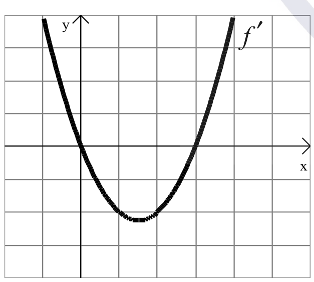
(c) Find the value of $\dfrac{d^2y}{dx^2}$ at the point P found in part (b). Does the curve have a local maximum, a local minimum, or neither at point P? Justify your answer.

On problems 7 – 8, the graph of the derivative, f', of a function f is shown.

(a) On what interval(s) is f increasing or decreasing? Justify your answer.

(b) At what value(s) of x does f have a local maximum or local minimum? Justify your answer.

7.



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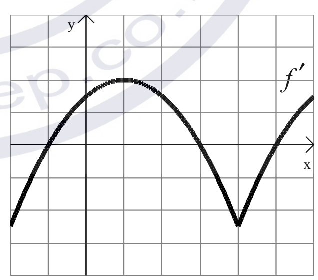

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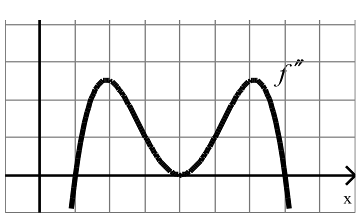
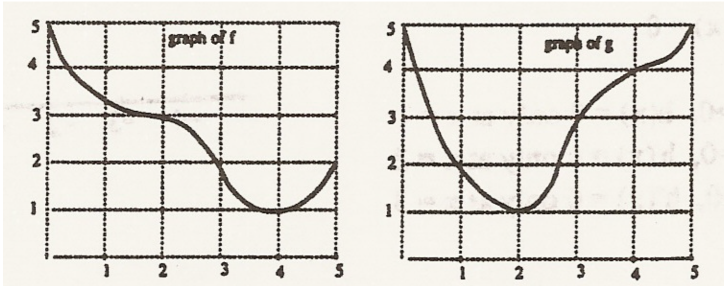
9. The graph of the second derivative, f'', of a function f is shown. State the x-coordinates of the inflection points of f. Justify your answer.



TURN->>>

10. The function h is defined by $h(x)=f(g(x))$, where f and g are the functions whose

graphs are shown below.



(a) Evaluate $h(2)$.

(b) Estimate $h'(1)$.

(c) Is the graph of the composite function h increasing or decreasing at $x = 3$? Show your reasoning.

(d) Find all values of x for which the graph of h has a horizontal tangent. Show your reasoning.

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11. For what values of a and b does the function $f(x) = x^3 + ax^2 + bx + 2$ have a local maximum when $x = -3$ and a local minimum when $x = -1$?

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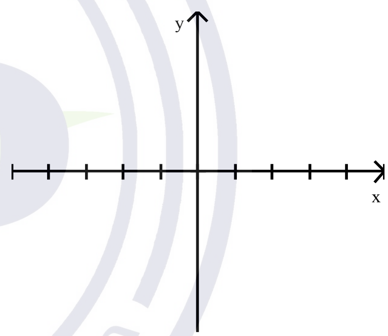
12. Sketch the graph of a function $f(x)$ that meets all of the following criteria:

1) The domain of f is $(-\infty, 0) \cup (0, \infty)$.
2) $f(-x) = -f(x)$
3) For $x > 0$, $f(x) = 0$ only at $x = 1$.
4) For $x > 0$, $f'(x) = 0$ only at $x = 2$.
5) For $x > 0$, $f''(x) = 0$ only at $x = 3$.
6) $\lim_{x \to 0^+} f(x) = \infty$
7) $\lim_{x \to \infty} f(x) = 0$



Answers to Worksheet on Second Derivative Test

1. Rel. max. at (0, 3), rel. min. at (2, - 1)
2. Rel. max. at $(-2, -4)$, rel. min. at (2, 4)
3. Rel. max. at $\left(\frac{3\pi}{4}, \sqrt{2}\right)$, rel. min. at $\left(\frac{7\pi}{4}, -\sqrt{2}\right)$
4. Rel. min. at $\left(\frac{\pi}{2}, 1\right)$ and $\left(\frac{3\pi}{2}, -3\right)$, rel. max. at $\left(\frac{\pi}{6}, \frac{3}{2}\right)$ and $\left(\frac{5\pi}{6}, \frac{3}{2}\right)$
5. (a) $y-2=-3(x+1)$

   (b) Local minimum at $x=-1$ because $g'(-1)=0$ and $g''(-1)=7>0$.
6. (a) $2x+8y\frac{dy}{dx}=3y+3x\frac{dy}{dx}$

   $(8y-3x)\frac{dy}{dx}=3y-2x$

   (b) $y=2$

   (c) Local max. since $\frac{dy}{dx}=0$ and $\frac{d^2y}{dx^2}=-\frac{2}{7}$
7. (a) incr. on $(-\infty, 0)\cup(3,\infty)$; decr. on (0, 3) (b) Rel. max. at x = 0, rel. min. at x = 3
8. (a) decr. on $(-\infty, -1)\cup(3,5)$; incr. on $(-1, 3)\cup(5,\infty)$

   (b) Rel. min. at x = − 1, x = 5; rel. max. at x = 3
9. x = 1 and x = 7
10. (a) 3.4 (b) $\frac{1}{4}$ (c) decr. (d) 2, 0.25, 4
11. a = 6, b = 9
12.