

SATPREP

Trigonometric Equation

$a \sin \theta + b \cos \theta$	$R \sin(\theta + \alpha)$	$\alpha = \arctan\left(\frac{b}{a}\right)$
$a \sin \theta - b \cos \theta$	$R \sin(\theta - \alpha)$	$\alpha = \arctan\left(\frac{b}{a}\right)$
$a \sin \theta + b \cos \theta$	$R \cos(\theta - \alpha)$	$\alpha = \arctan\left(\frac{a}{b}\right)$
$a \sin \theta - b \cos \theta$	$-R \cos(\theta + \alpha)$	$\alpha = \arctan\left(\frac{a}{b}\right)$

Exercises 1

Each of the following expressions can be written in the form $R \cos(x - \alpha)$ with $-\pi < \alpha < \pi$. In each case determine the values of R and α (in radians) correct to 3 decimal places.

- a) $5 \cos x + 12 \sin x$ b) $3 \cos x + \sin x$ c) $3 \cos x - \sin x$ d) $6 \cos x + 5 \sin x$
e) $-5 \cos x + 12 \sin x$ f) $4 \cos x - \sin x$ g) $-2 \cos x - 3 \sin x$ h) $-\cos x + 3 \sin x$
i) $\cos x + \sin x$ j) $\cos x - \sin x$ k) $\sin x - \cos x$ l) $-(\cos x + \sin x)$

Exercises 2

Solve the following equations for $0 < x < 2\pi$

- a) $2 \cos x + \sin x = 1$ b) $2 \cos x - \sin x = 1$ c) $-2 \cos x - \sin x = 1$
d) $\cos x - 2 \sin x = 1$ e) $\cos x + 2 \sin x = 1$ f) $-\cos x + 2 \sin x = 1$

Exercises 3

For each of the following functions determine the maximum value and the smallest positive angle (in radians, to three decimal places) at which the maximum value occurs.

- a) $f(x) = 6 + 3 \cos x + 4 \sin x$ b) $f(x) = 3 - 4 \cos x + 3 \sin x$
c) $f(x) = 1 - 3 \cos x - 4 \sin x$ d) $f(x) = 2 + \cos x - \sin x$

Solutions

Exercises 1

- a) 13, 1.176 b) $\sqrt{10}$, 0.322 c) $\sqrt{10}$, -0.322 d) $\sqrt{61}$, 0.695
e) 13, 1.966 f) $\sqrt{17}$, -0.245 g) $\sqrt{13}$, -2.159 h) $\sqrt{10}$, 1.893
i) $\sqrt{2}$, $\frac{\pi}{4}$ j) $\sqrt{2}$, $-\frac{\pi}{4}$ k) $\sqrt{2}$, $\frac{3\pi}{4}$ l) $\sqrt{2}$, $-\frac{3\pi}{4}$

Exercises 2

- a) 1.571, $\left(\frac{\pi}{2}\right)$ or 5.640 b) 0.644 or 4.712 $\left(\frac{3\pi}{2}\right)$ c) 2.498 or 4.712 $\left(\frac{3\pi}{2}\right)$
d) 4.069 e) 2.214 f) 3.142 (π) or 0.927

Exercises 3

- a) Max value 11 at 0.927 b) Max value 8 at 2.498
c) Max value 6 at 4.069 d) Max value $2 + \sqrt{2}$ at $\frac{7\pi}{4}$

