

Assignment : *Binomial Theorem*

1. Write out the following binomial expansions.

(i) $(x + 1)^4$

(ii) $(1 + x)^7$

(iii) $(x + 2)^5$

(iv) $(2x + 1)^6$

(v) $(2x - 3)^4$

(vi) $(2x + 3y)^3$

(vii) $\left(x - \frac{2}{x}\right)^3$

(viii) $\left(x + \frac{2}{x^2}\right)^4$

(ix) $\left(3x^2 - \frac{2}{x}\right)^5$

2. In these expansions, find the coefficient of these terms.

(i) x^5 in $(1 + x)^8$

(ii) x^4 in $(1 - x)^{10}$

(iii) x^6 in $(1 + 3x)^{12}$

(iv) x^7 in $(1 - 2x)^{15}$

(v) x^2 in $\left(x^2 + \frac{2}{x}\right)^{10}$

3. Find the first three terms, in descending powers of x , in the expansion

of $\left(2x - \frac{2}{x}\right)^4$.

4. Find the first three terms, in ascending powers of x , in the expansion $(2 + kx)^6$.

5. **(i)** Find the first three terms, in ascending powers of x , in the expansion $(1 - 2x)^6$.

(ii) Hence find the coefficients of x and x^2 in the expansion of $(4 - x)(2 - 4x)^6$.

6. **(i)** Find the first three terms, in descending powers of x , in the expansion

$\left(4x - \frac{k}{x^2}\right)^6$.

(ii) Given that the value of the term in the expansion which is independent of x is 240, find possible values of k .

7. **(i)** Find the first three terms, in descending powers of x , in the expansion of

$\left(x^2 - \frac{1}{x}\right)^6$.

(ii) Find the coefficient of x^3 in the expansion of $\left(x^2 - \frac{1}{x}\right)^6$.

8. **(i)** Find the first three terms, in descending powers of x , in the expansion of $\left(x - \frac{2}{x}\right)^5$.
- (ii)** Hence find the coefficient of x in the expansion of $\left(4 + \frac{1}{x^2}\right)\left(x - \frac{2}{x}\right)^5$.
9. **(i)** Show that $(2 + x)^4 = 16 + 32x + 24x^2 + 8x^3 + x^4$ for all x .
- (ii)** Find the values of x for which $(2 + x)^4 = 16 + 16x + x^4$.
10. The first three terms in the expansion of $(2 + ax)^n$, in ascending powers of x , are $32 - 40x + bx^2$. Find the values of the constants n , a and b .
11. **(i)** Find the first three terms in the expansion of $(2 - x)^6$ in ascending powers of x .
- (ii)** Find the value of k for which there is no term in x^2 in the expansion of $(1 + kx)(2 - x)^6$.
12. **(i)** Find the first three terms in the expansion of $(1 + ax)^5$ in ascending powers of x .
- (ii)** Given that there is no term in x in the expansion of $(1 - 2x)(1 + ax)^5$, find the value of the constant a .
- (iii)** For this value of a , find the coefficient of x^2 in the expansion of $(1 - 2x)(1 + ax)^5$.

Answer

1. (i) $x^4 + 4x^3 + 6x^2 + 4x + 1$
(ii) $1 + 7x + 21x^2 + 35x^3 + 35x^4 + 21x^5 + 7x^6 + x^7$
(iii) $x^5 + 10x^4 + 40x^3 + 80x^2 + 80x + 32$
(iv) $64x^6 + 192x^5 + 240x^4 + 160x^3 + 60x^2 + 12x + 1$
(v) $16x^4 - 96x^3 + 216x^2 - 216x + 81$
(vi) $8x^3 + 36x^2y + 54xy^2 + 27y^3$
(vii) $x^3 - 6x + \frac{12}{x} - \frac{8}{x^3}$
(viii) $x^4 + 8x + \frac{24}{x^2} + \frac{32}{x^5} + \frac{16}{x^8}$
(ix) $243x^{10} - 810x^7 + 1080x^4 - 720x + \frac{240}{x^2} - \frac{32}{x^5}$
2. (i) 56
(ii) 210
(iii) 673 596
(iv) -823 680
(v) 13 440
3. $16x^4 - 64x^2 + 96$
4. $64 + 192kx + 240k^2x^2$
5. (i) $1 - 12x + 60x^2$
(ii) -3136 and 16 128
6. (i) $4096x^6 - 6144kx^3 + 3840k^2$
(ii) $\pm \frac{1}{4}$
7. (i) $x^{12} - 6x^9 + 15x^6$
(ii) -20
8. (i) $x^5 - 10x^3 + 40x$
(ii) 150
9. (ii) $x = 0, -1$ and -2
10. $n = 5, a = -\frac{1}{2}, b = 20$
11. (i) $64 - 192x + 240x^2$
(ii) 1.25
12. (i) $1 + 5ax + 10a^2x^2$
(ii) $a = \frac{2}{5}$
(iii) -2.4