SATPREP

Assignment- Optmization

Solve each optimization problem.

1) A farmer wants to construct a rectangular pigpen using 500 ft of fencing. The pen will be built next to an existing stone wall, so only three sides of fencing need to be constructed to enclose the pen. What dimensions should the farmer use to construct the pen with the largest possible area?

2) A company has started selling a new type of smartphone at the price of $\$130 - 0.05x$ where x is the number of smartphones manufactured per day. The parts for each smartphone cost \$50 and the labor and overhead for running the plant cost \$6000 per day. How many smartphones should the company manufacture and sell per day to maximize profit?

3) A supermarket employee wants to construct an open-top box from a 14 by 30 in piece of cardboard. To do this, the employee plans to cut out squares of equal size from the four corners so the four sides can be bent upwards. What size should the squares be in order to create a box with the largest possible volume?

4) A rancher wants to construct two identical rectangular corrals using 400 ft of fencing. The rancher decides to build them adjacent to each other, so they share fencing on one side. What dimensions should the rancher use to construct each corral so that together, they will enclose the largest possible area?

5) Which points on the graph of $y = 3 - x^2$ are closest to the point $(0, 2)$?

6) Engineers are designing a box-shaped aquarium with a square bottom and an open top. The aquarium must hold 2048 ft³ of water. What dimensions should they use to create an acceptable aquarium with the least amount of glass?

7) An architect is designing a composite window by attaching a semicircular window on top of a rectangular window, so the diameter of the top window is equal to and aligned with the width of the bottom window. If the architect wants the perimeter of the composite window to be 14 ft, what dimensions should the bottom window be in order to create the composite window with the largest area?

8) Two vertical poles, one 15 ft high and the other 30 ft high, stand 24 feet apart on a flat field. A worker wants to support both poles by running rope from the ground to the top of each post. If the worker wants to stake both ropes in the ground at the same point, where should the stake be placed to use the least amount of rope?

Answers to Assignment- Optmization

1) 125 ft (perpendicular to wall) by 250 ft (parallel to wall)
2) 800
3) 3 in
4) 50 ft (non-adjacent sides) by $\frac{200}{3}$ ft (adjacent sides)
5) d = the distance from point $(0, 2)$ to a point on the parabola x = the x-coord. of a point on the parabola
   Function to minimize: $d = \sqrt{x^2 + (3 - x^2 - 2)^2}$ where $-\infty < x < \infty$
   Points on the parabola that are closest to the point $(0, 2)$: $\left(-\frac{\sqrt{2}}{2}, \frac{5}{2}\right), \left(\frac{\sqrt{2}}{2}, \frac{5}{2}\right)$
6) A = the area of the glass x = the length of the sides of the square bottom
   Function to minimize: $A = x^2 + 4x \cdot \frac{2048}{x^2}$ where $0 < x < \infty$
   Dimensions of the aquarium: 16 ft by 16 ft by 8 ft tall
7) A = the area of the composite window x = the width of the bottom window = the diameter of the top window
   Function to maximize: $A = x\left(\frac{14}{2} - \frac{x}{2} - \frac{\pi x}{4}\right) + \frac{1}{2}\pi \cdot \left(\frac{x}{2}\right)^2$ where $0 < x < \frac{56}{4 + \pi}$
   Dimensions of the bottom window: $\frac{28}{4 + \pi}$ ft (width) by $\frac{14}{4 + \pi}$ ft (height)
8) L = the total length of rope x = the horizontal distance from the short pole to the stake
   Function to minimize: $L = \sqrt{x^2 + 15^2} + \sqrt{(24 - x)^2 + 30^2}$ where $0 \le x \le 24$
   Stake should be placed: 8 ft from the short pole (or 16 ft from the long pole)