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Mathematics: analysis and approaches
Standard level
Paper 1

24 October 2024

Zone A afternoon | **Zone B** afternoon | **Zone C** afternoon

Candidate session number

1 hour 30 minutes

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Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches SL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.



Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 6]

Consider the following set of ordered data.

27	27	31	32	33	37	41	46	54	57
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(a) Write down

- (i) the mode;
- (ii) the range;
- (iii) the median.

[3]

(b) Find the interquartile range.

[3]

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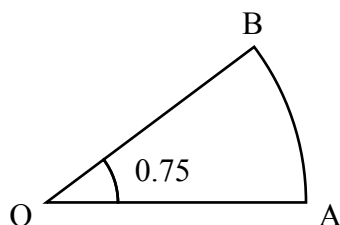


2. [Maximum mark: 5]

Points A and B lie on a circle with centre O and radius r cm, where $\angle AOB = 0.75$ radians.

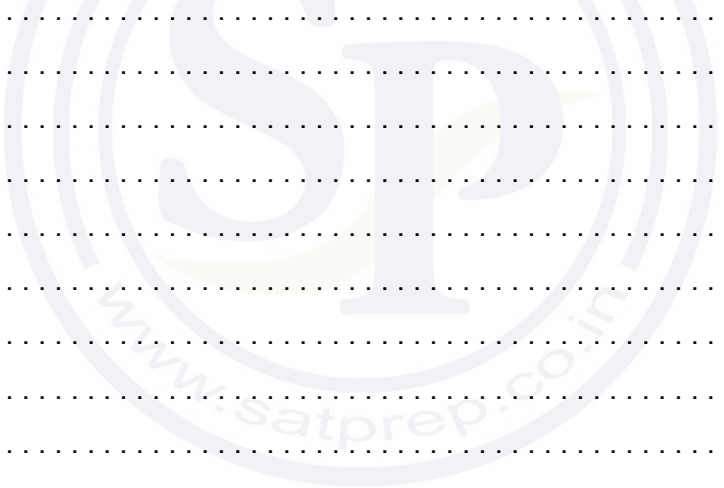
This is shown on the following diagram.

diagram not to scale



The area of sector OAB is 6 cm^2 .

- (a) Find the value of r . [3]
- (b) Hence, find the perimeter of sector OAB. [2]

A large, faint watermark of the Satprep logo is centered on the page. The logo consists of a circular seal with the letters 'SP' in the center, a green leaf-like shape behind them, and the text 'www.satprep.com' around the bottom inner edge of the seal.

3. [Maximum mark: 6]

Two events A and B are such that $P(A) = 0.45$, $P(B) = 0.65$ and $P(A \cup B) = 0.8$.

(a) Find $P(A \cap B)$. [3]

(b) Find $P(A'|B')$. [3]

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4. [Maximum mark: 4]

Prove that $(2n + 5)^2 - (2n - 5)^2$ is a multiple of 10 for all $n \in \mathbb{Z}^+$.

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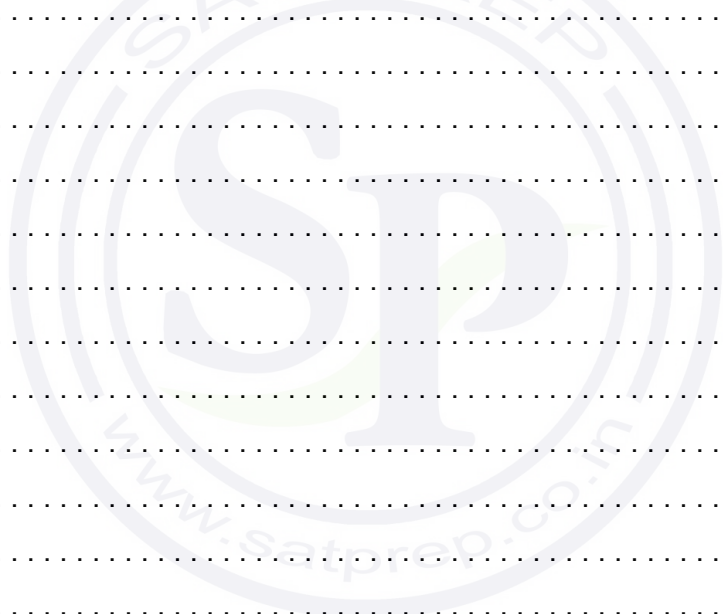
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The following diagram shows triangle ABC, where $AB = 3\text{ cm}$, $BC = 10\text{ cm}$, $AC = 8\text{ cm}$ and $\hat{A} = 2\theta$.

Diagram of a triangle ABC . Side AB is 3 cm , side BC is 10 cm , and side AC is 8 cm . The angle at vertex B is labeled 2θ .

Find the exact value of $\cos \theta$, giving your answer in the form $\frac{\sqrt{p}}{q}$, where $p, q \in \mathbb{Z}^+$.

A large, light gray watermark logo is centered on the page. It consists of a circular emblem with the letters 'SP' in the middle. The 'S' is on the left and the 'P' is on the right, both in a serif font. A green leaf-like shape is positioned behind the 'P'. The emblem is surrounded by concentric circles. Text is written around the inner circle: 'SAT PREP' at the top and 'www.satprep.co.in' at the bottom.

6. [Maximum mark: 6]

For a particular arithmetic sequence, $u_{10} = 14$ and $S_{25} = 200$.

Find the value of k such that $u_k = 0$.

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Section B

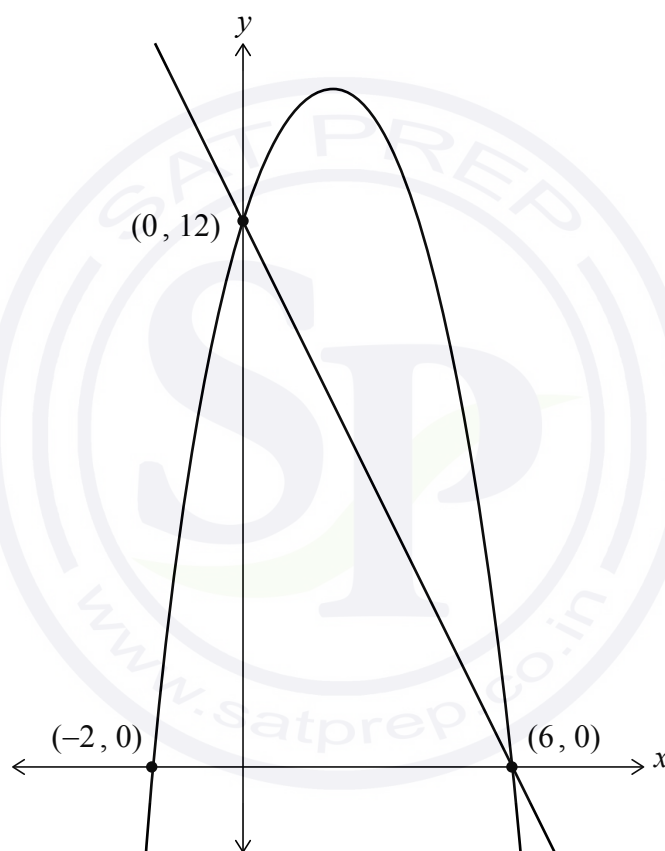
Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 16]

The following diagram shows parts of the graphs of two functions f and g .

The graph of f is linear, has an x -intercept at $(6, 0)$ and a y -intercept at $(0, 12)$.

The graph of g has x -intercepts at $(-2, 0)$ and $(6, 0)$ and a y -intercept at $(0, 12)$.



(a) Write down the equation for f in the form $f(x) = mx + c$.

[2]

(This question continues on the following page)



Do **not** write solutions on this page.

(Question 7 continued)

The function g is given by $g(x) = -x^2 + bx + 12$, where b is a real constant.

(b) Find the value of b . [3]

(c) Show that the area of the region enclosed by the graph of f and the graph of g can be represented by the definite integral $\int_0^6 (-x^2 + 6x)dx$. [2]

(d) Hence, find the area of the region enclosed by the graph of f and the graph of g . [4]

Point P is on the graph of g . The tangent to the graph of g at P is parallel to the graph of f .

(e) Find the coordinates of P . [5]



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8. [Maximum mark: 14]

The function f is defined as $f(x) = \log_2(8x)$, where $x > 0$.

(a) Find the value of

(i) $f(2)$;

(ii) $f\left(\frac{1}{8}\right)$. [3]

(b) Find an expression for $f^{-1}(x)$. [4]

(c) Hence, or otherwise, find $f^{-1}(0)$. [1]

The graph of $y = f(4x^2)$ can be obtained by translating and stretching the graph of $y = \log_2 x$.

(d) Describe these two transformations specifying the order in which they are to be applied. [6]



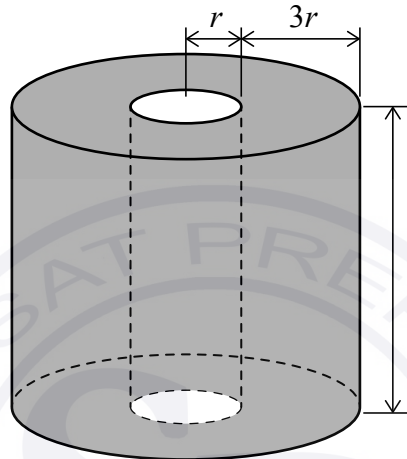
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9. [Maximum mark: 17]

Consider a cylinder of radius $4r$ and height h . A smaller cylinder of radius r is removed from the centre to form a hollow cylinder. This is shown in the following diagram.

All lengths are measured in centimetres.

diagram not to scale



The total surface area of the hollow cylinder, in cm^2 , is given by S .

The volume of the hollow cylinder, in cm^3 , is given by V .

(a) Show that $S = 30\pi r^2 + 10\pi rh$. [3]

(b) The total surface area of the hollow cylinder is $240\pi \text{ cm}^2$.

Show that $V = 360\pi r - 45\pi r^3$. [6]

(c) Find an expression for $\frac{dV}{dr}$. [2]

The hollow cylinder has its maximum volume when $r = p\sqrt{\frac{2}{3}}$, where $p \in \mathbb{Z}^+$.

(d) Find the value of p . [3]

(e) Hence, find this maximum volume, giving your answer in the form $q\pi\sqrt{\frac{2}{3}}$, where $q \in \mathbb{Z}^+$. [3]



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will not be marked.



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Mathematics: analysis and approaches

Standard level

Paper 1

24 October 2024

Zone A afternoon | Zone B afternoon | Zone C afternoon

Candidate session number

1 hour 30 minutes

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Instructions to candidates

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- The maximum mark for this examination paper is **[80 marks]**.



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Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 6]

Consider the following set of ordered data.

22	22	26	27	28	32	36	38	39	42
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(a) Write down

- (i) the mode;
- (ii) the range;
- (iii) the median.

[3]

(b) Find the interquartile range.

[3]

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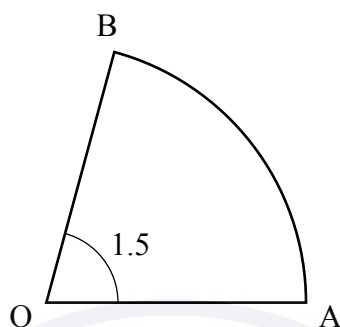


2. [Maximum mark: 5]

Points A and B lie on a circle with centre O and radius r cm, where $\angle AOB = 1.5$ radians.

This is shown on the following diagram.

diagram not to scale



The area of sector OAB is 48 cm^2 .

(a) Find the value of r . [3]

(b) Hence, find the perimeter of sector OAB . [2]

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3. [Maximum mark: 6]

Two events A and B are such that $P(A) = 0.65$, $P(B) = 0.45$ and $P(A \cup B) = 0.85$.

(a) Find $P(A \cap B)$. [3]

(b) Find $P(A' | B')$. [3]

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4. [Maximum mark: 4]

Prove that $(3n + 2)^2 - (3n - 2)^2$ is a multiple of 12 for all $n \in \mathbb{Z}^+$.

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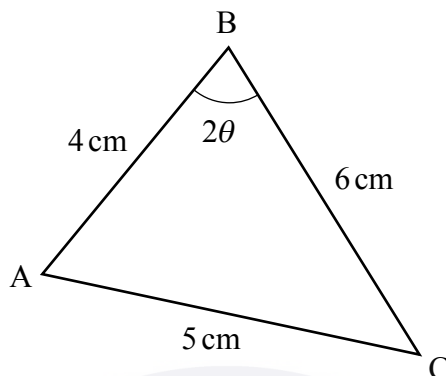
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5. [Maximum mark: 6]

The following diagram shows triangle ABC , where $AB = 4\text{ cm}$, $BC = 6\text{ cm}$, $AC = 5\text{ cm}$ and $\hat{ABC} = 2\theta$.

diagram not to scale



Find the exact value of $\cos \theta$, giving your answer in the form $\frac{p\sqrt{2}}{q}$, where $p, q \in \mathbb{Z}^+$.

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6. [Maximum mark: 6]

For a particular arithmetic sequence, $u_{10} = 16$ and $S_{25} = 100$.

Find the value of k such that $u_k = 0$.

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Section B

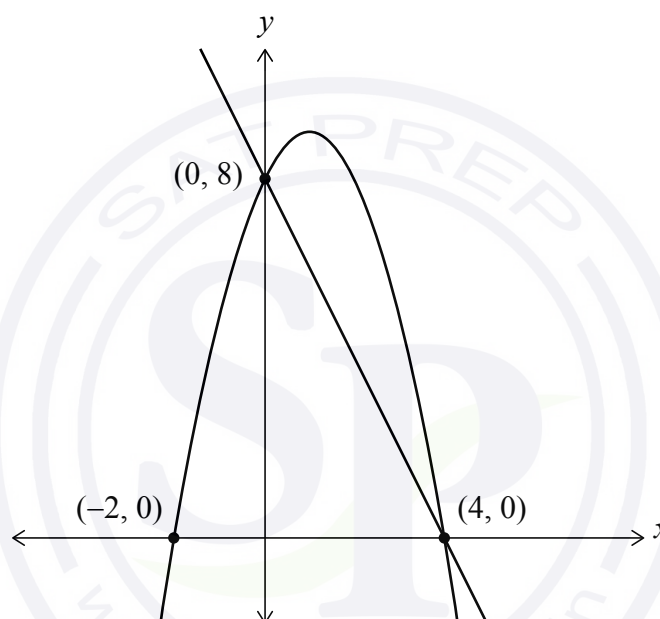
Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 16]

The following diagram shows parts of the graphs of two functions f and g .

The graph of f is linear, has an x -intercept at $(4, 0)$ and a y -intercept at $(0, 8)$.

The graph of g has x -intercepts at $(-2, 0)$ and $(4, 0)$ and a y -intercept at $(0, 8)$.



- (a) Write down the equation for f in the form $f(x) = mx + c$. [2]

The function g is given by $g(x) = -x^2 + bx + 8$, where b is a real constant.

- (b) Find the value of b . [3]

- (c) Show that the area of the region enclosed by the graph of f and the graph of g can be represented by the definite integral $\int_0^4 (-x^2 + 4x) dx$. [2]

- (d) Hence, find the area of the region enclosed by the graph of f and the graph of g . [4]

Point P is on the graph of g . The tangent to the graph of g at P is parallel to the graph of f .

- (e) Find the coordinates of P . [5]



Do **not** write solutions on this page.

8. [Maximum mark: 14]

The function f is defined as $f(x) = \log_2(4x)$, where $x > 0$.

(a) Find the value of

(i) $f(8)$;

(ii) $f\left(\frac{1}{4}\right)$. [3]

(b) Find an expression for $f^{-1}(x)$. [4]

(c) Hence, or otherwise, find $f^{-1}(0)$. [1]

The graph of $y = f(16x^3)$ can be obtained by translating and stretching the graph of $y = \log_2 x$.

(d) Describe these two transformations specifying the order in which they are to be applied. [6]



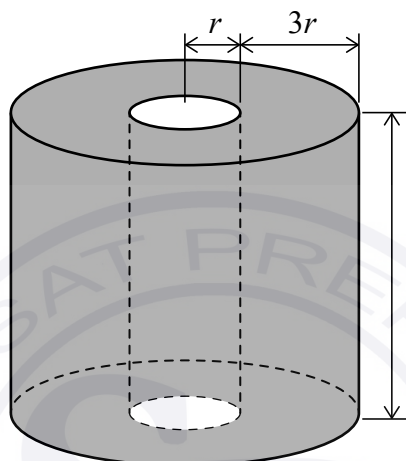
Do **not** write solutions on this page.

9. [Maximum mark: 17]

Consider a cylinder of radius $4r$ and height h . A smaller cylinder of radius r is removed from the centre to form a hollow cylinder. This is shown in the following diagram.

All lengths are measured in centimetres.

diagram not to scale



The total surface area of the hollow cylinder, in cm^2 , is given by S .

The volume of the hollow cylinder, in cm^3 , is given by V .

(a) Show that $S = 30\pi r^2 + 10\pi rh$. [3]

(b) The total surface area of the hollow cylinder is $240\pi \text{ cm}^2$.

Show that $V = 360\pi r - 45\pi r^3$. [6]

(c) Find an expression for $\frac{dV}{dr}$. [2]

The hollow cylinder has its maximum volume when $r = p\sqrt{\frac{2}{3}}$, where $p \in \mathbb{Z}^+$.

(d) Find the value of p . [3]

(e) Hence, find this maximum volume, giving your answer in the form $q\pi\sqrt{\frac{2}{3}}$, where $q \in \mathbb{Z}^+$. [3]



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Mathematics: analysis and approaches

Standard level

Paper 1

1 May 2024

Zone A afternoon | Zone B afternoon | Zone C afternoon

Candidate session number

1 hour 30 minutes

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Instructions to candidates

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- The maximum mark for this examination paper is **[80 marks]**.



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Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 4]

The second term of an arithmetic sequence is 10 and the fourth term is 22.

(a) Find the value of the common difference. [2]

(b) Find an expression for u_n , the n th term. [2]



2. [Maximum mark: 6]

Claire rolls a six-sided die 16 times.

The scores obtained are shown in the following frequency table.

Score	Frequency
1	p
2	q
3	4
4	2
5	0
6	3

It is given that the mean score is 3.

- (a) Find the value of p and the value of q . [5]

Each of Claire's scores is multiplied by 10 in order to determine the final score for a game she is playing.

- (b) Write down the mean final score. [1]

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3. [Maximum mark: 5]

It is given that $\log_{10} a = \frac{1}{3}$, where $a > 0$.

Find the value of

(a) $\log_{10}\left(\frac{1}{a}\right);$ [2]

(b) $\log_{1000} a$. [3]

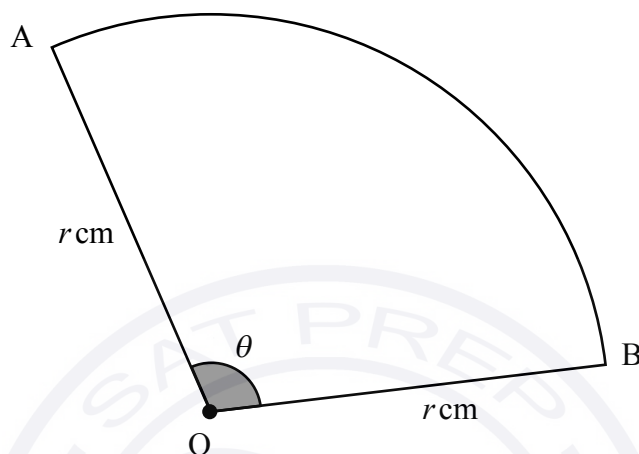
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4. [Maximum mark: 8]

Points A and B lie on the circumference of a circle of radius r cm with centre at O.

The sector OAB is shown on the following diagram. The angle AOB is denoted as θ and is measured in radians.

diagram not to scale



The perimeter of the sector is 10 cm and the area of the sector is 6.25 cm^2 .

- (a) Show that $4r^2 - 20r + 25 = 0$. [4]
- (b) Hence, or otherwise, find the value of r and the value of θ . [4]

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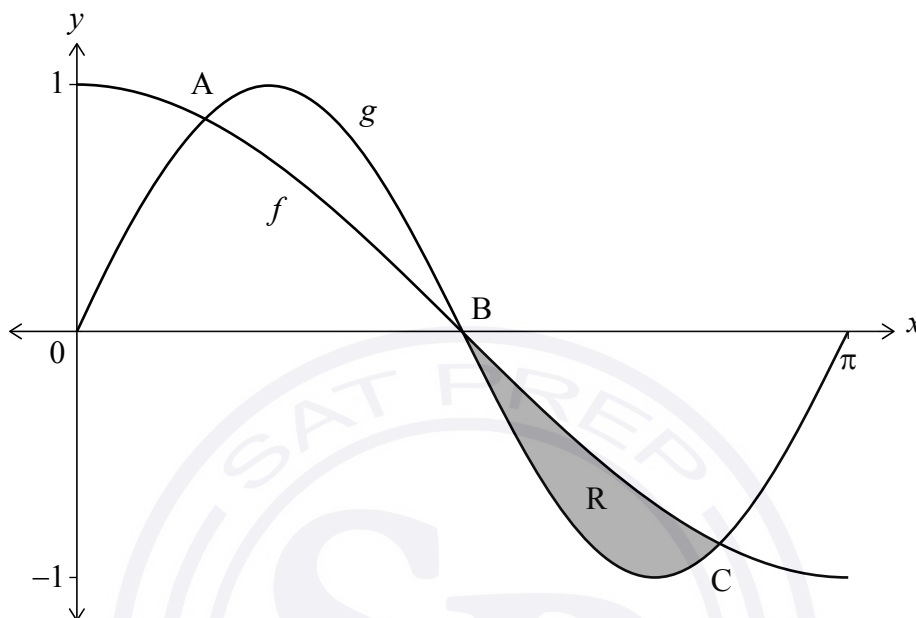
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5. [Maximum mark: 7]

Consider the functions $f(x) = \cos x$ and $g(x) = \sin 2x$, where $0 \leq x \leq \pi$.

The graph of f intersects the graph of g at the point A, the point B $\left(\frac{\pi}{2}, 0\right)$ and the point C as shown on the following diagram.



- (a) Find the x -coordinate of point A and the x -coordinate of point C. [3]

The shaded region R is enclosed by the graph of f and the graph of g between the points B and C.

- (b) Find the area of R. [4]

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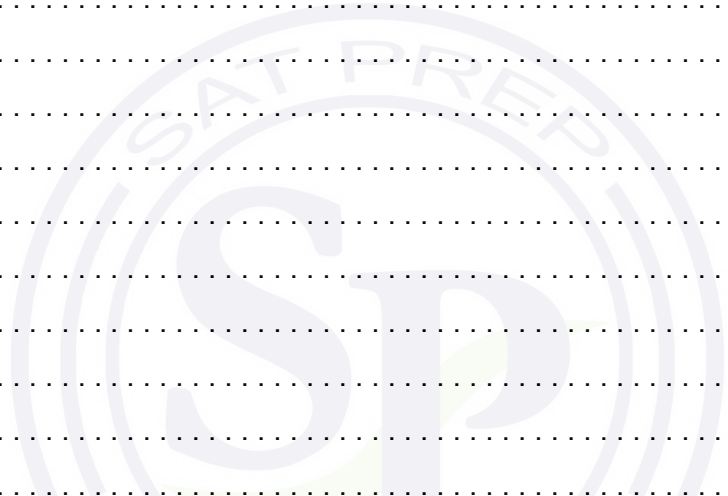
6. [Maximum mark: 5]

Consider a geometric sequence with first term 1 and common ratio 10.

S_n is the sum of the first n terms of the sequence.

- (a) Find an expression for S_n in the form $\frac{a^n - 1}{b}$, where $a, b \in \mathbb{Z}^+$. [1]

- (b) Hence, show that $S_1 + S_2 + S_3 + \dots + S_n = \frac{10(10^n - 1) - 9n}{81}$. [4]

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Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 14]

Consider the curve with equation $y = x^3 - x^2 - x + 1$.

(a) Find

(i) $\frac{dy}{dx}$;

(ii) $\frac{d^2y}{dx^2}$.

[3]

The curve has a local maximum at A.

(b) Find the coordinates of A, using your answer to part (a)(ii) to justify your answer.

[6]

The curve has a point of inflexion at B.

(c) Find the x -coordinate of B.

[2]

The line L is the normal to the curve at the point $(0, 1)$.

(d) Find the equation of L .

[3]



Do **not** write solutions on this page.

8. [Maximum mark: 14]

Consider the function $f(x) = \frac{4x+2}{x-2}$, $x \neq 2$.

- (a) Sketch the graph of $y = f(x)$. On your sketch, indicate the values of any axis intercepts and label any asymptotes with their equations. [5]
- (b) Write down the range of f . [1]

Consider the function $g(x) = x^2 + bx + c$. The graph of g has an axis of symmetry at $x = 2$.

The two roots of $g(x) = 0$ are $-\frac{1}{2}$ and p , where $p \in \mathbb{Q}$.

- (c) Show that $p = \frac{9}{2}$. [1]
- (d) Find the value of b and the value of c . [3]
- (e) Find the y -coordinate of the vertex of the graph of $y = g(x)$. [2]
- (f) Find the number of solutions of the equation $f(x) = g(x)$. [2]



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9. [Maximum mark: 17]

A bag contains buttons which are either red or blue.

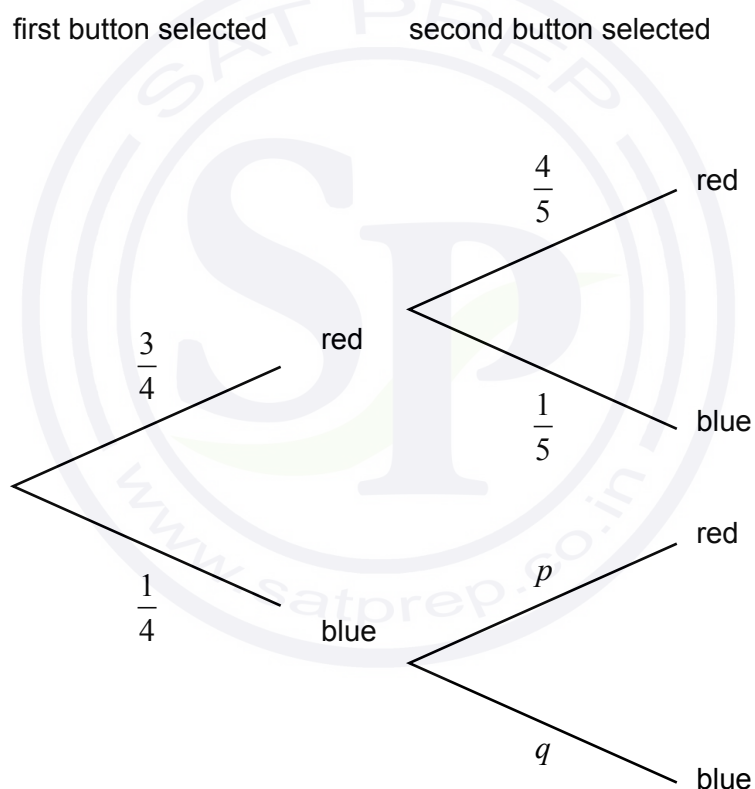
Initially, the bag contains three red buttons and one blue button.

Francine randomly selects one button from the bag. She then replaces the button and adds one extra button of the same colour.

For example, if she selects a red button, she then replaces it and adds one extra red button so that the bag then contains four red buttons and one blue button.

Francine then randomly selects a second button from the bag.

The following tree diagram represents the probabilities of the first two selections.



- (a) Find the value of p and the value of q . [2]
- (b) Show that the probability that Francine selects two buttons of the same colour is $\frac{7}{10}$. [2]
- (c) Given that Francine selects two buttons of the same colour, find the probability that she selects two red buttons. [3]

(This question continues on the following page)



Do **not** write solutions on this page.

(Question 9 continued)

The random variable X is defined as the number of red buttons selected by Francine.

The following table shows the probability distribution of X .

x	0	1	2
$P(X = x)$	$\frac{1}{10}$	a	b

(d) Find the value of a and the value of b . [2]

(e) Hence, find the expected number of red buttons selected by Francine. [2]

Francine restarts the process with three red buttons and one blue button in the bag. She selects buttons as before, replacing the button and adding one extra button of the same colour each time. She repeats this until she selects a blue button.

(f) Given that the first two buttons she selects are red, write down the probability that the next button she selects is blue. [1]

The probability that she selects the first blue button after n selections in total is $\frac{3}{56}$.

(g) Find the value of n . [5]



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Mathematics: analysis and approaches

Standard level

Paper 1

1 May 2024

Zone A afternoon | Zone B afternoon | Zone C afternoon

Candidate session number

1 hour 30 minutes

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Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches SL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.



Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

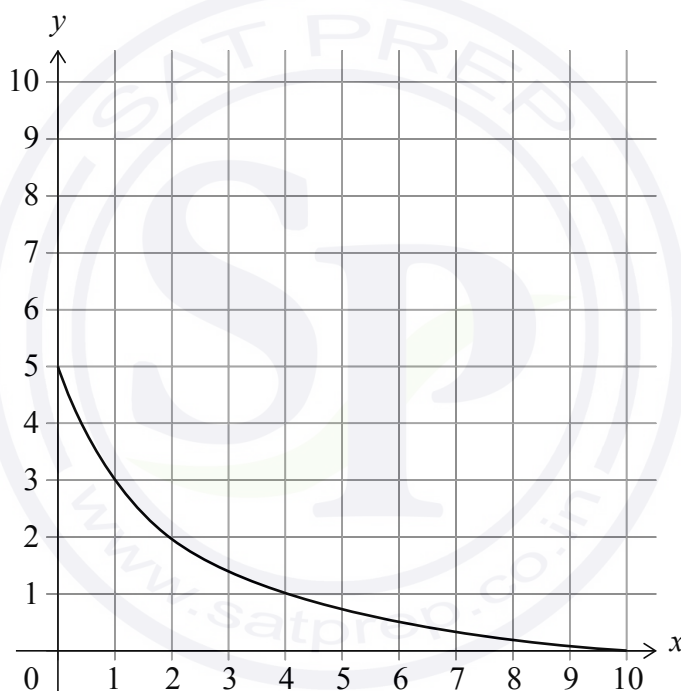
Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 5]

The graph of $y = f(x)$ for $0 \leq x \leq 10$ is shown in the following diagram.

The graph intercepts the axes at $(10, 0)$ and $(0, 5)$.



(a) Write down the value of

(i) $f(4)$;

(ii) $f \circ f(4)$;

(iii) $f^{-1}(3)$.

[3]

(b) On the axes above, sketch the graph of $y = f^{-1}(x)$. Show clearly where the graph intercepts the axes.

[2]

(This question continues on the following page)



(Question 1 continued)

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2. [Maximum mark:4]

Solve $\tan(2x - 5^\circ) = 1$ for $0^\circ \leq x \leq 180^\circ$.

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3. [Maximum mark: 5]

(a) Solve $3m^2 + 5m - 2 = 0$. [3]

(b) Hence or otherwise, solve $3 \times 9^x + 5 \times 3^x - 2 = 0$. [2]

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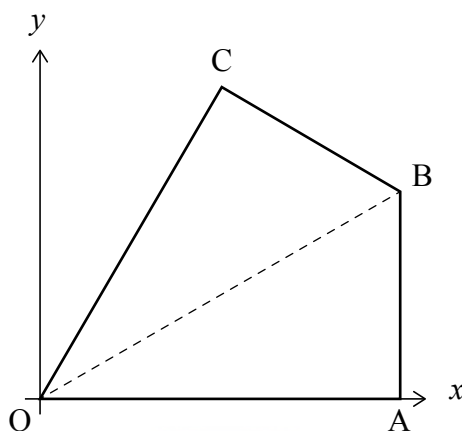
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4. [Maximum mark: 7]

Quadrilateral OABC is shown on the following set of axes.



OABC is symmetrical about [OB].


A has coordinates $(6, 0)$ and C has coordinates $(3, 3\sqrt{3})$.

- (a) (i) Write down the coordinates of the midpoint of $[AC]$.
- (ii) Hence or otherwise, find the equation of the line passing through the points O and B.

[4]

- (b) Given that $[OA]$ is perpendicular to $[AB]$, find the area of the quadrilateral $OABC$.

[3]



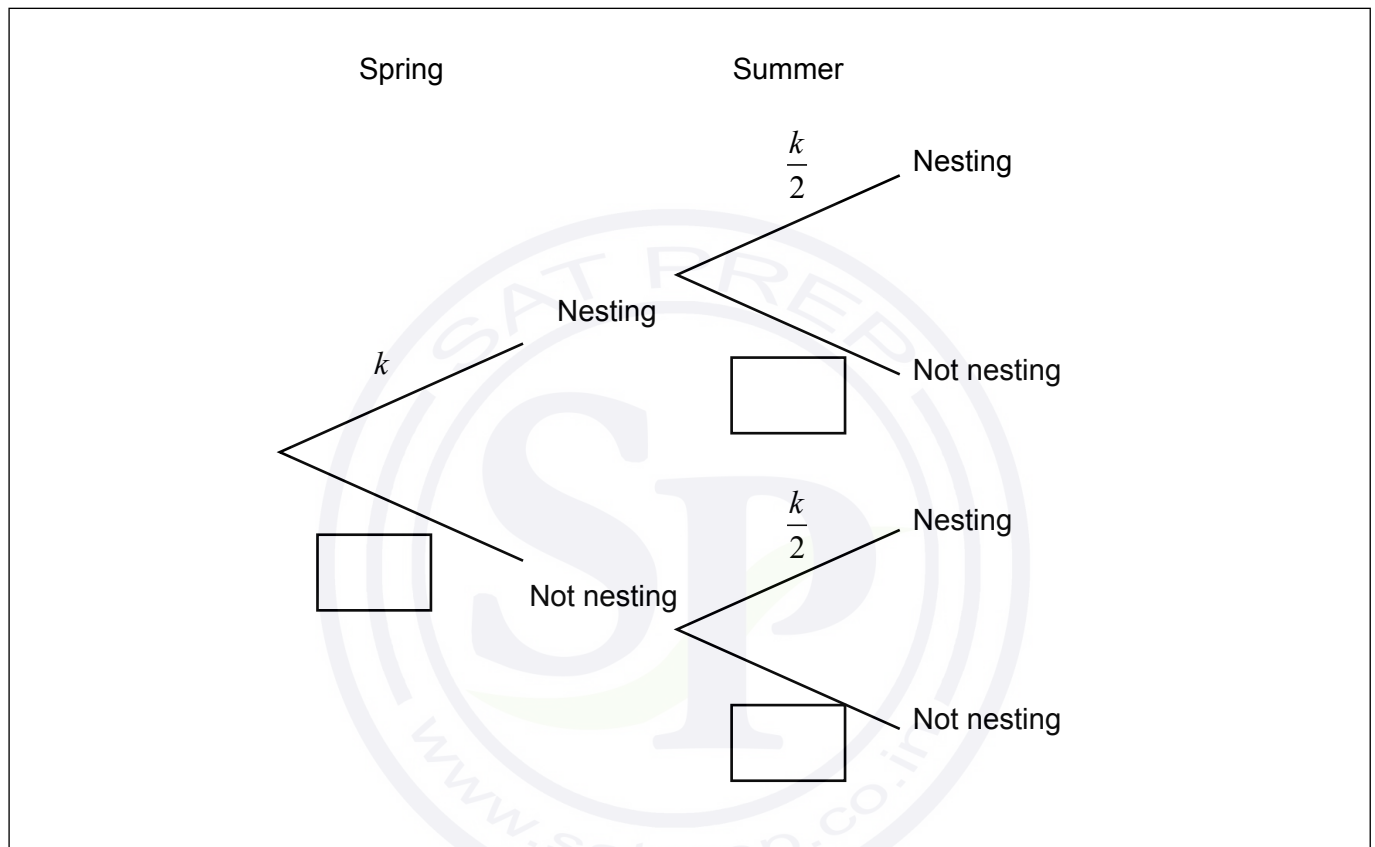
5. [Maximum mark: 6]

A species of bird can nest in two seasons: Spring and Summer.

The probability of nesting in Spring is k .

The probability of nesting in Summer is $\frac{k}{2}$.

This is shown in the following tree diagram.



- (a) Complete the tree diagram to show the probabilities of not nesting in each season. Write your answers in terms of k .

[2]

It is known that the probability of not nesting in Spring and not nesting in Summer is $\frac{5}{9}$.

- (b) (i) Show that $9k^2 - 27k + 8 = 0$.

- (ii) Both $k = \frac{1}{3}$ and $k = \frac{8}{3}$ satisfy $9k^2 - 27k + 8 = 0$.

State why $k = \frac{1}{3}$ is the only valid solution.

[4]

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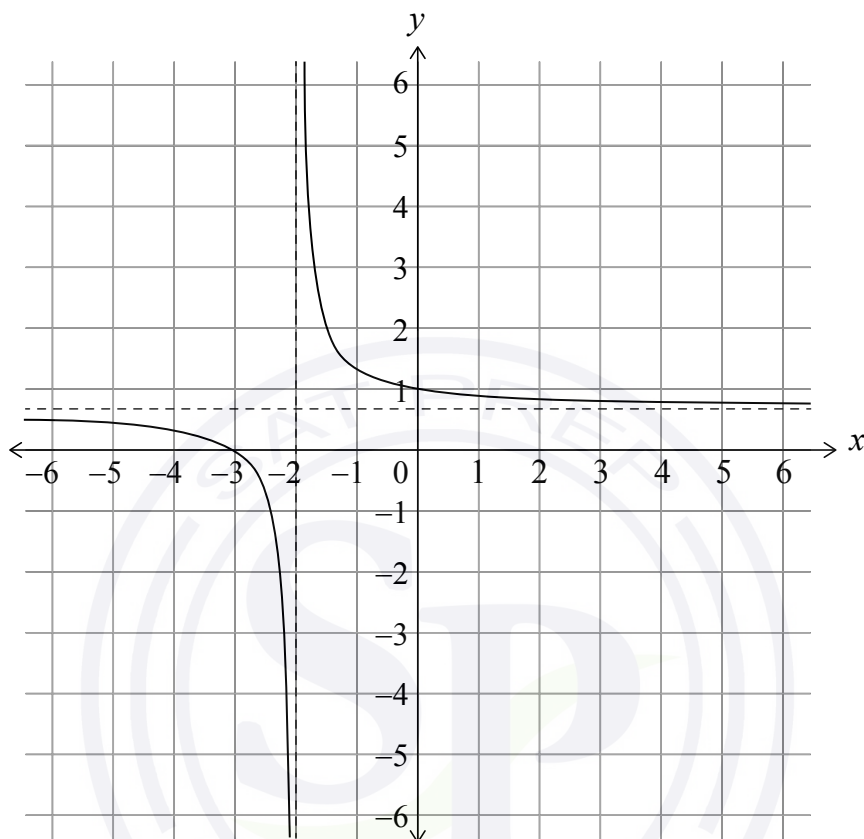
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6. [Maximum mark: 8]

A function f is defined by $f(x) = \frac{2(x+3)}{3(x+2)}$, where $x \in \mathbb{R}, x \neq -2$.

The graph $y = f(x)$ is shown below.



(a) Write down the equation of the horizontal asymptote. [1]

Consider $g(x) = mx + 1$, where $m \in \mathbb{R}, m \neq 0$.

- (b) (i) Write down the number of solutions to $f(x) = g(x)$ for $m > 0$.
 (ii) Determine the value of m such that $f(x) = g(x)$ has only one solution for x .
 (iii) Determine the range of values for m , where $f(x) = g(x)$ has two solutions for $x \geq 0$. [7]

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(Question 6 continued)

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Section B

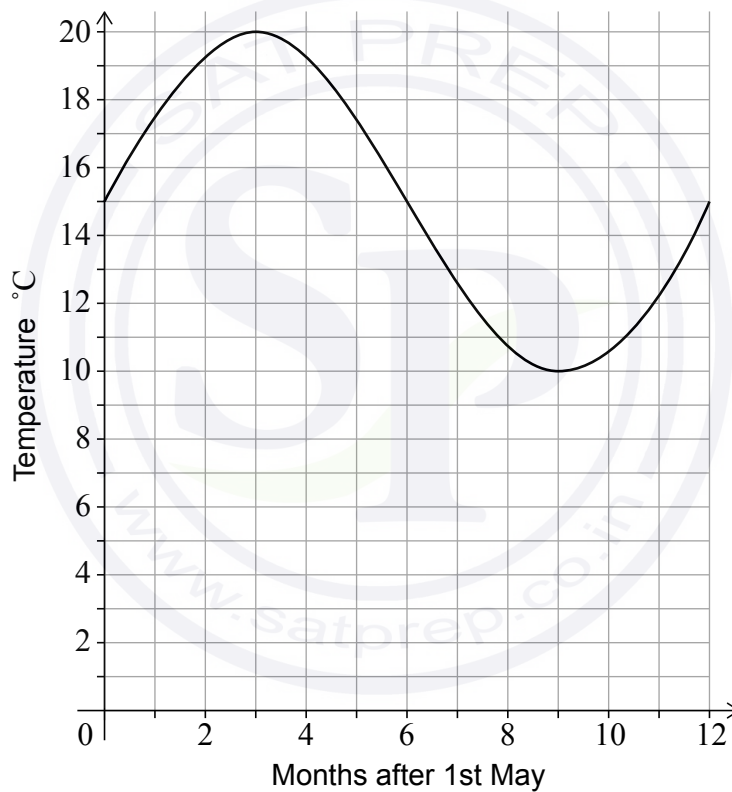
Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 12]

Alex only swims in the sea if the water temperature is at least 15°C . Alex goes into the sea close to home for the first time each year at the start of May when the water becomes warm enough.

Alex models the water temperature at midday with the function $f(x) = a \sin bx + c$ for $0 \leq x \leq 12$, where x is the number of months after 1st May and where $a, b, c > 0$.

The graph of $y = f(x)$ is shown in the following diagram.



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(Question 7 continued)

(a) Show that $b = \frac{\pi}{6}$. [1]

(b) Write down the value of

(i) a ;

(ii) c . [2]

Alex is going on holiday and models the water temperature at midday in the sea at the holiday destination with the function $g(x) = 3.5 \sin \frac{\pi}{6}x + 11$, where $0 \leq x \leq 12$ and x is the number of months after 1st May.

(c) Using this new model $g(x)$

(i) find the midday water temperature on 1st October, five months after 1st May.

(ii) show that the midday water temperature is never warm enough for Alex to swim. [6]

(d) Alex compares the two models and finds that $g(x) = 0.7f(x) + q$. Determine the value of q . [3]



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8. [Maximum mark: 17]

The derivative of a function f is given by $f'(x) = \frac{2x+2}{x^2+2x+2}$, for $x \in \mathbb{R}$.

- (a) (i) Show that $x^2 + 2x + 2 > 0$ for all values of x .
 (ii) Hence, find the values of x for which f is increasing. [3]
- (b) (i) Write down the value of x for which $f'(x) = 0$.
 (ii) Show that $f''(x) = \frac{-2x^2 - 4x}{(x^2 + 2x + 2)^2}$.
 (iii) Hence, justify that the value of x found in part (b)(i) corresponds to a local minimum point on the graph of f . [7]

It is given that $f(2) = 3 + \ln 10$.

- (c) Find an expression for $f(x)$. [4]
 (d) Find the equation of the normal to the graph of f at $(2, 3 + \ln 10)$. [3]



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9. [Maximum mark: 16]

Consider the arithmetic sequence a, p, q, \dots , where $a, p, q \neq 0$.

(a) Show that $2p - q = a$. [2]

Consider the geometric sequence a, s, t, \dots , where $a, s, t \neq 0$.

(b) Show that $s^2 = at$. [2]

The first term of both sequences is a .

It is given that $q = t = 1$.

(c) Show that $p > \frac{1}{2}$. [2]

Consider the case where $a = 9$, $s > 0$ and $q = t = 1$.

(d) Write down the first four terms of the

(i) arithmetic sequence;

(ii) geometric sequence. [4]

The arithmetic and the geometric sequence are used to form a new arithmetic sequence u_n .

The first three terms of u_n are $u_1 = 9 + \ln 9$, $u_2 = 5 + \ln 3$, and $u_3 = 1 + \ln 1$.

(e) (i) Find the common difference of the new sequence in terms of $\ln 3$.

(ii) Show that $\sum_{i=1}^{10} u_i = -90 - 25\ln 3$. [6]



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Mathematics: analysis and approaches
Standard level
Paper 1

30 October 2023

Zone A afternoon | **Zone B** afternoon | **Zone C** afternoon

Candidate session number

1 hour 30 minutes

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Instructions to candidates

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- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.



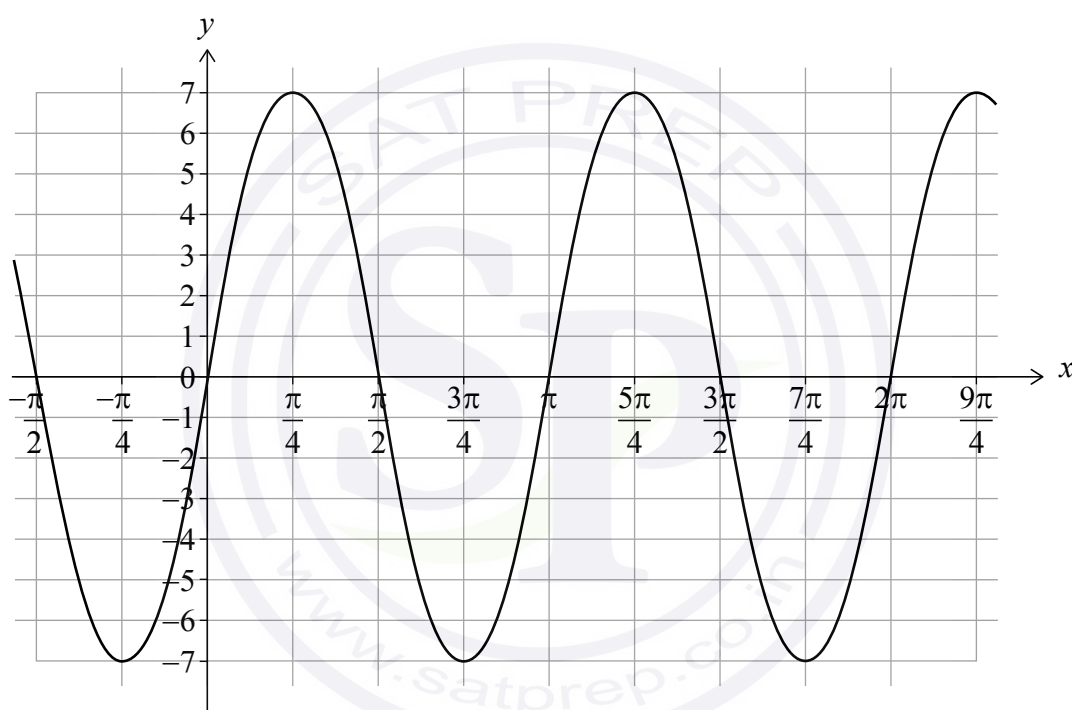
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Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 7]

Consider the function $f(x) = a \sin(bx)$ with $a, b \in \mathbb{Z}^+$. The following diagram shows part of the graph of f .



- (a) Write down the value of a . [1]
- (b) (i) Write down the period of f .
(ii) Hence, find the value of b . [3]
- (c) Find the value of $f\left(\frac{\pi}{12}\right)$. [3]

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(Question 1 continued)

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2. [Maximum mark: 5]

Consider the functions $f(x) = x + 2$ and $g(x) = x^2 - k^2$, where k is a real constant.

(a) Write down an expression for $(g \circ f)(x)$. [2]

(b) Given that $(g \circ f)(4) = 11$, find the possible values of k . [3]

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3. [Maximum mark: 4]

Events A and B are such that $P(A) = 0.7$, $P(B) = 0.75$ and $P(A \cap B) = 0.55$.

(a) Find $P(A \cup B)$. [2]

(b) Hence, or otherwise, find $P(A' \cap B')$. [2]

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4. [Maximum mark: 7]

The sum of the first n terms of an arithmetic sequence is given by $S_n = pn^2 - qn$, where p and q are positive constants.

It is given that $S_5 = 65$ and $S_6 = 96$.

- (a) Find the value of p and the value of q . [5]

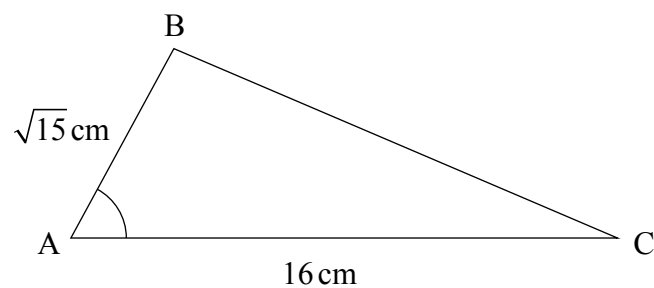
- (b) Find the value of u_6 . [2]

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5. [Maximum mark: 6]

In the following triangle ABC, $AB = \sqrt{15}$ cm, $AC = 16$ cm and $\cos \hat{BAC} = \frac{1}{4}$.

diagram not to scale



Find the area of triangle ABC.

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6. [Maximum mark: 6]

The binomial expansion of $(1 + kx)^n$ is given by $1 + \frac{9x}{2} + 15k^2x^2 + \dots + k^nx^n$, where $n \in \mathbb{Z}^+$ and $k \in \mathbb{Q}$.

Find the value of n and the value of k .

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Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 17]

A ballet company performs *The Sleeping Beauty* every year. Last year they gave a total of 60 performances at their theatre which has a maximum capacity of 800. The number of tickets sold, n , at each performance is shown in the following frequency table.

Number of tickets sold, n	Number of performances
$0 < n \leq 200$	3
$200 < n \leq 400$	p
$400 < n \leq 600$	18
$600 < n \leq 800$	30

- (a) (i) Find the value of p .
- (ii) Write down the modal class.

[2]

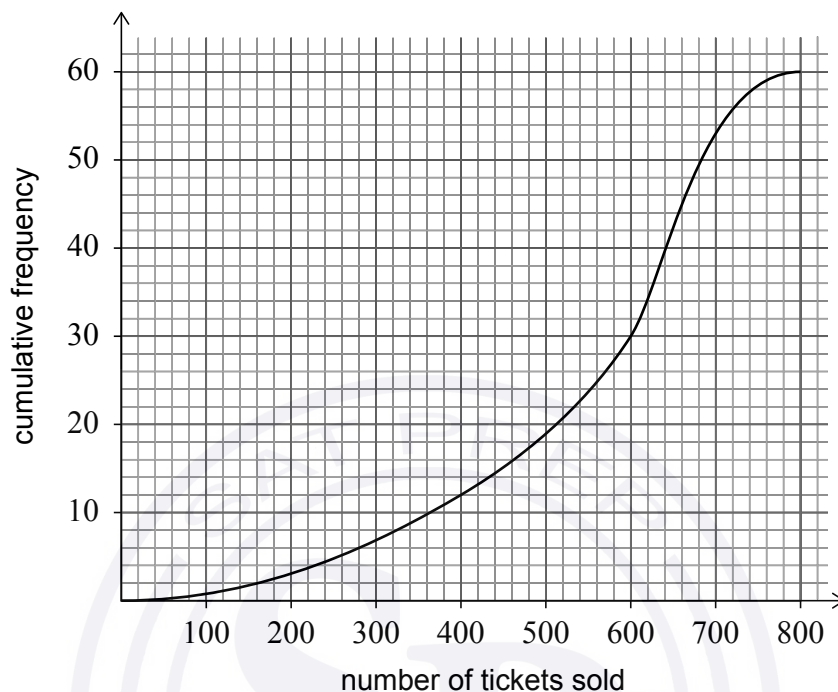
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(Question 7 continued)

The following cumulative frequency diagram also displays these data.



- (b) Use the cumulative frequency curve to estimate
- (i) the median number of tickets sold;
 - (ii) the number of performances where at least 80% of the tickets were sold.

[4]

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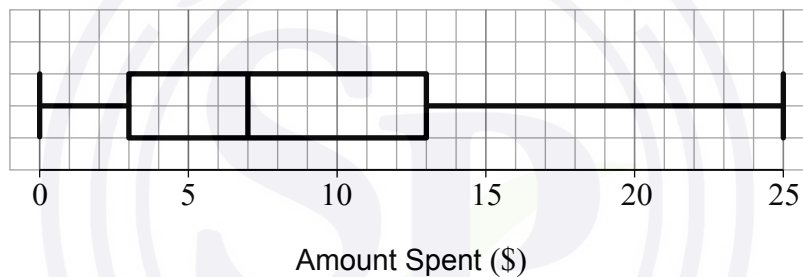
(Question 7 continued)

After a performance, the company decides to conduct a survey to obtain feedback from the audience.

- (c) (i) State one disadvantage of the company surveying only the first 5 % of the audience as they leave the theatre.
- (ii) Describe briefly how the company could collect feedback from 5 % of the audience using the systematic sampling method.
- (iii) State the sampling method which should be used if the survey is to be representative of the number of children and the number of adults in the audience. [4]

Last year 36 000 tickets were sold to *The Sleeping Beauty*.

- (d) The following box and whisker diagram displays the amount spent by the audience at the souvenir shop when they attended the performance.



- (i) Estimate the number of people who spent between \$3 and \$25.
- (ii) Half the audience spent less than \$ a . Estimate the value of a . [3]
- (e) This year the company will again give 60 performances and expects to sell 18 additional tickets for each performance.
- (i) Calculate the mean number of tickets the company expects to sell this year for each performance.
- (ii) State what effect, if any, this increase in ticket sales would have on the variance of the number of tickets sold for each performance. [4]



Do **not** write solutions on this page.

8. [Maximum mark: 15]

The functions f and g are defined by

$$f(x) = \ln(2x - 7), \text{ where } x > \frac{7}{2}$$

$$g(x) = 2 \ln x - \ln d, \text{ where } x > 0, d \in \mathbb{R}^+.$$

(a) State the equation of the vertical asymptote to the graph of $y = g(x)$. [1]

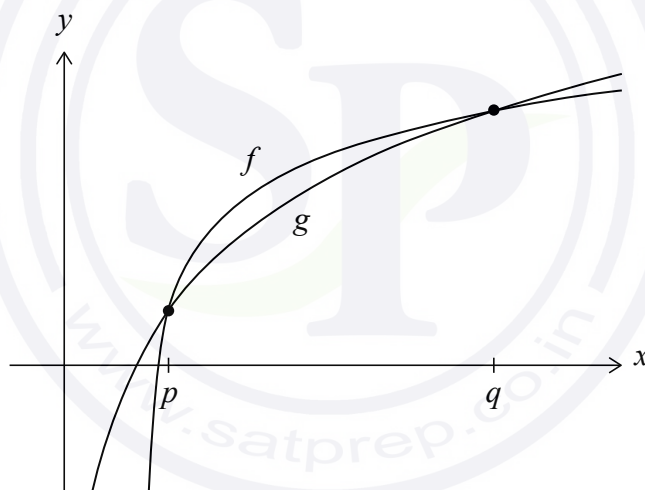
The graphs of $y = f(x)$ and $y = g(x)$ intersect at two distinct points.

(b) (i) Show that, at the points of intersection, $x^2 - 2dx + 7d = 0$.

(ii) Hence, show that $d^2 - 7d > 0$.

(iii) Find the range of possible values of d . [9]

The following diagram shows part of the graphs of $y = f(x)$ and $y = g(x)$.



The graphs intersect at $x = p$ and $x = q$, where $p < q$.

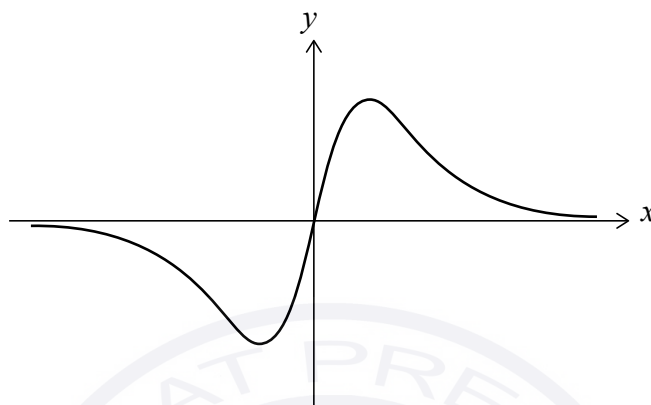
(c) In the case where $d = 10$, find the value of $q - p$. Express your answer in the form $a\sqrt{b}$, where $a, b \in \mathbb{Z}^+$. [5]



Do **not** write solutions on this page.

9. [Maximum mark: 13]

Consider the function f defined by $f(x) = \frac{12x}{(x^2 + 2)^3}$, where $x \in \mathbb{R}$. The graph of f is shown in the following diagram.



(a) Show that $f'(x) = \frac{12(2 - 5x^2)}{(x^2 + 2)^4}$. [4]

(b) Find $\int f(x) dx$. [4]

Consider a function $g(x)$ defined for $x \in \mathbb{R}$. The derivative of g is such that $g'(x) = f'(x)$, for all $x \in \mathbb{R}$.

Let R be the region enclosed by the graph of f , the graph of g , the line $x = 0$ and the line $x = 3$. The area of R is $\frac{21}{2}$.

(c) Find the two possible expressions for $g(x)$. [5]



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Mathematics: analysis and approaches

Standard level

Paper 1

30 October 2023

Zone A afternoon | Zone B afternoon | Zone C afternoon

Candidate session number

1 hour 30 minutes

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Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.



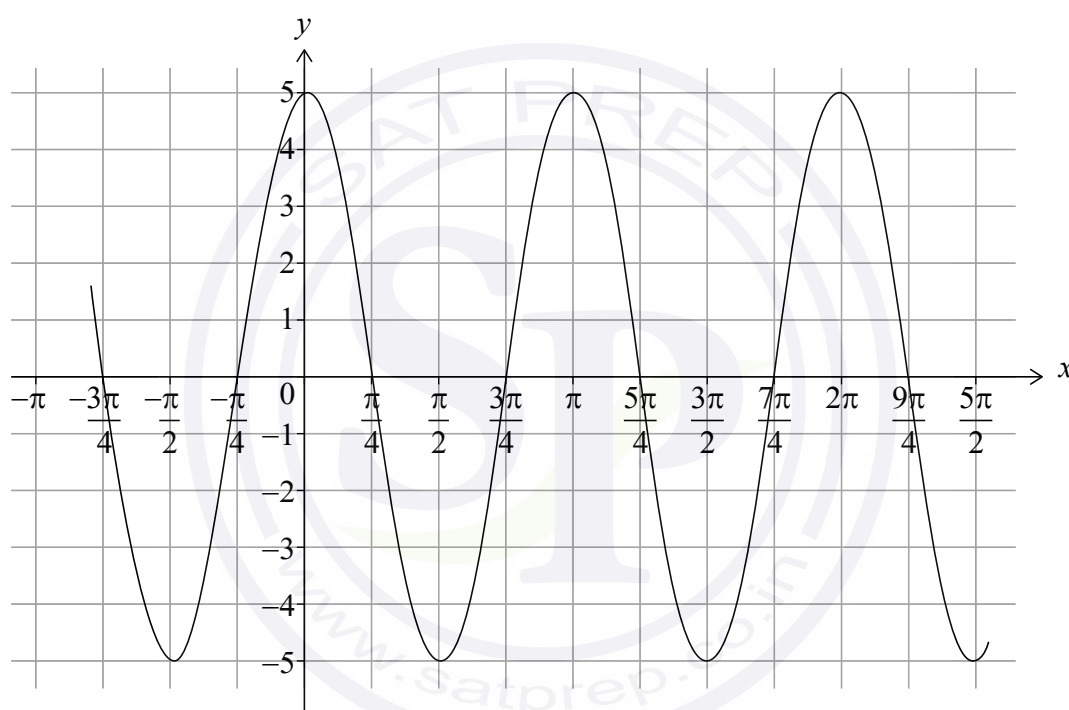
Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 7]

Consider the function $f(x) = a \cos(bx)$, with $a, b \in \mathbb{Z}^+$. The following diagram shows part of the graph of f .



- (a) Write down the value of a . [1]
- (b) (i) Write down the period of f . [3]
(ii) Hence, find the value of b .
- (c) Find the value of $f\left(\frac{\pi}{6}\right)$. [3]

(This question continues on the following page)



(Question 1 continued)

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2. [Maximum mark: 5]

Consider the functions $f(x) = x - 3$ and $g(x) = x^2 + k^2$, where k is a real constant.

(a) Write down an expression for $(g \circ f)(x)$. [2]

(b) Given that $(g \circ f)(2) = 10$, find the possible values of k . [3]

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3. [Maximum mark: 4]

Events A and B are such that $P(A) = 0.65$, $P(B) = 0.75$ and $P(A \cap B) = 0.6$.

(a) Find $P(A \cup B)$. [2]

(b) Hence, or otherwise, find $P(A' \cap B')$. [2]

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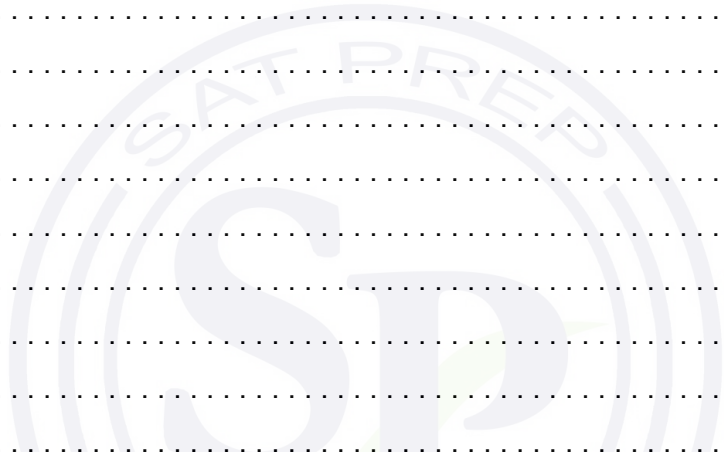


4. [Maximum mark: 7]

The sum of the first n terms of an arithmetic sequence is given by $S_n = pn^2 - qn$, where p and q are positive constants.

It is given that $S_4 = 40$ and $S_5 = 65$.

- (a) Find the value of p and the value of q . [5]
- (b) Find the value of u_5 . [2]

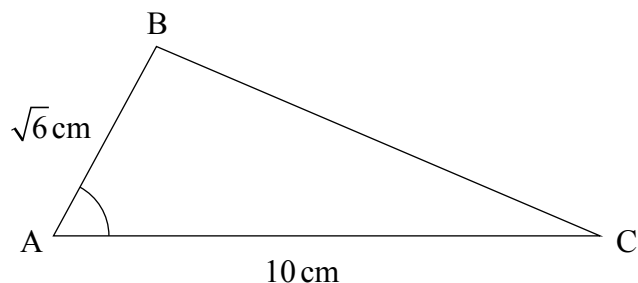




5. [Maximum mark: 6]

In the following triangle ABC , $AB = \sqrt{6}$ cm, $AC = 10$ cm and $\cos \hat{BAC} = \frac{1}{5}$.

diagram not to scale



Find the area of triangle ABC .

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6. [Maximum mark: 6]

The binomial expansion of $(1 + kx)^n$ is given by $1 + 12x + 28k^2x^2 + \dots + k^nx^n$ where $n \in \mathbb{Z}^+$ and $k \in \mathbb{Q}$.

Find the value of n and the value of k .

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Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 17]

A ballet company performs *The Nutcracker* every year. Last year they gave a total of 60 performances at their theatre which has a maximum capacity of 800. The number of tickets sold, n , at each performance is shown in the following frequency table.

Number of tickets sold, n	Number of performances
$0 < n \leq 200$	3
$200 < n \leq 400$	p
$400 < n \leq 600$	18
$600 < n \leq 800$	30

- (a) (i) Find the value of p .
- (ii) Write down the modal class.

[2]

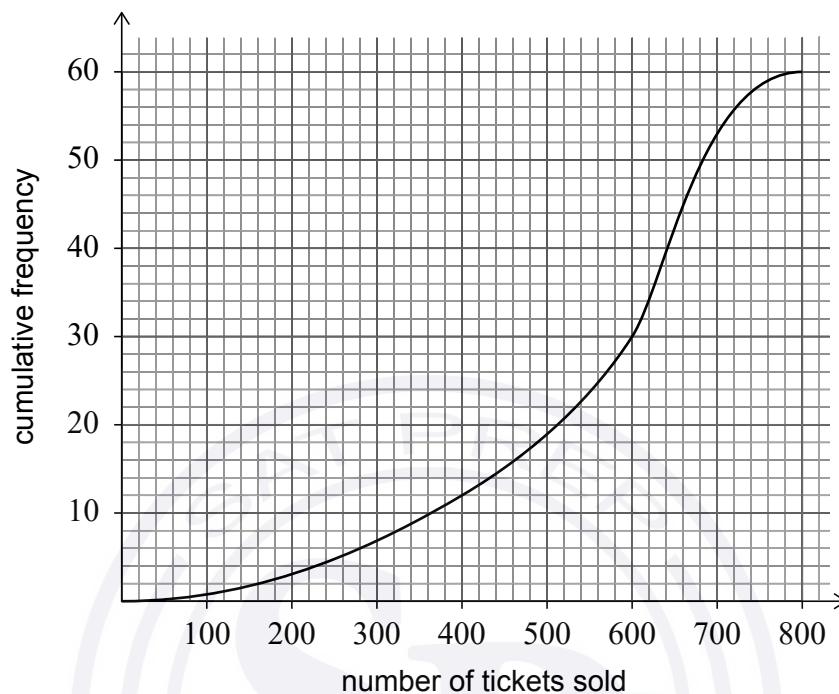
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(Question 7 continued)

The following cumulative frequency diagram also displays these data.



(b) Use the cumulative frequency curve to estimate

- (i) the median number of tickets sold;
- (ii) the number of performances where at least 80% of the tickets were sold.

[4]

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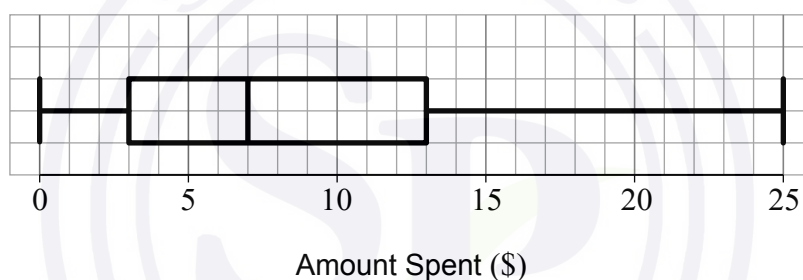
(Question 7 continued)

After a performance, the company decides to conduct a survey to obtain feedback from the audience.

- (c) (i) State one disadvantage of the company surveying only the first 5 % of the audience as they leave the theatre.
- (ii) Describe briefly how the company could collect feedback from 5 % of the audience using the systematic sampling method.
- (iii) State the sampling method which should be used if the survey is to be representative of the number of children and the number of adults in the audience. [4]

Last year 36 000 tickets were sold to *The Nutcracker*.

- (d) The following box and whisker diagram displays the amount spent by the audience at the souvenir shop when they attended the performance.



- (i) Estimate the number of people who spent between \$3 and \$25.
- (ii) Half the audience spent less than \$ a . Estimate the value of a . [3]
- (e) This year the company will again give 60 performances and expects to sell 17 additional tickets for each performance.
- (i) Calculate the mean number of tickets the company expects to sell this year for each performance.
- (ii) State what effect, if any, this increase in ticket sales would have on the variance of the number of tickets sold for each performance. [4]



Do **not** write solutions on this page.

8. [Maximum mark: 15]

The functions f and g are defined by

$$f(x) = \ln(2x - 9), \text{ where } x > \frac{9}{2}$$

$$g(x) = 2 \ln x - \ln d, \text{ where } x > 0, d \in \mathbb{R}^+.$$

(a) State the equation of the vertical asymptote to the graph of $y = g(x)$. [1]

The graphs of $y = f(x)$ and $y = g(x)$ intersect at two distinct points.

(b) (i) Show that, at the points of intersection, $x^2 - 2dx + 9d = 0$.

(ii) Hence show that $d^2 - 9d > 0$.

(iii) Find the range of possible values of d . [9]

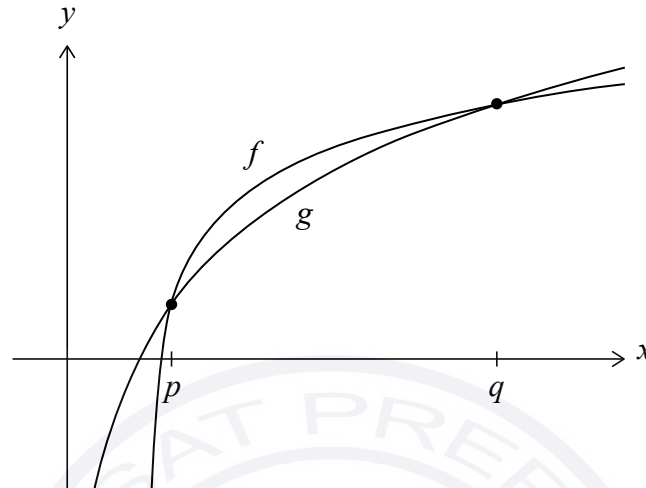
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(Question 8 continued)

The following diagram shows part of the graphs of $y = f(x)$ and $y = g(x)$.



The graphs intersect at $x = p$ and $x = q$, where $p < q$.

- (c) In the case where $d = 10$, find the value of $q - p$. Express your answer in the form $a\sqrt{b}$, where $a, b \in \mathbb{Z}^+$.

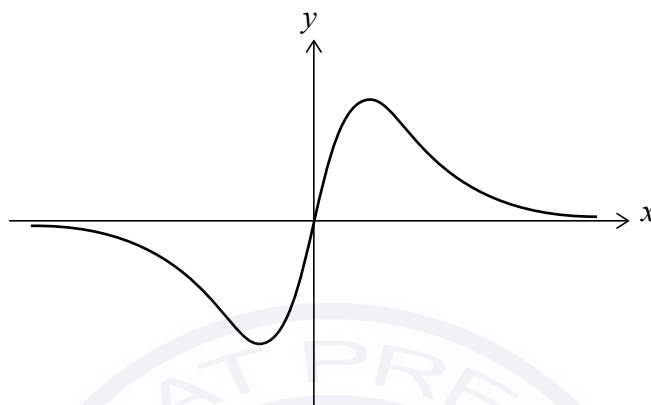
[5]



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9. [Maximum mark: 13]

Consider the function f defined by $f(x) = \frac{8x}{(x^2 + 1)^3}$, where $x \in \mathbb{R}$. The graph of f is shown in the following diagram.



(a) Show that $f'(x) = \frac{8(1-5x^2)}{(x^2 + 1)^4}$. [4]

(b) Find $\int f(x)dx$. [4]

Consider a function $g(x)$ defined for $x \in \mathbb{R}$. The derivative of g is such that $g'(x) = f'(x)$, for all $x \in \mathbb{R}$.

Let R be the region enclosed by the graph of f , the graph of g , the line $x = 0$ and the line $x = 3$. The area of R is $\frac{27}{2}$.

(c) Find the two possible expressions for $g(x)$. [5]



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Mathematics: analysis and approaches

Standard level

Paper 1

8 May 2023

Zone A afternoon | **Zone B** morning | **Zone C** afternoon

Candidate session number

1 hour 30 minutes

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Instructions to candidates

- Write your session number in the boxes above.
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- You are not permitted access to any calculator for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.



Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 5]

Point P has coordinates $(-3, 2)$, and point Q has coordinates $(15, -8)$. Point M is the midpoint of [PQ].

(a) Find the coordinates of M. [2]

Line L is perpendicular to [PQ] and passes through M.

(b) Find the gradient of L . [2]

(c) Hence, write down the equation of L . [1]

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2. [Maximum mark: 7]

The function f is defined by $f(x) = \frac{7x+7}{2x-4}$ for $x \in \mathbb{R}, x \neq 2$.

- (a) Find the zero of $f(x)$. [2]
- (b) For the graph of $y = f(x)$, write down the equation of
- (i) the vertical asymptote;
- (ii) the horizontal asymptote. [2]
- (c) Find $f^{-1}(x)$, the inverse function of $f(x)$. [3]

[illegible]

3. [Maximum mark: 6]

On a Monday at an amusement park, a sample of 40 visitors was randomly selected as they were leaving the park. They were asked how many times that day they had been on a ride called *The Dragon*. This information is summarized in the following frequency table.

Number of times on <i>The Dragon</i>	Frequency
0	6
1	16
2	13
3	2
4	3

It can be assumed that this sample is representative of all visitors to the park for the following day.

(a) For the following day, Tuesday, estimate

(i) the probability that a randomly selected visitor will ride *The Dragon*;

(ii) the expected number of times a visitor will ride *The Dragon*.

[4]

It is known that 1000 visitors will attend the amusement park on Tuesday. *The Dragon* can carry a maximum of 10 people each time it runs.

(b) Estimate the minimum number of times *The Dragon* must run to satisfy demand.

[2]

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4. [Maximum mark: 6]

(a) Show that the equation $\cos 2x = \sin x$ can be written in the form $2 \sin^2 x + \sin x - 1 = 0$. [1]

(b) Hence, solve $\cos 2x = \sin x$, where $-\pi \leq x \leq \pi$. [5]

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5. [Maximum mark: 6]

Find the range of possible values of k such that $e^{2x} + \ln k = 3e^x$ has at least one real solution.

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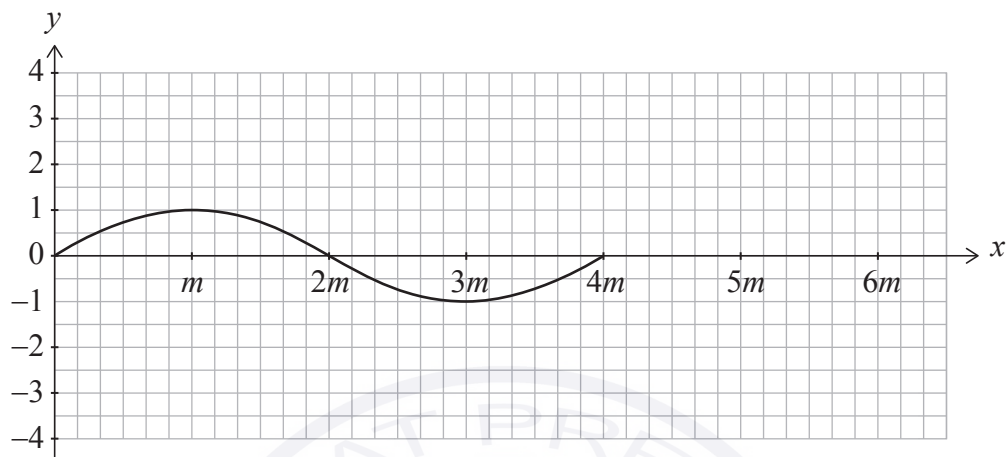
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6. [Maximum mark: 6]

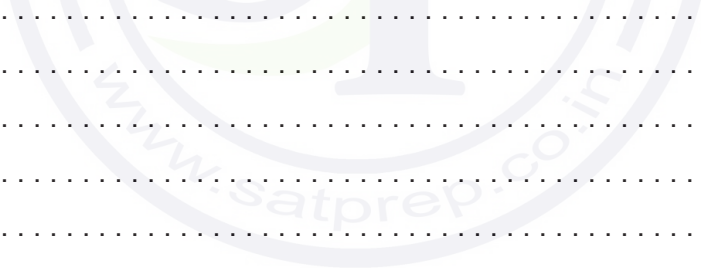
The function f is defined by $f(x) = \sin qx$, where $q > 0$. The following diagram shows part of the graph of f for $0 \leq x \leq 4m$, where x is in radians. There are x -intercepts at $x = 0, 2m$ and $4m$.



- (a) Find an expression for m in terms of q . [2]

The function g is defined by $g(x) = 3\sin\frac{2qx}{3}$, for $0 \leq x \leq 6m$.

- (b) On the axes above, sketch the graph of g . [4]



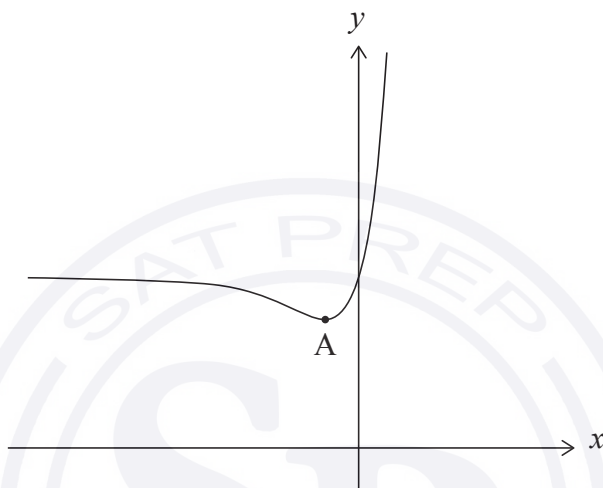
Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 13]

The function h is defined by $h(x) = 2xe^x + 3$, for $x \in \mathbb{R}$. The following diagram shows part of the graph of h , which has a local minimum at point A.



- (a) Find the value of the y -intercept. [2]
- (b) Find $h'(x)$. [2]
- (c) Hence, find the coordinates of A. [5]
- (d) (i) Show that $h''(x) = (2x + 4)e^x$.
- (ii) Find the values of x for which the graph of h is concave-up. [4]



Do **not** write solutions on this page.

8. [Maximum mark: 14]

Consider the arithmetic sequence u_1, u_2, u_3, \dots .

The sum of the first n terms of this sequence is given by $S_n = n^2 + 4n$.

- (a) (i) Find the sum of the first five terms. [4]
- (ii) Given that $S_6 = 60$, find u_6 . [2]
- (b) Find u_1 . [3]
- (c) Hence or otherwise, write an expression for u_n in terms of n . [3]

Consider a geometric sequence, v_n , where $v_2 = u_1$ and $v_4 = u_6$.

- (d) Find the possible values of the common ratio, r . [3]
- (e) Given that $v_{99} < 0$, find v_5 . [2]



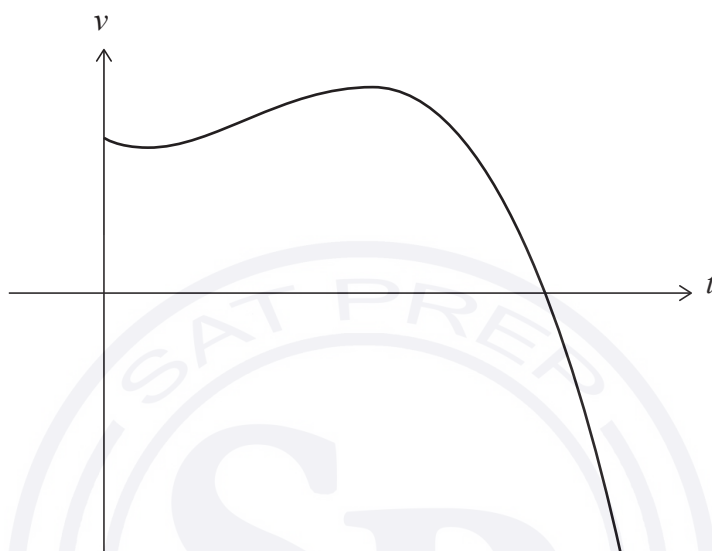
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9. [Maximum mark: 17]

An object moves along a straight line. Its velocity, $v \text{ m s}^{-1}$, at time t seconds is given by

$$v(t) = -t^3 + \frac{7}{2}t^2 - 2t + 6, \text{ for } 0 \leq t \leq 4. \text{ The object first comes to rest at } t = k.$$

The graph of v is shown in the following diagram.



At $t = 0$, the object is at the origin.

- (a) Find the displacement of the object from the origin at $t = 1$. [5]
- (b) Find an expression for the acceleration of the object. [2]
- (c) Hence, find the greatest speed reached by the object before it comes to rest. [5]
- (d) Find the greatest speed reached by the object for $0 \leq t \leq 4$. [2]
- (e) Write down an expression that represents the distance travelled by the object while its speed is increasing. Do not evaluate the expression. [3]

References:

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Mathematics: analysis and approaches
Standard level
Paper 1

8 May 2023

Zone A afternoon | **Zone B** morning | **Zone C** afternoon

Candidate session number

1 hour 30 minutes

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Instructions to candidates

- Write your session number in the boxes above.
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- You are not permitted access to any calculator for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.



Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

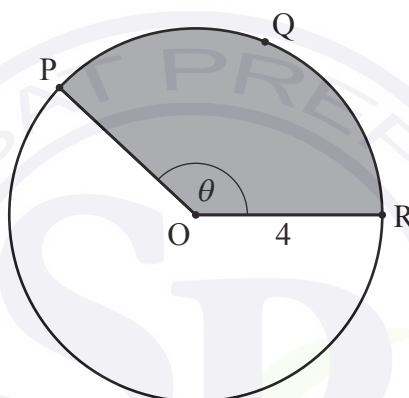
Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 6]

The following diagram shows a circle with centre O and radius 4 cm.

diagram not to scale



The points P , Q and R lie on the circumference of the circle and $\angle POR = \theta$, where θ is measured in radians.

The length of arc PQR is 10 cm.

- (a) Find the perimeter of the shaded sector. [2]
- (b) Find θ . [2]
- (c) Find the area of the shaded sector. [2]

(This question continues on the following page)



(Question 1 continued)

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2. [Maximum mark: 6]

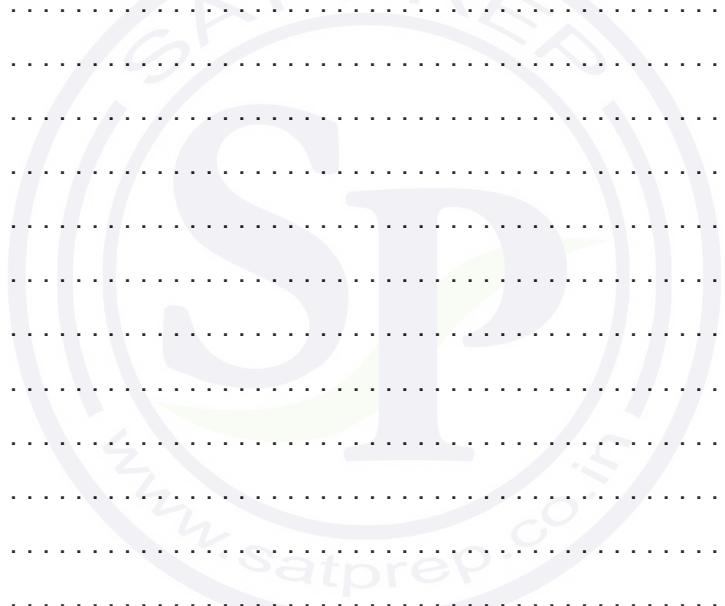
Consider an arithmetic sequence with $u_1 = 0.6$ and $u_4 = 0.15$.

(a) Find the common difference, d .

The following table shows the probability distribution of a discrete random variable X such that $P(X=n) = \frac{u_n}{k}$, where $n \in \mathbb{Z}^+$, $1 \leq n \leq 4$ and $k \in \mathbb{R}^+$.

n	1	2	3	4
$P(X = n)$	$\frac{0.6}{k}$	$\frac{u_2}{k}$	$\frac{u_3}{k}$	$\frac{0.15}{k}$

(b) Find the value of k .

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3. [Maximum mark: 5]

A function f is defined by $f(x) = 1 - \frac{1}{x-2}$, where $x \in \mathbb{R}$, $x \neq 2$.

- (a) The graph of $y = f(x)$ has a vertical asymptote and a horizontal asymptote.

Write down the equation of

- (i) the vertical asymptote;
- (ii) the horizontal asymptote.

[2]

- (b) Find the coordinates of the point where the graph of $y = f(x)$ intersects

- (i) the y -axis;
- (ii) the x -axis.

[2]

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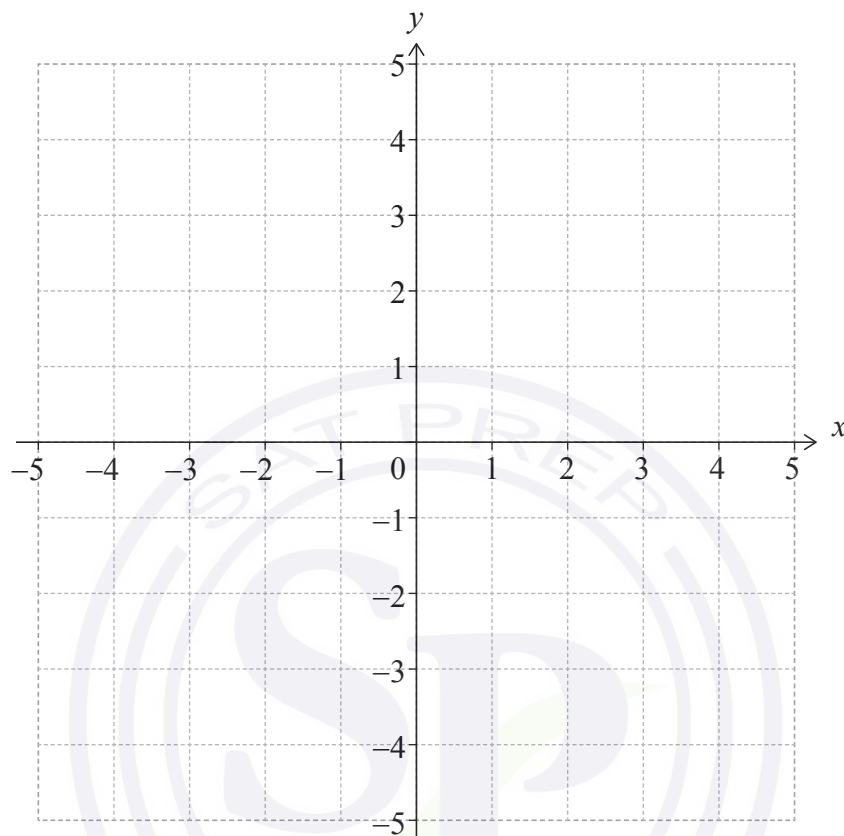
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(Question 3 continued)

- (c) On the following set of axes, sketch the graph of $y = f(x)$, showing all the features found in parts (a) and (b).

[1]



4. [Maximum mark: 5]

Events A and B are such that $P(A) = 0.4$, $P(A|B) = 0.25$ and $P(A \cup B) = 0.55$.

Find $P(B)$.

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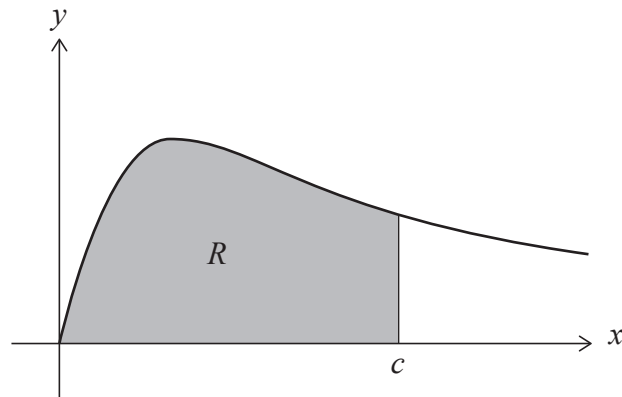
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5. [Maximum mark: 6]

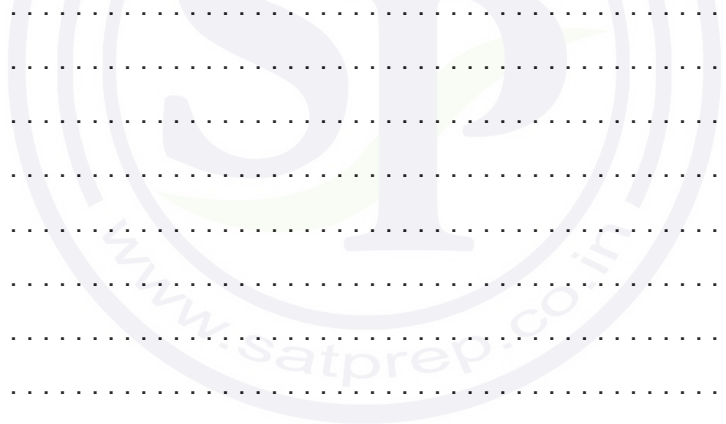
The following diagram shows part of the graph of $y = \frac{x}{x^2 + 2}$ for $x \geq 0$.



The shaded region R is bounded by the curve, the x -axis and the line $x = c$.

The area of R is $\ln 3$.

Find the value of c .



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6. [Maximum mark: 7]

The functions f and g are defined for $x \in \mathbb{R}$ by

$$f(x) = ax + b, \text{ where } a, b \in \mathbb{Z}$$

$$g(x) = x^2 + x + 3.$$

Find the two possible functions f such that $(g \circ f)(x) = 4x^2 - 14x + 15$.



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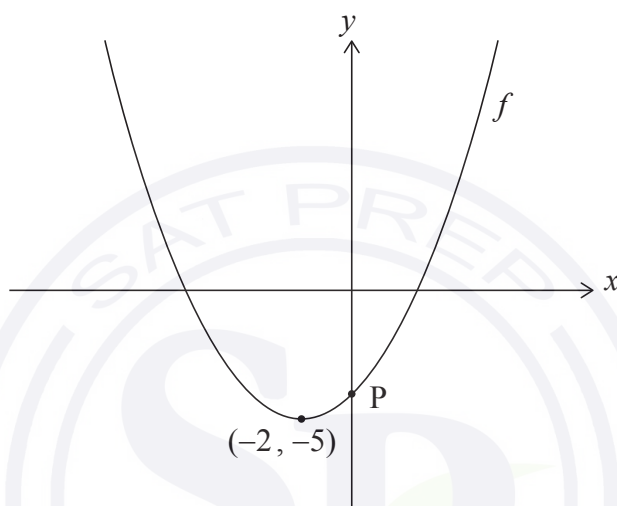
Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 17]

The following diagram shows part of the graph of a quadratic function f .

The vertex of the parabola is $(-2, -5)$ and the y -intercept is at point P.



(a) Write down the equation of the axis of symmetry.

[1]

The function can be written in the form $f(x) = \frac{1}{4}(x-h)^2 + k$, where $h, k \in \mathbb{Z}$.

(b) Write down the values of h and k .

[2]

(c) Find the y -coordinate of P.

[2]

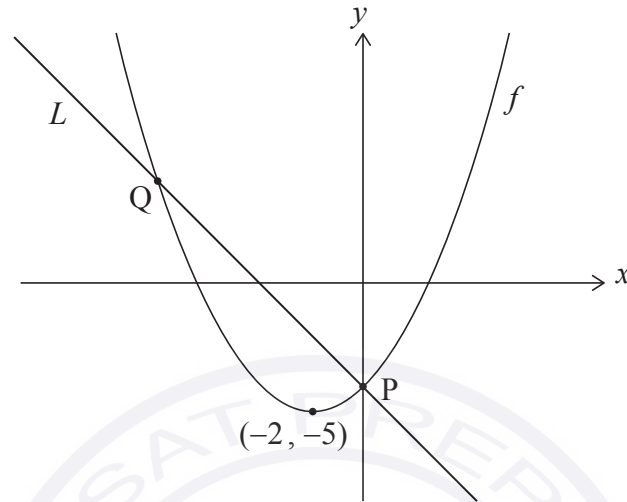
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(Question 7 continued)

In the following diagram, the line L is normal to the graph of f at point P .



- (d) Find the equation of the line L , in the form $y = ax + b$. [4]

The line L intersects the graph of f at a second point, Q , as shown above.

- (e) Calculate the distance between P and Q . [8]



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8. [Maximum mark: 14]

Consider an acute angle θ such that $\cos\theta = \frac{2}{3}$.

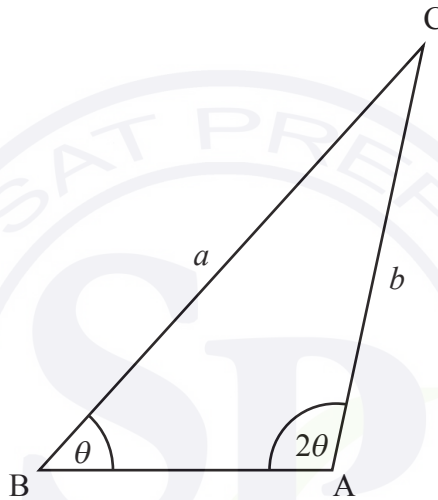
(a) Find the value of

(i) $\sin\theta$;

(ii) $\sin 2\theta$.

[4]

The following diagram shows triangle ABC , with $\hat{B} = \theta$, $\hat{A} = 2\theta$, $BC = a$ and $AC = b$.



(b) Show that $b = \frac{3a}{4}$.

[2]

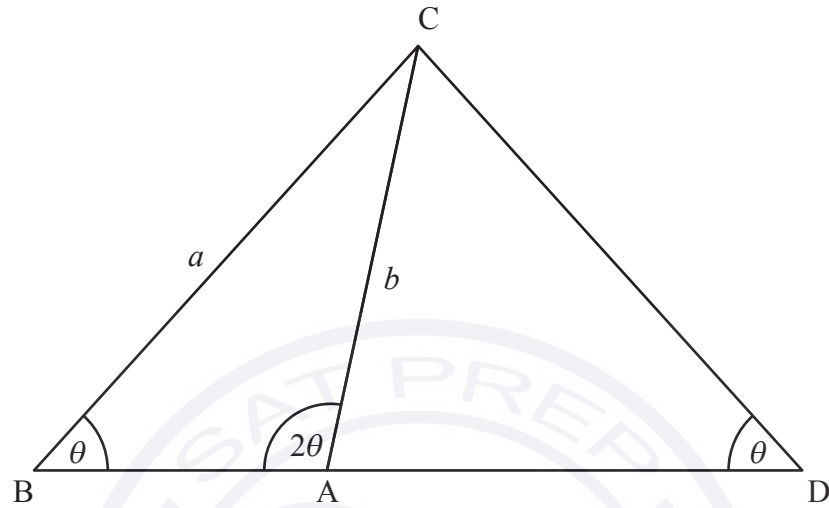
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(Question 8 continued)

[BA] is extended to form an isosceles triangle DAC, with $\hat{D} = \theta$, as shown in the following diagram.



- (c) Find the value of $\sin \hat{CAD}$. [3]
- (d) Find the area of triangle DAC, in terms of a . [5]

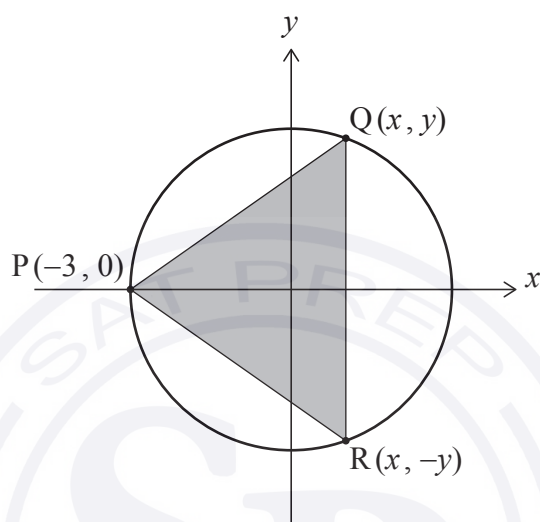


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9. [Maximum mark: 14]

A circle with equation $x^2 + y^2 = 9$ has centre $(0, 0)$ and radius 3.

A triangle, PQR, is inscribed in the circle with its vertices at $P(-3, 0)$, $Q(x, y)$ and $R(x, -y)$, where Q and R are variable points in the first and fourth quadrants respectively. This is shown in the following diagram.



- (a) For point Q, show that $y = \sqrt{9 - x^2}$. [1]
- (b) Hence, find an expression for A , the area of triangle PQR, in terms of x . [3]
- (c) Show that $\frac{dA}{dx} = \frac{9 - 3x - 2x^2}{\sqrt{9 - x^2}}$. [4]
- (d) Hence or otherwise, find the y -coordinate of R such that A is a maximum. [6]

References:

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Mathematics: analysis and approaches
Standard level
Paper 1

Monday 31 October 2022 (afternoon)

Candidate session number

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1 hour 30 minutes

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- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.



Section A



2. [Maximum mark: 4]

The function g is defined by $g(x) = e^{x^2+1}$, where $x \in \mathbb{R}$.

Find $g'(-1)$.

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3. [Maximum mark: 7]

Consider a circle with a diameter AB , where A has coordinates $(1, 4, 0)$ and B has coordinates $(-3, 2, -4)$.

(a) Find

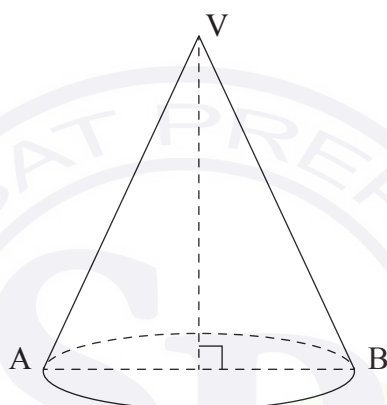
(i) the coordinates of the centre of the circle;

(ii) the radius of the circle.

[4]

The circle forms the base of a right cone whose vertex V has coordinates $(-1, -1, 0)$.

diagram not to scale



(b) Find the exact volume of the cone.

[3]

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4. [Maximum mark: 5]

Let a be a constant, where $a > 1$.

(a) Show that $a^2 + \left(\frac{a^2 - 1}{2}\right)^2 = \left(\frac{a^2 + 1}{2}\right)^2$. [3]

Consider a right-angled triangle with sides of length a , $\left(\frac{a^2-1}{2}\right)$ and $\left(\frac{a^2+1}{2}\right)$.

(b) Find an expression for the area of the triangle in terms of a . [2]



5. [Maximum mark: 5]

The derivative of the function f is given by $f'(x) = \frac{6x}{x^2 + 1}$.

The graph of $y = f(x)$ passes through the point $(1, 5)$. Find an expression for $f(x)$.

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6. [Maximum mark: 6]

Events A and B are such that $P(A) = 0.3$ and $P(B) = 0.8$.

- Determine the value of $P(A \cap B)$ in the case where the events A and B are independent. [1]
- Determine the minimum possible value of $P(A \cap B)$. [3]
- Determine the maximum possible value of $P(A \cap B)$, justifying your answer. [2]



Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 16]

- (a) The graph of a quadratic function f has its vertex at the point $(3, 2)$ and it intersects the x -axis at $x = 5$. Find f in the form $f(x) = a(x - h)^2 + k$. [3]

The quadratic function g is defined by $g(x) = px^2 + (t - 1)x - p$ where $x \in \mathbb{R}$ and $p, t \in \mathbb{R}, p \neq 0$.

- (b) In the case where $g(-3) = g(1) = 4$,

(i) find the value of p and the value of t ;

(ii) find the range of g . [7]

- (c) The linear function j is defined by $j(x) = -x + 3p$ where $x \in \mathbb{R}$ and $p \in \mathbb{R}, p \neq 0$.

Show that the graphs of $j(x) = -x + 3p$ and $g(x) = px^2 + (t - 1)x - p$ have two distinct points of intersection for every possible value of p and t . [6]

8. [Maximum mark: 15]

- (a) Calculate the value of each of the following logarithms:

(i) $\log_2 \frac{1}{16}$;

(ii) $\log_9 3$;

(iii) $\log_{\sqrt{3}} 81$. [7]

- (b) It is given that $\log_{ab} a = 3$, where $a, b \in \mathbb{R}^+, ab \neq 1$.

(i) Show that $\log_{ab} b = -2$.

(ii) Hence find the value of $\log_{ab} \frac{\sqrt[3]{a}}{\sqrt{b}}$. [8]



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9. [Maximum mark: 15]

The function f is defined by $f(x) = \cos^2 x - 3 \sin^2 x$, $0 \leq x \leq \pi$.

(a) Find the roots of the equation $f(x) = 0$. [5]

(b) (i) Find $f'(x)$.

(ii) Hence find the coordinates of the points on the graph of $y = f(x)$ where $f'(x) = 0$. [7]

(c) Sketch the graph of $y = f(x)$, clearly showing the coordinates of any points where $f'(x) = 0$ and any points where the graph meets the coordinate axes. [3]



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12EP10

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12EP12



Mathematics: analysis and approaches
Standard level
Paper 1

Friday 6 May 2022 (afternoon)

Candidate session number

1 hour 30 minutes

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Instructions to candidates

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- The maximum mark for this examination paper is **[80 marks]**.



Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 5]

The following table shows values of $f(x)$ and $g(x)$ for different values of x .

Both f and g are one-to-one functions.

x	–2	0	3	4
$f(x)$	8	4	0	–3
$g(x)$	–5	–2	4	0

- (a) Find $g(0)$. [1]
- (b) Find $(f \circ g)(0)$. [2]
- (c) Find the value of x such that $f(x) = 0$. [2]



2. [Maximum mark: 5]

The n^{th} term of an arithmetic sequence is given by $u_n = 15 - 3n$.

- (a) State the value of the first term, u_1 . [1]
- (b) Given that the n^{th} term of this sequence is -33 , find the value of n . [2]
- (c) Find the common difference, d . [2]

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3. [Maximum mark: 6]

Consider any three consecutive integers, $n - 1$, n and $n + 1$.

(a) Prove that the sum of these three integers is always divisible by 3. [2]

(b) Prove that the sum of the squares of these three integers is never divisible by 3. [4]

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4. [Maximum mark: 6]

A function f is defined by $f(x) = \frac{2x-1}{x+1}$, where $x \in \mathbb{R}$, $x \neq -1$.

(a) The graph of $y = f(x)$ has a vertical asymptote and a horizontal asymptote.

Write down the equation of

(i) the vertical asymptote;

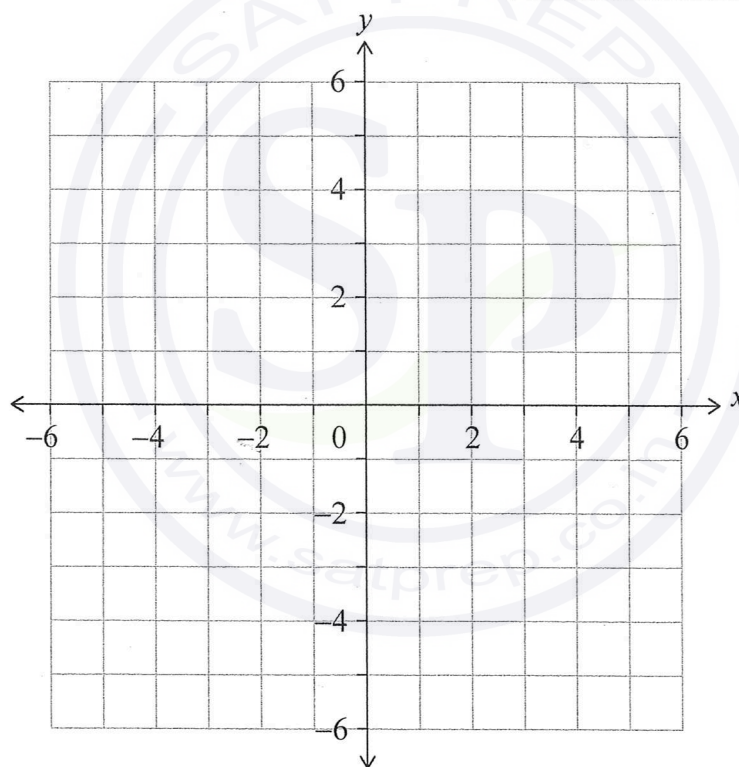
(ii) the horizontal asymptote.

[2]

(b) On the set of axes below, sketch the graph of $y = f(x)$.

On your sketch, clearly indicate the asymptotes and the position of any points of intersection with the axes.

[3]



(c) Hence, solve the inequality $0 < \frac{2x-1}{x+1} < 2$.

[1]

(This question continues on the following page)



(Question 4 continued)

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5. [Maximum mark: 5]

Find the least positive value of x for which $\cos\left(\frac{x}{2} + \frac{\pi}{3}\right) = \frac{1}{\sqrt{2}}$.

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6. [Maximum mark: 7]

Consider the binomial expansion $(x + 1)^7 = x^7 + ax^6 + bx^5 + 35x^4 + \dots + 1$ where $x \neq 0$ and $a, b \in \mathbb{Z}^+$.

[2]

The third term in the expansion is the mean of the second term and the fourth term in the expansion.

[5]

[illegible]

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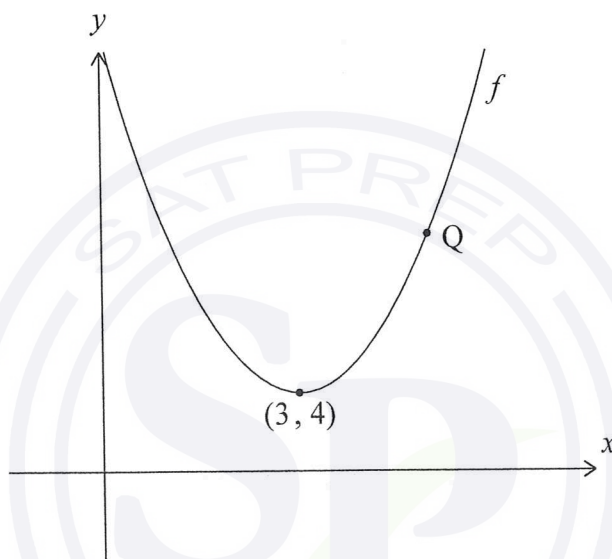
Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 15]

The following diagram shows part of the graph of a quadratic function f .

The graph of f has its vertex at $(3, 4)$, and it passes through point Q as shown.



- (a) Write down the equation of the axis of symmetry. [1]
- (b) The function can be written in the form $f(x) = a(x - h)^2 + k$.
 - (i) Write down the values of h and k .
 - (ii) Point Q has coordinates $(5, 12)$. Find the value of a . [4]

The line L is tangent to the graph of f at Q.

- (c) Find the equation of L . [4]

Now consider another function $y = g(x)$. The derivative of g is given by $g'(x) = f(x) - d$, where $d \in \mathbb{R}$.

- (d) Find the values of d for which g is an increasing function. [3]
- (e) Find the values of x for which the graph of g is concave-up. [3]

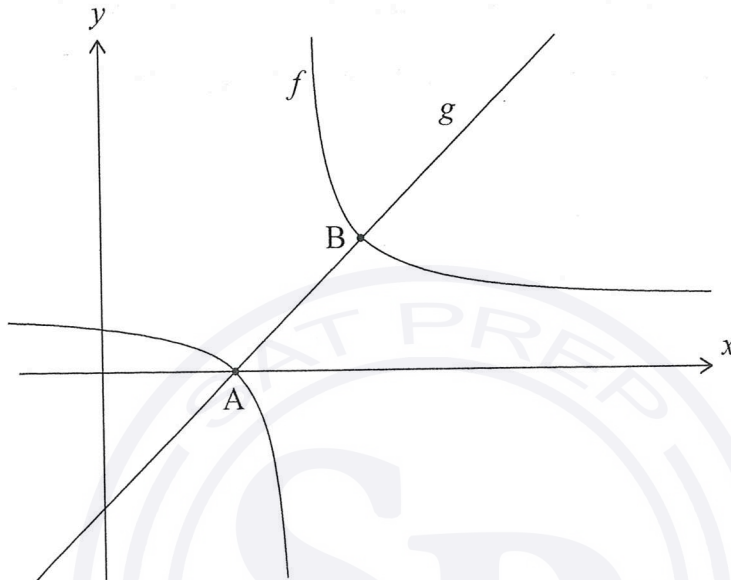


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8. [Maximum mark: 15]

Consider the functions $f(x) = \frac{1}{x-4} + 1$, for $x \neq 4$, and $g(x) = x - 3$ for $x \in \mathbb{R}$.

The following diagram shows the graphs of f and g .



The graphs of f and g intersect at points A and B. The coordinates of A are $(3, 0)$.

(a) Find the coordinates of B.

[5]

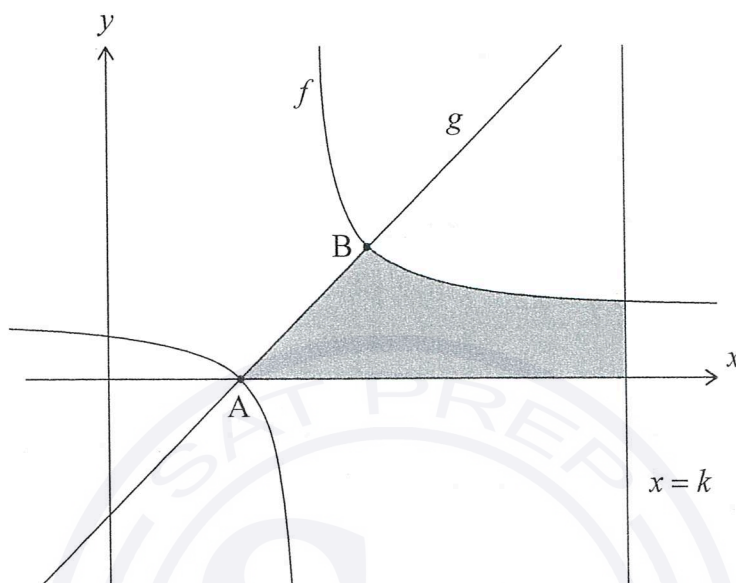
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(Question 8 continued)

In the following diagram, the shaded region is enclosed by the graph of f , the graph of g , the x -axis, and the line $x = k$, where $k \in \mathbb{Z}$.



The area of the shaded region can be written as $\ln(p) + 8$, where $p \in \mathbb{Z}$.

- (b) Find the value of k and the value of p .

[10]



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9. [Maximum mark: 16]

A biased four-sided die with faces labelled 1, 2, 3 and 4 is rolled and the result recorded. Let X be the result obtained when the die is rolled. The probability distribution for X is given in the following table where p and q are constants.

x	1	2	3	4
$P(X=x)$	p	0.3	q	0.1

For this probability distribution, it is known that $E(X) = 2$.

(a) Show that $p = 0.4$ and $q = 0.2$. [5]

(b) Find $P(X > 2)$. [2]

Nicky plays a game with this four-sided die. In this game she is allowed a maximum of five rolls. Her score is calculated by adding the results of each roll. Nicky wins the game if her score is at least ten.

After three rolls of the die, Nicky has a score of four.

(c) Assuming that rolls of the die are independent, find the probability that Nicky wins the game. [5]

David has two pairs of unbiased four-sided dice, a yellow pair and a red pair.

Both yellow dice have faces labelled 1, 2, 3 and 4. Let S represent the sum obtained by rolling the two yellow dice. The probability distribution for S is shown below.

s	2	3	4	5	6	7	8
$P(S=s)$	$\frac{1}{16}$	$\frac{2}{16}$	$\frac{3}{16}$	$\frac{4}{16}$	$\frac{3}{16}$	$\frac{2}{16}$	$\frac{1}{16}$

The first red die has faces labelled 1, 2, 2 and 3. The second red die has faces labelled 1, a , a and b , where $a < b$ and $a, b \in \mathbb{Z}^+$. The probability distribution for the sum obtained by rolling the red pair is the same as the distribution for the sum obtained by rolling the yellow pair.

(d) Determine the value of b . [2]

(e) Find the value of a , providing evidence for your answer. [2]





Mathematics: analysis and approaches
Standard level
Paper 1

Friday 6 May 2022 (afternoon)

Candidate session number

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1 hour 30 minutes

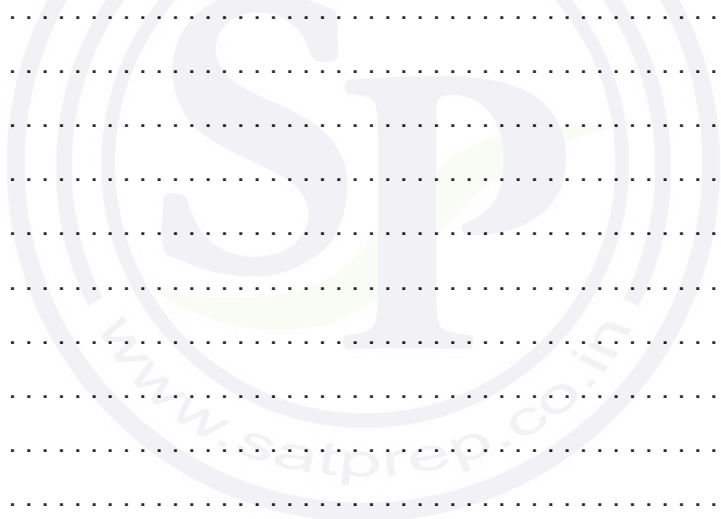
Instructions to candidates

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- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.



Section A

[2]

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2. [Maximum mark: 5]

(a) The expression $\frac{3\sqrt{x}-5}{\sqrt{x}}$ can be written as $3 - 5x^p$. Write down the value of p . [1]

(b) Hence, find the value of $\int_1^9 \left(\frac{3\sqrt{x}-5}{\sqrt{x}} \right) dx$. [4]

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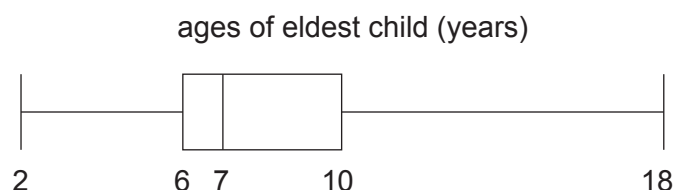


3. [Maximum mark: 7]

A survey at a swimming pool is given to one adult in each family. The age of the adult, a years old, and of their eldest child, c years old, are recorded.

The ages of the eldest child are summarized in the following box and whisker diagram.

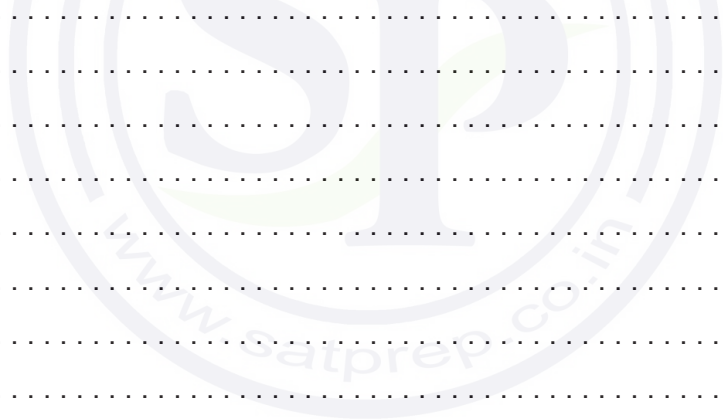
diagram not to scale



- (a) Find the largest value of c that would not be considered an outlier. [3]

The regression line of a on c is $a = \frac{7}{4}c + 20$. The regression line of c on a is $c = \frac{1}{2}a - 9$.

- (b) (i) One of the adults surveyed is 42 years old. Estimate the age of their eldest child.
- (ii) Find the mean age of all the adults surveyed.

A large, faint watermark of the Salprep logo is centered in the background. It features a circular emblem with the letters 'SP' in the center, where the 'S' is light blue and the 'P' is light green. Below the emblem, the text 'www.salprep.co.in' is written in a circular path. The entire watermark is semi-transparent.

4. [Maximum mark: 7]

Consider the functions $f(x) = \sqrt{3} \sin x + \cos x$ where $0 \leq x \leq \pi$ and $g(x) = 2x$ where $x \in \mathbb{R}$.

- (a) Find $(f \circ g)(x)$. [2]
- (b) Solve the equation $(f \circ g)(x) = 2 \cos 2x$ where $0 \leq x \leq \pi$. [5]


SAT PREP

SD



Consider the curve with equation $y = (2x - 1)e^{kx}$, where $x \in \mathbb{R}$ and $k \in \mathbb{Q}$.

Find the value of k .



6. [Maximum mark: 7]

Consider $f(x) = 4\sin x + 2.5$ and $g(x) = 4\sin\left(x - \frac{3\pi}{2}\right) + 2.5 + q$, where $x \in \mathbb{R}$ and $q > 0$.

The graph of g is obtained by two transformations of the graph of f .


(a) Describe these two transformations.

[2]

The y -intercept of the graph of g is at $(0, r)$.

(b) Given that $g(x) \geq 7$, find the smallest value of r .

[5]



Turn over

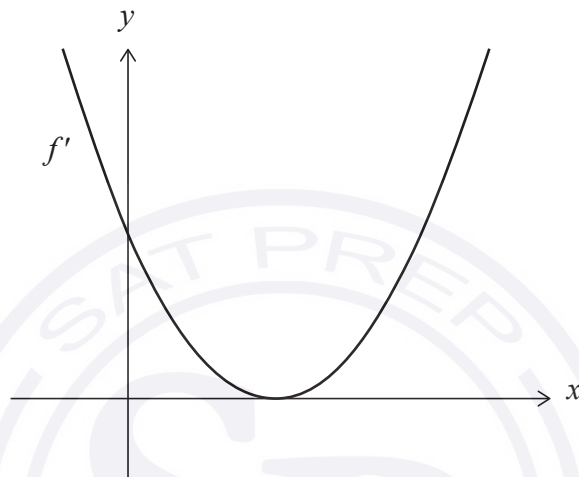
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Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 14]

A function, f , has its derivative given by $f'(x) = 3x^2 - 12x + p$, where $p \in \mathbb{R}$. The following diagram shows part of the graph of f' .



The graph of f' has an axis of symmetry $x = q$.

(a) Find the value of q . [2]

The vertex of the graph of f' lies on the x -axis.

(b) (i) Write down the value of the discriminant of f' .
(ii) Hence or otherwise, find the value of p . [4]

(c) Find the value of the gradient of the graph of f' at $x = 0$. [3]

(d) Sketch the graph of f'' , the second derivative of f . Indicate clearly the x -intercept and the y -intercept. [2]

The graph of f has a point of inflexion at $x = a$.

(e) (i) Write down the value of a .
(ii) Find the values of x for which the graph of f is concave-down. Justify your answer. [3]



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8. [Maximum mark: 15]

Consider the series $\ln x + p \ln x + \frac{1}{3} \ln x + \dots$, where $x \in \mathbb{R}$, $x > 1$ and $p \in \mathbb{R}$, $p \neq 0$.

(a) Consider the case where the series is geometric.

(i) Show that $p = \pm \frac{1}{\sqrt{3}}$.

(ii) Given that $p > 0$ and $S_{\infty} = 3 + \sqrt{3}$, find the value of x . [5]

(b) Now consider the case where the series is arithmetic with common difference d .

(i) Show that $p = \frac{2}{3}$.

(ii) Write down d in the form $k \ln x$, where $k \in \mathbb{Q}$.

(iii) The sum of the first n terms of the series is $-3 \ln x$.

Find the value of n . [10]

9. [Maximum mark: 15]

(a) (i) Expand and simplify $(1 - a)^3$ in ascending powers of a .

(ii) By using a suitable substitution for a , show that $1 - 3 \cos 2x + 3 \cos^2 2x - \cos^3 2x = 8 \sin^6 x$. [6]

Consider $f(x) = 4 \cos x (1 - 3 \cos 2x + 3 \cos^2 2x - \cos^3 2x)$.

(b) (i) Show that $\int_0^m f(x) dx = \frac{32}{7} \sin^7 m$, where m is a positive real constant.

(ii) It is given that $\int_m^{\frac{\pi}{2}} f(x) dx = \frac{127}{28}$, where $0 \leq m \leq \frac{\pi}{2}$. Find the value of m . [9]

References:

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Mathematics: analysis and approaches
Standard level
Paper 1

Monday 1 November 2021 (afternoon)

Candidate session number

1 hour 30 minutes

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Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
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- A clean copy of the **mathematics: analysis and approaches formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.



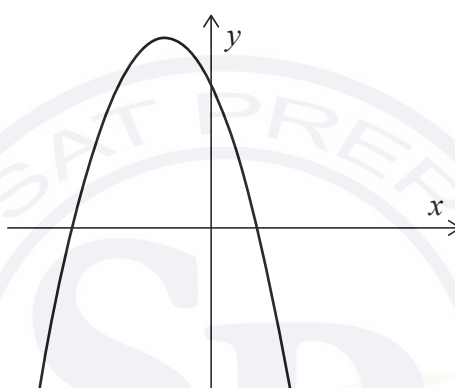
Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 7]

Consider the function $f(x) = -2(x - 1)(x + 3)$, for $x \in \mathbb{R}$. The following diagram shows part of the graph of f .



(a) For the graph of f

- (i) find the x -coordinates of the x -intercepts;
- (ii) find the coordinates of the vertex.

[5]

The function f can be written in the form $f(x) = -2(x - h)^2 + k$.

(b) Write down the value of h and the value of k .

[2]

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2. [Maximum mark: 4]

Given that $\frac{dy}{dx} = \cos\left(x - \frac{\pi}{4}\right)$ and $y = 2$ when $x = \frac{3\pi}{4}$, find y in terms of x .

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3. [Maximum mark: 5]

The function f is defined by $f(x) = \frac{2x+4}{3-x}$, where $x \in \mathbb{R}$, $x \neq 3$.

- (a) Write down the equation of
- (i) the vertical asymptote of the graph of f ;
 - (ii) the horizontal asymptote of the graph of f . [2]
- (b) Find the coordinates where the graph of f crosses
- (i) the x -axis;
 - (ii) the y -axis. [2]

[illegible]

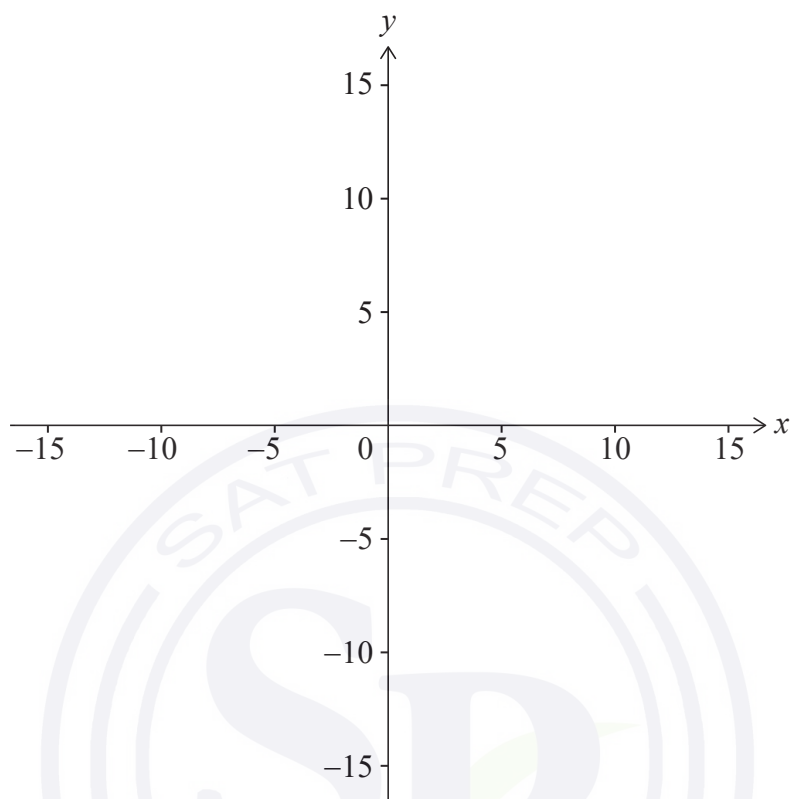
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(Question 3 continued)

(c) Sketch the graph of f on the axes below.

[1]



4. [Maximum mark: 5]

Box 1 contains 5 red balls and 2 white balls.

Box 2 contains 4 red balls and 3 white balls.

- (a) A box is chosen at random and a ball is drawn. Find the probability that the ball is red. [3]

Let A be the event that “box 1 is chosen” and let R be the event that “a red ball is drawn”.

- (b) Determine whether events A and R are independent. [2]

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5. [Maximum mark: 7]

The function f is defined for all $x \in \mathbb{R}$. The line with equation $y = 6x - 1$ is the tangent to the graph of f at $x = 4$.

- (a) Write down the value of $f'(4)$. [1]
- (b) Find $f(4)$. [1]

The function g is defined for all $x \in \mathbb{R}$ where $g(x) = x^2 - 3x$ and $h(x) = f(g(x))$.

- (c) Find $h(4)$. [2]
- (d) Hence find the equation of the tangent to the graph of h at $x = 4$. [3]

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6. [Maximum mark: 7]

(a) Show that $2x - 3 - \frac{6}{x-1} = \frac{2x^2 - 5x - 3}{x-1}$, $x \in \mathbb{R}$, $x \neq 1$. [2]

(b) Hence or otherwise, solve the equation $2\sin 2\theta - 3 - \frac{6}{\sin 2\theta - 1} = 0$ for $0 \leq \theta \leq \pi$, $\theta \neq \frac{\pi}{4}$. [5]

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Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 16]

A particle P moves along the x -axis. The velocity of P is $v \text{ m s}^{-1}$ at time t seconds, where $v(t) = 4 + 4t - 3t^2$ for $0 \leq t \leq 3$. When $t = 0$, P is at the origin O .

- (a) (i) Find the value of t when P reaches its maximum velocity. [7]
- (ii) Show that the distance of P from O at this time is $\frac{88}{27}$ metres. [7]
- (b) Sketch a graph of v against t , clearly showing any points of intersection with the axes. [4]
- (c) Find the total distance travelled by P . [5]

8. [Maximum mark: 15]

Consider the function $f(x) = a^x$ where $x, a \in \mathbb{R}$ and $x > 0, a > 1$.

The graph of f contains the point $\left(\frac{2}{3}, 4\right)$.

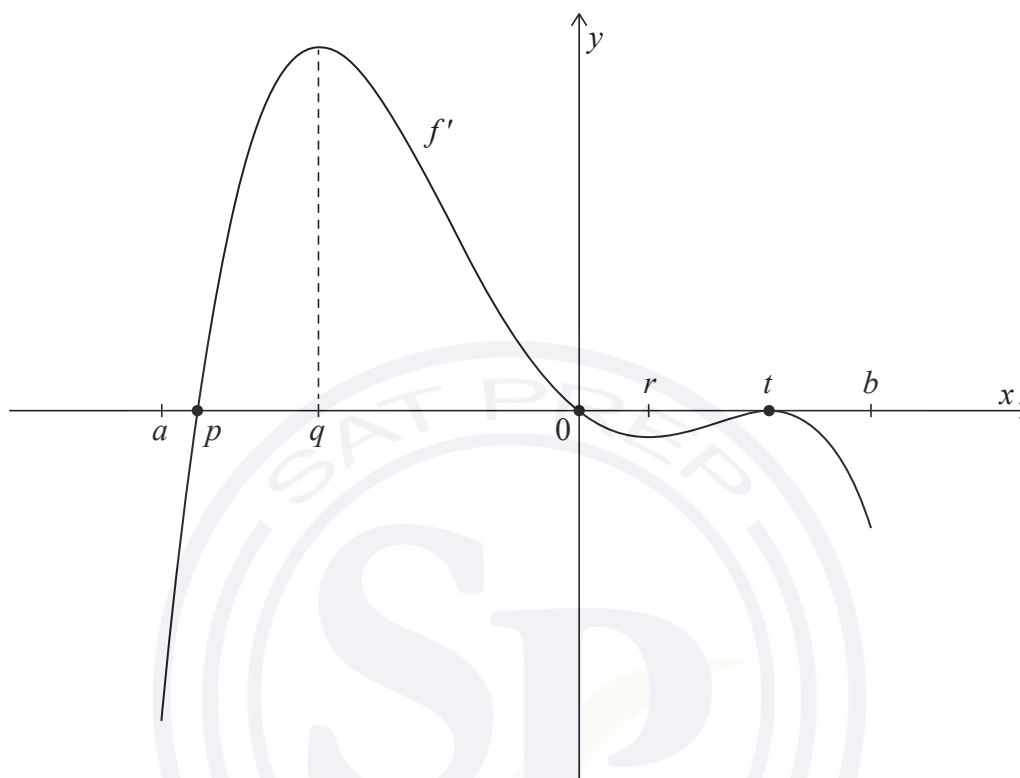
- (a) Show that $a = 8$. [2]
- (b) Write down an expression for $f^{-1}(x)$. [1]
- (c) Find the value of $f^{-1}(\sqrt{32})$. [3]
- (d) Consider the arithmetic sequence $\log_8 27, \log_8 p, \log_8 q, \log_8 125$, where $p > 1$ and $q > 1$.
 - (i) Show that $27, p, q$ and 125 are four consecutive terms in a geometric sequence.
 - (ii) Find the value of p and the value of q . [9]



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9. [Maximum mark: 14]

Consider a function f with domain $a < x < b$. The following diagram shows the graph of f' , the derivative of f .



The graph of f' , the derivative of f , has x -intercepts at $x = p$, $x = 0$ and $x = t$. There are local maximum points at $x = q$ and $x = t$ and a local minimum point at $x = r$.

- (a) Find all the values of x where the graph of f is increasing. Justify your answer. [2]
- (b) Find the value of x where the graph of f has a local maximum. [1]
- (c) (i) Find the value of x where the graph of f has a local minimum. Justify your answer.
- (ii) Find the values of x where the graph of f has points of inflexion. Justify your answer. [5]
- (d) The total area of the region enclosed by the graph of f' , the derivative of f , and the x -axis is 20.

Given that $f(p) + f(t) = 4$, find the value of $f(0)$. [6]

References:



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Mathematics: analysis and approaches
Standard level
Paper 1

Thursday 6 May 2021 (afternoon)

Candidate session number

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1 hour 30 minutes

Instructions to candidates

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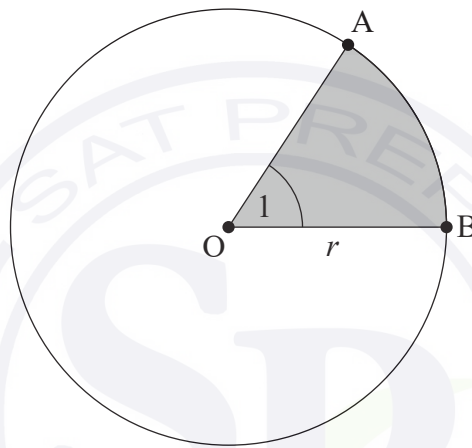
Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 6]

The following diagram shows a circle with centre O and radius r .

diagram not to scale



Points A and B lie on the circumference of the circle, and $\angle AOB = 1$ radian.

The perimeter of the shaded region is 12.

(a) Find the value of r . [3]

(b) Hence, find the exact area of the **non-shaded** region. [3]

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2. [Maximum mark: 4]

Consider two consecutive positive integers, n and $n + 1$.

Show that the difference of their squares is equal to the sum of the two integers.

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3. [Maximum mark: 6]

(a) Show that the equation $2 \cos^2 x + 5 \sin x = 4$ may be written in the form $2 \sin^2 x - 5 \sin x + 2 = 0$.

[1]

(b) Hence, solve the equation $2 \cos^2 x + 5 \sin x = 4$, $0 \leq x \leq 2\pi$.

[5]

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4. [Maximum mark: 5]

In the expansion of $(x + k)^7$, where $k \in \mathbb{R}$, the coefficient of the term in x^5 is 63.

Find the possible values of k .

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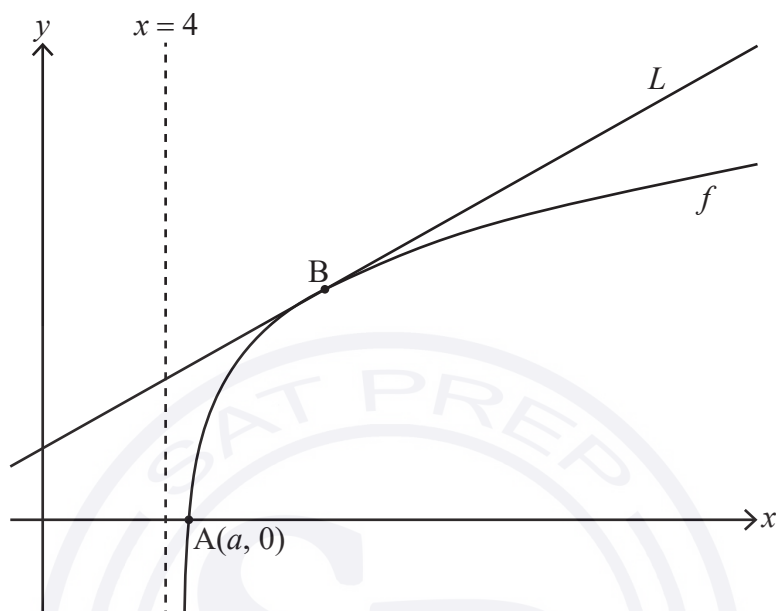
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5. [Maximum mark: 9]

Consider the function f defined by $f(x) = \ln(x^2 - 16)$ for $x > 4$.

The following diagram shows part of the graph of f which crosses the x -axis at point A, with coordinates $(a, 0)$. The line L is the tangent to the graph of f at the point B.



(a) Find the exact value of a . [3]

(b) Given that the gradient of L is $\frac{1}{3}$, find the x -coordinate of B. [6]

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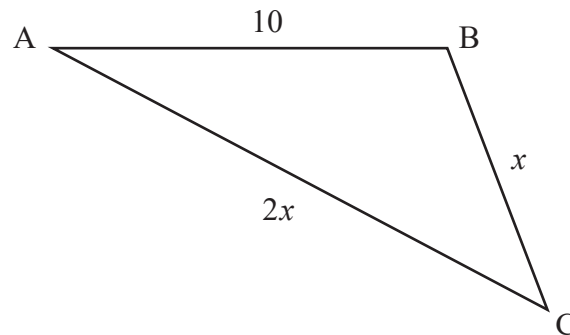
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6. [Maximum mark: 7]

The following diagram shows triangle ABC, with $AB = 10$, $BC = x$ and $AC = 2x$.

diagram not to scale



Given that $\cos \hat{C} = \frac{3}{4}$, find the area of the triangle.

Give your answer in the form $\frac{p\sqrt{q}}{2}$ where $p, q \in \mathbb{Z}^+$.

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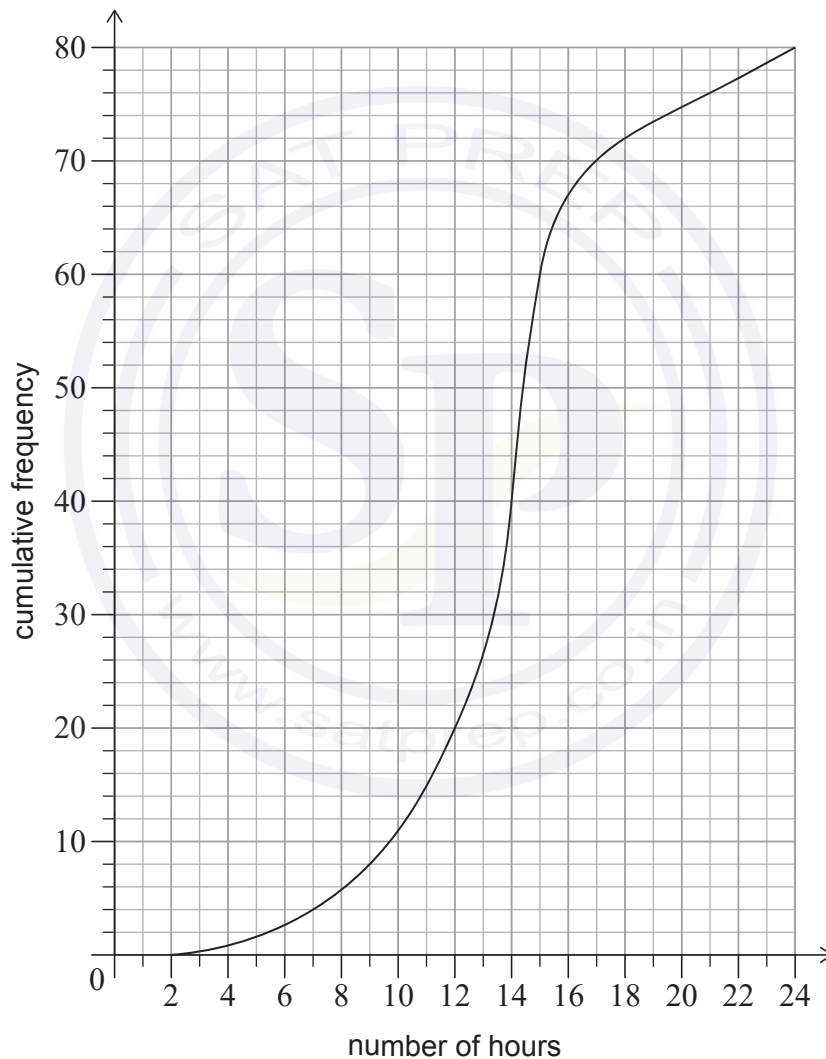
Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 14]

A large school has students from Year 6 to Year 12.

A group of 80 students in Year 12 were randomly selected and surveyed to find out how many hours per week they each spend doing homework. Their results are represented by the following cumulative frequency graph.



(This question continues on the following page)



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(Question 7 continued)

- (a) Find the median number of hours per week these Year 12 students spend doing homework. [2]
- (b) Given that 10% of these Year 12 students spend more than k hours per week doing homework, find the value of k . [3]

This same information is represented by the following table.

Hours (h) spent doing homework	$2 < h \leq 7$	$7 < h \leq 15$	$15 < h \leq 21$	$21 < h \leq 24$
Frequency	4	p	16	q

- (c) Find the value of p and the value of q . [4]

There are 320 students in Year 12 at this school.

- (d) Estimate the number of Year 12 students that spend more than 15 hours each week doing homework. [3]
- (e) (i) Explain why this sampling method might not provide an accurate representation of the amount of time **all** of the students in the school spend doing homework.
- (ii) Suggest a more appropriate sampling method. [2]

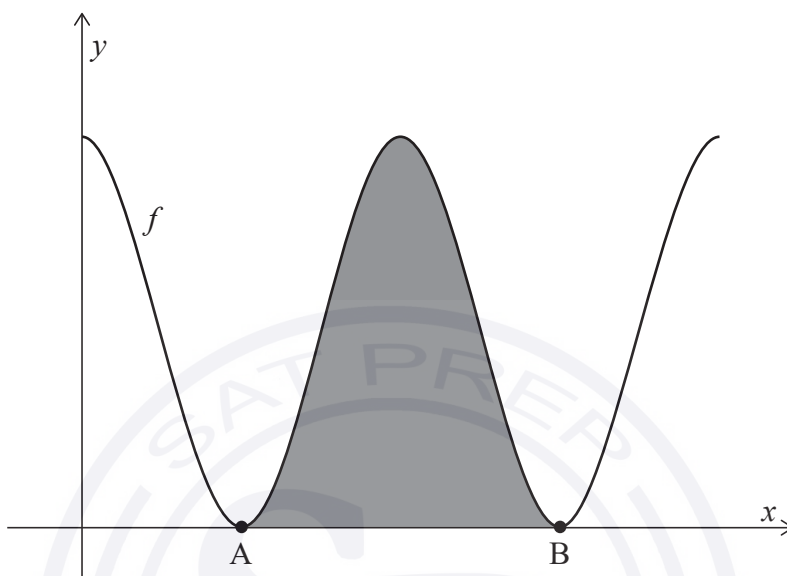


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8. [Maximum mark: 15]

Consider the function f defined by $f(x) = 6 + 6 \cos x$, for $0 \leq x \leq 4\pi$.

The following diagram shows the graph of $y = f(x)$.



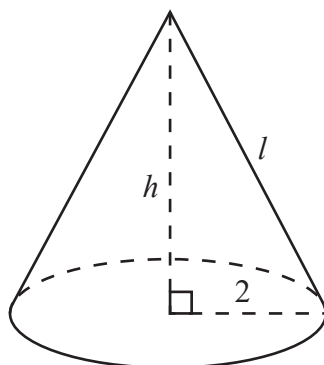
The graph of f touches the x -axis at points A and B, as shown. The shaded region is enclosed by the graph of $y = f(x)$ and the x -axis, between the points A and B.

- (a) Find the x -coordinates of A and B. [3]
- (b) Show that the area of the shaded region is 12π . [5]

The right cone in the following diagram has a total surface area of 12π , equal to the shaded area in the previous diagram.

The cone has a base radius of 2, height h , and slant height l .

diagram not to scale



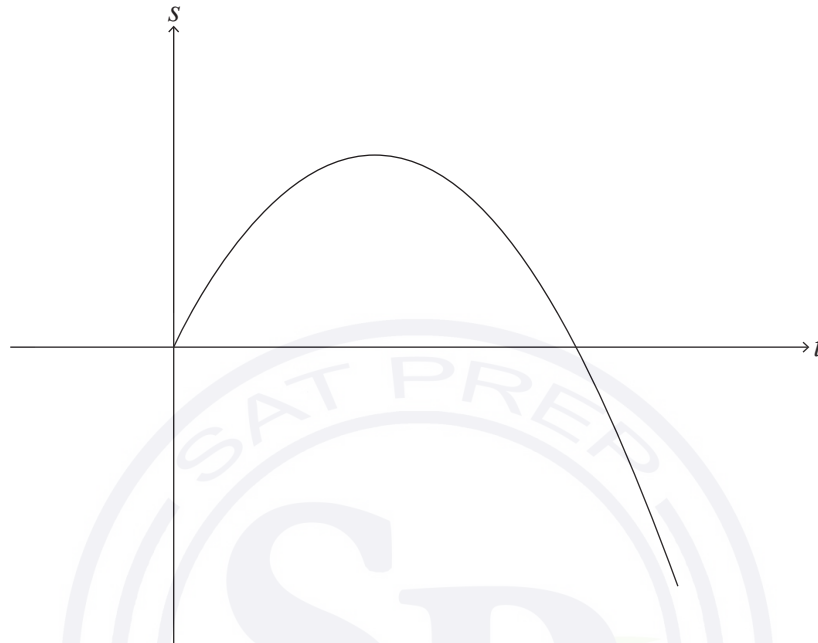
- (c) Find the value of l . [3]
- (d) Hence, find the volume of the cone. [4]



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9. [Maximum mark: 14]

Particle A travels in a straight line such that its displacement, s metres, from a fixed origin after t seconds is given by $s(t) = 8t - t^2$, for $0 \leq t \leq 10$, as shown in the following diagram.



Particle A starts at the origin and passes through the origin again when $t = p$.

(a) Find the value of p . [2]

Particle A changes direction when $t = q$.

(b) (i) Find the value of q .
(ii) Find the displacement of particle A from the origin when $t = q$. [4]

(c) Find the distance of particle A from the origin when $t = 10$. [2]

The total distance travelled by particle A is given by d .

(d) Find the value of d . [2]

A second particle, particle B, travels along the same straight line such that its velocity is given by $v(t) = 14 - 2t$, for $t \geq 0$.

When $t = k$, the distance travelled by particle B is equal to d .

(e) Find the value of k . [4]

References:

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12EP12



Mathematics: analysis and approaches
Standard level
Paper 1

Thursday 6 May 2021 (afternoon)

Candidate session number

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1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.



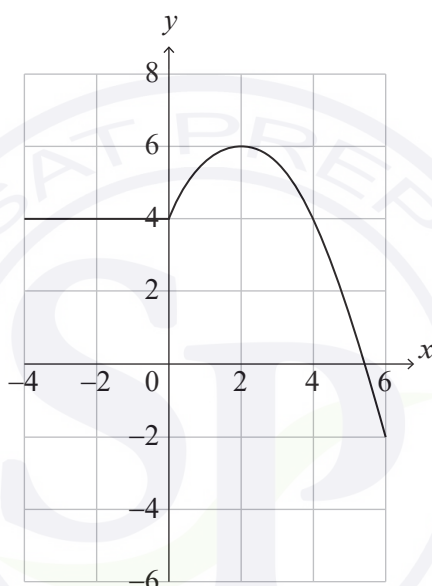
Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 5]

The graph of $y = f(x)$ for $-4 \leq x \leq 6$ is shown in the following diagram.



(a) Write down the value of

(i) $f(2)$;

(ii) $(f \circ f)(2)$.

[2]

(b) Let $g(x) = \frac{1}{2}f(x) + 1$ for $-4 \leq x \leq 6$. On the axes above, sketch the graph of g .

[3]

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2. [Maximum mark: 4]

The diameter of a spherical planet is $6 \times 10^4 \text{ km}$.


(a) Write down the radius of the planet.

[1]

The volume of the planet can be expressed in the form $\pi(a \times 10^k)\text{km}^3$ where $1 \leq a < 10$ and $k \in \mathbb{Z}$.

(b) Find the value of a and the value of k .

[3]





3. [Maximum mark: 5]

Consider an arithmetic sequence where $u_8 = S_8 = 8$. Find the value of the first term, u_1 , and the value of the common difference, d .

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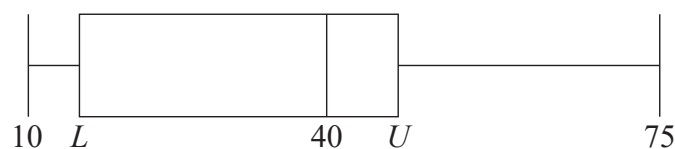
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4. [Maximum mark: 5]

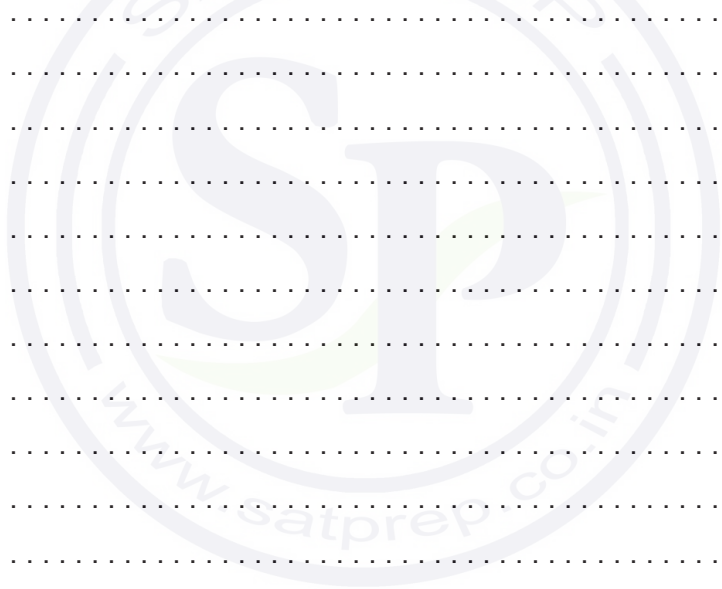
A research student weighed lizard eggs in grams and recorded the results. The following box and whisker diagram shows a summary of the results where L and U are the lower and upper quartiles respectively.

diagram not to scale



The interquartile range is 20 grams and there are no outliers in the results.

- (a) Find the minimum possible value of U . [3]
- (b) Hence, find the minimum possible value of L . [2]

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5. [Maximum mark: 7]

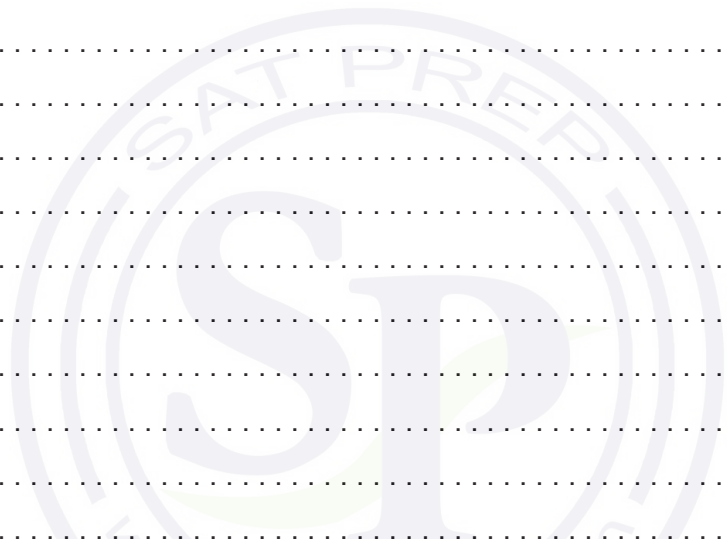
Consider the functions $f(x) = -(x - h)^2 + 2k$ and $g(x) = e^{x-2} + k$ where $h, k \in \mathbb{R}$.

(a) Find $f'(x)$. [1]

The graphs of f and g have a common tangent at $x = 3$.

(b) Show that $h = \frac{e+6}{2}$. [3]

(c) Hence, show that $k = e + \frac{e^2}{4}$. [3]

A large, light gray watermark logo for 'SAT PREP' is centered on the page. The logo consists of a circular seal with 'SAT PREP' written around the top inner edge. In the center of the seal is a large 'S' and 'P' with a green leaf-like shape between them. The background of the page is white with horizontal dotted lines.

6. [Maximum mark: 8]

(a) Show that $\sin 2x + \cos 2x - 1 = 2 \sin x (\cos x - \sin x)$. [2]

(b) Hence or otherwise, solve $\sin 2x + \cos 2x - 1 + \cos x - \sin x = 0$ for $0 < x < 2\pi$. [6]

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Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 14]

Let $f(x) = mx^2 - 2mx$, where $x \in \mathbb{R}$ and $m \in \mathbb{R}$. The line $y = mx - 9$ meets the graph of f at exactly one point.

(a) Show that $m = 4$. [6]

The function f can be expressed in the form $f(x) = 4(x - p)(x - q)$, where $p, q \in \mathbb{R}$.

(b) Find the value of p and the value of q . [2]

The function f can also be expressed in the form $f(x) = 4(x - h)^2 + k$, where $h, k \in \mathbb{R}$.

(c) Find the value of h and the value of k . [3]

(d) Hence find the values of x where the graph of f is both negative and increasing. [3]

8. [Maximum mark: 16]

Let $y = \frac{\ln x}{x^4}$ for $x > 0$.

(a) Show that $\frac{dy}{dx} = \frac{1 - 4 \ln x}{x^5}$. [3]

Consider the function defined by $f(x) = \frac{\ln x}{x^4}$ for $x > 0$ and its graph $y = f(x)$.

(b) The graph of f has a horizontal tangent at point P. Find the coordinates of P. [5]

(c) Given that $f''(x) = \frac{20 \ln x - 9}{x^6}$, show that P is a local maximum point. [3]

(d) Solve $f(x) > 0$ for $x > 0$. [2]

(e) Sketch the graph of f , showing clearly the value of the x -intercept and the approximate position of point P. [3]



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9. [Maximum mark: 16]

A biased four-sided die, A, is rolled. Let X be the score obtained when die A is rolled. The probability distribution for X is given in the following table.

x	1	2	3	4
$P(X = x)$	p	p	p	$\frac{1}{2}p$

(a) Find the value of p . [2]

(b) Hence, find the value of $E(X)$. [2]

A second biased four-sided die, B, is rolled. Let Y be the score obtained when die B is rolled. The probability distribution for Y is given in the following table.

y	1	2	3	4
$P(Y = y)$	q	q	q	r

(c) (i) State the range of possible values of r .

(ii) Hence, find the range of possible values of q . [3]

(d) Hence, find the range of possible values for $E(Y)$. [3]

Agnes and Barbara play a game using these dice. Agnes rolls die A once and Barbara rolls die B once. The probability that Agnes' score is less than Barbara's score is $\frac{1}{2}$.

(e) Find the value of $E(Y)$. [6]

References:

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12EP10

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12EP12

**Mathematics: analysis and approaches**
Standard level
Paper 1

Specimen

Candidate session number

1 hour 30 minutes

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Instructions to candidates

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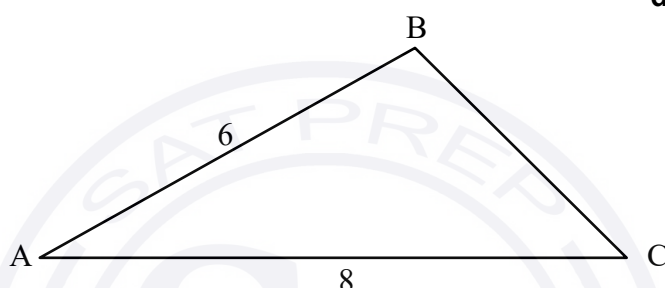
Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 5]

The following diagram shows triangle ABC, with $AB = 6$ and $AC = 8$.

diagram not to scale



- (a) Given that $\cos \hat{A} = \frac{5}{6}$, find the value of $\sin \hat{A}$. [3]
- (b) Find the area of triangle ABC. [2]

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2. [Maximum mark: 5]

Let A and B be events such that $P(A) = 0.5$, $P(B) = 0.4$ and $P(A \cup B) = 0.6$.
Find $P(A | B)$.

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3. [Maximum mark: 5]

- (a) Show that $(2n - 1)^2 + (2n + 1)^2 = 8n^2 + 2$, where $n \in \mathbb{Z}$. [2]
- (b) Hence, or otherwise, prove that the sum of the squares of any two consecutive odd integers is even. [3]

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4. [Maximum mark: 5]

Let $f'(x) = \frac{8x}{\sqrt{2x^2 + 1}}$. Given that $f(0) = 5$, find $f(x)$.

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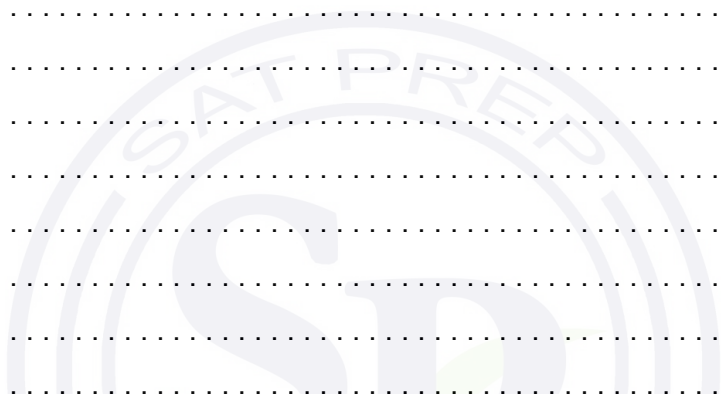


5. [Maximum mark: 5]

The functions f and g are defined such that $f(x) = \frac{x+3}{4}$ and $g(x) = 8x+5$.

- (a) Show that $(g \circ f)(x) = 2x + 11$. [2]

- (b) Given that $(g \circ f)^{-1}(a) = 4$, find the value of a . [3]

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6. [Maximum mark: 8]

- (a) Show that $\log_9(\cos 2x + 2) = \log_3 \sqrt{\cos 2x + 2}$. [3]
- (b) Hence or otherwise solve $\log_3(2 \sin x) = \log_9(\cos 2x + 2)$ for $0 < x < \frac{\pi}{2}$. [5]

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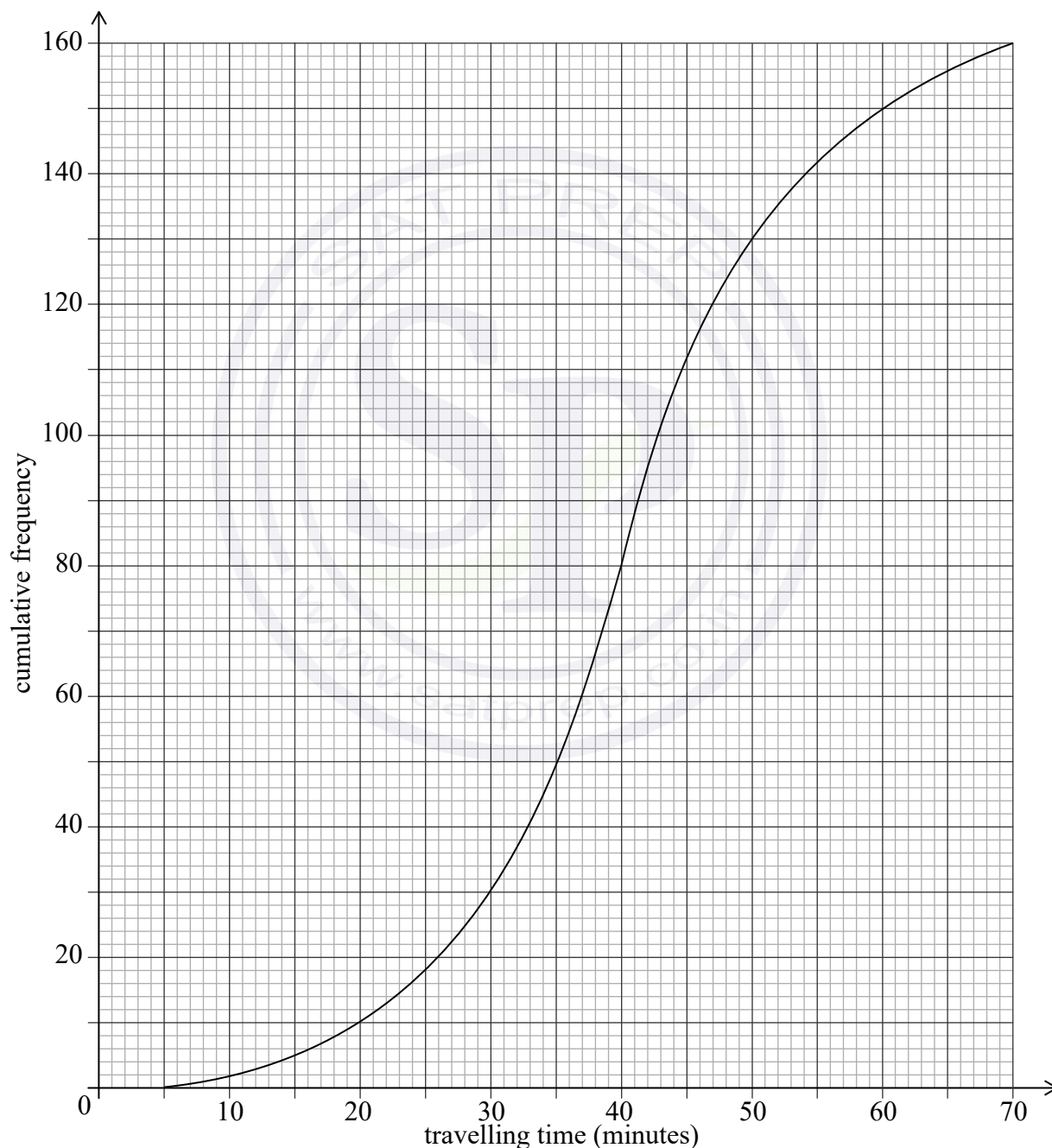
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Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 15]

A large company surveyed 160 of its employees to find out how much time they spend traveling to work on a given day. The results of the survey are shown in the following cumulative frequency diagram.



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(Question 7 continued)

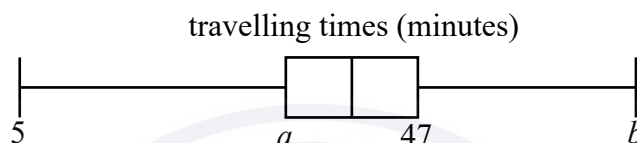
(a) Find the median number of minutes spent traveling to work. [2]

(b) Find the number of employees whose travelling time is within 15 minutes of the median. [3]

Only 10% of the employees spent more than k minutes traveling to work.

(c) Find the value of k . [3]

The results of the survey can also be displayed on the following box-and-whisker diagram.



(d) Write down the value of b . [1]

(e) (i) Find the value of a . [4]

(ii) Hence, find the interquartile range. [4]

Travelling times of less than p minutes are considered outliers.

(f) Find the value of p . [2]

8. [Maximum mark: 16]

Let $f(x) = \frac{1}{3}x^3 + x^2 - 15x + 17$.

(a) Find $f'(x)$. [2]

The graph of f has horizontal tangents at the points where $x = a$ and $x = b$, $a < b$.

(b) Find the value of a and the value of b . [3]

(c) (i) Sketch the graph of $y = f'(x)$. [2]

(ii) Hence explain why the graph of f has a local maximum point at $x = a$. [4]

(d) (i) Find $f''(b)$. [4]

(ii) Hence, use your answer to part (d)(i) to show that the graph of f has a local minimum point at $x = b$. [4]

The normal to the graph of f at $x = a$ and the tangent to the graph of f at $x = b$ intersect at the point (p, q) .

(e) Find the value of p and the value of q . [5]



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9. [Maximum mark: 16]

Let $f(x) = \frac{\ln 5x}{kx}$ where $x > 0$, $k \in \mathbb{R}^+$.

(a) Show that $f'(x) = \frac{1 - \ln 5x}{kx^2}$. [3]

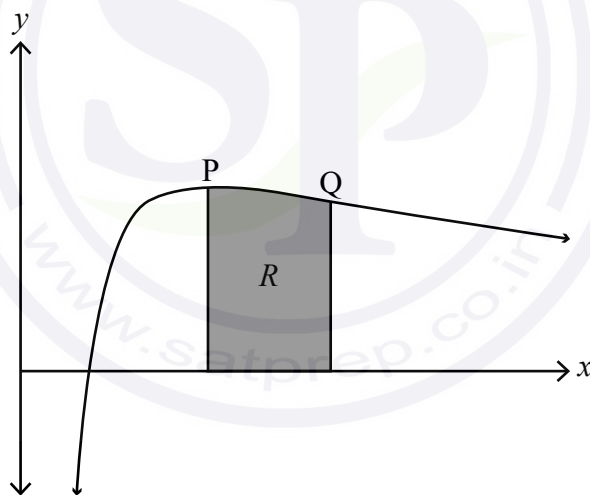
The graph of f has exactly one maximum point P.

(b) Find the x -coordinate of P. [3]

The second derivative of f is given by $f''(x) = \frac{2 \ln 5x - 3}{kx^3}$. The graph of f has exactly one point of inflexion Q.

(c) Show that the x -coordinate of Q is $\frac{1}{5}e^{\frac{3}{2}}$. [3]

The region R is enclosed by the graph of f , the x -axis, and the vertical lines through the maximum point P and the point of inflexion Q.



(d) Given that the area of R is 3, find the value of k . [7]

