Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: analysis and approaches
Standard level
Paper 1

15 May 2025

Zone A afternoon | **Zone B** afternoon | **Zone C** afternoon

1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
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- You are not permitted access to any calculator for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
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- A clean copy of the **mathematics: analysis and approaches SL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

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Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 5]

Write each of the following expressions in the form $\ln k$, where $k \in \mathbb{Z}^+$.

(a) $\ln 3 + \ln 4$ [1]

(b) $3 \ln 2$ [2]

(c) $-\ln \dfrac{1}{2}$ [2]

2. [Maximum mark: 5]

Consider the function $f(x) = \dfrac{4x^3}{3} - 16x$, where $x \in \mathbb{R}$.

The graph of $y = f(x)$ has a local minimum point at (p, q) where $p > 0$.

Find the value of p and the value of q.

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3. [Maximum mark: 7]

Bob invests 1000 dinar in an account which pays a nominal annual interest rate of 4% compounded **quarterly**.

The amount of money in the account after one complete year can be written as $1000(1 + k)^4$ where $k \in \mathbb{Q}$.

(a) Write down the value of k. [1]

(b) Expand and simplify $(1 + x)^4$. [2]

(c) Hence or otherwise, find the amount of money in the account after one complete year, giving your answer correct to the nearest dinar. [4]

4. [Maximum mark: 4]

Find the area completely enclosed by the curves $y = e^x$, $y = -e^x$, and the lines $x = -1$ and $x = 1$.

5. [Maximum mark: 6]

Consider events A and B such that $P(A') = P(A \cup B) = \dfrac{3}{4}$ and $P(B|A) = \dfrac{2}{3}$.

(a) Find $P(A \cap B)$. [3]

(b) Show that events A and B are independent. [3]

6. [Maximum mark: 8]

Consider a sequence of ten rectangular picture frames $F_1, F_2, \ldots, F_9, F_{10}$.

Picture frame F_1 has width $4\,\text{cm}$ and height $5\,\text{cm}$.

The width and height of picture frame F_n, are each increased by 50% to generate the width and height of the next picture frame F_{n+1}, for $n \in \mathbb{Z}^+$, $1 \le n \le 9$.

(a) (i) Show that the area of picture frame F_n is $20\left(\dfrac{9}{4}\right)^{n-1} \text{cm}^2$.

(ii) Hence, find the mean area of the ten picture frames, giving your answer in the

form $p\left(\left(\dfrac{9}{4}\right)^{a} - 1\right)\text{cm}^2$, where $p \in \mathbb{Q}^+$, $a \in \mathbb{Z}^+$. [5]

(b) Find the median area of the ten picture frames, giving your answer in

the form $q\left(\dfrac{9}{4}\right)^{4}\text{cm}^2$, where $q \in \mathbb{Q}^+$. [3]

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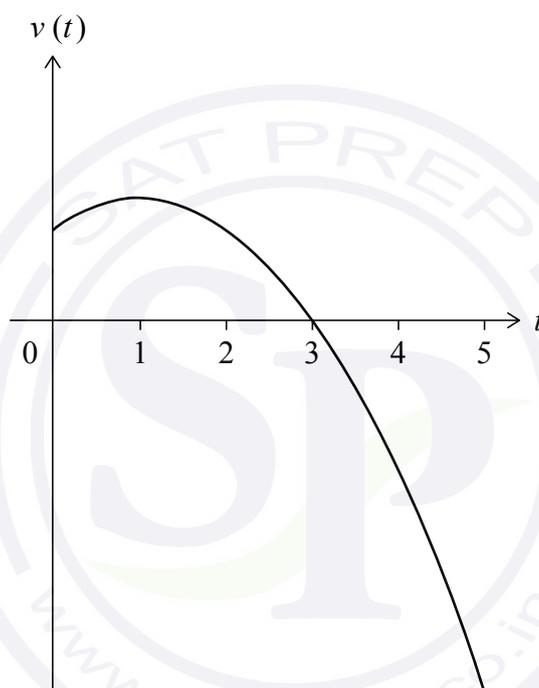Do **not** write solutions on this page

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 13]

An object moves in a straight line.

Its velocity $v\,\mathrm{m\,s}^{-1}$, at time t seconds, is given by $v(t) = 30 + 20t - 10t^2$ for $0 \le t \le 5$.

The graph of v is shown in the following diagram.

The graph of v has a local maximum point where $t = 1$ and intersects the t-axis at $t = 3$.

(a) Determine the object's

   (i) maximum velocity;

   (ii) maximum speed. [4]

At $t = T$, the object changes direction.

(b) (i) Write down the value of T.

   (ii) Find the distance travelled by the object in the first T seconds. [5]

(c) Determine whether the object returns to its initial position during the time period $0 \le t \le 5$, justifying your answer. [4]

Do **not** write solutions on this page

8. [Maximum mark: 15]

The function f is defined by $f(x) = 5(x+1)(x+3)$, where $x \in \mathbb{R}$.

(a) Write $f(x)$ in the form $a(x-h)^2 + k$, where $a, h, k \in \mathbb{Z}$. [4]

(b) Sketch the graph of $y = f(x)$, showing the values of any intercepts with the axes and the coordinates of the vertex. [4]

(c) Solve the inequality $f(x) \le 40$. [4]

The function g is defined by $g(x) = \ln x$, where $x \in \mathbb{R}$, $x > 0$.

(d) (i) Write down an expression for $(f \circ g)(x)$.

(ii) Solve the inequality $(f \circ g)(x) \le 40$. [3]

Turn over
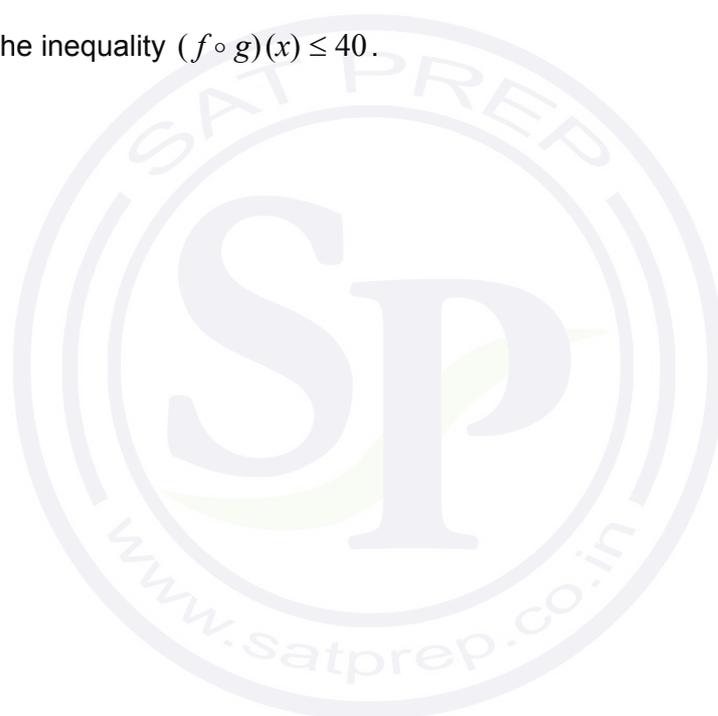
Do **not** write solutions on this page

9. [Maximum mark: 17]

A solid cylinder has height $h\,\text{cm}$ and base radius $R\,\text{cm}$.

The cylinder fits exactly inside a hollow sphere of radius $r\,\text{cm}$.

Points A, B and C are points where the surface of the cylinder touches the surface of the sphere.

The line segment $[AB]$ is a diameter of the sphere.

The line segment $[BC]$ is a diameter of the base of the cylinder and $A\hat{B}C = \theta$.

This information is shown on the following diagram.

diagram not to scale
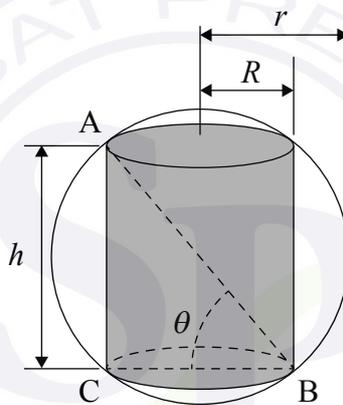
(a) (i) By considering triangle ABC, show that $R = r\cos\theta$.

    (ii) Find an expression for h in terms of r and θ.   [4]

(b) Hence or otherwise, show that the total surface area, $S\,\text{cm}^2$, of the cylinder is given by $S = 2\pi r^2(1 + 2\sin\theta\cos\theta - \sin^2\theta)$.   [4]

The external surface area of the sphere is $2S$.

(c) Show that $\tan\theta = 2$.   [4]

The volume of the cylinder is $V\,\text{cm}^3$.

(d) Find V, giving your answer in the form $p\pi r^3\sqrt{5}$, where $p \in \mathbb{Q}^+$.   [5]

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Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1.   [Maximum mark: 4]

Consider the function $f(x) = x^3 + 5x^2 - 8$, where $x \in \mathbb{R}$.

(a)   Find $f'(1)$. [2]

(b)   Find the equation of the tangent to the graph of f at $x = 1$. [2]

2. [Maximum mark: 5]

The derivative of a function g is given by $g'(x) = \cos x + e^{2x}$, where $x \in \mathbb{R}$.

Given that $g(0) = 7$, find $g(x)$.

Turn over

3. [Maximum mark: 5]

The function f is defined by $f(x) = \dfrac{3x-2}{2x+1}$ for $x \in \mathbb{R}$, $x \neq -\dfrac{1}{2}$.

The following diagram shows part of the graph of $y = f(x)$.
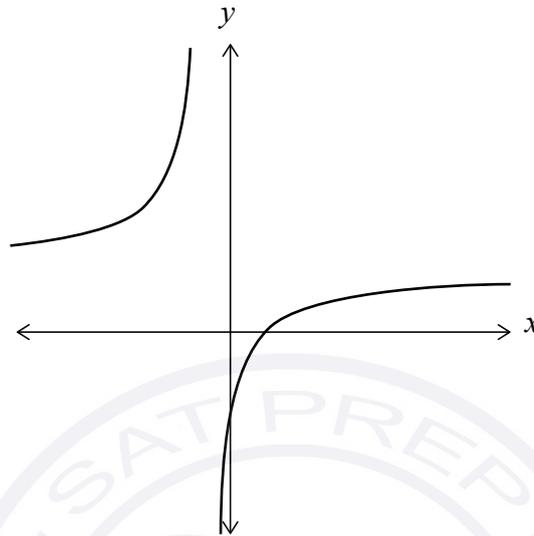
(a) Write down the value of $f(0)$. [1]

(b) Write down the equation of the horizontal asymptote. [1]

The function g is defined by $g(x) = -f(x)$ for $x \geq 0$.

(c) Find the range of g. [3]

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4.   [Maximum mark: 7]

The following diagram shows a non-right angled triangle ABC.

diagram not to scale
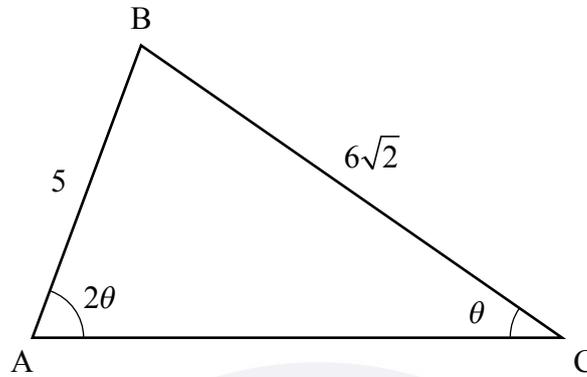
$AB = 5$, $BC = 6\sqrt{2}$, $A\hat{C}B = \theta$ and $B\hat{A}C = 2\theta$, where $0 < \theta < \dfrac{\pi}{2}$.

(a)   Using the sine rule, show that $\cos\theta = \dfrac{3\sqrt{2}}{5}$.   [3]

(b)   Hence, find $\sin\theta$.   [2]

Point D is located on [AC] such that the area of triangle BCD is $2\sqrt{14}$.

(c)   Find DC.   [2]

Turn over

5. [Maximum mark: 7]

The quadratic equation $x^2 + kx + 15 - k = 0$ has two distinct real roots.

(a) Find the possible values of k. [5]

(b) Find the possible values of k in the case where the two distinct real roots are both positive or both negative. [2]

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6. [Maximum mark: 7]

Consider the function $f(x) = \sqrt{x^2 \ln x + 4 - x^2}$, where $x \in \mathbb{R}$, $x > 0$.

(a) Show that the distance, l, between the origin and any point on the graph of f is given by $l = \sqrt{x^2 \ln x + 4}$. [1]

(b) Hence, find the x-coordinate of the point on the graph of f which is closest to the origin. [6]

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 13]

A discrete random variable, X, has the following probability distribution, where $a > 0$ and k is a constant.

x	0	a	$2a$	$3a$
$\mathrm{P}(X = x)$	k	$3k^2$	$2k^2$	k^2

(a) Show that $k = \dfrac{1}{3}$. [5]

(b) Find $\mathrm{P}(X < 3a)$. [2]

(c) Find $\mathrm{P}(X \geq a \,|\, X < 3a)$. [3]

(d) Given that $\mathrm{E}(X) = 20$, find the value of a. [3]

Do **not** write solutions on this page.

8. [Maximum mark: 15]

A Ferris wheel with diameter 72 metres rotates at a constant speed. The lowest point on the wheel is 8 metres above the ground, as shown on the following diagram. A seat starts at the lowest point.
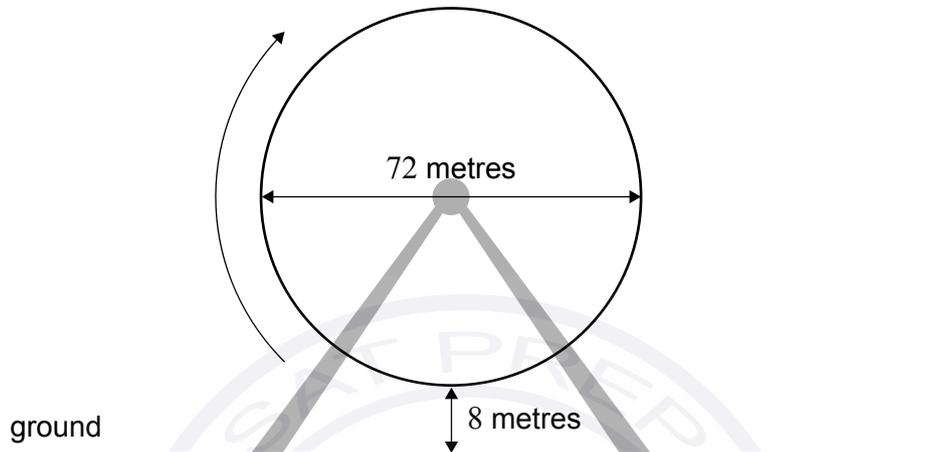
The wheel completes one revolution in 12 minutes.

After t minutes, the height, h metres, of the seat above the ground is given by $h(t) = a\cos(bt) + 44$, where a, $b \in \mathbb{R}$.

(a) Find the value of

    (i) a;

    (ii) b. [4]

(b) Sketch the graph of h, for $0 \le t \le 12$. Clearly label the coordinates of the maximum point and the end points. [2]

Consider one revolution of the Ferris wheel.

(c) The seat is at least $26\,\text{m}$ above the ground for T minutes. Find the value of T. [5]

(d) (i) Find $h'(t)$.

    (ii) Hence or otherwise, find the time at which the height of the seat above the ground is decreasing at its fastest rate. [4]

Turn over

Do **not** write solutions on this page.

9. [Maximum mark: 17]

The function f is defined by $f(x) = 4^x$, where $x \in \mathbb{R}$.

(a) Find $f^{-1}(8)$. Express your answer in the form $\dfrac{p}{q}$ where $p, q \in \mathbb{Z}$. [3]

The function g is defined by $g(x) = 1 + \log_2 x$, where $x \in \mathbb{R}^+$.
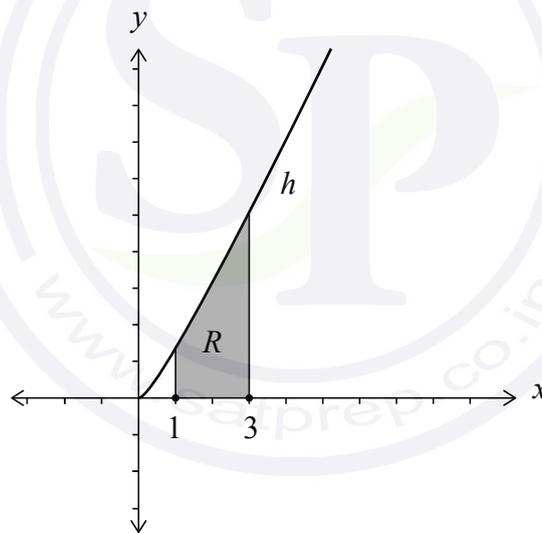
(b) (i) Find an expression for $g^{-1}(x)$.

(ii) Describe a sequence of transformations that transforms the graph of $y = g^{-1}(x)$ to the graph of $y = f(x)$. [4]

(c) Show that $(f \circ g)(x) = 4x^2$. [3]

The function h is defined by $h(x) = \dfrac{4x^2}{2x+1}$, $x \neq -\dfrac{1}{2}$.

The following diagram shows part of the graph of h. Let R be the region enclosed by the graph of h and the x-axis, between the lines $x = 1$ and $x = 3$.

(d) (i) Show that $2x - 1 + \dfrac{1}{2x+1} = \dfrac{4x^2}{2x+1}$.

(ii) Hence or otherwise, find the area of R, giving your answer in the form $p + q \ln r$, where $p, q, r \in \mathbb{Q}^+$. [7]

Please **do not** write on this page.

Answers written on this page
will not be marked.

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Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1.   [Maximum mark: 4]

The scores achieved by 80 golfers in a competition are summarized in the following box and whisker diagram.

score

(a)   Find the interquartile range.   [2]

(b)   Find the number of golfers that scored between 70 and 74.   [2]

2. [Maximum mark: 5]

Let $\log_{10} 2 = p$ and $\log_{10} 3 = q$.

(a) Find an expression for $\log_{10} 24$ in terms of p and q. [3]

(b) Find an expression for $\log_3 8$ in terms of p and q. [2]

3.   [Maximum mark: 5]

The following diagram shows the graph of $y = f(x)$, for $-6 \le x \le 5$.

(a)   Write down the value of $f(-3)$.   [1]

(b)   State the domain of f^{-1}, the inverse function of f.   [1]

(c)   Find the value of x that satisfies $f^{-1}(2x - 7) = -3$.   [3]

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4. [Maximum mark: 6]

(a) Show that $\cos^4 x - \sin^4 x = \cos 2x$. [3]

(b) Hence, find $\int (\cos^4 x - \sin^4 x)\,\mathrm{d}x$. [3]

5. [Maximum mark: 7]

Consider the curve $y = x^2 - x - 1$ and the line $y = mx - 3$, where $m \in \mathbb{R}$.

(a) Show that the curve and the line meet when $x^2 - (m + 1)x + 2 = 0$. [2]

(b) Hence, find the values of m when the line is tangent to the curve. [5]

6. [Maximum mark: 6]

The random variables X and Y are normally distributed with $X \sim N(7, a^2)$ and $Y \sim N(19, a^2)$, where $a > 0$.

(a) Find b such that $P(X > b) = P(Y > 22)$. [2]

(b) Write down the approximate value of $P(7 - a < X < 7 + a)$, correct to two significant figures. [1]

(c) Given that $a = 3$, calculate the approximate value of $P(Y < 22)$, correct to two significant figures. [3]

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7.   [Maximum mark: 15]

A gardener plans to enclose part of their garden with rope. The total area being enclosed is $60\,\text{m}^2$. This will be further divided by rope to make eight identical rectangular areas, each measuring x metres by y metres, where $x, y > 0$. This is shown in the following diagram.

diagram not to scale

garden
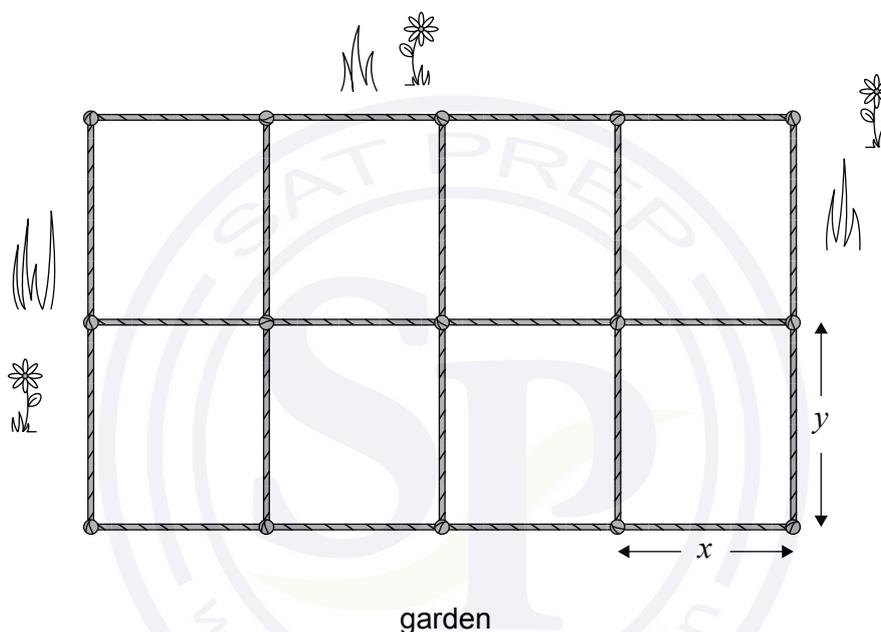
(a)   Find an expression for y in terms of x. [2]

(b)   Show that the total length, T metres, of rope required is given by

$$T = 12x + \frac{75}{x}.$$ [2]

(c)   Find an expression for $\dfrac{\mathrm{d}T}{\mathrm{d}x}$. [2]

(This question continues on the following page)

Do **not** write solutions on this page.

(Question 7 continued)

When $x = k$, $\dfrac{\mathrm{d}T}{\mathrm{d}x} = 0$.

(d) (i) Find the value of k.

(ii) Hence, calculate the value of T when $x = k$.

(iii) Find the value of y when $x = k$. [7]

(e) (i) Find an expression for $\dfrac{\mathrm{d}^2 T}{\mathrm{d}x^2}$.

(ii) Hence, justify whether T has a local minimum or a local maximum when $x = k$. [2]

Do **not** write solutions on this page.

8. [Maximum mark: 16]

Consider the sequence $\{u_n\}$, with nth term given by u_n. The first three terms are

$$u_1 = k - 5,\ u_2 = 3 - 2k \text{ and } u_3 = 5k + 3, \text{ where } k \in \mathbb{R}.$$

(a) Consider the case when $\{u_n\}$ is arithmetic.

    (i) Find the value of k.

    (ii) Hence, or otherwise, find u_3. [5]

(b) Consider the case where $k = 12$.

    (i) Show that the first three terms of $\{u_n\}$ form a geometric sequence.

    (ii) Given that $\{u_n\}$ is geometric, state a reason why the sum of an infinite number of terms of this sequence does not exist. [4]

(c) The sequence, $\{u_n\}$, is geometric for a second value of k.

    (i) Show that $k^2 - 10k - 24 = 0$.

    (ii) Find the first three terms of $\{u_n\}$ for this second value of k.

    (iii) Hence, write down the value of S_{2m}, the sum of the first $2m$ terms, for this second value of k. [7]

Do **not** write solutions on this page.

9. [Maximum mark: 16]

(a) (i) Solve $5 - 4x - x^2 = 0$.

(ii) Hence, find the values of x such that $5 - 4x - x^2 > 0$. [4]

Consider the function $f(x) = \log_k(5 - 4x - x^2)$, where $a < x < b$ and $k > 1$.
Part of the graph of f is shown in the following diagram.
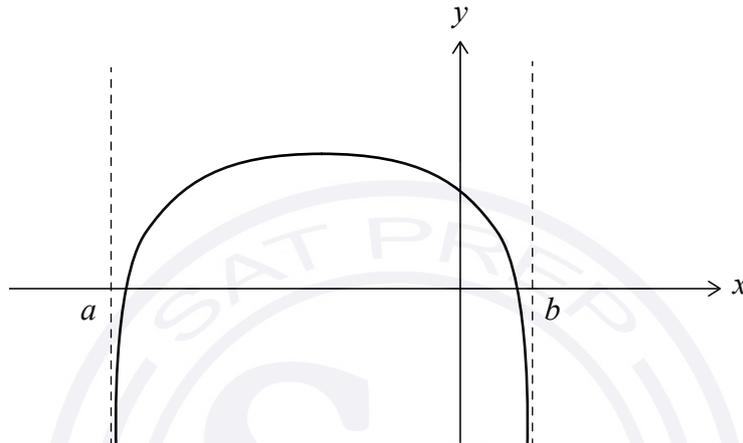
The graph of f has vertical asymptotes at $x = a$ and $x = b$.

(b) Write down the value of

(i) a;

(ii) b. [2]

(c) Find the exact values of x such that $f(x) = 0$. [4]

The graph of f has a maximum value of 2.

(d) Find the value of k. [6]

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Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 6]

   Consider the following set of ordered data.

27	27	31	32	33	37	41	46	54	57

   (a) Write down

       (i)   the mode;

       (ii)   the range;

       (iii) the median. [3]

   (b) Find the interquartile range. [3]

2. [Maximum mark: 5]

Points A and B lie on a circle with centre O and radius r cm, where $A\hat{O}B = 0.75$ radians.

This is shown on the following diagram.

diagram not to scale
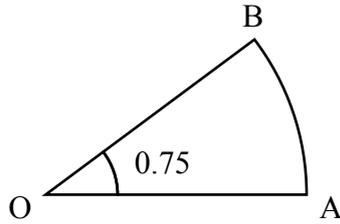
The area of sector OAB is $6\,\text{cm}^2$.

(a) Find the value of r. [3]

(b) Hence, find the perimeter of sector OAB. [2]

3. [Maximum mark: 6]

Two events A and B are such that $\mathrm{P}(A) = 0.45$, $\mathrm{P}(B) = 0.65$ and $\mathrm{P}(A \cup B) = 0.8$.

(a) Find $\mathrm{P}(A \cap B)$. [3]

(b) Find $\mathrm{P}(A'|B')$. [3]

4. [Maximum mark: 4]

Prove that $(2n + 5)^2 - (2n - 5)^2$ is a multiple of 10 for all $n \in \mathbb{Z}^+$.

5. [Maximum mark: 6]

The following diagram shows triangle ABC, where $AB = 3\,cm$, $BC = 10\,cm$, $AC = 8\,cm$ and $A\hat{B}C = 2\theta$.

diagram not to scale
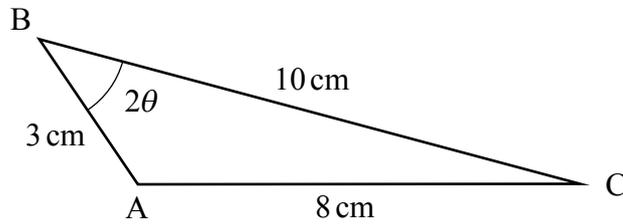
Find the exact value of $\cos\theta$, giving your answer in the form $\dfrac{\sqrt{p}}{q}$, where p, $q \in \mathbb{Z}^+$.

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6. [Maximum mark: 6]

For a particular arithmetic sequence, $u_{10} = 14$ and $S_{25} = 200$.

Find the value of k such that $u_k = 0$.

Turn over

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 16]

The following diagram shows parts of the graphs of two functions f and g.

The graph of f is linear, has an x-intercept at $(6, 0)$ and a y-intercept at $(0, 12)$.

The graph of g has x-intercepts at $(-2, 0)$ and $(6, 0)$ and a y-intercept at $(0, 12)$.
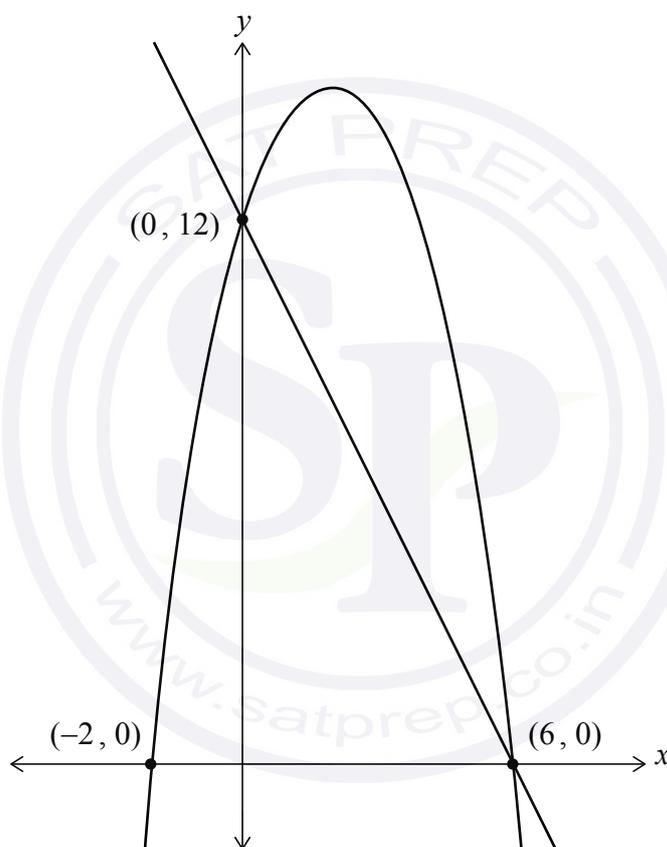
(a) Write down the equation for f in the form $f(x) = mx + c$. [2]

(This question continues on the following page)

Do **not** write solutions on this page.

(Question 7 continued)

The function g is given by $g(x) = -x^2 + bx + 12$, where b is a real constant.

(b) Find the value of b. [3]

(c) Show that the area of the region enclosed by the graph of f and the graph of g can be represented by the definite integral $\int_0^6 (-x^2 + 6x)\,\mathrm{d}x$. [2]

(d) Hence, find the area of the region enclosed by the graph of f and the graph of g. [4]

Point P is on the graph of g. The tangent to the graph of g at P is parallel to the graph of f.

(e) Find the coordinates of P. [5]

Do **not** write solutions on this page.

8. [Maximum mark: 14]

The function f is defined as $f(x) = \log_2(8x)$, where $x > 0$.

(a) Find the value of

(i) $f(2)$;

(ii) $f\left(\dfrac{1}{8}\right)$. [3]

(b) Find an expression for $f^{-1}(x)$. [4]

(c) Hence, or otherwise, find $f^{-1}(0)$. [1]

The graph of $y = f(4x^2)$ can be obtained by translating and stretching the graph of $y = \log_2 x$.

(d) Describe these two transformations specifying the order in which they are to be applied. [6]

Do **not** write solutions on this page.

9. [Maximum mark: 17]

Consider a cylinder of radius $4r$ and height h. A smaller cylinder of radius r is removed from the centre to form a hollow cylinder. This is shown in the following diagram.

All lengths are measured in centimetres.

diagram not to scale

The total surface area of the hollow cylinder, in cm^2, is given by S.

The volume of the hollow cylinder, in cm^3, is given by V.

(a) Show that $S = 30\pi r^2 + 10\pi rh$. [3]

(b) The total surface area of the hollow cylinder is $240\pi\,\text{cm}^2$.

Show that $V = 360\pi r - 45\pi r^3$. [6]

(c) Find an expression for $\dfrac{\mathrm{d}V}{\mathrm{d}r}$. [2]

The hollow cylinder has its maximum volume when $r = p\sqrt{\dfrac{2}{3}}$, where $p \in \mathbb{Z}^+$.

(d) Find the value of p. [3]

(e) Hence, find this maximum volume, giving your answer in the form $q\pi\sqrt{\dfrac{2}{3}}$, where $q \in \mathbb{Z}^+$. [3]

Please **do not** write on this page.

Answers written on this page
will not be marked.

Diploma Programme
Programme du diplôme
Programa del Diploma

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Mathematics: analysis and approaches
Standard level
Paper 1

24 October 2024

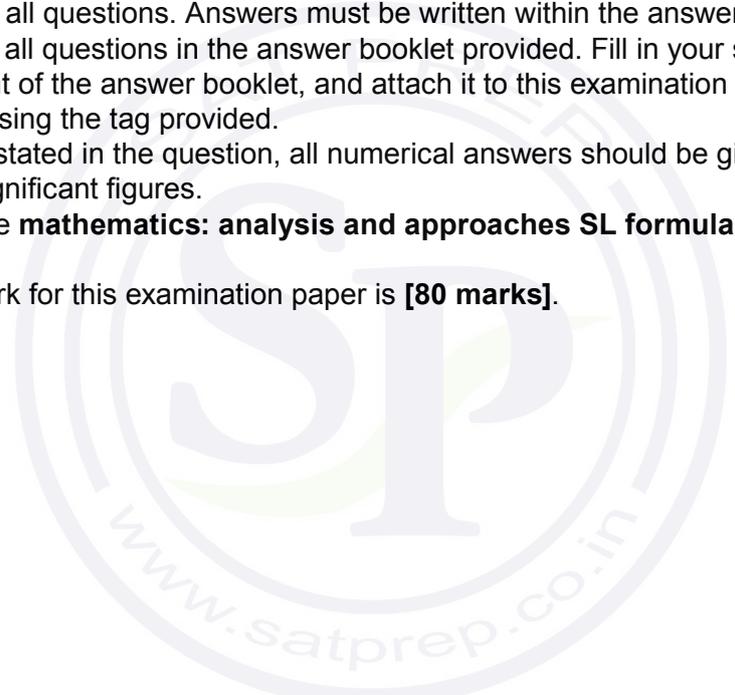**Zone A** afternoon │ **Zone B** afternoon │ **Zone C** afternoon

1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches SL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

12EP01

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Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 6]

Consider the following set of ordered data.

22	22	26	27	28	32	36	38	39	42

(a) Write down

    (i) the mode;

    (ii) the range;

    (iii) the median. [3]

(b) Find the interquartile range. [3]

2. [Maximum mark: 5]

Points A and B lie on a circle with centre O and radius r cm, where $A\hat{O}B = 1.5$ radians.

This is shown on the following diagram.

diagram not to scale
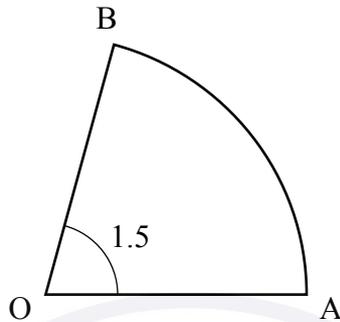
The area of sector OAB is $48\,\text{cm}^2$.

(a) Find the value of r. [3]

(b) Hence, find the perimeter of sector OAB. [2]

Turn over

3. [Maximum mark: 6]

Two events A and B are such that $\mathrm{P}(A) = 0.65$, $\mathrm{P}(B) = 0.45$ and $\mathrm{P}(A \cup B) = 0.85$.

(a) Find $\mathrm{P}(A \cap B)$. [3]

(b) Find $\mathrm{P}(A' \mid B')$. [3]

4. [Maximum mark: 4]

Prove that $(3n + 2)^2 - (3n - 2)^2$ is a multiple of 12 for all $n \in \mathbb{Z}^+$.

Turn over

5. [Maximum mark: 6]

The following diagram shows triangle ABC, where $AB = 4\,cm$, $BC = 6\,cm$, $AC = 5\,cm$ and $A\hat{B}C = 2\theta$.

diagram not to scale

Find the exact value of $\cos\theta$, giving your answer in the form $\dfrac{p\sqrt{2}}{q}$, where p, $q \in \mathbb{Z}^+$.

6. [Maximum mark: 6]

For a particular arithmetic sequence, $u_{10} = 16$ and $S_{25} = 100$.

Find the value of k such that $u_k = 0$.

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7.   [Maximum mark: 16]

The following diagram shows parts of the graphs of two functions f and g.

The graph of f is linear, has an x-intercept at $(4, 0)$ and a y-intercept at $(0, 8)$.

The graph of g has x-intercepts at $(-2, 0)$ and $(4, 0)$ and a y-intercept at $(0, 8)$.
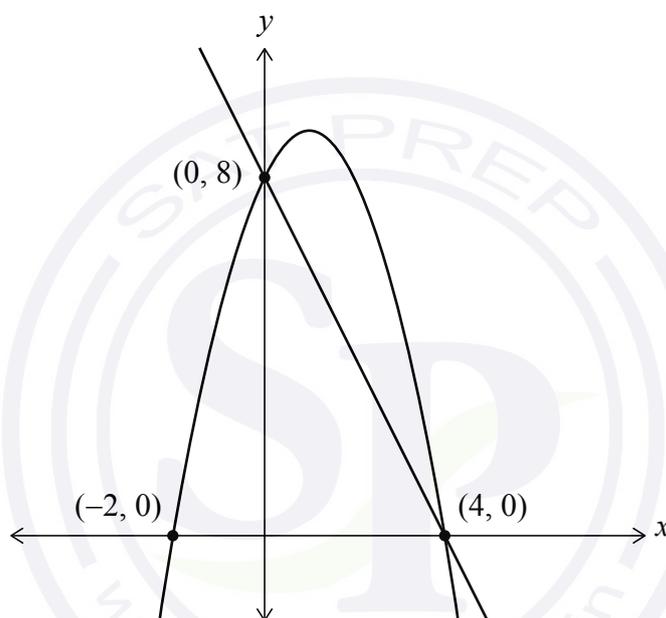
(a)   Write down the equation for f in the form $f(x) = mx + c$.   [2]

The function g is given by $g(x) = -x^2 + bx + 8$, where b is a real constant.

(b)   Find the value of b.   [3]

(c)   Show that the area of the region enclosed by the graph of f and the graph of g can be
        represented by the definite integral $\int_0^4 (-x^2 + 4x)\mathrm{d}x$.   [2]

(d)   Hence, find the area of the region enclosed by the graph of f and the graph of g.   [4]

Point P is on the graph of g. The tangent to the graph of g at P is parallel to the graph of f.

(e)   Find the coordinates of P.   [5]

Do **not** write solutions on this page.

8. [Maximum mark: 14]

The function f is defined as $f(x) = \log_2(4x)$, where $x > 0$.

(a) Find the value of

(i) $f(8)$;

(ii) $f\left(\dfrac{1}{4}\right)$. [3]

(b) Find an expression for $f^{-1}(x)$. [4]

(c) Hence, or otherwise, find $f^{-1}(0)$. [1]

The graph of $y = f(16x^3)$ can be obtained by translating and stretching the graph of $y = \log_2 x$.

(d) Describe these two transformations specifying the order in which they are to be applied. [6]

Turn over

Do **not** write solutions on this page.

9. [Maximum mark: 17]

Consider a cylinder of radius $4r$ and height h. A smaller cylinder of radius r is removed from the centre to form a hollow cylinder. This is shown in the following diagram.

All lengths are measured in centimetres.

diagram not to scale

The total surface area of the hollow cylinder, in cm^2, is given by S.

The volume of the hollow cylinder, in cm^3, is given by V.

(a) Show that $S = 30\pi r^2 + 10\pi rh$. [3]

(b) The total surface area of the hollow cylinder is $240\pi \, \text{cm}^2$.

Show that $V = 360\pi r - 45\pi r^3$. [6]

(c) Find an expression for $\dfrac{\mathrm{d}V}{\mathrm{d}r}$. [2]

The hollow cylinder has its maximum volume when $r = p\sqrt{\dfrac{2}{3}}$, where $p \in \mathbb{Z}^+$.

(d) Find the value of p. [3]

(e) Hence, find this maximum volume, giving your answer in the form $q\pi\sqrt{\dfrac{2}{3}}$, where $q \in \mathbb{Z}^+$. [3]

12EP10

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: analysis and approaches
Standard level
Paper 1

1 May 2024

Zone A afternoon | **Zone B** afternoon | **Zone C** afternoon

1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
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- Section A: answer all questions. Answers must be written within the answer boxes provided.
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- A clean copy of the **mathematics: analysis and approaches SL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

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Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 4]

The second term of an arithmetic sequence is 10 and the fourth term is 22.

(a) Find the value of the common difference. [2]

(b) Find an expression for u_n, the nth term. [2]

2. [Maximum mark: 6]

Claire rolls a six-sided die 16 times.

The scores obtained are shown in the following frequency table.

Score	Frequency
1	p
2	q
3	4
4	2
5	0
6	3

It is given that the mean score is 3.

(a) Find the value of p and the value of q. [5]

Each of Claire's scores is multiplied by 10 in order to determine the final score for a game she is playing.

(b) Write down the mean final score. [1]

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3.   [Maximum mark: 5]

It is given that $\log_{10} a = \dfrac{1}{3}$, where $a > 0$.

Find the value of

(a) $\log_{10}\left(\dfrac{1}{a}\right)$;   [2]

(b) $\log_{1000} a$.   [3]

4. [Maximum mark: 8]

Points A and B lie on the circumference of a circle of radius r cm with centre at O.

The sector OAB is shown on the following diagram. The angle AÔB is denoted as θ and is measured in radians.

diagram not to scale
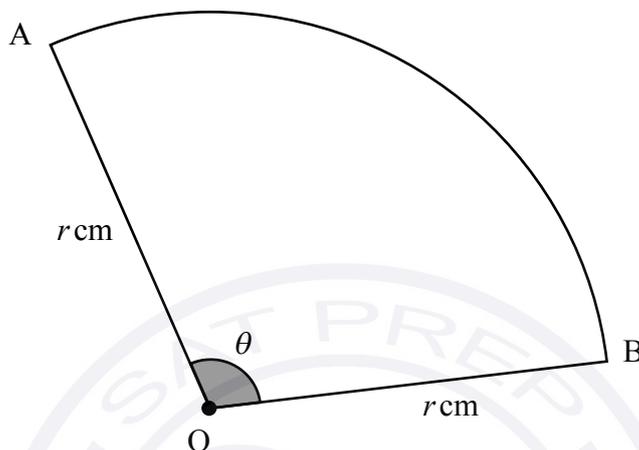
The perimeter of the sector is $10\,\text{cm}$ and the area of the sector is $6.25\,\text{cm}^2$.

(a)   Show that $4r^2 - 20r + 25 = 0$. [4]

(b)   Hence, or otherwise, find the value of r and the value of θ. [4]

Turn over

5.   [Maximum mark: 7]

Consider the functions $f(x) = \cos x$ and $g(x) = \sin 2x$, where $0 \leq x \leq \pi$.

The graph of f intersects the graph of g at the point A, the point B $\left(\dfrac{\pi}{2}, 0\right)$ and the point C as shown on the following diagram.
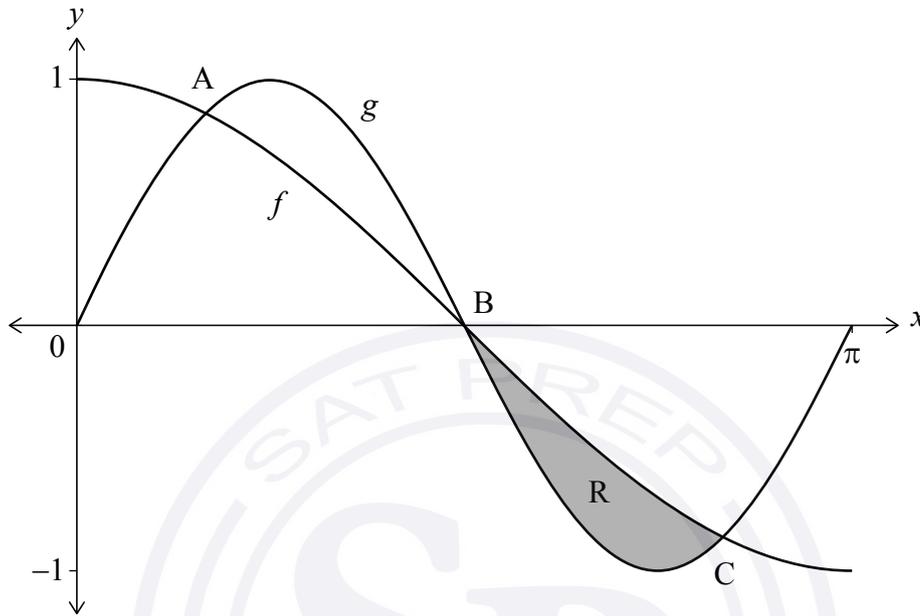
(a)   Find the x-coordinate of point A and the x-coordinate of point C.   [3]

The shaded region R is enclosed by the graph of f and the graph of g between the points B and C.

(b)   Find the area of R.   [4]

6. [Maximum mark: 5]

Consider a geometric sequence with first term 1 and common ratio 10.

S_n is the sum of the first n terms of the sequence.

(a) Find an expression for S_n in the form $\dfrac{a^n - 1}{b}$, where a, $b \in \mathbb{Z}^+$. [1]

(b) Hence, show that $S_1 + S_2 + S_3 + \ldots + S_n = \dfrac{10\left(10^n - 1\right) - 9n}{81}$. [4]

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 14]

Consider the curve with equation $y = x^3 - x^2 - x + 1$.

(a) Find

   (i) $\dfrac{dy}{dx}$;

   (ii) $\dfrac{d^2y}{dx^2}$. [3]

The curve has a local maximum at A.

(b) Find the coordinates of A, using your answer to part (a)(ii) to justify your answer. [6]

The curve has a point of inflexion at B.

(c) Find the x-coordinate of B. [2]

The line L is the normal to the curve at the point $(0, 1)$.

(d) Find the equation of L. [3]

Do **not** write solutions on this page.

8. [Maximum mark: 14]

Consider the function $f(x) = \dfrac{4x+2}{x-2}$, $x \neq 2$.

(a) Sketch the graph of $y = f(x)$. On your sketch, indicate the values of any axis intercepts and label any asymptotes with their equations. [5]

(b) Write down the range of f. [1]

Consider the function $g(x) = x^2 + bx + c$. The graph of g has an axis of symmetry at $x = 2$.

The two roots of $g(x) = 0$ are $-\dfrac{1}{2}$ and p, where $p \in \mathbb{Q}$.

(c) Show that $p = \dfrac{9}{2}$. [1]

(d) Find the value of b and the value of c. [3]

(e) Find the y-coordinate of the vertex of the graph of $y = g(x)$. [2]

(f) Find the number of solutions of the equation $f(x) = g(x)$. [2]
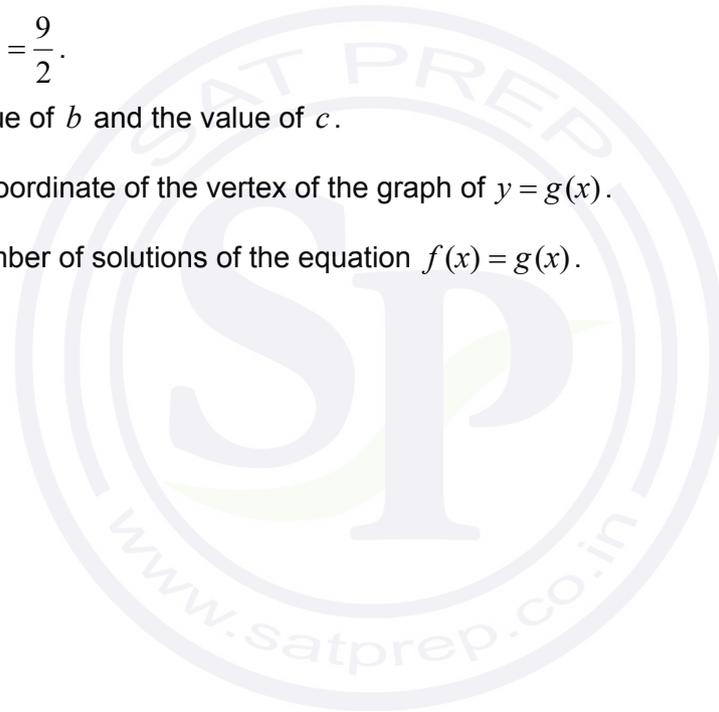
Do **not** write solutions on this page.

9. [Maximum mark: 17]

A bag contains buttons which are either red or blue.

Initially, the bag contains three red buttons and one blue button.

Francine randomly selects one button from the bag. She then replaces the button and adds one extra button of the same colour.

For example, if she selects a red button, she then replaces it and adds one extra red button so that the bag then contains four red buttons and one blue button.

Francine then randomly selects a second button from the bag.

The following tree diagram represents the probabilities of the first two selections.
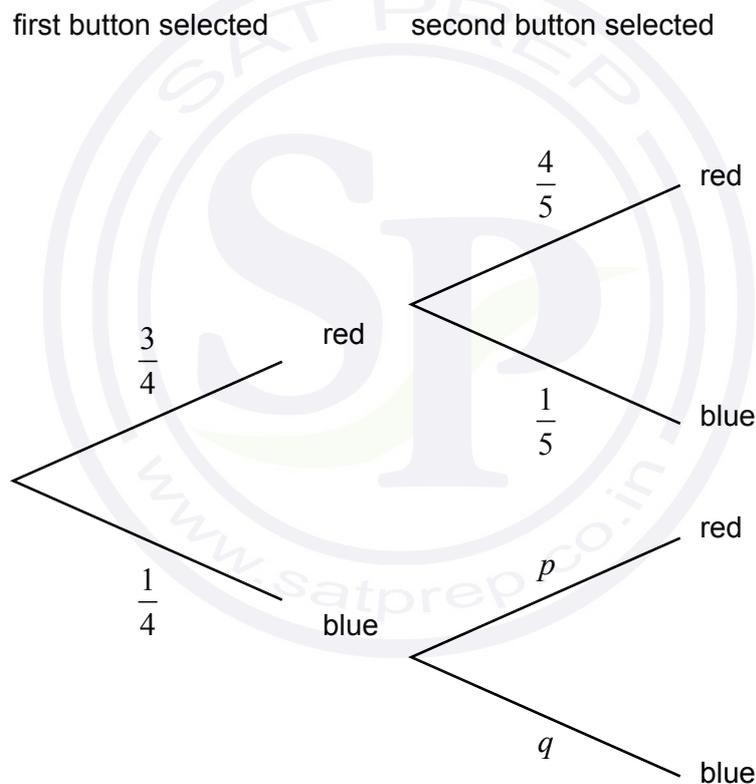
(a) Find the value of p and the value of q. [2]

(b) Show that the probability that Francine selects two buttons of the same colour is $\dfrac{7}{10}$. [2]

(c) Given that Francine selects two buttons of the same colour, find the probability that she selects two red buttons. [3]

(This question continues on the following page)

Do **not** write solutions on this page.

(Question 9 continued)

The random variable X is defined as the number of red buttons selected by Francine.

The following table shows the probability distribution of X.

x	0	1	2
$P(X = x)$	$\dfrac{1}{10}$	a	b

(d)   Find the value of a and the value of b. [2]

(e)   Hence, find the expected number of red buttons selected by Francine. [2]

Francine restarts the process with three red buttons and one blue button in the bag. She selects buttons as before, replacing the button and adding one extra button of the same colour each time. She repeats this until she selects a blue button.

(f)   Given that the first two buttons she selects are red, write down the probability that the next button she selects is blue. [1]

The probability that she selects the first blue button after n selections in total is $\dfrac{3}{56}$.

(g)   Find the value of n. [5]

Please **do not** write on this page.

Answers written on this page
will not be marked.

Diploma Programme
Programme du diplôme
Programa del Diploma

ib

Mathematics: analysis and approaches
Standard level
Paper 1

1 May 2024

Zone A afternoon │ **Zone B** afternoon │ **Zone C** afternoon

1 hour 30 minutes

Instructions to candidates

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- The maximum mark for this examination paper is **[80 marks]**.

16EP01

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Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 5]

The graph of $y = f(x)$ for $0 \le x \le 10$ is shown in the following diagram.

The graph intercepts the axes at $(10, 0)$ and $(0, 5)$.
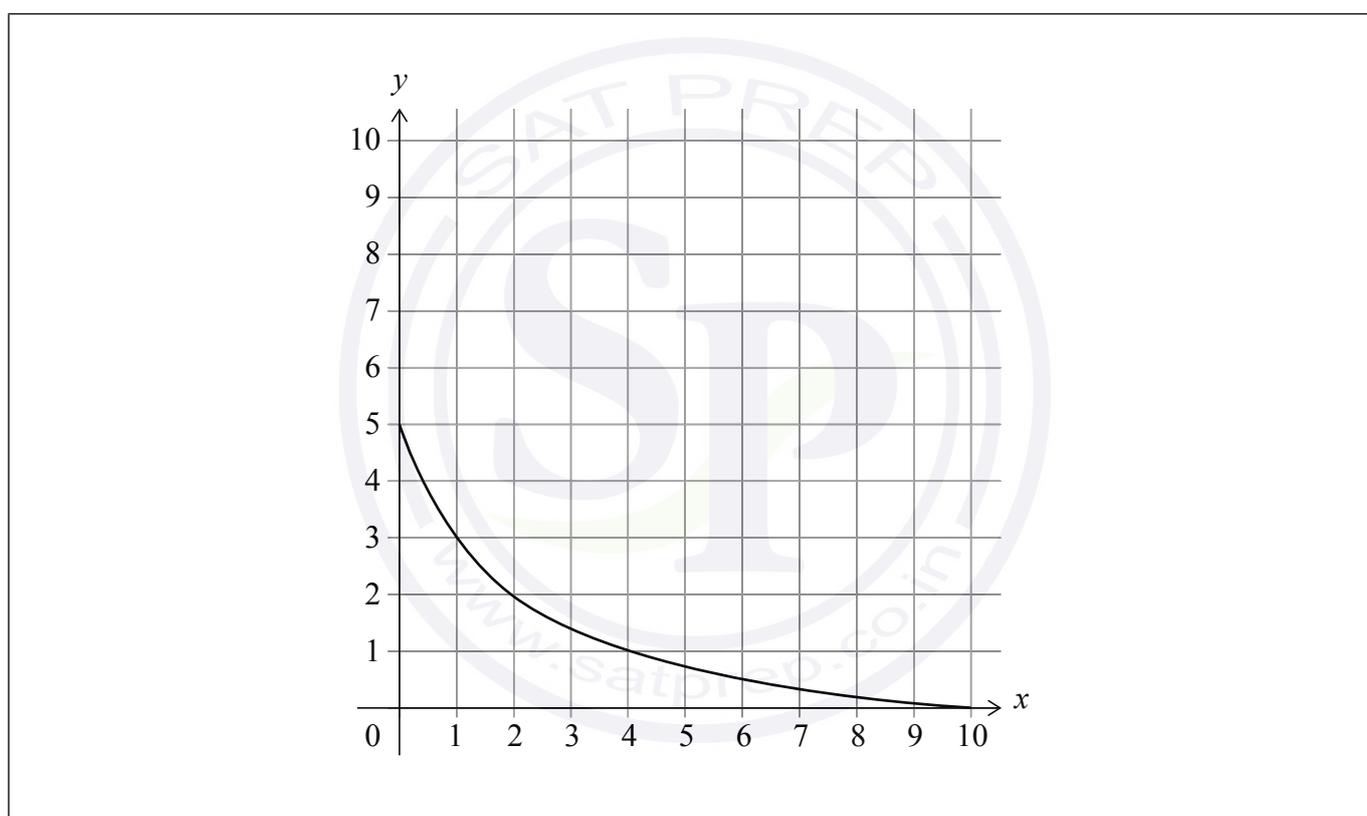
(a)   Write down the value of

(i) $f(4)$;

(ii) $f \circ f(4)$;

(iii) $f^{-1}(3)$. [3]

(b)   On the axes above, sketch the graph of $y = f^{-1}(x)$. Show clearly where the graph intercepts the axes. [2]

(This question continues on the following page)

(Question 1 continued)

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Turn over

2. [Maximum mark:4]

Solve $\tan(2x - 5°) = 1$ for $0° \leq x \leq 180°$.

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3. [Maximum mark: 5]

(a) Solve $3m^2 + 5m - 2 = 0$. [3]

(b) Hence or otherwise, solve $3 \times 9^x + 5 \times 3^x - 2 = 0$. [2]

Please **do not** write on this page.

Answers written on this page
will not be marked.

4. [Maximum mark: 7]

Quadrilateral OABC is shown on the following set of axes.
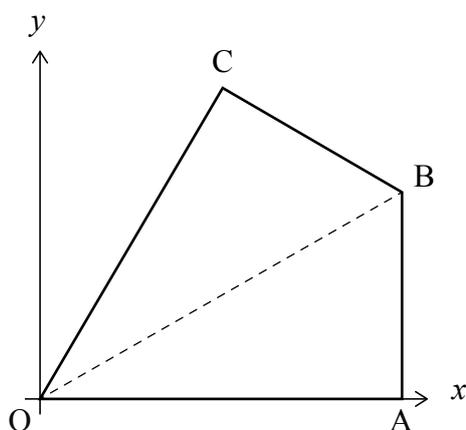
OABC is symmetrical about [OB].

A has coordinates $(6, 0)$ and C has coordinates $\left(3, 3\sqrt{3}\right)$.

(a) (i)   Write down the coordinates of the midpoint of [AC].

     (ii) Hence or otherwise, find the equation of the line passing through the points O
           and B .   [4]

(b) Given that [OA] is perpendicular to [AB], find the area of the quadrilateral OABC.   [3]

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5. [Maximum mark: 6]

A species of bird can nest in two seasons: Spring and Summer.

The probability of nesting in Spring is k.

The probability of nesting in Summer is $\dfrac{k}{2}$.

This is shown in the following tree diagram.
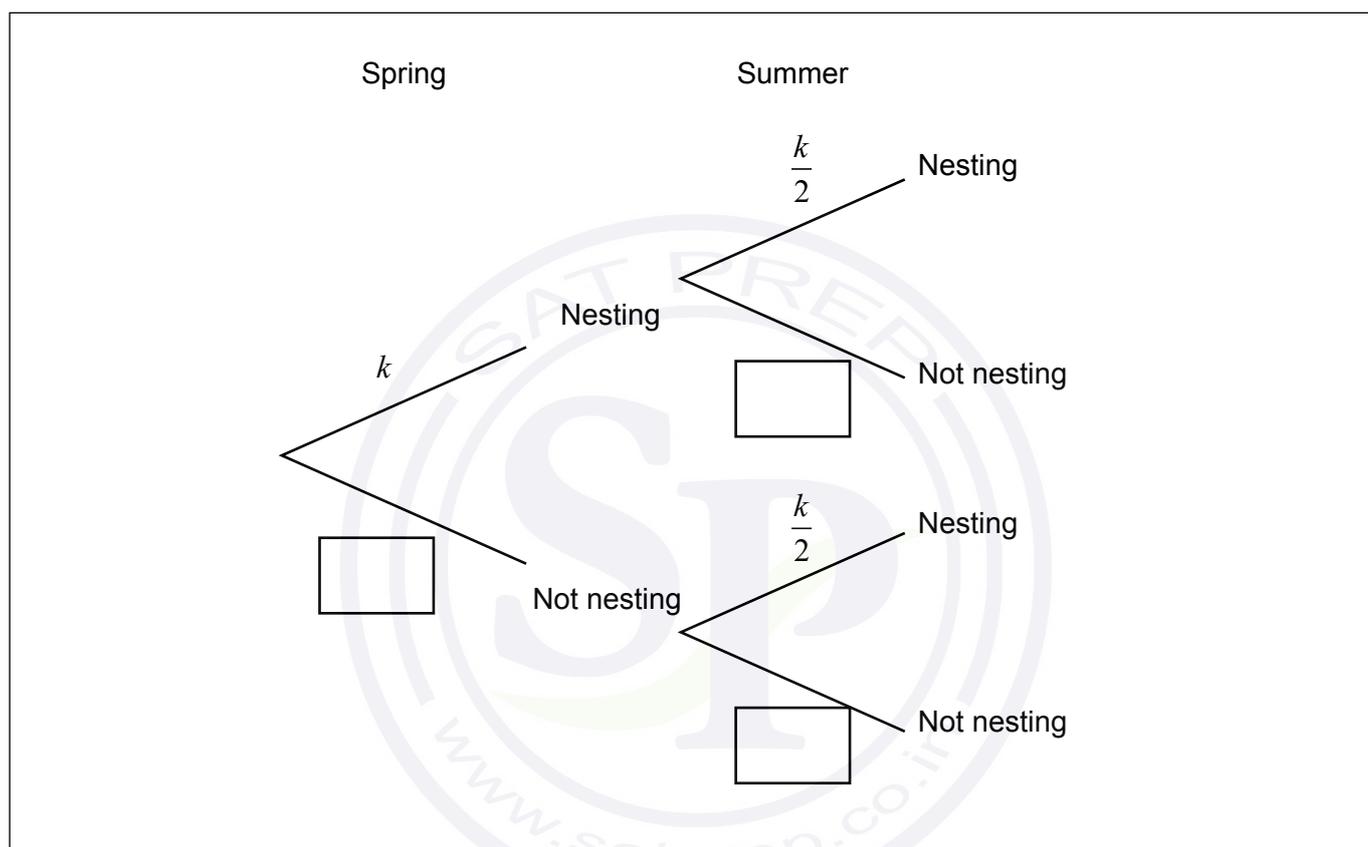
(a) Complete the tree diagram to show the probabilities of not nesting in each season. Write your answers in terms of k. [2]

It is known that the probability of not nesting in Spring and not nesting in Summer is $\dfrac{5}{9}$.

(b) (i) Show that $9k^2 - 27k + 8 = 0$.

(ii) Both $k = \dfrac{1}{3}$ and $k = \dfrac{8}{3}$ satisfy $9k^2 - 27k + 8 = 0$.

State why $k = \dfrac{1}{3}$ is the only valid solution. [4]

(This question continues on the following page)

(Question 5 continued)

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Turn over

6. [Maximum mark: 8]

A function f is defined by $f(x) = \dfrac{2(x+3)}{3(x+2)}$, where $x \in \mathbb{R}$, $x \neq -2$.

The graph $y = f(x)$ is shown below.
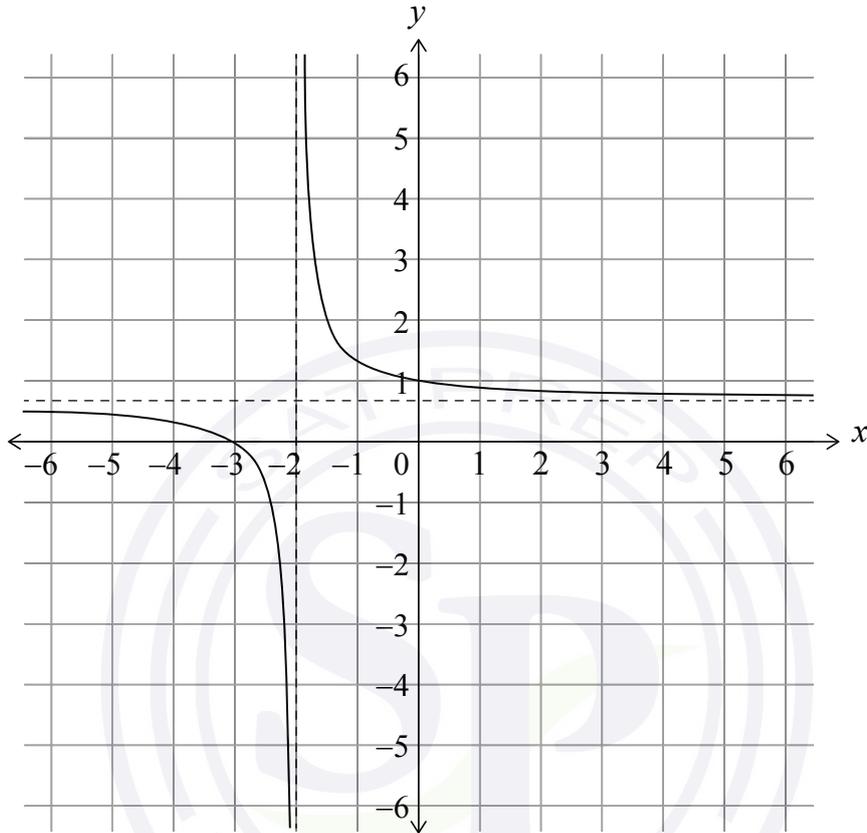
(a) Write down the equation of the horizontal asymptote. [1]

Consider $g(x) = mx + 1$, where $m \in \mathbb{R}$, $m \neq 0$.

(b) (i) Write down the number of solutions to $f(x) = g(x)$ for $m > 0$.

(ii) Determine the value of m such that $f(x) = g(x)$ has only one solution for x.

(iii) Determine the range of values for m, where $f(x) = g(x)$ has two solutions for $x \geq 0$. [7]

(This question continues on the following page)

(Question 6 continued)

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Turn over

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 12]

Alex only swims in the sea if the water temperature is at least $15°\text{C}$. Alex goes into the sea close to home for the first time each year at the start of May when the water becomes warm enough.

Alex models the water temperature at midday with the function $f(x) = a \sin bx + \text{c}$ for $0 \le x \le 12$, where x is the number of months after 1st May and where a, b, $c > 0$.

The graph of $y = f(x)$ is shown in the following diagram.
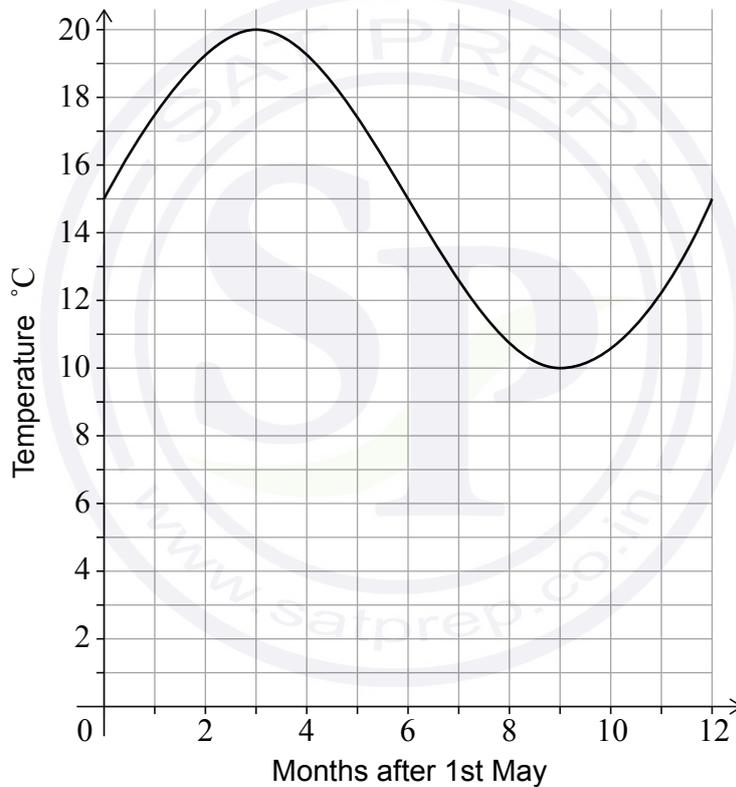
(This question continues on the following page)

Do **not** write solutions on this page.

(Question 7 continued)

(a) Show that $b = \dfrac{\pi}{6}$. [1]

(b) Write down the value of

    (i) a;

    (ii) c. [2]

Alex is going on holiday and models the water temperature at midday in the sea at the holiday destination with the function $g(x) = 3.5\sin\dfrac{\pi}{6}x + 11$, where $0 \le x \le 12$ and x is the number of months after 1st May.

(c) Using this new model $g(x)$

    (i) find the midday water temperature on 1st October, five months after 1st May.

    (ii) show that the midday water temperature is never warm enough for Alex to swim. [6]

(d) Alex compares the two models and finds that $g(x) = 0.7 f(x) + q$. Determine the value of q. [3]

Turn over

Do **not** write solutions on this page.

8. [Maximum mark: 17]

The derivative of a function f is given by $f'(x) = \dfrac{2x+2}{x^2+2x+2}$, for $x \in \mathbb{R}$.

(a) (i) Show that $x^2 + 2x + 2 > 0$ for all values of x.

(ii) Hence, find the values of x for which f is increasing. [3]

(b) (i) Write down the value of x for which $f'(x) = 0$.

(ii) Show that $f''(x) = \dfrac{-2x^2 - 4x}{(x^2 + 2x + 2)^2}$.

(iii) Hence, justify that the value of x found in part (b)(i) corresponds to a local minimum point on the graph of f. [7]

It is given that $f(2) = 3 + \ln 10$.

(c) Find an expression for $f(x)$. [4]

(d) Find the equation of the normal to the graph of f at $(2, 3 + \ln 10)$. [3]

Do **not** write solutions on this page.

9. [Maximum mark: 16]

Consider the arithmetic sequence a, p, $q \ldots$, where a, p, $q \neq 0$.

(a) Show that $2p - q = a$. [2]

Consider the geometric sequence a, s, $t \ldots$, where a, s, $t \neq 0$.

(b) Show that $s^2 = at$. [2]

The first term of both sequences is a.

It is given that $q = t = 1$.

(c) Show that $p > \dfrac{1}{2}$. [2]

Consider the case where $a = 9$, $s > 0$ and $q = t = 1$.

(d) Write down the first four terms of the

    (i) arithmetic sequence;

    (ii) geometric sequence. [4]

The arithmetic and the geometric sequence are used to form a new arithmetic sequence u_n.

The first three terms of u_n are $u_1 = 9 + \ln 9$, $u_2 = 5 + \ln 3$, and $u_3 = 1 + \ln 1$.

(e) (i) Find the common difference of the new sequence in terms of $\ln 3$.

    (ii) Show that $\displaystyle\sum_{i=1}^{10} u_i = -90 - 25\ln 3$. [6]

Please **do not** write on this page.

Answers written on this page
will not be marked.

Diploma Programme
Programme du diplôme
Programa del Diploma

**Mathematics: analysis and approaches
Standard level
Paper 1**

30 October 2023

Zone A afternoon | **Zone B** afternoon | **Zone C** afternoon

1 hour 30 minutes

Candidate session number

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

8823–7109
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16EP01

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Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 7]

    Consider the function $f(x) = a \sin(bx)$ with $a, b \in \mathbb{Z}^+$. The following diagram shows part of the graph of f.

    (a) Write down the value of a. [1]

    (b) (i) Write down the period of f.

         (ii) Hence, find the value of b. [3]

    (c) Find the value of $f\left(\dfrac{\pi}{12}\right)$. [3]

(This question continues on the following page)

16EP02

(Question 1 continued)

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2.   [Maximum mark: 5]

Consider the functions $f(x) = x + 2$ and $g(x) = x^2 - k^2$, where k is a real constant.

(a)   Write down an expression for $(g \circ f)(x)$.   [2]

(b)   Given that $(g \circ f)(4) = 11$, find the possible values of k.   [3]

3.   [Maximum mark: 4]

Events A and B are such that $P(A) = 0.7$, $P(B) = 0.75$ and $P(A \cap B) = 0.55$.

(a)   Find $P(A \cup B)$.   [2]

(b)   Hence, or otherwise, find $P(A' \cap B')$.   [2]

4.   [Maximum mark: 7]

The sum of the first n terms of an arithmetic sequence is given by $S_n = pn^2 - qn$, where p and q are positive constants.

It is given that $S_5 = 65$ and $S_6 = 96$.

(a)   Find the value of p and the value of q.   [5]

(b)   Find the value of u_6.   [2]

5.   [Maximum mark: 6]

In the following triangle ABC, $AB = \sqrt{15}$ cm, $AC = 16$ cm and $\cos B\hat{A}C = \dfrac{1}{4}$.

diagram not to scale

Find the area of triangle ABC.

Turn over

6. [Maximum mark: 6]

The binomial expansion of $(1 + kx)^n$ is given by $1 + \dfrac{9x}{2} + 15k^2x^2 + \ldots + k^n x^n$, where $n \in \mathbb{Z}^+$ and $k \in \mathbb{Q}$.

Find the value of n and the value of k.

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Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 17]

A ballet company performs *The Sleeping Beauty* every year. Last year they gave a total of 60 performances at their theatre which has a maximum capacity of 800. The number of tickets sold, n, at each performance is shown in the following frequency table.

Number of tickets sold, n	Number of performances
$0 < n \le 200$	3
$200 < n \le 400$	p
$400 < n \le 600$	18
$600 < n \le 800$	30

(a) (i) Find the value of p.

(ii) Write down the modal class. [2]

(This question continues on the following page)

Turn over

Do **not** write solutions on this page.

(Question 7 continued)

The following cumulative frequency diagram also displays these data.

(b)   Use the cumulative frequency curve to estimate

   (i)   the median number of tickets sold;

   (ii)   the number of performances where at least 80% of the tickets were sold.   [4]

(This question continues on the following page)

Do **not** write solutions on this page.

(Question 7 continued)

After a performance, the company decides to conduct a survey to obtain feedback from the audience.

(c) (i) State one disadvantage of the company surveying only the first 5% of the audience as they leave the theatre.

(ii) Describe briefly how the company could collect feedback from 5% of the audience using the systematic sampling method.

(iii) State the sampling method which should be used if the survey is to be representative of the number of children and the number of adults in the audience. [4]

Last year $36\,000$ tickets were sold to *The Sleeping Beauty*.

(d) The following box and whisker diagram displays the amount spent by the audience at the souvenir shop when they attended the performance.

Amount Spent ($)
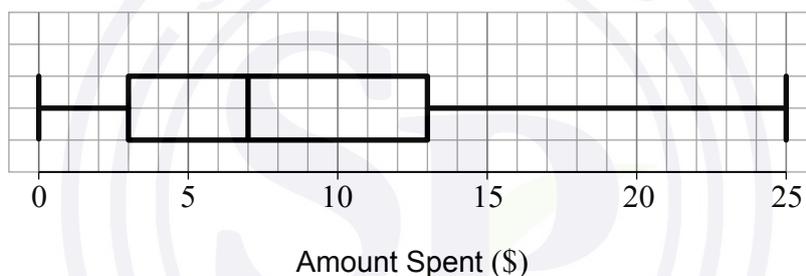
(i) Estimate the number of people who spent between $\$3$ and $\$25$.

(ii) Half the audience spent less than $\$a$. Estimate the value of a. [3]

(e) This year the company will again give 60 performances and expects to sell 18 additional tickets for each performance.

(i) Calculate the mean number of tickets the company expects to sell this year for each performance.

(ii) State what effect, if any, this increase in ticket sales would have on the variance of the number of tickets sold for each performance. [4]

Turn over

Do **not** write solutions on this page.

8. [Maximum mark: 15]

The functions f and g are defined by

$f(x) = \ln(2x - 7)$, where $x > \dfrac{7}{2}$

$g(x) = 2\ln x - \ln d$, where $x > 0$, $d \in \mathbb{R}^+$.

(a) State the equation of the vertical asymptote to the graph of $y = g(x)$. [1]

The graphs of $y = f(x)$ and $y = g(x)$ intersect at two distinct points.

(b) (i) Show that, at the points of intersection, $x^2 - 2dx + 7d = 0$.

(ii) Hence, show that $d^2 - 7d > 0$.

(iii) Find the range of possible values of d. [9]

The following diagram shows part of the graphs of $y = f(x)$ and $y = g(x)$.
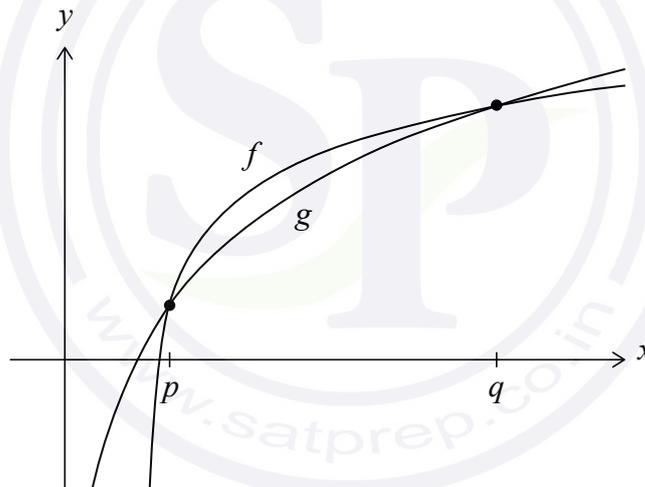
The graphs intersect at $x = p$ and $x = q$, where $p < q$.

(c) In the case where $d = 10$, find the value of $q - p$. Express your answer in the form $a\sqrt{b}$, where $a, b \in \mathbb{Z}^+$. [5]

Do **not** write solutions on this page.

9. [Maximum mark: 13]

Consider the function f defined by $f(x) = \dfrac{12x}{\left(x^2 + 2\right)^3}$, where $x \in \mathbb{R}$. The graph of f is shown in the following diagram.
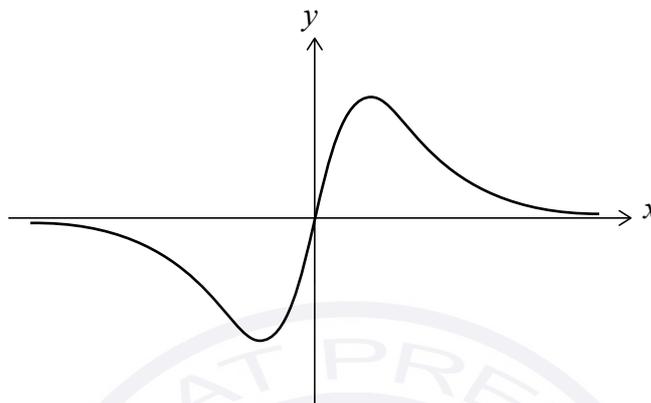
(a) Show that $f'(x) = \dfrac{12\left(2 - 5x^2\right)}{\left(x^2 + 2\right)^4}$. [4]

(b) Find $\int f(x)\,\mathrm{d}x$. [4]

Consider a function $g(x)$ defined for $x \in \mathbb{R}$. The derivative of g is such that $g'(x) = f'(x)$, for all $x \in \mathbb{R}$.

Let R be the region enclosed by the graph of f, the graph of g, the line $x = 0$ and the line $x = 3$. The area of R is $\dfrac{21}{2}$.

(c) Find the two possible expressions for $g(x)$. [5]

Please **do not** write on this page.

Answers written on this page
will not be marked.

Please **do not** write on this page.

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Diploma Programme
Programme du diplôme
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Mathematics: analysis and approaches
Standard level
Paper 1

30 October 2023

Zone A afternoon │ **Zone B** afternoon │ **Zone C** afternoon

1 hour 30 minutes

Candidate session number

Instructions to candidates

- Write your session number in the boxes above.
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Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 7]

Consider the function $f(x) = a\cos(bx)$, with $a, b \in \mathbb{Z}^+$. The following diagram shows part of the graph of f.

(a) Write down the value of a. [1]

(b) (i) Write down the period of f.

(ii) Hence, find the value of b. [3]

(c) Find the value of $f\left(\dfrac{\pi}{6}\right)$. [3]

(This question continues on the following page)

16EP02

(Question 1 continued)

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Answers written on this page
will not be marked.

2. [Maximum mark: 5]

Consider the functions $f(x) = x - 3$ and $g(x) = x^2 + k^2$, where k is a real constant.

(a) Write down an expression for $(g \circ f)(x)$. [2]

(b) Given that $(g \circ f)(2) = 10$, find the possible values of k. [3]

3. [Maximum mark: 4]

Events A and B are such that $P(A) = 0.65$, $P(B) = 0.75$ and $P(A \cap B) = 0.6$.

(a) Find $P(A \cup B)$. [2]

(b) Hence, or otherwise, find $P(A' \cap B')$. [2]

4.   [Maximum mark: 7]

The sum of the first n terms of an arithmetic sequence is given by $S_n = pn^2 - qn$, where p and q are positive constants.

It is given that $S_4 = 40$ and $S_5 = 65$.

(a)   Find the value of p and the value of q.   [5]

(b)   Find the value of u_5.   [2]

5. [Maximum mark: 6]

In the following triangle ABC, $AB = \sqrt{6}$ cm, $AC = 10$ cm and $\cos B\hat{A}C = \dfrac{1}{5}$.

diagram not to scale
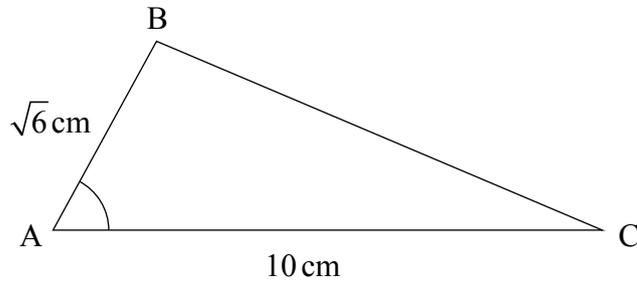
Find the area of triangle ABC.

6. [Maximum mark: 6]

The binomial expansion of $(1 + kx)^n$ is given by $1 + 12x + 28k^2x^2 + ... + k^nx^n$ where $n \in \mathbb{Z}^+$ and $k \in \mathbb{Q}$.

Find the value of n and the value of k.

Turn over

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7.   [Maximum mark: 17]

A ballet company performs *The Nutcracker* every year. Last year they gave a total of 60 performances at their theatre which has a maximum capacity of 800. The number of tickets sold, n, at each performance is shown in the following frequency table.

Number of tickets sold, n	Number of performances
$0 < n \leq 200$	3
$200 < n \leq 400$	p
$400 < n \leq 600$	18
$600 < n \leq 800$	30

(a)   (i)   Find the value of p.

(ii)   Write down the modal class.   [2]

(This question continues on the following page)

Do **not** write solutions on this page.

(Question 7 continued)

The following cumulative frequency diagram also displays these data.

(b)   Use the cumulative frequency curve to estimate

(i)   the median number of tickets sold;

(ii)   the number of performances where at least 80% of the tickets were sold.   [4]

(This question continues on page 13)

Turn over

Please **do not** write on this page.

Answers written on this page
will not be marked.

Do **not** write solutions on this page.

(Question 7 continued)

After a performance, the company decides to conduct a survey to obtain feedback from the audience.

(c) (i)   State one disadvantage of the company surveying only the first 5% of the audience as they leave the theatre.

(ii)   Describe briefly how the company could collect feedback from 5% of the audience using the systematic sampling method.

(iii) State the sampling method which should be used if the survey is to be representative of the number of children and the number of adults in the audience.   [4]

Last year $36\,000$ tickets were sold to *The Nutcracker*.

(d) The following box and whisker diagram displays the amount spent by the audience at the souvenir shop when they attended the performance.

Amount Spent ($)
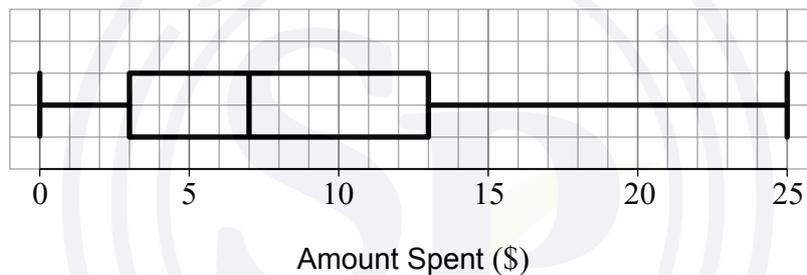
(i)   Estimate the number of people who spent between $\$3$ and $\$25$.

(ii)   Half the audience spent less than $\$a$. Estimate the value of a.   [3]

(e) This year the company will again give 60 performances and expects to sell 17 additional tickets for each performance.

(i)   Calculate the mean number of tickets the company expects to sell this year for each performance.

(ii)   State what effect, if any, this increase in ticket sales would have on the variance of the number of tickets sold for each performance.   [4]

Do **not** write solutions on this page.

8. [Maximum mark: 15]

The functions f and g are defined by

$f(x) = \ln(2x - 9)$, where $x > \dfrac{9}{2}$

$g(x) = 2\ln x - \ln d$, where $x > 0$, $d \in \mathbb{R}^+$.

(a) State the equation of the vertical asymptote to the graph of $y = g(x)$. [1]

The graphs of $y = f(x)$ and $y = g(x)$ intersect at two distinct points.

(b) (i) Show that, at the points of intersection, $x^2 - 2dx + 9d = 0$.

(ii) Hence show that $d^2 - 9d > 0$.

(iii) Find the range of possible values of d. [9]

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(Question 8 continued)

The following diagram shows part of the graphs of $y = f(x)$ and $y = g(x)$.
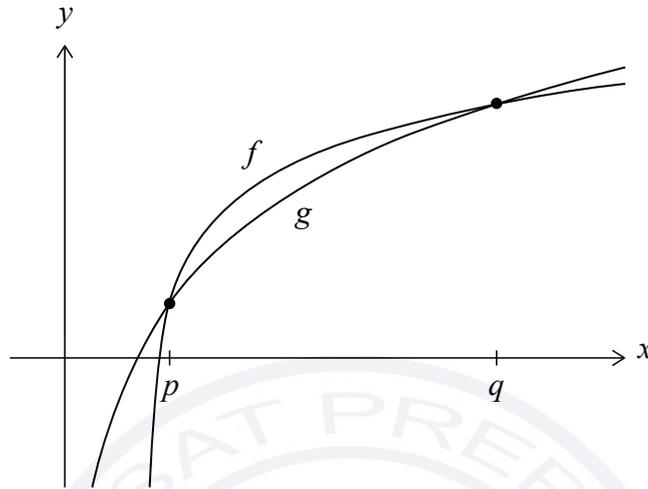
The graphs intersect at $x = p$ and $x = q$, where $p < q$.

(c)   In the case where $d = 10$, find the value of $q - p$. Express your answer in the form $a\sqrt{b}$, where $a, b \in \mathbb{Z}^+$. [5]

Do **not** write solutions on this page.

9. [Maximum mark: 13]

Consider the function f defined by $f(x) = \dfrac{8x}{\left(x^2+1\right)^3}$, where $x \in \mathbb{R}$. The graph of f is shown in the following diagram.
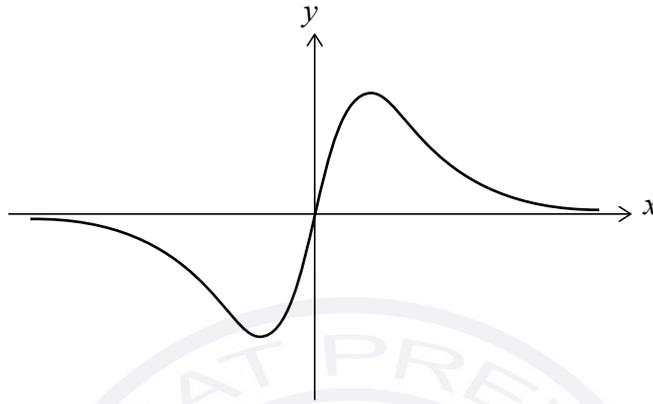
(a) Show that $f'(x) = \dfrac{8\left(1-5x^2\right)}{\left(x^2+1\right)^4}$. [4]

(b) Find $\int f(x)dx$. [4]

Consider a function $g(x)$ defined for $x \in \mathbb{R}$. The derivative of g is such that $g'(x) = f'(x)$, for all $x \in \mathbb{R}$.

Let R be the region enclosed by the graph of f, the graph of g, the line $x = 0$ and

the line $x = 3$. The area of R is $\dfrac{27}{2}$.

(c) Find the two possible expressions for $g(x)$. [5]

Mathematics: analysis and approaches
Standard level
Paper 1

8 May 2023

Zone A afternoon | **Zone B** morning | **Zone C** afternoon

Candidate session number

1 hour 30 minutes

Instructions to candidates

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- The maximum mark for this examination paper is **[80 marks]**.

12EP01

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Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 5]

Point P has coordinates $(-3, 2)$, and point Q has coordinates $(15, -8)$. Point M is the midpoint of $[PQ]$.

(a) Find the coordinates of M. [2]

Line L is perpendicular to $[PQ]$ and passes through M.

(b) Find the gradient of L. [2]

(c) Hence, write down the equation of L. [1]

2. [Maximum mark: 7]

The function f is defined by $f(x) = \dfrac{7x+7}{2x-4}$ for $x \in \mathbb{R}$, $x \neq 2$.

(a) Find the zero of $f(x)$. [2]

(b) For the graph of $y = f(x)$, write down the equation of

　　(i)　the vertical asymptote;

　　(ii)　the horizontal asymptote. [2]

(c) Find $f^{-1}(x)$, the inverse function of $f(x)$. [3]

3. [Maximum mark: 6]

On a Monday at an amusement park, a sample of 40 visitors was randomly selected as they were leaving the park. They were asked how many times that day they had been on a ride called *The Dragon*. This information is summarized in the following frequency table.

Number of times on *The Dragon*	Frequency
0	6
1	16
2	13
3	2
4	3

It can be assumed that this sample is representative of all visitors to the park for the following day.

(a) For the following day, Tuesday, estimate

    (i) the probability that a randomly selected visitor will ride *The Dragon*;

    (ii) the expected number of times a visitor will ride *The Dragon*. [4]

It is known that 1000 visitors will attend the amusement park on Tuesday. *The Dragon* can carry a maximum of 10 people each time it runs.

(b) Estimate the minimum number of times *The Dragon* must run to satisfy demand. [2]

4. [Maximum mark: 6]

(a) Show that the equation $\cos 2x = \sin x$ can be written in the form $2\sin^2 x + \sin x - 1 = 0$. [1]

(b) Hence, solve $\cos 2x = \sin x$, where $-\pi \le x \le \pi$. [5]

5. [Maximum mark: 6]

Find the range of possible values of k such that $e^{2x} + \ln k = 3e^x$ has at least one real solution.

6. [Maximum mark: 6]

The function f is defined by $f(x) = \sin qx$, where $q > 0$. The following diagram shows part of the graph of f for $0 \le x \le 4m$, where x is in radians. There are x-intercepts at $x = 0$, $2m$ and $4m$.

(a)   Find an expression for m in terms of q. [2]

The function g is defined by $g(x) = 3\sin\dfrac{2qx}{3}$, for $0 \le x \le 6m$.

(b)   On the axes above, sketch the graph of g. [4]

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Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 13]

The function h is defined by $h(x) = 2xe^x + 3$, for $x \in \mathbb{R}$. The following diagram shows part of the graph of h, which has a local minimum at point A.
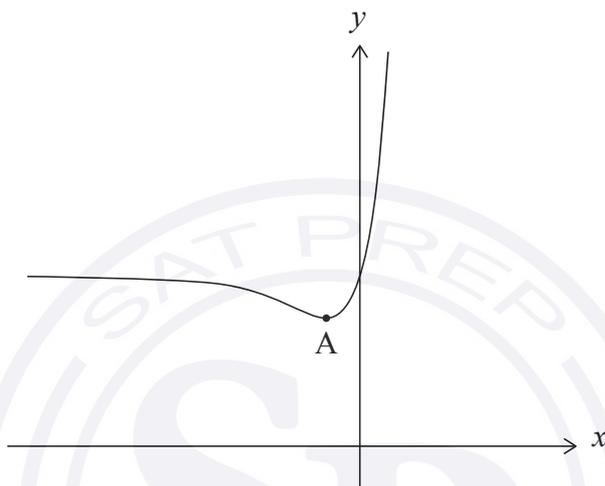
(a) Find the value of the y-intercept. [2]

(b) Find $h'(x)$. [2]

(c) Hence, find the coordinates of A. [5]

(d) (i) Show that $h''(x) = (2x + 4)e^x$.

(ii) Find the values of x for which the graph of h is concave-up. [4]

Do **not** write solutions on this page.

8.   [Maximum mark: 14]

Consider the arithmetic sequence $u_1, u_2, u_3, \ldots$.

The sum of the first n terms of this sequence is given by $S_n = n^2 + 4n$.

(a)   (i)   Find the sum of the first five terms.

(ii)   Given that $S_6 = 60$, find u_6.   [4]

(b)   Find u_1.   [2]

(c)   Hence or otherwise, write an expression for u_n in terms of n.   [3]

Consider a geometric sequence, v_n, where $v_2 = u_1$ and $v_4 = u_6$.

(d)   Find the possible values of the common ratio, r.   [3]

(e)   Given that $v_{99} < 0$, find v_5.   [2]

Do **not** write solutions on this page.

9.   [Maximum mark: 17]

An object moves along a straight line. Its velocity, $v\,\text{m}\,\text{s}^{-1}$, at time t seconds is given by
$v(t) = -t^3 + \dfrac{7}{2}t^2 - 2t + 6$, for $0 \le t \le 4$. The object first comes to rest at $t = k$.

The graph of v is shown in the following diagram.
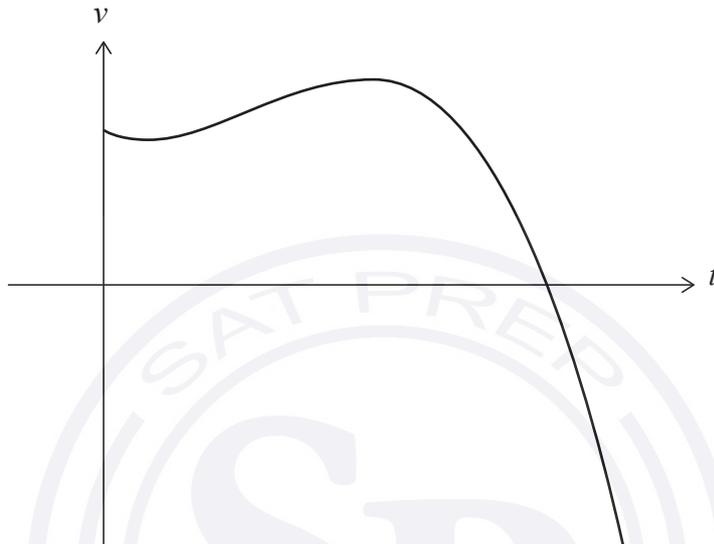
At $t = 0$, the object is at the origin.

(a)   Find the displacement of the object from the origin at $t = 1$. [5]

(b)   Find an expression for the acceleration of the object. [2]

(c)   Hence, find the greatest speed reached by the object before it comes to rest. [5]

(d)   Find the greatest speed reached by the object for $0 \le t \le 4$. [2]

(e)   Write down an expression that represents the distance travelled by the object while its speed is increasing. Do not evaluate the expression. [3]

12EP10

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**Mathematics: analysis and approaches
Standard level
Paper 1**

8 May 2023

Zone A afternoon │ **Zone B** morning │ **Zone C** afternoon

1 hour 30 minutes

Instructions to candidates

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- The maximum mark for this examination paper is **[80 marks]**.

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Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 6]

The following diagram shows a circle with centre O and radius $4\,cm$.

diagram not to scale
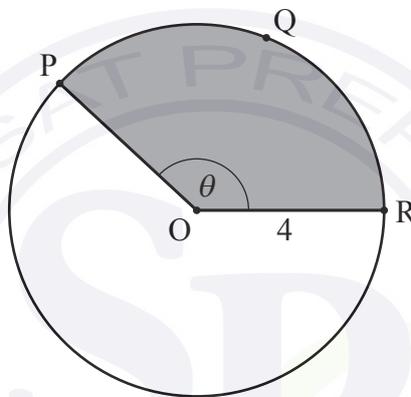
The points P, Q and R lie on the circumference of the circle and $P\hat{O}R = \theta$, where θ is measured in radians.

The length of arc PQR is $10\,cm$.

(a) Find the perimeter of the shaded sector. [2]

(b) Find θ. [2]

(c) Find the area of the shaded sector. [2]

(This question continues on the following page)

(Question 1 continued)

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Please **do not** write on this page.

Answers written on this page
will not be marked.

2. [Maximum mark: 6]

Consider an arithmetic sequence with $u_1 = 0.6$ and $u_4 = 0.15$.

(a) Find the common difference, d. [2]

The following table shows the probability distribution of a discrete random variable X such that $P(X = n) = \dfrac{u_n}{k}$, where $n \in \mathbb{Z}^+$, $1 \le n \le 4$ and $k \in \mathbb{R}^+$.

n	1	2	3	4
$P(X = n)$	$\dfrac{0.6}{k}$	$\dfrac{u_2}{k}$	$\dfrac{u_3}{k}$	$\dfrac{0.15}{k}$

(b) Find the value of k. [4]

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3.   [Maximum mark: 5]

A function f is defined by $f(x) = 1 - \dfrac{1}{x-2}$, where $x \in \mathbb{R}$, $x \neq 2$.

(a)   The graph of $y = f(x)$ has a vertical asymptote and a horizontal asymptote.

Write down the equation of

(i)   the vertical asymptote;

(ii)   the horizontal asymptote.   [2]

(b)   Find the coordinates of the point where the graph of $y = f(x)$ intersects

(i)   the y-axis;

(ii)   the x-axis.   [2]

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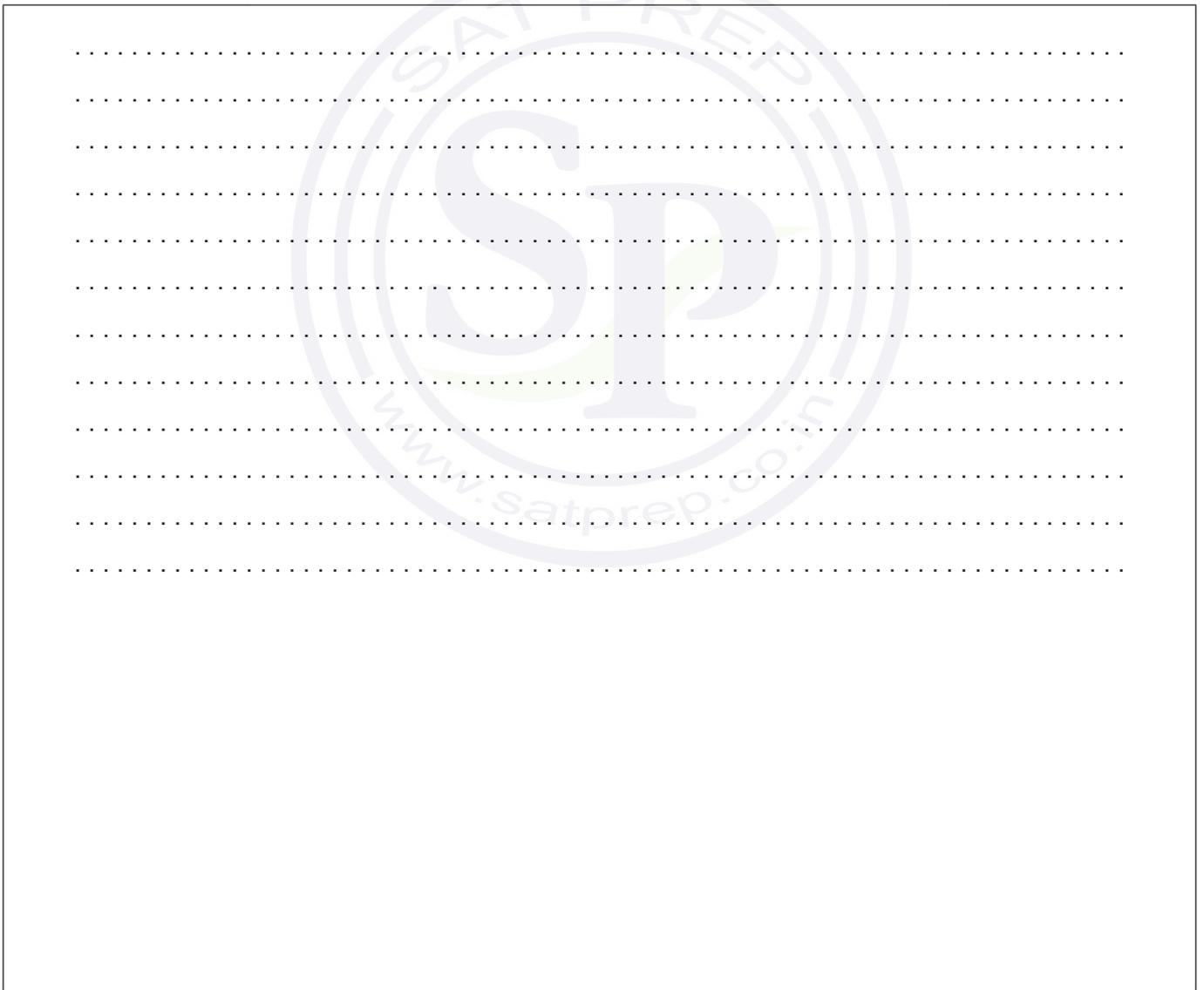
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(Question 3 continued)

    (c)   On the following set of axes, sketch the graph of $y = f(x)$, showing all the features found in parts (a) and (b).   [1]
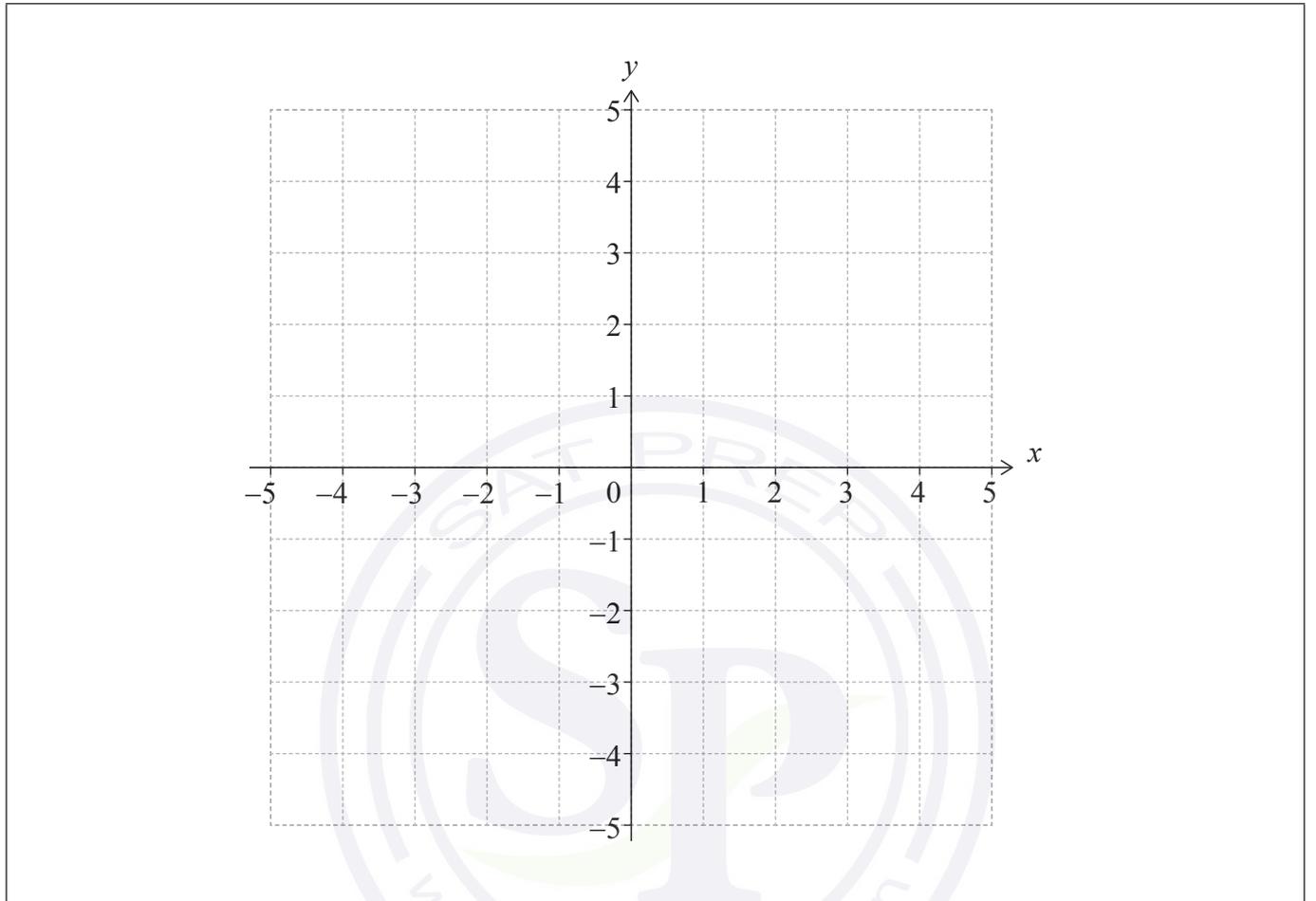
4. [Maximum mark: 5]

Events A and B are such that $\mathrm{P}(A) = 0.4$, $\mathrm{P}(A|B) = 0.25$ and $\mathrm{P}(A \cup B) = 0.55$.

Find $\mathrm{P}(B)$.

5.   [Maximum mark: 6]

The following diagram shows part of the graph of $y = \dfrac{x}{x^2 + 2}$ for $x \geq 0$.

The shaded region R is bounded by the curve, the x-axis and the line $x = c$.

The area of R is $\ln 3$.

Find the value of c.

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Turn over

Please **do not** write on this page.

Answers written on this page
will not be marked.

6. [Maximum mark: 7]

The functions f and g are defined for $x \in \mathbb{R}$ by

$$f(x) = ax + b, \text{ where } a, b \in \mathbb{Z}$$

$$g(x) = x^2 + x + 3.$$

Find the two possible functions f such that $(g \circ f)(x) = 4x^2 - 14x + 15$.

Turn over

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7.   [Maximum mark: 17]

The following diagram shows part of the graph of a quadratic function f.

The vertex of the parabola is $(-2, -5)$ and the y-intercept is at point P.
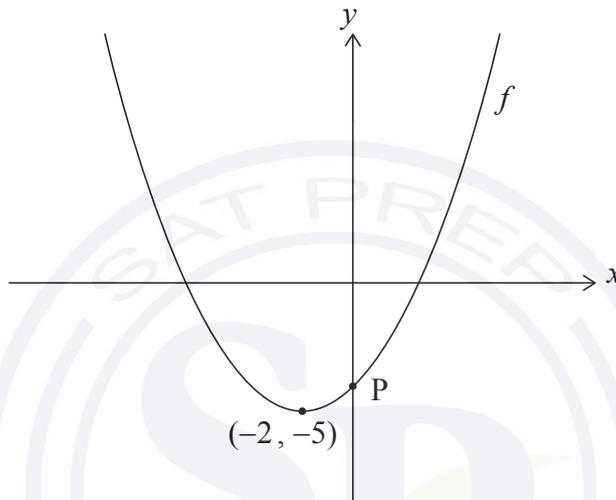
(a)   Write down the equation of the axis of symmetry.   [1]

The function can be written in the form $f(x) = \dfrac{1}{4}(x - h)^2 + k$, where $h, k \in \mathbb{Z}$.

(b)   Write down the values of h and k.   [2]

(c)   Find the y-coordinate of P.   [2]

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(Question 7 continued)

In the following diagram, the line L is normal to the graph of f at point P.
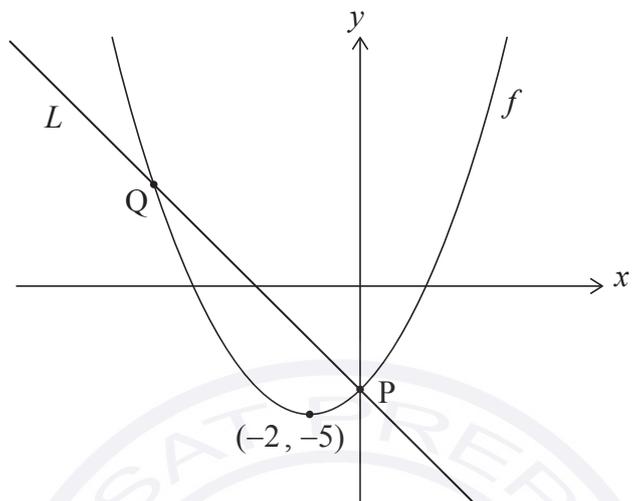
(d) Find the equation of the line L, in the form $y = ax + b$. [4]

The line L intersects the graph of f at a second point, Q, as shown above.

(e) Calculate the distance between P and Q. [8]

Turn over

Do **not** write solutions on this page.

8.   [Maximum mark: 14]

Consider an acute angle θ such that $\cos\theta = \dfrac{2}{3}$.

(a)   Find the value of

(i) $\sin\theta$;

(ii) $\sin 2\theta$.   [4]

The following diagram shows triangle ABC, with $\hat{\mathrm{B}} = \theta$, $\hat{\mathrm{A}} = 2\theta$, $\mathrm{BC} = a$ and $\mathrm{AC} = b$.
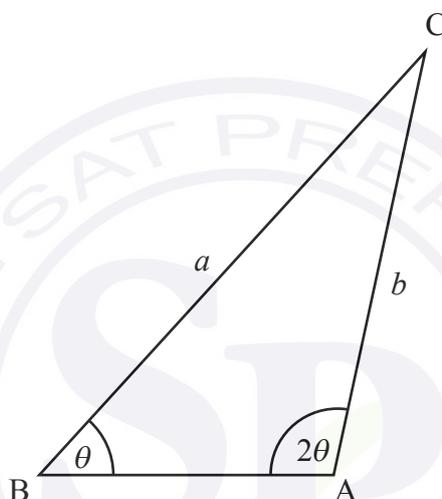
(b)   Show that $b = \dfrac{3a}{4}$.   [2]

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Do **not** write solutions on this page.

(Question 8 continued)

[BA] is extended to form an isosceles triangle DAC, with $\hat{D} = \theta$, as shown in the following diagram.
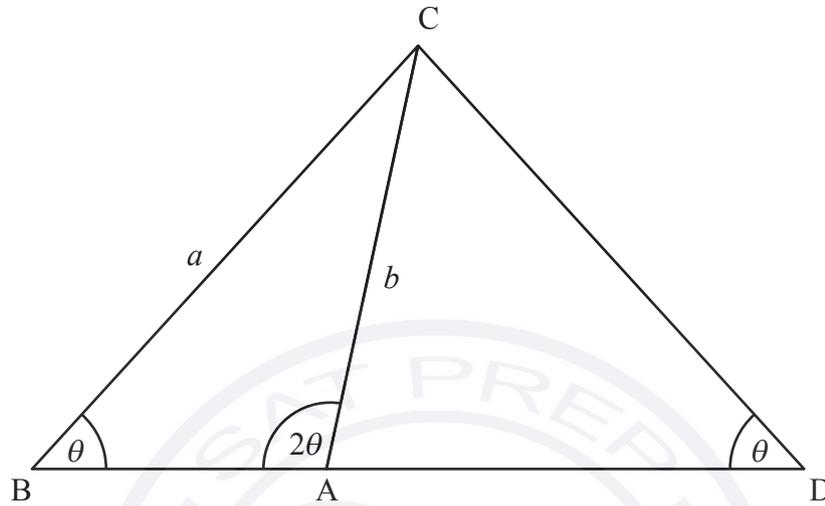
(c)   Find the value of $\sin C\hat{A}D$. [3]

(d)   Find the area of triangle DAC, in terms of a. [5]

Do **not** write solutions on this page.

9. [Maximum mark: 14]

A circle with equation $x^2 + y^2 = 9$ has centre $(0, 0)$ and radius 3.

A triangle, PQR, is inscribed in the circle with its vertices at $P(-3, 0)$, $Q(x, y)$ and $R(x, -y)$, where Q and R are variable points in the first and fourth quadrants respectively. This is shown in the following diagram.
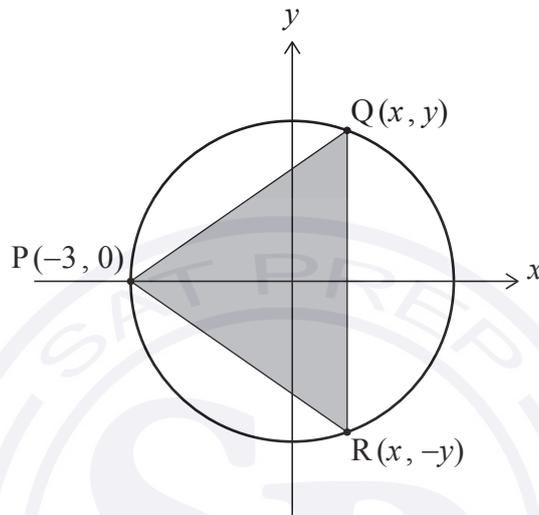
(a)   For point Q, show that $y = \sqrt{9 - x^2}$. [1]

(b)   Hence, find an expression for A, the area of triangle PQR, in terms of x. [3]

(c)   Show that $\dfrac{\mathrm{d}A}{\mathrm{d}x} = \dfrac{9 - 3x - 2x^2}{\sqrt{9 - x^2}}$. [4]

(d)   Hence or otherwise, find the y-coordinate of R such that A is a maximum. [6]

References:

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16EP16

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: analysis and approaches
Standard level
Paper 1

Monday 31 October 2022 (afternoon)

Candidate session number

1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

12EP01

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Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1.   [Maximum mark: 7]

Let $f(x) = -2x + 3$, for $x \in \mathbb{R}$.

(a)   The graph of a linear function g is parallel to the graph of f and passes through the origin. Find an expression for $g(x)$. [2]

(b)   The graph of a linear function h is perpendicular to the graph of f and passes through the point $(-1, 2)$. Find an expression for $h(x)$. [3]

(c)   Find $(g \circ h)(0)$. [2]

2. [Maximum mark: 4]

The function g is defined by $g(x) = e^{x^2 + 1}$, where $x \in \mathbb{R}$.

Find $g'(-1)$.

3. [Maximum mark: 7]

Consider a circle with a diameter AB, where A has coordinates $(1, 4, 0)$ and B has coordinates $(-3, 2, -4)$.

(a) Find

    (i) the coordinates of the centre of the circle;

    (ii) the radius of the circle. [4]

The circle forms the base of a right cone whose vertex V has coordinates $(-1, -1, 0)$.

diagram not to scale
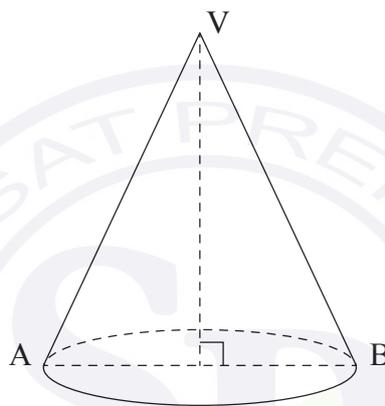
(b) Find the exact volume of the cone. [3]

4.   [Maximum mark: 5]

Let a be a constant, where $a > 1$.

(a)   Show that $a^2 + \left(\dfrac{a^2 - 1}{2} \right)^2 = \left(\dfrac{a^2 + 1}{2} \right)^2$.   [3]

Consider a right-angled triangle with sides of length a, $\left(\dfrac{a^2 - 1}{2} \right)$ and $\left(\dfrac{a^2 + 1}{2} \right)$.

(b)   Find an expression for the area of the triangle in terms of a.   [2]

Turn over

5. [Maximum mark: 5]

The derivative of the function f is given by $f'(x) = \dfrac{6x}{x^2 + 1}$.

The graph of $y = f(x)$ passes through the point $(1, 5)$. Find an expression for $f(x)$.

6. [Maximum mark: 6]

Events A and B are such that $P(A) = 0.3$ and $P(B) = 0.8$.

(a) Determine the value of $P(A \cap B)$ in the case where the events A and B are independent. [1]

(b) Determine the minimum possible value of $P(A \cap B)$. [3]

(c) Determine the maximum possible value of $P(A \cap B)$, justifying your answer. [2]

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 16]

(a) The graph of a quadratic function f has its vertex at the point $(3, 2)$ and it intersects the x-axis at $x = 5$. Find f in the form $f(x) = a(x - h)^2 + k$. [3]

The quadratic function g is defined by $g(x) = px^2 + (t - 1)x - p$ where $x \in \mathbb{R}$ and $p, t \in \mathbb{R}, p \neq 0$.

(b) In the case where $g(-3) = g(1) = 4$,

　(i) find the value of p and the value of t;

　(ii) find the range of g. [7]

(c) The linear function j is defined by $j(x) = -x + 3p$ where $x \in \mathbb{R}$ and $p \in \mathbb{R}, p \neq 0$.

　Show that the graphs of $j(x) = -x + 3p$ and $g(x) = px^2 + (t - 1)x - p$ have two distinct points of intersection for every possible value of p and t. [6]

8. [Maximum mark: 15]

(a) Calculate the value of each of the following logarithms:

　(i) $\log_2 \dfrac{1}{16}$;

　(ii) $\log_9 3$;

　(iii) $\log_{\sqrt{3}} 81$. [7]

(b) It is given that $\log_{ab} a = 3$, where $a, b \in \mathbb{R}^+, ab \neq 1$.

　(i) Show that $\log_{ab} b = -2$.

　(ii) Hence find the value of $\log_{ab} \dfrac{\sqrt[3]{a}}{\sqrt{b}}$. [8]

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9. [Maximum mark: 15]

The function f is defined by $f(x) = \cos^2 x - 3\sin^2 x$, $0 \le x \le \pi$.

(a) Find the roots of the equation $f(x) = 0$. [5]

(b) (i) Find $f'(x)$.

(ii) Hence find the coordinates of the points on the graph of $y = f(x)$ where $f'(x) = 0$. [7]

(c) Sketch the graph of $y = f(x)$, clearly showing the coordinates of any points where $f'(x) = 0$ and any points where the graph meets the coordinate axes. [3]

References:

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12EP09

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Mathematics: analysis and approaches
Standard level
Paper 1

Friday 6 May 2022 (afternoon)

1 hour 30 minutes

Candidate session number

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

16EP01

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 5]

    The following table shows values of $f(x)$ and $g(x)$ for different values of x.

    Both f and g are one-to-one functions.

x	−2	0	3	4
$f(x)$	8	4	0	−3
$g(x)$	−5	−2	4	0

    (a) Find $g(0)$. [1]

    (b) Find $(f \circ g)(0)$. [2]

    (c) Find the value of x such that $f(x) = 0$. [2]

2. [Maximum mark: 5]

The nth term of an arithmetic sequence is given by $u_n = 15 - 3n$.

(a) State the value of the first term, u_1. [1]

(b) Given that the nth term of this sequence is -33, find the value of n. [2]

(c) Find the common difference, d. [2]

3. [Maximum mark: 6]

Consider any three consecutive integers, $n-1$, n and $n+1$.

(a) Prove that the sum of these three integers is always divisible by 3. [2]

(b) Prove that the sum of the squares of these three integers is never divisible by 3. [4]

4. [Maximum mark: 6]

A function f is defined by $f(x) = \dfrac{2x-1}{x+1}$, where $x \in \mathbb{R}$, $x \neq -1$.

(a) The graph of $y = f(x)$ has a vertical asymptote and a horizontal asymptote.

     Write down the equation of

     (i)   the vertical asymptote;

     (ii)   the horizontal asymptote.   [2]

(b) On the set of axes below, sketch the graph of $y = f(x)$.

     On your sketch, clearly indicate the asymptotes and the position of any points of intersection with the axes.   [3]
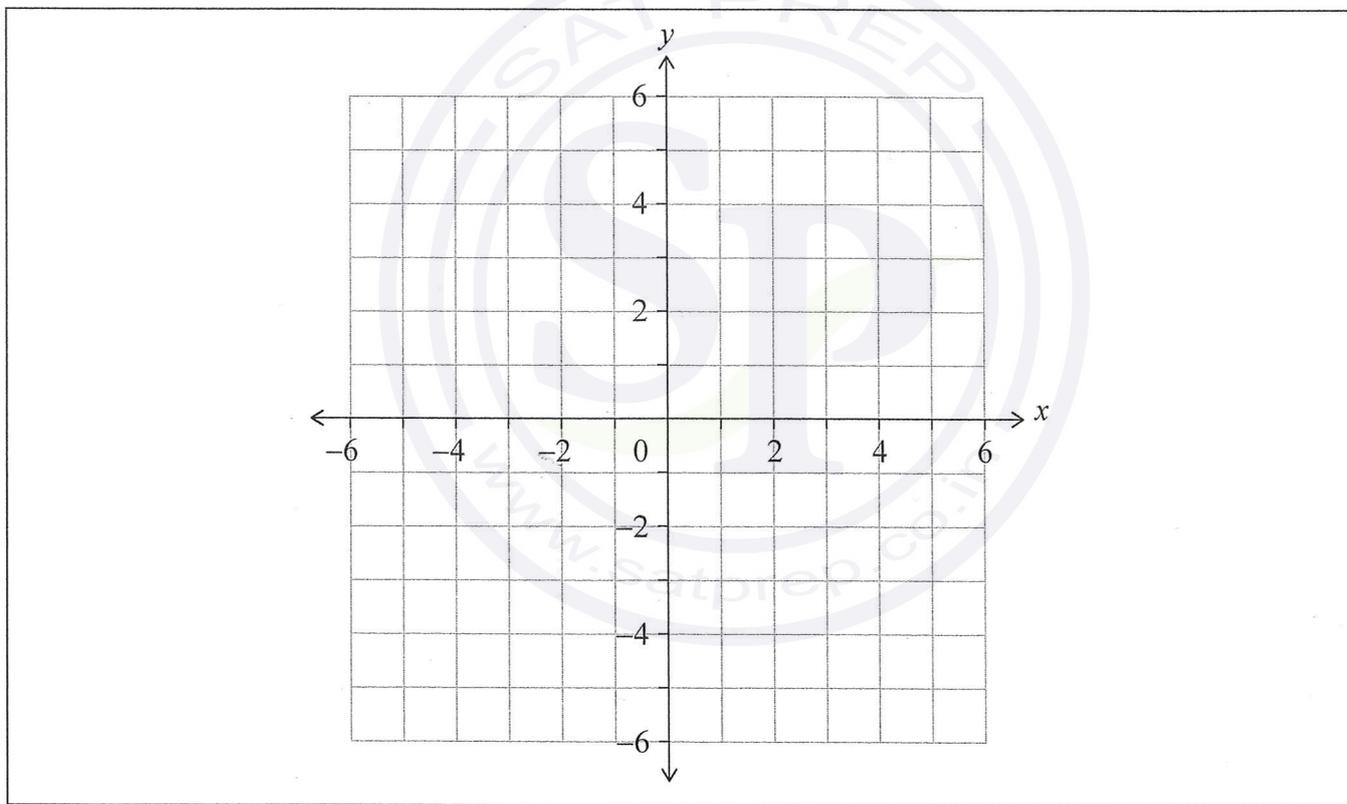
(c) Hence, solve the inequality $0 < \dfrac{2x-1}{x+1} < 2$.   [1]

(This question continues on the following page)

(Question 4 continued)

16EP07

5. [Maximum mark: 5]

Find the least positive value of x for which $\cos\left(\dfrac{x}{2} + \dfrac{\pi}{3}\right) = \dfrac{1}{\sqrt{2}}$.

6. [Maximum mark: 7]

Consider the binomial expansion $(x + 1)^7 = x^7 + ax^6 + bx^5 + 35x^4 + ... + 1$ where $x \neq 0$ and $a, b \in \mathbb{Z}^+$.

(a) Show that $b = 21$. [2]

The third term in the expansion is the mean of the second term and the fourth term in the expansion.

(b) Find the possible values of x. [5]

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Answers written on this page
will not be marked.

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7.   [Maximum mark: 15]

The following diagram shows part of the graph of a quadratic function f.

The graph of f has its vertex at $(3, 4)$, and it passes through point Q as shown.
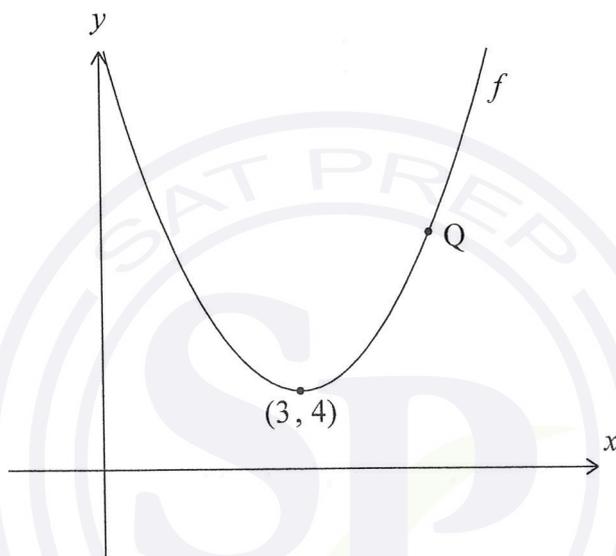
(a)   Write down the equation of the axis of symmetry.   [1]

(b)   The function can be written in the form $f(x) = a(x - h)^2 + k$.

   (i)   Write down the values of h and k.

   (ii)   Point Q has coordinates $(5, 12)$. Find the value of a.   [4]

The line L is tangent to the graph of f at Q.

(c)   Find the equation of L.   [4]

Now consider another function $y = g(x)$. The derivative of g is given by $g'(x) = f(x) - d$, where $d \in \mathbb{R}$.

(d)   Find the values of d for which g is an increasing function.   [3]

(e)   Find the values of x for which the graph of g is concave-up.   [3]

Turn over

Do **not** write solutions on this page.

8. [Maximum mark: 15]

Consider the functions $f(x) = \dfrac{1}{x-4} + 1$, for $x \neq 4$, and $g(x) = x - 3$ for $x \in \mathbb{R}$.

The following diagram shows the graphs of f and g.
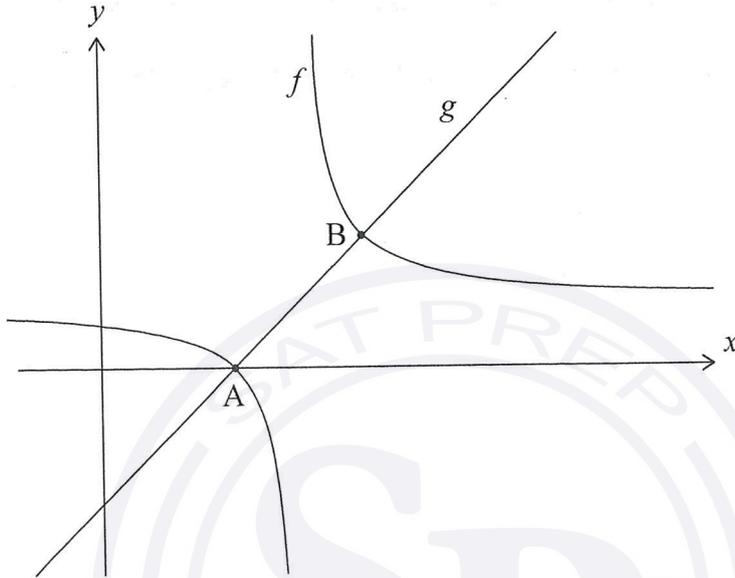
The graphs of f and g intersect at points A and B. The coordinates of A are $(3, 0)$.

(a) Find the coordinates of B.

[5]

(This question continues on the following page)

Do **not** write solutions on this page.

(Question 8 continued)

In the following diagram, the shaded region is enclosed by the graph of f, the graph of g, the x-axis, and the line $x = k$, where $k \in \mathbb{Z}$.
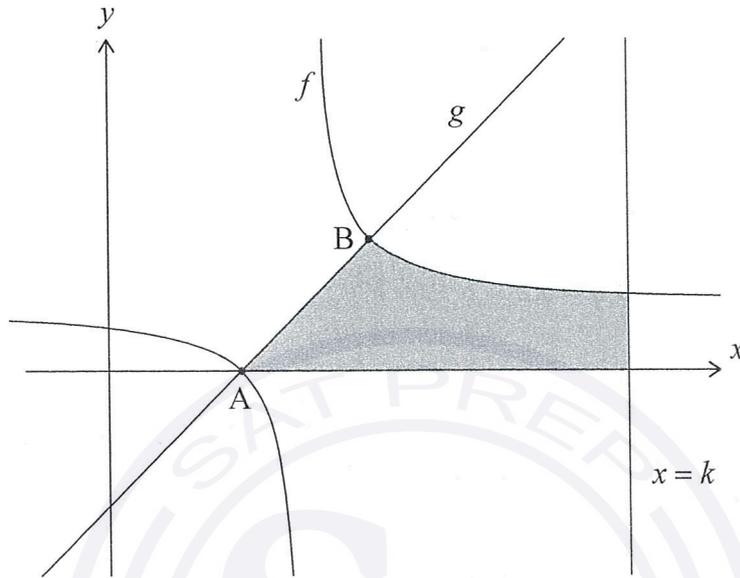
The area of the shaded region can be written as $\ln(p) + 8$, where $p \in \mathbb{Z}$.

(b)   Find the value of k and the value of p.   [10]

Turn over

Do **not** write solutions on this page.

9. [Maximum mark: 16]

A biased four-sided die with faces labelled $1, 2, 3$ and 4 is rolled and the result recorded. Let X be the result obtained when the die is rolled. The probability distribution for X is given in the following table where p and q are constants.

x	1	2	3	4
$P(X = x)$	p	0.3	q	0.1

For this probability distribution, it is known that $E(X) = 2$.

(a) Show that $p = 0.4$ and $q = 0.2$. [5]

(b) Find $P(X > 2)$. [2]

Nicky plays a game with this four-sided die. In this game she is allowed a maximum of five rolls. Her score is calculated by adding the results of each roll. Nicky wins the game if her score is at least ten.

After three rolls of the die, Nicky has a score of four.

(c) Assuming that rolls of the die are independent, find the probability that Nicky wins the game. [5]

David has two pairs of unbiased four-sided dice, a yellow pair and a red pair.

Both yellow dice have faces labelled $1, 2, 3$ and 4. Let S represent the sum obtained by rolling the two yellow dice. The probability distribution for S is shown below.

s	2	3	4	5	6	7	8
$P(S = s)$	$\dfrac{1}{16}$	$\dfrac{2}{16}$	$\dfrac{3}{16}$	$\dfrac{4}{16}$	$\dfrac{3}{16}$	$\dfrac{2}{16}$	$\dfrac{1}{16}$

The first red die has faces labelled $1, 2, 2$ and 3. The second red die has faces labelled $1, a, a$ and b, where $a < b$ and $a, b \in \mathbb{Z}^+$. The probability distribution for the sum obtained by rolling the red pair is the same as the distribution for the sum obtained by rolling the yellow pair.

(d) Determine the value of b. [2]

(e) Find the value of a, providing evidence for your answer. [2]

Diploma Programme
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Mathematics: analysis and approaches
Standard level
Paper 1

Friday 6 May 2022 (afternoon)

1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

12EP01

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 5]

Consider the points $A(-2, 20)$, $B(4, 6)$ and $C(-14, 12)$. The line L passes through the point A and is perpendicular to $[BC]$.

(a)   Find the equation of L. [3]

The line L passes through the point $(k, 2)$.

(b)   Find the value of k. [2]

2. [Maximum mark: 5]

(a) The expression $\dfrac{3\sqrt{x}-5}{\sqrt{x}}$ can be written as $3 - 5x^{p}$. Write down the value of p.   [1]

(b) Hence, find the value of $\displaystyle\int_{1}^{9}\left(\dfrac{3\sqrt{x}-5}{\sqrt{x}}\right)dx$.   [4]

3. [Maximum mark: 7]

A survey at a swimming pool is given to one adult in each family. The age of the adult, a years old, and of their eldest child, c years old, are recorded.

The ages of the eldest child are summarized in the following box and whisker diagram.

diagram not to scale

ages of eldest child (years)

2   6 7   10   18

(a) Find the largest value of c that would not be considered an outlier. [3]

The regression line of a on c is $a = \dfrac{7}{4}c + 20$. The regression line of c on a is $c = \dfrac{1}{2}a - 9$.

(b) (i) One of the adults surveyed is 42 years old. Estimate the age of their eldest child.

(ii) Find the mean age of all the adults surveyed. [4]

4.   [Maximum mark: 7]

Consider the functions $f(x) = \sqrt{3}\sin x + \cos x$ where $0 \le x \le \pi$ and $g(x) = 2x$ where $x \in \mathbb{R}$.

(a)   Find $(f \circ g)(x)$.   [2]

(b)   Solve the equation $(f \circ g)(x) = 2\cos 2x$ where $0 \le x \le \pi$.   [5]

5. [Maximum mark: 5]

Consider the curve with equation $y = (2x - 1)\mathrm{e}^{kx}$, where $x \in \mathbb{R}$ and $k \in \mathbb{Q}$.

The tangent to the curve at the point where $x = 1$ is parallel to the line $y = 5\mathrm{e}^k x$.

Find the value of k.

6.   [Maximum mark: 7]

Consider $f(x) = 4\sin x + 2.5$ and $g(x) = 4\sin\left(x - \dfrac{3\pi}{2}\right) + 2.5 + q$, where $x \in \mathbb{R}$ and $q > 0$.

The graph of g is obtained by two transformations of the graph of f.

(a)   Describe these two transformations.   [2]

The y-intercept of the graph of g is at $(0, r)$.

(b)   Given that $g(x) \geq 7$, find the smallest value of r.   [5]

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 14]

A function, f, has its derivative given by $f'(x) = 3x^2 - 12x + p$, where $p \in \mathbb{R}$. The following diagram shows part of the graph of f'.
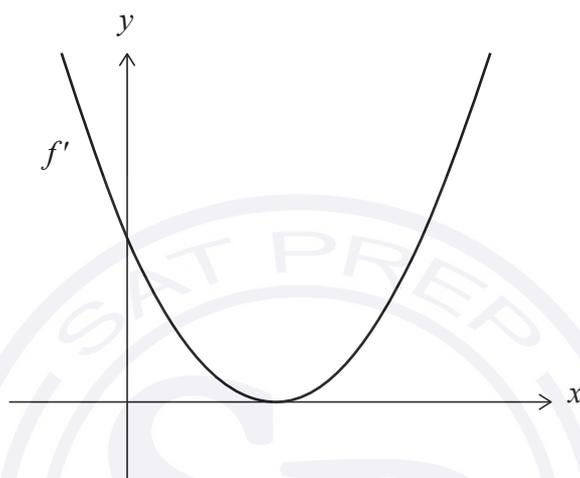
The graph of f' has an axis of symmetry $x = q$.

(a) Find the value of q. [2]

The vertex of the graph of f' lies on the x-axis.

(b) (i) Write down the value of the discriminant of f'.

   (ii) Hence or otherwise, find the value of p. [4]

(c) Find the value of the gradient of the graph of f' at $x = 0$. [3]

(d) Sketch the graph of f'', the second derivative of f. Indicate clearly the x-intercept and the y-intercept. [2]

The graph of f has a point of inflexion at $x = a$.

(e) (i) Write down the value of a.

   (ii) Find the values of x for which the graph of f is concave-down. Justify your answer. [3]

Do **not** write solutions on this page.

8. [Maximum mark: 15]

Consider the series $\ln x + p \ln x + \frac{1}{3} \ln x + ...$, where $x \in \mathbb{R}$, $x > 1$ and $p \in \mathbb{R}$, $p \neq 0$.

(a) Consider the case where the series is geometric.

    (i) Show that $p = \pm \frac{1}{\sqrt{3}}$.

    (ii) Given that $p > 0$ and $S_\infty = 3 + \sqrt{3}$, find the value of x.   [5]

(b) Now consider the case where the series is arithmetic with common difference d.

    (i) Show that $p = \frac{2}{3}$.

    (ii) Write down d in the form $k \ln x$, where $k \in \mathbb{Q}$.

    (iii) The sum of the first n terms of the series is $-3 \ln x$.

         Find the value of n.   [10]

9. [Maximum mark: 15]

(a) (i) Expand and simplify $(1 - a)^3$ in ascending powers of a.

    (ii) By using a suitable substitution for a, show that
    $1 - 3\cos 2x + 3\cos^2 2x - \cos^3 2x = 8\sin^6 x$.   [6]

Consider $f(x) = 4\cos x \left(1 - 3\cos 2x + 3\cos^2 2x - \cos^3 2x\right)$.

(b) (i) Show that $\int_0^m f(x)\, dx = \frac{32}{7}\sin^7 m$, where m is a positive real constant.

    (ii) It is given that $\int_m^{\frac{\pi}{2}} f(x)\, dx = \frac{127}{28}$, where $0 \le m \le \frac{\pi}{2}$. Find the value of m.   [9]

References:

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Please **do not** write on this page.

Answers written on this page
will not be marked.

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: analysis and approaches
Standard level
Paper 1

Monday 1 November 2021 (afternoon)

1 hour 30 minutes

Candidate session number

Instructions to candidates

- Write your session number in the boxes above.
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- You are not permitted access to any calculator for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

12EP01

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Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 7]

Consider the function $f(x) = -2(x - 1)(x + 3)$, for $x \in \mathbb{R}$. The following diagram shows part of the graph of f.
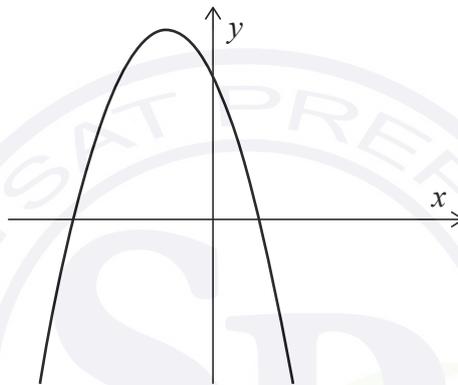
(a) For the graph of f

    (i) find the x-coordinates of the x-intercepts;

    (ii) find the coordinates of the vertex. [5]

The function f can be written in the form $f(x) = -2(x - h)^2 + k$.

(b) Write down the value of h and the value of k. [2]

2. [Maximum mark: 4]

Given that $\dfrac{dy}{dx} = \cos\left(x - \dfrac{\pi}{4}\right)$ and $y = 2$ when $x = \dfrac{3\pi}{4}$, find y in terms of x.

Turn over

3. [Maximum mark: 5]

The function f is defined by $f(x) = \dfrac{2x+4}{3-x}$, where $x \in \mathbb{R}$, $x \neq 3$.

(a) Write down the equation of

    (i) the vertical asymptote of the graph of f;

    (ii) the horizontal asymptote of the graph of f. [2]

(b) Find the coordinates where the graph of f crosses

    (i) the x-axis;

    (ii) the y-axis. [2]

(This question continues on the following page)

12EP04
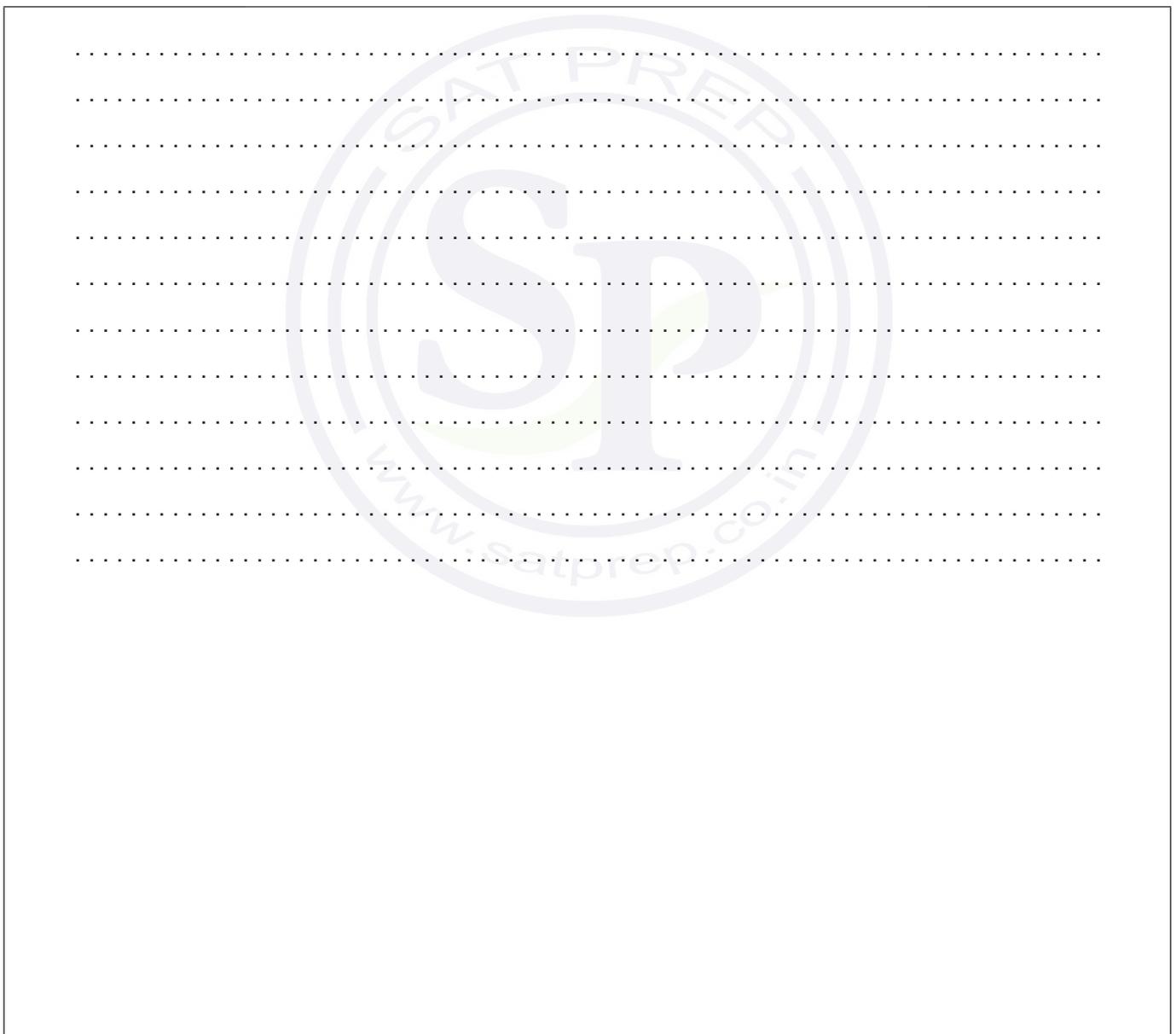
(Question 3 continued)

(c)   Sketch the graph of f on the axes below. [1]

Turn over

4. [Maximum mark: 5]

Box 1 contains 5 red balls and 2 white balls.
Box 2 contains 4 red balls and 3 white balls.

(a) A box is chosen at random and a ball is drawn. Find the probability that the ball is red. [3]

Let A be the event that "box 1 is chosen" and let R be the event that "a red ball is drawn".

(b) Determine whether events A and R are independent. [2]

5. [Maximum mark: 7]

The function f is defined for all $x \in \mathbb{R}$. The line with equation $y = 6x - 1$ is the tangent to the graph of f at $x = 4$.

(a) Write down the value of $f'(4)$. [1]

(b) Find $f(4)$. [1]

The function g is defined for all $x \in \mathbb{R}$ where $g(x) = x^2 - 3x$ and $h(x) = f(g(x))$.

(c) Find $h(4)$. [2]

(d) Hence find the equation of the tangent to the graph of h at $x = 4$. [3]

6.   [Maximum mark: 7]

(a)   Show that $2x - 3 - \dfrac{6}{x-1} = \dfrac{2x^2 - 5x - 3}{x-1}$, $x \in \mathbb{R}$, $x \neq 1$.   [2]

(b)   Hence or otherwise, solve the equation $2\sin 2\theta - 3 - \dfrac{6}{\sin 2\theta - 1} = 0$ for $0 \leq \theta \leq \pi$, $\theta \neq \dfrac{\pi}{4}$.   [5]

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 16]

A particle P moves along the x-axis. The velocity of P is $v\,\mathrm{m\,s^{-1}}$ at time t seconds, where $v(t) = 4 + 4t - 3t^2$ for $0 \le t \le 3$. When $t = 0$, P is at the origin O.

    (a)   (i)   Find the value of t when P reaches its maximum velocity.

            (ii)   Show that the distance of P from O at this time is $\dfrac{88}{27}$ metres. [7]

    (b)   Sketch a graph of v against t, clearly showing any points of intersection with the axes. [4]

    (c)   Find the total distance travelled by P. [5]

8. [Maximum mark: 15]

Consider the function $f(x) = a^x$ where $x, a \in \mathbb{R}$ and $x > 0$, $a > 1$.

The graph of f contains the point $\left(\dfrac{2}{3}, 4\right)$.

    (a)   Show that $a = 8$. [2]

    (b)   Write down an expression for $f^{-1}(x)$. [1]

    (c)   Find the value of $f^{-1}\left(\sqrt{32}\right)$. [3]

    (d)   Consider the arithmetic sequence $\log_8 27$, $\log_8 p$, $\log_8 q$, $\log_8 125$, where $p > 1$ and $q > 1$.

            (i)   Show that $27, p, q$ and 125 are four consecutive terms in a geometric sequence.

            (ii)   Find the value of p and the value of q. [9]

Do **not** write solutions on this page.

9. [Maximum mark: 14]

Consider a function f with domain $a < x < b$. The following diagram shows the graph of f', the derivative of f.
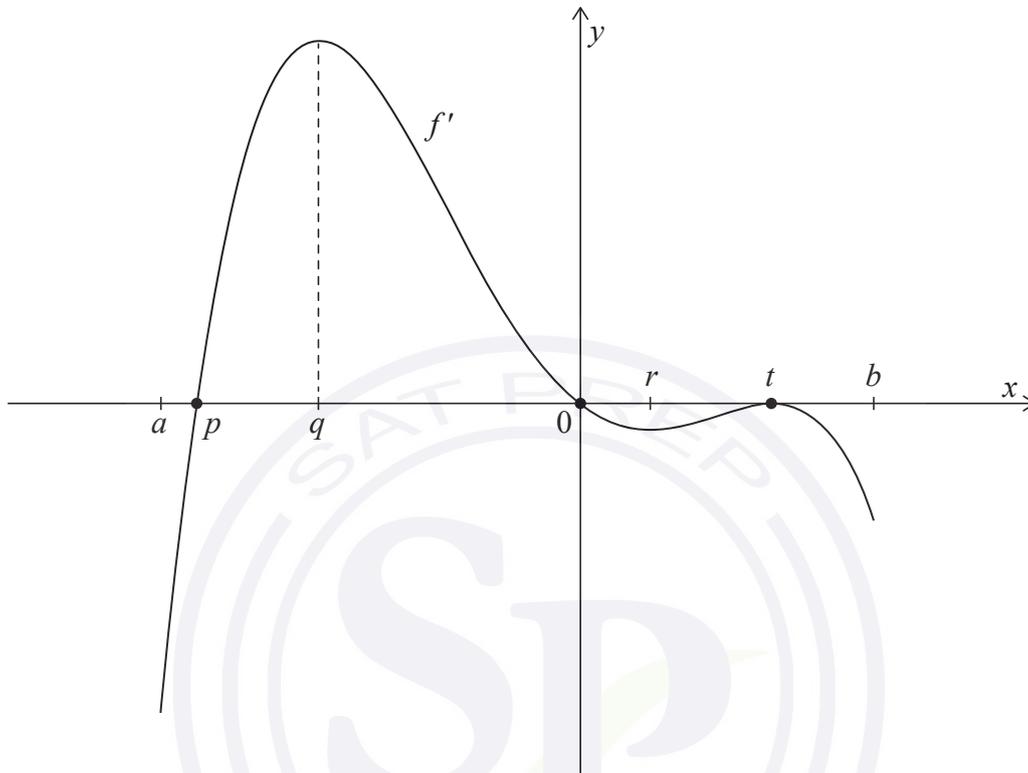
The graph of f', the derivative of f, has x-intercepts at $x = p$, $x = 0$ and $x = t$. There are local maximum points at $x = q$ and $x = t$ and a local minimum point at $x = r$.

(a) Find all the values of x where the graph of f is increasing. Justify your answer. [2]

(b) Find the value of x where the graph of f has a local maximum. [1]

(c) (i) Find the value of x where the graph of f has a local minimum. Justify your answer.

   (ii) Find the values of x where the graph of f has points of inflexion. Justify your answer. [5]

(d) The total area of the region enclosed by the graph of f', the derivative of f, and the x-axis is 20.

Given that $f(p) + f(t) = 4$, find the value of $f(0)$. [6]

References:

12EP10

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: analysis and approaches
Standard level
Paper 1

Thursday 6 May 2021 (afternoon)

1 hour 30 minutes

Candidate session number

Instructions to candidates

- Write your session number in the boxes above.
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- You are not permitted access to any calculator for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
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- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

12EP01

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Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 6]

The following diagram shows a circle with centre O and radius r.

diagram not to scale
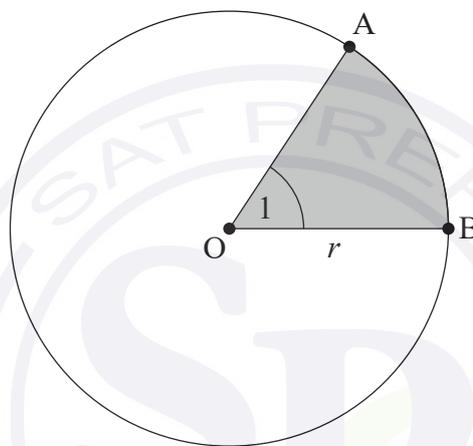
Points A and B lie on the circumference of the circle, and $A\hat{O}B = 1$ radian.

The perimeter of the shaded region is 12.

(a) Find the value of r. [3]

(b) Hence, find the exact area of the **non-shaded** region. [3]

2. [Maximum mark: 4]

Consider two consecutive positive integers, n and $n + 1$.

Show that the difference of their squares is equal to the sum of the two integers.

3. [Maximum mark: 6]

(a) Show that the equation $2\cos^2 x + 5\sin x = 4$ may be written in the form $2\sin^2 x - 5\sin x + 2 = 0$. [1]

(b) Hence, solve the equation $2\cos^2 x + 5\sin x = 4$, $0 \le x \le 2\pi$. [5]

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4. [Maximum mark: 5]

In the expansion of $(x + k)^7$, where $k \in \mathbb{R}$, the coefficient of the term in x^5 is 63.
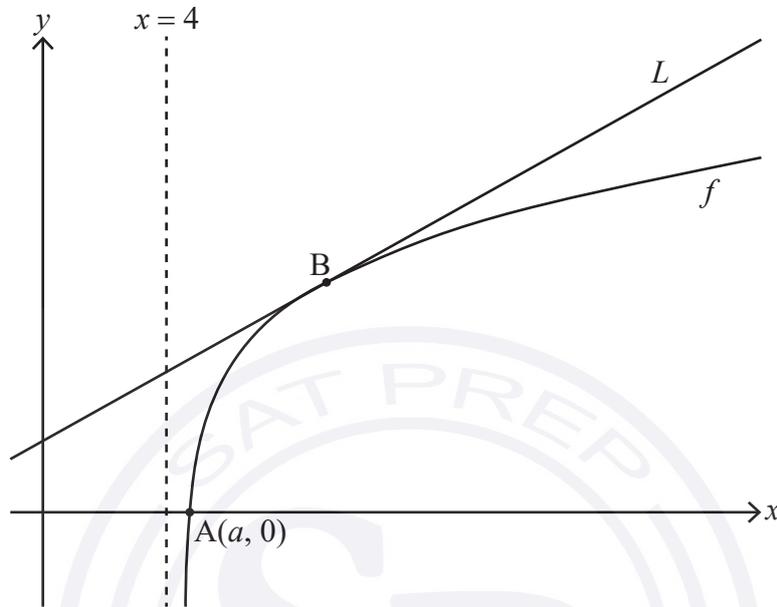
Find the possible values of k.

5. [Maximum mark: 9]

Consider the function f defined by $f(x) = \ln(x^2 - 16)$ for $x > 4$.

The following diagram shows part of the graph of f which crosses the x-axis at point A, with coordinates $(a, 0)$. The line L is the tangent to the graph of f at the point B.

(a) Find the exact value of a.   [3]

(b) Given that the gradient of L is $\dfrac{1}{3}$, find the x-coordinate of B.   [6]

6. [Maximum mark: 7]

The following diagram shows triangle ABC, with $AB = 10$, $BC = x$ and $AC = 2x$.

diagram not to scale

Given that $\cos\hat{C} = \dfrac{3}{4}$, find the area of the triangle.

Give your answer in the form $\dfrac{p\sqrt{q}}{2}$ where $p, q \in \mathbb{Z}^+$.

**Turn over**

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 14]

A large school has students from Year 6 to Year 12.

A group of 80 students in Year 12 were randomly selected and surveyed to find out how many hours per week they each spend doing homework. Their results are represented by the following cumulative frequency graph.
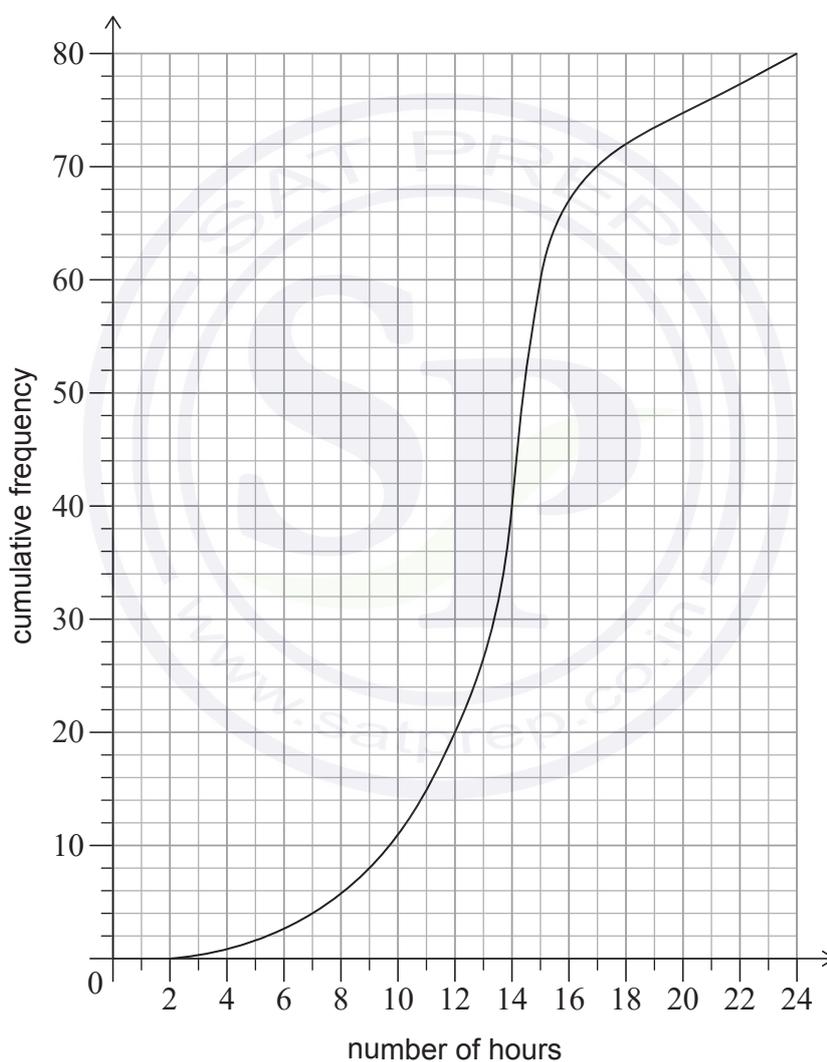
(This question continues on the following page)

Do **not** write solutions on this page.

(Question 7 continued)

(a) Find the median number of hours per week these Year 12 students spend doing homework. [2]

(b) Given that 10% of these Year 12 students spend more than k hours per week doing homework, find the value of k. [3]

This same information is represented by the following table.

Hours (h) spent doing homework	$2 < h \le 7$	$7 < h \le 15$	$15 < h \le 21$	$21 < h \le 24$
Frequency	4	p	16	q

(c) Find the value of p and the value of q. [4]

There are 320 students in Year 12 at this school.

(d) Estimate the number of Year 12 students that spend more than 15 hours each week doing homework. [3]

(e) (i) Explain why this sampling method might not provide an accurate representation of the amount of time **all** of the students in the school spend doing homework.

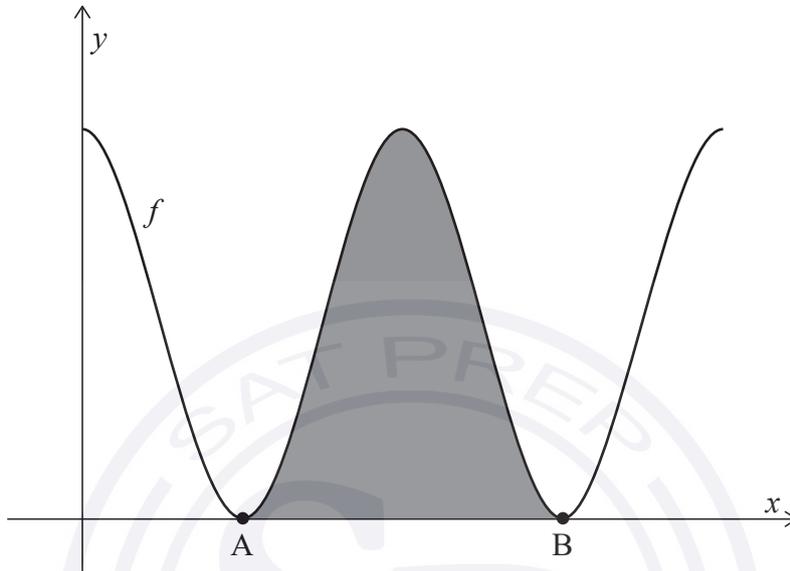(ii) Suggest a more appropriate sampling method. [2]

Do **not** write solutions on this page.

8. [Maximum mark: 15]

Consider the function f defined by $f(x) = 6 + 6\cos x$, for $0 \le x \le 4\pi$.

The following diagram shows the graph of $y = f(x)$.

The graph of f touches the x-axis at points A and B, as shown. The shaded region is enclosed by the graph of $y = f(x)$ and the x-axis, between the points A and B.

(a) Find the x-coordinates of A and B. [3]

(b) Show that the area of the shaded region is 12π. [5]

The right cone in the following diagram has a total surface area of 12π, equal to the shaded area in the previous diagram.

The cone has a base radius of 2, height h, and slant height l.

diagram not to scale
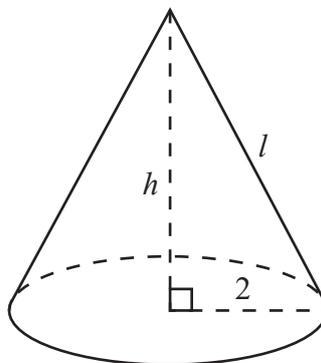
(c) Find the value of l. [3]

(d) Hence, find the volume of the cone. [4]

12EP10

Do **not** write solutions on this page.

9.   [Maximum mark: 14]

Particle A travels in a straight line such that its displacement, s metres, from a fixed origin after t seconds is given by $s(t) = 8t - t^2$, for $0 \leq t \leq 10$, as shown in the following diagram.

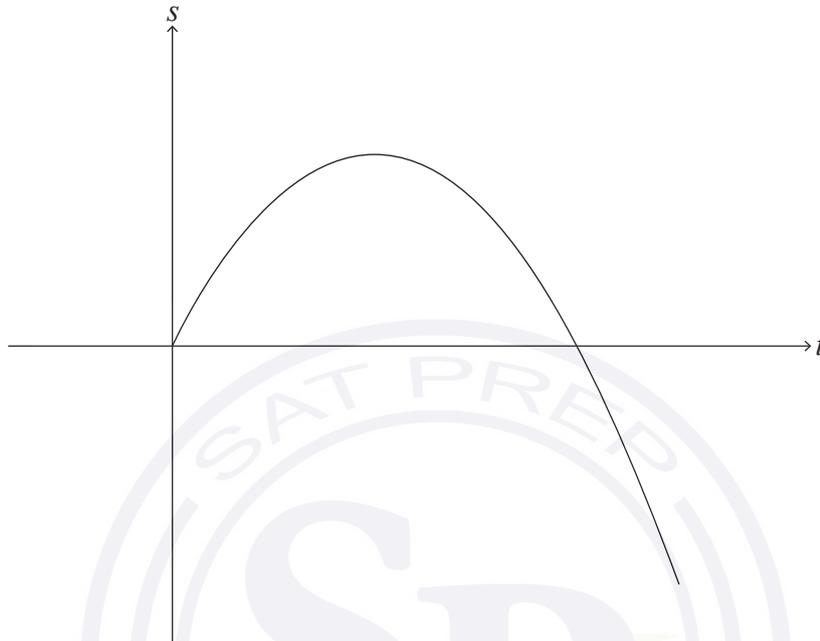Particle A starts at the origin and passes through the origin again when $t = p$.

(a)   Find the value of p.   [2]

Particle A changes direction when $t = q$.

(b)   (i)   Find the value of q.

         (ii)   Find the displacement of particle A from the origin when $t = q$.   [4]

(c)   Find the distance of particle A from the origin when $t = 10$.   [2]

The total distance travelled by particle A is given by d.

(d)   Find the value of d.   [2]

A second particle, particle B, travels along the same straight line such that its velocity is given by $v(t) = 14 - 2t$, for $t \geq 0$.

When $t = k$, the distance travelled by particle B is equal to d.

(e)   Find the value of k.   [4]

References:

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12EP11

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: analysis and approaches
Standard level
Paper 1

Thursday 6 May 2021 (afternoon)

Candidate session number

1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
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- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

12EP01

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Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 5]

The graph of $y = f(x)$ for $-4 \leq x \leq 6$ is shown in the following diagram.
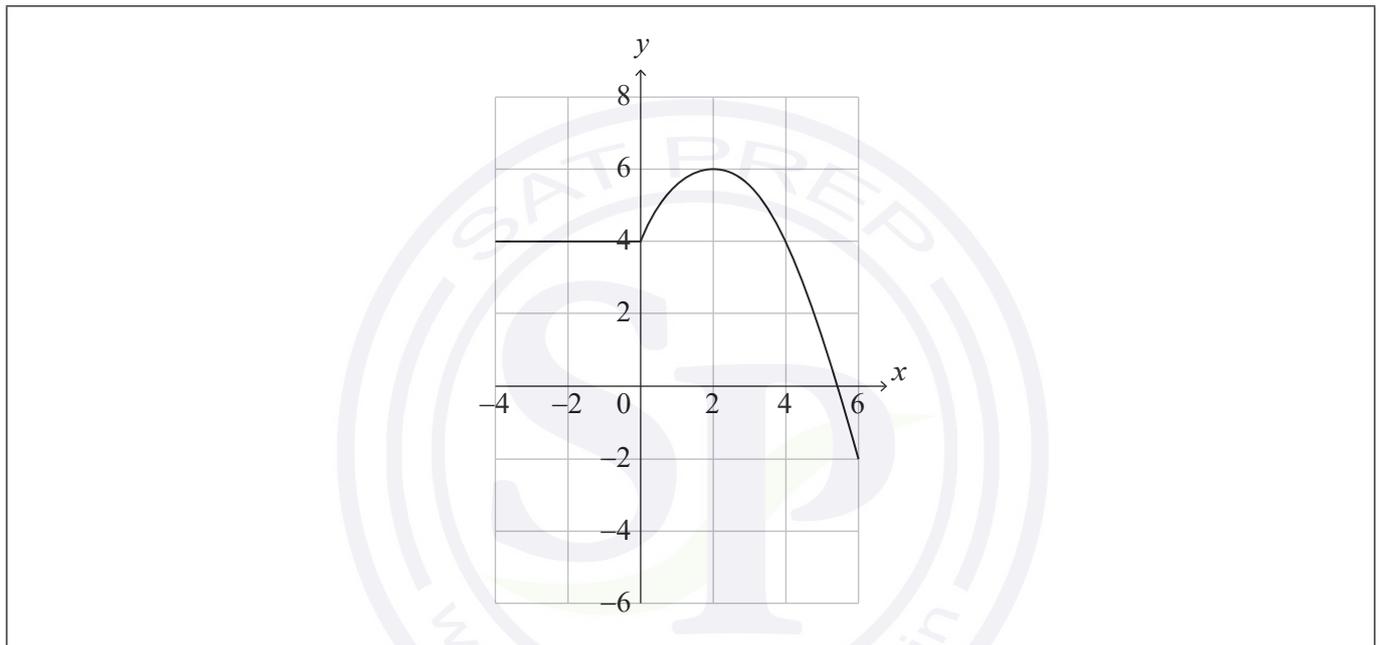
(a) Write down the value of

(i) $f(2)$;

(ii) $(f \circ f)(2)$. [2]

(b) Let $g(x) = \dfrac{1}{2}f(x) + 1$ for $-4 \leq x \leq 6$. On the axes above, sketch the graph of g. [3]

2. [Maximum mark: 4]

The diameter of a spherical planet is $6 \times 10^4 \, \text{km}$.

(a) Write down the radius of the planet. [1]

The volume of the planet can be expressed in the form $\pi(a \times 10^k) \, \text{km}^3$ where $1 \leq a < 10$ and $k \in \mathbb{Z}$.

(b) Find the value of a and the value of k. [3]

3. [Maximum mark: 5]

Consider an arithmetic sequence where $u_8 = S_8 = 8$. Find the value of the first term, u_1, and the value of the common difference, d.

4. [Maximum mark: 5]

A research student weighed lizard eggs in grams and recorded the results. The following box and whisker diagram shows a summary of the results where L and U are the lower and upper quartiles respectively.

diagram not to scale

The interquartile range is 20 grams and there are no outliers in the results.

(a) Find the minimum possible value of U. [3]

(b) Hence, find the minimum possible value of L. [2]

5.   [Maximum mark: 7]

Consider the functions $f(x) = -(x - h)^2 + 2k$ and $g(x) = e^{x-2} + k$ where h, $k \in \mathbb{R}$.

(a)   Find $f'(x)$.   [1]

The graphs of f and g have a common tangent at $x = 3$.

(b)   Show that $h = \dfrac{e + 6}{2}$.   [3]

(c)   Hence, show that $k = e + \dfrac{e^2}{4}$.   [3]

6. [Maximum mark: 8]

(a) Show that $\sin 2x + \cos 2x - 1 = 2\sin x (\cos x - \sin x)$. [2]

(b) Hence or otherwise, solve $\sin 2x + \cos 2x - 1 + \cos x - \sin x = 0$ for $0 < x < 2\pi$. [6]

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7.   [Maximum mark: 14]

Let $f(x) = mx^2 - 2mx$, where $x \in \mathbb{R}$ and $m \in \mathbb{R}$. The line $y = mx - 9$ meets the graph of f at exactly one point.

(a)   Show that $m = 4$.   [6]

The function f can be expressed in the form $f(x) = 4(x - p)(x - q)$, where p, $q \in \mathbb{R}$.

(b)   Find the value of p and the value of q.   [2]

The function f can also be expressed in the form $f(x) = 4(x - h)^2 + k$, where h, $k \in \mathbb{R}$.

(c)   Find the value of h and the value of k.   [3]

(d)   Hence find the values of x where the graph of f is both negative and increasing.   [3]

8.   [Maximum mark: 16]

Let $y = \dfrac{\ln x}{x^4}$ for $x > 0$.

(a)   Show that $\dfrac{dy}{dx} = \dfrac{1 - 4\ln x}{x^5}$.   [3]

Consider the function defined by $f(x) = \dfrac{\ln x}{x^4}$ for $x > 0$ and its graph $y = f(x)$.

(b)   The graph of f has a horizontal tangent at point P. Find the coordinates of P.   [5]

(c)   Given that $f''(x) = \dfrac{20\ln x - 9}{x^6}$, show that P is a local maximum point.   [3]

(d)   Solve $f(x) > 0$ for $x > 0$.   [2]

(e)   Sketch the graph of f, showing clearly the value of the x-intercept and the approximate position of point P.   [3]

Do **not** write solutions on this page.

9. [Maximum mark: 16]

A biased four-sided die, A, is rolled. Let X be the score obtained when die A is rolled. The probability distribution for X is given in the following table.

x	1	2	3	4
$P(X=x)$	p	p	p	$\frac{1}{2}p$

(a) Find the value of p. [2]

(b) Hence, find the value of $E(X)$. [2]

A second biased four-sided die, B, is rolled. Let Y be the score obtained when die B is rolled. The probability distribution for Y is given in the following table.

y	1	2	3	4
$P(Y=y)$	q	q	q	r

(c) (i) State the range of possible values of r.

 (ii) Hence, find the range of possible values of q. [3]

(d) Hence, find the range of possible values for $E(Y)$. [3]

Agnes and Barbara play a game using these dice. Agnes rolls die A once and Barbara rolls die B once. The probability that Agnes' score is less than Barbara's score is $\frac{1}{2}$.

(e) Find the value of $E(Y)$. [6]

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: analysis and approaches
Standard level
Paper 1

Specimen

1 hour 30 minutes

Candidate session number

Instructions to candidates

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- You are not permitted access to any calculator for this paper.
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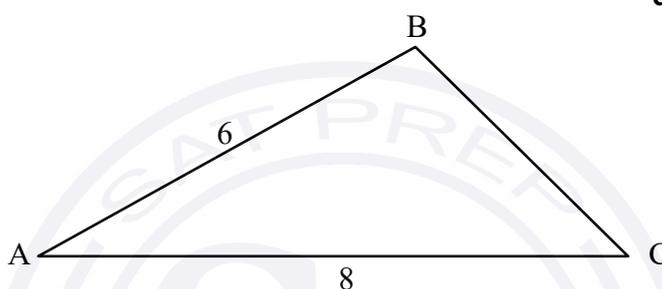
Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 5]

The following diagram shows triangle ABC, with $AB = 6$ and $AC = 8$.

diagram not to scale

(a) Given that $\cos \hat{A} = \dfrac{5}{6}$, find the value of $\sin \hat{A}$. [3]

(b) Find the area of triangle ABC. [2]

2. [Maximum mark: 5]

Let A and B be events such that $P(A) = 0.5$, $P(B) = 0.4$ and $P(A \cup B) = 0.6$.
Find $P(A \mid B)$.

3.　[Maximum mark: 5]

(a)　Show that $(2n - 1)^2 + (2n + 1)^2 = 8n^2 + 2$, where $n \in \mathbb{Z}$.　　　　[2]

(b)　Hence, or otherwise, prove that the sum of the squares of any two consecutive odd integers is even.　　　　[3]

4.   [Maximum mark: 5]

Let $f'(x) = \dfrac{8x}{\sqrt{2x^2 + 1}}$. Given that $f(0) = 5$, find $f(x)$.

5. [Maximum mark: 5]

The functions f and g are defined such that $f(x) = \dfrac{x+3}{4}$ and $g(x) = 8x + 5$.

(a) Show that $(g \circ f)(x) = 2x + 11$. [2]

(b) Given that $(g \circ f)^{-1}(a) = 4$, find the value of a. [3]

6. [Maximum mark: 8]

(a) Show that $\log_9 (\cos 2x + 2) = \log_3 \sqrt{\cos 2x + 2}$. [3]

(b) Hence or otherwise solve $\log_3 (2 \sin x) = \log_9 (\cos 2x + 2)$ for $0 < x < \dfrac{\pi}{2}$. [5]

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7.   [Maximum mark: 15]

A large company surveyed 160 of its employees to find out how much time they spend traveling to work on a given day. The results of the survey are shown in the following cumulative frequency diagram.
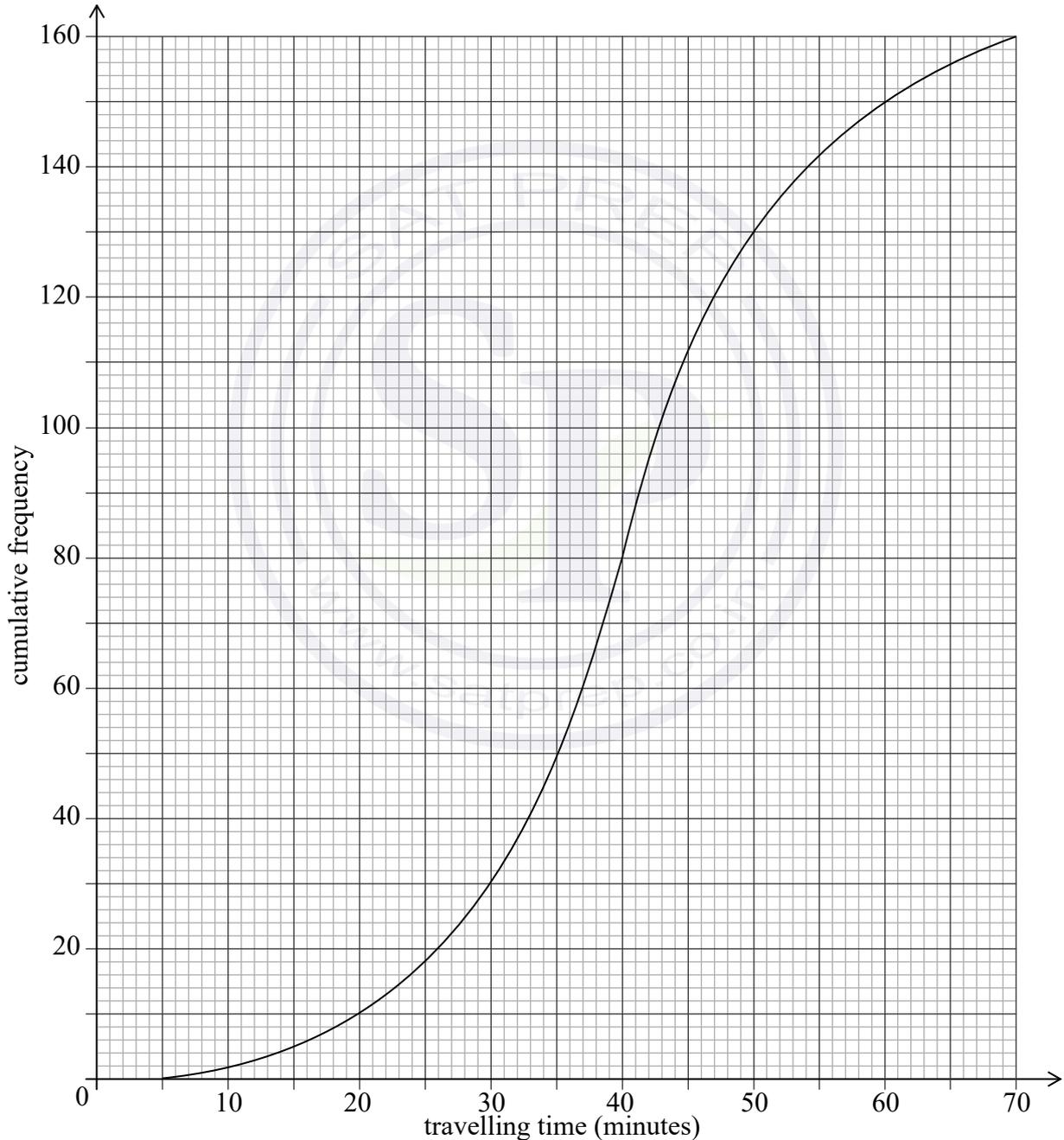
(This question continues on the following page)

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(Question 7 continued)

    (a)   Find the median number of minutes spent traveling to work.   [2]

    (b)   Find the number of employees whose travelling time is within 15 minutes of the median.   [3]

Only 10% of the employees spent more than k minutes traveling to work.

    (c)   Find the value of k.   [3]

The results of the survey can also be displayed on the following box-and-whisker diagram.

<center>travelling times (minutes)</center>
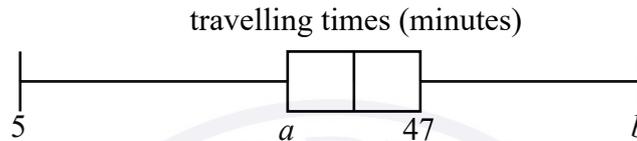
    (d)   Write down the value of b.   [1]

    (e)   (i)   Find the value of a.

            (ii)   Hence, find the interquartile range.   [4]

Travelling times of less than p minutes are considered outliers.

    (f)   Find the value of p.   [2]

8.   [Maximum mark: 16]

Let $f(x) = \dfrac{1}{3}x^3 + x^2 - 15x + 17$.

    (a)   Find $f'(x)$.   [2]

The graph of f has horizontal tangents at the points where $x = a$ and $x = b$, $a < b$.

    (b)   Find the value of a and the value of b.   [3]

    (c)   (i)   Sketch the graph of $y = f'(x)$.

            (ii)   Hence explain why the graph of f has a local maximum point at $x = a$.   [2]

    (d)   (i)   Find $f''(b)$.

            (ii)   Hence, use your answer to part (d)(i) to show that the graph of f has a local
                  minimum point at $x = b$.   [4]

The normal to the graph of f at $x = a$ and the tangent to the graph of f at $x = b$ intersect at
the point (p, q).

    (e)   Find the value of p and the value of q.   [5]

Do **not** write solutions on this page.

9. [Maximum mark: 16]

Let $f(x) = \dfrac{\ln 5x}{kx}$ where $x > 0$, $k \in \mathbb{R}^+$.

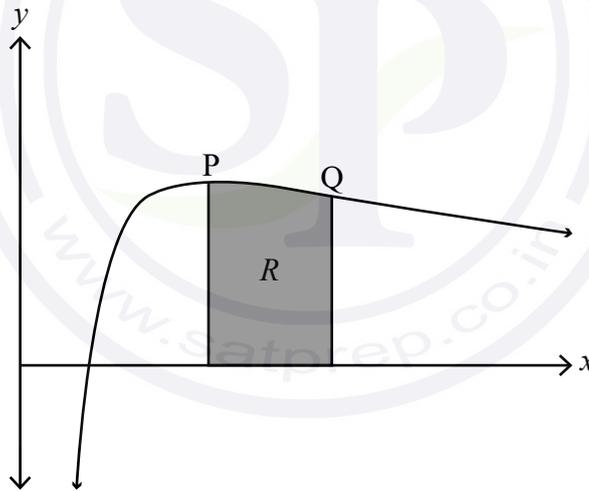(a) Show that $f'(x) = \dfrac{1 - \ln 5x}{kx^2}$. [3]

The graph of f has exactly one maximum point P.

(b) Find the x-coordinate of P. [3]

The second derivative of f is given by $f''(x) = \dfrac{2\ln 5x - 3}{kx^3}$. The graph of f has exactly one point of inflexion Q.

(c) Show that the x-coordinate of Q is $\dfrac{1}{5}e^{\frac{3}{2}}$. [3]

The region R is enclosed by the graph of f, the x-axis, and the vertical lines through the maximum point P and the point of inflexion Q.

(d) Given that the area of R is 3, find the value of k. [7]