8823 – 7114M

Markscheme

November 2023

Mathematics: analysis and approaches

Standard level

Paper 1 TZ2

19 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

R Marks awarded for clear **Reasoning**.

AG Answer given in the question and so no marks are awarded.

FT Follow through. The practice of awarding marks, despite candidate errors in previous parts, for their correct methods/answers using incorrect results.

Using the markscheme

1 General

Award marks using the annotations as noted in the markscheme *eg* ***M1, A2***.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***A3***, ***M2*** *etc*., do **not** split the marks, unless there is a note.
- The response to a "show that" question does not need to restate the ***AG*** line, unless a **Note** makes this explicit in the markscheme.
- Once a correct answer to a question or part question is seen, ignore further working even if this working is incorrect and/or suggests a misunderstanding of the question. This will encourage a uniform approach to marking, with less examiner discretion. Although some candidates may be advantaged for that specific question item, it is likely that these candidates will lose marks elsewhere too.
- An exception to the previous rule is when an incorrect answer from further working is used **in a subsequent part**. For example, when a correct exact value is followed by an incorrect decimal approximation in the first part and this approximation is then used in the second part. In this situation, award ***FT*** marks as appropriate but do not award the final ***A1*** in the first part.

Examples:

	Correct answer seen	Further working seen	Any FT issues?	Action
1.	$8\sqrt{2}$	$5.65685...$ *(incorrect decimal value)*	No. Last part in question.	Award ***A1*** for the final mark *(condone the incorrect further working)*
2.	$\frac{35}{72}$	0.468111… (incorrect decimal value)	Yes. Value is used in subsequent parts.	Award ***A0*** for the final mark *(and full **FT** is available in subsequent parts)*

3 Implied marks

Implied marks appear in **brackets e.g. *(M1)*** and can only be awarded if **correct** work is seen or implied by subsequent working/answer.

4 Follow through marks (only applied after an error is made)

Follow through (***FT***) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) (e.g. incorrect value from part (a) used in part (d) or incorrect value from part (c)(i) used in part (c)(ii)). Usually, to award ***FT*** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if all the marks awarded in a subsequent part are for the answer or are implied, then ***FT*** marks should be awarded for *their* correct answer, even when working is not present.

For example: following an incorrect answer to part (a) that is used in subsequent parts, where the markscheme for the subsequent part is ***(M1)A1***, it is possible to award full marks for *their* correct answer, **without working being seen.** For longer questions where all but the answer marks are implied this rule applies but may be overwritten by a **Note** in the Markscheme.

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks, by reflecting on what each mark is for and how that maps to the simplified version.
- If the error leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word "their" in a description, to indicate that candidates may be using an incorrect value.
- If the candidate's answer to the initial question clearly contradicts information given in the question, it is not appropriate to award any ***FT*** marks in the subsequent parts. This includes when candidates fail to complete a "show that" question correctly, and then in subsequent parts use their incorrect answer rather than the given value.
- Exceptions to these ***FT*** rules will be explicitly noted on the markscheme.
- If a candidate makes an error in one part but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the command term was "Hence".

5 Mis-read

If a candidate incorrectly copies values or information from the question, this is a mis-read (***MR***). A candidate should be penalized only once for a particular misread. Use the ***MR*** stamp to indicate that this has been a misread and do not award the first mark, even if this is an ***M*** mark, but award all others as appropriate.

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.
- If a candidate uses a correct answer, to a "show that" question, to a higher degree of accuracy than given in the question, this is NOT a misread and full marks may be scored in the subsequent part.
- ***MR*** can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If the command term is 'Hence' and not 'Hence or otherwise' then alternative methods are not permitted unless covered by a note in the mark scheme.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**.

7 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation** for example 1.9 and 1,9 or 1000 and 1,000 and 1.000.
- Do not accept final answers written using calculator notation. However, ***M*** marks and intermediate ***A*** marks can be scored, when presented using calculator notation, provided the evidence clearly reflects the demand of the mark.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, some **equivalent** answers will generally appear in brackets. Not all equivalent notations/answers/methods will be presented in the markscheme and examiners are asked to apply appropriate discretion to judge if the candidate work is equivalent.

8 Format and accuracy of answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. If the level of accuracy is not stated in the question, the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures.*

Where values are used in subsequent parts, the markscheme will generally use the exact value, however candidates may also use the correct answer to 3 sf in subsequent parts. The markscheme will often explicitly include the subsequent values that come "*from the use of 3 sf values*".

Simplification of final answers: Candidates are advised to give final answers using good mathematical form. In general, for an ***A*** mark to be awarded, arithmetic should be completed, and any values that lead to integers should be simplified; for example, $\sqrt{\frac{25}{4}}$ should be written as $\frac{5}{2}$. An exception to this is simplifying fractions, where lowest form is not required (although the numerator and the denominator must be integers); for example, $\frac{10}{4}$ may be left in this form or written as $\frac{5}{2}$. However, $\frac{10}{5}$ should be written as 2, as it simplifies to an integer.

Algebraic expressions should be simplified by completing any operations such as addition and multiplication, e.g. $4e^{2x} \times e^{3x}$ should be simplified to $4e^{5x}$, and $4e^{2x} \times e^{3x} - e^{4x} \times e^{x}$ should be simplified to $3e^{5x}$. Unless specified in the question, expressions do not need to be factorized, nor do factorized expressions need to be expanded, so $x(x+1)$ and $x^2 + x$ are both acceptable.

Please note: intermediate ***A*** marks do NOT need to be simplified.

9 Calculators

No calculator is allowed. The use of any calculator on this paper is malpractice and will result in no grade awarded. If you see work that suggests a candidate has used any calculator, please follow the procedures for malpractice.

10. Presentation of candidate work

Crossed out work: If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work unless an explicit note from the candidate indicates that they would like the work to be marked.

More than one solution: Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise. If the layout of the responses makes it difficult to judge, examiners should apply appropriate discretion to judge which is "first".

Section A

1. (a) $a=5$ ***A1***

[1 mark]

(b) (i) period = π ***A1***

(ii) $b=\frac{2\pi}{\pi}$ OR $\pi=\frac{2\pi}{b}$ ***(A1)***

$=2$ ***A1***

[3 marks]

(c) substituting $\frac{\pi}{6}$ into their $f(x)$ ***(M1)***

$f\left(\frac{\pi}{6}\right)=5\cos\left(\frac{\pi}{3}\right)$

$\cos\left(\frac{\pi}{3}\right)=\frac{1}{2}$ ***(A1)***

$=\frac{5}{2}$ ***A1***

[3 marks]

Total [7 marks]

2. (a) attempt to form $(g \circ f)(x)$ ***(M1)***

$((g \circ f)(x)) = (x-3)^2 + k^2 \quad (= x^2 - 6x + 9 + k^2)$ ***A1***

[2 marks]

(b) substituting $x = 2$ into their $(g \circ f)(x)$ and setting their expression $= 10$ ***(M1)***

$(2-3)^2 + k^2 = 10$ OR $2^2 - 6(2) + 9 + k^2 = 10$

$k^2 = 9$ ***(A1)***

$k = \pm 3$ ***A1***

[3 marks]

Total [5 marks]

3. (a) $(\mathrm{P}(A \cup B) =) 0.65 + 0.75 - 0.6$ OR $0.05 + 0.6 + 0.15$ ***(A1)***

$= 0.8$ ***A1***

[2 marks]

(b) recognition that $A' \cap B' = (A \cup B)'$ OR $A' \cap B' = 1 - A \cup B$
(region/value may be seen in a correctly shaded/labeled Venn diagram) ***(M1)***

$(= 1 - 0.8)$

$= 0.2$ ***A1***

Note: For the final mark, 0.2 must be stated as the candidate's answer, or labeled as $\mathrm{P}(A' \cap B')$ in their Venn diagram. Just seeing an unlabeled 0.2 in the correct region of their diagram earns ***M1A0***.

[2 marks]

Total [4 marks]

4. (a) **METHOD 1**

attempt to form at least one equation, using either S_4 or S_5 ***(M1)***

$65 = 25p - 5q \quad (13 = 5p - q)$ **and** $40 = 16p - 4q \quad (10 = 4p - q)$ ***(A1)***

valid attempt to solve simultaneous linear equations in p and q by substituting or eliminating one of the variables. ***(M1)***

$p = 3$, $q = 2$ ***A1A1***

Note: If candidate does not explicitly state their values of p and q, but gives $S_n = 3n^2 - 2n$, award final two marks as ***A1A0***.

METHOD 2

attempt to form at least one equation, using either S_4 or S_5 ***(M1)***

$65 = \frac{5}{2}(2u_1 + 4d) \quad (26 = 2u_1 + 4d)$ **and** $40 = 2(2u_1 + 3d) \quad (20 = 2u_1 + 3d)$ ***(A1)***

valid attempt to solve simultaneous linear equations in u_1 and d by substituting or eliminating one of the variables. ***(M1)***

$u_1 = 1$, $d = 6$ ***A1***

$S_n = \frac{n}{2}(2 + 6(n-1)) = 3n^2 - 2n$

$p = 3$ and $q = 2$ ***A1***

Note: If candidate does not explicitly state their values of p and q, do not award the final mark.

[5 marks]

(b) $u_5 = S_5 - S_4$ OR substituting their values of u_1 and d into $u_5 = u_1 + 4d$

OR substituting their value of u_1 into $65 = \frac{5}{2}(u_1 + u_5)$ ***(M1)***

$(u_5 =) 65 - 40$ OR $(u_5 =) 1 + 4 \times 6$ OR $65 = \frac{5}{2}(1 + u_5)$

$= 25$ ***A1***

[2 marks]

Total [7 marks]

5. METHOD 1

EITHER

attempt to use Pythagoras' theorem in a right-angled triangle. ***(M1)***

$\left(\sqrt{5^2 - 1^2} =\right)\sqrt{24}$ ***(A1)***

OR

attempt to use the Pythagorean identity $\cos^2\alpha + \sin^2\alpha = 1$ ***(M1)***

$\sin^2 \mathrm{B\hat{A}C} = 1 - \left(\frac{1}{5}\right)^2$ ***(A1)***

THEN

$\sin \mathrm{B\hat{A}C} = \frac{\sqrt{24}}{5}$ (may be seen in area formula) ***A1***

attempt to use 'Area $= \frac{1}{2}ab\sin C$' (must include their calculated value of $\sin \mathrm{B\hat{A}C}$) ***(M1)***

$= \frac{1}{2} \times 10 \times \sqrt{6} \times \frac{\sqrt{24}}{5}$ ***(A1)***

$= 12$ (cm²) ***A1***

[6 marks]

continued…

Question 5 continued

METHOD 2

attempt to find perpendicular height of triangle BAC ***(M1)***

EITHER

$\text{height} = \sqrt{6} \times \sin \text{B}\hat{\text{A}}\text{C}$

attempt to use the Pythagorean identity $\cos^2 \alpha + \sin^2 \alpha = 1$ ***(M1)***

$\text{height} = \sqrt{6} \times \sqrt{1 - \left(\frac{1}{5}\right)^2}$ ***(A1)***

$= \sqrt{6} \times \frac{\sqrt{24}}{5} \left(= \frac{12}{5} \right)$ ***A1***

OR

$\text{adjacent} = \frac{\sqrt{6}}{5}$ ***(A1)***

attempt to use Pythagoras' theorem in a right-angled triangle. ***(M1)***

$\text{height} = \sqrt{6 - \frac{6}{25}} \left(= \frac{12}{5} \right)$ (may be seen in area formula) ***(A1)***

THEN

attempt to use 'Area $= \frac{1}{2} \text{base} \times \text{height}$' (must include their calculated value for height) ***(M1)***

$= \frac{1}{2} \times 10 \times \frac{12}{5}$

$= 12$ (cm^2) ***A1***

[6 marks]

6. attempt to apply binomial expansion ***(M1)***

$(1+kx)^n = 1 + {}^nC_1 kx + {}^nC_2 k^2 x^2 + \ldots$ OR ${}^nC_1 k = 12$ OR ${}^nC_2 = 28$

$nk = 12$ ***(A1)***

$\frac{n(n-1)}{2} = 28$ OR $\frac{n!}{(n-2)!2!} = 28$ ***(A1)***

$n^2 - n - 56 = 0$ OR $n(n-1) = 56$

valid attempt to solve ***(M1)***

$(n-8)(n+7) = 0$ OR $8(8-1) = 56$ OR finding correct value in Pascal's triangle

$\Rightarrow n = 8$ ***A1***

$\Rightarrow k = \frac{3}{2}$ ***A1***

Note: If candidate finds $n = 8$ with no working shown, award ***M1A0A0M1A1A0.***
If candidate finds $n = 8$ and $k = \frac{3}{2}$ with no working shown, award ***M1A0A0M1A1A1***.

[6 marks]

Section B

7. (a) (i) $p = 9$ ***A1***

(ii) $600 < n \leq 800$ ***A1***

Note: Award ***A0*** if candidate answers 700.

[2 marks]

(b) (i) median = 600 ***A1***

(ii) $80\% \text{ of } 800 = 640$ ***(A1)***

40 (performances less than 80% of tickets sold) ***(A1)***

20 (performances) ***A1***

[4 marks]

(c) (i) any reasonable answer which suggests a biased sample (must include reason, do not accept reasons such as "sample size is too small", or answers that simply say "not representative of entire audience" without a valid reason) ***A1***

eg likely to come from the same part of the theatre OR be part of same group OR be from priority seating OR it is convenience sampling

(ii) every 20^{th} person ***A1A1***

Note: Award ***A1*** for recognizing that sampling occurs at regular intervals eg "every". Award ***A1*** for interval length is 20.

(iii) quota (sampling method) ***A1***

[4 marks]

continued…

Question 7 continued

(d) (i) 75% (of 36000 spent between \$3 and \$25) ***(M1)***

= 27000 ***A1***

(ii) $a = 7$ ***A1***

[3 marks]

(e) (i) **METHOD 1**

old mean is 600 (tickets) ***(A1)***

recognising new mean is old mean + 17 ***(M1)***

$600 + 17$

$= 617$ (tickets) ***A1***

METHOD 2

new total number of tickets = $36000 + 60 \times 17 (= 37020)$ ***(A1)***

new mean = $\frac{36000 + 60 \times 17}{60} \left(= \frac{37020}{60}\right)$ ***(M1)***

$= 617$ (tickets) ***A1***

(ii) no effect on the variance ***A1***

[4 marks]

Total [17 marks]

8. (a) $x = 0$ ***A1***

[1 mark]

(b) (i) setting $\ln(2x-9) = 2\ln x - \ln d$ ***M1***

attempt to use power rule ***(M1)***

$2\ln x = \ln x^2$ (seen anywhere)

attempt to use product/quotient rule for logs ***(M1)***

$\ln(2x-9) = \ln\dfrac{x^2}{d}$ OR $\ln\dfrac{x^2}{2x-9} = \ln d$ OR $\ln(2x-9)d = \ln x^2$

$\dfrac{x^2}{d} = 2x-9$ OR $\dfrac{x^2}{2x-9} = d$ OR $(2x-9)d = x^2$ ***A1***

$x^2 - 2dx + 9d = 0$ ***AG***

(ii) discriminant = $(-2d)^2 - 4\times1\times9d$ ***(A1)***

recognizing discriminant > 0 ***(M1)***

$(-2d)^2 - 4\times1\times9d > 0$ OR $(2d)^2 - 4\times9d > 0$ OR $4d^2 - 36d > 0$ ***A1***

$d^2 - 9d > 0$ ***AG***

(iii) setting $d(d-9) > 0$ OR $d(d-9) = 0$ OR sketch graph
OR sign test OR $d^2 > 9d$ ***(M1)***

$d < 0$ or $d > 9$, but $d \in \mathbb{R}^+$

$d > 9$ (or $]9, \infty[$) ***A1***

[9 marks]

continued…

Question 8 continued

(c) $x^2 - 20x + 90 \ (= 0)$ ***A1***

attempting to solve their 3 term quadratic equation ***(M1)***

$\left((x-10)^2 - 10 = 0\right)$ or $\left((x =) \dfrac{20 \pm \sqrt{(-20)^2 - 4 \times 1 \times 90}}{2}\right)$

$x = 10 - \sqrt{10} \, (= p)$ or $x = 10 + \sqrt{10} \, (= q)$ ***(A1)***

subtracting their values of x ***(M1)***

distance $= 2\sqrt{10}$ ***A1***

$(a = 2, b = 10)$

Note: Accept $1\sqrt{40}$ OR $\sqrt{40}$.

[5 marks]

Total [15 marks]

9. (a) attempt to use either the quotient or product rule ***(M1)***

$$\frac{8\left(x^2+1\right)^3-8x\times 3\times 2x\left(x^2+1\right)^2}{\left(x^2+1\right)^6} \quad \text{OR} \quad 8\left(x^2+1\right)^{-3}+8x\times(-3)\times 2x\left(x^2+1\right)^{-4}$$ ***A1A1***

Note: Award ***A1*** for correctly applying chain rule to $\left(x^2+1\right)^3$ and ***A1*** for everything else correct.

$$=\frac{8\left(x^2+1-6x^2\right)}{\left(x^2+1\right)^4} \text{ OR } \frac{8\left(x^2+1\right)^2\left(x^2+1-6x^2\right)}{\left(x^2+1\right)^6} \text{ OR } \frac{8-40x^2}{\left(x^2+1\right)^4} \text{ OR } \frac{8x^2+8-48x^2}{\left(x^2+1\right)^4}$$ ***A1***

$$=\frac{8\left(1-5x^2\right)}{\left(x^2+1\right)^4}$$ ***AG***

[4 marks]

continued…

Question 9 continued

(b) **EITHER**

attempts to integrate by substitution using $u = x^2 + 1$ or $u = x^2$ ***(M1)***

$u = x^2 + 1 \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = 2x$ OR $u = x^2 \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = 2x$

Note: If candidate simply states $u = x^2 + 1$ or $u = x^2$, but does not attempt to substitute into their integral, do not award the ***(M1)***.

$\int \frac{8x}{\left(x^2+1\right)^3}\mathrm{d}x = \int \frac{4}{u^3}\mathrm{d}u$ OR $\int \frac{8x}{\left(x^2+1\right)^3}\mathrm{d}x = \int \frac{4}{\left(u+1\right)^3}\mathrm{d}u$ ***(A1)***

= $-2u^{-2}(+c)$ OR $-2\left(u+1\right)^{-2}(+c)$ ***(A1)***

OR

attempts to apply integration by inspection ***(M1)***

$4\int \frac{2x}{\left(x^2+1\right)^3}\mathrm{d}x$

$= 4 \times \left(-\frac{1}{2}\right)\left(x^2+1\right)^{-2}(+c)$ ***(A1)(A1)***

Note: Award ***A1*** for correct power of $\left(x^2+1\right)$ and ***A1*** for $-\frac{1}{2}$.

THEN

$-2\left(x^2+1\right)^{-2} + c$ OR $-\frac{2}{\left(x^2+1\right)^2} + c$ (final answer must include $+c$) ***A1***

[4 marks]

continued…

Question 9 continued

(c) recognizing $g'(x) = f'(x) \Rightarrow g(x) = f(x) + k$ (may be seen in diagram/drawing) ***A1***

area of *R* is given by subtracting functions *f* and *g* in integral(s) ***(M1)***

$\pm\int_0^3 k\,\mathrm{d}x$ OR $=\int_0^3 |g - f|\,\mathrm{d}x$ OR $\int_0^3 f(x) + k$ - $f(x)\,\mathrm{d}x$ OR $\int_0^3 f(x)\,\mathrm{d}x - \int_0^3 g(x)\,\mathrm{d}x$

$= \pm[kx]_0^3$ OR $\left[-\frac{2}{(x^2+1)^2} + kx\right]_0^3 - \left[-\frac{2}{(x^2+1)^2}\right]_0^3$ OR $\left[-\frac{2}{(x^2+1)^2}\right]_0^3 - \left[-\frac{2}{(x^2+1)^2} + kx\right]_0^3$ ***(A1)***

$\pm 3k = \frac{27}{2}$ ***(A1)***

$k = \pm\frac{27}{6}\left(= \pm\frac{9}{2} = \pm 4.5\right)$

$g(x) = \frac{8x}{(x^2+1)^3} - \frac{9}{2}$ AND $g(x) = \frac{8x}{(x^2+1)^3} + \frac{9}{2}$ (accept $f(x) + \frac{9}{2}$ AND $f(x) - \frac{9}{2}$) ***A1***

[5 marks]

Total [13 marks]

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Follow through (***FT***) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) (e.g. incorrect value from part (a) used in part (d) or incorrect value from part (c)(i) used in part (c)(ii)). Usually, to award ***FT*** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if all the marks awarded in a subsequent part are for the answer or are implied, then ***FT*** marks should be awarded for *their* correct answer, even when working is not present.

For example: following an incorrect answer to part (a) that is used in subsequent parts, where the markscheme for the subsequent part is ***(M1)A1***, it is possible to award full marks for *their* correct answer, **without working being seen.** For longer questions where all but the answer marks are implied this rule applies but may be overwritten by a **Note** in the Markscheme.

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks, by reflecting on what each mark is for and how that maps to the simplified version.
- If the error leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word "their" in a description, to indicate that candidates may be using an incorrect value.
- If the candidate's answer to the initial question clearly contradicts information given in the question, it is not appropriate to award any ***FT*** marks in the subsequent parts. This includes when candidates fail to complete a "show that" question correctly, and then in subsequent parts use their incorrect answer rather than the given value.
- Exceptions to these ***FT*** rules will be explicitly noted on the markscheme.
- If a candidate makes an error in one part but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the command term was "Hence".

5 Mis-read

If a candidate incorrectly copies values or information from the question, this is a mis-read (***MR***). A candidate should be penalized only once for a particular misread. Use the ***MR*** stamp to indicate that this has been a misread and do not award the first mark, even if this is an ***M*** mark, but award all others as appropriate.

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.

- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).

- Miscopying of candidates' own work does **not** constitute a misread, it is an error.

- If a candidate uses a correct answer, to a "show that" question, to a higher degree of accuracy than given in the question, this is NOT a misread and full marks may be scored in the subsequent part.

- ***MR*** can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If the command term is 'Hence' and not 'Hence or otherwise' then alternative methods are not permitted unless covered by a note in the mark scheme.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**.

7 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation** for example 1.9 and 1,9 or 1000 and 1,000 and 1.000.

- Do not accept final answers written using calculator notation. However, ***M*** marks and intermediate ***A*** marks can be scored, when presented using calculator notation, provided the evidence clearly reflects the demand of the mark.

- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.

- In the markscheme, some **equivalent** answers will generally appear in brackets. Not all equivalent notations/answers/methods will be presented in the markscheme and examiners are asked to apply appropriate discretion to judge if the candidate work is equivalent.

8 Format and accuracy of answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. If the level of accuracy is not stated in the question, the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures.*

Where values are used in subsequent parts, the markscheme will generally use the exact value, however candidates may also use the correct answer to 3 sf in subsequent parts. The markscheme will often explicitly include the subsequent values that come "*from the use of 3 sf values*".

Simplification of final answers: Candidates are advised to give final answers using good mathematical form. In general, for an ***A*** mark to be awarded, arithmetic should be completed, and any values that lead to integers should be simplified; for example, $\sqrt{\frac{25}{4}}$ should be written as $\frac{5}{2}$. An exception to this is simplifying fractions, where lowest form is not required (although the numerator and the denominator must be integers); for example, $\frac{10}{4}$ may be left in this form or written as $\frac{5}{2}$. However, $\frac{10}{5}$ should be written as 2, as it simplifies to an integer.

Algebraic expressions should be simplified by completing any operations such as addition and multiplication, e.g. $4e^{2x} \times e^{3x}$ should be simplified to $4e^{5x}$, and $4e^{2x} \times e^{3x} - e^{4x} \times e^{x}$ should be simplified to $3e^{5x}$. Unless specified in the question, expressions do not need to be factorized, nor do factorized expressions need to be expanded, so $x(x+1)$ and $x^2 + x$ are both acceptable.

Please note: intermediate ***A*** marks do NOT need to be simplified.

9 Calculators

No calculator is allowed. The use of any calculator on this paper is malpractice and will result in no grade awarded. If you see work that suggests a candidate has used any calculator, please follow the procedures for malpractice.

10. Presentation of candidate work

Crossed out work: If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work unless an explicit note from the candidate indicates that they would like the work to be marked.

More than one solution: Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise. If the layout of the responses makes it difficult to judge, examiners should apply appropriate discretion to judge which is "first".

Section A

1. (a) $a = 7$ ***A1***

[1 mark]

(b) (i) period = π ***A1***

(ii) $b = \frac{2\pi}{\pi}$ OR $\pi = \frac{2\pi}{b}$ ***(A1)***

$= 2$ ***A1***

[3 marks]

(c) substituting $\frac{\pi}{12}$ into their $f(x)$ ***(M1)***

$f\left(\frac{\pi}{12}\right) = 7\sin\left(\frac{\pi}{6}\right)$

$\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$ ***(A1)***

$= \frac{7}{2}$ ***A1***

[3 marks]

Total [7 marks]

2. (a) attempt to form $(g \circ f)(x)$ ***(M1)***

$((g \circ f)(x)) = (x+2)^2 - k^2 \quad (= x^2 + 4x + 4 - k^2)$ ***A1***

[2 marks]

(b) substituting $x = 4$ into their $(g \circ f)(x)$ and setting their expression $= 11$ ***(M1)***

$(4+2)^2 - k^2 = 11$ OR $4^2 + 4(4) + 4 - k^2 = 11$

$k^2 = 25$ OR $-k^2 = -25$ ***(A1)***

$k = \pm 5$ ***A1***

[3 marks]

Total [5 marks]

3. (a) $(\mathrm{P}(A \cup B) =) 0.7 + 0.75 - 0.55$ ***(A1)***

$= 0.9$ ***A1***

[2 marks]

(b) recognition that $A' \cap B' = (A \cup B)'$ OR $A' \cap B' = 1 - A \cup B$
(region/value may be seen in a correctly shaded/labeled Venn diagram) ***(M1)***

$(= 1 - 0.9)$

$= 0.1$ ***A1***

Note: For the final mark, 0.1 must be stated as the candidate's answer, or labeled as $\mathrm{P}(A' \cap B')$ in their Venn diagram. Just seeing an unlabeled 0.1 in the correct region of their diagram earns ***M1A0***.

[2 marks]

Total [4 marks]

4. (a) **METHOD 1**

attempt to form at least one equation, using either S_5 or S_6 ***(M1)***

$65 = 25p - 5q \quad (13 = 5p - q)$ **and** $96 = 36p - 6q \quad (16 = 6p - q)$ ***(A1)***

valid attempt to solve simultaneous linear equations in p and q by substituting or eliminating one of the variables. ***(M1)***

$p = 3, \quad q = 2$ ***A1A1***

Note: If candidate does not explicitly state their values of p and q, but gives $S_n = 3n^2 - 2n$, award final two marks as ***A1A0***.

METHOD 2

attempt to form at least one equation, using either S_5 or S_6 ***(M1)***

$65 = \frac{5}{2}(2u_1 + 4d) \quad (26 = 2u_1 + 4d)$ **and** $96 = 3(2u_1 + 5d) \quad (32 = 2u_1 + 5d)$ ***(A1)***

valid attempt to solve simultaneous linear equations in u_1 and d by substituting or eliminating one of the variables. ***(M1)***

$u_1 = 1, \; d = 6$ ***A1***

$S_n = \frac{n}{2}(2 + 6(n-1)) = 3n^2 - 2n$

$p = 3$ and $q = 2$ ***A1***

Note: If candidate does not explicitly state their values of p and q, do not award the final mark.

[5 marks]

(b) $u_6 = S_6 - S_5$ OR substituting their values of u_1 and d into $u_6 = u_1 + 5d$

OR substituting their value of u_1 into $96 = \frac{6}{2}(u_1 + u_6)$ ***(M1)***

$(u_6 =) 96 - 65$ OR $(u_6 =) 1 + 5 \times 6$ OR $96 = 3(1 + u_6)$

$= 31$ ***A1***

[2 marks]

Total [7 marks]

5. METHOD 1

EITHER

attempt to use Pythagoras' theorem in a right-angled triangle. ***(M1)***

$\left(\sqrt{4^2 - 1^2} =\right)\sqrt{15}$ ***(A1)***

OR

attempt to use the Pythagorean identity $\cos^2\alpha + \sin^2\alpha = 1$ ***(M1)***

$\sin^2 \mathrm{B\hat{A}C} = 1 - \left(\frac{1}{4}\right)^2$ ***(A1)***

THEN

$\sin \mathrm{B\hat{A}C} = \frac{\sqrt{15}}{4}$ (may be seen in area formula) ***A1***

attempt to use 'Area $= \frac{1}{2}ab\sin C$' (must include their calculated value of $\sin \mathrm{B\hat{A}C}$) ***(M1)***

$= \frac{1}{2} \times 16 \times \sqrt{15} \times \frac{\sqrt{15}}{4}$ ***(A1)***

$= 30$ (cm^2) ***A1***

[6 marks]

continued…

Question 5 continued

METHOD 2

attempt to find perpendicular height of triangle BAC ***(M1)***

EITHER

$\text{height} = \sqrt{15} \times \sin \text{B}\hat{\text{A}}\text{C}$

attempt to use the Pythagorean identity $\cos^2 \alpha + \sin^2 \alpha = 1$ ***(M1)***

$\text{height} = \sqrt{15} \times \sqrt{1 - \left(\frac{1}{4}\right)^2}$ ***(A1)***

$= \sqrt{15} \times \frac{\sqrt{15}}{4} \left(= \frac{15}{4}\right)$ (may be seen in area formula) ***A1***

OR

$\text{adjacent} = \frac{\sqrt{15}}{4}$ ***(A1)***

attempt to use Pythagoras' theorem in a right-angled triangle. ***(M1)***

$\text{height} = \sqrt{15 - \frac{15}{16}} \left(= \frac{15}{4}\right)$ (may be seen in area formula) ***A1***

THEN

attempt to use 'Area $= \frac{1}{2} \text{base} \times \text{height}$' (must include their calculated value for height) ***(M1)***

$= \frac{1}{2} \times 16 \times \frac{15}{4}$

$= 30$ (cm^2) ***A1***

[6 marks]

6. attempt to apply binomial expansion *(M1)*

$(1+kx)^n = 1 + {}^nC_1 kx + {}^nC_2 k^2 x^2 + \ldots$ OR ${}^nC_1 k = \frac{9}{2}$ OR ${}^nC_2 = 15$

$nk = \frac{9}{2}$ *(A1)*

$\frac{n(n-1)}{2} = 15$ OR $\frac{n!}{(n-2)!2!} = 15$ *(A1)*

$(n^2 - n - 30 = 0)$ OR $n(n-1) = 30$

valid attempt to solve (***M1)***

$(n-6)(n+5) = 0$ OR $6(6-1) = 30$ OR finding correct value in Pascal's triangle

$\Rightarrow n = 6$ ***A1***

$\Rightarrow k = \frac{3}{4}$ ***A1***

Note: If candidate finds $n = 6$ with no working shown, award ***M1A0A0M1A1A0.***
If candidate finds $n = 6$ and $k = \frac{3}{4}$ with no working shown, award ***M1A0A0M1A1A1***.

[6 marks]

Section B

7. (a) (i) $p = 9$ ***A1***

(ii) $600 < n \leq 800$ ***A1***

Note: Award ***A0*** if candidate answers 700.

[2 marks]

(b) (i) median = 600 ***A1***

(ii) $80\%\text{ of }800 = 640$ ***(A1)***

40 (performances less than 80% of tickets sold) ***(A1)***

20 (performances) ***A1***

[4 marks]

(c) (i) any reasonable answer which suggests a biased sample (must include reason, do not accept reasons such as "sample size is too small", or answers that simply say "not representative of entire audience" without a valid reason) ***A1***

eg likely to come from the same part of the theatre OR be part of same group OR be from priority seating OR it is convenience sampling

(ii) every 20^{th} person ***A1A1***

Note: Award ***A1*** for recognizing that sampling occurs at regular intervals eg "every". Award ***A1*** for interval length is 20.

(iii) quota (sampling method) ***A1***

[4 marks]

continued…

Question 7 continued

(d) (i) 75% (of 36000 spent between \$$3$ and \$$25$) ***(M1)***

= 27000 ***A1***

(ii) $a = 7$ ***A1***

[3 marks]

(e) (i) **METHOD 1**

old mean is 600 (tickets) ***(A1)***

recognising new mean is old mean + 18 ***(M1)***

$600 + 18$

$= 618$ (tickets) ***A1***

METHOD 2

new total number of tickets = $36000 + 60 \times 18 (= 37080)$ ***(A1)***

$$\text{new mean} = \frac{36000 + 60 \times 18}{60} \left(= \frac{37080}{60} \right)$$ ***(M1)***

$= 618$ (tickets) ***A1***

(ii) no effect on the variance ***A1***

[4 marks]

Total [17 marks]

8. (a) $x = 0$ ***A1***

[1 mark]

(b) (i) setting $\ln(2x-7) = 2\ln x - \ln d$ ***M1***

attempt to use power rule ***(M1)***

$2\ln x = \ln x^2$ (seen anywhere)

attempt to use product/quotient rule for logs ***(M1)***

$\ln(2x-7) = \ln\dfrac{x^2}{d}$ OR $\ln\dfrac{x^2}{2x-7} = \ln d$ OR $\ln(2x-7)d = \ln x^2$

$\dfrac{x^2}{d} = 2x-7$ OR $\dfrac{x^2}{2x-7} = d$ OR $(2x-7)d = x^2$ ***A1***

$x^2 - 2dx + 7d = 0$ ***AG***

(ii) discriminant = $(-2d)^2 - 4\times 7d$ ***(A1)***

recognizing discriminant > 0 ***(M1)***

$(2d)^2 - 4\times 7d > 0$ OR $4d^2 - 28d > 0$ ***A1***

$d^2 - 7d > 0$ ***AG***

(iii) setting $d(d-7) > 0$ OR $d(d-7) = 0$ OR sketch graph
OR sign test OR $d^2 > 7d$ ***(M1)***

$d < 0$ or $d > 7$, but $d \in \mathbb{R}^+$

$d > 7$ (or $]7,\infty[$) ***A1***

[9 marks]

continued…

Question 8 continued

(c) $x^2 - 20x + 70 \ (= 0)$ ***A1***

attempting to solve their 3 term quadratic equation ***(M1)***

$\left((x-10)^2 - 30 = 0\right)$ or $\left((x =) \dfrac{20 \pm \sqrt{(-20)^2 - 4 \times 1 \times 70}}{2}\right)$

$x = 10 - \sqrt{30}\,(= p)$ or $x = 10 + \sqrt{30}\,(= q)$ ***(A1)***

subtracting their values of x ***(M1)***

distance $= 2\sqrt{30}$ $\left(\text{or } \sqrt{120}\right)$ ***A1***

$(a = 2, b = 30)$ $(\text{or } a = 1,\ b = 120)$

[5 marks]

Total [15 marks]

9. (a) attempt to use either the quotient or product rule ***(M1)***

$$\frac{12\left(x^2+2\right)^3-12x\times 3\times 2x\left(x^2+2\right)^2}{\left(x^2+2\right)^6} \text{ OR } 12\left(x^2+2\right)^{-3}+12x\times(-3)\times 2x\left(x^2+2\right)^{-4}$$ ***A1A1***

Note: Award ***A1*** for correctly applying chain rule to $\left(x^2+2\right)^3$ and ***A1*** for everything else correct.

$$=\frac{12\left(x^2+2-6x^2\right)}{\left(x^2+2\right)^4} \text{ OR } \frac{12\left(x^2+2\right)^2\left(x^2+2-6x^2\right)}{\left(x^2+2\right)^6} \text{ OR } \frac{24-60x^2}{\left(x^2+2\right)^4} \text{ OR } \frac{12x^2+24-72x^2}{\left(x^2+2\right)^4}$$ ***A1***

$$=\frac{12\left(2-5x^2\right)}{\left(x^2+2\right)^4}$$ ***AG***

[4 marks]

continued…

Question 9 continued

(b) **EITHER**

attempts to integrate by substitution using $u = x^2 + 2$ or $u = x^2$ ***(M1)***

$u = x^2 + 2 \Rightarrow \frac{du}{dx} = 2x$ OR $u = x^2 \Rightarrow \frac{du}{dx} = 2x$

Note: If candidate simply states $u = x^2 + 2$ or $u = x^2$, but does not attempt to substitute into their integral, do not award the ***(M1)***.

$\int \frac{12x}{\left(x^2+2\right)^3} dx = \int \frac{6}{u^3} du$ OR $\int \frac{12x}{\left(x^2+2\right)^3} dx = \int \frac{6}{(u+2)^3} du$ ***(A1)***

$= -3u^{-2} (+c)$ OR $-3(u+1)^{-2} (+c)$ ***(A1)***

OR

attempts to apply integration by inspection ***(M1)***

$6\int \frac{2x}{\left(x^2+2\right)^3} dx$

$= 6 \times \left(-\frac{1}{2}\right)\left(x^2+2\right)^{-2} (+c)$ ***(A1)(A1)***

Note: Award ***A1*** for correct power of $\left(x^2+2\right)$ and ***A1*** for $-\frac{1}{2}$.

THEN

$-3\left(x^2+2\right)^{-2} + c$ OR $-\frac{3}{\left(x^2+2\right)^2} + c$ (final answer must include $+c$) ***A1***

[4 marks]

continued…

Question 9 continued

(c) recognizing $g'(x) = f'(x) \Rightarrow g(x) = f(x) + k$ (may be seen in diagram/drawing) ***A1***

area of *R* is given by subtracting functions *f* and *g* in integral(s) ***(M1)***

$\pm\int_0^3 k\mathrm{d}x$ OR $= \int_0^3 |g - f|\mathrm{d}x$ OR $\int_0^3 f(x) + k$ - $f(x)\mathrm{d}x$ OR $\int_0^3 f(x)\mathrm{d}x - \int_0^3 g(x)\mathrm{d}x$

$= \pm[kx]_0^3$ OR $\left[-\frac{2}{\left(x^2+1\right)^2} + kx\right]_0^3 - \left[-\frac{2}{\left(x^2+1\right)^2}\right]_0^3$ OR $\left[-\frac{2}{\left(x^2+1\right)^2}\right]_0^3 - \left[-\frac{2}{\left(x^2+1\right)^2} + kx\right]_0^3$ ***(A1)***

$\pm 3k = \frac{21}{2}$ ***(A1)***

$k = \pm\frac{21}{6}\left(= \pm\frac{7}{2} = \pm 3.5\right)$

$g(x) = \frac{12x}{\left(x^2+2\right)^3} - \frac{7}{2}$ AND $g(x) = \frac{12x}{\left(x^2+2\right)^3} + \frac{7}{2}$ (accept $f(x) + \frac{7}{2}$ AND $f(x) - \frac{7}{2}$) ***A1***

[5 marks]

Total [13 marks]

Markscheme

May 2023

Mathematics: analysis and approaches

Standard level

Paper 1

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

R Marks awarded for clear **Reasoning**.

AG Answer given in the question and so no marks are awarded.

FT Follow through. The practice of awarding marks, despite candidate errors in previous parts, for their correct methods/answers using incorrect results.

Using the markscheme

1 General

Award marks using the annotations as noted in the markscheme *eg* ***M1*, *A2***.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***A3***, ***M2*** *etc*., do **not** split the marks, unless there is a note.
- The response to a “show that” question does not need to restate the ***AG*** line, unless a **Note** makes this explicit in the markscheme.
- Once a correct answer to a question or part question is seen, ignore further working even if this working is incorrect and/or suggests a misunderstanding of the question. This will encourage a uniform approach to marking, with less examiner discretion. Although some candidates may be advantaged for that specific question item, it is likely that these candidates will lose marks elsewhere too.
- An exception to the previous rule is when an incorrect answer from further working is used **in a subsequent part**. For example, when a correct exact value is followed by an incorrect decimal approximation in the first part and this approximation is then used in the second part. In this situation, award ***FT*** marks as appropriate but do not award the final ***A1*** in the first part.

Examples:

	Correct answer seen	Further working seen	Any FT issues?	Action
1.	$8\sqrt{2}$	$5.65685...$ *(incorrect decimal value)*	No. Last part in question.	Award ***A1*** for the final mark *(condone the incorrect further working)*
2.	$\frac{35}{72}$	0.468111… (incorrect decimal value)	Yes. Value is used in subsequent parts.	Award ***A0*** for the final mark *(and full **FT** is available in subsequent parts)*

3 Implied marks

Implied marks appear in **brackets e.g. *(M1)***,and can only be awarded if **correct** work is seen or implied by subsequent working/answer.

4 Follow through marks (only applied after an error is made)

Follow through (***FT***) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) (e.g. incorrect value from part (a) used in part (d) or incorrect value from part (c)(i) used in part (c)(ii)). Usually, to award ***FT*** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if all the marks awarded in a subsequent part are for the answer or are implied, then ***FT*** marks should be awarded for *their* correct answer, even when working is not present.

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- In the markscheme, some **equivalent** answers will generally appear in brackets. Not all equivalent notations/answers/methods will be presented in the markscheme and examiners are asked to apply appropriate discretion to judge if the candidate work is equivalent.

8 Format and accuracy of answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. If the level of accuracy is not stated in the question, the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures.*

Where values are used in subsequent parts, the markscheme will generally use the exact value, however candidates may also use the correct answer to 3 sf in subsequent parts. The markscheme will often explicitly include the subsequent values that come "*from the use of 3 sf values*".

Simplification of final answers: Candidates are advised to give final answers using good mathematical form. In general, for an ***A*** mark to be awarded, arithmetic should be completed, and any values that lead to integers should be simplified; for example, $\sqrt{\frac{25}{4}}$ should be written as $\frac{5}{2}$. An exception to this is simplifying fractions, where lowest form is not required (although the numerator and the denominator must be integers); for example, $\frac{10}{4}$ may be left in this form or written as $\frac{5}{2}$. However, $\frac{10}{5}$ should be written as 2, as it simplifies to an integer.

Algebraic expressions should be simplified by completing any operations such as addition and multiplication, e.g. $4e^{2x} \times e^{3x}$ should be simplified to $4e^{5x}$, and $4e^{2x} \times e^{3x} - e^{4x} \times e^{x}$ should be simplified to $3e^{5x}$. Unless specified in the question, expressions do not need to be factorized, nor do factorized expressions need to be expanded, so $x(x+1)$ and $x^2 + x$ are both acceptable.

Please note: intermediate ***A*** marks do NOT need to be simplified.

9 Calculators

No calculator is allowed. The use of any calculator on this paper is malpractice and will result in no grade awarded. If you see work that suggests a candidate has used any calculator, please follow the procedures for malpractice.

10. Presentation of candidate work

Crossed out work: If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work unless an explicit note from the candidate indicates that they would like the work to be marked.

More than one solution: Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise. If the layout of the responses makes it difficult to judge, examiners should apply appropriate discretion to judge which is "first".

Section A

1. (a) attempts to find perimeter ***(M1)***

$\text{arc} + 2 \times \text{radius}$ OR $10 + 4 + 4$

$= 18 \text{ (cm)}$ ***A1***

[2 marks]

(b) $10 = 4\theta$ ***(A1)***

$\theta = \frac{10}{4} \left(= \frac{5}{2}, 2.5\right)$ ***A1***

[2 marks]

(c) area $= \frac{1}{2}\left(\frac{10}{4}\right)\left(4^2\right) \ (= 1.25 \times 16)$ ***(A1)***

$= 20 \text{ (cm}^2\text{)}$ ***A1***

[2 marks]

Total [6 marks]

2. (a) $u_1 + 3d = u_4$ ***(M1)***

$0.6 + 3d = 0.15$

$d = -0.15$ ***A1***

[2 marks]

(b) **METHOD 1**

$u_2 = 0.45$ or $u_3 = 0.3$ (may be seen in their equation) ***(A1)***

summing their probabilities to 1 (seen anywhere) ***(M1)***

$$\frac{0.6}{k} + \frac{u_2}{k} + \frac{u_3}{k} + \frac{0.15}{k} = 1$$

$\frac{0.6}{k} + \frac{0.45}{k} + \frac{0.3}{k} + \frac{0.15}{k} = 1$ (or equivalent) ***(A1)***

$$\frac{1.5}{k} = 1$$

$k = 1.5$ ***A1***

[4 marks]

METHOD 2 (using S_n formula)

$S_4 = \frac{4}{2}\left(2(0.6) + (4-1)(-0.15)\right)$ OR $S_4 = \frac{4}{2}\left(2\left(\frac{0.6}{k}\right) + (4-1)\left(\frac{-0.15}{k}\right)\right)$

OR $S_4 = \frac{4}{2}(0.6 + 0.15)$ OR $S_4 = \frac{4}{2}\left(\frac{0.6}{k} + \frac{0.15}{k}\right)$ (or equivalent) ***(A1)***

summing their probabilities to 1 (seen anywhere) ***(M1)***

$\frac{u_1}{k} + \frac{u_2}{k} + \frac{u_3}{k} + \frac{u_4}{k} = 1$ OR $u_1 + u_2 + u_3 + u_4 = k$ OR $S_4 = 1$ OR $S_4 = k$

$\frac{4}{2}\left(2(0.6) + (4-1)(-0.15)\right) = k$ (or equivalent) ***(A1)***

$k = 1.5$ ***A1***

[4 marks]

Total [6 marks]

3. (a) (i) $x = 2$ ***A1***

(ii) $y = 1$ ***A1***

[2 marks]

(b) (i) $\left(0, \frac{3}{2}\right)$ ***A1***

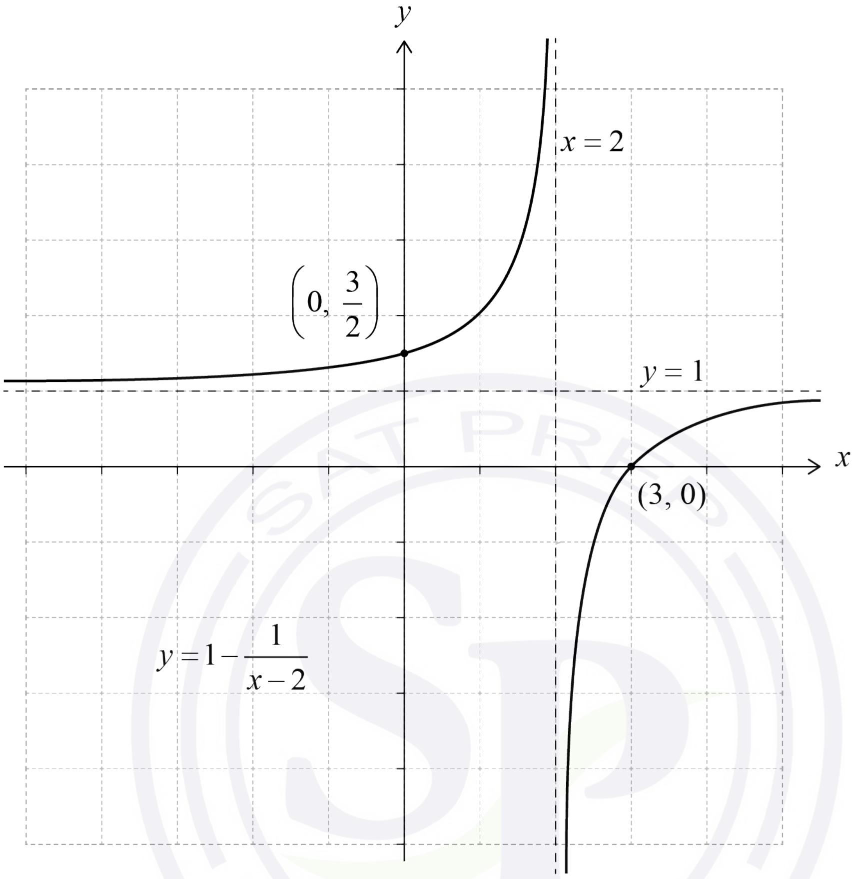
(ii) $(3, 0)$ ***A1***

[2 marks]

continued…

Question 3 continued

(c)



two correct branches with correct asymptotic behaviour and intercepts clearly shown ***A1***

[1 mark]

Total [5 marks]

4. substitutes into $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$ to form

$0.55 = 0.4 + \mathrm{P}(B) - \mathrm{P}(A \cap B)$ (or equivalent) ***(A1)***

substitutes into $\mathrm{P}(A \mid B) = \dfrac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$ to form $0.25 = \dfrac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$ (or equivalent) ***(A1)***

attempts to combine their two probability equations to form an equation in $\mathrm{P}(B)$ ***(M1)***

Note: The above two ***A*** marks are awarded independently.

correct equation in $\mathrm{P}(B)$ ***A1***

$0.55 = 0.4 + \mathrm{P}(B) - 0.25\mathrm{P}(B)$ OR $\dfrac{\mathrm{P}(B) - 0.15}{\mathrm{P}(B)} = 0.25$ OR $\mathrm{P}(B) - 0.15 = 0.25\mathrm{P}(B)$

(or equivalent)

$\mathrm{P}(B) = \dfrac{15}{75} \left(= \dfrac{1}{5} = 0.2 \right)$ ***A1***

Total [5 marks]

5. $A = \int_0^c \frac{x}{x^2+2}\,dx$

EITHER

attempts to integrate by inspection or substitution using $u = x^2 + 2$ or $u = x^2$ ***(M1)***

Note: If candidate simply states $u = x^2 + 2$ or $u = x^2$, but does not attempt to integrate, do not award the ***(M1)***.

Note: If candidate does not explicitly state the u-substitution, award the ***(M1)*** only for expressions of the form $k \ln u$ or $k \ln(u+2)$.

$\left[\frac{1}{2}\ln u\right]_2^{c^2+2}$ OR $\left[\frac{1}{2}\ln(u+2)\right]_0^{c^2}$ OR $\left[\frac{1}{2}\ln(x^2+2)\right]_0^{c}$ ***A1***

Note: Limits may be seen in the substitution step.

OR

attempts to integrate by inspection ***(M1)***

Note: Award the ***(M1)*** only for expressions of the form $k \ln(x^2+2)$.

$\left[\frac{1}{2}\ln(x^2+2)\right]_0^{c}$ ***A1***

Note: Limits may be seen in the substitution step.

THEN

correctly substitutes their limits into their integrated expression ***(M1)***

$\frac{1}{2}\left(\ln(c^2+2) - \ln 2\right) (= \ln 3)$ OR $\frac{1}{2}\ln(c^2+2) - \frac{1}{2}\ln 2 (= \ln 3)$

continued…

Question 5 continued

correctly applies at least one log law to their expression ***(M1)***

$\frac{1}{2}\ln\left(\frac{c^2+2}{2}\right)(=\ln 3)$ OR $\ln\sqrt{c^2+2}-\ln\sqrt{2}\,(=\ln 3)$ OR $\ln\left(\frac{c^2+2}{2}\right)=\ln 9$

OR $\ln\left(c^2+2\right)-\ln 2=\ln 9$ OR $\ln\sqrt{\frac{c^2+2}{2}}(=\ln 3)$ OR $\ln\frac{\sqrt{c^2+2}}{\sqrt{2}}(=\ln 3)$

Note: Condone the absence of $\ln 3$ up to this stage.

$\frac{c^2+2}{2}=9$ **OR** $\sqrt{\frac{c^2+2}{2}}=3$ ***A1***

$c^2=16$

$c=4$ ***A1***

Note: Award ***A0*** for $c=\pm 4$ as a final answer.

Total [6 marks]

6. attempts to form $(g \circ f)(x)$ ***(M1)***

$[f(x)]^2 + f(x) + 3$ OR $(ax+b)^2 + ax + b + 3$

$a^2x^2 + 2abx + b^2 + ax + b + 3 \left(= 4x^2 - 14x + 15\right)$ ***(A1)***

equates their corresponding terms to form at least one equation ***(M1)***

$a^2x^2 = 4x^2$ OR $a^2 = 4$ OR $2abx + ax = -14x$ OR $2ab + a = -14$ OR
$b^2 + b + 3 = 15$

$a = \pm 2$ (seen anywhere) ***A1***

attempt to use $2ab + a = -14$ to pair the correct values (seen anywhere) ***(M1)***

$f(x) = 2x - 4$ (accept $a = 2$ with $b = -4$), $f(x) = -2x + 3$ (accept $a = -2$ with $b = 3$) ***A1A1***

[7 marks]

Section B

7. (a) $x = -2$ (must be an equation) ***A1***

[1 mark]

(b) $h = -2$, $k = -5$ ***A1A1***

[2 marks]

(c) substituting $x = 0$ into $f(x)$ ***(M1)***

$y = \frac{1}{4}(0+2)^2 - 5$

$y = -4$ (accept $\mathrm{P}(0, -4)$) ***A1***

[2 marks]

(d) $f'(x) = \frac{1}{2}(x+2)\left(= \frac{1}{2}x + 1\right)$ ***(A1)***

substituting $x = 0$ into their derivative ***(M1)***

$f'(0) = 1$

gradient of normal is -1 (may be seen in their equation) ***A1***

$y = -x - 4$ (accept $a = -1$, $b = -4$) ***A1***

Note: Award ***A0*** for $L = -x - 4$ (without the $y =$).

[4 marks]

continued…

Question 7 continued

(e) equating their $f(x)$ to their L ***(M1)***

$\frac{1}{4}(x+2)^2-5=-x-4$

$\frac{1}{4}x^2+2x=0$ (or equivalent) ***(A1)***

valid attempt to solve their quadratic ***(M1)***

$\frac{1}{4}x(x+8)=0$ OR $x(x+8)=0$

$x=-8$ ***A1***

Note: Accept both solutions $x=-8$ and $x=0$ here, $x=-8$ may be seen in working to find coordinates of Q or distance.

substituting their value of x (not $x=0$) into their $f(x)$ or their L ***(M1)***

$y=-(-8)-4$ or $y=\frac{1}{4}(-8+2)^2-5$

$\mathrm{Q}(-8, 4)$ ***A1***

correct substitution into distance formula ***(A1)***

$\sqrt{(-8-0)^2+(4-(-4))^2}$

$\text{distance}=\sqrt{128}$ $(=8\sqrt{2})$ ***A1***

[8 marks]

Total [17 marks]

8. (a) (i) attempt to use Pythagoras ***(M1)***

$\sin^2\theta + \left(\frac{2}{3}\right)^2 = 1$ OR $x^2 + 2^2 = 3^2$ OR right triangle with side 2 and hypotenuse 3

$\sin\theta = \frac{\sqrt{5}}{3}$ ***A1***

(ii) attempt to substitute into double-angle identity using their value of $\sin\theta$ ***(M1)***

$\sin 2\theta = 2\left(\frac{\sqrt{5}}{3}\right)\left(\frac{2}{3}\right)$

$\sin 2\theta = \frac{4\sqrt{5}}{9}$ ***A1***

[4 marks]

(b) **METHOD 1 (using values from part (a))**

$\frac{b}{\sin\theta} = \frac{a}{\sin 2\theta}$

attempt to use sine rule with their values from part (a) ***(M1)***

$\frac{b}{\left(\frac{\sqrt{5}}{3}\right)} = \frac{a}{\left(\frac{4\sqrt{5}}{9}\right)}$ OR $\frac{\left(\frac{\sqrt{5}}{3}\right)}{b} = \frac{\left(\frac{4\sqrt{5}}{9}\right)}{a}$

correct working that leads to ***AG*** ***A1***

$\frac{\sqrt{5}}{3}a = \frac{4\sqrt{5}}{9}b$ OR $\frac{3b}{\sqrt{5}} = \frac{9a}{4\sqrt{5}}$ OR $\frac{a}{3} = \frac{4b}{9}$ (or equivalent)

$b = \frac{3a}{4}$ ***AG***

continued…

Question 8 continued

METHOD 2 (double-angle identity)

$$\frac{b}{\sin\theta}=\frac{a}{\sin 2\theta}$$

using double-angle identity **(A1)**

$$\frac{b}{\sin\theta}=\frac{a}{2\sin\theta\cos\theta} \quad \text{OR} \quad b=\frac{a\sin\theta}{2\sin\theta\cos\theta} \quad \text{OR} \quad b=\frac{a}{2\cos\theta}$$

correct working (involving substituting $\cos\theta=\frac{2}{3}$) that leads to ***AG*** **A1**

$$b=\frac{a\sin\theta}{2\sin\theta\left(\frac{2}{3}\right)} \quad \text{OR} \quad b=\frac{a\left(\frac{\sqrt{5}}{3}\right)}{2\left(\frac{\sqrt{5}}{3}\right)\left(\frac{2}{3}\right)} \quad \text{OR} \quad b=\frac{a}{2\left(\frac{2}{3}\right)} \quad \text{(or equivalent)}$$

$$b=\frac{3a}{4}$$ ***AG***

[2 marks]

(c) **METHOD 1 (using supplementary angles)**

recognizing $\text{C}\hat{\text{A}}\text{D}$ and $\text{B}\hat{\text{A}}\text{C}$ are supplementary ***(M1)***

recognizing supplementary angles have the same sine value ***(A1)***

$$\sin \text{C}\hat{\text{A}}\text{D}=\sin 2\theta$$

$$\sin \text{C}\hat{\text{A}}\text{D}=\frac{4\sqrt{5}}{9}$$ ***A1***

continued…

Question 8 continued

METHOD 2 (using sine rule)

recognizing $\text{CD} = a$ ***(M1)***

$$\frac{a}{\sin \text{C}\hat{\text{A}}\text{D}} = \frac{b}{\sin \theta}$$

correct substitution of $\sin \theta = \frac{\sqrt{5}}{3}$ and $b = \frac{3a}{4}$ into sine rule ***(A1)***

$$\frac{a}{\sin \text{C}\hat{\text{A}}\text{D}} = \frac{\left(\frac{3a}{4}\right)}{\left(\frac{\sqrt{5}}{3}\right)} \quad \textbf{OR} \quad \sin \text{C}\hat{\text{A}}\text{D} = \frac{a\left(\frac{\sqrt{5}}{3}\right)}{\left(\frac{3a}{4}\right)} \quad \text{(or equivalent)}$$

$$\sin \text{C}\hat{\text{A}}\text{D} = \frac{4\sqrt{5}}{9}$$ ***A1***

[3 marks]

(d) **METHOD 1 (using $\text{C}\hat{\text{A}}\text{D}$ in area formula)**

recognizing $\text{D}\hat{\text{C}}\text{A} = \theta$ ***(A1)***

recognizing $\text{AD} = b \left(= \frac{3a}{4}\right)$ ***(A1)***

correct substitution into area formula (must substitute expressions for two sides and name/expression/value for $\sin \text{C}\hat{\text{A}}\text{D}$) ***(M1)***

$$\text{area} = \frac{1}{2}(b)(b)\left(\frac{4\sqrt{5}}{9}\right) \quad \textbf{OR} \quad \text{area} = \frac{1}{2}(b)(b)\sin 2\theta \quad \textbf{OR} \quad \text{area} = \frac{1}{2}(b)(b)\sin \text{C}\hat{\text{A}}\text{D}$$

correct substitution in terms of a ***(A1)***

$$\text{area} = \frac{1}{2}\left(\frac{3a}{4}\right)\left(\frac{3a}{4}\right)\left(\frac{4\sqrt{5}}{9}\right)$$

$$\text{area} = \frac{\sqrt{5}\,a^2}{8}$$ ***A1***

continued…

Question 8 continued

METHOD 2 (using $\mathrm{A\hat{C}D}$ or $\mathrm{A\hat{D}C}$ in area formula)

recognizing $\mathrm{CD} = a$ ***(A1)***

recognizing $\mathrm{AD} = b \left(= \frac{3a}{4} \right)$ and/or $\mathrm{D\hat{C}A} = \theta$ ***(A1)***

correct substitution into area formula (must substitute expressions for two sides and name/expression/value for $\sin \mathrm{A\hat{D}C}$ or $\sin \mathrm{A\hat{C}D}$) ***(M1)***

$\text{area} = \frac{1}{2}(a)(b)\left(\frac{\sqrt{5}}{3}\right)$ OR $\text{area} = \frac{1}{2}(a)(b)\sin\theta$ OR $\text{area} = \frac{1}{2}(a)(b)\sin \mathrm{A\hat{D}C}$

OR $\text{area} = \frac{1}{2}(a)(b)\sin \mathrm{A\hat{C}D}$

correct substitution in terms of a ***(A1)***

$$\text{area} = \frac{1}{2}(a)\left(\frac{3a}{4}\right)\left(\frac{\sqrt{5}}{3}\right)$$

$$\text{area} = \frac{\sqrt{5}\,a^2}{8}$$ ***A1***

[5 marks]

Total [14 marks]

9. (a) $y^2 = 9 - x^2$ OR $y = \pm\sqrt{9-x^2}$ ***A1***

(since $y > 0$) $\Rightarrow y = \sqrt{9-x^2}$ ***AG***

[1 mark]

(b) $b = 2y$ $\left(= 2\sqrt{9-x^2}\right)$ or $h = x + 3$ ***(A1)***

attempts to substitute their base expression and height expression into $A = \frac{1}{2}bh$ ***(M1)***

$A = \sqrt{9-x^2}(x+3)$ (or equivalent) $\left(= \frac{2(x+3)\sqrt{9-x^2}}{2} = x\sqrt{9-x^2} + 3\sqrt{9-x^2} \right)$ ***A1***

[3 marks]

(c) attempts to use the product rule to find $\frac{\mathrm{d}A}{\mathrm{d}x}$ ***(M1)***

attempts to use the chain rule to find $\frac{\mathrm{d}}{\mathrm{d}x}\sqrt{9-x^2}$ ***(M1)***

$\left(\frac{\mathrm{d}A}{\mathrm{d}x} =\right) \sqrt{9-x^2} + (3+x)\left(\frac{1}{2}\right)\left(9-x^2\right)^{-\frac{1}{2}}(-2x)$ $\left(= \sqrt{9-x^2} - \frac{x^2+3x}{\sqrt{9-x^2}} \right)$ ***A1***

$\left(\frac{\mathrm{d}A}{\mathrm{d}x} =\right) \frac{9-x^2}{\sqrt{9-x^2}} - \frac{x^2+3x}{\sqrt{9-x^2}}$ $\left(= \frac{9-x^2-\left(x^2+3x\right)}{\sqrt{9-x^2}} \right)$ ***A1***

$\frac{\mathrm{d}A}{\mathrm{d}x} = \frac{9-3x-2x^2}{\sqrt{9-x^2}}$ ***AG***

[4 marks]

continued…

Question 9 continued

(d) $\frac{\mathrm{d}A}{\mathrm{d}x} = 0 \left(\frac{9-3x-2x^2}{\sqrt{9-x^2}} = 0 \right)$ ***(M1)***

attempts to solve $9-3x-2x^2=0$ (or equivalent) ***(M1)***

$-(2x-3)(x+3)(=0)$ OR $x = \frac{3 \pm \sqrt{(-3)^2 - 4(-2)(9)}}{2(-2)}$ (or equivalent) ***(A1)***

$x = \frac{3}{2}$ ***A1***

Note: Award the above ***A1*** if $x=-3$ is also given.

substitutes their value of x into either $y=\sqrt{9-x^2}$ or $y=-\sqrt{9-x^2}$ ***(M1)***

Note: Do not award the above ***(M1)*** if $x \le 0$.

$y = -\sqrt{9 - \left(\frac{3}{2}\right)^2}$

$= -\frac{\sqrt{27}}{2} \left(= -\frac{3\sqrt{3}}{2}, = -\sqrt{\frac{27}{4}}, = -\sqrt{6.75} \right)$ ***A1***

[6 marks]

Total [14 marks]

Markscheme

May 2023

Mathematics: analysis and approaches

Standard level

Paper 1

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

R Marks awarded for clear **Reasoning**.

AG Answer given in the question and so no marks are awarded.

FT Follow through. The practice of awarding marks, despite candidate errors in previous parts, for their correct methods/answers using incorrect results.

Using the markscheme

1 General

Award marks using the annotations as noted in the markscheme *eg* ***M1, A2.***

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***A3***, ***M2*** *etc*., do **not** split the marks, unless there is a note.
- The response to a "show that" question does not need to restate the ***AG*** line, unless a **Note** makes this explicit in the markscheme.
- Once a correct answer to a question or part question is seen, ignore further working even if this working is incorrect and/or suggests a misunderstanding of the question. This will encourage a uniform approach to marking, with less examiner discretion. Although some candidates may be advantaged for that specific question item, it is likely that these candidates will lose marks elsewhere too.
- An exception to the previous rule is when an incorrect answer from further working is used **in a subsequent part**. For example, when a correct exact value is followed by an incorrect decimal approximation in the first part and this approximation is then used in the second part. In this situation, award ***FT*** marks as appropriate but do not award the final ***A1*** in the first part.

Examples:

	Correct answer seen	Further working seen	Any FT issues?	Action
1.	$8\sqrt{2}$	$5.65685...$ *(incorrect decimal value)*	No. Last part in question.	Award ***A1*** for the final mark *(condone the incorrect further working)*
2.	$\frac{35}{72}$	0.468111… (incorrect decimal value)	Yes. Value is used in subsequent parts.	Award ***A0*** for the final mark *(and full **FT** is available in subsequent parts)*

3 Implied marks

Implied marks appear in **brackets e.g. *(M1)***,and can only be awarded if **correct** work is seen or implied by subsequent working/answer.

4 Follow through marks (only applied after an error is made)

Follow through (***FT***) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) (e.g. incorrect value from part (a) used in part (d) or incorrect value from part (c)(i) used in part (c)(ii)). Usually, to award ***FT*** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if all the marks awarded in a subsequent part are for the answer or are implied, then ***FT*** marks should be awarded for *their* correct answer, even when working is not present.

For example: following an incorrect answer to part (a) that is used in subsequent parts, where the markscheme for the subsequent part is ***(M1)A1***, it is possible to award full marks for *their* correct answer, **without working being seen.** For longer questions where all but the answer marks are implied this rule applies but may be overwritten by a **Note** in the Markscheme.

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate.

- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks, by reflecting on what each mark is for and how that maps to the simplified version.

- If the error leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).

- The markscheme may use the word "their" in a description, to indicate that candidates may be using an incorrect value.

- If the candidate's answer to the initial question clearly contradicts information given in the question, it is not appropriate to award any ***FT*** marks in the subsequent parts. This includes when candidates fail to complete a "show that" question correctly, and then in subsequent parts use their incorrect answer rather than the given value.

- Exceptions to these ***FT*** rules will be explicitly noted on the markscheme.

- If a candidate makes an error in one part but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the command term was "Hence".

5 Mis-read

If a candidate incorrectly copies values or information from the question, this is a mis-read (***MR***). A candidate should be penalized only once for a particular misread. Use the ***MR*** stamp to indicate that this has been a misread and do not award the first mark, even if this is an ***M*** mark, but award all others as appropriate.

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.
- If a candidate uses a correct answer, to a "show that" question, to a higher degree of accuracy than given in the question, this is NOT a misread and full marks may be scored in the subsequent part.
- ***MR*** can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If the command term is 'Hence' and not 'Hence or otherwise' then alternative methods are not permitted unless covered by a note in the mark scheme.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**.

7 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation** for example 1.9 and 1,9 or 1000 and 1,000 and 1.000.
- Do not accept final answers written using calculator notation. However, ***M*** marks and intermediate ***A*** marks can be scored, when presented using calculator notation, provided the evidence clearly reflects the demand of the mark.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.

- In the markscheme, some **equivalent** answers will generally appear in brackets. Not all equivalent notations/answers/methods will be presented in the markscheme and examiners are asked to apply appropriate discretion to judge if the candidate work is equivalent.

8 Format and accuracy of answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. If the level of accuracy is not stated in the question, the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures.*

Where values are used in subsequent parts, the markscheme will generally use the exact value, however candidates may also use the correct answer to 3 sf in subsequent parts. The markscheme will often explicitly include the subsequent values that come "*from the use of 3 sf values*".

Simplification of final answers: Candidates are advised to give final answers using good mathematical form. In general, for an ***A*** mark to be awarded, arithmetic should be completed, and any values that lead to integers should be simplified; for example, $\sqrt{\frac{25}{4}}$ should be written as $\frac{5}{2}$. An exception to this is simplifying fractions, where lowest form is not required (although the numerator and the denominator must be integers); for example, $\frac{10}{4}$ may be left in this form or written as $\frac{5}{2}$. However, $\frac{10}{5}$ should be written as 2, as it simplifies to an integer.

Algebraic expressions should be simplified by completing any operations such as addition and multiplication, e.g. $4e^{2x} \times e^{3x}$ should be simplified to $4e^{5x}$, and $4e^{2x} \times e^{3x} - e^{4x} \times e^{x}$ should be simplified to $3e^{5x}$. Unless specified in the question, expressions do not need to be factorized, nor do factorized expressions need to be expanded, so $x(x+1)$ and $x^2 + x$ are both acceptable.

Please note: intermediate ***A*** marks do NOT need to be simplified.

9 Calculators

No calculator is allowed. The use of any calculator on this paper is malpractice and will result in no grade awarded. If you see work that suggests a candidate has used any calculator, please follow the procedures for malpractice.

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Section A

1. (a) $\text{M}\ (6, -3)$ ***A1A1***

[2 marks]

(b) gradient of $[\text{PQ}] = -\frac{5}{9}$ ***(A1)***

gradient of $L = \frac{9}{5}$ ***A1***

[2 marks]

(c) $y + 3 = \frac{9}{5}(x - 6)$ OR $y = \frac{9}{5}x - \frac{69}{5}$ (or equivalent) ***A1***

Note: Do not accept $L = \frac{9}{5}x - \frac{69}{5}$.

[1 mark]

Total [5 marks]

2. (a) recognizing $f(x)=0$ ***(M1)***

$x=-1$ ***A1***

[2 marks]

(b) (i) $x=2$ (must be an equation with x) ***A1***

(ii) $y=\frac{7}{2}$ (must be an equation with y) ***A1***

[2 marks]

(c) **EITHER**

interchanging x and y ***(M1)***

$2xy-4x=7y+7$

correct working with y terms on the same side: $2xy-7y=4x+7$ ***(A1)***

OR

$2yx-4y=7x+7$

correct working with x terms on the same side: $2yx-7x=4y+7$ ***(A1)***

interchanging x and y OR making x the subject $x=\frac{4y+7}{2y-7}$ ***(M1)***

THEN

$f^{-1}(x)=\frac{4x+7}{2x-7}$ (or equivalent) $\left(x\neq\frac{7}{2}\right)$ ***A1***

[3 marks]

Total [7 marks]

3. (a) (i) summing frequencies of riders or finding complement ***(M1)***

probability $= \frac{34}{40}$ ***A1***

(ii) attempt to find expected value ***(M1)***

$\frac{16}{40}+\left(2\times\frac{13}{40}\right)+\left(3\times\frac{2}{40}\right)+\left(4\times\frac{3}{40}\right)$

$\frac{60}{40}(=1.5)$ ***A1***

[4 marks]

(b) evidence of **their** rides/visitor $\times 1000$ or $\div 10$ ***(M1)***

1500 OR 0.15

150 (times) ***A1***

[2 marks]

Total [6 marks]

4. (a) $1-2\sin^2 x=\sin x$ ***A1***

$2\sin^2 x+\sin x-1=0$ ***AG***

[1 mark]

(b) valid attempt to solve quadratic ***(M1)***

$(2\sin x-1)(\sin x+1)$ OR $\dfrac{-1\pm\sqrt{1-4(2)(-1)}}{2(2)}$

recognition to solve for $\sin x$ ***(M1)***

$\sin x=\dfrac{1}{2}$ OR $\sin x=-1$

any correct solution from $\sin x=-1$ ***A1***

any correct solution from $\sin x=\dfrac{1}{2}$ ***A1***

Note: The previous two marks may be awarded for degree or radian values, irrespective of domain.

$x=-\dfrac{\pi}{2},\dfrac{\pi}{6},\dfrac{5\pi}{6}$ ***A1***

Note: If no working shown, award no marks for a final value(s).
Award ***A0*** for $-\dfrac{\pi}{2},\dfrac{\pi}{6},\dfrac{5\pi}{6}$ if additional values also given.

[5 marks]

Total [6 marks]

5. recognition of quadratic in e^x ***(M1)***

$\left(\mathrm{e}^x\right)^2 - 3\mathrm{e}^x + \ln k\left(=0\right)$ OR $A^2 - 3A + \ln k\left(=0\right)$

recognizing discriminant ≥ 0 (seen anywhere) ***(M1)***

$\left(-3\right)^2 - 4(1)(\ln k)$ OR $9 - 4\ln k$ ***(A1)***

$\ln k \le \dfrac{9}{4}$ ***(A1)***

$\mathrm{e}^{9/4}$ (seen anywhere) ***A1***

$0 < k \le \mathrm{e}^{9/4}$ ***A1***

[6 marks]

6. (a) recognition that period is $4m$ OR substitution of a point on f (except the origin) ***(M1)***

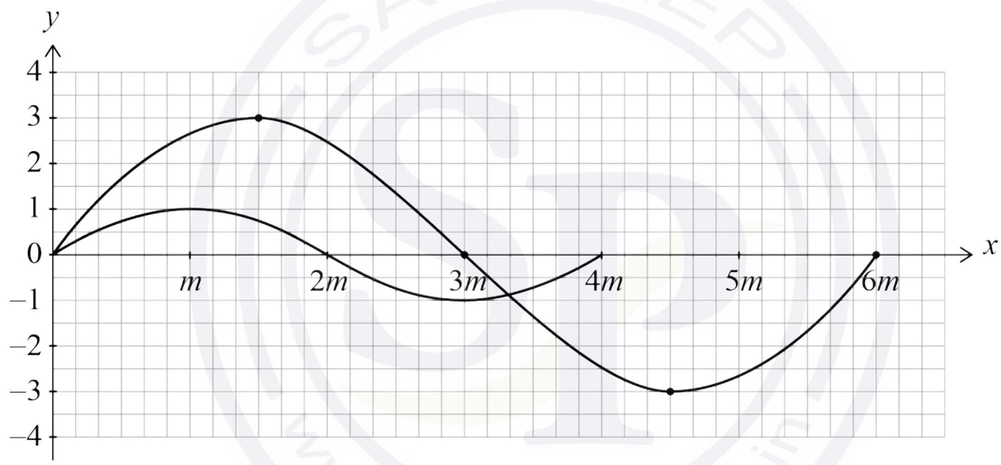
$4m = \frac{2\pi}{q}$ OR $1 = \sin qm$

$m = \frac{\pi}{2q}$ ***A1***

[2 marks]

(b) horizontal scale factor is $\frac{3}{2}$ (seen anywhere) ***(A1)***

Note: This ***(A1)*** may be earned by seeing a period of $6m$, half period of $3m$ or the correct x-coordinate of the maximum/minimum point.



A1A1A1

Note: Curve must be an approximate sinusoidal shape (sine or cosine).
Only in this case, award the following:
A1 for correct amplitude.
A1 for correct domain.
A1 for correct max and min points **and** correct x-intercepts.

[4 marks]

Total [6 marks]

Section B

7. (a) substitution of $x=0$ ***(M1)***

$(y=)\ 3$ (accept $(0, 3)$) ***A1***

[2 marks]

(b) evidence of using the product rule ***(M1)***

$h'(x)=2e^{x}+2xe^{x}$ ***A1***

[2 marks]

(c) setting their derivative equal to zero ***(M1)***

correct working ***(A1)***

$2e^{x}(1+x)\ (=0)$ OR $-2x=2$

$x=-1$ (seen anywhere, and must follow on from their derivative) ***A1***

substituting their value of x into $h(x)$ ***(M1)***

$y=-\dfrac{2}{e}+3\ \left(=-2e^{-1}+3\right)$ ***A1***

$\mathrm{A}\left(-1,-\dfrac{2}{e}+3\right)$

[5 marks]

continued…

Question 7 continued

(d) (i) $h''(x) = 2e^x + 2e^x + 2xe^x$ OR $2e^x + 2e^x(1+x)$ ***A1A1***

Note: Award ***A1*** for $\left(2e^x\right)' = 2e^x$, ***A1*** for $2e^x + 2xe^x$ or $\left(2x+2\right)e^x$

$h''(x) = \left(2x+4\right)e^x$ ***AG***

(ii) recognition that $h'' > 0$ OR attempt to find point of inflexion ***(M1)***

since $e^x > 0$, $2x+4>0$ OR $2x+4=0$ $\left(\Rightarrow x = -2\right)$

$x > -2$ ***A1***

[4 marks]

Total [13 marks]

8. (a) (i) recognition that $n = 5$ ***(M1)***

$S_5 = 45$ ***A1***

(ii) **METHOD 1**

recognition that $S_5 + u_6 = S_6$ ***(M1)***

$u_6 = 15$ ***A1***

METHOD 2

recognition that $60 = \frac{6}{2}(S_1 + u_6)$ ***(M1)***

$60 = 3(5 + u_6)$

$u_6 = 15$ ***A1***

METHOD 3

substituting their u_1 and d values into $u_1 + (n-1)d$ ***(M1)***

$u_6 = 15$ ***A1***

[4 marks]

(b) recognition that $u_1 = S_1$ (may be seen in (a)) OR substituting their u_6 into S_6 ***(M1)***

OR equations for S_5 and S_6 in terms of u_1 and d

$1 + 4$ OR $60 = \frac{6}{2}(u_1 + 15)$

$u_1 = 5$ ***A1***

[2 marks]

continued…

Question 8 continued

(c) **EITHER**

valid attempt to find d (may be seen in (a) or (b)) ***(M1)***

$d = 2$ ***(A1)***

OR

valid attempt to find $S_n - S_{n-1}$ ***(M1)***

$n^2 + 4n - \left(n^2 - 2n + 1 + 4n - 4\right)$ ***(A1)***

OR

equating $n^2 + 4n = \frac{n}{2}\left(5 + u_n\right)$ ***(M1)***

$2n + 8 = 5 + u_n$ (or equivalent) ***(A1)***

THEN

$u_n = 5 + 2(n-1)$ OR $u_n = 2n + 3$ ***A1***

[3 marks]

(d) recognition that $v_2 r^2 = v_4$ OR $(v_3)^2 = v_2 \times v_4$ ***(M1)***

$r^2 = 3$ OR $v_3 = (\pm)5\sqrt{3}$ ***(A1)***

$r = \pm\sqrt{3}$ ***A1***

Note: If no working shown, award ***M1A1A0*** for $\sqrt{3}$.

[3 marks]

(e) recognition that r is negative ***(M1)***

$v_5 = -15\sqrt{3} \quad \left(= -\frac{45}{\sqrt{3}}\right)$ ***A1***

[2 marks]

Total [14 marks]

9. (a) attempt to integrate v (integration of at least one term) ***(M1)***

$\left(s(t)=\right)-\frac{1}{4}t^4+\frac{7}{6}t^3-t^2+6t\ \left(+C\right)$ ***A2***

Note: Award ***A1*** for at least two correct terms.

substitution of $t=1$ into their integrated expression ***(M1)***

displacement = $5\frac{11}{12}\left(=\frac{71}{12}\right)$(m) ***A1***

[5 marks]

(b) attempt to differentiate v (differentiation of at least one term) ***(M1)***

$a\left(t\right)=-3t^2+7t-2$ ***A1***

[2 marks]

(c) setting their $v'\left(t\right)=0$ ***(M1)***

$-3t^2+7t-2=0$

valid attempt to solve quadratic ***(M1)***

$\left(3t-1\right)\left(t-2\right)=0$ OR $\frac{-7\pm\sqrt{49-4\left(-3\right)\left(-2\right)}}{-6}$

$t=\frac{1}{3},2$ ($t=\frac{1}{3}$ may be omitted) ***(A1)***

substitute their largest positive t-value into $v(t)$ ***(M1)***

greatest speed is 8 (ms^{-1}) ***A1***

[5 marks]

continued…

Question 9 continued

(d) attempt to check other boundary value at $t = 4$ ***(M1)***

$v(4) = -64 + 56 - 8 + 6 \quad (= -10)$

greatest speed is 10 ms^{-1} ***A1***

[2 marks]

(e) identifying correct intervals where speed increases (may be seen in integral) ***(A1)(A1)***

$t = \frac{1}{3}$ to $t = 2$ and $t = k$ to $t = 4$

$\int_{\frac{1}{3}}^{2} v(t)\,\mathrm{d}t + \int_{k}^{4} \left|v(t)\right| \mathrm{d}t$ OR $\int_{\frac{1}{3}}^{2} v\,\mathrm{d}t + \left|\int_{k}^{4} v\,\mathrm{d}t\right|$ OR $\int_{\frac{1}{3}}^{2} v(t)\,\mathrm{d}t - \int_{k}^{4} v(t)\,\mathrm{d}t$ ***A1***

Note: Condone missing $\mathrm{d}t$.

[3 marks]

Total [17 marks]

Markscheme

November 2022

Mathematics: analysis and approaches

Standard level

Paper 1

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

R Marks awarded for clear **Reasoning**.

AG Answer given in the question and so no marks are awarded.

FT Follow through. The practice of awarding marks, despite candidate errors in previous parts, for their correct methods/answers using incorrect results.

Using the markscheme

1 General

Award marks using the annotations as noted in the markscheme *eg* ***M1, A2***.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***A3***, ***M2*** *etc*., do **not** split the marks, unless there is a note.
- The response to a "show that" question does not need to restate the ***AG*** line, unless a **Note** makes this explicit in the markscheme.
- Once a correct answer to a question or part question is seen, ignore further working even if this working is incorrect and/or suggests a misunderstanding of the question. This will encourage a uniform approach to marking, with less examiner discretion. Although some candidates may be advantaged for that specific question item, it is likely that these candidates will lose marks elsewhere too.
- An exception to the previous rule is when an incorrect answer from further working is used **in a subsequent part**. For example, when a correct exact value is followed by an incorrect decimal approximation in the first part and this approximation is then used in the second part. In this situation, award ***FT*** marks as appropriate but do not award the final ***A1*** in the first part.

Examples:

	Correct answer seen	Further working seen	Any FT issues?	Action
1.	$8\sqrt{2}$	$5.65685...$ *(incorrect decimal value)*	No. Last part in question.	Award ***A1*** for the final mark *(condone the incorrect further working)*
2.	$\frac{35}{72}$	0.468111… (incorrect decimal value)	Yes. Value is used in subsequent parts.	Award ***A0*** for the final mark *(and full **FT** is available in subsequent parts)*

3 Implied marks

Implied marks appear in **brackets e.g. *(M1)***,and can only be awarded if **correct** work is seen or implied by subsequent working/answer.

4 Follow through marks (only applied after an error is made)

Follow through (***FT***) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) (e.g. incorrect value from part (a) used in part (d) or incorrect value from part (c)(i) used in part (c)(ii)). Usually, to award ***FT*** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if all the marks awarded in a subsequent part are for the answer or are implied, then ***FT*** marks should be awarded for *their* correct answer, even when working is not present.

For example: following an incorrect answer to part (a) that is used in subsequent parts, where the markscheme for the subsequent part is ***(M1)A1***, it is possible to award full marks for *their* correct answer, **without working being seen.** For longer questions where all but the answer marks are implied this rule applies but may be overwritten by a **Note** in the Markscheme.

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks, by reflecting on what each mark is for and how that maps to the simplified version.
- If the error leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word "their" in a description, to indicate that candidates may be using an incorrect value.
- If the candidate's answer to the initial question clearly contradicts information given in the question, it is not appropriate to award any ***FT*** marks in the subsequent parts. This includes when candidates fail to complete a "show that" question correctly, and then in subsequent parts use their incorrect answer rather than the given value.
- Exceptions to these ***FT*** rules will be explicitly noted on the markscheme.
- If a candidate makes an error in one part but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the command term was "Hence".

5 Mis-read

If a candidate incorrectly copies values or information from the question, this is a mis-read (***MR***). A candidate should be penalized only once for a particular misread. Use the ***MR*** stamp to indicate that this has been a misread and do not award the first mark, even if this is an ***M*** mark, but award all others as appropriate.

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.
- If a candidate uses a correct answer, to a "show that" question, to a higher degree of accuracy than given in the question, this is NOT a misread and full marks may be scored in the subsequent part.
- ***MR*** can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If the command term is 'Hence' and not 'Hence or otherwise' then alternative methods are not permitted unless covered by a note in the mark scheme.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**.

7 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation** for example 1.9 and 1,9 or 1000 and 1,000 and 1.000.
- Do not accept final answers written using calculator notation. However, ***M*** marks and intermediate ***A*** marks can be scored, when presented using calculator notation, provided the evidence clearly reflects the demand of the mark.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, some **equivalent** answers will generally appear in brackets. Not all equivalent notations/answers/methods will be presented in the markscheme and examiners are asked to apply appropriate discretion to judge if the candidate work is equivalent.

8 Format and accuracy of answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. If the level of accuracy is not stated in the question, the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures.*

Where values are used in subsequent parts, the markscheme will generally use the exact value, however candidates may also use the correct answer to 3 sf in subsequent parts. The markscheme will often explicitly include the subsequent values that come "*from the use of 3 sf values*".

Simplification of final answers: Candidates are advised to give final answers using good mathematical form. In general, for an ***A*** mark to be awarded, arithmetic should be completed, and any values that lead to integers should be simplified; for example, $\sqrt{\frac{25}{4}}$ should be written as $\frac{5}{2}$. An exception to this is simplifying fractions, where lowest form is not required (although the numerator and the denominator must be integers); for example, $\frac{10}{4}$ may be left in this form or written as $\frac{5}{2}$. However, $\frac{10}{5}$ should be written as 2, as it simplifies to an integer.

Algebraic expressions should be simplified by completing any operations such as addition and multiplication, e.g. $4e^{2x} \times e^{3x}$ should be simplified to $4e^{5x}$, and $4e^{2x} \times e^{3x} - e^{4x} \times e^{x}$ should be simplified to $3e^{5x}$. Unless specified in the question, expressions do not need to be factorized, nor do factorized expressions need to be expanded, so $x(x+1)$ and $x^2 + x$ are both acceptable.

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Section A

1. (a) gradient of g is -2 (may be seen in function, do not accept $-2x+3$) ***(A1)***

$g(x) = -2x$ ***A1***

[2 marks]

(b) gradient is $\frac{1}{2}$ (may be seen in function) ***(A1)***

attempt to substitute **their** gradient and $(-1, 2)$ into any form of equation for straight line ***(M1)***

$y - 2 = \frac{1}{2}(x+1)$ OR $2 = \frac{1}{2}\cdot(-1) + c$

$h(x) = \frac{1}{2}(x+1) + 2 \quad \left(= \frac{1}{2}x + \frac{5}{2}\right)$ ***A1***

[3 marks]

(c) $(g \circ h)(x) = -2\left(\frac{1}{2}x + \frac{5}{2}\right)$ OR $h(0) = \frac{5}{2}$ OR $g\left(\frac{5}{2}\right)$ ***(A1)***

$(g \circ h)(0) = -5$ ***A1***

[2 marks]

Total [7 marks]

2. $g'(x) = 2x\mathrm{e}^{x^2+1}$ ***(A2)***

substitute $x = -1$ into **their** derivative ***(M1)***

$g'(-1) = -2\mathrm{e}^2$ ***A1***

Note: Award ***A0M0A0*** in cases where candidate's incorrect derivative is $g'(x) = \mathrm{e}^{x^2+1}$.

[4 marks]

3. (a) (i) attempt to find midpoint of A and B ***(M1)***

centre $(-1,3,-2)$ (accept vector notation and/or missing brackets) ***A1***

(ii) attempt to find AB or half of AB or distance between the centre and A (or B) ***(M1)***

$$\frac{\sqrt{4^2+2^2+4^2}}{2} \text{ or } \sqrt{2^2+1^2+2^2}$$

$= 3$ ***A1***

[4 marks]

(b) attempt to find the distance between their centre and V
(the perpendicular height of the cone) ***(M1)***

$\sqrt{0^2+4^2+2^2}$ OR $\sqrt{(\text{their slant height})^2-(\text{their radius})^2}$

$=\sqrt{20}\left(=2\sqrt{5}\right)$ ***(A1)***

Volume $=\frac{1}{3}\pi 3^2\sqrt{20}$

$=3\pi\sqrt{20}\left(=6\pi\sqrt{5}\right)$ ***A1***

[3 marks]

Total [7 marks]

4. (a)

Note: Award a maximum of ***M1A0A0*** if the candidate manipulates both sides of the equation (such as moving terms from one side to the other).

METHOD 1 (working with LHS)

attempting to expand $\left(a^2-1\right)^2$ (do not accept a^4+1 or a^4-1) ***(M1)***

$\text{LHS} = a^2 + \frac{a^4-2a^2+1}{4}$ or $\frac{4a^2+a^4-2a^2+1}{4}$ ***A1***

$= \frac{a^4+2a^2+1}{4}$ ***A1***

$= \left(\frac{a^2+1}{2}\right)^2 \; (= \text{RHS})$ ***AG***

Note: Do not award the final ***A1*** if further working contradicts the AG.

METHOD 2 (working with RHS)

attempting to expand $\left(a^2+1\right)^2$ ***(M1)***

$\text{RHS} = \frac{a^4+2a^2+1}{4}$

$= \frac{4a^2+a^4-2a^2+1}{4}$ ***A1***

$= a^2 + \frac{a^4-2a^2+1}{4}$ ***A1***

$= a^2 + \left(\frac{a^2-1}{2}\right)^2 \; (=\text{LHS})$ ***AG***

Note: Do not award the final ***A1*** if further working contradicts the ***AG***.

[3 marks]

continued…

Question 4 continued

(b) recognise base and height as a and $\left(\frac{a^2-1}{2}\right)$ (may be seen in diagram) ***(M1)***

correct substitution into triangle area formula ***A1***

$$\text{Area} = \frac{a}{2}\left(\frac{a^2-1}{2}\right) \text{ (or equivalent) } \left(= \frac{a\left(a^2-1\right)}{4} = \frac{a^3-a}{4}\right)$$

[2 marks]

Total [5 marks]

5. recognizing need to integrate ***(M1)***

$\int \frac{6x}{x^2+1}\mathrm{d}x$ OR $u = x^2+1$ OR $\frac{\mathrm{d}u}{\mathrm{d}x} = 2x$

$\int \frac{3}{u}\mathrm{d}u$ OR $3\int \frac{2x}{x^2+1}\mathrm{d}x$ ***(A1)***

$= 3\ln\left(x^2+1\right)(+c)$ or $3\ln u(+c)$ ***A1***

correct substitution of $x=1$ and $f(x)=5$ or $x=1$ and $u=2$ into equation

using **their** integrated expression (must involve *c*) ***(M1)***

$5 = 3\ln 2 + c$

$f(x) = 3\ln\left(x^2+1\right) + 5 - 3\ln 2 \quad \left(= 3\ln\left(x^2+1\right) + 5 - \ln 8 = 3\ln\left(\frac{x^2+1}{2}\right) + 5\right)$

(or equivalent) ***A1***

Note: Accept the use of the modulus sign in working and the final answer.

[5 marks]

6. (a) $\mathrm{P}(A \cap B) = 0.24$ ***A1***

[1 mark]

(b) $\mathrm{P}(A \cup B) = 1.1 - \mathrm{P}(A \cap B)$ ***(A1)***

$(0 \le)\mathrm{P}(A \cup B) \le 1$ ***(M1)***

Note: This may be conveyed in a clearly labelled diagram or written explanation where $\mathrm{P}(A \cup B) = 1$

the minimum value of $\mathrm{P}(A \cap B)$ is 0.1 ***A1***

[3 marks]

(c) A is a subset of B (so $\mathrm{P}(A \cap B) = \mathrm{P}(A)$). ***R1***

Note: This may be conveyed in a clearly labelled diagram where A is completely inside B, or in a written explanation indicating that $\mathrm{P}(A \cap B) = \mathrm{P}(A)$

so the maximum value of $\mathrm{P}(A \cap B)$ is 0.3 ***A1***

Note: Do not award ***R0A1***.

[2 marks]

Total [6 marks]

Section B

7. (a) correct substitution of $h=3$ and $k=2$ into $f(x)$ ***(A1)***

$f(x)=a(x-3)^2+2$

correct substitution of $(5,0)$ ***(A1)***

$0=a(5-3)^2+2 \quad \left(a=-\frac{1}{2}\right)$

Note: The first two A marks are independent.

$f(x)=-\frac{1}{2}(x-3)^2+2$ ***A1***

[3 marks]

(b) (i) **METHOD 1**

correct substitution of $(1, 4)$ ***(A1)***

$p+(t-1)-p=4$

$t=5$ ***A1***

substituting their value of t into $9p-3(t-1)-p=4$ ***(M1)***

$8p-12=4$

$p=2$ ***A1***

METHOD 2

correct substitution of ONE of the coordinates $(-3,4)$ or $(1,4)$ ***(A1)***

$9p-3(t-1)-p=4$ OR $p+(t-1)-p=4$

valid attempt to solve their two equations ***(M1)***

$p=2, t=5$ ***A1A1***

$\left(g(x)=2x^2+4x-2\right)$

(ii) attempt to find the x-coordinate of the vertex ***(M1)***

$x=\frac{-3+1}{2}(=-1)$ OR $\frac{-4}{2\times 2}$ OR $4x+4=0$ OR $2(x+1)^2-4$

y-coordinate of the vertex $=-4$ ***(A1)***

correct range ***A1***

$[-4,+\infty[$ OR $y\geq -4$ OR $g\geq -4$ OR $[-4,\infty)$

[7 marks]

(c) equating the two functions or equations **(M1)**

$g(x)=j(x)$ OR $px^2+(t-1)x-p=-x+3p$

$px^2+tx-4p=0$ **(A1)**

attempt to find discriminant (do not accept only in quadratic formula) **(M1)**

$\Delta=t^2+16p^2$ **A1**

$\Delta=t^2+16p^2>0$, because $t^2\geq 0$ and $p^2>0$, therefore the sum will be positive **R1R1**

Note: Award ***R1*** for recognising that Δ is positive and ***R1*** for the reason.

There are two distinct points of intersection between the graphs of g and j. **AG**

[6 marks]

Total [16 marks]

8. (a) (i) valid approach to find the required logarithm ***(M1)***

$2^x = \frac{1}{16}$ OR $2^x = 2^{-4}$ OR $\frac{1}{16} = 2^{-4}$ OR $\log_2 1 - \log_2 16$

$\log_2 \frac{1}{16} = -4$ ***A1***

(ii) valid approach to find the required logarithm ***(M1)***

$9^x = 3$ OR $3^{2x} = 3$ OR $3 = 9^{\frac{1}{2}}$ OR $\frac{\log_3 3}{\log_3 9}$

$\log_9 3 = \frac{1}{2}$ ***A1***

(iii) $\left(\sqrt{3}\right)^x = 81$ OR $\frac{\log_3 81}{\log_3 \sqrt{3}}$ ***(A1)***

$(3)^{\frac{x}{2}} = 3^4$ OR $\frac{x}{2} = 4$ OR $\frac{4}{\frac{1}{2}}$ ***(A1)***

$x = 8$ ***A1***

[7 marks]

continued…

Question 8 continued

(b) (i)

Note: There are many valid approaches to the question, and the steps may be seen in different ways. Some possible methods are given here, but candidates may use a combination of one or more of these methods. In all methods, the final ***A*** mark is awarded for working which leads directly to the ***AG***.

METHOD 1

$(ab)^3 = a$ ***(A1)***

attempt to isolate b or a power of b ***(M1)***

correct working ***(A1)***

$b = \frac{a}{a^3b^2}$ OR $b^3 = a^{-2}$ OR $b^{-1} = (ab)^2$ OR $b^3 = \frac{1}{a^2}$

$b = \frac{1}{a^2b^2}$ OR $b = (ab)^{-2}$ OR $3\log_{ab} b = -2\log_{ab} a$ OR $-\log_{ab} b = 2\log_{ab} ab$ ***A1***

$\log_{ab} b = -2$ ***AG***

METHOD 2

$(ab)^3 = a$ ***(A1)***

taking logarithm to base ab on both sides ***(M1)***

$\log_{ab}(ab)^3 = \log_{ab} a$ OR $\log_{ab} a^3b^3 = \log_{ab} a$

correct application of log rules leading to equation in terms of $\log_{ab}$ ***(A1)***

$3\log_{ab} a + 3\log_{ab} b = \log_{ab} a$ OR $3\log_{ab} b = -2\log_{ab} a$ OR $\log_{ab} b^3 = \log_{ab} a^{-2}$

$\log_{ab} b = \log_{ab} a^{-\frac{2}{3}}$ OR $\log_{ab} b = -\frac{2}{3}\log_{ab} a$ OR $\log_{ab} b = -\frac{2}{3}(3)$ ***A1***

$\log_{ab} b = -2$ ***AG***

Note: Candidates may substitute $\log_{ab} a = 3$ at any point in their working.

continued…

Question 8 continued

METHOD 3

$\log_{ab} a = 3$

writing in terms of base a ***(M1)***

$$\frac{\log_a a}{\log_a ab}(=3)$$

correct application of log rules ***(A1)***

$$\frac{\log_a a}{\log_a a + \log_a b}(=3) \quad \text{OR} \quad \frac{1}{1+\log_a b}(=3) \quad \text{OR} \quad 3\log_a b = -2 \quad \text{OR}$$

$$\log_a b = -\frac{2}{3}$$

writing $\log_{ab} b$ in terms of base a ***(A1)***

$$\log_{ab} b = \frac{\log_a b}{\log_a a + \log_a b}$$

correct working ***A1***

$$\log_{ab} b = \frac{-\frac{2}{3}}{1-\frac{2}{3}} \quad \text{OR} \quad \frac{\left(-\frac{2}{3}\right)}{\left(\frac{1}{3}\right)}$$

$\log_{ab} b = -2$ ***AG***

continued…

Question 8 continued

METHOD 4

$\log_{ab} ab = 1$ ***A2***

$\log_{ab} a + \log_{ab} b = 1$ ***(A1)***

$3 + \log_{ab} b = 1$ ***A1***

$\log_{ab} b = -2$ ***AG***

(ii) applying the quotient rule or product rule for logs

$\log_{ab} \frac{\sqrt[3]{a}}{\sqrt{b}} = \log_{ab} \sqrt[3]{a} - \log_{ab} \sqrt{b}$ OR $\log_{ab} \frac{\sqrt[3]{a}}{\sqrt{b}} = \log_{ab} \sqrt[3]{a} + \log_{ab} \frac{1}{\sqrt{b}}$ ***(A1)***

correct working ***(A1)***

$= \frac{1}{3}\log_{ab} a - \frac{1}{2}\log_{ab} b$ OR $\log_{ab} ab - \log_{ab} \sqrt{b}$

$= \frac{1}{3} \cdot 3 - \frac{1}{2}(-2)$ ***(A1)***

$= 2$ ***A1***

Note: Award ***A1A0A0A1*** for a correct answer with no working.

[8 marks]

Total [15 marks]

9. (a) $\cos^2 x - 3\sin^2 x = 0$

valid attempt to reduce equation to one involving one trigonometric function ***(M1)***

$\frac{\sin^2 x}{\cos^2 x} = \frac{1}{3}$ OR $1 - \sin^2 x - 3\sin^2 x = 0$ OR $\cos^2 x - 3(1 - \cos^2 x) = 0$

OR $\cos 2x - 1 + \cos 2x = 0$

correct equation ***(A1)***

$\tan^2 x = \frac{1}{3}$ OR $\cos^2 x = \frac{3}{4}$ OR $\sin^2 x = \frac{1}{4}$ OR $\cos 2x = \frac{1}{2}$

$\tan x = \pm\frac{1}{\sqrt{3}}$ OR $\cos x = \pm\frac{\sqrt{3}}{2}$ OR $\sin x = (\pm)\frac{1}{2}$ OR $2x = \frac{\pi}{3}\left(, \frac{5\pi}{3}\right)$ ***(A1)***

$x = \frac{\pi}{6},\ x = \frac{5\pi}{6}$ ***A1A1***

Note: Award ***M1A1A0A1A0*** for candidates who omit the $\pm$ (for tan or cos) and give only $x = \frac{\pi}{6}$.

Award ***M1A1A0A0A0*** for candidates who omit the $\pm$ (for tan or cos) and give only $x = 30°$.

Award ***M1A1A1A1A0*** for candidates who give both answers in degrees.

Award ***M1A1A1A1A0*** for candidates who give both correct answers in radians, but who include additional solutions outside the domain.

Award a maximum of ***M1A0A0A1A1*** for correct answers with no working.

[5 marks]

continued…

Question 9 continued

(b) (i) attempt to use the chain rule (may be evidenced by at least one $\cos x \sin x$ term) ***(M1)***

$f'(x) = -2\cos x \sin x - 6\sin x \cos x \left(= -8\sin x \cos x = -4\sin 2x\right)$ ***A1***

(ii) valid attempt to solve their $f'(x) = 0$ ***(M1)***

At least 2 correct x-coordinates (may be seen in coordinates) ***(A1)***

$x = 0$, $x = \dfrac{\pi}{2}$, $x = \pi$

Note:	Accept additional correct solutions outside the domain. Award ***A0*** if any additional incorrect solutions are given.

correct coordinates (may be seen in graph for part (c)) ***A1A1A1***

$(0,1)$, $(\pi,1)$, $\left(\dfrac{\pi}{2},-3\right)$

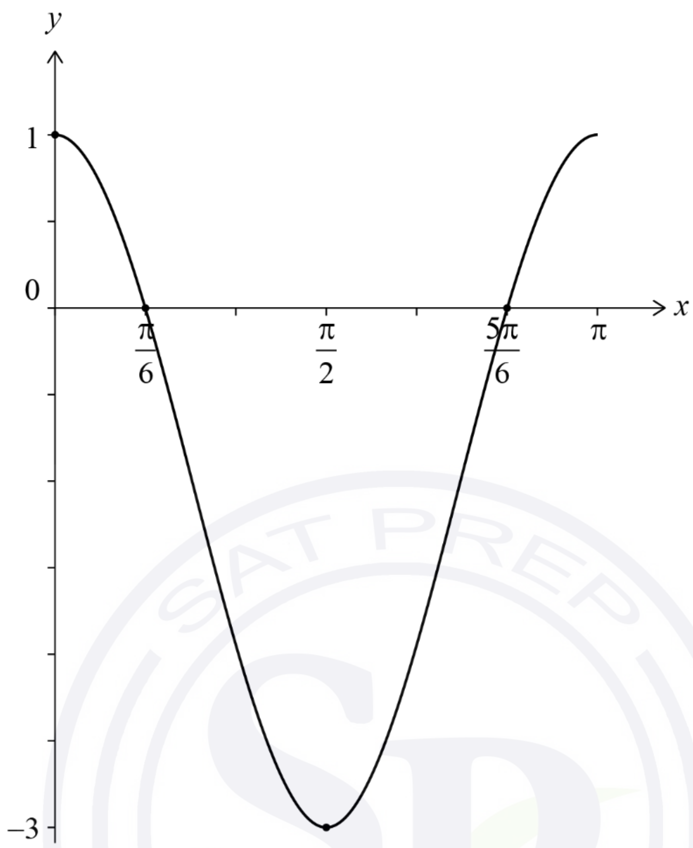
Note:	Award a maximum of ***M1A1A1A1A0*** if any additional solutions are given.
Note:	If candidates do not find at least two correct x-coordinates, it is possible to award the appropriate final marks for their correct coordinates, such as ***M1A0A0A1A0.***

[7 marks]

continued…

Question 9 continued

(c)



Note: In this question do not award follow through from incorrect values found in earlier parts.

approximately correct smooth curve with minimum at $\left(\frac{\pi}{2}, -3\right)$ ***A1***

Note: If candidates do not gain this mark then award no further marks.

endpoints at $(0,1)$, $(\pi,1)$, x-intercepts at $\frac{\pi}{6}$, $\frac{5\pi}{6}$ ***A1***

correct concavity clearly shown at $(0,1)$ and $(\pi,1)$ ***A1***

Note: The final two marks may be awarded independently of each other.

[3 marks]

Total [15 marks]

Markscheme

May 2022

Mathematics: analysis and approaches

Standard level

Paper 1

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

R Marks awarded for clear **Reasoning**.

AG Answer given in the question and so no marks are awarded.

FT Follow through. The practice of awarding marks, despite candidate errors in previous parts, for their correct methods/answers using incorrect results.

Using the markscheme

1 General

Award marks using the annotations as noted in the markscheme *eg* ***M1, A2***.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***A3***, ***M2*** *etc*., do **not** split the marks, unless there is a note.
- The response to a "show that" question does not need to restate the ***AG*** line, unless a **Note** makes this explicit in the markscheme.
- Once a correct answer to a question or part question is seen, ignore further working even if this working is incorrect and/or suggests a misunderstanding of the question. This will encourage a uniform approach to marking, with less examiner discretion. Although some candidates may be advantaged for that specific question item, it is likely that these candidates will lose marks elsewhere too.
- An exception to the previous rule is when an incorrect answer from further working is used **in a subsequent part**. For example, when a correct exact value is followed by an incorrect decimal approximation in the first part and this approximation is then used in the second part. In this situation, award ***FT*** marks as appropriate but do not award the final ***A1*** in the first part.

Examples:

	Correct answer seen	**Further working seen**	**Any FT issues?**	**Action**
1.	$8\sqrt{2}$	$5.65685...$ *(incorrect decimal value)*	No. Last part in question.	Award ***A1*** for the final mark *(condone the incorrect further working)*
2.	$\frac{35}{72}$	$0.468111\ldots$ (incorrect decimal value)	Yes. Value is used in subsequent parts.	Award ***A0*** for the final mark *(and full **FT** is available in subsequent parts)*

3 Implied marks

Implied marks appear in **brackets e.g. *(M1)***,and can only be awarded if **correct** work is seen or implied by subsequent working/answer.

4 Follow through marks (only applied after an error is made)

Follow through (***FT***) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) (e.g. incorrect value from part (a) used in part (d) or incorrect value from part (c)(i) used in part (c)(ii)). Usually, to award ***FT*** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if all the marks awarded in a subsequent part are for the answer or are implied, then ***FT*** marks should be awarded for *their* correct answer, even when working is not present.

For example: following an incorrect answer to part (a) that is used in subsequent parts, where the markscheme for the subsequent part is ***(M1)A1***, it is possible to award full marks for *their* correct answer, **without working being seen.** For longer questions where all but the answer marks are implied this rule applies but may be overwritten by a **Note** in the Markscheme.

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate.

- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks, by reflecting on what each mark is for and how that maps to the simplified version.

- If the error leads to an inappropriate value (*e.g*. probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).

- The markscheme may use the word “their” in a description, to indicate that candidates may be using an incorrect value.

- If the candidate's answer to the initial question clearly contradicts information given in the question, it is not appropriate to award any ***FT*** marks in the subsequent parts. This includes when candidates fail to complete a "show that" question correctly, and then in subsequent parts use their incorrect answer rather than the given value.

- Exceptions to these ***FT*** rules will be explicitly noted on the markscheme.

- If a candidate makes an error in one part but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the command term was "Hence".

5 Mis-read

If a candidate incorrectly copies values or information from the question, this is a mis-read (***MR***). A candidate should be penalized only once for a particular misread. Use the ***MR*** stamp to indicate that this has been a misread and do not award the first mark, even if this is an ***M*** mark, but award all others as appropriate.

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.

- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).

- Miscopying of candidates' own work does **not** constitute a misread, it is an error.

- If a candidate uses a correct answer, to a "show that" question, to a higher degree of accuracy than given in the question, this is NOT a misread and full marks may be scored in the subsequent part.

- ***MR*** can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If the command term is 'Hence' and not 'Hence or otherwise' then alternative methods are not permitted unless covered by a note in the mark scheme.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**.

7 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation** for example 1.9 and $1{,}9$ or 1000 and $1{,}000$ and 1.000.

- Do not accept final answers written using calculator notation. However, ***M*** marks and intermediate ***A*** marks can be scored, when presented using calculator notation, provided the evidence clearly reflects the demand of the mark.

- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.

- In the markscheme, some **equivalent** answers will generally appear in brackets. Not all equivalent notations/answers/methods will be presented in the markscheme and examiners are asked to apply appropriate discretion to judge if the candidate work is equivalent.

8 Format and accuracy of answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. If the level of accuracy is not stated in the question, the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures.*

Where values are used in subsequent parts, the markscheme will generally use the exact value, however candidates may also use the correct answer to 3 sf in subsequent parts. The markscheme will often explicitly include the subsequent values that come “*from the use of 3 sf values*”.

Simplification of final answers: Candidates are advised to give final answers using good mathematical form. In general, for an ***A*** mark to be awarded, arithmetic should be completed, and any values that lead to integers should be simplified; for example, $\sqrt{\frac{25}{4}}$ should be written as $\frac{5}{2}$. An exception to this is simplifying fractions, where lowest form is not required (although the numerator and the denominator must be integers); for example, $\frac{10}{4}$ may be left in this form or written as $\frac{5}{2}$. However, $\frac{10}{5}$ should be written as 2, as it simplifies to an integer.

Algebraic expressions should be simplified by completing any operations such as addition and multiplication, e.g. $4e^{2x} \times e^{3x}$ should be simplified to $4e^{5x}$, and $4e^{2x} \times e^{3x} - e^{4x} \times e^{x}$ should be simplified to $3e^{5x}$. Unless specified in the question, expressions do not need to be factorized, nor do factorized expressions need to be expanded, so $x(x+1)$ and $x^2 + x$ are both acceptable.

Please note: intermediate ***A*** marks do NOT need to be simplified.

9 Calculators

No calculator is allowed. The use of any calculator on this paper is malpractice and will result in no grade awarded. If you see work that suggests a candidate has used any calculator, please follow the procedures for malpractice.

10. Presentation of candidate work

Crossed out work: If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work unless an explicit note from the candidate indicates that they would like the work to be marked.

More than one solution: Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise. If the layout of the responses makes it difficult to judge, examiners should apply appropriate discretion to judge which is “first”.

Section A

1. (a) $g(0) = -2$ ***A1***

[1 mark]

(b) evidence of using composite function ***(M1)***

$f(g(0))$ OR $f(-2)$

$(f \circ g)(0) = 8$ ***A1***

[2 marks]

(c) $x = 3$ ***A2***

[2 marks]

Total [5 marks]

2. (a) $u_1 = 12$ ***A1***

[1 mark]

(b) $15 - 3n = -33$ ***(A1)***

$n = 16$ ***A1***

[2 marks]

(c) valid approach to find d ***(M1)***

$u_2 - u_1 = 9 - 12$ OR recognize gradient is -3 OR attempts to solve $-33 = 12 + 15d$

$d = -3$ ***A1***

[2 marks]

Total [5 marks]

3. (a) $(n-1)+n+(n+1)$ ***(A1)***

$=3n$ ***A1***

which is always divisible by 3 ***AG***

[2 marks]

(b) $(n-1)^2+n^2+(n+1)^2$ $\left(= n^2-2n+1+n^2+n^2+2n+1\right)$ ***A1***

attempts to expand either $(n-1)^2$ or $(n+1)^2$ (do not accept n^2-1 or n^2+1) ***(M1)***

$=3n^2+2$ ***A1***

demonstrating recognition that 2 is not divisible by 3 or $\frac{2}{3}$ seen after correct expression divided by 3 ***R1***

$3n^2$ is divisible by 3 and so $3n^2+2$ is never divisible by 3

OR the first term is divisible by 3, the second is not

OR $3\left(n^2+\frac{2}{3}\right)$ OR $\frac{3n^2+2}{3}=n^2+\frac{2}{3}$

hence the sum of the squares is never divisible by 3 ***AG***

[4 marks]

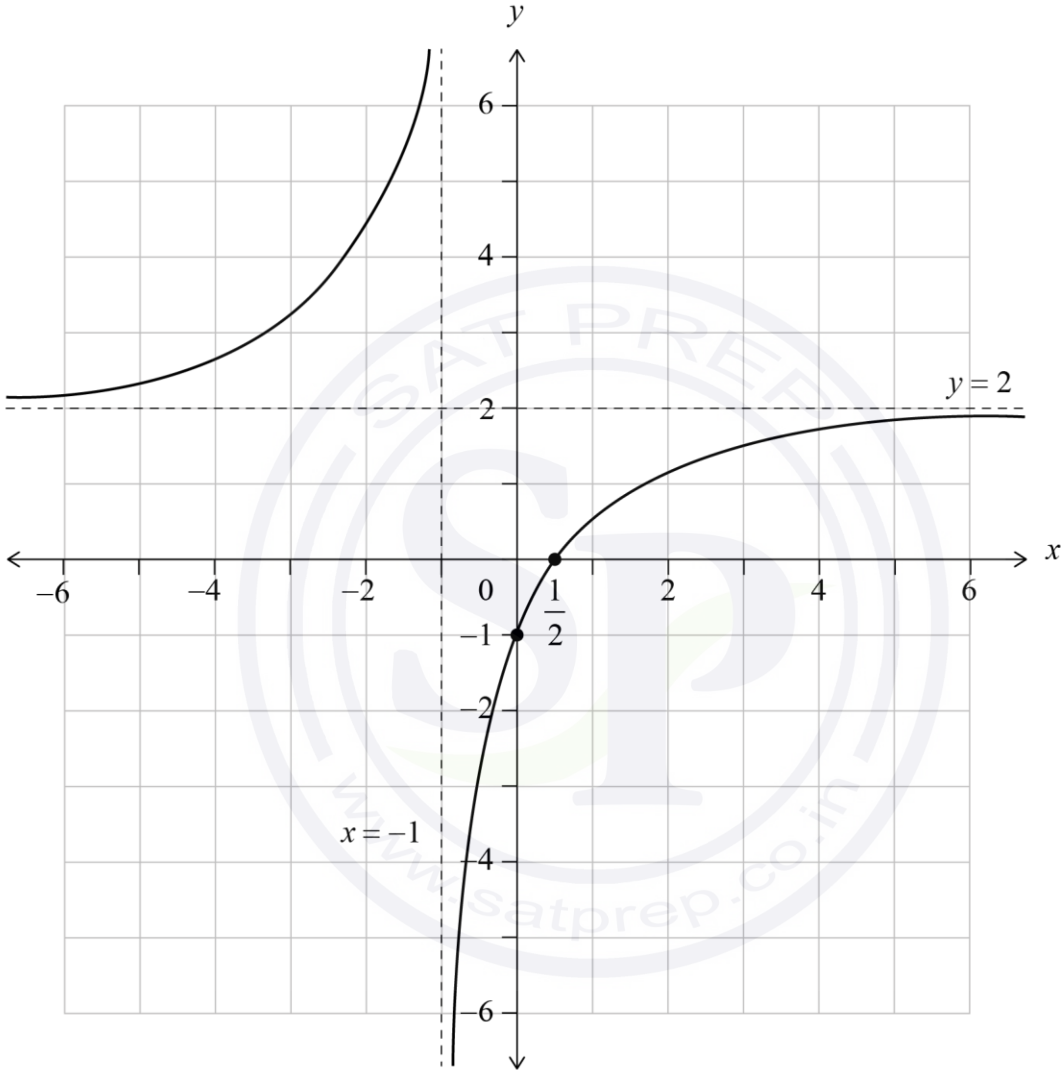
Total [6 marks]

4. (a) (i) $x = -1$ ***A1***

(ii) $y = 2$ ***A1***

[2 marks]

(b)



continued…

Question 4 continued

rational function shape with two branches in opposite quadrants, with two correctly positioned asymptotes and asymptotic behaviour shown ***A1***

Note: The equations of the asymptotes are not required on the graph provided there is a clear indication of asymptotic behaviour at $x=-1$ and $y=2$ (or at their FT asymptotes from part (a)).

axes intercepts clearly shown at $x=\frac{1}{2}$ and $y=-1$ ***A1A1***

[3 marks]

(c) $x>\frac{1}{2}$ ***A1***

Note: Accept correct alternative correct notation, such as $\left(\frac{1}{2}, \infty\right)$ and $\left]\frac{1}{2}, \infty\right[$.

[1 mark]

Total [6 marks]

5. determines $\frac{\pi}{4}$ (or $45°$) as the first quadrant (reference) angle ***(A1)***

attempts to solve $\frac{x}{2}+\frac{\pi}{3}=\frac{\pi}{4}$ ***(M1)***

Note: Award ***M1*** for attempting to solve $\frac{x}{2}+\frac{\pi}{3}=\frac{\pi}{4},\frac{7\pi}{4}(,...)$

$\frac{x}{2}+\frac{\pi}{3}=\frac{\pi}{4}\Rightarrow x<0$ and so $\frac{\pi}{4}$ is rejected ***(R1)***

$\frac{x}{2}+\frac{\pi}{3}=2\pi-\frac{\pi}{4}\ \left(=\frac{7\pi}{4}\right)$ ***A1***

$x=\frac{17\pi}{6}$ (must be in radians) ***A1***

[5 marks]

6. (a) **EITHER**

recognises the required term (or coefficient) in the expansion ***(M1)***

$bx^5 = {}^7C_2\, x^5 1^2$ OR $b = {}^7C_2$ OR ${}^7C_5$

$$b = \frac{7!}{2!5!} \left(= \frac{7!}{2!(7-2)!} \right)$$

correct working ***A1***

$$\frac{7\times6\times5\times4\times3\times2\times1}{2\times1\times5\times4\times3\times2\times1} \quad \text{OR} \quad \frac{7\times6}{2!} \quad \text{OR} \quad \frac{42}{2}$$

OR

lists terms from row 7 of Pascal's triangle ***(M1)***

$1, 7, 21, \ldots$ ***A1***

THEN

$b = 21$ ***AG***

[2 marks]

(b) $a = 7$ ***(A1)***

correct equation ***A1***

$$21x^5 = \frac{ax^6 + 35x^4}{2} \quad \text{OR} \quad 21x^5 = \frac{7x^6 + 35x^4}{2}$$

correct quadratic equation ***A1***

$7x^2 - 42x + 35 = 0$ OR $x^2 - 6x + 5 = 0$ (or equivalent)

valid attempt to solve **their** quadratic ***(M1)***

$$(x-1)(x-5) = 0 \quad \text{OR} \quad x = \frac{6 \pm \sqrt{(-6)^2 - 4(1)(5)}}{2(1)}$$

$x = 1,\ x = 5$ ***A1***

Note: Award final ***A0*** for obtaining $x = 0,\ x = 1, x = 5$.

[5 marks]

Total [7 marks]

Section B

7. (a) $x = 3$ ***A1***

Note: Must be an equation in the form " $x =$ ". Do not accept 3 or $\frac{-b}{2a} = 3$.

[1 mark]

(b) (i) $h = 3$, $k = 4$ (accept $a(x-3)^2 + 4$) ***A1A1***

(ii) attempt to substitute coordinates of Q ***(M1)***

$12 = a(5-3)^2 + 4$, $4a + 4 = 12$

$a = 2$ ***A1***

[4 marks]

(c) recognize need to find derivative of f ***(M1)***

$f'(x) = 4(x-3)$ or $f'(x) = 4x - 12$ ***A1***

$f'(5) = 8$ (may be seen as gradient in their equation) ***(A1)***

$y - 12 = 8(x-5)$ or $y = 8x - 28$ ***A1***

Note: Award ***A0*** for $L = 8x - 28$.

[4 marks]

continued…

Question 7 continued

(d) **METHOD 1**

Recognizing that for g to be increasing, $f(x)-d>0$, or $g'>0$ ***(M1)***

The vertex must be above the *x*-axis, $4-d>0$, $d-4<0$ ***(R1)***

$d<4$ ***A1***

[3 marks]

METHOD 2

attempting to find discriminant of g' ***(M1)***

$(-12)^2-4(2)(22-d)$

recognizing discriminant must be negative ***(R1)***

$-32+8d<0$ OR $\Delta<0$

$d<4$ ***A1***

[3 marks]

(e) recognizing that for g to be concave up, $g''>0$ ***(M1)***

$g''>0$ when $f'>0$, $4x-12>0$, $x-3>0$ ***(R1)***

$x>3$ ***A1***

[3 marks]

Total [15 marks]

8. (a) $\frac{1}{x-4}+1=x-3$ ***(M1)***

$x^2-8x+15=0$ OR $(x-4)^2=1$ ***(A1)***

valid attempt to solve **their** quadratic ***(M1)***

$(x-3)(x-5)=0$ OR $x=\frac{8\pm\sqrt{8^2-4(1)(15)}}{2(1)}$ OR $(x-4)=\pm1$

$x=5$ $(x=3,\ x=5)$ (may be seen in answer) ***A1***

$\mathrm{B}(5,\ 2)$ (accept $x=5$, $y=2$) ***A1***

[5 marks]

continued…

Question 8 continued

(b) recognizing two correct regions from $x=3$ to $x=5$ and from $x=5$ to $x=k$ ***(R1)***

$\text{triangle}+\int_5^k f(x)\text{d}x$ OR $\int_3^5 g(x)\text{d}x+\int_5^k f(x)\text{d}x$ OR $\int_3^5 (x-3)\text{d}x+\int_5^k\left(\frac{1}{x-4}+1\right)\text{d}x$

area of triangle is 2 OR $\frac{2\cdot 2}{2}$ OR $\left(\frac{5^2}{2}-3(5)\right)-\left(\frac{3^2}{2}-3(3)\right)$ ***(A1)***

correct integration ***(A1)(A1)***

$$\int\left(\frac{1}{x-4}+1\right)\text{d}x=\ln(x-4)+x\ (+C)$$

Note: Award ***A1*** for $\ln(x-4)$ and ***A1*** for x.
Note: The first three ***A*** marks may be awarded independently of the ***R*** mark.

substitution of **their** limits (for x) into **their** integrated function (in terms of x) ***(M1)***

$\ln(k-4)+k-(\ln 1+5)$

$[\ln(x-4)+x]_5^k=\ln(k-4)+k-5$ ***A1***

adding **their** two areas (in terms of k) and equating to $\ln p+8$ ***(M1)***

$2+\ln(k-4)+k-5=\ln p+8$

equating **their** non-log terms to 8 (equation must be in terms of k) ***(M1)***

$k-3=8$

$k=11$ ***A1***

$11-4=p$

$p=7$ ***A1***

[10 marks]

Total [15 marks]

9. (a) uses $\sum \mathrm{P}(X=x)=1$ to form a linear equation in p and q ***(M1)***

correct equation in terms of p and q from summing to 1 ***A1***

$p+0.3+q+0.1=1$ OR $p+q=0.6$ (or equivalent)

uses $\mathrm{E}(X)=2$ to form a linear equation in p and q ***(M1)***

correct equation in terms of p and q from $\mathrm{E}(X)=2$ ***A1***

$p+0.6+3q+0.4=2$ OR $p+3q=1$ (or equivalent)

Note: The marks for using $\sum \mathrm{P}(X=x)=1$ and the marks for using $\mathrm{E}(X)=2$ may be awarded independently of each other.

evidence of correctly solving these equations simultaneously ***A1***

for example, $2q=0.4 \Rightarrow q=0.2$ or $p+3\times(0.6-p)=1 \Rightarrow p=0.4$

so $p=0.4$ and $q=0.2$ ***AG***

[5 marks]

(b) valid approach ***(M1)***

$\mathrm{P}(X>2)=\mathrm{P}(X=3)+\mathrm{P}(X=4)$ OR $\mathrm{P}(X>2)=1-\mathrm{P}(X=1)-\mathrm{P}(X=2)$

$=0.3$ ***A1***

[2 marks]

continued…

Question 9 continued

(c) recognises at least one of the valid scores (6, 7, or 8) required to win the game ***(M1)***

Note: Award ***M0*** if candidate also considers scores other than 6, 7, or 8 (such as 5).

let T represent the score on the last two rolls

a score of 6 is obtained by rolling $(2,4),(4,2)$ or $(3,3)$

$\mathrm{P}(T=6)=2(0.3)(0.1)+(0.2)^2 \quad (=0.1)$ ***A1***

a score of 7 is obtained by rolling $(3,4)$ or $(4,3)$

$\mathrm{P}(T=7)=2(0.2)(0.1) \quad (=0.04)$ ***A1***

a score of 8 is obtained by rolling $(4,4)$

$\mathrm{P}(T=8)=(0.1)^2 \quad (=0.01)$ ***A1***

Note: The above 3 ***A1*** marks are independent of each other.

$\mathrm{P}(\text{Nicky wins})=0.1+0.04+0.01$

$=0.15$ ***A1***

[5 marks]

(d) $3+b=8$ ***(M1)***

$b=5$ ***A1***

[2 marks]

continued…

Question 9 continued

(e) **METHOD 1**

EITHER

$\mathrm{P}(S=5)=\frac{4}{16}$

$\mathrm{P}(S=a+2)=\frac{4}{16}$ ***A1***

$\Rightarrow a+2=5$

OR

$\mathrm{P}(S=6)=\frac{3}{16}$

$\mathrm{P}(S=a+3)=\frac{2}{16}$ and $\mathrm{P}(S=5+1)=\frac{1}{16}$ ***A1***

$\Rightarrow a+3=6$

OR

$\mathrm{P}(S=4)=\frac{3}{16}$

$\mathrm{P}(S=a+1)=\frac{2}{16}$ and $\mathrm{P}(S=1+3)=\frac{1}{16}$ ***A1***

$\Rightarrow a+1=4$

THEN

$\Rightarrow a=3$ ***A1***

Note: Award ***A0A0*** for $a=3$ obtained without working/reasoning/justification.

[2 marks]

continued…

Question 9 continued

METHOD 2

EITHER

correctly lists a relevant part of the sample space ***A1***

for example, $\{S=4\}=\{(3,1),(1,a),(1,a)\}$ or $\{S=5\}=\{(2,a),(2,a),(2,a),(2,a)\}$

or $\{S=6\}=\{(3,a),(3,a),(1,5)\}$

$a+3=6$

OR

eliminates possibilities (exhaustion) for $a<5$

convincingly shows that $a\neq 2,4$ ***A1***

$a\neq 4$, for example, $\mathrm{P}(S=7)=\dfrac{2}{16}$ from $(2,5),(2,5)$ and so

$(3,a),(3,a)\Rightarrow a+3\neq 7$

THEN

$\Rightarrow a=3$ ***A1***

[2 marks]

Total [16 marks]

M22/5/MATHX/SP1/ENG/TZ1/XX/M

Markscheme

May 2022

Mathematics: analysis and approaches

Standard level

Paper 1

25 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

R Marks awarded for clear **Reasoning**.

AG Answer given in the question and so no marks are awarded.

FT Follow through. The practice of awarding marks, despite candidate errors in previous parts, for their correct methods/answers using incorrect results.

Using the markscheme

1 General

Award marks using the annotations as noted in the markscheme *eg* ***M1, A2***.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***A3***, ***M2*** *etc*., do **not** split the marks, unless there is a note.
- The response to a “show that” question does not need to restate the ***AG*** line, unless a **Note** makes this explicit in the markscheme.
- Once a correct answer to a question or part question is seen, ignore further working even if this working is incorrect and/or suggests a misunderstanding of the question. This will encourage a uniform approach to marking, with less examiner discretion. Although some candidates may be advantaged for that specific question item, it is likely that these candidates will lose marks elsewhere too.
- An exception to the previous rule is when an incorrect answer from further working is used **in a subsequent part**. For example, when a correct exact value is followed by an incorrect decimal approximation in the first part and this approximation is then used in the second part. In this situation, award ***FT*** marks as appropriate but do not award the final ***A1*** in the first part.

Examples:

	Correct answer seen	**Further working seen**	**Any FT issues?**	**Action**
1.	$8\sqrt{2}$	$5.65685...$ *(incorrect decimal value)*	No. Last part in question.	Award ***A1*** for the final mark *(condone the incorrect further working)*
2.	$\frac{35}{72}$	$0.468111\ldots$ (incorrect decimal value)	Yes. Value is used in subsequent parts.	Award ***A0*** for the final mark *(and full **FT** is available in subsequent parts)*

3 Implied marks

Implied marks appear in **brackets e.g. *(M1)***,and can only be awarded if **correct** work is seen or implied by subsequent working/answer.

4 Follow through marks (only applied after an error is made)

Follow through (***FT***) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) (e.g. incorrect value from part (a) used in part (d) or incorrect value from part (c)(i) used in part (c)(ii)). Usually, to award ***FT*** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if all the marks awarded in a subsequent part are for the answer or are implied, then ***FT*** marks should be awarded for *their* correct answer, even when working is not present.

For example: following an incorrect answer to part (a) that is used in subsequent parts, where the markscheme for the subsequent part is ***(M1)A1***, it is possible to award full marks for *their* correct answer, **without working being seen.** For longer questions where all but the answer marks are implied this rule applies but may be overwritten by a **Note** in the Markscheme.

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks, by reflecting on what each mark is for and how that maps to the simplified version.
- If the error leads to an inappropriate value (*e.g*. probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word “their” in a description, to indicate that candidates may be using an incorrect value.

- If the candidate's answer to the initial question clearly contradicts information given in the question, it is not appropriate to award any ***FT*** marks in the subsequent parts. This includes when candidates fail to complete a "show that" question correctly, and then in subsequent parts use their incorrect answer rather than the given value.

- Exceptions to these ***FT*** rules will be explicitly noted on the markscheme.

- If a candidate makes an error in one part but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the command term was "Hence".

5 Mis-read

If a candidate incorrectly copies values or information from the question, this is a mis-read (***MR***). A candidate should be penalized only once for a particular misread. Use the ***MR*** stamp to indicate that this has been a misread and do not award the first mark, even if this is an ***M*** mark, but award all others as appropriate.

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.

- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).

- Miscopying of candidates' own work does **not** constitute a misread, it is an error.

- If a candidate uses a correct answer, to a "show that" question, to a higher degree of accuracy than given in the question, this is NOT a misread and full marks may be scored in the subsequent part.

- ***MR*** can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If the command term is 'Hence' and not 'Hence or otherwise' then alternative methods are not permitted unless covered by a note in the mark scheme.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**.

7 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation** for example 1.9 and $1{,}9$ or 1000 and $1{,}000$ and 1.000.

- Do not accept final answers written using calculator notation. However, ***M*** marks and intermediate ***A*** marks can be scored, when presented using calculator notation, provided the evidence clearly reflects the demand of the mark.

- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.

- In the markscheme, some **equivalent** answers will generally appear in brackets. Not all equivalent notations/answers/methods will be presented in the markscheme and examiners are asked to apply appropriate discretion to judge if the candidate work is equivalent.

8 Format and accuracy of answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. If the level of accuracy is not stated in the question, the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures.*

Where values are used in subsequent parts, the markscheme will generally use the exact value, however candidates may also use the correct answer to 3 sf in subsequent parts. The markscheme will often explicitly include the subsequent values that come "*from the use of 3 sf values*".

Simplification of final answers: Candidates are advised to give final answers using good mathematical form. In general, for an ***A*** mark to be awarded, arithmetic should be completed, and any values that lead to integers should be simplified; for example, $\sqrt{\frac{25}{4}}$ should be written as $\frac{5}{2}$. An exception to this is simplifying fractions, where lowest form is not required (although the numerator and the denominator must be integers); for example, $\frac{10}{4}$ may be left in this form or written as $\frac{5}{2}$. However, $\frac{10}{5}$ should be written as 2, as it simplifies to an integer.

Algebraic expressions should be simplified by completing any operations such as addition and multiplication, e.g. $4e^{2x} \times e^{3x}$ should be simplified to $4e^{5x}$, and $4e^{2x} \times e^{3x} - e^{4x} \times e^{x}$ should be simplified to $3e^{5x}$. Unless specified in the question, expressions do not need to be factorized, nor do factorized expressions need to be expanded, so $x(x+1)$ and $x^2 + x$ are both acceptable.

Please note: intermediate ***A*** marks do NOT need to be simplified.

9 Calculators

No calculator is allowed. The use of any calculator on this paper is malpractice and will result in no grade awarded. If you see work that suggests a candidate has used any calculator, please follow the procedures for malpractice.

10. Presentation of candidate work

Crossed out work: If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work unless an explicit note from the candidate indicates that they would like the work to be marked.

More than one solution: Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise. If the layout of the responses makes it difficult to judge, examiners should apply appropriate discretion to judge which is "first".

Section A

1. (a) $m_{\text{BC}} = \dfrac{12-6}{-14-4}\ \left(= -\dfrac{1}{3}\right)$ ***(A1)***

finding $m_L = \dfrac{-1}{m_{\text{BC}}}$ using their m_{BC} ***(M1)***

$m_L = 3$

$y - 20 = 3(x+2)$, $y = 3x + 26$ ***A1***

Note: Do not accept $L = 3x + 26$.

[3 marks]

(b) substituting $(k, 2)$ into their *L* ***(M1)***

$2 - 20 = 3(k+2)$ OR $2 = 3k + 26$

$k = -8$ ***A1***

[2 marks]

Total [5 marks]

2. (a) $\dfrac{3\sqrt{x}-5}{\sqrt{x}} = 3-5x^{-\frac{1}{2}}$ ***A1***

$p = -\dfrac{1}{2}$

[1 mark]

(b) $\displaystyle\int \frac{3\sqrt{x}-5}{\sqrt{x}}\mathrm{d}x = 3x-10x^{\frac{1}{2}}\ (+c)$ ***A1A1***

substituting limits into their integrated function and subtracting ***(M1)***

$3(9)-10(9)^{\frac{1}{2}}-\left(3(1)-10(1)^{\frac{1}{2}}\right)$ OR $27-10\times 3-(3-10)$

$=4$ ***A1***

[4 marks]

Total [5 marks]

3. (a) $\text{IQR} = 10 - 6\ (= 4)$ ***(A1)***

attempt to find $Q_3 + 1.5 \times \text{IQR}$ ***(M1)***

$10 + 6$

16 ***A1***

[3 marks]

(b) (i) choosing $c = \frac{1}{2}a - 9$ ***(M1)***

$\frac{1}{2} \times 42 - 9$

= 12 (years old) ***A1***

(ii) attempt to solve system by substitution or elimination ***(M1)***

34 (years old) ***A1***

[4 marks]

Total [7 marks]

4. (a) $(f \circ g)(x) = f(2x)$ ***(A1)***

$f(2x) = \sqrt{3}\sin 2x + \cos 2x$ ***A1***

[2 marks]

(b) $\sqrt{3}\sin 2x + \cos 2x = 2\cos 2x$

$\sqrt{3}\sin 2x = \cos 2x$

recognising to use $\tan$ or $\cot$ ***M1***

$\tan 2x = \frac{1}{\sqrt{3}}$ OR $\cot 2x = \sqrt{3}$ (values may be seen in right triangle) ***(A1)***

$\left(\arctan\left(\frac{1}{\sqrt{3}}\right) =\right) \frac{\pi}{6}$ (seen anywhere) (accept degrees) ***(A1)***

$2x = \frac{\pi}{6}, \frac{7\pi}{6}$

$x = \frac{\pi}{12}, \frac{7\pi}{12}$ ***A1A1***

Note:	Do not award the final ***A1*** if any additional solutions are seen. Award ***A1A0*** for correct answers in degrees. Award ***A0A0*** for correct answers in degrees with additional values.

[5 marks]

Total [7 marks]

5. evidence of using product rule ***(M1)***

$\frac{dy}{dx} = (2x-1)\times\left(ke^{kx}\right) + 2\times e^{kx} \quad \left(= e^{kx}(2kx - k + 2)\right)$ ***A1***

correct working for one of (seen anywhere) ***A1***

$\frac{dy}{dx}$ at $x = 1 \Rightarrow ke^{k} + 2e^{k}$

OR

slope of tangent is $5e^{k}$

their $\frac{dy}{dx}$ at $x = 1$ equals the *slope* of $y = 5e^{k}x$ (= $5e^{k}$) (seen anywhere) ***(M1)***

$ke^{k} + 2e^{k} = 5e^{k}$

$k = 3$ ***A1***

[5 marks]

6. (a) translation (shift) by $\frac{3\pi}{2}$ to the right/positive horizontal direction ***A1***

translation (shift) by q upwards/positive vertical direction ***A1***

Note: accept translation by $\begin{pmatrix} \frac{3\pi}{2} \\ q \end{pmatrix}$

Do not accept 'move' for translation/shift.

[2 marks]

(b) **METHOD 1**

minimum of $4\sin\left(x-\frac{3\pi}{2}\right)$ is -4 (may be seen in sketch) ***(M1)***

$-4+2.5+q \geq 7$

$q \geq 8.5$ (accept $q = 8.5$) ***A1***

substituting $x=0$ and their q $(=8.5)$ to find r ***(M1)***

$(r=)$ $4\sin\left(\frac{-3\pi}{2}\right)+2.5+8.5$

$4+2.5+8.5$ ***(A1)***

smallest value of r is 15 ***A1***

continued…

Question 6 continued

METHOD 2

substituting $x = 0$ to find an expression (for r) in terms of q ***(M1)***

$\left(g(0) = r =\right) \quad 4\sin\left(\frac{-3\pi}{2}\right) + 2.5 + q$

$\left(r =\right) \ 6.5 + q$ ***A1***

minimum of $4\sin\left(x - \frac{3\pi}{2}\right)$ is -4 ***(M1)***

$-4 + 2.5 + q \geq 7$

$-4 + 2.5 + (r - 6.5) \geq 7$ (accept =) ***(A1)***

smallest value of *r* is 15 ***A1***

METHOD 3

$4\sin\left(x - \frac{3\pi}{2}\right) + 2.5 + q = 4\cos x + 2.5 + q$ ***A1***

y-intercept of $4\cos x + 2.5 + q$ is a maximum ***(M1)***

amplitude of $g(x)$ is 4 ***(A1)***

attempt to find least maximum ***(M1)***

$r = 2 \times 4 + 7$

smallest value of *r* is 15 ***A1***

[5 marks]

Total [7 marks]

Section B

7. (a) **EITHER**

attempt to use $x = -\frac{b}{2a}$ ***(M1)***

$q = -\frac{-12}{2 \times 3}$

OR

attempt to complete the square ***(M1)***

$3(x-2)^2 - 12 + p$

OR

attempt to differentiate and equate to 0 ***(M1)***

$f'(x) = 6x - 12 = 0$

THEN

$q = 2$ ***A1***

[2 marks]

continued…

Question 7 continued

(b) (i) discriminant $= 0$ ***A1***

(ii) **EITHER**

attempt to substitute into $b^2 - 4ac$ ***(M1)***

$(-12)^2 - 4 \times 3 \times p = 0$ ***A1***

OR

$f'(2) = 0$ ***(M1)***

$-12 + p = 0$ ***A1***

THEN

$p = 12$ ***A1***

[4 marks]

(c) $f''(x) = 6x - 12$ ***A1***

attempt to find $f''(0)$ ***(M1)***

$= 6 \times 0 - 12$

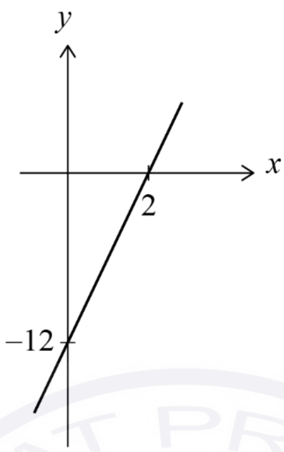
gradient = -12 ***A1***

[3 marks]

continued…

Question 7 continued

(d)



A1A1

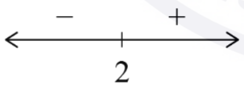
Note: Award ***A1*** for line with positive gradient, ***A1*** for correct intercepts.

[2 marks]

(e) (i) $a = 2$ ***A1***

(ii) $x < 2$ ***A1***

$f''(x) < 0$ (for $x < 2$) OR the f'' is below the x-axis (for $x < 2$)

OR 

f'' (sign diagram must be labelled f'') ***R1***

[3 marks]

Total [14 marks]

8. (a) (i) **EITHER**

attempt to use a ratio from consecutive terms ***M1***

$$\frac{p\ln x}{\ln x}=\frac{\frac{1}{3}\ln x}{p\ln x} \quad \text{OR} \quad \frac{1}{3}\ln x=(\ln x)r^2 \quad \text{OR} \quad p\ln x=\ln x\left(\frac{1}{3p}\right)$$

Note: Candidates may use $\ln x^1+\ln x^p+\ln x^{\frac{1}{3}}+\ldots$ and consider the powers of x in geometric sequence.

Award ***M1*** for $\frac{p}{1}=\frac{\frac{1}{3}}{p}$.

OR

$r=p$ and $r^2=\frac{1}{3}$ ***M1***

THEN

$p^2=\frac{1}{3}$ OR $r=\pm\frac{1}{\sqrt{3}}$ ***A1***

$p=\pm\frac{1}{\sqrt{3}}$ ***AG***

Note: Award ***M0A0*** for $r^2=\frac{1}{3}$ or $p^2=\frac{1}{3}$ with no other working seen.

(ii) $\frac{\ln x}{1-\frac{1}{\sqrt{3}}}$ $\left(=3+\sqrt{3}\right)$ ***(A1)***

$\ln x=3-\frac{3}{\sqrt{3}}+\sqrt{3}-\frac{\sqrt{3}}{\sqrt{3}}$ OR $\ln x=3-\sqrt{3}+\sqrt{3}-1$ $(\Rightarrow \ln x=2)$ ***A1***

$x=\mathrm{e}^2$ ***A1***

[5 marks]

continued…

Question 8 continued

(b) (i) **METHOD 1**

attempt to find a difference from consecutive terms or from u_2 ***M1***

correct equation ***A1***

$p\ln x - \ln x = \frac{1}{3}\ln x - p\ln x$ OR $\frac{1}{3}\ln x = \ln x + 2\left(p\ln x - \ln x\right)$

Note: Candidates may use $\ln x^1 + \ln x^p + \ln x^{\frac{1}{3}} + \ldots$ and consider the powers of x in arithmetic sequence.
Award ***M1A1*** for $p - 1 = \frac{1}{3} - p$.

$2p\ln x = \frac{4}{3}\ln x \quad \left(\Rightarrow 2p = \frac{4}{3}\right)$ ***A1***

$p = \frac{2}{3}$ ***AG***

METHOD 2

attempt to use arithmetic mean $u_2 = \frac{u_1 + u_3}{2}$ ***M1***

$p\ln x = \frac{\ln x + \frac{1}{3}\ln x}{2}$ ***A1***

$2p\ln x = \frac{4}{3}\ln x \quad \left(\Rightarrow 2p = \frac{4}{3}\right)$ ***A1***

$p = \frac{2}{3}$ ***AG***

continued…

Question 8 continued

METHOD 3

attempt to find difference using u_3 ***M1***

$$\frac{1}{3}\ln x = \ln x + 2d \quad \left(\Rightarrow d = -\frac{1}{3}\ln x\right)$$

$$u_2 = \ln x + \frac{1}{2}\left(\frac{1}{3}\ln x - \ln x\right) \text{ OR } p\ln x - \ln x = -\frac{1}{3}\ln x$$ ***A1***

$$p\ln x = \frac{2}{3}\ln x$$ ***A1***

$$p = \frac{2}{3}$$ ***AG***

(ii) $d = -\frac{1}{3}\ln x$ ***A1***

continued…

Question 8 continued

(iii) **METHOD 1**

$$S_n = \frac{n}{2}\left[2\ln x + (n-1)\times\left(-\frac{1}{3}\ln x\right)\right]$$

attempt to substitute into S_n and equate to $-3\ln x$ ***(M1)***

$$\frac{n}{2}\left[2\ln x + (n-1)\times\left(-\frac{1}{3}\ln x\right)\right] = -3\ln x$$

correct working with S_n (seen anywhere) ***(A1)***

$$\frac{n}{2}\left[2\ln x - \frac{n}{3}\ln x + \frac{1}{3}\ln x\right] \text{ OR } n\ln x - \frac{n(n-1)}{6}\ln x \text{ OR}$$
$$\frac{n}{2}\left(\ln x + \left(\frac{4-n}{3}\right)\ln x\right)$$

correct equation without $\ln x$ ***A1***

$$\frac{n}{2}\left(\frac{7}{3} - \frac{n}{3}\right) = -3 \text{ OR } n - \frac{n(n-1)}{6} = -3 \text{ or equivalent}$$

> **Note:** Award as above if the series $1 + p + \frac{1}{3} + \ldots$ is considered leading to
> $\frac{n}{2}\left(\frac{7}{3} - \frac{n}{3}\right) = -3$.

attempt to form a quadratic $= 0$ ***(M1)***

$$n^2 - 7n - 18 = 0$$

attempt to solve their quadratic ***(M1)***

$$(n-9)(n+2) = 0$$

$$n = 9$$ ***A1***

continued…

Question 8 continued

METHOD 2

listing the first 7 terms of the sequence ***(A1)***

$$\ln x + \frac{2}{3}\ln x + \frac{1}{3}\ln x + 0 - \frac{1}{3}\ln x - \frac{2}{3}\ln x - \ln x + \ldots$$

recognizing first 7 terms sum to 0 ***M1***

8th term is $-\frac{4}{3}\ln x$ ***(A1)***

9th term is $-\frac{5}{3}\ln x$ ***(A1)***

sum of 8th and 9th terms $= -3\ln x$ ***(A1)***

$n = 9$ ***A1***

[10 marks]

Total [15 marks]

9. (a) (i) **EITHER**

attempt to use binomial expansion ***(M1)***

$1 + {}^3C_1 \times 1 \times (-a) + {}^3C_2 \times 1 \times (-a)^2 + 1 \times (-a)^3$

OR

$(1-a)(1-a)(1-a)$

$= (1-a)(1-2a+a^2)$ ***(M1)***

THEN

$= 1 - 3a + 3a^2 - a^3$ ***A1***

(ii) $a = \cos 2x$ ***(A1)***

So, $1 - 3\cos 2x + 3\cos^2 2x - \cos^3 2x =$

$(1-\cos 2x)^3$ ***A1***

attempt to substitute any double angle rule for $\cos 2x$ into $(1-\cos 2x)^3$ ***(M1)***

$= (2\sin^2 x)^3$ ***A1***

$= 8\sin^6 x$ ***AG***

Note: Allow working RHS to LHS.

[6 marks]

continued…

Question 9 continued

(b) (i) recognizing to integrate $\int\left(4\cos x \times 8\sin^6 x\right)\mathrm{d}x$ ***(M1)***

EITHER

applies integration by inspection ***(M1)***

$32\int\left(\cos x \times (\sin x)^6\right)\mathrm{d}x$

$=\frac{32}{7}\sin^7 x\ (+c)$ ***A1***

$\left[\frac{32}{7}\sin^7 x\right]_0^m \quad \left(=\frac{32}{7}\sin^7 m-\frac{32}{7}\sin^7 0\right)$ ***A1***

OR

$u=\sin x \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x}=\cos x$ ***(M1)***

$\int 32\cos x\,(\sin^6 x)\,\mathrm{d}x=\int 32u^6\mathrm{d}u$

$=\frac{32}{7}u^7\ (+c)$ ***A1***

$\left[\frac{32}{7}\sin^7 x\right]_0^m$ OR $\left[\frac{32}{7}u^7\right]_0^{\sin m} \quad \left(=\frac{32}{7}\sin^7 m-\frac{32}{7}\sin^7 0\right)$ ***A1***

THEN

$=\frac{32}{7}\sin^7 m$ ***AG***

continued…

Question 9 continued

(ii) **EITHER**

$$\int_m^{\frac{\pi}{2}} f(x)\,\mathrm{d}x \left(=\left[\frac{32}{7}\sin^7 x\right]_m^{\frac{\pi}{2}}\right)=\frac{32}{7}\sin^7\frac{\pi}{2}-\frac{32}{7}\sin^7 m$$ ***M1***

$$\frac{32}{7}\sin^7\frac{\pi}{2}-\frac{32}{7}\sin^7 m=\frac{127}{28} \quad \text{OR} \quad \frac{32}{7}\left(1-\sin^7 m\right)=\frac{127}{28}$$ ***(M1)***

OR

$$\int_0^{\frac{\pi}{2}} f(x)\,\mathrm{d}x=\int_0^{m} f(x)\,\mathrm{d}x+\int_m^{\frac{\pi}{2}} f(x)\,\mathrm{d}x$$ ***M1***

$$\frac{32}{7}=\frac{32}{7}\sin^7 m+\frac{127}{28}$$ ***(M1)***

THEN

$$\sin^7 m=\frac{1}{128}\ \left(=\frac{1}{2^7}\right)$$ ***(A1)***

$$\sin m=\frac{1}{2}$$ ***(A1)***

$$m=\frac{\pi}{6}$$ ***A1***

[9 marks]

Total [15 marks]

Markscheme

November 2021

Mathematics: analysis and approaches

Standard level

Paper 1

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

R Marks awarded for clear **Reasoning**.

AG Answer given in the question and so no marks are awarded.

FT Follow through. The practice of awarding marks, despite candidate errors in previous parts, for their correct methods/answers using incorrect results.

Using the markscheme

1 General

Award marks using the annotations as noted in the markscheme *eg* ***M1, A2***.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***A3***, ***M2*** *etc*., do **not** split the marks, unless there is a note.
- The response to a “show that” question does not need to restate the ***AG*** line, unless a **Note** makes this explicit in the markscheme.
- Once a correct answer to a question or part question is seen, ignore further working even if this working is incorrect and/or suggests a misunderstanding of the question. This will encourage a uniform approach to marking, with less examiner discretion. Although some candidates may be advantaged for that specific question item, it is likely that these candidates will lose marks elsewhere too.
- An exception to the previous rule is when an incorrect answer from further working is used **in a subsequent part**. For example, when a correct exact value is followed by an incorrect decimal approximation in the first part and this approximation is then used in the second part. In this situation, award ***FT*** marks as appropriate but do not award the final ***A1*** in the first part. Examples:

	Correct answer seen	Further working seen	Any FT issues?	Action
1.	$8\sqrt{2}$	$5.65685...$ *(incorrect decimal value)*	No. Last part in question.	Award ***A1*** for the final mark *(condone the incorrect further working)*
2.	$\frac{35}{72}$	$0.468111\ldots$ (incorrect decimal value)	Yes. Value is used in subsequent parts.	Award ***A0*** for the final mark *(and full **FT** is available in subsequent parts)*

3 Implied marks

Implied marks appear in **brackets e.g. *(M1)***,and can only be awarded if **correct** work is seen or implied by subsequent working/answer.

4 Follow through marks (only applied after an error is made)

Follow through (***FT***) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) (e.g. incorrect value from part (a) used in part (d) or incorrect value from part (c)(i) used in part (c)(ii)). Usually, to award ***FT*** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if all the marks awarded in a subsequent part are for the answer or are implied, then ***FT*** marks should be awarded for *their* correct answer, even when working is not present.

For example: following an incorrect answer to part (a) that is used in subsequent parts, where the markscheme for the subsequent part is ***(M1)A1***, it is possible to award full marks for *their* correct answer, **without working being seen.** For longer questions where all but the answer marks are implied this rule applies but may be overwritten by a **Note** in the Markscheme.

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks, by reflecting on what each mark is for and how that maps to the simplified version.
- If the error leads to an inappropriate value (*e.g*. probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word “their” in a description, to indicate that candidates may be using an incorrect value.
- If the candidate’s answer to the initial question clearly contradicts information given in the question, it is not appropriate to award any ***FT*** marks in the subsequent parts. This includes when candidates fail to complete a “show that” question correctly, and then in subsequent parts use their incorrect answer rather than the given value.

- Exceptions to these ***FT*** rules will be explicitly noted on the markscheme.
- If a candidate makes an error in one part but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the command term was "Hence".

5 Mis-read

If a candidate incorrectly copies values or information from the question, this is a mis-read (***MR***). A candidate should be penalized only once for a particular misread. Use the ***MR*** stamp to indicate that this has been a misread and do not award the first mark, even if this is an ***M*** mark, but award all others as appropriate.

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.
- If a candidate uses a correct answer, to a "show that" question, to a higher degree of accuracy than given in the question, this is NOT a misread and full marks may be scored in the subsequent part.
- ***MR*** can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If the command term is 'Hence' and not 'Hence or otherwise' then alternative methods are not permitted unless covered by a note in the mark scheme.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**.

7 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation** for example 1.9 and $1,9$ or 1000 and $1,000$ and 1.000.
- Do not accept final answers written using calculator notation. However, ***M*** marks and intermediate ***A*** marks can be scored, when presented using calculator notation, provided the evidence clearly reflects the demand of the mark.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, some **equivalent** answers will generally appear in brackets. Not all equivalent notations/answers/methods will be presented in the markscheme and examiners are asked to apply appropriate discretion to judge if the candidate work is equivalent.

8 Format and accuracy of answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. If the level of accuracy is not stated in the question, the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures.*

Where values are used in subsequent parts, the markscheme will generally use the exact value, however candidates may also use the correct answer to 3 sf in subsequent parts. The markscheme will often explicitly include the subsequent values that come "*from the use of 3 sf values*".

Simplification of final answers: Candidates are advised to give final answers using good mathematical form. In general, for an ***A*** mark to be awarded, arithmetic should be completed, and any values that lead to integers should be simplified; for example, $\sqrt{\frac{25}{4}}$ should be written as $\frac{5}{2}$. An exception to this is simplifying fractions, where lowest form is not required (although the numerator and the denominator must be integers); for example, $\frac{10}{4}$ may be left in this form or written as $\frac{5}{2}$. However, $\frac{10}{5}$ should be written as 2, as it simplifies to an integer.

Algebraic expressions should be simplified by completing any operations such as addition and multiplication, e.g. $4e^{2x} \times e^{3x}$ should be simplified to $4e^{5x}$, and $4e^{2x} \times e^{3x} - e^{4x} \times e^{x}$ should be simplified to $3e^{5x}$. Unless specified in the question, expressions do not need to be factorized, nor do factorized expressions need to be expanded, so $x(x+1)$ and $x^2 + x$ are both acceptable.

Please note: intermediate ***A*** marks do NOT need to be simplified.

9 Calculators

No calculator is allowed. The use of any calculator on this paper is malpractice and will result in no grade awarded. If you see work that suggests a candidate has used any calculator, please follow the procedures for malpractice.

10. Presentation of candidate work

Crossed out work: If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work unless an explicit note from the candidate indicates that they would like the work to be marked.

More than one solution: Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise. If the layout of the responses makes it difficult to judge, examiners should apply appropriate discretion to judge which is "first".

Section A

1. (a) (i) setting $f(x)=0$ ***(M1)***

$x=1$, $x=-3$ (accept $(1,0),(-3,0)$) ***A1***

(ii) **METHOD 1**

$x=-1$ ***A1***

substituting their x-coordinate into f ***(M1)***

$y=8$ ***A1***

$(-1,8)$

METHOD 2

attempt to complete the square ***(M1)***

$-2\left((x+1)^2-4\right)$

$x=-1$, $y=8$ ***A1A1***

$(-1,8)$

[5 marks]

(b) $h=-1$ ***A1***

$k=8$ ***A1***

[2 marks]

Total [7 marks]

2. recognition that $y=\int\cos\left(x-\frac{\pi}{4}\right)\mathrm{d}x$ ***(M1)***

$y=\sin\left(x-\frac{\pi}{4}\right)\ (+c)$ ***(A1)***

substitute both x and y values into their integrated expression including c ***(M1)***

$2=\sin\frac{\pi}{2}+c$

$c=1$

$y=\sin\left(x-\frac{\pi}{4}\right)+1$ ***A1***

[4 marks]

3. (a) (i) $x=3$ ***A1***

(ii) $y=-2$ ***A1***

[2 marks]

(b) (i) $(-2,0)$ (accept $x=-2$) ***A1***

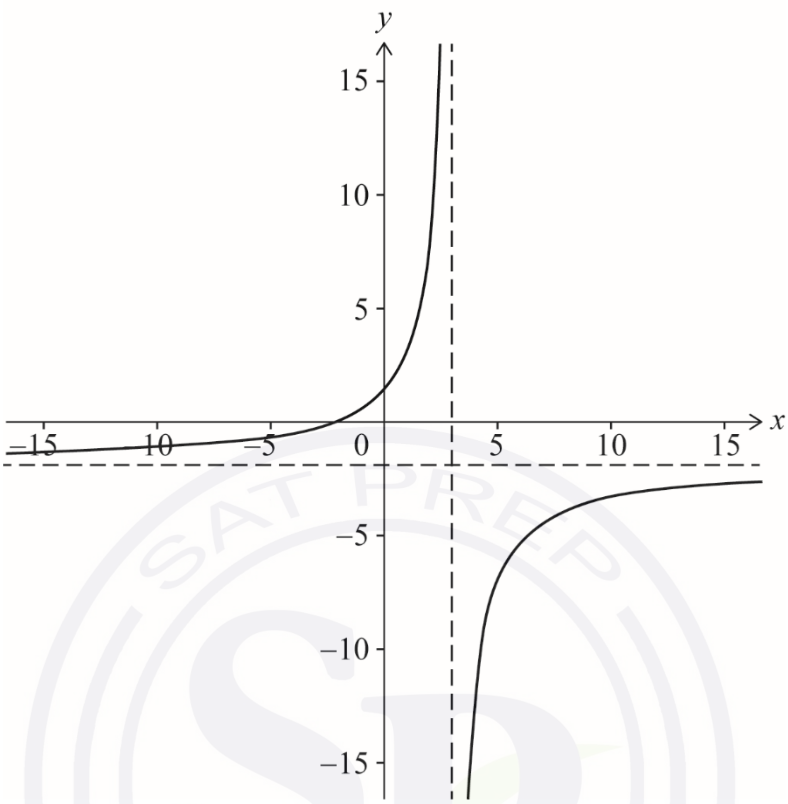
(ii) $\left(0,\frac{4}{3}\right)$ (accept $y=\frac{4}{3}$ and $f(0)=\frac{4}{3}$) ***A1***

[2 marks]

continued…

Question 3 continued.

(c)



A1

Note: Award ***A1*** for completely correct shape: two branches in correct quadrants with asymptotic behaviour.

[1 mark]

Total [5 marks]

4. (a) valid approach to find $\mathrm{P}(R)$ ***(M1)***

tree diagram (must include probabilty of picking box) with correct required probabilities

OR $\mathrm{P}(R \cap B_1) + \mathrm{P}(R \cap B_2)$ OR $\mathrm{P}(R \mid B_1)\mathrm{P}(B_1) + \mathrm{P}(R \mid B_2)\mathrm{P}(B_2)$

$\frac{5}{7} \cdot \frac{1}{2} + \frac{4}{7} \cdot \frac{1}{2}$ ***(A1)***

$\mathrm{P}(R) = \frac{9}{14}$ ***A1***

[3 marks]

(b) events A and R are not independent, since $\frac{9}{14} \cdot \frac{1}{2} \neq \frac{5}{14}$ OR $\frac{5}{7} \neq \frac{9}{14}$ OR $\frac{5}{9} \neq \frac{1}{2}$

OR an explanation e.g. different number of red balls in each box ***A2***

Note: Both conclusion and reasoning are required. Do not split the ***A2***.

[2 marks]

Total [5 marks]

5. (a) $f'(4) = 6$ ***A1***

[1 mark]

(b) $f(4) = 6 \times 4 - 1 = 23$ ***A1***

[1 mark]

(c) $h(4) = f\left(g(4)\right)$ ***(M1)***

$h(4) = f(4^2 - 3 \times 4) = f(4)$

$h(4) = 23$ ***A1***

[2 marks]

(d) attempt to use chain rule to find h' ***(M1)***

$f'\left(g\left(x\right)\right) \times g'\left(x\right)$ OR $\left(x^2 - 3x\right)' \times f'\left(x^2 - 3x\right)$

$h'(4) = \left(2 \times 4 - 3\right) f'\left(4^2 - 3 \times 4\right)$ ***A1***

$= 30$

$y - 23 = 30\left(x - 4\right)$ OR $y = 30x - 97$ ***A1***

[3 marks]

Total [7 marks]

6. (a) **METHOD 1**

attempt to write all LHS terms with a common denominator of $x-1$ ***(M1)***

$$2x-3-\frac{6}{x-1}=\frac{2x(x-1)-3(x-1)-6}{x-1} \text{ OR } \frac{(2x-3)(x-1)}{x-1}-\frac{6}{x-1}$$

$$=\frac{2x^2-2x-3x+3-6}{x-1} \text{ OR } \frac{2x^2-5x+3}{x-1}-\frac{6}{x-1}$$ ***A1***

$$=\frac{2x^2-5x-3}{x-1}$$ ***AG***

METHOD 2

attempt to use algebraic division on RHS ***(M1)***

correctly obtains quotient of $2x-3$ and remainder -6 ***A1***

$=2x-3-\dfrac{6}{x-1}$ as required. ***AG***

[2 marks]

continued…

Question 6 continued.

(b) consider the equation $\frac{2\sin^2 2\theta - 5\sin 2\theta - 3}{\sin 2\theta - 1} = 0$ ***(M1)***

$\Rightarrow 2\sin^2 2\theta - 5\sin 2\theta - 3 = 0$

EITHER

attempt to factorise in the form $(2\sin 2\theta + a)(\sin 2\theta + b)$ ***(M1)***

Note: Accept any variable in place of $\sin 2\theta$.

$(2\sin 2\theta + 1)(\sin 2\theta - 3) = 0$

OR

attempt to substitute into quadratic formula ***(M1)***

$\sin 2\theta = \frac{5 \pm \sqrt{49}}{4}$

THEN

$\sin 2\theta = -\frac{1}{2}$ or $\sin 2\theta = 3$ ***(A1)***

Note: Award ***A1*** for $\sin 2\theta = -\frac{1}{2}$ only.

one of $\frac{7\pi}{6}$ OR $\frac{11\pi}{6}$ (accept 210 or 330) ***(A1)***

$\theta = \frac{7\pi}{12}, \frac{11\pi}{12}$ (must be in radians) ***A1***

Note: Award ***A0*** if additional answers given.

[5 marks]

Total [7 marks]

Section B

7. (a) (i) valid approach to find turning point ($v' = 0$, $-\frac{b}{2a}$, average of roots) ***(M1)***

$4 - 6t = 0$ OR $-\frac{4}{2(-3)}$ OR $\frac{-\frac{2}{3}+2}{2}$

$t = \frac{2}{3}$ (s) ***A1***

(ii) attempt to integrate v ***(M1)***

$\int v \, \mathrm{d}t = \int \left(4 + 4t - 3t^2\right) \mathrm{d}t = 4t + 2t^2 - t^3 \left(+c\right)$ ***A1A1***

Note: Award ***A1*** for $4t + 2t^2$, ***A1*** for $-t^3$.

attempt to substitute their t into their solution for the integral ***(M1)***

distance $= 4\left(\frac{2}{3}\right) + 2\left(\frac{2}{3}\right)^2 - \left(\frac{2}{3}\right)^3$

$= \frac{8}{3} + \frac{8}{9} - \frac{8}{27}$ (or equivalent) ***A1***

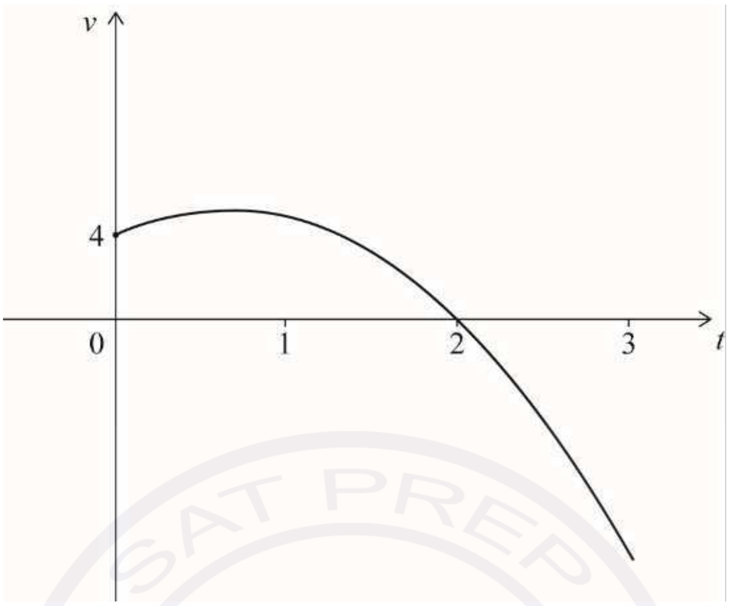
$= \frac{88}{27}$ (m) ***AG***

[7 marks]

continued…

Question 7 continued.

(b)



valid approach to solve $4+4t-3t^2=0$ (may be seen in part (a)) ***(M1)***

$(2-t)(2+3t)$ OR $\frac{-4\pm\sqrt{16+48}}{-6}$

correct x- intercept on the graph at $t=2$ ***A1***

Note: The following two ***A*** marks may only be awarded if the shape is a concave down parabola. These two marks are independent of each other and the ***(M1)***.

correct domain from 0 to 3 starting at $(0,4)$ ***A1***

Note: The 3 must be clearly indicated.

vertex in approximately correct place for $t=\frac{2}{3}$ and $v>4$ ***A1***

[4 marks]

continued…

Question 7 continued.

(c) recognising to integrate between 0 and 2, or 2 and 3 OR $\int_0^3 \left|4+4t-3t^2\right| \mathrm{d}t$ ***(M1)***

$\int_0^2 \left(4+4t-3t^2\right) \mathrm{d}t$

$=8$ ***A1***

$\int_2^3 \left(4+4t-3t^2\right) \mathrm{d}t$

$=-5$ ***A1***

valid approach to sum the two areas (seen anywhere) ***(M1)***

$\int_0^2 v \,\mathrm{d}t - \int_2^3 v \,\mathrm{d}t$ OR $\int_0^2 v \,\mathrm{d}t + \left|\int_2^3 v \,\mathrm{d}t\right|$

total distance travelled $=13$ (m) ***A1***

[5 marks]

Total [16 marks]

8. (a) $f\left(\frac{2}{3}\right)=4$ OR $a^{\frac{2}{3}}=4$ ***(M1)***

$a=4^{\frac{3}{2}}$ OR $a=\left(2^2\right)^{\frac{3}{2}}$ OR $a^2=64$ OR $\sqrt[3]{a}=2$ ***A1***

$a=8$ ***AG***

[2 marks]

(b) $f^{-1}(x)=\log_8 x$ ***A1***

> **Note**: Accept $f^{-1}(x)=\log_a x$.
>
> Accept any equivalent expression for f^{-1} e.g. $f^{-1}(x)=\frac{\ln x}{\ln 8}$.

[1 mark]

(c) correct substitution ***(A1)***

$\log_8\sqrt{32}$ OR $8^x=32^{\frac{1}{2}}$

correct working involving log/index law ***(A1)***

$\frac{1}{2}\log_8 32$ OR $\frac{5}{2}\log_8 2$ OR $\log_8 2=\frac{1}{3}$ OR $\log_2 2^{\frac{5}{2}}$ OR $\log_2 8=3$ OR $\frac{\ln 2^{\frac{5}{2}}}{\ln 2^3}$ OR $2^{3x}=2^{\frac{5}{2}}$

$f^{-1}\left(\sqrt{32}\right)=\frac{5}{6}$ ***A1***

[3 marks]

continued…

Question 8 continued.

(d) (i) **METHOD 1**

equating a pair of differences ***(M1)***

$u_2 - u_1 = u_4 - u_3 (= u_3 - u_2)$

$\log_8 p - \log_8 27 = \log_8 125 - \log_8 q$

$\log_8 125 - \log_8 q = \log_8 q - \log_8 p$

$\log_8\left(\frac{p}{27}\right) = \log_8\left(\frac{125}{q}\right),\ \log_8\left(\frac{125}{q}\right) = \log_8\left(\frac{q}{p}\right)$ ***A1A1***

$\frac{p}{27} = \frac{125}{q}$ and $\frac{125}{q} = \frac{q}{p}$ ***A1***

$27,\ p,\ q$ and 125 are in geometric sequence ***AG***

Note: If candidate assumes the sequence is geometric, award no marks for part (i). If $r = \frac{5}{3}$ has been found, this will be awarded marks in part (ii).

METHOD 2

expressing a pair of consecutive terms, in terms of d ***(M1)***

$p = 8^d \times 27$ and $q = 8^{2d} \times 27$ OR $q = 8^{2d} \times 27$ and $125 = 8^{3d} \times 27$

two correct pairs of consecutive terms, in terms of d ***A1***

$\frac{8^d \times 27}{27} = \frac{8^{2d} \times 27}{8^d \times 27} = \frac{8^{3d} \times 27}{8^{2d} \times 27}$ (must include 3 ratios) ***A1***

all simplify to 8^d ***A1***

$27,\ p,\ q$ and 125 are in geometric sequence ***AG***

continued…

Question 8 continued.

(ii) **METHOD 1 (geometric, finding *r*)**

$u_4 = u_1 r^3$ OR $125 = 27(r)^3$ ***(M1)***

$r = \frac{5}{3}$ (seen anywhere) ***A1***

$p = 27r$ OR $\frac{125}{q} = \frac{5}{3}$ ***(M1)***

$p = 45,\ q = 75$ ***A1A1***

METHOD 2 (arithmetic)

$u_4 = u_1 + 3d$ OR $\log_8 125 = \log_8 27 + 3d$ ***(M1)***

$d = \log_8\left(\frac{5}{3}\right)$ (seen anywhere) ***A1***

$\log_8 p = \log_8 27 + \log_8\left(\frac{5}{3}\right)$ OR $\log_8 q = \log_8 27 + 2\log_8\left(\frac{5}{3}\right)$ ***(M1)***

$p = 45,\ q = 75$ ***A1A1***

METHOD 3 (geometric using proportion)

recognizing proportion ***(M1)***

$pq = 125 \times 27$ OR $q^2 = 125p$ OR $p^2 = 27q$

two correct proportion equations ***A1***

attempt to eliminate either p or q ***(M1)***

$q^2 = 125 \times \frac{125 \times 27}{q}$ OR $p^2 = 27 \times \frac{125 \times 27}{p}$

$p = 45,\ q = 75$ ***A1A1***

[9 marks]

Total [15 marks]

Special note: In this question if candidates use the word 'gradient' in their reasoning. e.g. gradient is positive, it must be clear whether this is the gradient of f or the gradient of f' to earn the ***R*** mark.

9. (a) f increases when $p < x < 0$ ***A1***

f increases when $f'(x) > 0$ OR f' is above the x-axis ***R1***

Note: Do not award ***A0R1.***

[2 marks]

(b) $x = 0$ ***A1***

[1 mark]

(c) (i) f is minimum when $x = p$ ***A1***

because $f'(p) = 0$, $f'(x) < 0$ when $x < p$ and $f'(x) > 0$ when $x > p$
(may be seen in a sign diagram clearly labelled as f')
OR because f' changes from negative to positive at $x = p$
OR $f'(p) = 0$ and slope of f' is positive at $x = p$ ***R1***

Note: Do not award ***A0 R1***

(ii) f has points of inflexion when $x = q$, $x = r$ and $x = t$ ***A2***

f' has turning points at $x = q$, $x = r$ and $x = t$
OR
$f''(q) = 0$, $f''(r) = 0$ and $f''(t) = 0$ and f' changes from increasing to decreasing or vice versa at each of these x-values (may be seen in a sign diagram clearly labelled as f'' and f') ***R1***

Note: Award ***A0*** if any incorrect answers are given. Do not award ***A0R1.***

[5 marks]

continued…

Question 9 continued.

(d) recognizing area from p to t (seen anywhere) ***M1***

$$\int_p^t |f'(x)|\,\mathrm{d}x$$

recognizing to negate integral for area below x-axis ***(M1)***

$$\int_p^0 f'(x)\,\mathrm{d}x - \int_0^t f'(x)\,\mathrm{d}x \quad \text{OR} \quad \int_p^0 f'(x)\,\mathrm{d}x + \int_t^0 f'(x)\,\mathrm{d}x$$

$\int_m^n f'(x)\,\mathrm{d}x = f(n) - f(m)$ (for any integral) ***(M1)***

$f(0) - f(p) - \left[f(t) - f(0)\right]$ OR $f(0) - f(p) + f(0) - f(t)$ ***(A1)***

$2f(0) - \left[f(t) + f(p)\right] = 20$, $2f(0) - 4 = 20$ ***(A1)***

$f(0) = 12$ ***A1***

[6 marks]
Total [14 marks]

M21/5/MATHX/SP1/ENG/TZ2/XX/M

Markscheme

May 2021

Mathematics: analysis and approaches

Standard level

Paper 1

20 pages

Instructions to Examiners

Abbreviations

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1 General

Award marks using the annotations as noted in the markscheme *eg* ***M1***, ***A2***.

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	Correct answer seen	**Further working seen**	**Any FT issues?**	**Action**
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Implied marks appear in **brackets e.g. *(M1)***,and can only be awarded if **correct** work is seen or implied by subsequent working/answer.

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5 Mis-read

If a candidate incorrectly copies values or information from the question, this is a mis-read (***MR***). A candidate should be penalized only once for a particular misread. Use the ***MR*** stamp to indicate that this has been a misread and do not award the first mark, even if this is an ***M*** mark, but award all others as appropriate.

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.
- If a candidate uses a correct answer, to a "show that" question, to a higher degree of accuracy than given in the question, this is NOT a misread and full marks may be scored in the subsequent part.
- ***MR*** can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If the command term is 'Hence' and not 'Hence or otherwise' then alternative methods are not permitted unless covered by a note in the mark scheme.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**.

7 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation** for example 1.9 and $1,9$ or 1000 and $1,000$ and 1.000.
- Do not accept final answers written using calculator notation. However, ***M*** marks and intermediate ***A*** marks can be scored, when presented using calculator notation, provided the evidence clearly reflects the demand of the mark.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, some **equivalent** answers will generally appear in brackets. Not all equivalent notations/answers/methods will be presented in the markscheme and examiners are asked to apply appropriate discretion to judge if the candidate work is equivalent.

8 Format and accuracy of answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. If the level of accuracy is not stated in the question, the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures.*

Where values are used in subsequent parts, the markscheme will generally use the exact value, however candidates may also use the correct answer to 3 sf in subsequent parts. The markscheme will often explicitly include the subsequent values that come "*from the use of 3 sf values*".

Simplification of final answers: Candidates are advised to give final answers using good mathematical form. In general, for an ***A*** mark to be awarded, arithmetic should be completed, and any values that lead to integers should be simplified; for example, $\sqrt{\frac{25}{4}}$ should be written as $\frac{5}{2}$. An exception to this is simplifying fractions, where lowest form is not required (although the numerator and the denominator must be integers); for example, $\frac{10}{4}$ may be left in this form or written as $\frac{5}{2}$. However, $\frac{10}{5}$ should be written as 2, as it simplifies to an integer.

Algebraic expressions should be simplified by completing any operations such as addition and multiplication, e.g. $4e^{2x} \times e^{3x}$ should be simplified to $4e^{5x}$, and $4e^{2x} \times e^{3x} - e^{4x} \times e^{x}$ should be simplified to $3e^{5x}$. Unless specified in the question, expressions do not need to be factorized, nor do factorized expressions need to be expanded, so $x(x+1)$ and $x^2 + x$ are both acceptable.

Please note: intermediate ***A*** marks do NOT need to be simplified.

9 Calculators

No calculator is allowed. The use of any calculator on this paper is malpractice and will result in no grade awarded. If you see work that suggests a candidate has used any calculator, please follow the procedures for malpractice.

10. Presentation of candidate work

Crossed out work: If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work unless an explicit note from the candidate indicates that they would like the work to be marked.

More than one solution: Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise. If the layout of the responses makes it difficult to judge, examiners should apply appropriate discretion to judge which is "first".

Section A

1. (a) minor arc AB has length r ***(A1)***

recognition that perimeter of shaded sector is $3r$ ***(A1)***

$3r = 12$

$r = 4$ ***A1***

[3 marks]

(b) **EITHER**

$\theta = 2\pi - \mathrm{A\hat{O}B} \left(= 2\pi - 1\right)$ ***(M1)***

Area of non-shaded region $= \frac{1}{2}(2\pi - 1)\left(4^2\right)$ ***(A1)***

OR

area of circle - area of shaded sector ***(M1)***

$16\pi - \left(\frac{1}{2} \times 1 \times 4^2\right)$ ***(A1)***

THEN

$\text{area} = 16\pi - 8 \ \left(= 8(2\pi - 1)\right)$ ***A1***

[3 marks]

Total [6 marks]

2. attempt to subtract squares of integers ***(M1)***

$(n+1)^2 - n^2$

EITHER

correct order of subtraction and correct expansion of $(n+1)^2$, seen anywhere ***A1A1***

$= n^2 + 2n + 1 - n^2 \left(= 2n+1\right)$

OR

correct order of subtraction and correct factorization of difference of squares ***A1A1***

$= (n+1-n)(n+1+n)(= 2n+1)$

THEN

$= n + n + 1$ =RHS ***A1***

Note: Do not award final ***A1*** unless all previous working is correct.

which is the sum of n and $n+1$ ***AG***

Note: If expansion and order of subtraction are correct, award full marks for candidates who find the sum of the integers as $2n+1$ and then show that the difference of the squares (subtracted in the correct order) is $2n+1$.

[4 marks]

3. (a) **METHOD 1**

correct substitution of $\cos^2 x = 1 - \sin^2 x$ ***A1***

$2\left(1 - \sin^2 x\right) + 5\sin x = 4$

$2\sin^2 x - 5\sin x + 2 = 0$ ***AG***

METHOD 2

correct substitution using double-angle identities ***A1***

$\left(2\cos^2 x - 1\right) + 5\sin x = 3$

$1 - 2\sin^2 x + 5\sin x = 3$

$2\sin^2 x - 5\sin x + 2 = 0$ ***AG***

[1 mark]

continued…

Question 3 continued

(b) **EITHER**

attempting to factorise ***M1***

$(2\sin x - 1)(\sin x - 2)$ ***A1***

OR

attempting to use the quadratic formula ***M1***

$\sin x = \frac{5 \pm \sqrt{5^2 - 4 \times 2 \times 2}}{4} \left(= \frac{5 \pm 3}{4} \right)$ ***A1***

THEN

$\sin x = \frac{1}{2}$ ***(A1)***

$x = \frac{\pi}{6}, \frac{5\pi}{6}$ ***A1A1***

[5 marks]

Total [6 marks]

4. **EITHER**

attempt to use the binomial expansion of $(x+k)^7$ ***(M1)***

${}^7C_0x^7k^0 + {}^7C_1x^6k^1 + {}^7C_2x^5k^2 + \ldots$ (or ${}^7C_0k^7x^0 + {}^7C_1k^5x^1 + {}^7C_2k^5x^2 + \ldots$)

identifying the correct term ${}^7C_2x^5k^2$ (or ${}^7C_5k^2x^5$) ***(A1)***

OR

attempt to use the general term ${}^7C_rx^rk^{7-r}$ (or ${}^7C_rk^rx^{7-r}$) ***(M1)***

$r=2$ (or $r=5$) ***(A1)***

THEN

${}^7C_2=21$ (or ${}^7C_5=21$) (seen anywhere) ***(A1)***

$21x^5k^2=63x^5$ ($21k^2=63$, $k^2=3$) ***A1***

$k=\pm\sqrt{3}$ ***A1***

Note: If working shown, award ***M1A1A1A1A0*** for $k=\sqrt{3}$.

[5 marks]

5. (a) $\ln(x^2 - 16) = 0$ ***(M1)***

$e^0 = x^2 - 16 (= 1)$

$x^2 = 17$ OR $x = \pm\sqrt{17}$ ***(A1)***

$a = \sqrt{17}$ ***A1***

[3 marks]

(b) attempt to differentiate (must include $2x$ and/or $\frac{1}{x^2 - 16}$) ***(M1)***

$f'(x) = \frac{2x}{x^2 - 16}$ ***A1***

setting their derivative $= \frac{1}{3}$ ***M1***

$\frac{2x}{x^2 - 16} = \frac{1}{3}$

$x^2 - 16 = 6x$ OR $x^2 - 6x - 16 = 0$ (or equivalent) ***A1***

valid attempt to solve their quadratic ***(M1)***

$x = 8$ ***A1***

Note: Award ***A0*** if the candidate's final answer includes additional solutions (such as $x = -2, 8$).

[6 marks]

Total [9 marks]

6. METHOD 1

attempt to use the cosine rule to find the value of x ***(M1)***

$100 = x^2 + 4x^2 - 2(x)(2x)\left(\frac{3}{4}\right)$ ***A1***

$2x^2 = 100$

$x^2 = 50$ OR $x = \sqrt{50}$ $\left(= 5\sqrt{2}\right)$ ***A1***

attempt to find $\sin\hat{C}$ (seen anywhere) ***(M1)***

$\sin^2\hat{C} + \left(\frac{3}{4}\right)^2 = 1$ OR $x^2 + 3^2 = 4^2$ or right triangle with side 3 and hypotenuse 4

$\sin\hat{C} = \frac{\sqrt{7}}{4}$ ***(A1)***

Note: The marks for finding $\sin\hat{C}$ may be awarded independently of the first three marks for finding x.

correct substitution into the area formula using their value of x (or x^2) and their value of $\sin\hat{C}$ ***(M1)***

$A = \frac{1}{2} \times 5\sqrt{2} \times 10\sqrt{2} \times \frac{\sqrt{7}}{4}$ or $A = \frac{1}{2} \times \sqrt{50} \times 2\sqrt{50} \times \frac{\sqrt{7}}{4}$

$A = \frac{25\sqrt{7}}{2}$ ***A1***

continued…

Question 6 continued

METHOD 2

attempt to find the height, h, of the triangle in terms of x ***(M1)***

$h^2+\left(\frac{3}{4}x\right)^2=x^2$ OR $h^2+\left(\frac{5}{4}x\right)^2=10^2$ OR $h=\frac{\sqrt{7}}{4}x$ ***A1***

equating their expressions for either h^2 or h ***(M1)***

$x^2-\left(\frac{3}{4}x\right)^2=10^2-\left(\frac{5}{4}x\right)^2$ OR $\sqrt{100-\frac{25}{16}x^2}=\frac{\sqrt{7}}{4}x$ (or equivalent) ***A1***

$x^2=50$ OR $x=\sqrt{50}$ $\left(=5\sqrt{2}\right)$ ***A1***

correct substitution into the area formula using their value of x (or x^2) ***(M1)***

$A=\frac{1}{2}\times 2\sqrt{50}\times\frac{\sqrt{7}}{4}\sqrt{50}$ OR $A=\frac{1}{2}\left(2\times 5\sqrt{2}\right)\left(\frac{\sqrt{7}}{4}5\sqrt{2}\right)$

$A=\frac{25\sqrt{7}}{2}$ ***A1***

Total [7 marks]

Section B

7. (a) evidence of median position *(M1)*

40 students

median $=14$ (hours) ***A1***

[2 marks]

(b) recognizing there are 8 students in the top 10% *(M1)*

72 students spent less than k hours *(A1)*

$k=18$ (hours) ***A1***

[3 marks]

(c) 15 hours is 60 students OR $p=60-4$ *(M1)*

$p=56$ ***A1***

21 hours is 76 students OR $q=80-76$ OR $q=80-4-56-16$ *(A1)*

$q=4$ ***A1***

[4 marks]

(d) 20 of the 80 students OR $\frac{1}{4}$ spend more than 15 hours doing homework *(A1)*

$\frac{20}{80}=\frac{x}{320}$ OR $\frac{1}{4}\times 320$ OR 4×20 *(A1)*

80 (students) ***A1***

[3 marks]

(e) (i) only year 12 students surveyed OR amount of homework might be different for different year levels ***R1***

(ii) stratified sampling OR survey students in all years ***R1***

[2 marks]

Total [14 marks]

8. (a) $6+6\cos x=0$ (or setting their $f'(x)=0$) ***(M1)***

$\cos x=-1$ (or $\sin x=0$)

$x=\pi$, $x=3\pi$ ***A1A1***

[3 marks]

(b) attempt to integrate $\int_{\pi}^{3\pi}(6+6\cos x)\mathrm{d}x$ ***(M1)***

$=[6x+6\sin x]_{\pi}^{3\pi}$ ***A1A1***

substitute their limits into their integrated expression and subtract ***(M1)***

$=(18\pi+6\sin 3\pi)-(6\pi+6\sin\pi)$

$=(6(3\pi)+0)-(6\pi+0)$ $(=18\pi-6\pi)$ ***A1***

area $=12\pi$ ***AG***

[5 marks]

continued…

Question 8 continued

(c) attempt to substitute into formula for surface area (including base) ***(M1)***

$\pi\left(2^2\right)+\pi(2)(l)=12\pi$ ***(A1)***

$4\pi+2\pi l=12\pi$

$2\pi l=8\pi$

$l=4$ ***A1***

[3 marks]

(d) valid attempt to find the height of the cone ***(M1)***

e.g. $2^2+h^2=(\text{their } l)^2$

$h=\sqrt{12}\ \left(=2\sqrt{3}\right)$ ***(A1)***

attempt to use $V=\frac{1}{3}\pi r^2 h$ with their values substituted ***M1***

$\left(\frac{1}{3}\pi\left(2^2\right)\left(\sqrt{12}\right)\right)$

$\text{volume}=\frac{4\pi\sqrt{12}}{3}\left(=\frac{8\pi\sqrt{3}}{3}=\frac{8\pi}{\sqrt{3}}\right)$ ***A1***

[4 marks]

Total [15 marks]

9. (a) setting $s(t)=0$ ***(M1)***

$8t-t^2=0$

$t(8-t)=0$

$p=8$ (accept $t=8$, $(8,0)$) ***A1***

> **Note:** Award ***A0*** if the candidate's final answer includes additional solutions (such as $p=0, 8$).

[2 marks]

(b) (i) recognition that when particle changes direction $v=0$ OR local maximum on graph of s OR vertex of parabola ***(M1)***

$q=4$ (accept $t=4$) ***A1***

(ii) substituting their value of q into $s(t)$ OR integrating $v(t)$ from $t=0$ to $t=4$ ***(M1)***

$\text{displacement}=16$ (m) ***A1***

[4 marks]

(c) $s(10)=-20$ OR $\text{distance}=|s(t)|$ OR integrating $v(t)$ from $t=0$ to $t=10$ ***(M1)***

$\text{distance}=20$ (m) ***A1***

[2 marks]

(d) 16 forward + 36 backward OR $16+16+20$ OR $\int_0^{10}|v(t)|dt$ ***(M1)***

$d=52$ (m) ***A1***

[2 marks]

continued…

Question 9 continued

(e) **METHOD 1**

graphical method with triangles on $v(t)$ graph ***M1***

$49+\left(\dfrac{x(2x)}{2}\right)$ ***(A1)***

$49+x^2=52$, $x=\sqrt{3}$ ***(A1)***

$k=7+\sqrt{3}$ ***A1***

[4 marks]

METHOD 2

recognition that $\text{distance}=\int\left|v(t)\right|\mathrm{d}t$ ***M1***

$\int_0^7(14-2t)\,\mathrm{d}t+\int_7^k(2t-14)\,\mathrm{d}t$

$\left[14t-t^2\right]_0^7+\left[t^2-14t\right]_7^k$ ***(A1)***

$14(7)-7^2+\left(\left(k^2-14k\right)-\left(7^2-14(7)\right)\right)=52$ ***(A1)***

$k=7+\sqrt{3}$ ***A1***

[4 marks]

Total [14 marks]

Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

May 2021

Mathematics: analysis and approaches

Standard level

Paper 1

18 pages

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If a candidate incorrectly copies values or information from the question, this is a mis-read (***MR***). A candidate should be penalized only once for a particular misread. Use the ***MR*** stamp to indicate that this has been a misread and do not award the first mark, even if this is an ***M*** mark, but award all others as appropriate.

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.
- If a candidate uses a correct answer, to a "show that" question, to a higher degree of accuracy than given in the question, this is NOT a misread and full marks may be scored in the subsequent part.
- ***MR*** can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If the command term is 'Hence' and not 'Hence or otherwise' then alternative methods are not permitted unless covered by a note in the mark scheme.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**.

7 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation** for example 1.9 and $1{,}9$ or 1000 and $1{,}000$ and 1.000.

- Do not accept final answers written using calculator notation. However, ***M*** marks and intermediate ***A*** marks can be scored, when presented using calculator notation, provided the evidence clearly reflects the demand of the mark.

- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.

- In the markscheme, some **equivalent** answers will generally appear in brackets. Not all equivalent notations/answers/methods will be presented in the markscheme and examiners are asked to apply appropriate discretion to judge if the candidate work is equivalent.

8 Format and accuracy of answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. If the level of accuracy is not stated in the question, the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures.*

Where values are used in subsequent parts, the markscheme will generally use the exact value, however candidates may also use the correct answer to 3 sf in subsequent parts. The markscheme will often explicitly include the subsequent values that come "*from the use of 3 sf values*".

Simplification of final answers: Candidates are advised to give final answers using good mathematical form. In general, for an ***A*** mark to be awarded, arithmetic should be completed, and any values that lead to integers should be simplified; for example, $\sqrt{\frac{25}{4}}$ should be written as $\frac{5}{2}$. An exception to this is simplifying fractions, where lowest form is not required (although the numerator and the denominator must be integers); for example, $\frac{10}{4}$ may be left in this form or written as $\frac{5}{2}$. However, $\frac{10}{5}$ should be written as 2, as it simplifies to an integer.

Algebraic expressions should be simplified by completing any operations such as addition and multiplication, e.g. $4e^{2x} \times e^{3x}$ should be simplified to $4e^{5x}$, and $4e^{2x} \times e^{3x} - e^{4x} \times e^{x}$ should be simplified to $3e^{5x}$. Unless specified in the question, expressions do not need to be factorized, nor do factorized expressions need to be expanded, so $x(x+1)$ and $x^2 + x$ are both acceptable.

Please note: intermediate ***A*** marks do NOT need to be simplified.

9 Calculators

No calculator is allowed. The use of any calculator on this paper is malpractice and will result in no grade awarded. If you see work that suggests a candidate has used any calculator, please follow the procedures for malpractice.

10. Presentation of candidate work

Crossed out work: If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work unless an explicit note from the candidate indicates that they would like the work to be marked.

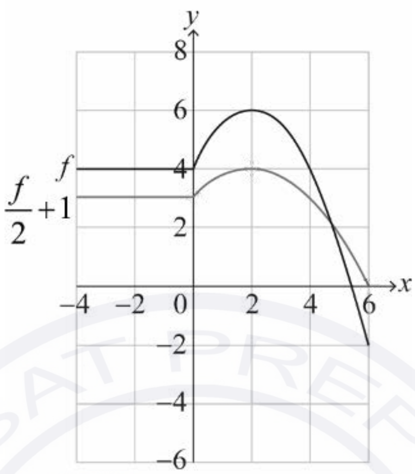
More than one solution: Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise. If the layout of the responses makes it difficult to judge, examiners should apply appropriate discretion to judge which is "first".

1. (a) (i) $f(2)=6$ ***A1***

(ii) $(f \circ f)(2)=-2$ ***A1***

[2 marks]

(b)



M1A1A1

Note: Award ***M1*** for an attempt to apply any vertical stretch or vertical translation,
A1 for a correct horizontal line segment between –4 and 0 (located roughly at $y=3$),
A1 for a correct concave down parabola including max point at (2,4) and for correct end points at (0,3) and (6,0) (within circles). Points do not need to be labelled.

[3 marks]
Total [5 marks]

2. (a) 3×10^4 OR 30000 (km) (accept $3\bullet10^4$) ***A1***

[1 mark]

(b) $\frac{4}{3}\pi\left(3\times10^4\right)^3$ OR $\frac{4}{3}\pi\left(30000\right)^3$ ***(A1)***

$=\frac{4}{3}\pi\times27\times10^{12}\left(=\pi\left(36\times10^{12}\right)\right)$ OR $=\frac{4}{3}\pi\times27000000000000$ ***(A1)***

$=\pi\left(3.6\times10^{13}\right)$ (km^3) OR $a=3.6, k=13$ ***A1***

[3 marks]
Total [4 marks]

3. **METHOD 1 (finding u_1 first, from S_8)**

$4(u_1+8)=8$ ***(A1)***

$u_1=-6$ ***A1***

$u_1+7d=8$ OR $4(2u_1+7d)=8$ (may be seen with their value of u_1) ***(A1)***

attempt to substitute their u_1 ***(M1)***

$d=2$ ***A1***

METHOD 2 (solving simultaneously)

$u_1+7d=8$ ***(A1)***

$4(u_1+8)=8$ OR $4(2u_1+7d)=8$ OR $u_1=-3d$ ***(A1)***

attempt to solve linear or simultaneous equations ***(M1)***

$u_1=-6, d=2$ ***A1A1***

[5 marks]

4. (a) attempt to use definition of outlier

$1.5\times 20+Q_3$ ***(M1)***

$1.5\times 20+U\geq 75$ $(\Rightarrow U\geq 45$, accept $U>45)$ OR $1.5\times 20+Q_3=75$ ***A1***

minimum value of $U=45$ ***A1***

[3 marks]

(b) attempt to use interquartile range ***(M1)***

$U-L=20$ (may be seen in part (a)) OR $L\geq 25$ (accept $L>25$)

minimum value of $L=25$ ***A1***

[2 marks]

Total [5 marks]

5. (a) $f'(x) = -2(x-h)$ ***A1***

[1 mark]

(b) $g'(x) = e^{x-2}$ OR $g'(3) = e^{3-2}$ (may be seen anywhere) ***A1***

Note: The derivative of g must be explicitly seen, either in terms of x or 3.

recognizing $f'(3) = g'(3)$ ***(M1)***

$-2(3-h) = e^{3-2}$ $(= e)$

$-6 + 2h = e$ OR $3 - h = -\frac{e}{2}$ ***A1***

Note: The final ***A1*** is dependent on one of the previous marks being awarded.

$h = \frac{e+6}{2}$ ***AG***

[3 marks]

(c) $f(3) = g(3)$ ***(M1)***

$-(3-h)^2 + 2k = e^{3-2} + k$

correct equation in k

EITHER

$-\left(3 - \frac{e+6}{2}\right)^2 + 2k = e^{3-2} + k$ ***A1***

$k = e + \left(\frac{6-e-6}{2}\right)^2 \left(= e + \left(\frac{-e}{2}\right)^2\right)$ ***A1***

OR

$k = e + \left(3 - \frac{e+6}{2}\right)^2$ ***A1***

$k = e + 9 - 3e - 18 + \frac{e^2 + 12e + 36}{4}$ ***A1***

THEN

$k = e + \frac{e^2}{4}$ ***AG***

[3 marks]
Total [7 marks]

6. (a)

> **Note:** Do not award the final ***A1*** for proofs which work from both sides to find a common expression other than $2\sin x\cos x - 2\sin^2 x$.

METHOD 1 (LHS to RHS)

attempt to use double angle formula for $\sin 2x$ or $\cos 2x$ ***M1***

LHS= $2\sin x\cos x + \cos 2x - 1$ OR

$\sin 2x + 1 - 2\sin^2 x - 1$ OR

$2\sin x\cos x + 1 - 2\sin^2 x - 1$

$= 2\sin x\cos x - 2\sin^2 x$ ***A1***

$\sin 2x + \cos 2x - 1 = 2\sin x(\cos x - \sin x)$ =RHS ***AG***

METHOD 2 (RHS to LHS)

RHS $= 2\sin x\cos x - 2\sin^2 x$

attempt to use double angle formula for $\sin 2x$ or $\cos 2x$ ***M1***

$= \sin 2x + 1 - 2\sin^2 x - 1$ ***A1***

$= \sin 2x + \cos 2x - 1 =$ LHS ***AG***

[2 marks]

(b) attempt to factorise ***M1***

$(\cos x - \sin x)(2\sin x + 1) = 0$ ***A1***

recognition of $\cos x = \sin x \Rightarrow \frac{\sin x}{\cos x} = \tan x = 1$ OR $\sin x = -\frac{1}{2}$ ***(M1)***

one correct reference angle seen anywhere, accept degrees ***(A1)***

$\frac{\pi}{4}$ OR $\frac{\pi}{6}$ (accept $-\frac{\pi}{6}, \frac{7\pi}{6}$)

> **Note:** This ***(M1)(A1)*** is independent of the previous ***M1A1.***

$x = \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{\pi}{4}, \frac{5\pi}{4}$ ***A2***

> **Note:** Award ***A1*** for any two correct (radian) answers.
> Award ***A1A0*** if additional values given with the four correct (radian) answers.
>
> Award ***A1A0*** for four correct answers given in degrees.

[6 marks]

Total [8 marks]

7. (a) **METHOD 1 (discriminant)**

$mx^2 - 2mx = mx - 9$ ***(M1)***

$mx^2 - 3mx + 9 = 0$

recognizing $\Delta = 0$ (seen anywhere) ***M1***

$\Delta = (-3m)^2 - 4(m)(9)$ (do not accept only in quadratic formula for x) ***A1***

valid approach to solve quadratic for m ***(M1)***

$9m(m-4) = 0$ OR $m = \dfrac{36 \pm \sqrt{36^2 - 4 \times 9 \times 0}}{2 \times 9}$

both solutions $m = 0, 4$ ***A1***

$m \neq 0$ with a valid reason ***R1***

the two graphs would not intersect OR $0 \neq -9$

$m = 4$ ***AG***

METHOD 2 (equating slopes)

$mx^2 - 2mx = mx - 9$ (seen anywhere) ***(M1)***

$f'(x) = 2mx - 2m$ ***A1***

equating slopes, $f'(x) = m$ (seen anywhere) ***M1***

$2mx - 2m = m$

$x = \dfrac{3}{2}$ ***A1***

substituting their x value ***(M1)***

$\left(\dfrac{3}{2}\right)^2 m - 2m \times \dfrac{3}{2} = m \times \dfrac{3}{2} - 9$

$\dfrac{9}{4}m - \dfrac{12}{4}m = \dfrac{6}{4}m - 9$ ***A1***

$\dfrac{-9m}{4} = -9$

$m = 4$ ***AG***

continued…

Question 7(a) continued

METHOD 3 (using $\frac{-b}{2a}$)

$mx^2 - 2mx = mx - 9$ ***(M1)***

$mx^2 - 3mx + 9 = 0$

attempt to find x-coord of vertex using $\frac{-b}{2a}$ ***(M1)***

$\frac{-(-3m)}{2m}$ ***A1***

$x = \frac{3}{2}$ ***A1***

substituting their x value ***(M1)***

$\left(\frac{3}{2}\right)^2 m - 3m \times \frac{3}{2} + 9 = 0$

$\frac{9}{4}m - \frac{9}{2}m + 9 = 0$ ***A1***

$-9m = -36$

$m = 4$ ***AG***

[6 marks]

(b) $4x(x-2)$ ***(A1)***

$p = 0$ and $q = 2$ OR $p = 2$ and $q = 0$ ***A1***

[2 marks]

(c) attempt to use valid approach ***(M1)***

$\frac{0+2}{2}$, $\frac{-(-8)}{2\times 4}$, $f(1)$, $8x - 8 = 0$ OR $4\left(x^2 - 2x + 1 - 1\right)\left(= 4(x-1)^2 - 4\right)$

$h = 1,\ k = -4$ ***A1A1***

[3 marks]

(d) **EITHER**

recognition $x = h$ to 2 (may be seen on sketch) ***(M1)***

OR

recognition that $f(x) < 0$ and $f'(x) > 0$ ***(M1)***

THEN

$1 < x < 2$ ***A1A1***

Note: Award ***A1*** for two correct values, ***A1*** for correct inequality signs.

[3 marks]

Total [14 marks]

8. (a) attempt to use quotient or product rule ***(M1)***

$$\frac{dy}{dx} = \frac{x^4\left(\frac{1}{x}\right) - (\ln x)\left(4x^3\right)}{\left(x^4\right)^2} \text{ OR } (\ln x)\left(-4x^{-5}\right) + \left(x^{-4}\right)\left(\frac{1}{x}\right)$$ ***A1***

correct working ***A1***

$$= \frac{x^3(1-4\ln x)}{x^8} \text{ OR cancelling } x^3 \text{ OR } \frac{-4\ln x}{x^5} + \frac{1}{x^5}$$

$$= \frac{1-4\ln x}{x^5}$$ ***AG***

[3 marks]

(b) $f'(x) = \frac{dy}{dx} = 0$ ***(M1)***

$$\frac{1-4\ln x}{x^5} = 0$$

$$\ln x = \frac{1}{4}$$ ***(A1)***

$$x = e^{\frac{1}{4}}$$ ***A1***

substitution of their x to find y ***(M1)***

$$y = \frac{\ln e^{\frac{1}{4}}}{\left(e^{\frac{1}{4}}\right)^4}$$

$$= \frac{1}{4e}\left(= \frac{1}{4}e^{-1}\right)$$ ***A1***

$$P\left(e^{\frac{1}{4}}, \frac{1}{4e}\right)$$

[5 marks]

continued…

Question 8 continued

(c) $f''\left(e^{\frac{1}{4}}\right) = \dfrac{20\ln e^{\frac{1}{4}} - 9}{\left(e^{\frac{1}{4}}\right)^6}$ ***(M1)***

$= \dfrac{5-9}{e^{1.5}} \quad \left(= -\dfrac{4}{e^{1.5}}\right)$ ***A1***

which is negative ***R1***

hence P is a local maximum ***AG***

Note: The ***R1*** is dependent on the previous ***A1*** being awarded.

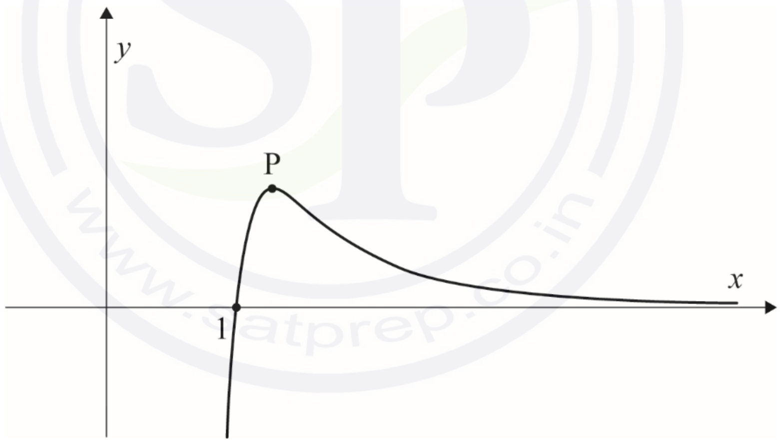
[3 marks]

(d) $\ln x > 0$ ***(A1)***

$x > 1$ ***A1***

[2 marks]

(e)



A1A1A1

Note: Award ***A1*** for one x-intercept only, located at 1
A1 for local maximum, P, in approximately correct position
A1 for curve approaching x-axis as $x \to \infty$ (including change in concavity).

[3 marks]
Total [16 marks]

9. (a) recognising probabilities sum to 1 ***(M1)***

$$p + p + p + \frac{1}{2}p = 1$$

$$p = \frac{2}{7}$$ ***A1***

[2 marks]

(b) valid attempt to find $\mathrm{E}(X)$ ***(M1)***

$$1 \times p + 2 \times p + 3 \times p + 4 \times \frac{1}{2}p \left(= 8p\right)$$

$$\mathrm{E}(X) = \frac{16}{7}$$ ***A1***

[2 marks]

(c) (i) $0 \le r \le 1$ ***A1***

(ii) attempt to find a value of q ***(M1)***

$0 \le 1 - 3q \le 1$ OR $r = 0 \Rightarrow q = \frac{1}{3}$ OR $r = 1 \Rightarrow q = 0$

$$0 \le q \le \frac{1}{3}$$ ***A1***

[3 marks]

(d) $\mathrm{E}(Y) = 1 \times q + 2 \times q + 3 \times q + 4 \times r \left(= 2 + 2r \text{ OR } 4 - 6q\right)$ ***(A1)***

one correct boundary value ***A1***

$1 \times \frac{1}{3} + 2 \times \frac{1}{3} + 3 \times \frac{1}{3} + 4 \times 0 \left(= 2\right)$ OR

$1 \times 0 + 2 \times 0 + 3 \times 0 + 4 \times 1 \left(= 4\right)$ OR

$2 + 2(0) \left(= 2\right)$ OR

$2 + 2(1) \left(= 4\right)$ OR

$4 - 6(0) \left(= 4\right)$ OR $4 - 6\left(\frac{1}{3}\right)\left(= 2\right)$

$$2 \le \mathrm{E}(Y) \le 4$$ ***A1***

[3 marks]

continued…

Question 9 continued

(e) **METHOD 1**

evidence of choosing at least four correct outcomes from
1&2, 1&3, 1&4, 2&3, 2&4, 3&4 ***(M1)***

$\frac{6}{7}q+\frac{6}{7}r$ OR $3pq+3pr$ OR $pq+pq+p(1-3q)+pq+p(1-3q)+p(1-3q)$ ***(A1)***

solving for either q or r ***M1***

$\frac{6}{7}(q+1-3q)=\frac{1}{2}$ OR $\frac{6}{7}\left(\frac{1-r}{3}+r\right)=\frac{1}{2}$ OR $3pq+3p(1-3q)=\frac{1}{2}$

OR $3p\left(\frac{1-r}{3}\right)+3pr=\frac{1}{2}$

EITHER two correct values

$q=\frac{5}{24}$ and $r=\frac{3}{8}$ ***A1A1***

OR one correct value

$q=\frac{5}{24}$ OR $r=\frac{3}{8}$ ***A1***

substituting their value for q or r ***A1***

$4-6\left(\frac{5}{24}\right)$ OR $2+2\left(\frac{3}{8}\right)$

THEN

$\mathrm{E}(Y)=\frac{11}{4}$ ***A1***

[6 marks]

continued…

Question 9 continued

METHOD 2 (solving for $\mathrm{E}(Y)$)
evidence of choosing at least four correct outcomes from
1&2, 1&3, 1&4, 2&3, 2&4, 3&4 ***(M1)***

$\frac{6}{7}q+\frac{6}{7}r$ OR $3pq+3pr$ OR $pq+pq+p(1-3q)+pq+p(1-3q)+p(1-3q)$ ***(A1)***

rearranging to make q the subject ***M1***

$$q=\frac{4-\mathrm{E}(Y)}{6}$$

$$3pq+3p(1-3q)=\frac{1}{2}$$ ***M1***

$$\frac{6}{7}\times\left(\frac{4-\mathrm{E}(Y)}{6}\right)+\frac{6}{7}\left(1-3\left(\frac{4-\mathrm{E}(Y)}{6}\right)\right)=\frac{1}{2}$$ ***A1***

$$\frac{2(\mathrm{E}(Y)-1)}{7}=\frac{1}{2}$$

$$\mathrm{E}(Y)=\frac{11}{4}$$ ***A1***

[6 marks]
Total [16 marks]

Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

Specimen paper

Mathematics: analysis and approaches

Standard level

Paper 1

11 pages

International Baccalaureate
Baccalauréat International
Bachillerato Internacional

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

R Marks awarded for clear **Reasoning**.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

*Award marks using the annotations as noted in the markscheme eg **M1**, **A2**.*

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***M2***, ***A2***, *etc*., do **not** split the marks, unless there is a note.
- Once a correct answer to a question or part-question is seen, ignore further correct working. However, if further working indicates a lack of mathematical understanding do not award the final ***A1***. An exception to this may be in numerical answers, where a correct exact value is followed by an incorrect decimal. However, if the incorrect decimal is carried through to a subsequent part, and correct working shown, award ***FT*** marks as appropriate but do not award the final ***A1*** in that part.

Examples

	Correct answer seen	**Further working seen**	**Action**
1.	$8\sqrt{2}$	5.65685... *(incorrect decimal value)*	Award the final ***A1*** *(ignore the further working)*
2.	$\frac{1}{4}\sin 4x$	$\sin x$	Do not award the final ***A1***
3.	$\log a - \log b$	$\log(a-b)$	Do not award the final ***A1***

3 Implied marks

*Implied marks appear in **brackets e.g. (M1)**, and can only be awarded if **correct** work is seen or if implied in subsequent working.*

- Normally the correct work is seen or implied in the next line.
- Marks **without** brackets can only be awarded for work that is **seen**.

4 Follow through marks (only applied after an error is made)

*Follow through (**FT**) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) or subpart(s). Usually, to award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if the only marks awarded in a subpart are for the answer (i.e. there is no working expected), then **FT** marks should be awarded if appropriate.*

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*e.g.* probability greater than 1, use of $r>1$ for the sum of an infinite GP, $\sin\theta=1.5$, non integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word “their” in a description, to indicate that candidates may be using an incorrect value.
- Exceptions to this rule will be explicitly noted on the markscheme.
- If a candidate makes an error in one part, but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the question says hence. It is often possible to use a different approach in subsequent parts that does not depend on the answer to previous parts.

5 Mis-read

*If a candidate incorrectly copies information from the question, this is a mis-read (**MR**). Apply a **MR** penalty of 1 mark to that question*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta=1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates’ own work does **not** constitute a misread, it is an error.
- The ***MR*** penalty can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for part-questions are indicated by **EITHER** . . . **OR**.

7 Alternative forms

*Unless the question specifies otherwise, **accept** equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

8 Accuracy of Answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. There are two types of accuracy errors, and the final answer mark should not be awarded if these errors occur.

- **Rounding errors**: only applies to final answers not to intermediate steps.
- **Level of accuracy**: when this is not specified in the question the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures*.

9 Calculators

No calculator is allowed. The use of any calculator on paper 1 is malpractice, and will result in no grade awarded. If you see work that suggests a candidate has used any calculator, please follow the procedures for malpractice. Examples: finding an angle, given a trig ratio of 0.4235.

Section A

1. (a) valid approach using Pythagorean identity ***(M1)***

$\sin^2 A + \left(\frac{5}{6}\right)^2 = 1$ (or equivalent) ***(A1)***

$\sin A = \frac{\sqrt{11}}{6}$ ***A1***

[3 marks]

(b) $\frac{1}{2} \times 8 \times 6 \times \frac{\sqrt{11}}{6}$ (or equivalent) ***(A1)***

$\text{area} = 4\sqrt{11}$ ***A1***

[2 marks]

Total [5 marks]

2. attempt to substitute into $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ ***(M1)***

Note: Accept use of Venn diagram or other valid method.

$0.6 = 0.5 + 0.4 - P(A \cap B)$ ***(A1)***

$P(A \cap B) = 0.3$ (seen anywhere) ***A1***

attempt to substitute into $P(A|B) = \frac{P(A \cap B)}{P(B)}$ ***(M1)***

$= \frac{0.3}{0.4}$

$P(A|B) = 0.75 \left(= \frac{3}{4}\right)$ ***A1***

Total [5 marks]

3. (a) attempting to expand the LHS ***(M1)***

$\text{LHS} = (4n^2 - 4n + 1) + (4n^2 + 4n + 1)$ ***A1***

$= 8n^2 + 2 (= \text{RHS})$ ***AG***

[2 marks]

(b) ***METHOD 1***

recognition that $2n-1$ and $2n+1$ represent two consecutive odd integers (for $n \in \mathbb{Z}$) ***R1***

$8n^2 + 2 = 2(4n^2 + 1)$ ***A1***

valid reason *eg* divisible by 2 (2 is a factor) ***R1***

so the sum of the squares of any two consecutive odd integers is even ***AG***

[3 marks]

METHOD 2

recognition, *eg* that n and $n+2$ represent two consecutive odd integers (for $n \in \mathbb{Z}$) ***R1***

$n^2 + (n+2)^2 = 2(n^2 + 2n + 2)$ ***A1***

valid reason *eg* divisible by 2 (2 is a factor) ***R1***

so the sum of the squares of any two consecutive odd integers is even ***AG***

[3 marks]

Total [5 marks]

4. attempt to integrate ***(M1)***

$u = 2x^2 + 1 \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = 4x$

$\int \frac{8x}{\sqrt{2x^2+1}}\,\mathrm{d}x = \int \frac{2}{\sqrt{u}}\,\mathrm{d}u$ ***(A1)***

EITHER

$= 4\sqrt{u}\,(+C)$ ***A1***

OR

$= 4\sqrt{2x^2+1}\,(+C)$ ***A1***

THEN

correct substitution into **their** integrated function (must have C) ***(M1)***

$5 = 4 + C \Rightarrow C = 1$

$f(x) = 4\sqrt{2x^2+1} + 1$ ***A1***

Total [5 marks]

5. (a) attempt to form composition *M1*

correct substitution $g\left(\frac{x+3}{4}\right)=8\left(\frac{x+3}{4}\right)+5$ *A1*

$(g\circ f)(x)=2x+11$ *AG*

[2 marks]

(b) attempt to substitute 4 (seen anywhere) *(M1)*

correct equation $a=2\times 4+11$ *(A1)*

$a=19$ *A1*

[3 marks]

Total [5 marks]

6. (a) attempting to use the change of base rule *M1*

$\log_9(\cos 2x+2)=\frac{\log_3(\cos 2x+2)}{\log_3 9}$ *A1*

$=\frac{1}{2}\log_3(\cos 2x+2)$ *A1*

$=\log_3\sqrt{\cos 2x+2}$ *AG*

[3 marks]

(b) $\log_3(2\sin x)=\log_3\sqrt{\cos 2x+2}$

$2\sin x=\sqrt{\cos 2x+2}$ *M1*

$4\sin^2 x=\cos 2x+2$ (or equivalent) *A1*

use of $\cos 2x=1-2\sin^2 x$ *(M1)*

$6\sin^2 x=3$

$\sin x=(\pm)\frac{1}{\sqrt{2}}$ *A1*

$x=\frac{\pi}{4}$ *A1*

Note: Award ***A0*** if solutions other than $x=\frac{\pi}{4}$ are included.

[5 marks]

Total [8 marks]

Section B

7. (a) evidence of median position *(M1)*
80th employee
40 minutes *A1*
[2 marks]

(b) valid attempt to find interval $(25-55)$ *(M1)*
18 (employees), 142 (employees) *A1*
124 *A1*
[3 marks]

(c) recognising that there are 16 employees in the top 10% *(M1)*
144 employees travelled more than k minutes *(A1)*
$k = 56$ *A1*
[3 marks]

(d) $b = 70$ *A1*
[1 mark]

(e) (i) recognizing a is first quartile value *(M1)*
40 employees
$a = 33$ *A1*

(ii) $47 - 33$ *(M1)*
$\text{IQR} = 14$ *A1*
[4 marks]

(f) attempt to find $1.5 \times$ **their** IQR *(M1)*
$33 - 21$
12 *(A1)*
[2 marks]

[Total 15 marks]

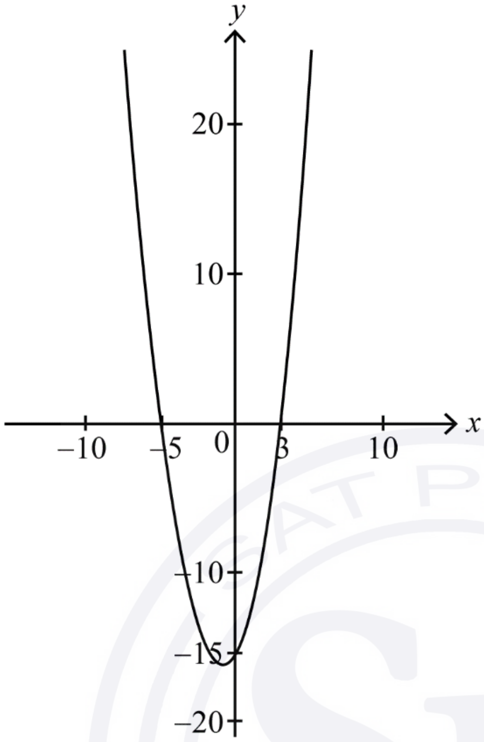
8. (a) $f'(x) = x^2 + 2x - 15$ *(M1)A1*
[2 marks]

(b) correct reasoning that $f'(x) = 0$ (seen anywhere) *(M1)*
$x^2 + 2x - 15 = 0$
valid approach to solve quadratic *M1*
$(x-3)(x+5)$, quadratic formula
correct values for x
$3, -5$
correct values for a and b
$a = -5$ and $b = 3$ *A1*
[3 marks]

continued…

Question 8 continued

(c) (i)



A1

(ii) first derivative changes from positive to negative at $x=a$ ***A1***

so local maximum at $x=a$ ***AG***

[2 marks]

(d) (i) $f''(x) = 2x + 2$ ***A1***

substituting **their** b into **their** second derivative ***(M1)***

$f''(3) = 2 \times 3 + 2$

$f''(b) = 8$ ***(A1)***

(ii) $f''(b)$ is positive so graph is concave up ***R1***

so local minimum at $x = b$ ***AG***

[4 marks]

(e) normal to f at $x=a$ is $x = -5$ (seen anywhere) ***(A1)***

attempt to find y-coordinate at their value of b ***(M1)***

$f(3) = -10$ ***(A1)***

tangent at $x = b$ has equation $y = -10$ (seen anywhere) ***A1***

intersection at $(-5, -10)$

$p = -5$ and $q = -10$ ***A1***

[5 marks]

[Total 16 marks]

9. (a) attempt to use quotient rule *(M1)*
correct substitution into quotient rule

$$f'(x) = \frac{5kx\left(\frac{1}{5x}\right) - k\ln 5x}{(kx)^2} \text{ (or equivalent)}$$ ***A1***

$$= \frac{k - k\ln 5x}{k^2x^2}, \left(k \in \mathbb{R}^+\right)$$ ***A1***

$$= \frac{1-\ln 5x}{kx^2}$$ ***AG***

[3 marks]

(b) $f'(x) = 0$ ***M1***

$$\frac{1-\ln 5x}{kx^2} = 0$$

$\ln 5x = 1$ ***(A1)***

$x = \frac{\mathrm{e}}{5}$ ***A1***

[3 marks]

(c) $f''(x) = 0$ ***M1***

$$\frac{2\ln 5x - 3}{kx^3} = 0$$

$\ln 5x = \frac{3}{2}$ ***A1***

$5x = \mathrm{e}^{\frac{3}{2}}$ ***A1***

so the point of inflexion occurs at $x = \frac{1}{5}\mathrm{e}^{\frac{3}{2}}$ ***AG***

[3 marks]

continued…

Question 9 continued

(d) attempt to integrate ***(M1)***

$u = \ln 5x \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = \frac{1}{x}$

$\int \frac{\ln 5x}{kx}\,\mathrm{d}x = \frac{1}{k}\int u\ \mathrm{d}u$ ***(A1)***

EITHER

$= \frac{u^2}{2k}$ ***A1***

so $\frac{1}{k}\int_1^{\frac{3}{2}} u\ \mathrm{d}u = \left[\frac{u^2}{2k}\right]_1^{\frac{3}{2}}$ ***A1***

OR

$= \frac{(\ln 5x)^2}{2k}$ ***A1***

so $\int_{\frac{e}{5}}^{\frac{1}{5}e^{\frac{3}{2}}} \frac{\ln 5x}{kx}\,\mathrm{d}x = \left[\frac{(\ln 5x)^2}{2k}\right]_{\frac{e}{5}}^{\frac{1}{5}e^{\frac{3}{2}}}$ ***A1***

THEN

$= \frac{1}{2k}\left(\frac{9}{4} - 1\right)$

$= \frac{5}{8k}$ ***A1***

setting **their** expression for area equal to 3 ***M1***

$\frac{5}{8k} = 3$

$k = \frac{5}{24}$ ***A1***

[7 marks]

Total [16 marks]