Diploma Programme
Programme du diplôme
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Mathematics: analysis and approaches
Standard level
Paper 2

31 October 2023

Zone A afternoon | **Zone B** afternoon | **Zone C** afternoon

Candidate session number

1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

8823 – 7115

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Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 7]

Consider the function defined by $f(x) = x^2 - 8x$. The graph of f passes through the point $\mathrm{A}(3, -15)$.

(a) (i) Find the gradient of the tangent to the graph of f at the point A.

(ii) Hence, write down the gradient of the normal to the graph of f at point A. [3]

(b) Write down the equation of the normal to the graph of f at point A. [1]

The normal to the graph of f at point A intersects the graph of f again at a second point B.

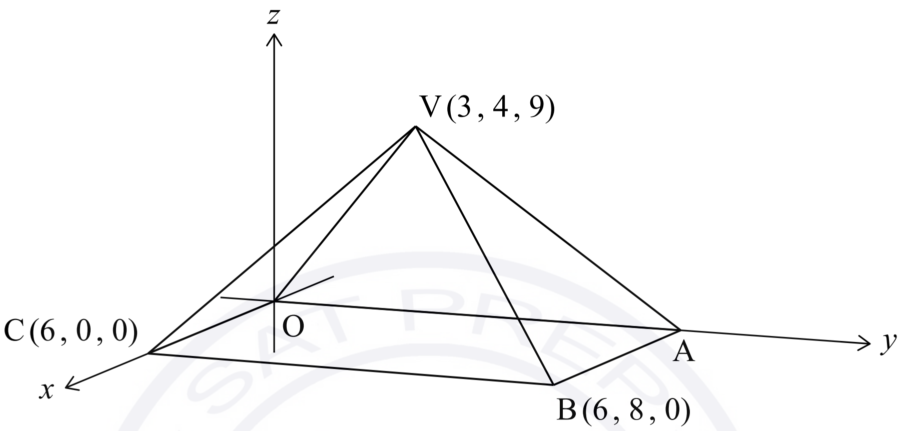
(c) Find the coordinates of B. [3]

2. [Maximum mark: 6]

The following diagram shows a pyramid with vertex V and rectangular base OABC.

Point B has coordinates $(6, 8, 0)$, point C has coordinates $(6, 0, 0)$ and point V has coordinates $(3, 4, 9)$.

diagram not to scale



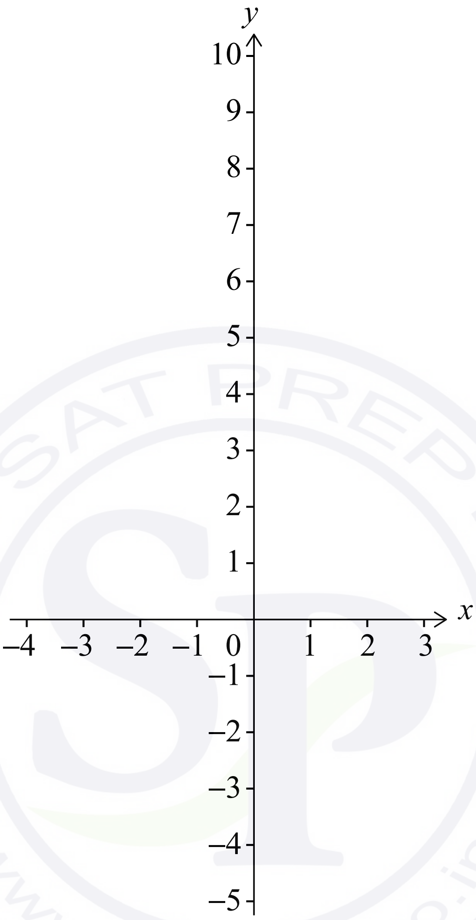
(a) Find BV. [2]

(b) Find the size of $B\hat{V}C$. [4]

3. [Maximum mark: 5]

Consider the function $f(x) = e^{x} - 3x - 4$.

(a) On the following axes, sketch the graph of f for $-4 \le x \le 3$. [3]



The function g is defined by $g(x) = e^{2x} - 6x - 7$.

(b) The graph of g is obtained from the graph of f by a horizontal stretch with scale factor k, followed by a vertical translation of c units.

Find the value of k and the value of c. [2]

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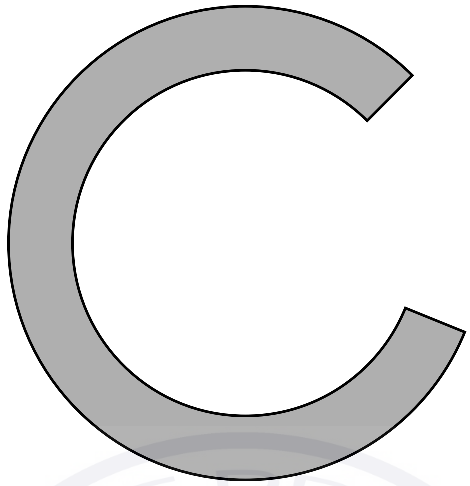
(Question 3 continued)

4. [Maximum mark: 7]

A company is designing a new logo in the shape of a letter “C”.



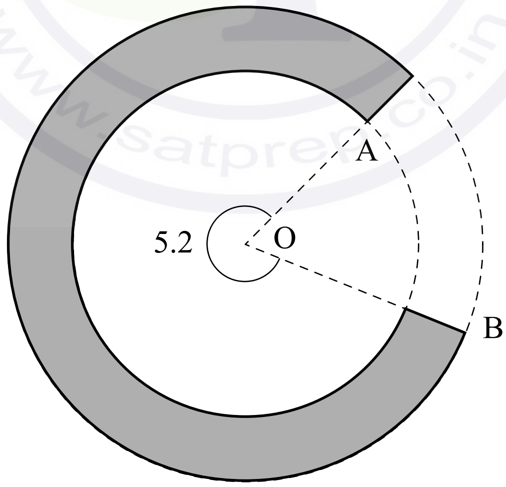
The letter “C” is formed between two circles with centre O.

The point A lies on the circumference of the inner circle with radius $r\,\mathrm{cm}$, where $r < 10$.

The point B lies on the circumference of the outer circle with radius $10\,\mathrm{cm}$.

The reflex angle $\mathrm{A\hat{O}B}$ is 5.2 radians. The letter “C” is shown by the shaded area in the following diagram.

diagram not to scale



(This question continues on the following page)

(Question 4 continued)

(a) Show that the area of the "C" is given by $260 - 2.6r^2$. [2]

The area of the "C" is $64\,\text{cm}^2$.

(b) (i) Find the value of r.

(ii) Find the perimeter of the "C". [5]

5. [Maximum mark: 5]

A particle moves along a straight line. Its displacement, s metres, from a fixed point O after time t seconds is given by $s(t) = 4.3\sin\left(\sqrt{3t+5}\right)$, where $0 \le t \le 10$.

The particle first comes to rest after q seconds.

(a) Find the value of q. [2]

(b) Find the total distance that the particle travels in the first q seconds. [3]

6. [Maximum mark: 5]

The following table shows the probability distribution of a discrete random variable X, where a, $k \in \mathbb{R}^{+}$.

x	1	2	3	4
$\mathrm{P}(X=x)$	k	k^2	a	k^3

Given that $\mathrm{E}(X) = 2.3$, find the value of a.

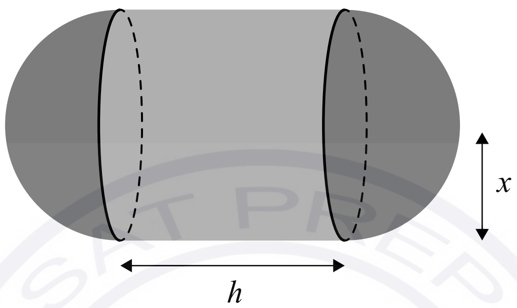
Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 14]

The solid shown in the following diagram is comprised of a cylinder and two hemispheres. The cylinder has height $h\,\text{cm}$ and radius $x\,\text{cm}$. The hemispheres fit exactly onto either end of the cylinder.



The volume of the cylinder is $45\,\text{cm}^3$.

(a) Show that the total surface area, $S\,\text{cm}^2$, of the solid is given by $S = \frac{90}{x} + 4\pi x^2$. [3]

The total surface area of the solid has a local maximum or a local minimum value when $x = a$.

(b) (i) Find an expression for $\frac{\mathrm{d}S}{\mathrm{d}x}$.

(ii) Hence, find the **exact** value of a. [5]

(c) (i) Find an expression for $\frac{\mathrm{d}^2S}{\mathrm{d}x^2}$.

(ii) Use the second derivative of S to justify that S is a minimum when $x = a$.

(iii) Find the minimum surface area of the solid. [6]

Do **not** write solutions on this page.

8. [Maximum mark: 15]

Give your answers to parts (a)(ii), (c)(i) and (d) correct to two decimal places.

Daniela and Sorin have each recently received some money. Daniela won a cash prize and Sorin received an inheritance.

Daniela had two options to choose from to receive her winnings. In both options she receives a payment on the first day of each month for three years.

Option A Each payment is $\$5500$.

Option B The first payment is $\$2000$. In each month which follows, the payment is 6% more than the previous month.

(a) Find the total amount Daniela would receive if she chooses

(i) Option A;

(ii) Option B. [5]

Sorin received an inheritance of $\$120\,000$. Sorin invested his inheritance in an account that pays a nominal annual interest rate of 4% per annum, compounded monthly. The interest is added on the last day of each month.

(b) Write down an expression for the value of Sorin's investment after n years. [1]

Daniela chose Option B and received her first payment on 1st January 2023. Sorin invested his inheritance on the same day.

(c) (i) Find the **total** value of Daniela's winnings and Sorin's investment on the last day of the sixth month.

(ii) Find the minimum number of complete months before the total value of Daniela's winnings and Sorin's investment is at least $\$250\,000$. [6]

At the end of the three years, Daniela invested $\$40\,000$ for a further six years in a second account that pays a nominal interest rate of $r\%$ per annum compounded quarterly.

(d) Find the value of r if this investment grows to $\$53\,000$ after six years. [3]

Do **not** write solutions on this page.

9. [Maximum mark: 16]

A farmer is growing a field of wheat plants. The height, $H\,\text{cm}$, of each plant can be modelled by a normal distribution with mean μ and standard deviation σ.

It is known that $\mathrm{P}(H < 94.6) = 0.288$ and $\mathrm{P}(H > 98.1) = 0.434$.

(a) Find the probability that the height of a randomly selected plant is between $94.6\,\text{cm}$ and $98.1\,\text{cm}$. [2]

(b) Find the value of μ and the value of σ. [5]

The farmer measures 100 randomly selected plants. Any plant with a height greater than $98.1\,\text{cm}$ is considered ready to harvest. Heights of plants are independent of each other.

(c) (i) Find the probability that exactly 34 plants are ready to harvest.

(ii) Given that fewer than 49 plants are ready to harvest, find the probability that exactly 34 plants are ready to harvest. [6]

In another field, the farmer is growing the same variety of wheat, but is using a different fertilizer. The heights of these plants, $F\,\text{cm}$, are normally distributed with mean 98.6 and standard deviation d. The farmer finds the interquartile range to be $4.82\,\text{cm}$.

(d) Find the value of d. [3]

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2223 – 7110

11 pages

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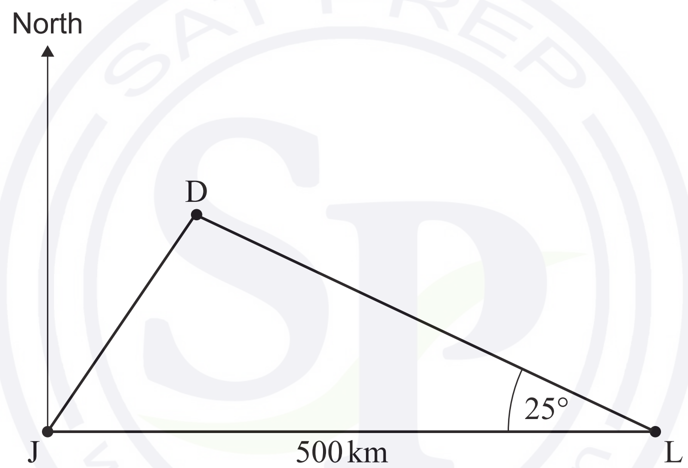
Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 5]

The cities Lucknow (L), Jaipur (J) and Delhi (D) are represented in the following diagram. Lucknow lies $500\,\mathrm{km}$ directly east of Jaipur, and $\mathrm{J\hat{L}D} = 25°$.

diagram not to scale



The bearing of D from J is $034°$.

(a) Find $\mathrm{J\hat{D}L}$. [2]

(b) Find the distance between Lucknow and Delhi. [3]

2. [Maximum mark: 6]

The value of a car is given by the function $C = 40\,000(0.91)^t$, where t is in years since 1 January 2023 and C is in $\text{USD}\,(\$)$.

(a) Write down the annual rate of depreciation of the car. [1]

(b) Find the value of the car on 1 January 2028. [2]

Alvie wants to buy this car. On 1 January 2023, he invested $\$15\,000$ in an account that earns 3% annual interest compounded yearly. He makes no further deposits to, or withdrawals from, the account.

Alvie wishes to buy this car for its value on 1 January 2028. In addition to the money in his account, he will need an extra $\$M$.

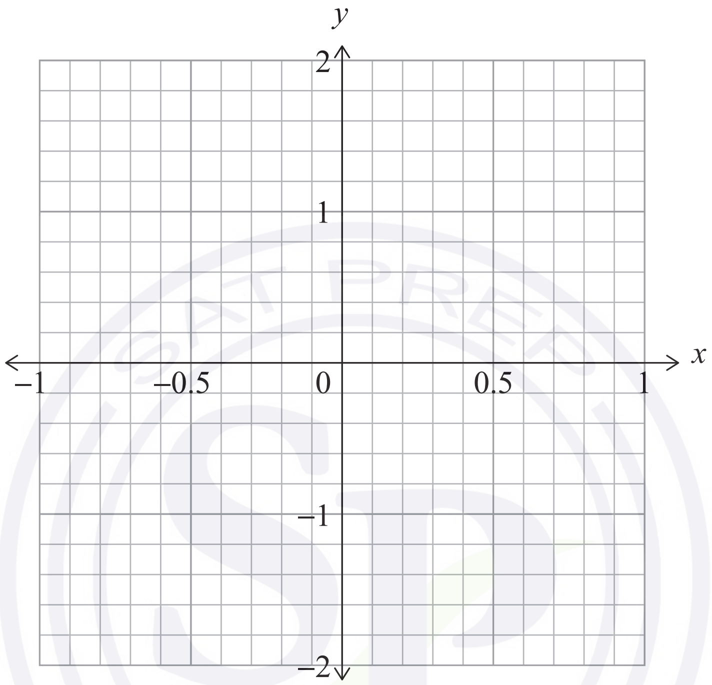
(c) Find the value of M. [3]

3. [Maximum mark: 5]

The functions f and g are defined by $f(x) = 2x - x^3$ and $g(x) = \tan x$.

(a) Find $(f \circ g)(x)$. [2]

(b) On the following grid, sketch the graph of $y = (f \circ g)(x)$ for $-1 \le x \le 1$. Write down and clearly label the coordinates of any local maximum or minimum points. [3]



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4. [Maximum mark: 7]

The total number of children, y, visiting a park depends on the highest temperature, T, in degrees Celsius (°C). A park official predicts the total number of children visiting his park on any given day using the model $y = -0.6T^2 + 23T + 110$, where $10 \le T \le 35$.

(a) Use this model to estimate the number of children in the park on a day when the highest temperature is $25\,°\mathrm{C}$. [2]

An ice cream vendor investigates the relationship between the total number of children visiting the park and the number of ice creams sold, x. The following table shows the data collected on five different days.

Total number of children (y)	81	175	202	346	360
Ice creams sold (x)	15	27	23	35	46

(b) Find an appropriate regression equation that will allow the vendor to predict the number of ice creams sold on a day when there are y children in the park. [3]

(c) Hence, use your regression equation to predict the number of ice creams that the vendor sells on a day when the highest temperature is $25\,°\mathrm{C}$. [2]

5. [Maximum mark: 5]

A company manufactures metal tubes for bicycle frames. The diameters of the tubes, D mm, are normally distributed with mean 32 and standard deviation σ. The interquartile range of the diameters is 0.28.

Find the value of σ.

6. [Maximum mark: 7]

The coefficient of x^6 in the expansion of $(ax^3 + b)^8$ is 448.

The coefficient of x^6 in the expansion of $(ax^3 + b)^{10}$ is 2880.

Find the value of a and the value of b, where a, $b > 0$.

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 13]

The temperature of a cup of tea, t minutes after it is poured, can be modelled by $H(t) = 21 + 75e^{-0.08t}$, $t \geq 0$. The temperature is measured in degrees Celsius ($°C$).

(a) (i) Find the initial temperature of the tea.

(ii) Find the temperature of the tea three minutes after it is poured. [2]

(b) Write down the value of $H'(3)$. [2]

(c) Interpret the meaning of your answer to part (b) in the given context. [2]

(d) After k minutes, the tea will be below $67\,°C$ and cool enough to drink.

Find the least possible value of k, where $k \in \mathbb{Z}^+$. [3]

As the tea cools, $H(t)$ approaches the temperature of the room, which is constant.

(e) Find the temperature of the room. [2]

(f) Find the limit of $H'(t)$ as t approaches infinity. [2]

Do **not** write solutions on this page.

8. [Maximum mark: 17]

In a large city, 160 people were surveyed. Of those, 60 were children (C) and the rest adults (A).

Each person in the survey was asked whether they preferred milk chocolate (M) or dark chocolate (D). It was found that 48 of the children preferred milk chocolate. This information is shown in the following table.

	M **(milk chocolate)**	***D*** **(dark chocolate)**	**Total**
C **(children)**	48	p	60
A **(adults)**	x	y	q

(a) Find the value of

(i) p;

(ii) q. [2]

(b) Three people are chosen at random from those surveyed. Find the probability that all three are adults. [4]

(c) (i) Given that $\mathrm{P}(A \mid M) = \frac{1}{3}$, find the value of x.

(ii) A person is chosen at random from those surveyed. Write down the probability that they are an adult who prefers milk chocolate. [4]

(d) Determine if the events A and M are independent. Justify your answer. [3]

It can be assumed that the survey results are representative of the population of the city.

(e) Ten people in the city are chosen at random. Find the probability that at least five of them prefer dark chocolate. [4]

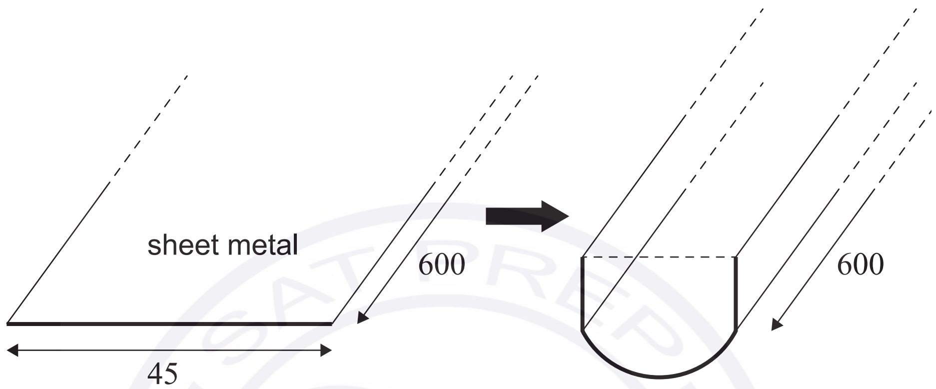
Do **not** write solutions on this page.

9. [Maximum mark: 15]

An engineer is designing a gutter to catch rainwater from the roof of a house.

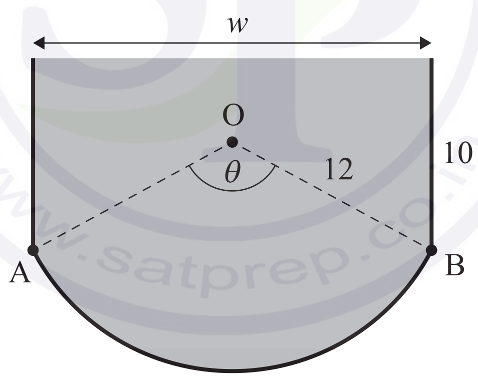
The gutter will be open at the top and is made by folding a piece of sheet metal $45\,\text{cm}$ wide and $600\,\text{cm}$ long.

diagram not to scale



The cross-section of the gutter is shaded in the following diagram.

diagram not to scale



The height of both vertical sides is $10\,\text{cm}$. The width of the gutter is $w\,\text{cm}$.

Arc AB lies on the circumference of a circle with centre O and radius $12\,\text{cm}$.

(This question continues on the following page)

Do **not** write solutions on this page.

(Question 9 continued)

Let $\text{A}\hat{\text{O}}\text{B} = \theta$ radians, where $0 < \theta < \pi$.

(a) Show that $\theta = 2.08$, correct to three significant figures. [3]

(b) Find the area of the cross-section of the gutter. [7]

In a storm, the total volume, in cm^3, of rainwater that enters the gutter can be modelled by a function $R(t)$, where t is the time, in seconds, since the start of the storm.

It was determined that the **rate** at which rainwater entered the gutter could be modelled by

$$R'(t) = 50\cos\left(\frac{2\pi t}{5}\right) + 3000,\ t \geq 0.$$

During any 60-second period, if the volume of rainwater entering the gutter is greater than the volume of the gutter, it will overflow.

(c) Determine whether the gutter overflowed in this storm. Justify your answer. [5]

References:

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Standard level
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12 pages

2223 – 7115

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Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 5]

A botanist is conducting an experiment which studies the growth of plants.

The heights of the plants are measured on seven different days.

The following table shows the number of days, d, that the experiment has been running and the average height, h cm, of the plants on each of those days.

Number of days (d)	2	5	13	24	33	37	42
Average height (h)	10	16	30	59	76	79	82

(a) The regression line of h on d for this data can be written in the form $h = ad + b$.

Find the value of a and the value of b. [2]

(b) Write down the value of the Pearson's product-moment correlation coefficient, r. [1]

(c) Use your regression line to estimate the average height of the plants when the experiment has been running for 20 days. [2]

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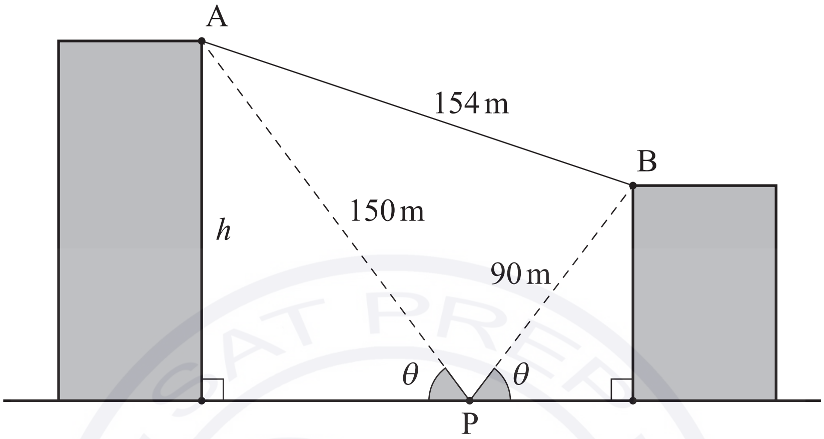
2. [Maximum mark: 6]

The following diagram shows two buildings situated on level ground.

From point P on the ground directly between the two buildings, the angle of elevation to the top of each building is θ.

diagram not to scale



The distance from point P to point A at the top of the taller building is 150 metres.

The distance from point P to point B at the top of the shorter building is 90 metres.

The distance between A and B is 154 metres.

(a) Find the measure of $\text{A}\hat{\text{P}}\text{B}$. [3]

(b) Find the height, h, of the taller building. [3]

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3. [Maximum mark: 5]

The amount of a drug, in milligrams (mg), in a patient's body can be modelled by the function $A(t) = 500\mathrm{e}^{-kt}$, where k is a positive constant and t is the time in hours after the initial dose is given.

(a) Write down the amount of the drug in the patient's body when $t = 0$. [1]

After three hours, the amount of the drug in the patient's body has decreased to $280\,\mathrm{mg}$.

(b) Find the value of k. [2]

The second dose is given T hours after the initial dose, when the amount of the drug in the patient's body is $140\,\mathrm{mg}$.

(c) Find the value of T. [2]

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4. [Maximum mark: 8]

The weights, W grams, of bags of rice packaged in a factory can be modelled by a normal distribution with mean 204 grams and standard deviation 5 grams.

(a) A bag of rice is selected at random.

Find the probability that it weighs more than 210 grams. [2]

According to this model, 80% of the bags of rice weigh between w grams and 210 grams.

(b) Find the probability that a randomly selected bag of rice weighs less than w grams. [2]

(c) Find the value of w. [2]

(d) Ten bags of rice are selected at random.

Find the probability that exactly one of the bags weighs less than w grams. [2]

5. [Maximum mark: 7]

The expansion of $(x+h)^8$, where $h>0$, can be written as $x^8+ax^7+bx^6+cx^5+dx^4+...+h^8$, where $a, b, c, d, ... \in \mathbb{R}$.

(a) Find an expression, in terms of h, for

(i) a;

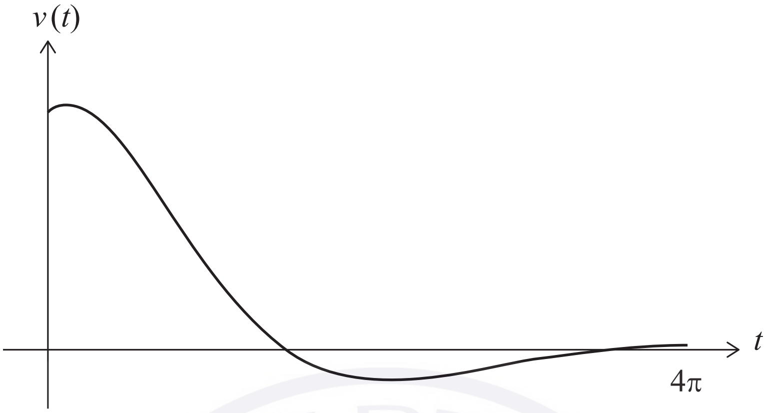
(ii) b;

(iii) d. [4]

(b) Given that a, b, and d are the first three terms of a geometric sequence, find the value of h. [3]

6. [Maximum mark: 6]

A particle moves in a straight line such that its velocity, $v\,\mathrm{m\,s^{-1}}$, at time t seconds is given by $v(t) = 4e^{-\frac{t}{3}}\cos\left(\frac{t}{2} - \frac{\pi}{4}\right)$, for $0 \le t \le 4\pi$. The graph of v is shown in the following diagram.



Let t_1 be the first time when the particle's **acceleration** is zero.

(a) Find the value of t_1. [2]

Let t_2 be the **second** time when the particle is instantaneously at rest.

(b) Find the value of t_2. [2]

(c) Find the distance travelled by the particle between $t = t_1$ and $t = t_2$. [2]

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Please **do not** write on this page.

Answers written on this page
will not be marked.

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 15]

Consider the function $h(x) = \sqrt{4x-2}$, for $x \geq \frac{1}{2}$.

(a) (i) Find $h^{-1}(x)$, the inverse of $h(x)$, and state its domain.

(ii) Write down the range of $h^{-1}(x)$. [5]

(b) The graph of h intersects the graph of h^{-1} at two points.

Find the x-coordinates of these two points. [3]

(c) Find the area enclosed by the graph of h and the graph of h^{-1}. [2]

(d) Find $h'(x)$. [2]

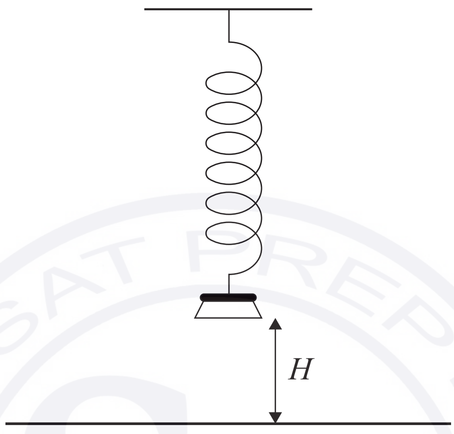
(e) Find the value of x for which the graph of h and the graph of h^{-1} have the same gradient. [3]

Do **not** write solutions on this page.

8. [Maximum mark: 13]

A weight suspended on a spring is pulled down and released, so that it moves up and down vertically.

The height, H metres, of the base of the weight above the ground can be modelled by the function $H(t) = a\cos(7.8t) + b$, for $a, b \in \mathbb{R}$ and $0 \le t \le 10$, where t is the time in seconds after the weight is released.



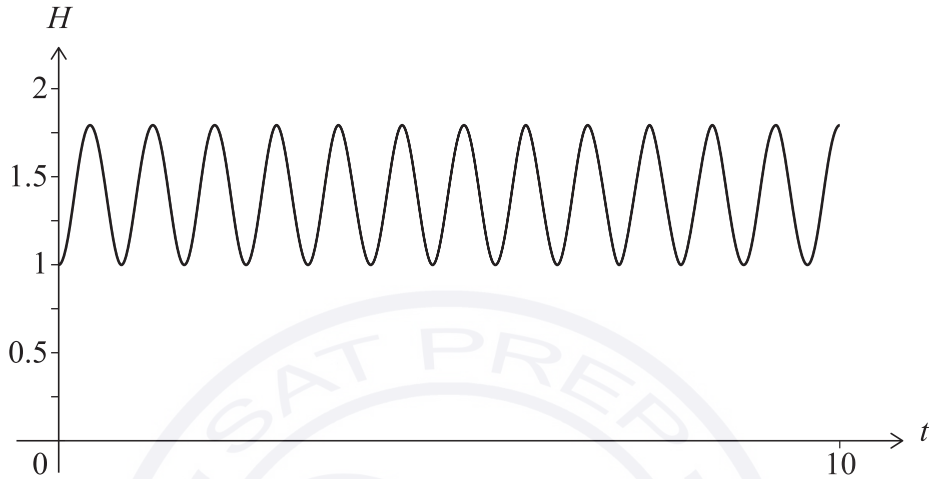
(a) Find the period of the function. [2]

(This question continues on the following page)

Do **not** write solutions on this page.

(Question 8 continued)

The weight is released when its base is at a minimum height of 1 metre above the ground, and it reaches a maximum height of 1.8 metres above the ground. The graph of H is shown in the following diagram.



(b) Find the value of

(i) a;

(ii) b. [3]

(c) Find the number of times that the weight reaches its maximum height in the first five seconds of its motion. [2]

(d) Find the first time that the base of the weight reaches a height of 1.5 metres. [2]

A camera is set to take a picture of the weight at a random time during the first five seconds of its motion.

(e) Find the probability that the height of the base of the weight is greater than 1.5 metres at the time the picture is taken. [4]

Do **not** write solutions on this page.

9. [Maximum mark: 15]

A bag contains n balls. It is known that ten of the balls are green, and the rest of the balls are red. Balls are drawn from the bag, one after the other, without replacement.

(a) Find, in terms of n, the probability that

(i) the first ball drawn is green;

(ii) the first two balls are green. [3]

For the following parts of this question, let $n = 25$.

(b) Show that the probability that the first two balls are red is 0.35. [2]

(c) Find the probability that the first three balls are all red. [2]

(d) Find the probability that at least one of the first three balls is green. [2]

A game is played where **four** balls are drawn, one after the other, from the bag of 25 balls, without replacement. A player earns points based on when the first green ball is drawn. At the end of each game, the four balls are put back in the bag.

A player earns zero points if no green ball is picked, or if the first green ball is picked on the first or second draw.

A player earns 10 points if the first green ball is picked on the third draw and earns 50 points if the first green ball is picked on the fourth draw.

Millie plays this game k times. She finds her score by adding together her points from each game.

(e) Find the least value of k such that Millie's expected score is greater than 100. [6]

References:

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: analysis and approaches
Standard level
Paper 2

9 May 2023

Zone A afternoon | **Zone B** morning | **Zone C** afternoon

Candidate session number

1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

2223 – 7110

11 pages

International Baccalaureate
Baccalauréat International
Bachillerato Internacional

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

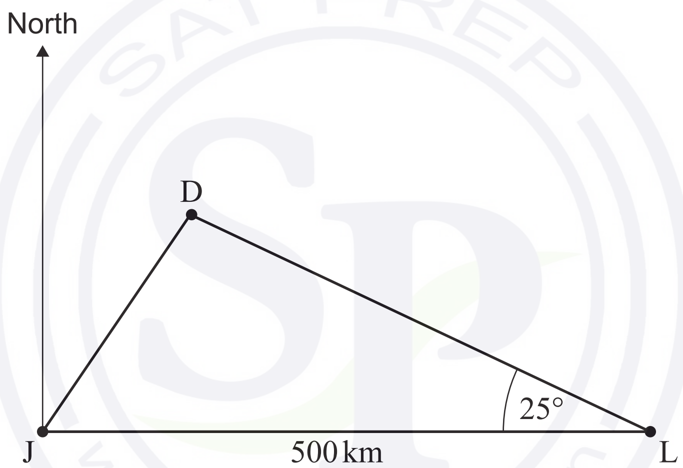
Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 5]

The cities Lucknow (L), Jaipur (J) and Delhi (D) are represented in the following diagram. Lucknow lies $500\,\mathrm{km}$ directly east of Jaipur, and $\mathrm{J\hat{L}D} = 25°$.

diagram not to scale



The bearing of D from J is $034°$.

(a) Find $\mathrm{J\hat{D}L}$. [2]

(b) Find the distance between Lucknow and Delhi. [3]

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2. [Maximum mark: 6]

The value of a car is given by the function $C = 40\,000(0.91)^t$, where t is in years since 1 January 2023 and C is in $\text{USD}\,(\$)$.

(a) Write down the annual rate of depreciation of the car. [1]

(b) Find the value of the car on 1 January 2028. [2]

Alvie wants to buy this car. On 1 January 2023, he invested $\$15\,000$ in an account that earns 3% annual interest compounded yearly. He makes no further deposits to, or withdrawals from, the account.

Alvie wishes to buy this car for its value on 1 January 2028. In addition to the money in his account, he will need an extra $\$M$.

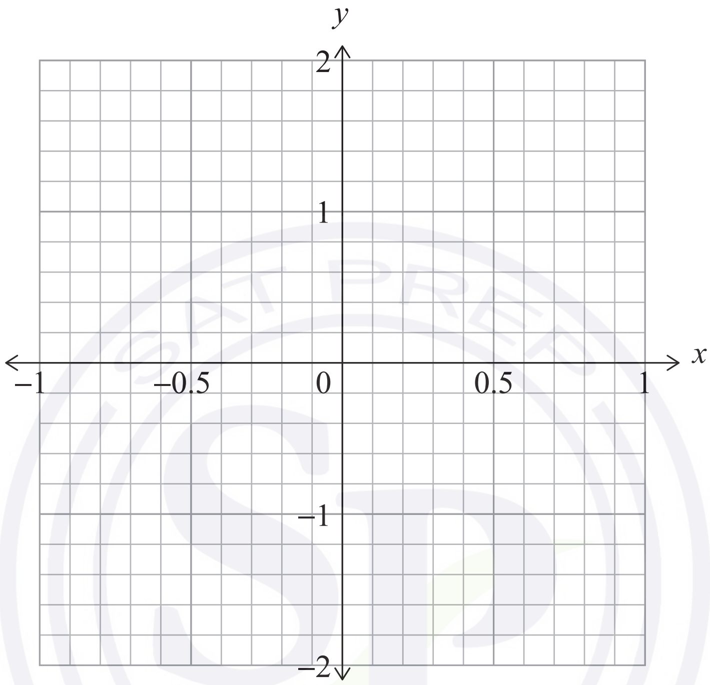
(c) Find the value of M. [3]

3. [Maximum mark: 5]

The functions f and g are defined by $f(x) = 2x - x^3$ and $g(x) = \tan x$.

(a) Find $(f \circ g)(x)$. [2]

(b) On the following grid, sketch the graph of $y = (f \circ g)(x)$ for $-1 \le x \le 1$. Write down and clearly label the coordinates of any local maximum or minimum points. [3]



4. [Maximum mark: 7]

The total number of children, y, visiting a park depends on the highest temperature, T, in degrees Celsius (°C). A park official predicts the total number of children visiting his park on any given day using the model $y = -0.6T^2 + 23T + 110$, where $10 \le T \le 35$.

(a) Use this model to estimate the number of children in the park on a day when the highest temperature is $25\,°C$. [2]

An ice cream vendor investigates the relationship between the total number of children visiting the park and the number of ice creams sold, x. The following table shows the data collected on five different days.

Total number of children (y)	81	175	202	346	360
Ice creams sold (x)	15	27	23	35	46

(b) Find an appropriate regression equation that will allow the vendor to predict the number of ice creams sold on a day when there are y children in the park. [3]

(c) Hence, use your regression equation to predict the number of ice creams that the vendor sells on a day when the highest temperature is $25\,°C$. [2]

5. [Maximum mark: 5]

A company manufactures metal tubes for bicycle frames. The diameters of the tubes, D mm, are normally distributed with mean 32 and standard deviation σ. The interquartile range of the diameters is 0.28.

Find the value of σ.

6. [Maximum mark: 7]

The coefficient of x^6 in the expansion of $(ax^3 + b)^8$ is 448.

The coefficient of x^6 in the expansion of $(ax^3 + b)^{10}$ is 2880.

Find the value of a and the value of b, where a, $b > 0$.

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 13]

The temperature of a cup of tea, t minutes after it is poured, can be modelled by $H(t) = 21 + 75e^{-0.08t}$, $t \geq 0$. The temperature is measured in degrees Celsius $(°C)$.

(a) (i) Find the initial temperature of the tea.

(ii) Find the temperature of the tea three minutes after it is poured. [2]

(b) Write down the value of $H'(3)$. [2]

(c) Interpret the meaning of your answer to part (b) in the given context. [2]

(d) After k minutes, the tea will be below $67\,°C$ and cool enough to drink.

Find the least possible value of k, where $k \in \mathbb{Z}^+$. [3]

As the tea cools, $H(t)$ approaches the temperature of the room, which is constant.

(e) Find the temperature of the room. [2]

(f) Find the limit of $H'(t)$ as t approaches infinity. [2]

Do **not** write solutions on this page.

8. [Maximum mark: 17]

In a large city, 160 people were surveyed. Of those, 60 were children (C) and the rest adults (A).

Each person in the survey was asked whether they preferred milk chocolate (M) or dark chocolate (D). It was found that 48 of the children preferred milk chocolate. This information is shown in the following table.

	M **(milk chocolate)**	***D*** **(dark chocolate)**	**Total**
C **(children)**	48	p	60
A **(adults)**	x	y	q

(a) Find the value of

(i) p;

(ii) q. [2]

(b) Three people are chosen at random from those surveyed. Find the probability that all three are adults. [4]

(c) (i) Given that $\mathrm{P}(A \mid M) = \frac{1}{3}$, find the value of x.

(ii) A person is chosen at random from those surveyed. Write down the probability that they are an adult who prefers milk chocolate. [4]

(d) Determine if the events A and M are independent. Justify your answer. [3]

It can be assumed that the survey results are representative of the population of the city.

(e) Ten people in the city are chosen at random. Find the probability that at least five of them prefer dark chocolate. [4]

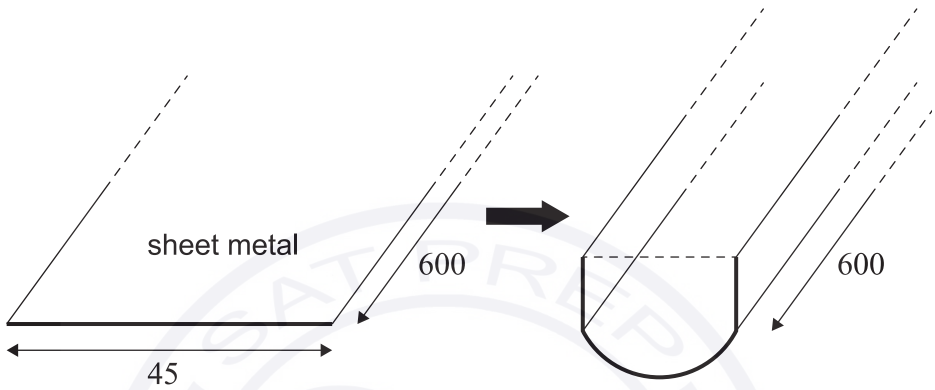
Do **not** write solutions on this page.

9. [Maximum mark: 15]

An engineer is designing a gutter to catch rainwater from the roof of a house.

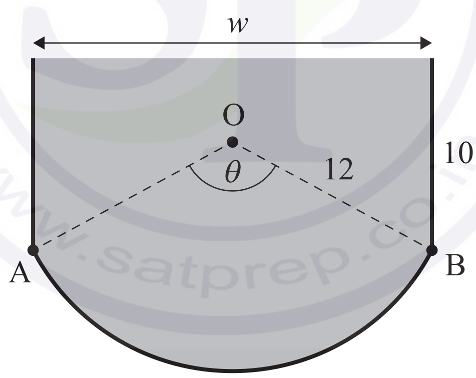
The gutter will be open at the top and is made by folding a piece of sheet metal 45 cm wide and 600 cm long.

diagram not to scale



The cross-section of the gutter is shaded in the following diagram.

diagram not to scale



The height of both vertical sides is 10 cm. The width of the gutter is $w\text{ cm}$.

Arc AB lies on the circumference of a circle with centre O and radius 12 cm.

(This question continues on the following page)

Do **not** write solutions on this page.

(Question 9 continued)

Let $\text{A}\hat{\text{O}}\text{B} = \theta$ radians, where $0 < \theta < \pi$.

(a) Show that $\theta = 2.08$, correct to three significant figures. [3]

(b) Find the area of the cross-section of the gutter. [7]

In a storm, the total volume, in cm^3, of rainwater that enters the gutter can be modelled by a function $R(t)$, where t is the time, in seconds, since the start of the storm.

It was determined that the **rate** at which rainwater entered the gutter could be modelled by

$$R'(t) = 50\cos\left(\frac{2\pi t}{5}\right) + 3000\,,\ t \geq 0\,.$$

During any 60-second period, if the volume of rainwater entering the gutter is greater than the volume of the gutter, it will overflow.

(c) Determine whether the gutter overflowed in this storm. Justify your answer. [5]

References:

Please **do not** write on this page.

Answers written on this page
will not be marked.

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: analysis and approaches
Standard level
Paper 2

Tuesday 1 November 2022 (morning)

Candidate session number

1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
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- A clean copy of the **mathematics: analysis and approaches formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

8822 – 7105

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 5]

The following table shows the Mathematics test scores (x) and the Science test scores (y) for a group of eight students.

Mathematics scores (x)	64	68	72	75	80	82	85	86
Science scores (y)	67	72	77	76	84	83	89	91

The regression line of y on x for this data can be written in the form $y = ax + b$.

(a) Find the value of a and the value of b. [2]

(b) Write down the value of the Pearson's product-moment correlation coefficient, r. [1]

(c) Use the equation of your regression line to predict the Science test score for a student who has a score of 78 on the Mathematics test. Express your answer to the nearest integer. [2]

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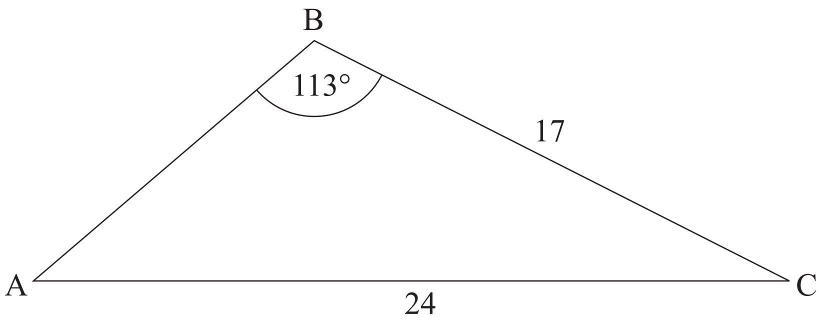
2. [Maximum mark: 6]

The following diagram shows triangle ABC, with $\text{AC} = 24$, $\text{BC} = 17$, and $\text{A}\hat{\text{B}}\text{C} = 113°$.

diagram not to scale

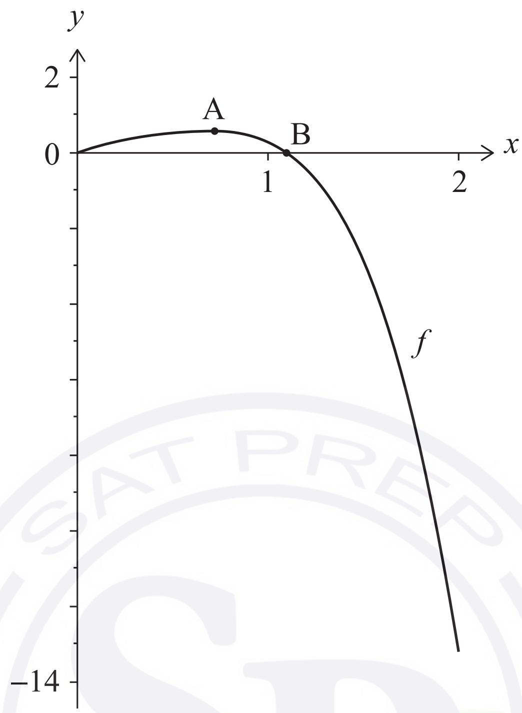


(a) Find $\text{B}\hat{\text{A}}\text{C}$. [3]

(b) Find AB. [3]

3. [Maximum mark: 6]

The function f is defined as $f(x) = \ln(xe^x + 1) - x^4$, for $0 \le x \le 2$. The graph of f is shown in the following diagram.



The graph of f has a local maximum at point A. The graph intersects the x-axis at the origin and at point B.

(a) Find the coordinates of A. [2]

(b) Find the x-coordinate of B. [1]

(c) Find the total area enclosed by the graph of f, the x-axis and the line $x = 2$. [3]

4. [Maximum mark: 5]

A geometric sequence has a first term of 50 and a fourth term of 86.4.

The sum of the first n terms of the sequence is S_n.

Find the smallest value of n such that $S_n > 33\,500$.

5. [Maximum mark: 7]

The population of a town t years after 1 January 2014 can be modelled by the function

$$P(t) = 15\,000\mathrm{e}^{kt}\text{, where } k < 0 \text{ and } t \geq 0.$$

It is known that between 1 January 2014 and 1 January 2022 the population decreased by 11%.

Use this model to estimate the population of this town on 1 January 2041.

6. [Maximum mark: 6]

Consider the expansion of $\dfrac{(ax+1)^9}{21x^2}$, where $a \neq 0$. The coefficient of the term in x^4 is $\dfrac{8}{7}a^5$.

Find the value of a.

Please **do not** write on this page.

Answers written on this page
will not be marked.

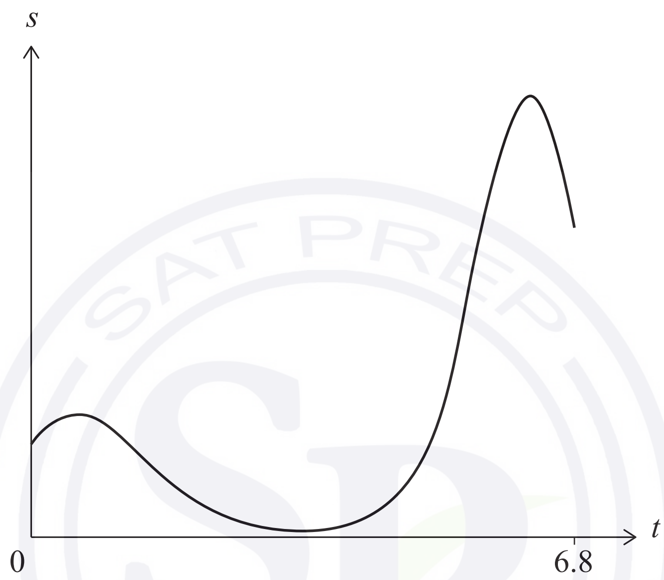
Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 16]

A particle moves in a straight line. Its displacement, s metres, from a fixed point P at time t seconds is given by $s(t) = 3(t+2)^{\cos t}$, for $0 \le t \le 6.8$, as shown in the following graph.



(a) Find the particle's initial displacement from the point P. [2]

(b) Find the particle's velocity when $t = 2$. [2]

(c) Determine the intervals of time when the particle is moving away from the point P. [5]

The acceleration of the particle is zero when $t = b$ and $t = c$, where $b < c$.

(d) Find the value of b and the value of c. [4]

(e) Find the total distance travelled by the particle for $b \le t \le c$. [3]

Do **not** write solutions on this page.

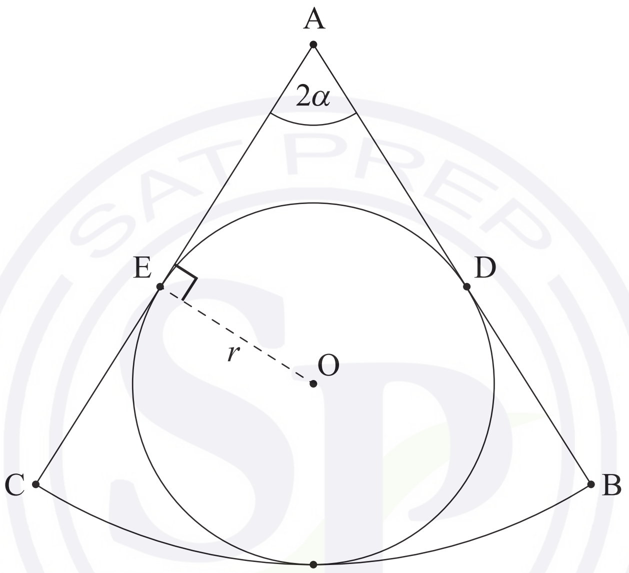
8. [Maximum mark: 13]

The following diagram shows a sector ABC of a circle with centre A. The angle $\mathrm{B\hat{A}C} = 2\alpha$, where $0 < \alpha < \frac{\pi}{2}$, and $\mathrm{O\hat{E}A} = \frac{\pi}{2}$.

A circle with centre O and radius r is inscribed in sector ABC.

AB and AC are both tangent to the circle at points D and E respectively.

diagram not to scale



(a) Show that the area of the quadrilateral ADOE is $\dfrac{r^2}{\tan\alpha}$. [4]

(This question continues on the following page)

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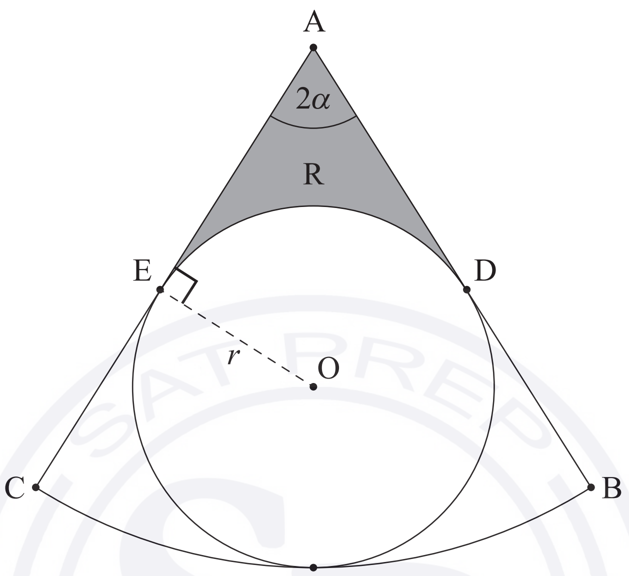
(Question 8 continued)

R represents the shaded region shown in the following diagram.

diagram not to scale



(b) (i) Find $\mathrm{D\hat{O}E}$ in terms of α.

(ii) Hence or otherwise, find an expression for the area of R. [5]

(c) Find the value of α for which the area of R is equal to the area of the circle of centre O and radius r. [4]

Do **not** write solutions on this page.

9. [Maximum mark: 16]

The time worked, T, in hours per week by employees of a large company is normally distributed with a mean of 42 and standard deviation 10.7.

(a) Find the probability that an employee selected at random works more than 40 hours per week. [2]

(b) A group of four employees is selected at random. Each employee is asked in turn whether they work more than 40 hours per week. Find the probability that the fourth employee is the only one in the group who works more than 40 hours per week. [3]

(c) A large group of employees work more than 40 hours per week.

(i) An employee is selected at random from this large group.

Find the probability that this employee works less than 55 hours per week.

(ii) Ten employees are selected at random from this large group.

Find the probability that exactly five of them work less than 55 hours per week. [7]

It is known that $\mathrm{P}(a \le T \le b) = 0.904$ and that $\mathrm{P}(T > b) = 2\mathrm{P}(T < a)$, where a and b are numbers of hours worked per week. An employee who works fewer than a hours per week is considered to be a part-time employee.

(d) Find the maximum time, in hours per week, that an employee can work and still be considered part-time. [4]

References:

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: analysis and approaches
Standard level
Paper 2

Monday 9 May 2022 (morning)

1 hour 30 minutes

Candidate session number

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
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- A clean copy of the **mathematics: analysis and approaches formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

2222–7115

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

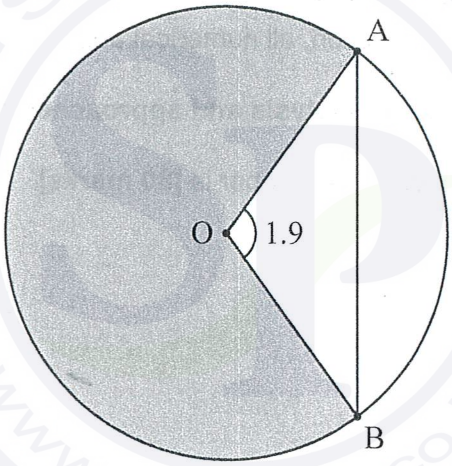
Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 6]

The following diagram shows a circle with centre O and radius 5 metres.

Points A and B lie on the circle and $A\hat{O}B = 1.9$ radians.

diagram not to scale



(a) Find the length of the chord [AB]. [3]

(b) Find the area of the shaded sector. [3]

2. [Maximum mark: 5]

The derivative of a function g is given by $g'(x) = 3x^2 + 5e^x$, where $x \in \mathbb{R}$. The graph of g passes through the point $(0, 4)$. Find $g(x)$.

3. [Maximum mark: 6]

Gemma and Kaia started working for different companies on January 1st 2011.

Gemma's starting annual salary was $\$45\,000$, and her annual salary increases 2% on January 1st each year after 2011.

(a) Find Gemma's annual salary for the year 2021, to the nearest dollar. [3]

Kaia's annual salary is based on a yearly performance review. Her salary for the years 2011, 2013, 2014, 2018, and 2022 is shown in the following table.

year (x)	2011	2013	2014	2018	2022
annual salary ($\$S$)	$45\,000$	$47\,200$	$48\,500$	$53\,000$	$57\,000$

(b) Assuming Kaia's annual salary can be approximately modelled by the equation $S = ax + b$, show that Kaia had a higher salary than Gemma in the year 2021, according to the model. [3]

4. [Maximum mark: 6]

Events A and B are independent and $\mathrm{P}(A) = 3\mathrm{P}(B)$.

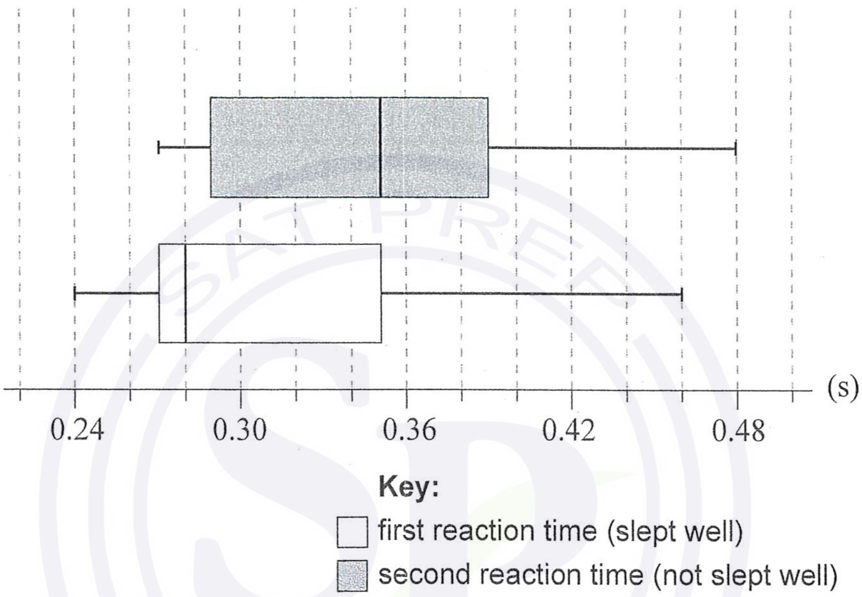
Given that $\mathrm{P}(A \cup B) = 0.68$, find $\mathrm{P}(B)$.

5. [Maximum mark: 6]

A random sample of nine adults were selected to see whether sleeping well affected their reaction times to a visual stimulus. Each adult's reaction time was measured twice.

The first measurement for reaction time was taken on a morning after the adult had slept well. The second measurement was taken on a morning after the same adult had not slept well.

The box and whisker diagrams for the reaction times, measured in seconds, are shown below.



Consider the box and whisker diagram representing the reaction times after sleeping well.

(a) State the median reaction time after sleeping well. [1]

(b) Verify that the measurement of 0.46 seconds is not an outlier. [3]

(c) State why it appears that the mean reaction time is greater than the median reaction time. [1]

Now consider the two box and whisker diagrams.

(d) Comment on whether these box and whisker diagrams provide any evidence that might suggest that not sleeping well causes an increase in reaction time. [1]

(This question continues on the following page)

(Question 5 continued)

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6. [Maximum mark: 7]

A particle moves in a straight line such that its velocity, $v\,\text{m s}^{-1}$, at time t seconds is given by

$$v = \frac{(t^2+1)\cos t}{4},\ 0 \le t \le 3.$$

(a) Determine when the particle changes its direction of motion. [2]

(b) Find the times when the particle's acceleration is $-1.9\,\text{m s}^{-2}$. [3]

(c) Find the particle's acceleration when its speed is at its greatest. [2]

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Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

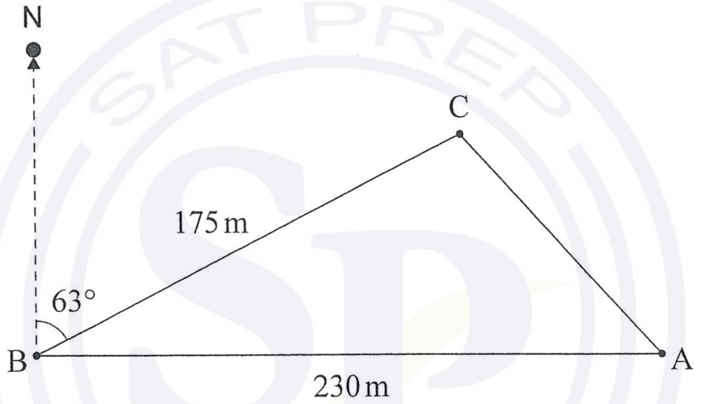
7. [Maximum mark: 14]

A farmer is placing posts at points A, B, and C in the ground to mark the boundaries of a triangular piece of land on his property.

From point A, he walks due west 230 metres to point B.
From point B, he walks 175 metres on a bearing of 063° to reach point C.

This is shown in the following diagram.

diagram not to scale



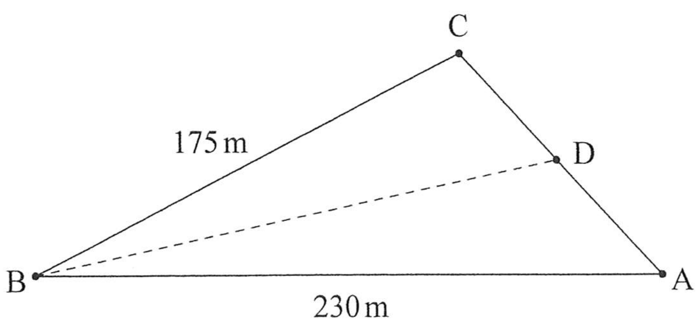
(a) Find the distance from point A to point C. [4]

(b) Find the area of this piece of land. [2]

(c) Find $\hat{CAB}$. [3]

The farmer wants to divide the piece of land into two sections. He will put a post at point D, which is between A and C. He wants the boundary BD to divide the piece of land such that the sections have equal area. This is shown in the following diagram.

diagram not to scale



(d) Find the distance from point B to point D. [5]

Do **not** write solutions on this page.

8. [Maximum mark: 12]

A scientist conducted a nine-week experiment on two plants, A and B, of the same species. He wanted to determine the effect of using a new plant fertilizer. Plant A was given fertilizer regularly, while Plant B was not.

The scientist found that the height of Plant A, h_A cm, at time t weeks can be modelled by the function $h_A(t) = \sin(2t + 6) + 9t + 27$, where $0 \le t \le 9$.

The scientist found that the height of Plant B, h_B cm, at time t weeks can be modelled by the function $h_B(t) = 8t + 32$, where $0 \le t \le 9$.

(a) Use the scientist's models to find the initial height of

(i) Plant B;

(ii) Plant A correct to three significant figures. [3]

(b) Find the values of t when $h_A(t) = h_B(t)$. [3]

(c) For $0 \le t \le 9$, find the total amount of time when the rate of growth of Plant B was greater than the rate of growth of Plant A. [6]

Do **not** write solutions on this page.

9. [Maximum mark: 18]

The time it takes Suzi to drive from home to work each morning is normally distributed with a mean of 35 minutes and a standard deviation of σ minutes.

On 25% of days, it takes Suzi longer than 40 minutes to drive to work.

(a) Find the value of σ. [4]

(b) On a randomly selected day, find the probability that Suzi's drive to work will take longer than 45 minutes. [2]

Suzi will be late to work if it takes her longer than 45 minutes to drive to work. The time it takes to drive to work each day is independent of any other day.

Suzi will work five days next week.

(c) Find the probability that she will be late to work at least one day next week. [3]

(d) Given that Suzi will be late to work at least one day next week, find the probability that she will be late less than three times. [5]

Suzi will work 22 days this month. She will receive a bonus if she is on time at least 20 of those days.

So far this month, she has worked 16 days and been on time 15 of those days.

(e) Find the probability that Suzi will receive a bonus. [4]

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: analysis and approaches
Standard level
Paper 2

Monday 9 May 2022 (morning)

Candidate session number

1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

2222 – 7110

11 pages

International Baccalaureate
Baccalauréat International
Bachillerato Internacional

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 6]

In this question, give all answers correct to two decimal places.

Sam invests $\$1700$ in a savings account that pays a nominal annual rate of interest of 2.74%, compounded half-yearly. Sam makes no further payments to, or withdrawals from, this account.

(a) Find the amount that Sam will have in his account after 10 years. [3]

David also invests $\$1700$ in a savings account that pays an annual rate of interest of $r\%$, compounded yearly. David makes no further payments or withdrawals from this account.

(b) Find the value of r required so that the amount in David's account after 10 years will be equal to the amount in Sam's account. [2]

(c) Find the interest David will earn over the 10 years. [1]

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2. [Maximum mark: 4]

The number of hours spent exercising each week by a group of students is shown in the following table.

Exercising time (in hours)	**Number of students**
2	5
3	1
4	4
5	3
6	x

The median is 4.5 hours.

(a) Find the value of x. [2]

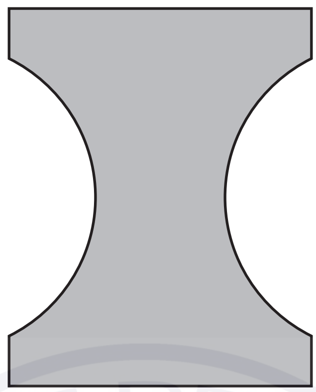
(b) Find the standard deviation. [2]

Turn over

3. [Maximum mark: 6]

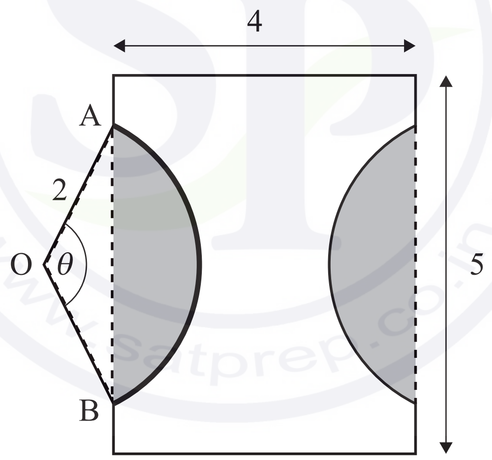
A company is designing a new logo. The logo is created by removing two equal segments from a rectangle, as shown in the following diagram.

diagram not to scale



The rectangle measures $5\,\text{cm}$ by $4\,\text{cm}$. The points A and B lie on a circle, with centre O and radius $2\,\text{cm}$, such that $\text{A}\hat{\text{O}}\text{B} = \theta$, where $0 < \theta < \pi$. This information is shown in the following diagram.

diagram not to scale



(a) Find the area of one of the shaded segments in terms of θ. [3]

(b) Given that the area of the logo is $13.4\,\text{cm}^2$, find the value of θ. [3]

(This question continues on the following page)

(Question 3 continued)

4. [Maximum mark: 6]

A discrete random variable, X, has the following probability distribution:

x	0	1	2	3
$\mathrm{P}(X=x)$	0.41	$k-0.28$	0.46	$0.29-2k^2$

(a) Show that $2k^2-k+0.12=0$. [1]

(b) Find the value of k, giving a reason for your answer. [3]

(c) Hence, find $\mathrm{E}(X)$. [2]

5. [Maximum mark: 7]

A particle moves along a straight line so that its velocity, $v\,\mathrm{m\,s^{-1}}$, after t seconds is given by $v(t) = \mathrm{e}^{\sin t} + 4\sin t$ for $0 \le t \le 6$.

(a) Find the value of t when the particle is at rest. [2]

(b) Find the acceleration of the particle when it changes direction. [3]

(c) Find the total distance travelled by the particle. [2]

6. [Maximum mark: 6]

Let A and B be two independent events such that $\mathrm{P}(A \cap B') = 0.16$ and $\mathrm{P}(A' \cap B) = 0.36$.

(a) Given that $\mathrm{P}(A \cap B) = x$, find the value of x. [4]

(b) Find $\mathrm{P}(A' \mid B')$. [2]

Do **not** write solutions on this page.

Section B

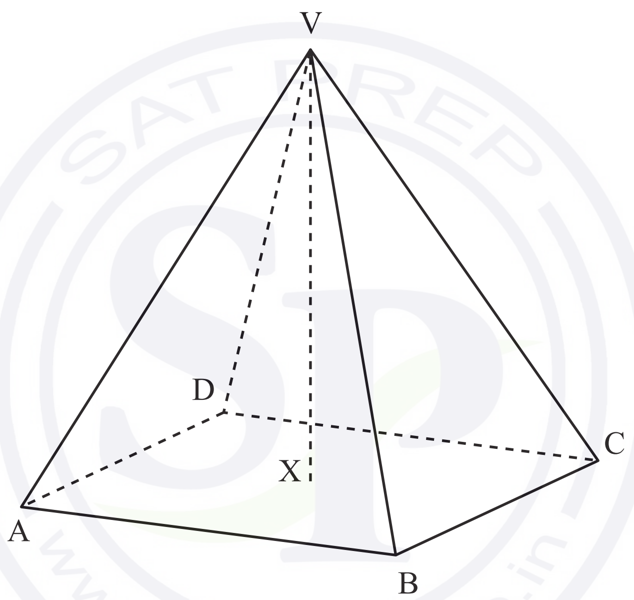
Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 13]

All lengths in this question are in centimetres.

A solid metal ornament is in the shape of a right pyramid, with vertex V and square base ABCD. The centre of the base is X. Point V has coordinates $(1, 5, 0)$ and point A has coordinates $(-1, 1, 6)$.

diagram not to scale



(a) Find AV. [2]

(b) Given that $\mathrm{A\hat{V}B} = 40^\circ$, find AB. [3]

The volume of the pyramid is $57.2\,\mathrm{cm}^3$, correct to three significant figures.

(c) Find the height of the pyramid, VX. [3]

A second ornament is in the shape of a cuboid with a rectangular base of length $2x\,\mathrm{cm}$, width $x\,\mathrm{cm}$ and height $y\,\mathrm{cm}$. The cuboid has the same volume as the pyramid.

(d) The cuboid has a minimum surface area of $S\,\mathrm{cm}^2$. Find the value of S. [5]

Do **not** write solutions on this page.

8. [Maximum mark: 16]

The function f is defined by $f(x) = \frac{4x+1}{x+4}$, where $x \in \mathbb{R}$, $x \neq -4$.

(a) For the graph of f

(i) write down the equation of the vertical asymptote;

(ii) find the equation of the horizontal asymptote. [3]

(b) (i) Find $f^{-1}(x)$.

(ii) Using an algebraic approach, show that the graph of f^{-1} is obtained by a reflection of the graph of f in the y-axis followed by a reflection in the x-axis. [8]

The graphs of f and f^{-1} intersect at $x = p$ and $x = q$, where $p < q$.

(c) (i) Find the value of p and the value of q.

(ii) Hence, find the area enclosed by the graph of f and the graph of f^{-1}. [5]

Do **not** write solutions on this page

9. [Maximum mark: 16]

A bakery makes two types of muffins: chocolate muffins and banana muffins.

The weights, C grams, of the chocolate muffins are normally distributed with a mean of 62 g and standard deviation of 2.9 g.

(a) Find the probability that a randomly selected chocolate muffin weighs less than 61 g. [2]

(b) In a random selection of 12 chocolate muffins, find the probability that exactly 5 weigh less than 61 g. [2]

The weights, B grams, of the banana muffins are normally distributed with a mean of 68 g and standard deviation of 3.4 g.

Each day 60% of the muffins made are chocolate.

On a particular day, a muffin is randomly selected from all those made at the bakery.

(c) (i) Find the probability that the randomly selected muffin weighs less than 61 g.

(ii) Given that a randomly selected muffin weighs less than 61 g, find the probability that it is chocolate. [7]

The machine that makes the chocolate muffins is adjusted so that the mean weight of the chocolate muffins remains the same but their standard deviation changes to $\sigma\text{ g}$. The machine that makes the banana muffins is not adjusted. The probability that the weight of a randomly selected muffin from these machines is less than 61 g is now 0.157.

(d) Find the value of σ. [5]

References:

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Answers written on this page
will not be marked.

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: analysis and approaches
Standard level
Paper 2

Tuesday 2 November 2021 (morning)

Candidate session number

1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

8821 – 7105

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 5]

In Lucy's music academy, eight students took their piano diploma examination and achieved scores out of 150. For her records, Lucy decided to record the average number of hours per week each student reported practising in the weeks prior to their examination. These results are summarized in the table below.

Average weekly practice time (h)	28	13	45	33	17	29	39	36
Diploma score (D)	115	82	120	116	79	101	110	121

(a) Find Pearson's product-moment correlation coefficient, r, for these data. [2]

(b) The relationship between the variables can be modelled by the regression equation $D = ah + b$. Write down the value of a and the value of b. [1]

(c) One of these eight students was disappointed with her result and wished she had practised more. Based on the given data, determine how her score could have been expected to alter had she practised an extra five hours per week. [2]

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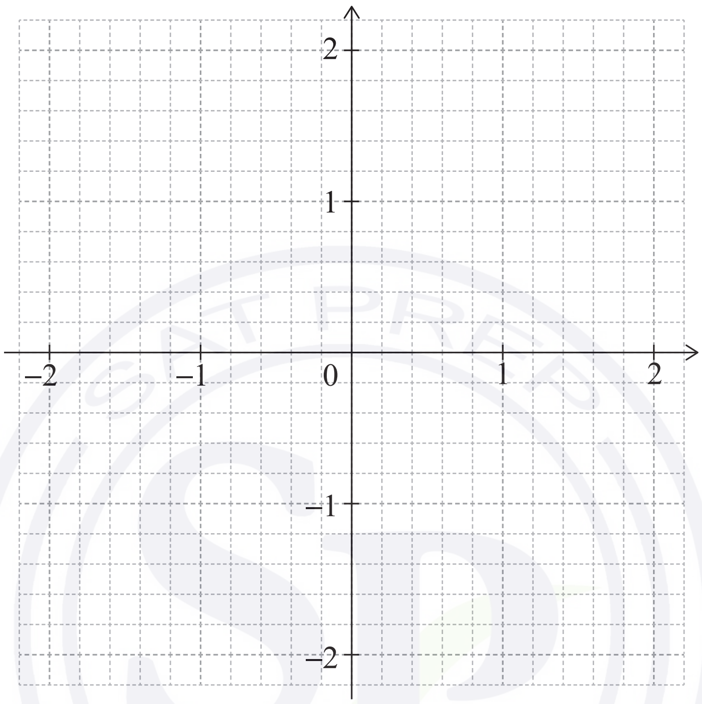
2. [Maximum mark: 5]

Consider the function $f(x) = e^{-x^2} - 0.5$, for $-2 \le x \le 2$.

(a) Find the values of x for which $f(x) = 0$. [2]

(b) Sketch the graph of f on the following grid. [3]



3. [Maximum mark: 5]

Consider a triangle ABC, where $\text{AC} = 12$, $\text{CB} = 7$ and $\text{B}\hat{\text{A}}\text{C} = 25°$.

Find the smallest possible perimeter of triangle ABC.

4. [Maximum mark: 7]

A factory manufactures lamps. It is known that the probability that a lamp is found to be defective is 0.05. A random sample of 30 lamps is tested.

(a) Find the probability that there is at least one defective lamp in the sample. [3]

(b) Given that there is at least one defective lamp in the sample, find the probability that there are at most two defective lamps. [4]

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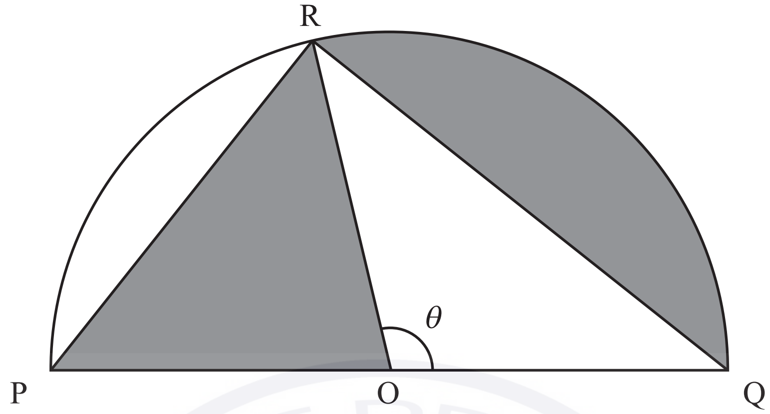
5. [Maximum mark: 6]

The following diagram shows a semicircle with centre O and radius r. Points P, Q and R lie on the circumference of the circle, such that $\mathrm{PQ} = 2r$ and $\mathrm{R\hat{O}Q} = \theta$, where $0 < \theta < \pi$.



(a) Given that the areas of the two shaded regions are equal, show that $\theta = 2\sin\theta$. [5]

(b) Hence determine the value of θ. [1]

6. [Maximum mark: 9]

The sum of the first n terms of a geometric sequence is given by $S_n = \sum_{r=1}^{n} \frac{2}{3}\left(\frac{7}{8}\right)^r$.

(a) Find the first term of the sequence, u_1. [2]

(b) Find S_∞. [3]

(c) Find the least value of n such that $S_\infty - S_n < 0.001$. [4]

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 14]

Points A and P lie on opposite banks of a river, such that AP is the shortest distance across the river. Point B represents the centre of a city which is located on the riverbank. $\text{PB} = 215\,\text{km}$, $\text{AP} = 65\,\text{km}$ and $\text{A}\hat{\text{P}}\text{B} = 90^\circ$.

The following diagram shows this information.

A boat travels at an average speed of $42\,\text{km}\,\text{h}^{-1}$. A bus travels along the straight road between P and B at an average speed of $84\,\text{km}\,\text{h}^{-1}$.

(a) Find the travel time, in hours, from A to B given that

(i) the boat is taken from A to P, and the bus from P to B;

(ii) the boat travels directly to B. [4]

There is a point D, which lies on the road from P to B, such that $\text{BD} = x\,\text{km}$. The boat travels from A to D, and the bus travels from D to B.

(b) (i) Find an expression, in terms of x for the travel time T, from A to B, passing through D.

(ii) Find the value of x so that T is a minimum.

(iii) Write down the minimum value of T. [6]

(c) An excursion involves renting the boat and the bus. The cost to rent the boat is $\$200$ per hour, and the cost to rent the bus is $\$150$ per hour.

(i) Find the new value of x so that the total cost C to travel from A to B via D is a minimum.

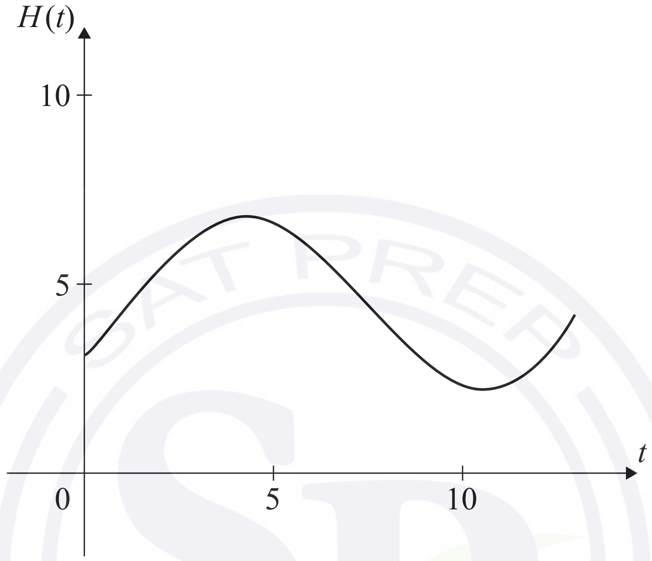
(ii) Write down the minimum total cost for this journey. [4]

Do **not** write solutions on this page.

8. [Maximum mark: 13]

The height of water, in metres, in Dungeness harbour is modelled by the function $H(t) = a\sin\left(b(t-c)\right) + d$, where t is the number of hours after midnight, and a, b, c and d are constants, where $a > 0$, $b > 0$ and $c > 0$.

The following graph shows the height of the water for 13 hours, starting at midnight.



The first high tide occurs at 04:30 and the next high tide occurs 12 hours later. Throughout the day, the height of the water fluctuates between $2.2\,\text{m}$ and $6.8\,\text{m}$.

All heights are given correct to one decimal place.

(a) Show that $b = \frac{\pi}{6}$. [1]

(b) Find the value of a. [2]

(c) Find the value of d. [2]

(d) Find the smallest possible value of c. [3]

(e) Find the height of the water at 12:00. [2]

(f) Determine the number of hours, over a 24-hour period, for which the tide is higher than 5 metres. [3]

Do **not** write solutions on this page.

9. [Maximum mark: 16]

The random variable X follows a normal distribution with mean μ and standard deviation σ.

(a) Find $\mathrm{P}(\mu - 1.5\sigma < X < \mu + 1.5\sigma)$. [3]

The avocados grown on a farm have weights, in grams, that are normally distributed with mean μ and standard deviation σ. Avocados are categorized as small, medium, large or premium, according to their weight. The following table shows the probability an avocado grown on the farm is classified as small, medium, large or premium.

Category	Small	Medium	Large	Premium
Probability	0.04	0.576	0.288	0.096

The maximum weight of a small avocado is 106.2 grams.

The minimum weight of a premium avocado is 182.6 grams.

(b) Find the value of μ and of σ. [5]

A supermarket purchases all the avocados from the farm that weigh more than 106.2 grams.

(c) Find the probability that an avocado chosen at random from this purchase is categorized as

(i) medium;

(ii) large;

(iii) premium. [4]

The selling prices of the different categories of avocado at this supermarket are shown in the following table:

Category	Medium	Large	Premium
Selling price ($) per avocado	1.10	1.29	1.96

The supermarket pays the farm $\$200$ for the avocados and assumes it will then sell them in exactly the same proportion as purchased from the farm.

(d) According to this model, find the minimum number of avocados that must be sold so that the net profit for the supermarket is at least $\$438$. [4]

References:

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Answers written on this page
will not be marked.

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Answers written on this page
will not be marked.

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: analysis and approaches
Standard level
Paper 2

Friday 7 May 2021 (morning)

Candidate session number

1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

2221 – 7115

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 6]

At a café, the waiting time between ordering and receiving a cup of coffee is dependent upon the number of customers who have already ordered their coffee and are waiting to receive it.

Sarah, a regular customer, visited the café on five consecutive days. The following table shows the number of customers, x, ahead of Sarah who have already ordered and are waiting to receive their coffee and Sarah's waiting time, y minutes.

Number of customers (x)	3	9	11	10	5
Sarah's waiting time (y)	6	10	12	11	6

The relationship between x and y can be modelled by the regression line of y on x with equation $y = ax + b$.

(a) (i) Find the value of a and the value of b.

(ii) Write down the value of Pearson's product-moment correlation coefficient, r. [3]

(b) Interpret, in context, the value of a found in part (a)(i). [1]

On another day, Sarah visits the café to order a coffee. Seven customers have already ordered their coffee and are waiting to receive it.

(c) Use the result from part (a)(i) to estimate Sarah's waiting time to receive her coffee. [2]

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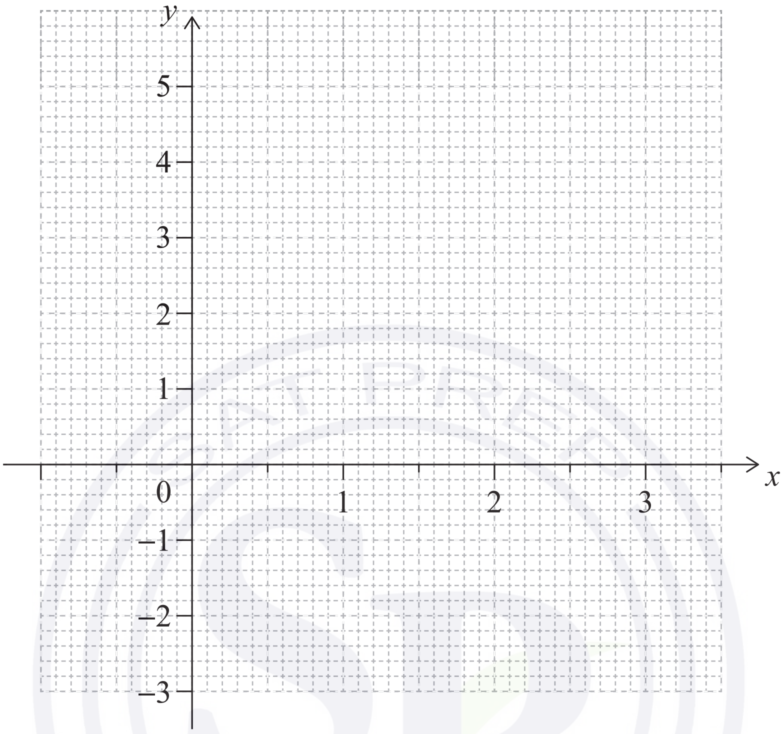
2. [Maximum mark: 5]

Let $f(x) = 3x - 4^{0.15x^2}$ for $0 \le x \le 3$.

(a) Sketch the graph of f on the grid below. [3]



(b) Find the value of x for which $f'(x) = 0$. [2]

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3. [Maximum mark: 5]

An arithmetic sequence has first term 60 and common difference -2.5.

(a) Given that the kth term of the sequence is zero, find the value of k. [2]

Let S_n denote the sum of the first n terms of the sequence.

(b) Find the maximum value of S_n. [3]

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4. [Maximum mark: 8]

At a school, 70% of the students play a sport and 20% of the students are involved in theatre. 18% of the students do neither activity.

A student is selected at random.

(a) Find the probability that the student plays a sport and is involved in theatre. [2]

(b) Find the probability that the student is involved in theatre, but does not play a sport. [2]

At the school 48% of the students are girls, and 25% of the girls are involved in theatre.

A student is selected at random. Let G be the event "the student is a girl" and let T be the event "the student is involved in theatre".

(c) Find $\mathrm{P}(G \cap T)$. [2]

(d) Determine if the events G and T are independent. Justify your answer. [2]

5. [Maximum mark: 6]

The functions f and g are defined for $x \in \mathbb{R}$ by $f(x) = 6x^2 - 12x + 1$ and $g(x) = -x + c$, where $c \in \mathbb{R}$.

(a) Find the range of f. [2]

(b) Given that $(g \circ f)(x) \le 0$ for all $x \in \mathbb{R}$, determine the set of possible values for c. [4]

6. [Maximum mark: 7]

All living plants contain an isotope of carbon called carbon-14. When a plant dies, the isotope decays so that the amount of carbon-14 present in the remains of the plant decreases. The time since the death of a plant can be determined by measuring the amount of carbon-14 still present in the remains.

The amount, A, of carbon-14 present in a plant t years after its death can be modelled by $A = A_0 e^{-kt}$ where $t \geq 0$ and A_0, k are positive constants.

At the time of death, a plant is defined to have 100 units of carbon-14.

(a) Show that $A_0 = 100$. [1]

The time taken for half the original amount of carbon-14 to decay is known to be 5730 years.

(b) Show that $k = \frac{\ln 2}{5730}$. [3]

(c) Find, correct to the nearest 10 years, the time taken after the plant's death for 25% of the carbon-14 to decay. [3]

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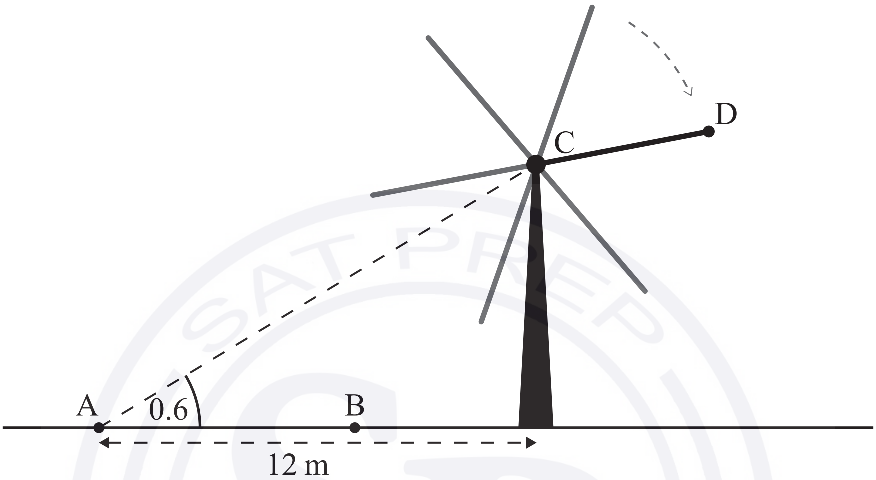
Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 13]

The six blades of a windmill rotate around a centre point C. Points A and B and the base of the windmill are on level ground, as shown in the following diagram.



From point A the angle of elevation of point C is 0.6 radians.

(a) Given that point A is 12 metres from the base of the windmill, find the height of point C above the ground. [2]

An observer walks 7 metres from point A to point B.

(b) Find the angle of elevation of point C from point B. [2]

The observer keeps walking until he is standing directly under point C. The observer has a height of 1.8 metres, and as the blades of the windmill rotate, the end of each blade passes 2.5 metres over his head.

(c) Find the length of each blade of the windmill. [2]

One of the blades is painted a different colour than the others. The end of this blade is labelled point D. The height h, in metres, of point D above the ground can be modelled by the function $h(t) = p\cos\left(\frac{3\pi}{10}t\right) + q$, where t is in seconds and $p, q \in \mathbb{R}$. When $t = 0$, point D is at its maximum height.

(d) Find the value of p and the value of q. [4]

If the observer stands directly under point C for one minute, point D will pass over his head n times.

(e) Find the value of n. [3]

Do **not** write solutions on this page.

8. [Maximum mark: 15]

The flight times, T minutes, between two cities can be modelled by a normal distribution with a mean of 75 minutes and a standard deviation of σ minutes.

(a) Given that 2% of the flight times are longer than 82 minutes, find the value of σ. [3]

(b) Find the probability that a randomly selected flight will have a flight time of more than 80 minutes. [2]

(c) Given that a flight between the two cities takes longer than 80 minutes, find the probability that it takes less than 82 minutes. [4]

On a particular day, there are 64 flights scheduled between these two cities.

(d) Find the expected number of flights that will have a flight time of more than 80 minutes. [3]

(e) Find the probability that more than 6 of the flights on this particular day will have a flight time of more than 80 minutes. [3]

Do **not** write solutions on this page.

9. [Maximum mark: 15]

All answers in this question should be given to four significant figures.

In a local weekly lottery, tickets cost $\$2$ each.

In the first week of the lottery, a player will receive $\$D$ for each ticket, with the probability distribution shown in the following table. For example, the probability of a player receiving $\$10$ is 0.03. The grand prize in the first week of the lottery is $\$1000$.

d	0	2	10	50	Grand Prize
$\mathrm{P}(D=d)$	0.85	c	0.03	0.002	0.0001

(a) Find the value of c. [2]

(b) Determine whether this lottery is a fair game in the first week. Justify your answer. [4]

If nobody wins the grand prize in the first week, the probabilities will remain the same, but the value of the grand prize will be $\$2000$ in the second week, and the value of the grand prize will continue to double each week until it is won. All other prize amounts will remain the same.

(c) Given that the grand prize is not won and the grand prize continues to double, write an expression in terms of n for the value of the grand prize in the nth week of the lottery. [2]

The wth week is the first week in which the player is expected to make a profit. Ryan knows that if he buys a lottery ticket in the wth week, his expected profit is $\$p$.

(d) Find the value of p. [7]

References:

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Answers written on this page
will not be marked.

Please **do not** write on this page.

Answers written on this page will not be marked.

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: analysis and approaches
Standard level
Paper 2

Friday 7 May 2021 (morning)

Candidate session number

1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

2221 – 7110

International Baccalaureate
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Please **do not** write on this page.

Answers written on this page
will not be marked.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 6]

(a) Find $\int (6x+7)\,\mathrm{d}x$. [3]

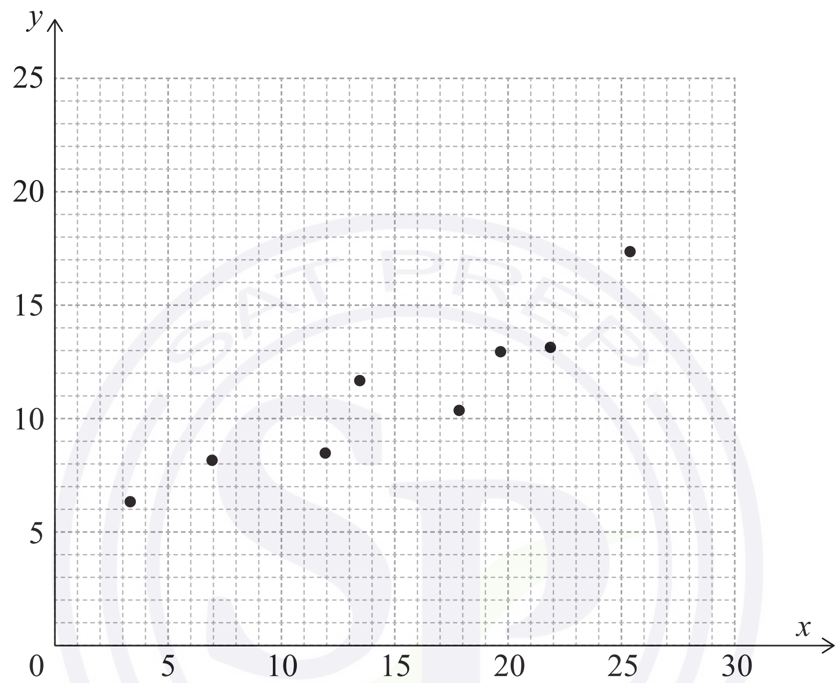
(b) Given $f'(x) = 6x + 7$ and $f(1.2) = 7.32$, find $f(x)$. [3]

2. [Maximum mark: 7]

The following table shows the data collected from an experiment.

x	3.3	6.9	11.9	13.4	17.8	19.6	21.8	25.3
y	6.3	8.1	8.4	11.6	10.3	12.9	13.1	17.3

The data is also represented on the following scatter diagram.



The relationship between x and y can be modelled by the regression line of y on x with equation $y = ax + b$, where $a, b \in \mathbb{R}$.

(a) Write down the value of a and the value of b. [2]

(b) Use this model to predict the value of y when $x = 18$. [2]

(c) Write down the value of $\bar{x}$ and the value of $\bar{y}$. [1]

(d) Draw the line of best fit on the scatter diagram. [2]

(This question continues on the following page)

(Question 2 continued)

3. [Maximum mark: 6]

A company produces bags of sugar whose masses, in grams, can be modelled by a normal distribution with mean 1000 and standard deviation 3.5. A bag of sugar is rejected for sale if its mass is less than 995 grams.

(a) Find the probability that a bag selected at random is rejected. [2]

(b) Estimate the number of bags which will be rejected from a random sample of 100 bags. [1]

(c) Given that a bag is not rejected, find the probability that it has a mass greater than 1005 grams. [3]

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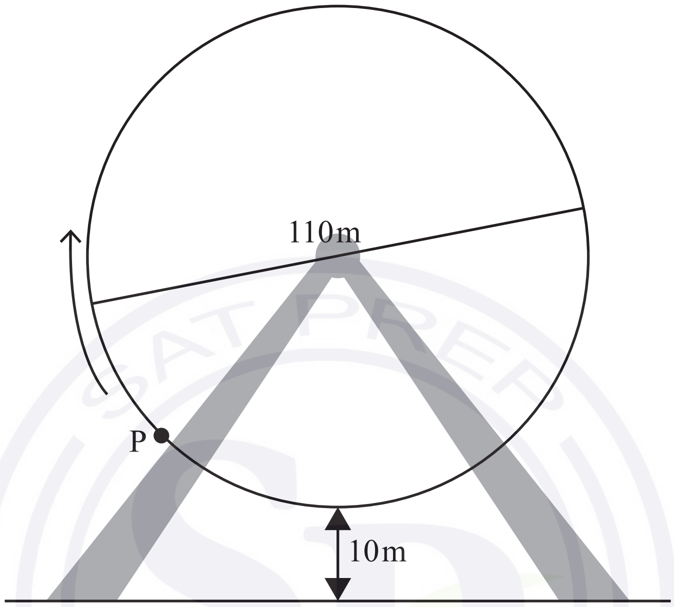
4. [Maximum mark: 5]

A Ferris wheel with diameter 110 metres rotates at a constant speed. The lowest point on the wheel is 10 metres above the ground, as shown on the following diagram. P is a point on the wheel. The wheel starts moving with P at the lowest point and completes one revolution in 20 minutes.

diagram not to scale



The height, h metres, of P above the ground after t minutes is given by $h(t) = a\cos(bt) + c$, where $a,\ b,\ c \in \mathbb{R}$.

Find the values of a, b and c.

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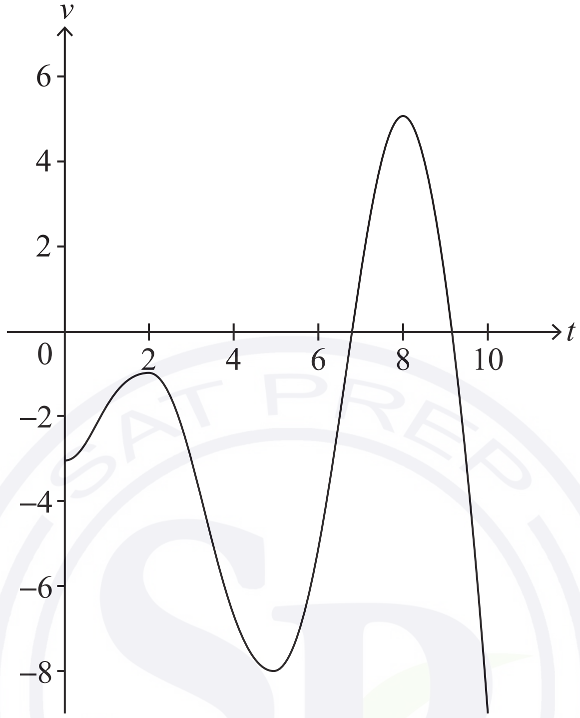
5. [Maximum mark: 6]

A particle moves in a straight line. The velocity, $v\,\text{ms}^{-1}$, of the particle at time t seconds is given by $v(t) = t\sin t - 3$, for $0 \le t \le 10$.

The following diagram shows the graph of v.



(a) Find the smallest value of t for which the particle is at rest. [2]

(b) Find the total distance travelled by the particle. [2]

(c) Find the acceleration of the particle when $t = 7$. [2]

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6. [Maximum mark: 5]

Consider the expansion of $(3 + x^2)^{n+1}$, where $n \in \mathbb{Z}^+$.

Given that the coefficient of x^4 is $20\,412$, find the value of n.

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

7. [Maximum mark: 16]

Two friends Amelia and Bill, each set themselves a target of saving $\$20\,000$. They each have $\$9000$ to invest.

(a) Amelia invests her $\$9000$ in an account that offers an interest rate of 7% per annum compounded **annually**.

(i) Find the value of Amelia's investment after 5 years to the nearest hundred dollars.

(ii) Determine the number of years required for Amelia's investment to reach the target. [5]

(b) Bill invests his $\$9000$ in an account that offers an interest rate of $r\%$ per annum compounded **monthly**, where r is set to two decimal places.

Find the minimum value of r needed for Bill to reach the target after 10 years. [3]

(c) A third friend Chris also wants to reach the $\$20\,000$ target. He puts his money in a safe where he does not earn any interest. His system is to add more money to this safe each year. Each year he will add half the amount added in the previous year.

(i) Show that Chris will never reach the target if his initial deposit is $\$9000$.

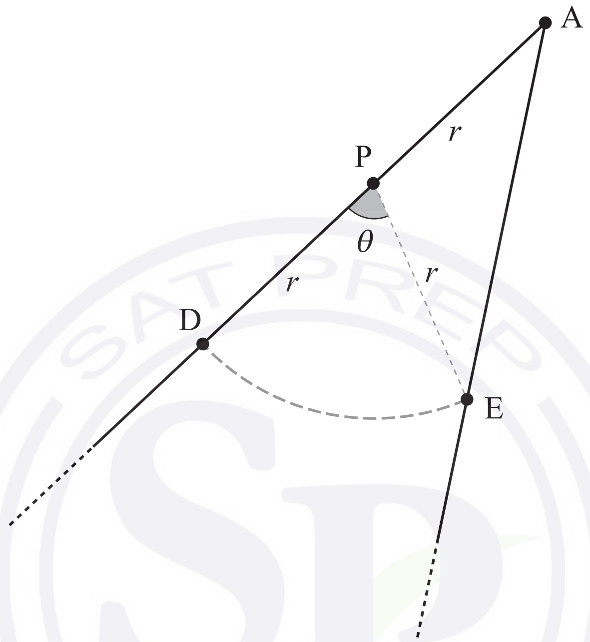
(ii) Find the amount Chris needs to deposit initially in order to reach the target after 5 years. Give your answer to the nearest dollar. [8]

Do **not** write solutions on this page.

8. [Maximum mark: 14]

Two straight fences meet at point A and a field lies between them.

A horse is tied to a post, P, by a rope of length r metres. Point D is on one fence and point E is on the other, such that $\mathrm{PD} = \mathrm{PE} = \mathrm{PA} = r$ and $\mathrm{D\hat{P}E} = \theta$ radians. This is shown in the following diagram.



The length of the arc DE shown in the diagram is $28\,\mathrm{m}$.

(a) Write down an expression for r in terms of θ. [1]

(b) Show that the area of the field that the horse can reach is $\dfrac{392}{\theta^2}(\theta + \sin\theta)$. [4]

(c) The area of field that the horse can reach is $460\,\mathrm{m}^2$. Find the value of θ. [2]

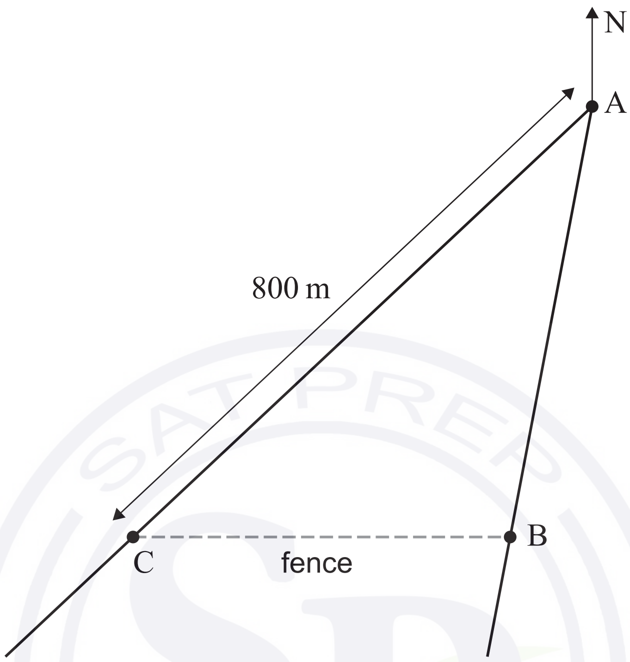
(d) Hence, find the size of $\mathrm{D\hat{A}E}$. [2]

(This question continues on the following page)

Do **not** write solutions on this page.

(Question 8 continued)

A new fence is to be constructed between points B and C which will enclose the field, as shown in the following diagram.



Point C is due west of B and $AC = 800\,\text{m}$. The bearing of B from A is $195°$.

(e) (i) Find the size of $A\hat{B}C$.

(ii) Find the length of new fence required. [5]

Do **not** write solutions on this page.

9. [Maximum mark: 15]

Consider the function f defined by $f(x) = 90\,e^{-0.5x}$ for $x \in \mathbb{R}^{+}$.

The graph of f and the line $y = x$ intersect at point P.

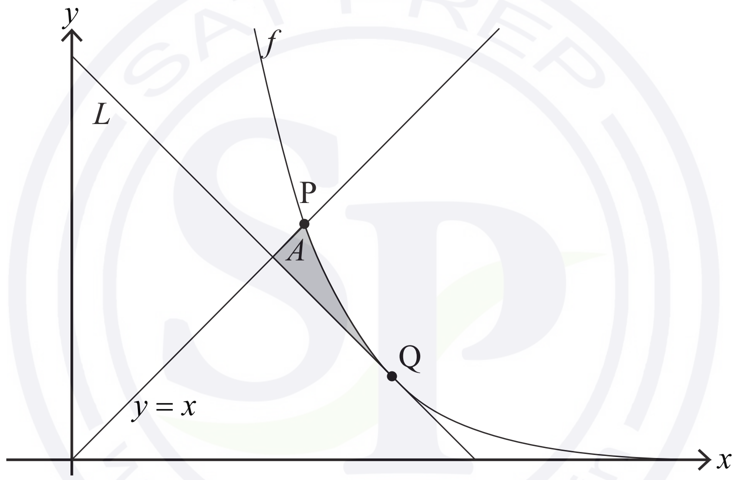
(a) Find the x-coordinate of P. [2]

The line L has a gradient of -1 and is a tangent to the graph of f at the point Q.

(b) Find the exact coordinates of Q. [4]

(c) Show that the equation of L is $y = -x + 2\ln 45 + 2$. [2]

The shaded region A is enclosed by the graph of f and the lines $y = x$ and L.



(d) (i) Find the x-coordinate of the point where L intersects the line $y = x$.

(ii) Hence, find the area of A. [5]

(This question continues on the following page)

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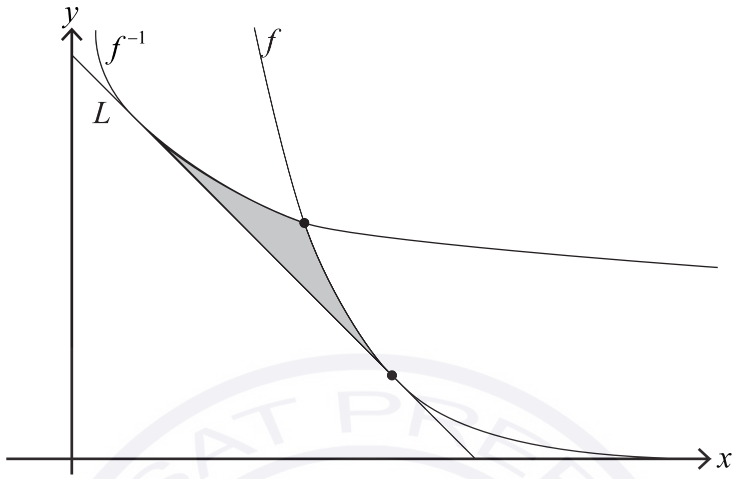
(Question 9 continued)

The line L is tangent to the graphs of both f and the inverse function f^{-1}.



(e) Find the shaded area enclosed by the graphs of f and f^{-1} and the line L. [2]

References:

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Answers written on this page
will not be marked.

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: analysis and approaches
Standard level
Paper 2

Specimen

Candidate session number

1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 6]

A metal sphere has a radius $12.7\,\text{cm}$.

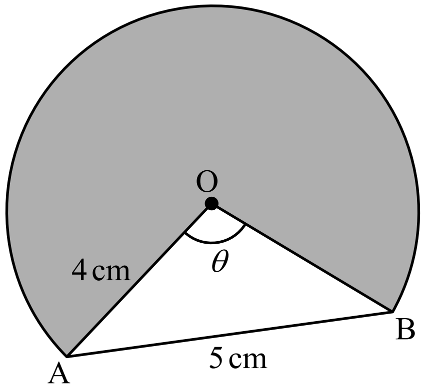
(a) Find the volume of the sphere expressing your answer in the form $a \times 10^k$, $1 \le a < 10$ and $k \in \mathbb{Z}$. [3]

The sphere is to be melted down and remoulded into the shape of a cone with a height of $14.8\,\text{cm}$.

(b) Find the radius of the base of the cone, correct to 2 significant figures. [3]

2. [Maximum mark: 6]

The following diagram shows part of a circle with centre O and radius $4\,\mathrm{cm}$.



Chord AB has a length of $5\,\mathrm{cm}$ and $\mathrm{A\hat{O}B} = \theta$.

(a) Find the value of θ, giving your answer in radians. [3]

(b) Find the area of the shaded region. [3]

3. [Maximum mark: 6]

On 1st January 2020, Laurie invests $\$P$ in an account that pays a nominal annual interest rate of 5.5%, compounded **quarterly**.

The amount of money in Laurie's account **at the end of each year** follows a geometric sequence with common ratio, r.

(a) Find the value of r, giving your answer to four significant figures. [3]

Laurie makes no further deposits to or withdrawals from the account.

(b) Find the year in which the amount of money in Laurie's account will become double the amount she invested. [3]

4. [Maximum mark: 6]

A six-sided biased die is weighted in such a way that the probability of obtaining a “six” is $\frac{7}{10}$.

The die is tossed five times. Find the probability of obtaining

(a) at most three “sixes”. [3]

(b) the third “six” on the fifth toss. [3]

5. [Maximum mark: 5]

The following table below shows the marks scored by seven students on two different mathematics tests.

Test 1 (x)	15	23	25	30	34	34	40
Test 2 (y)	20	26	27	32	35	37	35

Let L_1 be the regression line of x on y. The equation of the line L_1 can be written in the form $x = ay + b$.

(a) Find the value of a and the value of b. [2]

Let L_2 be the regression line of y on x. The lines L_1 and L_2 pass through the same point with coordinates (p, q).

(b) Find the value of p and the value of q. [3]

6. [Maximum mark: 7]

The displacement, in centimetres, of a particle from an origin, O, at time t seconds, is given by $s(t) = t^2 \cos t + 2t \sin t$, $0 \le t \le 5$.

(a) Find the maximum distance of the particle from O. [3]

(b) Find the acceleration of the particle at the instant it first changes direction. [4]

Do **not** write solutions on this page.

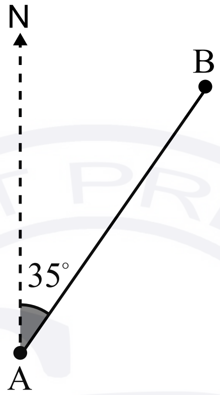
Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

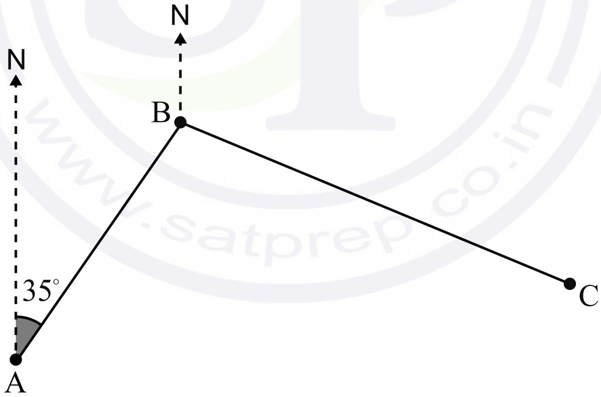
7. [Maximum mark: 16]

Adam sets out for a hike from his camp at point A. He hikes at an average speed of $4.2\,\mathrm{km/h}$ for 45 minutes, on a bearing of 035° from the camp, until he stops for a break at point B.

(a) Find the distance from point A to point B. [2]



Adam leaves point B on a bearing of 114° and continues to hike for a distance of $4.6\,\mathrm{km}$ until he reaches point C.



(b) (i) Show that $\mathrm{A\hat{B}C}$ is 101°.

(ii) Find the distance from the camp to point C. [5]

(c) Find $\mathrm{B\hat{C}A}$. [3]

Adam's friend Jacob wants to hike directly from the camp to meet Adam at point C.

(d) Find the bearing that Jacob must take to point C. [3]

Jacob hikes at an average speed of $3.9\,\mathrm{km/h}$.

(e) Find, to the nearest minute, the time it takes for Jacob to reach point C. [3]

Do **not** write solutions on this page.

8. [Maximum mark: 15]

The length, X mm, of a certain species of seashell is normally distributed with mean 25 and variance, σ^2.

The probability that X is less than 24.15 is 0.1446.

(a) Find $\mathrm{P}(24.15 < X < 25)$. [2]

(b) (i) Find σ, the standard deviation of X.

(ii) Hence, find the probability that a seashell selected at random has a length greater than 26 mm. [5]

A random sample of 10 seashells is collected on a beach. Let Y represent the number of seashells with lengths greater than 26 mm.

(c) Find $\mathrm{E}(Y)$. [3]

(d) Find the probability that exactly three of these seashells have a length greater than 26 mm. [2]

A seashell selected at random has a length less than 26 mm.

(e) Find the probability that its length is between 24.15 mm and 25 mm. [3]

Do **not** write solutions on this page.

9. [Maximum mark: 13]

Consider a function f, such that $f(x) = 5.8\sin\left(\frac{\pi}{6}(x+1)\right) + b$, $0 \le x \le 10$, $b \in \mathbb{R}$.

(a) Find the period of f. [2]

The function f has a local maximum at the point $(2, 21.8)$, and a local minimum at $(8, 10.2)$.

(b) (i) Find the value of b.

(ii) Hence, find the value of $f(6)$. [4]

A second function g is given by $g(x) = p\sin\left(\frac{2\pi}{9}(x - 3.75)\right) + q$, $0 \le x \le 10$; $p, q \in \mathbb{R}$.

The function g passes through the points $(3, 2.5)$ and $(6, 15.1)$.

(c) Find the value of p and the value of q. [5]

(d) Find the value of x for which the functions have the greatest difference. [2]