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Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: applications and interpretation
Higher level
Paper 1

15 May 2025

Zone A afternoon | **Zone B** afternoon | **Zone C** afternoon

Candidate session number

2 hours

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all questions.
- Answers must be written within the answer boxes provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.

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2225 – 7306

25 pages

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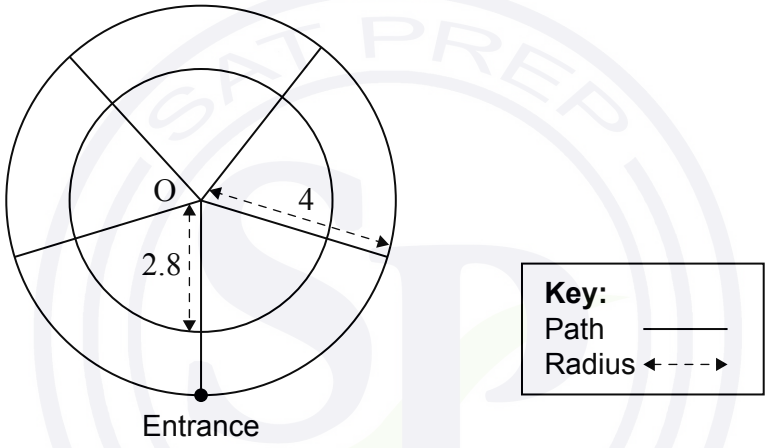
Answers must be written within the answer boxes provided. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 6]

A child's game is played by making paths in the snow. First, two circular paths are made using the same centre, O. The radius of the smaller circle is $2.8\,\mathrm{m}$, and the radius of the larger circle is $4\,\mathrm{m}$. Additional paths are then made from O to the outer edge of the larger circle, dividing each circle into 5 equal sectors, as shown in the following diagram.

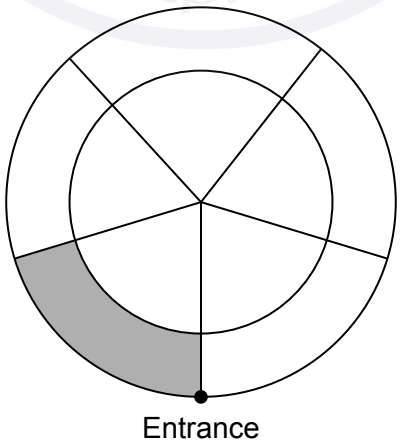
For your calculations, ignore the widths of the paths.

diagram not to scale



Each region **between** paths will be coloured with environmentally friendly dye. The shaded region below will be coloured orange.

diagram not to scale



Maureen has enough orange dye to cover an area of $6\,\mathrm{m}^2$.

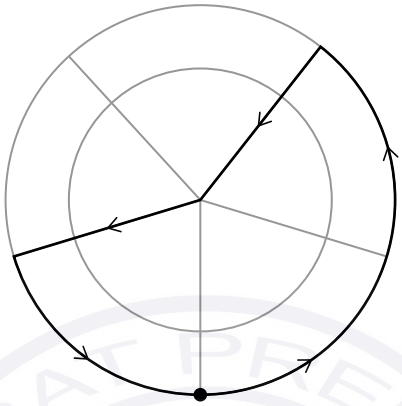
(This question continues on the following page)

(Question 1 continued)

(a) Show that Maureen has enough orange dye to cover the shaded region. [3]

During the game, the players start at the entrance and must travel only along the paths made in the snow. Maureen travels from the entrance along the path shown in the following diagram.

diagram not to scale



Entrance

(b) Calculate the distance Maureen travels along this path, starting from the entrance and returning to the entrance. [3]

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2. [Maximum mark: 7]

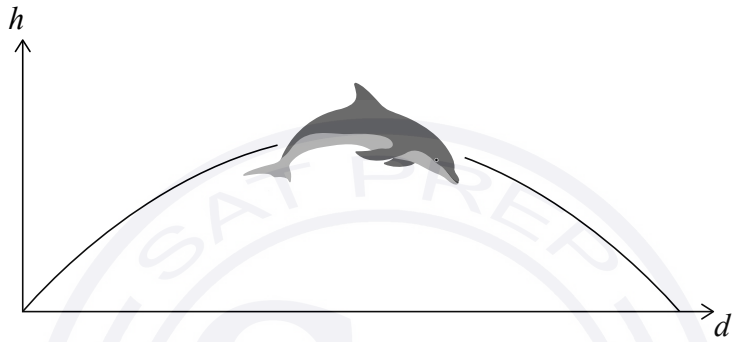
A dolphin jumps out of water. The path of its jump can be modelled by a quadratic function using the following variables:

d is the horizontal distance from the point where the dolphin leaves the water

h is the height of the dolphin above the surface of the water.

All distances are measured in metres, from the point where the dolphin leaves the water.

diagram not to scale



On a particular jump, the dolphin first reaches a height of $5.1\,\text{m}$ above the surface of the water when the horizontal distance is $3\,\text{m}$ and again when the horizontal distance is $8.5\,\text{m}$.

(a) Write down the equation of the axis of symmetry of the quadratic function. [2]

The quadratic function that models the dolphin's jump is $h(d) = -0.2d^2 + bd$, where $\text{d} > 0$ and b is a constant.

(b) Calculate the value of b. [2]

(c) Find the horizontal distance when the dolphin re-enters the water. [2]

(d) Explain what a negative h-value would mean in this context. [1]

(This question continues on the following page)

(Question 2 continued)

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3. [Maximum mark: 8]

Raul wants to buy a new truck for $42\,695$ US dollars (USD). Raul is offered two different loan options to pay for the truck. For both options, Raul will pay back the loan over a period of 6 years.

The two options are shown in the following table.

Loan option	**Size of loan (% purchase price)**	**Nominal annual interest rate (compounded monthly)**
A	100% of purchase price (after no initial payment)	7.15%
B	80% of purchase price (after a 20% initial payment from Raul)	6.40%

For both loan options, Raul will make monthly payments, at the end of each month, until the loan is completely paid off.

(a) Calculate the amount of Raul's monthly payment, to two decimal places, if he chooses

(i) option A.

(ii) option B. [6]

(b) State a reason why Raul might choose

(i) option A.

(ii) option B. [2]

(This question continues on the following page)

(Question 3 continued)

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4. [Maximum mark: 7]

A manufacturer designs hats that businesses can order.

For an order of 15 hats or fewer, the price per hat is p euros (EUR). For larger orders, the price of **each** hat after the first 15 ordered is reduced by $5\,\text{EUR}$.

$C(n)$ represents the total cost of purchasing n hats from the manufacturer.

(a) Write down an expression in terms of p for the total cost of ordering 15 hats. [1]

(b) Write down an equation for $C(n)$, in terms of p and n, for the total cost of ordering n hats when $n > 15$. [3]

A company decides to order 100 hats. The mean price per hat for this order is $25.73\,\text{EUR}$.

(c) (i) Calculate the exact total cost of the order.

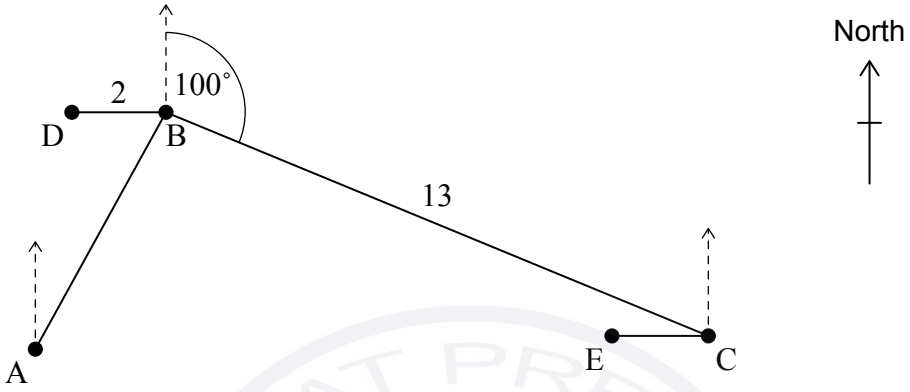
(ii) Hence, determine the value of p, correct to 2 decimal places. [3]

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5. [Maximum mark: 9]

Ticky is sailing a boat in the ocean. For the first part of her trip, she plans to start at point A and travel to point B. She then plans to turn and travel $13\,\text{km}$ on a bearing of $100°$ to point C as shown in the following diagram.

diagram not to scale



Due to navigational error in the first part of the trip, Ticky arrives at point D instead of point B. Point D is $2\,\text{km}$ due west of point B.

(a) Determine

(i) the distance from point D to point C.

(ii) the bearing Ticky must use to travel directly from point D to point C. [7]

Due to navigational error in the second part of the trip, Ticky arrives at point E instead of point C. Point E is $2\,\text{km}$ due west of point C.

(b) Write down the distance between point D and point E. Justify your answer. [2]

(This question continues on the following page)

(Question 5 continued)

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6. [Maximum mark: 6]

Walkea Tree is a furniture store. They offer a 30% discount on all chairs priced over 250 Canadian dollars (CAD).

A different furniture store, LuxeCraft, offer customers a reduction of $150\,\text{CAD}$ on all chairs priced over $250\,\text{CAD}$.

(a) Given that x is the original price of a chair and $x > 250$,

(i) write down a function $f(x)$ that models the final price of a chair at Walkea Tree, after the discount is applied.

(ii) write down a function $g(x)$ that models the final price of a chair at LuxeCraft, after the reduction is applied. [2]

A third store, Opulent Furnishings, decides to offer both a 30% discount **and** a reduction of $150\,\text{CAD}$ on all their chairs priced over $250\,\text{CAD}$.

(b) Describe the meaning of $(f \circ g)(x)$, in context. [2]

(c) State, with justification, whether $(f \circ g)(x)$ or $(g \circ f)(x)$ is better for the **customer** when they buy a chair with an original price of $450\,\text{CAD}$. [2]

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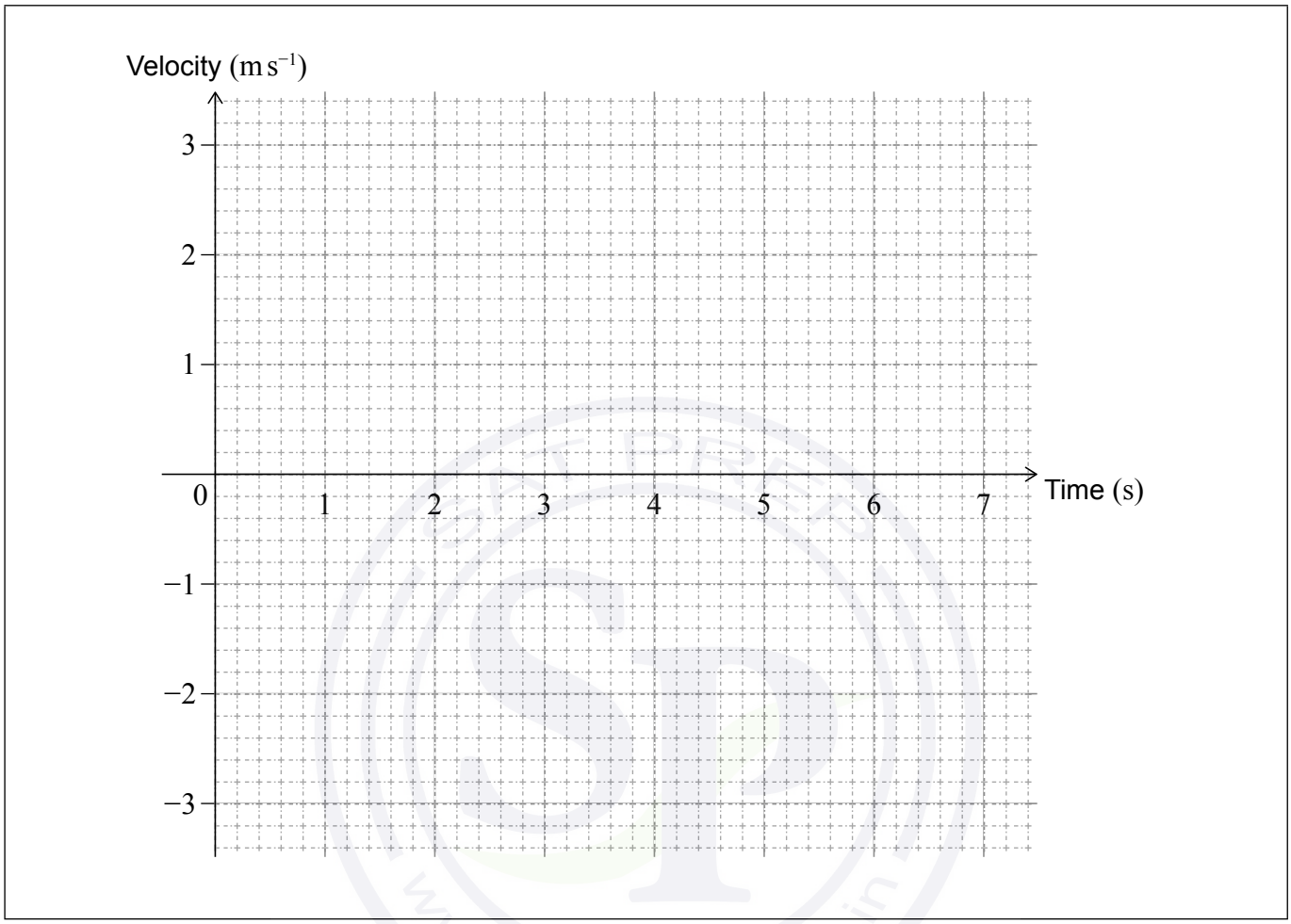
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7. [Maximum mark: 8]

Conrad is investigating the motion of a particle. The velocity of the particle, in m s^{-1}, is given by $v(t) = 2\cos t + \sin 2t - 0.2$ where t is the time, in seconds, after the investigation begins.



(a) On the axes,

(i) sketch the graph of $v(t)$ for $0 \le t \le 6$.

(ii) label, with coordinates, the point(s) where the particle has zero acceleration. [5]

(b) (i) Write down an integral expression for the distance the particle travels during the first 6 seconds of the investigation.

(ii) Hence, find the distance the particle travels during the first 6 seconds of the investigation. [3]

(This question continues on the following page)

(Question 7 continued)

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8. [Maximum mark: 7]

Consider the function $f(x) = x^3 - 9x^2 + 23x - 15$ for $0 \le x \le 5$.

The graph of $f(x)$ crosses the x-axis when $x = 1$, $x = 5$ and $x = a$.

(a) Find the value of a. [2]

(b) Use your graphic display calculator to find the area enclosed by the curve $y = f(x)$ and the x-axis, when $1 \le x \le a$. [2]

Another function is defined as $g(x) = -x^2 + 6x - 5$.

The area enclosed by the curve $y = f(x)$ and the x-axis, when $1 \le x \le b$, is equal to the area enclosed by the curve $y = g(x)$ and the x-axis, when $1 \le x \le b$.

(c) Find the value of b, where $1 < b < 3$. [3]

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9. [Maximum mark: 7]

In a year, Louis attends school for a total of 180 school days. Louis leaves his house at exactly $7{:}45\,\text{am}$ every day and takes public transportation from his house to school.

His travel times to school are normally distributed with a mean of 35.5 minutes and a standard deviation of 5.2 minutes. The travel times are independent of each other.

He is late if he arrives at school after $8{:}30\,\text{am}$.

(a) Find the probability that Louis is late on at least 10 days in the year. [4]

Mrs. Beale is responsible for recording students who arrive late. However, Mrs. Beale does not always accurately mark lateness.

On days when Louis is late, there is a $35\,\%$ chance that Mrs. Beale will still mark him as being on time. Mrs. Beale's accuracy for one school day is independent of her accuracy for any other school day.

Louis will receive a detention if he is marked late to school at least 4 times during the year.

(b) Find the probability of Louis receiving a detention in the year. [3]

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10. [Maximum mark: 9]

Consider the function $f(x) = \ln(2 - x) + \ln(-2 - x) - 2\ln(-x)$, where $x < -2$.

The function $f(x)$ can be rewritten in the form $\ln(g(x))$.

(a) Find an expression for $g(x)$. [2]

(b) Write down the range of $f^{-1}(x)$. [2]

The function $f^{-1}(x)$ can be written in the form $\frac{a}{\sqrt{1 - e^x}}$.

(c) Find the value of a. [5]

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11. [Maximum mark: 8]

Nanthana is completing an exploration on multiplication using complex numbers. She considers a geometric sequence where $u_1 = 9$ and $r = \frac{2}{3} + \frac{2}{3}\mathrm{i}$.

(a) Write down the value of

(i) u_2.

(ii) u_3. [2]

Nanthana claims that the sequence $|u_1|, |u_2|, |u_3|, \ldots$ also forms a geometric sequence.

(b) Show that Nanthana's claim is correct, stating the exact value of the common ratio for this sequence. [4]

(c) Hence, find the sum of the infinite sequence $|u_1|, |u_2|, |u_3|, \ldots$. [2]

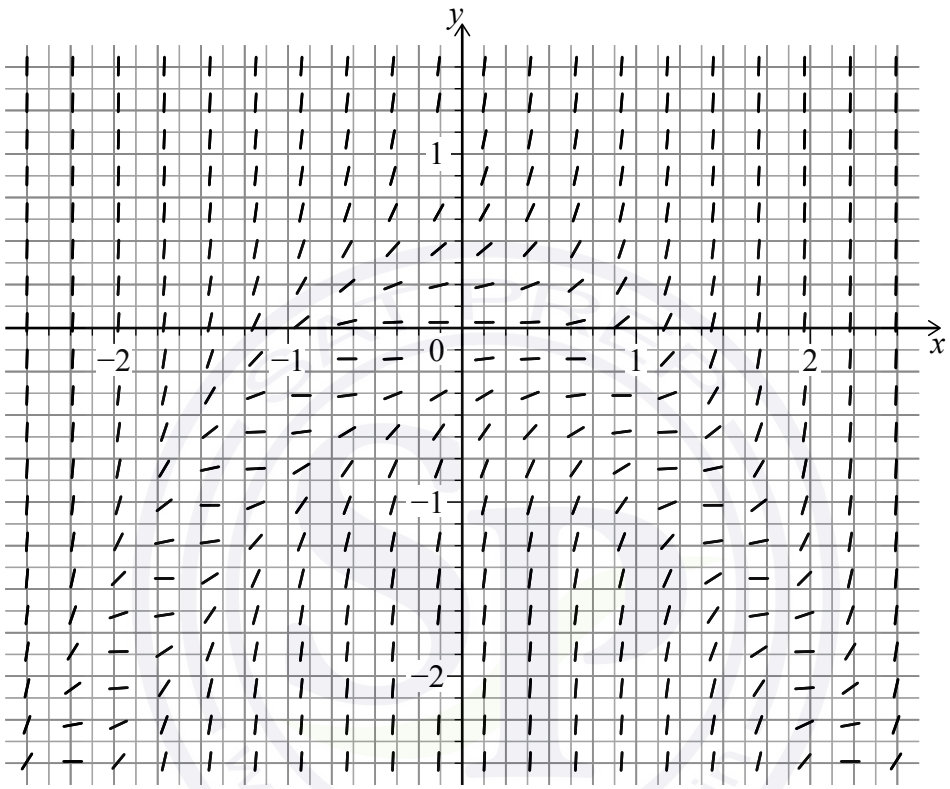
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12. [Maximum mark: 7]

The following graph shows the slope field for the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = x^4 + 4x^2y + 4y^2.$$



(a) On the slope field, sketch the solution to the differential equation that passes through the point $(0, 1)$. [2]

Some of the solutions to the differential equation have a point at which the gradient is zero. The curve on which all these points lie is given by the equation $y = ax^2$, where a is a constant to be found.

(b) Sketch this curve on the slope field. [2]

(c) Use the differential equation to find the value of a. [3]

(This question continues on the following page)

(Question 12 continued)

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13. [Maximum mark: 7]

The points $\mathrm{R}(-2, 2)$, $\mathrm{S}(3, -5)$, and $\mathrm{T}(5, 1)$ are the vertices of a triangle. The triangle goes through a sequence of transformations in the following order:

I. a transformation using the matrix $\boldsymbol{A} = \begin{bmatrix} \cos\frac{\pi}{3} & -\sin\frac{\pi}{3} \\ \sin\frac{\pi}{3} & \cos\frac{\pi}{3} \end{bmatrix}$

II. a transformation using the matrix $\boldsymbol{B} = \begin{bmatrix} -0.8 & 0.6 \\ 0.6 & 0.8 \end{bmatrix}$.

(a) Describe the nature of the transformation represented by $\boldsymbol{A}$. [2]

Instead of using this sequence of transformations, the image of triangle RST can be found using only a reflection of triangle RST in line $y = ax$.

(b) Given that $a > 0$, find the value of a. [5]

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14. [Maximum mark: 6]

Consider the following functions:

$$L_1(x) = 14.8\sin(0.35x + 1.27)$$

$$L_2(x) = 18.5\sin(0.35x - 2.36)$$

(a) Write down both functions in the form $\mathrm{Im}(a\mathrm{e}^{bxi}\,\mathrm{e}^{ci})$. [2]

The sum of $L_1(x)$ and $L_2(x)$ can be expressed in the form $L_3(x) = q\sin(0.35x + r)$, where $0 \le r < 2\pi$.

(b) Use your answer to part (a) to find the value of q and of r. [4]

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15. [Maximum mark: 8]

Quentin is investigating the motion of a mass hanging on a large spring.

From research, Quentin thinks the displacement, x, in metres, of the mass at time t, in seconds, will satisfy the differential equation

$$9.8 = 2\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + 2.4\frac{\mathrm{d}x}{\mathrm{d}t} + 8x\,.$$

Quentin begins his investigation when the mass is at a displacement of 0.5 metres and is moving such that $\frac{\mathrm{d}x}{\mathrm{d}t} = 9.8$.

(a) By using Euler's method with $h = 0.1$ to estimate the motion of the mass after being released, find:

(i) the displacement of the mass after 1 second,

(ii) the first estimated time the mass reaches $x = 0$. [5]

Using technology, Quentin correctly measures the displacement of the mass, after 1 second. His software gives him a value of 385 centimetres, which is accurate to the nearest centimetre.

(b) Find the **maximum possible** percentage error between your estimate from part (a)(i) and the exact displacement after 1 second. [3]

(This question continues on the following page)

(Question 15 continued)

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References:

2. fitie, 2017. *Vector illustration of dolphins* [image online] Available at: https://www.gettyimages.co.uk/detail/illustration/vector-illustration-of-dolphins-royalty-free-illustration/638431854 [Accessed 19 April 2024] Source adapted.

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Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: applications and interpretation
Higher level
Paper 1

15 May 2025

Zone A afternoon | **Zone B** afternoon | **Zone C** afternoon

Candidate session number

2 hours

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all questions.
- Answers must be written within the answer boxes provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.

SATPREP www.satprep.co.in

2225 – 7311

19 pages

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Answers must be written within the answer boxes provided. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 7]

Give answers to this question correct to two decimal places.

Pierre invests 1500 euros (EUR) at the end of each month for 10 years into a savings plan that pays a nominal annual interest rate of 3.6% compounded monthly.

(a) Calculate the value of Pierre's savings plan at the end of the 10 years. [3]

At the end of the 10 years, Pierre withdraws $100\,000$ EUR from the savings plan to use as a deposit on a house.

Pierre invests the remainder into another account for 15 years at a nominal annual interest rate of 4.5% compounded quarterly.

(b) Calculate the amount in Pierre's account at the end of this time. [4]

(This question continues on the following page)

(Question 1 continued)

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2. [Maximum mark: 9]

The point A has coordinates $(1, 2, 1)$ and the point B has coordinates $(3, 5, 2)$.

(a) Find AB. [2]

Triangle ABC is right-angled with its right angle at B. The point C has coordinates $(2, 8, k)$.

(b) Find the value of k. [4]

(c) Calculate the size of $B\hat{A}C$. [3]

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3. [Maximum mark: 6]

Two judges, Brett and Clarence, rank the skill levels of eight sheepdogs in a competition. The sheepdogs are labelled A to H and the judges rank the dogs as shown in the table.

Rank	1	2	3	4	5	6	7	8
Brett	A	C	D	B	E	F	G	H
Clarence	A	B	D	C	E	G	F	H

(a) Write down the rank that Brett awards sheepdog B. [1]

(b) Calculate Spearman's rank correlation coefficient for these data. [4]

(c) Comment on your answer to part (b) in terms of the ranks awarded by Brett and Clarence. [1]

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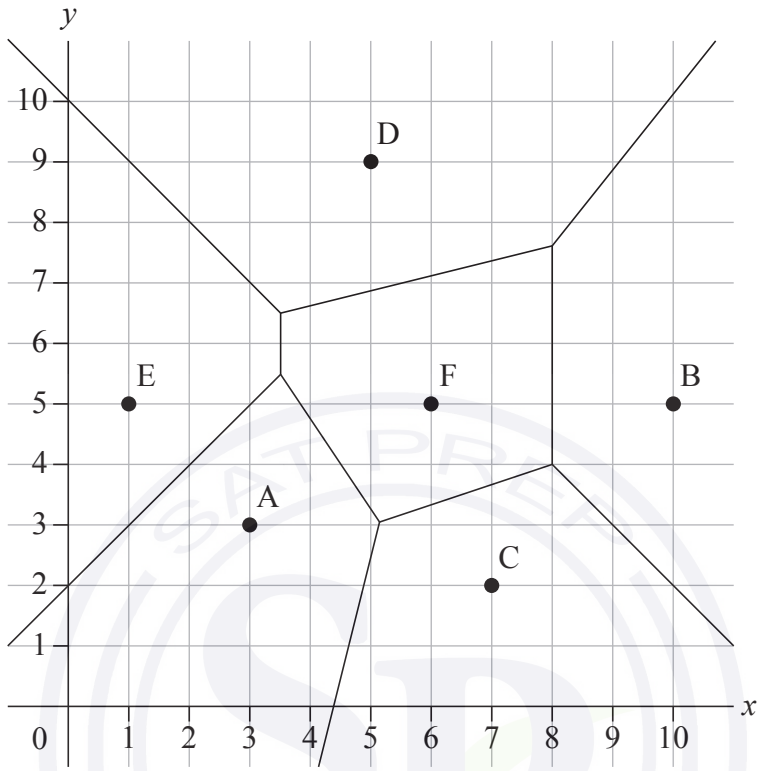
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4. [Maximum mark: 7]

Consider the Voronoi diagram which shows the sites $A(3, 3)$, $B(10, 5)$, $C(7, 2)$, $D(5, 9)$, $E(1, 5)$ and $F(6, 5)$. The diagram also shows the cells formed by each site and their boundaries.



Vertex X is equidistant from sites B, C and F.

(a) (i) Write down the coordinates of X.

(ii) The exact value of BX is $\sqrt{n}$. Write down the value of n. [2]

Vertex $Y(a, b)$ is equidistant from sites B, D and F.

(b) (i) Write down the value of a.

(ii) Find the exact value of b. [5]

(This question continues on the following page)

(Question 4 continued)

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5. [Maximum mark: 6]

A speed camera is used to determine whether a car is exceeding a speed limit of $8.3\,\mathrm{m\,s^{-1}}$.

An exact distance of $10\,\mathrm{m}$ is marked out.

The car travels this $10\,\mathrm{m}$ distance in 1.2 seconds, measured to the nearest 0.1 second.

Determine whether it is certain that the car was exceeding the speed limit of $8.3\,\mathrm{m\,s^{-1}}$.

Justify your answer.

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6. [Maximum mark: 5]

The temperatures at 5 different locations on a coral reef were measured. The mean of this sample was $20.1°\text{C}$ and the standard deviation of this sample, s_n, was $3.2°\text{C}$.

(a) Find an unbiased estimate of the population variance. [2]

(b) Assuming that the temperatures are normally distributed, find a 95% confidence interval for the mean temperature on the coral reef. [2]

(c) Using your answer to part (b), determine if it is plausible that the mean temperature on the coral reef could be $17°\text{C}$. [1]

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7. [Maximum mark: 7]

(a) Find the indefinite integral $\int xe^{-x^2}dx$. [4]

(b) Hence find the area bounded by the x-axis, the curve $y = xe^{-x^2}$ and the line $x = k$. Give your answer in terms of k. [3]

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8. [Maximum mark: 6]

A mapping system stores the connections between 5 towns, labelled A, B, C, D and E, in an adjacency matrix. The adjacency matrix, with rows and columns in alphabetical order, is

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \end{pmatrix}.$$

(a) Draw and label a graph to represent the adjacency matrix. [2]

(b) Determine the number of walks of length 4 which start and end at the same town. [4]

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9. [Maximum mark: 7]

A climate scientist is modelling an ice sheet as a rectangle.

She believes that the width $(x\,\text{km})$ is increasing at a constant rate of $10\,\text{km}$ per year and the length $(y\,\text{km})$ is decreasing at a constant rate of $5\,\text{km}$ per year.

The time, t, is measured in years, and the area, A, is measured in km^2.

When $t = 0$ then $x = 75$ and $y = 40$.

(a) Find $\frac{\mathrm{d}A}{\mathrm{d}t}$ when $t = 0$. [3]

(b) State, with justification, whether the area of the ice sheet is increasing or decreasing when $t = 0$. [1]

(c) Find the change in the area of the ice sheet between $t = 0$ and $t = 1$. [3]

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10. [Maximum mark: 5]

Consider the following function, $f(x)$, defined on the domain of integers from 0 to 4 inclusive.

x	0	1	2	3	4
$f(x)$	3	1	0	4	2

(a) Solve $f(x) = 4$. [1]

(b) Solve $f(x) = x$. [1]

(c) Complete the following table.

x	0	1	2	3	4
$f^{-1}(x)$					

[3]

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11. [Maximum mark: 6]

A biologist believes that there is a relationship between the possible population size of a group of birds (p thousand) and the population of a colony of wasps (w thousand). Based on her research she believes that the relationship is

$$w = p^3 - 4p^2 + 3p.$$

(a) When $w = 0$, find the possible values of p. [2]

(b) Determine the positive values of w for which there is only one positive value of p. [4]

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12. [Maximum mark: 9]

An engineer's model for an object's motion is that its acceleration, $\frac{\mathrm{d}v}{\mathrm{d}t}$, is proportional to $v^{1.5}$, where v is its velocity measured in $\mathrm{m\,s^{-1}}$.

(a) Write down a differential equation based on the engineer's belief. [1]

The initial velocity of the object is $4\,\mathrm{m\,s^{-1}}$ and its initial acceleration is $-3\,\mathrm{m\,s^{-2}}$.

(b) Use the engineer's model to find an expression for the velocity of the object after t seconds. [8]

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13. [Maximum mark: 9]

A biologist uses a wire frame to count the number of worms in a $1\,\text{m}^2$ section.

She models the number of worms found in each $1\,\text{m}^2$ section as following a Poisson distribution with mean 1.2.

(a) Find the probability of observing exactly one worm in one $1\,\text{m}^2$ section. [1]

(b) Find the probability of observing at least one worm in one $1\,\text{m}^2$ section. [1]

The biologist looks at 5 independent $1\,\text{m}^2$ sections.

(c) Find the probability of observing a total of five worms in 5 sections. [2]

(d) Find the probability of observing exactly one worm in all 5 sections. [2]

(e) Find the probability of observing at least one worm in exactly 3 of the 5 sections. [3]

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14. [Maximum mark: 6]

A vet wants to find a relationship between the age in days of a breed of puppy (d) and its weight in $\text{kg}(w)$.

To do this he collects a large quantity of data and plots two graphs.

He finds the regression line for each graph. His results are summarized in the table.

	Horizontal axis	Vertical axis	Gradient	Intercept on vertical axis	R^2
Graph 1	d	$\ln w$	0.00571	1.54	0.72
Graph 2	$\ln d$	$\ln w$	0.302	0.693	0.95

Based on these results, find the best of the two possible relationships between w and d.

Express your relationship in the form $w = f(d)$ where f is a simplified expression.

Justify your choice of expression.

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15. [Maximum mark: 8]

An astronomer models the shape of a parabolic mirror using the equation $y = x^2$.

(a) Find the equation of the normal to the mirror at the point $(2, 4)$. [3]

A ray of light comes from an object at coordinates $(0, 10)$ and hits the mirror at the point $(2, 4)$.

(b) Find the gradient of the ray of light. [2]

(c) Find the angle between the ray of light and the normal to the mirror. [3]

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16. [Maximum mark: 7]

An electrical engineer models a circuit using the equation

$$z^2 + 2tz + 8t = 0$$

where t is the time in seconds and $0 \leq t \leq 2$.

(a) When $t = 1$, find the value of z which satisfies $\frac{\pi}{2} < \arg z < \pi$. Give your answer in the form $a + bi$. [2]

The power in the circuit is given by $|z|^2$.

(b) Find the value of t in the interval $0 \leq t \leq 2$ for which the power is maximized. [5]

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Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: applications and interpretation
Higher level
Paper 1

15 May 2025

Zone A afternoon | **Zone B** afternoon | **Zone C** afternoon

Candidate session number

2 hours

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- The maximum mark for this examination paper is **[110 marks]**.

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29 pages

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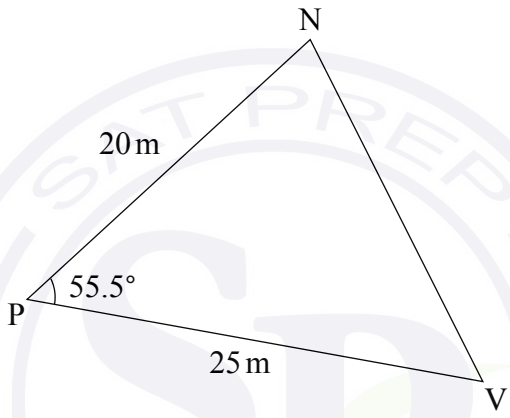
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1. [Maximum mark: 9]

Three points N, P, and V are shown on the following diagram. NP is 20 metres, PV is 25 metres and $\mathrm{V\hat{P}N}$ is $55.5°$.

diagram not to scale



(a) Find NV. [3]

(b) Find $\mathrm{P\hat{N}V}$. [3]

(c) Hence or otherwise, find the shortest distance between P and $[\mathrm{NV}]$. [3]

(This question continues on the following page)

(Question 1 continued)

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2. [Maximum mark: 5]

The loudness of a sound, L, measured in decibels (dB) is determined by the intensity of the sound, I, measured in watts per square metre ($\mathrm{Wm^{-2}}$). The relationship between loudness and intensity can be expressed using the logarithmic function

$$L = 10\log_{10}\left(\frac{I}{I_0}\right),\ I > 0$$

where I_0 is the reference intensity (the intensity of the least audible sound to the human ear).

The reference intensity I_0 is $10^{-12}\,\mathrm{Wm^{-2}}$.

The intensity of sound on a busy street is $10^{-5}\,\mathrm{Wm^{-2}}$.

(a) Calculate the loudness of the sound. [2]

The sound of a jet engine reaches a loudness of $185\,\mathrm{dB}$.

(b) Determine the intensity of its sound. Give your answer in the form $a \times 10^k$ where $1 \leq a < 10$, $k \in \mathbb{Z}$. [3]

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3. [Maximum mark: 7]

Prakash is the leader of a customer service team and is interested in determining whether there is a relationship between a customer's satisfaction level and the type of service interaction they have experienced.

He collects data from a random sample of 250 customers and tracks their satisfaction level after three types of service interactions: in-person, online chat bots and website contact forms.

He categorizes the satisfaction levels as satisfied, neutral and dissatisfied.

He records the data in the following table.

		Satisfaction level		
		Satisfied	Neutral	Dissatisfied
Type of service interaction	In-person	35	30	23
	Online chat bots	31	39	23
	Website contact forms	19	28	22

Prakash performs a χ^2 test for independence at the 5% significance level.

The critical value is 9.488.

The null hypothesis, H_0, is the satisfaction level and the type of service interaction are independent.

(a) State the alternative hypothesis for this test. [1]

(b) Find the degrees of freedom for this test. [1]

(c) Find χ^2_{calc}, the chi-squared test statistic. [2]

Prakash concludes that there is sufficient evidence to reject the null hypothesis.

(d) (i) State whether Prakash is correct. Justify your answer.

(ii) Write down the conclusion for this test in context. [3]

(This question continues on the following page)

(Question 3 continued)

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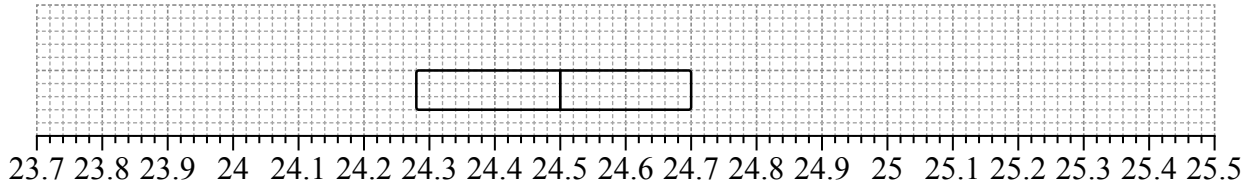
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4. [Maximum mark: 7]

The times, in seconds, for the fastest 16 women in a $50\,\text{m}$ freestyle swimming championship event were recorded. All swimmers recorded different times.

Part of a box and whisker diagram for these times is shown in the following diagram.



Swimming times in seconds

(a) Write down the number of swimmers who took more than 24.70 seconds to complete the race. [1]

(b) Find the interquartile range (IQR) for the data. [2]

An outlier is defined as a value that satisfies one of the following:

- more than $1.5 \times$ IQR below the lower quartile
- more than $1.5 \times$ IQR above the upper quartile.

Of the 16 women, the two fastest swimmers took 23.96 and 24.12 seconds and the two slowest women took 25.12 and 25.40 seconds to complete the race.

(c) (i) Show that only one of these times is an outlier.

(ii) Complete the box and whisker diagram above. [4]

(This question continues on the following page)

(Question 4 continued)

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5. [Maximum mark: 8]

Sofia takes out a loan for $50\,000$ euros. She pays a nominal interest rate of 4.2% per year, compounded monthly. She must pay back the loan in 25 years through regular monthly payments, made at the end of each month.

(a) Find the amount Sofia must pay back each month. Give your answer correct to two decimal places. [2]

At the end of eight years, Sofia wins the lottery and pays back the remainder of the loan early in one single payment.

(b) (i) Find how much Sofia pays back in this single payment. Give your answer correct to two decimal places.

(ii) Find the amount of money saved by Sofia by paying back the loan early. [6]

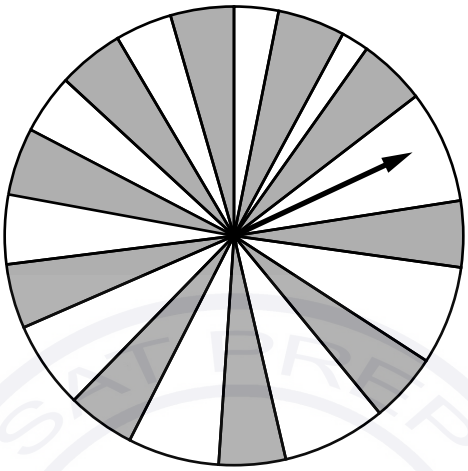
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6. [Maximum mark: 6]

A circular disk has 10 white sectors and 10 shaded sectors. The size of the angles, made at the centre of the circle by the white sectors, form an arithmetic sequence. The common difference of this sequence is $3°$ and the first term is $6°$.

An arrow is attached to the centre of the circle as shown in the following diagram.



The arrow can be spun and has an equal chance of stopping at any point.

(a) Find the probability that the arrow will stop in a white sector. [3]

In a game, the arrow is spun 4 times and the player wins if the arrow stops in a white sector at least 3 times.

(b) Find the probability that the player wins the game. [3]

(This question continues on the following page)

(Question 6 continued)

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7. [Maximum mark: 7]

A team of researchers is using a model to predict the relative happiness of different countries. To do this, a value x is calculated based on easily measured parameters, for example, life expectancy, or available social support. It is assumed that higher values of x indicate greater happiness.

To test the model a survey is conducted in six countries, A, B, C, D, E and F. In these countries the level of happiness is assessed directly using questionnaires and given a score y, out of 10, with higher scores indicating greater happiness.

To select the countries for the survey, all countries are divided into three equal groups based on wealth and two countries are chosen randomly from each group.

(a) Write down the name of this type of sampling. [1]

The results of the survey, along with the value obtained from the model, are given in the following table.

Country	A	B	C	D	E	F
Value from the model (x)	12.3	15.2	14.1	18.5	20.1	19.2
Happiness score (y)	5.2	7.3	6.2	6.9	8.0	7.2

The researchers will accept the model is a valid predictor of happiness score if the Pearson's product-moment correlation coefficient, r, is greater than 0.8.

(b) (i) Find the value of r.

(ii) Hence state whether the model can be regarded as a valid predictor of happiness score. [3]

(c) Find the equation of the regression line y on x. [1]

For a particular country $x = 17.2$.

(d) Use the regression line to predict the happiness score for this country. [2]

(This question continues on the following page)

(Question 7 continued)

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8. [Maximum mark: 5]

The matrix $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ transforms a point A to a point B.

The matrix $\begin{pmatrix} 3 & 1 \\ 0 & 1 \end{pmatrix}$ transforms the point B to a point C with coordinates $(4, 7)$.

Find the coordinates of A.

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9. [Maximum mark: 6]

Consider the curve $y = 4x\sqrt{3x-2}$ for $x \geq \frac{2}{3}$.

(a) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$. [3]

(b) Hence show that the equation of the tangent to the curve at the point $(6, 96)$ is $y = 25x - 54$. [3]

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10. [Maximum mark: 8]

A satellite orbiting the Earth experiences a gravitational force, $F(\mathrm{N})$, which varies inversely as the square of its distance, d (kilometres), from the centre of the Earth.

When $d = 7000$, $F = 40\,000$.

(a) (i) Find an equation for F in terms of d.

(ii) Find the value of F when d is $3500\,\mathrm{km}$. [4]

(b) (i) Identify which **two** of the following graphs would form a straight line.

A. F against d

B. F against d^2

C. F against d^{-2}

D. $\log_{10} F$ against $\log_{10} d$

E. $\log_{10} F$ against d

F. F against $\log_{10} d$

(ii) For each of these two graphs, find the value of the gradient of the line formed. [4]

(This question continues on the following page)

(Question 10 continued)

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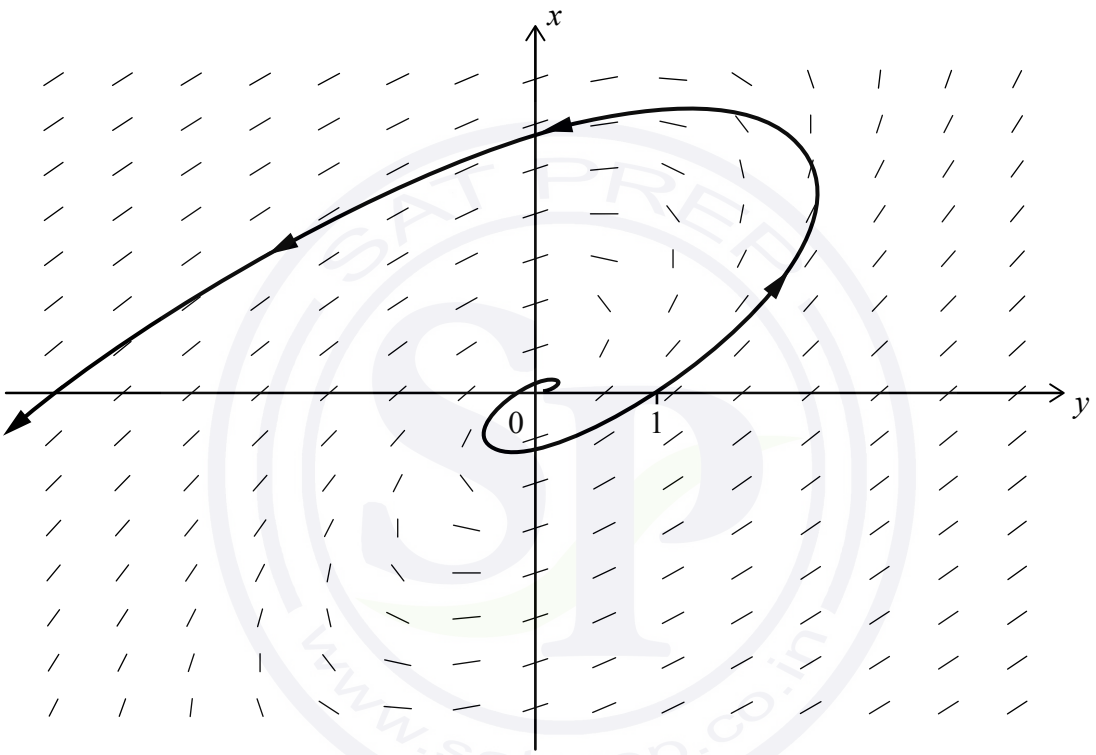
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11. [Maximum mark: 6]

Consider the system given by the coupled differential equations

$$\begin{aligned}\frac{\mathrm{d}x}{\mathrm{d}t} &= 3x - 2y \\ \frac{\mathrm{d}y}{\mathrm{d}t} &= 4x - y\end{aligned}.$$

The following phase portrait shows the trajectory that passes through the point $(1, 0)$ at time $t = 5$.



(a) Write down which one of the following could be an eigenvalue for the system. [1]

A. 1

B. $2\mathrm{i}$

C. $1 + 2\mathrm{i}$

D. $-1 + 2\mathrm{i}$

(b) Use Euler's method with a step length of 0.1 to find the approximate values of x and y when $t = 5.5$. [5]

(This question continues on the following page)

(Question 11 continued)

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12. [Maximum mark: 5]

Tajinder believes that the number of cars arriving at an intersection every minute follows a Poisson distribution. Under this assumption he decides to test the null hypothesis that the mean is 12.4 cars per minute against the alternative hypothesis that the mean is greater than 12.4.

The test consists of observing the intersection for 10 minutes and then rejecting the null hypothesis if more than 130 cars arrive.

(a) (i) State the meaning of a Type I error.

(ii) Find the probability of a Type I error for Tajinder's test. [4]

(b) Write down one reason why a Poisson distribution might not be a suitable model for the number of cars arriving at the intersection. [1]

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13. [Maximum mark: 9]

The growth of a fungus on a dish can be modelled by the differential equation

$$\frac{\mathrm{d}P}{\mathrm{d}t} = 0.2(1-P),\ 0 \le P < 1$$

where P is the proportion of the dish that is covered with fungus and t is the time in days from when the fungus began to form.

(a) Assuming that $P = 0$ when $t = 0$, show that $P = 1 - \mathrm{e}^{-0.2t}$. [7]

(b) Find the time at which the fungus covers 90% of the dish. [2]

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14. [Maximum mark: 8]

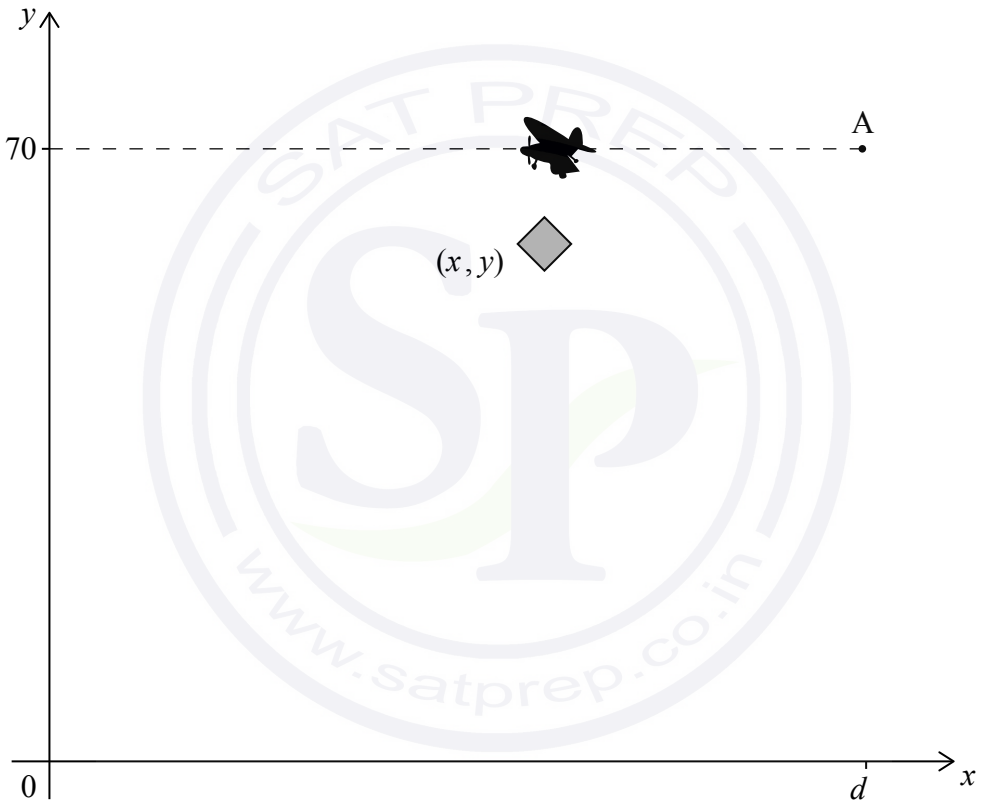
A plane needs to drop a relief package for a remote village. The villagers have marked out a point O in the middle of a field where the package should land.

The package is released from the plane at point A when it is a horizontal distance of d metres from O.

The position vector of the package t seconds after release is given by the vector $\begin{pmatrix} x \\ y \end{pmatrix}$,

where x is the horizontal displacement of the package from O and y is the vertical height of the package above the ground. This is shown in the following diagram.

diagram not to scale



(This question continues on the following page)

(Question 14 continued)

The acceleration of the package is $\begin{pmatrix} \ddot{x} \\ \ddot{y} \end{pmatrix} = \begin{pmatrix} 0 \\ -9.8 \end{pmatrix} \text{ms}^{-2}$ and its velocity at the point of release is $\begin{pmatrix} -20 \\ 0 \end{pmatrix} \text{ms}^{-1}$.

(a) Find the velocity vector of the package $\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix}$ at time t. [2]

The vertical height of the plane above the ground at the moment the package is released is $70\,\text{m}$.

(b) Find the value of d, given that the package lands at O. [6]

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15. [Maximum mark: 6]

A Markov chain has two states, A and B. The transition matrix for the Markov chain is

$$\begin{matrix} & \mathrm{A} & \mathrm{B} \\ \mathrm{A} & p & 1-q \\ \mathrm{B} & 1-p & q \end{matrix}.$$

One of the eigenvalues for this matrix is equal to 1.

(a) Find the eigenvector corresponding to the eigenvalue of 1. [3]

(b) Hence, find an expression, in terms of p and q, for the proportion of time spent in state A. [2]

(c) Hence or otherwise, find the proportion of time spent in state A when $p = 0.3$ and $q = 0.6$. [1]

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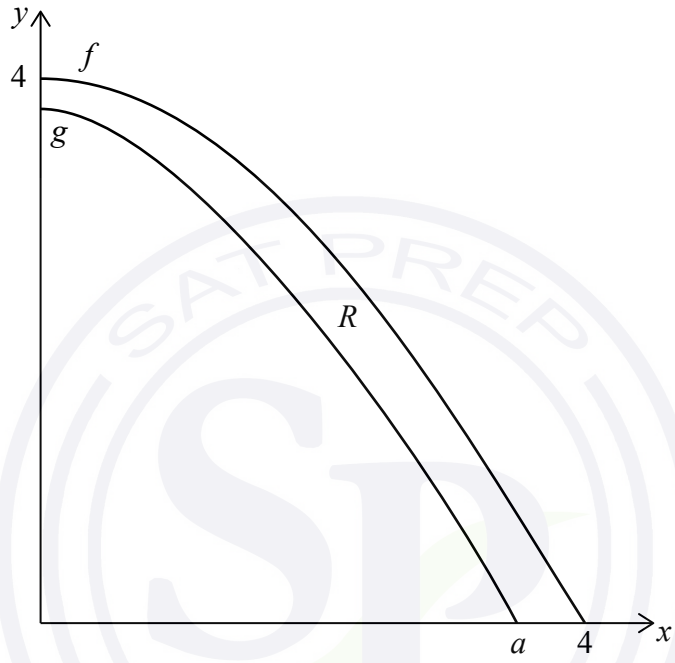
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16. [Maximum mark: 8]

The graph of f is defined by $f(x) = 4\cos\left(\frac{\pi}{8}x\right)$, $0 \le x \le 4$.

The graph of $y = g(x)$ is formed by translating the graph of f 0.3 units to the left and 0.2 units down. The graph of g intersects the x-axis at $(a, 0)$ and its domain is then restricted to $0 \le x \le a$.

These curves are shown on the following graph where units are in centimetres.



(a) Write down an expression for $g(x)$. [2]

(b) Find the value of a. [1]

A wall-mounted light shade can be modelled by the solid formed when the region between the two curves, R, is rotated through π radians about the x-axis.

(c) (i) Write down an expression for the volume of the solid formed.

(ii) Hence find the volume of material used in the light shade. [5]

(This question continues on the following page)

(Question 16 continued)

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Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: applications and interpretation
Higher level
Paper 1

24 October 2024

Zone A afternoon | **Zone B** afternoon | **Zone C** afternoon

Candidate session number

2 hours

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all questions.
- Answers must be written within the answer boxes provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.

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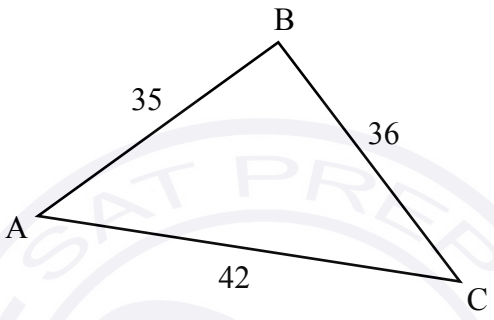
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Answers must be written within the answer boxes provided. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 5]

Consider the following triangle, ABC, such that $AB = 35\,\text{cm}$, $BC = 36\,\text{cm}$, and $CA = 42\,\text{cm}$.

diagram not to scale



(a) Find the value of $C\hat{A}B$. [3]

(b) Find the area of the triangle ABC. [2]

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2. [Maximum mark: 6]

Radioactive carbon is a material that decays over time.

The mass, $m(t)$ (in nanograms), of radioactive carbon in a fossil of a plant, after t years, can be modelled by the function

$$m(t) = 120\mathrm{e}^{-0.000121t}$$

where t is the time since the plant died.

(a) Write down the initial mass of the radioactive carbon. [1]

(b) Find the mass of the radioactive carbon after $20\,000$ years. [2]

(c) Calculate the smallest number of complete years it takes for more than half the sample to decay. [3]

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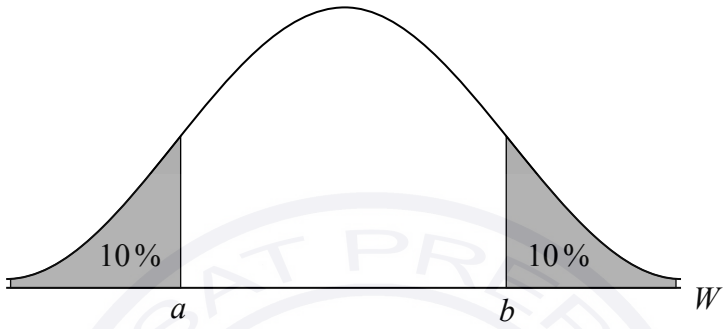
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3. [Maximum mark: 7]

The mass, W, of Manx cats is normally distributed with a mean of $4.5\,\text{kg}$ and a standard deviation of $0.4\,\text{kg}$.

(a) A Manx cat is selected at random. Calculate the probability this cat's mass is more than $3.5\,\text{kg}$. [2]

The following curve represents this distribution. It is known that $\text{P}(W < a) = 0.1$ and $\text{P}(W > b) = 0.1$.



(b) Find the value of

(i) a

(ii) b. [3]

(c) Two Manx cats are selected at random from a large population. Find the probability that they both have a mass less than $3.5\,\text{kg}$. [2]

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4. [Maximum mark: 5]

On 1 January in a particular year, Eva invests $\$25\,000$ in a new bank account. The account earns 5% simple interest, on the original $\$25\,000$, at the start of each subsequent year.

The amounts in the account at the start of each year form an arithmetic sequence.

(a) Find the common difference of this sequence. [2]

After k complete years, the amount in Eva's account will be greater than $\$44\,000$ for the first time.

(b) Find the value of k. [3]

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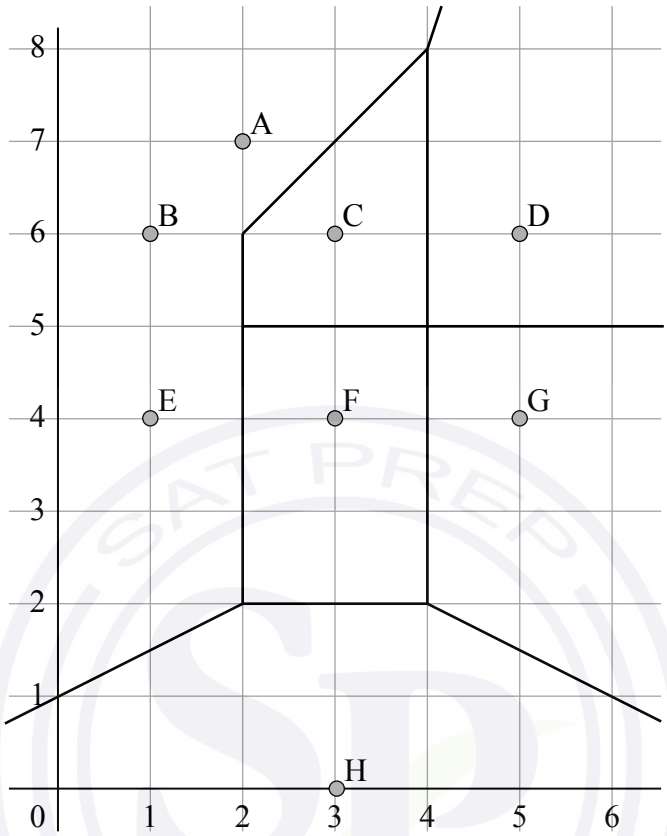
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5. [Maximum mark: 6]

The sites in the Voronoi diagram represent eight hospitals in a city.



Each site has integer coordinates. Two edges are missing from the diagram.

(a) Draw the missing edges on the diagram. [2]

One square unit on the diagram represents $4\,\text{km}^2$ in the city.

(b) Find the area, in km^2, of the cell containing

(i) site F

(ii) site C. [3]

The hospitals at sites C and F have the same number of patients each year.

(c) Suggest a reason why the number of patients is not proportional to the area of the cell. [1]

(This question continues on the following page)

(Question 5 continued)

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6. [Maximum mark: 4]

A museum has an annual membership fee of $\$200$, which includes 10 free visits. Any additional visits are charged at $\$30$ each. The total cost, $\$C$, of n visits during the year can be modelled by

$$C(n) = \begin{cases} 200, & n < p \\ an + b, & n \geq p \end{cases}, \text{ where } a, b, p, n \in \mathbb{Z}.$$

(a) Write down the value of

(i) a

(ii) p. [2]

(b) Find the value of b. [2]

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7. [Maximum mark: 6]

When Daniel retires, he invests $\$400\,000$ in an annuity fund that earns interest at a nominal rate of 4.5% per year, compounded monthly.

Daniel then withdraws $\$3600$ at the end of every month to pay for his living expenses.

(a) Find how much is in the annuity fund after 5 years. [3]

(b) Calculate how many times Daniel is able to make these withdrawals. [3]

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8. [Maximum mark: 7]

The amount of daylight, L (in hours), in London in 2024 can be modelled by

$$L = a\sin\left(b\left(t - c\right)\right) + d,$$

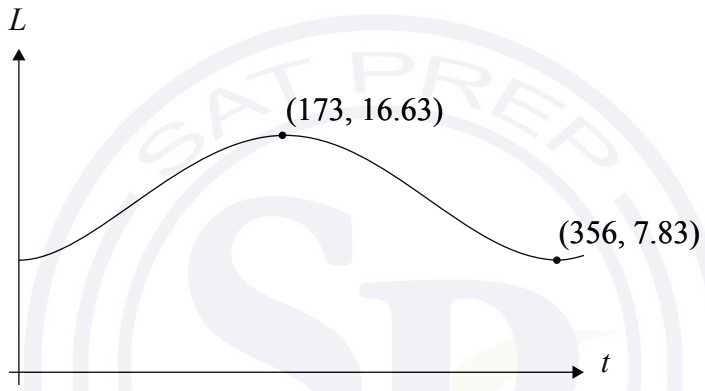
where $a, b, c, d > 0$ and t is the day of the year.

For example, day $1 = 1$ January, day $2 = 2$ January, and so on.

The maximum value of L is 16.63 hours on day 173 (21 June 2024).

The minimum value of L is 7.83 hours on day 356 (21 December 2024).

This information is shown in the following diagram.



Find the value of

(a) d [2]

(b) a [1]

(c) b [2]

(d) c. [2]

(This question continues on the following page)

(Question 8 continued)

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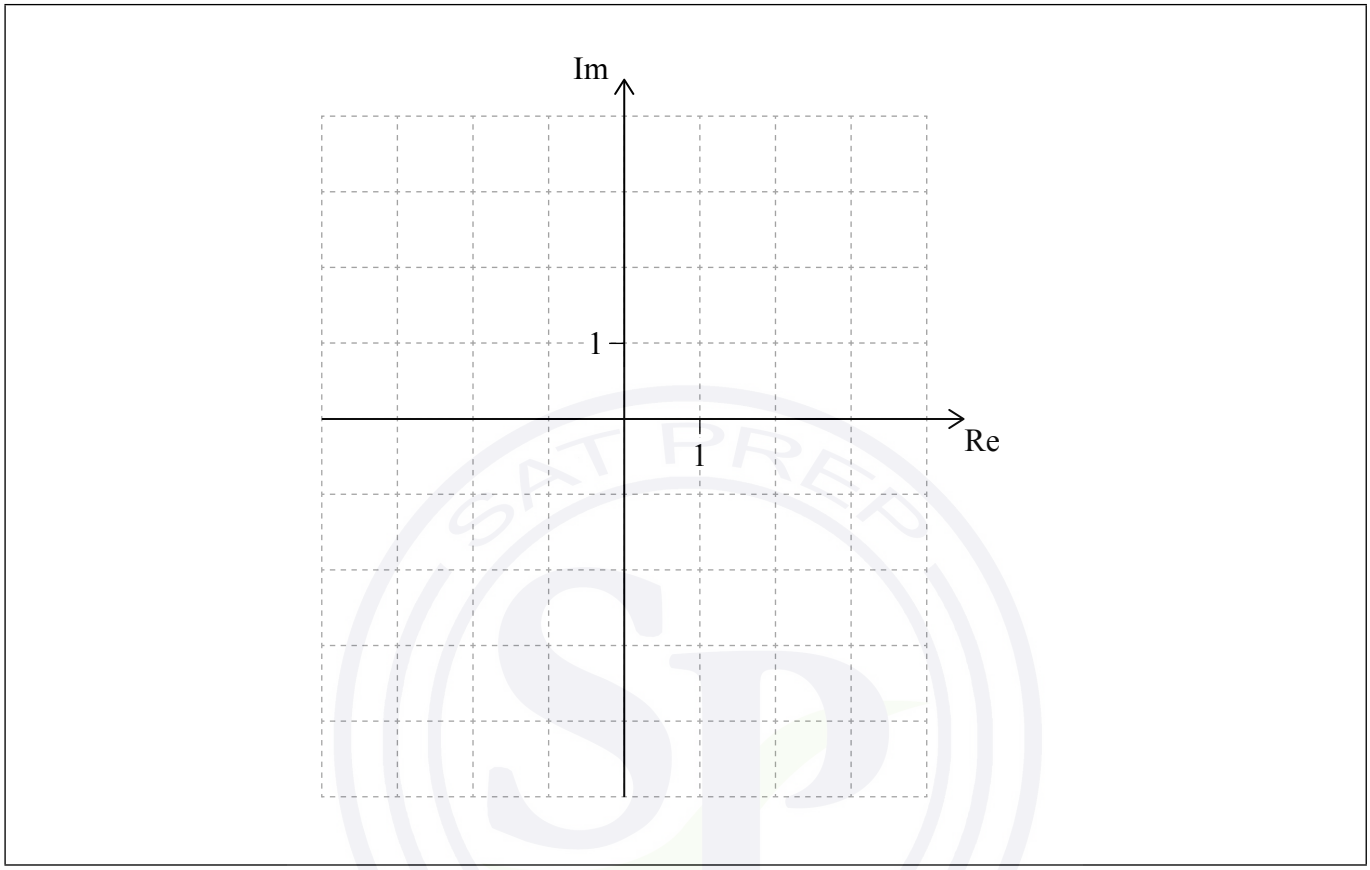
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9. [Maximum mark: 5]

Let $z = 2 - 3\mathrm{i}$.

(a) Plot z on the Argand diagram. [1]



z can be written in the form $r\,\mathrm{cis}\,\theta$, where $r > 0$ and $-\pi < \theta \leq \pi$.

(b) Find the value of

(i) r

(ii) θ. [2]

(c) Find the value of $z\mathrm{i}$ in the form $a + b\mathrm{i}$, where $a, b \in \mathbb{Z}$. [1]

$z\mathrm{i}$ can be obtained from z by a geometric transformation.

(d) By plotting $z\mathrm{i}$ on the Argand diagram, or otherwise, describe fully this transformation. [1]

(This question continues on the following page)

(Question 9 continued)

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10. [Maximum mark: 8]

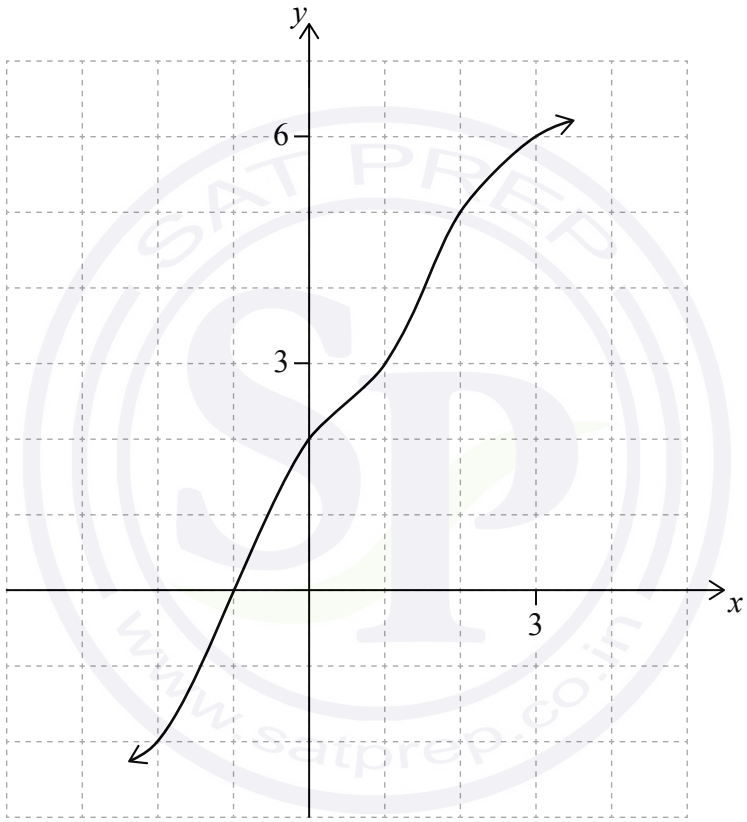
The function f is defined by $f(x) = 4\ln(2x - 3)$, where $x > \frac{3}{2}$.

The graph of $y = f(x)$ is obtained from the graph of $y = \ln x$ by a sequence of three transformations.

(a) Describe fully these transformations, including the order in which they occur. [3]

The graph of $y = g(x)$ is shown.

The graph passes through the points $(-2, -2)$, $(-1, 0)$, $(0, 2)$, $(1, 3)$, $(2, 5)$ and $(3, 6)$.



(b) Find $(f \circ g)(2)$. [2]

(c) Solve $(f \circ g)(x) = 2\ln 9$. [3]

(This question continues on the following page)

(Question 10 continued)

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11. [Maximum mark: 8]

Let $\boldsymbol{M} = \begin{pmatrix} -4 & 2 \\ -3 & 3 \end{pmatrix}$.

(a) Find the eigenvalues of $\boldsymbol{M}$. [3]

$\boldsymbol{M}$ can be written in the form $\boldsymbol{M} = \boldsymbol{PDP}^{-1}$, where $\boldsymbol{D}$ is a diagonal matrix.

(b) (i) Write down $\boldsymbol{D}$.

(ii) Find $\boldsymbol{P}$. [5]

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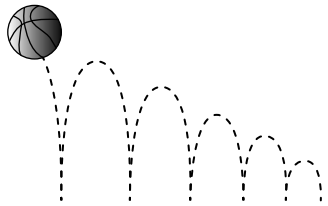
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12. [Maximum mark: 7]

Andreas drops a ball and records a video of the ball bouncing.



He uses the video to find the maximum height, in metres, after each of the first four bounces. His results are shown in the table.

Bounce number, n	Maximum height, h
1	0.613
2	0.514
3	0.439
4	0.377

Andreas thinks the maximum height can be modelled by the function

$$h(n) = 0.613\left(\frac{0.514}{0.613}\right)^{n-1}, \text{ where } n \in \mathbb{Z}^+.$$

(a) Complete the following table. [2]

Bounce number, n	Maximum height, h, according to the model
1	0.613
2	0.514
3	
4	

(b) Hence, calculate the sum of square residuals (SS_{res}). [2]

Andreas' friend thinks a better model for h could be found using an exponential least squares regression curve.

(c) (i) Find the equation of this model.

(ii) Use this model to estimate the height from which the ball was originally dropped. [3]

(This question continues on the following page)

(Question 12 continued)

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13. [Maximum mark: 6]

In 2008, there were no beavers in Scotland, as they had become extinct. In 2009, beavers were introduced from Norway. Surveys were carried out in the region of Tayside to monitor the beaver population.

Figure 1: A beaver



It is believed that the population, P, grows according to a logistic model

$$P = \frac{L}{1 + C\mathrm{e}^{-0.28t}},$$

where t is the number of years after 2008, $t \geq 1$.

A survey in 2017 estimated the population to be 433.

A survey in 2020 estimated the population to be 954.

Find the carrying capacity, L, for the number of beavers in Tayside, according to this model.

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14. [Maximum mark: 8]

While playing on the beach, Sabine builds a mound of sand. The shape of the cross section through the centre of this mound can be modelled by the function

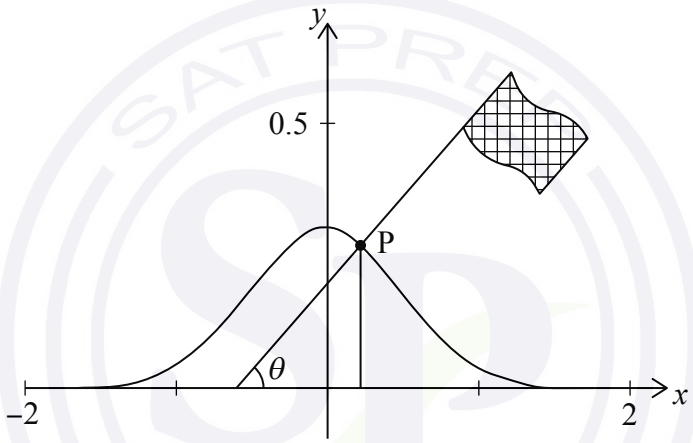
$$f(x) = 0.3\mathrm{e}^{-2x^2},$$

where $-2 < x < 2$.

Sabine places a flagpole into this mound at a point, P, which is $0.25\,\mathrm{m}$ above the level beach. The pole is perpendicular to the mound of sand at point P.

This information is shown in the diagram, where the x-axis represents the level beach and the y-axis represents the height above the beach. The scale on the axes is 1 unit $= 1$ metre.

diagram not to scale



The coordinates of P are $(a, 0.25)$, where $a > 0$.

(a) Calculate the value of a. [2]

(b) Find an expression for $f'(x)$. [2]

(c) Hence, or otherwise, find the angle, θ, the flagpole makes with the level beach. [4]

(This question continues on the following page)

(Question 14 continued)

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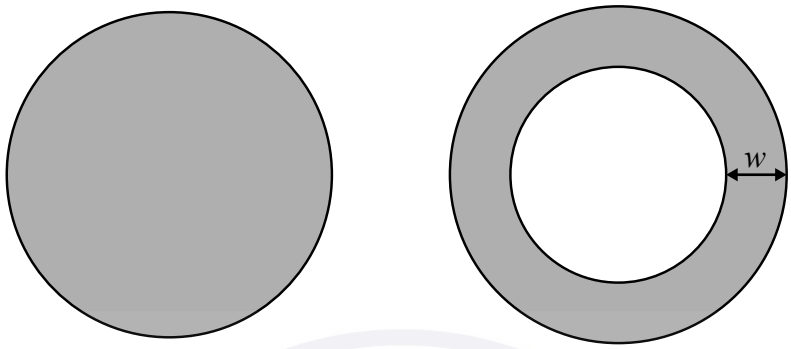
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15. [Maximum mark: 8]

A metal ring, of width w, is made by first cutting a circular disc and then cutting a circular hole exactly in the centre of this disc. This is shown in the diagram.

diagram not to scale



disc ring

A machine produces many of these rings each day.

The diameters of the discs are normally distributed with mean $30\,\text{cm}$ and standard deviation $0.8\,\text{cm}$.

(a) Find the distribution of the radii of the discs. [3]

The radii of the **holes** are normally distributed with mean $12\,\text{cm}$ and standard deviation $0.25\,\text{cm}$.

(b) Calculate the probability that the width of a randomly chosen ring is less than $2.5\,\text{cm}$. [5]

(This question continues on the following page)

(Question 15 continued)

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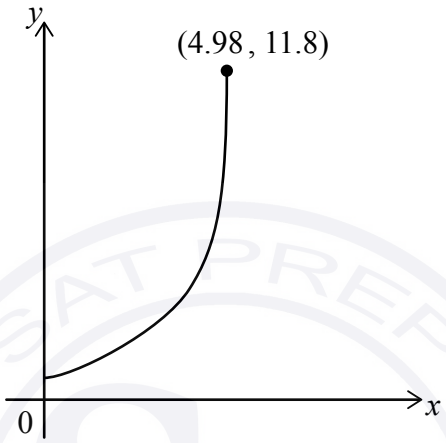
16. [Maximum mark: 7]

As part of his mathematics exploration, Jules models the shape of part of a wine glass to find the capacity (volume) of the glass.

He finds that the edge of half the glass can be modelled by the function

$$f(x) = 4 - 2\ln(5 - x), \text{ where } 0 \le x \le 4.98.$$

A graph of $y = f(x)$ is shown, with a scale of 1 unit $= 1\,\text{cm}$.



(a) Find the y-intercept. [1]

The point $(4.98, 11.8)$ represents a point at the top of the glass.

Let R be the region enclosed by the graph of f, the y-axis and the line $y = 11.8$.

Jules finds the capacity by rotating the region, R, $360°$ about the y-axis.

(b) Calculate the capacity of the glass that Jules obtains. [6]

(This question continues on the following page)

(Question 16 continued)

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17. [Maximum mark: 7]

The fish in a lake feed on insects. Let F be the population of fish and L be the population of insects at time t (weeks).

The populations can be modelled using the coupled differential equations

$$\frac{\mathrm{d}F}{\mathrm{d}t} = 0.000004FL - 0.2F$$
$$\frac{\mathrm{d}L}{\mathrm{d}t} = 0.06L - 0.00003FL$$

where $F > 0$ and $L > 0$.

When $t = 0$, it is estimated that $F = 5000$ and $L = 80\,000$.

(a) For $0 \leq t \leq 52$, use Euler's method, with a step length $h = 4$, to find an approximate value for

(i) the maximum value of F

(ii) the minimum value of L. [5]

(b) (i) State what will happen to the number of fish in the lake, as predicted by the model.

(ii) Suggest a reason why this prediction may not occur. [2]

(This question continues on the following page)

(Question 17 continued)

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Figure 1 animatedfunk, n.d. *Beaver Illustrations – stock illustration* [image online] Available at: https://www.gettyimages.co.uk/detail/illustration/beaver-illustrations-royalty-free-illustration/1453231718 [Accessed 30 November 2023]. Source adapted.

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Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: applications and interpretation
Higher level
Paper 1

1 May 2024

Zone A afternoon | **Zone B** afternoon | **Zone C** afternoon

Candidate session number

2 hours

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all questions.
- Answers must be written within the answer boxes provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.

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2224 – 7201

25 pages

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Answers must be written within the answer boxes provided. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 6]

Maan deposited $\$100\,000$ into a savings account with a nominal annual interest rate of $I\,\%$ **compounded monthly**. At the end of the eighth year, the amount in the account had increased to $\$150\,000$.

(a) Find the value of I. [3]

Maan withdraws the $\$150\,000$ and places it in an annuity, earning a nominal annual interest rate of $6.1\,\%$ **compounded monthly**. At the end of each month, Maan will receive a payment of $\$1000$.

(b) Find the amount of money remaining in the annuity at the end of 10 years. Express your answer to the nearest dollar. [3]

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2. [Maximum marks: 7]

After taking a mathematics test, Fatima wonders how many more marks she would have achieved if she had spent an extra 1.5 hours studying.

To find out, she randomly selects five students from her class who took the same test and asks them how many hours (t) they spent studying for the test and the marks (m) they achieved. Their responses are shown in the following table.

Hours, *t*	0	1.2	1.6	2.5	4
Marks, *m*	45	54	61	72	86

Fatima believes there might be a linear relationship between the time spent studying and the results obtained.

(a) (i) Find the Pearson's product moment correlation coefficient, r, for this data.

(ii) Find the least squares regression line of m on t for this data. [4]

(b) According to her model, find how many more marks Fatima would have achieved if she spent an extra 1.5 hours studying. [2]

(c) State one reason why the value obtained in part (b) might not be valid. [1]

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3. [Maximum mark: 6]

The formula $F = 1.8C + 32$ is used to convert a temperature in degrees Celsius, C, to degrees Fahrenheit, F.

(a) (i) Find a formula for converting a temperature in degrees Fahrenheit to degrees Celsius.

(ii) Find the temperature in degrees Celsius that is recorded as 77 degrees Fahrenheit. [3]

Over one year, the mean daily temperature in Mexico City was calculated to be 17 degrees Celsius with a standard deviation of 9 degrees Celsius.

(b) For the same year, find in degrees Fahrenheit

(i) the mean daily temperature in Mexico City.

(ii) the standard deviation of the daily temperature in Mexico City. [3]

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4. [Maximum mark: 4]

Consider the differential equation $\frac{dy}{dx} = \log_{10}(x + y)$, where $x \geq 0$ and $y > 0$.

Given that $y = 1$ when $x = 0$, use Euler's method with a step length of 0.1 to find an approximate value for y when $x = 2$.

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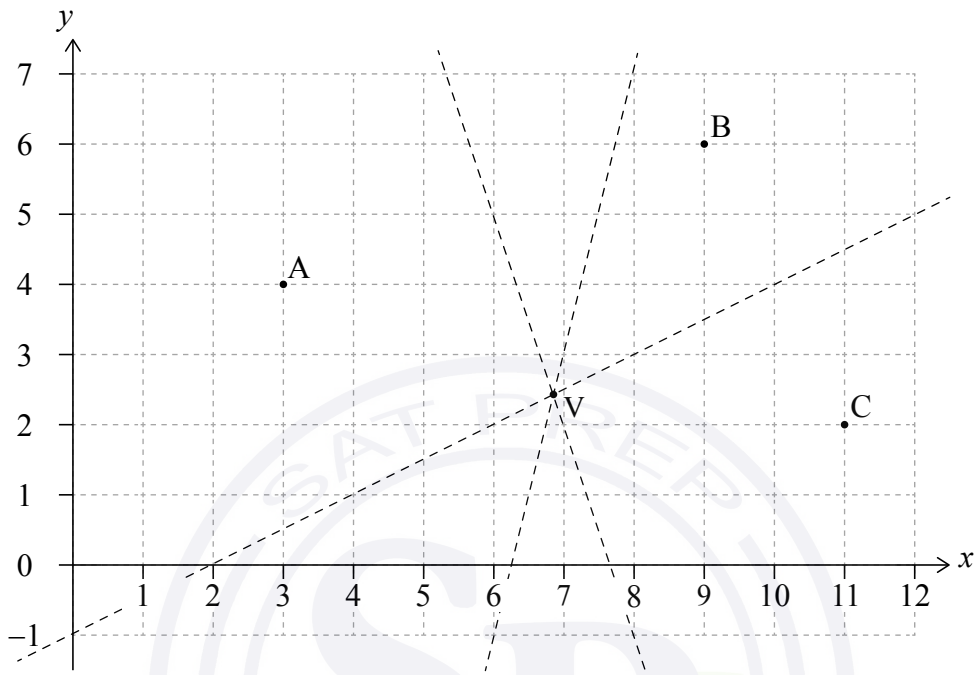
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5. [Maximum mark: 6]

Points $A(3, 4)$, $B(9, 6)$ and $C(11, 2)$ are shown on the following diagram, along with the perpendicular bisectors of [AB], [AC] and [BC].



The perpendicular bisector of [BC] intercepts the axes at coordinates $(0, -1)$ and $(2, 0)$.

(a) Write down the equation of the perpendicular bisector of [BC]. [2]

The equation of the perpendicular bisector of [AB] is $y = -3x + 23$.

(b) Find the coordinates of point V where the perpendicular bisectors meet. Give your answer to four significant figures. [2]

A Voronoi diagram is constructed with points A, B and C as the three sites.

(c) Draw, clearly, the edges of the Voronoi diagram on the given diagram. [2]

(This question continues on the following page)

(Question 5 continued)

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6. [Maximum mark: 7]

On 1 January 2025, the Faber Car Company will release a new car to global markets. The company expects to sell 40 cars in January 2025. The number of cars sold each month can be modelled by a geometric sequence where $r = 1.1$.

(a) Use this model to find the number of cars that will be sold in December 2025. [2]

(b) Use this model to find the total number of cars that will be sold in the year

(i) 2025.

(ii) 2026. [5]

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7. [Maximum mark: 8]

When studying big cats, researchers use a model in which the mass (m kilograms) of an animal is directly proportional to the cube of its shoulder height (h metres).



Cheetah



Lion

A cheetah has a mass of $64\,\text{kg}$ and shoulder height of 0.8 metres.

(a) (i) Use the model to find an expression for m in terms of h.

(ii) Hence find the mass of a different cheetah, with a shoulder height of 0.75 metres. [4]

'Rubner's law' states that the energy needs of an animal (E) are directly proportional to the square of h.

The energy needs of a lion of mass $220\,\text{kg}$ are k times the energy needs of a cheetah of mass $64\,\text{kg}$.

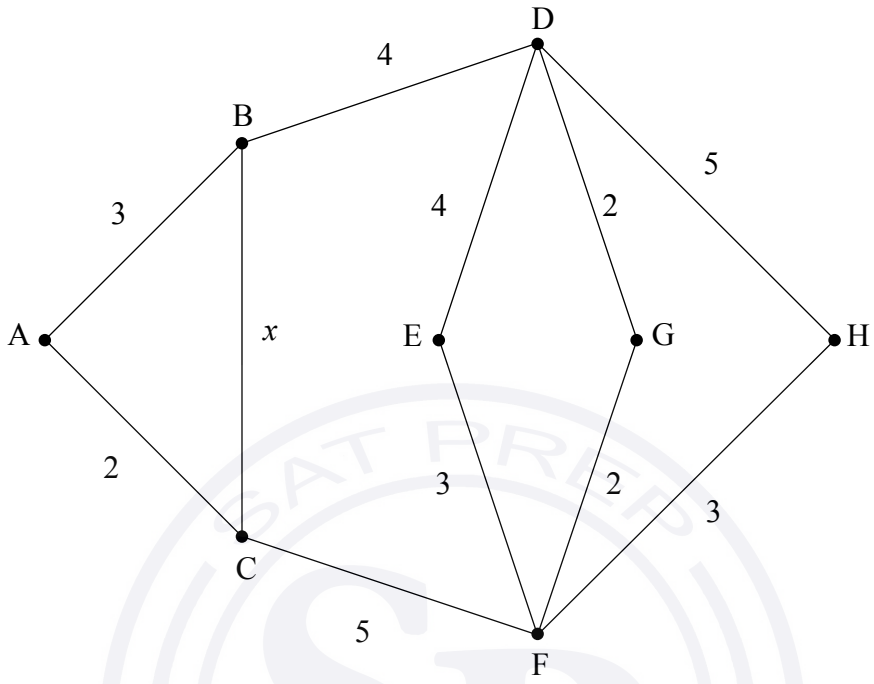
(b) Find the value of k. [4]

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8. [Maximum mark: 5]

The weights on the following graph represent the lengths of different roads in kilometres.

diagram not to scale



(a) Write down the vertices with odd degree. [1]

The total length of the roads is $33 + x$ km.

(b) Find **two** expressions, in terms of x, for the shortest distance required to walk along all of the paths, beginning and ending at the same vertex.
Include in your answer the interval of values of x for which each expression is valid. [4]

(This question continues on the following page)

(Question 8 continued)

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9. [Maximum mark: 7]

(a) Find $\int \frac{8}{2x+3}\,\mathrm{d}x$. [3]

(b) Hence find the exact area between the curve $y = \frac{8}{2x+3}$, the x-axis and the lines $x = 0$ and $x = 6$. Give your answer in the form $a \ln b$, where $a, b \in \mathbb{N}$. [4]

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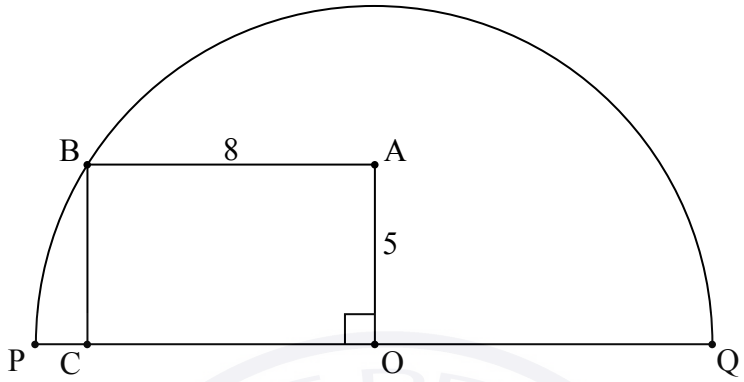
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10. [Maximum mark: 5]

The following diagram shows a semicircle with centre O and diameter PQ. A rectangle $OABC$ is also shown, such that $AB = 8$ and $OA = 5$.

diagram not to scale



Find the length of the arc BQ.

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11. [Maximum mark: 8]

Let $\boldsymbol{R}(\alpha)$ be the matrix representing a rotation, counter-clockwise (anticlockwise) about the origin, through an angle of α.

(a) Write down $\boldsymbol{R}(2\alpha)$ as a 2×2 matrix. [2]

(b) Calculate $\boldsymbol{R}(\alpha) \times \boldsymbol{R}(\alpha)$. [2]

(c) Use your answers from part (a) and part (b) to

(i) explain why $\sin(2\alpha) = 2\sin(\alpha)\cos(\alpha)$.

(ii) show that $\cos(2\alpha) = 1 - 2\sin^2(\alpha)$. [4]

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12. [Maximum mark: 8]

Roma is told by the manufacturers of their phone that, when charging from 0%, the percentage P of charge can be modelled by $P = 100 - 100 \times 2^{-t}$, where t is the number of hours since the phone started charging.

(a) Find the time taken for the amount of charge to go from 0% to 40%, according to this model. [2]

Roma decides to test the model. They charge the phone from 0% and record the charge of the phone for different values of t. The results are shown in the following table.

t **hours**	1	2	3	4
P **percent**	48	74	86	91

(b) Find the sum of the square residuals (SS_{res}) when using $P = 100 - 100 \times 2^{-t}$ to fit this data. [4]

Roma now uses a quadratic function to model the charge in the phone. They calculate that, when using this model, the sum of the square residuals for these four points is 2.45.

(c) State one reason why Roma might prefer to use

(i) the quadratic model.

(ii) $P = 100 - 100 \times 2^{-t}$. [2]

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13. [Maximum mark: 6]

Let $z_1 = 4 + 5\mathrm{i}$.

(a) (i) Find $|z_1|$.

(ii) Find $\arg(z_1)$. [2]

Let $z_2 = 3\mathrm{e}^{2\mathrm{i}}$.

(b) Find the area of the triangle on an Argand diagram with vertices 0, z_1 and z_2. [4]

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14. [Maximum mark: 4]

Two model cars are driving along the same straight track. Car A leaves point O at $t = 0$ and its displacement (x_A metres) from O at t seconds is given by the equation

$$x_A = 4\sqrt{t}\text{, for } 0 \le t \le 9.$$

A faster car, B, leaves O at $t = 3$.

After starting its motion, the time taken by car B to reach any point on the track is half the time that was taken by A.

(a) Write down the equation for the displacement of B, x_B, in terms of t. [2]

(b) Find the value of t at which B catches up with A. [2]

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15. [Maximum mark: 6]

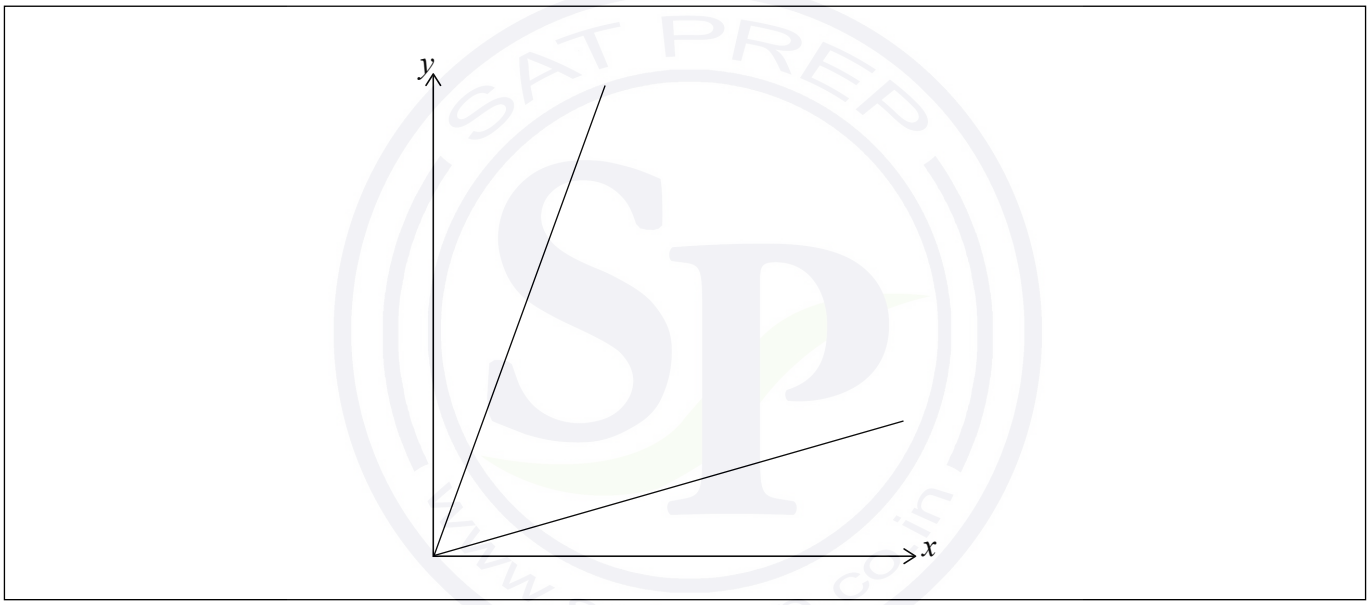
A system of differential equations of the form $\frac{\mathrm{d}x}{\mathrm{d}t} = ax + by$, $\frac{\mathrm{d}y}{\mathrm{d}t} = cx + dy$ has eigenvalues $\lambda = -1$ and $\lambda = 2$ with corresponding eigenvectors $\begin{pmatrix} 3 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 3 \end{pmatrix}$.

The following incomplete phase portrait for this system, with $x, y \geq 0$, shows lines through $(0, 0)$ parallel to the eigenvectors.

(a) On the phase portrait

(i) show the direction of motion along the eigenvectors.

(ii) sketch one trajectory in each of the three regions. [3]



In the system described above, x and y are the population sizes of two species, X and Y. The population of Y is vulnerable, so it will be increased by adding more animals from a different area. Currently, $x = 252$ and $y = 60$.

(b) Find the minimum number of new animals from species Y that need to be added for the population not to reduce to 0 over time. [3]

(This question continues on the following page)

(Question 15 continued)

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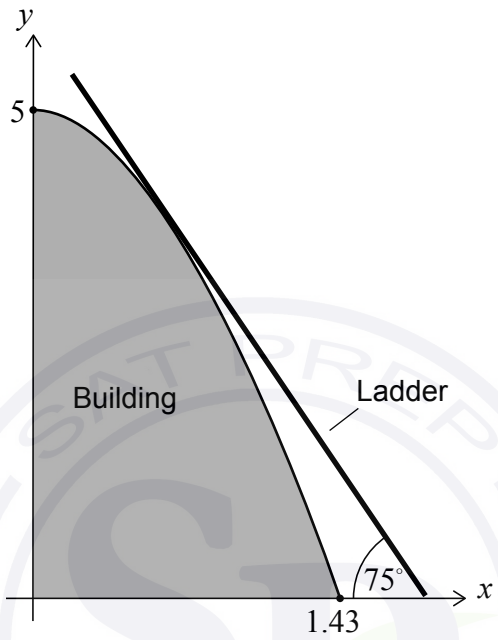
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16. [Maximum mark: 8]

The cross section of the side of a building can be modelled by a curve with equation $y = 5\cos(1.1x), 0 \le x \le 1.43$, as shown in the following diagram. Distances are measured in metres.

diagram not to scale



A builder leans a straight ladder against the building to do repairs. For safety reasons, the angle between the ladder and the horizontal ground must be 75°.

Find the height above the ground at which the ladder touches the building.

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17. [Maximum mark: 9]

Phoebe opens a coffee shop, near to a well-established Apollo coffee shop.

After being open for a few months, Phoebe notices that

- 10% of customers who preferred the Apollo coffee shop in one month preferred her coffee shop the following month.
- 25% of customers who preferred her coffee shop in one month preferred the Apollo coffee shop the following month.

She decides to show these changes in the following transition matrix.

$$\begin{pmatrix} 0.9 & 0.25 \\ 0.1 & 0.75 \end{pmatrix}$$

The two eigenvalues for this matrix are 1 and 0.65. An eigenvector corresponding to the eigenvalue of 1 is $\begin{pmatrix} 5 \\ 2 \end{pmatrix}$.

(a) Find an eigenvector corresponding to the eigenvalue of 0.65. [2]

A diagonal matrix of eigenvalues is $\boldsymbol{D} = \begin{pmatrix} 0.65 & 0 \\ 0 & 1 \end{pmatrix}$.

(b) Write down an expression for $\boldsymbol{D}^n$, giving your answer as a 2×2 matrix in terms of n. [1]

When Phoebe's coffee shop first opened, the Apollo shop had 7000 customers the previous month.

(c) Assuming all 7000 customers continue to go to one of these coffee shops, find an expression for the number that will favour Phoebe's coffee shop after n months. [6]

(This question continues on the following page)

(Question 17 continued)

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Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: applications and interpretation
Higher level
Paper 1

1 May 2024

Zone A afternoon | **Zone B** afternoon | **Zone C** afternoon

Candidate session number

2 hours

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all questions.
- Answers must be written within the answer boxes provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.

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Answers must be written within the answer boxes provided. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 9]

Imani invests $\$3000$ in a bank that pays a nominal annual interest rate of 1.25% compounded monthly.

(a) Calculate the amount of money Imani will have in the bank at the end of 6 years. Give your answer correct to two decimal places. [3]

(b) Calculate the number of months it takes until Imani has at least $\$3550$ in the bank. [2]

Imani uses the $\$3550$ as a partial payment for a used car costing $\$22\,000$. For the remainder she takes out a loan from a bank.

(c) Write down the amount of money that Imani takes out as a loan. [1]

The loan is for 8 years and the nominal annual interest rate is 12.6% compounded monthly. Imani will pay the loan in fixed monthly instalments at the end of each month.

(d) Calculate the amount, correct to the nearest dollar, that Imani will have to pay the bank each month. [3]

(This question continues on the following page)

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(Question 1 continued)

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2. [Maximum mark: 6]

Jerry makes handcrafted chocolates. On average, 1 in 25 of the chocolates that Jerry makes is flawed. Whether or not a chocolate is flawed is independent of all other chocolates.

(a) In a batch of 20 chocolates, chosen at random, find the probability that

(i) two are flawed.

(ii) more than two are flawed. [4]

Jerry sells the perfect chocolates for 50 pesos each and the flawed ones for 15 pesos each.

(b) Calculate the expected number of pesos Jerry makes from selling a batch of 20 randomly selected chocolates. [2]

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3. [Maximum mark: 7]

The pH scale is a measure of the acidity of a solution. Its value is given by the formula

$$\text{pH} = -\log_{10}[\text{H}^+],$$

where $[\text{H}^+]$ is the concentration of hydrogen ions in the solution (measured in moles per litre).

(a) Calculate the pH value if the concentration of hydrogen ions is 0.0003. [2]

The pH of milk is 6.6.

(b) Calculate the concentration of hydrogen ions in milk. [2]

The strength of an acid is measured by its concentration of hydrogen ions.

A lemon has a pH value of 2 and a tomato has a pH value of 4.5.

(c) Calculate how many times stronger the acid in a lemon is when compared to the acid in a tomato. [3]

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4. [Maximum mark: 7]

Gustav plays a game in which he first tosses an unbiased coin and then rolls an unbiased six-sided die.

If the coin shows tails, the score on the die is Gustav's final number of points.

If the coin shows heads, one is added to the score on the die for Gustav's final number of points.

(a) Find the probability that Gustav's final number of points is 7. [2]

(b) Complete the following table. [3]

Final number of points	1	2	3	4	5	6	7
Probability							

(c) Calculate the expected value of Gustav's final number of points. [2]

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5. [Maximum mark: 8]

The annual growth of a tree is $80\,\%$ of its growth during the previous year.

This year the tree is $42\,\text{m}$ in height and one year ago its height was $37\,\text{m}$.

(a) Calculate the annual growth of the tree in the coming year. [2]

(b) Calculate the height of the tree 6 years from now. Give your answer correct to the nearest cm. [4]

If the tree continues to follow this pattern of growth, its height will never exceed k metres.

(c) Find the smallest possible value of k. [2]

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6. [Maximum mark: 6]

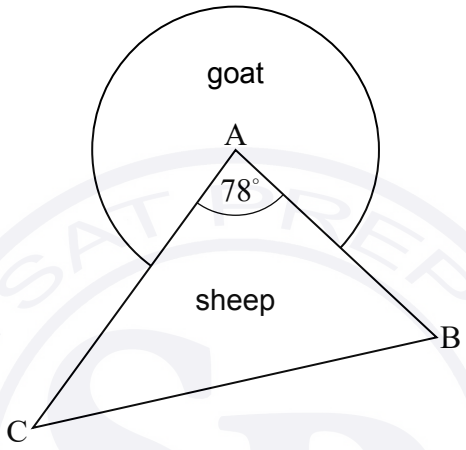
A sheep is in a field in the shape of a triangle, ABC.

$AC = 21$ metres, $AB = 15$ metres and $C\hat{A}B = 78^\circ$.

A goat is in an adjacent field in the shape of a sector of a circle with centre, A, and radius 8 metres.

The fields are shown in the diagram.

diagram not to scale



Determine which animal, the sheep or the goat, is in the field with the larger area, and state how many extra square metres are in this larger field.

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7. [Maximum mark: 4]

Consider the differential equation $\frac{dy}{dx} = xy - 1$, given that $y = 2$ when $x = 1$.

Use Euler's method with step size 0.1 to find the approximate value of y when $x = 1.5$.

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8. [Maximum mark: 10]

The quadrilateral $ABCD$ has coordinates $A(1, 3, 5)$, $B(4, 7, 5)$, $C(5, 8, 7)$ and $D(2, 4, 7)$.

(a) Write down $\vec{AD}$. [1]

(b) Calculate

(i) the size of $B\hat{A}D$.

(ii) the area of triangle BAD. [7]

(c) Show that $ABCD$ is a parallelogram. [2]

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9. [Maximum mark: 6]

The number of traffic accidents at a road junction is modelled by a Poisson distribution with a mean of 0.76 accidents per week.

(a) Under this model, calculate the probability that

(i) there are at least 2 accidents in a particular week.

(ii) there will be exactly 3 accidents in a particular 4-week period. [4]

The local traffic authority wishes to determine the probability that, in an 8-week period, fewer than 2 accidents occur in a week on exactly 5 occasions. It assumes that the weekly occurrence of accidents is independent of the week in which these occur.

(b) State the appropriate model that the traffic authority should use to determine this probability. [2]

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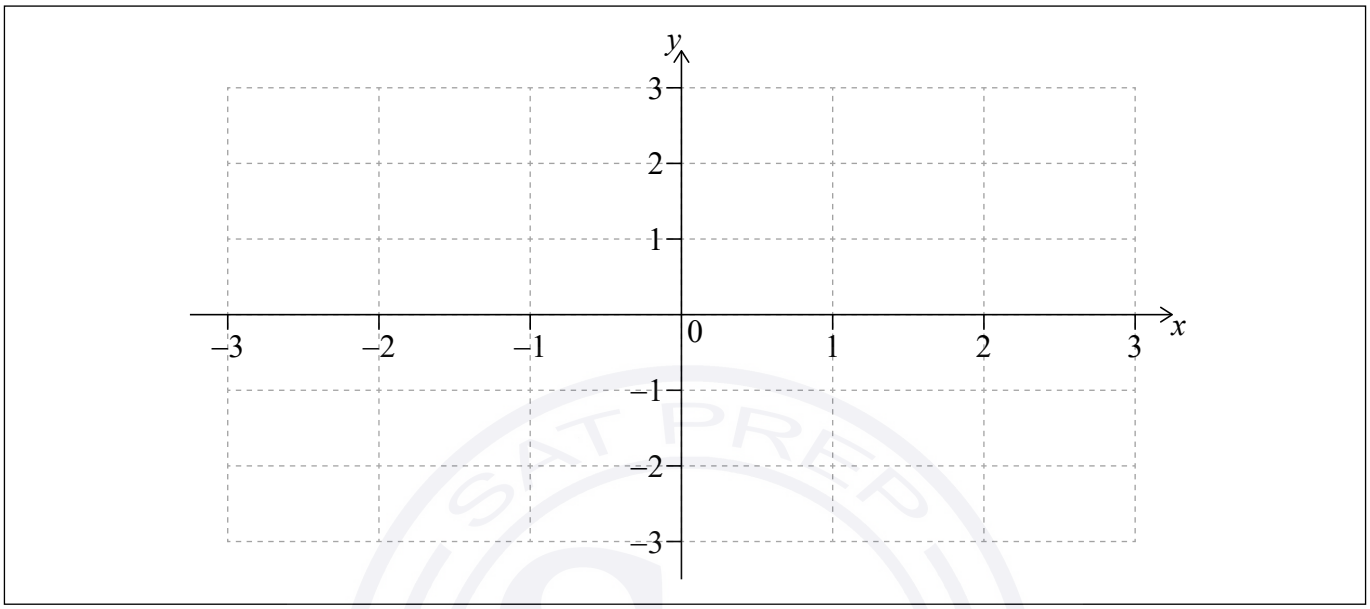
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10. [Maximum mark: 9]

Consider the function $f(x) = x\sqrt{3 - x^2}$, $-\sqrt{3} \leq x \leq \sqrt{3}$.

(a) Sketch the graph of $y = f(x)$ on the following pair of axes. [2]



The area between the graph of $y = f(x)$ and the x-axis is rotated through 360° about the x-axis.

(b) (i) Write down an integral that represents this volume.

(ii) Calculate the value of this integral. [4]

The graph of the function f is transformed, to give the graph of the function g, in the following way:

- It is first stretched by scale factor 2, parallel to the x-axis with the y-axis invariant.
- It is then stretched by scale factor 0.5, parallel to the y-axis with the x-axis invariant.

(c) Find the volume obtained when the area between the graph of $y = g(x)$ and the x-axis is rotated through 360° about the x-axis. [3]

(This question continues on the following page)

(Question 10 continued)

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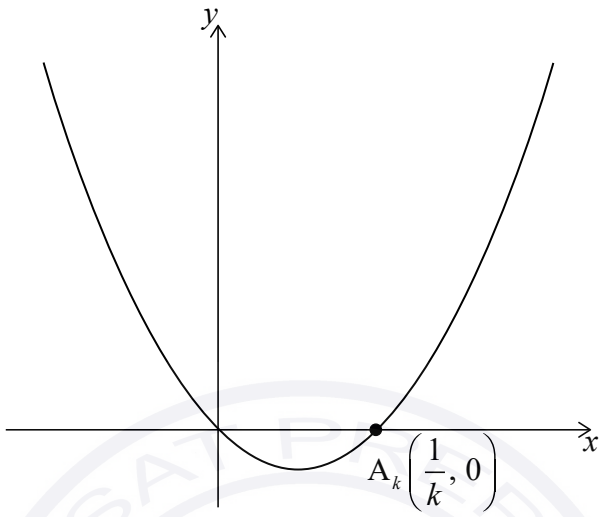
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11. [Maximum mark: 9]

The diagram shows the curve with equation $y_k = kx^2 - x$, $k > 0$, which intersects the x-axis at the origin and at the point $\mathrm{A}_k\left(\frac{1}{k}, 0\right)$.



The normal to the curve at A_k intersects the curve again at point B_k.

(a) Show that the x-coordinate of B_k is $-\frac{1}{k}$. [6]

Consider the case where $k = 2$.

(b) Calculate the finite area of the region between the curve with equation $y_2 = 2x^2 - x$ and the normal at A_2. [3]

(This question continues on the following page)

(Question 11 continued)

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12. [Maximum mark: 7]

A duck is sitting in a duck pond at point $\mathrm{A}(7, 4, 0)$ relative to an origin O, where lengths are measured in metres and time, t, is measured in seconds. It takes off and flies in a straight line with vector equation

$$\boldsymbol{d} = \begin{pmatrix} 7 \\ 4 \\ 0 \end{pmatrix} + t \begin{pmatrix} 6 \\ 6 \\ 3 \end{pmatrix}.$$

(a) Find the speed of the duck through the air (in $\mathrm{m\,s^{-1}}$). [2]

A hawk hovering at position vector $\begin{pmatrix} -38 \\ 134 \\ 315 \end{pmatrix}$, relative to O, sees the duck take off and immediately dives from its position with constant velocity vector $\begin{pmatrix} 15 \\ -20 \\ -60 \end{pmatrix}$ to intercept the duck.

(b) Write down the vector equation for $\boldsymbol{h}$, that models the flight of the hawk. [1]

(c) Find the position vector at which the hawk intercepts the duck. [4]

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13. [Maximum mark: 8]

A particle starts from rest at point O and moves in a straight line with velocity, v, given by

$$v = 3\sin(t)\left(1+\cos(t)\right),\ t \geq 0$$

where v is measured in metres per second and time, t (radians), is measured in seconds.

The particle next comes to instantaneous rest when $t = a$.

(a) Determine the value of a. [2]

(b) Find the maximum velocity of the particle during the interval $0 \leq t \leq a$. [2]

(c) By finding the total distance travelled between $t = 0$ and $t = a$, find the average speed of the particle during the interval $0 \leq t \leq a$. [4]

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14. [Maximum mark: 6]

A cup of hot water is placed in a room and is left to cool for half an hour. Its temperature, measured in $^{\circ}\mathrm{C}$, is recorded every 5 minutes. The results are shown in the table.

Time (mins)	5	10	15	20	25	30
Temperature (°C)	59	52	46	40	35	29

Akira uses the power function $T(t) = at^b + 25$ to model the temperature, T, of the water t minutes after it was placed in the room.

(a) State what the value of 25 represents in this context. [1]

(b) Use your graphic display calculator to find the value of a and of b. [3]

Soo Min models the temperature, T, of the water t minutes after it was placed in the room as $T(t) = kc^t + 25$.

(c) Find the value of k and of c. [1]

(d) State a reason why Soo Min's model of the temperature is a better fit for the data than Akira's model. [1]

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15. [Maximum mark: 8]

Consider the complex number $z = -1 + \mathrm{i}$.

(a) Express z in the form $r\mathrm{e}^{\mathrm{i}\theta}$ where $-\pi < \theta \leq \pi$. [2]

A and B are the points on the Argand diagram that represent the complex numbers z and z^2, respectively.

A is mapped onto B by the composition of a rotation and an enlargement.

(b) (i) Describe fully this mapping of A onto B, stating the scale factor of the enlargement and the angle of rotation.

(ii) Find and simplify a matrix that maps A onto B. [5]

(c) Find the smallest positive integer, n, for which z^n is real and positive. [1]

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Answers written on this page
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Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: applications and interpretation
Higher level
Paper 1

30 October 2023

Zone A afternoon | **Zone B** afternoon | **Zone C** afternoon

Candidate session number

2 hours

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all questions.
- Answers must be written within the answer boxes provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.

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Answers must be written within the answer boxes provided. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 7]

The growth of a particular type of seashell is being studied by Manon. At the end of each month Manon records the increase in the width of a seashell since the end of the previous month.

She models the monthly increase in the width of the seashell by a geometric sequence with common ratio 0.8. In the first month, the width of the seashell increases by $4\,\text{mm}$.

(a) Find by how much the width of the seashell will increase during the third month, according to her model. [2]

(b) Find the total increase in the width of the seashell, predicted by Manon's model, during the first year. [2]

Manon's seashell had a width of $25\,\text{mm}$ at the beginning of the first month.

(c) Find the maximum possible width of the seashell, predicted by Manon's model. [3]

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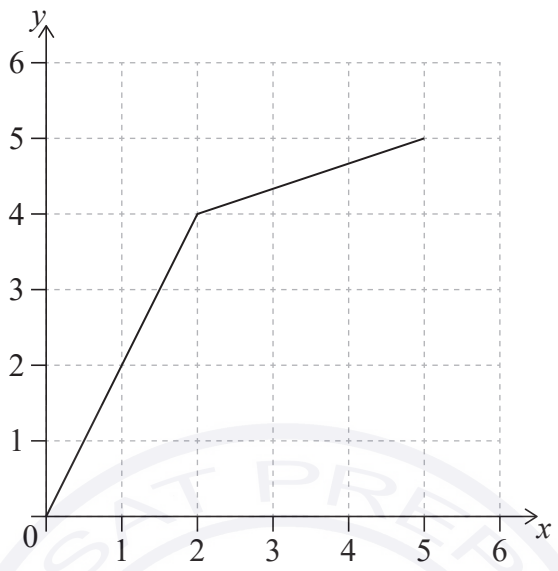
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2. [Maximum mark: 7]

The graph of the function f is given in the following diagram.



(a) Write down $f(2)$. [1]

(b) On the axes, sketch $y = f^{-1}(x)$. [2]

The function g is defined as $g(x) = 3x - 1$.

(c) Find an expression for $g^{-1}(x)$. [2]

(d) Find a value of x where $f^{-1}(x) = g^{-1}(x)$. [2]

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3. [Maximum mark: 6]

The Great Pyramid of Giza is the oldest of the Seven Wonders of the Ancient World. When it was built, 4500 years ago, the measurements of the pyramid were in Royal Egyptian Cubits (REC).



[Source: Nina Aldin Thune. https://en.wikipedia.org/wiki/Great_Pyramid_of_Giza#/media/File:Kheops-Pyramid.jpg. Licensed under CC BY 2.5 https://creativecommons.org/licenses/by/2.5/#. Image adapted.]

Viktor reads online that $1\,\text{REC}$ is equal to 0.52 metres, rounded to two decimal places.

(a) Write down the upper and lower bounds of $1\,\text{REC}$ in metres. [2]

The Great Pyramid of Giza has a square base with side lengths of $440\,\text{REC}$ and a height of $280\,\text{REC}$. Viktor assumes that these two measurements are exact and that the Great Pyramid can be modelled as a square-based pyramid with smooth faces.

(b) Find the minimum possible volume of the pyramid in cubic metres. [4]

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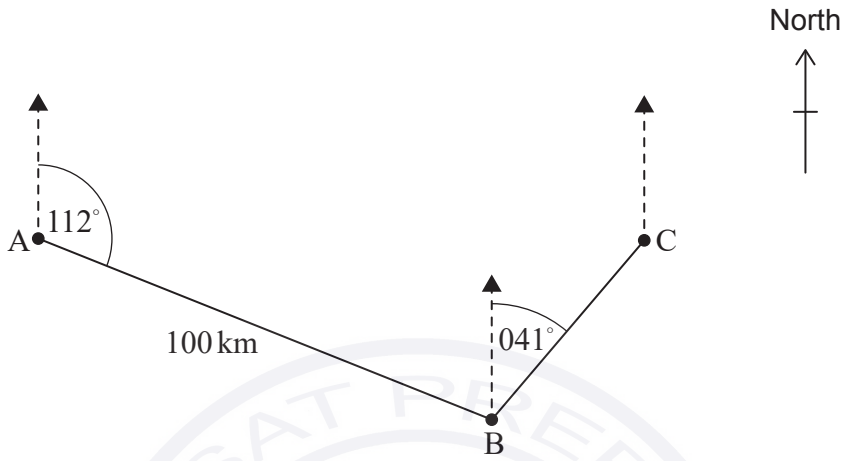
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4. [Maximum mark: 6]

Jason sails his boat from point A for a distance of $100\,\text{km}$, on a bearing of 112°, to arrive at point B. He then sails on a bearing of 041° to point C. Jason's journey is shown in the diagram.

diagram not to scale



(a) Find $A\hat{B}C$. [2]

Point C is directly east of point A.

(b) Calculate the distance that Jason sails to return directly from point C to point A. [4]

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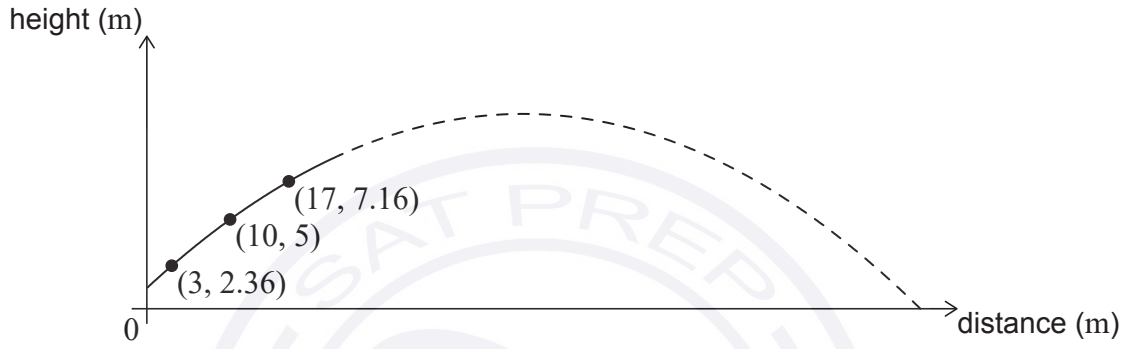
5. [Maximum mark: 9]

A sports player on a horizontal athletic field hits a ball. The height of the ball above the field, in metres, after it is hit can be modelled using a quadratic function of the form $f(x) = ax^2 + bx + c$, where x represents the horizontal distance, in metres, that the ball has travelled from the player.

A specialized camera tracks the initial path of the ball after it is hit by the player. The camera records that the ball travels through the three points $(3, 2.36)$, $(10, 5)$ and $(17, 7.16)$, as shown in **Diagram 1**.

diagram not to scale

Diagram 1



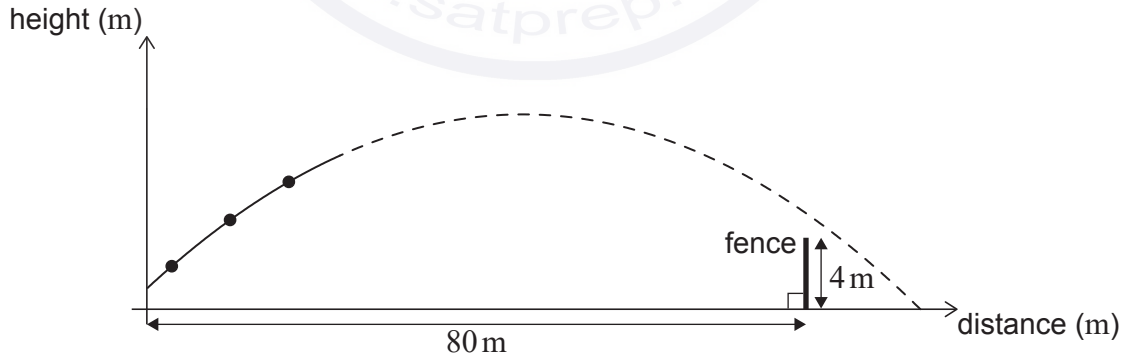
(a) Use the coordinates $(3, 2.36)$ to write down an equation in terms of a, b, and c. [1]

(b) Use your answer to part (a) and two similar equations to find the equation of the quadratic model for the height of the ball. [3]

A 4-metre-high fence is 80 metres from where the player hit the ball, as shown in **Diagram 2**.

diagram not to scale

Diagram 2



(c) Show that the model predicts that the ball will go over the fence. [3]

(d) Find the horizontal distance that the ball will travel, from the player until it first hits the ground, according to this model. [2]

(This question continues on the following page)

(Question 5 continued)

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6. [Maximum mark: 8]

On the following Voronoi diagram, the coordinates of three farmhouses are $A(0, 3)$, $B(8, 3)$ and $C(8, 13)$, where distances are measured in kilometres. Each farmhouse owns the land that is closest to it, and their boundaries are defined by the dotted lines on **Diagram 1**.

Diagram 1

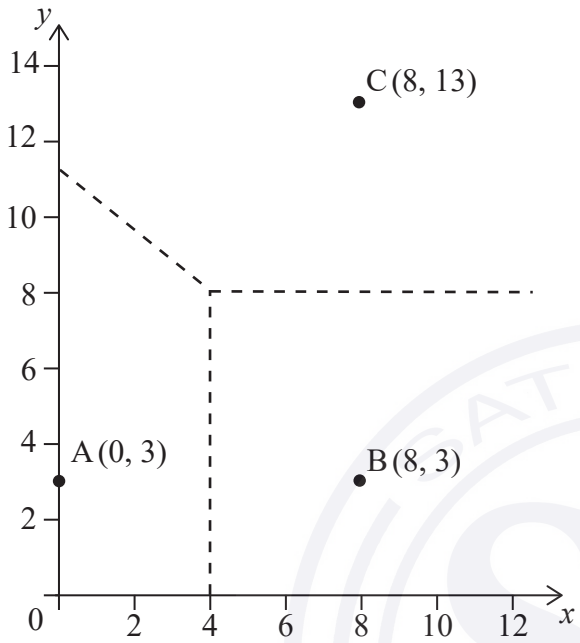
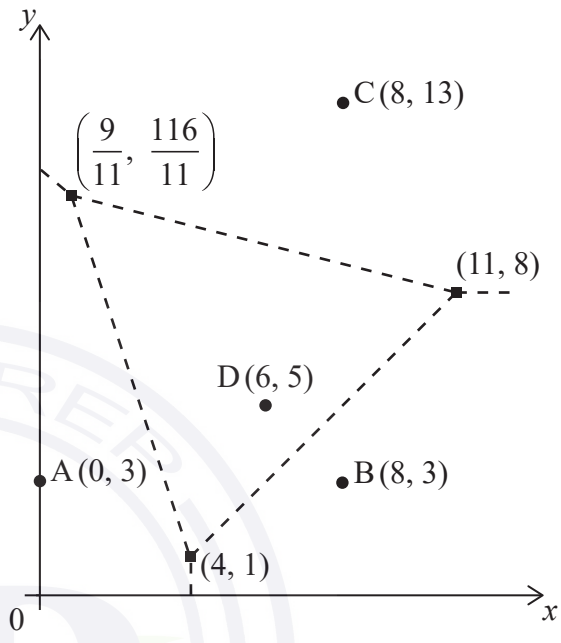


Diagram 2



To provide water to the farms it is decided to construct a well at the point where the boundaries meet on **Diagram 1**.

(a) Write down the coordinates of this point. [1]

(b) Find the equation of the perpendicular bisector of $[AC]$. [3]

An additional farmhouse $D(6, 5)$ is built on the land. The Voronoi diagram has been redrawn to show the new boundaries. The coordinates of the vertices of these boundaries are indicated on **Diagram 2**.

A wind turbine is to be built at one of the vertices.

(c) The wind turbine should be as far from the nearest farmhouses as possible.

(i) By calculating appropriate distances, find the location of the wind turbine.

(ii) Hence, write down the distance of the wind turbine to the nearest farmhouse. [4]

(This question continues on the following page)

(Question 6 continued)

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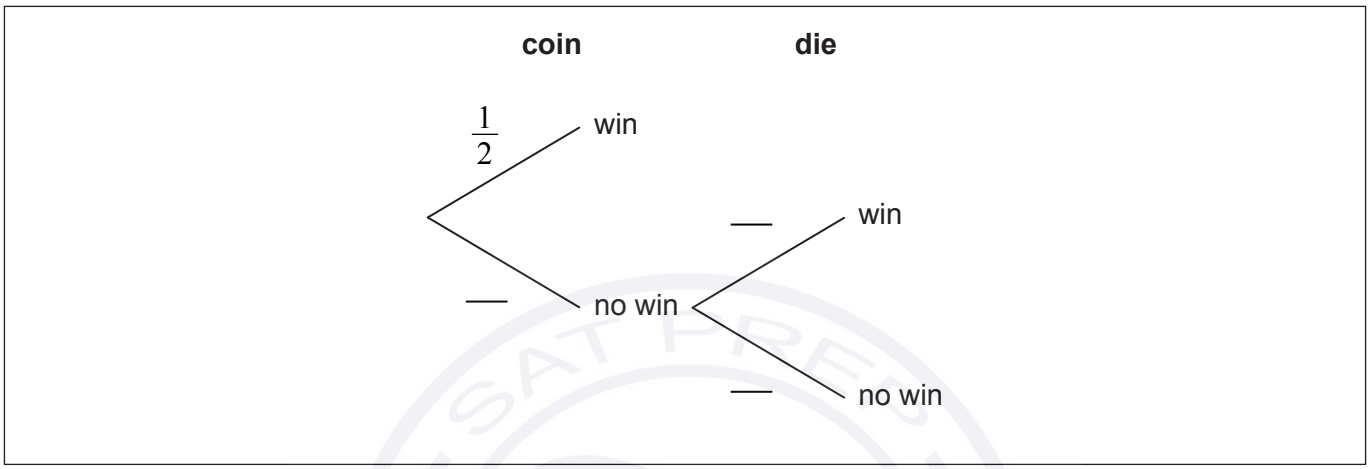
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7. [Maximum mark: 7]

Michèle is playing a game. In the game, she must first flip a fair coin which will result in the coin landing on heads or tails. If the coin lands on heads, then she wins a prize. If it lands on tails, then she has another chance but this time she must roll a fair six-sided dice and get a five or six in order to win a prize.

(a) Complete the tree diagram by writing in the three missing probabilities. [2]



(b) Find the probability that Michèle does **not** win a prize. [2]

(c) Given that Michèle won a prize, find the probability that the coin landed on heads. [3]

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8. [Maximum mark: 6]

Given $z = \sqrt{3} - \mathrm{i}$.

(a) Write z in the form $z = r\mathrm{e}^{\theta \mathrm{i}}$, where $r \in \mathbb{R}^+$, $-\pi < \theta \leq \pi$. [2]

Let $z_1 = \mathrm{e}^{2t\mathrm{i}}$ and $z_2 = 2\mathrm{e}^{\left(2t - \frac{\pi}{6}\right)\mathrm{i}}$.

(b) Find $\mathrm{Im}(z_1 + z_2)$ in the form $p\sin(2t + q)$, where $p > 0$, $t \in \mathbb{R}$ and $-\pi \leq q \leq \pi$. [4]

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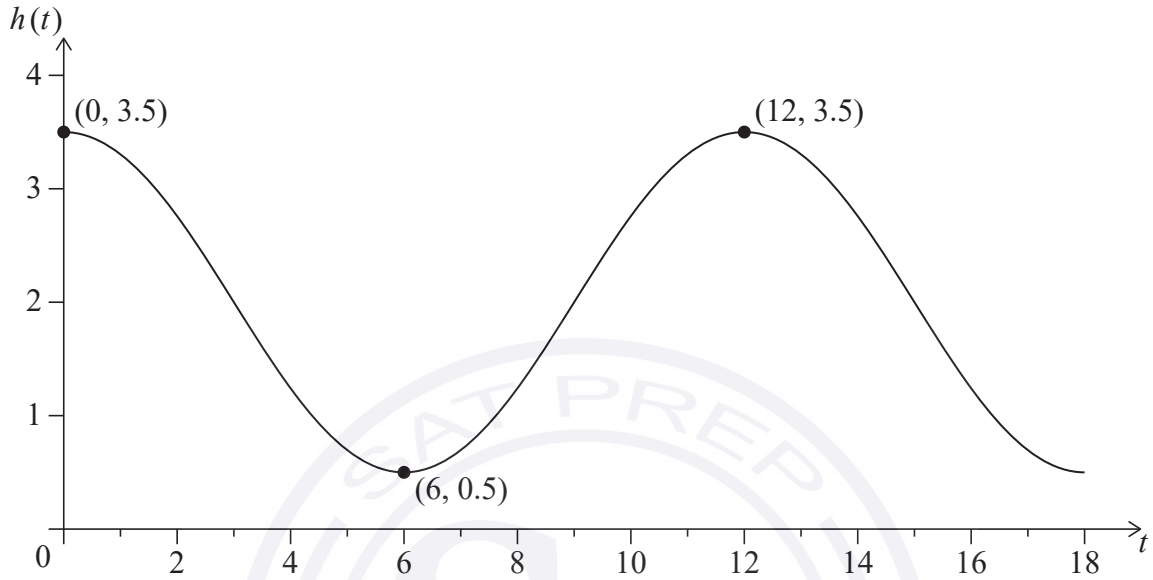
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9. [Maximum mark: 8]

Joon is a keen surfer and wants to model waves passing a particular point P, which is off the shore of his favourite beach. Joon sets up a model of the waves in terms of $h(t)$, the height of the water in metres, and t, the time in seconds from when he begins recording the height of the water at point P.



The function has the form $h(t) = p\cos\left(\frac{\pi}{6}t\right) + q$, $t \geq 0$.

(a) Find the values of p and q. [2]

(b) Find

(i) $h'(t)$.

(ii) $h''(t)$. [3]

Joon will begin to surf the wave when the rate of change of h with respect to t, at P, is at its maximum. This will first occur when $t = k$.

(c) (i) Find the value of k.

(ii) Find the height of the water at this time. [3]

(This question continues on the following page)

(Question 9 continued)

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10. [Maximum mark: 7]

The decay of a chemical isotope over five years is recorded in **Table 1**. The mass of the chemical M is measured to the nearest gram at the beginning of each year t of the experiment.

Table 1

Time t (years)	1	2	3	4	5
Mass M (grams)	1000	660	517	435	381

It is believed that the decay of the isotope can be modelled by an equation of the form $M = a \times t^b$.

(a) Use power regression on your graphic display calculator to find the value of a and the value of b. [2]

The values of t and M can be transformed such that $x = \ln t$ and $y = \ln M$. **Table 2** shows data for x and y to three decimal places.

Table 2

x	0	0.693	1.099	1.386	1.609
y	6.908	6.492	6.248	6.075	5.943

(b) Find the linear regression equation of y on x, in the form $y = cx + d$. Give the values of c and d to three decimal places. [2]

(c) Hence, show that this linear regression is equivalent to the power regression found in part (a). [3]

(This question continues on the following page)

(Question 10 continued)

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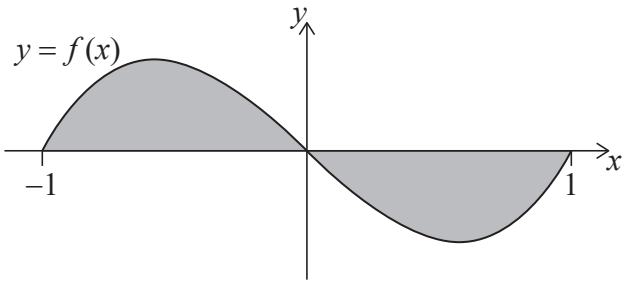
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11. [Maximum mark: 7]

Consider the function $f(x) = x^3 - x$, for $-1 \le x \le 1$. The shaded region, R, is bounded by the graph of $y = f(x)$ and the x-axis.

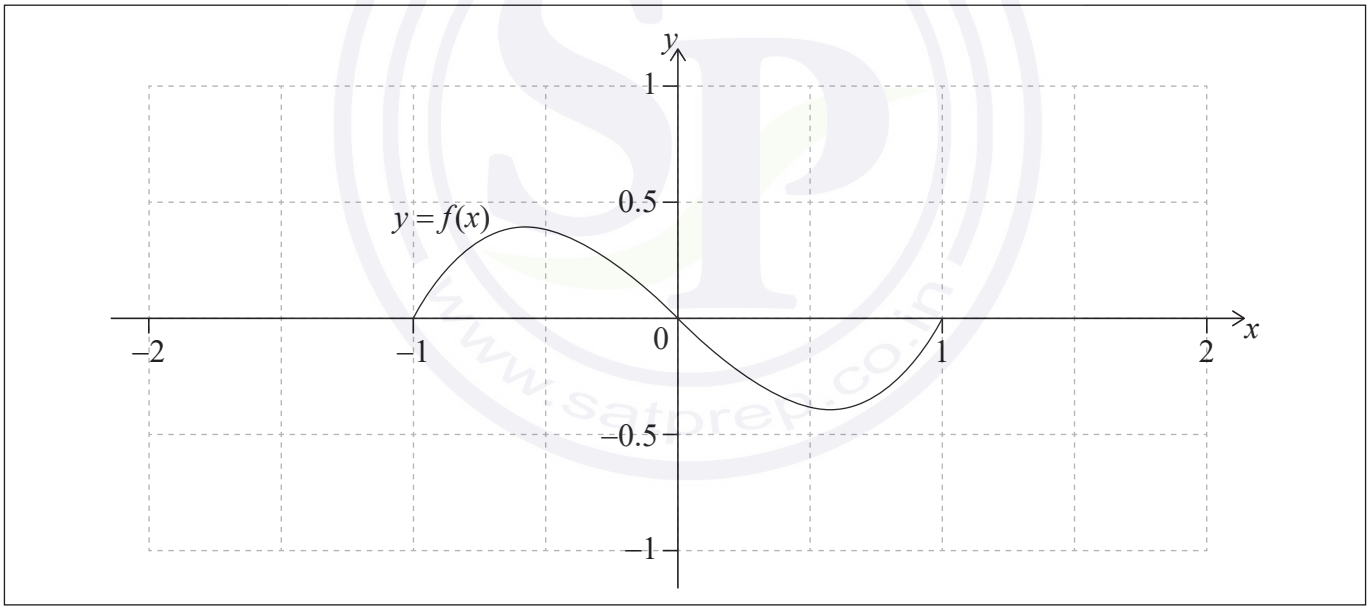


(a) (i) Write down an integral that represents the area of R.

(ii) Find the area of R. [2]

Another function, g, is defined such that $g(x) = 2f(x-1)$.

(b) On the following set of axes, the graph of $y = f(x)$ has been drawn. On the same set of axes, sketch the graph of $y = g(x)$. [2]



The region R from the original graph $y = f(x)$ is rotated through 2π radians about the x-axis to form a solid.

(c) Find the volume of the solid. [3]

(This question continues on the following page)

(Question 11 continued)

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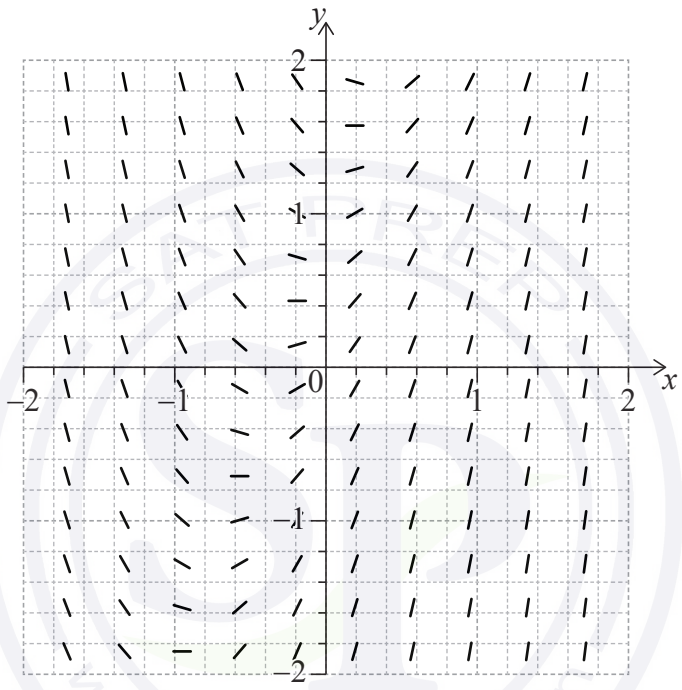
12. [Maximum mark: 4]

Consider the differential equation $\frac{dy}{dx} = 3x - y + 1$.

(a) Find the equation of the tangent to the solution curve at the point $(-1, -1)$ in the form $ax + by + c = 0$. [2]

The slope field for this differential equation is shown in the following diagram.

(b) Sketch the solution curve that passes through the point $(-1, -1)$. [2]



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13. [Maximum mark: 8]

The velocity v of a particle at time t, as it moves along a straight line, can be modelled by the piecewise function

$$v(t) = \begin{cases} u_1(t), & 0 \le t \le T \\ u_2(t), & t \ge T \end{cases}$$

where $u_1(t) = 2t^2 - t^3$ and $u_2(t) = 8 - 4t$. It is required that $u_1(T) = u_2(T)$.

(a) Find the value of T. [2]

(b) Show that $u_1'(T) = u_2'(T)$. [2]

The displacement of the particle at time $t = 0$ is zero.

(c) Find the time when the particle returns to its initial position. [4]

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14. [Maximum mark: 7]

A straight line L has vector equation $\boldsymbol{r} = \begin{pmatrix} 1 \\ 3 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$ and point Q has coordinates $(11, -1, 3)$.

Point P is the point on L closest to Q.

(a) Find the coordinates of P. [4]

(b) Find a vector that is perpendicular to both L and the line passing through points P and Q. [3]

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15. [Maximum mark: 7]

The eating habits of students in a school are studied over a number of months. The focus of the study is whether non-vegetarians become vegetarians, and whether vegetarians remain vegetarians.

Each month, students choose between the vegetarian or non-vegetarian lunch options. Once they have chosen for the month, they cannot change the option until the next month.

In any month during the study, it is noticed that the probability of a non-vegetarian becoming vegetarian the following month is 0.1, and that the probability of a vegetarian remaining a vegetarian the following month is 0.8.

This situation can be represented by the transition matrix

$$\boldsymbol{T} = \begin{pmatrix} 0.8 & 0.1 \\ 0.2 & 0.9 \end{pmatrix}.$$

(a) Interpret the value 0.9 in $\boldsymbol{T}$ in terms of the changes in the eating habits of the students in the school. [1]

(b) Find the eigenvalues of matrix $\boldsymbol{T}$. [3]

One of the eigenvectors of $\boldsymbol{T}$ is $\begin{pmatrix} -1 \\ 1 \end{pmatrix}$.

(c) Find another, non-parallel, eigenvector and interpret it in context. [3]

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16. [Maximum mark: 6]

When Jef plays basketball, the number of shots he takes during any 6 minutes of play can be modelled by a Poisson distribution with mean 2.5.

(a) Find the probability that Jef takes less than 7 shots during any 12 minutes of play. [2]

It can be assumed that the outcomes of the shots are independent of each other, and the probability of success of a shot is constant. The probability that Jef is successful with a shot is 0.4.

It can be assumed that the probability of Jef's success with a shot is independent of the number of shots that he takes.

(b) Find the probability that during any 6 minutes of play Jef takes fewer than 4 shots and is successful at least once. [4]

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Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: applications and interpretation
Higher level
Paper 1

8 May 2023

Zone A afternoon | **Zone B** morning | **Zone C** afternoon

Candidate session number

2 hours

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all questions.
- Answers must be written within the answer boxes provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.

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Answers must be written within the answer boxes provided. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 7]

A player throws a basketball. The height of the basketball is modelled by

$$h(t) = -4.75t^2 + 8.75t + 1.5\,,\ t \geq 0\,,$$

where h is the height of the basketball above the ground, in metres, and t is the time, in seconds, after it was thrown.

(a) Find how long it takes for the basketball to reach its maximum height. [2]

(b) Assuming that no player catches the basketball, find how long it would take for the basketball to hit the ground. [2]

Another player catches the basketball when it is at a height of 1.2 metres.

(c) Find the value of t when this player catches the basketball. [2]

(d) Write down one limitation of using $h(t)$ to model the height of the basketball. [1]

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2. [Maximum mark: 4]

A company that owns many restaurants wants to determine if there are differences in the quality of the food cooked for three different meals: breakfast, lunch and dinner.

Their quality assurance team randomly selects 500 items of food to inspect. The quality of this food is classified as perfect, satisfactory, or poor. The data is summarized in the following table.

		Quality			
		Perfect	**Satisfactory**	**Poor**	**Total**
Meal	**Breakfast**	101	124	7	**232**
	Lunch	68	81	5	**154**
	Dinner	35	69	10	**114**
	Total	**204**	**274**	**22**	**500**

A χ^2 test at the $5\,\%$ significance level is carried out to determine if there is significant evidence of a difference in the quality of the food cooked for the three meals.

The critical value for this test is 9.488.

The hypotheses for this test are:
H_0: The quality of the food and the type of meal are independent.
H_1: The quality of the food and the type of meal are not independent.

(a) Find the χ^2 statistic. [2]

(b) State, with justification, the conclusion for this test. [2]

(This question continues on the following page)

(Question 2 continued)

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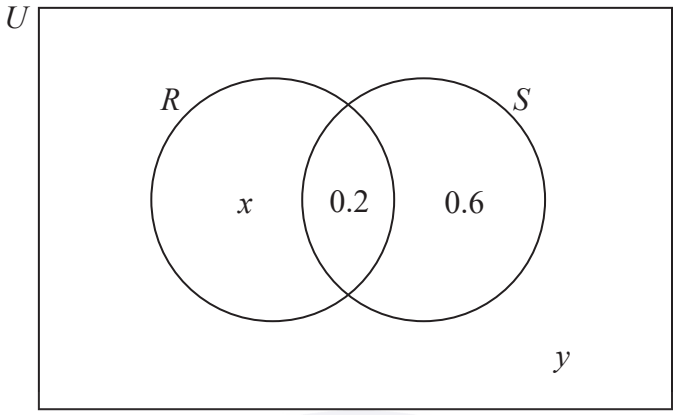
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3. [Maximum mark: 7]

The following Venn diagram shows two independent events, R and S. The values in the diagram represent probabilities.



(a) Find the value of x. [3]

(b) Find the value of y. [2]

(c) Find $\mathrm{P}(R' \mid S')$. [2]

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4. [Maximum mark: 6]

Angel has $\$520$ in his savings account. Angel considers investing the money for 5 years with a bank. The bank offers an annual interest rate of 1.2% compounded quarterly.

(a) Calculate the amount of money Angel would have at the end of 5 years with the bank. Give your answer correct to two decimal places. [3]

Instead of investing the money, Angel decides to buy a phone that costs $\$520$. At the end of 5 years, the phone will have a value of $\$30$. It may be assumed that the depreciation rate per year is constant.

(b) Calculate the annual depreciation rate of the phone. [3]

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5. [Maximum mark: 6]

The lengths of the seeds from a particular mango tree are approximated by a normal distribution with a mean of $4\,\text{cm}$ and a standard deviation of $0.25\,\text{cm}$.

A seed from this mango tree is chosen at random.

(a) Calculate the probability that the length of the seed is less than $3.7\,\text{cm}$. [2]

It is known that $30\,\%$ of the seeds have a length greater than $k\,\text{cm}$.

(b) Find the value of k. [2]

For a seed of length $d\,\text{cm}$, chosen at random, $\text{P}(4 - m < d < 4 + m) = 0.6$.

(c) Find the value of m. [2]

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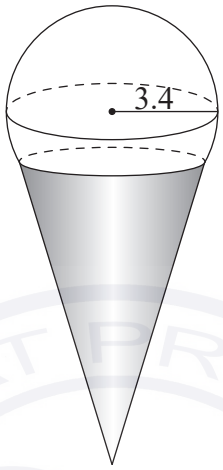
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6. [Maximum mark: 5]

Ruhi buys a scoop of ice cream in the shape of a sphere with a radius of $3.4\,\text{cm}$. The ice cream is served in a cone, and it may be assumed that $\frac{1}{5}$ of the volume of the ice cream is inside the cone. This is shown in the following diagram.

diagram not to scale



(a) Calculate the volume of ice cream that is not inside the cone. [3]

The cone has a slant height of $11\,\text{cm}$ and a radius of $3\,\text{cm}$.

The outside of the cone is covered with chocolate.

(b) Calculate the surface area of the cone that is covered with chocolate. Give your answer correct to the nearest cm^2. [2]

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7. [Maximum mark: 6]

Akar starts a new job in Australia and needs to travel daily from Wollongong to Sydney and back. He travels to work for 28 consecutive days and therefore makes 56 single journeys. Akar makes all journeys by bus.

The probability that he is successful in getting a seat on the bus for any single journey is 0.86.

(a) Determine the expected number of these 56 journeys for which Akar gets a seat on the bus. [1]

(b) Find the probability that Akar gets a seat on at least 50 journeys during these 28 days. [3]

The probability that Akar gets a seat on at most n journeys is at least 0.25.

(c) Find the smallest possible value of n. [2]

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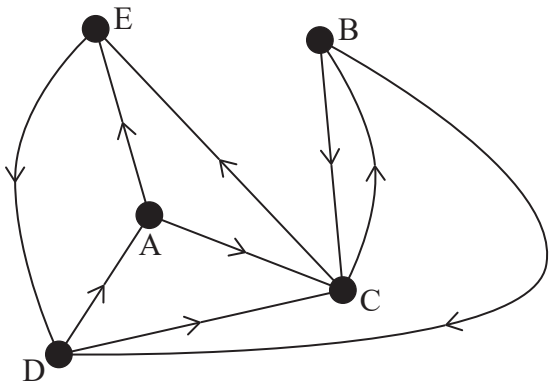
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8. [Maximum mark: 7]

The following directed, unweighted, graph shows a simplified road network on an island, connecting five small villages marked A to E.



(a) Construct the adjacency matrix $\boldsymbol{M}$ for this network. [3]

Beatriz the bus driver starts at village E and drives to seven villages, such that the seventh village is A.

(b) (i) Determine how many possible routes Beatriz could have taken, to travel from E to A.

(ii) Describe one possible route taken by Beatriz, by listing the villages visited in order. [4]

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9. [Maximum mark: 9]

At a running club, Sung-Jin conducts a test to determine if there is any association between an athlete's age and their best time taken to run $100\,\text{m}$. Eight athletes are chosen at random, and their details are shown below.

Athlete	A	B	C	D	E	F	G	H
Age (years)	13	17	22	18	19	25	11	36
Time (seconds)	13.4	14.6	13.4	12.9	12.0	11.8	17.0	13.1

Sung-Jin decides to calculate the Spearman's rank correlation coefficient for his set of data.

(a) Complete the table of ranks. [2]

Athlete	A	B	C	D	E	F	G	H
Age rank			3					
Time rank							1	

(b) Calculate the Spearman's rank correlation coefficient, r_s. [2]

(c) Interpret this value of r_s in the context of the question. [1]

(d) Suggest a mathematical reason why Sung-Jin may have decided not to use Pearson's product-moment correlation coefficient with his data from the original table. [1]

(e) (i) Find the coefficient of determination for the data from the original table.

(ii) Interpret this value in the context of the question. [3]

(This question continues on the following page)

(Question 9 continued)

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10. [Maximum mark: 6]

A chocolate company plans to produce chocolate bars with special flavours. They survey 246 people to determine if there is any particular preference for one of the flavours.

The table below shows the information collected.

Hot chilli	Almond crunch	Spiced Chai	Ginger'n'lime
75	59	46	66

A χ^2 goodness of fit test at the 5% significance level is carried out on the data.

The critical value for the test is 7.82.

(a) State the null and alternative hypotheses for this test. [2]

(b) Perform the test and give your conclusion in context. [4]

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11. [Maximum mark: 6]

Two AC (alternating current) electrical sources of equal frequencies are combined.

The voltage of the first source is modelled by the equation $V = 30\sin(t + 60^\circ)$.

The voltage of the second source is modelled by the equation $V = 60\sin(t + 10^\circ)$.

(a) Determine the maximum voltage of the combined sources. [2]

(b) Using your graphic display calculator, find a suitable equation for the combined voltages, giving your answer in the form $V = V_0\sin(at + b)$, where a, b and V_0 are constants, $a > 0$ and $0^\circ \leq b < 180^\circ$. [4]

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12. [Maximum mark: 5]

A spherical balloon is being inflated such that its volume is increasing at a rate of $15\,\text{cm}^3\,\text{s}^{-1}$.

(a) Find the radius of the balloon when its volume is $288\pi\,\text{cm}^3$. [2]

(b) Hence or otherwise, find the rate of change of the radius at this instant. [3]

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13. [Maximum mark: 7]

The matrices $\boldsymbol{P} = \begin{pmatrix} 3 & 1 \\ 0 & 2 \end{pmatrix}$ and $\boldsymbol{Q} = \begin{pmatrix} -4 & 1 \\ 1 & 3 \end{pmatrix}$ represent two transformations.

A triangle T is transformed by $\boldsymbol{P}$, and this image is then transformed by $\boldsymbol{Q}$ to form a new triangle, T'.

(a) Find the single matrix that represents the transformation $T' \rightarrow T$, which will undo the transformation described above. [4]

The area of T' is $273\,\text{cm}^2$.

(b) Using your answer to part (a), or otherwise, determine the area of T. [3]

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14. [Maximum mark: 8]

In this question, $\boldsymbol{i}$ denotes a unit vector due east, and $\boldsymbol{j}$ denotes a unit vector due north.

Two ships, A and B, are each moving with constant velocities.

The position vector of ship A, at time t hours, is given as $\boldsymbol{r}_A = (1 + 2t)\boldsymbol{i} + (3 - 3t)\boldsymbol{j}$.

The position vector of ship B, at time t hours, is given as $\boldsymbol{r}_B = (-2 + 4t)\boldsymbol{i} + (-4 + t)\boldsymbol{j}$.

(a) Find the bearing on which ship A is sailing. [3]

(b) Find the value of t when ship B is directly south of ship A. [2]

(c) Find the value of t when ship B is directly south-east of ship A. [3]

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15. [Maximum mark: 6]

A random sample of eight packets of Apollo coffee granules are selected from a supermarket shelf.

The weights of the coffee granules present in each packet are as follows:

$222\,\text{g}$ $226\,\text{g}$ $221\,\text{g}$ $228\,\text{g}$ $227\,\text{g}$ $225\,\text{g}$ $222\,\text{g}$ $223\,\text{g}$

(a) (i) Find an unbiased estimate for the mean weight of coffee granules in a packet of Apollo coffee.

(ii) Calculate a $95\,\%$ confidence interval for the population mean. Give your answer to four significant figures. [3]

(b) State one assumption you have made in order for your interval to be valid. [1]

(c) The label of each packet has a description which includes the phrase: "contains $226\,\text{g}$ of coffee granules".

Using your answer to part (a)(ii), briefly comment on the claim on the label. [2]

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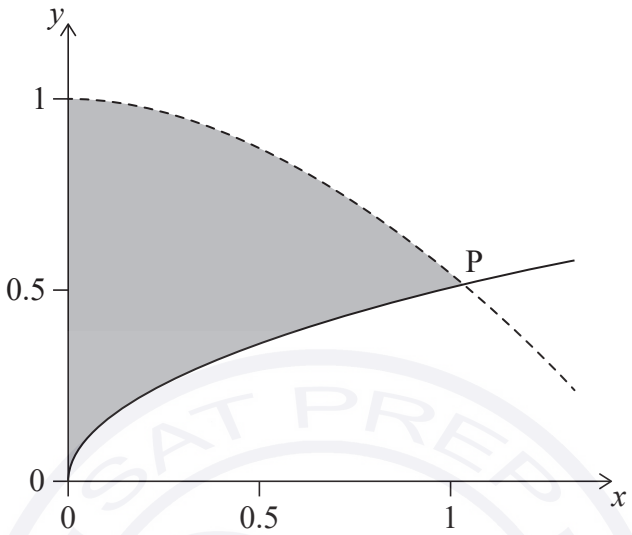
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16. [Maximum mark: 9]

The following diagram shows parts of the curves of $y = \cos x$ and $y = \frac{\sqrt{x}}{2}$.

P is the point of intersection of the two curves.



(a) Use your graphic display calculator to find the coordinates of P. [2]

The shaded region is rotated 360° **about the y-axis** to form a volume of revolution V.

(b) Express V as the sum of two definite integrals. [5]

(c) Hence find the value of V. [2]

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17. [Maximum mark: 6]

Consider the differential equation

$$\left(x^2+1\right)\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{x}{2y-2}\text{, for } x\geq 0,\ y\geq 1,$$

where $y=1$ when $x=0$.

(a) Explain why Euler's method cannot be used to find an approximate value for y when $x=0.1$. [1]

(b) By solving the differential equation, show that $y=1+\sqrt{\frac{\ln\left(x^2+1\right)}{2}}$. [4]

(c) Hence deduce the value of y when $x=0.1$. [1]

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Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: applications and interpretation
Higher level
Paper 1

8 May 2023

Zone A afternoon | **Zone B** morning | **Zone C** afternoon

Candidate session number

2 hours

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all questions.
- Answers must be written within the answer boxes provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.

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2223 – 7206

23 pages

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Answers must be written within the answer boxes provided. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 5]

On 1 January 2022, Mina deposited $\$1000$ into a bank account with an annual interest rate of 4%, compounded monthly. At the end of January, and the end of every month after that, she deposits $\$100$ into the same account.

(a) Calculate the amount of money in her account at the start of 2024. Give your answer to two decimal places. [3]

(b) Find how many complete months, counted from 1 January 2022, it will take for Mina to have more than $\$5000$ in her account. [2]

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2. [Maximum mark: 6]

Carys believes that, on a memory retention test, the mean score of bilingual people (μ_b) will be higher than the mean score of monolingual people (μ_m). Carys gave a memory retention test to a random sample of students in her class. The results are shown in the two tables.

	Scores									
Bilingual	100	94	100	90	100	94	98	98	98	98

	Scores							
Monolingual	97	92	88	98	88	94	100	100

Carys performs a one-tailed t-test at a 5% level of significance. It is assumed that the scores are normally distributed and the samples have equal variances.

(a) State the null and alternative hypotheses. [2]

(b) Calculate the p-value for this test. [2]

(c) State the conclusion of the test in the context of the question. Justify your answer. [2]

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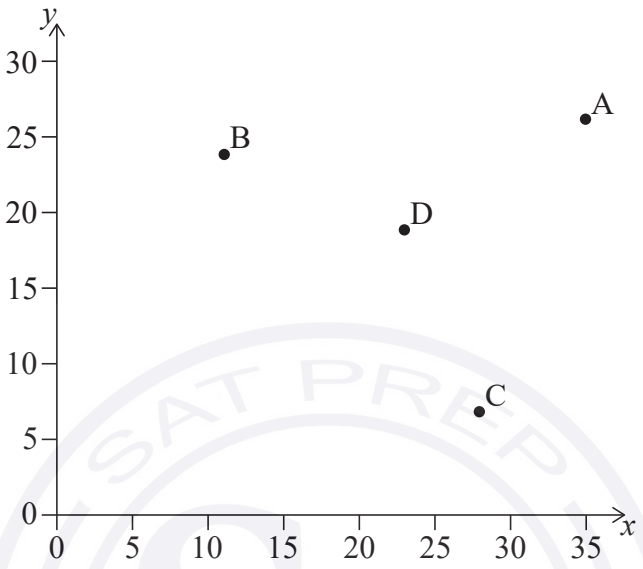
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3. [Maximum mark: 5]

Three towns have positions $A(35, 26)$, $B(11, 24)$, and $C(28, 7)$ according to the coordinate system shown where distances are measured in miles.

Dominique's farm is located at the position $D(24, 19)$.



(a) Find AD. [2]

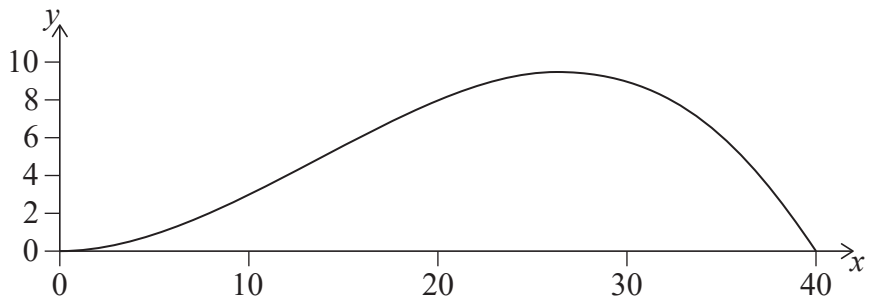
On a particular day, the mean temperatures recorded in each of towns A, B and C are $34\,°C$, $29\,°C$ and $30\,°C$ respectively.

(b) Use nearest neighbour interpolation to estimate the temperature at Dominique's farm on that particular day. [3]

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4. [Maximum mark: 8]

The cross section of a scale model of a hill is modelled by the following graph.



The heights of the model are measured at horizontal intervals and are given in the table.

Horizontal distance, x cm	0	10	20	30	40
Vertical distance, y cm	0	3	8	9	0

(a) Use the trapezoidal rule with $h = 10$ to find an approximation for the cross-sectional area of the model. [2]

It is given that the equation of the curve is $y = 0.04x^2 - 0.001x^3,\ 0 \le x \le 40$.

(b) (i) Write down an integral to find the exact cross-sectional area.

(ii) Calculate the value of the cross-sectional area to two decimal places. [4]

(c) Find the percentage error in the area found using the trapezoidal rule. [2]

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5. [Maximum mark: 5]

A boat travels $8\,\text{km}$ on a bearing of $315°$ and then a further $6\,\text{km}$ on a bearing of $045°$.
Find the bearing on which the boat should travel to return directly to the starting point.

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6. [Maximum mark: 7]

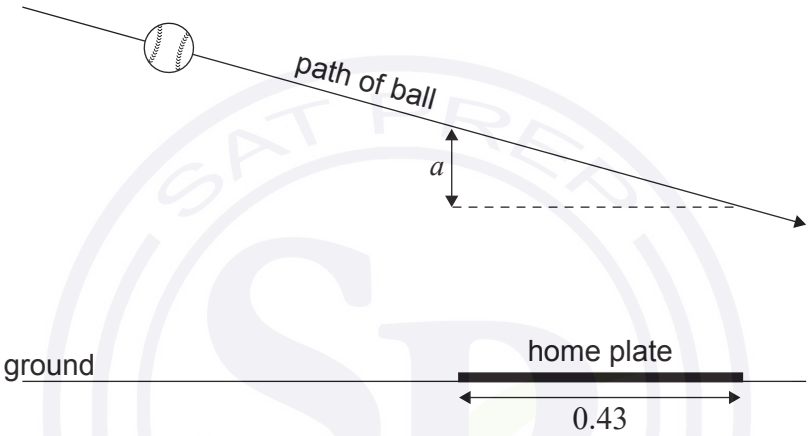
In a baseball game, Sakura is the batter standing beside home plate. The ball is thrown towards home plate along a path that can be modelled by the following function.

$$y = -0.045x + 2$$

In this model, x is the horizontal distance of the ball from the point the ball is thrown and y is the vertical height of the ball above the ground. Both measured in metres.

The outcome of the throw is called a strike if the height of the ball is between $0.53\,\text{m}$ and $1.24\,\text{m}$ at some point while it travels over home plate. The length of home plate is $0.43\,\text{m}$.

diagram not to scale



When the ball reaches the front of home plate, the height of the ball above the ground is $1.25\,\text{m}$. The height of the ball changes by a metres as the ball travels over the length of home plate.

(a) (i) Find the value of a.

(ii) Justify why this throw is a strike. [4]

On the next throw, Sakura hits the ball towards a wall that is 5 metres high. The horizontal distance of the wall from the point where the ball was hit is 96 metres. The path of the ball after it is hit can be modelled by the function $h(d)$.

$$h(d) = -0.01d^2 + 1.04d + 0.66, \text{ for } h,\ d > 0$$

In this model, h is the height of the ball above the ground and d is the horizontal distance of the ball from the point where it was hit. Both h and d are measured in metres.

(b) Determine whether the ball will go over the wall. Justify your answer. [3]

(This question continues on the following page)

(Question 6 continued)

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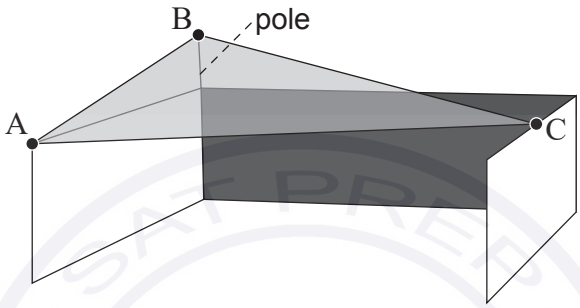
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7. [Maximum mark: 9]

A triangular cover is positioned over a walled garden to provide shade. It is anchored at points A and C, located at the top of a $2\,\text{m}$ wall, and at a point B, located at the top of a $1\,\text{m}$ vertical pole fixed to a top corner of the wall.

The three edges of the cover can be represented by the vectors $\overrightarrow{AB} = \begin{pmatrix} 0 \\ 6 \\ 1 \end{pmatrix}$, $\overrightarrow{AC} = \begin{pmatrix} 7 \\ 3 \\ 0 \end{pmatrix}$ and $\overrightarrow{BC} = \begin{pmatrix} 7 \\ -3 \\ -1 \end{pmatrix}$, where distances are measured in metres.



(a) Calculate the vector product $\overrightarrow{AB} \times \overrightarrow{AC}$. [2]

(b) Hence find the area of the triangular cover. [2]

The point X on $[AC]$ is such that $[BX]$ is perpendicular to $[AC]$.

(c) Use your answer to part (b) to find the distance BX. [3]

(d) Find the angle the cover makes with the horizontal plane. [2]

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8. [Maximum mark: 7]

The random variables (X, Y) follow a bivariate normal distribution with product-moment correlation coefficient ρ. The values of six random observations of (X, Y) are shown in the table.

x	6.3	4.1	5.6	9.2	7.8	8.2
y	9.2	4.9	8.9	10.3	8.9	9.8

(a) State null and alternative hypotheses which could be used to test whether there is a linear correlation between X and Y. [2]

(b) Determine the value of

(i) the product-moment correlation coefficient, r, of the sample.

(ii) the corresponding p-value. [3]

(c) State whether your result from part (b)(ii) indicates there is sufficient evidence to claim that, at the 5% significance level, X and Y are not linearly correlated.

Give a reason for your answer. [2]

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9. [Maximum mark: 5]

On a specific day, the speed of cars as they pass a speed camera can be modelled by a normal distribution with a mean of $67.3\,\text{km h}^{-1}$.

A speed of $75.7\,\text{km h}^{-1}$ is two standard deviations from the mean.

(a) Find the standard deviation for the speed of the cars. [2]

It is found that 82% of cars on this road travel at speeds between $p\,\text{km h}^{-1}$ and $q\,\text{km h}^{-1}$, where $p < q$. This interval includes cars travelling at a speed of $74\,\text{km h}^{-1}$.

(b) Show that the region of the normal distribution between p and q is **not** symmetrical about the mean. [3]

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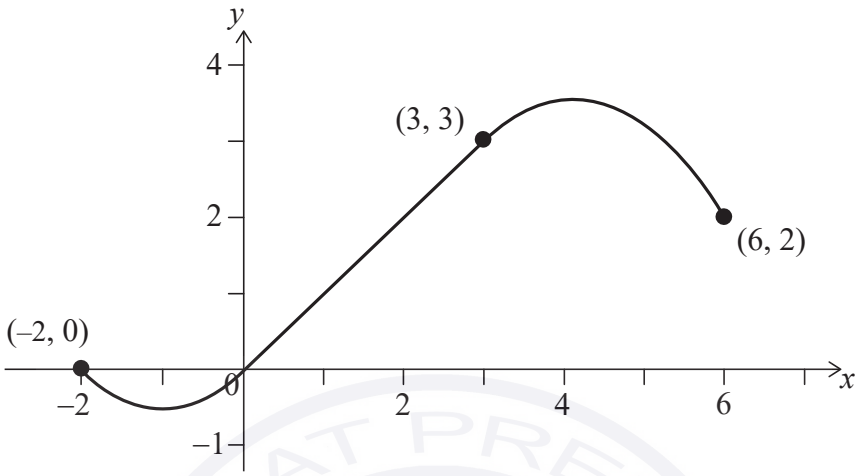
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10. [Maximum mark: 9]

A decorative hook can be modelled by the curve with equation $y = f(x)$. The graph of $y = f(x)$ is shown and consists of a line segment from $(0, 0)$ to $(3, 3)$ and two sections formed by quadratic curves.



(a) Write down the equation of the line segment for $0 \leq x \leq 3$. [1]

The quadratic curve, with endpoints $(-2, 0)$ and $(0, 0)$, has the same gradient at $(0, 0)$ as the line segment.

(b) Find the equation of the curve between $(-2, 0)$ and $(0, 0)$. [3]

The second quadratic curve, with endpoints $(3, 3)$ and $(6, 2)$, has the same gradient at $(3, 3)$ as the line segment.

(c) Find the equation of this curve. [4]

(d) Write down f as a piecewise function. [1]

(This question continues on the following page)

(Question 10 continued)

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11. [Maximum mark: 6]

A shop sells oranges and lemons. The weights of the oranges are assumed to be normally distributed with mean 205 grams and standard deviation 5 grams. The weights of the lemons are assumed to be normally distributed with mean 105 grams and standard deviation 3 grams.

Nelia selects 1 orange and 2 lemons at random and independent of each other. Calculate the probability that the weight of her orange exceeds the combined weight of her lemons.

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12. [Maximum mark: 5]

Two AC (alternating current) electrical sources with the same frequencies are combined. The voltages from these sources can be expressed as $V_1 = 6\sin(at + 30°)$ and $V_2 = 6\sin(at + 90°)$.

The combined total voltage can be expressed in the form $V_1 + V_2 = V\sin(at + \theta°)$.

Determine the value of V and the value of θ.

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13. [Maximum mark: 6]

The displacement, $x\,(\text{cm})$, of the end of a spring, at time t (seconds), is given by

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + 2\frac{\mathrm{d}x}{\mathrm{d}t} + 10x = 0\,.$$

At $t = 0$, $x = 0.75$ and $\frac{\mathrm{d}x}{\mathrm{d}t} = 0$.

Use Euler's method, with a step length 0.1 seconds, to estimate the value of x when $t = 0.5$.

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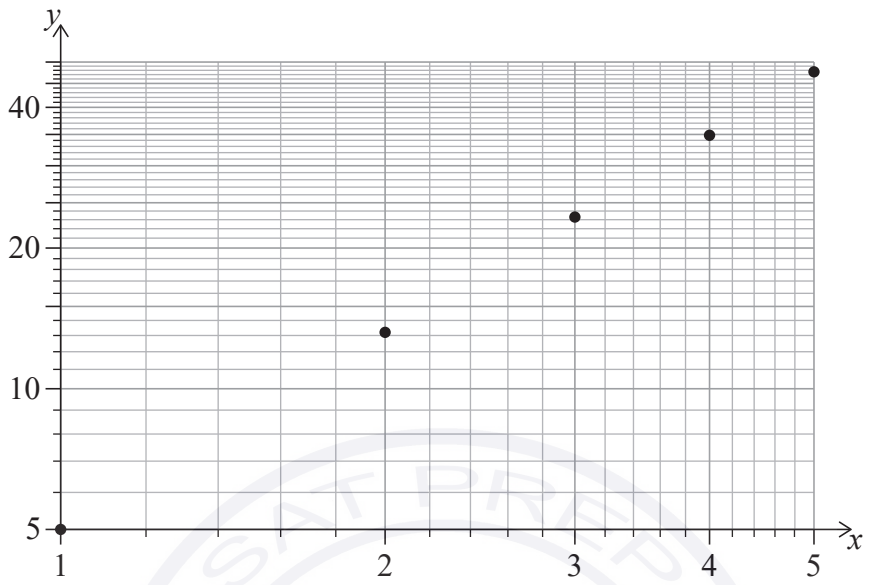
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14. [Maximum mark: 6]

Petra examines two quantities, x and y, and plots data points on a log-log graph.



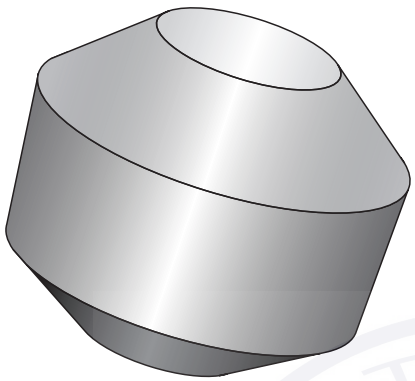
She observes that on this graph the data points follow a perfect straight line. Given that the line passes through the points $(2, 13.1951)$ and $(4, 34.822)$, find the equation of the relationship connecting x and y. Your final answer should not include logarithms.

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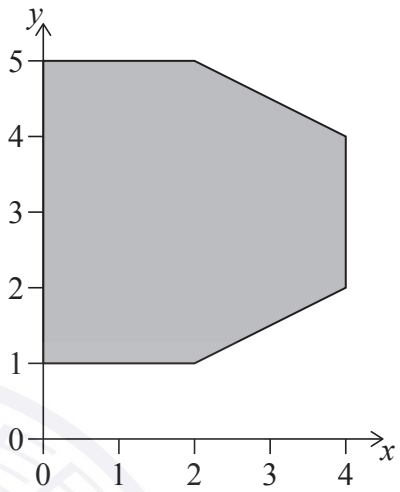
15. [Maximum mark: 6]

The solid shown is formed by rotating the hexagon with vertices $(2, 1)$, $(0, 1)$, $(0, 5)$, $(2, 5)$, $(4, 4)$ and $(4, 2)$ about the y-axis.

Solid



Hexagon



Find the volume of this solid.

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16. [Maximum mark: 6]

The relationship between the intensity, I, of a light source and the distance, d, from the light source can be modelled by $I = \frac{k}{d^2}$.

Pablo measures the intensity of a light source at different distances. The data collected is shown in the table.

d (m)	1	2	5
I (lm)	42	11	1.5

Pablo finds the sum of square residuals in the form $1.0641k^2 - 89.62k + c$.

(a) Find the exact value of c. [4]

(b) Hence find the least squares regression curve of the form $I = \frac{k}{d^2}$. [2]

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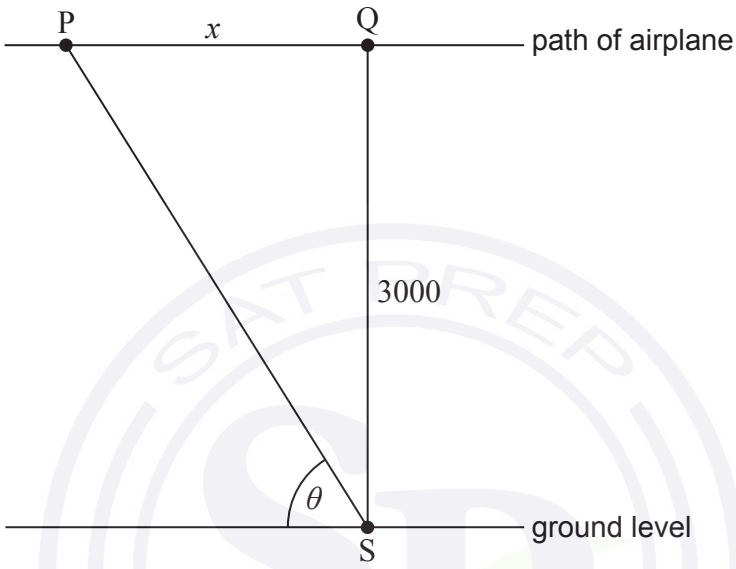
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17. [Maximum mark: 9]

An airplane, P, is flying at a constant altitude of $3000\,\mathrm{m}$ at a speed of $250\,\mathrm{m\,s^{-1}}$. Its path passes over a tracking station, S, at ground level. Let Q be the point $3000\,\mathrm{m}$ directly above the tracking station.

At a particular time, T, as the airplane is flying towards Q, the angle of elevation, θ, of the airplane from S is increasing at a rate of 0.075 radians per second. The distance from Q to P is given by x.



(a) Use related rates to show that, at time T, $\dfrac{\mathrm{d}x}{\mathrm{d}\theta} = -\dfrac{10\,000}{3}$. [2]

(b) Find $x(\theta)$, x as a function of θ. [1]

(c) Find an expression for $\dfrac{\mathrm{d}x}{\mathrm{d}\theta}$ in terms of $\sin\theta$. [3]

(d) Hence find the horizontal distance from the station to the plane at time T. [3]

(This question continues on the following page)

(Question 17 continued)

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Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: applications and interpretation
Higher level
Paper 1

Monday 31 October 2022 (afternoon)

Candidate session number

2 hours

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all questions.
- Answers must be written within the answer boxes provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.

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8822 – 7201

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Answers must be written within the answer boxes provided. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 5]

Sergio is interested in whether an adult's favourite breakfast berry depends on their income level. He obtains the following data for 341 adults and decides to carry out a χ^2 test for independence, at the 10% significance level.

		Income level		
		Low	**Medium**	**High**
Favourite berry	**Strawberry**	21	39	30
	Blueberry	39	67	42
	Other berry	32	45	26

(a) Write down the null hypothesis. [1]

(b) Find the value of the χ^2 statistic. [2]

The critical value of this χ^2 test is 7.78.

(c) Write down Sergio's conclusion to the test in context. Justify your answer. [2]

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2. [Maximum mark: 5]

Celeste heated a cup of coffee and then let it cool to room temperature. Celeste found the coffee's temperature, T, measured in $^{\circ}\mathrm{C}$, could be modelled by the following function,

$$T(t) = 71\,\mathrm{e}^{-0.0514t} + 23\,,\ t \geq 0\,,$$

where t is the time, in minutes, after the coffee started to cool.

(a) Find the coffee's temperature 16 minutes after it started to cool. [2]

(b) Write down the room temperature. [1]

(c) Given that $T^{-1}(50) = k$, find the value of k. [2]

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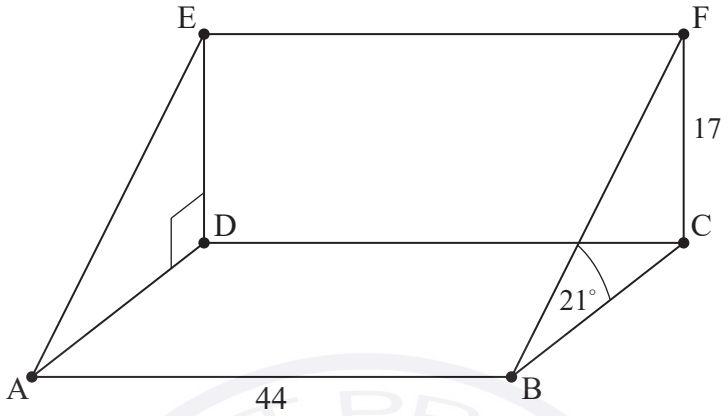
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3. [Maximum mark: 5]

An artificial ski slope can be modelled as a triangular prism, as shown in the diagram. Rectangle ABCD is horizontal, and rectangle CDEF is vertical.

diagram not to scale



The maximum height of the ski slope, CF, is 17 metres and the steepest angle of the ski slope, $F\hat{B}C$, is 21°.

(a) Calculate the length of [BF]. [2]

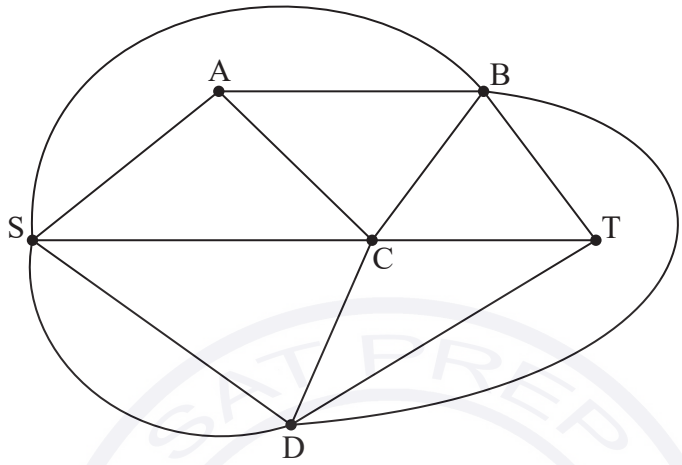
The width of the base of the ski slope, AB, is 44 metres. Mayumi skis in a straight line, starting from point E and finishing at the base of the ski slope.

(b) Find the value of the least steep angle that Mayumi can ski. [3]

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4. [Maximum mark: 7].

In a competition, a contestant has to move through a maze to find treasure. A graph of the maze is shown below, where each edge represents a corridor in the maze. The contestant starts at S and the treasure is located at T.



(a) Complete the adjacency matrix, $\boldsymbol{M}$, for the graph. [2]

$$\begin{array}{c} \\ \mathrm{S} \\ \mathrm{A} \\ \mathrm{B} \\ \mathrm{C} \\ \mathrm{D} \\ \mathrm{T} \end{array} \begin{array}{c} \begin{array}{cccccc} \mathrm{S} & \mathrm{A} & \mathrm{B} & \mathrm{C} & \mathrm{D} & \mathrm{T} \end{array} \\ \begin{pmatrix} 0 & 1 & 1 & 1 & \square & 0 \\ 1 & 0 & 1 & 1 & \square & 0 \\ 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 \\ \square & \square & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{pmatrix} \end{array}$$

The competition rules state that the contestant can walk along a maximum of four corridors.

(b) Find the number of walks from S to T with a maximum of 4 edges. [4]

(c) Explain why the number of ways the contestant can reach the treasure is less than the answer to part (b). [1]

(This question continues on the following page)

(Question 4 continued)

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5. [Maximum mark: 7]

Taizo plays a game where he throws one ball at two bottles that are sitting on a table. The probability of knocking over bottles, in any given game, is shown in the following table.

Number of bottles knocked over	0	1	2
Probability	0.5	0.4	0.1

(a) Taizo plays two games that are independent of each other. Find the probability that Taizo knocks over a **total** of two bottles. [4]

In any given game, Taizo will win k points if he knocks over two bottles, win 4 points if he knocks over one bottle and lose 8 points if no bottles are knocked over.

(b) Find the value of k such that the game is fair. [3]

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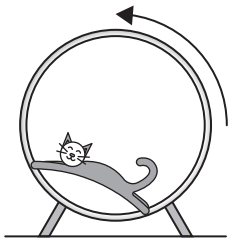
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6. [Maximum mark: 6]

A cat runs inside a circular exercise wheel, making the wheel spin at a constant rate in an anticlockwise direction. The height, h cm, of a fixed point, P, on the wheel can be modelled by $h(t) = a\sin(bt) + c$ where t is the time in seconds and a, b, $c \in \mathbb{R}^+$.



When $t = 0$, point P is at a height of 78 cm.

(a) Write down the value of c. [1]

When $t = 4$, point P first reaches its maximum height of 143 cm.

(b) Find the value of

(i) a.

(ii) b. [3]

(c) Write down the minimum height of point P. [1]

Later, the cat is tired, and it takes twice as long for point P to complete one revolution at a new constant rate.

(d) Write down the new value of b. [1]

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7. [Maximum mark: 7]

On 1 December 2022, Laviola invests 800 euros (EUR) into a savings account which pays a nominal annual interest rate of $7.5\,\%$ compounded monthly. At the end of each month, Laviola deposits an additional $\text{EUR}\ 500$ into the savings account.

At the end of k months, Laviola will have saved enough money to withdraw $\text{EUR}\ 10\,000$.

(a) Find the smallest possible value of k, for $k \in \mathbb{Z}^+$. [4]

(b) For this value of k, find the interest earned in the savings account.
Express your answer correct to the nearest EUR. [3]

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8. [Maximum mark: 7]

Line L_1 has a vector equation $\boldsymbol{r} = \begin{pmatrix} 3p+4 \\ 2p-1 \\ p+9 \end{pmatrix}$, where $p \in \mathbb{R}$.

Line L_2 has a vector equation $\boldsymbol{r} = \begin{pmatrix} q-2 \\ 1-q \\ 2q+1 \end{pmatrix}$, where $q \in \mathbb{R}$.

The two lines intersect at point M.

(a) Find the coordinates of M. [3]

(b) Find the acute angle between the two lines. [4]

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9. [Maximum mark: 8]

The transformation T is represented by the matrix $\boldsymbol{M} = \begin{pmatrix} 2 & -4 \\ 3 & 1 \end{pmatrix}$.

A pentagon with an area of $12\,\text{cm}^2$ is transformed by T.

(a) Find the area of the image of the pentagon. [2]

Under the transformation T, the image of point X has coordinates $(2t - 3,\ 6 - 5t)$, where $t \in \mathbb{R}$.

(b) Find, in terms of t, the coordinates of X. [6]

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10. [Maximum mark: 7]

Stars are classified by their brightness. The brightest stars in the sky have a magnitude of 1. The magnitude, m, of another star can be modelled as a function of its brightness, b, relative to a star of magnitude 1, as shown by the following equation.

$$m = 1 - 2.5 \log_{10}(b)$$

The star called Acubens has a brightness of 0.0525.

(a) Find the magnitude of Acubens. [2]

Ceres has a magnitude of 7 and is the least bright star visible without magnification.

(b) Find the brightness of Ceres. [2]

The star Proxima Centauri has a greater magnitude than the planet Neptune. The difference in their magnitudes is 3.2.

(c) Find how many times brighter Neptune is compared to Proxima Centauri. [3]

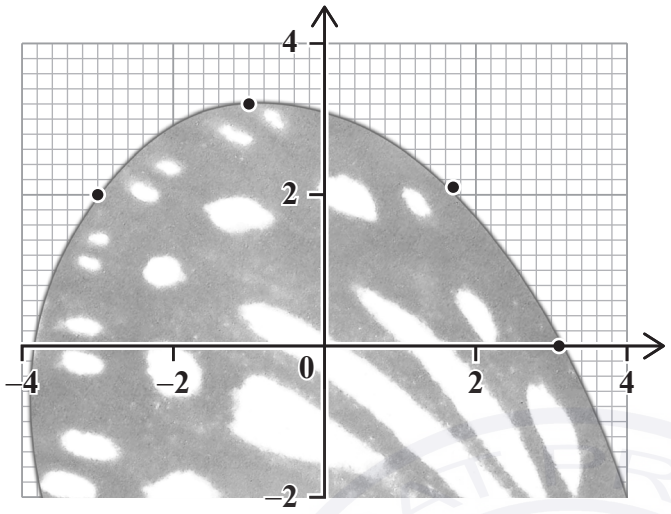
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Answers written on this page
will not be marked.

11. [Maximum mark: 5]

Gloria wants to model the curved edge of a butterfly wing. She inserts a photo of the wing into her graphing software and finds the coordinates of four points on the edge of the wing.



x	y
–3	2
–1	3.2
1.7	2.1
3.1	0

Gloria thinks a cubic curve will be a good model for the butterfly wing.

(a) Find the equation of the cubic regression curve for this data. [2]

For the photo of a second butterfly wing, Gloria finds the equation of the regression curve is $y = 0.0083x^3 - 0.075x^2 - 0.58x + 2.2$.

Gloria realizes that her photo of the second butterfly is an enlargement of the life-size butterfly, scale factor 2 and centred on $(0, 0)$.

(b) Find the equation of the cubic curve that models the life-size wing. [3]

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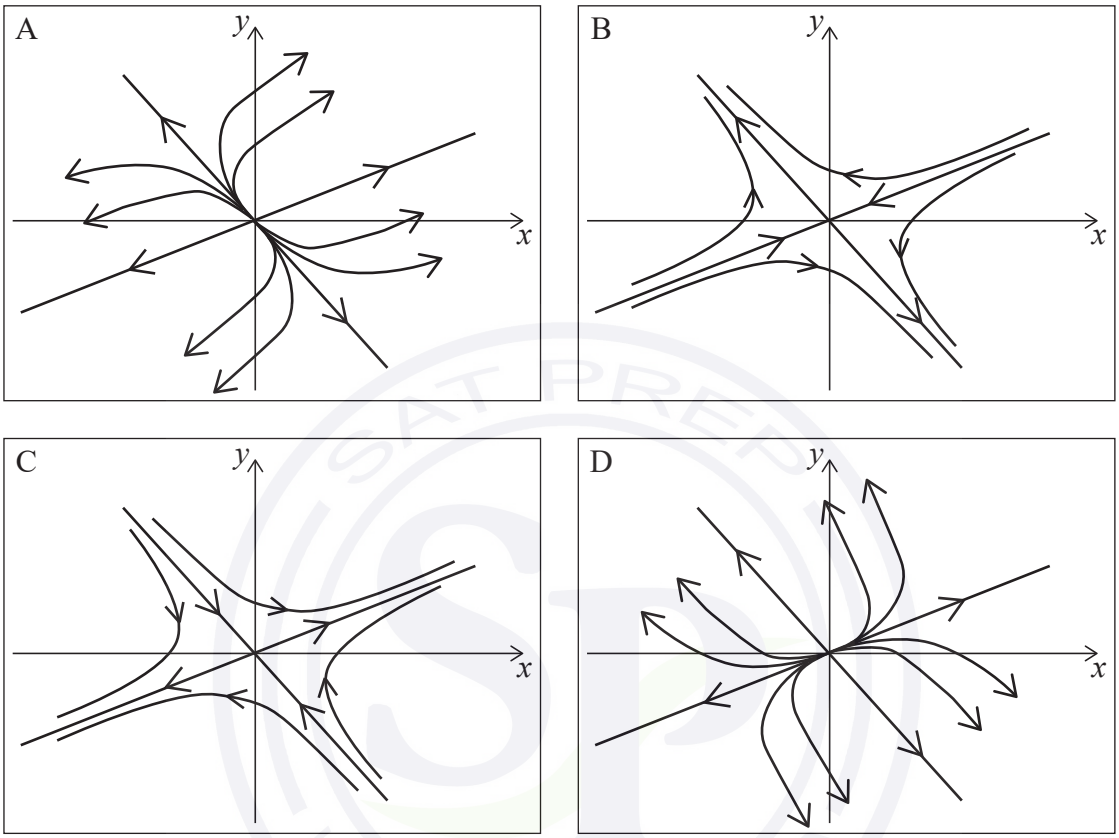
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12. [Maximum mark: 5]

Four possible phase portraits for the coupled differential equations $\frac{\mathrm{d}x}{\mathrm{d}t} = ax + by$ and $\frac{\mathrm{d}y}{\mathrm{d}t} = cx + dy$ are shown, labelled A, B, C and D.



The matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ has eigenvalues λ_1 and λ_2.

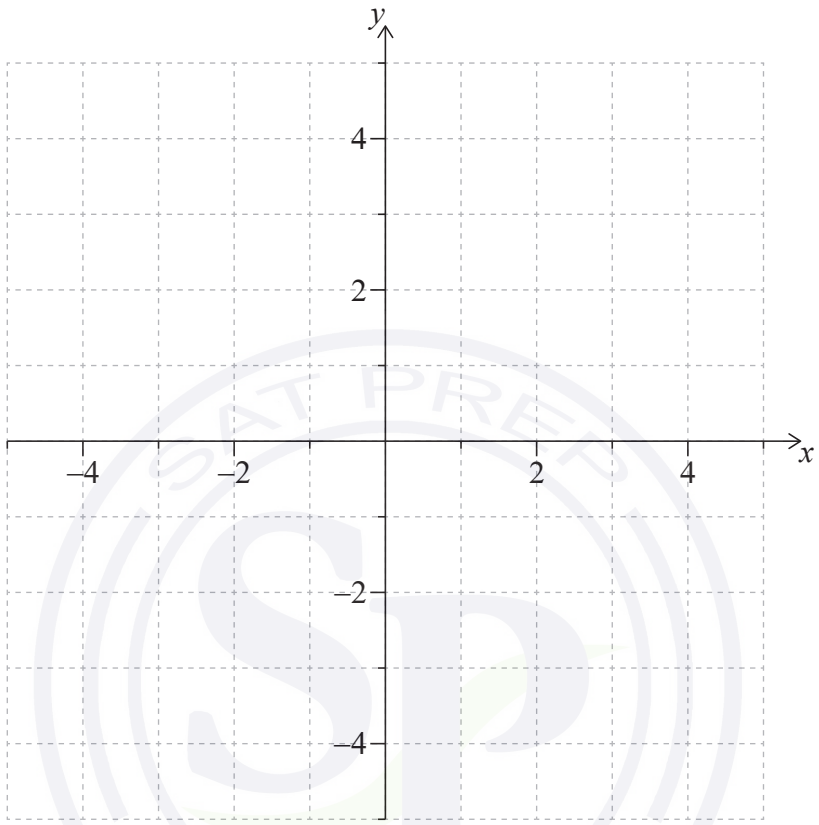
(a) Complete the following table by writing down the letter of the phase portrait that best matches the description. [3]

Description	Phase portrait
$\lambda_1 = 2$ with eigenvector $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and $\lambda_2 = 3$ with eigenvector $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$	
$\lambda_1 = 2$ with eigenvector $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and $\lambda_2 = -3$ with eigenvector $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$	
$\lambda_1 = -2$ with eigenvector $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and $\lambda_2 = 3$ with eigenvector $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$	

(This question continues on the following page)

(Question 12 continued)

(b) On the following axes, sketch the phase portrait that corresponds to $\lambda_1 = -2 + 3\mathrm{i}$ and $\lambda_2 = -2 - 3\mathrm{i}$, given that $\frac{\mathrm{d}y}{\mathrm{d}t} = -12$ at $(3, 0)$. [2]



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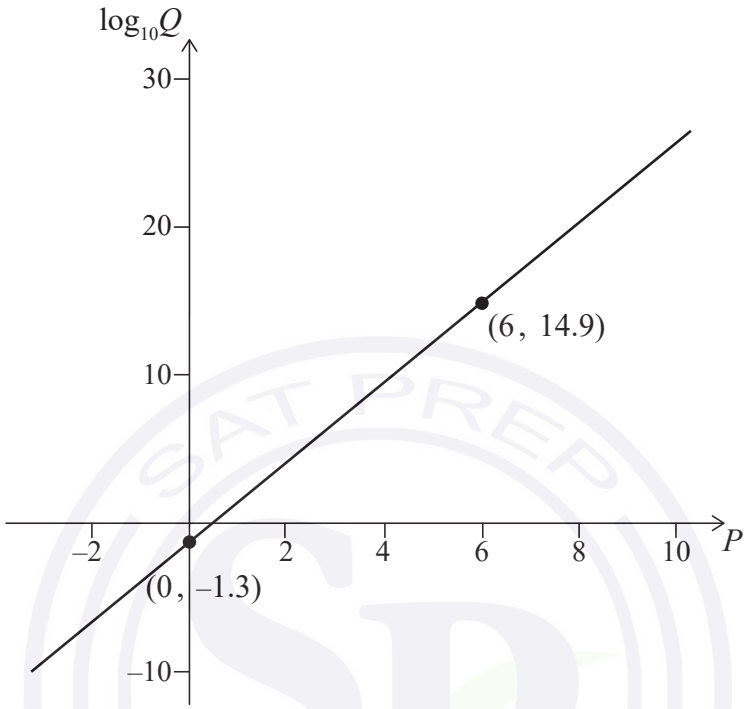
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13. [Maximum mark: 6]

Gen is investigating the relationship between two sets of data, labelled P and Q, that she collected. She created a scatter plot with P on the x-axis and $\log_{10} Q$ on the y-axis. Gen noticed that the points had a strong linear correlation, so she drew a line of best fit, as shown in the diagram. The line passes through the points $(0, -1.3)$ and $(6, 14.9)$.



(a) Find an equation for Q in terms of P. [3]

Gen also investigates the relationship between the same data, Q, and some new data, R. She believes that the data can be modelled by $Q = a \ln R + b$ and she decides to create a scatter plot to verify her belief.

(b) State what expression Gen should plot on each axis to verify her belief. [1]

The scatter plot has a linear relationship and Gen finds $a = 4.3$ and $b = 12.1$.

(c) Find an equation for P in terms of R. [2]

(This question continues on the following page)

(Question 13 continued)

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14. [Maximum mark: 9]

A particle moves such that its velocity, v metres per second, at time t seconds, is given by $v = t\sin(t^2)$.

(a) Find an expression for the acceleration of the particle. [2]

(b) Hence, or otherwise, find its greatest acceleration for $0 \leq t \leq 8$. [2]

The particle starts at the origin.

(c) Find an expression for the displacement of the particle. [3]

(d) Hence show that the particle never has a negative displacement. [2]

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15. [Maximum mark: 5]

An electrical circuit contains a capacitor. The charge on the capacitor, q Coulombs, at time t seconds, satisfies the differential equation

$$\frac{\mathrm{d}^2q}{\mathrm{d}t^2} + 5\frac{\mathrm{d}q}{\mathrm{d}t} + 20q = 200\,.$$

Initially $q = 1$ and $\frac{\mathrm{d}q}{\mathrm{d}t} = 8$.

Use Euler's method with $h = 0.1$ to estimate the maximum charge on the capacitor during the first second.

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16. [Maximum mark: 8]

The principal of a school is concerned that only 30% of her students are choosing healthy options from the school canteen. She organizes a campaign to promote healthy eating and decides to test if the campaign has increased the number of students choosing healthy options. She assumes that a student's choice is independent of other students' choices.

(a) Write down suitable hypotheses for this test. [2]

The principal decides to take a random sample of 80 students. She will reject the null hypothesis if at least 31 students choose a healthy option.

(b) Find the probability that she makes a Type I error. [3]

In fact, the campaign led to 40% of her students choosing a healthy option.

(c) Find the probability that she makes a Type II error. [3]

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17. [Maximum mark: 8]

The time of sunrise, R hours after midnight, in Taipei can be modelled by

$$R = 1.08\cos(0.0165t + 0.413) + 4.94,$$

where t is the day of the year 2021 (for example, $t = 2$ represents 2 January 2021).

The time of sunset, S hours after midnight, in Taipei can be modelled by

$$S = 1.15\cos(0.0165t - 2.97) + 18.9.$$

The number of daylight hours, D, in Taipei during 2021 can be modelled by

$$D = a\cos(0.0165t + b) + c.$$

(a) Find the value of a, of b and of c. [6]

(b) Hence, or otherwise, find the largest number of daylight hours in Taipei during 2021 and the day of the year on which this occurs. [2]

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References:

11. Fleur, 2019. *photo-1560263816-d704d83cce0f.* [image online] Available at: https://unsplash.com/photos/SE2zTdS1MNo [Accessed 8 February 2022]. Source adapted.

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Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: applications and interpretation
Higher level
Paper 1

Friday 6 May 2022 (afternoon)

Candidate session number

2 hours

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all questions.
- Answers must be written within the answer boxes provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.

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25 pages

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1. [Maximum mark: 6]

A group of 130 applicants applied for admission into either the Arts programme or the Sciences programme at a university. The outcomes of their applications are shown in the following table.

	Accepted	**Rejected**
Arts programme	17	24
Sciences programme	25	64

(a) Find the probability that a randomly chosen applicant from this group was accepted by the university. [1]

An applicant is chosen at random from this group. It is found that they were accepted into the programme of their choice.

(b) Find the probability that the applicant applied for the Arts programme. [2]

Two different applicants are chosen at random from the original group.

(c) Find the probability that both applicants applied to the Arts programme. [3]

(This question continues on the following page)

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(Question 1 continued)

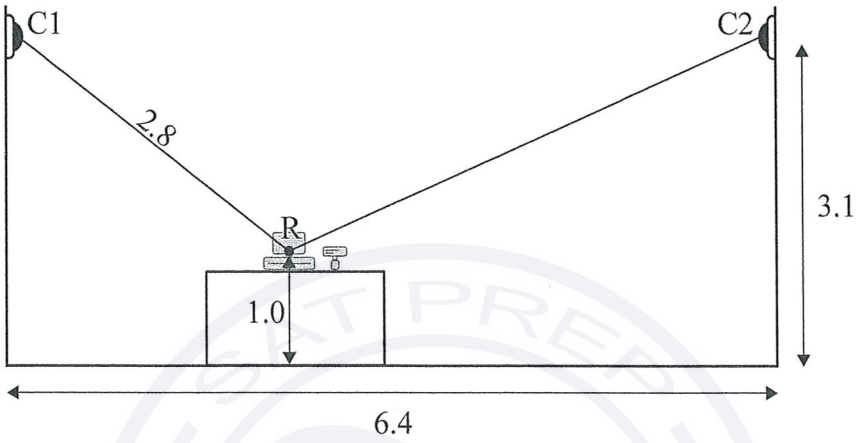
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2. [Maximum mark: 8]

The owner of a convenience store installs two security cameras, represented by points C1 and C2. Both cameras point towards the centre of the store's cash register, represented by the point R.

The following diagram shows this information on a cross-section of the store.

diagram not to scale



The cameras are positioned at a height of $3.1\,\text{m}$, and the horizontal distance between the cameras is $6.4\,\text{m}$. The cash register is sitting on a counter so that its centre, R, is $1.0\,\text{m}$ above the floor.

The distance from Camera 1 to the centre of the cash register is $2.8\,\text{m}$.

(a) Determine the angle of depression from Camera 1 to the centre of the cash register. Give your answer in degrees. [2]

(b) Calculate the distance from Camera 2 to the centre of the cash register. [4]

(c) Without further calculation, determine which camera has the largest angle of depression to the centre of the cash register. Justify your response. [2]

(This question continues on the following page)

(Question 2 continued)

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Answers written on this page
will not be marked.

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3. [Maximum mark: 7]

A polygraph test is used to determine whether people are telling the truth or not, but it is not completely accurate. When a person tells the truth, they have a 20% chance of failing the test. Each test outcome is independent of any previous test outcome.

10 people take a polygraph test and all 10 tell the truth.

(a) Calculate the expected number of people who will pass this polygraph test. [2]

(b) Calculate the probability that exactly 4 people will fail this polygraph test. [2]

(c) Determine the probability that fewer than 7 people will pass this polygraph test. [3]

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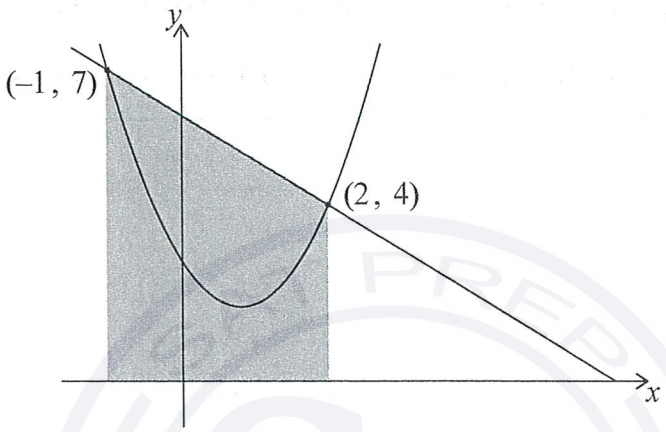
4. [Maximum mark: 7]

The graphs of $y = 6 - x$ and $y = 1.5x^2 - 2.5x + 3$ intersect at $(2, 4)$ and $(-1, 7)$, as shown in the following diagrams.

In **diagram 1**, the region enclosed by the lines $y = 6 - x$, $x = -1$, $x = 2$ and the x-axis has been shaded.

diagram not to scale

Diagram 1

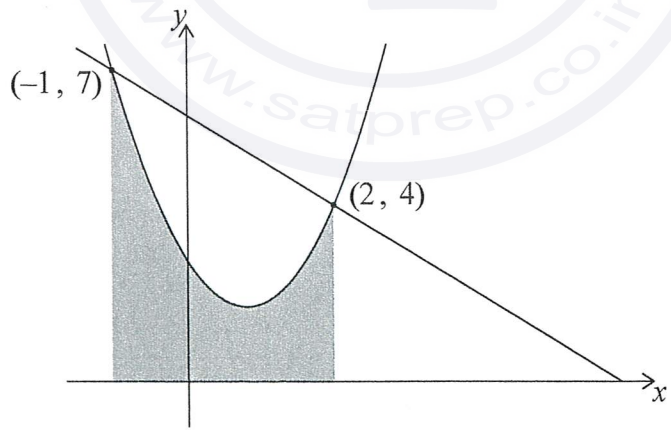


(a) Calculate the area of the shaded region in **diagram 1**. [2]

In **diagram 2**, the region enclosed by the curve $y = 1.5x^2 - 2.5x + 3$, and the lines $x = -1$, $x = 2$ and the x-axis has been shaded.

diagram not to scale

Diagram 2



(b) (i) Write down an integral for the area of the shaded region in **diagram 2**.

(ii) Calculate the area of this region. [3]

(c) Hence, determine the area enclosed between $y = 6 - x$ and $y = 1.5x^2 - 2.5x + 3$. [2]

(This question continues on the following page)

(Question 4 continued)

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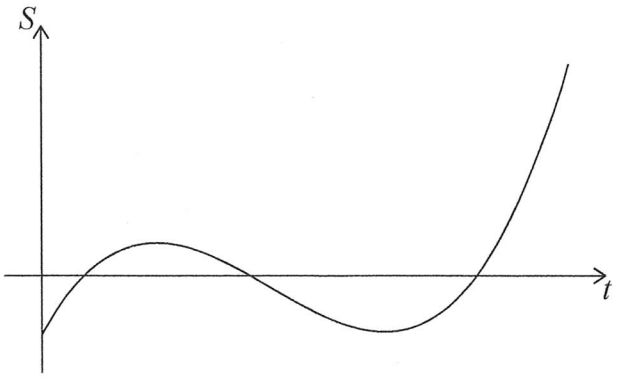
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5. [Maximum mark: 8]

The graph below shows the average savings, S thousand dollars, of a group of university graduates as a function of t, the number of years after graduating from university.



(a) Write down one feature of this graph which suggests a cubic function might be appropriate to model this scenario. [1]

The equation of the model can be expressed in the form $S = at^3 + bt^2 + ct + d$, where a, b, c and d are real constants.

The graph of the model must pass through the following four points.

t	0	1	2	3
S	−5	3	−1	−5

(b) (i) Write down the value of d.

(ii) Write down three simultaneous equations for a, b and c.

(iii) Hence, or otherwise, find the values of a, b and c. [4]

A negative value of S indicates that a graduate is expected to be in debt.

(c) Use the model to determine the total length of time, in years, for which a graduate is expected to be in debt after graduating from university. [3]

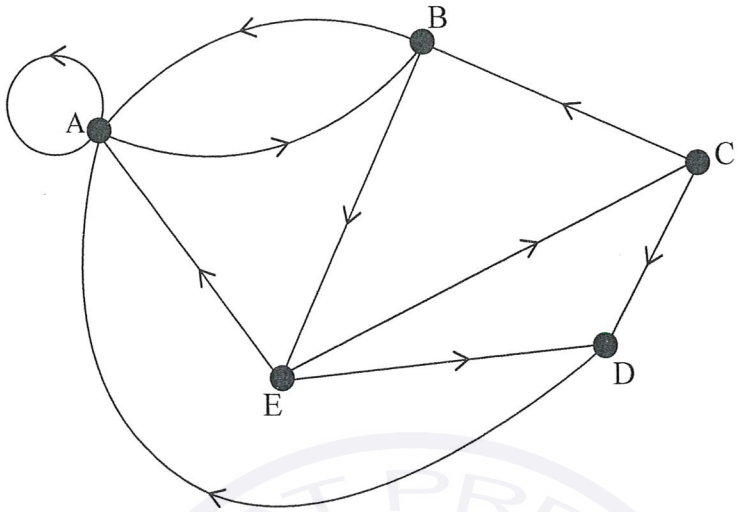
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(Question 5 continued)

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6. [Maximum mark: 5]

Consider the following directed network.



(a) Write down the adjacency matrix for this network. [2]

(b) Determine the number of different walks of length 5 that start and end at the same vertex. [3]

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7. [Maximum mark: 5]

The sum of an infinite geometric sequence is 9.

The first term is 4 more than the second term.

Find the third term. Justify your answer.

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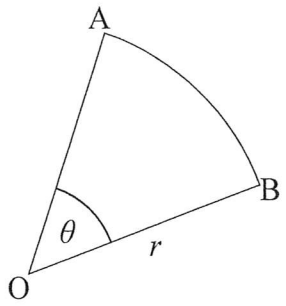
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8. [Maximum mark: 8]

The diagram shows a sector, OAB, of a circle with centre O and radius r, such that $\hat{AOB} = \theta$.



Sam measured the value of r to be $2\,\text{cm}$ and the value of θ to be 30°.

(a) Use Sam's measurements to calculate the area of the sector. Give your answer to four significant figures. [2]

It is found that Sam's measurements are accurate to only one significant figure.

(b) Find the upper bound and lower bound of the area of the sector. [3]

(c) Find, with justification, the largest possible percentage error if the answer to part (a) is recorded as the area of the sector. [3]

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9. [Maximum mark: 8]

A psychologist records the number of digits (d) of π that a sample of IB Mathematics higher level candidates could recall.

d	2	3	4	5	6	7
Frequency	2	6	24	21	11	3

(a) Find an unbiased estimate of the population mean of d. [1]

(b) Find an unbiased estimate of the population variance of d. [2]

The psychologist has read that in the general population people can remember an average of 4.4 digits of π. The psychologist wants to perform a statistical test to see if IB Mathematics higher level candidates can remember more digits than the general population.

(c) $H_0: \mu = 4.4$ is the null hypothesis for this test.

(i) State the alternative hypothesis.

(ii) Given that all assumptions for this test are satisfied, carry out an appropriate hypothesis test. State and justify your conclusion. Use a 5% significance level. [5]

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10. [Maximum mark: 5]

The function $f(x) = \ln\left(\frac{1}{x-2}\right)$ is defined for $x > 2$, $x \in \mathbb{R}$.

(a) Find an expression for $f^{-1}(x)$. You are not required to state a domain. [3]

(b) Solve $f(x) = f^{-1}(x)$. [2]

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11. [Maximum mark: 6]

Juliana plans to invest money for 10 years in an account paying 3.5% interest, compounded annually. She expects the annual inflation rate to be 2% per year throughout the 10-year period.

Juliana would like her investment to be worth a real value of $\$4000$, compared to current values, at the end of the 10-year period. She is considering two options.

Option 1: Make a one-time investment at the start of the 10-year period.

Option 2: Invest $\$1000$ at the start of the 10-year period and then invest $\$x$ into the account at the end of each year (including the first and last years).

(a) For option 1, determine the minimum amount Juliana would need to invest. Give your answer to the nearest dollar. [3]

(b) For option 2, find the minimum value of x that Juliana would need to invest each year. Give your answer to the nearest dollar. [3]

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12. [Maximum mark: 6]

The sex of cuttlefish is difficult to determine visually, so it is often found by weighing the cuttlefish.

The weights of adult male cuttlefish are known to be normally distributed with mean $10\,\text{kg}$ and standard deviation $0.5\,\text{kg}$.

The weights of adult female cuttlefish are known to be normally distributed with mean $12\,\text{kg}$ and standard deviation $1\,\text{kg}$.

A zoologist uses the null hypothesis that in the absence of information a cuttlefish is male.

If the weight is found to be above $11.5\,\text{kg}$ the cuttlefish is classified as female.

(a) Find the probability of making a Type I error when weighing a male cuttlefish. [2]

(b) Find the probability of making a Type II error when weighing a female cuttlefish. [2]

90% of adult cuttlefish are male.

(c) Find the probability of making an error using the zoologist's method. [2]

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13. [Maximum mark: 5]

An electric circuit has two power sources. The voltage, V_1, provided by the first power source, at time t, is modelled by

$$V_1 = \mathrm{Re}(2\mathrm{e}^{3t\mathrm{i}}).$$

The voltage, V_2, provided by the second power source is modelled by

$$V_2 = \mathrm{Re}(5\mathrm{e}^{(3t+4)\mathrm{i}}).$$

The total voltage in the circuit, V_T, is given by

$$V_T = V_1 + V_2.$$

(a) Find an expression for V_T in the form $A\cos(Bt + C)$, where A, B and C are real constants. [4]

(b) Hence write down the maximum voltage in the circuit. [1]

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14. [Maximum mark: 4]

The shape of a vase is formed by rotating a curve about the y-axis.

The vase is $10\,\text{cm}$ high. The internal radius of the vase is measured at $2\,\text{cm}$ intervals along the height:

Height (cm)	Radius (cm)
0	4
2	6
4	8
6	7
8	3
10	5

Use the trapezoidal rule to estimate the volume of water that the vase can hold.

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15. [Maximum mark: 7]

The equation of the line $y = mx + c$ can be expressed in vector form $\boldsymbol{r} = \boldsymbol{a} + \lambda\boldsymbol{b}$.

(a) Find the vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ in terms of m and/or c. [2]

The matrix $\boldsymbol{M}$ is defined by $\begin{pmatrix} 6 & 3 \\ 4 & 2 \end{pmatrix}$.

(b) Find the value of $\det \boldsymbol{M}$. [1]

The line $y = mx + c$ (where $m \neq -2$) is transformed into a new line using the transformation described by matrix $\boldsymbol{M}$.

(c) Show that the equation of the resulting line does not depend on m or c. [4]

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16. [Maximum mark: 7]

The position vector of a particle, P, relative to a fixed origin O at time t is given by

$$\vec{OP} = \begin{pmatrix} \sin(t^2) \\ \cos(t^2) \end{pmatrix}.$$

(a) Find the velocity vector of P. [2]

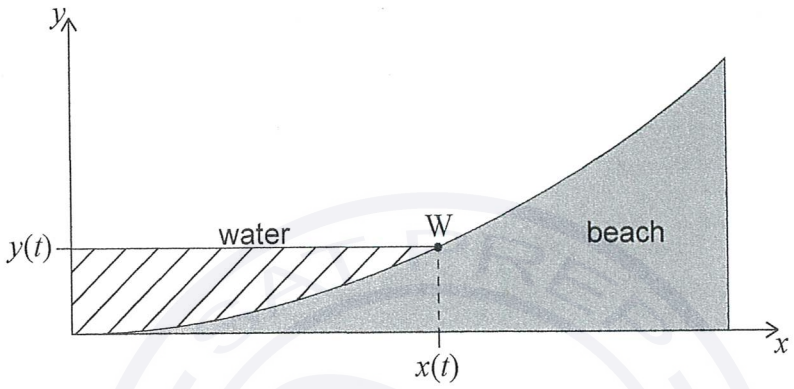
(b) Show that the acceleration vector of P is never parallel to the position vector of P. [5]

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17. [Maximum mark: 8]

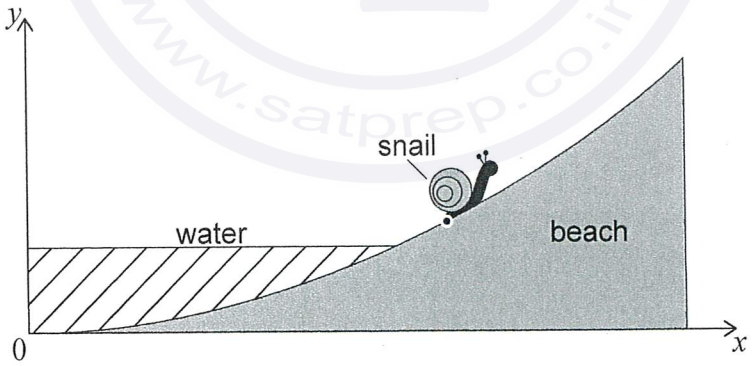
The cross-section of a beach is modelled by the equation $y = 0.02x^2$ for $0 \le x \le 10$ where y is the height of the beach (in metres) at a horizontal distance x metres from an origin. t is the time in hours after low tide.

At $t = 0$ the water is at the point $(0, 0)$. The height of the water rises at a rate of 0.2 metres per hour. The point $\mathrm{W}(x(t), y(t))$ indicates where the water level meets the beach at time t.



(a) When W has an x-coordinate equal to 1, find the horizontal component of the velocity of W. [3]

A snail is modelled as a single point. At $t = 0$ it is positioned at $(1, 0.02)$. The snail travels away from the incoming water at a speed of 1 metre per hour in the direction along the curve of the cross-section of the beach. The following diagram shows this for a value of t, such that $t > 0$.



(b) (i) Find the time taken for the snail to reach the point $(10, 2)$.

(ii) Hence show that the snail reaches the point $(10, 2)$ before the water does. [5]

(This question continues on the following page)

(Question 17 continued)

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Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: applications and interpretation
Higher level
Paper 1

Friday 6 May 2022 (afternoon)

Candidate session number

2 hours

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all questions.
- Answers must be written within the answer boxes provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.

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Answers must be written within the answer boxes provided. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 5]

The height of a baseball after it is hit by a bat is modelled by the function

$$h(t) = -4.8t^2 + 21t + 1.2$$

where $h(t)$ is the height in metres above the ground and t is the time in seconds after the ball was hit.

(a) Write down the height of the ball above the ground at the instant it is hit by the bat. [1]

(b) Find the value of t when the ball hits the ground. [2]

(c) State an appropriate domain for t in this model. [2]

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2. [Maximum mark: 5]

The ticket prices for a concert are shown in the following table.

Ticket Type	Price (in Australian dollars, $)
Adult	15
Child	10
Student	12

- A total of 600 tickets were sold.
- The total amount of money from ticket sales was $\$7816$.
- There were twice as many adult tickets sold as child tickets.

Let the number of adult tickets sold be x, the number of child tickets sold be y, and the number of student tickets sold be z.

(a) Write down three equations that express the information given above. [3]

(b) Find the number of each type of ticket sold. [2]

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3. [Maximum mark: 7]

Leo is investigating whether a six-sided die is fair. He rolls the die 60 times and records the observed frequencies in the following table:

Number on die	1	2	3	4	5	6
Observed frequency	8	7	6	15	12	12

Leo carries out a χ^2 goodness of fit test at a 5% significance level.

(a) Write down the null and alternative hypotheses. [1]

(b) Write down the degrees of freedom. [1]

(c) Write down the expected frequency of rolling a 1. [1]

(d) Find the p-value for the test. [2]

(e) State the conclusion of the test. Give a reason for your answer. [2]

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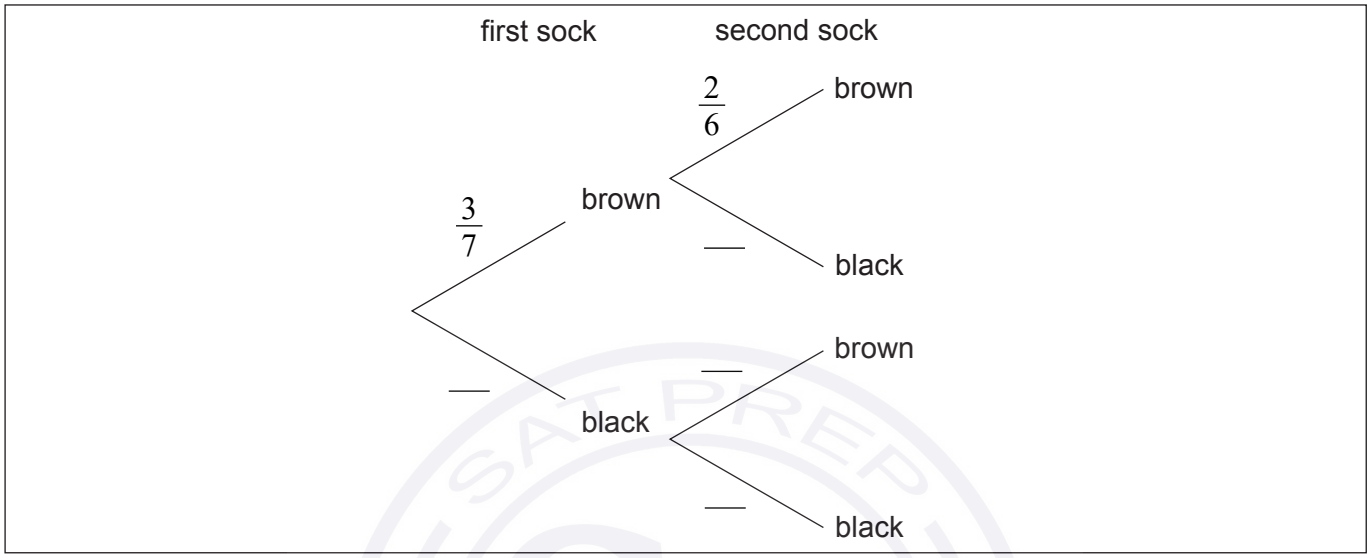
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4. [Maximum mark: 7]

Karl has three brown socks and four black socks in his drawer. He takes two socks at random from the drawer.

(a) Complete the tree diagram. [2]



(b) Find the probability that Karl takes two socks of the same colour. [2]

(c) Given that Karl has two socks of the same colour find the probability that he has two brown socks. [3]

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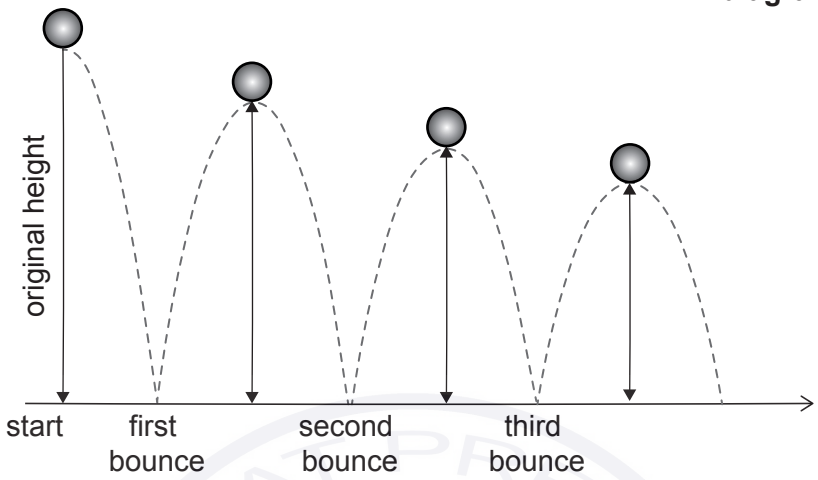
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5. [Maximum mark: 7]

A ball is dropped from a height of 1.8 metres and bounces on the ground. The maximum height reached by the ball, after each bounce, is 85% of the previous maximum height.

diagram not to scale



(a) Show that the maximum height reached by the ball after it has bounced for the sixth time is $68\,\text{cm}$, to the nearest cm. [2]

(b) Find the number of times, after the first bounce, that the maximum height reached is greater than $10\,\text{cm}$. [2]

(c) Find the total **vertical** distance travelled by the ball from the point at which it is dropped until the fourth bounce. [3]

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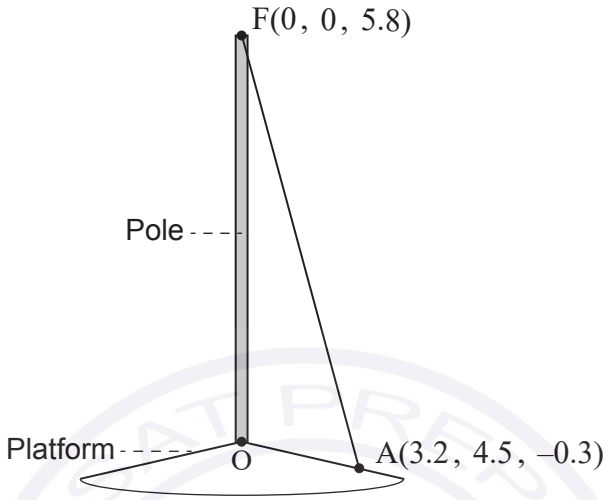
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6. [Maximum mark: 8]

A vertical pole stands on a sloped platform. The bottom of the pole is used as the origin, O, of a coordinate system in which the top, F, of the pole has coordinates $(0, 0, 5.8)$. All units are in metres.

diagram not to scale



The pole is held in place by ropes attached at F.

One of these ropes is attached to the platform at point $A(3.2, 4.5, -0.3)$. The rope forms a straight line from A to F.

(a) Find $\vec{AF}$. [1]

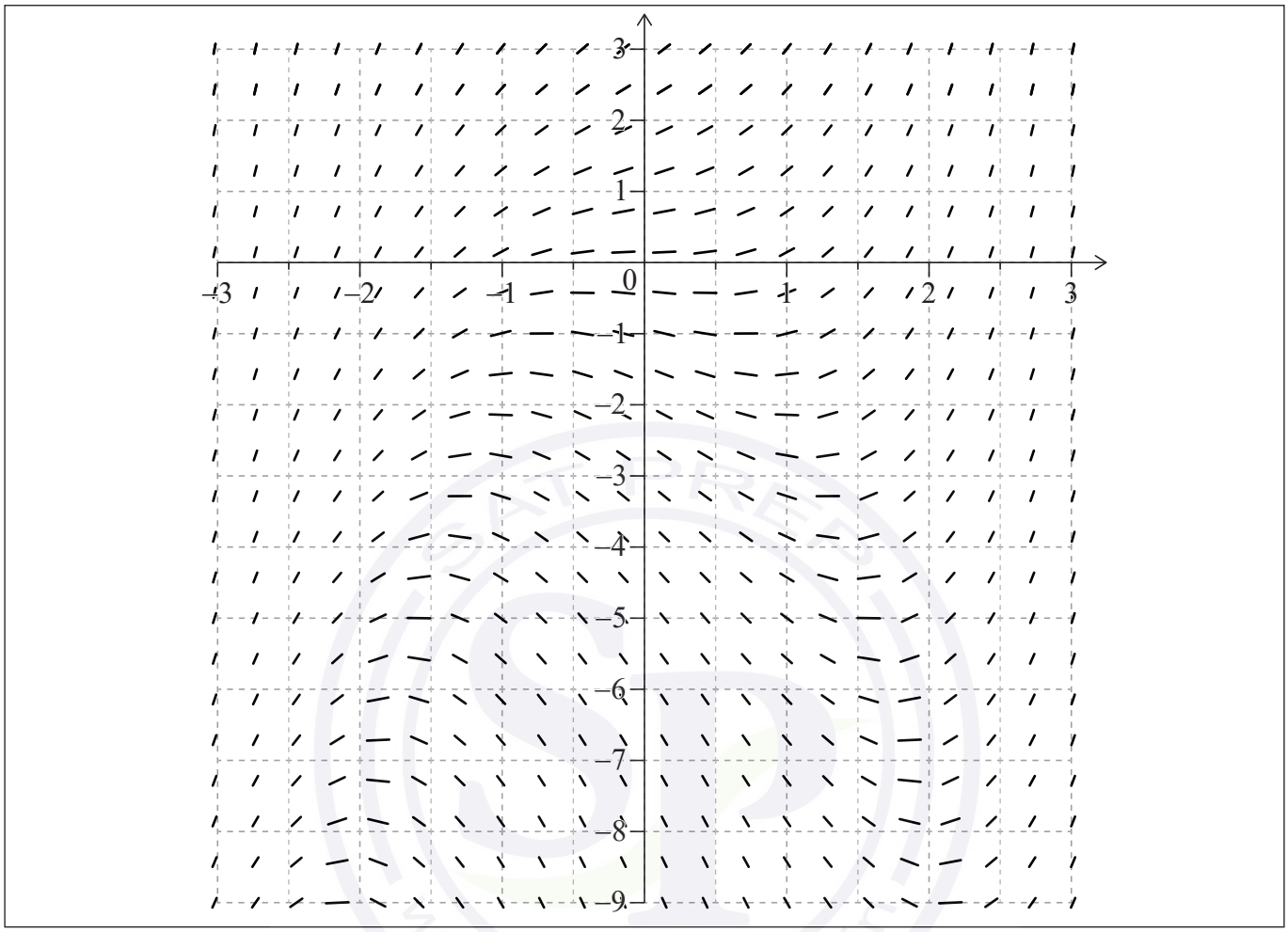
(b) Find the length of the rope. [2]

(c) Find $F\hat{A}O$, the angle the rope makes with the platform. [5]

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7. [Maximum mark: 4]

A slope field for the differential equation $\frac{dy}{dx} = x^2 + \frac{y}{2}$ is shown.



Some of the solutions to the differential equation have a local maximum point and a local minimum point.

(a) (i) Write down the equation of the curve on which all these maximum and minimum points lie.

(ii) Sketch this curve on the slope field. [2]

The solution to the differential equation that passes through the point $(0, -2)$ has both a local maximum point and a local minimum point.

(b) On the slope field, sketch the solution to the differential equation that passes through $(0, -2)$. [2]

(This question continues on the following page)

(Question 7 continued)

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8. [Maximum mark: 6]

Consider the curve $y = 2x(4 - e^x)$.

(a) Find

(i) $\frac{dy}{dx}$.

(ii) $\frac{d^2y}{dx^2}$. [4]

The curve has a point of inflexion at (a, b).

(b) Find the value of a. [2]

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9. [Maximum mark: 6]

A company produces bags of sugar with a labelled weight of $1\,\text{kg}$. The weights of the bags are normally distributed with a mean of $1\,\text{kg}$ and a standard deviation of $100\,\text{g}$. In an inspection, if the weight of a randomly chosen bag is less than $950\,\text{g}$ then the company fails the inspection.

(a) Find the probability that the company fails the inspection. [2]

A statistician in the company suggests it would be fairer if the company passes the inspection when the mean weight of five randomly chosen bags is greater than $950\,\text{g}$.

(b) Find the probability of passing the inspection if the statistician's suggestion is followed. [4]

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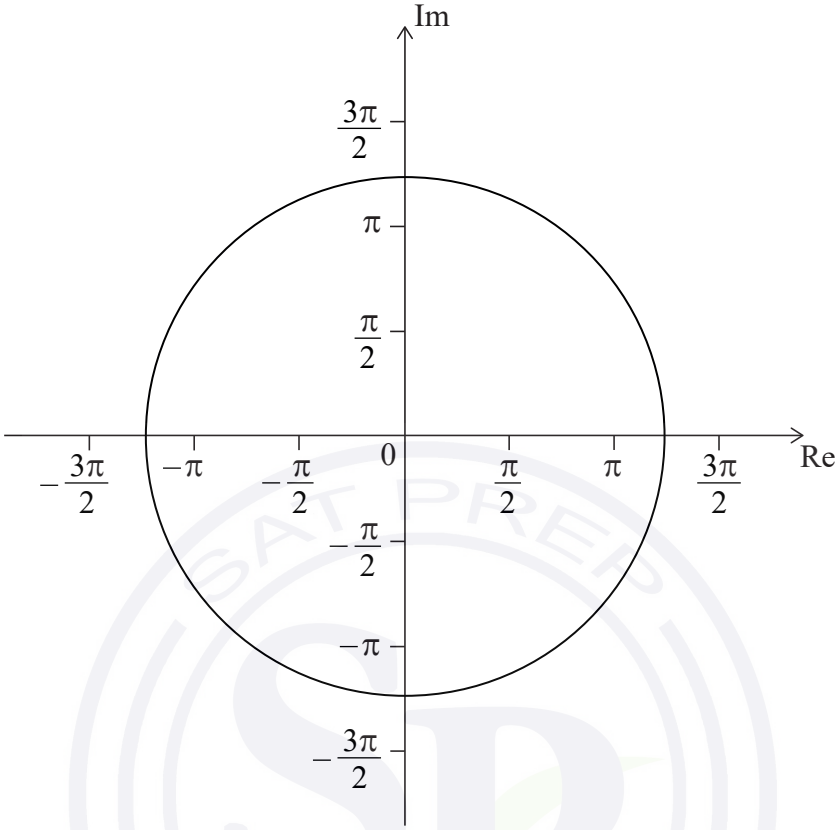
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10. [Maximum mark: 7]

The following Argand diagram shows a circle centre 0 with a radius of 4 units.



A set of points, $\{z_\theta\}$, on the Argand plane are defined by the equation

$$z_\theta = \frac{1}{2}\theta e^{\theta i}, \text{ where } \theta \geq 0.$$

(a) Plot on the Argand diagram the points corresponding to

(i) $\theta = \frac{\pi}{2}$.

(ii) $\theta = \pi$.

(iii) $\theta = \frac{3\pi}{2}$. [3]

Consider the case where $|z_\theta| = 4$.

(b) (i) Find this value of θ.

(ii) For this value of θ, plot the approximate position of z_θ on the Argand diagram. [4]

(This question continues on the following page)

(Question 10 continued)

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11. [Maximum mark: 5]

The matrix $\boldsymbol{M} = \begin{pmatrix} 0.2 & 0.7 \\ 0.8 & 0.3 \end{pmatrix}$ has eigenvalues -0.5 and 1.

(a) Find an eigenvector corresponding to the eigenvalue of 1. Give your answer in the form $\begin{pmatrix} a \\ b \end{pmatrix}$, where $a, b \in \mathbb{Z}$. [3]

A switch has two states, A and B. Each second it either remains in the same state or moves according to the following rule: If it is in state A it will move to state B with a probability of 0.8 and if it is in state B it will move to state A with a probability of 0.7.

(b) Using your answer to (a), or otherwise, find the long-term probability of the switch being in state A. Give your answer in the form $\frac{c}{d}$, where $c, d \in \mathbb{Z}^+$. [2]

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12. [Maximum mark: 8]

The strength of earthquakes is measured on the Richter magnitude scale, with values typically between 0 and 8 where 8 is the most severe.

The Gutenberg–Richter equation gives the average number of earthquakes per year, N, which have a magnitude of at least M. For a particular region the equation is

$$\log_{10} N = a - M, \text{ for some } a \in \mathbb{R}.$$

This region has an average of 100 earthquakes per year with a magnitude of at least 3.

(a) Find the value of a. [2]

The equation for this region can also be written as $N = \frac{b}{10^M}$.

(b) Find the value of b. [2]

Within this region the most severe earthquake recorded had a magnitude of 7.2.

(c) Find the average number of earthquakes in a year with a magnitude of at least 7.2. [1]

The number of earthquakes in a given year with a magnitude of at least 7.2 can be modelled by a Poisson distribution, with mean N. The number of earthquakes in one year is independent of the number of earthquakes in any other year.

Let Y be the number of years between the earthquake of magnitude 7.2 and the next earthquake of at least this magnitude.

(d) Find $\mathrm{P}(Y > 100)$. [3]

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13. [Maximum mark: 5]

At $1{:}00\ \text{pm}$ a ship is $1\,\text{km}$ east and $4\,\text{km}$ north of a harbour. A coordinate system is defined with the harbour at the origin. The position vector of the ship at $1{:}00\ \text{pm}$ is given by $\begin{pmatrix}1\\4\end{pmatrix}$.

The ship has a constant velocity of $\begin{pmatrix}1.2\\-0.6\end{pmatrix}$ kilometres per hour (km h^{-1}).

(a) Write down an expression for the position vector $\boldsymbol{r}$ of the ship, t hours after $1{:}00\ \text{pm}$. [1]

(b) Find the time at which the bearing of the ship from the harbour is 045°. [4]

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14. [Maximum mark: 8]

(a) (i) Expand $\left(\frac{1}{u}+1\right)^2$.

(ii) Find $\int\left(\frac{1}{(x+2)}+1\right)^2 \mathrm{d}x$.

[4]

The region bounded by $y=\frac{1}{(x+2)}+1$, $x=0$, $x=2$ and the x-axis is rotated through 2π about the x-axis to form a solid.

(b) Find the volume of the solid formed. Give your answer in the form $\frac{\pi}{4}(a+b\ln(c))$, where a, b, $c\in\mathbb{Z}$.

[4]

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15. [Maximum mark: 7]

The number of cars arriving at a junction in a particular town in any given minute between $9{:}00\ \text{am}$ and $10{:}00\ \text{am}$ is historically known to follow a Poisson distribution with a mean of 5.4 cars per minute.

A new road is built near the town. It is claimed that the new road has decreased the number of cars arriving at the junction.

To test the claim, the number of cars, X, arriving at the junction between $9{:}00\ \text{am}$ and $10{:}00\ \text{am}$ on a particular day will be recorded. The test will have the following hypotheses:

H_0: the mean number of cars arriving at the junction has not changed,
H_1: the mean number of cars arriving at the junction has decreased.

The alternative hypothesis will be accepted if $X \leq 300$.

(a) Assuming the null hypothesis to be true, state the distribution of X. [1]

(b) Find the probability of a Type I error. [2]

(c) Find the probability of a Type II error, if the number of cars now follows a Poisson distribution with a mean of 4.5 cars per minute. [4]

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16. [Maximum mark: 7]

The wind chill index W is a measure of the temperature, in $^{\circ}\mathrm{C}$, felt when taking into account the effect of the wind.

When Frieda arrives at the top of a hill, the relationship between the wind chill index and the speed of the wind v in kilometres per hour $(\mathrm{km\,h^{-1}})$ is given by the equation

$$W = 19.34 - 7.405v^{0.16}$$

(a) Find an expression for $\frac{\mathrm{d}W}{\mathrm{d}v}$. [2]

When Frieda arrives at the top of a hill, the speed of the wind is 10 kilometres per hour and increasing at a rate of $5\,\mathrm{km\,h^{-1}\,minute^{-1}}$.

(b) Find the rate of change of W at this time. [5]

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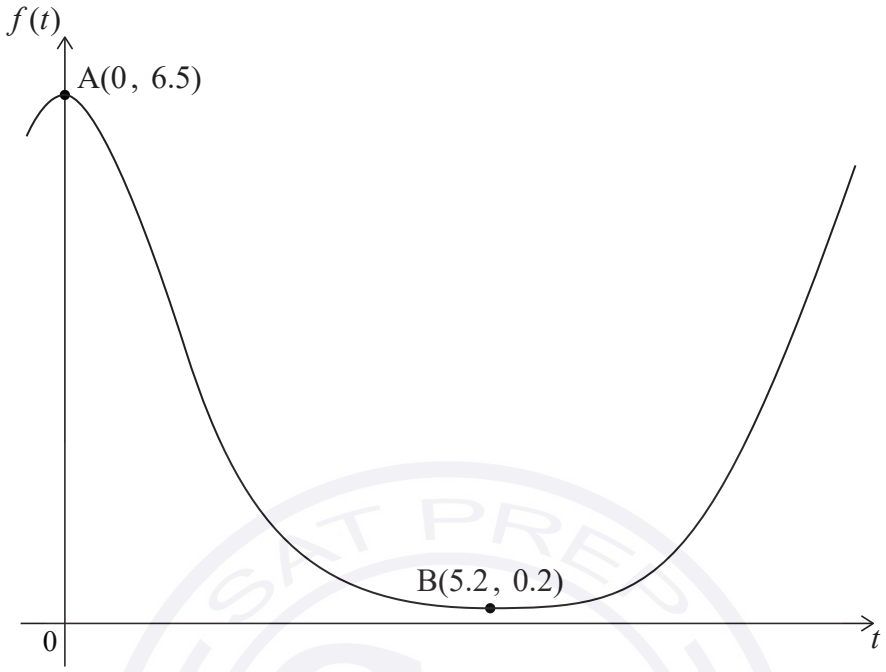
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17. [Maximum mark: 8]

A function f is of the form $f(t) = pe^{q\cos(rt)}$, $p, q, r \in \mathbb{R}^+$. Part of the graph of f is shown.



The points A and B have coordinates $A(0, 6.5)$ and $B(5.2, 0.2)$, and lie on f.

The point A is a local maximum and the point B is a local minimum.

Find the value of p, of q and of r.

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Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: applications and interpretation
Higher level
Paper 1

Monday 1 November 2021 (afternoon)

Candidate session number

2 hours

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all questions.
- Answers must be written within the answer boxes provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.

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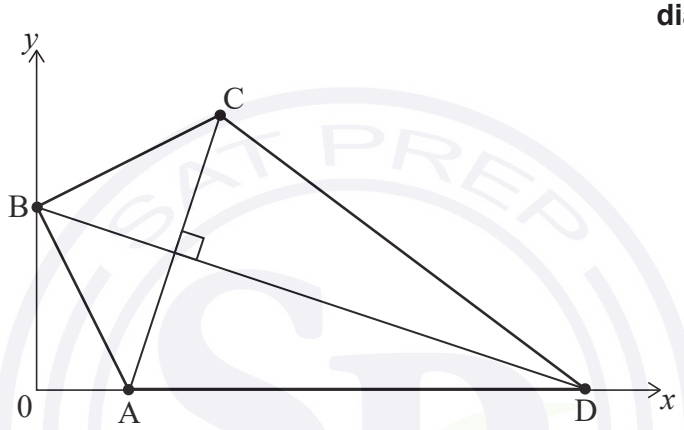
Answers must be written within the answer boxes provided. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 6]

Dilara is designing a kite ABCD on a set of coordinate axes in which one unit represents $10\,\text{cm}$.

The coordinates of A, B and C are $(2,\ 0)$, $(0,\ 4)$ and $(4,\ 6)$ respectively. Point D lies on the x-axis. $[\text{AC}]$ is perpendicular to $[\text{BD}]$. This information is shown in the following diagram.

diagram not to scale



(a) Find the gradient of the line through A and C. [2]

(b) Write down the gradient of the line through B and D. [1]

(c) Find the equation of the line through B and D. Give your answer in the form $ax + by + d = 0$, where a, b and d are integers. [2]

(d) Write down the x-coordinate of point D. [1]

(This question continues on the following page)

(Question 1 continued)

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2. [Maximum mark: 5]

Inspectors are investigating the carbon dioxide emissions of a power plant. Let R be the rate, in tonnes per hour, at which carbon dioxide is being emitted and t be the time in hours since the inspection began.

When R is plotted against t, the total amount of carbon dioxide produced is represented by the area between the graph and the horizontal t-axis.

The rate, R, is measured over the course of two hours. The results are shown in the following table.

t	0	0.4	0.8	1.2	1.6	2
R	30	50	60	40	20	50

(a) Use the trapezoidal rule with an interval width of 0.4 to estimate the total amount of carbon dioxide emitted during these two hours. [3]

The real amount of carbon dioxide emitted during these two hours was 72 tonnes.

(b) Find the percentage error of the estimate found in part (a). [2]

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3. [Maximum mark: 7]

Let the function $h(x)$ represent the height in centimetres of a cylindrical tin can with diameter $x\,\text{cm}$.

$$h(x) = \frac{640}{x^2} + 0.5 \text{ for } 4 \le x \le 14.$$

(a) Find the range of h. [3]

The function h^{-1} is the inverse function of h.

(b) (i) Find $h^{-1}(10)$.

(ii) In the context of the question, interpret your answer to part (b)(i).

(iii) Write down the range of h^{-1}. [4]

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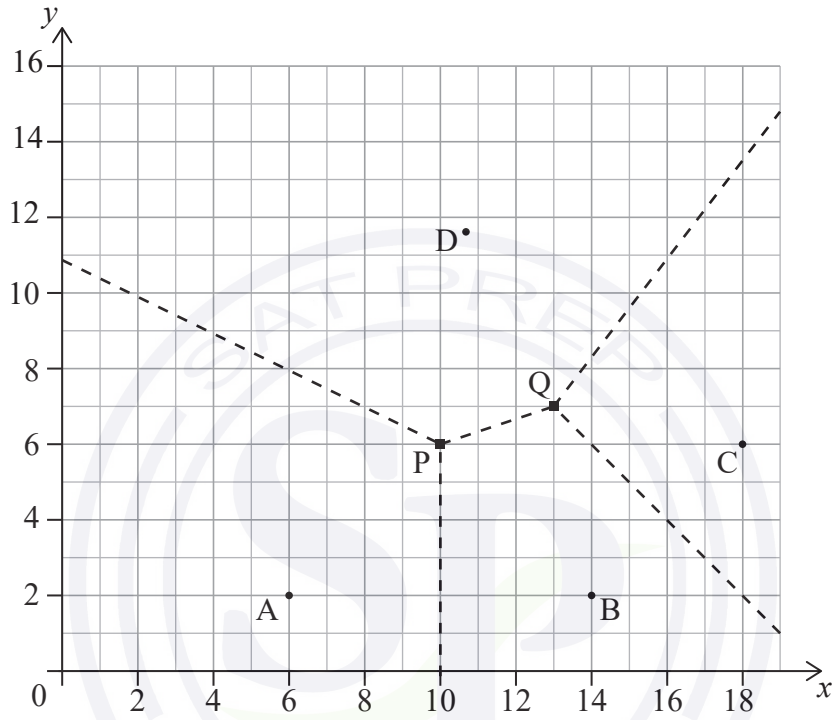
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4. [Maximum mark: 6]

There are four stations used by the fire wardens in a national forest.

On the following Voronoi diagram, the coordinates of the stations are $A(6, 2)$, $B(14, 2)$, $C(18, 6)$ and $D(10.8, 11.6)$ where distances are measured in kilometres.

The dotted lines represent the boundaries of the regions patrolled by the fire warden at each station. The boundaries meet at $P(10, 6)$ and $Q(13, 7)$.



To reduce the areas of the regions that the fire wardens patrol, a new station is to be built within the quadrilateral $ABCD$. The new station will be located so that it is as far as possible from the nearest existing station.

(a) Show that the new station should be built at P. [3]

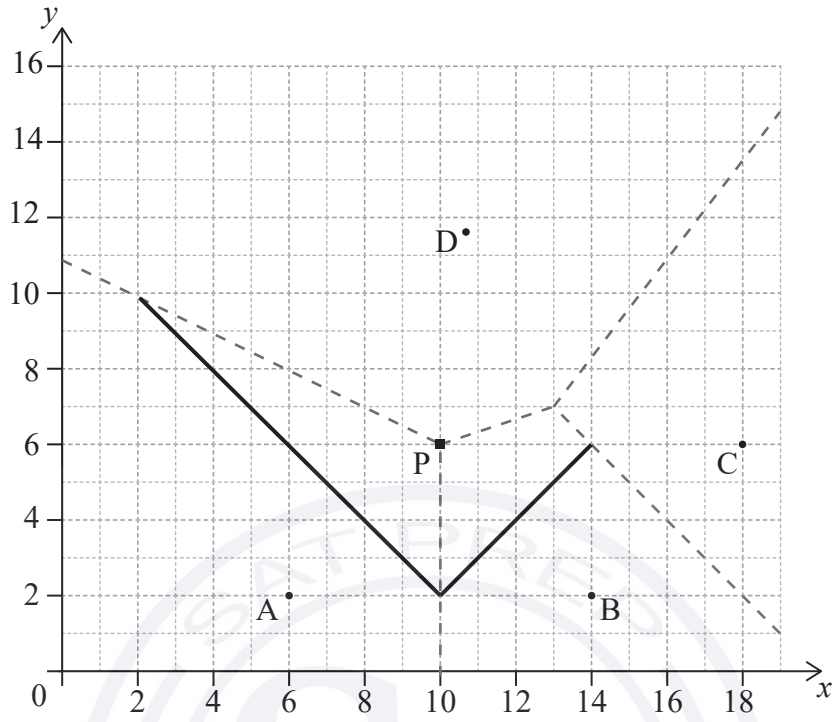
The Voronoi diagram is to be updated to include the region around the new station at P. The edges defined by the perpendicular bisectors of $[AP]$ and $[BP]$ have been added to the following diagram.

(b) (i) Write down the equation of the perpendicular bisector of $[PC]$.

(ii) Hence draw the missing boundaries of the region around P on the following diagram. [3]

(This question continues on the following page)

(Question 4 continued)



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5. [Maximum mark: 9]

In this question, give all answers correct to 2 decimal places.

Raul and Rosy want to buy a new house and they need a loan of 170 000 Australian dollars (AUD) from a bank. The loan is for 30 years and the annual interest rate for the loan is 3.8 %, compounded monthly. They will pay the loan in fixed monthly instalments at the end of each month.

(a) Find the amount they will pay the bank each month. [3]

(b) (i) Find the amount Raul and Rosy will still owe the bank at the end of the first 10 years.

(ii) Using your answers to parts (a) and (b)(i), calculate how much interest they will have paid in total during the first 10 years. [6]

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6. [Maximum mark: 5]

An infinite geometric sequence, with terms u_n, is such that $u_1 = 2$ and $\sum_{k=1}^{\infty} u_k = 10$.

(a) Find the common ratio, r, for the sequence. [2]

(b) Find the least value of n such that $u_n < \frac{1}{2}$. [3]

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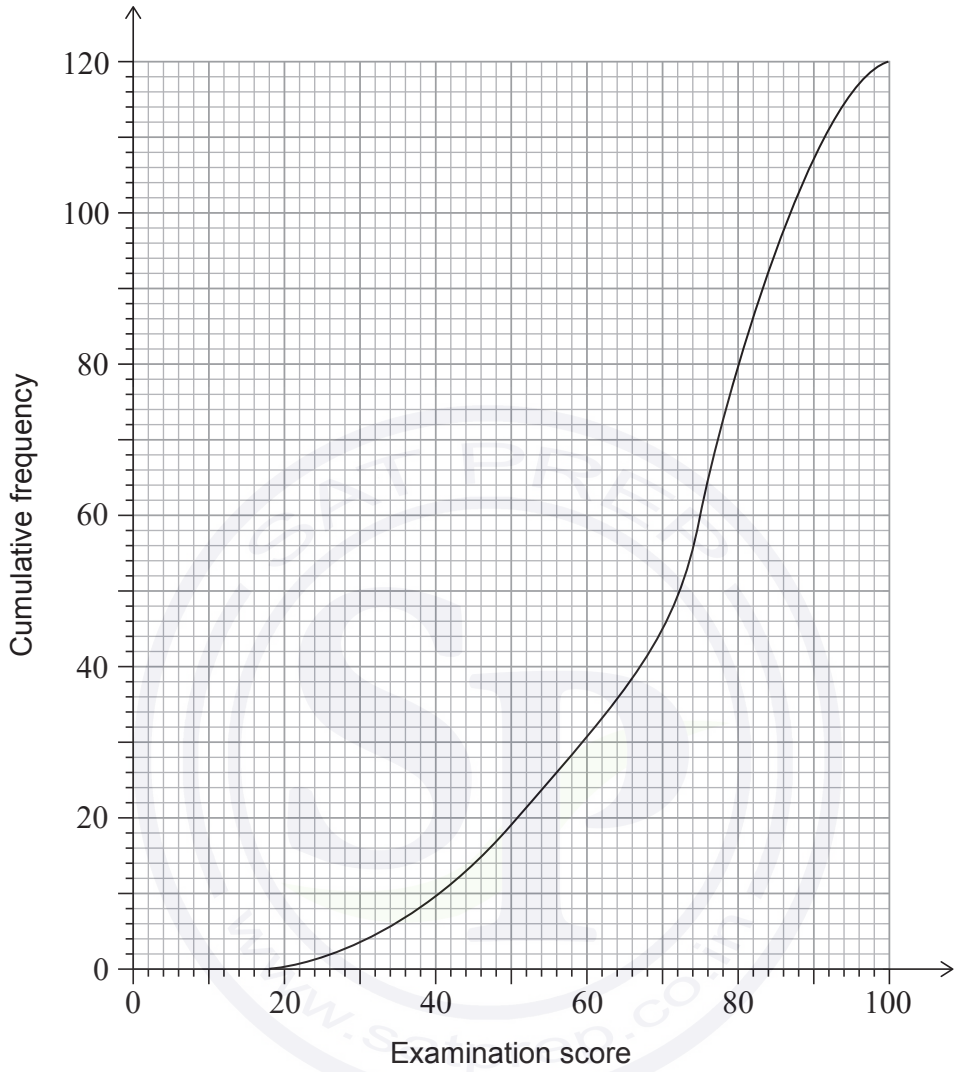
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7. [Maximum mark: 8]

A group of 120 students sat a history exam. The cumulative frequency graph shows the scores obtained by the students.



(a) Find the median of the scores obtained. [1]

The students were awarded a grade from 1 to 5, depending on the score obtained in the exam. The number of students receiving each grade is shown in the following table.

Grade	1	2	3	4	5
Number of students	6	13	26	a	b

(b) Find an expression for a in terms of b. [2]

(This question continues on the following page)

(Question 7 continued)

(c) The mean grade for these students is 3.65.

(i) Find the number of students who obtained a grade 5.

(ii) Find the minimum score needed to obtain a grade 5. [5]

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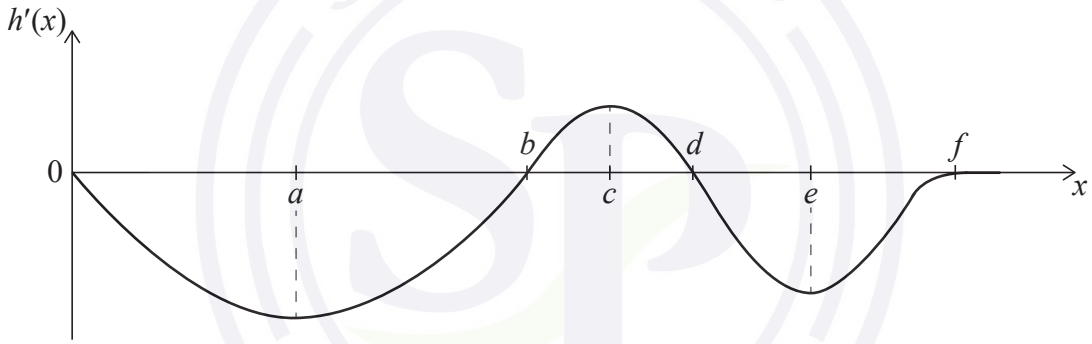
8. [Maximum mark: 5]

Juri skis from the top of a hill to a finishing point at the bottom of the hill. She takes the shortest route, heading directly to the finishing point (F).



Let $h(x)$ define the height of the hill above F at a horizontal distance x from the starting point at the top of the hill.

The graph of the **derivative** of $h(x)$ is shown below. The graph of $h'(x)$ has local minima and maxima when x is equal to a, c and e. The graph of $h'(x)$ intersects the x-axis when x is equal to b, d, and f.



(a) (i) Identify the x value of the point where $|h'(x)|$ has its maximum value.

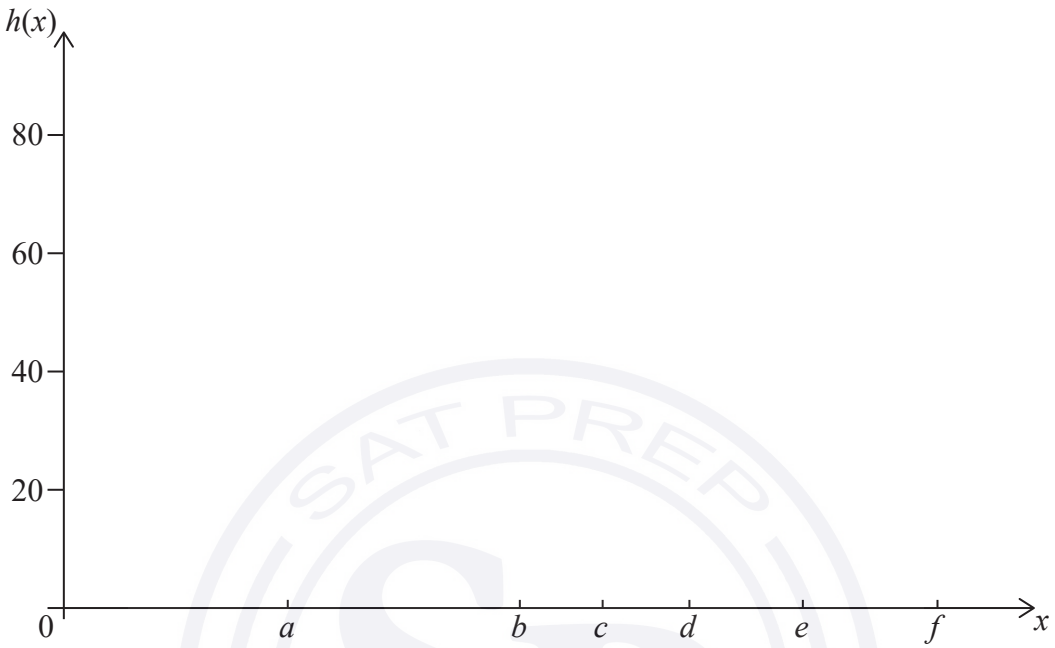
(ii) Interpret this point in the given context. [2]

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(Question 8 continued)

Juri starts at a height of 60 metres and finishes at F, where $x = f$.

(b) Sketch a possible diagram of the hill on the following pair of coordinate axes. [3]



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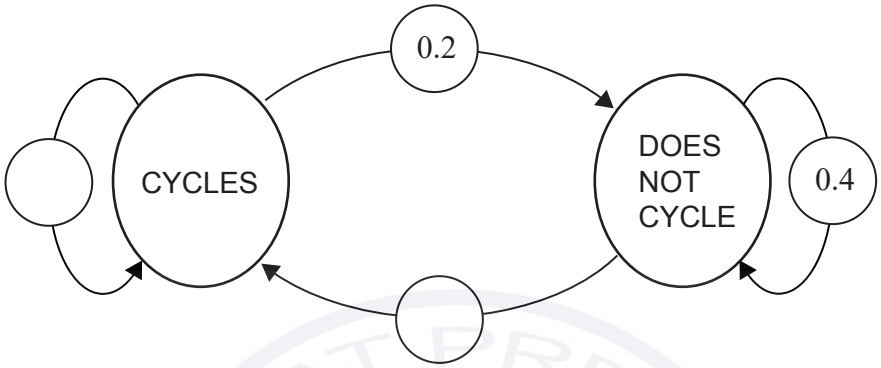
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9. [Maximum mark: 5]

Katie likes to cycle to work as much as possible. If Katie cycles to work one day then she has a probability of 0.2 of not cycling to work on the next work day. If she does not cycle to work one day then she has a probability of 0.4 of not cycling to work on the next work day.

(a) Complete the following transition diagram to represent this information. [2]



Katie works for 180 days in a year.

(b) Find the probability that Katie cycles to work on her final working day of the year. [3]

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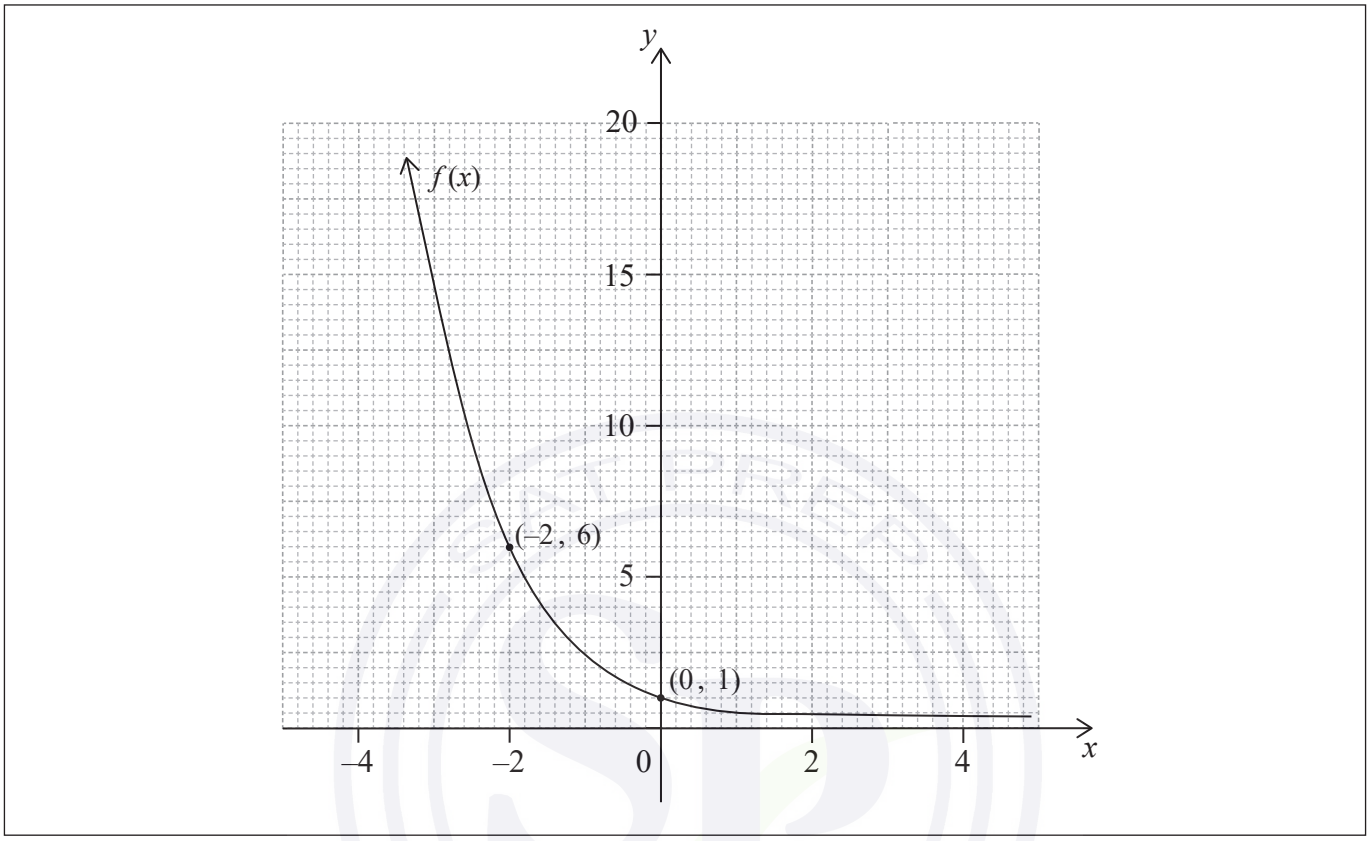
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10. [Maximum mark: 4]

The graph of $y = f(x)$ is given on the following set of axes. The graph passes through the points $(-2, 6)$ and $(0, 1)$, and has a horizontal asymptote at $y = 0$.



Let $g(x) = 2f(x-2) + 4$.

(a) Find $g(0)$. [2]

(b) On the same set of axes draw the graph of $y = g(x)$, showing any intercepts and asymptotes. [2]

(This question continues on the following page)

(Question 10 continued)

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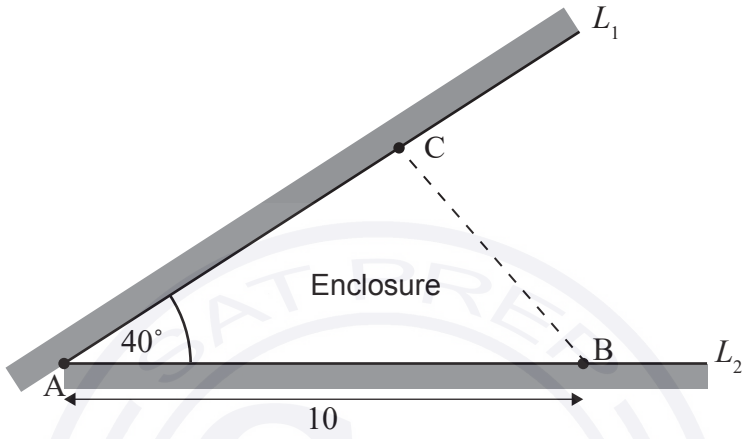
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11. [Maximum mark: 6]

The following diagram shows a corner of a field bounded by two walls defined by lines L_1 and L_2. The walls meet at a point A, making an angle of 40°.

Farmer Nate has $7\,\text{m}$ of fencing to make a triangular enclosure for his sheep. One end of the fence is positioned at a point B on L_2, $10\,\text{m}$ from A. The other end of the fence will be positioned at some point C on L_1, as shown on the diagram.

diagram not to scale



He wants the enclosure to take up as little of the current field as possible.

Find the minimum possible area of the triangular enclosure ABC. [6]

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12. [Maximum mark: 5]

The following table shows the time, in days, from December 1st and the percentage of Christmas trees in stock at a shop on the beginning of that day.

Days since December 1st (d)	1	3	6	9	12	15	18
Percentage of Christmas trees left in stock (x)	100	51	29	21	18	16	14

The following table shows the natural logarithm of both d and x on these days to 2 decimal places.

ln (d)	0	1.10	1.79	2.20	2.48	2.71	2.89
ln (x)	4.61	3.93	3.37	3.04	2.89	2.77	2.64

(a) Use the data in the second table to find the value of m and the value of b for the regression line, $\ln x = m(\ln d) + b$. [2]

(b) Assuming that the model found in part (a) remains valid, estimate the percentage of trees in stock when $d = 25$. [3]

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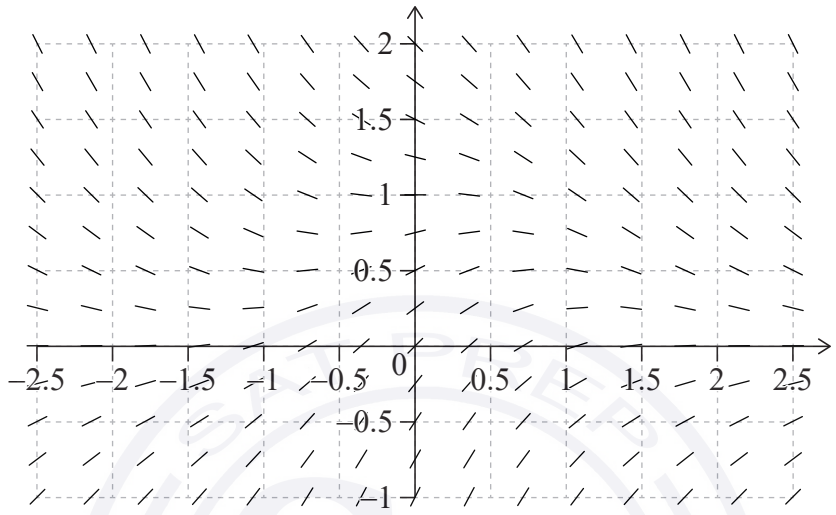
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13. [Maximum mark: 7]

The slope field for the differential equation $\frac{dy}{dx} = e^{-x^2} - y$ is shown in the following two graphs.

(a) Calculate the value of $\frac{dy}{dx}$ at the point $(0, 1)$. [1]

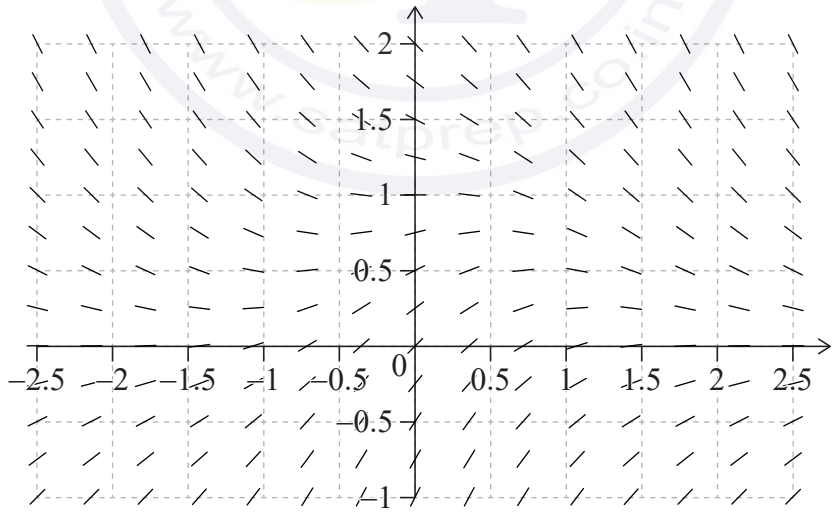
(b) Sketch, on the first graph, a curve that represents the points where $\frac{dy}{dx} = 0$. [2]



(c) On the second graph,

(i) sketch the solution curve that passes through the point $(0, 0)$.

(ii) sketch the solution curve that passes through the point $(0, 0.75)$. [4]



(This question continues on the following page)

(Question 13 continued)

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14. [Maximum mark: 7]

On Paul's farm, potatoes are packed in sacks labelled $50\,\text{kg}$. The weights of the sacks of potatoes can be modelled by a normal distribution with mean weight $49.8\,\text{kg}$ and standard deviation $0.9\,\text{kg}$.

(a) Find the probability that a sack is under its labelled weight. [2]

(b) Find the lower quartile of the weights of the sacks of potatoes. [2]

The sacks of potatoes are transported in crates. There are 10 sacks in each crate and the weights of the sacks of potatoes are independent of each other.

(c) Find the probability that the total weight of the sacks of potatoes in a crate exceeds $500\,\text{kg}$. [3]

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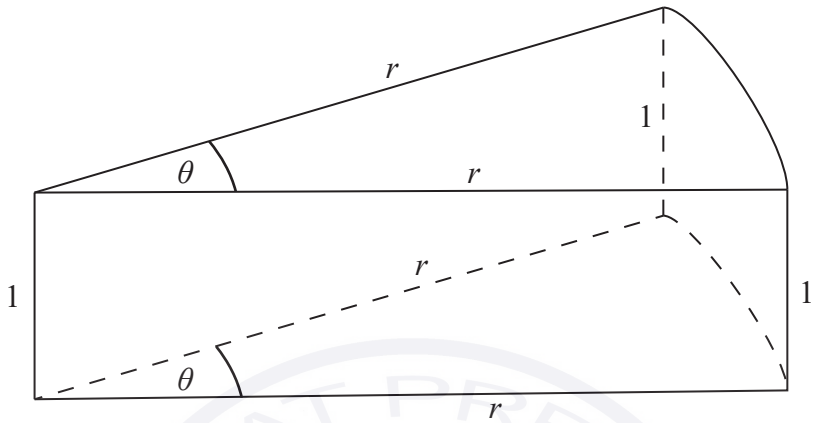
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15. [Maximum mark: 9]

The following diagram shows a frame that is made from wire. The total length of wire is equal to $15\,\text{cm}$. The frame is made up of two identical sectors of a circle that are parallel to each other. The sectors have angle θ radians and radius $r\,\text{cm}$. They are connected by $1\,\text{cm}$ lengths of wire perpendicular to the sectors. This is shown in the diagram below.



(a) Show that $r = \frac{6}{2+\theta}$. [2]

The faces of the frame are covered by paper to enclose a volume, V.

(b) (i) Find an expression for V in terms of θ.

(ii) Find the expression $\frac{\mathrm{d}V}{\mathrm{d}\theta}$.

(iii) Solve algebraically $\frac{\mathrm{d}V}{\mathrm{d}\theta} = 0$ to find the value of θ that will maximize the volume, V. [7]

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16. [Maximum mark: 9]

A ship S is travelling with a constant velocity, $\boldsymbol{v}$, measured in kilometres per hour, where

$$\boldsymbol{v} = \begin{pmatrix} -12 \\ 15 \end{pmatrix}.$$

At time $t = 0$ the ship is at a point $A(300, 100)$ relative to an origin O, where distances are measured in kilometres.

(a) Find the position vector $\overrightarrow{OS}$ of the ship at time t hours. [1]

A lighthouse is located at a point $(129, 283)$.

(b) Find the value of t when the ship will be closest to the lighthouse. [6]

An alarm will sound if the ship travels within 20 kilometres of the lighthouse.

(c) State whether the alarm will sound. Give a reason for your answer. [2]

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17. [Maximum mark: 7]

The sides of a bowl are formed by rotating the curve $y = 6\ln x,\ 0 \le y \le 9$, about the y-axis, where x and y are measured in centimetres. The bowl contains water to a height of $h\,\text{cm}$.

(a) Show that the volume of water, V, in terms of h is $V = 3\pi\left(e^{\frac{h}{3}} - 1\right)$. [5]

(b) Hence find the maximum capacity of the bowl in cm^3. [2]

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Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: applications and interpretation
Higher level
Paper 1

Thursday 6 May 2021 (afternoon)

Candidate session number

2 hours

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all questions.
- Answers must be written within the answer boxes provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.

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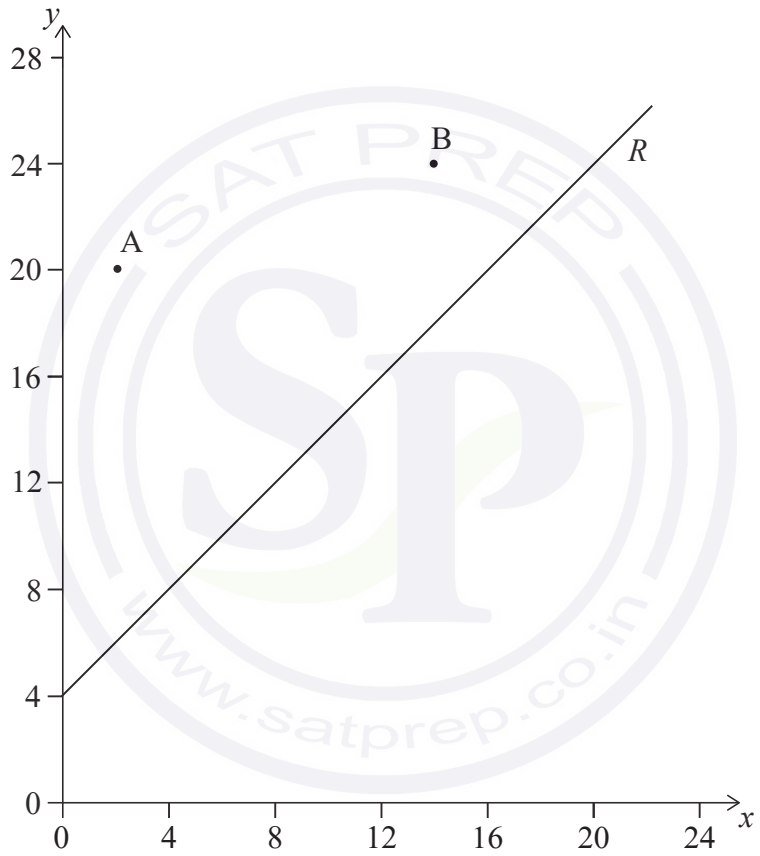
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Answers must be written within the answer boxes provided. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 7]

Two schools are represented by points $\mathrm{A}(2,\ 20)$ and $\mathrm{B}(14,\ 24)$ on the graph below. A road, represented by the line R with equation $-x + y = 4$, passes near the schools. An architect is asked to determine the location of a new bus stop on the road such that it is the same distance from the two schools.



(a) Find the equation of the perpendicular bisector of $[\mathrm{AB}]$. Give your equation in the form $y = mx + c$. [5]

(b) Determine the coordinates of the point on R where the bus stop should be located. [2]

(This question continues on the following page)

(Question 1 continued)

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2. [Maximum mark: 7]

A function is defined by $f(x) = 2 - \frac{12}{x+5}$ for $-7 \leq x \leq 7$, $x \neq -5$.

(a) Find the range of f. [3]

(b) Find an expression for the inverse function $f^{-1}(x)$. The domain is not required. [3]

(c) Write down the range of $f^{-1}(x)$. [1]

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3. [Maximum mark: 6]

At Springfield University, the weights, in kg, of 10 chinchilla rabbits and 10 sable rabbits were recorded. The aim was to find out whether chinchilla rabbits are generally heavier than sable rabbits. The results obtained are summarized in the following table.

Weight of chinchilla rabbits, kg	4.9	4.2	4.1	4.4	4.3	4.6	4.0	4.7	4.5	4.4
Weight of sable rabbits, kg	4.2	4.1	4.1	4.2	4.5	4.4	4.5	3.9	4.2	4.0

A t-test is to be performed at the 5% significance level.

(a) Write down the null and alternative hypotheses. [2]

(b) Find the p-value for this test. [2]

(c) Write down the conclusion to the test. Give a reason for your answer. [2]

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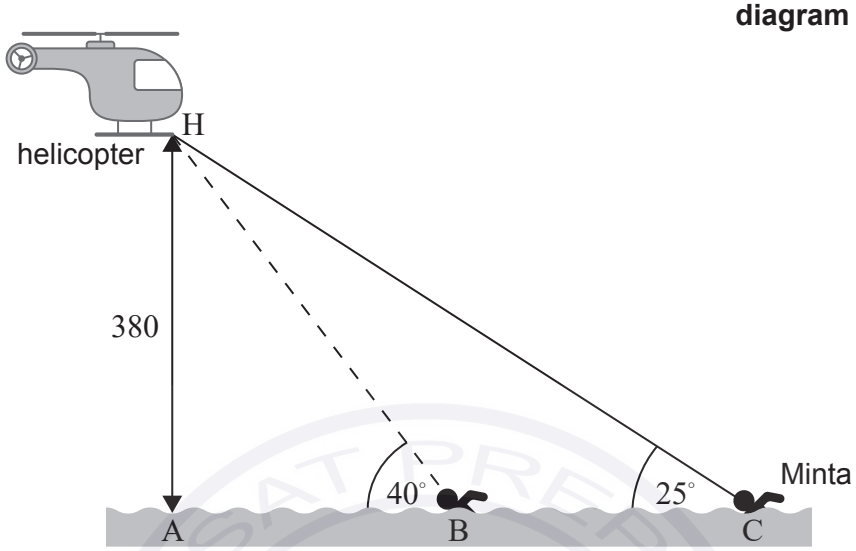
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4. [Maximum mark: 6]

The diagram below shows a helicopter hovering at point H, 380 m vertically above a lake. Point A is the point on the surface of the lake, directly below the helicopter.

diagram not to scale



Minta is swimming at a constant speed in the direction of point A. Minta observes the helicopter from point C as she looks upward at an angle of 25°. After 15 minutes, Minta is at point B and she observes the same helicopter at an angle of 40°.

(a) Find the distance from A to C. [2]

(b) Find the distance from B to C. [3]

(c) Find Minta's speed, in metres per hour. [1]

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5. [Maximum mark: 5]

Roger buys a new laptop for himself at a cost of £495. At the same time, he buys his daughter Chloe a higher specification laptop at a cost of £2200.

It is anticipated that Roger's laptop will depreciate at a rate of 10% per year, whereas Chloe's laptop will depreciate at a rate of 15% per year.

(a) Estimate the value of Roger's laptop after 5 years. [2]

Roger and Chloe's laptops will have the same value k years after they were purchased.

(b) Find the value of k. [2]

(c) Comment on the validity of your answer to part (b). [1]

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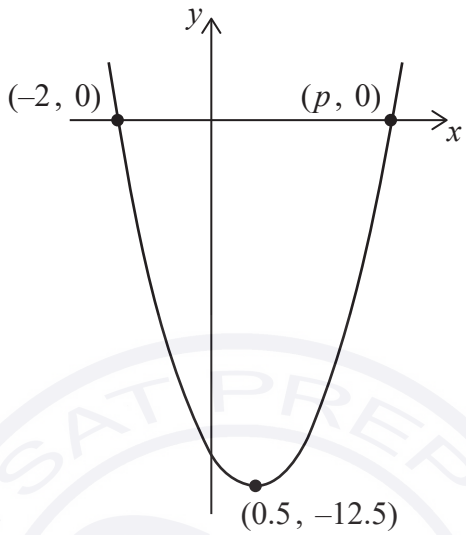
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6. [Maximum mark: 7]

Consider the function $f(x) = ax^2 + bx + c$. The graph of $y = f(x)$ is shown in the diagram. The vertex of the graph has coordinates $(0.5, -12.5)$. The graph intersects the x-axis at two points, $(-2, 0)$ and $(p, 0)$.

diagram not to scale



(a) Find the value of p. [1]

(b) Find the value of

(i) a.

(ii) b.

(iii) c. [5]

(c) Write down the equation of the axis of symmetry of the graph. [1]

(This question continues on the following page)

(Question 6 continued)

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7. [Maximum mark: 6]

A meteorologist models the height of a hot air balloon launched from the ground. The model assumes the balloon travels vertically upwards and travels 450 metres in the first minute.

Due to the decrease in temperature as the balloon rises, the balloon will continually slow down. The model suggests that each minute the balloon will travel only 82% of the distance travelled in the previous minute.

(a) Find how high the balloon will travel in the first 10 minutes after it is launched. [3]

(b) The balloon is required to reach a height of at least 2520 metres.

Determine whether it will reach this height. [2]

(c) Suggest a limitation of the given model. [1]

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8. [Maximum mark: 7]

Two lines L_1 and L_2 are given by the following equations, where $p \in \mathbb{R}$.

$$L_1 : r = \begin{pmatrix} 2 \\ p+9 \\ -3 \end{pmatrix} + \lambda \begin{pmatrix} p \\ 2p \\ 4 \end{pmatrix}$$

$$L_2 : r = \begin{pmatrix} 14 \\ 7 \\ p+12 \end{pmatrix} + \mu \begin{pmatrix} p+4 \\ 4 \\ -7 \end{pmatrix}$$

It is known that L_1 and L_2 are perpendicular.

(a) Find the possible value(s) for p. [3]

(b) In the case that $p < 0$, determine whether the lines intersect. [4]

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9. [Maximum mark: 8]

A newspaper vendor in Singapore is trying to predict how many copies of *The Straits Times* they will sell. The vendor forms a model to predict the number of copies sold each weekday. According to this model, they expect the same number of copies will be sold each day.

To test the model, they record the number of copies sold each weekday during a particular week. This data is shown in the table.

Day	Monday	Tuesday	Wednesday	Thursday	Friday
Number of copies sold	74	97	91	86	112

A goodness of fit test at the 5% significance level is used on this data to determine whether the vendor's model is suitable. The critical value for the test is 9.49.

(a) Find an estimate for how many copies the vendor expects to sell each day. [1]

(b) (i) State the null and alternative hypotheses for this test.

(ii) Write down the degrees of freedom for this test.

(iii) Write down the conclusion to the test. Give a reason for your answer. [7]

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10. [Maximum mark: 6]

A manufacturer of chocolates produces them in individual packets, claiming to have an average of 85 chocolates per packet.

Talha bought 30 of these packets in order to check the manufacturer's claim.

Given that the number of individual chocolates is x, Talha found that, from his 30 packets, $\sum x = 2506$ and $\sum x^2 = 209\,738$.

(a) Find an unbiased estimate for the mean number (μ) of chocolates per packet. [1]

(b) Use the formula $s_{n-1}^2 = \dfrac{\sum x^2 - \frac{(\sum x)^2}{n}}{n-1}$ to determine an unbiased estimate for the variance of the number of chocolates per packet. [2]

(c) Find a $95\,\%$ confidence interval for μ. You may assume that all conditions for a confidence interval have been met. [2]

(d) Suggest, with justification, a valid conclusion that Talha could make. [1]

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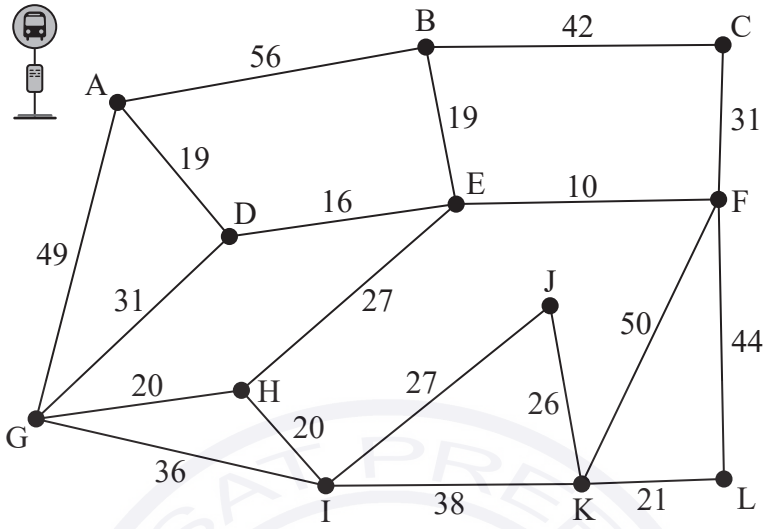
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11. [Maximum mark: 7]

The diagram below shows a network of roads in a small village with the weights indicating the distance of each road, in metres, and junctions indicated with letters.



Musab is required to deliver leaflets to every house on each road. He wishes to minimize his total distance.

(a) Musab starts and finishes from the village bus-stop at A. Determine the total distance Musab will need to walk. [5]

Instead of having to catch the bus to the village, Musab's sister offers to drop him off at any junction and pick him up at any other junction of his choice.

(b) Explain which junctions Musab should choose as his starting and finishing points. [2]

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12. [Maximum mark: 8]

It is given that $z_1 = 3\,\text{cis}\left(\frac{3\pi}{4}\right)$ and $z_2 = 2\,\text{cis}\left(\frac{n\pi}{16}\right)$, $n \in \mathbb{Z}^+$.

(a) In parts (a)(i) and (a)(ii), give your answers in the form $re^{i\theta}$, $r \geq 0$, $-\pi < \theta \leq \pi$.

(i) Find the value of $z_1^{\,3}$.

(ii) Find the value of $\left(\frac{z_1}{z_2}\right)^4$ for $n = 2$. [5]

(b) Find the least value of n such that $z_1 z_2 \in \mathbb{R}^+$. [3]

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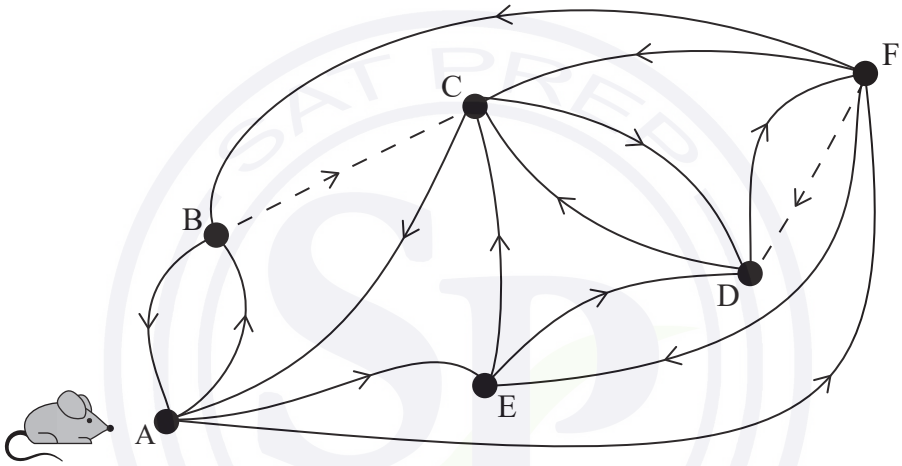
13. [Maximum mark: 8]

The graph below shows a small maze, in the form of a network of directed routes. The vertices A to F show junctions in the maze and the edges show the possible paths available from one vertex to another.

A mouse is placed at vertex A and left to wander the maze freely. The routes shown by dashed lines indicate paths sprinkled with sugar.

When the mouse reaches any junction, she rests for a constant time before continuing.

At any junction, it may also be assumed that
- the mouse chooses any available normal path with equal probability
- if the junction includes a path sprinkled with sugar, the probability of choosing this path is twice that of a normal path.



(a) Determine the transition matrix for this graph. [3]

(b) If the mouse was left to wander indefinitely, use your graphic display calculator to estimate the percentage of time that the mouse would spend at point F. [3]

(c) Comment on your answer to part (b), referring to at least one limitation of the model. [2]

(This question continues on the following page)

(Question 13 continued)

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14. [Maximum mark: 7]

A geometric transformation $T: \begin{pmatrix} x \\ y \end{pmatrix} \mapsto \begin{pmatrix} x' \\ y' \end{pmatrix}$ is defined by

$$T: \begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 7 & -10 \\ 2 & -3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} -5 \\ 4 \end{pmatrix}.$$

(a) Find the coordinates of the image of the point $(6, -2)$. [2]

(b) Given that $T: \begin{pmatrix} p \\ q \end{pmatrix} \mapsto 2\begin{pmatrix} p \\ q \end{pmatrix}$, find the value of p and the value of q. [3]

(c) A triangle L with vertices lying on the xy plane is transformed by T.

Explain why both L and its image will have exactly the same area. [2]

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15. [Maximum mark: 7]

The number of coffees sold per hour at an independent coffee shop is modelled by a Poisson distribution with a mean of 22 coffees per hour.

Sheila, the shop's owner wants to increase the number of coffees sold in the shop. She decides to offer a discount to customers who buy more than one coffee.

To test how successful this strategy is, Sheila records the number of coffees sold over a single 5-hour period. Sheila decides to use a 5% level of significance in her test.

(a) State the null and alternative hypotheses for the test. [1]

(b) Find the probability that Sheila will make a type I error in her test conclusion. [4]

Sheila finds 126 coffees were sold during the 5-hour period.

(c) State Sheila's conclusion to the test. Justify your answer. [2]

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16. [Maximum mark: 8]

A particle P moves in a straight line, such that its displacement x at time $t\,(t \geq 0)$ is defined by the differential equation $\dot{x} = x\cos t\left(\mathrm{e}^{-\sin t}\right)$. At time $t = 0$, $x = \frac{1}{\mathrm{e}}$.

(a) By using Euler's method with a step length of 0.1, find an approximate value for x when $t = 0.3$. [3]

(b) By solving the differential equation, find the percentage error in your approximation for x when $t = 0.3$. [5]

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References:

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Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: applications and interpretation
Higher level
Paper 1

Thursday 6 May 2021 (afternoon)

Candidate session number

2 hours

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all questions.
- Answers must be written within the answer boxes provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.

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2221 – 7206

24 pages

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Answers must be written within the answer boxes provided. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 4]

George goes fishing. From experience he knows that the mean number of fish he catches per hour is 1.1. It is assumed that the number of fish he catches can be modelled by a Poisson distribution.

On a day in which George spends 8 hours fishing, find the probability that he will catch more than 9 fish.

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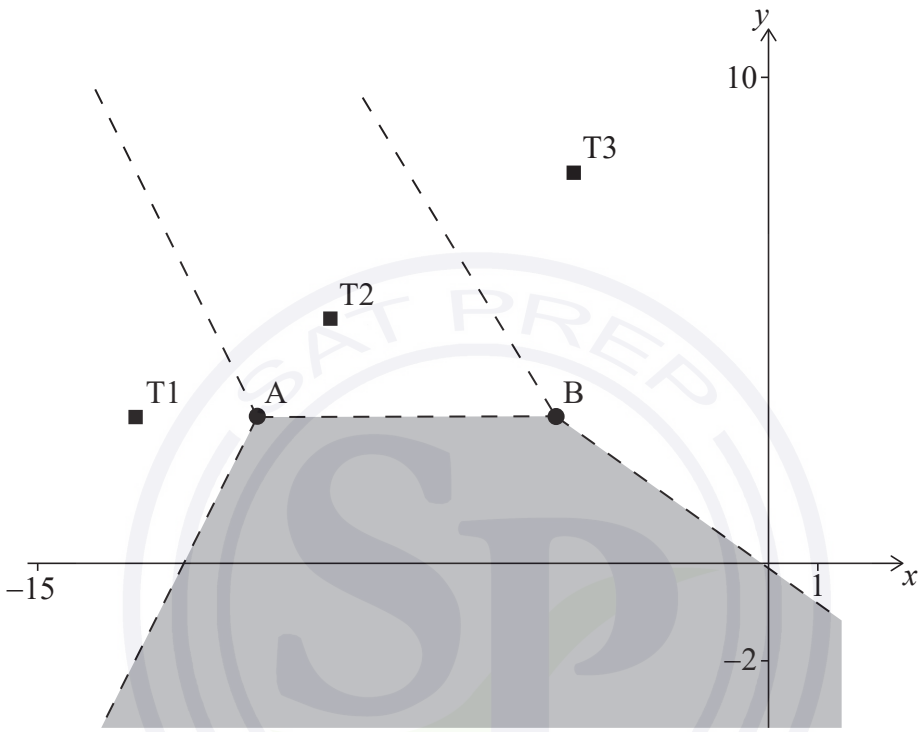
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2. [Maximum mark: 6]

The Voronoi diagram below shows three identical cellular phone towers, T1, T2 and T3. A fourth cellular phone tower, T4 is located in the shaded region. The dashed lines in the diagram below represent the edges in the Voronoi diagram.

Horizontal scale: 1 unit represents 1 km.
Vertical scale: 1 unit represents 1 km.



Tim stands inside the shaded region.

(a) Explain why Tim will receive the strongest signal from tower T4. [1]

Tower T2 has coordinates $(-9, 5)$ and the edge connecting vertices A and B has equation $y = 3$.

(b) Write down the coordinates of tower T4. [2]

Tower T1 has coordinates $(-13, 3)$.

(c) Find the gradient of the edge of the Voronoi diagram between towers T1 and T2. [3]

(This question continues on the following page)

(Question 2 continued)

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3. [Maximum mark: 8]

Charlie and Daniella each began a fitness programme. On day one, they both ran $500\,\text{m}$. On each subsequent day, Charlie ran $100\,\text{m}$ more than the previous day whereas Daniella increased her distance by 2% of the distance ran on the previous day.

(a) Calculate how far

(i) Charlie ran on day 20 of his fitness programme.

(ii) Daniella ran on day 20 of her fitness programme. [5]

On day n of the fitness programmes Daniella runs more than Charlie for the first time.

(b) Find the value of n. [3]

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4. [Maximum mark: 6]

Professor Wei observed that students have difficulty remembering the information presented in his lectures.

He modelled the percentage of information retained, R, by the function $R(t) = 100\,e^{-pt}$, $t \geq 0$, where t is the number of days after the lecture.

He found that 1 day after a lecture, students had forgotten 50% of the information presented.

(a) Find the value of p. [2]

(b) Use this model to find the percentage of information retained by his students 36 hours after Professor Wei's lecture. [2]

Based on his model, Professor Wei believes that his students will always retain some information from his lecture.

(c) State a mathematical reason why Professor Wei might believe this. [1]

(d) Write down one possible limitation of the **domain** of the model. [1]

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5. [Maximum mark: 6]

A garden has a triangular sunshade suspended from three points $A(2, 0, 2)$, $B(8, 0, 2)$ and $C(5, 4, 3)$, relative to an origin in the corner of the garden. All distances are measured in metres.

(a) (i) Find $\overrightarrow{CA}$.

(ii) Find $\overrightarrow{CB}$. [2]

(b) Find $\overrightarrow{CA} \times \overrightarrow{CB}$. [2]

(c) Hence find the area of the triangle ABC. [2]

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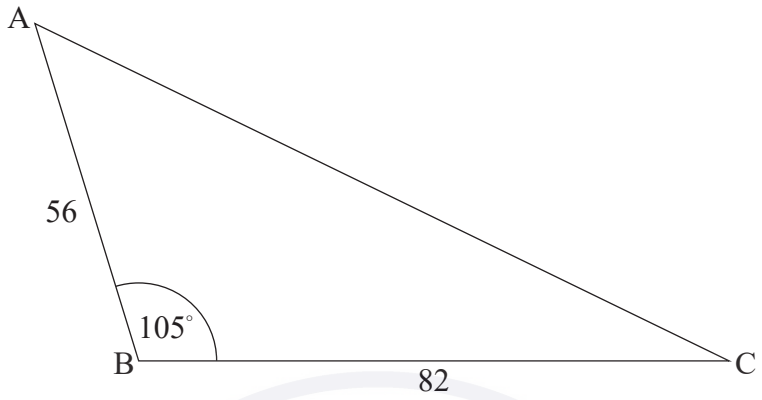
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6. [Maximum mark: 5]

A triangular field ABC is such that $AB = 56\,\text{m}$ and $BC = 82\,\text{m}$, each measured correct to the nearest metre, and the angle at B is equal to 105°, measured correct to the nearest 5°.

diagram not to scale



Calculate the maximum possible area of the field.

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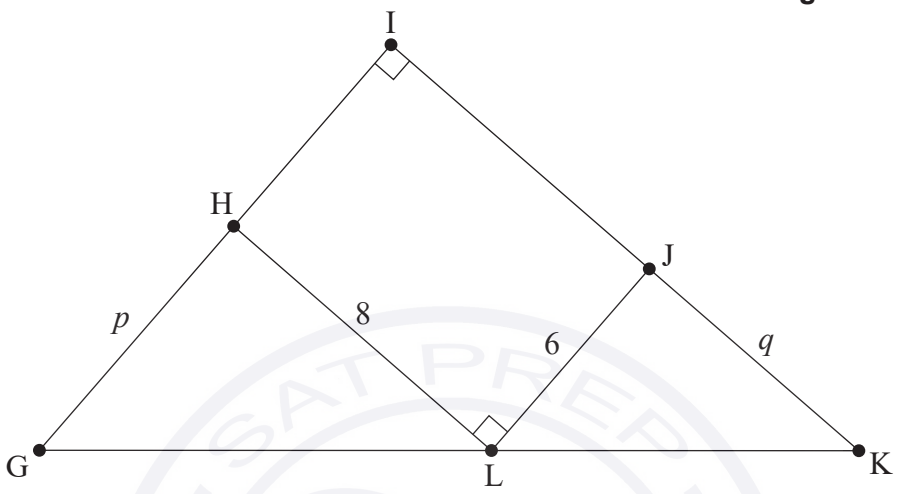
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7. [Maximum mark: 8]

Ellis designs a gift box. The top of the gift box is in the shape of a right-angled triangle GIK.

A rectangular section HIJL is inscribed inside this triangle. The lengths of GH, JK, HL, and LJ are p cm, q cm, 8 cm and 6 cm respectively.

diagram not to scale



The area of the top of the gift box is A cm^2.

(a) (i) Find A in terms of p and q.

(ii) Show that $A = \frac{192}{q} + 3q + 48$. [4]

(b) Find $\frac{dA}{dq}$. [2]

Ellis wishes to find the value of q that will minimize the area of the top of the gift box.

(c) (i) Write down an equation Ellis could solve to find this value of q.

(ii) Hence, or otherwise, find this value of q. [2]

(This question continues on the following page)

(Question 7 continued)

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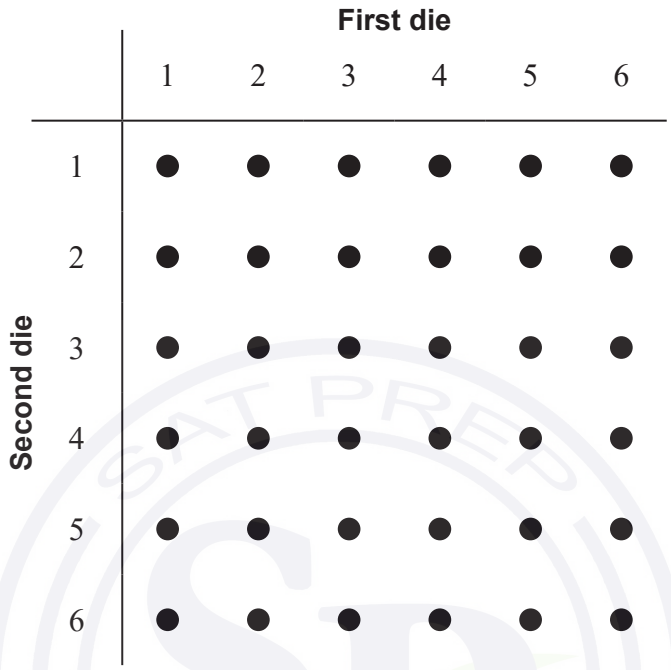
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8. [Maximum mark: 7]

A game is played where two unbiased dice are rolled and the score in the game is the greater of the two numbers shown. If the two numbers are the same, then the score in the game is the number shown on one of the dice. A diagram showing the possible outcomes is given below.



Let T be the random variable "the score in a game".

(a) Complete the table to show the probability distribution of T. [2]

t	1	2	3	4	5	6
$\mathbf{P}(T = t)$						

(b) Find the probability that

(i) a player scores at least 3 in a game.

(ii) a player scores 6, given that they scored at least 3. [3]

(c) Find the expected score of a game. [2]

(This question continues on the following page).

(Question 8 continued)

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9. [Maximum mark: 8]

Consider $w = \mathrm{i}z + 1$, where $w, z \in \mathbb{C}$.

(a) Find w when

(i) $z = 2\mathrm{i}$.

(ii) $z = 1 + \mathrm{i}$. [3]

Point z on the Argand diagram can be transformed to point w by two transformations.

(b) Describe these two transformations and give the order in which they are applied. [3]

(c) Hence, or otherwise, find the value of z when $w = 2 - \mathrm{i}$. [2]

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10. [Maximum mark: 7]

An engineer plans to visit six oil rigs (A–F) in the Gulf of Mexico, starting and finishing at A. The travelling time, in minutes, between each of the rigs is shown in the table.

	A	**B**	**C**	**D**	**E**	**F**
A		55	63	79	87	93
B	55		46	58	88	92
C	63	46		87	77	66
D	79	58	87		23	70
E	87	88	77	23		47
F	93	92	66	70	47	

The data above can be represented by a graph G.

(a) (i) Use Prim's algorithm to find the weight of the minimum spanning tree of the subgraph of G obtained by deleting A and starting at B. List the order in which the edges are selected.

(ii) Hence find a lower bound for the travelling time needed to visit all the oil rigs. [6]

(b) Describe how an improved lower bound might be found. [1]

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11. [Maximum mark: 7]

A factory, producing plastic gifts for a fast food restaurant's Jolly meals, claims that just 1% of the toys produced are faulty.

A restaurant manager wants to test this claim. A box of 200 toys is delivered to the restaurant. The manager checks all the toys in this box and four toys are found to be faulty.

(a) Identify the type of sampling used by the restaurant manager. [1]

The restaurant manager performs a one-tailed hypothesis test, at the 10% significance level, to determine whether the factory's claim is reasonable. It is known that faults in the toys occur independently.

(b) Write down the null and alternative hypotheses. [2]

(c) Find the p-value for the test. [2]

(d) State the conclusion of the test. Give a reason for your answer. [2]

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12. [Maximum mark: 8]

A tank of water initially contains 400 litres. Water is leaking from the tank such that after 10 minutes there are 324 litres remaining in the tank.

The volume of water, V litres, remaining in the tank after t minutes, can be modelled by the differential equation

$$\frac{\mathrm{d}V}{\mathrm{d}t} = -k\sqrt{V}, \text{ where } k \text{ is a constant.}$$

(a) Show that $V = \left(20 - \frac{t}{5}\right)^2$. [6]

(b) Find the time taken for the tank to empty. [2]

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13. [Maximum mark: 7]

A submarine is located in a sea at coordinates $(0.8, 1.3, -0.3)$ relative to a ship positioned at the origin O. The x direction is due east, the y direction is due north and the z direction is vertically upwards.

All distances are measured in kilometres.

The submarine travels with direction vector $\begin{pmatrix} -2 \\ -3 \\ 1 \end{pmatrix}$.

(a) Assuming the submarine travels in a straight line, write down an equation for the line along which it travels. [2]

The submarine reaches the surface of the sea at the point P.

(b) (i) Find the coordinates of P.

(ii) Find OP. [5]

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14. [Maximum mark: 6]

The weights of apples from Tony's farm follow a normal distribution with mean $158\,\text{g}$ and standard deviation $13\,\text{g}$. The apples are sold in bags that contain six apples.

(a) Find the mean weight of a bag of apples. [2]

(b) Find the standard deviation of the weights of these bags of apples. [2]

(c) Find the probability that a bag selected at random weighs more than $1\,\text{kg}$. [2]

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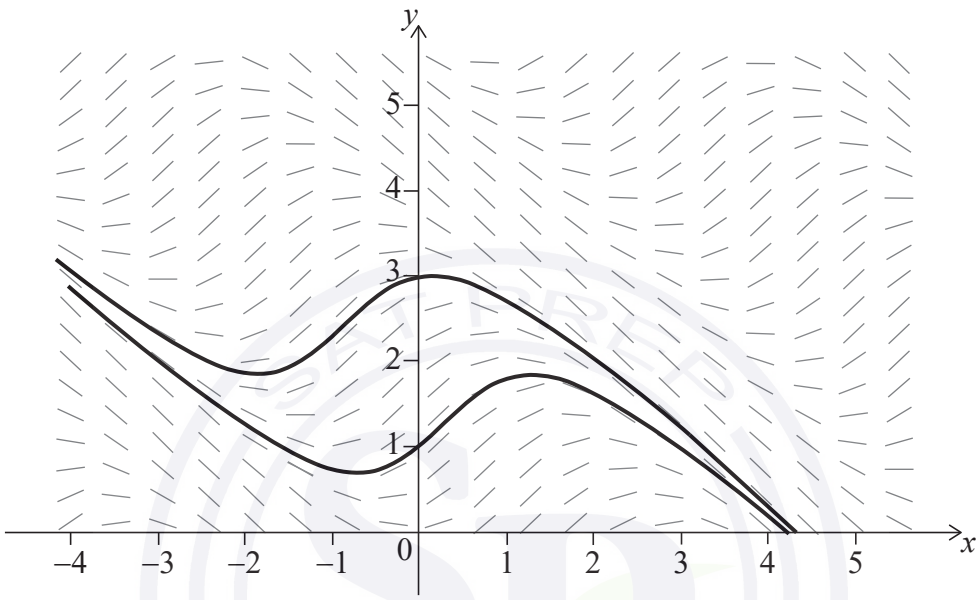
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15. [Maximum mark: 5]

The diagram shows the slope field for the differential equation

$$\frac{dy}{dx} = \sin(x + y),\ -4 \le x \le 5,\ 0 \le y \le 5.$$

The graphs of the two solutions to the differential equation that pass through points $(0, 1)$ and $(0, 3)$ are shown.



For the two solutions given, the local minimum points lie on the straight line L_1.

(a) Find the equation of L_1, giving your answer in the form $y = mx + c$. [3]

For the two solutions given, the local maximum points lie on the straight line L_2.

(b) Find the equation of L_2. [2]

(This question continues on the following page)

(Question 15 continued)

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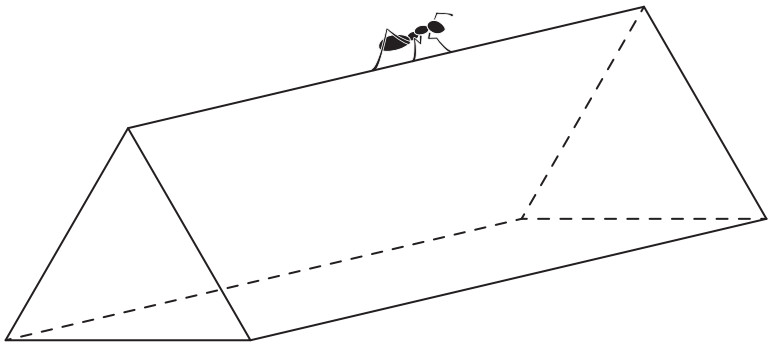
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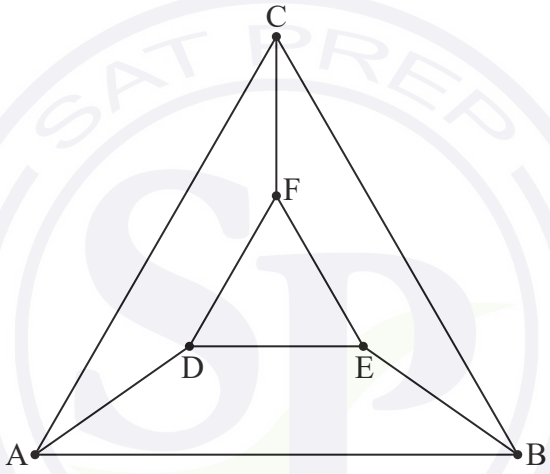
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16. [Maximum mark: 5]

An ant is walking along the edges of a wire frame in the shape of a triangular prism.



The vertices and edges of this frame can be represented by the graph below.



(a) Write down the adjacency matrix, $\boldsymbol{M}$, for this graph. [3]

(b) Find the number of ways that the ant can start at the vertex A, and walk along exactly 6 edges to return to A. [2]

(This question continues on the following page)

(Question 16 continued)

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17. [Maximum mark: 7]

The graph of the function $f(x) = \ln x$ is translated by $\begin{pmatrix} a \\ b \end{pmatrix}$ so that it then passes through the points $(0, 1)$ and $(e^3, 1 + \ln 2)$.

Find the value of a and the value of b.

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Diploma Programme
Programme du diplôme
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Mathematics: applications and interpretation
Higher level
Paper 1

Specimen paper

Candidate session number

2 hours

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- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all questions.
- Answers must be written within the answer boxes provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.

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1. [Maximum mark: 6]

At the end of a school day, the Headmaster conducted a survey asking students in how many classes they had used the internet.

The data is shown in the following table.

Number of classes in which the students used the internet	0	1	2	3	4	5	6
Number of students	20	24	30	k	10	3	1

(a) State whether the data is discrete or continuous. [1]

The mean number of classes in which a student used the internet is 2.

(b) Find the value of k. [4]

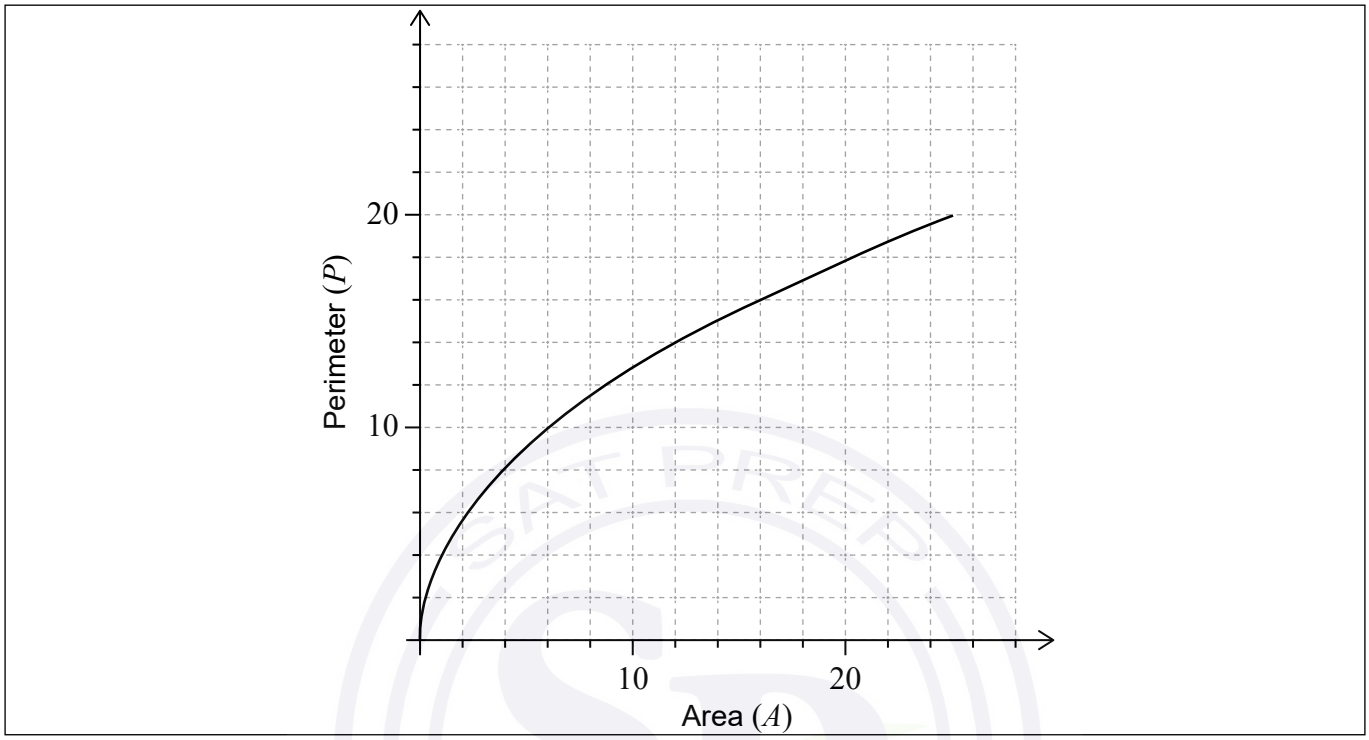
It was not possible to ask every person in the school, so the Headmaster arranged the student names in alphabetical order and then asked every 10th person on the list.

(c) Identify the sampling technique used in the survey. [1]

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2. [Maximum mark: 5]

The perimeter of a given square P can be represented by the function $P(A) = 4\sqrt{A}$, $A \geq 0$, where A is the area of the square. The graph of the function P is shown for $0 \leq A \leq 25$.



(a) Write down the value of $P(25)$. [1]

(b) On the axes above, draw the graph of the inverse function, P^{-1}. [3]

(c) In the context of the question, explain the meaning of $P^{-1}(8) = 4$. [1]

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3. [Maximum mark: 6]

Professor Vinculum investigated the migration season of the Bulbul bird from their natural wetlands to a warmer climate.

He found that during the migration season their population, P could be modelled by $P = 1350 + 400(1.25)^{-t}$, $t \geq 0$, where t is the number of days since the start of the migration season.

(a) Find the population of the Bulbul birds,

(i) at the start of the migration season.

(ii) in the wetlands after 5 days. [3]

(b) Calculate the time taken for the population to decrease below 1400. [2]

(c) According to this model, find the smallest possible population of Bulbul birds during the migration season. [1]

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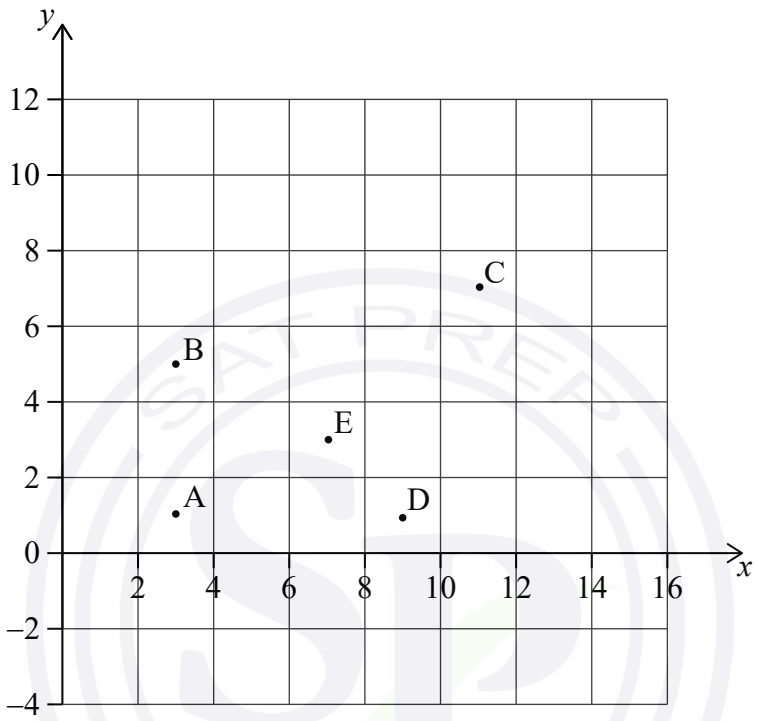
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4. [Maximum mark: 6]

Points $A(3, 1)$, $B(3, 5)$, $C(11, 7)$, $D(9, 1)$ and $E(7, 3)$ represent snow shelters in the Blackburn National Forest. These snow shelters are illustrated in the following coordinate axes.

Horizontal scale: 1 unit represents $1\,\text{km}$.

Vertical scale: 1 unit represents $1\,\text{km}$.

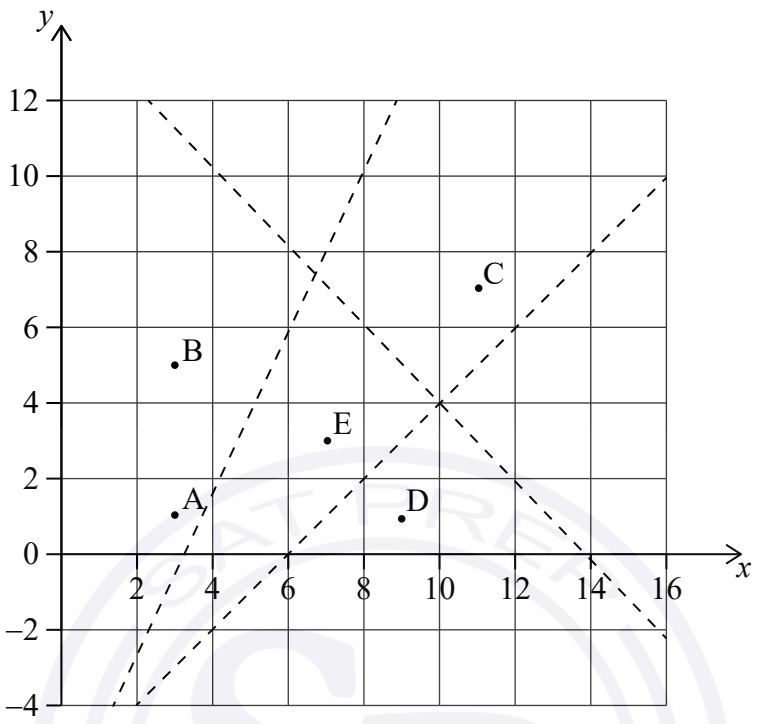


(a) Calculate the gradient of the line segment AE. [2]

(This question continues on the following page)

(Question 4 continued)

The Park Ranger draws three straight lines to form an incomplete Voronoi diagram.



(b) Find the equation of the line which would complete the Voronoi cell containing site E.
Give your answer in the form $ax + by + d = 0$ where $a, b, d \in \mathbb{Z}$. [3]

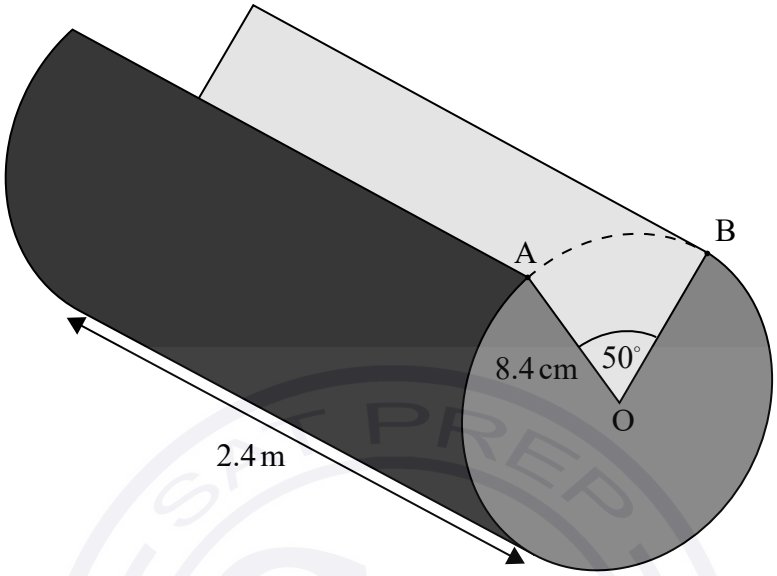
(c) In the context of the question, explain the significance of the Voronoi cell containing site E. [1]

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5. [Maximum mark: 5]

Helen is building a cabin using cylindrical logs of length $2.4\,\text{m}$ and radius $8.4\,\text{cm}$. A wedge is cut from one log and the cross-section of this log is illustrated in the following diagram.

diagram not to scale



(a) Find 50° in radians. [1]

(b) Find the volume of this log. [4]

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6. [Maximum mark: 6]

Jae Hee plays a game involving a biased six-sided die.
The faces of the die are labelled -3, -1, 0, 1, 2 and 5.
The score for the game, X, is the number which lands face up after the die is rolled.
The following table shows the probability distribution for X.

Score x	-3	-1	0	1	2	5
$\mathbf{P}(X = x)$	$\frac{1}{18}$	p	$\frac{3}{18}$	$\frac{1}{18}$	$\frac{2}{18}$	$\frac{7}{18}$

(a) Find the exact value of p. [1]

Jae Hee plays the game once.

(b) Calculate the expected score. [2]

Jae Hee plays the game twice and adds the two scores together.

(c) Find the probability Jae Hee has a **total** score of -3. [3]

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7. [Maximum mark: 5]

A particle, A, moves so that its velocity ($v\,\text{ms}^{-1}$) at time t is given by $v = 2\sin t,\ t \geq 0$.

The kinetic energy (E) of the particle A is measured in joules (J) and is given by $E = 5v^2$.

(a) Write down an expression for E as a function of time. [1]

(b) Hence find $\frac{\mathrm{d}E}{\mathrm{d}t}$. [2]

(c) Hence or otherwise find the first time at which the kinetic energy is changing at a rate of $5\,\text{J}\,\text{s}^{-1}$. [2]

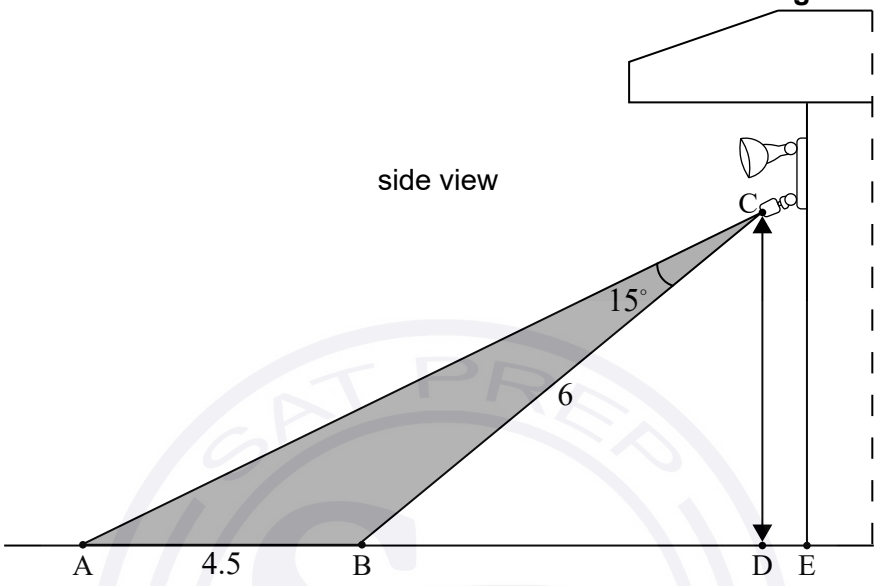
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8. [Maximum mark: 8]

Ollie has installed security lights on the side of his house that are activated by a sensor. The sensor is located at point C directly above point D. The area covered by the sensor is shown by the shaded region enclosed by triangle ABC. The distance from A to B is $4.5\,\text{m}$ and the distance from B to C is $6\,\text{m}$. Angle $A\hat{C}B$ is 15°.

diagram not to scale



(a) Find $C\hat{A}B$. [3]

Point B on the ground is $5\,\text{m}$ from point E at the entrance to Ollie's house. He is $1.8\,\text{m}$ tall and is standing at point D, below the sensor. He walks towards point B.

(b) Find the distance Ollie is **from the entrance to his house** when he first activates the sensor. [5]

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9. [Maximum mark: 5]

A manager wishes to check the mean weight of flour put into bags in his factory. He randomly samples 10 bags and finds the mean weight is $1.478\,\text{kg}$ and the standard deviation of the sample is $0.0196\,\text{kg}$.

(a) Find s_{n-1} for this sample. [2]

(b) Find a $95\,\%$ confidence interval for the population mean, giving your answer to 4 significant figures. [2]

(c) The bags are labelled as being $1.5\,\text{kg}$ weight. Comment on this claim with reference to your answer in part (b). [1]

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10. [Maximum mark: 6]

In a coffee shop, the time it takes to serve a customer can be modelled by a normal distribution with a mean of 1.5 minutes and a standard deviation of 0.4 minutes.

Two customers enter the shop together. They are served one at a time.

Find the probability that the total time taken to serve both customers will be less than 4 minutes.

Clearly state any assumptions you have made.

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11. [Maximum mark: 6]

A particle P moves with velocity $\boldsymbol{v} = \begin{pmatrix} -15 \\ 2 \\ 4 \end{pmatrix}$ in a magnetic field, $\boldsymbol{B} = \begin{pmatrix} 0 \\ d \\ 1 \end{pmatrix}$, $d \in \mathbb{R}$.

(a) Given that $\boldsymbol{v}$ is perpendicular to $\boldsymbol{B}$, find the value of d. [2]

The force, $\boldsymbol{F}$, produced by P moving in the magnetic field is given by the vector equation $\boldsymbol{F} = a\boldsymbol{v} \times \boldsymbol{B}$, $a \in \mathbb{R}^+$.

(b) Given that $|\boldsymbol{F}| = 14$, find the value of a. [4]

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12. [Maximum mark: 7]

Product research leads a company to believe that the revenue (R) made by selling its goods at a price (p) can be modelled by the equation.

$$R(p) = cpe^{dp},\ c,\ d \in \mathbb{R}$$

There are two competing models, A and B with different values for the parameters c and d.

Model A has $c = 3$, $d = -0.5$ and model B has $c = 2.5$, $d = -0.6$.

The company experiments by selling the goods at three different prices in three similar areas and the results are shown in the following table.

Area	Price (p)	Revenue (R)
1	1	1.5
2	2	1.8
3	3	1.5

The company will choose the model with the smallest value for the sum of square residuals.

Determine which model the company chose.

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13. [Maximum mark: 6]

The rates of change of the area covered by two types of fungi, X and Y,on a particular tree are given by the following equations, where x is the area covered by X and y is the area covered by Y.

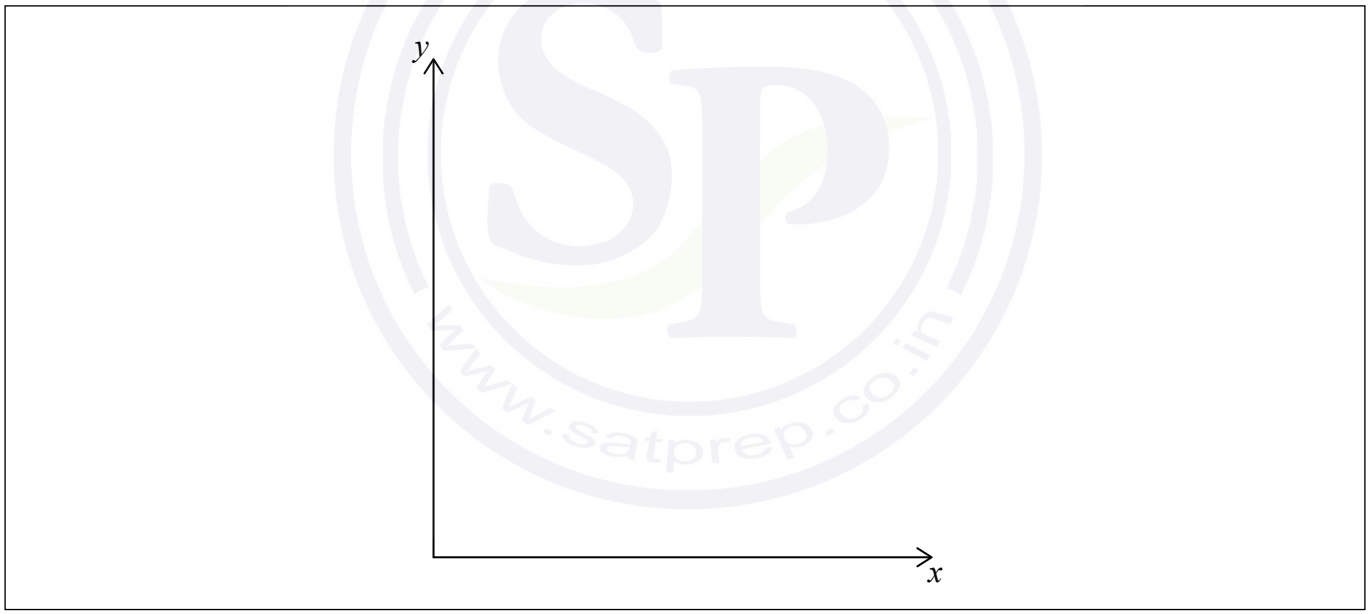
$$\frac{\mathrm{d}x}{\mathrm{d}t} = 3x - 2y$$
$$\frac{\mathrm{d}y}{\mathrm{d}t} = 2x - 2y$$

The matrix $\begin{pmatrix} 3 & -2 \\ 2 & -2 \end{pmatrix}$ has eigenvalues of 2 and -1 with corresponding eigenvectors $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$.

Initially $x = 8\,\mathrm{cm}^2$ and $y = 10\,\mathrm{cm}^2$.

(a) Find the value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ when $t = 0$. [2]

(b) On the following axes, sketch a possible trajectory for the growth of the two fungi, making clear any asymptotic behaviour. [4]



(This question continues on the following page)

(Question 13 continued)

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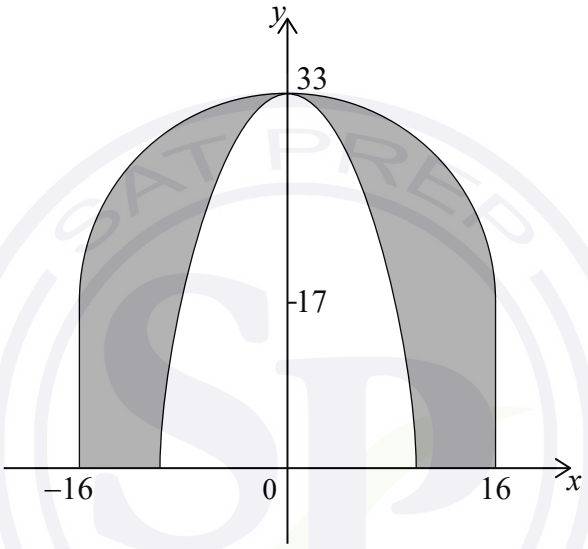
14. [Maximum mark: 8]

(a) The graph of $y = -x^3$ is transformed onto the graph of $y = 33 - 0.08x^3$ by a translation of a units vertically and a stretch parallel to the x-axis of scale factor b.

(i) Write down the value of a.

(ii) Find the value of b. [3]

(b) The outer dome of a large cathedral has the shape of a hemisphere of diameter $32\,\text{m}$, supported by vertical walls of height $17\,\text{m}$. It is also supported by an inner dome which can be modelled by rotating the curve $y = 33 - 0.08x^3$ through 360° about the y-axis between $y = 0$ and $y = 33$, as indicated in the diagram.



Find the volume of the space between the two domes. [5]

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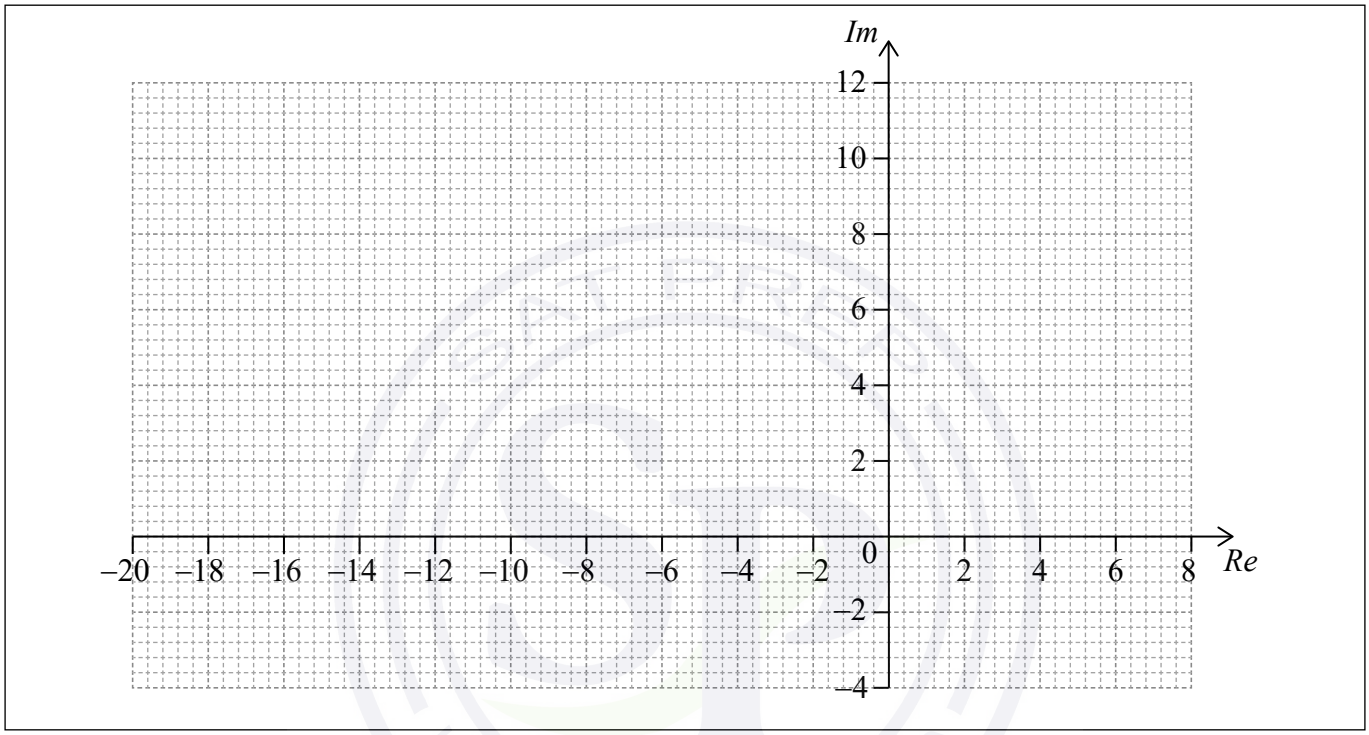
15. [Maximum mark: 7]

Let $w = ae^{\frac{\pi}{4}i}$, where $a \in \mathbb{R}^+$.

(a) For $a = 2$,

(i) find the values of w^2, w^3, and w^4;

(ii) draw w, w^2, w^3 and w^4 on the following Argand diagram. [5]



Let $z = \frac{w}{2 - i}$.

(b) Find the value of a for which successive powers of z lie on a circle. [2]

(This question continues on the following page)

(Question 15 continued)

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16. [Maximum mark: 6]

The number of fish that can be caught in one hour from a particular lake can be modelled by a Poisson distribution.

The owner of the lake, Emily, states in her advertising that the average number of fish caught in an hour is three.

Tom, a keen fisherman, is not convinced and thinks it is less than three. He decides to set up the following test. Tom will fish for one hour and if he catches fewer than two fish he will reject Emily's claim.

(a) State a suitable null and alternative hypotheses for Tom's test. [1]

(b) Find the probability of a Type I error. [2]

The average number of fish caught in an hour is actually 2.5.

(c) Find the probability of a Type II error. [3]

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17. [Maximum mark: 6]

Mr Burke teaches a mathematics class with 15 students. In this class there are 6 female students and 9 male students.

Each day Mr Burke randomly chooses one student to answer a homework question.

In the first month, Mr Burke will teach his class 20 times.

(a) Find the probability he will choose a female student 8 times. [2]

The Head of Year, Mrs Smith, decides to select a student at random from the year group to read the notices in assembly. There are 80 students in total in the year group. Mrs Smith calculates the probability of picking a male student 8 times in the first 20 assemblies is 0.153357 correct to 6 decimal places.

(b) Find the number of male students in the year group. [4]

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18. [Maximum mark: 6]

The rate, A, of a chemical reaction at a fixed temperature is related to the concentration of two compounds, B and C, by the equation

$$A = kB^x C^y, \text{ where } x,\ y,\ k \in \mathbb{R}.$$

A scientist measures the three variables three times during the reaction and obtains the following values.

Experiment	A (mol l^{-1} s^{-1})	B (mol l^{-1})	C (mol l^{-1})
1	5.74	2.1	3.4
2	2.88	1.5	2.4
3	0.980	0.8	1.9

Find x, y and k.

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