Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: applications and interpretation
Higher level
Paper 1

30 October 2023

Zone A afternoon | **Zone B** afternoon | **Zone C** afternoon

Candidate session number

2 hours

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all questions.
- Answers must be written within the answer boxes provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.

8823 – 7201

Answers must be written within the answer boxes provided. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 7]

The growth of a particular type of seashell is being studied by Manon. At the end of each month Manon records the increase in the width of a seashell since the end of the previous month.

She models the monthly increase in the width of the seashell by a geometric sequence with common ratio 0.8. In the first month, the width of the seashell increases by $4\,\text{mm}$.

(a) Find by how much the width of the seashell will increase during the third month, according to her model. [2]

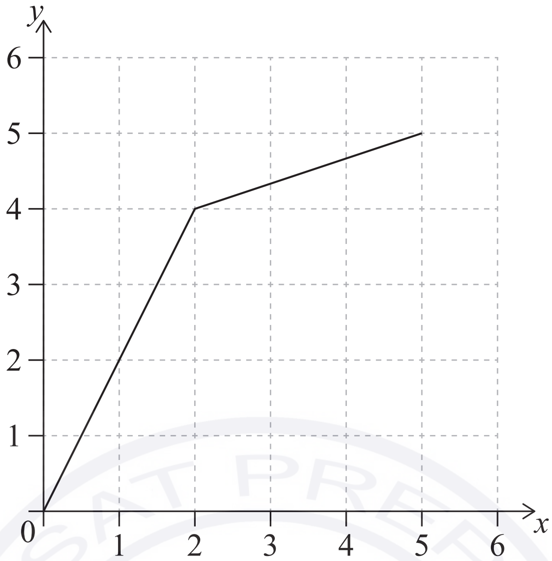
(b) Find the total increase in the width of the seashell, predicted by Manon's model, during the first year. [2]

Manon's seashell had a width of $25\,\text{mm}$ at the beginning of the first month.

(c) Find the maximum possible width of the seashell, predicted by Manon's model. [3]

2. [Maximum mark: 7]

The graph of the function f is given in the following diagram.



(a) Write down $f(2)$. [1]

(b) On the axes, sketch $y = f^{-1}(x)$. [2]

The function g is defined as $g(x) = 3x - 1$.

(c) Find an expression for $g^{-1}(x)$. [2]

(d) Find a value of x where $f^{-1}(x) = g^{-1}(x)$. [2]

3. [Maximum mark: 6]

The Great Pyramid of Giza is the oldest of the Seven Wonders of the Ancient World. When it was built, 4500 years ago, the measurements of the pyramid were in Royal Egyptian Cubits (REC).

[Source: Nina Aldin Thune. https://en.wikipedia.org/wiki/Great_Pyramid_of_Giza#/media/File:Kheops-Pyramid.jpg. Licensed under CC BY 2.5 https://creativecommons.org/licenses/by/2.5/#. Image adapted.]

Viktor reads online that $1\,\text{REC}$ is equal to 0.52 metres, rounded to two decimal places.

(a) Write down the upper and lower bounds of $1\,\text{REC}$ in metres. [2]

The Great Pyramid of Giza has a square base with side lengths of $440\,\text{REC}$ and a height of $280\,\text{REC}$. Viktor assumes that these two measurements are exact and that the Great Pyramid can be modelled as a square-based pyramid with smooth faces.

(b) Find the minimum possible volume of the pyramid in cubic metres. [4]

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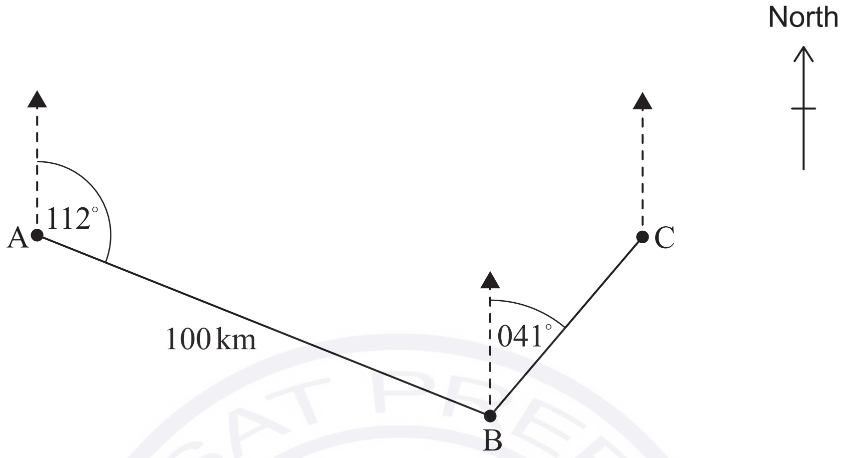
4. [Maximum mark: 6]

Jason sails his boat from point A for a distance of $100\,\text{km}$, on a bearing of 112°, to arrive at point B. He then sails on a bearing of 041° to point C. Jason's journey is shown in the diagram.

diagram not to scale



(a) Find $A\hat{B}C$. [2]

Point C is directly east of point A.

(b) Calculate the distance that Jason sails to return directly from point C to point A. [4]

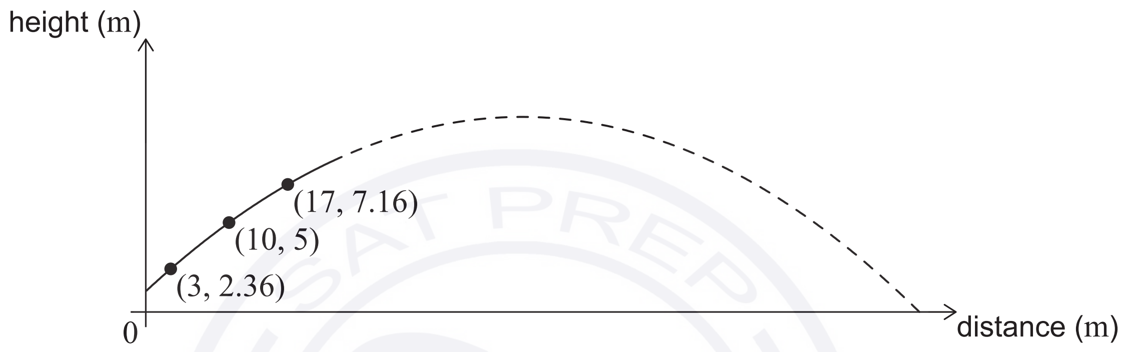
5. [Maximum mark: 9]

A sports player on a horizontal athletic field hits a ball. The height of the ball above the field, in metres, after it is hit can be modelled using a quadratic function of the form $f(x) = ax^2 + bx + c$, where x represents the horizontal distance, in metres, that the ball has travelled from the player.

A specialized camera tracks the initial path of the ball after it is hit by the player. The camera records that the ball travels through the three points $(3, 2.36)$, $(10, 5)$ and $(17, 7.16)$, as shown in **Diagram 1**.

diagram not to scale

Diagram 1



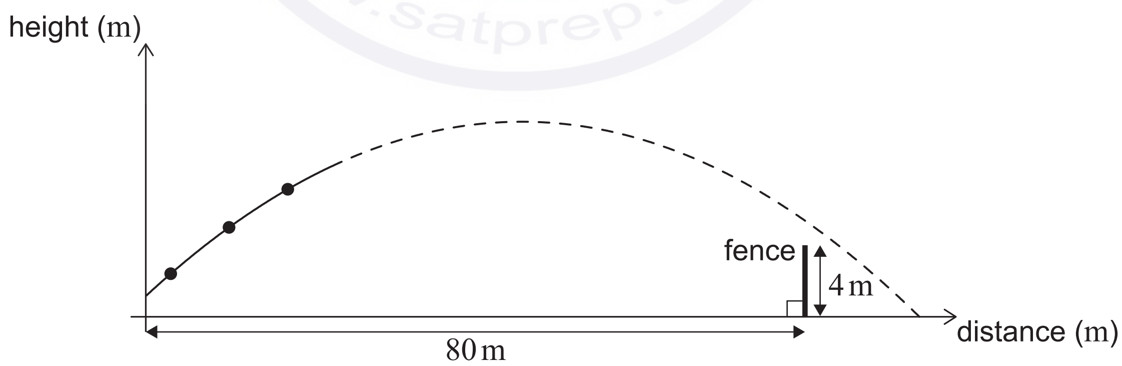
(a) Use the coordinates $(3, 2.36)$ to write down an equation in terms of a, b, and c. [1]

(b) Use your answer to part (a) and two similar equations to find the equation of the quadratic model for the height of the ball. [3]

A 4-metre-high fence is 80 metres from where the player hit the ball, as shown in **Diagram 2**.

diagram not to scale

Diagram 2



(c) Show that the model predicts that the ball will go over the fence. [3]

(d) Find the horizontal distance that the ball will travel, from the player until it first hits the ground, according to this model. [2]

(This question continues on the following page)

(Question 5 continued)

6. [Maximum mark: 8]

On the following Voronoi diagram, the coordinates of three farmhouses are $\text{A}(0, 3)$, $\text{B}(8, 3)$ and $\text{C}(8, 13)$, where distances are measured in kilometres. Each farmhouse owns the land that is closest to it, and their boundaries are defined by the dotted lines on **Diagram 1**.

Diagram 1

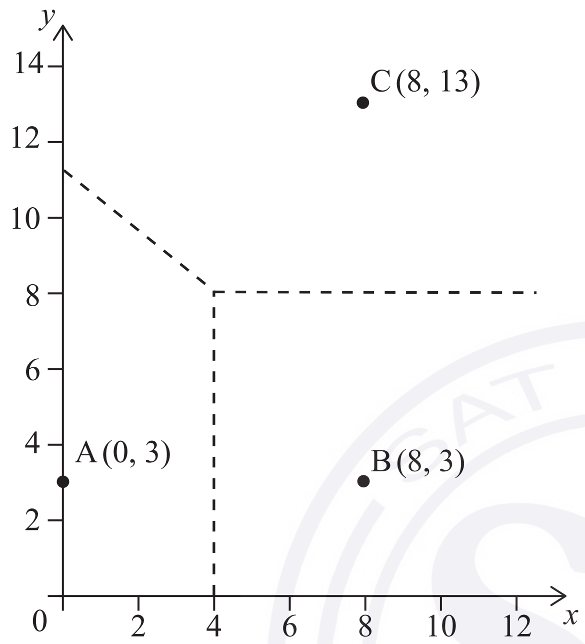
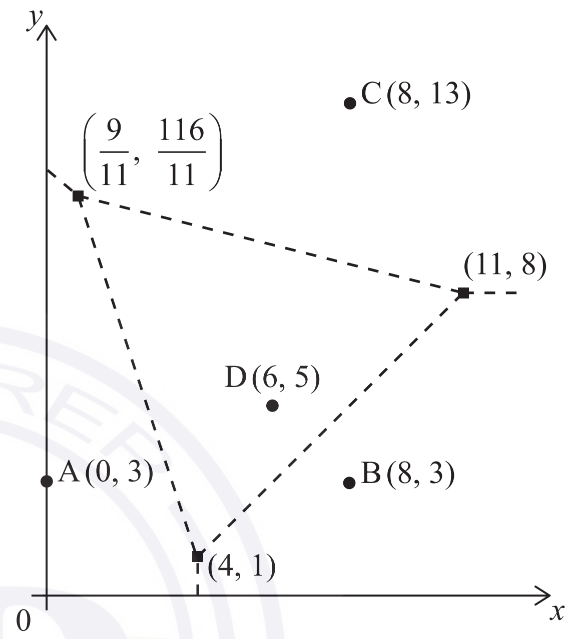


Diagram 2



To provide water to the farms it is decided to construct a well at the point where the boundaries meet on **Diagram 1**.

(a) Write down the coordinates of this point. [1]

(b) Find the equation of the perpendicular bisector of $[\text{AC}]$. [3]

An additional farmhouse $\text{D}(6, 5)$ is built on the land. The Voronoi diagram has been redrawn to show the new boundaries. The coordinates of the vertices of these boundaries are indicated on **Diagram 2**.

A wind turbine is to be built at one of the vertices.

(c) The wind turbine should be as far from the nearest farmhouses as possible.

(i) By calculating appropriate distances, find the location of the wind turbine.

(ii) Hence, write down the distance of the wind turbine to the nearest farmhouse. [4]

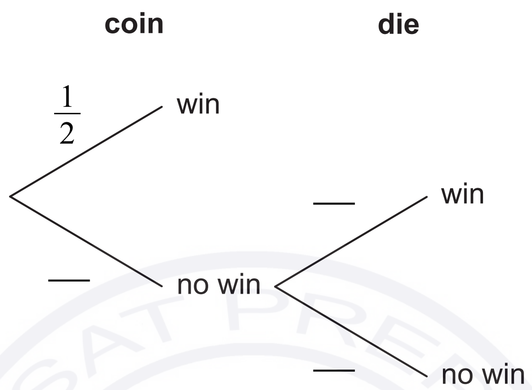
(This question continues on the following page)

(Question 6 continued)

7. [Maximum mark: 7]

Michèle is playing a game. In the game, she must first flip a fair coin which will result in the coin landing on heads or tails. If the coin lands on heads, then she wins a prize. If it lands on tails, then she has another chance but this time she must roll a fair six-sided dice and get a five or six in order to win a prize.

(a) Complete the tree diagram by writing in the three missing probabilities. [2]



(b) Find the probability that Michèle does **not** win a prize. [2]

(c) Given that Michèle won a prize, find the probability that the coin landed on heads. [3]

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8. [Maximum mark: 6]

Given $z = \sqrt{3} - \mathrm{i}$.

(a) Write z in the form $z = r\mathrm{e}^{\theta \mathrm{i}}$, where $r \in \mathbb{R}^+$, $-\pi < \theta \leq \pi$. [2]

Let $z_1 = \mathrm{e}^{2t\mathrm{i}}$ and $z_2 = 2\mathrm{e}^{\left(2t - \frac{\pi}{6}\right)\mathrm{i}}$.

(b) Find $\mathrm{Im}(z_1 + z_2)$ in the form $p\sin(2t + q)$, where $p > 0$, $t \in \mathbb{R}$ and $-\pi \leq q \leq \pi$. [4]

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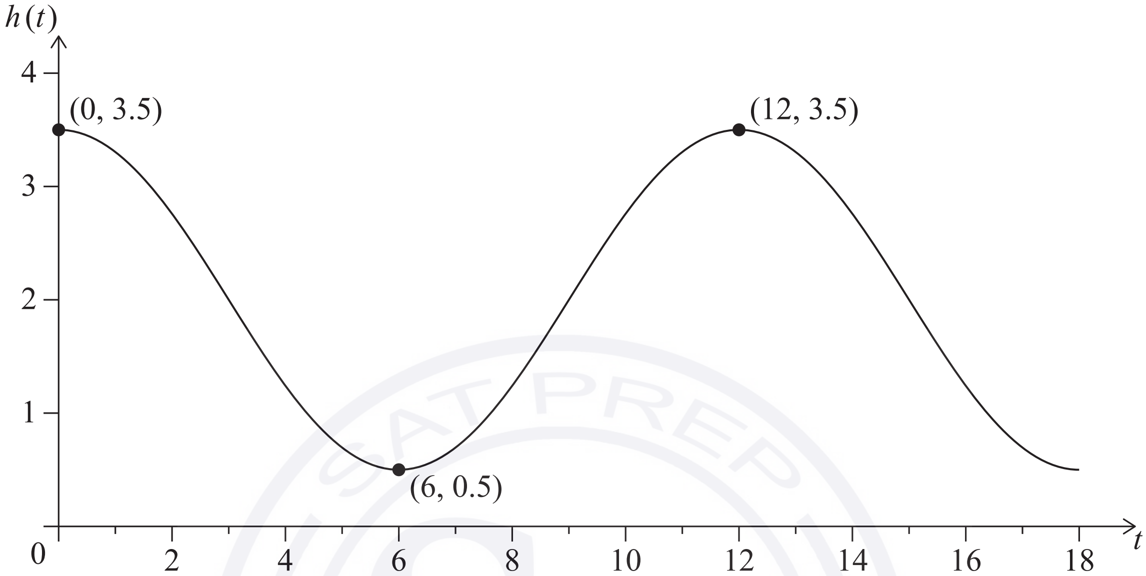
9. [Maximum mark: 8]

Joon is a keen surfer and wants to model waves passing a particular point P, which is off the shore of his favourite beach. Joon sets up a model of the waves in terms of $h(t)$, the height of the water in metres, and t, the time in seconds from when he begins recording the height of the water at point P.



The function has the form $h(t) = p\cos\left(\frac{\pi}{6}t\right) + q$, $t \geq 0$.

(a) Find the values of p and q. [2]

(b) Find

(i) $h'(t)$.

(ii) $h''(t)$. [3]

Joon will begin to surf the wave when the rate of change of h with respect to t, at P, is at its maximum. This will first occur when $t = k$.

(c) (i) Find the value of k.

(ii) Find the height of the water at this time. [3]

(This question continues on the following page)

(Question 9 continued)

10. [Maximum mark: 7]

The decay of a chemical isotope over five years is recorded in **Table 1**. The mass of the chemical M is measured to the nearest gram at the beginning of each year t of the experiment.

Table 1

Time t (years)	1	2	3	4	5
Mass M (grams)	1000	660	517	435	381

It is believed that the decay of the isotope can be modelled by an equation of the form $M = a \times t^b$.

(a) Use power regression on your graphic display calculator to find the value of a and the value of b. [2]

The values of t and M can be transformed such that $x = \ln t$ and $y = \ln M$. **Table 2** shows data for x and y to three decimal places.

Table 2

x	0	0.693	1.099	1.386	1.609
y	6.908	6.492	6.248	6.075	5.943

(b) Find the linear regression equation of y on x, in the form $y = cx + d$. Give the values of c and d to three decimal places. [2]

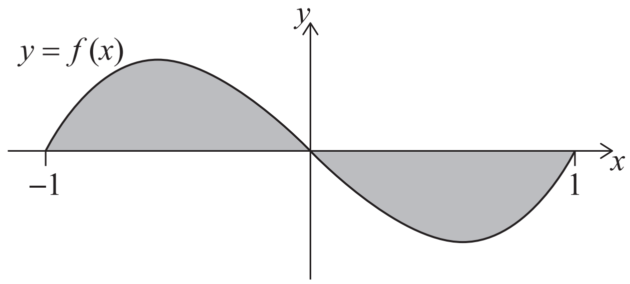
(c) Hence, show that this linear regression is equivalent to the power regression found in part (a). [3]

(This question continues on the following page)

(Question 10 continued)

11. [Maximum mark: 7]

Consider the function $f(x) = x^3 - x$, for $-1 \le x \le 1$. The shaded region, R, is bounded by the graph of $y = f(x)$ and the x-axis.

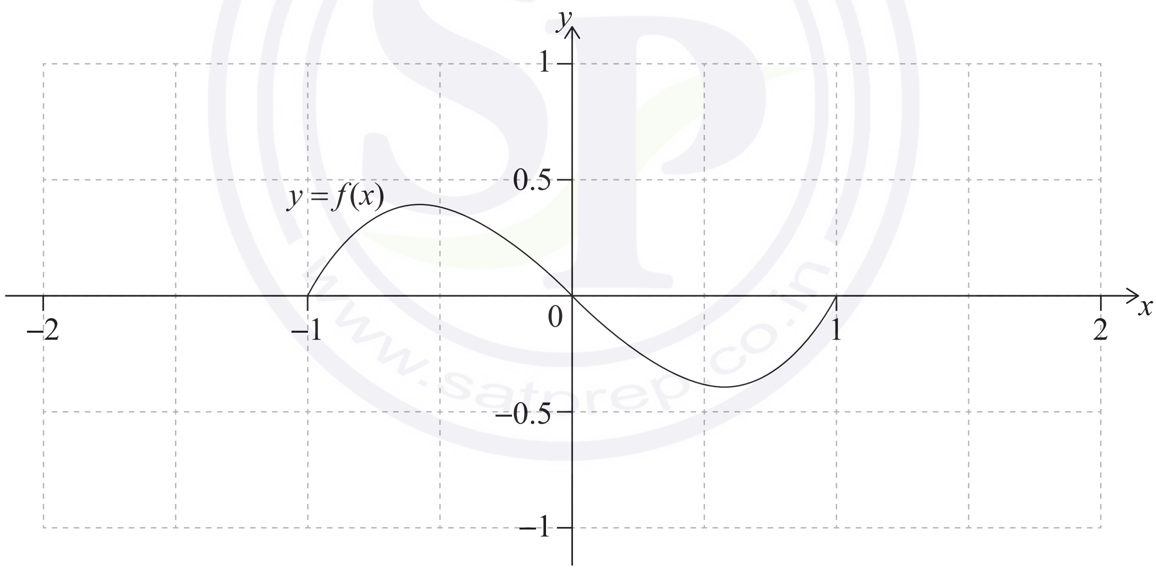


(a) (i) Write down an integral that represents the area of R.

(ii) Find the area of R. [2]

Another function, g, is defined such that $g(x) = 2f(x-1)$.

(b) On the following set of axes, the graph of $y = f(x)$ has been drawn. On the same set of axes, sketch the graph of $y = g(x)$. [2]



The region R from the original graph $y = f(x)$ is rotated through 2π radians about the x-axis to form a solid.

(c) Find the volume of the solid. [3]

(This question continues on the following page)

(Question 11 continued)

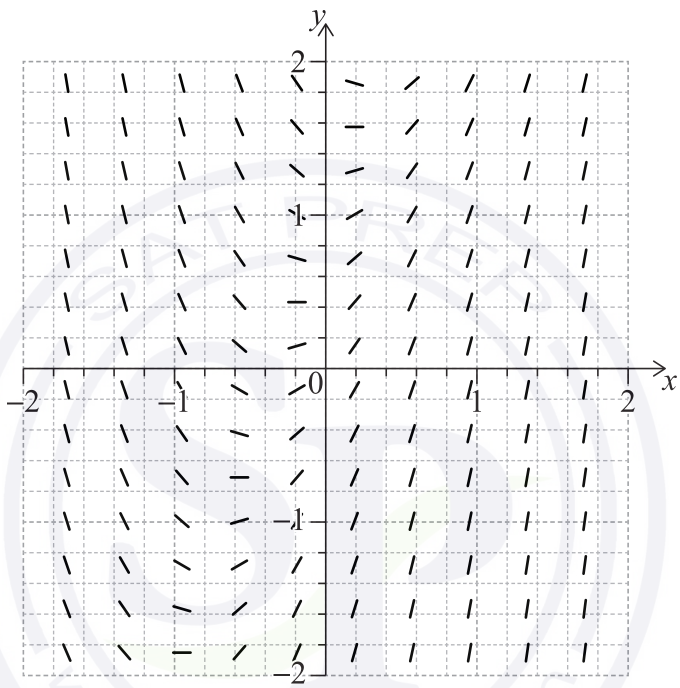
12. [Maximum mark: 4]

Consider the differential equation $\frac{dy}{dx} = 3x - y + 1$.

(a) Find the equation of the tangent to the solution curve at the point $(-1, -1)$ in the form $ax + by + c = 0$. [2]

The slope field for this differential equation is shown in the following diagram.

(b) Sketch the solution curve that passes through the point $(-1, -1)$. [2]



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13. [Maximum mark: 8]

The velocity v of a particle at time t, as it moves along a straight line, can be modelled by the piecewise function

$$v(t) = \begin{cases} u_1(t), & 0 \le t \le T \\ u_2(t), & t \ge T \end{cases}$$

where $u_1(t) = 2t^2 - t^3$ and $u_2(t) = 8 - 4t$. It is required that $u_1(T) = u_2(T)$.

(a) Find the value of T. [2]

(b) Show that $u_1'(T) = u_2'(T)$. [2]

The displacement of the particle at time $t = 0$ is zero.

(c) Find the time when the particle returns to its initial position. [4]

14. [Maximum mark: 7]

A straight line L has vector equation $\boldsymbol{r} = \begin{pmatrix} 1 \\ 3 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$ and point Q has coordinates $(11, -1, 3)$.

Point P is the point on L closest to Q.

(a) Find the coordinates of P. [4]

(b) Find a vector that is perpendicular to both L and the line passing through points P and Q. [3]

15. [Maximum mark: 7]

The eating habits of students in a school are studied over a number of months. The focus of the study is whether non-vegetarians become vegetarians, and whether vegetarians remain vegetarians.

Each month, students choose between the vegetarian or non-vegetarian lunch options. Once they have chosen for the month, they cannot change the option until the next month.

In any month during the study, it is noticed that the probability of a non-vegetarian becoming vegetarian the following month is 0.1, and that the probability of a vegetarian remaining a vegetarian the following month is 0.8.

This situation can be represented by the transition matrix

$$\boldsymbol{T} = \begin{pmatrix} 0.8 & 0.1 \\ 0.2 & 0.9 \end{pmatrix}.$$

(a) Interpret the value 0.9 in $\boldsymbol{T}$ in terms of the changes in the eating habits of the students in the school. [1]

(b) Find the eigenvalues of matrix $\boldsymbol{T}$. [3]

One of the eigenvectors of $\boldsymbol{T}$ is $\begin{pmatrix} -1 \\ 1 \end{pmatrix}$.

(c) Find another, non-parallel, eigenvector and interpret it in context. [3]

16. [Maximum mark: 6]

When Jef plays basketball, the number of shots he takes during any 6 minutes of play can be modelled by a Poisson distribution with mean 2.5.

(a) Find the probability that Jef takes less than 7 shots during any 12 minutes of play. [2]

It can be assumed that the outcomes of the shots are independent of each other, and the probability of success of a shot is constant. The probability that Jef is successful with a shot is 0.4.

It can be assumed that the probability of Jef's success with a shot is independent of the number of shots that he takes.

(b) Find the probability that during any 6 minutes of play Jef takes fewer than 4 shots and is successful at least once. [4]

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References:

3. Nina Aldin Thune. https://en.wikipedia.org/wiki/Great_Pyramid_of_Giza#/media/File:Kheops-Pyramid.jpg. Licensed under CC BY 2.5 https://creativecommons.org/licenses/by/2.5/#. Image adapted.

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Mathematics: applications and interpretation
Higher level
Paper 1

8 May 2023

Zone A afternoon | **Zone B** morning | **Zone C** afternoon

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1. [Maximum mark: 7]

A player throws a basketball. The height of the basketball is modelled by

$$h(t) = -4.75t^2 + 8.75t + 1.5,\ t \geq 0,$$

where h is the height of the basketball above the ground, in metres, and t is the time, in seconds, after it was thrown.

(a) Find how long it takes for the basketball to reach its maximum height. [2]

(b) Assuming that no player catches the basketball, find how long it would take for the basketball to hit the ground. [2]

Another player catches the basketball when it is at a height of 1.2 metres.

(c) Find the value of t when this player catches the basketball. [2]

(d) Write down one limitation of using $h(t)$ to model the height of the basketball. [1]

2. [Maximum mark: 4]

A company that owns many restaurants wants to determine if there are differences in the quality of the food cooked for three different meals: breakfast, lunch and dinner.

Their quality assurance team randomly selects 500 items of food to inspect. The quality of this food is classified as perfect, satisfactory, or poor. The data is summarized in the following table.

		Quality			
		Perfect	**Satisfactory**	**Poor**	**Total**
Meal	**Breakfast**	101	124	7	**232**
	Lunch	68	81	5	**154**
	Dinner	35	69	10	**114**
	Total	**204**	**274**	**22**	**500**

A χ^2 test at the 5% significance level is carried out to determine if there is significant evidence of a difference in the quality of the food cooked for the three meals.

The critical value for this test is 9.488.

The hypotheses for this test are:

H_0: The quality of the food and the type of meal are independent.

H_1: The quality of the food and the type of meal are not independent.

(a) Find the χ^2 statistic. [2]

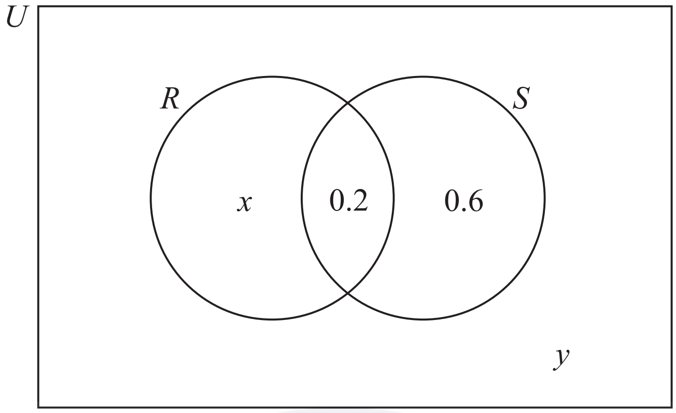
(b) State, with justification, the conclusion for this test. [2]

(This question continues on the following page)

(Question 2 continued)

3. [Maximum mark: 7]

The following Venn diagram shows two independent events, R and S. The values in the diagram represent probabilities.



(a) Find the value of x. [3]

(b) Find the value of y. [2]

(c) Find $\mathrm{P}(R' \mid S')$. [2]

4. [Maximum mark: 6]

Angel has $\$520$ in his savings account. Angel considers investing the money for 5 years with a bank. The bank offers an annual interest rate of 1.2% compounded quarterly.

(a) Calculate the amount of money Angel would have at the end of 5 years with the bank. Give your answer correct to two decimal places. [3]

Instead of investing the money, Angel decides to buy a phone that costs $\$520$. At the end of 5 years, the phone will have a value of $\$30$. It may be assumed that the depreciation rate per year is constant.

(b) Calculate the annual depreciation rate of the phone. [3]

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5. [Maximum mark: 6]

The lengths of the seeds from a particular mango tree are approximated by a normal distribution with a mean of $4\,\text{cm}$ and a standard deviation of $0.25\,\text{cm}$.

A seed from this mango tree is chosen at random.

(a) Calculate the probability that the length of the seed is less than $3.7\,\text{cm}$. [2]

It is known that 30% of the seeds have a length greater than $k\,\text{cm}$.

(b) Find the value of k. [2]

For a seed of length $d\,\text{cm}$, chosen at random, $\text{P}(4 - m < d < 4 + m) = 0.6$.

(c) Find the value of m. [2]

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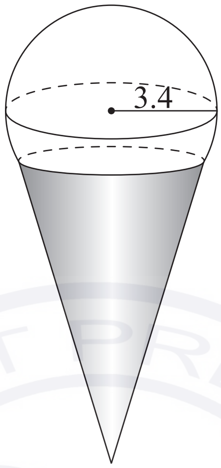
6. [Maximum mark: 5]

Ruhi buys a scoop of ice cream in the shape of a sphere with a radius of $3.4\,\text{cm}$. The ice cream is served in a cone, and it may be assumed that $\frac{1}{5}$ of the volume of the ice cream is inside the cone. This is shown in the following diagram.

diagram not to scale



(a) Calculate the volume of ice cream that is not inside the cone. [3]

The cone has a slant height of $11\,\text{cm}$ and a radius of $3\,\text{cm}$.

The outside of the cone is covered with chocolate.

(b) Calculate the surface area of the cone that is covered with chocolate. Give your answer correct to the nearest cm^2. [2]

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7. [Maximum mark: 6]

Akar starts a new job in Australia and needs to travel daily from Wollongong to Sydney and back. He travels to work for 28 consecutive days and therefore makes 56 single journeys. Akar makes all journeys by bus.

The probability that he is successful in getting a seat on the bus for any single journey is 0.86.

(a) Determine the expected number of these 56 journeys for which Akar gets a seat on the bus. [1]

(b) Find the probability that Akar gets a seat on at least 50 journeys during these 28 days. [3]

The probability that Akar gets a seat on at most n journeys is at least 0.25.

(c) Find the smallest possible value of n. [2]

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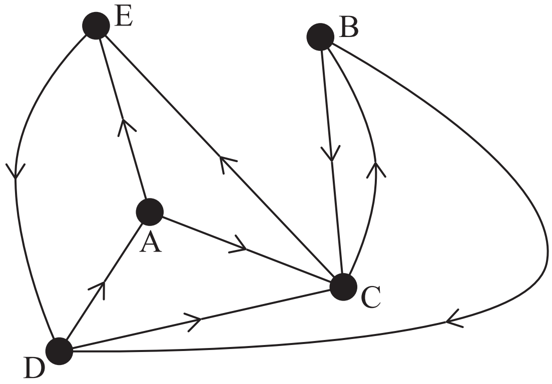
8. [Maximum mark: 7]

The following directed, unweighted, graph shows a simplified road network on an island, connecting five small villages marked A to E.



(a) Construct the adjacency matrix $\boldsymbol{M}$ for this network. [3]

Beatriz the bus driver starts at village E and drives to seven villages, such that the seventh village is A.

(b) (i) Determine how many possible routes Beatriz could have taken, to travel from E to A.

(ii) Describe one possible route taken by Beatriz, by listing the villages visited in order. [4]

9. [Maximum mark: 9]

At a running club, Sung-Jin conducts a test to determine if there is any association between an athlete's age and their best time taken to run 100 m. Eight athletes are chosen at random, and their details are shown below.

Athlete	A	B	C	D	E	F	G	H
Age (years)	13	17	22	18	19	25	11	36
Time (seconds)	13.4	14.6	13.4	12.9	12.0	11.8	17.0	13.1

Sung-Jin decides to calculate the Spearman's rank correlation coefficient for his set of data.

(a) Complete the table of ranks. [2]

Athlete	A	B	C	D	E	F	G	H
Age rank			3					
Time rank							1	

(b) Calculate the Spearman's rank correlation coefficient, r_s. [2]

(c) Interpret this value of r_s in the context of the question. [1]

(d) Suggest a mathematical reason why Sung-Jin may have decided not to use Pearson's product-moment correlation coefficient with his data from the original table. [1]

(e) (i) Find the coefficient of determination for the data from the original table.

(ii) Interpret this value in the context of the question. [3]

(This question continues on the following page)

(Question 9 continued)

10. [Maximum mark: 6]

A chocolate company plans to produce chocolate bars with special flavours. They survey 246 people to determine if there is any particular preference for one of the flavours.

The table below shows the information collected.

Hot chilli	Almond crunch	Spiced Chai	Ginger'n'lime
75	59	46	66

A χ^2 goodness of fit test at the 5% significance level is carried out on the data.

The critical value for the test is 7.82.

(a) State the null and alternative hypotheses for this test. [2]

(b) Perform the test and give your conclusion in context. [4]

11. [Maximum mark: 6]

Two AC (alternating current) electrical sources of equal frequencies are combined.

The voltage of the first source is modelled by the equation $V = 30\sin(t + 60^\circ)$.

The voltage of the second source is modelled by the equation $V = 60\sin(t + 10^\circ)$.

(a) Determine the maximum voltage of the combined sources. [2]

(b) Using your graphic display calculator, find a suitable equation for the combined voltages, giving your answer in the form $V = V_0 \sin(at + b)$, where a, b and V_0 are constants, $a > 0$ and $0^\circ \le b < 180^\circ$. [4]

12. [Maximum mark: 5]

A spherical balloon is being inflated such that its volume is increasing at a rate of $15\,\text{cm}^3\,\text{s}^{-1}$.

(a) Find the radius of the balloon when its volume is $288\,\pi\,\text{cm}^3$. [2]

(b) Hence or otherwise, find the rate of change of the radius at this instant. [3]

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13. [Maximum mark: 7]

The matrices $\boldsymbol{P}=\begin{pmatrix}3 & 1\\ 0 & 2\end{pmatrix}$ and $\boldsymbol{Q}=\begin{pmatrix}-4 & 1\\ 1 & 3\end{pmatrix}$ represent two transformations.

A triangle T is transformed by $\boldsymbol{P}$, and this image is then transformed by $\boldsymbol{Q}$ to form a new triangle, T'.

(a) Find the single matrix that represents the transformation $T' \rightarrow T$, which will undo the transformation described above. [4]

The area of T' is $273\,\text{cm}^2$.

(b) Using your answer to part (a), or otherwise, determine the area of T. [3]

14. [Maximum mark: 8]

In this question, $\boldsymbol{i}$ denotes a unit vector due east, and $\boldsymbol{j}$ denotes a unit vector due north.

Two ships, A and B, are each moving with constant velocities.

The position vector of ship A, at time t hours, is given as $\boldsymbol{r}_A = (1 + 2t)\boldsymbol{i} + (3 - 3t)\boldsymbol{j}$.

The position vector of ship B, at time t hours, is given as $\boldsymbol{r}_B = (-2 + 4t)\boldsymbol{i} + (-4 + t)\boldsymbol{j}$.

(a) Find the bearing on which ship A is sailing. [3]

(b) Find the value of t when ship B is directly south of ship A. [2]

(c) Find the value of t when ship B is directly south-east of ship A. [3]

15. [Maximum mark: 6]

A random sample of eight packets of Apollo coffee granules are selected from a supermarket shelf.

The weights of the coffee granules present in each packet are as follows:

$222\,g \quad 226\,g \quad 221\,g \quad 228\,g \quad 227\,g \quad 225\,g \quad 222\,g \quad 223\,g$

(a) (i) Find an unbiased estimate for the mean weight of coffee granules in a packet of Apollo coffee.

(ii) Calculate a 95% confidence interval for the population mean. Give your answer to four significant figures. [3]

(b) State one assumption you have made in order for your interval to be valid. [1]

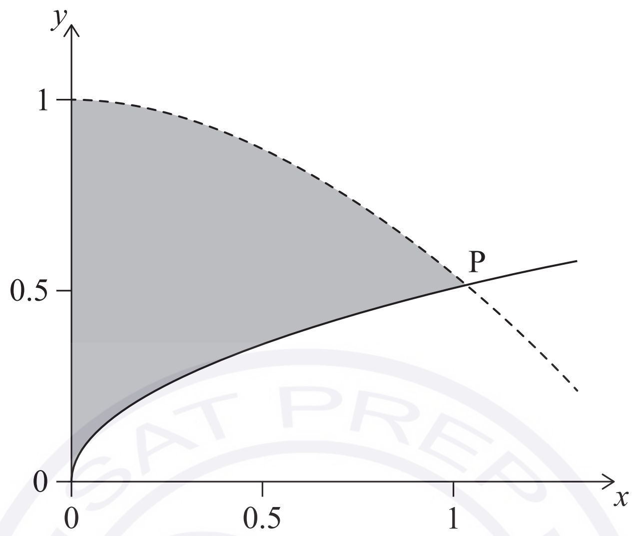
(c) The label of each packet has a description which includes the phrase: “contains $226\,g$ of coffee granules”.

Using your answer to part (a)(ii), briefly comment on the claim on the label. [2]

16. [Maximum mark: 9]

The following diagram shows parts of the curves of $y = \cos x$ and $y = \dfrac{\sqrt{x}}{2}$.

P is the point of intersection of the two curves.



(a) Use your graphic display calculator to find the coordinates of P. [2]

The shaded region is rotated 360° **about the y-axis** to form a volume of revolution V.

(b) Express V as the sum of two definite integrals. [5]

(c) Hence find the value of V. [2]

17. [Maximum mark: 6]

Consider the differential equation

$$\left(x^2+1\right)\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{x}{2y-2}\text{, for } x\ge 0\text{, } y\ge 1\text{,}$$

where $y=1$ when $x=0$.

(a) Explain why Euler's method cannot be used to find an approximate value for y when $x=0.1$. [1]

(b) By solving the differential equation, show that $y=1+\sqrt{\frac{\ln\left(x^2+1\right)}{2}}$. [4]

(c) Hence deduce the value of y when $x=0.1$. [1]

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References:

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Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: applications and interpretation
Higher level
Paper 1

8 May 2023

Zone A afternoon | **Zone B** morning | **Zone C** afternoon

Candidate session number

2 hours

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all questions.
- Answers must be written within the answer boxes provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.

2223 – 7206

23 pages

International Baccalaureate
Baccalauréat International
Bachillerato Internacional

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Answers must be written within the answer boxes provided. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 5]

On 1 January 2022, Mina deposited $\$1000$ into a bank account with an annual interest rate of 4%, compounded monthly. At the end of January, and the end of every month after that, she deposits $\$100$ into the same account.

(a) Calculate the amount of money in her account at the start of 2024. Give your answer to two decimal places. [3]

(b) Find how many complete months, counted from 1 January 2022, it will take for Mina to have more than $\$5000$ in her account. [2]

2. [Maximum mark: 6]

Carys believes that, on a memory retention test, the mean score of bilingual people (μ_b) will be higher than the mean score of monolingual people (μ_m). Carys gave a memory retention test to a random sample of students in her class. The results are shown in the two tables.

	Scores									
Bilingual	100	94	100	90	100	94	98	98	98	98

	Scores							
Monolingual	97	92	88	98	88	94	100	100

Carys performs a one-tailed t-test at a 5% level of significance. It is assumed that the scores are normally distributed and the samples have equal variances.

(a) State the null and alternative hypotheses. [2]

(b) Calculate the p-value for this test. [2]

(c) State the conclusion of the test in the context of the question. Justify your answer. [2]

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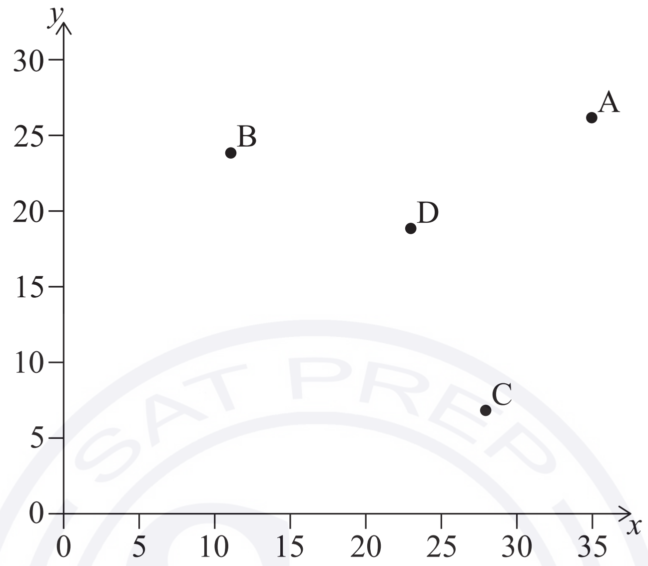
3. [Maximum mark: 5]

Three towns have positions $A(35, 26)$, $B(11, 24)$, and $C(28, 7)$ according to the coordinate system shown where distances are measured in miles.

Dominique's farm is located at the position $D(24, 19)$.



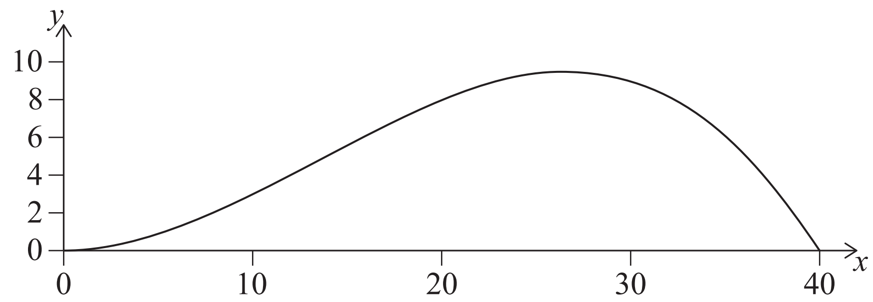
(a) Find AD. [2]

On a particular day, the mean temperatures recorded in each of towns A, B and C are $34\,°C$, $29\,°C$ and $30\,°C$ respectively.

(b) Use nearest neighbour interpolation to estimate the temperature at Dominique's farm on that particular day. [3]

4. [Maximum mark: 8]

The cross section of a scale model of a hill is modelled by the following graph.



The heights of the model are measured at horizontal intervals and are given in the table.

Horizontal distance, x cm	0	10	20	30	40
Vertical distance, y cm	0	3	8	9	0

(a) Use the trapezoidal rule with $h = 10$ to find an approximation for the cross-sectional area of the model. [2]

It is given that the equation of the curve is $y = 0.04x^2 - 0.001x^3,\ 0 \le x \le 40$.

(b) (i) Write down an integral to find the exact cross-sectional area.

(ii) Calculate the value of the cross-sectional area to two decimal places. [4]

(c) Find the percentage error in the area found using the trapezoidal rule. [2]

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5. [Maximum mark: 5]

A boat travels $8\,\text{km}$ on a bearing of $315°$ and then a further $6\,\text{km}$ on a bearing of $045°$. Find the bearing on which the boat should travel to return directly to the starting point.

6. [Maximum mark: 7]

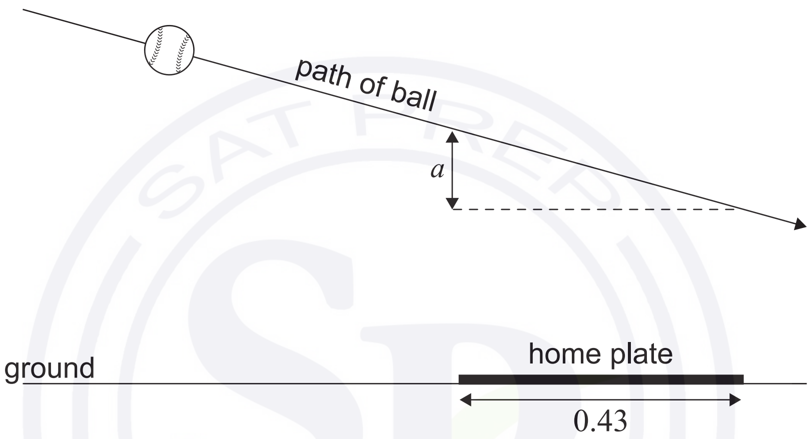
In a baseball game, Sakura is the batter standing beside home plate. The ball is thrown towards home plate along a path that can be modelled by the following function.

$$y = -0.045x + 2$$

In this model, x is the horizontal distance of the ball from the point the ball is thrown and y is the vertical height of the ball above the ground. Both measured in metres.

The outcome of the throw is called a strike if the height of the ball is between 0.53 m and 1.24 m at some point while it travels over home plate. The length of home plate is 0.43 m.

diagram not to scale



When the ball reaches the front of home plate, the height of the ball above the ground is 1.25 m. The height of the ball changes by a metres as the ball travels over the length of home plate.

(a) (i) Find the value of a.

(ii) Justify why this throw is a strike. [4]

On the next throw, Sakura hits the ball towards a wall that is 5 metres high. The horizontal distance of the wall from the point where the ball was hit is 96 metres. The path of the ball after it is hit can be modelled by the function $h(d)$.

$$h(d) = -0.01d^2 + 1.04d + 0.66\text{, for } h,\ d > 0$$

In this model, h is the height of the ball above the ground and d is the horizontal distance of the ball from the point where it was hit. Both h and d are measured in metres.

(b) Determine whether the ball will go over the wall. Justify your answer. [3]

(This question continues on the following page)

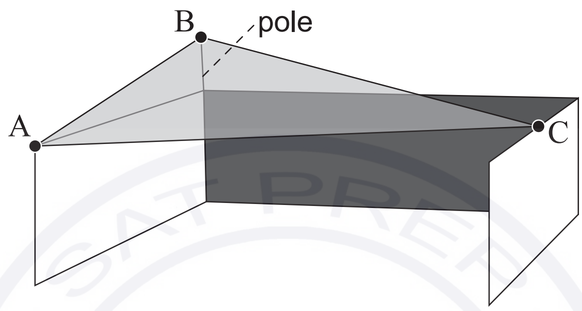
(Question 6 continued)

7. [Maximum mark: 9]

A triangular cover is positioned over a walled garden to provide shade. It is anchored at points A and C, located at the top of a 2 m wall, and at a point B, located at the top of a 1 m vertical pole fixed to a top corner of the wall.

The three edges of the cover can be represented by the vectors

$\vec{AB} = \begin{pmatrix} 0 \\ 6 \\ 1 \end{pmatrix}$, $\vec{AC} = \begin{pmatrix} 7 \\ 3 \\ 0 \end{pmatrix}$ and $\vec{BC} = \begin{pmatrix} 7 \\ -3 \\ -1 \end{pmatrix}$, where distances are measured in metres.



(a) Calculate the vector product $\vec{AB} \times \vec{AC}$. [2]

(b) Hence find the area of the triangular cover. [2]

The point X on [AC] is such that [BX] is perpendicular to [AC].

(c) Use your answer to part (b) to find the distance BX. [3]

(d) Find the angle the cover makes with the horizontal plane. [2]

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8. [Maximum mark: 7]

The random variables (X, Y) follow a bivariate normal distribution with product-moment correlation coefficient ρ. The values of six random observations of (X, Y) are shown in the table.

x	6.3	4.1	5.6	9.2	7.8	8.2
y	9.2	4.9	8.9	10.3	8.9	9.8

(a) State null and alternative hypotheses which could be used to test whether there is a linear correlation between X and Y. [2]

(b) Determine the value of

(i) the product-moment correlation coefficient, r, of the sample.

(ii) the corresponding p-value. [3]

(c) State whether your result from part (b)(ii) indicates there is sufficient evidence to claim that, at the 5% significance level, X and Y are not linearly correlated.

Give a reason for your answer. [2]

9. [Maximum mark: 5]

On a specific day, the speed of cars as they pass a speed camera can be modelled by a normal distribution with a mean of $67.3\,\text{km}\,\text{h}^{-1}$.

A speed of $75.7\,\text{km}\,\text{h}^{-1}$ is two standard deviations from the mean.

(a) Find the standard deviation for the speed of the cars. [2]

It is found that 82% of cars on this road travel at speeds between $p\,\text{km}\,\text{h}^{-1}$ and $q\,\text{km}\,\text{h}^{-1}$, where $p < q$. This interval includes cars travelling at a speed of $74\,\text{km}\,\text{h}^{-1}$.

(b) Show that the region of the normal distribution between p and q is **not** symmetrical about the mean. [3]

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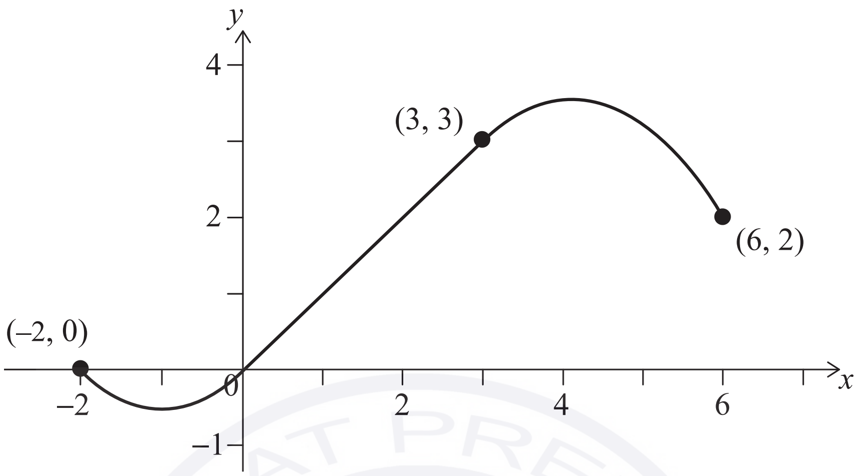
10. [Maximum mark: 9]

A decorative hook can be modelled by the curve with equation $y = f(x)$. The graph of $y = f(x)$ is shown and consists of a line segment from $(0, 0)$ to $(3, 3)$ and two sections formed by quadratic curves.



(a) Write down the equation of the line segment for $0 \le x \le 3$. [1]

The quadratic curve, with endpoints $(-2, 0)$ and $(0, 0)$, has the same gradient at $(0, 0)$ as the line segment.

(b) Find the equation of the curve between $(-2, 0)$ and $(0, 0)$. [3]

The second quadratic curve, with endpoints $(3, 3)$ and $(6, 2)$, has the same gradient at $(3, 3)$ as the line segment.

(c) Find the equation of this curve. [4]

(d) Write down f as a piecewise function. [1]

(This question continues on the following page)

(Question 10 continued)

11. [Maximum mark: 6]

A shop sells oranges and lemons. The weights of the oranges are assumed to be normally distributed with mean 205 grams and standard deviation 5 grams. The weights of the lemons are assumed to be normally distributed with mean 105 grams and standard deviation 3 grams.

Nelia selects 1 orange and 2 lemons at random and independent of each other. Calculate the probability that the weight of her orange exceeds the combined weight of her lemons.

12. [Maximum mark: 5]

Two AC (alternating current) electrical sources with the same frequencies are combined. The voltages from these sources can be expressed as $V_1 = 6\sin(at + 30°)$ and $V_2 = 6\sin(at + 90°)$.

The combined total voltage can be expressed in the form $V_1 + V_2 = V\sin(at + \theta°)$.

Determine the value of V and the value of θ.

13. [Maximum mark: 6]

The displacement, $x\,(\text{cm})$, of the end of a spring, at time t (seconds), is given by

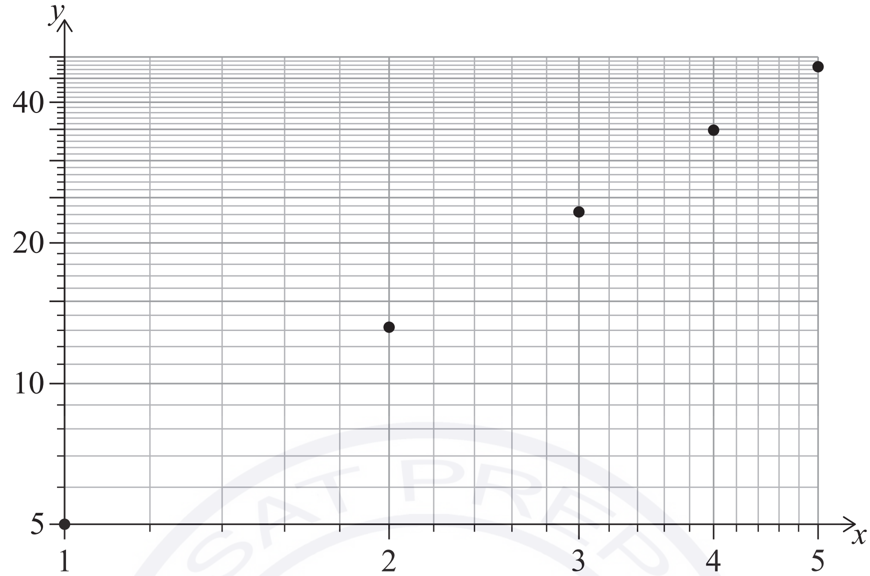
$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 10x = 0.$$

At $t = 0$, $x = 0.75$ and $\frac{dx}{dt} = 0$.

Use Euler's method, with a step length 0.1 seconds, to estimate the value of x when $t = 0.5$.

14. [Maximum mark: 6]

Petra examines two quantities, x and y, and plots data points on a log-log graph.

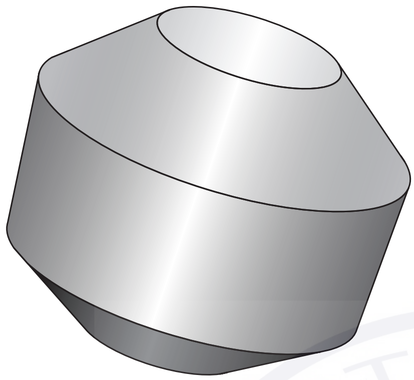


She observes that on this graph the data points follow a perfect straight line. Given that the line passes through the points $(2, 13.1951)$ and $(4, 34.822)$, find the equation of the relationship connecting x and y. Your final answer should not include logarithms.

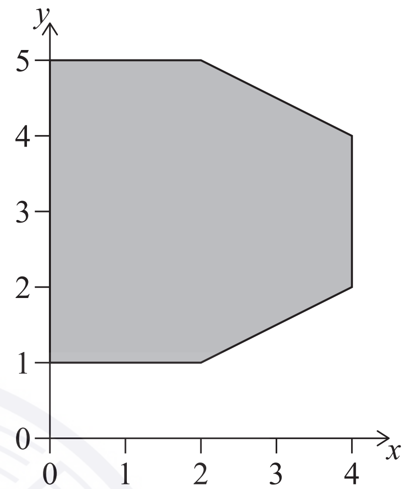
15. [Maximum mark: 6]

The solid shown is formed by rotating the hexagon with vertices $(2, 1)$, $(0, 1)$, $(0, 5)$, $(2, 5)$, $(4, 4)$ and $(4, 2)$ about the y-axis.

Solid



Hexagon



Find the volume of this solid.

16. [Maximum mark: 6]

The relationship between the intensity, I, of a light source and the distance, d, from the light source can be modelled by $I = \frac{k}{d^2}$.

Pablo measures the intensity of a light source at different distances. The data collected is shown in the table.

d (m)	1	2	5
I (lm)	42	11	1.5

Pablo finds the sum of square residuals in the form $1.0641k^2 - 89.62k + c$.

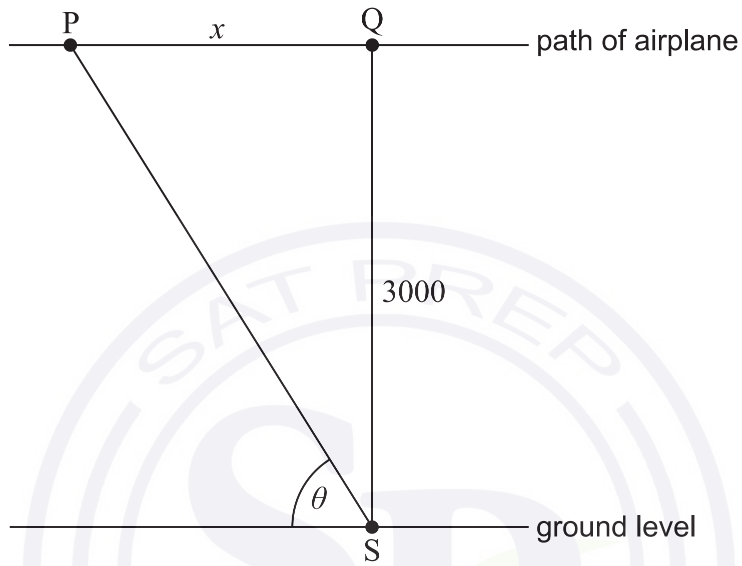
(a) Find the exact value of c. [4]

(b) Hence find the least squares regression curve of the form $I = \frac{k}{d^2}$. [2]

17. [Maximum mark: 9]

An airplane, P, is flying at a constant altitude of $3000\,\mathrm{m}$ at a speed of $250\,\mathrm{m\,s^{-1}}$. Its path passes over a tracking station, S, at ground level. Let Q be the point $3000\,\mathrm{m}$ directly above the tracking station.

At a particular time, T, as the airplane is flying towards Q, the angle of elevation, θ, of the airplane from S is increasing at a rate of 0.075 radians per second. The distance from Q to P is given by x.



(a) Use related rates to show that, at time T, $\frac{\mathrm{d}x}{\mathrm{d}\theta} = -\frac{10\,000}{3}$. [2]

(b) Find $x(\theta)$, x as a function of θ. [1]

(c) Find an expression for $\frac{\mathrm{d}x}{\mathrm{d}\theta}$ in terms of $\sin\theta$. [3]

(d) Hence find the horizontal distance from the station to the plane at time T. [3]

(This question continues on the following page)

(Question 17 continued)

References:

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Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: applications and interpretation
Higher level
Paper 1

Monday 31 October 2022 (afternoon)

Candidate session number

2 hours

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all questions.
- Answers must be written within the answer boxes provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.

8822 – 7201

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Answers must be written within the answer boxes provided. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 5]

Sergio is interested in whether an adult's favourite breakfast berry depends on their income level. He obtains the following data for 341 adults and decides to carry out a χ^2 test for independence, at the 10% significance level.

		Income level		
		Low	**Medium**	**High**
Favourite berry	**Strawberry**	21	39	30
	Blueberry	39	67	42
	Other berry	32	45	26

(a) Write down the null hypothesis. [1]

(b) Find the value of the χ^2 statistic. [2]

The critical value of this χ^2 test is 7.78.

(c) Write down Sergio's conclusion to the test in context. Justify your answer. [2]

2. [Maximum mark: 5]

Celeste heated a cup of coffee and then let it cool to room temperature. Celeste found the coffee's temperature, T, measured in $^{\circ}\mathrm{C}$, could be modelled by the following function,

$$T(t) = 71\,\mathrm{e}^{-0.0514t} + 23\,,\ t \geq 0\,,$$

where t is the time, in minutes, after the coffee started to cool.

(a) Find the coffee's temperature 16 minutes after it started to cool. [2]

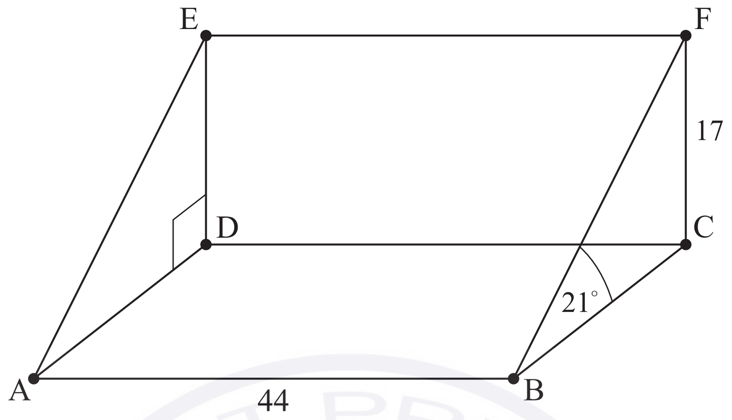
(b) Write down the room temperature. [1]

(c) Given that $T^{-1}(50) = k$, find the value of k. [2]

3. [Maximum mark: 5]

An artificial ski slope can be modelled as a triangular prism, as shown in the diagram. Rectangle ABCD is horizontal, and rectangle CDEF is vertical.

diagram not to scale



The maximum height of the ski slope, CF, is 17 metres and the steepest angle of the ski slope, $\text{F}\hat{\text{B}}\text{C}$, is 21°.

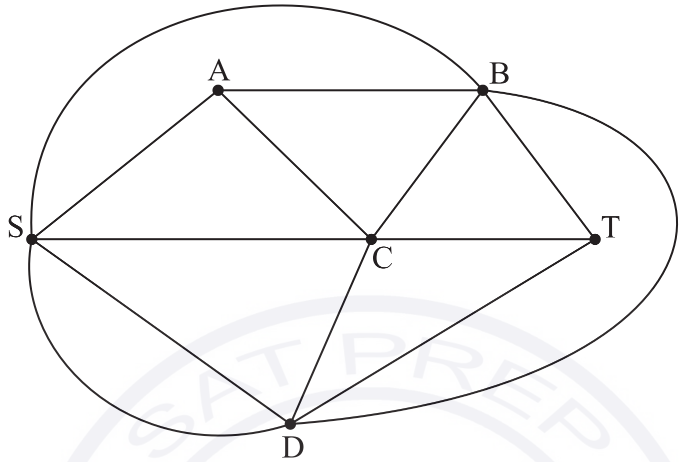
(a) Calculate the length of [BF]. [2]

The width of the base of the ski slope, AB, is 44 metres. Mayumi skis in a straight line, starting from point E and finishing at the base of the ski slope.

(b) Find the value of the least steep angle that Mayumi can ski. [3]

4. [Maximum mark: 7].

In a competition, a contestant has to move through a maze to find treasure. A graph of the maze is shown below, where each edge represents a corridor in the maze. The contestant starts at S and the treasure is located at T.



(a) Complete the adjacency matrix, $\boldsymbol{M}$, for the graph. [2]

$$\begin{array}{c} \\ S \\ A \\ B \\ C \\ D \\ T \end{array}
\begin{array}{c} \begin{array}{cccccc} S & A & B & C & D & T \end{array} \\
\begin{pmatrix} 0 & 1 & 1 & 1 & \square & 0 \\ 1 & 0 & 1 & 1 & \square & 0 \\ 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 \\ \square & \square & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{pmatrix} \end{array}$$

The competition rules state that the contestant can walk along a maximum of four corridors.

(b) Find the number of walks from S to T with a maximum of 4 edges. [4]

(c) Explain why the number of ways the contestant can reach the treasure is less than the answer to part (b). [1]

(This question continues on the following page)

(Question 4 continued)

5. [Maximum mark: 7]

Taizo plays a game where he throws one ball at two bottles that are sitting on a table. The probability of knocking over bottles, in any given game, is shown in the following table.

Number of bottles knocked over	0	1	2
Probability	0.5	0.4	0.1

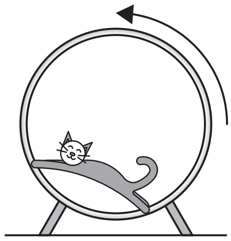
(a) Taizo plays two games that are independent of each other. Find the probability that Taizo knocks over a **total** of two bottles. [4]

In any given game, Taizo will win k points if he knocks over two bottles, win 4 points if he knocks over one bottle and lose 8 points if no bottles are knocked over.

(b) Find the value of k such that the game is fair. [3]

6. [Maximum mark: 6]

A cat runs inside a circular exercise wheel, making the wheel spin at a constant rate in an anticlockwise direction. The height, h cm, of a fixed point, P, on the wheel can be modelled by $h(t) = a\sin(bt) + c$ where t is the time in seconds and $a,\ b,\ c \in \mathbb{R}^+$.



When $t = 0$, point P is at a height of 78 cm.

(a) Write down the value of c. [1]

When $t = 4$, point P first reaches its maximum height of 143 cm.

(b) Find the value of

(i) a.

(ii) b. [3]

(c) Write down the minimum height of point P. [1]

Later, the cat is tired, and it takes twice as long for point P to complete one revolution at a new constant rate.

(d) Write down the new value of b. [1]

7. [Maximum mark: 7]

On 1 December 2022, Laviola invests 800 euros (EUR) into a savings account which pays a nominal annual interest rate of 7.5% compounded monthly. At the end of each month, Laviola deposits an additional $\text{EUR } 500$ into the savings account.

At the end of k months, Laviola will have saved enough money to withdraw $\text{EUR } 10\,000$.

(a) Find the smallest possible value of k, for $k \in \mathbb{Z}^+$. [4]

(b) For this value of k, find the interest earned in the savings account. Express your answer correct to the nearest EUR. [3]

8. [Maximum mark: 7]

Line L_1 has a vector equation $\boldsymbol{r} = \begin{pmatrix} 3p+4 \\ 2p-1 \\ p+9 \end{pmatrix}$, where $p \in \mathbb{R}$.

Line L_2 has a vector equation $\boldsymbol{r} = \begin{pmatrix} q-2 \\ 1-q \\ 2q+1 \end{pmatrix}$, where $q \in \mathbb{R}$.

The two lines intersect at point M.

(a) Find the coordinates of M. [3]

(b) Find the acute angle between the two lines. [4]

9. [Maximum mark: 8]

The transformation T is represented by the matrix $\boldsymbol{M} = \begin{pmatrix} 2 & -4 \\ 3 & 1 \end{pmatrix}$.

A pentagon with an area of $12\,\text{cm}^2$ is transformed by T.

(a) Find the area of the image of the pentagon. [2]

Under the transformation T, the image of point X has coordinates $(2t - 3,\ 6 - 5t)$, where $t \in \mathbb{R}$.

(b) Find, in terms of t, the coordinates of X. [6]

10. [Maximum mark: 7]

Stars are classified by their brightness. The brightest stars in the sky have a magnitude of 1. The magnitude, m, of another star can be modelled as a function of its brightness, b, relative to a star of magnitude 1, as shown by the following equation.

$$m = 1 - 2.5\log_{10}(b)$$

The star called Acubens has a brightness of 0.0525.

(a) Find the magnitude of Acubens. [2]

Ceres has a magnitude of 7 and is the least bright star visible without magnification.

(b) Find the brightness of Ceres. [2]

The star Proxima Centauri has a greater magnitude than the planet Neptune. The difference in their magnitudes is 3.2.

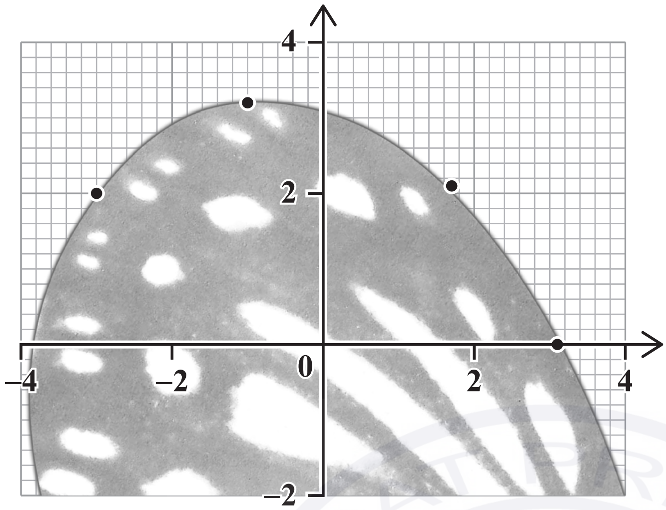
(c) Find how many times brighter Neptune is compared to Proxima Centauri. [3]

Please **do not** write on this page.

Answers written on this page will not be marked.

11. [Maximum mark: 5]

Gloria wants to model the curved edge of a butterfly wing. She inserts a photo of the wing into her graphing software and finds the coordinates of four points on the edge of the wing.



x	y
−3	2
−1	3.2
1.7	2.1
3.1	0

Gloria thinks a cubic curve will be a good model for the butterfly wing.

(a) Find the equation of the cubic regression curve for this data. [2]

For the photo of a second butterfly wing, Gloria finds the equation of the regression curve is $y = 0.0083x^3 - 0.075x^2 - 0.58x + 2.2$.

Gloria realizes that her photo of the second butterfly is an enlargement of the life-size butterfly, scale factor 2 and centred on $(0, 0)$.

(b) Find the equation of the cubic curve that models the life-size wing. [3]

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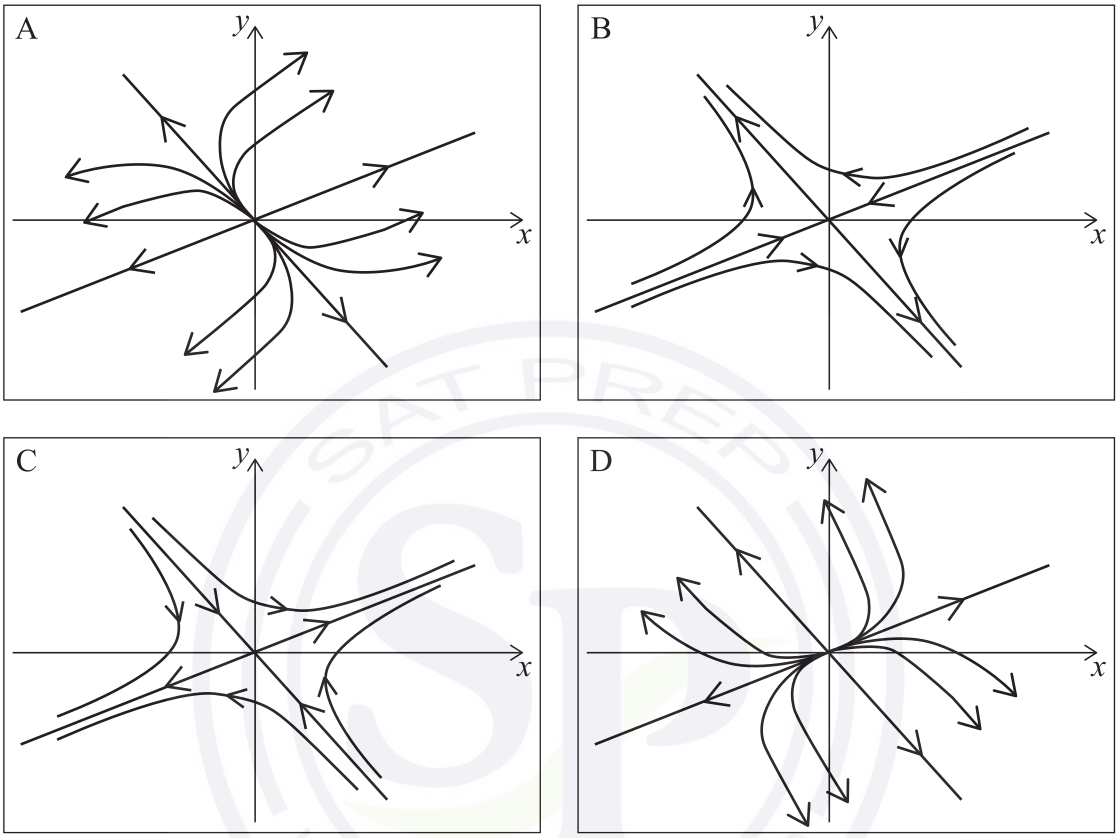
12. [Maximum mark: 5]

Four possible phase portraits for the coupled differential equations $\frac{dx}{dt} = ax + by$ and $\frac{dy}{dt} = cx + dy$ are shown, labelled A, B, C and D.



The matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ has eigenvalues λ_1 and λ_2.

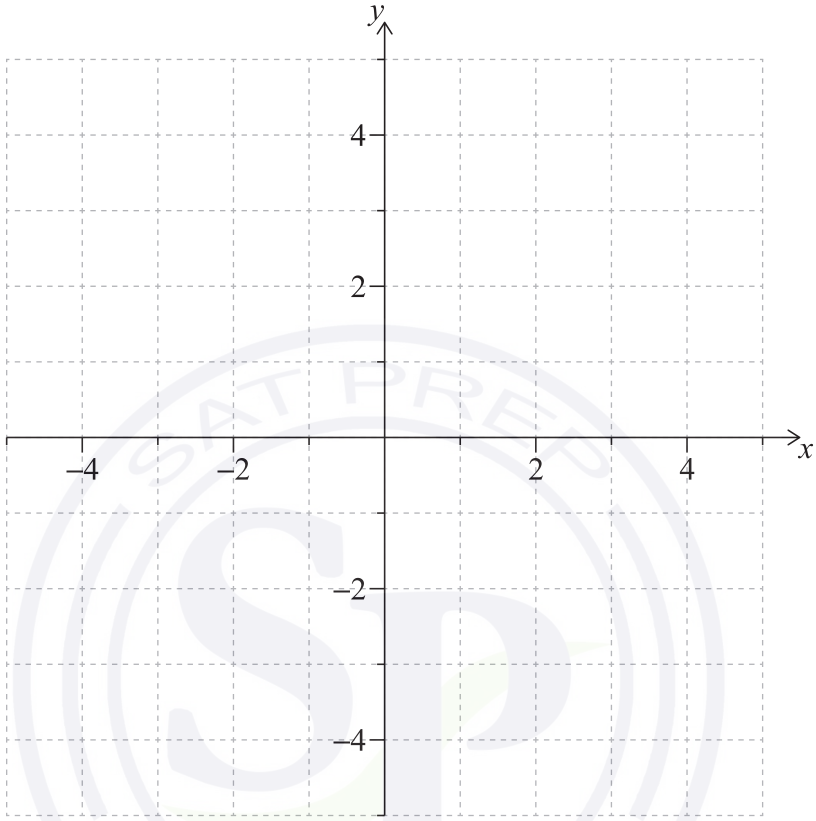
(a) Complete the following table by writing down the letter of the phase portrait that best matches the description. [3]

Description	**Phase portrait**
$\lambda_1 = 2$ with eigenvector $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and $\lambda_2 = 3$ with eigenvector $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$	
$\lambda_1 = 2$ with eigenvector $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and $\lambda_2 = -3$ with eigenvector $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$	
$\lambda_1 = -2$ with eigenvector $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and $\lambda_2 = 3$ with eigenvector $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$	

(This question continues on the following page)

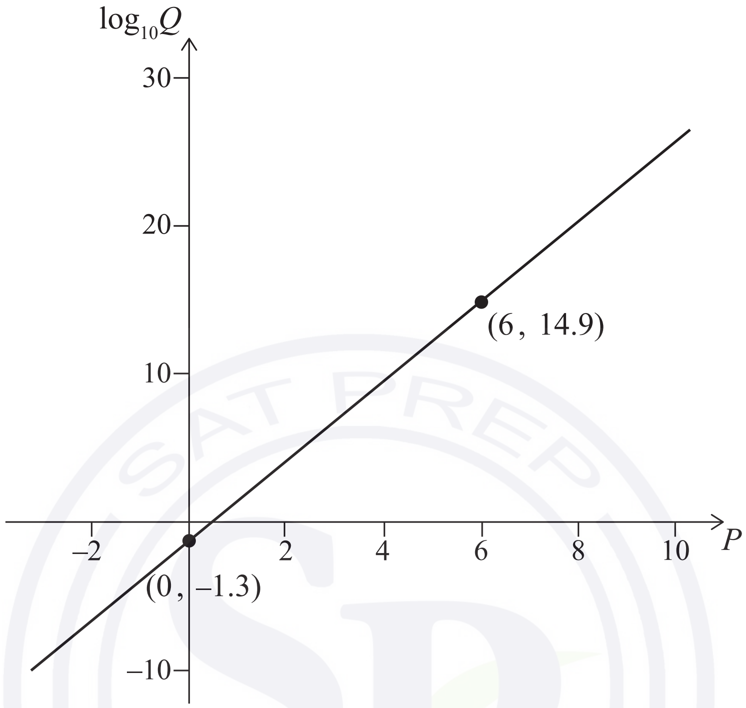
(Question 12 continued)

(b) On the following axes, sketch the phase portrait that corresponds to $\lambda_1 = -2 + 3\mathrm{i}$ and $\lambda_2 = -2 - 3\mathrm{i}$, given that $\frac{\mathrm{d}y}{\mathrm{d}t} = -12$ at $(3, 0)$. [2]



13. [Maximum mark: 6]

Gen is investigating the relationship between two sets of data, labelled P and Q, that she collected. She created a scatter plot with P on the x-axis and $\log_{10}Q$ on the y-axis. Gen noticed that the points had a strong linear correlation, so she drew a line of best fit, as shown in the diagram. The line passes through the points $(0, -1.3)$ and $(6, 14.9)$.



(a) Find an equation for Q in terms of P. [3]

Gen also investigates the relationship between the same data, Q, and some new data, R. She believes that the data can be modelled by $Q = a\ln R + b$ and she decides to create a scatter plot to verify her belief.

(b) State what expression Gen should plot on each axis to verify her belief. [1]

The scatter plot has a linear relationship and Gen finds $a = 4.3$ and $b = 12.1$.

(c) Find an equation for P in terms of R. [2]

(This question continues on the following page)

(Question 13 continued)

14. [Maximum mark: 9]

A particle moves such that its velocity, v metres per second, at time t seconds, is given by $v = t\sin(t^2)$.

(a) Find an expression for the acceleration of the particle. [2]

(b) Hence, or otherwise, find its greatest acceleration for $0 \le t \le 8$. [2]

The particle starts at the origin.

(c) Find an expression for the displacement of the particle. [3]

(d) Hence show that the particle never has a negative displacement. [2]

15. [Maximum mark: 5]

An electrical circuit contains a capacitor. The charge on the capacitor, q Coulombs, at time t seconds, satisfies the differential equation

$$\frac{d^2q}{dt^2} + 5\frac{dq}{dt} + 20q = 200.$$

Initially $q = 1$ and $\frac{dq}{dt} = 8$.

Use Euler's method with $h = 0.1$ to estimate the maximum charge on the capacitor during the first second.

16. [Maximum mark: 8]

The principal of a school is concerned that only 30% of her students are choosing healthy options from the school canteen. She organizes a campaign to promote healthy eating and decides to test if the campaign has increased the number of students choosing healthy options. She assumes that a student's choice is independent of other students' choices.

(a) Write down suitable hypotheses for this test. [2]

The principal decides to take a random sample of 80 students. She will reject the null hypothesis if at least 31 students choose a healthy option.

(b) Find the probability that she makes a Type I error. [3]

In fact, the campaign led to 40% of her students choosing a healthy option.

(c) Find the probability that she makes a Type II error. [3]

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17. [Maximum mark: 8]

The time of sunrise, R hours after midnight, in Taipei can be modelled by

$$R = 1.08\cos(0.0165t + 0.413) + 4.94,$$

where t is the day of the year 2021 (for example, $t = 2$ represents 2 January 2021).

The time of sunset, S hours after midnight, in Taipei can be modelled by

$$S = 1.15\cos(0.0165t - 2.97) + 18.9.$$

The number of daylight hours, D, in Taipei during 2021 can be modelled by

$$D = a\cos(0.0165t + b) + c.$$

(a) Find the value of a, of b and of c. [6]

(b) Hence, or otherwise, find the largest number of daylight hours in Taipei during 2021 and the day of the year on which this occurs. [2]

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References:

11. Fleur, 2019. *photo-1560263816-d704d83cce0f.* [image online] Available at: https://unsplash.com/photos/SE2zTdS1MNo [Accessed 8 February 2022]. Source adapted.

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: applications and interpretation
Higher level
Paper 1

Friday 6 May 2022 (afternoon)

Candidate session number

2 hours

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all questions.
- Answers must be written within the answer boxes provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.

2222–7211

25 pages

Answers must be written within the answer boxes provided. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 6]

A group of 130 applicants applied for admission into either the Arts programme or the Sciences programme at a university. The outcomes of their applications are shown in the following table.

	Accepted	**Rejected**
Arts programme	17	24
Sciences programme	25	64

(a) Find the probability that a randomly chosen applicant from this group was accepted by the university. [1]

An applicant is chosen at random from this group. It is found that they were accepted into the programme of their choice.

(b) Find the probability that the applicant applied for the Arts programme. [2]

Two different applicants are chosen at random from the original group.

(c) Find the probability that both applicants applied to the Arts programme. [3]

(This question continues on the following page)

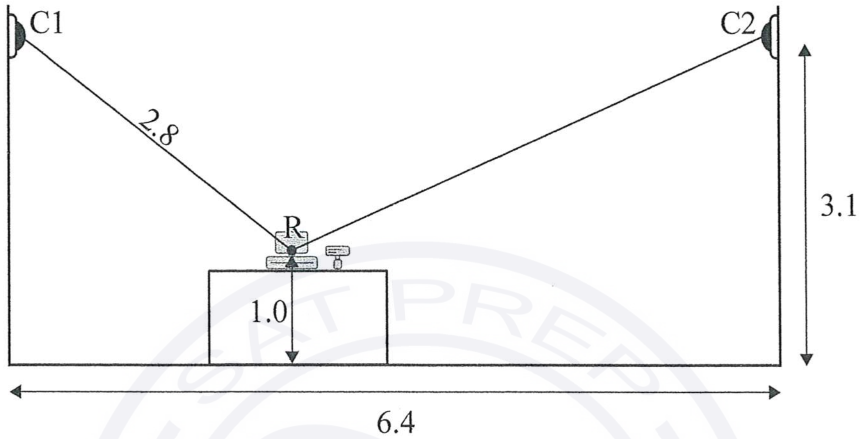
(Question 1 continued)

2. [Maximum mark: 8]

The owner of a convenience store installs two security cameras, represented by points C1 and C2. Both cameras point towards the centre of the store's cash register, represented by the point R.

The following diagram shows this information on a cross-section of the store.

diagram not to scale



The cameras are positioned at a height of 3.1 m, and the horizontal distance between the cameras is 6.4 m. The cash register is sitting on a counter so that its centre, R, is 1.0 m above the floor.

The distance from Camera 1 to the centre of the cash register is 2.8 m.

(a) Determine the angle of depression from Camera 1 to the centre of the cash register. Give your answer in degrees. [2]

(b) Calculate the distance from Camera 2 to the centre of the cash register. [4]

(c) Without further calculation, determine which camera has the largest angle of depression to the centre of the cash register. Justify your response. [2]

(This question continues on the following page)

(Question 2 continued)

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Answers written on this page
will not be marked.

3. [Maximum mark: 7]

A polygraph test is used to determine whether people are telling the truth or not, but it is not completely accurate. When a person tells the truth, they have a 20% chance of failing the test. Each test outcome is independent of any previous test outcome.

10 people take a polygraph test and all 10 tell the truth.

(a) Calculate the expected number of people who will pass this polygraph test. [2]

(b) Calculate the probability that exactly 4 people will fail this polygraph test. [2]

(c) Determine the probability that fewer than 7 people will pass this polygraph test. [3]

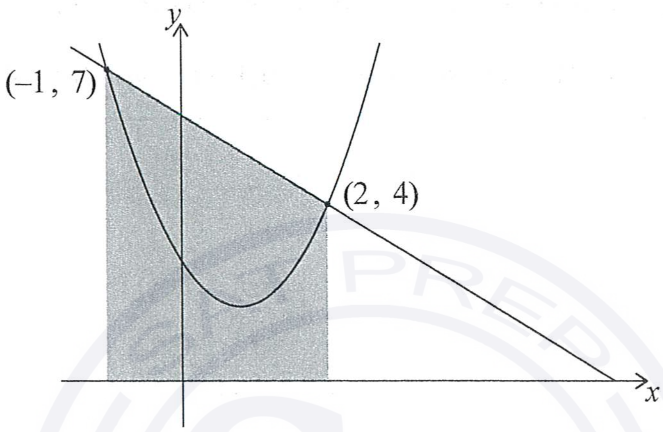
4. [Maximum mark: 7]

The graphs of $y = 6 - x$ and $y = 1.5x^2 - 2.5x + 3$ intersect at $(2, 4)$ and $(-1, 7)$, as shown in the following diagrams.

In **diagram 1**, the region enclosed by the lines $y = 6 - x$, $x = -1$, $x = 2$ and the x-axis has been shaded.

diagram not to scale

Diagram 1

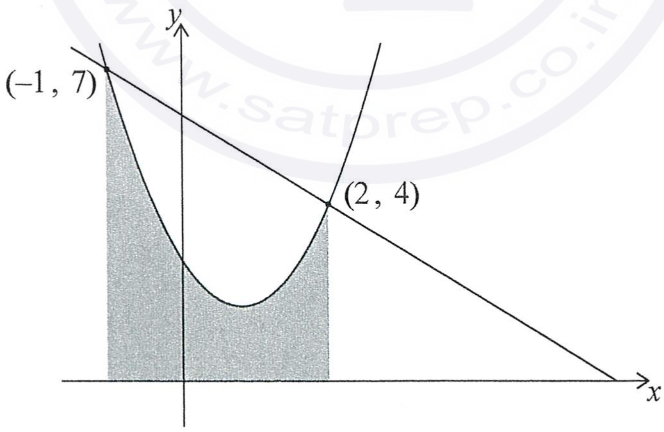


(a) Calculate the area of the shaded region in **diagram 1**. [2]

In **diagram 2**, the region enclosed by the curve $y = 1.5x^2 - 2.5x + 3$, and the lines $x = -1$, $x = 2$ and the x-axis has been shaded.

diagram not to scale

Diagram 2



(b) (i) Write down an integral for the area of the shaded region in **diagram 2**.

(ii) Calculate the area of this region. [3]

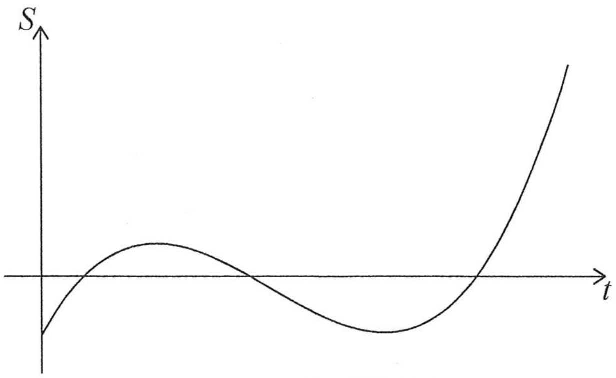
(c) Hence, determine the area enclosed between $y = 6 - x$ and $y = 1.5x^2 - 2.5x + 3$. [2]

(This question continues on the following page)

(Question 4 continued)

5. [Maximum mark: 8]

The graph below shows the average savings, S thousand dollars, of a group of university graduates as a function of t, the number of years after graduating from university.



(a) Write down one feature of this graph which suggests a cubic function might be appropriate to model this scenario. [1]

The equation of the model can be expressed in the form $S = at^3 + bt^2 + ct + d$, where a, b, c and d are real constants.

The graph of the model must pass through the following four points.

t	0	1	2	3
S	–5	3	–1	–5

(b) (i) Write down the value of d.

(ii) Write down three simultaneous equations for a, b and c.

(iii) Hence, or otherwise, find the values of a, b and c. [4]

A negative value of S indicates that a graduate is expected to be in debt.

(c) Use the model to determine the total length of time, in years, for which a graduate is expected to be in debt after graduating from university. [3]

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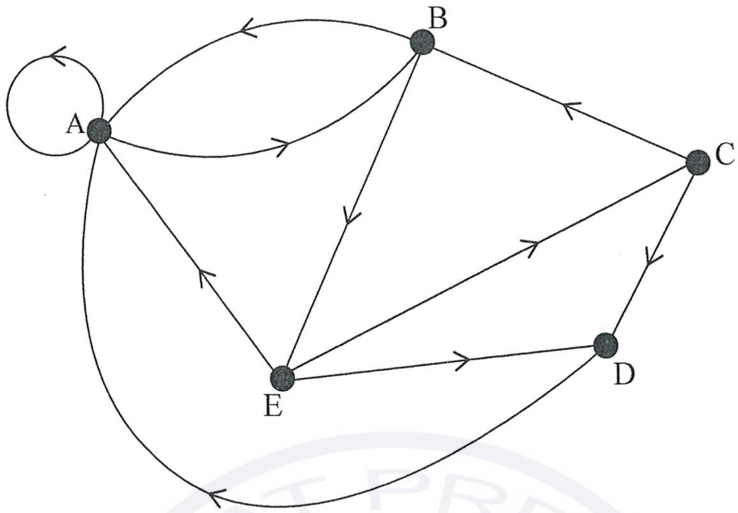
(Question 5 continued)

6. [Maximum mark: 5]

Consider the following directed network.



(a) Write down the adjacency matrix for this network. [2]

(b) Determine the number of different walks of length 5 that start and end at the same vertex. [3]

7. [Maximum mark: 5]

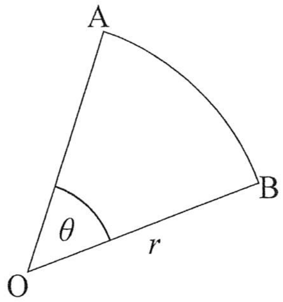
The sum of an infinite geometric sequence is 9.

The first term is 4 more than the second term.

Find the third term. Justify your answer.

8. [Maximum mark: 8]

The diagram shows a sector, OAB, of a circle with centre O and radius r, such that $\mathrm{A\hat{O}B} = \theta$.



Sam measured the value of r to be $2\,\mathrm{cm}$ and the value of θ to be 30°.

(a) Use Sam's measurements to calculate the area of the sector. Give your answer to four significant figures. [2]

It is found that Sam's measurements are accurate to only one significant figure.

(b) Find the upper bound and lower bound of the area of the sector. [3]

(c) Find, with justification, the largest possible percentage error if the answer to part (a) is recorded as the area of the sector. [3]

9. [Maximum mark: 8]

A psychologist records the number of digits (d) of π that a sample of IB Mathematics higher level candidates could recall.

d	2	3	4	5	6	7
Frequency	2	6	24	21	11	3

(a) Find an unbiased estimate of the population mean of d. [1]

(b) Find an unbiased estimate of the population variance of d. [2]

The psychologist has read that in the general population people can remember an average of 4.4 digits of π. The psychologist wants to perform a statistical test to see if IB Mathematics higher level candidates can remember more digits than the general population.

(c) $H_0: \mu = 4.4$ is the null hypothesis for this test.

(i) State the alternative hypothesis.

(ii) Given that all assumptions for this test are satisfied, carry out an appropriate hypothesis test. State and justify your conclusion. Use a 5% significance level. [5]

10. [Maximum mark: 5]

The function $f(x) = \ln\left(\frac{1}{x-2}\right)$ is defined for $x > 2$, $x \in \mathbb{R}$.

(a) Find an expression for $f^{-1}(x)$. You are not required to state a domain. [3]

(b) Solve $f(x) = f^{-1}(x)$. [2]

11. [Maximum mark: 6]

Juliana plans to invest money for 10 years in an account paying 3.5% interest, compounded annually. She expects the annual inflation rate to be 2% per year throughout the 10-year period.

Juliana would like her investment to be worth a real value of $\$4000$, compared to current values, at the end of the 10-year period. She is considering two options.

Option 1: Make a one-time investment at the start of the 10-year period.

Option 2: Invest $\$1000$ at the start of the 10-year period and then invest $\$x$ into the account at the end of each year (including the first and last years).

(a) For option 1, determine the minimum amount Juliana would need to invest. Give your answer to the nearest dollar. [3]

(b) For option 2, find the minimum value of x that Juliana would need to invest each year. Give your answer to the nearest dollar. [3]

12. [Maximum mark: 6]

The sex of cuttlefish is difficult to determine visually, so it is often found by weighing the cuttlefish.

The weights of adult male cuttlefish are known to be normally distributed with mean $10\,\text{kg}$ and standard deviation $0.5\,\text{kg}$.

The weights of adult female cuttlefish are known to be normally distributed with mean $12\,\text{kg}$ and standard deviation $1\,\text{kg}$.

A zoologist uses the null hypothesis that in the absence of information a cuttlefish is male.

If the weight is found to be above $11.5\,\text{kg}$ the cuttlefish is classified as female.

(a) Find the probability of making a Type I error when weighing a male cuttlefish. [2]

(b) Find the probability of making a Type II error when weighing a female cuttlefish. [2]

$90\,\%$ of adult cuttlefish are male.

(c) Find the probability of making an error using the zoologist's method. [2]

13. [Maximum mark: 5]

An electric circuit has two power sources. The voltage, V_1, provided by the first power source, at time t, is modelled by

$$V_1 = \mathrm{Re}(2e^{3t\mathrm{i}}).$$

The voltage, V_2, provided by the second power source is modelled by

$$V_2 = \mathrm{Re}(5e^{(3t+4)\mathrm{i}}).$$

The total voltage in the circuit, V_T, is given by

$$V_T = V_1 + V_2.$$

(a) Find an expression for V_T in the form $A\cos(Bt + C)$, where A, B and C are real constants. [4]

(b) Hence write down the maximum voltage in the circuit. [1]

14. [Maximum mark: 4]

The shape of a vase is formed by rotating a curve about the y-axis.

The vase is $10\,\text{cm}$ high. The internal radius of the vase is measured at $2\,\text{cm}$ intervals along the height:

Height (cm)	Radius (cm)
0	4
2	6
4	8
6	7
8	3
10	5

Use the trapezoidal rule to estimate the volume of water that the vase can hold.

15. [Maximum mark: 7]

The equation of the line $y = mx + c$ can be expressed in vector form $\boldsymbol{r} = \boldsymbol{a} + \lambda\boldsymbol{b}$.

(a) Find the vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ in terms of m and/or c. [2]

The matrix $\boldsymbol{M}$ is defined by $\begin{pmatrix} 6 & 3 \\ 4 & 2 \end{pmatrix}$.

(b) Find the value of $\det \boldsymbol{M}$. [1]

The line $y = mx + c$ (where $m \neq -2$) is transformed into a new line using the transformation described by matrix $\boldsymbol{M}$.

(c) Show that the equation of the resulting line does not depend on m or c. [4]

Please **do not** write on this page.

Answers written on this page will not be marked.

16. [Maximum mark: 7]

The position vector of a particle, P, relative to a fixed origin O at time t is given by

$$\vec{\mathrm{OP}} = \begin{pmatrix} \sin(t^2) \\ \cos(t^2) \end{pmatrix}.$$

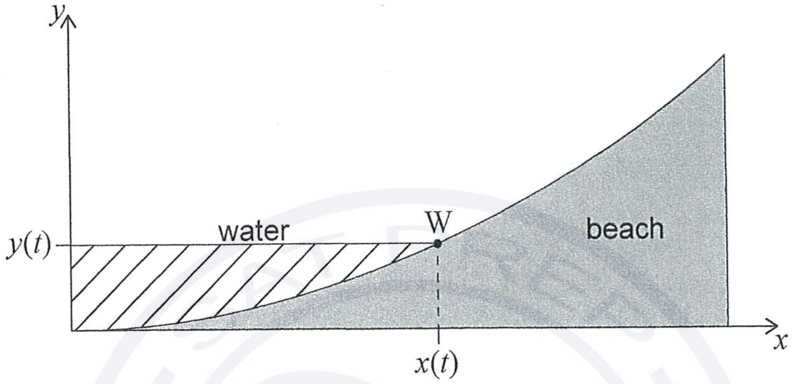
(a) Find the velocity vector of P. [2]

(b) Show that the acceleration vector of P is never parallel to the position vector of P. [5]

17. [Maximum mark: 8]

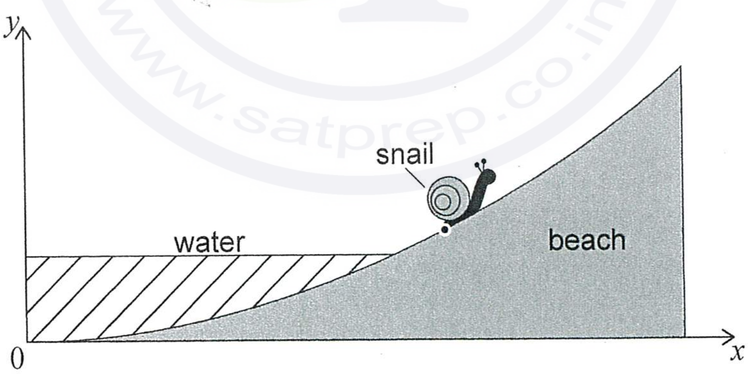
The cross-section of a beach is modelled by the equation $y = 0.02x^2$ for $0 \le x \le 10$ where y is the height of the beach (in metres) at a horizontal distance x metres from an origin. t is the time in hours after low tide.

At $t = 0$ the water is at the point $(0, 0)$. The height of the water rises at a rate of 0.2 metres per hour. The point $\text{W}(x(t), y(t))$ indicates where the water level meets the beach at time t.



(a) When W has an x-coordinate equal to 1, find the horizontal component of the velocity of W. [3]

A snail is modelled as a single point. At $t = 0$ it is positioned at $(1, 0.02)$. The snail travels away from the incoming water at a speed of 1 metre per hour in the direction along the curve of the cross-section of the beach. The following diagram shows this for a value of t, such that $t > 0$.



(b) (i) Find the time taken for the snail to reach the point $(10, 2)$.

(ii) Hence show that the snail reaches the point $(10, 2)$ before the water does. [5]

(This question continues on the following page)

(Question 17 continued)

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: applications and interpretation
Higher level
Paper 1

Friday 6 May 2022 (afternoon)

Candidate session number

2 hours

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all questions.
- Answers must be written within the answer boxes provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.

2222 – 7206

Answers must be written within the answer boxes provided. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 5]

The height of a baseball after it is hit by a bat is modelled by the function

$$h(t) = -4.8t^2 + 21t + 1.2$$

where $h(t)$ is the height in metres above the ground and t is the time in seconds after the ball was hit.

(a) Write down the height of the ball above the ground at the instant it is hit by the bat. [1]

(b) Find the value of t when the ball hits the ground. [2]

(c) State an appropriate domain for t in this model. [2]

2. [Maximum mark: 5]

The ticket prices for a concert are shown in the following table.

Ticket Type	Price (in Australian dollars, $)
Adult	15
Child	10
Student	12

- A total of 600 tickets were sold.
- The total amount of money from ticket sales was $\$7816$.
- There were twice as many adult tickets sold as child tickets.

Let the number of adult tickets sold be x, the number of child tickets sold be y, and the number of student tickets sold be z.

(a) Write down three equations that express the information given above. [3]

(b) Find the number of each type of ticket sold. [2]

3. [Maximum mark: 7]

Leo is investigating whether a six-sided die is fair. He rolls the die 60 times and records the observed frequencies in the following table:

Number on die	1	2	3	4	5	6
Observed frequency	8	7	6	15	12	12

Leo carries out a χ^2 goodness of fit test at a 5% significance level.

(a) Write down the null and alternative hypotheses. [1]

(b) Write down the degrees of freedom. [1]

(c) Write down the expected frequency of rolling a 1. [1]

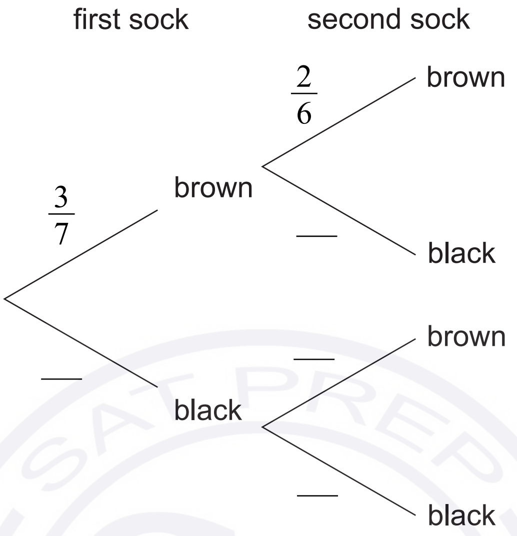
(d) Find the p-value for the test. [2]

(e) State the conclusion of the test. Give a reason for your answer. [2]

4. [Maximum mark: 7]

Karl has three brown socks and four black socks in his drawer. He takes two socks at random from the drawer.

(a) Complete the tree diagram. [2]



(b) Find the probability that Karl takes two socks of the same colour. [2]

(c) Given that Karl has two socks of the same colour find the probability that he has two brown socks. [3]

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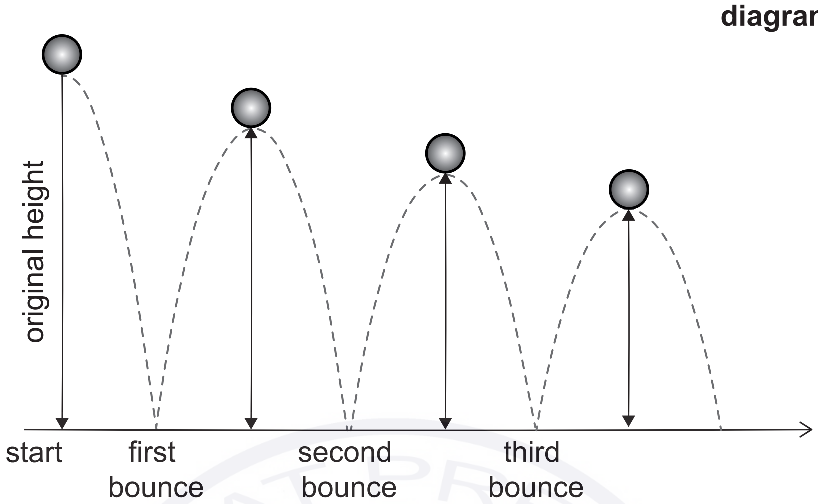
5. [Maximum mark: 7]

A ball is dropped from a height of 1.8 metres and bounces on the ground. The maximum height reached by the ball, after each bounce, is 85% of the previous maximum height.

diagram not to scale



(a) Show that the maximum height reached by the ball after it has bounced for the sixth time is $68\,\text{cm}$, to the nearest cm. [2]

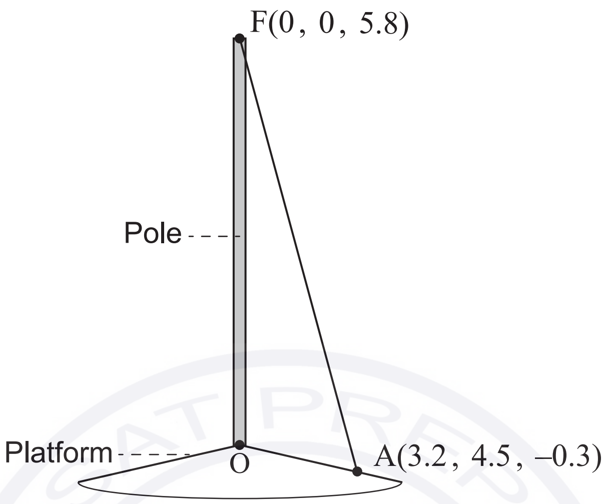
(b) Find the number of times, after the first bounce, that the maximum height reached is greater than $10\,\text{cm}$. [2]

(c) Find the total **vertical** distance travelled by the ball from the point at which it is dropped until the fourth bounce. [3]

6. [Maximum mark: 8]

A vertical pole stands on a sloped platform. The bottom of the pole is used as the origin, O, of a coordinate system in which the top, F, of the pole has coordinates $(0, 0, 5.8)$. All units are in metres.

diagram not to scale



The pole is held in place by ropes attached at F.

One of these ropes is attached to the platform at point $A(3.2, 4.5, -0.3)$. The rope forms a straight line from A to F.

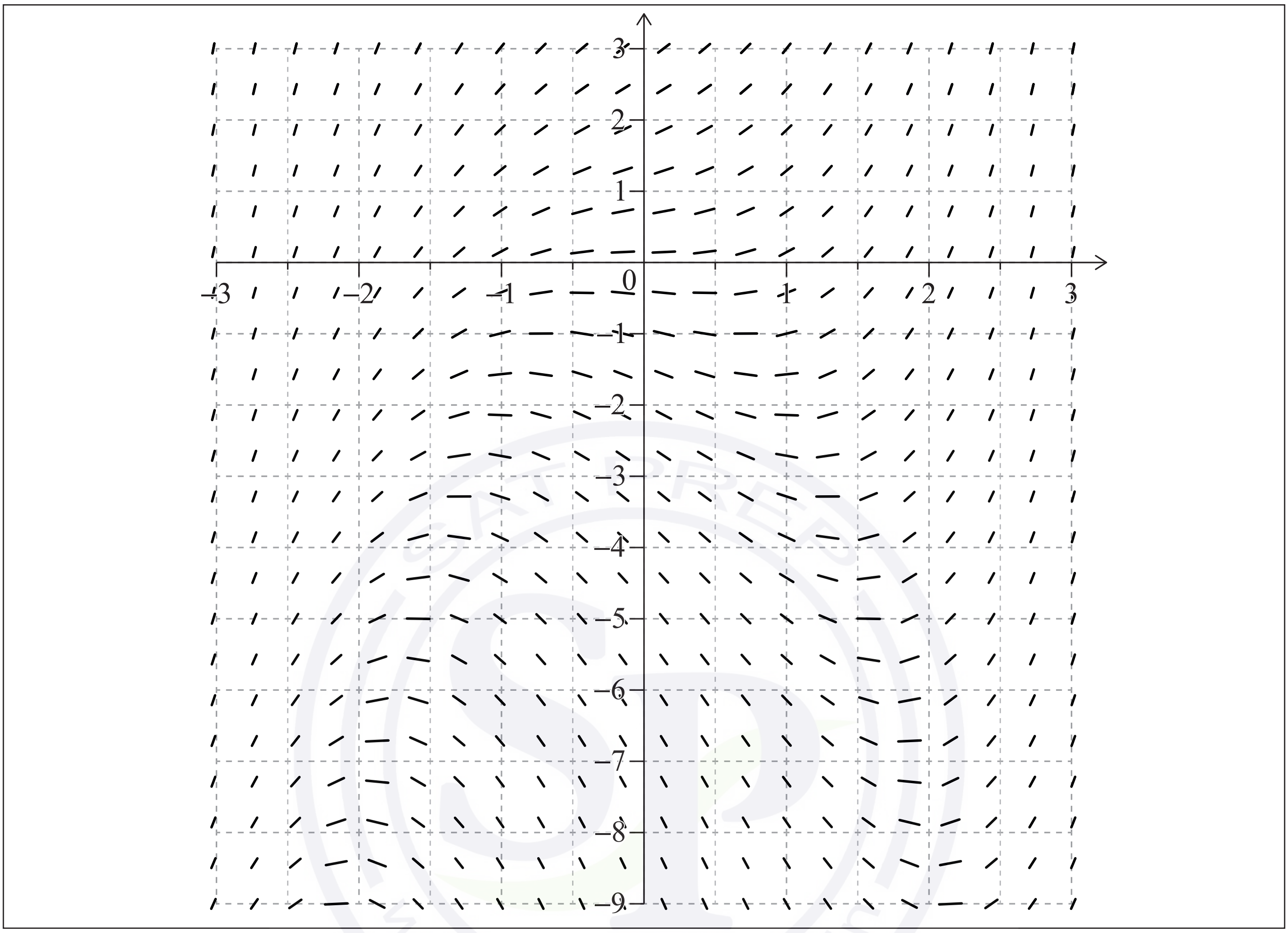
(a) Find $\overrightarrow{AF}$. [1]

(b) Find the length of the rope. [2]

(c) Find $F\hat{A}O$, the angle the rope makes with the platform. [5]

7. [Maximum mark: 4]

A slope field for the differential equation $\frac{dy}{dx} = x^2 + \frac{y}{2}$ is shown.



Some of the solutions to the differential equation have a local maximum point and a local minimum point.

(a) (i) Write down the equation of the curve on which all these maximum and minimum points lie.

(ii) Sketch this curve on the slope field. [2]

The solution to the differential equation that passes through the point $(0, -2)$ has both a local maximum point and a local minimum point.

(b) On the slope field, sketch the solution to the differential equation that passes through $(0, -2)$. [2]

(This question continues on the following page)

(Question 7 continued)

8. [Maximum mark: 6]

Consider the curve $y = 2x(4 - e^x)$.

(a) Find

(i) $\frac{dy}{dx}$.

(ii) $\frac{d^2y}{dx^2}$. [4]

The curve has a point of inflexion at (a, b).

(b) Find the value of a. [2]

9. [Maximum mark: 6]

A company produces bags of sugar with a labelled weight of $1\,\text{kg}$. The weights of the bags are normally distributed with a mean of $1\,\text{kg}$ and a standard deviation of $100\,\text{g}$. In an inspection, if the weight of a randomly chosen bag is less than $950\,\text{g}$ then the company fails the inspection.

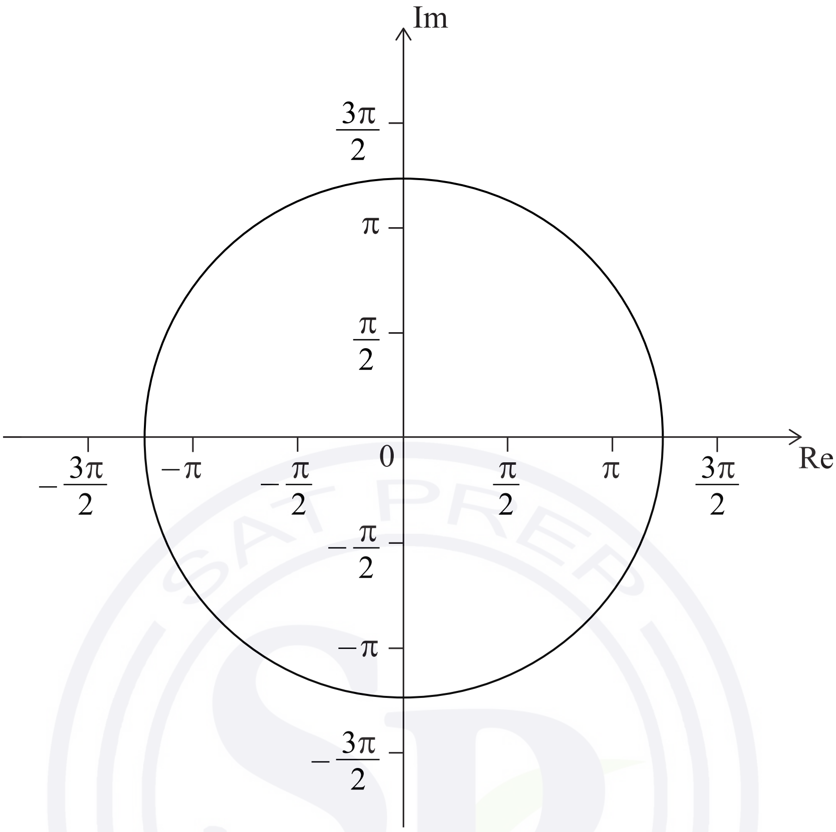
(a) Find the probability that the company fails the inspection. [2]

A statistician in the company suggests it would be fairer if the company passes the inspection when the mean weight of five randomly chosen bags is greater than $950\,\text{g}$.

(b) Find the probability of passing the inspection if the statistician's suggestion is followed. [4]

10. [Maximum mark: 7]

The following Argand diagram shows a circle centre 0 with a radius of 4 units.



A set of points, $\{z_\theta\}$, on the Argand plane are defined by the equation

$$z_\theta = \frac{1}{2}\theta e^{\theta i}, \text{ where } \theta \geq 0.$$

(a) Plot on the Argand diagram the points corresponding to

(i) $\theta = \frac{\pi}{2}$.

(ii) $\theta = \pi$.

(iii) $\theta = \frac{3\pi}{2}$. [3]

Consider the case where $|z_\theta| = 4$.

(b) (i) Find this value of θ.

(ii) For this value of θ, plot the approximate position of z_θ on the Argand diagram. [4]

(This question continues on the following page)

(Question 10 continued)

11. [Maximum mark: 5]

The matrix $\boldsymbol{M} = \begin{pmatrix} 0.2 & 0.7 \\ 0.8 & 0.3 \end{pmatrix}$ has eigenvalues -0.5 and 1.

(a) Find an eigenvector corresponding to the eigenvalue of 1. Give your answer in the form $\begin{pmatrix} a \\ b \end{pmatrix}$, where $a,\ b \in \mathbb{Z}$. [3]

A switch has two states, A and B. Each second it either remains in the same state or moves according to the following rule: If it is in state A it will move to state B with a probability of 0.8 and if it is in state B it will move to state A with a probability of 0.7.

(b) Using your answer to (a), or otherwise, find the long-term probability of the switch being in state A. Give your answer in the form $\frac{c}{d}$, where $c,\ d \in \mathbb{Z}^{+}$. [2]

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12. [Maximum mark: 8]

The strength of earthquakes is measured on the Richter magnitude scale, with values typically between 0 and 8 where 8 is the most severe.

The Gutenberg–Richter equation gives the average number of earthquakes per year, N, which have a magnitude of at least M. For a particular region the equation is

$$\log_{10} N = a - M \text{, for some } a \in \mathbb{R}.$$

This region has an average of 100 earthquakes per year with a magnitude of at least 3.

(a) Find the value of a. [2]

The equation for this region can also be written as $N = \frac{b}{10^M}$.

(b) Find the value of b. [2]

Within this region the most severe earthquake recorded had a magnitude of 7.2.

(c) Find the average number of earthquakes in a year with a magnitude of at least 7.2. [1]

The number of earthquakes in a given year with a magnitude of at least 7.2 can be modelled by a Poisson distribution, with mean N. The number of earthquakes in one year is independent of the number of earthquakes in any other year.

Let Y be the number of years between the earthquake of magnitude 7.2 and the next earthquake of at least this magnitude.

(d) Find $\mathrm{P}(Y > 100)$. [3]

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13. [Maximum mark: 5]

At $1{:}00$ pm a ship is $1\,\text{km}$ east and $4\,\text{km}$ north of a harbour. A coordinate system is defined with the harbour at the origin. The position vector of the ship at $1{:}00$ pm is given by $\begin{pmatrix} 1 \\ 4 \end{pmatrix}$.

The ship has a constant velocity of $\begin{pmatrix} 1.2 \\ -0.6 \end{pmatrix}$ kilometres per hour (km h^{-1}).

(a) Write down an expression for the position vector $\boldsymbol{r}$ of the ship, t hours after $1{:}00$ pm. [1]

(b) Find the time at which the bearing of the ship from the harbour is 045°. [4]

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14. [Maximum mark: 8]

(a) (i) Expand $\left(\frac{1}{u}+1\right)^2$.

(ii) Find $\int\left(\frac{1}{(x+2)}+1\right)^2 \mathrm{d}x$. [4]

The region bounded by $y=\frac{1}{(x+2)}+1$, $x=0$, $x=2$ and the x-axis is rotated through 2π about the x-axis to form a solid.

(b) Find the volume of the solid formed. Give your answer in the form $\frac{\pi}{4}(a+b\ln(c))$, where a, b, $c \in \mathbb{Z}$. [4]

15. [Maximum mark: 7]

The number of cars arriving at a junction in a particular town in any given minute between 9:00 am and 10:00 am is historically known to follow a Poisson distribution with a mean of 5.4 cars per minute.

A new road is built near the town. It is claimed that the new road has decreased the number of cars arriving at the junction.

To test the claim, the number of cars, X, arriving at the junction between 9:00 am and 10:00 am on a particular day will be recorded. The test will have the following hypotheses:

H_0: the mean number of cars arriving at the junction has not changed,
H_1: the mean number of cars arriving at the junction has decreased.

The alternative hypothesis will be accepted if $X \le 300$.

(a) Assuming the null hypothesis to be true, state the distribution of X. [1]

(b) Find the probability of a Type I error. [2]

(c) Find the probability of a Type II error, if the number of cars now follows a Poisson distribution with a mean of 4.5 cars per minute. [4]

16. [Maximum mark: 7]

The wind chill index W is a measure of the temperature, in $^{\circ}\mathrm{C}$, felt when taking into account the effect of the wind.

When Frieda arrives at the top of a hill, the relationship between the wind chill index and the speed of the wind v in kilometres per hour $(\mathrm{km\,h^{-1}})$ is given by the equation

$$W = 19.34 - 7.405v^{0.16}$$

(a) Find an expression for $\frac{\mathrm{d}W}{\mathrm{d}v}$. [2]

When Frieda arrives at the top of a hill, the speed of the wind is 10 kilometres per hour and increasing at a rate of $5\,\mathrm{km\,h^{-1}\,minute^{-1}}$.

(b) Find the rate of change of W at this time. [5]

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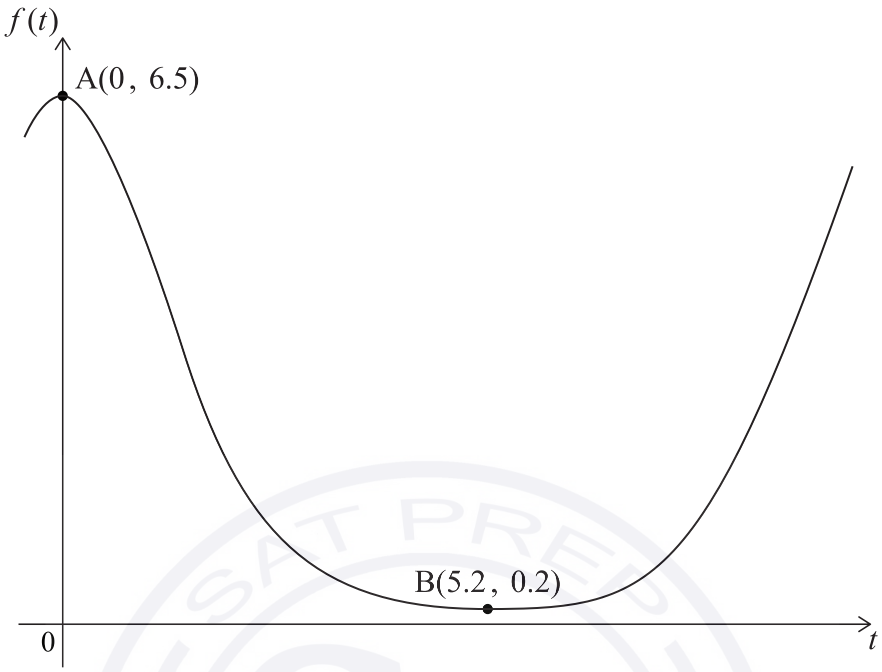
17. [Maximum mark: 8]

A function f is of the form $f(t) = p\mathrm{e}^{q\cos(rt)}$, $p, q, r \in \mathbb{R}^+$. Part of the graph of f is shown.



The points A and B have coordinates $\mathrm{A}(0, 6.5)$ and $\mathrm{B}(5.2, 0.2)$, and lie on f.

The point A is a local maximum and the point B is a local minimum.

Find the value of p, of q and of r.

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References:

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: applications and interpretation
Higher level
Paper 1

Monday 1 November 2021 (afternoon)

Candidate session number

2 hours

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all questions.
- Answers must be written within the answer boxes provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.

25 pages
8821 – 7201

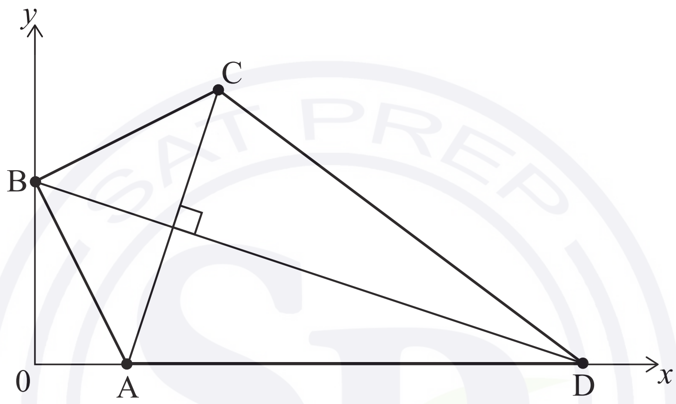
Answers must be written within the answer boxes provided. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 6]

Dilara is designing a kite ABCD on a set of coordinate axes in which one unit represents 10 cm.

The coordinates of A, B and C are $(2, 0)$, $(0, 4)$ and $(4, 6)$ respectively. Point D lies on the x-axis. $[AC]$ is perpendicular to $[BD]$. This information is shown in the following diagram.

diagram not to scale



(a) Find the gradient of the line through A and C. [2]

(b) Write down the gradient of the line through B and D. [1]

(c) Find the equation of the line through B and D. Give your answer in the form $ax + by + d = 0$, where a, b and d are integers. [2]

(d) Write down the x-coordinate of point D. [1]

(This question continues on the following page)

28EP02

(Question 1 continued)

2. [Maximum mark: 5]

Inspectors are investigating the carbon dioxide emissions of a power plant. Let R be the rate, in tonnes per hour, at which carbon dioxide is being emitted and t be the time in hours since the inspection began.

When R is plotted against t, the total amount of carbon dioxide produced is represented by the area between the graph and the horizontal t-axis.

The rate, R, is measured over the course of two hours. The results are shown in the following table.

t	0	0.4	0.8	1.2	1.6	2
R	30	50	60	40	20	50

(a) Use the trapezoidal rule with an interval width of 0.4 to estimate the total amount of carbon dioxide emitted during these two hours. [3]

The real amount of carbon dioxide emitted during these two hours was 72 tonnes.

(b) Find the percentage error of the estimate found in part (a). [2]

3. [Maximum mark: 7]

Let the function $h(x)$ represent the height in centimetres of a cylindrical tin can with diameter x cm.

$$h(x) = \frac{640}{x^2} + 0.5 \text{ for } 4 \le x \le 14.$$

(a) Find the range of h. [3]

The function h^{-1} is the inverse function of h.

(b) (i) Find $h^{-1}(10)$.

(ii) In the context of the question, interpret your answer to part (b)(i).

(iii) Write down the range of h^{-1}. [4]

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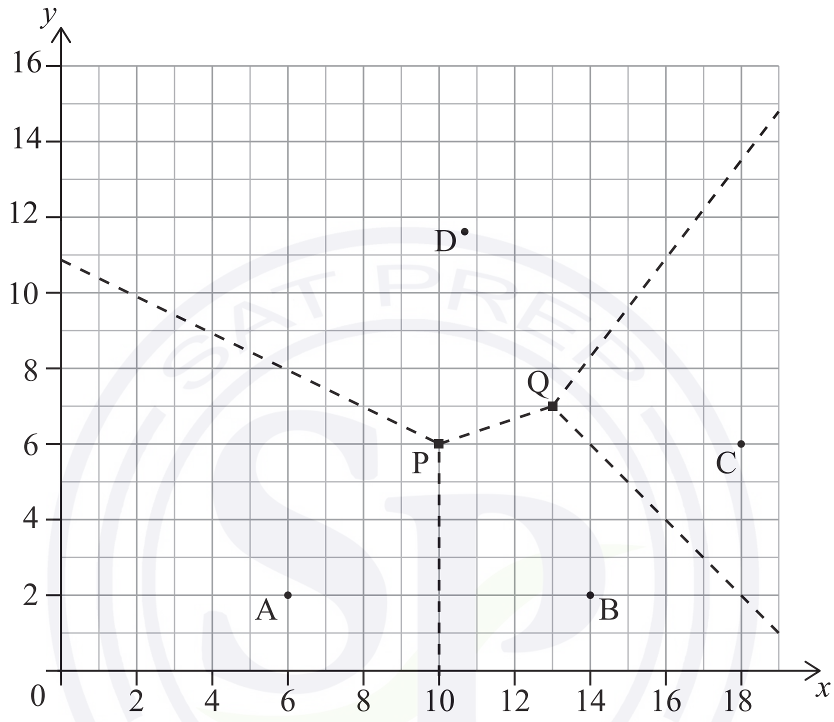
4. [Maximum mark: 6]

There are four stations used by the fire wardens in a national forest.

On the following Voronoi diagram, the coordinates of the stations are $A(6, 2)$, $B(14, 2)$, $C(18, 6)$ and $D(10.8, 11.6)$ where distances are measured in kilometres.

The dotted lines represent the boundaries of the regions patrolled by the fire warden at each station. The boundaries meet at $P(10, 6)$ and $Q(13, 7)$.



To reduce the areas of the regions that the fire wardens patrol, a new station is to be built within the quadrilateral ABCD. The new station will be located so that it is as far as possible from the nearest existing station.

(a) Show that the new station should be built at P. [3]

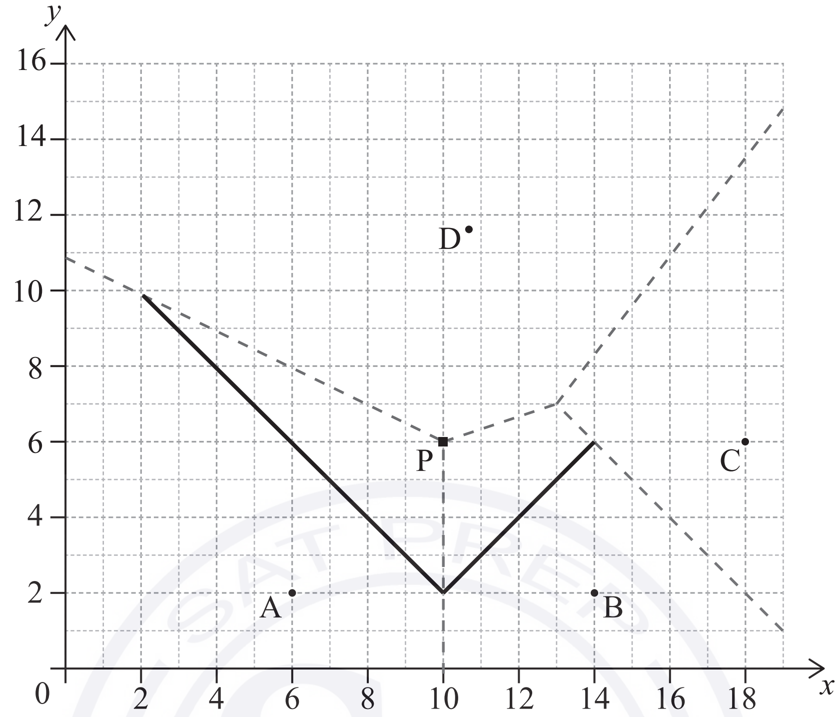
The Voronoi diagram is to be updated to include the region around the new station at P. The edges defined by the perpendicular bisectors of [AP] and [BP] have been added to the following diagram.

(b) (i) Write down the equation of the perpendicular bisector of [PC].

(ii) Hence draw the missing boundaries of the region around P on the following diagram. [3]

(This question continues on the following page)

(Question 4 continued)



5. [Maximum mark: 9]

In this question, give all answers correct to 2 decimal places.

Raul and Rosy want to buy a new house and they need a loan of 170 000 Australian dollars (AUD) from a bank. The loan is for 30 years and the annual interest rate for the loan is 3.8%, compounded monthly. They will pay the loan in fixed monthly instalments at the end of each month.

(a) Find the amount they will pay the bank each month. [3]

(b) (i) Find the amount Raul and Rosy will still owe the bank at the end of the first 10 years.

(ii) Using your answers to parts (a) and (b)(i), calculate how much interest they will have paid in total during the first 10 years. [6]

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6. [Maximum mark: 5]

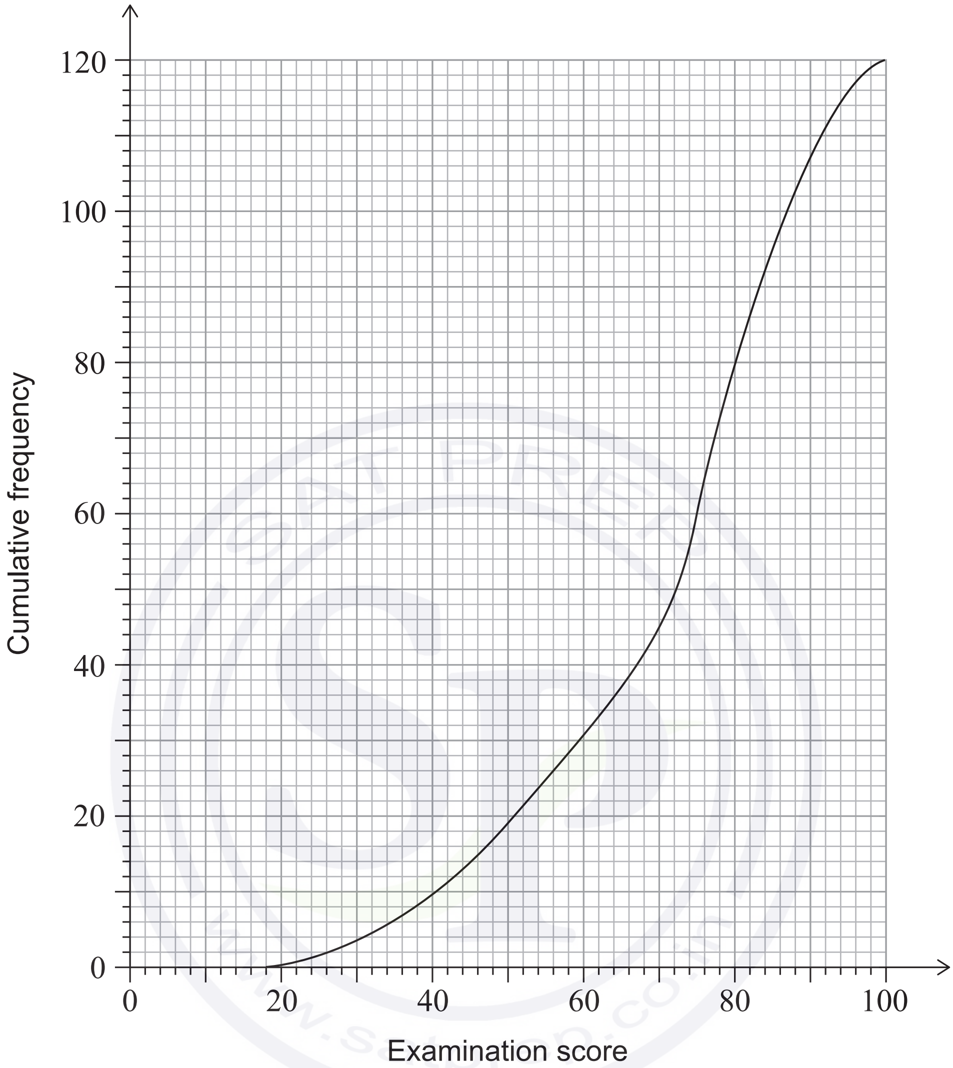
An infinite geometric sequence, with terms u_n, is such that $u_1 = 2$ and $\sum_{k=1}^{\infty} u_k = 10$.

(a) Find the common ratio, r, for the sequence. [2]

(b) Find the least value of n such that $u_n < \frac{1}{2}$. [3]

7. [Maximum mark: 8]

A group of 120 students sat a history exam. The cumulative frequency graph shows the scores obtained by the students.



(a) Find the median of the scores obtained. [1]

The students were awarded a grade from 1 to 5, depending on the score obtained in the exam. The number of students receiving each grade is shown in the following table.

Grade	1	2	3	4	5
Number of students	6	13	26	a	b

(b) Find an expression for a in terms of b. [2]

(This question continues on the following page)

(Question 7 continued)

(c) The mean grade for these students is 3.65.

(i) Find the number of students who obtained a grade 5.

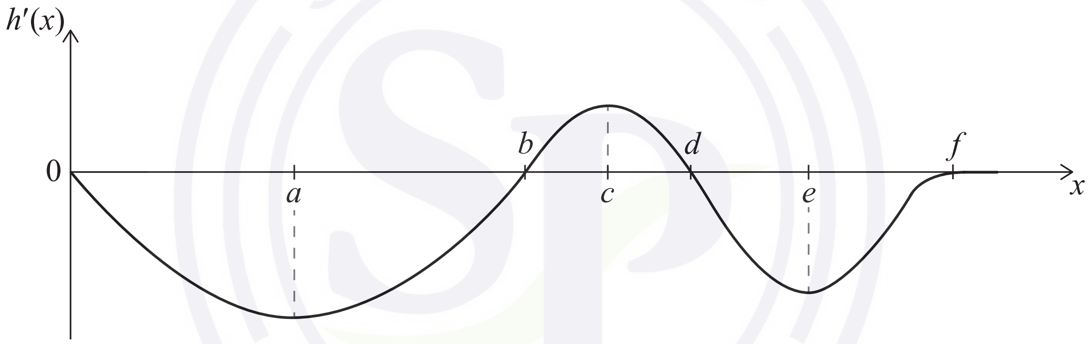
(ii) Find the minimum score needed to obtain a grade 5. [5]

8. [Maximum mark: 5]

Juri skis from the top of a hill to a finishing point at the bottom of the hill. She takes the shortest route, heading directly to the finishing point (F).

Let $h(x)$ define the height of the hill above F at a horizontal distance x from the starting point at the top of the hill.

The graph of the **derivative** of $h(x)$ is shown below. The graph of $h'(x)$ has local minima and maxima when x is equal to a, c and e. The graph of $h'(x)$ intersects the x-axis when x is equal to b, d, and f.



(a) (i) Identify the x value of the point where $|h'(x)|$ has its maximum value.

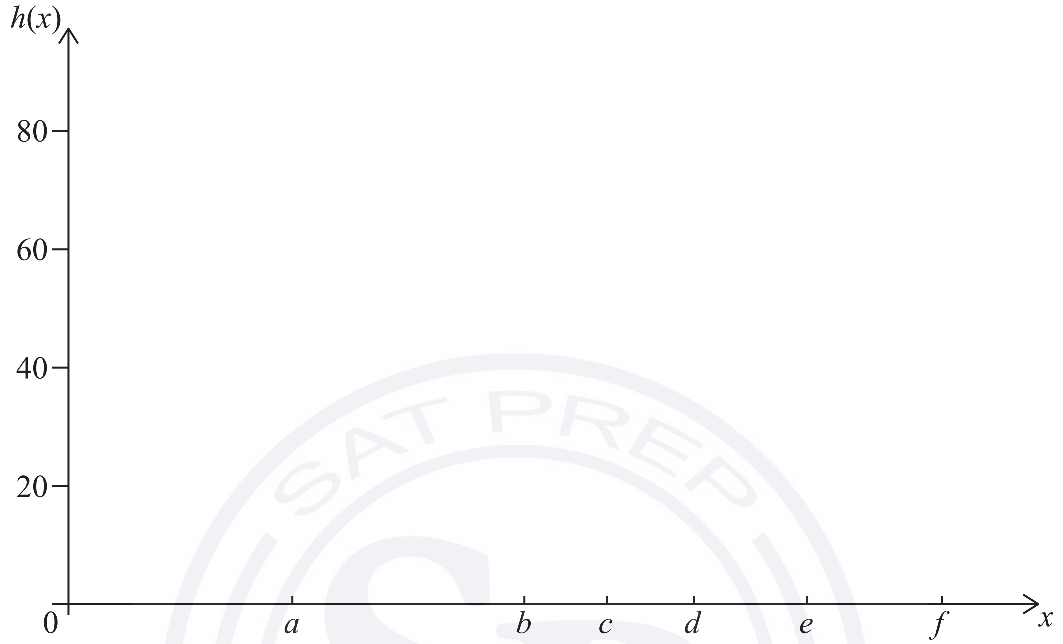
(ii) Interpret this point in the given context. [2]

(This question continues on the following page)

(Question 8 continued)

Juri starts at a height of 60 metres and finishes at F, where $x = f$.

(b) Sketch a possible diagram of the hill on the following pair of coordinate axes. [3]



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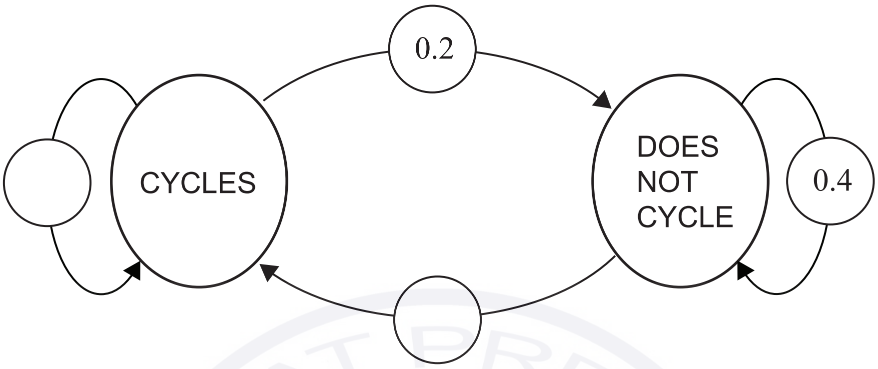
will not be marked.

9. [Maximum mark: 5]

Katie likes to cycle to work as much as possible. If Katie cycles to work one day then she has a probability of 0.2 of not cycling to work on the next work day. If she does not cycle to work one day then she has a probability of 0.4 of not cycling to work on the next work day.

(a) Complete the following transition diagram to represent this information. [2]



Katie works for 180 days in a year.

(b) Find the probability that Katie cycles to work on her final working day of the year. [3]

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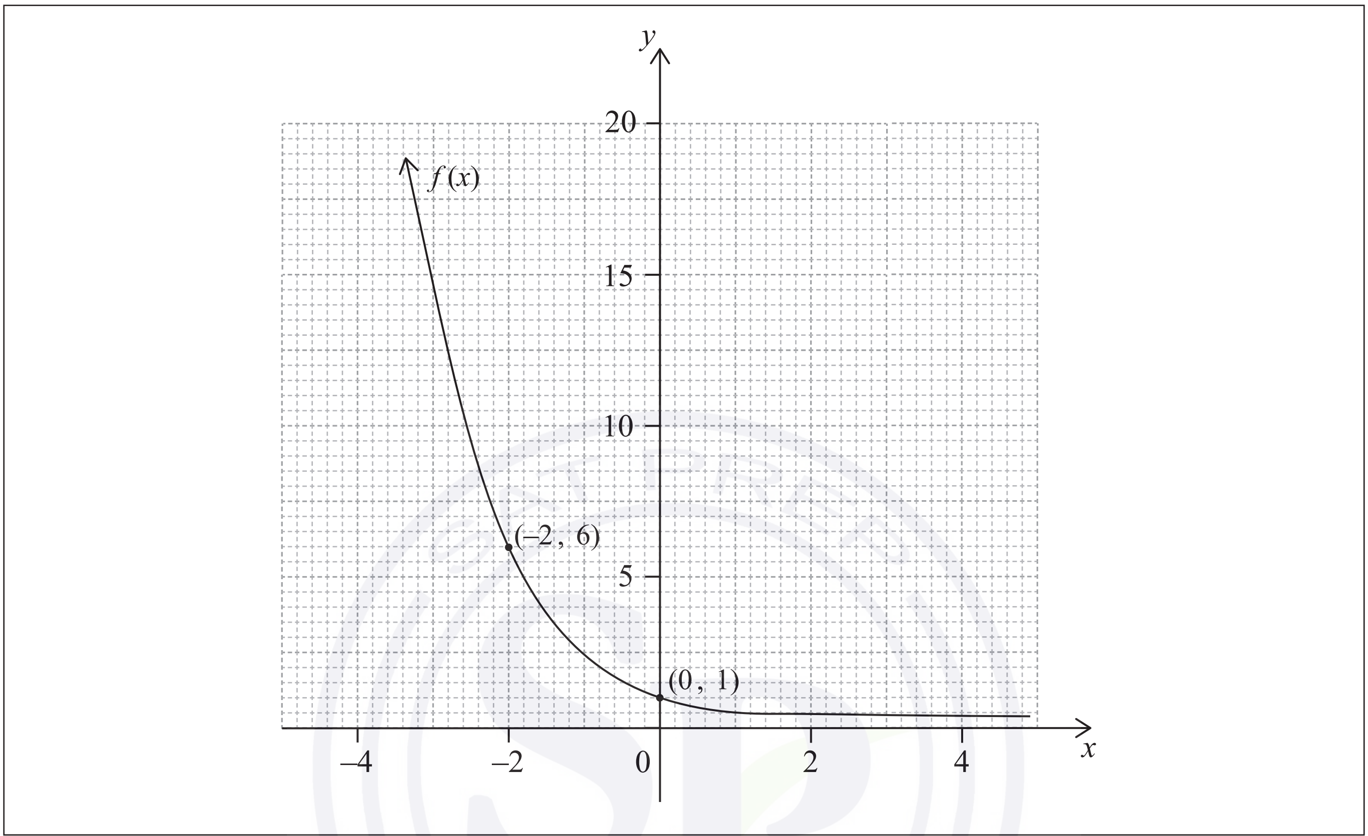
10. [Maximum mark: 4]

The graph of $y = f(x)$ is given on the following set of axes. The graph passes through the points $(-2, 6)$ and $(0, 1)$, and has a horizontal asymptote at $y = 0$.



Let $g(x) = 2f(x-2) + 4$.

(a) Find $g(0)$. [2]

(b) On the same set of axes draw the graph of $y = g(x)$, showing any intercepts and asymptotes. [2]

(This question continues on the following page)

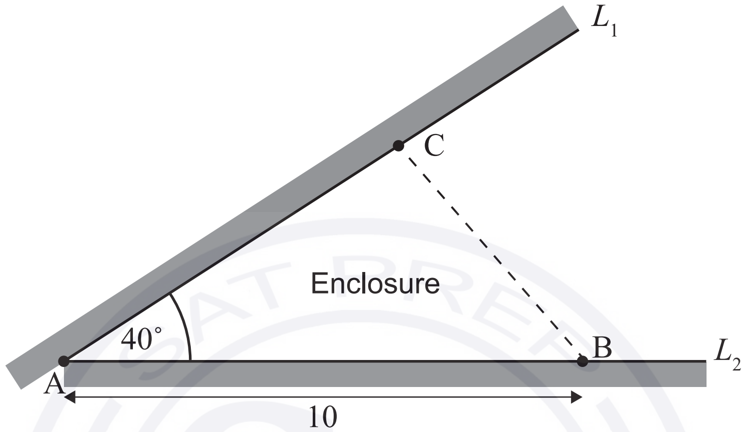
(Question 10 continued)

11. [Maximum mark: 6]

The following diagram shows a corner of a field bounded by two walls defined by lines L_1 and L_2. The walls meet at a point A, making an angle of 40°.

Farmer Nate has $7\,\text{m}$ of fencing to make a triangular enclosure for his sheep. One end of the fence is positioned at a point B on L_2, $10\,\text{m}$ from A. The other end of the fence will be positioned at some point C on L_1, as shown on the diagram.

diagram not to scale



He wants the enclosure to take up as little of the current field as possible.

Find the minimum possible area of the triangular enclosure ABC. [6]

12. [Maximum mark: 5]

The following table shows the time, in days, from December 1st and the percentage of Christmas trees in stock at a shop on the beginning of that day.

Days since December 1st (d)	1	3	6	9	12	15	18
Percentage of Christmas trees left in stock (x)	100	51	29	21	18	16	14

The following table shows the natural logarithm of both d and x on these days to 2 decimal places.

ln (d)	0	1.10	1.79	2.20	2.48	2.71	2.89
ln (x)	4.61	3.93	3.37	3.04	2.89	2.77	2.64

(a) Use the data in the second table to find the value of m and the value of b for the regression line, $\ln x = m(\ln d) + b$. [2]

(b) Assuming that the model found in part (a) remains valid, estimate the percentage of trees in stock when $d = 25$. [3]

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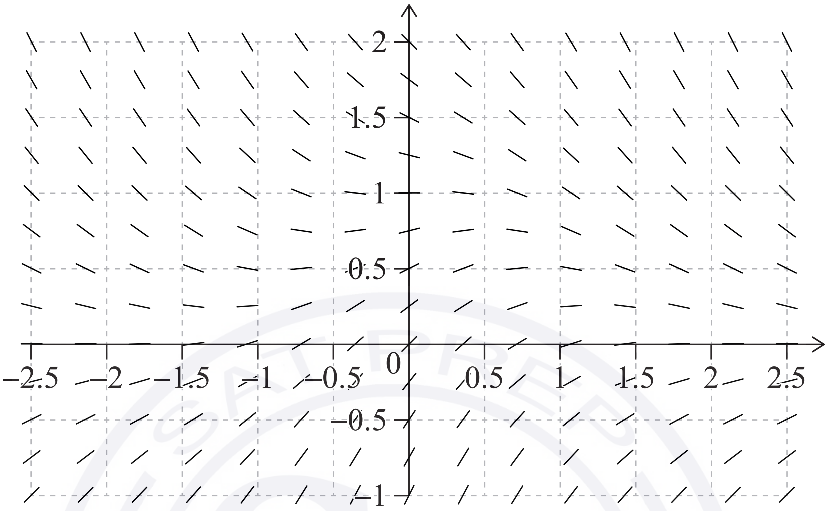
13. [Maximum mark: 7]

The slope field for the differential equation $\frac{dy}{dx} = e^{-x^2} - y$ is shown in the following two graphs.

(a) Calculate the value of $\frac{dy}{dx}$ at the point $(0, 1)$. [1]

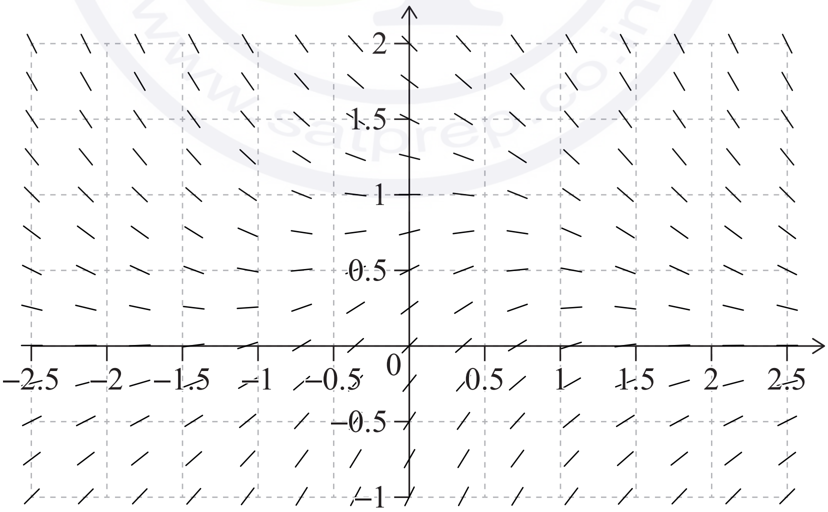
(b) Sketch, on the first graph, a curve that represents the points where $\frac{dy}{dx} = 0$. [2]



(c) On the second graph,

(i) sketch the solution curve that passes through the point $(0, 0)$.

(ii) sketch the solution curve that passes through the point $(0, 0.75)$. [4]



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(Question 13 continued)

14. [Maximum mark: 7]

On Paul's farm, potatoes are packed in sacks labelled $50\,\text{kg}$. The weights of the sacks of potatoes can be modelled by a normal distribution with mean weight $49.8\,\text{kg}$ and standard deviation $0.9\,\text{kg}$.

(a) Find the probability that a sack is under its labelled weight. [2]

(b) Find the lower quartile of the weights of the sacks of potatoes. [2]

The sacks of potatoes are transported in crates. There are 10 sacks in each crate and the weights of the sacks of potatoes are independent of each other.

(c) Find the probability that the total weight of the sacks of potatoes in a crate exceeds $500\,\text{kg}$. [3]

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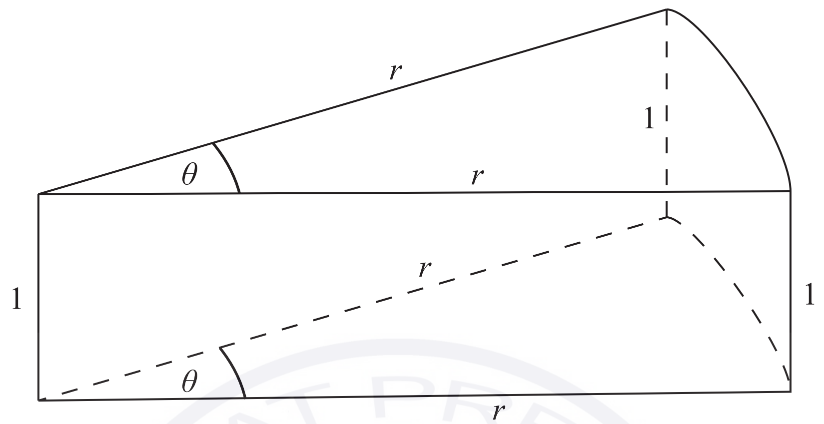
15. [Maximum mark: 9]

The following diagram shows a frame that is made from wire. The total length of wire is equal to $15\,\text{cm}$. The frame is made up of two identical sectors of a circle that are parallel to each other. The sectors have angle θ radians and radius $r\,\text{cm}$. They are connected by $1\,\text{cm}$ lengths of wire perpendicular to the sectors. This is shown in the diagram below.



(a) Show that $r = \dfrac{6}{2+\theta}$. [2]

The faces of the frame are covered by paper to enclose a volume, V.

(b) (i) Find an expression for V in terms of θ.

(ii) Find the expression $\dfrac{dV}{d\theta}$.

(iii) Solve algebraically $\dfrac{dV}{d\theta} = 0$ to find the value of θ that will maximize the volume, V. [7]

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16. [Maximum mark: 9]

A ship S is travelling with a constant velocity, $\boldsymbol{v}$, measured in kilometres per hour, where

$$\boldsymbol{v} = \begin{pmatrix} -12 \\ 15 \end{pmatrix}.$$

At time $t = 0$ the ship is at a point A(300, 100) relative to an origin O, where distances are measured in kilometres.

(a) Find the position vector $\overrightarrow{OS}$ of the ship at time t hours. [1]

A lighthouse is located at a point (129, 283).

(b) Find the value of t when the ship will be closest to the lighthouse. [6]

An alarm will sound if the ship travels within 20 kilometres of the lighthouse.

(c) State whether the alarm will sound. Give a reason for your answer. [2]

17. [Maximum mark: 7]

The sides of a bowl are formed by rotating the curve $y = 6\ln x$, $0 \le y \le 9$, about the y-axis, where x and y are measured in centimetres. The bowl contains water to a height of $h\,\text{cm}$.

(a) Show that the volume of water, V, in terms of h is $V = 3\pi\left(e^{\frac{h}{3}} - 1\right)$. [5]

(b) Hence find the maximum capacity of the bowl in cm^3. [2]

References:

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Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: applications and interpretation
Higher level
Paper 1

Thursday 6 May 2021 (afternoon)

Candidate session number

2 hours

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all questions.
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- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
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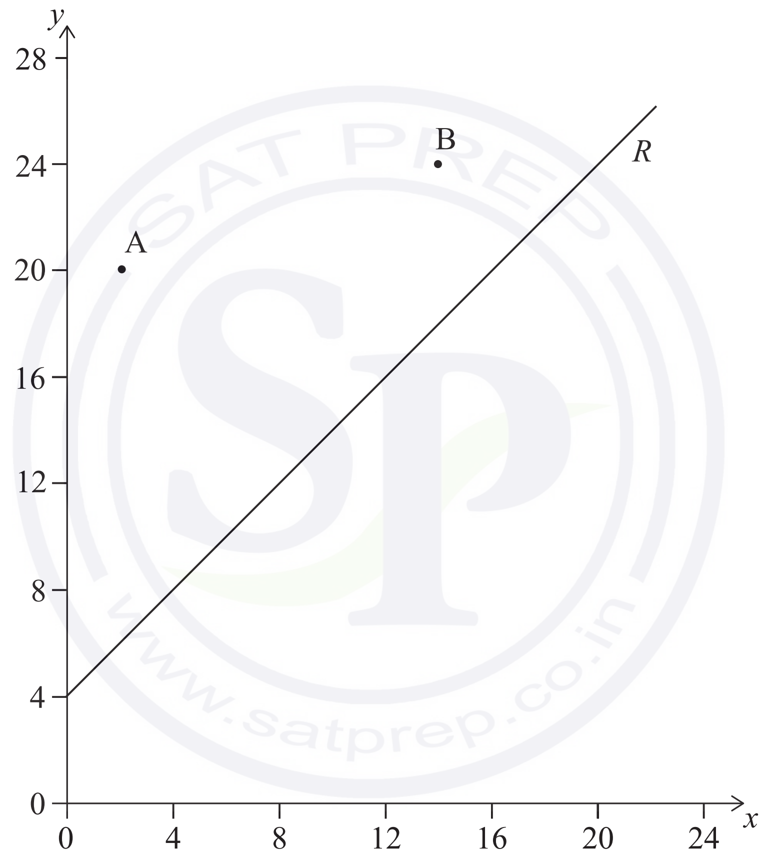
- The maximum mark for this examination paper is **[110 marks]**.

Answers must be written within the answer boxes provided. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 7]

Two schools are represented by points $\mathrm{A}(2, 20)$ and $\mathrm{B}(14, 24)$ on the graph below. A road, represented by the line R with equation $-x + y = 4$, passes near the schools. An architect is asked to determine the location of a new bus stop on the road such that it is the same distance from the two schools.



(a) Find the equation of the perpendicular bisector of $[\mathrm{AB}]$. Give your equation in the form $y = mx + c$. [5]

(b) Determine the coordinates of the point on R where the bus stop should be located. [2]

(This question continues on the following page)

(Question 1 continued)

2. [Maximum mark: 7]

A function is defined by $f(x) = 2 - \frac{12}{x+5}$ for $-7 \le x \le 7$, $x \ne -5$.

(a) Find the range of f. [3]

(b) Find an expression for the inverse function $f^{-1}(x)$. The domain is not required. [3]

(c) Write down the range of $f^{-1}(x)$. [1]

3. [Maximum mark: 6]

At Springfield University, the weights, in kg, of 10 chinchilla rabbits and 10 sable rabbits were recorded. The aim was to find out whether chinchilla rabbits are generally heavier than sable rabbits. The results obtained are summarized in the following table.

Weight of chinchilla rabbits, kg	4.9	4.2	4.1	4.4	4.3	4.6	4.0	4.7	4.5	4.4
Weight of sable rabbits, kg	4.2	4.1	4.1	4.2	4.5	4.4	4.5	3.9	4.2	4.0

A t-test is to be performed at the 5% significance level.

(a) Write down the null and alternative hypotheses. [2]

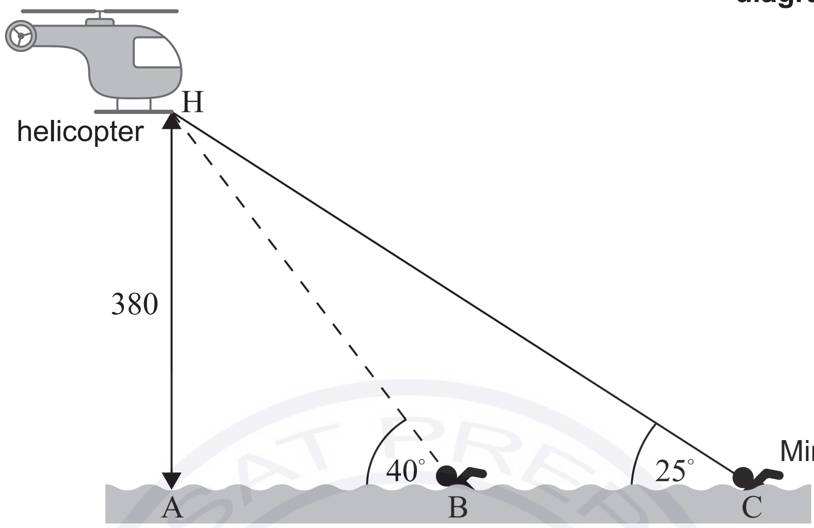
(b) Find the p-value for this test. [2]

(c) Write down the conclusion to the test. Give a reason for your answer. [2]

4. [Maximum mark: 6]

The diagram below shows a helicopter hovering at point H, $380\,\text{m}$ vertically above a lake. Point A is the point on the surface of the lake, directly below the helicopter.

diagram not to scale



Minta is swimming at a constant speed in the direction of point A. Minta observes the helicopter from point C as she looks upward at an angle of 25°. After 15 minutes, Minta is at point B and she observes the same helicopter at an angle of 40°.

(a) Find the distance from A to C. [2]

(b) Find the distance from B to C. [3]

(c) Find Minta's speed, in metres per hour. [1]

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5. [Maximum mark: 5]

Roger buys a new laptop for himself at a cost of £495. At the same time, he buys his daughter Chloe a higher specification laptop at a cost of £2200.

It is anticipated that Roger's laptop will depreciate at a rate of 10% per year, whereas Chloe's laptop will depreciate at a rate of 15% per year.

(a) Estimate the value of Roger's laptop after 5 years. [2]

Roger and Chloe's laptops will have the same value k years after they were purchased.

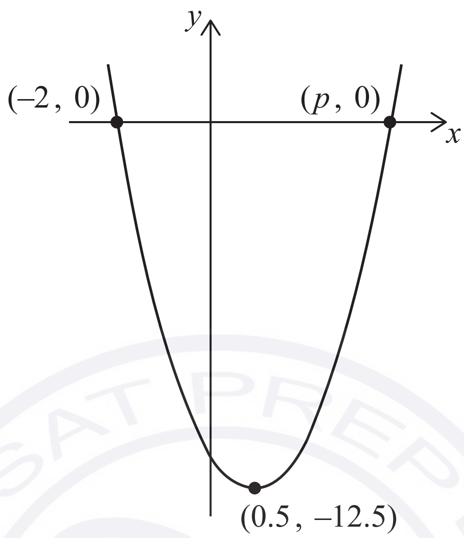
(b) Find the value of k. [2]

(c) Comment on the validity of your answer to part (b). [1]

6. [Maximum mark: 7]

Consider the function $f(x) = ax^2 + bx + c$. The graph of $y = f(x)$ is shown in the diagram. The vertex of the graph has coordinates $(0.5, -12.5)$. The graph intersects the x-axis at two points, $(-2, 0)$ and $(p, 0)$.

diagram not to scale



(a) Find the value of p. [1]

(b) Find the value of

(i) a.

(ii) b.

(iii) c. [5]

(c) Write down the equation of the axis of symmetry of the graph. [1]

(This question continues on the following page)

(Question 6 continued)

7. [Maximum mark: 6]

A meteorologist models the height of a hot air balloon launched from the ground. The model assumes the balloon travels vertically upwards and travels 450 metres in the first minute.

Due to the decrease in temperature as the balloon rises, the balloon will continually slow down. The model suggests that each minute the balloon will travel only 82% of the distance travelled in the previous minute.

(a) Find how high the balloon will travel in the first 10 minutes after it is launched. [3]

(b) The balloon is required to reach a height of at least 2520 metres.

Determine whether it will reach this height. [2]

(c) Suggest a limitation of the given model. [1]

8. [Maximum mark: 7]

Two lines L_1 and L_2 are given by the following equations, where $p \in \mathbb{R}$.

$$L_1: \ r = \begin{pmatrix} 2 \\ p+9 \\ -3 \end{pmatrix} + \lambda \begin{pmatrix} p \\ 2p \\ 4 \end{pmatrix}$$

$$L_2: \ r = \begin{pmatrix} 14 \\ 7 \\ p+12 \end{pmatrix} + \mu \begin{pmatrix} p+4 \\ 4 \\ -7 \end{pmatrix}$$

It is known that L_1 and L_2 are perpendicular.

(a) Find the possible value(s) for p. [3]

(b) In the case that $p < 0$, determine whether the lines intersect. [4]

9. [Maximum mark: 8]

A newspaper vendor in Singapore is trying to predict how many copies of *The Straits Times* they will sell. The vendor forms a model to predict the number of copies sold each weekday. According to this model, they expect the same number of copies will be sold each day.

To test the model, they record the number of copies sold each weekday during a particular week. This data is shown in the table.

Day	Monday	Tuesday	Wednesday	Thursday	Friday
Number of copies sold	74	97	91	86	112

A goodness of fit test at the 5% significance level is used on this data to determine whether the vendor's model is suitable. The critical value for the test is 9.49.

(a) Find an estimate for how many copies the vendor expects to sell each day. [1]

(b) (i) State the null and alternative hypotheses for this test.

(ii) Write down the degrees of freedom for this test.

(iii) Write down the conclusion to the test. Give a reason for your answer. [7]

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10. [Maximum mark: 6]

A manufacturer of chocolates produces them in individual packets, claiming to have an average of 85 chocolates per packet.

Talha bought 30 of these packets in order to check the manufacturer's claim.

Given that the number of individual chocolates is x, Talha found that, from his 30 packets, $\sum x = 2506$ and $\sum x^2 = 209\,738$.

(a) Find an unbiased estimate for the mean number (μ) of chocolates per packet. [1]

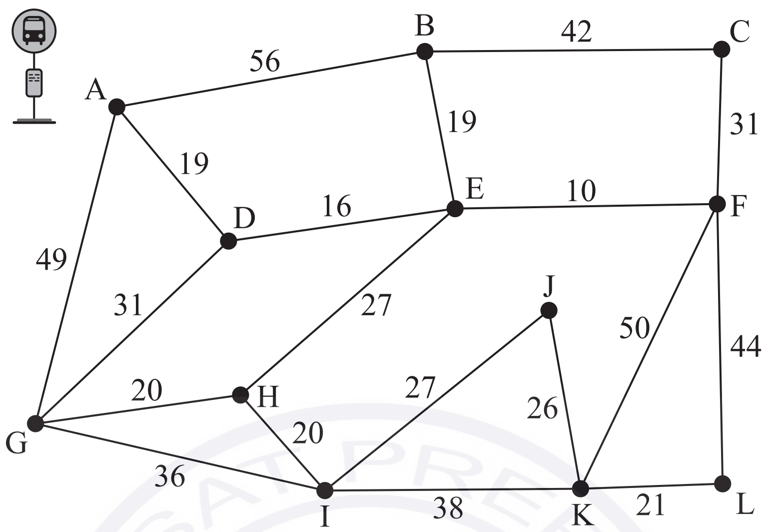
(b) Use the formula $s_{n-1}^2 = \dfrac{\sum x^2 - \frac{(\sum x)^2}{n}}{n-1}$ to determine an unbiased estimate for the variance of the number of chocolates per packet. [2]

(c) Find a 95% confidence interval for μ. You may assume that all conditions for a confidence interval have been met. [2]

(d) Suggest, with justification, a valid conclusion that Talha could make. [1]

11. [Maximum mark: 7]

The diagram below shows a network of roads in a small village with the weights indicating the distance of each road, in metres, and junctions indicated with letters.



Musab is required to deliver leaflets to every house on each road. He wishes to minimize his total distance.

(a) Musab starts and finishes from the village bus-stop at A. Determine the total distance Musab will need to walk. [5]

Instead of having to catch the bus to the village, Musab's sister offers to drop him off at any junction and pick him up at any other junction of his choice.

(b) Explain which junctions Musab should choose as his starting and finishing points. [2]

12. [Maximum mark: 8]

It is given that $z_1 = 3\,\mathrm{cis}\left(\frac{3\pi}{4}\right)$ and $z_2 = 2\,\mathrm{cis}\left(\frac{n\pi}{16}\right)$, $n \in \mathbb{Z}^+$.

(a) In parts (a)(i) and (a)(ii), give your answers in the form $re^{i\theta}$, $r \geq 0$, $-\pi < \theta \leq \pi$.

(i) Find the value of $z_1{}^3$.

(ii) Find the value of $\left(\frac{z_1}{z_2}\right)^4$ for $n = 2$. [5]

(b) Find the least value of n such that $z_1 z_2 \in \mathbb{R}^+$. [3]

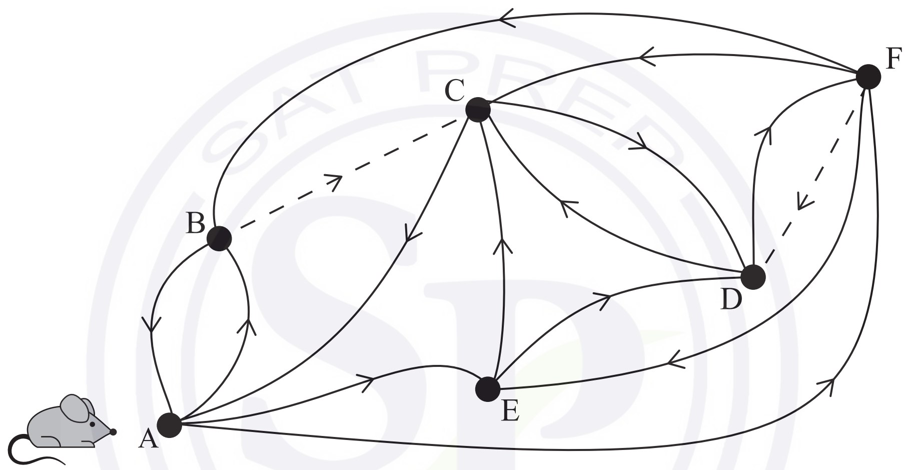
13. [Maximum mark: 8]

The graph below shows a small maze, in the form of a network of directed routes. The vertices A to F show junctions in the maze and the edges show the possible paths available from one vertex to another.

A mouse is placed at vertex A and left to wander the maze freely. The routes shown by dashed lines indicate paths sprinkled with sugar.

When the mouse reaches any junction, she rests for a constant time before continuing.

At any junction, it may also be assumed that
- the mouse chooses any available normal path with equal probability
- if the junction includes a path sprinkled with sugar, the probability of choosing this path is twice that of a normal path.



(a) Determine the transition matrix for this graph. [3]

(b) If the mouse was left to wander indefinitely, use your graphic display calculator to estimate the percentage of time that the mouse would spend at point F. [3]

(c) Comment on your answer to part (b), referring to at least one limitation of the model. [2]

(This question continues on the following page)

(Question 13 continued)

14. [Maximum mark: 7]

A geometric transformation $T:\begin{pmatrix}x\\y\end{pmatrix} \mapsto \begin{pmatrix}x'\\y'\end{pmatrix}$ is defined by

$$T:\begin{pmatrix}x'\\y'\end{pmatrix} = \begin{pmatrix}7 & -10\\2 & -3\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix} + \begin{pmatrix}-5\\4\end{pmatrix}.$$

(a) Find the coordinates of the image of the point $(6\,,\,-2)$. [2]

(b) Given that $T:\begin{pmatrix}p\\q\end{pmatrix} \mapsto 2\begin{pmatrix}p\\q\end{pmatrix}$, find the value of p and the value of q. [3]

(c) A triangle L with vertices lying on the xy plane is transformed by T.

Explain why both L and its image will have exactly the same area. [2]

15. [Maximum mark: 7]

The number of coffees sold per hour at an independent coffee shop is modelled by a Poisson distribution with a mean of 22 coffees per hour.

Sheila, the shop's owner wants to increase the number of coffees sold in the shop. She decides to offer a discount to customers who buy more than one coffee.

To test how successful this strategy is, Sheila records the number of coffees sold over a single 5-hour period. Sheila decides to use a 5% level of significance in her test.

(a) State the null and alternative hypotheses for the test. [1]

(b) Find the probability that Sheila will make a type I error in her test conclusion. [4]

Sheila finds 126 coffees were sold during the 5-hour period.

(c) State Sheila's conclusion to the test. Justify your answer. [2]

16. [Maximum mark: 8]

A particle P moves in a straight line, such that its displacement x at time t $(t \geq 0)$ is defined by the differential equation $\dot{x} = x\cos t\left(e^{-\sin t}\right)$. At time $t = 0$, $x = \frac{1}{e}$.

(a) By using Euler's method with a step length of 0.1, find an approximate value for x when $t = 0.3$. [3]

(b) By solving the differential equation, find the percentage error in your approximation for x when $t = 0.3$. [5]

References:

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: applications and interpretation
Higher level
Paper 1

Thursday 6 May 2021 (afternoon)

Candidate session number

2 hours

Instructions to candidates

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1. [Maximum mark: 4]

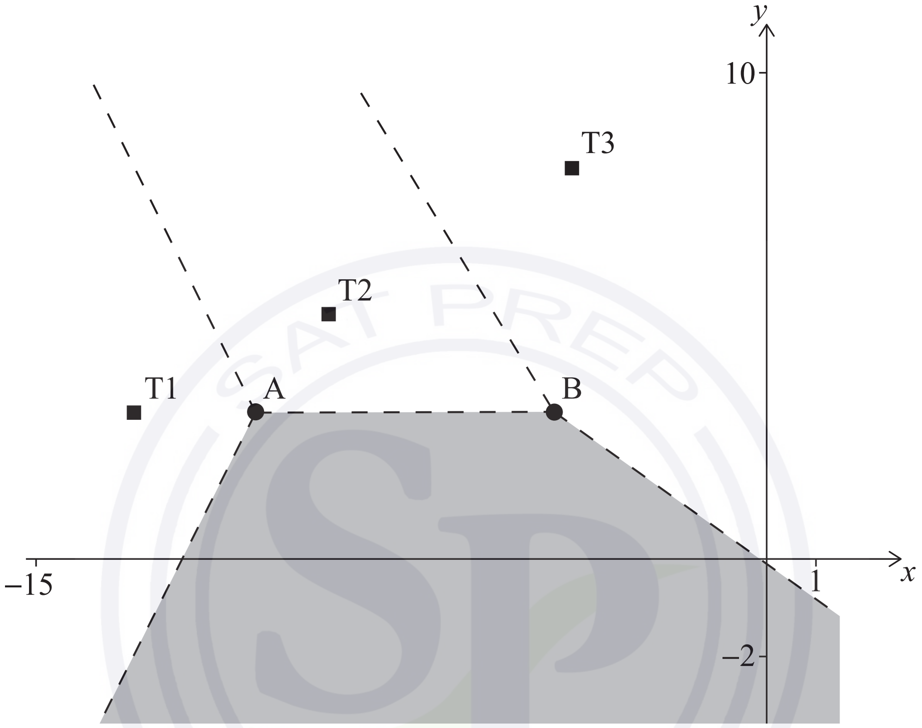
George goes fishing. From experience he knows that the mean number of fish he catches per hour is 1.1. It is assumed that the number of fish he catches can be modelled by a Poisson distribution.

On a day in which George spends 8 hours fishing, find the probability that he will catch more than 9 fish.

2. [Maximum mark: 6]

The Voronoi diagram below shows three identical cellular phone towers, T1, T2 and T3. A fourth cellular phone tower, T4 is located in the shaded region. The dashed lines in the diagram below represent the edges in the Voronoi diagram.

Horizontal scale: 1 unit represents 1 km.
Vertical scale: 1 unit represents 1 km.



Tim stands inside the shaded region.

(a) Explain why Tim will receive the strongest signal from tower T4. [1]

Tower T2 has coordinates $(-9, 5)$ and the edge connecting vertices A and B has equation $y = 3$.

(b) Write down the coordinates of tower T4. [2]

Tower T1 has coordinates $(-13, 3)$.

(c) Find the gradient of the edge of the Voronoi diagram between towers T1 and T2. [3]

(This question continues on the following page)

(Question 2 continued)

3. [Maximum mark: 8]

Charlie and Daniella each began a fitness programme. On day one, they both ran $500\,\text{m}$. On each subsequent day, Charlie ran $100\,\text{m}$ more than the previous day whereas Daniella increased her distance by 2% of the distance ran on the previous day.

(a) Calculate how far

(i) Charlie ran on day 20 of his fitness programme.

(ii) Daniella ran on day 20 of her fitness programme. [5]

On day n of the fitness programmes Daniella runs more than Charlie for the first time.

(b) Find the value of n. [3]

4. [Maximum mark: 6]

Professor Wei observed that students have difficulty remembering the information presented in his lectures.

He modelled the percentage of information retained, R, by the function $R(t) = 100\,\mathrm{e}^{-pt}$, $t \geq 0$, where t is the number of days after the lecture.

He found that 1 day after a lecture, students had forgotten 50% of the information presented.

(a) Find the value of p. [2]

(b) Use this model to find the percentage of information retained by his students 36 hours after Professor Wei's lecture. [2]

Based on his model, Professor Wei believes that his students will always retain some information from his lecture.

(c) State a mathematical reason why Professor Wei might believe this. [1]

(d) Write down one possible limitation of the **domain** of the model. [1]

5. [Maximum mark: 6]

A garden has a triangular sunshade suspended from three points $\mathrm{A}(2, 0, 2)$, $\mathrm{B}(8, 0, 2)$ and $\mathrm{C}(5, 4, 3)$, relative to an origin in the corner of the garden. All distances are measured in metres.

(a) (i) Find $\overrightarrow{\mathrm{CA}}$.

(ii) Find $\overrightarrow{\mathrm{CB}}$. [2]

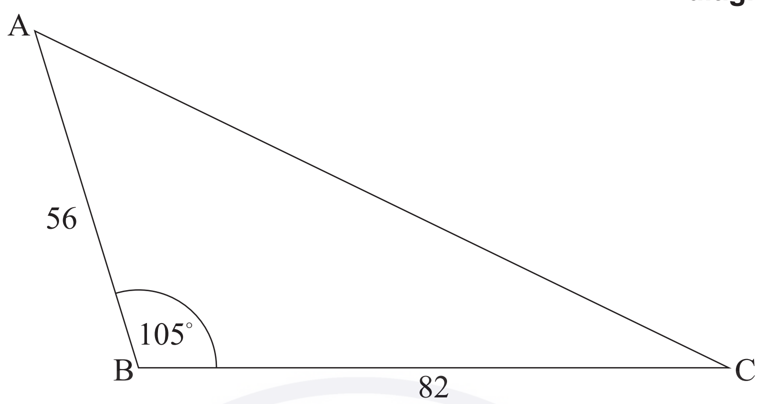
(b) Find $\overrightarrow{\mathrm{CA}} \times \overrightarrow{\mathrm{CB}}$. [2]

(c) Hence find the area of the triangle ABC. [2]

6. [Maximum mark: 5]

A triangular field ABC is such that $AB = 56\,m$ and $BC = 82\,m$, each measured correct to the nearest metre, and the angle at B is equal to 105°, measured correct to the nearest 5°.

diagram not to scale



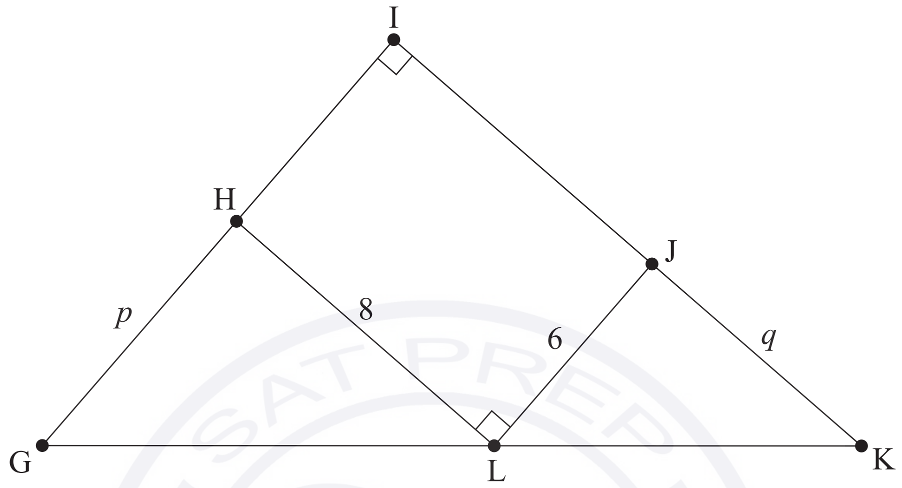
Calculate the maximum possible area of the field.

7. [Maximum mark: 8]

Ellis designs a gift box. The top of the gift box is in the shape of a right-angled triangle GIK.

A rectangular section HIJL is inscribed inside this triangle. The lengths of GH, JK, HL, and LJ are p cm, q cm, 8 cm and 6 cm respectively.

diagram not to scale



The area of the top of the gift box is $A\,\text{cm}^2$.

(a) (i) Find A in terms of p and q.

(ii) Show that $A = \dfrac{192}{q} + 3q + 48$. [4]

(b) Find $\dfrac{dA}{dq}$. [2]

Ellis wishes to find the value of q that will minimize the area of the top of the gift box.

(c) (i) Write down an equation Ellis could solve to find this value of q.

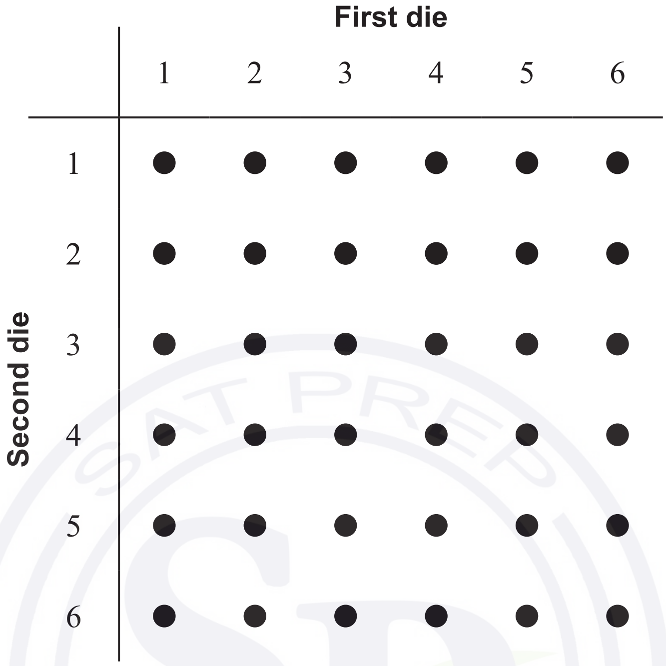
(ii) Hence, or otherwise, find this value of q. [2]

(This question continues on the following page)

(Question 7 continued)

8. [Maximum mark: 7]

A game is played where two unbiased dice are rolled and the score in the game is the greater of the two numbers shown. If the two numbers are the same, then the score in the game is the number shown on one of the dice. A diagram showing the possible outcomes is given below.



Let T be the random variable "the score in a game".

(a) Complete the table to show the probability distribution of T. [2]

t	1	2	3	4	5	6
$\mathrm{P}(T = t)$						

(b) Find the probability that

(i) a player scores at least 3 in a game.

(ii) a player scores 6, given that they scored at least 3. [3]

(c) Find the expected score of a game. [2]

(This question continues on the following page).

(Question 8 continued)

9. [Maximum mark: 8]

Consider $w = \mathrm{i}z + 1$, where $w, z \in \mathbb{C}$.

(a) Find w when

(i) $z = 2\mathrm{i}$.

(ii) $z = 1 + \mathrm{i}$. [3]

Point z on the Argand diagram can be transformed to point w by two transformations.

(b) Describe these two transformations and give the order in which they are applied. [3]

(c) Hence, or otherwise, find the value of z when $w = 2 - \mathrm{i}$. [2]

10. [Maximum mark: 7]

An engineer plans to visit six oil rigs (A–F) in the Gulf of Mexico, starting and finishing at A. The travelling time, in minutes, between each of the rigs is shown in the table.

	A	B	C	D	E	F
A		55	63	79	87	93
B	55		46	58	88	92
C	63	46		87	77	66
D	79	58	87		23	70
E	87	88	77	23		47
F	93	92	66	70	47	

The data above can be represented by a graph G.

(a) (i) Use Prim's algorithm to find the weight of the minimum spanning tree of the subgraph of G obtained by deleting A and starting at B. List the order in which the edges are selected.

(ii) Hence find a lower bound for the travelling time needed to visit all the oil rigs. [6]

(b) Describe how an improved lower bound might be found. [1]

11. [Maximum mark: 7]

A factory, producing plastic gifts for a fast food restaurant's Jolly meals, claims that just 1% of the toys produced are faulty.

A restaurant manager wants to test this claim. A box of 200 toys is delivered to the restaurant. The manager checks all the toys in this box and four toys are found to be faulty.

(a) Identify the type of sampling used by the restaurant manager. [1]

The restaurant manager performs a one-tailed hypothesis test, at the 10% significance level, to determine whether the factory's claim is reasonable. It is known that faults in the toys occur independently.

(b) Write down the null and alternative hypotheses. [2]

(c) Find the p-value for the test. [2]

(d) State the conclusion of the test. Give a reason for your answer. [2]

12. [Maximum mark: 8]

A tank of water initially contains 400 litres. Water is leaking from the tank such that after 10 minutes there are 324 litres remaining in the tank.

The volume of water, V litres, remaining in the tank after t minutes, can be modelled by the differential equation

$$\frac{\mathrm{d}V}{\mathrm{d}t} = -k\sqrt{V}, \text{ where } k \text{ is a constant.}$$

(a) Show that $V = \left(20 - \frac{t}{5}\right)^2$. [6]

(b) Find the time taken for the tank to empty. [2]

13. [Maximum mark: 7]

A submarine is located in a sea at coordinates $(0.8, 1.3, -0.3)$ relative to a ship positioned at the origin O. The x direction is due east, the y direction is due north and the z direction is vertically upwards.

All distances are measured in kilometres.

The submarine travels with direction vector $\begin{pmatrix} -2 \\ -3 \\ 1 \end{pmatrix}$.

(a) Assuming the submarine travels in a straight line, write down an equation for the line along which it travels. [2]

The submarine reaches the surface of the sea at the point P.

(b) (i) Find the coordinates of P.

(ii) Find OP. [5]

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14. [Maximum mark: 6]

The weights of apples from Tony's farm follow a normal distribution with mean 158 g and standard deviation 13 g. The apples are sold in bags that contain six apples.

(a) Find the mean weight of a bag of apples. [2]

(b) Find the standard deviation of the weights of these bags of apples. [2]

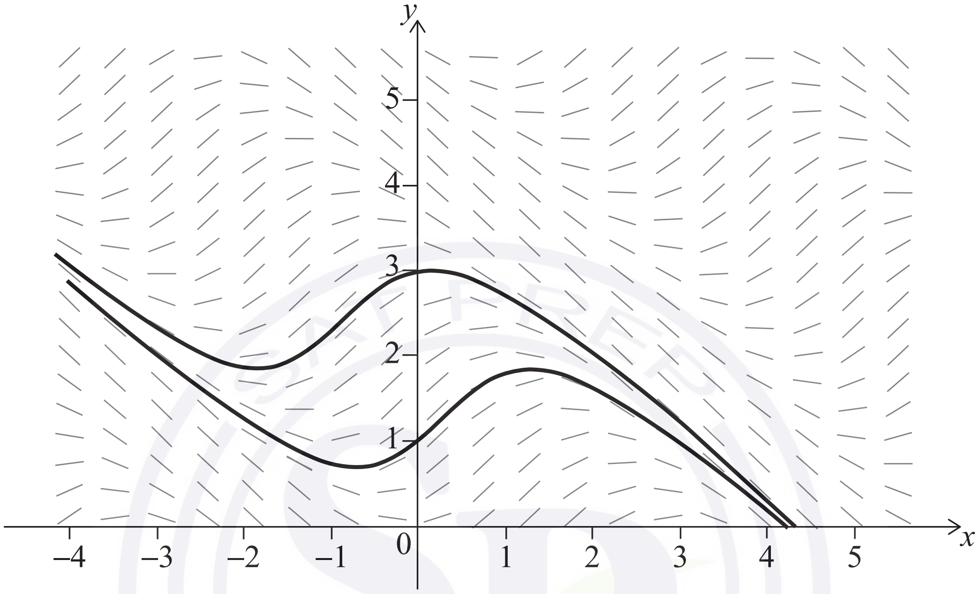
(c) Find the probability that a bag selected at random weighs more than 1 kg. [2]

15. [Maximum mark: 5]

The diagram shows the slope field for the differential equation

$$\frac{dy}{dx} = \sin(x + y),\ -4 \le x \le 5,\ 0 \le y \le 5.$$

The graphs of the two solutions to the differential equation that pass through points $(0, 1)$ and $(0, 3)$ are shown.



For the two solutions given, the local minimum points lie on the straight line L_1.

(a) Find the equation of L_1, giving your answer in the form $y = mx + c$. [3]

For the two solutions given, the local maximum points lie on the straight line L_2.

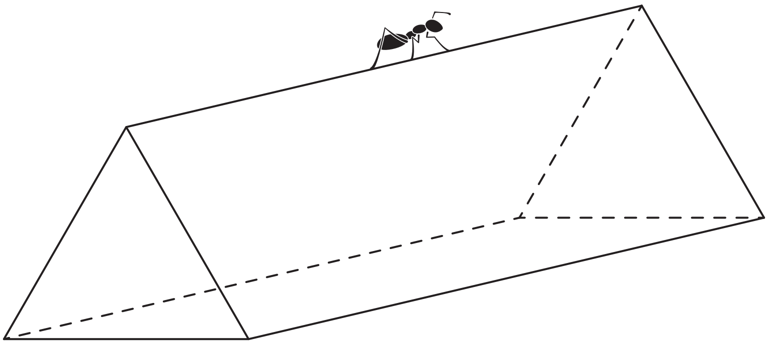
(b) Find the equation of L_2. [2]

(This question continues on the following page)

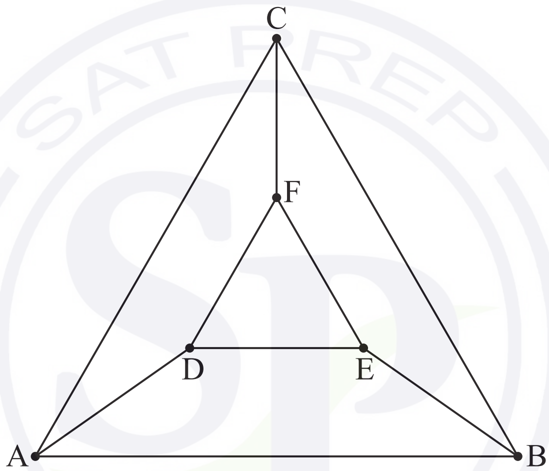
(Question 15 continued)

16. [Maximum mark: 5]

An ant is walking along the edges of a wire frame in the shape of a triangular prism.



The vertices and edges of this frame can be represented by the graph below.



(a) Write down the adjacency matrix, $\boldsymbol{M}$, for this graph. [3]

(b) Find the number of ways that the ant can start at the vertex A, and walk along exactly 6 edges to return to A. [2]

(This question continues on the following page)

(Question 16 continued)

17. [Maximum mark: 7]

The graph of the function $f(x) = \ln x$ is translated by $\begin{pmatrix} a \\ b \end{pmatrix}$ so that it then passes through the points $(0, 1)$ and $(e^3, 1 + \ln 2)$.

Find the value of a and the value of b.

References:

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: applications and interpretation
Higher level
Paper 1

Specimen paper

Candidate session number

2 hours

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all questions.
- Answers must be written within the answer boxes provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.

Please **do not** write on this page.

Answers written on this page
will not be marked.

Answers must be written within the answer boxes provided. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 6]

At the end of a school day, the Headmaster conducted a survey asking students in how many classes they had used the internet.

The data is shown in the following table.

Number of classes in which the students used the internet	0	1	2	3	4	5	6
Number of students	20	24	30	k	10	3	1

(a) State whether the data is discrete or continuous. [1]

The mean number of classes in which a student used the internet is 2.

(b) Find the value of k. [4]

It was not possible to ask every person in the school, so the Headmaster arranged the student names in alphabetical order and then asked every 10th person on the list.

(c) Identify the sampling technique used in the survey. [1]

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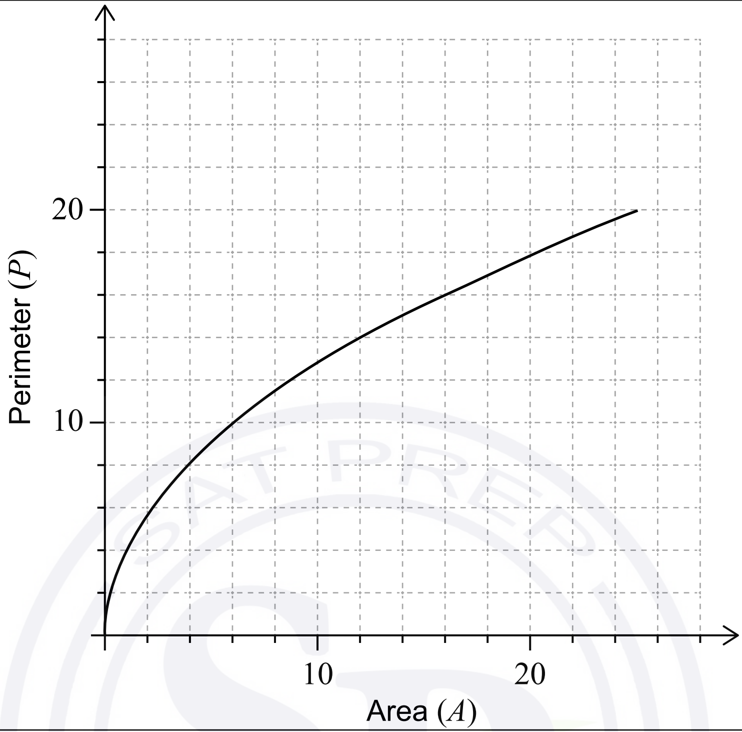
2. [Maximum mark: 5]

The perimeter of a given square P can be represented by the function $P(A) = 4\sqrt{A}$, $A \geq 0$, where A is the area of the square. The graph of the function P is shown for $0 \leq A \leq 25$.



(a) Write down the value of $P(25)$. [1]

(b) On the axes above, draw the graph of the inverse function, P^{-1}. [3]

(c) In the context of the question, explain the meaning of $P^{-1}(8) = 4$. [1]

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3. [Maximum mark: 6]

Professor Vinculum investigated the migration season of the Bulbul bird from their natural wetlands to a warmer climate.

He found that during the migration season their population, P could be modelled by $P = 1350 + 400(1.25)^{-t}$, $t \geq 0$, where t is the number of days since the start of the migration season.

(a) Find the population of the Bulbul birds,

(i) at the start of the migration season.

(ii) in the wetlands after 5 days. [3]

(b) Calculate the time taken for the population to decrease below 1400. [2]

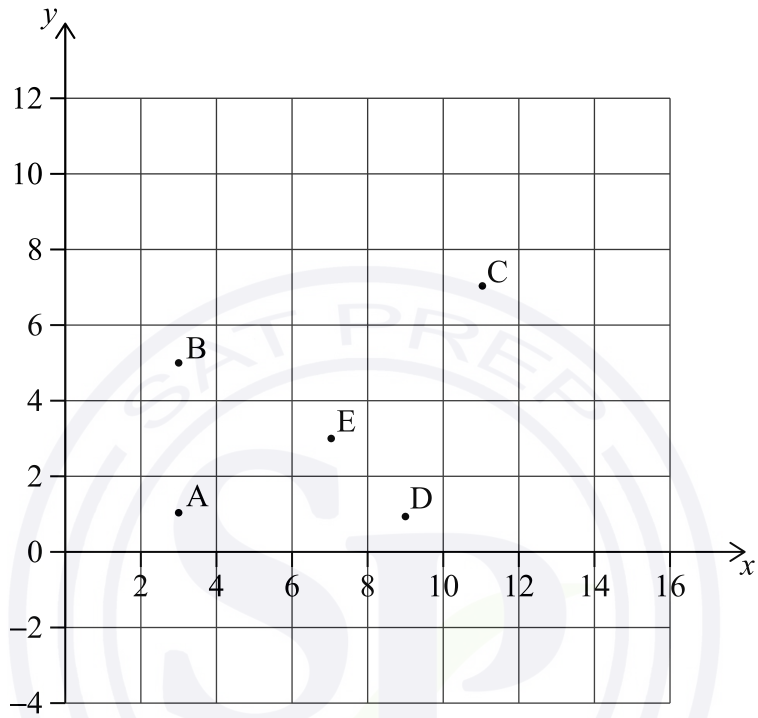
(c) According to this model, find the smallest possible population of Bulbul birds during the migration season. [1]

4. [Maximum mark: 6]

Points $A(3, 1)$, $B(3, 5)$, $C(11, 7)$, $D(9, 1)$ and $E(7, 3)$ represent snow shelters in the Blackburn National Forest. These snow shelters are illustrated in the following coordinate axes.

Horizontal scale: 1 unit represents $1\,km$.

Vertical scale: 1 unit represents $1\,km$.

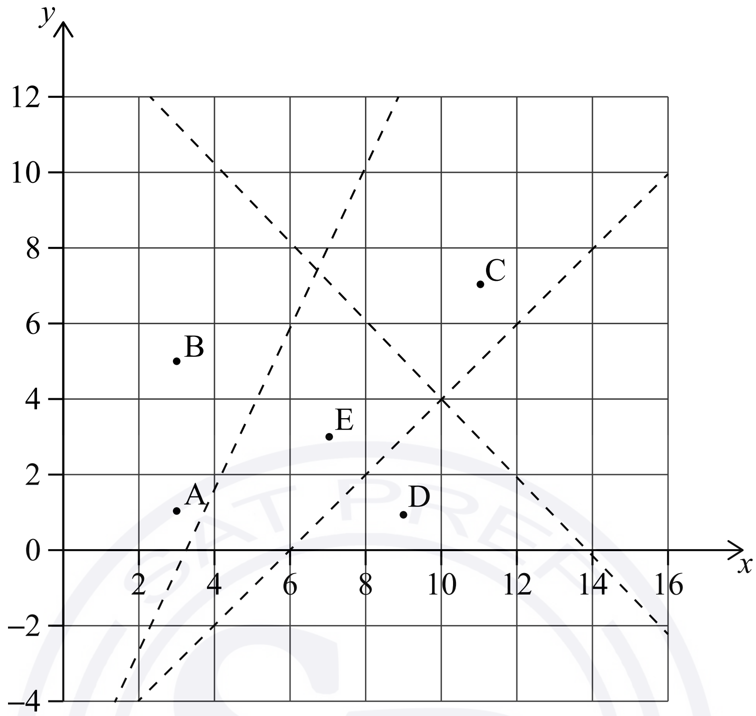


(a) Calculate the gradient of the line segment AE. [2]

(This question continues on the following page)

(Question 4 continued)

The Park Ranger draws three straight lines to form an incomplete Voronoi diagram.



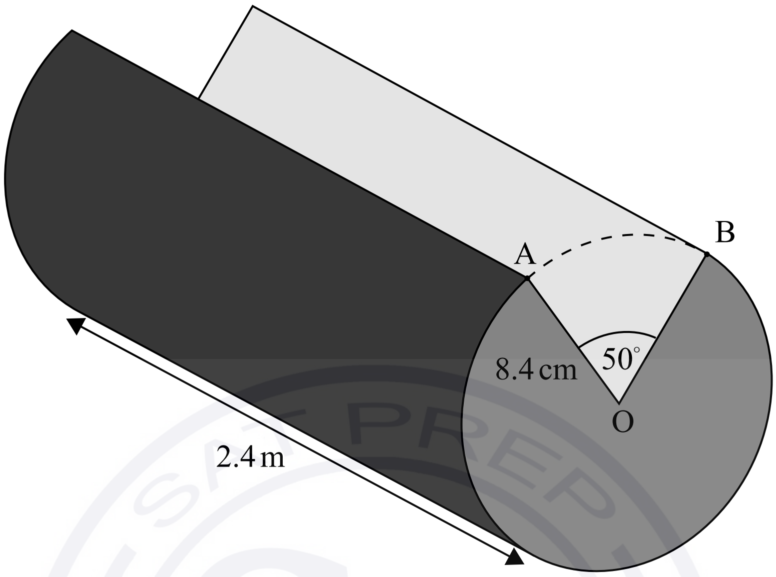
(b) Find the equation of the line which would complete the Voronoi cell containing site E. Give your answer in the form $ax + by + d = 0$ where a, b, $d \in \mathbb{Z}$. [3]

(c) In the context of the question, explain the significance of the Voronoi cell containing site E. [1]

5. [Maximum mark: 5]

Helen is building a cabin using cylindrical logs of length 2.4 m and radius 8.4 cm. A wedge is cut from one log and the cross-section of this log is illustrated in the following diagram.

diagram not to scale



(a) Find 50° in radians. [1]

(b) Find the volume of this log. [4]

6. [Maximum mark: 6]

Jae Hee plays a game involving a biased six-sided die.
The faces of the die are labelled -3, -1, 0, 1, 2 and 5.
The score for the game, X, is the number which lands face up after the die is rolled.
The following table shows the probability distribution for X.

Score x	-3	-1	0	1	2	5
$\mathbf{P(X=x)}$	$\frac{1}{18}$	p	$\frac{3}{18}$	$\frac{1}{18}$	$\frac{2}{18}$	$\frac{7}{18}$

(a) Find the exact value of p. [1]

Jae Hee plays the game once.

(b) Calculate the expected score. [2]

Jae Hee plays the game twice and adds the two scores together.

(c) Find the probability Jae Hee has a **total** score of -3. [3]

7. [Maximum mark: 5]

A particle, A, moves so that its velocity ($v\,\text{m}\,\text{s}^{-1}$) at time t is given by $v = 2\sin t,\ t \geq 0$.

The kinetic energy (E) of the particle A is measured in joules (J) and is given by $E = 5v^2$.

(a) Write down an expression for E as a function of time. [1]

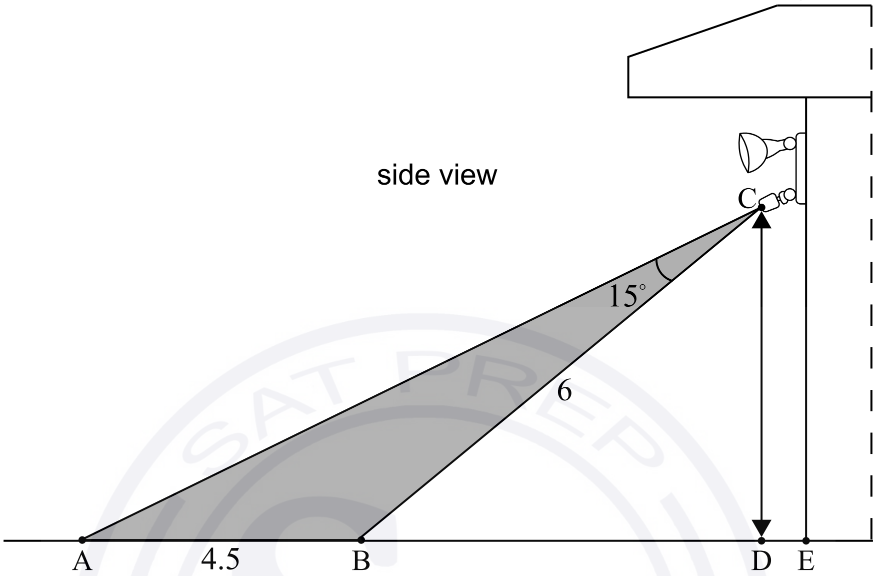
(b) Hence find $\frac{\mathrm{d}E}{\mathrm{d}t}$. [2]

(c) Hence or otherwise find the first time at which the kinetic energy is changing at a rate of $5\,\text{J}\,\text{s}^{-1}$. [2]

8. [Maximum mark: 8]

Ollie has installed security lights on the side of his house that are activated by a sensor. The sensor is located at point C directly above point D. The area covered by the sensor is shown by the shaded region enclosed by triangle ABC. The distance from A to B is 4.5 m and the distance from B to C is 6 m. Angle $\hat{ACB}$ is 15°.

diagram not to scale



(a) Find $\hat{CAB}$. [3]

Point B on the ground is 5 m from point E at the entrance to Ollie's house. He is 1.8 m tall and is standing at point D, below the sensor. He walks towards point B.

(b) Find the distance Ollie is **from the entrance to his house** when he first activates the sensor. [5]

. . . .

9. [Maximum mark: 5]

A manager wishes to check the mean weight of flour put into bags in his factory. He randomly samples 10 bags and finds the mean weight is $1.478\,\text{kg}$ and the standard deviation of the sample is $0.0196\,\text{kg}$.

(a) Find s_{n-1} for this sample. [2]

(b) Find a 95% confidence interval for the population mean, giving your answer to 4 significant figures. [2]

(c) The bags are labelled as being $1.5\,\text{kg}$ weight. Comment on this claim with reference to your answer in part (b). [1]

10. [Maximum mark: 6]

In a coffee shop, the time it takes to serve a customer can be modelled by a normal distribution with a mean of 1.5 minutes and a standard deviation of 0.4 minutes.

Two customers enter the shop together. They are served one at a time.

Find the probability that the total time taken to serve both customers will be less than 4 minutes.

Clearly state any assumptions you have made.

11. [Maximum mark: 6]

A particle P moves with velocity $\boldsymbol{v} = \begin{pmatrix} -15 \\ 2 \\ 4 \end{pmatrix}$ in a magnetic field, $\boldsymbol{B} = \begin{pmatrix} 0 \\ d \\ 1 \end{pmatrix}$, $d \in \mathbb{R}$.

(a) Given that $\boldsymbol{v}$ is perpendicular to $\boldsymbol{B}$, find the value of d. [2]

The force, $\boldsymbol{F}$, produced by P moving in the magnetic field is given by the vector equation $\boldsymbol{F} = a\boldsymbol{v} \times \boldsymbol{B}$, $a \in \mathbb{R}^{+}$.

(b) Given that $|\boldsymbol{F}| = 14$, find the value of a. [4]

12. [Maximum mark: 7]

Product research leads a company to believe that the revenue (R) made by selling its goods at a price (p) can be modelled by the equation.

$$R(p) = cpe^{dp},\ c,\ d \in \mathbb{R}$$

There are two competing models, A and B with different values for the parameters c and d.

Model A has $c = 3$, $d = -0.5$ and model B has $c = 2.5$, $d = -0.6$.

The company experiments by selling the goods at three different prices in three similar areas and the results are shown in the following table.

Area	**Price (p)**	**Revenue (R)**
1	1	1.5
2	2	1.8
3	3	1.5

The company will choose the model with the smallest value for the sum of square residuals.

Determine which model the company chose.

13. [Maximum mark: 6]

The rates of change of the area covered by two types of fungi, X and Y,on a particular tree are given by the following equations, where x is the area covered by X and y is the area covered by Y.

$$\frac{\mathrm{d}x}{\mathrm{d}t} = 3x - 2y$$
$$\frac{\mathrm{d}y}{\mathrm{d}t} = 2x - 2y$$

The matrix $\begin{pmatrix} 3 & -2 \\ 2 & -2 \end{pmatrix}$ has eigenvalues of 2 and -1 with corresponding eigenvectors $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$.

Initially $x = 8\,\mathrm{cm}^2$ and $y = 10\,\mathrm{cm}^2$.

(a) Find the value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ when $t = 0$. [2]

(b) On the following axes, sketch a possible trajectory for the growth of the two fungi, making clear any asymptotic behaviour. [4]

(This question continues on the following page)

(Question 13 continued)

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will not be marked.

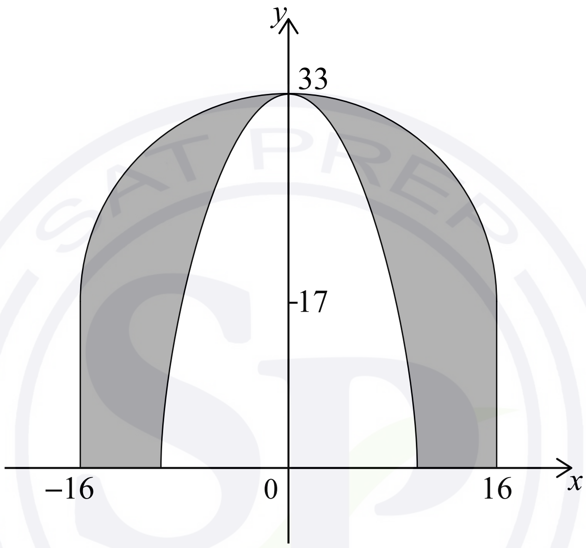
14. [Maximum mark: 8]

(a) The graph of $y = -x^3$ is transformed onto the graph of $y = 33 - 0.08x^3$ by a translation of a units vertically and a stretch parallel to the x-axis of scale factor b.

(i) Write down the value of a.

(ii) Find the value of b. [3]

(b) The outer dome of a large cathedral has the shape of a hemisphere of diameter 32 m, supported by vertical walls of height 17 m. It is also supported by an inner dome which can be modelled by rotating the curve $y = 33 - 0.08x^3$ through 360° about the y-axis between $y = 0$ and $y = 33$, as indicated in the diagram.



Find the volume of the space between the two domes. [5]

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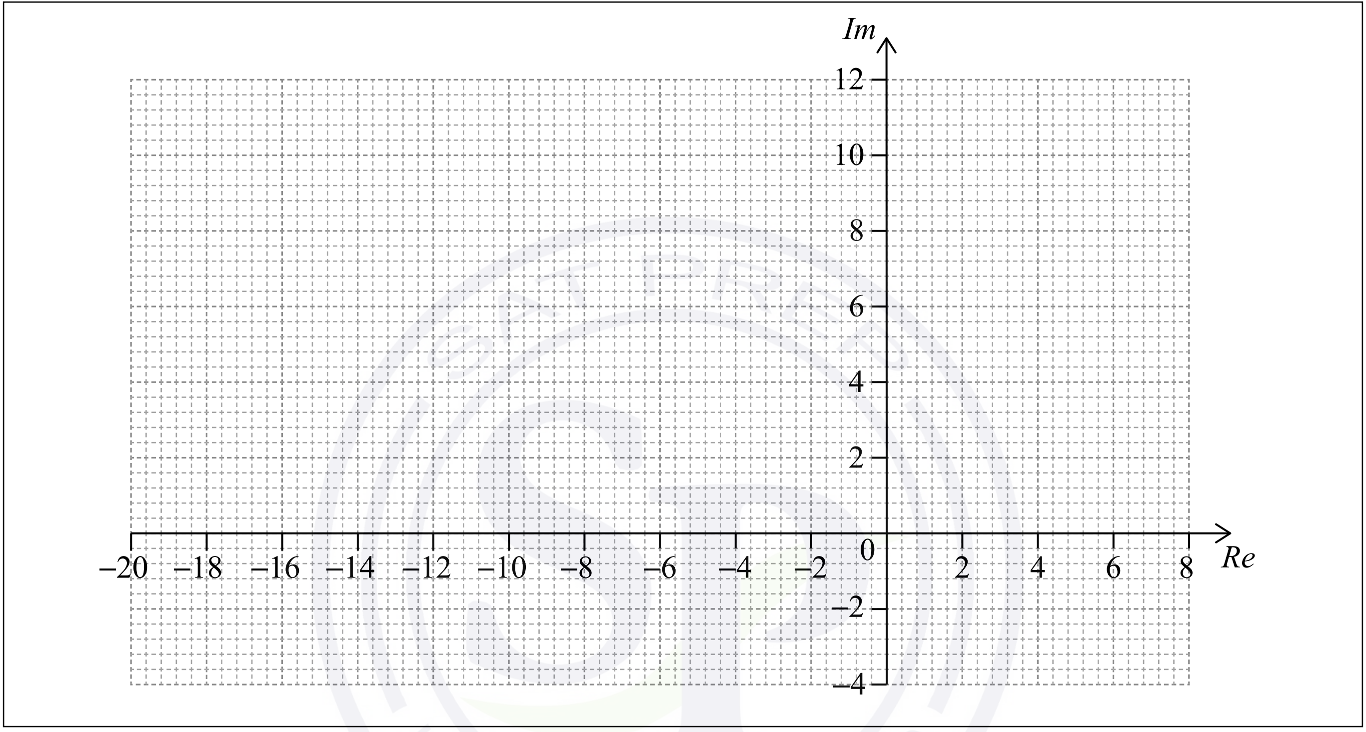
15. [Maximum mark: 7]

Let $w = a\mathrm{e}^{\frac{\pi}{4}\mathrm{i}}$, where $a \in \mathbb{R}^+$.

(a) For $a = 2$,

(i) find the values of w^2, w^3, and w^4;

(ii) draw w, w^2, w^3 and w^4 on the following Argand diagram. [5]



Let $z = \dfrac{w}{2 - \mathrm{i}}$.

(b) Find the value of a for which successive powers of z lie on a circle. [2]

(This question continues on the following page)

(Question 15 continued)

16. [Maximum mark: 6]

The number of fish that can be caught in one hour from a particular lake can be modelled by a Poisson distribution.

The owner of the lake, Emily, states in her advertising that the average number of fish caught in an hour is three.

Tom, a keen fisherman, is not convinced and thinks it is less than three. He decides to set up the following test. Tom will fish for one hour and if he catches fewer than two fish he will reject Emily's claim.

(a) State a suitable null and alternative hypotheses for Tom's test. [1]

(b) Find the probability of a Type I error. [2]

The average number of fish caught in an hour is actually 2.5.

(c) Find the probability of a Type II error. [3]

17. [Maximum mark: 6]

Mr Burke teaches a mathematics class with 15 students. In this class there are 6 female students and 9 male students.

Each day Mr Burke randomly chooses one student to answer a homework question.

In the first month, Mr Burke will teach his class 20 times.

(a) Find the probability he will choose a female student 8 times. [2]

The Head of Year, Mrs Smith, decides to select a student at random from the year group to read the notices in assembly. There are 80 students in total in the year group. Mrs Smith calculates the probability of picking a male student 8 times in the first 20 assemblies is 0.153357 correct to 6 decimal places.

(b) Find the number of male students in the year group. [4]

18. [Maximum mark: 6]

The rate, A, of a chemical reaction at a fixed temperature is related to the concentration of two compounds, B and C, by the equation

$$A = kB^x C^y \text{, where } x,\ y,\ k \in \mathbb{R}.$$

A scientist measures the three variables three times during the reaction and obtains the following values.

Experiment	A **(mol l^{-1} s^{-1})**	B **(mol l^{-1})**	C **(mol l^{-1})**
1	5.74	2.1	3.4
2	2.88	1.5	2.4
3	0.980	0.8	1.9

Find x, y and k.