8823 – 7201M

Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

November 2023

Mathematics: applications and interpretation

Higher level

Paper 1

20 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

R Marks awarded for clear **Reasoning**.

AG Answer given in the question and so no marks are awarded.

FT Follow through. The practice of awarding marks, despite candidate errors in previous parts, for their correct methods/answers using incorrect results.

Using the markscheme

1 General

Award marks using the annotations as noted in the markscheme eg ***M1***, ***A2***.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***A3***, ***M2*** *etc.*, do **not** split the marks, unless there is a note.
- The response to a "show that" question does not need to restate the ***AG*** line, unless a **Note** makes this explicit in the markscheme.
- Once a correct answer to a question or part question is seen, ignore further working even if this working is incorrect and/or suggests a misunderstanding of the question. This will encourage a uniform approach to marking, with less examiner discretion. Although some candidates may be advantaged for that specific question item, it is likely that these candidates will lose marks elsewhere too.
- An exception to the previous rule is when an incorrect answer from further working is used **in a subsequent part**. For example, when a correct exact value is followed by an incorrect decimal approximation in the first part and this approximation is then used in the second part. In this situation, award ***FT*** marks as appropriate but do not award the final ***A1*** in the first part. Examples:

	Correct answer seen	Further working seen	Any FT issues?	Action
1.	$8\sqrt{2}$	5.65685... *(incorrect decimal value)*	No. Last part in question.	Award ***A1*** for the final mark *(condone the incorrect further working)*
2.	$\frac{35}{72}$	0.468111… (incorrect decimal value)	Yes. Value is used in subsequent parts.	Award ***A0*** for the final mark *(and full **FT** is available in subsequent parts)*

3 Implied marks

Implied marks appear in **brackets e.g. *(M1)***,and can only be awarded if **correct** work is seen or implied by subsequent working/answer.

4 Follow through marks (only applied after an error is made)

Follow through (***FT***) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) (e.g. incorrect value from part (a) used in part (d) or incorrect value from part (c)(i) used in part (c)(ii)). Usually, to award ***FT*** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if all the marks awarded in a subsequent part are for the answer or are implied, then ***FT*** marks should be awarded for *their* correct answer, even when working is not present.

For example: following an incorrect answer to part (a) that is used in subsequent parts, where the markscheme for the subsequent part is ***(M1)A1***, it is possible to award full marks for *their* correct answer, **without working being seen.** For longer questions where all but the answer marks are implied this rule applies but may be overwritten by a **Note** in the Markscheme.

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks, by reflecting on what each mark is for and how that maps to the simplified version.
- If the error leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word “their” in a description, to indicate that candidates may be using an incorrect value.
- If the candidate’s answer to the initial question clearly contradicts information given in the question, it is not appropriate to award any ***FT*** marks in the subsequent parts. This includes when candidates fail to complete a “show that” question correctly, and then in subsequent parts use their incorrect answer rather than the given value.
- Exceptions to these ***FT*** rules will be explicitly noted on the markscheme.
- If a candidate makes an error in one part but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the command term was “Hence”.

5 Mis-read

If a candidate incorrectly copies values or information from the question, this is a mis-read (***MR***). A candidate should be penalized only once for a particular misread. Use the ***MR*** stamp to indicate that this has been a misread and do not award the first mark, even if this is an ***M*** mark, but award all others as appropriate.

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.
- If a candidate uses a correct answer, to a "show that" question, to a higher degree of accuracy than given in the question, this is NOT a misread and full marks may be scored in the subsequent part.
- ***MR*** can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If the command term is 'Hence' and not 'Hence or otherwise' then alternative methods are not permitted unless covered by a note in the mark scheme.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**.

7 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation** for example 1.9 and $1{,}9$ or 1000 and $1{,}000$ and 1.000.
- Do not accept final answers written using calculator notation. However, ***M*** marks and intermediate ***A*** marks can be scored, when presented using calculator notation, provided the evidence clearly reflects the demand of the mark.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, some **equivalent** answers will generally appear in brackets. Not all equivalent notations/answers/methods will be presented in the markscheme and examiners are asked to apply appropriate discretion to judge if the candidate work is equivalent.

8 Format and accuracy of answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. If the level of accuracy is not stated in the question, the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures.*

Where values are used in subsequent parts, the markscheme will generally use the exact value, however candidates may also use the correct answer to a "correct" level of accuracy (e.g 3 sf) in subsequent parts. The markscheme will often explicitly include the subsequent values that come "*from the use of 3 sf values*".

Simplification of final answers: Candidates are advised to give final answers using good mathematical form. In general, for an ***A*** mark to be awarded, arithmetic should be completed, and any values that lead to integers should be simplified; for example, $\sqrt{\frac{25}{4}}$ should be written as $\frac{5}{2}$. An exception to this is simplifying fractions, where lowest form is not required (although the numerator and the denominator must be integers); for example, $\frac{10}{4}$ may be left in this form or written as $\frac{5}{2}$. However, $\frac{10}{5}$ should be written as 2, as it simplifies to an integer.

Algebraic expressions should be simplified by completing any operations such as addition and multiplication, e.g. $4e^{2x} \times e^{3x}$ should be simplified to $4e^{5x}$, and $4e^{2x} \times e^{3x} - e^{4x} \times e^{x}$ should be simplified to $3e^{5x}$. Unless specified in the question, expressions do not need to be factorized, nor do factorized expressions need to be expanded, so $x(x+1)$ and $x^2 + x$ are both acceptable.

Please note: intermediate ***A*** marks do NOT need to be simplified.

9 Calculators

A GDC is required for this paper, but If you see work that suggests a candidate has used any calculator not approved for IB DP examinations (eg CAS enabled devices), please follow the procedures for malpractice.

10. Presentation of candidate work

Crossed out work: If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work unless an explicit note from the candidate indicates that they would like the work to be marked.

More than one solution: Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise. If the layout of the responses makes it difficult to judge, examiners should apply appropriate discretion to judge which is "first".

1. (a) attempt to use the term formula for a geometric sequence ***(M1)***

$4\times(0.8)^2$

$=2.56\ (\text{mm})\ \left(\frac{64}{25}\right)$ ***A1***

[2 marks]

(b) attempt to use the sum formula for a geometric sequence ***(M1)***

$\frac{4\times(1-0.8^{12})}{1-0.8}$

$=18.6\ (\text{mm})\ \ (18.6256...)$ ***A1***

[2 marks]

(c) attempt to use infinite geometric sum ***(M1)***

e.g. $\frac{4}{1-0.8}$ **OR** 20

adding 25 to their expression or value ***(M1)***

(maximum width $=25+20$)

$45\ (\text{mm})$ ***A1***

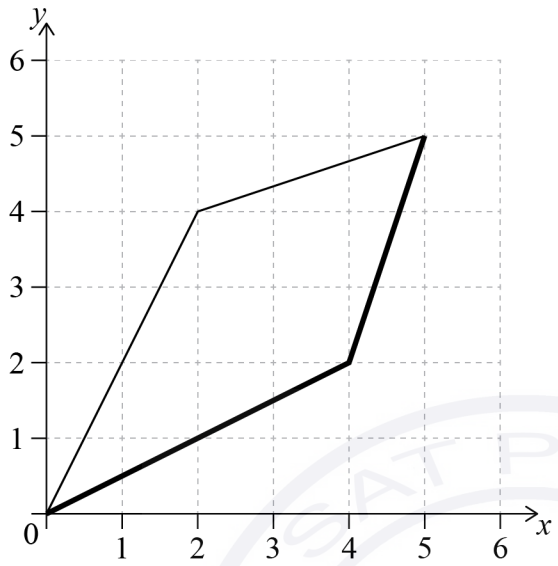
[3 marks]

[Total 7 marks]

2. (a) 4 ***A1***

[1 mark]

(b)



A1A1

Note: Award ***A1*** for passing through $(0, 0)$ and $(4, 2)$, ***A1*** for passing through $(4, 2)$ and $(5, 5)$.

[2 marks]

(c) attempt to solve $y = 3x - 1$ for x **OR** changing variables ***(M1)***

$\left(g^{-1}(x)\right) = \frac{x+1}{3}$ ***A1***

[2 marks]

(d) sketch of $g(x)$ or $g^{-1}(x)$, algebraic approach ***(M1)***

$\frac{1}{2}x = \frac{x+1}{3}$

$(x =)\ 2$ ***A1***

[2 marks]

[Total 7 marks]

3. (a) (upper bound =) $0.525\ (\text{m})$ ***A1***
(lower bound =) $0.515\ (\text{m})$ ***A1***

Note: Accept an answer in interval notation or written as an inequality.

[2 marks]

(b) **METHOD 1 Convert REC to linear metres**
attempt to convert REC to metres using their lower bound ***(M1)***
$440\times0.515\ (=226.6)$ **OR** $280\times0.515\ (=144.2)$ seen

attempt to use the formula for the volume of a right pyramid ***(M1)***

$(V=)\ \frac{1}{3}(440\times0.515)^2(280\times0.515)$ ***(A1)***

$2\,470\,000\ (\text{m}^3)\ \ (2\,468\,106.051\ldots,\ 2.47\times10^6)$ ***A1***

METHOD 2 Convert REC to cubic metres
attempt to use the formula for the volume of a right pyramid ***(M1)***

$(V=)\ \frac{1}{3}(440)^2(280)\ \ (=18\,069\,333.33...)$
attempt to convert 1 cubic REC to cubic metres using their lower bound ***(M1)***
(1 cubic REC =) 0.515^3

$(V=)\ \frac{1}{3}(440)^2(280)\times(0.515)^3$ ***(A1)***
$2\,470\,000\ (\text{m}^3)\ \ (2\,468\,106.051\ldots,\ 2.47\times10^6)$ ***A1***

[4 marks]
[Total 6 marks]

4. (a) recognizing supplementary angles or acute angles in right-triangles ***(M1)***
$(\text{A}\hat{\text{B}}\text{C}=)\ 41^\circ+(180^\circ-112^\circ),\ 41^\circ+(90^\circ-22^\circ)$

Note: Values may be seen on diagram.

$\text{A}\hat{\text{B}}\text{C}=109^\circ$ ***A1***

[2 marks]

(b) $\text{A}\hat{\text{C}}\text{B}=49^\circ$ (may be seen in part (a)) ***(A1)***
attempt to substitute into the sine rule (or equivalent) ***(M1)***
$\frac{\text{AC}}{\sin109^\circ}=\frac{100}{\sin49^\circ}$ ***(A1)***

$\text{AC}=125\ (\text{km})\ \ (=125.282...)$ ***A1***

[4 marks]
[Total 6 marks]

5. (a) $2.36 = a(3)^2 + b(3) + c$ **OR** $2.36 = 9a + 3b + c$ ***A1***

[1 mark]

(b) finding other equations to solve simultaneously ***(M1)***

$5 = a(10)^2 + b(10) + c$ **AND** $7.16 = a(17)^2 + b(17) + c$
OR $5 = 100a + 10b + c$ **AND** $7.16 = 289a + 17b + c$

any one coefficient in equation correct ***(A1)***
$f(x) = -0.00490x^2 + 0.441x + 1.08$ ***A1***

$\left(f(x) = -0.00489795\ldots x^2 + 0.440816\ldots x + 1.08163\ldots\right)$
$\left(f(x) = -\frac{6}{1225}x^2 + \frac{108}{245}x + \frac{53}{49}\right)$

Note: Award at most ***(M1)(A1)A0*** if answer is not expressed as an equation.

[3 marks]

(c) attempt to substitute 80 into their equation ***(M1)***

$\left(f(80) =\right) 5$ ***A1***

$5 > 4$ **OR** therefore the ball will go over the fence ***R1***

Note: Do not award ***A0R1***; their value must be seen to credit a correct conclusion.

[3 marks]

(d) setting their equation equal to zero, graph ***(M1)***

$0 = -0.00489795\ldots x^2 + 0.440816\ldots x + 1.08163\ldots$ **OR** $f(x) = 0$

92.4 (92.3902…) (m) ***A1***

[2 marks]
[Total 9 marks]

6. (a) $(4, 8)$ ***A1***

[1 mark]

(b) attempt to find the gradient of AC ***(M1)***

$\frac{13-3}{8-0}, \frac{10}{8}, \left(\frac{5}{4}\right), (1.25)$

attempt to substitute **their** coordinates and the negative reciprocal of **their** gradient into the equation of a straight line ***(M1)***

$y-8=-\frac{4}{5}(x-4)$ **OR** $8=-\frac{4}{5}(4)+c$ **OR** $c=11.2$

$y-8=-\frac{4}{5}(x-4) \quad (y=-0.8x+11.2,\ 4x+5y-56=0)$ ***A1***

[3 marks]

(c) (i) attempt to find one distance from a farm to any closest vertex ***M1***

finding a correct distance from at least two distinct vertices ***A1***

$7.58968\ldots, 4.472135\ldots \left(\sqrt{20}\right), 5.830951\ldots \left(\sqrt{34}\right)$

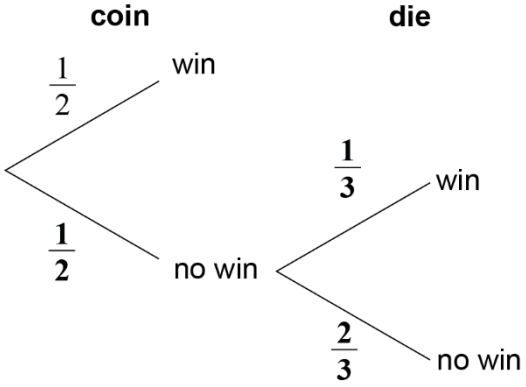
$\left(\frac{9}{11}, \frac{116}{11}\right)$ (is furthest) ***A1***

(ii) 7.59 (km) $(=7.58968...)$ ***A1***

[4 marks]

[Total 8 marks]

7. (a)



A1A1

Note: Award ***A1*** for completing first set of branches, ***A1*** for completing second set of branches.

[2 marks]

(b) attempt to multiply along the branches ***(M1)***

$\frac{1}{2} \times \frac{2}{3}$

$= \frac{1}{3} \ (= 0.333...)$ ***A1***

[2 marks]

(c) **EITHER**

$$\frac{\frac{1}{2}}{\frac{1}{2} + \left(\frac{1}{2} \times \frac{1}{3}\right)}$$ ***M1A1***

Note: Award ***M1*** for recognizing conditional probability, ***A1*** for correct substitution.

OR

$$\frac{\frac{1}{2}}{1 - \frac{1}{3}}$$ ***M1A1***

Note: Award ***M1*** for recognizing conditional probability, ***A1*** for correct substitution.

THEN

$= \frac{3}{4}$ ***A1***

[3 marks]
[Total 7 marks]

8. (a) $z = 2e^{-0.524i} \left(= 2e^{-\frac{\pi}{6}i} \right)$ ***A1A1***

Note: Award ***A1*** for the correct modulus and ***A1*** for the correct argument.

[2 marks]

(b) **METHOD 1**

$z_1 + z_2 = e^{2ti}\left(1 + 2e^{-\frac{\pi}{6}i}\right)$ ***(M1)***

$= e^{2ti} \times 2.90931...e^{-0.350879...i}$ ***(A1)***

$\text{Im}(z_1 + z_2) = 2.91\sin(2t - 0.351)$ ***A1A1***

METHOD 2

attempt to find the modulus of $\left(1 + 2e^{-\frac{\pi}{6}i}\right)$ ***(M1)***

$p = \left|1 + 2e^{-\frac{\pi}{6}i}\right|$

attempt to find the argument of $\left(1 + 2e^{-\frac{\pi}{6}i}\right)$ ***(M1)***

$q = \arg\left(1 + 2e^{-\frac{\pi}{6}i}\right)$

$\text{Im}(z_1 + z_2) = 2.91\sin(2t - 0.351)$ ***A1A1***

METHOD 3

sketching $\text{Im}(z_1 + z_2)$ ***(M1)***

max is $(0.961, 2.91)$ ***(A1)***

first root is 0.1754 ***(A1)***

$\text{Im}(z_1 + z_2) = 2.91\sin(2t - 0.351)$ ***A1***

[4 marks]

[Total 6 marks]

9. (a) $p = 1.5;\ q = 2$ ***A1A1***

[2 marks]

(b) attempt at using chain rule ***(M1)***

(i) $h'(t) = -\frac{\pi}{4}\sin\left(\frac{\pi}{6}t\right)\ \left(= -0.785\sin\left(\frac{\pi}{6}t\right)\right)$ ***A1***

(ii) $h''(t) = -\frac{\pi^2}{24}\cos\left(\frac{\pi}{6}t\right)\ \left(= -0.411233...\cos\left(\frac{\pi}{6}t\right)\right)$ ***A1***

[3 marks]

(c) (i) attempt to locate points of inflexion or max value of $h'(t)$ ***(M1)***

$h''(t) = -\frac{\pi^2}{24}\cos\left(\frac{\pi}{6}t\right) = 0$ **OR** sketch on graph **OR** $t = 3$ **OR** $\frac{\pi}{6}k = \frac{3\pi}{2}$

$(k =)\ 9$ ***A1***

(ii) $(h(9) =)\ 2$ (m) ***A1***

[3 marks]

[Total 8 marks]

10. (a) $M = 1000 \times t^{-0.6}$

$a = 1000 \left(= 999.972...\right)$ $b = -0.600$ $\left(-0.599991...\right)$ ***A1A1***

[2 marks]

(b) $y = -0.600\,x + 6.908$

$c = -0.600$ $d = 6.908$ ***A1A1***

Note: Long answer for c is $0.599991\ldots$ and for d is $6.90772\ldots$ If both answers are correct but not given to 3 decimal places award ***A1A0***.

[2 marks]

(c) **METHOD 1 (starting with the result in part (b))**

attempt to apply addition (or subtraction) log laws ***M1***

attempt to apply inverse log ***M1***

Note: These ***M1*** marks can be applied in either order depending on the approach.

e.g. $\ln M = \ln t^{-0.600} + \ln \mathrm{e}^{6.908}$ then $\ln M = \ln\left(\mathrm{e}^{6.908} \times t^{-0.600}\right)$

OR $\ln \dfrac{M}{t^{-0.600}} = 6.908$ then $\dfrac{M}{t^{-0.600}} = \mathrm{e}^{6.908}$

$M = \mathrm{e}^{6.908} t^{-0.600}$ ***A1***

$(M = 1000.24 t^{-0.6})$

$M = 1000 t^{-0.6}$ and hence (close enough to be) equivalent ***AG***

Note: The ***AG*** line (or something which approximates it) must be seen for the final ***A1*** to be awarded. If 3 sf answers are used from part (b), the coefficient is 1002; this can be condoned in the working, as it equals 1000 when rounded to 3 sf.

METHOD 2 (starting with the result in part (a))

attempt to apply log ***M1***

$\ln M = \ln\left(1000 \times t^{-0.6}\right)$

attempt to apply addition (or subtraction) log laws ***M1***

$\ln M = \ln 1000 + \ln t^{-0.6}$

$\ln M = 6.90775... - 0.6 \ln t$ ***A1***

$(\ln M = y,\ \ln t = x)$

$y = -0.600\,x + 6.908$ and hence (close enough to be) equivalent ***AG***

Note: The ***AG*** line (or something which approximates it) must be seen for the final ***A1*** to be awarded. Condone $b = -0.6$.

[3 marks]

[Total 7 marks]

11. (a) (i) **EITHER**

$(\text{area of } R =) \int_{-1}^{1} \left| x^3 - x \right| \mathrm{d}x$ ***A1***

OR

$(\text{area of } R =) 2 \times \int_{-1}^{0} x^3 - x \,\mathrm{d}x$ **OR** $(\text{area of } R =) -2 \times \int_{0}^{1} x^3 - x \,\mathrm{d}x$ ***A1***

OR

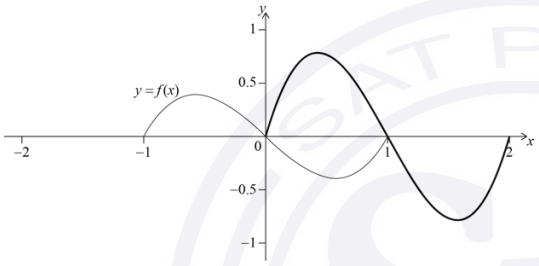
$(\text{area of } R =) \int_{-1}^{0} x^3 - x \,\mathrm{d}x - \int_{0}^{1} x^3 - x \,\mathrm{d}x$ ***A1***

(ii) $(\text{area of } R =) \ 0.5$ ***A1***

Note: Follow through from part (a)(i) only if answer is greater than zero.

[2 marks]

(b)



A1A1

Note: Award ***A1*** for sketch with correct shape on $[0, 2]$, ***A1*** for vertical stretch x2. Condone max/min of g extending to 1 / -1.

[2 marks]

(c) attempt to use $\pi \int y^2 \mathrm{d}x$ ***(M1)***

volume $= \pi \int_{-1}^{1} \left(x^3 - x \right)^2 \mathrm{d}x$ ***(A1)***

volume $= 0.479$ (cubic units) $\left(= 0.478718..., \frac{16\pi}{105} \right)$ ***A1***

[3 marks]
[Total 7 marks]

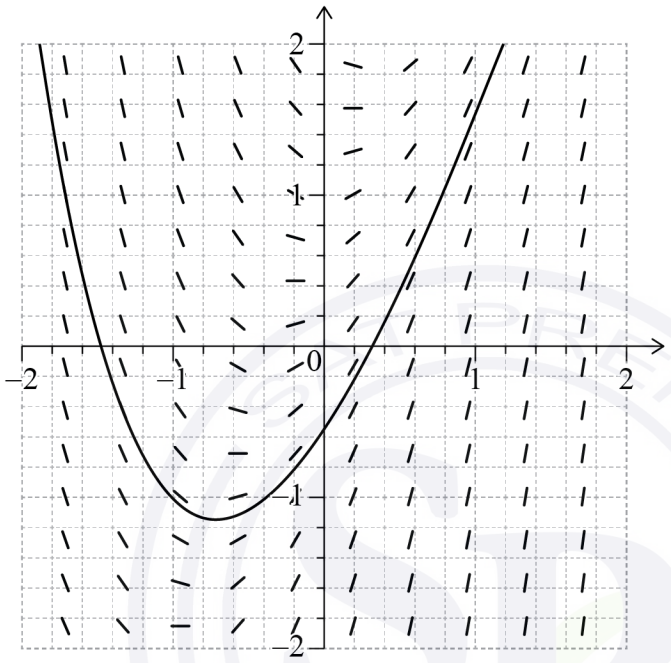
12. (a) gradient $(=-3+1+1)=-1$ ***A1***

$y+1=-1(x+1)$

$x+y+2=0$ ***A1***

[2 marks]

(b)



A1A1

Note: Award ***A1*** for (approximately) intersecting $(-1, -1)$ and with correct gradient, ***A1*** for generally plausible shape (e.g. not crossing over LOTS of isoclines).

[2 marks]
[Total 4 marks]

13. (a) attempt to solve $u_1(t) = u_2(t)$ **OR** sketch of two graphs ***(M1)***

$(T =)\ 2$ ***A1***

Note: Award ***(M1)A0*** if additional values of T are seen **OR** if $T = -2$ is their final answer.

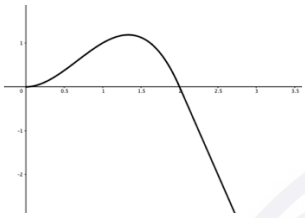
[2 marks]

(b) $u_1'(t) = 4t - 3t^2$ ***A1***

$u_1'(2) = -4 = u_2'(2)$ ***R1***

[2 marks]

(c)



recognition of integrating **AND** equating to zero ***(M1)***

$\int_0^2 2t^2 - t^3 \, \mathrm{d}t + \int_2^k 8 - 4t \, \mathrm{d}t = 0$ ***(A1)***

$\frac{4}{3} + (8k - 2k^2) - 8 = 0$ **OR** $1.18350\ldots \left(\frac{6-\sqrt{6}}{3}\right)$ seen ***(A1)***

Note: Award ***(M1)(A1)A0*** if integration done correctly but limits are not substituted.

$(k =)\ 2.82 \left(= 2.81649..., \frac{6+\sqrt{6}}{3}\right)$ ***A1***

[4 marks]
[Total 8 marks]

14. (a) vector from Q to any point in L or vice versa

$$= \begin{pmatrix} 1+\lambda \\ 3+\lambda \\ 2\lambda \end{pmatrix} - \begin{pmatrix} 11 \\ -1 \\ 3 \end{pmatrix} = \begin{pmatrix} -10+\lambda \\ 4+\lambda \\ 2\lambda - 3 \end{pmatrix}$$ ***(M1)***

EITHER (scalar product)
attempt to use scalar product ***(M1)***

$$\begin{pmatrix} -10+\lambda \\ 4+\lambda \\ 2\lambda - 3 \end{pmatrix} \bullet \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} = 0$$

$$-10+\lambda+4+\lambda+4\lambda-6=0$$

OR (distance formula)
attempt to use distance formula ***(M1)***
minimizing $(-10+\lambda)^2+(4+\lambda)^2+(-3+2\lambda)^2$

THEN
$\lambda = 2$ ***(A1)***

point $\mathrm{P}(3, 5, 4)$ ***A1***

Note: Do not award final ***A1*** for P given as a vector.

[4 marks]

(b) $\overrightarrow{\mathrm{PQ}} = \begin{pmatrix} 8 \\ -6 \\ -1 \end{pmatrix}$ ***(A1)***

attempt to use vector product ***(M1)***

(perpendicular vector =) $\begin{pmatrix} 8 \\ -6 \\ -1 \end{pmatrix} \times \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$

$\begin{pmatrix} -11 \\ -17 \\ 14 \end{pmatrix}$ ***A1***

Note: Award final ***A1*** for any multiple (positive or negative) of the answer given here.

[3 marks]
[Total 7 marks]

15. (a) probability of non veg remaining non veg ***A1***

[1 mark]

(b) attempt to use $\det(\boldsymbol{A}-\lambda\boldsymbol{I})=0$ ***(M1)***

$$\begin{vmatrix} 0.8-\lambda & 0.1 \\ 0.2 & 0.9-\lambda \end{vmatrix} = 0$$

$(0.8-\lambda)(0.9-\lambda)-0.1\times0.2=0$ ***(A1)***

$\lambda=1;\ \lambda=0.7$ ***A1***

[3 marks]

(c) $-2a+b=0$ ***M1***

$\boldsymbol{v}_1=\begin{pmatrix}1\\2\end{pmatrix}$ (accept any multiples of this answer) ***A1***

$\boldsymbol{v}_1$ means that in the long term the ratio of veg to non-veg is $1:2$

(in the long term one-third of students will be veg and two-thirds will not) ***A1***

[3 marks]

[Total 7 marks]

16. (a) Let X be the random variable number of shots taken in a 12 minute period

$X\sim\text{Po}(5)$ ***(A1)***

$\text{P}(X\leq6)=0.762\ \ (=0.762183...)$ ***A1***

[2 marks]

(b) $\text{P}(\text{less than 4 shots}\cap\text{success at least once})$

METHOD 1

$=\text{P}(\text{less than 4 shots})-\text{P}(\text{less than 4 shots}\cap\text{zero success})$ ***(M1)***

Note: Might be communicated in Venn diagram.

attempt to multiply by different powers of 0.6 ***(M1)***

$=\text{P}(X\leq3)-(\text{P}(X=0)\times(0.6)^0+\text{P}(X=1)\times(0.6)^1+\text{P}(X=2)\times(0.6)^2+\text{P}(X=3)\times(0.6)^3)$

(A1)

$=0.414\ \ (=0.413845...)$ ***A1***

METHOD 2

attempt to multiply by different powers of 0.4 ***(M1)***

$=\text{P}(X=1)\times(0.4)^1+\text{P}(X=2)\times\left((0.4)^2+2\times0.4\times0.6\right)+\text{P}(X=3)\times\left((0.4)^3+3\times0.4^2\times0.6+3\times0.4\times0.6^2\right)$

(M1)(A1)

Note: Award ***M1*** for recognizing the six different cases, e.g. $2\times0.4\times0.6$ (etc.) or equivalent seen, ***A1*** for completely correct expression.

$=0.414\ \ (=0.413845...)$ ***A1***

[4 marks]

[Total 6 marks]

Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

May 2023

Mathematics: applications and interpretation

Higher level

Paper 1

23 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

R Marks awarded for clear **Reasoning**.

AG Answer given in the question and so no marks are awarded.

FT Follow through. The practice of awarding marks, despite candidate errors in previous parts, for their correct methods/answers using incorrect results.

Using the markscheme

1 General

Award marks using the annotations as noted in the markscheme eg ***M1***, ***A2***.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***A3***, ***M2*** *etc.*, do **not** split the marks, unless there is a note.
- The response to a "show that" question does not need to restate the ***AG*** line, unless a **Note** makes this explicit in the markscheme.
- Once a correct answer to a question or part question is seen, ignore further working even if this working is incorrect and/or suggests a misunderstanding of the question. This will encourage a uniform approach to marking, with less examiner discretion. Although some candidates may be advantaged for that specific question item, it is likely that these candidates will lose marks elsewhere too.
- An exception to the previous rule is when an incorrect answer from further working is used **in a subsequent part**. For example, when a correct exact value is followed by an incorrect decimal approximation in the first part and this approximation is then used in the second part. In this situation, award ***FT*** marks as appropriate but do not award the final ***A1*** in the first part. Examples:

	Correct answer seen	Further working seen	Any FT issues?	Action
1.	$8\sqrt{2}$	5.65685... *(incorrect decimal value)*	No. Last part in question.	Award ***A1*** for the final mark *(condone the incorrect further working)*
2.	$\frac{35}{72}$	0.468111… (incorrect decimal value)	Yes. Value is used in subsequent parts.	Award ***A0*** for the final mark *(and full **FT** is available in subsequent parts)*

3 Implied marks

Implied marks appear in **brackets e.g. *(M1)***,and can only be awarded if **correct** work is seen or implied by subsequent working/answer.

4 Follow through marks (only applied after an error is made)

Follow through (***FT***) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) (e.g. incorrect value from part (a) used in part (d) or incorrect value from part (c)(i) used in part (c)(ii)). Usually, to award ***FT*** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if all the marks awarded in a subsequent part are for the answer or are implied, then ***FT*** marks should be awarded for *their* correct answer, even when working is not present.

For example: following an incorrect answer to part (a) that is used in subsequent parts, where the markscheme for the subsequent part is ***(M1)A1***, it is possible to award full marks for *their* correct answer, **without working being seen.** For longer questions where all but the answer marks are implied this rule applies but may be overwritten by a **Note** in the Markscheme.

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks, by reflecting on what each mark is for and how that maps to the simplified version.
- If the error leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word “their” in a description, to indicate that candidates may be using an incorrect value.
- If the candidate’s answer to the initial question clearly contradicts information given in the question, it is not appropriate to award any ***FT*** marks in the subsequent parts. This includes when candidates fail to complete a “show that” question correctly, and then in subsequent parts use their incorrect answer rather than the given value.
- Exceptions to these ***FT*** rules will be explicitly noted on the markscheme.
- If a candidate makes an error in one part but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the command term was “Hence”.

5 Mis-read

If a candidate incorrectly copies values or information from the question, this is a mis-read (***MR***). A candidate should be penalized only once for a particular misread. Use the ***MR*** stamp to indicate that this has been a misread and do not award the first mark, even if this is an ***M*** mark, but award all others as appropriate.

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.
- If a candidate uses a correct answer, to a "show that" question, to a higher degree of accuracy than given in the question, this is NOT a misread and full marks may be scored in the subsequent part.
- ***MR*** can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If the command term is 'Hence' and not 'Hence or otherwise' then alternative methods are not permitted unless covered by a note in the mark scheme.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**.

7 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation** for example 1.9 and $1{,}9$ or 1000 and $1{,}000$ and 1.000.
- Do not accept final answers written using calculator notation. However, ***M*** marks and intermediate ***A*** marks can be scored, when presented using calculator notation, provided the evidence clearly reflects the demand of the mark.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, some **equivalent** answers will generally appear in brackets. Not all equivalent notations/answers/methods will be presented in the markscheme and examiners are asked to apply appropriate discretion to judge if the candidate work is equivalent.

8 Format and accuracy of answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. If the level of accuracy is not stated in the question, the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures*.

Where values are used in subsequent parts, the markscheme will generally use the exact value, however candidates may also use the correct answer to a "correct" level of accuracy (e.g 3 sf) in subsequent parts. The markscheme will often explicitly include the subsequent values that come "*from the use of 3 sf values*".

Simplification of final answers: Candidates are advised to give final answers using good mathematical form. In general, for an ***A*** mark to be awarded, arithmetic should be completed, and any values that lead to integers should be simplified; for example, $\sqrt{\frac{25}{4}}$ should be written as $\frac{5}{2}$. An exception to this is simplifying fractions, where lowest form is not required (although the numerator and the denominator must be integers); for example, $\frac{10}{4}$ may be left in this form or written as $\frac{5}{2}$. However, $\frac{10}{5}$ should be written as 2, as it simplifies to an integer.

Algebraic expressions should be simplified by completing any operations such as addition and multiplication, e.g. $4e^{2x} \times e^{3x}$ should be simplified to $4e^{5x}$, and $4e^{2x} \times e^{3x} - e^{4x} \times e^{x}$ should be simplified to $3e^{5x}$. Unless specified in the question, expressions do not need to be factorized, nor do factorized expressions need to be expanded, so $x(x+1)$ and $x^2 + x$ are both acceptable.

Please note: intermediate ***A*** marks do NOT need to be simplified.

9 Calculators

A GDC is required for this paper, but If you see work that suggests a candidate has used any calculator not approved for IB DP examinations (eg CAS enabled devices), please follow the procedures for malpractice.

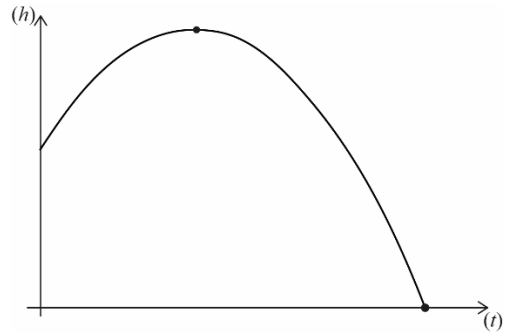
10. Presentation of candidate work

Crossed out work: If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work unless an explicit note from the candidate indicates that they would like the work to be marked.

More than one solution: Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise. If the layout of the responses makes it difficult to judge, examiners should apply appropriate discretion to judge which is "first".

1. (a) **METHOD 1**

correct sketch with some indication of maximum point ***(M1)***



0.921 (seconds) $\left(0.921052..., \frac{35}{38}\right)$ ***A1***

METHOD 2

correct substitution into equation for line of symmetry ***(M1)***

$(t =) \ -\frac{8.75}{2\times -4.75}$

0.921 (seconds) $\left(0.921052..., \frac{35}{38}\right)$ ***A1***

METHOD 3

equating the correct derivative to 0 ***(M1)***

$-9.5t + 8.75 = 0$

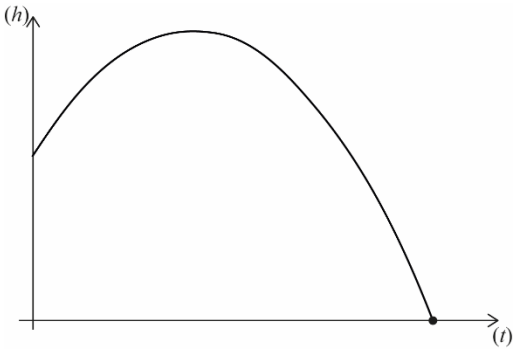
0.921 (seconds) $\left(0.921052..., \frac{35}{38}\right)$ ***A1***

Note: Award ***M1A0*** for a final answer of 0.92 seen with no working.

[2 marks]

(b) **METHOD 1**

correct sketch with some indication of x-intercept



(M1)

Note: May be seen in part (a).

2 (seconds) ***A1***

continued…

Question 1 continued

METHOD 2
setting the equation to zero ***(M1)***

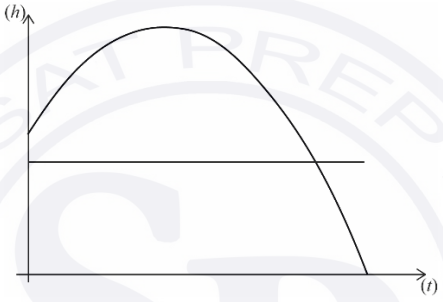
$-4.75t^2 + 8.75t + 1.5 = 0$

2 (seconds) ***A1***

Note: If both roots are given, with or without working, award ***(M1)A0***.

[2 marks]

(c) **METHOD 1**
correct sketch of quadratic function and a straight line in approximate correct position ***(M1)***



1.88 (seconds) (1.87577…(seconds)) ***A1***

METHOD 2
setting the equation equal to 1.2 ***(M1)***
$-4.75t^2 + 8.75t + 1.5 = 1.2$
1.88 (seconds) (1.87577…(seconds)) ***A1***

Note: Award ***(M1)A0*** if -0.0336702… seen as (part of) a final answer.
Award ***M1A0*** for answer of 1.9 seen without working.

[2 marks]

(d) *Award **R1** for a sensible reason in the context of the question:* ***R1***
e.g.
The model ignores air resistance (or wind)
The model treats the ball as a point
The model assumes gravity is constant
The model assumes that the ball continues to follow the trajectory even after hitting the ground
This model ignores the bouncing back of the ball after hitting the ground

Note: Do not accept generic criticisms of any mathematical model, such as:
There are assumptions being made
Models are never accurate / It is only a model

[1 mark]

Total [7 marks]

2. (a) 11.0 $(11.0212\ldots)$ ***A2***

Note: Award ***A1*** for a final answer of 11 if no unrounded answer is seen.

[2 marks]

(b) **EITHER**
$11.0 > 9.488$ $(11.0212\ldots > 9.488)$ ***R1***

OR
$0.0263 < 0.05$ $(0.0263264\ldots < 0.05)$ ***R1***

THEN

EITHER
(there is significant evidence to) reject H_0 ***A1***

OR
(there is significant evidence that) the (food) quality and the type of meal are not independent ***A1***

Note: Do not award ***R0A1.***
Award ***R1*** for $\chi^2_{\text{calc}} > \chi^2_{\text{crit}}$, provided the calculated value is explicitly seen in part (b).

Accept "p-value < significance level" provided their p-value is seen and their p-value is between 0 and 1.

[2 marks]
Total [4 marks]

3. (a) attempting to use $\mathrm{P}(R \cap S) = \mathrm{P}(R)\mathrm{P}(S)$ ***(M1)***

$0.2 = 0.8(0.2 + x)$ ***(A1)***

$x = 0.05$ ***A1***

[3 marks]

(b) $x + 0.2 + 0.6 + y = 1$ ***(M1)***

$y = 0.15$ ***A1***

[2 marks]

(c) **METHOD 1**

attempting to apply $\mathrm{P}(R' \mid S') = \dfrac{\mathrm{P}(R' \cap S')}{\mathrm{P}(S')}$ ***(M1)***

$\dfrac{0.15}{0.2}$

$= \dfrac{3}{4}$ ***A1***

METHOD 2

$\mathrm{P}(R' \mid S') = \mathrm{P}(R')$ (because R, S are independent) ***(M1)***

$= 1 - 0.25 = 0.75$ ***A1***

Note: ***FT*** from their values of x or y.

[2 marks]

Total [7 marks]

4. (a) **METHOD 1 (use of financial app in GDC)**

$N = 5$ **OR** $N = 20$
$I\% = 1.2$ $I\% = 1.2$
$PV = \pm 520$ $PV = \pm 520$
$P/Y = 1$ $P/Y = 4$
$C/Y = 4$ $C/Y = 4$ ***(M1)(A1)***

Note: Award ***M1*** for evidence of using the financial app on the calculator, ***A1*** for all correct entries.

($) 552.11 ***A1***

Note: Award at most ***(M1)(A1)A0*** if correct answer is not given to two decimal places.

METHOD 2 (use of formula)

attempt to substitute into compound interest formula ***(M1)***

$$520 \times \left(1 + \frac{1.2}{100 \times 4}\right)^{5 \times 4}$$ ***(A1)***

($) 552.11 ***A1***

Note: Award at most ***(M1)(A1)A0*** if correct answer is not given to two decimal places.

[3 marks]

(b) **EITHER**

$N = 5$
$I\% = 43.5$ $(43.4772...(\%))$
$PV = \pm 520$
$FV = \mp 30$ ***(M1)(A1)A1***

Note: Award ***M1*** for evidence of using the finance app on the calculator, ***A1*** for all correct entries, ***A1*** for correct final answer. Condone missing -/+ sign if the correct final answer is seen.

OR

$$30 = 520\left(1 - \frac{r}{100}\right)^5$$ (or equivalent) ***(M1)(A1)***

$(r =)$ 43.5% $(43.477...\%)$ ***A1***

Note: Award ***M1*** for using the compound interest formula, ***A1*** for correct substitutions and for equating to 30, ***A1*** for correct final answer. Accept $(r =) -43.5\%$.
Award ***M1A1A0*** for a final answer of 56.5%.

[3 marks]
Total [6 marks]

5. (a) $X \sim \mathrm{N}(4, 0.25^2)$

EITHER

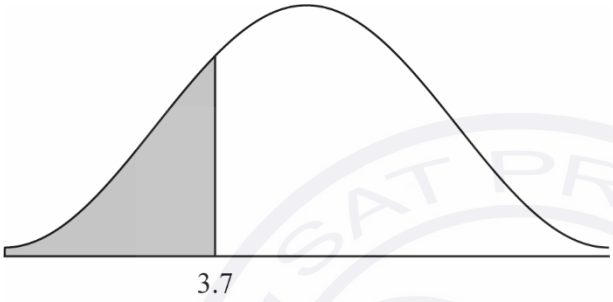
correct probability expression ***(M1)***

$\mathrm{P}(X < 3.7)$

Note: Accept a weak or strict inequality, and any label instead of X, e.g. length or L.

OR

normal curve with vertical line, left of mean, labelled 3.7, and shaded region ***(M1)***



THEN

0.115 $(0.115069\ldots, 11.5\%)$ ***A1***

Note: Award ***M1A0*** for 0.12 if no previous working.

[2 marks]

(b) **EITHER**

Correct probability expression ***(M1)***

$(\mathrm{P}(X < k) = 0.7$ **OR** $\mathrm{P}(X > k) = 0.3$

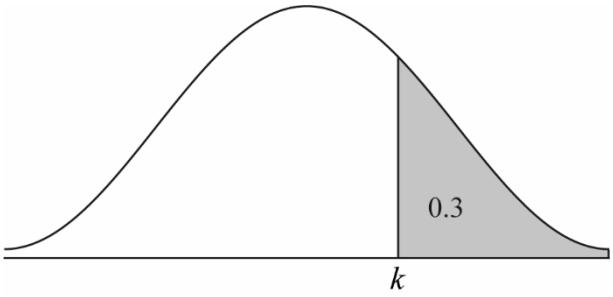
Note: Accept a weak or strict inequality, and any label instead of X *e.g.*, length or L.

continued…

Question 5 continued

OR

normal curve with vertical line to the right of the mean and shaded region, correctly labelled either 0.3 or 0.7 ***(M1)***



THEN

$(k =)\ 4.13\ (4.13110\ldots)$ ***A1***

Note: Award ***M1A0*** for 4.1 if no previous working.

[2 marks]

(c) **EITHER**

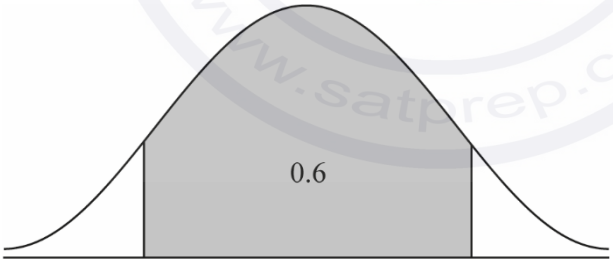
correct probability equation ***(M1)***

$\mathrm{P}(\text{length} < 4 + m) = 0.8$ **OR** $\mathrm{P}(\text{length} < 4 - m) = 0.2$

Note: Accept any letter instead of "length" *e.g.*, X or L.

OR

normal curve with vertical lines symmetrical about the mean line with a correct indication of an area of 0.6 or 0.2 or 0.8 ***(M1)***



THEN

$0.210\ (0.210405\ldots)$ ***A1***

Note: Award ***(M1)A0*** for an answer of 3.7895 or 4.2105 seen without working. Condone 0.21 seen and award ***(M1)A1***.

[2 marks]

Total [6 marks]

6. (a) **EITHER**

$\frac{4}{3}\pi(3.4)^3$ ***(A1)***

Multiplying their volume by $\frac{4}{5}$ ***(M1)***

OR

$\frac{4}{3}\pi(3.4)^3$ ***(A1)***

Subtracting $\frac{1}{5}$ of their volume ***(M1)***

$\left(\frac{4}{3}\pi(3.4)^3 - \frac{1}{5}\times\frac{4}{3}\pi(3.4)^3\right)$

Note: The ***M1*** can be awarded for a final answer of $32.9272\ldots$ seen without working.

THEN

$132\ \text{cm}^3\ \ (131.708\ldots\ \text{cm}^3)$ ***A1***

[3 marks]

(b) $\pi\times 3\times 11$ ***(A1)***

$103.672\ldots\ (\text{cm}^2)$ **OR** $33\pi\ (\text{cm}^2)$

$104\ (\text{cm}^2)$ ***A1***

[2 marks]

Total [5 marks]

7. (a) $(56\times 0.86) = 48.2\ \ (48.16)$ ***A1***

Note: Accept 48.

[1 mark]

(b) recognizing binomial distribution (may be seen in (a)) ***(M1)***

e.g. $X \sim \text{B}(56,\ 0.86)$

$\left(\text{P}(X \geq 50) =\right)\ 0.316$ ***A2***

[3 marks]

(c) $\text{P}(X \leq n) \geq 0.25$

$n = 46$ ***A2***

[2 marks]

Total [6 marks]

8. (a) attempt to create a 5x5 adjacency matrix ***(M1)***

$$\boldsymbol{M} = \begin{pmatrix} 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$ ***A2***

Note: Allow the transposed matrix. Award ***A2*** for all entries correct, ***A1*** if one or two entries are incorrect, ***A0*** otherwise.
Answer presented in markscheme assumes ABCDE ordering of rows and columns; accept other orders provided they are clearly communicated.
Award ***A1*** if the zeroes are replaced by blank cells.

[3 marks]

(b) (i) recognizing need to find $\boldsymbol{M}^7$ ***(M1)***

$$\boldsymbol{M}^7 = \begin{pmatrix} 8 & 8 & 17 & 8 & 13 \\ 8 & 10 & 19 & 17 & 14 \\ 6 & 11 & 16 & 10 & 17 \\ 11 & 8 & 19 & 14 & 10 \\ 2 & 6 & 8 & 11 & 8 \end{pmatrix}$$

2 (routes) ***A1***

(ii) vertices visited in order are
EITHER
$E \to D \to C \to B \to C \to B \to D \to A$ ***A2***

OR
$E \to D \to C \to B \to C \to E \to D \to A$ ***A2***

[4 marks]
Total [7 marks]

9. (a)

Athlete	A	B	C	D	E	F	G	H
Age rank	7	6	3	5	4	2	8	1
Time rank	3.5	2	3.5	6	7	8	1	5

A1A1

Note: Award ***A1*** for each correct row.

[2 marks]

(b) $r_s = -0.671\ (-0.670670...)$ ***A2***

Note: Only follow through from an incorrect table provided the ranks are all between 1 and 8.
Award ***A1*** for -0.67 **OR** for the omission of the negative sign, e.g. 0.671 $(0.670670...)$ or 0.67

[2 marks]

(c) (A value of $r_S = -0.671$) indicates a negative correlation between a person's age and the best time they take to run 100m. ***R1***

Note: Condone any comment that includes "weak" or "strong" etc. Accept an interpretation in words, but only if there is a general link described and not a rule: "The older a person gets, the faster they *tend to* run". Answer must be in context.

[1 mark]

(d) *Award **R1** for any sensible reason:* ***R1***
The correlation, such that it is, is unlikely to be linear for this type of data.
Spearman's CC is less sensitive to outliers
Sung-Jin is not sure the data is drawn from a bivariate normal distribution
There are outliers/extreme data
Same time for two athletes with significantly different ages

[1 mark]

(e) (i) $0.264\ (0.263762...)$ ***A2***

Note: Award ***A1*** for 0.26 with no working. Given that the exact model is not specific in the question, accept correct r^2 values from other regression models: 0.631, 0.650, 0.759 and 0.256.

(ii) approximately 26% of the variability in the times taken can be explained by the runner's age. ***R1***

[3 marks]
Total [9 marks]

10. (a) H_0 : there is no particular preference for any of the flavours ***A1***

H_1 : there is a particular preference for some of the flavours ***A1***

Note: Accept equivalent statements such as "H_0 : the population ratio of flavour preferences is $1:1:1:1$" or "H_0 : the population proportions are equal for each flavour" or "H_0 : the data is drawn from a uniform distribution".

[2 marks]

(b) **EITHER**

$p\text{-value} = 0.0629 \ \ (0.0629034...)$ ***A2***

$0.0629 > 0.05$ ***R1***

OR

$\chi^2_{\text{calc}} = 7.30$ ***A2***

$7.30 < 7.82$ ***R1***

Note: Award ***A2*** for either $p\text{-value} = 0.063$ or $\chi^2_{\text{calc}} = 7.3$ seen. Award ***R1*** for a correct comparison involve their $p\text{-value}$ or χ^2_{calc}, and follow through for their conclusion.

THEN

so there is insufficient evidence to reject H_0, i.e.

there is no particular preference for any of the flavours. ***A1***

Note: Do not award ***R0A1***.

[4 marks]

Total [6 marks]

11. (a) $30\sin(t+60^\circ)+60\sin(t+10^\circ)$ ***(M1)***

finding maximum graphically

$82.5 \text{ (V)} \ \ (82.5471...)$ ***A1***

Note: Award ***M1A0*** for 83.

[2 marks]

(b) recognizing that a is still 1 ***A1***

$V_0 = 82.5$ ***A1***

attempt to find an x-intercept of combined voltage ***(M1)***

$b = 26.2^\circ \ \ (26.1643...^\circ)$ **OR** any other correct x-intercept ***A1***

Note: May be seen in the final answer. Award ***M1A0*** for $b = 26$ with no working.

$\left(V_{\text{TOT}} = 82.5\sin(t+26.2^\circ) \ \ \left(82.5471...\sin(t+26.1643...^\circ)\right)\right)$

Note: Award at most ***(M1)A1(A1)A0*** if phase shift of $-153.835\ldots$ is seen in the final answer. In part (b), candidates may use $\arg\left(30e^{60i}+60e^{10i}\right)$ to determine the new phase shift, and hence could be awarded ***M1*** for this valid method.

[4 marks]

Total [6 marks]

12. (a) equating volume of sphere formula to 288π ***(M1)***

$$\frac{4}{3}\pi r^3 = 288\pi$$

$$\Rightarrow r = 6 \text{ (cm)}$$ ***A1***

[2 marks]

(b) $\frac{dV}{dr} = 4\pi r^2$ (seen anywhere) ***(A1)***

$$\frac{dV}{dt} = \frac{dV}{dr}\frac{dr}{dt}$$ ***(M1)***

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$15 = 4\pi \times 6^2 \times \frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{15}{144\pi} (\text{cm s}^{-1}) \ (0.0332, 0.0331572...)$$ ***A1***

[3 marks]

Total [5 marks]

13. (a) **METHOD 1 (find product of matrices first)**

$T \to T'$ is represented by $\boldsymbol{QP} = \begin{pmatrix} -4 & 1 \\ 1 & 3 \end{pmatrix}\begin{pmatrix} 3 & 1 \\ 0 & 2 \end{pmatrix}$ ***(M1)***

$= \begin{pmatrix} -12 & -2 \\ 3 & 7 \end{pmatrix}$ ***(A1)***

recognizing need to find their $(\boldsymbol{QP})^{-1}$ ***(M1)***

$(\boldsymbol{QP})^{-1} = \begin{pmatrix} -12 & -2 \\ 3 & 7 \end{pmatrix}^{-1}$

$= -\frac{1}{78}\begin{pmatrix} 7 & 2 \\ -3 & -12 \end{pmatrix}$ **OR** $= \begin{pmatrix} -0.0897435... & -0.0256410... \\ 0.0384615... & 0.153846... \end{pmatrix}$ ***A1***

METHOD 2 (find inverses of both matrices first)

recognizing need to find inverse of both $\boldsymbol{P}$ and $\boldsymbol{Q}$ ***(M1)***

$\boldsymbol{P}^{-1} = \begin{pmatrix} \frac{1}{3} & -\frac{1}{6} \\ 0 & \frac{1}{2} \end{pmatrix}$ **AND** $\boldsymbol{Q}^{-1} = \begin{pmatrix} -\frac{3}{13} & \frac{1}{13} \\ \frac{1}{13} & \frac{4}{13} \end{pmatrix}$ ***(A1)***

$T' \to T$ is represented by $\boldsymbol{P}^{-1}\boldsymbol{Q}^{-1} = \begin{pmatrix} 3 & 1 \\ 0 & 2 \end{pmatrix}^{-1}\begin{pmatrix} -4 & 1 \\ 1 & 3 \end{pmatrix}^{-1}$ ***(M1)***

$= -\frac{1}{78}\begin{pmatrix} 7 & 2 \\ -3 & -12 \end{pmatrix}$ **OR** $= \begin{pmatrix} -0.0897435... & -0.0256410... \\ 0.0384615... & 0.153846... \end{pmatrix}$ ***A1***

Note: In METHOD 1, award ***M1A0M1A0*** if they multiply the matrices in the wrong order.
In METHOD 2, award ***M1A1M1A0*** if they multiply the matrices in the wrong order.

[4 marks]

(b) $\left(\det\left[-\frac{1}{78}\begin{pmatrix} 7 & 2 \\ -3 & -12 \end{pmatrix}\right] =\right) -\frac{1}{78}$ **OR** $\left(\det\begin{pmatrix} -12 & -2 \\ 3 & 7 \end{pmatrix} =\right) -78$ ***(A1)***

area of $T' = |\det \boldsymbol{QP}| \times$ area of T **OR** area of $T = \left|\det(\boldsymbol{QP})^{-1}\right| \times$ area of T' ***(M1)***

$\Rightarrow$ area of $T = 273 \times \frac{1}{78}$

$= 3.5$ (cm^2) ***A1***

Note: Award ***(A1)(M0)A0*** for an answer of -3.5 (cm^2) with or without working. Accept an answer of 4.04 (cm^2) from use of 3sf values in their answer to part (a).

[3 marks]
Total [7 marks]

14. (a) $\boldsymbol{v}_B = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$ ***(A1)***

attempt to find any relevant angle ***(M1)***

$\tan^{-1}\left(\frac{3}{2}\right)$ $\left(= 56.3099...^\circ\right)$

$(90^\circ + 56.3099...^\circ =)$ 146° $(146.3099...^\circ)$ ***A1***

[3 marks]

(b) setting $1 + 2t = -2 + 4t$ ***(M1)***

$t = 1.5$ (hrs.) ***A1***

[2 marks]

(c) $\boldsymbol{r}_B - \boldsymbol{r}_A = (-3 + 2t)\boldsymbol{i} + (-7 + 4t)\boldsymbol{j}$ ***(M1)***

$-3 + 2t = -(-7 + 4t)$ ***(M1)***

$t = 1.67$ $\left(\text{hrs.}\right)$ $\left(1.66666..., \frac{5}{3}\right)$ ***A1***

[3 marks]

Total [8 marks]

15. (a) (i) 224 g (224.25 g) ***A1***

(ii) $[222.1, 226.4]$ ***A1A1***

Note: Award ***A1*** for each correct end of the interval. Accept open or closed (weak or strict) interval notation. Inequalities involving μ would also be accepted, but not involving $\bar{x}$.
Award ***A1A0*** for correct answers not given correct to 4 sf.

[3 marks]

(b) **EITHER**
the (population) weight of granules of Apollo coffee is normally distributed. ***R1***
OR
the readings are independent ***R1***

[1 mark]

(c) 226g lies within the confidence interval, ***R1***
so there is no evidence to dispute the claim on the label. ***A1***

Note: Do not award ***R0A1***.

[2 marks]

Total [6 marks]

16. (a) $(1.04,\ 0.509)$ $((1.03667...,0.509085...))$ ***A1A1***

[2 marks]

(b) attempt to make x the subject for either function ***(M1)***

$x = 4y^2$, $x = \cos^{-1} y$ ***A1A1***

attempt to use $V = \pi\int x^2 \mathrm{d}y$ ***(M1)***

$$V = \pi \int_0^{0.509085...} \left(4y^2\right)^2 \mathrm{d}y + \pi \int_{0.509085...}^{1} \left(\cos^{-1} y\right)^2 \mathrm{d}y$$ ***A1***

[5 marks]

(c) $= 1.15$ (units^3) ***A2***

Note: Do not ***FT*** from part (b) to part (c).
Award ***A1*** for 1.1. with no previous working.

[2 marks]
Total [9 marks]

17. (a) $\frac{\mathrm{d}y}{\mathrm{d}x}$ is undetermined at $(0, 1)$ ***R1***

$\left(\text{so cannot use } y_n = y_{n-1} + h\left(\frac{x}{(x^2+1)(2y-2)}\right)\right)$

Note: Accept "undefined", "indeterminate" or "division by zero" in place of "undetermined".

[1 mark]

(b) $\int (2y-2)\,\mathrm{d}y = \int \frac{x}{x^2+1}\mathrm{d}x$ ***M1***

$y^2 - 2y = \frac{1}{2}\ln(x^2+1) + c$ ***A1***

substituting $x = 0,\ y = 1$ ***M1***

$c = -1$

$y^2 - 2y + 1 = \frac{1}{2}\ln(x^2+1)$

$(y-1)^2 = \frac{1}{2}\ln(x^2+1)$ ***A1***

$y - 1 = \sqrt{\frac{1}{2}\ln(x^2+1)}$ (where positive root required as $y \geq 1$)

$y = 1 + \sqrt{\frac{\ln(x^2+1)}{2}}$ ***AG***

[4 marks]

(c) (when $x = 0.1$) $y = 1.07$ $(1.07053...)$ ***A1***

[1 mark]

Total [6 marks]

Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

May 2023

Mathematics: applications and interpretation

Higher level

Paper 1

32 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

R Marks awarded for clear **Reasoning**.

AG Answer given in the question and so no marks are awarded.

FT Follow through. The practice of awarding marks, despite candidate errors in previous parts, for their correct methods/answers using incorrect results.

Using the markscheme

1 General

Award marks using the annotations as noted in the markscheme eg ***M1***, ***A2***.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***A3***, ***M2*** *etc*., do **not** split the marks, unless there is a note.
- The response to a "show that" question does not need to restate the ***AG*** line, unless a **Note** makes this explicit in the markscheme.
- Once a correct answer to a question or part question is seen, ignore further working even if this working is incorrect and/or suggests a misunderstanding of the question. This will encourage a uniform approach to marking, with less examiner discretion. Although some candidates may be advantaged for that specific question item, it is likely that these candidates will lose marks elsewhere too.
- An exception to the previous rule is when an incorrect answer from further working is used **in a subsequent part**. For example, when a correct exact value is followed by an incorrect decimal approximation in the first part and this approximation is then used in the second part. In this situation, award ***FT*** marks as appropriate but do not award the final ***A1*** in the first part. Examples:

	Correct answer seen	Further working seen	Any FT issues?	Action
1.	$8\sqrt{2}$	5.65685... *(incorrect decimal value)*	No. Last part in question.	Award ***A1*** for the final mark *(condone the incorrect further working)*
2.	$\frac{35}{72}$	0.468111… (incorrect decimal value)	Yes. Value is used in subsequent parts.	Award ***A0*** for the final mark *(and full **FT** is available in subsequent parts)*

3 Implied marks

Implied marks appear in **brackets e.g. *(M1)***,and can only be awarded if **correct** work is seen or implied by subsequent working/answer.

4 Follow through marks (only applied after an error is made)

Follow through (***FT***) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) (e.g. incorrect value from part (a) used in part (d) or incorrect value from part (c)(i) used in part (c)(ii)). Usually, to award ***FT*** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if all the marks awarded in a subsequent part are for the answer or are implied, then ***FT*** marks should be awarded for *their* correct answer, even when working is not present.

For example: following an incorrect answer to part (a) that is used in subsequent parts, where the markscheme for the subsequent part is ***(M1)A1***, it is possible to award full marks for *their* correct answer, **without working being seen.** For longer questions where all but the answer marks are implied this rule applies but may be overwritten by a **Note** in the Markscheme.

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks, by reflecting on what each mark is for and how that maps to the simplified version.
- If the error leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word “their” in a description, to indicate that candidates may be using an incorrect value.
- If the candidate’s answer to the initial question clearly contradicts information given in the question, it is not appropriate to award any ***FT*** marks in the subsequent parts. This includes when candidates fail to complete a “show that” question correctly, and then in subsequent parts use their incorrect answer rather than the given value.
- Exceptions to these ***FT*** rules will be explicitly noted on the markscheme.
- If a candidate makes an error in one part but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the command term was “Hence”.

5 Mis-read

If a candidate incorrectly copies values or information from the question, this is a mis-read (***MR***). A candidate should be penalized only once for a particular misread. Use the ***MR*** stamp to indicate that this has been a misread and do not award the first mark, even if this is an ***M*** mark, but award all others as appropriate.

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.
- If a candidate uses a correct answer, to a "show that" question, to a higher degree of accuracy than given in the question, this is NOT a misread and full marks may be scored in the subsequent part.
- ***MR*** can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If the command term is 'Hence' and not 'Hence or otherwise' then alternative methods are not permitted unless covered by a note in the mark scheme.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**.

7 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation** for example 1.9 and $1{,}9$ or 1000 and $1{,}000$ and 1.000.
- Do not accept final answers written using calculator notation. However, ***M*** marks and intermediate ***A*** marks can be scored, when presented using calculator notation, provided the evidence clearly reflects the demand of the mark.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, some **equivalent** answers will generally appear in brackets. Not all equivalent notations/answers/methods will be presented in the markscheme and examiners are asked to apply appropriate discretion to judge if the candidate work is equivalent.

8 Format and accuracy of answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. If the level of accuracy is not stated in the question, the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures*.

Where values are used in subsequent parts, the markscheme will generally use the exact value, however candidates may also use the correct answer to a "correct" level of accuracy (e.g 3 sf) in subsequent parts. The markscheme will often explicitly include the subsequent values that come "*from the use of 3 sf values*".

Simplification of final answers: Candidates are advised to give final answers using good mathematical form. In general, for an ***A*** mark to be awarded, arithmetic should be completed, and any values that lead to integers should be simplified; for example, $\sqrt{\frac{25}{4}}$ should be written as $\frac{5}{2}$. An exception to this is simplifying fractions, where lowest form is not required (although the numerator and the denominator must be integers); for example, $\frac{10}{4}$ may be left in this form or written as $\frac{5}{2}$. However, $\frac{10}{5}$ should be written as 2, as it simplifies to an integer.

Algebraic expressions should be simplified by completing any operations such as addition and multiplication, e.g. $4e^{2x} \times e^{3x}$ should be simplified to $4e^{5x}$, and $4e^{2x} \times e^{3x} - e^{4x} \times e^{x}$ should be simplified to $3e^{5x}$. Unless specified in the question, expressions do not need to be factorized, nor do factorized expressions need to be expanded, so $x(x+1)$ and $x^2 + x$ are both acceptable.

Please note: intermediate ***A*** marks do NOT need to be simplified.

9 Calculators

A GDC is required for this paper, but If you see work that suggests a candidate has used any calculator not approved for IB DP examinations (eg CAS enabled devices), please follow the procedures for malpractice.

10. Presentation of candidate work

Crossed out work: If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work unless an explicit note from the candidate indicates that they would like the work to be marked.

More than one solution: Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise. If the layout of the responses makes it difficult to judge, examiners should apply appropriate discretion to judge which is "first".

1. (a) $N = 24$

$I = 4$

$PV = \pm 1000$

$PMT = \pm 100$

$P/Y = 12$

$C/Y = 12$ ***(M1)(A1)***

Note: Award ***M1*** for an attempt to use a financial app in their technology (i.e. at least three entries seen, but not necessarily correct).
Approaches that use the compound interest formula receive no marks.
Award ***A1*** for correct values of PV and PMT (signs must be the same) **and** a correct value of N.

$FV = (\$)3577.43$ ***A1***

Note: Award at most ***(M1)(A1)A0*** if the final answer is negative or not rounded to 2 dp.

[3 marks]

(b) $N = 36.5 \ (36.4689\ldots)$ ***(A1)***

$N = 37$ (months) ***A1***

Note: Allow ***FT*** from incorrect GDC inputs seen in part (a) for the first ***A1*** providing that PV and FV have opposite signs and the resulting value of N is positive.

[2 marks]

[Total: 5 marks]

2. (a) $H_0 : \mu_b = \mu_m$ ***A1***

$H_1 : \mu_b > \mu_m$ ***A1***

Note: Accept equivalent statements in words such as "the **mean** score of bilingual people equals the **mean** score of monolingual people".

[2 marks]

(b) $0.119 \ (0.119395\ldots)$ ***A2***

[2 marks]

(c) $0.119395\ldots > 0.05 \ \ (11.9395\ldots\% > 5\%)$ ***R1***

(fail to reject H_0) there is insufficient evidence to suggest that bilingual people have better memory retention than monolingual people ***A1***

Note: Do not award ***R0A1***.
The answer to part (c) MUST be consistent with **their** hypotheses and **their** p-value.

[2 marks]

[Total: 6 marks]

3. (a) attempt to use distance formula for points D and A ***(M1)***

$\mathrm{DA} = \sqrt{11^2 + 7^2}$

$= 13.0$ (miles) $(13.0384\ldots, \sqrt{170})$ ***A1***

Note: Accept 13 miles. Award ***M0A0*** for finding the equation of the line DA.
DA may be seen in part (b) but this should not be accepted as answer for part (a).

[2 marks]

(b) $\left(\mathrm{DB} = \sqrt{13^2 + 5^2} =\right)\ 13.9\ \left(13.9283..., \sqrt{194}\right)$ **AND**

$\left(\mathrm{DC} = \sqrt{4^2 + 12^2} =\right)\ 12.6\ \left(12.6491..., \sqrt{160}\right)$ ***A1***

recognizing closest town is best estimate ***(M1)***

(town C is closest)

$30\,^{\circ}\mathrm{C}$ ***A1***

Note: If their DA from part (a) is the shortest length, then allow ***FT*** in (b).

[3 marks]

[Total: 5 marks]

4. (a) attempt to substitute $h = 10$ and at least two different values of y into the trapezoidal rule ***(M1)***

$$\frac{10}{2}\big((0+0)+2(3+8+9)\big)$$

$$= 200\ (\text{cm}^2)$$ ***A1***

[2 marks]

(b) (i) $\int_0^{40} 0.04x^2 - 0.001x^3 \,\text{d}x$ **OR** $\int_0^{40} y \,\text{d}x$ ***A1A1***

Note: Award ***A1*** for a correct integral (including $\text{d}x$), ***A1*** for correct limits in the correct location.

(ii) $213.33\ (\text{cm}^2)$ ***A2***

Note: Answer must be given to 2 decimal places to award ***A2***. Award ***A1A0*** for a correct answer given to an incorrect accuracy of at least 3 significant figures, e.g. $213\ (\text{cm}^2)$.

[4 marks]

(c) attempt to substitute their parts (a) and (b)(ii) into percentage error formula ***(M1)***

$$\left|\frac{213.333... - 200}{213.333...}\right| \times 100$$

$$= 6.25(\%)\ \ (6.24999...(\%))$$ ***A1***

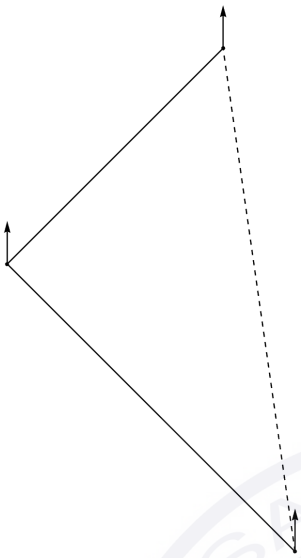
Note: Award ***(M1)A0*** for a final answer of $-6.25(\%)$ or 0.0625.

[2 marks]

[Total: 8 marks]

5. METHOD 1

diagram showing (approximately) correct directions (and order) for the 315° and 045° ***(A1)***



recognizing right angle triangle ***(M1)***

correct expression to find second angle in triangle ***(A1)***

e.g. $\arctan\left(\frac{6}{8}\right)$ **OR** $\arctan\left(\frac{8}{6}\right)$

correct expression to find bearing ***(A1)***

e.g. $\arctan\left(\frac{6}{8}\right)+135^\circ$ **OR** $360^\circ-\left(\arctan\left(\frac{8}{6}\right)+135^\circ\right)$

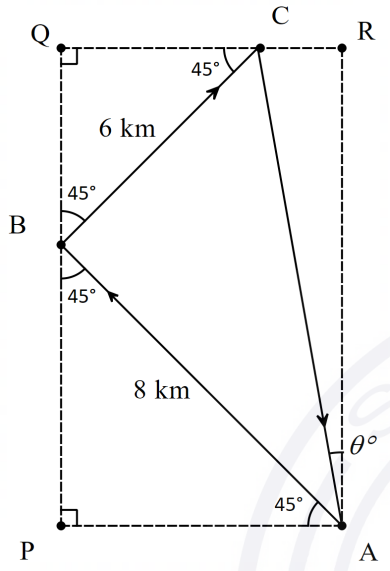
$=172^\circ \ (171.869...^\circ)$ ***A1***

continued…

Question 5 continued

METHOD 2

diagram showing (approximately) correct directions (and order) for the $315°$ and $045°$ (these may be shown in reverse as the return journey) ***(A1)***



finding the lengths marked AP, BP, CQ and BQ in the diagram ***(M1)***

$$\text{AP} = \text{BP} = 8\frac{\sqrt{2}}{2} = 5.6568...$$

$$\text{CQ} = \text{BQ} = 6\frac{\sqrt{2}}{2} = 4.2426...$$

Note: This may be done using a vector approach.

using $\tan\theta° = \dfrac{\text{AP} - \text{CQ}}{\text{PB} + \text{BQ}}$ or equivalent to find the direction of AC ***(A1)***

correct expression to find bearing ***(A1)***

$$180° - \arctan\left(\frac{8\frac{\sqrt{2}}{2} + 6\frac{\sqrt{2}}{2}}{8\frac{\sqrt{2}}{2} - 6\frac{\sqrt{2}}{2}}\right)$$

$$= 172° \quad (171.869...°)$$ ***A1***

[Total: 5 marks]

6. (a) (i) **METHOD 1**

attempt to find change in height of the ball using gradient ***(M1)***

$\frac{a}{0.43} = (-)\,0.045$

$a = (-)\,0.045 \times 0.43$

$a = (-)\,0.0194\,(\text{m})\ \ (0.01935\ (\text{m}))$ ***A1***

METHOD 2

attempt to find height at back of home plate ***(M1)***

horizontal distance to the front of the home plate $= 16.6666\ldots\ (\text{m})$

height at the back of the home plate $= -0.045(16.6666\ldots + 0.43) + 2$

$(= 1.23065\ (\text{m}))$

Note: The ***M1*** can be awarded for $16.6666\ldots + 0.43$ seen at some point.

$(a = 1.25 - 1.23065\ldots)$

$(a =)\ (-)0.0194\ (\text{m})\ \ (0.01935\ (\text{m}))$ ***A1***

(ii) $1.25 - 0.01935 = 1.23065$ (may be seen in part (a)(i)) ***A1***

$0.53 < 1.23065 < 1.24$ ***R1***

therefore a strike ***AG***

Note: Do not award ***A0R1***.

[4 marks]

continued…

Question 6 continued

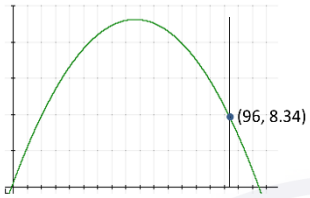
(b) **METHOD 1**

indication of $d = 96$ in the function $h(d)$ or its graph ***(M1)***

EITHER

$(h(96) =) -0.01(96)^2 + 1.04(96) + 0.66$

OR



THEN

$(h(96) =)\ 8.34\ (\text{m})$ ***A1***

$8.34 > 5$ so the ball will go over the wall. ***A1***

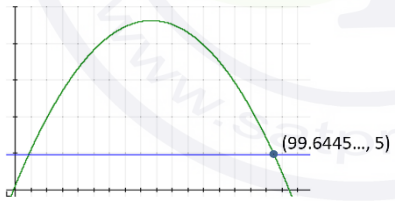
METHOD 2

indication of $h = 5$ in the function $h(d)$ or its graph ***(M1)***

EITHER

$5 = -0.01d^2 + 1.04d + 0.66$

OR



THEN

$d = 99.6\ (\text{m})\ (99.6445\ldots\ (\text{m}))\ (d = 4.35548\ldots\ (\text{m})$ may also be seen$)$ ***A1***

$96 < 99.6445\ldots$ so the ball will go over the wall. ***A1***

[3 marks]

[Total: 7 marks]

7. (a) attempt to find the vector product (e.g. one term correct) ***(M1)***

$$\begin{pmatrix}0\\6\\1\end{pmatrix}\times\begin{pmatrix}7\\3\\0\end{pmatrix}=\begin{pmatrix}-3\\7\\-42\end{pmatrix}$$ ***A1***

[2 marks]

(b) **METHOD 1**

attempt to use the vector product formula for the area of triangle

(condone incorrect signs and missing $\frac{1}{2}$) ***(M1)***

$\text{area} = \frac{1}{2}\sqrt{3^2+7^2+42^2}$

$= 21.3\ (\text{m}^2)\ \ (21.3424..., \frac{1}{2}\sqrt{1822})$ ***A1***

METHOD 2

find θ using $\vec{\text{AB}}\times\vec{\text{AC}} = \left|\vec{\text{AB}}\right|\left|\vec{\text{AC}}\right|\sin\theta$ ***(M1)***

$\theta = 67.1^\circ\ \ (67.1350^\circ\ldots, 1.171728\ldots.\ \text{radians})$

then $\text{area} = \frac{1}{2}\left|\vec{\text{AB}}\right|\left|\vec{\text{AC}}\right|\sin\theta$

$= 21.3\ (\text{m}^2)\ \left(21.3424\ldots, \frac{1}{2}\sqrt{1822}\right)$ ***A1***

[2 marks]

continued…

Question 7 continued

(c) $AC = 7.61577... \left(\sqrt{58}\right)$ ***(A1)***

setting the area formula $\frac{1}{2} \times \text{base} \times \text{height}$ equal to their part (b) ***(M1)***

$$BX = \frac{2 \times 21.3424...}{\sqrt{58}}$$

$= 5.60 \ (5.60480...)$ ***A1***

Note: Award ***A1*** for 5.6.

Award ***A1*** for 5.59 $(5.5936\ldots)$ from the use of 21.3 to 3 sf.

[3 marks]

(d) attempting to set up a trig ratio ***(M1)***

angle is $\arcsin\left(\frac{1}{BX}\right)$

10.3° $(10.2776...^\circ, 0.179378\ldots$ radians) ***A1***

[2 marks]

[Total: 9 marks]

8. (a) H_0: X and Y are not (linearly) correlated **OR** $\rho = 0$ ***A1***

H_1: X and Y are (linearly) correlated **OR** $\rho \neq 0$ ***A1***

Note: Accept "independent" or "not associated" in place of "not correlated".
If H_0 and H_1 are reversed, then award ***A0A1***.

[2 marks]

(b) (i) $r = 0.849 \ (0.848886...)$ ***A1***

(ii) p-value $= 0.0325 \ (0.0325277...)$ ***A2***

Note: Award ***A1*** for p-value $= 0.033$ or p-value $= 0.03$.
Award ***FT*** for $\rho > 0$ or $\rho < 0$ in part (a), p-value $= 0.0163 \ (0.0162638\ldots)$
or p-value $= 0.984 \ (0.983736\ldots)$
Award the full marks for seeing the values of r and p-value from the markscheme when H_0 and H_1 are reversed in part (a).

[3 marks]

(c) $0.0325 < 0.05$ ***R1***

(so we reject H_0 in favour of H_1)

(there is sufficient evidence to suggest) X and Y are (linearly) correlated ***A1***

Note: Their conclusion must be consistent with their p-value and their hypotheses and it must be in context.

[2 marks]

[Total: 7 marks]

9. (a) attempt to find the difference between 75.7 and 67.3 ***(M1)***

$\frac{75.7 - 67.3}{2}$

$4.2\ (\text{km h}^{-1})$ ***A1***

[2 marks]

(b) **METHOD 1 (Comparing areas above and below the mean)**

$\text{P}(67.3 < \text{speed} < 74)$ **OR** Normal CDF$(67.3, 74, 67.3, 4.2)$ **OR** sketch of normal distribution with 67.3 and 74 labelled and shaded between ***(M1)***

area of region between mean and q is at least $0.445\ \ (0.444670\ldots)$ ***A1***

Hence no more than $0.375\ \ (0.375329\ldots)$ between mean and p ***R1***

The region between p and q is not symmetrical ***AG***

METHOD 2 (Comparing areas in the tails)

attempt to calculate probability that speed $< p$ and speed $> q$ with $q = 74$ ***(M1)***

$\text{P}(\text{speed} < 74) = 0.944670\ldots$

$\text{P}(\text{speed} < p) = (0.944670\ldots - 0.82 =)\ 0.124670\ldots$

$\text{P}(\text{speed} > q) = (1 - 0.944670\ldots =)\ 0.0553295\ldots$ ***A1***

if $q \geq 74$, then $\text{P}(\text{speed} > q) \leq 0.0553295$ and $\text{P}(\text{speed} < p) \geq 0.124670$ so

$\text{P}(\text{speed} > q)$ will never equal $\text{P}(\text{speed} < p)$ ***R1***

the region between p and q is not symmetrical ***AG***

continued…

Question 9 continued

METHOD 3 (Assumption of symmetry comparing speeds)

attempt to calculate area below q assuming distribution is symmetrical ***(M1)***

e.g. $\mathrm{P}(\text{speed} < q) = 0.82 + ½ \times 0.18 \quad (0.91)$

EITHER

$(q =)\ 72.9 \quad (72.9311\ldots)$ ***A1***

$72.9 < 74$ so 74 would not be in the region ***R1***

the region between p and q is not symmetrical ***AG***

OR

$\mathrm{P}(\text{speed} < 74) = 0.945 \quad (0.944670\ldots)$ ***A1***

$0.945 > 0.91$ so 74 would not be in the region ***R1***

the region between p and q is not symmetrical ***AG***

METHOD 4 (Assumption of symmetry comparing areas)

attempt to calculate symmetrical area with 74 as a boundary ***(M1)***

$\mathrm{P}(60.6 < \text{speed} < 74)$ **OR** Normal CDF(60.6, 74, 67.3, 4.2) **OR**

$\mathrm{P}(67.3 < \text{speed} < 74)$ **OR** Normal CDF(67.3, 74, 67.3, 4.2)

EITHER

$0.889 \quad (0.889340\ldots)$ ***A1***

$0.889 > 0.82$ so 74 would not be in the region ***R1***

the region between p and q is not symmetrical ***AG***

OR

$0.445 \quad (0.444670\ldots)$ ***A1***

$0.445 > 0.82 \div 2$ so 74 would not be in the region ***R1***

the region between p and q is not symmetrical ***AG***

[3 marks]

[Total: 5 marks]

10. (a) $y = x$ ***A1***

[1 mark]

(b) **METHOD 1**

equation has the form $y = ax^2 + bx + c$

when $x = 0,\ y = 0$ so $c = 0$

$\frac{\mathrm{d}y}{\mathrm{d}x} = 2ax + b$

attempt to find the value of b by setting *their* derivative equal to 1 when x is 0 ***(M1)***

$2a(0) + b = 1$

$b = 1$ ***(A1)***

when $x = -2,\ y = 0$

$a = \frac{1}{2}$ (and hence $y = \frac{1}{2}x^2 + x$) ***A1***

METHOD 2

equation has the form $y = ax(x+2)$ **OR** $y = ax^2 + 2ax$ ***A1***

$\frac{\mathrm{d}y}{\mathrm{d}x} = 2ax + 2a$

attempt to find the value of a by setting *their* derivative equal to 1 when x is 0 ***(M1)***

$a = \frac{1}{2}$ (and hence $y = \frac{1}{2}x^2 + x$) ***A1***

Note: Writing $y = x(x+2)$ is incorrect and gains no marks.

[3 marks]

continued…

Question 10 continued

(c) equation is $y = ax^2 + bx + c$

finding an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ with unknown coefficients ***(M1)***

$\frac{\mathrm{d}y}{\mathrm{d}x} = 2ax + b$

setting up two equations using two points AND/OR one equation using the gradient function ***(M1)***

three correct equations ***(A1)***

$9a + 3b + c = 3$
$36a + 6b + c = 2$
$6a + b = 1$

$a = -\frac{4}{9},\ b = \frac{11}{3},\ c = -4$ $(a = -0.444444...,\ b = 3.66666...,\ c = -4)$ ***A1***

(and hence $y = -\frac{4}{9}x^2 + \frac{11}{3}x - 4$)

[4 marks]

(d) $f(x) = \begin{cases} \frac{1}{2}x^2 + x & , \quad -2 \le x < 0 \\ x & , \quad 0 \le x \le 3 \\ -\frac{4}{9}x^2 + \frac{11}{3}x - 4 & , \quad 3 < x \le 6 \end{cases}$ ***A1***

Note: Condone open or closed endpoints for all intervals.
Condone y in place of $f(x)$.
Allow ***FT*** from parts (a), (b) and (c).

[1 mark]

[Total: 9 marks]

11. Let $D = O - L - L$ ***(A1)***

$(\text{mean} =)\ 205 - 105 - 105\ (= -5)$ ***(A1)***

manipulating variances (not standard deviations) ***(M1)***

$(\text{variance} =)\ 25 + 9 + 9\ (= 43)$ **OR** $(\text{SD} =)\ 6.55743...$ ***(A1)***

$D \sim \text{N}(205 - 105 - 105,\ 25 + 9 + 9)$

attempt to find the probability that $D > 0$ ***(M1)***

$\text{P}(D > 0)$

$= 0.223\ (0.222882...)$ ***A1***

Note: If $D = O - 2L$ is seen or implied, award at most ***(A0)A1(M0)(A0)M1A0***.

[Total: 6 marks]

12. METHOD 1 Analytical approach

attempt to express V_1 or V_2 in exponential form ***(M1)***

e.g. $V_1 = \text{Im}(6\text{e}^{\text{i}\left(at+\frac{\pi}{6}\right)})$, $V_2 = \text{Im}(6\text{e}^{\text{i}\left(at+\frac{\pi}{2}\right)})$

Note: Accept angles in radians or degrees.

$(V_1 + V_2 =)\ 6\text{e}^{\text{i}\times\frac{\pi}{6}} + 6\text{e}^{\text{i}\times\frac{\pi}{2}}$ ***(A1)***

Note: This mark can be awarded even if seen as part of a correct larger expression.

$= 10.4\text{e}^{1.05\text{i}}\ \left(6\sqrt{3}\,\text{e}^{\frac{\text{i}\pi}{3}}\right)$ ***(A1)***

so V is 10.4 $(10.3923..., 6\sqrt{3})$ and θ is 60 (degrees) ***A1A1***

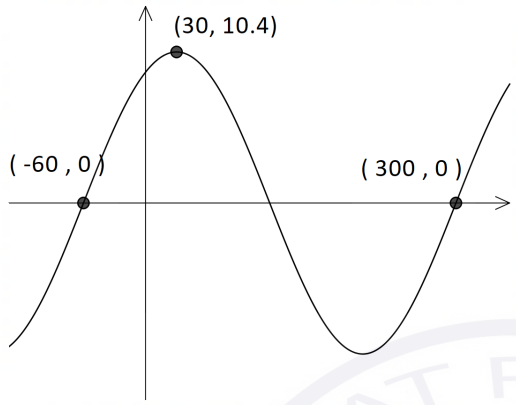
Note: Accept any value for θ that rounds to a 2sf answer of 60.
Do **not** accept a final answer for an angle in radians.
Do **not** award ***A1*** for answer of 60° resulting from incorrect working.

continued…

Question 12 continued

METHOD 2 Graphical approach

let $at = x$ and plot $V_1 + V_2$ curves on GDC ***(M1)***



attempt to find maximum ***(M1)***

$V = 10.4$ ***A1***

attempt to find any x-axis intercept (either -60 or 300) ***(M1)***

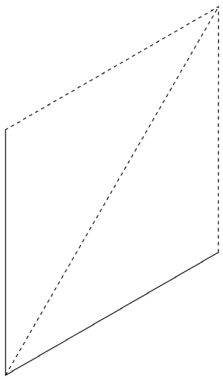
$\theta = 60$ (degrees) $\left(\theta = -300 \text{ (degrees)}\right)$ ***A1***

continued…

Question 12 continued

METHOD 3 Geometric approach

considering the rhombus



(M1)

$V = \sqrt{6^2 + 6^2 - 2 \times 6 \times 6 \cos 120^\circ}$ ***(M1)***

$\left(= \sqrt{108} = 6\sqrt{3}\right) = 10.4 \ \ (10.3923\ldots)$ ***A1***

$\theta = 60$ (degrees) ***A2***

Note: An answer of $\theta = -300$. is most likely to be seen in METHOD 2, but should be condoned in METHODS 1 and 3 if seen there.

[Total: 5 marks]

13. $\frac{\mathrm{d}x}{\mathrm{d}t} = y$ ***(A1)***

$\frac{\mathrm{d}y}{\mathrm{d}t} = -10x - 2y$ ***(A1)***

Note: Writing $\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -10x - 2\frac{\mathrm{d}x}{\mathrm{d}t}$ is a valid approach and should be awarded ***A1A1***.

attempt to use the Euler equations shown by finding either a correct x_{n+1} or y_{n+1} ***(M1)***

correct equations for both x_{n+1} and y_{n+1} ***(A1)***

$x_{n+1} = x_n + 0.1(y_n)$, $\quad y_{n+1} = y_n + 0.1(-10x_n - 2y_n)$ (accept equivalent notation)

$(t_{n+1} = t_n + 0.1)$

Note: All of the above marks can be implied by a correct second row in a table **OR** by a correct f_1 and f_2 clearly identified for use in Euler's method formula.

T	x	y
0	0.75	0
0.1	**0.75**	**-0.75**
0.2	0.675	-1.35
0.3	0.54	-1.755
0.4	0.3645	-1.944
0.5	0.1701	

so estimate is 0.170 ***A2***

Note: Accept 0.17 rounded to 2 sf.

[Total: 6 marks]

14. METHOD 1 Analytical approach

recognizing that the linear equation must be expressed in log form ***(M1)***

$\log y = m\log x + \log c$ (or $\log y = m\log x + C$)

EITHER

use of slope formula (must involve logs) ***(M1)***

$m = \dfrac{\log(34.822) - \log(13.1951)}{\log(4) - \log(2)} = 1.4$ ***A1***

attempt to substitute a value ***(M1)***

$\log c = \log(13.1951) - 1.4\log 2 \,(= 0.69897...)$

$\Rightarrow c = 5$ ***A1***

OR

$y = c.x^m$ ***(A1)***

attempt to set up two equations involving power functions ***(M1)***

$13.1951 = c \times 2^m$ and $34.822 = c \times 4^m$

$2^m = \dfrac{34.822}{13.1951} = 2.639... \Rightarrow m = \log_2 2.639... = 1.4$ ***A1***

$c = \dfrac{13.1951}{2.639...} = 5$ ***A1***

THEN

(so the equation is) $y = 5 \times x^{1.4}$ ***A1***

METHOD 2 Regression analysis

recognizing that a log-log graph results in a power function model ***(M1)***

$y = a \times x^b$

attempt to find a power regression model using the given two points ***(M1)***

$a = 5$ and $b = 1.4$ ***(A1)(A1)***

(so the equation is) $y = 5 \times x^{1.4}$ ***A2***

[Total: 6 marks]

15. METHOD 1 Using the volume formula

volume of a "full" or "half" cylinder (seen anywhere) ***(A1)***

$\pi\int_2^4 4^2\,\mathrm{d}y$, $\pi\times 4^2\times 2$, 32π $(100.53\ldots)$ **OR**

$\pi\int_2^3 4^2\,\mathrm{d}y$, $\pi\times 4^2\times 1$, 16π $(50.265\ldots)$

one correct equation for the diagonal lines (seen anywhere) ***(A1)***

$y=\frac{1}{2}x$ or $y=6-\frac{1}{2}x$

attempt to write one equation x in terms of y ***(M1)***

$x=2y$, $x=12-2y$

EITHER (symmetry plus the volume of the "half" cylinder)

recognition of symmetry between $y=1$ and $y=3$ ***(M1)***

$2\pi\left(\int_1^2 (2y)^2\,\mathrm{d}y+\int_2^3 4^2\,\mathrm{d}y\right)$ ***(A1)***

OR (symmetry plus volume of the "full" cylinder)

recognition of symmetry between $y=1$ and $y=2$ ***(M1)***

$2\pi\left(\int_1^2 (2y)^2\,\mathrm{d}y\right)+\int_2^4 4^2\,\mathrm{d}y$ ***(A1)***

OR (calculation of separate parts) ***(M1)***

$\pi\left(\int_1^2 (2y)^2\,\mathrm{d}y+\int_2^4 4^2\,\mathrm{d}y+\int_4^5 (-2y+12)^2\,\mathrm{d}y\right)$ ***(A1)***

THEN

(volume of the solid=) $159\ \left(159.174\ldots,\frac{152\pi}{3}\right)$ ***A1***

continued…

Question 15 continued

METHOD 2 Geometric approach using cones and cylinders

volume of a cylinder (seen anywhere) ***(A1)***

$\pi \times 4^2 \times 2,\ 32\pi\ (100.53\ldots)$ (a *full* cylinder) **OR**

$\pi \times 4^2 \times 1,\ 16\pi\ (50.265\ldots)$ (a *half* cylinder)

using volume of cone formula to find the volume of the truncated cone ***(M1)***

correct expression to find the volume of the truncated cone (seen anywhere) ***(A1)***

$\frac{1}{3}\left(\pi \times 4^2 \times 2 - \pi \times 2^2 \times 1\right)$

attempt to find an expression for total volume using symmetry or individual parts ***(M1)***

correct expression for total volume ***(A1)***

$2\left(\frac{1}{3}\left(\pi 4^2 \times 2 - \pi 2^2 \times 1\right) + \pi 4^2 \times 1\right)$ **OR** $\frac{1}{3}\left(\pi 4^2 \times 2 - \pi 2^2 \times 1\right) + \pi 4^2 \times 2 + \frac{1}{3}\left(\pi 4^2 \times 2 - \pi 2^2 \times 1\right)$

(volume of the solid=) $159\ \left(159.174..., \frac{152\pi}{3}\right)$ ***A1***

Note: There are other valid approaches possible.

[Total: 6 marks]

16. (a) attempt to find what the model predicts in terms of k ***(M1)***

$k, \frac{k}{4}, \frac{k}{25}$

correct expression for sum of square residuals ***(A1)***

$(k-42)^2+\left(\frac{k}{4}-11\right)^2+\left(\frac{k}{25}-1.5\right)^2$

valid attempt to find c by expanding or recognizing the constant terms ***(M1)***

$c = 42^2+11^2+1.5^2$

$=1887.25$ ***A1***

[4 marks]

(b) valid method to find the k value at the minimum ***(M1)***

$k=\frac{89.62}{2\times 1.0641}$ $(=42.1107...)$, graph, completing the square

(so least squares regression is) $I=\frac{42.1}{d^2}$ ***A1***

[2 marks]

[Total: 6 marks]

17. (a) attempt to use the chain rule to set up a related rate ***(M1)***

correct expression ***A1***

$\frac{\mathrm{d}x}{\mathrm{d}\theta} = \frac{\mathrm{d}x}{\mathrm{d}t} \div \frac{\mathrm{d}\theta}{\mathrm{d}t}$ **OR** $\frac{-250}{0.075}$

$= -\frac{10000}{3}$ ***AG***

[2 marks]

(b) $x(\theta) = \frac{3000}{\tan\theta}$ ***A1***

[1 mark]

(c) attempt to use chain rule **OR** quotient rule ***(M1)***

$\frac{-3000}{\tan^2\theta \times \cos^2\theta}$, $\frac{-3000\left(\sin\theta(-\sin\theta) - \cos^2\theta\right)}{\sin^2\theta}$ ***(A1)***

$= -\frac{3000}{\sin^2\theta}$ ***A1***

[3 marks]

(d) setting their equation in part (c) equal to the given expression in part (a) ***(M1)***

$-\frac{3000}{\sin^2\theta} = -\frac{10000}{3}$

$\theta = 1.24904...$ ***(A1)***

$x(1.24904...) = 1000\,\mathrm{m}$ ***A1***

[3 marks]

[Total: 9 marks]

Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

November 2022

Mathematics: applications and interpretation

Higher level

Paper 1

20 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**.
A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.
R Marks awarded for clear **Reasoning**.
AG Answer given in the question and so no marks are awarded.
FT Follow through. The practice of awarding marks, despite candidate errors in previous parts, for their correct methods/answers using incorrect results.

Using the markscheme

1 General

Award marks using the annotations as noted in the markscheme *eg* ***M1, A2.***

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***A3***, ***M2*** *etc.*, do **not** split the marks, unless there is a note.
- The response to a "show that" question does not need to restate the ***AG*** line, unless a **Note** makes this explicit in the markscheme.
- Once a correct answer to a question or part question is seen, ignore further working even if this working is incorrect and/or suggests a misunderstanding of the question. This will encourage a uniform approach to marking, with less examiner discretion. Although some candidates may be advantaged for that specific question item, it is likely that these candidates will lose marks elsewhere too.
- An exception to the previous rule is when an incorrect answer from further working is used **in a subsequent part**. For example, when a correct exact value is followed by an incorrect decimal approximation in the first part and this approximation is then used in the second part. In this situation, award ***FT*** marks as appropriate but do not award the final ***A1*** in the first part. Examples:

	Correct answer seen	**Further working seen**	**Any FT issues?**	**Action**
1.	$8\sqrt{2}$	5.65685... *(incorrect decimal value)*	No. Last part in question.	Award ***A1*** for the final mark *(condone the incorrect further working)*
2.	$\frac{35}{72}$	0.468111… (incorrect decimal value)	Yes. Value is used in subsequent parts.	Award ***A0*** for the final mark *(and full **FT** is available in subsequent parts)*

3 Implied marks

Implied marks appear in **brackets e.g.** ***(M1)***,and can only be awarded if **correct** work is seen or implied by subsequent working/answer.

4 Follow through marks (only applied after an error is made)

Follow through (***FT***) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) (e.g. incorrect value from part (a) used in part (d) or incorrect value from part (c)(i) used in part (c)(ii)). Usually, to award ***FT*** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if all the marks awarded in a subsequent part are for the answer or are implied, then ***FT*** marks should be awarded for *their* correct answer, even when working is not present.

For example: following an incorrect answer to part (a) that is used in subsequent parts, where the markscheme for the subsequent part is ***(M1)A1***, it is possible to award full marks for *their* correct answer, **without working being seen.** For longer questions where all but the answer marks are implied this rule applies but may be overwritten by a **Note** in the Markscheme.

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks, by reflecting on what each mark is for and how that maps to the simplified version.
- If the error leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word “their” in a description, to indicate that candidates may be using an incorrect value.
- If the candidate’s answer to the initial question clearly contradicts information given in the question, it is not appropriate to award any ***FT*** marks in the subsequent parts. This includes when candidates fail to complete a “show that” question correctly, and then in subsequent parts use their incorrect answer rather than the given value.
- Exceptions to these ***FT*** rules will be explicitly noted on the markscheme.
- If a candidate makes an error in one part but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the command term was “Hence”.

5 Mis-read

If a candidate incorrectly copies values or information from the question, this is a mis-read (***MR***). A candidate should be penalized only once for a particular misread. Use the ***MR*** stamp to indicate that this has been a misread and do not award the first mark, even if this is an ***M*** mark, but award all others as appropriate.

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.
- If a candidate uses a correct answer, to a "show that" question, to a higher degree of accuracy than given in the question, this is NOT a misread and full marks may be scored in the subsequent part.
- ***MR*** can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If the command term is 'Hence' and not 'Hence or otherwise' then alternative methods are not permitted unless covered by a note in the mark scheme.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**.

7 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation** for example 1.9 and $1{,}9$ or 1000 and $1{,}000$ and 1.000.
- Do not accept final answers written using calculator notation. However, ***M*** marks and intermediate ***A*** marks can be scored, when presented using calculator notation, provided the evidence clearly reflects the demand of the mark.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, some **equivalent** answers will generally appear in brackets. Not all equivalent notations/answers/methods will be presented in the markscheme and examiners are asked to apply appropriate discretion to judge if the candidate work is equivalent.

8 Format and accuracy of answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. If the level of accuracy is not stated in the question, the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures*.

Where values are used in subsequent parts, the markscheme will generally use the exact value, however candidates may also use the correct answer in subsequent parts. The markscheme will often explicitly include the subsequent values that come "*from the use of 3 sf values*".

Simplification of final answers: Candidates are advised to give final answers using good mathematical form. In general, for an ***A*** mark to be awarded, arithmetic should be completed, and any values that lead to integers should be simplified; for example, $\sqrt{\frac{25}{4}}$ should be written as $\frac{5}{2}$. An exception to this is simplifying fractions, where lowest form is not required (although the numerator and the denominator must be integers); for example, $\frac{10}{4}$ may be left in this form or written as $\frac{5}{2}$. However, $\frac{10}{5}$ should be written as 2, as it simplifies to an integer.

Algebraic expressions should be simplified by completing any operations such as addition and multiplication, e.g. $4e^{2x} \times e^{3x}$ should be simplified to $4e^{5x}$, and $4e^{2x} \times e^{3x} - e^{4x} \times e^{x}$ should be simplified to $3e^{5x}$. Unless specified in the question, expressions do not need to be factorized, nor do factorized expressions need to be expanded, so $x(x+1)$ and $x^2 + x$ are both acceptable.

Please note: intermediate ***A*** marks do NOT need to be simplified.

9 Calculators

A GDC is required for this paper, but If you see work that suggests a candidate has used any calculator not approved for IB DP examinations (eg CAS enabled devices), please follow the procedures for malpractice.

10. Presentation of candidate work

Crossed out work: If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work unless an explicit note from the candidate indicates that they would like the work to be marked.

More than one solution: Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise. If the layout of the responses makes it difficult to judge, examiners should apply appropriate discretion to judge which is "first".

1. (a) The favourite breakfast/berry (of adults) is independent of (their) income (level). ***A1***

[1 mark]

(b) $\chi^2 = 2.27 \ (2.26821...)$ ***A2***

[2 marks]

(c) **EITHER**

$2.27 < 7.78$ **OR** $2.27 <$ critical value ***R1***

OR

$0.687 > 0.1$ (using p-value)

THEN

(Do not reject H_0)

Insufficient evidence (at the 10% significance level) that the favourite berry depends on income level. ***A1***

Note: Do not award ***R0A1***. Accept "χ^2" in place of their "2.27", provided an answer was seen in part (b). Their conclusion must be consistent with their χ^2 (or a correct p-value) and their hypothesis.

[2 marks]

Total [5 marks]

2. (a) $71\mathrm{e}^{-0.0514(16)} + 23$ ***(M1)***

$54.2\ ^\circ\mathrm{C} \ (54.1956...)$ ***A1***

[2 marks]

(b) $23\ ^\circ\mathrm{C}$ ***A1***

[1 mark]

(c) $50 = 71\mathrm{e}^{-0.0514(k)} + 23$ ***(M1)***

$k = 18.8 \ \left(\dfrac{-5000}{257}\ln\left(\dfrac{27}{71}\right), 18.8101...\right)$ ***A1***

Note: Award ***M1*** for a sketch showing a point of intersection between the exponential function and $y = 50$.

[2 marks]

Total [5 marks]

3. (a) $\sin(21^\circ) = \frac{17}{BF}$ ***(M1)***

$BF = 47.4 \text{ m } (47.4372...)$ ***A1***

[2 marks]

(b) **EITHER**

$BE = \sqrt{47.4372...^2 + 44^2} = 64.7015...$ ***(A1)***

$\sin^{-1}\left(\frac{17}{BE}\right)$ ***(M1)***

$= 15.2^\circ \ (15.2329...^\circ) \quad \left(\text{or } 0.266 \text{ radians } (0.265866...)\right)$ ***A1***

OR

$AD = \sqrt{47.4372...^2 - 17^2} = 44.2865...$

$DB = \sqrt{64.7015...^2 + 44^2} = 62.4832...$ ***(A1)***

$\tan^{-1}\left(\frac{17}{62.4832...}\right)$ ***(M1)***

$= 15.2^\circ \ (15.2329...^\circ) \quad \left(\text{or } 0.266 \text{ radians } (0.265866...)\right)$ ***A1***

[3 marks]

Total [5 marks]

4. (a)

$$\begin{array}{c} \\ \mathrm{S} \\ \mathrm{A} \\ \mathrm{B} \\ \mathrm{C} \\ \mathrm{D} \\ \mathrm{T} \end{array} \begin{array}{c} \begin{array}{cccccc} \mathrm{S} & \mathrm{A} & \mathrm{B} & \mathrm{C} & \mathrm{D} & \mathrm{T} \end{array} \\ \begin{pmatrix} 0 & 1 & 1 & 1 & \boxed{2} & 0 \\ 1 & 0 & 1 & 1 & \boxed{0} & 0 \\ 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 \\ \boxed{2} & \boxed{0} & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{pmatrix} \end{array}$$

$\mathrm{SD} = \mathrm{DS} = 2$ ***A1***
$\mathrm{AD} = \mathrm{DA} = 0$ ***A1***
[2 marks]

(b) attempt to calculate at least one of $\boldsymbol{M}^2$, $\boldsymbol{M}^3$ and $\boldsymbol{M}^4$ ***(M1)***
attempt to calculate all of $\boldsymbol{M}^2$, $\boldsymbol{M}^3$ and $\boldsymbol{M}^4$ ***(M1)***
finding at least one of the top right entries, $4, 10, 64$ ***(A1)***
78 walks ***A1***

Note: If $\mathrm{SD} = \mathrm{DS} = 1$ is their answer in part (a), their ***FT*** answer is $(3 + 8 + 41 =)\ 52$ walks.

[4 marks]

(c) because some of the walks will pass through T, before returning to T ***R1***
[1 mark]
Total [7 marks]

5. (a) $0.5 \times 0.1 + 0.4 \times 0.4 + 0.1 \times 0.5$ ***(M1)(M1)(M1)***

Note: Award ***M1*** for 0.5×0.1 or 0.1×0.5, ***M1*** for 0.4×0.4, ***M1*** for adding three correct products.

0.26 ***A1***
[4 marks]

(b) $0 = -8 \times 0.5 + 4 \times 0.4 + 0.1k$ ***(M1)(M1)***

Note: Award ***M1*** for correct substitution into the formula for expected value, award ***M1*** for the expected value formula equated to zero.

$(k =)\ 24$ (points) ***A1***
[3 marks]
Total [7 marks]

6. (a) 78 ***A1***

[1 mark]

(b) (i) 65 ***A1***

(ii) **EITHER**

(period $=$) 16 (could be seen on sketch) ***(M1)***

$b = \frac{2\pi}{16}$ **OR** $b = \frac{360^\circ}{16}$

$(b =)\ 0.393\ \left(0.392699..., \frac{\pi}{8}\right)$ **OR** $(b =)\ 22.5^\circ$ ***A1***

OR

$143 = 65\sin(4b) + 78$ ***(M1)***

$(\sin(4b) = 1)$

$(4b = \frac{\pi}{2}$ **OR** $4b = 90^\circ)$

$(b =)\ 0.393\ \left(0.392699..., \frac{\pi}{8}\right)$ **OR** $(b =)\ 22.5^\circ$ ***A1***

[3 marks]

(c) 13 ***A1***

Note: Apply follow through marking only if their final answer is positive.

[1 mark]

(d) $(b =)\ 0.196\ \left(0.196349..., \frac{\pi}{16}\right)$ **OR** $(b =) 11.3^\circ\ (11.25^\circ)$ ***A1***

[1 mark]
Total [6 marks]

7. (a) $I\% = 7.5$
$PV = \mp 800$
$PMT = \mp 500$
$FV = \pm 10\,000$
$P/Y = 12$
$C/Y = 12$ ***(M1)(A1)***

Note: Award ***M1*** for an attempt to use a financial app in their technology (e.g. at least four rows seen, but not necessarily correct), award ***A1*** for $\text{PMT} = -500$ or $\text{PMT} = 500$, with same sign to PV and opposite sign to FV.

$17.3070\ldots$ ***(A1)***
$(k =)\ 18$ ***A1***

Note: Award ***(M0)(A0)(A0)A0*** for a final answer of 17 with no working. The final answer must be an integer.

[4 marks]

(b) $10389 - (18 \times 500 + 800)$ **OR** $10389 - (9800)$ ***(A1)(M1)***

Note: Award ***(A1)*** for 10389 $(10389.38\ldots)$ seen. Award ***(M1)*** for subtraction of their $(18 \times 500 + 800)$ from FV. ***FT*** from their value of k. Award ***A0M1A0*** for $10\,000 - (18 \times 500 + 800)$. Do not award the final ***A1FT*** if their answer is negative.

589 EUR ***A1***

Note: Final answer must be to the nearest euro.

[3 marks]
Total [7 marks]

8. (a) setting up at least two simultaneous equations ***(M1)***

$p = -0.8$ **OR** $q = 3.6$ ***(A1)***

M has coordinates $(1.6, -2.6, 8.2)$ ***A1***

[3 marks]

(b) using vectors $\begin{pmatrix}3\\2\\1\end{pmatrix}$ and $\begin{pmatrix}1\\-1\\2\end{pmatrix}$ ***(M1)***

$\begin{pmatrix}3\\2\\1\end{pmatrix} \bullet \begin{pmatrix}1\\-1\\2\end{pmatrix} = 3$ ***(A1)***

$\cos\theta = \dfrac{\begin{pmatrix}3\\2\\1\end{pmatrix} \bullet \begin{pmatrix}1\\-1\\2\end{pmatrix}}{\sqrt{3^2+2^2+1^2}\sqrt{1^2+(-1)^2+2^2}}$ $\left(\cos\theta = \dfrac{3}{\sqrt{14}\sqrt{6}}\right)$ ***(M1)***

Note: Accept correct use of vector product.

$(\theta =)$ 1.24 radians $(1.23732\ldots)$ $(70.9^\circ$ $(70.8933...))$ ***A1***

[4 marks]

Total [7 marks]

9. (a) attempt to find $\det(\boldsymbol{M})$ ***(M1)***

$= 14$

$(12 \times 14) = 168 \text{ cm}^2$ ***A1***

[2 marks]

(b) let X have coordinates (x, y)

METHOD 1

$\boldsymbol{M}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2t-3 \\ 6-5t \end{pmatrix}$ ***(M1)***

$\begin{pmatrix} x \\ y \end{pmatrix} = \boldsymbol{M}^{-1}\begin{pmatrix} 2t-3 \\ 6-5t \end{pmatrix}$ ***(A1)***

$\boldsymbol{M}^{-1} = \frac{1}{14}\begin{pmatrix} 1 & 4 \\ -3 & 2 \end{pmatrix}$ ***A1***

$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{14}\begin{pmatrix} 2t-3+24-20t \\ -6t+9+12-10t \end{pmatrix}$ ***(M1)***

$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{14}\begin{pmatrix} 21-18t \\ 21-16t \end{pmatrix}$ **OR** $\left(\frac{21-18t}{14}, \frac{21-16t}{14}\right)$ ***A1A1***

METHOD 2

writing two simultaneous equations ***(M1)***

$2x - 4y = 2t - 3$ ***(A1)***

$3x + y = 6 - 5t$ ***(A1)***

attempting to solve the equations ***(M1)***

$(x, y) = \left(\frac{3}{2} - \frac{9t}{7}, \frac{3}{2} - \frac{8t}{7}\right)$ ***A1A1***

[6 marks]

Total [8 marks]

10. (a) $m = 1 - 2.5\log_{10}(0.0525)$ ***(M1)***

$= 4.20 \ (4.19960\ldots)$ ***A1***

[2 marks]

(b) attempt to solve $7 = 1 - 2.5\log_{10}(b)$ ***(M1)***

Note: Accept a sketch from their GDC as an attempt to solve $7 = 1 - 2.5\log_{10}(b)$.

$b = 0.00398 \ (0.00398107...)$ ***A1***

[2 marks]

(c) $-3.2 = \left(1 - 2.5\log_{10}(b_n)\right) - \left(1 - 2.5\log_{10}(b_p)\right)$ ***M1***

$-3.2 = -2.5\log_{10}\left(\frac{b_n}{b_p}\right)$ ***A1***

$\frac{b_n}{b_p} = 19.1 \ (19.0546...)$ ***A1***

[3 marks]

Total [7 marks]

11. (a) $y = -0.00855x^3 - 0.234x^2 - 0.225x + 3.20$ ***A2***

$(y = -0.00854819...x^3 - 0.234002...x^2 - 0.224884...x + 3.20056...)$

Note: Award ***A0A1*** for at least two terms correct.

[2 marks]

(b) $y(2x)$ (for horizontal stretch) ***(A1)***

attempt to stretch vertically by factor $\frac{1}{2}$ ***(M1)***

$y = 0.0332x^3 - 0.15x^2 - 0.58x \ (+1.1)$ ***A1***

Note: Award ***A0M1A0*** for a vertical stretch, factor 2. Although a d value of 1.1 is preferred, technically this value can be wrong/omitted and the question is still answered (hence it is presented in brackets).

[3 marks]

Total [5 marks]

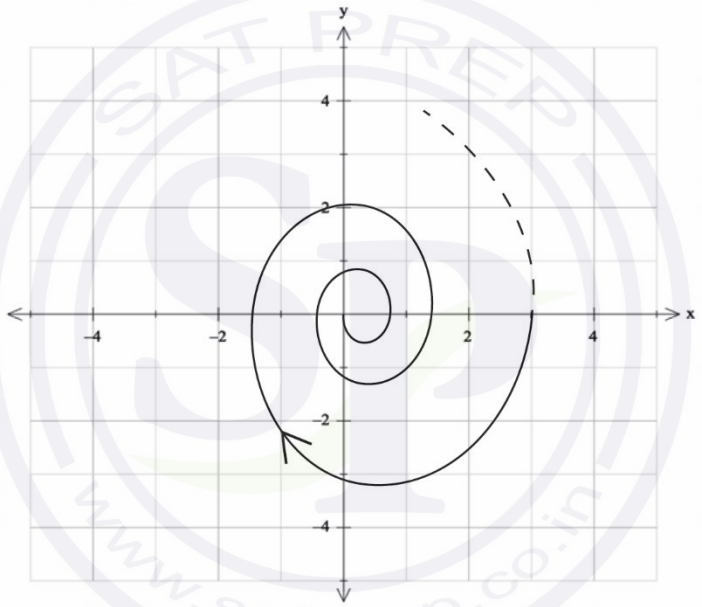
12. (a)

Description	Phase portrait
$\lambda_1 = 2$ with eigenvector $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and $\lambda_2 = 3$ with eigenvector $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$	**D**
$\lambda_1 = 2$ with eigenvector $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and $\lambda_2 = -3$ with eigenvector $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$	**C**
$\lambda_1 = -2$ with eigenvector $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and $\lambda_2 = 3$ with eigenvector $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$	**B**

A1A1A1

[3 marks]

(b)



spiral (crossing x-axis at least twice), centre at origin ***A1***

arrow indicating clockwise, passing through or starting from $(3, 0)$ ***A1***

[2 marks]

Total [5 marks]

13. (a) Gradient $= \frac{14.9+1.3}{6}$ $(=2.7)$ ***(M1)***

$\log_{10} Q = 2.7P - 1.3$ ***(A1)***

$Q = 10^{2.7P-1.3}$ **OR** $Q = 0.0501 \times 10^{2.7P}$ $(= 0.0501187... \times 10^{2.7P})$ ***A1***

[3 marks]

(b) $\ln R$ on one axis and Q on the other axis ***A1***

[1 mark]

(c) $\log_{10}(4.3\ln R + 12.1) = 2.7P - 1.3$ **OR** $10^{2.7P-1.3} = 4.3\ln R + 12.1$ ***(M1)***

$P = \frac{\log_{10}(4.3\ln R + 12.1) + 1.3}{2.7}$ ***A1***

[2 marks]

Total [6 marks]

14. (a) attempt to use product rule ***(M1)***

$a = 2t^2\cos(t^2) + \sin(t^2)$ ***A1***

[2 marks]

(b) graph of a ***(M1)***

$126\ (\text{m s}^{-2})$ $(125.699...)$ ***A1***

[2 marks]

(c) attempt at integration by substitution or inspection ***(M1)***

$s = -\frac{1}{2}\cos(t^2)$ $(+c)$ ***A1***

$(s = 0$ when $t = 0) \Rightarrow c = \frac{1}{2}$ ***A1***

$\left(s = -\frac{1}{2}\cos(t^2) + \frac{1}{2}\right)$

[3 marks]

(d) $\cos(t^2) \leq 1$ ***A1***

$-\frac{1}{2}\cos(t^2) \geq -\frac{1}{2}$

so $\frac{1}{2} - \frac{1}{2}\cos(t^2) \geq 0$ ***R1***

hence the particle never has a negative displacement. ***AG***

Note: Do not accept reasoning based on a sketch of the graph.

[2 marks]

Total [9 marks]

15. EITHER

$q_{n+1} = q_n + 0.1\left(\frac{\mathrm{d}q}{\mathrm{d}t}\right)_n$

$\left(\frac{\mathrm{d}q}{\mathrm{d}t}\right)_{n+1} = \left(\frac{\mathrm{d}q}{\mathrm{d}t}\right)_n + 0.1\left(\frac{\mathrm{d}^2q}{\mathrm{d}t^2}\right)_n$ ***(M1)***

OR

let $\frac{\mathrm{d}q}{\mathrm{d}t} = y$

$q_{n+1} = q_n + 0.1y_n$

$y_{n+1} = y_n + 0.1\left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)_n$ ***(M1)***

THEN

EITHER

$\frac{\mathrm{d}y}{\mathrm{d}t} = 200 - 5y - 20q$ ***(A1)***

OR

$\frac{\mathrm{d}^2q}{\mathrm{d}t^2} = 200 - 5\frac{\mathrm{d}q}{\mathrm{d}t} - 20q$ ***(A1)***

THEN

evidence of using Euler's method (e.g.) ***(M1)***

0	1	8	140
0.1	1.8	22	54

maximum charge $= 12.7$ (Coulombs, at $t = 0.7$) ***A2***

Note: Award ***A0A1*** for a final answer of 10.8, from reading the value at $t = 1$.

Total [5 marks]

16. (a) (let p be the probability of a student choosing healthy options)

$\mathrm{H}_0 : p = 0.3$ ***A1***

$\mathrm{H}_1 : p > 0.3$ ***A1***

Note: Award ***A0A1*** for correct hypotheses with μ in place of p. Accept equivalent hypotheses in words.

[2 marks]

(b) a type I error is rejecting H_0 when H_0 is true ***(M1)***

(let N = number of students choosing a healthy option)

$N \sim \mathrm{B}(80, 0.3)$

$\mathrm{P}(31 \leq N \leq 80)$ **OR** $\mathrm{P}(N \geq 31)$ **OR** $1 - \mathrm{P}(N \leq 30)$ ***(M1)***

Note: Do not accept the use of the Normal approximation.

$0.0587 \ (0.0587481\ldots)$ ***A1***

[3 marks]

(c) a type II error is accepting H_0 when H_0 is not true ***(M1)***

$N \sim \mathrm{B}(80, 0.4)$

$\mathrm{P}(0 \leq N \leq 30)$ **OR** $\mathrm{P}(N \leq 30)$ ***(M1)***

$0.369 \ (0.368726\ldots)$ ***A1***

[3 marks]

Total [8 marks]

17. (a) $D = S - R$

METHOD 1

$= \text{Re}(1.15\text{e}^{(0.0165t-2.97)\text{i}}) - \text{Re}(1.08\text{e}^{(0.0165t+0.413)\text{i}})$ $(+18.9 - 4.94)$ ***(M1)(A1)***

$= \text{Re}\left(\text{e}^{0.0165t\text{i}}(1.15\text{e}^{-2.97\text{i}} - 1.08\text{e}^{0.413\text{i}})\right)$ $(+13.96)$ ***(M1)***

$= \text{Re}\left(\text{e}^{0.0165t\text{i}}(2.21379...\text{e}^{-2.85310...\text{i}})\right)$ $(+13.96)$ ***(A1)***

$= 2.21\cos(0.0165t - 2.85) + 13.96$ $\left(2.21379...\cos(0.0165t - 2.85310...) + 13.96\right)$ ***A1A1***

$(a = 2.21,\ b = -2.85,\ c = 13.96)$

Note: Award ***A1*** for $2.21\cos(0.0165t - 2.85)$ and ***A1*** for "$+13.96$". The ***A1*** for 13.96 is independent of the previous marks.

METHOD 2

$c = 13.9 - 4.94\ldots = 13.96$ ***A1***

using a graph of D ***M1***

maximum $(172.915\ldots, 16.1738\ldots)$ ***(A1)***

minimum $(-17.4842\ldots, 11.7462\ldots)$ ***(A1)***

EITHER

amplitude $16.1738\ldots - 11.7462\ldots = 4.4276\ldots$

$a = 2.21$ $(2.2138\ldots)$ ***A1***

OR

$a = 16.1738\ldots - 13.96 = 2.21$ $(2.2138\ldots)$ ***A1***

THEN

EITHER

when $t = 0$, $D = 11.8377\ldots$

$11.8377\ldots = 2.2138\cos(b) + 13.96$

$b = -2.85$ $(2.85309\ldots)$ ***A1***

OR

$b = -0.0165 \times 172.915\ldots = -2.85$ $(2.85309\ldots)$ ***A1***

[6 marks]

(b) 16.2 $(16.1737\ldots)$ hours on day 173 ***A1A1***

Note: Accept an answer of "day 172" for the second ***A1***.

[2 marks]

Total [8 marks]

Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

May 2022

Mathematics: applications and interpretation

Higher level

Paper 1

24 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**.
A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.
R Marks awarded for clear **Reasoning**.
AG Answer given in the question and so no marks are awarded.
FT Follow through. The practice of awarding marks, despite candidate errors in previous parts, for their correct methods/answers using incorrect results.

Using the markscheme

1 General

Award marks using the annotations as noted in the markscheme *eg* ***M1, A2.***

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***A3***, ***M2*** *etc*., do **not** split the marks, unless there is a note.
- The response to a "show that" question does not need to restate the ***AG*** line, unless a **Note** makes this explicit in the markscheme.
- Once a correct answer to a question or part question is seen, ignore further working even if this working is incorrect and/or suggests a misunderstanding of the question. This will encourage a uniform approach to marking, with less examiner discretion. Although some candidates may be advantaged for that specific question item, it is likely that these candidates will lose marks elsewhere too.
- An exception to the previous rule is when an incorrect answer from further working is used **in a subsequent part**. For example, when a correct exact value is followed by an incorrect decimal approximation in the first part and this approximation is then used in the second part. In this situation, award ***FT*** marks as appropriate but do not award the final ***A1*** in the first part. Examples:

	Correct answer seen	**Further working seen**	**Any FT issues?**	**Action**
1.	$8\sqrt{2}$	5.65685... *(incorrect decimal value)*	No. Last part in question.	Award ***A1*** for the final mark *(condone the incorrect further working)*
2.	$\frac{35}{72}$	0.468111… (incorrect decimal value)	Yes. Value is used in subsequent parts.	Award ***A0*** for the final mark *(and full **FT** is available in subsequent parts)*

3 Implied marks

Implied marks appear in **brackets e.g.** ***(M1)***, and can only be awarded if **correct** work is seen or implied by subsequent working/answer.

4 Follow through marks (only applied after an error is made)

Follow through (***FT***) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) (e.g. incorrect value from part (a) used in part (d) or incorrect value from part (c)(i) used in part (c)(ii)). Usually, to award ***FT*** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if all the marks awarded in a subsequent part are for the answer or are implied, then ***FT*** marks should be awarded for *their* correct answer, even when working is not present.

For example: following an incorrect answer to part (a) that is used in subsequent parts, where the markscheme for the subsequent part is ***(M1)A1***, it is possible to award full marks for *their* correct answer, **without working being seen.** For longer questions where all but the answer marks are implied this rule applies but may be overwritten by a **Note** in the Markscheme.

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks, by reflecting on what each mark is for and how that maps to the simplified version.
- If the error leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word “their” in a description, to indicate that candidates may be using an incorrect value.
- If the candidate’s answer to the initial question clearly contradicts information given in the question, it is not appropriate to award any ***FT*** marks in the subsequent parts. This includes when candidates fail to complete a “show that” question correctly, and then in subsequent parts use their incorrect answer rather than the given value.
- Exceptions to these ***FT*** rules will be explicitly noted on the markscheme.
- If a candidate makes an error in one part but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the command term was “Hence”.

5 Mis-read

If a candidate incorrectly copies values or information from the question, this is a mis-read (***MR***). A candidate should be penalized only once for a particular misread. Use the ***MR*** stamp to indicate that this has been a misread and do not award the first mark, even if this is an ***M*** mark, but award all others as appropriate.

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.
- If a candidate uses a correct answer, to a "show that" question, to a higher degree of accuracy than given in the question, this is NOT a misread and full marks may be scored in the subsequent part.
- ***MR*** can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If the command term is 'Hence' and not 'Hence or otherwise' then alternative methods are not permitted unless covered by a note in the mark scheme.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**.

7 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation** for example 1.9 and $1{,}9$ or 1000 and $1{,}000$ and 1.000.
- Do not accept final answers written using calculator notation. However, ***M*** marks and intermediate ***A*** marks can be scored, when presented using calculator notation, provided the evidence clearly reflects the demand of the mark.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, some **equivalent** answers will generally appear in brackets. Not all equivalent notations/answers/methods will be presented in the markscheme and examiners are asked to apply appropriate discretion to judge if the candidate work is equivalent.

8 Format and accuracy of answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. If the level of accuracy is not stated in the question, the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures.*

Where values are used in subsequent parts, the markscheme will generally use the exact value, however candidates may also use the correct answer in subsequent parts. The markscheme will often explicitly include the subsequent values that come "*from the use of 3 sf values*".

Simplification of final answers: Candidates are advised to give final answers using good mathematical form. In general, for an ***A*** mark to be awarded, arithmetic should be completed, and any values that lead to integers should be simplified; for example, $\sqrt{\frac{25}{4}}$ should be written as $\frac{5}{2}$. An exception to this is simplifying fractions, where lowest form is not required (although the numerator and the denominator must be integers); for example, $\frac{10}{4}$ may be left in this form or written as $\frac{5}{2}$. However, $\frac{10}{5}$ should be written as 2, as it simplifies to an integer.

Algebraic expressions should be simplified by completing any operations such as addition and multiplication, e.g. $4\mathrm{e}^{2x} \times \mathrm{e}^{3x}$ should be simplified to $4\mathrm{e}^{5x}$, and $4\mathrm{e}^{2x} \times \mathrm{e}^{3x} - \mathrm{e}^{4x} \times \mathrm{e}^{x}$ should be simplified to $3\mathrm{e}^{5x}$. Unless specified in the question, expressions do not need to be factorized, nor do factorized expressions need to be expanded, so $x(x+1)$ and $x^2 + x$ are both acceptable.

Please note: intermediate ***A*** marks do NOT need to be simplified.

9 Calculators

A GDC is required for this paper, but If you see work that suggests a candidate has used any calculator not approved for IB DP examinations (eg CAS enabled devices), please follow the procedures for malpractice.

10. Presentation of candidate work

Crossed out work: If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work unless an explicit note from the candidate indicates that they would like the work to be marked.

More than one solution: Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise. If the layout of the responses makes it difficult to judge, examiners should apply appropriate discretion to judge which is "first".

1. (a) $\left(\frac{17+25}{130}=\right)\ \frac{42}{130}\ \ \left(\frac{21}{65}, 0.323076\ldots\right)$ ***A1***

[1 mark]

(b) $\left(\frac{17}{17+25}=\right)\ \frac{17}{42}\ \ (0.404761\ldots)$ ***A1A1***

Note: Award ***A1*** for correct numerator and ***A1*** for correct denominator. Award ***A1A0*** for working of $\frac{17/130}{\text{their answer to (a)}}$ if followed by an incorrect answer.

[2 marks]

(c) $\frac{41}{130}\times\frac{40}{129}$ ***A1M1***

Note: Award ***A1*** for two correct fractions seen, ***M1*** for multiplying their fractions.

$=\frac{1640}{16770}\approx 0.0978\ \left(0.0977936\ldots,\ \frac{164}{1677}\right)$ ***A1***

[3 marks]
Total [6 marks]

2. (a) $\sin\theta = \frac{2.1}{2.8}$ **OR** $\tan\theta = \frac{2.1}{1.85202\ldots}$ ***(M1)***

$(\theta =)\ 48.6\ °\ \ (48.5903\ldots°)$ ***A1***

[2 marks]

(b) **METHOD 1**

$\sqrt{2.8^2 - 2.1^2}$ **OR** $2.8\cos(48.5903...)$ **OR** $\frac{2.1}{\tan(48.5903\ldots)}$ ***(M1)***

Note: Award ***M1*** for attempt to use Pythagorean Theorem with 2.1 seen or for attempt to use cosine or tangent ratio.

$1.85\ \text{m}\ \ (1.85202\ldots)$ ***(A1)***

Note: Award the ***M1A1*** if 1.85 is seen in part (a).

$(6.4 - 1.85202\ldots)$
$4.55\ \text{m}\ \ (4.54797\ldots)$ ***(A1)***

Note: Award ***A1*** for 4.55 or equivalent seen, either as a separate calculation or in Pythagorean Theorem.

$\sqrt{(4.54797\ldots)^2 + 2.1^2}$
$5.01\ \text{m}\ \ (5.00939\ldots\text{m})$ ***A1***

METHOD 2
attempt to use cosine rule ***(M1)***
$(c^2 =)\ 2.8^2 + 6.4^2 - 2(2.8)(6.4)\cos(48.5903\ldots)$ ***(A1)(A1)***

Note: Award ***A1*** for $48.5903...°$ substituted into cosine rule formula, ***A1*** for correct substitution.

$(c =)\ 5.01\ \text{m}\ \ (5.00939\ldots\text{m})$ ***A1***

[4 marks]

(c) camera 1 is closer to the cash register than camera 2 (and both cameras are at the same height on the wall) ***R1***
the larger angle of depression is from camera 1 ***A1***

Note: Do not award ***R0A1***. Award ***R0A0*** if additional calculations are completed and used in their justification, as per the question. Accept "$1.85<4.55$" or "$2.8<5.01$" as evidence for the ***R1***.

[2 marks]
Total [8 marks]

3. (a) $(\mathrm{E}(X) =) 10 \times 0.8$ ***(M1)***

8 (people) ***A1***

[2 marks]

(b) recognition of binomial probability ***(M1)***

$0.0881 \quad (0.0880803\ldots)$ ***A1***

[2 marks]

(c) 0.8 and 6 seen **OR** 0.2 and 3 seen ***(A1)***

attempt to use binomial probability ***(M1)***

$0.121 \quad (0.120873\ldots)$ ***A1***

[3 marks]

Total [7 marks]

4. (a) **EITHER**

attempt to substitute 3, 4 and 7 into area of a trapezoid formula ***(M1)***

$(A =) \frac{1}{2}(7+4)(3)$

OR

given area expressed as an integral ***(M1)***

$(A =) \int_{-1}^{2} (6 - x)\ \mathrm{d}x$

OR

attempt to sum area of rectangle and area of triangle ***(M1)***

$(A =) 4 \times 3 + \frac{1}{2}(3)(3)$

THEN

16.5 (square units) ***A1***

[2 marks]

(b) (i) $(A =) \int_{-1}^{2} 1.5x^2 - 2.5x + 3\ \mathrm{d}x$ ***A1A1***

Note: Award ***A1*** for the limits $x = -1$, $x = 2$ in correct location. Award ***A1*** for an integral of the quadratic function, $\mathrm{d}x$ must be included. Do not accept "y" in place of the function, given that two equations are in the question.

(ii) 9.75 (square units) ***A1***

[3 marks]

(c) $16.5 - 9.75$ ***(M1)***

6.75 (square units) ***A1***

[2 marks]

Total [7 marks]

5. (a) *Accept any one of the following (or equivalent):*
one minimum and one maximum point
three x-intercepts or three roots (or zeroes)
one point of inflexion ***R1***

Note: Do not accept "S shape" as a justification.

[1 mark]

(b) (i) $(d =) -5$ ***A1***

(ii) $8 = a + b + c$
$4 = 8a + 4b + 2c$
$0 = 27a + 9b + 3c$ ***A2***

Note: Award ***A2*** if all three equations are correct.
Award ***A1*** if at least one is correct. Award ***A1*** for three correct equations that include the letter "d".

(iii) $a = 2,\ b = -12,\ c = 18$ ***A1***

[4 marks]

(c) equating found expression to zero ***(M1)***
$0 = 2t^3 - 12t^2 + 18t - 5$
$t = 0.358216\ldots,\ 1.83174\ldots,\ 3.81003\ldots$ ***(A1)***

(so total time in debt is $3.81003\ldots - 1.83174\ldots + 0.358216 \approx$)
$2.34\ (2.33650\ldots)$ years ***A1***

[3 marks]
Total [8 marks]

6. (a)

$$\begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \end{pmatrix}$$ ***A2***

Note: Award ***A2*** for the transposed matrix. Presentation in markscheme assumes columns/rows ordered A-E; accept a matrix with rows and/or columns in a different order only if appropriately communicated. Do not ***FT*** from part (a) into part (b).

[2 marks]

(b) raising their matrix to a power of 5 ***(M1)***

$$\boldsymbol{M}^5 = \begin{pmatrix} 17 & 9 & 2 & 3 & 5 \\ 17 & 10 & 3 & 4 & 4 \\ 13 & 6 & 2 & 2 & 4 \\ 8 & 5 & 1 & 2 & 2 \\ 18 & 11 & 2 & 4 & 5 \end{pmatrix}$$ ***(A1)***

Note: The numbers along the diagonal are sufficient to award ***M1A1***.

(the required number is $17+10+2+2+5=)\ 36$ ***A1***

[3 marks]
Total [5 marks]

7. METHOD 1

$\frac{u_1}{1-r} = 9$ ***A1***

therefore $u_1 = 9 - 9r$

$u_1 = 4 + u_1 r$ ***A1***

substitute or solve graphically: ***M1***

$9 - 9r = 4 + (9 - 9r)r$ **OR** $\frac{4}{(1-r)^2} = 9$

$9r^2 - 18r + 5 = 0$

$r = \frac{1}{3}$ or $r = \frac{5}{3}$

only $r = \frac{1}{3}$ is possible as the sum to infinity exists ***R1***

then $u_1 = 9 - \left(9 \times \frac{1}{3}\right) = 6$

$u_3 = 6 \times \frac{1}{3}^2 = \frac{2}{3}$ ***A1***

METHOD 2

$\frac{u_1}{1-r} = 9$ ***A1***

$r = \frac{u_1 - 4}{u_1}$ ***A1***

attempt to solve ***M1***

$$\frac{u_1}{1 - \left(\frac{u_1 - 4}{u_1}\right)} = 9$$

$$\frac{u_1}{\left(\frac{4}{u_1}\right)} = 9$$

$(u_1)^2 = 36$

$u_1 = \pm 6$

attempting to solve both possible sequences

$6, 2, \ldots$ or $-6, -10 \ldots$

$r = \frac{1}{3}$ or $r = \frac{5}{3}$

only $r = \frac{1}{3}$ is possible as the sum to infinity exists ***R1***

$u_3 = 6 \times \left(\frac{1}{3}\right)^2 = \frac{2}{3}$ ***A1***

Total [5 marks]

8. (a) $\pi \times 2^2 \times \frac{30}{360}$ ***(M1)***

$= 1.047 \text{ cm}^2$ ***A1***

Note: Do not award the final mark if the answer is not correct to 4 sf.

[2 marks]

(b) attempt to substitute any two values from $1.5, 2.5, 25$ or 35 into area of sector formula ***(M1)***

$\left(\text{upper bound} = \pi \times 2.5^2 \times \frac{35}{360} = \right) \ 1.91 \ \text{cm}^2 \ \ (1.90895\ldots)$ ***A1***

$\left(\text{lower bound} = \pi \times 1.5^2 \times \frac{25}{360} = \right) \ 0.491 \ \text{cm}^2 \ \ (0.490873\ldots)$ ***A1***

Note: Given the nature of the question, accept correctly rounded **OR** correctly truncated 3 significant figure answers.

[3 marks]

(c) $\left(\left|\frac{1.047 - 1.90895\ldots}{1.90895\ldots}\right| \times 100 = \right) \ 45.2 \ (\%) \ \ (45.1532\ldots)$ ***A1***

$\left(\left|\frac{1.047 - 0.490873\ldots}{0.490873\ldots}\right| \times 100 = \right) \ 113 \ (\%) \ \ (113.293\ldots)$ ***A1***

so the largest percentage error is 113 % ***A1***

Note: Accept 45.1 (%) (45.1428), from use of full accuracy answers. Given the nature of the question, accept correctly rounded **OR** correctly truncated 3 significant figure answers. Award ***A0A1A0*** if 113% is the only value found.

[3 marks]

Total [8 marks]

9. (a) $\bar{x} = 4.63 \ (4.62686\ldots)$ ***A1***

[1 mark]

(b) $s_{n-1} = 1.098702$ ***(A1)***

$s_{n-1}^2 = 1.21 \ (1.207146\ldots)$ ***A1***

Note: Award ***A0A0*** for an answer of 1.19 from biased estimate.

[2 marks]

(c) (i) $H_1 : \mu > 4.4$ ***A1***

(ii) **METHOD 1**

using a z-test ***(M1)***

$p = 0.0454992\ldots$ ***A1***

$p < 0.05$ ***R1***

reject null hypothesis ***A1***

(therefore there is significant evidence that the IB HL math students know more digits of π than the population in general)

Note: Do not award ***R0A1***. Allow ***R1A1*** for consistent conclusion following on from their p-value.

METHOD 2

using a t-test ***(M1)***

$p = 0.0478584\ldots$ ***A1***

$p < 0.05$ ***R1***

reject null hypothesis ***A1***

(therefore there is significant evidence that the IB HL math students know more digits of π than the population in general)

Note: Do not award ***R0A1***. Allow ***R1A1*** for consistent conclusion following on from their p-value.

[5 marks]

Total [8 marks]

10. (a) $y = \ln\left(\frac{1}{x-2}\right)$

an attempt to isolate x (or y if switched) ***(M1)***

$e^y = \frac{1}{x-2}$

$x - 2 = e^{-y}$

$x = e^{-y} + 2$

switching x and y (seen anywhere) ***M1***

$f^{-1}(x) = e^{-x} + 2$ ***A1***

[3 marks]

(b) sketch of $f(x)$ and $f^{-1}(x)$ ***(M1)***

$x = 2.12 \ \ (2.12002\ldots)$ ***A1***

[2 marks]

Total [5 marks]

11. (a) **METHOD 1** – *(With FV=4000)*

EITHER
N= 10
I=1.5
FV= 4000
P/Y= 1
C/Y= 1 ***(A1)(M1)***

Note: Award ***A1*** for $(3.5-2=)\ 1.5$ seen and ***M1*** for all other entries correct.

OR
$4000 = A(1+0.015)^{10}$ ***(A1)(M1)***

Note: Award ***A1*** for 1.5 or 0.015 seen, ***M1*** for attempt to substitute into compound interest formula **and** equating to 4000.

THEN
(PV =) \$3447 ***A1***

Note: Award ***A0*** if not rounded to a whole number or a negative sign given.

METHOD 2 – *(With FV including inflation)*
calculate FV with inflation
4000×1.02^{10} ***(A1)***
(=4875.977…)

EITHER
$4000\times1.02^{10} = \text{PV}\times1.035^{10}$ ***(A1)***
OR
N= 10
I= 3.5
FV= 4875.977…
P/Y= 1
C/Y= 1 ***(M1)***

Note: Award ***M1*** for *their* FV and all other entries correct.

THEN
(PV =) \$3457 ***A1***

Note: Award ***A0*** if not rounded to a whole number or a negative sign given.

continued…

Question 11 continued

METHOD 3 – *(Using formula to calculate real rate of return)*
(real rate of return =) $1.47058\ldots(\%)$ ***(A1)***

EITHER
$4000 = \text{PV} \times 1.0147058\ldots^{10}$ ***(A1)***
OR
N= 10
I= 1.47058…
FV= 4000
P/Y= 1
C/Y= 1 ***(M1)***

Note: Award ***M1*** for all entries correct.

THEN
(PV =) \$3457 ***A1***

[3 marks]

(b) **METHOD 1** – *(Finding the future value of the investment using PV from part (a))*
N= 10
I=3.5
PV= 3446.66…(from Method 1) **OR** 3456.67…(from Methods 2, 3)
P/Y= 1
C/Y= 1 ***(M1)***

Note: Award ***M1*** for interest rate 3.5 **and** answer to part (a) as PV.

(FV=) \$4861.87 **OR** \$4875.97 ***(A1)***
so payment required (from TVM) will be \$294 **OR** \$295 ***A1***

Note: Award ***A0*** if a negative sign given, unless already penalized in part (a).

METHOD 2 – *(Using FV)*
N= 10
I=3.5
PV= −1000
FV= 4875.977…
P/Y= 1
C/Y= 1 ***(A1)(M1)***

Note: Award ***A1*** for I=3.5 **and** FV= ±4875.977…, ***M1*** for all other entries correct **and** opposite PV and FV signs.

(PMT =) \$295 (295.393) ***A1***

Note: Correct 3sf answer is 295, however accept an answer of 296 given that the context supports rounding up. Award ***A0*** if a negative sign given, unless already penalized in part (a).

[3 marks]
Total [6 marks]

12. (a) $\mathrm{P}(\text{Type I error}) = \mathrm{P}(\text{stating female when male})$

$= \mathrm{P}(W_{Male} > 11.5)$ ***(M1)***

$= 0.00135 \quad (0.00134996\ldots)$ ***A1***

[2 marks]

(b) $\mathrm{P}(\text{Type II error}) = \mathrm{P}(\text{stating male when female})$

$= \mathrm{P}(W_{Female} < 11.5)$ ***(M1)***

$= 0.309 \quad (0.308537\ldots)$ ***A1***

[2 marks]

(c) attempt to use the total probability ***(M1)***

$\mathrm{P}(\text{error}) = 0.9 \times 0.00134996\ldots + 0.1 \times 0.308537\ldots$

$= 0.0321 \quad (0.0320687\ldots)$ ***A1***

[2 marks]

Total [6 marks]

13. (a) **METHOD 1**

recognizing that the real part is distributive ***(M1)***

$V_T = \mathrm{Re}\left(2\mathrm{e}^{3ti} + 5\mathrm{e}^{3ti+4\mathrm{i}}\right)$

$= \mathrm{Re}\left(\mathrm{e}^{3ti}\left(2 + 5\mathrm{e}^{4\mathrm{i}}\right)\right)$ ***(A1)***

(from the GDC) $2 + 5\mathrm{e}^{4\mathrm{i}} = 3.99088\ldots\mathrm{e}^{-1.89418\ldots\mathrm{i}}$ ***(A1)***

Note: Accept arguments differing by 2π e.g. $4.38900\ldots$).

therefore $V_T = 3.99\cos(3t - 1.89) \quad \left(3.99088\ldots\cos(3t - 1.89418\ldots)\right)$ ***A1***

Note: Award the last ***A1*** for the correct values of A, B and C seen either in the required form or not. If method used is unclear and answer is partially incorrect, assume Method 2 and award appropriate marks eg. ***(M1)A1A0A0*** if only A value is correct.

continued…

Question 13 continued

METHOD 2

converting given expressions to cos form ***(M1)***

$V_T = 2\cos 3t + 5\cos(3t+4)$

(from graph) $A = 3.99$ $(3.99088\ldots)$ ***A1***

$V_T = 3.99\cos(Bt+C)$

either by considering transformations or inserting points

$B = 3$ ***A1***

$C = -1.89$ $(-1.89418\ldots)$ ***A1***

Note: Accept arguments differing by 2π e.g. $4.38900\ldots$.

(so, $V_T = 3.99\cos(3t-1.89)$ $\left(3.99088\ldots\cos(3t-1.89418\ldots)\right)$)

Note: It is possible to have $A = 3.99$, $B = -3$ with $C = 1.89$ **OR** $A = -3.99$, $B = 3$ with $C = 1.25$ **OR** $A = -3.99$, $B = -3$ with $C = -1.25$ due to properties of the cosine curve.

[4 marks]

(b) maximum voltage is 3.99 $(3.99088\ldots)$ (units) ***A1***

[1 mark]

Total [5 marks]

14. $V = \pi\int_0^{10} y^2\,\mathrm{d}x$ **OR** $\pi\int_0^{10} x^2\,\mathrm{d}y$ ***(M1)***

$h = 2$

$\approx \pi\times\frac{1}{2}\times 2\times\left(\left(4^2+5^2\right)+2\times\left(6^2+8^2+7^2+3^2\right)\right)$ ***M1A1***

$= 1120\ \mathrm{cm}^3$ $(1121.548\ldots)$ ***A1***

Note: Do not award the second ***M1*** If the terms are not squared.

Total [4 marks]

15. (a) (one vector to the line is $\begin{pmatrix}0\\c\end{pmatrix}$ therefore) $\boldsymbol{a}=\begin{pmatrix}0\\c\end{pmatrix}$ ***A1***

the line goes m up for every 1 across

(so the direction vector is) $\boldsymbol{b}=\begin{pmatrix}1\\m\end{pmatrix}$ ***A1***

Note: Although these are the most likely answers, many others are possible.

[2 marks]

(b) (from GDC **OR** $6\times2-4\times3$) $|M|=0$ ***A1***

[1 mark]

(c) **METHOD 1**

$\begin{pmatrix}X\\Y\end{pmatrix}=\begin{pmatrix}6&3\\4&2\end{pmatrix}\begin{pmatrix}x\\mx+c\end{pmatrix}=\begin{pmatrix}6x+3mx+3c\\4x+2mx+2c\end{pmatrix}$ ***M1A1***

$=\begin{pmatrix}3(2x+mx+c)\\2(2x+mx+c)\end{pmatrix}$ ***A1***

therefore the new line has equation $3Y=2X$ ***A1***

which is independent of m or c ***AG***

Note: The ***AG*** line (or equivalent) must be seen for the final ***A1*** line to be awarded.

METHOD 2

take two points on the line, e.g $(0,\ c)$ and $(1,\ m+c)$ ***M1***

these map to $\begin{pmatrix}6&3\\4&2\end{pmatrix}\begin{pmatrix}0\\c\end{pmatrix}=\begin{pmatrix}3c\\2c\end{pmatrix}$

and $\begin{pmatrix}6&3\\4&2\end{pmatrix}\begin{pmatrix}1\\m+c\end{pmatrix}=\begin{pmatrix}6+3m+3c\\4+2m+2c\end{pmatrix}$ ***A1***

therefore a direction vector is $\begin{pmatrix}6+3m\\4+2m\end{pmatrix}=(2+m)\begin{pmatrix}3\\2\end{pmatrix}$

(since $m\neq-2$) a direction vector is $\begin{pmatrix}3\\2\end{pmatrix}$

the line passes through $\begin{pmatrix}3c\\2c\end{pmatrix}-c\begin{pmatrix}3\\2\end{pmatrix}=\begin{pmatrix}0\\0\end{pmatrix}$ therefore it always has the origin as a jump-on vector ***A1***

the vector equation is therefore $\boldsymbol{r}=\mu\begin{pmatrix}3\\2\end{pmatrix}$ ***A1***

which is independent of m or c ***AG***

Note: The ***AG*** line (or equivalent) must be seen for the final ***A1*** line to be awarded.

continued…

Question 15 continued

METHOD 3

$$\boldsymbol{r} = \begin{pmatrix} 6 & 3 \\ 4 & 2 \end{pmatrix}\left(\begin{pmatrix} 0 \\ c \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ m \end{pmatrix}\right) = \begin{pmatrix} 3c \\ 2c \end{pmatrix} + \lambda \begin{pmatrix} 6+3m \\ 4+2m \end{pmatrix} \quad \textbf{M1A1}$$

$$= c\begin{pmatrix} 3 \\ 2 \end{pmatrix} + (2+m)\lambda \begin{pmatrix} 3 \\ 2 \end{pmatrix} \quad \textbf{A1}$$

$$= \mu \begin{pmatrix} 3 \\ 2 \end{pmatrix}$$

where $\mu = c + (2+m)\lambda$ is an arbitrary parameter. **A1**

which is independent of m or c (as μ can take any value) **AG**

Note: The ***AG*** line (or equivalent) must be seen for the final ***A1*** line to be awarded.

[4 marks]

Total [7 marks]

16. (a) attempt at chain rule ***(M1)***

$$\left(\boldsymbol{v} = \frac{\mathrm{d}OP}{\mathrm{d}t} = \right) \begin{pmatrix} 2t\cos t^2 \\ -2t\sin t^2 \end{pmatrix} \quad \textbf{A1}$$

[2 marks]

(b) attempt at product rule ***(M1)***

$$\boldsymbol{a} = \begin{pmatrix} 2\cos t^2 - 4t^2 \sin t^2 \\ -2\sin t^2 - 4t^2 \cos t^2 \end{pmatrix} \quad \textbf{A1}$$

METHOD 1

let $S = \sin t^2$ and $C = \cos t^2$

finding $\cos\theta$ using

$$\boldsymbol{a} \bullet \overrightarrow{\mathrm{OP}} = 2SC - 4t^2S^2 - 2SC - 4t^2C^2 = -4t^2 \quad \textbf{M1}$$

$$|\overrightarrow{\mathrm{OP}}| = 1$$

$$|\boldsymbol{a}| = \sqrt{\left(2C - 4t^2S\right)^2 + \left(-2S - 4t^2C\right)^2}$$

$$= \sqrt{4 + 16t^4} > 4t^2$$

if θ is the angle between them, then

$$\cos\theta = -\frac{4t^2}{\sqrt{4+16t^4}} \quad \textbf{A1}$$

so $-1 < \cos\theta < 0$ therefore the vectors are never parallel ***R1***

continued…

Question 16 continued

METHOD 2

solve

$$\begin{pmatrix} 2\cos t^2 - 4t^2 \sin t^2 \\ -2\sin t^2 - 4t^2 \cos t^2 \end{pmatrix} = k \begin{pmatrix} \sin t^2 \\ \cos t^2 \end{pmatrix}$$ ***M1***

then

$$(k =)\ \frac{2\cos t^2 - 4t^2 \sin t^2}{\sin t^2} = \frac{-2\sin t^2 - 4t^2 \cos t^2}{\cos t^2}$$

Note: Condone candidates not excluding the division by zero case here. Some might go straight to the next line.

$$2\cos^2 t^2 - 4t^2 \cos t^2 \sin t^2 = -2\sin^2 t^2 - 4t^2 \cos t^2 \sin t^2$$

$$2\cos^2 t^2 + 2\sin^2 t^2 = 0$$

$$2 = 0$$ ***A1***

this is never true so the two vectors are never parallel ***R1***

METHOD 3

embedding vectors in a 3d space and taking the cross product: ***M1***

$$\begin{pmatrix} \sin t^2 \\ \cos t^2 \\ 0 \end{pmatrix} \times \begin{pmatrix} 2\cos t^2 - 4t^2 \sin t^2 \\ -2\sin t^2 - 4t^2 \cos t^2 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -2\sin^2 t^2 - 4t^2 \cos t^2 \sin t^2 - 2\cos^2 t^2 + 4t^2 \cos t^2 \sin t^2 \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ 0 \\ -2 \end{pmatrix}$$ ***A1***

since the cross product is never zero, the two vectors are never parallel ***R1***

[5 marks]

Total [7 marks]

17. (a) use of chain rule ***(M1)***

$\frac{\mathrm{d}y}{\mathrm{d}t} = \frac{\mathrm{d}y}{\mathrm{d}x}\frac{\mathrm{d}x}{\mathrm{d}t}$

attempt to find $\frac{\mathrm{d}y}{\mathrm{d}x}$ at $x=1$ ***(M1)***

$0.2 = 0.04 \times \frac{\mathrm{d}x}{\mathrm{d}t}$

$\left(\frac{\mathrm{d}x}{\mathrm{d}t} =\right)\ 5\ \mathrm{m\,h^{-1}}$ ***A1***

[3 marks]

(b) (i) if the position of the snail is (X, Y)

from part (a) $\frac{\mathrm{d}X}{\mathrm{d}t} = \frac{1}{0.04X}\frac{\mathrm{d}Y}{\mathrm{d}t}$

since speed is 1:

finding modulus of velocity vector and equating to 1 ***(M1)***

$1 = \sqrt{\left(\frac{\dot{Y}}{0.04X}\right)^2 + \dot{Y}^2}$ **OR** $1 = \sqrt{\dot{X}^2 + 0.0016X^2\dot{X}^2}$

$1 = \dot{Y}^2\left(\frac{1}{0.0016X^2} + 1\right)$ **OR** $1 = \dot{X}^2(1 + 0.0016X^2)$

$\dot{Y} = \sqrt{\frac{1}{\frac{1}{0.08Y} + 1}}$ **OR** $\dot{X} = \sqrt{\frac{1}{1 + 0.0016X^2}}$ ***(A1)***

$\int_{0.02}^{2} \sqrt{\frac{1}{0.08Y} + 1}\ \mathrm{d}Y = \int_{0}^{T} \mathrm{d}t$ **OR** $\int_{1}^{10} \sqrt{1 + 0.0016X^2}\ \mathrm{d}X = \int_{0}^{T} \mathrm{d}t$ ***(M1)***

$T = 9.26$ hours ***A1***

(ii) **EITHER**

time for water to reach top is $\frac{2}{0.2} = 10$ hours (seen anywhere) ***A1***

OR

or at time $t = 9.26$, height of water is $0.2 \times 9.26 = 1.852$ ***A1***

THEN

so the water will not reach the snail ***AG***

[5 marks]

Total [8 marks]

Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

May 2022

Mathematics: applications and interpretation

Higher level

Paper 1

23 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**.
A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.
R Marks awarded for clear **Reasoning**.
AG Answer given in the question and so no marks are awarded.
FT Follow through. The practice of awarding marks, despite candidate errors in previous parts, for their correct methods/answers using incorrect results.

Using the markscheme

1 General

Award marks using the annotations as noted in the markscheme *eg* ***M1, A2.***

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***A3***, ***M2*** *etc.*, do **not** split the marks, unless there is a note.
- The response to a "show that" question does not need to restate the ***AG*** line, unless a **Note** makes this explicit in the markscheme.
- Once a correct answer to a question or part question is seen, ignore further working even if this working is incorrect and/or suggests a misunderstanding of the question. This will encourage a uniform approach to marking, with less examiner discretion. Although some candidates may be advantaged for that specific question item, it is likely that these candidates will lose marks elsewhere too.
- An exception to the previous rule is when an incorrect answer from further working is used **in a subsequent part**. For example, when a correct exact value is followed by an incorrect decimal approximation in the first part and this approximation is then used in the second part. In this situation, award ***FT*** marks as appropriate but do not award the final ***A1*** in the first part. Examples:

	Correct answer seen	**Further working seen**	**Any FT issues?**	**Action**
1.	$8\sqrt{2}$	5.65685... *(incorrect decimal value)*	No. Last part in question.	Award ***A1*** for the final mark *(condone the incorrect further working)*
2.	$\frac{35}{72}$	0.468111… (incorrect decimal value)	Yes. Value is used in subsequent parts.	Award ***A0*** for the final mark *(and full **FT** is available in subsequent parts)*

3 Implied marks

Implied marks appear in **brackets e.g.** ***(M1)***,and can only be awarded if **correct** work is seen or implied by subsequent working/answer.

4 Follow through marks (only applied after an error is made)

Follow through (***FT***) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) (e.g. incorrect value from part (a) used in part (d) or incorrect value from part (c)(i) used in part (c)(ii)). Usually, to award ***FT*** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if all the marks awarded in a subsequent part are for the answer or are implied, then ***FT*** marks should be awarded for *their* correct answer, even when working is not present.

For example: following an incorrect answer to part (a) that is used in subsequent parts, where the markscheme for the subsequent part is ***(M1)A1***, it is possible to award full marks for *their* correct answer, **without working being seen.** For longer questions where all but the answer marks are implied this rule applies but may be overwritten by a **Note** in the Markscheme.

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks, by reflecting on what each mark is for and how that maps to the simplified version.
- If the error leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word “their” in a description, to indicate that candidates may be using an incorrect value.
- If the candidate’s answer to the initial question clearly contradicts information given in the question, it is not appropriate to award any ***FT*** marks in the subsequent parts. This includes when candidates fail to complete a “show that” question correctly, and then in subsequent parts use their incorrect answer rather than the given value.
- Exceptions to these ***FT*** rules will be explicitly noted on the markscheme.
- If a candidate makes an error in one part but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the command term was “Hence”.

5 Mis-read

If a candidate incorrectly copies values or information from the question, this is a mis-read (***MR***). A candidate should be penalized only once for a particular misread. Use the ***MR*** stamp to indicate that this has been a misread and do not award the first mark, even if this is an ***M*** mark, but award all others as appropriate.

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.
- If a candidate uses a correct answer, to a "show that" question, to a higher degree of accuracy than given in the question, this is NOT a misread and full marks may be scored in the subsequent part.
- ***MR*** can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If the command term is 'Hence' and not 'Hence or otherwise' then alternative methods are not permitted unless covered by a note in the mark scheme.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**.

7 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation** for example 1.9 and $1{,}9$ or 1000 and $1{,}000$ and 1.000.
- Do not accept final answers written using calculator notation. However, ***M*** marks and intermediate ***A*** marks can be scored, when presented using calculator notation, provided the evidence clearly reflects the demand of the mark.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, some **equivalent** answers will generally appear in brackets. Not all equivalent notations/answers/methods will be presented in the markscheme and examiners are asked to apply appropriate discretion to judge if the candidate work is equivalent.

8 Format and accuracy of answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. If the level of accuracy is not stated in the question, the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures*.

Where values are used in subsequent parts, the markscheme will generally use the exact value, however candidates may also use the correct answer in subsequent parts. The markscheme will often explicitly include the subsequent values that come "*from the use of 3 sf values*".

Simplification of final answers: Candidates are advised to give final answers using good mathematical form. In general, for an ***A*** mark to be awarded, arithmetic should be completed, and any values that lead to integers should be simplified; for example, $\sqrt{\frac{25}{4}}$ should be written as $\frac{5}{2}$. An exception to this is simplifying fractions, where lowest form is not required (although the numerator and the denominator must be integers); for example, $\frac{10}{4}$ may be left in this form or written as $\frac{5}{2}$. However, $\frac{10}{5}$ should be written as 2, as it simplifies to an integer.

Algebraic expressions should be simplified by completing any operations such as addition and multiplication, e.g. $4e^{2x} \times e^{3x}$ should be simplified to $4e^{5x}$, and $4e^{2x} \times e^{3x} - e^{4x} \times e^{x}$ should be simplified to $3e^{5x}$. Unless specified in the question, expressions do not need to be factorized, nor do factorized expressions need to be expanded, so $x(x+1)$ and $x^2 + x$ are both acceptable.

Please note: intermediate ***A*** marks do NOT need to be simplified.

9 Calculators

A GDC is required for this paper, but If you see work that suggests a candidate has used any calculator not approved for IB DP examinations (eg CAS enabled devices), please follow the procedures for malpractice.

10. Presentation of candidate work

Crossed out work: If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work unless an explicit note from the candidate indicates that they would like the work to be marked.

More than one solution: Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise. If the layout of the responses makes it difficult to judge, examiners should apply appropriate discretion to judge which is "first".

1. (a) 1.2 metres ***A1***

[1 mark]

(b) $-4.8t^2 + 21t + 1.2 = 0$ ***(M1)***

$(t =)\ 4.43\ \text{s}\ \ (4.431415...\ s)$ ***A1***

Note: If both values for t are seen do not award the ***A1*** mark unless the negative is explicitly excluded.

[2 marks]

(c) $0 \le t \le 4.43$ **OR** $[0,\ 4.43]$ ***A1A1***

Note: Award ***A1*** for correct endpoints and ***A1*** for expressing answer with correct notation. Award at most ***A1A0*** for use of x instead of t.

[2 marks]
[Total 5 marks]

2. (a) $x + y + z = 600$ ***A1***

$15x + 10y + 12z = 7816$ ***A1***

$x = 2y$ ***A1***

Note: Condone other labelling if clear, e.g. a (adult), c (child) and s (student). Accept equivalent, distinct equations e.g. $2y + y + z = 600$.

[3 marks]

(b) $x = 308,\ y = 154,\ z = 138$ ***A1A1***

Note: Award ***A1*** for all three correct values seen, ***A1*** for correctly labelled as x, y or z. Accept answers written in words: e.g. 308 adult tickets.

[2 marks]
[Total 5 marks]

3. (a) H_0: The die is fair **OR** $\mathrm{P}(\text{any number}) = \frac{1}{6}$ **OR** probabilities are equal

H_1: The die is not fair **OR** $\mathrm{P}(\text{any number}) \neq \frac{1}{6}$ **OR** probabilities are not equal ***A1***

[1 mark]

(b) 5 ***A1***

[1 mark]

(c) 10 ***A1***

[1 mark]

(d) $(p\text{-value} =)\ 0.287\ \ (0.28724163\ldots.)$ ***A2***

[2 marks]

(e) $0.287 > 0.05$ ***R1***

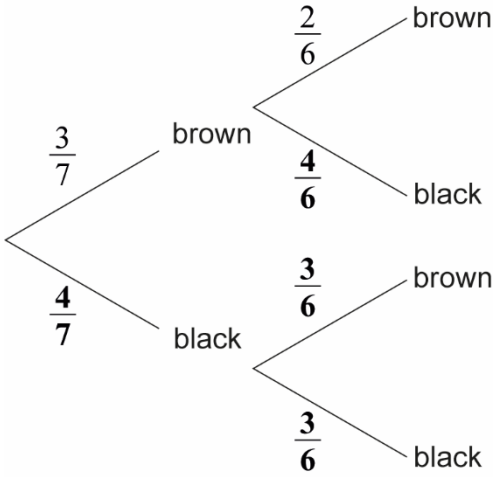
EITHER
Insufficient evidence to reject the null hypothesis ***A1***

OR
Insufficient evidence to reject that the die is fair ***A1***

Note: Do not award ***R0A1***. Condone "*accept* the null hypothesis" or "the die is fair". Their conclusion must be consistent with their p-value and their hypothesis.

[2 marks]
[Total 7 marks]

4. (a)



A1A1

Note: Award ***A1*** for $\frac{4}{7}$ and $\frac{4}{6}$ correctly placed, ***A1*** for final two probabilities correct.

[2 marks]

(b) multiplying along branches and then adding outcomes ***(M1)***

$\frac{3}{7}\times\frac{2}{6}+\frac{4}{7}\times\frac{3}{6}$

$=\frac{18}{42}\left(=\frac{3}{7}\approx 0.429\ (42.9\%)\right)$ ***A1***

[2 marks]

(c) use of conditional probability formula ***M1***

$\frac{\left(\frac{3}{7}\times\frac{2}{6}\right)}{\left(\frac{3}{7}\right)}$ ***A1***

$=\frac{6}{18}\left(=\frac{1}{3}\right)\ \left(\frac{252}{756},\ 0.333,\ 33.3\%\right)$ ***A1***

[3 marks]
[Total 7 marks]

5. (a) use of geometric sequence with $r = 0.85$ ***M1***

EITHER

$(0.85)^6(1.8)$ **OR** $0.678869\ldots$ **OR** $(0.85)^5(1.53)$ ***A1***

$= 0.68 \text{ m}$

$= 68 \text{ cm}$ ***AG***

OR

$(0.85)^6(180)$ **OR** $(0.85)^5(153)$ ***A1***

$= 68 \text{ cm}$ ***AG***

[2 marks]

(b) **EITHER**

$(0.85)^n(1.8) > 0.1$ **OR** $(0.85)^{n-1}(1.53) > 0.1$ ***(M1)***

Note: If 1.8 m (or 180 cm) is used then ***(M1)*** only awarded for use of n in $(0.85)^n(1.8) > 0.1$.
If 1.53 m (or 153 cm) is used then ***(M1)*** only awarded for use of $n-1$ in $(0.85)^{n-1}(1.53) > 0.1$.

17 ***A1***

OR

$(0.85)^{17}(1.8) = 0.114 \text{ m}$ and $(0.85)^{18}(1.8) = 0.0966 \text{ m}$ ***(M1)***

17 ***A1***

OR

solving $(0.85)^n(1.8) = 0.1$ to find $n = 17.8$ ***(M1)***

17 ***A1***

Note: Evidence of solving may be a graph **OR** the "solver" function **OR** use of logs to solve the equation. Working may use cm.

[2 marks]

continued…

Question 5 continued

(c) **EITHER**

distance (in one direction) travelled between first and fourth bounce

$= \frac{(1.8 \times 0.85)(1 - 0.85^3)}{1 - 0.85}$ $(= 3.935925)$ ***(A1)***

recognizing distances are travelled twice except first distance ***(M1)***

$1.8 + 2(3.935925)$

$= 9.67 \text{ m}$ $(9.67185... \text{ m})$ ***A1***

OR

distance (in one direction) travelled between drop and fourth bounce

$= \frac{(1.8)(1 - 0.85^4)}{1 - 0.85}$ $(= 5.735925)$ ***(A1)***

recognizing distances are travelled twice except first distance ***(M1)***

$2(5.735925) - 1.8$

$= 9.67 \text{ m}$ $(9.67185... \text{ m})$ ***A1***

OR

distance (in one direction) travelled between first and fourth bounce

$(0.85)(1.8) + (0.85)^2(1.8) + (0.85)^3(1.8)$ $(= 3.935925...)$ ***(A1)***

recognizing distances are travelled twice except first distance ***(M1)***

$1.8 + 2(0.85)(1.8) + 2(0.85)^2(1.8) + 2(0.85)^3(1.8)$

$= 9.67 \text{ m}$ $(9.67185... \text{ m})$ ***A1***

Note: Answers may be given in cm.

[3 marks]

[Total 7 marks]

6. (a) $\begin{pmatrix} -3.2 \\ -4.5 \\ 6.1 \end{pmatrix}$ ***A1***

[1 mark]

(b) $\sqrt{(-3.2)^2 + (-4.5)^2 + 6.1^2}$ ***(M1)***

$8.22800\ldots \approx 8.23 \text{ m}$ ***A1***

[2 marks]

(c) **EITHER**

$\vec{AO} = \begin{pmatrix} -3.2 \\ -4.5 \\ 0.3 \end{pmatrix}$ ***A1***

$\cos\theta = \dfrac{\vec{AO} \bullet \vec{AF}}{\left|\vec{AO}\right|\left|\vec{AF}\right|}$

$\vec{AO} \bullet \vec{AF} = (-3.2)^2 + (-4.5)^2 + (0.3 \times 6.1) \ (= 32.32)$ ***(A1)***

$\cos\theta = \dfrac{32.32}{\sqrt{3.2^2 + 4.5^2 + 0.3^2} \times 8.22800...}$ ***(M1)***

$= 0.710326\ldots$ ***(A1)***

Note: If $\vec{OA}$ is used in place of $\vec{AO}$ then $\cos\theta$ will be negative.
Award ***A1(A1)(M1)(A1)*** as above. In order to award the final ***A1***, some justification for changing the resulting obtuse angle to its supplementary angle **must** be seen.

OR

$AO = \sqrt{3.2^2 + 4.5^2 + 0.3^2} \ (= 5.52991...)$ ***(A1)***

$\cos\theta = \dfrac{8.22800...^2 + 5.52991...^2 - 5.8^2}{2 \times 8.22800... \times 5.52991...}$ ***(M1)(A1)***

$= 0.710326\ldots$ ***(A1)***

THEN

$\theta = 0.780833... \approx 0.781$ **OR** $44.7384\ldots^\circ \approx 44.7^\circ$ ***A1***

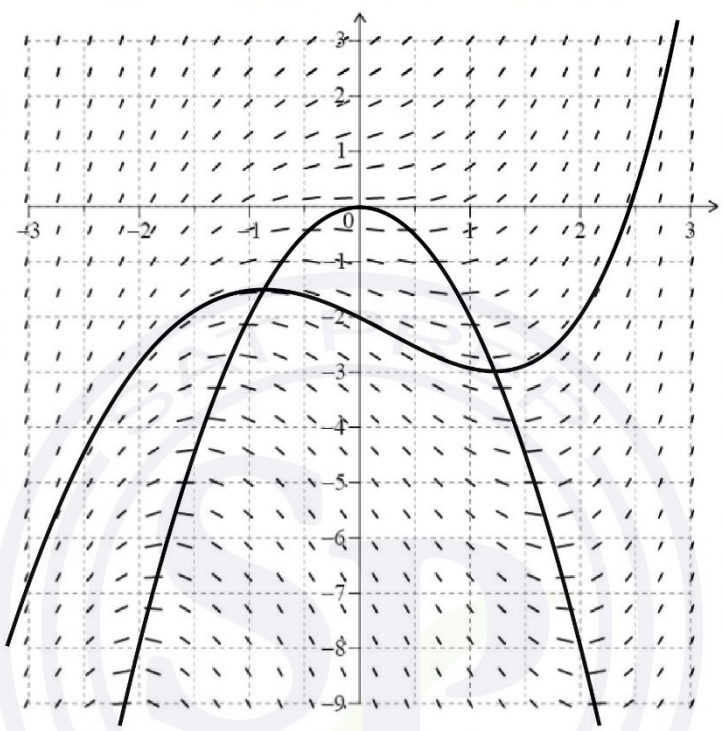
[5 marks]
[Total 8 marks]

7. (a) (i) $x^2 + \frac{y}{2} = 0 \ (y = -2x^2)$ ***A1***

(ii) $y = -2x^2$ drawn on diagram (correct shape with a maximum at $(0, 0)$) ***A1***

[2 marks]

(b)



correct shape with a local maximum and minimum, passing through $(0, -2)$ ***A1***

local maximum and minimum on the graph of $y = -2x^2$ ***A1***

[2 marks]
[Total 4 marks]

8. (a) (i) use of product rule ***(M1)***

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 2\left(4 - \mathrm{e}^{x}\right) + 2x\left(-\mathrm{e}^{x}\right) \quad \textit{A1}$$

$$= 8 - 2\mathrm{e}^{x} - 2x\mathrm{e}^{x}$$

(ii) use of product rule ***(M1)***

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = -2\mathrm{e}^{x} - 2\mathrm{e}^{x} - 2x\mathrm{e}^{x} \quad \textit{A1}$$

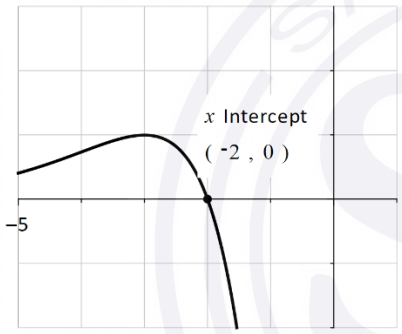
$$= -4\mathrm{e}^{x} - 2x\mathrm{e}^{x}$$

$$= -2(2 + x)\mathrm{e}^{x}$$

[4 marks]

(b) $-2(2+a)\mathrm{e}^{a} = 0$ **OR** sketch of $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ with x-intercept indicated

OR finding the local maximum of $\frac{\mathrm{d}y}{\mathrm{d}x}$ at $(-2, 8.27)$ ***(M1)***



$(a =)\ -2$ ***A1***

[2 marks]
[Total 6 marks]

9. (a) let X be the weight of sugar in the bag

$\mathrm{P}(X < 950) = 0.308537... \approx 0.309$ ***(M1)A1***

[2 marks]

(b) **METHOD 1**

let $\bar{X}$ be the mean weight of 5 bags of sugar

$\mathrm{E}(\bar{X}) = 1000$ ***(A1)***

use of $\mathrm{Var}(\bar{X}) = \frac{\sigma^2}{n}$ ***(M1)***

$\mathrm{Var}(\bar{X}) = \frac{100^2}{5} \ (= 2000)$ ***(A1)***

$\bar{X} \sim \mathrm{N}(1000,\ 2000)$

$\mathrm{P}(\bar{X} > 950) = 0.868223... \approx 0.868 \ (86.8\%)$ ***A1***

METHOD 2

let T be the total weight of 5 bags of sugar

$\mathrm{E}(T) = 5000$ ***(A1)***

use of $\mathrm{Var}(X_1 + X_2) = \mathrm{Var}(X_1) + \mathrm{Var}(X_2)$ for independent random variables ***(M1)***

$\mathrm{Var}(T) = 5 \times 100^2 \ (= 50\,000)$ ***(A1)***

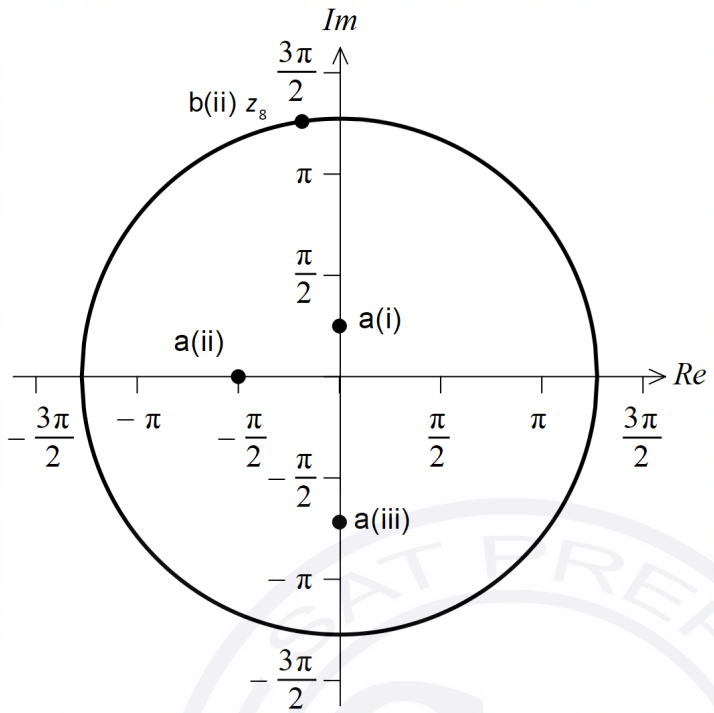
$T \sim \mathrm{N}(5000,\ 50\,000)$

$\mathrm{P}(T > 4750) = 0.868223... \approx 0.868 \ (86.8\%)$ ***A1***

[4 marks]

[Total 6 marks]

10. (a)



A1A1A1

Note: Award ***A1*** for correct modulus and ***A1*** for correct argument for part (a)(i), and ***A1*** for other two points correct.
The points may not be labelled, and they may be shown by line segments.

[3 marks]

(b) (i) $\frac{1}{2}\theta = 4$ ***(M1)***

$\Rightarrow \theta = 8$ ***A1***

(ii) z_8 is shown in the diagram above ***A1A1***

Note: Award ***A1*** for a point plotted on the circle and ***A1*** for a point plotted in the second quadrant.

[4 marks]
[Total 7 marks]

11. (a) $\lambda = 1$

$\begin{pmatrix} -0.8 & 0.7 \\ 0.8 & -0.7 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ **OR** $\begin{pmatrix} 0.2 & 0.7 \\ 0.8 & 0.3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$ ***(M1)***

$0.8x = 0.7y$ ***(A1)***

an eigenvector is $\begin{pmatrix} 7 \\ 8 \end{pmatrix}$ (or equivalent with integer values) ***A1***

[3 marks]

(b) **EITHER**
(the long-term probability matrix is given by the eigenvector corresponding to the eigenvalue equal to 1, scaled so that the sum of the entries is 1)
$8 + 7 = 15$ ***(M1)***

OR
$\begin{pmatrix} 0.2 & 0.7 \\ 0.8 & 0.3 \end{pmatrix}\begin{pmatrix} p \\ 1-p \end{pmatrix} = \begin{pmatrix} p \\ 1-p \end{pmatrix}$ ***(M1)***

OR

considering high powers of the matrix *e.g.* $\begin{pmatrix} 0.2 & 0.7 \\ 0.8 & 0.3 \end{pmatrix}^{50}$ ***(M1)***

$\begin{pmatrix} \frac{7}{15} & \frac{7}{15} \\ \frac{8}{15} & \frac{8}{15} \end{pmatrix}$

THEN

probability of being in state A is $\frac{7}{15}$ ***A1***

[2 marks]
[Total 5 marks]

12. (a) $\log_{10} 100 = a - 3$ ***(M1)***

$a = 5$ ***A1***

[2 marks]

(b) **EITHER**

$N = 10^{5-M}$ ***(M1)***

$= \frac{10^5}{10^M} \left(= \frac{100000}{10^M} \right)$

OR

$100 = \frac{b}{10^3}$ ***(M1)***

THEN

$b = 100000 \ \ (= 10^5)$ ***A1***

[2 marks]

(c) $N = \frac{10^5}{10^{7.2}} = 0.00631 \ \ (0.0063095...)$ ***A1***

Note: Do not accept an answer of $10^{-2.2}$.

[1 mark]

(d) **METHOD 1**

$Y > 100 \Rightarrow$ no earthquakes in the first 100 years ***(M1)***

EITHER

let X be the number of earthquakes of at least magnitude 7.2 in a year

$X \sim \text{Po}(0.0063095\ldots)$

$\left(\text{P}(X = 0)\right)^{100}$ ***(M1)***

OR

let X be the number of earthquakes in 100 years

$X \sim \text{Po}(0.0063095\ldots \times 100)$ ***(M1)***

$\text{P}(X = 0)$

THEN

$0.532 \ (0.532082\ldots)$ ***A1***

METHOD 2

$Y > 100 \Rightarrow$ no earthquakes in the first 100 years ***(M1)***

let X be the number of earthquakes in 100 years

since n is large and p is small

$X \sim \text{B}(100, \ 0.0063095\ldots)$ ***(M1)***

$\text{P}(X = 0)$

$0.531 \ \ (0.531019\ldots)$ ***A1***

[3 marks]

[Total 8 marks]

13. (a) $(\boldsymbol{r} =) \begin{pmatrix} 1 \\ 4 \end{pmatrix} + t \begin{pmatrix} 1.2 \\ -0.6 \end{pmatrix}$ ***A1***

Note: Do not condone the use of λ or any other variable apart from t.

[1 mark]

(b) when the bearing from the port is $045°$, the distance east from the port is equal to the distance north from the port ***(M1)***

$1 + 1.2t = 4 - 0.6t$ ***(A1)***

$1.8t = 3$

$t = \frac{5}{3}$ $\left(1.6666..., 1 \text{ hour } 40 \text{ minutes}\right)$ ***(A1)***

time is 2:40 pm (14:40) ***A1***

[4 marks]

[Total 5 marks]

14. (a) (i) $\frac{1}{u^2} + \frac{2}{u} + 1$ ***A1***

(ii) $\int \left(\frac{1}{(x+2)} + 1 \right)^2 \mathrm{d}x$

$= \int \left(\frac{1}{(x+2)^2} + \frac{2}{x+2} + 1 \right) \mathrm{d}x$ **OR** $\int \left(\frac{1}{u^2} + \frac{2}{u} + 1 \right) \mathrm{d}u$ ***(M1)***

$= -\frac{1}{(x+2)} + 2\ln|x+2| + x(+c)$ ***A1A1***

Note: Award ***A1*** for first expression, ***A1*** for second two expressions.
Award ***A1A0*** for a final answer of $= -\frac{1}{u} + 2\ln(u) + u(+c)$.

[4 marks]

continued…

Question 14 continued

(b) volume $= \pi\left[-\frac{1}{(x+2)} + 2\ln(x+2) + x\right]_0^2$ ***M1***

$= \pi\left(-\frac{1}{4} + 2\ln(4) + 2 + \frac{1}{2} - 2\ln 2\right)$ ***A1***

$= \pi\left(\frac{9}{4} + 2\ln(4) - 2\ln 2\right)$

use of log laws seen, for example ***M1***

$\pi\left(\frac{9}{4} + 4\ln(2) - 2\ln 2\right)$ **OR** $\pi\left(\frac{9}{4} + 2\ln\left(\frac{4}{2}\right)\right)$

$= \frac{\pi}{4}\left(9 + 8\ln(2)\right)$ **OR** $a = 9$, $b = 8$ and $c = 2$ ***A1***

Note: Other correct integer solutions are possible and should be accepted for example $a = 9$, $b = c = 4$.

[4 marks]
Total [8 marks]

15. (a) $X \sim \text{Po}(324)$ ***A1***

Note: Both distribution and mean must be seen for ***A1*** to be awarded.

[1 mark]

(b) $\text{P}(X \leq 300)$ ***(M1)***

$= 0.0946831... \approx 0.0947$ ***A1***

[2 marks]

(c) (mean number of cars =) $4.5 \times 60 = 270$ ***(A1)***

$\text{P}(X > 300 \mid \lambda = 270)$ ***(M1)***

Note: Award ***M1*** for using $\lambda = 270$ to evaluate a probability.

$\text{P}(X \geq 301)$ **OR** $1 - \text{P}(X \leq 300)$ ***(M1)***

$= 0.0334207... \approx 0.0334$ ***A1***

[4 marks]
[Total 7 marks]

16. (a) use of power rule ***(M1)***

$\frac{dW}{dv} = -1.1848v^{-0.84}$ **OR** $-1.18v^{-0.84}$ ***A1***

[2 marks]

(b) $\frac{dv}{dt} = 5$ ***(A1)***

$\frac{dW}{dt} = \frac{dv}{dt} \times \frac{dW}{dv}$ ***(M1)***

$\left(\frac{dW}{dt} = -5 \times 1.1848v^{-0.84}\right)$

when $v = 10$

$\frac{dW}{dt} = -5 \times 1.1848 \times 10^{-0.84}$ ***(M1)***

-0.856 $(-0.856278\ldots)^{\circ}\text{C}\,\text{min}^{-1}$ ***A2***

Note: Accept a negative answer communicated in words, "decreasing at a rate of…".
Accept a final answer of $-0.852809\ldots\,^{\circ}\text{C}\,\text{min}^{-1}$ from use of -1.18.
Accept 51.4 (or 51.2) $^{\circ}\text{C}\,\text{hour}^{-1}$.

[5 marks]
[Total 7 marks]

17. substitute coordinates of A

$f(0) = p\,\mathrm{e}^{q\cos(0)} = 6.5$

$6.5 = p\,\mathrm{e}^{q}$ ***(A1)***

substitute coordinates of B

$f(5.2) = p\mathrm{e}^{q\cos(5.2r)} = 0.2$

EITHER

$f'(t) = -pqr\sin(rt)\,\mathrm{e}^{q\cos(rt)}$ ***(M1)***

minimum occurs when $-pqr\sin(5.2r)\,\mathrm{e}^{q\cos(5.2r)} = 0$

$\sin(rt) = 0$

$r \times 5.2 = \pi$ ***(A1)***

OR

minimum value occurs when $\cos(rt) = -1$ ***(M1)***

$r \times 5.2 = \pi$ ***(A1)***

OR

period = $2 \times 5.2 = 10.4$ ***(A1)***

$r = \dfrac{2\pi}{10.4}$ ***(M1)***

THEN

$r = \dfrac{\pi}{5.2} = 0.604152...\ (0.604)$ ***A1***

$0.2 = p\,\mathrm{e}^{-q}$ ***(A1)***

eliminate p or q ***(M1)***

$\mathrm{e}^{2q} = \dfrac{6.5}{0.2}$ **OR** $0.2 = \dfrac{p^2}{6.5}$

$q = 1.74\ (1.74062...)$ ***A1***

$p = 1.14017...\ (1.14)$ ***A1***

[Total 8 marks]

Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

November 2021

Mathematics: applications and interpretation

Higher level

Paper 1

21 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**.
A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.
R Marks awarded for clear **Reasoning**.
AG Answer given in the question and so no marks are awarded.
FT Follow through. The practice of awarding marks, despite candidate errors in previous parts, for their correct methods/answers using incorrect results.

Using the markscheme

1 General

Award marks using the annotations as noted in the markscheme *eg* ***M1***, ***A2***.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***A3***, ***M2*** *etc.*, do **not** split the marks, unless there is a note.
- The response to a "show that" question does not need to restate the ***AG*** line, unless a **Note** makes this explicit in the markscheme.
- Once a correct answer to a question or part question is seen, ignore further working even if this working is incorrect and/or suggests a misunderstanding of the question. This will encourage a uniform approach to marking, with less examiner discretion. Although some candidates may be advantaged for that specific question item, it is likely that these candidates will lose marks elsewhere too.
- An exception to the previous rule is when an incorrect answer from further working is used **in a subsequent part**. For example, when a correct exact value is followed by an incorrect decimal approximation in the first part and this approximation is then used in the second part. In this situation, award ***FT*** marks as appropriate but do not award the final ***A1*** in the first part. Examples:

	Correct answer seen	**Further working seen**	**Any FT issues?**	**Action**
1.	$8\sqrt{2}$	5.65685... *(incorrect decimal value)*	No. Last part in question.	Award ***A1*** for the final mark *(condone the incorrect further working)*
2.	$\frac{35}{72}$	0.468111… (incorrect decimal value)	Yes. Value is used in subsequent parts.	Award ***A0*** for the final mark *(and full **FT** is available in subsequent parts)*

3 Implied marks

Implied marks appear in **brackets e.g.** ***(M1)***,and can only be awarded if **correct** work is seen or implied by subsequent working/answer.

4 Follow through marks (only applied after an error is made)

Follow through (***FT***) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) (e.g. incorrect value from part (a) used in part (d) or incorrect value from part (c)(i) used in part (c)(ii)). Usually, to award ***FT*** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if all the marks awarded in a subsequent part are for the answer or are implied, then ***FT*** marks should be awarded for *their* correct answer, even when working is not present.

For example: following an incorrect answer to part (a) that is used in subsequent parts, where the markscheme for the subsequent part is ***(M1)A1***, it is possible to award full marks for *their* correct answer, **without working being seen.** For longer questions where all but the answer marks are implied this rule applies but may be overwritten by a **Note** in the Markscheme.

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks, by reflecting on what each mark is for and how that maps to the simplified version.
- If the error leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word “their” in a description, to indicate that candidates may be using an incorrect value.
- If the candidate’s answer to the initial question clearly contradicts information given in the question, it is not appropriate to award any ***FT*** marks in the subsequent parts. This includes when candidates fail to complete a “show that” question correctly, and then in subsequent parts use their incorrect answer rather than the given value.
- Exceptions to these ***FT*** rules will be explicitly noted on the markscheme.
- If a candidate makes an error in one part but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the command term was “Hence”.

5 Mis-read

If a candidate incorrectly copies values or information from the question, this is a mis-read (***MR***). A candidate should be penalized only once for a particular misread. Use the ***MR*** stamp to indicate that this has been a misread and do not award the first mark, even if this is an ***M*** mark, but award all others as appropriate.

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.
- If a candidate uses a correct answer, to a "show that" question, to a higher degree of accuracy than given in the question, this is NOT a misread and full marks may be scored in the subsequent part.
- ***MR*** can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If the command term is 'Hence' and not 'Hence or otherwise' then alternative methods are not permitted unless covered by a note in the mark scheme.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**.

7 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation** for example 1.9 and $1{,}9$ or 1000 and $1{,}000$ and 1.000.

- Do not accept final answers written using calculator notation. However, ***M*** marks and intermediate ***A*** marks can be scored, when presented using calculator notation, provided the evidence clearly reflects the demand of the mark.

- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.

- In the markscheme, some **equivalent** answers will generally appear in brackets. Not all equivalent notations/answers/methods will be presented in the markscheme and examiners are asked to apply appropriate discretion to judge if the candidate work is equivalent.

8 Format and accuracy of answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. If the level of accuracy is not stated in the question, the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures.*

Where values are used in subsequent parts, the markscheme will generally use the exact value, however candidates may also use the correct answer to 3 sf in subsequent parts. The markscheme will often explicitly include the subsequent values that come "*from the use of 3 sf values*".

Simplification of final answers: Candidates are advised to give final answers using good mathematical form. In general, for an ***A*** mark to be awarded, arithmetic should be completed, and any values that lead to integers should be simplified; for example, $\sqrt{\frac{25}{4}}$ should be written as $\frac{5}{2}$. An exception to this is simplifying fractions, where lowest form is not required (although the numerator and the denominator must be integers); for example, $\frac{10}{4}$ may be left in this form or written as $\frac{5}{2}$. However, $\frac{10}{5}$ should be written as 2, as it simplifies to an integer.

Algebraic expressions should be simplified by completing any operations such as addition and multiplication, e.g. $4e^{2x} \times e^{3x}$ should be simplified to $4e^{5x}$, and $4e^{2x} \times e^{3x} - e^{4x} \times e^{x}$ should be simplified to $3e^{5x}$. Unless specified in the question, expressions do not need to be factorized, nor do factorized expressions need to be expanded, so $x(x+1)$ and $x^2 + x$ are both acceptable.

Please note: intermediate ***A*** marks do NOT need to be simplified.

9 Calculators

A GDC is required for this paper, but If you see work that suggests a candidate has used any calculator not approved for IB DP examinations (eg CAS enabled devices), please follow the procedures for malpractice.

10. Presentation of candidate work

Crossed out work: If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work unless an explicit note from the candidate indicates that they would like the work to be marked.

More than one solution: Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise. If the layout of the responses makes it difficult to judge, examiners should apply appropriate discretion to judge which is “first”.

1. (a) $m = \frac{6-0}{4-2} = 3$ ***(M1)A1***

[2 marks]

(b) $(m =)\ -\frac{1}{3}\ (-0.333, -0.333333...)$ ***A1***

[1 mark]

(c) an equation of line with a correct intercept and either of their gradients from (a) or (b) ***(M1)***

e.g. $y = -\frac{1}{3}x + 4$ **OR** $y - 4 = -\frac{1}{3}(x - 0)$

Note: Award ***(M1)*** for substituting either of their gradients from parts (a) or (b) and point B or $(3, 3)$ into equation of a line.

$x + 3y - 12 = 0$ or any integer multiple ***A1***

[2 marks]

(d) $(x =)\ 12$ ***A1***

[1mark]

Total: [6 marks]

2. (a) attempt at using trapezoidal rule formula ***(M1)***

$\frac{1}{2}\left(\frac{2-0}{5}\right)(30 + 50 + 2(50 + 60 + 40 + 20))$ ***A1***

$(\text{total carbon} =)\ 84$ tonnes ***A1***

[3 marks]

(b) $\left|\frac{84-72}{72}\right| \times 100\%$ ***(M1)***

Note: Award ***(M1)*** for correct substitution of final answer in part (a) into percentage error formula.

$= 16.7\%\ (16.6666...\%)$ ***A1***

[2 marks]

Total: [5 marks]

3. (a) $h(4)=\frac{640}{4^2}+0.5$ **OR** $h(14)=\frac{640}{14^2}+0.5$ ***(M1)***

Note: Award ***(M1)*** for substituting 4 or 14 into h. This can be implicit from seeing 3.77 $(3.76530\ldots)$ or 40.5.

$3.77 \le h(x) \le 40.5$ $(3.76530... \le h(x) \le 40.5)$ ***A1A1***

Note: Award ***A1*** for both correct endpoints seen, ***A1*** for the endpoints in a correct interval.

[3 marks]

(b) (i) $h(x)=10$ **OR** $h^{-1}(x)=\sqrt{\frac{640}{x-0.5}}$ **OR** $h^{-1}(10)=\sqrt{\frac{640}{10-0.5}}$ ***(M1)***

$(x=)$ 8.21 cm $(8.20782...)$ ***A1***

(ii) a tin that is 10 cm high will have a diameter of 8.21 cm $(8.20782...)$ ***A1***

Note: Condone a correct answer expressed as the converse.

(iii) $4 \le h^{-1} \le 14$ ***A1***

Note: Accept $4 \le y \le 14$. Do not ***FT*** in this part.

[4 marks]
Total: [7 marks]

4. (a) (the best placement is either point P or point Q)
attempt at using the distance formula ***(M1)***

$\text{AP}=\sqrt{(10-6)^2+(6-2)^2}$ **OR**

$\text{BP}=\sqrt{(10-14)^2+(6-2)^2}$ **OR**

$\text{DP}=\sqrt{(10-10.8)^2+(6-11.6)^2}$ **OR**

$\text{BQ}=\sqrt{(13-14)^2+(7-2)^2}$ **OR**

$\text{CQ}=\sqrt{(13-18)^2+(7-6)^2}$ **OR**

$\text{DQ}=\sqrt{(13-10.8)^2+(7-11.6)^2}$

(AP or BP or DP $=$) $\sqrt{32}=5.66$ $(5.65685...)$ **AND**
(BQ or CQ or DQ $=$) $\sqrt{26}=5.10$ $(5.09901...)$ ***A1***

$\sqrt{32}>\sqrt{26}$ **OR** AP (or BP or DP) is greater than BQ (or CQ or DQ) ***A1***

point P is the furthest away ***AG***

Note: Follow through from their values provided their AP (or BP or DP) is greater than their BQ (or CQ or DQ).

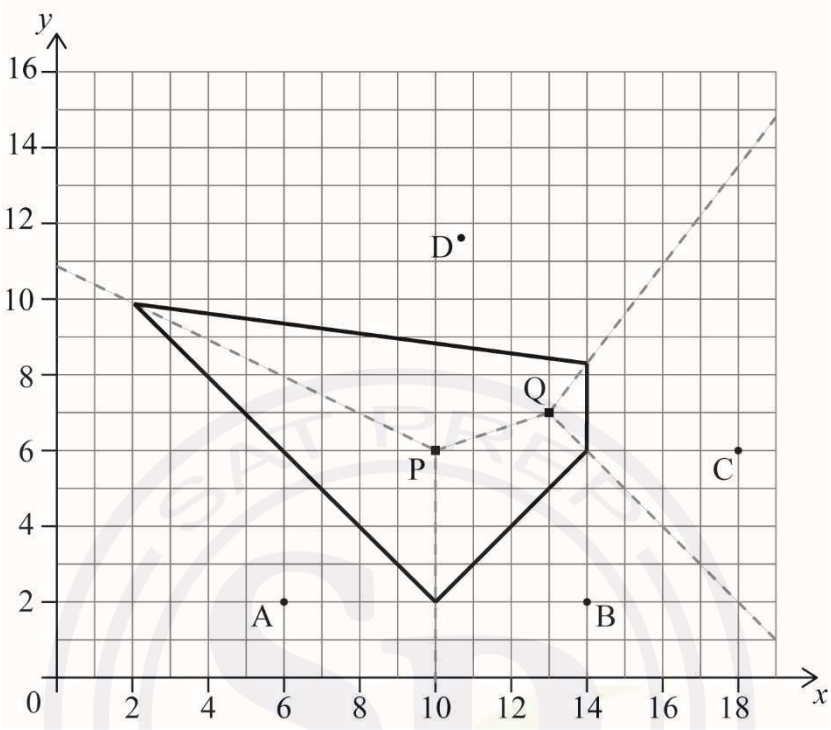
[3 marks]

continued…

Question 4 continued

(b) (i) $x = 14$ ***A1***

(ii)



A1A1

Note: Award ***A1*** for each correct straight line. Do not ***FT*** from their part b(i).

[3 marks]
Total: [6 marks]

5. (a) $N = 360$
$I\% = 3.8$
$PV = (\pm)170\,000$
$FV = 0$
$P/Y = 12$
$C/Y = 12$ ***(M1)(A1)***

Note: Award ***(M1)*** for an attempt to use a financial app in their technology with at least two entries seen, award ***A1*** for all entries correct. Accept a positive or negative value for PV.

$(PMT =)\ 792.13\ \text{AUD}$ ***A1***

Note: Accept an answer of -792.13. Do not award final ***A1*** if answer is not given correct to 2 dp

[3 marks]

(b) (i) $N = 120$
$I\% = 3.8$
$PV = (\pm)170\,000$
$PMT = (\mp)792.13$
$P/Y = 12$
$C/Y = 12$ ***(M1)(A1)***

Note: Award ***(M1)*** for an attempt to use a financial app in their technology with a least two entries seen, award ***A1*** for all entries correct. PV and PMT must have opposite signs.

$(FV =)\ 133019.94\ \text{AUD}$ ***A1***

Note: Do not award final ***A1*** if answer is not given correct to 2 dp, unless already penalized in part (a). Accept 133020.30 from use of exact value for PMT.

(ii) amount of money paid: $120 \times 792.13\ (= 95055.60)$ ***(M1)***
loan paid off: $170\,000 - 133019.94\ (= 36980.06)$ ***(M1)***
interest paid: $(95\,055.60 - 36\,980.06 =)\ 58\,075.54\ \text{AUD}$ ***A1***

Note: Allow 58075.60 or 58075.90 from use of some exact values from parts (a) and (b)(i). If their answer to part (b)(i) is greater than 170 000 then award at most ***(M1)(M1)(A0)*** for follow through in part (b)(ii).

[6 marks]
Total: [9 marks]

6. (a) $10 = \frac{2}{1-r}$ ***(M1)***

$r = 0.8$ ***A1***

[2 marks]

(b) $2\times(0.8)^{n-1} < 0.5$ **OR** $2\times(0.8)^{n-1} = 0.5$ ***(M1)***

$(n >)\ 7.212.....$ ***(A1)***

$n = 8$ ***A1***

Note: If $n = 7$ is seen, with or without seeing the value $7.212\ldots$ then award ***M1A1A0***.

[3 marks]

Total: [5 marks]

7. (a) 75 ***A1***

[1 mark]

(b) recognition that all entries add up to 120 ***(M1)***

$a = 120 - 6 - 13 - 26 - b$ **OR** $a = 75 - b$ ***A1***

[2 marks]

(c) (i) $\frac{6\times1+13\times2+26\times3+(75-b)\times4+b\times5}{120} = 3.65$ ***(M1)(A1)***

Note: Award ***(M1)*** for attempt to substitute into mean formula, LHS expression is sufficient for the ***M*** mark. Award ***(A1)*** for correct substitutions in one variable **OR** in two variables, followed by evidence of solving simultaneously with $a + b = 75$.

$(b =)\ 28$ ***A1***

(ii) $120 -$ their part (c)(i) seen (e.g. 92 indicated on graph) ***(M1)***

84 ***A1***

[5 marks]

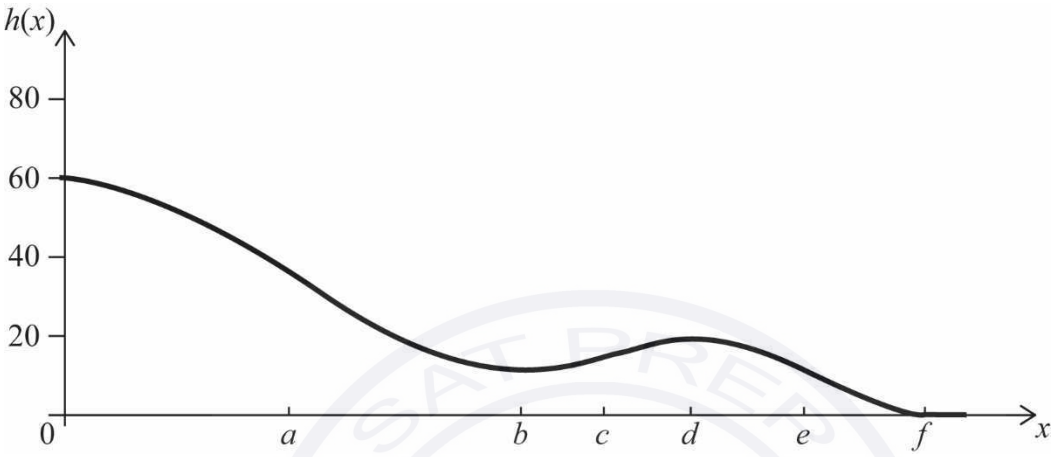
Total: [8 marks]

8. (a) (i) a ***A1***

(ii) the hill is at its steepest / largest slope of hill ***A1***

[2 marks]

(b)



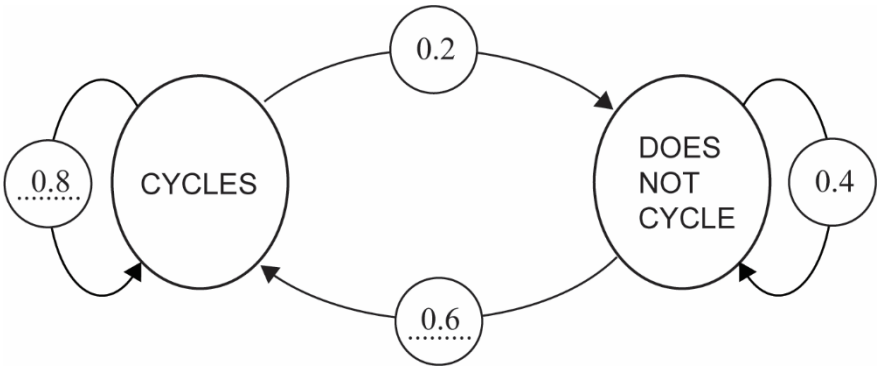
A1A1A1

Note: Award ***(A1)*** for decreasing function from 0 to b and d to f and increasing from b to d; ***(A1)*** for minimum at b and max at d; ***(A1)*** for starting at height of 60 and finishing at a height of 0 at f. If reasonable curvature not evident on graph (i.e. only straight lines used) award ***A1A0A1***.

[3 marks]

Total: [5 marks]

9. (a)



A1A1

[2 marks]

(b) $\boldsymbol{A} = \begin{pmatrix} 0.8 & 0.6 \\ 0.2 & 0.4 \end{pmatrix}$ ***(A1)***

$\boldsymbol{A}^{180} = \begin{pmatrix} 0.75 & 0.75 \\ 0.25 & 0.25 \end{pmatrix}$ ***(M1)***

0.75 ***A1***

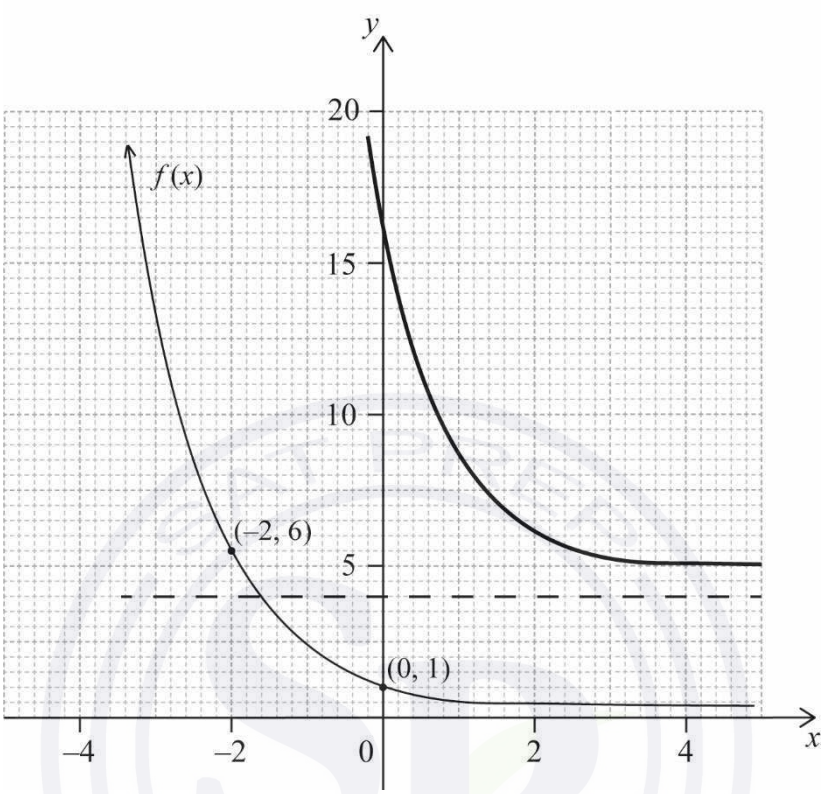
[3 marks]

Total: [5 marks]

10. (a) $g(0) = 16$ ***M1A1***

[2 marks]

(b)



y-asymptote $(y = 4)$ ***A1***

concave up decreasing curve and passing through $(0, 16)$ ***A1***

[2 marks]

Total: [4 marks]

11. METHOD 1

attempt to find AC using cosine rule ***M1***

$7^2 = 10^2 + \mathrm{AC}^2 - 2\times 10\times \mathrm{AC}\times \cos 40^\circ$ ***(A1)***

attempt to solve a quadratic equation ***(M1)***

$\mathrm{AC} = 4.888...$ **AND** $10.432...$ ***(A1)***

Note: At least $\mathrm{AC} = 4.888...$ must be seen, or implied by subsequent working.

minimum area = $\frac{1}{2}\times 10\times 4.888...\times \sin(40^\circ)$ ***M1***

Note: Do not award ***M1*** if incorrect value for minimizing the area has been chosen.

$= 15.7\ \mathrm{m}^2$ ***A1***

METHOD 2

attempt to find $\mathrm{A\hat{C}B}$ using the sine Rule ***M1***

$\frac{\sin C}{10} = \frac{\sin 40}{7}$ ***(A1)***

$C = 66.674...^\circ$ **OR** $113.325....^\circ$ ***(A1)***

EITHER

$B = 180 - 40 - 113.325...$

$B = 26.675...^\circ$ ***(A1)***

area $= \frac{1}{2}\times 10\times 7\times \sin(26.675...^\circ)$ ***M1***

OR

sine rule or cosine rule to find $\mathrm{AC} = 4.888...$ ***(A1)***

minimum area = $\frac{1}{2}\times 10\times 4.888...\times \sin(40^\circ)$ ***M1***

THEN

$= 15.7\ \mathrm{m}^2$ ***A1***

Note: Award ***A0M1A0*** if the wrong length AC or the wrong angle B selected but used correctly finding a value of $33.5\ \mathrm{m}^2$ for the area.

Total: [6 marks]

12. (a) $m = -0.695\ (-0.695383...)$; $b = 4.63\ (4.62974...)$ ***A1A1***

[2 marks]

(b) $\ln x = -0.695(\ln 25) + 4.63$ ***M1***

$\ln x = 2.39288...$ ***(A1)***

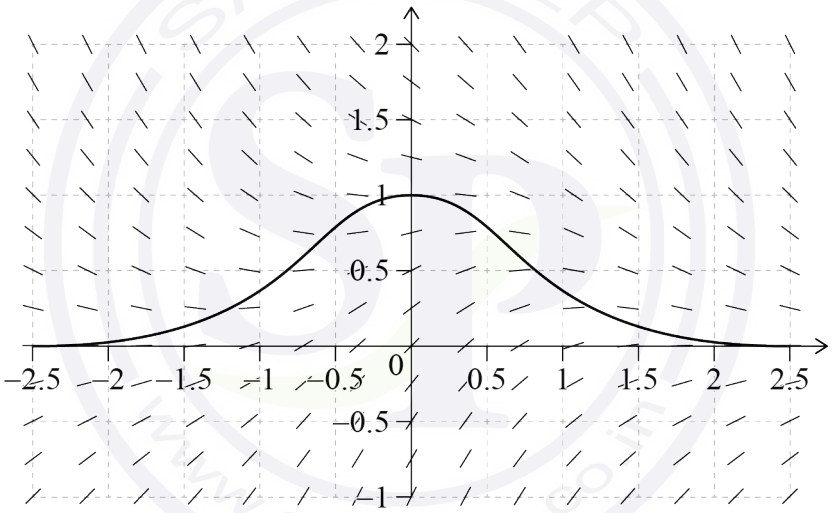
$x = 10.9\%$ ***A1***

[3 marks]
Total: [5 marks]

13. (a) $\left(\frac{dy}{dx} = e^0 - 1\right) = 0$ ***A1***

[1 mark]

(b)



gradient = 0 at (0, 1) ***A1***
correct shape ***A1***

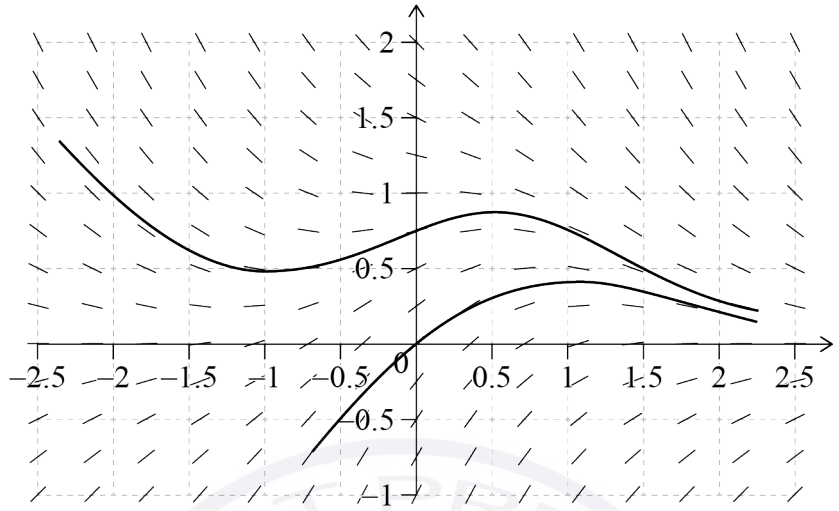
Note:	Award second ***A1*** for horizontal asymptote of $y = 0$, and general symmetry about the y-axis.

[2 marks]

continued…

Question 13 continued

(c)



(i) positive gradient at origin ***A1***
correct shape ***A1***

Note: Award second ***A1*** for a single maximum in 1st quadrant and tending toward an asymptote.

(ii) positive gradient at $(0, 0.75)$ ***A1***
correct shape ***A1***

Note: Award second ***A1*** for a single minimum in 2nd quadrant, single maximum in 1st quadrant and tending toward an asymptote.

[4 marks]
Total: [7 marks]

14. (a) let X be the random variable "the weight of a sack of potatoes"
$\mathrm{P}(X < 50)$ ***(M1)***
$= 0.588 \text{ kg} \quad (0.587929...)$ ***A1***
[2 marks]

(b) $\mathrm{P}(X < l) = 0.25$ ***(M1)***
$49.2 \text{ kg} \ (49.1929\ldots)$ ***A1***
[2 marks]

(c) attempt to sum 10 independent random variables ***(M1)***
$Y = \sum_{i=1}^{10} X_i \sim \mathrm{N}\left(498, 10 \times 0.9^2\right)$ ***(A1)***

$\mathrm{P}(Y > 500) = 0.241$ ***A1***
[3 marks]
Total: [7 marks]

15. (a) $15 = 3 + 4r + 2r\theta$ ***M1***

$12 = 2r(2 + \theta)$ ***A1***

Note: Award ***A1*** for any reasonable working leading to expected result e,g, factorizing r.

$r = \dfrac{6}{2 + \theta}$ ***AG***

[2 marks]

(b) (i) attempt to use sector area to find volume ***(M1)***

volume $= \dfrac{1}{2} r^2 \theta \times 1$

$= \dfrac{1}{2} \times \dfrac{36}{(2+\theta)^2} \times \theta \quad \left(= \dfrac{18\theta}{(2+\theta)^2} \right)$ ***A1***

(ii) $\dfrac{\mathrm{d}V}{\mathrm{d}\theta} = \dfrac{(2+\theta)^2 \times 18 - 36\theta(2+\theta)}{(2+\theta)^4}$ ***M1A1A1***

$\dfrac{\mathrm{d}V}{\mathrm{d}\theta} = \dfrac{36 - 18\theta}{(2+\theta)^3}$

(iii) $\dfrac{\mathrm{d}V}{\mathrm{d}\theta} = \dfrac{36 - 18\theta}{(2+\theta)^3} = 0$ ***M1***

Note: Award this ***M1*** for simplified version equated to zero. The simplified version may have been seen in part (b)(ii).

$\theta = 2$ ***A1***

[7 marks]

Total: [9 marks]

16. (a) $\overrightarrow{OS} = \begin{pmatrix} 300 \\ 100 \end{pmatrix} + t\begin{pmatrix} -12 \\ 15 \end{pmatrix}$ ***A1***

[1 mark]

(b) attempt to find the vector from L to S ***(M1)***

$\overrightarrow{LS} = \begin{pmatrix} 171 \\ -183 \end{pmatrix} + t\begin{pmatrix} -12 \\ 15 \end{pmatrix}$ ***A1***

EITHER

$\left|\overrightarrow{LS}\right| = \sqrt{(171-12t)^2 + (15t-183)^2}$ ***(M1)(A1)***

minimize to find t on GDC ***(M1)***

OR

S closest when $\overrightarrow{LS} \bullet \begin{pmatrix} -12 \\ 15 \end{pmatrix} = 0$ ***(M1)***

$\left(\begin{pmatrix} 171 \\ -183 \end{pmatrix} + t\begin{pmatrix} -12 \\ 15 \end{pmatrix}\right) \bullet \begin{pmatrix} -12 \\ 15 \end{pmatrix} = 0$

$-2052 + 144t - 2745 + 225t = 0$ ***(M1)(A1)***

OR

S closest when $\overrightarrow{LS} \bullet \begin{pmatrix} -12 \\ 15 \end{pmatrix} = 0$ ***(M1)***

$\overrightarrow{LS} = \begin{pmatrix} 5k \\ 4k \end{pmatrix}$

$\overrightarrow{OS} = \begin{pmatrix} 129 + 5k \\ 283 + 4k \end{pmatrix}$ ***(A1)***

$\begin{pmatrix} 129 + 5k \\ 283 + 4k \end{pmatrix} = \begin{pmatrix} 300 - 12t \\ 100 + 15t \end{pmatrix}$

Solving simultaneously ***(M1)***

THEN

$t = 13$ ***A1***

[6 marks]

(c) the alarm will sound ***A1***

$|\overrightarrow{LS}| = 19.2.... < 20$ ***R1***

Note: Do not award ***A1R0***.

[2 marks]

Total: [9 marks]

17. (a) attempt to use $V = \pi\int_a^b x^2\,\mathrm{d}y$ ***(M1)***

$x = \mathrm{e}^{\frac{y}{6}}$ or any reasonable attempt to find x in terms of y ***(M1)***

$V = \pi\int_0^h \mathrm{e}^{\frac{y}{3}}\,\mathrm{d}y$ ***A1***

Note: Correct limits must be seen for the ***A1*** to be awarded.

$= \pi\left[3\mathrm{e}^{\frac{y}{3}}\right]_0^h$ ***(A1)***

Note: Condone the absence of limits for this ***A1*** mark.

$= 3\pi\left[\mathrm{e}^{\frac{h}{3}} - \mathrm{e}^0\right]$ ***A1***

$= 3\pi\left[\mathrm{e}^{\frac{h}{3}} - 1\right]$ ***AG***

Note: If the variable used in the integral is x instead of y (i.e. $V = \pi\int_0^h \mathrm{e}^{\frac{x}{3}}\mathrm{d}x$) and the candidate has not stated that they are interchanging x and y then award at most ***M1M1A0A1A1AG***.

[5 marks]

(b) maximum volume when $h = 9$ cm ***(M1)***

max volume $= 180$ cm^3 ***A1***

[2 marks]
Total: [7 marks]

Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

May 2021

Mathematics: applications and interpretation

Higher level

Paper 1

22 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**.
A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.
R Marks awarded for clear **Reasoning**.
AG Answer given in the question and so no marks are awarded.
FT Follow through. The practice of awarding marks, despite candidate errors in previous parts, for their correct methods/answers using incorrect results.

Using the markscheme

1 General

Award marks using the annotations as noted in the markscheme *eg* ***M1***, ***A2***.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***A3***, ***M2*** *etc.*, do **not** split the marks, unless there is a note.
- The response to a "show that" question does not need to restate the ***AG*** line, unless a **Note** makes this explicit in the markscheme.
- Once a correct answer to a question or part question is seen, ignore further working even if this working is incorrect and/or suggests a misunderstanding of the question. This will encourage a uniform approach to marking, with less examiner discretion. Although some candidates may be advantaged for that specific question item, it is likely that these candidates will lose marks elsewhere too.
- An exception to the previous rule is when an incorrect answer from further working is used **in a subsequent part**. For example, when a correct exact value is followed by an incorrect decimal approximation in the first part and this approximation is then used in the second part. In this situation, award ***FT*** marks as appropriate but do not award the final ***A1*** in the first part. Examples:

	Correct answer seen	**Further working seen**	**Any FT issues?**	**Action**
1.	$8\sqrt{2}$	5.65685... *(incorrect decimal value)*	No. Last part in question.	Award ***A1*** for the final mark *(condone the incorrect further working)*
2.	$\frac{35}{72}$	0.468111… (incorrect decimal value)	Yes. Value is used in subsequent parts.	Award ***A0*** for the final mark *(and full **FT** is available in subsequent parts)*

3 Implied marks

Implied marks appear in **brackets e.g.** ***(M1)***,and can only be awarded if **correct** work is seen or implied by subsequent working/answer.

4 Follow through marks (only applied after an error is made)

Follow through (***FT***) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) (e.g. incorrect value from part (a) used in part (d) or incorrect value from part (c)(i) used in part (c)(ii)). Usually, to award ***FT*** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if all the marks awarded in a subsequent part are for the answer or are implied, then ***FT*** marks should be awarded for *their* correct answer, even when working is not present.

For example: following an incorrect answer to part (a) that is used in subsequent parts, where the markscheme for the subsequent part is ***(M1)A1***, it is possible to award full marks for *their* correct answer, **without working being seen.** For longer questions where all but the answer marks are implied this rule applies but may be overwritten by a **Note** in the Markscheme.

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks, by reflecting on what each mark is for and how that maps to the simplified version.
- If the error leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word “their” in a description, to indicate that candidates may be using an incorrect value.
- If the candidate’s answer to the initial question clearly contradicts information given in the question, it is not appropriate to award any ***FT*** marks in the subsequent parts. This includes when candidates fail to complete a “show that” question correctly, and then in subsequent parts use their incorrect answer rather than the given value.
- Exceptions to these ***FT*** rules will be explicitly noted on the markscheme.
- If a candidate makes an error in one part but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the command term was “Hence”.

5 Mis-read

If a candidate incorrectly copies values or information from the question, this is a mis-read (***MR***). A candidate should be penalized only once for a particular misread. Use the ***MR*** stamp to indicate that this has been a misread and do not award the first mark, even if this is an ***M*** mark, but award all others as appropriate.

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.
- If a candidate uses a correct answer, to a "show that" question, to a higher degree of accuracy than given in the question, this is NOT a misread and full marks may be scored in the subsequent part.
- ***MR*** can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If the command term is 'Hence' and not 'Hence or otherwise' then alternative methods are not permitted unless covered by a note in the mark scheme.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**.

7 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation** for example 1.9 and $1{,}9$ or 1000 and $1{,}000$ and 1.000.

- Do not accept final answers written using calculator notation. However, ***M*** marks and intermediate ***A*** marks can be scored, when presented using calculator notation, provided the evidence clearly reflects the demand of the mark.

- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.

- In the markscheme, some **equivalent** answers will generally appear in brackets. Not all equivalent notations/answers/methods will be presented in the markscheme and examiners are asked to apply appropriate discretion to judge if the candidate work is equivalent.

8 Format and accuracy of answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. If the level of accuracy is not stated in the question, the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures.*

Where values are used in subsequent parts, the markscheme will generally use the exact value, however candidates may also use the correct answer to 3 sf in subsequent parts. The markscheme will often explicitly include the subsequent values that come "*from the use of 3 sf values*".

Simplification of final answers: Candidates are advised to give final answers using good mathematical form. In general, for an ***A*** mark to be awarded, arithmetic should be completed, and any values that lead to integers should be simplified; for example, $\sqrt{\frac{25}{4}}$ should be written as $\frac{5}{2}$. An exception to this is simplifying fractions, where lowest form is not required (although the numerator and the denominator must be integers); for example, $\frac{10}{4}$ may be left in this form or written as $\frac{5}{2}$. However, $\frac{10}{5}$ should be written as 2, as it simplifies to an integer.

Algebraic expressions should be simplified by completing any operations such as addition and multiplication, e.g. $4e^{2x} \times e^{3x}$ should be simplified to $4e^{5x}$, and $4e^{2x} \times e^{3x} - e^{4x} \times e^{x}$ should be simplified to $3e^{5x}$. Unless specified in the question, expressions do not need to be factorized, nor do factorized expressions need to be expanded, so $x(x+1)$ and $x^2 + x$ are both acceptable.

Please note: intermediate ***A*** marks do NOT need to be simplified.

9 Calculators

A GDC is required for this paper, but If you see work that suggests a candidate has used any calculator not approved for IB DP examinations (eg CAS enabled devices), please follow the procedures for malpractice.

10. Presentation of candidate work

Crossed out work: If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work unless an explicit note from the candidate indicates that they would like the work to be marked.

More than one solution: Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise. If the layout of the responses makes it difficult to judge, examiners should apply appropriate discretion to judge which is "first".

1. (a) gradient $\text{AB} = \frac{4}{12}\ \left(\frac{1}{3}\right)$ ***(A1)***

midpoint $\text{AB}: (8, 22)$ ***(A1)***

gradient of bisector $= -\frac{1}{\text{gradient AB}} = -3$ ***(M1)***

perpendicular bisector: $22 = -3 \times 8 + b$ **OR** $(y - 22) = -3(x - 8)$ ***(M1)***

perpendicular bisector: $y = -3x + 46$ ***A1***

[5 marks]

(b) attempt to solve simultaneous equations ***(M1)***

$x + 4 = -3x + 46$

$(10.5, 14.5)$ ***A1***

[2 marks]

Total [7 marks]

2. (a) $\left(f(-7) =\right)\ 8$ and $\left(f(7) =\right)\ 1$ ***(A1)***

range is $f(x) \leq 1,\ f(x) \geq 8$ ***A1A1***

Note: Award at most ***A1A1A0*** if strict inequalities are used.

[3 marks]

(b) interchanging x, y at any stage ***(A1)***

$y = 2 - \frac{12}{x + 5}$

$\frac{12}{x + 5} = 2 - y$

$\frac{12}{2 - y} = x + 5$ ***(A1)***

$\frac{12}{2 - y} - 5 = x$

$\left(f^{-1}(x) =\right)\ \frac{12}{2 - x} - 5\ \left(= \frac{2 + 5x}{2 - x}\right)$ ***A1***

[3 marks]

(c) range is $-7 \leq f^{-1}(x) \leq 7,\ f^{-1}(x) \neq -5$ ***A1***

[1 mark]

Total [7 marks]

3. (a) (let $\mu_c =$ population mean for chinchilla rabbits, $\mu_s =$ population mean for sable rabbits)

$H_0 : \mu_c = \mu_s$ ***A1***

$H_1 : \mu_c > \mu_s$ ***A1***

Note: Accept an equivalent statement in words, must include mean and reference to "**population** mean" / "mean for **all** chinchilla rabbits" for the first ***A1*** to be awarded. The terms "*on average"* and "*generally"* are also acceptable to indicate populations.
Do not accept an imprecise *"the means are equal".*

[2 marks]

(b) p-value $= 0.0408$ $(0.0408065...)$ ***A2***

Note: Award ***A1*** for an answer of $0.041565\ldots$, from "unpooled" settings on GDC.

[2 marks]

(c) $0.0408 < 0.05$. ***R1***

(there is sufficient evidence to) reject (or not accept) H_0 ***A1***

(there is sufficient evidence to suggest that chinchilla rabbits are (generally) heavier than sable rabbits)

Note: Do not award ***R0A1***. Accept 'accept H_1'.

[2 marks]

Total [6 marks]

4. (a) $\text{AC} = \dfrac{380}{\tan 25^\circ}$ **OR** $\text{AC} = \sqrt{\left(\dfrac{380}{\sin 25^\circ}\right)^2 - 380^2}$ **OR** $\dfrac{380}{\sin 25^\circ} = \dfrac{\text{AC}}{\sin 65^\circ}$ ***(M1)***

$\text{AC} = 815 \text{ m } (814.912\ldots)$ ***A1***

[2 marks]

(b) **METHOD 1**

attempt to find AB ***(M1)***

$\text{AB} = \dfrac{380}{\tan 40^\circ}$

$= 453 \text{ m } (452.866\ldots)$ ***(A1)***

$\text{BC} = 814.912\ldots - 452.866\ldots$

$= 362 \text{ m } (362.046\ldots)$ ***A1***

METHOD 2

attempt to find HB ***(M1)***

$\text{HB} = \dfrac{380}{\sin 40^\circ}$

$591 \text{ m } (= 591.175...)$ ***(A1)***

$\text{BC} = \dfrac{591.175... \times \sin 15^\circ}{\sin 25^\circ}$

$= 362 \text{ m } (362.046...)$ ***A1***

[3 marks]

(c) $362.046\ldots \times 4$

$= 1450 \text{ m h}^{-1} \ (1448.18\ldots)$ ***A1***

[1 mark]

Total [6 marks]

5. (a) $£495 \times 0.9^5 = £292 \ (£292.292...)$ ***(M1)A1***

[2 marks]

(b) $495 \times 0.9^k = 2200 \times 0.85^k$ ***(M1)***

$k = 26.1 \ (26.0968...)$ ***A1***

Note: Award ***M1A0*** for $k - 1$ in place of k.

[2 marks]

(c) depreciation rates unlikely to be constant (especially over a long time period) ***R1***

Note: Accept reasonable answers based on the magnitude of k or the fact that "value" depends on factors other than time.

[1 mark]

Total [5 marks]

6. (a) 3 ***A1***

Note: Accept $(3, 0)$ seen.

[1 mark]

(b) **METHOD 1**

$0 = 4a - 2b + c$, $0 = 9a + 3b + c$, $-\frac{25}{2} = \frac{1}{4}a + \frac{1}{2}b + c$ ***(M1)(A1)***

(i) 2 ***A1***

(ii) -2 ***A1***

(iii) -12 ***A1***

Note: Award the ***(M1)(A1)*** if at least one correct value is seen.
Do not apply ***FT*** form part (a) if workings are not shown.

METHOD 2

$-12.5 = a(0.5 + 2)(0.5 - 3)$ ***(M1)***

(i) $a = 2$ ***A1***

$0 = 2x(3)^2 + 3b + c$
$0 = 2x(-2)^2 + (-2)b + c$ ***(M1)***

(ii) $b = -2$ ***A1***

(iii) $c = -12$ ***A1***

[5 marks]

(c) $x = 0.5$ ***A1***

Note: Do not ***FT*** from their part (b), this is a contradiction with the diagram.

[1 mark]

Total [7 marks]

7. (a) recognition of geometric sequence *eg* $r = 0.82$ ***(M1)***

$$S_{10} = \frac{450\left(1 - 0.82^{10}\right)}{1 - 0.82}$$ ***(A1)***

$= 2160 \text{ m} \ (2156.37...)$ ***A1***

[3 marks]

(b) $S_{\infty} = \frac{450}{1 - 0.82}$ ***(M1)***

$= 2500 < 2520$ so the balloon will not reach the required height. ***A1***

[2 marks]

(c) horizontal motion not taken into account,
rate of cooling will not likely be linear,
balloon is considered a point mass / size of balloon not considered,
effects of wind/weather unlikely to be consistent,
a discrete model has been used, whereas a continuous one may offer greater accuracy ***R1***

Note: Accept any other sensible answer.

[1 mark]

Total [6 marks]

8. (a) setting a dot product of the direction vectors equal to zero ***(M1)***

$$\begin{pmatrix} p \\ 2p \\ 4 \end{pmatrix} . \begin{pmatrix} p+4 \\ 4 \\ -7 \end{pmatrix} = 0$$

$p(p+4)+8p-28=0$ ***(A1)***

$p^2+12p-28=0$

$(p+14)(p-2)=0$

$p=-14,\ p=2$ ***A1***

[3 marks]

(b) $p=-14 \Rightarrow$

$$L_1 : r = \begin{pmatrix} 2 \\ -5 \\ -3 \end{pmatrix} + \lambda \begin{pmatrix} -14 \\ -28 \\ 4 \end{pmatrix}$$

$$L_2 : r = \begin{pmatrix} 14 \\ 7 \\ -2 \end{pmatrix} + \mu \begin{pmatrix} -10 \\ 4 \\ -7 \end{pmatrix}$$

a common point would satisfy the equations

$2-14\lambda = 14-10\mu$

$-5-28\lambda = 7+4\mu$ ***(M1)***

$-3+4\lambda = -2-7\mu$

METHOD 1

solving the first two equations simultaneously

$\lambda = -\frac{1}{2}, \mu = \frac{1}{2}$ ***A1***

substitute into the third equation: ***M1***

$$-3+4\left(-\frac{1}{2}\right) \neq -2+\frac{1}{2}(-7)$$

so lines do not intersect. ***R1***

Note: Accept equivalent methods based on the order in which the equations are considered.

METHOD 2

attempting to solve the equations using a GDC ***M1***

GDC indicates no solution ***A1***

so lines do not intersect ***R1***

[4 marks]

Total [7 marks]

9. (a) $\left(\frac{74+97+91+86+112}{5}\right)=92$ ***A1***

[1 mark]

(b) (i) H_0 : The data satisfies the model ***A1***

H_1 : The data does not satisfy the model ***A1***

Note: Do not accept "H_0 : The same number of copies will be sold each day" but accept a similar statement if the word 'expect' or 'expected' is included. Similarly for H_1.

(ii) 4 ***A1***

(iii) $\chi^2_{\text{calc}} = 8.54 \ (8.54347...)$ **OR** $p\text{-value} = 0.0736 \ (0.0735802...)$ ***A2***

$8.54 < 9.49$ **OR** $0.0736 > 0.05$ ***R1***

therefore there is insufficient evidence to reject H_0 ***A1***

(i.e. the data satisfies the model)

Note: Do not award ***R0A1***. Accept "accept" or "do not reject" in place of "insufficient evidence to reject".
Award the ***R1*** for comparing their p-value with 0.05 or their χ^2 value with 9.49 and then ***FT*** their final conclusion.

[7 marks]

Total [8 marks]

10. (a) $\overline{x} = \frac{\sum x}{n} = \frac{2506}{30} = 83.5 \ \ (83.5333\ldots)$ ***A1***

[1 mark]

(b) $\left(s_{n-1}^2 = \frac{\sum x^2 - \frac{\left(\sum x\right)^2}{n}}{n-1} = \right) \ \frac{209738 - \frac{2506^2}{30}}{29}$ ***(M1)***

$= 13.9 \ \ (13.9126...)$ ***A1***

[2 marks]

(c) $\left(82.1,\ 84.9\right) \ \ (82.1405...,\ 84.9261...)$ ***A2***

[2 marks]

(d) 85 is outside the confidence interval and therefore Talha would suggest that the manufacturer's claim is incorrect ***R1***

Note: The conclusion must refer back to the original claim.

Allow use of a two sided t-test giving a p-value rounding to $0.04 < 0.05$ and therefore Talha would suggest that the manufacturer's claims in incorrect.

[1 mark]

Total [6 marks]

11. (a) Odd vertices are A, B, D, H ***A1***

Consider pairings: ***M1***

Note: Award ***(M1)*** if there are four vertices not necessarily all correct.

AB DH has shortest route AD, DE, EB and DE, EH,
so repeated edges $(19+16+19)+(16+27)=97$

Note: Condone AB in place of AD, DE, EB giving $56+(16+27)=99$.

AD BH has shortest route AD and BE, EH,
so repeated edges $19+(19+27)=65$

AH BD has shortest route AD, DE, EH and BE, ED,
so repeated edges $(19+16+27)+(19+16)=97$ ***A2***

Note: Award ***A1*** if only one or two pairings are correctly considered.

so best pairing is AD, BH
weight of route is therefore $582+65=647$ ***A1***

[5 marks]

(b) least value of the pairings is 19 therefore repeat AD ***R1***

B and H ***A1***

Note: Do not award ***R0A1***.

[2 marks]

Total [7 marks]

12. (a) (i) $z_1^3 = 27e^{\frac{i\pi}{4}} (= 27e^{0.785398...i})$ ***A1A1***

Note: Award ***A1*** for 27 and ***A1*** for the angle in the correct form.

(ii) $\left(\frac{z_1}{z_2}\right)^4 = \left(\frac{81}{16}\right)e^{\frac{i\pi}{2}} (= 5.0625e^{1.57079...i})$ ***A1A2***

Note: Award ***A1*** for $\frac{81}{16}$, ***A2*** for the angle in the correct form and ***A1*** for the angle in incorrect form e.g. $\text{cis}\frac{\pi}{2}$ and/or $\frac{5\pi}{2}$.

Award ***A1*** if i is given in place of $\text{cis}\frac{\pi}{2}$.

[5 marks]

(b) $z_1 z_2 = 6\,\text{cis}\left(\frac{3\pi}{4} + \frac{n\pi}{16}\right)$ ***(M1)***

$= 6\,\text{cis}\left(\frac{12\pi + n\pi}{16}\right)$

$12\pi + n\pi = 32\pi$ ***(M1)***

$n = 20$ ***A1***

[3 marks]

Total [8 marks]

13.

(a) transition matrix is

$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \\ F \end{array}\begin{array}{c} \begin{array}{cccccc} A & B & C & D & E & F \end{array} \\ \begin{pmatrix} 0 & \frac{1}{3} & \frac{1}{2} & 0 & 0 & 0 \\ \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{5} \\ 0 & \frac{2}{3} & 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{5} \\ 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & \frac{2}{5} \\ \frac{1}{3} & 0 & 0 & 0 & 0 & \frac{1}{5} \\ \frac{1}{3} & 0 & 0 & \frac{1}{2} & 0 & 0 \end{pmatrix} \end{array}$$ ***M1A1A1***

Note: Allow the transposed matrix.

Award ***M1*** for a 6x6 matrix with all values between 0 and 1, and all columns (or rows if transposed) adding up to 1, award ***A1*** for one correct row (or column if transposed) and ***A1*** for all rows (or columns if transposed) correct.

[3 marks]

(b) attempting to raise the transition matrix to a large power ***(M1)***

steady state vector is $\begin{pmatrix} (0.157) \\ (0.0868) \\ (0.256) \\ (0.241) \\ (0.0868) \\ 0.173 \end{pmatrix}$ ***(A1)***

so percentage of time spent at vertex F is 17.3% ***A1***

Note: Accept 17.2%.

[3 marks]

(c) the model assumes instantaneous travel from junction to junction, ***R1***
and hence the answer obtained would be an overestimate ***R1***
OR
the mouse may eat the sugar over time ***R1***
and hence the probabilities would change ***R1***

Note: Accept any other sensible answer.

[2 marks]

Total [7 marks]

14. (a) $\begin{pmatrix} 7 & -10 \\ 2 & -3 \end{pmatrix}\begin{pmatrix} 6 \\ -2 \end{pmatrix}+\begin{pmatrix} -5 \\ 4 \end{pmatrix}$ ***(M1)***

$=\begin{pmatrix} 57 \\ 22 \end{pmatrix}$ **OR** $(57,\ 22)$ ***A1***

[2 marks]

(b) $\begin{pmatrix} 2p \\ 2q \end{pmatrix}=\begin{pmatrix} 7 & -10 \\ 2 & -3 \end{pmatrix}\begin{pmatrix} p \\ q \end{pmatrix}+\begin{pmatrix} -5 \\ 4 \end{pmatrix}$ ***(M1)***

$7p-10q-5=2p$

$2p-3q+4=2q$ ***(A1)***

solve simultaneously:

$p=13,\ q=6$ ***A1***

Note: Award ***A0*** if 13 and 6 are not labelled or are labelled the other way around.

[3 marks]

(c) $\det\begin{pmatrix} 7 & -10 \\ 2 & -3 \end{pmatrix}=-1$ **OR** $\left|\det\begin{pmatrix} 7 & -10 \\ 2 & -3 \end{pmatrix}\right|=1$ ***A1***

scale factor of image area is therefore $(|-1|=)1$ (and the translation does not affect the area)

A1

[2 marks]

Total [7 marks]

15. (a) $H_0 : m = 110$, $H_1 : m > 110$ ***A1***

Note: Accept other appropriate variables for the mean.
Accept 22 in place of 110.

[1 mark]

(b) $P(X \geq 128) = 0.05024$ ***(M1)(A1)***

$P(X \geq 129) = 0.04153$ ***(M1)***

(probability of making a type I error is) 0.0415 ***A1***

Note: If other probabilities are seen, the final ***A1*** cannot be awarded unless 0.0415 is clearly identified as the final answer.

[4 marks]

(c) $X \sim \text{Po}(110)$

$P(X \geq 126) = 0.072 > 0.05$ **OR** recognizing $126 < 129$ or ≤ 128 ***R1***

so there is insufficient evidence to reject H_0 ***A1***

(*ie* there is insufficient evidence to suggest that the number of coffees being sold has increased)

Note: Accept 'Accept H_0'.
Do not award ***R0A1***.

[2 marks]

Total [7 marks]

16. (a) $x_n = x_{n-1} + h\,f(x_{n-1},\, t_{n-1})$

$h = 0.1,\ f(x,\, t) = x\cos t\,(\mathrm{e}^{-\sin t})$

$x_n = x_{n-1} + 0.1x_{n-1}\cos t_{n-1}(\mathrm{e}^{-\sin t_{n-1}})$ ***(M1)***

Note: Award ***M1*** for a valid start.

n	t_n	x_n
0	0	0.367879
1	0.1	0.404667
2	0.2	0.441106
3	0.3	0.476548

(A1)

Note: Award ***A1*** for a correct x value when $n = 1$.

$x(0.3) \approx 0.477\ \ (0.476548...)$ ***A1***

[3 marks]

(b) **EITHER**

$\displaystyle\int \frac{\mathrm{d}x}{x} = \int \cos t(\mathrm{e}^{-\sin t})\ \mathrm{d}t\ \ (+c)$ ***M1***

$\ln x = -\mathrm{e}^{-\sin t} + c$ ***A1***

$t = 0,\ x = \dfrac{1}{\mathrm{e}} \Rightarrow c = 0$ ***M1***

$x = \mathrm{e}^{(-\mathrm{e}^{-\sin t})}$

$x(0.3) \approx 0.475140...$ ***A1***

OR

$\displaystyle\int_{1/\mathrm{e}}^{x} \frac{\mathrm{d}u}{u} = \int_{0}^{0.3} \cos t(\mathrm{e}^{-\sin t})\,\mathrm{d}t$ ***M1***

$[\ln u]_{1/\mathrm{e}}^{x} = 0.255855...$ (from GDC) ***A1***

$\ln x + 1 = 0.255855...$

$\ln x = -0.744145...$ ***A1***

$x = \mathrm{e}^{-0.744145} = 0.475140....$ ***A1***

THEN

percentage error $= \left|\dfrac{0.476548... - 0.475140...}{0.475140...}\right| \times 100 = 0.296\%\ \ (2.96192...)$ ***A1***

Note: If candidates do not attempt to find c, they may score ***M1A0M0A1A1***.

[5 marks]

Total [8 marks]

Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

May 2021

Mathematics: applications and interpretation

Higher level

Paper 1

19 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**.
A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.
R Marks awarded for clear **Reasoning**.
AG Answer given in the question and so no marks are awarded.
FT Follow through. The practice of awarding marks, despite candidate errors in previous parts, for their correct methods/answers using incorrect results.

Using the markscheme

1 General

Award marks using the annotations as noted in the markscheme *eg* ***M1***, ***A2***.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***A3***, ***M2*** *etc.*, do **not** split the marks, unless there is a note.
- The response to a "show that" question does not need to restate the ***AG*** line, unless a **Note** makes this explicit in the markscheme.
- Once a correct answer to a question or part question is seen, ignore further working even if this working is incorrect and/or suggests a misunderstanding of the question. This will encourage a uniform approach to marking, with less examiner discretion. Although some candidates may be advantaged for that specific question item, it is likely that these candidates will lose marks elsewhere too.
- An exception to the previous rule is when an incorrect answer from further working is used **in a subsequent part**. For example, when a correct exact value is followed by an incorrect decimal approximation in the first part and this approximation is then used in the second part. In this situation, award ***FT*** marks as appropriate but do not award the final ***A1*** in the first part. Examples:

	Correct answer seen	**Further working seen**	**Any FT issues?**	**Action**
1.	$8\sqrt{2}$	5.65685... *(incorrect decimal value)*	No. Last part in question.	Award ***A1*** for the final mark *(condone the incorrect further working)*
2.	$\frac{35}{72}$	0.468111… (incorrect decimal value)	Yes. Value is used in subsequent parts.	Award ***A0*** for the final mark *(and full **FT** is available in subsequent parts)*

3 Implied marks

Implied marks appear in **brackets e.g.** ***(M1)***,and can only be awarded if **correct** work is seen or implied by subsequent working/answer.

4 Follow through marks (only applied after an error is made)

Follow through (***FT***) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) (e.g. incorrect value from part (a) used in part (d) or incorrect value from part (c)(i) used in part (c)(ii)). Usually, to award ***FT*** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if all the marks awarded in a subsequent part are for the answer or are implied, then ***FT*** marks should be awarded for *their* correct answer, even when working is not present.

For example: following an incorrect answer to part (a) that is used in subsequent parts, where the markscheme for the subsequent part is ***(M1)A1***, it is possible to award full marks for *their* correct answer, **without working being seen.** For longer questions where all but the answer marks are implied this rule applies but may be overwritten by a **Note** in the Markscheme.

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate.

- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks, by reflecting on what each mark is for and how that maps to the simplified version.

- If the error leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).

- The markscheme may use the word “their” in a description, to indicate that candidates may be using an incorrect value.

- If the candidate’s answer to the initial question clearly contradicts information given in the question, it is not appropriate to award any ***FT*** marks in the subsequent parts. This includes when candidates fail to complete a “show that” question correctly, and then in subsequent parts use their incorrect answer rather than the given value.

- Exceptions to these ***FT*** rules will be explicitly noted on the markscheme.

- If a candidate makes an error in one part but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the command term was “Hence”.

5 Mis-read

If a candidate incorrectly copies values or information from the question, this is a mis-read (***MR***). A candidate should be penalized only once for a particular misread. Use the ***MR*** stamp to indicate that this has been a misread and do not award the first mark, even if this is an ***M*** mark, but award all others as appropriate.

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.
- If a candidate uses a correct answer, to a "show that" question, to a higher degree of accuracy than given in the question, this is NOT a misread and full marks may be scored in the subsequent part.
- ***MR*** can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If the command term is 'Hence' and not 'Hence or otherwise' then alternative methods are not permitted unless covered by a note in the mark scheme.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**.

7 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation** for example 1.9 and $1{,}9$ or 1000 and $1{,}000$ and 1.000.

- Do not accept final answers written using calculator notation. However, ***M*** marks and intermediate ***A*** marks can be scored, when presented using calculator notation, provided the evidence clearly reflects the demand of the mark.

- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.

- In the markscheme, some **equivalent** answers will generally appear in brackets. Not all equivalent notations/answers/methods will be presented in the markscheme and examiners are asked to apply appropriate discretion to judge if the candidate work is equivalent.

8 Format and accuracy of answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. If the level of accuracy is not stated in the question, the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures.*

Where values are used in subsequent parts, the markscheme will generally use the exact value, however candidates may also use the correct answer to 3 sf in subsequent parts. The markscheme will often explicitly include the subsequent values that come "*from the use of 3 sf values*".

Simplification of final answers: Candidates are advised to give final answers using good mathematical form. In general, for an ***A*** mark to be awarded, arithmetic should be completed, and any values that lead to integers should be simplified; for example, $\sqrt{\frac{25}{4}}$ should be written as $\frac{5}{2}$. An exception to this is simplifying fractions, where lowest form is not required (although the numerator and the denominator must be integers); for example, $\frac{10}{4}$ may be left in this form or written as $\frac{5}{2}$. However, $\frac{10}{5}$ should be written as 2, as it simplifies to an integer.

Algebraic expressions should be simplified by completing any operations such as addition and multiplication, e.g. $4\mathrm{e}^{2x} \times \mathrm{e}^{3x}$ should be simplified to $4\mathrm{e}^{5x}$, and $4\mathrm{e}^{2x} \times \mathrm{e}^{3x} - \mathrm{e}^{4x} \times \mathrm{e}^{x}$ should be simplified to $3\mathrm{e}^{5x}$. Unless specified in the question, expressions do not need to be factorized, nor do factorized expressions need to be expanded, so $x(x+1)$ and $x^2 + x$ are both acceptable.

Please note: intermediate ***A*** marks do NOT need to be simplified.

9 Calculators

A GDC is required for this paper, but If you see work that suggests a candidate has used any calculator not approved for IB DP examinations (eg CAS enabled devices), please follow the procedures for malpractice.

10. Presentation of candidate work

Crossed out work: If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work unless an explicit note from the candidate indicates that they would like the work to be marked.

More than one solution: Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise. If the layout of the responses makes it difficult to judge, examiners should apply appropriate discretion to judge which is "first".

1. $X \sim \text{Po}(8.8)$ ***(M1)***

Note: Award ***(M1)*** for calculating the mean, 8.8, of the distribution

$\text{P}(X > 9) = \text{P}(X \geq 10)$ **OR** $\text{P}(X > 9) = 1 - \text{P}(X \leq 9)$ ***(M1)***

$\text{P}(X > 9) = 0.386 \ (0.386260...)$ ***(M1)A1***

Note: Award ***(M1)(M0)(M1)A0*** for finding $\text{P}(X \geq 9) = 0.518 \ (0.517719...)$ **OR** $\text{P}(X \leq 9) = 0.614 \ (0.613740...)$.

Total [4 marks]

2. (a) every point in the shaded region is closer to tower T4 ***R1***

Note: Specific reference must be made to the closeness of tower T4.

[1 mark]

(b) $(-9, 1)$ ***A1A1***

Note: Award ***A1*** for each correct coordinate. Accept $x = -9$ and $y = 1$. Award at most ***A0A1*** if parentheses are missing.

[2 marks]

(c) correct use of gradient formula ***(M1)***

e.g. $(m =) \dfrac{5-3}{-9--13} \left(= \dfrac{1}{2}\right)$

taking negative reciprocal of **their** m (at any point) ***(M1)***

edge gradient $= -2$ ***A1***

[3 marks]

Total [6 marks]

3. (a) (i) attempt to find u_{20} using an arithmetic sequence ***(M1)***

e.g. $u_1 = 500$ and $d = 100$ **OR** $u_{20} = 500 + 1900$ **OR** $500, 600, 700, \ldots$

(Charlie ran) 2400 m ***A1***

(ii) $(r =)\ 1.02$ ***(A1)***

attempt to find u_{20} using a geometric sequence ***(M1)***

e.g. identifying $u_1 = 500$ and a value for r **OR** $500 \times r^{19}$ **OR** $500,\ 510,\ 520.2, \ldots$

(Daniella ran) 728 m $(728.405...)$ ***A1***

[5 marks]

(b) $500 \times 1.02^{n-1} > 500 + (n-1) \times 100$ ***(M1)***

attempt to solve inequality ***(M1)***

$n > 184.215...$

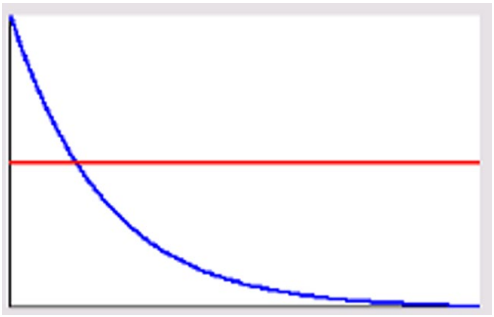
$n = 185$ ***A1***

[3 marks]

Total [8 marks]

4. (a) $50 = 100\,e^{-1\times p}$ **OR** $0.5 = e^{-1\times p}$ ***(M1)***

OR



(M1)

0.693 $(0.693147..., \ln 2)$ ***A1***

[2 marks]

(b) $R(1.5) = 100\,e^{-0.693147...\times 1.5}$ ***(M1)***

$35.4\,(\%)$ $(35.3553...)$ ***A1***

[2 marks]

(c) $R(t) > 0$ **OR** $R(t)$ has a horizontal asymptote ***R1***

[1 mark]

(d) *Award **A1** for **one** reasonable limitation of the domain:* ***A1***

small values of t produce unrealistic results
$R(0) = 100\%$
large values of t are not possible
people do not live forever
model is not valid at small or large values of t

The reason should focus on the domain $t \geq 0$. Do not accept answers such as:

recollection varies for different people
memories are discrete not continuous
the nature of the information will change how easily it is recalled
emotional/physical stress can affect recollection/concentration

Note: Do not accept $t \geq 0$ as this is a limitation that has been given in the question.

[1 mark]

Total [6 marks]

5. (a) (i) $\overrightarrow{CA} = \begin{pmatrix} -3 \\ -4 \\ -1 \end{pmatrix}$ ***A1***

(ii) $\overrightarrow{CB} = \begin{pmatrix} 3 \\ -4 \\ -1 \end{pmatrix}$ ***A1***

[2 marks]

(b) $\overrightarrow{CA} \times \overrightarrow{CB} = \begin{pmatrix} 0 \\ -6 \\ 24 \end{pmatrix}$ ***(M1)A1***

Note: Do not award ***(M1)*** if less than 2 entries are correct.

[2 marks]

(c) area is $\frac{1}{2}\sqrt{6^2 + 24^2} = 12.4 \text{ m}^2 \ \ (12.3693..., 3\sqrt{17})$ ***(M1)A1***

[2 marks]

Total [6 marks]

6. attempt to find any relevant maximum value ***(M1)***
largest sides are 56.5 and 82.5 ***(A1)***
smallest possible angle is 102.5 ***(A1)***

attempt to substitute into area of a triangle formula ***(M1)***
$\frac{1}{2} \times 56.5 \times 82.5 \times \sin(102.5^\circ)$

$= 2280 \text{ m}^2 \ \ (2275.37...)$ ***A1***

Total [5 marks]

7. (a) (i) $A = \frac{1}{2} \times 6 \times q + \frac{1}{2} \times 8 \times p + 48$ **OR** $A = \frac{1}{2}(p+6)(q+8)$ **OR**
$A = 3q + 4p + 48$ ***A1***

(ii) valid attempt to link p and q, using tangents, similar triangles or other method ***(M1)***

eg. $\tan\theta = \frac{8}{p}$ and $\tan\theta = \frac{q}{6}$ **OR** $\tan\theta = \frac{p}{8}$ and $\tan\theta = \frac{6}{q}$ **OR** $\frac{8}{p} = \frac{q}{6}$

correct equation linking p and q ***A1***

eg. $pq = 48$ **OR** $p = \frac{48}{q}$ **OR** $q = \frac{48}{p}$

substitute $p = \frac{48}{q}$ into a correct area expression ***M1***

eg. $(A =)\frac{1}{2} \times 6 \times q + \frac{1}{2} \times 8 \times \frac{48}{q} + 48$ **OR** $(A =)\frac{1}{2}\left(\frac{48}{q} + 6\right)(q+8)$

$A = 3q + \frac{192}{q} + 48$ ***AG***

Note: The ***AG*** line must be seen with no incorrect, intermediate working, for the final ***M1*** to be awarded.

[4 marks]

(b) $\frac{-192}{q^2} + 3$ ***A1A1***

Note: Award ***A1*** for $\frac{-192}{q^2}$, ***A1*** for 3. Award ***A1A0*** if extra terms are seen.

[2 marks]

(c) (i) $\frac{-192}{q^2} + 3 = 0$ ***A1***

(ii) $q = 8$ cm ***A1***

[2 marks]

Total [8 marks]

8. (a)

t	1	2	3	4	5	6
$\mathrm{P}(T=t)$	$\frac{1}{36}$ (0.027777…)	$\frac{3}{36}$ (0.083333…)	$\frac{5}{36}$ (0.138888…)	$\frac{7}{36}$ (0.194444…)	$\frac{9}{36}$ (0.25)	$\frac{11}{36}$ (0.305555…)

A2

Note: Award ***A1*** if three to five probabilities are correct.

[2 marks]

(b) (i) $\frac{32}{36}$ $\left(\frac{8}{9}, 0.888888..., 88.9\%\right)$ ***(A1)***

(ii) use of conditional probability ***(M1)***
e.g. denominator of 32 **OR** denominator of $0.888888\ldots$, etc.
$\frac{11}{32}$ $(0.34375, 34.4\%)$ ***A1***

[3 marks]

(c) $\frac{1\times1+3\times2+5\times3+...+11\times6}{36}$ ***(M1)***

$=\frac{161}{36}$ $\left(4\frac{17}{36}, 4.47, 4.47222...\right)$ ***A1***

[2 marks]

Total [7 marks]

9. (a) (i) $\mathrm{i}^2 = -1$ ***(M1)***

$w = -2 + 1 = -1$ ***A1***

(ii) $w = -1 + \mathrm{i} + 1 = \mathrm{i}$ ***A1***

[3 marks]

(b) **EITHER**

rotation of 90° (anticlockwise, centre at the origin) ***A1A1***

Note: Award ***A1*** for "rotation" and ***A1*** for "90°".

followed by a translation of $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ ***A1***

OR

translation of $\begin{pmatrix} 0 \\ -1 \end{pmatrix}$ ***A1***

followed by rotation of 90° (anticlockwise, centre at the origin) ***A1A1***

Note: Award ***A1*** for "rotation" and ***A1*** for "90°".

[3 marks]

(c) **EITHER**

move 1 to left to $1 - \mathrm{i}$ ***(M1)***

then rotate by -90° to

$-1 - \mathrm{i}$ ***A1***

OR

$\mathrm{i}z + 1 = 2 - \mathrm{i}$

$\mathrm{i}z = 1 - \mathrm{i}$

$z = \dfrac{1 - \mathrm{i}}{\mathrm{i}}$ ***(M1)***

$-1 - \mathrm{i}$ ***A1***

[2 marks]

Total [8 marks]

10. (a) (i) use of Prim's algorithm ***M1***

BC 46 ***A1***

BD 58 ***A1***

DE 23

EF 47

Total 174 ***A1***

Note: Award ***M0A0A0A1*** for 174 without correct working e.g. use of Kruskal's, or with no working.
Award ***M1A0A0A1*** for 174 by using Prim's from an incorrect starting point.

(ii) $AB + AC = 55 + 63 = 118$ ***(M1)***

$174 + 118 = 292$ minutes ***A1***

[6 marks]

(b) delete a different vertex ***A1***

[1 mark]

Total [7 marks]

11. (a) Convenience ***A1***

[1 mark]

(b) H_0: 1% of the toys produced are faulty ***A1***

H_1: More than 1% are faulty ***A1***

[2 marks]

(c) $X \sim B(200, 0.01)$ ***(M1)***

$P(X \geq 4) = 0.142$ ***A1***

Note: Any attempt using Normal approximation to find p-value is awarded ***M0A0***.

[2 marks]

(d) $14\% > 10\%$ ***R1***

so there is insufficient evidence to reject H_0. ***A1***

Note: Do not award ***R0A1***. Accept "fail to reject H_0" or "accept H_0".

[2 marks]

Total [7 marks]

12. (a) $\frac{\mathrm{d}V}{\mathrm{d}t} = -kV^{\frac{1}{2}}$

use of separation of variables ***(M1)***

$\Rightarrow \int V^{-\frac{1}{2}}\,\mathrm{d}V = \int -k\,\mathrm{d}t$ ***A1***

$2V^{\frac{1}{2}} = -kt\ (+c)$ ***A1***

considering initial conditions $40 = c$ ***A1***

$2\sqrt{324} = -10k + 40$

$\Rightarrow k = 0.4$ ***A1***

$2\sqrt{V} = -0.4t + 40$

$\Rightarrow \sqrt{V} = 20 - 0.2t$ ***A1***

Note: Award ***A1*** for any correct intermediate step that leads to the ***AG***.

$\Rightarrow V = \left(20 - \frac{t}{5}\right)^2$ ***AG***

Note: Do not award the final ***A1*** if the ***AG*** line is not stated.

[6 marks]

(b) $0 = \left(20 - \frac{t}{5}\right)^2 \Rightarrow t = 100$ minutes ***(M1)A1***

[2 marks]

Total [8 marks]

13. (a) $\boldsymbol{r} = \begin{pmatrix} 0.8 \\ 1.3 \\ -0.3 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ -3 \\ 1 \end{pmatrix}$ ***A1A1***

Note: Award ***A1*** for each correct vector. Award ***A0A1*** if their "$\boldsymbol{r} =$" is omitted.

[2 marks]

(b) (i) $-0.3 + \lambda = 0$ ***(M1)***

$\Rightarrow \lambda = 0.3$

$\boldsymbol{r} = \begin{pmatrix} 0.8 \\ 1.3 \\ -0.3 \end{pmatrix} + 0.3 \begin{pmatrix} -2 \\ -3 \\ 1 \end{pmatrix} = \begin{pmatrix} 0.2 \\ 0.4 \\ 0 \end{pmatrix}$ ***(M1)***

P has coordinates $(0.2,\ 0.4,\ 0)$ ***A1***

Note: Accept the coordinates of P in vector form.

(ii) $\sqrt{0.2^2 + 0.4^2}$ ***(M1)***

$= 0.447 \text{ km}$ $(=447 \text{ m})$ ***A1***

[5 marks]

Total [7 marks]

14. (a) $158 \times 6 = 948 (\text{g})$ ***(M1)A1***

[2 marks]

(b) variance 6×13^2 ***(M1)***

$\text{SD} = 31.8 (\text{g})$ $(13\sqrt{6},\ 31.8433\ldots)$ ***A1***

[2 marks]

(c) $X \sim \text{N}(948,\ 31.8433...^2)$

$\text{P}(X > 1000) = 0.0512$ $(0.0512350\ldots)$ ***(M1)A1***

Note: Accept 0.0510 $(0.0510014\ldots)$ if 3 sf value 31.8 is used.

Award ***(M1)A1FT*** if the answer is correct for their SD, even if no working is shown. e.g. If the SD is 78 then accept 0.252.

[2 marks]

Total [6 marks]

15. (a) $\sin(x+y)=0$ ***A1***

$\Rightarrow x+y=0$ ***(M1)***

(the equation of L_1 is) $y=-x$ ***A1***

[3 marks]

(b) $x+y=\pi$ **OR** $y=-x+\pi$ ***(M1)A1***

[2 marks]

Total [5 marks]

16. (a) $\boldsymbol{M}=\begin{pmatrix}0&1&1&1&0&0\\1&0&1&0&1&0\\1&1&0&0&0&1\\1&0&0&0&1&1\\0&1&0&1&0&1\\0&0&1&1&1&0\end{pmatrix}$ ***A1A1A1***

Note: Award ***A1*** for each two correct rows.

[3 marks]

(b) calculating $\boldsymbol{M}^6$ ***(M1)***

143 ***A1***

[2 marks]

Total [6 marks]

17. new function is $f(x-a)+b\left(=\ln(x-a)+b\right)$ ***(M1)***

$f(0)=\ln(-a)+b=1$ ***A1***

$f(\mathrm{e}^3)=\ln(\mathrm{e}^3-a)+b=1+\ln 2$ ***A1***

$\ln(-a)=\ln(\mathrm{e}^3-a)-\ln 2$ ***(M1)***

$$\ln(-a)=\ln\left(\frac{\mathrm{e}^3-a}{2}\right)$$

$$-a=\frac{\mathrm{e}^3-a}{2}$$

$-2a=\mathrm{e}^3-a$

$a=-\mathrm{e}^3 \quad (=-20.0855...)$ ***A1***

$b=1-\ln\mathrm{e}^3=1-3=-2$ ***(M1)A1***

Total [7 marks]

Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

Specimen paper

Mathematics: applications and interpretation

Higher level

Paper 1

15 pages

International Baccalaureate®
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Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

R Marks awarded for clear **Reasoning**.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Award marks using the annotations as noted in the markscheme eg ***M1****,* ***A2****.*

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***M2***, ***A3***, *etc.*, do **not** split the marks, unless there is a note.
- Once a correct answer to a question or part-question is seen, ignore further correct working. However, if further working indicates a lack of mathematical understanding do not award the final ***A1***. An exception to this may be in numerical answers, where a correct exact value is followed by an incorrect decimal. However, if the incorrect decimal is carried through to a subsequent part, and correct ***FT*** working shown, award ***FT*** marks as appropriate but do not award the final ***A1*** in that part.

Examples

	Correct answer seen	Further working seen	Action
1.	$8\sqrt{2}$	5.65685... *(incorrect decimal value)*	Award the final ***A1*** *(ignore the further working)*
2.	$\frac{1}{4}\sin 4x$	$\sin x$	Do not award the final ***A1***
3.	$\log a - \log b$	$\log(a-b)$	Do not award the final ***A1***

3 Implied marks

*Implied marks appear in **brackets e.g. (M1)**, and can only be awarded if **correct** work is seen or if implied in subsequent working.*

- Normally the correct work is seen or implied in the next line.
- Marks **without** brackets can only be awarded for work that is **seen**.

4 Follow through marks (only applied after an error is made)

*Follow through (**FT**) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) or subpart(s). Usually, to award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if the only marks awarded in a subpart are for the answer (i.e. there is no working expected), then **FT** marks should be awarded if appropriate.*

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*e.g.* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word “their” in a description, to indicate that candidates may be using an incorrect value.
- Exceptions to this rule will be explicitly noted on the markscheme.
- If a candidate makes an error in one part, but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the question says hence. It is often possible to use a different approach in subsequent parts that does not depend on the answer to previous parts.

5 Mis-read

*If a candidate incorrectly copies information from the question, this is a mis-read (**MR**). Apply a **MR** penalty of 1 mark to that question*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates’ own work does **not** constitute a misread, it is an error.
- The ***MR*** penalty can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme

- Alternative methods for complete questions are indicated by ***METHOD 1***, ***METHOD 2***, *etc*.
- Alternative solutions for part-questions are indicated by ***EITHER*** . . . ***OR***.

7 Alternative forms

Unless the question specifies otherwise, ***accept*** *equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

8 Accuracy of Answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. There are two types of accuracy errors, and the final answer mark should not be awarded if these errors occur.

- **Rounding errors**: only applies to final answers not to intermediate steps.
- **Level of accuracy**: when this is not specified in the question the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures*.

9 Calculators

A GDC is required for this examination, but calculators with symbolic manipulation features/ CAS functionality are not allowed.

Calculator notation
The subject guide says:

> *Students must always use correct mathematical notation, not calculator notation.*

Do **not** accept final answers written using calculator notation. However, do not penalize the use of calculator notation in the working.

1. (a) discrete ***A1***

[1 mark]

(b) $\frac{24 + 60 + 3k + 40 + 15 + 6}{88 + k} = 2$ ***M1A1***

Note: Award ***M1*** for substitution into the formula for the mean, award ***A1*** for a correct equation.

attempt to solve their equation ***(M1)***

$k = 31$ ***A1***

[4 marks]

(c) systematic ***A1***

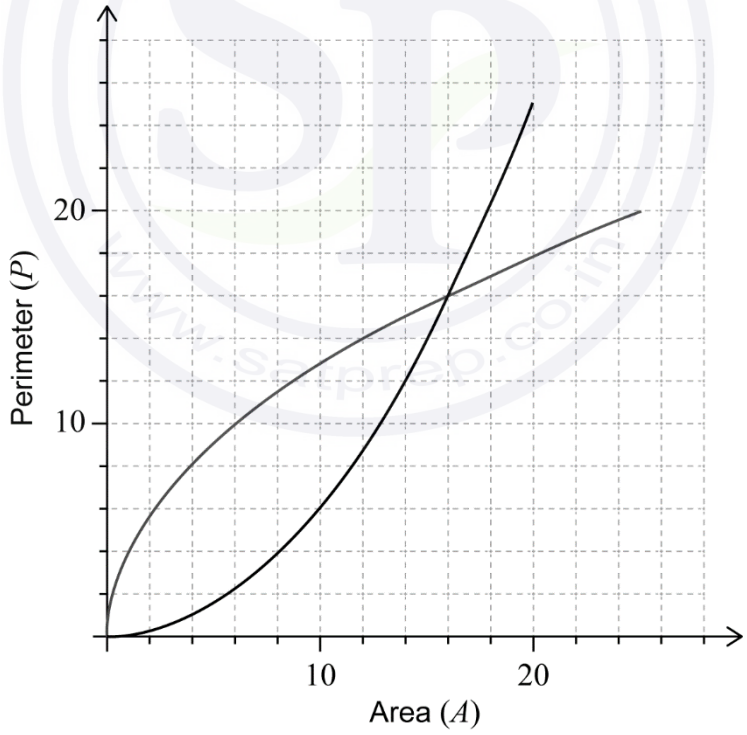
[1 mark]

Total [6 marks]

2. (a) 20 ***A1***

[1 mark]

(b)



(M1)A1A1

Note: Award ***(M1)*** for reflection in the line $P = A$, award ***A1*** for endpoint at $(20, 25)$, award ***A1*** for passing through $(16, 16)$.

[3 marks]

(c) when the perimeter is 8, the area is 4 ***A1***

[1 mark]

Total [5 marks]

3. (a) (i) 1750 ***A1***

(ii) $1350 + 400\,(1.25)^{-5}$ ***(M1)***

$= 1480$ ***A1***

Note: Accept 1481.

[3 marks]

(b) $1400 = 1350 + 400\,(1.25)^{-t}$ ***(M1)***

9.32 (days $(9.31885\ldots)$ (days)) ***A1***

[2 marks]

(c) 1350 ***A1***

Note: Accept 1351 as a valid interpretation of the model as $P = 1350$ is an asymptote.

[1 mark]

Total [6 marks]

4. (a) $\dfrac{3-1}{7-3}$ ***(M1)***

$= 0.5$ ***A1***

[2 marks]

(b) $y - 2 = -2(x - 5)$ ***(A1)(M1)***

Note: Award ***(A1)*** for their -2 seen, award ***(M1)*** for the correct substitution of $(5, 2)$ and their normal gradient in equation of a line.

$2x + y - 12 = 0$ ***A1***

[3 marks]

(c) every point in the cell is closer to E than any other snow shelter ***A1***

[1 mark]

Total [6 marks]

5. (a) $\frac{50 \times \pi}{180} = 0.873\ (0.872664\ldots)$ ***A1***

[1 mark]

(b) volume $= 240\left(\pi \times 8.4^2 - \frac{1}{2} \times 8.4^2 \times 0.872664\ldots\right)$ ***M1M1M1***

Note: Award ***M1*** $240 \times$ area , award ***M1*** for correctly substituting area sector formula, award ***M1*** for subtraction of the angles or their areas.

$= 45800\ (= 45811.96071)$ ***A1***

[4 marks]

Total [5 marks]

6. (a) $\frac{4}{18}\left(\frac{2}{9}\right)$ ***A1***

[1 mark]

(b) $-3 \times \frac{1}{18} + (-1) \times \frac{4}{18} + 0 \times \frac{3}{18} + \ldots + 5 \times \frac{7}{18}$ ***(M1)***

Note: Award ***(M1)*** for their correct substitution into the formula for expected value.

$= 1.83\left(\frac{33}{18}, 1.83333\ldots\right)$ ***A1***

[2 marks]

(c) $2 \times \frac{1}{18} \times \frac{3}{18}$ ***(M1)(M1)***

Note: Award ***(M1)*** for $\frac{1}{18} \times \frac{3}{18}$, award ***(M1)*** for multiplying their product by 2.

$= \frac{1}{54}\left(\frac{6}{324}, 0.0185185\ldots, 1.85\%\right)$ ***A1***

[3 marks]

Total [6 marks]

7. (a) $E = 5(2\sin t)^2 \left(= 20\sin^2 t\right)$ ***A1***

[1 mark]

(b) $\frac{\mathrm{d}E}{\mathrm{d}t} = 40\sin t\cos t$ ***(M1)A1***

[2 marks]

(c) $t = 0.126$ ***(M1)A1***

[2 marks]

Total [5 marks]

8. (a) $\frac{\sin \mathrm{C\hat{A}B}}{6} = \frac{\sin 15^\circ}{4.5}$ ***(M1)(A1)***

$\mathrm{C\hat{A}B} = 20.2^\circ$ (20.187415…) ***A1***

Note: Award ***(M1)*** for substituted sine rule formula and award ***(A1)*** for correct substitutions.

[3 marks]

(b) $\mathrm{C\hat{B}D} = 20.2 + 15 = 35.2^\circ$ ***A1***

(let X be the point on BD where Ollie activates the sensor)

$\tan 35.18741\ldots^\circ = \frac{1.8}{\mathrm{BX}}$ ***(M1)***

Note: Award ***A1*** for their correct angle $\mathrm{C\hat{B}D}$. Award ***M1*** for correctly substituted trigonometric formula.

$\mathrm{BX} = 2.55285\ldots$ ***A1***

$5 - 2.55285\ldots$ ***(M1)***

$= 2.45\,(\mathrm{m})$ (2.44714…) ***A1***

[5 marks]

Total [8 marks]

9. (a) $s_{n-1} = \sqrt{\frac{10}{9}} \times 0.0196 = 0.02066\ldots$ ***(M1)A1***

[2 marks]

(b) $(1.463\,,\,1.493)$ ***(M1)A1***

Note: If s_n used answer is $(1.464\,,\,1.492)$, award ***M1A0.***

[2 marks]

(c) $95\,\%$ of the time these results would be produced by a population with mean of less than $1.5\,\text{kg}$, so it is likely the mean weight is less than $1.5\,\text{kg}$ ***R1***

[1 mark]

Total [5 marks]

10. let T be the time to serve both customers and T_i the time to serve the ith customer

assuming independence of T_1 and T_2 ***R1***

T is normally distributed and $T = T_1 + T_2$ ***(M1)***

$E(T) = 1.5 + 1.5 = 3$ ***A1***

$\text{Var}(T) = 0.4^2 + 0.4^2 = 0.32$ ***M1A1***

$P(T < 4) = 0.961$ ***A1***

Total [6 marks]

11. (a) $15 \times 0 + 2d + 4 = 0$ ***(M1)***

$d = -2$ ***A1***

[2 marks]

(b) $a\begin{pmatrix} -15 \\ 2 \\ 4 \end{pmatrix} \times \begin{pmatrix} 0 \\ -2 \\ 1 \end{pmatrix}$ ***(M1)***

$= a\begin{pmatrix} 10 \\ 15 \\ 30 \end{pmatrix} \left(= 5a \begin{pmatrix} 2 \\ 3 \\ 6 \end{pmatrix} \right)$ ***A1***

magnitude is $5a\sqrt{2^2 + 3^2 + 6^2} = 14$ ***M1***

$a = \frac{14}{35} \ (= 0.4)$ ***A1***

[4 marks]

Total [6 marks]

12. *(Model A)*

$R = 3pe^{-0.5p}$ ***M1***

predicted values

p	R
1	1.8196
2	2.2073
3	2.0082

(A1)

$SS_{res} = (1.8196 - 1.5)^2 + (2.2073 - 1.8)^2 + (2.0082 - 1.5)^2$ ***(M1)***

$= 0.5263\ldots$ ***A1***

(Model B)

$R = 2.5pe^{-0.6p}$

predicted values

p	R
1	1.372
2	1.506
3	1.2397

(A1)

$SS_{res} = 0.170576\ldots$ ***A1***

chose model B ***A1***

Note: Method marks can be awarded if seen for either model A or model B. Award final ***A1*** if it is a correct deduction from their calculated values for A and B.

Total [7 marks]

13. (a) $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{16-20}{24-20}$ ***M1***

$= -1$ ***A1***

[2 marks]

(b) asymptote of trajectory along $\boldsymbol{r} = k\begin{pmatrix}2\\1\end{pmatrix}$ ***M1A1***

Note: Award ***M1A0*** if asymptote along $\begin{pmatrix}1\\2\end{pmatrix}$.

trajectory begins at $(8, 10)$ with negative gradient ***A1A1***

[4 marks]

Total [6 marks]

14. (a) (i) $a = 33$ ***A1***

(ii) $\frac{1}{\sqrt[3]{0.08}} = 2.32$ ***M1A1***

[3 marks]

(b) volume within outer dome

$\frac{2}{3}\pi \times 16^3 + \pi \times 16^2 \times 17 = 22\,250.85$ ***M1A1***

volume within inner dome

$\pi\int_0^{33}\left(\frac{33-y}{0.08}\right)^{\frac{2}{3}}\mathrm{d}y = 3446.92$ ***M1A1***

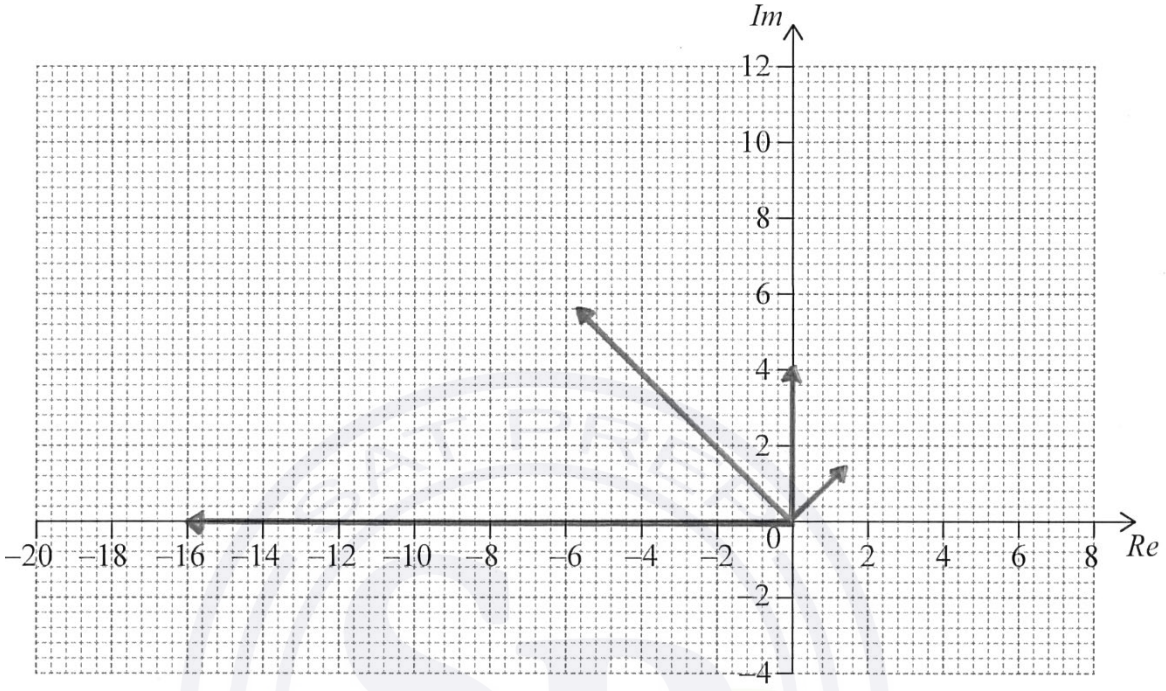
volume between $= 22\,250.85 - 3446.92 = 18\,803.93\,\mathrm{m}^3$ ***A1***

[5 marks]

Total [8 marks]

15. (a) (i) $4e^{\frac{\pi}{2}i}, 8e^{\frac{3\pi}{4}i}, 16e^{\pi i}$ $\left(= 4i, -4\sqrt{2} + 4\sqrt{2}i, -16\right)$ ***(M1)A1***

(ii)



A3

Note: Award ***A1*** for correct arguments, award ***A1*** for $4i$ and -16 clearly indicated, award ***A1*** for $|w| < 4$ and $4 < |w^3| < 16$.

[5 marks]

(b) $2^2 + 1^2 = a^2$ ***M1***

$a = \sqrt{5}$ $(= 2.24)$ ***A1***

[2 marks]

Total [7 marks]

16. (a) $H_0 : m = 3, H_1 : m < 3$ ***A1***

Note: Accept equivalent statements in words.

[1 mark]

(b) *(let X be the number of fish caught)*
$P(X \leq 1 \mid m = 3) = 0.199$ ***M1A1***

[2 marks]

(c) $P(X \geq 2 \mid m = 2.5)$ $(= 1 - P(X \leq 1 \mid m = 2.5))$ ***M1A1***

Note: Award ***M1*** for using $m = 2.5$ to evaluate a probability, award ***A1*** for also having $X \geq 2$.

$= 0.713$ ***A1***

[3 marks]

Total [6 marks]

17. (a) $P(X = 8)$ ***(M1)***

Note: Award ***(M1)*** for evidence of recognizing binomial probability.
eg, $P(X = 8), X \sim B\left(20, \frac{6}{15}\right)$.

$= 0.180\ (0.179705\ldots)$ ***A1***

[2 marks]

(b) let x be the number of male students

recognize that probability of selecting a male is equal to $\frac{x}{80}$ ***(A1)***

$\left(\text{set up equation } {}^{20}C_8 \left(\frac{x}{80}\right)^8 \left(\frac{80-x}{80}\right)^{12} = \right) 0.153357$ ***(M1)***

number of male students $= 37$ ***(M1)A1***

Note: Award ***(M1)A0*** for 27.

[4 marks]

Total [6 marks]

18. $\log A = x\log B + y\log C + \log k$ ***(M1)***

$\log 5.74 = x\log 2.1 + y\log 3.4 + \log k$
$\log 2.88 = x\log 1.5 + y\log 2.4 + \log k$
$\log 0.980 = x\log 0.8 + y\log 1.9 + \log k$ ***M1A1***

Note: Allow any consistent base, allow numerical equivalents.

attempting to solve their system of equations ***(M1)***

$x = 1.53,\ y = 0.505$ ***A1***

$k = 0.997$ ***A1***

Total [6 marks]