Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

November 2023

Mathematics: applications and interpretation

Higher level

Paper 3

13 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

R Marks awarded for clear **Reasoning**.

AG Answer given in the question and so no marks are awarded.

FT Follow through. The practice of awarding marks, despite candidate errors in previous parts, for their correct methods/answers using incorrect results.

Using the markscheme

1 General

Award marks using the annotations as noted in the markscheme eg ***M1***, ***A2***.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.

- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.

- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.

- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***. If ***A1*** marks are on separate lines, they are assumed to be dependent and hence ***A0A1*** is unlikely to be awarded. However, where such marks are *independent* (e.g. the markscheme is presenting them in sequence, but in the solution one does not lead directly to the other) this should be communicated via a note, and hence ***A0A1*** (for example) can be awarded.

- Where the markscheme specifies ***A3***, ***M2*** *etc*., do **not** split the marks, unless there is a note.

- The response to a "show that" question does not need to restate the ***AG*** line, unless a **Note** makes this explicit in the markscheme.

- Once a correct answer to a question or part question is seen, ignore further working even if this working is incorrect and/or suggests a misunderstanding of the question. This will encourage a uniform approach to marking, with less examiner discretion. Although some candidates may be advantaged for that specific question item, it is likely that these candidates will lose marks elsewhere too.

- An exception to the previous rule is when an incorrect answer from further working is used **in a subsequent part**. For example, when a correct exact value is followed by an incorrect decimal

approximation in the first part and this approximation is then used in the second part. In this situation, award ***FT*** marks as appropriate but do not award the final ***A1*** in the first part. Examples:

	Correct answer seen	**Further working seen**	**Any FT issues?**	**Action**
1.	$8\sqrt{2}$	5.65685... *(incorrect decimal value)*	No. Last part in question.	Award ***A1*** for the final mark *(condone the incorrect further working)*
2.	$\frac{35}{72}$	0.468111… (incorrect decimal value)	Yes. Value is used in subsequent parts.	Award ***A0*** for the final mark *(and full **FT** is available in subsequent parts)*

3 Implied marks

Implied marks appear in **brackets e.g. *(M1)***,and can only be awarded if **correct** work is seen or implied by subsequent working/answer.

4 Follow through marks (only applied after an error is made)

Follow through (***FT***) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) (e.g. incorrect value from part (a) used in part (d) or incorrect value from part (c)(i) used in part (c)(ii)). Usually, to award ***FT*** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if all the marks awarded in a subsequent part are for the answer or are implied, then ***FT*** marks should be awarded for *their* correct answer, even when working is not present.

For example: following an incorrect answer to part (a) that is used in subsequent parts, where the markscheme for the subsequent part is ***(M1)A1***, it is possible to award full marks for *their* correct answer, **without working being seen.** For longer questions where all but the answer marks are implied this rule applies but may be overwritten by a **Note** in the Markscheme.

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks, by reflecting on what each mark is for and how that maps to the simplified version
- If the error leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word “their” in a description, to indicate that candidates may be using an incorrect value.
- If the candidate’s answer to the initial question clearly contradicts information given in the question, it is not appropriate to award any ***FT*** marks in the subsequent parts. This includes when candidates fail to complete a “show that” question correctly, and then in subsequent parts use their incorrect answer rather than the given value.
- Exceptions to these ***FT*** rules will be explicitly noted on the markscheme.
- If a candidate makes an error in one part but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the command term was “Hence”.

5 Mis-read

If a candidate incorrectly copies values or information from the question, this is a mis-read (***MR***). A candidate should be penalized only once for a particular misread. Use the ***MR*** stamp to indicate that this has been a misread and do not award the first mark, even if this is an ***M*** mark, but award all others as appropriate.

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.
- If a candidate uses a correct answer, to a "show that" question, to a higher degree of accuracy than given in the question, this is NOT a misread and full marks may be scored in the subsequent part.
- ***MR*** can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If the command term is 'Hence' and not 'Hence or otherwise' then alternative methods are not permitted unless covered by a note in the mark scheme.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**.

7 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation** for example 1.9 and $1{,}9$ or 1000 and 1,000 and 1.000.
- Do not accept final answers written using calculator notation. However, ***M*** marks and intermediate ***A*** marks can be scored, when presented using calculator notation, provided the evidence clearly reflects the demand of the mark.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, some **equivalent** answers will generally appear in brackets. Not all equivalent notations/answers/methods will be presented in the markscheme and examiners are asked to apply appropriate discretion to judge if the candidate work is equivalent.

8 Format and accuracy of answers

Final answers will generally not need to restate the variable and/or units to be considered correct. To help examiners, the markscheme will include variables and units, where appropriate. However, their omission from a candidate's final answer should only be penalized if explicitly instructed in a markscheme note.

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. If the level of accuracy is not stated in the question, , the general rule applies to final answers: unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures.

Where values are used in subsequent parts, the markscheme will generally use the exact value, however candidates may also use the correct answer to a "correct" level of accuracy (e.g 3 sf) in subsequent parts. The markscheme will often explicitly include the subsequent values that come "*from the use of 3 sf values*".

Simplification of final answers: Candidates are advised to give final answers using good mathematical form. In general, for an ***A*** mark to be awarded, arithmetic should be completed, and any values that lead to integers should be simplified; for example, $\sqrt{\frac{25}{4}}$ should be written as $\frac{5}{2}$. An exception to this is simplifying fractions, where lowest form is not required (although the numerator and the denominator must be integers); for example, $\frac{10}{4}$ may be left in this form or written as $\frac{5}{2}$. However, $\frac{10}{5}$ should be written as 2, as it simplifies to an integer.

Algebraic expressions should be simplified by completing any operations such as addition and multiplication, e.g. $4e^{2x} \times e^{3x}$ should be simplified to $4e^{5x}$, and $4e^{2x} \times e^{3x} - e^{4x} \times e^{x}$ should be simplified to $3e^{5x}$. Unless specified in the question, expressions do not need to be factorized, nor do factorized expressions need to be expanded, so $x(x+1)$ and $x^2 + x$ are both acceptable.

Please note: intermediate ***A*** marks do NOT need to be simplified.

9 Calculators

A GDC is required for this paper, but If you see work that suggests a candidate has used any calculator not approved for IB DP examinations (eg CAS enabled devices), please follow the procedures for malpractice.

10. Presentation of candidate work

Crossed out work: If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work unless an explicit note from the candidate indicates that they would like the work to be marked.

More than one solution: Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise. If the layout of the responses makes it difficult to judge, examiners should apply appropriate discretion to judge which is "first".

1. (a) (i) $\frac{dv}{dt} = \frac{ds}{dt} \times \frac{dv}{ds}$ ***A1***

$(v = \frac{ds}{dt})$

$\frac{dv}{dt} = v\frac{dv}{ds}$ ***AG***

[1 mark]

(ii) $v\frac{dv}{ds} = g$

attempt to separate variables ***M1***

$\int v\,dv = \int g\,ds$

$\frac{v^2}{2} = gs\ (+c)$ ***A1***

using initial conditions (can be done at any point) ***M1***

$50 = c$

so $v = \sqrt{2gs + 100}$ ***A1***

Note: Marks are intentionally unimplied to ensure on-syllabus techniques are used.

[4 marks]

(iii) **EITHER**

attempt to use their part (a)(ii) to find a value of s when $v = 330$ ***(M1)***

$330 = \sqrt{2gs + 100}$

therefore $s = 5551.02...$ ***A1***

$(5551.02 < 40000)$

so (the model does predict) he will reach the speed of sound ***A1***

OR

attempt to use their part (a)(ii) to find a value of v when $s = 40000$ ***(M1)***

$v = \sqrt{2g(40000) + 100}$

$= 885 \quad (885.49...)$ ***A1***

$(885 > 330)$

so (the model does predict) he will reach the speed of sound (before $s = 40000$) ***A1***

Note: For the **OR** method, accept any large s that leads to $v = 330$.
FT from $\sqrt{2gs}$ gives 885 $(885.437\ldots)$ for v and 5560 $(5556.12\ldots)$ for s
FT from their *v* or their *s* for the final A1, provided M1 is awarded

[3 marks]

(b) (i) $v = gt\ (+c)$ OR gradient is a constant ***(M1)***

so the graph should be a straight line ***A1***

[2 marks]

(ii) the graph is not a straight line / only (approx.) straight for small t, so the model does not appear to be valid ***R1***

Note: Award ***R1*** for recognising that the graph is non-linear **AND** stating that the model does not appear to be valid

[1 mark]

(c) (i) $v\dfrac{\mathrm{d}v}{\mathrm{d}s} = g - kv^2$

separating variables ***(M1)***

$\displaystyle\int \frac{v}{g - kv^2}\,\mathrm{d}v = \int \mathrm{d}s$

$-\dfrac{1}{2k}\ln(g - kv^2) = s\ (+c)$ **OR** $-\dfrac{1}{2k}\ln\left|g - kv^2\right| = s\ (+c)$ ***(A1)***

rearranging to make v the subject ***(M1)***

Note: Award ***(M1)*** for making v the subject of their equation and not just an attempt, or an erroneous equation with v also on the RHS.

$g - kv^2 = A\mathrm{e}^{-2ks}$

$v = \sqrt{\dfrac{g - A\mathrm{e}^{-2ks}}{k}}$

applying initial conditions (here or elsewhere) ***(M1)***

$100 = \dfrac{g - A}{k}$

$A = g - 100k$

so

$v = \sqrt{\dfrac{g - (g - 100k)\mathrm{e}^{-2ks}}{k}}$ ***A1***

[5 marks]

(ii) $9.672 = 9.8 - 1600k$ ***A1A1***

Note: Award ***A1*** for correct left-hand side and ***A1*** for correct right-hand side.

$k = \dfrac{9.8 - 9.672}{1600}$

$k = 8 \times 10^{-5}$ ***AG***

Note: Award ***A1A0*** for $k = 8\times10^{-5}$ substituted into the right-hand side of the expression, leading to 9.672.

[2 marks]

(iii) $s \to \infty, \mathrm{e}^{-2ks} \to 0$ **OR** $\frac{\mathrm{d}v}{\mathrm{d}t} = 0$ **OR** graph/table ***(M1)***

$(v_{\max} = \sqrt{\frac{g}{k}} =) 350 \quad (ms^{-1})$ ***A1***

[2 marks]

(iv) upper limit occurs when $s = 40000$ ***(M1)***

Note: The ***M1*** can be implied by 40000 substituted into their part (c)(i).

$349.7\ (\mathrm{ms}^{-1})$ ***A1***

Note: Answer must be to 4 sf.

[2 marks]

(d) $s_{n+1} = s_n + 4000$ ***(A1)***

$v_{n+1} = v_n + 4000\times\left(\frac{3.98\times10^{14}}{v_n(6.41\times10^6 - s_n)^2} - (8\times10^{-5})v_n\right)$ ***(M1)(A1)***

Note: Award ***(M1)*** for attempt to use Euler method formula **AND** dividing through by v.

if $v_0 = 10$, then $v_{10} = 361\ (360.658\ldots)$ ***A1***

[4 marks]

(e) (i) Use a smaller step length ***R1***
OR
Use a better method such as Runge-Kutta ***R1***
OR
(Try to) solve the equation exactly ***R1***

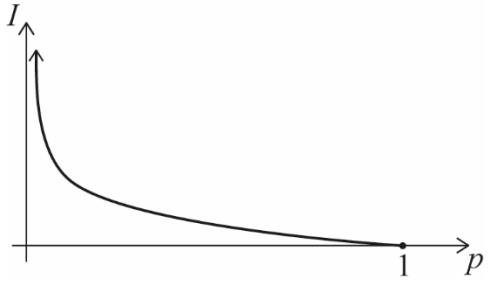
[1 mark]

(ii) Any reasonable response: ***R1***
For example:
Ignoring parachute / end point of motion / only valid for certain domain.
Treating Felix as a point object.
Ignoring weather / wind / air currents.
Assuming path is directly downwards.
Assuming perfect measurement of initial speed.

[1 mark]

[Total: 28 marks]

2. (a) (i)



approximately correct shape (decreasing and convex) ***A1***
asymptotic behaviour to the I -axis labelled (e.g. arrow) /
equation of asymptote $(p=0)$ seen ***A1***
p -intercept labelled at $p=1$ and graph does not extend beyond ***A1***

Note: Condone I and p being labelled as y and x.
Do not award second ***A1*** if y -intercept label is seen.

[3 marks]

(ii) $\frac{\mathrm{d}I}{\mathrm{d}p}=-\frac{1}{p}$ ***A1A1***

Note: Award ***A1*** for $\frac{\mathrm{d}I}{\mathrm{d}p}$ or equivalent (e.g. I') and ***A1*** for $-\frac{1}{p}$. Do not accept $\frac{\mathrm{d}y}{\mathrm{d}x}$ for the first ***A1*** unless followed by $-\frac{1}{x}$, which can earn ***A1A0.***

(for $0<p\leq 1$) we therefore have $\frac{\mathrm{d}I}{\mathrm{d}p}<0$ ***R1***

hence the function is decreasing ***AG***
[3 marks]

(iii) Any plausible interpretation IN CONTEXT. ***R1***
For example:
More information is gained from a rarer event.
Less information is gained from a more common event.
Information (gained) decreases as probability increases.

[1 mark]

(b) (i) $\frac{5}{10}$ $(=\frac{1}{2})$ ***A1***

[1 mark]

(ii) attempt to substitute $p=$ their (b)(i) into $I=-\ln p$ ***(M1)***

$=0.693$ $(0.693147...,-\ln\left(\frac{1}{2}\right))$ ***A1***

[2 marks]

(iii) $\frac{9}{10}$ ***(A1)***

$= 0.105 \quad (0.105360..., -\ln\left(\frac{9}{10}\right))$ ***A1***

[2 marks]

(c) (i) attempt to substitute into the formula for $\mathrm{E}(I)$ and recognise that $n = 2$ (or two terms are needed) ***M1***

$\mathrm{E}(I) = -\frac{1}{10}\ln\left(\frac{1}{10}\right) - \frac{9}{10}\ln\left(\frac{9}{10}\right)$ ***A1***

$0.325 \quad (0.325082...)$ ***AG***

[2 marks]

(ii) $\mathrm{E}(I) = -\frac{1}{2}\ln\left(\frac{1}{2}\right) - \frac{1}{2}\ln\left(\frac{1}{2}\right)$ ***A1***

$0.693 \quad (0.693147..., \ln(2))$ ***A1***

$0.693 > 0.325$ ***AG***

[2 marks]

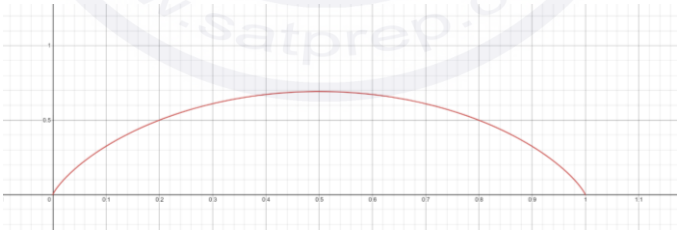
(d) (i) $(I =) -\ln(1-p)$ ***A1***

[1 mark]

(ii) $(\mathrm{E}(I) =) -p\ln p - (1-p)\ln(1-p)$ ***A1***

[1 mark]

(iii) attempt to use graphical method or calculus to maximize $\mathrm{E}(I)$ ***M1***



maximum occurs when $p = \frac{1}{2}$ ***A1***

[2 marks]

(e) (i) $(x=)\frac{6}{12}\ \left(=\frac{1}{2}\right)$ ***A1***

EITHER
for the scales to balance, the odd ball must be in the six balls not chosen ***R1***

OR
for the scales to balance, all the balls chosen must be of equal weight and hence
$\frac{11}{12}\times\frac{10}{11}\times\frac{9}{10}\times...\times\frac{6}{7}$ ***R1***

[2 marks]

(ii) **EITHER**
recognition that the sum of the probabilities on the third row of the table equals 1 ***(M1)***
e.g. $x+2y=1$

OR

for one side to be heavier, the odd ball must be one of six balls chosen $\left(\frac{6}{12}\right)$ and half the time this will result in left-side being heavier,

therefore $y=\frac{6}{12}\times\frac{1}{2}$ ***(M1)***

$y=\frac{1}{4}$ ***A1***

[2 marks]

(iii) $z=-\frac{1}{6}\ln\frac{1}{6}-\frac{1}{6}\ln\frac{1}{6}-\frac{2}{3}\ln\frac{2}{3}$ ***(M1)***

$=0.868\quad(0.867563...)$ ***A1***

[2 marks]

(iv) 4 balls on each side because that configuration has the largest $\mathrm{E}(I)$ ***R1***

Note: Award ***R1*** for giving a correct reason **AND** stating "4 balls on each side"

[1 mark]

[Total: 27 marks]

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2.	$\frac{35}{72}$	0.468111… (incorrect decimal value)	Yes. Value is used in subsequent parts.	Award ***A0*** for the final mark *(and full **FT** is available in subsequent parts)*

3 Implied marks

Implied marks appear in **brackets e.g. *(M1)***,and can only be awarded if **correct** work is seen or implied by subsequent working/answer.

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Please note: intermediate ***A*** marks do NOT need to be simplified.

9 Calculators

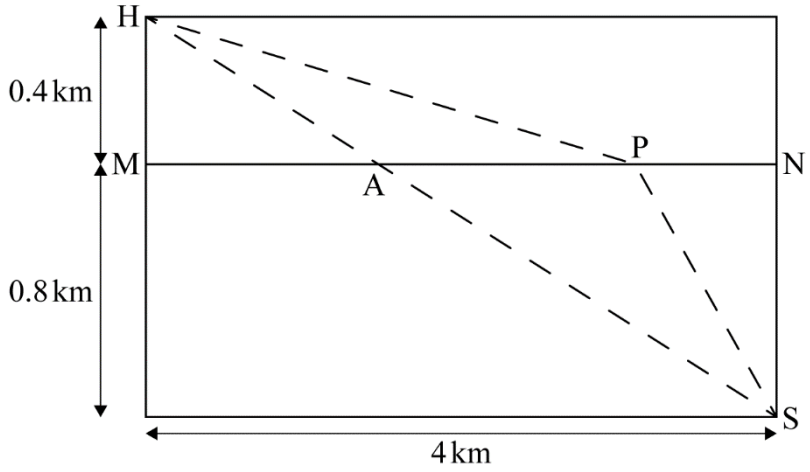
A GDC is required for this paper, but If you see work that suggests a candidate has used any calculator not approved for IB DP examinations (eg CAS enabled devices), please follow the procedures for malpractice.

10. Presentation of candidate work

Crossed out work: If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work unless an explicit note from the candidate indicates that they would like the work to be marked.

More than one solution: Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise. If the layout of the responses makes it difficult to judge, examiners should apply appropriate discretion to judge which is "first".

1. (a)



METHOD 1

$\hat{\text{MHS}} = \left(\tan^{-1}\frac{4}{1.2} =\right)\ 73.3007...^{\circ}$ OR $1.27933...$ ***(A1)***

use of trigonometry to find HA or AS ***(M1)***

$\text{HA} = \frac{0.4}{\cos \hat{\text{MHS}}}$ **AND** $\text{AS} = \frac{0.8}{\cos \hat{\text{MHS}}}$ ***(A1)***

$(\text{HA} = 1.39204...$ and $\text{AS} = 2.78408...)$

use of $\text{time} = \frac{\text{distance}}{\text{speed}}$ for either of their distances ***(M1)***

time taken $= \left(\frac{\text{AH}}{15} + \frac{\text{AS}}{5}\right)$

$0.649618...$ (hours) ***(A1)***

(38.97712… minutes)

therefore 39 (mins) ***A1FT***

Note: Allow ***FT***, within the question part, from their time in hours for the final ***A1***.

METHOD 2

EITHER

use of similar triangles to identify either length MA or AN ***(M1)***

$\left(\frac{4}{3}\text{ or }\frac{8}{3}\right)$

attempt to use Pythagoras for either triangle AMH or ANS ***(M1)***

$\mathrm{AH}^2 = 0.4^2 + \left(\frac{4}{3}\right)^2$ **AND** $\mathrm{AS}^2 = 0.8^2 + \left(\frac{8}{3}\right)^2$ ***(A1)***

OR

attempt to use Pythagoras for larger triangle ***(M1)***

$\mathrm{SH}^2 = 4^2 + 1.2^2$

$\mathrm{AH} = \frac{1}{3}\sqrt{4^2 + 1.2^2}$ **AND** $\mathrm{AS} = \frac{2}{3}\sqrt{4^2 + 1.2^2}$ ***(M1)(A1)***

THEN

$(\mathrm{HA} = 1.39204...$ and $\mathrm{AS} = 2.78408...)$

use of $\text{time} = \frac{\text{distance}}{\text{speed}}$ for either of THEIR distances ***(M1)***

time taken $= \left(\frac{\mathrm{AH}}{15} + \frac{\mathrm{AS}}{5}\right)$

$0.649618...$ (hours) ***(A1)***

(38.97712… minutes)

therefore 39 (mins) ***A1FT***

Note: Allow ***FT***, within the question part, from their time in hours for the final ***A1***.

[6 marks]

(b) (i) $\text{PH}^2 = 0.4^2 + x^2$ **AND** $\text{PS}^2 = 0.8^2 + (4-x)^2$ ***A1***

Note: This ***A1*** can be implied by a clear expression for the time in each region coming from distance / speed below.

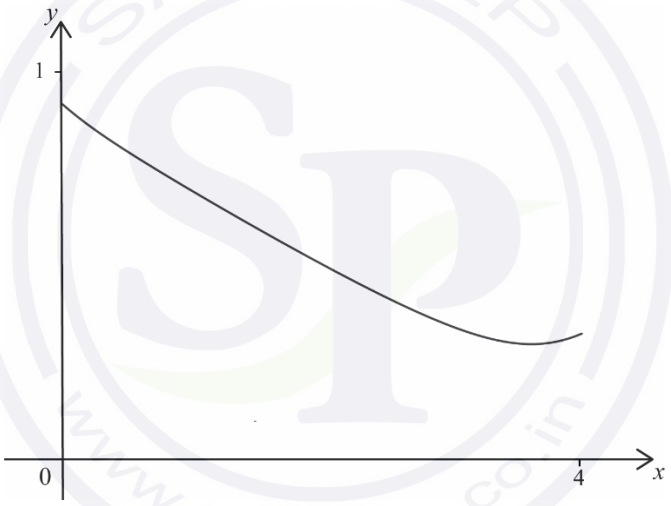
$$T(x) = \frac{\text{PH}}{15} + \frac{\text{PS}}{5}$$ ***(M1)***

$$T(x) = \frac{\sqrt{0.4^2 + x^2}}{15} + \frac{\sqrt{0.8^2 + (4-x)^2}}{5}$$ ***A1***

$$T(x) = \frac{\sqrt{0.4^2 + x^2} + 3\sqrt{0.8^2 + (4-x)^2}}{15}$$ ***AG***

[3 marks]

(ii)



correct shape with minimum point nearer $x = 4$ than $x = 0$ ***A1***
correct (approximate) y-intercept, $0.843\ldots$ (must be clearly below 1) ***A1***

[2 marks]

(iii) using the GDC, at the minimum $x = 3.72$ $(3.71898...)$ ***A1***

Note: Do not accept coordinates of the minimum point.

[1 mark]

(iv) finding their $T(x)$ for their value of x ***M1***
$T(x) = 0.418946\ldots$
so time saved $(= 38.97712\ldots - 25.1367\ldots \text{mins})$ = 14 (mins) ***A1***

[2 marks]

(c) (i) attempt at chain rule ***M1***

$$T'(x) = \frac{1}{15}\left(\frac{x}{\sqrt{0.4^2 + x^2}} - \frac{3(4-x)}{\sqrt{0.8^2 + (4-x)^2}}\right)$$ ***A1A1***

Note: Award ***A1*** for each correct term. Accept any equivalent form i.e. condone fractions not simplified.

[3 marks]

(ii) setting their $T'(x) = 0$ ***M1***

Note: This requires more than just a statement that the derivative equals zero – they must use their attempt at $T'(x)$.

$$\frac{1}{15}\left(\frac{x}{\sqrt{0.4^2 + x^2}} - \frac{3(4-x)}{\sqrt{0.8^2 + (4-x)^2}}\right) = 0$$

$$\frac{x}{\sqrt{0.16 + x^2}} = \frac{3(4-x)}{\sqrt{0.64 + (4-x)^2}}$$ ***AG***

[1 mark]

(iii) **METHOD 1**

$\cos \hat{\text{HPM}} = \frac{x}{\sqrt{0.16 + x^2}}$ **AND** $\cos \hat{\text{SPN}} = \frac{4-x}{\sqrt{0.64 + (4-x)^2}}$ ***A1***

substituting in the above equation and rearranging ***M1***

$\cos \hat{\text{HPM}} = 3\cos \hat{\text{SPN}}$ leading to $\frac{\cos \hat{\text{HPM}}}{\cos \hat{\text{SPN}}} = 3 = \left(\frac{15}{5}\right)$

verifying the result ***AG***

METHOD 2

$$\frac{x}{\sqrt{0.16 + x^2}} = \frac{3(4-x)}{\sqrt{0.64 + (4-x)^2}}$$

attempt to rearrange into a quotient ***M1***

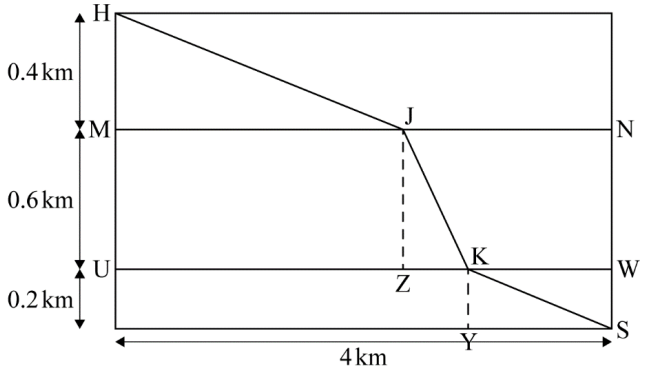
$$\left(\frac{15}{5} = 3 =\right) \frac{\frac{x}{\sqrt{0.16 + x^2}}}{\frac{4-x}{\sqrt{0.64 + (4-x)^2}}}$$

$$= \frac{\cos \hat{\text{HPM}}}{\cos \hat{\text{SPN}}}$$ ***A1***

verifying the result ***AG***

[2 marks]

(d) **METHOD 1**



let $\mathrm{MJ} = y$ km and W and Z be the points on the new boundary directly below N and J

attempt to find ZK in terms of MJ ***M1***

$(\mathrm{KW} = 0.5y)$

$\mathrm{ZK} = (4 - 1.5y)$ km. ***A1***

attempt to use the result from (c)(iii) at J ***M1***

$$\frac{\cos \mathrm{H\hat{J}M}}{\cos \mathrm{Z\hat{K}J}} = \frac{y}{\sqrt{y^2 + 0.4^2}} \div \frac{(4-1.5y)}{\sqrt{(4-1.5y)^2 + 0.6^2}} = \frac{15}{5} \quad \textit{A1}$$

Note: Accept $\cos \mathrm{N\hat{J}K}$ in place of $\cos \mathrm{Z\hat{K}J}$.

$$\left(\text{leading to } \frac{y}{\sqrt{y^2 + 0.16}} = \frac{3(4-1.5y)}{\sqrt{(4-1.5y)^2 + 0.36}} \right)$$

valid method for solving this equation, eg drawing graphs of both sides of the equation, using SOLVER, etc. ***(M1)***

solution is $y = 2.53$ ***A1***

METHOD 2

combining the field into one region with height 0.6 km ***M1***

$$\cos \mathrm{H\hat{P}M} = \frac{x}{\sqrt{0.36 + x^2}}$$

$$\cos \mathrm{S\hat{P}N} = \frac{4-x}{\sqrt{0.36 + (4-x)^2}} \quad \textit{A1}$$

Note: Both expressions, or their ratio, are required for the ***A1*** to be awarded.

therefore

$$\frac{x\sqrt{0.36 + (4-x)^2}}{(4-x)\sqrt{0.36 + x^2}} = 3 \quad \textit{A1}$$

valid method for solving ***(M1)***

attempting to find MJ in terms of x e.g. $\mathrm{MJ} = \frac{2}{3}x$ ***M1***

so $\mathrm{MJ} = 2.53$ ***A1***

[6 marks]

Total [26 marks]

2. (a) (i) because the (population) standard deviation(s) are unknown ***A1***

Note: Ignore any references to sample size.

[1 mark]

(ii) **EITHER**
he has no idea beforehand which way the difference would be if there is a difference ***A1***

OR
he is only interested that there is a difference (not the direction) ***A1***

[1 mark]

(b) (i) **EITHER**
$H_0: \mu_F = \mu_G;\ H_1: \mu_F \neq \mu_G$ ***A1***

OR
$H_0: \mu_D = 0;\ H_1: \mu_D \neq 0$ ***A1***

Note: Accept an equivalent statement in words, must include mean and reference to "population mean" / "mean for all those taking the French exam" etc. for the first ***A1*** to be awarded. The terms "on average" and "generally" are also acceptable to indicate populations. Do not accept an imprecise "the means are equal".

Do not accept "There is (no) (significant) evidence of a difference between μ_F and μ_G" for either hypothesis or "There is (no) significant difference between marks in French and German".

[1 mark]

(ii) Generate a third column giving French mark – German mark or German mark – French mark. ***(M1)***
e.g.

French mark	German mark	Difference
42	39	3
65	66	–1
82	71	11
…	…	…

p-value $= 0.153$. ***A1***

[2 marks]

(iii) The p-value gives the probability of seeing the observed difference in means (or a larger difference) assuming H_0 to be true. ***A1***

Note: Do not accept "the probability that the data occurs by chance" or similar.

[1 mark]

(iv) because $0.153 > 0.05$ ***R1***

EITHER

there is not (significant evidence of) a difference between the (population) means ***A1***

OR

fail to reject H_0 (accept "accept H_0") ***A1***

Note: Do not award ***R0A1***.
Remember to ***FT*** from part (b)(ii).
Do not award the final ***A1*** if the null hypothesis in part (b)(i) is logically wrong (i.e. if the null and the alternative have been reversed or are nonsense) but this can be awarded if part (b)(i) is just poorly communicated.

[2 marks]

(c) (i) $\mathrm{H}_0 : \rho = 0 ;\ \mathrm{H}_1 : \rho > 0$ ***A1***

Note: Condone $\mathrm{H}_0 : \rho \leq 0$.

[1 mark]

(ii) p-value $= 0.00286$ ***A2***

$0.00286 < 0.05$ ***R1***

he should conclude that the two sets of marks are (generally) positively correlated ***A1***

Note: Allow ***FT*** from any test for correlation.
Do not award ***R0A1***.
The final ***R1A1*** should follow through from their p-value.
Do not award the final ***A1*** if the null hypothesis in part (c)(i) is wrong (i.e. if the null and the alternative have been reversed or are nonsense), but this can be awarded if part (c)(i) is just poorly communicated.
The final conclusion must be in context.

[4 marks]

(d) (i) the regression line of German on French is

$\text{German} = 10.2393... + 0.737495...\text{French}$ ***(A1)***

EITHER

substituting $\text{French} = 58$ into their regression line ***(M1)***

OR

sketch showing regression line and $x = 58$ ***(M1)***

THEN

Paul's German mark $= 53$ ***A1***

Note: Accept an answer of 53.0 $(53.0140...)$ or 52.9 as integer results are not explicitly stated in the question.
Regression lines may be written in terms of y and x.

[3 marks]

(ii) recognizing need to use line French on German
$\text{French} = 4.04116... + 1.01122...\text{German}$ ***(A1)***
putting $\text{French} = 71$, Sue's German mark $= 66$ ***A1***

Note: Accept an answer of 66.2 $(66.2158...)$ or 66.3 as integer results are not explicitly stated in the question.
Although not required in the markscheme as presented, candidates may have considered $\text{French} = 70.5$ and $\text{French} = 71.5$; this is valid and will lead to the correct answer.
If the line German on French is used in part (d)(ii) the answer is 63; award ***A0A0***.

[2 marks]

(e) (i) **EITHER**
the maximum value of τ occurs when all pairs are concordant so max $= +1$
the minimum value of τ occurs when all pairs are discordant so min $= -1$
A1

OR

when all concordant $C - D = \frac{n(n-1)}{2}$, and when all discordant $C - D = -\frac{n(n-1)}{2}$
A1

OR

when all concordant $C = \frac{n(n-1)}{2}$, $D = 0$ and when all discordant $C = 0$, $D = \frac{n(n-1)}{2}$
A1

THEN
hence the range is $[-1, +1]$ ***AG***

Note: Accept an answer which is just based on $n = 6$.

[1 mark]

(ii) $(53 - 76)(41 - 70) > 0$ ***A1***
Hence concordant ***AG***

[1 mark]

(iii) Evidence of a valid method, eg ***M1***
P_1: C, D, C, C, C
P_2 : C, C, C, D
P_3 : D, C, D
P_4 : D, C
P_5 : D
P_6 :

Note: At least one pair beyond (P_1, P_2) needs to be compared to award ***M1***.

any evidence (a statement or a list) that 15 pairs need to be considered ***A1***

$C = 9,\ D = 6$ ***A1***

using their stated C and D values in given formula with $n = 6$ ***M1***
$\frac{2(9-6)}{6(6-1)}$ **OR** $\frac{9-6}{15}$
$\tau = 0.2$ ***AG***

[4 marks]

(f) (i) H_0: There is no (underlying) association (or correlation) between the two sets of marks
H_1: There is an (underlying) association (or correlation) between the two sets of marks ***A1***

Note: Do not accept independence in the hypotheses.

[1 mark]

(ii) τ does not lie in the critical region **OR** $0.2<0.733$ ***R1***
EITHER
there is insufficient evidence to indicate that there is an association between the two sets of marks ***A1***

OR
fail to reject H_0 (accept "accept H_0") ***A1***

Note: Do not award ***R0A1***.
In this question the final ***A1*** mark can be awarded for "fail to reject H_0" or "accept H_0" even if the hypotheses in (f)(i) are the wrong way round as the critical region is given.

[2 marks]

(g) no ***A1***
because scaling the marks will not affect the concordances/ discordances ***R1***

Note: Do not award ***A1R0***.

[2 marks]
Total [29 marks]

Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

May 2023

Mathematics: applications and interpretation

Higher level

Paper 3

15 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

R Marks awarded for clear **Reasoning**.

AG Answer given in the question and so no marks are awarded.

FT Follow through. The practice of awarding marks, despite candidate errors in previous parts, for their correct methods/answers using incorrect results.

Using the markscheme

1 General

Award marks using the annotations as noted in the markscheme eg ***M1***, ***A2***.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***A3***, ***M2*** *etc.*, do **not** split the marks, unless there is a note.
- The response to a "show that" question does not need to restate the ***AG*** line, unless a **Note** makes this explicit in the markscheme.
- Once a correct answer to a question or part question is seen, ignore further working even if this working is incorrect and/or suggests a misunderstanding of the question. This will encourage a uniform approach to marking, with less examiner discretion. Although some candidates may be advantaged for that specific question item, it is likely that these candidates will lose marks elsewhere too.
- An exception to the previous rule is when an incorrect answer from further working is used **in a subsequent part**. For example, when a correct exact value is followed by an incorrect decimal approximation in the first part and this approximation is then used in the second part. In this situation, award ***FT*** marks as appropriate but do not award the final ***A1*** in the first part. Examples:

	Correct answer seen	Further working seen	Any FT issues?	Action
1.	$8\sqrt{2}$	5.65685... *(incorrect decimal value)*	No. Last part in question.	Award ***A1*** for the final mark *(condone the incorrect further working)*
2.	$\frac{35}{72}$	0.468111… (incorrect decimal value)	Yes. Value is used in subsequent parts.	Award ***A0*** for the final mark *(and full **FT** is available in subsequent parts)*

3 Implied marks

Implied marks appear in **brackets e.g. *(M1)***,and can only be awarded if **correct** work is seen or implied by subsequent working/answer.

4 Follow through marks (only applied after an error is made)

Follow through (***FT***) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) (e.g. incorrect value from part (a) used in part (d) or incorrect value from part (c)(i) used in part (c)(ii)). Usually, to award ***FT*** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if all the marks awarded in a subsequent part are for the answer or are implied, then ***FT*** marks should be awarded for *their* correct answer, even when working is not present.

For example: following an incorrect answer to part (a) that is used in subsequent parts, where the markscheme for the subsequent part is ***(M1)A1***, it is possible to award full marks for *their* correct answer, **without working being seen.** For longer questions where all but the answer marks are implied this rule applies but may be overwritten by a **Note** in the Markscheme.

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks, by reflecting on what each mark is for and how that maps to the simplified version.
- If the error leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word “their” in a description, to indicate that candidates may be using an incorrect value.
- If the candidate’s answer to the initial question clearly contradicts information given in the question, it is not appropriate to award any ***FT*** marks in the subsequent parts. This includes when candidates fail to complete a “show that” question correctly, and then in subsequent parts use their incorrect answer rather than the given value.
- Exceptions to these ***FT*** rules will be explicitly noted on the markscheme.
- If a candidate makes an error in one part but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the command term was “Hence”.

5 Mis-read

If a candidate incorrectly copies values or information from the question, this is a mis-read (***MR***). A candidate should be penalized only once for a particular misread. Use the ***MR*** stamp to indicate that this has been a misread and do not award the first mark, even if this is an ***M*** mark, but award all others as appropriate.

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.
- If a candidate uses a correct answer, to a "show that" question, to a higher degree of accuracy than given in the question, this is NOT a misread and full marks may be scored in the subsequent part.
- ***MR*** can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If the command term is 'Hence' and not 'Hence or otherwise' then alternative methods are not permitted unless covered by a note in the mark scheme.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**.

7 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation** for example 1.9 and $1{,}9$ or 1000 and $1{,}000$ and 1.000.
- Do not accept final answers written using calculator notation. However, ***M*** marks and intermediate ***A*** marks can be scored, when presented using calculator notation, provided the evidence clearly reflects the demand of the mark.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, some **equivalent** answers will generally appear in brackets. Not all equivalent notations/answers/methods will be presented in the markscheme and examiners are asked to apply appropriate discretion to judge if the candidate work is equivalent.

8 Format and accuracy of answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. If the level of accuracy is not stated in the question, the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures*.

Where values are used in subsequent parts, the markscheme will generally use the exact value, however candidates may also use the correct answer to a "correct" level of accuracy (e.g 3 sf) in subsequent parts. The markscheme will often explicitly include the subsequent values that come "*from the use of 3 sf values*".

Simplification of final answers: Candidates are advised to give final answers using good mathematical form. In general, for an ***A*** mark to be awarded, arithmetic should be completed, and any values that lead to integers should be simplified; for example, $\sqrt{\frac{25}{4}}$ should be written as $\frac{5}{2}$. An exception to this is simplifying fractions, where lowest form is not required (although the numerator and the denominator must be integers); for example, $\frac{10}{4}$ may be left in this form or written as $\frac{5}{2}$. However, $\frac{10}{5}$ should be written as 2, as it simplifies to an integer.

Algebraic expressions should be simplified by completing any operations such as addition and multiplication, e.g. $4e^{2x} \times e^{3x}$ should be simplified to $4e^{5x}$, and $4e^{2x} \times e^{3x} - e^{4x} \times e^{x}$ should be simplified to $3e^{5x}$. Unless specified in the question, expressions do not need to be factorized, nor do factorized expressions need to be expanded, so $x(x+1)$ and $x^2 + x$ are both acceptable.

Please note: intermediate ***A*** marks do NOT need to be simplified.

9 Calculators

A GDC is required for this paper, but If you see work that suggests a candidate has used any calculator not approved for IB DP examinations (eg CAS enabled devices), please follow the procedures for malpractice.

10. Presentation of candidate work

Crossed out work: If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work unless an explicit note from the candidate indicates that they would like the work to be marked.

More than one solution: Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise. If the layout of the responses makes it difficult to judge, examiners should apply appropriate discretion to judge which is "first".

1. (a) $C = kd$ ***(M1)***

$0.80 = 0.5k$ **OR** $\frac{C}{d} = \frac{0.8}{0.5}$ ***(A1)***

$k = 1.6$

$C = 1.6d$ **OR** $C = \frac{d}{0.625}$ ***A1***

Note: For the final ***A1*** do not accept $C = \frac{0.8}{0.5}d$ or a correct equation which does not have C as the subject.

[3 marks]

(b) $d = \frac{0.96}{1.6}$ ***M1***

Note: Award ***M1*** for the substitution of $C = 0.96$ into a correct equation, award ***M0*** for substitution of $d = 0.6$.

$= 0.6$ ***AG***

[1 mark]

(c) attempt at using Pythagoras ***M1***

$d^2 = h^2 + (2r)^2$ ***(A1)***

$r^2 = \frac{1}{4}\left(d^2 - h^2\right)$ ***A1***

[3 marks]

(d) $V = \pi r^2(2h)$ ***(A1)***

$V = \frac{\pi}{4}\left(d^2 - h^2\right)(2h)$ ***M1***

Note: The ***M1*** is for the substitution of their expression for r^2 into their formula for V.

$V = \frac{\pi}{2}\left(d^2h - h^3\right)$ ***AG***

[2 marks]

(e) (i) $V = \frac{\pi}{2}\left(d^2h - h^3\right)$

$= \frac{\pi}{2}\left(0.6^2 \times 0.4 - 0.4^3\right)$ ***(M1)***

Note: The ***M1*** is for substituting correct values of both d and h in the formula from part (d).

$= 0.126\ (\text{m}^3)\ \left(0.12566\ldots, \frac{\pi}{25}, 0.04\pi\right)$ ***A1***

[2 marks]

(ii) $V = \frac{\pi}{2}\left(0.36h - h^3\right)$ ***M1***

Note: Award ***M1*** for substitution of $d = 0.6$. This may be seen anywhere.

$\frac{\mathrm{d}V}{\mathrm{d}h} = \frac{\pi}{2}\left(0.36 - 3h^2\right) = 0$ ***A1***

Note: Award ***A1*** for correct differentiation.

$h^2 = \frac{0.36}{3}\ (= 0.12)$ ***A1***

Note: Do not award the final ***A1*** if the working is done with approximate values or if $\sqrt{0.12}$ is substituted into $\frac{\mathrm{d}V}{\mathrm{d}h}$.

$h = \sqrt{0.12}$ ***AG***

[3 marks]

(iii) substituting $h = \sqrt{0.12}$ into equation for V **OR** use of graph ***(M1)***

$V = \frac{\pi}{2}(0.36 - 0.12)\sqrt{0.12}$

$0.131\ (\text{m}^3)\ (0.130593..., 0.0416\pi)$ ***A1***

[2 marks]

(f) x-coordinate of S is 0.4 ***(A1)***

let the y-coordinate be y_S

attempt at Pythagoras ***(M1)***

$(y_S + 0.2)^2 = 0.6^2 - 0.4^2$

$y_S + 0.2 = \sqrt{0.2}$

$y_S = 0.247\ (0.247213...)$ ***(A1)***

Note: The ***(M1)*** mark can be implied by a y-coordinate of 0.447 or 0.647 seen.

any valid method to find equation ***(M1)***

e.g. quadratic regression, vertex form, simultaneous equations

Note: Award only if the student has found three points on the curve.

EITHER

$y = -0.295x^2 + 0.236x + 0.2$ ***A2***

$(y = -0.295081...x^2 + 0.236065...x + 0.2)$

Note: Award ***A1*** if one coefficient ($-0.295081...$ or $0.236065...$) is correct or if "$y =$" is missing, ***A2*** for completely correct equation.
Award ***A1*** for $y = -0.294x^2 + 0.235x + 0.2$ obtained from using the 3 sf value of y_S.

OR

$y = -0.295(x - 0.4)^2 + 0.247$ ***A2***

$(y = -0.295081...(x - 0.4)^2 + 0.247213...)$

Note: Award ***A1*** for -0.295, ***A2*** for completely correct equation.

[6 marks]

(g) volume $= \pi\int_0^{0.8} \left(-0.295081...x^2 + 0.236065...x + 0.2\right)^2 \mathrm{d}x$ ***M1***

Note: Award ***M1*** for the minimum of an integral with the correct limits and their function squared.

$= 0.135\ (0.135161\ldots)$ ***A1***

$0.135 > 0.131$ ***R1***

Note: Award ***R1*** independently of the previous marks for a correct comparison of their (clearly stated) volume with their answer to part (e)(iii).

Hence the volume is greater than any cylinder volume ***AG***

[3 marks]

(h) *Award **A1** for at least one reasonable answer, for example:*

the barrel is full of wine when sold
the barrel/stick is constructed with zero thickness
the stick is straight and inflexible
the hole has no diameter
volume of wine is the only important factor in value
aesthetics are not important

Note: Do not accept statements that relate to the barrel having circular cross-section, for example, 'there are no deformities', 'it is perfectly smooth' as these assumptions have already been made with the chosen model.

[1 mark]

[Total 26 marks]

2. (a) $\vec{PQ} = \begin{pmatrix} -115 \\ 115 \\ 0 \end{pmatrix}$ **OR** $\sqrt{(215-100)^2+(-197+82)^2+0^2}$ ***(M1)***

Note: Accept working using "A and B" in place of "P and Q".

$\sqrt{115^2+115^2+0^2}$

$= 163 \text{ (km)} \ (162.634..., \sqrt{26450})$ ***A1***

[2 marks]

(b) speed of each aircraft = $\sqrt{640^2+640^2+0^2}$ ***(M1)***

$= 905 \ (\text{km h}^{-1}) \ \ (905.096\ldots, \sqrt{819200})$ **OR** $251 \ \text{m s}^{-1} \ (251.388...)$ ***A1***

[2 marks]

(c) time to travel $162.634\ldots$ km is $\frac{162.634...}{905.096...}$ $(0.179687...)$ ***(M1)***

Note: Accept $\frac{115}{640}$ from a consideration of the component directions.
Accept use of the 3 significant figure answer from parts (a) and (b):
$\frac{163}{905} = 0.180110\ldots$

EITHER

(multiply by 60 to get) $10.8 \ (10.7812...)$ (minutes) or 10 minutes 48 seconds ***A1***

$(10.8 > 10)$ hence not in conflict ***R1***

Note: Award ***R1*** for a correct comparison of their time, in minutes, with 10 minutes.

OR

(convert 10 minutes into hours) $0.167 \ (0.166666...)$ (hours) ***A1***

$(0.180 > 0.167)$ hence not in conflict ***R1***

Note: Award ***R1*** for a correct comparison of their time, in hours, with 0.167 hours.

[3 marks]

(d) $\left(\boldsymbol{r}_A = \right) \begin{pmatrix} 100 \\ -82 \\ 10.7 \end{pmatrix} + t \begin{pmatrix} -640 \\ 640 \\ 0 \end{pmatrix}$ ***A1***

[1 mark]

(e) (i) $\boldsymbol{r}_A - \boldsymbol{r}_C = \begin{pmatrix} (100-640t)-(-400-140t) \\ (-82+640t)-(-41+604t) \\ 10.7-(9.1+2t) \end{pmatrix}$ ***(M1)(A1)***

$= \begin{pmatrix} 500-500t \\ -41+36t \\ 1.6-2t \end{pmatrix}$

Note: Award ***M1*** for an attempt to subtract their $\boldsymbol{r}_A$ and $\boldsymbol{r}_C$ in either order, ***(A1)*** for a correct expression, which does not need to be simplfiied and which may be seen in the line below.

$|\boldsymbol{r}_A - \boldsymbol{r}_C| = \sqrt{(500-500t)^2 + (-41+36t)^2 + (1.6-2t)^2}$ ***(M1)***

Note: Award ***(M1)*** for a correct attempt to find the modulus of their $\boldsymbol{r}_A - \boldsymbol{r}_C$.

this is equal to $10\ \text{km}$ when $t = 0.983\ \ (0.983441...)$ and $1.02\ \ (1.01799...)$

A1A1

[5 marks]

(ii) **METHOD 1**

consideration of the vertical component of their $\boldsymbol{r}_A - \boldsymbol{r}_C$ from part (e)(i) ***M1***

$1.6-2t$

$-0.3 < 1.6-2t < 0.3$ ***A1***

Note: Award ***A1*** for relating their expression to -0.3 **and** 0.3; accept an equality.

$(0.65 <)\ t < 0.95$ ***A1***

Note: Award ***A1*** for 0.95 seen.

interval is outside of interval from part (e)(i) (the two conditions are never broken at the same time) ***R1***

Note: The ***R1*** can only be awarded if there is a clear consideration of intervals.

hence regulations are not broken ***A1***

Note: Do not award ***R0A1***.

METHOD 2

consideration of the vertical component of their $\boldsymbol{r}_A - \boldsymbol{r}_C$ from part (e)(i) ***M1***

$1.6 - 2t$

when $t = 0.983441...$ the difference in height is $1.6 - 2 \times 0.983441...$

$= (-)\ 0.366882\ldots \text{ km}$ ***A1***

EITHER

as time increases the vertical displacement between the aircraft also increases ***R2***

OR

when $t = 1.01799...$ the difference in height is $1.6 - 2 \times 1.01799...$

$= (-)\ 0.435970\ldots (\text{km})$ ***A1***

because it is a linear function, the difference in height is also greater than 0.3 km between these values of t. ***R1***

Note: Accept an argument from a graph.

THEN

so regulations are not broken ***A1***

Note: Do not award ***R0A1***.

[5 marks]

(f) circle ***A1***

radius of $6.4\ (\text{km})$ ***A1***

centred on $\text{R} / (0, 0)$ ***A1***

Note: A description that includes only one or two of the points above can be awarded a further ***A1*** for a further correct statement, for example: "the speed is $243.2\ \text{km h}^{-1}$" **OR** "it travels anticlockwise when viewed from above" **OR** "it travels clockwise when viewed from below".

[3 marks]

(g) (i) attempt at scalar product for the correct two vectors ***(M1)***

Note: Award ***(M1)*** for a product and sum of components leading to a single scalar expression.

$\vec{\text{RE}} \bullet \boldsymbol{b} = -1(20-\lambda)+(10+\lambda)$

$= -10+2\lambda$ ***A1***

[2 marks]

(ii) $-10+2\lambda = 0$ ***(M1)***

Note: Award ***(M1)*** for setting their scalar product equal to 0, but only if their scalar product is a single expression.

$\lambda = 5$ ***A1***

[2 marks]

(iii) **METHOD 1**

$\vec{\text{RE}} = \begin{pmatrix} 20 \\ 10 \end{pmatrix} + 5\begin{pmatrix} -1 \\ 1 \end{pmatrix} \left(= \begin{pmatrix} 15 \\ 15 \end{pmatrix} \right)$ ***(M1)***

$\left|\vec{\text{RE}}\right|_{\min} = \sqrt{15^2+15^2} = 21.2 \ \ (21.2132..., 15\sqrt{2}, \sqrt{450})$ (km) ***A1***

METHOD 2

$\left|\vec{\text{RE}}\right| = \sqrt{(20-\lambda)^2+(10+\lambda)^2}$ ***(M1)***

$\left|\vec{\text{RE}}\right|_{\min} = 21.2 \ \ (21.2132..., 15\sqrt{2}, \sqrt{450})$ km ***A1***

[2 marks]

(iv) consideration of at least two distances from (their) 21.2, 6.4 and 10 ***M1***

EITHER

$6.4+10 = 16.4 < 21.2$ ***R1***

OR

$21.2-10 = 11.2 > 6.4$ ***R1***

OR

$21.2-6.4 = 14.8 > 10$ ***R1***

Note: Accept equivalent reasoning in words.

THEN

hence they do not break regulations ***AG***

[2 marks]

[Total 29 marks]

Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

November 2022

Mathematics: applications and interpretation

Higher level

Paper 3

14 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**.
A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.
R Marks awarded for clear **Reasoning**.
AG Answer given in the question and so no marks are awarded.
FT Follow through. The practice of awarding marks, despite candidate errors in previous parts, for their correct methods/answers using incorrect results.

Using the markscheme

1 General

Award marks using the annotations as noted in the markscheme *eg* ***M1, A2.***

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***A3***, ***M2*** *etc.*, do **not** split the marks, unless there is a note.
- The response to a "show that" question does not need to restate the ***AG*** line, unless a **Note** makes this explicit in the markscheme.
- Once a correct answer to a question or part question is seen, ignore further working even if this working is incorrect and/or suggests a misunderstanding of the question. This will encourage a uniform approach to marking, with less examiner discretion. Although some candidates may be advantaged for that specific question item, it is likely that these candidates will lose marks elsewhere too.
- An exception to the previous rule is when an incorrect answer from further working is used **in a subsequent part**. For example, when a correct exact value is followed by an incorrect decimal approximation in the first part and this approximation is then used in the second part. In this situation, award ***FT*** marks as appropriate but do not award the final ***A1*** in the first part. Examples:

	Correct answer seen	**Further working seen**	**Any FT issues?**	**Action**
1.	$8\sqrt{2}$	5.65685... *(incorrect decimal value)*	No. Last part in question.	Award ***A1*** for the final mark *(condone the incorrect further working)*
2.	$\frac{35}{72}$	0.468111… (incorrect decimal value)	Yes. Value is used in subsequent parts.	Award ***A0*** for the final mark *(and full **FT** is available in subsequent parts)*

3 Implied marks

Implied marks appear in **brackets e.g.** ***(M1)***,and can only be awarded if **correct** work is seen or implied by subsequent working/answer.

4 Follow through marks (only applied after an error is made)

Follow through (***FT***) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) (e.g. incorrect value from part (a) used in part (d) or incorrect value from part (c)(i) used in part (c)(ii)). Usually, to award ***FT*** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if all the marks awarded in a subsequent part are for the answer or are implied, then ***FT*** marks should be awarded for *their* correct answer, even when working is not present.

For example: following an incorrect answer to part (a) that is used in subsequent parts, where the markscheme for the subsequent part is ***(M1)A1***, it is possible to award full marks for *their* correct answer, **without working being seen.** For longer questions where all but the answer marks are implied this rule applies but may be overwritten by a **Note** in the Markscheme.

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks, by reflecting on what each mark is for and how that maps to the simplified version.
- If the error leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word “their” in a description, to indicate that candidates may be using an incorrect value.
- If the candidate’s answer to the initial question clearly contradicts information given in the question, it is not appropriate to award any ***FT*** marks in the subsequent parts. This includes when candidates fail to complete a “show that” question correctly, and then in subsequent parts use their incorrect answer rather than the given value.
- Exceptions to these ***FT*** rules will be explicitly noted on the markscheme.
- If a candidate makes an error in one part but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the command term was “Hence”.

5 Mis-read

If a candidate incorrectly copies values or information from the question, this is a mis-read (***MR***). A candidate should be penalized only once for a particular misread. Use the ***MR*** stamp to indicate that this has been a misread and do not award the first mark, even if this is an ***M*** mark, but award all others as appropriate.

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.
- If a candidate uses a correct answer, to a "show that" question, to a higher degree of accuracy than given in the question, this is NOT a misread and full marks may be scored in the subsequent part.
- ***MR*** can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If the command term is 'Hence' and not 'Hence or otherwise' then alternative methods are not permitted unless covered by a note in the mark scheme.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**.

7 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation** for example 1.9 and $1{,}9$ or 1000 and $1{,}000$ and 1.000.
- Do not accept final answers written using calculator notation. However, ***M*** marks and intermediate ***A*** marks can be scored, when presented using calculator notation, provided the evidence clearly reflects the demand of the mark.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, some **equivalent** answers will generally appear in brackets. Not all equivalent notations/answers/methods will be presented in the markscheme and examiners are asked to apply appropriate discretion to judge if the candidate work is equivalent.

8 Format and accuracy of answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. If the level of accuracy is not stated in the question, the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures*.

Where values are used in subsequent parts, the markscheme will generally use the exact value, however candidates may also use the correct answer in subsequent parts. The markscheme will often explicitly include the subsequent values that come "*from the use of 3 sf values*".

Simplification of final answers: Candidates are advised to give final answers using good mathematical form. In general, for an ***A*** mark to be awarded, arithmetic should be completed, and any values that lead to integers should be simplified; for example, $\sqrt{\frac{25}{4}}$ should be written as $\frac{5}{2}$. An exception to this is simplifying fractions, where lowest form is not required (although the numerator and the denominator must be integers); for example, $\frac{10}{4}$ may be left in this form or written as $\frac{5}{2}$. However, $\frac{10}{5}$ should be written as 2, as it simplifies to an integer.

Algebraic expressions should be simplified by completing any operations such as addition and multiplication, e.g. $4e^{2x} \times e^{3x}$ should be simplified to $4e^{5x}$, and $4e^{2x} \times e^{3x} - e^{4x} \times e^{x}$ should be simplified to $3e^{5x}$. Unless specified in the question, expressions do not need to be factorized, nor do factorized expressions need to be expanded, so $x(x+1)$ and $x^2 + x$ are both acceptable.

Please note: intermediate ***A*** marks do NOT need to be simplified.

9 Calculators

A GDC is required for this paper, but If you see work that suggests a candidate has used any calculator not approved for IB DP examinations (eg CAS enabled devices), please follow the procedures for malpractice.

10. Presentation of candidate work

Crossed out work: If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work unless an explicit note from the candidate indicates that they would like the work to be marked.

More than one solution: Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise. If the layout of the responses makes it difficult to judge, examiners should apply appropriate discretion to judge which is "first".

1. (a) $\frac{12}{5}\left(\frac{144}{60}, 2.4\right)$ ***(M1)A1***

[2 marks]

(b) $\frac{3}{5}\left(\frac{144}{240}, 0.6\right)$ ***A1***

[1 mark]

(c) (i) wins $\sim \mathrm{B}\left(4, \frac{3}{5}\right)$

$\mathrm{P}(\text{wins} = 0) = 0.0256$ ***A1***

Note: Allow ***FT*** from use of their probability in part (b) but only when used with $n = 4$.

(ii) expected frequency $= 60 \times 0.0256$ ***(M1)***

$= 1.536$ ***A1***

[3 marks]

(d) (i) H_0: data follows a Binomial distribution with $n = 4$ ***A1***

(ii) $(\mathrm{df} = 4 - 1 - 1 =)\ 2$ ***A1***

(iii) p-value $= 0.954\ \ (0.953872...)$ ***(M1)A1***

(iv) $0.954 > 0.05$ ***R1***

insufficient evidence to reject H_0 ***A1***

Note: Condone "accept H_0". Follow through from their p-value in part (d)(iii) if the reasoning is correct and correct conclusions are made. Do NOT award ***R0A1***.

[6 marks]

(e) wins $\sim \mathrm{B}\left(4, \frac{3}{5}\right)$ **OR** $1 - 0.0256$ ***(M1)***

$\mathrm{P}(\text{wins} \geq 1) = 0.974\ \ (0.9744)$ ***A1***

[2 marks]

(f) (i) there are 145 transitions that start with Argentina, ***(M1)***

(of which 85 lead to Argentina winning,) so the probability is $\frac{85}{145}$ ***A1***

$= \frac{17}{29}$ ***AG***

(ii) $\begin{pmatrix} \frac{17}{29} & \frac{31}{47} \\ \frac{12}{29} & \frac{16}{47} \end{pmatrix} \left(= \begin{pmatrix} 0.586 & 0.660 \\ 0.414 & 0.340 \end{pmatrix} \right)$ ***A1A1***

Note: Accept the transposed matrix as correct.
Award ***A1*** for $\frac{17}{29}$ placed in a leading diagonal.
Award ***A1*** for all other values correct and in correct position in the matrix.

[4 marks]

(g) (i) write their matrix with λ subtracted from the leading diagonal ***(M1)***
equate determinant to zero ***(M1)***

$$\det \begin{pmatrix} \frac{17}{29} - \lambda & \frac{31}{47} \\ \frac{12}{29} & \frac{16}{47} - \lambda \end{pmatrix} = 0$$

$\left(\frac{17}{29} - \lambda\right)\left(\frac{16}{47} - \lambda\right) - \frac{12}{29} \times \frac{31}{47} = 0$ ***A1***

correct intermediate step
$1363\lambda^2 - 1263\lambda - 100 = 0$ ***AG***

Note: Do not award ***A1*** if there is no intermediate step leading from determinant to given answer.
Solving $\boldsymbol{T}\begin{pmatrix} x \\ y \end{pmatrix} = \lambda \begin{pmatrix} x \\ y \end{pmatrix}$ for x and y may be seen and is a valid alternative method.
Accept working in the form $\det(\lambda I - T) = 0$.

(ii) $\lambda = 1, -\frac{100}{1363}$ $(-0.0733675...)$ ***A1***

(iii) attempt to solve $\boldsymbol{T}\begin{pmatrix} x \\ y \end{pmatrix} = \lambda\begin{pmatrix} x \\ y \end{pmatrix}$ ***(M1)***

$y = \frac{564}{899}x$

eigenvector for $\lambda = 1$ is $\begin{pmatrix} 1 \\ \frac{564}{899} \end{pmatrix}$ $\left(= \begin{pmatrix} 1 \\ 0.627 \end{pmatrix}\right)$ ***A1***

eigenvector for $\lambda = -\frac{100}{1363}$ is $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ ***A1***

Note: Allow correct multiples of the eigenvectors.
If eigenvector $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ is stated without the second eigenvector, or any other working, then award ***M0A0A1***.

[7 marks]

(h) **EITHER**

solution found using $\begin{pmatrix} 1 \\ \frac{564}{899} \end{pmatrix}$ $\left(= \begin{pmatrix} 1 \\ 0.627 \end{pmatrix}\right)$

$x + \frac{564}{899}x = 1$ ***(M1)***

$x = 0.614$ $\left(0.614490..., \frac{899}{1463}\right)$ ***(A1)***

OR

solution can be found from high power of transition matrix

$\begin{pmatrix} \frac{17}{29} & \frac{31}{47} \\ \frac{12}{29} & \frac{16}{47} \end{pmatrix}^{50} = \begin{pmatrix} 0.614 & 0.614 \\ 0.386 & 0.386 \end{pmatrix}$ ***(M1)***

Note: Accept the transposed matrix if consistent with their answer to part (f)(ii).

probability $= 0.614$ $\left(0.614490..., \frac{899}{1463}\right)$ ***(A1)***

THEN

$\mathrm{P}(3 \text{ wins}) = 0.614 \times 0.586^2$ $\left(= \frac{899}{1463} \times \left(\frac{17}{29}\right)^2\right)$ ***(M1)***

$= 0.211$ $\left(0.211162..., \frac{8959}{42427}\right)$ ***A1***

[4 marks]
[Total: 29 marks]

2. (a) attempt to separate variable ***M1***

$\int \frac{1}{C}\,\mathrm{d}C = \int -k\,\mathrm{d}t$

$\ln|C| = -kt\ (+c)$ ***A1***

$C = A\mathrm{e}^{-kt}$

substituting $t = 0$, $C = d$

$A = d$ ***A1***

Note: To award the ***A1***, $t = 0$ must be seen.

$C = d\mathrm{e}^{-kt}$ ***AG***

[3 marks]

(b) $0.05d = d\mathrm{e}^{-0.2t}$ ***(A1)***

$15.0\ \ (14.9786\ldots)$ ***A1***

[2 marks]

(c) **EITHER**

first dose: when $t = 2T$ then $C_1 = d\mathrm{e}^{-0.4T}$ ***A1***

second dose: when $t = T$ then $C_2 = d\mathrm{e}^{-0.2T}$ ***A1***

third dose: when $t = 0$ then $C_3 = d$ ***A1***

sum the three doses ***M1***

Note: A timing of the dose and the expression must be clearly indicted and correct to award ***A1***. Condone absence of d for ***A1A1A1***.

OR

considering values of C before and after a dose ***M1***

immediately before the second dose, $C = d\,\mathrm{e}^{-0.2T}$

immediately after the second dose, $C = d\,\mathrm{e}^{-0.2T} + d$ ***A1***

immediately before the third dose, $C = (d\,\mathrm{e}^{-0.2T} + d)\,\mathrm{e}^{-0.2T}$ ***(A1)***

immediately after the third dose, $C = (d\,\mathrm{e}^{-0.2T} + d)\,\mathrm{e}^{-0.2T} + d$ ***A1***

THEN

$\Rightarrow C = d\,(1 + \mathrm{e}^{-0.2T} + \mathrm{e}^{-0.4T})$ ***AG***

Note: Do not accept any response using the general formula given before part (d). Award ***M1A1A1A0*** if d not included in the final formula.

[4 marks]

(d) $r = \mathrm{e}^{-0.2T}$ and $u_1 = d$ ***A1***

correct substitution into geometric series formula ***A1***

$$C = \frac{d\left(1-\left(\mathrm{e}^{-0.2T}\right)^n\right)}{1-\mathrm{e}^{-0.2T}}$$

$$= d\left(\frac{1-\mathrm{e}^{-0.2nT}}{1-\mathrm{e}^{-0.2T}}\right)$$ ***AG***

[2 marks]

(e) (i) as $n \to \infty$, $\mathrm{e}^{-0.2nT} \to 0$ (since $T > 0$) ***(M1)***

$$H_\infty = \frac{d}{1-\mathrm{e}^{-0.2T}}$$ ***A1***

(ii) **METHOD 1**

$$\lim_{n\to\infty} L_n = \lim_{n\to\infty} H_n \times \mathrm{e}^{-0.2T}$$ ***(M1)(A1)***

$$L_\infty = \left(\frac{d}{1-\mathrm{e}^{-0.2T}}\right)\mathrm{e}^{-0.2T} \quad \textbf{OR} \quad = \frac{d}{\mathrm{e}^{0.2T}-1}$$ ***A1***

METHOD 2

L_n occurs immediately before the $(n+1)^{\text{th}}$ dose ***(M1)***

$$L_n = d\left(\frac{1-\mathrm{e}^{-0.2nT}}{1-\mathrm{e}^{-0.2T}}\right)\mathrm{e}^{-0.2T}$$ ***(A1)***

as $n \to \infty$, $\mathrm{e}^{-0.2nT} \to 0$ (since $T > 0$)

$$L_\infty = \left(\frac{d}{1-\mathrm{e}^{-0.2T}}\right)\mathrm{e}^{-0.2T} \quad \textbf{OR} \quad = \frac{d}{\mathrm{e}^{0.2T}-1}$$ ***A1***

[5 marks]

(f) (i) **METHOD 1**

$H_\infty - L_\infty = \dfrac{d}{1-\mathrm{e}^{-0.2T}} - \dfrac{d\mathrm{e}^{-0.2T}}{1-\mathrm{e}^{-0.2T}}$ ***M1***

$= \dfrac{d(1-\mathrm{e}^{-0.2T})}{1-\mathrm{e}^{-0.2T}}$ ***A1***

$= d$ ***AG***

METHOD 2

$H_\infty - L_\infty = \dfrac{d}{1-\mathrm{e}^{-0.2T}} - \dfrac{d}{\mathrm{e}^{0.2T}-1}$ ***M1***

$= \dfrac{d\,\mathrm{e}^{0.2T} - d - d + d\,\mathrm{e}^{-0.2T}}{\mathrm{e}^{0.2T} - 1 - \mathrm{e}^{0} + \mathrm{e}^{-0.2T}}$

$= \dfrac{d(\mathrm{e}^{0.2T} + \mathrm{e}^{-0.2T} - 2)}{\mathrm{e}^{0.2T} + \mathrm{e}^{-0.2T} - 2}$ ***A1***

$= d$ ***AG***

Note: Award ***M1A0*** for the use of their formulae from part (e).
Award ***M0A0*** if at no point $n \to \infty$ in their response.

(ii) **EITHER**

$\ln\left(\dfrac{H_\infty}{L_\infty}\right) = \ln\left(\dfrac{\dfrac{d}{1-\mathrm{e}^{-0.2T}}}{\dfrac{d\mathrm{e}^{-0.2T}}{1-\mathrm{e}^{-0.2T}}}\right)$ ***M1***

$= \ln(\mathrm{e}^{0.2T})$ ***A1***

$= 0.2T$ ***A1***

OR

$H_\infty \mathrm{e}^{-0.2T} = L_\infty$ ***(M1)***

$\ln\left(\dfrac{L_\infty}{H_\infty}\right) = -0.2T$ ***A1***

$\ln\left(\dfrac{L_\infty}{H_\infty}\right)^{-1} = 0.2T$ ***A1***

THEN

$5\ln\left(\dfrac{H_\infty}{L_\infty}\right) = T$ ***AG***

Note: Award ***M1A0*** for the use of their incorrect formulae from part (e).
Award ***M0A0*** if at no point $n \to \infty$ in their response.

[5 marks]

(g) (i) $d = 0.22$ ***A1***

(ii) $T = 7.70$ ***A1***

Note: Accept $T = 7.7$.

[2 marks]

(h) $0.06 = 0.22\,\mathrm{e}^{-0.2t}$

proportion of time $= \dfrac{6.49641...}{7.702...}$ ***(M1)***

84.3% ***A1***

[2 marks]

(i) **EITHER**

rounding to 8 hours leads to 3 times a day ***R1***

OR

7.7 hours is difficult to schedule ***R1***

OR

a tolerance may be necessary to not be too close to unsafe boundary ***R1***

OR

avoid giving too much of the drug and giving the patient an overdose ***R1***

Note: Accept any reasonable argument that engages with the model in a practical sense.
Do not accept an answer based on part (h) along the lines “it should be given more often”.

[1 mark]

[Total: 26 mark]

Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

May 2022

Mathematics: applications and interpretation

Higher level

Paper 3

16 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**.
A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.
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AG Answer given in the question and so no marks are awarded.
FT Follow through. The practice of awarding marks, despite candidate errors in previous parts, for their correct methods/answers using incorrect results.

Using the markscheme

1 General

Award marks using the annotations as noted in the markscheme *eg* ***M1, A2.***

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- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
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- An exception to the previous rule is when an incorrect answer from further working is used **in a subsequent part**. For example, when a correct exact value is followed by an incorrect decimal approximation in the first part and this approximation is then used in the second part. In this situation, award ***FT*** marks as appropriate but do not award the final ***A1*** in the first part. Examples:

	Correct answer seen	**Further working seen**	**Any FT issues?**	**Action**
1.	$8\sqrt{2}$	5.65685... *(incorrect decimal value)*	No. Last part in question.	Award ***A1*** for the final mark *(condone the incorrect further working)*
2.	$\frac{35}{72}$	0.468111… (incorrect decimal value)	Yes. Value is used in subsequent parts.	Award ***A0*** for the final mark *(and full **FT** is available in subsequent parts)*

3 Implied marks

Implied marks appear in **brackets e.g.** ***(M1)***,and can only be awarded if **correct** work is seen or implied by subsequent working/answer.

4 Follow through marks (only applied after an error is made)

Follow through (***FT***) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) (e.g. incorrect value from part (a) used in part (d) or incorrect value from part (c)(i) used in part (c)(ii)). Usually, to award ***FT*** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if all the marks awarded in a subsequent part are for the answer or are implied, then ***FT*** marks should be awarded for *their* correct answer, even when working is not present.

For example: following an incorrect answer to part (a) that is used in subsequent parts, where the markscheme for the subsequent part is ***(M1)A1***, it is possible to award full marks for *their* correct answer, **without working being seen.** For longer questions where all but the answer marks are implied this rule applies but may be overwritten by a **Note** in the Markscheme.

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks, by reflecting on what each mark is for and how that maps to the simplified version.
- If the error leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word “their” in a description, to indicate that candidates may be using an incorrect value.
- If the candidate’s answer to the initial question clearly contradicts information given in the question, it is not appropriate to award any ***FT*** marks in the subsequent parts. This includes when candidates fail to complete a “show that” question correctly, and then in subsequent parts use their incorrect answer rather than the given value.
- Exceptions to these ***FT*** rules will be explicitly noted on the markscheme.
- If a candidate makes an error in one part but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the command term was “Hence”.

5 Mis-read

If a candidate incorrectly copies values or information from the question, this is a mis-read (***MR***). A candidate should be penalized only once for a particular misread. Use the ***MR*** stamp to indicate that this has been a misread and do not award the first mark, even if this is an ***M*** mark, but award all others as appropriate.

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.
- If a candidate uses a correct answer, to a "show that" question, to a higher degree of accuracy than given in the question, this is NOT a misread and full marks may be scored in the subsequent part.
- ***MR*** can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If the command term is 'Hence' and not 'Hence or otherwise' then alternative methods are not permitted unless covered by a note in the mark scheme.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**.

7 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation** for example 1.9 and $1{,}9$ or 1000 and $1{,}000$ and 1.000.
- Do not accept final answers written using calculator notation. However, ***M*** marks and intermediate ***A*** marks can be scored, when presented using calculator notation, provided the evidence clearly reflects the demand of the mark.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, some **equivalent** answers will generally appear in brackets. Not all equivalent notations/answers/methods will be presented in the markscheme and examiners are asked to apply appropriate discretion to judge if the candidate work is equivalent.

8 Format and accuracy of answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. If the level of accuracy is not stated in the question, the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures.*

Where values are used in subsequent parts, the markscheme will generally use the exact value, however candidates may also use the correct answer in subsequent parts. The markscheme will often explicitly include the subsequent values that come "*from the use of 3 sf values*".

Simplification of final answers: Candidates are advised to give final answers using good mathematical form. In general, for an ***A*** mark to be awarded, arithmetic should be completed, and any values that lead to integers should be simplified; for example, $\sqrt{\frac{25}{4}}$ should be written as $\frac{5}{2}$. An exception to this is simplifying fractions, where lowest form is not required (although the numerator and the denominator must be integers); for example, $\frac{10}{4}$ may be left in this form or written as $\frac{5}{2}$. However, $\frac{10}{5}$ should be written as 2, as it simplifies to an integer.

Algebraic expressions should be simplified by completing any operations such as addition and multiplication, e.g. $4\mathrm{e}^{2x} \times \mathrm{e}^{3x}$ should be simplified to $4\mathrm{e}^{5x}$, and $4\mathrm{e}^{2x} \times \mathrm{e}^{3x} - \mathrm{e}^{4x} \times \mathrm{e}^{x}$ should be simplified to $3\mathrm{e}^{5x}$. Unless specified in the question, expressions do not need to be factorized, nor do factorized expressions need to be expanded, so $x(x+1)$ and $x^2 + x$ are both acceptable.

Please note: intermediate ***A*** marks do NOT need to be simplified.

9 Calculators

A GDC is required for this paper, but If you see work that suggests a candidate has used any calculator not approved for IB DP examinations (eg CAS enabled devices), please follow the procedures for malpractice.

10. Presentation of candidate work

Crossed out work: If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work unless an explicit note from the candidate indicates that they would like the work to be marked.

More than one solution: Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise. If the layout of the responses makes it difficult to judge, examiners should apply appropriate discretion to judge which is "first".

1. (a) (i) mean $= 4.23\ (4.23333\ldots)$ ***A1***

variance $= 4.27\ (4.26777...)$ ***A1***

[2 marks]

(ii) mean is close to the variance ***A1***

[1 mark]

(b) *One of the following:*
the number of bags sold each day is independent of any other day
the sale of one bag is independent of any other bag sold
the sales of bags of rice (each day) occur at a *constant mean* rate ***A1***

Note: Award ***A1*** for a correct answer in context. Any statement referring to independence must refer to either the independence of each bag sold or the independence of the number of bags sold each day. If the third option is seen, the statement must refer to a "constant mean" or "constant average". Do not accept "the number of bags sold each day is constant".

[1 mark]

(c) attempt to find Poisson probabilities and multiply by 90 ***(M1)***
$a = 7.018$ ***A1***
$b = 17.498$ ***A1***

EITHER
$90 \times \mathrm{P}(X \geq 8) = 90 \times \left(1 - \mathrm{P}(X \leq 7)\right)$ ***(M1)***
$c = 5.755$ ***A1***

OR
$90 - 7.018 - 11.903 - 16.665 - 17.498 - 14.698 - 10.289 - 6.173$ ***(M1)***
$c = 5.756$ ***A1***

Note: Do not penalize the omission of clear a, b and c labelling as this will be penalized later if correct values are interchanged.

[5 marks]

(d) (i) 7 ***A1***

[1 mark]

(ii) H_0 : The number of bags of rice sold each day follows a Poisson distribution with mean 4.2. ***A1***

H_1 : The number of bags of rice sold each day does not follow a Poisson distribution with mean 4.2. ***A1***

Note: Award ***A1A1*** for **both** hypotheses correctly stated and in correct order. Award ***A1A0*** if reference to the data and/or "mean 4.2" is not included in the hypotheses, but otherwise correct.

evidence of attempting to group data to obtain the observed frequencies for ≤ 1 and ≥ 8 ***(M1)***

p-value $= 0.728$ $(0.728100...)$ ***A2***

0.728 $(0.728100...) > 0.05$ ***R1***

the result is not significant so there is no reason to reject H_0
(the number of bags sold each day follows a Poisson distribution) ***A1***

Note: Do not award ***R0A1***. The conclusion MUST follow through from their hypotheses. If no hypotheses are stated, the final ***A1*** can still be awarded for a correct conclusion as long as it is in context (e.g. therefore the data follows a Poisson distribution).

[7 marks]

(e) (i) **METHOD 1**

evidence of multiplying 4.2×60 (seen anywhere) ***M1***

$H_0 : \mu = 252$

$H_1 : \mu > 252$ ***A1***

Note: Accept $H_0 : \mu = 4.2$ and $H_1 : \mu > 4.2$ for the ***A1***.

evidence of finding probabilities around critical region ***(M1)***

Note: Award ***(M1)*** for any of these values seen:
$P(X \geq 277) = 0.0630518\ldots$ **OR** $P(X \leq 276) = 0.936948\ldots$
$P(X \geq 278) = 0.0558415\ldots$ **OR** $P(X \leq 277) = 0.944158\ldots$
$P(X \geq 279) = 0.0493055\ldots$ **OR** $P(X \leq 278) = 0.950694\ldots$.

critical value $= 279$ ***A1***

$282 \geq 279$, ***R1***

the null hypothesis is rejected ***A1***

(the advertising increased the number of bags sold during the 60 days)

Note: Do not award ***R0A1***. Accept statements referring to the advertising being effective for ***A1*** as long as the ***R*** mark is satisfied. For the ***R1A1***, follow through within the part from their critical value.

METHOD 2

evidence of dividing 282 by 60 (or 4.7 seen anywhere) ***M1***

$H_0 : \mu = 4.2$

$H_1 : \mu > 4.2$ ***A1***

attempt to find critical value using central limit theorem ***(M1)***

(e.g. sample standard deviation $= \sqrt{\frac{4.2}{60}}$, $\bar{X} \sim N\left(4.2, \sqrt{\frac{4.2}{60}}\right)$, etc.)

Note: Award ***(M1)*** for a p-value of $0.0293907\ldots$ seen.

critical value $= 4.63518\ldots$ ***A1***

$4.7 > 4.63518\ldots$ ***R1***

the null hypothesis is rejected ***A1***

(the advertising increased the number of bags sold during the 60 days)

Note: Do not award ***R0A1***. Accept statements referring to the advertising being effective for ***A1*** as long as the ***R*** mark is satisfied. For the ***R1A1***, follow through within the part from their critical value.

[6 marks]

(ii) $\left(P(X \geq 279 \mid \mu = 252) =\right)$ 0.0493 $(0.0493055\ldots)$ ***A1***

Note: If a candidate uses **METHOD 2** in part (e)(i), allow an ***FT*** answer of 0.05 for this part but only if the candidate has attempted to find a p-value.

[1 mark]

(f) attempt to compare profit *difference* with cost of advertising ***(M1)***

Note: Award ***(M1)*** for evidence of candidate mathematically comparing a profit difference with the cost of the advertising.

EITHER
(comparing profit from 30 extra bags of rice with cost of advertising)
$14850 < 18000$ ***A1***

OR
(comparing total profit with and without advertising)
$121590 < 124740$ ***A1***

OR
(comparing increase of average daily profit with daily advertising cost)
$247.50 < 300$ ***A1***

THEN

EITHER
Even though the number of bags of rice increased, the advertising is not worth it as the overall profit did not increase. ***R1***

OR
The advertising is worth it even though the cost is less than the increased profit, since the number of customers increased (possibly buying other products and/or returning in the future after advertising stops) ***R1***

Note: Follow through within the part for correct reasoning consistent with their comparison.

[3 marks]

[Total 27 marks]

2. (a) $AF^2 = 89.2^2 + 104.9^2 - 2(89.2)(104.9)\cos 83$ ***(M1)(A1)***

Note: Award ***(M1)*** for substitution into the cosine rule and ***(A1)*** for correct substitution.

$AF = 129 \text{ m} \quad (129.150...)$ ***A1***

[3 marks]

(b) $21310 \div 129.150\ldots$ ***(M1)***

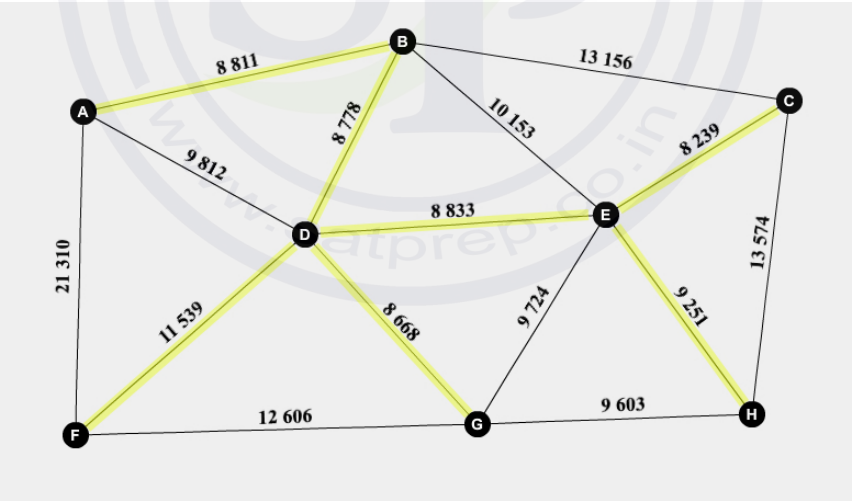
$\$ 165$ ***A1***

[2 marks]

(c) any reasonable statement referring to the lake ***R1***
(eg. there is a lake between A and F, the cables would need to be installed under/over/around the lake, special waterproof cables are needed for lake, etc.)

[1 mark]

(d) (i) edges (or weights) are chosen in the order
CE (8239)
DG (8668)
BD (8778)
AB (8811)
DE (8833)
EH (9251)
DF (11 539) ***A1A1A1***



Note: Award ***A1*** for the first two edges chosen in the correct order. Award ***A1A1*** for the first six edges chosen in the correct order. Award ***A1A1A1*** for all seven edges chosen in the correct order. Accept a diagram as an answer, provided the order of edges is communicated**.**

[3 marks]

(ii) Finding the sum of the weights of their edges ***(M1)***
$8239 + 8668 + 8778 + 8811 + 8833 + 9251 + 11539$

total cost = $\$64\,119$ ***A1***

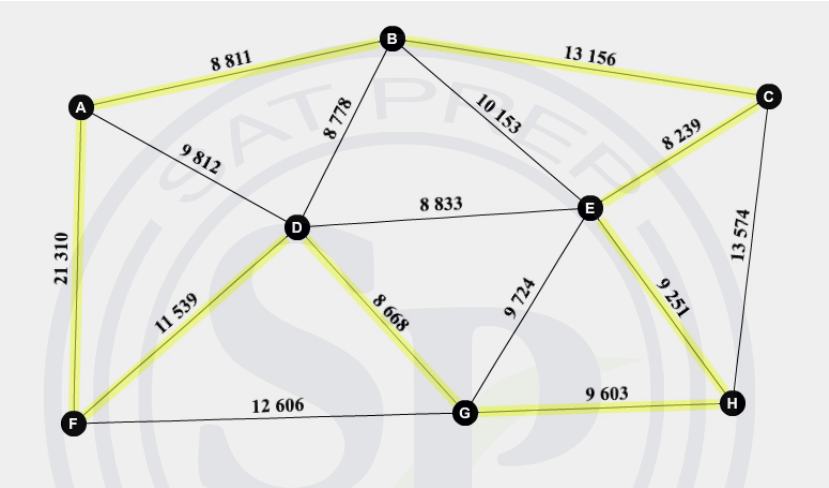
[2 marks]

(e) a Hamiltonian cycle is not always an Eulerian circuit as it does not have to include all edges of the graph (only all vertices) ***R1***

[1 mark]

(f) edges (or weights) are chosen in the order

DG (8668)
GH (9603)
HE (9251)
EC (8239)
CB (13 156)
BA (8811)
AF (21 310)
FD (11 539) ***A1A1A1***



Note: Award ***A1*** for the first two edges chosen in the correct order. Award ***A1A1*** for the first five edges chosen in the correct order. Award ***A1A1A1*** for all eight edges chosen in the correct order. Accept a diagram as an answer, provided the order of edges is communicated.

finding the sum of the weights of their edges ***(M1)***

$8668+9603+9251+8239+13156+8811+21310+11539$

upper bound = \$90 577 ***A1***

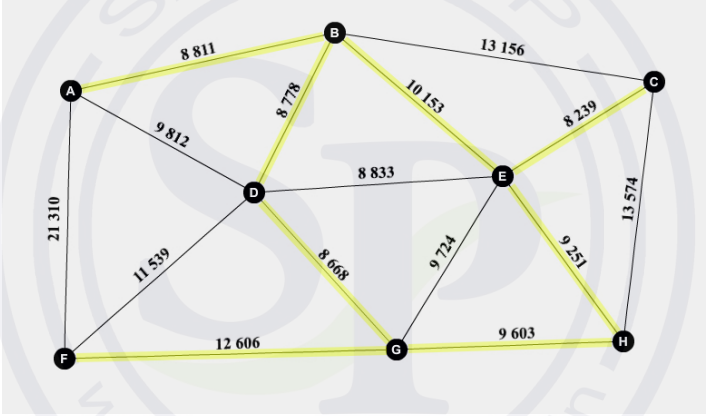
[5 marks]

(g) attempt to find MST after deleting vertex D ***(M1)***
these edges (or weights) (in any order)
CE (8239)
AB (8811)
EH (9251)
GH (9603)
BE (10 153)
FG (12 606) ***A1***

Note: Prim's or Kruskal's algorithm could be used at this stage.

reconnect D to MST with two different edges ***(M1)***
DG (8668)
BD (8778) ***A1***

Note: This ***A1*** is independent of the first ***A*** mark and can be awarded if both DG and BD are chosen to reconnect D to the MST, even if the MST is incorrect.



finding the sum of the weights of their edges ***(M1)***
$8239+8811+9251+9603+10153+12606+8668+8778$

Note: For candidates with an incorrect MST or no MST, the weights of at least seven of the edges being summed (two of which must connect to D) must be shown to award this ***(M1)***.

lower bound = \$76 109 ***A1***

[6 marks]

(h) **METHOD 1**

recognition of a binomial distribution ***(M1)***

$X \sim \mathrm{B}(2,\ 0.014)$

finding the probability that a cable fails (at least one of its connections fails)

$\mathrm{P}(X > 0) = 0.027804$ **OR** $1 - \mathrm{P}(X = 0) = 0.027804$ ***A1***

recognition that **two** cables must fail for the network to go offline ***M1***

recognition of binomial distribution for network, $Y \sim \mathrm{B}(8,\ 0.027804)$ ***(M1)***

$\mathrm{P}(Y \geq 2) = 0.0194\ \ (0.0193602...)$ **OR** $1 - \mathrm{P}(Y < 2) = 0.0194\ \ (0.0193602\ldots)$ ***A1***

therefore, the diagram satisfies the requirement since $1.94\% < 2\%$ ***AG***

Note: Evidence of binomial distribution may be seen as combinations.

METHOD 2

recognition of a binomial distribution ***(M1)***

$X \sim \mathrm{B}(16,\ 0.014)$

finding the probability that at least **two** connections fail

$\mathrm{P}(X \geq 2) = 0.0206473\ldots$ **OR** $1 - \mathrm{P}(X < 2) = 0.0206473\ldots$ ***A1***

recognition that the previous answer is an overestimate ***M1***

finding probability of two ends of the same cable failing, $F \sim \mathrm{B}(2,\ 0.014)$,

and the ends of the other 14 cables not failing, $S \sim \mathrm{B}(14, 0.014)$

$\mathrm{P}(F = 2) \times \mathrm{P}(S = 0) = 0.0000160891\ldots$ ***(A1)***

$0.0000160891\ldots \times 8 = 0.00128713\ldots$

$0.0206473\ldots - 0.00128713\ldots = 0.0194\ \ (0.0193602\ldots)$ ***A1***

therefore, the diagram satisfies the requirement since $1.94\% < 2\%$ ***AG***

METHOD 3

recognition of a binomial distribution ***M1***

$X \sim \mathrm{B}(16,\ 0.014)$

finding the probability that the network remains secure if 0 or 1 connections fail or if 2 connections fail provided that the second failed connection occurs at the other end of the cable with the first failure ***(M1)***

$\mathrm{P}(\text{remains secure}) = \mathrm{P}(X \leq 1) + \frac{1}{15} \times \mathrm{P}(X = 2)$ ***A1***

$= 0.9806397625$ ***A1***

$\mathrm{P}(\text{network fails}) = 1 - 0.9806397625 = 0.0194\ \ (0.0193602...)$ ***A1***

therefore, the diagram satisfies the requirement since $1.94\% < 2\%$ ***AG***

METHOD 4

$\mathrm{P}(\text{network failing})$

$= 1 - \mathrm{P}(0 \text{ connections failing}) - \mathrm{P}(1 \text{ connection failing})$

$\quad - \mathrm{P}(2 \text{ connections on the same cable failing})$ ***M1***

$= 1 - 0.986^{16} - {}^{16}C_1 \times 0.014 \times 0.986^{15} - {}^{8}C_1 \times 0.014^2 \times 0.986^{14}$ ***A1A1A1***

Note: Award ***A1*** for each of 2nd, 3rd and last terms.

$= 0.0194 \ \ (0.0193602\ldots)$ ***A1***

therefore, the diagram satisfies the requirement since $1.94\% < 2\%$ ***AG***

[5 marks]

[Total 28 marks]

Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

May 2022

Mathematics: applications and interpretation

Higher level

Paper 3

15 pages

Instructions to Examiners

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- An exception to the previous rule is when an incorrect answer from further working is used **in a subsequent part**. For example, when a correct exact value is followed by an incorrect decimal approximation in the first part and this approximation is then used in the second part. In this situation, award ***FT*** marks as appropriate but do not award the final ***A1*** in the first part. Examples:

	Correct answer seen	**Further working seen**	**Any FT issues?**	**Action**
1.	$8\sqrt{2}$	5.65685... *(incorrect decimal value)*	No. Last part in question.	Award ***A1*** for the final mark *(condone the incorrect further working)*
2.	$\frac{35}{72}$	0.468111… (incorrect decimal value)	Yes. Value is used in subsequent parts.	Award ***A0*** for the final mark *(and full **FT** is available in subsequent parts)*

3 Implied marks

Implied marks appear in **brackets e.g.** ***(M1)***,and can only be awarded if **correct** work is seen or implied by subsequent working/answer.

4 Follow through marks (only applied after an error is made)

Follow through (***FT***) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) (e.g. incorrect value from part (a) used in part (d) or incorrect value from part (c)(i) used in part (c)(ii)). Usually, to award ***FT*** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if all the marks awarded in a subsequent part are for the answer or are implied, then ***FT*** marks should be awarded for *their* correct answer, even when working is not present.

For example: following an incorrect answer to part (a) that is used in subsequent parts, where the markscheme for the subsequent part is ***(M1)A1***, it is possible to award full marks for *their* correct answer, **without working being seen.** For longer questions where all but the answer marks are implied this rule applies but may be overwritten by a **Note** in the Markscheme.

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5 Mis-read

If a candidate incorrectly copies values or information from the question, this is a mis-read (***MR***). A candidate should be penalized only once for a particular misread. Use the ***MR*** stamp to indicate that this has been a misread and do not award the first mark, even if this is an ***M*** mark, but award all others as appropriate.

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- Miscopying of candidates' own work does **not** constitute a misread, it is an error.
- If a candidate uses a correct answer, to a "show that" question, to a higher degree of accuracy than given in the question, this is NOT a misread and full marks may be scored in the subsequent part.
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Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If the command term is 'Hence' and not 'Hence or otherwise' then alternative methods are not permitted unless covered by a note in the mark scheme.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
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Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation** for example 1.9 and $1{,}9$ or 1000 and $1{,}000$ and 1.000.
- Do not accept final answers written using calculator notation. However, ***M*** marks and intermediate ***A*** marks can be scored, when presented using calculator notation, provided the evidence clearly reflects the demand of the mark.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
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Simplification of final answers: Candidates are advised to give final answers using good mathematical form. In general, for an ***A*** mark to be awarded, arithmetic should be completed, and any values that lead to integers should be simplified; for example, $\sqrt{\frac{25}{4}}$ should be written as $\frac{5}{2}$.

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Algebraic expressions should be simplified by completing any operations such as addition and multiplication, e.g. $4\mathrm{e}^{2x} \times \mathrm{e}^{3x}$ should be simplified to $4\mathrm{e}^{5x}$, and $4\mathrm{e}^{2x} \times \mathrm{e}^{3x} - \mathrm{e}^{4x} \times \mathrm{e}^{x}$ should be simplified to $3\mathrm{e}^{5x}$. Unless specified in the question, expressions do not need to be factorized, nor do factorized expressions need to be expanded, so $x(x+1)$ and $x^2 + x$ are both acceptable.

Please note: intermediate ***A*** marks do NOT need to be simplified.

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1. (a) (i) $Q(t) = 3090t - 54000$ $(3094.27...t - 54042.3...)$ ***A1A1***

Note: Award at most ***A1A0*** if answer is not an equation. Award ***A1A0*** for an answer including either x or y.

[2 marks]

(ii) 0.755 $(0.754741\ldots)$ ***A1***

[1 mark]

(iii) t is not a random variable **OR** it is not a (bivariate) normal distribution
OR data is not a sample from a population
OR data appears nonlinear
OR r only measures linear correlation ***R1***

Note: Do not accept "r is not large enough".

[1 mark]

(b) (i) attempt to separate variables ***(M1)***

$$\int \frac{1}{Q}\,\mathrm{d}Q = \int \beta N\,\mathrm{d}t$$

$$\ln|Q| = \beta Nt + c$$ ***A1A1A1***

Note: Award ***A1*** for LHS, ***A1*** for βNt, and ***A1*** for $+c$.
Award full marks for $Q = \mathrm{e}^{\beta Nt + c}$ **OR** $Q = A\mathrm{e}^{\beta Nt}$.
Award ***M1A1A1A0*** for $Q = \mathrm{e}^{\beta Nt}$

[4 marks]

(ii) attempt at exponential regression ***(M1)***

$Q = 1.15\mathrm{e}^{0.292t}$ $(Q = 1.14864...\mathrm{e}^{0.292055...t})$ ***A1***

OR

attempt at exponential regression ***(M1)***

$Q = 1.15 \times 1.34^{t}$ $(1.14864... \times 1.33917...^{t})$ ***A1***

Note: Condone answers involving y or x. Condone absence of "$Q =$"
Award ***M1A0*** for an incorrect answer in correct format.

[2 marks]

(iii) 0.999 $(0.999431...)$ ***A1***

[1 mark]

(iv) comparing something to do with R^2 and something to do with r ***M1***

Note: Examples of where the ***M1*** should be awarded:
$R^2 > r$
$R > r$
$0.999 > 0.755$
$0.999 > 0.755^2 \quad (= 0.563)$
The "correlation coefficient" in the exponential model is larger.
Model B has a larger R^2
Examples of where the ***M1*** should **not** be awarded:
The exponential model shows better correlation (since not clear how it is being measured)
Model 2 has a better fit
Model 2 is more correlated

an unambiguous comparison between R^2 and r^2 or R and r leading to the conclusion that the model in part (b) is more suitable / better ***A1***

Note: Condone candidates claiming that R is the "correlation coefficient" for the non-linear model.

[2 marks]

(v) it suggests that there will be more infected computers than the entire population ***R1***

Note: Accept any response that recognizes unlimited growth.

[1 mark]

(c) $1.15\,e^{0.292t} = 2.3$ **OR** $1.15 \times 1.34^t = 2.3$ **OR** $t = \dfrac{\ln 2}{0.292}$ **OR** using the model to find two specific times with values of $Q(t)$ which double ***M1***

$t = 2.37$ (days) ***A1***

Note: Do not ***FT*** from a model which is not exponential. Award ***M0A0*** for an answer of 2.13 which comes from using $(10, 20)$ from the data or any other answer which finds a doubling time from figures given in the table.

[2 marks]

(d) an attempt to calculate β for city X ***(M1)***

$\beta = \dfrac{0.292055...}{2.6 \times 10^6}$ **OR** $\beta = \dfrac{\ln 1.33917...}{2.6 \times 10^6}$

$= 1.12328... \times 10^{-7}$ ***A1***

this is larger than 9.64×10^{-8} so the virus spreads more easily in city X ***R1***

Note: It is possible to award ***M1A0R1***.
Condone "so the virus spreads faster in city X" for the final ***R1***.

[3 marks]

(e) $a = 38.3,\ b = 3086.1$ ***A1A1***

Note: Award ***A1A0*** if values are correct but not to 1 dp.

[2 marks]

(f) (i) $\dfrac{Q'}{Q} = 0.42228 - 2.5561\times10^{-6}Q$ ***(A1)(A1)***

Note: Award ***A1*** for each coefficient seen – not necessarily in the equation. Do not penalize seeing in the context of y and x.

identifying that the constant is k **OR** that the gradient is $-\dfrac{k}{L}$ ***(M1)***

therefore $k = 0.422 \quad (0.422228...)$ ***A1***

$\dfrac{k}{L} = 2.5561\times10^{-6}$

$L = 165000 \quad (165205)$ ***A1***

Note: Accept a value of L of 164843 from use of 3 sf value of k, or any other value from plausible pre-rounding.
Allow follow-through **within** the question part, from the equation of their line to the final two ***A1*** marks.

[5 marks]

(ii) recognizing that their L is the eventual number of infected ***(M1)***

$\dfrac{165205...}{2600000} = 6.35\% \quad (6.35403...\%)$ ***A1***

Note: Accept any final answer consistent with their answer to part (f)(i) unless their L is less than 120146 in which case award at most ***M1A0***.

[2 marks]

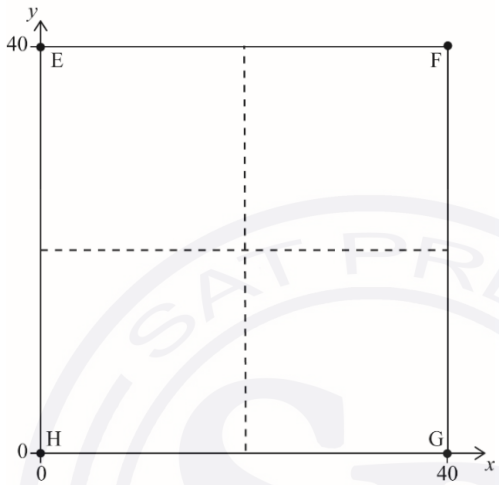
[Total 28 marks]

2. (a) the size of each town is small (in comparison with the distance between the towns)
OR
if towns have an identifiable centre
OR
the centre of the town is at that point ***R1***

Note: Accept a geographical landmark in place of "centre", e.g. "town hall" or "capitol".

[1 mark]

(b)



A1

Note: There is no need for a scale / coordinates here. Condone boundaries extending beyond the metropolitan area.

[1 mark]

(c) (i) the gradient of IF is $\frac{40-20}{40-30}=2$ ***(A1)***

negative reciprocal of any gradient ***(M1)***

gradient of perpendicular bisector $=-\frac{1}{2}$

Note: Seeing $-\frac{2}{3}$ (for example) used clearly as a gradient anywhere is evidence of the "negative reciprocal" method despite being applied to an inappropriate gradient.

midpoint is $\left(\frac{40+30}{2}, \frac{40+20}{2}\right)=(35,\ 30)$ ***(A1)***

equation of perpendicular bisector is $y-30=-\frac{1}{2}(x-35)$ ***A1***

Note: Accept equivalent forms *e.g.* $y=-\frac{1}{2}x+\frac{95}{2}$ or $2y+x-95=0$.
Allow ***FT*** for the final ***A1*** from their midpoint and gradient of perpendicular bisector, as long as the ***M1*** has been awarded.

[4 marks]

(ii) the perpendicular bisector of EH is $y = 20$ ***(A1)***

Note: Award this ***A1*** if seen in the y-coordinate of any final answer or if 20 is used as the y-value in the equation of any other perpendicular bisector.

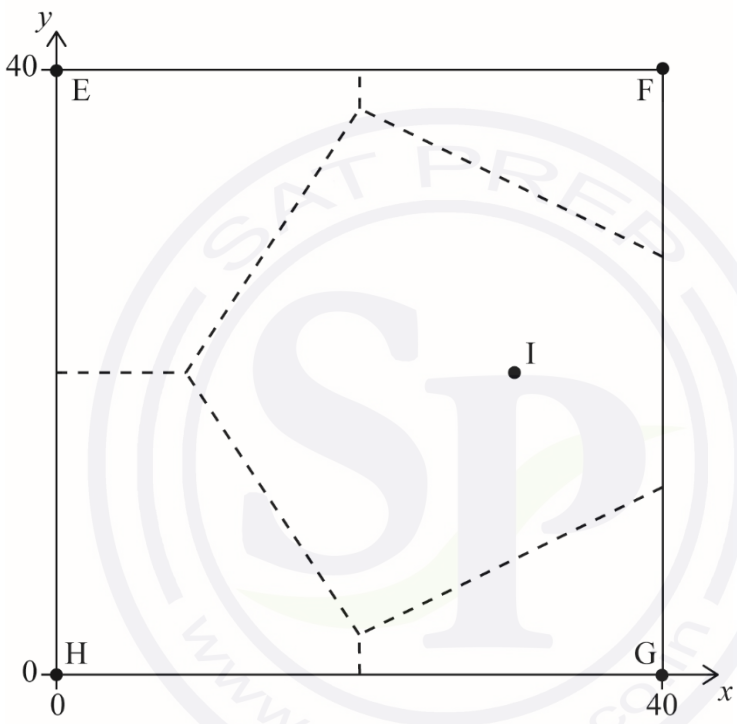
attempt to use symmetry **OR** intersecting two perpendicular bisectors ***(M1)***

$\left(\frac{25}{3}, 20\right)$ ***A1***

$(20, 2.5)$ ***A1***

[4 marks]

(iii)



M1A1

Note: Award ***M1*** for exactly four perpendicular bisectors around I (IE, IF, IG and IH) seen, even if not in exactly the right place.
Award ***A1*** for a completely correct diagram. Scale / coordinates are NOT necessary. Vertices should be in approximately the correct positions but only penalized if clearly wrong (condone northern and southern vertices appearing to be very close to the boundary).

Condone the Voronoi diagram extending outside of the square.
Do not award follow-though marks in this part.

[2 marks]

(d) 30% of 40 is 12 ***(A1)***

recognizing line intersects bisectors at $y = c$ (or equivalent) but different x-values ***(M1)***

$c = \frac{3}{2}x_1 + \frac{15}{2}$ and $c = -\frac{1}{2}x_2 + \frac{95}{2}$

finding an expression for the distance in Isaacopolis in terms of one variable ***(M1)***

$x_2 - x_1 = (95 - 2c) - \frac{2c - 15}{3} = 100 - \frac{8c}{3}$

equating their expression to 12

$100 - \frac{8c}{3} = 0.3 \times 40 = 12$

$c = 33$

distance $= 33$ (km) ***A1***

[4 marks]

(e) (i) must be a vertex (award if vertex given as a final answer) ***(R1)***
attempt to calculate the distance of at least one town from a vertex ***(M1)***

Note: This must be seen as a calculation or a value.

correct calculation of distances ***A1***

$\frac{65}{3}$ **OR** 21.7 **AND** $\sqrt{406.25}$ **OR** 20.2

$\left(\frac{25}{3}, 20\right)$ ***A1***

Note: Award ***R1M0A0A0*** for a vertex written with no other supporting calculations.
Award ***R1M0A0A1*** for correct vertex with no other supporting calculations.
The final ***A1*** is not dependent on the previous ***A1***. There is no follow-through for the final ***A1***.

Do not accept an answer based on "uniqueness" in the question.

[4 marks]

(ii) *For example, any one of the following:*
decision does not take into account the different population densities
closer to a city will reduce travel time/help employees
it is closer to some cites than others ***R1***

Note: Accept any correct reason that engages with the scenario.
Do not accept any answer to do with ethical issues about whether toxic waste should ever be dumped, or dumped in a metropolitan area.

[1 mark]

(f) (i) **METHOD 1**

attempting $\boldsymbol{M}^3$ ***M1***

attempting $\boldsymbol{M}^4$ ***M1***

e.g.

last row/column of $\boldsymbol{M}^3 = \begin{pmatrix} 3 & 5 & 1 & 6 & 0 & 7 \end{pmatrix}$

last row/column of $\boldsymbol{M}^4 = \begin{pmatrix} 10 & 12 & 4 & 16 & 1 & 18 \end{pmatrix}$

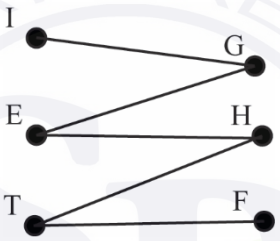
hence Isaacopolis is the last city to be polluted ***A1***

Note: Do not award the ***A1*** unless both $\boldsymbol{M}^3$ and $\boldsymbol{M}^4$ are considered.
Award ***M1M0A0*** for a claim that the shortest distance is from T to I and that it is 4, without any support.

METHOD 2

attempting to translate $\boldsymbol{M}$ to a graph or a list of cities polluted on each day ***(M1)***

correct graph or list ***A1***



hence Isaacopolis is the last city to be polluted ***A1***

Note: Award ***M1A1A1*** for a clear description of the graph in words leading to the correct answer.

[3 marks]

(ii) it takes 4 days ***A1***

[1 mark]

(iii) **EITHER**

the orders of the different vertices are:

E	2
F	1
G	2
H	2
I	1
T	2

(A1)

Note: Accept a list where each order is 2 greater than listed above.

OR

a correct diagram/graph showing the connections between the locations ***(A1)***

Note: Accept a diagram with loops at each vertex.
This mark should be awarded if candidate is clearly using their correct diagram from the previous part.

THEN

"Start at F and end at I" **OR** *"Start at I and end at F"* ***A1***

Note: Award ***A1A0*** for *"it could start at either F or I"*.
Award ***A1A1*** for *"$IGEHTF$"* **OR** *"$FTHEGI$"*.
Award ***A1A1*** for "*F and I*" **OR** "*I and F*".

[2 marks]

[Total 27 marks]

Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

November 2021

Mathematics: applications and interpretation

Higher level

Paper 3

12 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**.
A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.
R Marks awarded for clear **Reasoning**.
AG Answer given in the question and so no marks are awarded.
FT Follow through. The practice of awarding marks, despite candidate errors in previous parts, for their correct methods/answers using incorrect results.

Using the markscheme

1 General
Award marks using the annotations as noted in the markscheme *eg **M1**, **A2**.*

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***A3***, ***M2*** *etc.*, do **not** split the marks, unless there is a note.
- The response to a "show that" question does not need to restate the ***AG*** line, unless a **Note** makes this explicit in the markscheme.
- Once a correct answer to a question or part question is seen, ignore further working even if this working is incorrect and/or suggests a misunderstanding of the question. This will encourage a uniform approach to marking, with less examiner discretion. Although some candidates may be advantaged for that specific question item, it is likely that these candidates will lose marks elsewhere too.
- An exception to the previous rule is when an incorrect answer from further working is used **in a subsequent part**. For example, when a correct exact value is followed by an incorrect decimal approximation in the first part and this approximation is then used in the second part. In this situation, award ***FT*** marks as appropriate but do not award the final ***A1*** in the first part. Examples:

	Correct answer seen	**Further working seen**	**Any FT issues?**	**Action**
1.	$8\sqrt{2}$	5.65685... *(incorrect decimal value)*	No. Last part in question.	Award ***A1*** for the final mark *(condone the incorrect further working)*
2.	$\frac{35}{72}$	0.468111… (incorrect decimal value)	Yes. Value is used in subsequent parts.	Award ***A0*** for the final mark *(and full **FT** is available in subsequent parts)*

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Implied marks appear in **brackets e.g.** ***(M1)***,and can only be awarded if **correct** work is seen or implied by subsequent working/answer.

4 Follow through marks (only applied after an error is made)

Follow through (***FT***) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) (e.g. incorrect value from part (a) used in part (d) or incorrect value from part (c)(i) used in part (c)(ii)). Usually, to award ***FT*** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if all the marks awarded in a subsequent part are for the answer or are implied, then ***FT*** marks should be awarded for *their* correct answer, even when working is not present.

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Please note: intermediate ***A*** marks do NOT need to be simplified.

9 Calculators

No calculator is allowed. The use of any calculator on this paper is malpractice and will result in no grade awarded. If you see work that suggests a candidate has used any calculator, please follow the procedures for malpractice.

OR

A GDC is required for this paper, but If you see work that suggests a candidate has used any calculator not approved for IB DP examinations (eg CAS enabled devices), please follow the procedures for malpractice.

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1. (a) e.g. ABCDEGHFA ***A1***

Note: Accept any other correct answers starting at any vertex.

(b) (i) 7 vertices, so 6 edges required for MST ***(M1)***

Note: To award ***(M1)***, their 6 edges should not form a cycle.

Order selected	Edge
1	BC
2	CD
3	DE
4	BH
5	GH
6	FG

M1
A1
A1

Note: Award ***M1*** for the first three edges in correct order, ***A1*** for BH in correct order and ***A1*** for all of the edges correct.

weight of MST = 33 ***A1***

Note: The final ***A1*** can be awarded independently of previous marks.

(ii) lower bound = $33+3+x$ ***(M1)***
$=36+x$ ***A1***

(c) (i) $p=13$ ***A1***

(ii) $q=17$ ***A1***

(iii) $r=14$ ***A1***

(d) (i) attempt to use nearest neighbour algorithm ***(M1)***

any two correct cycles from
ABCDEGHFA, AFGHBCDE(F)A, AB(A)FGHCDE(F)A ***A1A1***

Note: Bracketed vertices may be omitted in candidate's answer.
Award ***M1A0A1*** for candidates who list two correct sequences of vertices, but omit the final vertex A.

(ii) use ABCDEGHFA **OR** their shortest cycle from (d)(i) ***(M1)***
upper bound = 43 ***A1***

continued…

Question 1 continued

(e) (i) cycle starts: ABCDEGHF
return to A has two options, $FA = 18$ or x ***(M1)***
hence least value of $x = 19$ ***A1***

(ii) upper bound = 58 ***A2***

(f) recognition that edges will be repeated / there are odd vertices ***(M1)***
$BH + DG = 21,\ BD + GH = 15, BG + DH = 21$ **OR** $18 + x$ ***A1***
recognizing BD and GH is lowest weight and is repeated ***(M1)***
solution to CPP $= 107 + x$ ***A1***
$x = 13$ ***A1***

Note: Award ***M1A0M0A1A1*** if only pairing BD and GH is considered, leading to a correct answer.

2. (a) (i) $h(t) = -0.134t + 3.1$ ***A1A1***

Note: Award ***A1*** for an equation in h and t and ***A1*** for the coefficient -0.134 and constant 3.1.

(ii) **EITHER**
the rate of change of height (of water in metres per minute) ***A1***

Note: Accept "rate of decrease" or "rate of increase" in place of "rate of change".

OR
the (average) amount that the height (of the water) decreases each minute ***A1***

(iii) **EITHER**
unreliable to use h on t equation to estimate t ***A1***

OR
unreliable to extrapolate from original data ***A1***

OR
rate of change (of height) might not remain constant (as the water drains out) ***A1***

(b) (i) $h(t) = 0.002t^2 - 0.174t + 3.2$ ***A1***

(ii) $0.002t^2 - 0.174t + 3.2 = 0$ ***(M1)***
26.4 $(26.4046\ldots)$ ***A1***

(iii) **EITHER**
$(0 \leq)\ t \leq 26.4$ $(t \leq 26.4046...)$ ***A1***

OR
$(0 \leq)\ t \leq 20$ (due to range of original data / interpolation) ***A1***

continued…

Question 2 continued

(c) $V = \pi(1)^2 h$ ***(A1)***

EITHER

$\frac{\mathrm{d}V}{\mathrm{dt}} = \pi\frac{\mathrm{d}h}{\mathrm{dt}}$ ***M1***

OR

attempt to use chain rule ***M1***

$\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{\mathrm{d}h}{\mathrm{d}V} \times \frac{\mathrm{d}V}{\mathrm{d}t}$

THEN

$\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{1}{\pi} \times -\pi R^2\sqrt{70560h}$ ***A1***

$\frac{\mathrm{d}h}{\mathrm{d}t} = -R^2\sqrt{70560h}$ ***AG***

(d) attempt to separate variables ***M1***

$\int \frac{1}{\sqrt{70560\,h}}\mathrm{d}h = \int -R^2\mathrm{d}t$ ***A1***

$\frac{2\sqrt{h}}{\sqrt{70560}} = -R^2t + c$ ***A1A1***

Note: Award ***A1*** for each correct side of the equation.

$\sqrt{h} = \frac{\sqrt{70560}}{2}\left(c - R^2t\right)$ ***A1***

Note: Award the final ***A1*** for any correct intermediate step that clearly leads to the given equation.

$h = 17640\left(c - R^2t\right)^2$ ***AG***

(e) $t = 0 \Rightarrow 3.2 = 17640c^2$ ***(M1)***

$c = 0.0134687...$ ***(A1)***

substituting $h = 0$ and their non-zero value of c ***(M1)***

$T = \frac{c}{R^2} = \frac{0.0134687...}{0.023^2}$

$= 25.5$ (minutes) $(25.4606\ldots)$ ***A1***

(f) $h = 0 \Rightarrow c = R^2t$

$c = 0.023^2 \times 15 \ (= 0.007935)$ ***(A1)***

$t = 0 \Rightarrow h = 17640(0.023^2 \times 15)^2$ ***(M1)***

$h = 1.11$ (metres) $(1.11068\ldots)$ ***A1***

continued…

Question 2 continued

(g) (i) let h be the height of water in the highest container
from parts (d) and (e) we get

$\frac{\mathrm{d}h}{\mathrm{d}t} = -35280R^2\left(0.0134687... - R^2t\right)$ ***(M1)(A1)***

so $\frac{\mathrm{d}H}{\mathrm{d}t} = 35280R^2\left(0.0135 - R^2t\right) - R^2\sqrt{70560H}$ ***M1A1***

$\left(\frac{\mathrm{d}H}{\mathrm{d}t} = 18.6631...\left(0.0134687... - 0.000529t\right) - 0.000529\sqrt{70560H}\right)$

$\left(\frac{\mathrm{d}H}{\mathrm{d}t} = 0.251367... - 0.0987279... - 0.140518...\sqrt{H}\right)$

$\frac{\mathrm{d}H}{\mathrm{d}t} \approx 0.2514 - 0.009873t - 0.1405\sqrt{H}$ ***AG***

(ii) evidence of using Euler's method correctly
e.g. $y_1 = 1.05545...$ ***(A1)***

maximum value of $H = 1.45$ (metres) (at 8.5 minutes) ***A2***
($1.44678\ldots$ metres)

Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

May 2021

Mathematics: applications and interpretation

Higher level

Paper 3

16 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**.
A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.
R Marks awarded for clear **Reasoning**.
AG Answer given in the question and so no marks are awarded.
FT Follow through. The practice of awarding marks, despite candidate errors in previous parts, for their correct methods/answers using incorrect results.

Using the markscheme

1 General

Award marks using the annotations as noted in the markscheme *eg* ***M1***, ***A2***.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***A3***, ***M2*** *etc.*, do **not** split the marks, unless there is a note.
- The response to a "show that" question does not need to restate the ***AG*** line, unless a **Note** makes this explicit in the markscheme.
- Once a correct answer to a question or part question is seen, ignore further working even if this working is incorrect and/or suggests a misunderstanding of the question. This will encourage a uniform approach to marking, with less examiner discretion. Although some candidates may be advantaged for that specific question item, it is likely that these candidates will lose marks elsewhere too.
- An exception to the previous rule is when an incorrect answer from further working is used **in a subsequent part**. For example, when a correct exact value is followed by an incorrect decimal approximation in the first part and this approximation is then used in the second part. In this situation, award ***FT*** marks as appropriate but do not award the final ***A1*** in the first part. Examples:

	Correct answer seen	**Further working seen**	**Any FT issues?**	**Action**
1.	$8\sqrt{2}$	5.65685... *(incorrect decimal value)*	No. Last part in question.	Award ***A1*** for the final mark *(condone the incorrect further working)*
2.	$\frac{35}{72}$	0.468111… (incorrect decimal value)	Yes. Value is used in subsequent parts.	Award ***A0*** for the final mark *(and full **FT** is available in subsequent parts)*

3 Implied marks

Implied marks appear in **brackets e.g.** ***(M1)***,and can only be awarded if **correct** work is seen or implied by subsequent working/answer.

4 Follow through marks (only applied after an error is made)

Follow through (***FT***) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) (e.g. incorrect value from part (a) used in part (d) or incorrect value from part (c)(i) used in part (c)(ii)). Usually, to award ***FT*** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if all the marks awarded in a subsequent part are for the answer or are implied, then ***FT*** marks should be awarded for *their* correct answer, even when working is not present.

For example: following an incorrect answer to part (a) that is used in subsequent parts, where the markscheme for the subsequent part is ***(M1)A1***, it is possible to award full marks for *their* correct answer, **without working being seen.** For longer questions where all but the answer marks are implied this rule applies but may be overwritten by a **Note** in the Markscheme.

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks, by reflecting on what each mark is for and how that maps to the simplified version.
- If the error leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word “their” in a description, to indicate that candidates may be using an incorrect value.
- If the candidate’s answer to the initial question clearly contradicts information given in the question, it is not appropriate to award any ***FT*** marks in the subsequent parts. This includes when candidates fail to complete a “show that” question correctly, and then in subsequent parts use their incorrect answer rather than the given value.
- Exceptions to these ***FT*** rules will be explicitly noted on the markscheme.
- If a candidate makes an error in one part but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the command term was “Hence”.

5 Mis-read

If a candidate incorrectly copies values or information from the question, this is a mis-read (***MR***). A candidate should be penalized only once for a particular misread. Use the ***MR*** stamp to indicate that this has been a misread and do not award the first mark, even if this is an ***M*** mark, but award all others as appropriate.

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.
- If a candidate uses a correct answer, to a "show that" question, to a higher degree of accuracy than given in the question, this is NOT a misread and full marks may be scored in the subsequent part.
- ***MR*** can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If the command term is 'Hence' and not 'Hence or otherwise' then alternative methods are not permitted unless covered by a note in the mark scheme.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**.

7 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation** for example 1.9 and $1{,}9$ or 1000 and $1{,}000$ and 1.000.
- Do not accept final answers written using calculator notation. However, ***M*** marks and intermediate ***A*** marks can be scored, when presented using calculator notation, provided the evidence clearly reflects the demand of the mark.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, some **equivalent** answers will generally appear in brackets. Not all equivalent notations/answers/methods will be presented in the markscheme and examiners are asked to apply appropriate discretion to judge if the candidate work is equivalent.

8 Format and accuracy of answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. If the level of accuracy is not stated in the question, the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures.*

Where values are used in subsequent parts, the markscheme will generally use the exact value, however candidates may also use the correct answer to 3 sf in subsequent parts. The markscheme will often explicitly include the subsequent values that come "*from the use of 3 sf values*".

Simplification of final answers: Candidates are advised to give final answers using good mathematical form. In general, for an ***A*** mark to be awarded, arithmetic should be completed, and any values that lead to integers should be simplified; for example, $\sqrt{\frac{25}{4}}$ should be written as $\frac{5}{2}$. An exception to this is simplifying fractions, where lowest form is not required (although the numerator and the denominator must be integers); for example, $\frac{10}{4}$ may be left in this form or written as $\frac{5}{2}$. However, $\frac{10}{5}$ should be written as 2, as it simplifies to an integer.

Algebraic expressions should be simplified by completing any operations such as addition and multiplication, e.g. $4e^{2x} \times e^{3x}$ should be simplified to $4e^{5x}$, and $4e^{2x} \times e^{3x} - e^{4x} \times e^{x}$ should be simplified to $3e^{5x}$. Unless specified in the question, expressions do not need to be factorized, nor do factorized expressions need to be expanded, so $x(x+1)$ and $x^2 + x$ are both acceptable.

Please note: intermediate ***A*** marks do NOT need to be simplified.

9 Calculators

A GDC is required for this paper, but If you see work that suggests a candidate has used any calculator not approved for IB DP examinations (eg CAS enabled devices), please follow the procedures for malpractice.

10. Presentation of candidate work

Crossed out work: If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work unless an explicit note from the candidate indicates that they would like the work to be marked.

More than one solution: Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise. If the layout of the responses makes it difficult to judge, examiners should apply appropriate discretion to judge which is "first".

1. (a) (i) Any one from: ***R1***
increase sample size / increase response rate / repeat process
check whether sample is representative
test-retest participants or do a parallel test
use a stratified sample
use a random sample

Note: Do not condone:
Ask different types of doctor
Ask for proof of income
Ask for proof of being a doctor
Remove anonymity
Remove response K.

[1 mark]

(ii) Any one from: ***R1***
non-random sampling means a subset of population might be responding
self-reported happiness is not the same as happiness
happiness is not a constant / cannot be quantified / is difficult to measure
income might include external sources
Juliet is only sampling doctors in her city
correlation does not imply causation
sample might be biased

Note: Do not condone the following common but vague responses unless they make a clear link to validity:
Sample size is too small
Result is not generalizable
There may be other variables Juliet is ignoring
Sample might not be representative

[1 mark]

(b) because the income is very different / implausible / clearly contrived ***R1***

Note: Answers must explicitly reference "income" to get credit.

[1 mark]

continued…

Question 1 continued

(c) (i) $(\$)\ 90\ 200$ ***(M1)A1***

[2 marks]

(ii) $r = 0.558\ \ (0.557723...)$ ***A2***

[2 marks]

(d) (i) **EITHER**

only looking for change in one direction ***R1***

OR

only looking for greater happiness with greater income ***R1***

OR

only looking for evidence of positive correlation ***R1***

[1 mark]

(ii) $H_0 : \rho = 0;\ H_1 : \rho > 0$ ***A1A1***

Note: Award ***A1*** for ρ seen (do not accept r), ***A1*** for both correct hypotheses, using **their** ρ or r. Accept an equivalent statement in words, however reference to "correlation for the population" or "association for the population" must be explicit for the first ***A1*** to be awarded.

Watch out for a null hypothesis in words similar to "Annual income is not associated with greater happiness". This is effectively saying $\rho \leq 0$ and should not be condoned.

[2 marks]

continued…

Question 1 continued

(iii) **METHOD 1 – using critical value of *r***

$0.558 > 0.549$ $(0.557723... > 0.549)$ ***R1***

(therefore significant evidence of) a positive correlation ***A1***

Note: Do not award ***R0A1***.

METHOD 2 – using *p*-value

$0.0469 < 0.05$ $(0.0469463... < 0.05)$ ***A1***

Note: Follow through from their *r*-value from part (c)(ii).

(therefore significant evidence of) a positive correlation ***A1***

Note: Do not award ***A0A1***.

[2 marks]

(e) (i) $a = 0.000126$ $(0.000125842...)$, $b = 41.1$ $(41.1490...)$ ***A1***

[1 mark]

(ii) **EITHER**

the amount the happiness score increases for every \$1 increase in (annual) income ***A1***

OR

rate of change of happiness with respect to (annual) income ***A1***

Note: Accept equivalent responses e.g. an increase of 1.26 in happiness for every $\$10000$ increase in salary.

[1 mark]

(iii) $c = -2.06 \times 10^{-9}$ $(-2.06191... \times 10^{-9})$,

$d = 7.05 \times 10^{-4}$ $(7.05272... \times 10^{-4})$,

$e = 12.6$ $(12.5878...)$ ***A1***

[1 mark]

continued…

Question 1 continued

(iv) for quadratic model: $R^2 = 0.659 \ (0.659145...)$ ***A1***

for linear model: $R^2 = 0.311 \ (0.311056...)$ ***A1***

Note: Follow through from their r value from part (c)(ii).

[2 marks]

(v) **EITHER**

quadratic model is a better fit to the data / more accurate ***A1***

OR

quadratic model explains a higher proportion of the variance ***A1***

[1 mark]

(vi) **EITHER**

not valid, R^2 not a useful measure to compare models with different numbers of parameters ***A1***

OR

not valid, quadratic model will always have a better fit than a linear model ***A1***

Note: Accept any other sensible critique of the validity of the method. Do not accept any answers which focus on the conclusion rather than the method of model selection.

[1 mark]

continued…

Question 1 continued

(f) (i) (single sample) t-test ***A1***

[1 mark]

(ii) **EITHER**

$H_0 : \mu = 80\,000;\ H_1 : \mu \neq 80\,000$ ***A1***

OR

H_0 : (sample is drawn from a population where) the population mean is $\$80\,000$

H_1 : the population mean is not $\$80\,000$ ***A1***

Note: Do not allow ***FT*** from an incorrect test in part (f)(i) other than a z-test.

[1 mark]

(iii) $p = 0.610\ \ (0.610322...)$ ***A1***

Note: For a z-test follow through from part (f)(i), either 0.578 (from biased estimate of variance) or 0.598 (from unbiased estimate of variance).

$0.610 > 0.05$ ***R1***

EITHER

no (significant) evidence that mean differs from $\$80\,000$ ***A1***

OR

the sample could plausibly have been drawn from the quoted population ***A1***

Note: Allow ***R1FTA1FT*** from an incorrect p-value, but the final ***A1*** must still be in the context of the original research question.

[3 marks]

[Total 24 marks]

2. (a) (i) population growth rate / birth rate of sharks (due to eating mackerel) ***A1***

[1 mark]

(ii) (net) death rate of sharks ***A1***

[1 mark]

(b) (i) $\gamma MS - \delta S = 0$ ***A1***

since $S \neq 0$ ***R1***

Note: Accept $S > 0$.

getting to given answer without further error by either cancelling or factorizing ***A1***

$M = \frac{\delta}{\gamma}$ ***AG***

[3 marks]

(ii) $\frac{\mathrm{d}M}{\mathrm{d}t} = 0$

$\alpha M - \beta MS = 0$ ***(M1)***

(since $M \neq 0$) $S = \frac{\alpha}{\beta}$ ***A1***

[2 marks]

(c) (i) $M_{eq} = \frac{\delta}{\gamma} \Rightarrow \frac{\delta}{\frac{1}{2}\gamma} = 2M_{eq}$ ***M1***

Note: Accept equivalent in words.

Doubles ***A1***

Note: Do not accept "increases".

[2 marks]

(ii) $M_{eq} = \frac{\delta}{\gamma}$ is not dependent on α ***R1***

Note: Award ***R0*** for any contextual argument.

no change ***A1***

Note: Do not award ***R0A1***.

[2 marks]

continued…

Question 2 continued

(d) (i) $\frac{\mathrm{d}M}{\mathrm{d}t} = \alpha M$ ***A1***

[1 mark]

(ii) $\int \frac{1}{M}\,\mathrm{d}M = \int \alpha\,\mathrm{d}t$ ***M1***

Note: Award ***M1*** is for an attempt to separate variables. This means getting to the point $\int f(M)\,\mathrm{d}M = \int g(t)\,\mathrm{d}t$ where the integral can be seen or implied by further work.

$\ln|M| = \alpha t + c$ ***A1***

Note: Accept $\ln M$. Condone missing constant of integration for this mark.

$M = k\mathrm{e}^{\alpha t}$

when $t = 0$, $M_0 = k$ ***M1***

Note: Award ***M1*** for a clear attempt at using initial conditions to find a constant of integration. Only possible if the constant of integration exists. $t = 0$ or "initially" or similar must be seen. Substitution may appear earlier, following the integration.

initial conditions and all other manipulations correct and clearly communicated to get to the final answer ***A1***

$M = M_0\mathrm{e}^{\alpha t}$ ***AG***

[4 marks]

(iii) $M = 3M_0$ seen anywhere ***(A1)***

substituting $t = 2$, $M = 3M_0$ into equation $M = M_0\mathrm{e}^{\alpha t}$ ***(M1)***

$3M_0 = M_0\mathrm{e}^{2\alpha}$

$\alpha = \frac{1}{2}\ln 3$ **OR** $0.549306...$ ***A1***

Note: The ***A1*** requires either the exact answer or an answer to at least 4 sf.

≈ 0.549 ***AG***

[3 marks]

continued…

Question 2 continued

(e) (i) an attempt to set up one recursive equation ***(M1)***

Note: Must include **two** given parameters **and** M_n and S_n **and** M_{n+1} or S_{n+1} for the ***(M1)*** to be awarded.

$M_{n+1} = M_n + 0.1(0.549M_n - 0.236M_nS_n)$ ***A1***

$S_{n+1} = S_n + 0.1(0.244M_nS_n - 1.39S_n)$ ***A1***

[3 marks]

(ii) **EITHER**

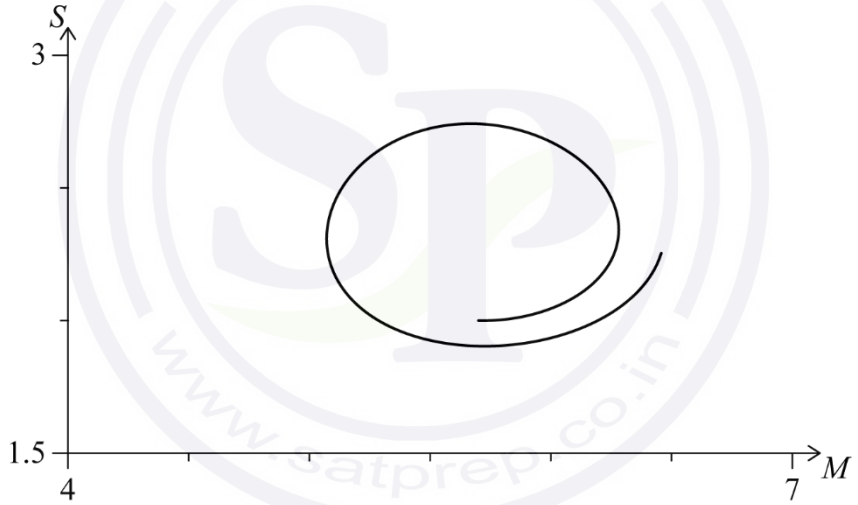
$6.12 \ (6.11609...)$ ***A2***

OR

$6120 \ (6116.09...)$ (mackerel per km^3) ***A2***

[2 marks]

(f) (i)



spiral or closed loop shape ***A1***

approximately 1.25 rotations (can only be awarded if a spiral) ***A1***

correct shape, in approximately correct position (centred at approx. $(5.5, 2.5)$) ***A1***

Note: Award ***A0A0A0*** for any plot of S or M against t.

[3 marks]

continued…

Question 2 continued

(ii) **EITHER**

approximate minimum is $(5.07223\ldots)\ 5.07$ (which is greater than 5) ***A1***

OR

the line $M = 5$ clearly labelled on their phase portrait ***A1***

THEN

(the density will not fall below 5000) hence sufficient for sustainable fishing

A1

Note: Do not award ***A0A1***. Only if the minimum point is labelled on the sketch then a statement here that *"the mackerel population is always above 5000*" would be sufficient. Accept the value 5.07 seen within a table of values.

[2 marks]

(iii) Any two from: ***A1A1***

- Current values / parameters are only an estimate,
- The Euler method is only an approximate method / choosing $h = 0.1$ might be too large.
- There might be random variation / the model has no stochastic component
- Conditions / parameters might change over the nine years,
- A discrete system is being approximated by a continuous system,

Allow any other sensible critique.

If a candidate identifies factors which the model ignores, award ***A1*** per factor identified. These factors could include:

- Other predators
- Seasonality
- Temperature
- The effect of fishing
- Environmental catastrophe
- Migration

Note: Do not allow:
"You cannot have 5.07 mackerel".
It is only a model (as this is too vague).
Some factors have been ignored (without specifically identifying the factors).
Values do not always follow the equation / model. (as this is too vague)

[2 marks]

[Total 31 marks]

Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

May 2021

Mathematics: applications and interpretation

Higher level

Paper 3

13 pages

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- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***A3***, ***M2 etc***., do **not** split the marks, unless there is a note.
- The response to a "show that" question does not need to restate the ***AG*** line, unless a **Note** makes this explicit in the markscheme.
- Once a correct answer to a question or part question is seen, ignore further working even if this working is incorrect and/or suggests a misunderstanding of the question. This will encourage a uniform approach to marking, with less examiner discretion. Although some candidates may be advantaged for that specific question item, it is likely that these candidates will lose marks elsewhere too.
- An exception to the previous rule is when an incorrect answer from further working is used **in a subsequent part**. For example, when a correct exact value is followed by an incorrect decimal approximation in the first part and this approximation is then used in the second part. In this situation, award ***FT*** marks as appropriate but do not award the final ***A1*** in the first part. Examples:

	Correct answer seen	**Further working seen**	**Any FT issues?**	**Action**
1.	$8\sqrt{2}$	5.65685... *(incorrect decimal value)*	No. Last part in question.	Award ***A1*** for the final mark *(condone the incorrect further working)*
2.	$\frac{35}{72}$	0.468111… (incorrect decimal value)	Yes. Value is used in subsequent parts.	Award ***A0*** for the final mark *(and full **FT** is available in subsequent parts)*

3 Implied marks

Implied marks appear in **brackets e.g.** ***(M1)***,and can only be awarded if **correct** work is seen or implied by subsequent working/answer.

4 Follow through marks (only applied after an error is made)

Follow through (***FT***) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) (e.g. incorrect value from part (a) used in part (d) or incorrect value from part (c)(i) used in part (c)(ii)). Usually, to award ***FT*** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if all the marks awarded in a subsequent part are for the answer or are implied, then ***FT*** marks should be awarded for *their* correct answer, even when working is not present.

For example: following an incorrect answer to part (a) that is used in subsequent parts, where the markscheme for the subsequent part is ***(M1)A1***, it is possible to award full marks for *their* correct answer, **without working being seen.** For longer questions where all but the answer marks are implied this rule applies but may be overwritten by a **Note** in the Markscheme.

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks, by reflecting on what each mark is for and how that maps to the simplified version.
- If the error leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word “their” in a description, to indicate that candidates may be using an incorrect value.
- If the candidate’s answer to the initial question clearly contradicts information given in the question, it is not appropriate to award any ***FT*** marks in the subsequent parts. This includes when candidates fail to complete a “show that” question correctly, and then in subsequent parts use their incorrect answer rather than the given value.
- Exceptions to these ***FT*** rules will be explicitly noted on the markscheme.
- If a candidate makes an error in one part but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the command term was “Hence”.

5 Mis-read

If a candidate incorrectly copies values or information from the question, this is a mis-read (***MR***). A candidate should be penalized only once for a particular misread. Use the ***MR*** stamp to indicate that this has been a misread and do not award the first mark, even if this is an ***M*** mark, but award all others as appropriate.

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.
- If a candidate uses a correct answer, to a "show that" question, to a higher degree of accuracy than given in the question, this is NOT a misread and full marks may be scored in the subsequent part.
- ***MR*** can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If the command term is 'Hence' and not 'Hence or otherwise' then alternative methods are not permitted unless covered by a note in the mark scheme.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**.

7 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation** for example 1.9 and $1{,}9$ or 1000 and $1{,}000$ and 1.000.

- Do not accept final answers written using calculator notation. However, ***M*** marks and intermediate ***A*** marks can be scored, when presented using calculator notation, provided the evidence clearly reflects the demand of the mark.

- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.

- In the markscheme, some **equivalent** answers will generally appear in brackets. Not all equivalent notations/answers/methods will be presented in the markscheme and examiners are asked to apply appropriate discretion to judge if the candidate work is equivalent.

8 Format and accuracy of answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. If the level of accuracy is not stated in the question, the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures.*

Where values are used in subsequent parts, the markscheme will generally use the exact value, however candidates may also use the correct answer to 3 sf in subsequent parts. The markscheme will often explicitly include the subsequent values that come "*from the use of 3 sf values*".

Simplification of final answers: Candidates are advised to give final answers using good mathematical form. In general, for an ***A*** mark to be awarded, arithmetic should be completed, and any values that lead to integers should be simplified; for example, $\sqrt{\frac{25}{4}}$ should be written as $\frac{5}{2}$. An exception to this is simplifying fractions, where lowest form is not required (although the numerator and the denominator must be integers); for example, $\frac{10}{4}$ may be left in this form or written as $\frac{5}{2}$. However, $\frac{10}{5}$ should be written as 2, as it simplifies to an integer.

Algebraic expressions should be simplified by completing any operations such as addition and multiplication, e.g. $4e^{2x} \times e^{3x}$ should be simplified to $4e^{5x}$, and $4e^{2x} \times e^{3x} - e^{4x} \times e^{x}$ should be simplified to $3e^{5x}$. Unless specified in the question, expressions do not need to be factorized, nor do factorized expressions need to be expanded, so $x(x+1)$ and $x^2 + x$ are both acceptable.

Please note: intermediate ***A*** marks do NOT need to be simplified.

9 Calculators

A GDC is required for this paper, but If you see work that suggests a candidate has used any calculator not approved for IB DP examinations (eg CAS enabled devices), please follow the procedures for malpractice.

10. Presentation of candidate work

Crossed out work: If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work unless an explicit note from the candidate indicates that they would like the work to be marked.

More than one solution: Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise. If the layout of the responses makes it difficult to judge, examiners should apply appropriate discretion to judge which is "first".

1. (a) recognition that period $= 669$ ***(M1)***

$b = \frac{2\pi}{669}$ **OR** $b = 0.00939190\ldots$ ***A1***

Note: Award ***A1*** for a correct expression leading to the given value or for a correct value of b to 4 sf or greater accuracy.

$b \approx 0.00939$ ***AG***

[2 marks]

(b) length of day = $24\frac{2}{3}$ hours ***(A1)***

Note: Award ***A1*** for $\frac{2}{3}$, $0.666...$, $0.\overline{6}$ or 0.667.

$\frac{2\pi}{24\frac{2}{3}}$ ***(M1)***

Note: Accept $\left(\frac{360}{24\frac{2}{3}}\right)$.

$= 0.255$ radians $\left(= 0.254723..., \frac{3\pi}{37}, 14.5945...^{\circ}\right)$ ***A1***

[3 marks]

(c) (i) substitution of either value of δ into equation ***(M1)***

correct use of arccos to find a value for ω ***(M1)***

Note: Both ***(M1)*** lines may be seen in either part (c)(i) or part (c)(ii).

$\cos\omega = 0.839\tan(-0.440)$ ***A1***

$\omega = 1.97684...$

≈ 1.98 ***AG***

Note: For substitution of 1.98 award ***M0A0***.

[3 marks]

(ii) $\delta = 0.440$

$\omega = 1.16$ $(1.16474...)$ ***A1***

[1 mark]

continued…

Question 1 continued

(d) (i) $R_{\max} = \dfrac{1.97684...}{0.25472...}$ ***(M1)***

$= 7.76$ hours $(7.76075...)$ ***A1***

Note: Accept 7.70 from use of 1.98.

[2 marks]

(ii) $R_{\min} = \dfrac{1.16474...}{0.25472...}$

$= 4.57$ hours $(4.57258...)$ ***A1***

[1 mark]

Note: Accept 4.55 and 4.56 from use of rounded values.

(e) $a = \dfrac{7.76075... - 4.57258...}{2}$ ***M1***

$\approx 1.59408...$ ***A1***

Note: Award ***M1*** for substituting their values into a correct expression.
Award ***A1*** for a correct value of a from their expression which has at least 3 significant figures and rounds correctly to 1.6.

≈ 1.6 (correct to 2 sf) ***AG***

[2 marks]

(f) **EITHER**

$c = \dfrac{7.76075... + 4.57258...}{2} \left(= \dfrac{12.333...}{2} \right)$ ***(M1)***

OR

$c = 4.57258... + 1.59408...$ or $c = 7.76075... - 1.59408...$

THEN

$= 6.17$ $(6.16666...)$ ***A1***

Note: Accept 6.16 from use of rounded values.
Follow through on their answers to part (d) and 1.6.

[2 marks]

(g) $d = 18.65 - 6.16666...$ ***(M1)***

$= 12.5$ $(12.4833...)$ ***A1***

Note: Follow through for 18.65 minus their answer to part (f).

[2 marks]

continued…

Question 1 continued

(h) (i) at least one expression in the form $re^{g(t)i}$ ***(M1)***

$z_1 = 1.5e^{(0.00939t+2.83)i}$, $z_2 = 1.6e^{(0.00939t)i}$ ***A1A1***

[3 marks]

(ii) **EITHER**

$z_1 - z_2 = 1.5e^{(0.00939t+2.83)i} - 1.6e^{(0.00939t)i}$

$= e^{0.00939ti}\left(1.5e^{2.83i} - 1.6\right)$ ***(M1)***

$= e^{0.00939ti}\left(3.06249...e^{2.99086...i}\right)$ ***(A1)(A1)***

OR

graph of L or f

$p = 3.06249...$ ***(A1)***

$r = -0.150729...$ **OR** $r = 2.99086...$ ***(M1)(A1)***

Note: The p and r variables (or equivalent) must be seen.

THEN

$L(t) = 3.06\sin(0.00939t + 2.99) + 12.5$ ***A1***

$\left(L(t) = 3.06248...\sin(0.00939t + 2.99086...) + 12.4833...\right)$

Note: Accept equivalent forms, e.g. $L(t) = 3.06\sin(0.00939t - 0.151) + 12.5$.
Follow through on their answer to part (g) replacing 12.5.

[4 marks]

(iii) shortest time between sunrise and sunset

$12.4833... - 3.06249...$ ***(M1)***

$= 9.42$ hours $(9.420843...)$ ***A1***

Note: Accept 9.44 from use of 3 sf values.

[2 marks]

[Total 27 marks]

2. (a) Use of χ^2 test for independence ***(M1)***

H_0 : Staying (or leaving) the firm and interview rating are independent.

H_1 : Staying (or leaving) the firm and interview rating are not independent ***A1***

Note: For H_1 accept '…are dependent' in place of '…not independent'.

p-value $= 0.487$ $(0.487221...)$ ***A2***

Note: Award ***A1*** for $\chi^2 = 1.438...$ if p-value is omitted or incorrect.

$0.487 > 0.05$ ***R1***

(the result is not significant at the 5% level)
insufficient evidence to reject the H_0 (or "accept H_0") ***A1***

Note: Do not award ***R0A1***.
The final ***R1A1*** can follow through from their incorrect p-value

[6 marks]

(b) $\frac{55}{91} \times 18 = 10.9$ $(10.8791...)$ ***M1A1***

Note: Award ***A1*** for anything that rounds to 10.9.

≈ 11 ***AG***

[2 marks]

continued…

Question 2 continued

(c) (i) there seems to be a difference between the two departments ***(A1)***

the international department manager seems to be less generous than the national department manager ***R1***

Note: The ***A1*** is for commenting there is a difference between the two departments and the ***R1*** is for correctly commenting on the direction of the difference

[2 marks]

(ii)

	L	M	N	O	P	Q	R
Written assessment rank	1	2	3	4	5	6	7
Manager score rank	1	2.5	4.5	2.5	4.5	6	7

(M1)(A1)

Note: Award ***(M1)*** for an attempt to rank the data, and ***(A1)*** for correct ranks for both variables. Accept either set of rankings in reverse.

$r_s = 0.909 \ \ (0.909241.....)$ ***(M1)A1***

Note: The ***(M1)*** is for calculating the PMCC for their ranks.

Note: If a final answer of 0.9107 is seen, from use of $1-\frac{6\sum d^2}{n(n^2-1)}$, award ***(M1)(A1)A1***.

Accept -0.909 if one set of ranks has been ordered in reverse.

[4 marks]

(iii) **EITHER**

there is a (strong) association between the written assessment mark and the manager scores. ***A1***

OR

there is a (strong) agreement in the rank order of the written assessment marks and the rank order of the manager scores. ***A1***

OR

there is a (strong linear) correlation between the rank order of the written assessment marks and the rank order of the manager scores. ***A1***

Note: Follow through on a value for their value of r_s in c(ii).

THEN

the written assessment is likely to be a valid measure (of the level of employee performance) ***R1***

[2 marks]

continued…

Question 2 continued

(d) (i) test-retest ***A1***

[1 mark]

(ii) p-value $= 0.00209 \ (0.0020939...)$ ***A2***

$0.00209 < 0.05$ ***R1***

(the result is significant at the 5% level)

(there is sufficient evidence to) reject H_0 ***A1***

Note: Do not award ***R0A1***. Accept "accept H_1".
The final ***R1A1*** can follow through from their incorrect p-value.

[4 marks]

(iii) the test seems reliable ***A1***

Note: Follow through from their answer in part (d)(ii). Do not award if there is no conclusion in d(ii).

[1 mark]

(e) (i) 25 ***A1***

[1 mark]

(ii) probability of significant result given no correlation is 0.05 ***(M1)***

probability of at least one significant result in 25 tests is

$1 - 0.95^{25}$ ***(M1)(A1)***

Note: Award ***(M1)*** for use of $1 - \mathrm{P}(0)$ or the binomial distribution with any value of p.

[1 mark]

$= 0.723 \ (0.722610...)$ ***A1***

[4 marks]

(iii) (though the result is significant) it is very likely that one significant result would be achieved by chance, so it should be disregarded or further evidence sought

R1

[1 mark]

[Total 28 marks]

Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

Specimen paper

Mathematics: applications and interpretation

Higher level

Paper 3

9 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

R Marks awarded for clear **Reasoning**.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Award marks using the annotations as noted in the markscheme eg ***M1****,* ***A2****.*

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***M2***, ***A3***, *etc.*, do **not** split the marks, unless there is a note.
- Once a correct answer to a question or part-question is seen, ignore further correct working. However, if further working indicates a lack of mathematical understanding do not award the final ***A1***. An exception to this may be in numerical answers, where a correct exact value is followed by an incorrect decimal. However, if the incorrect decimal is carried through to a subsequent part, and correct ***FT*** working shown, award ***FT*** marks as appropriate but do not award the final ***A1*** in that part.

Examples

	Correct answer seen	Further working seen	Action
1.	$8\sqrt{2}$	5.65685... *(incorrect decimal value)*	Award the final ***A1*** *(ignore the further working)*
2.	$\frac{1}{4}\sin 4x$	$\sin x$	Do not award the final ***A1***
3.	$\log a - \log b$	$\log(a-b)$	Do not award the final ***A1***

3 Implied marks

*Implied marks appear in **brackets e.g. (M1)**, and can only be awarded if **correct** work is seen or if implied in subsequent working.*

- Normally the correct work is seen or implied in the next line.
- Marks **without** brackets can only be awarded for work that is **seen**.

4 Follow through marks (only applied after an error is made)

*Follow through (**FT**) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) or subpart(s). Usually, to award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if the only marks awarded in a subpart are for the answer (i.e. there is no working expected), then **FT** marks should be awarded if appropriate.*

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*e.g.* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word “their” in a description, to indicate that candidates may be using an incorrect value.
- Exceptions to this rule will be explicitly noted on the markscheme.
- If a candidate makes an error in one part, but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the question says hence. It is often possible to use a different approach in subsequent parts that does not depend on the answer to previous parts.

5 Mis-read

*If a candidate incorrectly copies information from the question, this is a mis-read (**MR**). Apply a **MR** penalty of 1 mark to that question*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates’ own work does **not** constitute a misread, it is an error.
- The ***MR*** penalty can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme

- Alternative methods for complete questions are indicated by ***METHOD 1***, ***METHOD 2***, *etc*.
- Alternative solutions for part-questions are indicated by ***EITHER*** . . . ***OR***.

7 Alternative forms

Unless the question specifies otherwise, ***accept*** *equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

8 Accuracy of Answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. There are two types of accuracy errors, and the final answer mark should not be awarded if these errors occur.

- **Rounding errors**: only applies to final answers not to intermediate steps.
- **Level of accuracy**: when this is not specified in the question the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures*.

9 Calculators

A GDC is required for this examination, but calculators with symbolic manipulation features/ CAS functionality are not allowed.

Calculator notation
The subject guide says:

> *Students must always use correct mathematical notation, not calculator notation.*

Do **not** accept final answers written using calculator notation. However, do not penalize the use of calculator notation in the working.

1. (a) χ^2 (goodness of fit) ***A1***

[1 mark]

(b) ***EITHER***

because aim is to measure improvement

OR

because the students may be of different ability in the two schools ***R1***

[1 mark]

(c) (i) 0.1875 (accept 0.188, 0.19) ***A1***

(ii) 2.46 ***(M1)A1***

Note: Award ***(M1)A0*** for 2.63.

[3 marks]

(d) H_0: there has been no improvement

H_1: there has been an improvement ***A1***

attempt at a one-tailed paired t-test ***(M1)***

p-value $= 0.423$ ***A1***

there is no significant evidence that the students have improved ***R1***

Note: If the hypotheses are not stated award a maximum of ***A0M1A1R0***.

[4 marks]

(e) (i) H_0: there is no difference between the schools

H_1: school B did better than school A ***A1***

one-tailed 2 sample t-test ***(M1)***

p-value $= 0.0984$ ***A1***

$0.0984 > 0.05$ (not significant at the 5% level) so do not reject the null hypothesis ***R1A1***

Note: The final ***A1*** cannot be awarded following an incorrect reason.
The final ***R1A1*** can follow through from their incorrect p-value.
Award a maximum of ***A1(M1)A0R1A1*** for p-value $= 0.0993$.

(ii) sample too small for the central limit theorem to apply (and t-tests assume normal distribution) ***R1***

[6 marks]

continued…

Question 1 continued

(f) (i) $H_0: \rho = 0$

$H_1: \rho > 0$ ***A1***

Note: Allow hypotheses to be expressed in words.

p-value $= 0.00157$ ***A1***

$(0.00157 < 0.01)$ there is a significant evidence of a (linear) correlation between effort and improvement (so it is reasonable to assume a linear relationship) ***R1***

(ii) (gradient of line of regression =) 6.6 ***A1***

[4 marks]

(g) H_0: improvement and gender are independent

H_1: improvement and gender are not independent ***A1***

choice of χ^2 test for independence ***(M1)***

groups first two columns as expected values in first column less than 5 ***M1***

new observed table

	$(f-p)<0$	$0 \le (f-p) < 2$	$(f-p) \ge 2$
Male	14	10	9
Female	11	14	8

(A1)

p-value $= 0.581$ ***A1***

no significant evidence that gender and improvement are dependent ***R1***

[6 marks]

(h) *For example:*
larger samples / include data from whole school
take equal numbers of boys and girls in each sample
have a similar range of abilities in each sample
(if possible) have similar ranges of effort ***R1R1***

Note: Award ***R1*** for each reasonable suggestion to improve the validity of the test.

[2 marks]

Total [27 marks]

2. (a) (i) 2000 ***(M1)A1***

(ii) because the value of $\frac{\mathrm{d}x}{\mathrm{d}t}$ is positive (for $x > 0$) ***R1***

[3 marks]

(b) (i) substitute $x = 800,\ y = 600$ into both equations ***M1***

both equations equal 0 ***A1***

hence an equilibrium point ***AG***

(ii) $x = 0,\ y = 0$ ***A1***

$x = 2000,\ y = 0,\ x = 0,\ y = 3000$ ***M1A1A1***

Note: Award ***M1*** for an attempt at solving the system provided some values of x and y are found.

[6 marks]

(c) (i) $\int \frac{1}{x}\mathrm{d}x = \int 2\,\mathrm{d}t$ ***M1***

$\ln x = 2t + c$ ***A1A1***

Note: Award ***A1*** for RHS, ***A1*** for LHS.

$x = \mathrm{e}^{c}\mathrm{e}^{2t}$ ***M1***

$x = A\mathrm{e}^{2t}\ \left(\text{where } A = \mathrm{e}^{c}\right)$ ***AG***

(ii) $y = B\mathrm{e}^{3t}$ ***A1***

Note: Allow any letter for the constant term, including A.

(iii) $x = 15,\ y = 18$ ***(M1)A1***

[7 marks]

continued…

Question 2 continued

(d) (i) $x_{n+1} = x_n + 0.2\frac{x_n}{1000}(2000 - x_n - 2y_n)$

$y_{n+1} = y_n + 0.2\frac{y_n}{1000}(3000 - 3x_n - y_n)$ ***M1A1***

Note: Accept equivalent forms.

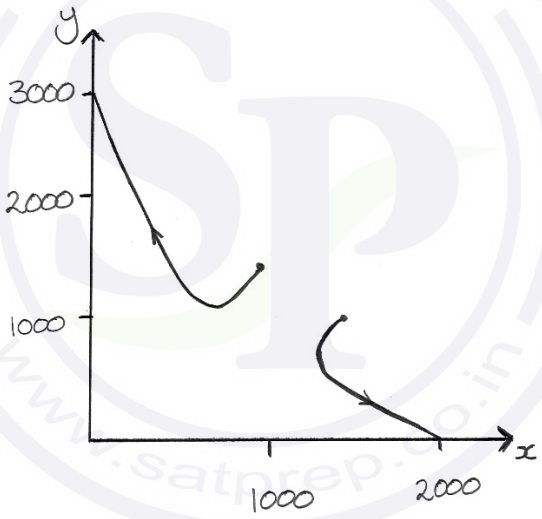
(ii) $x = 319,\ y = 617$ ***(M1)A1A1***

(iii) number of brown squirrels go down to 0,
number of black squirrels to a population of 3000 ***A1***

(iv) number of brown squirrels go to 2000,
number of black squirrels goes down to 0 ***A1***

[7 marks]

(e) (i) ***AND*** (ii)



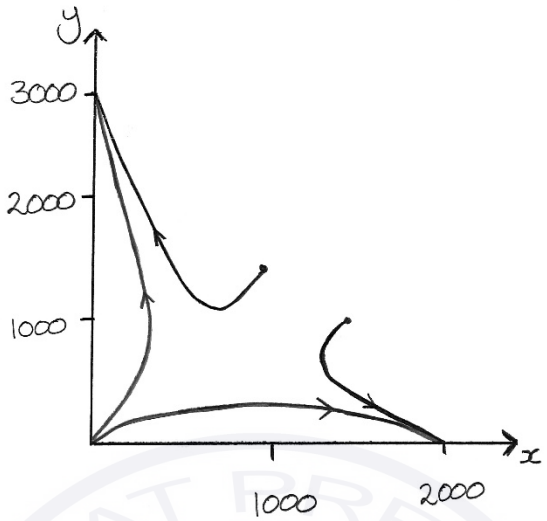
M1A1A1

[3 marks]

continued…

Question 2 continued

(f)



A1A1

Note: Award ***A1*** for a trajectory beginning close to $(0, 0)$ and going to $(0, 3000)$ and ***A1*** for a trajectory beginning close to $(0, 0)$ and going to $(2000, 0)$ in approximately the correct places.

[2 marks]

Total [28 marks]