Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: applications and interpretation
Standard level
Paper 1

30 October 2023

Zone A afternoon | **Zone B** afternoon | **Zone C** afternoon

Candidate session number

1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all questions.
- Answers must be written within the answer boxes provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

8823 – 7214

Answers must be written within the answer boxes provided. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 7]

Joel is a keen cyclist who keeps a record of his performance. The following table shows the time, in minutes, it takes him to ride one kilometre on hills with different gradients. The gradient of each hill is constant.

Gradient G (%)	0	4	10	15	20
Time T (min.)	2.11	5.39	10.56	13.20	18.58

(a) (i) Find the equation of the regression line of T on G.

(ii) Describe the correlation between T and G with reference to the value of r, the Pearson's product-moment correlation coefficient. [4]

On Saturday, Joel intends to ride a hill with a gradient of 17%.

(b) Estimate the time it will take Joel to ride one kilometre up the hill. [2]

This morning, Joel rode one kilometre up a hill, and it took 22 minutes.

(c) Explain why it would be inappropriate to use the equation found in part (a) to estimate the gradient of this hill. [1]

(This question continues on the following page)

(Question 1 continued)

2. [Maximum mark: 6]

The Great Pyramid of Giza is the oldest of the Seven Wonders of the Ancient World. When it was built, 4500 years ago, the measurements of the pyramid were in Royal Egyptian Cubits (REC).

[Source: Nina Aldin Thune. https://en.wikipedia.org/wiki/Great_Pyramid_of_Giza#/media/File:Kheops-Pyramid.jpg. Licensed under CC BY 2.5 https://creativecommons.org/licenses/by/2.5/#. Image adapted.]

Viktor reads online that $1\,\text{REC}$ is equal to 0.52 metres, rounded to two decimal places.

(a) Write down the upper and lower bounds of $1\,\text{REC}$ in metres. [2]

The Great Pyramid of Giza has a square base with side lengths of $440\,\text{REC}$ and a height of $280\,\text{REC}$. Viktor assumes that these two measurements are exact and that the Great Pyramid can be modelled as a square-based pyramid with smooth faces.

(b) Find the minimum possible volume of the pyramid in cubic metres. [4]

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3. [Maximum mark: 6]

Consider the graph of the following function:

$$g(x)=\frac{8}{x}+\frac{x^2}{2}\text{, for } x\neq 0.$$

(a) Write down the equation of the vertical asymptote of $g(x)$. [1]

(b) Find $g'(x)$. [3]

(c) Write down the interval in which $g(x)$ is increasing. [2]

4. [Maximum mark: 8]

On the following Voronoi diagram, the coordinates of three farmhouses are $\mathrm{A}(0, 3)$, $\mathrm{B}(8, 3)$ and $\mathrm{C}(8, 13)$, where distances are measured in kilometres. Each farmhouse owns the land that is closest to it, and their boundaries are defined by the dotted lines on **Diagram 1**.

Diagram 1

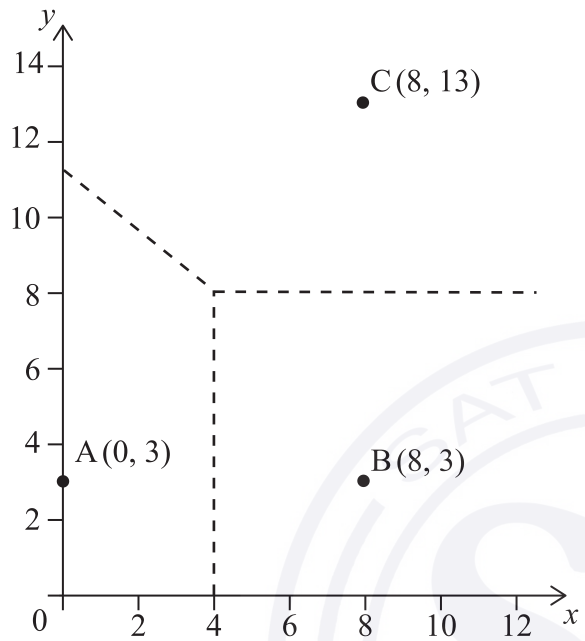
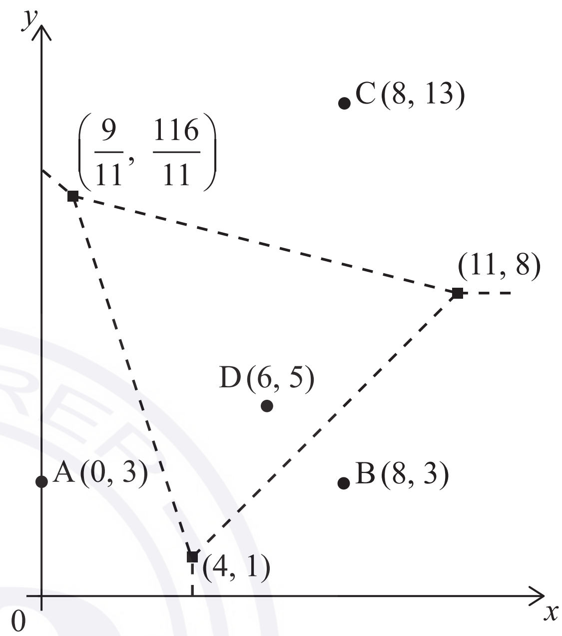


Diagram 2



To provide water to the farms it is decided to construct a well at the point where the boundaries meet on **Diagram 1**.

(a) Write down the coordinates of this point. [1]

(b) Find the equation of the perpendicular bisector of $[\mathrm{AC}]$. [3]

An additional farmhouse $\mathrm{D}(6, 5)$ is built on the land. The Voronoi diagram has been redrawn to show the new boundaries. The coordinates of the vertices of these boundaries are indicated on **Diagram 2**.

A wind turbine is to be built at one of the vertices.

(c) The wind turbine should be as far from the nearest farmhouses as possible.

(i) By calculating appropriate distances, find the location of the wind turbine.

(ii) Hence, write down the distance of the wind turbine to the nearest farmhouse. [4]

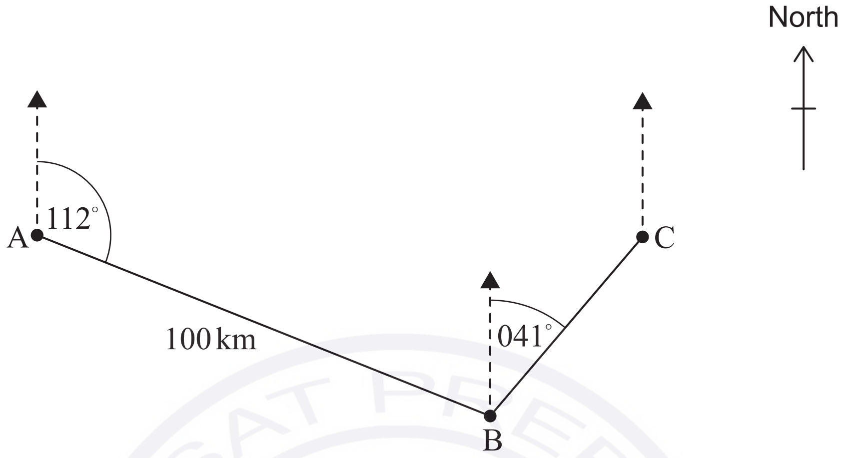
(This question continues on the following page)

(Question 4 continued)

5. [Maximum mark: 6]

Jason sails his boat from point A for a distance of $100\,\text{km}$, on a bearing of 112°, to arrive at point B. He then sails on a bearing of 041° to point C. Jason's journey is shown in the diagram.

diagram not to scale



(a) Find $A\hat{B}C$. [2]

Point C is directly east of point A.

(b) Calculate the distance that Jason sails to return directly from point C to point A. [4]

6. [Maximum mark: 4]

Consider the following function:

$$h(x) = \frac{2}{\sqrt{x-1}} + \frac{1}{2}\text{, for } x > 1\,.$$

(a) Find $h^{-1}(1)$. [2]

(b) Find the domain of $h^{-1}(x)$. [2]

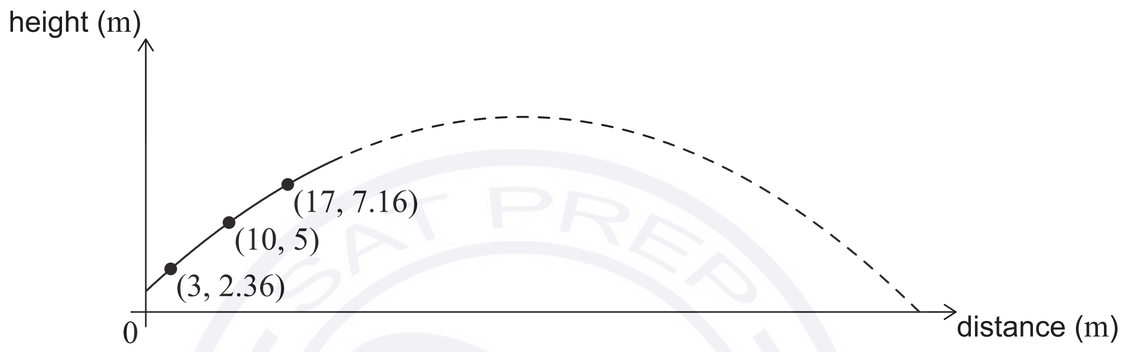
7. [Maximum mark: 9]

A sports player on a horizontal athletic field hits a ball. The height of the ball above the field, in metres, after it is hit can be modelled using a quadratic function of the form $f(x) = ax^2 + bx + c$, where x represents the horizontal distance, in metres, that the ball has travelled from the player.

A specialized camera tracks the initial path of the ball after it is hit by the player. The camera records that the ball travels through the three points $(3, 2.36)$, $(10, 5)$ and $(17, 7.16)$, as shown in **Diagram 1**.

diagram not to scale

Diagram 1



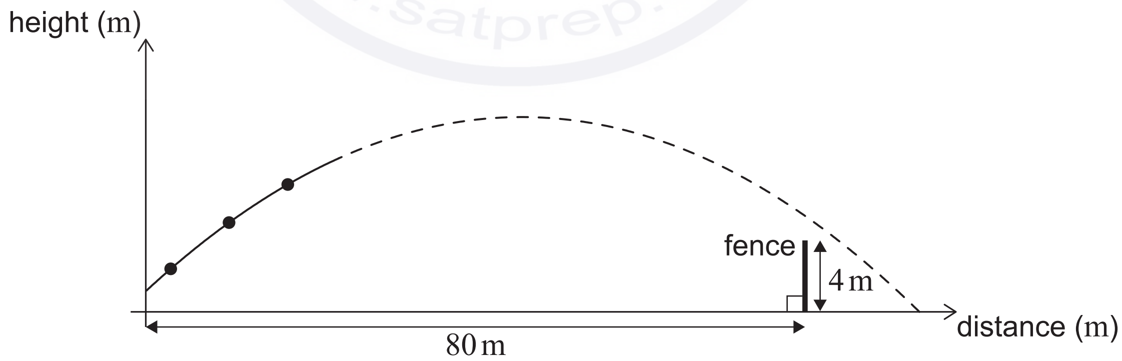
(a) Use the coordinates $(3, 2.36)$ to write down an equation in terms of a, b, and c. [1]

(b) Use your answer to part (a) and two similar equations to find the equation of the quadratic model for the height of the ball. [3]

A 4-metre-high fence is 80 metres from where the player hit the ball, as shown in **Diagram 2**.

diagram not to scale

Diagram 2



(c) Show that the model predicts that the ball will go over the fence. [3]

(d) Find the horizontal distance that the ball will travel, from the player until it first hits the ground, according to this model. [2]

(This question continues on the following page)

(Question 7 continued)

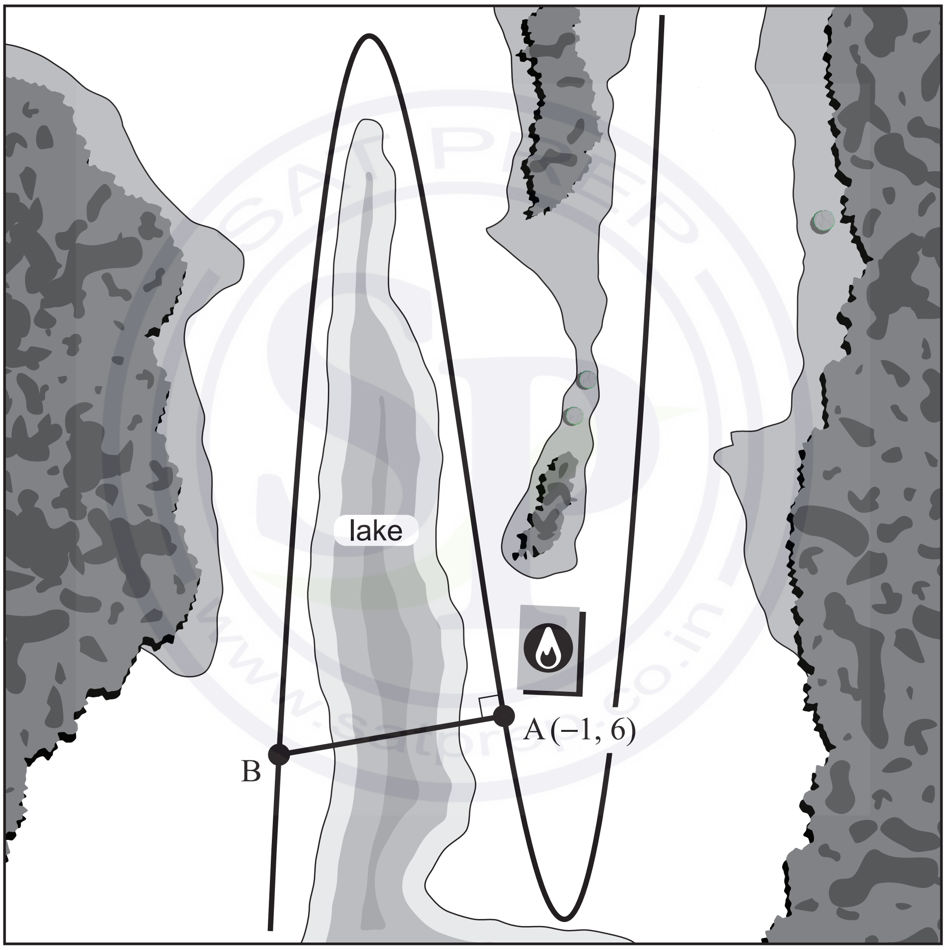
8. [Maximum mark: 7]

The diagram shows a map containing a long, winding road passing a lake. The shape of the road can be modelled by the function $r(x) = (x+1)^3 + 2x^2 - 4x$. All distances in the map are in kilometres.

The local fire station is located at point A, which has coordinates $(-1, 6)$.

To save time during emergencies, the local community is planning the construction of a bridge over the lake. The bridge will be built such that it is normal to the road at point A and will connect the fire station to point B.

diagram not to scale



(a) Using your graphic display calculator, find the value of $r'(-1)$. [2]

(b) Find the equation of the line normal to $r(x)$ at point A, which can be used to model the new bridge. [2]

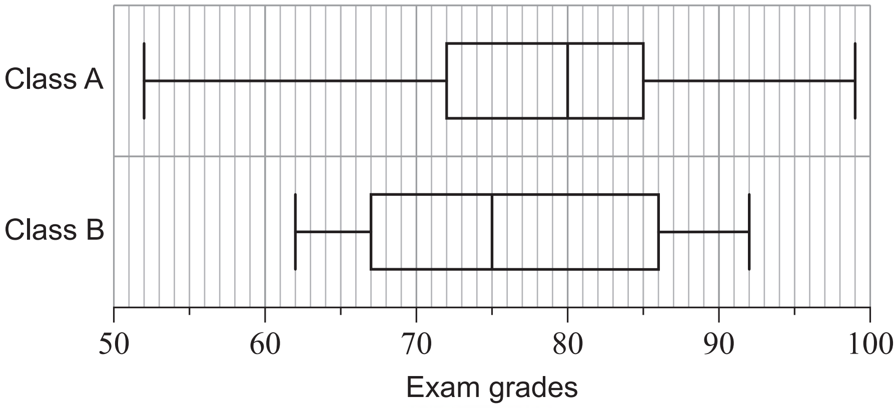
(c) Hence, determine the length of the new bridge. [3]

(This question continues on the following page)

(Question 8 continued)

9. [Maximum mark: 8]

Mr Kelly is a history teacher. After grading his final exams, he creates the following box and whisker diagram to compare the grades of his two classes.



(a) Identify which **two** of the following statements **must** be true according to the box and whisker diagram. Indicate your choices by placing tick marks in the second column of the following table. [2]

Statement	True (✓)
A higher percentage of students in Class B received a grade less than 70 on the exam, than in Class A.	
The data for Class B is normally distributed.	
More students in Class A received a grade greater than 90 on the exam than in Class B.	
The interquartile range for Class A is less than the interquartile range for Class B.	

At the end of the year, Mr Kelly surveyed a random sample of students from each of his two large classes to determine how satisfied they were with his teaching.

Each student independently selected a value from 1 to 10, with 1 meaning that they were not satisfied at all and 10 meaning that they were very satisfied.

His collected data from the student surveys is shown.

Class A	6	9	8	10	1	9	10	9	8	4

Class B	7	5	3	4	3	8	6	7

(This question continues on the following page)

(Question 9 continued)

Mr Kelly believes that there was no difference in the general satisfaction between the two classes. He assumes that the data is drawn from a population that can be modelled by a normal distribution and proposes to conduct a t-test at the 5% significance level.

(b) Write down the null and alternative hypotheses for his test. [2]

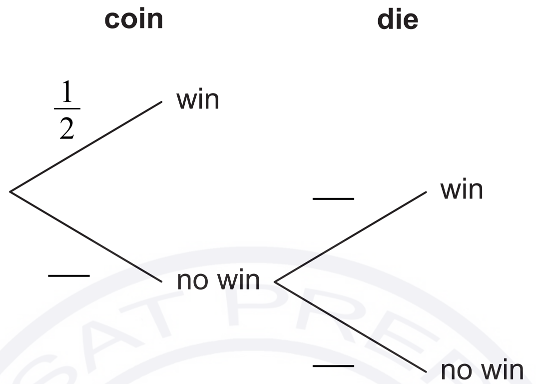
(c) Find the p-value for his test. [2]

(d) Write down the conclusion to the test. Give a reason for your answer. [2]

10. [Maximum mark: 7]

Michèle is playing a game. In the game, she must first flip a fair coin which will result in the coin landing on heads or tails. If the coin lands on heads, then she wins a prize. If it lands on tails, then she has another chance but this time she must roll a fair six-sided die and get a five or six in order to win a prize.

(a) Complete the tree diagram by writing in the three missing probabilities. [2]



(b) Find the probability that Michèle does **not** win a prize. [2]

(c) Given that Michèle won a prize, find the probability that the coin landed on heads. [3]

11. [Maximum mark: 6]

Toktam works at a local bakery 5 days each week. She drives an old car to work that has a 65% probability of starting on any given morning. The probability of the car starting on a given morning is independent of it starting on any other morning.

(a) Find the probability that Toktam's car starts on exactly two mornings in a particular 5 day workweek. [2]

Toktam walks to work on mornings when her car does not start and it is **not** raining. Toktam takes a taxi to work on mornings when her car does not start and it is raining.

Where Toktam lives, there is a 45% probability of rain on any given morning, independent of any other morning. The probability of Toktam's car starting is independent of the weather.

(b) Find the probability that Toktam will **not** have to take a taxi in a particular workweek. [4]

12. [Maximum mark: 6]

Thurston believes that more popular musical artists sell more albums.

He begins to investigate this belief by randomly selecting eight musical artists and collecting data on the number of followers each of the artists has on a particular social media platform. He then collects data on the number of albums each artist sold in the first week after releasing an album. His data is shown in **Table 1**.

Table 1

	Artist 1	Artist 2	Artist 3	Artist 4	Artist 5	Artist 6	Artist 7	Artist 8
Number of social media followers (in thousands)	11 500	12 400	1300	2300	674	49 500	315	94 400
Number of albums sold in first week (in thousands)	123	62.4	17.4	94.9	52.5	27	21.6	595.5

Thurston decides to calculate the Spearman's rank correlation coefficient.

(a) Complete the table of ranks shown in **Table 2**. [1]

Table 2

	Artist 1	Artist 2	Artist 3	Artist 4	Artist 5	Artist 6	Artist 7	Artist 8
Rank – social media followers	4	3	6	5	7	2	8	1
Rank – albums sold in first week								1

(This question continues on the following page)

(Question 12 continued)

(b) Calculate the value of r_s, Spearman's rank correlation coefficient. [2]

Thurston believes that artists with a higher number of social media followers sell more albums in the first week. He carries out a hypothesis test using a 10% significance level with the following null hypothesis:

H_0: In the population, there is no monotonic relationship between the number of social media followers and the number of albums sold in the first week.

(c) Write down Thurston's alternative hypothesis. [1]

The critical value of r_s for this test is 0.643.

(d) State the conclusion of the hypothesis test, giving a reason. [2]

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References:

2. Nina Aldin Thune. https://en.wikipedia.org/wiki/Great_Pyramid_of_Giza#/media/File:Kheops-Pyramid.jpg. Licensed under CC BY 2.5 https://creativecommons.org/licenses/by/2.5/#. Image adapted.

Mathematics: applications and interpretation
Standard level
Paper 1

30 October 2023

Zone A afternoon | **Zone B** afternoon | **Zone C** afternoon

Candidate session number

1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all questions.
- Answers must be written within the answer boxes provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

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Answers must be written within the answer boxes provided. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 7]

Billy is a keen walker who keeps a record of his performance. The following table shows the time, in minutes, it takes him to walk one kilometre up hills with different gradients. The gradient of each hill is constant.

Gradient G (%)	0	4	10	15	20
Time T (min.)	6.85	8.42	11.20	14.49	17.88

(a) (i) Find the equation of the regression line of T on G.

(ii) Describe the correlation between T and G with reference to the value of r, the Pearson's product-moment correlation coefficient. [4]

On Sunday, Billy intends to walk up a hill with a gradient of 13%.

(b) Estimate the time it will take Billy to walk one kilometre up the hill. [2]

This morning, Billy walked one kilometre up a hill, and it took 22 minutes.

(c) Explain why it would be inappropriate to use the equation found in part (a) to estimate the gradient of this hill. [1]

(This question continues on the following page)

(Question 1 continued)

2. [Maximum mark: 6]

The Great Pyramid of Giza is the oldest of the Seven Wonders of the Ancient World. When it was built, 4500 years ago, the measurements of the pyramid were in Royal Egyptian Cubits (REC).

[Source: Nina Aldin Thune. https://en.wikipedia.org/wiki/Great_Pyramid_of_Giza#/media/File:Kheops-Pyramid.jpg. Licensed under CC BY 2.5 https://creativecommons.org/licenses/by/2.5/#. Image adapted.]

Viktor reads online that $1\,\text{REC}$ is equal to 0.52 metres, rounded to two decimal places.

(a) Write down the upper and lower bounds of $1\,\text{REC}$ in metres. [2]

The Great Pyramid of Giza has a square base with side lengths of $440\,\text{REC}$ and a height of $280\,\text{REC}$. Viktor assumes that these two measurements are exact and that the Great Pyramid can be modelled as a square-based pyramid with smooth faces.

(b) Find the minimum possible volume of the pyramid in cubic metres. [4]

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3. [Maximum mark: 6]

Consider the graph of the following function:

$$f(x) = \frac{16}{x} + \frac{x^2}{8}, \text{ for } x \neq 0.$$

(a) Write down the equation of the vertical asymptote of $f(x)$. [1]

(b) Find $f'(x)$. [3]

(c) Write down the interval in which $f(x)$ is increasing. [2]

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4. [Maximum mark: 8]

On the following Voronoi diagram, the coordinates of three farmhouses are $A(0, 3)$, $B(8, 3)$ and $C(8, 13)$, where distances are measured in kilometres. Each farmhouse owns the land that is closest to it, and their boundaries are defined by the dotted lines on **Diagram 1**.

Diagram 1

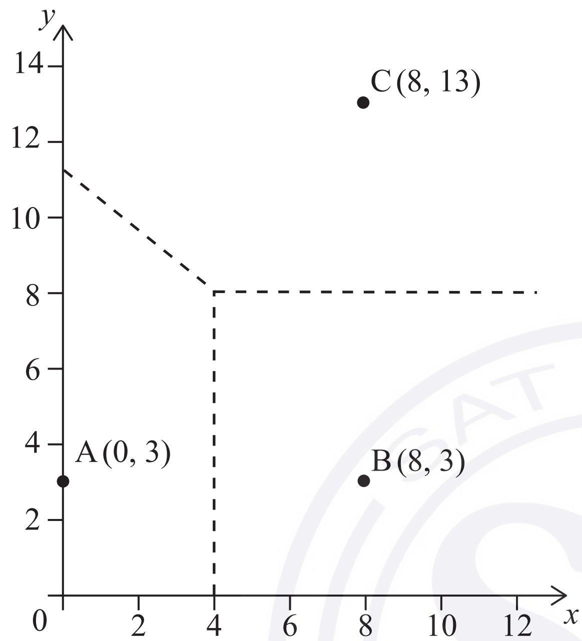
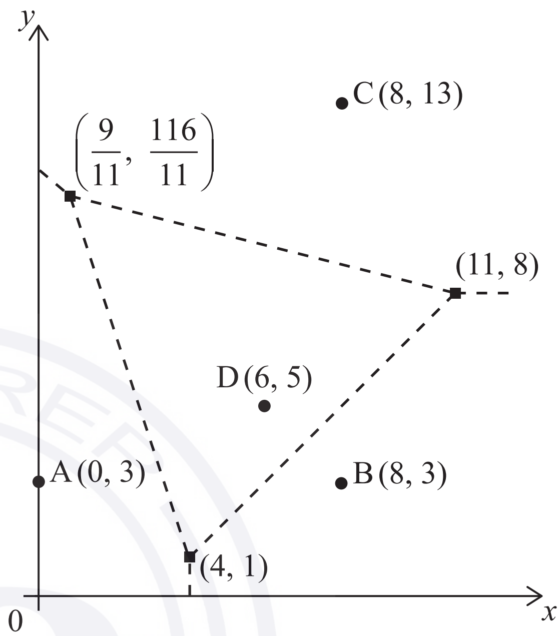


Diagram 2



To provide water to the farms it is decided to construct a well at the point where the boundaries meet on **Diagram 1**.

(a) Write down the coordinates of this point. [1]

(b) Find the equation of the perpendicular bisector of $[AC]$. [3]

An additional farmhouse $D(6, 5)$ is built on the land. The Voronoi diagram has been redrawn to show the new boundaries. The coordinates of the vertices of these boundaries are indicated on **Diagram 2**.

A wind turbine is to be built at one of the vertices.

(c) The wind turbine should be as far from the nearest farmhouses as possible.

(i) By calculating appropriate distances, find the location of the wind turbine.

(ii) Hence, write down the distance of the wind turbine to the nearest farmhouse. [4]

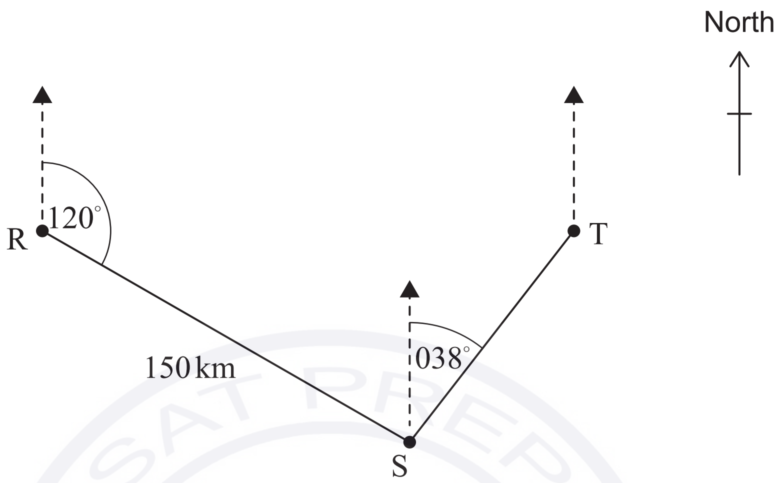
(This question continues on the following page)

(Question 4 continued)

5. [Maximum mark: 6]

Ron sails his boat from point R for a distance of $150\,\text{km}$, on a bearing of 120°, to arrive at point S. He then sails on a bearing of 038° to point T. Ron's journey is shown in the diagram.

diagram not to scale



(a) Find $\hat{RST}$. [2]

Point T is directly east of point R.

(b) Calculate the distance that Ron sails to return directly from point T to point R. [4]

6. [Maximum mark: 4]

Consider the following function:

$$h(x) = \frac{2}{\sqrt{x-1}} + \frac{1}{2}\text{, for } x > 1\text{.}$$

(a) Find $h^{-1}(1)$. [2]

(b) Find the domain of $h^{-1}(x)$. [2]

7. [Maximum mark: 9]

An athlete on a horizontal athletic field throws a discus. The height of the discus above the field, in metres, after it is thrown can be modelled using a quadratic function of the form $f(x) = ax^2 + bx + c$, where x represents the horizontal distance, in metres, that the discus has travelled from the athlete.

A specialized camera tracks the initial path of the discus after it is thrown by the athlete. The camera records that the discus travels through the three points $(3, 2.82)$, $(6, 4.25)$ and $(9, 5.30)$, as shown in **Diagram 1**.

diagram not to scale

Diagram 1

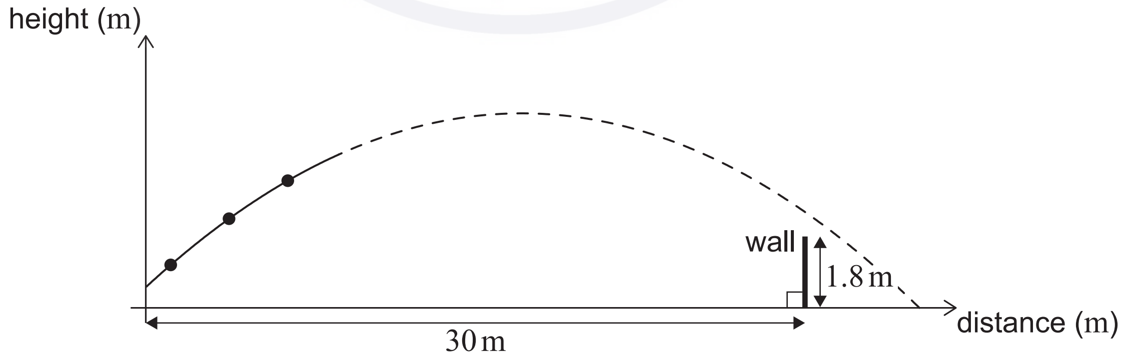
(a) Use the coordinates $(3, 2.82)$ to write down an equation in terms of a, b and c. [1]

(b) Use your answer to part (a) and two similar equations to find the equation of the quadratic model for the height of the discus. [3]

A 1.8-metre-high wall is 30 metres from where the athlete threw the discus, as shown in **Diagram 2**.

diagram not to scale

Diagram 2



(c) Show that the model predicts that the discus will go over the wall. [3]

(d) Find the horizontal distance that the discus will travel, from the athlete until it first hits the ground, according to this model. [2]

(This question continues on the following page)

(Question 7 continued)

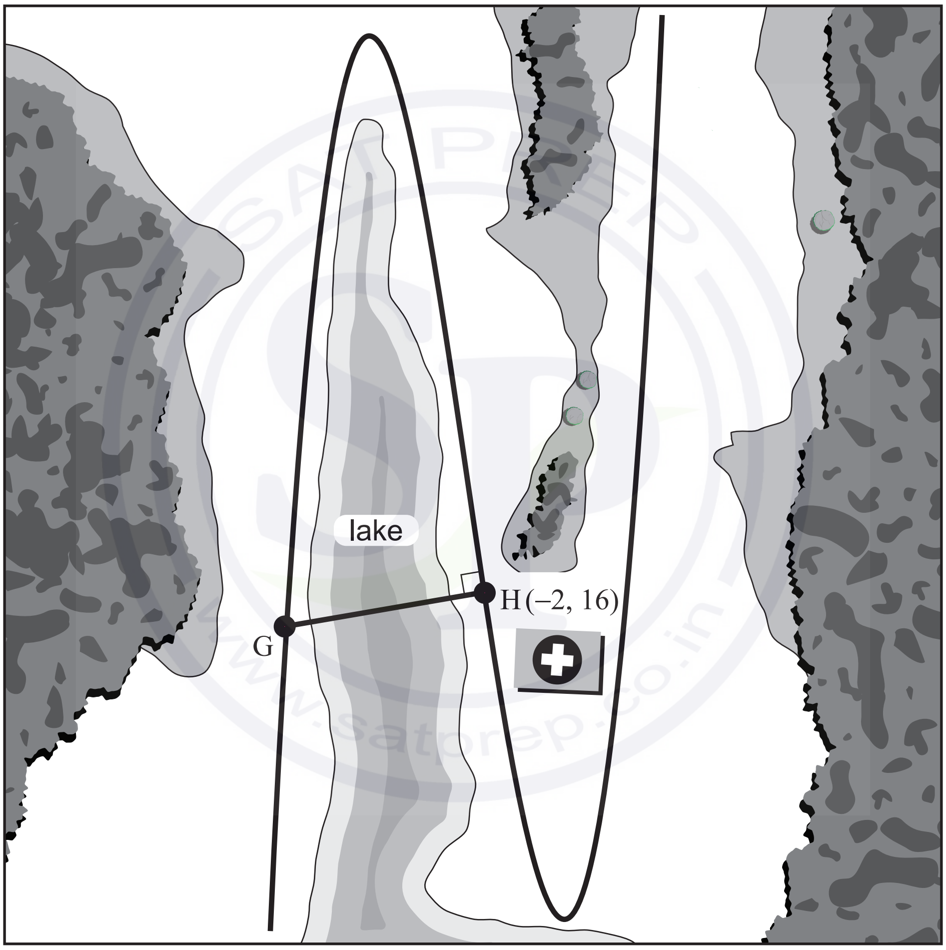
8. [Maximum mark: 7]

The diagram shows a map containing a long, winding road passing a lake. The shape of the road can be modelled by the function $r(x) = (x+2)^3 + 3x^2 - 2x$. All distances in the map are in kilometres.

The local hospital is located at point H, which has coordinates $(-2, 16)$.

To save time during emergencies, the local community is planning the construction of a bridge over the lake. The bridge will be built such that it is normal to the road at point H and will connect the hospital to point G.

diagram not to scale



(a) Using your graphic display calculator, find the value of $r'(-2)$. [2]

(b) Find the equation of the line normal to $r(x)$ at point H, which can be used to model the new bridge. [2]

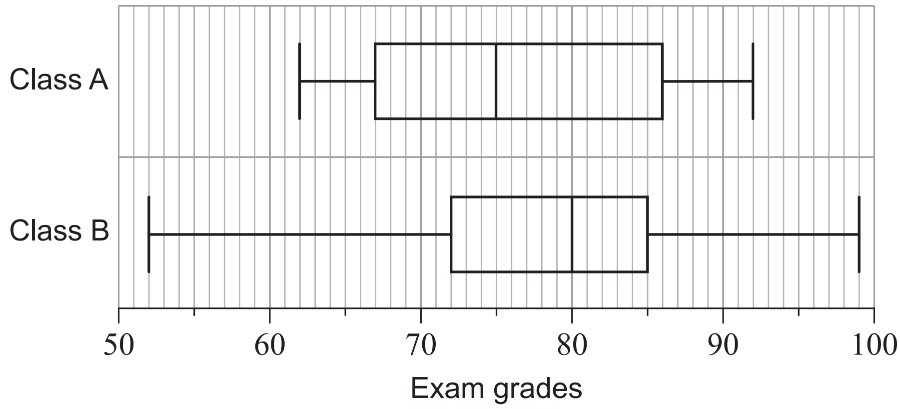
(c) Hence, determine the length of the new bridge. [3]

(This question continues on the following page)

(Question 8 continued)

9. [Maximum mark: 8]

Mrs Whitehouse is a chemistry teacher. After grading her final exams, she creates the following box and whisker diagram to compare the grades of her two classes.



(a) Identify which **two** of the following statements **must** be true according to the box and whisker diagram. Indicate your choices by placing tick marks in the second column of the following table. [2]

Statement	**True (✓)**
The data for Class A is normally distributed.	
A higher percentage of students in Class A received a grade less than 70 on the exam, than in Class B.	
More students in Class B received a grade greater than 90 on the exam than in Class A.	
The interquartile range for Class B is less than the interquartile range for Class A.	

At the end of the year, Mrs Whitehouse surveyed a random sample of students from each of her two large classes to determine how satisfied they were with her teaching.

Each student independently selected a value from 1 to 10, with 1 meaning that they were not satisfied at all and 10 meaning that they were very satisfied.

Her collected data from the student surveys is shown.

Class A	7	5	3	4	3	8	6	5

Class B	6	9	8	10	1	9	10	9	8	3

(This question continues on the following page)

(Question 9 continued)

Mrs Whitehouse believes that there was no difference in the general satisfaction between the two classes. She assumes that the data is drawn from a population that can be modelled by a normal distribution and proposes to conduct a t-test at the 5% significance level.

(b) Write down the null and alternative hypotheses for her test. [2]

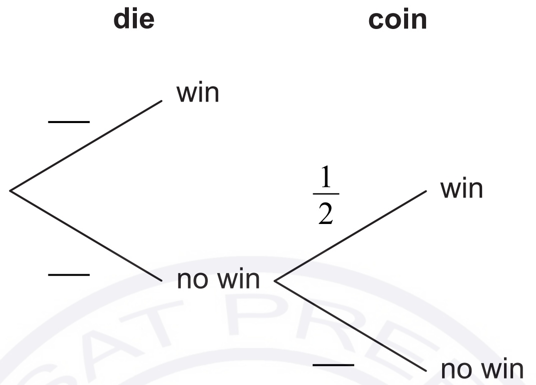
(c) Find the p-value for her test. [2]

(d) Write down the conclusion to the test. Give a reason for your answer. [2]

10. [Maximum mark: 7]

Rita is playing a game. In the game, she must roll a fair six-sided die. If she gets a five or six then she wins a prize. If not, then she has another chance but this time she must flip a fair coin which will result in the coin landing on heads or tails. If the coin lands on heads, then Rita wins a prize.

(a) Complete the tree diagram by writing in the three missing probabilities. [2]



(b) Find the probability that Rita does **not** win a prize. [2]

(c) Given that Rita won a prize, find the probability that she got a five or six when she rolled the die. [3]

11. [Maximum mark: 6]

Nicole works at a local school 5 days each week. She drives an old car to work that has a 72% probability of starting on any given morning. The probability of the car starting on a given morning is independent of it starting on any other morning.

(a) Find the probability that Nicole's car starts on exactly three mornings in a particular 5 day workweek. [2]

Nicole walks to work on mornings when her car does not start and it is **not** raining. Nicole takes the bus to work on mornings when her car does not start and it is raining.

Where Nicole lives, there is a 42% probability of rain on any given morning, independent of any other morning. The probability of Nicole's car starting is independent of the weather.

(b) Find the probability that Nicole will **not** have to take the bus in a particular workweek. [4]

12. [Maximum mark: 6]

Thurston believes that more popular musical artists sell more albums.

He begins to investigate this belief by randomly selecting eight musical artists and collecting data on the number of followers each of the artists has on a particular social media platform. He then collects data on the number of albums each artist sold in the first week after releasing an album. His data is shown in **Table 1**.

Table 1

	Artist 1	Artist 2	Artist 3	Artist 4	Artist 5	Artist 6	Artist 7	Artist 8
Number of social media followers (in thousands)	11 500	12 400	1300	2300	674	49 500	315	94 400
Number of albums sold in first week (in thousands)	123	62.4	17.4	94.9	52.5	27	21.6	595.5

Thurston decides to calculate the Spearman's rank correlation coefficient.

(a) Complete the table of ranks shown in **Table 2**. [1]

Table 2

	Artist 1	Artist 2	Artist 3	Artist 4	Artist 5	Artist 6	Artist 7	Artist 8
Rank – social media followers	4	3	6	5	7	2	8	1
Rank – albums sold in first week								1

(This question continues on the following page)

(Question 12 continued)

(b) Calculate the value of r_s, Spearman's rank correlation coefficient. [2]

Thurston believes that artists with a higher number of social media followers sell more albums in the first week. He carries out a hypothesis test using a 10% significance level with the following null hypothesis:

H_0: In the population, there is no monotonic relationship between the number of social media followers and the number of albums sold in the first week.

(c) Write down Thurston's alternative hypothesis. [1]

The critical value of r_s for this test is 0.643.

(d) State the conclusion of the hypothesis test, giving a reason. [2]

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Disclaimer:

References:

2. Nina Aldin Thune. https://en.wikipedia.org/wiki/Great_Pyramid_of_Giza#/media/File:Kheops-Pyramid.jpg. Licensed under CC BY 2.5 https://creativecommons.org/licenses/by/2.5/#. Image adapted.

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: applications and interpretation
Standard level
Paper 1

8 May 2023

Zone A afternoon | **Zone B** morning | **Zone C** afternoon

Candidate session number

1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all questions.
- Answers must be written within the answer boxes provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

2223 – 7214

Answers must be written within the answer boxes provided. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 5]

Zaha is designing a bridge to cross a river. She believes that the weight of the steel needed for this bridge is approximately $53\,632\,000\,\text{kg}$.

The exact weight of the steel needed for the bridge is $55\,625\,000\,\text{kg}$.

(a) Find the percentage error in Zaha's approximation. [2]

Zaha's design is used to build five identical bridges.

(b) (i) Find the weight of the steel needed for these five bridges, **to three significant figures**.

(ii) Write down your answer to part (b)(i) in the form $a \times 10^k$, where $1 \leq a < 10$, $k \in \mathbb{Z}$. [3]

2. [Maximum mark: 6]

Angel has $\$ 520$ in his savings account. Angel considers investing the money for 5 years with a bank. The bank offers an annual interest rate of 1.2% compounded quarterly.

(a) Calculate the amount of money Angel would have at the end of 5 years with the bank. Give your answer correct to two decimal places. [3]

Instead of investing the money, Angel decides to buy a phone that costs $\$ 520$. At the end of 5 years, the phone will have a value of $\$ 30$. It may be assumed that the depreciation rate per year is constant.

(b) Calculate the annual depreciation rate of the phone. [3]

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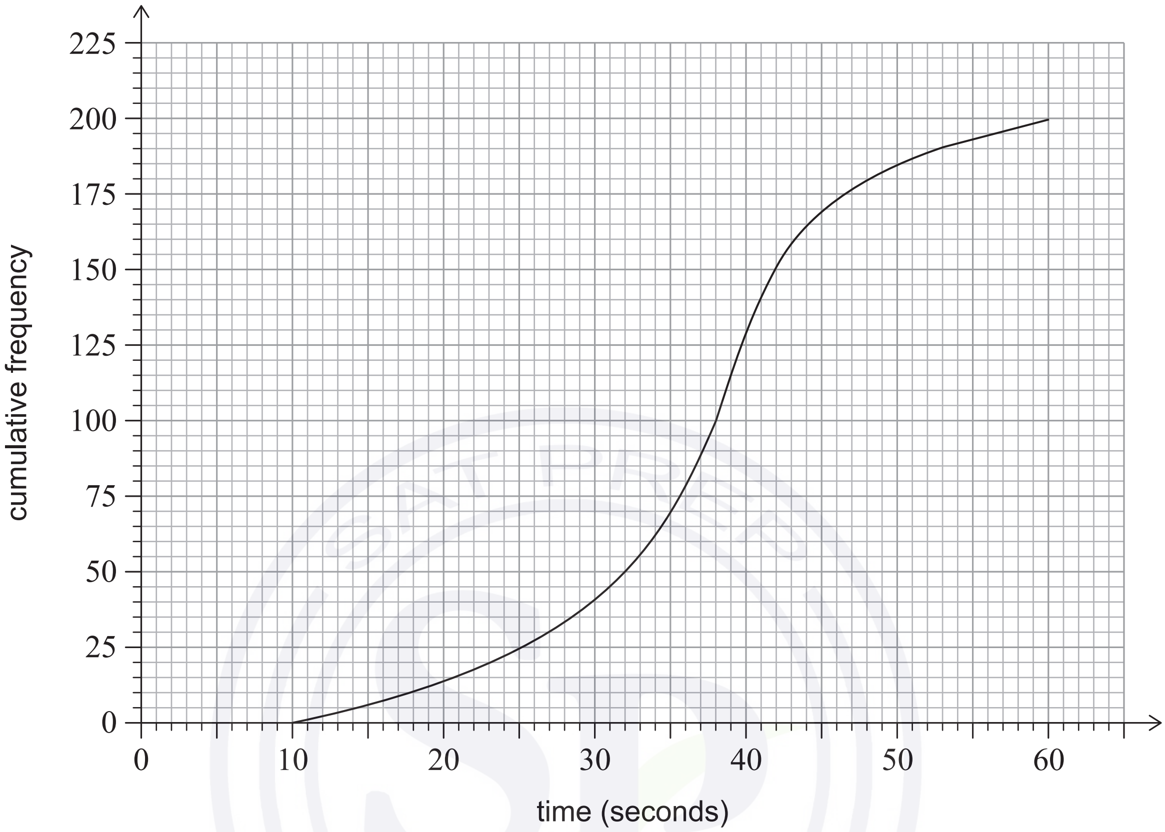
3. [Maximum mark: 7]

In a school, 200 students solved a problem in a mathematics competition. Their times to solve the problem were recorded and the following cumulative frequency graph was produced.



(a) Use the graph to find

(i) the median time;

(ii) the lower quartile;

(iii) the upper quartile;

(iv) the interquartile range. [4]

Cedric took 14 seconds to solve the problem.

(b) Determine whether Cedric's time is an outlier. [3]

(This question continues on the following page)

(Question 3 continued)

4. [Maximum mark: 6]

At a running club, Sung-Jin conducts a test to determine if there is any association between an athlete's age and their best time taken to run $100\,\text{m}$. Eight athletes are chosen at random, and their details are shown below.

Athlete	A	B	C	D	E	F	G	H
Age (years)	13	17	22	18	19	25	11	36
Time (seconds)	13.4	14.6	13.4	12.9	12.0	11.8	17.0	13.1

Sung-Jin decides to calculate the Spearman's rank correlation coefficient for his set of data.

(a) Complete the table of ranks. [2]

Athlete	A	B	C	D	E	F	G	H
Age rank			3					
Time rank							1	

(b) Calculate the Spearman's rank correlation coefficient, r_s. [2]

(c) Interpret this value of r_s in the context of the question. [1]

(d) Suggest a mathematical reason why Sung-Jin may have decided not to use Pearson's product-moment correlation coefficient with his data from the original table. [1]

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5. [Maximum mark: 4]

The following frequency distribution table shows the test grades for a group of students.

Grade	1	2	3	4	5	6	7
Frequency	1	4	7	9	p	9	4

For this distribution, the mean grade is 4.5.

(a) Write down the total number of students in terms of p. [1]

(b) Calculate the value of p. [3]

6. [Maximum mark: 6]

A company that owns many restaurants wants to determine if there are differences in the quality of the food cooked for three different meals: breakfast, lunch and dinner.

Their quality assurance team randomly selects 500 items of food to inspect. The quality of this food is classified as perfect, satisfactory, or poor. The data is summarized in the following table.

		Quality			
		Perfect	**Satisfactory**	**Poor**	**Total**
Meal	**Breakfast**	101	124	7	**232**
	Lunch	68	81	5	**154**
	Dinner	35	69	10	**114**
	Total	**204**	**274**	**22**	**500**

An item of food is chosen at random from these 500.

(a) Find the probability that its quality is not perfect, given that it is from breakfast. [2]

A χ^2 test at the 5% significance level is carried out to determine if there is significant evidence of a difference in the quality of the food cooked for the three meals.

The critical value for this test is 9.488.

The hypotheses for this test are:

H_0: The quality of the food and the type of meal are independent.

H_1: The quality of the food and the type of meal are not independent.

(b) Find the χ^2 statistic. [2]

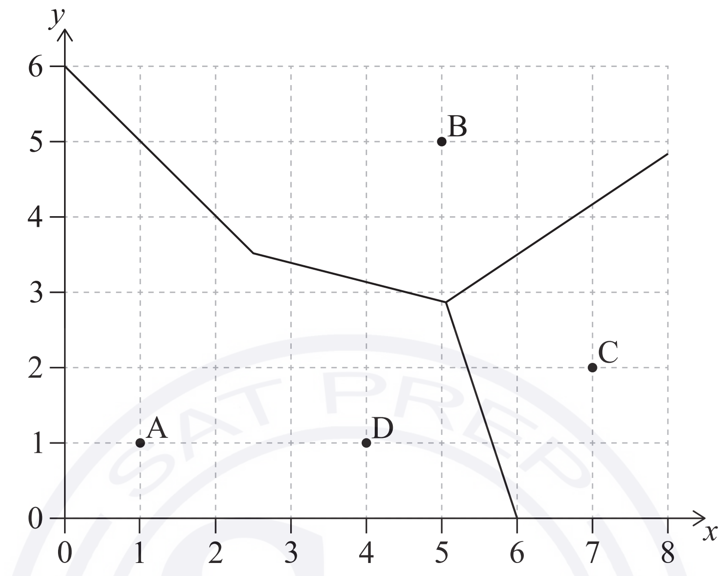
(c) State, with justification, the conclusion for this test. [2]

(This question continues on the following page)

(Question 6 continued)

7. [Maximum mark: 6]

Ani owns four cafes represented by points A, B, C and D. Ani wants to divide the area into delivery regions. This process has been started in the following incomplete Voronoi diagram, where 1 unit represents 1 kilometre.



The midpoint of CD is $(5.5,\ 1.5)$.

(a) Show that the equation of the perpendicular bisector of [CD] is $y = -3x + 18$. [3]

(b) Complete the Voronoi diagram shown above. [1]

Ani opens an office equidistant from three of the cafes, B, C and D. The equation of the perpendicular bisector of [BC] is $3y = 2x - 1.5$.

(c) Find the coordinates of the office. [2]

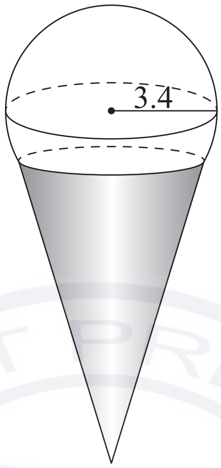
(This question continues on the following page)

(Question 7 continued)

8. [Maximum mark: 5]

Ruhi buys a scoop of ice cream in the shape of a sphere with a radius of $3.4\,\text{cm}$. The ice cream is served in a cone, and it may be assumed that $\frac{1}{5}$ of the volume of the ice cream is inside the cone. This is shown in the following diagram.

diagram not to scale



(a) Calculate the volume of ice cream that is not inside the cone. [3]

The cone has a slant height of $11\,\text{cm}$ and a radius of $3\,\text{cm}$.

The outside of the cone is covered with chocolate.

(b) Calculate the surface area of the cone that is covered with chocolate. Give your answer correct to the nearest cm^2. [2]

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9. [Maximum mark: 6]

The lengths of the seeds from a particular mango tree are approximated by a normal distribution with a mean of $4\,\text{cm}$ and a standard deviation of $0.25\,\text{cm}$.

A seed from this mango tree is chosen at random.

(a) Calculate the probability that the length of the seed is less than $3.7\,\text{cm}$. [2]

It is known that 30% of the seeds have a length greater than $k\,\text{cm}$.

(b) Find the value of k. [2]

For a seed of length $d\,\text{cm}$, chosen at random, $\text{P}(4 - m < d < 4 + m) = 0.6$.

(c) Find the value of m. [2]

10. [Maximum mark: 8]

A player throws a basketball. The height of the basketball is modelled by

$$h(t) = -4.75t^2 + 8.75t + 1.5\,,\ t \geq 0\,,$$

where h is the height of the basketball above the ground, in metres, and t is the time, in seconds, after it was thrown.

(a) Find how long it takes for the basketball to reach its maximum height. [2]

(b) Assuming that no player catches the basketball, find how long it would take for the basketball to hit the ground. [2]

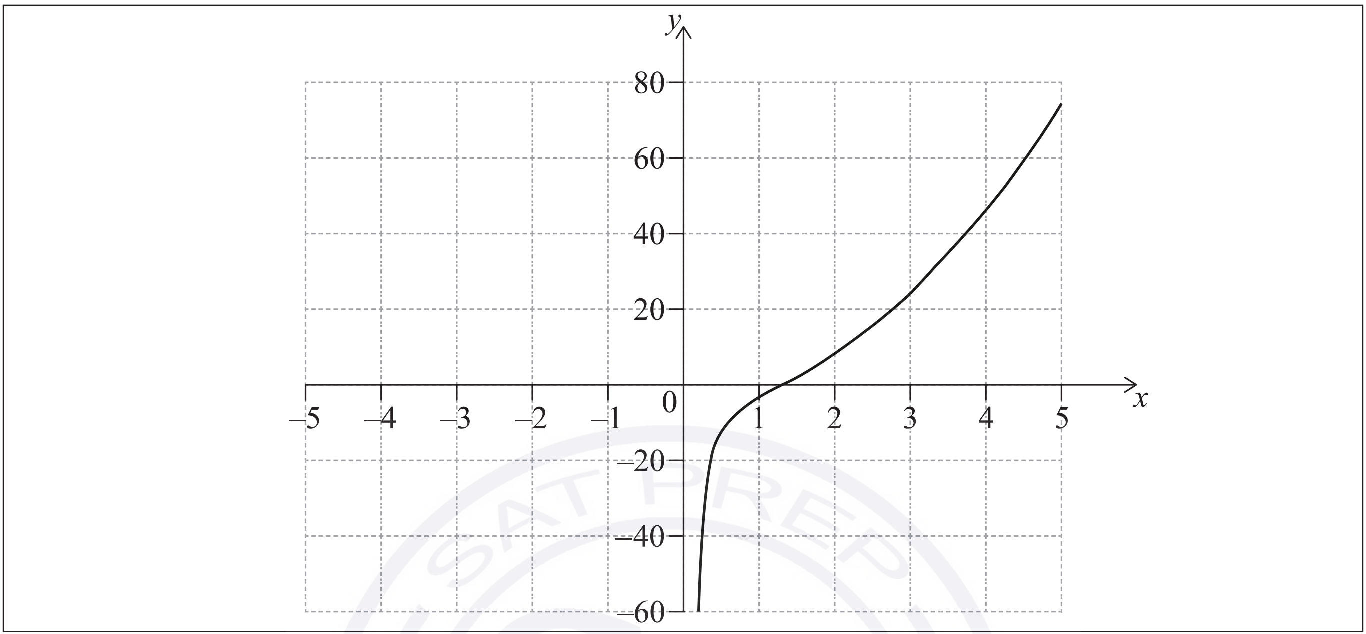
Another player catches the basketball when it is at a height of 1.2 metres.

(c) Find the value of t when this player catches the basketball. [2]

(d) Write down two limitations of using $h(t)$ to model the height of the basketball. [2]

11. [Maximum mark: 7]

Consider $f(x) = 3x^2 - \frac{5}{x}$, $x \neq 0$. The graph of f for $0 < x \leq 5$ is shown on the following axes.



(a) (i) Sketch the graph of f, for $-5 \leq x < 0$, on the same axes.

(ii) Write down the x-coordinate of the local minimum point. [4]

(b) Use your graphic display calculator to find the solutions to the equation $f(x) = 20$. [2]

(c) Write down the equation of the vertical asymptote for the graph of f. [1]

Please **do not** write on this page.

Answers written on this page
will not be marked.

12. [Maximum mark: 5]

In a game, balls are thrown to hit a target. The random variable X is the number of times the target is hit in five attempts. The probability distribution for X is shown in the following table.

x	0	1	2	3	4	5
$\mathrm{P}(X=x)$	0.15	0.2	k	0.16	$2k$	0.25

(a) Find the value of k. [2]

The player has a chance to win money based on how many times they hit the target.

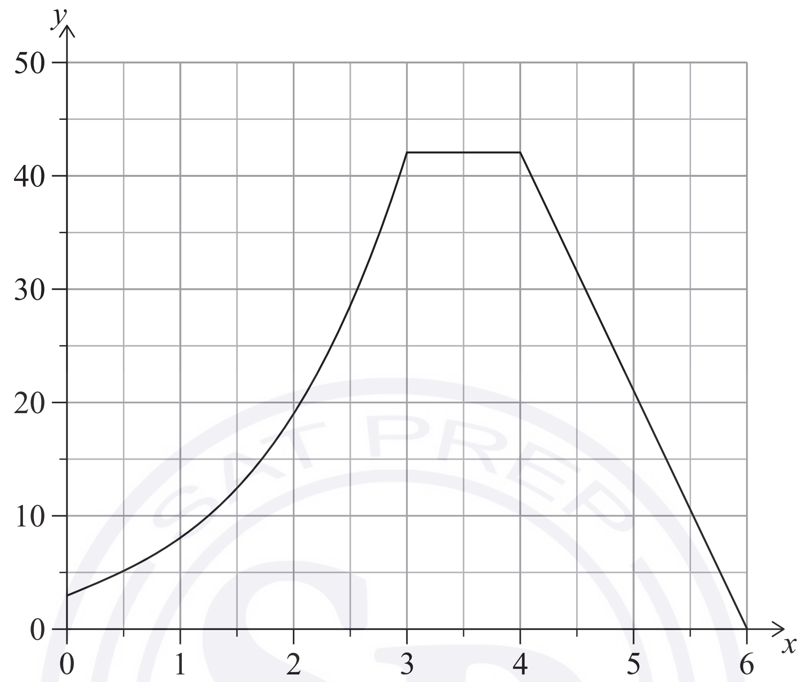
The gain for the player, in \$, is shown in the following table, where a negative gain means that the player loses money.

x	0	1	2	3	4	5
Player's gain (\$)	–4	–3	–1	0	1	4

(b) Determine whether this game is fair. Justify your answer. [3]

13. [Maximum mark: 9]

An engineer wants to calculate the cross-sectional area of a dam. The cross-section of the dam can be modelled by a curve and two straight lines as shown in the following diagram, where distances are measured in metres.



The curve is modelled by a function $f(x)$. The following table gives values of $f(x)$ for different values of x in the interval $0 \le x \le 3$.

x	0	0.5	1	1.5	2	2.5	3
$y = f(x)$	3	5.13	8	12.4	19	28.6	42

(a) Calculate an estimate for the area in the interval $0 \le x \le 3$ by using the trapezoidal rule with three equal intervals. [2]

It is known that $f'(x) = 3x^2 + 4$ in the domain $0 < x < 3$.

(b) Find an expression for $f(x)$, in the domain $0 < x < 3$. [4]

(c) **Hence** find the actual area of the **entire** cross-section. [3]

(This question continues on the following page)

(Question 13 continued)

References:

Please **do not** write on this page.

Answers written on this page
will not be marked.

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: applications and interpretation
Standard level
Paper 1

8 May 2023

Zone A afternoon | **Zone B** morning | **Zone C** afternoon

Candidate session number

1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all questions.
- Answers must be written within the answer boxes provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

19 pages

2223 – 7209

Answers must be written within the answer boxes provided. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 6]

The decathlon is a competition where athletes compete in ten events. Two of those events are long jump and high jump. In both events, a greater distance means a better ranking.

The table shows results for these two events at the World Championships.

	Event		**Rank**	
Athlete's Country	**Long Jump (m)**	**High Jump (m)**	**Long Jump Rank**	**High Jump Rank**
Germany	7.64	2.11	1	
France	7.52	2.08	2	
Estonia	7.49	1.84	3	
Canada	7.44	2.02	4	
Netherlands	7.33	2.05	5	
Ukraine	7.28	2.02	6	
Algeria	7.22	1.90	7	
Austria	7.11	1.87	8	
Grenada	6.98	1.99	9	
Japan	6.64	1.96	10	

The Spearman's rank correlation coefficient is used to determine if there is a linear correlation between an athlete's ranking in long jump and their ranking in high jump.

(a) Complete the table to show the athletes' rankings in high jump. [2]

(b) Find the value of the Spearman's rank correlation coefficient r_s. [2]

(This question continues on the following page)

(Question 1 continued)

The following guide is used by the coach to determine the strength of the correlation between the ranks for long jump and high jump.

$\lvert r_s \rvert$	**Strength**
0.000 to 0.199	Very weak
0.200 to 0.399	Weak
0.400 to 0.599	Moderate
0.600 to 0.799	Strong
0.800 to 1.000	Very strong

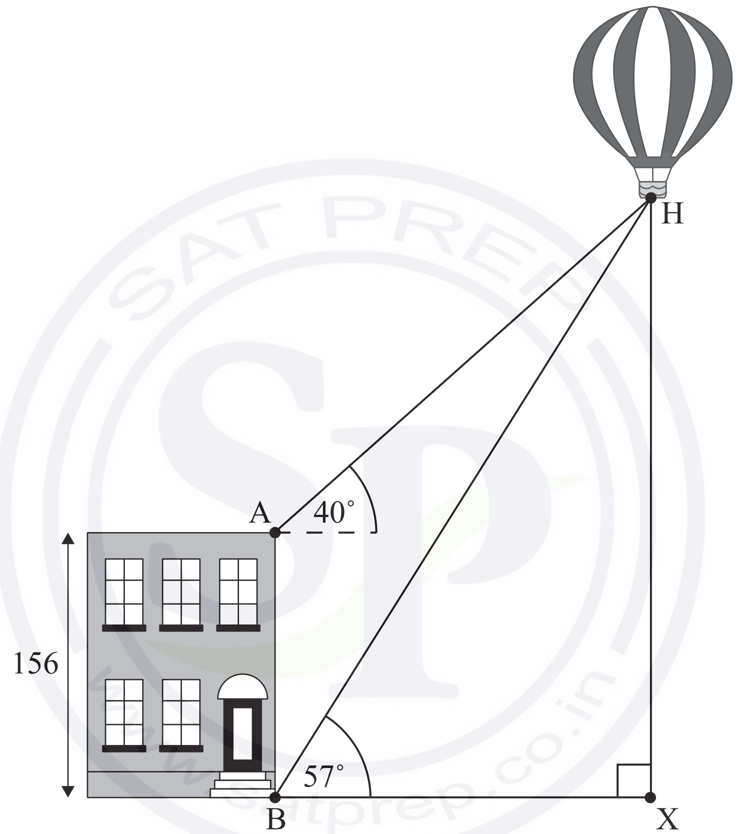
(c) State the strength of the correlation between the rankings as indicated by the table and interpret this in the context of the question. [2]

2. [Maximum mark: 6]

Point H on a hot-air balloon is sighted at the same time by two observers. One observer is at the top of a vertical building that is 156 metres tall. The other observer is at the base of the building.

The angle of elevation from point A (at the top of the building) to H is $40°$, and the angle of elevation from point B (at the base of the building) to H is $57°$. Point X is the ground directly below point H. This information is shown in the diagram.

diagram not to scale



(a) Find the size of angle $\mathrm{A\hat{H}B}$. [2]

(b) Calculate the distance from point B to point H. [3]

The hot-air balloon remains at a constant height as it moves further away from the building.

(c) Describe, in words, the change in the angle of depression from point H to point B as the horizontal distance between the balloon and the building increases. [1]

(This question continues on the following page)

(Question 2 continued)

3. [Maximum mark: 5]

On 1 January 2022, Mina deposited $\$1000$ into a bank account with an annual interest rate of 4%, compounded monthly. At the end of January, and the end of every month after that, she deposits $\$100$ into the same account.

(a) Calculate the amount of money in her account at the start of 2024. Give your answer to two decimal places. [3]

(b) Find how many complete months, counted from 1 January 2022, it will take for Mina to have more than $\$5000$ in her account. [2]

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4. [Maximum mark: 6]

Carys believes that, on a memory retention test, the mean score of bilingual people (μ_b) will be higher than the mean score of monolingual people (μ_m). Carys gave a memory retention test to a random sample of students in her class. The results are shown in the two tables.

	Scores									
Bilingual	100	94	100	90	100	94	98	98	98	98

	Scores							
Monolingual	97	92	88	98	88	94	100	100

Carys performs a one-tailed t-test at a 5% level of significance. It is assumed that the scores are normally distributed and the samples have equal variances.

(a) State the null and alternative hypotheses. [2]

(b) Calculate the p-value for this test. [2]

(c) State the conclusion of the test in the context of the question. Justify your answer. [2]

5. [Maximum mark: 5]

Line L_1 is tangent to the graph of a function $f(x)$ at the point $\mathrm{P}(3, -1)$. Line L_2 is given by the equation $y = -\frac{1}{2}x - \frac{5}{2}$ and is perpendicular to L_1.

(a) Write down the gradient of L_1. [1]

(b) Find the equation of L_1 in the form $y = mx + c$. [2]

(c) Show that L_2 is not the line that is normal to $f(x)$ at point P. [2]

6. [Maximum mark: 5]

When the brakes of a car are fully applied the car will continue to travel some distance before it completely stops. This stopping distance, d, in metres is directly proportional to the square of the speed of the car, v, in kilometres per hour (km h^{-1}).

When a car is travelling at a speed of $50\,\text{km h}^{-1}$ it will travel $12.3\,\text{m}$ after the brakes are fully applied before it completely stops.

(a) Determine an equation for d in terms of v. [2]

The police can use this equation to estimate if cars are exceeding the speed limit.

A car is found to have travelled $33\,\text{m}$, after fully applying its brakes, before it completely stopped.

(b) Use your equation from part (a) to estimate the speed at which this car was travelling before the brakes were applied. [2]

(c) After the brakes have been fully applied, identify one other variable besides speed that could affect stopping distance. [1]

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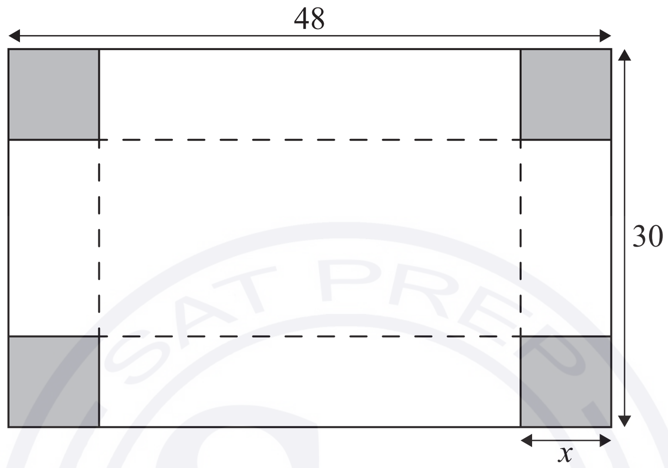
7. [Maximum mark: 6]

A rectangular box, with an open top, is to be constructed from a piece of cardboard that measures $48\,\text{cm}$ by $30\,\text{cm}$.

Squares of equal size will be cut from the corners of the cardboard, as indicated by the shading in the diagram. The sides will then be folded along the dotted lines to form the box.

diagram not to scale



The volume of the box, in cubic centimetres, can be modelled by the function $V(x) = (48 - 2x)(30 - 2x)(x)$, for $0 < x < k$, where x is the length of the sides of the squares removed in centimetres.

(a) Write down the maximum possible value of k in this context. [1]

(b) Find the value of x that maximizes the volume of the box. [2]

A second piece of $48\,\text{cm}$ by $30\,\text{cm}$ cardboard is damaged and a strip $2\,\text{cm}$ wide must be removed from all four sides. A box will then be constructed in a similar manner from the remaining cardboard.

(c) Calculate the maximum possible volume of the box made from the second piece of cardboard. [3]

(This question continues on the following page)

(Question 7 continued)

8. [Maximum mark: 7]

"Password entropy" is a measure of the predictability of a computer password. The higher the entropy, the more difficult it is to guess the password.

The relationship between the password entropy, p, (measured in bits) and the number of guesses, G, required to decode the password is given by $0.301p = \log_{10} G$.

(a) Calculate the value of p for a password that takes 5000 guesses to decode. [2]

(b) Write down G as a function of p. [1]

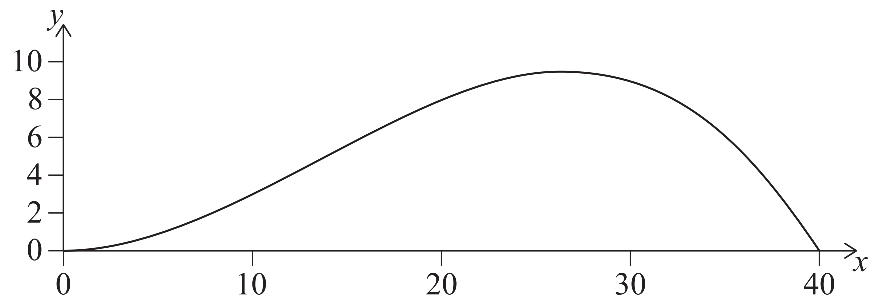
(c) Find the number of guesses required to decode a password that has an entropy of 28 bits. Write your answer in the form $a \times 10^k$, where $1 \le a < 10$, $k \in \mathbb{Z}$. [3]

There is a point on the graph of the function $G(p)$ with coordinates $(0, 1)$.

(d) Explain what these coordinate values mean in the context of computer passwords. [1]

9. [Maximum mark: 8]

The cross section of a scale model of a hill is modelled by the following graph.



The heights of the model are measured at horizontal intervals and are given in the table.

Horizontal distance, x cm	0	10	20	30	40
Vertical distance, y cm	0	3	8	9	0

(a) Use the trapezoidal rule with $h = 10$ to find an approximation for the cross-sectional area of the model. [2]

It is given that the equation of the curve is $y = 0.04x^2 - 0.001x^3,\ 0 \le x \le 40$.

(b) (i) Write down an integral to find the exact cross-sectional area.

(ii) Calculate the value of the cross-sectional area to two decimal places. [4]

(c) Find the percentage error in the area found using the trapezoidal rule. [2]

10. [Maximum mark: 7]

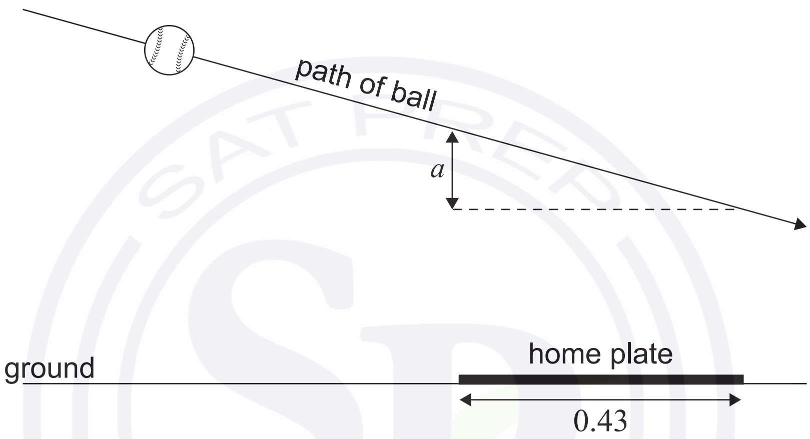
In a baseball game, Sakura is the batter standing beside home plate. The ball is thrown towards home plate along a path that can be modelled by the following function

$$y = -0.045x + 2.$$

In this model, x is the horizontal distance of the ball from the point the ball is thrown and y is the vertical height of the ball above the ground. Both measured in metres.

The outcome of the throw is called a strike if the height of the ball is between 0.53 m and 1.24 m at some point while it travels over home plate. The length of home plate is 0.43 m.

diagram not to scale



When the ball reaches the front of home plate, the height of the ball above the ground is 1.25 m. The height of the ball changes by a metres as the ball travels over the length of home plate.

(a) (i) Find the value of a.

(ii) Justify why this throw is a strike. [4]

On the next throw, Sakura hits the ball towards a wall that is 5 metres high. The horizontal distance of the wall from the point where the ball was hit is 96 metres. The path of the ball after it is hit can be modelled by the function $h(d)$.

$$h(d) = -0.01d^2 + 1.04d + 0.66, \text{ for } h,\ d > 0$$

In this model, h is the height of the ball above the ground and d is the horizontal distance of the ball from the point where it was hit. Both h and d are measured in metres.

(b) Determine whether the ball will go over the wall. Justify your answer. [3]

(This question continues on the following page)

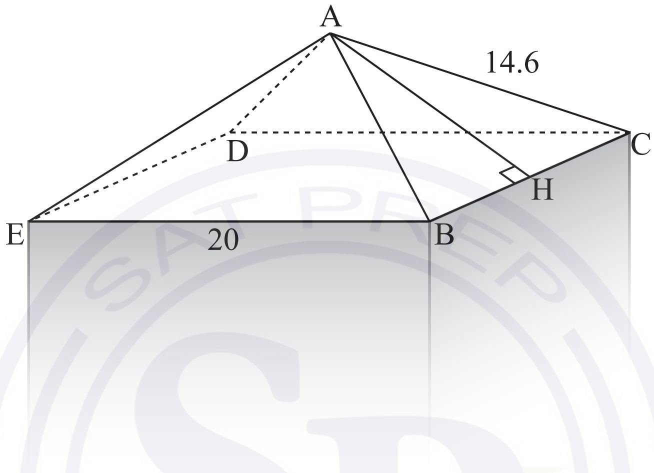
(Question 10 continued)

11. [Maximum mark: 7]

Vertical posts are to be placed around the outer edge of a children's park. Each post is formed from a cuboid with a right square-based pyramid on top.

The cuboid part of the post is machine-made such that its width, and hence the width of the pyramid, is exactly 20 cm. The length from the apex of the pyramid, A, to any corner of the base of the pyramid is 14.6 cm, **but** this is only accurate to the nearest tenth of a centimetre. The post is shown in the diagram.

diagram not to scale



(a) Write down the upper bound and lower bound for the possible lengths of edge AC. [2]

Point H is the midpoint of BC.

(b) Determine the upper bound and lower bound for AH, the slant height of the pyramid. [3]

For the post to be safe for children, the angle between the slant height and the base of the pyramid must be less than $22°$.

(c) Show that this post is safe for children. Justify your answer. [2]

(This question continues on the following page)

(Question 11 continued)

12. [Maximum mark: 7]

On a specific day, the speed of cars as they pass a speed camera can be modelled by a normal distribution with a mean of $67.3\,\text{km h}^{-1}$.

A speed of $75.7\,\text{km h}^{-1}$ is two standard deviations from the mean.

(a) Find the standard deviation for the speed of the cars. [2]

Speeding tickets are issued to all drivers travelling at a speed greater than $72\,\text{km h}^{-1}$.

(b) Find the probability that a randomly selected driver who passes the speed camera receives a speeding ticket. [2]

It is found that 82% of cars on this road travel at speeds between $p\,\text{km h}^{-1}$ and $q\,\text{km h}^{-1}$, where $p < q$. This interval includes cars travelling at a speed of $74\,\text{km h}^{-1}$.

(c) Show that the region of the normal distribution between p and q is **not** symmetrical about the mean. [3]

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13. [Maximum mark: 5]

A boat travels $8\,\text{km}$ on a bearing of $315°$ and then a further $6\,\text{km}$ on a bearing of $045°$. Find the bearing on which the boat should travel to return directly to the starting point.

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References:

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will not be marked.

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: applications and interpretation
Standard level
Paper 1

Monday 31 October 2022 (afternoon)

Candidate session number

1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all questions.
- Answers must be written within the answer boxes provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

8822 – 7204

17 pages

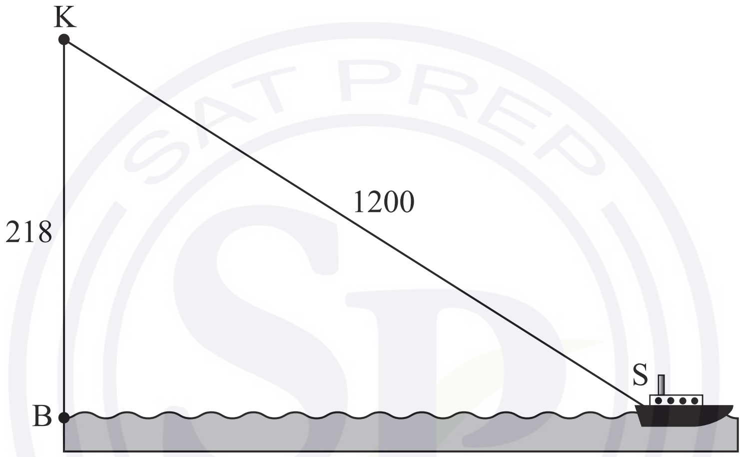
Answers must be written within the answer boxes provided. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 6]

Kacheena stands at point K, the top of a $218\,\text{m}$ vertical cliff. The base of the cliff is located at point B. A ship is located at point S, $1200\,\text{m}$ from Kacheena.

This information is shown in the following diagram.

diagram not to scale



(a) Find the angle of elevation from the ship to Kacheena. [2]

(b) Find the horizontal distance from the base of the cliff to the ship. [2]

(c) Write down your answer to part (b) in the form $a \times 10^k$ where $1 \le a < 10$ and $k \in \mathbb{Z}$. [2]

(This question continues on the following page)

(Question 1 continued)

2. [Maximum mark: 7]

In the first month of a reforestation program, the town of Neerim plants 85 trees. Each subsequent month the number of trees planted will increase by an additional 30 trees.

The number of trees to be planted in each of the first three months are shown in the following table.

Month	**Trees planted**
1	85
2	115
3	145

(a) Find the number of trees to be planted in the 15th month. [3]

(b) Find the total number of trees to be planted in the first 15 months. [2]

(c) Find the mean number of trees planted per month during the first 15 months. [2]

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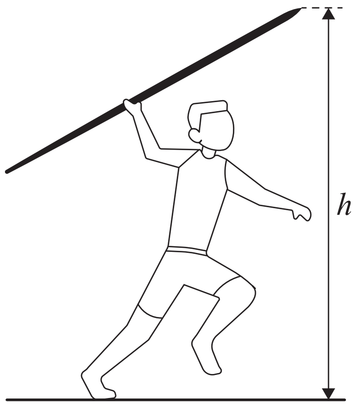
3. [Maximum mark: 5]

DeVaughn throws a javelin in a school track and field competition.



The height, h, of the front tip of the javelin above the ground, in metres, is modelled by the following quadratic function,

$$h(t) = -3.6t^2 + 10.8t + 1.8, \; t \geq 0$$

where t is the time in seconds after the javelin is thrown.

(a) Write down the height of the front tip of the javelin at the time it is thrown. [1]

(b) Find the value of t when the front tip of the javelin reaches its maximum height. [2]

(c) Find the value of t when the front tip of the javelin strikes the ground. [2]

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4. [Maximum mark: 5]

Sergio is interested in whether an adult's favourite breakfast berry depends on their income level. He obtains the following data for 341 adults and decides to carry out a χ^2 test for independence, at the 10% significance level.

		Income level		
		Low	**Medium**	**High**
Favourite berry	**Strawberry**	21	39	30
	Blueberry	39	67	42
	Other berry	32	45	26

(a) Write down the null hypothesis. [1]

(b) Find the value of the χ^2 statistic. [2]

The critical value of this χ^2 test is 7.78.

(c) Write down Sergio's conclusion to the test in context. Justify your answer. [2]

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5. [Maximum mark: 6]

Celeste heated a cup of coffee and then let it cool to room temperature. Celeste found the coffee's temperature, T, measured in $^\circ\text{C}$, could be modelled by the following function,

$$T(t) = 71\,\mathrm{e}^{-0.0514t} + 23\,,\ t \geq 0\,,$$

where t is the time, in minutes, after the coffee started to cool.

(a) Find the coffee's temperature 16 minutes after it started to cool. [2]

The graph of T has a horizontal asymptote.

(b) Write down the equation of the horizontal asymptote. [1]

(c) Write down the room temperature. [1]

(d) Given that $T^{-1}(50) = k$, find the value of k. [2]

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6. [Maximum mark: 5]

Manny and Annabelle, mathematics teachers at Burnham High School, give their students the same examination. A random sample of the examination scores were collected from each of their classes.

Examination scores from Manny's class	76	77	82	84	88	90	91	98
Examination scores from Annabelle's class	68	79	81	89	91	92	92	95

Annabelle uses these scores to conduct a two-tailed t-test to compare the means of the two classes, at the 5% level of significance. It is assumed the examination scores for both classes have the same variance and are normally distributed.

The null hypothesis is $\mu_1 = \mu_2$, where μ_1 is the mean examination score from Manny's class and μ_2 is the mean examination score from Annabelle's class.

(a) Write down the alternative hypothesis. [1]

(b) Find the p-value for this test. Give your answer correct to five decimal places. [2]

Annabelle concludes there is insufficient evidence to reject the null hypothesis.

(c) State whether Annabelle's conclusion is correct. Give a reason for your answer. [2]

7. [Maximum mark: 7]

On 1 December 2022, Laviola invests 800 euros (EUR) into a savings account which pays a nominal annual interest rate of 7.5% compounded monthly. At the end of each month, Laviola deposits an additional $\text{EUR } 500$ into the savings account.

At the end of k months, Laviola will have saved enough money to withdraw $\text{EUR } 10\,000$.

(a) Find the smallest possible value of k, for $k \in \mathbb{Z}^+$. [4]

(b) For this value of k, find the interest earned in the savings account.
Express your answer correct to the nearest EUR. [3]

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8. [Maximum mark: 5]

Roy is a member of a motorsport club and regularly drives around the Port Campbell racetrack.

The times he takes to complete a lap are normally distributed with mean 59 seconds and standard deviation 3 seconds.

(a) Find the probability that Roy completes a lap in less than 55 seconds. [2]

Roy will complete a 20 lap race. It is expected that 8.6 of the laps will take more than t seconds.

(b) Find the value of t. [3]

9. [Maximum mark: 7]

Taizo plays a game where he throws one ball at two bottles that are sitting on a table. The probability of knocking over bottles, in any given game, is shown in the following table.

Number of bottles knocked over	0	1	2
Probability	0.5	0.4	0.1

(a) Taizo plays two games that are independent of each other. Find the probability that Taizo knocks over a **total** of two bottles. [4]

In any given game, Taizo will win k points if he knocks over two bottles, win 4 points if he knocks over one bottle and lose 8 points if no bottles are knocked over.

(b) Find the value of k such that the game is fair. [3]

10. [Maximum mark: 6]

Stars are classified by their brightness. The brightest stars in the sky have a magnitude of 1. The magnitude, m, of another star can be modelled as a function of its brightness, b, relative to a star of magnitude 1, as shown by the following equation.

$$m = 1 - 2.5 \log_{10}(b)$$

The star called Acubens has a brightness of 0.0525.

(a) Find the magnitude of Acubens. [2]

Ceres has a magnitude of 7 and is the least bright star visible without magnification.

(b) Find the brightness of Ceres. [2]

(c) Find how many times brighter Acubens is compared to Ceres. [2]

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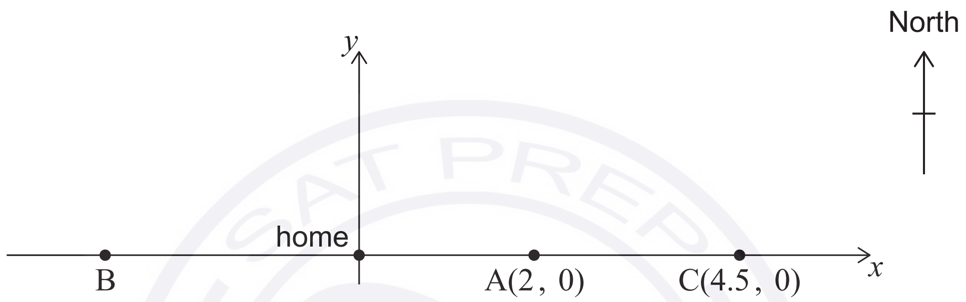
11. [Maximum mark: 7]

Kristi's house is located on a long straight road which traverses east–west. The road can be modelled by the equation $y = 0$, and her home is located at the origin $(0, 0)$.

She is training for a marathon by running from her home to a point on the road and then returning to her home by bus.

- The first day Kristi runs 2 kilometres east to point $A(2, 0)$.
- The second day Kristi runs west to point B.
- The third day Kristi runs 4.5 kilometres east to point $C(4.5, 0)$.

This information is represented in the following diagram.



Each day Kristi increases the distance she runs. The point she reaches each day can be represented by an x-coordinate. These x-coordinates form a geometric sequence.

(a) Show that the common ratio, r, is -1.5. [2]

On the 6th day, Kristi runs to point F.

(b) Find the location of point F. [2]

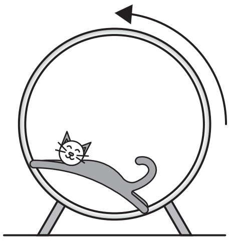
(c) Find the total distance Kristi runs during the first 7 days of training. [3]

(This question continues on the following page)

(Question 11 continued)

12. [Maximum mark: 6]

A cat runs inside a circular exercise wheel, making the wheel spin at a constant rate in an anticlockwise direction. The height, h cm, of a fixed point, P, on the wheel can be modelled by $h(t) = a\sin(bt) + c$ where t is the time in seconds and a, b, $c \in \mathbb{R}^+$.



When $t = 0$, point P is at a height of 78 cm.

(a) Write down the value of c. [1]

When $t = 4$, point P first reaches its maximum height of 143 cm.

(b) Find the value of

(i) a.

(ii) b. [3]

(c) Write down the minimum height of point P. [1]

Later, the cat is tired, and it takes twice as long for point P to complete one revolution at a new constant rate.

(d) Write down the new value of b. [1]

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13. [Maximum mark: 8]

Giles charges a customer per hour to hire his boat. It is known that

$$\frac{dP}{dt} = 20 - \frac{980}{t^2},\ 0 < t \le 12$$

where P is the cost per hour, in Norwegian krone (NOK), that the customer is charged and t is the time, in hours, spent on the boat.

The cost per hour has a local minimum when the boat is hired for h hours.

(a) Find the value of h. [2]

Sandra hired Giles' boat for 5 hours and was charged NOK 328 per hour. Yvonne hires Giles' boat for 7 hours.

(b) Show that the cost per hour for Yvonne is NOK 312. [6]

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Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: applications and interpretation
Standard level
Paper 1

Friday 6 May 2022 (afternoon)

1 hour 30 minutes

Candidate session number

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all questions.
- Answers must be written within the answer boxes provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

2222–7214

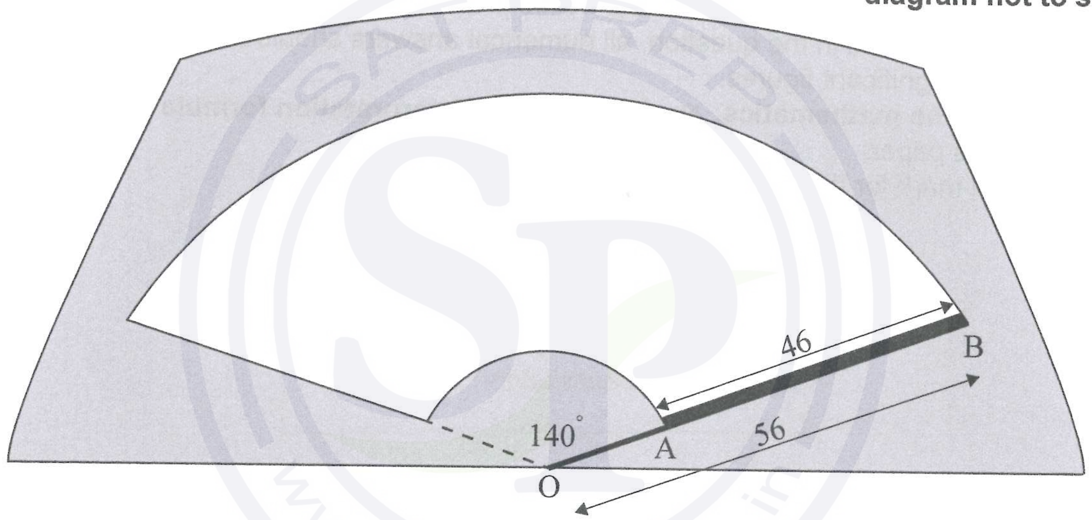
Answers must be written within the answer boxes provided. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 5]

The straight metal arm of a windscreen wiper on a car rotates in a circular motion from a pivot point, O, through an angle of $140°$. The windscreen is cleared by a rubber blade of length $46\,\text{cm}$ that is attached to the metal arm between points A and B. The total length of the metal arm, OB, is $56\,\text{cm}$.

The part of the windscreen cleared by the rubber blade is shown unshaded in the following diagram.

diagram not to scale



(a) Calculate the length of the arc made by B, the end of the rubber blade. [2]

(b) Determine the area of the windscreen that is cleared by the rubber blade. [3]

(This question continues on the following page)

(Question 1 continued)

2. [Maximum mark: 6]

A group of 130 applicants applied for admission into either the Arts programme or the Sciences programme at a university. The outcomes of their applications are shown in the following table.

	Accepted	**Rejected**
Arts programme	17	24
Sciences programme	25	64

(a) Find the probability that a randomly chosen applicant from this group was accepted by the university. [1]

An applicant is chosen at random from this group. It is found that they were accepted into the programme of their choice.

(b) Find the probability that the applicant applied for the Arts programme. [2]

Two different applicants are chosen at random from the original group.

(c) Find the probability that both applicants applied to the Arts programme. [3]

(This question continues on the following page)

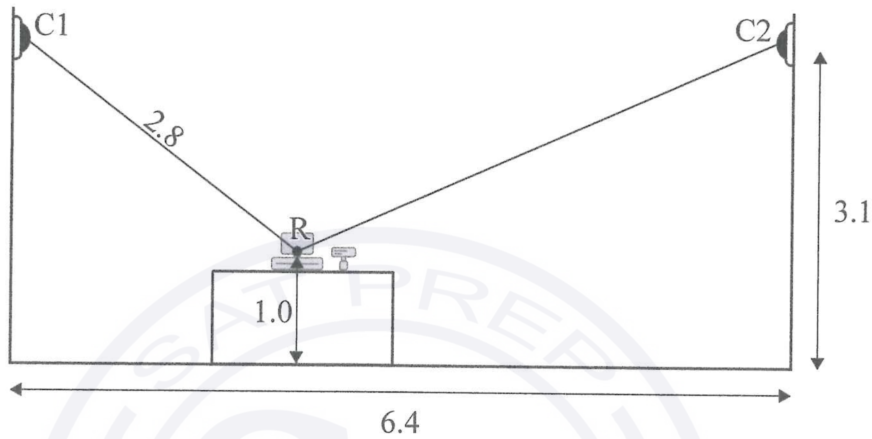
(Question 2 continued)

3. [Maximum mark: 8]

The owner of a convenience store installs two security cameras, represented by points C1 and C2. Both cameras point towards the centre of the store's cash register, represented by the point R.

The following diagram shows this information on a cross-section of the store.

diagram not to scale



The cameras are positioned at a height of $3.1\,\text{m}$, and the horizontal distance between the cameras is $6.4\,\text{m}$. The cash register is sitting on a counter so that its centre, R, is $1.0\,\text{m}$ above the floor.

The distance from Camera 1 to the centre of the cash register is $2.8\,\text{m}$.

(a) Determine the angle of depression from Camera 1 to the centre of the cash register. Give your answer in degrees. [2]

(b) Calculate the distance from Camera 2 to the centre of the cash register. [4]

(c) Without further calculation, determine which camera has the largest angle of depression to the centre of the cash register. Justify your response. [2]

(This question continues on the following page)

(Question 3 continued)

4. [Maximum mark: 5]

The pH of a solution measures its acidity and can be determined using the formula pH $= -\log_{10}C$, where C is the concentration of hydronium ions in the solution, measured in moles per litre. A lower pH indicates a more acidic solution.

The concentration of hydronium ions in a particular type of coffee is 1.3×10^{-5} moles per litre.

(a) Calculate the pH of the coffee. [2]

A different, unknown, liquid has 10 times the concentration of hydronium ions of the coffee in part (a).

(b) Determine whether the unknown liquid is more or less acidic than the coffee. Justify your answer mathematically. [3]

5. [Maximum mark: 7]

A polygraph test is used to determine whether people are telling the truth or not, but it is not completely accurate. When a person tells the truth, they have a 20% chance of failing the test. Each test outcome is independent of any previous test outcome.

10 people take a polygraph test and all 10 tell the truth.

(a) Calculate the expected number of people who will pass this polygraph test. [2]

(b) Calculate the probability that exactly 4 people will fail this polygraph test. [2]

(c) Determine the probability that fewer than 7 people will pass this polygraph test. [3]

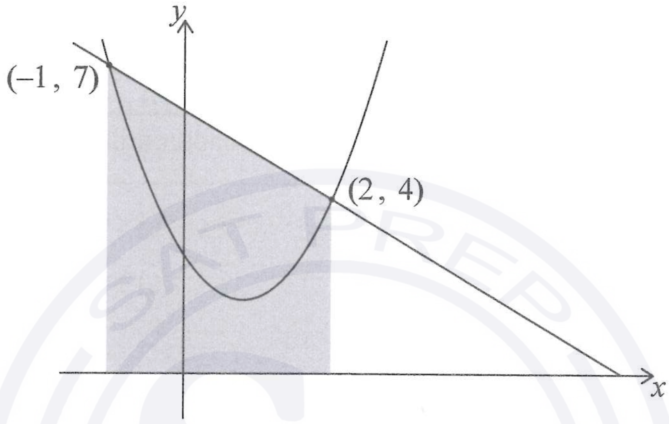
6. [Maximum mark: 7]

The graphs of $y = 6 - x$ and $y = 1.5x^2 - 2.5x + 3$ intersect at $(2, 4)$ and $(-1, 7)$, as shown in the following diagrams.

In **diagram 1**, the region enclosed by the lines $y = 6 - x$, $x = -1$, $x = 2$ and the x-axis has been shaded.

diagram not to scale

Diagram 1

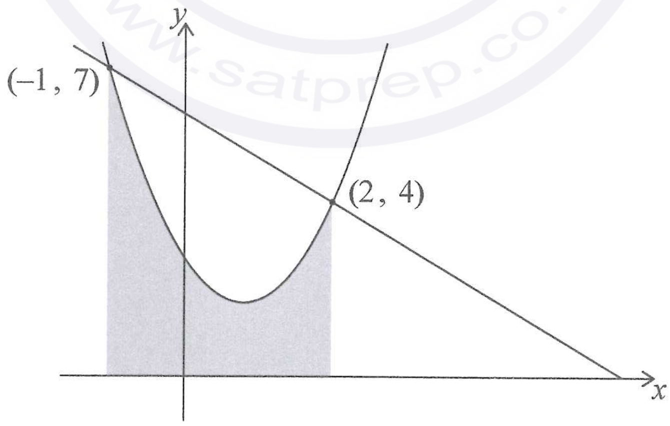


(a) Calculate the area of the shaded region in **diagram 1**. [2]

In **diagram 2**, the region enclosed by the curve $y = 1.5x^2 - 2.5x + 3$, and the lines $x = -1$, $x = 2$ and the x-axis has been shaded.

diagram not to scale

Diagram 2



(b) (i) Write down an integral for the area of the shaded region in **diagram 2**.

(ii) Calculate the area of this region. [3]

(c) Hence, determine the area enclosed between $y = 6 - x$ and $y = 1.5x^2 - 2.5x + 3$. [2]

(This question continues on the following page)

(Question 6 continued)

7. [Maximum mark: 5]

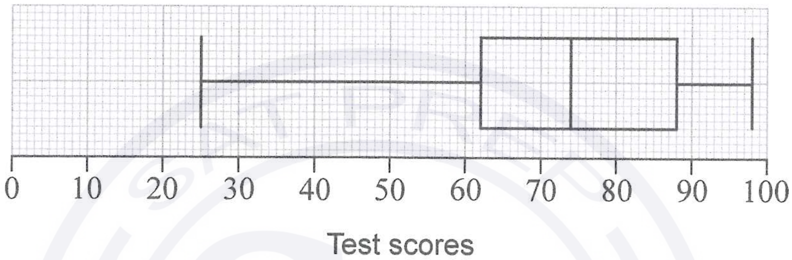
A college runs a mathematics course in the morning. Scores for a test from this class are shown below.

$$25 \quad 33 \quad 51 \quad 62 \quad 63 \quad 63 \quad 70 \quad 74 \quad 79 \quad 79 \quad 81 \quad 88 \quad 90 \quad 90 \quad 98$$

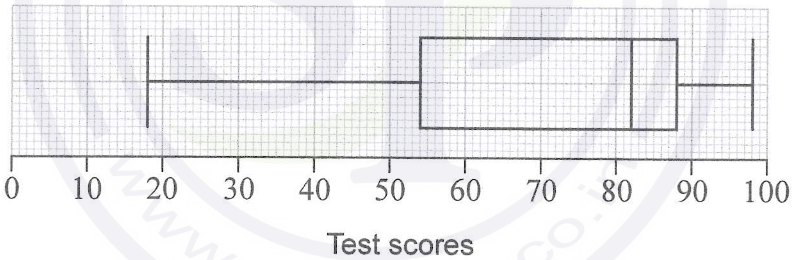
For these data, the lower quartile is 62 and the upper quartile is 88.

(a) Show that the test score of 25 would not be considered an outlier. [3]

The box and whisker diagram showing these scores is given below.



Another mathematics class is run by the college during the evening. A box and whisker diagram showing the scores from this class for the same test is given below.



A researcher reviews the box and whisker diagrams and believes that the evening class performed better than the morning class.

(b) With reference to the box and whisker diagrams, state one aspect that may support the researcher's opinion and one aspect that may counter it. [2]

(This question continues on the following page)

(Question 7 continued)

8. [Maximum mark: 6]

A study was conducted to investigate whether the mean reaction time of drivers who are talking on mobile phones is the same as the mean reaction time of drivers who are talking to passengers in the vehicle. Two independent groups were randomly selected for the study.

To gather data, each driver was put in a car simulator and asked to either talk on a mobile phone or talk to a passenger. Each driver was instructed to apply the brakes as soon as they saw a red light appear in front of the car. The reaction times of the drivers, in seconds, were recorded, as shown in the following table.

Talking on mobile phone	Talking to passenger
0.69	0.67
0.87	0.86
0.98	0.60
1.04	0.81
0.79	0.76
0.87	0.71
0.71	0.74

At the 10% level of significance, a t-test was used to compare the mean reaction times of the two groups. Each data set is assumed to be normally distributed, and the population variances are assumed to be the same.

Let μ_1 and μ_2 be the population means for the two groups. The null hypothesis for this test is $H_0: \mu_1 - \mu_2 = 0$.

(a) State the alternative hypothesis. [1]

(b) Calculate the p-value for this test. [2]

(c) (i) State the conclusion of the test. Justify your answer.

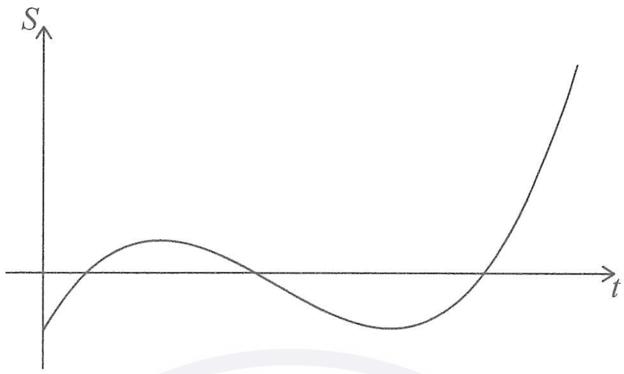
(ii) State what your conclusion means in context. [3]

(This question continues on the following page)

(Question 8 continued)

9. [Maximum mark: 8]

The graph below shows the average savings, S thousand dollars, of a group of university graduates as a function of t, the number of years after graduating from university.



(a) Write down one feature of this graph which suggests a cubic function might be appropriate to model this scenario. [1]

The equation of the model can be expressed in the form $S = at^3 + bt^2 + ct + d$, where a, b, c and d are real constants.

The graph of the model must pass through the following four points.

t	0	1	2	3
S	−5	3	−1	−5

(b) (i) Write down the value of d.

(ii) Write down three simultaneous equations for a, b and c.

(iii) Hence, or otherwise, find the values of a, b and c. [4]

A negative value of S indicates that a graduate is expected to be in debt.

(c) Use the model to determine the total length of time, in years, for which a graduate is expected to be in debt after graduating from university. [3]

(This question continues on the following page)

(Question 9 continued)

10. [Maximum mark: 5]

The masses of Fuji apples are normally distributed with a mean of $163\,\text{g}$ and a standard deviation of $6.83\,\text{g}$.

When Fuji apples are picked, they are classified as small, medium, large or extra large depending on their mass. Large apples have a mass of between $172\,\text{g}$ and $183\,\text{g}$.

(a) Determine the probability that a Fuji apple selected at random will be a large apple. [2]

Approximately $68\,\%$ of Fuji apples have a mass within the medium-sized category, which is between k and $172\,\text{g}$.

(b) Find the value of k. [3]

11. [Maximum mark: 7]

Consider the function $f(x) = x^2 - \frac{3}{x}$, $x \neq 0$.

(a) Find $f'(x)$. [2]

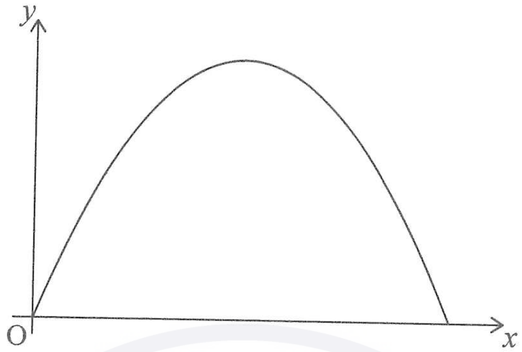
Line L is a tangent to $f(x)$ at the point $(1, -2)$.

(b) Use your answer to part (a) to find the gradient of L. [2]

(c) Determine the number of lines parallel to L that are tangent to $f(x)$. Justify your answer. [3]

12. [Maximum mark: 5]

The cross-section of an arched entrance into the ballroom of a hotel is in the shape of a parabola. This cross-section can be modelled by part of the graph $y = -1.6x^2 + 4.48x$, where y is the height of the archway, in metres, at a horizontal distance, x metres, from the point O, in the bottom corner of the archway.



(a) Determine an equation for the axis of symmetry of the parabola that models the archway. [2]

To prepare for an event, a square-based crate that is $1.6\,\text{m}$ wide and $2.0\,\text{m}$ high is to be moved through the archway into the ballroom. The crate must remain upright while it is being moved.

(b) Determine whether the crate will fit through the archway. Justify your answer. [3]

13. [Maximum mark: 6]

Juliana plans to invest money for 10 years in an account paying 3.5% interest, compounded annually. She expects the annual inflation rate to be 2% per year throughout the 10-year period.

Juliana would like her investment to be worth a real value of $\$4000$, compared to current values, at the end of the 10-year period. She is considering two options.

Option 1: Make a one-time investment at the start of the 10-year period.

Option 2: Invest $\$1000$ at the start of the 10-year period and then invest $\$x$ into the account at the end of each year (including the first and last years).

(a) For option 1, determine the minimum amount Juliana would need to invest. Give your answer to the nearest dollar. [3]

(b) For option 2, find the minimum value of x that Juliana would need to invest each year. Give your answer to the nearest dollar. [3]

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: applications and interpretation
Standard level
Paper 1

Friday 6 May 2022 (afternoon)

1 hour 30 minutes

Candidate session number

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all questions.
- Answers must be written within the answer boxes provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

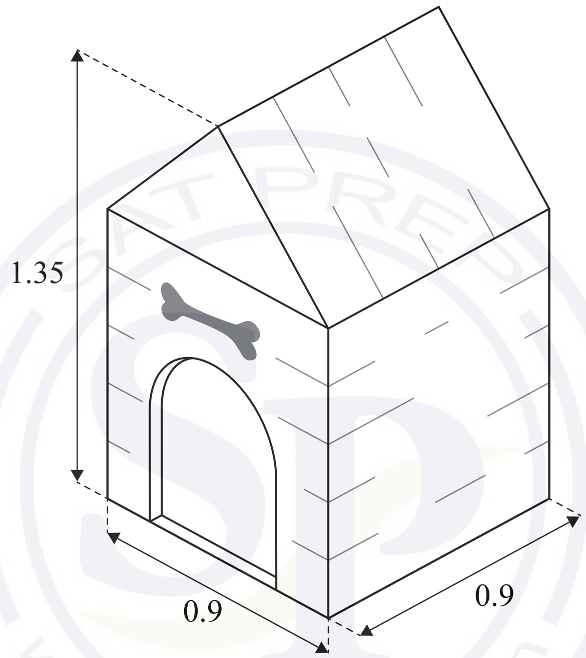
2222 – 7209

Answers must be written within the answer boxes provided. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 5]

The front view of a doghouse is made up of a square with an isosceles triangle on top. The doghouse is 1.35 m high and 0.9 m wide, and sits on a square base.

diagram not to scale



The top of the rectangular surfaces of the roof of the doghouse are to be painted.

Find the area to be painted.

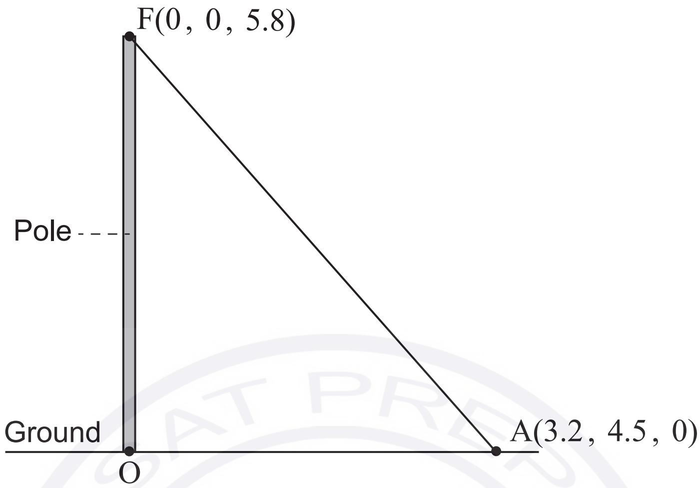
(This question continues on the following page)

(Question 1 continued)

2. [Maximum mark: 4]

A vertical pole stands on horizontal ground. The bottom of the pole is taken as the origin, O, of a coordinate system in which the top, F, of the pole has coordinates $(0, 0, 5.8)$. All units are in metres.

diagram not to scale



The pole is held in place by ropes attached at F.

One of the ropes is attached to the ground at a point A with coordinates $(3.2, 4.5, 0)$. The rope forms a straight line from A to F.

(a) Find the length of the rope connecting A to F. [2]

(b) Find $\text{F}\hat{\text{A}}\text{O}$, the angle the rope makes with the ground. [2]

3. [Maximum mark: 5]

The height of a baseball after it is hit by a bat is modelled by the function

$$h(t) = -4.8t^2 + 21t + 1.2$$

where $h(t)$ is the height in metres above the ground and t is the time in seconds after the ball was hit.

(a) Write down the height of the ball above the ground at the instant it is hit by the bat. [1]

(b) Find the value of t when the ball hits the ground. [2]

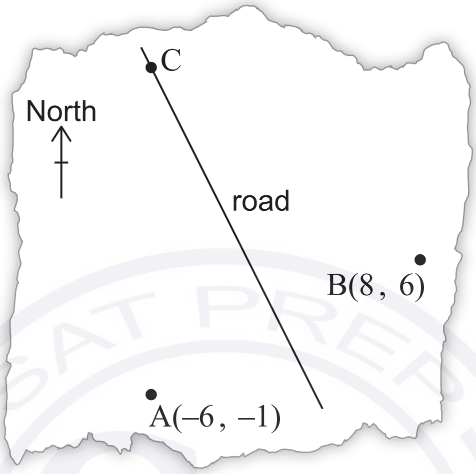
(c) State an appropriate domain for t in this model. [2]

4. [Maximum mark: 7]

Three towns, A, B and C are represented as coordinates on a map, where the x and y axes represent the distances east and north of an origin, respectively, measured in kilometres.

Town A is located at $(-6, -1)$ and town B is located at $(8, 6)$. A road runs along the perpendicular bisector of [AB]. This information is shown in the following diagram.

diagram not to scale



(a) Find the equation of the line that the road follows. [5]

Town C is due north of town A and the road passes through town C.

(b) Find the y-coordinate of town C. [2]

5. [Maximum mark: 5]

The ticket prices for a concert are shown in the following table.

Ticket Type	Price (in Australian dollars, \$)
Adult	15
Child	10
Student	12

- A total of 600 tickets were sold.
- The total amount of money from ticket sales was $\$7816$.
- There were twice as many adult tickets sold as child tickets.

Let the number of adult tickets sold be x, the number of child tickets sold be y, and the number of student tickets sold be z.

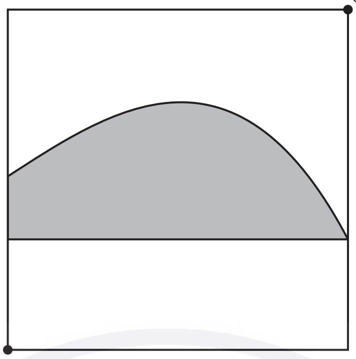
(a) Write down three equations that express the information given above. [3]

(b) Find the number of each type of ticket sold. [2]

6. [Maximum mark: 7]

A modern art painting is contained in a square frame. The painting has a shaded region bounded by a smooth curve and a horizontal line.

diagram not to scale



When the painting is placed on a coordinate axes such that the bottom left corner of the painting has coordinates $(-1, -1)$ and the top right corner has coordinates $(2, 2)$, the curve can be modelled by $y = f(x)$ and the horizontal line can be modelled by the x-axis. Distances are measured in metres.

(a) Use the trapezoidal rule, with the values given in the following table, to approximate the area of the shaded region. [3]

x	-1	0	1	2
y	0.6	1.2	1.2	0

The artist used the equation $y = \dfrac{-x^3 - 3x^2 + 4x + 12}{10}$ to draw the curve.

(b) Find the exact area of the shaded region in the painting. [2]

(c) Find the area of the unshaded region in the painting. [2]

(This question continues on the following page)

(Question 6 continued)

7. [Maximum mark: 7]

Leo is investigating whether a six-sided die is fair. He rolls the die 60 times and records the observed frequencies in the following table:

Number on die	1	2	3	4	5	6
Observed frequency	8	7	6	15	12	12

Leo carries out a χ^2 goodness of fit test at a 5% significance level.

(a) Write down the null and alternative hypotheses. [1]

(b) Write down the degrees of freedom. [1]

(c) Write down the expected frequency of rolling a 1. [1]

(d) Find the p-value for the test. [2]

(e) State the conclusion of the test. Give a reason for your answer. [2]

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8. [Maximum mark: 6]

A factory produces bags of sugar with a labelled weight of $500\,\text{g}$. The weights of the bags are normally distributed with a mean of $500\,\text{g}$ and a standard deviation of $3\,\text{g}$.

(a) Write down the percentage of bags that weigh more than $500\,\text{g}$. [1]

A bag that weighs less than $495\,\text{g}$ is rejected by the factory for being underweight.

(b) Find the probability that a randomly chosen bag is rejected for being underweight. [2]

A bag that weighs more than k grams is rejected by the factory for being overweight.
The factory rejects $2\,\%$ of bags for being overweight.

(c) Find the value of k. [3]

9. [Maximum mark: 7]

The function f is defined by $f(x)=\frac{2}{x}+3x^2-3$, $x\neq 0$.

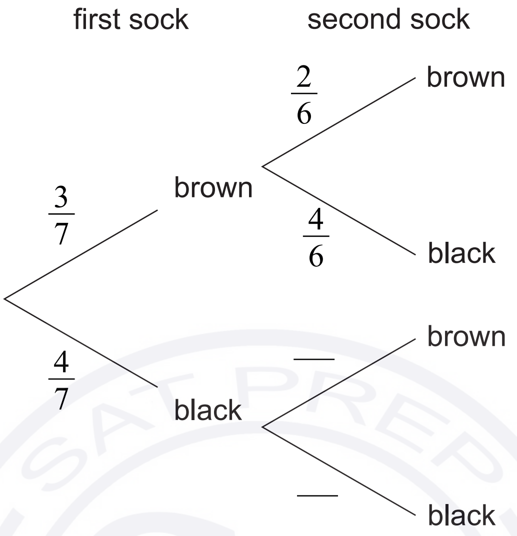
(a) Find $f'(x)$. [3]

(b) Find the equation of the normal to the curve $y=f(x)$ at $(1,\ 2)$ in the form $ax+by+d=0$, where $a,\ b,\ d\in\mathbb{Z}$. [4]

10. [Maximum mark: 6]

Karl has three brown socks and four black socks in his drawer. He takes two socks at random from the drawer.

(a) Complete the tree diagram. [1]



(b) Find the probability that Karl takes two socks of the same colour. [2]

(c) Given that Karl has two socks of the same colour find the probability that he has two brown socks. [3]

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11. [Maximum mark: 8]

The strength of earthquakes is measured on the Richter magnitude scale, with values typically between 0 and 8 where 8 is the most severe.

The Gutenberg–Richter equation gives the average number of earthquakes per year, N, which have a magnitude of at least M. For a particular region the equation is

$$\log_{10} N = a - M, \text{ for some } a \in \mathbb{R}.$$

This region has an average of 100 earthquakes per year with a magnitude of at least 3.

(a) Find the value of a. [2]

The equation for this region can also be written as $N = \frac{b}{10^M}$.

(b) Find the value of b. [2]

(c) Given $0 < M < 8$, find the range for N. [2]

The expected length of time, in years, between earthquakes with a magnitude of at least M is $\frac{1}{N}$.

Within this region the most severe earthquake recorded had a magnitude of 7.2.

(d) Find the expected length of time between this earthquake and the next earthquake of at least this magnitude. Give your answer to the nearest year. [2]

12. [Maximum mark: 6]

A company's profit per year was found to be changing at a rate of

$$\frac{dP}{dt} = 3t^2 - 8t$$

where P is the company's profit in thousands of dollars and t is the time since the company was founded, measured in years.

(a) Determine whether the profit is increasing or decreasing when $t = 2$. [2]

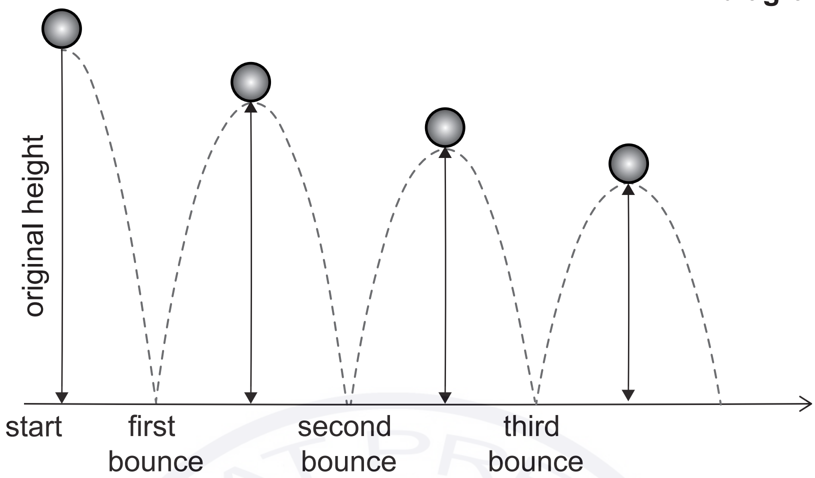
One year after the company was founded, the profit was 4 thousand dollars.

(b) Find an expression for $P(t)$, when $t \geq 0$. [4]

13. [Maximum mark: 7]

A ball is dropped from a height of 1.8 metres and bounces on the ground. The maximum height reached by the ball, after each bounce, is 85% of the previous maximum height.

diagram not to scale



(a) Show that the maximum height reached by the ball after it has bounced for the sixth time is $68\,\text{cm}$, to the nearest cm. [2]

(b) Find the number of times, after the first bounce, that the maximum height reached is greater than $10\,\text{cm}$. [2]

(c) Find the total **vertical** distance travelled by the ball from the point at which it is dropped until the fourth bounce. [3]

References:

Mathematics: applications and interpretation
Standard level
Paper 1

Monday 1 November 2021 (afternoon)

Candidate session number

1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all questions.
- Answers must be written within the answer boxes provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

8821 – 7204

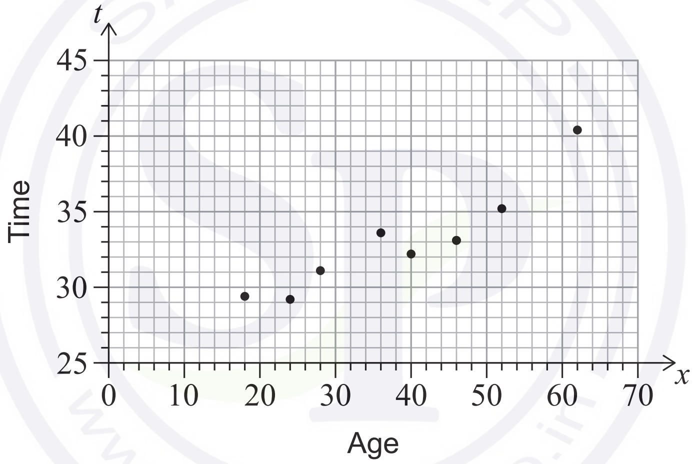
Answers must be written within the answer boxes provided. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 6]

Eduardo believes that there is a linear relationship between the age of a male runner and the time it takes them to run 5000 metres.

To test this, he recorded the age, x years, and the time, t minutes, for eight males in a single 5000 m race. His results are presented in the following table and scatter diagram.

***x*, years**	18	24	28	36	40	46	52	62
***t*, minutes**	29.4	29.2	31.1	33.6	32.2	33.1	35.2	40.4



(a) For this data, find the value of the Pearson's product-moment correlation coefficient, r. [2]

Eduardo looked in a sports science text book. He found that the following information about r was appropriate for athletic performance.

Value of $\lvert r \rvert$	Description of the correlation
$0 \le \lvert r \rvert < 0.4$	weak
$0.4 \le \lvert r \rvert < 0.8$	moderate
$0.8 \le \lvert r \rvert \le 1$	strong

(b) Comment on your answer to part (a), using the information that Eduardo found. [1]

(c) Write down the equation of the regression line of t on x, in the form $t = ax + b$. [1]

(This question continues on the following page)

(Question 1 continued)

A 57-year-old male also ran in the 5000 m race.

(d) Use the equation of the regression line to estimate the time he took to complete the 5000 m race. [2]

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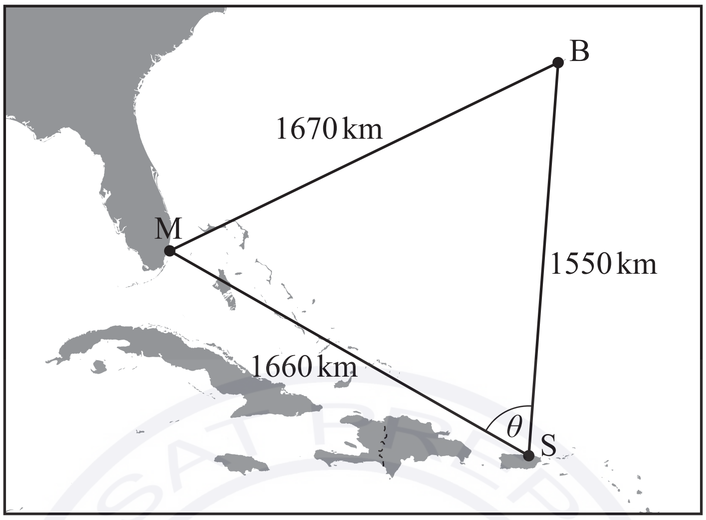
2. [Maximum mark: 5]

The Bermuda Triangle is a region of the Atlantic Ocean with Miami (M), Bermuda (B), and San Juan (S) as vertices, as shown on the diagram.

diagram not to scale



The distances between M, B and S are given in the following table, correct to three significant figures.

Distance between Miami and Bermuda	$1670\,km$
Distance between Bermuda and San Juan	$1550\,km$
Distance between San Juan and Miami	$1660\,km$

(a) Calculate the value of θ, the measure of angle $M\hat{S}B$. [3]

(b) Find the area of the Bermuda Triangle. [2]

(This question continues on the following page)

(Question 2 continued)

Please **do not** write on this page.

Answers written on this page will not be marked.

3. [Maximum mark: 4]

Natasha carries out an experiment on the growth of mould. She believes that the growth can be modelled by an exponential function

$$P(t) = Ae^{kt},$$

where P is the area covered by mould in mm^2, t is the time in days since the start of the experiment and A and k are constants.

The area covered by mould is $112\,\text{mm}^2$ at the start of the experiment and $360\,\text{mm}^2$ after 5 days.

(a) Write down the value of A. [1]

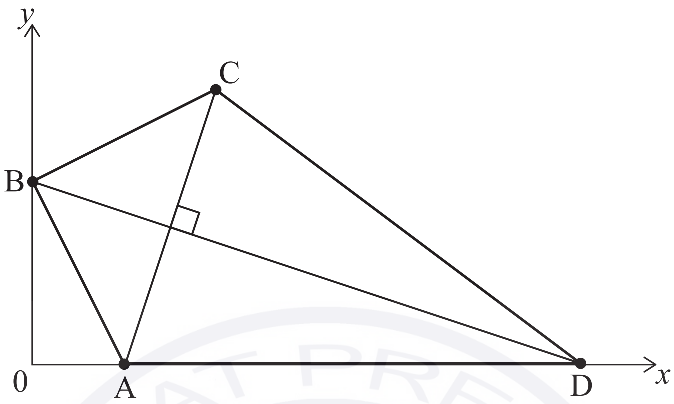
(b) Find the value of k. [3]

4. [Maximum mark: 6]

Dilara is designing a kite ABCD on a set of coordinate axes in which one unit represents 10 cm.

The coordinates of A, B and C are $(2, 0)$, $(0, 4)$ and $(4, 6)$ respectively. Point D lies on the x-axis. [AC] is perpendicular to [BD]. This information is shown in the following diagram.

diagram not to scale



(a) Find the gradient of the line through A and C. [2]

(b) Write down the gradient of the line through B and D. [1]

(c) Find the equation of the line through B and D. Give your answer in the form $ax + by + d = 0$, where a, b and d are integers. [2]

(d) Write down the x-coordinate of point D. [1]

(This question continues on the following page)

(Question 4 continued)

5. [Maximum mark: 7]

Let the function $h(x)$ represent the height in centimetres of a cylindrical tin can with diameter x cm.

$$h(x) = \frac{640}{x^2} + 0.5 \text{ for } 4 \le x \le 14.$$

(a) Find the range of h. [3]

The function h^{-1} is the inverse function of h.

(b) (i) Find $h^{-1}(10)$.

(ii) In the context of the question, interpret your answer to part (b)(i).

(iii) Write down the range of h^{-1}. [4]

6. [Maximum mark: 5]

Inspectors are investigating the carbon dioxide emissions of a power plant. Let R be the rate, in tonnes per hour, at which carbon dioxide is being emitted and t be the time in hours since the inspection began.

When R is plotted against t, the total amount of carbon dioxide produced is represented by the area between the graph and the horizontal t-axis.

The rate, R, is measured over the course of two hours. The results are shown in the following table.

t	0	0.4	0.8	1.2	1.6	2
R	30	50	60	40	20	50

(a) Use the trapezoidal rule with an interval width of 0.4 to estimate the total amount of carbon dioxide emitted during these two hours. [3]

The real amount of carbon dioxide emitted during these two hours was 72 tonnes.

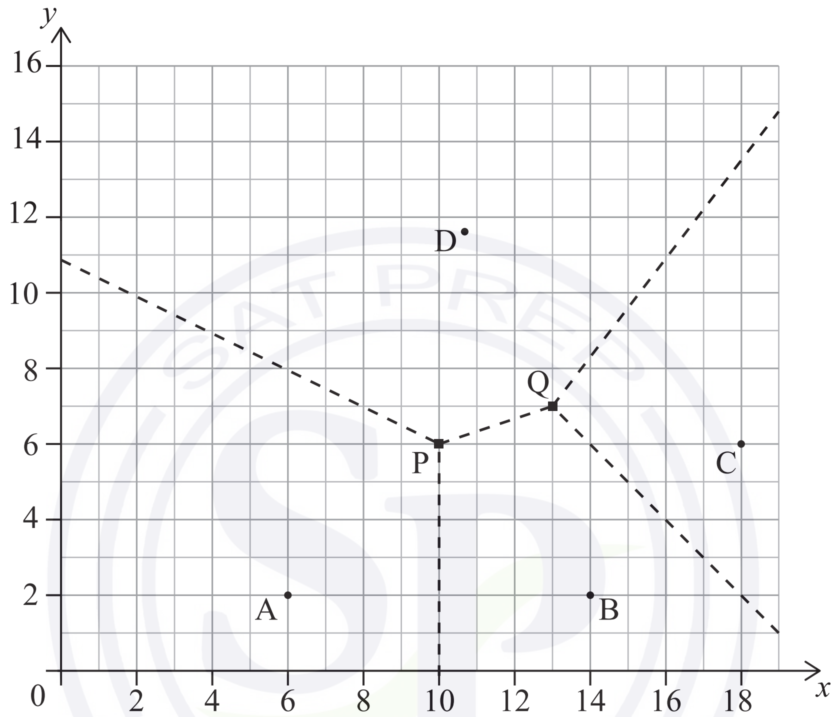
(b) Find the percentage error of the estimate found in part (a). [2]

7. [Maximum mark: 6]

There are four stations used by the fire wardens in a national forest.

On the following Voronoi diagram, the coordinates of the stations are $A(6, 2)$, $B(14, 2)$, $C(18, 6)$ and $D(10.8, 11.6)$ where distances are measured in kilometres.

The dotted lines represent the boundaries of the regions patrolled by the fire warden at each station. The boundaries meet at $P(10, 6)$ and $Q(13, 7)$.



To reduce the areas of the regions that the fire wardens patrol, a new station is to be built within the quadrilateral ABCD. The new station will be located so that it is as far as possible from the nearest existing station.

(a) Show that the new station should be built at P. [3]

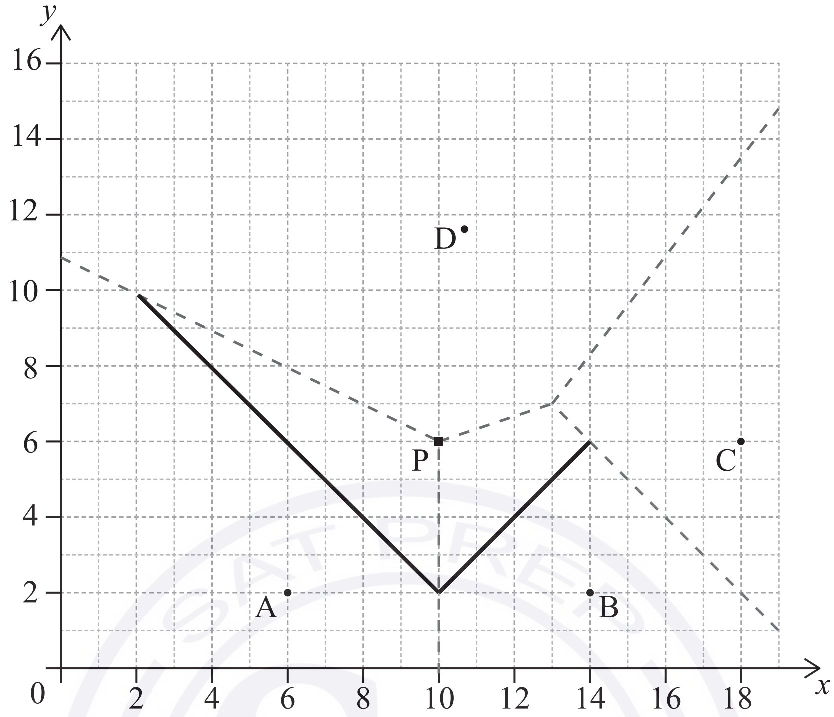
The Voronoi diagram is to be updated to include the region around the new station at P. The edges defined by the perpendicular bisectors of [AP] and [BP] have been added to the following diagram.

(b) (i) Write down the equation of the perpendicular bisector of [PC].

(ii) Hence draw the missing boundaries of the region around P on the following diagram. [3]

(This question continues on the following page)

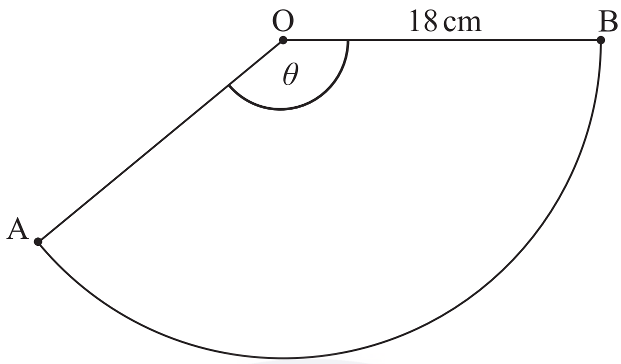
(Question 7 continued)



8. [Maximum mark: 5]

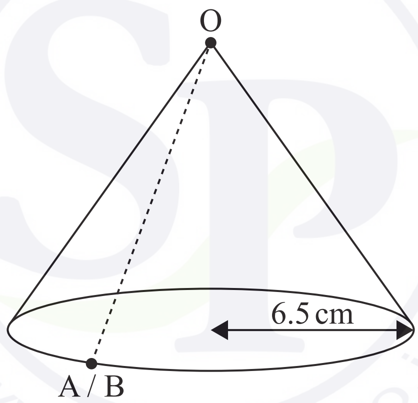
Joey is making a party hat in the form of a cone. The hat is made from a sector, AOB, of a circular piece of paper with a radius of $18\,\text{cm}$ and $\text{A}\hat{\text{O}}\text{B} = \theta$ as shown in the diagram.

diagram not to scale



To make the hat, sides [OA] and [OB] are joined together. The hat has a base radius of $6.5\,\text{cm}$.

diagram not to scale



(a) (i) Write down the perimeter of the base of the hat in terms of π.

(ii) Find the value of θ. [3]

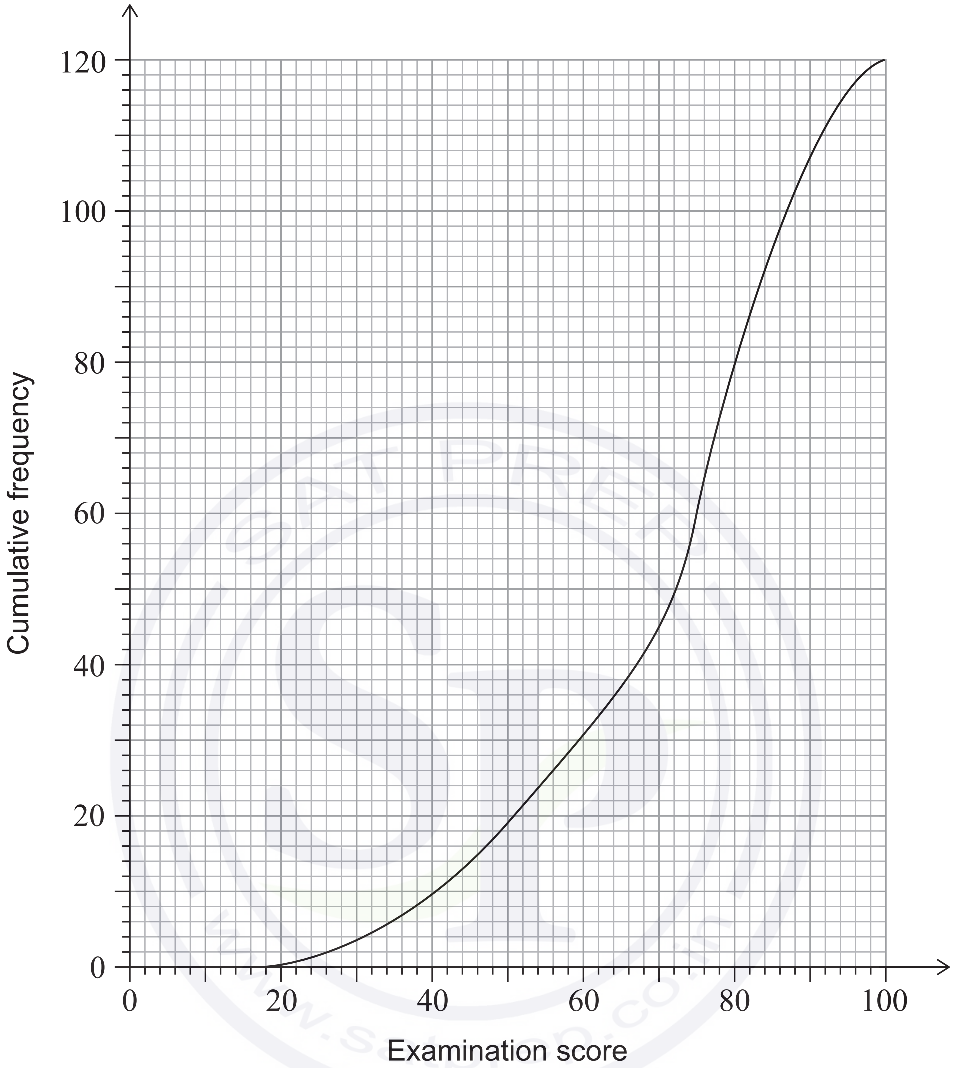
(b) Find the surface area of the outside of the hat. [2]

(This question continues on the following page)

(Question 8 continued)

9. [Maximum mark: 8]

A group of 120 students sat a history exam. The cumulative frequency graph shows the scores obtained by the students.



(a) Find the median of the scores obtained. [1]

The students were awarded a grade from 1 to 5, depending on the score obtained in the exam. The number of students receiving each grade is shown in the following table.

Grade	1	2	3	4	5
Number of students	6	13	26	a	b

(b) Find an expression for a in terms of b. [2]

(This question continues on the following page)

(Question 9 continued)

(c) The mean grade for these students is 3.65.

(i) Find the number of students who obtained a grade 5.

(ii) Find the minimum score needed to obtain a grade 5. [5]

Please **do not** write on this page.

Answers written on this page
will not be marked.

10. [Maximum mark: 9]

In this question, give all answers correct to 2 decimal places.

Raul and Rosy want to buy a new house and they need a loan of $170\,000$ Australian dollars (AUD) from a bank. The loan is for 30 years and the annual interest rate for the loan is $3.8\,\%$, compounded monthly. They will pay the loan in fixed monthly instalments at the end of each month.

(a) Find the amount they will pay the bank each month. [3]

(b) (i) Find the amount Raul and Rosy will still owe the bank at the end of the first 10 years.

(ii) Using your answers to parts (a) and (b)(i), calculate how much interest they will have paid in total during the first 10 years. [6]

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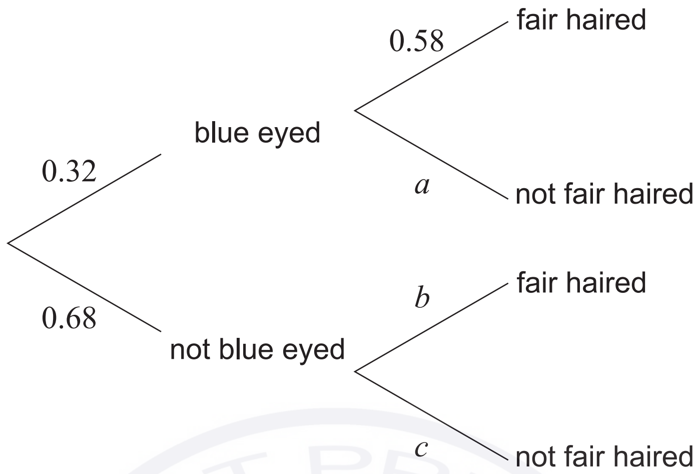
11. [Maximum mark: 5]

In a city, 32% of people have blue eyes. If someone has blue eyes, the probability that they also have fair hair is 58%. This information is represented in the following tree diagram.



(a) Write down the value of a. [1]

(b) Find an expression, in terms of b, for the probability of a person not having blue eyes **and** having fair hair. [1]

It is known that 41% of people in this city have fair hair.

(c) Calculate the value of

(i) b.

(ii) c. [3]

(This question continues on the following page)

(Question 11 continued)

12. [Maximum mark: 6]

The surface area of an open box with a volume of $32\,\text{cm}^3$ and a square base with sides of length $x\,\text{cm}$ is given by $S(x) = x^2 + \frac{128}{x}$ where $x > 0$.

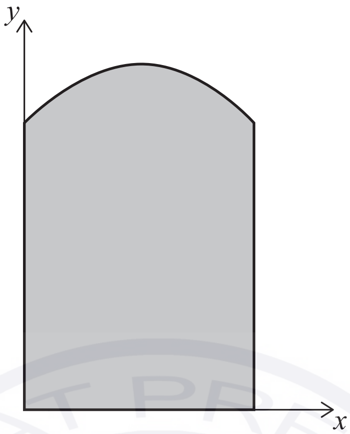
(a) Find $S'(x)$. [3]

(b) (i) Solve $S'(x) = 0$.

(ii) Interpret your answer to (b)(i) in context. [3]

13. [Maximum mark: 8]

Irina uses a set of coordinate axes to draw her design of a window. The base of the window is on the x-axis, the upper part of the window is in the form of a quadratic curve and the sides are vertical lines, as shown on the diagram. The curve has end points $(0,\ 10)$ and $(8,\ 10)$ and its vertex is $(4,\ 12)$. Distances are measured in centimetres.



The quadratic curve can be expressed in the form $y = ax^2 + bx + c$ for $0 \le x \le 8$.

(a) (i) Write down the value of c.

(ii) Hence form two equations in terms of a and b.

(iii) Hence find the equation of the quadratic curve. [5]

(b) Find the area of the shaded region in Irina's design. [3]

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(This question continues on the following page)

(Question 13 continued)

References:

2. Bermuda Triangle map [online] Available at: https://commons.wikimedia.org/wiki/File:Bermuda_Triangle_map_(de).svg Thomas Römer. This file is licensed under the Creative Commons Attribution-Share Alike 3.0 Unported license. (CC BY-SA 3.0) https://creativecommons.org/licenses/by-sa/3.0/deed.en [Accessed 17 December 2020] Source adapted.

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: applications and interpretation
Standard level
Paper 1

Thursday 6 May 2021 (afternoon)

Candidate session number

1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all questions.
- Answers must be written within the answer boxes provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

2221 – 7209

19 pages

Answers must be written within the answer boxes provided. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 4]

Katya approximates π, correct to four decimal places, by using the following expression.

$$3 + \frac{1}{6 + \frac{13}{16}}$$

(a) Calculate Katya's approximation of π, correct to four decimal places. [2]

(b) Calculate the percentage error in using Katya's four decimal place approximation of π, compared to the exact value of π in your calculator. [2]

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2. [Maximum mark: 4]

Deb used a thermometer to record the maximum daily temperature over ten consecutive days. Her results, in degrees Celsius ($^\circ C$), are shown below.

$$14, 15, 14, 11, 10, 9, 14, 15, 16, 13$$

For this data set, find the value of

(a) the mode. [1]

(b) the mean. [2]

(c) the standard deviation. [1]

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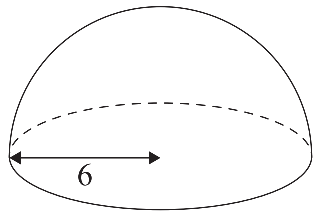
3. [Maximum mark: 6]

A piece of candy is made in the shape of a solid hemisphere. The radius of the hemisphere is 6 mm.



(a) Calculate the **total** surface area of one piece of candy. [4]

The total surface of the candy is coated in chocolate. It is known that 1 gram of the chocolate covers an area of $240\,\text{mm}^2$.

(b) Calculate the weight of chocolate required to coat one piece of candy. [2]

4. [Maximum mark: 7]

The price of gas at Leon's gas station is $\$1.50$ per litre. If a customer buys a minimum of 10 litres, a discount of $\$5$ is applied.

This can be modelled by the following function, L, which gives the total cost when buying a minimum of 10 litres at Leon's gas station.

$$L(x) = 1.50x - 5, \ x \geq 10$$

where x is the number of litres of gas that a customer buys.

(a) Find the total cost of buying 40 litres of gas at Leon's gas station. [2]

(b) Find $L^{-1}(70)$. [2]

The price of gas at Erica's gas station is $\$1.30$ per litre. A customer must buy a minimum of 10 litres of gas. The total cost at Erica's gas station is cheaper than Leon's gas station when $x > k$.

(c) Find the minimum value of k. [3]

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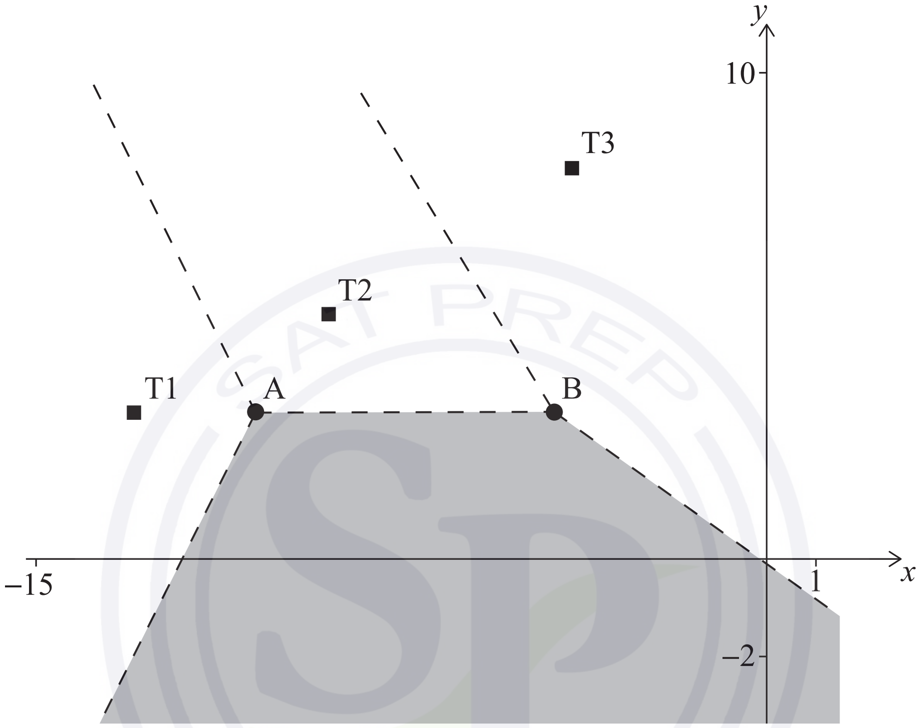
5. [Maximum mark: 6]

The Voronoi diagram below shows three identical cellular phone towers, T1, T2 and T3. A fourth cellular phone tower, T4 is located in the shaded region. The dashed lines in the diagram below represent the edges in the Voronoi diagram.

Horizontal scale: 1 unit represents 1 km.
Vertical scale: 1 unit represents 1 km.



Tim stands inside the shaded region.

(a) Explain why Tim will receive the strongest signal from tower T4. [1]

Tower T2 has coordinates $(-9, 5)$ and the edge connecting vertices A and B has equation $y = 3$.

(b) Write down the coordinates of tower T4. [2]

Tower T1 has coordinates $(-13, 3)$.

(c) Find the gradient of the edge of the Voronoi diagram between towers T1 and T2. [3]

(This question continues on the following page)

(Question 5 continued)

6. [Maximum mark: 5]

Arriane has geese on her farm. She claims the mean weight of eggs from her black geese is less than the mean weight of eggs from her white geese.

She recorded the weights of eggs, in grams, from a random selection of geese. The data is shown in the table.

Weights of eggs from black geese	136	134	142	141	128	126
Weights of eggs from white geese	135	138	141	140	136	134

In order to test her claim, Arriane performs a t-test at a 10% level of significance. It is assumed that the weights of eggs are normally distributed and the samples have equal variances.

(a) State, in words, the null hypothesis. [1]

(b) Calculate the p-value for this test. [2]

(c) State whether the result of the test supports Arriane's claim. Justify your reasoning. [2]

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7. [Maximum mark: 6]

Professor Wei observed that students have difficulty remembering the information presented in his lectures.

He modelled the percentage of information retained, R, by the function $R(t) = 100\,e^{-pt}$, $t \geq 0$, where t is the number of days after the lecture.

He found that 1 day after a lecture, students had forgotten 50% of the information presented.

(a) Find the value of p. [2]

(b) Use this model to find the percentage of information retained by his students 36 hours after Professor Wei's lecture. [2]

Based on his model, Professor Wei believes that his students will always retain some information from his lecture.

(c) State a mathematical reason why Professor Wei might believe this. [1]

(d) Write down one possible limitation of the **domain** of the model. [1]

8. [Maximum mark: 8]

Charlie and Daniella each began a fitness programme. On day one, they both ran $500\,\text{m}$. On each subsequent day, Charlie ran $100\,\text{m}$ more than the previous day whereas Daniella increased her distance by 2% of the distance ran on the previous day.

(a) Calculate how far

(i) Charlie ran on day 20 of his fitness programme.

(ii) Daniella ran on day 20 of her fitness programme. [5]

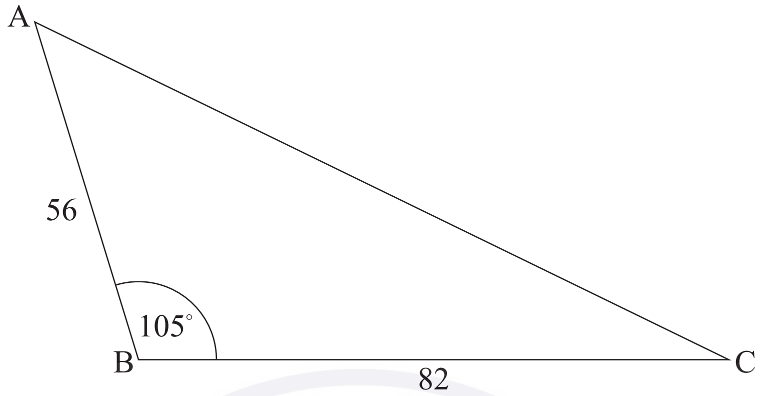
On day n of the fitness programmes Daniella runs more than Charlie for the first time.

(b) Find the value of n. [3]

9. [Maximum mark: 5]

A triangular field ABC is such that $AB = 56\,m$ and $BC = 82\,m$, each measured correct to the nearest metre, and the angle at B is equal to 105°, measured correct to the nearest 5°.

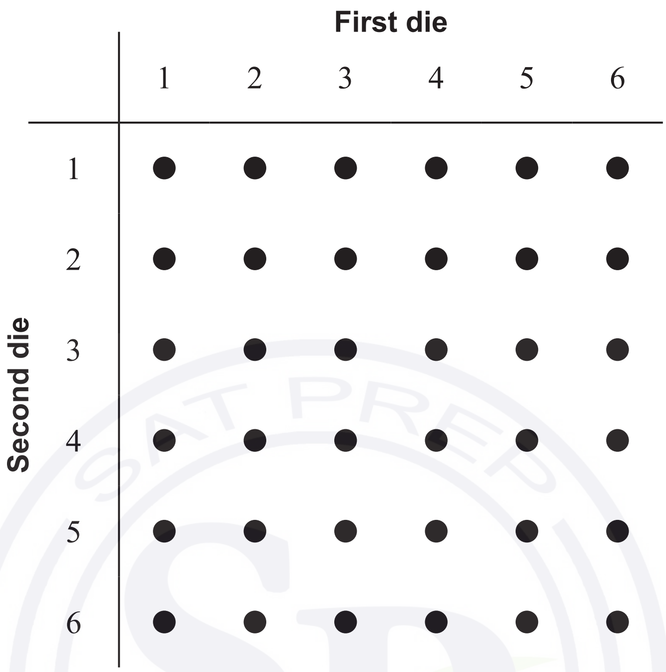
diagram not to scale



Calculate the maximum possible area of the field.

10. [Maximum mark: 7]

A game is played where two unbiased dice are rolled and the score in the game is the greater of the two numbers shown. If the two numbers are the same, then the score in the game is the number shown on one of the dice. A diagram showing the possible outcomes is given below.



Let T be the random variable "the score in a game".

(a) Complete the table to show the probability distribution of T. [2]

t	1	2	3	4	5	6
$P(T = t)$						

(b) Find the probability that

(i) a player scores at least 3 in a game.

(ii) a player scores 6, given that they scored at least 3. [3]

(c) Find the expected score of a game. [2]

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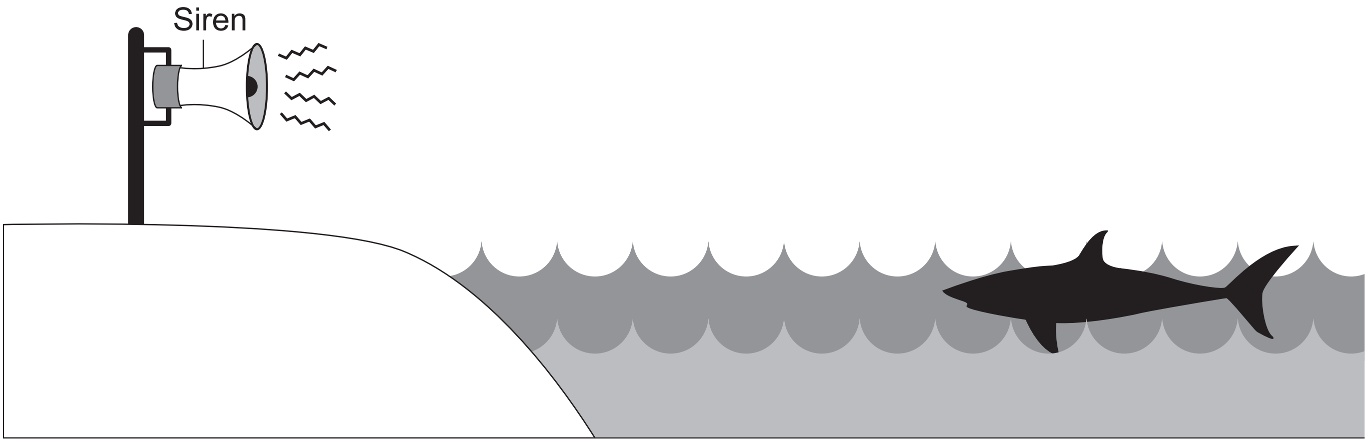
(Question 10 continued)

11. [Maximum mark: 6]

If a shark is spotted near to Brighton beach, a lifeguard will activate a siren to warn swimmers.

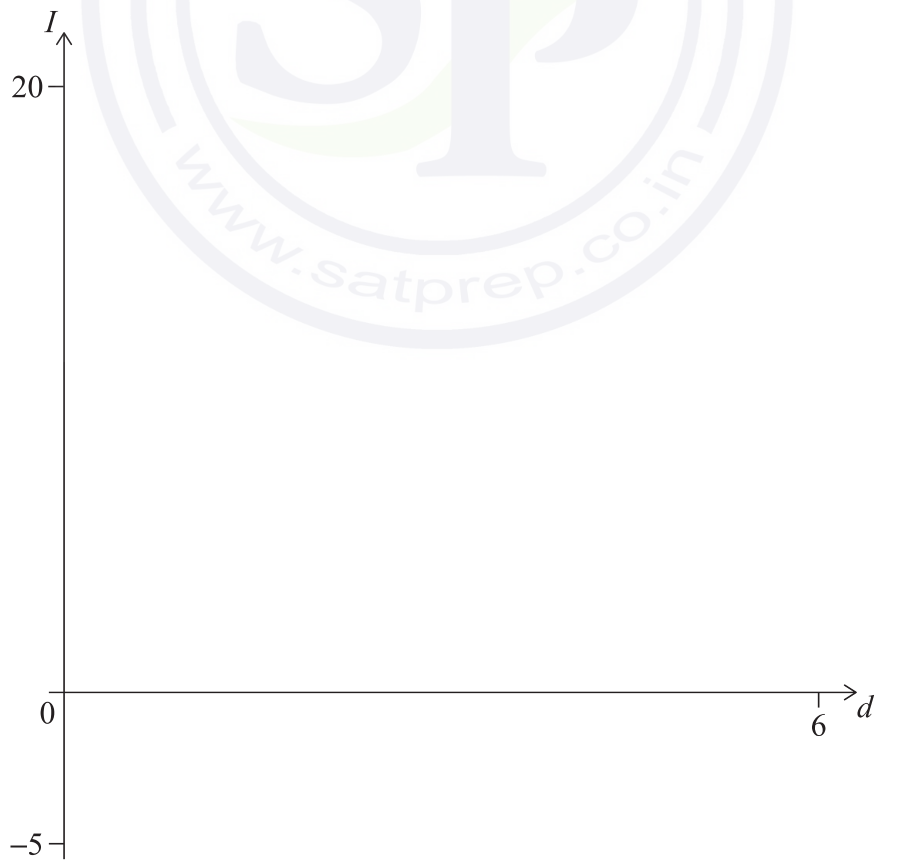


The sound intensity, I, of the siren varies inversely with the square of the distance, d, from the siren, where $d > 0$.

It is known that at a distance of 1.5 metres from the siren, the sound intensity is 4 watts per square metre (W m^{-2}).

(a) Show that $I = \frac{9}{d^2}$. [2]

(b) Sketch the curve of I on the axes below showing clearly the point $(1.5, 4)$. [2]



(This question continues on the following page)

(Question 11 continued)

Whilst swimming, Scarlett can hear the siren only if the sound intensity at her location is greater than $1.5 \times 10^{-6}\,\mathrm{W m^{-2}}$.

(c) Find the values of d where Scarlett cannot hear the siren. [2]

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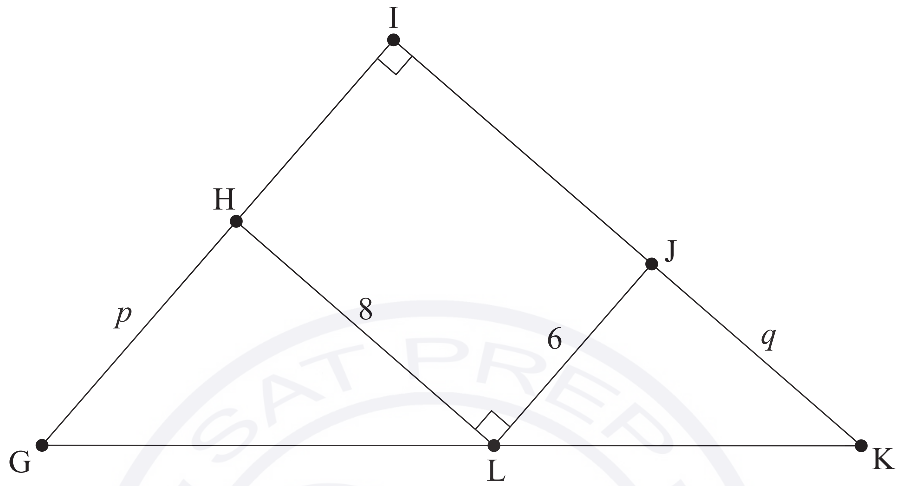
12. [Maximum mark: 8]

Ellis designs a gift box. The top of the gift box is in the shape of a right-angled triangle GIK.

A rectangular section HIJL is inscribed inside this triangle. The lengths of GH, JK, HL, and LJ are p cm, q cm, 8 cm and 6 cm respectively.

diagram not to scale



The area of the top of the gift box is $A\,\text{cm}^2$.

(a) (i) Find A in terms of p and q.

(ii) Show that $A = \frac{192}{q} + 3q + 48$. [4]

(b) Find $\frac{dA}{dq}$. [2]

Ellis wishes to find the value of q that will minimize the area of the top of the gift box.

(c) (i) Write down an equation Ellis could solve to find this value of q.

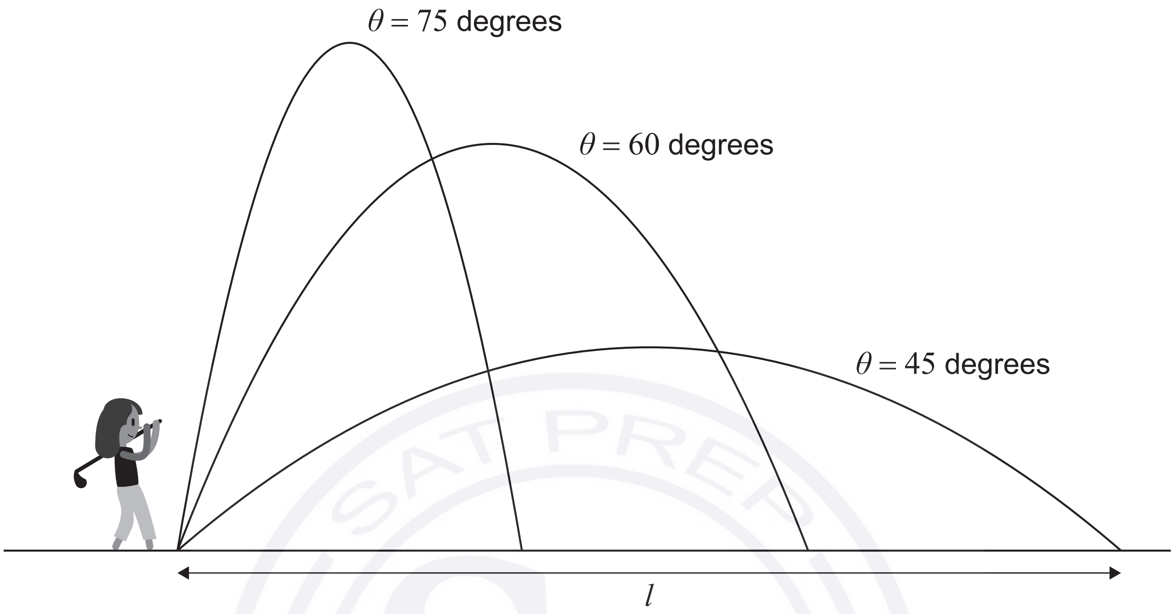
(ii) Hence, or otherwise, find this value of q. [2]

(This question continues on the following page)

(Question 12 continued)

13. [Maximum mark: 8]

Sieun hits golf balls into the air. Each time she hits a ball she records θ, the angle at which the ball is launched into the air, and l, the horizontal distance, in metres, which the ball travels from the point of contact to the first time it lands. The diagram below represents this information.



Sieun analyses her results and concludes:

$$\frac{dl}{d\theta} = -0.2\theta + 9,\ 35^\circ \le \theta \le 75^\circ.$$

(a) Determine whether the graph of l against θ is increasing or decreasing at $\theta = 50^\circ$. [3]

Sieun observes that when the angle is 40°, the ball will travel a horizontal distance of $205.5\,\text{m}$.

(b) Find an expression for the function $l(\theta)$. [5]

(This question continues on the following page)

(Question 13 continued)

References:

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Answers written on this page
will not be marked.

Mathematics: applications and interpretation
Standard level
Paper 1

Thursday 6 May 2021 (afternoon)

Candidate session number

1 hour 30 minutes

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2221 – 7214

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1. [Maximum mark: 5]

The amount, in milligrams, of a medicinal drug in the body t hours after it was injected is given by $D(t) = 23(0.85)^t$, $t \geq 0$. Before this injection, the amount of the drug in the body was zero.

(a) Write down

(i) the initial dose of the drug.

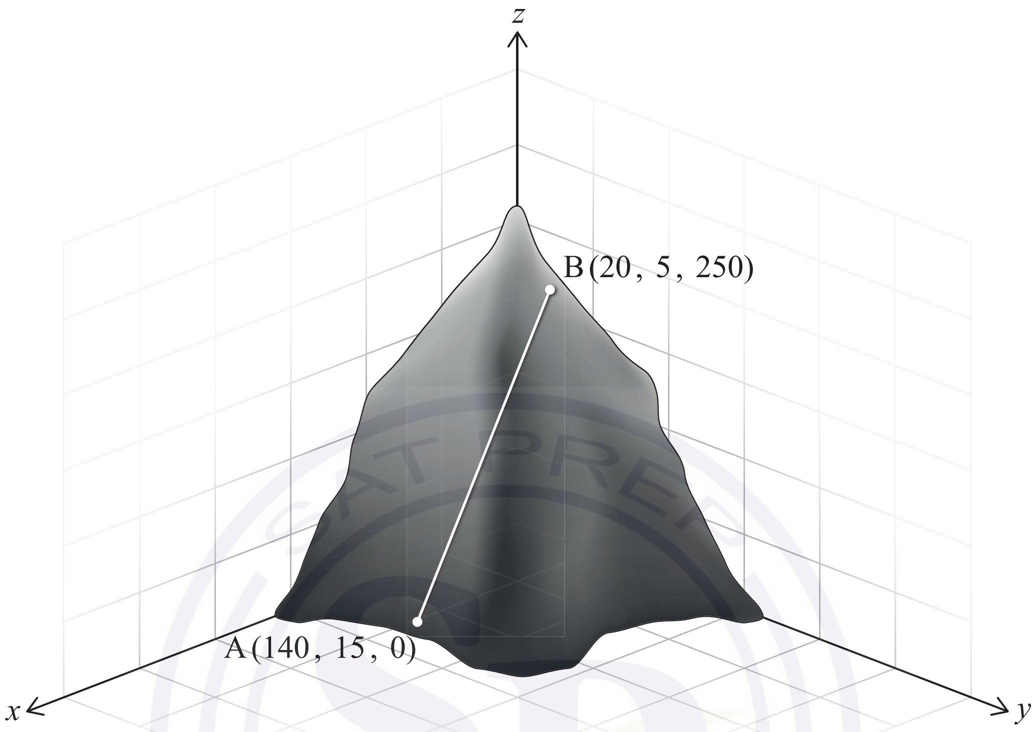
(ii) the percentage of the drug that leaves the body each hour. [3]

(b) Calculate the amount of the drug remaining in the body 10 hours after the injection. [2]

2. [Maximum mark: 5]

An inclined railway travels along a straight track on a steep hill, as shown in the diagram.

diagram not to scale



The locations of the stations on the railway can be described by coordinates in reference to x, y, and z-axes, where the x and y axes are in the horizontal plane and the z-axis is vertical.

The ground level station A has coordinates $(140, 15, 0)$ and station B, located near the top of the hill, has coordinates $(20, 5, 250)$. All coordinates are given in metres.

(a) Find the distance between stations A and B. [2]

Station M is to be built halfway between stations A and B.

(b) Find the coordinates of station M. [2]

(c) Write down the height of station M, in metres, above the ground. [1]

(This question continues on the following page)

(Question 2 continued)

Please **do not** write on this page.

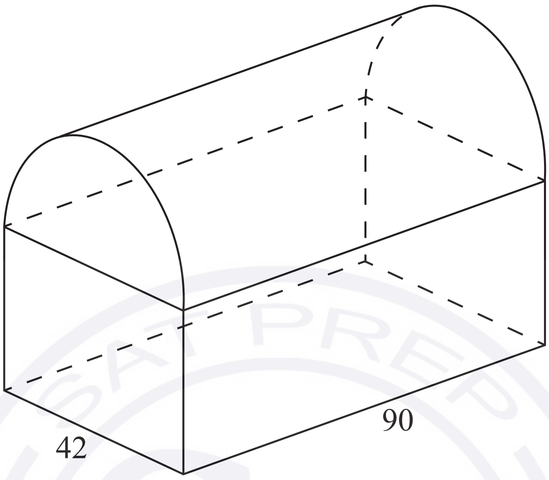
Answers written on this page
will not be marked.

3. [Maximum mark: 7]

A storage container consists of a box of length $90\,\text{cm}$, width $42\,\text{cm}$ and height $34\,\text{cm}$, and a lid in the shape of a half-cylinder, as shown in the diagram. The lid fits the top of the box exactly. The total exterior surface of the storage container is to be painted.

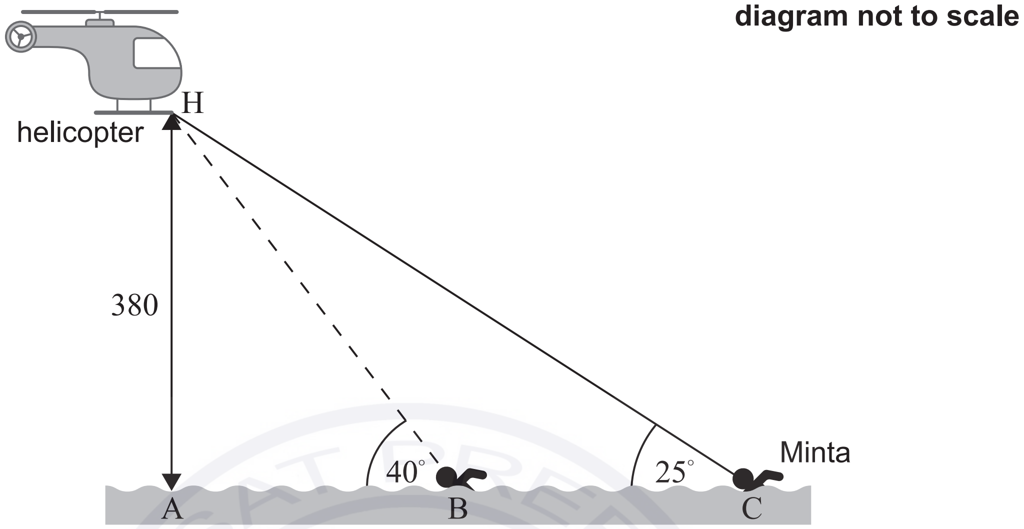
Find the area to be painted.

diagram not to scale



4. [Maximum mark: 7]

The diagram below shows a helicopter hovering at point H, 380 m vertically above a lake. Point A is the point on the surface of the lake, directly below the helicopter.



Minta is swimming at a constant speed in the direction of point A. Minta observes the helicopter from point C as she looks upward at an angle of 25°. After 15 minutes, Minta is at point B and she observes the same helicopter at an angle of 40°.

(a) Write down the size of the angle of depression from H to C. [1]

(b) Find the distance from A to C. [2]

(c) Find the distance from B to C. [3]

(d) Find Minta's speed, in metres per hour. [1]

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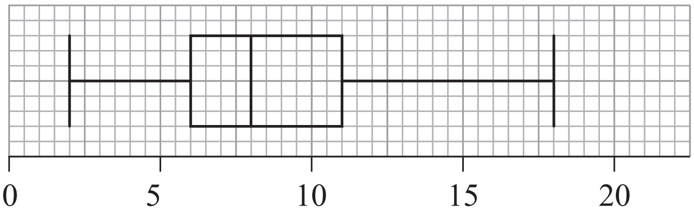
5. [Maximum mark: 5]

The number of sick days taken by each employee in a company during a year was recorded. The data was organized in a box and whisker diagram as shown below:



(a) For this data, write down

(i) the minimum number of sick days taken during the year.

(ii) the lower quartile.

(iii) the median. [3]

Paul claims that this box and whisker diagram can be used to infer that the percentage of employees who took fewer than six sick days is smaller than the percentage of employees who took more than eleven sick days.

(b) State whether Paul is correct. Justify your answer. [2]

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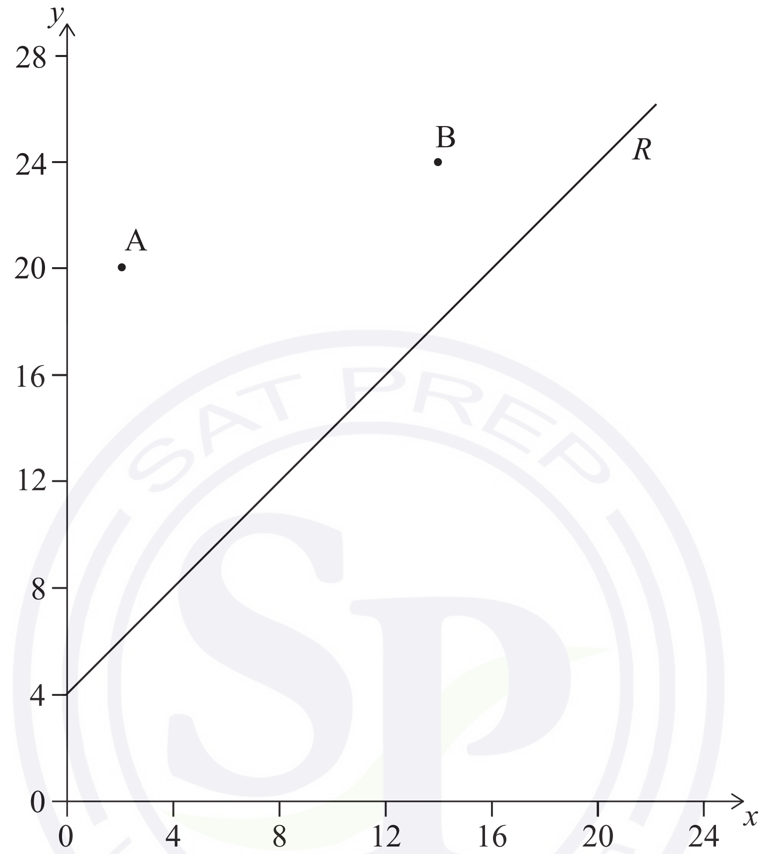
6. [Maximum mark: 7]

Two schools are represented by points $A(2, 20)$ and $B(14, 24)$ on the graph below. A road, represented by the line R with equation $-x + y = 4$, passes near the schools. An architect is asked to determine the location of a new bus stop on the road such that it is the same distance from the two schools.



(a) Find the equation of the perpendicular bisector of $[AB]$. Give your equation in the form $y = mx + c$. [5]

(b) Determine the coordinates of the point on R where the bus stop should be located. [2]

(This question continues on the following page)

(Question 6 continued)

7. [Maximum mark: 5]

A function is defined by $f(x) = 2 - \frac{12}{x+5}$ for $-7 \le x \le 7$, $x \ne -5$.

(a) Find the range of f. [3]

(b) Find the value of $f^{-1}(0)$. [2]

8. [Maximum mark: 6]

At Springfield University, the weights, in kg, of 10 chinchilla rabbits and 10 sable rabbits were recorded. The aim was to find out whether chinchilla rabbits are generally heavier than sable rabbits. The results obtained are summarized in the following table.

Weight of chinchilla rabbits, kg	4.9	4.2	4.1	4.4	4.3	4.6	4.0	4.7	4.5	4.4
Weight of sable rabbits, kg	4.2	4.1	4.1	4.2	4.5	4.4	4.5	3.9	4.2	4.0

A t-test is to be performed at the 5% significance level.

(a) Write down the null and alternative hypotheses. [2]

(b) Find the p-value for this test. [2]

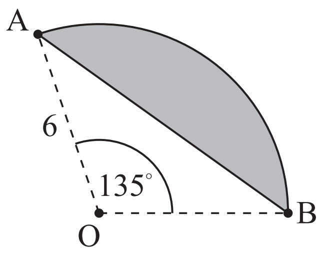
(c) Write down the conclusion to the test. Give a reason for your answer. [2]

9. [Maximum mark: 7]

A garden includes a small lawn. The lawn is enclosed by an arc AB of a circle with centre O and radius $6\,\text{m}$, such that $\text{A}\hat{\text{O}}\text{B} = 135^\circ$. The straight border of the lawn is defined by chord [AB].

The lawn is shown as the shaded region in the following diagram.

diagram not to scale



(a) A footpath is to be laid around the curved side of the lawn. Find the length of the footpath. [3]

(b) Find the area of the lawn. [4]

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10. [Maximum mark: 6]

Tommaso and Pietro have each been given 1500 euro to save for college.

Pietro invests his money in an account that pays a nominal annual interest rate of 2.75%, **compounded half-yearly**.

(a) Calculate the amount Pietro will have in his account after 5 years. Give your answer correct to 2 decimal places. [3]

Tommaso wants to invest his money in an account such that his investment will increase to 1.5 times the initial amount in 5 years. Assume the account pays a nominal annual interest of $r\%$ **compounded quarterly**.

(b) Determine the value of r. [3]

Please **do not** write on this page.

Answers written on this page will not be marked.

11. [Maximum mark: 6]

A newspaper vendor in Singapore is trying to predict how many copies of *The Straits Times* they will sell. The vendor forms a model to predict the number of copies sold each weekday. According to this model, they expect the same number of copies will be sold each day.

To test the model, they record the number of copies sold each weekday during a particular week. This data is shown in the table.

Day	Monday	Tuesday	Wednesday	Thursday	Friday
Number of copies sold	74	97	91	86	112

A goodness of fit test at the 5% significance level is used on this data to determine whether the vendor's model is suitable.

The critical value for the test is 9.49 and the hypotheses are

H_0 : The data satisfies the model.
H_1 : The data does not satisfy the model.

(a) Find an estimate for how many copies the vendor expects to sell each day. [1]

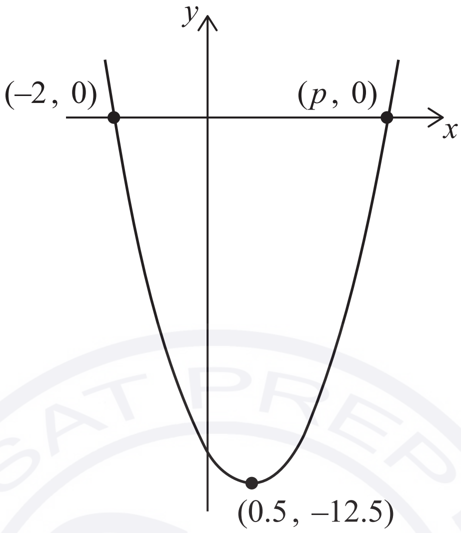
(b) (i) Write down the degrees of freedom for this test.

(ii) Write down the conclusion to the test. Give a reason for your answer. [5]

12. [Maximum mark: 7]

Consider the function $f(x) = ax^2 + bx + c$. The graph of $y = f(x)$ is shown in the diagram. The vertex of the graph has coordinates $(0.5, -12.5)$. The graph intersects the x-axis at two points, $(-2, 0)$ and $(p, 0)$.

diagram not to scale



(a) Find the value of p. [1]

(b) Find the value of

(i) a.

(ii) b.

(iii) c. [5]

(c) Write down the equation of the axis of symmetry of the graph. [1]

(This question continues on the following page)

(Question 12 continued)

13. [Maximum mark: 7]

A company produces and sells electric cars. The company's profit, P, in thousands of dollars, changes based on the number of cars, x, they produce per month.

The rate of change of their profit from producing x electric cars is modelled by

$$\frac{\mathrm{d}P}{\mathrm{d}x} = -1.6x + 48, \ x \geq 0.$$

The company makes a profit of 260 (thousand dollars) when they produce 15 electric cars.

(a) Find an expression for P in terms of x. [5]

The company regularly increases the number of cars it produces.

(b) Describe how their profit changes if they increase production to over 30 cars per month and up to 50 cars per month. Justify your answer. [2]

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References:

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: applications and interpretation
Standard level
Paper 1

Specimen paper

Candidate session number

1 hour 30 minutes

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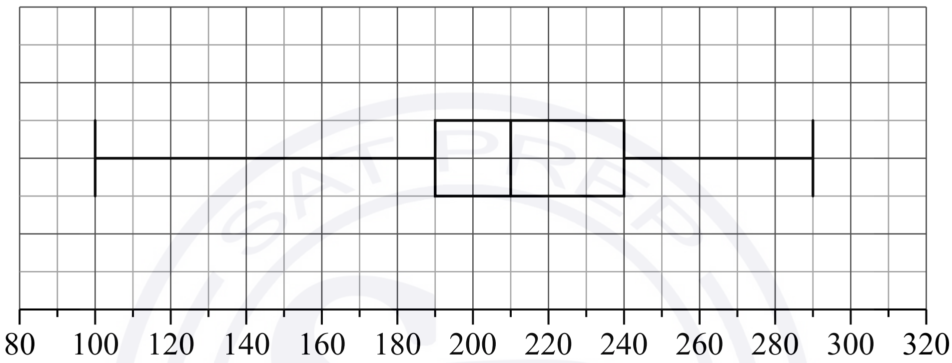
1. [Maximum mark: 6]

Palvinder breeds Labrador puppies at his farm. Over many years he recorded the weight (g) of the puppies.

The data is illustrated in the following box and whisker diagram.



(a) Write down the median weight of the puppies. [1]

(b) Write down the upper quartile. [1]

(c) Find the interquartile range. [2]

The weights of these Labrador puppies are normally distributed.

(d) Find the weight of the heaviest possible puppy that is not an outlier. [2]

(This question continues on the following page)

(Question 1 continued)

2. [Maximum mark: 6]

The Osaka Tigers basketball team play in a multilevel stadium.

The most expensive tickets are in the first row. The ticket price, in Yen (¥), for each row forms an arithmetic sequence. Prices for the first three rows are shown in the following table.

Ticket pricing per game	
1st row	6800 Yen
2nd row	6550 Yen
3rd row	6300 Yen

(a) Write down the value of the common difference, d [1]

(b) Calculate the price of a ticket in the 16th row. [2]

(c) Find the total cost of buying 2 tickets in each of the first 16 rows. [3]

3. [Maximum mark: 6]

At the end of a school day, the Headmaster conducted a survey asking students in how many classes they had used the internet.

The data is shown in the following table.

Number of classes in which the students used the internet	0	1	2	3	4	5	6
Number of students	20	24	30	k	10	3	1

(a) State whether the data is discrete or continuous. [1]

The mean number of classes in which a student used the internet is 2.

(b) Find the value of k. [4]

It was not possible to ask every person in the school, so the Headmaster arranged the student names in alphabetical order and then asked every 10th person on the list.

(c) Identify the sampling technique used in the survey. [1]

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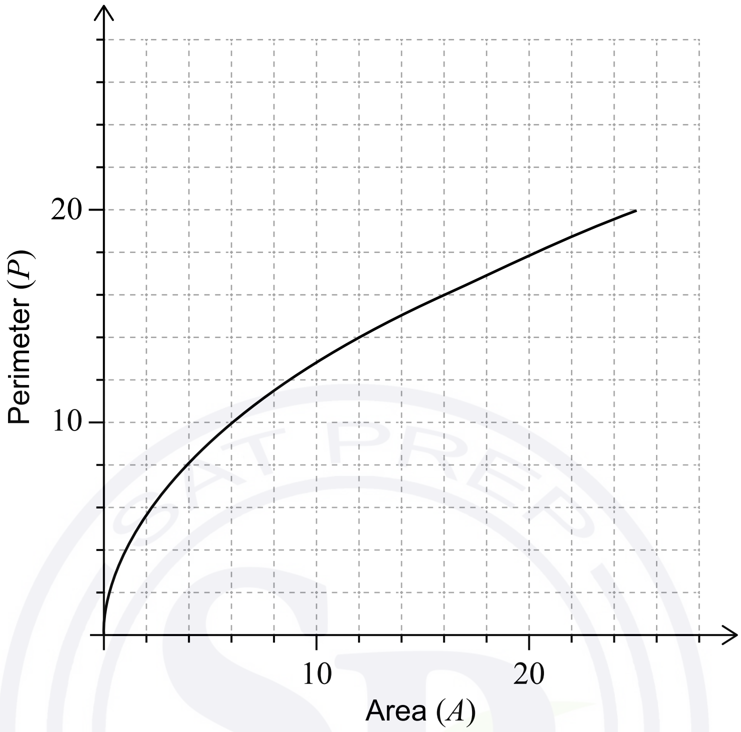
4. [Maximum mark: 6]

The perimeter of a given square P can be represented by the function $P(A) = 4\sqrt{A}$, $A \geq 0$, where A is the area of the square. The graph of the function P is shown for $0 \leq A \leq 25$.



(a) Write down the value of $P(25)$. [1]

The range of $P(A)$ is $0 \leq P(A) \leq n$.

(b) Hence write down the value of n. [1]

(c) On the axes above, draw the graph of the inverse function, P^{-1}. [3]

(d) In the context of the question, explain the meaning of $P^{-1}(8) = 4$. [1]

(This question continues on the following page)

(Question 4 continued)

5. [Maximum mark: 6]

Professor Vinculum investigated the migration season of the Bulbul bird from their natural wetlands to a warmer climate.

He found that during the migration season their population, P could be modelled by $P = 1350 + 400(1.25)^{-t}$, $t \geq 0$, where t is the number of days since the start of the migration season.

(a) Find the population of the Bulbul birds,

(i) at the start of the migration season.

(ii) in the wetlands after 5 days. [3]

(b) Calculate the time taken for the population to decrease below 1400. [2]

(c) According to this model, find the smallest possible population of Bulbul birds during the migration season. [1]

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6. [Maximum mark: 5]

As part of a study into healthy lifestyles, Jing visited Surrey Hills University. Jing recorded a person's position in the university and how frequently they ate a salad. Results are shown in the table.

	Salad meals per week			
	0	**1 – 2**	**3 – 4**	**> 4**
Students	45	26	18	6
Professors	15	8	5	12
Staff and Administration	16	13	10	6

Jing conducted a χ^2 test for independence at a 5% level of significance.

(a) State the null hypothesis. [1]

(b) Calculate the p-value for this test. [2]

(c) State, giving a reason, whether the null hypothesis should be accepted. [2]

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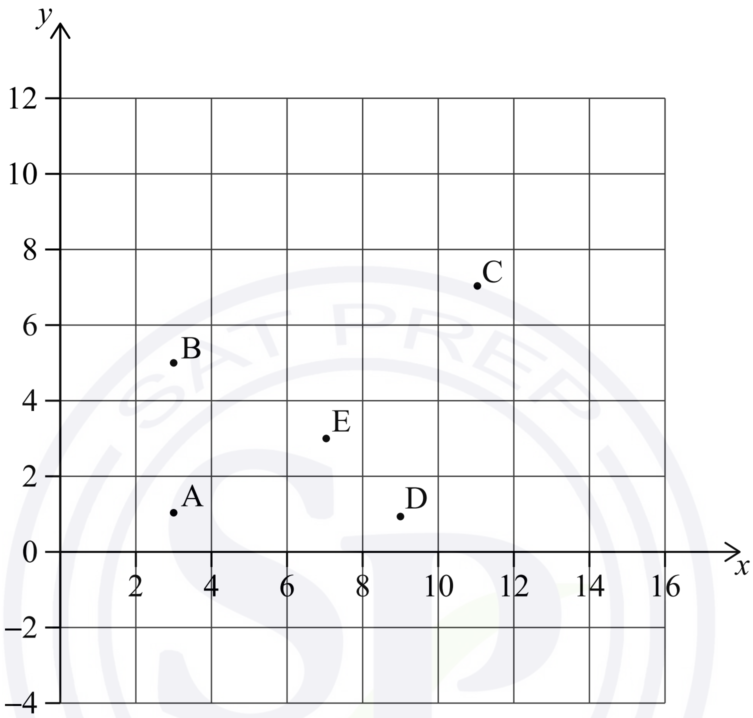
7. [Maximum mark: 6]

Points $A(3, 1)$, $B(3, 5)$, $C(11, 7)$, $D(9, 1)$ and $E(7, 3)$ represent snow shelters in the Blackburn National Forest. These snow shelters are illustrated in the following coordinate axes.

Horizontal scale: 1 unit represents 1 km.

Vertical scale: 1 unit represents 1 km.

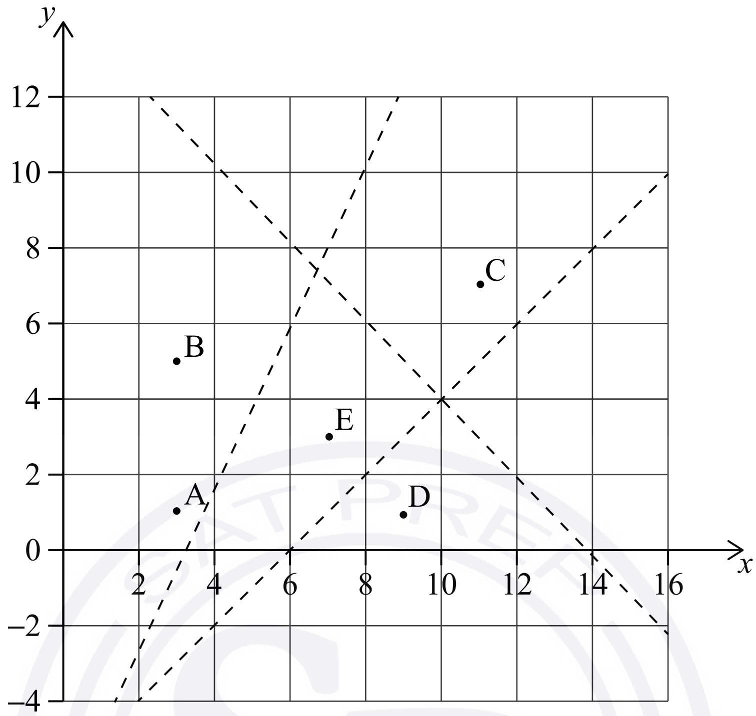


(a) Calculate the gradient of the line segment AE. [2]

(This question continues on the following page)

(Question 7 continued)

The Park Ranger draws three straight lines to form an incomplete Voronoi diagram.



(b) Find the equation of the line which would complete the Voronoi cell containing site E. Give your answer in the form $ax + by + d = 0$ where $a, b, d \in \mathbb{Z}$. [3]

(c) In the context of the question, explain the significance of the Voronoi cell containing site E. [1]

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8. [Maximum mark: 4]

The intensity level of sound, L measured in decibels (dB), is a function of the sound intensity, S watts per square metre $(\mathrm{W\,m^{-2}})$. The intensity level is given by the following formula.

$$L = 10\log_{10}(S \times 10^{12}),\ S \geq 0$$

(a) An orchestra has a sound intensity of $6.4 \times 10^{-3}\,\mathrm{W\,m^{-2}}$. Calculate the intensity level, L of the orchestra. [2]

(b) A rock concert has an intensity level of $112\,\mathrm{dB}$. Find the sound intensity, S. [2]

9. [Maximum mark: 6]

Ms Calhoun measures the heights of students in her mathematics class. She is interested to see if the mean height of male students, μ_1, is the same as the mean height of female students, μ_2. The information is recorded in the table.

Male height (cm)	150	148	143	152	151	149	147	
Female height (cm)	148	152	154	147	146	153	152	150

At the 10% level of significance, a t-test was used to compare the means of the two groups. The data is assumed to be normally distributed and the standard deviations are equal between the two groups.

(a) (i) State the null hypothesis.

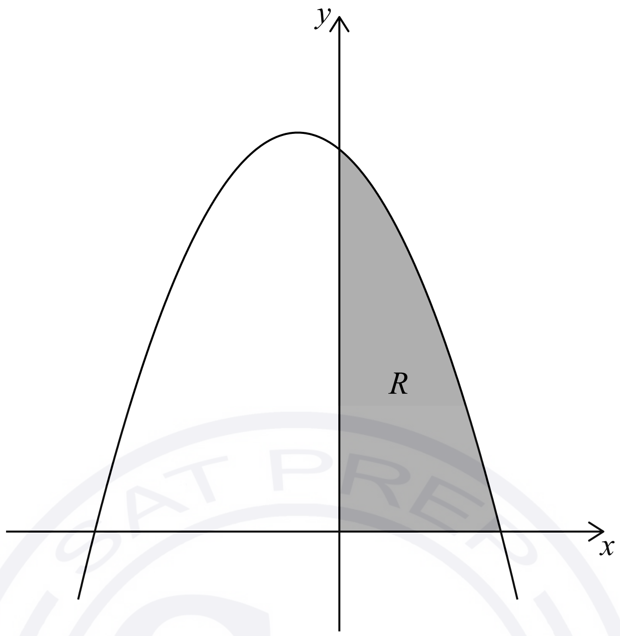
(ii) State the alternative hypothesis. [2]

(b) Calculate the p-value for this test. [2]

(c) State, giving a reason, whether Ms Calhoun should accept the null hypothesis. [2]

10. [Maximum mark: 5]

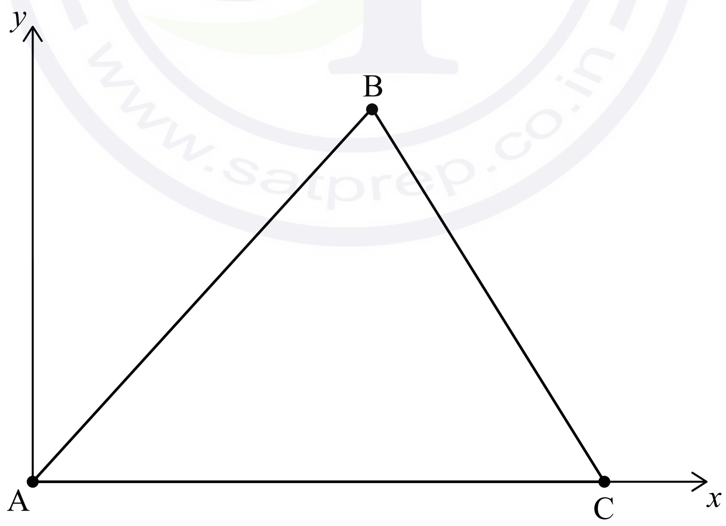
The following diagram shows part of the graph of $f(x) = (6 - 3x)(4 + x)$, $x \in \mathbb{R}$. The shaded region R is bounded by the x-axis, y-axis and the graph of f.



(a) Write down an integral for the area of region R. [2]

(b) Find the area of region R. [1]

The three points $\mathrm{A}(0, 0)$, $\mathrm{B}(3, 10)$ and $\mathrm{C}(a, 0)$ define the vertices of a triangle.



(c) Find the value of a, the x-coordinate of C, such that the area of the triangle is equal to the area of region R. [2]

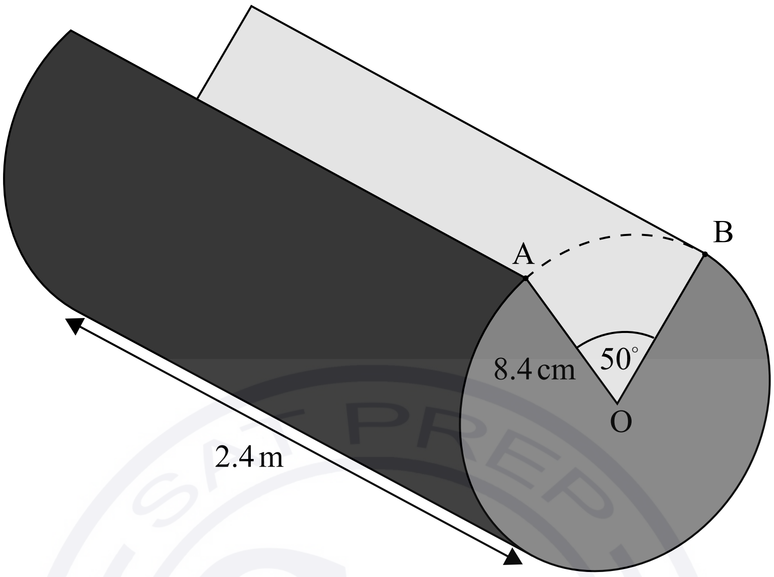
(This question continues on the following page)

(Question 10 continued)

11. [Maximum mark: 4]

Helen is building a cabin using cylindrical logs of length $2.4\,\text{m}$ and radius $8.4\,\text{cm}$. A wedge is cut from one log and the cross-section of this log is illustrated in the following diagram.

diagram not to scale



Find the volume of this log.

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12. [Maximum mark: 6]

Jae Hee plays a game involving a biased six-sided die.
The faces of the die are labelled -3, -1, 0, 1, 2 and 5.
The score for the game, X, is the number which lands face up after the die is rolled.
The following table shows the probability distribution for X.

Score x	-3	-1	0	1	2	5
$\mathbf{P}(X=x)$	$\frac{1}{18}$	p	$\frac{3}{18}$	$\frac{1}{18}$	$\frac{2}{18}$	$\frac{7}{18}$

(a) Find the exact value of p. [1]

Jae Hee plays the game once.

(b) Calculate the expected score. [2]

Jae Hee plays the game twice and adds the two scores together.

(c) Find the probability Jae Hee has a **total** score of -3. [3]

13. [Maximum mark: 6]

Mr Burke teaches a mathematics class with 15 students. In this class there are 6 female students and 9 male students.

Each day Mr Burke randomly chooses one student to answer a homework question.

(a) Find the probability that on any given day Mr Burke chooses a female student to answer a question. [1]

In the first month, Mr Burke will teach his class 20 times.

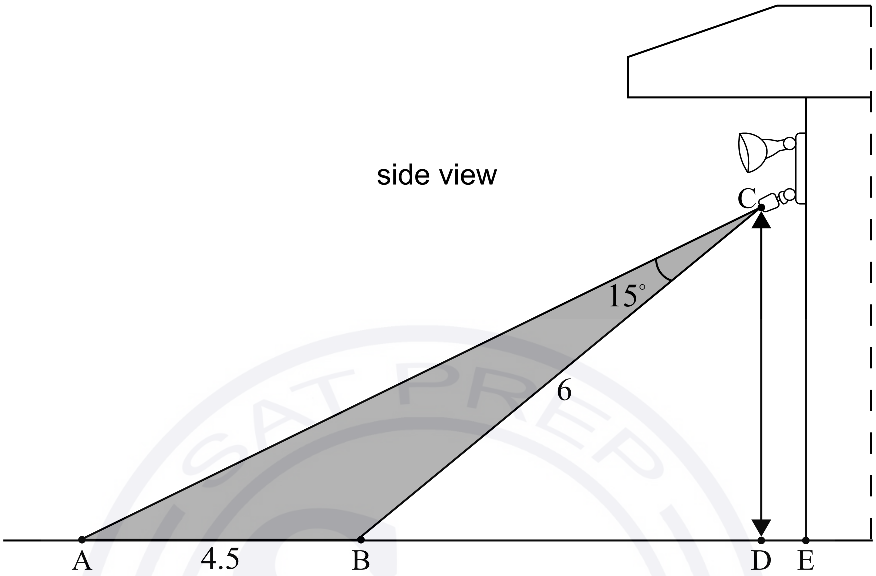
(b) Find the probability he will choose a female student 8 times. [2]

(c) Find the probability he will choose a male student at most 9 times. [3]

14. [Maximum mark: 8]

Ollie has installed security lights on the side of his house that are activated by a sensor. The sensor is located at point C directly above point D. The area covered by the sensor is shown by the shaded region enclosed by triangle ABC. The distance from A to B is 4.5 m and the distance from B to C is 6 m. Angle $\text{A}\hat{\text{C}}\text{B}$ is 15°.

diagram not to scale



(a) Find $\text{C}\hat{\text{A}}\text{B}$. [3]

Point B on the ground is 5 m from point E at the entrance to Ollie's house. He is 1.8 m tall and is standing at point D, below the sensor. He walks towards point B.

(b) Find the distance Ollie is **from the entrance to his house** when he first activates the sensor. [5]