8823 – 7214M

Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

November 2023

Mathematics: applications and interpretation

Standard level

Paper 1

14 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

R Marks awarded for clear **Reasoning**.

AG Answer given in the question and so no marks are awarded.

FT Follow through. The practice of awarding marks, despite candidate errors in previous parts, for their correct methods/answers using incorrect results.

Using the markscheme

1 General

Award marks using the annotations as noted in the markscheme eg ***M1***, ***A2***.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
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- The response to a "show that" question does not need to restate the ***AG*** line, unless a **Note** makes this explicit in the markscheme.
- Once a correct answer to a question or part question is seen, ignore further working even if this working is incorrect and/or suggests a misunderstanding of the question. This will encourage a uniform approach to marking, with less examiner discretion. Although some candidates may be advantaged for that specific question item, it is likely that these candidates will lose marks elsewhere too.
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	Correct answer seen	Further working seen	Any FT issues?	Action
1.	$8\sqrt{2}$	5.65685... *(incorrect decimal value)*	No. Last part in question.	Award ***A1*** for the final mark *(condone the incorrect further working)*
2.	$\frac{35}{72}$	0.468111… (incorrect decimal value)	Yes. Value is used in subsequent parts.	Award ***A0*** for the final mark *(and full **FT** is available in subsequent parts)*

3 Implied marks

Implied marks appear in **brackets e.g. *(M1)***,and can only be awarded if **correct** work is seen or implied by subsequent working/answer.

4 Follow through marks (only applied after an error is made)

Follow through (***FT***) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) (e.g. incorrect value from part (a) used in part (d) or incorrect value from part (c)(i) used in part (c)(ii)). Usually, to award ***FT*** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if all the marks awarded in a subsequent part are for the answer or are implied, then ***FT*** marks should be awarded for *their* correct answer, even when working is not present.

For example: following an incorrect answer to part (a) that is used in subsequent parts, where the markscheme for the subsequent part is ***(M1)A1***, it is possible to award full marks for *their* correct answer, **without working being seen.** For longer questions where all but the answer marks are implied this rule applies but may be overwritten by a **Note** in the Markscheme.

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- Exceptions to these ***FT*** rules will be explicitly noted on the markscheme.
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- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
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- ***MR*** can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If the command term is 'Hence' and not 'Hence or otherwise' then alternative methods are not permitted unless covered by a note in the mark scheme.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**.

7 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation** for example 1.9 and $1{,}9$ or 1000 and $1{,}000$ and 1.000.
- Do not accept final answers written using calculator notation. However, ***M*** marks and intermediate ***A*** marks can be scored, when presented using calculator notation, provided the evidence clearly reflects the demand of the mark.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, some **equivalent** answers will generally appear in brackets. Not all equivalent notations/answers/methods will be presented in the markscheme and examiners are asked to apply appropriate discretion to judge if the candidate work is equivalent.

8 Format and accuracy of answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. If the level of accuracy is not stated in the question, the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures*.

Where values are used in subsequent parts, the markscheme will generally use the exact value, however candidates may also use the correct answer to a "correct" level of accuracy (e.g 3 sf) in subsequent parts. The markscheme will often explicitly include the subsequent values that come "*from the use of 3 sf values*".

Simplification of final answers: Candidates are advised to give final answers using good mathematical form. In general, for an ***A*** mark to be awarded, arithmetic should be completed, and any values that lead to integers should be simplified; for example, $\sqrt{\frac{25}{4}}$ should be written as $\frac{5}{2}$. An exception to this is simplifying fractions, where lowest form is not required (although the numerator and the denominator must be integers); for example, $\frac{10}{4}$ may be left in this form or written as $\frac{5}{2}$. However, $\frac{10}{5}$ should be written as 2, as it simplifies to an integer.

Algebraic expressions should be simplified by completing any operations such as addition and multiplication, e.g. $4e^{2x} \times e^{3x}$ should be simplified to $4e^{5x}$, and $4e^{2x} \times e^{3x} - e^{4x} \times e^{x}$ should be simplified to $3e^{5x}$. Unless specified in the question, expressions do not need to be factorized, nor do factorized expressions need to be expanded, so $x(x+1)$ and $x^2 + x$ are both acceptable.

Please note: intermediate ***A*** marks do NOT need to be simplified.

9 Calculators

A GDC is required for this paper, but If you see work that suggests a candidate has used any calculator not approved for IB DP examinations (eg CAS enabled devices), please follow the procedures for malpractice.

10. Presentation of candidate work

Crossed out work: If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work unless an explicit note from the candidate indicates that they would like the work to be marked.

More than one solution: Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise. If the layout of the responses makes it difficult to judge, examiners should apply appropriate discretion to judge which is "first".

1. (a) (i) $T = 0.799G + 2.14 \quad (= 0.798803...G + 2.13972...)$ ***A1A1***

Note: Award ***A1*** for **correct** values of a and b, ***A1*** for an equation using these **correct** values.

(ii) $(r =)\ 0.996\ (= 0.996247...)$ ***A1***
(there is a very) strong positive linear correlation ***R1***

Note: If r is missing award ***A0R0***.

[4 marks]

(b) attempt to substitute 17 into their regression equation ***(M1)***
$0.798803...(17) + 2.13972...$
$15.7\ (\text{mins})\ (= 15.7193...)$ ***A1***

[2 marks]

(c) **EITHER**
using the T on G regression line cannot (always) reliably make a prediction for G ***R1***
OR
equation is for Time on Gradient, not Gradient on Time ***R1***
OR
this estimate is an extrapolation ***R1***
OR
there is no reason to assume this new hill has constant gradient ***R1***

[1 mark]

[Total 7 marks]

2. (a) (upper bound =) $0.525\ (\text{m})$ ***A1***
(lower bound =) $0.515\ (\text{m})$ ***A1***

Note: Accept an answer in interval notation or written as an inequality.

[2 marks]

(b) **METHOD 1 Convert REC to linear metres**
attempt to convert REC to metres using their lower bound ***(M1)***
$440\times0.515\ (=226.6)$ **OR** $280\times0.515\ (=144.2)$ seen

attempt to use the formula for the volume of a right pyramid ***(M1)***

$(V=)\ \frac{1}{3}(440\times0.515)^2(280\times0.515)$ ***(A1)***

$2\,470\,000\ (\text{m}^3)\ \ (2\,468\,106.051\ldots,\ 2.47\times10^6)$ ***A1***

METHOD 2 Convert REC to cubic metres
attempt to use the formula for the volume of a right pyramid ***(M1)***

$(V=)\ \frac{1}{3}(440)^2(280)\ \ (=18\,069\,333.33...)$
attempt to convert 1 cubic REC to cubic metres using their lower bound ***(M1)***
(1 cubic REC =) 0.515^3

$(V=)\ \frac{1}{3}(440)^2(280)\times(0.515)^3$ ***(A1)***
$2\,470\,000\ (\text{m}^3)\ \ (2\,468\,106.051\ldots,\ 2.47\times10^6)$ ***A1***

[4 marks]
[Total 6 marks]

3. (a) $x=0$ ***A1***

Note: Answer must be an equation; an answer of "0" or "the y-axis" is awarded ***A0***.

[1 mark]

(b) $\left(g'(x)=\right)-8x^{-2}+x$ ***A1A1A1***

Note: Award ***A1*** for -8 seen, ***A1*** for x^{-2} $\left(\text{or } \frac{1}{x^2}\right)$ and ***A1*** for second term being x.
Award at most ***A1A1A0*** if additional terms are seen.

(c) $x>2$ **OR** $(2,\ \infty)$ **OR** $2<x<\infty$ ***A1A1***

Note: Award ***A1*** for 2 seen and award ***A1*** for **correct** inequality.

[2 marks]
[Total 6 marks]

4. (a) $(4, 8)$ ***A1***

[1 mark]

(b) attempt to find the gradient of AC ***(M1)***

$\frac{13-3}{8-0}, \frac{10}{8}, \left(\frac{5}{4}\right), (1.25)$

attempt to substitute **their** coordinates and the negative reciprocal of **their** gradient into the equation of a straight line ***(M1)***

$y-8=-\frac{4}{5}(x-4)$ **OR** $8=-\frac{4}{5}(4)+c$ **OR** $c=11.2$

$y-8=-\frac{4}{5}(x-4) \quad (y=-0.8x+11.2,\ 4x+5y-56=0)$ ***A1***

[3 marks]

(c) (i) attempt to find one distance from a farm to any closest vertex ***M1***

finding a correct distance from at least two distinct vertices ***A1***

$7.58968\ldots, 4.472135\ldots \left(\sqrt{20}\right), 5.830951\ldots \left(\sqrt{34}\right)$

$\left(\frac{9}{11}, \frac{116}{11}\right)$ (is furthest) ***A1***

(ii) 7.59 (km) $(=7.58968...)$ ***A1***

[4 marks]

[Total 8 marks]

5. (a) recognizing supplementary angles or acute angles in right-triangles ***(M1)***

$(\hat{ABC}=)\ 41^\circ+(180^\circ-112^\circ),\ 41^\circ+(90^\circ-22^\circ)$

$\hat{ABC}=109^\circ$ ***A1***

[2 marks]

(b) $\hat{ACB}=49^\circ$ (may be seen in part (a)) ***(A1)***

attempt to substitute into the sine rule (or equivalent) ***(M1)***

$\frac{AC}{\sin 109^\circ}=\frac{100}{\sin 49^\circ}$ ***(A1)***

$AC=125$ (km) $(=125.282...)$ ***A1***

[4 marks]

[Total 6 marks]

6. (a) setting $h(x) = 1$ ***(M1)***

$\left(h^{-1}(1) =\right)\ 17$ ***A1***

[2 marks]

(b) $x > \frac{1}{2}$ ***A1A1***

Note: Award ***A1*** for $\frac{1}{2}$ seen, ***A1*** for completely correct answer.

[2 marks]
[Total 4 marks]

7. (a) $2.36 = a(3)^2 + b(3) + c$ **OR** $2.36 = 9a + 3b + c$ ***A1***

[1 mark]

(b) finding other equations to solve simultaneously ***(M1)***

$5 = a(10)^2 + b(10) + c$ **AND** $7.16 = a(17)^2 + b(17) + c$
OR $5 = 100a + 10b + c$ **AND** $7.16 = 289a + 17b + c$

any one coefficient in equation correct ***(A1)***

$f(x) = -0.00490x^2 + 0.441x + 1.08$ ***A1***

Note: Condone the 2 sf answer for the coefficient of x^2 i.e. -0.0049.

$\left(f(x) = -0.00489795\ldots x^2 + 0.440816\ldots x + 1.08163\ldots\right)$

$\left(f(x) = -\frac{6}{1225}x^2 + \frac{108}{245}x + \frac{53}{49}\right)$

Note: Award at most ***(M1)(A1)A0*** if answer is not expressed as an equation.

[3 marks]

(c) attempt to substitute 80 into their equation ***(M1)***

$\left(f(80) =\right)\ 5$ ***A1***

$5 > 4$ **OR** therefore the ball will go over the fence ***R1***

Note: Do not award ***A0R1***; their value must be seen to credit a correct conclusion.

[3 marks]

(d) setting their equation equal to zero, graph ***(M1)***

$0 = -0.00489795\ldots x^2 + 0.440816\ldots x + 1.08163\ldots$ **OR** $f(x) = 0$

$92.4\ \ (92.3902\ldots)\ \text{(m)}$ ***A1***

[2 marks]
[Total 9 marks]

8. (a) $\left(r'(-1)=\right)\ -8$ ***A2***

[2 marks]

(b) $\frac{1}{8}$ seen ***(A1)***

$y-6=\frac{1}{8}(x+1)$ **OR** $y=0.125x+6.13\ \ (6.125)$ **OR**

$y=\frac{x}{8}+\frac{49}{8}$ **OR** $x-8y+49=0$ ***A1***

[2 marks]

(c) attempt to find coordinates of point B ***(M1)***
e.g. $(-5.02,\ 5.50)$ **OR** $(1.02,\ 6.25)$ seen

attempt to substitute into distance formula ***(M1)***
$\left(d=\sqrt{(-1+5.02076\ldots)^2+(6-5.49740\ldots)^2}\right)$

$(d=)\ 4.05\ (\text{km})\ \ (4.05205\ldots)$ ***A1***

[3 marks]
[Total 7 marks]

9. (a)

Statement	True (✓)
A higher percentage of students in Class B received a grade less than 70 on the exam than Class A.	✓
The data for Class B is normally distributed.	
More students in Class A received a grade greater than 90 on the exam than Class B.	
The interquartile range for Class A is less than the interquartile range for Class B.	✓

A1A1

Note: Award ***A0A0*** if three or four statements are selected.

[2 marks]

(b) **EITHER**

$H_0 : \mu_1 = \mu_2$ ***A1***

$H_1 : \mu_1 \neq \mu_2$ ***A1***

OR

$H_0 : \mu_A = \mu_B$ ***A1***

$H_1 : \mu_A \neq \mu_B$ ***A1***

Note: Accept an equivalent statement in words, but must include reference to "population mean" / "mean for class A and class B" for the ***A1*** to be awarded.

Do not accept an imprecise "the means are equal".

[2 marks]

(c) p-value $= 0.111 \ (0.110700\ldots)$ ***A2***

[2 marks]

(d) $0.111 > 0.05$ ***R1***

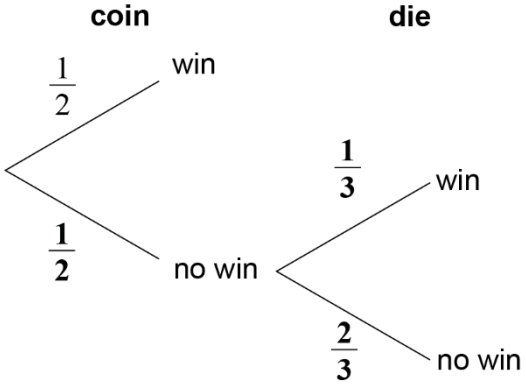
there is insufficient evidence to reject H_0 ***A1***

Note: Do not award ***R0A1***. The answer to part (d) MUST follow through if hypotheses are incorrect/reversed etc., the answer to part (d) must reflect this in order for the ***A1*** to be credited.

[2 marks]

[Total 8 marks]

10. (a)



A1A1

Note: Award ***A1*** for completing first set of branches, ***A1*** for completing second set of branches.

[2 marks]

(b) attempt to multiply along the branches ***(M1)***

$\frac{1}{2} \times \frac{2}{3}$

$= \frac{1}{3} \ (= 0.333...)$ ***A1***

[2 marks]

(c) **EITHER**

$$\frac{\frac{1}{2}}{\frac{1}{2} + \left(\frac{1}{2} \times \frac{1}{3}\right)}$$ ***M1A1***

Note: Award ***M1*** for recognizing conditional probability, ***A1*** for correct substitution.

OR

$$\frac{\frac{1}{2}}{1 - \frac{1}{3}}$$ ***M1A1***

Note: Award ***M1*** for recognizing conditional probability, ***A1*** for correct substitution.

THEN

$= \frac{3}{4}$ ***A1***

[3 marks]
[Total 7 marks]

11. (a) evidence of using binomial distribution ***(M1)***

Note: Evidence is $X \sim \mathrm{B}(5,\ 0.65)$ or binomial with $n = 5,\ p = 0.65$.

$0.181\ \ (0.181146\ldots)$ ***A1***

[2 marks]

(b) attempt to find the probability of taking a taxi, (or *not* taking a taxi); ***(M1)***
$\mathrm{P}(\text{take taxi}) = 0.35 \times 0.45$, $\mathrm{P}(\text{not take taxi}) = 0.65 + 0.35 \times 0.55$

0.1575 or 0.8425 seen ***(A1)***
EITHER
correct use of binomial distribution with their probability
$X \sim \mathrm{B}(5,\ 0.1575)$, $X = 0$ **OR** $X \sim \mathrm{B}(5,\ 0.8425)$, $X = 5$ ***(A1)***

OR
$(1 - 0.1575)^5$ **OR** $(0.8425)^5$ seen ***(A1)***

THEN
$0.424\ \ (0.424472\ldots)$ ***A1***

[4 marks]
[Total 6 marks]

12. (a) ***A1***

	Artist 1	Artist 2	Artist 3	Artist 4	Artist 5	Artist 6	Artist 7	Artist 8
Rank – social media followers	4	3	6	5	7	2	8	1
Rank – albums sold in first week	**2**	**4**	**8**	**3**	**5**	**6**	**7**	1

[1 mark]

(b) $(r_s =)\ 0.595\ \ (0.595238\ldots)$ ***A2***

[2 marks]

(c) $(\mathrm{H}_1 :)$ In the population, there is a **positive** monotonic relationship between the number of social media followers and the number of albums sold in the first week. ***A1***

[1 mark]

(d) $0.595 < 0.643$ ***R1***
there is insufficient evidence to reject H_0 ***A1***

Note: Do not award ***R0A1***.

[2 marks]
[Total 6 marks]

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Algebraic expressions should be simplified by completing any operations such as addition and multiplication, e.g. $4e^{2x} \times e^{3x}$ should be simplified to $4e^{5x}$, and $4e^{2x} \times e^{3x} - e^{4x} \times e^{x}$ should be simplified to $3e^{5x}$. Unless specified in the question, expressions do not need to be factorized, nor do factorized expressions need to be expanded, so $x(x+1)$ and $x^2 + x$ are both acceptable.

Please note: intermediate ***A*** marks do NOT need to be simplified.

9 Calculators

A GDC is required for this paper, but If you see work that suggests a candidate has used any calculator not approved for IB DP examinations (eg CAS enabled devices), please follow the procedures for malpractice.

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More than one solution: Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise. If the layout of the responses makes it difficult to judge, examiners should apply appropriate discretion to judge which is "first".

1. (a) (i) $T = 0.552G + 6.36 \quad (= 0.552139...G + 6.35703...)$ ***A1A1***

Note: Award ***A1*** for **correct** values of a and b, ***A1*** for an equation using these **correct** values.

(ii) $(r =)\ 0.994\ (= 0.993910...)$ ***A1***

there is a (very) strong positive linear correlation ***R1***

Note: If r is missing award ***A0R0***.

[4 marks]

(b) attempt to substitute 13 into their regression equation ***(M1)***

$T = 0.552139...(13) + 6.35703...$

$13.5\ (\text{mins})\ (= 13.5348...)$ ***A1***

[2 marks]

(c) **EITHER**

using the T on G regression line cannot (always) reliably make a prediction for G ***R1***

OR

equation is for Time on Gradient not Gradient on Time. ***R1***

OR

this estimate is an extrapolation ***R1***

OR

there is no reason to assume this new hill has constant gradient ***R1***

[1 mark]

[Total 7 marks]

2. (a) (upper bound =) $0.525\ (\text{m})$ ***A1***
(lower bound =) $0.515\ (\text{m})$ ***A1***

Note: Accept an answer in interval notation or written as an inequality.

[2 marks]

(b) **METHOD 1 Convert REC to linear metres**
attempt to convert REC to metres using their lower bound ***(M1)***
$440\times0.515\ (=226.6)$ **OR** $280\times0.515\ (=144.2)$ seen

attempt to use the formula for the volume of a right pyramid ***(M1)***

$(V=)\ \frac{1}{3}(440\times0.515)^2(280\times0.515)$ ***(A1)***

$2\,470\,000\ (\text{m}^3)\ \ (2\,468\,106.051\ldots,\ 2.47\times10^6)$ ***A1***

METHOD 2 Convert REC to cubic metres
attempt to use the formula for the volume of a right pyramid ***(M1)***

$(V=)\ \frac{1}{3}(440)^2(280)\ \ (=18\,069\,333.33...)$
attempt to convert 1 cubic REC to cubic metres using their lower bound ***(M1)***
(1 cubic REC =) 0.515^3

$(V=)\ \frac{1}{3}(440)^2(280)\times(0.515)^3$ ***(A1)***
$2\,470\,000\ (\text{m}^3)\ \ (2\,468\,106.051\ldots,\ 2.47\times10^6)$ ***A1***

[4 marks]
[Total 6 marks]

3. (a) $x=0$ ***A1***

Note: Answer must be an equation; an answer of "0" or "the y-axis" is awarded ***A0***.

[1 mark]

(b) $\left(f'(x)=\right)-16x^{-2}+\frac{x}{4}$ ***A1A1A1***

Note: Award ***A1*** for -16 seen, ***A1*** for x^{-2} $\left(\text{or } \frac{1}{x^2}\right)$ and ***A1*** for second term being $\frac{x}{4}$.
Award at most ***A1A1A0*** if additional terms are seen.

[3 marks]

(c) $x>4$ **OR** $(4,\ \infty)$ **OR** $4<x<\infty$ ***A1A1***

Note: Award ***A1*** for 4 seen and award ***A1*** for **correct** inequality.

[2 marks]
[Total 6 marks]

4. (a) $(4, 8)$ ***A1***

[1 mark]

(b) attempt to find the gradient of AC ***(M1)***

$\frac{13-3}{8-0}, \frac{10}{8}, \left(\frac{5}{4}\right), (1.25)$

attempt to substitute **their** coordinates and the negative reciprocal of **their** gradient into the equation of a straight line ***(M1)***

$y-8=-\frac{4}{5}(x-4)$ **OR** $8=-\frac{4}{5}(4)+c$ **OR** $c=11.2$

$y-8=-\frac{4}{5}(x-4) \quad (y=-0.8x+11.2,\ 4x+5y-56=0)$ ***A1***

[3 marks]

(c) (i) attempt to find one distance from a farm to any closest vertex ***M1***

finding a correct distance from at least two distinct vertices ***A1***

$7.58968\ldots, 4.472135\ldots \left(\sqrt{20}\right), 5.830951\ldots \left(\sqrt{34}\right)$

$\left(\frac{9}{11}, \frac{116}{11}\right)$ (is furthest) ***A1***

(ii) $7.59 \text{ (km)} \ (=7.58968...)$ ***A1***

[4 marks]

[Total 8 marks]

5. (a) recognizing supplementary angles or acute angles in right-triangles ***(M1)***

$(\hat{RST}=)\ 38^\circ+(180^\circ-120^\circ),\ 38^\circ+(90^\circ-30^\circ)$

$\hat{RST}=98^\circ$ ***A1***

[2 marks]

(b) $\hat{RTS}=52^\circ$ (may be seen in part (a)) ***(A1)***

attempt to substitute into the sine rule (or equivalent) ***(M1)***

$\frac{RT}{\sin 98^\circ}=\frac{150}{\sin 52^\circ}$ ***(A1)***

$RT=189 \text{ (km)} \ (=188.500...)$ ***A1***

[4 marks]

[Total 6 marks]

6. (a) setting $h(x) = 1$ ***(M1)***

$\left(h^{-1}(1) =\right) 17$ ***A1***

[2 marks]

(b) $x > \frac{1}{2}$ ***A1A1***

Note: Award ***A1*** for $\frac{1}{2}$ seen, ***A1*** for completely correct answer.

[2 marks]
[Total 4 marks]

7. (a) $2.82 = a(3)^2 + b(3) + c$ **OR** $2.82 = 9a + 3b + c$ ***A1***

[1 mark]

(b) finding other equations to solve simultaneously ***(M1)***

$4.25 = a(6)^2 + b(6) + c$ **AND** $5.30 = a(9)^2 + b(9) + c$
OR $4.25 = 36a + 6b + c$ **AND** $5.30 = 81a + 9b + c$

any one coefficient in equation correct ***(A1)***

$f(x) = -0.0211x^2 + 0.667x + 1.01$ ***A1***
$\left(f(x) = -0.0211111\ldots x^2 + 0.666666\ldots x + 1.01\right)$
$\left(f(x) = -\frac{19}{900}x^2 + \frac{2}{3}x + \frac{101}{100}\right)$

Note: Award at most ***(M1)(A1)A0*** if answer is not expressed as an equation.

[3 marks]

(c) attempt to substitute 30 into their equation ***(M1)***

$\left(f(30) =\right)\ 2.01$ ***A1***

$2.01 > 1.8$ **OR** therefore the discus will go over the wall ***R1***

Note: Do not award ***A0R1***; their value must be seen to credit a correct conclusion.

[3 marks]

(d) setting their equation equal to zero **OR** graph with the zero indicated ***(M1)***

$0 = -0.0211111\ldots x^2 + 0.666666\ldots x + 1.01\ldots$ **OR** $f(x) = 0$

$33.0\ \ (33.0275\ldots)\ \text{(m)}$ ***A1***

[2 marks]
[Total 9 marks]

8. (a) $(r'(-2) =)\ -14$ ***A2***

[2 marks]

(b) $\frac{1}{14}$ seen ***(A1)***

$y - 16 = \frac{1}{14}(x + 2)$ **OR** $y = 0.0714x + 16.1$ $(y = 0.0714285...x + 16.1428...)$ **OR**

$y = \frac{x}{14} + \frac{113}{7}$ **OR** $x - 14y + 226 = 0$ ***A1***

[2 marks]

(c) attempt to find coordinates of point G ***(M1)***

e.g. $(-7.53997\ldots, 15.6042\ldots)$ **OR** $(0.539978..., 13.1814\ldots)$ seen

attempt to substitute into distance formula ***(M1)***

$\left(d = \sqrt{(-2 + 7.53997\ldots)^2 + (16 - 15.6042\ldots)^2}\right)$

$(d =)\ 5.55\,(\text{km})\ (5.55409\ldots)$ ***A1***

[3 marks]

[Total 7 marks]

9. (a)

Statement	True (✓)
The data for Class A is normally distributed.	
A higher percentage of students in Class A received a grade less than 70 on the exam, than in Class B.	✓
More students in Class B received a grade greater than 90 on the exam than in Class A.	
The interquartile range for Class B is less than the interquartile range for Class A.	✓

A1A1

Note: Award ***A0A0*** if three or four statements are selected.

[2 marks]

(b) **EITHER**

$H_0 : \mu_1 = \mu_2$ ***A1***

$H_1 : \mu_1 \neq \mu_2$ ***A1***

OR

$H_0 : \mu_A = \mu_B$ ***A1***

$H_1 : \mu_A \neq \mu_B$ ***A1***

Note: Accept an equivalent statement in words, but must include reference to "population mean" / "mean for class A and class B" for the ***A1*** to be awarded.

Do not accept an imprecise "the means are equal".

[2 marks]

(c) p-value $= 0.0952 \ \ (0.0952085\ldots)$ ***A2***

[2 marks]

(d) $0.0952 > 0.05$ ***R1***

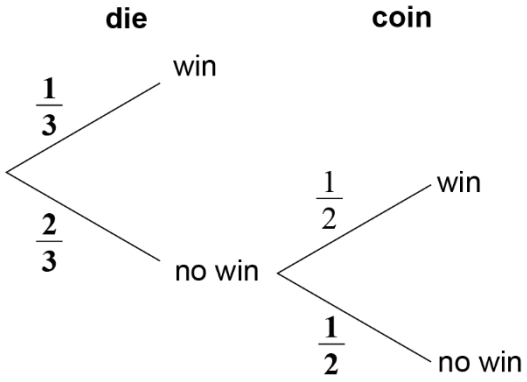
there is insufficient evidence to reject H_0 ***A1***

Note: Do not award ***R0A1***. The answer to part (d) MUST follow through from their hypotheses seen in part (b) and their p-value seen in part (c); if hypotheses are incorrect/reversed, etc., the answer to part (d) must reflect this in order for the ***A1*** to be credited.

[2 marks]

[Total 8 marks]

10. (a)



A1A1

Note: Award ***A1*** for completing first set of branches, ***A1*** for completing second set of branches.

[2 marks]

(b) attempt to multiply along the branches ***(M1)***

$\frac{2}{3}\times\frac{1}{2}$

$=\frac{1}{3}\ \ (=0.333...)$ ***A1***

[2 marks]

(c) **EITHER**

$$\frac{\frac{1}{3}}{\frac{1}{3}+\left(\frac{2}{3}\times\frac{1}{2}\right)}$$ ***M1A1***

Note: Award ***M1*** for recognizing conditional probability, ***A1*** for correct substitution.

OR

$$\frac{\frac{1}{3}}{1-\frac{1}{3}}$$ ***M1A1***

Note: Award ***M1*** for recognizing conditional probability, ***A1*** for correct substitution.

THEN

$=\frac{1}{2}$ ***A1***

[3 marks]
[Total 7 marks]

11. (a) evidence of using binomial distribution ***(M1)***

Note: Evidence is $X \sim \mathrm{B}(5,\ 0.72)$ or binomial with $n = 5,\ p = 0.72$.

$0.293\quad (0.292626\ldots)$ ***A1***

[2 marks]

(b) attempt to find the probability of taking a bus, (or *not* taking a bus); ***(M1)***
$\mathrm{P}(\text{take bus}) = 0.28 \times 0.42,\ \mathrm{P}(\text{not take bus}) = 0.72 + 0.28 \times 0.58$

0.1176 or 0.8824 seen ***(A1)***

EITHER
correct use of binomial distribution with their probability
$X \sim \mathrm{B}(5,\ 0.1176),\ X = 0$ **OR** $X \sim \mathrm{B}(5,\ 0.8824),\ X = 5$ ***(A1)***
OR
$(1 - 0.1176)^5$ **OR** $(0.8824)^5$ seen ***(A1)***

THEN
$0.535\quad (0.534967\ldots)$ ***A1***

[4 marks]
[Total 6 marks]

12. (a) ***A1***

	Artist 1	Artist 2	Artist 3	Artist 4	Artist 5	Artist 6	Artist 7	Artist 8
Rank – social media followers	4	3	6	5	7	2	8	1
Rank – albums sold in first week	**2**	**4**	**8**	**3**	**5**	**6**	**7**	1

[1 mark]

(b) $(r_s =)\ 0.595\ \ (0.595238\ldots)$ ***A2***

[2 marks]

(c) $(\mathrm{H}_1 :)$ In the population, there is a **positive** monotonic relationship between the number of social media followers and the number of albums sold in the first week. ***A1***

[1 mark]

(d) $0.595 < 0.643$ ***R1***

there is insufficient evidence to reject H_0 ***A1***

Note: Do not award ***R0A1***.

[2 marks]
[Total 6 marks]

Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

May 2023

Mathematics: applications and interpretation

Standard level

Paper 1

20 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

R Marks awarded for clear **Reasoning**.

AG Answer given in the question and so no marks are awarded.

FT Follow through. The practice of awarding marks, despite candidate errors in previous parts, for their correct methods/answers using incorrect results.

Using the markscheme

1 General

Award marks using the annotations as noted in the markscheme eg ***M1***, ***A2***.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***A3***, ***M2*** *etc.*, do **not** split the marks, unless there is a note.
- The response to a "show that" question does not need to restate the ***AG*** line, unless a **Note** makes this explicit in the markscheme.
- Once a correct answer to a question or part question is seen, ignore further working even if this working is incorrect and/or suggests a misunderstanding of the question. This will encourage a uniform approach to marking, with less examiner discretion. Although some candidates may be advantaged for that specific question item, it is likely that these candidates will lose marks elsewhere too.
- An exception to the previous rule is when an incorrect answer from further working is used **in a subsequent part**. For example, when a correct exact value is followed by an incorrect decimal approximation in the first part and this approximation is then used in the second part. In this situation, award ***FT*** marks as appropriate but do not award the final ***A1*** in the first part. Examples:

	Correct answer seen	Further working seen	Any FT issues?	Action
1.	$8\sqrt{2}$	5.65685... *(incorrect decimal value)*	No. Last part in question.	Award ***A1*** for the final mark *(condone the incorrect further working)*
2.	$\frac{35}{72}$	0.468111… (incorrect decimal value)	Yes. Value is used in subsequent parts.	Award ***A0*** for the final mark *(and full **FT** is available in subsequent parts)*

3 Implied marks

Implied marks appear in **brackets e.g. *(M1)***,and can only be awarded if **correct** work is seen or implied by subsequent working/answer.

4 Follow through marks (only applied after an error is made)

Follow through (***FT***) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) (e.g. incorrect value from part (a) used in part (d) or incorrect value from part (c)(i) used in part (c)(ii)). Usually, to award ***FT*** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if all the marks awarded in a subsequent part are for the answer or are implied, then ***FT*** marks should be awarded for *their* correct answer, even when working is not present.

For example: following an incorrect answer to part (a) that is used in subsequent parts, where the markscheme for the subsequent part is ***(M1)A1***, it is possible to award full marks for *their* correct answer, **without working being seen.** For longer questions where all but the answer marks are implied this rule applies but may be overwritten by a **Note** in the Markscheme.

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks, by reflecting on what each mark is for and how that maps to the simplified version.
- If the error leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word “their” in a description, to indicate that candidates may be using an incorrect value.
- If the candidate’s answer to the initial question clearly contradicts information given in the question, it is not appropriate to award any ***FT*** marks in the subsequent parts. This includes when candidates fail to complete a “show that” question correctly, and then in subsequent parts use their incorrect answer rather than the given value.
- Exceptions to these ***FT*** rules will be explicitly noted on the markscheme.
- If a candidate makes an error in one part but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the command term was “Hence”.

5 Mis-read

If a candidate incorrectly copies values or information from the question, this is a mis-read (***MR***). A candidate should be penalized only once for a particular misread. Use the ***MR*** stamp to indicate that this has been a misread and do not award the first mark, even if this is an ***M*** mark, but award all others as appropriate.

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.
- If a candidate uses a correct answer, to a "show that" question, to a higher degree of accuracy than given in the question, this is NOT a misread and full marks may be scored in the subsequent part.
- ***MR*** can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If the command term is 'Hence' and not 'Hence or otherwise' then alternative methods are not permitted unless covered by a note in the mark scheme.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**.

7 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation** for example 1.9 and $1{,}9$ or 1000 and $1{,}000$ and 1.000.
- Do not accept final answers written using calculator notation. However, ***M*** marks and intermediate ***A*** marks can be scored, when presented using calculator notation, provided the evidence clearly reflects the demand of the mark.
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Algebraic expressions should be simplified by completing any operations such as addition and multiplication, e.g. $4e^{2x} \times e^{3x}$ should be simplified to $4e^{5x}$, and $4e^{2x} \times e^{3x} - e^{4x} \times e^{x}$ should be simplified to $3e^{5x}$. Unless specified in the question, expressions do not need to be factorized, nor do factorized expressions need to be expanded, so $x(x+1)$ and $x^2 + x$ are both acceptable.

Please note: intermediate ***A*** marks do NOT need to be simplified.

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More than one solution: Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise. If the layout of the responses makes it difficult to judge, examiners should apply appropriate discretion to judge which is "first".

1. (a) attempt to substitute into percentage error formula ***(M1)***

$$\left|\frac{53632000-55625000}{55625000}\right|\times 100$$

$3.58\ (\%)\ \ (3.58292\ldots(\%))$ ***A1***

Note: Award ***(M1)A0*** for a final answer of $-3.58(\%)$ or 0.0358.

[2 marks]

(b) (i) $278\,000\,000$ ***A1***

(ii) 2.78×10^{8} ***A1A1***

Note: Award ***A1*** for correct mantissa, consistent with their answer in part (b)(i).
Award ***A1*** for a correct exponent, consistent with their answer in part (b)(i).
Award ***A0A0*** for answers such as 27.8×10^{7}.

[3 marks]
Total [5 marks]

2. (a) **METHOD 1 (use of financial app in GDC)**

$N = 5$	**OR**	$N = 20$
$I\% = 1.2$		$I\% = 1.2$
$PV = \pm 520$		$PV = \pm 520$
$P/Y = 1$		$P/Y = 4$
$C/Y = 4$		$C/Y = 4$

(M1)(A1)

Note: Award ***M1*** for evidence of using the financial app on the calculator, ***A1*** for all correct entries.

($) 552.11 ***A1***

Note: Award at most ***(M1)(A1)A0*** if correct answer is not given to two decimal places.

METHOD 2 (use of formula)

attempt to substitute into compound interest formula ***(M1)***

$$520 \times \left(1 + \frac{1.2}{100 \times 4}\right)^{5 \times 4} \quad \textit{(A1)}$$

($) 552.11 ***A1***

Note: Award at most ***(M1)(A1)A0*** if correct answer is not given to two decimal places.

[3 marks]

(b) **EITHER**

$N = 5$

$I\% = 43.5 \ (43.4772...(\%))$

$PV = \pm 520$

$FV = \mp 30$ ***(M1)(A1)A1***

Note: Award ***M1*** for evidence of using the finance app on the calculator, ***A1*** for all correct entries, ***A1*** for correct final answer. Condone missing -/+ sign if the correct final answer is seen.

OR

$$30 = 520\left(1 - \frac{r}{100}\right)^5 \quad \text{(or equivalent)} \quad \textit{(M1)(A1)}$$

$(r =) \ 43.5\% \ (43.477...\%)$ ***A1***

Note: Award ***M1*** for using the compound interest formula, ***A1*** for correct substitutions and for equating to 30, ***A1*** for correct final answer. Accept $(r =) \ -43.5\%$.
Award ***M1A1A0*** for a final answer of 56.5%.

[3 marks]

Total [6 marks]

3. (a) (i) 38 (s) ***A1***

(ii) 32 (s) ***A1***

(iii) 42 (s) ***A1***

(iv) 10 (s) ***A1***

Note: Accept a tolerance of ± 0.5 for parts (a)(i)-(iii).

[4 marks]

(b) 1.5 x IQR ***(M1)***

$(32 - 1.5 \times 10 =)\ 17$ (s) ***A1***

$14 < 17$, therefore it is an outlier ***R1***

Note: Do not award the ***R1*** unless an explicit comparison of 14 and their 17 is seen.
e.g. $14 < 17$
14 is outside the interval $[17, 57]$.

[3 marks]
Total [7 marks]

4. (a)

Athlete	A	B	C	D	E	F	G	H
Age rank	7	6	3	5	4	2	8	1
Time rank	3.5	2	3.5	6	7	8	1	5

A1A1

Note: Award ***A1*** for each correct row.

[2 marks]

(b) $r_s = -0.671 \ (-0.670670...)$ ***A2***

Note: Only follow through from an incorrect table provided the ranks are all between 1 and 8.
Award ***A1*** for -0.67 **OR** for the omission of the negative sign, e.g. 0.671 $(0.670670...)$ or 0.67

[2 marks]

(c) (A value of $r_S = -0.671$) indicates a negative correlation between a person's age and the best time they take to run 100m. ***R1***

Note: Condone any comment that includes "weak" or "strong" etc. Accept an interpretation in words, but only if there is a general link described and not a rule: "The older a person gets, the faster they *tend to* run".
Answer must be in context.

[1 mark]

(d) *Award **R1** for any sensible reason:* ***R1***
The correlation, such that it is, is unlikely to be linear for this type of data.
Spearman's CC is less sensitive to outliers
Sung-Jin is not sure the data is drawn from a bivariate normal distribution
There are outliers/extreme data
Same time for two athletes with significantly different ages

[1 mark]
Total [6 marks]

5. (a) $34+p$ ***A1***

[1 mark]

(b) attempt to substitute into the mean formula, equating to 4.5 ***(M1)***

$$\frac{1\times1+2\times4...5\times p+6\times9+7\times4}{34+p}=4.5$$ ***A1***

$(p=)\ 10$ ***A1***

Note: Do not award the final ***A1*** if final answer is not an integer.
Award ***(M1)A0A1*** for an unsupported answer of $(p=)\ 10$.

[3 marks]
Total [4 marks]

6. (a) $0.565\ \left(0.564655..., \frac{131}{232}, 56.4655...\%\right)$ ***A1A1***

Note: Award ***A1*** for correct numerator, ***A1*** for correct denominator.

[2 marks]

(b) $11.0\ (11.0212\ldots)$ ***A2***

Note: Award ***A1*** for a final answer of 11 if no unrounded answer is seen.

[2 marks]

(c) **EITHER**
$11.0 > 9.488\ (11.0212\ldots > 9.488\ )$ ***R1***

OR
$0.0263 < 0.05\ (0.0263264\ldots < 0.05\ )$ ***R1***

THEN

EITHER
(there is significant evidence to) reject H_0 ***A1***

OR
(there is significant evidence that) the (food) quality and the type of meal are not independent ***A1***

Note: Do not award ***R0A1.***
Award ***R1*** for $\chi^2_{\text{calc}} > \chi^2_{\text{crit}}$, provided the calculated value is explicitly seen in part (b).

Accept "p-value < significance level" provided their p-value is seen and their p-value is between 0 and 1.

[2 marks]
Total [6 marks]

7. (a) attempt to find gradient of CD ***(M1)***

gradient of $\mathrm{CD} = \frac{1}{3}$ therefore perpendicular gradient $\mathrm{CD} = -3$ ***A1***

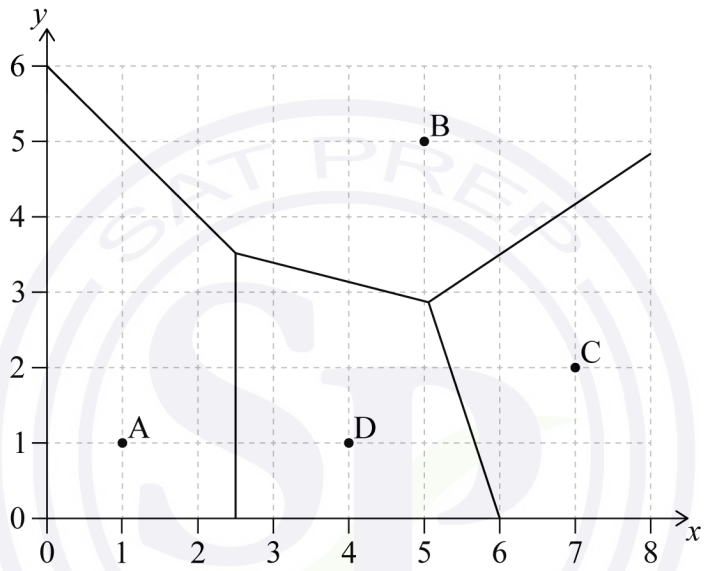
$y - 1.5 = -3(x - 5.5)$ **OR** $1.5 = -3(5.5) + c$ ***M1***

Note: Award ***M1*** for substituting the gradient and midpoint into equation of line, provided further work is seen leading to a correct answer.

$y = -3x + 18$ ***AG***

[3 marks]

(b)



perpendicular bisector AD: a vertical line with x intercept 2.5 ***A1***

Note: The perpendicular bisector should not go beyond the intersection point (should not enter site B).

[1 mark]

(c) attempt to solve simultaneous equations: $3y = 2x - 1.5$ and $y = -3x + 18$ ***(M1)***

$(5.05, 2.86)$ $((5.04545\ldots, 2.86363\ldots))$ ***A1***

Note: Accept $x = 5.05$ $(5.04545...)$, $y = 2.86$ $(2.86363...)$ in place of coordinates.
Accept $(5.05, 2.87)$ and $(5.05, 2.85)$ for using their 3 sf or 4 sf x-value to find y from any of the two equations.

[2 marks]
Total [6 marks]

8. (a) **EITHER**

$\frac{4}{3}\pi(3.4)^3$ ***(A1)***

multiplying their volume by $\frac{4}{5}$ ***(M1)***

OR

$\frac{4}{3}\pi(3.4)^3$ ***(A1)***

Subtracting $\frac{1}{5}$ of their volume ***(M1)***

$\left(\frac{4}{3}\pi(3.4)^3 - \frac{1}{5}\times\frac{4}{3}\pi(3.4)^3\right)$

Note: The ***M1*** can be awarded for a final answer of $32.9272\ldots$ seen without working.

THEN

$132\ \text{cm}^3$ $(131.708\ldots\ \text{cm}^3)$ ***A1***

[3 marks]

(b) $\pi\times 3\times 11$ ***(A1)***

$103.672\ldots\ (\text{cm}^2)$ **OR** $33\pi\ (\text{cm}^2)$

$104\ (\text{cm}^2)$ ***A1***

[2 marks]

Total [5 marks]

9. (a) $X \sim \mathrm{N}(4, 0.25^2)$

EITHER

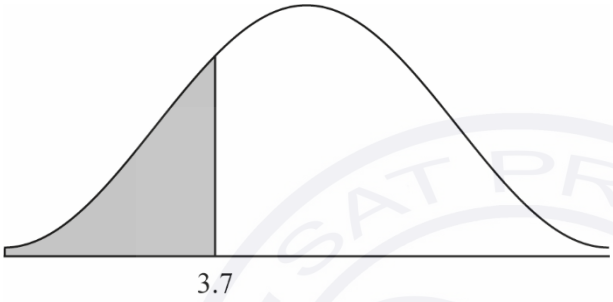
correct probability expression ***(M1)***

$\mathrm{P}(X < 3.7)$

Note: Accept a weak or strict inequality, and any label instead of X, e.g. length or L.

OR

normal curve with vertical line, left of mean, labelled 3.7, and shaded region ***(M1)***



THEN

0.115 $(0.115069\ldots, 11.5\%)$ ***A1***

Note: Award ***M1A0*** for 0.12 if no previous working.

[2 marks]

(b) **EITHER**

Correct probability expression ***(M1)***

$(\mathrm{P}(X < k) = 0.7$ **OR** $\mathrm{P}(X > k) = 0.3$

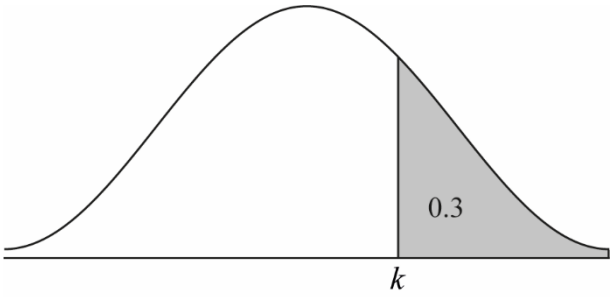
Note: Accept a weak or strict inequality, and any label instead of X *e.g.*, length or L.

continued…

Question 9 continued

OR

normal curve with vertical line to the right of the mean and shaded region, correctly labelled either 0.3 or 0.7 ***(M1)***



THEN

$(k =)\ 4.13\ (4.13110\ldots)$ ***A1***

Note: Award ***M1A0*** for 4.1 if no previous working.

[2 marks]

(c) **EITHER**

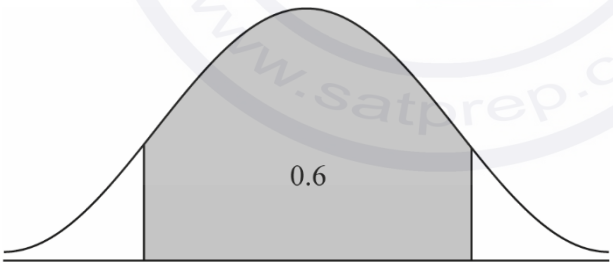
correct probability equation ***(M1)***

$\mathrm{P}(\text{length} < 4 + m) = 0.8$ **OR** $\mathrm{P}(\text{length} < 4 - m) = 0.2$

Note: Accept any letter instead of "length" *e.g.*, X or L.

OR

normal curve with vertical lines symmetrical about the mean line with a correct indication of an area of 0.6 or 0.2 or 0.8 ***(M1)***



THEN

$0.210\ (0.210405\ldots)$ ***A1***

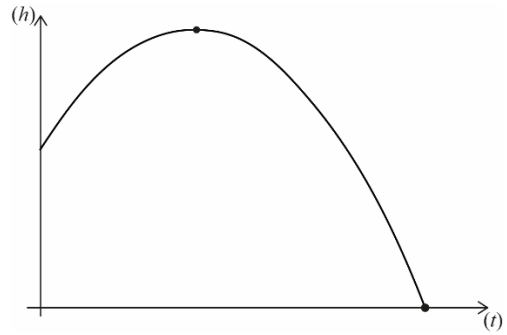
Note: Award ***(M1)A0*** for an answer of 3.7895 or 4.2105 seen without working.

[2 marks]

Total [6 marks]

10. (a) **METHOD 1**

correct sketch with some indication of maximum point ***(M1)***



0.921 (seconds) $\left(0.921052..., \frac{35}{38}\right)$ ***A1***

METHOD 2

correct substitution into equation for line of symmetry ***(M1)***

$(t =) \ -\frac{8.75}{2\times -4.75}$

0.921 (seconds) $\left(0.921052..., \frac{35}{38}\right)$ ***A1***

METHOD 3

equating the correct derivative to 0 ***(M1)***

$-9.5t + 8.75 = 0$

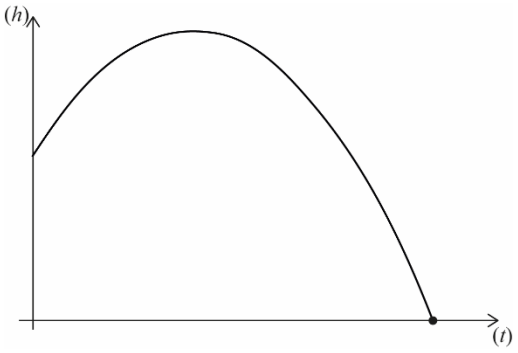
0.921 (seconds) $\left(0.921052..., \frac{35}{38}\right)$ ***A1***

Note: Award ***M1A0*** for a final answer of 0.92 seen with no working.

[2 marks]

(b) **METHOD 1**

correct sketch with some indication of x-intercept



(M1)

Note: May be seen in part (a).

2 (seconds) ***A1***

continued…

Question 10 continued

METHOD 2
setting the equation to zero ***(M1)***

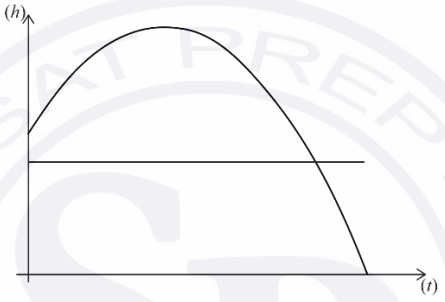
$-4.75t^2 + 8.75t + 1.5 = 0$

2 (seconds) ***A1***

Note: If both roots are given, with or without working, award ***(M1)A0***.

[2 marks]

(c) **METHOD 1**
correct sketch of quadratic function and a straight line in approximate correct position ***(M1)***



1.88 (seconds) (1.87577…(seconds)) ***A1***

METHOD 2
setting the equation equal to 1.2 ***(M1)***
$-4.75t^2 + 8.75t + 1.5 = 1.2$
1.88 (seconds) (1.87577…(seconds)) ***A1***

Note: Award ***(M1)A0*** if -0.0336702… seen as (part of) a final answer.
Award ***M1A0*** for answer of 1.9 seen without working.

[2 marks]

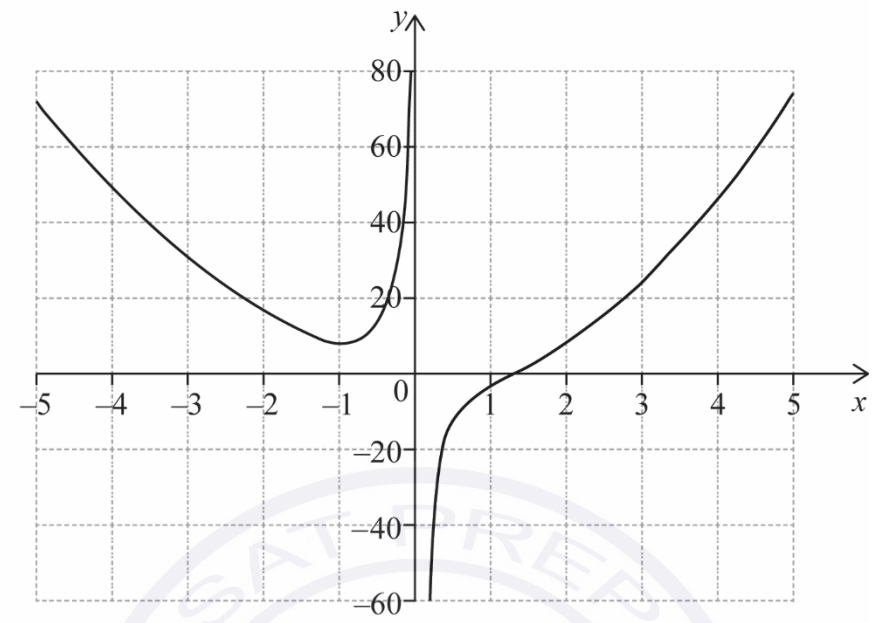
(d) *Award **R1** for each sensible reason, in the context of the question:* ***R1R1***
e.g.
The model ignores air resistance (or wind)
The model treats the ball as a point
The model assumes gravity is constant
The model assumes that the ball continues to follow the trajectory even after hitting the ground
This model ignores the bouncing back of the ball after hitting the ground

Note: Do not accept generic criticisms of any mathematical model, such as:
There are assumptions being made
Models are never accurate / It is only a model

[2 mark]

Total [8 marks]

11. (a) (i)



A1A1A1

Note: Award ***A1*** for correct shape (curve must be smooth) in the second quadrant only, ***A1*** for asymptotic behaviour, ***A1*** for minimum point in approximately correct position AND left-most point around $(-5, 76)$ (allow a tolerance of half a square for these two points).

(ii) $-0.941\ (-0.941035...)$ ***A1***

[4 marks]

(b) $-2.45\ \ (-2.44651\ldots),\ -0.252\ \ (-0.252412),\ 2.70\ \ (2.69892\ldots)$ ***A2***

Note: Award ***A2*** for all three correct, ***A1*** for two correct and ***A0*** otherwise.
Award at most ***A1*** if additional solutions are seen.
Award ***A1A0*** for a final answer given as coordinates $(-2.45, 20)$, $(-0.252, 20)$, $(2.70, 20)$ or $(2.7, 20)$.
Award ***A1A0*** for three correct answers given in 2 sf.

[2 marks]

(c) $x = 0$ ***A1***

[1 mark]
Total [7 marks]

12. (a) $0.15+0.2+k+0.16+2k+0.25=1$ ***(M1)***

$k=0.08$ ***A1***

[2 marks]

(b) $(-4\times0.15)+(-3\times0.2)+(-1\times0.08)+(0\times0.16)+(1\times0.16)+(4\times0.25)$ ***(M1)***

$=-0.12$ ***A1***

$\mathrm{E}(X)\neq0$ therefore the game is not fair ***R1***

Note: Do not award ***A0R1*** without an explicit value for $\mathrm{E}(X)$ seen. The ***R1*** can be awarded for comparing **their** $\mathrm{E}(X)$ to zero provided working is shown.

[3 marks]

Total [5 marks]

13. (a) attempt at using the trapezoidal rule ***(M1)***

$\text{area}=\frac{1}{2}\left(3+2(8+19)+42\right)$

$=49.5\ (\mathrm{m}^2)$ ***A1***

[2 marks]

(b) recognition of need to integrate (***e.g.*** reverse power rule or integral symbol) ***(M1)***

$\int 3x^2+4\,\mathrm{d}x=x^3+4x+c$ ***(A1)(A1)***

Note: Award ***A1*** for each correct term.

$f(x)=x^3+4x+3$ ***A1***

Note: Award ***A1*** for simplified correct answer including the value of c. Accept a value of c of 3.005 or 3.025 or 2.975 for using the non-integer x-values and their corresponding y-values.

[4 marks]

(c) **METHOD 1**

forming expression for sum of integral and deconstructing the trapezoid into a rectangle and triangle ***(M1)***

$\int_0^3 x^3+4x+3\ \mathrm{d}x(=47.25)+42\times1+\frac{1}{2}\times2\times42\ (=84)$ ***(A1)***

$=131\ (\mathrm{m}^2)\ (131.25)$ ***A1***

METHOD 2

forming expression for sum of integral and trapezoid ***(M1)***

$\int_0^3 x^3+4x+3\ \mathrm{d}x(=47.25)\ +\frac{1}{2}\times4\times42\ (=84)$ ***(A1)***

$=131\ (\mathrm{m}^2)\ (131.25)$ ***A1***

Note: Award ***(A1)*** for their integral with the correct limits added to 84 or their 47.25 added to 84.

[3 marks]

Total [9 marks]

Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

May 2023

Mathematics: applications and interpretation

Standard level

Paper 1

24 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

R Marks awarded for clear **Reasoning**.

AG Answer given in the question and so no marks are awarded.

FT Follow through. The practice of awarding marks, despite candidate errors in previous parts, for their correct methods/answers using incorrect results.

Using the markscheme

1 General

Award marks using the annotations as noted in the markscheme eg ***M1***, ***A2***.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***A3***, ***M2*** *etc.*, do **not** split the marks, unless there is a note.
- The response to a "show that" question does not need to restate the ***AG*** line, unless a **Note** makes this explicit in the markscheme.
- Once a correct answer to a question or part question is seen, ignore further working even if this working is incorrect and/or suggests a misunderstanding of the question. This will encourage a uniform approach to marking, with less examiner discretion. Although some candidates may be advantaged for that specific question item, it is likely that these candidates will lose marks elsewhere too.
- An exception to the previous rule is when an incorrect answer from further working is used **in a subsequent part**. For example, when a correct exact value is followed by an incorrect decimal approximation in the first part and this approximation is then used in the second part. In this situation, award ***FT*** marks as appropriate but do not award the final ***A1*** in the first part. Examples:

	Correct answer seen	Further working seen	Any FT issues?	Action
1.	$8\sqrt{2}$	5.65685... *(incorrect decimal value)*	No. Last part in question.	Award ***A1*** for the final mark *(condone the incorrect further working)*
2.	$\frac{35}{72}$	0.468111… (incorrect decimal value)	Yes. Value is used in subsequent parts.	Award ***A0*** for the final mark *(and full **FT** is available in subsequent parts)*

3 Implied marks

Implied marks appear in **brackets e.g.** ***(M1)***,and can only be awarded if **correct** work is seen or implied by subsequent working/answer.

4 Follow through marks (only applied after an error is made)

Follow through (***FT***) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) (e.g. incorrect value from part (a) used in part (d) or incorrect value from part (c)(i) used in part (c)(ii)). Usually, to award ***FT*** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if all the marks awarded in a subsequent part are for the answer or are implied, then ***FT*** marks should be awarded for *their* correct answer, even when working is not present.

For example: following an incorrect answer to part (a) that is used in subsequent parts, where the markscheme for the subsequent part is ***(M1)A1***, it is possible to award full marks for *their* correct answer, **without working being seen.** For longer questions where all but the answer marks are implied this rule applies but may be overwritten by a **Note** in the Markscheme.

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks, by reflecting on what each mark is for and how that maps to the simplified version.
- If the error leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word “their” in a description, to indicate that candidates may be using an incorrect value.
- If the candidate’s answer to the initial question clearly contradicts information given in the question, it is not appropriate to award any ***FT*** marks in the subsequent parts. This includes when candidates fail to complete a “show that” question correctly, and then in subsequent parts use their incorrect answer rather than the given value.
- Exceptions to these ***FT*** rules will be explicitly noted on the markscheme.
- If a candidate makes an error in one part but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the command term was “Hence”.

5 Mis-read

If a candidate incorrectly copies values or information from the question, this is a mis-read (***MR***). A candidate should be penalized only once for a particular misread. Use the ***MR*** stamp to indicate that this has been a misread and do not award the first mark, even if this is an ***M*** mark, but award all others as appropriate.

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.
- If a candidate uses a correct answer, to a "show that" question, to a higher degree of accuracy than given in the question, this is NOT a misread and full marks may be scored in the subsequent part.
- ***MR*** can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If the command term is 'Hence' and not 'Hence or otherwise' then alternative methods are not permitted unless covered by a note in the mark scheme.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**.

7 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation** for example 1.9 and $1{,}9$ or 1000 and $1{,}000$ and 1.000.
- Do not accept final answers written using calculator notation. However, ***M*** marks and intermediate ***A*** marks can be scored, when presented using calculator notation, provided the evidence clearly reflects the demand of the mark.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, some **equivalent** answers will generally appear in brackets. Not all equivalent notations/answers/methods will be presented in the markscheme and examiners are asked to apply appropriate discretion to judge if the candidate work is equivalent.

8 Format and accuracy of answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. If the level of accuracy is not stated in the question, the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures*.

Where values are used in subsequent parts, the markscheme will generally use the exact value, however candidates may also use the correct answer to a "correct" level of accuracy (e.g 3 sf) in subsequent parts. The markscheme will often explicitly include the subsequent values that come "*from the use of 3 sf values*".

Simplification of final answers: Candidates are advised to give final answers using good mathematical form. In general, for an ***A*** mark to be awarded, arithmetic should be completed, and any values that lead to integers should be simplified; for example, $\sqrt{\frac{25}{4}}$ should be written as $\frac{5}{2}$. An exception to this is simplifying fractions, where lowest form is not required (although the numerator and the denominator must be integers); for example, $\frac{10}{4}$ may be left in this form or written as $\frac{5}{2}$. However, $\frac{10}{5}$ should be written as 2, as it simplifies to an integer.

Algebraic expressions should be simplified by completing any operations such as addition and multiplication, e.g. $4e^{2x} \times e^{3x}$ should be simplified to $4e^{5x}$, and $4e^{2x} \times e^{3x} - e^{4x} \times e^{x}$ should be simplified to $3e^{5x}$. Unless specified in the question, expressions do not need to be factorized, nor do factorized expressions need to be expanded, so $x(x+1)$ and $x^2 + x$ are both acceptable.

Please note: intermediate ***A*** marks do NOT need to be simplified.

9 Calculators

A GDC is required for this paper, but If you see work that suggests a candidate has used any calculator not approved for IB DP examinations (eg CAS enabled devices), please follow the procedures for malpractice.

10. Presentation of candidate work

Crossed out work: If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work unless an explicit note from the candidate indicates that they would like the work to be marked.

More than one solution: Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise. If the layout of the responses makes it difficult to judge, examiners should apply appropriate discretion to judge which is “first”.

1. (a)

	Event		Rank	
Country	**Long Jump (m)**	**High Jump (m)**	**Long Jump Rank**	**High Jump Rank**
Germany	7.64	2.11	1	**1**
France	7.52	2.08	2	**2**
Estonia	7.49	1.84	3	**10**
Canada	7.44	2.02	4	**4.5**
Netherlands	7.33	2.05	5	**3**
Ukraine	7.28	2.02	6	**4.5**
Algeria	7.22	1.90	7	**8**
Austria	7.11	1.87	8	**9**
Grenada	6.98	1.99	9	**6**
Japan	6.64	1.96	10	**7**

A1A1

Note: Award ***A1*** for ranking of tied heights, ***A1*** for correct ranking of non-tied heights.

[2 marks]

(b) $(r_s =)\ 0.541 \quad (0.541035\ldots)$ ***A2***

Note: Award ***A2*** for an answer of $0.539\ (0.539393\ldots)$ from use of the formula for Spearman's rank correlation coefficient when data has tied ranks.

[2 marks]

(c) moderate (correlation) ***A1***

as long jump ranking increases, high jump ranking will (likely) increase ***A1***

[2 marks]

[Total: 6 marks]

2. (a) attempt to calculate $\hat{AHB}$ using 33 **OR** use of alternate angles ***(M1)***

e.g., $180-(33+130)$ **OR** $90-(33+40)$ **OR** $57-40$

$17\ (°)$ ***A1***

[2 marks]

(b) attempt to use sine rule ***(M1)***

$$\frac{BH}{\sin(130^\circ)}=\frac{156}{\sin(17^\circ)}$$ ***(A1)***

$(BH=)\ 409\ (m)\ (408.736\ldots)$ ***A1***

Note: If radians are used, answer is $151\ (150.922\ldots)$; award at most ***(M1)(A1)A0***.

[3 marks]

(c) (the angle of depression from the hot air balloon) gets smaller ***A1***
(as the horizontal distance increases)

[1 mark]

[Total: 6 marks]

3. (a) $N = 24$

$I = 4$

$PV = \pm 1000$

$PMT = \pm 100$

$P/Y = 12$

$C/Y = 12$ ***(M1)(A1)***

Note: Award ***M1*** for an attempt to use a financial app in their technology (i.e. at least three entries seen, but not necessarily correct).
Approaches that use the compound interest formula receive no marks.
Award ***A1*** for correct values of PV and PMT (signs must be the same) **and** a correct value of N.

$FV = (\$)3577.43$ ***A1***

Note: Award at most ***(M1)(A1)A0*** if the final answer is negative or not rounded to 2 dp.

[3 marks]

(b) $N = 36.5 \ \ (36.4689\ldots)$ ***(A1)***

$N = 37$ (months) ***A1***

Note: Allow ***FT*** from incorrect GDC inputs seen in part (a) for the first ***A1*** providing that PV and FV have opposite signs and the resulting value of N is positive.

[2 marks]

[Total: 5 marks]

4. (a) $H_0 : \mu_b = \mu_m$ ***A1***

$H_1 : \mu_b > \mu_m$ ***A1***

Note: Accept equivalent statements in words such as "the **mean** score of bilingual people equals the **mean** score of monolingual people".

[2 marks]

(b) $0.119 \ (0.119395\ldots)$ ***A2***

[2 marks]

(c) $0.119395\ldots > 0.05 \ (11.9395\ldots\% > 5\%)$ ***R1***

(fail to reject H_0) there is insufficient evidence to suggest that bilingual people have better memory retention than monolingual people ***A1***

Note: Do not award ***R0A1***.
The answer to part (c) MUST be consistent with **their** hypotheses and **their** p-value.

[2 marks]

[Total: 6 marks]

5. (a) 2 ***A1***

[1 mark]

(b) attempt to substitute their part (a) and point $(3, -1)$ into the slope-intercept form or point-slope form of an equation ***(M1)***

$-1 = 2 \times 3 + c$ **OR** $y + 1 = 2(x - 3)$

$y = 2x - 7$ ***A1***

Note: Equation must be in the form $y = mx + c$ for ***A1*** to be awarded.

[2 marks]

continued…

Question 5 continued

(c) **METHOD 1**

attempt to show that P does not lie on L_2 ***(M1)***

e.g. $-\frac{1}{2}(3)-\frac{5}{2}$ **OR** graph showing L_2 and P in approximate correct locations

$-1 \neq -\frac{1}{2}(3)-\frac{5}{2}$ $(-1 \neq -4)$ **OR** $(3, -1)$ does not lie on the graph of L_2 ***R1***

hence L_2 is not the normal line to $f(x)$ at point P ***AG***

METHOD 2

attempt to find the equation of the normal line at $(3, -1)$ ***(M1)***

$(-1 = -\frac{1}{2}(3)+c$ **OR** $y+1 = -\frac{1}{2}(x-3))$

the normal line is $y = -\frac{1}{2}x+\frac{1}{2}$ ***R1***

hence L_2 is not the normal line to $f(x)$ at point P ***AG***

METHOD 3

attempt to find the intersection of L_1 and L_2 ***(M1)***

Intersection of $y = 2x-7$ and $y = -\frac{1}{2}x-\frac{5}{2}$ is $(1.8, -3.4)$

$x = 1.8 \neq 3$ **OR** $y = -3.4 \neq -1$ ***R1***

hence L_2 is not the normal line to $f(x)$ at point P ***AG***

Note: Accept equivalent written arguments provided values are seen.
Methods 1 and 2 are independent of the answers in (a) and (b) but FT marks can be given for Method 3.

[2 marks]

[Total: 5 marks]

6. (a) attempt to set up a direct variation equation that includes a constant, k, or the calculation of a constant using 12.3 and 50 ***(M1)***

e.g., $d = kv^2$ **OR** $12.3 = k \times 50^2$

$(k =)\ 0.00492 \quad \left(\frac{1}{203.252...}\right)$

$d = 0.00492v^2$ **OR** $d = \frac{v^2}{203}$ ***A1***

[2 marks]

(b) substituting 33 for d in their part (a) ***(A1)***

$33 = 0.00492 \times v^2$ **OR** $33 = \frac{v^2}{203.252...}$

$(v =)\ 81.9\ (\text{km h}^{-1})\ \ (81.8982\ldots\ (\text{km h}^{-1}))$ ***A1***

[2 marks]

(c) *Award **R1** for a reasonable variable that exists after the brakes are applied such as:*

- road material
- weather conditions
- condition/type of brakes
- weight/type of vehicle
- gradient/incline of road
- traction
- wind resistance
- friction

R1

Note: Do not accept a variable that refers to the timing of the brakes being applied such as:

- slow reaction time
- inexperienced driver

[1 mark]

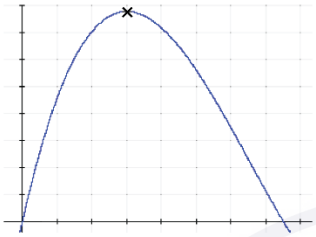
[Total: 5 marks]

7. (a) $(k =)\ 15$ ***A1***

[1 mark]

(b) **EITHER**

attempt to sketch the function $V(x)$ with indication of maximum ***(M1)***



OR

recognition of setting the derivative to 0 ***(M1)***

e.g. $V'(x) = 0$

THEN

$(x =)\ 6\ (\text{cm})$ ***A1***

Note: Award ***(M1)A0*** for the maximum given as a coordinate pair.

[2 marks]

(c) 44 or 26 seen ***(A1)***

attempting to adjust the constant(s) in the given volume formula ***(M1)***

volume of second box $= (44 - 2x)(26 - 2x)(x)$

(New maximum volume $=)\ 2730\ \text{cm}^3\quad (2726.13...\ \text{cm}^3)$ ***A1***

Note: Units must be seen to award the final ***A1***. Award ***(A1)(M1)A0*** for the maximum given as a coordinate pair.

[3 marks]

[Total: 6 marks]

8. (a) attempt to substitute 5000 for G ***(M1)***

$0.301p = \log_{10} 5000$

$(p =)\ 12.3$ (bits) $(12.2889\ldots)$ ***A1***

[2 marks]

(b) $(G =) 10^{0.301p}$ **OR** 2^p ***A1***

[1 mark]

(c) attempt to substitute 28 for p in given equation or $G(p)$ ***(M1)***

$0.301 \times 28 = \log_{10} G$ **OR** $(G =)\ 10^{0.301\times 28}$

$(G =)\ 2.68\times 10^8$ $(2.67916\ldots\times 10^8)$ ***A1A1***

Note: Award ***A1*** for 2.68, ***A1*** for 10^8. Award ***M1A1A0*** for a correct final answer not written in scientific notation or written incorrectly in scientific notation (e.g., $268\ 000\ 000$ or 26.8×10^7 or 2.68E08).

[3 marks]

(d) if a password has an **entropy of 0** (bits), then the password can be **guessed in one** try / then the **password is known** ***R1***

Note: Reference must be made to both entropy and number of guesses/password known for ***R1*** to be awarded.
Do not accept "no password" as this contradicts the context.

[1 mark]

Total [7 marks]

9. (a) attempt to substitute $h = 10$ and at least two different values of y into the trapezoidal rule ***(M1)***

$$\frac{10}{2}\big((0+0)+2(3+8+9)\big)$$

$$= 200\ (\text{cm}^2)$$ ***A1***

[2 marks]

(b) (i) $\int_0^{40} 0.04x^2 - 0.001x^3 \,\text{d}x$ **OR** $\int_0^{40} y \,\text{d}x$ ***A1A1***

Note: Award ***A1*** for a correct integral (including $\text{d}x$), ***A1*** for correct limits in the correct location.

(ii) $213.33\ (\text{cm}^2)$ ***A2***

Note: Answer must be given to 2 decimal places to award ***A2***. Award ***A1A0*** for a correct answer given to an incorrect accuracy of at least 3 significant figures, e.g. $213\ (\text{cm}^2)$.

[4 marks]

(c) attempt to substitute their parts (a) and (b)(ii) into percentage error formula ***(M1)***

$$\left|\frac{213.333... - 200}{213.333...}\right| \times 100$$

$$= 6.25(\%)\ \ (6.24999...(\%))$$ ***A1***

Note: Award ***(M1)A0*** for a final answer of $-6.25(\%)$ or 0.0625.

[2 marks]

[Total: 8 marks]

10. (a) (i) **METHOD 1**

attempt to find change in height of the ball using gradient ***(M1)***

$\frac{a}{0.43} = (-)0.045$

$a = (-)0.045 \times 0.43$

$a = (-)0.0194\,(\text{m})\ \ (0.01935\ (\text{m}))$ ***A1***

METHOD 2

attempt to find height at back of home plate ***(M1)***

horizontal distance to the front of the home plate $= 16.6666\ldots\ (\text{m})$

height at the back of the home plate $= -0.045(16.6666\ldots + 0.43) + 2$

$(= 1.23065\ (\text{m}))$

Note: The ***M1*** can be awarded for $16.6666\ldots + 0.43$ seen at some point.

$(a = 1.25 - 1.23065\ldots)$

$(a =)\ (-)0.0194\ (\text{m})\ \ (0.01935\ (\text{m}))$ ***A1***

(ii) $1.25 - 0.01935 = 1.23065$ (may be seen in part (a)(i)) ***A1***

$0.53 < 1.23065 < 1.24$ ***R1***

therefore a strike ***AG***

Note: Do not award ***A0R1***.

[4 marks]

continued…

Question 10 continued

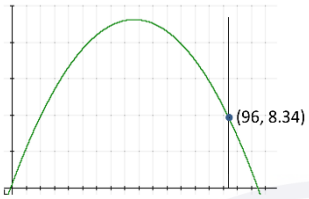
(b) **METHOD 1**

indication of $d = 96$ in the function $h(d)$ or its graph ***(M1)***

EITHER

$(h(96) =) -0.01(96)^2 + 1.04(96) + 0.66$

OR



THEN

$(h(96) =)\ 8.34\ (\text{m})$ ***A1***

$8.34 > 5$ so the ball will go over the wall. ***A1***

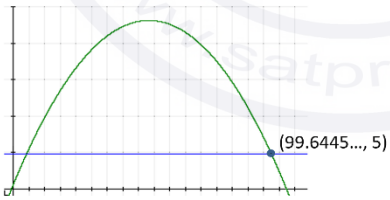
METHOD 2

indication of $h = 5$ in the function $h(d)$ or its graph ***(M1)***

EITHER

$5 = -0.01d^2 + 1.04d + 0.66$

OR



THEN

$d = 99.6\ (\text{m})\ (99.6445\ldots\ (\text{m}))\ (d = 4.35548\ldots\ (\text{m})$ may also be seen) ***A1***

$96 < 99.6445\ldots$ so the ball will go over the wall. ***A1***

[3 marks]

[Total: 7 marks]

11. (a) 14.55 (cm) to 14.65 (cm) ***A1A1***

Note: Award ***A1*** for each value. Accept $14.55 \leq AC < 14.65$.

[2 marks]

(b) attempt to use Pythagorean theorem **OR** trig ratio to find slant height ***(M1)***

a correct expression for either the **upper** or **lower** bound ***(A1)***

$\sqrt{14.55^2 - 10^2}$ **OR** $\sqrt{14.65^2 - 10^2}$ **OR**

$\sin(46.5844...^\circ) = \frac{AH}{14.55}$ **OR** $\sin(46.9533...^\circ) = \frac{AH}{14.65}$

(lower bound =) 10.6 (cm) (10.5689…) **AND**

(upper bound =) 10.7 (cm) (10.7061…) ***A1***

[3 marks]

continued…

Question 11 continued

(c) **METHOD 1**

attempt to find the maximum angle measure of the post using trigonometry ***(M1)***

e.g. $\cos\theta = \frac{10}{10.7061...}$ **OR** $\frac{\sin\theta}{3.82393\ldots} = \frac{\sin(90^\circ)}{10.7061\ldots}$

Note: Accept an inequality.

$(\theta =)\ 20.9\ (^\circ)\quad (20.9265\ldots(^\circ))$ ***A1***

and hence the post is safe ***AG***

Note: Use of radians gives an answer of $0.365\ \ (0.365237\ldots)$; award at most ***(M1)A0*** since this value cannot be directly compared to 22°.

Award at most ***(M1)A0*** for an angle calculated using their lower bound from part (b).

METHOD 2

attempt to find the longest slant height for angle to be a maximum of 22° ***(M1)***

e.g. $\cos(22^\circ) = \frac{10}{x}$

$(x = 10.7853\ldots)$

$10.7061\ldots < 10.7853\ldots$ ***A1***

and hence the post is safe ***AG***

Note: A comparison to their upper bound from part (b) is required for ***A1*** to be awarded. Use of radians gives an unreasonable answer of $-10.0003...$; award at most ***(M1)A0***.

[2 marks]

Total [7 marks]

12. (a) attempt to find the difference between 75.7 and 67.3 ***(M1)***

$\frac{75.7-67.3}{2}$

$4.2\ (\text{km h}^{-1})$ ***A1***

[2 marks]

(b) recognition of normal distribution that includes 72 ***(M1)***

e.g., sketch of normal distribution curve with 72 labelled to the right of the mean **OR**
Normal CDF calculation using 72

$0.132\quad (0.131559\ldots, 13.2\%, 13.1559\ldots\%)$ ***A1***

[2 marks]

(c) **METHOD 1 (Comparing areas above and below the mean)**

$\text{P}(67.3 < \text{speed} < 74)$ **OR** Normal CDF$(67.3, 74, 67.3, 4.2)$ **OR** sketch of normal distribution with 67.3 and 74 labelled and shaded between ***(M1)***

area of region between mean and q is at least $0.445\ \ (0.444670\ldots)$ ***A1***

Hence no more than $0.375\ \ (0.375329\ldots)$ between mean and p ***R1***

The region between p and q is not symmetrical ***AG***

METHOD 2 (Comparing areas in the tails)

attempt to calculate probability that speed $< p$ and speed $> q$ with $q = 74$ ***(M1)***

$\text{P}(\text{speed} < 74) = 0.944670\ldots$

$\text{P}(\text{speed} < p) = (0.944670\ldots - 0.82 =)\ \ 0.124670\ldots$

$\text{P}(\text{speed} > q) = (1 - 0.944670\ldots =)\ \ 0.0553295\ldots$ ***A1***

if $q \geq 74$, then $\text{P}(\text{speed} > q) \leq 0.0553295$ and $\text{P}(\text{speed} < p) \geq 0.124670$ so

$\text{P}(\text{speed} > q)$ will never equal $\text{P}(\text{speed} < p)$ ***R1***

the region between p and q is not symmetrical ***AG***

continued…

Question 12 continued

METHOD 3 (Assumption of symmetry comparing speeds)

attempt to calculate area below q assuming distribution is symmetrical ***(M1)***

e.g. $\mathrm{P}(\text{speed} < q) = 0.82 + ½ \times 0.18 \quad (0.91)$

EITHER

$(q =)\ 72.9 \quad (72.9311\ldots)$ ***A1***

$72.9 < 74$ so 74 would not be in the region ***R1***

the region between p and q is not symmetrical ***AG***

OR

$\mathrm{P}(\text{speed} < 74) = 0.945 \quad (0.944670\ldots)$ ***A1***

$0.945 > 0.91$ so 74 would not be in the region ***R1***

the region between p and q is not symmetrical ***AG***

METHOD 4 (Assumption of symmetry comparing areas)

attempt to calculate symmetrical area with 74 as a boundary ***(M1)***

$\mathrm{P}(60.6 < \text{speed} < 74)$ **OR** Normal CDF(60.6, 74, 67.3, 4.2) **OR**

$\mathrm{P}(67.3 < \text{speed} < 74)$ **OR** Normal CDF(67.3, 74, 67.3, 4.2)

EITHER

$0.889 \quad (0.889340\ldots)$ ***A1***

$0.889 > 0.82$ so 74 would not be in the region ***R1***

the region between p and q is not symmetrical ***AG***

OR

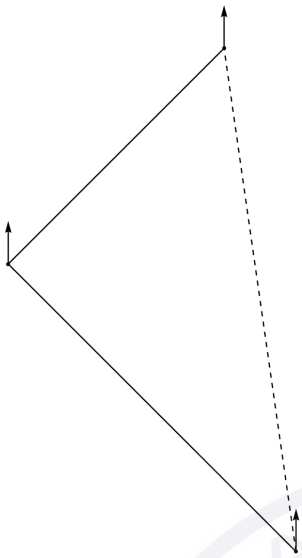
$0.445 \quad (0.444670\ldots)$ ***A1***

$0.445 > 0.82 \div 2$ so 74 would not be in the region ***R1***

the region between p and q is not symmetrical ***AG***

[3 marks]
[Total: 7 marks]

13. diagram showing (approximately) correct directions (and order) for the 315° and 045° ***(A1)***



Note: Values do not need to be seen on the diagram to award the ***A1***.

recognizing right angle triangle ***(M1)***

correct expression to find second angle in triangle ***(A1)***

e.g. $\arctan\left(\frac{6}{8}\right)$ **OR** $\arctan\left(\frac{8}{6}\right)$

correct expression to find bearing ***(A1)***

e.g. $\arctan\left(\frac{6}{8}\right)+135^\circ$ **OR** $360^\circ-\left(\arctan\left(\frac{8}{6}\right)+135^\circ\right)$

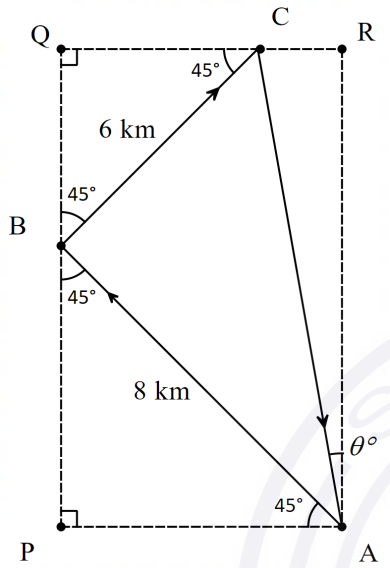
$=172^\circ$ $(171.869...^\circ)$ ***A1***

continued…

Question 13 continued

METHOD 2

diagram showing (approximately) correct directions (and order) for the $315°$ and $045°$ (these may be shown in reverse as the return journey) ***(A1)***



finding the lengths marked $\mathrm{AP}, \mathrm{BP}, \mathrm{CQ}$ and BQ in the diagram ***(M1)***

$$\mathrm{AP} = \mathrm{BP} = 8\frac{\sqrt{2}}{2} = 5.6568...$$

$$\mathrm{CQ} = \mathrm{BQ} = 6\frac{\sqrt{2}}{2} = 4.2426...$$

Note: This may be done using a vector approach.

using $\tan\theta° = \dfrac{\mathrm{AP} - \mathrm{CQ}}{\mathrm{PB} + \mathrm{BQ}}$ or equivalent to find the direction of AC ***(A1)***

correct expression to find bearing ***(A1)***

$$180° - \arctan\left(\frac{8\frac{\sqrt{2}}{2} + 6\frac{\sqrt{2}}{2}}{8\frac{\sqrt{2}}{2} - 6\frac{\sqrt{2}}{2}}\right)$$

$= 172° \quad (171.869...°)$ ***A1***

[Total: 5 marks]

Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

November 2022

Mathematics: applications and interpretation

Standard level

Paper 1

15 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**.
A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.
R Marks awarded for clear **Reasoning**.
AG Answer given in the question and so no marks are awarded.
FT Follow through. The practice of awarding marks, despite candidate errors in previous parts, for their correct methods/answers using incorrect results.

Using the markscheme

1 General

Award marks using the annotations as noted in the markscheme *eg* ***M1, A2.***

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***A3***, ***M2*** *etc.*, do **not** split the marks, unless there is a note.
- The response to a "show that" question does not need to restate the ***AG*** line, unless a **Note** makes this explicit in the markscheme.
- Once a correct answer to a question or part question is seen, ignore further working even if this working is incorrect and/or suggests a misunderstanding of the question. This will encourage a uniform approach to marking, with less examiner discretion. Although some candidates may be advantaged for that specific question item, it is likely that these candidates will lose marks elsewhere too.
- An exception to the previous rule is when an incorrect answer from further working is used **in a subsequent part**. For example, when a correct exact value is followed by an incorrect decimal approximation in the first part and this approximation is then used in the second part. In this situation, award ***FT*** marks as appropriate but do not award the final ***A1*** in the first part. Examples:

	Correct answer seen	**Further working seen**	**Any FT issues?**	**Action**
1.	$8\sqrt{2}$	5.65685... *(incorrect decimal value)*	No. Last part in question.	Award ***A1*** for the final mark *(condone the incorrect further working)*
2.	$\frac{35}{72}$	0.468111… (incorrect decimal value)	Yes. Value is used in subsequent parts.	Award ***A0*** for the final mark *(and full **FT** is available in subsequent parts)*

3 Implied marks

Implied marks appear in **brackets e.g.** ***(M1)***,and can only be awarded if **correct** work is seen or implied by subsequent working/answer.

4 Follow through marks (only applied after an error is made)

Follow through (***FT***) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) (e.g. incorrect value from part (a) used in part (d) or incorrect value from part (c)(i) used in part (c)(ii)). Usually, to award ***FT*** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if all the marks awarded in a subsequent part are for the answer or are implied, then ***FT*** marks should be awarded for *their* correct answer, even when working is not present.

For example: following an incorrect answer to part (a) that is used in subsequent parts, where the markscheme for the subsequent part is ***(M1)A1***, it is possible to award full marks for *their* correct answer, **without working being seen.** For longer questions where all but the answer marks are implied this rule applies but may be overwritten by a **Note** in the Markscheme.

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks, by reflecting on what each mark is for and how that maps to the simplified version.
- If the error leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word “their” in a description, to indicate that candidates may be using an incorrect value.
- If the candidate’s answer to the initial question clearly contradicts information given in the question, it is not appropriate to award any ***FT*** marks in the subsequent parts. This includes when candidates fail to complete a “show that” question correctly, and then in subsequent parts use their incorrect answer rather than the given value.
- Exceptions to these ***FT*** rules will be explicitly noted on the markscheme.
- If a candidate makes an error in one part but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the command term was “Hence”.

5 Mis-read

If a candidate incorrectly copies values or information from the question, this is a mis-read (***MR***). A candidate should be penalized only once for a particular misread. Use the ***MR*** stamp to indicate that this has been a misread and do not award the first mark, even if this is an ***M*** mark, but award all others as appropriate.

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.
- If a candidate uses a correct answer, to a "show that" question, to a higher degree of accuracy than given in the question, this is NOT a misread and full marks may be scored in the subsequent part.
- ***MR*** can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If the command term is 'Hence' and not 'Hence or otherwise' then alternative methods are not permitted unless covered by a note in the mark scheme.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**.

7 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation** for example 1.9 and $1{,}9$ or 1000 and $1{,}000$ and 1.000.
- Do not accept final answers written using calculator notation. However, ***M*** marks and intermediate ***A*** marks can be scored, when presented using calculator notation, provided the evidence clearly reflects the demand of the mark.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, some **equivalent** answers will generally appear in brackets. Not all equivalent notations/answers/methods will be presented in the markscheme and examiners are asked to apply appropriate discretion to judge if the candidate work is equivalent.

8 Format and accuracy of answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. If the level of accuracy is not stated in the question, the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures*.

Where values are used in subsequent parts, the markscheme will generally use the exact value, however candidates may also use the correct answer in subsequent parts. The markscheme will often explicitly include the subsequent values that come "*from the use of 3 sf values*".

Simplification of final answers: Candidates are advised to give final answers using good mathematical form. In general, for an ***A*** mark to be awarded, arithmetic should be completed, and any values that lead to integers should be simplified; for example, $\sqrt{\frac{25}{4}}$ should be written as $\frac{5}{2}$. An exception to this is simplifying fractions, where lowest form is not required (although the numerator and the denominator must be integers); for example, $\frac{10}{4}$ may be left in this form or written as $\frac{5}{2}$. However, $\frac{10}{5}$ should be written as 2, as it simplifies to an integer.

Algebraic expressions should be simplified by completing any operations such as addition and multiplication, e.g. $4\mathrm{e}^{2x} \times \mathrm{e}^{3x}$ should be simplified to $4\mathrm{e}^{5x}$, and $4\mathrm{e}^{2x} \times \mathrm{e}^{3x} - \mathrm{e}^{4x} \times \mathrm{e}^{x}$ should be simplified to $3\mathrm{e}^{5x}$. Unless specified in the question, expressions do not need to be factorized, nor do factorized expressions need to be expanded, so $x(x+1)$ and $x^2 + x$ are both acceptable.

Please note: intermediate ***A*** marks do NOT need to be simplified.

9 Calculators

A GDC is required for this paper, but If you see work that suggests a candidate has used any calculator not approved for IB DP examinations (eg CAS enabled devices), please follow the procedures for malpractice.

10. Presentation of candidate work

Crossed out work: If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work unless an explicit note from the candidate indicates that they would like the work to be marked.

More than one solution: Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise. If the layout of the responses makes it difficult to judge, examiners should apply appropriate discretion to judge which is "first".

1. (a) $\sin(\hat{BSK}) = \frac{218}{1200}$ **OR** $\frac{\sin(\hat{BSK})}{218} = \frac{\sin(90^\circ)}{1200}$ ***(M1)***

Note: Award ***M1*** for a correct trig formula. Accept other variables representing $\hat{BSK}$.

$(\hat{BSK} =)\ 10.5^\circ\ (10.4668...)$ ***A1***

Note: Award ***A1*** for the radian answer, $0.182681...$. Award ***M1A0*** if the candidate finds the correct angle of elevation but then uses it to find a complementary angle as their final answer.

[2 marks]

(b) $SB^2 + 218^2 = 1200^2$ **OR** $\cos(10.4668...) = \frac{SB}{1200}$ **OR** $\tan(10.4668...) = \frac{218}{SB}$ **OR**

$\frac{BS}{\sin(79.5331...^\circ)} = \frac{1200}{\sin(90^\circ)}$ ***(M1)***

1180 (m) $\left(\sqrt{1392476}, 1180.03...\right)$ ***A1***

[2 marks]

(c) 1.18×10^3 ***A1A1***

Note: Award ***A1*** for 1.18
Award ***A1*** for 10^3
Accept their rounded answer to part (b).
Award ***A0A0*** for answers of the type: 11.8×10^2.

[2 marks]
Total [6 marks]

2. (a) use of the n^{th} term of an arithmetic sequence formula ***(M1)***

$u_{15} = 85 + (15-1)\times30$ ***(A1)***

505 ***A1***

[3 marks]

(b) use of the sum of n terms of an arithmetic sequence formula ***(M1)***

$S_{15} = \frac{15}{2}(85+505)$ **OR** $\frac{15}{2}\left(2\times85+(15-1)\times30\right)$

4430 (4425) ***A1***

[2 marks]

(c) $\frac{4425}{15}$ **OR** $85+(8-1)\times30$ ***(M1)***

295 ***A1***

Note: Accept $295.333\ldots$ from use of 3sf value from part (b).

[2 marks]
Total [7 marks]

3. (a) 1.8 (m) ***A1***

[1 mark]

(b) **EITHER**

$\frac{-10.8}{2(-3.6)}$ ***(M1)***

OR

$-7.2(t)+10.8=0$ ***(M1)***

OR

sketch indicating maximum ***(M1)***

THEN

$(t=)$ 1.5 seconds ***A1***

Note: Award ***(M1)A0*** for $(1.5, 9.9)$ seen.

[2 marks]

(c) **EITHER**

$0=-3.6t^2+10.8t+1.8$ ***(M1)***

OR

sketch indicating a root ***(M1)***

THEN

$(t=)$ 3.16 seconds $(3.15831...)$ ***A1***

Note: Award at most ***M1A0*** if -0.158 $(-0.158312..)$ is part of the final answer unless clearly rejected.

[2 marks]

Total [5 marks]

4. (a) The favourite breakfast/berry (of adults) is independent of (their) income (level). ***A1***

[1 mark]

(b) $\chi^2 = 2.27 \ (2.26821...)$ ***A2***

[2 marks]

(c) **EITHER**

$2.27 < 7.78$ **OR** $2.27 <$ critical value ***R1***

OR

$0.687 > 0.1$ (using p-value)

THEN

(Do not reject H_0)

Insufficient evidence (at the 10% significance level) that the favourite berry depends on income level. ***A1***

Note: Do not award ***R0A1***. Accept "χ^2" in place of their "2.27", provided an answer was seen in part (b). Their conclusion must be consistent with their χ^2 (or a correct p-value) and their hypothesis.

[2 marks]

Total [5 marks]

5. (a) $71\mathrm{e}^{-0.0514(16)} + 23$ ***(M1)***

$54.2 \ ^\circ\mathrm{C} \ (54.1956...)$ ***A1***

[2 marks]

(b) $T = 23$ ***A1***

Note: Condone $y = 23$.

[1 mark]

(c) $23 \ ^\circ\mathrm{C}$ ***A1***

[1 mark]

(d) $50 = 71\mathrm{e}^{-0.0514(k)} + 23$ ***(M1)***

$k = 18.8 \ \left(\dfrac{-5000}{257}\ln\left(\dfrac{27}{71}\right), 18.8101...\right)$ ***A1***

Note: Award ***M1*** for a sketch showing a point of intersection between the exponential function and $y = 50$.

[2 marks]

Total [6 marks]

6. (a) $(H_1:)\ \mu_1 \neq \mu_2$ ***A1***

Note: Accept an equivalent statement in words referring to μ_1 and μ_2 as defined in the question.

[1 mark]

(b) $0.97652\ (0.976516...)$ ***A2***

[2 marks]

(c) $0.97652 > 0.05\ (0.977 > 0.05)$ ***R1***

Annabelle's conclusion is correct. ***A1***

Note: Do not award ***R0A1***. Answer must reference Annabelle's conclusion; do not accept an answer, without context, of "fail to reject H_0" for the ***A1*** mark.

[2 marks]

Total [5 marks]

7. (a) $I\% = 7.5$
$PV = \mp 800$
$PMT = \mp 500$
$FV = \pm 10\,000$
$P/Y = 12$
$C/Y = 12$ ***(M1)(A1)***

Note: Award ***M1*** for an attempt to use a financial app in their technology (e.g. at least four rows seen, but not necessarily correct), award ***A1*** for $\text{PMT} = -500$ or $\text{PMT} = 500$, with same sign to PV and opposite sign to FV.

$17.3070\ldots$ ***(A1)***

$(k =)\ 18$ ***A1***

Note: Award ***(M0)(A0)(A0)A0*** for a final answer of 17 with no working. The final answer must be an integer.

[4 marks]

(b) $10389 - (18 \times 500 + 800)$ **OR** $10389 - (9800)$ ***(A1)(M1)***

Note: Award ***(A1)*** for $10389\ (10389.38\ldots)$ seen. Award ***(M1)*** for subtraction of their $(18 \times 500 + 800)$ from FV. ***FT*** from their value of k. Award ***A0M1A0*** for $10\,000 - (18 \times 500 + 800)$. Do not award the final ***A1FT*** if their answer is negative.

589 EUR ***A1***

Note: Final answer must be to the nearest euro.

[3 marks]

Total [7 marks]

8. (a) $\mathrm{P}(T < 55)$ ***(M1)***

$0.0912\ (0.0912112...)$ ***A1***

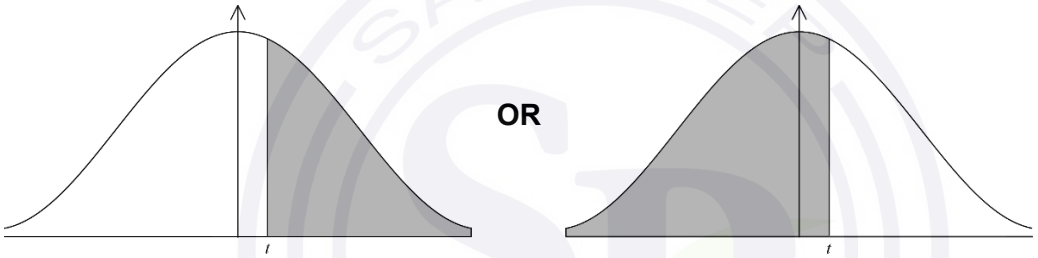
Note: Award ***M1*** for a correct calculator notation such as $\text{normal cdf}(0, 55, 59, 3)$ or $\text{normal cdf}(-1^{99}, 55, 59, 3)$.

[2 marks]

(b) correct use of expected value
$8.6 = 20 \times p$ **OR** $(p =)\ 0.43$ seen ***(M1)***

EITHER
correct probability statement ***(M1)***
$\mathrm{P}(T > t) = 0.43$ **OR** $\mathrm{P}(T < t) = 0.57$

OR
t indicated on sketch to communicate correct area ***(M1)***



OR

THEN
$(t =)\ 59.5$ (seconds) (59.5291...) ***A1***

[3 marks]
Total [5 marks]

9. (a) $0.5 \times 0.1 + 0.4 \times 0.4 + 0.1 \times 0.5$ ***(M1)(M1)(M1)***

Note: Award ***M1*** for 0.5×0.1 or 0.1×0.5, ***M1*** for 0.4×0.4, ***M1*** for adding three correct products.

0.26 ***A1***

[4 marks]

(b) $0 = -8 \times 0.5 + 4 \times 0.4 + 0.1k$ ***(M1)(M1)***

Note: Award ***M1*** for correct substitution into the formula for expected value, award ***M1*** for the expected value formula equated to zero.

$(k =)\ 24$ (points) ***A1***

[3 marks]
Total [7 marks]

10. (a) $m = 1 - 2.5\log_{10}(0.0525)$ ***(M1)***

$= 4.20 \ (4.19960\ldots)$ ***A1***

[2 marks]

(b) attempt to solve $7 = 1 - 2.5\log_{10}(b)$ ***(M1)***

Note: Accept a sketch from their GDC as an attempt to solve $7 = 1 - 2.5\log_{10}(b)$.

$b = 0.00398 \ (0.00398107...)$ ***A1***

[2 marks]

(c) $\dfrac{0.0525}{0.00398107}$ ***(M1)***

$= 13.2 \ (13.1874\ldots)$ ***A1***

[2 marks]

Total [6 marks]

11. (a) $4.5 = 2(r)^{3-1}$ ***M1***

$r = \pm 1.5$, ***R1***

(Some x-values are negative or direction from house changes each day)

$r = -1.5$ ***AG***

Note: Award ***M0R0AG*** for a verification approach $4.5 = 2(-1.5)^{3-1}$.

[2 marks]

(b) $2(-1.5)^{6-1}$ ***(M1)***

EITHER

$(-15.2, 0) \ (-15.1875..., 0)$ ***A1***

OR

$x = -15.2$ km ***A1***

OR

15.2 km west (of the origin) ***A1***

Note: Award ***(M1)A0*** for an answer of "-15.2 (km)" without indicating that it is the x-value.

[2 marks]

(c) choosing $r = 1.5$ ***(A1)***

$\dfrac{2\left((1.5)^7 - 1\right)}{1.5 - 1}$ ***(M1)***

Note: Award ***M1*** for an attempt at a substituted GP formula with $n = 7$. Award ***A0M1A0*** for substitution of $r = -1.5$, with $n = 7$ (this can be implied from a final answer of $14.4687...$).

64.3... km $(64.3437...)$ ***A1***

[3 marks]

Total [7 marks]

12. (a) 78 ***A1***

[1 mark]

(b) (i) 65 ***A1***

(ii) **EITHER**

(period $=$) 16 (could be seen on sketch) ***(M1)***

$b = \frac{2\pi}{16}$ **OR** $b = \frac{360^\circ}{16}$

$(b =)\ 0.393\ \left(0.392699..., \frac{\pi}{8}\right)$ **OR** $(b =)\ 22.5^\circ$ ***A1***

OR

$143 = 65\sin(4b) + 78$ ***(M1)***

$(\sin(4b) = 1)$

$(4b = \frac{\pi}{2}$ **OR** $4b = 90^\circ)$

$(b =)\ 0.393\ \left(0.392699..., \frac{\pi}{8}\right)$ **OR** $(b =)\ 22.5^\circ$ ***A1***

[3 marks]

(c) 13 ***A1***

Note: Apply follow through marking only if their final answer is positive.

[1 mark]

(d) $(b =)\ 0.196\ \left(0.196349..., \frac{\pi}{16}\right)$ **OR** $(b =) 11.3^\circ\ (11.25^\circ)$ ***A1***

[1 mark]
Total [6 marks]

13. (a) $0 = 20 - \frac{980}{t^2}$ **OR** $\frac{\mathrm{d}P}{\mathrm{d}t} = 0$ ***(M1)***

Note: Accept equivalent information presented in a labelled sketch.

$(h =)\ 7$ hours ***A1***

Note: Award ***M1A0*** for an answer of $(7, 280)$.

[2 marks]

(b) recognition of need to integrate (e.g. reverse power rule or integral symbol) ***(M1)***

$P(t) = 20t + \frac{980}{t}\ (+c)$ ***A1A1***

$328 = 20 \times 5 + \frac{980}{5} + c$ ***(M1)***

Note: Award ***(M1)*** for substitution of $P = 328$ and $t = 5$ into their $P(t)$. A constant of integration must be seen (can be implied by a correct answer).

$c = 32$ ***A1***

$P(7) = 20 \times 7 + \frac{980}{7} + 32$ ***M1***

Note: Award ***M1*** for substituting 7 and their 32 into their $P(t)$.
Do not award the final ***M*** mark if their substituted values do not lead to 312.

312 NOK ***AG***

[6 marks]
Total [8 marks]

Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

May 2022

Mathematics: applications and interpretation

Standard level

Paper 1

19 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**.
A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.
R Marks awarded for clear **Reasoning**.
AG Answer given in the question and so no marks are awarded.
FT Follow through. The practice of awarding marks, despite candidate errors in previous parts, for their correct methods/answers using incorrect results.

Using the markscheme

1 General

Award marks using the annotations as noted in the markscheme *eg* ***M1, A2.***

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***A3***, ***M2*** *etc*., do **not** split the marks, unless there is a note.
- The response to a "show that" question does not need to restate the ***AG*** line, unless a **Note** makes this explicit in the markscheme.
- Once a correct answer to a question or part question is seen, ignore further working even if this working is incorrect and/or suggests a misunderstanding of the question. This will encourage a uniform approach to marking, with less examiner discretion. Although some candidates may be advantaged for that specific question item, it is likely that these candidates will lose marks elsewhere too.
- An exception to the previous rule is when an incorrect answer from further working is used **in a subsequent part**. For example, when a correct exact value is followed by an incorrect decimal approximation in the first part and this approximation is then used in the second part. In this situation, award ***FT*** marks as appropriate but do not award the final ***A1*** in the first part. Examples:

	Correct answer seen	**Further working seen**	**Any FT issues?**	**Action**
1.	$8\sqrt{2}$	5.65685... *(incorrect decimal value)*	No. Last part in question.	Award ***A1*** for the final mark *(condone the incorrect further working)*
2.	$\frac{35}{72}$	0.468111… (incorrect decimal value)	Yes. Value is used in subsequent parts.	Award ***A0*** for the final mark *(and full **FT** is available in subsequent parts)*

3 Implied marks

Implied marks appear in **brackets e.g.** ***(M1)***, and can only be awarded if **correct** work is seen or implied by subsequent working/answer.

4 Follow through marks (only applied after an error is made)

Follow through (***FT***) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) (e.g. incorrect value from part (a) used in part (d) or incorrect value from part (c)(i) used in part (c)(ii)). Usually, to award ***FT*** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if all the marks awarded in a subsequent part are for the answer or are implied, then ***FT*** marks should be awarded for *their* correct answer, even when working is not present.

For example: following an incorrect answer to part (a) that is used in subsequent parts, where the markscheme for the subsequent part is ***(M1)A1***, it is possible to award full marks for *their* correct answer, **without working being seen.** For longer questions where all but the answer marks are implied this rule applies but may be overwritten by a **Note** in the Markscheme.

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks, by reflecting on what each mark is for and how that maps to the simplified version.
- If the error leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word “their” in a description, to indicate that candidates may be using an incorrect value.
- If the candidate’s answer to the initial question clearly contradicts information given in the question, it is not appropriate to award any ***FT*** marks in the subsequent parts. This includes when candidates fail to complete a “show that” question correctly, and then in subsequent parts use their incorrect answer rather than the given value.
- Exceptions to these ***FT*** rules will be explicitly noted on the markscheme.
- If a candidate makes an error in one part but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the command term was “Hence”.

5 Mis-read

If a candidate incorrectly copies values or information from the question, this is a mis-read (***MR***). A candidate should be penalized only once for a particular misread. Use the ***MR*** stamp to indicate that this has been a misread and do not award the first mark, even if this is an ***M*** mark, but award all others as appropriate.

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.
- If a candidate uses a correct answer, to a "show that" question, to a higher degree of accuracy than given in the question, this is NOT a misread and full marks may be scored in the subsequent part.
- ***MR*** can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If the command term is 'Hence' and not 'Hence or otherwise' then alternative methods are not permitted unless covered by a note in the mark scheme.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**.

7 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation** for example 1.9 and $1{,}9$ or 1000 and $1{,}000$ and 1.000.
- Do not accept final answers written using calculator notation. However, ***M*** marks and intermediate ***A*** marks can be scored, when presented using calculator notation, provided the evidence clearly reflects the demand of the mark.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, some **equivalent** answers will generally appear in brackets. Not all equivalent notations/answers/methods will be presented in the markscheme and examiners are asked to apply appropriate discretion to judge if the candidate work is equivalent.

8 Format and accuracy of answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. If the level of accuracy is not stated in the question, the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures.*

Where values are used in subsequent parts, the markscheme will generally use the exact value, however candidates may also use the correct answer in subsequent parts. The markscheme will often explicitly include the subsequent values that come "*from the use of 3 sf values*".

Simplification of final answers: Candidates are advised to give final answers using good mathematical form. In general, for an ***A*** mark to be awarded, arithmetic should be completed, and any values that lead to integers should be simplified; for example, $\sqrt{\frac{25}{4}}$ should be written as $\frac{5}{2}$.

An exception to this is simplifying fractions, where lowest form is not required (although the numerator and the denominator must be integers); for example, $\frac{10}{4}$ may be left in this form or written as $\frac{5}{2}$. However, $\frac{10}{5}$ should be written as 2, as it simplifies to an integer.

Algebraic expressions should be simplified by completing any operations such as addition and multiplication, e.g. $4e^{2x} \times e^{3x}$ should be simplified to $4e^{5x}$, and $4e^{2x} \times e^{3x} - e^{4x} \times e^{x}$ should be simplified to $3e^{5x}$. Unless specified in the question, expressions do not need to be factorized, nor do factorized expressions need to be expanded, so $x(x+1)$ and $x^2 + x$ are both acceptable.

Please note: intermediate ***A*** marks do NOT need to be simplified.

9 Calculators

A GDC is required for this paper, but If you see work that suggests a candidate has used any calculator not approved for IB DP examinations (eg CAS enabled devices), please follow the procedures for malpractice.

10. Presentation of candidate work

Crossed out work: If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work unless an explicit note from the candidate indicates that they would like the work to be marked.

More than one solution: Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise. If the layout of the responses makes it difficult to judge, examiners should apply appropriate discretion to judge which is "first".

1. (a) attempt to substitute into length of arc formula ***(M1)***

$\frac{140°}{360°} \times 2\pi \times 56$

$137 \text{ cm} \quad \left(136.833\ldots, \frac{392\pi}{9} \text{ cm}\right)$ ***A1***

[2 marks]

(b) subtracting two substituted area of sectors formulae ***(M1)***

$\left(\frac{140°}{360°} \times \pi \times 56^2\right) - \left(\frac{140°}{360°} \times \pi \times 10^2\right)$ **OR** $\frac{140°}{360°} \times \pi \times \left(56^2 - 10^2\right)$ ***(A1)***

$3710 \text{ cm}^2 \quad (3709.17\ldots \text{ cm}^2)$ ***A1***

[3 marks]

Total [5 marks]

2. (a) $\left(\frac{17+25}{130} =\right) \frac{42}{130} \quad \left(\frac{21}{65}, 0.323076\ldots\right)$ ***A1***

[1 mark]

(b) $\left(\frac{17}{17+25} =\right) \frac{17}{42} \quad (0.404761\ldots)$ ***A1A1***

Note: Award ***A1*** for correct numerator and ***A1*** for correct denominator. Award ***A1A0*** for working of $\frac{17/130}{\text{their answer to (a)}}$ if followed by an incorrect answer.

[2 marks]

(c) $\frac{41}{130} \times \frac{40}{129}$ ***A1M1***

Note: Award ***A1*** for two correct fractions seen, ***M1*** for multiplying their fractions.

$= \frac{1640}{16770} \approx 0.0978 \left(0.0977936\ldots, \frac{164}{1677}\right)$ ***A1***

[3 marks]

Total [6 marks]

3. (a) $\sin\theta = \frac{2.1}{2.8}$ **OR** $\tan\theta = \frac{2.1}{1.85202\ldots}$ ***(M1)***

$(\theta =)\ 48.6\ °\quad (48.5903\ldots°)$ ***A1***

[2 marks]

(b) **METHOD 1**

$\sqrt{2.8^2 - 2.1^2}$ **OR** $2.8\cos(48.5903...)$ **OR** $\frac{2.1}{\tan(48.5903\ldots)}$ ***(M1)***

Note: Award ***M1*** for attempt to use Pythagorean Theorem with 2.1 seen or for attempt to use cosine or tangent ratio.

$1.85\ (\text{m})\quad (1.85202\ldots)$ ***(A1)***

Note: Award the ***M1A1*** if 1.85 is seen in part (a).

$(6.4 - 1.85202\ldots)$
$4.55\ \text{m}\quad (4.54797\ldots)$ ***(A1)***

Note: Award ***A1*** for 4.55 or equivalent seen, either as a separate calculation or in Pythagorean Theorem.

$\sqrt{(4.54797\ldots)^2 + 2.1^2}$
$5.01\ \text{m}\quad (5.00939\ldots\text{m})$ ***A1***

METHOD 2
attempt to use cosine rule ***(M1)***
$(c^2 =)\ 2.8^2 + 6.4^2 - 2(2.8)(6.4)\cos(48.5903\ldots)$ ***(A1)(A1)***

Note: Award ***A1*** for $48.5903...°$ substituted into cosine rule formula, ***A1*** for correct substitution.

$(c =)\ 5.01\ \text{m}\quad (5.00939\ldots\text{m})$ ***A1***

[4 marks]

(c) camera 1 is closer to the cash register (than camera 2 and both cameras are at the same height on the wall) ***R1***
the larger angle of depression is from camera 1 ***A1***

Note: Do not award ***R0A1***. Award ***R0A0*** if additional calculations are completed and used in their justification, as per the question. Accept "$1.85<4.55$" or "$2.8<5.01$" as evidence for the ***R1***.

[2 marks]
Total [8 marks]

4. (a) $(\text{pH} =) \ -\log_{10}(1.3\times10^{-5})$ ***(M1)***

$4.89 \quad (4.88605\ldots)$ ***A1***

[2 marks]

(b) **EITHER**

calculating pH

$(\text{pH} =) \ -\log_{10}(10\times1.3\times10^{-5})$ ***(M1)***

$3.89 \quad (3.88605\ldots)$ ***A1***

$(3.89 < 4.89$, therefore) the unknown liquid is more acidic (than coffee). ***A1***

Note: Follow through within the part for the final ***A1***. A correct conclusion must be supported by a mathematical justification linking the C-value to the pH level to earn the final ***A1***; a comparison of C-values only earns ***M0A0A0***.

OR

referencing the graph

The graph of $y = -\log_{10}(x)$ shows that as the value of x increases, the value of y decreases. ***M1***

Since the C-value (x-value) of the unknown liquid is larger than that of the coffee, the pH level (y-value) is lower. ***R1***

The unknown liquid is more acidic (than coffee). ***A1***

Note: Follow through within the part for the final ***A1***. A correct conclusion must be supported by a mathematical justification linking the C-value to the pH level to earn the final ***A1***; a comparison of C-values only earns ***M0R0A0***.

[3 marks]

Total [5 marks]

5. (a) $\left(\text{E}(X) =\right) 10\times0.8$ ***(M1)***

8 (people) ***A1***

[2 marks]

(b) recognition of binomial probability ***(M1)***

$0.0881 \quad (0.0880803\ldots)$ ***A1***

[2 marks]

(c) 0.8 and 6 seen **OR** 0.2 and 3 seen ***(A1)***

attempt to use binomial probability ***(M1)***

$0.121 \quad (0.120873\ldots)$ ***A1***

[3 marks]

Total [7 marks]

6. (a) **EITHER**

attempt to substitute 3, 4 and 7 into area of a trapezoid formula ***(M1)***

$(A =) \frac{1}{2}(7+4)(3)$

OR

given line expressed as an integral ***(M1)***

$(A =) \int_{-1}^{2}(6-x)\ \mathrm{d}x$

OR

attempt to sum area of rectangle and area of triangle ***(M1)***

$(A =) 4 \times 3 + \frac{1}{2}\ (3)(3)$

THEN

16.5 (square units) ***A1***

[2 marks]

(b) (i) $(A =) \int_{-1}^{2} 1.5x^2 - 2.5x + 3\ \mathrm{d}x$ ***A1A1***

Note: Award ***A1*** for the limits $x = -1$, $x = 2$ in correct location. Award ***A1*** for an integral of the quadratic function, $\mathrm{d}x$ must be included. Do not accept "y" in place of the function, given that two equations are in the question.

(ii) 9.75 (square units) ***A1***

[3 marks]

(c) $16.5 - 9.75$ ***(M1)***

6.75 (square units) ***A1***

[2 marks]

Total [7 marks]

7. (a) $(88 - 62) \times 1.5$ **OR** 26×1.5 seen anywhere **OR** 39 seen anywhere ***(M1)***

$62 - 39$
23 ***A1***

$25 > 23$ ***R1***
so is not an outlier ***AG***

[3 marks]

(b) The median score for the evening class is higher than the median score for the morning class. ***A1***

THEN
but the scores are more spread out in the evening class than in the morning class ***A1***

OR
the scores are more inconsistent in the evening class ***A1***
OR
the lowest scores are in the evening class ***A1***
OR
the interquartile range is lower in the morning class ***A1***
OR
the lower quartile is lower in the evening class ***A1***

Note: If an incorrect comparison is also made, award at most ***A1A0***.

Award ***A0*** for a comparison that references "the mean score" unless working is shown for the estimated means of the data sets, calculated from the mid-points of the 4 intervals. The estimated mean for the morning class is 71.375 and the estimated mean for the evening class is 70.5.

[2 marks]
Total [5 marks]

8. (a) $(H_1:)\ \mu_1 - \mu_2 \neq 0 \quad (\mu_1 \neq \mu_2)$ ***A1***

Note: Accept an equivalent statement in words, however reference to "**population** mean" must be explicit for ***A1*** to be awarded.

[1 mark]

(b) $0.0778 \quad (0.0778465\ldots)$ ***A2***

Note: Award ***A1*** for an answer of $0.0815486\ldots$ from not using a pooled estimate of the variance.

[2 marks]

(c) (i) $0.0778 < 0.1$ ***R1***
reject the null hypothesis ***A1***

Note: Do not award ***R0A1***.

(ii) there is (significant evidence of) a difference between the (population) **mean** reaction times ***A1***

Note: Their conclusion in (c)(ii) must match their conclusion in (c)(i) to earn ***A1***. Award ***A0*** if their conclusion refers to mean reaction times in the sample.

[3 marks]
Total [6 marks]

9. (a) *Accept any one of the following (or equivalent):*
one minimum and one maximum point
three x-intercepts or three roots (or zeroes)
one point of inflexion ***R1***

Note: Do not accept "S shape" as a justification.

[1 mark]

(b) (i) $(d =) -5$ ***A1***

(ii) $8 = a + b + c$
$4 = 8a + 4b + 2c$
$0 = 27a + 9b + 3c$ ***A2***

Note: Award ***A2*** if all three equations are correct.
Award ***A1*** if at least one is correct. Award ***A1*** for three correct equations that include the letter "d".

(iii) $a = 2,\ b = -12,\ c = 18$ ***A1***

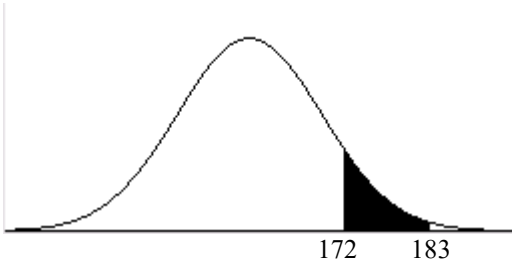
[4 marks]

(c) equating found expression to zero ***(M1)***
$0 = 2t^3 - 12t^2 + 18t - 5$
$t = 0.358216\ldots,\ 1.83174\ldots,\ 3.81003\ldots$ ***(A1)***

(so total time in debt is $3.81003\ldots - 1.83174\ldots + 0.358216 \approx$)
$2.34\ (2.33650\ldots)$ years ***A1***

[3 marks]
Total [8 marks]

10. (a) sketch of normal curve with shaded region to the right of the mean and correct values ***(M1)***



$0.0921 \quad (0.0920950\ldots)$ ***A1***

[2 marks]

(b) **EITHER**

$\left(\mathrm{P}(x < 172)\right)$

$0.906200\ldots$ ***(A1)***

$(0.906200\ldots - 0.68)$

$0.226200\ldots$ ***(A1)***

OR

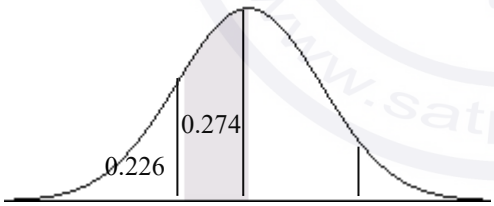
$\left(\mathrm{P}(163 < x < 172)\right)$

$0.406200\ldots$ ***(A1)***

$0.5 - (0.68 - 0.406200\ldots)$ **OR** $0.5 + (0.68 - 0.406200\ldots)$

$0.226200\ldots$ **OR** $0.773799\ldots$ ***(A1)***

OR



(A1)(A1)

Note: Award ***A1*** for a normal distribution curve with a vertical line on each side of the mean and a correct probability of either 0.406 or 0.274 or 0.906 shown, ***A1*** for a probability of 0.226 seen.

THEN

$(k =)\ 158$ g $(157.867\ldots \text{g})$ ***A1***

[3 marks]

Total [5 marks]

11. (a) $\left(f'(x)=\right)\ 2x+\frac{3}{x^2}$ ***A1A1***

Note: Award ***A1*** for $2x$, ***A1*** for $+\frac{3}{x^2}$ **OR** $+3x^{-2}$.

[2 marks]

(b) attempt to substitute 1 into their part (a) ***(M1)***

$\left(f'(1)=\right)\ 2(1)+\frac{3}{1^2}$

5 ***A1***

[2 marks]

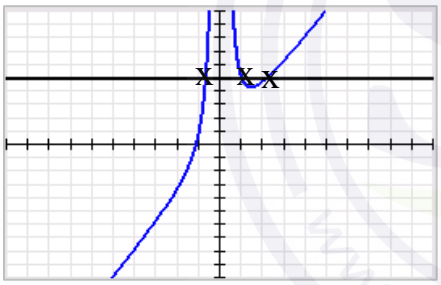
(c) **EITHER**

$5=2x+\frac{3}{x^2}$ ***M1***

$x=-0.686,\ 1,\ 2.19\ \ (-0.686140...,\ 1,\ 2.18614\ldots)$ ***A1***

OR

sketch of $y=f'(x)$ with line $y=5$ ***M1***



three points of intersection marked on this graph ***A1***
(and it can be assumed no further intersections occur outside of this window)

THEN

there are two other tangent lines to $f(x)$ that are parallel to L ***A1***

Note: The final ***A1*** can be awarded provided two solutions other than $x=1$ are shown **OR** three points of intersection are marked on the graph.

Award ***M1A1A1*** for an answer of "3 lines" where L is considered to be parallel with itself (given guide definition of parallel lines), but only if working is shown.

[3 marks]

Total [7 marks]

12. (a) $(x =)\ -\frac{4.48}{2(-1.6)}$ **OR** coordinates of maximum point $(1.4, 3.136)$ ***(M1)***

$x = 1.4$ ***A1***

[2 marks]

(b) **METHOD 1**

the cart is centred in the archway when it is between $x = 0.6$ and $x = 2.2$, ***A1***

where $y \geq 2.112\ (\text{m})$ (which is greater than 2) ***R1***

the archway is tall enough for the crate ***A1***

Note: Do not award ***R0A1***.

METHOD 2

the height of the archway is greater or equal to 2.0 between $x = 0.557385\ldots$ and $x = 2.24261\ldots$ ***A1***

width of this section of archway =

$(2.24261\ldots - 0.557385\ldots =)\ 1.68522\ldots\ (\text{m})$ (which is greater than 1.6) ***R1***

the archway is wide enough for the crate ***A1***

Note: Do not award ***R0A1***.

[3 marks]

Total [5 marks]

13. (a) **METHOD 1** – *(with FV = 4000)*

EITHER

N= 10

I=1.5

FV= 4000

P/Y= 1

C/Y= 1 ***(A1)(M1)***

Note: Award ***A1*** for $(3.5 - 2 =)\ 1.5$ seen and ***M1*** for all other entries correct.

OR

$4000 = A(1 + 0.015)^{10}$ ***(A1)(M1)***

Note: Award ***A1*** for 1.5 or 0.015 seen, ***M1*** for attempt to substitute into compound interest formula **and** equating to 4000.

THEN

(PV =) $3447 ***A1***

Note: Award ***A0*** if not rounded to a whole number or a negative sign given.

continued…

Question 13 continued

METHOD 2 – *(With FV including inflation)*
calculate FV with inflation
4000×1.02^{10} ***(A1)***
(=4875.977…)

EITHER
$4000 \times 1.02^{10} = \text{PV} \times 1.035^{10}$ ***(A1)***
OR
N= 10
I= 3.5
FV= 4875.977…
P/Y= 1
C/Y= 1 ***(M1)***

Note: Award ***M1*** for *their* FV and all other entries correct.

THEN
(PV =) \$3457 ***A1***

Note: Award ***A0*** if not rounded to a whole number or a negative sign given.

METHOD 3 – *(Using formula to calculate real rate of return)*
(real rate of return =) 1.47058…(%) ***(A1)***

EITHER
$4000 = \text{PV} \times 1.0147058\ldots^{10}$ ***(A1)***
OR
N= 10
I= 1.47058…
FV= 4000
P/Y= 1
C/Y= 1 ***(M1)***

Note: Award ***M1*** for all entries correct.

THEN
(PV =) \$3457 ***A1***

[3 marks]

continued…

Question 13 continued

(b) **METHOD 1** – *(Finding the future value of the investment using PV from part (a))*
N= 10
I=3.5
PV= 3446.66…(from Method 1) **OR** 3456.67…(from Methods 2, 3)
P/Y= 1
C/Y= 1 ***(M1)***

Note: Award ***M1*** for interest rate 3.5 **and** answer to part (a) as PV.

(FV=) \$4861.87 **OR** \$4875.97 ***(A1)***
so payment required (from TVM) will be \$294 **OR** \$295 ***A1***

Note: Award ***A0*** if a negative sign given, unless already penalized in part (a).

METHOD 2 – *(Using FV)*
N= 10
I=3.5
PV= –1000
FV= 4875.977…
P/Y= 1
C/Y= 1 ***(A1)(M1)***

Note: Award ***A1*** for I=3.5 **and** FV= ±4875.977…, ***M1*** for all other entries correct **and** opposite PV and FV signs.

(PMT =) \$295 (295.393) ***A1***

Note: Correct 3sf answer is 295, however accept an answer of 296 given that the context supports rounding up. Award ***A0*** if a negative sign given, unless already penalized in part (a).

[3 marks]
Total [6 marks]

Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

May 2022

Mathematics: applications and interpretation

Standard level

Paper 1

16 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**.
A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.
R Marks awarded for clear **Reasoning**.
AG Answer given in the question and so no marks are awarded.
FT Follow through. The practice of awarding marks, despite candidate errors in previous parts, for their correct methods/answers using incorrect results.

Using the markscheme

1 General

Award marks using the annotations as noted in the markscheme *eg* ***M1, A2.***

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***A3***, ***M2*** *etc.*, do **not** split the marks, unless there is a note.
- The response to a "show that" question does not need to restate the ***AG*** line, unless a **Note** makes this explicit in the markscheme.
- Once a correct answer to a question or part question is seen, ignore further working even if this working is incorrect and/or suggests a misunderstanding of the question. This will encourage a uniform approach to marking, with less examiner discretion. Although some candidates may be advantaged for that specific question item, it is likely that these candidates will lose marks elsewhere too.
- An exception to the previous rule is when an incorrect answer from further working is used **in a subsequent part**. For example, when a correct exact value is followed by an incorrect decimal approximation in the first part and this approximation is then used in the second part. In this situation, award ***FT*** marks as appropriate but do not award the final ***A1*** in the first part. Examples:

	Correct answer seen	**Further working seen**	**Any FT issues?**	**Action**
1.	$8\sqrt{2}$	5.65685... *(incorrect decimal value)*	No. Last part in question.	Award ***A1*** for the final mark *(condone the incorrect further working)*
2.	$\frac{35}{72}$	0.468111… (incorrect decimal value)	Yes. Value is used in subsequent parts.	Award ***A0*** for the final mark *(and full **FT** is available in subsequent parts)*

3 Implied marks

Implied marks appear in **brackets e.g.** ***(M1)***,and can only be awarded if **correct** work is seen or implied by subsequent working/answer.

4 Follow through marks (only applied after an error is made)

Follow through (***FT***) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) (e.g. incorrect value from part (a) used in part (d) or incorrect value from part (c)(i) used in part (c)(ii)). Usually, to award ***FT*** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if all the marks awarded in a subsequent part are for the answer or are implied, then ***FT*** marks should be awarded for *their* correct answer, even when working is not present.

For example: following an incorrect answer to part (a) that is used in subsequent parts, where the markscheme for the subsequent part is ***(M1)A1***, it is possible to award full marks for *their* correct answer, **without working being seen.** For longer questions where all but the answer marks are implied this rule applies but may be overwritten by a **Note** in the Markscheme.

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks, by reflecting on what each mark is for and how that maps to the simplified version.
- If the error leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word “their” in a description, to indicate that candidates may be using an incorrect value.
- If the candidate’s answer to the initial question clearly contradicts information given in the question, it is not appropriate to award any ***FT*** marks in the subsequent parts. This includes when candidates fail to complete a “show that” question correctly, and then in subsequent parts use their incorrect answer rather than the given value.
- Exceptions to these ***FT*** rules will be explicitly noted on the markscheme.
- If a candidate makes an error in one part but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the command term was “Hence”.

5 Mis-read

If a candidate incorrectly copies values or information from the question, this is a mis-read (***MR***). A candidate should be penalized only once for a particular misread. Use the ***MR*** stamp to indicate that this has been a misread and do not award the first mark, even if this is an ***M*** mark, but award all others as appropriate.

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.
- If a candidate uses a correct answer, to a "show that" question, to a higher degree of accuracy than given in the question, this is NOT a misread and full marks may be scored in the subsequent part.
- ***MR*** can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If the command term is 'Hence' and not 'Hence or otherwise' then alternative methods are not permitted unless covered by a note in the mark scheme.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**.

7 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation** for example 1.9 and $1{,}9$ or 1000 and $1{,}000$ and 1.000.
- Do not accept final answers written using calculator notation. However, ***M*** marks and intermediate ***A*** marks can be scored, when presented using calculator notation, provided the evidence clearly reflects the demand of the mark.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, some **equivalent** answers will generally appear in brackets. Not all equivalent notations/answers/methods will be presented in the markscheme and examiners are asked to apply appropriate discretion to judge if the candidate work is equivalent.

8 Format and accuracy of answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. If the level of accuracy is not stated in the question, the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures*.

Where values are used in subsequent parts, the markscheme will generally use the exact value, however candidates may also use the correct answer in subsequent parts. The markscheme will often explicitly include the subsequent values that come "*from the use of 3 sf values*".

Simplification of final answers: Candidates are advised to give final answers using good mathematical form. In general, for an ***A*** mark to be awarded, arithmetic should be completed, and any values that lead to integers should be simplified; for example, $\sqrt{\frac{25}{4}}$ should be written as $\frac{5}{2}$. An exception to this is simplifying fractions, where lowest form is not required (although the numerator and the denominator must be integers); for example, $\frac{10}{4}$ may be left in this form or written as $\frac{5}{2}$. However, $\frac{10}{5}$ should be written as 2, as it simplifies to an integer.

Algebraic expressions should be simplified by completing any operations such as addition and multiplication, e.g. $4e^{2x} \times e^{3x}$ should be simplified to $4e^{5x}$, and $4e^{2x} \times e^{3x} - e^{4x} \times e^{x}$ should be simplified to $3e^{5x}$. Unless specified in the question, expressions do not need to be factorized, nor do factorized expressions need to be expanded, so $x(x+1)$ and $x^2 + x$ are both acceptable.

Please note: intermediate ***A*** marks do NOT need to be simplified.

9 Calculators

A GDC is required for this paper, but If you see work that suggests a candidate has used any calculator not approved for IB DP examinations (eg CAS enabled devices), please follow the procedures for malpractice.

10. Presentation of candidate work

Crossed out work: If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work unless an explicit note from the candidate indicates that they would like the work to be marked.

More than one solution: Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise. If the layout of the responses makes it difficult to judge, examiners should apply appropriate discretion to judge which is "first".

1. height of triangle at roof $= 1.35 - 0.9 = 0.45$ ***(A1)***

Note: Award ***A1*** for 0.45 (height of triangle) seen on the diagram.

slant height $= \sqrt{0.45^2 + 0.45^2}$ **OR** $\sin(45^\circ) = \dfrac{0.45}{\text{slant height}}$ ***(M1)***

$= \sqrt{0.405}$ $(0.636396..., 0.45\sqrt{2})$ ***A1***

Note: If using $\sin(45^\circ) = \dfrac{0.45}{\text{slant height}}$ then ***(A1)*** for angle of 45°, ***(M1)*** for a correct trig statement.

area of one rectangle on roof $= \sqrt{0.405} \times 0.9$ $(= 0.572756...)$ ***M1***

area painted $= (2 \times \sqrt{0.405} \times 0.9 = 2 \times 0.572756...)$

1.15 m^2 $(1.14551... \text{ m}^2, 0.81\sqrt{2} \text{ m}^2)$ ***A1***

[Total 5 marks]

2. (a) $\sqrt{3.2^2 + 4.5^2 + 5.8^2}$ ***(M1)***

$= 8.01$ $(8.00812...)$ m ***A1***

[2 marks]

(b) $\text{F}\hat{\text{A}}\text{O} = \sin^{-1}\left(\dfrac{5.8}{8.00812...}\right)$ **OR** $\cos^{-1}\left(\dfrac{5.52177...}{8.00812...}\right)$ **OR** $\tan^{-1}\left(\dfrac{5.8}{5.52177...}\right)$ ***(M1)***

46.4° $(46.4077...^\circ)$ ***A1***

[2 marks]

[Total 4 marks]

3. (a) 1.2 metres ***A1***

[1 mark]

(b) $-4.8t^2 + 21t + 1.2 = 0$ ***(M1)***

$(t =)$ 4.43 s $(4.431415...$ s$)$ ***A1***

Note: If both values for t are seen do not award the ***A1*** mark unless the negative is explicitly excluded.

[2 marks]

(c) $0 \le t \le 4.43$ **OR** $[0, 4.43]$ ***A1A1***

Note: Award ***A1*** for correct endpoints and ***A1*** for expressing answer with correct notation. Award at most ***A1A0*** for use of x instead of t.

[2 marks]

[Total 5 marks]

4. (a) midpoint $(1, 2.5)$ ***A1***

$m_{AB} = \frac{6-(-1)}{8-(-6)} = \frac{1}{2}$ ***(M1)A1***

Note: Accept equivalent gradient statements including using midpoint.

$m_{\perp} = -2$ ***M1***

Note: Award ***M1*** for finding the negative reciprocal of their gradient.

$y - 2.5 = -2(x-1)$ **OR** $y = -2x + \frac{9}{2}$ **OR** $4x + 2y - 9 = 0$ ***A1***

[5 marks]

(b) substituting $x = -6$ into their equation from part (a) ***(M1)***

$y = -2(-6) + \frac{9}{2}$

$y = 16.5$ ***A1***

Note: Award ***M1A0*** for $(-6, 16.5)$ as their final answer.

[2 marks]
[Total 7 marks]

5. (a) $x + y + z = 600$ ***A1***

$15x + 10y + 12z = 7816$ ***A1***

$x = 2y$ ***A1***

Note: Condone other labelling if clear, e.g. a (adult), c (child) and s (student). Accept equivalent, distinct equations e.g. $2y + y + z = 600$.

[3 marks]

(b) $x = 308,\ y = 154,\ z = 138$ ***A1A1***

Note: Award ***A1*** for all three correct values seen, ***A1*** for correctly labelled as x, y or z. Accept answers written in words: e.g. 308 adult tickets.

[2 marks]
[Total 5 marks]

6. (a) $\frac{1}{2}\left(0.6+0+2(1.2+1.2)\right)$ ***(A1)(M1)***

Note: Award ***A1*** for evidence of $h=1$, ***M1*** for a correct substitution into trapezoidal rule (allow for an incorrect h only). The zero can be omitted in the working.

$2.7\ \text{m}^2$ ***A1***

[3 marks]

(b) $\int_{-1}^{2}\frac{-x^3-3x^2+4x+12}{10}\,\text{d}x$ **OR** $\int_{-1}^{2}f(x)\,\text{d}x$ ***(M1)***

Note: Award ***M1*** for using definite integration with correct limits.

$2.925\ \text{m}^2$ ***A1***

Note: Question requires exact answer, do not award final ***A1*** for 2.93.

[2 marks]

(c) $9-2.925$ ***(M1)***

Note: Award ***M1*** for 9 seen as part of a subtraction.

$=6.08\ \text{m}^2\ \ (6.075)$ ***A1***

[2 marks]
[Total 7 marks]

7. (a) H_0 : The die is fair **OR** $P(\text{any number}) = \frac{1}{6}$ **OR** probabilities are equal

H_1 : The die is not fair **OR** $P(\text{any number}) \neq \frac{1}{6}$ **OR** probabilities are not equal ***A1***

[1 mark]

(b) 5 ***A1***

[1 mark]

(c) 10 ***A1***

[1 mark]

(d) $(p\text{-value} =)\ 0.287\ (0.28724163\ldots.)$ ***A2***

[2 marks]

(e) $0.287 > 0.05$ ***R1***

EITHER
Insufficient evidence to reject the null hypothesis ***A1***

OR
Insufficient evidence to reject that the die is fair ***A1***

Note: Do not award ***R0A1***. Condone "*accept* the null hypothesis" or "the die is fair". Their conclusion must be consistent with their p-value and their hypothesis.

[2 marks]
[Total 7 marks]

8. (a) 50% ***A1***

Note: Do not accept 0.5 or $\frac{1}{2}$.

[1 mark]

(b) $0.0478\ (0.0477903\ldots, 4.78\%)$ ***A2***

[2 marks]

(c) $P(X < k) = 0.98$ **OR** $P(X > k) = 0.02$ ***(M1)***

Note: Award ***(M1)*** for a sketch with correct region identified.

$506\text{ g}\ (506.161\ldots)$ ***A2***

[3 marks]
[Total 6 marks]

9. (a) $f'(x) = -2x^{-2} + 6x$ **OR** $f'(x) = -\frac{2}{x^2} + 6x$ ***A1(M1)A1***

Note: Award ***A1*** for $6x$ seen, and ***(M1)*** for expressing $\frac{1}{x}$ as x^{-1} (this can be implied from either x^{-2} or $\frac{2}{x^2}$ seen in their final answer), ***A1*** for $-\frac{2}{x^2}$. Award at most ***A1(M1)A0*** if any additional terms are seen.

[3 marks]

(b) finding gradient at $x = 1$

$\left.\frac{\mathrm{d}y}{\mathrm{d}x}\right|_{x=1} = 4$ ***A1***

finding the perpendicular gradient ***M1***

$m_{\perp} = -\frac{1}{4}$

$2 = -\frac{1}{4}(1) + c$ **OR** $y - 2 = -\frac{1}{4}(x - 1)$ ***M1***

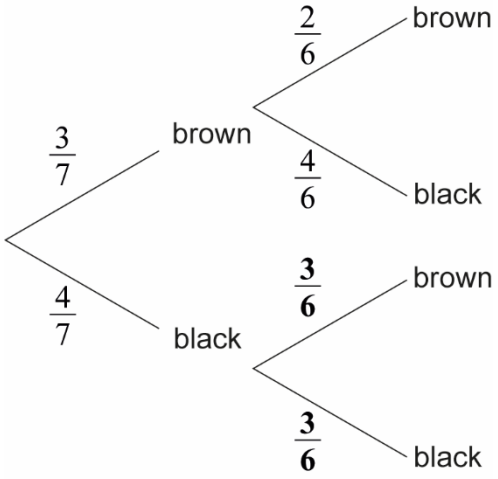
Note: Award ***M1*** for correctly substituting $x = 1$ and $y = 2$ and their $m_{\perp}$.

$x + 4y - 9 = 0$ ***A1***

Note: Do not award the final ***A1*** if the answer is not in the required form. Accept integer multiples of the equation.

[4 marks]
[Total 7 marks]

10. (a)



A1

Note: Award ***A1*** for both missing probabilities correct.

[1 mark]

(b) multiplying along branches and then adding outcomes ***(M1)***

$$\frac{3}{7}\times\frac{2}{6}+\frac{4}{7}\times\frac{3}{6}$$

$$=\frac{18}{42}\left(=\frac{3}{7}\approx 0.429\ (42.9\%)\right)$$ ***A1***

[2 marks]

(c) use of conditional probability formula ***M1***

$$\frac{\left(\frac{3}{7}\times\frac{2}{6}\right)}{\left(\frac{3}{7}\right)}$$ ***A1***

$$=\frac{6}{18}\left(=\frac{1}{3}\right)\ \left(\frac{252}{756},\ 0.333,\ 33.3\%\right)$$ ***A1***

[3 marks]
[Total 6 marks]

11. (a) $\log_{10} 100 = a - 3$ ***(M1)***

$a = 5$ ***A1***

[2 marks]

(b) **EITHER**

$N = 10^{5-M}$ ***(M1)***

$= \frac{10^5}{10^M} \left(= \frac{100000}{10^M} \right)$

OR

$100 = \frac{b}{10^3}$ ***(M1)***

THEN

$b = 100000 \ (= 10^5)$ ***A1***

[2 marks]

(c) $0.001 < N < 100000 \ (10^{-3} < N < 10^5)$ ***A1A1***

Note: Award ***A1*** for correct endpoints and ***A1*** for correct inequalities/interval notation.

[2 marks]

(d) $N = \frac{10^5}{10^{7.2}} \ (= 0.0063095...)$ ***(M1)***

length of time = $\frac{1}{0.0063095...} = 10^{2.2}$

= 158 years ***A1***

[2 marks]

[Total 8 marks]

12. (a) **METHOD 1**

(when $t = 2$)

$\frac{\mathrm{d}P}{\mathrm{d}t} = -4$ **OR** $\frac{\mathrm{d}P}{\mathrm{d}t} < 0$ (equivalent in words) **OR** $3(2)^2 - 8(2) = -4$ ***M1***

therefore P is decreasing ***A1***

METHOD 2

sketch with $t = 2$ indicated in 4th quadrant **OR** t-intercepts identified ***M1***

therefore P is decreasing ***A1***

[2 marks]

(b) $(P(t) =)\ t^3 - 4t^2\ (+c)$ ***A1A1***

$4 = 1^3 - 4(1)^2 + c$ ***(M1)***

Note: Award ***M1*** for substituting $(1, 4)$ into their equation with $+c$ seen.

$c = 7$

$P(t) = t^3 - 4t^2 + 7$ ***A1***

[4 marks]

[Total 6 marks]

13. (a) use of geometric sequence with $r = 0.85$ ***M1***

EITHER

$(0.85)^6(1.8)$ **OR** $0.678869\ldots$ **OR** $(0.85)^5(1.53)$ ***A1***

$= 0.68$ m

$= 68$ cm ***AG***

OR

$(0.85)^6(180)$ **OR** $(0.85)^5(153)$ ***A1***

$= 68$ cm ***AG***

[2 marks]

continued…

Question 13 continued

(b) **EITHER**

$(0.85)^n(1.8) > 0.1$ **OR** $(0.85)^{n-1}(1.53) > 0.1$ ***(M1)***

Note: If 1.8 m (or 180 cm) is used then ***(M1)*** only awarded for use of n in $(0.85)^n(1.8) > 0.1$.
If 1.53 m (or 153 cm) is used then ***(M1)*** only awarded for use of $n-1$ in $(0.85)^{n-1}(1.53) > 0.1$.

17 ***A1***

OR

$(0.85)^{17}(1.8) = 0.114$ m and $(0.85)^{18}(1.8) = 0.0966$ m ***(M1)***

17 ***A1***

OR

solving $(0.85)^n(1.8) = 0.1$ to find $n = 17.8$ ***(M1)***

17 ***A1***

Note: Evidence of solving may be a graph **OR** the "solver" function **OR** use of logs to solve the equation. Working may use cm.

[2 marks]

(c) **EITHER**

distance (in one direction) travelled between first and fourth bounce

$= \dfrac{(1.8 \times 0.85)(1-0.85^3)}{1-0.85}$ $(= 3.935925)$ ***(A1)***

recognizing distances are travelled twice except first distance ***(M1)***

$1.8 + 2(3.935925)$

$= 9.67$ m $(9.67185...$ m$)$ ***A1***

OR

distance (in one direction) travelled between drop and fourth bounce

$= \dfrac{(1.8)(1-0.85^4)}{1-0.85}$ $(= 5.735925)$ ***(A1)***

recognizing distances are travelled twice except first distance ***(M1)***

$2(5.735925) - 1.8$

$= 9.67$ m $(9.67185...$ m$)$ ***A1***

OR

distance (in one direction) travelled between first and fourth bounce

$(0.85)(1.8) + (0.85)^2(1.8) + (0.85)^3(1.8)$ $(= 3.935925...)$ ***(A1)***

recognizing distances are travelled twice except first distance ***(M1)***

$1.8 + 2(0.85)(1.8) + 2(0.85)^2(1.8) + 2(0.85)^3(1.8)$

$= 9.67$ m $(9.67185...$ m$)$ ***A1***

Note: Answers may be given in cm.

[3 marks]
[Total 7 marks]

Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

November 2021

Mathematics: analysis and approaches

Standard level

Paper 1

23 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

R Marks awarded for clear **Reasoning**.

AG Answer given in the question and so no marks are awarded.

FT Follow through. The practice of awarding marks, despite candidate errors in previous parts, for their correct methods/answers using incorrect results.

Using the markscheme

1 General

Award marks using the annotations as noted in the markscheme *eg* ***M1, A2****.*

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***A3***, ***M2*** *etc*., do **not** split the marks, unless there is a note.
- The response to a "show that" question does not need to restate the ***AG*** line, unless a **Note** makes this explicit in the markscheme.
- Once a correct answer to a question or part question is seen, ignore further working even if this working is incorrect and/or suggests a misunderstanding of the question. This will encourage a uniform approach to marking, with less examiner discretion. Although some candidates may be advantaged for that specific question item, it is likely that these candidates will lose marks elsewhere too.
- An exception to the previous rule is when an incorrect answer from further working is used **in a subsequent part**. For example, when a correct exact value is followed by an incorrect decimal approximation in the first part and this approximation is then used in the second part. In this situation, award ***FT*** marks as appropriate but do not award the final ***A1*** in the first part. Examples:

	Correct answer seen	Further working seen	Any FT issues?	Action
1.	$8\sqrt{2}$	5.65685... *(incorrect decimal value)*	No. Last part in question.	Award ***A1*** for the final mark *(condone the incorrect further working)*
2.	$\frac{35}{72}$	0.468111… (incorrect decimal value)	Yes. Value is used in subsequent parts.	Award ***A0*** for the final mark *(and full **FT** is available in subsequent parts)*

3 Implied marks

Implied marks appear in **brackets e.g.** ***(M1)***,and can only be awarded if **correct** work is seen or implied by subsequent working/answer.

4 Follow through marks (only applied after an error is made)

Follow through (***FT***) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) (e.g. incorrect value from part (a) used in part (d) or incorrect value from part (c)(i) used in part (c)(ii)). Usually, to award ***FT*** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if all the marks awarded in a subsequent part are for the answer or are implied, then ***FT*** marks should be awarded for *their* correct answer, even when working is not present.

For example: following an incorrect answer to part (a) that is used in subsequent parts, where the markscheme for the subsequent part is ***(M1)A1***, it is possible to award full marks for *their* correct answer, **without working being seen.** For longer questions where all but the answer marks are implied this rule applies but may be overwritten by a **Note** in the Markscheme.

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate.

- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks, by reflecting on what each mark is for and how that maps to the simplified version.

- If the error leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).

- The markscheme may use the word “their” in a description, to indicate that candidates may be using an incorrect value.

- If the candidate’s answer to the initial question clearly contradicts information given in the question, it is not appropriate to award any ***FT*** marks in the subsequent parts. This includes when candidates fail to complete a “show that” question correctly, and then in subsequent parts use their incorrect answer rather than the given value.

- Exceptions to these ***FT*** rules will be explicitly noted on the markscheme.

- If a candidate makes an error in one part but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the command term was "Hence".

5 Mis-read

If a candidate incorrectly copies values or information from the question, this is a mis-read (***MR***). A candidate should be penalized only once for a particular misread. Use the ***MR*** stamp to indicate that this has been a misread and do not award the first mark, even if this is an ***M*** mark, but award all others as appropriate.

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.

- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).

- Miscopying of candidates' own work does **not** constitute a misread, it is an error.

- If a candidate uses a correct answer, to a "show that" question, to a higher degree of accuracy than given in the question, this is NOT a misread and full marks may be scored in the subsequent part.

- ***MR*** can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If the command term is 'Hence' and not 'Hence or otherwise' then alternative methods are not permitted unless covered by a note in the mark scheme.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**.

7 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation** for example 1.9 and $1{,}9$ or 1000 and $1{,}000$ and 1.000.

- Do not accept final answers written using calculator notation. However, ***M*** marks and intermediate ***A*** marks can be scored, when presented using calculator notation, provided the evidence clearly reflects the demand of the mark.

- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.

- In the markscheme, some **equivalent** answers will generally appear in brackets. Not all equivalent notations/answers/methods will be presented in the markscheme and examiners are asked to apply appropriate discretion to judge if the candidate work is equivalent.

8 Format and accuracy of answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. If the level of accuracy is not stated in the question, the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures.*

Where values are used in subsequent parts, the markscheme will generally use the exact value, however candidates may also use the correct answer to 3 sf in subsequent parts. The markscheme will often explicitly include the subsequent values that come "*from the use of 3 sf values*".

Simplification of final answers: Candidates are advised to give final answers using good mathematical form. In general, for an ***A*** mark to be awarded, arithmetic should be completed, and any values that lead to integers should be simplified; for example, $\sqrt{\frac{25}{4}}$ should be written as $\frac{5}{2}$. An exception to this is simplifying fractions, where lowest form is not required (although the numerator and the denominator must be integers); for example, $\frac{10}{4}$ may be left in this form or written as $\frac{5}{2}$. However, $\frac{10}{5}$ should be written as 2, as it simplifies to an integer.

Algebraic expressions should be simplified by completing any operations such as addition and multiplication, e.g. $4e^{2x} \times e^{3x}$ should be simplified to $4e^{5x}$, and $4e^{2x} \times e^{3x} - e^{4x} \times e^{x}$ should be simplified to $3e^{5x}$. Unless specified in the question, expressions do not need to be factorized, nor do factorized expressions need to be expanded, so $x(x+1)$ and $x^2 + x$ are both acceptable.

Please note: intermediate ***A*** marks do NOT need to be simplified.

9 Calculators

No calculator is allowed. The use of any calculator on this paper is malpractice and will result in no grade awarded. If you see work that suggests a candidate has used any calculator, please follow the procedures for malpractice.

10. Presentation of candidate work

Crossed out work: If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work unless an explicit note from the candidate indicates that they would like the work to be marked.

More than one solution: Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise. If the layout of the responses makes it difficult to judge, examiners should apply appropriate discretion to judge which is "first".

Section A

1. (a) (i) setting $f(x) = 0$ ***(M1)***

$x = 1$, $x = -3$ (accept $(1,0), (-3,0)$) ***A1***

(ii) **METHOD 1**

$x = -1$ ***A1***

substituting their x-coordinate into f ***(M1)***

$y = 8$ ***A1***

$(-1,8)$

METHOD 2

attempt to complete the square ***(M1)***

$-2\left((x+1)^2 - 4\right)$

$x = -1$, $y = 8$ ***A1A1***

$(-1,8)$

[5 marks]

(b) $h = -1$ ***A1***

$k = 8$ ***A1***

[2 marks]

Total [7 marks]

2. recognition that $y = \int \cos\left(x - \frac{\pi}{4}\right) \mathrm{d}x$ ***(M1)***

$y = \sin\left(x - \frac{\pi}{4}\right) \ (+c)$ ***(A1)***

substitute both x and y values into their integrated expression including c (***M1)***

$2 = \sin\frac{\pi}{2} + c$

$c = 1$

$y = \sin\left(x - \frac{\pi}{4}\right) + 1$ ***A1***

[4 marks]

3. (a) (i) $x = 3$ ***A1***

(ii) $y = -2$ ***A1***

[2 marks]

(b) (i) $(-2, 0)$ (accept $x = -2$) ***A1***

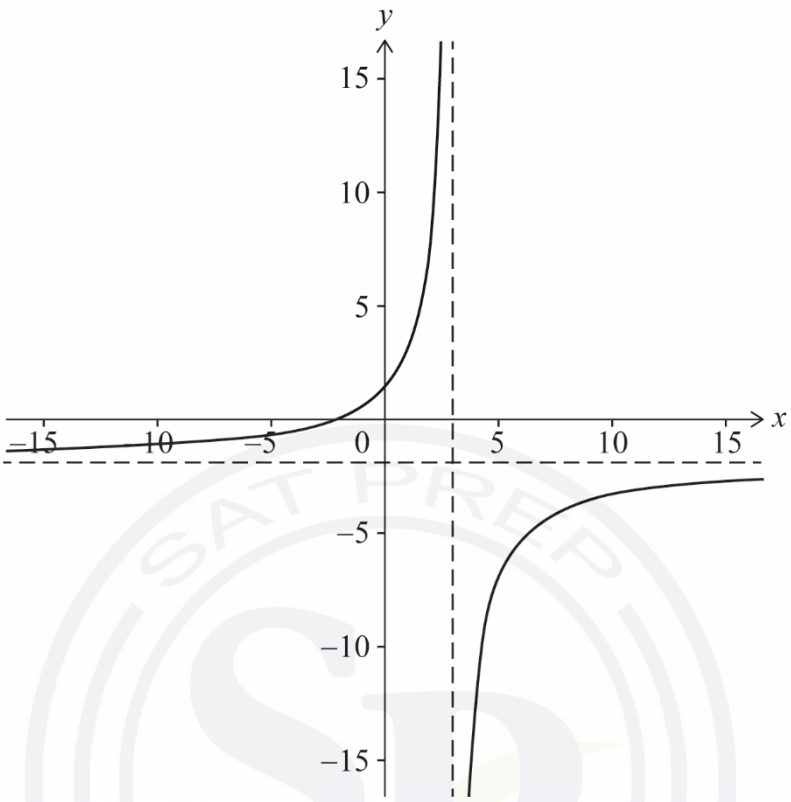
(ii) $\left(0, \frac{4}{3}\right)$ (accept $y = \frac{4}{3}$ and $f(0) = \frac{4}{3}$) ***A1***

[2 marks]

continued…

Question 3 continued.

(c)



A1

Note: Award ***A1*** for completely correct shape: two branches in correct quadrants with asymptotic behaviour.

[1 mark]

Total [5 marks]

4. (a) valid approach to find $\mathrm{P}(R)$ ***(M1)***

tree diagram (must include probabilty of picking box) with correct required probabilities

OR $\mathrm{P}(R \cap B_1) + \mathrm{P}(R \cap B_2)$ OR $\mathrm{P}(R \mid B_1)\mathrm{P}(B_1) + \mathrm{P}(R \mid B_2)\mathrm{P}(B_2)$

$\frac{5}{7} \cdot \frac{1}{2} + \frac{4}{7} \cdot \frac{1}{2}$ ***(A1)***

$\mathrm{P}(R) = \frac{9}{14}$ ***A1***

[3 marks]

(b) events A and R are not independent, since $\frac{9}{14} \cdot \frac{1}{2} \neq \frac{5}{14}$ OR $\frac{5}{7} \neq \frac{9}{14}$ OR $\frac{5}{9} \neq \frac{1}{2}$

OR an explanation e.g. different number of red balls in each box ***A2***

Note: Both conclusion and reasoning are required. Do not split the ***A2***.

[2 marks]

Total [5 marks]

5. (a) $f'(4) = 6$ ***A1***

[1 mark]

(b) $f(4) = 6 \times 4 - 1 = 23$ ***A1***

[1 mark]

(c) $h(4) = f\left(g(4)\right)$ ***(M1)***

$h(4) = f(4^2 - 3 \times 4) = f(4)$

$h(4) = 23$ ***A1***

[2 marks]

(d) attempt to use chain rule to find h' ***(M1)***

$f'\left(g\left(x\right)\right) \times g'\left(x\right)$ OR $\left(x^2 - 3x\right)' \times f'\left(x^2 - 3x\right)$

$h'(4) = \left(2 \times 4 - 3\right) f'\left(4^2 - 3 \times 4\right)$ ***A1***

$= 30$

$y - 23 = 30\left(x - 4\right)$ OR $y = 30x - 97$ ***A1***

[3 marks]

Total [7 marks]

6. (a) **METHOD 1**

attempt to write all LHS terms with a common denominator of $x-1$ ***(M1)***

$2x-3-\frac{6}{x-1}=\frac{2x(x-1)-3(x-1)-6}{x-1}$ OR $\frac{(2x-3)(x-1)}{x-1}-\frac{6}{x-1}$

$=\frac{2x^2-2x-3x+3-6}{x-1}$ OR $\frac{2x^2-5x+3}{x-1}-\frac{6}{x-1}$ ***A1***

$=\frac{2x^2-5x-3}{x-1}$ ***AG***

METHOD 2

attempt to use algebraic division on RHS ***(M1)***

correctly obtains quotient of $2x-3$ and remainder -6 ***A1***

$=2x-3-\frac{6}{x-1}$ as required. ***AG***

[2 marks]

continued…

Question 6 continued.

(b) consider the equation $\frac{2\sin^2 2\theta - 5\sin 2\theta - 3}{\sin 2\theta - 1} = 0$ ***(M1)***

$\Rightarrow 2\sin^2 2\theta - 5\sin 2\theta - 3 = 0$

EITHER

attempt to factorise in the form $(2\sin 2\theta + a)(\sin 2\theta + b)$ ***(M1)***

Note: Accept any variable in place of $\sin 2\theta$.

$(2\sin 2\theta + 1)(\sin 2\theta - 3) = 0$

OR

attempt to substitute into quadratic formula ***(M1)***

$\sin 2\theta = \frac{5 \pm \sqrt{49}}{4}$

THEN

$\sin 2\theta = -\frac{1}{2}$ or $\sin 2\theta = 3$ ***(A1)***

Note: Award ***A1*** for $\sin 2\theta = -\frac{1}{2}$ only.

one of $\frac{7\pi}{6}$ OR $\frac{11\pi}{6}$ (accept 210 or 330) ***(A1)***

$\theta = \frac{7\pi}{12}, \frac{11\pi}{12}$ (must be in radians) ***A1***

Note: Award ***A0*** if additional answers given.

[5 marks]

Total [7 marks]

Section B

7. (a) (i) valid approach to find turning point ($v' = 0$, $-\frac{b}{2a}$, average of roots) ***(M1)***

$4 - 6t = 0$ OR $-\frac{4}{2(-3)}$ OR $\frac{-\frac{2}{3}+2}{2}$

$t = \frac{2}{3}$ (s) ***A1***

(ii) attempt to integrate v ***(M1)***

$\int v \, \mathrm{d}t = \int \left(4 + 4t - 3t^2\right) \mathrm{d}t = 4t + 2t^2 - t^3 \left(+c\right)$ ***A1A1***

Note: Award ***A1*** for $4t + 2t^2$, ***A1*** for $-t^3$.

attempt to substitute their t into their solution for the integral ***(M1)***

distance $= 4\left(\frac{2}{3}\right) + 2\left(\frac{2}{3}\right)^2 - \left(\frac{2}{3}\right)^3$

$= \frac{8}{3} + \frac{8}{9} - \frac{8}{27}$ (or equivalent) ***A1***

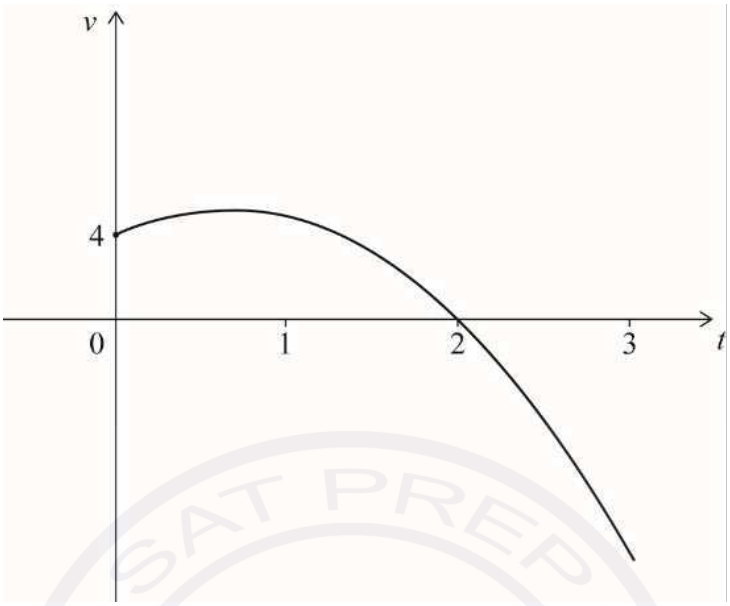
$= \frac{88}{27}$ (m) ***AG***

[7 marks]

continued…

Question 7 continued.

(b)



valid approach to solve $4+4t-3t^2=0$ (may be seen in part (a)) ***(M1)***

$(2-t)(2+3t)$ OR $\dfrac{-4\pm\sqrt{16+48}}{-6}$

correct x- intercept on the graph at $t=2$ ***A1***

Note: The following two ***A*** marks may only be awarded if the shape is a concave down parabola. These two marks are independent of each other and the ***(M1)***.

correct domain from 0 to 3 starting at $(0,4)$ ***A1***

Note: The 3 must be clearly indicated.

vertex in approximately correct place for $t=\dfrac{2}{3}$ and $v>4$ ***A1***

[4 marks]

continued…

Question 7 continued.

(c) recognising to integrate between 0 and 2, or 2 and 3 OR $\int_0^3 \left|4+4t-3t^2\right| \, \mathrm{d}t$ ***(M1)***

$\int_0^2 \left(4+4t-3t^2\right) \, \mathrm{d}t$

$=8$ ***A1***

$\int_2^3 \left(4+4t-3t^2\right) \, \mathrm{d}t$

$=-5$ ***A1***

valid approach to sum the two areas (seen anywhere) ***(M1)***

$\int_0^2 v \, \mathrm{d}t - \int_2^3 v \, \mathrm{d}t$ OR $\int_0^2 v \, \mathrm{d}t + \left|\int_2^3 v \, \mathrm{d}t\right|$

total distance travelled $=13$ (m) ***A1***

[5 marks]

Total [16 marks]

8. (a) $f\left(\frac{2}{3}\right)=4$ OR $a^{\frac{2}{3}}=4$ ***(M1)***

$a=4^{\frac{3}{2}}$ OR $a=\left(2^2\right)^{\frac{3}{2}}$ OR $a^2=64$ OR $\sqrt[3]{a}=2$ ***A1***

$a=8$ ***AG***

[2 marks]

(b) $f^{-1}(x)=\log_8 x$ ***A1***

Note: Accept $f^{-1}(x)=\log_a x$.

Accept any equivalent expression for f^{-1} e.g. $f^{-1}(x)=\frac{\ln x}{\ln 8}$.

[1 mark]

(c) correct substitution ***(A1)***

$\log_8\sqrt{32}$ OR $8^x=32^{\frac{1}{2}}$

correct working involving log/index law ***(A1)***

$\frac{1}{2}\log_8 32$ OR $\frac{5}{2}\log_8 2$ OR $\log_8 2=\frac{1}{3}$ OR $\log_2 2^{\frac{5}{2}}$ OR $\log_2 8=3$ OR $\frac{\ln 2^{\frac{5}{2}}}{\ln 2^3}$ OR $2^{3x}=2^{\frac{5}{2}}$

$f^{-1}\left(\sqrt{32}\right)=\frac{5}{6}$ ***A1***

[3 marks]

continued…

Question 8 continued.

(d) (i) **METHOD 1**

equating a pair of differences ***(M1)***

$u_2 - u_1 = u_4 - u_3 (= u_3 - u_2)$

$\log_8 p - \log_8 27 = \log_8 125 - \log_8 q$

$\log_8 125 - \log_8 q = \log_8 q - \log_8 p$

$\log_8\left(\frac{p}{27}\right) = \log_8\left(\frac{125}{q}\right),\ \log_8\left(\frac{125}{q}\right) = \log_8\left(\frac{q}{p}\right)$ ***A1A1***

$\frac{p}{27} = \frac{125}{q}$ and $\frac{125}{q} = \frac{q}{p}$ ***A1***

$27,\ p,\ q$ and 125 are in geometric sequence ***AG***

Note: If candidate assumes the sequence is geometric, award no marks for part (i). If $r = \frac{5}{3}$ has been found, this will be awarded marks in part (ii).

METHOD 2

expressing a pair of consecutive terms, in terms of d ***(M1)***

$p = 8^d \times 27$ and $q = 8^{2d} \times 27$ OR $q = 8^{2d} \times 27$ and $125 = 8^{3d} \times 27$

two correct pairs of consecutive terms, in terms of d ***A1***

$\frac{8^d \times 27}{27} = \frac{8^{2d} \times 27}{8^d \times 27} = \frac{8^{3d} \times 27}{8^{2d} \times 27}$ (must include 3 ratios) ***A1***

all simplify to 8^d ***A1***

$27,\ p,\ q$ and 125 are in geometric sequence ***AG***

continued…

Question 8 continued.

(ii) **METHOD 1 (geometric, finding r)**

$u_4 = u_1 r^3$ OR $125 = 27(r)^3$ ***(M1)***

$r = \frac{5}{3}$ (seen anywhere) ***A1***

$p = 27r$ OR $\frac{125}{q} = \frac{5}{3}$ ***(M1)***

$p = 45,\ q = 75$ ***A1A1***

METHOD 2 (arithmetic)

$u_4 = u_1 + 3d$ OR $\log_8 125 = \log_8 27 + 3d$ ***(M1)***

$d = \log_8\left(\frac{5}{3}\right)$ (seen anywhere) ***A1***

$\log_8 p = \log_8 27 + \log_8\left(\frac{5}{3}\right)$ OR $\log_8 q = \log_8 27 + 2\log_8\left(\frac{5}{3}\right)$ ***(M1)***

$p = 45,\ q = 75$ ***A1A1***

METHOD 3 (geometric using proportion)

recognizing proportion ***(M1)***

$pq = 125 \times 27$ OR $q^2 = 125p$ OR $p^2 = 27q$

two correct proportion equations ***A1***

attempt to eliminate either p or q ***(M1)***

$q^2 = 125 \times \frac{125 \times 27}{q}$ OR $p^2 = 27 \times \frac{125 \times 27}{p}$

$p = 45,\ q = 75$ ***A1A1***

[9 marks]
Total [15 marks]

Special note: In this question if candidates use the word 'gradient' in their reasoning. e.g. gradient is positive, it must be clear whether this is the gradient of f or the gradient of f' to earn the ***R*** mark.

9. (a) f increases when $p < x < 0$ ***A1***

f increases when $f'(x) > 0$ OR f' is above the x-axis ***R1***

Note: Do not award ***A0R1.***

[2 marks]

(b) $x = 0$ ***A1***

[1 mark]

(c) (i) f is minimum when $x = p$ ***A1***

because $f'(p) = 0$, $f'(x) < 0$ when $x < p$ and $f'(x) > 0$ when $x > p$

(may be seen in a sign diagram clearly labelled as f')

OR because f' changes from negative to positive at $x = p$

OR $f'(p) = 0$ and slope of f' is positive at $x = p$ ***R1***

Note: Do not award ***A0 R1***

(ii) f has points of inflexion when $x = q$, $x = r$ and $x = t$ ***A2***

f' has turning points at $x = q$, $x = r$ and $x = t$

OR

$f''(q) = 0$, $f''(r) = 0$ and $f''(t) = 0$ and f' changes from increasing to decreasing or vice versa at each of these x-values (may be seen in a sign diagram clearly labelled as f'' and f') ***R1***

Note: Award ***A0*** if any incorrect answers are given. Do not award ***A0R1.***

[5 marks]

continued…

Question 9 continued.

(d) recognizing area from p to t (seen anywhere) ***M1***

$\int_p^t |f'(x)| \mathrm{d}x$

recognizing to negate integral for area below x-axis ***(M1)***

$\int_p^0 f'(x)\mathrm{d}x - \int_0^t f'(x)\mathrm{d}x$ OR $\int_p^0 f'(x)\mathrm{d}x + \int_t^0 f'(x)\mathrm{d}x$

$\int_m^n f'(x)\mathrm{d}x = f(n) - f(m)$ (for any integral) ***(M1)***

$f(0) - f(p) - [f(t) - f(0)]$ OR $f(0) - f(p) + f(0) - f(t)$ ***(A1)***

$2f(0) - [f(t) + f(p)] = 20$, $2f(0) - 4 = 20$ ***(A1)***

$f(0) = 12$ ***A1***

[6 marks]
Total [14 marks]

Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

May 2021

Mathematics: applications and interpretation

Standard level

Paper 1

17 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**.
A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.
R Marks awarded for clear **Reasoning**.
AG Answer given in the question and so no marks are awarded.
FT Follow through. The practice of awarding marks, despite candidate errors in previous parts, for their correct methods/answers using incorrect results.

Using the markscheme

1 General
Award marks using the annotations as noted in the markscheme *eg **M1**, **A2**.*

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***A3***, ***M2*** *etc.*, do **not** split the marks, unless there is a note.
- The response to a "show that" question does not need to restate the ***AG*** line, unless a **Note** makes this explicit in the markscheme.
- Once a correct answer to a question or part question is seen, ignore further working even if this working is incorrect and/or suggests a misunderstanding of the question. This will encourage a uniform approach to marking, with less examiner discretion. Although some candidates may be advantaged for that specific question item, it is likely that these candidates will lose marks elsewhere too.
- An exception to the previous rule is when an incorrect answer from further working is used **in a subsequent part**. For example, when a correct exact value is followed by an incorrect decimal approximation in the first part and this approximation is then used in the second part. In this situation, award ***FT*** marks as appropriate but do not award the final ***A1*** in the first part. Examples:

	Correct answer seen	**Further working seen**	**Any FT issues?**	**Action**
1.	$8\sqrt{2}$	5.65685... *(incorrect decimal value)*	No. Last part in question.	Award ***A1*** for the final mark *(condone the incorrect further working)*
2.	$\frac{35}{72}$	0.468111… (incorrect decimal value)	Yes. Value is used in subsequent parts.	Award ***A0*** for the final mark *(and full **FT** is available in subsequent parts)*

3 Implied marks

Implied marks appear in **brackets e.g.** ***(M1)***,and can only be awarded if **correct** work is seen or implied by subsequent working/answer.

4 Follow through marks (only applied after an error is made)

Follow through (***FT***) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) (e.g. incorrect value from part (a) used in part (d) or incorrect value from part (c)(i) used in part (c)(ii)). Usually, to award ***FT*** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if all the marks awarded in a subsequent part are for the answer or are implied, then ***FT*** marks should be awarded for *their* correct answer, even when working is not present.

For example: following an incorrect answer to part (a) that is used in subsequent parts, where the markscheme for the subsequent part is ***(M1)A1***, it is possible to award full marks for *their* correct answer, **without working being seen.** For longer questions where all but the answer marks are implied this rule applies but may be overwritten by a **Note** in the Markscheme.

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks, by reflecting on what each mark is for and how that maps to the simplified version.
- If the error leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word “their” in a description, to indicate that candidates may be using an incorrect value.
- If the candidate’s answer to the initial question clearly contradicts information given in the question, it is not appropriate to award any ***FT*** marks in the subsequent parts. This includes when candidates fail to complete a “show that” question correctly, and then in subsequent parts use their incorrect answer rather than the given value.
- Exceptions to these ***FT*** rules will be explicitly noted on the markscheme.
- If a candidate makes an error in one part but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the command term was “Hence”.

5 Mis-read

If a candidate incorrectly copies values or information from the question, this is a mis-read (***MR***). A candidate should be penalized only once for a particular misread. Use the ***MR*** stamp to indicate that this has been a misread and do not award the first mark, even if this is an ***M*** mark, but award all others as appropriate.

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.
- If a candidate uses a correct answer, to a "show that" question, to a higher degree of accuracy than given in the question, this is NOT a misread and full marks may be scored in the subsequent part.
- ***MR*** can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If the command term is 'Hence' and not 'Hence or otherwise' then alternative methods are not permitted unless covered by a note in the mark scheme.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**.

7 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation** for example 1.9 and $1{,}9$ or 1000 and $1{,}000$ and 1.000.

- Do not accept final answers written using calculator notation. However, ***M*** marks and intermediate ***A*** marks can be scored, when presented using calculator notation, provided the evidence clearly reflects the demand of the mark.

- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.

- In the markscheme, some **equivalent** answers will generally appear in brackets. Not all equivalent notations/answers/methods will be presented in the markscheme and examiners are asked to apply appropriate discretion to judge if the candidate work is equivalent.

8 Format and accuracy of answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. If the level of accuracy is not stated in the question, the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures.*

Where values are used in subsequent parts, the markscheme will generally use the exact value, however candidates may also use the correct answer to 3 sf in subsequent parts. The markscheme will often explicitly include the subsequent values that come "*from the use of 3 sf values*".

Simplification of final answers: Candidates are advised to give final answers using good mathematical form. In general, for an ***A*** mark to be awarded, arithmetic should be completed, and any values that lead to integers should be simplified; for example, $\sqrt{\frac{25}{4}}$ should be written as $\frac{5}{2}$. An exception to this is simplifying fractions, where lowest form is not required (although the numerator and the denominator must be integers); for example, $\frac{10}{4}$ may be left in this form or written as $\frac{5}{2}$. However, $\frac{10}{5}$ should be written as 2, as it simplifies to an integer.

Algebraic expressions should be simplified by completing any operations such as addition and multiplication, e.g. $4e^{2x} \times e^{3x}$ should be simplified to $4e^{5x}$, and $4e^{2x} \times e^{3x} - e^{4x} \times e^{x}$ should be simplified to $3e^{5x}$. Unless specified in the question, expressions do not need to be factorized, nor do factorized expressions need to be expanded, so $x(x+1)$ and $x^2 + x$ are both acceptable.

Please note: intermediate ***A*** marks do NOT need to be simplified.

9 Calculators

A GDC is required for this paper, but If you see work that suggests a candidate has used any calculator not approved for IB DP examinations (eg CAS enabled devices), please follow the procedures for malpractice.

10. Presentation of candidate work

Crossed out work: If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work unless an explicit note from the candidate indicates that they would like the work to be marked.

More than one solution: Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise. If the layout of the responses makes it difficult to judge, examiners should apply appropriate discretion to judge which is "first".

1. (a) $\pi \approx 3 + \dfrac{1}{6 + \dfrac{13}{16}}$

$= 3.14678... \left(\dfrac{343}{109}, 3\frac{16}{109} \right)$ ***(A1)***

$= 3.1468$ ***A1***

Note: Award ***A1*** for correct rounding to 4 decimal places. Follow through within this part.

[2 marks]

(b) $\left| \dfrac{3.1468 - \pi}{\pi} \right| \times 100$ ***(M1)***

Note: Award ***M1*** for substitution of their final answer in part (a) into the percentage error formula. Candidates should use the exact value of π from their GDC.

$= 0.166\,(\%)\ \ (0.165754...)$ ***A1***

[2 marks]

Total [4 marks]

2. (a) 14 ***A1***

[1 mark]

(b) $\dfrac{14 + 15 + ...}{10}$ ***(M1)***

$= 13.1$ ***A1***

[2 marks]

(c) $2.21\ \ (2.21133...)$ ***A1***

[1 mark]

Total [4 marks]

3. (a) $\frac{1}{2}\times 4\times\pi\times 6^2+\pi\times 6^2$ **OR** $3\times\pi\times 6^2$ ***(M1)(A1)(M1)***

Note: Award ***M1*** for use of surface area of a sphere formula (or curved surface area of a hemisphere), ***A1*** for substituting correct values into hemisphere formula, ***M1*** for adding the area of the circle.

$=339\ \text{mm}^2\ \ (108\pi,\ 339.292...)$ ***A1***

[4 marks]

(b) $\frac{339.292...}{240}$ ***(M1)***

$=1.41\ (\text{g})\ \left(\frac{9\pi}{20},\ 0.45\pi,\ 1.41371...\right)$ ***A1***

[2 marks]

Total [6 marks]

4. (a) $L(40)=1.50\times 40-5$ ***(M1)***

$=\$\ 55$ ***A1***

[2 marks]

(b) $70=1.50x-5$ ***(M1)***

$(x=)\ 50$ litres ***A1***

[2 marks]

(c) $1.30x$ ***(A1)***

$1.30x<1.50x-5$ ***(M1)***

Note: Award ***M1*** for a graph showing two intersecting linear functions, provided one function has a y-intercept of 0 and the other function has a negative y-intercept.

(minimum value of $k=)\ 25$ ***A1***

Note: Accept $x>25$.

[3 marks]

Total [7 marks]

5. (a) every point in the shaded region is closer to tower $\mathrm{T4}$ ***R1***

Note: Specific reference must be made to the closeness of tower T4.

[1 mark]

(b) $(-9, 1)$ ***A1A1***

Note: Award ***A1*** for each correct coordinate. Award at most ***A0A1*** if parentheses are missing.

[2 marks]

(c) correct use of gradient formula ***(M1)***

e.g. $(m =) \frac{5-3}{-9--13} \left(= \frac{1}{2}\right)$

taking negative reciprocal of **their** m (at any point) ***(M1)***

edge gradient $= -2$ ***A1***

[3 marks]

Total [6 marks]

6. (a) **EITHER**

H_0: The population mean weight of eggs from (her/the) black geese is equal to/the same as the population mean weight of eggs from (her/the) white geese.

OR

H_0: The population mean weight of eggs from (her/the) black geese is not less than the population mean weight of eggs from (her/the) white geese. ***A1***

Note: Reference to the "population mean weight" must be explicit for the ***A1*** to be awarded. The term “population” can be implied by use of “all” or “on average” or “generally” when relating to the weight of eggs e.g. “the mean weight of eggs for all (her/the) black geese”.
Award ***A0*** if reference is made to the mean weights from the sample or the table.
Award ***A0*** for a null hypothesis written in symbolic form.

[1 mark]

(b) p-value $= 0.177$ $(0.176953...)$ ***A2***

Note: Award ***A1*** for an answer of $0.18221\ldots$, from “unpooled” settings on GDC.

[2 marks]

(c) $0.177 > 0.1$ ***R1***

(insufficient evidence to reject H_0)

Arriane’s claim is not supported by the evidence ***A1***

Note: Accept $p > 0.1$ or $p >$ *significance level* provided p is explicitly seen in part (b).
Award ***A1*** only if reference is specifically made to Arriane's claim.
Do not award ***R0A1***.

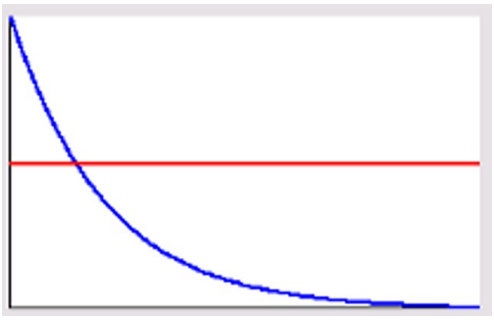
[2 marks]

Total [5 marks]

7. (a) **EITHER**

$50 = 100\,e^{-1\times p}$ **OR** $0.5 = e^{-1\times p}$ ***(M1)***

OR



(M1)

THEN

0.693 $(0.693147..., \ln 2)$ ***A1***

[2 marks]

(b) $R(1.5) = 100\,e^{-0.693147...\times 1.5}$ ***(M1)***

$35.4\,(\%)$ $(35.3553...)$ ***A1***

[2 marks]

(c) $R(t) > 0$ **OR** $R(t)$ has a horizontal asymptote ***R1***

[1 mark]

(d) *Award **A1** for **one** reasonable limitation of the domain:* ***A1***

small values of t produce unrealistic results

$R(0) = 100\%$

large values of t are not possible

people do not live forever

model is not valid at small or large values of t

The reason should focus on the domain $t \geq 0$. Do not accept answers such as:

recollection varies for different people

memories are discrete not continuous

the nature of the information will change how easily it is recalled

emotional/physical stress can affect recollection/concentration

Note: Do not accept $t \geq 0$ as this is a limitation that has been given in the question.

[1 mark]

Total [6 marks]

8. (a) (i) attempt to find u_{20} using an arithmetic sequence ***(M1)***
e.g. $u_1 = 500$ and $d = 100$ **OR** $u_{20} = 500 + 1900$ **OR** $500, 600, 700, \ldots$

(Charlie ran) 2400 m ***A1***

(ii) $(r =)\ 1.02$ ***(A1)***
attempt to find u_{20} using a geometric sequence ***(M1)***
e.g. identifying $u_1 = 500$ and a value for r **OR** $500 \times r^{19}$ **OR** $500,\ 510,\ 520.2, \ldots$

(Daniella ran) $728 \text{ m}\ \ (728.405...)$ ***A1***
[5 marks]

(b) $500 \times 1.02^{n-1} > 500 + (n-1) \times 100$ ***(M1)***

attempt to solve inequality ***(M1)***
$n > 184.215...$
$n = 185$ ***A1***
[3 marks]

Total [8 marks]

9. attempt to find any relevant maximum value ***(M1)***
largest sides are 56.5 and 82.5 ***(A1)***
smallest possible angle is 102.5 ***(A1)***

attempt to substitute into area of a triangle formula ***(M1)***
$\frac{1}{2} \times 56.5 \times 82.5 \times \sin(102.5^\circ)$

$= 2280 (\text{m}^2)\ \ (2275.37...)$ ***A1***
Total [5 marks]

10. (a)

t	1	2	3	4	5	6
$\mathrm{P}(T=t)$	$\frac{1}{36}$ (0.027777…)	$\frac{3}{36}$ (0.083333…)	$\frac{5}{36}$ (0.138888…)	$\frac{7}{36}$ (0.194444…)	$\frac{9}{36}$ (0.25)	$\frac{11}{36}$ (0.305555…)

A2

Note: Award ***A1*** if three to five probabilities are correct.

[2 marks]

(b) (i) $\frac{32}{36}$ $\left(\frac{8}{9}, 0.888888..., 88.9\%\right)$ ***(A1)***

(ii) use of conditional probability ***(M1)***
e.g. denominator of 32 **OR** denominator of $0.888888\ldots$, etc.
$\frac{11}{32}$ $(0.34375, 34.4\%)$ ***A1***

[3 marks]

(c) $\frac{1\times1+3\times2+5\times3+...+11\times6}{36}$ ***(M1)***

$=\frac{161}{36}$ $\left(4\frac{17}{36}, 4.47, 4.47222...\right)$ ***A1***

[2 marks]

Total [7 marks]

11. (a) $I = \frac{k}{d^2}$ ***(M1)***

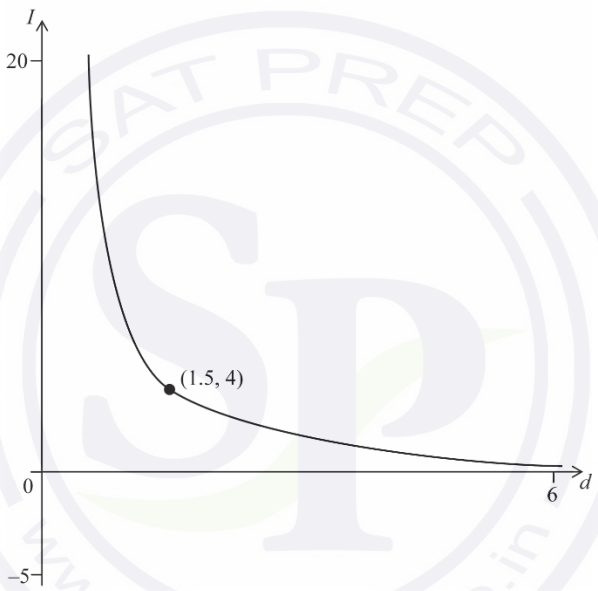
$4 = \frac{k}{1.5^2}$ ***M1***

$I = \frac{9}{d^2}$ ***AG***

Note: The ***AG*** line must be seen for the second ***M1*** to be awarded.
Award no marks for substituting 1.5 and 4 into $I = \frac{9}{d^2}$ (i.e., working backwards).

[2 marks]

(b)



A1A1

Note: Award ***A1*** for correct general shape (concave up) with no I-intercept, passing through the marked point $(1.5, 4)$; the point must be labelled with either the coordinates or the values 1.5 and 4 on the x and y axes. Award ***A1*** for the curve showing asymptotic behavior (i.e. I tends to 0, as d tends to infinity), extending to at least $d = 6$; the curve must not cross nor veer away from the horizontal asymptote.

[2 marks]

(c) $1.5 \times 10^{-6} \geq \frac{9}{d^2}$ ***(M1)***

Note: Award ***(M1)*** for a correct inequality.

$d \geq 2450$ (m) $(2449.48...)$ ***A1***

Note: Award ***A0*** for $d = 2450$.

[2 marks]

Total [6 marks]

12. (a) (i) $A = \frac{1}{2} \times 6 \times q + \frac{1}{2} \times 8 \times p + 48$ ***OR*** $A = \frac{1}{2}(p+6)(q+8)$ ***OR***
$A = 3q + 4p + 48$ ***A1***

(ii) valid attempt to link p and q, using tangents, similar triangles or other method ***(M1)***

eg. $\tan\theta = \frac{8}{p}$ and $\tan\theta = \frac{q}{6}$ **OR** $\tan\theta = \frac{p}{8}$ and $\tan\theta = \frac{6}{q}$ **OR** $\frac{8}{p} = \frac{q}{6}$

correct equation linking p and q ***A1***

eg. $pq = 48$ **OR** $p = \frac{48}{q}$ **OR** $q = \frac{48}{p}$

substitute $p = \frac{48}{q}$ into a correct area expression ***M1***

eg. $(A =)\frac{1}{2} \times 6 \times q + \frac{1}{2} \times 8 \times \frac{48}{q} + 48$ **OR** $(A =)\frac{1}{2}\left(\frac{48}{q} + 6\right)(q+8)$

$A = 3q + \frac{192}{q} + 48$ ***AG***

Note: The ***AG*** line must be seen with no incorrect, intermediate working, for the final ***M1*** to be awarded.

[4 marks]

(b) $\frac{-192}{q^2} + 3$ ***A1A1***

Note: Award ***A1*** for $\frac{-192}{q^2}$, ***A1*** for 3. Award ***A1A0*** if extra terms are seen.

[2 marks]

(c) (i) $\frac{-192}{q^2} + 3 = 0$ ***A1***

(ii) $q = 8$ cm ***A1***

[2 marks]

Total [8 marks]

13. (a) $l'(50) = -0.2 \times 50 + 9$ ***(M1)***

$= -1$ ***A1***

the curve is decreasing at $\theta = 50^\circ$. ***A1***

Note: For the final ***A1***, follow through within this question part for their $l'(50)$ value. Award ***A0*** for an answer of "decreasing" with no work shown.

[3 marks]

(b) recognition of need to integrate (e.g. reverse power rule or integral symbol or integrating at least one term correctly) ***(M1)***

$l(\theta) = -0.1\theta^2 + 9\theta \ (+c)$ ***A1A1***

$205.5 = -0.1 \times (40)^2 + 9 \times (40) + c$ ***(M1)***

Note: Award ***M1*** for correct substitution of $\theta = 40^\circ$ **and** $l = 205.5$. A constant of integration must be seen (can be implied by a correct answer).

$c = 5.5$

$\left(l(\theta) =\right) \ -0.1\theta^2 + 9\theta + 5.5$ ***A1***

Note: Accept any variable in the working, but for the final ***A1***, the variable θ must be used in the expression.

[5 marks]

Total [8 marks]

Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

May 2021

Mathematics: applications and interpretation

Standard level

Paper 1

16 pages

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A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.
R Marks awarded for clear **Reasoning**.
AG Answer given in the question and so no marks are awarded.
FT Follow through. The practice of awarding marks, despite candidate errors in previous parts, for their correct methods/answers using incorrect results.

Using the markscheme

1 General

Award marks using the annotations as noted in the markscheme *eg* ***M1, A2****.*

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***A3***, ***M2*** *etc.*, do **not** split the marks, unless there is a note.
- The response to a "show that" question does not need to restate the ***AG*** line, unless a **Note** makes this explicit in the markscheme.
- Once a correct answer to a question or part question is seen, ignore further working even if this working is incorrect and/or suggests a misunderstanding of the question. This will encourage a uniform approach to marking, with less examiner discretion. Although some candidates may be advantaged for that specific question item, it is likely that these candidates will lose marks elsewhere too.
- An exception to the previous rule is when an incorrect answer from further working is used **in a subsequent part**. For example, when a correct exact value is followed by an incorrect decimal approximation in the first part and this approximation is then used in the second part. In this situation, award ***FT*** marks as appropriate but do not award the final ***A1*** in the first part. Examples:

	Correct answer seen	**Further working seen**	**Any FT issues?**	**Action**
1.	$8\sqrt{2}$	5.65685... *(incorrect decimal value)*	No. Last part in question.	Award ***A1*** for the final mark *(condone the incorrect further working)*
2.	$\frac{35}{72}$	0.468111… (incorrect decimal value)	Yes. Value is used in subsequent parts.	Award ***A0*** for the final mark *(and full **FT** is available in subsequent parts)*

3 Implied marks

Implied marks appear in **brackets e.g.** ***(M1)***,and can only be awarded if **correct** work is seen or implied by subsequent working/answer.

4 Follow through marks (only applied after an error is made)

Follow through (***FT***) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) (e.g. incorrect value from part (a) used in part (d) or incorrect value from part (c)(i) used in part (c)(ii)). Usually, to award ***FT*** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if all the marks awarded in a subsequent part are for the answer or are implied, then ***FT*** marks should be awarded for *their* correct answer, even when working is not present.

For example: following an incorrect answer to part (a) that is used in subsequent parts, where the markscheme for the subsequent part is ***(M1)A1***, it is possible to award full marks for *their* correct answer, **without working being seen.** For longer questions where all but the answer marks are implied this rule applies but may be overwritten by a **Note** in the Markscheme.

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks, by reflecting on what each mark is for and how that maps to the simplified version.
- If the error leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word “their” in a description, to indicate that candidates may be using an incorrect value.
- If the candidate’s answer to the initial question clearly contradicts information given in the question, it is not appropriate to award any ***FT*** marks in the subsequent parts. This includes when candidates fail to complete a “show that” question correctly, and then in subsequent parts use their incorrect answer rather than the given value.
- Exceptions to these ***FT*** rules will be explicitly noted on the markscheme.
- If a candidate makes an error in one part but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the command term was “Hence”.

5 Mis-read

If a candidate incorrectly copies values or information from the question, this is a mis-read (***MR***). A candidate should be penalized only once for a particular misread. Use the ***MR*** stamp to indicate that this has been a misread and do not award the first mark, even if this is an ***M*** mark, but award all others as appropriate.

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.
- If a candidate uses a correct answer, to a "show that" question, to a higher degree of accuracy than given in the question, this is NOT a misread and full marks may be scored in the subsequent part.
- ***MR*** can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If the command term is 'Hence' and not 'Hence or otherwise' then alternative methods are not permitted unless covered by a note in the mark scheme.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**.

7 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation** for example 1.9 and $1{,}9$ or 1000 and $1{,}000$ and 1.000.

- Do not accept final answers written using calculator notation. However, ***M*** marks and intermediate ***A*** marks can be scored, when presented using calculator notation, provided the evidence clearly reflects the demand of the mark.

- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.

- In the markscheme, some **equivalent** answers will generally appear in brackets. Not all equivalent notations/answers/methods will be presented in the markscheme and examiners are asked to apply appropriate discretion to judge if the candidate work is equivalent.

8 Format and accuracy of answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. If the level of accuracy is not stated in the question, the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures.*

Where values are used in subsequent parts, the markscheme will generally use the exact value, however candidates may also use the correct answer to 3 sf in subsequent parts. The markscheme will often explicitly include the subsequent values that come "*from the use of 3 sf values*".

Simplification of final answers: Candidates are advised to give final answers using good mathematical form. In general, for an ***A*** mark to be awarded, arithmetic should be completed, and any values that lead to integers should be simplified; for example, $\sqrt{\frac{25}{4}}$ should be written as $\frac{5}{2}$. An exception to this is simplifying fractions, where lowest form is not required (although the numerator and the denominator must be integers); for example, $\frac{10}{4}$ may be left in this form or written as $\frac{5}{2}$. However, $\frac{10}{5}$ should be written as 2, as it simplifies to an integer.

Algebraic expressions should be simplified by completing any operations such as addition and multiplication, e.g. $4\mathrm{e}^{2x} \times \mathrm{e}^{3x}$ should be simplified to $4\mathrm{e}^{5x}$, and $4\mathrm{e}^{2x} \times \mathrm{e}^{3x} - \mathrm{e}^{4x} \times \mathrm{e}^{x}$ should be simplified to $3\mathrm{e}^{5x}$. Unless specified in the question, expressions do not need to be factorized, nor do factorized expressions need to be expanded, so $x(x+1)$ and $x^2 + x$ are both acceptable.

Please note: intermediate ***A*** marks do NOT need to be simplified.

9 Calculators

A GDC is required for this paper, but If you see work that suggests a candidate has used any calculator not approved for IB DP examinations (eg CAS enabled devices), please follow the procedures for malpractice.

10. Presentation of candidate work

Crossed out work: If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work unless an explicit note from the candidate indicates that they would like the work to be marked.

More than one solution: Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise. If the layout of the responses makes it difficult to judge, examiners should apply appropriate discretion to judge which is "first".

1. (a) (i) 23 mg ***A1***

(ii) $1-0.85$ **OR** $\frac{23-19.55}{23}$ **OR** 0.15 ***(M1)***

15 (%) ***A1***

[3 marks]

(b) $23(0.85)^{10}$ ***(M1)***

4.53 mg $(4.52811\ldots)$ ***A1***

[2 marks]

Total [5 marks]

2. (a) attempt at substitution into 3D distance formula ***(M1)***

$\text{AB} = \sqrt{(140-20)^2+(15-5)^2+250^2} \quad \left(=\sqrt{77\,000}\right)$

$= 277$ m $\left(10\sqrt{770},\ 277.488...\right)$ ***A1***

[2 marks]

(b) attempt at substitution in the midpoint formula ***(M1)***

$\left(\frac{140+20}{2}, \frac{15+5}{2}, \frac{0+250}{2}\right)$

$(80,\ 10,\ 125)$ ***A1***

[2 marks]

(c) 125 m ***A1***

[1 mark]

Total [5 marks]

3. $2\times90\times34\ (=6120)$ **AND** $2\times42\times34\ (=2856)$ ***(A1)***

$90\times42\ (=3780)$ ***(A1)***

$r = 21$ ***(A1)***

$\pi\times21^2\ (=441\pi,\ 1385.44...)$ ***(M1)***

use of curved surface area formula ***(M1)***

$21\pi\times90\ (=1890\pi,\ 5937.61...)$ ***(A1)***

20100 cm^2 $(20079.0\ldots)$ ***A1***

Total [7 marks]

4. (a) 25° ***A1***

[1 mark]

(b) $\text{AC} = \dfrac{380}{\tan 25^\circ}$ **OR** $\text{AC} = \sqrt{\left(\dfrac{380}{\sin 25^\circ}\right)^2 - 380^2}$ **OR** $\dfrac{380}{\sin 25^\circ} = \dfrac{\text{AC}}{\sin 65^\circ}$ ***(M1)***

$\text{AC} = 815 \text{ m } (814.912\ldots)$ ***A1***

[2 marks]

(c) **METHOD 1**

attempt to find AB ***(M1)***

$\text{AB} = \dfrac{380}{\tan 40^\circ}$

$= 453 \text{ m } (452.866\ldots)$ ***(A1)***

$\text{BC} = 814.912\ldots - 452.866\ldots$

$= 362 \text{ m } (362.046\ldots)$ ***A1***

METHOD 2

attempt to find HB ***(M1)***

$\text{HB} = \dfrac{380}{\sin 40^\circ}$

$591 \text{ m } (= 591.175...)$ ***(A1)***

$\text{BC} = \dfrac{591.175... \times \sin 15^\circ}{\sin 25^\circ}$

$= 362 \text{ m } (362.046...)$ ***A1***

[3 marks]

(d) $362.046\ldots \times 4$

$= 1450 \text{ m h}^{-1} \ (1448.18\ldots)$ ***A1***

[1 mark]

Total [7 marks]

5. (a) (i) 2 ***A1***

(ii) 6 ***A1***

(iii) 8 ***A1***

[3 marks]

(b) **EITHER**
Each of these percentages represent approximately 25% of the employees. ***R1***

OR
The diagram is not explicit enough to show what is happening at the quartiles regarding 6 and 11 / we do not have the data points ***R1***

OR
Discrete data not clear how to interpret "fewer". ***R1***

THEN
Hence, Paul is not correct (**OR** no such inference can be made). ***A1***

Note: Do not award ***R0A1***.

[2 marks]

Total [5 marks]

6. (a) gradient $\text{AB} = \frac{4}{12}\left(\frac{1}{3}\right)$ ***(A1)***

midpoint $\text{AB}: (8, 22)$ ***(A1)***

gradient of bisector $= -\frac{1}{\text{gradient AB}} = -3$ ***(M1)***

perpendicular bisector: $22 = -3 \times 8 + b$ **OR** $(y - 22) = -3(x - 8)$ ***(M1)***

perpendicular bisector: $y = -3x + 46$ ***A1***

[5 marks]

(b) attempt to solve simultaneous equations ***(M1)***
$x + 4 = -3x + 46$
$(10.5, 14.5)$ ***A1***

[2 marks]

Total [7 marks]

7. (a) $(f(-7) =)\ 8$ and $(f(7) =)\ 1$ ***(A1)***

range is $f(x) \leq 1,\ f(x) \geq 8$ ***A1A1***

Note: Award at most ***A1A1A0*** if strict inequalities are used.

[3 marks]

(b) **EITHER**

sketch of f and $y = 0$ or sketch of f^{-1} and $x = 0$ ***(M1)***

OR

finding the correct expression of $f^{-1}(x) = \dfrac{-2-5x}{x-2}$ ***(M1)***

OR

$f^{-1}(0) = \dfrac{-2-5(0)}{0-2}$ ***(M1)***

OR

$f(x) = 0$ ***(M1)***

THEN

$f^{-1}(0) = 1$ ***A1***

[2 marks]

Total [5 marks]

8. (a) (let $\mu_c =$ population mean for chinchilla rabbits, $\mu_s =$ population mean for sable rabbits)

$H_0 : \mu_c = \mu_s$ ***A1***

$H_1 : \mu_c > \mu_s$ ***A1***

Note: Accept an equivalent statement in words, must include mean and reference to "**population** mean" / "mean for **all** chinchilla rabbits" for the first ***A1*** to be awarded.
Do not accept an imprecise *"the means are equal".*

[2 marks]

(b) p-value $= 0.0408$ $(0.0408065...)$ ***A2***

Note: Award ***A1*** for an answer of $0.041565\ldots$, from "unpooled" settings on GDC.

[2 marks]

(c) $0.0408 < 0.05$. ***R1***

(there is sufficient evidence to) reject (or not accept) H_0 ***A1***

(there is sufficient evidence to suggest that chinchilla rabbits are heavier than sable rabbits)

Note: Do not award ***R0A1***. Accept 'accept H_1'.

[2 marks]
Total [6 marks]

9. (a) $135^\circ \times \frac{12\pi}{360^\circ}$ ***(M1)(A1)***

14.1 (m) $(14.1371\ldots)$ ***A1***

[3 marks]

(b) evidence of splitting region into two areas ***(M1)***

$135^\circ \times \frac{\pi 6^2}{360^\circ} - \frac{6 \times 6 \times \sin 135^\circ}{2}$ ***(M1)(M1)***

Note: Award ***M1*** for correctly-substituting into area of sector formula, ***M1*** for evidence of substituting into area of triangle formula.

$42.4115\ldots - 12.7279\ldots$

$29.7\ \text{m}^2$ $(29.6835\ldots)$ ***A1***

[4 marks]
Total [7 marks]

10. (a) **METHOD 1**

$N = 5$	**OR**	$N = 10$
$I\% = 2.75$		$I\% = 2.75$
$PV = -1500$		$PV = -1500$
$PMT = 0$		$PMT = 0$
$P/Y = 1$		$P/Y = 2$
$C/Y = 2$		$C/Y = 2$

(M1)(A1)

Note: Award ***M1*** for an attempt to use a financial app in their technology, ***A1*** for all entries correct.

METHOD 2

$1500\left(1+\frac{2.75}{2\times 100}\right)^{2\times 5}$ ***(M1)(A1)***

1719.49 euro ***A1***

[3 marks]

(b) **METHOD 1**

$N = 5$	**OR**	$N = 20$
$PV = \pm 1500$		$PV = \pm 1500$
$FV = \mp 2250$		$FV = \mp 2250$
$PMT = 0$		$PMT = 0$
$P/Y = 1$		$P/Y = 4$
$C/Y = 4$		$C/Y = 4$

(M1)(A1)

Note: Award ***M1*** for an attempt to use a financial app in their technology, ***A1*** for all entries correct. PV and FV must have opposite signs.

METHOD 2

$1500\left(1+\frac{r}{4\times 100}\right)^{4\times 5} = 2250$ **OR** $\left(1+\frac{r}{4\times 100}\right)^{4\times 5} = 1.5$ ***(M1)(A1)***

Note: Award ***M1*** for substitution in compound interest formula, ***A1*** for correct substitution and for equating to 2250 (if using LHS equation) or to 1.5 (if using RHS equation).

$r = 8.19$ $(8.19206...)$ ***A1***

Note: Accept $r = 8.19\%$.
Accept a trial and error method which leads to $r = 8.19$.

[3 marks]

Total [6 marks]

11. (a) $\left(\frac{74+97+91+86+112}{5}\right)=92$ ***A1***

[1 mark]

(b) (i) 4 ***A1***

(ii) $\chi^2_{\text{calc}}=8.54\ \ (8.54347...)$ **OR** $p\text{-value}=0.0736\ \ (0.0735802...)$ ***A2***

$8.54<9.49$ **OR** $0.0736>0.05$ ***R1***

therefore there is insufficient evidence to reject H_0 ***A1***

(i.e. the data satisfies the model)

Note: Do not award ***R0A1***. Accept "accept" or "do not reject" in place of "insufficient evidence to reject".
Award the ***R1*** for comparing their p-value with 0.05 or their χ^2 value with 9.49 and then ***FT*** their final conclusion.

[5 marks]

Total [6 marks]

12. (a) 3 ***A1***

Note: Accept $(3, 0)$ seen.

[1 mark]

(b) **METHOD 1**

$0 = 4a - 2b + c$, $0 = 9a + 3b + c$, $-\frac{25}{2} = \frac{1}{4}a + \frac{1}{2}b + c$ ***(M1)(A1)***

(i) 2 ***A1***

(ii) -2 ***A1***

(iii) -12 ***A1***

Note: Award the ***(M1)(A1)*** if at least one correct value is seen.
Do not apply ***FT*** form part (a) if workings are not shown.

METHOD 2

$-12.5 = a(0.5 + 2)(0.5 - 3)$ ***(M1)***

(i) $a = 2$ ***A1***

$0 = 2 \times (3)^2 + 3b + c$
$0 = 2 \times (-2)^2 + (-2)b + c$ ***(M1)***

(ii) $b = -2$ ***A1***

(iii) $c = -12$ ***A1***

[5 marks]

(c) $x = 0.5$ ***A1***

Note: Do not ***FT*** from their part (b), this is a contradiction with the diagram.

[1 mark]

Total [7 marks]

13. (a) recognition of need to integrate (*eg* reverse power rule or integral symbol) ***(M1)***

$P(x) = -0.8x^2 + 48x\ (+c)$ ***A1A1***

$260 = -0.8\times(15)^2 + 48\times(15) + c$ ***(M1)***

Note: Award ***M1*** for correct substitution of $x = 15$ and $P = 260$. A constant of integration must be seen (can be implied by a correct answer).

$c = -280$

$P(x) = -0.8x^2 + 48x - 280$ ***A1***

[5 marks]

(b) profit will decrease (with each new car produced) ***A1***

EITHER

because the profit function is decreasing / the gradient is negative / the rate of change of P is negative ***R1***

OR

$\int_{30}^{50} -1.6x + 48\ (\mathrm{d}x) = -320$ ***R1***

OR

evidence of finding $P(30) = 440$ and $P(50) = 120$ ***R1***

Note: Award at most ***R1A0*** if $P(30)$ or $P(50)$ or both have incorrect values.

[2 marks]

Total [7 marks]

Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

Specimen paper

Mathematics: applications and interpretation

Standard level

Paper 1

13 pages

International Baccalaureate®
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Bachillerato Internacional

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

R Marks awarded for clear **Reasoning**.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Award marks using the annotations as noted in the markscheme eg ***M1****,* ***A2****.*

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***M2***, ***A3***, *etc*., do **not** split the marks, unless there is a note.
- Once a correct answer to a question or part-question is seen, ignore further correct working. However, if further working indicates a lack of mathematical understanding do not award the final ***A1***. An exception to this may be in numerical answers, where a correct exact value is followed by an incorrect decimal. However, if the incorrect decimal is carried through to a subsequent part, and correct ***FT*** working shown, award ***FT*** marks as appropriate but do not award the final ***A1*** in that part.

Examples

	Correct answer seen	Further working seen	Action
1.	$8\sqrt{2}$	5.65685... *(incorrect decimal value)*	Award the final ***A1*** *(ignore the further working)*
2.	$\frac{1}{4}\sin 4x$	$\sin x$	Do not award the final ***A1***
3.	$\log a - \log b$	$\log(a-b)$	Do not award the final ***A1***

3 Implied marks

*Implied marks appear in **brackets e.g. (M1)**, and can only be awarded if **correct** work is seen or if implied in subsequent working.*

- Normally the correct work is seen or implied in the next line.
- Marks **without** brackets can only be awarded for work that is **seen**.

4 Follow through marks (only applied after an error is made)

*Follow through (**FT**) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) or subpart(s). Usually, to award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if the only marks awarded in a subpart are for the answer (i.e. there is no working expected), then **FT** marks should be awarded if appropriate.*

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*e.g.* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word “their” in a description, to indicate that candidates may be using an incorrect value.
- Exceptions to this rule will be explicitly noted on the markscheme.
- If a candidate makes an error in one part, but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the question says hence. It is often possible to use a different approach in subsequent parts that does not depend on the answer to previous parts.

5 Mis-read

*If a candidate incorrectly copies information from the question, this is a mis-read (**MR**). Apply a **MR** penalty of 1 mark to that question*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates’ own work does **not** constitute a misread, it is an error.
- The ***MR*** penalty can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme

- Alternative methods for complete questions are indicated by ***METHOD 1***, ***METHOD 2***, *etc*.
- Alternative solutions for part-questions are indicated by ***EITHER*** . . . ***OR***.

7 Alternative forms

Unless the question specifies otherwise, ***accept*** *equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

8 Accuracy of Answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. There are two types of accuracy errors, and the final answer mark should not be awarded if these errors occur.

- **Rounding errors**: only applies to final answers not to intermediate steps.
- **Level of accuracy**: when this is not specified in the question the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures*.

9 Calculators

A GDC is required for this examination, but calculators with symbolic manipulation features/ CAS functionality are not allowed.

Calculator notation
The subject guide says:

> *Students must always use correct mathematical notation, not calculator notation.*

Do **not** accept final answers written using calculator notation. However, do not penalize the use of calculator notation in the working.

1. (a) $210\,\text{g}$ ***A1***

[1 mark]

(b) $240\,\text{g}$ ***A1***

[1 mark]

(c) $240 - 190$ ***(M1)***

$= 50\,\text{g}$ ***A1***

[2 marks]

(d) $240 + 1.5 \times (50)$ ***M1***

$= 315\,\text{g}$ ***A1***

[2 marks]

Total [6 marks]

2. (a) $(d =) - 250$ ***A1***

[1 mark]

(b) $(u_{16} =) 6800 + (16 - 1)(-250)$ ***M1***

$(¥) 3050$ ***A1***

[2 marks]

(c) $(S_{16} =)\left(\frac{16}{2}\right)(2 \times 6800 + (16 - 1)(-250)) \times 2$ ***M1M1***

Note: Award ***M1*** for correct substitution into arithmetic series formula.
Award ***M1*** for multiplication by 2 seen.

OR

$(S_{16} =)\left(\frac{16}{2}\right)(6800 + 3050) \times 2$ ***M1M1***

Note: Award ***M1*** for correct substitution into arithmetic series formula.
Award ***M1*** for multiplication by 2 seen.

$(¥) 158\,000\ (157\,600)$ ***A1***

[3 marks]

Total [6 marks]

3. (a) discrete ***A1***

[1 mark]

(b) $\frac{24 + 60 + 3k + 40 + 15 + 6}{88 + k} = 2$ ***M1A1***

Note: Award ***M1*** for substitution into the formula for the mean, award ***A1*** for a correct equation.

attempt to solve their equation ***(M1)***

$k = 31$ ***A1***

[4 marks]

(c) systematic ***A1***

[1 mark]

Total [6 marks]

4. (a) 20 ***A1***

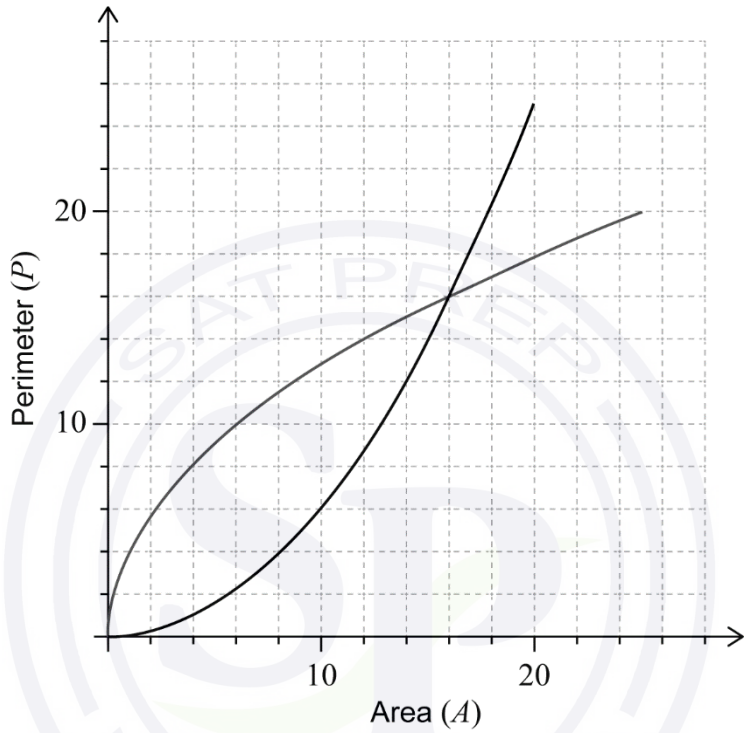
[1 mark]

(b) $n = 20$ ***A1***

Note: Follow through from part (a).

[1 mark]

(c)



(M1)A1A1

Note: Award ***(M1)*** for reflection in the line $P = A$, award ***A1*** for endpoint at $(20, 25)$, award ***A1*** for passing through $(16, 16)$.

[3 marks]

(d) when the perimeter is 8, the area is 4 ***A1***

[1 mark]

Total [6 marks]

5. (a) (i) 1750 ***A1***

(ii) $1350 + 400\,(1.25)^{-5}$ ***(M1)***

$= 1480$ ***A1***

Note: Accept 1481.

[3 marks]

(b) $1400 = 1350 + 400(1.25)^{-t}$ ***(M1)***

9.32 (days $(9.31885\ldots)$ (days)) ***A1***

[2 marks]

(c) 1350 ***A1***

Note: Accept 1351 as a valid interpretation of the model as $P = 1350$ is an asymptote.

[1 mark]

Total [6 marks]

6. (a) number of salad meals per week is independent of a person's position in the university ***A1***

Note: Accept "not associated" instead of independent.

[1 mark]

(b) 0.0201 $(0.0201118\ldots)$ ***A2***

[2 marks]

(c) $0.0201 < 0.05$ ***R1***

the null hypothesis is rejected ***A1***

[2 marks]

Note: Award ***(R1)*** for a correct comparison of their p-value to the test level, award ***(A1)*** for the correct interpretation from that comparison. Do not award ***(R0)(A1)***.

Total [5 marks]

7. (a) $\frac{3-1}{7-3}$ ***(M1)***

$= 0.5$ ***A1***

[2 marks]

(b) $y-2=-2(x-5)$ ***(A1)(M1)***

Note: Award ***(A1)*** for their -2 seen, award ***(M1)*** for the correct substitution of $(5, 2)$ and their normal gradient in equation of a line.

$2x + y - 12 = 0$ ***A1***

[3 marks]

(c) every point in the cell is closer to E than any other snow shelter ***A1***

[1 mark]

Total [6 marks]

8. (a) $10\log_{10}\left(6.4\times10^{-3}\times10^{12}\right)$ ***(M1)***

$= 98.1(\text{dB})\ (98.06179\ldots)$ ***A1***

[2 marks]

(b) $112 = 10\log_{10}\left(S\times10^{12}\right)$ ***(M1)***

$0.158\left(\text{W m}^{-2}\right)\left(0.158489\ldots\left(\text{W m}^{-2}\right)\right)$ ***A1***

[2 marks]

Total [4 marks]

9. (a) (i) $\mu_1 - \mu_2 = 0$ ***A1***

(ii) $\mu_1 - \mu_2 \neq 0$ ***A1***

Note: Accept equivalent statements in words.

[2 marks]

(b) $0.296\ (0.295739\ldots)$ ***A2***

[2 marks]

(c) $0.296 > 0.1$ ***R1***

fail to reject the null hypothesis, there is no difference between the mean height of male and female students ***A1***

Note: Award ***(R1)*** for a correct comparison of their p-value to the test level, award ***(A1)*** for the correct interpretation from that comparison. Do not award ***R0A1***.

[2 marks]

Total [6 marks]

10. (a) $A = \int_0^2 (6-3x)(4+x)\mathrm{d}x$ ***A1A1***

Note: Award ***A1*** for the limits $x = 0,\ x = 2$. Award ***A1*** for an integral of $f(x)$.

[2 marks]

(b) 28 ***A1***

[1 mark]

(c) $28 = 0.5 \times a \times 10$ ***M1***

$5.6\left(\frac{28}{5}\right)$ ***A1***

[2 marks]

Total [5 marks]

11. volume $= 240\left(\pi \times 8.4^2 - \frac{1}{2} \times 8.4^2 \times 0.872664\ldots\right)$ ***M1M1M1***

Note: Award ***M1*** $240 \times$ area, award ***M1*** for correctly substituting area sector formula, award ***M1*** for subtraction of their area of the sector from area of circle.

$= 45800\ (= 45811.96071)$ ***A1***

Total [4 marks]

12. (a) $\frac{4}{18}\left(\frac{2}{9}\right)$ ***A1***

[1 mark]

(b) $-3 \times \frac{1}{18} + (-1) \times \frac{4}{18} + 0 \times \frac{3}{18} + \ldots + 5 \times \frac{7}{18}$ ***(M1)***

Note: Award ***(M1)*** for their correct substitution into the formula for expected value.

$= 1.83\left(\frac{33}{18}, 1.83333\ldots\right)$ ***A1***

[2 marks]

(c) $2 \times \frac{1}{18} \times \frac{3}{18}$ ***(M1)(M1)***

Note: Award ***(M1)*** for $\frac{1}{18} \times \frac{3}{18}$, award ***(M1)*** for multiplying their product by 2.

$= \frac{1}{54}\left(\frac{6}{324}, 0.0185185\ldots, 1.85\%\right)$ ***A1***

[3 marks]

Total [6 marks]

13. (a) $\frac{6}{15}\left(0.4, \frac{2}{5}\right)$ ***A1***

[1 mark]

(b) $\mathrm{P}(X = 8)$ ***(M1)***

Note: Award ***(M1)*** for evidence of recognizing binomial probability.
eg, $\mathrm{P}(X = 8)$, $X \sim \mathrm{B}\left(20, \frac{6}{15}\right)$.

$0.180\ (0.179705\ldots)$ ***A1***

[2 marks]

(c) $\mathrm{P}(\text{male}) = \frac{9}{15}\ (0.6)$ ***A1***

$\mathrm{P}(X \leq 9) = 0.128\ (0.127521\ldots)$ ***(M1)A1***

Note: Award ***(M1)*** for evidence of correct approach ***eg***, $\mathrm{P}(X \leq 9)$.

[3 marks]

Total [6 marks]

14. (a) $\frac{\sin C\hat{A}B}{6} = \frac{\sin 15^\circ}{4.5}$ ***(M1)(A1)***

$C\hat{A}B = 20.2^\circ$ (20.187415…) ***A1***

Note: Award ***(M1)*** for substituted sine rule formula and award ***(A1)*** for correct substitutions.

[3 marks]

(b) $C\hat{B}D = 20.2 + 15 = 35.2^\circ$ ***A1***

(let X be the point on BD where Ollie activates the sensor)

$\tan 35.18741\ldots^\circ = \frac{1.8}{BX}$ ***(M1)***

Note: Award ***A1*** for their correct angle $C\hat{B}D$. Award ***M1*** for correctly substituted trigonometric formula.

$BX = 2.55285\ldots$ ***A1***

$5 - 2.55285\ldots$ ***(M1)***

$= 2.45\,(\text{m})$ (2.44714…) ***A1***

[5 marks]

Total [8 marks]