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Mathematics: applications and interpretation

Standard level

Paper 2

16 May 2025

Zone A morning | Zone B morning | Zone C morning

1 hour 30 minutes

Instructions to candidates

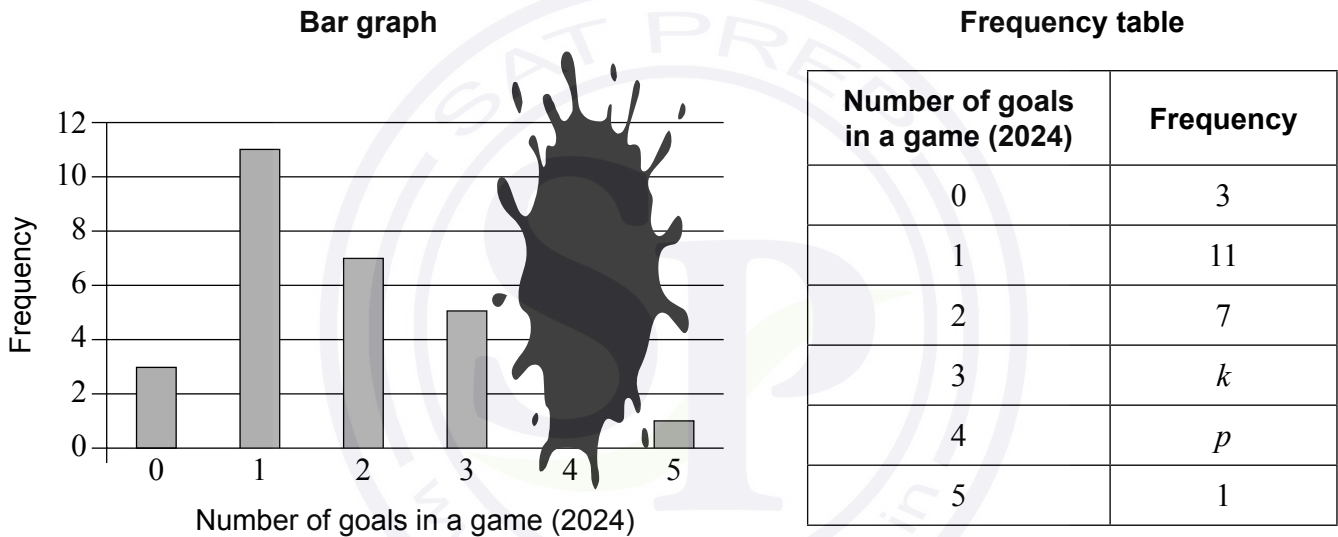
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all the questions in the answer booklet provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation SL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

Answer **all** questions in the answer booklet provided. Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 15]

Paul has a bar graph for the total number of goals scored in each game of a soccer tournament in 2024. The bar graph is shown below, however the frequency of 4 goals in a game is unreadable.

Paul uses this bar graph to create a frequency table.



(a) Write down the value of k . [1]

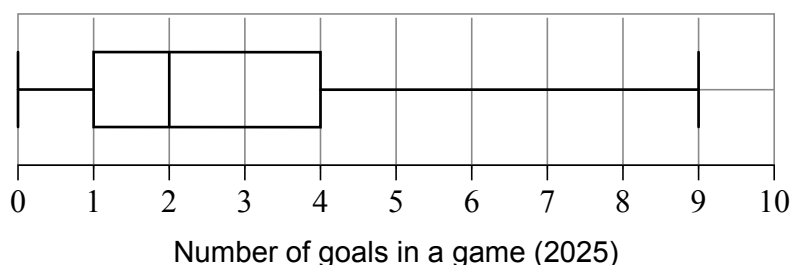
Paul knows that the mean number of goals per game scored during the tournament was 2.2.

- (b) (i) Write down an equation for the mean in terms of p .
- (ii) Determine the value of p . [3]

(This question continues on the following page)

(Question 1 continued)

Data for the number of goals per game in the **2025 soccer tournament** are shown in the following box and whisker diagram.



After comparing the box and whisker diagram from the 2025 tournament with the frequency table from the 2024 tournament, Paul concludes that the distribution of goals is consistent between the two tournaments.

- (c) State two observations that support Paul’s conclusion using values from the data to compare any **two** of:

range, symmetry, median, and interquartile range.

[3]

Paul plans to watch all the games from the **2024** tournament in a random order. He will watch each game once.

For the first game he watches, he defines event F as:

“scoring either 0 goals or exactly 1 goal”.

- (d) Write down the event(s) from the table that are equivalent to F . There may be more than one correct event.

[2]

Event	Description
A	Scoring exactly 2 goals in this game
B	Scoring more than 1 goal in this game
C	Scoring at least 2 goals in this game
D	Scoring either 0 goals or exactly 1 goal in all games except this game
E	Not scoring 0 goals or exactly 1 goal in any game

- (e) If exactly 1 goal was scored in the first game Paul watches, write down the probability that exactly 1 goal was scored in the second game he watches. Give your answer as a fraction.

[2]

- (f) Calculate the probability that 5 goals were scored in the first game that Paul watches **and** 0 goals were scored in the second game he watches.

[4]

2. [Maximum mark: 15]

Give all answers in this question to the nearest dollar.

Anika invested 10 000 Canadian dollars (CAD) in an account which earns a nominal annual interest rate of 3.4%, compounded monthly.

(a) Calculate the value of Anika’s investment after 6 years. [3]

Anika wants to purchase a house that costs 225 000 CAD. Anika uses the value obtained in part (a) to make an initial payment on the house and takes out a loan to pay the rest.

(b) Determine the amount of Anika’s loan. [1]

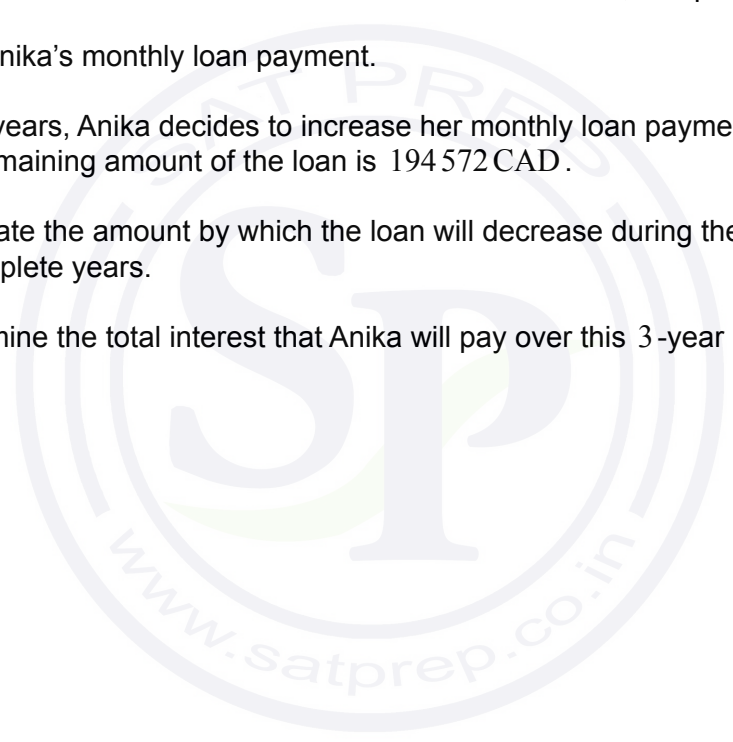
Anika amortizes her loan over 15 years. She arranges to make a loan payment at the end of each month. The nominal annual interest rate for the loan is 6.4%, compounded half-yearly.

(c) Determine Anika’s monthly loan payment. [5]

After 2 complete years, Anika decides to increase her monthly loan payment to 2200 CAD. At the time, the remaining amount of the loan is 194 572 CAD.

(d) (i) Calculate the amount by which the loan will decrease during the next 3 complete years.

(ii) Determine the total interest that Anika will pay over this 3-year period. [6]



3. [Maximum mark: 17]

Felix is training for a competition consisting of running and swimming. He plans to train for 12 weeks to prepare for the competition.

The running part of the competition is a 5 km race. During the 12 weeks, Felix runs on **3 days each week**. On the first day of training, Felix runs 2 km. Each subsequent day that he runs, he increases his distance by 5%.

(a) Explain why the distances of each of Felix’s runs form a geometric sequence and state the common ratio. [2]

(b) Find the total distance of Felix’s first 15 runs. [2]

A published recommendation states that, on the last day of training, an athlete should run a distance that is at least twice as far as the race distance.

(c) Show that Felix’s final run of his 12-week training plan meets this recommendation. [3]

For the swimming part of the competition, competitors must swim 375 m out to a marker in the water and then 375 m back to the shore.



Felix plans to swim **2 times per week** to train for the swimming portion of the competition. On each day he swims in the 1st week, Felix swims out to a marker that is 80 m from the shore, and then swims back to shore.

At the start of each subsequent week, the marker is moved to increase the distance between the marker and the shore by a constant amount. Felix swims a total distance of 704 m in the 4th week.

The distances that Felix swims each week form an arithmetic sequence, where each term, u_n , is the **total distance** he swims in the n th week.

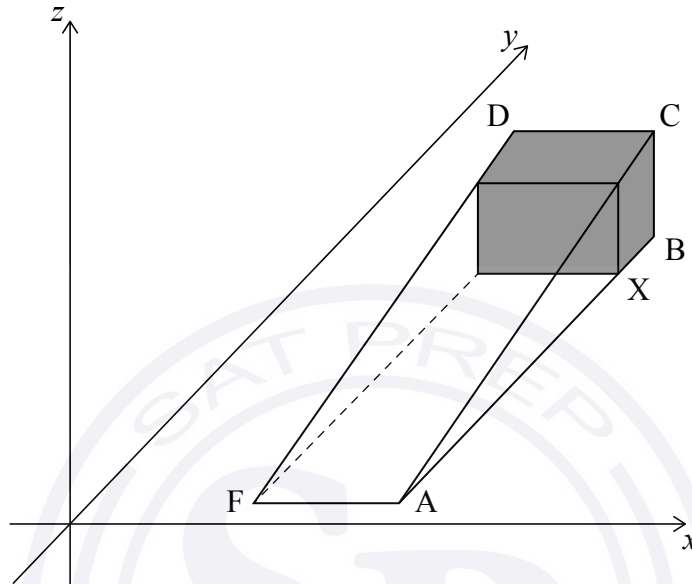
(d) (i) Find u_1 .
 (ii) Determine the increase in the total distance Felix swims each week.
 (iii) Hence calculate the distance between the marker and the shore in the 2nd week of training. [7]

(e) Calculate the distance of Felix’s final swim in his 12th week of training. [3]

(Question 4 continued)

To lower the cost of the doorstop, AirFlow decides to reduce its volume to 625 cm^3 by removing the shaded section shown in the following diagram. The remaining triangular faces are still right-angled triangles.

diagram not to scale



The length of the smaller doorstop, AX , can be calculated by

$$AX = AB \times \sqrt{\frac{V_s}{V_o}}$$

where V_s is the volume of the smaller doorstop, and V_o is the volume of the original doorstop, both in cubic centimetres.

(e) Find AX .

[2]

The material needed to make the doorstop costs 0.025 US dollars (USD) per cubic centimetre. The production of each doorstop requires 10% more material than its final volume, due to wastage in the production process.

To make a profit, the company will sell the doorstop for 20% more than the cost of the material.

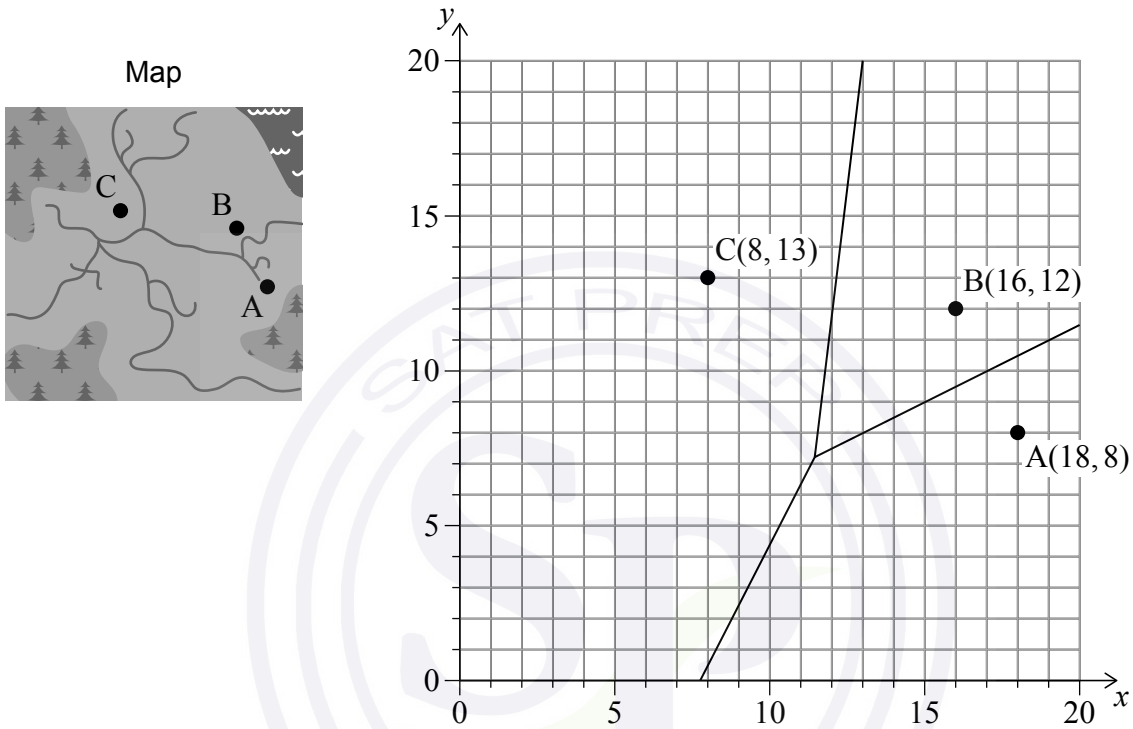
(f) Determine the price, to two decimal places, the company will charge for the **new** doorstop.

[3]

5. [Maximum mark: 19]

The locations of three fire stations within a 50 by 50 kilometre rural area of Japan are shown on the following map.

A Voronoi diagram can be used to determine the part of the rural area for which each fire station is responsible. The Voronoi diagram for the three fire stations is shown on the coordinate grid beside the map, where 1 unit represents 2.5 km.



- (a) (i) Find the midpoint of [BC].
- (ii) Find the gradient of the perpendicular bisector of [BC].
- (iii) Hence find the equation of the edge between sites B and C. [6]
- (b) (i) Identify the fire station that is expected to respond, based on the Voronoi diagram, if a fire is reported at a location with coordinates (14, 10). Justify your response.
- (ii) Suggest a reason why a different fire station might respond to this fire. [2]

(This question continues on the following page)

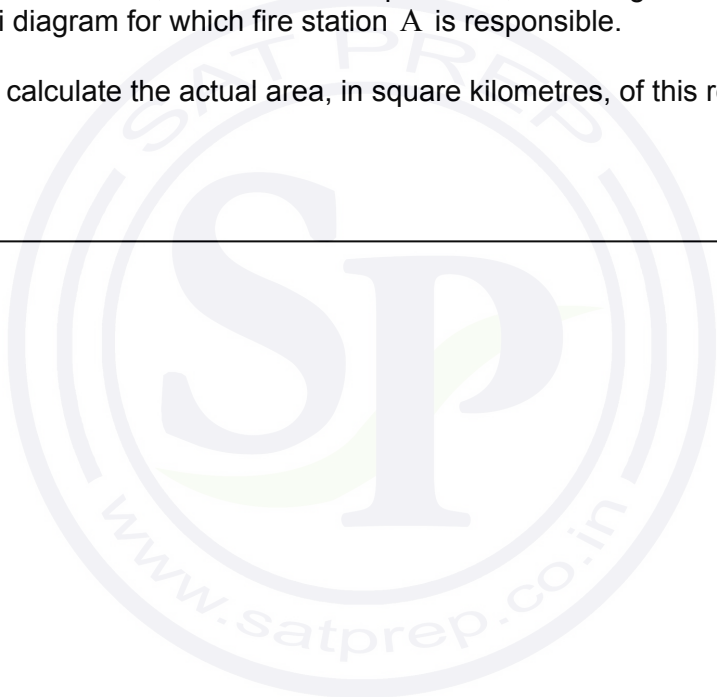
(Question 5 continued)

A fire is reported at a location D with coordinates $\left(11\frac{1}{3}, 7\frac{1}{6}\right)$. The distance of D from A on the Voronoi diagram, to six significant figures, is 6.71855 units (16.7964 km on the map).

- (c) (i) Show that the distance of D from B , to six significant figures, is also 6.71855 units.
- (ii) Show that any of the three fire stations would be expected to respond to the fire reported at D . [4]

The equation of the edge between fire stations A and B is $y = 0.5x + 1.5$. The edge between fire stations A and C has an x -intercept of 7.75.

- (d) (i) Determine the area, to the nearest square unit, of the region within the given Voronoi diagram for which fire station A is responsible.
- (ii) Hence, calculate the actual area, in square kilometres, of this region. [7]



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Mathematics: applications and interpretation

Standard level

Paper 2

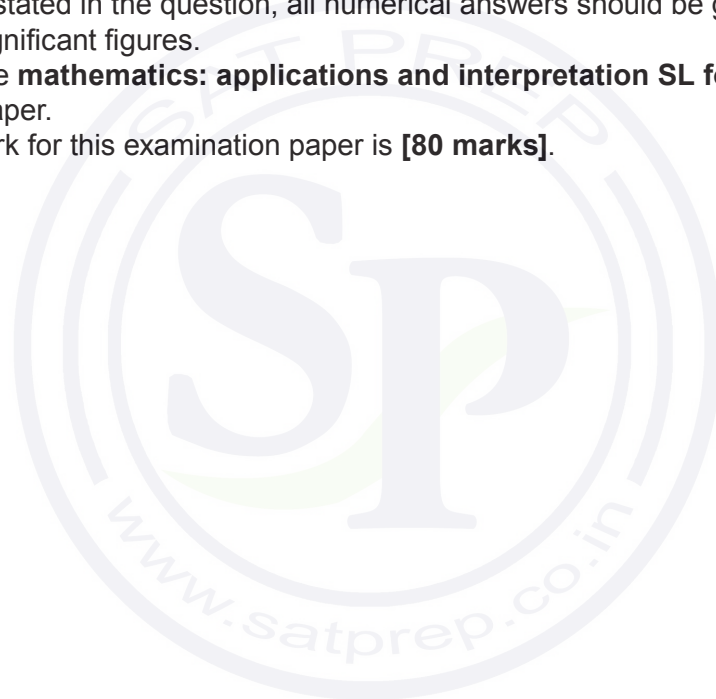
16 May 2025

Zone A morning | Zone B morning | Zone C morning

1 hour 30 minutes

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all the questions in the answer booklet provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation SL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

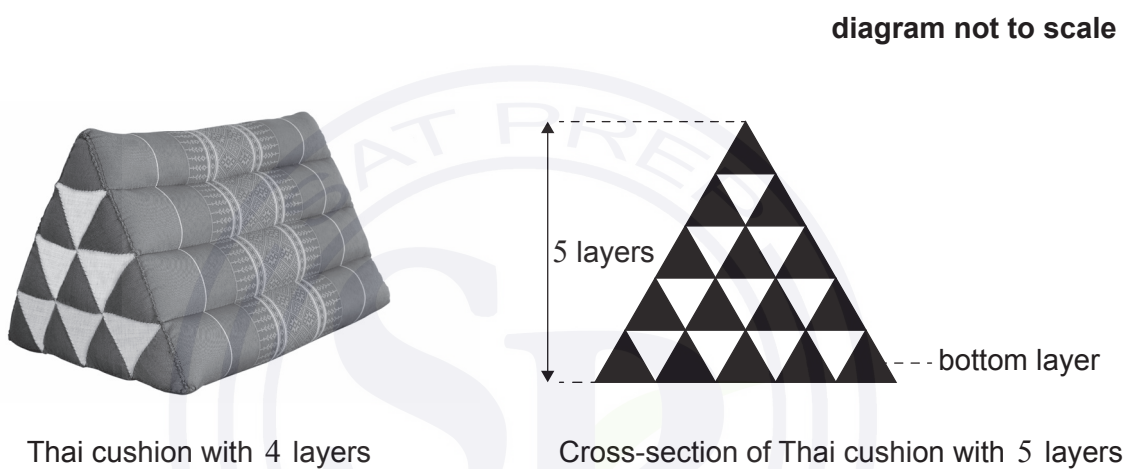


Answer **all** questions in the answer booklet provided. Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 16]

Thai cushions are designed with a triangular cross-section and are made from layers of smaller cushions. These cushions can be modelled as triangular prisms.

This is shown in the diagram.



(a) Write down the number of triangular prisms in the bottom layer of the cushion with

- (i) 4 layers.
- (ii) 5 layers.

[2]

Mayumi notices that the number of triangular prisms in the bottom layer of the cushions forms an arithmetic sequence.

- (b) (i) Write down the common difference of this sequence.
- (ii) Find an expression for the number of triangular prisms in the bottom layer of a cushion with n layers.

[3]

(This question continues on the following page)

(Question 1 continued)

Mayumi wants to extend this design to create a cushion with 9 layers.

- (c) (i) Find the number of triangular prisms in the bottom layer of Mayumi’s cushion. [3]
- (ii) Calculate the **total** number of triangular prisms in Mayumi’s cushion. [3]
- (d) Find an expression for the **total** number of triangular prisms in a cushion with n layers, giving your answer in its simplest form. [2]

The cross-section of the cushion consists of black triangles and white triangles.



This cushion with 4 layers has a total of 6 white triangles.

This cushion with 5 layers has 4 white triangles in its bottom layer.

- (e) Write down the total number of black triangles in a cushion with 4 layers. [1]

The number of black triangles in each layer forms an arithmetic sequence.

- (f) Find and simplify an expression for the total number of black triangles in a cushion with n layers. [2]

The total number of white triangles in a cushion with n layers is $\frac{n(n-1)}{2}$.

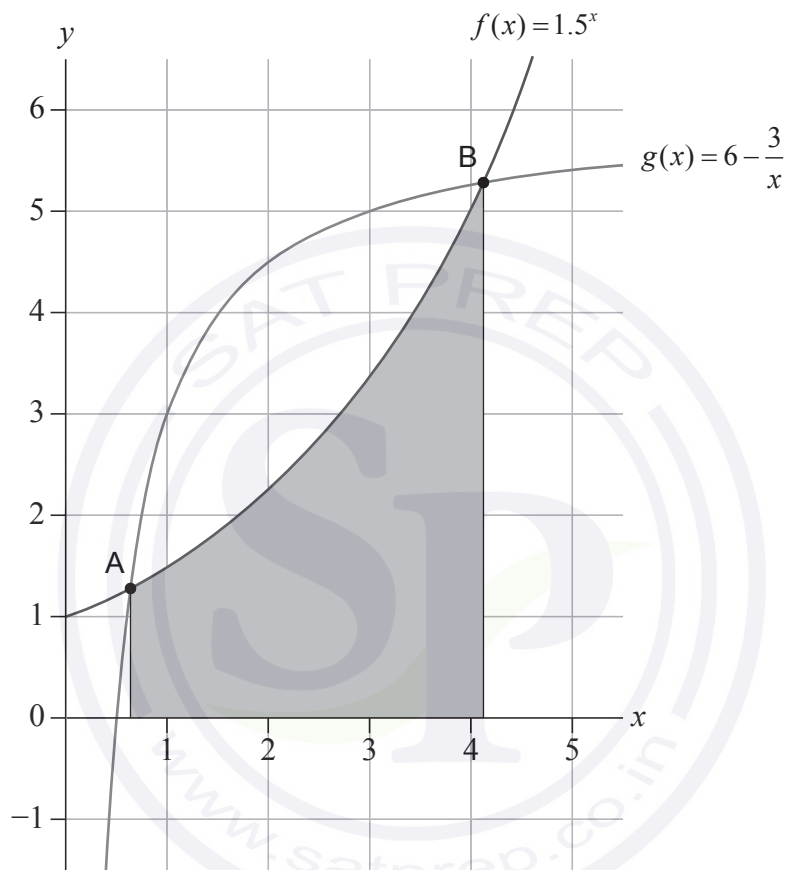
- (g) Using both the given expression and your answer to part (f), find and simplify an expression for the total number of black and white triangles in a cushion with n layers. [3]

2. [Maximum mark: 12]

The diagram shows part of the graphs of the functions

$$f(x) = 1.5^x \quad x \geq 0$$

$$g(x) = 6 - \frac{3}{x} \quad x > 0.$$



- (a) Solve $f(x) = g(x)$. [3]
- (b) (i) Write down the integral that represents the area of the shaded region.
 (ii) Calculate the area of this shaded region.
 (iii) Hence, or otherwise, calculate the area of the region enclosed between the curves $y = f(x)$ and $y = g(x)$. [6]

The tangent to the graph of $y = f(x)$ is parallel to the tangent to the graph of $y = g(x)$ at $x = k$.

- (c) Find the value of k . [3]

3. [Maximum mark: 20]

According to the timetable, the evening train from Cardiff arrives in London at 6:00 pm. Records show that the arrival times for this train are normally distributed with a mean arrival time of 6:20 pm and a standard deviation of 10 minutes.

Let X be the random variable “the number of minutes late the train arrives”.

(a) Write down

(i) the mean of X .

(ii) the variance of X .

[2]

(b) Find the probability that the evening train arrives in London between 15 and 25 minutes late.

[2]

If the train arrives more than 30 minutes late, the train company must pay a fine.

(c) Calculate the expected number of days, in a 365-day year, that the train company pays a fine.

[3]

Janine arrives in London on the evening train from Cardiff. She then has to catch a connecting train.

She allows 10 minutes to change platforms at the station.

She requires the minimum probability of arriving at the platform to catch the connecting train to be 99%.

(d) Find the earliest time, correct to the nearest minute, that the connecting train could depart to meet Janine’s requirements.

[4]

David, an employee of the train company, is concerned about the number of trains being late.

He spends 7 days in London noting the arrivals of trains.

It can be assumed that train arrivals are independent.

(e) Find the probability that the Cardiff to London evening train arrives late

(i) on the first day that David is in London.

(ii) on 5 of the 7 days that David is in London.

(iii) on at least 5 consecutive days that David is in London.

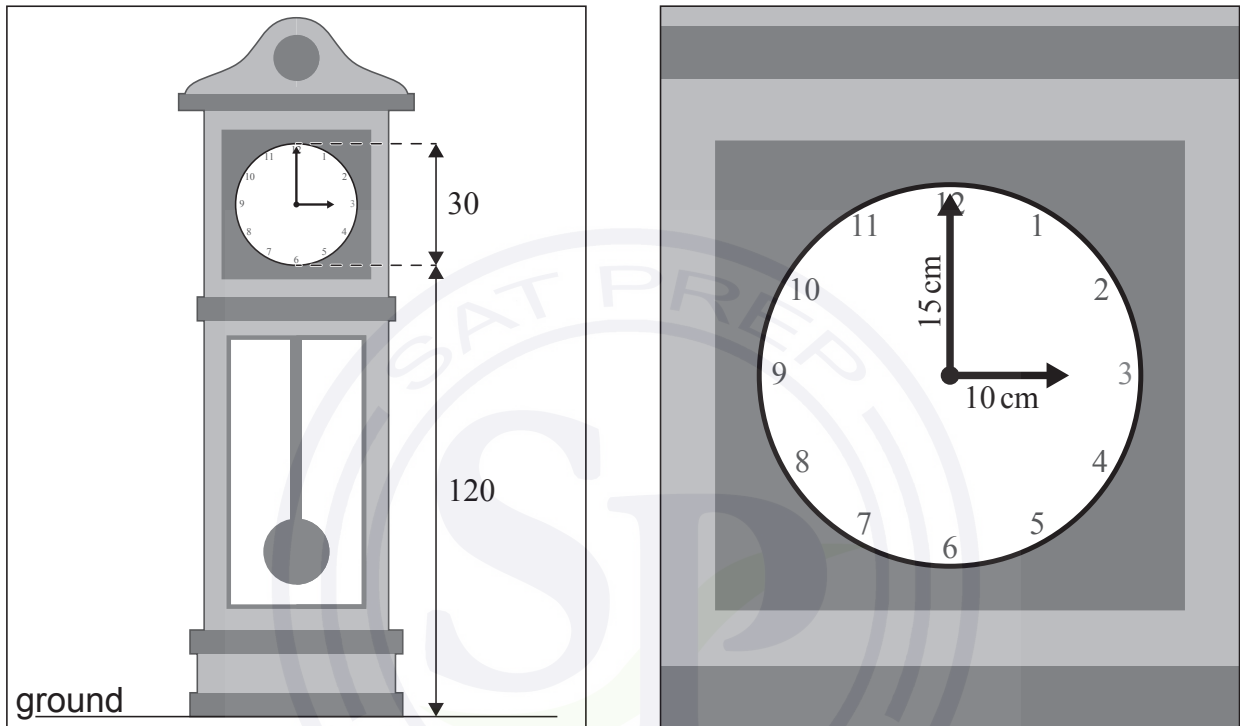
[9]

4. [Maximum mark: 16]

The circular clock face of a grandfather clock is 30 cm in diameter and its lowest point is 120 cm above the ground. The length of the minute hand is 15 cm and the length of the hour hand is 10 cm. This is shown in the diagram.

The clock is started at 3 pm precisely.

diagram not to scale



- (a) (i) Write down the size of the angle between the hour hand and the minute hand of the clock at 3 pm.
- (ii) Calculate the distance between the tips of the two hands of the clock at 3 pm. [3]

(This question continues on the following page)

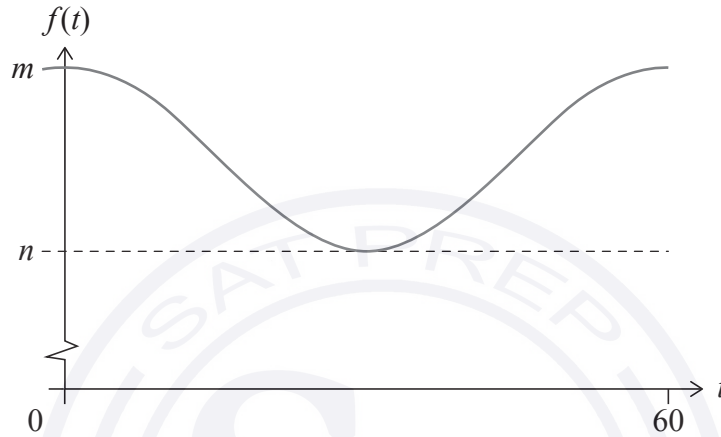
(Question 4 continued)

The height of the tip of the minute hand, $f(t)$, above the ground can be modelled by the cosine function

$$f(t) = a \cos(bt) + d, \quad a, b > 0$$

where the angle bt is measured in degrees and t is the number of minutes after 3 pm.

A sketch of the graph of $y = f(t)$ as time passes from 3 pm to 4 pm is shown.



(b) Write down the value of

(i) m .

(ii) n .

[2]

(c) Hence find the value of

(i) a .

(ii) b .

(iii) d .

[3]

(This question continues on page 9)



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(Question 4 continued)

The height of the tip of the hour hand, $g(t)$, above the ground can be modelled by the sine function

$$g(t) = p \sin(qt) + d$$

where qt is measured in degrees and t is the number of minutes after 3 pm.

- (d) (i) Show that $q = 0.5$.
 - (ii) Find the value of p . [3]
 - (e) Find the height of the tip of the hour hand above the ground at 4 pm. [2]
 - (f) Find the first time after 3 pm at which the tips of the two hands of the clock are at the same height above the ground.
- Give your answer correct to the nearest minute. [3]



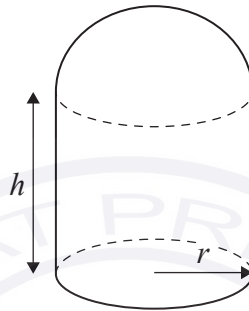
5. [Maximum mark: 16]

Ju Shen designs a plastic cover, in the shape of a cylinder combined with a hemisphere on top, as shown in the diagram.

The plastic used to make the cover forms the curved surface of both the hemisphere and the cylinder; there is no bottom to the cover, however it stands on a flat horizontal surface.

Let the height of the cylinder be h cm and the internal radius of its base be r cm.

diagram not to scale



- (a) Find an expression for the total internal surface area, A cm², of the cover in terms of r and h . [2]

- (b) Show that the total volume, V cm³, enclosed by the cover and the horizontal surface is given by the expression

$$V = \frac{2\pi r^3 + 3\pi r^2 h}{3}. \quad [2]$$

The total volume enclosed by the cover is 10 000 cm³.

- (c) Hence show that $h = \frac{30\,000 - 2\pi r^3}{3\pi r^2}$. [2]

Ju Shen uses the total internal surface area to model the amount of plastic used to construct the cover.

- (d) Show that A is given by the expression

$$A = \frac{2\pi r^2}{3} + \frac{20\,000}{r}. \quad [2]$$

(This question continues on the following page)

(Question 5 continued)

Ju Shen wants to use the minimum amount of plastic in the construction of the cover.

- (e) Find an expression for $\frac{dA}{dr}$. [3]
 - (f) Find the value of r and the value of h that minimizes the use of plastic. [4]
 - (g) By interpreting your answer to part (f), suggest the best shape for Ju Shen's plastic cover. [1]
-





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References:

1. naisupakit, 2016. *Triangle Pillow tradition native Thai style pillow*. [image online] Available at: <https://www.gettyimages.co.uk/detail/photo/triangle-pillow-tradition-native-thai-style-pillow-royalty-free-image/623127206> [Accessed 9 April 2024]. Source adapted.
4. bortonia, 2020. *Antique Grandfather Clock Icon on Transparent Background*. [image online] Available at: <https://www.gettyimages.co.uk/detail/illustration/antique-grandfather-clock-icon-on-royalty-free-illustration/1284116855> [Accessed 9 April 2024]. Source adapted.

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Mathematics: applications and interpretation

Standard level

Paper 2

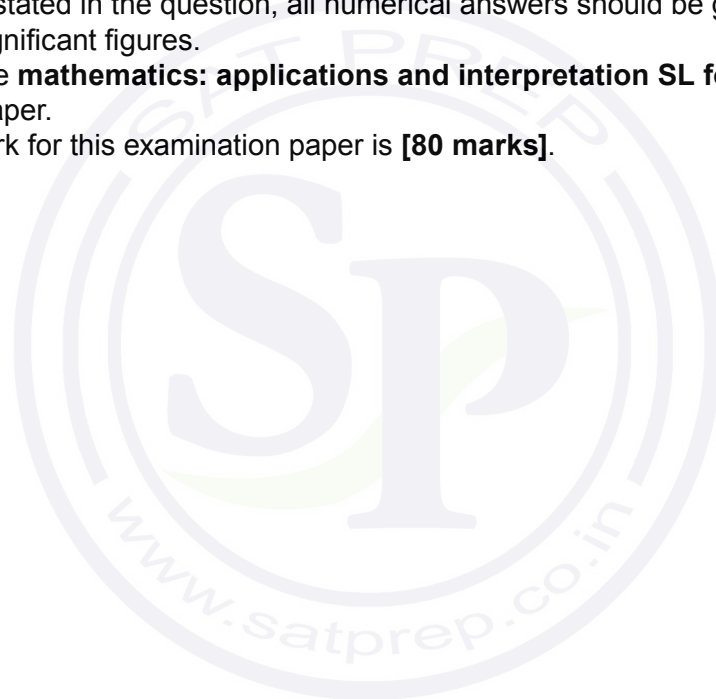
16 May 2025

Zone A morning | Zone B morning | Zone C morning

1 hour 30 minutes

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all the questions in the answer booklet provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation SL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.



Answer **all** questions in the answer booklet provided. Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 18]

Cathie is a financial analyst studying the growth of two investment accounts, Account 1 and Account 2, for a new client.

Account 1 has an initial amount of 5000 US Dollars (USD). Interest is added to the amount in Account 1 at the end of each year in the following manner: 200USD at the end of the first year, 260USD at the end of the second year, 320USD at the end of the third year, 380USD at the end of the fourth year and 440USD at the end of the fifth year.

Assume the amount of interest continues to increase each year so that it follows an arithmetic sequence.

- (a) Find
- (i) the common difference.
 - (ii) the amount of interest, in USD, added at the end of the 10th year. [3]
- (b) Show that the amount of money in Account 1 after n years may be expressed as
- $$5000 + \frac{n}{2}(340 + 60n). \quad [3]$$
- (c) Hence or otherwise, find the amount of money in Account 1 at the end of 10 years. [2]

Account 2 has the same initial amount of 5000USD. Account 2 pays 6.5% interest compounded annually. The interest is added to the amount in the account at the end of each year.

The amount in Account 2 after n years can be expressed as $5000 \times B^n$ where $B \in \mathbb{R}$.

- (d) (i) Write down the value of B .
- (ii) Hence or otherwise, show that Account 1 will have more money than Account 2 at the end of 10 years. [4]

The client is interested in a longer-term investment. Cathie finds that it will take at least m complete years for the amount in Account 2 to exceed the amount in Account 1.

- (e) Find the value of m . [3]
- (f) Determine the total interest added to Account 2 at the end of m years. Give your answer correct to the nearest dollar. [3]

2. [Maximum mark: 17]

A company produces electronic components on a large scale. They carry out quality control tests to determine whether the components meet the company's standards.

Zaakir, the owner of the company, wants the quality control team to analyse the distribution of the weights of the components.

Based on historical data, the quality control team knows that the weights of the components follow a normal distribution with a mean of 2.5 grams and a standard deviation of 0.15 grams.

- (a) Find the probability that the weight of a component selected at random is greater than 2.8 grams. [2]

The probability that the weight of a component selected at random is greater than w grams is 0.8.

- (b) (i) Sketch a diagram of a normal curve to show the area represented by this probability.
(ii) Find the value of w . [4]

To pass Test 1, the weight of a component must be between 2.3 grams and 2.7 grams.

- (c) (i) Find the probability that a randomly selected component passes Test 1.
(ii) Find the expected number of components in a box of 200 that will pass Test 1. [3]

Zaakir asks the quality control team to conduct a more in-depth analysis by performing a new test, Test 2. The probability of a component passing Test 2 is 0.95. The team randomly selects one box and tests each of the 200 components.

- (d) Find the probability that exactly 190 components pass Test 2. [2]
(e) Find the probability that at least 188 components pass Test 2. [2]

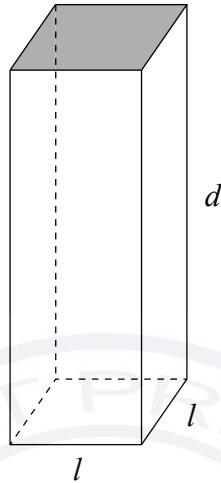
Instead of testing all 200 components, Zaakir now decides to test a random sample of 12 components from a box of 200 components. He decides that the box will only be dispatched if at least 10 of the 12 components pass both Test 1 and Test 2. The results of Test 1 and Test 2 are independent.

- (f) Find the probability that the box is dispatched. [4]

3. [Maximum mark: 20]

Kailash manufactures drink containers in the shape of a cuboid. The container has a square top and a square base of length, l cm. Its height, d cm, is three times the length of the base.

diagram not to scale



(a) Write down an expression for d in terms of l . [1]

The container can hold 375 cm^3 of drink.

(b) Find the value of l and d . [3]

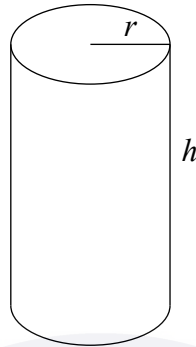
(c) Calculate the total external surface area of the container. [3]

(This question continues on the following page)

(Question 3 continued)

To reduce environmental impact, Kailash is trying to minimize the amount of material needed for the production of the 375 cm^3 container.

He is willing to change the shape to a cylinder with radius $r \text{ cm}$, and height $h \text{ cm}$, as shown below.



The cylindrical container of drink must also hold 375 cm^3 .

(d) Find an expression for the height, h , of the container in terms of r . [2]

Let the total external surface area be $A \text{ cm}^2$.

(e) Show that $A = 2\pi r^2 + \frac{750}{r}$. [2]

(f) Find $\frac{dA}{dr}$. [3]

(g) Hence or otherwise

(i) find the value of r that will minimize A .

(ii) find the minimum value of A needed for the cylinder. [3]

To produce the containers, additional material is required:

- 10% additional surface area for the cuboid
- 25% additional surface area for the cylinder.

Kailash will choose the container that requires the least total amount of material.

(h) Determine which container Kailash should choose. Justify your answer. [3]

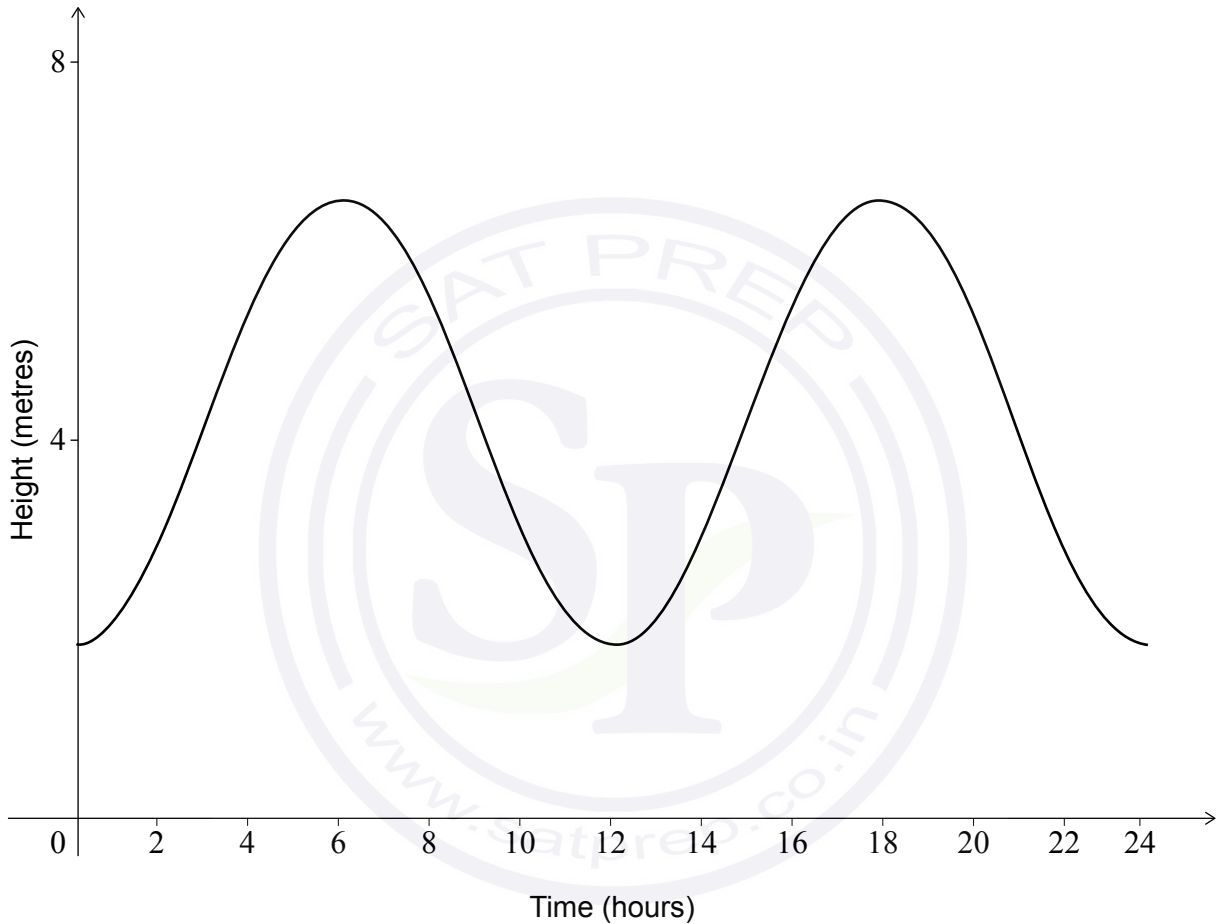
4. [Maximum mark: 13]

On a particular day the height of the tide, h , in metres, at Albion harbour can be modelled by the function

$$h(t) = -2.5 \cos(bt^\circ) + 4.5, \text{ where } b \in \mathbb{R}, 0 \leq t \leq 24$$

and t represents the number of hours after midnight.

The graph of h is shown in the following diagram.



- (a) Show that the value of b is 30. [1]
- (b) Find the height of the tide when $t = 5$. [2]
- (c) Write down
 - (i) the amplitude of h .
 - (ii) the equation of the principal axis. [3]

(This question continues on the following page)

(Question 4 continued)

Boats can only leave or return to Albion harbour when $h(t) \geq 2.65$. Robin wants to leave the harbour to go fishing as soon as possible after the time is 12:00.

- (d) Determine the earliest possible time that Robin could leave the harbour.
Give your answer to the nearest minute. [3]

The boat will take 15 minutes to travel from the harbour to the fishing site. Robin intends to return to the harbour on the same day.

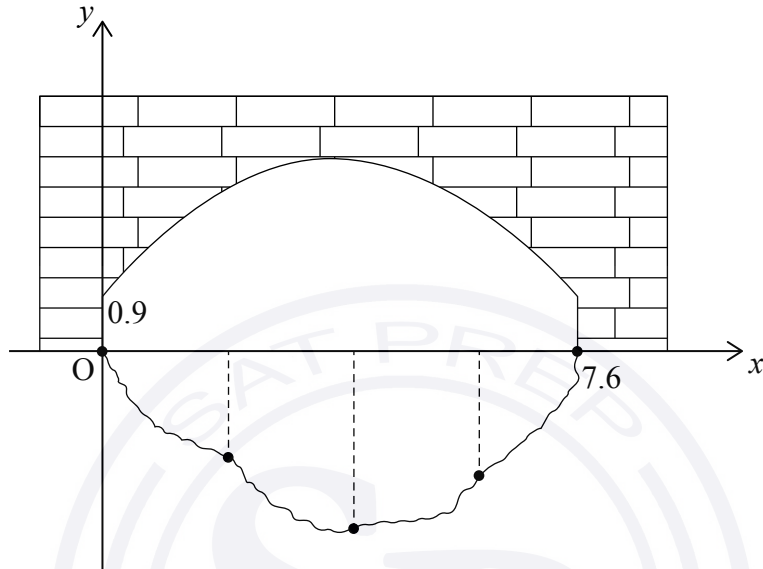
- (e) Determine the maximum length of time he could spend at the fishing site, in hours, and still be certain he will be able to enter the harbour on his return. [4]



5. [Maximum mark: 12]

The diagram shows the cross-section of a bridge and a river. A coordinate system has been added with the origin, O , at the point where the bridge meets the water on one side. All units are in metres.

diagram not to scale



A researcher wants to calculate the volume of water that flows under the bridge. To do this he takes measurements of the depth every 1.9 m from O . The depths are shown in the following table.

Horizontal distance from O in metres	0	1.9	3.8	5.7	7.6
Vertical depth of water in metres	0	1.68	2.81	2.32	0

- (a) Use the trapezoidal rule to find the cross-sectional area of the river as it passes under the bridge. [3]

The water flows under the bridge at a rate of 0.3 m s^{-1} .

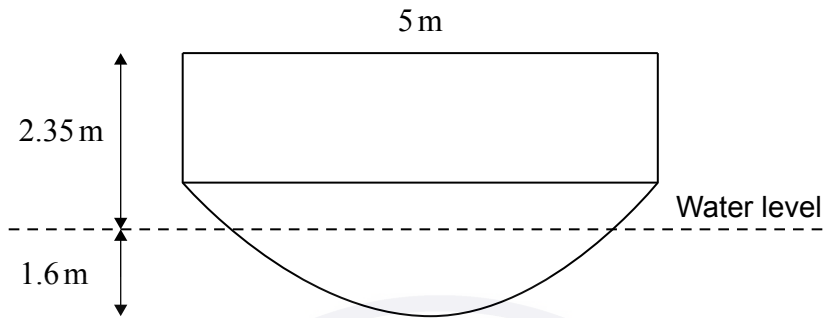
- (b) Find the volume of water that passes under the bridge each second. [2]

(This question continues on the following page)

(Question 5 continued)

A boat is travelling along the river. The cross-section of the boat and the water level is shown in the following diagram.

The top of the boat is parallel to the water level and has a width of 5 m. The height of the boat is 2.35 m above the water level and the lowest part of the boat is 1.6 m below the water level.



The boat is travelling down the centre of the river.

- (c) Find the vertical distance between the lowest part of the boat and the bottom of the river as it passes under the bridge. [1]

The curved arch of the bridge can be modelled by the equation

$$y = -0.15x^2 + 1.14x + 0.9, \quad 0 \leq x \leq 7.6.$$

- (d) Find the maximum height of the curved arch above the water level. [2]
- (e) Determine whether the top of the boat will be able to pass under the bridge. [4]



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Mathematics: applications and interpretation

Standard level

Paper 2

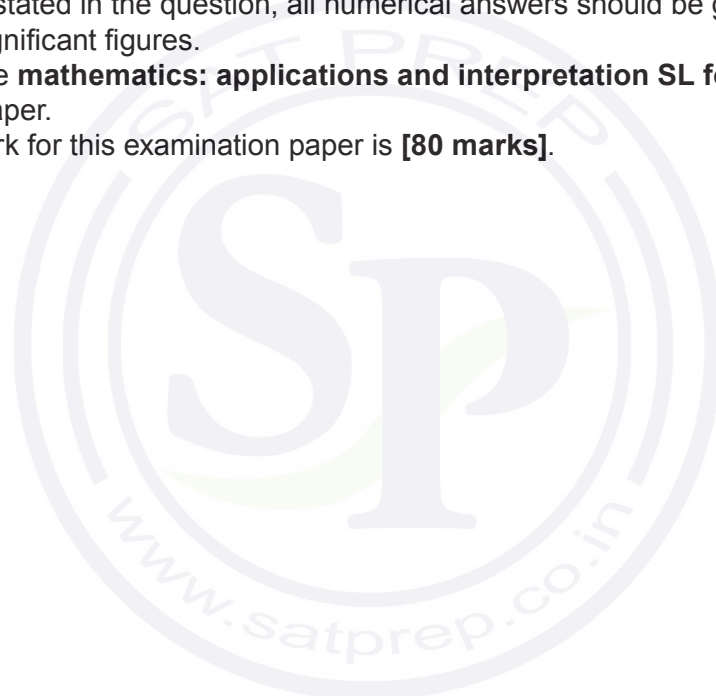
25 October 2024

Zone A morning | Zone B morning | Zone C morning

1 hour 30 minutes

Instructions to candidates

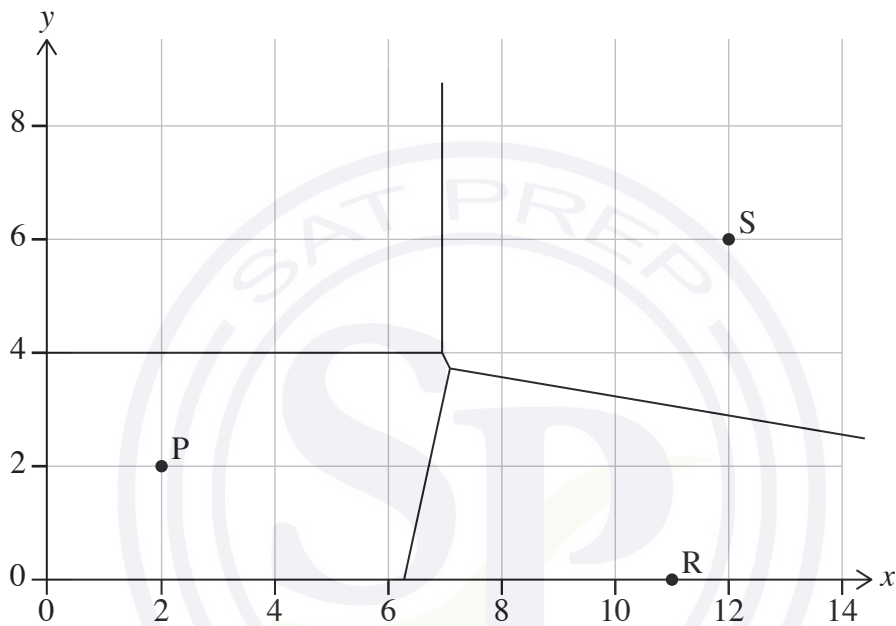
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all the questions in the answer booklet provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation SL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.



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1. [Maximum mark: 13]

The following grid shows a restaurant's floorplan. There are four food stations centred at points P, Q, R and S. The Voronoi diagram for these four points is shown. Point Q is not shown.



One unit represents 1 metre.

Point P is located at (2, 2).

The equation of the perpendicular bisector of [PQ] is $y = 4$.

(a) Write down the coordinates of Q. [1]

Points R and S are located at (11, 0), and (12, 6), respectively.

(b) Find

(i) the coordinates of the midpoint of [RS]

(ii) the equation of the perpendicular bisector of [RS]. Give your answer in the form $y = mx + c$. [6]

(This question continues on the following page)

(Question 1 continued)

Customers in the restaurant take their food from the nearest food station.

The following table shows the average waiting time, in minutes, for each food station.

Food station	P	Q	R	S
Average waiting time (minutes)	8	4	9	6

- (c) Using nearest-neighbour interpolation, find the average waiting time for a customer at point (6, 8). [1]

The restaurant owner wishes to determine whether customers spend more money during the weekend. She decides to use a two-sample t -test at a 5% level of significance.

For this test, the null hypothesis is:

$$H_0 : \mu_1 = \mu_2$$

where μ_1 is the mean amount of money spent by all customers on weekdays, and μ_2 is the mean amount of money spent by all customers on weekends.

- (d) State the alternative hypothesis. [1]

The owner considers collecting her sample data from 2 weekdays and 1 weekend by recording the amount spent on the first 12 orders on each of these days.

- (e) (i) State which one of the following methods best describes the owner's planned sampling:
convenience, systematic, or stratified.
(ii) State one disadvantage of this sampling technique. [2]

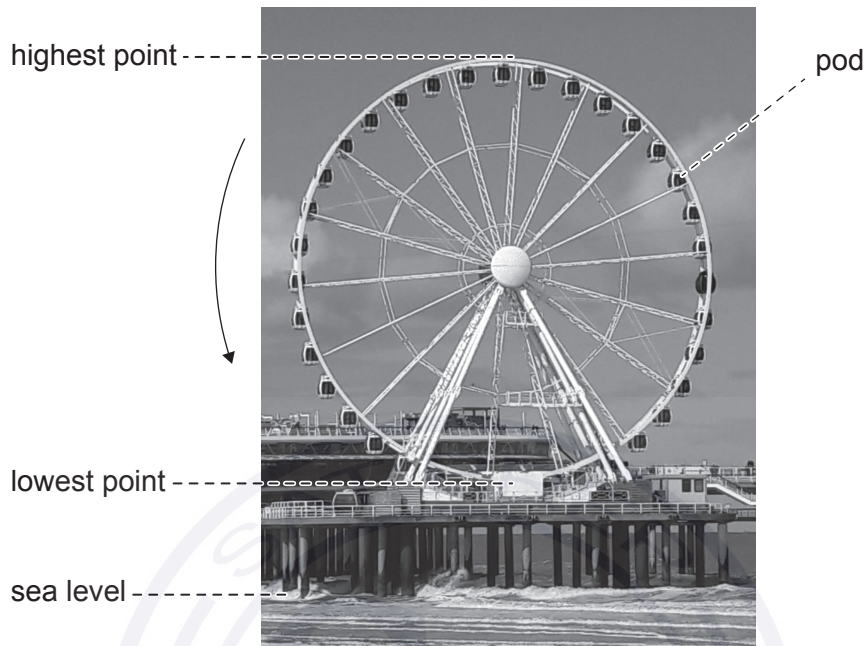
The restaurant owner instead collects the data using simple random sampling.

The p -value is 0.0897 for the sampled data.

- (f) State the conclusion for the test. Justify your answer. [2]

2. [Maximum mark: 16]

The Scheveningen Ferris wheel's lowest point is 8 m above sea level, and its highest point is 45 m above sea level.



- (a) (i) Show that the radius of the Ferris wheel is 18.5 m.
- (ii) Calculate the circumference of the Ferris wheel. [2]

There are pods, equally spaced around the wheel, that carry passengers.

- (b) When the wheel rotates 10° , find the distance that a pod travels along the circumference. [3]

(This question continues on the following page)

(Question 2 continued)

The height in metres, above sea level, of a particular pod is modelled by the function:

$$h(t) = a \sin(bt) + d, \text{ for } a, b > 0,$$

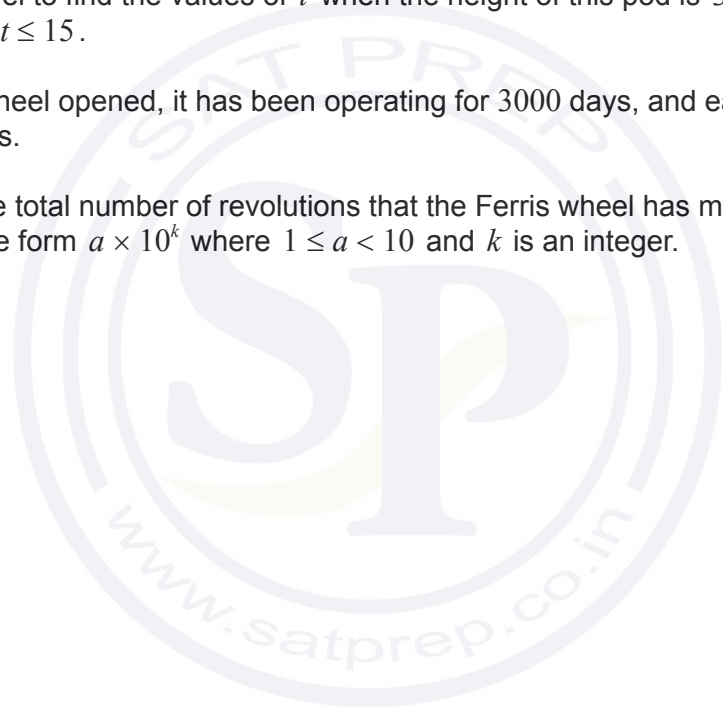
where t is the time, measured in minutes.

The wheel takes 15 minutes to complete 1 revolution.

- (c) (i) Find the value of b .
 - (ii) Find the value of d .
 - (iii) Hence, write down the equation of the sinusoidal model. [5]
- (d) Use the model to find the values of t when the height of this pod is 33 m above sea level for $0 \leq t \leq 15$. [3]

Since the Ferris wheel opened, it has been operating for 3000 days, and each day it rotates nonstop for 8 hours.

- (e) Calculate the total number of revolutions that the Ferris wheel has made. Give your answer in the form $a \times 10^k$ where $1 \leq a < 10$ and k is an integer. [3]



3. [Maximum mark: 14]

A survey was answered by 20 000 expatriates (people living in a country that is not their own). The data ranked countries in order of the country they felt was best for expatriates. The highest-ranked country was Switzerland.

These results were compared to happiness scores taken from *The World Happiness Report 2022*. The following table shows this data for the top 10 expatriate countries.

Country	Switzerland	New Zealand	Spain	Australia	Cyprus	Portugal	Ireland	United Arab Emirates	France	Netherlands
Expatriate country rank	1	2	3	4	5	6	7	8	9	10
Happiness score	7.5	7.2	6.5	7.2	6.2	6.0	7.0	6.6	6.7	7.4

(a) For the **happiness score**, find

(i) the upper quartile

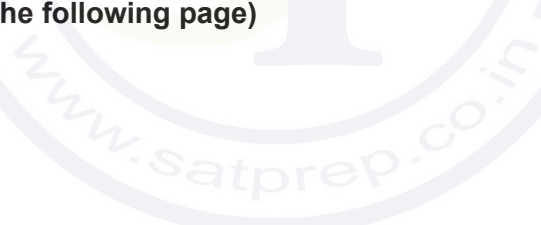
(ii) the interquartile range.

[4]

(b) Show that Switzerland's happiness score is not an outlier for this data.

[3]

(This question continues on the following page)



(Question 3 continued)

The happiness scores were ranked to calculate Spearman's rank correlation coefficient, r_s . These ranks are shown in the following table.

Country	Switzerland	New Zealand	Spain	Australia	Cyprus	Portugal	Ireland	United Arab Emirates	France	Netherlands
Happiness score	7.5	7.2	6.5	7.2	6.2	6.0	7.0	6.6	6.7	7.4
Happiness rank	1	a	b	c	9	10	5	7	6	2

(c) Write down the value of

(i) a

(ii) b

(iii) c .

[3]

(d) (i) Find r_s .

(ii) If France's happiness score is upgraded to 6.9, explain why the value of r_s does not change.

[3]

Jose concludes from this data that countries with high happiness scores are likely to be favourite expatriate countries.

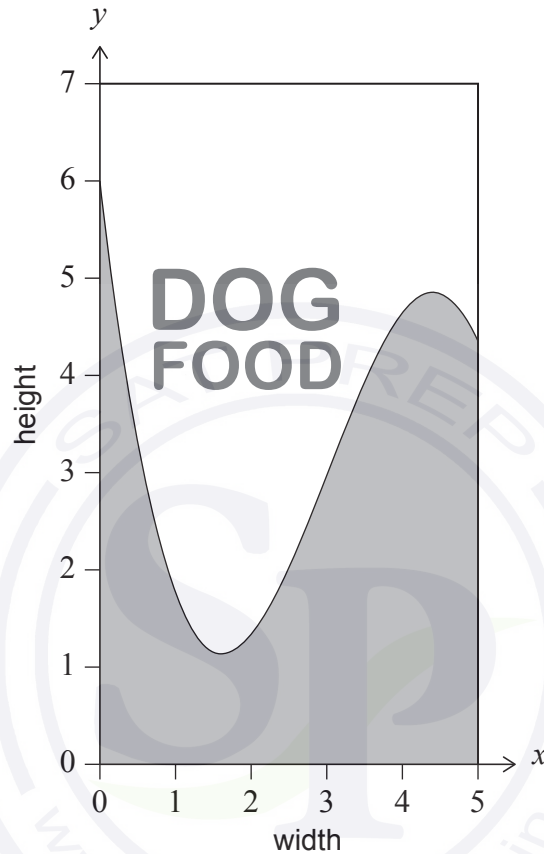
(e) State, with a reason, whether Jose's conclusion is appropriate.

[1]

4. [Maximum mark: 20]

A type of dog food is sold in cylindrical containers. A new label for the container is being considered. The label will be a rectangle that is 5 cm wide and 7 cm high.

The design on the label is a curve, as shown on the following axes, where one unit represents 1 cm for both axes.



The values in the table approximate points on the curve, correct to one decimal place.

Width, x	0	1	2	3	4	5
Height, y	6	1.7	1.3	3.0	4.7	4.3

- (a) Use the trapezoidal rule with five intervals, and the values given in the table, to estimate the shaded area below the curve. [3]

The curve used in the label design can be modelled by:

$$y = -\frac{x^3}{3} + 3x^2 - 7x + 6, \text{ for } 0 \leq x \leq 5.$$

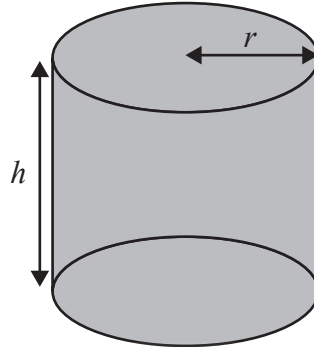
- (b) (i) Write down the integral that represents the area of the shaded region.
 (ii) Hence, find the area of the shaded region. [4]

(This question continues on the following page)

(Question 4 continued)

The dog food is sold in closed cylindrical containers, with radius r and height h .

diagram not to scale



The whole container is made from one type of material, and it is assumed that the thickness of the material is negligible.

Each container has a volume of 400 cm^3 .

- (c) Write down an equation, in terms of r and h , that shows this information. [1]

The amount of material used for each container can be modelled by the external surface area of the container.

The external surface area, A , of the container can be expressed as

$$A = 2\pi r^2 + \frac{k}{r}, \text{ where } r > 0.$$

- (d) Find the value of k . [4]

- (e) (i) Find $\frac{dA}{dr}$.
(ii) Given that A has a minimum value, find the value of r that will minimize the material used. [5]

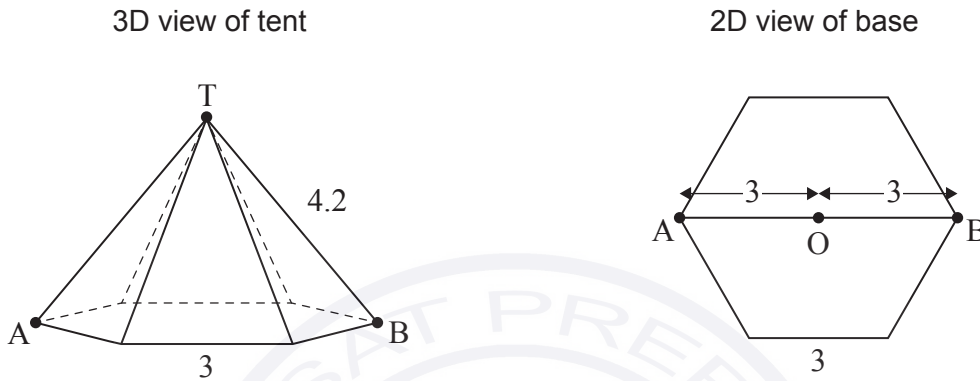
The containers are made so that the surface area is minimized. The 5 cm by 7 cm rectangular label is to be glued to the curved surface of the container.

- (f) Show that the label will fit on the container. [3]

5. [Maximum mark: 17]

Jamali is designing a tent in the shape of a right pyramid with a regular hexagonal base, centre O . The length of each side of the base is 3 m, the length of each sloping edge is 4.2 m, and the distance between each vertex on the base and O is 3 m, as shown in the diagrams.

diagrams not to scale



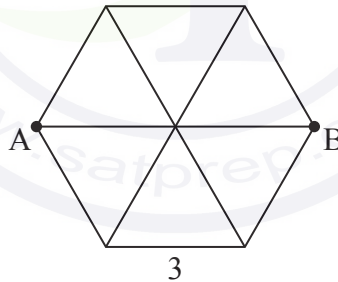
The top of the tent, T , will be supported by a vertical pole from O .

(a) Find the length of the pole, OT .

[2]

The hexagonal base can be divided into six equilateral triangles.

diagram not to scale



(b) Find

(i) the area of the base

(ii) the volume of the tent.

[5]

(c) Find the value of \hat{OAT} .

[2]

(This question continues on the following page)

(Question 5 continued)

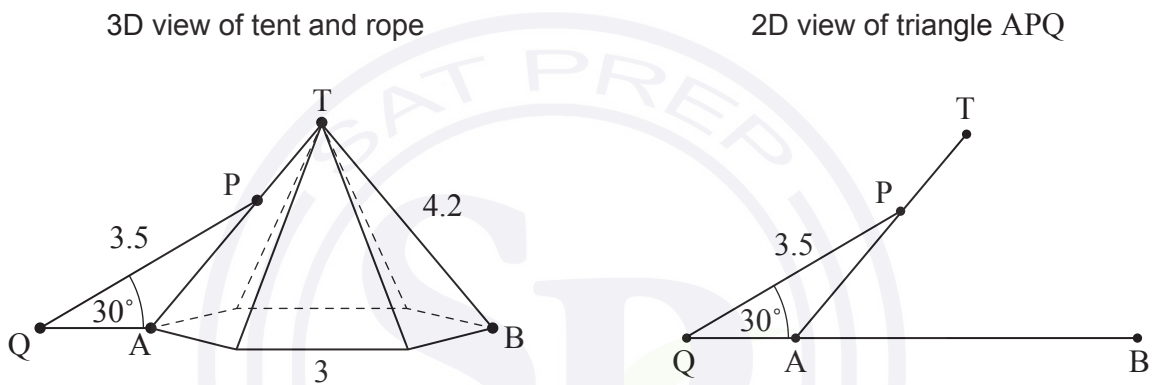
For extra support, Jamali decides to attach a rope, with length 3.5 m, to the tent at a point, P, on the edge AT.

The rope will be fixed to the ground at point Q, such that:

- the rope, [PQ], is straight
- points Q, A and B lie on a straight line
- $\hat{AQP} = 30^\circ$.

This is shown in the diagrams.

diagrams not to scale



(d) Find AQ. [4]

For decoration at night, Jamali wants to fix a strip of lights from point A to a point, R, along the rope [PQ].

The strip of lights, [AR], is straight and has length 0.8m.

(e) Find the two possible values of QR. [4]



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References:

3. InterNations. 2022. *Expat Insider 2022*. [online] Available at: <https://www.internations.org/expat-insider/2022/>. Source adapted.
Heliwell, J. F., Huang, H., Wang, S. and Norton, M., 2022. Statistical Appendix for "Happiness, benevolence, and trust during COVID-19 and beyond," Chapter 2 of *World Happiness Report 2022*. [pdf online] Available at: <https://worldhappiness.report/ed/2022/happiness-benevolence-and-trust-during-covid-19-and-beyond/> [Accessed 13 November 2023]. Source adapted.

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Mathematics: applications and interpretation

Standard level

Paper 2

25 October 2024

Zone A morning | Zone B morning | Zone C morning

1 hour 30 minutes

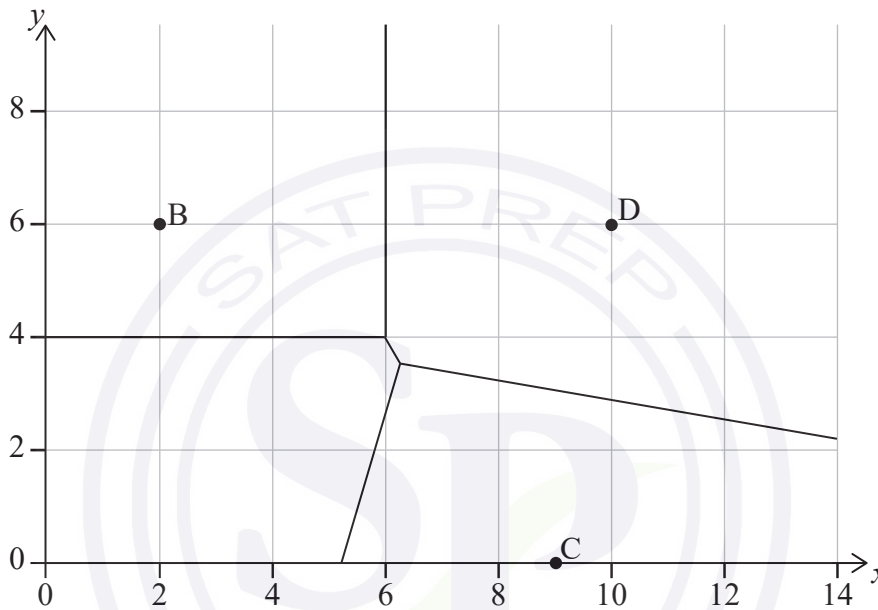
Instructions to candidates

- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all the questions in the answer booklet provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation SL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

Answer **all** questions in the answer booklet provided. Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 13]

The following grid shows a restaurant's floorplan. There are four food stations centred at points A, B, C and D. The Voronoi diagram for these four points is shown. Point A is not shown.



One unit represents 1 metre.

Point B is located at (2, 6).

The equation of the perpendicular bisector of [AB] is $y = 4$.

(a) Write down the coordinates of A. [1]

Points C and D are located at (9, 0), and (10, 6), respectively.

(b) Find

(i) the coordinates of the midpoint of [CD]

(ii) the equation of the perpendicular bisector of [CD]. Give your answer in the form $y = mx + c$. [6]

(This question continues on the following page)

(Question 1 continued)

Customers in the restaurant take their food from the nearest food station.

The following table shows the average waiting time, in minutes, for each food station.

Food station	A	B	C	D
Average waiting time (minutes)	9	5	10	7

- (c) Using nearest-neighbour interpolation, find the average waiting time for a customer at point (4, 6). [1]

The restaurant owner wishes to determine whether customers spend more money during the weekend. She decides to use a two-sample t -test at a 5% level of significance.

For this test, the null hypothesis is:

$$H_0 : \mu_1 = \mu_2$$

where μ_1 is the mean amount of money spent by all customers on weekdays, and μ_2 is the mean amount of money spent by all customers on weekends.

- (d) State the alternative hypothesis. [1]

The owner considers collecting her sample data from 2 weekdays and 1 weekend by recording the amount spent on the first 12 orders on each of these days.

- (e) (i) State which one of the following methods best describes the owner's planned sampling:
convenience, systematic, or stratified.
(ii) State one disadvantage of this sampling technique. [2]

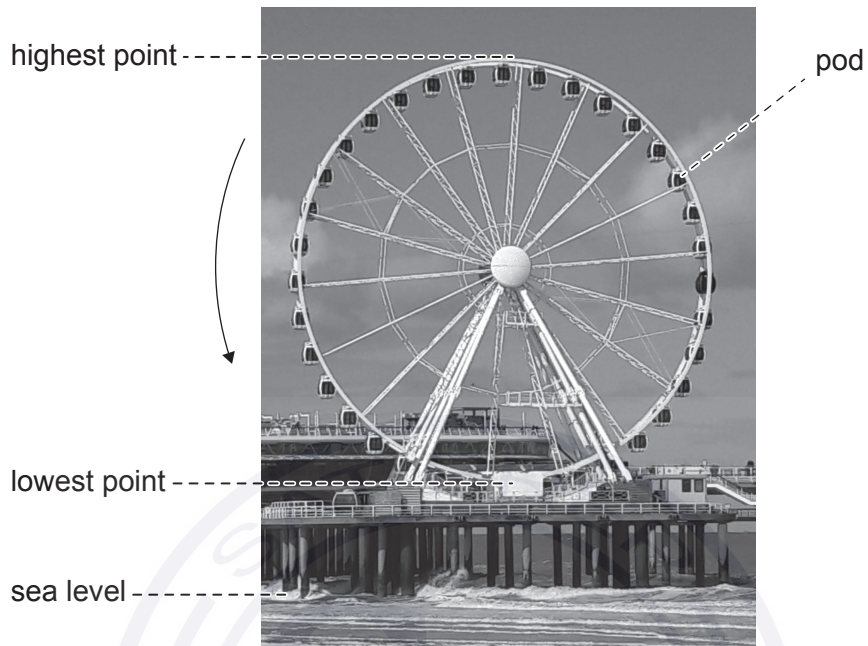
The restaurant owner instead collects the data using simple random sampling.

The p -value is 0.0897 for the sampled data.

- (f) State the conclusion for the test. Justify your answer. [2]

2. [Maximum mark: 16]

The Scheveningen Ferris wheel's lowest point is 8 m above sea level, and its highest point is 45 m above sea level.



- (a) (i) Show that the radius of the Ferris wheel is 18.5 m.
- (ii) Calculate the circumference of the Ferris wheel. [2]

There are pods, equally spaced around the wheel, that carry passengers.

- (b) When the wheel rotates 10° , find the distance that a pod travels along the circumference. [3]

(This question continues on the following page)

(Question 2 continued)

The height in metres, above sea level, of a particular pod is modelled by the function:

$$h(t) = a \sin(bt) + d, \text{ for } a, b > 0,$$

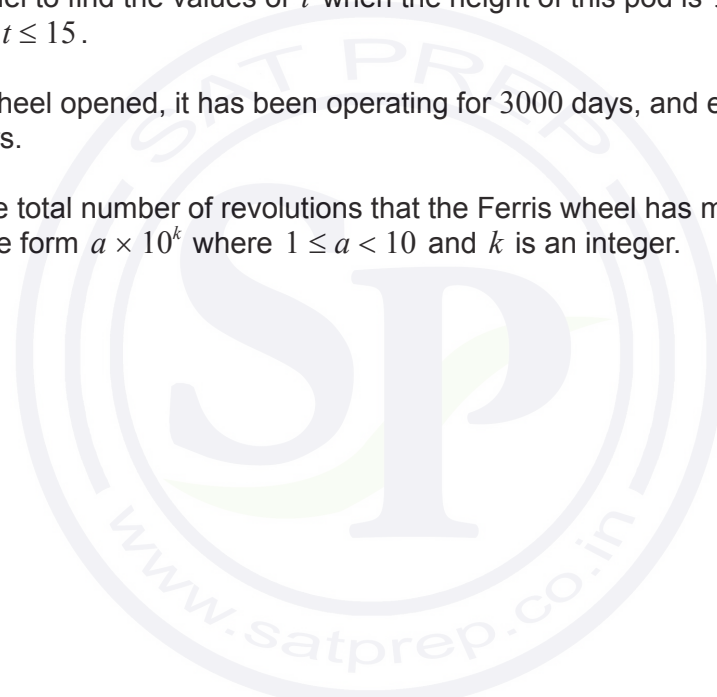
where t is the time, measured in minutes.

The wheel takes 15 minutes to complete 1 revolution.

- (c) (i) Find the value of b .
 - (ii) Find the value of d .
 - (iii) Hence, write down the equation of the sinusoidal model. [5]
- (d) Use the model to find the values of t when the height of this pod is 33 m above sea level for $0 \leq t \leq 15$. [3]

Since the Ferris wheel opened, it has been operating for 3000 days, and each day it rotates nonstop for 8 hours.

- (e) Calculate the total number of revolutions that the Ferris wheel has made. Give your answer in the form $a \times 10^k$ where $1 \leq a < 10$ and k is an integer. [3]



3. [Maximum mark: 14]

A survey was answered by 20 000 expatriates (people living in a country that is not their own). The data ranked countries in order of the country they felt was best for expatriates. The highest-ranked country was Switzerland.

These results were compared to happiness scores taken from *The World Happiness Report 2022*. The following table shows this data for the top 10 expatriate countries.

Country	Switzerland	New Zealand	Spain	Australia	Cyprus	Portugal	Ireland	United Arab Emirates	France	Netherlands
Expatriate country rank	1	2	3	4	5	6	7	8	9	10
Happiness score	7.5	7.2	6.5	7.2	6.2	6.0	7.0	6.6	6.7	7.4

(a) For the **happiness score**, find

(i) the upper quartile

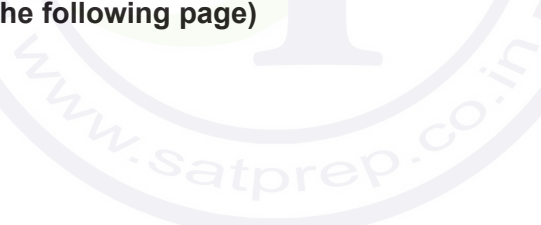
(ii) the interquartile range.

[4]

(b) Show that Switzerland's happiness score is not an outlier for this data.

[3]

(This question continues on the following page)



(Question 3 continued)

The happiness scores were ranked to calculate Spearman's rank correlation coefficient, r_s . These ranks are shown in the following table.

Country	Switzerland	New Zealand	Spain	Australia	Cyprus	Portugal	Ireland	United Arab Emirates	France	Netherlands
Happiness score	7.5	7.2	6.5	7.2	6.2	6.0	7.0	6.6	6.7	7.4
Happiness rank	1	a	b	c	9	10	5	7	6	2

(c) Write down the value of

(i) a

(ii) b

(iii) c .

[3]

(d) (i) Find r_s .

(ii) If France's happiness score is upgraded to 6.9, explain why the value of r_s does not change.

[3]

Jose concludes from this data that countries with high happiness scores are likely to be favourite expatriate countries.

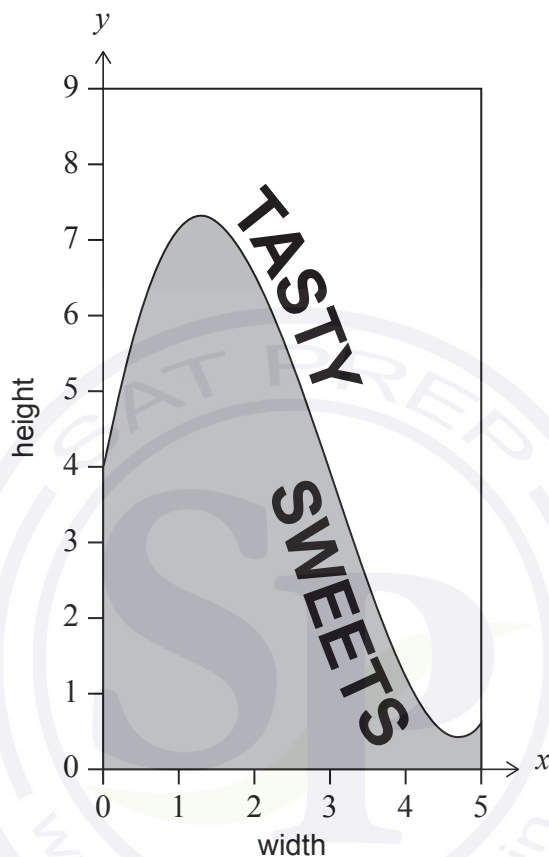
(e) State, with a reason, whether Jose's conclusion is appropriate.

[1]

4. [Maximum mark: 20]

Sweets are sold in cylindrical containers. A new label for the container is being considered. The label will be a rectangle that is 5 cm wide and 9 cm high.

The design on the label is a curve, as shown on the following axes, where one unit represents 1 cm for both axes.



The values in the table approximate points on the curve, correct to one decimal place.

Width, x	0	1	2	3	4	5
Height, y	4	7.3	6.7	4.0	1.3	0.7

- (a) Use the trapezoidal rule with five intervals, and the values given in the table, to estimate the shaded area below the curve. [3]

The curve used in the label design can be modelled by:

$$y = \frac{x^3}{3} - 3x^2 + 6x + 4, \text{ for } 0 \leq x \leq 5.$$

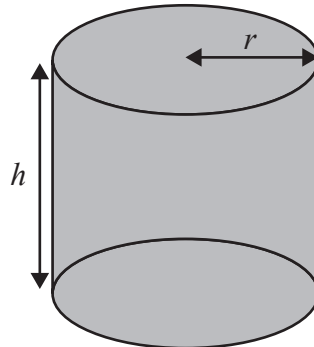
- (b) (i) Write down the integral that represents the area of the shaded region.
 (ii) Hence, find the area of the shaded region. [4]

(This question continues on the following page)

(Question 4 continued)

The sweets are sold in closed cylindrical containers, with radius r and height h .

diagram not to scale



The whole container is made from one type of material, and it is assumed that the thickness of the material is negligible.

Each container has a volume of 600 cm^3 .

- (c) Write down an equation, in terms of r and h , that shows this information. [1]

The amount of material used for each container can be modelled by the external surface area of the container.

The external surface area, A , of the container can be expressed as

$$A = 2\pi r^2 + \frac{k}{r}, \text{ where } r > 0.$$

- (d) Find the value of k . [4]

- (e) (i) Find $\frac{dA}{dr}$.
 (ii) Given that A has a minimum value, find the value of r that will minimize the material used. [5]

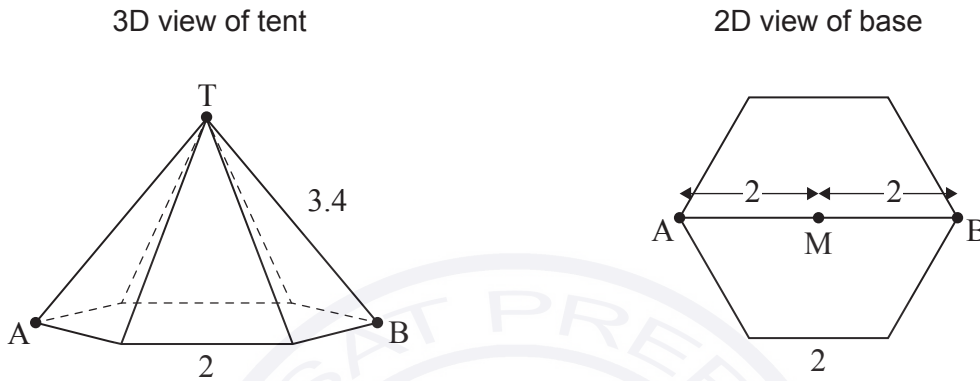
The containers are made so that the surface area is minimized. The 5 cm by 9 cm rectangular label is to be glued to the curved surface of the container.

- (f) Show that the label will fit on the container. [3]

5. [Maximum mark: 17]

Gaurika is designing a tent in the shape of a right pyramid with a regular hexagonal base, centre M . The length of each side of the base is 2 m , the length of each sloping edge is 3.4 m , and the distance between each vertex on the base and M is 2 m , as shown in the diagrams.

diagrams not to scale

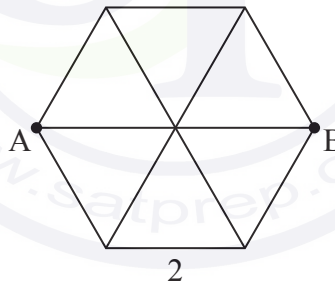


The top of the tent, T , will be supported by a vertical pole from M .

- (a) Find the length of the pole, MT . [2]

The hexagonal base can be divided into six equilateral triangles.

diagram not to scale



- (b) Find
- (i) the area of the base
 - (ii) the volume of the tent. [5]
- (c) Find the value of $\hat{M}AT$. [2]

(This question continues on the following page)

(Question 5 continued)

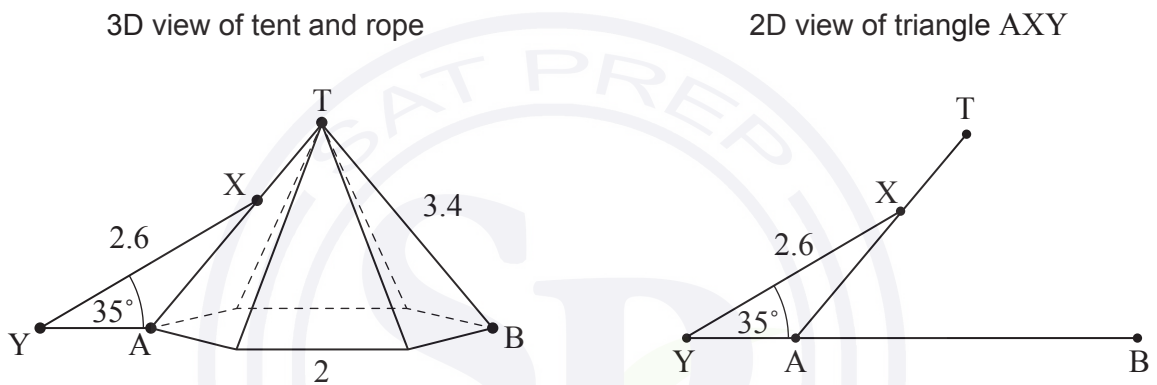
For extra support, Gaurika decides to attach a rope, with length 2.6 m, to the tent at a point, X, on the edge AT.

The rope will be fixed to the ground at point Y, such that:

- the rope, [XY], is straight
- points Y, A and B lie on a straight line
- $\hat{A}YX = 35^\circ$.

This is shown in the diagrams.

diagrams not to scale



(d) Find AY. [4]

For decoration at night, Gaurika wants to fix a strip of lights from point A to a point, Z, along the rope [XY].

The strip of lights, [AZ], is straight and has length 0.9 m.

(e) Find the two possible values of YZ. [4]



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References:

3. InterNations. 2022. *Expat Insider 2022*. [online] Available at: <https://www.internations.org/expat-insider/2022/>. Source adapted.
Heliwell, J. F., Huang, H., Wang, S. and Norton, M., 2022. Statistical Appendix for "Happiness, benevolence, and trust during COVID-19 and beyond," Chapter 2 of *World Happiness Report 2022*. [pdf online] Available at: <https://worldhappiness.report/ed/2022/happiness-benevolence-and-trust-during-covid-19-and-beyond/> [Accessed 13 November 2023]. Source adapted.

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Mathematics: applications and interpretation

Standard level

Paper 2

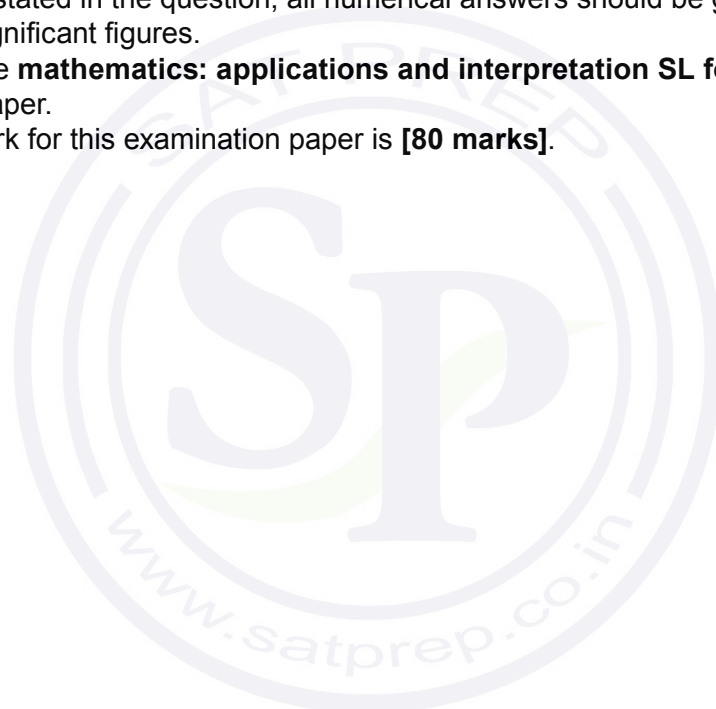
2 May 2024

Zone A morning | Zone B morning | Zone C morning

1 hour 30 minutes

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all the questions in the answer booklet provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation SL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.



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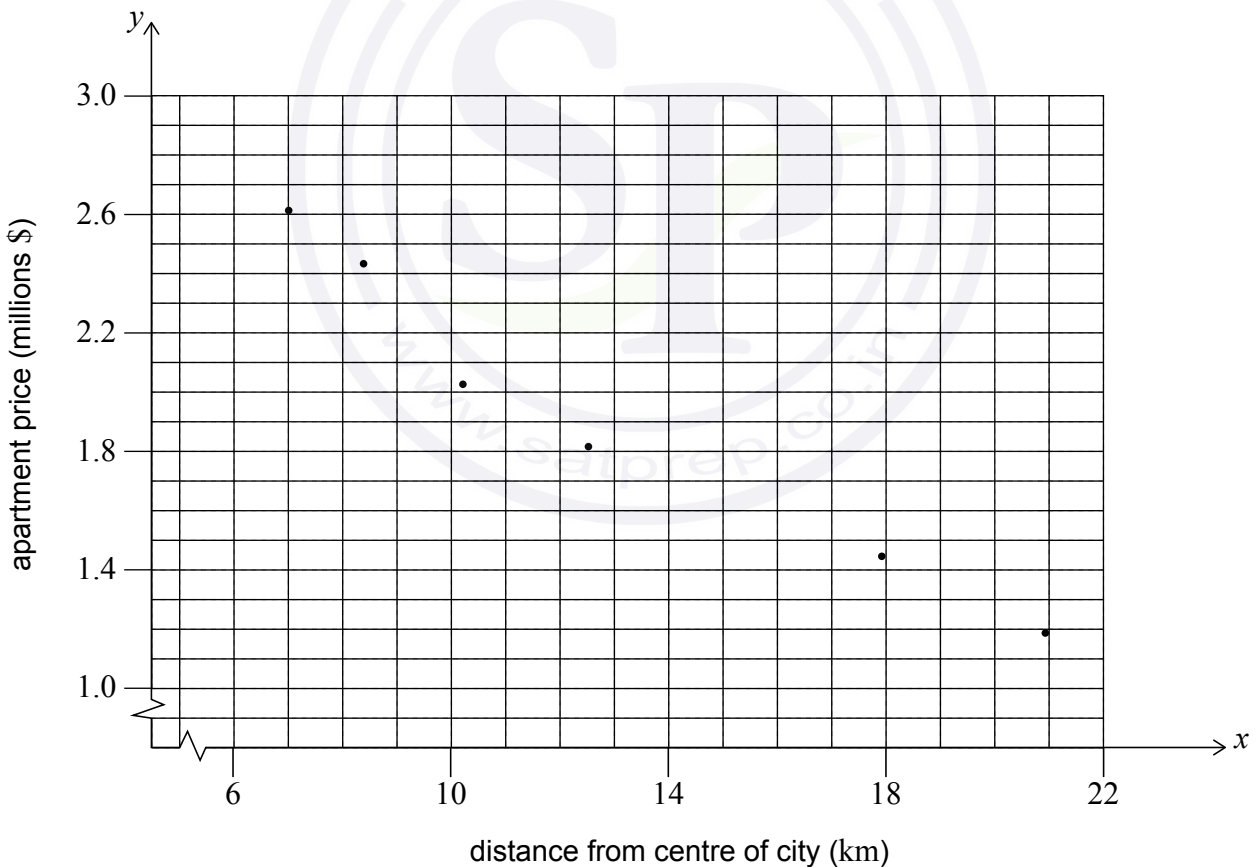
1. [Maximum mark: 19]

Xavie conducted a study to see if there is a relationship between the price of an apartment, y , and its distance, x , from the city centre of Melbourne.

They took a random sample of six typical apartments along a train line in the city. Xavie obtained the data shown in the following table.

x (kilometres)	7.0	8.4	10.3	12.5	17.8	20.9
y (millions of dollars)	2.61	2.44	2.03	1.81	1.45	1.18

A plot of these data is seen in the following graph.



(This question continues on the following page)

(Question 1 continued)

- (a) Write down the value of the Spearman’s rank correlation coefficient, r_s . [1]
- (b) (i) Find the Pearson’s product-moment correlation coefficient, r .
- (ii) Use your value of r to state which **two** of the following would best describe the correlation between the variables.

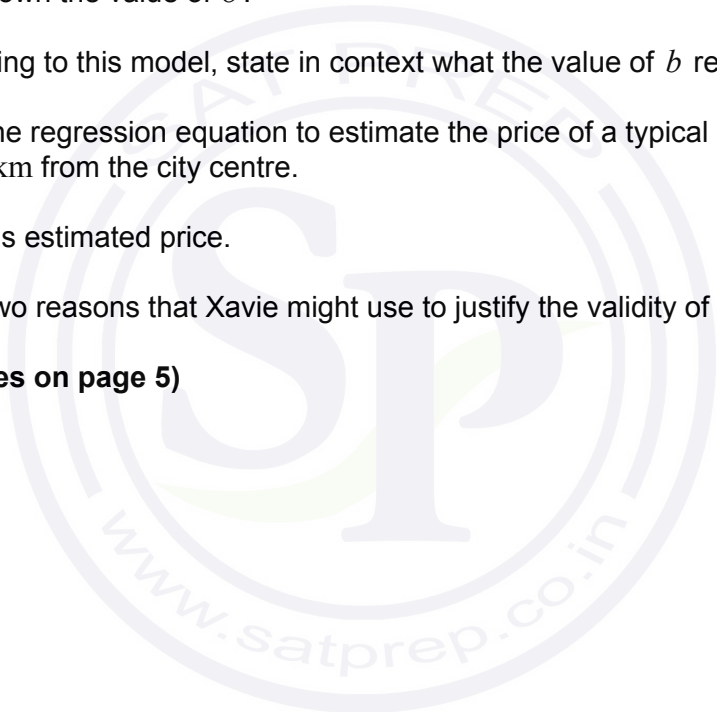
Positive	Negative	Strong	Weak	No correlation
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 [4]

The relationship between the variables can be modelled by the regression equation $y = ax + b$.

- (c) (i) Write down the value of a .
 - (ii) Write down the value of b .
 - (iii) According to this model, state in context what the value of b represents. [3]
- (d) Xavie uses the regression equation to estimate the price of a typical apartment located 19.6 km from the city centre.
- (i) Find this estimated price.
 - (ii) State two reasons that Xavie might use to justify the validity of this estimate. [5]

(This question continues on page 5)





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(Question 1 continued)

To verify whether this relationship applies in a different direction from the city centre, Xavie considers two locations, A and B, both an equal distance from the city centre. They take a random sample of seven apartments from each location and record the prices (in millions of dollars) in the following tables.

Apartment price in location A						
1.21	1.25	1.31	1.32	1.58	1.95	2.13

Apartment price in location B						
1.51	1.58	1.69	2.61	2.72	2.81	2.95

Xavie conducts a *t*-test, at the 5 % level of significance, to see if the mean apartment price in location A is different to the mean apartment price in location B. They assume the population variances are the same.

For this test, Xavie takes the null hypothesis to be $\mu_A = \mu_B$.

- (e) Write down the alternative hypothesis. [1]
- (f) Find the *p*-value for this test. [2]
- (g) State the conclusion of the test. Justify your answer. [2]
- (h) State one additional assumption Xavie has made about the distributions to conduct this test. [1]

2. [Maximum mark: 15]

The company Fred Express delivers packages. From past experience, the time taken, T , to deliver a package follows a normal distribution with mean 64 hours and standard deviation 12 hours.

(a) State $P(T < 64)$. [1]

(b) Find $P(44 < T < 64)$. [2]

30% of packages are delivered in less than k hours.

(c) (i) Sketch a diagram of this normal distribution, shading the region that represents $P(T < k)$.

(ii) Find the value of k . [4]

For quality control, the manager randomly selects five outgoing packages. These selections are independent.

(d) Find the probability that exactly two of these packages are delivered in less than k hours. [3]

Fred Express charges a fixed amount of \$4.50 for any package weighing 1 kg or less. Heavier packages are charged an additional fee of \$2.00 per kg. This fee is applied for any weight **in excess of** 1 kg. For example, a 1.5 kg package is charged an additional \$1.00.

(e) Write down an expression for the amount charged to deliver a package of weight x kg, where $x > 1$. [2]

(f) Find the amount Fred Express charges for a 5.3 kg package. [1]

Meiling is charged \$7.20 for the delivery of a package.

(g) Find the weight of Meiling's package. [2]

3. [Maximum mark: 13]

A shop uses the following model to estimate n , the number of smoothies sold per day, in terms of x , the price of a single smoothie in pesos.

$$n = \frac{40\,000}{x^2}$$

The maximum number of smoothies the shop can make in a day is 400.

- (a) Find the maximum price they could charge per smoothie for the shop to sell 400 in one day. [2]
- (b) On a day when the shop sells smoothies at 50 pesos each, use the model to find
 - (i) the number of smoothies sold.
 - (ii) the total income from the smoothies sold. [2]

The cost of making each smoothie is 20 pesos. The profit per day (P) is the total income from the sale of smoothies that day minus the cost of making them.

- (c) (i) Show that, according to the model, $P = \frac{40\,000}{x} - \frac{800\,000}{x^2}$.
- (ii) Find $\frac{dP}{dx}$.
- (iii) Find the value of x for which $\frac{dP}{dx} = 0$.
- (iv) Find the number of smoothies sold when the profit is maximized. [9]

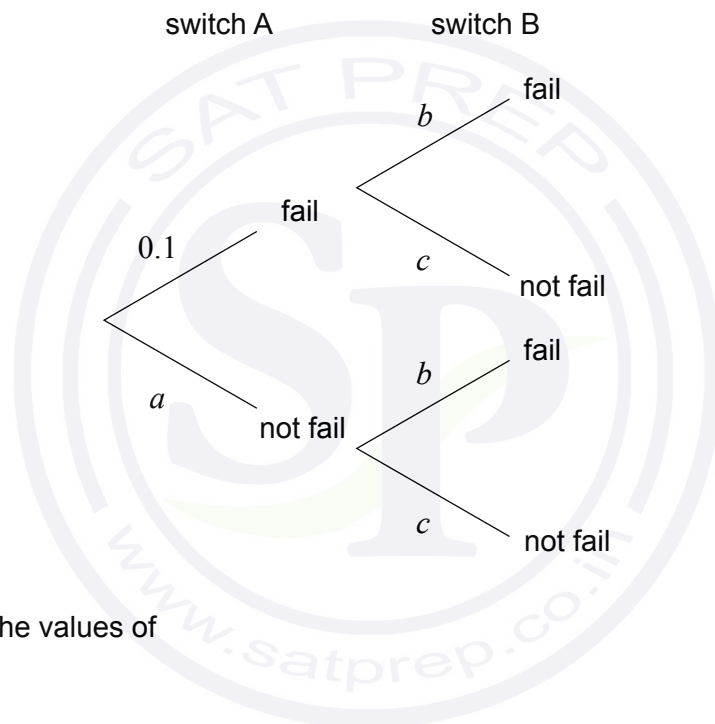
4. [Maximum mark: 12]

A type of generator will only function if a particular switch is working. The generator has a main switch, A, and a 'back up' switch, B.

The manufacturer claims the probability of switch A failing within one month of being fitted is 0.1 and the probability of the cheaper switch B failing within one month is 0.3. Whether or not a switch fails is independent of the state of the other switch.

If both switches fail, the generator needs to shut down to replace the switches. Both switches are replaced after a month of use (whether they have failed or not) or whenever the generator needs to be shut down.

The following tree diagram shows the probabilities of a switch failing within one month of them both being replaced, assuming the manufacturer's claim is correct.



(a) Write down the values of

(i) a .

(ii) b .

(iii) c .

[2]

(b) Hence find the probability that the generator needs to shut down within one month of the switches being replaced.

[1]

(This question continues on the following page)

(Question 4 continued)

The owner of the generator is suspicious of the switch manufacturer’s claims, so they look back through the past 200 occasions when the switches were replaced. The records show whether no switches, one switch or two switches had failed.

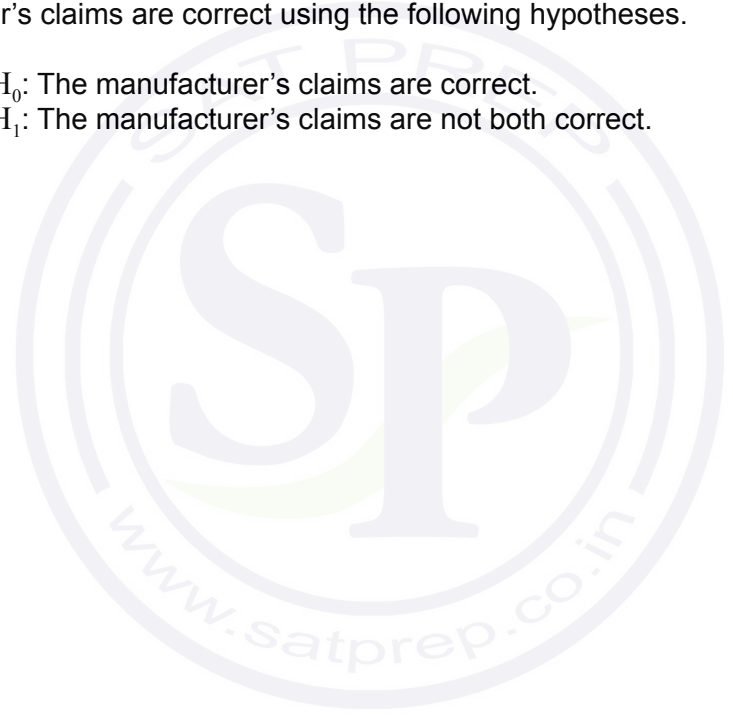
The data the owner collected are shown in the following table.

No switch fails	One switch fails	Two switches fail
118	72	10

(c) Show that the expected value of no switches failing in the generator, during the last 200 occasions when the switches were replaced, is 126. [2]

(d) Perform a χ^2 goodness of fit test at the 5% significance level to test whether the manufacturer’s claims are correct using the following hypotheses.

H_0 : The manufacturer’s claims are correct.
 H_1 : The manufacturer’s claims are not both correct. [7]



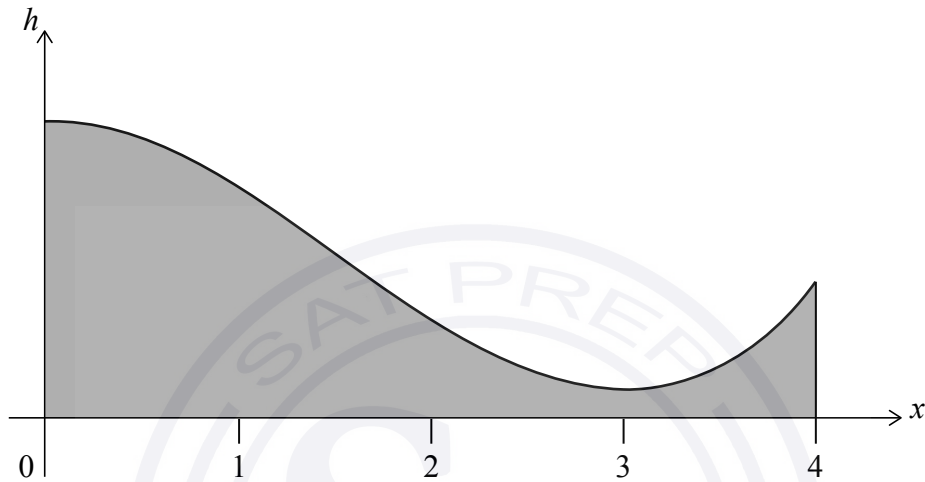
5. [Maximum mark: 21]

Andrew designs a fibreglass water slide for a water park.

Let x be the horizontal distance, in metres, from the start of the water slide.

Let h be the height, in metres, of the water slide above the horizontal ground.

The following diagram shows a cross section of the water slide. The water slide will be supported by a concrete structure, represented by the shaded region in the diagram.



It is known that $h'(x) = 0.6x^2 - 1.8x$.

(a) Find the gradient of the water slide when $x = 2$. [2]

At the exit point of the water slide, $x = 4$ and $h = 1.4$.

(b) Find an expression for $h(x)$. [4]

(c) (i) Write down an integral that can be used to find the area of the shaded region.

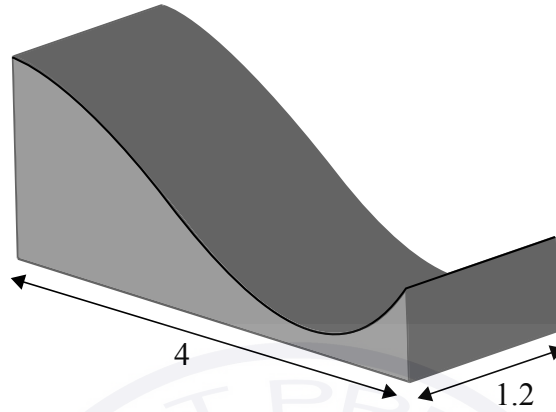
(ii) Find the cross-sectional area of the concrete structure. [4]

(This question continues on the following page)

(Question 5 continued)

The water slide and concrete structure have a uniform cross section and a width of 1.2 metres, as shown in the following diagram.

diagram not to scale



- (d) Find the volume of the concrete structure. [1]

To comply with safety regulations, Andrew updates this design. The start of the design will stay the same. Then, over the domain $2 \leq x \leq 4$, the updated design must satisfy the following conditions.

$$h(x) = ax^2 + bx + c \quad h(4) = 1 \quad h'(2) = -1.2 \quad h'(3) = 0$$

- (e) Write down **three** equations, in terms of a , b and c , using the given conditions. [6]
- (f) Hence find the values of a , b and c . [2]

Andrew claims the updated design will still provide a smooth path for a person using the slide.

- (g) Explain why Andrew's claim is correct at the point where $x = 2$. [2]
-

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Mathematics: applications and interpretation

Standard level

Paper 2

2 May 2024

Zone A morning | Zone B morning | Zone C morning

1 hour 30 minutes

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all the questions in the answer booklet provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation SL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

Answer **all** questions in the answer booklet provided. Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 16]

In a given week, the number of students in a particular primary school that were absent due to headlice (H), influenza (I) and/or chickenpox (C) were recorded as follows.

The primary school has 500 students.

- 35 students had headlice only
- 20 students had influenza only
- 5 students had chickenpox only
- 4 students had headlice and influenza but not chickenpox
- 2 students had headlice and chickenpox but not influenza
- 3 students had influenza and chickenpox but not headlice
- 1 student had headlice, influenza and chickenpox

- (a) Draw a Venn diagram to represent this information. [3]
- (b) Calculate the number of students who did not have headlice or influenza or chickenpox. [2]

A student is chosen at random from all the students in the school.

- (c) Find the probability that this student has
 - (i) headlice.
 - (ii) influenza given that the student has headlice. [4]

(This question continues on the following page)

(Question 1 continued)

Diego is a teacher in the school. He believes that the number of students, n , who have had influenza during the first t days of the school year, can be modelled by the function

$$n(t) = 250 - 240(2)^{kt}, k \in \mathbb{R}.$$

- (d) Use Diego’s model to calculate the number of students who started the school year with influenza. [2]

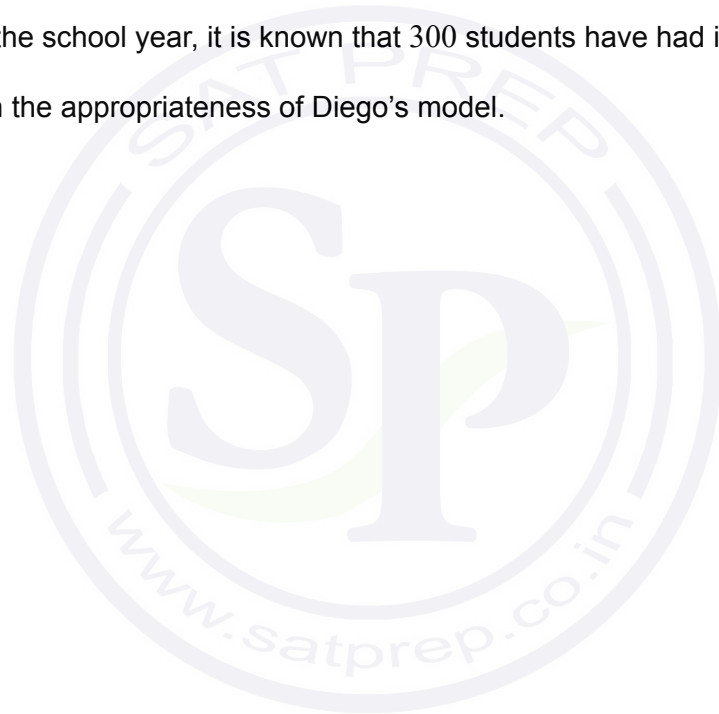
It is known that 130 students have had influenza during the first 10 days of the school year.

- (e) Find the value of k . [2]

- (f) Using this model, calculate how many days it will take for 200 students to have had influenza since the start of the school year. [2]

By the last day of the school year, it is known that 300 students have had influenza.

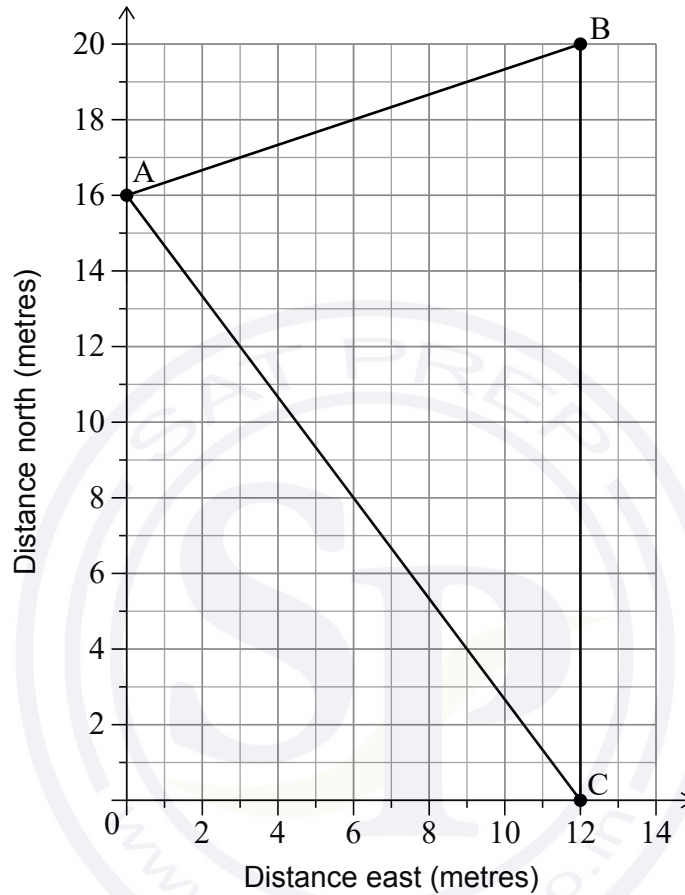
- (g) Comment on the appropriateness of Diego’s model. [1]



2. [Maximum mark: 14]

Mai is at an amusement park. A map of part of the amusement park is represented on the following coordinate axes.

Mai's favourite three attractions are positioned at $A(0, 16)$, $B(12, 20)$ and $C(12, 0)$. All measurements are in metres.



- (a) Write down the distance between B and C. [1]
- (b) Calculate the distance between A and B. [2]

(This question continues on the following page)

(Question 2 continued)

Mai is standing at the attraction at B and wants to walk directly to the attraction at A.

- (c) Calculate the bearing of A from B. [3]

A drinking fountain is to be installed at a point that is an equal distance from each of the attractions at A, B and C.

- (d) (i) Write down the gradient of [AC].
(ii) Write down the mid-point of [AC].
(iii) Hence calculate the coordinates of the drinking fountain. [8]

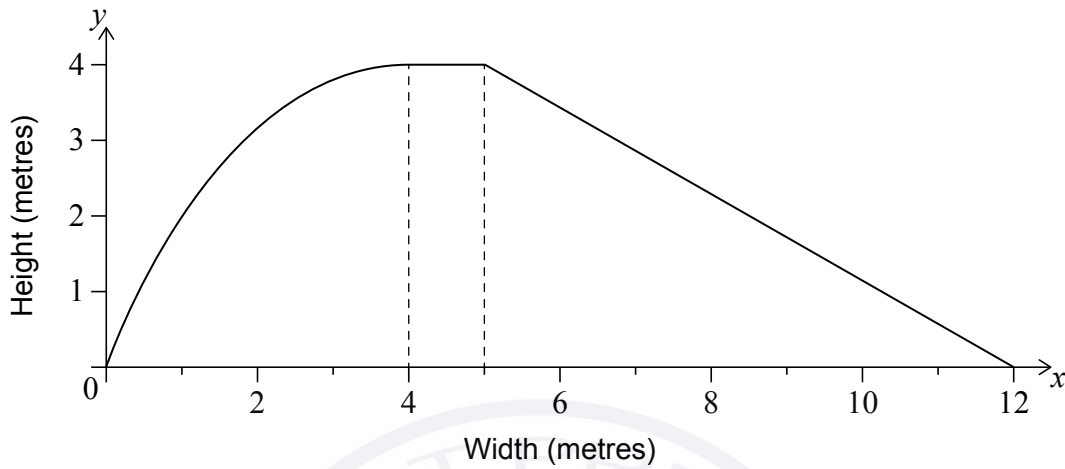




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3. [Maximum mark: 15]

The following diagram shows a model of the side view of a water slide. All lengths are measured in metres.



The curved edge of the slide is modelled by

$$f(x) = -\frac{1}{4}x^2 + 2x \text{ for } 0 \leq x \leq 4.$$

The remainder of the slide is modelled by

$$g(x) = \begin{cases} 4, & \text{for } 4 \leq x \leq 5 \\ \frac{48}{7} - \frac{4x}{7}, & \text{for } 5 \leq x \leq 12 \end{cases}$$

- (a) Use the trapezoidal rule with an interval width of 1 to calculate the approximate area under the model of the slide in the interval $0 \leq x \leq 4$. [5]
- (b) Find $\int \left(-\frac{1}{4}x^2 + 2x \right) dx$. [3]
- (c) Calculate the exact area under the entire model of the slide, for $0 \leq x \leq 12$. [4]
- (d) Find the percentage error in the **total** area under the entire model of the slide when using the approximate value from part (a). [3]

4. [Maximum mark: 19]

A recent study found that the heights of Dutch women can be modelled by a normal distribution with mean 170.7 cm and standard deviation 6.3 cm.

A Dutch woman is chosen at random.

(a) Calculate the probability that her height is

(i) less than 160 cm.

(ii) between 160 cm and 170 cm.

[4]

27% of Dutch women have a height of more than h metres.

(b) Calculate the value of h .

[2]

Janneke selects a random sample of 200 Dutch women from Amsterdam and measures their heights. She wants to determine whether this sample could have been chosen from a normally distributed population with mean of 170.7 cm and standard deviation of 6.3 cm.

She performs a χ^2 goodness of fit test at the 5% significance level. She begins by creating the following frequency table.

Height, h	Observed frequency	Expected frequency
$h \leq 160$	6	8.943
$160 < h \leq 170$	85	a
$170 < h \leq 180$	92	b
$h > 180$	17	13.99

(c) Calculate, correct to four significant figures, the value of

(i) a .

(ii) b .

[2]

(This question continues on the following page)

(Question 4 continued)

The hypotheses for Janneke’s test are

H_0 : the heights are drawn from a normally distributed population with mean 170.7 cm and standard deviation 6.3 cm

H_1 : the heights are not drawn from a normally distributed population with mean 170.7 cm and standard deviation 6.3 cm

- (d) Write down the degrees of freedom for this test. [1]

The critical value for this test is 7.815.

- (e) Perform the χ^2 goodness of fit test and state your conclusion, justifying your reasoning. [4]

Gundega claims that, on average, Latvian women are taller than Dutch women.

Random samples of 10 Latvian women and 10 Dutch women are chosen, and their heights are measured.

Heights of Latvian women (cm)									
171	163	180	159	169	182	166	168	171	170

Heights of Dutch women (cm)									
173	182	181	166	175	161	169	165	172	169

Gundega performs a t -test at the 5% significance level. It is assumed that the populations are normally distributed and have equal variances.

- (f) Write down the null and alternative hypotheses for this test. [2]
- (g) Perform the t -test and state the conclusion, justifying your reasoning. [4]

5. [Maximum mark: 16]

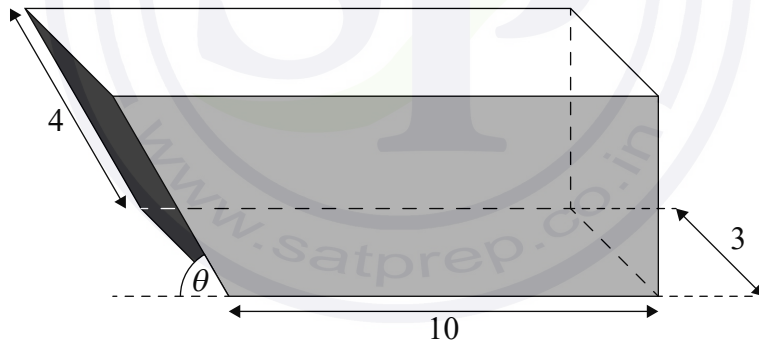
A skip is a container used to carry garbage away from a construction site. For safety reasons the garbage must not extend beyond the top of the skip. The maximum volume of garbage to be removed is therefore equal to the volume of the skip.



A particular design of skip can be modelled as a prism with a trapezoidal cross section. For the skip to be transported, it must have a rectangular base of length 10 m and width 3 m. The length of the sloping edge is fixed at 4 m, and makes an angle of θ with the horizontal.

The following diagram shows such a skip.

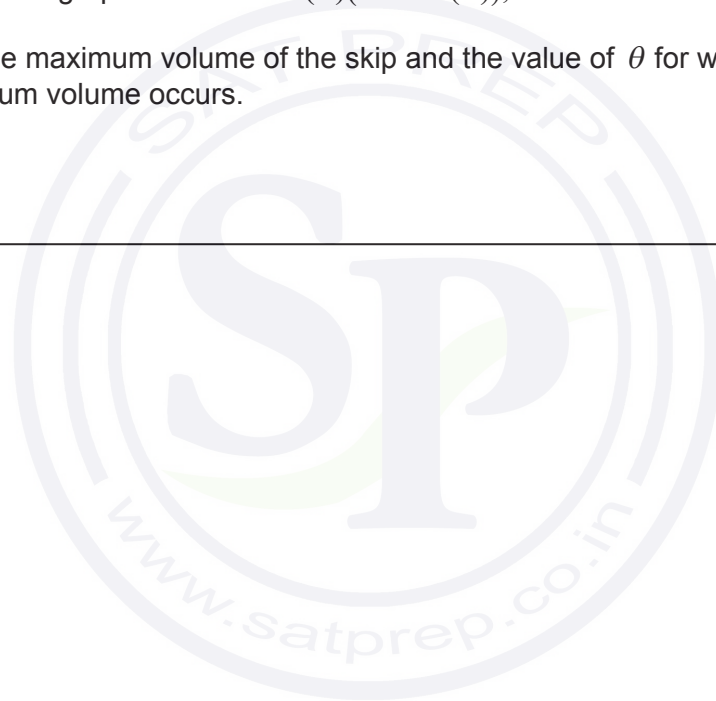
diagram not to scale



(This question continues on the following page)

(Question 5 continued)

- (a) Find the volume of this skip,
- (i) if the length of the top edge of the skip is 11 m.
 - (ii) if the height of the skip is 3.2 m.
 - (iii) if θ is 60° . [9]
- (b) Show that the volume, $V \text{ m}^3$, of the skip is given by
- $$24 \sin(\theta)(5 + \cos(\theta)).$$
- [2]
- (c) Explain, in context, why $\theta \neq 0$. [1]
- (d) (i) Sketch the graph of $V = 24 \sin(\theta)(5 + \cos(\theta))$, $0 < \theta < 90^\circ$.
- (ii) Find the maximum volume of the skip and the value of θ for which this maximum volume occurs. [4]



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References:

5. Andyqwe, n.d. *Dumpster truck* [image online] Available at: <https://www.gettyimages.co.uk/detail/photo/dumpster-truck-royalty-free-image/157611454> [Accessed 18 April 2023] Source adapted.

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Mathematics: applications and interpretation

Standard level

Paper 2

31 October 2023

Zone A afternoon | Zone B afternoon | Zone C afternoon

1 hour 30 minutes

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all the questions in the answer booklet provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

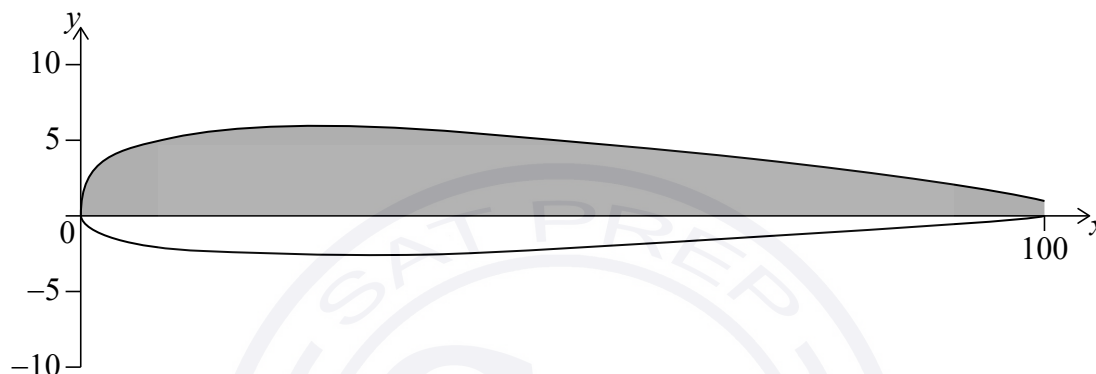


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Answer **all** questions in the answer booklet provided. Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 12]

Gabriel is investigating the shape of model airplane wings. A cross-section of one of the wings is shown, graphed on the coordinate axes.



The shaded part of the cross-section is the area between the x -axis and the curve with equation

$$y = 2\sqrt{x} - \frac{x}{5} + 1, \text{ for } 0 \leq x \leq 100$$

where x is the distance, in cm, from the front of the wing and y is the height, in cm, above the horizontal axis through the wing, as shown in the diagram.

- (a) Find the values of a , b and c , shown in the table. [3]

x (cm)	0	25	50	75	100
y (cm)	1	a	b	c	1

Gabriel uses the trapezoidal rule with four intervals to estimate the shaded area of the cross-section of the wing.

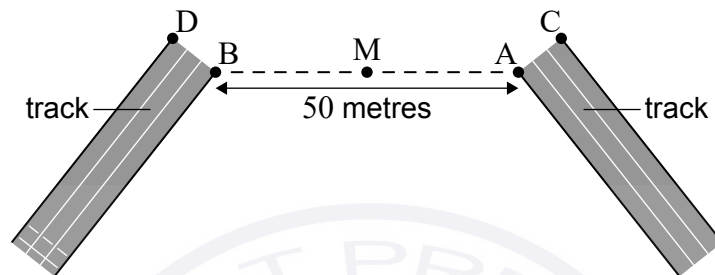
- (b) Find Gabriel's estimate of the shaded area of the cross-section. [3]
- (c) (i) Write down the integral that Gabriel can use to find the exact area of the shaded part of the cross-section.
- (ii) Hence, use your graphic display calculator to find the area of the shaded part of the cross-section. Give your answer correct to one decimal place. [4]
- (d) Calculate the percentage error of Gabriel's estimate in part (b). [2]

2. [Maximum mark: 15]

Madhu is designing a jogging track for the campus of her school. The following diagram shows an incomplete portion of the track.

Madhu wants to design the track such that the inner edge is a smooth curve from point A to point B, and the other edge is a smooth curve from point C to point D. The distance between points A and B is 50 metres.

diagram not to scale



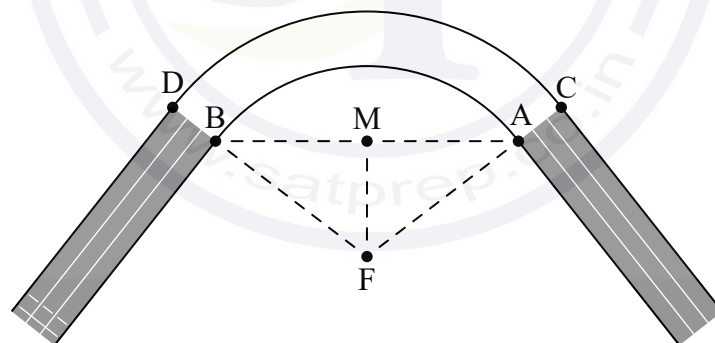
To create a smooth curve, Madhu first walks to M, the midpoint of [AB].

(a) Write down the length of [BM].

[1]

Madhu then walks 20 metres in a direction perpendicular to [AB] to get from point M to point F. Point F is the centre of a circle whose arc will form the smooth curve between points A and B on the track, as shown in the following diagram.

diagram not to scale



(b) (i) Find the length of [BF].

(ii) Find \hat{BFM} .

[4]

(c) Hence, find the length of arc AB.

[3]

(This question continues on the following page)

(Question 2 continued)

The outer edge of the track, from C to D, is also a circular arc with centre F, such that the track is 2 metres wide.

- (d) Calculate the area of the curved portion of the track, ABDC. [4]

The base of the track will be made of concrete that is 12 cm deep.

- (e) Calculate the volume of concrete needed to create the curved portion of the track. [3]



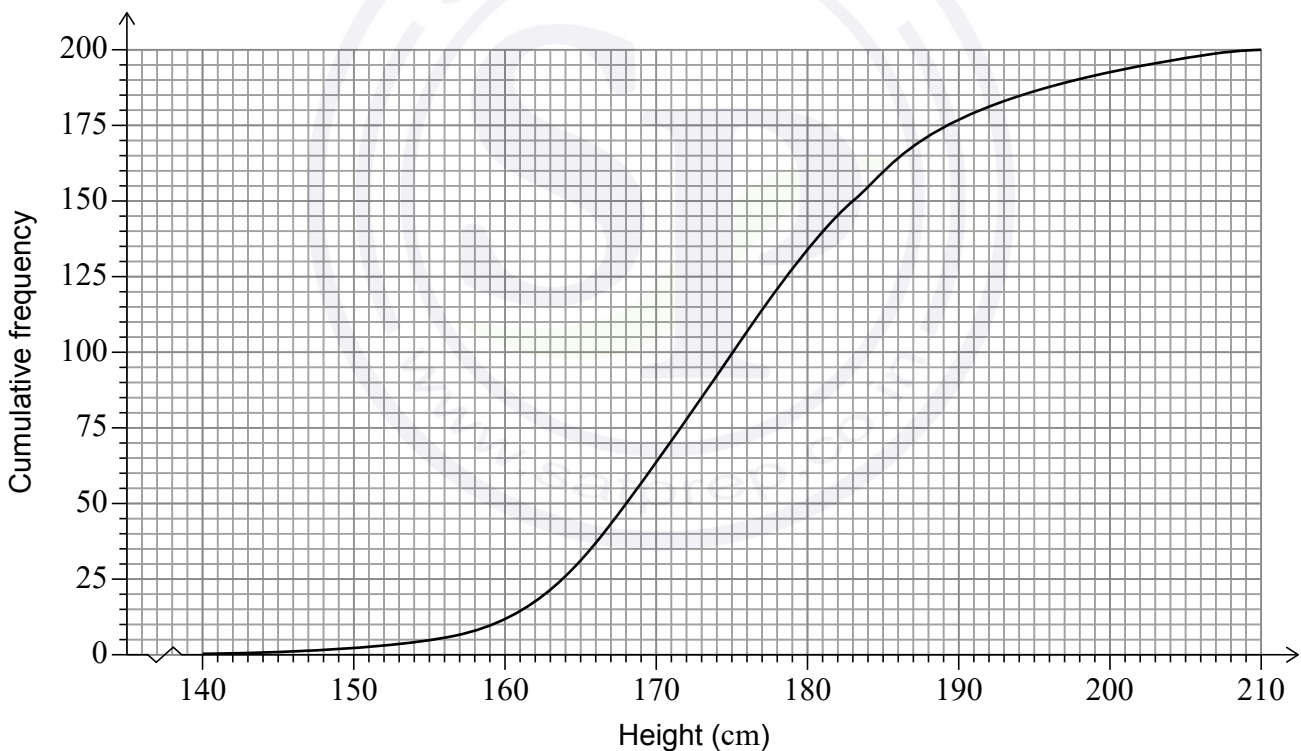
3. [Maximum mark: 18]

The heights, h , of 200 university students are recorded in the following table.

Height (cm)	Frequency
$140 \leq h < 160$	11
$160 \leq h < 170$	51
$170 \leq h < 180$	68
$180 \leq h < 190$	47
$190 \leq h < 210$	23

- (a) (i) Write down the mid-interval value of $140 \leq h < 160$.
- (ii) Calculate an estimate of the mean height of the 200 students. [3]

This table is used to create the following cumulative frequency graph.



- (b) Use the cumulative frequency curve to estimate the interquartile range. [2]

Laszlo is a student in the data set and his height is 204 cm.

- (c) Use your answer to part (b) to estimate whether Laszlo's height is an outlier for this data. Justify your answer. [3]

(This question continues on the following page)

(Question 3 continued)

It is believed that the heights of university students follow a normal distribution with mean 176 cm and standard deviation 13.5 cm.

It is decided to perform a χ^2 goodness of fit test on the data to determine whether this sample of 200 students could have plausibly been drawn from an underlying distribution $N(176, 13.5^2)$.

(d) Write down the null and the alternative hypotheses for the test. [2]

As part of the test, the following table is created.

Height of student (cm)	Observed frequency	Expected frequency
$h < 160$	11	23.6
$160 \leq h < 170$	51	42.1
$170 \leq h < 180$	68	a
$180 \leq h < 190$	47	46.7
$190 \leq h$	23	b

(e) (i) Find the value of a and the value of b .
(ii) Hence, perform the test to a 5% significance level, clearly stating the conclusion in context. [8]

4. [Maximum mark: 16]

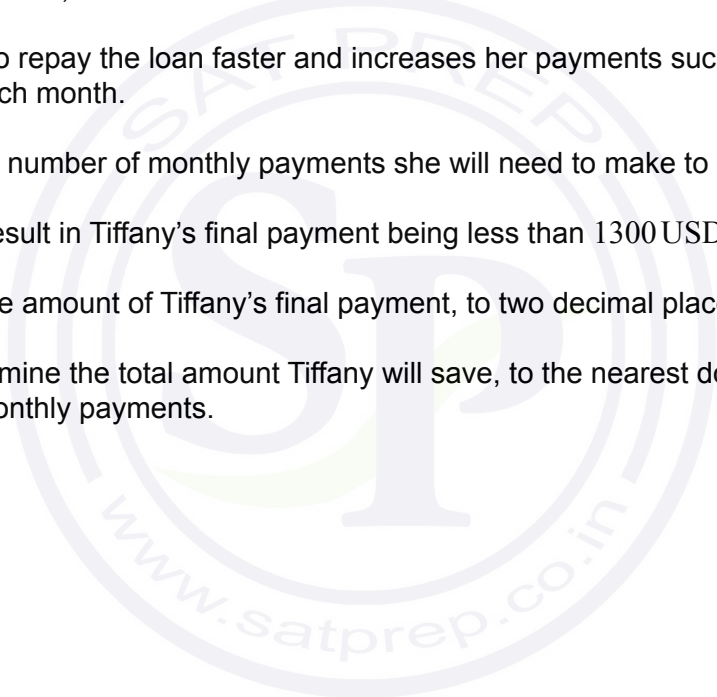
Tiffany wants to buy a house for a price of 285 000 US Dollars (USD). She goes to a bank to get a loan to buy the house. To be eligible for the loan, Tiffany must make an initial down payment equal to 15 % of the price of the house.

The bank offers her a 30-year loan for the remaining balance, with a 4 % nominal interest rate per annum, compounded monthly. Tiffany will pay the loan in fixed payments at the end of each month.

- (a) (i) Find the original amount of the loan after the down payment is paid.
Give the exact answer.
- (ii) Calculate Tiffany's monthly payment for this loan, to two decimal places. [5]
- (b) Using your answer from part (a)(ii), calculate the total amount Tiffany will pay over the life of the loan, to the nearest dollar. Do **not** include the initial down payment. [2]

Tiffany would like to repay the loan faster and increases her payments such that she pays 1300 USD each month.

- (c) Find the total number of monthly payments she will need to make to pay off the loan. [2]
- This strategy will result in Tiffany's final payment being less than 1300 USD.
- (d) Determine the amount of Tiffany's final payment, to two decimal places. [4]
 - (e) Hence, determine the total amount Tiffany will save, to the nearest dollar, by making the higher monthly payments. [3]



5. [Maximum mark: 19]

Howell Industries is an international company that sells travel coffee mugs. Their market research predicts that the average number of mugs they will sell each month is modelled by the equation

$$n = 20\,000 - 1000x$$

where x represents the selling price, in euro (EUR), of each mug.

A salesperson suggests that Howell Industries should sell the mugs for 16 EUR each.

- (a) Find the average number of mugs that this model predicts Howell Industries will sell at this price. [2]
- (b) Calculate Howell Industries' average monthly income, before any expenses, at this selling price. [2]
- (c) Hence, write down the function $R(x)$ that can be used to predict Howell Industries' average monthly income, before expenses, at any selling price, x . [1]

Howell Industries has 10 000 EUR of fixed monthly operational costs. Additionally, Howell Industries must pay their mug supplier 10 EUR for each coffee mug.

- (d) Calculate Howell Industries' average monthly profit if they sell each mug at a price of 16 EUR. [3]
- (e) Show that the average monthly profit for any selling price, x , can be found using the function $P(x) = -1000x^2 + 30\,000x - 210\,000$. [2]
- (f) (i) Find $P'(x)$.
(ii) Show that the salesperson's selling price does not maximize their average monthly profit. [4]

Howell Industries negotiates a new deal with their mug supplier. Under the new deal, the supplier agrees to discount the cost of each mug based on the number of mugs purchased by Howell Industries. The cost charged by the supplier for each mug can be found using the function

$$C(n) = 10 - 0.0001n$$

where n represents the number of mugs sold by Howell Industries.

- (g) Find the function that can be used to find Howell Industries' average monthly profit using the new deal from the supplier. [3]
- (h) Hence, find the selling price, per mug, that Howell Industries should choose in order to maximize their average monthly profit under the new deal. [2]

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Mathematics: applications and interpretation

Standard level

Paper 2

31 October 2023

Zone A afternoon | Zone B afternoon | Zone C afternoon

1 hour 30 minutes

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all the questions in the answer booklet provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

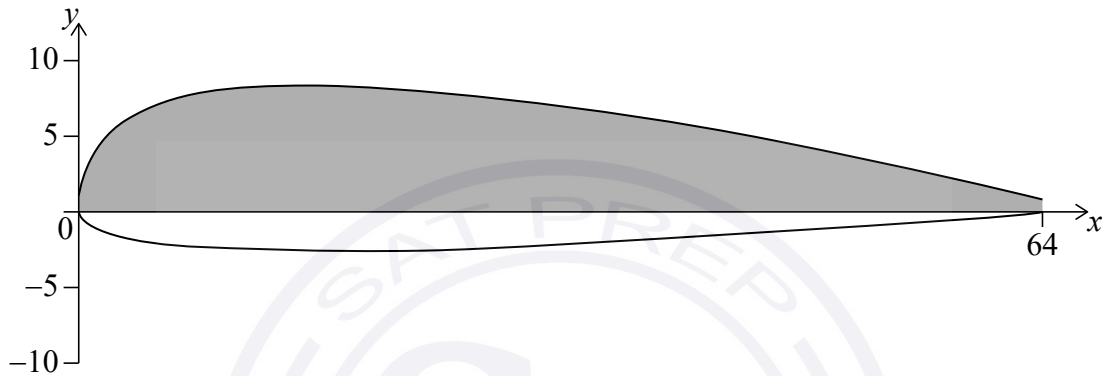


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Answer **all** questions in the answer booklet provided. Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 12]

Jan is investigating the shape of model helicopter propeller blades. A cross-section of one of the blades is shown, graphed on the coordinate axes.



The shaded part of the cross-section is the area between the x -axis and the curve with equation

$$y = 4\sqrt{x} - \frac{x}{2} + 1, \text{ for } 0 \leq x \leq 64$$

where x is the distance, in mm, from the edge of the blade and y is the height, in mm, above the horizontal axis through the blade, as shown in the diagram.

- (a) Find the values of a , b and c , shown in the table. [3]

x (mm)	0	16	32	48	64
y (mm)	1	a	b	c	1

Jan uses the trapezoidal rule with four intervals to estimate the shaded area of the cross-section of the blade.

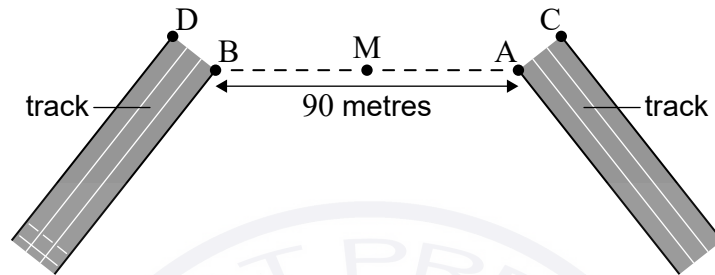
- (b) Find Jan's estimate of the shaded area of the cross-section. [3]
- (c) (i) Write down the integral that Jan can use to find the exact area of the shaded part of the cross-section.
- (ii) Hence, use your graphic display calculator to find the area of the shaded part of the cross-section. Give your answer correct to one decimal place. [4]
- (d) Calculate the percentage error of Jan's estimate in part (b). [2]

2. [Maximum mark: 15]

Ansel is designing a racing track for a local bicycle club. The following diagram shows an incomplete portion of the track.

Ansel wants to design the track such that the inner edge is a smooth curve from point A to point B, and the other edge is a smooth curve from point C to point D. The distance between points A and B is 90 metres.

diagram not to scale

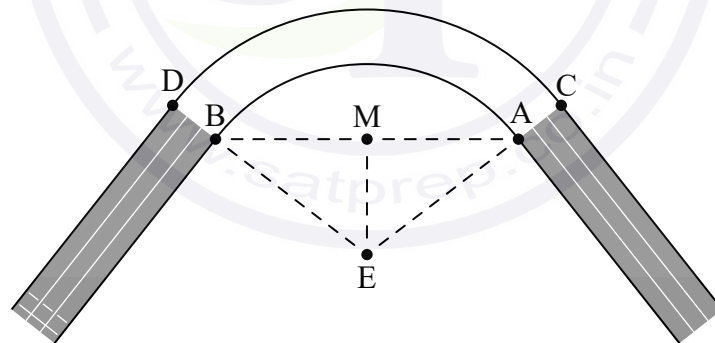


To create a smooth curve, Ansel first walks to M, the midpoint of [AB].

- (a) Write down the length of [BM]. [1]

Ansel then walks 32 metres in a direction perpendicular to [AB] to get from point M to point E. Point E is the centre of a circle whose arc will form the smooth curve between points A and B on the track, as shown in the following diagram.

diagram not to scale



- (b) (i) Find the length of [BE].
(ii) Find \hat{BEM} . [4]
- (c) Hence, find the length of arc AB. [3]

(This question continues on the following page)

(Question 2 continued)

The outer edge of the track, from C to D, is also a circular arc with centre E, such that the track is 4 metres wide.

- (d) Calculate the area of the curved portion of the track, ABDC. [4]

The base of the track will be made of concrete that is 15 cm deep.

- (e) Calculate the volume of concrete needed to create the curved portion of the track. [3]



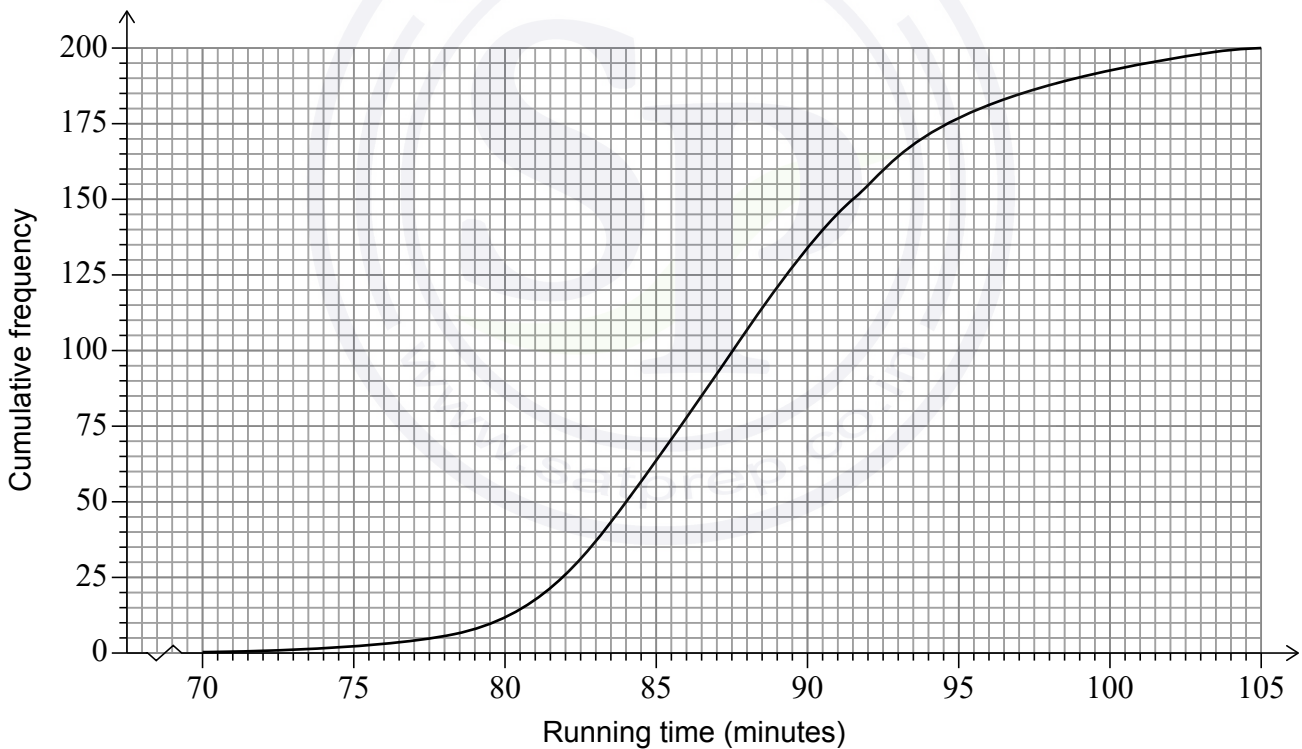
3. [Maximum mark: 18]

The running time, t (minutes), of 200 family movies are recorded in the following table.

Running time (minutes)	Frequency
$70 \leq t < 80$	11
$80 \leq t < 85$	51
$85 \leq t < 90$	68
$90 \leq t < 95$	47
$95 \leq t < 105$	23

- (a) (i) Write down the mid-interval value of $70 \leq t < 80$.
- (ii) Calculate an estimate of the mean running time of the 200 movies. [3]

This table is used to create the following cumulative frequency graph.



- (b) Use the cumulative frequency curve to estimate the interquartile range. [2]

“Star Feud” is a movie in the data set and its running time is 100 minutes.

- (c) Use your answer to part (b) to estimate whether “Star Feud’s” running time is an outlier for this data. Justify your answer. [3]

(This question continues on the following page)

(Question 3 continued)

It is believed that the running times of family movies follow a normal distribution with mean 88 minutes and standard deviation 6.75 minutes.

It is decided to perform a χ^2 goodness of fit test on the data to determine whether this sample of 200 movies could have plausibly been drawn from an underlying distribution $N(88, 6.75^2)$.

- (d) Write down the null and the alternative hypotheses for the test. [2]

As part of the test, the following table is created.

Movie running time (minutes)	Observed frequency	Expected frequency
$t < 80$	11	23.6
$80 \leq t < 85$	51	42.1
$85 \leq t < 90$	68	a
$90 \leq t < 95$	47	46.7
$95 \leq t$	23	b

- (e) (i) Find the value of a and the value of b .
(ii) Hence, perform the test to a 5% significance level, clearly stating the conclusion in context. [8]

4. [Maximum mark: 16]

Ruben wants to buy a car for a price of 285 000 South African rand (ZAR). He goes to a bank to get a loan to buy the car. To be eligible for the loan, Ruben must make an initial down payment equal to 25% of the price of the car.

The bank offers him a 5-year loan for the remaining balance, with a 4.5% nominal interest rate per annum, compounded monthly. Ruben will pay the loan in fixed payments at the end of each month.

- (a) (i) Find the original amount of the loan after the down payment is paid.
Give the exact answer.
- (ii) Calculate Ruben's monthly payment for this loan, to two decimal places. [5]
- (b) Using your answer from part (a)(ii), calculate the total amount Ruben will pay over the life of the loan, to the nearest ZAR. Do **not** include the initial down payment. [2]

Ruben would like to repay the loan faster and increases his payments such that he pays 4600 ZAR each month.

- (c) Find the total number of monthly payments he will need to make to pay off the loan. [2]
- This strategy will result in Ruben's final payment being less than 4600 ZAR.
- (d) Determine the amount of Ruben's final payment, to two decimal places. [4]
 - (e) Hence, determine the total amount Ruben will save, to the nearest ZAR, by making the higher monthly payments. [3]

5. [Maximum mark: 19]

Mosaic Industries is an international company that sells cell phone cases. Their market research predicts that the average number of cases they will sell each month is modelled by the equation

$$n = 20\,000 - 1000x$$

where x represents the selling price, in euro (EUR), of each case.

A salesperson suggests that Mosaic Industries should sell the cases for 16 EUR each.

- (a) Find the average number of cases that this model predicts Mosaic Industries will sell at this price. [2]
- (b) Calculate Mosaic Industries' average monthly income, before any expenses, at this selling price. [2]
- (c) Hence, write down the function $R(x)$ that can be used to predict Mosaic Industries' average monthly income, before expenses, at any selling price, x . [1]

Mosaic Industries has 10 000 EUR of fixed monthly operational costs. Additionally, Mosaic Industries must pay their phone case supplier 10 EUR for each case.

- (d) Calculate Mosaic Industries' average monthly profit if they sell each case at a price of 16 EUR. [3]
- (e) Show that the average monthly profit for any selling price, x , can be found using the function $P(x) = -1000x^2 + 30\,000x - 210\,000$. [2]
- (f) (i) Find $P'(x)$.
(ii) Show that the salesperson's selling price does not maximize their average monthly profit. [4]

Mosaic Industries negotiates a new deal with their phone case supplier. Under the new deal, the supplier agrees to discount the cost of each case based on the number of cases purchased by Mosaic Industries. The cost charged by the supplier for each case can be found using the function

$$C(n) = 10 - 0.0001n$$

where n represents the number of cases sold by Mosaic Industries.

- (g) Find the function that can be used to find Mosaic Industries' average monthly profit using the new deal from the supplier. [3]
- (h) Hence, find the selling price, per case, that Mosaic Industries should choose in order to maximize their average monthly profit under the new deal. [2]

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Mathematics: applications and interpretation

Standard level

Paper 2

9 May 2023

Zone A afternoon | Zone B morning | Zone C afternoon

1 hour 30 minutes

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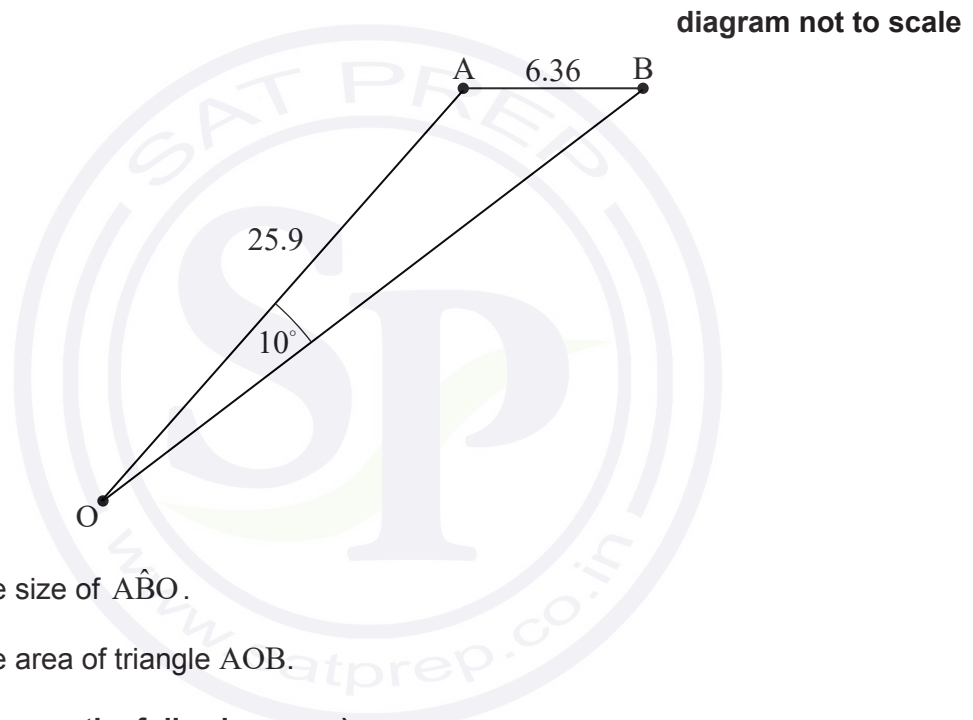
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1. [Maximum mark: 17]

The diagram shows points in a park viewed from above, at a specific moment in time.

The distance between two trees, at points A and B, is 6.36 m.

Odette is playing football in the park and is standing at point O, such that $\hat{A}OB = 10^\circ$, $OA = 25.9\text{ m}$ and $\hat{O}AB$ is obtuse.



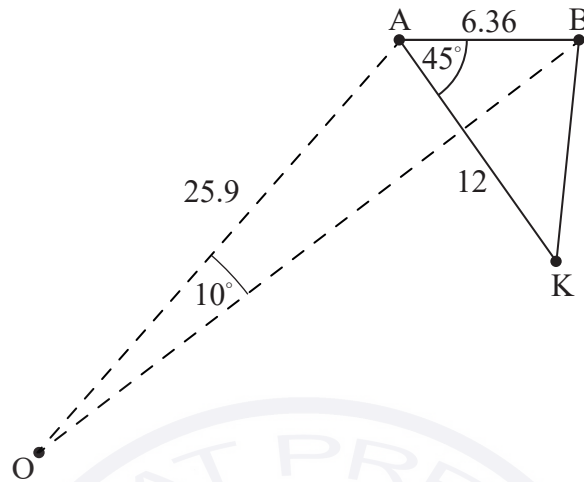
- (a) Calculate the size of $\hat{A}BO$. [3]
- (b) Calculate the area of triangle AOB. [4]

(This question continues on the following page)

(Question 1 continued)

Odette's friend, Khemil, is standing at point K such that he is 12 m from A and $\hat{KAB} = 45^\circ$.

diagram not to scale

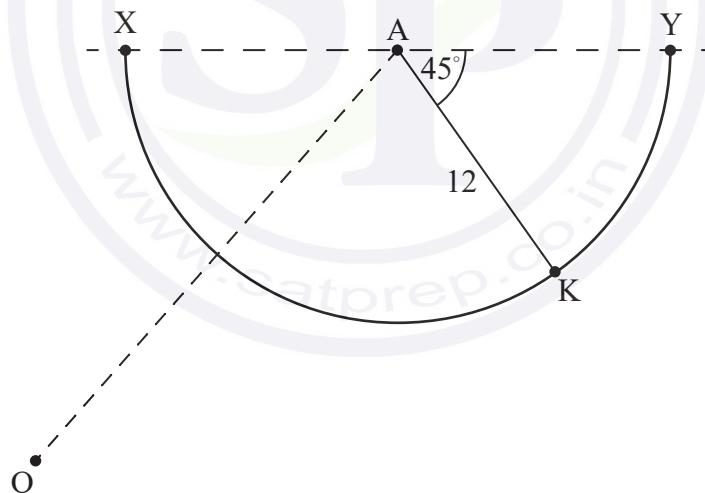


(c) Calculate Khemil's distance from B.

[3]

XY is a semicircular path in the park with centre A, such that $\hat{KAY} = 45^\circ$. Khemil is standing on the path and Odette's football is at point X. This is shown in the diagram below.

diagram not to scale



The length $KX = 22.2$ m, $\hat{KOX} = 53.8^\circ$ and $\hat{OKX} = 51.1^\circ$.

(d) Find whether Odette or Khemil is closer to the football.

[4]

Khemil runs along the semicircular path to pick up the football.

(e) Calculate the distance that Khemil runs.

[3]

2. [Maximum mark: 15]

Daina makes pendulums to sell at a market. She plans to make 10 pendulums on the first day and, on each subsequent day, make 6 more than she did the day before.

(a) Calculate the number of pendulums she would make on the 12th day. [3]

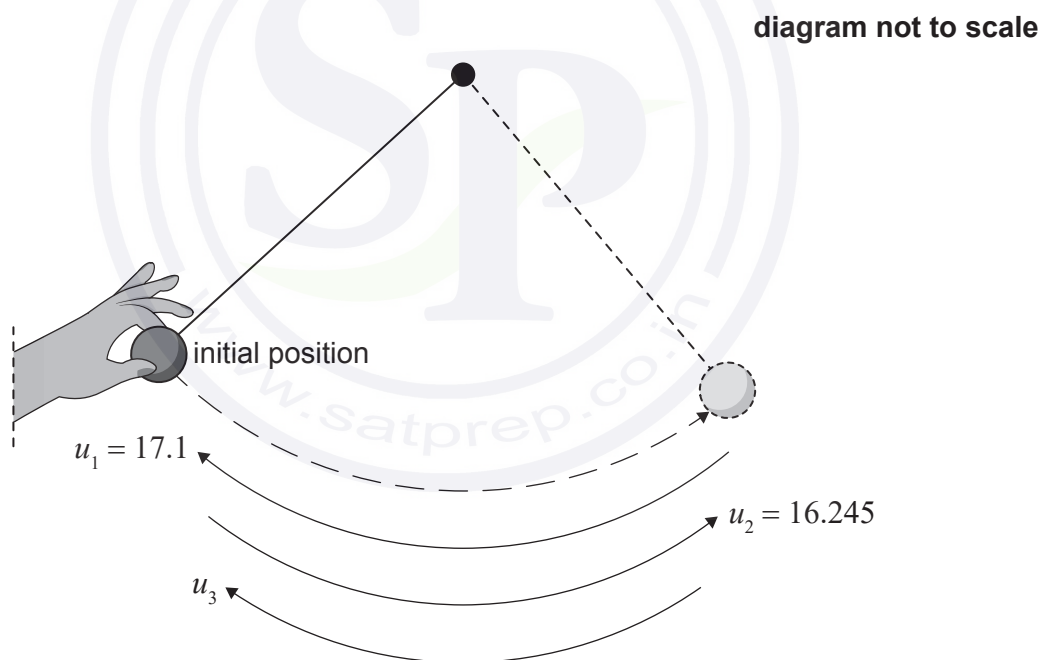
She plans to make pendulums for a **total** of 15 days in preparation for going to the market.

(b) Calculate the total number of pendulums she would have available at the market. [2]

Daina would like to have at least 1000 pendulums available to sell at the market and therefore decides to increase her production. She still plans to make 10 pendulums on the first day, but on each subsequent day, she will make x more than she did the day before.

(c) Given that she will still make pendulums for a total of 15 days, calculate the minimum integer value of x required for her to reach her target. [3]

Daina tests one of her pendulums. She releases the ball at the end of the pendulum to swing freely. The point at which she releases it is shown as the initial position on the left side of the following diagram. Daina begins recording the distances travelled by the ball **after** it has reached the extreme position, represented by the right-hand side of the diagram.



(This question continues on the following page)

(Question 2 continued)

On each successive swing, the distance that the ball travelled was 95% of its previous distance. During the first swing that Daina recorded, the ball travelled a distance of 17.1 cm. During the second swing that she recorded, it travelled a distance of 16.245 cm.

- (d) Calculate the distance that the ball travelled during the 5th recorded swing. [3]
- (e) Calculate the total distance that the ball travelled during the first 16 recorded swings. [2]
- (f) Calculate the distance that the ball travelled before Daina started recording. [2]



3. [Maximum mark: 15]

A scientist is conducting an experiment on the growth of a certain species of bacteria.

The population of the bacteria, P , can be modelled by the function

$$P(t) = 1200 \times k^t, \quad t \geq 0,$$

where t is the number of hours since the experiment began, and k is a positive constant.

(a) (i) Write down the value of $P(0)$.

(ii) Interpret what this value means in this context. [2]

3 hours after the experiment began, the population of the bacteria is 18 750.

(b) Find the value of k . [2]

(c) Find the population of the bacteria 1 hour and 30 minutes after the experiment began. [2]

The scientist conducts a second experiment with a different species of bacteria.

The population of this bacteria, S , can be modelled by the function

$$S(t) = 5000 \times 1.65^t, \quad t \geq 0,$$

where t is the number of hours since both experiments began.

(d) Find the value of t when the two populations of bacteria are equal. [2]

It takes 2 hours and m minutes for the number of bacteria in the second experiment to reach 19 000.

(e) Find the value of m , giving your answer as an integer value. [4]

The bacteria in the second experiment are growing inside a container. The scientist models the volume of each bacterium in the second experiment to be $1 \times 10^{-18} \text{ m}^3$, and the available volume inside the container is $2.1 \times 10^{-5} \text{ m}^3$.

(f) Determine how long it would take for the bacteria to fill the container. [3]

4. [Maximum mark: 17]

It is claimed that a new remedy cures 82% of the patients with a particular medical problem.

This remedy is to be used by 115 patients, and it is assumed that the 82% claim is true.

- (a) Find the probability that exactly 90 of these patients will be cured. [3]
- (b) Find the probability that at least 95 of these patients will be cured. [2]
- (c) Find the variance in the possible number of patients that will be cured. [2]

The probability that at least n patients will be cured is less than 30%.

- (d) Find the least value of n . [3]

A clinic is interested to see if the mean recovery time of their patients who tried the new remedy is less than that of their patients who continued with an older remedy. The clinic randomly selects some of their patients and records their recovery time in days. The results are shown in the table below.

	Recovery time (days)								
Group N (new remedy)	12	13	9	13	14	15	17		

	Recovery time (days)								
Group O (old remedy)	17	11	10	18	20	22	14	15	18

The data is assumed to follow a normal distribution and the population variance is the same for the two groups. A t -test is used to compare the means of the two groups at the 10% significance level.

- (e) State the appropriate null and alternative hypotheses for this t -test. [2]
- (f) Find the p -value for this test. [2]
- (g) State the conclusion for this test. Give a reason for your answer. [2]
- (h) Explain what the p -value represents. [1]

5. [Maximum mark: 16]

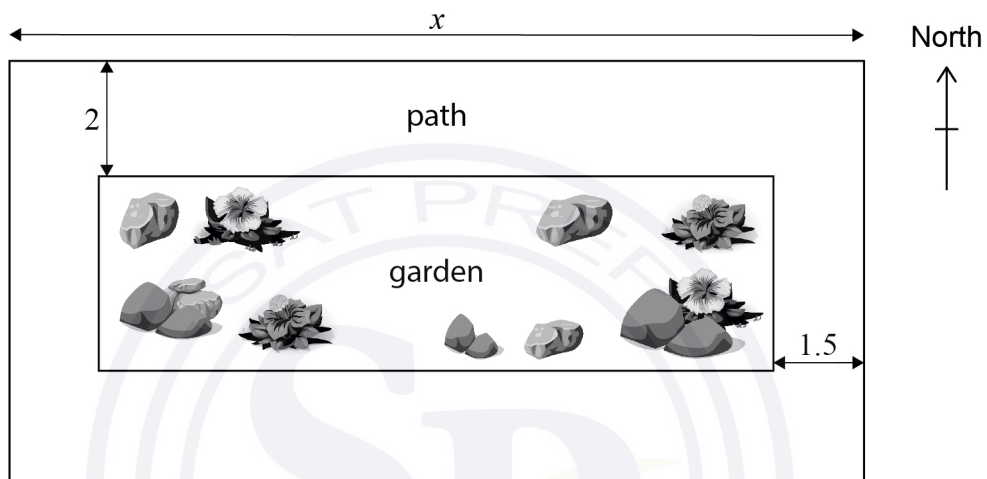
A particular park consists of a rectangular garden, of area $A \text{ m}^2$, and a concrete path surrounding it. The park has a total area of 1200 m^2 .

The width of the path at the north and south side of the park is 2 m.

The width of the path at the west and east side of the park is 1.5 m.

The length of the park (along the north and south sides) is x metres, $3 < x < 300$.

diagram not to scale



- (a) (i) Write down the length of the garden in terms of x .
- (ii) Find an expression for the width of the garden in terms of x .
- (iii) Hence show that $A = 1212 - 4x - \frac{3600}{x}$. [5]
- (b) Find the possible dimensions of the park if the area of the garden is 800 m^2 . [4]
- (c) Find an expression for $\frac{dA}{dx}$. [3]
- (d) Use your answer from part (c) to find the value of x that will maximize the area of the garden. [2]
- (e) Find the maximum possible area of the garden. [2]

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Standard level

Paper 2

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1. [Maximum mark: 15]

The mean annual temperatures for Earth, recorded at fifty-year intervals, are shown in the table.

Year (x)	1708	1758	1808	1858	1908	1958	2008
Temperature °C (y)	8.73	9.22	9.10	9.12	9.13	9.45	9.76

Tami creates a linear model for this data by finding the equation of the straight line passing through the points with coordinates (1708, 8.73) and (1958, 9.45).

- (a) Calculate the gradient of the straight line that passes through these two points. [2]
- (b) (i) Interpret the meaning of the gradient in the context of the question.
 (ii) State appropriate units for the gradient. [2]
- (c) Find the equation of this line giving your answer in the form $y = mx + c$. [2]
- (d) Use Tami's model to estimate the mean annual temperature in the year 2000. [2]

Thandizo uses linear regression to obtain a model for the data.

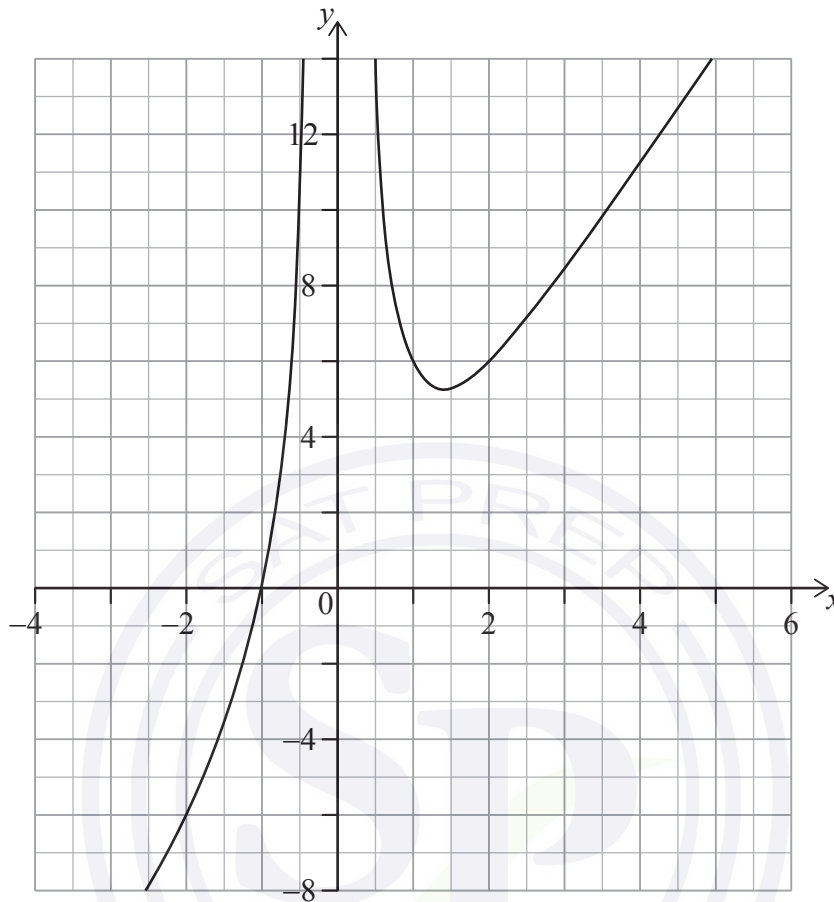
- (e) (i) Find the equation of the regression line y on x .
 (ii) Find the value of r , the Pearson's product-moment correlation coefficient. [3]
- (f) Use Thandizo's model to estimate the mean annual temperature in the year 2000. [2]

Thandizo uses his regression line to predict the year when the mean annual temperature will first exceed 15 °C.

- (g) State two reasons why Thandizo's prediction may not be valid. [2]

2. [Maximum mark: 16]

Consider the function $f(x) = 3x - 1 + 4x^{-2}$. Part of the graph of $y = f(x)$ is shown below.



The function is defined for all values of x except for $x = a$.

(a) Write down the value of a . [1]

(b) Use your graphic display calculator to find the coordinates of the local minimum. [2]

The equation $f(x) = w$, where $w \in \mathbb{R}$, has three solutions.

(c) Identify one possible value for w . [1]

The line $y = mx - \frac{1}{4}$ is tangent to $f(x)$ when $x = -4$.

(d) Write down whether the value of m is positive or negative. Justify your answer. [2]

(This question continues on the following page)

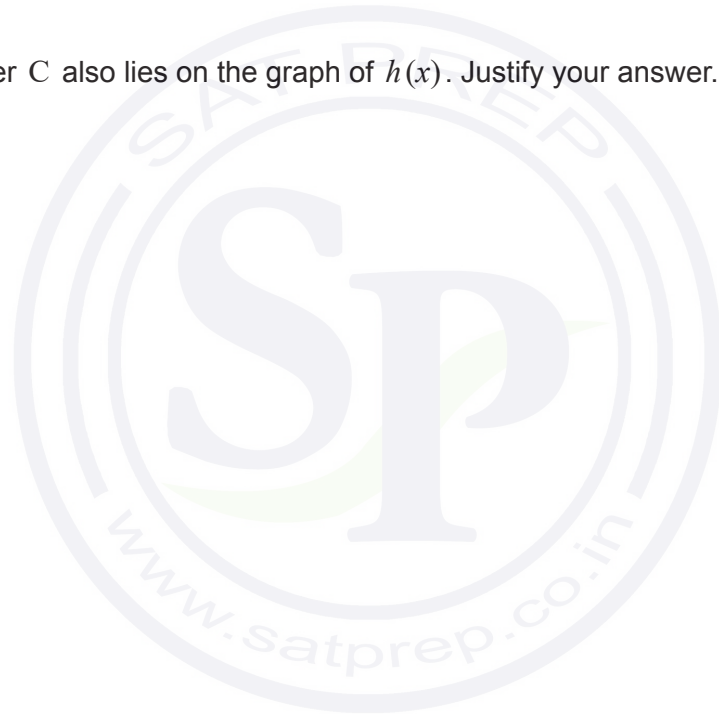
(Question 2 continued)

A second function is given by $g(x) = kp^x - 9$, where $p > 0$. The graph of $y = g(x)$ intersects the y -axis at point $A(0, -5)$ and passes through point $B(3, 4.5)$.

- (e) Find the value of
 - (i) k ;
 - (ii) p . [4]
- (f) Write down the equation of the horizontal asymptote of $y = g(x)$. [2]
- (g) Find the solution of $f(x) = g(x)$ when $x > 0$. [2]

Consider a third function, h , where $h(x) = f(x) + g(x)$. The point $C(-1, q)$ lies on the graph of $g(x)$.

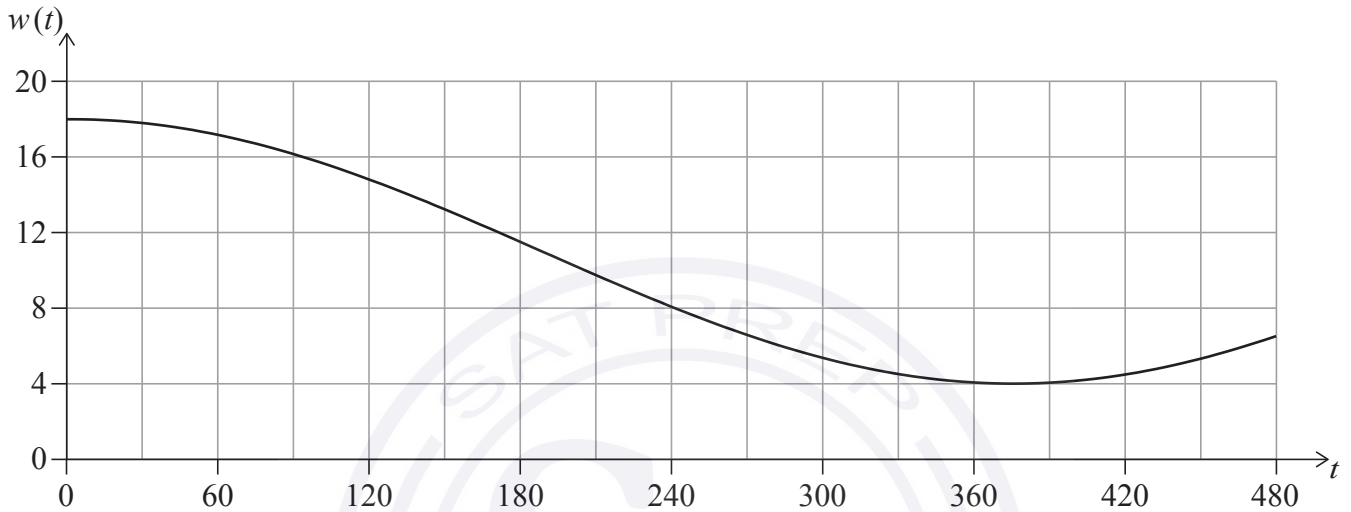
- (h) State whether C also lies on the graph of $h(x)$. Justify your answer. [2]



3. [Maximum mark: 15]

The depth of water, w metres, in a particular harbour can be modelled by the function $w(t) = a \cos(bt^\circ) + d$ where t is the length of time, in minutes, after 06:00.

On 20 January, the first high tide occurs at 06:00, at which time the depth of water is 18 m. The following low tide occurs at 12:15 when the depth of water is 4 m. This is shown in the diagram.



- (a) Find the value of a . [2]
- (b) Find the value of d . [2]
- (c) Find the period of the function in minutes. [3]
- (d) Find the value of b . [2]

Naomi is sailing to the harbour on the morning of 20 January. Boats can enter or leave the harbour only when the depth of water is at least 6 m.

- (e) Find the latest time before 12:00, to the nearest minute, that Naomi can enter the harbour. [4]
- (f) Find the length of time (in minutes) between 06:00 and 15:00 on 20 January during which Naomi **cannot** enter or leave the harbour. [2]

4. [Maximum mark: 17]

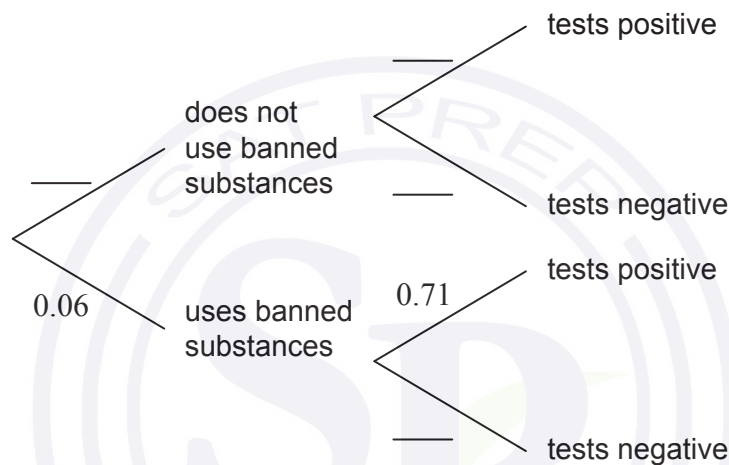
A large international sports tournament tests their athletes for banned substances. They interpret a positive test result as meaning that the athlete uses banned substances. A negative result means that they do not.

The probability that an athlete uses banned substances is estimated to be 0.06.

If an athlete **uses** banned substances, the probability that they will test positive is 0.71.

If an athlete does **not use** banned substances, the probability that they will test negative is 0.98.

- (a) Using the information given, **copy** (into your answer booklet) and complete the following tree diagram. [2]



- (b) (i) Determine the probability that a randomly selected athlete does not use banned substances and tests negative. [4]
- (ii) If two athletes are selected at random, calculate the probability that both athletes do not use banned substances and both test negative. [4]
- (c) (i) Calculate the probability that a randomly selected athlete will receive an **incorrect** test result. [5]
- (ii) A random sample of 1300 athletes at the tournament are selected for testing. Calculate the expected number of athletes in the sample that will receive an incorrect test result. [5]

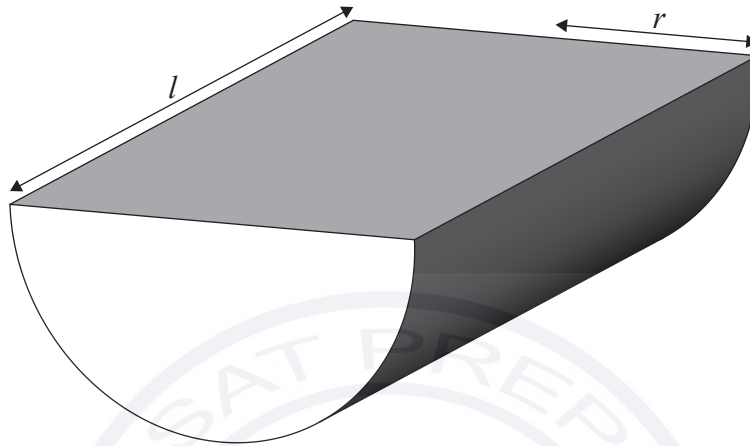
Team X are competing in the tournament. There are 20 athletes in this team. It is known that none of the athletes in Team X use banned substances.

- (d) Calculate the probability that none of the athletes in Team X will test positive. [4]
- (e) Determine the probability that more than 2 athletes in Team X will test positive. [2]

5. [Maximum mark: 17]

A large closed container, in the shape of a half cylinder with a rectangular lid, is to be constructed with a volume of 0.8 m^3 . The container has a length of l metres and a radius of r metres.

diagram not to scale



- (a) Find an exact expression for l in terms of r and π . [2]

The container will be constructed using two different materials. The material for both the curved surface and the rectangular lid of the container costs \$4.40 per square metre. The material for the semicircular ends of the container costs \$ p per square metre.

The cost, C , of the materials to construct the container can be written in terms of r and p (where $p > 0$ and $r > 0$).

- (b) Show that $C = 7.04r^{-1} + \frac{14.08}{\pi}r^{-1} + p\pi r^2$. [4]

- (c) Find $\frac{dC}{dr}$. [3]

The cost of materials to construct the container is minimized when the radius of the container, r , is 0.7 m.

- (d) Find the value of p . [3]

In total, 350 containers will be constructed at this minimum cost.

- (e) Calculate the cost of materials, to the nearest dollar, to construct all 350 containers. [3]

(This question continues on the following page)

(Question 5 continued)

The materials for constructing the containers can be purchased at a discount according to the information in the table.

Cost of materials (\$ C) before discount	Discount applied to entire order
$1000 \leq C < 2500$	1%
$2500 \leq C < 5000$	4%
$5000 \leq C < 10\,000$	8%
$C \geq 10\,000$	10%

- (f) Determine the cost of materials for 350 containers after the discount is applied. [2]



References:

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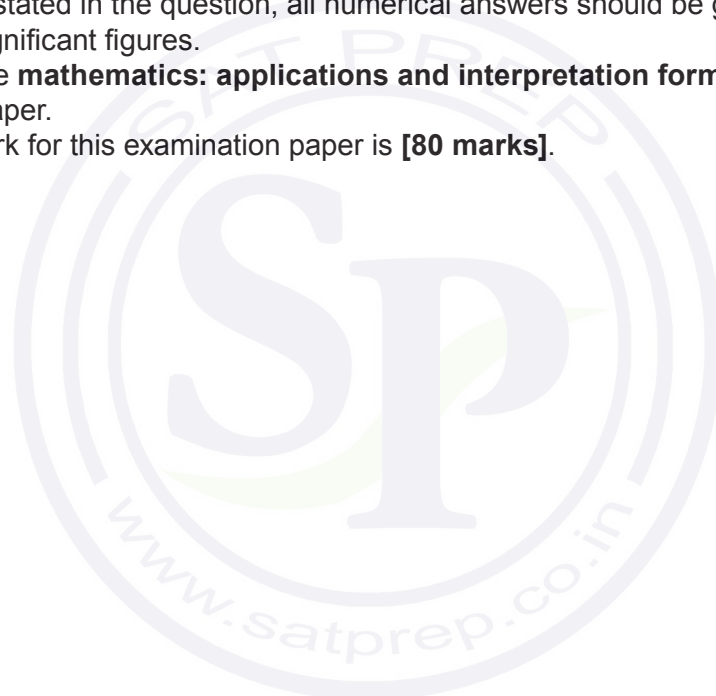
Mathematics: applications and interpretation
Standard level
Paper 2

Tuesday 1 November 2022 (morning)

1 hour 30 minutes

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all the questions in the answer booklet provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.



Answer **all** questions in the answer booklet provided. Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 17]

Elsie, a librarian, wants to investigate the length of time, T minutes, that people spent in her library on a particular day.

- (a) State whether the variable T is discrete or continuous. [1]

Elsie's data for 160 people who visited the library on that particular day is shown in the following table.

T (minutes)	$0 \leq T < 20$	$20 \leq T < 40$	$40 \leq T < 60$	$60 \leq T < 80$	$80 \leq T < 100$
Frequency	50	62	k	14	8

- (b) Find the value of k . [2]
- (c) (i) Write down the modal class.
(ii) Write down the mid-interval value for this class. [2]
- (d) Use Elsie's data to calculate an estimate of the mean time that people spent in the library. [2]
- (e) Using the table, write down the maximum possible number of people who spent 35 minutes or less in the library on that day. [1]

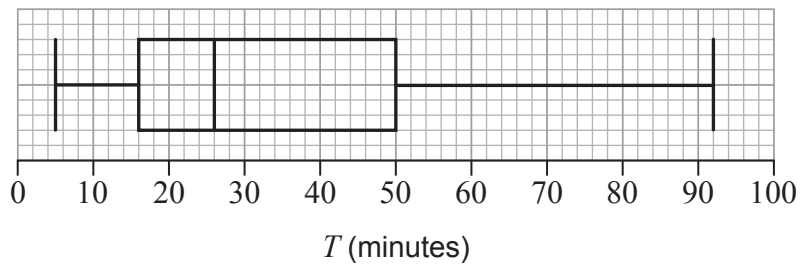
Elsie assumes her data to be representative of future visitors to the library.

- (f) Find the probability a visitor spends at least 60 minutes in the library. [2]

(This question continues on the following page)

(Question 1 continued)

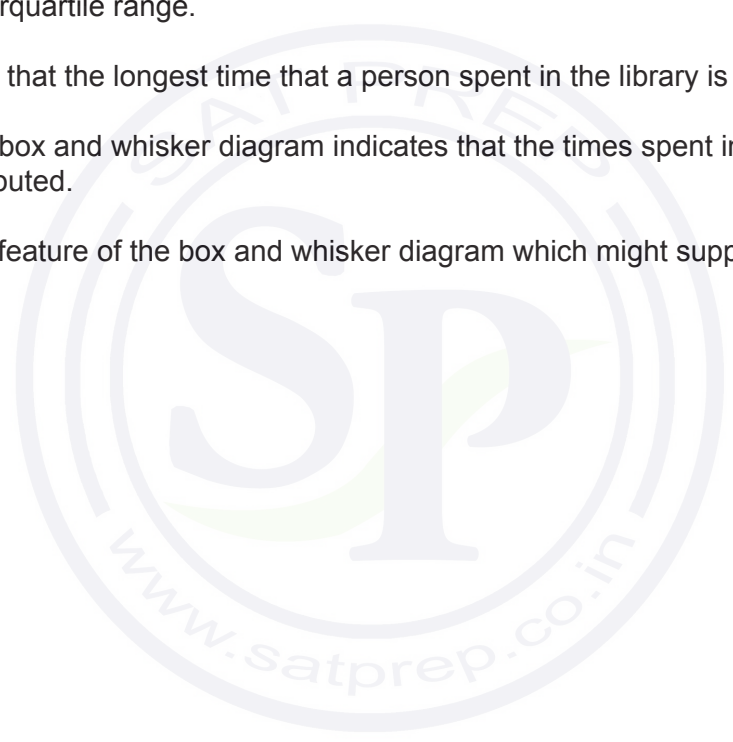
The following box and whisker diagram shows the times, in minutes, that the 160 visitors spent in the library.



- (g) Write down the median time spent in the library. [1]
- (h) Find the interquartile range. [2]
- (i) Hence show that the longest time that a person spent in the library is not an outlier. [3]

Elsie believes the box and whisker diagram indicates that the times spent in the library are not normally distributed.

- (j) Identify one feature of the box and whisker diagram which might support Elsie's belief. [1]

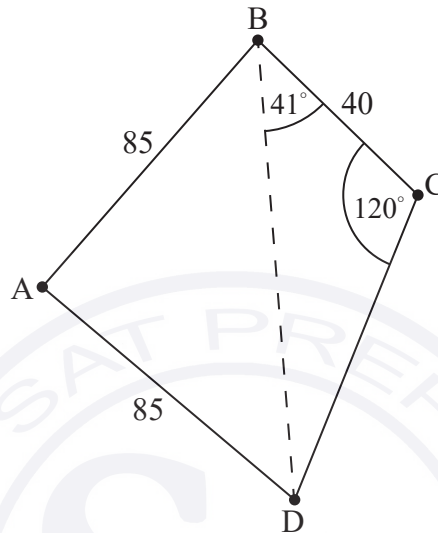


2. [Maximum mark: 17]

The following diagram shows a park bounded by a fence in the shape of a quadrilateral ABCD. A straight path crosses through the park from B to D.

$$AB = 85 \text{ m}, AD = 85 \text{ m}, BC = 40 \text{ m}, \hat{C}BD = 41^\circ, \hat{B}CD = 120^\circ$$

diagram not to scale



- (a) (i) Write down the value of angle BDC. [4]
- (ii) Hence use triangle BDC to find the length of path BD. [4]
- (b) Calculate the size of angle $\hat{B}AD$, correct to five significant figures. [3]

The size of angle $\hat{B}AD$ rounds to 77° , correct to the nearest degree. Use $\hat{B}AD = 77^\circ$ for the rest of this question.

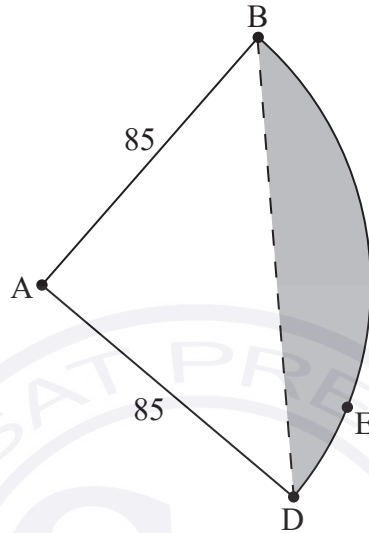
- (c) Find the area bounded by the path BD, and fences AB and AD. [3]

(This question continues on the following page)

(Question 2 continued)

A landscaping firm proposes a new design for the park. Fences BC and CD are to be replaced by a fence in the shape of a circular arc BED with center A. This is illustrated in the following diagram.

diagram not to scale



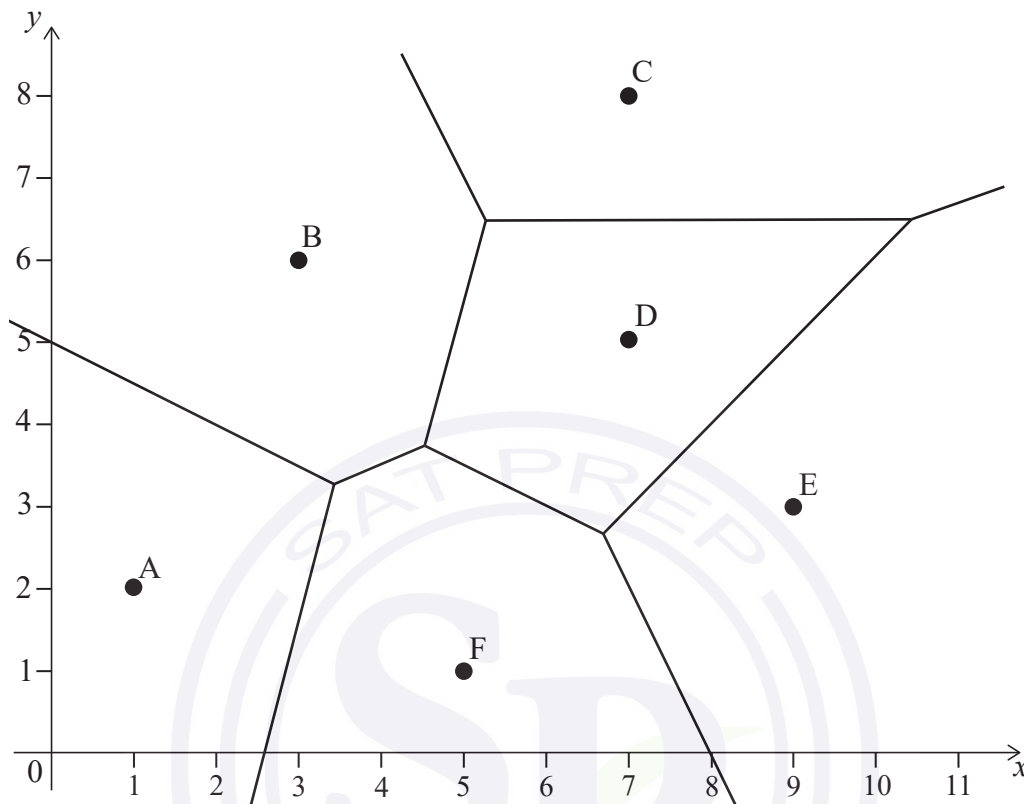
- (d) Write down the distance from A to E. [1]
- (e) Find the perimeter of the proposed park, ABED. [3]
- (f) Find the area of the shaded region in the proposed park. [3]



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3. [Maximum mark: 13]

Six restaurant locations (labelled A, B, C, D, E and F) are shown, together with their Voronoi diagram. All distances are measured in kilometres.



(a) Elena wants to eat at the closest restaurant to her. Write down the restaurant she should go to, if she is at

(i) $(2, 7)$.

(ii) $(0, 1)$, when restaurant A is closed.

[2]

Restaurant C is at $(7, 8)$ and restaurant D is at $(7, 5)$.

(b) Find the equation of the perpendicular bisector of CD.

[2]

Restaurant B is at $(3, 6)$.

(c) Find the equation of the perpendicular bisector of BC.

[5]

(d) Hence find

(i) the coordinates of the point which is of equal distance from B, C and D.

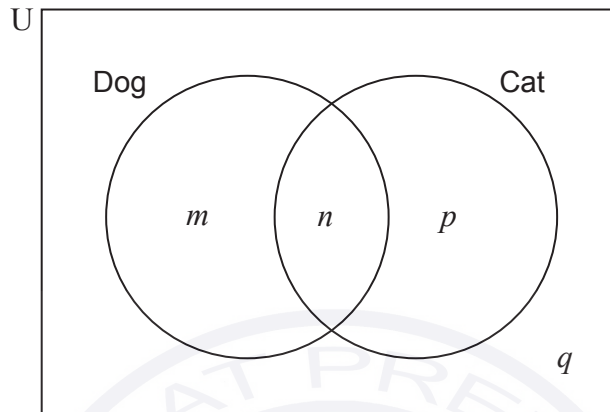
(ii) the distance of this point from D.

[4]

4. [Maximum mark: 16]

At Mirabooka Primary School, a survey found that 68% of students have a dog and 36% of students have a cat. 14% of students have both a dog and a cat.

This information can be represented in the following Venn diagram, where m , n , p and q represent the percentage of students within each region.



- (a) Find the value of
 - (i) m .
 - (ii) n .
 - (iii) p .
 - (iv) q . [4]
- (b) Find the percentage of students who have a dog or a cat or both. [1]
- (c) Find the probability that a randomly chosen student
 - (i) has a dog but does not have a cat.
 - (ii) has a dog given that they do not have a cat. [3]

(This question continues on the following page)

(Question 4 continued)

Each year, one student is chosen randomly to be the school captain of Mirabooka Primary School.

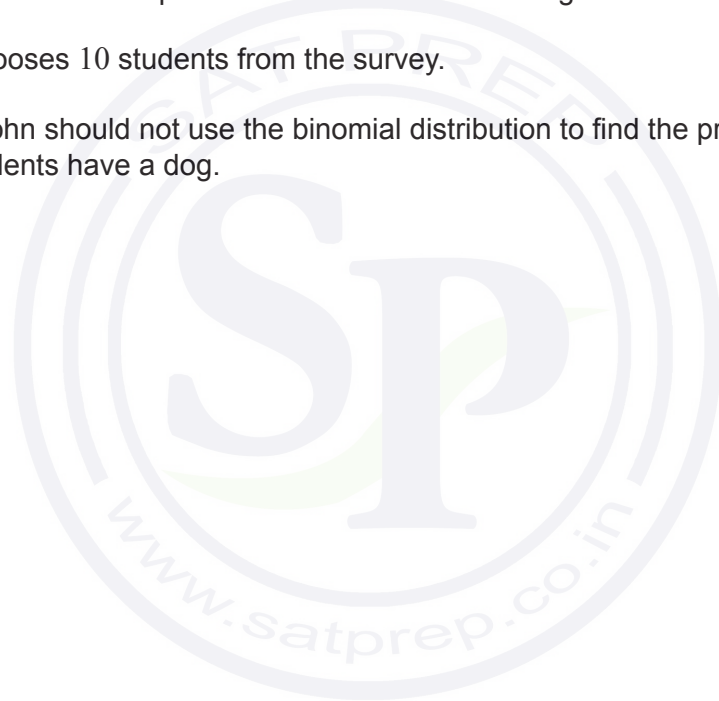
Tim is using a binomial distribution to make predictions about how many of the next 10 school captains will own a dog. He assumes that the percentages found in the survey will remain constant for future years and that the events “being a school captain” and “having a dog” are independent.

Use Tim’s model to find the probability that in the next 10 years

- (d) (i) 5 school captains have a dog.
- (ii) more than 3 school captains have a dog.
- (iii) exactly 9 school captains in succession have a dog. [7]

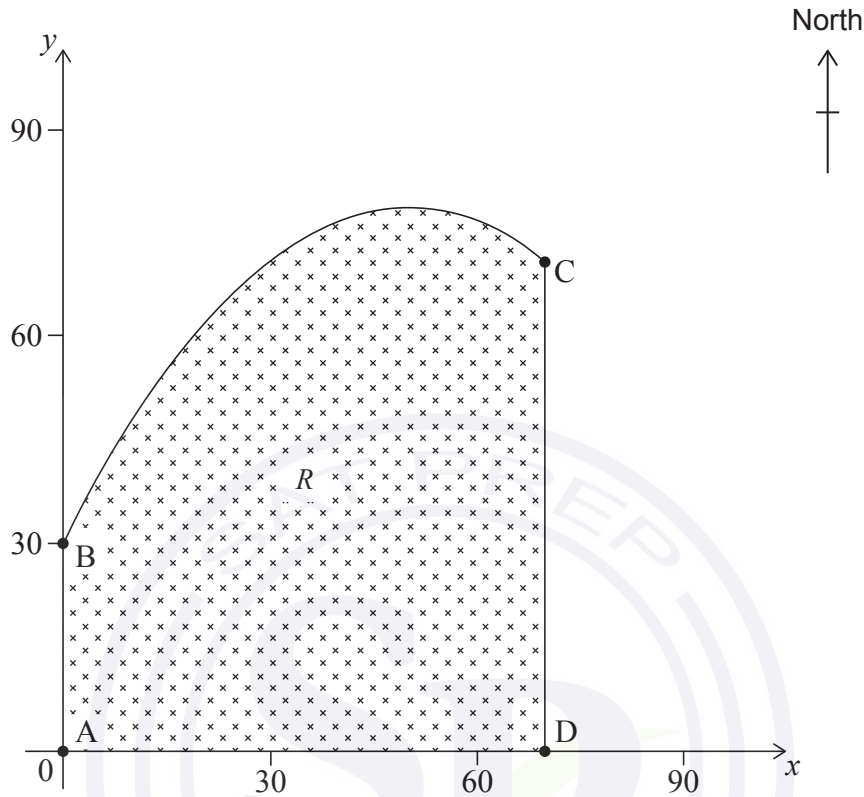
John randomly chooses 10 students from the survey.

- (e) State why John should not use the binomial distribution to find the probability that 5 of these students have a dog. [1]



5. [Maximum mark: 17]

Linda owns a field, represented by the shaded region R . The plan view of the field is shown in the following diagram, where both axes represent distance and are measured in metres.



The segments $[AB]$, $[CD]$ and $[AD]$ respectively represent the western, eastern and southern boundaries of the field. The function, $f(x)$, models the northern boundary of the field between points B and C and is given by

$$f(x) = \frac{-x^2}{50} + 2x + 30, \text{ for } 0 \leq x \leq 70.$$

- (a) (i) Find $f'(x)$.
 (ii) Hence find the coordinates of the point on the field that is furthest north. [5]

Point A has coordinates $(0, 0)$, point B has coordinates $(0, 30)$, point C has coordinates $(70, 72)$ and point D has coordinates $(70, 0)$.

- (b) (i) Write down the integral which can be used to find the area of the shaded region R .
 (ii) Find the area of Linda's field. [4]

(This question continues on the following page)

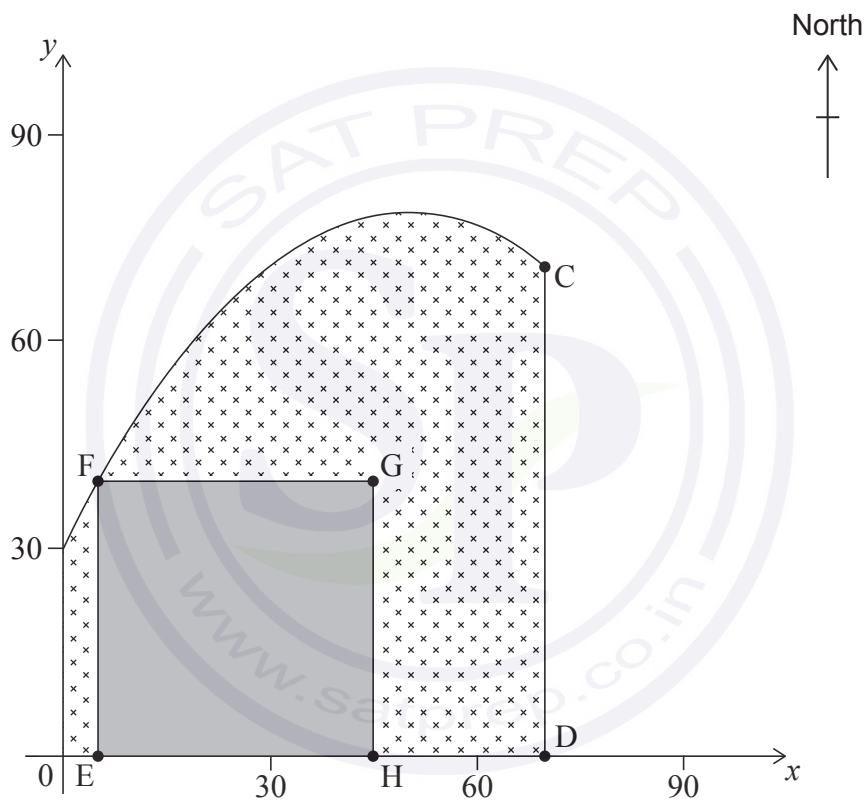
(Question 5 continued)

Linda used the trapezoidal rule with ten intervals to estimate the area. This calculation underestimated the area by 11.4m^2 .

- (c) (i) Calculate the percentage error in Linda's estimate.
- (ii) Suggest how Linda might be able to reduce the error whilst still using the trapezoidal rule.

[3]

Linda would like to construct a building on her field. The **square** foundation of the building, EFGH, will be located such that [EH] is on the southern boundary and point F is on the northern boundary of the property. A possible location of the foundation of the building is shown in the following diagram.



The area of the square foundation will be largest when [GH] lies on [CD].

- (d) (i) Find the x -coordinate of point E for the largest area of the square foundation of building EFGH.
- (ii) Find the largest area of the foundation.

[5]

References:

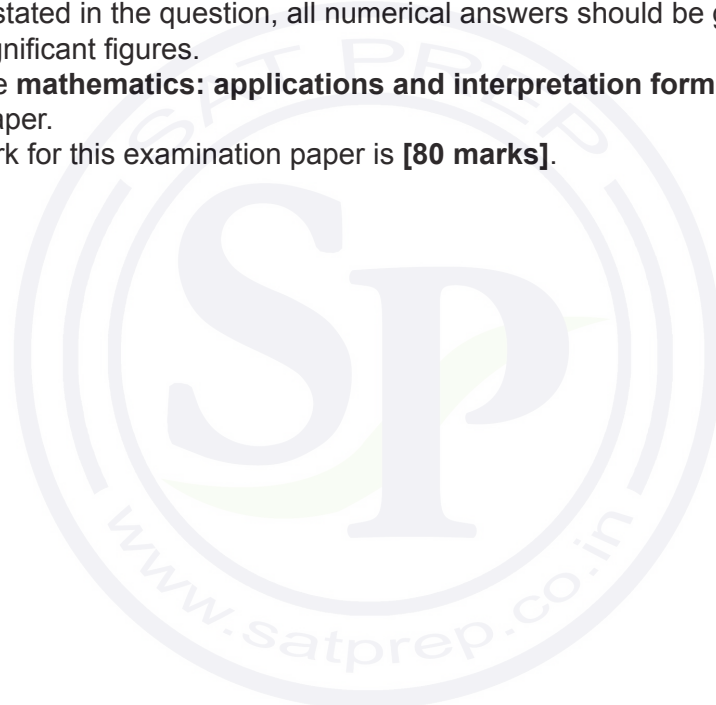
Mathematics: applications and interpretation
Standard level
Paper 2

Monday 9 May 2022 (morning)

1 hour 30 minutes

Instructions to candidates

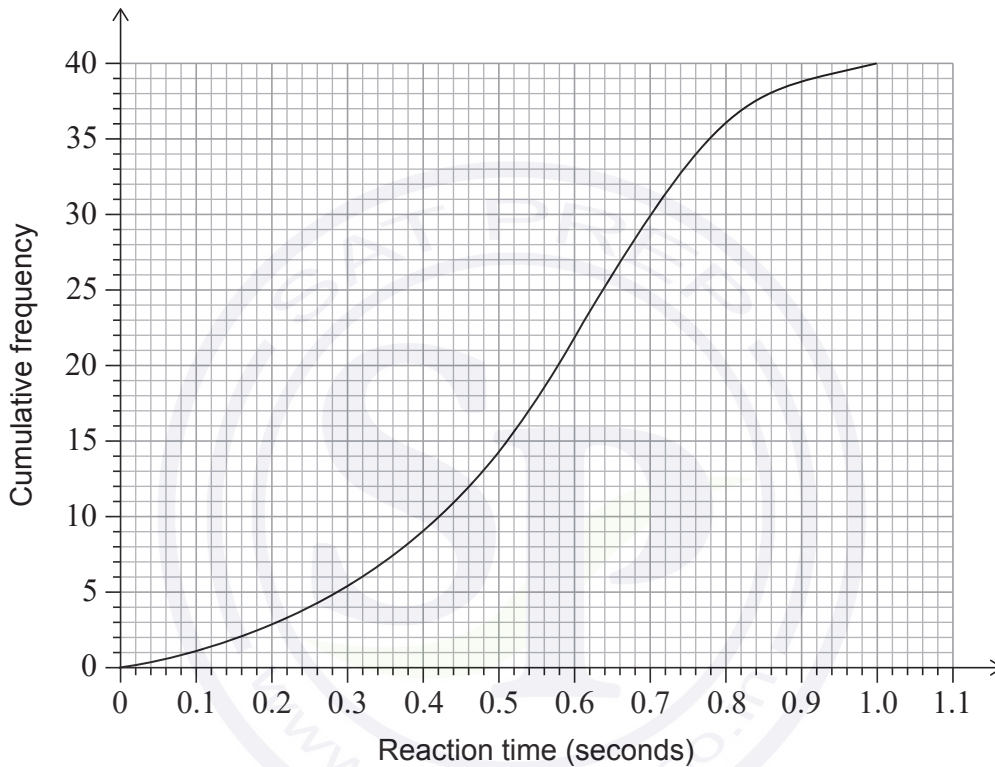
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- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
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- The maximum mark for this examination paper is **[80 marks]**.



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1. [Maximum mark: 17]

Mackenzie conducted an experiment on the reaction times of teenagers. The results of the experiment are displayed in the following cumulative frequency graph.



- (a) Use the graph to estimate the
 - (i) median reaction time;
 - (ii) interquartile range of the reaction times. [4]
- (b) Find the estimated number of teenagers who have a reaction time greater than 0.4 seconds. [2]
- (c) Determine the 90th percentile of the reaction times from the cumulative frequency graph. [2]

(This question continues on the following page)

(Question 1 continued)

Mackenzie created the cumulative frequency graph using the following grouped frequency table.

Reaction time, t (s)	Frequency
$0 < t \leq 0.2$	3
$0.2 < t \leq 0.4$	a
$0.4 < t \leq 0.6$	13
$0.6 < t \leq 0.8$	14
$0.8 < t \leq 1.0$	b

- (d) Write down the value of
- (i) a ;
 - (ii) b . [2]
- (e) Write down the modal class from the table. [1]
- (f) Use your graphic display calculator to find an estimate of the mean reaction time. [2]
- Upon completion of the experiment, Mackenzie realized that some values were grouped incorrectly in the frequency table. Some reaction times recorded in the interval $0 < t \leq 0.2$ should have been recorded in the interval $0.2 < t \leq 0.4$.
- (g) Suggest how, if at all, the estimated mean and estimated median reaction times will change if the errors are corrected. Justify your response. [4]

2. [Maximum mark: 13]

Scott purchases food for his dog in large bags and feeds the dog the same amount of dog food each day. The amount of dog food left in the bag at the end of each day can be modelled by an arithmetic sequence.

On a particular day, Scott opened a new bag of dog food and fed his dog. By the end of the third day there were 115.5 cups of dog food remaining in the bag and at the end of the eighth day there were 108 cups of dog food remaining in the bag.

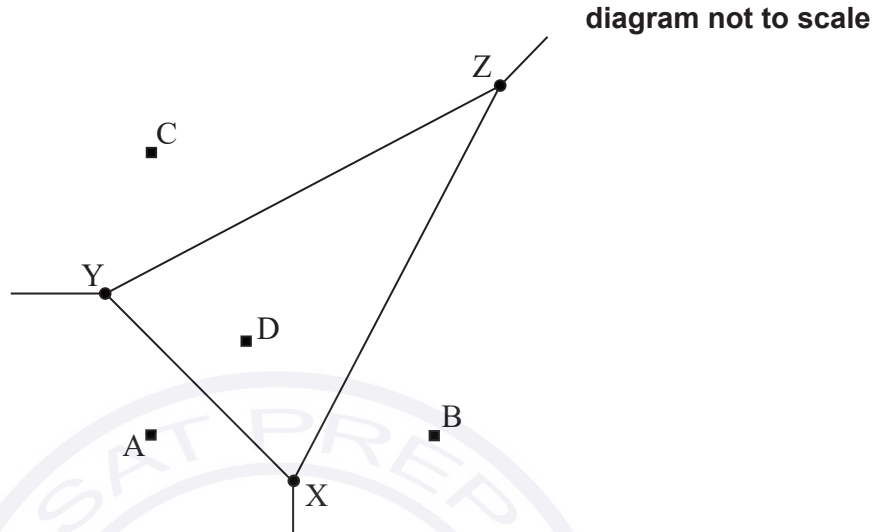
- (a) Find the number of cups of dog food
- (i) fed to the dog per day;
 - (ii) remaining in the bag at the end of the first day. [4]
- (b) Calculate the number of days that Scott can feed his dog with one bag of food. [2]

In 2021, Scott spent \$625 on dog food. Scott expects that the amount he spends on dog food will increase at an annual rate of 6.4%.

- (c) Determine the amount that Scott expects to spend on dog food in 2025. Round your answer to the nearest dollar. [3]
- (d) (i) Calculate the value of $\sum_{n=1}^{10} (625 \times 1.064^{(n-1)})$.
- (ii) Describe what the value in part (d)(i) represents in this context. [3]
- (e) Comment on the appropriateness of modelling this scenario with a geometric sequence. [1]

3. [Maximum mark: 18]

The Voronoi diagram below shows four supermarkets represented by points with coordinates $A(0, 0)$, $B(6, 0)$, $C(0, 6)$ and $D(2, 2)$. The vertices X , Y , Z are also shown. All distances are measured in kilometres.



(a) Find the midpoint of $[BD]$. [2]

(b) Find the equation of (XZ) . [4]

The equation of (XY) is $y = 2 - x$ and the equation of (YZ) is $y = 0.5x + 3.5$.

(c) Find the coordinates of X . [3]

The coordinates of Y are $(-1, 3)$ and the coordinates of Z are $(7, 7)$.

(d) Determine the exact length of $[YZ]$. [2]

(e) Given that the exact length of $[XY]$ is $\sqrt{32}$, find the size of \hat{XYZ} in degrees. [4]

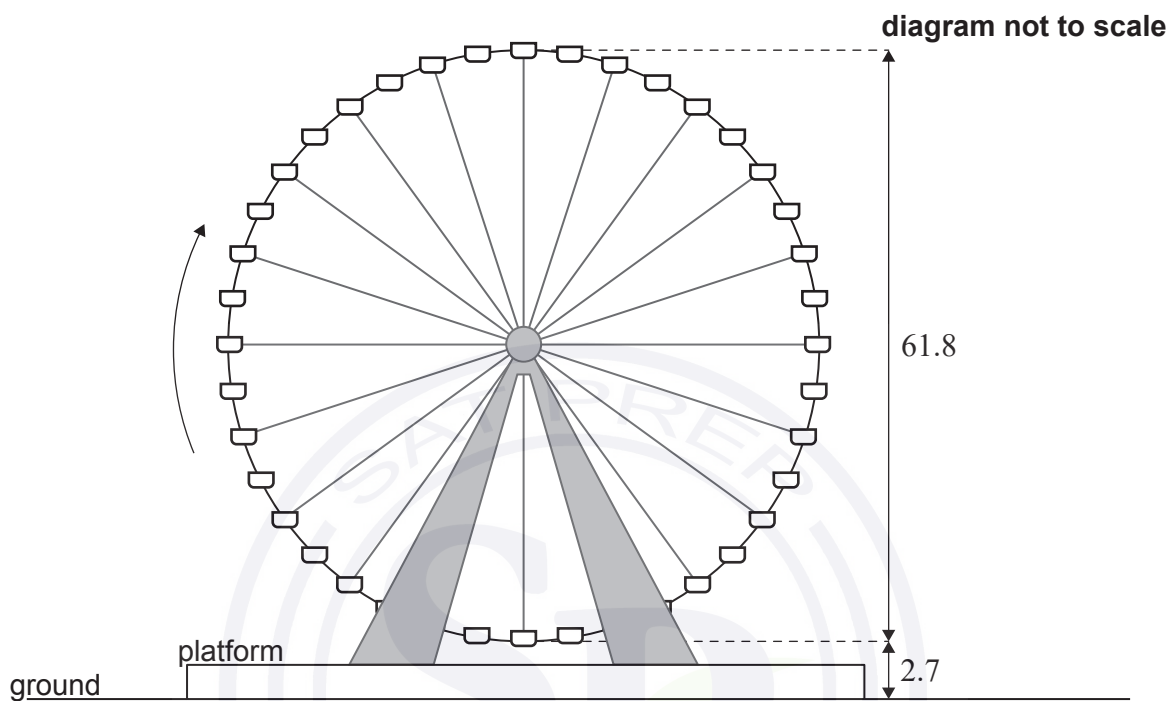
(f) Hence find the area of triangle XYZ . [2]

A town planner believes that the larger the area of the Voronoi cell XYZ , the more people will shop at supermarket D .

(g) State one criticism of this interpretation. [1]

4. [Maximum mark: 17]

The Texas Star is a Ferris wheel at the state fair in Dallas. The Ferris wheel has a diameter of 61.8 m. To begin the ride, a passenger gets into a chair at the lowest point on the wheel, which is 2.7 m above the ground, as shown in the following diagram. A ride consists of multiple revolutions, and the Ferris wheel makes 1.5 revolutions per minute.



The height of a chair above the ground, h , measured in metres, during a ride on the Ferris wheel can be modelled by the function $h(t) = -a \cos(bt) + d$, where t is the time, in seconds, since a passenger began their ride.

- (a) Calculate the value of
- (i) a ;
 - (ii) b ;
 - (iii) d .
- [6]

A ride on the Ferris wheel lasts for 12 minutes in total.

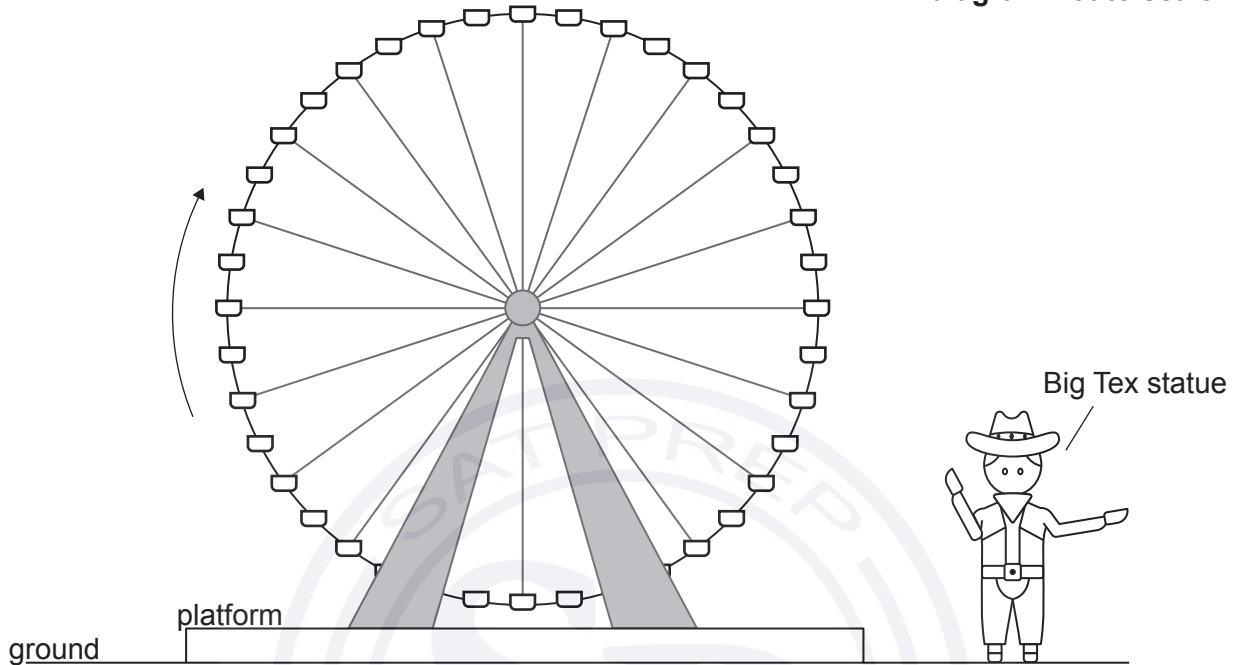
- (b) Calculate the number of revolutions of the Ferris wheel per ride. [2]
- (c) For exactly one ride on the Ferris wheel, suggest
- (i) an appropriate domain for $h(t)$;
 - (ii) an appropriate range for $h(t)$.
- [3]

(This question continues on the following page)

(Question 4 continued)

Big Tex is a 16.7 metre-tall cowboy statue that stands on the horizontal ground next to the Ferris wheel.

diagram not to scale



- (d) By considering the graph of $h(t)$, determine the length of time during one revolution of the Ferris wheel for which the chair is higher than the cowboy statue. [3]

There is a plan to relocate the Texas Star Ferris wheel onto a taller platform which will increase the maximum height of the Ferris wheel to 65.2 m. This will change the value of one parameter, a , b or d , found in part (a).

- (e) (i) Identify which parameter will change.
- (ii) Find the new value of the parameter identified in part (e)(i). [3]

5. [Maximum mark: 15]

A cafe makes x litres of coffee each morning. The cafe’s profit each morning, C , measured in dollars, is modelled by the following equation

$$C = \frac{x}{10} \left(k^2 - \frac{3}{100} x^2 \right)$$

where k is a positive constant.

(a) Find an expression for $\frac{dC}{dx}$ in terms of k and x . [3]

(b) Hence find the maximum value of C in terms of k . Give your answer in the form pk^3 , where p is a constant. [4]

The cafe’s manager knows that the cafe makes a profit of \$426 when 20 litres of coffee are made in a morning.

(c) (i) Find the value of k .
 (ii) Use the model to find how much coffee the cafe should make each morning to maximize its profit. [3]

(d) Sketch the graph of C against x , labelling the maximum point and the x -intercepts with their coordinates. [3]

The manager of the cafe wishes to serve as many customers as possible.

(e) Determine the maximum amount of coffee the cafe can make that will not result in a loss of money for the morning. [2]

References:

4. Aline Escobar., n.d. Cowboy. [image online] Available at: <https://thenounproject.com/search/?q=cowboy&i=1080130>
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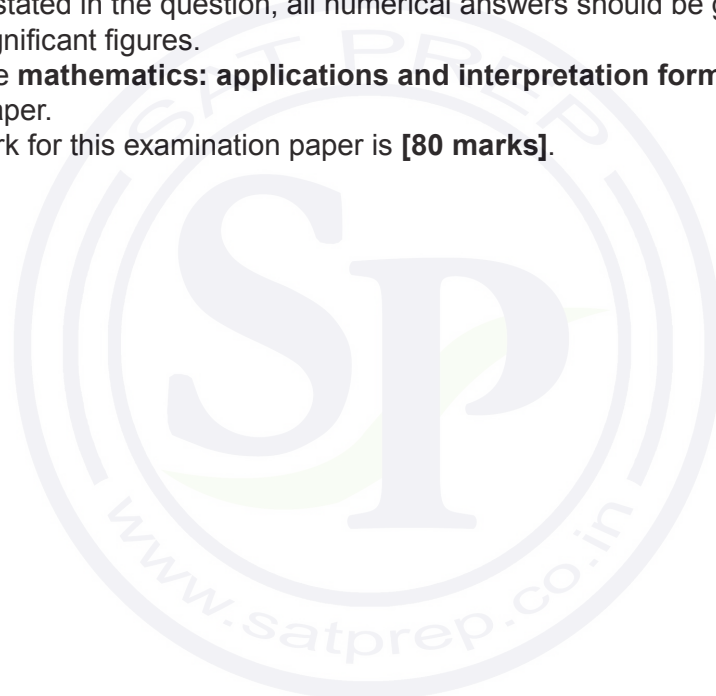
Mathematics: applications and interpretation
Standard level
Paper 2

Monday 9 May 2022 (morning)

1 hour 30 minutes

Instructions to candidates

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- Answer all the questions in the answer booklet provided.
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- A clean copy of the **mathematics: applications and interpretation formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

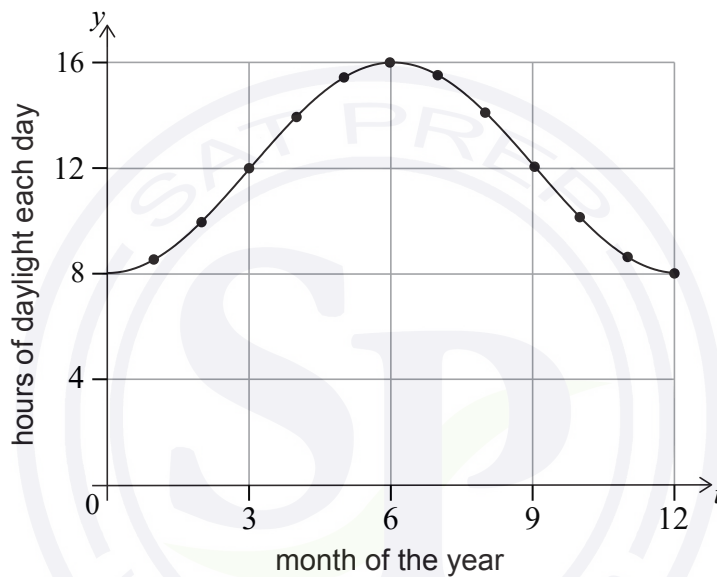


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1. [Maximum mark: 15]

Boris recorded the number of daylight hours on the first day of each month in a northern hemisphere town.

This data was plotted onto a scatter diagram. The points were then joined by a smooth curve, with minimum point (0, 8) and maximum point (6, 16) as shown in the following diagram.



Let the curve in the diagram be $y = f(t)$, where t is the time, measured in months, since Boris first recorded these values.

Boris thinks that $f(t)$ might be modelled by a quadratic function.

- (a) Write down one reason why a quadratic function would not be a good model for the number of hours of daylight per day, across a number of years.

[1]

(This question continues on the following page)

(Question 1 continued)

Paula thinks that a better model is $f(t) = a \cos(bt) + d$, $t \geq 0$, for specific values of a , b and d .

(b) For Paula’s model, use the diagram to write down

(i) the amplitude.

(ii) the period.

(iii) the equation of the principal axis.

[4]

(c) Hence or otherwise find the equation of this model in the form:

[3]

$$f(t) = a \cos(bt) + d$$

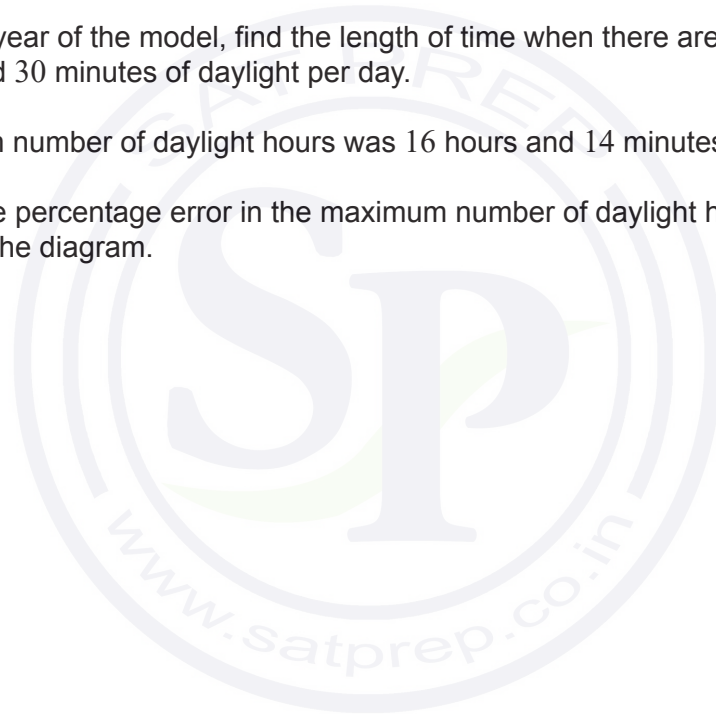
(d) For the first year of the model, find the length of time when there are more than 10 hours and 30 minutes of daylight per day.

[4]

The true maximum number of daylight hours was 16 hours and 14 minutes.

(e) Calculate the percentage error in the maximum number of daylight hours Boris recorded in the diagram.

[3]

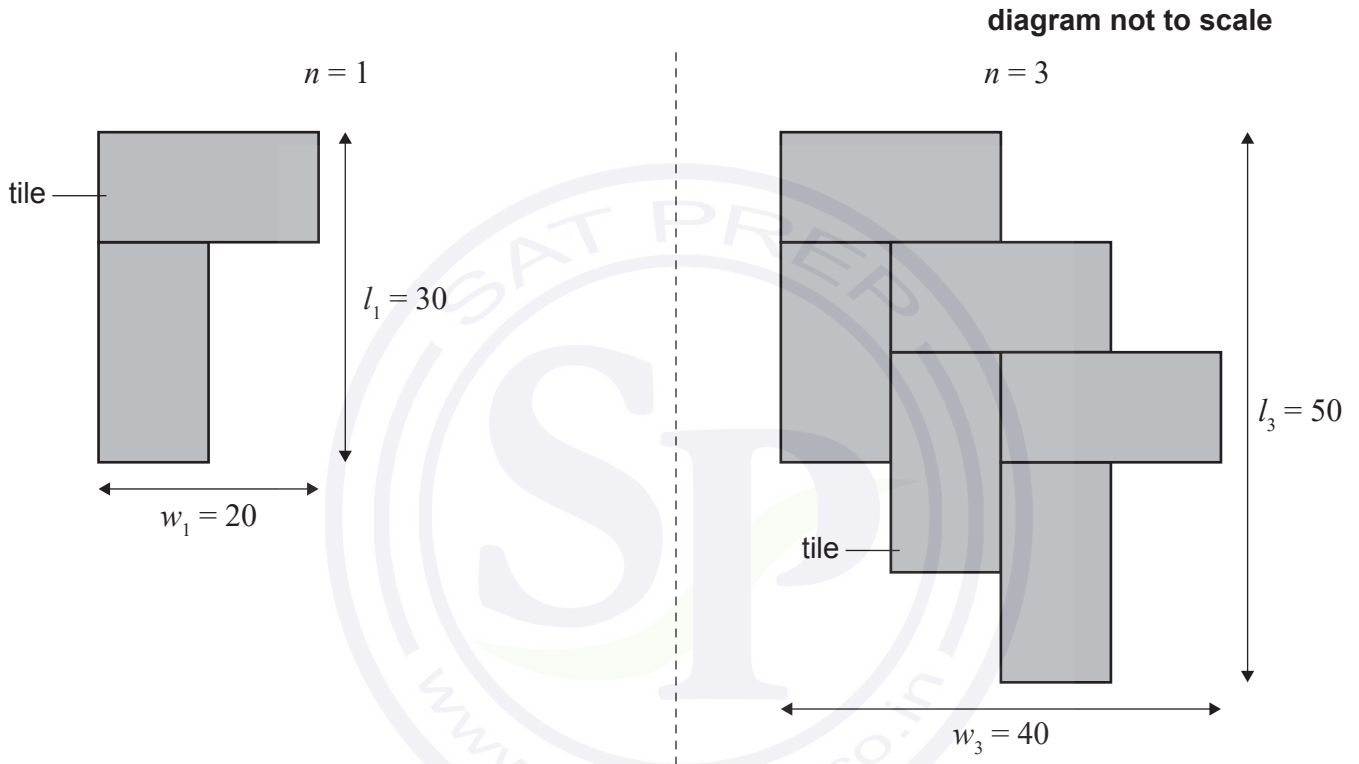


2. [Maximum mark: 19]

Eddie decides to construct a path across his rectangular grass lawn using pairs of tiles.

Each tile is 10 cm wide and 20 cm long. The following diagrams show the path after Eddie has laid one pair and three pairs of tiles. This pattern continues until Eddie reaches the other side of his lawn. When n pairs of tiles are laid, the path has a width of w_n centimetres and a length l_n centimetres.

The following diagrams show this pattern for one pair of tiles and for three pairs of tiles, where the white space around each diagram represents Eddie's lawn.



The following table shows the values of w_n and l_n for the first three values of n .

Number of pairs of tiles, n	Width of lawn crossed by path, w_n (cm)	Length of lawn crossed by path, l_n (cm)
1	20	30
2	a	b
3	40	50

(This question continues on the following page)

(Question 2 continued)

- (a) Find the value of
- (i) a .
 - (ii) b . [2]
- (b) Write down an expression in terms of n for
- (i) w_n .
 - (ii) l_n . [3]

Eddie’s lawn has a length 740 cm.

- (c) (i) Show that Eddie needs 144 tiles.
- (ii) Find the value of w_n for this path. [3]
- (d) Find the total area of the tiles in Eddie’s path. Give your answer in the form $a \times 10^k$ where $1 \leq a < 10$ and k is an integer. [3]

The tiles cost \$24.50 per square metre and are sold in packs of five tiles.

- (e) Find the cost of a single pack of five tiles. [3]

To allow for breakages Eddie wants to have at least 8% more tiles than he needs.

- (f) Find the minimum number of packs of tiles Eddie will need to order. [3]

There is a fixed delivery cost of \$35.

- (g) Find the total cost for Eddie’s order. [2]



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3. [Maximum mark: 16]

The scores of the eight highest scoring countries in the 2019 Eurovision song contest are shown in the following table.

	Eurovision score
Netherlands	498
Italy	472
Russia	370
Switzerland	364
Sweden	334
Norway	331
North Macedonia	305
Azerbaijan	302

- (a) For this data, find
 - (i) the upper quartile.
 - (ii) the interquartile range. [4]
- (b) Determine if the Netherlands' score is an outlier for this data. Justify your answer. [3]

(This question continues on the following page)

(Question 3 continued)

Chester is investigating the relationship between the highest-scoring countries' Eurovision score and their population size to determine whether population size can reasonably be used to predict a country's score.

The populations of the countries, to the nearest million, are shown in the table.

	Population (x) (millions)	Eurovision score (y)
Netherlands	17	498
Italy	60	472
Russia	145	370
Switzerland	9	364
Sweden	10	334
Norway	5	331
North Macedonia	2	305
Azerbaijan	10	302

Chester finds that, for this data, the Pearson's product moment correlation coefficient is $r = 0.249$.

- (c) State whether it would be appropriate for Chester to use the equation of a regression line for y on x to predict a country's Eurovision score. Justify your answer. [2]

Chester then decides to find the Spearman's rank correlation coefficient for this data, and creates a table of ranks.

	Population rank (to the nearest million)	Eurovision score rank
Netherlands	3	1
Italy	2	2
Russia	1	3
Switzerland	a	4
Sweden	b	5
Norway	7	6
North Macedonia	8	7
Azerbaijan	c	8

(This question continues on the following page)

(Question 3 continued)

(d) Write down the value of:

(i) a ,

(ii) b ,

(iii) c .

[3]

(e) (i) Find the value of the Spearman's rank correlation coefficient r_s .

(ii) Interpret the value obtained for r_s .

[3]

(f) When calculating the ranks, Chester incorrectly read the Netherlands' score as 478. Explain why the value of the Spearman's rank correlation r_s does not change despite this error.

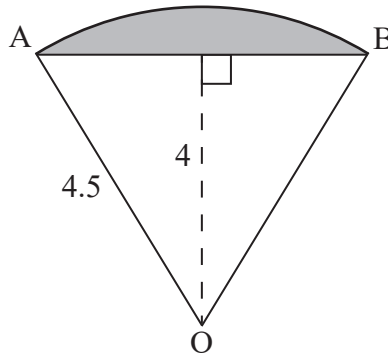
[1]



4. [Maximum mark: 15]

A sector of a circle, centre O and radius 4.5 m, is shown in the following diagram.

diagram not to scale



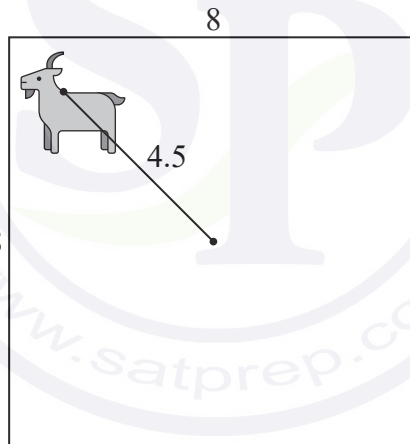
(a) (i) Find the angle \widehat{AOB} .

(ii) Find the area of the shaded segment.

[8]

A square field with side 8 m has a goat tied to a post in the centre by a rope such that the goat can reach all parts of the field up to 4.5 m from the post.

diagram not to scale



(b) (i) Find the area of a circle with radius 4.5 m.

(ii) Find the area of the field that can be reached by the goat.

[5]

Let V be the volume of grass eaten by the goat, in cubic metres, and t be the length of time, in hours, that the goat has been in the field.

The goat eats grass at the rate of $\frac{dV}{dt} = 0.3te^{-t}$.

(c) Find the value of t at which the goat is eating grass at the greatest rate.

[2]

5. [Maximum mark: 15]

The aircraft for a particular flight has 72 seats. The airline’s records show that historically for this flight only 90% of the people who purchase a ticket arrive to board the flight. They assume this trend will continue and decide to sell extra tickets and hope that no more than 72 passengers will arrive.

The number of passengers that arrive to board this flight is assumed to follow a binomial distribution with a probability of 0.9.

- (a) The airline sells 74 tickets for this flight. Find the probability that more than 72 passengers arrive to board the flight. [3]
- (b) (i) Write down the expected number of passengers who will arrive to board the flight if 72 tickets are sold. [2]
- (ii) Find the maximum number of tickets that could be sold if the expected number of passengers who arrive to board the flight must be less than or equal to 72. [2]

Each passenger pays \$150 for a ticket. If too many passengers arrive, then the airline will give \$300 in compensation to each passenger that cannot board.

- (c) Find, to the nearest integer, the expected increase or decrease in the money made by the airline if they decide to sell 74 tickets rather than 72. [8]

References:

- 2. mynamepong, n.d. Goat [image online] Available at: <https://thenounproject.com/term/goat/1761571/>
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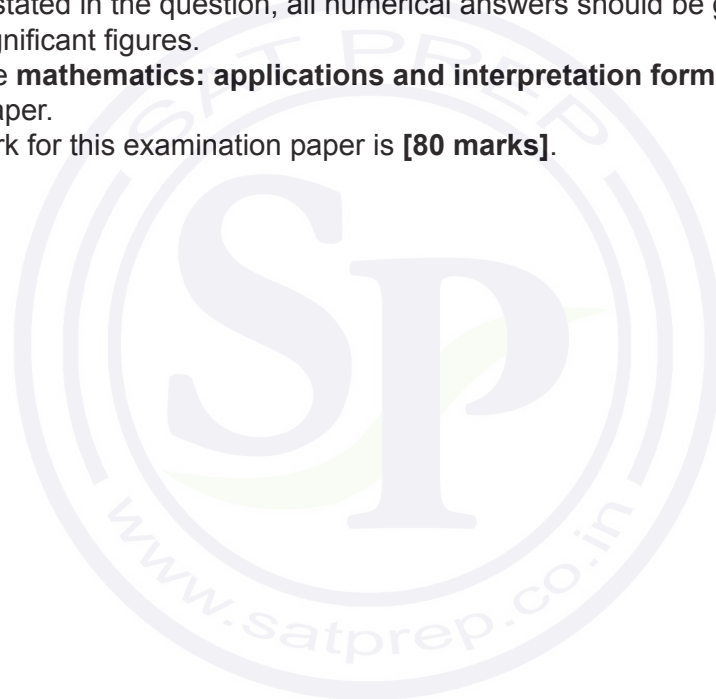
Mathematics: applications and interpretation
Standard level
Paper 2

Tuesday 2 November 2021 (morning)

1 hour 30 minutes

Instructions to candidates

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- Answer all the questions in the answer booklet provided.
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1. [Maximum mark: 16]

A group of 1280 students were asked which electronic device they preferred. The results per age group are given in the following table.

Preferred device	Age			Total
	11–13	14–16	17–18	
Laptop	143	160	153	456
Tablet	205	224	131	560
Mobile phone	72	128	64	264
Total	420	512	348	1280

- (a) A student from the group is chosen at random. Calculate the probability that the student
- (i) prefers a tablet.
 - (ii) is 11–13 years old and prefers a mobile phone.
 - (iii) prefers a laptop **given that** they are 17–18 years old.
 - (iv) prefers a tablet or is 14–16 years old. [9]

A χ^2 test for independence was performed on the collected data at the 1% significance level. The critical value for the test is 13.277.

- (b) State the null and alternative hypotheses. [1]
- (c) Write down the number of degrees of freedom. [1]
- (d) (i) Write down the χ^2 test statistic.
- (ii) Write down the p -value.
- (iii) State the conclusion for the test in context. Give a reason for your answer. [5]

2. [Maximum mark: 16]

The admissions team at a new university are trying to predict the number of student applications they will receive each year.

Let n be the number of years that the university has been open. The admissions team collect the following data for the first two years.

Year, n	Number of applications received in year n
1	12 300
2	12 669

- (a) Calculate the percentage increase in applications from the first year to the second year. [2]

It is assumed that the number of students that apply to the university each year will follow a geometric sequence, u_n .

- (b) (i) Write down the common ratio of the sequence.
 (ii) Find an expression for u_n .
 (iii) Find the number of student applications the university expects to receive when $n = 11$. Express your answer to the nearest integer. [4]

In the first year there were 10 380 places at the university available for applicants. The admissions team announce that the number of places available will increase by 600 every year.

Let v_n represent the number of places available at the university in year n .

- (c) Write down an expression for v_n . [2]

For the first 10 years that the university is open, all places are filled. Students who receive a place each pay an \$80 acceptance fee.

- (d) Calculate the total amount of acceptance fees paid to the university in the first 10 years. [3]

When $n = k$, the number of places available will, for the first time, exceed the number of students applying.

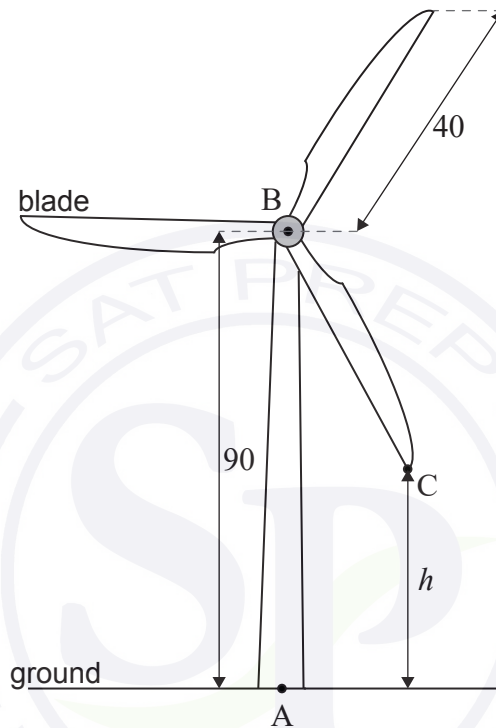
- (e) Find k . [3]
 (f) State whether, for all $n > k$, the university will have places available for all applicants. Justify your answer. [2]

3. [Maximum mark: 20]

A wind turbine is designed so that the rotation of the blades generates electricity. The turbine is built on horizontal ground and is made up of a vertical tower and three blades.

The point A is on the base of the tower directly below point B at the top of the tower. The height of the tower, AB, is 90 m. The blades of the turbine are centred at B and are each of length 40 m. This is shown in the following diagram.

diagram not to scale



The end of one of the blades of the turbine is represented by point C on the diagram. Let h be the height of C above the ground, measured in metres, where h varies as the blade rotates.

- (a) Find the
 - (i) maximum value of h .
 - (ii) minimum value of h . [2]

The blades of the turbine complete 12 rotations per minute under normal conditions, moving at a constant rate.

- (b) (i) Find the time, in seconds, it takes for the blade [BC] to make one complete rotation under these conditions.
- (ii) Calculate the angle, in degrees, that the blade [BC] turns through in one second. [3]

(This question continues on the following page)

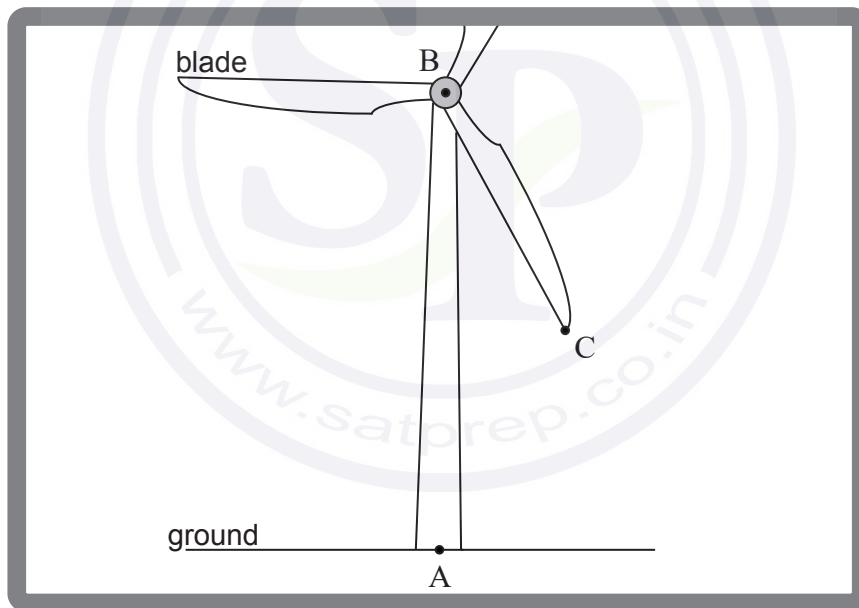
(Question 3 continued)

The height, h , of point C can be modelled by the following function. Time, t , is measured from the instant when the blade [BC] first passes [AB] and is measured in seconds.

$$h(t) = 90 - 40 \cos(72t^\circ), t \geq 0$$

- (c) (i) Write down the amplitude of the function.
- (ii) Find the period of the function. [2]
- (d) Sketch the function $h(t)$ for $0 \leq t \leq 5$, clearly labelling the coordinates of the maximum and minimum points. [3]
- (e) (i) Find the height of C above the ground when $t = 2$.
- (ii) Find the time, in seconds, that point C is above a height of 100 m, during each complete rotation. [5]

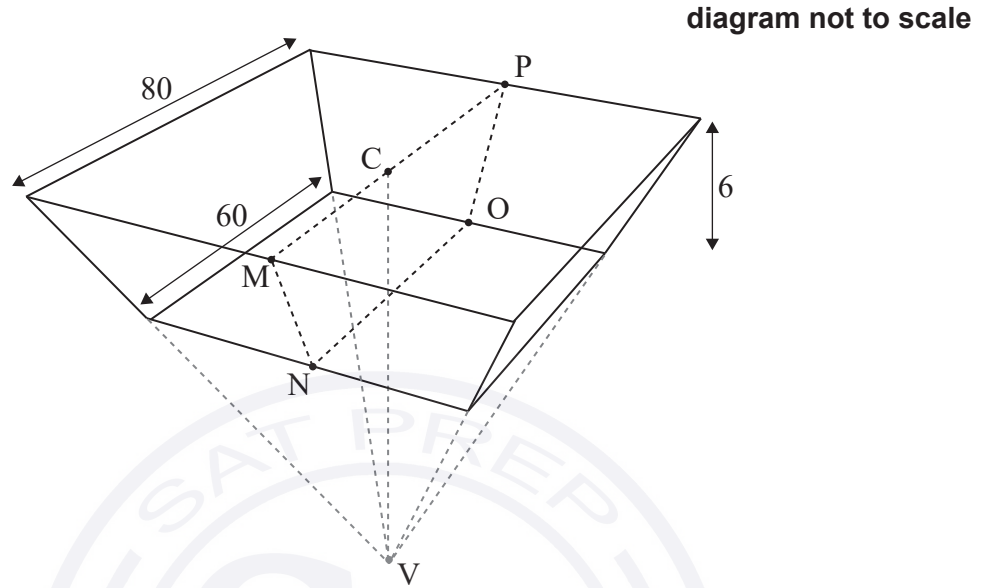
Looking through his window, Tim has a partial view of the rotating wind turbine. The position of his window means that he cannot see any part of the wind turbine that is **more than 100 m** above the ground. This is illustrated in the following diagram.



- (f) (i) At any given instant, find the probability that point C is visible from Tim's window.
- The wind speed increases. The blades rotate at twice the speed, but still at a constant rate.
- (ii) At any given instant, find the probability that Tim can see point C from his window. Justify your answer. [5]

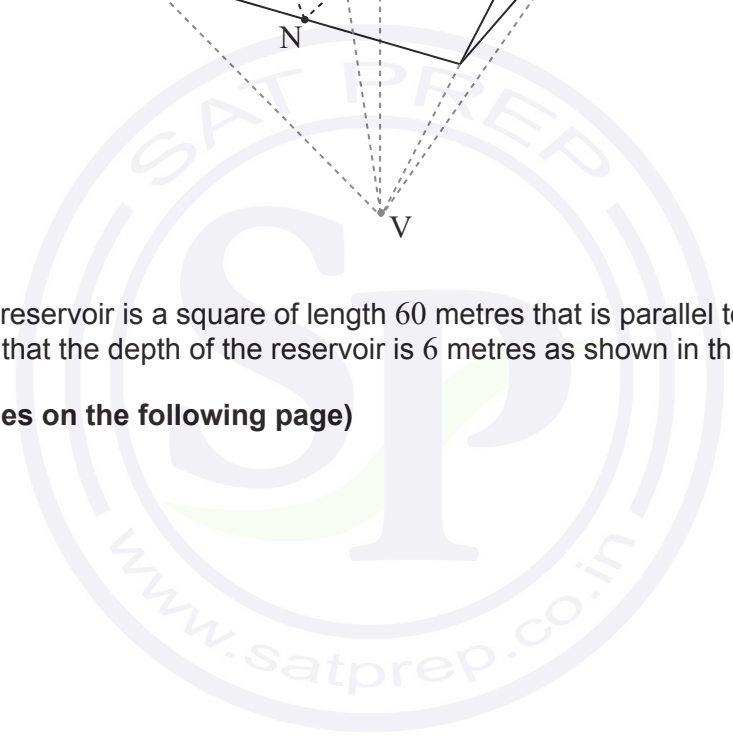
4. [Maximum mark: 14]

A large water reservoir is built in the form of part of an upside-down right pyramid with a horizontal square base of length 80 metres. The point C is the centre of the square base and point V is the vertex of the pyramid.



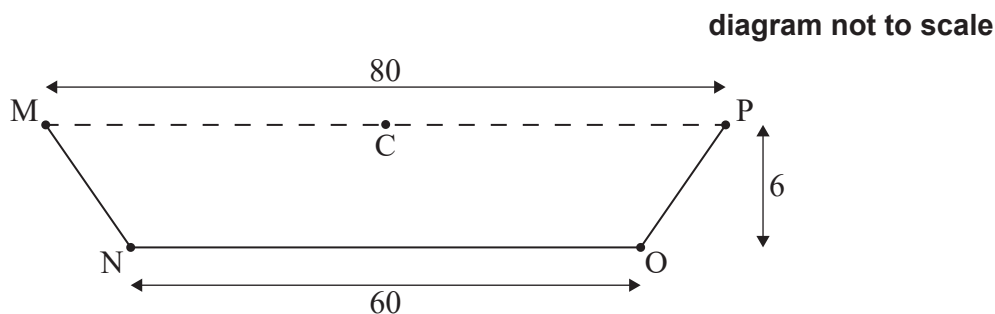
The bottom of the reservoir is a square of length 60 metres that is parallel to the base of the pyramid, such that the depth of the reservoir is 6 metres as shown in the diagram.

(This question continues on the following page)



(Question 4 continued)

The second diagram shows a vertical cross section, MNOPC, of the reservoir.



- (a) Find the angle of depression from M to N. [2]
- (b) (i) Find CV. [5]
(ii) Hence or otherwise, show that the volume of the reservoir is $29\,600\text{ m}^3$.

Every day 80 m^3 of water from the reservoir is used for irrigation.

Joshua states that, if no other water enters or leaves the reservoir, then when it is full there is enough irrigation water for at least one year.

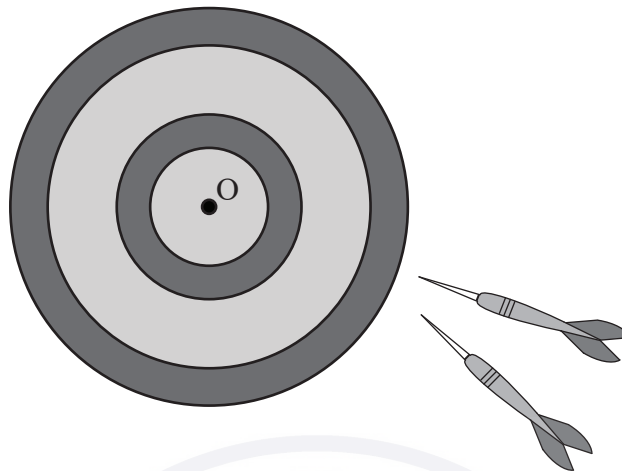
- (c) By finding an appropriate value, determine whether Joshua is correct. [2]

To avoid water leaking into the ground, the five interior sides of the reservoir have been painted with a watertight material.

- (d) Find the area that was painted. [5]

5. [Maximum mark: 14]

Arianne plays a game of darts.



The distance that her darts land from the centre, O, of the board can be modelled by a normal distribution with mean 10 cm and standard deviation 3 cm.

- (a) Find the probability that
- (i) a dart lands less than 13 cm from O.
 - (ii) a dart lands more than 15 cm from O. [3]

Each of Arianne's throws is independent of her previous throws.

- (b) Find the probability that Arianne throws two consecutive darts that land more than 15 cm from O. [2]

In a competition a player has three darts to throw on each turn. A point is scored if a player throws **all** three darts to land within a central area around O. When Arianne throws a dart the probability that it lands within this area is 0.8143.

- (c) Find the probability that Arianne does **not** score a point on a turn of three darts. [2]

In the competition Arianne has ten turns, each with three darts.

- (d) (i) Find the probability that Arianne scores at least 5 points in the competition.
 (ii) Find the probability that Arianne scores at least 5 points and less than 8 points.
 (iii) Given that Arianne scores at least 5 points, find the probability that Arianne scores less than 8 points. [7]

References:

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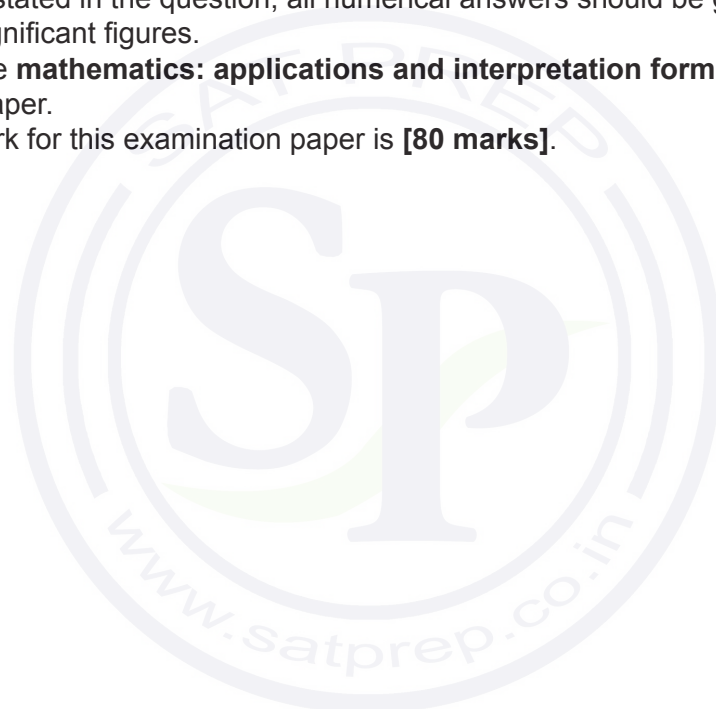
Mathematics: applications and interpretation
Standard level
Paper 2

Friday 7 May 2021 (morning)

1 hour 30 minutes

Instructions to candidates

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- Answer all the questions in the answer booklet provided.
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1. [Maximum mark: 19]

A medical centre is testing patients for a certain disease. This disease occurs in 5% of the population.

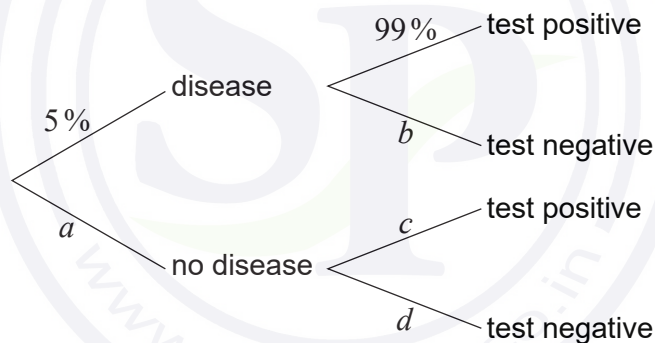
They test every patient who comes to the centre on a particular day.

(a) State the sampling method being used. [1]

It is intended that if a patient has the disease, they test “positive”, and if a patient does not have the disease, they test “negative”.

However, the tests are not perfect, and only 99% of people who have the disease test positive. Also, 2% of people who **do not** have the disease test positive.

The tree diagram shows some of this information.



(b) Write down the value of

- (i) a .
- (ii) b .
- (iii) c .
- (iv) d .

[4]

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(Question 1 continued)

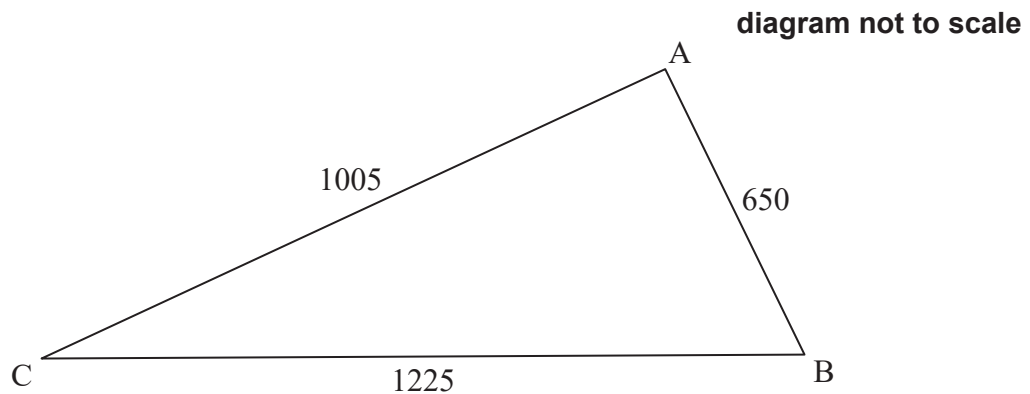
- (c) Use the tree diagram to find the probability that a patient selected at random
- (i) will not have the disease and will test positive.
 - (ii) will test negative.
 - (iii) has the disease given that they tested negative. [8]
- (d) The medical centre finds the actual number of positive results in their sample is different than predicted by the tree diagram. Explain why this might be the case. [1]

The staff at the medical centre looked at the care received by all visiting patients on a randomly chosen day. All the patients received at least one of these services: they had medical tests (M), were seen by a nurse (N), or were seen by a doctor (D). It was found that:

- 78 had medical tests,
 - 45 were seen by a nurse;
 - 30 were seen by a doctor;
 - 9 had medical tests and were seen by a doctor and a nurse;
 - 18 had medical tests and were seen by a doctor but were not seen by a nurse;
 - 11 patients were seen by a nurse and had medical tests but were not seen by a doctor;
 - 2 patients were seen by a doctor without being seen by nurse and without having medical tests.
- (e) Draw a Venn diagram to illustrate this information, placing all relevant information on the diagram. [3]
- (f) Find the total number of patients who visited the centre during this day. [2]

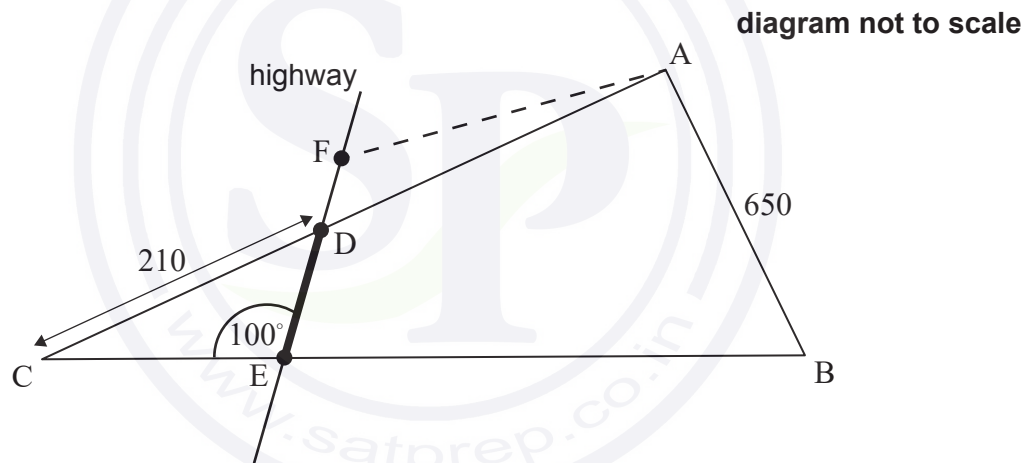
2. [Maximum mark: 15]

A farmer owns a field in the shape of a triangle ABC such that $AB = 650\text{ m}$, $AC = 1005\text{ m}$ and $BC = 1225\text{ m}$.



(a) Find the size of \hat{ACB} . [3]

The local town is planning to build a highway that will intersect the borders of the field at points D and E, where $DC = 210\text{ m}$ and $\hat{CED} = 100^\circ$, as shown in the diagram below.



(b) Find DE. [3]

The town wishes to build a carpark here. They ask the farmer to exchange the part of the field represented by triangle DCE. In return the farmer will get a triangle of equal area ADF, where F lies on the same line as D and E, as shown in the diagram above.

(c) Find the area of triangle DCE. [5]

(d) Estimate DF. You may assume the highway has a width of zero. [4]

3. [Maximum mark: 16]

A new concert hall was built with 14 seats in the first row. Each subsequent row of the hall has two more seats than the previous row. The hall has a total of 20 rows.

(a) Find:

(i) the number of seats in the last row.

(ii) the total number of seats in the concert hall.

[5]

The concert hall opened in 2019. The average number of visitors per concert during that year was 584. In 2020, the average number of visitors per concert increased by 1.2%.

(b) Find the average number of visitors per concert in 2020.

[2]

The concert organizers use this data to model future numbers of visitors. It is assumed that the average number of visitors per concert will continue to increase each year by 1.2%.

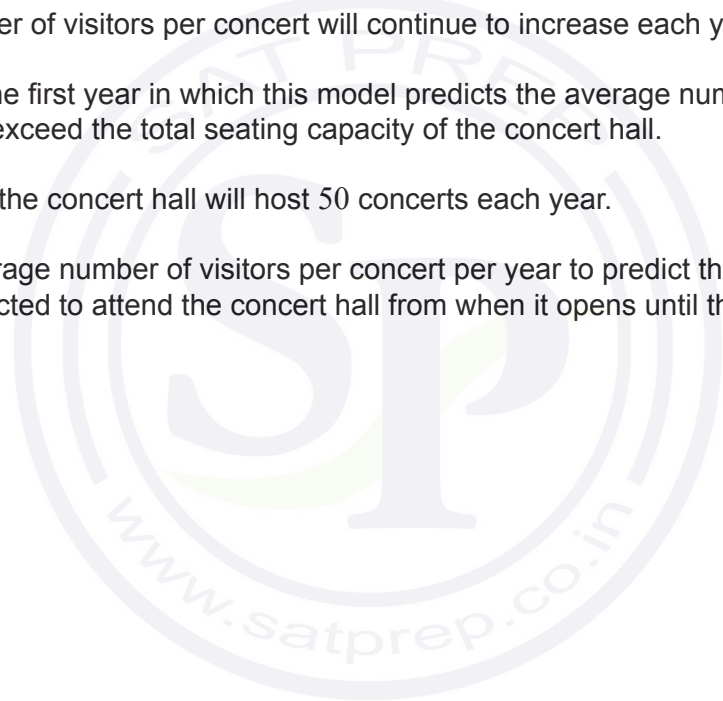
(c) Determine the first year in which this model predicts the average number of visitors per concert will exceed the total seating capacity of the concert hall.

[5]

It is assumed that the concert hall will host 50 concerts each year.

(d) Use the average number of visitors per concert per year to predict the **total** number of people expected to attend the concert hall from when it opens until the end of 2025.

[4]



4. [Maximum mark: 14]

It is known that the weights of male Persian cats are normally distributed with mean 6.1 kg and variance 0.5^2 kg^2 .

(a) Sketch a diagram showing the above information. [2]

(b) Find the proportion of male Persian cats weighing between 5.5 kg and 6.5 kg. [2]

A group of 80 male Persian cats are drawn from this population.

(c) Determine the expected number of cats in this group that have a weight of less than 5.3 kg. [3]

(d) It is found that 12 of the cats weigh more than x kg. Estimate the value of x . [3]

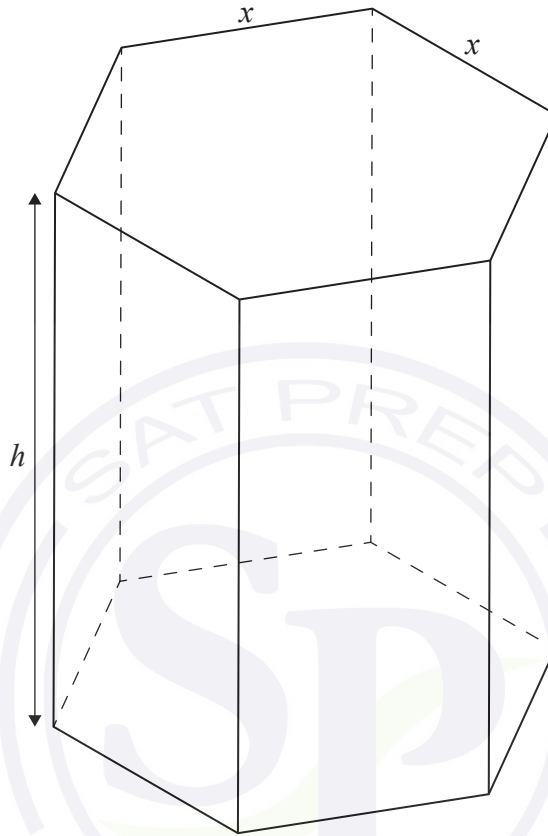
(e) Ten of the cats are chosen at random. Find the probability that exactly one of them weighs over 6.25 kg. [4]



5. [Maximum mark: 16]

A hollow chocolate box is manufactured in the form of a right prism with a regular hexagonal base. The height of the prism is h cm, and the top and base of the prism have sides of length x cm.

diagram not to scale



- (a) Given that $\sin 60^\circ = \frac{\sqrt{3}}{2}$, show that the area of the base of the box is equal to $\frac{3\sqrt{3}x^2}{2}$. [2]
- (b) Given that the total external surface area of the box is 1200 cm^2 , show that the volume of the box may be expressed as $V = 300\sqrt{3}x - \frac{9}{4}x^3$. [5]
- (c) Sketch the graph of $V = 300\sqrt{3}x - \frac{9}{4}x^3$, for $0 \leq x \leq 16$. [2]
- (d) Find an expression for $\frac{dV}{dx}$. [2]
- (e) Find the value of x which maximizes the volume of the box. [2]
- (f) Hence, or otherwise, find the maximum possible volume of the box. [2]

The box will contain spherical chocolates. The production manager assumes that they can calculate the exact number of chocolates in each box by dividing the volume of the box by the volume of a single chocolate and then rounding down to the nearest integer.

- (g) Explain why the production manager is incorrect. [1]

References:

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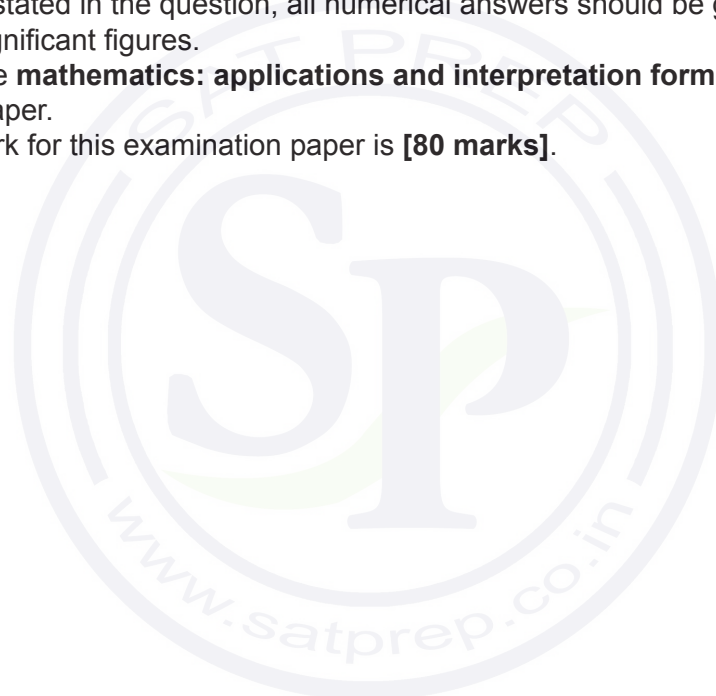
Mathematics: applications and interpretation
Standard level
Paper 2

Friday 7 May 2021 (morning)

1 hour 30 minutes

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all the questions in the answer booklet provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.



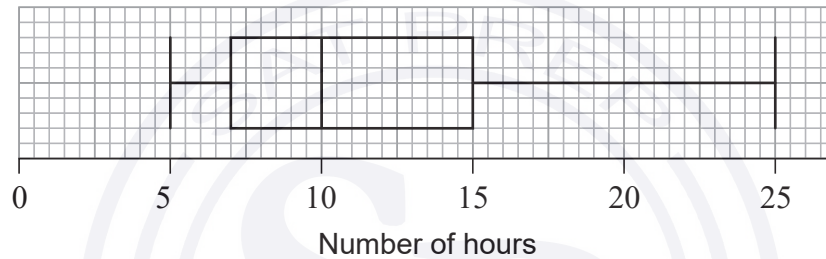
Answer **all** questions in the answer booklet provided. Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 18]

As part of his mathematics exploration about classic books, Jason investigated the time taken by students in his school to read the book *The Old Man and the Sea*. He collected his data by stopping and asking students in the school corridor, until he reached his target of 10 students from **each** of the literature classes in his school.

- (a) State which of the two sampling methods, systematic or quota, Jason has used. [1]

Jason constructed the following box and whisker diagram to show the number of hours students in the sample took to read this book.

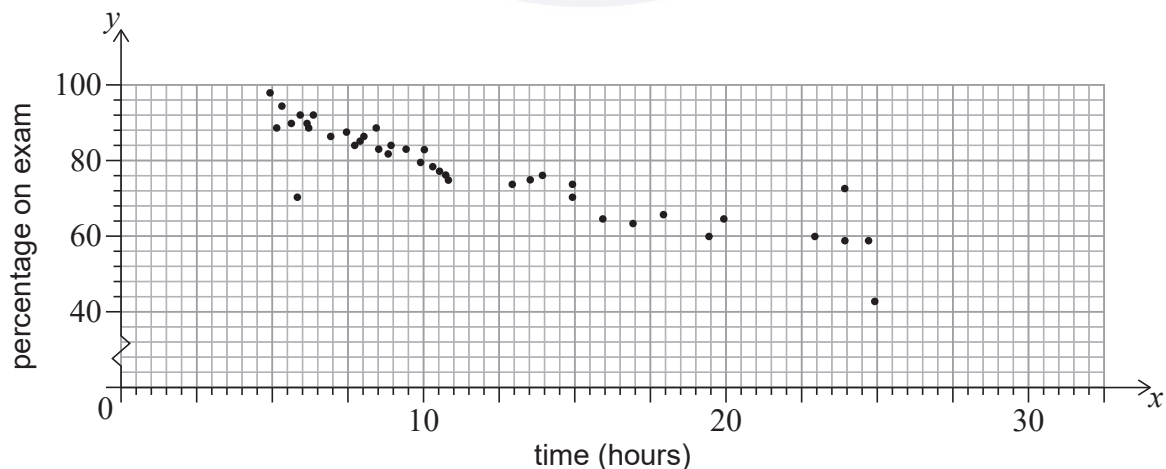


- (b) Write down the median time to read the book. [1]
 (c) Calculate the interquartile range. [2]

Mackenzie, a member of the sample, took 25 hours to read the novel. Jason believes Mackenzie's time is not an outlier.

- (d) Determine whether Jason is correct. Support your reasoning. [4]

For each student interviewed, Jason recorded the time taken to read *The Old Man and the Sea* (x), measured in hours, and paired this with their percentage score on the final exam (y). These data are represented on the scatter diagram.



- (e) Describe the correlation. [1]

(This question continues on the following page)

(Question 1 continued)

Jason correctly calculates the equation of the regression line y on x for these students to be

$$y = -1.54x + 98.8.$$

He uses the equation to estimate the percentage score on the final exam for a student who read the book in 1.5 hours.

- (f) Find the percentage score calculated by Jason. [2]
- (g) State whether it is valid to use the regression line y on x for Jason's estimate. Give a reason for your answer. [2]

Jason found a website that rated the 'top 50' classic books. He randomly chose eight of these classic books and recorded the number of pages. For example, Book H is rated 44th and has 281 pages. These data are shown in the table.

Book	A	B	C	D	E	F	G	H
Number of pages (n)	4215	863	585	1225	366	209	624	281
Top 50 rating (t)	1	2	5	7	13	22	40	44

Jason intends to analyse the data using Spearman's rank correlation coefficient, r_s .

- (h) Copy and complete the information in the following table. [2]

Book	A	B	C	D	E	F	G	H
Rank – Number of pages	1							
Rank – Top 50 Rating	1							

- (i) (i) Calculate the value of r_s .
- (ii) Interpret your result. [3]

2. [Maximum mark: 17]

The diagram below shows a circular clockface with centre O. The clock's minute hand has a length of 10 cm. The clock's hour hand has a length of 6 cm.

At 4:00 pm the endpoint of the minute hand is at point A and the endpoint of the hour hand is at point B.

diagram not to scale

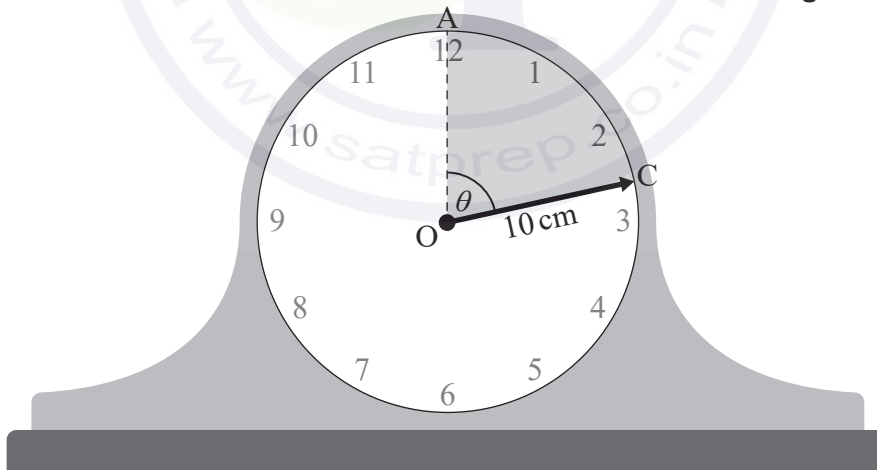


(a) Find the size of angle $\hat{A}OB$ in degrees. [2]

(b) Find the distance between points A and B. [3]

Between 4:00 pm and 4:13 pm, the endpoint of the **minute hand** rotates through an angle, θ , from point A to point C. This is illustrated in the diagram.

diagram not to scale



(c) Find the size of angle θ in degrees. [2]

(d) Calculate the length of arc AC. [2]

(e) Calculate the area of the shaded sector, AOC. [2]

(This question continues on the following page)

(Question 2 continued)

A **second** clock is illustrated in the diagram below. The clock face has radius 10 cm with minute and hour hands both of length 10 cm. The time shown is 6:00 am. The bottom of the clock face is located 3 cm above a horizontal bookshelf.

diagram not to scale



- (f) Write down the height of the endpoint of the minute hand above the bookshelf at 6:00 am. [1]

The height, h centimetres, of the endpoint of the minute hand above the bookshelf is modelled by the function

$$h(\theta) = 10 \cos \theta + 13, \theta \geq 0,$$

where θ is the angle rotated by the minute hand from 6:00 am.

- (g) Find the value of h when $\theta = 160^\circ$. [2]

The height, g centimetres, of the endpoint of the **hour hand** above the bookshelf is modelled by the function

$$g(\theta) = -10 \cos \left(\frac{\theta}{12} \right) + 13, \theta \geq 0,$$

where θ is the angle in degrees rotated by the minute hand from 6:00 am.

- (h) Write down the amplitude of $g(\theta)$. [1]

The endpoints of the minute hand and hour hand meet when $\theta = k$.

- (i) Find the smallest possible value of k . [2]

3. [Maximum mark: 19]

Give your answers in parts (a), (d)(i), (e) and (f) to the nearest dollar.

Daisy invested 37 000 Australian dollars (AUD) in a fixed deposit account with an annual interest rate of 6.4% compounded **quarterly**.

(a) Calculate the value of Daisy’s investment after 2 years. [3]

After m months, the amount of money in the fixed deposit account has appreciated to more than 50 000 AUD.

(b) Find the minimum value of m , where $m \in \mathbb{N}$. [4]

Daisy is saving to purchase a new apartment. The price of the apartment is 200 000 AUD.

Daisy makes an initial payment of 25% and takes out a loan to pay the rest.

(c) Write down the amount of the loan. [1]

The loan is for 10 years, compounded monthly, with equal monthly payments of 1700 AUD made by Daisy at the end of each month.

(d) For this loan, find

(i) the amount of interest paid by Daisy.

(ii) the annual interest rate of the loan. [5]

After 5 years of paying off this loan, Daisy decides to pay the **remainder** in one final payment.

(e) Find the amount of Daisy’s final payment. [3]

(f) Find how much money Daisy saved by making one final payment after 5 years. [3]

4. [Maximum mark: 13]

The stopping distances for bicycles travelling at 20 km h^{-1} are assumed to follow a normal distribution with mean 6.76 m and standard deviation 0.12 m .

(a) Under this assumption, find, correct to four decimal places, the probability that a bicycle chosen at random travelling at 20 km h^{-1} manages to stop

(i) in less than 6.5 m .

(ii) in more than 7 m .

[3]

1000 randomly selected bicycles are tested and their stopping distances when travelling at 20 km h^{-1} are measured.

(b) Find, correct to four significant figures, the expected number of bicycles tested that stop between

(i) 6.5 m and 6.75 m .

(ii) 6.75 m and 7 m .

[3]

The measured stopping distances of the 1000 bicycles are given in the table.

Measured stopping distance	Number of bicycles
Less than 6.5 m	12
Between 6.5 m and 6.75 m	428
Between 6.75 m and 7 m	527
More than 7 m	33

It is decided to perform a χ^2 goodness of fit test at the 5% level of significance to decide whether the stopping distances of bicycles travelling at 20 km h^{-1} can be modelled by a normal distribution with mean 6.76 m and standard deviation 0.12 m .

(c) State the null and alternative hypotheses.

[2]

(d) Find the p -value for the test.

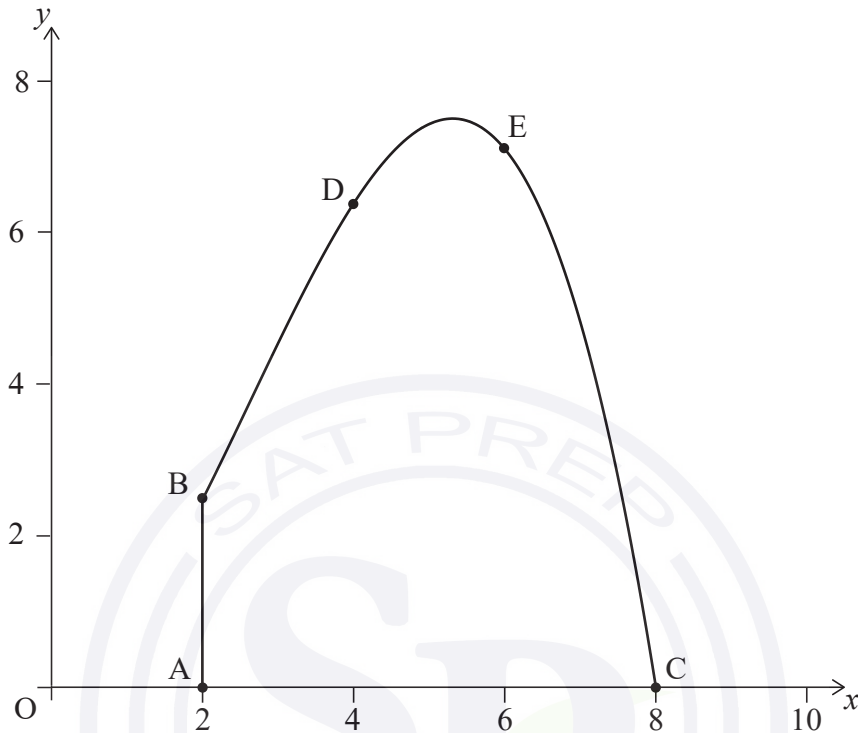
[3]

(e) State the conclusion of the test. Give a reason for your answer.

[2]

5. [Maximum mark: 13]

The cross-sectional view of a tunnel is shown on the axes below. The line $[AB]$ represents a vertical wall located at the left side of the tunnel. The height, in metres, of the tunnel above the horizontal ground is modelled by $y = -0.1x^3 + 0.8x^2$, $2 \leq x \leq 8$, relative to an origin O .



Point A has coordinates $(2, 0)$, point B has coordinates $(2, 2.4)$, and point C has coordinates $(8, 0)$.

- (a) (i) Find $\frac{dy}{dx}$.
- (ii) Hence find the maximum height of the tunnel. [6]

When $x = 4$ the height of the tunnel is 6.4 m and when $x = 6$ the height of the tunnel is 7.2 m. These points are shown as D and E on the diagram, respectively.

- (b) Use the trapezoidal rule, with three intervals, to estimate the cross-sectional area of the tunnel. [3]
- (c) (i) Write down the integral which can be used to find the cross-sectional area of the tunnel.
- (ii) Hence find the cross-sectional area of the tunnel. [4]

References:

Mathematics: applications and interpretation
Standard level
Paper 2

Specimen paper

1 hour 30 minutes

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
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Answer **all** questions in the answer booklet provided. Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 17]

In this question, give all answers to two decimal places.

Bryan decides to purchase a new car with a price of €14 000, but cannot afford the full amount. The car dealership offers two options to finance a loan.

Finance option A:

A 6 year loan at a nominal annual interest rate of 14% **compounded quarterly**. No deposit required and repayments are made each quarter.

- (a) (i) Find the repayment made each quarter.
- (ii) Find the total amount paid for the car.
- (iii) Find the interest paid on the loan. [7]

Finance option B:

A 6 year loan at a nominal annual interest rate of $r\%$ **compounded monthly**. Terms of the loan require a 10% deposit and monthly repayments of €250.

- (b) (i) Find the amount to be borrowed for this option.
- (ii) Find the annual interest rate, r . [5]
- (c) State which option Bryan should choose. Justify your answer. [2]

Bryan's car depreciates at an annual rate of 25% per year.

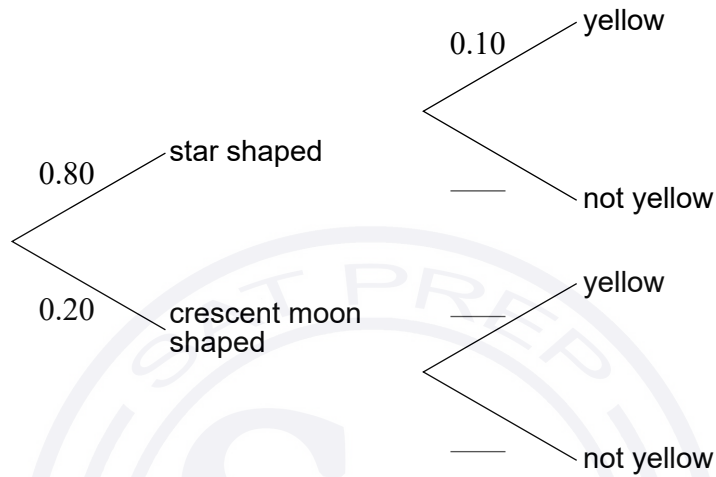
- (d) Find the value of Bryan's car six years after it is purchased. [3]

2. [Maximum mark: 14]

Slugworth Candy Company sell a variety pack of colourful, shaped sweets.

The sweets are produced such that 80% are star shaped and 20% are shaped like a crescent moon. It is known that 10% of the stars and 30% of the crescent moons are coloured yellow.

(a) Using the given information, **copy** and complete the following tree diagram. [2]



(b) A sweet is selected at random.

(i) Find the probability that the sweet is yellow.

(ii) Given that the sweet is yellow, find the probability it is star shaped. [4]

(This question continues on the following page)

(Question 2 continued)

According to manufacturer specifications, the colours in each variety pack should be distributed as follows.

Colour	Brown	Red	Green	Orange	Yellow	Purple
Percentage (%)	15	25	20	20	10	10

Mr Slugworth opens a pack of 80 sweets and records the frequency of each colour.

Colour	Brown	Red	Green	Orange	Yellow	Purple
Observed Frequency	10	20	16	18	12	4

To investigate if the sample is consistent with manufacturer specifications, Mr Slugworth conducts a χ^2 goodness of fit test. The test is carried out at a 5% significance level.

- (c) Write down the null hypothesis for this test. [1]
- (d) **Copy** and complete the following table in your answer booklet. [2]

Colour	Brown	Red	Green	Orange	Yellow	Purple
Expected Frequency						

- (e) Write down the number of degrees of freedom. [1]
- (f) Find the p -value for the test. [2]
- (g) State the conclusion of the test. Give a reason for your answer. [2]

3. [Maximum mark: 17]

The Malvern Aquatic Center hosted a 3 metre spring board diving event. The judges, Stan and Minsun awarded 8 competitors a score out of 10. The raw data is collated in the following table.

Competitors	A	B	C	D	E	F	G	H
Stan's score (x)	4.1	3	4.3	6	7.1	6	7.5	6
Minsun's score (y)	4.7	4.6	4.8	7.2	7.8	9	9.5	7.2

- (a) (i) Write down the value of the Pearson's product-moment correlation coefficient, r .
 (ii) Using the value of r , interpret the relationship between Stan's score and Minsun's score. [4]
- (b) Write down the equation of the regression line y on x . [2]
- (c) (i) Use your regression equation from part (b) to estimate Minsun's score when Stan awards a perfect 10.
 (ii) State whether this estimate is reliable. Justify your answer. [4]

The Commissioner for the event would like to find the Spearman's rank correlation coefficient.

- (d) **Copy** and complete the information in the following table. [2]

Competitors	A	B	C	D	E	F	G	H
Stan's Rank		8					1	4
Minsun's Rank		8					1	4.5

- (e) (i) Find the value of the Spearman's rank correlation coefficient, r_s .
 (ii) Comment on the result obtained for r_s . [4]

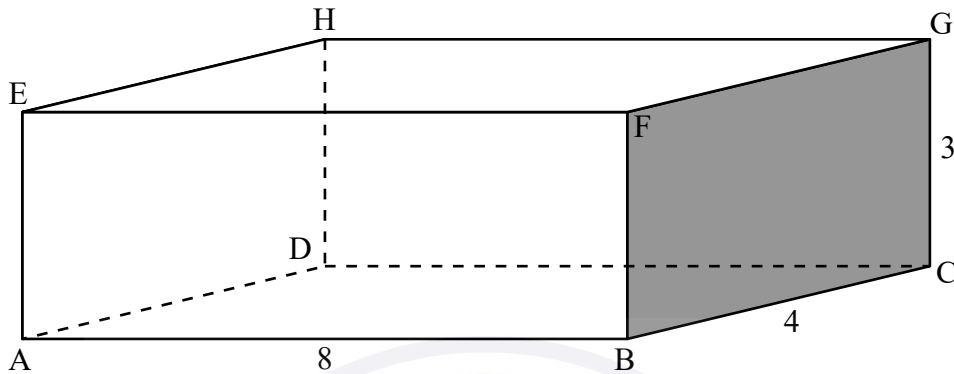
The Commissioner believes Minsun's score for competitor G is too high and so decreases the score from 9.5 to 9.1.

- (f) Explain why the value of the Spearman's rank correlation coefficient r_s does not change. [1]

4. [Maximum mark: 15]

The Happy Straw Company manufactures drinking straws.

The straws are packaged in small closed rectangular boxes, each with length 8 cm, width 4 cm and height 3 cm. The information is shown in the diagram.



- (a) Calculate the surface area of the box in cm^2 . [2]
- (b) Calculate the length AG. [2]

Each week, the Happy Straw Company sells x boxes of straws. It is known that $\frac{dP}{dx} = -2x + 220$, $x \geq 0$, where P is the weekly profit, in dollars, from the sale of x thousand boxes.

- (c) Find the number of boxes that should be sold each week to maximize the profit. [3]

The profit from the sale of 20 000 boxes is \$1700.

- (d) Find $P(x)$. [5]
- (e) Find the least number of boxes which must be sold each week in order to make a profit. [3]

5. [Maximum mark: 17]

The braking distance of a vehicle is defined as the distance travelled from where the brakes are applied to the point where the vehicle comes to a complete stop.

The speed, $s \text{ m s}^{-1}$, and braking distance, $d \text{ m}$, of a truck were recorded. This information is summarized in the following table.

Speed, $s \text{ m s}^{-1}$	0	6	10
Braking distance, $d \text{ m}$	0	12	60

This information was used to create Model A, where d is a function of s , $s \geq 0$.

$$\text{Model A: } d(s) = ps^2 + qs, \text{ where } p, q \in \mathbb{Z}$$

At a speed of 6 m s^{-1} , Model A can be represented by the equation $6p + q = 2$.

- (a) (i) Write down a second equation to represent Model A, when the speed is 10 m s^{-1} .
 (ii) Find the values of p and q [4]
- (b) Find the coordinates of the vertex of the graph of $y = d(s)$. [2]
- (c) Using the values in the table and your answer to part (b), sketch the graph of $y = d(s)$ for $0 \leq s \leq 10$ and $-10 \leq d \leq 60$, clearly showing the vertex. [3]
- (d) Hence, identify why Model A may not be appropriate at lower speeds. [1]

Additional data was used to create Model B, **a revised model** for the braking distance of a truck.

$$\text{Model B: } d(s) = 0.95s^2 - 3.92s$$

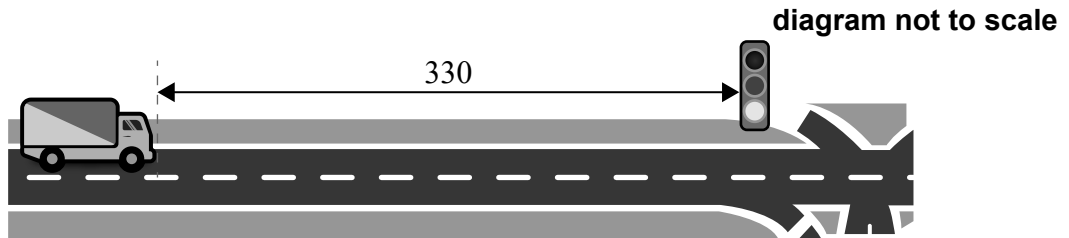
- (e) Use Model B to calculate an estimate for the braking distance at a speed of 20 m s^{-1} . [2]
- The actual braking distance at 20 m s^{-1} is 320 m .
- (f) Calculate the percentage error in the estimate in part (e). [2]

(This question continues on the following page)

(Question 5 continued)

It is found that once a driver realizes the need to stop their vehicle, 1.6 seconds will elapse, on average, before the brakes are engaged. During this reaction time, the vehicle will continue to travel at its original speed.

A truck approaches an intersection with speed $s \text{ m s}^{-1}$. The driver notices the intersection's traffic lights are red and they must stop the vehicle within a distance of 330 m.



- (g) Using model B and taking reaction time into account, calculate the maximum possible speed of the truck if it is to stop before the intersection.

[3]

