Mathematics: applications and interpretation
Standard level
Paper 2

31 October 2023

Zone A afternoon | **Zone B** afternoon | **Zone C** afternoon

1 hour 30 minutes

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all the questions in the answer booklet provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

9 pages

8823 – 7215

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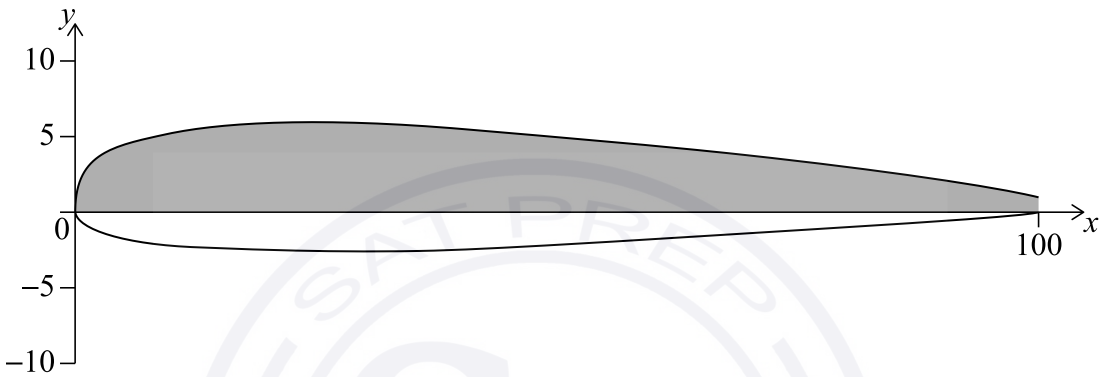
Answer **all** questions in the answer booklet provided. Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 12]

Gabriel is investigating the shape of model airplane wings. A cross-section of one of the wings is shown, graphed on the coordinate axes.



The shaded part of the cross-section is the area between the x-axis and the curve with equation

$$y = 2\sqrt{x} - \frac{x}{5} + 1, \text{ for } 0 \le x \le 100$$

where x is the distance, in cm, from the front of the wing and y is the height, in cm, above the horizontal axis through the wing, as shown in the diagram.

(a) Find the values of a, b and c, shown in the table. [3]

***x* (cm)**	0	25	50	75	100
***y* (cm)**	1	a	b	c	1

Gabriel uses the trapezoidal rule with four intervals to estimate the shaded area of the cross-section of the wing.

(b) Find Gabriel's estimate of the shaded area of the cross-section. [3]

(c) (i) Write down the integral that Gabriel can use to find the exact area of the shaded part of the cross-section.

(ii) Hence, use your graphic display calculator to find the area of the shaded part of the cross-section. Give your answer correct to one decimal place. [4]

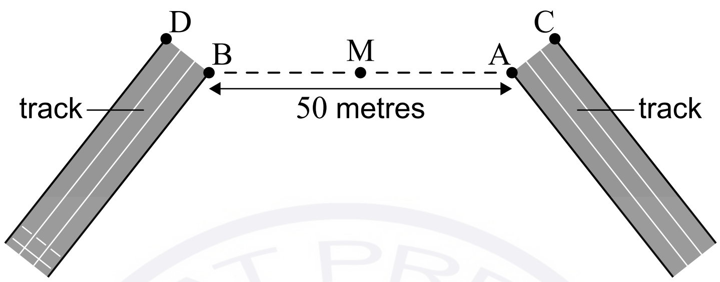
(d) Calculate the percentage error of Gabriel's estimate in part (b). [2]

2. [Maximum mark: 15]

Madhu is designing a jogging track for the campus of her school. The following diagram shows an incomplete portion of the track.

Madhu wants to design the track such that the inner edge is a smooth curve from point A to point B, and the other edge is a smooth curve from point C to point D. The distance between points A and B is 50 metres.

diagram not to scale

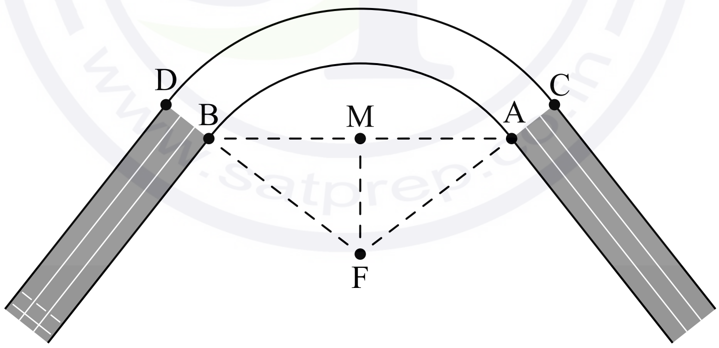


To create a smooth curve, Madhu first walks to M, the midpoint of [AB].

(a) Write down the length of [BM]. [1]

Madhu then walks 20 metres in a direction perpendicular to [AB] to get from point M to point F. Point F is the centre of a circle whose arc will form the smooth curve between points A and B on the track, as shown in the following diagram.

diagram not to scale



(b) (i) Find the length of [BF].

(ii) Find $B\hat{F}M$. [4]

(c) Hence, find the length of arc AB. [3]

(This question continues on the following page)

(Question 2 continued)

The outer edge of the track, from C to D, is also a circular arc with centre F, such that the track is 2 metres wide.

(d) Calculate the area of the curved portion of the track, ABDC. [4]

The base of the track will be made of concrete that is $12\,\mathrm{cm}$ deep.

(e) Calculate the volume of concrete needed to create the curved portion of the track. [3]

3. [Maximum mark: 18]

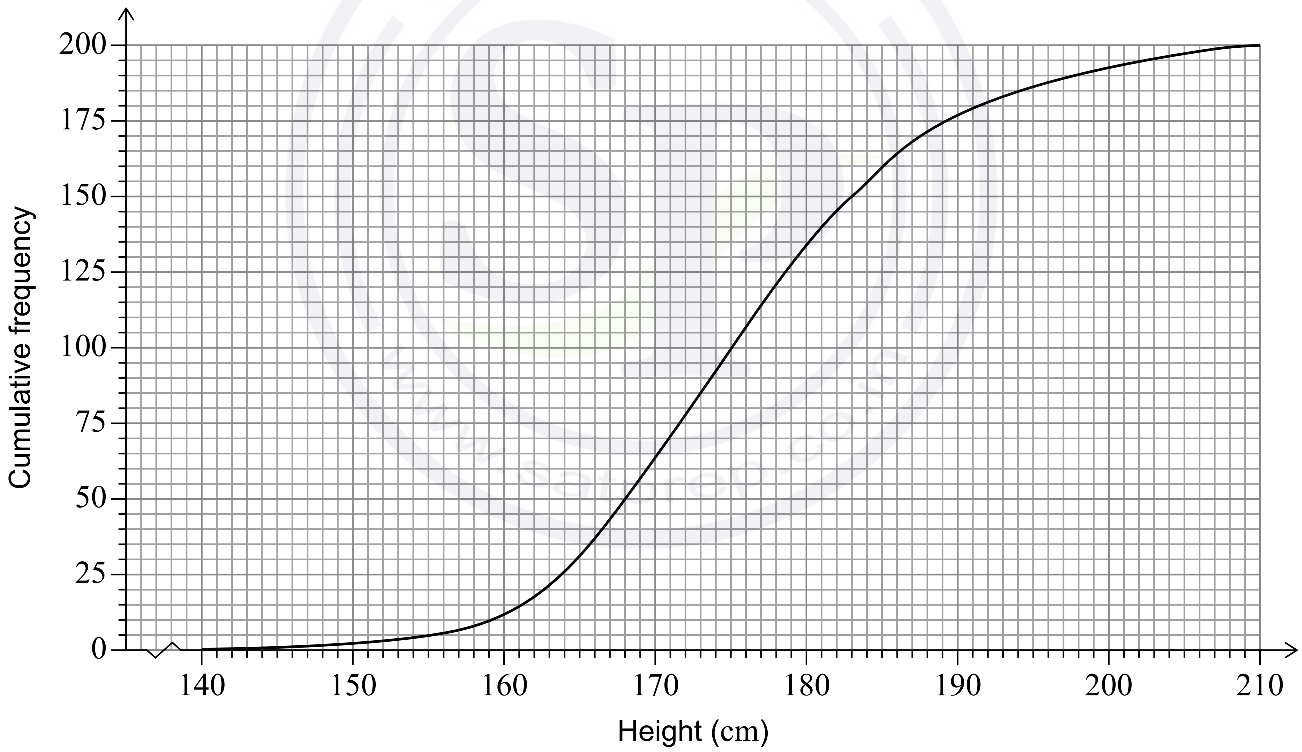
The heights, h, of 200 university students are recorded in the following table.

Height (cm)	**Frequency**
$140 \le h < 160$	11
$160 \le h < 170$	51
$170 \le h < 180$	68
$180 \le h < 190$	47
$190 \le h < 210$	23

(a) (i) Write down the mid-interval value of $140 \le h < 160$.

(ii) Calculate an estimate of the mean height of the 200 students. [3]

This table is used to create the following cumulative frequency graph.



(b) Use the cumulative frequency curve to estimate the interquartile range. [2]

Laszlo is a student in the data set and his height is $204\,\text{cm}$.

(c) Use your answer to part (b) to estimate whether Laszlo's height is an outlier for this data. Justify your answer. [3]

(This question continues on the following page)

(Question 3 continued)

It is believed that the heights of university students follow a normal distribution with mean $176\,\text{cm}$ and standard deviation $13.5\,\text{cm}$.

It is decided to perform a χ^2 goodness of fit test on the data to determine whether this sample of 200 students could have plausibly been drawn from an underlying distribution $\text{N}(176,\ 13.5^2)$.

(d) Write down the null and the alternative hypotheses for the test. [2]

As part of the test, the following table is created.

Height of student (cm)	**Observed frequency**	**Expected frequency**
$h < 160$	11	23.6
$160 \le h < 170$	51	42.1
$170 \le h < 180$	68	a
$180 \le h < 190$	47	46.7
$190 \le h$	23	b

(e) (i) Find the value of a and the value of b.

(ii) Hence, perform the test to a 5% significance level, clearly stating the conclusion in context. [8]

4. [Maximum mark: 16]

Tiffany wants to buy a house for a price of $285\,000$ US Dollars (USD). She goes to a bank to get a loan to buy the house. To be eligible for the loan, Tiffany must make an initial down payment equal to $15\,\%$ of the price of the house.

The bank offers her a 30-year loan for the remaining balance, with a $4\,\%$ nominal interest rate per annum, compounded monthly. Tiffany will pay the loan in fixed payments at the end of each month.

(a) (i) Find the original amount of the loan after the down payment is paid. Give the exact answer.

(ii) Calculate Tiffany's monthly payment for this loan, to two decimal places. [5]

(b) Using your answer from part (a)(ii), calculate the total amount Tiffany will pay over the life of the loan, to the nearest dollar. Do **not** include the initial down payment. [2]

Tiffany would like to repay the loan faster and increases her payments such that she pays $1300\,\text{USD}$ each month.

(c) Find the total number of monthly payments she will need to make to pay off the loan. [2]

This strategy will result in Tiffany's final payment being less than $1300\,\text{USD}$.

(d) Determine the amount of Tiffany's final payment, to two decimal places. [4]

(e) Hence, determine the total amount Tiffany will save, to the nearest dollar, by making the higher monthly payments. [3]

5. [Maximum mark: 19]

Howell Industries is an international company that sells travel coffee mugs. Their market research predicts that the average number of mugs they will sell each month is modelled by the equation

$$n = 20\,000 - 1000x$$

where x represents the selling price, in euro (EUR), of each mug.

A salesperson suggests that Howell Industries should sell the mugs for $16\,\text{EUR}$ each.

(a) Find the average number of mugs that this model predicts Howell Industries will sell at this price. [2]

(b) Calculate Howell Industries' average monthly income, before any expenses, at this selling price. [2]

(c) Hence, write down the function $R(x)$ that can be used to predict Howell Industries' average monthly income, before expenses, at any selling price, x. [1]

Howell Industries has $10\,000\,\text{EUR}$ of fixed monthly operational costs. Additionally, Howell Industries must pay their mug supplier $10\,\text{EUR}$ for each coffee mug.

(d) Calculate Howell Industries' average monthly profit if they sell each mug at a price of $16\,\text{EUR}$. [3]

(e) Show that the average monthly profit for any selling price, x, can be found using the function $P(x) = -1000x^2 + 30\,000x - 210\,000$. [2]

(f) (i) Find $P'(x)$.

(ii) Show that the salesperson's selling price does not maximize their average monthly profit. [4]

Howell Industries negotiates a new deal with their mug supplier. Under the new deal, the supplier agrees to discount the cost of each mug based on the number of mugs purchased by Howell Industries. The cost charged by the supplier for each mug can be found using the function

$$C(n) = 10 - 0.0001n$$

where n represents the number of mugs sold by Howell Industries.

(g) Find the function that can be used to find Howell Industries' average monthly profit using the new deal from the supplier. [3]

(h) Hence, find the selling price, per mug, that Howell Industries should choose in order to maximize their average monthly profit under the new deal. [2]

Diploma Programme
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8823 – 7210

International Baccalaureate
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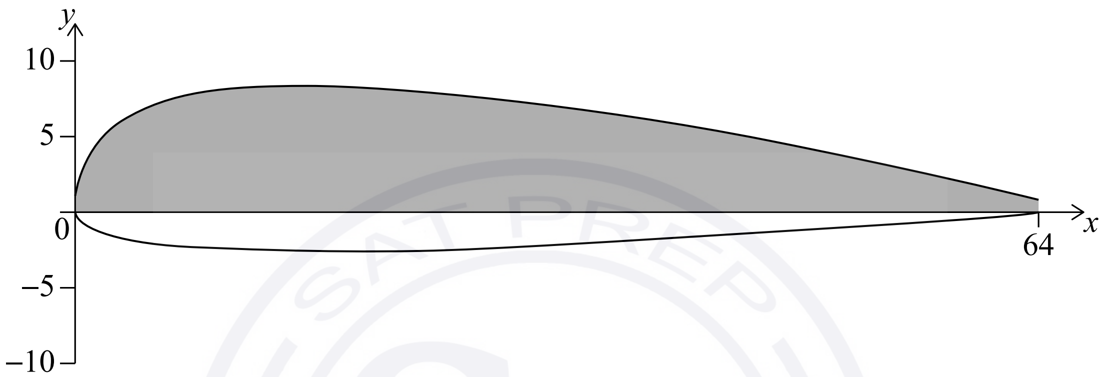
1. [Maximum mark: 12]

Jan is investigating the shape of model helicopter propeller blades. A cross-section of one of the blades is shown, graphed on the coordinate axes.



The shaded part of the cross-section is the area between the x-axis and the curve with equation

$$y = 4\sqrt{x} - \frac{x}{2} + 1, \text{ for } 0 \le x \le 64$$

where x is the distance, in mm, from the edge of the blade and y is the height, in mm, above the horizontal axis through the blade, as shown in the diagram.

(a) Find the values of a, b and c, shown in the table. [3]

x (mm)	0	16	32	48	64
y (mm)	1	a	b	c	1

Jan uses the trapezoidal rule with four intervals to estimate the shaded area of the cross-section of the blade.

(b) Find Jan's estimate of the shaded area of the cross-section. [3]

(c) (i) Write down the integral that Jan can use to find the exact area of the shaded part of the cross-section.

(ii) Hence, use your graphic display calculator to find the area of the shaded part of the cross-section. Give your answer correct to one decimal place. [4]

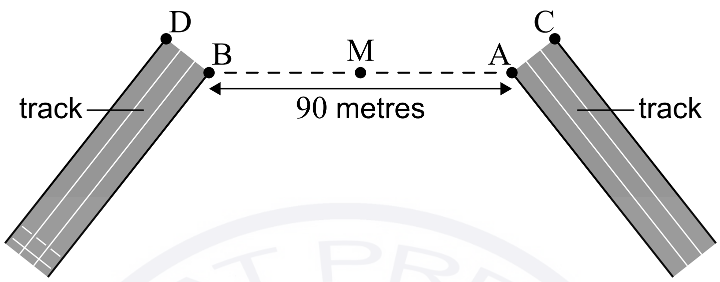
(d) Calculate the percentage error of Jan's estimate in part (b). [2]

2. [Maximum mark: 15]

Ansel is designing a racing track for a local bicycle club. The following diagram shows an incomplete portion of the track.

Ansel wants to design the track such that the inner edge is a smooth curve from point A to point B, and the other edge is a smooth curve from point C to point D. The distance between points A and B is 90 metres.

diagram not to scale

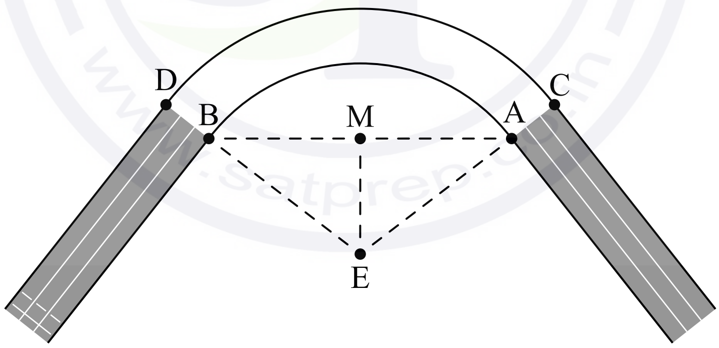


To create a smooth curve, Ansel first walks to M, the midpoint of $[\mathrm{AB}]$.

(a) Write down the length of $[\mathrm{BM}]$. [1]

Ansel then walks 32 metres in a direction perpendicular to $[\mathrm{AB}]$ to get from point M to point E. Point E is the centre of a circle whose arc will form the smooth curve between points A and B on the track, as shown in the following diagram.

diagram not to scale



(b) (i) Find the length of $[\mathrm{BE}]$.

(ii) Find $\mathrm{B\hat{E}M}$. [4]

(c) Hence, find the length of arc AB. [3]

(This question continues on the following page)

(Question 2 continued)

The outer edge of the track, from C to D, is also a circular arc with centre E, such that the track is 4 metres wide.

(d) Calculate the area of the curved portion of the track, ABDC. [4]

The base of the track will be made of concrete that is 15 cm deep.

(e) Calculate the volume of concrete needed to create the curved portion of the track. [3]

3. [Maximum mark: 18]

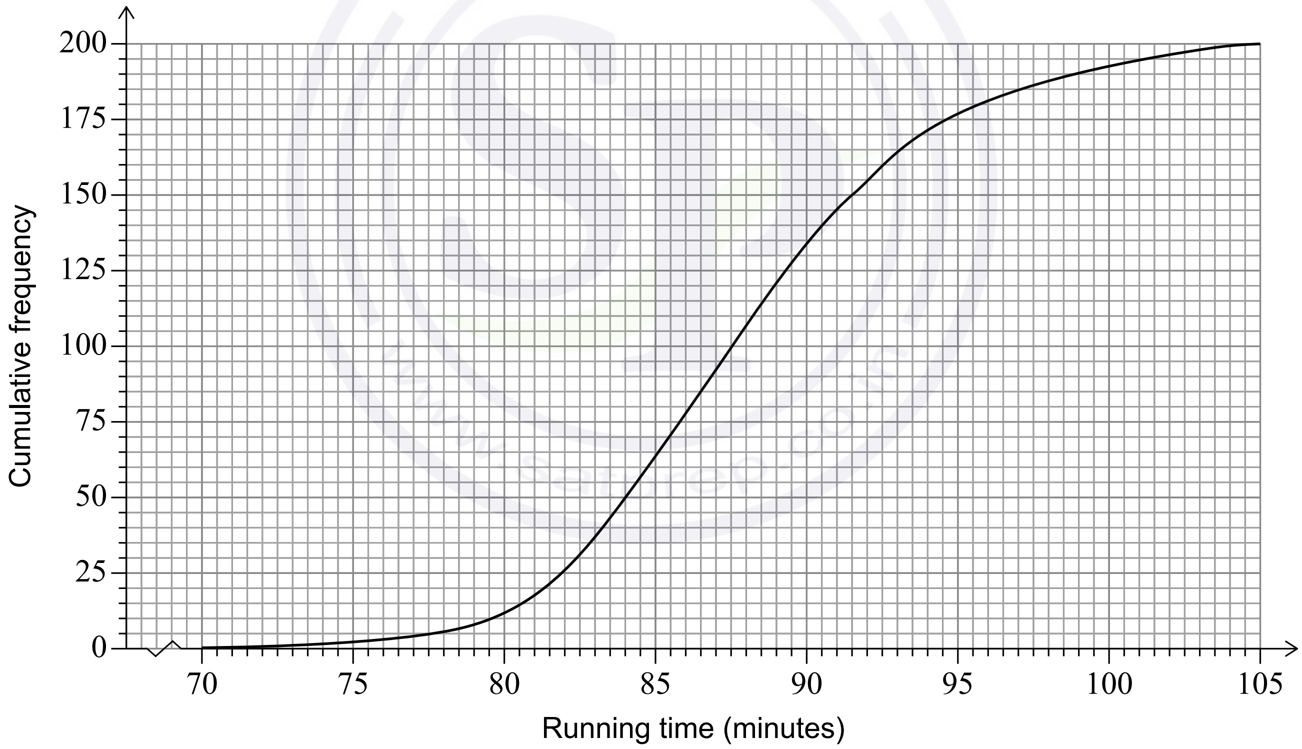
The running time, t (minutes), of 200 family movies are recorded in the following table.

Running time (minutes)	**Frequency**
$70 \le t < 80$	11
$80 \le t < 85$	51
$85 \le t < 90$	68
$90 \le t < 95$	47
$95 \le t < 105$	23

(a) (i) Write down the mid-interval value of $70 \le t < 80$.

(ii) Calculate an estimate of the mean running time of the 200 movies. [3]

This table is used to create the following cumulative frequency graph.



(b) Use the cumulative frequency curve to estimate the interquartile range. [2]

"Star Feud" is a movie in the data set and its running time is 100 minutes.

(c) Use your answer to part (b) to estimate whether "Star Feud's" running time is an outlier for this data. Justify your answer. [3]

(This question continues on the following page)

(Question 3 continued)

It is believed that the running times of family movies follow a normal distribution with mean 88 minutes and standard deviation 6.75 minutes.

It is decided to perform a χ^2 goodness of fit test on the data to determine whether this sample of 200 movies could have plausibly been drawn from an underlying distribution $N(88, 6.75^2)$.

(d) Write down the null and the alternative hypotheses for the test. [2]

As part of the test, the following table is created.

Movie running time (minutes)	Observed frequency	Expected frequency
$t < 80$	11	23.6
$80 \le t < 85$	51	42.1
$85 \le t < 90$	68	a
$90 \le t < 95$	47	46.7
$95 \le t$	23	b

(e) (i) Find the value of a and the value of b.

(ii) Hence, perform the test to a 5% significance level, clearly stating the conclusion in context. [8]

4. [Maximum mark: 16]

Ruben wants to buy a car for a price of $285\,000$ South African rand (ZAR). He goes to a bank to get a loan to buy the car. To be eligible for the loan, Ruben must make an initial down payment equal to $25\,\%$ of the price of the car.

The bank offers him a 5-year loan for the remaining balance, with a $4.5\,\%$ nominal interest rate per annum, compounded monthly. Ruben will pay the loan in fixed payments at the end of each month.

(a) (i) Find the original amount of the loan after the down payment is paid. Give the exact answer.

(ii) Calculate Ruben's monthly payment for this loan, to two decimal places. [5]

(b) Using your answer from part (a)(ii), calculate the total amount Ruben will pay over the life of the loan, to the nearest ZAR. Do **not** include the initial down payment. [2]

Ruben would like to repay the loan faster and increases his payments such that he pays $4600\,\text{ZAR}$ each month.

(c) Find the total number of monthly payments he will need to make to pay off the loan. [2]

This strategy will result in Ruben's final payment being less than $4600\,\text{ZAR}$.

(d) Determine the amount of Ruben's final payment, to two decimal places. [4]

(e) Hence, determine the total amount Ruben will save, to the nearest ZAR, by making the higher monthly payments. [3]

5. [Maximum mark: 19]

Mosaic Industries is an international company that sells cell phone cases. Their market research predicts that the average number of cases they will sell each month is modelled by the equation

$$n = 20\,000 - 1000x$$

where x represents the selling price, in euro (EUR), of each case.

A salesperson suggests that Mosaic Industries should sell the cases for $16\,\text{EUR}$ each.

(a) Find the average number of cases that this model predicts Mosaic Industries will sell at this price. [2]

(b) Calculate Mosaic Industries' average monthly income, before any expenses, at this selling price. [2]

(c) Hence, write down the function $R(x)$ that can be used to predict Mosaic Industries' average monthly income, before expenses, at any selling price, x. [1]

Mosaic Industries has $10\,000\,\text{EUR}$ of fixed monthly operational costs. Additionally, Mosaic Industries must pay their phone case supplier $10\,\text{EUR}$ for each case.

(d) Calculate Mosaic Industries' average monthly profit if they sell each case at a price of $16\,\text{EUR}$. [3]

(e) Show that the average monthly profit for any selling price, x, can be found using the function $P(x) = -1000x^2 + 30\,000x - 210\,000$. [2]

(f) (i) Find $P'(x)$.

(ii) Show that the salesperson's selling price does not maximize their average monthly profit. [4]

Mosaic Industries negotiates a new deal with their phone case supplier. Under the new deal, the supplier agrees to discount the cost of each case based on the number of cases purchased by Mosaic Industries. The cost charged by the supplier for each case can be found using the function

$$C(n) = 10 - 0.0001n$$

where n represents the number of cases sold by Mosaic Industries.

(g) Find the function that can be used to find Mosaic Industries' average monthly profit using the new deal from the supplier. [3]

(h) Hence, find the selling price, per case, that Mosaic Industries should choose in order to maximize their average monthly profit under the new deal. [2]

Diploma Programme
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Programa del Diploma

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2223 – 7215

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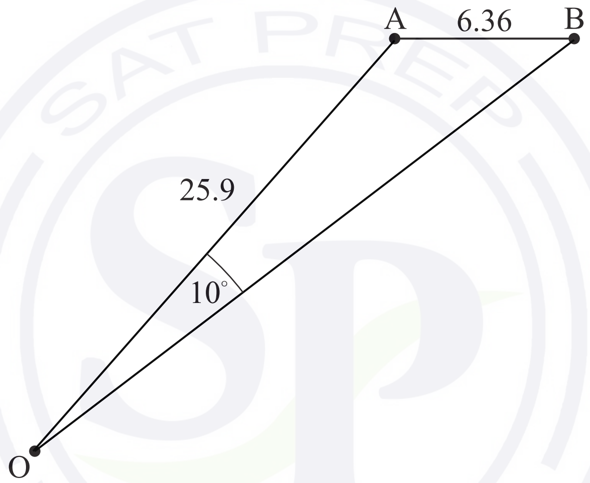
1. [Maximum mark: 17]

The diagram shows points in a park viewed from above, at a specific moment in time.

The distance between two trees, at points A and B, is $6.36\,\text{m}$.

Odette is playing football in the park and is standing at point O, such that $\text{A}\hat{\text{O}}\text{B} = 10^\circ$, $\text{OA} = 25.9\,\text{m}$ and $\text{O}\hat{\text{A}}\text{B}$ is obtuse.

diagram not to scale



(a) Calculate the size of $\text{A}\hat{\text{B}}\text{O}$. [3]

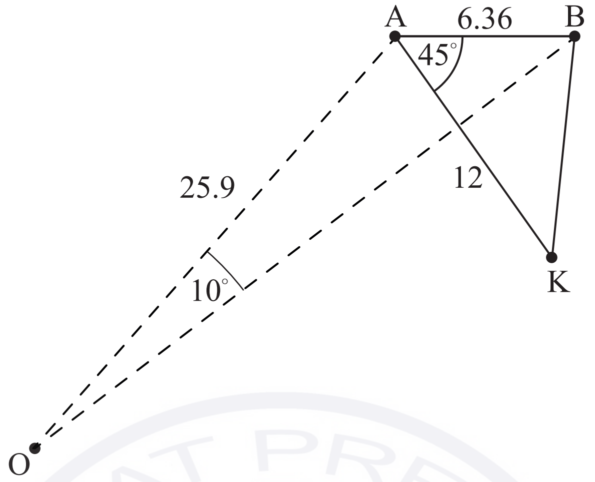
(b) Calculate the area of triangle AOB. [4]

(This question continues on the following page)

(Question 1 continued)

Odette's friend, Khemil, is standing at point K such that he is $12\,m$ from A and $K\hat{A}B = 45^\circ$.

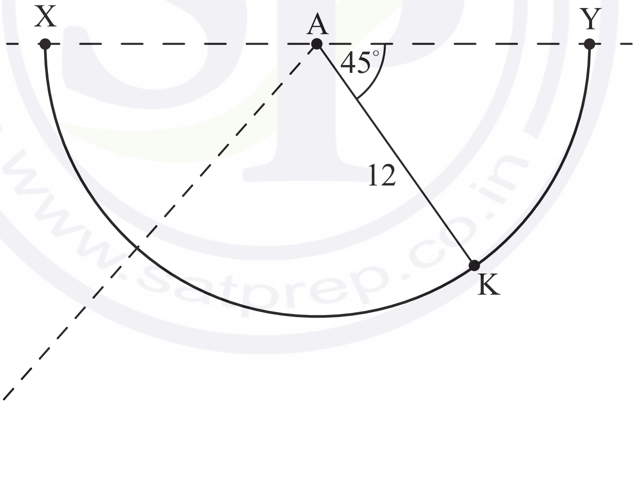
diagram not to scale



(c) Calculate Khemil's distance from B. [3]

XY is a semicircular path in the park with centre A, such that $K\hat{A}Y = 45^\circ$. Khemil is standing on the path and Odette's football is at point X. This is shown in the diagram below.

diagram not to scale



The length $KX = 22.2\,m$, $K\hat{O}X = 53.8^\circ$ and $O\hat{K}X = 51.1^\circ$.

(d) Find whether Odette or Khemil is closer to the football. [4]

Khemil runs along the semicircular path to pick up the football.

(e) Calculate the distance that Khemil runs. [3]

2. [Maximum mark: 15]

Daina makes pendulums to sell at a market. She plans to make 10 pendulums on the first day and, on each subsequent day, make 6 more than she did the day before.

(a) Calculate the number of pendulums she would make on the 12th day. [3]

She plans to make pendulums for a **total** of 15 days in preparation for going to the market.

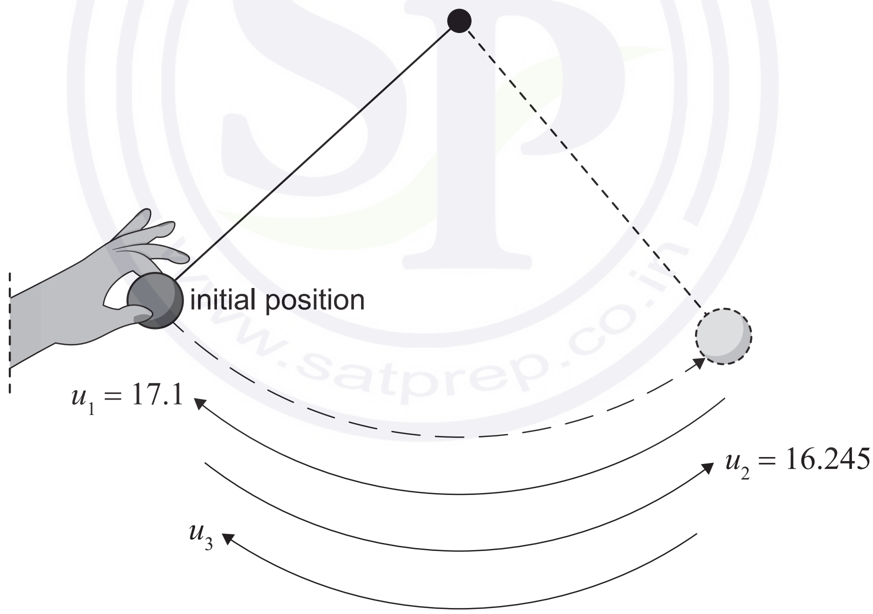
(b) Calculate the total number of pendulums she would have available at the market. [2]

Daina would like to have at least 1000 pendulums available to sell at the market and therefore decides to increase her production. She still plans to make 10 pendulums on the first day, but on each subsequent day, she will make x more than she did the day before.

(c) Given that she will still make pendulums for a total of 15 days, calculate the minimum integer value of x required for her to reach her target. [3]

Daina tests one of her pendulums. She releases the ball at the end of the pendulum to swing freely. The point at which she releases it is shown as the initial position on the left side of the following diagram. Daina begins recording the distances travelled by the ball **after** it has reached the extreme position, represented by the right-hand side of the diagram.

diagram not to scale



(This question continues on the following page)

(Question 2 continued)

On each successive swing, the distance that the ball travelled was 95% of its previous distance. During the first swing that Daina recorded, the ball travelled a distance of 17.1 cm. During the second swing that she recorded, it travelled a distance of 16.245 cm.

(d) Calculate the distance that the ball travelled during the 5th recorded swing. [3]

(e) Calculate the total distance that the ball travelled during the first 16 recorded swings. [2]

(f) Calculate the distance that the ball travelled before Daina started recording. [2]

3. [Maximum mark: 15]

A scientist is conducting an experiment on the growth of a certain species of bacteria.

The population of the bacteria, P, can be modelled by the function

$$P(t) = 1200 \times k^{t},\ t \geq 0,$$

where t is the number of hours since the experiment began, and k is a positive constant.

(a) (i) Write down the value of $P(0)$.

(ii) Interpret what this value means in this context. [2]

3 hours after the experiment began, the population of the bacteria is $18\,750$.

(b) Find the value of k. [2]

(c) Find the population of the bacteria 1 hour and 30 minutes after the experiment began. [2]

The scientist conducts a second experiment with a different species of bacteria.
The population of this bacteria, S, can be modelled by the function

$$S(t) = 5000 \times 1.65^{t},\ t \geq 0,$$

where t is the number of hours since both experiments began.

(d) Find the value of t when the two populations of bacteria are equal. [2]

It takes 2 hours and m minutes for the number of bacteria in the second experiment to reach $19\,000$.

(e) Find the value of m, giving your answer as an integer value. [4]

The bacteria in the second experiment are growing inside a container. The scientist models the volume of each bacterium in the second experiment to be $1 \times 10^{-18}\,\text{m}^3$, and the available volume inside the container is $2.1 \times 10^{-5}\,\text{m}^3$.

(f) Determine how long it would take for the bacteria to fill the container. [3]

4. [Maximum mark: 17]

It is claimed that a new remedy cures 82% of the patients with a particular medical problem.

This remedy is to be used by 115 patients, and it is assumed that the 82% claim is true.

(a) Find the probability that exactly 90 of these patients will be cured. [3]

(b) Find the probability that at least 95 of these patients will be cured. [2]

(c) Find the variance in the possible number of patients that will be cured. [2]

The probability that at least n patients will be cured is less than 30%.

(d) Find the least value of n. [3]

A clinic is interested to see if the mean recovery time of their patients who tried the new remedy is less than that of their patients who continued with an older remedy. The clinic randomly selects some of their patients and records their recovery time in days. The results are shown in the table below.

	Recovery time (days)								
Group N (new remedy)	12	13	9	13	14	15	17		

	Recovery time (days)								
Group O (old remedy)	17	11	10	18	20	22	14	15	18

The data is assumed to follow a normal distribution and the population variance is the same for the two groups. A t-test is used to compare the means of the two groups at the 10% significance level.

(e) State the appropriate null and alternative hypotheses for this t-test. [2]

(f) Find the p-value for this test. [2]

(g) State the conclusion for this test. Give a reason for your answer. [2]

(h) Explain what the p-value represents. [1]

5. [Maximum mark: 16]

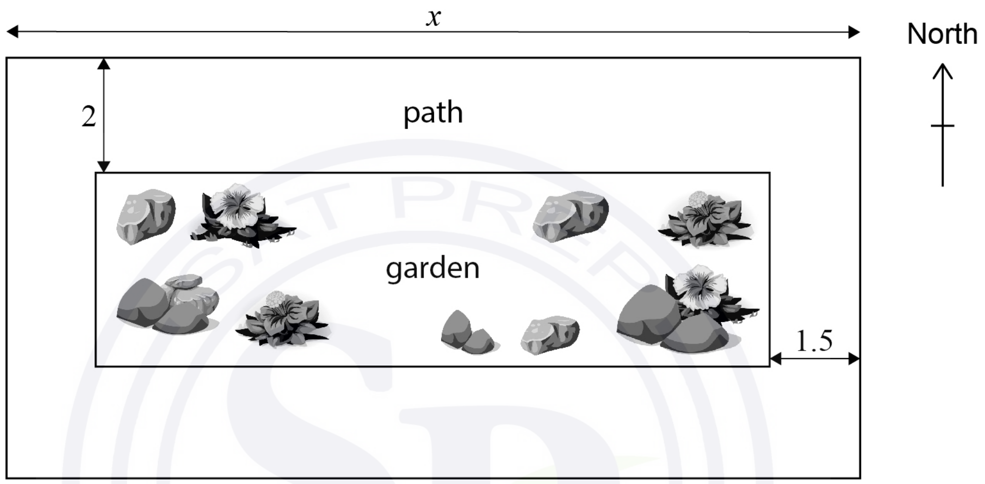
A particular park consists of a rectangular garden, of area $A\,\text{m}^2$, and a concrete path surrounding it. The park has a total area of $1200\,\text{m}^2$.

The width of the path at the north and south side of the park is $2\,\text{m}$.

The width of the path at the west and east side of the park is $1.5\,\text{m}$.

The length of the park (along the north and south sides) is x metres, $3 < x < 300$.

diagram not to scale



(a) (i) Write down the length of the garden in terms of x.

(ii) Find an expression for the width of the garden in terms of x.

(iii) Hence show that $A = 1212 - 4x - \dfrac{3600}{x}$. [5]

(b) Find the possible dimensions of the park if the area of the garden is $800\,\text{m}^2$. [4]

(c) Find an expression for $\dfrac{\text{d}A}{\text{d}x}$. [3]

(d) Use your answer from part (c) to find the value of x that will maximize the area of the garden. [2]

(e) Find the maximum possible area of the garden. [2]

References:

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: applications and interpretation
Standard level
Paper 2

9 May 2023

Zone A afternoon | **Zone B** morning | **Zone C** afternoon

1 hour 30 minutes

Instructions to candidates

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- A graphic display calculator is required for this paper.
- Answer all the questions in the answer booklet provided.
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- A clean copy of the **mathematics: applications and interpretation formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

2223 – 7210

9 pages

International Baccalaureate
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Answer **all** questions in the answer booklet provided. Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 15]

The mean annual temperatures for Earth, recorded at fifty-year intervals, are shown in the table.

Year (x)	1708	1758	1808	1858	1908	1958	2008
Temperature °C (y)	8.73	9.22	9.10	9.12	9.13	9.45	9.76

Tami creates a linear model for this data by finding the equation of the straight line passing through the points with coordinates $(1708, 8.73)$ and $(1958, 9.45)$.

(a) Calculate the gradient of the straight line that passes through these two points. [2]

(b) (i) Interpret the meaning of the gradient in the context of the question.

(ii) State appropriate units for the gradient. [2]

(c) Find the equation of this line giving your answer in the form $y = mx + c$. [2]

(d) Use Tami's model to estimate the mean annual temperature in the year 2000. [2]

Thandizo uses linear regression to obtain a model for the data.

(e) (i) Find the equation of the regression line y on x.

(ii) Find the value of r, the Pearson's product-moment correlation coefficient. [3]

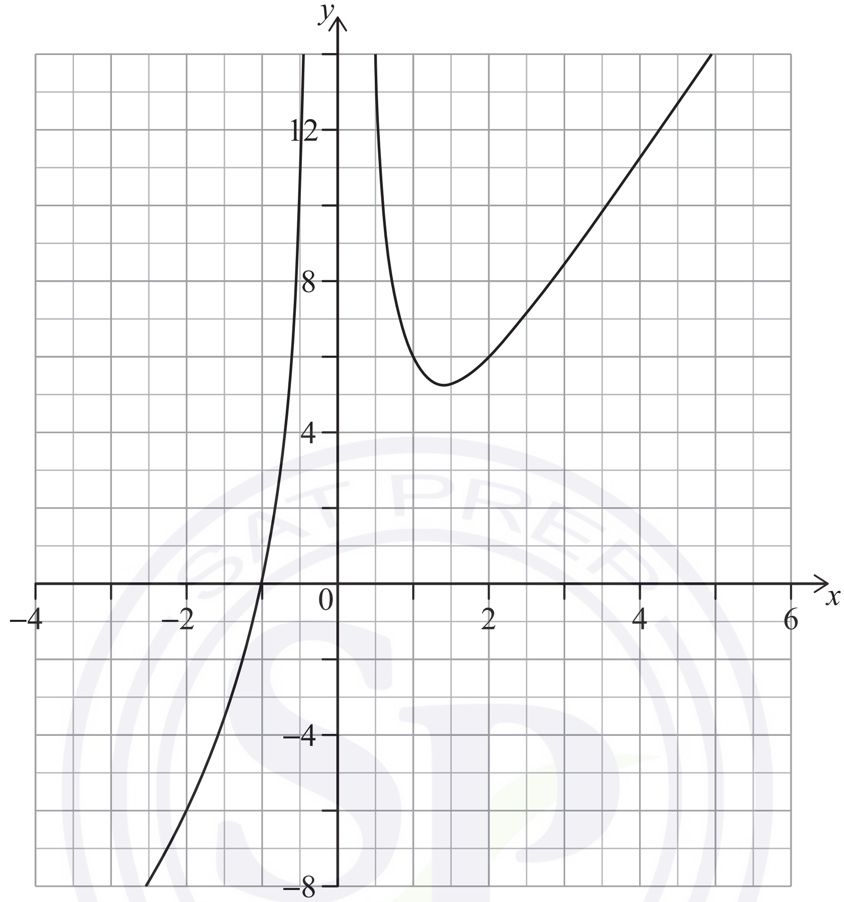
(f) Use Thandizo's model to estimate the mean annual temperature in the year 2000. [2]

Thandizo uses his regression line to predict the year when the mean annual temperature will first exceed $15\,°C$.

(g) State two reasons why Thandizo's prediction may not be valid. [2]

2. [Maximum mark: 16]

Consider the function $f(x) = 3x - 1 + 4x^{-2}$. Part of the graph of $y = f(x)$ is shown below.



The function is defined for all values of x except for $x = a$.

(a) Write down the value of a. [1]

(b) Use your graphic display calculator to find the coordinates of the local minimum. [2]

The equation $f(x) = w$, where $w \in \mathbb{R}$, has three solutions.

(c) Identify one possible value for w. [1]

The line $y = mx - \frac{1}{4}$ is tangent to $f(x)$ when $x = -4$.

(d) Write down whether the value of m is positive or negative. Justify your answer. [2]

(This question continues on the following page)

(Question 2 continued)

A second function is given by $g(x) = kp^x - 9$, where $p > 0$. The graph of $y = g(x)$ intersects the y-axis at point $\text{A}(0, -5)$ and passes through point $\text{B}(3, 4.5)$.

(e) Find the value of

(i) k;

(ii) p. [4]

(f) Write down the equation of the horizontal asymptote of $y = g(x)$. [2]

(g) Find the solution of $f(x) = g(x)$ when $x > 0$. [2]

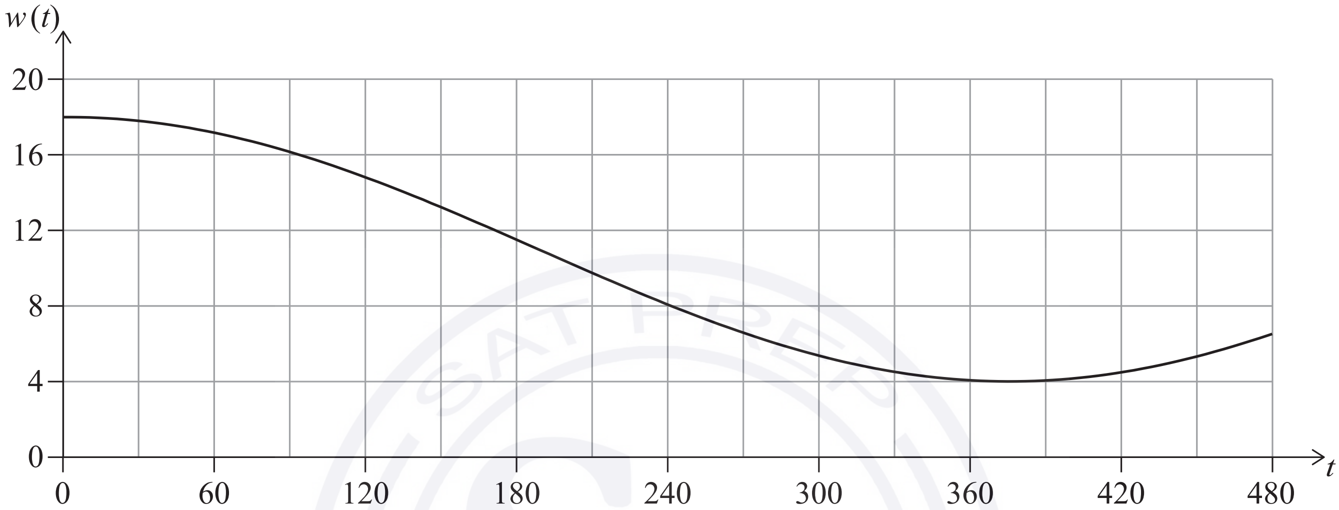
Consider a third function, h, where $h(x) = f(x) + g(x)$. The point $\text{C}(-1, q)$ lies on the graph of $g(x)$.

(h) State whether C also lies on the graph of $h(x)$. Justify your answer. [2]

3. [Maximum mark: 15]

The depth of water, w metres, in a particular harbour can be modelled by the function $w(t) = a\cos(bt^{\circ}) + d$ where t is the length of time, in minutes, after 06:00.

On 20 January, the first high tide occurs at 06:00, at which time the depth of water is $18\,\text{m}$. The following low tide occurs at 12:15 when the depth of water is $4\,\text{m}$. This is shown in the diagram.



(a) Find the value of a. [2]

(b) Find the value of d. [2]

(c) Find the period of the function in minutes. [3]

(d) Find the value of b. [2]

Naomi is sailing to the harbour on the morning of 20 January. Boats can enter or leave the harbour only when the depth of water is at least $6\,\text{m}$.

(e) Find the latest time before 12:00, to the nearest minute, that Naomi can enter the harbour. [4]

(f) Find the length of time (in minutes) between 06:00 and 15:00 on 20 January during which Naomi **cannot** enter or leave the harbour. [2]

4. [Maximum mark: 17]

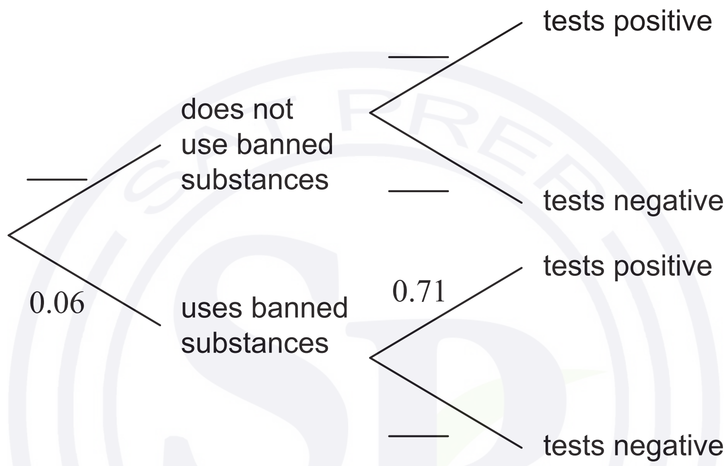
A large international sports tournament tests their athletes for banned substances.
They interpret a positive test result as meaning that the athlete uses banned substances.
A negative result means that they do not.

The probability that an athlete uses banned substances is estimated to be 0.06.

If an athlete **uses** banned substances, the probability that they will test positive is 0.71.

If an athlete does **not use** banned substances, the probability that they will test negative is 0.98.

(a) Using the information given, **copy** (into your answer booklet) and complete the following tree diagram. [2]



(b) (i) Determine the probability that a randomly selected athlete does not use banned substances and tests negative.

(ii) If two athletes are selected at random, calculate the probability that both athletes do not use banned substances and both test negative. [4]

(c) (i) Calculate the probability that a randomly selected athlete will receive an **incorrect** test result.

(ii) A random sample of 1300 athletes at the tournament are selected for testing. Calculate the expected number of athletes in the sample that will receive an incorrect test result. [5]

Team X are competing in the tournament. There are 20 athletes in this team. It is known that none of the athletes in Team X use banned substances.

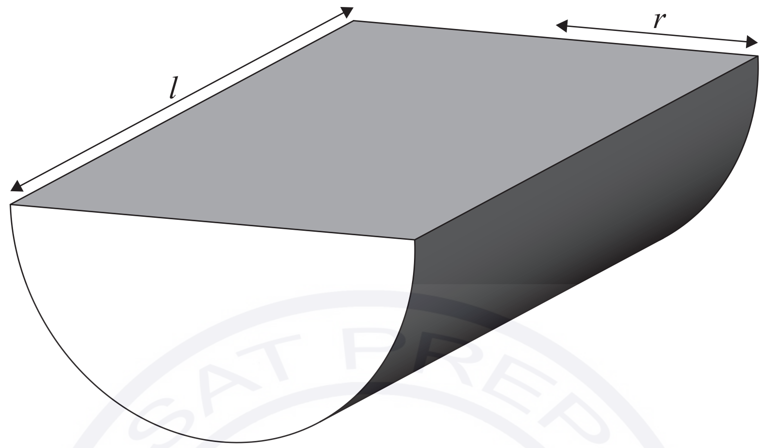
(d) Calculate the probability that none of the athletes in Team X will test positive. [4]

(e) Determine the probability that more than 2 athletes in Team X will test positive. [2]

5. [Maximum mark: 17]

A large closed container, in the shape of a half cylinder with a rectangular lid, is to be constructed with a volume of $0.8\,\text{m}^3$. The container has a length of l metres and a radius of r metres.

diagram not to scale



(a) Find an exact expression for l in terms of r and π. [2]

The container will be constructed using two different materials. The material for both the curved surface and the rectangular lid of the container costs $\$4.40$ per square metre. The material for the semicircular ends of the container costs $\$p$ per square metre.

The cost, C, of the materials to construct the container can be written in terms of r and p (where $p > 0$ and $r > 0$).

(b) Show that $C = 7.04r^{-1} + \dfrac{14.08}{\pi}r^{-1} + p\pi r^2$. [4]

(c) Find $\dfrac{\mathrm{d}C}{\mathrm{d}r}$. [3]

The cost of materials to construct the container is minimized when the radius of the container, r, is $0.7\,\text{m}$.

(d) Find the value of p. [3]

In total, 350 containers will be constructed at this minimum cost.

(e) Calculate the cost of materials, to the nearest dollar, to construct all 350 containers. [3]

(This question continues on the following page)

(Question 5 continued)

The materials for constructing the containers can be purchased at a discount according to the information in the table.

Cost of materials (\$ C) before discount	**Discount applied to entire order**
$1000 \le C < 2500$	1%
$2500 \le C < 5000$	4%
$5000 \le C < 10\,000$	8%
$C \ge 10\,000$	10%

(f) Determine the cost of materials for 350 containers after the discount is applied. [2]

References:

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: applications and interpretation
Standard level
Paper 2

Tuesday 1 November 2022 (morning)

1 hour 30 minutes

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all the questions in the answer booklet provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

8822 – 7205

Answer **all** questions in the answer booklet provided. Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 17]

Elsie, a librarian, wants to investigate the length of time, T minutes, that people spent in her library on a particular day.

(a) State whether the variable T is discrete or continuous. [1]

Elsie's data for 160 people who visited the library on that particular day is shown in the following table.

T **(minutes)**	$0 \le T < 20$	$20 \le T < 40$	$40 \le T < 60$	$60 \le T < 80$	$80 \le T < 100$
Frequency	50	62	k	14	8

(b) Find the value of k. [2]

(c) (i) Write down the modal class.

(ii) Write down the mid-interval value for this class. [2]

(d) Use Elsie's data to calculate an estimate of the mean time that people spent in the library. [2]

(e) Using the table, write down the maximum possible number of people who spent 35 minutes or less in the library on that day. [1]

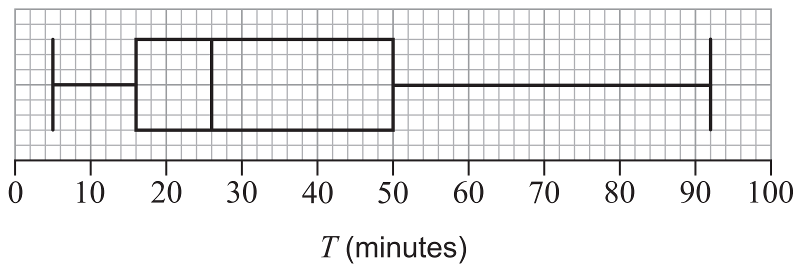
Elsie assumes her data to be representative of future visitors to the library.

(f) Find the probability a visitor spends at least 60 minutes in the library. [2]

(This question continues on the following page)

(Question 1 continued)

The following box and whisker diagram shows the times, in minutes, that the 160 visitors spent in the library.



(g) Write down the median time spent in the library. [1]

(h) Find the interquartile range. [2]

(i) Hence show that the longest time that a person spent in the library is not an outlier. [3]

Elsie believes the box and whisker diagram indicates that the times spent in the library are not normally distributed.

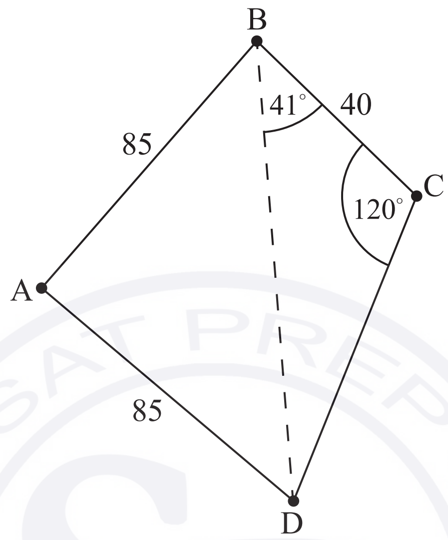
(j) Identify one feature of the box and whisker diagram which might support Elsie's belief. [1]

2. [Maximum mark: 17]

The following diagram shows a park bounded by a fence in the shape of a quadrilateral ABCD. A straight path crosses through the park from B to D.

$$\mathrm{AB} = 85\,\mathrm{m},\ \mathrm{AD} = 85\,\mathrm{m},\ \mathrm{BC} = 40\,\mathrm{m},\ \mathrm{C\hat{B}D} = 41^\circ,\ \mathrm{B\hat{C}D} = 120^\circ$$

diagram not to scale



(a) (i) Write down the value of angle BDC.

(ii) Hence use triangle BDC to find the length of path BD. [4]

(b) Calculate the size of angle $\mathrm{B\hat{A}D}$, correct to five significant figures. [3]

The size of angle $\mathrm{B\hat{A}D}$ rounds to 77°, correct to the nearest degree. Use $\mathrm{B\hat{A}D} = 77^\circ$ for the rest of this question.

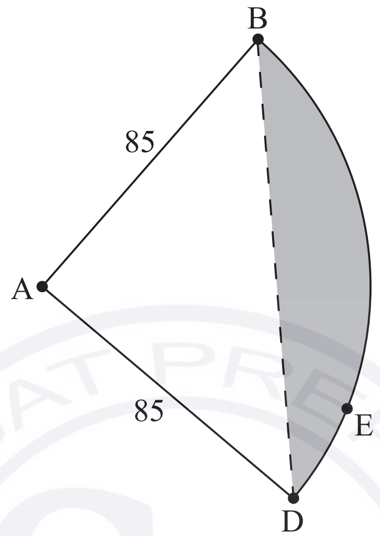
(c) Find the area bounded by the path BD, and fences AB and AD. [3]

(This question continues on the following page)

(Question 2 continued)

A landscaping firm proposes a new design for the park. Fences BC and CD are to be replaced by a fence in the shape of a circular arc BED with center A. This is illustrated in the following diagram.

diagram not to scale



(d) Write down the distance from A to E. [1]

(e) Find the perimeter of the proposed park, ABED. [3]

(f) Find the area of the shaded region in the proposed park. [3]

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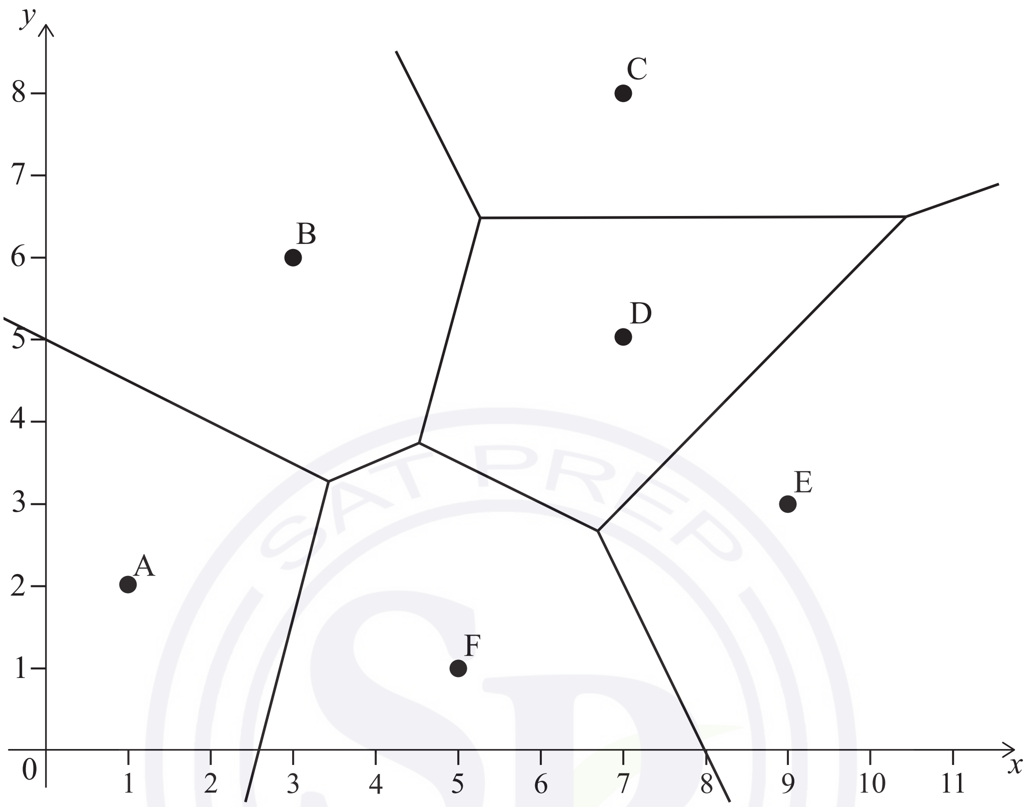
3. [Maximum mark: 13]

Six restaurant locations (labelled A, B, C, D, E and F) are shown, together with their Voronoi diagram. All distances are measured in kilometres.



(a) Elena wants to eat at the closest restaurant to her. Write down the restaurant she should go to, if she is at

(i) $(2, 7)$.

(ii) $(0, 1)$, when restaurant A is closed. [2]

Restaurant C is at $(7, 8)$ and restaurant D is at $(7, 5)$.

(b) Find the equation of the perpendicular bisector of CD. [2]

Restaurant B is at $(3, 6)$.

(c) Find the equation of the perpendicular bisector of BC. [5]

(d) Hence find

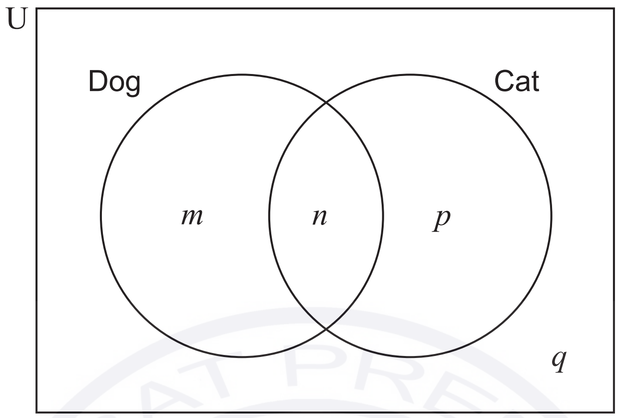
(i) the coordinates of the point which is of equal distance from B, C and D.

(ii) the distance of this point from D. [4]

4. [Maximum mark: 16]

At Mirabooka Primary School, a survey found that 68% of students have a dog and 36% of students have a cat. 14% of students have both a dog and a cat.

This information can be represented in the following Venn diagram, where m, n, p and q represent the percentage of students within each region.



(a) Find the value of

(i) m.

(ii) n.

(iii) p.

(iv) q. [4]

(b) Find the percentage of students who have a dog or a cat or both. [1]

(c) Find the probability that a randomly chosen student

(i) has a dog but does not have a cat.

(ii) has a dog given that they do not have a cat. [3]

(This question continues on the following page)

(Question 4 continued)

Each year, one student is chosen randomly to be the school captain of Mirabooka Primary School.

Tim is using a binomial distribution to make predictions about how many of the next 10 school captains will own a dog. He assumes that the percentages found in the survey will remain constant for future years and that the events “being a school captain” and “having a dog” are independent.

Use Tim’s model to find the probability that in the next 10 years

(d) (i) 5 school captains have a dog.

(ii) more than 3 school captains have a dog.

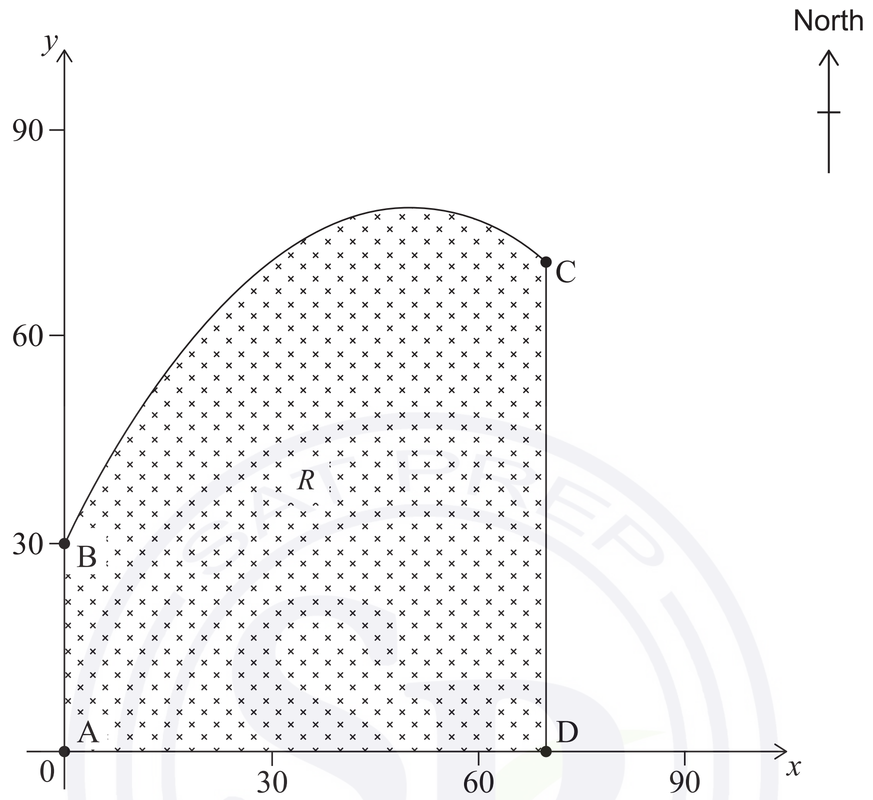
(iii) exactly 9 school captains in succession have a dog. [7]

John randomly chooses 10 students from the survey.

(e) State why John should not use the binomial distribution to find the probability that 5 of these students have a dog. [1]

5. [Maximum mark: 17]

Linda owns a field, represented by the shaded region R. The plan view of the field is shown in the following diagram, where both axes represent distance and are measured in metres.



The segments [AB], [CD] and [AD] respectively represent the western, eastern and southern boundaries of the field. The function, $f(x)$, models the northern boundary of the field between points B and C and is given by

$$f(x) = \frac{-x^2}{50} + 2x + 30 \text{, for } 0 \le x \le 70.$$

(a) (i) Find $f'(x)$.

(ii) Hence find the coordinates of the point on the field that is furthest north. [5]

Point A has coordinates $(0, 0)$, point B has coordinates $(0, 30)$, point C has coordinates $(70, 72)$ and point D has coordinates $(70, 0)$.

(b) (i) Write down the integral which can be used to find the area of the shaded region R.

(ii) Find the area of Linda's field. [4]

(This question continues on the following page)

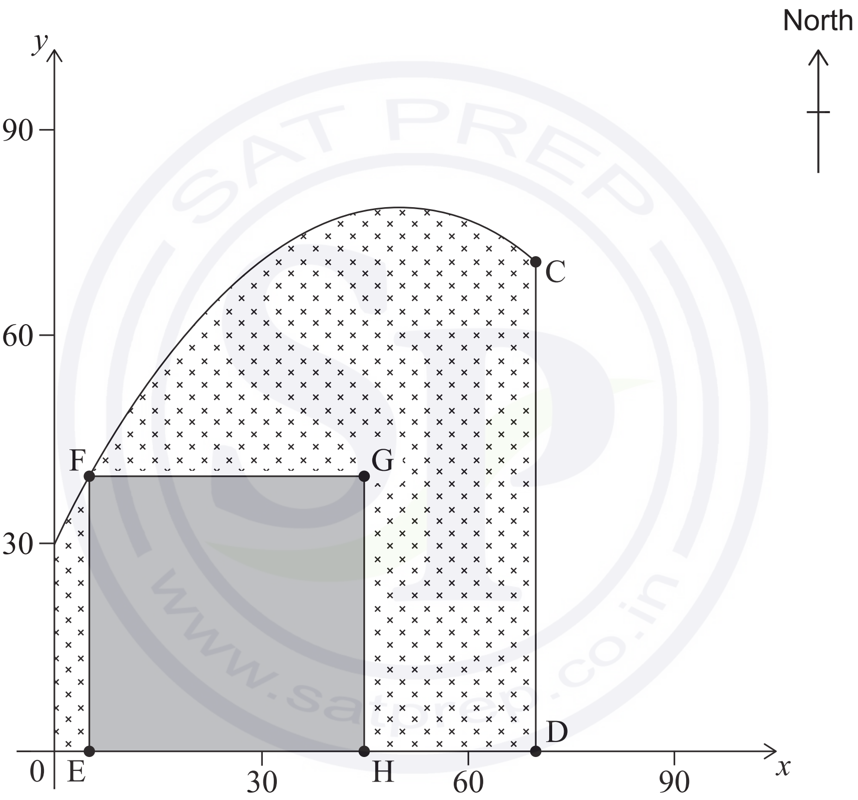
(Question 5 continued)

Linda used the trapezoidal rule with ten intervals to estimate the area. This calculation underestimated the area by $11.4\,\text{m}^2$.

(c) (i) Calculate the percentage error in Linda's estimate.

(ii) Suggest how Linda might be able to reduce the error whilst still using the trapezoidal rule. [3]

Linda would like to construct a building on her field. The **square** foundation of the building, EFGH, will be located such that [EH] is on the southern boundary and point F is on the northern boundary of the property. A possible location of the foundation of the building is shown in the following diagram.



The area of the square foundation will be largest when [GH] lies on [CD].

(d) (i) Find the x-coordinate of point E for the largest area of the square foundation of building EFGH.

(ii) Find the largest area of the foundation. [5]

References:

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: applications and interpretation
Standard level
Paper 2

Monday 9 May 2022 (morning)

1 hour 30 minutes

Instructions to candidates

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2222 – 7215

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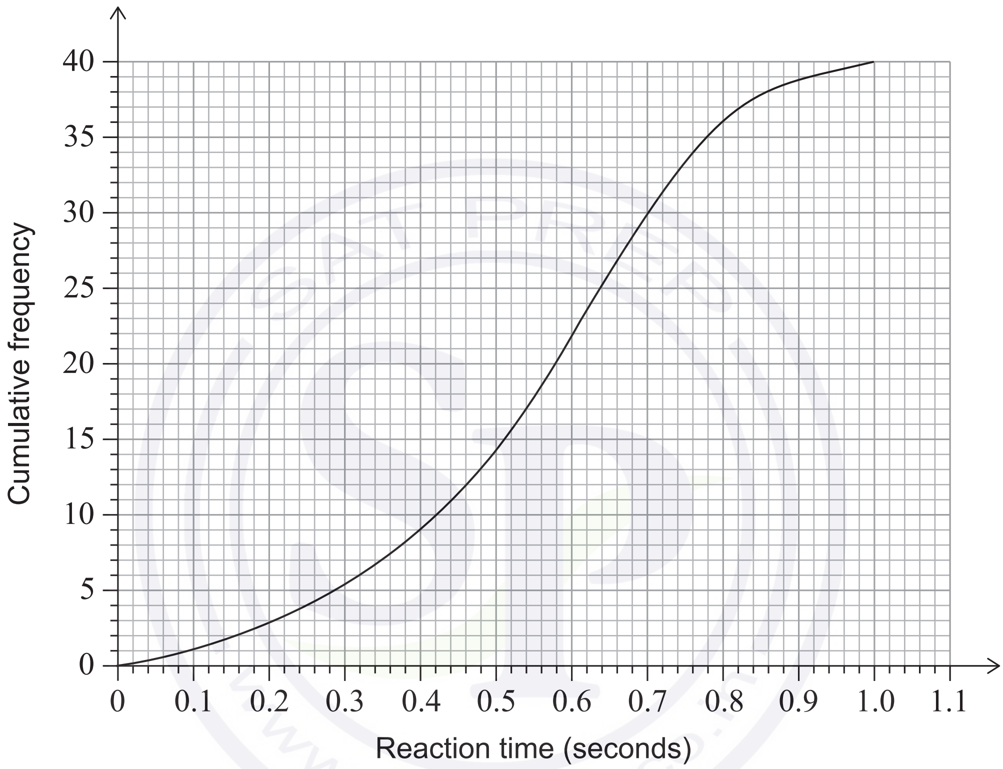
1. [Maximum mark: 17]

Mackenzie conducted an experiment on the reaction times of teenagers. The results of the experiment are displayed in the following cumulative frequency graph.



(a) Use the graph to estimate the

(i) median reaction time;

(ii) interquartile range of the reaction times. [4]

(b) Find the estimated number of teenagers who have a reaction time greater than 0.4 seconds. [2]

(c) Determine the 90th percentile of the reaction times from the cumulative frequency graph. [2]

(This question continues on the following page)

(Question 1 continued)

Mackenzie created the cumulative frequency graph using the following grouped frequency table.

Reaction time, t (s)	**Frequency**
$0 < t \leq 0.2$	3
$0.2 < t \leq 0.4$	a
$0.4 < t \leq 0.6$	13
$0.6 < t \leq 0.8$	14
$0.8 < t \leq 1.0$	b

(d) Write down the value of

(i) a;

(ii) b. [2]

(e) Write down the modal class from the table. [1]

(f) Use your graphic display calculator to find an estimate of the mean reaction time. [2]

Upon completion of the experiment, Mackenzie realized that some values were grouped incorrectly in the frequency table. Some reaction times recorded in the interval $0 < t \leq 0.2$ should have been recorded in the interval $0.2 < t \leq 0.4$.

(g) Suggest how, if at all, the estimated mean and estimated median reaction times will change if the errors are corrected. Justify your response. [4]

2. [Maximum mark: 13]

Scott purchases food for his dog in large bags and feeds the dog the same amount of dog food each day. The amount of dog food left in the bag at the end of each day can be modelled by an arithmetic sequence.

On a particular day, Scott opened a new bag of dog food and fed his dog. By the end of the third day there were 115.5 cups of dog food remaining in the bag and at the end of the eighth day there were 108 cups of dog food remaining in the bag.

(a) Find the number of cups of dog food

(i) fed to the dog per day;

(ii) remaining in the bag at the end of the first day. [4]

(b) Calculate the number of days that Scott can feed his dog with one bag of food. [2]

In 2021, Scott spent $\$625$ on dog food. Scott expects that the amount he spends on dog food will increase at an annual rate of 6.4%.

(c) Determine the amount that Scott expects to spend on dog food in 2025. Round your answer to the nearest dollar. [3]

(d) (i) Calculate the value of $\sum_{n=1}^{10}(625 \times 1.064^{(n-1)})$.

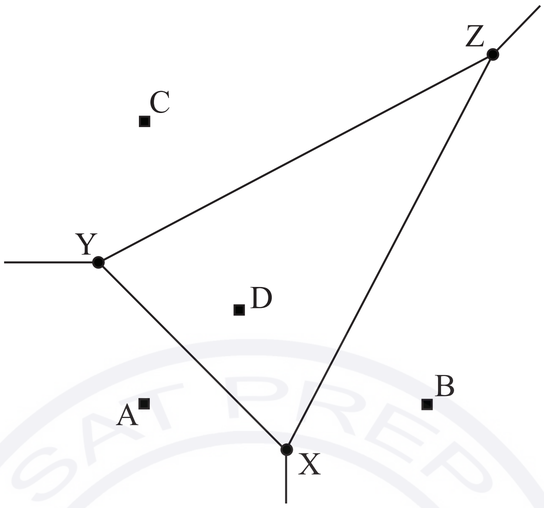
(ii) Describe what the value in part (d)(i) represents in this context. [3]

(e) Comment on the appropriateness of modelling this scenario with a geometric sequence. [1]

3. [Maximum mark: 18]

The Voronoi diagram below shows four supermarkets represented by points with coordinates $\mathrm{A}(0\,,\ 0)$, $\mathrm{B}(6\,,\ 0)$, $\mathrm{C}(0\,,\ 6)$ and $\mathrm{D}(2\,,\ 2)$. The vertices X, Y, Z are also shown. All distances are measured in kilometres.

diagram not to scale



(a) Find the midpoint of [BD]. [2]

(b) Find the equation of (XZ). [4]

The equation of (XY) is $y = 2 - x$ and the equation of (YZ) is $y = 0.5x + 3.5$.

(c) Find the coordinates of X. [3]

The coordinates of Y are $(-1\,,\ 3)$ and the coordinates of Z are $(7\,,\ 7)$.

(d) Determine the exact length of [YZ]. [2]

(e) Given that the exact length of [XY] is $\sqrt{32}$, find the size of $\mathrm{X\hat{Y}Z}$ in degrees. [4]

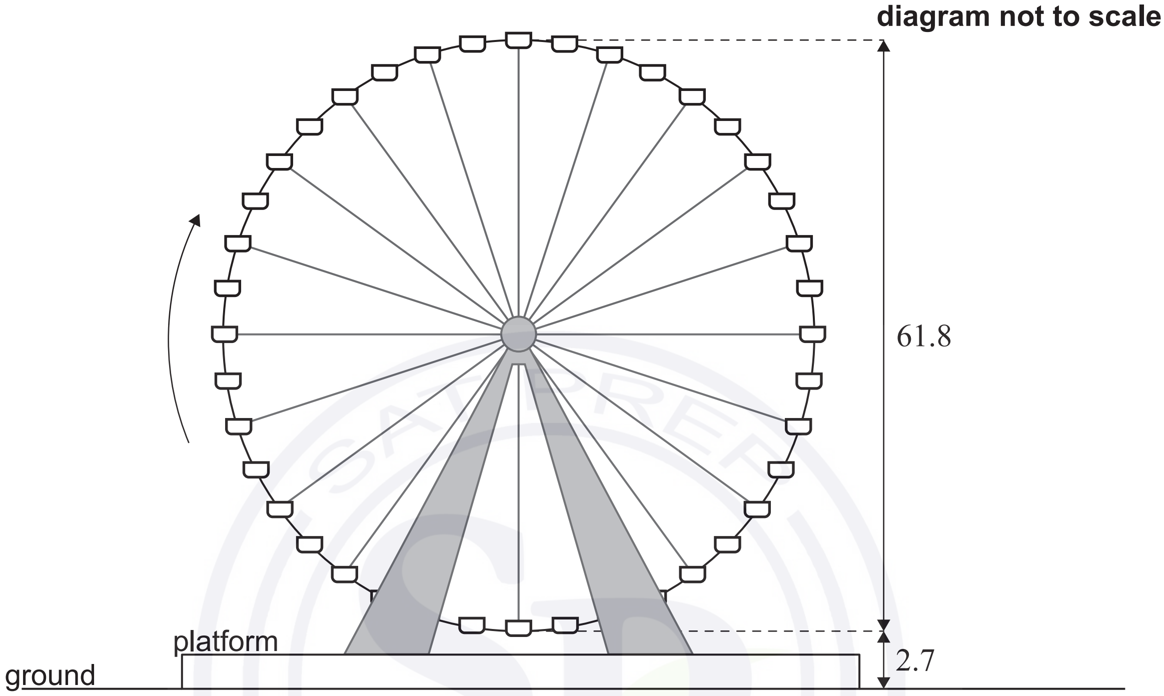
(f) Hence find the area of triangle XYZ. [2]

A town planner believes that the larger the area of the Voronoi cell XYZ, the more people will shop at supermarket D.

(g) State one criticism of this interpretation. [1]

4. [Maximum mark: 17]

The Texas Star is a Ferris wheel at the state fair in Dallas. The Ferris wheel has a diameter of 61.8 m. To begin the ride, a passenger gets into a chair at the lowest point on the wheel, which is 2.7 m above the ground, as shown in the following diagram. A ride consists of multiple revolutions, and the Ferris wheel makes 1.5 revolutions per minute.



The height of a chair above the ground, h, measured in metres, during a ride on the Ferris wheel can be modelled by the function $h(t) = -a\cos(bt) + d$, where t is the time, in seconds, since a passenger began their ride.

(a) Calculate the value of

(i) a;

(ii) b;

(iii) d. [6]

A ride on the Ferris wheel lasts for 12 minutes in total.

(b) Calculate the number of revolutions of the Ferris wheel per ride. [2]

(c) For exactly one ride on the Ferris wheel, suggest

(i) an appropriate domain for $h(t)$;

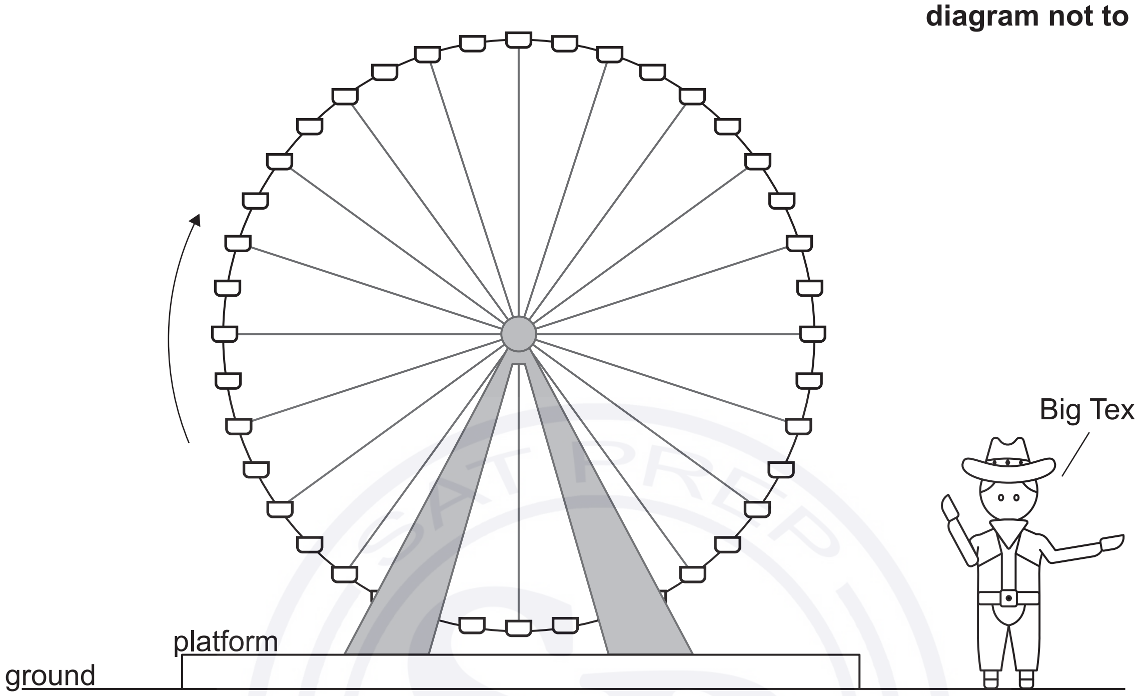
(ii) an appropriate range for $h(t)$. [3]

(This question continues on the following page)

(Question 4 continued)

Big Tex is a 16.7 metre-tall cowboy statue that stands on the horizontal ground next to the Ferris wheel.

diagram not to scale



(d) By considering the graph of $h(t)$, determine the length of time during one revolution of the Ferris wheel for which the chair is higher than the cowboy statue. [3]

There is a plan to relocate the Texas Star Ferris wheel onto a taller platform which will increase the maximum height of the Ferris wheel to $65.2\,\text{m}$. This will change the value of one parameter, a, b or d, found in part (a).

(e) (i) Identify which parameter will change.

(ii) Find the new value of the parameter identified in part (e)(i). [3]

5. [Maximum mark: 15]

A cafe makes x litres of coffee each morning. The cafe's profit each morning, C, measured in dollars, is modelled by the following equation

$$C = \frac{x}{10}\left(k^2 - \frac{3}{100}x^2\right)$$

where k is a positive constant.

(a) Find an expression for $\frac{\mathrm{d}C}{\mathrm{d}x}$ in terms of k and x. [3]

(b) Hence find the maximum value of C in terms of k. Give your answer in the form pk^3, where p is a constant. [4]

The cafe's manager knows that the cafe makes a profit of $\$426$ when 20 litres of coffee are made in a morning.

(c) (i) Find the value of k.

(ii) Use the model to find how much coffee the cafe should make each morning to maximize its profit. [3]

(d) Sketch the graph of C against x, labelling the maximum point and the x-intercepts with their coordinates. [3]

The manager of the cafe wishes to serve as many customers as possible.

(e) Determine the maximum amount of coffee the cafe can make that will not result in a loss of money for the morning. [2]

References:

4. Aline Escobar., n.d. Cowboy. [image online] Available at: https://thenounproject.com/search/?q=cowboy&i=1080130 This file is licensed under the Creative Commons Attribution-ShareAlike 3.0 Unported (CC BY-SA 3.0) https://creativecommons.org/licenses/by-sa/3.0/deed.en [Accessed 13/05/2021]. Source adapted.

Mathematics: applications and interpretation
Standard level
Paper 2

Monday 9 May 2022 (morning)

1 hour 30 minutes

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all the questions in the answer booklet provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

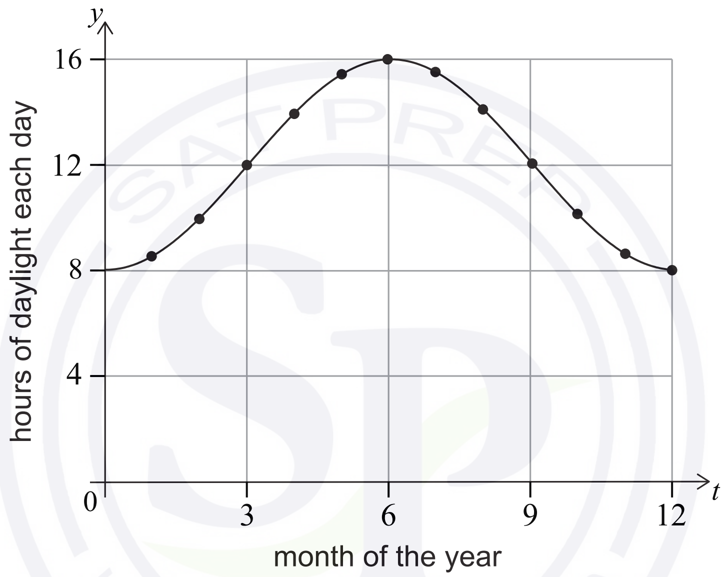
2222 – 7210

Answer **all** questions in the answer booklet provided. Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 15]

Boris recorded the number of daylight hours on the first day of each month in a northern hemisphere town.

This data was plotted onto a scatter diagram. The points were then joined by a smooth curve, with minimum point $(0, 8)$ and maximum point $(6, 16)$ as shown in the following diagram.



Let the curve in the diagram be $y = f(t)$, where t is the time, measured in months, since Boris first recorded these values.

Boris thinks that $f(t)$ might be modelled by a quadratic function.

(a) Write down one reason why a quadratic function would not be a good model for the number of hours of daylight per day, across a number of years. [1]

(This question continues on the following page)

(Question 1 continued)

Paula thinks that a better model is $f(t) = a\cos(bt) + d$, $t \geq 0$, for specific values of a, b and d.

(b) For Paula's model, use the diagram to write down

(i) the amplitude.

(ii) the period.

(iii) the equation of the principal axis. [4]

(c) Hence or otherwise find the equation of this model in the form: [3]

$$f(t) = a\cos(bt) + d$$

(d) For the first year of the model, find the length of time when there are more than 10 hours and 30 minutes of daylight per day. [4]

The true maximum number of daylight hours was 16 hours and 14 minutes.

(e) Calculate the percentage error in the maximum number of daylight hours Boris recorded in the diagram. [3]

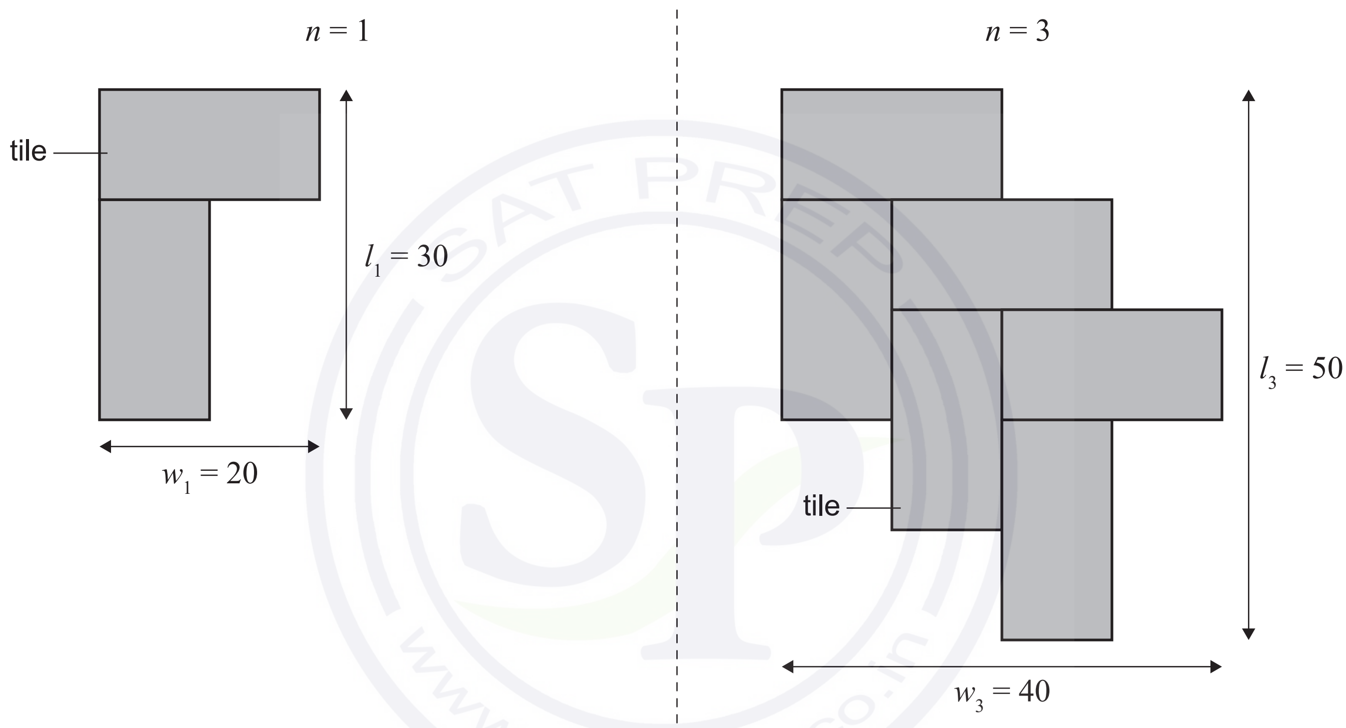
2. [Maximum mark: 19]

Eddie decides to construct a path across his rectangular grass lawn using pairs of tiles.

Each tile is $10\,\text{cm}$ wide and $20\,\text{cm}$ long. The following diagrams show the path after Eddie has laid one pair and three pairs of tiles. This pattern continues until Eddie reaches the other side of his lawn. When n pairs of tiles are laid, the path has a width of w_n centimetres and a length l_n centimetres.

The following diagrams show this pattern for one pair of tiles and for three pairs of tiles, where the white space around each diagram represents Eddie's lawn.

diagram not to scale



The following table shows the values of w_n and l_n for the first three values of n.

Number of pairs of tiles, n	Width of lawn crossed by path, w_n (cm)	Length of lawn crossed by path, l_n (cm)
1	20	30
2	a	b
3	40	50

(This question continues on the following page)

(Question 2 continued)

(a) Find the value of

(i) a.

(ii) b. [2]

(b) Write down an expression in terms of n for

(i) w_n.

(ii) l_n. [3]

Eddie's lawn has a length $740\,\text{cm}$.

(c) (i) Show that Eddie needs 144 tiles.

(ii) Find the value of w_n for this path. [3]

(d) Find the total area of the tiles in Eddie's path. Give your answer in the form $a \times 10^k$ where $1 \le a < 10$ and k is an integer. [3]

The tiles cost $\$24.50$ per square metre and are sold in packs of five tiles.

(e) Find the cost of a single pack of five tiles. [3]

To allow for breakages Eddie wants to have at least 8% more tiles than he needs.

(f) Find the minimum number of packs of tiles Eddie will need to order. [3]

There is a fixed delivery cost of $\$35$.

(g) Find the total cost for Eddie's order. [2]

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3. [Maximum mark: 16]

The scores of the eight highest scoring countries in the 2019 Eurovision song contest are shown in the following table.

	Eurovision score
Netherlands	498
Italy	472
Russia	370
Switzerland	364
Sweden	334
Norway	331
North Macedonia	305
Azerbaijan	302

(a) For this data, find

(i) the upper quartile.

(ii) the interquartile range. [4]

(b) Determine if the Netherlands' score is an outlier for this data. Justify your answer. [3]

(This question continues on the following page)

(Question 3 continued)

Chester is investigating the relationship between the highest-scoring countries' Eurovision score and their population size to determine whether population size can reasonably be used to predict a country's score.

The populations of the countries, to the nearest million, are shown in the table.

	Population (x) (millions)	**Eurovision score (y)**
Netherlands	17	498
Italy	60	472
Russia	145	370
Switzerland	9	364
Sweden	10	334
Norway	5	331
North Macedonia	2	305
Azerbaijan	10	302

Chester finds that, for this data, the Pearson's product moment correlation coefficient is $r = 0.249$.

(c) State whether it would be appropriate for Chester to use the equation of a regression line for y on x to predict a country's Eurovision score. Justify your answer. [2]

Chester then decides to find the Spearman's rank correlation coefficient for this data, and creates a table of ranks.

	Population rank (to the nearest million)	**Eurovision score rank**
Netherlands	3	1
Italy	2	2
Russia	1	3
Switzerland	a	4
Sweden	b	5
Norway	7	6
North Macedonia	8	7
Azerbaijan	c	8

(This question continues on the following page)

(Question 3 continued)

(d) Write down the value of:

(i) a,

(ii) b,

(iii) c. [3]

(e) (i) Find the value of the Spearman's rank correlation coefficient r_s.

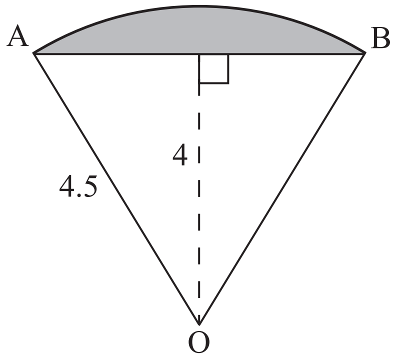
(ii) Interpret the value obtained for r_s. [3]

(f) When calculating the ranks, Chester incorrectly read the Netherlands' score as 478. Explain why the value of the Spearman's rank correlation r_s does not change despite this error. [1]

4. [Maximum mark: 15]

A sector of a circle, centre O and radius 4.5 m, is shown in the following diagram.

diagram not to scale

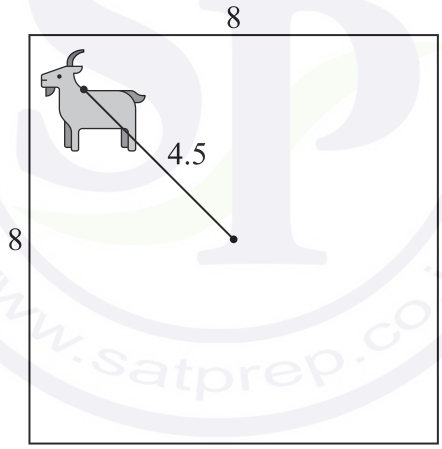


(a) (i) Find the angle $\text{A}\hat{\text{O}}\text{B}$.

(ii) Find the area of the shaded segment. [8]

A square field with side 8 m has a goat tied to a post in the centre by a rope such that the goat can reach all parts of the field up to 4.5 m from the post.

diagram not to scale



(b) (i) Find the area of a circle with radius 4.5 m.

(ii) Find the area of the field that can be reached by the goat. [5]

Let V be the volume of grass eaten by the goat, in cubic metres, and t be the length of time, in hours, that the goat has been in the field.

The goat eats grass at the rate of $\frac{\mathrm{d}V}{\mathrm{d}t} = 0.3\,t\mathrm{e}^{-t}$.

(c) Find the value of t at which the goat is eating grass at the greatest rate. [2]

5. [Maximum mark: 15]

The aircraft for a particular flight has 72 seats. The airline's records show that historically for this flight only 90% of the people who purchase a ticket arrive to board the flight. They assume this trend will continue and decide to sell extra tickets and hope that no more than 72 passengers will arrive.

The number of passengers that arrive to board this flight is assumed to follow a binomial distribution with a probability of 0.9.

(a) The airline sells 74 tickets for this flight. Find the probability that more than 72 passengers arrive to board the flight. [3]

(b) (i) Write down the expected number of passengers who will arrive to board the flight if 72 tickets are sold. [2]

(ii) Find the maximum number of tickets that could be sold if the expected number of passengers who arrive to board the flight must be less than or equal to 72. [2]

Each passenger pays $\$150$ for a ticket. If too many passengers arrive, then the airline will give $\$300$ in compensation to each passenger that cannot board.

(c) Find, to the nearest integer, the expected increase or decrease in the money made by the airline if they decide to sell 74 tickets rather than 72. [8]

References:

2. mynamepong, n.d. Goat [image online] Available at: https://thenounproject.com/term/goat/1761571/
This file is licensed under the Creative Commons Attribution-ShareAlike 3.0 Unported (CC BY-SA 3.0) https://creativecommons.org/licenses/by-sa/3.0/deed.en [Accessed 22 April 2010] Source adapted.

Mathematics: applications and interpretation
Standard level
Paper 2

Tuesday 2 November 2021 (morning)

1 hour 30 minutes

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all the questions in the answer booklet provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

8821 – 7205

Answer **all** questions in the answer booklet provided. Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 16]

A group of 1280 students were asked which electronic device they preferred. The results per age group are given in the following table.

	Age			
Preferred device	**11–13**	**14–16**	**17–18**	**Total**
Laptop	143	160	153	456
Tablet	205	224	131	560
Mobile phone	72	128	64	264
Total	420	512	348	1280

(a) A student from the group is chosen at random. Calculate the probability that the student

(i) prefers a tablet.

(ii) is $11–13$ years old and prefers a mobile phone.

(iii) prefers a laptop **given that** they are $17–18$ years old.

(iv) prefers a tablet or is $14–16$ years old. [9]

A χ^2 test for independence was performed on the collected data at the 1% significance level. The critical value for the test is 13.277.

(b) State the null and alternative hypotheses. [1]

(c) Write down the number of degrees of freedom. [1]

(d) (i) Write down the χ^2 test statistic.

(ii) Write down the p-value.

(iii) State the conclusion for the test in context. Give a reason for your answer. [5]

2. [Maximum mark: 16]

The admissions team at a new university are trying to predict the number of student applications they will receive each year.

Let n be the number of years that the university has been open. The admissions team collect the following data for the first two years.

Year, n	**Number of applications received in year n**
1	12 300
2	12 669

(a) Calculate the percentage increase in applications from the first year to the second year. [2]

It is assumed that the number of students that apply to the university each year will follow a geometric sequence, u_n.

(b) (i) Write down the common ratio of the sequence.

(ii) Find an expression for u_n.

(iii) Find the number of student applications the university expects to receive when $n = 11$. Express your answer to the nearest integer. [4]

In the first year there were $10\,380$ places at the university available for applicants. The admissions team announce that the number of places available will increase by 600 every year.

Let v_n represent the number of places available at the university in year n.

(c) Write down an expression for v_n. [2]

For the first 10 years that the university is open, all places are filled. Students who receive a place each pay an $\$80$ acceptance fee.

(d) Calculate the total amount of acceptance fees paid to the university in the first 10 years. [3]

When $n = k$, the number of places available will, for the first time, exceed the number of students applying.

(e) Find k. [3]

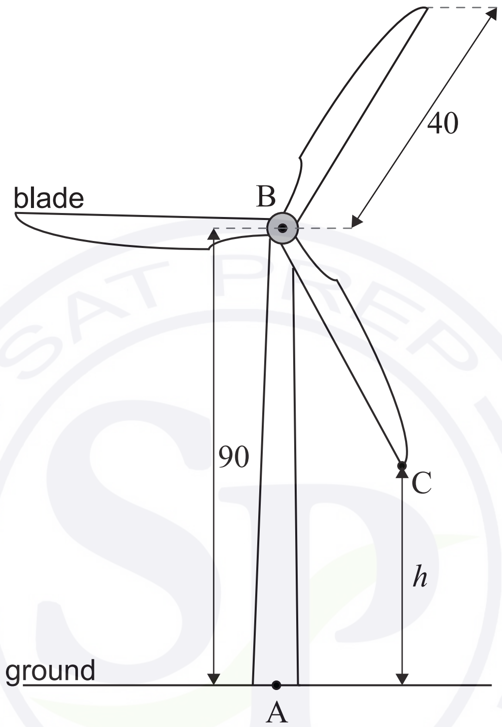
(f) State whether, for all $n > k$, the university will have places available for all applicants. Justify your answer. [2]

3. [Maximum mark: 20]

A wind turbine is designed so that the rotation of the blades generates electricity. The turbine is built on horizontal ground and is made up of a vertical tower and three blades.

The point A is on the base of the tower directly below point B at the top of the tower. The height of the tower, AB, is $90\,\text{m}$. The blades of the turbine are centred at B and are each of length $40\,\text{m}$. This is shown in the following diagram.

diagram not to scale



The end of one of the blades of the turbine is represented by point C on the diagram. Let h be the height of C above the ground, measured in metres, where h varies as the blade rotates.

(a) Find the

(i) maximum value of h.

(ii) minimum value of h. [2]

The blades of the turbine complete 12 rotations per minute under normal conditions, moving at a constant rate.

(b) (i) Find the time, in seconds, it takes for the blade [BC] to make one complete rotation under these conditions.

(ii) Calculate the angle, in degrees, that the blade [BC] turns through in one second. [3]

(This question continues on the following page)

(Question 3 continued)

The height, h, of point C can be modelled by the following function. Time, t, is measured from the instant when the blade [BC] first passes [AB] and is measured in seconds.

$$h(t) = 90 - 40\cos(72t^{\circ}),\ t \geq 0$$

(c) (i) Write down the amplitude of the function.

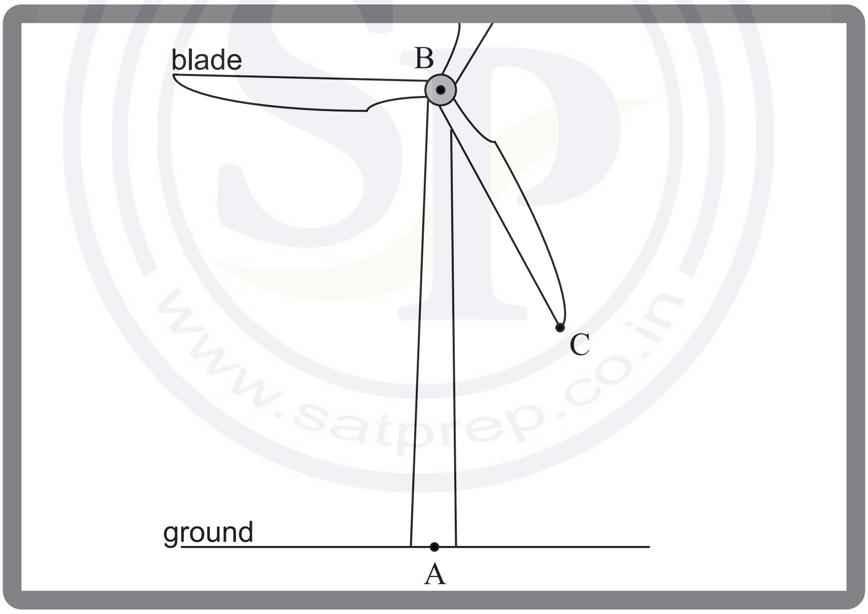
(ii) Find the period of the function. [2]

(d) Sketch the function $h(t)$ for $0 \leq t \leq 5$, clearly labelling the coordinates of the maximum and minimum points. [3]

(e) (i) Find the height of C above the ground when $t = 2$.

(ii) Find the time, in seconds, that point C is above a height of 100 m, during each complete rotation. [5]

Looking through his window, Tim has a partial view of the rotating wind turbine. The position of his window means that he cannot see any part of the wind turbine that is **more than 100 m** above the ground. This is illustrated in the following diagram.



(f) (i) At any given instant, find the probability that point C is visible from Tim's window.

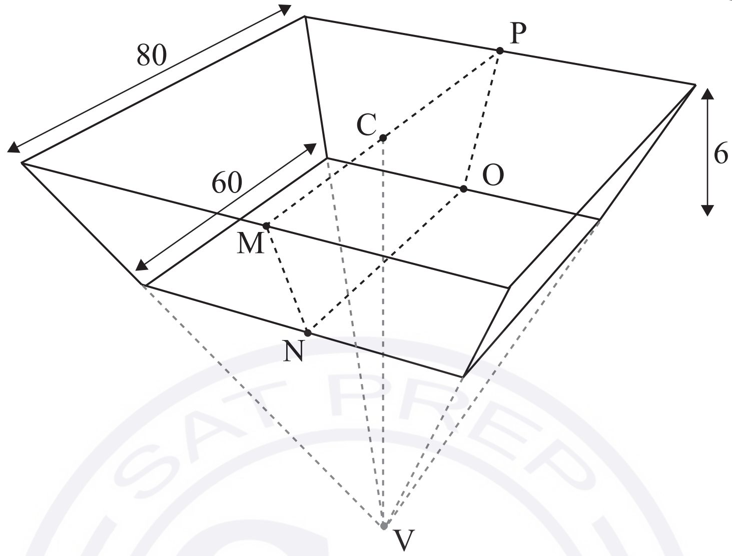
The wind speed increases. The blades rotate at twice the speed, but still at a constant rate.

(ii) At any given instant, find the probability that Tim can see point C from his window. Justify your answer. [5]

4. [Maximum mark: 14]

A large water reservoir is built in the form of part of an upside-down right pyramid with a horizontal square base of length 80 metres. The point C is the centre of the square base and point V is the vertex of the pyramid.

diagram not to scale



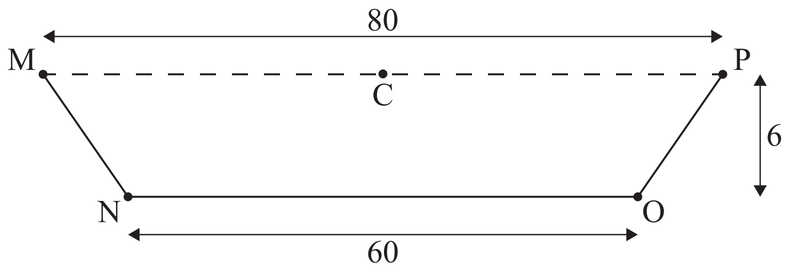
The bottom of the reservoir is a square of length 60 metres that is parallel to the base of the pyramid, such that the depth of the reservoir is 6 metres as shown in the diagram.

(This question continues on the following page)

(Question 4 continued)

The second diagram shows a vertical cross section, MNOPC, of the reservoir.

diagram not to scale



(a) Find the angle of depression from M to N. [2]

(b) (i) Find CV.

(ii) Hence or otherwise, show that the volume of the reservoir is $29\,600\,\text{m}^3$. [5]

Every day $80\,\text{m}^3$ of water from the reservoir is used for irrigation.

Joshua states that, if no other water enters or leaves the reservoir, then when it is full there is enough irrigation water for at least one year.

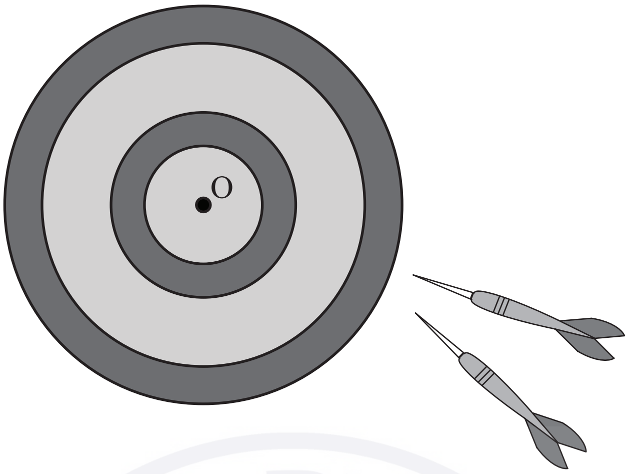
(c) By finding an appropriate value, determine whether Joshua is correct. [2]

To avoid water leaking into the ground, the five interior sides of the reservoir have been painted with a watertight material.

(d) Find the area that was painted. [5]

5. [Maximum mark: 14]

Arianne plays a game of darts.



The distance that her darts land from the centre, O, of the board can be modelled by a normal distribution with mean $10\,\text{cm}$ and standard deviation $3\,\text{cm}$.

(a) Find the probability that

(i) a dart lands less than $13\,\text{cm}$ from O.

(ii) a dart lands more than $15\,\text{cm}$ from O. [3]

Each of Arianne's throws is independent of her previous throws.

(b) Find the probability that Arianne throws two consecutive darts that land more than $15\,\text{cm}$ from O. [2]

In a competition a player has three darts to throw on each turn. A point is scored if a player throws **all** three darts to land within a central area around O. When Arianne throws a dart the probability that it lands within this area is 0.8143.

(c) Find the probability that Arianne does **not** score a point on a turn of three darts. [2]

In the competition Arianne has ten turns, each with three darts.

(d) (i) Find the probability that Arianne scores at least 5 points in the competition.

(ii) Find the probability that Arianne scores at least 5 points and less than 8 points.

(iii) Given that Arianne scores at least 5 points, find the probability that Arianne scores less than 8 points. [7]

References:

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: applications and interpretation
Standard level
Paper 2

Friday 7 May 2021 (morning)

1 hour 30 minutes

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all the questions in the answer booklet provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

2221 – 7215

Answer **all** questions in the answer booklet provided. Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 19]

A medical centre is testing patients for a certain disease. This disease occurs in 5% of the population.

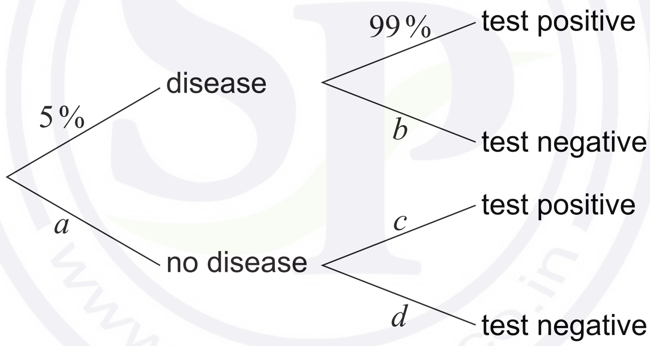
They test every patient who comes to the centre on a particular day.

(a) State the sampling method being used. [1]

It is intended that if a patient has the disease, they test "positive", and if a patient does not have the disease, they test "negative".

However, the tests are not perfect, and only 99% of people who have the disease test positive. Also, 2% of people who **do not** have the disease test positive.

The tree diagram shows some of this information.



(b) Write down the value of

(i) a.

(ii) b.

(iii) c.

(iv) d. [4]

(This question continues on the following page)

(Question 1 continued)

(c) Use the tree diagram to find the probability that a patient selected at random

(i) will not have the disease and will test positive.

(ii) will test negative.

(iii) has the disease given that they tested negative. [8]

(d) The medical centre finds the actual number of positive results in their sample is different than predicted by the tree diagram. Explain why this might be the case. [1]

The staff at the medical centre looked at the care received by all visiting patients on a randomly chosen day. All the patients received at least one of these services: they had medical tests (M), were seen by a nurse (N), or were seen by a doctor (D). It was found that:

- 78 had medical tests,
- 45 were seen by a nurse;
- 30 were seen by a doctor;
- 9 had medical tests and were seen by a doctor and a nurse;
- 18 had medical tests and were seen by a doctor but were not seen by a nurse;
- 11 patients were seen by a nurse and had medical tests but were not seen by a doctor;
- 2 patients were seen by a doctor without being seen by nurse and without having medical tests.

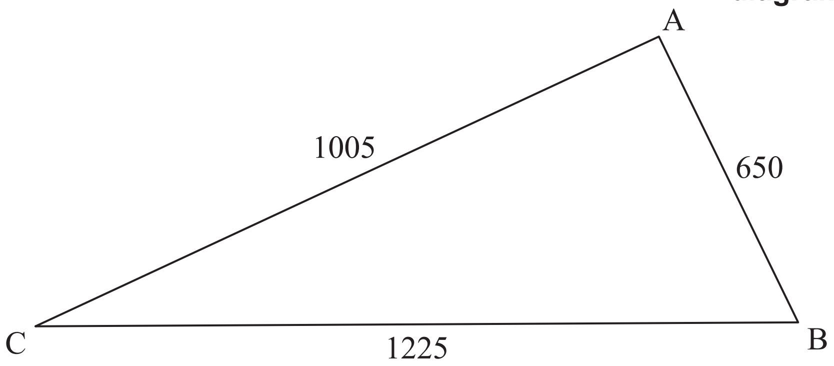
(e) Draw a Venn diagram to illustrate this information, placing all relevant information on the diagram. [3]

(f) Find the total number of patients who visited the centre during this day. [2]

2. [Maximum mark: 15]

A farmer owns a field in the shape of a triangle ABC such that $AB = 650\,\text{m}$, $AC = 1005\,\text{m}$ and $BC = 1225\,\text{m}$.

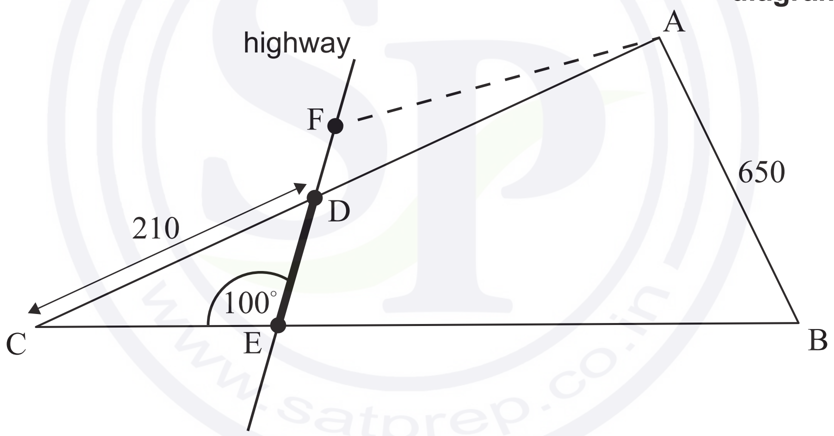
diagram not to scale



(a) Find the size of $A\hat{C}B$. [3]

The local town is planning to build a highway that will intersect the borders of the field at points D and E, where $DC = 210\,\text{m}$ and $C\hat{E}D = 100^\circ$, as shown in the diagram below.

diagram not to scale



(b) Find DE. [3]

The town wishes to build a carpark here. They ask the farmer to exchange the part of the field represented by triangle DCE. In return the farmer will get a triangle of equal area ADF, where F lies on the same line as D and E, as shown in the diagram above.

(c) Find the area of triangle DCE. [5]

(d) Estimate DF. You may assume the highway has a width of zero. [4]

3. [Maximum mark: 16]

A new concert hall was built with 14 seats in the first row. Each subsequent row of the hall has two more seats than the previous row. The hall has a total of 20 rows.

(a) Find:

(i) the number of seats in the last row.

(ii) the total number of seats in the concert hall. [5]

The concert hall opened in 2019. The average number of visitors per concert during that year was 584. In 2020, the average number of visitors per concert increased by 1.2%.

(b) Find the average number of visitors per concert in 2020. [2]

The concert organizers use this data to model future numbers of visitors. It is assumed that the average number of visitors per concert will continue to increase each year by 1.2%.

(c) Determine the first year in which this model predicts the average number of visitors per concert will exceed the total seating capacity of the concert hall. [5]

It is assumed that the concert hall will host 50 concerts each year.

(d) Use the average number of visitors per concert per year to predict the **total** number of people expected to attend the concert hall from when it opens until the end of 2025. [4]

4. [Maximum mark: 14]

It is known that the weights of male Persian cats are normally distributed with mean $6.1\,\text{kg}$ and variance $0.5^2\,\text{kg}^2$.

(a) Sketch a diagram showing the above information. [2]

(b) Find the proportion of male Persian cats weighing between $5.5\,\text{kg}$ and $6.5\,\text{kg}$. [2]

A group of 80 male Persian cats are drawn from this population.

(c) Determine the expected number of cats in this group that have a weight of less than $5.3\,\text{kg}$. [3]

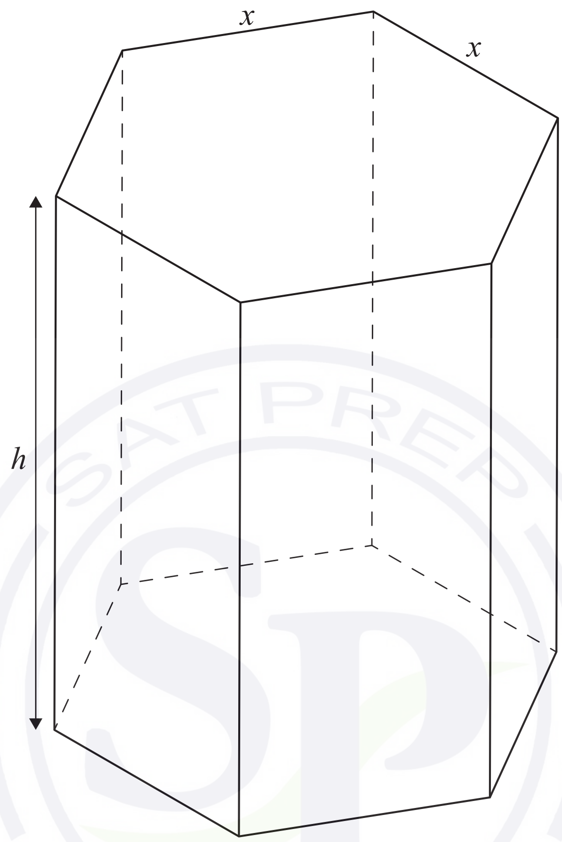
(d) It is found that 12 of the cats weigh more than $x\,\text{kg}$. Estimate the value of x. [3]

(e) Ten of the cats are chosen at random. Find the probability that exactly one of them weighs over $6.25\,\text{kg}$. [4]

5. [Maximum mark: 16]

A hollow chocolate box is manufactured in the form of a right prism with a regular hexagonal base. The height of the prism is h cm, and the top and base of the prism have sides of length x cm.

diagram not to scale



(a) Given that $\sin 60^\circ = \frac{\sqrt{3}}{2}$, show that the area of the base of the box is equal to $\frac{3\sqrt{3}x^2}{2}$. [2]

(b) Given that the total external surface area of the box is $1200\,\text{cm}^2$, show that the volume of the box may be expressed as $V = 300\sqrt{3}\,x - \frac{9}{4}x^3$. [5]

(c) Sketch the graph of $V = 300\sqrt{3}\,x - \frac{9}{4}x^3$, for $0 \le x \le 16$. [2]

(d) Find an expression for $\frac{\mathrm{d}V}{\mathrm{d}x}$. [2]

(e) Find the value of x which maximizes the volume of the box. [2]

(f) Hence, or otherwise, find the maximum possible volume of the box. [2]

The box will contain spherical chocolates. The production manager assumes that they can calculate the exact number of chocolates in each box by dividing the volume of the box by the volume of a single chocolate and then rounding down to the nearest integer.

(g) Explain why the production manager is incorrect. [1]

References:

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: applications and interpretation
Standard level
Paper 2

Friday 7 May 2021 (morning)

1 hour 30 minutes

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all the questions in the answer booklet provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

2221 – 7210

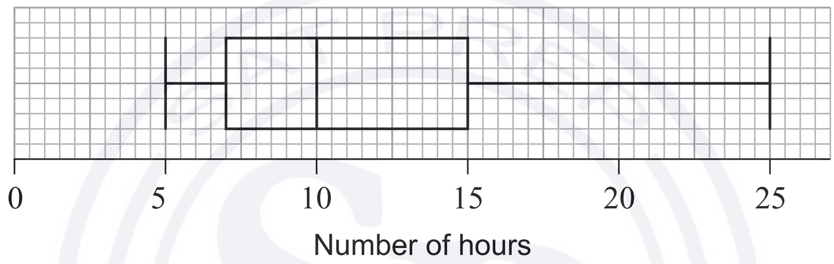
Answer **all** questions in the answer booklet provided. Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 18]

As part of his mathematics exploration about classic books, Jason investigated the time taken by students in his school to read the book *The Old Man and the Sea*. He collected his data by stopping and asking students in the school corridor, until he reached his target of 10 students from **each** of the literature classes in his school.

(a) State which of the two sampling methods, systematic or quota, Jason has used. [1]

Jason constructed the following box and whisker diagram to show the number of hours students in the sample took to read this book.



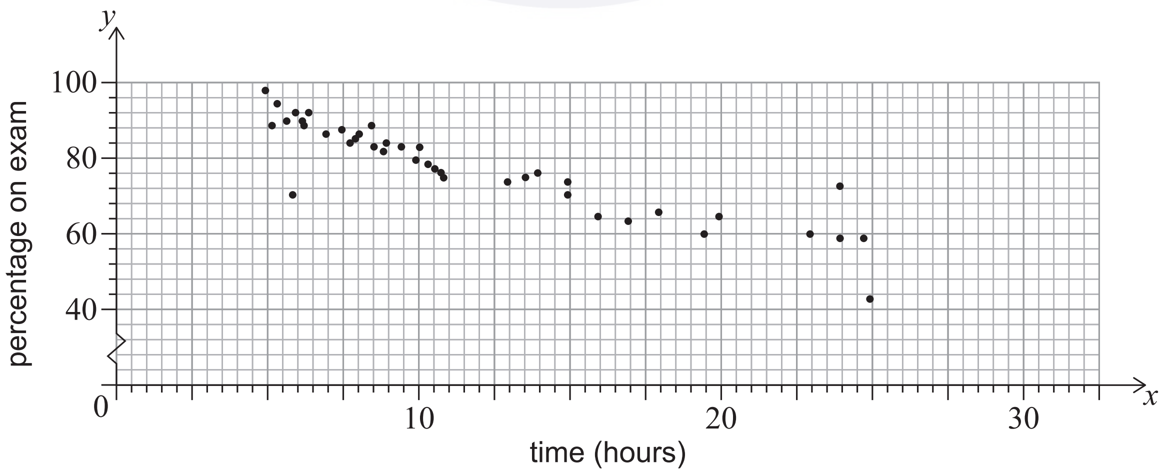
(b) Write down the median time to read the book. [1]

(c) Calculate the interquartile range. [2]

Mackenzie, a member of the sample, took 25 hours to read the novel. Jason believes Mackenzie's time is not an outlier.

(d) Determine whether Jason is correct. Support your reasoning. [4]

For each student interviewed, Jason recorded the time taken to read *The Old Man and the Sea* (x), measured in hours, and paired this with their percentage score on the final exam (y). These data are represented on the scatter diagram.



(e) Describe the correlation. [1]

(This question continues on the following page)

(Question 1 continued)

Jason correctly calculates the equation of the regression line y on x for these students to be

$$y = -1.54x + 98.8\,.$$

He uses the equation to estimate the percentage score on the final exam for a student who read the book in 1.5 hours.

(f) Find the percentage score calculated by Jason. [2]

(g) State whether it is valid to use the regression line y on x for Jason's estimate. Give a reason for your answer. [2]

Jason found a website that rated the 'top 50' classic books. He randomly chose eight of these classic books and recorded the number of pages. For example, Book H is rated 44th and has 281 pages. These data are shown in the table.

Book	A	B	C	D	E	F	G	H
Number of pages (n)	4215	863	585	1225	366	209	624	281
Top 50 rating (t)	1	2	5	7	13	22	40	44

Jason intends to analyse the data using Spearman's rank correlation coefficient, r_s.

(h) Copy and complete the information in the following table. [2]

Book	A	B	C	D	E	F	G	H
Rank – Number of pages	1							
Rank – Top 50 Rating	1							

(i) (i) Calculate the value of r_s.

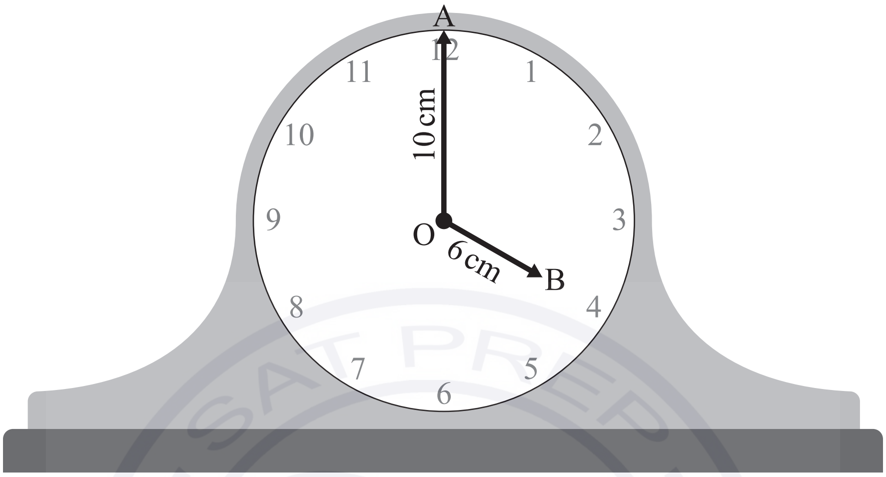
(ii) Interpret your result. [3]

2. [Maximum mark: 17]

The diagram below shows a circular clockface with centre O. The clock's minute hand has a length of $10\,\text{cm}$. The clock's hour hand has a length of $6\,\text{cm}$.

At 4:00 pm the endpoint of the minute hand is at point A and the endpoint of the hour hand is at point B.

diagram not to scale

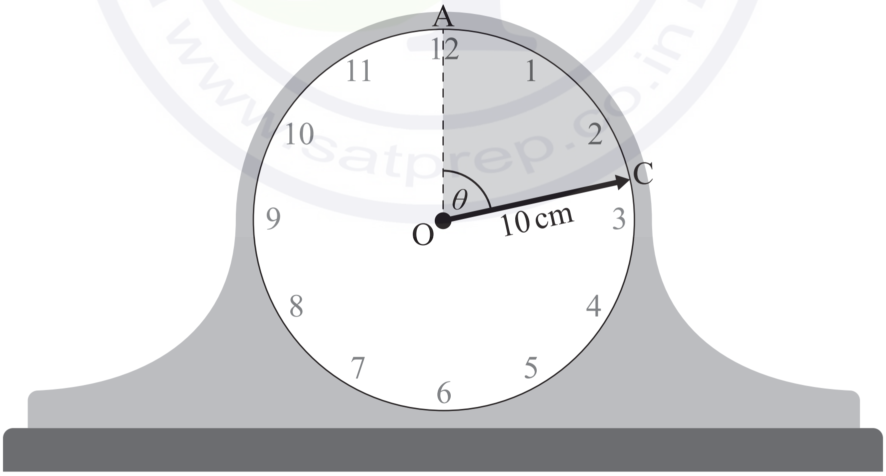


(a) Find the size of angle $\text{A}\hat{\text{O}}\text{B}$ in degrees. [2]

(b) Find the distance between points A and B. [3]

Between 4:00 pm and 4:13 pm, the endpoint of the **minute hand** rotates through an angle, θ, from point A to point C. This is illustrated in the diagram.

diagram not to scale



(c) Find the size of angle θ in degrees. [2]

(d) Calculate the length of arc AC. [2]

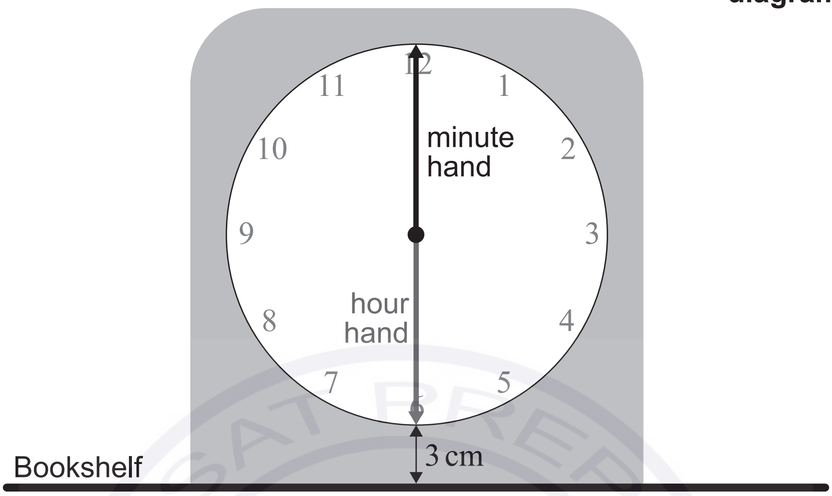
(e) Calculate the area of the shaded sector, AOC. [2]

(This question continues on the following page)

(Question 2 continued)

A **second** clock is illustrated in the diagram below. The clock face has radius $10\,\text{cm}$ with minute and hour hands both of length $10\,\text{cm}$. The time shown is $6{:}00$ am. The bottom of the clock face is located $3\,\text{cm}$ above a horizontal bookshelf.

diagram not to scale



(f) Write down the height of the endpoint of the minute hand above the bookshelf at $6{:}00$ am. [1]

The height, h centimetres, of the endpoint of the minute hand above the bookshelf is modelled by the function

$$h(\theta) = 10\cos\theta + 13\,,\ \theta \ge 0\,,$$

where θ is the angle rotated by the minute hand from $6{:}00$ am.

(g) Find the value of h when $\theta = 160^\circ$. [2]

The height, g centimetres, of the endpoint of the **hour hand** above the bookshelf is modelled by the function

$$g(\theta) = -10\cos\left(\frac{\theta}{12}\right) + 13\,,\ \theta \ge 0\,,$$

where θ is the angle in degrees rotated by the minute hand from $6{:}00$ am.

(h) Write down the amplitude of $g(\theta)$. [1]

The endpoints of the minute hand and hour hand meet when $\theta = k$.

(i) Find the smallest possible value of k. [2]

3. [Maximum mark: 19]

Give your answers in parts (a), (d)(i), (e) and (f) to the nearest dollar.

Daisy invested $37\,000$ Australian dollars (AUD) in a fixed deposit account with an annual interest rate of 6.4% compounded **quarterly**.

(a) Calculate the value of Daisy's investment after 2 years. [3]

After m months, the amount of money in the fixed deposit account has appreciated to more than $50\ 000\ \text{AUD}$.

(b) Find the minimum value of m, where $m \in \mathbb{N}$. [4]

Daisy is saving to purchase a new apartment. The price of the apartment is $200\,000\ \text{AUD}$.

Daisy makes an initial payment of 25% and takes out a loan to pay the rest.

(c) Write down the amount of the loan. [1]

The loan is for 10 years, compounded monthly, with equal monthly payments of $1700\ \text{AUD}$ made by Daisy at the end of each month.

(d) For this loan, find

(i) the amount of interest paid by Daisy.

(ii) the annual interest rate of the loan. [5]

After 5 years of paying off this loan, Daisy decides to pay the **remainder** in one final payment.

(e) Find the amount of Daisy's final payment. [3]

(f) Find how much money Daisy saved by making one final payment after 5 years. [3]

4. [Maximum mark: 13]

The stopping distances for bicycles travelling at $20\,\text{km}\,\text{h}^{-1}$ are assumed to follow a normal distribution with mean $6.76\,\text{m}$ and standard deviation $0.12\,\text{m}$.

(a) Under this assumption, find, correct to four decimal places, the probability that a bicycle chosen at random travelling at $20\,\text{km}\,\text{h}^{-1}$ manages to stop

(i) in less than $6.5\,\text{m}$.

(ii) in more than $7\,\text{m}$. [3]

1000 randomly selected bicycles are tested and their stopping distances when travelling at $20\,\text{km}\,\text{h}^{-1}$ are measured.

(b) Find, correct to four significant figures, the expected number of bicycles tested that stop between

(i) $6.5\,\text{m}$ and $6.75\,\text{m}$.

(ii) $6.75\,\text{m}$ and $7\,\text{m}$. [3]

The measured stopping distances of the 1000 bicycles are given in the table.

Measured stopping distance	**Number of bicycles**
Less than $6.5\,\text{m}$	12
Between $6.5\,\text{m}$ and $6.75\,\text{m}$	428
Between $6.75\,\text{m}$ and $7\,\text{m}$	527
More than $7\,\text{m}$	33

It is decided to perform a χ^2 goodness of fit test at the 5% level of significance to decide whether the stopping distances of bicycles travelling at $20\,\text{km}\,\text{h}^{-1}$ can be modelled by a normal distribution with mean $6.76\,\text{m}$ and standard deviation $0.12\,\text{m}$.

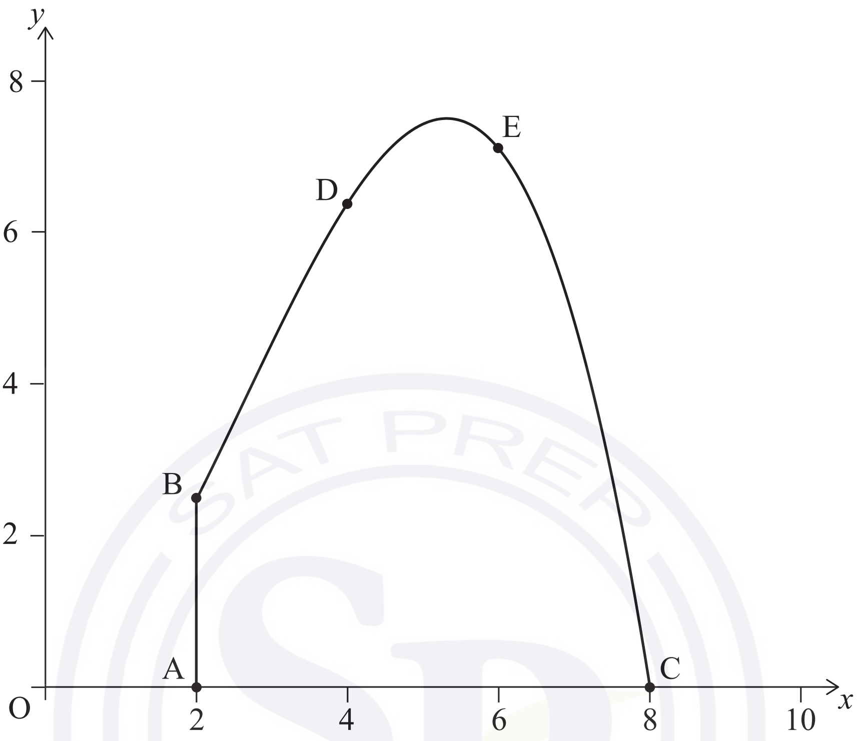
(c) State the null and alternative hypotheses. [2]

(d) Find the p-value for the test. [3]

(e) State the conclusion of the test. Give a reason for your answer. [2]

5. [Maximum mark: 13]

The cross-sectional view of a tunnel is shown on the axes below. The line [AB] represents a vertical wall located at the left side of the tunnel. The height, in metres, of the tunnel above the horizontal ground is modelled by $y = -0.1x^3 + 0.8x^2$, $2 \le x \le 8$, relative to an origin O.



Point A has coordinates $(2, 0)$, point B has coordinates $(2, 2.4)$, and point C has coordinates $(8, 0)$.

(a) (i) Find $\frac{dy}{dx}$.

(ii) Hence find the maximum height of the tunnel. [6]

When $x = 4$ the height of the tunnel is 6.4 m and when $x = 6$ the height of the tunnel is 7.2 m. These points are shown as D and E on the diagram, respectively.

(b) Use the trapezoidal rule, with three intervals, to estimate the cross-sectional area of the tunnel. [3]

(c) (i) Write down the integral which can be used to find the cross-sectional area of the tunnel.

(ii) Hence find the cross-sectional area of the tunnel. [4]

References:

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: applications and interpretation
Standard level
Paper 2

Specimen paper

1 hour 30 minutes

Instructions to candidates

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- The maximum mark for this examination paper is **[80 marks]**.

9 pages

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Answer **all** questions in the answer booklet provided. Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 17]

In this question, give all answers to two decimal places.

Bryan decides to purchase a new car with a price of €$14\,000$, but cannot afford the full amount. The car dealership offers two options to finance a loan.

Finance option A:

A 6 year loan at a nominal annual interest rate of 14% **compounded quarterly**. No deposit required and repayments are made each quarter.

(a) (i) Find the repayment made each quarter.

(ii) Find the total amount paid for the car.

(iii) Find the interest paid on the loan. [7]

Finance option B:

A 6 year loan at a nominal annual interest rate of $r\,\%$ **compounded monthly**. Terms of the loan require a 10% deposit and monthly repayments of €250.

(b) (i) Find the amount to be borrowed for this option.

(ii) Find the annual interest rate, r. [5]

(c) State which option Bryan should choose. Justify your answer. [2]

Bryan's car depreciates at an annual rate of 25% per year.

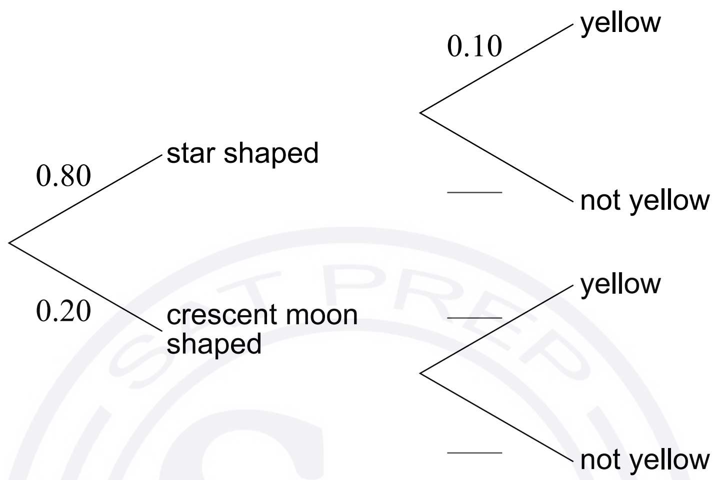
(d) Find the value of Bryan's car six years after it is purchased. [3]

2. [Maximum mark: 14]

Slugworth Candy Company sell a variety pack of colourful, shaped sweets.

The sweets are produced such that 80% are star shaped and 20% are shaped like a crescent moon. It is known that 10% of the stars and 30% of the crescent moons are coloured yellow.

(a) Using the given information, **copy** and complete the following tree diagram. [2]



(b) A sweet is selected at random.

(i) Find the probability that the sweet is yellow.

(ii) Given that the sweet is yellow, find the probability it is star shaped. [4]

(This question continues on the following page)

(Question 2 continued)

According to manufacturer specifications, the colours in each variety pack should be distributed as follows.

Colour	Brown	Red	Green	Orange	Yellow	Purple
Percentage (%)	15	25	20	20	10	10

Mr Slugworth opens a pack of 80 sweets and records the frequency of each colour.

Colour	Brown	Red	Green	Orange	Yellow	Purple
Observed Frequency	10	20	16	18	12	4

To investigate if the sample is consistent with manufacturer specifications, Mr Slugworth conducts a χ^2 goodness of fit test. The test is carried out at a 5% significance level.

(c) Write down the null hypothesis for this test. [1]

(d) **Copy** and complete the following table in your answer booklet. [2]

Colour	Brown	Red	Green	Orange	Yellow	Purple
Expected Frequency						

(e) Write down the number of degrees of freedom. [1]

(f) Find the p-value for the test. [2]

(g) State the conclusion of the test. Give a reason for your answer. [2]

3. [Maximum mark: 17]

The Malvern Aquatic Center hosted a 3 metre spring board diving event. The judges, Stan and Minsun awarded 8 competitors a score out of 10. The raw data is collated in the following table.

Competitors	A	B	C	D	E	F	G	H
Stan's score (x)	4.1	3	4.3	6	7.1	6	7.5	6
Minsun's score (y)	4.7	4.6	4.8	7.2	7.8	9	9.5	7.2

(a) (i) Write down the value of the Pearson's product–moment correlation coefficient, r.

(ii) Using the value of r, interpret the relationship between Stan's score and Minsun's score. [4]

(b) Write down the equation of the regression line y on x. [2]

(c) (i) Use your regression equation from part (b) to estimate Minsun's score when Stan awards a perfect 10.

(ii) State whether this estimate is reliable. Justify your answer. [4]

The Commissioner for the event would like to find the Spearman's rank correlation coefficient.

(d) **Copy** and complete the information in the following table. [2]

Competitors	A	B	C	D	E	F	G	H
Stan's Rank		8					1	4
Minsun's Rank		8					1	4.5

(e) (i) Find the value of the Spearman's rank correlation coefficient, r_s.

(ii) Comment on the result obtained for r_s. [4]

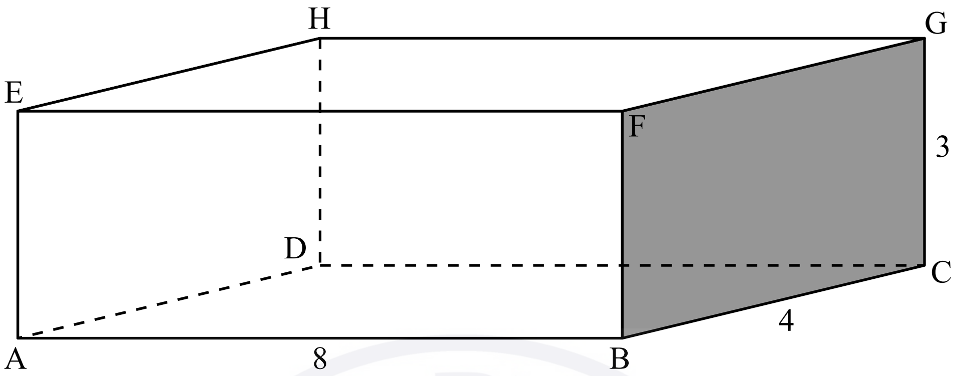
The Commissioner believes Minsun's score for competitor G is too high and so decreases the score from 9.5 to 9.1.

(f) Explain why the value of the Spearman's rank correlation coefficient r_s does not change. [1]

4. [Maximum mark: 15]

The Happy Straw Company manufactures drinking straws.

The straws are packaged in small closed rectangular boxes, each with length $8\,\text{cm}$, width $4\,\text{cm}$ and height $3\,\text{cm}$. The information is shown in the diagram.



(a) Calculate the surface area of the box in cm^2. [2]

(b) Calculate the length AG. [2]

Each week, the Happy Straw Company sells x boxes of straws. It is known that $\frac{\text{d}P}{\text{d}x} = -2x + 220$, $x \geq 0$, where P is the weekly profit, in dollars, from the sale of x thousand boxes.

(c) Find the number of boxes that should be sold each week to maximize the profit. [3]

The profit from the sale of $20\,000$ boxes is $\$1700$.

(d) Find $P(x)$. [5]

(e) Find the least number of boxes which must be sold each week in order to make a profit. [3]

5. [Maximum mark: 17]

The braking distance of a vehicle is defined as the distance travelled from where the brakes are applied to the point where the vehicle comes to a complete stop.

The speed, $s\ \text{m s}^{-1}$, and braking distance, d m, of a truck were recorded. This information is summarized in the following table.

Speed, s m s^{-1}	0	6	10
Braking distance, d m	0	12	60

This information was used to create Model A, where d is a function of s, $s \geq 0$.

Model A: $d(s) = ps^2 + qs$, where $p, q \in \mathbb{Z}$

At a speed of $6\ \text{m s}^{-1}$, Model A can be represented by the equation $6p + q = 2$.

(a) (i) Write down a second equation to represent Model A, when the speed is $10\ \text{m s}^{-1}$.

(ii) Find the values of p and q [4]

(b) Find the coordinates of the vertex of the graph of $y = d(s)$. [2]

(c) Using the values in the table and your answer to part (b), sketch the graph of $y = d(s)$ for $0 \leq s \leq 10$ and $-10 \leq d \leq 60$, clearly showing the vertex. [3]

(d) Hence, identify why Model A may not be appropriate at lower speeds. [1]

Additional data was used to create Model B, **a revised model** for the braking distance of a truck.

Model B: $d(s) = 0.95s^2 - 3.92s$

(e) Use Model B to calculate an estimate for the braking distance at a speed of $20\ \text{m s}^{-1}$. [2]

The actual braking distance at $20\ \text{m s}^{-1}$ is 320 m.

(f) Calculate the percentage error in the estimate in part (e). [2]

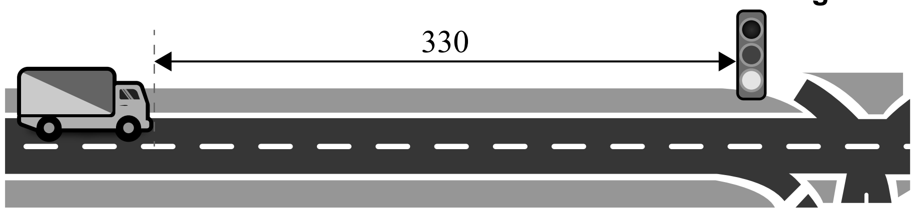
(This question continues on the following page)

(Question 5 continued)

It is found that once a driver realizes the need to stop their vehicle, 1.6 seconds will elapse, on average, before the brakes are engaged. During this reaction time, the vehicle will continue to travel at its original speed.

A truck approaches an intersection with speed $s\ \mathrm{m\,s^{-1}}$. The driver notices the intersection's traffic lights are red and they must stop the vehicle within a distance of $330\,\mathrm{m}$.

diagram not to scale



(g) Using model B and taking reaction time into account, calculate the maximum possible speed of the truck if it is to stop before the intersection. [3]