2225 – 7310M

Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

May 2025

Mathematics: applications and interpretation

Standard level

Paper 2

19 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

R Marks awarded for clear **Reasoning**.

AG Answer given in the question and so no marks are awarded.

FT Follow through. The practice of awarding marks, despite candidate errors in previous parts, for their correct methods/answers using incorrect results.

Using the markscheme

1 General

Award marks using the annotations as noted in the markscheme *eg* ***M1, A2.***

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***A3***, ***M2*** *etc.*, do **not** split the marks, unless there is a note.
- The response to a "show that" question does not need to restate the ***AG*** line, unless a **Note** makes this explicit in the markscheme.
- Once a correct answer to a question or part question is seen, ignore further working even if this working is incorrect and/or suggests a misunderstanding of the question. This will encourage a uniform approach to marking, with less examiner discretion. Although some candidates may be advantaged for that specific question item, it is likely that these candidates will lose marks elsewhere too.
- An exception to the previous rule is when an incorrect answer from further working is used **in a subsequent part**. For example, when a correct exact value is followed by an incorrect decimal approximation in the first part and this approximation is then used in the second part. In this situation, award ***FT*** marks as appropriate but do not award the final ***A1*** in the first part.

Examples:

	Correct answer seen	**Further working seen**	**Any FT issues?**	**Action**
1.	$8\sqrt{2}$	5.65685... *(incorrect decimal value)*	No. Last part in question.	Award ***A1*** for the final mark *(condone the incorrect further working)*
2.	$\frac{35}{72}$	0.468111… (incorrect decimal value)	Yes. Value is used in subsequent parts.	Award ***A0*** for the final mark *(and full **FT** is available in subsequent parts)*

3 Implied marks

Implied marks appear in **brackets e.g.** ***(M1)***,and can only be awarded if **correct** work is seen or implied by subsequent working/answer.

4 Follow through marks (only applied after an error is made)

Follow through (***FT***) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) (e.g. incorrect value from part (a) used in part (d) or incorrect value from part (c)(i) used in part (c)(ii)). Usually, to award ***FT*** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if all the marks awarded in a subsequent part are for the answer or are implied, then ***FT*** marks should be awarded for *their* correct answer, even when working is not present.

For example: following an incorrect answer to part (a) that is used in subsequent parts, where the markscheme for the subsequent part is ***(M1)A1***, it is possible to award full marks for *their* correct answer, **without working being seen.** For longer questions where all but the answer marks are implied this rule applies but may be overwritten by a **Note** in the Markscheme.

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate.

- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks, by reflecting on what each mark is for and how that maps to the simplified version.

- If the error leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).

- The markscheme may use the word “their” in a description, to indicate that candidates may be using an incorrect value.

- If the candidate’s answer to the initial question clearly contradicts information given in the question, it is not appropriate to award any ***FT*** marks in the subsequent parts. This includes when candidates fail to complete a “show that” question correctly, and then in subsequent parts use their incorrect answer rather than the given value.

- Exceptions to these ***FT*** rules will be explicitly noted on the markscheme.

- If a candidate makes an error in one part but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the command term was “Hence”.

5 Mis-read

If a candidate incorrectly copies values or information from the question, this is a mis-read (***MR***). A candidate should be penalized only once for a particular misread. Use the ***MR*** stamp to indicate that this has been a misread and do not award the first mark, even if this is an ***M*** mark, but award all others as appropriate.

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.
- If a candidate uses a correct answer, to a "show that" question, to a higher degree of accuracy than given in the question, this is NOT a misread and full marks may be scored in the subsequent part.
- ***MR*** can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If the command term is 'Hence' and not 'Hence or otherwise' then alternative methods are not permitted unless covered by a note in the mark scheme.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**.

7 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation** for example 1.9 and 1,9 or 1000 and 1,000 and 1.000.
- Do not accept final answers written using calculator notation. However, ***M*** marks and intermediate ***A*** marks can be scored, when presented using calculator notation, provided the evidence clearly reflects the demand of the mark.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, some **equivalent** answers will generally appear in brackets. Not all equivalent notations/answers/methods will be presented in the markscheme and examiners are asked to apply appropriate discretion to judge if the candidate work is equivalent.

8 Format and accuracy of answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. If the level of accuracy is not stated in the question, the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures.*

Where values are used in subsequent parts, the markscheme will generally use the exact value, however candidates may also use the correct answer to 3 sf in subsequent parts. The markscheme will often explicitly include the subsequent values that come "*from the use of 3 sf values*".

Simplification of final answers: Candidates are advised to give final answers using good mathematical form. In general, for an ***A*** mark to be awarded, arithmetic should be completed, and any values that lead to integers should be simplified; for example, $\sqrt{\frac{25}{4}}$ should be written as $\frac{5}{2}$. An exception to this is simplifying fractions, where lowest form is not required (although the numerator and the denominator must be integers); for example, $\frac{10}{4}$ may be left in this form or written as $\frac{5}{2}$. However, $\frac{10}{5}$ should be written as 2, as it simplifies to an integer.

Algebraic expressions should be simplified by completing any operations such as addition and multiplication, e.g. $4e^{2x} \times e^{3x}$ should be simplified to $4e^{5x}$, and $4e^{2x} \times e^{3x} - e^{4x} \times e^{x}$ should be simplified to $3e^{5x}$. Unless specified in the question, expressions do not need to be factorized, nor do factorized expressions need to be expanded, so $x(x+1)$ and $x^2 + x$ are both acceptable.

Please note: intermediate ***A*** marks do NOT need to be simplified.

9 Calculators

A GDC is required for this paper, but if you see work that suggests a candidate has used any calculator not approved for IB DP examinations (eg CAS enabled devices), please follow the procedures for malpractice.

10 Presentation of candidate work

Crossed out work: If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work unless an explicit note from the candidate indicates that they would like the work to be marked.

More than one solution: Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise. If the layout of the responses makes it difficult to judge, examiners should apply appropriate discretion to judge which is "first".

1. (a) $(k =)\ 5$ ***A1***

[1 mark]

(b) (i) attempt to substitute into the mean formula with a sum of at least two correct products **and** equating to 2.2 (or $2.2\ (p+27)$ if denominator not seen) or equating to "mean". Equation may be seen in part b(ii). ***(M1)***

$$2.2 = \frac{0\times 3+1\times 11+2\times 7+3\times 5+4\times p+5\times 1}{3+11+7+5+p+1} \quad \textbf{OR} \quad 2.2 = \frac{45+4p}{p+27}$$ ***A1***

(ii) $(p =)\ 8$ ***A1***

Note: Follow through from part (b)(i) only if their value of p is a whole number.

[3 marks]

(c) finding the median $(=2)$, quartiles ($Q_1 = 1$ and $Q_3 = 4$), or interquartile range of the data $IQR = 3$ from the 2024 tournament ***(A1)***

Note: Award ***(A1)*** for any correctly identified/labelled value referring to 2024 data as listed in the question. Accept "Range from 0 to 5" without reference to 2024.

Any two of the following statements: ***R1R1***

The median number of goals per game **in both years** is 2.
In both years, (the lower quartile is 1 and the upper quartile is 4) the IQR is 3.
Both distributions are (roughly) skewed to the right

Note: Award ***R1*** for each correct statement. Each statement must reference values from both the box and whisker diagram **and** the data from the 2024 tournament to award ***R1***. (e.g. "the median is 2" earns ***R0***, "the median for both years is 2" earns ***R1***.) If an incorrect statement is made, award at most ***R1R0***.

[3 marks]

(d) B and C ***A1A1***

Note: Award ***A1*** for each correct event, if one or two events are selected.
Award ***A1A0*** for two correct events and one additional incorrect event selected.
Award ***A0A1*** for an answer of "A, D, and E" from misreading the meaning of F'.
Award ***A0A0*** for any other combination of events selected.

[2 marks]

(e) $\frac{5}{17}\left(\frac{10}{34}\right)$ ***A1A1***

Note: Award ***A1*** for correct numerator, ***A1*** for correct denominator. Award at most ***A1A0*** for a decimal answer $(0.294117\ldots)$ if no fraction is shown. Do not award ***A1ft*** for their denominator if their answer to part (a) or part (b)(ii) is not a whole number.

[2 marks]

(f) $\frac{1}{35}\times\frac{3}{34}$ ***(A1)(A1)(M1)***

Note: Award ***A1*** for each correct probability, ***M1*** for multiplying their two probabilities. Do not award any marks if their answer to part (a) or part (b)(ii) is not a whole number.

0.00252 $(0.00252100\ldots, \frac{3}{1190}, 0.252\%)$ ***A1***

[4 marks]
Total [15 marks]

2. (a) **EITHER**

$N = 72$ ***(M1)(A1)***

$I\% = 3.4$

$PV = \pm 10\,000$

$P/Y = 12$

$C/Y = 12$

Note: Award ***M1*** for an attempt to use a financial app in their technology (i.e., at least three entries seen, but not necessarily correct), ***A1*** for all entries correct.

OR

$N = 72$ ***(M1)(A1)***

$I\% = 3.4/12$

$PV = \pm 10\,000$

$P/Y = 1$

$C/Y = 1$

Note: Award ***M1*** for an attempt to use a financial app in their technology (i.e., at least three entries seen, but not necessarily correct), ***A1*** for all entries correct.

OR

attempt to substitute into the compound interest formula ***(M1)***

$FV = 10\,000 \times \left(1 + \frac{3.4}{100 \times 12}\right)^{6 \times 12}$ ***(A1)***

THEN

($FV =$) 12 259 (CAD) ***A1***

[3 marks]

(b) 212 741 (CAD) ***A1***

[1 mark]

(c) $N = 180$ ***(M1)(A1)(A1)(A1)***
$FV = 0$
$I\% = 6.4$
$PV = \pm 212\,741$
$P/Y = 12$
$C/Y = 2$

Note: Award ***M1*** for attempt to use a financial app with at least three entries seen, but not necessarily correct, ***A1*** for N **and** I, ***A1*** for P/Y **and** C/Y, ***A1*** for their PV from (b) **and** $FV = 0$. Approaches that use the compound interest formula receive no marks.

$(PMT =)\ 1832$ (CAD) ***A1***

Note: Accept -1832 (CAD).

[5 marks]

(d) (i) $N = 36$ ***(M1)(A1)***
$PV = \pm 194\,572$
$PMT = 2200$

Note: Award ***M1*** for adjusting at least one of N, PV, or PMT; ***A1*** for correct entries (PV and PMT must have opposite signs). Allow ***FT*** from incorrect entries for P/Y and/or C/Y from part (c).

$(FV =) 148\,099$ (CAD) ***(A1)***
$(194\,572 - 148\,099)$

46 473 (CAD) ***A1***

(ii) $2200 \times 12 \times 3\ (= 79\,200)$ ***(A1)***
$(79\,200 - 46\,473 =)$

32 727 (CAD) ***A1***

[6 marks]
Total [15 marks]

3. (a) **EITHER**
The ratio of consecutive terms is constant. ***R1***

OR
Each term of the sequence is found by multiplying the previous term by a constant. ***R1***

Note: Do not accept "it has a common ratio" unless it is stated or illustrated that the ratio is between consecutive terms. Do not accept "because $r = 1.05$" or " because each subsequent day that he runs he increases his distance by 5%" for the explanation. The explanation must refer to a constant ratio or repeated multiplication by a constant.

THEN
$r = 1.05$ ***A1***

Note: Award ***R0A1*** for $r = 1.05$ seen with incorrect or no explanation.

[2 marks]

(b) attempt to substitute into the sum formula for geometric sequence **OR**
attempt to write the sum in sigma notation using the term formula **OR** a sum of geometric sequence terms ***(M1)***

$(S_{15} =) \dfrac{2\left(1.05^{15} - 1\right)}{1.05 - 1}$ **OR** $\sum_1^{15}(2 \times 1.05^{n-1})$ **OR** $2 + 2 \times 1.05 + 2 \times 1.05^2 + ...$

$43.2\ (\text{km})$ $(43.1571\ldots\ (\text{km}))$ ***A1***
[2 marks]

(c) choosing $n = 36$ ***(A1)***

attempt to substitute into term formula for geometric sequence ***(M1)***
$(u_{36} =)\ 2 \times 1.05^{36-1}$

$11.0\ (11.0320\ldots) > 5 \times 2$ **OR** 11.0 verbally compared to 5×2 ***A1***

Note: Award at most ***A1M1A0*** if a final distance (11.0 or 11) is not given or if the comparison to 10 (5×2) is not shown. Accept 11 for u_{36} (correct 2sf answer).

[3 marks]

(d) (i) recognition of swimming out and back **AND/OR** swimming two times per week ***(M1)***

$(u_1 =)\ 80 \times 2 \times 2$

$(u_1 =)\ 320\,(\text{m})$ ***A1***

(ii) attempt to substitute into term formula of arithmetic sequence ***(M1)***

$704 = 320 + (4 - 1)\,d$ **OR** $\frac{704-320}{3}$ ***(A1)***

Note: Equating to 704 must be seen to award the ***(A1)***.

$(d =)\ 128\ (\text{m})$ ***A1***

(iii) **METHOD 1: Using increased swimming distance**

$(128 \div 2 \div 2 =)\ 32\ (\text{m})$ (further each week) ***(A1)***

$(80 + 32 =)\ 112\ (\text{m})$ (from the shore in week 2) ***A1***

METHOD 2: Using total swimming distance for week 2

$(320 + 128 =)\ 448\ (\text{m})$ (swam in week 2) ***(A1)***

$(448 \div 2 \div 2 =)\ 112\ (\text{m})$ (from the shore in week 2) ***A1***

[7 marks]

(e) **METHOD 1: Using sequence for distance swam per week (i.e. 320, 448, 576…)**

attempt to substitute *their* answers from (d)(i) and (d)(ii) into term formula ***(M1)***

$(u_{12} =)\ 320 + (12 - 1) \times 128$

$1728\ (\text{m})$ (for the week) ***(A1)***

$864\ (\text{m})$ (on a day in week 12) ***A1***

METHOD 2: Using a sequence (t_n) for distance swam per day each week (i.e. 160, 224, 288…)

recognizing $t_1 = 160$ and $d = 64$ ***(A1)***

attempt to substitute 160 and 64 into term formula ***(M1)***

$(t_{12} =)\ 160 + (12 - 1) \times 64$

$864\ (\text{m})$ (on a day in week 12) ***A1***

METHOD 3: **Using a sequence (v_n) for distance of marker from shore each week (i.e. 80, 112, 144…)**

attempt to substitute 80 and 32 into term formula ***(M1)***

$(v_{12} =)\ 80 + (12 - 1) \times 32$

(marker is) $432\ (\text{m})$ (from shore) ***(A1)***

$(432 \times 2 =)\ 864\ (\text{m})$ (on a day in week 12) ***A1***

[3 marks]

[Total: 17 marks]

4. (a) $(4, 1, 0)$ ***A1A1***

Note: Award ***A1*** for 4 seen as the x-coordinate or 1 seen as the y- coordinate, ***A1*** for all coordinates correct and in the correct order. Accept $x = 4, y = 1, z = 0$.

[2 marks]

(b) correct substitution into distance formula ***(A1)***
$\sqrt{(13-13)^2 + (25-1)^2 + (7-0)^2}$ **OR** $\sqrt{0^2 + 24^2 + 7^2}$

Note: Accept omission of $(13-13)^2$ or 0^2.

$(\text{AC} =)\ 25\ (\text{cm})$ ***A1***

[2 marks]

(c) **Method 1**
24 and 7 seen ***A1***

$24^2 + 7^2 = 25^2$ **OR** $625 = 625$ **OR** (angle $\text{B} =)\ \cos^{-1}\left(\frac{7^2 + 24^2 - 25^2}{2(7)(24)}\right) = 90^\circ$ ***R1***

Note: Accept $\text{AB}^2 + \text{BC}^2 = 625$ as long as the length of AB and of BC are defined.

therefore ΔABC must be a right-angled triangle ***AG***

Method 2

A relevant statement describing $\overrightarrow{\text{AB}}$ and $\overrightarrow{\text{BC}}$, such as:
$\text{AB} \parallel y$-axis **AND** $\text{BC} \parallel z$-axis
OR
x-coordinates are the same for A and B **AND** y-coordinates are the same for B and C ***A1***
AB perpendicular to BC since x-axis is perpendicular to z-axis ***R1***
therefore ΔABC must be a right-angled triangle ***AG***

[2 marks]

(d) width of 9 or correct cross-section area (84) seen ***(A1)***

correct substitution into volume formula ***(A1)***
$V = \frac{1}{2}(24)(7)(9)$

$(\text{Volume} =) 756\ \text{cm}^3$ ***A1***

Note: Do not award final ***A1*** if correct units are not included.

[3 marks]

(e) correct substitution using *their* AB from part (c) or (d) and *their* original volume from part (d) ***(A1)***

$(\text{AX} =)\ 24 \times \sqrt{\frac{625}{756}}$

$21.8\ (\text{cm})\quad (21.8217\ldots\ (\text{cm}))$ ***A1***

[2 marks]

(f) multiplying 625 by 1.1 or 1.2 or 0.025 ***(M1)***

$625 \times 1.1 \times 0.025 \times 1.2$ ***(A1)***

(price $=$) $20.63\ \ (20.625)$ (USD) ***A1***

[3 marks]

[Total 14 marks]

5. (a) (i) attempt to substitute any given coordinates into midpoint formula ***(M1)***

$(12, 12.5)$ or $(12, \frac{25}{2})$ ***A1***

Note: Award ***A0*** if parentheses are omitted.

(ii) attempt to find gradient of BC using coordinates of B and C ***(M1)***

(gradient of $\mathrm{BC}=$) $\frac{12-13}{16-8}\left(=-\frac{1}{8}=-0.125\right)$ ***(A1)***

(gradient of $\perp=$) 8 ***A1***

Note: Follow through within question part for final ***A1*** from their gradient of BC only if the gradient is seen.

(iii) **EITHER**

$8(x-12)=y-12.5 \quad \left(8(x-12)=y-12\frac{1}{2}\right)$ ***A1***

OR

$y=8x-83.5 \quad \left(y=8x-83\frac{1}{2}\right)$ ***A1***

[6 marks]

(b) (i) **EITHER**

B is expected to respond since $(14, 10)$ lies in the same cell (of the Voronoi diagram) as fire station B ***R1***

OR

B is expected to respond since $(14, 10)$ is closer to B than to any other fire station ***R1***

OR

B is expected to respond since it's the closest (nearest responder) to $(14, 10)$ ***R1***

OR

Fire station B will respond as the fire is within B's boundaries/area/region. ***R1***

Note: Do not award ***R1*** unless the statement refers to both the distance (closer) to B (or located in the B cell) **and** the responder (point B or fire station B) For example, award ***R0*** for "Point B is closer to $(14, 10)$" as there is no reference to a responder.

(ii) *Award **R1** for any viable reason such as:*

- fire station B might have already responded to a different fire
- a different fire station might have more manpower or equipment available to fight the fire
- a different fire station might respond to assist fire station B if the fire is large
- the Voronoi diagram only considers straight-line distances, so a different fire station might have a shorter driving route to the fire ***R1***

[2 marks]

(c) (i) attempt to calculate the length DB ***(M1)***

a correct expression for DB ***A1***

$(\mathrm{DB}=)\ \sqrt{\left(11\frac{1}{3}-16\right)^2+\left(7\frac{1}{6}-12\right)^2}$

$\mathrm{DB}=6.71855$ ***AG***

(ii) **METHOD 1**

$(\mathrm{DC}=)\ \sqrt{\left(11\frac{1}{3}-8\right)^2+\left(7\frac{1}{6}-13\right)^2}$ ***A1***

$\mathrm{DC}=6.71855$ therefore, the fire is equidistant from all 3 fire stations ***R1***

so, any of the fire stations would be expected to respond ***AG***

Note: Do not award ***A0R1***. Do not award ***R1*** if ***AG*** statement is not seen.

METHOD 2

The intersection of bisector AB and bisector BC:

$y=8x-83.5$ and $y=0.5x+1.5$ have an intersection at $\mathrm{D}\left(11\frac{1}{3},\ 7\frac{1}{6}\right)$ ***A1***

Note: Calculator solve is acceptable but D must be clearly identified from a calculation of intersecting lines.

Since D is the intersection of the bisector lines, the fire at D is equidistant from all 3 fire stations ***R1***

so, any of the fire stations would be expected to respond ***AG***

Note: Do not award ***A0R1.*** Do not award ***R1*** if ***AG*** statement is not seen.

[4 marks]

(d) (i) **METHOD 1: Dividing into two triangles**

$(0.5\times 20+1.5=)\ 11.5$ seen in an area calculation ***(A1)***

$(20-7.75=)\ 12.25$ or $(20-11.3333\ldots=)\ 8.66666\ldots$ seen ***(A1)***

substitution into triangle area formula for either triangle ***(M1)***

$(A=)\ 0.5\times(20-7.75)\times 7.16666\ldots+0.5\times 11.5\times(20-11.3333\ldots)$ ***(A1)***

$93.7291\ldots$ (sq. units)

94 (sq. units) ***A1***

Note: Answer must be given as a whole number to award final ***A1***.
Award ***(A1)(A1)(M1)(A1)A0*** for an answer of 93.7 without working.

METHOD 2: Dividing vertically into triangle and trapezoid

$(0.5\times 20+1.5=)\ 11.5$ seen in an area calculation ***(A1)***

attempt to substitute into area of trapezoid formula ***(M1)***
$(A=)\ 0.5\times(7.16666\ldots+11.5)\times(20-11.3333\ldots)$

Note: Lengths of parallel sides must be seen to award the ***M1***.

correct calculation of right triangle area ***(A1)***
$(A=)\ 0.5\times(11.3333\ldots-7.75)\times 7.16666\ldots$

$(A=)\ 0.5\times(11.3333\ldots-7.75)\times 7.16666\ldots+0.5\times(7.16666\ldots+11.5)\times(20-11.3333\ldots)$ ***(A1)***

$93.7291\ldots$ (sq. units)

94 (sq. units) ***A1***

Note: Answer must be given as a whole number to award final ***A1***.
Award ***(A1)(M1)(A1)(A1)A0*** for an answer of 93.7 without working.

METHOD 3: Dividing horizontally into triangle and trapezoid

$(0.5\times 20+1.5=)\ 11.5$ seen in an area calculation ***(A1)***

attempt to substitute into area of trapezoid formula ***(M1)***

$(A=)\ 0.5\times\left[(20-7.75)+(20-11.3333\ldots)\right]\times 7.16666\ldots$

Note: Lengths of parallel sides must be seen to award the ***M1***.

correct calculation of right triangle area ***(A1)***

$(A=)\ 0.5\times(20-11.3333\ldots)\times(11.5-7.16666\ldots)$

$(A=)\ 0.5\times(20-11.3333\ldots)\times(11.5-7.16666\ldots)+0.5\times[(20-7.75)+(20-11.3333\ldots)]\times 7.16666\ldots$ ***(A1)***

$93.7291\ldots$ (sq. units)

94 (sq. units) ***A1***

Note: Answer must be given as a whole number to award final ***A1***.
Award ***(A1)(M1)(A1)(A1)A0*** for an answer of 93.7 without working.

METHOD 4: Dividing into two triangles and rectangle

$(0.5\times 20+1.5=)11.5$ seen in an area calculation ***(A1)***

$(20-11.3333...)$ **OR** $(11.3333...-7.75)$ **OR** $(11.5-7.16666...)$ seen ***(A1)***

substitution into triangle area formula. ***(M1)***

$0.5\times(11.3333...-7.75)\times 7.16666...+(20-11.3333...)\times 7.16666...+$
$+0.5\times(11.5-7.16666...)\times(20-11.3333...)$

(A1)

$93.7291\ldots$ (sq. units))

94 (sq. units) ***A1***

Note: Answer must be given as a whole number to award final ***A1***.
Award ***(A1)(A1)(M1)(A1)A0*** for an answer of 93.7 without working.

METHOD 5: Calculus

A and C mid point $(13, 10.5)$

(gradient of AC=) $\frac{13-8}{8-18} = -\frac{5}{10}$

(gradient of $\perp$=) 2

AC bisector $2(x-13) = y-10.5$ **OR** $y = 2x-15.5$ ***A1***

attempt to integrate their equation and given equation ***(M1)***

$\text{area} = \int_{7.75}^{11.3} 2x-15.5\,\mathrm{d}x + \int_{11.3}^{20} 0.5x+1.5\,\mathrm{d}x$ ***(A1)(A1)***

Note: Award ***A1*** for each correct expression with correct limits.

93.7291… (sq. units))

94 (sq. units) ***A1***

Note: Answer must be given as a whole number to award final ***A1***.
Award ***A1(M1)(A1)(A1)A0*** for an answer of 93.7 without working.

(ii) (1 sq unit = 2.5 km $\times$ 2.5 km =) 6.25 (km^2) ***(A1)***

Note: Award ***A1*** for 6.25 seen.

$(94 \times 6.25\,\text{km}^2)$

588 (587.5) (km^2) ***A1***

Note: Accept: 586 (585.807…) (km^2) from (93.7291… $\times$ 6.25 km^2).

[7 marks]
[Total: 19 marks]

2225 – 7315M

Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

May 2025

Mathematics: applications and interpretation

Standard level

Paper 2

12 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

R Marks awarded for clear **Reasoning**.

AG Answer given in the question and so no marks are awarded.

FT Follow through. The practice of awarding marks, despite candidate errors in previous parts, for their correct methods/answers using incorrect results.

Using the markscheme

1 General

Award marks using the annotations as noted in the markscheme *eg* ***M1, A2****.*

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***A3***, ***M2*** *etc*., do **not** split the marks, unless there is a note.
- The response to a "show that" question does not need to restate the ***AG*** line, unless a **Note** makes this explicit in the markscheme.
- Once a correct answer to a question or part question is seen, ignore further working even if this working is incorrect and/or suggests a misunderstanding of the question. This will encourage a uniform approach to marking, with less examiner discretion. Although some candidates may be advantaged for that specific question item, it is likely that these candidates will lose marks elsewhere too.
- An exception to the previous rule is when an incorrect answer from further working is used **in a subsequent part**. For example, when a correct exact value is followed by an incorrect decimal approximation in the first part and this approximation is then used in the second part. In this situation, award ***FT*** marks as appropriate but do not award the final ***A1*** in the first part.

Examples:

	Correct answer seen	Further working seen	Any FT issues?	Action
1.	$8\sqrt{2}$	5.65685... *(incorrect decimal value)*	No. Last part in question.	Award ***A1*** for the final mark *(condone the incorrect further working)*
2.	$\frac{35}{72}$	0.468111… (incorrect decimal value)	Yes. Value is used in subsequent parts.	Award ***A0*** for the final mark *(and full **FT** is available in subsequent parts)*

3 Implied marks

Implied marks appear in **brackets e.g.** ***(M1)***,and can only be awarded if **correct** work is seen or implied by subsequent working/answer.

4 Follow through marks (only applied after an error is made)

Follow through (***FT***) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) (e.g. incorrect value from part (a) used in part (d) or incorrect value from part (c)(i) used in part (c)(ii)). Usually, to award ***FT*** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if all the marks awarded in a subsequent part are for the answer or are implied, then ***FT*** marks should be awarded for *their* correct answer, even when working is not present.

For example: following an incorrect answer to part (a) that is used in subsequent parts, where the markscheme for the subsequent part is ***(M1)A1***, it is possible to award full marks for *their* correct answer, **without working being seen.** For longer questions where all but the answer marks are implied this rule applies but may be overwritten by a **Note** in the Markscheme.

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate.

- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks, by reflecting on what each mark is for and how that maps to the simplified version.

- If the error leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).

- The markscheme may use the word “their” in a description, to indicate that candidates may be using an incorrect value.

- If the candidate’s answer to the initial question clearly contradicts information given in the question, it is not appropriate to award any ***FT*** marks in the subsequent parts. This includes when candidates fail to complete a “show that” question correctly, and then in subsequent parts use their incorrect answer rather than the given value.

- Exceptions to these ***FT*** rules will be explicitly noted on the markscheme.
- If a candidate makes an error in one part but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the command term was "Hence".

5 Mis-read

If a candidate incorrectly copies values or information from the question, this is a mis-read (***MR***). A candidate should be penalized only once for a particular misread. Use the ***MR*** stamp to indicate that this has been a misread and do not award the first mark, even if this is an ***M*** mark, but award all others as appropriate.

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.

- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).

- Miscopying of candidates' own work does **not** constitute a misread, it is an error.

- If a candidate uses a correct answer, to a "show that" question, to a higher degree of accuracy than given in the question, this is NOT a misread and full marks may be scored in the subsequent part.

- ***MR*** can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If the command term is 'Hence' and not 'Hence or otherwise' then alternative methods are not permitted unless covered by a note in the mark scheme.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**.

7 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation** for example 1.9 and $1{,}9$ or 1000 and $1{,}000$ and 1.000.

- Do not accept final answers written using calculator notation. However, ***M*** marks and intermediate ***A*** marks can be scored, when presented using calculator notation, provided the evidence clearly reflects the demand of the mark.

- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.

- In the markscheme, some **equivalent** answers will generally appear in brackets. Not all equivalent notations/answers/methods will be presented in the markscheme and examiners are asked to apply appropriate discretion to judge if the candidate work is equivalent.

8 Format and accuracy of answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. If the level of accuracy is not stated in the question, the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures.*

Where values are used in subsequent parts, the markscheme will generally use the exact value, however candidates may also use the correct answer to 3 sf in subsequent parts. The markscheme will often explicitly include the subsequent values that come "*from the use of 3 sf values*".

Simplification of final answers: Candidates are advised to give final answers using good mathematical form. In general, for an ***A*** mark to be awarded, arithmetic should be completed, and any values that lead to integers should be simplified; for example, $\sqrt{\frac{25}{4}}$ should be written as $\frac{5}{2}$. An exception to this is simplifying fractions, where lowest form is not required (although the numerator and the denominator must be integers); for example, $\frac{10}{4}$ may be left in this form or written as $\frac{5}{2}$. However, $\frac{10}{5}$ should be written as 2, as it simplifies to an integer.

Algebraic expressions should be simplified by completing any operations such as addition and multiplication, e.g. $4e^{2x} \times e^{3x}$ should be simplified to $4e^{5x}$, and $4e^{2x} \times e^{3x} - e^{4x} \times e^{x}$ should be simplified to $3e^{5x}$. Unless specified in the question, expressions do not need to be factorized, nor do factorized expressions need to be expanded, so $x(x+1)$ and $x^2 + x$ are both acceptable.

Please note: intermediate ***A*** marks do NOT need to be simplified.

9 Calculators

A GDC is required for this paper, but if you see work that suggests a candidate has used any calculator not approved for IB DP examinations (eg CAS enabled devices), please follow the procedures for malpractice.

10. Presentation of candidate work

Crossed out work: If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work unless an explicit note from the candidate indicates that they would like the work to be marked.

More than one solution: Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise. If the layout of the responses makes it difficult to judge, examiners should apply appropriate discretion to judge which is “first”.

1. (a) (i) 7 ***A1***

(ii) 9 ***A1***

[2 marks]

(b) (i) 2 ***A1***

(ii) $1+2(n-1)\left(=2n-1\right)$ ***(M1)A1***

Note: ***(M1)*** for substituting their (b)(i) into the arithmetic sequence formula.

[3 marks]

(c) (i) 17 ***A1***

(ii) $\frac{9}{2}(1+17)$ OR $\frac{9}{2}(2\times1+(9-1)\times2)$ OR $1+3+\ldots+15+17$ ***(M1)***

$=81$ ***A1***

[3 marks]

(d) $\frac{n}{2}(1+(2n-1))$ OR $\frac{n}{2}(2\times1+(n-1)\times2)$ ***(M1)***

$=n^2$ ***A1***

[2 marks]

(e) 10 ***A1***

[1 mark]

(f) $1+2+3+...+n$ ***(M1)***

$=\frac{n}{2}(1+n)$ ***A1***

Note: Award ***(M1)*** for attempt for use of sum to n terms of an arithmetic sequence.

[2 marks]

(g) $\frac{n(1+n)}{2}+\frac{n(n-1)}{2}$ ***M1***

Note: Award ***M1*** for addition of answers to parts (f) and given expression.

$=\frac{n}{2}\left[\left(1+n\right)+\left(n-1\right)\right]$ OR $\frac{n+n^2+n^2-n}{2}$ ***M1***

Note: Award ***M1*** for valid non-trivial attempt at further simplification of their sum.

$=\frac{n}{2}(2n)$ OR $\frac{2n^2}{2}$

$=n^2$ ***A1***

Note: Award a maximum of ***M1M1A0*** for **ft** from an incorrect part (f).

[3 marks]

Total [16 marks]

2. (a) $0.638\ (0.637623...)$ and $4.10\ (4.09811...)$ ***A2A1***

Note: Award ***A2*** for one correct answer, accept 4.1. Award ***A2A0*** if coordinate pair is given.

[3 marks]

(b) (i) $\int_{0.638}^{4.1} 1.5^x \,\mathrm{d}x$ OR $\int_{0.638}^{4.1} f(x)\,\mathrm{d}x$ ***A1A1***

Note: ***A1*** for function and $\mathrm{d}x$, ***A1*** for limits. Award ***A1*** for $\int_{0.638}^{4.1} y\mathrm{d}x$.
Condone the use of modulus within the integrand.

(ii) $9.81\ (9.80788...)$ (from 3sf)

$9.80\ (9.79844...)$ (from full) ***A2***

Note: Accept 9.8 for ***A2***.

(iii) correct expression for area ***(A1)***

$\int_{0.638}^{4.1} g(x) - f(x)\,\mathrm{d}x$ **OR** $\int_{0.638}^{4.1} 6 - \frac{3}{x} - 1.5^x \,\mathrm{d}x$

$5.38\ (5.38290...)$ ***A1***

Note: Accept 5.39

[6 marks]

(c) $f'(x) = g'(x)$ ***(M1)***

correct graphs of $f'(x)$ and $g'(x)$ ***(M1)***

Note: Award ***M1*** if $\frac{3}{x^2}$ seen.

$(k =) 1.86\ (1.86406...)$ ***A1***

[3 marks]
Total [12 marks]

3. (a) (i) 20 (min) ***A1***

(ii) 100 (min^2) (accept 10^2) ***A1***

[2 marks]

(b) $\text{P}(15 < X < 25)$ ***(M1)***

Note: Condone calculator syntax.

0.383 $(0.382924...)$ (38.3%) ***A1***

[2 marks]

(c) $\text{P}(X > 30) \times 365$ ***(M1)(M1)***

Note: Award ***M1*** for attempt to find the probability $\text{P}(X > 30)$ and ***M1*** for their probability multiplied by 365. Condone calculator syntax

57.9 $(57.9091...)$ ***A1***

Note: For a final answer of 57 or 58 irrespective of 57.9 seen award ***M1M1A0***.

[3 marks]

(d) $\text{P}(X < k) = 0.99$ ***(M1)***

Note: Condone calculator syntax.

$43.2634... + 10 = 53.2634...$ ***(M1)(A1)***

time is $6{:}54$ pm $(18{:}54)$ ***A1***

Note: Award ***M1M1A1A0*** for a final answer of $6{:}53$ $(18{:}53)$

[4 marks]

(e) (i) $\text{P}(X > 0)$ ***(M1)***

Note: Condone calculator syntax.

0.977 $(0.9772499...)$ ***A1***

(ii) recognition of binomial ***(M1)***

$n = 7$, $p = 0.9772499...$ ***(A1)***

Note: Condone calculator syntax.

5 successes

0.00969 $(0.00968755...)$ ***A1***

Note: Accept 0.00989 from use of 3sf answers

(iii) three terms ***(M1)***

correct powers of probabilities ***(M1)***

correct coefficients ***(M1)***

$1(0.9772499...)^7 + 2(0.9772499...)^6(0.02275) + 3(0.9772499...)^5(0.02275)^2$

0.892 $(0.892231...)$ ***A1***

Note: Accept 0.891 from use of 3sf answers only if working is seen.

[9 marks]

Total [20 marks]

4. (a) (i) 90° ***A1***

(ii) $\sqrt{10^2+15^2}$ ***(M1)***

$18(.0\ )(\text{cm}\ )\quad(18.0277...)\quad\left(\sqrt{325}\right)$ ***A1***

[3 marks]

(b) (i) 150 ***A1***

(ii) 120 ***A1***

Note: Award ***A0A1*FT** for (0, 150) and (0, 120)

[2 marks]

(c) (i) 15 ***A1***

(ii) $\left(\frac{360}{60}=\right)6$ ***A1***

(iii) 135 ***A1***

[3 marks]

(d) (i) $(q=)\frac{360}{(60\times12)}=0.5$ or in 60 minutes, 30 degrees travelled so $(q=)\frac{30}{60}=\frac{1}{2}$ ***A1AG***

(ii) -10 ***A2***

Note: Award ***A1*** for 10.

[3 marks]

(e) substitution of $t=60$ into their $g(t)$ ***(M1)***

130 (cm) ***A1***

[2 marks]

(f) $15\cos(6t)+135=-10\sin(0.5t)+135$ ***(M1)***

$(t=)15.8806...$ ***(A1)***

(time =) 3:16 (pm) (15:16) ***A1***

Note: If the answer to part (d)(ii) is 10, then $t=14.2115$ and the time is 3:14 (pm) (15:14)

[3 marks]

Total [16 marks]

5. (a) $(A=)\ 2\pi r^2 + 2\pi rh$ ***A1A1***

Note: Award ***A1*** for either term correct (even if unsimplified) and ***A1*** for all correct (including simplification and no extra term).

[2 marks]

(b) $V = \dfrac{2\pi r^3}{3} + \pi r^2 h$ ***A1A1***

$V = \dfrac{2\pi r^3 + 3\pi r^2 h}{3}$ ***AG***

Note: Award ***A1*** for one correct term, ***A1*** for both and completion to required form.

[2 marks]

(c) 10000 substituted into (b) ***M1***

correct rearrangement with at least one line of intermediate working leading to the given value.

M1

$h = \dfrac{30000 - 2\pi r^3}{3\pi r^2}$ ***AG***

[2 marks]

(d) h substituted into their A ***M1***

$(A=)\ 2\pi r^2 + 2\pi r\left(\dfrac{30000 - 2\pi r^3}{3\pi r^2}\right)$

further consistent rearrangement that leads to **given** A ***A1***

[2 marks]

(e) $\dfrac{4\pi r}{3} - \dfrac{20000}{r^2}$ ***A1A1A1***

Note: Award ***A1*** for $\dfrac{4\pi r}{3}$, ***A1*** for -20000 seen, and ***A1*** for a correct index for r in second term.

[3 marks]

(f) $\dfrac{\mathrm{d}A}{\mathrm{d}r} = 0$ ***(M1)***

EITHER

rearrangement leading to $r^3 = \dfrac{15000}{\pi}$ ***(M1)***

OR

graph of $\dfrac{\mathrm{d}A}{\mathrm{d}r}$ ***(M1)***

THEN

$r = 16.8\ (16.8389...)$ OR $\sqrt[3]{\dfrac{15000}{\pi}}$ ***A1***

Note: Accept correct graph of Area function leading to correct minimum value for r for ***M1M1A1***.

$h = 0$ ***A1***

[4 marks]

(g) hemisphere ***A1***

Note: Do not ***FT*** from incorrect answers to part (f).

[1 mark]

Total [16 marks]

2225 – 7320M

Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

May 2025

Mathematics: applications and interpretation

Standard level

Paper 2

18 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

R Marks awarded for clear **Reasoning**.

AG Answer given in the question and so no marks are awarded.

FT Follow through. The practice of awarding marks, despite candidate errors in previous parts, for their correct methods/answers using incorrect results.

Using the markscheme

1 General

Award marks using the annotations as noted in the markscheme *eg* ***M1***, ***A2***.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***A3***, ***M2*** *etc.*, do **not** split the marks, unless there is a note.
- The response to a "show that" question does not need to restate the ***AG*** line, unless a **Note** makes this explicit in the markscheme.
- Once a correct answer to a question or part question is seen, ignore further working even if this working is incorrect and/or suggests a misunderstanding of the question. This will encourage a uniform approach to marking, with less examiner discretion. Although some candidates may be advantaged for that specific question item, it is likely that these candidates will lose marks elsewhere too.
- An exception to the previous rule is when an incorrect answer from further working is used **in a subsequent part**. For example, when a correct exact value is followed by an incorrect decimal approximation in the first part and this approximation is then used in the second part. In this situation, award ***FT*** marks as appropriate but do not award the final ***A1*** in the first part.

Examples:

	Correct answer seen	**Further working seen**	**Any FT issues?**	**Action**
1.	$8\sqrt{2}$	5.65685... *(incorrect decimal value)*	No. Last part in question.	Award ***A1*** for the final mark *(condone the incorrect further working)*
2.	$\frac{35}{72}$	0.468111… (incorrect decimal value)	Yes. Value is used in subsequent parts.	Award ***A0*** for the final mark *(and full **FT** is available in subsequent parts)*

3 Implied marks

Implied marks appear in **brackets e.g.** ***(M1)***,and can only be awarded if **correct** work is seen or implied by subsequent working/answer.

4 Follow through marks (only applied after an error is made)

Follow through (***FT***) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) (e.g. incorrect value from part (a) used in part (d) or incorrect value from part (c)(i) used in part (c)(ii)). Usually, to award ***FT*** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if all the marks awarded in a subsequent part are for the answer or are implied, then ***FT*** marks should be awarded for *their* correct answer, even when working is not present.

For example: following an incorrect answer to part (a) that is used in subsequent parts, where the markscheme for the subsequent part is ***(M1)A1***, it is possible to award full marks for *their* correct answer, **without working being seen.** For longer questions where all but the answer marks are implied this rule applies but may be overwritten by a **Note** in the Markscheme.

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate.

- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks, by reflecting on what each mark is for and how that maps to the simplified version.

- If the error leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).

- The markscheme may use the word “their” in a description, to indicate that candidates may be using an incorrect value.

- If the candidate’s answer to the initial question clearly contradicts information given in the question, it is not appropriate to award any ***FT*** marks in the subsequent parts. This includes when candidates fail to complete a “show that” question correctly, and then in subsequent parts use their incorrect answer rather than the given value.

- Exceptions to these ***FT*** rules will be explicitly noted on the markscheme.

- If a candidate makes an error in one part but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the command term was “Hence”.

5 Mis-read

If a candidate incorrectly copies values or information from the question, this is a mis-read (***MR***). A candidate should be penalized only once for a particular misread. Use the ***MR*** stamp to indicate that this has been a misread and do not award the first mark, even if this is an ***M*** mark, but award all others as appropriate.

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.
- If a candidate uses a correct answer, to a "show that" question, to a higher degree of accuracy than given in the question, this is NOT a misread and full marks may be scored in the subsequent part.
- ***MR*** can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If the command term is 'Hence' and not 'Hence or otherwise' then alternative methods are not permitted unless covered by a note in the mark scheme.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**.

7 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation** for example 1.9 and 1,9 or 1000 and 1,000 and 1.000.
- Do not accept final answers written using calculator notation. However, ***M*** marks and intermediate ***A*** marks can be scored, when presented using calculator notation, provided the evidence clearly reflects the demand of the mark.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, some **equivalent** answers will generally appear in brackets. Not all equivalent notations/answers/methods will be presented in the markscheme and examiners are asked to apply appropriate discretion to judge if the candidate work is equivalent.

8 Format and accuracy of answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. If the level of accuracy is not stated in the question, the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures.*

Where values are used in subsequent parts, the markscheme will generally use the exact value, however candidates may also use the correct answer to 3 sf in subsequent parts. The markscheme will often explicitly include the subsequent values that come "*from the use of 3 sf values*".

Simplification of final answers: Candidates are advised to give final answers using good mathematical form. In general, for an ***A*** mark to be awarded, arithmetic should be completed, and any values that lead to integers should be simplified; for example, $\sqrt{\frac{25}{4}}$ should be written as $\frac{5}{2}$. An exception to this is simplifying fractions, where lowest form is not required (although the numerator and the denominator must be integers); for example, $\frac{10}{4}$ may be left in this form or written as $\frac{5}{2}$. However, $\frac{10}{5}$ should be written as 2, as it simplifies to an integer.

Algebraic expressions should be simplified by completing any operations such as addition and multiplication, e.g. $4e^{2x} \times e^{3x}$ should be simplified to $4e^{5x}$, and $4e^{2x} \times e^{3x} - e^{4x} \times e^{x}$ should be simplified to $3e^{5x}$. Unless specified in the question, expressions do not need to be factorized, nor do factorized expressions need to be expanded, so $x(x+1)$ and $x^2 + x$ are both acceptable.

Please note: intermediate ***A*** marks do NOT need to be simplified.

9 Calculators

A GDC is required for this paper, but if you see work that suggests a candidate has used any calculator not approved for IB DP examinations (eg CAS enabled devices), please follow the procedures for malpractice.

10. Presentation of candidate work

Crossed out work: If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work unless an explicit note from the candidate indicates that they would like the work to be marked.

More than one solution: Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise. If the layout of the responses makes it difficult to judge, examiners should apply appropriate discretion to judge which is "first".

Q1.

(a) (i) 60 (USD) ***A1***

(ii) attempt to substitute into the arithmetic sequence formula ***(M1)***
$(u_{10} =)(200 + 9 \times 60)$
740 (USD) ***A1***
[3 marks]

(b) attempt to substitute into the sum of an arithmetic sequence formula ***M1***

$\frac{n}{2}(2 \times 200 + (n-1) \times 60)$ ***A1***

$5000 + \frac{n}{2}(400 + 60n - 60)$ ***A1***

$5000 + \frac{n}{2}(340 + 60n)$ ***AG***

Note: Award ***A1*** for adding 5000 and a correct step in simplifying the expression which leads to the given answer. The final line need not be seen. Award ***M0A0A0*** if the student works backwards, i.e. $5000 + \frac{10}{2}(340 + 60 \times 10)$.

[3 marks]

(c) attempt to substitute $n = 10$ into $5000 + \frac{n}{2}(340 + 60n)$ ***(M1)***

$5000 + \frac{10}{2}(340 + 60 \times 10)$
$= 9700$ (USD) ***A1***
[2 marks]

(d) (i) $B = 1.065$ ***A1***

(ii) **EITHER**

use of their B from part (i), in given formula ***(M1)***

$(u_{11} =) 5000 \times 1.065^{10}$ ***(A1)***

OR

$N = 10, I\% = 6.5, PV = \pm 5000, P/Y = 1, C/Y = 1$ ***(M1)(A1)***

Note: Award ***M1*** for using the finance app on the GDC, ***A1*** for all correct entries.

continued…

Question 1 continued.

THEN

$9390\ (\text{USD})\ (9385.68\ldots)$ ***A1***

hence Account 1> Account 2 (after 10 years) OR $9700 > 9390$ ***AG***

Note: The given answer (Account 1> Account 2) or numerical comparison must be seen to award final ***A1***. Rounding of B to 1.07 (7% in TVM) will lead to Account 2> Account 1. Award at most ***M1A1A0***. If $B = 6.5$, the final answer is 6.73×10^{11}. If $B = 0.065$, the final answer is 6.73×10^{-9}. These answers are unrealistic, award at most ***M1A1A0***.

[4 marks]

(e) attempt at comparing the accounts after m years (e.g. graph, table, solver, etc..) ***(M1)***

$5000\times1.065^{m} > 5000+\frac{m}{2}(340+60\times m)$ ***(A1)***

$(m=)38.0462\ldots$

$(m=)39$ (years) ***A1***

Note: Follow through in (e) for their equation (account 2), if explicitly seen in d(ii).

[3 marks]

(f) $FV = 58\ 291.42\ldots$ after their 39 years ***(A1)***

attempt to find the interest by subtracting 5000 from their FV after 39 years ***(M1)***

$58\ 291.42\ldots - 5000$

$53\ 291$ (USD) ***A1***

Note: Award at most ***(A1)(M1)A0*** if the final answer is not rounded to the nearest dollar. Award at most ***A0M1A0*** for a non-integer exponent, even if it follows through from part (e).

[3 marks]
Total [18 marks]

Q2.

(a) let W be the "weights of components" $W \sim \mathrm{N}\left(2.5, 0.15^2\right)$

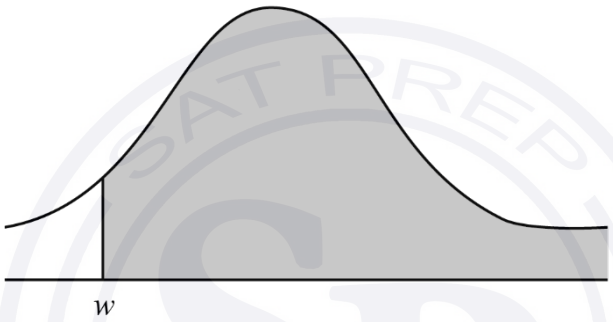
$\mathrm{P}(W > 2.8)$ ***(M1)***

$= 0.0228\left(0.0227500...\right)$ ***A1***

Note: Award ***M1*** for correct calculator notation seen eg normcdf(2.8, 9E99, 2.5, 0.15).

[2 marks]

(b) (i)



A1A1

Award ***A1*** for normal curve with reasonable shape

Note: Reasonable shape = approximately symmetric, change of curvature (not parabolic), approximately asymptotic (no clear upward direction in extreme values).

Award ***A1*** for area shaded to the right of w, with left hand boundary to the left of centre of the curve.

continued…

Question 2 continued.

(ii) $\mathrm{P}(X > w) = 0.8$ or $\mathrm{P}(X < w) = 0.2$ ***(M1)***

$= 2.37\ (2.37375\ldots)$ ***A1***

Note: Award ***M1*** for correct calculator notation that represents the intended shaded region eg $(m =)$ InvNorm$(0.2, 2.5, 0.15)$
Award ***M1A0*** for $2.63\ (2.62624\ldots)$ seen. If their diagram shows w located right of centre, award ***M1A1FT*** for $w = 2.63$.

[4 marks]

(c) (i) $(\mathrm{P}(2.3 < W < 2.7) =) 0.818 (= 0.817577...)$ ***A1***

(ii) $200 \times \mathrm{P}(2.3 < W < 2.7)$ ***(M1)***

$164\ (163.515\ldots)$ ***A1***

Note: Accept 163 and correct non-integer values i.e. $163.515...$

[3 marks]

(d) recognition of binomial ***(M1)***

X is the random variable "number of components passing the test" $X \sim \mathrm{B}(200, 0.95)$

$0.128\ (0.128357\ldots)$ ***A1***

[2 marks]

(e) $\mathrm{P}(X \geq 188)$ OR $1 - \mathrm{P}(X \leq 187)$ ***(M1)***

$0.796 (0.796484...)$ ***A1***

Note: Award ***M1*** for calculator notation communicating correct parameters and range.
eg binomcdf(200, 0.95, 188, 200) OR 1-binomcdf(200, 0.95, 187) OR binomcdf(200, 0.05, 12)

[2 marks]

continued…

Question 2 continued.

(f) multiplication of probabilities for independent events ***(M1)***

let $\mathrm{P}(T_1)$ and $\mathrm{P}(T_2)$ be the probabilities of passing Test 1 and Test 2, respectively,

$\mathrm{P}(T_1 \cap T_2) = 0.817577... \times 0.95$

$= 0.776698...$ ***A1***

recognition of a binomial distribution with $n = 12, p = 0.776698...$ ***(M1)***

let Y be the random variable "number of components which pass both tests,

$Y \sim \mathrm{B}(12, 0.776698...)$

$\mathrm{P}(Y \geq 10)$

$0.477 (0.477419..)$ ***A1***

[4 marks]

Total [17 marks]

Q3.

(a) $(d =)3l$ ***A1***

[1 mark]

(b) substitution into volume of cuboid formula $V = l^2 \times 3l\,(=375)$ ***(M1)***

attempting to solve their $3l^3 = 375$ ***(M1)***

$l^3 = 125$ OR $l = \sqrt[3]{125}$ OR $l^3 = \dfrac{375}{3}$

$l = 5,\ d = 15$ ***A1***

[3 marks]

(c) finding the area of a square and the area of a (non-square) rectangle 5×5 and 5×15 ***(M1)***

$(5 \times 5 + 5 \times 15 + 5 \times 15) \times 2$ ***(A1)***

$= 350\text{cm}^2$ ***A1***

Note: The final answer is 350cm^2. Units are required.

[3 marks]

(d) $\pi r^2 h = 375$ ***(A1)***

$h = \dfrac{375}{\pi r^2}$ ***A1***

Note: Award at most ***A0A1*** for a final answer $h = \dfrac{V}{\pi r^2}$.

[2 marks]

(e) attempt to substitute their h into the curved surface area $2\pi rh$ ***M1***

$A = 2\pi r^2 + 2\pi r\left(\dfrac{375}{\pi r^2}\right)$ ***A1***

$A = 2\pi r^2 + \dfrac{750}{r}$ ***AG***

[2 marks]

(f) $\left(\dfrac{\mathrm{d}A}{\mathrm{d}r} =\right) 4\pi r - 750r^{-2}$ ***A1A1A1***

Note: Award ***A1*** for $4\pi r$, ***A1*** for -750, ***A1*** for r^{-2} or equivalent. Award at most ***A1A1A0*** for additional terms seen.

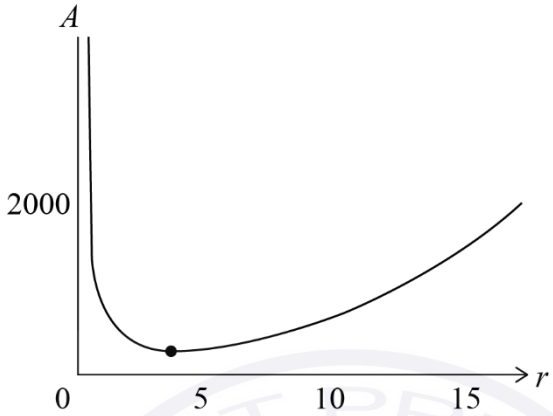
[3 marks]

continued…

Question 3 continued.

(g) (i) **METHOD 1**

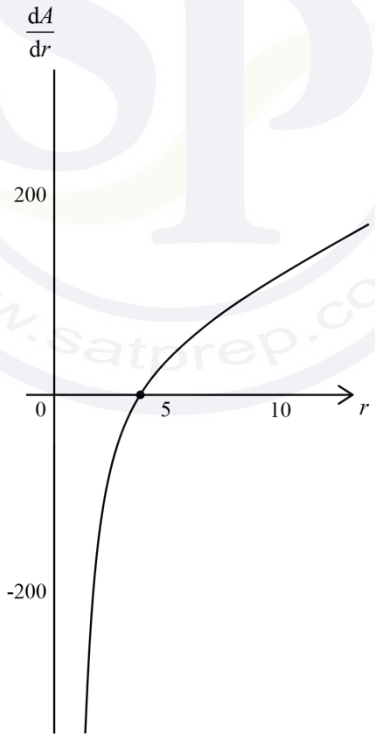
correct sketch with some indication of min point (need not be labelled) ***(M1)***



$(r =) 3.91 (\text{cm}) (3.90796...)$ ***A1***

METHOD 2

attempt to solve $4\pi r - \frac{750}{r^2} = 0$, graphically or otherwise ***(M1)***



$(r =) 3.91 (\text{cm}) (3.90796...)$ ***A1***

(ii) $288 (\text{cm}^2)$ (287.873...) ***A1***

[3 marks]

continued…

Question 3 continued.

(h) cuboid: 1.10×350 and cylinder: $1.25 \times 287.873...$ ***(M1)***

cuboid: 385 and cylinder: $359.842...$ ***R1***
he should choose the cylinder ***A1***

Note: Do not award ***R0A1.***

[3 marks]

Total [20 marks]

Q4.

(a) $\frac{360}{12}$ ***M1***

$= 30$ ***AG***

Note: Award ***M0*** for use of given value, 30.

[1 mark]

(b) substitution of $t = 5$ into model $\left(-2.5\cos(30 \times 5) + 4.5\right)$ ***(M1)***

$h(5) = 6.67\left(6.66506...\right)$ ***A1***

Note: Award ***M1A0*** for $2.75187...$ (radian answer).

[2 marks]

(c) (i) 2.5 (m) ***A1***

Note: Award ***A0*** for -2.5 (m)

(ii) $h(t) = 4.5$ ***A1A1***

Note: Award ***A1*** for an equation of a horizontal line $h =$ constant (accept $y =$ constant), ***A1*** for that constant being 4.5. Second ***A1*** is dependent on first ***A1***. Do not award ***A0A1***.

[3 marks]

(d) attempt at finding point of intersection between $h(t) = 2.65$ and the curve. ***(M1)***

13.4 $(13.4089\ldots)$ ***(A1)***
$13{:}25$ (1:25 p.m.) ***A1***

[3 marks]

Note: Award ***M1A1A1FT*** for consistent use of radians 12.1720… 12:10, 0:10 p.m.
Award at most ***M1A0A0*** if the student works in the first cycle $\left(t = 1.40895\ldots\right)$

(e) $22.5910\ldots - 13.4089\ldots. - \ 0.5$ ***(A1)(M1)(M1)***

Note: Award ***A1*** for $22.5910\ldots$seen, ***M1*** for subtracting their two times for harbour, ***M1*** for subtracting 0.5 from their answer at any stage of the work.

8.68 (hours)$(8.68209\ldots)$ (hours) ***A1***

[4 marks]

Note: Award at most ***A0M0M1A0*** for consistent use of radians. There is no 15 minute window for the ship to reach nor return from the fishing site.

Total [13 marks]

Q5.

(a) attempt to substitute into trapezoidal formula ***(M1)***

(area of cross-section) $\approx \frac{1.9}{2}\left(0+0+2\left(1.68+2.81+2.32\right)\right)$ ***(A1)***

$= 12.9\ (\text{m}^2)(12.939)$ ***A1***

[3 marks]

(b) attempt to find the volume of water in one second, their (a) $\times 0.3$ ***(M1)***

12.939×0.3

$= 3.88\left(3.8817\right)(\text{m}^3)$ ***A1***

Note: Accept 3.87 for use of 3sf answer to part (a)12.9.

[2 marks]

(c) $\left(2.81-1.6=\right)1.21$ (m) ***A1***

[1 mark]

(d) attempt to find maximum eg graphically or $\frac{\mathrm{d}y}{\mathrm{d}x}=0$ or using height at $x=3.8$ ***(M1)***

3.07 (m) OR 3.066 (m) ***A1***

Note: Award ***M1A0*** for coordinate pair $(3.8, 3.066)$

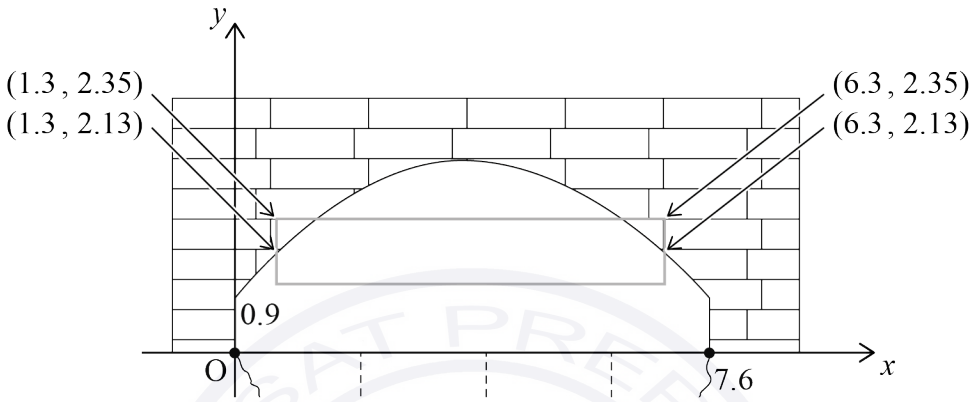
[2 marks]

continued…

Question 5 continued.

(e) **METHOD 1**

height of bridge < height of boat at $x = 1.3$ or $x = 6.3$ (5 m wide boat)



use of $3.8 + 2.5$ or $3.8 - 2.5$ ***(M1)***

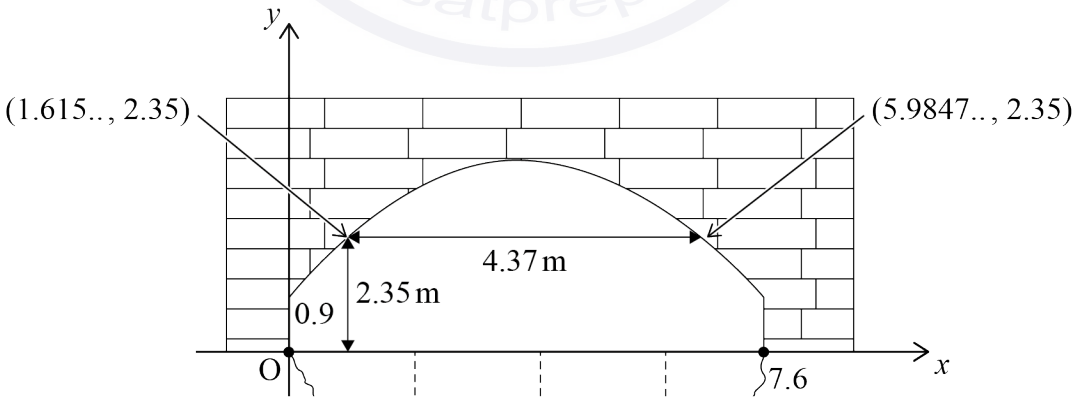
attempt to find the height of the bridge at either 1.3 or 6.3 m ***(M1)***

$2.13(\text{m})(2.1285)$ ***A1***

$(2.13 < 2.35)$ the boat cannot pass under the bridge ***A1***

METHOD 2

boat is wider (5 m) than the width between the arch at $h = 2.35$ m (height of boat)



attempt to find the values of x ($1.61520\ldots$ and $5.98479\ldots$) when the bridge is 2.35 m above the water ***(M1)***

continued…

Question 5 continued.

subtract their two values to find the width of bridge at that point at this height ***(M1)***

$5.984795... - 1.615204...$ OR $3.8 - 1.615204\ldots$ OR $5.98479\ldots - 3.8$

$4.37(4.36959...)(\text{m})$ OR $2.18\ (2.18479\ldots)(\text{m})$ ***A1***

$(4.37 < 5)$ OR $(2.18 < 2.5)$ the boat cannot pass under the bridge ***A1***

METHOD 3 (see the diagram for method 2)

use of $3.8 + 2.5\ \ (6.3)$ or $3.8 - 2.5\ \ (1.3)$ ***(M1)***

attempt to find the values of x when the bridge is 2.35 m above the water ***(M1)***

$5.98479\ldots$ or $1.61520\ldots$ (m) ***A1***

$6.3 > 5.98\ldots(5.98479\ldots)$ or $1.3 <\ 1.62\ (1.61520\ldots)$ the boat cannot pass under the bridge ***A1***

Note: In all 3 methods the final ***A1*** is dependent on the previous ***A1.***

[4 marks]

[Total 12 marks]

Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

November 2024

Mathematics: applications and interpretation

Standard level

Paper 2

15 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

R Marks awarded for clear **Reasoning**.

AG Answer given in the question and so no marks are awarded.

FT Follow through. The practice of awarding marks, despite candidate errors in previous parts, for their correct methods/answers using incorrect results.

Using the markscheme

1 General

Award marks using the annotations as noted in the markscheme *eg* ***M1, A2****.*

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***A3***, ***M2*** *etc.*, do **not** split the marks, unless there is a note.
- The response to a "show that" question does not need to restate the ***AG*** line, unless a **Note** makes this explicit in the markscheme.
- Once a correct answer to a question or part question is seen, ignore further working even if this working is incorrect and/or suggests a misunderstanding of the question. This will encourage a uniform approach to marking, with less examiner discretion. Although some candidates may be advantaged for that specific question item, it is likely that these candidates will lose marks elsewhere too.
- An exception to the previous rule is when an incorrect answer from further working is used **in a subsequent part**. For example, when a correct exact value is followed by an incorrect decimal approximation in the first part and this approximation is then used in the second part. In this situation, award ***FT*** marks as appropriate but do not award the final ***A1*** in the first part.

Examples:

	Correct answer seen	Further working seen	Any FT issues?	Action
1.	$8\sqrt{2}$	5.65685... *(incorrect decimal value)*	No. Last part in question.	Award ***A1*** for the final mark *(condone the incorrect further working)*
2.	$\frac{35}{72}$	0.468111… (incorrect decimal value)	Yes. Value is used in subsequent parts.	Award ***A0*** for the final mark *(and full **FT** is available in subsequent parts)*

3 Implied marks

Implied marks appear in **brackets e.g.** ***(M1)***,and can only be awarded if **correct** work is seen or implied by subsequent working/answer.

4 Follow through marks (only applied after an error is made)

Follow through (***FT***) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) (e.g. incorrect value from part (a) used in part (d) or incorrect value from part (c)(i) used in part (c)(ii)). Usually, to award ***FT*** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if all the marks awarded in a subsequent part are for the answer or are implied, then ***FT*** marks should be awarded for *their* correct answer, even when working is not present.

For example: following an incorrect answer to part (a) that is used in subsequent parts, where the markscheme for the subsequent part is ***(M1)A1***, it is possible to award full marks for *their* correct answer, **without working being seen.** For longer questions where all but the answer marks are implied this rule applies but may be overwritten by a **Note** in the Markscheme.

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate.

- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks, by reflecting on what each mark is for and how that maps to the simplified version.

- If the error leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).

- The markscheme may use the word “their” in a description, to indicate that candidates may be using an incorrect value.

- If the candidate’s answer to the initial question clearly contradicts information given in the question, it is not appropriate to award any ***FT*** marks in the subsequent parts. This includes when candidates fail to complete a “show that” question correctly, and then in subsequent parts use their incorrect answer rather than the given value.

- Exceptions to these ***FT*** rules will be explicitly noted on the markscheme.

- If a candidate makes an error in one part but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the command term was “Hence”.

5 Mis-read

If a candidate incorrectly copies values or information from the question, this is a mis-read (***MR***). A candidate should be penalized only once for a particular misread. Use the ***MR*** stamp to indicate that this has been a misread and do not award the first mark, even if this is an ***M*** mark, but award all others as appropriate.

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.
- If a candidate uses a correct answer, to a "show that" question, to a higher degree of accuracy than given in the question, this is NOT a misread and full marks may be scored in the subsequent part.
- ***MR*** can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If the command term is 'Hence' and not 'Hence or otherwise' then alternative methods are not permitted unless covered by a note in the mark scheme.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**.

7 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation** for example 1.9 and $1{,}9$ or 1000 and $1{,}000$ and 1.000.

- Do not accept final answers written using calculator notation. However, ***M*** marks and intermediate ***A*** marks can be scored, when presented using calculator notation, provided the evidence clearly reflects the demand of the mark.

- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.

- In the markscheme, some **equivalent** answers will generally appear in brackets. Not all equivalent notations/answers/methods will be presented in the markscheme and examiners are asked to apply appropriate discretion to judge if the candidate work is equivalent.

8 Format and accuracy of answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. If the level of accuracy is not stated in the question, the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures.*

Where values are used in subsequent parts, the markscheme will generally use the exact value, however candidates may also use the correct answer to 3 sf in subsequent parts. The markscheme will often explicitly include the subsequent values that come "*from the use of 3 sf values*".

Simplification of final answers: Candidates are advised to give final answers using good mathematical form. In general, for an ***A*** mark to be awarded, arithmetic should be completed, and any values that lead to integers should be simplified; for example, $\sqrt{\frac{25}{4}}$ should be written as $\frac{5}{2}$. An exception to this is simplifying fractions, where lowest form is not required (although the numerator and the denominator must be integers); for example, $\frac{10}{4}$ may be left in this form or written as $\frac{5}{2}$. However, $\frac{10}{5}$ should be written as 2, as it simplifies to an integer.

Algebraic expressions should be simplified by completing any operations such as addition and multiplication, e.g. $4e^{2x} \times e^{3x}$ should be simplified to $4e^{5x}$, and $4e^{2x} \times e^{3x} - e^{4x} \times e^{x}$ should be simplified to $3e^{5x}$. Unless specified in the question, expressions do not need to be factorized, nor do factorized expressions need to be expanded, so $x(x+1)$ and $x^2 + x$ are both acceptable.

Please note: intermediate ***A*** marks do NOT need to be simplified.

9 Calculators

A GDC is required for this paper, but if you see work that suggests a candidate has used any calculator not approved for IB DP examinations (eg CAS enabled devices), please follow the procedures for malpractice.

10. Presentation of candidate work

Crossed out work: If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work unless an explicit note from the candidate indicates that they would like the work to be marked.

More than one solution: Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise. If the layout of the responses makes it difficult to judge, examiners should apply appropriate discretion to judge which is "first".

1. (a) $(2, 6)$ ***A1***

Note: Award ***A0*** if parentheses are omitted.

[1 mark]

(b) (i) attempt to use midpoint formula (at least one correct) ***(M1)***

$\left(\frac{11+12}{2}, \frac{0+6}{2}\right)$

$(11.5,\ 3)$ ***A1***

(ii) $\left(m = \frac{6-0}{12-11} =\right) 6$ ***(A1)***

finding negative reciprocal of their gradient ***(M1)***

$m_{\perp} = -\frac{1}{6}$

attempt to substitute their midpoint and their gradient into equation of straight line ***(M1)***

eg. $y-3 = -\frac{1}{6}\left(x-\frac{23}{2}\right)$ **OR** $3 = -\frac{1}{6}(11.5)+c$

$y = -0.167x + 4.92\left(y = -0.166666\ldots x + 4.91666\ldots, y = -\frac{1}{6}x + \frac{59}{12}\right)$ ***A1***

Note: Substituting $m = 6$ (no negative reciprocal) and their point into the equation of straight line would receive at most ***A1M0M1A0***.

[6 marks]

(c) (closest to Q so) 4 ***A1***

[1 mark]

(d) **EITHER**

$.(\mathrm{H}_1:)\ \mu_1 < \mu_2.$ ***A1***

OR

mean amount of money spent by all customers on weekdays is less than the **mean** amount of money spent by all customers on weekends ***A1***

Note: Award ***A0*** for 'sample mean' or if 'mean" is omitted. For example, 'money spent on weekdays is less than the money spent on weekends'.

[1 mark]

continue…

Question 1 continued.

(e) (i) convenience ***A1***

(ii) not representative of the whole population ***A1***

Do not accept the following reasons
- disadvantages of other sampling methods
- there are five days during the week, only two on a weekend
- anything based on speculation/information not presented in the question
- people spend different amounts of money
- this is a sample (doesn't collect every data point) more data should have been collected

[2 marks]

(f) $p > 0.05 \ (\alpha)$ **OR** $0.0897\ldots. > 0.05$ ***R1***

There is insufficient evidence to reject H_0 ***A1***

Note: Do not award ***R0A1***.

[2 marks]
[Total 13 marks]

2. (a) (i) $\frac{45-8}{2}$ ***A1***

$18.5\ (\text{m})$ ***AG***

Note: Use of 18.5 in the calculation is reverse engineering, and falls short of the demand of the command term "Show that'. In such cases, award ***A0***.

(ii) $(2\pi \times 18.5 =)\ 116\ \ (116.2389\ldots,\ 37\pi)\ (\text{m})$. ***A1***

[2 marks]

(b) recognition the distance is the arc length ***(M1)***

$\frac{10°}{360°} \times 2\pi \times 18.5$ **OR** $\frac{116.2389\ldots}{36}$ ***(A1)***

$= 3.23\ \left(3.228859\ldots,\ \frac{37\pi}{36}\right)(\text{m})$ ***A1***

[3 marks]

(c) (i) $\left(b = \frac{360}{15} =\right)\ 24$ ***A1***

(ii) $\frac{\max+\min}{2}$ ***(A1)***

$(d =)\ \frac{45+8}{2}$ **OR** $45 - 18.5$ **OR** $8 + 18.5$

$= 26.5$ ***A1***

(iii) $h(t) = 18.5\sin(24t) + 26.5$ ***A1A1***

Note: Award ***A1*** for 18.5 seen as parameter a, ***A1*** for a completely correct equation (including LHS).

[5 marks]

(d) attempt to equate 33 with $h(t)$ **OR** sketch graph of curve and line ***(M1)***

$h(t)(= 18.5\sin(24t) + 26.5) = 33$

$t = 0.857\ \ (0.857083\ldots)$ and $t = 6.64\ \ (6.64291\ldots)$ (minutes) ***A1A1***

[3 marks]

(e) 4 (revolutions per hour) seen **OR** 32 (revolutions per day) seen ***(A1)***

$\left(4 \times 8 \times 3000 =\right)\ 96000$ ***(A1)***

9.6×10^4 ***A1***

[3 marks]

[Total 16 marks]

3. (a) (i) $(Q_3 =)\ 7.2$ ***A2***

Note: Award ***A1A0*** for a correct ordered list of happiness scores, when the correct Q_3 is not seen.

(ii) $Q_1 = 6.5$ ***(A1)***

$IQR = 7.2 - 6.5$

$= 0.7$ ***A1***

[4 marks]

(b) $Q_3 + 1.5 \times IQR$ ***(A1)***

$(7.2 + 1.5 \times 0.7 =)\ 8.25$ ***A1***

since $7.5 < 8.25$ ***R1***

Switzerland is not an outlier ***AG***

Note: Do not award ***A0A0R1***.

[3 marks]

(c) (i) $a = 3.5$ ***A1***

(ii) $b = 8$ ***A1***

(iii) $c = 3.5$ ***A1***

[3 marks]

(d) (i) $(r_s =)\ 0.164\ (0.164134\ldots)$ ***A2***

(ii) France rank (of sixth) is unchanged (so the r_s is unchanged) ***R1***

[3 marks]

(e) Because r_s is too close to zero and hence Jose's conclusion is not appropriate ***R1***

Note: Award ***R1*** for a comment on the value r_s and "not appropriate" as a conclusion.
Accept "r_s indicates a weak correlation of the ranks".
Do not accept "r_s indicates a weak correlation of happiness score and country rank".

[1 mark]

[Total 14 marks]

4. (a) $\frac{1}{2}\times 1\times\left((6+4.3)+2(1.7+1.3+3+4.7)\right)$ ***A1A1***

$=15.9\ \ (15.85)\ (\text{cm}^2)$ ***A1***

[3 marks]

(b) (i) $\int_0^5\left(-\frac{x^3}{3}+3x^2-7x+6\right)\text{d}x$ ***A2***

Note: Award ***A2*** for an integral with correct integrand and correct limits.

(ii) $15.4\ (15.4166\ldots,\frac{185}{12})\ \ (\text{cm}^2)$ ***A2***

Note: If their limits are outside the domain, award at most ***A1A0FT***.

[4 marks]

(c) $400=\pi r^2 h$ ***A1***

(d) **METHOD 1** (Substitution of $h=\frac{400}{\pi r^2}$ in $A=2\pi r^2+2\pi rh$)

$A=2\pi r^2+2\pi rh$ ***(A1)***

attempt to isolate h ***(M1)***

$h=\frac{400}{\pi r^2}$

correct substitution of their h into correct expression ***(M1)***

$A=2\pi r^2+2\pi r\left(\frac{400}{\pi r^2}\right)\left(A=2\pi r^2+\frac{800}{r}\right)$

$k=800$ ***A1***

METHOD 2 (Equating $2\pi rh$ and $\frac{k}{r}$)

$A=2\pi r^2+2\pi rh$ **OR** $2\pi rh=\frac{k}{r}$ ***(A1)***

attempt to isolate h ***(M1)***

$h=\frac{400}{\pi r^2}$

correct substitution of their h into $2\pi r^2+2\pi rh=2\pi r^2+\frac{k}{r}$ ***(M1)***

$2\pi r\left(\frac{400}{\pi r^2}\right)=\frac{k}{r}$

$k=800$ ***A1***

[4 marks]

continue…

Question 4 continued.

(e) (i) $\frac{\mathrm{d}A}{\mathrm{d}r} = 4\pi r - 800r^{-2}$ ***A1(M1)A1***

Note: Award ***A1*** for $4\pi r$ seen, and ***(M1)*** for expressing $\frac{800}{r}$ as $800r^{-1}$ (can be implied through $\mp\frac{800}{r^2}$ seen), ***A1*** for $-800r^{-2}$. Award at most ***A1(M1)A0*** if any additional terms are seen.

(ii) $0 = 4\pi r - 800r^{-2}$ ***(M1)***

$r = 3.99 \quad \left(3.99294\ldots, \sqrt[3]{\frac{200}{\pi}}\right)$ (cm) ***A1***

Note: Award at most ***M1A0*** if k is not found.

[5 marks]

(f) $(h =)\ 7.99\ (7.98589\ldots)$ (cm) ***A1***

$(C =)\ 2\pi\ (3.99294...) \approx 25.1\ (25.0884\ldots)$(cm) ***A1***

EITHER

the longest dimension of the label (7 cm) is less than both values and hence the label will fit (in any rotation) ***R1***

OR

$7 < 7.99$ and $5 < 25.1$ ***R1***

Note: Do not accept an argument based on the comparison of areas.

[3 marks]

[Total 20 marks]

5. (a) attempt to use Pythagoras' theorem ***(M1)***

$\sqrt{4.2^2 - 3^2}$

$= 2.94\ (2.93938\ldots)$ (m) ***A1***

[2 marks]

(b) (i) **METHOD 1** (Use of $\frac{1}{2} \times a \times b \times \sin(\theta)$)

$60°$ ***(A1)***

attempt to find area of one triangle using $\frac{1}{2} \times a \times b \times \sin(\theta)$ ***(M1)***

$\frac{1}{2} \times 3 \times 3 \times \sin(60°)$

$\left(6 \times \frac{1}{2} \times 3 \times 3 \times \sin(60°)\right) = 23.4\ \ (23.3826..., \frac{27\sqrt{3}}{2})\ (\text{m}^2)$ ***A1***

Note: Award ***A0M0A0*** for $\frac{1}{2} \times 3 \times 3$ or equivalent.

METHOD 2 (Use of altitude)

(altitude is) $\frac{3}{2}\sqrt{3}\ (2.59807\ldots)$ ***(A1)***

attempt to find area of one triangle using $\frac{1}{2} \times b \times h$ with their altitude ***(M1)***

$\frac{1}{2} \times 3 \times \frac{3}{2}\sqrt{3}$

$\left(6 \times \frac{1}{2} \times 3 \times \frac{3}{2}\sqrt{3}\right) = 23.4\ \left(23.3826\ldots, \frac{27\sqrt{3}}{2}\right)\ (\text{m}^2)$ ***A1***

Note: Award ***A0M0A0*** for $\frac{1}{2} \times 2 \times 2$ or equivalent.

METHOD 3 (Finding the area of a trapezoid)

(altitude of one trapezoid is) $\frac{3}{2}\sqrt{3}\ (2.59807\ldots)$ ***(A1)***

attempt to find area of one trapezoid using $\frac{1}{2} \times (a+b)h$ ***(M1)***

$\frac{1}{2}(3+6)\left(\frac{3}{2}\sqrt{3}\right)$ **OR** $\frac{27}{4}\sqrt{3}$

$\left(2 \times \frac{1}{2}(3+6)\left(\frac{3}{2}\sqrt{3}\right)\right) = 23.4\ \left(23.3826\ldots, \frac{27\sqrt{3}}{2}\right)\ (\text{m}^2)$ ***A1***

continue…

Question 5 continued.

(ii) $\frac{1}{3} \times 23.3826... \times 2.93938...$ ***(A1)***

$= 22.9 \text{ m}^3 \ (22.9102\ldots)$ ***A1***

Note: Units must be seen.

[5 marks]

(c) $\cos(\hat{OAT}) = \frac{3}{4.2}$ or correct equivalent ***(A1)***

$(\hat{OAT} =)\ 44.4^\circ$ (44.4153…, 0.775193… radians) ***A1***

[2 marks]

(d) Angle QAP = 180 – 44.4... = 135.584…° ***(A1)***

Angle QPA = 180 – 30 – 135.584... = 14.4153…° ***(A1)***

Note: These angles may be seen in the sine rule.

Attempt to substitute into sine rule ***(M1)***

$$\frac{AQ}{\sin(14.4153...)} = \frac{3.5}{\sin(135.584\ldots)}$$

$AQ = 1.25 \ \ (1.24500\ldots)$ (m) ***A1***

Note: Accept 1.24, from use of a 3sf value for $\hat{QAP}$ and $\hat{QPA}$.

[4 marks]

(e) attempt to substitute into cosine rule to form a quadratic for QR ***(M1)***

$0.8^2 = QR^2 + 1.24500...^2 - 2 \times 1.24500... \times QR \times \cos(30)$ ***(A1)***

$QR = 0.576\ (0.575717\ldots)$ (m) , $1.58\ \ (1.58068\ldots)$ (m) ***A1A1***

[4 marks]

[Total: 17 marks]

Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

November 2024

Mathematics: applications and interpretation

Standard level

Paper 2

15 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

R Marks awarded for clear **Reasoning**.

AG Answer given in the question and so no marks are awarded.

FT Follow through. The practice of awarding marks, despite candidate errors in previous parts, for their correct methods/answers using incorrect results.

Using the markscheme

1 General

Award marks using the annotations as noted in the markscheme *eg* ***M1, A2****.*

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***A3***, ***M2*** *etc*., do **not** split the marks, unless there is a note.
- The response to a "show that" question does not need to restate the ***AG*** line, unless a **Note** makes this explicit in the markscheme.
- Once a correct answer to a question or part question is seen, ignore further working even if this working is incorrect and/or suggests a misunderstanding of the question. This will encourage a uniform approach to marking, with less examiner discretion. Although some candidates may be advantaged for that specific question item, it is likely that these candidates will lose marks elsewhere too.
- An exception to the previous rule is when an incorrect answer from further working is used **in a subsequent part**. For example, when a correct exact value is followed by an incorrect decimal approximation in the first part and this approximation is then used in the second part. In this situation, award ***FT*** marks as appropriate but do not award the final ***A1*** in the first part.

Examples:

	Correct answer seen	Further working seen	Any FT issues?	Action
1.	$8\sqrt{2}$	5.65685... *(incorrect decimal value)*	No. Last part in question.	Award ***A1*** for the final mark *(condone the incorrect further working)*
2.	$\frac{35}{72}$	0.468111… (incorrect decimal value)	Yes. Value is used in subsequent parts.	Award ***A0*** for the final mark *(and full **FT** is available in subsequent parts)*

3 Implied marks

Implied marks appear in **brackets e.g. *(M1)***,and can only be awarded if **correct** work is seen or implied by subsequent working/answer.

4 Follow through marks (only applied after an error is made)

Follow through (***FT***) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) (e.g. incorrect value from part (a) used in part (d) or incorrect value from part (c)(i) used in part (c)(ii)). Usually, to award ***FT*** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if all the marks awarded in a subsequent part are for the answer or are implied, then ***FT*** marks should be awarded for *their* correct answer, even when working is not present.

For example: following an incorrect answer to part (a) that is used in subsequent parts, where the markscheme for the subsequent part is ***(M1)A1***, it is possible to award full marks for *their* correct answer, **without working being seen.** For longer questions where all but the answer marks are implied this rule applies but may be overwritten by a **Note** in the Markscheme.

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate.

- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks, by reflecting on what each mark is for and how that maps to the simplified version.

- If the error leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).

- The markscheme may use the word “their” in a description, to indicate that candidates may be using an incorrect value.

- If the candidate’s answer to the initial question clearly contradicts information given in the question, it is not appropriate to award any ***FT*** marks in the subsequent parts. This includes when candidates fail to complete a “show that” question correctly, and then in subsequent parts use their incorrect answer rather than the given value.

- Exceptions to these ***FT*** rules will be explicitly noted on the markscheme.

- If a candidate makes an error in one part but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the command term was “Hence”.

5 Mis-read

If a candidate incorrectly copies values or information from the question, this is a mis-read (***MR***). A candidate should be penalized only once for a particular misread. Use the ***MR*** stamp to indicate that this has been a misread and do not award the first mark, even if this is an ***M*** mark, but award all others as appropriate.

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.
- If a candidate uses a correct answer, to a "show that" question, to a higher degree of accuracy than given in the question, this is NOT a misread and full marks may be scored in the subsequent part.
- ***MR*** can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If the command term is 'Hence' and not 'Hence or otherwise' then alternative methods are not permitted unless covered by a note in the mark scheme.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**.

7 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation** for example 1.9 and $1{,}9$ or 1000 and $1{,}000$ and 1.000.

- Do not accept final answers written using calculator notation. However, ***M*** marks and intermediate ***A*** marks can be scored, when presented using calculator notation, provided the evidence clearly reflects the demand of the mark.

- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.

- In the markscheme, some **equivalent** answers will generally appear in brackets. Not all equivalent notations/answers/methods will be presented in the markscheme and examiners are asked to apply appropriate discretion to judge if the candidate work is equivalent.

8 Format and accuracy of answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. If the level of accuracy is not stated in the question, the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures.*

Where values are used in subsequent parts, the markscheme will generally use the exact value, however candidates may also use the correct answer to 3 sf in subsequent parts. The markscheme will often explicitly include the subsequent values that come "*from the use of 3 sf values*".

Simplification of final answers: Candidates are advised to give final answers using good mathematical form. In general, for an ***A*** mark to be awarded, arithmetic should be completed, and any values that lead to integers should be simplified; for example, $\sqrt{\frac{25}{4}}$ should be written as $\frac{5}{2}$. An exception to this is simplifying fractions, where lowest form is not required (although the numerator and the denominator must be integers); for example, $\frac{10}{4}$ may be left in this form or written as $\frac{5}{2}$. However, $\frac{10}{5}$ should be written as 2, as it simplifies to an integer.

Algebraic expressions should be simplified by completing any operations such as addition and multiplication, e.g. $4e^{2x} \times e^{3x}$ should be simplified to $4e^{5x}$, and $4e^{2x} \times e^{3x} - e^{4x} \times e^{x}$ should be simplified to $3e^{5x}$. Unless specified in the question, expressions do not need to be factorized, nor do factorized expressions need to be expanded, so $x(x+1)$ and $x^2 + x$ are both acceptable.

Please note: intermediate ***A*** marks do NOT need to be simplified.

9 Calculators

A GDC is required for this paper, but if you see work that suggests a candidate has used any calculator not approved for IB DP examinations (eg CAS enabled devices), please follow the procedures for malpractice.

10. Presentation of candidate work

Crossed out work: If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work unless an explicit note from the candidate indicates that they would like the work to be marked.

More than one solution: Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise. If the layout of the responses makes it difficult to judge, examiners should apply appropriate discretion to judge which is "first".

1. (a) $(2, 2)$ ***A1***

Note: Award ***A0*** if paretheses are omitted.

[1 mark]

(b) (i) attempt to use midpoint formula (at least one correct) ***(M1)***

$\left(\frac{9+10}{2}, \frac{0+6}{2}\right)$

$(9.5, 3)$ ***A1***

(ii) $\left(m = \frac{6-0}{10-9} = \right)6$ ***(A1)***

finding negative reciprocal of their gradient ***(M1)***

$m_{\perp} = -\frac{1}{6}$

attempt to substitute their midpoint and their gradient into equation of straight line ***(M1)***

eg. $y - 3 = -\frac{1}{6}\left(x - \frac{19}{2}\right)$ **OR** $3 = -\frac{1}{6}(9.5) + c$

$y = -\frac{1}{6}x + \frac{55}{12}$ **OR** $y = -0.167x + 4.58 \quad \left(y = -0.166666...x + 4.58333...\right)$ ***A1***

Note: Substituting $m = 6$ (no negative reciprocal) and their point into the equation of straight line would receive at most ***A1M0M1A0***.

[6 marks]

(c) (closest to B so) 5 ***A1***

[1 mark]

(d) **EITHER**

$(\mathrm{H}_1 :)\ \mu_1 < \mu_2$ ***A1***

OR

mean amount of money spent by all customers on weekdays, is less than the **mean** amount of money spent by all customers on weekends ***A1***

Note: Award ***A0*** for "sample mean" or if 'mean' is omitted. Eg "money spent on weekdays, is less than the money spent on weekends".

[1 mark]

continue…

Question 1 continued.

(e) (i) convenience ***A1***

(ii) not representative of the whole population ***A1***

Do not accept the following reasons

- disadvantages of other sampling methods
- there are five days during the week, only two on a weekend
- anything based on speculation/information not presented in the question
- people spend different amounts of money
- this is a sample (doesn't collect every data point) more data should have been collected

[2 marks]

(f) $p > 0.05\ (\alpha)$ **OR** $0.0897\ldots. > 0.05$ ***R1***

There is insufficient evidence to reject H_0. ***A1***

Note: Do not award ***R0A1***.

[2 marks]
[Total 13 marks]

2. (a) (i) $\frac{45-8}{2}$ ***A1***

$18.5\ (\text{m})$ ***AG***

Note: Use of 18.5 in the calculation is reverse engineering, and falls short of the demand of the command term "Show that'. In such cases, award ***A0***.

(ii) $(2\pi \times 18.5 =)\ 116\ \ (116.2389\ldots,\ 37\pi)\ (\text{m})$ ***A1***

[2 marks]

(b) recognition the distance is the arc length ***(M1)***

$\frac{10°}{360°} \times 2\pi \times 18.5$ **OR** $\frac{116.2389\ldots}{36}$ ***(A1)***

$= 3.23\ \left(3.228859\ldots,\ \frac{37\pi}{36}\right)(\text{m})$ ***A1***

[3 marks]

(c) (i) $\left(b = \frac{360}{15} =\right)\ 24$ ***A1***

(ii) $\frac{\max+\min}{2}$ ***(A1)***

$(d =)\ \frac{45+8}{2}$ **OR** $45 - 18.5$ **OR** $8 + 18.5$

$= 26.5$ ***A1***

(iii) $h(t) = 18.5\sin(24t) + 26.5$ ***A1A1***

Note: Award ***A1*** for 18.5 seen as parameter a, ***A1*** for a completely correct equation (including LHS).

[5 marks]

(d) attempt to equate 33 with $h(t)$ **OR** sketch graph of curve and line ***(M1)***

$h(t)(= 18.5\sin(24t) + 26.5) = 33$

$t = 0.857\ \ (0.857083\ldots)$ and $t = 6.64\ \ (6.64291\ldots)$ (minutes) ***A1A1***

[3 marks]

(e) 4 (revolutions per hour) seen **OR** 32 (revolutions per day) seen ***(A1)***

$\left(4 \times 8 \times 3000 =\right)\ 96000$ ***(A1)***

9.6×10^4 ***A1***

[3 marks]

[Total 16 marks]

3. (a) (i) $(Q_3 =)\ 7.2$ ***A2***

Note: Award ***A1A0*** for a correct ordered list of happiness scores, when the correct Q_3 is not seen.

(ii) $Q_1 = 6.5$ ***(A1)***

$IQR = 7.2 - 6.5$

$= 0.7$ ***A1***

[4 marks]

(b) $Q_3 + 1.5 \times IQR$ ***(A1)***

$(7.2 + 1.5 \times 0.7 =)\ 8.25$ ***A1***

since $7.5 < 8.25$ ***R1***

Switzerland is not an outlier ***AG***

Note: Do not award ***A0A0R1***.

[3 marks]

(c) (i) $a = 3.5$ ***A1***

(ii) $b = 8$ ***A1***

(iii) $c = 3.5$ ***A1***

[3 marks]

(d) (i) $(r_s =)\ 0.164\ (0.164134\ldots)$ ***A2***

(ii) France rank (of sixth) is unchanged (so the r_s is unchanged) ***R1***

[3 marks]

(e) Because r_s is too close to zero and hence Jose's conclusion is not appropriate ***R1***

Note: Award ***R1*** for a comment on the value r_s and "not appropriate" as a conclusion.
Accept "r_s indicates a weak correlation of the ranks".
Do not accept "r_s indicates a weak correlation of happiness score and country rank".

[1 mark]

[Total 14 marks]

4. (a) $\frac{1}{2}\times 1\times\left((4+0.7)+2(7.3+6.7+4+1.3)\right)$ ***(A1)(A1)***

$=21.7\ \ (21.65)\ \ (\text{cm}^2)$ ***A1***

[3 marks]

(b) (i) $\int_0^5\left(\frac{x^3}{3}-3x^2+6x+4\right)\mathrm{d}x$ ***A2***

Note: Award ***A2*** for an integral with correct integrand and correct limits.

(ii) $22.1\ (22.08333\ldots,\frac{265}{12})\ \ (\text{cm}^2)$ ***A2***

Note: If their limits are outside the domain, award at most ***A1A0FT***.

[4 marks]

(c) $600=\pi r^2 h$ ***A1***

[1 mark]

(d) **METHOD 1** (Substitution of $h=\frac{600}{\pi r^2}$ in $A=2\pi r^2+2\pi rh$)

$A=2\pi r^2+2\pi rh$ ***(A1)***

attempt to isolate h ***(M1)***

$h=\frac{600}{\pi r^2}$

correct substitution of their h into correct expression ***(M1)***

$A=2\pi r^2+2\pi r\left(\frac{600}{\pi r^2}\right)\quad\left(A=2\pi r^2+\frac{1200}{r}\right)$

$k=1200$ ***A1***

METHOD 2 (Equating $2\pi rh$ and $\frac{k}{r}$)

$A=2\pi r^2+2\pi rh$ **OR** $2\pi rh=\frac{k}{r}$ ***(A1)***

attempt to isolate h ***(M1)***

$h=\frac{600}{\pi r^2}$

correct substitution of their h into $2\pi r^2+2\pi rh=2\pi r^2+\frac{k}{r}$ ***(M1)***

$2\pi r\left(\frac{600}{\pi r^2}\right)=\frac{k}{r}$

$k=1200$ ***A1***

[4 marks]

continue…

Question 4 continued.

(e) (i) $\frac{\mathrm{d}A}{\mathrm{d}r} = 4\pi r - 1200r^{-2}$ ***A1(M1)A1***

Note: Award ***A1*** for $4\pi r$ seen, and ***(M1)*** for expressing $\frac{1200}{r}$ as $1200r^{-1}$ (can be implied through $\mp\frac{1200}{r^2}$ seen), ***A1*** for $-1200r^{-2}$. Award at most ***A1(M1)A0*** if any additional terms are seen.

(ii) $0 = 4\pi r - 1200r^{-2}$ ***(M1)***

$r = 4.57 \left(4.570781\ldots, \sqrt[3]{\frac{300}{\pi}} \right)$ (cm) ***A1***

Note: Award at most ***M1A0*** if the final answer is in terms of k.

[5 marks]

(f) $(h =)$ 9.14 (9.14156…..) (cm) ***A1***

$(C =)$ 2π $(4.570781\ldots) \approx 28.7$ $\left(28.7190\ldots\right)$(cm) ***A1***

EITHER

the longest dimension of the label (9 cm) is less than both values and hence the label will fit (in any rotation) ***R1***

OR

$9 < 9.14$ and $5 < 28.7$ ***R1***

Note: Do not accept an argument based on the comparison of areas.

[3 marks]

[Total 20 marks]

5. (a) attempt to use Pythagoras' theorem ***(M1)***

$\sqrt{3.4^2 - 2^2}$

$= 2.75\ (2.74954\ldots)$ (m) ***A1***

[2 marks]

(b) (i) **METHOD 1** (Use of $\frac{1}{2} \times a \times b \times \sin(\theta)$)

$60°$ ***(A1)***

attempt to find area of one triangle using $\frac{1}{2} \times a \times b \times \sin(\theta)$ ***(M1)***

$\frac{1}{2} \times 2 \times 2 \times \sin(60°)$

$\left(6 \times \frac{1}{2} \times 2 \times 2 \times \sin(60^\circ)\right) = 10.4\ \ (10.3923..., 6\sqrt{3})\ (\text{m}^2)$ ***A1***

Note: Award ***A0M0A0*** for $\frac{1}{2} \times 2 \times 2$ or equivalent.

METHOD 2 (Use of altitude)

(altitude is) $\sqrt{3}$ ***(A1)***

attempt to find the area of one triangle using $\frac{1}{2} \times b \times h$ with their altitude. ***(M1)***

$\frac{1}{2} \times 2 \times \sqrt{3}$

$\left(6 \times \frac{1}{2} \times 2 \times \sqrt{3}\right) = 10.4\ \ (10.3923..., 6\sqrt{3})\ (\text{m}^2)$ ***A1***

Note: Award ***A0M0A0*** for $\frac{1}{2} \times 2 \times 2$ or equivalent.

METHOD 3 (Finding the area of a trapezoid)

(altitude of one trapezoid is) $\sqrt{3}$ ***(A1)***

attempt to find area of one trapezoid using $\frac{1}{2} \times (a + b)h$ ***(M1)***

$\frac{1}{2} \times (2 + 4)\sqrt{3} \quad \left(3\sqrt{3}\right)$

$\left(2 \times \frac{1}{2} \times (2 + 4)\sqrt{3}\right) = 10.4\ \ (10.3923..., 6\sqrt{3})\ (\text{m}^2)$ ***A1***

continue…

Question 5 continued.

(ii) $\frac{1}{3} \times 10.3923... \times 2.74954...$ ***(A1)***

$= 9.52 \text{ m}^3 \ (9.52470\ldots)$ ***A1***

Note: Units must be seen.

[5 marks]

(c) $\cos(\text{M}\hat{\text{A}}\text{T}) = \frac{2}{3.4}$ or correct equivalent ***(A1)***

$(\text{M}\hat{\text{A}}\text{T} =) \ 54.0^\circ \quad (53.9681\ldots)$ ***A1***

[2 marks]

(d) Angle YAX = 180 – 53.9681... = 126.031…° ***(A1)***

Angle YXA = 180 – 35 – 126.031... = 18.9681…° ***(A1)***

Note: These angles may be seen in the sine rule.

Attempt to substitute into sine rule ***(M1)***

$$\frac{\text{AY}}{\sin(18.9681...)} = \frac{2.6}{\sin(126.031)}$$

$\text{AY} = 1.05 \quad (1.04503\ldots) \ (\text{m})$ ***A1***

[4 marks]

(e) attempt to substitute into cosine rule to form a quadratic for YZ ***(M1)***

$0.9^2 = \text{YZ}^2 + 1.04503...^2 - 2 \times 1.04503... \times \text{YZ} \times \cos(35)$ ***(A1)***

$\text{YZ} = 0.185 \ (0.184692\ldots) \ (\text{m}) \ , \ 1.53 \ (1.52739\ldots) \ (\text{m})$ ***A1A1***

[4 marks]

[Total: 17 marks]

2224 – 7205M

Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

May 2024

Mathematics: Applications and interpretation

Standard level

Paper 2

12 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

R Marks awarded for clear **Reasoning**.

AG Answer given in the question and so no marks are awarded.

FT Follow through. The practice of awarding marks, despite candidate errors in previous parts, for their correct methods/answers using incorrect results.

Using the markscheme

1 General

Award marks using the annotations as noted in the markscheme *eg* ***M1, A2.***

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***A3***, ***M2*** *etc.*, do **not** split the marks, unless there is a note.
- The response to a "show that" question does not need to restate the ***AG*** line, unless a **Note** makes this explicit in the markscheme.
- Once a correct answer to a question or part question is seen, ignore further working even if this working is incorrect and/or suggests a misunderstanding of the question. This will encourage a uniform approach to marking, with less examiner discretion. Although some candidates may be advantaged for that specific question item, it is likely that these candidates will lose marks elsewhere too.
- An exception to the previous rule is when an incorrect answer from further working is used **in a subsequent part**. For example, when a correct exact value is followed by an incorrect decimal approximation in the first part and this approximation is then used in the second part. In this situation, award ***FT*** marks as appropriate but do not award the final ***A1*** in the first part.

Examples:

	Correct answer seen	**Further working seen**	**Any FT issues?**	**Action**
1.	$8\sqrt{2}$	5.65685...	No.	Award ***A1*** for the final mark

		(incorrect decimal value)	Last part in question.	*(condone the incorrect further working)*
2.	$\frac{35}{72}$	0.468111… (incorrect decimal value)	Yes. Value is used in subsequent parts.	Award ***A0*** for the final mark *(and full **FT** is available in subsequent parts)*

3 Implied marks

Implied marks appear in **brackets e.g.** ***(M1)***,and can only be awarded if **correct** work is seen or implied by subsequent working/answer.

4 Follow through marks (only applied after an error is made)

Follow through (***FT***) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) (e.g. incorrect value from part (a) used in part (d) or incorrect value from part (c)(i) used in part (c)(ii)). Usually, to award ***FT*** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if all the marks awarded in a subsequent part are for the answer or are implied, then ***FT*** marks should be awarded for *their* correct answer, even when working is not present.

For example: following an incorrect answer to part (a) that is used in subsequent parts, where the markscheme for the subsequent part is ***(M1)A1***, it is possible to award full marks for *their* correct answer, **without working being seen.** For longer questions where all but the answer marks are implied this rule applies but may be overwritten by a **Note** in the Markscheme.

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate.

- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks, by reflecting on what each mark is for and how that maps to the simplified version.

- If the error leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).

- The markscheme may use the word “their” in a description, to indicate that candidates may be using an incorrect value.

- If the candidate’s answer to the initial question clearly contradicts information given in the question, it is not appropriate to award any ***FT*** marks in the subsequent parts. This includes when candidates fail to complete a “show that” question correctly, and then in subsequent parts use their incorrect answer rather than the given value.

- Exceptions to these ***FT*** rules will be explicitly noted on the markscheme.

- If a candidate makes an error in one part but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the command term was “Hence”.

5 Mis-read

If a candidate incorrectly copies values or information from the question, this is a mis-read (***MR***). A candidate should be penalized only once for a particular misread. Use the ***MR*** stamp to indicate that this has been a misread and do not award the first mark, even if this is an ***M*** mark, but award all others as appropriate.

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.

- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).

- Miscopying of candidates' own work does **not** constitute a misread, it is an error.

- If a candidate uses a correct answer, to a "show that" question, to a higher degree of accuracy than given in the question, this is NOT a misread and full marks may be scored in the subsequent part.

- ***MR*** can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If the command term is 'Hence' and not 'Hence or otherwise' then alternative methods are not permitted unless covered by a note in the mark scheme.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**.

7 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation** for example 1.9 and 1,9 or 1000 and 1,000 and 1.000.

- Do not accept final answers written using calculator notation. However, ***M*** marks and intermediate ***A*** marks can be scored, when presented using calculator notation, provided the evidence clearly reflects the demand of the mark.

- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.

- In the markscheme, some **equivalent** answers will generally appear in brackets. Not all equivalent notations/answers/methods will be presented in the markscheme and examiners are asked to apply appropriate discretion to judge if the candidate work is equivalent.

8 Format and accuracy of answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. If the level of accuracy is not stated in the question, the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures.*

Where values are used in subsequent parts, the markscheme will generally use the exact value, however candidates may also use the correct answer to 3 sf in subsequent parts. The markscheme will often explicitly include the subsequent values that come "*from the use of 3 sf values*".

Simplification of final answers: Candidates are advised to give final answers using good mathematical form. In general, for an ***A*** mark to be awarded, arithmetic should be completed, and any values that lead to integers should be simplified; for example, $\sqrt{\frac{25}{4}}$ should be written as $\frac{5}{2}$. An exception to this is simplifying fractions, where lowest form is not required (although the numerator and the denominator must be integers); for example, $\frac{10}{4}$ may be left in this form or written as $\frac{5}{2}$. However, $\frac{10}{5}$ should be written as 2, as it simplifies to an integer.

Algebraic expressions should be simplified by completing any operations such as addition and multiplication, e.g. $4e^{2x} \times e^{3x}$ should be simplified to $4e^{5x}$, and $4e^{2x} \times e^{3x} - e^{4x} \times e^{x}$ should be simplified to $3e^{5x}$. Unless specified in the question, expressions do not need to be factorized, nor do factorized expressions need to be expanded, so $x(x+1)$ and $x^2 + x$ are both acceptable.

Please note: intermediate ***A*** marks do NOT need to be simplified.

9 Calculators

A GDC is required for this paper, but if you see work that suggests a candidate has used any calculator not approved for IB DP examinations (eg CAS enabled devices), please follow the procedures for malpractice.

10. Presentation of candidate work

Crossed out work: If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work unless an explicit note from the candidate indicates that they would like the work to be marked.

More than one solution: Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise. If the layout of the responses makes it difficult to judge, examiners should apply appropriate discretion to judge which is "first".

1. (a) $r_s = -1$ ***A1***

[1 mark]

(b) (i) $r = -0.979 \quad (-0.979191\ldots)$ ***A2***

(ii) strong **AND** negative ***A1A1***

Note: Award at most ***A1A0*** if additional answers are seen.
Due to the demand of the question, do not accept "negative (from the graph)" if their r value is positive.

[4 marks]

(c) (i) $a = -0.0992 \qquad (a = -0.0992075\ldots)$ ***A1***

(ii) $b = 3.19 \qquad (b = 3.19150\ldots)$ ***A1***

(iii) b represents the (typical) price of an apartment in the centre (of the city) ***A1***

Note: To award the ***A1***, some reference to "centre" or "zero distance from the city" needs to be seen.

[3 marks]

(d) (i) attempt to substitute 19.6 for x ***(M1)***

$y = -0.0992075\ldots \times 19.6 + 3.19150\ldots$

$= 1.25 \qquad (1.24704\ldots)$ ***A1***

price $= 1.25\,\text{million (AUD)} \quad (1.24704\ldots\,\text{million})$ ***A1***

(ii) interpolation ***R1***

strong correlation. ***R1***

[5 marks]

(e) $\mu_A \neq \mu_B$ ***A1***

[1 mark]

(f) $p = 0.0224 \quad (0.0223977\ldots)$ ***A2***

Note: Award ***A1*** for 0.022 (2sf)
Award ***A1*** for an answer of $p = 0.0265 \quad (0.0265017\ldots)$, from use of unpooled GDC settings.

[2 marks]

(g) $0.0223977\ldots < 0.05$ ***R1***

(there is sufficient evidence to) reject the null hypothesis ***A1***

Note: Do not award ***R0A1***.

[2 marks]

(h) (the two populations are) normally distributed ***A1***

Note: Do not accept "independent" as that applies to the samples, not the populations.

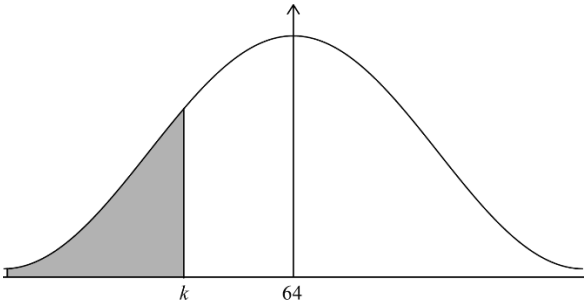
[1 mark]

[Total 19 marks]

2. (a) 0.5 ***A1***
[1 mark]

(b) $0.452 \ (0.452209\ldots)$ ***A2***
[2 marks]

(c) (i)



A1A1

Note: Award ***A1*** for a normal curve (with symmetry and some evidence of change of curvature towards the extreme values).
Award ***A1*** for a shaded region $x < k$, where k<mean.

(ii) $\mathrm{P}(T < k) = 0.3$
solving a cumulative distribution function **OR**
use of inverse function on GDC ***(M1)***
$k = 57.7 \ \ (57.7071\ldots)$ ***A1***
[4 marks]

(d) recognizing binomial distribution ***(M1)***
$\mathrm{B}(5,\ 0.3) \ \ \left(\mathrm{P}(X = 2)\right)$ ***(A1)***

$0.309 \ \ (0.3087)$ ***A1***
[3 marks]

(e) $2(x-1)+4.5$ **OR** $2x+2.5$ ***A1A1***

Note: Award ***A1*** for a linear expression with a gradient of 2,
A1 for a completely correct expression in x.

[2 marks]

(f) $(\$)13.10$ (accept 13.1) ***A1***
[1 mark]

(g) attempt to solve $2(x-1)+4.5=7.2$ **OR** $2x+2.5=7.2$ ***(M1)***
2.35 (kg) ***A1***

Note: Award ***M1A1FT*** for an answer of 1.35 (kg) from $2x+4.5$ seen in (e).

[2 marks]
[Total 15 marks]

3. (a) $\frac{40000}{x^2} = 400$ ***(M1)***

$x = 10$ (pesos) (since x is positive) ***A1***

[2 marks]

(b) (i) $\left(\frac{40000}{50^2} =\right) 16$ ***A1***

(ii) $(50 \times 16 =)\ 800$ (pesos) ***A1***

[2 marks]

(c) (i) **EITHER**

profit for each smoothie = $x - 20$ ***(M1)***

$P = \frac{40000}{x^2} \times (x - 20)$ ***A1***

OR

profit = revenue – costs = $nx - 20n$ ***(M1)***

$P = x \times \frac{40000}{x^2} - 20 \times \frac{40000}{x^2}$ ***A1***

Note: Do not award ***A1*** if $\frac{40000}{x}$ seen as first term unless explained (in part (a) or (b)), as it is given in question.

THEN

$P = \frac{40000}{x} - \frac{800000}{x^2}$ ***AG***

(ii) attempt to express P ready for power rule ***(M1)***

$P = 40000x^{-1} - 800000x^{-2}$

$\frac{dP}{dx} = -\frac{40000}{x^2} + \frac{1600000}{x^3}$ **OR** $\frac{dP}{dx} = -40000\,x^{-2} + 1600000\,x^{-3}$ ***A1A1***

Note: The ***(M1)*** can be awarded for either of the correct terms seen.
A1 for each correct term.
At most ***M1A1A0*** if additional terms seen.

continued...

Question 3 continued

(iii) attempt to find x-value ***(M1)***

e.g. sketch of $\frac{\mathrm{d}P}{\mathrm{d}x}$ with x-intercept indicated **OR** recognition that it occurs at the maximum of P **OR** algebraic approach (requires multiplication by x^3)

$x = 40$ ***A1***

Note: $\frac{-40000}{x^2} + \frac{1600000}{x^3} = 0$ is insufficient to award ***M1***, this is given in the question. There must be an "attempt to find x-value".
Award ***M1A0*** for a coordinate pair $(40, 500)$.

(iv) attempt to substitue their x-value into equation for n ***(M1)***

$n = \frac{40000}{40^2}$

$= 25$ ***A1***

Note: Given the nature of the function *P*, the local maximum is also the global maximum. This is often the case in examinations, but should not always be assumed.

[9 marks]
[Total 13 marks]

4. (a) (i) 0.9 (ii) 0.3 (iii) 0.7 ***A2***

Note: Award ***A1A0*** if one of the values is incorrect, ***A0A0*** otherwise.

[2 marks]

(b) $(0.1 \times 0.3 =)\ 0.03$ ***A1***

[1 mark]

(c) $\mathrm{P}(\text{no fail}) = 0.63$ ***A1***

multiplying by 200 ***M1***

$= 126$ ***AG***

Note: Award ***A0M0*** for a flawed approach to find $\mathrm{P}(\text{no fail}) = 0.63$, e.g. $\frac{126}{200} = 0.63$, which is reverse engineering.

[2 marks]

continued...

Question 4 continued

(d) **EITHER**

attempt to find probability one switch failing ***(M1)***

$\mathrm{P}(\text{one failing}) = 0.34$ ***(A1)***

OR

expected value for two switches failing $= 6$ ***(A1)***

expected value for one switch failing $= 200 - 126 - 6$ ***(M1)***

THEN

No switch fails	One switch fails	Two switches fail
126	68	6

(A1)

degrees of freedom $= 2$ ***(A1)***

Note*:* Award ***A1*** for df = 2 seen anywhere and may be awarded independent of the ***M1*** mark.
The df cannot be implied from chi sq statistic = 3.40989

p-value 0.182 $(0.181781\ldots)$ ***A1***

$0.182 > 0.05$ ***R1***

hence insufficient evidence to reject H_0 (that the manufacturers claims are correct) ***A1***

Note: The ***R1A1*** can be awarded as follow through within part (d) from their (explicitly labelled) incorrect p-value.
An unrealistic p-value ($p \geq 1$) should preclude awarding the final ***R1A1***.
Accept either a conclusion to not reject the null hypothesis or the manufacturers claims are correct.
Do not award ***R0A1***.

[7 marks]

[Total 12 marks]

5. (a) attempt to substitute 2 into the DE ***(M1)***

$0.6 \times 2^2 - 1.8 \times 2$

-1.2 ***A1***

[2 marks]

(b) recognition of need to integrate ***(M1)***

Note: An increase of powers (expressions with $x^2 + x^3$) is sufficient evidence of the need to integrate.

$0.2x^3 - 0.9x^2 \ \ (+c)$ (***A1***)

attempt to substitute given condition to find c ***(M1)***

$0.2(4)^3 - 0.9(4)^2 + c = 1.4$ OR $c = 3$

$\big(h(x) =\big) \ \ 0.2x^3 - 0.9x^2 + 3$ ***A1***

[4 marks]

continued…

Question 5 continued

(c) (i) $\int_0^4 0.2x^3 - 0.9x^2 + 3\mathrm{d}x$ **OR** $\int_0^4 h\,\mathrm{d}x$ ***A1A1***

Note: Award ***A1*** for a correct integral, ***A1*** for correct limits in the correct location.
Award at most ***A0A1*** if $\mathrm{d}x$ is omitted.
Award at most ***A0A1*** for "integral $y\mathrm{d}x$ " or integral $f(x)\mathrm{d}x$, this is not the correct integrand.

(ii) $5.6\ (\mathrm{m}^2)$ ***A2***

[4 marks]

Note: A negative area for their integrand is unrealistic. Award at most ***A1A0*** for their ***FT*** answer expressed as negative area or a negative area converted to a positive answer."

(d) $(5.6\times 1.2 =)\ 6.72\ \mathrm{m}^3$ ***A1***

Note: Correct unit must be seen for the ***A1*** to be awarded.

[1 mark]

(e) attempt to substitute one of the given conditions, both x and h, into either the function or the derivative ***(M1)***

$1 = 16a + 4b + c$ ***A1***

attempt to differentiate $h(x)$ ***(M1)***

$h'(x) = 2a(x) + b$ ***(A1)***

$-1.2 = 4a + b$ ***A1***

$0 = 6a + b$ **OR** $3 = \frac{-b}{2a}$ ***A1***

Note: The equations can be found in any order, and hence the associated mark(s) should be awarded independently.
Accept $1 = 4a + 2b + c$ and $1.2 = 8a + b$ as correct equations, from symmetry.
Award at most ***M1A1M1A1A1A0*** if the equations are not simplified to integer values.
E.g . $1 = 4^2 a + 4b + c$, $-1.2 = 2\times 2a + b$, $0 = 2\times 3a + b$

[6 marks]

(f) $a = 0.6\left(\frac{3}{5}\right),\ b = -3.6\left(-\frac{18}{5}\right),\ c = 5.8\left(\frac{29}{5}\right)$ ***A2***

Note: Award ***A1*** if only two are correct, ***A0*** otherwise.
Only follow through from three explicit equations given in part (e).

[2 marks]

(g) gradients are the same **OR** $h'(2) = -1.2$ for both curves ***A1***
heights are the same **OR** $h(2) = 1$ for both curves ***A1***

[2 marks]

[Total 21 marks]

2224 – 7210M

Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

May 2024

Mathematics: applications and interpretation

Standard level

Paper 2

15 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

R Marks awarded for clear **Reasoning**.

AG Answer given in the question and so no marks are awarded.

FT Follow through. The practice of awarding marks, despite candidate errors in previous parts, for their correct methods/answers using incorrect results.

Using the markscheme

1 General

Award marks using the annotations as noted in the markscheme eg ***M1***, ***A2***.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***A3***, ***M2*** *etc.*, do **not** split the marks, unless there is a note.
- The response to a "show that" question does not need to restate the ***AG*** line, unless a **Note** makes this explicit in the markscheme.
- Once a correct answer to a question or part question is seen, ignore further working even if this working is incorrect and/or suggests a misunderstanding of the question. This will encourage a uniform approach to marking, with less examiner discretion. Although some candidates may be advantaged for that specific question item, it is likely that these candidates will lose marks elsewhere too.
- An exception to the previous rule is when an incorrect answer from further working is used **in a subsequent part**. For example, when a correct exact value is followed by an incorrect decimal approximation in the first part and this approximation is then used in the second part. In this situation, award ***FT*** marks as appropriate but do not award the final ***A1*** in the first part. Examples:

	Correct answer seen	Further working seen	Any FT issues?	Action
1.	$8\sqrt{2}$	5.65685... *(incorrect decimal value)*	No. Last part in question.	Award ***A1*** for the final mark *(condone the incorrect further working)*
2.	$\frac{35}{72}$	0.468111… (incorrect decimal value)	Yes. Value is used in subsequent parts.	Award ***A0*** for the final mark *(and full **FT** is available in subsequent parts)*

3 Implied marks

Implied marks appear in **brackets e.g. *(M1)***,and can only be awarded if **correct** work is seen or implied by subsequent working/answer.

4 Follow through marks (only applied after an error is made)

Follow through (***FT***) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) (e.g. incorrect value from part (a) used in part (d) or incorrect value from part (c)(i) used in part (c)(ii)). Usually, to award ***FT*** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if all the marks awarded in a subsequent part are for the answer or are implied, then ***FT*** marks should be awarded for *their* correct answer, even when working is not present.

For example: following an incorrect answer to part (a) that is used in subsequent parts, where the markscheme for the subsequent part is ***(M1)A1***, it is possible to award full marks for *their* correct answer, **without working being seen.** For longer questions where all but the answer marks are implied this rule applies but may be overwritten by a **Note** in the Markscheme.

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks, by reflecting on what each mark is for and how that maps to the simplified version.
- If the error leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word “their” in a description, to indicate that candidates may be using an incorrect value.
- If the candidate’s answer to the initial question clearly contradicts information given in the question, it is not appropriate to award any ***FT*** marks in the subsequent parts. This includes when candidates fail to complete a “show that” question correctly, and then in subsequent parts use their incorrect answer rather than the given value.
- Exceptions to these ***FT*** rules will be explicitly noted on the markscheme.
- If a candidate makes an error in one part but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the command term was “Hence”.

5 Mis-read

If a candidate incorrectly copies values or information from the question, this is a mis-read (***MR***). A candidate should be penalized only once for a particular misread. Use the ***MR*** stamp to indicate that this has been a misread and do not award the first mark, even if this is an ***M*** mark, but award all others as appropriate.

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.
- If a candidate uses a correct answer, to a "show that" question, to a higher degree of accuracy than given in the question, this is NOT a misread and full marks may be scored in the subsequent part.
- ***MR*** can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If the command term is 'Hence' and not 'Hence or otherwise' then alternative methods are not permitted unless covered by a note in the mark scheme.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**.

7 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation** for example 1.9 and $1{,}9$ or 1000 and $1{,}000$ and 1.000.
- Do not accept final answers written using calculator notation. However, ***M*** marks and intermediate ***A*** marks can be scored, when presented using calculator notation, provided the evidence clearly reflects the demand of the mark.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, some **equivalent** answers will generally appear in brackets. Not all equivalent notations/answers/methods will be presented in the markscheme and examiners are asked to apply appropriate discretion to judge if the candidate work is equivalent.

8 Format and accuracy of answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. If the level of accuracy is not stated in the question, the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures.*

Where values are used in subsequent parts, the markscheme will generally use the exact value, however candidates may also use the correct answer to a "correct" level of accuracy (e.g 3 sf) in subsequent parts. The markscheme will often explicitly include the subsequent values that come "*from the use of 3 sf values*".

Simplification of final answers: Candidates are advised to give final answers using good mathematical form. In general, for an ***A*** mark to be awarded, arithmetic should be completed, and any values that lead to integers should be simplified; for example, $\sqrt{\frac{25}{4}}$ should be written as $\frac{5}{2}$. An exception to this is simplifying fractions, where lowest form is not required (although the numerator and the denominator must be integers); for example, $\frac{10}{4}$ may be left in this form or written as $\frac{5}{2}$. However, $\frac{10}{5}$ should be written as 2, as it simplifies to an integer.

Algebraic expressions should be simplified by completing any operations such as addition and multiplication, e.g. $4e^{2x} \times e^{3x}$ should be simplified to $4e^{5x}$, and $4e^{2x} \times e^{3x} - e^{4x} \times e^{x}$ should be simplified to $3e^{5x}$. Unless specified in the question, expressions do not need to be factorized, nor do factorized expressions need to be expanded, so $x(x+1)$ and $x^2 + x$ are both acceptable.

Please note: intermediate ***A*** marks do NOT need to be simplified.

9 Calculators

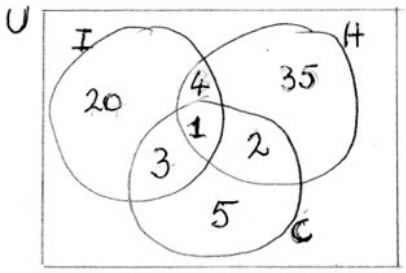
A GDC is required for this paper, but If you see work that suggests a candidate has used any calculator not approved for IB DP examinations (eg CAS enabled devices), please follow the procedures for malpractice.

10. Presentation of candidate work

Crossed out work: If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work unless an explicit note from the candidate indicates that they would like the work to be marked.

More than one solution: Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise. If the layout of the responses makes it difficult to judge, examiners should apply appropriate discretion to judge which is "first".

1. (a)



A3

Note: Award ***A1*** for 1 in correct place, ***A1*** for 3, 2 and 4 correct, ***A1*** for 35, 20 and 5 correct. Award at most ***A0A1A1*** if the rectangle is omitted. Condone the omission of the 430, as explicitly asked for in part (b).

[3 marks]

(b) $35+4+20+3+1+2+5$ ***(M1)***

430 ***A1***

Note: The 430 may be seen in the Venn diagram.

[2 marks]

(c) (i) $\frac{42}{500}$ $(\frac{21}{250}, 0.084, 8.4\%)$ ***A2***

Note: Award ***A1*** for numerator, ***A1*** for denominator.

(ii) $\frac{5}{42}$ $\left(0.119,\ 0.119047\ldots, 11.9\%\right)$ ***A2***

Note: ***A1*** for numerator, ***A1*** for denominator.
The first ***A1*** can be awarded for an attempt to use conditional probability with 0.084 on the denominator.

[4 marks]

(d) substituting $t=0$ into given expression ***(M1)***

10 ***A1***

[2 marks]

(e) $130=250-240(2)^{10k}$ ***(M1)***

$k=-0.1$ ***A1***

[2 marks]

(f) $200=250-240(2)^{-0.1t}$ ***(M1)***

$t=22.6$ $(22.6303\ldots, 23)$ (days) ***A1***

[2 marks]

(g) **EITHER**
model does not predict n to go above 250 / reach 300 ***A1***

OR
$250 - 240 \times 2^{-0.1 \times 365} = 250$, so does not reach 300 ***A1***

OR
there is no solution to $n(t) = 300$ ***A1***

OR
correct sketch graph, with 250 and/or 300 labelled, and a supporting comment ***A1***

THEN
hence Diego's model is not appropriate.

Note: Do not credit reasoning based on selecting arbitrary high values of t and finding the associated n value.

[1 mark]
[Total: 16 marks]

2. (a) $\text{BC} = 20\ (\text{m})$ ***A1***

[1 mark]

(b) use of Pythagoras ***(M1)***

$\text{AB} = \sqrt{12^2 + 4^2}$

$= 12.6\ (\text{m})\quad (12.6491\ldots,\ \sqrt{160})$ ***A1***

[2 marks]

(c) **METHOD 1 – finding angle ABC**

correct use of a trig ratio to find $\text{A}\hat{\text{B}}\text{C}$ (or finding the bearing of B from A) ***(A1)***

e.g. $\tan\left(\text{A}\hat{\text{B}}\text{C}\right) = \frac{12}{4},\ \cos\text{A}\hat{\text{B}}\text{C} = \frac{20^2 + 12.649^2 - 20^2}{2 \times 20 \times 12.649},\ \cos\text{A}\hat{\text{B}}\text{C} = \frac{6.3245}{20}$

$\text{A}\hat{\text{B}}\text{C} = 71.6\ (71.5650\ldots)$ ***(A1)***

Note: Angle $\text{A}\hat{\text{B}}\text{C}$ can be 71.5 or 72.2 depending on their working out. Bearings should be given in degrees.

$180 + 71.5650\ldots = 252°\ (251.565\ldots)$ ***A1***

Note: The final ***A1*** can be awarded for 180 plus their 71.6. If radians used, award ***A1A1*** for $1.24904\ldots$ or $4.39063\ldots$ seen, and then ***A0*** for the radian answer.

METHOD 2 – finding angle that AB makes with the horizontal (angle *H*)

correct use of a trig ratio to find H, the angle AB makes with horizontal ***(A1)***

e.g. $\tan\hat{H} = \frac{4}{12},\ \cos\hat{H} = \frac{12^2 + 12.649^2 - 4^2}{2 \times 12 \times 12.649}$

$\hat{H} = 18.4\ (18.4349...)$ ***(A1)***

Note: Accept $18.5\ (18.5078\ldots)$ from use of 3sf answer from part (b). Bearings should be given in degrees.

$270 - 18.4348... = 252°\ (251.565\ldots)$ ***A1***

Note: The final ***A1*** can be awarded for 270 minus their 18.4. If radians used, award ***A1A1*** for $0.321750\ldots$ or $4.39063\ldots$ seen, and then ***A0*** for the radian answer.

[3 marks]

(d) (i) $-\frac{4}{3}$ $\left(-\frac{16}{12}\right)$ ***A1***

(ii) $(6, 8)$ ***A1A1***

Note: Award ***A1A0*** if parentheses are missing.

(iii) gradient of (their) perp line = $\frac{3}{4}$ ***(M1)***

equation of perpendicular bisector of AC ***(A1)***

e.g. $(y-8)=\frac{3}{4}(x-6)$ **OR** $y=\frac{3}{4}x+3.5$

EITHER

equation of perpendicular bisector of BC is $y=10$ ***(A1)***

OR

equation of perpendicular bisector of AB is $y=-3x+36$ ***(A1)***

Note: The ***A1*** is for either equation of perpendicular bisector of BC or AB.

point of intersection $\left(8\frac{2}{3}, 10\right)$ **OR** $(8.67, 10)$ $\left[(8.666\ldots, 10)\right]$ ***(M1)A1***

Note: Award ***M1*** for an attempt to equate their perpendicular bisectors
Award the final ***A1*** for the correct coordinate pair – parentheses omitted or not.

[8 marks]

[Total: 14 marks]

3. (a) heights, 0, 4, 1.75, 3 and 3.75 seen ***(A2)***

Note: Award ***A1A0*** if **two** of 1.75, 3 or 3.75 are seen.

attempt to use trapezoidal rule formula for their heights ***(M1)***

$\frac{1}{2}\times 1\times\{0+4+2(1.75+3+3.75)\}$ ***(A1)***

Note: Award ***(M1)(A1)*** for correctly expressing this as 3 trapezoids and a triangle. The "$\times 1$" need not be seen.

$=10.5\ (\text{m}^2)$ ***A1***

[5 marks]

(b) $-\frac{1}{12}x^3+x^2+c$ ***A1A1A1***

[3 marks]

(c) $\int_0^4\left(-\frac{1}{4}x^2+2x\right)\text{d}x+1\times 4+\frac{1}{2}\times 7\times 4$ ***(A1)(M1)(A1)***

Note: Award ***A1*** for correct area of rectangle **OR** triangle, ***M1*** for substituting correct limits into given integral (may be seen in part (b)), and ***A1*** for entire expression correct.

$=10.6666\ldots. + 4 + 14$

$= 28\frac{2}{3}\ (\text{m}^2)\left(\frac{86}{3}\right)$ ***A1***

Note: The answer must be **exact** for the ***A1*** to be awarded. For an answer of 28.7 or 28.66 award ***(A1)(M1)(A1)A0***.

[4 marks]

(d) (Total area using part (a) =) 28.5 ***(A1)***

Percentage error $=\left|\frac{28.5-28.6666\ldots}{28.6666\ldots}\right|\times 100$ ***(M1)***

Note: if their trapezoid value is incorrect but is used correctly in the percentage error formula, award at most ***A0M1A0***. If it is clear from the answer that ×100 has been used, then condone the omission and award the ***M*** mark.

$=0.581\ (\%)\ (0.581395\ldots)$ ***A1***

(accept 0.697 from use of 28.7)

[3 marks]

[Total: 15 marks]

4. (a) (i) $\mathrm{P}(X < 160)$ **OR** labelled sketch of region **OR** calc syntax with correct bounds ***(M1)***

Note: Accept either zero or a large negative value as the lower bound.

$= 0.0447$ $(0.0447149\ldots, 4.47\%)$ ***A1***

(ii) $\mathrm{P}(160 < X < 170)$ **OR** labelled sketch of region **OR** calc syntax with correct bounds ***(M1)***

$= 0.411$ $(0.411049\ldots, 41.1\%)$ ***A1***

Note: Award ***A0A2*** for answers of 0.045 and 0.41 both given to 2 sf.

[4 marks]

(b) $\mathrm{P}(X > h) = 0.27$ **OR** labelled sketch of region **OR** calc syntax with correct bounds ***(M1)***

$= 1.75$ (m) $(1.74560\ldots)$ ***A1***

Note: Accept 175 (cm).

[2 marks]

(c) (i) 82.21 ***A1***

(ii) 94.86 ***A1***

Note: Follow through from an incorrect part (c)(i) if fourth value is found by subtracting first three values from 200. Award at most ***A0A1*** if both answers are not given to four significant figures.
Award ***A0A1*** for an answer of $a = 82.2$ and $b = 94.8$.

[2 marks]

(d) 3 ***A1***

[1 mark]

(e) p-value $= 0.616$ $(0.615583\ldots.)$ **OR** $\chi^2 = 1.80$ $(1.79702\ldots.)$ ***A2***

Note: Award ***A1A0*** if the p-value or χ^2-value is given correct to 2 dp.

$0.615583\ldots > 0.05$ **OR** $1.79702\ldots. < 7.815$ ***R1***

EITHER

fail to reject the null hypothesis ***A1***

OR

the heights are normally distributed with mean 170.7 cm and standard deviation 6.3 cm ***A1***

Note: Do not award ***R0A1***. Condone "accept" in place of "fail to reject".

The ***R1A1*** can be awarded as follow through within part (e) from their (explicitly labelled) p-value or χ^2-value. Accept comparison in words.

[4 marks]

(f) **EITHER**

$H_0 : \mu_L = \mu_D$ ***A1***

$H_1 : \mu_L > \mu_D$ ***A1***

OR

H_0 : The (population) mean height of Latvian women is equal to the (population) mean height of Dutch women ***A1***

H_1 : The (population) mean height of Latvian women is greater than the (population) mean height of Dutch women ***A1***

Note: Award at most ***A0A1*** if the hypotheses explicitly refer to the "sample" and not the population. For $H_0 : m_1 = m_2$ and $H_1 : m_1 > m_2$ award ***A0A1***.

[2 marks]

(g) p-value $= 0.673$ $(0.673205\ldots)$ ***A2***

Note: In this question the p-value is the same 3 sf value for unpooled GDC settings so will be awarded ***A2***.
If using a two-tailed test, the answer is p-value $= 0.654$ $(0.653589\ldots)$; award ***A1*** if alternative hypothesis was correct or ***A2*** if it follows through correctly from their alternative hypothesis (i.e. two-tailed test was penalized in part (f)).

$0.673205 > 0.05$ ***R1***

fail to reject the null hypothesis (Gundega is not correct) ***A1***

Note: Do not award ***R0A1***. Condone "accept" in place of "fail to reject".

The ***R1A1*** can be awarded as follow through within part (g) from their (explicitly labelled) p-value. Accept comparison in words.

[4 marks]

[Total: 19 marks]

5. (a) (i) correct approach to find missing length ***(A1)***

$\sqrt{4^2-1^2}\quad(=\sqrt{15})$

attempt to find cross-section ***(M1)***

e.g. use of area of trapezoid formula or rectangle+triangle or rectangle – triangle

use of volume of prism formula ***(M1)***

(their cross-section multiplied by 3)

$3\left[\frac{1}{2}(10+11)\left(\sqrt{4^2-1^2}\right)\right]$

$=122\,(\text{m}^3)\ \ (121.998\ldots)$ ***A1***

(ii) correct approach to find missing height ***(A1)***

$\sqrt{4^2-3.2^2}\quad(=2.4)$

attempt to find volume ***(M1)***

(multiplication by 3.2 and 3 seen)

$3\left[\frac{1}{2}\left(10+10+\sqrt{4^2-3.2^2}\right)(3.2)\right]$

$=108\,(\text{m}^3)\ \ (107.52...)$ ***A1***

(iii) correct approach to find missing lengths ***(A1)***

$\sin(60^\circ)$ and $\cos(60^\circ)$ **OR** $\sin(60^\circ)$ and Pythagoras etc seen in work

$3\left[\frac{1}{2}(10+10+4\cos(60^\circ))4\sin(60^\circ)\right]$

$=114\,(\text{m}^3)\ \ (114.315\ldots)$ ***A1***

[9 marks]

(b) $V=3\left[\frac{1}{2}(10+10+4\cos(\theta))4\sin(\theta)\right]$ ***A1***

all correct intermediate working leading to given answer ***A1***

e.g. $V=6\sin(\theta)(20+4\cos(\theta))$

$V=24\sin(\theta)(5+\cos(\theta))$ ***AG***

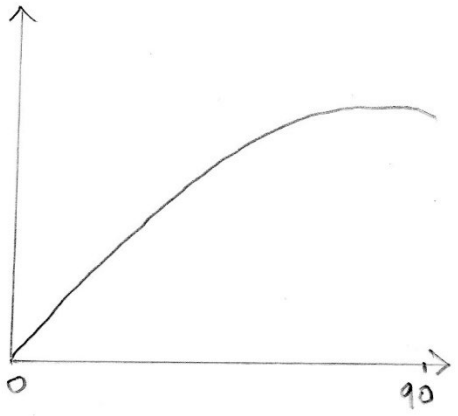
Note: The ***AG*** line must be seen for the final ***A1*** to be awarded.

[2 marks]

(c) *accept any reasoning along the lines:* "skip would have zero volume" or "if the angle is zero, then the contents would fall out" ***R1***

[1 mark]

(d) (i)



A1A1

Note: Award ***A1*** for the correct shape and ***A1*** for the graph on the correct, labelled, domain. Condone omission of θ / V labels (or x/y).

(ii) $\theta = 79.3^\circ$ $(79.2723...^\circ)$ $(1.38$ $(1.38356...))$, $V_{\max} = 122$ $(122.292...)$ ***A1A1***

Note: Award ***A0A1*** if values are reversed and ***A0A0*** for a coordinate pair.

[4 marks]

[Total: 16 marks]

Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

November 2023

Mathematics: applications and interpretation

Standard level

Paper 2

15 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

R Marks awarded for clear **Reasoning**.

AG Answer given in the question and so no marks are awarded.

FT Follow through. The practice of awarding marks, despite candidate errors in previous parts, for their correct methods/answers using incorrect results.

Using the markscheme

1 General

Award marks using the annotations as noted in the markscheme eg ***M1***, ***A2***.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***A3***, ***M2*** *etc*., do **not** split the marks, unless there is a note.
- The response to a "show that" question does not need to restate the ***AG*** line, unless a **Note** makes this explicit in the markscheme.
- Once a correct answer to a question or part question is seen, ignore further working even if this working is incorrect and/or suggests a misunderstanding of the question. This will encourage a uniform approach to marking, with less examiner discretion. Although some candidates may be advantaged for that specific question item, it is likely that these candidates will lose marks elsewhere too.
- An exception to the previous rule is when an incorrect answer from further working is used **in a subsequent part**. For example, when a correct exact value is followed by an incorrect decimal approximation in the first part and this approximation is then used in the second part. In this situation, award ***FT*** marks as appropriate but do not award the final ***A1*** in the first part. Examples:

	Correct answer seen	Further working seen	Any FT issues?	Action
1.	$8\sqrt{2}$	5.65685... *(incorrect decimal value)*	No. Last part in question.	Award ***A1*** for the final mark *(condone the incorrect further working)*
2.	$\frac{35}{72}$	0.468111… (incorrect decimal value)	Yes. Value is used in subsequent parts.	Award ***A0*** for the final mark *(and full **FT** is available in subsequent parts)*

3 Implied marks

Implied marks appear in **brackets e.g. *(M1)***,and can only be awarded if **correct** work is seen or implied by subsequent working/answer.

4 Follow through marks (only applied after an error is made)

Follow through (***FT***) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) (e.g. incorrect value from part (a) used in part (d) or incorrect value from part (c)(i) used in part (c)(ii)). Usually, to award ***FT*** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if all the marks awarded in a subsequent part are for the answer or are implied, then ***FT*** marks should be awarded for *their* correct answer, even when working is not present.

For example: following an incorrect answer to part (a) that is used in subsequent parts, where the markscheme for the subsequent part is ***(M1)A1***, it is possible to award full marks for *their* correct answer, **without working being seen.** For longer questions where all but the answer marks are implied this rule applies but may be overwritten by a **Note** in the Markscheme.

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks, by reflecting on what each mark is for and how that maps to the simplified version.
- If the error leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word “their” in a description, to indicate that candidates may be using an incorrect value.
- If the candidate’s answer to the initial question clearly contradicts information given in the question, it is not appropriate to award any ***FT*** marks in the subsequent parts. This includes when candidates fail to complete a “show that” question correctly, and then in subsequent parts use their incorrect answer rather than the given value.
- Exceptions to these ***FT*** rules will be explicitly noted on the markscheme.
- If a candidate makes an error in one part but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the command term was “Hence”.

5 Mis-read

If a candidate incorrectly copies values or information from the question, this is a mis-read (***MR***). A candidate should be penalized only once for a particular misread. Use the ***MR*** stamp to indicate that this has been a misread and do not award the first mark, even if this is an ***M*** mark, but award all others as appropriate.

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.
- If a candidate uses a correct answer, to a "show that" question, to a higher degree of accuracy than given in the question, this is NOT a misread and full marks may be scored in the subsequent part.
- ***MR*** can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If the command term is 'Hence' and not 'Hence or otherwise' then alternative methods are not permitted unless covered by a note in the mark scheme.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**.

7 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation** for example 1.9 and $1{,}9$ or 1000 and $1{,}000$ and 1.000.
- Do not accept final answers written using calculator notation. However, ***M*** marks and intermediate ***A*** marks can be scored, when presented using calculator notation, provided the evidence clearly reflects the demand of the mark.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, some **equivalent** answers will generally appear in brackets. Not all equivalent notations/answers/methods will be presented in the markscheme and examiners are asked to apply appropriate discretion to judge if the candidate work is equivalent.

8 Format and accuracy of answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. If the level of accuracy is not stated in the question, the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures.*

Where values are used in subsequent parts, the markscheme will generally use the exact value, however candidates may also use the correct answer to a "correct" level of accuracy (e.g 3 sf) in subsequent parts. The markscheme will often explicitly include the subsequent values that come "*from the use of 3 sf values*".

Simplification of final answers: Candidates are advised to give final answers using good mathematical form. In general, for an ***A*** mark to be awarded, arithmetic should be completed, and any values that lead to integers should be simplified; for example, $\sqrt{\frac{25}{4}}$ should be written as $\frac{5}{2}$. An exception to this is simplifying fractions, where lowest form is not required (although the numerator and the denominator must be integers); for example, $\frac{10}{4}$ may be left in this form or written as $\frac{5}{2}$. However, $\frac{10}{5}$ should be written as 2, as it simplifies to an integer.

Algebraic expressions should be simplified by completing any operations such as addition and multiplication, e.g. $4e^{2x} \times e^{3x}$ should be simplified to $4e^{5x}$, and $4e^{2x} \times e^{3x} - e^{4x} \times e^{x}$ should be simplified to $3e^{5x}$. Unless specified in the question, expressions do not need to be factorized, nor do factorized expressions need to be expanded, so $x(x+1)$ and $x^2 + x$ are both acceptable.

Please note: intermediate ***A*** marks do NOT need to be simplified.

9 Calculators

A GDC is required for this paper, but If you see work that suggests a candidate has used any calculator not approved for IB DP examinations (eg CAS enabled devices), please follow the procedures for malpractice.

10. Presentation of candidate work

Crossed out work: If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work unless an explicit note from the candidate indicates that they would like the work to be marked.

More than one solution: Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise. If the layout of the responses makes it difficult to judge, examiners should apply appropriate discretion to judge which is "first".

1. (a) $(a =)\ 6$ ***A1***

$(b =)\ 5.14\ (5.14213...)$ ***A1***

$(c =)\ 3.32\ (3.32050\ldots)$ ***A1***

[3 marks]

(b) attempt to use the trapezoidal rule ***(M1)***

(area =) $\frac{1}{2}(25)\big((1+1)+2(6+5.14213...+3.32050\ldots)\big)$ ***(A1)***

(area =) $387\,(\text{cm}^2)\ (386.566\ldots)$ ***A1***

[3 marks]

(c) (i) $\int_0^{100}\left(2\sqrt{x}-\frac{x}{5}+1\right)\text{d}x$ ***A1A1***

Note: Award ***A1*** for correct function seen within the integral and ***A1*** for correct limits in the correct location and the inclusion of the $\text{d}x$.

(ii) $433.3\,(\text{cm}^2)$ ***A2***

[4 marks]

(d) attempt to substitute **their** area values into the percentage error formula ***(M1)***

$$\left|\frac{386.566\ldots-433.3}{433.3}\right|\times 100$$

$10.8\ (\%)\ (10.7855...)$ ***A1***

Note: Accept an answer of 10.7 from use of 387 from part (b).

[2 marks]

[Total 12 marks]

2. (a) 25 (m) ***A1***

[1 mark]

(b) (i) recognition of need to use Pythagoras theorem ***(M1)***

$BF^2 = 20^2 + 25^2$

$(BF =)\ 32.0\ \ (32.0156\ldots,\ \sqrt{1025},\ 5\sqrt{41})$ (m) ***A1***

(ii) correct use of trig ratio for $B\hat{F}M$ ***(M1)***

$(B\hat{F}M =)\ \tan^{-1}\left(\frac{25}{20}\right)$ or equivalent

$(B\hat{F}M =)\ 51.3\ (51.3401...)$ ***A1***

Note: Accept an answer of 51.4 from use of 3sf answer to part (b)(i) and then either cosine rule or inverse sine.

[4 marks]

(c) attempt to use arc length formula ***(M1)***

(arc length =) $\frac{2\times 51.3401...}{360}\times 2\pi(32.0156\ldots)$ ***(A1)***

(arc length =) $57.4\ \ (57.3755\ldots)$ (m) ***A1***

Note: Accept 57.3 from use of 3 sf. values of their answers from parts (b)(i) and (b)(ii).

[3 marks]

(d) $34.0156\ldots$ (seen anywhere) ***(A1)***

use of area of sector formula ***(M1)***

recognition of subtracting areas of two sectors ***(M1)***

(area =) $\frac{102.680\ldots}{360}\times\pi\left((34.0156\ldots)^2-(32.0156\ldots)^2\right)$

(area =) $118\ (m^2)\ \ (118.335\ldots)$ ***A1***

[4 marks]

(e) multiplying their area from part (d) by 0.12 or 12 ***(M1)***
$0.12\ (\text{m})$ seen **OR** $1183350\ (\text{cm}^2)$ seen ***(A1)***
$118.335\ldots \times 0.12$ **OR** 1183350×12
$14.2\ \ (14.2002\ldots)\ \text{m}^3$ **OR** $14200000\ (14200236)\ \text{cm}^3$ ***A1***

[3 marks]
[Total 15 marks]

3. (a) (i) $150\ (\text{cm})$ ***A1***

(ii) attempt to substitute values in the mean formula with at least one mid-interval value multiplied by a corresponding frequency ***(M1)***

(mean $=$) $176\ (176.3)\ (\text{cm})$ ***A1***

[3 marks]

(b) 183 **OR** 168 seen ***(A1)***

Note: These values may be seen in the working for part (c).

$(\text{IQR} = 183 - 168 =)\ 15\ (\text{cm})$ ***A1***

[2 marks]

(c) (upper bound =) $183 + 1.5 \times 15$ **OR** 205.5 seen ***A1***
$205.5 > 204$ **OR** $204 - 183 < 22.5$ **OR** $204 - 22.5 < 183$ ***R1***
Laszlo's height is not an outlier ***A1***

Note: Do not award ***R0A1***.

[3 marks]

(d) H_0 : The heights of the students can be modelled by $\text{N}(176, 13.5^2)$
H_1 : The heights of the students cannot be modelled by $\text{N}(176, 13.5^2)$ ***A1A1***

Note: Award ***A1*** for each correct hypothesis that includes a reference to normal distribution with a mean of 176 and a standard deviation of 13.5 (or variance of 13.5^2). "Correlation", "independence", "association", and "relationship" are incorrect.

Award at most ***A0A1*** for correctly worded hypotheses that include a reference to a normal distribution but omit the distribution's parameters in one or both hypotheses.
Award ***A0A1*** for correct hypotheses that are reversed.

[2 marks]

(e) (i) $h \sim \mathrm{N}(176, 13.5^2)$

attempt to find normal probability in either correct range ***(M1)***
$\mathrm{P}(170 \leq h < 180)$ **OR** $\mathrm{P}(h \geq 190)$

recognition of multiplying either of their probabilities by 200 ***(M1)***
$0.288137... \times 200$ **OR** $0.149859... \times 200$

$a = 57.6$ $(57.6274...)$, $b = 30.0$ $(29.9718...)$ ***A1A1***

(ii) $\mathrm{df} = 4$ ***(A1)***

$(p =)$ 0.0166 $(= 0.0166282\ldots)$ ***A1***

comparing their p-value to 0.05 ***R1***
$0.0166 < 0.05$

Note: Accept p value of 0.0165 $(= 0.0164693...)$ from using a and b to 3 sf.

(Reject H_0, There is sufficient evidence to say that) the data has not been drawn from the $(\mathrm{N}(176, 13.5^2))$ distribution. ***A1***

Note: Do not award ***R0A1***.

The conclusion to part (e)(ii) **MUST** follow through from their hypotheses seen in part (d); if hypotheses are incorrect/reversed etc., the answer to part (e)(ii) must reflect this in order for the ***A1*** to be credited.

[8 marks]
[Total 18 marks]

4. (a) (i) attempt to find 15% or 85% of 285000 ***(M1)***

285000×0.85

$242\,250$ (USD) ***A1***

Note: Do not award ***A1*** if answer is not given exact.

(ii) $N = 360$

$I\% = 4$

$PV = (\pm)\,242\,250$

$FV = 0$

$P/Y = 12$

$C/Y = 12$ ***(M1)(A1)***

Note: Award ***M1*** for an attempt to use a financial app in their technology with at least two entries seen, award ***A1*** for all entries correct.

$(PMT =)\ 1156.54$ (USD) ***A1***

Note: Do not award final ***A1*** if answer is not given to 2 dp.

[5 marks]

(b) 1156.54×360 ***(M1)***

416354 (USD) ***A1***

Note: Do not award ***A1*** if answer is not given to the nearest dollar, unless already penalized in part (a)(ii).

[2 marks]

(c) $I\% = 4$

$PV = (\pm)\,242\,250$

$PMT = (\mp)1300$

$FV = 0$

$P/Y = 12$

$C/Y = 12$ ***(A1)***

Note: Award ***A1*** for $PMT = (\mp)1300$

$(N =)\ 292$ ***A1***

[2 marks]

(d) **METHOD 1**

$N = 291$
$I\% = 4$
$PV = (\pm)\,242\,250$
$PMT = (\mp)1300$
$P/Y = 12$
$F/Y = 12$ ***(A1)***

Note: Award ***A1*** for $N = 291$ seen.

$(FV =)\ 871.91\ (871.908\ldots)$ ***A1***

valid attempt to find interest in final month (e.g. $N = 1$ **OR** $PV = 871.91$) ***(M1)***

$N = 1$
$I\% = 4$
$PV = 871.91\ (871.908\ldots)$
$FV = 0$
$P/Y = 12$
$F/Y = 12$

$(PMT =)\ 874.82$ (USD) ***A1***

Note: Do not award ***A1*** if answer is not given correct to 2dp, unless already penalized previously.

METHOD 2

$N = 292$
$I\% = 4$
$PV = (\pm)\,242\,250$
$PMT = (\mp)1300$
$P/Y = 12$
$F/Y = 12$ ***(A1)***

Note: Award ***A1*** for $N = 292$ seen.

$(FV =)\ 425.185\ldots$ ***A1***

$1300 - 425.185...$ ***(A1)***
$(PMT =)\ 874.82$ (USD) ***A1***

Note: Accept 874.81. Do not award ***A1*** if answer is not given correct to 2dp, unless already penalized previously.

[4 marks]

(e) $291 \times 1300 + 874.82$ ***(M1)***

379174.82

attempt to find difference between their value and their part (b) ***(M1)***
$(416354 - 379174.82)$

37179 (USD) ***A1***

Note: Accept 37180 (USD) from using the 2 dp. answer from part (b). Do not penalize for not rounding to nearest dollar if this has already been penalized in part (b).

[3 marks]
[Total 16 marks]

5. (a) attempt to substitute 16 into the given formula ***(M1)***

$n = 20000 - 1000(16)$
$n = 4000$ ***A1***

[2 marks]

(b) multiplying their answer to part (a) by 16 ***(M1)***
(average monthly income =) 16×4000
64000 (EUR) ***A1***

[2 marks]

(c) $R(x) = x(20000 - 1000x)$ **OR** $R(x) = 20000x - 1000x^2$ ***A1***

[1 mark]

(d) **EITHER**
attempt to find total costs (both fixed and variable) **AND** subtract from their 64000 ***(M1)***
$64000 - (10000 + 10 \times 4000)$ ***(A1)***

OR
attempt to find total profit from mugs **AND** subtract fixed costs ***(M1)***
$(16 - 10) \times 4000 - 10000$ ***(A1)***

THEN
$= 14000$ (EUR) ***A1***

[3 marks]

(e) **METHOD 1**
attempt to subtract total costs in terms of x from their $R(x)$ ***(M1)***
$\left(P(x) = \right)\left(20000x - 1000x^2\right) - \left(10000 + 10(20000 - 1000x)\right)$

correct intermediate step leading to given answer ***A1***
(e.g. correct expansion of $10(20000 - 1000x)$)
$P(x) = -1000x^2 + 30000x - 210000$ ***AG***

Note: Do not award the ***A1*** mark if the ***AG*** line is not stated.

METHOD 2
attempt to express profit per mug, and then subtract fixed monthly costs ***(M1)***
$\left(P(x) = \right)(x - 10)(20000 - 1000x) - 10000$
correct expansion leading to the given answer ***A1***
$P(x) = -1000x^2 + 30000x - 210000$ ***AG***

Note: Do not award the ***A1*** mark if the ***AG*** line is not stated.

[2 marks]

(f) (i) $(P'(x) =) \ -2000x + 30000$ ***A1A1***

Note: Award at most ***A1A0*** if additional terms are seen.

(ii) **METHOD 1**

$P'(x) = 0$ **OR** sketch of $P(x)$ **OR** use of $x = -\frac{b}{2a}$

$x = 15$ is the maximum, not 16 ***A1R1***

hence salesman's price is not the optimum ***AG***

Note: Award ***A1*** for $x = 15$, and ***R1*** for either comparing it to 16 **OR** making a statement that is some version of the ***AG*** line. It is possible to award ***A1R0***.

METHOD 2

$P'(16) = -2000 \neq 0$ ***A1R1***

hence salesman's price is not the optimum ***AG***

Note: Award ***A1*** for finding an appropriate value, and ***R1*** for comparing it to zero. It is possible to award ***A1R0***. To award the ***R1*** a statement that is some version of the ***AG*** line must also be given.

METHOD 3

finding $P(x)$ for any value from $14 < x < 16$ ***A1***

comparing this value to their part (d) ***R1***

hence salesman's price is not the optimum ***AG***

Note: It is possible to award ***A1R0***. To award the ***R1*** a statement that is some version of the ***AG*** line must also be given.

[4 marks]

(g) substituting the expression for n into cost function, $C(n)$. ***(M1)***

(cost =) $10 - 0.0001(20000 - 1000x) = 8 + 0.1x$

substituting $C(x)$ into the total cost expression and subtracting for $R(x)$ ***(M1)***

(New $P(x)$ =) $\left(20000x - 1000x^2\right) - \left(10000 + (8 + 0.1x)(20000 - 1000x)\right)$ ***A1***

((New $P(x)$ =) $-900x^2 + 26000x - 170000$)

[3 marks]

(h) 14.4 (EUR) $(14.4444\ldots, \frac{130}{9})$ ***A2***

[2 marks]

[Total 19 marks]

8823 – 7210M

Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

November 2023

Mathematics: applications and interpretation

Standard level

Paper 2

15 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

R Marks awarded for clear **Reasoning**.

AG Answer given in the question and so no marks are awarded.

FT Follow through. The practice of awarding marks, despite candidate errors in previous parts, for their correct methods/answers using incorrect results.

Using the markscheme

1 General

Award marks using the annotations as noted in the markscheme eg ***M1***, ***A2***.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***A3***, ***M2*** *etc*., do **not** split the marks, unless there is a note.
- The response to a "show that" question does not need to restate the ***AG*** line, unless a **Note** makes this explicit in the markscheme.
- Once a correct answer to a question or part question is seen, ignore further working even if this working is incorrect and/or suggests a misunderstanding of the question. This will encourage a uniform approach to marking, with less examiner discretion. Although some candidates may be advantaged for that specific question item, it is likely that these candidates will lose marks elsewhere too.
- An exception to the previous rule is when an incorrect answer from further working is used **in a subsequent part**. For example, when a correct exact value is followed by an incorrect decimal approximation in the first part and this approximation is then used in the second part. In this situation, award ***FT*** marks as appropriate but do not award the final ***A1*** in the first part. Examples:

	Correct answer seen	Further working seen	Any FT issues?	Action
1.	$8\sqrt{2}$	5.65685... *(incorrect decimal value)*	No. Last part in question.	Award ***A1*** for the final mark *(condone the incorrect further working)*
2.	$\frac{35}{72}$	0.468111… (incorrect decimal value)	Yes. Value is used in subsequent parts.	Award ***A0*** for the final mark *(and full **FT** is available in subsequent parts)*

3 Implied marks

Implied marks appear in **brackets e.g. *(M1)***,and can only be awarded if **correct** work is seen or implied by subsequent working/answer.

4 Follow through marks (only applied after an error is made)

Follow through (***FT***) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) (e.g. incorrect value from part (a) used in part (d) or incorrect value from part (c)(i) used in part (c)(ii)). Usually, to award ***FT*** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if all the marks awarded in a subsequent part are for the answer or are implied, then ***FT*** marks should be awarded for *their* correct answer, even when working is not present.

For example: following an incorrect answer to part (a) that is used in subsequent parts, where the markscheme for the subsequent part is ***(M1)A1***, it is possible to award full marks for *their* correct answer, **without working being seen.** For longer questions where all but the answer marks are implied this rule applies but may be overwritten by a **Note** in the Markscheme.

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks, by reflecting on what each mark is for and how that maps to the simplified version.
- If the error leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word “their” in a description, to indicate that candidates may be using an incorrect value.
- If the candidate’s answer to the initial question clearly contradicts information given in the question, it is not appropriate to award any ***FT*** marks in the subsequent parts. This includes when candidates fail to complete a “show that” question correctly, and then in subsequent parts use their incorrect answer rather than the given value.
- Exceptions to these ***FT*** rules will be explicitly noted on the markscheme.
- If a candidate makes an error in one part but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the command term was “Hence”.

5 Mis-read

If a candidate incorrectly copies values or information from the question, this is a mis-read (***MR***). A candidate should be penalized only once for a particular misread. Use the ***MR*** stamp to indicate that this has been a misread and do not award the first mark, even if this is an ***M*** mark, but award all others as appropriate.

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.
- If a candidate uses a correct answer, to a "show that" question, to a higher degree of accuracy than given in the question, this is NOT a misread and full marks may be scored in the subsequent part.
- ***MR*** can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If the command term is 'Hence' and not 'Hence or otherwise' then alternative methods are not permitted unless covered by a note in the mark scheme.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**.

7 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation** for example 1.9 and $1{,}9$ or 1000 and $1{,}000$ and 1.000.
- Do not accept final answers written using calculator notation. However, ***M*** marks and intermediate ***A*** marks can be scored, when presented using calculator notation, provided the evidence clearly reflects the demand of the mark.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, some **equivalent** answers will generally appear in brackets. Not all equivalent notations/answers/methods will be presented in the markscheme and examiners are asked to apply appropriate discretion to judge if the candidate work is equivalent.

8 Format and accuracy of answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. If the level of accuracy is not stated in the question, the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures.*

Where values are used in subsequent parts, the markscheme will generally use the exact value, however candidates may also use the correct answer to a "correct" level of accuracy (e.g 3 sf) in subsequent parts. The markscheme will often explicitly include the subsequent values that come "*from the use of 3 sf values*".

Simplification of final answers: Candidates are advised to give final answers using good mathematical form. In general, for an ***A*** mark to be awarded, arithmetic should be completed, and any values that lead to integers should be simplified; for example, $\sqrt{\frac{25}{4}}$ should be written as $\frac{5}{2}$. An exception to this is simplifying fractions, where lowest form is not required (although the numerator and the denominator must be integers); for example, $\frac{10}{4}$ may be left in this form or written as $\frac{5}{2}$. However, $\frac{10}{5}$ should be written as 2, as it simplifies to an integer.

Algebraic expressions should be simplified by completing any operations such as addition and multiplication, e.g. $4e^{2x} \times e^{3x}$ should be simplified to $4e^{5x}$, and $4e^{2x} \times e^{3x} - e^{4x} \times e^{x}$ should be simplified to $3e^{5x}$. Unless specified in the question, expressions do not need to be factorized, nor do factorized expressions need to be expanded, so $x(x+1)$ and $x^2 + x$ are both acceptable.

Please note: intermediate ***A*** marks do NOT need to be simplified.

9 Calculators

A GDC is required for this paper, but If you see work that suggests a candidate has used any calculator not approved for IB DP examinations (eg CAS enabled devices), please follow the procedures for malpractice.

10. Presentation of candidate work

Crossed out work: If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work unless an explicit note from the candidate indicates that they would like the work to be marked.

More than one solution: Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise. If the layout of the responses makes it difficult to judge, examiners should apply appropriate discretion to judge which is "first".

1. (a) $(a =)\ 9$ ***A1***

$(b =)\ 7.63\ (7.62741...)$ ***A1***

$(c =)\ 4.71\ (4.71281...)$ ***A1***

[3 marks]

(b) attempt to use the trapezoidal rule ***(M1)***

(area =) $\frac{1}{2}(16)\big((1+1)+2(9+7.62741...+4.71281\ldots)\big)$ ***(A1)***

(area =) $357\,(\text{mm}^2)\ (357.443\ldots)$ ***A1***

[3 marks]

(c) (i) $\int_0^{64}\left(4\sqrt{x}-\frac{x}{2}+1\right)\mathrm{d}x$ ***A1A1***

Note: Award ***A1*** for correct function seen within the integral and ***A1*** for correct limits in the correct location and the inclusion of the $\mathrm{d}x$.

(ii) $405.3\,(\text{mm}^2)$ ***A2***

[4 marks]

(d) attempt to substitute **their** area values into the percentage error formula ***(M1)***

$$\left|\frac{357.443\ldots-405.3}{405.3}\right|\times 100$$

$11.8\,(\%)\ (11.8076...)$ ***A1***

Note: Accept an answer of 11.9 from use of 357 from part (b).

[2 marks]

[Total 12 marks]

2. (a) 45 (m) ***A1***

[1 mark]

(b) (i) recognition of need to use Pythagoras theorem ***(M1)***

$BE^2 = 32^2 + 45^2$

$(BE =)\ 55.2\ (55.2177\ldots, \sqrt{3049})$ (m) ***A1***

(ii) correct use of trig ratio for $B\hat{E}M$ ***(M1)***

$(B\hat{E}M =)\ \tan^{-1}\left(\frac{45}{32}\right)$ or equivalent

$(B\hat{E}M =)\ 54.6\ (54.5829...)$ ***A1***

[4 marks]

(c) attempt to use arc length formula ***(M1)***

(arc length =) $\frac{2\times 54.5829...}{360}\times 2\pi(55.2177\ldots)$ ***(A1)***

(arc length =) $105\ (105.206\ldots)$ (m) ***A1***

[3 marks]

(d) $59.2177\ldots$ (seen anywhere) ***(A1)***

use of area of sector formula ***(M1)***

recognition of subtracting areas of two sectors ***(M1)***

(area =) $\frac{109.165\ldots}{360}\times\pi\left((59.2177\ldots)^2-(55.2177\ldots)^2\right)$

(area =) $436\ (m^2)\ (436.068\ldots)$ ***A1***

[4 marks]

(e) multiplying their area from part (d) by 0.15 or 15 ***(M1)***
0.15 (m) seen **OR** 4360688 (cm^2) seen ***(A1)***
$436.068\ldots \times 0.15$ **OR** 4360688×15
65.4 $(65.4103\ldots)$ m^3 **OR** 65400000 (65410332) cm^3 ***A1***

[3 marks]
[Total 15 marks]

3. (a) (i) 75 (minutes) ***A1***

(ii) attempt to substitute values in the mean formula with at least one mid-interval value multiplied by a corresponding frequency ***(M1)***

(mean $=$) 88.2 (88.15) (minutes) ***A1***

[3 marks]

(b) 91.5 **OR** 84 seen ***(A1)***

Note: These values may be seen in the working for part (c).

$(\text{IQR} = 91.5 - 84 =)$ 7.5 (minutes) ***A1***

[2 marks]

(c) (upper bound =) $91.5 + 1.5 \times 7.5$ **OR** 102.75 seen ***A1***
$102.75 > 100$ **OR** $100 - 91.5 < 11.25$ **OR** $100 - 11.25 < 91.5$) ***R1***
Star Feud is not an outlier ***A1***

Note: Do not award ***R0A1***.

[3 marks]

(d) H_0 : The running times of the movies can be modelled by $N(88, 6.75^2)$
H_1 : The running times of the movies cannot be modelled by $N(88, 6.75^2)$ ***A1A1***

Note: Award ***A1*** for each correct hypothesis that includes a reference to normal distribution with a mean of 88 and a standard deviation of 6.75 (or variance of 6.75^2). "Correlation", "independence", "association", and "relationship" are incorrect.

Award at most ***A0A1*** for correctly worded hypotheses that include a reference to a normal distribution but omit the distribution's parameters in one or both hypotheses. Award ***A0A1*** for correct hypotheses that are reversed.

[2 marks]

(e) (i) $T \sim \mathrm{N}(88,\ 6.75^2)$

attempt to find normal probability in either correct range $\mathrm{P}(85 \leq T < 90)$ **OR** $\mathrm{P}(T \geq 95)$ ***(M1)***

recognition of multiplying either of their probabilities by 200 $0.288137... \times 200$ **OR** $0.149859... \times 200$ ***(M1)***

$a = 57.6\ \ (57.6274...)$, $\ b = 30.0\ \ (29.9718...)$ ***A1A1***

(ii) $\mathrm{df} = 4$ ***(A1)***

$(p =)\ 0.0166\ (= 0.0166282\ldots)$ ***A1***

comparing their p-value to 0.05 $0.0166 < 0.05$ ***R1***

Note: Accept p value of $0.0165\ (= 0.0164693...)$ from using a and b to 3 sf.

(Reject H_0, There is sufficient evidence to say that) the data has not been drawn from the ($\mathrm{N}(88,\ 6.75^2)$) distribution. ***A1***

Note: Do not award ***R0A1***.

The conclusion to part (e)(ii) **MUST** follow through from their hypotheses seen in part (d); if hypotheses are incorrect/reversed etc., the answer to part (e)(ii) must reflect this in order for the ***A1*** to be credited.

[8 marks]

[Total 18 marks]

4. (a) (i) attempt to find 25% or 75% of 285000 ***(M1)***

285000×0.75

213750 (ZAR) ***A1***

Note: Do not award ***A1*** if answer is not given exact.

(ii) $N = 60$

$I\% = 4.5$

$PV = (\pm) 213750$

$FV = 0$

$P/Y = 12$

$C/Y = 12$ ***(M1)(A1)***

Note: Award ***M1*** for an attempt to use a financial app in their technology with at least two entries seen, award ***A1*** for all entries correct.

$(PMT =)\ 3984.95$ (ZAR) ***A1***

Note: Do not award final ***A1*** if answer is not given to 2 dp.

[5 marks]

(b) 3984.95×60 ***(M1)***

239097 (ZAR) ***A1***

Note: Do not award ***A1*** if answer is not given to the nearest rand, unless already penalized in part (a)(ii).

[2 marks]

(c) $I\% = 4.5$

$PV = (\pm) 213750$

$PMT = (\mp) 4600$

$FV = 0$

$P/Y = 12$

$C/Y = 12$ ***(A1)***

Note: Award ***A1*** for $PMT = (\mp) 4\,600$.

$(N =)\ 52$ ***A1***

[2 marks]

(d) **METHOD 1**

$N = 51$
$I\% = 4.5$
$PV = (\pm)213750$
$PMT = (\mp)4600$
$P/Y = 12$
$F/Y = 12$ ***(A1)***

Note: Award ***A1*** for $N = 51$ seen.

$(FV =)\ 704.156...$ ***A1***

valid attempt to find interest in final month (e.g. $N = 1$ **OR** $PV = 704.156...$) ***(M1)***

$N = 1$
$I\% = 4.5$
$PV = 704.16\ \ (704.156...)$
$FV = 0$
$P/Y = 12$
$F/Y = 12$

$(PMT =)\ \ 706.80$ (ZAR) ***A1***

Note: Do not award ***A1*** if answer is not given correct to 2dp, unless already penalized previously.

[4 marks]

METHOD 2

$N = 52$
$I\% = 4.5$
$PV = (\pm)213750$
$PMT = (\mp)4600$
$P/Y = 12$
$F/Y = 12$ ***(A1)***

Note: Award ***A1*** for $N = 52$ seen.

$(FV =)\ 3893.20...$ ***A1***

$4600 - 3893.20...$ ***(A1)***
$(PMT =)\ \ 706.80$ (ZAR) ***A1***

Note: Do not award ***A1*** if answer is not given correct to 2dp, unless already penalized previously.

[4 marks]

(e) $51 \times 4600 + 706.80$ ***(M1)***

235306.80

attempt to find difference between their value and their part (b) ***(M1)***
$(239097 - 235306.80)$

3790 (ZAR) ***A1***

Note: Do not penalize for not rounding to nearest rand if this has already been penalized in part (b).

[3 marks]
[Total 16 marks]

5. (a) attempt to substitute 16 into the given formula ***(M1)***

$n = 20000 - 1000(16)$
$n = 4000$ ***A1***
[2 marks]

(b) multiplying their answer to part (a) by 16 ***(M1)***
(average monthly income =) 16×4000
64000 (EUR) ***A1***
[2 marks]

(c) $R(x) = x(20000 - 1000x)$ **OR** $R(x) = 20000x - 1000x^2$ ***A1***
[1 mark]

(d) **EITHER**

attempt to find total costs (both fixed and variable) **AND** subtract from their 64000 ***(M1)***
$64000 - (10000 + 10 \times 4000)$ ***(A1)***

OR
attempt to find total profit from cases **AND** subtract fixed costs ***(M1)***
$(16 - 10) \times 4000 - 10000$ ***(A1)***

THEN
$= 14000$ (EUR) ***A1***
[3 marks]

(e) **METHOD 1**

attempt to subtract total costs in terms of x from their $R(x)$ ***(M1)***

$(P(x) =)(20000x - 1000x^2) - (10000 + 10(20000 - 1000x))$

correct intermediate step leading to given answer ***A1***
(e.g. correct expansion of $10(20000 - 1000x)$)

$P(x) = -1000x^2 + 30000x - 210000$ ***AG***

Note: Do not award the ***A1*** mark if the ***AG*** line is not stated.

METHOD 2

attempt to express profit per case, and then subtract fixed monthly costs ***(M1)***

$(P(x) =)(x - 10)(20000 - 1000x) - 10000$

correct expansion leading to the given answer ***A1***

$P(x) = -1000x^2 + 30000x - 210000$ ***AG***

Note: Do not award the ***A1*** mark if the ***AG*** line is not stated.

[2 marks]

(f) (i) $(P'(x) =) \ -2000x + 30000$ ***A1A1***

Note: Award at most ***A1A0*** if additional terms are seen.

(ii) **METHOD 1**

$P'(x) = 0$ **OR** sketch of $P(x)$ **OR** use of $x = -\frac{b}{2a}$

$x = 15$ is the maximum, not 16 ***A1R1***

hence salesman's price is not the optimum ***AG***

Note: Award ***A1*** for $x = 15$, and ***R1*** for comparing it to 16 **OR** making a statement that is some version of the ***AG*** line. It is possible to award ***A1R0***.

METHOD 2

$P'(16) = -2000 \neq 0$ ***A1R1***

hence salesman's price is not the optimum ***AG***

Note: Award ***A1*** for finding an appropriate value, and ***R1*** for comparing it to zero. It is possible to award ***A1R0***. To award the ***R1*** a statement that is some version of the ***AG*** line must also be given.

METHOD 3

finding $P(x)$ for any value from $14 < x < 16$ ***A1***

comparing this value to their part (d) ***R1***

hence salesman's price is not the optimum ***AG***

Note: It is possible to award ***A1R0***. To award the ***R1*** a statement that is some version of the ***AG*** line must also be given.

[4 marks]

(g) substituting the expression for n into cost function, $C(n)$. ***(M1)***

(cost =) $10 - 0.0001(20000 - 1000x) = 8 + 0.1x$

substituting $C(x)$ into the total cost expression and subtracting for $R(x)$ ***(M1)***

(New $P(x)$ =) $\left(20000x - 1000x^2\right) - \left(10000 + (8 + 0.1x)(20000 - 1000x)\right)$ ***A1***

((New $P(x)$ =) $-900x^2 + 26000x - 170000$)

[3 marks]

(h) 14.4 (EUR) $(14.4444\ldots, \frac{130}{9})$ ***A2***

[2 marks]
[Total 19 marks]

Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

May 2023

Mathematics: applications and interpretation

Standard level

Paper 2

16 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

R Marks awarded for clear **Reasoning**.

AG Answer given in the question and so no marks are awarded.

FT Follow through. The practice of awarding marks, despite candidate errors in previous parts, for their correct methods/answers using incorrect results.

Using the markscheme

1 General

Award marks using the annotations as noted in the markscheme eg ***M1***, ***A2***.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***A3***, ***M2*** *etc.*, do **not** split the marks, unless there is a note.
- The response to a "show that" question does not need to restate the ***AG*** line, unless a **Note** makes this explicit in the markscheme.
- Once a correct answer to a question or part question is seen, ignore further working even if this working is incorrect and/or suggests a misunderstanding of the question. This will encourage a uniform approach to marking, with less examiner discretion. Although some candidates may be advantaged for that specific question item, it is likely that these candidates will lose marks elsewhere too.
- An exception to the previous rule is when an incorrect answer from further working is used **in a subsequent part**. For example, when a correct exact value is followed by an incorrect decimal approximation in the first part and this approximation is then used in the second part. In this situation, award ***FT*** marks as appropriate but do not award the final ***A1*** in the first part. Examples:

	Correct answer seen	Further working seen	Any FT issues?	Action
1.	$8\sqrt{2}$	5.65685... *(incorrect decimal value)*	No. Last part in question.	Award ***A1*** for the final mark *(condone the incorrect further working)*
2.	$\frac{35}{72}$	0.468111… (incorrect decimal value)	Yes. Value is used in subsequent parts.	Award ***A0*** for the final mark *(and full **FT** is available in subsequent parts)*

3 Implied marks

Implied marks appear in **brackets e.g.** ***(M1)***,and can only be awarded if **correct** work is seen or implied by subsequent working/answer.

4 Follow through marks (only applied after an error is made)

Follow through (***FT***) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) (e.g. incorrect value from part (a) used in part (d) or incorrect value from part (c)(i) used in part (c)(ii)). Usually, to award ***FT*** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if all the marks awarded in a subsequent part are for the answer or are implied, then ***FT*** marks should be awarded for *their* correct answer, even when working is not present.

For example: following an incorrect answer to part (a) that is used in subsequent parts, where the markscheme for the subsequent part is ***(M1)A1***, it is possible to award full marks for *their* correct answer, **without working being seen.** For longer questions where all but the answer marks are implied this rule applies but may be overwritten by a **Note** in the Markscheme.

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks, by reflecting on what each mark is for and how that maps to the simplified version.
- If the error leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word “their” in a description, to indicate that candidates may be using an incorrect value.
- If the candidate’s answer to the initial question clearly contradicts information given in the question, it is not appropriate to award any ***FT*** marks in the subsequent parts. This includes when candidates fail to complete a “show that” question correctly, and then in subsequent parts use their incorrect answer rather than the given value.
- Exceptions to these ***FT*** rules will be explicitly noted on the markscheme.
- If a candidate makes an error in one part but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the command term was “Hence”.

5 Mis-read

If a candidate incorrectly copies values or information from the question, this is a mis-read (***MR***). A candidate should be penalized only once for a particular misread. Use the ***MR*** stamp to indicate that this has been a misread and do not award the first mark, even if this is an ***M*** mark, but award all others as appropriate.

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.
- If a candidate uses a correct answer, to a "show that" question, to a higher degree of accuracy than given in the question, this is NOT a misread and full marks may be scored in the subsequent part.
- ***MR*** can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If the command term is 'Hence' and not 'Hence or otherwise' then alternative methods are not permitted unless covered by a note in the mark scheme.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**.

7 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation** for example 1.9 and $1{,}9$ or 1000 and $1{,}000$ and 1.000.
- Do not accept final answers written using calculator notation. However, ***M*** marks and intermediate ***A*** marks can be scored, when presented using calculator notation, provided the evidence clearly reflects the demand of the mark.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, some **equivalent** answers will generally appear in brackets. Not all equivalent notations/answers/methods will be presented in the markscheme and examiners are asked to apply appropriate discretion to judge if the candidate work is equivalent.

8 Format and accuracy of answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. If the level of accuracy is not stated in the question, the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures*.

Where values are used in subsequent parts, the markscheme will generally use the exact value, however candidates may also use the correct answer to a "correct" level of accuracy (e.g 3 sf) in subsequent parts. The markscheme will often explicitly include the subsequent values that come "*from the use of 3 sf values*".

Simplification of final answers: Candidates are advised to give final answers using good mathematical form. In general, for an ***A*** mark to be awarded, arithmetic should be completed, and any values that lead to integers should be simplified; for example, $\sqrt{\frac{25}{4}}$ should be written as $\frac{5}{2}$. An exception to this is simplifying fractions, where lowest form is not required (although the numerator and the denominator must be integers); for example, $\frac{10}{4}$ may be left in this form or written as $\frac{5}{2}$. However, $\frac{10}{5}$ should be written as 2, as it simplifies to an integer.

Algebraic expressions should be simplified by completing any operations such as addition and multiplication, e.g. $4e^{2x} \times e^{3x}$ should be simplified to $4e^{5x}$, and $4e^{2x} \times e^{3x} - e^{4x} \times e^{x}$ should be simplified to $3e^{5x}$. Unless specified in the question, expressions do not need to be factorized, nor do factorized expressions need to be expanded, so $x(x+1)$ and $x^2 + x$ are both acceptable.

Please note: intermediate ***A*** marks do NOT need to be simplified.

9 Calculators

A GDC is required for this paper, but If you see work that suggests a candidate has used any calculator not approved for IB DP examinations (eg CAS enabled devices), please follow the procedures for malpractice.

10. Presentation of candidate work

Crossed out work: If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work unless an explicit note from the candidate indicates that they would like the work to be marked.

More than one solution: Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise. If the layout of the responses makes it difficult to judge, examiners should apply appropriate discretion to judge which is "first".

1. (a) attempt to use sine rule ***(M1)***

$\frac{\sin \hat{ABO}}{25.9} = \frac{\sin 10^\circ}{6.36}$ ***(A1)***

45.0° $(45.0036...^\circ)$ ***A1***

Note: Accept an answer of 45° for full marks.

[3 marks]

(b) $(\hat{OAB} =)\ 124.996...^\circ$ ***(A1)***

attempt to use area of triangle formula ***(M1)***

$\frac{1}{2} \times 25.9 \times 6.36 \times \sin(124.996...^\circ)$ ***(A1)***

$67.5\ \text{m}^2$ $(67.4700...\ \text{m}^2)$ ***A1***

Note: Units are required. The final ***A1*** is only awarded if the correct units are seen in their answer; hence award ***(A1)(M1)(A1)A0*** for an unsupported answer of 67.5. Accept $67.4670\ldots\text{m}^2$ from use of 3 sf values.

Full follow through marks can be awarded for this part even if their $\hat{OAB}$ is not obtuse, provided that all working is shown.

[4 marks]

(c) attempt to use cosine rule ***(M1)***

$(BK =)\sqrt{12^2 + 6.36^2 - 2 \times 12 \times 6.36 \times \cos 45^\circ}$ ***(A1)***

8.75 (m) $(8.74738\ldots(\text{m}))$ ***A1***

Note: Award ***(M1)(A1)(A0)*** for radian answer of 10.2 (m) $(10.2109\ldots(\text{m}))$ with or without working shown.

[3 marks]

continued…

Question 1 continued

(d) **METHOD 1**

attempt to use sine rule with measurements from triangle OKX ***(M1)***

$\frac{\mathrm{OX}}{\sin 51.1^\circ} = \frac{22.2}{\sin 53.8^\circ}$ ***(A1)***

$(\mathrm{OX} =)\ 21.4\ (\mathrm{m})\ (21.4099\ldots)(\mathrm{m})$ ***A1***

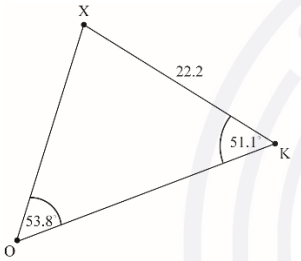
$(21.4\ (\mathrm{m}) < 22.2\ (\mathrm{m}))$

Odette is closer to the football / Khemil is further from the football ***A1***

Note: For the final ***A1*** to be given, $21.4\ (21.4099\ldots)$ must be seen. Follow through within question part for final ***A1*** for a consistent comparison with their OX.

METHOD 2

sketch of triangle OXK with vertices, angles and lengths ***(A1)***



51.1° is smallest angle in triangle OXK ***R1***

opposite side (OX) is smallest length ***R1***

therefore Odette is closest ***A1***

[4 marks]

(e) attempt to use length of arc formula ***(M1)***

$\frac{135}{360} \times 2\pi \times 12$ ***(A1)***

$28.3\,(\mathrm{m})\ (9\pi,\ 28.2743...)\ (\mathrm{m})$ ***A1***

[3 marks]

Total [17 marks]

2. (a) recognizing arithmetic sequence (may be seen in part (b)) ***(M1)***

$(u_{12} =)\ 10+(12-1)\times 6$ ***(A1)***

76 ***A1***

[3 marks]

(b) correct substitution into either arithmetic series formula ***(A1)***

$(S_{15} =)\ \frac{15}{2}\left(2\times 10+(15-1)\times 6\right)$ **OR** $(S_{15} =)\ \frac{15}{2}(10+94)$

780 ***A1***

[2 marks]

(c) attempt to use either arithmetic series formula equated to 1000 ***(M1)***

$\frac{15}{2}\left(2\times 10+(15-1)\times x\right)=1000$ **OR** $\frac{15}{2}(10+u_{15})=1000$

$x=8.09523...$ ***(A1)***

$x=9$ ***A1***

Note: Follow through within question part for final ***A1*** for candidates correctly rounding their value of x up to the nearest integer. Award ***(M0)(A0)A0*** for a response of $x=8$ with no working shown.

[3 marks]

(d) recognizing geometric sequence (may be seen in part (e)) ***(M1)***

$17.1\times 0.95^{5-1}$ ***(A1)***

$13.9\ (\text{cm})(13.9280\ldots)$ ***A1***

[3 marks]

(e) correct substitution into geometric series formula ***(A1)***

$\frac{17.1(1-0.95^{16})}{1-0.95}$

$191\ (\text{cm})\ (191.476\ldots(\text{cm}))$ ***A1***

[2 marks]

(f) correct method to find u_0 ***(M1)***

$u_0=17.1\times(0.95)^{0-1}$ **OR** $17.1=0.95x$ **OR** $\frac{17.1}{0.95}$ (seen)

Note: Award ***(M0)A0*** for any attempt to find answer using 0.05 or 1.05.

$18\ (\text{cm})$ ***A1***

[2 marks]

Total [15 marks]

3. (a) (i) 1200 ***A1***

(ii) the initial population of the bacteria ***A1***

[2 marks]

(b) $1200 \times k^3 = 18750$ ***(A1)***

$(k =)\ 2.5$ ***A1***

[2 marks]

(c) $1200 \times 2.5^{1.5}$ ***(A1)***

$4740\ (4743.41\ldots)$ ***A1***

Note: Do not penalize if final answer is not given as an integer. Award ***(A1)A0*** for an answer of $3950\ (3949.14\ldots)$ from use of 1.3 in the exponent, but only if working is shown.

[2 marks]

(d) equating $P(t)$ and $S(t)$ **OR** equating each function to a common variable ***(M1)***

$1200 \times 2.5^t = 5000 \times 1.65^t$; $1200 \times 2.5^t = x$ and $5000 \times 1.65^t = x$

$t = 3.43$ (hours) $(3.43456...)$ ***A1***

[2 marks]

continued…

Question 3 continued

(e) **METHOD 1**

$5000 \times 1.65^t = 19000$ ***(M1)***

$(t =)\ 2.66586...$ **OR** $(t - 2 =)\ 0.66586...$ (seen) ***(A1)***

multiplying by 60 seen to convert to minutes ***(M1)***
$(m = 39.9521\ldots)$

$(m =)\ 40$ (minutes) **OR** 2 hours and 40 minutes ***A1***

METHOD 2

equating an expression for $S(t)$ to 19000 ***(M1)***

expressing t as $2 + \frac{m}{60}$ ***(A1)***

$5000 \times 1.65^{2+\frac{m}{60}} = 19000$

$2 + \frac{m}{60} = 2.66586\ldots$ ***A1***

$(m =)\ 40$ (minutes) **OR** 2 hours and 40 minutes ***A1***

Note: Award ***(M1)(A1)(M1)A0*** for an answer of $39.9521...$ or 39 with or without working.

[4 marks]

(f) **EITHER (find volume of all bacteria)**

multiplying total population of bacteria by the volume of bacterium ***(M1)***

$1 \times 10^{-18} \times \left(5000 \times 1.65^t\right)$

setting expression equal to 2.1×10^{-5} and attempt to solve (e.g. sketch) ***(M1)***

OR (find total number of bacteria to fill container)

attempting to find the total number of bacteria to fill container ***(M1)***

$2.1 \times 10^{-5} = n \times 1 \times 10^{-18}$ **OR** $\frac{2.1 \times 10^{-5}}{1 \times 10^{-18}}$ **OR** 2.1×10^{13}

setting value equal to $S(t)$ and attempt to solve (e.g. sketch) ***(M1)***

Note: If $\frac{2.1 \times 10^{-5}}{1 \times 10^{-18}}$ is seen but candidate has an incorrect total number of bacteria, the second ***(M1)*** can still be awarded for setting their incorrect value equal to $S(t)$ and attempting to solve.

THEN

$t = 44.2$ (hours) $(44.2480...)$ ***A1***

[3 marks]

Total [15 marks]

4. (a) recognition of binomial distribution ***(M1)***
e.g. $X \sim \mathrm{B}(115,\ 0.82)$ **OR** binompdf$(115,\ 0.82,\ 90)$ etc.

$(\mathrm{P}(X = 90) =)\ 0.0535\ (0.0535325...)$ ***A2***

Note: Award ***(M1)A1A0*** for an answer of 0.054 with or without working shown.

[3 marks]

(b) selecting correct region of distribution ***(M1)***
e.g. $\mathrm{P}(X \geq 95)$ **OR** $1 - \mathrm{P}(X \leq 94)$ **OR** $1 -$ binomcdf$(115,\ 0.82,\ 94)$

$0.491\ (0.491036\ldots)$ ***A1***

[2 marks]

(c) substitution in the variance formula for binomial distribution ***(M1)***
$115 \times 0.82 \times 0.18$

$17.0\ \ (16.974)$ ***A1***

Note: Allow 17 for the final answer.

[2 marks]

(d) **METHOD 1**
attempt to write an expression containing n inside the brackets of $P(\ )$ **AND** including 0.3 or 0.7 ***(M1)***

$\mathrm{P}(X \geq n) < 0.3$ **OR** $\mathrm{P}(X \leq n-1) > 0.7$ ***(A1)***
$n = 98$ ***A1***

METHOD 2
using binomcdf in GDC for at least two different values of n greater than 90 ***(M1)***

EITHER
$(\mathrm{P}(X < 97) =)\ 0.696683\ldots$ **AND** $(\mathrm{P}(X < 98) =)\ 0.778249\ldots$ (seen) ***(A1)***

OR
$(\mathrm{P}(X > 97) =) 0.303316\ldots$ **AND** $(\mathrm{P}(X > 98) =) 0.221750\ldots$ (seen) ***(A1)***

THEN
$n = 98$ ***A1***

[3 marks]

continued…

Question 4 continued

(e) (μ_1 : population mean recovery time for new remedy)

(μ_2 : population mean recovery time for old remedy)

$H_0 : \mu_1 = \mu_2 \quad (H_0 : \mu_1 - \mu_2 = 0)$ ***A1***

$H_1 : \mu_1 < \mu_2 \quad (H_1 : \mu_1 - \mu_2 < 0)$ ***A1***

Note: Accept an equivalent statement in words, must include mean and reference to "**population** mean", e.g. "mean for **all** patients on old remedy", for the first ***A1*** to be awarded.

Do not accept an imprecise *"the means are equal".*

Award ***A0A1*** for reversed hypotheses ($H_0 : \mu_1 < \mu_2$, $H_1 : \mu_1 = \mu_2$).

[2 marks]

(f) 0.0620 $(0.0620061\ldots)$ ***A2***

Note: Allow 0.062 as final answer. Award ***A1*** for an answer of 0.06. Award ***A1*** for an answer of $0.0527756\ldots$ from use of unpooled setting.

Follow through from an incorrect alternative hypothesis as long as their p-value matches their alternative hypothesis.

[2 marks]

(g) $0.0620 < 0.1$ ***R1***

(sufficient evidence to) reject H_0 ***A1***

Note: Do not award ***R0A1***. Accept "p-value is less than 0.1" provided an answer was seen in part (f).

[2 marks]

(h) the probability of obtaining results (at least as extreme) as those observed given that the null hypothesis is true ***A1***

[1 mark]

Total [17 marks]

5. (a) (i) $x-3$ ***A1***

(ii) attempt to use 1200 to find width of park in terms of only x ***(M1)***

$\frac{1200}{x}$ (seen) **OR** $1200 = x \times$ park width **OR** $1200 = x \times$ (garden width $+4$)

$\frac{1200}{x} - 4$ ***A1***

(iii) $A = (x-3) \times \left(\frac{1200}{x} - 4 \right)$ ***A1***

$= 1200 - 4x - \frac{3600}{x} + 12$ ***A1***

Note: Award first ***A1FT*** for multiplying *their* garden length and width and second ***A1*** for a simplified (parentheses removed) expression for A that leads to the given answer. The given answer must be shown for the second ***A1*** mark to be awarded

$= 1212 - 4x - \frac{3600}{x}$ ***AG***

[5 marks]

(b) setting $1212 - 4x - \frac{3600}{x} = 800$ (accept a sketch) ***(M1)***

$x = 9.64 \ (9.64011\ldots)$ (m) **OR** $x = 93.4 \ (93.3598\ldots)$ (m) ***A1***

(width $=$) $124 \ (124.479\ldots)$ (m) ***A1***

(width $=$) $12.9 \ (12.8534\ldots)$ (m) ***A1***

Note: To award the final ***A1*** both values of x **and** both values of the width must be seen. Accept 12.8 for second value of width from candidate dividing 1200 by 3 sf value of 93.4.

[4 marks]

(c) $\left(\frac{dA}{dx} = \right) -4 + \frac{3600}{x^2}$ **OR** $-4 + 3600x^{-2}$ ***A1A1A1***

Note: Award ***A1*** for -4, ***A1*** for $+3600$, and ***A1*** for x^{-2} or x^2 in denominator.

[3 marks]

(d) setting *their* $\frac{dA}{dx}$ equal to 0 **OR** sketch of *their* $\frac{dA}{dx}$ with x-intercept highlighted ***M1***

$(x =) \ 30$ (m) ***A1***

Note: To award ***A1FT*** the candidate's value of x must be within the domain given in the problem $(3 < x < 300)$.

[2 marks]

continued…

Question 5 continued

(e) **EITHER**

evidence of using GDC to find maximum of graph of $A = 1212 - 4x - \frac{3600}{x}$ ***(M1)***

OR

substitution of *their* x into A ***(M1)***

OR

dividing 1200 by *their* x to find width of park **and** subtracting 3 from *their* x and 4 from the width to find park dimensions ***(M1)***

Note: For the last two methods, only follow through if $3 <$ their $x < 300$.

THEN

$(A =)\ 972\ (\text{m}^2)$ ***A1***

[2 marks]

Total [16 marks]

Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

May 2023

Mathematics: applications and interpretation

Standard level

Paper 2

21 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

R Marks awarded for clear **Reasoning**.

AG Answer given in the question and so no marks are awarded.

FT Follow through. The practice of awarding marks, despite candidate errors in previous parts, for their correct methods/answers using incorrect results.

Using the markscheme

1 General

Award marks using the annotations as noted in the markscheme *eg* ***M1, A2.***

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***A3***, ***M2*** *etc*., do **not** split the marks, unless there is a note.
- The response to a "show that" question does not need to restate the ***AG*** line, unless a **Note** makes this explicit in the markscheme.
- Once a correct answer to a question or part question is seen, ignore further working even if this working is incorrect and/or suggests a misunderstanding of the question. This will encourage a uniform approach to marking, with less examiner discretion. Although some candidates may be advantaged for that specific question item, it is likely that these candidates will lose marks elsewhere too.
- An exception to the previous rule is when an incorrect answer from further working is used **in a subsequent part**. For example, when a correct exact value is followed by an incorrect decimal approximation in the first part and this approximation is then used in the second part. In this situation, award ***FT*** marks as appropriate but do not award the final ***A1*** in the first part.

Examples:

	Correct answer seen	Further working seen	Any FT issues?	Action
1.	$8\sqrt{2}$	5.65685... *(incorrect decimal value)*	No. Last part in question.	Award ***A1*** for the final mark *(condone the incorrect further working)*
2.	$\frac{35}{72}$	0.468111… (incorrect decimal value)	Yes. Value is used in subsequent parts.	Award ***A0*** for the final mark *(and full **FT** is available in subsequent parts)*

3 Implied marks

Implied marks appear in **brackets e.g. *(M1)***,and can only be awarded if **correct** work is seen or implied by subsequent working/answer.

4 Follow through marks (only applied after an error is made)

Follow through (***FT***) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) (e.g. incorrect value from part (a) used in part (d) or incorrect value from part (c)(i) used in part (c)(ii)). Usually, to award ***FT*** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if all the marks awarded in a subsequent part are for the answer or are implied, then ***FT*** marks should be awarded for *their* correct answer, even when working is not present.

For example: following an incorrect answer to part (a) that is used in subsequent parts, where the markscheme for the subsequent part is ***(M1)A1***, it is possible to award full marks for *their* correct answer, **without working being seen.** For longer questions where all but the answer marks are implied this rule applies but may be overwritten by a **Note** in the Markscheme.

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate.

- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks, by reflecting on what each mark is for and how that maps to the simplified version.

- If the error leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).

- The markscheme may use the word “their” in a description, to indicate that candidates may be using an incorrect value.

- If the candidate’s answer to the initial question clearly contradicts information given in the question, it is not appropriate to award any ***FT*** marks in the subsequent parts. This includes when candidates fail to complete a “show that” question correctly, and then in subsequent parts use their incorrect answer rather than the given value.

- Exceptions to these ***FT*** rules will be explicitly noted on the markscheme.

- If a candidate makes an error in one part but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the command term was "Hence".

5 Mis-read

If a candidate incorrectly copies values or information from the question, this is a mis-read (***MR***). A candidate should be penalized only once for a particular misread. Use the ***MR*** stamp to indicate that this has been a misread and do not award the first mark, even if this is an ***M*** mark, but award all others as appropriate.

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.

- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).

- Miscopying of candidates' own work does **not** constitute a misread, it is an error.

- If a candidate uses a correct answer, to a "show that" question, to a higher degree of accuracy than given in the question, this is NOT a misread and full marks may be scored in the subsequent part.

- ***MR*** can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If the command term is 'Hence' and not 'Hence or otherwise' then alternative methods are not permitted unless covered by a note in the mark scheme.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**.

7 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation** for example 1.9 and 1,9 or 1000 and 1,000 and 1.000.

- Do not accept final answers written using calculator notation. However, ***M*** marks and intermediate ***A*** marks can be scored, when presented using calculator notation, provided the evidence clearly reflects the demand of the mark.

- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.

- In the markscheme, some **equivalent** answers will generally appear in brackets. Not all equivalent notations/answers/methods will be presented in the markscheme and examiners are asked to apply appropriate discretion to judge if the candidate work is equivalent.

8 Format and accuracy of answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. If the level of accuracy is not stated in the question, the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures.*

Where values are used in subsequent parts, the markscheme will generally use the exact value, however candidates may also use the correct answer to 3 sf in subsequent parts. The markscheme will often explicitly include the subsequent values that come "*from the use of 3 sf values*".

Simplification of final answers: Candidates are advised to give final answers using good mathematical form. In general, for an ***A*** mark to be awarded, arithmetic should be completed, and any values that lead to integers should be simplified; for example, $\sqrt{\frac{25}{4}}$ should be written as $\frac{5}{2}$. An exception to this is simplifying fractions, where lowest form is not required (although the numerator and the denominator must be integers); for example, $\frac{10}{4}$ may be left in this form or written as $\frac{5}{2}$. However, $\frac{10}{5}$ should be written as 2, as it simplifies to an integer.

Algebraic expressions should be simplified by completing any operations such as addition and multiplication, e.g. $4e^{2x} \times e^{3x}$ should be simplified to $4e^{5x}$, and $4e^{2x} \times e^{3x} - e^{4x} \times e^{x}$ should be simplified to $3e^{5x}$. Unless specified in the question, expressions do not need to be factorized, nor do factorized expressions need to be expanded, so $x(x+1)$ and $x^2 + x$ are both acceptable.

Please note: intermediate ***A*** marks do NOT need to be simplified.

9 Calculators

A GDC is required for this paper, but if you see work that suggests a candidate has used any calculator not approved for IB DP examinations (eg CAS enabled devices), please follow the procedures for malpractice.

10. Presentation of candidate work

Crossed out work: If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work unless an explicit note from the candidate indicates that they would like the work to be marked.

More than one solution: Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise. If the layout of the responses makes it difficult to judge, examiners should apply appropriate discretion to judge which is "first".

1. (a) $\frac{9.45-8.73}{1958-1708}$ ***(M1)***

$= 0.00288 \left(\frac{9}{3125}\right)$ ***A1***

[2 marks]

(b) (i) the (mean) yearly change in (mean annual) temperature ***A1***

Note: Accept equivalent statements, e.g. "rate of change of temperature".

(ii) $^{\circ}\text{C / year}$ **OR** degrees C per year ***A1***

Note: Do not follow through from part (b)(i) into (b)(ii).

[2 marks]

(c) attempt to substitute point and gradient into appropriate formula ***(M1)***

$8.73 = 0.00288 \times 1708 + c \Rightarrow c = 3.81096...$

or

$9.45 = 0.00288 \times 1958 + c \Rightarrow c = 3.81096$.

equation is $y = 0.00288x + 3.81$ ***A1***

[2 marks]

(d) attempt to substitute 2000 into their part (c) ***(M1)***

$0.00288 \times 2000 + 3.81096...$

$= 9.57\ (^{\circ}\text{C})\ \ (9.57096...)$ ***A1***

[2 marks]

continued…

Question 1 continued

(e) (i) $y = 0.00256x + 4.46$ $(0.00255714...x + 4.46454...)$ ***(M1)A1***

Note: Award ***(M1)A0*** for answers that show the correct method, but are presented incorrectly (e.g. no "$y =$" or truncated values etc.). Accept 4.465 as the correct answer to 4 sf.

(ii) 0.861 $(0.861333...)$ ***A1***

[3 marks]

(f) attempt to substitute 2000 into their part (e)(i) ***(M1)***

$0.00255714... \times 2000 + 4.46454...$

$= 9.58(^{\circ}\text{C})$ $(9.57882...(^{\circ}\text{C}))$ ***A1***

Note: Award ***A1*** for 9.57 from $0.00255714 \times 2000 + 4.46$.

[2 marks]

(g) cannot (always reliably) make a prediction of x from a value of y, when using a y on x line / regression line is not x on y ***A1***

extrapolation ***A1***

[2 marks]

[Total: 15 marks]

2. (a) $(a =)\ 0$ ***A1***

[1 mark]

(b) $(1.39,\ 5.24)$ $((1.38672\ldots,\ 5.24025\ldots)$ ***A1A1***

Note: Award ***A1*** for each correct coordinate; do not accept 2 sf values. Award at most ***A0A1*** if parentheses are missing. Accept $x = 1.39$, $y = 5.24$.

[2 marks]

(c) any value greater than $5.24025\ldots$ ***A1***

Note: Accept a specific value **OR** a description of the correct interval for all values.

[1 mark]

continued…

Question 2 continued

(d) **METHOD 1**

attempt to relate gradient of function to graph of f ***R1***

e.g. the function is increasing at $x = -4$

$m > 0$ ***A1***

METHOD 2

attempt to find value of $f'(-4)$ ***R1***

$\left(= \frac{25}{8} = 3.125 \right)$

$m > 0$ ***A1***

Note: The ***A1*** can only be awarded if the correct value of $f'(-4)$ or m is found

METHOD 3

attempt to substitute $(-4, -12.75)$ into equation of tangent

$-12.75 = m(-4) - \frac{1}{4}$ so $m = 3.125$ ***R1***

$m > 0$ ***A1***

Note: Do not award ***R0A1***.

[2 marks]

continued…

Question 2 continued

(e) (i) attempt to substitute $(0, -5)$ or $(3, 4.5)$ into $g(x)$ ***(M1)***

$-5 = k \times p^0 - 9$

$(k =) 4$ ***A1***

(ii) attempt to substitute $(3, 4.5)$ and their part (e)(i) into $g(x)$ ***(M1)***

$4.5 = 4 \times p^3 - 9$

$p^3 = 3.375$

$(p =) 1.5$ ***A1***

[4 marks]

(f) $y = -9$

A2

Note: Award ***A1*** for -9 seen, ***A2*** for a completely correct equation.
Condone $g(x) = -9$

[2 marks]

(g) $(x =) 4$ ***(M1)A1***

Note: Award ***M1*** for equating $f(x)$ to **their** $g(x)$
Award ***(M1)A0*** for the y-coordinate (11.25) as the answer or part of the answer (e.g. coordinates).
Award ***(M1)A0*** for an answer of $(x =) -2.39421\ldots$

[2 marks]

continued…

Question 2 continued

(h) **METHOD 1:**

$f(-1) = 0 \Rightarrow h(-1) = g(-1)$ (This justification could be graphical.) ***R1***

yes ***A1***

Note: Award ***R1*** for any reasoning based on the fact that -1 is the x-intercept of f. Accept equivalent arguments in words. Do not award ***R0A1***.

METHOD 2:

$g(-1) = -\frac{19}{3} \quad (-6.33333\ldots)$

$h(x) = 3x - 1 + 4x^{-2} + 4 \times 1.5^{x} - 9$

$h(-1) = 3(-1) - 1 + 4(-1)^{-2} + 4 \times 1.5^{-1} - 9$

$h(-1) = -\frac{19}{3} \quad (-6.33333\ldots)$ ***R1***

yes ***A1***

Note: Award ***R1*** for showing that $g(-1) = -\frac{19}{3}$ and $h(-1) = -\frac{19}{3}$. Do not award ***R0A1***.

[2 marks]

[Total: 16 marks]

3. (a) $\frac{18-4}{2}$ ***(M1)***

$(a) = 7$ ***A1***

[2 marks]

(b) $\frac{18+4}{2}$ **OR** $18-7$ **OR** $4+7$ ***(M1)***

$(d) = 11$ ***A1***

[2 marks]

(c) (time between high and low tide is) $6\text{h}15\text{m}$ **OR** 375 minutes ***(A1)***

multiplying by 2 ***(M1)***

750 minutes ***A1***

[3 marks]

(d) **EITHER**

$\frac{360^\circ}{b} = 750$ ***(A1)***

OR

$7\cos(b \times 375) + 11 = 4$ ***(A1)***

THEN

$(b =)\ 0.48$ ***A1***

Note: Award ***A1A0*** for an answer of $\frac{2\pi}{750}\left(= \frac{\pi}{375} = 0.00837758...\right)$

[2 marks]

(e) equating their cos function to 6 or graphing their cos function and 6

(M1)

$7\cos(0.48t) + 11 = 6$

$\Rightarrow t = 282.468...$ (minutes) ***(A1)***

$=4.70780\ldots$ (hr) **OR** 4hr 42 mins (4hr $42.4681\ldots$ mins) ***(A1)***

so the time is 10:42 ***A1***

[4 marks]

continued…

Question 3 continued

(f) next solution is $t = 467.531...$ ***(A1)***

$467.531... - 282.468...$

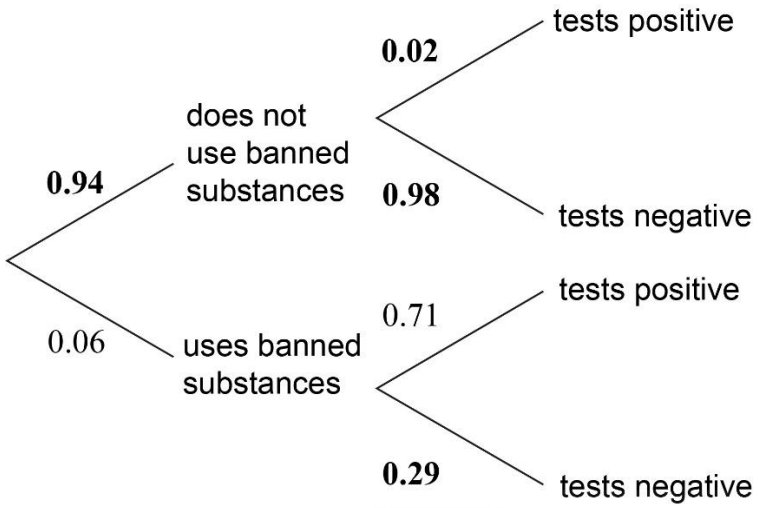
$185\ (\text{mins})\ (185.063...)$ ***A1***

Note: Accept an (unsupported) answer of 186 (from correct 3 sf values for ***t)***

[2 marks]

[Total: 15 marks]

4. (a)



A1A1

Note: Award ***A1*** for any one value correct, ***A1*** for other three values correct. Accept percentage responses as equivalent forms on **all** branches.

[2 marks]

(b) (i) multiplication of two probabilities along the tree diagram ***(M1)***

0.94×0.98

$= 0.921 \ (0.9212, 92.1\%, 92.12\%)$ ***A1***

Note: Do not accept the 2sf value for the final ***A1***.

(ii) $(0.9212)^2$ ***(A1)***

$= 0.849 \ (0.848609\ldots, 84.9\%, 84.8609...\%)$ ***A1***

Note: Accept an answer of 0.848 (0.848241) from use of 3 sf answer from part (b)(i).

[4 marks]

continued…

Question 4 continued

(c) (i) $0.94\times0.02+0.06\times0.29$ ***(A1)(M1)***

Note: Award ***A1*** for two correct products from their tree diagram seen, ***M1*** for the addition of their two products.

0.0362 (3.62%) ***A1***

(ii) multiplying their part(c)(i) by 1300

0.0362×1300 ***(M1)***

47.1 (47.06) ***A1***

Note: accept the 2 sf value of 47 for the final ***A1***

[5 marks]

(d) $p=0.02$ **OR** $p=0.98$ ***(A1)***

recognition of binomial probability with $n=20$ ***(M1)***

$\mathrm{P}(X=0)$ **OR** $\mathrm{P}(X=20)$ ***(M1)***

0.668 $(0.667607\ldots)$ ***A1***

Note: Award ***(A1)(M1)(M1)A0*** for an answer of 0.667.

$0.98^{20}=0.668$ $(0.667607\ldots)$ is awarded full marks.

[4 marks]

(e) $\mathrm{P}(X\geq3)$ **OR** $\mathrm{P}(X\leq17)$ ***(M1)***

0.00707 $(0.00706869\ldots)$ ***A1***

Note: Award ***(M1)A0*** for an answer of 0.00706.

FT from their value of p in part (d)

[2 marks]

[Total: 17 marks]

5. (a) equating a volume of a half cylinder (or cylinder) to 0.8 ***(M1)***

$$0.8 = \frac{1}{2}\pi r^2 l$$

$$l = \frac{1.6}{\pi r^2}$$ ***A1***

Note: Do not accept decimal approximation of π for the ***A1*** given the demand of question.
Condone the use of h for l for the ***M1***

[2 marks]

(b) calculating area in terms of r and l ***M1***

$$C = 2lr + \pi r^2 + \pi rl$$

area with l replaced by $\frac{1.6}{\pi r^2}$ ***M1***

apply costs to correct part of each surface ***M1***

a correct substitution into an expression for C, leading to given answer ***A1***

e.g. $(C =)\ 4.40 \times \pi r\left(\frac{1.6}{\pi r^2}\right) + 4.40 \times 2r\left(\frac{1.6}{\pi r^2}\right) + p \times \pi r^2$

$$(C =)\ 7.04r^{-1} + \frac{14.08}{\pi}r^{-1} + p\pi r^2$$ ***AG***

Note: The ***AG*** line must be seen to award the final ***A1***.
No incorrect working should be seen after the correct substitution

[4 marks]

continued…

Question 5 continued

(c) **EITHER**

$$\left(\frac{\mathrm{d}C}{\mathrm{d}r} =\right) -7.04r^{-2} - \frac{14.08}{\pi}r^{-2} + 2p\pi r$$ ***A1A1A1***

OR

$-7.04r^{-2} - 4.48r^{-2} + 6.28pr \quad \left(-7.04r^{-2} - (4.48180...)r^{-2} + 6.28318...pr\right)$

A1A1A1

OR

$-11.5r^{-2} + 6.28pr \quad \left((-11.5218...)r^{-2} + 6.28318...pr\right)$ ***A2A1***

Note: Award ***A1*** for each correct term.
Award at most ***A1A1A0*** if extra terms are seen.

[3 marks]

continued…

Question 5 continued

(d) recognition of setting $\frac{\mathrm{d}C}{\mathrm{d}r}$ to zero ***(M1)***

attempt to substitute 0.7 in for r in their derivative ***(M1)***

$$0 = -7.04(0.7)^{-2} - \frac{14.08}{\pi} \times (0.7)^{-2} + 2p\pi \times 0.7$$

(p =) ($)5.35 (per square metre) (($) 5.34621…) ***A1***

Note: Accept $5.34, as this will also lead to a radius of 0.7 (to 3sf).

[3 marks]

(e) attempt to calculate the cost of one container ***(M1)***

$$(C =)\ 7.04(0.7)^{-1} + \frac{14.08}{\pi}(0.7)^{-1} + 5.34621...\pi \times 0.7^2$$ ***(A1)***

Note: May be shown within a calculation of the cost of all containers.

$(C =)$ 24.6895…

$24.6895… \times 350$

= ($) 8641 ***A1***

Note: Answer must be rounded to the nearest dollar to award the final ***A1.***
Accept answers between 8641 and 8645 (inclusive), due to rounding the value of p and/or the cost of one container to the nearest cent.
Award ***(M1)(A1)A0*** for an answer rounded to 3sf (e.g. ($)8640) or to 2dp (e.g., ($)8641.35).
Accept an answer of ($)8638 from use of $5.34 in their cost calculation.

[3 marks]

continued…

Question 5 continued

(f) attempt to apply a discount of 8% to their part (e) ***(M1)***

Note: the discount percentage will depend on their answer to part (e) e.g. $8641.35... \times 0.92$ **OR** $8641.35... \times 0.08$

($)7950 (($)7950.04…) ***A1***

[2 marks]

[Total: 17 marks]

Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

November 2022

Mathematics: applications and interpretation

Standard level

Paper 2

15 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**.
A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.
R Marks awarded for clear **Reasoning**.
AG Answer given in the question and so no marks are awarded.
FT Follow through. The practice of awarding marks, despite candidate errors in previous parts, for their correct methods/answers using incorrect results.

Using the markscheme

1 General

Award marks using the annotations as noted in the markscheme *eg* ***M1, A2.***

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***A3***, ***M2*** *etc*., do **not** split the marks, unless there is a note.
- The response to a "show that" question does not need to restate the ***AG*** line, unless a **Note** makes this explicit in the markscheme.
- Once a correct answer to a question or part question is seen, ignore further working even if this working is incorrect and/or suggests a misunderstanding of the question. This will encourage a uniform approach to marking, with less examiner discretion. Although some candidates may be advantaged for that specific question item, it is likely that these candidates will lose marks elsewhere too.
- An exception to the previous rule is when an incorrect answer from further working is used **in a subsequent part**. For example, when a correct exact value is followed by an incorrect decimal approximation in the first part and this approximation is then used in the second part. In this situation, award ***FT*** marks as appropriate but do not award the final ***A1*** in the first part. Examples:

	Correct answer seen	**Further working seen**	**Any FT issues?**	**Action**
1.	$8\sqrt{2}$	5.65685... *(incorrect decimal value)*	No. Last part in question.	Award ***A1*** for the final mark *(condone the incorrect further working)*
2.	$\frac{35}{72}$	0.468111… (incorrect decimal value)	Yes. Value is used in subsequent parts.	Award ***A0*** for the final mark *(and full **FT** is available in subsequent parts)*

3 Implied marks

Implied marks appear in **brackets e.g.** ***(M1)***,and can only be awarded if **correct** work is seen or implied by subsequent working/answer.

4 Follow through marks (only applied after an error is made)

Follow through (***FT***) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) (e.g. incorrect value from part (a) used in part (d) or incorrect value from part (c)(i) used in part (c)(ii)). Usually, to award ***FT*** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if all the marks awarded in a subsequent part are for the answer or are implied, then ***FT*** marks should be awarded for *their* correct answer, even when working is not present.

For example: following an incorrect answer to part (a) that is used in subsequent parts, where the markscheme for the subsequent part is ***(M1)A1***, it is possible to award full marks for *their* correct answer, **without working being seen.** For longer questions where all but the answer marks are implied this rule applies but may be overwritten by a **Note** in the Markscheme.

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks, by reflecting on what each mark is for and how that maps to the simplified version.
- If the error leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word “their” in a description, to indicate that candidates may be using an incorrect value.
- If the candidate’s answer to the initial question clearly contradicts information given in the question, it is not appropriate to award any ***FT*** marks in the subsequent parts. This includes when candidates fail to complete a “show that” question correctly, and then in subsequent parts use their incorrect answer rather than the given value.
- Exceptions to these ***FT*** rules will be explicitly noted on the markscheme.
- If a candidate makes an error in one part but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the command term was “Hence”.

5 Mis-read

If a candidate incorrectly copies values or information from the question, this is a mis-read (***MR***). A candidate should be penalized only once for a particular misread. Use the ***MR*** stamp to indicate that this has been a misread and do not award the first mark, even if this is an ***M*** mark, but award all others as appropriate.

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.
- If a candidate uses a correct answer, to a "show that" question, to a higher degree of accuracy than given in the question, this is NOT a misread and full marks may be scored in the subsequent part.
- ***MR*** can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If the command term is 'Hence' and not 'Hence or otherwise' then alternative methods are not permitted unless covered by a note in the mark scheme.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**.

7 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation** for example 1.9 and $1{,}9$ or 1000 and $1{,}000$ and 1.000.
- Do not accept final answers written using calculator notation. However, ***M*** marks and intermediate ***A*** marks can be scored, when presented using calculator notation, provided the evidence clearly reflects the demand of the mark.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, some **equivalent** answers will generally appear in brackets. Not all equivalent notations/answers/methods will be presented in the markscheme and examiners are asked to apply appropriate discretion to judge if the candidate work is equivalent.

8 Format and accuracy of answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. If the level of accuracy is not stated in the question, the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures*.

Where values are used in subsequent parts, the markscheme will generally use the exact value, however candidates may also use the correct answer in subsequent parts. The markscheme will often explicitly include the subsequent values that come "*from the use of 3 sf values*".

Simplification of final answers: Candidates are advised to give final answers using good mathematical form. In general, for an ***A*** mark to be awarded, arithmetic should be completed, and any values that lead to integers should be simplified; for example, $\sqrt{\frac{25}{4}}$ should be written as $\frac{5}{2}$. An exception to this is simplifying fractions, where lowest form is not required (although the numerator and the denominator must be integers); for example, $\frac{10}{4}$ may be left in this form or written as $\frac{5}{2}$. However, $\frac{10}{5}$ should be written as 2, as it simplifies to an integer.

Algebraic expressions should be simplified by completing any operations such as addition and multiplication, e.g. $4e^{2x} \times e^{3x}$ should be simplified to $4e^{5x}$, and $4e^{2x} \times e^{3x} - e^{4x} \times e^{x}$ should be simplified to $3e^{5x}$. Unless specified in the question, expressions do not need to be factorized, nor do factorized expressions need to be expanded, so $x(x+1)$ and $x^2 + x$ are both acceptable.

Please note: intermediate ***A*** marks do NOT need to be simplified.

9 Calculators

A GDC is required for this paper, but If you see work that suggests a candidate has used any calculator not approved for IB DP examinations (eg CAS enabled devices), please follow the procedures for malpractice.

10. Presentation of candidate work

Crossed out work: If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work unless an explicit note from the candidate indicates that they would like the work to be marked.

More than one solution: Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise. If the layout of the responses makes it difficult to judge, examiners should apply appropriate discretion to judge which is "first".

1. (a) continuous ***A1***

[1 mark]

(b) $160-50-62-14-8$ ***(M1)***

$(k=)\ 26$ ***A1***

[2 marks]

(c) (i) $20 \leq T < 40$ ***A1***

(ii) 30 ***A1***

[2 marks]

(d) 33.5 minutes ***A2***

Note: ***FT*** from their value of k and their mid-interval value. Follow through from part (c)(ii) but only if mid-interval value lies in their interval.

[2 marks]

(e) 112 ***A1***

[1 mark]

(f) $\frac{22}{160}\ \left[0.138,\ 0.1375,\ 13.75\%,\ \frac{11}{80}\right]$ ***A1A1***

Note: Award ***A1*** for correct numerator, ***A1*** for correct denominator.

[2 marks]

(g) 26 minutes ***A1***

[1 mark]

(h) $50-16$ ***(M1)***

Note: Award ***M1*** for both correct quartiles seen.

34 minutes ***A1***

[2 marks]

continued…

Question 1 continued

(i) correct substitution into outlier formula ***(M1)***

$50+1.5\times 34$

$=101$ ***A1***

$92<101$ **OR** highest value on diagram <101 ***R1***

not an outlier ***AG***

Note: Award ***R1*** for their correct comparison. Follow through from their part (h).
Award ***R0*** if their conclusion is "it is an outlier", this contradicts Elsie's belief.

[3 marks]

(j) **EITHER**

the diagram is not symmetric or equivalent

e.g the median is not in the center of the box or

the lengths of the whiskers are (very) different or (positive or right) skew

OR

the mean and median are (very) different; ***A1***

[1 mark]

Total [17 marks]

2. (a) (i) 19° ***A1***

(ii) $\frac{\text{BD}}{\sin 120^\circ} = \frac{40}{\sin 19^\circ}$ ***(M1)(A1)***

Note: Award ***M1*** for substituted sine rule for BCD, ***A1*** for their correct substitution.

$(\text{BD} =) 106 \text{ m} \quad (106.401...)$ ***A1***

[4 marks]

(b) **METHOD 1 (cosine rule)**

$\cos \text{BAD} = \frac{85^2 + 85^2 - 106.401...^2}{2 \times 85 \times 85}$ ***(M1)(A1)***

Note: Award ***M1*** for substituted cosine rule, ***A1*** for their correct substitution.

77.495 ***A1***

Note: Accept an answer of 77.149 from use of 3 sf answer from part (a). The final answer must be correct to five significant figures.

METHOD 2 (right angled trig/isosceles triangles)

$\sin\left(\frac{\text{BAD}}{2}\right) = \frac{53.2008...}{85}$ ***(A1)(M1)***

Note: Award ***A1*** for $53.2008...$ seen. Award ***M1*** for correctly substituted trig ratio. Follow through from part (a).

$77.495...$ ***A1***

Note: Use of 3 sf answer from part (a), results in 77.149.

[3 marks]

(c) **EITHER**

$(\text{Area} =) \ \frac{1}{2} \times 85 \times 85 \times \sin(77^\circ)$ ***(M1)(A1)***

Note: Award ***M1*** for substituted area formula, ***A1*** for correct substitution. Award at most ***(M1)(A1)A0*** if an angle other than 77° is used.

OR

$(\text{Area} =) \ \frac{1}{2} \times \left(2 \times 85 \times \sin(38.5^\circ)\right) \times \left(85 \times \cos(38.5^\circ)\right)$ ***(M1)(A1)***

Note: Award ***M1*** for substituted area formula $A = \frac{1}{2}bh$, ***A1*** for correct substitution.

$3520 \text{ m}^2 \ (3519.91...)$ ***A1***

[3 marks]

continued…

Question 2 continued

(d) 85 m ***A1***

[1 mark]

(e) $85+85+\frac{77}{360}\times 2\pi\times 85$ ***(M1)(M1)***

Note: Award ***M1*** for correctly substituted into $\frac{\theta}{360}\times 2\pi\times r$, ***M1*** for addition of AB and AD.

284 m (284.231...) ***A1***

[3 marks]

(f) $\frac{77}{360}\times\pi\times(85)^2-3519.91...$ ***(M1)(M1)***

Note: Award ***M1*** for correctly substituted area of sector formula, ***M1*** for subtraction of their area from part (c).

1330 m^2 (1334.93...) ***A1***

[3 marks]

Total [17 marks]

3. (a) (i) B ***A1***

(ii) F ***A1***

[2 marks]

(b) correct substitution into the midpoint formula ***(M1)***

$\frac{8+5}{2}$

$y = 6.5$ ***A1***

Note: Answer must be an equation for the ***A1*** to be awarded.

[2 marks]

(c) midpoint $= (5, 7)$ ***(A1)***

correct use of gradient formula ***(M1)***

$\frac{8-6}{7-3}$

gradient of $\text{BC} = 0.5$ ***(A1)***

negative reciprocal of gradient ***(M1)***

perpendicular gradient $= -2$

$y - 7 = -2(x - 5)$ (or $y = -2x + 17$) ***A1***

[5 marks]

(d) (i) attempt to find the intersection of two perpendicular bisectors (BC & CD) ***(M1)***

Note: This may be seen graphically or algebraically.

$6.5 - 7 = -2(x - 5)$ **OR** $6.5 = -2x + 17$

Note: Accept equivalent methods using the perpendicular bisector of BD, $y - 5.5 = 4(x - 5)$ **OR** $y = 4x - 14.5$

$x = 5.25, y = 6.5$ **OR** $(5.25, 6.5)$ ***A1***

Note: The x-coordinate must be exact or expressed to at least 3 sf.

(ii) their correct substitution into distance formula ***(M1)***

$\sqrt{(5.25 - 7)^2 + (6.5 - 5)^2}$

$= 2.30 \text{ km} \left(2.30488..., \frac{\sqrt{85}}{4} \right)$ ***A1***

[4 marks]

Total [13 marks]

4. (a) (i) $(m =)\ 54(\%)$ ***A1***

(ii) $(n =)\ 14(\%)$ ***A1***

(iii) $(p =)\ 22(\%)$ ***A1***

(iv) $(q =)\ 10(\%)$ ***A1***

Note: Based on their n, follow through for parts (i) and (iii), but only if it does not contradict the given information. Follow through for part (iv) but only if the total is 100%.

[4 marks]

(b) $90\ (\%)$ ***A1***

Note: Award ***A0*** for a decimal answer.

[1 mark]

(c) (i) $0.54\ \left(\frac{54}{100}, \frac{27}{50}, 54\%\right)$ ***A1***

(ii) $\frac{54}{64}\ \left(0.844, \frac{27}{32}, 84.4\%, 0.84375\right)$ ***A1A1***

Note: Award ***A1*** for a correct denominator (0.64 or 64 seen), ***A1*** for the correct final answer.

[3 marks]

(d) (i) recognizing Binomial distribution with correct parameters ***(M1)***

$X \sim \mathrm{B}(10,\ 0.68)$

$\left(\mathrm{P}(X = 5) =\right)\ 0.123\ \ (0.122940..., 12.3\%)$ ***A1***

(ii) $1 - \mathrm{P}(X \leq 3)$ **OR** $\mathrm{P}(X \geq 4)$ **OR** $\mathrm{P}(4 \leq X \leq 10)$ ***(M1)***

$0.984\ (0.984497..., 98.4\%)$ ***A1***

(iii) $(0.68)^9 \times 0.32$ ***(M1)***

recognition of two possible cases ***(M1)***

$2 \times \left((0.68)^9 \times 0.32\right)$

$0.0199\ \ (0.0198957..., 1.99\%)$ ***A1***

[7 marks]

(e) **EITHER**

the probability is not constant ***A1***

OR

the events are not independent ***A1***

OR

the events should be modelled by the hypergeometric distribution instead ***A1***

[1 mark]

Total [16 marks]

5. (a) (i) $f'(x) = \frac{-2x}{50} + 2 \quad \left(= \frac{-x}{25} + 2, -0.04x + 2 \right)$ ***A1A1***

Note: Award ***A1*** for each correct term. Award at most ***A0A1*** if extra terms are seen.

(ii) $0 = \frac{-x}{25} + 2$ **OR** sketch of $f'(x)$ with x-intercept indicated ***M1***

$x = 50$ ***A1***

$y = 80$ ***A1***

$(50, 80)$

Note: Award ***M0A0A1*** for the coordinate $(50, 80)$ seen either with no working or found from a graph of $f(x)$.

[5 marks]

(b) (i) $\int_0^{70} \frac{-x^2}{50} + 2x + 30 \, \mathrm{d}x$ ***A1A1***

Note: Award ***A1*** for a correct integral, ***A1*** for correct limits in the correct location. Award at most ***A0A1*** if $\mathrm{d}x$ is omitted.

(ii) (Area =) $4710 \text{ m}^2 \quad \left(4713.33\ldots, \frac{14140}{3} \right)$ ***A2***

[4 marks]

(c) (i) $\frac{11.4}{4713.33...} \times 100\%$ **OR** $\left| \frac{4701.93... - 4713.33...}{4713.33...} \right| \times 100\%$ ***(M1)***

Note: Award ***(M1)*** for their correct substitution into the percentage error formula.

$0.242\% \quad (0.241867...\%)$ ***A1***

Note: Percentage sign is required. Accept $0.242038...\%$ if 4710 is used.

(ii) **EITHER**

reduce the width of the intervals (trapezoids) ***A1***

OR

increase the number of intervals (trapezoids) ***A1***

Note: Accept equivalent statements. Award ***A0*** for the ambiguous answer "increase the intervals".

[3 marks]

continued…

Question 5 continued

(d) (i) width of the square is $70-x$ **OR** the length of the square is $\frac{-x^2}{50}+2x+30$

(M1)

Note: Award ***(M1)*** for $70-x$ seen anywhere. Accept $\frac{-x^2}{50}+2x+30$ but only if this expression is explicitly identified as a dimension of the square.

in term of x, equating the length to the width ED ***(M1)***

$\frac{-x^2}{50}+2x+30=70-x$

$(x=14.7920...$ or $135.21)$

$(x=)$ 14.8 m $(14.7920...)$ ***A1***

Note: Award ***M0M0A0*** for an unsupported answer of 15. Award at most ***M1M0A0*** for an approach which leads to $A'(x)=0$. This will lead to a square base which extends beyond the east boundary of the property. Similar for any solution where F is not on the northern boundary, or GH is not on the east boundary.

(ii) **EITHER**

$(70-14.7920...)^2$ ***(M1)***

OR

$(55.2079...)^2$ ***(M1)***

OR

$\left(\frac{-(14.7920...)^2}{50}+2(14.7920...)+30\right)^2$ ***(M1)***

THEN

(Area =) 3050 m^2 $(3047.92...)$ ***A1***

Note: Follow through from part (d)(i), provided x is between 0 and 70. Award at most ***M1A0*** if their answer is outside the range of their $[0, 4713.33\ldots]$ from part (b).

[5 marks]

Total [17 marks]

Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

May 2022

Mathematics: applications and interpretation

Standard level

Paper 2

16 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**.
A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.
R Marks awarded for clear **Reasoning**.
AG Answer given in the question and so no marks are awarded.
FT Follow through. The practice of awarding marks, despite candidate errors in previous parts, for their correct methods/answers using incorrect results.

Using the markscheme

1 General

Award marks using the annotations as noted in the markscheme *eg* ***M1, A2.***

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***A3***, ***M2*** *etc.*, do **not** split the marks, unless there is a note.
- The response to a "show that" question does not need to restate the ***AG*** line, unless a **Note** makes this explicit in the markscheme.
- Once a correct answer to a question or part question is seen, ignore further working even if this working is incorrect and/or suggests a misunderstanding of the question. This will encourage a uniform approach to marking, with less examiner discretion. Although some candidates may be advantaged for that specific question item, it is likely that these candidates will lose marks elsewhere too.
- An exception to the previous rule is when an incorrect answer from further working is used **in a subsequent part**. For example, when a correct exact value is followed by an incorrect decimal approximation in the first part and this approximation is then used in the second part. In this situation, award ***FT*** marks as appropriate but do not award the final ***A1*** in the first part. Examples:

	Correct answer seen	**Further working seen**	**Any FT issues?**	**Action**
1.	$8\sqrt{2}$	5.65685... *(incorrect decimal value)*	No. Last part in question.	Award ***A1*** for the final mark *(condone the incorrect further working)*
2.	$\frac{35}{72}$	0.468111… (incorrect decimal value)	Yes. Value is used in subsequent parts.	Award ***A0*** for the final mark *(and full **FT** is available in subsequent parts)*

3 Implied marks

Implied marks appear in **brackets e.g.** ***(M1)***,and can only be awarded if **correct** work is seen or implied by subsequent working/answer.

4 Follow through marks (only applied after an error is made)

Follow through (***FT***) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) (e.g. incorrect value from part (a) used in part (d) or incorrect value from part (c)(i) used in part (c)(ii)). Usually, to award ***FT*** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if all the marks awarded in a subsequent part are for the answer or are implied, then ***FT*** marks should be awarded for *their* correct answer, even when working is not present.

For example: following an incorrect answer to part (a) that is used in subsequent parts, where the markscheme for the subsequent part is ***(M1)A1***, it is possible to award full marks for *their* correct answer, **without working being seen.** For longer questions where all but the answer marks are implied this rule applies but may be overwritten by a **Note** in the Markscheme.

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks, by reflecting on what each mark is for and how that maps to the simplified version.
- If the error leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word “their” in a description, to indicate that candidates may be using an incorrect value.
- If the candidate’s answer to the initial question clearly contradicts information given in the question, it is not appropriate to award any ***FT*** marks in the subsequent parts. This includes when candidates fail to complete a “show that” question correctly, and then in subsequent parts use their incorrect answer rather than the given value.
- Exceptions to these ***FT*** rules will be explicitly noted on the markscheme.
- If a candidate makes an error in one part but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the command term was “Hence”.

5 Mis-read

If a candidate incorrectly copies values or information from the question, this is a mis-read (***MR***). A candidate should be penalized only once for a particular misread. Use the ***MR*** stamp to indicate that this has been a misread and do not award the first mark, even if this is an ***M*** mark, but award all others as appropriate.

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.
- If a candidate uses a correct answer, to a "show that" question, to a higher degree of accuracy than given in the question, this is NOT a misread and full marks may be scored in the subsequent part.
- ***MR*** can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If the command term is 'Hence' and not 'Hence or otherwise' then alternative methods are not permitted unless covered by a note in the mark scheme.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**.

7 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation** for example 1.9 and $1{,}9$ or 1000 and $1{,}000$ and 1.000.
- Do not accept final answers written using calculator notation. However, ***M*** marks and intermediate ***A*** marks can be scored, when presented using calculator notation, provided the evidence clearly reflects the demand of the mark.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, some **equivalent** answers will generally appear in brackets. Not all equivalent notations/answers/methods will be presented in the markscheme and examiners are asked to apply appropriate discretion to judge if the candidate work is equivalent.

8 Format and accuracy of answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. If the level of accuracy is not stated in the question, the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures.*

Where values are used in subsequent parts, the markscheme will generally use the exact value, however candidates may also use the correct answer in subsequent parts. The markscheme will often explicitly include the subsequent values that come "*from the use of 3 sf values*".

Simplification of final answers: Candidates are advised to give final answers using good mathematical form. In general, for an ***A*** mark to be awarded, arithmetic should be completed, and any values that lead to integers should be simplified; for example, $\sqrt{\frac{25}{4}}$ should be written as $\frac{5}{2}$. An exception to this is simplifying fractions, where lowest form is not required (although the numerator and the denominator must be integers); for example, $\frac{10}{4}$ may be left in this form or written as $\frac{5}{2}$. However, $\frac{10}{5}$ should be written as 2, as it simplifies to an integer.

Algebraic expressions should be simplified by completing any operations such as addition and multiplication, e.g. $4\mathrm{e}^{2x} \times \mathrm{e}^{3x}$ should be simplified to $4\mathrm{e}^{5x}$, and $4\mathrm{e}^{2x} \times \mathrm{e}^{3x} - \mathrm{e}^{4x} \times \mathrm{e}^{x}$ should be simplified to $3\mathrm{e}^{5x}$. Unless specified in the question, expressions do not need to be factorized, nor do factorized expressions need to be expanded, so $x(x+1)$ and $x^2 + x$ are both acceptable.

Please note: intermediate ***A*** marks do NOT need to be simplified.

9 Calculators

A GDC is required for this paper, but If you see work that suggests a candidate has used any calculator not approved for IB DP examinations (eg CAS enabled devices), please follow the procedures for malpractice.

10. Presentation of candidate work

Crossed out work: If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work unless an explicit note from the candidate indicates that they would like the work to be marked.

More than one solution: Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise. If the layout of the responses makes it difficult to judge, examiners should apply appropriate discretion to judge which is "first".

1. (a) (i) 0.58 (s) ***A1***

(ii) $0.7 - 0.42$ ***(A1)(M1)***

Note: Award ***A1*** for correct quartiles seen, ***M1*** for subtraction of their quartiles.

0.28 s ***A1***

[4 marks]

(b) 9 (people have reaction time ≤ 0.4) ***(A1)***

31 (people have reaction time > 0.4) ***A1***

[2 marks]

(c) $(90\% \times 40 =)\ 36$ **OR** 4 ***(A1)***

0.8 s ***A1***

[2 marks]

(d) (i) $(a =)\ 6$ ***A1***

(ii) $(b =)\ 4$ ***A1***

[2 marks]

(e) $0.6 < t \leq 0.8$ ***A1***

[1 mark]

(f) 0.55 s ***A2***

[2 marks]

(g) the mean will increase ***A1***

because the incorrect reaction times are moving from a lower interval to a higher interval which will increase the numerator of the mean calculation ***R1***

the median will stay the same ***A1***

because the median or middle of the data is greater than both intervals being changed ***R1***

Note: Do not award ***A1R0***.

[4 marks]

Total [17 marks]

2. (a) (i) **EITHER**

$115.5 = u_1 + (3-1)\times d \quad (115.5 = u_1 + 2d)$

$108 = u_1 + (8-1)\times d \quad (108 = u_1 + 7d)$ ***(M1)(A1)***

Note: Award ***M1*** for attempting to use the arithmetic sequence term formula, ***A1*** for both equations correct. Working for ***M1*** and ***A1*** can be found in parts (i) or (ii).

$(d = -1.5)$

1.5 (cups/day) ***A1***

Note: Answer must be written as a positive value to award ***A1***.

OR

$(d =) \ \dfrac{115.5-108}{5}$ ***(M1)(A1)***

Note: Award ***M1*** for attempting a calculation using the difference between term 3 and term 8; ***A1*** for a correct substitution.

$(d =) \ 1.5$ (cups/day) ***A1***

(ii) $(u_1 =) \ 118.5$ (cups) ***A1***

[4 marks]

(b) attempting to substitute their values into the term formula for arithmetic sequence equated to zero ***(M1)***

$0 = 118.5 + (n-1)\times(-1.5)$

$(n =) \ 80$ days ***A1***

Note: Follow through from part (a) only if their answer is positive.

[2 marks]

(c) $(t_5 =) \ 625\times 1.064^{(5-1)}$ ***(M1)(A1)***

Note: Award ***M1*** for attempting to use the geometric sequence term formula; ***A1*** for a correct substitution

$\$ 801$ ***A1***

Note: The answer must be rounded to a whole number to award the final ***A1***.

[3 marks]

continued…

Question 2 continued

(d) (i) $(S_{10} =)$ (\$) 8390 (8394.39...) ***A1***

(ii) **EITHER**

the total cost (of dog food) ***R1***
for 10 years beginning in 2021 **OR** 10 years before 2031 ***R1***

OR

the total cost (of dog food) ***R1***
from 2021 to 2030 (inclusive) **OR** from 2021 to (the start of) 2031 ***R1***

[3 marks]

(e) **EITHER**
According to the model, the cost of dog food per year will eventually be too high to keep a dog.
OR
The model does not necessarily consider changes in inflation rate.
OR
The model is appropriate as long as inflation increases at a similar rate.
OR
The model does not account for changes in the amount of food the dog eats as it ages/becomes ill/stops growing.
OR
The model is appropriate since dog food bags can only be bought in discrete quantities. ***R1***

Note: Accept reasonable answers commenting on the appropriateness of the model for the specific scenario. There should be a reference to the given context. A reference to the geometric model must be clear: either "model" is mentioned specifically, or other mathematical terms such as "increasing" or "discrete quantities" are seen. Do not accept a contextual argument in isolation, e.g. "The dog will eventually die".

[1 mark]
Total [13 marks]

3. (a) $\left(\frac{2+6}{2}, \frac{2+0}{2}\right)$ ***(M1)***

$(4, 1)$ ***A1***

Note: Award ***A0*** if parentheses are omitted in the final answer.

[2 marks]

(b) attempt to substitute values into gradient formula ***(M1)***

$\left(\frac{0-2}{6-2} = \right) -\frac{1}{2}$ ***(A1)***

therefore the gradient of perpendicular bisector is 2 ***(M1)***

so $y-1=2(x-4)$ $(y=2x-7)$ ***A1***

[4 marks]

(c) identifying the correct equations to use: ***(M1)***

$y=2-x$ and $y=2x-7$

evidence of solving their correct equations or of finding intersection point graphically ***(M1)***

$(3, -1)$ ***A1***

Note: Accept an answer expressed as "$x=3$, $y=-1$".

[3 marks]

(d) attempt to use distance formula ***(M1)***

$\mathrm{YZ} = \sqrt{(7-(-1))^2 + (7-3)^2}$

$= \sqrt{80}$ $\left(4\sqrt{5}\right)$ ***A1***

[2 marks]

(e) **METHOD 1 (cosine rule)**

length of XZ is $\sqrt{80}$ $\left(4\sqrt{5}, 8.94427...\right)$ ***(A1)***

Note: Accept 8.94 and 8.9.

attempt to substitute into cosine rule ***(M1)***

$\cos \mathrm{X\hat{Y}Z} = \frac{80+32-80}{2\times\sqrt{80}\sqrt{32}}$ $(=0.316227\ldots)$ ***(A1)***

Note: Award ***A1*** for correct substitution of XZ, YZ, $\sqrt{32}$ values in the cos rule. Exact values do not need to be used in the substitution.

$\left(\mathrm{X\hat{Y}Z} = \right) 71.6°$ $(71.5650\ldots°)$ ***A1***

Note: Last ***A1*** mark may be lost if prematurely rounded values of XZ, YZ and/or XY are used.

continued…

Question 3 continued

METHOD 2 (splitting isosceles triangle in half)

length of XZ is $\sqrt{80}$ $\left(4\sqrt{5}, 8.94427...\right)$ ***(A1)***

Note: Accept 8.94 and 8.9.

required angle is $\cos^{-1}\left(\frac{\sqrt{32}}{2\sqrt{80}}\right)$ ***(M1)(A1)***

Note: Award ***A1*** for correct substitution of XZ (or YZ), $\frac{\sqrt{32}}{2}$ values in the cos rule.
Exact values do not need to be used in the substitution.

$\left(\mathrm{X\hat{Y}Z} =\right)\ 71.6°\ \ (71.5650°)$ ***A1***

Note: Last ***A1*** mark may be lost if prematurely rounded values of XZ, YZ and/or XY are used.

[4 marks]

(f) (area =) $\frac{1}{2}\sqrt{80}\sqrt{32}\sin 71.5650\ldots$ **OR** (area =) $\frac{1}{2}\sqrt{32}\sqrt{72}$ ***(M1)***

$= 24\ \text{km}^2$ ***A1***

[2 marks]

(g) *Any sensible answer such as:*
There might be factors other than proximity which influence shopping choices.
A larger area does not necessarily result in an increase in population.
The supermarkets might be specialized / have a particular clientele who visit even if other shops are closer.
Transport links might not be represented by Euclidean distances.
etc. ***R1***

[1 mark]

Total [18 marks]

4. (a) (i) an attempt to find the amplitude ***(M1)***

$\frac{61.8}{2}$ **OR** $\frac{64.5-2.7}{2}$

$(a=)\ 30.9\ \text{m}$ ***A1***

Note: Accept an answer of $(a=)\ -30.9\ \text{m}$.

(ii) $(\text{period}=\frac{60}{1.5}=)\ 40\ (\text{s})$ ***(A1)***

$((b=)\ \frac{360^\circ}{40})$

$(b=)\ 9$ ***A1***

Note: Accept an answer of $(b=)\ -9$.

(iii) attempt to find d ***(M1)***

$(d=)\ 30.9+2.7$ **OR** $\frac{64.5+2.7}{2}$

$(d=)\ 33.6\ \text{m}$ ***A1***

[6 marks]

(b) 12×1.5 **OR** $\frac{12\times60}{40}$ ***(M1)***

18 (revolutions per ride) ***A1***

[2 marks]

(c) (i) $0\le t\le 720$ ***A1***

(ii) $2.7\le h\le 64.5$ ***A1A1***

Note: Award ***A1*** for correct endpoints of domain and ***A1*** for correct endpoints of range. Award ***A1*** for correct direction of both inequalities.

[3 marks]

(d) graph of $h(t)$ and $y=16.7$ **OR** $h(t)=16.7$ ***(M1)***

6.31596… and 33.6840… ***(A1)***

27.4 (s) (27.3680…) ***A1***

[3 marks]

continued…

Question 4 continued

(e) (i) d ***A1***

(ii) **EITHER**

$d + 30.9 = 65.2$ ***(A1)***

OR

$65.2 - (61.8 + 2.7) = 0.7$ ***(A1)***

OR

3.4 (new platform height) ***(A1)***

THEN

(d =) 34.3 m ***A1***

[3 marks]

Total [17 marks]

5. (a) attempt to expand given expression ***(M1)***

$$C = \frac{xk^2}{10} - \frac{3x^3}{1000}$$

$$\frac{\mathrm{d}C}{\mathrm{d}x} = \frac{k^2}{10} - \frac{9x^2}{1000}$$ ***M1A1***

Note: Award ***M1*** for power rule correctly applied to at least one term and ***A1*** for correct answer.

[3 marks]

(b) equating their $\frac{\mathrm{d}C}{\mathrm{d}x}$ to zero ***(M1)***

$$\frac{k^2}{10} - \frac{9x^2}{1000} = 0$$

$$x^2 = \frac{100k^2}{9}$$

$$x = \frac{10k}{3}$$ ***(A1)***

substituting their x back into given expression ***(M1)***

$$C_{\max} = \frac{10k}{30}\left(k^2 - \frac{300k^2}{900}\right)$$

$$C_{\max} = \frac{2k^3}{9}\left(0.222\ldots k^3\right)$$ ***A1***

[4 marks]

(c) (i) substituting 20 into given expression and equating to 426 ***M1***

$$426 = \frac{20}{10}\left(k^2 - \frac{3}{100}(20)^2\right)$$

$$k = 15$$ ***A1***

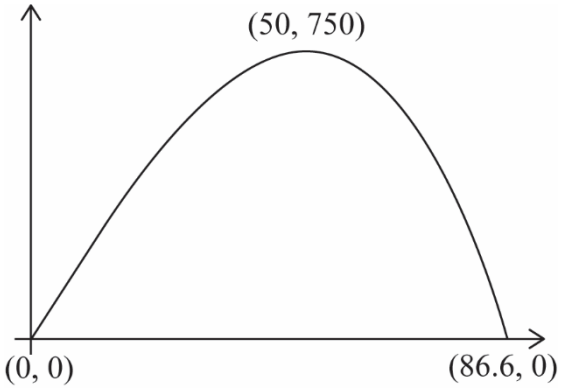
(ii) 50 ***A1***

[3 marks]

continued…

Question 5 continued

(d)



A1A1A1

Note: Award ***A1*** for graph drawn for positive x indicating an increasing and then decreasing function, ***A1*** for maximum labelled and ***A1*** for graph passing through the origin and 86.6, marked on the x-axis or whose coordinates are given.

[3 marks]

(e) setting their expression for C to zero **OR** choosing correct x-intercept on their graph of C ***(M1)***

$x_{\max} = 86.6 \quad (86.6025\ldots)$ litres ***A1***

[2 marks]

Total [15 marks]

Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

May 2022

Mathematics: applications and interpretation

Standard level

Paper 2

15 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**.
A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.
R Marks awarded for clear **Reasoning**.
AG Answer given in the question and so no marks are awarded.
FT Follow through. The practice of awarding marks, despite candidate errors in previous parts, for their correct methods/answers using incorrect results.

Using the markscheme

1 General

Award marks using the annotations as noted in the markscheme *eg* ***M1, A2****.*

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***A3***, ***M2*** *etc*., do **not** split the marks, unless there is a note.
- The response to a "show that" question does not need to restate the ***AG*** line, unless a **Note** makes this explicit in the markscheme.
- Once a correct answer to a question or part question is seen, ignore further working even if this working is incorrect and/or suggests a misunderstanding of the question. This will encourage a uniform approach to marking, with less examiner discretion. Although some candidates may be advantaged for that specific question item, it is likely that these candidates will lose marks elsewhere too.
- An exception to the previous rule is when an incorrect answer from further working is used **in a subsequent part**. For example, when a correct exact value is followed by an incorrect decimal approximation in the first part and this approximation is then used in the second part. In this situation, award ***FT*** marks as appropriate but do not award the final ***A1*** in the first part. Examples:

	Correct answer seen	**Further working seen**	**Any FT issues?**	**Action**
1.	$8\sqrt{2}$	5.65685... *(incorrect decimal value)*	No. Last part in question.	Award ***A1*** for the final mark *(condone the incorrect further working)*
2.	$\frac{35}{72}$	0.468111… (incorrect decimal value)	Yes. Value is used in subsequent parts.	Award ***A0*** for the final mark *(and full **FT** is available in subsequent parts)*

3 Implied marks

Implied marks appear in **brackets e.g.** ***(M1)***,and can only be awarded if **correct** work is seen or implied by subsequent working/answer.

4 Follow through marks (only applied after an error is made)

Follow through (***FT***) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) (e.g. incorrect value from part (a) used in part (d) or incorrect value from part (c)(i) used in part (c)(ii)). Usually, to award ***FT*** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if all the marks awarded in a subsequent part are for the answer or are implied, then ***FT*** marks should be awarded for *their* correct answer, even when working is not present.

For example: following an incorrect answer to part (a) that is used in subsequent parts, where the markscheme for the subsequent part is ***(M1)A1***, it is possible to award full marks for *their* correct answer, **without working being seen.** For longer questions where all but the answer marks are implied this rule applies but may be overwritten by a **Note** in the Markscheme.

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks, by reflecting on what each mark is for and how that maps to the simplified version.
- If the error leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word “their” in a description, to indicate that candidates may be using an incorrect value.
- If the candidate’s answer to the initial question clearly contradicts information given in the question, it is not appropriate to award any ***FT*** marks in the subsequent parts. This includes when candidates fail to complete a “show that” question correctly, and then in subsequent parts use their incorrect answer rather than the given value.
- Exceptions to these ***FT*** rules will be explicitly noted on the markscheme.
- If a candidate makes an error in one part but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the command term was “Hence”.

5 Mis-read

If a candidate incorrectly copies values or information from the question, this is a mis-read (***MR***). A candidate should be penalized only once for a particular misread. Use the ***MR*** stamp to indicate that this has been a misread and do not award the first mark, even if this is an ***M*** mark, but award all others as appropriate.

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.
- If a candidate uses a correct answer, to a "show that" question, to a higher degree of accuracy than given in the question, this is NOT a misread and full marks may be scored in the subsequent part.
- ***MR*** can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If the command term is 'Hence' and not 'Hence or otherwise' then alternative methods are not permitted unless covered by a note in the mark scheme.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**.

7 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation** for example 1.9 and $1{,}9$ or 1000 and $1{,}000$ and 1.000.
- Do not accept final answers written using calculator notation. However, ***M*** marks and intermediate ***A*** marks can be scored, when presented using calculator notation, provided the evidence clearly reflects the demand of the mark.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, some **equivalent** answers will generally appear in brackets. Not all equivalent notations/answers/methods will be presented in the markscheme and examiners are asked to apply appropriate discretion to judge if the candidate work is equivalent.

8 Format and accuracy of answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. If the level of accuracy is not stated in the question, the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures.*

Where values are used in subsequent parts, the markscheme will generally use the exact value, however candidates may also use the correct answer in subsequent parts. The markscheme will often explicitly include the subsequent values that come "*from the use of 3 sf values*".

Simplification of final answers: Candidates are advised to give final answers using good mathematical form. In general, for an ***A*** mark to be awarded, arithmetic should be completed, and any values that lead to integers should be simplified; for example, $\sqrt{\frac{25}{4}}$ should be written as $\frac{5}{2}$. An exception to this is simplifying fractions, where lowest form is not required (although the numerator and the denominator must be integers); for example, $\frac{10}{4}$ may be left in this form or written as $\frac{5}{2}$. However, $\frac{10}{5}$ should be written as 2, as it simplifies to an integer.

Algebraic expressions should be simplified by completing any operations such as addition and multiplication, e.g. $4e^{2x} \times e^{3x}$ should be simplified to $4e^{5x}$, and $4e^{2x} \times e^{3x} - e^{4x} \times e^{x}$ should be simplified to $3e^{5x}$. Unless specified in the question, expressions do not need to be factorized, nor do factorized expressions need to be expanded, so $x(x+1)$ and $x^2 + x$ are both acceptable.

Please note: intermediate ***A*** marks do NOT need to be simplified.

9 Calculators

A GDC is required for this paper, but If you see work that suggests a candidate has used any calculator not approved for IB DP examinations (eg CAS enabled devices), please follow the procedures for malpractice.

10. Presentation of candidate work

Crossed out work: If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work unless an explicit note from the candidate indicates that they would like the work to be marked.

More than one solution: Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise. If the layout of the responses makes it difficult to judge, examiners should apply appropriate discretion to judge which is "first".

1. (a) **EITHER**
annual cycle for daylight length ***R1***

OR
there is a minimum length for daylight (cannot be negative) ***R1***

OR
a quadratic could not have a maximum and a minimum or equivalent ***R1***

Note: Do not accept "Paula's model is better".

[1 mark]

(b) (i) 4 ***A1***

(ii) 12 ***A1***

(iii) $y = 12$ ***A1A1***

Note: Award ***A1*** "$y =$ (a constant)" and ***A1*** for that constant being 12.

[4 marks]

(c) $f(t) = -4\cos(30t) + 12$ **OR** $f(t) = -4\cos(-30t) + 12$ ***A1A1A1***

Note: Award ***A1*** for $b = 30$ (or $b = -30$), ***A1*** for $a = -4$, and ***A1*** for $d = 12$. Award at most ***A1A1A0*** if extra terms are seen or form is incorrect. Award at most ***A1A1A0*** if x is used instead of t.

[3 marks]

(d) $10.5 = -4\cos(30t) + 12$ ***(M1)***

EITHER
$t_1 = 2.26585\ldots,\ t_2 = 9.73414\ldots$ ***(A1)(A1)***

OR
$t_1 = \frac{1}{30}\cos^{-1}\frac{3}{8}$ ***(A1)***
$t_2 = 12 - t_1$ ***(A1)***

THEN
$9.73414\ldots - 2.26585\ldots$
7.47 $(7.46828\ldots)$ months $(0.622356\ldots$years$)$ ***A1***

Note: Award ***M1A1A1A0*** for an unsupported answer of 7.46. If there is only one intersection point, award ***M1A1A0A0***.

[4 marks]

continued…

Question 1 continued

(e) $\left|\dfrac{16-\left(16+\dfrac{14}{60}\right)}{16+\dfrac{14}{60}}\right|\times 100\%$ ***(M1)(M1)***

Note: Award ***M1*** for correct values and absolute value signs, ***M1*** for $\times 100$.

$=1.44\%\ \ (1.43737...\%)$ ***A1***

[3 marks]

[Total 15 marks]

2. (a) (i) 30 ***A1***

(ii) 40 ***A1***

[2 marks]

(b) arithmetic formula chosen ***(M1)***

(i) $w_n = 20+(n-1)10 \quad (=10+10n)$ ***A1***

(ii) $l_n = 30+(n-1)10 \quad (=20+10n)$ ***A1***

[3 marks]

(c) (i) $740 = 30+(n-1)10$ **OR** $740 = 20+10n$ ***M1***

$n = 72$ ***A1***

144 tiles ***AG***

Note: The ***AG*** line must be stated for the final ***A1*** to be awarded.

(ii) $w_{72} = 730$ ***A1***

[3 marks]

(d) $(10\times 20)\times 144$ ***(M1)***

$=28800$ ***(A1)***

$2.88\times 10^4 \ \ \text{cm}^2$ ***A1***

Note: Follow through within the question for correctly converting *their* intermediate value into standard form (but only if the pre-conversion value is seen).

[3 marks]

continued…

Question 2 continued

(e) **EITHER**

1 square metre $= 100\,\text{cm} \times 100\,\text{cm}$ ***(M1)***

(so, 50 tiles) and hence 10 packs of tiles in a square metre ***(A1)***

(so each pack is $\frac{\$24.50}{10 \text{ packs}}$)

OR

area covered by one pack of tiles is $(0.2\,\text{m} \times 0.1\,\text{m} \times 5 =)\ 0.1\ \text{m}^2$ ***(A1)***

24.5×0.1 ***(M1)***

THEN

$2.45 per pack (of 5 tiles) ***A1***

[3 marks]

(f) $\frac{1.08 \times 144}{5}\ (= 31.104)$ ***(M1)(M1)***

Note: Award ***M1*** for correct numerator, ***M1*** for correct denominator.

32 (packs of tiles) ***A1***

[3 marks]

(g) $35 + (32 \times 2.45)$ ***(M1)***

$113 (113.4) ***A1***

[2 marks]

[Total 19 marks]

3. (a) (i) $\frac{370+472}{2}$ ***(M1)***

Note: This ***(M1)*** can also be awarded for either a correct Q_3 or a correct Q_1 in part (a)(ii).

$Q_3 = 421$ ***A1***

(ii) their part (a)(i) – their Q_1 (clearly stated) ***(M1)***

IQR $= (421 - 318 =)\ 103$ ***A1***

[4 marks]

(b) $(Q_3 + 1.5(\text{IQR}) =)\ 421 + (1.5 \times 103)$ ***(M1)***

$= 575.5$

since $498 < 575.5$ ***R1***

Netherlands is not an outlier ***A1***

Note: The ***R1*** is dependent on the ***(M1)***. Do not award ***R0A1***.

[3 marks]

(c) not appropriate ("no" is sufficient) ***A1***

as r is too close to zero / too weak a correlation ***R1***

[2 marks]

(d) (i) 6 ***A1***

(ii) 4.5 ***A1***

(iii) 4.5 ***A1***

[3 marks]

(e) (i) $r_s = 0.683\ \ (0.682646...)$ ***A2***

(ii) **EITHER**

there is a (positive) association between the population size and the score ***A1***

OR

there is a (positive) linear correlation between the ranks of the population size and the ranks of the scores (when compared with the PMCC of 0.249). ***A1***

[3 marks]

(f) lowering the top score by 20 does not change its rank so r_s is unchanged ***R1***

Note: Accept "this would not alter the rank" or "Netherlands still top rank" or similar. Condone any statement that clearly implies the ranks have not changed, for example: "The Netherlands still has the highest score."

[1 mark]

[Total 16 marks]

4. (a) (i) $\left(\frac{1}{2}\text{AÔB} =\right) \arccos\left(\frac{4}{4.5}\right) = 27.266...$ ***(M1)(A1)***

$\text{AÔB} = 54.532... \approx 54.5^\circ \ (0.951764... \approx 0.952 \text{ radians})$ ***A1***

Note: Other methods may be seen; award ***(M1)(A1)*** for use of a correct trigonometric method to find an appropriate angle and then ***A1*** for the correct answer.

(ii) finding area of triangle

EITHER

area of triangle = $\frac{1}{2} \times 4.5^2 \times \sin(54.532...)$ ***(M1)***

Note: Award ***M1*** for correct substitution into formula.

$= 8.24621... \approx 8.25 \text{ m}^2$ ***(A1)***

OR

$\text{AB} = 2 \times \sqrt{4.5^2 - 4^2} = 4.1231\ldots$

area triangle $= \frac{4.1231\ldots \times 4}{2}$ ***(M1)***

$= 8.24621... \approx 8.25 \text{ m}^2$ ***(A1)***

finding area of sector

EITHER

area of sector = $\frac{54.532...}{360} \times \pi \times 4.5^2$ ***(M1)***

$= 9.63661... \approx 9.64 \text{ m}^2$ ***(A1)***

OR

area of sector = $\frac{1}{2} \times 0.9517641.. \times 4.5^2$ ***(M1)***

$= 9.63661... \approx 9.64 \text{ m}^2$ ***(A1)***

THEN

area of segment = $9.63661... - 8.24621...$

$= 1.39 \text{ m}^2 \ (1.39040...)$ ***A1***

[8 marks]

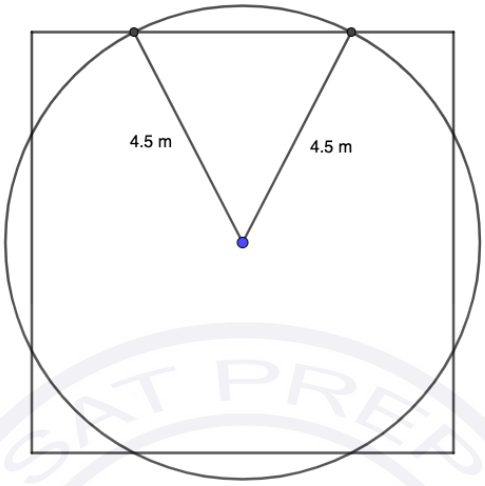
continued…

Question 4 continued

(b) (i) $\pi \times 4.5^2$ ***(M1)***

$63.6\ \text{m}^2\ \ (63.6172\ldots\ \text{m}^2)$ ***A1***

(ii) **METHOD 1**



$4 \times 1.39040...\ \ (5.56160)$ ***(A1)***

subtraction of four segments from area of circle ***(M1)***

$= 58.1\ \text{m}^2\ \ (58.055...)$ ***A1***

METHOD 2

$4(0.5 \times 4.5^2 \times \sin 54.532\ldots) + 4\left(\frac{35.4679}{360} \times \pi \times 4.5^2\right)$ ***(M1)***

$= 32.9845\ldots + 25.0707$ ***(A1)***

$= 58.1\ \text{m}^2\ \ (58.055\ldots)$ ***A1***

[5 marks]

(c) sketch of $\frac{\text{d}V}{\text{d}t}$ **OR** $\frac{\text{d}V}{\text{d}t} = 0.110363...$ **OR** attempt to find where $\frac{\text{d}^2V}{\text{d}t^2} = 0$ ***(M1)***

$t = 1$ hour ***A1***

[2 marks]

[Total 15 marks]

5. (a) (let T be the number of passengers who arrive)

$\left(\mathrm{P}(T > 72) =\right)\ \mathrm{P}(T \geq 73)$ **OR** $1 - \mathrm{P}(T \leq 72)$ ***(A1)***

$T \sim \mathrm{B}(74,\ 0.9)$ **OR** $n = 74$ ***(M1)***

$= 0.00379\ \ (0.00379124...)$ ***A1***

Note: Using the distribution $\mathrm{B}(74,\ 0.1)$, to work with the 10% that do not arrive for the flight, here and throughout this question, is a valid approach.

[3 marks]

(b) (i) 72×0.9 ***(M1)***

64.8 ***A1***

(ii) $n \times 0.9 = 72$ ***(M1)***

80 ***A1***

[4 marks]

(c) **METHOD 1**

EITHER

when selling 74 tickets

	$T \leq 72$	$T = 73$	$T = 74$
Income minus compensation (I)	11100	10800	10500
Probability	0.9962…	0.003380…	0.0004110…

top row ***A1A1***

bottom row ***A1A1***

Note: Award ***A1A1*** for each row correct. Award ***A1*** for one correct entry and ***A1*** for the remaining entries correct.

$\mathrm{E}(I) = 11100 \times 0.9962... + 10800 \times 0.00338... + 10500 \times 0.000411 \approx 11099$ ***(M1)A1***

OR

income is $74 \times 150 = 11100$ ***(A1)***

expected compensation is

$0.003380... \times 300 + 0.0004110... \times 600\ \ (= 1.26070...)$ ***(M1)A1A1***

expected income when selling 74 tickets is $11100 - 1.26070\ldots$ ***(M1)***

$= 11098.73..\ \ (= \$11099)$ ***A1***

THEN

income for 72 tickets $= 72 \times 150 = 10800$ ***(A1)***

so expected gain $\approx 11099 - 10800 = \$299$ ***A1***

continued…

Question 5 continued

METHOD 2

for 74 tickets sold, let C be the compensation paid out

$\mathrm{P}(T = 73) = 0.00338014\ldots,\ \mathrm{P}(T = 74) = 0.000411098\ldots$ ***A1A1***

$\mathrm{E}(C) = 0.003380... \times 300 + 0.0004110... \times 600\ \ (= 1.26070...)$ ***(M1)A1A1***

extra expected revenue $= 300 - 1.01404... - 0.246658\ldots\ \ (300 - 1.26070\ldots)$ ***(A1)(M1)***

Note: Award ***A1*** for the 300 and ***M1*** for the subtraction.

$= \$299$ (to the nearest dollar) ***A1***

METHOD 3

let D be the change in income when selling 74 tickets.

	$T \leq 72$	$T = 73$	$T = 74$
Change in income	300	0	–300

(A1)(A1)

Note: Award ***A1*** for one error, however award ***A1A1*** if there is no explicit mention that $T = 73$ would result in $D = 0$ and the other two are correct.

$\mathrm{P}(T \leq 73) = 0.9962...,\ \mathrm{P}(T = 74) = 0.000411098\ldots$ ***A1A1***

$\mathrm{E}(D) = 300 \times 0.9962... + 0 \times 0.003380... - 300 \times 0.0004110$ ***(M1)A1A1***

$= \$299$ ***A1***

[8 marks]

[Total 15 marks]

Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

November 2021

Mathematics: applications and interpretation

Standard level

Paper 2

15 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**.
A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.
R Marks awarded for clear **Reasoning**.
AG Answer given in the question and so no marks are awarded.
FT Follow through. The practice of awarding marks, despite candidate errors in previous parts, for their correct methods/answers using incorrect results.

Using the markscheme

1 General
Award marks using the annotations as noted in the markscheme *eg* ***M1, A2.***

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***A3***, ***M2*** *etc.*, do **not** split the marks, unless there is a note.
- The response to a "show that" question does not need to restate the ***AG*** line, unless a **Note** makes this explicit in the markscheme.
- Once a correct answer to a question or part question is seen, ignore further working even if this working is incorrect and/or suggests a misunderstanding of the question. This will encourage a uniform approach to marking, with less examiner discretion. Although some candidates may be advantaged for that specific question item, it is likely that these candidates will lose marks elsewhere too.
- An exception to the previous rule is when an incorrect answer from further working is used **in a subsequent part**. For example, when a correct exact value is followed by an incorrect decimal approximation in the first part and this approximation is then used in the second part. In this situation, award ***FT*** marks as appropriate but do not award the final ***A1*** in the first part. Examples:

	Correct answer seen	**Further working seen**	**Any FT issues?**	**Action**
1.	$8\sqrt{2}$	5.65685... *(incorrect decimal value)*	No. Last part in question.	Award ***A1*** for the final mark *(condone the incorrect further working)*
2.	$\frac{35}{72}$	0.468111… (incorrect decimal value)	Yes. Value is used in subsequent parts.	Award ***A0*** for the final mark *(and full **FT** is available in subsequent parts)*

3 Implied marks

Implied marks appear in **brackets e.g.** ***(M1)***,and can only be awarded if **correct** work is seen or implied by subsequent working/answer.

4 Follow through marks (only applied after an error is made)

Follow through (***FT***) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) (e.g. incorrect value from part (a) used in part (d) or incorrect value from part (c)(i) used in part (c)(ii)). Usually, to award ***FT*** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if all the marks awarded in a subsequent part are for the answer or are implied, then ***FT*** marks should be awarded for *their* correct answer, even when working is not present.

For example: following an incorrect answer to part (a) that is used in subsequent parts, where the markscheme for the subsequent part is ***(M1)A1***, it is possible to award full marks for *their* correct answer, **without working being seen.** For longer questions where all but the answer marks are implied this rule applies but may be overwritten by a **Note** in the Markscheme.

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks, by reflecting on what each mark is for and how that maps to the simplified version.
- If the error leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word “their” in a description, to indicate that candidates may be using an incorrect value.
- If the candidate’s answer to the initial question clearly contradicts information given in the question, it is not appropriate to award any ***FT*** marks in the subsequent parts. This includes when candidates fail to complete a “show that” question correctly, and then in subsequent parts use their incorrect answer rather than the given value.
- Exceptions to these ***FT*** rules will be explicitly noted on the markscheme.
- If a candidate makes an error in one part but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the command term was “Hence”.

5 Mis-read

If a candidate incorrectly copies values or information from the question, this is a mis-read (***MR***). A candidate should be penalized only once for a particular misread. Use the ***MR*** stamp to indicate that this has been a misread and do not award the first mark, even if this is an ***M*** mark, but award all others as appropriate.

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.
- If a candidate uses a correct answer, to a "show that" question, to a higher degree of accuracy than given in the question, this is NOT a misread and full marks may be scored in the subsequent part.
- ***MR*** can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If the command term is 'Hence' and not 'Hence or otherwise' then alternative methods are not permitted unless covered by a note in the mark scheme.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**.

7 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation** for example 1.9 and $1{,}9$ or 1000 and $1{,}000$ and 1.000.

- Do not accept final answers written using calculator notation. However, ***M*** marks and intermediate ***A*** marks can be scored, when presented using calculator notation, provided the evidence clearly reflects the demand of the mark.

- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.

- In the markscheme, some **equivalent** answers will generally appear in brackets. Not all equivalent notations/answers/methods will be presented in the markscheme and examiners are asked to apply appropriate discretion to judge if the candidate work is equivalent.

8 Format and accuracy of answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. If the level of accuracy is not stated in the question, the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures.*

Where values are used in subsequent parts, the markscheme will generally use the exact value, however candidates may also use the correct answer to 3 sf in subsequent parts. The markscheme will often explicitly include the subsequent values that come "*from the use of 3 sf values*".

Simplification of final answers: Candidates are advised to give final answers using good mathematical form. In general, for an ***A*** mark to be awarded, arithmetic should be completed, and any values that lead to integers should be simplified; for example, $\sqrt{\frac{25}{4}}$ should be written as $\frac{5}{2}$. An exception to this is simplifying fractions, where lowest form is not required (although the numerator and the denominator must be integers); for example, $\frac{10}{4}$ may be left in this form or written as $\frac{5}{2}$. However, $\frac{10}{5}$ should be written as 2, as it simplifies to an integer.

Algebraic expressions should be simplified by completing any operations such as addition and multiplication, e.g. $4\mathrm{e}^{2x} \times \mathrm{e}^{3x}$ should be simplified to $4\mathrm{e}^{5x}$, and $4\mathrm{e}^{2x} \times \mathrm{e}^{3x} - \mathrm{e}^{4x} \times \mathrm{e}^{x}$ should be simplified to $3\mathrm{e}^{5x}$. Unless specified in the question, expressions do not need to be factorized, nor do factorized expressions need to be expanded, so $x(x+1)$ and $x^2 + x$ are both acceptable.

Please note: intermediate ***A*** marks do NOT need to be simplified.

9 Calculators

A GDC is required for this paper, but If you see work that suggests a candidate has used any calculator not approved for IB DP examinations (eg CAS enabled devices), please follow the procedures for malpractice.

10. Presentation of candidate work

Crossed out work: If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work unless an explicit note from the candidate indicates that they would like the work to be marked.

More than one solution: Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise. If the layout of the responses makes it difficult to judge, examiners should apply appropriate discretion to judge which is "first".

1. (a) (i) $\frac{560}{1280}\left(\frac{7}{16}, 0.4375\right)$ ***A1A1***

Note: Award ***A1*** for correct numerator, ***A1*** for correct denominator.

(ii) $\frac{72}{1280}\left(\frac{9}{160}, 0.05625\right)$ ***A1A1***

Note: Award ***A1*** for correct numerator, ***A1*** for correct denominator.

(iii) $\frac{153}{348}\left(\frac{51}{116}, 0.439655...\right)$ ***A1A1***

Note: Award ***A1*** for correct numerator, ***A1*** for correct denominator.

(iv) $160+224+128+205+131$ **OR** $560+512-224$ ***(M1)***

$\frac{848}{1280}\left(\frac{53}{80}, 0.6625\right)$ ***A1A1***

Note: Award ***A1*** for correct denominator (1280) seen, ***(M1)*** for correct calculation of the numerator, ***A1*** for the correct answer.

[9 marks]

(b) H_0 : the variables are independent

H_1 : the variables are dependent ***A1***

Note: Award ***A1*** for both hypotheses correct. Do not accept "not correlated" or "not related" in place of "independent".

[1 mark]

(c) 4 ***A1***

[1 mark]

(d) (i) $(\chi^2 =)\ 23.3\ (23.3258...)$ ***A2***

(ii) $0.000109\ (0.000108991...)$ **OR** 1.09×10^{-4} ***A1***

(iii) **EITHER**

$23.3 > 13.277$ ***R1***

OR

$0.000109 < 0.01$ ***R1***

THEN

(there is sufficient evidence to accept H_1 that) preferred device and age group are not independent ***A1***

Note: For the final ***A1*** the answer must be in context. Do not award ***A1R0***.

[5 marks]

Total: [16 marks]

2. (a) $\frac{12669-12300}{12300}\times 100$ ***(M1)***

3% ***A1***

[2 marks]

(b) (i) 1.03 ***A1***

Note: Follow through from part (a).

(ii) $(u_n =)\ 12300\times 1.03^{n-1}$ ***A1***

(iii) $(u_{11} =) 12300\times 1.03^{10}$ ***(M1)***

16530 ***A1***

Note: Answer must be to the nearest integer. Do not accept 16500.

[4 marks]

(c) $(v_n =)\ 10380+600(n-1)$ **OR** $600n+9780$ ***M1A1***

Note: Award ***M1*** for substituting into arithmetic sequence formula, ***A1*** for correct substitution.

[2 marks]

(d) $80\times\frac{10}{2}\left(2(10380)+9(600)\right)$ ***(M1)(M1)***

Note: Award ***(M1)*** for multiplying by 80 and ***(M1)*** for substitution into sum of arithmetic sequence formula.

$\$10\,500\,000\ (\$10\,464\,000)$ ***A1***

[3 marks]

continued…

Question 2 continued

(e) $12300\times1.03^{n-1} < 10380 + 600(n-1)$ or equivalent ***(M1)***

Note: Award ***M1*** for equating their expressions from parts (b) and (c).

EITHER

graph showing $y = 12300\times1.03^{n-1}$ and $y = 10380 + 600(n-1)$ ***(M1)***

OR

graph showing $y = 12300\times1.03^{n-1} - \left(10380 + 600(n-1)\right)$ ***(M1)***

OR

list of values including, $(u_n =)\ 17537$ and $(v_n =)\ 17580$ ***(M1)***

OR

$12.4953...$ from graphical method or solving numerical equality ***(M1)***

Note: Award ***(M1)*** for a valid attempt to solve.

THEN

$(k =)\ 13$ ***A1***

[3 marks]

(f) this will not guarantee enough places. ***A1***

EITHER

A written statement that $u_n > v_n$, with range of n. ***R1***

Example: "when $n = 24$ (or greater), the number of applications will exceed the number of places again" ("$u_n > v_n$, $n \geq 24$").

OR

exponential growth will always exceed linear growth ***R1***

Note: Accept an equivalent sketch. Do not award ***A1R0***.

[2 marks]

Total: [16 marks]

3. (a) (i) maximum $h = 130$ metres ***A1***

(ii) minimum $h = 50$ metres ***A1***

[2 marks]

(b) (i) $(60 \div 12 =)\ 5$ seconds ***A1***

(ii) $360 \div 5$ ***(M1)***

Note: Award ***(M1)*** for 360 divided by their time for one revolution.

$= 72^\circ$ ***A1***

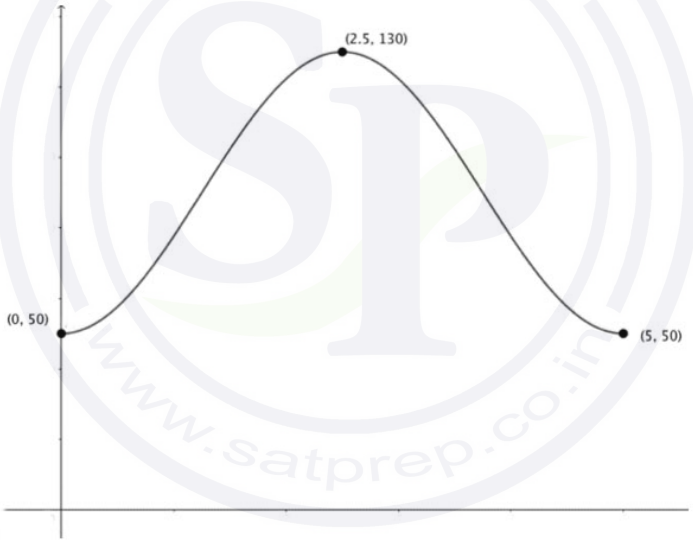
[3 marks]

(c) (i) (amplitude $=)\ 40$ ***A1***

(ii) (period $= \frac{360}{72} =\ )\ 5$ ***A1***

[2 marks]

(d)



Maximum point labelled with correct coordinates. ***A1***

At least one minimum point labelled. Coordinates seen for any minimum points must be correct. ***A1***

Correct shape with an attempt at symmetry and “concave up" evident as it approaches the minimum points. Graph must be drawn in the given domain. ***A1***

[3 marks]

continued…

Question 3 continued

(e) (i) $h = 90 - 40\cos(144^\circ)$ ***(M1)***

$(h =)\ 122\,\text{m}\ \ (122.3606.....)$ ***A1***

(ii) evidence of $h = 100$ on graph **OR** $100 = 90 - 40\cos(72t)$ ***(M1)***

t coordinates $3.55\ (3.54892...)$ **OR** $1.45\ (1.45107...)$ or equivalent ***(A1)***

Note: Award ***A1*** for either t-coordinate seen.

$= 2.10$ seconds $(2.09784\ldots)$ ***A1***

[5 marks]

(f) (i) $5 - 2.09784\ldots$ ***(M1)***

$\dfrac{(2.902153...)}{5}$ ***(M1)***

$0.580\ \ (0.580430...)$ ***A1***

(ii) **METHOD 1**

changing the frequency/dilation of the graph will not change the proportion of time that point C is visible. ***A1***

$0.580\ \ (0.580430...)$ ***A1***

METHOD 2

correct calculation of relevant found values

$\dfrac{(2.902153...)/2}{5/2}$ ***A1***

$0.580\ \ (0.580430...)$ ***A1***

Note: Award ***A0A1*** for an unsupported correct probability.

[5 marks]

Total: [20 marks]

4. (a) $\tan(\theta) = \frac{6}{10}$ ***(M1)***

$(\theta =)\ 31.0^\circ\ (30.9637...^\circ)$ **OR** $0.540\ (0.540419\ldots)$ ***A1***

[2 marks]

(b) (i) $(\text{CV} =)\ 40\tan(\theta)$ **OR** $(\text{CV} =)\ 4\times 6$ ***(M1)***

Note: Award ***(M1)*** for an attempt at trigonometry or similar triangles (e.g. ratios).

$(\text{CV} =)\ 24\ \text{m}$ ***A1***

(ii) $(V =)\ \frac{1}{3}80^2\times 24 - \frac{1}{3}60^2\times 18$ ***M1A1A1***

Note: Award ***M1*** for finding the difference between the volumes of two pyramids, ***A1*** for each correct volume expression. The final ***A1*** is contingent on correct working leading to the given answer.
If the correct final answer is not seen, award at most ***M1A1A0***. Award ***M0A0A0*** for any height derived from $V = 29\,600$, including 18.875 or 13.875.

$(V =)\ 29\,600\ \text{m}^3$ ***AG***

[5 marks]

(c) **METHOD 1**

$\left(\frac{29\,600}{80} =\right)\ 370$ (days) ***A1***

$(370 > 366)$ Joshua is correct ***A1***

Note: Award ***A0A0*** for unsupported answer of "Joshua is correct". Accept $1.01... > 1$ for the first ***A1*** mark.

METHOD 2

$80\times 366 = 29280\ \text{m}^3$ **OR** $80\times 365 = 29200\ \text{m}^3$ ***A1***

$(29280 < 29600)$ Joshua is correct ***A1***

Note: The second ***A1*** can be awarded for an answer consistent with their result.

[2 marks]

continued…

Question 4 continued

(d) height of trapezium is $\sqrt{10^2+6^2}$ $(=11.6619...)$ ***(M1)***

area of trapezium is $\frac{80+60}{2}\times\sqrt{10^2+6^2}$ $(=816.333...)$ ***(M1)(A1)***

$(SA=)$ $4\times\left(\frac{80+60}{2}\times\sqrt{10^2+6^2}\right)+60^2$ ***(M1)***

Note: Award ***M1*** for adding 4 times their (MNOP) trapezium area to the area of the (60×60) base.

$(SA=)$ $6870\ \text{m}^2$ $(6865.33\ \text{m}^2)$ ***A1***

Note: No marks are awarded if the correct shape is not identified.

[5 marks]
Total: [14 marks]

5. (a) (i) Let X be the random variable "distance from O".

$X \sim \mathrm{N}(10, 3^2)$

$\mathrm{P}(X < 13) = 0.841 \ (0.841344...)$ ***(M1)A1***

(ii) $\left(\mathrm{P}(X > 15) =\right) \ 0.0478 \ (0.0477903)$ ***A1***

[3 marks]

(b) $\mathrm{P}(X > 15) \times \mathrm{P}(X > 15)$ ***(M1)***

$= 0.00228 \ (0.00228391...)$ ***A1***

[2 marks]

(c) $1 - (0.8143)^3$ ***(M1)***

$0.460 \ (0.460050...)$ ***A1***

[2 marks]

(d) (i) **METHOD 1**

let Y be the random variable "number of points scored"

evidence of use of binomial distribution ***(M1)***

$Y \sim \mathrm{B}(10, 0.539949...)$ ***(A1)***

$\left(\mathrm{P}(Y \geq 5) =\right) \ 0.717 \ (0.716650...)$. ***A1***

METHOD 2

let Q be the random variable "number of times a point is not scored"

evidence of use of binomial distribution ***(M1)***

$Q \sim \mathrm{B}(10, 0.460050...)$ ***(A1)***

$\left(\mathrm{P}(Q \leq 5) =\right) \ 0.717 \ (0.716650...)$ ***A1***

(ii) $\mathrm{P}(5 \leq Y < 8)$ ***(M1)***

$0.628 \ (0.627788...)$ ***A1***

Note: Award ***M1*** for a correct probability statement or indication of correct lower and upper bounds, 5 and 7.

(iii) $\dfrac{\mathrm{P}(5 \leq Y < 8)}{\mathrm{P}(Y \geq 5)} \left(= \dfrac{0.627788...}{0.716650...}\right)$ ***(M1)***

$0.876 \ (0.876003...)$ ***A1***

[7 marks]

Total: [14 marks]

Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

May 2021

Mathematics: applications and interpretation

Standard level

Paper 2

17 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**.
A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.
R Marks awarded for clear **Reasoning**.
AG Answer given in the question and so no marks are awarded.
FT Follow through. The practice of awarding marks, despite candidate errors in previous parts, for their correct methods/answers using incorrect results.

Using the markscheme

1 General

Award marks using the annotations as noted in the markscheme *eg* ***M1, A2.***

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***A3***, ***M2*** *etc.*, do **not** split the marks, unless there is a note.
- The response to a "show that" question does not need to restate the ***AG*** line, unless a **Note** makes this explicit in the markscheme.
- Once a correct answer to a question or part question is seen, ignore further working even if this working is incorrect and/or suggests a misunderstanding of the question. This will encourage a uniform approach to marking, with less examiner discretion. Although some candidates may be advantaged for that specific question item, it is likely that these candidates will lose marks elsewhere too.
- An exception to the previous rule is when an incorrect answer from further working is used **in a subsequent part**. For example, when a correct exact value is followed by an incorrect decimal approximation in the first part and this approximation is then used in the second part. In this situation, award ***FT*** marks as appropriate but do not award the final ***A1*** in the first part. Examples:

	Correct answer seen	**Further working seen**	**Any FT issues?**	**Action**
1.	$8\sqrt{2}$	5.65685... *(incorrect decimal value)*	No. Last part in question.	Award ***A1*** for the final mark *(condone the incorrect further working)*
2.	$\frac{35}{72}$	0.468111… (incorrect decimal value)	Yes. Value is used in subsequent parts.	Award ***A0*** for the final mark *(and full **FT** is available in subsequent parts)*

3 Implied marks

Implied marks appear in **brackets e.g.** ***(M1)***,and can only be awarded if **correct** work is seen or implied by subsequent working/answer.

4 Follow through marks (only applied after an error is made)

Follow through (***FT***) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) (e.g. incorrect value from part (a) used in part (d) or incorrect value from part (c)(i) used in part (c)(ii)). Usually, to award ***FT*** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if all the marks awarded in a subsequent part are for the answer or are implied, then ***FT*** marks should be awarded for *their* correct answer, even when working is not present.

For example: following an incorrect answer to part (a) that is used in subsequent parts, where the markscheme for the subsequent part is ***(M1)A1***, it is possible to award full marks for *their* correct answer, **without working being seen.** For longer questions where all but the answer marks are implied this rule applies but may be overwritten by a **Note** in the Markscheme.

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate.

- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks, by reflecting on what each mark is for and how that maps to the simplified version.

- If the error leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).

- The markscheme may use the word “their” in a description, to indicate that candidates may be using an incorrect value.

- If the candidate’s answer to the initial question clearly contradicts information given in the question, it is not appropriate to award any ***FT*** marks in the subsequent parts. This includes when candidates fail to complete a “show that” question correctly, and then in subsequent parts use their incorrect answer rather than the given value.

- Exceptions to these ***FT*** rules will be explicitly noted on the markscheme.

- If a candidate makes an error in one part but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the command term was “Hence”.

5 Mis-read

If a candidate incorrectly copies values or information from the question, this is a mis-read (***MR***). A candidate should be penalized only once for a particular misread. Use the ***MR*** stamp to indicate that this has been a misread and do not award the first mark, even if this is an ***M*** mark, but award all others as appropriate.

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.
- If a candidate uses a correct answer, to a "show that" question, to a higher degree of accuracy than given in the question, this is NOT a misread and full marks may be scored in the subsequent part.
- ***MR*** can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If the command term is 'Hence' and not 'Hence or otherwise' then alternative methods are not permitted unless covered by a note in the mark scheme.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**.

7 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation** for example 1.9 and $1{,}9$ or 1000 and $1{,}000$ and 1.000.

- Do not accept final answers written using calculator notation. However, ***M*** marks and intermediate ***A*** marks can be scored, when presented using calculator notation, provided the evidence clearly reflects the demand of the mark.

- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.

- In the markscheme, some **equivalent** answers will generally appear in brackets. Not all equivalent notations/answers/methods will be presented in the markscheme and examiners are asked to apply appropriate discretion to judge if the candidate work is equivalent.

8 Format and accuracy of answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. If the level of accuracy is not stated in the question, the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures.*

Where values are used in subsequent parts, the markscheme will generally use the exact value, however candidates may also use the correct answer to 3 sf in subsequent parts. The markscheme will often explicitly include the subsequent values that come "*from the use of 3 sf values*".

Simplification of final answers: Candidates are advised to give final answers using good mathematical form. In general, for an ***A*** mark to be awarded, arithmetic should be completed, and any values that lead to integers should be simplified; for example, $\sqrt{\frac{25}{4}}$ should be written as $\frac{5}{2}$.

An exception to this is simplifying fractions, where lowest form is not required (although the numerator and the denominator must be integers); for example, $\frac{10}{4}$ may be left in this form or written as $\frac{5}{2}$. However, $\frac{10}{5}$ should be written as 2, as it simplifies to an integer.

Algebraic expressions should be simplified by completing any operations such as addition and multiplication, e.g. $4e^{2x} \times e^{3x}$ should be simplified to $4e^{5x}$, and $4e^{2x} \times e^{3x} - e^{4x} \times e^{x}$ should be simplified to $3e^{5x}$. Unless specified in the question, expressions do not need to be factorized, nor do factorized expressions need to be expanded, so $x(x+1)$ and $x^2 + x$ are both acceptable.

Please note: intermediate ***A*** marks do NOT need to be simplified.

9 Calculators

A GDC is required for this paper, but If you see work that suggests a candidate has used any calculator not approved for IB DP examinations (eg CAS enabled devices), please follow the procedures for malpractice.

10. Presentation of candidate work

Crossed out work: If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work unless an explicit note from the candidate indicates that they would like the work to be marked.

More than one solution: Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise. If the layout of the responses makes it difficult to judge, examiners should apply appropriate discretion to judge which is "first".

1. (a) convenience sampling ***A1***

[1 mark]

(b) (i) 95% ***A1***

(ii) 1% ***A1***

(iii) 2% ***A1***

(iv) 98% ***A1***

[4 marks]

(c) (i) 0.95×0.02 ***(M1)***

0.019 ***A1***

(ii) $0.05 \times 0.01 + 0.95 \times 0.98$ ***(M1)(M1)***

Note: Award ***M1*** for summing two products and ***M1*** for correct products seen.

$0.932 \quad (0.9315)$ ***A1***

(iii) recognition of conditional probability ***(M1)***

$$\frac{0.05 \times 0.01}{0.05 \times 0.01 + 0.95 \times 0.98}$$ ***A1***

$0.000537 \quad (0.000536768\ldots)$ ***A1***

Note: Accept 0.000536 if 0.932 used.

[8 marks]

(d) **EITHER**

sample may not be representative of population ***A1***

OR

sample is not randomly selected ***A1***

OR

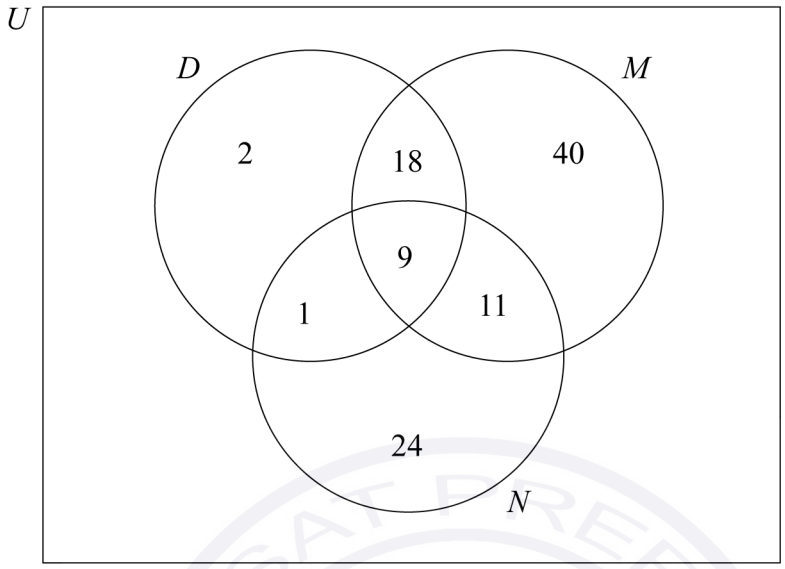
unrealistic to think expected and observed values will be exactly equal ***A1***

[1 mark]

continued…

Question 1 continued

(e)



A1A1A1

Note: Award ***A1*** for rectangle and 3 labelled circles and 9 in centre region; ***A1*** for 2, 40, 24; ***A1*** for 18, 1, and 11.

[3 marks]

(f) $18+9+1+11+2+40+24$ ***(M1)***

105 ***A1***

Note: Follow through from the entries on their Venn diagram in part (e). Working required for ***FT***.

[2 marks]

Total [19 marks]

2. (a) use of cosine rule ***(M1)***

$\hat{ACB} = \cos^{-1}\left(\frac{1005^2 + 1225^2 - 650^2}{2 \times 1005 \times 1225}\right)$ ***(A1)***

$= 32.0^\circ \ (31.9980...)$ ***A1***

[3 marks]

(b) use of sine rule ***(M1)***

$\frac{DE}{\sin 31.9980...^\circ} = \frac{210}{\sin 100^\circ}$ ***(A1)***

$(DE =)\ 113 \text{ m}\ (112.9937...)$ ***A1***

[3 marks]

(c) **METHOD 1**

$180^\circ - (100^\circ + \text{ their part } (a))$ ***(M1)***

$= 48.0019\ldots^\circ$ **OR** $0.837791...$ ***(A1)***

substituted area of triangle formula ***(M1)***

$\frac{1}{2} \times 112.9937... \times 210 \times \sin 48.002^\circ$ ***(A1)***

$8820 \text{ m}^2\ (8817.18\ldots)$ ***A1***

METHOD 2

$\frac{CE}{\sin(180 - 100 - \text{their part } (a))} = \frac{210}{\sin 100}$ ***(M1)***

$(CE =)\ 158.472...$ ***(A1)***

substituted area of triangle formula ***(M1)***

EITHER

$\frac{1}{2} \times 112.993... \times 158.472... \times \sin 100$ ***(A1)***

OR

$\frac{1}{2} \times 210... \times 158.472... \times \sin(\text{their part } (a))$ ***(A1)***

THEN

$8820 \text{ m}^2\ (8817.18\ldots)$ ***A1***

continued…

Question 2 continued

METHOD 3

$\text{CE}^2 = 210^2 + 112.993...^2 - \left(2 \times 210 \times 112.993... \times \cos(180 - 100 - \text{their part } (a))\right)$ ***(M1)***

$(\text{CE} =)\ 158.472\ldots$ ***(A1)***

substituted area of triangle formula ***(M1)***

$\frac{1}{2} \times 112.993... \times 158.472... \times \sin 100$ ***(A1)***

$8820\ \text{m}^2\ \ (8817.18\ldots)$ ***A1***

[5 marks]

(d) $1005 - 210$ **OR** 795 ***(A1)***

equating answer to part (c) to area of a triangle formula ***(M1)***

$8817.18\ldots = \frac{1}{2} \times \text{DF} \times (1005 - 210) \times \sin 48.002...^\circ$ ***(A1)***

$(\text{DF} =)\ 29.8\ \text{m}\ \ (29.8473...)$ ***A1***

[4 marks]

Total [15 marks]

3. (a) (i) recognition of arithmetic sequence with common difference 2 ***(M1)***
use of arithmetic sequence formula ***(M1)***
$14+2(20-1)$
52 ***A1***

(ii) use of arithmetic series formula ***(M1)***
$\frac{14+52}{2}\times 20$
660 ***A1***
[5 marks]

(b) $584+(584\times 0.012)$ **OR** $584\times(1.012)^1$ ***(M1)***
$591\ (591.008)$ ***A1***

Note: Award ***M0A0*** if incorrect r used in part (b), and ***FT*** with their r in parts (c) and (d).

[2 marks]

(c) recognition of geometric sequence ***(M1)***
equating their nth geometric sequence term to their 660 ***(M1)***

Note: Accept inequality.

METHOD 1

EITHER
$660=584\times(1.012)^{x-1}$ ***A1***
$(x-1=)\ 10.3\ (10.2559\ldots)$
$x=11.3\ (11.2559\ldots)$ ***A1***
2030 ***A1***

OR
$660=584\times(1.012)^{x}$ ***A1***
$x=10.3\ (10.2559\ldots)$ ***A1***
2030 ***A1***

continued…

Question 3 continued

METHOD 2

11^{th} term 658 $(657.987...)$ ***(M1)A1***
12^{th} term 666 $(665.883...)$ ***(M1)A1***
2030 ***A1***

Note: The last mark can be awarded if both their 11^{th} and 12^{th} correct terms are seen.

[5 marks]

(d) 7 seen ***(A1)***

EITHER

$584\left(\frac{1.012^7-1}{1.012-1}\right)$ ***(M1)***

multiplying their sum by 50 ***(M1)***

OR

sum of the number of visitors for their r and their seven years ***(M1)***
multiplying their sum by 50 ***(M1)***

OR

$29\,200\left(\frac{1.012^7-1}{1.012-1}\right)$ ***(M1)(M1)***

THEN

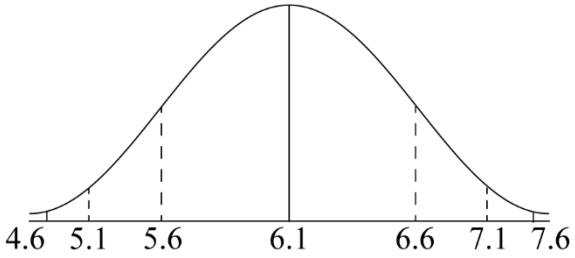
212000 $(211907.3\ldots)$ ***A1***

Note: Follow though from their r from part (b).

[4 marks]

Total [16 marks]

4. (a)



A1A1

Note: Award ***A1*** for a normal curve with mean labelled 6.1 or μ, ***A1*** for indication of SD (0.5): marks on horizontal axis at 5.6 and/or 6.6 **OR** $\mu - 0.5$ and/or $\mu + 0.5$ on the correct side and approximately correct position.

[2 marks]

(b) $X \sim \mathrm{N}(6.1,\ 0.5^2)$

$\mathrm{P}(5.5 < X < 6.5)$ **OR** labelled sketch of region ***(M1)***

$= 0.673\ \ (0.673074...)$ ***A1***

[2 marks]

(c) $(\mathrm{P}(X < 5.3) =)\ \ 0.0547992...$ ***(A1)***

$0.0547992... \times 80$ ***(M1)***

$= 4.38\ \ (4.38393...)$ ***A1***

[3 marks]

(d) 0.15 **OR** 0.85 ***(A1)***

$\mathrm{P}(X > x) = 0.15$ **OR** $\mathrm{P}(X < x) = 0.85$ **OR** labelled sketch of region ***(M1)***

$6.62\ \ (6.61821\ldots)$ ***A1***

[3 marks]

(e) $(\mathrm{P}(X > 6.25) =)\ \ 0.382088...$ ***(A1)***

recognition of binomial ***(M1)***

e.g. $\mathrm{B}(10,\ 0.382088...)$

$0.0502\ \ (0.0501768...)$ ***A2***

[4 marks]

Total [14 marks]

5. (a) evidence of splitting diagram into equilateral triangles ***M1***

area $= 6\left(\frac{1}{2}x^2 \sin 60^\circ\right)$ ***A1***

$= \frac{3\sqrt{3}x^2}{2}$ ***AG***

Note: The ***AG*** line must be seen for the final ***A1*** to be awarded.

[2 marks]

(b) total surface area of prism $1200 = 2\left(3x^2 \frac{\sqrt{3}}{2}\right) + 6xh$ ***M1A1***

Note: Award ***M1*** for expressing total surface areas as a sum of areas of rectangles and hexagons, and ***A1*** for a correctly substituted formula, equated to 1200.

$h = \frac{400 - \sqrt{3}x^2}{2x}$ ***A1***

volume of prism $= \frac{3\sqrt{3}}{2}x^2 \times h$ ***(M1)***

$= \frac{3\sqrt{3}}{2}x^2\left(\frac{400 - \sqrt{3}x^2}{2x}\right)$ ***A1***

$= 300\sqrt{3}x - \frac{9}{4}x^3$ ***AG***

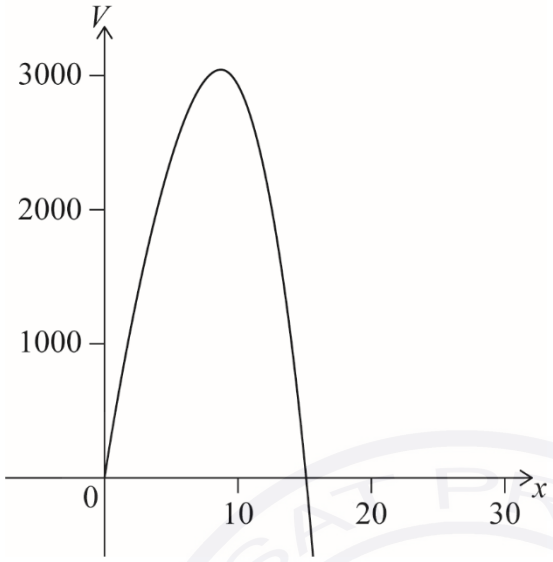
Note: The ***AG*** line must be seen for the final ***A1*** to be awarded.

[5 marks]

continued…

Question 5 continued

(c)



A1A1

Note: Award ***A1*** for correct shape, ***A1*** for roots in correct place with some indication of scale (indicated by a labelled point).

[2 marks]

(d) $\frac{\mathrm{d}V}{\mathrm{d}x} = 300\sqrt{3} - \frac{27}{4}x^2$ ***A1A1***

Note: Award ***A1*** for a correct term.

[2 marks]

(e) from the graph of V or $\frac{\mathrm{d}V}{\mathrm{d}x}$ **OR** solving $\frac{\mathrm{d}V}{\mathrm{d}x} = 0$ ***(M1)***

$x = 8.77 \ \ (8.77382...)$ ***A1***

[2 marks]

continued…

Question 5 continued

(f) from the graph of V **OR** substituting their value for x into V ***(M1)***

$V_{\max} = 3040 \text{ cm}^3$ $(3039.34...)$ ***A1***

[2 marks]

(g) **EITHER**

wasted space / spheres do not pack densely (tesselate) ***A1***

OR

the model uses exterior values / assumes infinite thinness of materials and hence the modelled volume is not the true volume ***A1***

[1 mark]

Total [16 marks]

Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

May 2021

Mathematics: applications and interpretation

Standard level

Paper 2

16 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**.
A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.
R Marks awarded for clear **Reasoning**.
AG Answer given in the question and so no marks are awarded.
FT Follow through. The practice of awarding marks, despite candidate errors in previous parts, for their correct methods/answers using incorrect results.

Using the markscheme

1 General

Award marks using the annotations as noted in the markscheme *eg* ***M1***, ***A2***.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***A3***, ***M2*** *etc.*, do **not** split the marks, unless there is a note.
- The response to a "show that" question does not need to restate the ***AG*** line, unless a **Note** makes this explicit in the markscheme.
- Once a correct answer to a question or part question is seen, ignore further working even if this working is incorrect and/or suggests a misunderstanding of the question. This will encourage a uniform approach to marking, with less examiner discretion. Although some candidates may be advantaged for that specific question item, it is likely that these candidates will lose marks elsewhere too.
- An exception to the previous rule is when an incorrect answer from further working is used **in a subsequent part**. For example, when a correct exact value is followed by an incorrect decimal approximation in the first part and this approximation is then used in the second part. In this situation, award ***FT*** marks as appropriate but do not award the final ***A1*** in the first part. Examples:

	Correct answer seen	**Further working seen**	**Any FT issues?**	**Action**
1.	$8\sqrt{2}$	5.65685... *(incorrect decimal value)*	No. Last part in question.	Award ***A1*** for the final mark *(condone the incorrect further working)*
2.	$\frac{35}{72}$	0.468111… (incorrect decimal value)	Yes. Value is used in subsequent parts.	Award ***A0*** for the final mark *(and full **FT** is available in subsequent parts)*

3 Implied marks

Implied marks appear in **brackets e.g.** ***(M1)***,and can only be awarded if **correct** work is seen or implied by subsequent working/answer.

4 Follow through marks (only applied after an error is made)

Follow through (***FT***) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) (e.g. incorrect value from part (a) used in part (d) or incorrect value from part (c)(i) used in part (c)(ii)). Usually, to award ***FT*** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if all the marks awarded in a subsequent part are for the answer or are implied, then ***FT*** marks should be awarded for *their* correct answer, even when working is not present.

For example: following an incorrect answer to part (a) that is used in subsequent parts, where the markscheme for the subsequent part is ***(M1)A1***, it is possible to award full marks for *their* correct answer, **without working being seen.** For longer questions where all but the answer marks are implied this rule applies but may be overwritten by a **Note** in the Markscheme.

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks, by reflecting on what each mark is for and how that maps to the simplified version.
- If the error leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word “their” in a description, to indicate that candidates may be using an incorrect value.
- If the candidate’s answer to the initial question clearly contradicts information given in the question, it is not appropriate to award any ***FT*** marks in the subsequent parts. This includes when candidates fail to complete a “show that” question correctly, and then in subsequent parts use their incorrect answer rather than the given value.
- Exceptions to these ***FT*** rules will be explicitly noted on the markscheme.
- If a candidate makes an error in one part but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the command term was “Hence”.

5 Mis-read

If a candidate incorrectly copies values or information from the question, this is a mis-read (***MR***). A candidate should be penalized only once for a particular misread. Use the ***MR*** stamp to indicate that this has been a misread and do not award the first mark, even if this is an ***M*** mark, but award all others as appropriate.

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.
- If a candidate uses a correct answer, to a "show that" question, to a higher degree of accuracy than given in the question, this is NOT a misread and full marks may be scored in the subsequent part.
- ***MR*** can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If the command term is 'Hence' and not 'Hence or otherwise' then alternative methods are not permitted unless covered by a note in the mark scheme.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**.

7 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation** for example 1.9 and $1{,}9$ or 1000 and $1{,}000$ and 1.000.

- Do not accept final answers written using calculator notation. However, ***M*** marks and intermediate ***A*** marks can be scored, when presented using calculator notation, provided the evidence clearly reflects the demand of the mark.

- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.

- In the markscheme, some **equivalent** answers will generally appear in brackets. Not all equivalent notations/answers/methods will be presented in the markscheme and examiners are asked to apply appropriate discretion to judge if the candidate work is equivalent.

8 Format and accuracy of answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. If the level of accuracy is not stated in the question, the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures.*

Where values are used in subsequent parts, the markscheme will generally use the exact value, however candidates may also use the correct answer to 3 sf in subsequent parts. The markscheme will often explicitly include the subsequent values that come "*from the use of 3 sf values*".

Simplification of final answers: Candidates are advised to give final answers using good mathematical form. In general, for an ***A*** mark to be awarded, arithmetic should be completed, and any values that lead to integers should be simplified; for example, $\sqrt{\frac{25}{4}}$ should be written as $\frac{5}{2}$.

An exception to this is simplifying fractions, where lowest form is not required (although the numerator and the denominator must be integers); for example, $\frac{10}{4}$ may be left in this form or written as $\frac{5}{2}$. However, $\frac{10}{5}$ should be written as 2, as it simplifies to an integer.

Algebraic expressions should be simplified by completing any operations such as addition and multiplication, e.g. $4e^{2x} \times e^{3x}$ should be simplified to $4e^{5x}$, and $4e^{2x} \times e^{3x} - e^{4x} \times e^{x}$ should be simplified to $3e^{5x}$. Unless specified in the question, expressions do not need to be factorized, nor do factorized expressions need to be expanded, so $x(x+1)$ and $x^2 + x$ are both acceptable.

Please note: intermediate ***A*** marks do NOT need to be simplified.

9 Calculators

A GDC is required for this paper, but If you see work that suggests a candidate has used any calculator not approved for IB DP examinations (eg CAS enabled devices), please follow the procedures for malpractice.

10. Presentation of candidate work

Crossed out work: If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work unless an explicit note from the candidate indicates that they would like the work to be marked.

More than one solution: Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise. If the layout of the responses makes it difficult to judge, examiners should apply appropriate discretion to judge which is "first".

1. (a) Quota sampling ***A1***

[1 mark]

(b) 10 (hours) ***A1***

[1 mark]

(c) $15-7$ ***(M1)***

Note: Award ***M1*** for 15 and 7 seen.

8 ***A1***

[2 marks]

(d) indication of a valid attempt to find the upper fence ***(M1)***

$15+1.5\times 8$

27 ***A1***

$25<27$ (accept equivalent answer in words) ***R1***

Jason is correct ***A1***

Note: Do not award ***R0A1***. Follow through **within** this part from *their* 27, but only if their value is supported by a valid attempt **or** clearly and correctly explains what their value represents.

[4 marks]

(e) "negative" seen ***A1***

Note: Strength cannot be inferred visually; ignore "strong" or "weak".

[1 mark]

(f) correct substitution ***(M1)***

$y=-1.54\times 1.5+98.8$

96.5 (%) (96.49) ***A1***

[2 marks]

(g) not reliable ***A1***

extrapolation **OR** outside the given range of the data ***R1***

Note: Do not award ***A1R0.*** Only accept reasoning that includes reference to the range of the data. Do not accept a contextual reason such as 1.5 hours is too short to read the book.

[2 marks]

continued…

Question 1 continued

(h)

	Book							
	A	**B**	**C**	**D**	**E**	**F**	**G**	**H**
Rank – Number of pages	**1**	3	5	2	6	8	4	7
Rank – Top 50 Rating	**1**	2	3	4	5	6	7	8

A1A1

Note: Award ***A1*** for correct ranks for 'number of pages'. Award ***A1*** for correct ranks for 'top 50 rating'.

[2 marks]

(i) (i) 0.714 $(0.714285...)$ ***A2***

Note: ***FT*** from their table.

(ii) **EITHER**
there is a (strong/moderate) positive association between the number of pages and the top 50 rating. ***A1***

OR
there is a (strong/moderate) agreement between the rank order of number of pages and the rank order top 50 rating. ***A1***

OR
there is a (strong/moderate) positive (linear) correlation between the rank order of number of pages and the rank order top 50 rating. ***A1***

Note: Follow through from their value of r_s .

[3 marks]

Total [18 marks]

2. (a) $4 \times \frac{360^\circ}{12}$ **OR** $4 \times 30^\circ$ ***(M1)***

120° ***A1***

[2 marks]

(b) substitution in cosine rule ***(M1)***

$AB^2 = 10^2 + 6^2 - 2 \times 10 \times 6 \times \cos(120^\circ)$ ***(A1)***

$AB = 14 \text{ cm}$ ***A1***

Note: Follow through marks in part (b) are contingent on working seen.

[3 marks]

(c) $\theta = 13 \times 6$ ***(M1)***

$= 78^\circ$ ***A1***

[2 marks]

(d) substitution into the formula for arc length ***(M1)***

$l = \frac{78}{360} \times 2 \times \pi \times 10$ **OR** $l = \frac{13\pi}{30} \times 10$

$= 13.6 \text{ cm} \left(13.6135..., 4.33\pi, \frac{13\pi}{3} \right)$ ***A1***

[2 marks]

(e) substitution into the area of a sector ***(M1)***

$A = \frac{78}{360} \times \pi \times 10^2$ **OR** $l = \frac{1}{2} \times \frac{13\pi}{30} \times 10^2$

$= 68.1 \text{ cm}^2 \left(68.0678..., 21.7\pi, \frac{65\pi}{3} \right)$ ***A1***

[2 marks]

(f) 23 ***A1***

[1 mark]

continued…

Question 2 continued

(g) correct substitution ***(M1)***

$h = 10\cos(160^\circ) + 13$

$= 3.60 \text{ cm } (3.60307...)$ ***A1***

[2 marks]

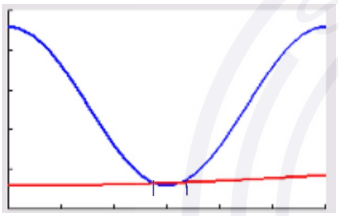
(h) 10 ***A1***

[1 mark]

(i) **EITHER**

$10 \times \cos(\theta) + 13 = -10 \times \cos\left(\frac{\theta}{12}\right) + 13$ ***(M1)***

OR



(M1)

Note: Award ***M1*** for equating the functions. Accept a sketch of $h(\theta)$ and $g(\theta)$ with point(s) of intersection marked.

THEN

$k = 196^\circ \ (196.363...)$ ***A1***

Note: The answer $166.153\ldots$ is incorrect but the correct method is implicit. Award ***(M1)A0***.

[2 marks]

Total [17 marks]

3. (a) **EITHER**

$N = 2$
$PV = -37\,000$
$I\% = 6.4$
$P/Y = 1$
$C/Y = 4$ ***(M1)(A1)***

Note: Award ***M1*** for an attempt to use a financial app in their technology, award ***A1*** for all entries correct.

OR

$N = 8$
$PV = -37\,000$
$I\% = 6.4$
$P/Y = 4$
$C/Y = 4$ ***(M1)(A1)***

Note: Award ***M1*** for an attempt to use a financial app in their technology, award ***A1*** for all entries correct.

OR

$$FV = 37\,000 \times \left(1 + \frac{6.4}{100 \times 4}\right)^{4\times 2}$$ ***(M1)(A1)***

Note: Award ***M1*** for substitution into compound interest formula, ***(A1)*** for correct substitution.

$= 42\,010 \text{ AUD}$ ***A1***

Note: Award ***(M1)(A1)A0*** for unsupported 42009.87.

[3 marks]

(b) **EITHER**

$PV = -37\,000$
$FV = 50\,000$
$I\% = 6.4$
$P/Y = 1$
$C/Y = 4$ ***(M1)(A1)***

Note: Award ***M1*** for an attempt to use a financial app in their technology, award ***A1*** for all entries correct. The final mark can still be awarded for the correct number of months (multiple of 3).

continued…

Question 3 continued

OR

$PV = -37\,000$
$FV = 50\,000$
$I\% = 6.4$
$P/Y = 4$
$C/Y = 4$ ***(M1)(A1)***

Note: Award ***M1*** for an attempt to use a financial app in their technology, award ***A1*** for all entries correct.

OR

$50000 < 37\,000 \times \left(1 + \frac{6.4}{100 \times 4}\right)^{4 \times n}$ **OR** $50\,000 < 37\,000 \times \left(1 + \frac{6.4}{100 \times 4}\right)^{n}$ ***(M1)(A1)***

Note: Award ***M1*** for the correct inequality, $50\,000$ and substituted compound interest formula. Allow an equation. Award ***A1*** for correct substitution.

THEN

$N = 4.74$ (years) $(4.74230...)$ **OR** $N = 18.9692...$ (quarters) ***(A1)***

$m = 57$ months ***A1***

Note: Award ***A1*** for rounding their m to the correct number of months. The final answer must be a multiple of 3. Follow through within this part.

[4 marks]

(c) $150\,000$ AUD ***A1***

[1 mark]

continued…

Question 3 continued

(d) (i) $120 \times 1700 - 150\,000$ ***(M1)***

$= 54\,000$ AUD ***A1***

(ii) $N = 120$
$PV = -150\,000$
$PMT = 1700$
$FV = 0$
$P/Y = 12$
$C/Y = 12$ ***(M1)(A1)***

Note: Award ***M1*** for an attempt to use a financial app in their technology or an attempt to use an annuity formula or $FV = 0$ seen. If a compound interest formula is equated to zero, award ***M1***, otherwise award ***M0*** for a substituted compound interest formula.
Award ***A1*** for all entries correct in financial app or correct substitution in annuity formula, but award ***A0*** for a substituted compound interest formula.
Follow through marks in part (d)(ii) are contingent on working seen.

$r = 6.46$ (%) (6.45779...) ***A1***

[5 marks]

(e) $N = 60$
$I = 6.46$ (6.45779...)
$PV = -150\,000$
$PMT = 1700$
$P/Y = 12$
$C/Y = 12$ ***(M1)(A1)***

Note: Award ***M1*** for an attempt to use a financial app in their technology or an attempt to use an annuity formula. Award ***(M0)*** for a substituted compound interest formula. Award ***A1*** for all entries correct. Follow through marks in part (e) are contingent on working seen.

$FV = 86\,973$ AUD ***A1***

[3 marks]

(f) $204\,000 - (60 \times 1700 + 86\,973)$ **OR** $204\,000 - 188\,973$ ***(M1)(M1)***

Note: Award ***M1*** for 60×1700. Award ***M1*** for subtracting their $(60 \times 1700 + 86\,973)$ from their $(204\,000)$. Award at most ***M1M0*** for their $204\,000 - (60 \times 1700)$ or ***M0M0*** for their $204\,000 - (86\,973)$. Follow through from parts (d)(i) and (e).
Follow through marks in part (f) are contingent on working seen.

15 027 AUD ***A1***

[3 marks]

Total [19 marks]

4. (a) (i) evidence of correct probability ***(M1)***
e.g sketch **OR** correct probability statement, $\mathrm{P}(X < 6.5)$

0.0151 ***A1***

(ii) 0.0228 ***A1***

Note: Answers should be given to 4 decimal place.

[3 marks]

(b) (i) multiplying **their** probability by 1000 ***(M1)***
451.7 ***A1***

(ii) 510.5 ***A1***

[3 marks]

Note: Answers should be given to 4 sf.

(c) H_0 : stopping distances can be modelled by $\mathrm{N}(6.76, 0.12^2)$
H_1 : stopping distances cannot be modelled by $\mathrm{N}(6.76, 0.12^2)$ ***A1A1***

Note: Award ***A1*** for correct H_0, including reference to the mean and standard deviation.
Award ***A1*** for the negation of their H_0.

[2 marks]

(d) 15.1 or 22.8 seen ***(M1)***

0.0727 $(0.0726542\ldots, 7.27\%)$ ***A2***

[3 marks]

(e) $0.05 < 0.0727$ ***R1***
there is insufficient evidence to reject H_0 (or "accept H_0") ***A1***

Note: Do not award ***R0A1***.

[2 marks]
Total [13 marks]

5. (a) (i) evidence of power rule (at least one correct term seen) ***(M1)***

$\frac{\mathrm{d}y}{\mathrm{d}x} = -0.3x^2 + 1.6x$ ***A1***

(ii) $-0.3x^2 + 1.6x = 0$ ***M1***

$x = 5.33 \left(5.33333..., \frac{16}{3} \right)$ ***A1***

$y = -0.1 \times 5.33333...^3 + 0.8 \times 5.33333...^2$ ***(M1)***

Note: Award ***M1*** for substituting their zero for $\frac{\mathrm{d}y}{\mathrm{d}x}$ $(5.333\ldots)$ into y.

$7.59 \text{ m } (7.58519...)$ ***A1***

Note: Award ***M0A0M0A0*** for an unsupported 7.59.
Award at most ***M0A0M1A0*** if only the last two lines in the solution are seen.
Award at most ***M1A0M1A1*** if their $x = 5.33$ is not seen.

[6 marks]

(b) $A = \frac{1}{2} \times 2\big((2.4 + 0) + 2(6.4 + 7.2)\big)$ ***(A1)(M1)***

Note: Award ***A1*** for $h = 2$ seen. Award ***M1*** for correct substitution into the trapezoidal rule (the zero can be omitted in working).

$= 29.6 \text{ m}^2$ ***A1***

[3 marks]

(c) (i) $A = \int_2^8 -0.1x^3 + 0.8x^2 \, \mathrm{d}x$ **OR** $A = \int_2^8 y \, \mathrm{d}x$ ***A1A1***

Note: Award ***A1*** for a correct integral, ***A1*** for correct limits in the correct location.
Award at most ***A0A1*** if $\mathrm{d}x$ is omitted.

(ii) $A = 32.4 \text{ m}^2$ ***A2***

Note: As per the marking instructions, ***FT*** from their integral in part (c)(i).
Award at most ***A1FTA0*** if their area is >48, this is outside the constraints of the question (a 6x8 rectangle).

[4 marks]

Total [13 marks]

Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

Specimen paper

Mathematics: applications and interpretation

Standard level

Paper 2

12 pages

International Baccalaureate®
Baccalauréat International
Bachillerato Internacional

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

R Marks awarded for clear **Reasoning**.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Award marks using the annotations as noted in the markscheme eg ***M1****,* ***A2****.*

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***M2***, ***A3***, *etc.*, do **not** split the marks, unless there is a note.
- Once a correct answer to a question or part-question is seen, ignore further correct working. However, if further working indicates a lack of mathematical understanding do not award the final ***A1***. An exception to this may be in numerical answers, where a correct exact value is followed by an incorrect decimal. However, if the incorrect decimal is carried through to a subsequent part, and correct ***FT*** working shown, award ***FT*** marks as appropriate but do not award the final ***A1*** in that part.

Examples

	Correct answer seen	Further working seen	Action
1.	$8\sqrt{2}$	5.65685... *(incorrect decimal value)*	Award the final ***A1*** *(ignore the further working)*
2.	$\frac{1}{4}\sin 4x$	$\sin x$	Do not award the final ***A1***
3.	$\log a - \log b$	$\log(a-b)$	Do not award the final ***A1***

3 Implied marks

*Implied marks appear in **brackets e.g. (M1)**, and can only be awarded if **correct** work is seen or if implied in subsequent working.*

- Normally the correct work is seen or implied in the next line.
- Marks **without** brackets can only be awarded for work that is **seen**.

4 Follow through marks (only applied after an error is made)

*Follow through (**FT**) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) or subpart(s). Usually, to award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if the only marks awarded in a subpart are for the answer (i.e. there is no working expected), then **FT** marks should be awarded if appropriate.*

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*e.g.* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word “their” in a description, to indicate that candidates may be using an incorrect value.
- Exceptions to this rule will be explicitly noted on the markscheme.
- If a candidate makes an error in one part, but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the question says hence. It is often possible to use a different approach in subsequent parts that does not depend on the answer to previous parts.

5 Mis-read

*If a candidate incorrectly copies information from the question, this is a mis-read (**MR**). Apply a **MR** penalty of 1 mark to that question*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates’ own work does **not** constitute a misread, it is an error.
- The ***MR*** penalty can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme

- Alternative methods for complete questions are indicated by ***METHOD 1***, ***METHOD 2***, *etc*.
- Alternative solutions for part-questions are indicated by ***EITHER*** . . . ***OR***.

7 Alternative forms

Unless the question specifies otherwise, ***accept*** *equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

8 Accuracy of Answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. There are two types of accuracy errors, and the final answer mark should not be awarded if these errors occur.

- **Rounding errors**: only applies to final answers not to intermediate steps.
- **Level of accuracy**: when this is not specified in the question the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures*.

9 Calculators

A GDC is required for this examination, but calculators with symbolic manipulation features/ CAS functionality are not allowed.

Calculator notation
The subject guide says:

> *Students must always use correct mathematical notation, not calculator notation.*

Do **not** accept final answers written using calculator notation. However, do not penalize the use of calculator notation in the working.

1. (a) (i) $N = 24$

$I\% = 14$

$PV = -14000$

$FV = 0$

$P/Y = 4$

$C/Y = 4$ ***(M1)(A1)***

Note: Award ***M1*** for an attempt to use a financial app in their technology, award ***A1*** for all entries correct. Accept $PV = 14000$.

$(€)871.82$ ***A1***

(ii) $4 \times 6 \times 871.82$ ***(M1)***

$(€)20923.68$ ***A1***

(iii) $20923.68 - 14000$ ***(M1)***

$(€)6923.68$ ***A1***

[7 marks]

(b) (i) 0.9×14000 $(= 14000 - 0.10 \times 14000)$ ***M1***

$(€)12600.00$ ***A1***

(ii) $N = 72$

$PV = 12600$

$PMT = -250$

$FV = 0$

$P/Y = 12$

$C/Y = 12$ ***(M1)(A1)***

Note: Award ***M1*** for an attempt to use a financial app in their technology, award ***A1*** for all entries correct. Accept $PV = -12600$ provided $PMT = 250$.

$12.56(\%)$ ***A1***

[5 marks]

continued…

Question 1 continued

(c) ***EITHER***

Bryan should choose Option A ***A1***
no deposit is required ***R1***

Note: Award ***R1*** for stating that no deposit is required. Award ***A1*** for the correct choice from that fact. Do not award ***R0A1.***

OR

Bryan should choose Option B ***A1***
cost of Option A $(6923.69) >$ cost of Option B $(72 \times 250 - 12\,600 = 5400)$ ***R1***

Note: Award ***R1*** for a correct comparison of costs. Award ***A1*** for the correct choice from that comparison. Do not award ***R0A1***.

[2 marks]

(d) $14\,000\left(1-\frac{25}{100}\right)^{6}$ ***(M1)(A1)***

Note: Award ***M1*** for substitution into compound interest formula. Award ***A1*** for correct substitutions.

$= 2491.70 \text{ (USD)}$ ***A1***

OR

$N = 6$

$I\% = -25$

$PV = \pm 14\,000$

$P/Y = 1$

$C/Y = 1$ ***(A1)(M1)***

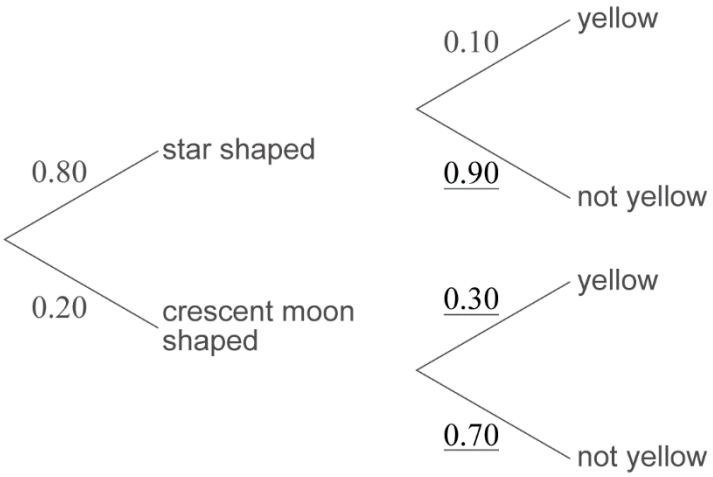
Note: Award ***A1*** for $PV = \pm 14\,000$, ***M1*** for other entries correct.

2491.70 (USD) ***A1***

[3 marks]

Total [17 marks]

2. (a)



A1A1

Note: Award ***A1*** for each correct pair of branches. Accept decimal or percentage responses as equivalent forms on branches.

[2 marks]

(b) (i) $\mathrm{P}(Y) = 0.8 \times 0.1 + 0.2 \times 0.3$ ***M1***

$= 0.14$ ***A1***

(ii) $\mathrm{P}(\text{Star} \mid Y) = \dfrac{0.8 \times 0.1}{0.14}$ ***M1***

$= 0.571\left(\dfrac{4}{7}, 0.571428\ldots\right)$ ***A1***

[4 marks]

(c) the colours of the sweets are distributed according to manufacturer specifications ***A1***

[1 mark]

(d)

Colour	Brown	Red	Green	Orange	Yellow	Purple
Expected Frequency	12	20	16	16	8	8

A2

Note: Award ***A2*** for all 6 correct expected values, ***A1*** for 4 or 5 correct values, ***A0*** otherwise.

[2 marks]

(e) 5 ***A1***

[1 mark]

(f) 0.469 (0.4688117…) ***A2***

[2 marks]

continued…

Question 2 continued

(g) since $0.469 > 0.05$ ***R1***

fail to reject the null hypothesis. There is insufficient evidence to reject the manufacturer's specifications ***A1***

Note: Award ***R1*** for a correct comparison of their correct p-value to the test level, award ***A1*** for the correct result from that comparison. Do not award ***R0A1***.

[2 marks]

Total [14 marks]

3. (a) (i) $0.909\ (0.909181\ldots)$ ***A2***

(ii) (very) strong and positive ***A1A1***

Note: Award ***A1*** for (very) strong ***A1*** for positive.

[4 marks]

(b) $y = 1.14x + 0.578\ (y = 1.14033\ldots\ x + 0.578183\ldots)$ ***A1A1***

Note: Award ***A1*** for $1.14x$, ***A1*** for 0.578. Award a maximum of ***A1A0*** if the answer is not an equation in the form $y = mx + c$.

[2 marks]

(c) (i) $1.14 \times 10 + 0.578$ ***M1***

$12.0\ (11.9814\ldots)$ ***A1***

(ii) no the estimate is not reliable ***A1***

outside the known data range ***R1***

OR

a score greater than 10 is not possible ***R1***

Note: Do not award ***A1R0***.

[4 marks]

(d)

Competitors	A	B	C	D	E	F	G	H
Stan's rank	7	8	6	4	2	4	1	4
Minsun's rank	7	8	6	4.5	3	2	1	4.5

A1A1

Note: Award ***A1*** for correct ranks for Stan. Award ***A1*** for correct ranks for Minsun.

[2 marks]

(e) (i) $0.933\ (0.932673\ldots)$ ***A2***

(ii) Stan and Minsun strongly agree on the ranking of competitors. ***A1A1***

Note: Award ***A1*** for "strongly agree", ***A1*** for reference to a rank order.

[4 marks]

(f) decreasing the score to 9.1, does not change the rank of competitor G ***A1***

[1 mark]

Total [17 marks]

4. (a) $2(8\times4+3\times4+3\times8)$ ***M1***

$=136\ (\text{cm}^2)$ ***A1***

[2 marks]

(b) $\sqrt{8^2+4^2+3^2}$ ***M1***

$(\text{AG}=)\ 9.43\ (\text{cm})\ (9.4339\ldots, \sqrt{89})$ ***A1***

[2 marks]

(c) $-2x+220=0$ ***M1***

$x=110$ ***A1***

$110\,000$ (boxes) ***A1***

[3 marks]

(d) $P(x)=\int -2x+220\,\text{d}x$ ***M1***

Note: Award ***M1*** for evidence of integration.

$P(x)=-x^2+220x+c$ ***A1A1***

Note: Award ***A1*** for either $-x^2$ or $220x$ award ***A1*** for both correct terms and constant of integration.

$1700=-(20)^2+220(20)+c$ ***M1***

$c=-2300$

$P(x)=-x^2+220x-2300$ ***A1***

[5 marks]

(e) $-x^2+220x-2300=0$ ***M1***

$x=11.005$ ***A1***

$11\,006$ (boxes) ***A1***

Note: Award ***M1*** for their $P(x)=0$, award ***A1*** for their correct solution to x. Award the final ***A1*** for expressing their solution to the minimum number of boxes. Do not accept $11\,005$, the nearest integer, nor $11\,000$, the answer expressed to 3 significant figures, as these will not satisfy the demand of the question.

[3 marks]

Total [15 marks]

5. (a) (i) $p(10)^2 + q(10) = 60$ ***M1***

$10p + q = 6 \ (100p + 10q = 60)$ ***A1***

(ii) $p = 1, \ q = -4$ ***A1A1***

Note: If p and q are both incorrect then award ***M1A0*** for an attempt to solve simultaneous equations.

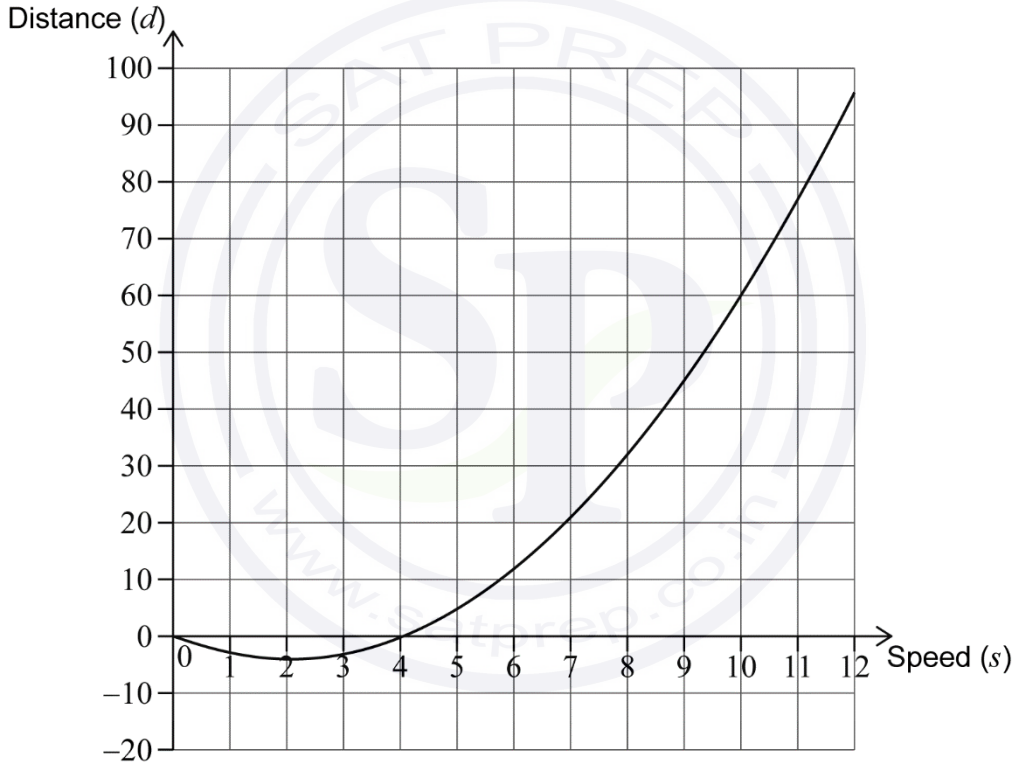
[4 marks]

(b) $(2, -4)$ ***A1A1***

Note: Award ***A1*** for each correct coordinate. Award ***A0A1*** if parentheses are missing.

[2 marks]

(c)



A3

Note: Award ***A1*** for smooth quadratic curve on labelled axes and within correct window. Award ***A1*** for the curve passing through $(0, 0)$ and $(10, 60)$. Award ***A1*** for the curve passing through their vertex. Follow through from part (b).

[3 marks]

(d) the graph indicates there are negative stopping distances (for low speeds) ***R1***

Note: Award ***R1*** for identifying that a feature of their graph results in negative stopping distances (vertex, range of stopping distances…).

[1 mark]

continued…

Question 5 continued

(e) $0.95\times 20^2 - 3.92\times 20$ ***(M1)***

$= 302\,(\text{m})$ (301.6…) ***A1***

[2 marks]

(f) $\left|\dfrac{301.6-320}{320}\right|\times 100$ ***M1***

$= 5.75\,(\%)$ ***A1***

[2 marks]

(g) $330 = 1.6\times s + 0.95\times s^2 - 3.92\times s$ ***M1A1***

Note: Award ***M1*** for an attempt to find an expression including stopping distance (model B) and reaction distance, equated to 330.
Award ***A1*** for a completely correct equation.

$19.9\,(\text{m s}^{-1})$ (19.8988…) ***A1***

[3 marks]

Total [17 marks]