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MATHEMATICS **9709/11**

Paper 1 Pure Mathematics 1 **October/November 2024**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2024 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

This document consists of **15** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles

1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.

2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.

3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.

4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).

5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.

6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	Identify correct term and obtain $6(kx)^2 \cdot \left(\frac{2}{x}\right)^2$	**M1**	Needs numerical coefficient or $\frac{4!}{2!2!}$, not ${}^4C_2$.
	Equate to 150 and obtain $k = \frac{5}{2}$	**A1**	Ignore $-\frac{5}{2}$
	Identify correct term $4(kx)^3 \cdot \left(\frac{2}{x}\right)$ with their value of k	**M1**	Needs numerical coefficient or $\frac{4!}{3!1!}$.
	Obtain coefficient 125	**A1**	Accept $125x^2$ as final answer.
		4	

Question	Answer	Marks	Guidance
2	Differentiate to obtain $2x + ax^{-2}$ or equivalent	**B1**	
	Equate first derivative to zero, substitute $x = -3$ and attempt value of a	**M1**	Must be an attempt at differentiation.
	Obtain $a = 54$	**A1**	
	Obtain $b = 27$	**A1**	
		4	

Question	Answer	Marks	Guidance
3	Use correct sector area formula	**M1**	
	Obtain $\frac{1}{2}\times 15^2 \times \frac{2}{5}\pi - \frac{1}{2}\times x^2 \times \frac{2}{5}\pi = \frac{209}{5}\pi$ or equivalent	**A1**	
	Obtain $[x]=4$	**A1**	AWRT 4.00.
	Use correct arc length formula twice	**M1**	
	Obtain $22+\frac{38}{5}\pi$	**A1**	OE. Must be in terms of π. Like terms must be collected. Not from a rounded value of x.
		5	

Question	Answer	Marks	Guidance
4	Substitute for y (or x) in first equation and simplify	***M1**	All terms to one side and brackets expanded.
	Obtain $10x^2+3kx-40$ [= 0] (or $10y^2+11ky+k^2-360[=0]$)	**A1**	
	Attempt b^2-4ac for 3-term quadratic involving k	**DM1**	Not in quadratic formula unless b^2-4ac is isolated.
	Obtain $9k^2+1600$ (or $81k^2+14400$)	**A1**	
	$9k^2+1600>0$	**A1 FT**	FT for $ak^2+b>0$ with a, $b>0$.
		5	

Question	Answer	Marks	Guidance
5(a)	Attempt correct process for solving 3-term quadratic equation in $\sqrt{x}$	**M1**	Accept $8y^2 - 6y - 9 \rightarrow (2y - 3)(4y + 3)$, if $y = \sqrt{x}$ specified.
	Obtain at least $2\sqrt{x} - 3 = 0$ or equivalent	**A1**	Ignore $4\sqrt{x} + 3 = 0$. **SC B1** for $\sqrt{x} = \frac{3}{2}$ with no method shown for solving the 3-term quadratic.
	Conclude $x = \frac{9}{4}$ ignore $\frac{9}{16}$	**A1**	**SC B1** if no method shown for solving the 3-term quadratic.
	Alternative Method for Q5(a)		
	$3\sqrt{x} = 4x - \frac{9}{2} \rightarrow 16x^2 - 45x + \frac{81}{4}$ o.e and attempt correct process to solve	**M1**	
	Obtain $x = \frac{9}{4}$ or $\frac{9}{16}$	**A1**	**SC B1** if no method shown for solving the 3-term quadratic.
	$x = \frac{9}{4}$ ignore $\frac{9}{16}$	**A1**	**SC B1** if no method shown for solving the 3-term quadratic.
		3	

Question	Answer	Marks	Guidance
5(b)	Integrate to obtain form $k_1x^2 + k_2x^{\frac{3}{2}} + k_3x$ where $k_1k_2k_3 \neq 0$	M1	
	Obtain correct $2x^2 - 2x^{\frac{3}{2}} + x$ or equivalent	A1	Allow unsimplified.
	Substitute $x = 4$ and $y = 11$ to attempt value of c	M1	Dependent on at least 2 correct terms involving x.
	Obtain $y = 2x^2 - 2x^{\frac{3}{2}} + x - 9$	A1	Must be simplified. Allow ‘f(x) =’. Allow y missing if y appears previously.
		4	

Question	Answer	Marks	Guidance
6(a)	State or imply centre of C_1 is $(-3, 5)$	B1	
	State or imply centre of C_2 is $(9, -4)$	B1	
	Attempt correct process for finding distance between centres	M1	
	Obtain 15	A1	
		4	

Question	Answer	Marks	Guidance
6(b)	$R = 4$ and $R = 8$	**B1**	
	Obtain least or greatest distance	**B1 FT**	'15' – R_1 – R_2 or '15' + R_1 + R_2.
	Obtain 3 and 27	**B1 FT**	'15' – R_1 – R_2 and '15' + R_1 + R_2.
		3	

Question	Answer	Marks	Guidance
7(a)	Differentiate to obtain form $k_1(2x+1)^{-\frac{4}{3}}$	**M1**	
	Obtain correct $-8(2x+1)^{-\frac{4}{3}}$ or unsimplified equivalent	**A1**	
	Attempt equation of tangent at $\left(\frac{7}{2}, 6\right)$ with numerical gradient	**M1**	Gradient must come from a differentiated expression.
	Obtain $y = -\frac{1}{2}x + \frac{31}{4}$ or equivalent of requested form	**A1**	
		4	

Question	Answer	Marks	Guidance
7(b)	Integrate to obtain form $k_2(2x+1)^{\frac{2}{3}}$	**M1**	
	Obtain correct $9(2x+1)^{\frac{2}{3}}$ or unsimplified equivalent	**A1**	
	Use correct limits correctly to find area	**M1**	Substitute correct limits into an integrated expression. 36 – 9 minimum working required.
	Obtain 27	**A1**	**SC B1** if M1 A1 M0 scored.
		4	

Question	Answer	Marks	Guidance
8(a)	Use $\tan^2\beta = \dfrac{\sin^2\beta}{\cos^2\beta}$	**B1**	E.g. $\tan^2\beta = \dfrac{\sin^2\beta}{\cos^2\beta}$ and then replaces $\sin^2\beta$ with a^2 or $\cos^2\beta$ with $1 - a^2$.
	$\cos\beta = -\sqrt{1-a^2}$	**B1**	
	Obtain $\dfrac{a^2}{1-a^2} + 3a\sqrt{1-a^2}$	**B1**	
		3	

Question	Answer	Marks	Guidance
8(b)	Use correct identity to obtain 3-term quadratic equation in $\sin\theta$	*M1	
	Obtain $\sin^2\theta + 4\sin\theta + 1[=0]$	A1	
	Attempt to solve quadratic	DM1	At least as far as $\frac{-4\pm\sqrt{12}}{2}$. -15.5^{o} implies attempt at solving quadratic.
	Obtain 195.5	A1	
	Obtain 344.5	A1FT	Following first answer; and no others for $0° < \theta < 360°$ but must be in 4^{th} quadrant. **SC B1** for 3.41^c and 6.01^c.
		5	

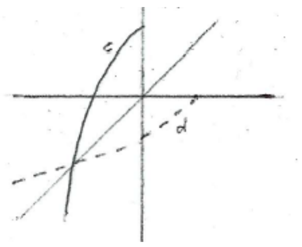
Question	Answer	Marks	Guidance
9(a)	Differentiate to obtain $5 + 12x - 9x^2$	B1	
	Attempt to find two critical values by solving quadratic equation or inequality	M1	
	Obtain values $-\frac{1}{3}$ and $\frac{5}{3}$	A1	**SC B1** if no method for solving the quadratic.
	Conclude $x < -\frac{1}{3},\ x > \frac{5}{3}$	A1FT	**SC B1** if no method for solving the quadratic.
		4	

Question	Answer	Marks	Guidance
9(b)	Equate first derivative to 9 and simplify to 3 term quadratic	*M1	
	Obtain $x = \frac{2}{3}$	A1	**SC B1** for solving $5 + 12x - 9x^2 = 9$ without simplifying to a 3-term quadratic.
	Use *x*-value and corresponding *y*-value to determine value of *k*	DM1	
	Obtain $k = \frac{28}{9}$	A1	**SC B1** for $k = \frac{28}{9}$ from solving $5 + 12x - 9x^2 = 9$ without simplifying to a 3-term quadratic.
		4	

Question	Answer	Marks	Guidance
10(a)	State or imply that first 3 terms of GP are $5 + d,\ 5 + 4d,\ 5 + 10d$	B1	
	Form equation $(5 + 4d)^2 = (5 + d)(5 + 10d)$ or equivalent	M1	
	Obtain $d = 2.5$	A1	Ignore 0 as a solution. **SC B1** Obtain $d = 2.5$ and 7.5, 15, 30 by trial and improvement www.
	Alternative Method for Question 10(a):		
	State or imply that first 3 terms of GP are $5 + d,\ 5 + 4d,\ 5 + 10d$	B1	
	$(5 + d)R = 5 + 4d \rightarrow d = \frac{5 - 5R}{R - 4},\ (5 + d)R^2 = 5 + 10d \rightarrow R^2 - 3R + 2$ [= 0]	M1	OE Eliminates *d*.
	Obtain $d = 2.5$	A1	
		3	

Question	Answer	Marks	Guidance
10(b)	Use correct formula for sum of AP with their value of d	M1	
	Obtain or imply 7700	A1	
	State or imply GP is $7.5,\ 15,\ 30, \ldots$	B1	
	Use correct formula for sum of GP with their common ratio	M1	
	Obtain $S_{77} - G_{10} = 27.5$	A1	
		5	

Question	Answer	Marks	Guidance
11(a)	Obtain $b = 2$ and $c = \frac{3}{2}$	B1	
	Obtain $\frac{15}{2} - 2\left(x - \frac{3}{2}\right)^2$	B1	
	State range is $y \leqslant \frac{15}{2}$ or $f\left(x\right) \leqslant \frac{15}{2}$ with $\leqslant$ given or clearly implied (not $<$)	B1 FT	Following *their* value of a.
		3	
11(b)	State that reflection is in x-axis	B1	Accept transformations in any order.
	State or imply that translation is by $\begin{pmatrix} -\frac{3}{2} \\ \frac{15}{2} \end{pmatrix}$ or equivalent	B1 FT	Following *their* values of a and c in part **(a)**. Accept transformations in any order.
		2	

Question	Answer	Marks	Guidance
11(c)	Sketch the correct graph appearing in second and third quadrants only	**B1**	
	State that each y-value is associated with a single x-value or equivalent	**B1**	Accept passes the horizontal line test. Ignore passes the vertical line test.
		2	
11(d)	Sketch the correct graph with suitable labelling to distinguish the two curves	**B1**	Appearing in third and fourth quadrants only.
	Draw the line $y = x$	**B1**	See above; no need to label the line.
	Attempt correct process for finding the inverse function	**M1**	Allowing use of $\pm$ and y so far.
	Obtain $\frac{3}{2} - \sqrt{\frac{15}{4} - \frac{1}{2}x}$ or equivalent	**A1**	Must involve x at the conclusion.
		4	

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AWRT	Answer Which Rounds To

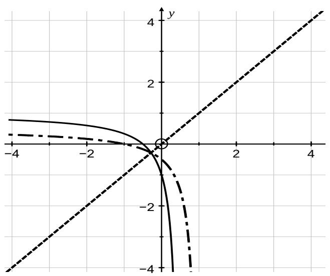
Question	Answer	Marks	Guidance
1(a)	$a = 4$	B1	Allow $4\sin(2x) + 3$ if values of a, b and c are not stated.
	$b = 2$	B1	
	$c = 3$	B1	
		3	
1(b)(i)	5	B1	Ignore attempts at finding solutions.
		1	
1(b)(ii)	1	B1	Ignore attempts at finding solutions.
		1	

Question	Answer	Marks	Guidance
2(a)	$\frac{20}{2}\left(2\times -20+\left(20-1\right)\times 5\right)$ or $\frac{20}{2}\left(-20+75\right)$	**M1**	Correct use of either S_{20} formula with a = –20 and d = 5.
	550	**A1**	
		2	
2(b)	$\frac{2k}{2}\left(-40+\left(2k-1\right)\times 5\right)$ or $\frac{k}{2}\left(-40+\left(k-1\right)\times 5\right)$	**M1***	Correct use of S_n formula with $a=-20$, $d=5$ and either k or $2k$. This mark can be awarded for clear use of $\frac{n}{2}\left(a+l\right)$ when correct values of a and d are used.
	$\left[-40k+10k^2-5k=-200k+25k^2-25k \Rightarrow\right] 15k^2-180k=0$	**DM1**	Equating their S_{2k} to 10 × *their* S_k and reaching a 2-term quadratic or 2 term linear equation if k has been cancelled. Condone errors in simplification.
	$k=12$	**A1**	Condone extra solution $k=0$.
		3	

Question	Answer	Marks	Guidance
3(a)	$\left[f(2+h)=\right]\ 2(2+h)^2-3$	B1	SOI
	$\dfrac{\left(2(2+h)^2-3\right)-5}{(2+h)-2}$ $\left[=\dfrac{2h^2+8h}{h}\right]$	M1	$\dfrac{\left\{their\left(2(2+h)^2-3\right)\right\}-their5}{(2+h)-2}$ can be implied by the simplified expression or the correct answer. *Their* 5 must come from $2(2)^2-3$.
	$2h+8$ or $2(h+4)$	A1	
		3	
3(b)	$h\to0$, or chord [AB] $\to$ tangent [at A]	B1	Either of these statements or any sight of $h = 0$.
	8	B1FT	Could come from anywhere except wrong working. Either correct or FT their linear expression from **(a)**.
		2	

Question	Answer	Marks	Guidance
4(a)	15 or $\binom{6}{2}\times x^4\left(\dfrac{3}{x^2}\right)^2$ or $x^6\times\dfrac{6\times5}{2}\left(\dfrac{3}{x^3}\right)^2$	B1	OE May be in a list. Allow $\binom{6}{4}$.
	135	B1	Correct term must be identified if in a list. Allow $135x^0$.
		2	

Question	Answer	Marks	Guidance
4(b)	20 or $\binom{6}{3} \times x^3 \left(\frac{3}{x^2}\right)^3$ or $x^6 \times \frac{6.5.4}{3!}\left(\frac{3}{x^3}\right)^3$	**B1**	OE May be in a list.
	$= \frac{540}{x^3}$	**B1**	Identifying $\frac{1}{x^3}$ term. This can be implied by sight of 2160 as part of the constant term.
	$4 \times 540 - 5 \times 135$	**M1**	$4 \times \textit{their}\, 540 - 5 \times \textit{their}\, 135$
	1485	**A1**	Allow $1485x^0$.
		4	

Question	Answer	Marks	Guidance
5(a)(i)	$\left[\mathrm{f}(-1) = \right] \frac{1}{3}$	**B1**	Condone 0.333.
		1	
5(a)(ii)		**B1**	For showing the correct mirror line.
		B1	For correct shape: the curves should intersect in the first square in the third quadrant. To the left of the point of intersection, the reflection is below the original and crosses the x-axis. To the right of the point of intersection, the reflection is to the right the original.
		2	

Question	Answer	Marks	Guidance
5(a)(iii)	$\dfrac{2x+1}{2x-1}=y \Rightarrow 2x+1=y(2x-1)$	M1*	Equating y to the given function and clearing of fractions. x and y may be interchanged at this stage.
	$2xy-2x=y+1$	DM1	Condone ± errors during simplification.
	$\dfrac{x+1}{2(x-1)}$, $\dfrac{-x-1}{2-2x}$	A1	Allow ‘ f^{-1} ’or ‘y =’ but NOT ‘x =’, nor fractions within fractions.
	[Domain of f^{-1} is] $x<1$	B1	Accept $-\infty < x < 1$ or $(-\infty, 1)$, condone $[-\infty, 1)$.
	Alternative Method for Question 5(a)(iii)		
	$y=1+\dfrac{2}{2x-1} \Rightarrow y-1=\dfrac{2}{2x-1}$	M1*	Equating y to the given function after division by $2x-1$. Isolating the term in x. x and y may be interchanged at this stage.
	$2x=\dfrac{2}{y-1}+1$	DM1	Condone ± errors during simplification.
	$\dfrac{1}{x-1}+\dfrac{1}{2}$	A1	OE Allow ‘ f^{-1} ’or ‘y =’ but NOT ‘x =’, nor fractions within fractions.
	[Domain of f^{-1} is] $x<1$	B1	Accept $-\infty < x < 1$ or $(-\infty, 1)$, condone $[-\infty, 1)$.
		4	

Question	Answer	Marks	Guidance
5(b)	$gf\left(\frac{1}{4}\right) = -7$	B1	
	$\frac{2x+1}{2x-1} = -7$	M1	Equating $\frac{2x+1}{2x-1}$ to *their* $gf\left(\frac{1}{4}\right)$.
	$[x =]\ \frac{3}{8}$	A1	OE
	Alternative solution for Question 5(b)		
	$gf\left(\frac{1}{4}\right) = -7$	B1	
	$x = f^{-1}(-7)$	M1	$x = f^{-1}\left(\textit{their } gf\left(\frac{1}{4}\right)\right)$
	$[x =]\ \frac{3}{8}$	A1	OE
		3	

Question	Answer	Marks	Guidance
6	[Perimeter =] $r + r\theta + r + 2r \times 2\theta + r + r\theta + r$ $[= 4r + 6r\theta]$	**B1**	
	[Area =] $\frac{1}{2}r^2\theta + \frac{1}{2}(2r)^2 \times 2\theta + \frac{1}{2}r^2\theta$ $[=5r^2\theta]$	**B1**	
	$4r + 6r\theta = 14$ and $5r^2\theta = 10$	**M1***	$ar + br\theta = 14$ and $cr^2\theta = 10$ where a, b and c are constants $\neq 0$. Terms may be uncollected.
	EITHER		
	$5r^2\frac{14-4r}{6r} = 10$ or $4r + 6r\left(\frac{10}{5r^2}\right) = 14$	**DM1**	Eliminate θ to get an equation in r.
	$\left[\Rightarrow 2r^2 - 7r + 6 = 0 \Rightarrow\right]$ $(r-2)(2r-3) = 0$	**DM1**	Factorise or other accepted method for solving their 3-term quadratic.
	OR		
	$5\left(\frac{14}{4+6\theta}\right)^2\theta = 10$ or $4\left(\sqrt{\frac{10}{5\theta}}\right) + 6\left(\sqrt{\frac{10}{5\theta}}\right)\theta = 14$	**DM1**	Eliminate r to get an equation in θ.
	$[\Rightarrow 18\theta^2 - 25\theta + 8 = 0 \Rightarrow]$ $(9\theta - 8)(2\theta - 1) = 0$	**DM1**	Factorise or other accepted method for solving their 3-term quadratic.
	Then		
	$r = 2$ and $\theta = 0.5$	**B1**	Condone extra answers $r = \frac{3}{2}$ and $\theta = \frac{8}{9}$.
		6	

Question	Answer	Marks	Guidance
7(a)	$-2\left((x \pm p)^2 \pm q\right)$ or $-2(x \pm p)^2 \pm q$	**M1***	$p \neq 0$.
	$-2\left((x-2)^2 \pm q\right)$ or $-2(x-2)^2 \pm q$	**DM1**	
	$-2(x-2)^2 + 19$ and (2, 19)	**A1**	Accept $x = 2, y = 19$ or 2, 19.
		3	

Question	Answer	Marks	Guidance
7(b)	**Method 1**		
	$\left[x=\right]\pm 1$	**B1***	Both x co-ordinates for the points of intersection.
	Subtract and attempt to integrate	**M1***	
	$\left[\int\left(-2x^2+2\right)\mathrm{d}x\right]-\frac{2}{3}x^3+2x$	**B1***	Both terms correct.
	$\left(-\frac{2}{3}+2\right)-\left(\frac{2}{3}-2\right)$	**M1**	Apply *their* limits, one positive and one negative, obtained from equating the line and the curve to their integrated expression.
	$=\frac{8}{3},\ 2\frac{2}{3}$	**DB1**	AWRT 2.67 WWW. Condone $\frac{-8}{3}\to\frac{8}{3}$. **SC B1** for mistaking triangle for trapezium leading to $\frac{11}{3}$, i.e. a total of 2/5.
	Method 2		
	$\left[x=\right]\pm 1$	**B1***	Both x co-ordinates for the points of intersection.
	Attempt to integrate and subtract	**M1***	The second integral can be replaced with what is clearly their area of a trapezium.
	$\left\{\frac{-2x^3}{3}+\frac{8}{2}x^2+11x\right\}[-]\left\{\frac{8}{2}x^2+9x\right\}$	**B1***	OE All terms correct. The second integral can be replaced by $\frac{1}{2}\left(1+17\right)\times 2$ OE.

Question	Answer	Marks	Guidance
7(b)	$\left\{\left(\frac{-2}{3}+4+11\right)-\left(\frac{2}{3}+4-11\right)\right\}$ [−] $\{(4+9)-(4-9)\}$	**M1**	Apply *their* limits, one positive and one negative, obtained from equating the line and the curve, to *their* integrated expressions. If the trapezium has been used, the second integral can be replaced by *their* 18.
	$=\frac{8}{3},\ 2\frac{2}{3}$	**DB1**	AWRT 2.67 WWW. Condone $\frac{-8}{3}\to\frac{8}{3}$. **SC B1** for mistaking triangle for trapezium leading to $\frac{11}{3}$, i.e. a total of 2/5.
	Method 3		
	$[x=]\pm1$	**B1***	Both x co-ordinates for the points of intersection.
	Subtract and attempt to integrate	**M1***	
	$-\frac{2}{3}(x-2)^3-\frac{8}{2}x^2+10x$	**B1***	All terms correct.
	$\left(\frac{2}{3}-4+10\right)-(18-4-10)$	**M1**	Apply *their* limits, one positive and one negative, obtained from equating the line and the curve, to *their* integrated expression.
	$=\frac{8}{3},\ 2\frac{2}{3}$	**DB1**	AWRT 2.67 WWW.

Question	Answer	Marks	Guidance
7(b)	**Method 4**		
	$[x=]\pm 1$	**B1***	Both x co-ordinates for the points of intersection.
	Attempt to integrate and subtract	**M1***	The second integral can be replaced with what is clearly *their* area of a trapezium.
	$\left\{-\frac{2}{3}(x-2)^3+19x\right\}$ $[-]$ $\left\{\frac{8}{2}x^2+9x\right\}$	**B1***	All terms correct. The second integral can be replaced with $\frac{1}{2}(1+17)\times 2$ OE.
	$\left\{\left(\frac{2}{3}+19\right)-(18-19)\right\}$ $[-]$ $\{(4+9)-(4-9)\}$	**M1**	Apply *their* limits, one positive and one negative, obtained from equating the line and the curve, to *their* integrated expression. If the trapezium has been used the second integral can be replaced with *their* 18 OE.
	$=\frac{8}{3},\ 2\frac{2}{3}$	**DB1**	AWRT 2.67 WWW. Condone $\frac{-8}{3}\to\frac{8}{3}$. **SC B1** for mistaking triangle for trapezium leading to $\frac{11}{3}$, i.e. a total of 2/5.
		5	

Question	Answer	Marks	Guidance
8(a)	$\left(x-\left(-\frac{1}{2}p\right)\right)^2+\left(y-(-1)\right)^2$ OE	B1*	Allow $a=-\frac{1}{2}p$ and $b=-1$, or centre is $\left(-\frac{1}{2}p,-1\right)$.
	$\left(x-\left(-\frac{1}{2}p\right)\right)^2+\left(y-(-1)\right)^2=-q+1+\left(-\frac{1}{2}P\right)^2$ OE	DB1	
		2	
8(b)(i)	[Gradient of tangent =] $-\frac{1}{2}$	B1	OE SOI
	[Gradient of normal =] 2	M1	Use of $m_1m_2=-1$ with *their* numeric tangent gradient.
	$\frac{y-3}{x-4}=2$ $[y=2x-5]$	A1	OE ISW Allow $y=2x+c,\ 3=2\times4+c\ \Rightarrow c=-5$.
		3	

Question	Answer	Marks	Guidance
8(b)(ii)	**Method 1 for the first two marks:**		
	$-1-3=2\left(-\frac{1}{2}p-4\right)$ or $-1=-p-5$	**M1***	Using *their* stated centre or $\left(\frac{\pm p}{2},\ \pm 1\right)$ in *their* equation of the normal.
	$p=-4$	**A1**	
	Method 2 for the first two marks:		
	$-1=2x-5 \Rightarrow x=2 \ \Rightarrow \ -\frac{1}{2}p=2$	**M1***	Using their normal equation and *their* stated centre or $\left(\frac{\pm p}{2},\ \pm 1\right)$.
	$p=-4$	**A1**	
	Method 3 for the first two marks:		
	$2x+2y\frac{dy}{dx}+p+2\frac{dy}{dx}=0 \quad \left[\Rightarrow p=-8-8\frac{dy}{dx}\right]$	**M1***	
	$\left[\frac{dy}{dx}=-\frac{1}{2}\Rightarrow\right] \ p=-4$	**A1**	

Question	Answer	Marks	Guidance
8(b)(ii)	**Method 1 for the last 3 marks:**		
	$r^2 = (4-2)^2 + (3-(-1))^2$ $[=20]$	**M1***	Using (4, 3) and *their* centre or $\left(\frac{\pm their\ p}{2}, \pm 1\right)$ to find r^2 or r.
	$-q+1+\frac{1}{4}p^2 = 20$	**DM1**	OE Using *their* expression for r^2 (from **(a)**) equated to *their* 20.
	$q = -15$	**A1**	
	Method 2 for the last 3 marks:		
	$r = \frac{\lvert 2-2-10 \rvert}{\sqrt{5}}$ $\left[= \frac{10}{\sqrt{5}}\right]$	**M1***	Using $(2,-1)$ and $x+2y-10=0$ (distance from a point to a line).
	$-q+1+\frac{1}{4}p^2 = \left(\frac{10}{\sqrt{5}}\right)^2$	**DM1**	OE Using *their* expression for r^2 equated to *their* $\left(\frac{10}{\sqrt{5}}\right)^2$.
	$q = -15$	**A1**	
	Method 3 for the last 3 marks:		
	$4^2+3^2+4p+6+q=0$ $[\Rightarrow\ 4p+q+31=0]$ OR $\left(4-\left(-\frac{1}{2}p\right)\right)^2 + (3-(-1))^2 = -q+1+\left(-\frac{1}{2}p\right)^2$	**M1***	Substituting $(4,3)$ into their circle equation.
	$4(-4)+q+31=0$	**DM1**	Substituting *their* $p=-4$.
	$q = -15$	**A1**	

Question	Answer	Marks	Guidance
8(b)(ii)	**Alternative Method for Question 8(b)(ii)**		
	$4^2+3^2+4p+6+q=0$ $x^2+(2x-5)^2+px+2(2x-5)+q=0$ with $x=4$ $x^2+\left(\frac{10-x}{2}\right)^2+px+2\left(\frac{10-x}{2}\right)+q=0$ with $x=4$ $\left(\frac{y+5}{2}\right)^2+y^2+p\left(\frac{y+5}{2}\right)+2y+q=0$ with $y=3$ $(10-2y)^2+y^2+p(10-2y)+2y+q=0$ with $y=3$ {Each of these $\Rightarrow$ $4p+q+31=0$}	**M1***	Substituting $(4, 3)$ into *their* circle equation, or replacing y with $2x-5$ from the normal equation, or replacing y with $\frac{10-x}{2}$ from the tangent equation, or replacing x with $\frac{y+5}{2}$ from the normal equation, or replacing x with $10-2y$ from the tangent equation, and using either $x=4$ or $y=3$ to form an equation in p and q.
	$\frac{5}{4}x^2+(p-6)x+35+q=0 \Rightarrow (p-6)^2-4\times\frac{5}{4}\times(35+q)=0$ OR $5y^2-y(38+2p)+100+10p+q=0 \Rightarrow (38+2p)^2-4\times5\times(100+10p+q)=0$ {Each of these $\Rightarrow p^2-12p-139-5q=0$}	**M1***	Solving the tangent and circle equations simultaneously to form a quadratic equation in either x or y. Then using $b^2-4ac=0$ on their quadratic to form an equation in p and q.
	Solving the equations simultaneously to find p or q	**DM1**	
	$p=-4$	**A1**	
	$q=-15$	**A1**	
		5	

Question	Answer	Marks	Guidance
9(a)	$\left[\frac{1}{2}k^2 \times \frac{25}{4} - 2k \times \frac{5}{2} + 2 = \frac{1}{2}\right.$ OR $\left.\frac{1}{2}k^2 \times \frac{25}{4} - 2k \times \frac{5}{2} + 2 = k \times \frac{5}{2} + \left(\frac{1}{2} - \frac{5}{2}k\right)\right]$ $25k^2 - 40k + 12\ [=0]$	**M1***	Using $\left(\frac{5}{2}, \frac{1}{2}\right)$ in the curve equation or equating the line and the curve and then using $x = \frac{5}{2}$ and $p = \frac{1}{2} - \frac{5}{2}k$. Simplify to get a three-term quadratic in *k*. Condone errors in simplification.
	$k = \frac{2}{5}$	**A1**	OE Condone inclusion of $k = \frac{6}{5}$.
	$\frac{1}{2} = \left(their\ \frac{2}{5}\right)\left(\frac{5}{2}\right) + p \Rightarrow p =$	**DM1***	Using $\left(\frac{5}{2}, \frac{1}{2}\right)$ and *their k* in an equation in *p*. Either the line (as shown) or $4p^2 + 12p + 5 = 0$ are the most likely and solving for *p*.
	$p = -\frac{1}{2}$	**A1**	OE Condone inclusion of $p = -\frac{5}{2}$.
	$\frac{2}{25}x^2 - \frac{6}{5}x + \frac{5}{2}\ [=0]\ \left[4x^2 - 60x + 125\ [=0]\right]$	**DM1**	Equating the line and curve using *their k* and *p* and simplify to get a three-term quadratic [= 0].
	$\left(\frac{25}{2}, \frac{9}{2}\right)$	**A1 A1**	OE Accept $x = \frac{25}{2}, y = \frac{9}{2}$.

Question	Answer	Marks	Guidance
9(a)	**Alternative Method for Question 9(a)**		
	$\left[\frac{1}{2}k^2 \times \frac{25}{4} - 2k \times \frac{5}{2} + 2 = k \times \frac{5}{2} + p\right]$ $4p^2 + 12p + 5\ [=0]$	**M1***	OE Using $\left(\frac{5}{2}, \frac{1}{2}\right)$ in the curve equation or equating the line and the curve and then using $x = \frac{5}{2}$ and $k = \frac{1}{5} - \frac{2}{5}p$. Simplify to get a three-term quadratic in p [= 0].
	$p = -\frac{1}{2}$ OE	**A1**	Condone inclusion of $p = -\frac{5}{2}$.
	$\frac{1}{2} = \left(\frac{5}{2}k\right) + \left(\textit{their} - \frac{1}{2}\right) \Rightarrow k =$	**DM1***	Using $\left(\frac{5}{2}, \frac{1}{2}\right)$ and *their* p in the line equation and solving for k.
	$k = \frac{2}{5}$	**A1**	OE Condone inclusion of $k = \frac{6}{5}$.
	$\frac{2}{25}x^2 - \frac{6}{5}x + \frac{5}{2}[=0]\ \left[4x^2 - 60x + 125[=0]\right]$	**DM1**	Equating the line and curve using *their* k and p and simplify to get a three-term quadratic [= 0].
	$\left(\frac{25}{2}, \frac{9}{2}\right)$	**A1 A1**	OE Accept $x = \frac{25}{2}$, $y = \frac{9}{2}$.
		7	

Question	Answer	Marks	Guidance
9(b)	$\left[\frac{1}{2}k^2x^2 - 2kx + 2 = kx + p \Rightarrow\right] \frac{1}{2}k^2x^2 - 3kx + 2 - p$	**M1***	Equate the original equations of the curve and the line and collect like terms; k and p must still be present.
	$9k^2 - 4\times\frac{1}{2}k^2(2-p)$	**DM1**	Use of $b^2 - 4ac$ for their quadratic in x to give an expression in k and p. This expression can come from *their* equation in **(a)**.
	$p < -\frac{5}{2}$	**A1**	
		3	

Question	Answer	Marks	Guidance
10(a)	–18	**B1**	SOI
	$\frac{1}{18}$	**M1**	Use of $m_1m_2 = -1$ from $f'(x)$ with $x = 1$.
	$\frac{y\ [-\ 0]}{x-1} = \frac{1}{18}$	**A1**	OE ISW
		3	

Question	Answer	Marks	Guidance
10(b)	$\left[f(x)=\right]\left\{8(2x-3)^{\frac{4}{3}}.\frac{1}{2}.\frac{1}{\frac{4}{3}}\right\}\left\{-10x^{\frac{5}{3}}.\frac{1}{\frac{5}{3}}\right\}[+c]$ $\left[3(2x-3)^{\frac{4}{3}}-6x^{\frac{5}{3}}+c\right]$	**B1B1**	B1 for each unsimplified {}. Can be implied by equivalent simplified or partly simplified versions.
	$0=3(2(1)-3)^{\frac{4}{3}}-6(1)^{\frac{5}{3}}+c \quad [0=3-6+c]$	**M1**	Use of $x=1$ and $y=0$ in *their* integrated $f'(x)$, defined as an expression with at least one correct power, which must contain + c.
	$[f(x) \text{ or } y=]\ 3(2x-3)^{\frac{4}{3}}-6x^{\frac{5}{3}}+3$	**A1**	Only condone c = 3 as their final answer if all coefficients have previously been simplified in a correct statement.
		4	
10(c)	$b^2-4ac=128^2-4\times125\times192$ and stating "< 0" OR use of the quadratic formula and stating "No solutions" OR completing the square for the given quadratic and stating positive or > 0. OR sketch of the given quadratic and stating positive.	**M1***	$b^2-4ac=-79616$ can be accepted in place of working.
	No turning points [in the original function.]	**DM1**	
	Decreasing because $f'(\text{any positive } x \text{ value})<0$	**A1**	WWW e.g. $f'(1)=-18$.
		3	

CAMBRIDGE
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MATHEMATICS **9709/13**

Paper 1 Pure Mathematics 1 **October/November 2024**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2024 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

This document consists of **17** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles

1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.

2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.

3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.

4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).

5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.

6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	$a+3d=15$ and $a+7d=25$	**M1**	Or forming any valid equations which can be used to find *d* or *a*.
	Finding *a* and *d* $\left[d=\frac{5}{2},\ a=\frac{15}{2}\right]$	**DM1**	Or any valid method to find a using *their* d or d using *their* a or finding u_{30} directly from either u_4 or u_8 and *d*.
	$u_{30}=\frac{15}{2}+29\times\frac{5}{2}=80$	**A1**	
		3	

Question	Answer	Marks	Guidance
2	$\cos\left(\frac{\pi}{6}\right)+\tan 2x+\frac{\sqrt{3}}{2}=0 \Rightarrow \tan 2x=-\sqrt{3}$	**M1**	Making $\tan 2x$ the subject. $\tan 2x=0$ is M0. Accept decimals and one sign error.
	$\Rightarrow 2x=-\frac{\pi}{3} \Rightarrow x=-\frac{\pi}{6}$	**A1**	May come from non-exact working. Ignore answers outside the given range.
		2	

Question	Answer	Marks	Guidance
3(a)	$x^3:\ \binom{5}{3}3^2(-ax)^3\left[-10\times 3^2\times a^3\right]$ or $x^4:\ \binom{5}{4}3(-ax)^4\left[5\times 3\times a^4\right]$	**M1**	Allow for either term, allow sign error and combination notation.
	$x^3:\ -90a^3$	**A1**	Allow in the full expansion.
	$x^4: 15a^4$	**A1**	Allow in the full expansion.
		3	

Question	Answer	Marks	Guidance
3(b)	Coefficient of x^4 is $a \times \textit{their} -90a^3 + 7 \times \textit{their}\ 15a^4$ $\left[=15a^4\right]$	**M1**	Must select two appropriate terms only.
	$15a^4 = 240$	**DM1**	Reducing to a simple quartic equation in a.
	$\Rightarrow a^4 = 16 \Rightarrow a = 2$	**A1**	A0 if $a = -2$ is given as a solution.
		3	

Question	Answer	Marks	Guidance
4	Let $x = \sin^2\theta$ $(2x+7)(2x-1)=0$ or $(2\sin^2\theta+7)(2\sin^2\theta-1)$	**M1**	Or equivalent method.
	$\Rightarrow \sin^2\theta = \frac{1}{2} \Rightarrow \sin\theta = [\pm]\frac{1}{\sqrt{2}}$	**M1**	Finding $\sin^2\theta$ and then $\sin\theta$ (may be implied).
	$\theta = 45°,\ 135°,\ 225°,\ 315°$	**A1 A1**	A1 for any two correct values. A1 for all correct and no others within the range. For answers in radians, A1 only for all 4 angles. If no (correct) working, then **SC B1** for all 4 solutions.
		4	

Question	Answer	Marks	Guidance
5(a)	Reflection [in] y-axis	**B1 B1**	B1 for reflection B1 mention of y-axis, OE. **SC B2** for stretch, SF –1, parallel to x-axis.
	Translation or shift $\begin{pmatrix}-1\\0\end{pmatrix}$	**B1***	B1 for 'translation' and a correct vector/description. Do not accept 'left'/'right'. If two translations then B0 and B0 for the order.
	Stretch, factor 2, parallel to y-axis	**B2,1,0**	B2 all correct OE. B1 any 2 parts correct. This can be at any point in the sequence.
	Correct order and three correctly named transformations only	**DB1**	If a fourth transformation is given this mark is not awarded and no marks are given for the two transformations of the same type, except where the reflection is described as a stretch. If any transformation is incorrectly named this cannot be given. If translation is not $\begin{pmatrix}-1\\0\end{pmatrix}$ or $\begin{pmatrix}1\\0\end{pmatrix}$ then DB0 is given.
	Alternative Solution for first 3 marks		
	Translation or shift $\begin{pmatrix}1\\0\end{pmatrix}$	**B1***	B1 for 'translation' and correct vector/description.
	Reflection [in] y-axis	**B1 B1**	B1 for 'reflection', B1 for 'in y-axis'.
	Alternative solutions		
	There are alternative solutions which can be marked in the same way e.g. the given stretch, translation $\begin{pmatrix}-4\\0\end{pmatrix}$, reflect in $x=-2.5$		
		6	

Question	Answer	Marks	Guidance
5(b)	$g(x) = 2f(-x-1)$ or $a = 2,\ b = -1,\ c = -1$	B1	First B1 for $a = 2$ and no additional terms added to the function. $a = -2$ is B0.
		B1	Second B1 for $b = -1$ and $c = -1$.
		2	

Question	Answer	Marks	Guidance
6	$\dfrac{\dfrac{10(1-r^8)}{1-r}}{\dfrac{10(1-r^4)}{1-r}} = \dfrac{17}{16}$ $[a\dfrac{(1-r^8)}{(1-r)} = \dfrac{17}{16} \times a\dfrac{(1-r^4)}{(1-r)}]$	M1*	OE, i.e. substituting *p* and *q* expressions into ratio $\dfrac{17}{16}$. $16 = a\dfrac{(1-r^4)}{(1-r)},\ 17 = a\dfrac{(1-r^8)}{(1-r)}$ gets M0 unless recovered later.
	Simplifying to $16r^8 - 17r^4 + 1\ [=0]$ (or equivalent form)	DM1	Or $\dfrac{(1-r^8)}{(1-r^4)} = (1+r^4) = \dfrac{17}{16}$.
	$[(16r^4-1)(r^4-1)=0] \Rightarrow r = \pm\dfrac{1}{2}$	A1	Or $r^4 = \dfrac{1}{16} \Rightarrow r = \pm\dfrac{1}{2}$ (condone extra $r = \pm 1$ solution).
	$S_\infty = \dfrac{10}{1-\left([\pm]\dfrac{1}{2}\right)}$	DM1	Use of correct sum to infinity formula with either of *their r* values providing $\lvert r \rvert < 1$.
	$S_\infty = 20$ and $\dfrac{20}{3}$	A1	Allow 6.67 or better. A0 if there is only one or more than two S_∞ values.
		5	

Question	Answer	Marks	Guidance
7(a)	Area of sector *BOF* = $\frac{1}{2}\times 20^2\times(2\pi-2.4)$ $[=776.63\ldots]$	**M1**	Or combination of large semi-circle and small sector: $\frac{1}{2}\times 20^2\times\pi+\frac{1}{2}\times 20^2\times(\pi-2.4)$.
	Length *BD* = *DF* = $2\times 20\sin 0.6$ or $\sqrt{20^2+20^2-2\times 20\times 20\cos 1.2}$ $[=22.58\ldots]$	**M1***	Length of radius of small circles is acceptable for M1.
	Area of two semicircles = $\pi\times(20\sin 0.6)^2$ $[=400.64\ldots]$	**DM1**	
	Area of triangles = $2\times\frac{1}{2}\times 20\times 20\sin 1.2$ $[=372.81\ldots]$	**M1**	
	Total area = $1550\,[\text{cm}^2]$	**A1**	Expect 1550.09 but accept AWRT to 3sf.
		5	
7(b)	$\frac{1}{2}\pi r^2=50\pi \Rightarrow r=10$	**B1**	May be seen as $20\sin\frac{\theta}{2}$, where $\theta=\frac{\pi}{3}$.
	$\Rightarrow \theta=\frac{\pi}{3}$	**M1***	OE Finding θ using *their r*. Allow working in degrees.
	Arc length of sector *BOF* = $20\times\left(2\pi-\textit{their}\,\frac{2\pi}{3}\right)$	**DM1**	
	Total perimeter = $20\times\left(2\pi-\textit{their}\,\frac{2\pi}{3}\right)+2\pi\times\textit{their}\,10$	**DM1**	Dependent on the first dM1.
	$\frac{140\pi}{3}$ or $46\frac{2}{3}\pi$	**A1**	Must be a single exact term.
		5	

Question	Answer	Marks	Guidance
8(a)	$3(x-2)^2+2$ or $a=-2,\ b=2$	**B1 B1**	
		2	
8(b)	2 or $k=2$ or $k \geq 2$	**B1FT**	FT on *their a*. Do not accept $x = 2$ or $x \geqslant 2$.
		1	
8(c)	$3(x-2)^2+14-12=y \Rightarrow (x-2)^2=\frac{y-2}{3}$	**M1**	Using *their* completed square form.
	$x=[\pm]\sqrt{\frac{y-2}{3}}+2$	**DM1**	
	$f^{-1}(x)=\sqrt{\frac{x-2}{3}}+2$	**A1**	OE, e.g. $y=\frac{\sqrt{3x-6}}{3}+2$.
		3	

Question	Answer	Marks	Guidance
8(d)	Finding $f^{-1}(29)$ [= 5]	**M1**	Or solving f(x) = 29 [using *their* completed square form, OE].
	Finding $f^{-1}($ *their* $5)$	**M1**	Or solving f(x) = *their* 5.
	$x=3$	**A1**	If using f(x) method, x = 1 must be discarded.
	Alternative solution for Question 8(d)		
	$3\left(3(x-2)^2+2\right)-2)^2+2=29$ using *their* completed square form	**M1**	Or $3\left(3x^2-12x+14\right)^2-12\left(3x^2-12x+14\right)+14=29$. Allow if the $'=29'$ appears later in the working.
	Solving as far as $9(x-2)^4=9$ or $x^2-4x+3=0$	**DM1**	OE Or $[27]\left(x^4-8x^3+24x^2-32x+15\right)=0$.
	$x=3$ only	**A1**	WWW Only dependent on the first M1.
		3	

Question	Answer	Marks	Guidance
9(a)	$y=x^3-3x+3$ and $y=2x^3-4x^2+3 \Rightarrow x^3-4x^2+3x[=0]$	**M1**	Reducing to 3-term cubic or quadratic if x cancelled.
	$[x](x-1)(x-3)[=0]$	**DM1**	Factorising the cubic or quadratic.
	$x=0$, 1 and 3 {x = 0 may be seen in the working}	**A1**	**SC B1** for $x=1,\ 3$ only, with no M marks awarded.
		3	

Question	Answer	Marks	Guidance
9(b)	Attempt at integration of both functions. Can be before or after subtraction of the functions or integrals	M1	Expect integration of $\int\left(\left(x^3-3x+3\right)-\left(2x^3-4x^2+3\right)\right)dx$ or $\int\left(-x^3+4x^2-3x\right)dx$. At this stage, subtraction can be done either way.
	$=\pm\left(-\frac{x^4}{4}+\frac{4x^3}{3}-\frac{3x^2}{2}\right)$ or $\pm\left\{\left(\frac{x^4}{4}-\frac{3}{2}x^2+3x\right)-\left(\frac{2}{4}x^4-\frac{4}{3}x^3+3x\right)\right\}$	A1	OE ± covers A1 being awarded to those who subtract the 'other' way.
	$=\left[\left(-\frac{81}{4}+\frac{108}{3}-\frac{27}{2}\right)-\left(-\frac{1}{4}+\frac{4}{3}-\frac{3}{2}\right)\right]$, or $\left(\frac{81}{4}-\frac{27}{2}+9\right)-\left(\frac{1}{4}-\frac{3}{2}+3\right)-\left\{\left(\frac{81}{2}-\frac{108}{3}+9\right)-\left(\frac{1}{2}-\frac{4}{3}+3\right)\right\}$	DM1	OE Minimum required is $\left(\frac{63}{4}-\frac{7}{4}\right)-\left(\frac{27}{2}-\frac{13}{6}\right)$, i.e. four fractions. Correctly apply limits *their* 1 and 3. Do not allow if $x=0$ used. Need at least one correct substitution in every bracket. If two integrals, need to see substitution into both. Allow one sign error only in each expression, if brackets are not shown.
	$=\frac{8}{3}$	A1	Accept if this comes from use of limits $f(1)-f(3)$ or $\int\left(x^3-4x^2+3x\right)dx$, if $\left\|\frac{-8}{3}\right\|$ used. Only dependent on the first method mark. Accept AWRT 2.67.
		4	

Question	Answer	Marks	Guidance
10(a)	Gradient of $AB = \frac{-5-3}{8-4}$ $[=-2]$	**M1***	
	Midpoint $AB = \left(\frac{8+4}{2}, \frac{-5+3}{2}\right)$ $[(6,-1)]$	**M1**	
	Gradient of normal $= -\frac{1}{-2}\left[=\frac{1}{2}\right]$ and an attempt to find the required equation	**DM1**	Must be used to find equation of perpendicular through *their* $(6,-1)$.
	Equation of perpendicular bisector is $y+1=\frac{1}{2}(x-6)$, so $y=\frac{1}{2}x-4$	**A1**	WWW AG – working involving the perpendicular bisector must be seen.
	Alternative Method for Question 10(a)		
	$AC^2=(a-4)^2+(b-3)^2$, $BC^2=(a-8)^2+(b+5)^2$ both expanded	**M1***	
	Solving $AC = BC$ [= 10]	**DM1**	Only allow a single sign error.
	Eliminating a^2 and b^2	**DM1**	May be awarded before the previous DM1.
	$a=2b+8$, concluding $y=\frac{x}{2}-4$	**A1**	WWW
		4	

Question	Answer	Marks	Guidance
10(b)	Using the centre as $\left(a,\ \frac{1}{2}a-4\right)$	**M1**	May see centre as $(2y+8, y)$ OE. May be seen in an incorrect equation.
	$(4-a)^2+(3-0.5a+4)^2=100$	**M1**	Sub in (4, 3) or (8, −5). Could use circle with $(6,-1)$ and $r=\sqrt{80}$.
	$1.25a^2-15a-35\ [=0]\ \Rightarrow\ a^2-12a-28\ [=0]\ \left(\text{or } b^2+2b-15\ [=0]\right)$	**DM1**	Obtain a 3-term quadratic in *their* x or y.
	$[(a-14)(a+2)=0]\ \Rightarrow\ a=14,\ a=-2$	**A1**	Or $[(b-3)(b+5)=[0]]\ \Rightarrow\ b=3,\ b=-5$.
	$\Rightarrow\ (x-14)^2+(y-3)^2=100$ and $(x+2)^2+(y+5)^2=100$	**A1**	
	Alternative Method 1 for the first 3 marks:		
	Make a or b the subject from a circle centre (a,b) using A or B	**M1**	E.g. $b=\sqrt{100-(y-3)^2}+4$ from circle through A. These equations may have been found in part **(a)**.
	Form an equation in a or b only	**M1**	Substitute *their* a or b into their second circle equation.
	Simplify to a quadratic in a or b	**DM1**	Expect $a^2-12a-28=0$ or $b^2+2b-15=0$, OE.
	Alternative Method 2 for the first 3 marks:		
	Obtaining CM (C, centre; M, mid-point of AB)	**M1**	Expect $\sqrt{80}$. Must be clear this is CM, not AB.
	Using the triangle CMT, where CT is parallel to the x-axis, to find the vertical distance of C from M, MT	**DM1**	Expect $MT=4$.
	Using the triangle CMT, where MT is parallel to the y-axis, to find the horizontal distance of C from M, CT	**DM1**	Expect $CT=8$.
		5	

Question	Answer	Marks	Guidance
11(a)	$\frac{dy}{dx}=\frac{1}{2}kx^{-\frac{1}{2}}-8x$	**B1**	
	$\frac{d^2y}{dx^2}=-\frac{1}{4}kx^{-\frac{3}{2}}-8$	**B1**	
		2	
11(b)	$x^{-\frac{1}{2}}-8x=0 \Rightarrow \mathbf{1-8x^{\frac{3}{2}}=0}$ or $x^{-1}=64x^2$ $\left[\Rightarrow x^3=\frac{1}{64}\text{ or } 8x^{\frac{3}{2}}=1\right]$ Setting their $\frac{dy}{dx}$ to zero and solving, providing their only error(s) are incorrect coefficients	**M1**	OE Award if working leads to $x=\frac{1}{4}$ WWW. Squaring $x^{-\frac{1}{2}}-8x^2=0$ to $x^{-1}-64x^2=0$ gets M0.
	$x=\frac{1}{4}$ only	**A1**	If $x=0$ included, A0 and max of 3/4. **SC B1** only for $x=\frac{1}{4}$ only from squaring $x^{-\frac{1}{2}}-8x^2=0$ directly to $x^{-1}-64x^2=0$ (**SC B1** replacing the M1A1).
	$y=\frac{11}{4}$	**A1**	**SC B1** for $y=\frac{11}{4}$ from squaring $x^{-\frac{1}{2}}-8x^2=0$ to $x^{-1}-64x^2=0$.
	$\frac{d^2y}{dx^2}=-\frac{1}{2}x^{-\frac{3}{2}}-8$ which is negative, so maximum	**B1 FT**	WWW FT *their* x-value and *their* $\frac{d^2y}{dx^2}$. No FT if $x=0$ is the only solution.
		4	

Question	Answer	Marks	Guidance
11(c)	When $x=1$, attempting to find $y=k-2$ and gradient = $\frac{1}{2}k-8$	**M1***	OE **SC B1** if both correct gradients only, or both correct *y*-coordinates only.
	Equation of tangent is $y-k+2=\left(\frac{1}{2}k-8\right)(x-1)$	**A1**	OE, e.g. $y=\left(\frac{k}{2}-8\right)x+\frac{k}{2}+6$ or $y=\frac{k}{2}x-8x+\frac{k}{2}+6$.
	When $x=\frac{1}{4}$,attempting to find $y=\frac{1}{2}k+1.75$ and gradient = $k-2$	**M1***	OE
	Equation of tangent is $y-\frac{1}{2}k-1.75=(k-2)(x-0.25)$	**A1**	OE, e.g. $y=(k-2)x+\frac{k}{4}+\frac{9}{4}$ or $y=kx-2x+\frac{k}{4}+\frac{9}{4}$.
	Meet at $\left(\frac{1}{2}k-8\right)(0.6-1)+k-2=(k-2)(0.6-0.25)+\frac{1}{2}k+1.75$ Equate two tangent equations and substitute $x=0.6$	**DM1**	OE, e.g. $\left(\frac{k}{2}-8\right)0.6+\frac{k}{2}+6=(k-2)0.6+\frac{k}{4}+\frac{9}{4}$. M0 if constants in both equations are the same.
	$\Rightarrow [-0.2k+k+3.2-2=0.35k-0.7+0.5k+1.75]$ $\Rightarrow 0.05k=0.15$ $k=3$	**A1**	
		6	

Cambridge International AS & A Level

MATHEMATICS **9709/11**

Paper 1 Pure Mathematics 1 **May/June 2024**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2024 series for most Cambridge IGCSE, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles

1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.

2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.

3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.

4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).

5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.

6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1(a)	$3(y-2)^2-27$ or $a=-2,\ b=-27$	**B1 B1**	
		2	
1(b)	$(x^2-2)^2=9$ leading to $x^2-2=\pm 3$	**M1**	Must be x^2 unless substitution is clear.
	$x^2=-1$ or $x^2=5$	**M1**	Allow omission of -1 if ±3 seen.
	$x=\pm\sqrt{5}$	**A1**	**B1 SC** if M1M1 not awarded. Ignore ± i, i, –i, √–1. Use of calculator with no working scores 0/3.
	Alternative method for Question 1(b)		
	$3x^4-12x^2-15=0$ leading to $3(x^2-5)(x^2+1)[=0]$	**(M1)**	
	$x^2=-1$ or $x^2=5$	**(M1)**	Allow omission of –1 if factors seen. Factorising or other valid method.
	$x=\pm\sqrt{5}$	**(A1)**	**B1 SC** if M1M1 not scored. Ignore ± i, i, –i, √–1. Use of calculator with no working scores 0/3.
		3	

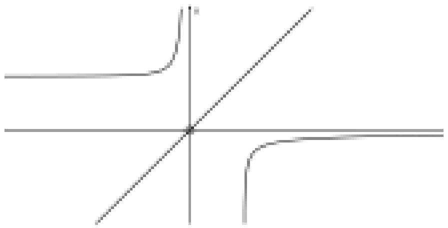
Question	Answer	Marks	Guidance
2(a)	{Stretch} {factor 3} { in y-direction}	**B2,1,0**	2 out of 3 scores B1.
	{Translation} $\begin{pmatrix} \{0\} \\ \{-2\} \end{pmatrix}$	**B2,1,0**	Accept shift.
	Alternative Method for Question 2(a)		
	{Translation} $\begin{pmatrix} \{0\} \\ \left\{-\frac{2}{3}\right\} \end{pmatrix}$	**(B2,1,0)**	2 out of 3 scores B1. Accept shift.
	{Stretch} {factor 3} { in y-direction}	**(B2,1,0)**	
		4	
2(b)	$[\mathrm{f}(x)] = \{-3\sin x\}\{-2\}$	**B1 B1**	No marks awarded if extra terms seen.
		2	

Question	Answer	Marks	Guidance
3(a)	$20 \times 27 \times a^3 [=160]$	**M1**	Allow 6C3 $\times$ $3^3 \times a^3 [=160]$. Accept $540a^3$ with no other working for M1.
	$[a] = \frac{2}{3}$	**A1**	Allow 0.667 AWRT. **SC B1** is $a = \frac{2}{3}$ with no other working.
		2	

Question	Answer	Marks	Guidance
3(b)	Coefficient of x^2 is $15 \times 81 \times \left(their\,\frac{2}{3}\right)^2\ [=540]$	B1 FT	May be in a list. 6C2 and 3^4 must be evaluated but may be implied by later work. Condone 540 with no working.
	$160 \times 1 - 2 \times their\,540$	M1	
	$= -920$	A1	Condone $-920x^3$.
		3	

Question	Answer	Marks	Guidance
4(a)	[k] = 4.00063	B1	CAO
		1	
4(b)	[Gradient AE] = 6.3566	B1	CAO
		1	
4(c)	Suggests that $\left[f'(2)\right] = 6.25$	B1	CAO
		1	

Question	Answer	Marks	Guidance
5(a)	$\frac{\sin^2 x - \cos x - 1}{1+\cos x} = \frac{1-\cos^2 x - \cos x - 1}{1+\cos x}$ or $\frac{-\cos^2 x - \cos x}{1+\cos x}$	**M1**	For use of $\sin^2 x + \cos^2 x = 1$. Allow use of s, c, t or omission of x throughout.
	$= \frac{-\cos x(1+\cos x)}{1+\cos x}$	**M1**	For factorising.
	$= -\cos x$	**A1**	
		3	
5(b)	$-\frac{1}{2}\cos x = \frac{1}{4} \Rightarrow x = \cos^{-1}\left(-\frac{1}{2}\right)$	**M1**	
	$x = 120°$ or $x = 240°$	**A1**	
		A1 FT	FT for 360 – *their* answer. A1 A0 if extra solution(s) in range. **SC B1** if answer in radians for both $\frac{2\pi}{3}, \frac{4\pi}{3}$.
		3	

Question	Answer	Marks	Guidance
6(a)		**B1**	For curve in correct quadrant.
		B1	Fully correct including line $y = x$. Horizontal asymptote closer to x axis than vertical asymptote is to y axis.
		2	

Question	Answer	Marks	Guidance
6(b)	$x=\frac{2}{y^2}+4$ leading to $y^2(x-4)=2$ or $y^2=\frac{2}{(x-4)}$	**M1**	Allow x and y swapped around.
	$y^2=\frac{2}{(x-4)}$ leading to $y=[\pm]\sqrt{\frac{2}{x-4}}$ or $x=[\pm]\sqrt{\frac{2}{y-4}}$	**M1**	
	$\left[f^{-1}(x)\right]=-\sqrt{\frac{2}{x-4}}$	**A1**	
		3	
6(c)	$[x]=-2$	**B1**	
		1	
6(d)	Because f^{-1} is always negative and f is always positive or curves do not intersect	**B1**	Accept other correct answers e.g. 'f is only defined for positive values of x and f^{-1} is only defined for negative values of x' or 'domains do not overlap' or 'the y values cannot be the same' or 'the x values cannot be the same'.
		1	

Question	Answer	Marks	Guidance
7(a)	Angle $\theta=\frac{\pi}{2}-\cos^{-1}\frac{10}{15}$ or $\sin^{-1}\frac{10}{15}=0.7297$	**B1**	Condone working in degrees if converted to radians at the end. AG
		1	

Question	Answer	Marks	Guidance
7(b)	$BC = \sqrt{15^2 - 10^2}\ [= 11.18... \text{ or } 5\sqrt{5}]$	**B1**	
	Arc $AB = 15 \times 0.7297 [= 10.9455]$	**B1**	
	Perimeter = *their* BC + *their* arc AB + 25 + 5π	**M1**	
	Perimeter = 62.8	**A1**	AWRT
	Area sector $AOB = \frac{1}{2} \times 15^2 \times 0.7297 [= 82.09]$	**B1**	
	Area = $\frac{1}{2} \times 10 \times$ *their* BC + *their* sector AOB + $\frac{\pi}{4} \times 10^2$	**M1**	
	Area = 217	**A1**	AWRT
		7	

Question	Answer	Marks	Guidance
8(a)	$2(4p-1)=25+13-p$	*M1	
	$p=\frac{40}{9}$	A1	
	$d=\left(4\times\frac{40}{9}-1\right)-25\left[=-\frac{74}{9}\right]$	DM1	Using *their* p to find d.
	10^{th} term = $25+9d=-49$	A1	
	Alternative Method for first 3 marks of Question 8(a)		
	$d=4p-26$, $d=14-5p$, $p+2d=-12$ Any two	(*M1)	Allow unsimplified or equivalent.
	Solving simultaneously to find p or d	(DM1)	
	$\left[p=\frac{40}{9}\right]$, $d=-\frac{74}{9}$	(A1)	
		4	

Question	Answer	Marks	Guidance
8(b)	$(4q-1)^2 = 25(13-q) \Rightarrow 16q^2 + 17q - 324[=0]$	M1	
	$(q-4)(16q+81)[=0]$ leading to $[\Rightarrow q=4]$	M1	Solve 3 term quadratic with real solutions.
	$[r=]\frac{3}{5}$	A1	Ignore $\frac{-17}{20}$.
	Sum to infinity $= \frac{25}{1-\frac{3}{5}} = \frac{125}{2}$	A1	Ignore extra solution. **SC B1** if no method shown for solving quadratic.
	Alternative Method for Question 8(b)		
	$25r = 4q - 1$, $25r^2 = 13 - q$ leading to $100r^2 + 25r - 51 = 0$	(M1)	
	$(5r-3)(20r+17)=0$	(M1)	Solve 3 term quadratic with real solutions.
	$r = \frac{3}{5}$	(A1)	Ignore $\frac{-17}{20}$.
	Sum to infinity $= \frac{25}{1-\frac{3}{5}} = \frac{125}{2}$	(A1)	Ignore extra solution. **SC B1** if no method shown for solving quadratic.
		4	

Question	Answer	Marks	Guidance
9	Volume of cylinder = $\pi \times 1^2 \times \frac{7}{5} = \frac{7}{5}\pi$	**B1**	May be done using $\int_1^{2.4} 1$. This would be the only mark available if candidate integrates *y*.
	Volume under curve $=[\pi]\int \frac{1}{(5x-4)^{\frac{2}{3}}}\mathrm{d}x$	**M1**	No further marks available if $\int y$.
	$=[\pi]\left\{\frac{3}{5}\right\}\left\{(5x-4)^{\frac{1}{3}}\right\}$	**B1 B1**	Calculator used for integration scores no further marks.
	$=[\pi]\frac{3}{5}\left(8^{\frac{1}{3}}-1\right)\ \left[=\frac{3}{5}\pi\right]$	**M1**	Uses limits 1, 2.4 in an integral of y^2.
	Volume = $\frac{7}{5}\pi - \frac{3}{5}\pi\ = \frac{4}{5}\pi$	**A1**	**SC B1** if the only error is not showing substitution.
		6	

Question	Answer	Marks	Guidance
10	$(x-3)^2+y^2=18 \quad y=mx-9$ leading to $(x-3)^2+(mx-9)^2=18$	M1	Finding equation of tangent and substituting into circle equation. Must be $mx-9$.
	$x^2-6x+9+m^2x^2-18mx+81=18$ leading to $(m^2+1)x^2-(6+18m)x+72[=0]$	M1	Brackets expanded and all terms collected on one side of the equation. May be implied in the discriminant. *m* cannot be numeric.
	$(6+18m)^2-4(m^2+1)\times 72[=0]$	*M1	Use of b^2-4ac. Not in quadratic formula. *m* cannot be numeric, *c* must be numeric.
	$36m^2+216m-252[=0]$ [leading to $m^2+6m-7=0$]	DM1	Simplifies to 3 term quadratic.
	$m=1$ or $m=-7$	A1	Condone no method for solving quadratic shown.
	$m=1$ leading to $2x^2-24x+72=0$ leading to $x=6$	DM1	Must be correct *x* for *their* quadratic.
	$m=-7$ leading to $50x^2+120x+72=0$ leading to $x=-\frac{6}{5}$	DM1	Must be correct *x* for *their* quadratic.
	$(6,-3),\left(-\frac{6}{5},-\frac{3}{5}\right)$	A1	

Question	Answer	Marks	Guidance
10	**Alternative Method 1 for first 4 marks of Question 10**		
	$\dfrac{\lvert 3m-1(0)-9\rvert}{\sqrt{m^2+1}}$	(M1)	Use of the formula for the length of a perpendicular from a point to a line.
	$\dfrac{\lvert 3m-1(0)-9\rvert}{\sqrt{m^2+1}}=\sqrt{18}$	(M1)	Equates length of a perpendicular from a point to a line to the radius.
	$(3m-9)^2=18(m^2+1)$	(M1)	Squares and clears the fraction.
	$9m^2-54m+81=0$ $\left[\text{leading to } m^2+6m-7=0\right]$	(M1)	
	Alternative Method 2 for first 3 marks of Question 10		
	$(3-x)(9+6x-x^2)^{-1/2}=m$	(M1)	OE Differentiates implicitly or otherwise and equates $\dfrac{dy}{dx}$ to m.
	$(1+m^2)x^2-6(1+m^2)x+9(1-m^2)[=0]$	(M1)	Brackets expanded and all terms collected on one side of the equation. May be implied in the discriminant.
	$36(1+m^2)^2-4(1+m^2)\times 9(1-m^2)[=0]$	(M1)	Use of b^2-4ac.
		8	

Question	Answer	Marks	Guidance
11(a)	$\frac{dy}{dx} = -\frac{12}{x^4} + \frac{3}{x^2}$	B1	
	$\frac{dy}{dx} = -\frac{12}{x^4} + \frac{3}{x^2} = 0$ leading to $3x^4 - 12x^2 = 0$ or $-12 + 3x^2 = 0$	M1	Set = 0 or uses $<$, $\leqslant$ and simplifies. Must be from $\frac{dy}{dx} = \frac{A}{x^4} + \frac{B}{x^2}$.
	$3x^2(x^2 - 4) = 0$ leading to $x = \pm 2$ only	A1	**SC B1** for $x = \pm 2$ if M0 scored.
	$-2 < x < 0$ and $0 < x < 2$ or (-2, 0) and (0, 2) or $-2 < x < 2$ and $x \neq 0$	B1FT	Allow and/or.
		B1FT	Allow $-2 \leqslant x < 0$ and / or $0 < x \leqslant 2$ but only B1B0 if 0 included in either or both. Allow [–2, 0) and (0, 2]. Allow B1B0 for $-2 < x < 2$ or (–2, 2). Must be from $\frac{dy}{dx} = \frac{A}{x^4} + \frac{B}{x^2}$.
		5	B marks only available if $\frac{dy}{dx} = \frac{A}{x^4} + \frac{B}{x^2}$.

Question	Answer	Marks	Guidance
11(b)	[At $x=1$] $y=3$ and $m\tan=-9$	*M1	Using *their* $\frac{dy}{dx}$.
	$m\text{norm}=-\frac{1}{-9}=\frac{1}{9}$	DM1	
	Equation of normal is $y-3=\frac{1}{9}(x-1)$ $\left[\text{leading to } y=\frac{1}{9}x+\frac{26}{9}\right]$	A1	
	At $x=-1, y=1, m=-9$	M1	
	Equation of tangent is $y-1=-9(x+1)$ $[\text{leading to } y=-9x-8]$	A1	
	Meet when $\frac{1}{9}x+\frac{26}{9}=-9x-8$ $\left[\text{leading to } x=-1.19512\ldots,\frac{-49}{41}\right]$	M1	Equates *their* tangent and *their* normal.
	Area $=\frac{1}{2}\times \textit{their}\, 1.19512\ldots\times \textit{their}\left(\frac{26}{9}+8\right)$	M1	If $\int y_2-y_1$ is used integration must be correct and substitution shown.
	6.51	A1	AWRT Accept fraction wrt 6.51
		8	

Cambridge International AS & A Level

MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **May/June 2024**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2024 series for most Cambridge IGCSE, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles

1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.

2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.

3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.

4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).

5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.

6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	$240\left[x^2\right]$ or $80a^2\,[x^2]$	**B1**	May be seen in an expansion.
	$240 = 12 \times 80a^2$	**M1**	*Their* 240 equated to 12 × *their* $80a^2$ which must contain a^2.
	0.5	**A1**	OE Condone ± 0.5
		3	

Question	Answer	Marks	Guidance
2	Stretch **factor** 4 in y-direction/parallel to the y axis/vertically.	**B1**	Allow use of SF in place of factor. Allow in/on/along the y axis or 'the x axis is invariant.'
	Translation $\begin{pmatrix}3\\0\end{pmatrix}$ or 3 parallel to the x axis or in the x direction, allow horizontally. $\begin{pmatrix}0\\-8\end{pmatrix}$ or -8 parallel to the y axis or in the y direction, allow vertically.	**B2**	Condone 'Shift'. These translations can be combined as $\begin{pmatrix}3\\-8\end{pmatrix}$, this counts as 2 elements. Give priority to a correct vector over any incorrect wording. B2 for all 3 B1 for 2 out of 3
	Two translations, one in each direction, and a stretch only.	**M1**	Condone inaccurate terminology, such as up, down, left and right, if the intention is clear.
	Correct order of operations. The stretch which must be in the in the y direction must come before the translation in the y direction.	**A1**	Condone inaccurate terminology if the intention is clear but numerical values must be correct.

Question	Answer	Marks	Guidance
2	**Alternative Method for Question 2**		
	Translation $\begin{pmatrix}3\\0\end{pmatrix}$ or 3 parallel to the x axis or in the x direction, allow horizontally. $\begin{pmatrix}0\\-2\end{pmatrix}$ or -2 parallel to the y axis or in the y direction, allow vertically.	**(B2)**	Condone 'Shift'. These translations can be combined as $\begin{pmatrix}3\\-2\end{pmatrix}$, this counts as 2 elements. Give priority to a correct vector over any incorrect wording. B2 for all 3. B1 for 2 out of 3.
	Stretch **factor** 4 in y-direction/parallel to the y axis/vertically.	**(B1)**	Allow use of SF in place of factor. Allow in/on/along the y axis or "the x axis is invariant."
	Two translations, one in each direction, and a stretch only.	**(M1)**	Condone inaccurate terminology, such as transform, move, up, down, left and right, if the intention is clear.
	Correct order of operations. The stretch which must be in the in the y direction must come after the translation in the y direction.	**(A1)**	Condone inaccurate terminology if the intention is clear but numerical values must be correct.
		5	

Question	Answer	Marks	Guidance
3(a)	$7\dfrac{\sin\theta}{\cos\theta} \div \cos\theta + 12[=0]$ $\left[\text{leading to } 7\dfrac{\sin\theta}{\cos\theta} + 12\cos\theta = 0\right]$	M1*	OE Use of $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$.
	$7\sin\theta + 12\left(1 - \sin^2\theta\right)[=0]$	DM1	Use of $s^2 + c^2 = 1$.
	$\Rightarrow 12\sin^2\theta - 7\sin\theta - 12 = 0$	A1	AG, WWW Condone use of s, c and t and/or omission of θ throughout working but the A1 is for cao.
		3	
3(b)	$\left[12\sin^2\theta - 7\sin\theta - 12 = 0 \text{ leading to}\right]\left(4\sin\theta + 3\right)\left(3\sin\theta - 4\right)$	M1	
	$\sin\theta = -\dfrac{3}{4}$ $\left[\text{or } \dfrac{4}{3}\right]$	B1	OE, WWW Can be implied by a correct value for $\sin^{-1}\left(-\dfrac{3}{4}\right)$ e.g. $-48.6°$.
	$[\theta =]228.6°, 311.4°$	B1	AWRT, WWW No others in the range $0° \le \theta \le 360°$. Ignore any answers outside this range. Condone 229°, 311°.
		3	

Question	Answer	Marks	Guidance
4(a)	$\left[f^{-1}(x)=\right](x+1)^2$	**B1**	ISW Condone '$y=$'.
		1	
4(b)	$0 < g(x) \leqslant \frac{1}{2}$ or $g(x) > 0$ and $g(x) \leqslant \frac{1}{2}$ or $\left(0, \frac{1}{2}\right]$	**B1**	Do not allow $g(x) > 0, g(x) \leqslant \frac{1}{2}$. Do not allow $g(x) > 0$ or $g(x) \leqslant \frac{1}{2}$. Condone g or y in place of $g(x)$.
	g^{-1} does not exist because it is one to many or g^{-1} does not exist because it is not one to one. Or g^{-1} does not exist because g is not one to one or g^{-1} does not exist because g is many to one or g^{-1} does not exist because g fails the horizontal line test.	**B1**	g^{-1} can be replaced by 'It' throughout. A correct statement followed by any further incorrect explanation can be awarded B1.
		2	

Question	Answer	Marks	Guidance
4(c)	$f\left(\frac{25}{16}\right)=\frac{1}{4}$	B1	SOI
	$\frac{1}{\left(\sqrt{x}-1\right)^2+2}=\frac{1}{4}$	M1	Equating $\frac{1}{\left(\sqrt{x}-1\right)^2+2}$, or *their* 'simplified' version, to *their* $f\left(\frac{25}{16}\right)$.
	$\left[\left(\sqrt{x}-1\right)^2+2=4 \text{ leading to}\right]$ $\sqrt{x}-1=\sqrt{2}$ leading to $x=\left(1\pm\sqrt{2}\right)^2$ Or $\left[x-2\sqrt{x}+1+2=4 \text{ leading to}\right]$ $x-2\sqrt{x}-1=0$ leading to $x=\left(1\pm\sqrt{2}\right)^2$ Or $\left[x-1=2\sqrt{x} \text{ leading to}\right]$ $x^2-6x+1=0$ leading to $x=\frac{6\pm\sqrt{36-4}}{2}$	A1	Simplification as far as $x=\ldots$ Allow just + in the results because − can be disregarded at this stage. Can be implied by the final answer. Note: $x=1\pm\sqrt{2}$ scores A0.
	$3+2\sqrt{2}$	A1	Must discount the solution $3-2\sqrt{2}$.
		4	

Question	Answer	Marks	Guidance
5(a)	$[d=]\sin\theta-\tan\theta$	**M1**	For subtraction of *their* first term from *their* second term. Condone incorrect evaluation before subtraction.
	$[d=]\frac{\sqrt{2}}{2}-1$	**A1**	OE Sight of -0.29 AWRT can be awarded M1A1.
	$[S_{40}=]\frac{40}{2}\left(2\times\tan\theta+39(\sin\theta-\tan\theta)\right)$	**M1**	Use of a correct formula for S_{40}. Condone use of *their*, clearly identified, incorrect values for *a* and *d* for this mark.
	$390\sqrt{2}-740$ or $\frac{780}{\sqrt{2}}-740$	**A1**	ISW If A0 then sight of -188 AWRT, -188.5 or -189 should be awarded M1A1M1A0.
		4	
5(b)(i)	$r=\frac{\sin\theta}{\tan\theta}$ $[=\cos\theta]$	**B1**	Condone omission of θ.
	$S_\infty=\frac{\tan\theta}{1-\cos\theta}$ or $\frac{\sin\theta}{\cos\theta-cos^2\theta}$ or $\frac{\tan^2\theta}{\tan\theta-\sin\theta}$	**B1**	ISW Do not allow fractions within fractions nor omission of θ.
		2	

Question	Answer	Marks	Guidance
5(b)(ii)	$a=\sqrt{3}\,(1.73..)$ and $r=\dfrac{1}{2}$	B1	OE, SOI.
	$[S_{10}=]\sqrt{3}\left(\dfrac{1-\left(\frac{1}{2}\right)^{10}}{1-\frac{1}{2}}\right)$	M1	This mark can be awarded for a correct formula with *their* values for a and r or $a=\tan\theta$ and $r=\dfrac{\sin\theta}{\tan\theta}$ or $\cos\theta$. Condone $\dfrac{1}{2}^{10}$.
	$=3.46$	A1	AWRT Condone $\dfrac{1023\sqrt{3}}{512}$.
		3	Note: S_9 gives the same answer but scores B1M0A0.

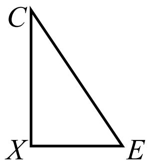
Question	Answer	Marks	Guidance
6(a)	$\frac{dy}{dx} = 2 - \frac{1}{2} \times 8x^{-\frac{1}{2}}$	B1	
	$2 - 4x^{-\frac{1}{2}} = 0$	M1	Equating *their* two term $\frac{dy}{dx}$, with at least one term correct, to 0.
	[A is] $(4, -8)$ or $x = 4, y = -8$	A1	
	[B is] $(16, 0)$ or $x = 16, y = 0$	B1	
		4	Note: Correct answers without use of $\frac{dy}{dx}$ can be awarded 4/4.

Question	Answer	Marks	Guidance
6(b)	$[\pm]\ \dfrac{2x^2}{2} - \dfrac{8}{\frac{3}{2}}x^{\frac{3}{2}}\ \ [+C]$	B1	Seen correct in unsimplified form or better.
	$[\pm]\dfrac{x^2-32x}{3}$ or $\dfrac{(2x-32)^2}{12}\ [+C]$	B1	Seen correct in unsimplified form or better.
	Attempt to integrate, defined by at least one correct power in each expression, and then subtract.	M1	Multiplying by 3 before integration scores M0.
	$\left\{\left(16^2 - \dfrac{8}{\frac{3}{2}}.16^{\frac{3}{2}}\right) - \left(4^2 - \dfrac{8}{\frac{3}{2}}.4^{\frac{3}{2}}\right)\right\}\ [-]\ \left\{\left(\dfrac{16^2-32\times16}{3}\right) - \left(\dfrac{4^2-32\times4}{3}\right)\right\}$	M1	Use of *their* x values, > 0, from **(a)** as limits in *their* integrated expressions. Allow, for correct limits, sight of $\pm\left(\left\{\left(-\dfrac{256}{3}\right)-\left(-\dfrac{80}{3}\right)\right\}[-]\left\{\left(-\dfrac{256}{3}\right)-\left(-\dfrac{112}{3}\right)\right\}\right)$. If incorrect limits are used, then clear substitution must be seen.

Question	Answer	Marks	Guidance
6(b)	**Alternative Method 1 for first 4 marks of Question 6(b)**		
	$\left[[\pm]\int 2x - 8x^{\frac{1}{2}}\,dx =\right]\quad x^2 - \frac{8}{\frac{3}{2}}x^{\frac{3}{2}}\quad [+C]$	**(B1)**	Seen correct in unsimplified form or better.
	[Area of triangle =] 48	**(B1)**	
	Attempt to integrate, defined by at least one correct power, and then subtract *their* triangle area.	**(M1)**	
	$\left\{\left(16^2 - \frac{8}{\frac{3}{2}}.16^{\frac{3}{2}}\right) - \left(4^2 - \frac{8}{\frac{3}{2}}.4^{\frac{3}{2}}\right)\right\}$	**(M1)**	Use of *their* x values, > 0, from **(a)** as limits in *their* integrated expression. Allow sight of $\pm\left(\left\{\left(-\frac{256}{3}\right) - \left(-\frac{80}{3}\right)\right\}\right)$. If incorrect limits are used, then clear substitution must be seen.

Question	Answer	Marks	Guidance
6(b)	**Alternative Method 2 for first 4 marks of Question 6(b)**		
	Subtract and then integrate, defined by at least two correct powers. Condone functions being the wrong way round.	**(M1)**	If terms in x have not been combined use the first scheme.
	$[\pm]\left(\frac{4}{3\times 2}x^2-\frac{8}{\frac{3}{2}}x^{\frac{3}{2}}+\frac{32x}{3}\right)$	**(B2,1,0)**	B2 for 3 correct terms, B1 for any 2 correct terms.
	$[\pm]\left(\left(\frac{4}{3\times 2}\times 16^2-\frac{8}{\frac{3}{2}}\times 16^{\frac{3}{2}}+\frac{32\times 16}{3}\right)-\left(\frac{4}{3\times 2}\times 4^2-\frac{8}{\frac{3}{2}}\times 4^{\frac{3}{2}}+\frac{32\times 4}{3}\right)\right)$	**(M1)**	Use of *their* x values, >0, from **(a)** as limits in *their* integrated expression. Allow sight of $\pm\left(0-\frac{32}{3}\right)$. If incorrect limits are used, then clear substitution must be seen.
	$\frac{32}{3}$, $10\frac{2}{3}$ or 10.7	**(B1)**	AWRT Allow $-\frac{32}{3}$ or $-\frac{32}{3}$ changed to $+\frac{32}{3}$ for this mark.
		(5)	Condone the inclusion of π for the first 4 marks but use of $\int y^2$ scores a maximum of B1 for the triangle.

Question	Answer	Marks	Guidance
7(a)	$(x-6)^2 + (2a-x+a)^2 = 18$	**M1***	Replacing y with $2a-x$ in the circle equation, condone incorrect expansion before substitution.
	$2x^2 - 12x - 6ax + 9a^2 + 36 - 18[=0]$	**A1**	All terms collected on one side of the equation. May be implied by the discriminant.
	$(12+6a)^2 - 4\times2\times(9a^2+18)\ [=0]$	**DM1**	Correct use of "$b^2 - 4ac$" from *their* 3 term quadratic equation in x, with an x term of the form $(m+na)x$ with both m and $n \neq 0$.
	$-36a^2 + 144a[+0=0]$	**A1**	
	$a=0,\ a=4$	**A1**	
		5	
7(b)	[Centre is] (6, −4) or [Point of intersection is] (9, −1)	**B1**	
	[Gradient of diameter] $=1$	**B1**	
	$y+4=x-6$ or $y+1=x-9$ [leading to $y=x-10$]	**B1FT**	FT on *their* point of intersection or *their* centre with an x co-ordinate of ±6 and gradient = 1.
		3	

Question	Answer	Marks	Guidance
8(a)(i)	 $X\hat{C}E = \frac{\pi}{6}$ $C\hat{E}X = \frac{\pi}{3}$		
	$\frac{XE}{0.4} = \sin\frac{\pi}{6}$ or $\frac{XE}{0.4} = \cos\frac{\pi}{3}$ $[XE = 0.2]$	**M1**	A correct trig expression involving *XE*. Do not condone a mixture of degrees and radians.
	Length $EF = 2 + 2 \times 0.2 = 2.4$	**A1**	AG
		2	

Question	Answer	Marks	Guidance
8(a)(ii)	$[CX =]0.4\cos\frac{\pi}{6}$ or $0.4\sin\frac{\pi}{3}$ or $\sqrt{0.4^2 - 0.2^2}$	**B1**	OE, SOI Expect $\frac{\sqrt{3}}{5}$ or 0.3464.
	$[\text{Sector} =]\frac{1}{2}\times(0.4)^2\times\frac{\pi}{3}$	**B1**	SOI Expect 0.0838 or $\frac{2\pi}{75}$. Allow use of $\frac{60}{360}\pi(0.4)^2$.
	Either Area of *their* (rectangle + two triangles + two sectors) Or Area of *their* (trapezium + two sectors)	**M1**	Either implied by a correct answer or areas clearly labelled. Expect 0.6928 + 0.06928 + 0.1676 or $\frac{2\sqrt{3}}{5}+\frac{\sqrt{3}}{25}+\frac{4\pi}{75}$. Or 0.7621 + 0.1676 or $\frac{11\sqrt{3}}{25}+\frac{4\pi}{75}$.
	0.930	**A1**	AWRT Condone $\frac{11\sqrt{3}}{25}+\frac{4\pi}{75}$.
		4	

Question	Answer	Marks	Guidance
8(b)	[Length AD =] $2+2r$	**B1**	Must be seen alone or part of a list and not part of a product.
	[Arc length =] $r\times\frac{\pi}{3}$	**B1**	May be implied by $r\times\frac{\pi}{3}\times 2$. Must be seen alone or part of a list.
	$[EF =]2+2r\sin\frac{\pi}{6}$ or $2+2r\cos\frac{\pi}{3}$ or $2+r$	**B1**	Must be seen alone or part of a list and not part of a product.
	[$4+3r+\frac{2\pi r}{3}=6$ leading to] 0.393	**B1**	AWRT Condone $\frac{6}{2\pi+9}$. NB: Using EF = 2.4 gives 0.391.
		4	

Question	Answer	Marks	Guidance
9(a)	$6(2x-3)^2 - 6x < 0$ or $= 0$	**B1***	Condone $\leqslant 0$. If $6(2x-3)^2$ only or $6(2x-3)^2 - 6$ is used, do not treat as a MR.
	$24x^2 - 78x + 54$ or $4x^2 - 13x + 9$ or $(x-1)(4x-9)$ OR $6(2x-3)^2 < 6x$ leading to $(2x-3) < \sqrt{x}$ leading to $2x - \sqrt{x} - 3$	**M1**	Expanding brackets and collecting terms to arrive at a three term quadratic, only condone sign errors.
	$[x =]1, \frac{9}{4}$	**B1**	
	$1 < x < \frac{9}{4}$ or $x > 1$ and $x < \frac{9}{4}$ or $\left(1, \frac{9}{4}\right)$	**DB1FT**	OE Condone consistent use of $\leqslant$ and $\geqslant$ or [ ]. Do not allow $x > 1$ or $x < \frac{9}{4}$ nor $x > 1, x < \frac{9}{4}$. FT on *their* values coming from a correct initial statement.
		4	

Question	Answer	Marks	Guidance
9(b)	$\left[\mathrm{f}(x)=\right]\left\{\frac{6}{3\times 2}(2x-3)^3\right\}\left\{-\frac{6}{2}x^2\right\}\quad[+C]$	**B1 B1**	B1 for each $\{$Correct integral$\}$.
	$-1=(-1)^3-3\times 1^2+C$	**M1**	$\mathrm{f}(x)=-1$ equated to *their* integrated expression, defined by two terms with at least one correct power + C, with $x = 1$.
	$\left[f(x)=\right](2x-3)^3-3x^2+3$	**A1**	CAO Only condone $C = 3$ as final answer if coefficients have been simplified earlier. Do not ISW if the result is of the form $y=mx+c$.
	Alternative method for Question 9(b)		
	$\left[f'(x)=24x^2-78x+54 \text{ leading to}\right]\left[\mathrm{f}(x)=\right]8x^3-39x^2+54x[+C]$	**(B2,1,0)**	B2 completely correct, B1 any two correct terms.
	$-1=8-39+54+C$	**(M1)**	$\mathrm{f}(x)=-1$ equated to *their* integrated expression, defined by three terms with at least one correct power + C, with $x = 1$.
	$\left[f(x)=\right]\ 8x^3-39x^2+54x-24$	**(A1)**	Only condone $C = -24$ as final answer if coefficients have been simplified earlier. Do not ISW if the result is of the form $y=mx+c$.
		4	

Question	Answer	Marks	Guidance
10(a)	$[x=]-2$	**B1**	
	$\frac{dy}{dx}=k(5-2x)^{\frac{1}{2}}$ $\left[=-2\times\frac{3}{2}(5-2x)^{\frac{1}{2}}\right]$	**M1***	OE Differentiating to get $k(5-2x)^{\frac{1}{2}}$ only.
	$\left[\frac{dy}{dx}=\frac{dy}{dt}\times\frac{dt}{dx} \text{ leading to}\right]$ $-9=\pm 5\times\frac{dt}{dx}$	**DM1**	Correct statement linking *their* numerical expression for $\frac{dy}{dx}$ with $\frac{dt}{dx}$ and ± 5.
	$\frac{5}{9}$ or 0.556 =	**A1**	AWRT
		4	

Question	Answer	Marks	Guidance
10(b)	$k(5-2x)^{\frac{1}{2}} = -3$	M1	Equating *their* $\frac{dy}{dx}$ of the form $k(5-2x)^{\frac{1}{2}}$ to -3.
	[B is] $(2,6)$	A1	
	Gradient $AB = m_1 = \frac{32-6}{-2-2}$,gradient of perpendicular$= -\frac{1}{m_1} = \frac{4}{26}$	M1*	For A, y must be 32. Clear use of $\frac{\text{difference in y co-ordinates}}{\text{difference in x co-ordinates}}$ for points A and B, condone inconsistent order, and using $m_1 m_2 = -1$. If incorrect values or another complete method used, then working must be clear.
	Mid point is $\left(\frac{2-2}{2}, \frac{6+32}{2}\right) = (0,19)$	M1*	Finding the midpoint of AB using A and B. If incorrect values used then all working must be clear. For A, y must be 32.
	$y - 19 = \frac{2}{13}(x-0)$	DM1	Finding the equation of the perpendicular bisector using *their* midpoint and *their* perpendicular gradient.
	$2x - 13y + 247 = 0$ or $\pm$ integer multiples of this.	A1	
		6	

Cambridge International AS & A Level

MATHEMATICS **9709/13**

Paper 1 Pure Mathematics 1 **May/June 2024**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2024 series for most Cambridge IGCSE, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles

1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.

2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.

3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.

4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).

5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.

6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	Correct second term $30x$ in expansion of $(1+3x)^{10}$	**B1**	WWW, may be implied later.
	Correct third term $+405x^2$	**B1**	Ignore subsequent terms, may be implied later.
	Multiply $(2-5x)$ by *their* $30x+405x^2$ to obtain two x^2 terms only	**M1**	Expect $-150x^2, 810x^2$.
	Coefficient is 660	**A1**	Must be clearly identified. Allow final answer $660x^2$.
		4	

Question	Answer	Marks	Guidance
2(a)	State $(\frac{5}{3}\pi,\ 0)$ for point A	**B1**	Or exact equivalent. Allow $x=\dfrac{5}{3}\pi$ or exact equivalent.
	$x=\dfrac{19}{6}\pi$ for point B	**B1**	Or exact equivalent. May be implied in coordinate or vector form.
	$y=-k$ for point B	**B1**	May be implied in coordinate or vector form.
		3	
2(b)	Solve at least as far as $\sin^{-1}3t=k\pi$ with correct value for $\cos^{-1}\left(\frac{1}{2}\sqrt{2}\right)$	**M1**	Allow use of $\pi=3.14...$. Allow $\sin^{-1}3t=30$.
	$\sin^{-1}3t=\frac{1}{6}\pi$ and hence $t=\frac{1}{6}$	**A1**	Or exact equivalent. Can use degrees if consistent.
		2	

Question	Answer	Marks	Guidance
3(a)	State $2r + r\theta = 65$ and $\frac{1}{2}r^2\theta = 225$	**B1**	
	Form a 3-term quadratic or cubic in r or θ or $r\theta$ from correct arc and sector formula	***M1**	Condone sign errors.
	Solve *their* 3 term quadratic or cubic to obtain values of r or θ	**DM1**	Expect $2r^2 - 65r + 450 = (2r-45)(r-10)$ or $18\theta^2 - 97\theta + 72 = (9\theta - 8)(2\theta - 9)$.
	$r = 10$ and $\theta = 4.5$ ignore $r = 22.5$ and $\theta = \frac{8}{9}$, do not ignore $r = 0$	**A1**	**B1 SC** if no quadratic or cubic solution. If $r = 0$ included A0 or B0 SC.
		4	
3(b)	Use correct formula for area of triangle with clear use of angle being $2\pi -$ their θ	**M1**	Expect 1.783 or 102.2°, *their* θ must be reflex.
	48.9	**A1**	AWRT, WWW or a second answer. Or greater accuracy; condone absence of units.
		2	

Question	Answer	Marks	Guidance
4(a)	Use identity $\tan\theta = \frac{\sin\theta}{\cos\theta}$	**M1**	
	Use identity $\cos^2\theta = 1 - \sin^2\theta$	**M1**	
	$\pm(5\sin^2\theta + 7\sin\theta - 6 = 0)$	**A1**	
		3	

Question	Answer	Marks	Guidance
4(b)	Attempt solution of *their* 3 term equation and correct process to find at least 1 value of $\sin x$ or $\sin 2x$ or $\sin\theta$	**M1**	Expect $(5s-3)(s+2)=0,\ s=3/5$.
	$x=18.4$	**A1**	Or greater accuracy. **B1 SC** if no solution to the quadratic.
	$x=71.6$ or (90 – *their* 18.4) or greater accuracy; and no other solutions for $0° < x < 180°$	**A1FT**	WWW **B1 SC** FT if no solution to the quadratic. **B1 SC** both correct in radians, 0.322, 1.25.
		3	

Question	Answer	Marks	Guidance
5(a)	Differentiate to obtain $4x+\frac{1}{2}x^{-2}$	**B1**	OE Condone '+c'.
	Equate first derivative to zero and solve $4x+\dfrac{K}{x^2}=0$ as far as $x^3=k$, K and k non-zero	**M1**	Not given if '+c' used.
	$x=-\frac{1}{2}$ and $y=\frac{9}{2}$	**A1**	OE **B1 SC** if no visible solution of the cubic.
		3	

Question	Answer	Marks	Guidance
5(b)	Differentiate *their* first derivative, substitute *their* x value. Substitution may be implied by a correct inequality or correct value,	**M1**	Must differentiate one term correctly. Expect $4 - x^{-3} = 12$ at $x = \frac{-1}{2}$ Alternative: substitute values of x into $\frac{\mathrm{d}y}{\mathrm{d}x}$. One value $x < -\frac{1}{2}$ and one value $-\frac{1}{2} < x < 0$.
	conclude minimum	**A1**	Following correct work only
		2	
5(c)	State increasing …	**B1**	
	… with clear reference to first derivative always being positive [for $x > 0$]	**B1**	Dependent on first derivative being correct. It is not sufficient to substitute values of x.
		2	

Question	Answer	Marks	Guidance
6(a)	Integrate to obtain form $k(5x-3)^{-1}$	*M1	OE
	$4(5x-3)^{-1}$	A1	Or unsimplified equivalent. Condone absence of $...+c$ so far.
	Substitute $x=\frac{4}{5}$ and $y=-3$ to attempt value of c	DM1	DM0 for substituting $\left(-3,\frac{4}{5}\right)$.
	$y=4(5x-3)^{-1}-7$ allow f(x) *or* $f=4(5x-3)^{-1}-7$	A1	OE Condone $c=-7$ as the final answer providing $y=\text{or } f(x)=\frac{4}{(5x-3)}+c$ OE is seen earlier. Attempts to write equation in $y=mx+c$ form scores A0. Do not ISW. Gains max 3/4.
		4	
6(b)	Carry out stretch by replacing x by $2x$ in *their* equation	M1	Award if given as the second transformation. Do not ignore sign errors.
	Carry out translation by replacing x by $x-2$ and y by $y-10$	M1	OE Award if given as the first transformation. Do not ignore sign errors.
	$y=\frac{4}{10x-23}+3$	A1	Or similarly simplified equivalent, WWW.
		3	

Question	Answer	Marks	Guidance
7(a)	$5(3+9d)=127.5$	**B1**	OE
	$d=2.5$	**B1**	
		2	
7(b)	Attempt to find either the first term or the last term in the set by considering $1.5+2.5(n-1)>25$ or $1.5+2.5(n-1)<100$ or equivalent equations	**M1**	Using *their d*. May be implied by correct answers.
	State or imply that 11th term or 26.5 is the first in the set	**A1**	
	State or imply that 40th term or 99 is the last in the set	**A1**	
	<u>Either use</u> $S_{40}-S_{10}$ <u>Or</u> use $\frac{1}{2}n(a+l)$ with correct results for *their* d <u>Or</u> use $\frac{1}{2}n[2a+(n-1)d]$ with correct results for *their* d	**DM1**	*Their* 40 and 10 from correct working with *their d.* Correct values 30, 26.5 and 99 respectively. Correct values 30, 26.5 and 2.5 respectively.
	Obtain 1882.5	**A1**	OE
		5	

Question	Answer	Marks	Guidance
8	Substitute $x=0$ and attempt solution of 3-term quadratic equation in y	M1	If $y=0$ used can score a maximum of M0 A0 B1 M1 A0 A1FT DM1 A0, i.e. 4/8.
	-5 and 3	A1	**B1 SC** if no working to solve the quadratic.
	State or imply centre of circle is $(3,\ -1)$	B1	Condone errors which don't affect finding centre. May be implied by the correct final y coordinate.
	Attempt gradient of AC or BC	*M1	
	$-\frac{4}{3}$ or $\frac{4}{3}$	A1	
	State or imply gradient of tangent is $\frac{3}{4}$ or $-\frac{3}{4}$	A1FT	Following *their* gradient of radius. Only FT when previous 2 marks are M1 A0.
	<u>Either</u> solve simultaneous equations (of 2 tangent equations) to find x- coordinate <u>Or</u> Substitute y-value of centre into either tangent equation	DM1	
	$x=-\frac{16}{3},\ y=-1$	A1	
	Alternative Method 1: for the 4th and 5th marks		
	Rearrange and differentiate the circle equation or differentiate implicitly	(M1)	Replaces the second M1.
	$\frac{dy}{dx}=\frac{3-x}{y+1}$ or $\frac{dy}{dx}=\frac{3-x}{\left(25-(x-3)^2\right)^{\frac{1}{2}}}$	(A1)	Replaces the second A1.

Question	Answer	Marks	Guidance
8	**Alternative Method 2: for the last 5 marks**		
	$\widehat{ACP}=\widehat{MAP}\ =\tan^{-1}\frac{4}{3}$ or identifying similar triangles *PMA* and *AMC*	**(M1A1)**	*C* is the circle centre, *P* is intersection of the two tangents, *M* is intersection of *PC* and the *y*-axis.
	$\tan MAP=\frac{PM}{4},\frac{4}{3}=\frac{PM}{4},PM=\frac{16}{3}$ or use of similar triangles	**(M1A1)**	
	P is $\left(\frac{-16}{3},-1\right)$	**(A1)**	
	Alternative Method 3: for the last 5 marks		
	Pythagoras on triangle *PAC*, $PC^2=PA^2+AC^2$,	**(M1)**	Identifies the required 3 sides and sets up formula.
	$PC^2=(PM+3)^2$, $PA^2=PM^2+4^2$, $AC=\text{radius}=5$	**(A1)**	Finds each side with two in terms of *PM* OE.
	$(PM+3)^2=PM^2+4^2+5^2$ leads to $6PM=32$, $PM=\frac{16}{3}$	**(M1A1)**	Sets up and solves equation.
	P is $\left(\frac{-16}{3},-1\right)$	**(A1)**	
		8	

Question	Answer	Marks	Guidance
9(a)	Differentiate to obtain form $kx^2(2x^3+10)^{-\frac{1}{2}}$	M1	OE
	$3x^2(2x^3+10)^{-\frac{1}{2}}$	A1	Or unsimplified equivalent.
	Substitute $x=3$ in first derivative and evaluate to find gradient	*M1	Expect $\frac{27}{8}$. Allow if first derivative of forms $k(2x^3+10)^{\frac{-1}{2}}$, $kx(2x^3+10)^{\frac{-1}{2}}$ or $kx^2(2x^3+10)^{\frac{-1}{2}}$.
	Attempt equation of tangent at $(3,8)$ with numerical gradient	DM1	Use of gradient of the normal is DM0.
	$[\pm](27x-8y-17)=0$ or integer multiples	A1	
		5	
9(b)	State or imply volume is $\pi\int(2x^3+10)\,dx$	B1	Implied if π appears only at the end. Do not allow an unsimplified: $\pi\int\left(\left(2x^3+10\right)^{1/2}\right)^2$.
	Integrate to obtain $k_1x^4+k_2x$ and evaluate using limits 1 and 3	M1	Where $k_1k_2\neq 0$.
	60π	A1	OE Allow from a correct integral and sight of limits. Allow numerical answers in the range 188-189.
		3	

Question	Answer	Marks	Guidance
10(a)	Substitute to obtain equation $9r^4 + 14r^2 - 8 = 0$	**B1**	OE
	Attempt solution of quadratic equation in r^2 to obtain at least one value of r or r^2	**M1**	Expect $(9r^2 - 4)(r^2 + 2)$.
	$r = \frac{2}{3}$ only	**A1**	**SC B1** answer without working.
		3	
10(b)	Substitute $a = 2$ and *their* r in correct formula and attempt to evaluate	**M1**	Expect $\dfrac{2\left(1-\left(\frac{2}{3}\right)^{20}\right)}{\left(1-\frac{2}{3}\right)}$ or $\dfrac{2\left(\left(\frac{2}{3}\right)^{20}-1\right)}{\left(\frac{2}{3}-1\right)}$.
	5.998	**A1**	AWRT and no other value.
		2	
10(c)	Identify $a_2 = \dfrac{4}{3}$ and common ratio as $\dfrac{8}{27}$.	**B1 FT**	Following *their* r provided $\lvert r \rvert < 1$. May be implied in the sum to infinity. Allow $\left(\dfrac{2}{3}\right)^3$.
	Substitute *their* new a and r in correct formula for sum to infinity and evaluate	**M1**	$\lvert r \rvert < 1$ otherwise M0.
	$\dfrac{36}{19}$	**A1**	OE Accept 1.89 or better from 1.894736…..
		3	

Question	Answer	Marks	Guidance
11(a)	Express $f(x)$ as: $a-(x-3)^2$ or $a-(3-x)^2$ where $a=\pm 19$ or ± 1	**M1**	OE If the form $-f(x)=(x^2-6x-10)$ is used the form must be returned to $f(x)=\ldots$ Completed square form must give $-x^2$. Answers must come from completion of the square (not calculus or graphs).
	$19-(3-x)^2$ or $19-(x-3)^2$	**A1**	OE
	$f(x) \leqslant 19$ or $y \leqslant 19$ with $\leqslant$, not $<$ or $-\infty < f(x) \leqslant 19$ or $-\infty \leqslant f(x) \leqslant 19$ or $(-\infty, 19]$ or $[-\infty, 19]$	**A1 FT**	Using *their* constant following the award of M1. **SC B1** answer only or answer from a method not involving completion of the square.
		3	

Question	Answer	Marks	Guidance
11(b)	$g^{-1}(x) = \frac{1}{4}(x-k)$	**B1**	
	$g^{-1}f(x) = \frac{1}{4}\left(10+6x-x^2-k\right) = 4x+k$	**M1**	OE May use *their* completed square form for f(x).
	Simplify the quadratic equation obtained from $g^{-1}f(x) = g(x)$ provided *k* is present and apply $b^2-4ac=0$ to this quadratic equation	***M1**	Expect $x^2+10x-10+5k=0$.
	Obtain $100-4(5k-10)=0$ and hence $k=7$	**A1**	
	Use *their* *k* to form and solve a quadratic in x	**DM1**	Allow if *their* quadratic has two solutions.
	$(-5,-13)$ only	**A1**	**SC B1** if no method seen.
	Alternative Method for first 4 marks		
	State $f(x) = gg(x)$	**(B1)**	
	$gg(x) = 16x+5k$	**(M1)**	
	Apply $b^2-4ac=0$ to quadratic equation obtained from $f(x) = gg(x)$	**(*M1)**	Provided *k* is present.
	$100-4(5k-10)=0$ and hence $k=7$	**(A1)**	
		6	

CAMBRIDGE
International Education

Cambridge International AS & A Level

MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **February/March 2024**

MARK SCHEME

Maximum Mark: 75

Published

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Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the February/March 2024 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

This document consists of **14** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles

1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.

2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.

3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.

4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).

5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.

6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	Integrate to obtain $-2x^{-1}$	**B1**	OE
	Substitute limits correctly with clear indication seen that upper limit gives 0	**M1**	For integral of form $-k\,x^{-n}$, where $k>0$, $n>0$.
	Obtain $\frac{2}{3}$	**A1**	WWW Accept 0.667.
		3	

Question	Answer	Marks	Guidance
2(a)	State $(3\pi,\ -k)$	**B1**	
		1	
2(b)	Obtain equation of form $[y=]c \pm k\sin\frac{1}{2}x$	**M1**	Any non-zero *c*.
	Obtain correct equation $[y=]2-k\sin\frac{1}{2}x$	**A1**	OE
	State $(3\pi,\ 2+k)$	**B1 FT**	Following part **(a)**, i.e. (*their x*, 2 – *their y*).
		3	

Question	Answer	Marks	Guidance
3	Integrate to obtain form $k(4x+5)^{\frac{3}{2}}$	*M1	
	Obtain correct $\frac{1}{2}(4x+5)^{\frac{3}{2}}$	A1	Or (unsimplified) equivalent. Condone missing … $+c$ so far.
	Substitute $x=1,\ y=9$ to form an equation in c	DM1	
	Obtain or imply $[y=]\ \dfrac{1}{2}(4x+5)^{\frac{3}{2}}-\dfrac{9}{2}$	A1	May be implied by $[a=]\dfrac{1}{2}\left(4(1)+5\right)^{\frac{3}{2}}-\dfrac{9}{2}$.
	Substitute $x=5$ to obtain $a=58$	A1	
		5	

Question	Answer	Marks	Guidance
4(a)	Expand bracket to obtain 3 terms and use correct identity	M1	θ may be missing or another symbol used.
	Use identity $\dfrac{\sin\theta}{\cos\theta}=\tan\theta$	M1	Does not require any further explanation. θ may be missing or another symbol used.
	Conclude with $2\tan\theta$	A1	WWW AG
		3	

Question	Answer	Marks	Guidance
4(b)	Attempt solution of $5\tan^3\theta = 2\tan\theta$ to obtain at least one value of $\tan\theta$	**M1**	SOI Can be awarded if $\tan\theta$ is cancelled and ignored.
	Obtain at least two of $0,\ \pm 32.3$	**A1**	Or greater accuracy. **SC B1** if no method shown.
	Obtain all three values	**A1**	Or greater accuracy; and no others in $-90° < \theta < 90°$ range. Other units **SC B1** only for all 3 angles. **SC B1** if no method shown.
		3	

Question	Answer	Marks	Guidance
5	Differentiate to obtain form $kx(2x^2-5)^{-2}$	**M1**	
	Obtain correct $-12x(2x^2-5)^{-2}$	**A1**	OE
	Substitute (2, 1) to obtain gradient $-\frac{24}{9}$	**A1**	OE e.g. $-\dfrac{8}{3}$. Allow -2.67.
	Apply negative reciprocal to *their* numerical gradient to obtain gradient of normal	***M1**	Must have been some attempt at differentiation. Expect $\dfrac{3}{8}$
	Attempt equation of normal using *their* gradient of the normal and (2, 1)	**DM1**	Expect $y-1=\dfrac{3}{8}(x-2)$.
	Obtain $3x-8y+2=0$ (allow multiples)	**A1**	Or equivalent of requested form e.g. $8y-3x-2=0$.
		6	

Question	Answer	Marks	Guidance
6	$\binom{4}{2}2^2(ax)^2$, $\binom{4}{3}2^{[1]}(ax)^3$	**B1 B1**	OE Expect $24a^2x^2, 8a^3x^3$ (may be seen in an expansion).
	Multiply terms involving x^2 and x^3 by $5-ax$ to obtain x^3 term	***M1**	Must find two products only (may be seen in an expansion).
	Equate coefficient of x^3 to 432 and solve for a	**DM1**	Ignore inclusion of x^3 at this stage.
	Obtain $a=3$ only	**A1**	
		5	

Question	Answer	Marks	Guidance
7(a)	Attempt substitution for y in quadratic equation	***M1**	Or substitution for x …
	Obtain $5x^2+30x+75-k[=0]$ or $5y^2-20y+50-k[=0]$	**A1**	OE e.g. $x^2+6x+15-\frac{k}{5}$ (all terms gathered together).
	Use $b^2-4ac=0$ with *their* a, b and c	**DM1**	‘ = 0’ may be implied in subsequent working or the answer.
	Obtain $900-20(75-k)=0$ or equivalent and hence $k=30$	**A1**	… obtaining $400-20(50-k)=0$ and $k=30$.
		4	
7(b)	Substitute *their* value of k in equation from part **(a)** and attempt solution	**M1**	Expect $5x^2+30x+45[=0]$ or $5y^2-20y+20[=0]$.
	Obtain coordinates $(-3,\ 2)$	**A1**	**SC B1** only $(-3,\ 2)$ without attempt at quadratic solution.
		2	

Question	Answer	Marks	Guidance
8(a)	Substitute $n=10$ and $a=6$ into $u_n = a+(n-1)d$	*M1	Expect $6+9d=19.5$ or equivalent.
	$[d=]1.5$	A1	
	Substitute $a=6$ and *their d* into correct formula for the sum of 100 terms	DM1	
	Obtain 8025	A1	
		4	
8(b)	Obtain $S=48$	B1	
	Identify for S_E first term 12 and common ratio $\frac{1}{4}$	B1	
	Attempt sum to infinity, S_E, with at least one of first term and common ratio correct	M1	Only awarded if $\|r\| < 1$.
	Obtain $S_E = 16$	A1	
		4	

Question	Answer	Marks	Guidance
9(a)	Attempt to form expression for $\text{gf}(x)$	*M1	Expect $5\left((3x-2)^2+k\right)-1$; fg($x$) is M0. Do not allow algebraic errors.
	Obtain $5(3x-2)^2+5k-1$	A1	OE e.g. $45x^2-60x+5k+19$.
	Their $5k-1$ = 39 or $5k-1 \geqslant 39$	DM1	Or use $b^2-4ac=0$ (must be '= 0', could be implied later) on $45x^2-60x+5k+19-39 \geqslant 0$ OE.
	Obtain $k=8$	A1	Do not accept $k \geqslant 8$.
		4	
9(b)	Obtaining $\left(3(5x-1)-2\right)^2+\textit{their}\ k$	M1	May simplify and/or use *k* at this stage; *k* may have come from an inequality in **(a)**.
	Conclude $\left[\text{fg}(x)\right] \geqslant 8$ allow $[y] \geqslant 8$	A1 FT	OE Following *their* value of *k*; must be ⩾, not >. Allow an accurate written description.
		2	
9(c)	State $\text{g}^{-1}(x)=\frac{1}{5}(x+1)$	B1	OE $\dfrac{1}{5}(x+1)$ must be indicated as the inverse.
	$\left[\text{h}(x)=\right]7x+4$	B1B1	If $7x+4$ only, it must be clear that this is $\text{h}(x)$.
		3	

Question	Answer	Marks	Guidance
10(a)	Obtain gradient of relevant radius is –2	**B1**	
	Using $m_1m_2=-1$ obtain the gradient of the tangent and use it to form a straight line equation for a line containing (–6, 9)	**M1**	m_1 must be from an attempt to find the gradient of the radius using the centre and the given point.
	Obtain $y=\frac{1}{2}x+12$	**A1**	OE e.g. $y-9=\dfrac{1}{2}(x+6)$.
		3	
10(b)	State or imply $(x+4)^2+(y-5)^2=20$	**B1**	If $x^2+y^2-2gx-2fy+c=0$ is used correctly with $(-g,-f)=(-4,5)$ and $c=g^2+f^2-r^2$ then M1.
	Obtain $x^2+y^2+8x-10y+21=0$	**B1**	A1 if above method used.
		2	
10(c)	Substitute $x=0$ in equation of circle to find y-values 3 and 7 or state C to AB = 4	**B1**	May be implied by AB = 4 or use of |x-coordinate of C|.
	Attempt value of θ either using cosine rule or via $\frac{1}{2}\theta$ using right-angled triangle	**M1**	Using *their* AB. If $\theta/2$ used, must be multiplied by 2.
	Obtain $\theta=0.9273$	**A1**	Or greater accuracy. A correct answer implies the M1.
		3	

Question	Answer	Marks	Guidance
10(d)	Attempt arc length using $r\theta$ formula with *their* θ (not *their* $\theta/2$) and $r=\sqrt{20}$	**M1**	Expect 4.15.
	Obtain perimeter = 8.15 or greater accuracy	**A1**	Condone missing units or incorrect units.
	Attempt area using $\frac{1}{2}r^2(\theta-\sin\theta)$ formula or equivalent with *their* θ and $r=\sqrt{20}$	**M1**	If sector – triangle used, both formulae must be correct. If triangle *ACM* used, area must be multiplied by 2.
	Obtain area = 1.27 or greater accuracy	**A1**	Condone missing units or incorrect units.
		4	

Question	Answer	Marks	Guidance
11(a)	Differentiate to obtain $-\frac{4}{3}x^{-\frac{5}{3}}+x^{-\frac{4}{3}}$ **or** rewrite as a quadratic equation in $x^{-\frac{1}{3}}$ or $x^{\frac{1}{3}}$	**B1**	Expect quadratic $2\left(x^{-\frac{1}{3}}\right)^2-3x^{-\frac{1}{3}}+1$ OE Allow $2x^2-3x+1$.
	Equate first derivative to zero and reach a solution for $x^{-\frac{1}{3}}$ or $x^{\frac{1}{3}}$ with no error in use of indices **or** complete square to find minimum point $2\left(a-\frac{3}{4}\right)^2-\frac{1}{8}$ where $a=x^{-\frac{1}{3}}$	**M1**	Substitution SOI if dealt with correctly later
	Obtain $x=\frac{64}{27}$	**A1**	Or exact equivalent. **SC B1** if no working shown. Ignore extra solution $x=0$.
	$y=-\frac{1}{8}$ seen	**B1**	Or exact equivalent. Allow -0.125.
		4	

Question	Answer	Marks	Guidance
11(b)	Recognise equation as quadratic in $x^{-\frac{1}{3}}$ or equivalent and attempt solution	**M1**	$2a^2-3a+1[=0]$ where $a=x^{-\frac{1}{3}}$.
	Obtain $x^{-\frac{1}{3}}=1$ and $x^{-\frac{1}{3}}=\frac{1}{2}$	**A1**	OE **SC B1** if no M mark awarded.
	Obtain 1 and 8	**A1**	**SC B1** if no M mark awarded.
	Integrate to obtain form $k_1x^{\frac{1}{3}}+k_2x^{\frac{2}{3}}+x$ or 2 out of 3 correct terms	***M1**	Expect $6x^{\frac{1}{3}}-\dfrac{9}{2}x^{\frac{2}{3}}+x$.
	Obtain correct $6x^{\frac{1}{3}}-\frac{9}{2}x^{\frac{2}{3}}+x$	**A1**	No other terms from a second integral.
	Apply *their* limits correctly	**DM1**	*Their* limits must be from *their* working.
	[Obtain –0.5 and conclude area is] 0.5	**A1**	
		7	

Cambridge International AS & A Level

MATHEMATICS **9709/11**

Paper 1 Pure Mathematics 1 **October/November 2023**

MARK SCHEME

Maximum Mark: 75

Published

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WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1(a)	$1+18x+135x^2$	**B2, 1, 0**	Accept $1,\ 18x,\ 135x^2$ listed horizontally or vertically or $1x^0+18x+135x^2$.
		2	
1(b)	Coefficient of x^2 is $135-7\times18+1=10$	**M1 A1**	3 products, allow $10x^2$. If full expansion given, like terms must be collected for M1.
		2	

Question	Answer	Marks	Guidance
2	$cx^2+3x-c=2cx+3$ leading to $cx^2+(3-2c)x-(c+3)\ \ [=0]$	**M1**	Forming a 3-term quadratic, all terms on one side.
	$b^2-4ac=(3-2c)^2+4c(c+3)$	**M1**	2nd M1 for b^2-4ac correct for *their* <u>a</u>, b, c i.e. no sign errors.
	$=8c^2+9$	**A1**	
	> 0 [for all values of c] leading to B [Intersects for all values of c]	**A1**	WWW
		4	

Question	Answer	Marks	Guidance
3	$\dfrac{dV}{dx} = 3x^2$	**B1**	SOI
	$\dfrac{dV}{dt}\left[= \dfrac{dV}{dx} \times \dfrac{dx}{dt}\right] = 3 \times 20^2 \times 0.01$	**M1**	Correct use of chain rule with x = 20 substituted into $\dfrac{dV}{dx}$.
	12	**A1**	
		3	

Question	Answer	Marks	Guidance
4(a)	$\{-(x-3)^2\}\ \{-1\}$	**B1 B1**	OE. Must be a quadratic e.g. $3x - 1$ B0 B0. **SC B1** for correct use of generalised function notation.
		2	
4(b)	$\{-(x-3)^2\}\ \{+1\}$	**B1 B1**	OE. Must be a quadratic. **SC B1** for correct use of generalised function notation.
		2	
4(c)	$\{\text{Translation}\}\ \begin{pmatrix}\{0\}\\\{2\}\end{pmatrix}$	**B2, 1, 0**	FT from **(a)** and **(b)** if a translation parallel to the y axis. B2 for fully correct, B1 with two elements correct. {} indicates different elements.
		2	

Question	Answer	Marks	Guidance
5(a)	$4\sin^2 x + 5\cos x + 2 \quad [=0]$	*M1	Multiply by $\sin x$ (or writing as a single fraction) and using $\tan x = \dfrac{\sin x}{\cos x}$.
	$4\left(1-\cos^2 x\right) + 5\cos x + 2 \quad [=0]$	DM1	Correctly obtaining a quadratic in $\cos x$ (allow sign errors).
	$4\cos^2 x - 5\cos x - 6 = 0$	A1	Condone missing x. Must be = 0 unless 0 appears on RHS earlier.
		3	
5(b)	$(4\cos x + 3)(\cos x - 2) \quad [=0]$	M1	Or use of formula or completing square.
	138.6°, 221.4°	A1 B1 FT	FT on 360° – 1st solution from quadratic in $\cos x$. Use of radians (2.42) A0 but allow B1 FT for 2π: 1st solution if use of radians is clear. **SC** If M0 scored **SC B1 B1** for correct final answer(s). If extra incorrect solutions in the range $0 \rightarrow 360°$ are given award A1 B0.
		3	

Question	Answer	Marks	Guidance
6(a)	$k = \dfrac{2}{3}$	B1	Allow ACB = $\dfrac{2\pi}{3}$.
		1	
6(b)	Perimeter of shaded area $= 2\pi r$	B1	
		1	

Question	Answer	Marks	Guidance
6(c)	Major sector $OAB = \frac{1}{2}r^2 \times \frac{4\pi}{3}$	*M1	Expect $\frac{2}{3}\pi r^2$. Finds area of any relevant sector or triangle. Can be embedded in segment formula.
	One or both segments $= [2]\times\left(\frac{1}{2}r^2 \times \frac{\pi}{3} - \frac{1}{2}r^2 \sin\frac{\pi}{3}\right)$	*M1	
	$= [2]\left(r^2\frac{\pi}{6} - r^2\frac{\sqrt{3}}{4}\right)$	A1	
	Shaded area $= \frac{2}{3}\pi r^2 - 2\left(\frac{1}{6}\pi r^2 - \frac{r^2\sqrt{3}}{4}\right)$	DM1	
	$= \frac{\pi r^2}{3} + \frac{r^2\sqrt{3}}{2}$	A1	

Question	Answer	Marks	Guidance
6(c)	**Alternative method for Question 6(c)**		
	Sector *CAOB* $=[2]\times\frac{1}{2}r^2 their\frac{1}{3}\pi$	***M1**	Expect [2] × $\frac{1}{6}\pi r^2$. Can be embedded in segment formula.
	One or both segments $=[2]\times\left(\frac{1}{2}r^2\times\frac{\pi}{3}-\frac{1}{2}r^2\sin\frac{\pi}{3}\right)$	***M1**	
	$= [2]\left(r^2\frac{\pi}{6}-r^2\frac{\sqrt{3}}{4}\right)$	**A1**	
	Shaded area = $\pi r^2 - \left\{\frac{1}{3}\pi r^2+2\left(r^2\frac{\pi}{6}-r^2\frac{\sqrt{3}}{4}\right)\right\}$	**DM1**	
	$=\frac{\pi r^2}{3}+\frac{r^2\sqrt{3}}{2}$	**A1**	

Question	Answer	Marks	Guidance
6(c)	**Alternative method for Question 6(c)**		
	Area of rhombus AOBC $= [2]\times\frac{1}{2}r^2\sin\frac{\pi}{3}$	**M1**	Expect [2] $\times\ \frac{\sqrt{3}}{4}$. Can be embedded in segment formula.
	One or both segments $= [2]\times\left(\frac{1}{2}r^2\times\frac{\pi}{3}-\frac{1}{2}r^2\sin\frac{\pi}{3}\right)$	**M1**	
	$= [2]\left(r^2\frac{\pi}{6}-r^2\frac{\sqrt{3}}{4}\right)$	**A1**	
	Shaded area = $\pi r^2 - \left\{\frac{\sqrt{3}}{2}r^2-4\left(r^2\frac{\pi}{6}-r^2\frac{\sqrt{3}}{4}\right)\right\}$	**DM1**	
	$=\frac{\pi r^2}{3}+\frac{r^2\sqrt{3}}{2}$	**A1**	
		5	

Question	Answer	Marks	Guidance
7	$a(1+r)=15$	**B1**	Accept $\frac{a(1-r^2)}{1-r}=15$ for first B1.
	$\frac{a}{1-r}=\frac{125}{7}$	**B1**	
	$\frac{125}{7}(1-r)(1+r)=15$	**M1**	Eliminate a.
	$1-r^2=\frac{105}{125}$	**M1**	
	$r^2=\frac{4}{25}$ leading to $r=-\frac{2}{5}$	**A1**	Condone $\frac{2}{5}$ or $\pm\frac{2}{5}$.
	$a=\frac{125}{7}\times\frac{7}{5}=25$	**A1**	Ignore 2nd answer.
	3rd term $=25\times\frac{4}{25}=4$	**A1**	CAO

Question	Answer	Marks	Guidance
7	**Alternative method for Question 7**		
	$a(1+r)=15$	**B1**	
	$\frac{a}{1-r}=\frac{125}{7}$	**B1**	
	$7(15\text{-}15r) = (125 - 125r)(1 - r^2)$	**M1**	
	$125r^3 - 125r^2 - 20r + 20 = 0$	**M1**	
	$r = \frac{-2}{5}$ $[1, \frac{2}{5}]$	**A1**	Condone extra 'answer' of r = 1.
	a = 25	**A1**	Ignore 2nd answer.
	3rd term = 4	**A1**	CAO

Question	Answer	Marks	Guidance
7	**Alternative method for Question 7**		
	$a(1+r)=15$	**B1**	
	$\frac{a}{1-r}=\frac{125}{7}$	**B1**	
	$\frac{a}{1-\left(\frac{15}{a}-1\right)}=\frac{125}{7}$	**M1**	Eliminate *r*.
	$7a^2 - 250a + 1875$ [= 0]	**M1**	
	a = 25, $\left[\frac{75}{7}\right]$	**A1**	Condone extra 'answer' of $r = \left[\frac{75}{7}\right]$.
	r = $\frac{-2}{5}$	**A1**	Ignore 2nd answer.
	3rd term $=25\times\frac{4}{25}=4$	**A1**	CAO
		7	

Question	Answer	Marks	Guidance
8(a)	$u=2x-3$ leading to $2u^4=u^2+1$ leading to $2u^4-u^2-1[=0]$	**B1**	
	$(2u^2+1)(u^2-1)[=0]$	**M1**	Factors or formula or completing square must be shown.
	$u=\pm1$ leading to $2x-3=\pm1$ leading to x = 1 or 2	**A1**	
	(1, 2), (2, 2)	**A1**	**Special case:** If B1 M0 scored then **SC B2** can be awarded for correct coordinates or **SC B1** for correct x values only.
			Special case $2(2x-3)^4=(2x-3)^2+1$ $32x^4-192x^3+428x^2-420x+152=0$ x = 1, 2 finding both from a correct quartic **SC B1** (1, 2), (2, 2) **SC DB1** **Special case:** Trial and improvement without quartic. Both x values correct B1, both coordinates correct B2.
		4	

Question	Answer	Marks	Guidance
8(b)	$\left\{\dfrac{(2x-3)^3}{3\times 2}+x\right\}[-]\left\{\dfrac{2(2x-3)^5}{5\times 2}\right\}$	**B1 B1**	Integrate the 2 functions.
	$\left(\dfrac{1}{6}+2\right)-\left(-\dfrac{1}{6}+1\right)-\left\{\dfrac{1}{5}-\left(-\dfrac{1}{5}\right)\right\}$	**M1**	Apply *their* limits $1 \to 2$ (must be shown) to an integral. Some evidence of substitution. Minimum $(\frac{13}{6}-\frac{5}{6})-(\frac{1}{5}+\frac{1}{5})$ or equivalent. Allow 1 sign error for 1st M1.
	$\dfrac{4}{3}-\dfrac{2}{5}$	**M1**	Subtract (at some point) the 2 areas. Must subtract **areas** and not just integrals.
	$\dfrac{14}{15}$	**A1**	**Special case:** If M0 for substitution of limits can award **SC B1** for correct answer. Condone $-\dfrac{14}{15}$ if corrected.
			If subtraction is the wrong way round award B1 B1 M1 M1 A0. $\int y^2\,dx$ or $\int x\,dy$ scores 0 /5. $\pi\int y\,dx$ used. Award B1 B1 M1 M1 A0.

Question	Answer	Marks	Guidance
8(b)	**Alternative method for Question 8(b)**		
	u = 2x – 3 $\int\left(u^2+1-2u^4\right)\mathrm{d}u$ $\left\{\frac{1}{2}\right\}\left(\left\{\frac{1}{3}u^3+u\right\}-\left\{\frac{2}{5}u^5\right\}\right)$	**B2,1,0**	
	$\frac{1}{2}\left(\left(\frac{1}{3}+1-\frac{2}{5}\right)-\left(\frac{-1}{3}-1+\frac{2}{5}\right)\right)$	**M1**	Applies limits –1 → 1.
		M1	Subtract (at some point) the 2 areas.
	$\frac{1}{2}\left(\frac{14}{15}+\frac{14}{15}\right)$ $\frac{14}{15}$	**A1**	
		5	

Question	Answer	Marks	Guidance
9(a)	$(2x-3)^2+4$	**B1 B1**	Or a = –3, b = 4.
		2	

Question	Answer	Marks	Guidance
9(b)	*their* $(2x-3)^2+4[<]8$ OR $4x^2-12x+13[<]8$	*M1	Linking quadratic with 8.
	$(2x-3)^2<4$ leading to $-2<2x-3<2$ OR $4x^2-12x+5<0$ leading to $(2x-1)(2x-5)<0$	DM1	Simplify to 3-term quadratic and solve. Condone no method shown.
	$\frac{1}{2}<x<2\frac{1}{2}$ leading to [LEAST] $p=\frac{1}{2}$, [GREATEST] $q=2\frac{1}{2}$	A1	
		3	
9(c)	$\mathrm{gf}(x)=12x^2-36x+40$	B1	OE $\mathrm{gf}(x)=3(2x-3)^2+13$.
		1	
9(d)	$y=(2x-3)^2+4$ leading to $(2x-3)^2=y-4$ leading to $2x-3=[\pm]\sqrt{y-4}$	*M1	
	$2x=3[\pm]\sqrt{y-4}$ leading to $x=\frac{3}{2}[\pm]\frac{\sqrt{y-4}}{2}$	DM1	
	$\mathrm{h}^{-1}(x)=\frac{3}{2}-\frac{\sqrt{x-4}}{2}$	A1	
		3	

Question	Answer	Marks	Guidance
10(a)	$\frac{\mathrm{d}y}{\mathrm{d}x}=3x^2 \ [+c]$	**B1**	
	$3\times 2^2+c=0$	**M1**	Substitute x = 2 and $\frac{\mathrm{d}y}{\mathrm{d}x}=0$ into an integral (c must be present).
	$\frac{\mathrm{d}y}{\mathrm{d}x}=3x^2 \ -12$	**A1**	
		3	
10(b)	$y=x^3-12x \ [+k]$	**B1 FT**	FT on *their* non-zero c (dependent on c being found at some stage).
	$-10=2^3-12\times 2+k$	**M1**	Substitute x = 2, y = –10 (k present).
	$y=x^3-12x+6$	**A1**	Must be y = (unless $y=x^3-12x+k$ stated earlier).
		3	

Question	Answer	Marks	Guidance
10(c)	$3x^2 - 12 = 0$ [leading to $x = -2$]	**M1**	Set *their* two term $\frac{dy}{dx} = 0$. Expect $x = -2$. Ignore x = 2 given in addition.
	$y = (-2)^3 - 12 \times (-2) + 6 = 22$ leading to (–2, 22)	**A1**	
	When $x = -2$, $\frac{d^2y}{dx^2} < 0$ (or –12) hence Maximum	**A1**	Can be from correct conclusion from $\frac{dy}{dx}$ **sign** diagram if $\frac{dy}{dx}$ calculated correctly. Do not allow concave downward for final A1. Can be awarded if the only error is incorrect or missing *y*-coordinate.
		3	
10(d)	At $x = 0$, $\frac{dy}{dx} = -12$, $y = 6$	**M1**	Both required. FT on *their* $\frac{dy}{dx}$ and *y*.
	$y - 6 = -12x$	**A1**	OE
		2	

Question	Answer	Marks	Guidance
11(a)	Gradient of *AB* = –1	**B1**	SOI
	Centre of circle = (4, –1)	**B1**	SOI
	Equation of *AB* is $y + 1 = -1(x - 4)$ leading to $y = -x + 3$	**B1 FT**	FT *their* centre with gradient –1.
		3	

Question	Answer	Marks	Guidance
11(b)	$(x-4)^2+(-x+3+1)^2=40$	*M1	Substitute *their AB* into circle equation.
	$2(x-4)^2=40$ OR [2](x^2-8x-4) leading to $\frac{8\pm\sqrt{64+16}}{2}$ or $\frac{16\pm\sqrt{256+64}}{4}$	DM1	Forming and solving 3-term quadratic.
	$x=4[\pm]\sqrt{20}$	A1	OE. No fractions.
	$(4-\sqrt{20},-1+\sqrt{20})$	A1	OE **Special case:** If M1 M0 scored then **SCB2** can be awarded for correct coordinates or **SCB1** for correct x values only. Ignore other coordinate
		4	
11(c)	$y-their(-1+\sqrt{20})=1\{x-their(4-\sqrt{20})\}$	M1	OE
	$y=x-5+2\sqrt{20}$ or $y=x-5+\sqrt{80}$ or $y=x-5+4\sqrt{5}$	A1	
		2	

Cambridge Assessment International Education

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MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **October/November 2023**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2023 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles

1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.

2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.

3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.

4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).

5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.

6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	[Coefficient of x^3 from $(3+2ax)^5=$] $10\times9\times8a^3$ $\left[=720a^3\right]$	**B1**	May be seen in an expansion or with x^3.
	[Coefficient of x^2 from $(2+ax)^6=$] $15\times16\times a^2$ $\left[=240a^2\right]$	**B1**	May be seen in an expansion or with x^2.
	$\textit{their}\left(10\times9\times8a^3\right)=6\times\textit{their}\left(15\times16\times a^2\right)$ $\left[\Rightarrow 720a^3=1440a^2\right]$	**M1**	OE Equating *their* <u>coefficient</u> of x^3 and 6 × *their* <u>coefficient</u> of x^2.
	$\boldsymbol{a}$ **= 2**	**A1**	Condone extra solution a = 0.
		4	

Question	Answer	Marks	Guidance
2	$[\tan^{-1}4x=]\left(\textit{their}-\frac{\pi}{6}\right)\pm\frac{\pi}{6}$ $\left[\tan^{-1}4x=\pm\frac{\pi}{3},\pm1.047\text{ or }0\right]$	**M1**	OE Evaluating $\left(-\cos^{-1}\frac{\sqrt{3}}{2}\right)$ in rad and adding or subtracting $\frac{\pi}{6}$. Allow working with **both** angles in degrees.
	$\left[4x=-\sqrt{3},\ x=\right]-\frac{\sqrt{3}}{4}$	**A1**	**Note:** answer of $-0.43\text{ or }\frac{\sqrt{3}}{4}$ implies M1
		2	

Question	Answer	Marks	Guidance
3(a)	[Gradient of normal =] $\dfrac{-1}{\text{Their}\,\frac{11}{2}}$ $\left[\dfrac{-1}{\frac{11}{2}}=-\dfrac{2}{11}\right]$	**M1**	Tangent gradient must come from $x = 2$ substituted into the given expression.
	$\dfrac{y-8}{x-2}=-\dfrac{2}{11}$ or $11y+2x=92$ or $y=-\dfrac{2x}{11}+\dfrac{92}{11}$	**A1**	OE
		2	
3(b)	$[y=]\left\{\dfrac{1}{2}x^2\div 2\right\}\left\{+\dfrac{72}{x^3}\div -3\right\}$ $[+c]$ $\left[\dfrac{x^2}{4}-\dfrac{24}{x^3}+c\right]$	**B1, B1**	One mark for each correct unsimplified { }.
	$8=\dfrac{1}{4}\times 4-\dfrac{24}{8}+c$	**M1**	Substitution of $x = 2$, $y = 8$ into *their* integrated expression, defined by at least one correct power. Two terms and + c needed.
	$y=\left(\dfrac{1}{4}\text{ or }0.25\right)x^2-\dfrac{24}{x^3}+10$	**A1**	Both coefficients must be simplified but allow x^{-3}. Condone $c = 10$ as line as long as either y or f(x) = is seen elsewhere.
		4	

Question	Answer	Marks	Guidance
4(a)	$[\text{Arc length} =]$ $2\times\dfrac{\pi}{3}$ or $\dfrac{60}{360}\times 2\pi\times 2$	**B1**	Finding one correct arc length – may be implied by correct final answer.
	$[\text{Perimeter} =]\,2\pi$ or 6.28	**B1**	AWRT
		2	

Question	Answer	Marks	Guidance
4(b)	[Area of one sector =] $\frac{1}{2}\times 2^2\times\frac{\pi}{3}$ or $\frac{60}{360}\times\pi\times 2^2\left[=\frac{2\pi}{3}\text{ or }2.09\right]$	B1	SOI AWRT
	[Area of triangle =] $\frac{1}{2}\times 2^2\times\sin\left(\frac{\pi}{3}\right)$ or other valid method $\left[=\sqrt{3}\text{ or }1.73\right]$	B1	AWRT Allow use of $60°$
	$\left[\text{Area of coin} = 3\text{ segments} + \text{triangle} \Rightarrow\right]\ 3\left(\frac{2\pi}{3}-\sqrt{3}\right)+\sqrt{3}$ [= 2.82]	M1	OE Or 3 sectors – 2 triangles $\left(3\times\frac{2\pi}{3}-2\times\sqrt{3}\right)$ or Sector + 2 segments $\left(\frac{2\pi}{3}+2\left(\frac{2\pi}{3}-\sqrt{3}\right)\right)$
	$2\pi-2\sqrt{3}$ or $2(\pi-\sqrt{3})$	A1	Must be one of these simplified versions but equivalent decimal answers can score B1B1M1
		4	

Question	Answer	Marks	Guidance
5(a)	$\frac{\cos\theta}{\sin\theta}=\frac{2-\sin\theta}{\cos\theta}$ leading to $\cos^2\theta[\sin\theta]=\sin\theta(2-\sin\theta)[\sin\theta]$	*M1	OE. Forming a correct equation in θ only using the terms of the GP and an attempt to clear fractions.
	$\cos^2\theta+\sin^2\theta=2\sin\theta$ leading to $\sin\theta=\left[\frac{1}{2}\right]$	DM1	Correct use of $\cos^2\theta+\sin^2\theta=1$ and attempt to solve for $\sin\theta$.
	$[\theta=]\frac{\pi}{6}$ or 0.524	A1	AWRT A0 for $\theta=30°$. Condone inclusion of $\frac{5\pi}{6}$ and/or
		3	

Question	Answer	Marks	Guidance
5(b)	$a=\frac{1}{2}\ \ r=\sqrt{3}$	**B1**	OE SOI Trigonometric values need to have been evaluated but allow decimal equivalents (0.5 and 1.73 AWRT)
	$S_{10}=\sin\left(their\frac{\pi}{6}\right)\left(\frac{1-\left(their\sqrt{3}\right)^{10}}{1-\left(their\sqrt{3}\right)}\right)$	**M1**	Use of a correct formula for S_{10}, with *their* value of θ. Their $\sqrt{3}$ needs to come from $\frac{\cos(their\theta)}{\sin(their\theta)}$ or $\frac{2-sin(their\theta)}{cos(their\theta)}$ OE
	$[S_{10}=]\frac{121}{\sqrt{3}-1}$	**A1**	$\frac{-121}{1-\sqrt{3}}$ or 165 AWRT scores B1M1A0**.**
		3	

Question	Answer	Marks	Guidance
6(a)	$\frac{\mathrm{d}}{\mathrm{d}x}\left(x^2-8x+5\right)=0\ \left[2x-8=0\right]$	**M1**	Correct differentiation of x^2 and equating their $\frac{\mathrm{d}y}{\mathrm{d}x}$ to 0.
	Alternative method 1 for first mark of Question 6(a)		
	$y=\left(x-4\right)^2-11$	**M1**	Attempt to complete the square as far as $y=\left(x-4\right)^2\pm k$.
	Alternative method 2 for first mark of Question 6(a)		
	$x=\frac{-b}{2a}=\frac{\pm 8}{2}$	**M1**	
	$x=4, y=-11$	**A1**	Answers from $x=\frac{8\pm\sqrt{64-20}}{2}$ leading to $x=4\pm\sqrt{11}$ scores M0A0
		2	
6(b)	$x=\left(\textit{their } x \text{ value from } a\right)+4$ [=8]	**B1 FT**	Can be from finding the equation of the transformed curve, differentiating and putting $\frac{\mathrm{d}y}{\mathrm{d}x}=0$.
	$y=\left\{\left(\textit{their } y \text{ value from } a\right)\times 2\right\}+1$ [-21]	**B1 FT**	Can be from putting x = 8 in the equation of the transformed curve.
		2	If B0B0 scored, **SC B1** for sight of $\left(4,-22\right)$.

Question	Answer	Marks	Guidance
6(c)	$2\left(x^2-8x+5\right)$ or $2\{(x-4)^2-11\}$	**B1**	Can be implied if both transformations done together: $2\left((x-4)^2-8(x-4)+5\right)+1$ OE.
	$\left((x-4)^2-8(x-4)+5\right)+1$ or $\left\{(x-4-4)^2-\mathit{their}11\right\}+1$	**M1**	For the x translation, each x becomes $(x-4)$.
		M1	For the y translation of +1.
	$y=2x^2-32x+107$ or $a=2, b=-32, c=107$	**A1**	Evidence to support *their* answer may be in **(b)** but answer must be seen in **(c).**
		4	

Question	Answer	Marks	Guidance
7(a)	$(2x-1)\left(4x^2+2x-1\right)=8x^3+4x^2-2x-4x^2-2x+1=8x^3-4x+1$	**B1**	AG Six correct terms leading to the correct answer.
		1	

Question	Answer	Marks	Guidance
7(b)	Starting with the LHS $\dfrac{\dfrac{\sin^2\theta}{\cos^2\theta}+1}{\dfrac{\sin^2\theta}{\cos^2\theta}-1}$ $\left[=\dfrac{\sin^2\theta+\cos^2\theta}{\sin^2\theta-\cos^2\theta}\right]$	*M1	For use of $\tan\theta=\dfrac{\sin\theta}{\cos\theta}$ in the numerator and denominator.
	$=\dfrac{1}{1-\cos^2\theta-\cos^2\theta}$ need to see clear evidence of this step	DM1	For use of $\sin^2\theta+\cos^2\theta=1$ twice, in a correct expression, resulting in an expression in $\cos^2\theta$.
	$=\dfrac{1}{1-2\cos^2\theta}$	A1	AG
	Alternative method 1 for Question 7(b)		
	Starting with the RHS $\dfrac{\sin^2\theta+\cos^2\theta}{\sin^2\theta+\cos^2\theta-2\cos^2\theta}$ $\left[=\dfrac{\sin^2\theta+\cos^2\theta}{\sin^2\theta-\cos^2\theta}\right]$	*M1	For use of $\sin^2\theta+\cos^2\theta=1$ twice.
	$=\dfrac{\dfrac{\sin^2\theta}{\cos^2\theta}+1}{\dfrac{\sin^2\theta}{\cos^2\theta}-1}$ need to see clear evidence of this step	DM1	Dividing throughout by $\cos^2\theta$.
	$=\dfrac{\tan^2\theta+1}{\tan^2\theta-1}$	A1	AG

Question	Answer	Marks	Guidance
7(b)	**Alternative method 2 for Question 7(b)**		
	Starting with the LHS $\dfrac{\sec^2\theta}{\sec^2\theta - 2}$	***M1**	For use of $1+\tan^2\theta = \sec^2\theta$ twice.
	Clear statement $\Rightarrow \dfrac{1}{1-2\cos^2\theta}$	**DM1**	AG For multiplying throughout by $\cos^2\theta$ to give the RHS.
		A1	
		3	

Question	Answer	Marks	Guidance
7(c)	$\dfrac{1}{1-2\cos^2\theta}=4\cos\theta$ leading to $1=\ 4\cos\theta\left(1-2\cos^2\theta\right)$ $\left[8\cos^3\theta-4\cos\theta+1=0\right]$	**B1**	Replace LHS with RHS from **(b)** and clear fractions.
	$\left(2\cos\theta-1\right)\left(4\cos^2\theta+2\cos\theta-1\right)[=0]$	***B1**	Use of the expression from **(a)** with $x=\cos\theta$.
	$[x\text{ or }\cos\theta=]\dfrac{1}{2}\text{ and}\dfrac{-2\pm\sqrt{4+16}}{8}$ OR $0.31,-0.81$ AWRT	**DB1**	OE For all three values.
	$[\theta=]60°,72°,144°$	**B2,1,0**	B2 for three correct answers only, B1 for two correct answers and no others (but allow $36°$ instead of $144°$) in the given range or 3 correct answers plus other values in the given range. Ignore answers outside of the given range. Accept AWRT 72.0, 144.0 . **SC B1** for all 3 correct answers in radians and no others: $\dfrac{\pi}{3},\dfrac{2\pi}{5}\text{ and}\dfrac{4\pi}{5}$.
		5	

Question	Answer	Marks	Guidance
8(a)	$y=(x+a)^2-a$ leading to $(x+a)^2=y\pm a$	*M1	x and y may be interchanged initially. Allow ± errors for these method marks.
	$x=[\pm]\sqrt{y\pm a}\pm a$	DM1	
	Alternative method for first 2 marks of Question 8(a)		
	$x=(y+a)^2-a$ leading to $y^2+2ay+a^2-a-x[=0]$	*M1	Allow ± errors for this method mark.
	$y=\dfrac{-2a\pm\sqrt{4a^2-4\left(a^2-a-x\right)}}{2}$	DM1	
	[y or $f^{-1}(x)$ =] $-\sqrt{x+a}-a$	A1	OE Must choose negative root.
		3	
8(b)(i)	$x\geqslant -a$	B1	Ignore infinity limit if included.
		1	
8(b)(ii)	y or $f^{-1}[(x)]\leqslant -a$	B1	Ignore negative infinity limit if included.
		1	

Question	Answer	Marks	Guidance
8(c)	$\left[\text{gf}\left(\frac{7}{2}\right)=\right]2\left(\left(x+\frac{7}{2}\right)^2-\frac{7}{2}\right)-1$ or $2x^2+4\left(\frac{7}{2}\right)x+2\left(\frac{7}{2}\right)^2-2\left(\frac{7}{2}\right)-1[=0]$	**B1**	OE Alternatively, $\left[\text{gf}(x)=0\Rightarrow\right]\text{f}(x)=\frac{1}{2}$.
	$[x=]-\frac{7}{2}\pm 2$ or $\frac{-14\pm\sqrt{14^2-4\times 2\times\frac{33}{2}}}{4}$ $\left[\frac{-14\pm\sqrt{64}}{4}\right]$ or factorising	**M1**	OE Solving their three term quadratic equation as far as two solutions or correctly selecting the negative root only. Alternatively, $\pm\sqrt{\frac{1}{2}+\frac{7}{2}}-\frac{7}{2}$.
	$[x=]-\frac{11}{2}$	**A1**	If B1M0 scored then award **SCB1** for the correct final answer.
		3	

Question	Answer	Marks	Guidance
9(a)	$2x^{\frac{1}{2}}+13x^{-\frac{1}{2}}=3x^{-\frac{1}{2}}+12$ all $\times x^{\frac{1}{2}}$ $\Rightarrow x-6x^{\frac{1}{2}}+5=0$	*M1	OE Equating the two expressions in x and then multiplying each term by $x^{\frac{1}{2}}$ or by their substitution for $x^{\frac{1}{2}}$. Coefficients need to be retained but condone +/– sign errors. Allow $x^{\frac{1}{2}}$ replaced by x.
	$\left(x^{\frac{1}{2}}-1\right)\left(x^{\frac{1}{2}}-5\right)$ [= 0] or [x=] $\frac{6\pm\sqrt{36-4\times1\times5}}{2}$	DM1	OE Solving their three-term quadratic.
	Alternative method for first 2 marks of Question 9(a)		
	$2x^{\frac{1}{2}}+13x^{-\frac{1}{2}}=3x^{-\frac{1}{2}}+12$ all $\times x^{\frac{1}{2}}$ leading to $2x+10=12x^{\frac{1}{2}}$	*M1	Equating the two expressions in x and isolating their term in $x^{\frac{1}{2}}$.
	$(2x+10)^2=144x$ leading to $[4]\left(x^2-26x+25\right)[=0]$ leading to $[4](x-25)(x-1)$ [= 0] or [x=] $\frac{26\pm\sqrt{676-4\times1\times25}}{2}$	DM1	OE Squaring both sides, rearranging and solving a three-term quadratic.
	$x=1$ and 25, $y=15$ and $12\frac{3}{5}$	A1, A1	A1 for both x-values and A1 for both y values. If M1DM0 scored then **SCB1B1** is available for final answers.
		4	Answers without working score 0/4

Question	Answer	Marks	Guidance
9(b)	Area $= \int \left(3x^{-\frac{1}{2}} + 12\right) - \left(2x^{\frac{1}{2}} + 13x^{-\frac{1}{2}}\right)[dx] \quad \left[= -2x^{\frac{1}{2}} + 12 - 10x^{-\frac{1}{2}}\right]$	**M1**	Attempt to integrate, defined by at least one correct fractional power, and subtract – condone the wrong way round.
	$= \left\{-\dfrac{2x^{\frac{3}{2}}}{\frac{3}{2}}\right\} + 12x \left\{-\dfrac{10x^{\frac{1}{2}}}{\frac{1}{2}}\right\}$	**B1 B1**	B1 for either { }. B1 for completely correct integration of their expression following through +/– sign errors from the subtraction.
	$\left(-\frac{4}{3}(\textit{their } 25)^{\frac{3}{2}} + 12(\textit{their } 25) - 20(\textit{their } 25)^{\frac{1}{2}}\right) -$ $\left(-\frac{4}{3}(\textit{their } 1)^{\frac{3}{2}} + 12(\textit{their } 1) - 20(\textit{their } 1)^{\frac{1}{2}}\right)$	**M1**	OE Substitution of *their* positive limits from part **(a)** in *their* integrated expression, defined by at least one correct fractional power, and subtraction.

Question	Answer	Marks	Guidance
9(b)	**Alternative method for first 4 marks of Question 9(b)**		
	Area = $\int\left(3x^{-\frac{1}{2}}+12\right)[\mathrm{d}x]-\int\left(2x^{\frac{1}{2}}+13x^{-\frac{1}{2}}\right)[\mathrm{d}x]$	**M1**	Attempt to integrate, defined by at least one correct fractional power, and subtract – condone the wrong way round.
	$=\left\{\frac{3x^{\frac{1}{2}}}{\frac{1}{2}}+12x\right\}[-]\left\{\frac{2x^{\frac{3}{2}}}{\frac{3}{2}}+\frac{13x^{\frac{1}{2}}}{\frac{1}{2}}\right\}$	**B1 B1**	OE One mark for each correct expression.
	$\left(\left(6(\textit{their}\ 25)^{\frac{1}{2}}+12(\textit{their}\ 25)\right)-\left(6(\textit{their}\ 1)^{\frac{1}{2}}+12(\textit{their}\ 1)\right)\right)$ [–] $\left(\left(\frac{4}{3}(\textit{their}\ 25)^{\frac{3}{2}}+26(\textit{their}\ 25)^{\frac{1}{2}}\right)-\left(\frac{4}{3}(\textit{their}\ 1)^{\frac{3}{2}}+26(\textit{their}\ 1)^{\frac{1}{2}}\right)\right)$	**M1**	OE Substitution of *their* positive limits from part **(a)** in both of their integrated expressions, defined by at least one correct fractional power, and subtraction.
	[Area =] $\frac{128}{3}, 42\frac{2}{3}$, 42.7	**A1**	AWRT If M1B1B1M0 then **SC B1** available for correct final answer. Condone negative answer if corrected.
		5	Condone the presence of π for the first 4 marks but use of $\int y^2$ scores 0/5

Question	Answer	Marks	Guidance
10(a)	$\dfrac{dy}{dx} = \left\{\dfrac{5}{3}(4x-3)^{\frac{2}{3}}\right\}\{\times 4\}\left\{-\dfrac{20}{3}\right\}$	B2,1,0	B2 Three correct unsimplified { } and no others. B1 Two correct { }or three correct { } and an additional term e.g. + *c*. B0 More than one error.
	$\left[\dfrac{20}{3}(4x-3)^{\frac{2}{3}} - \dfrac{20}{3} = 0\right]$ leading to $(4x-3)^2 = k,\ k > 0$ leading to $4x-3 = \pm m$	M1	Equating *their* $\dfrac{dy}{dx}$ to 0 and using a valid method to arrive at 2 answers.
	$[4x-3=\pm 1]\ \ [x=]\dfrac{1}{2}, 1$	A1	
	$\dfrac{d^2y}{dx^2} = \dfrac{40}{9}(4x-3)^{-\frac{1}{3}} \times 4$	B1	OE
	$\left[x=\dfrac{1}{2}\right]\ \dfrac{d^2y}{dx^2} = \left(\dfrac{160}{9}\right)(4x-3)^{-\frac{1}{3}} < 0$ or $-\dfrac{160}{9}$ or -17.8 so max $[x=1]\ \dfrac{d^2y}{dx^2} = \left(\dfrac{160}{9}\right)(4x-3)^{-\frac{1}{3}} > 0$ or $\dfrac{160}{9}$ or 17.8 so min	B1	If $\dfrac{d^2y}{dx^2}$ evaluated the answers for both must be correct OR Clear use of change in sign of $\dfrac{dy}{dx}$ correctly for both B1's. If B1M1A0B0B0 scored then **SCB1** can be awarded for: $\dfrac{dy}{dx} = \left\{\dfrac{5}{3}(4x-3)^{\frac{2}{3}}\right\} - \left\{\dfrac{20}{3}\right\}$ leading to $(4x-3)^2 = 64$ leading to $x = -\dfrac{5}{4}, \dfrac{11}{4}$. $\dfrac{d^2y}{dx^2} = \dfrac{10}{9}(4x-3)^{-\frac{1}{3}}$, $x = -\dfrac{5}{4}$, $\dfrac{d^2y}{dx^2} < 0$ so max, $x = \dfrac{11}{4}$, $\dfrac{d^2y}{dx^2} > 0$ so min.
		6	

Question	Answer	Marks	Guidance
10(b)	$x<\frac{1}{2},\ x>1$	**B1**	Allow $\leqslant$ and/or $\geqslant$. FT only from special case $x<-\frac{5}{4},\ x>\frac{11}{4}$ Condone: $1<x<\frac{1}{2}$.
		1	

Question	Answer	Marks	Guidance
11(a)	$\left(their\,\frac{7-4}{p-6}\right)\times\left(their\,\frac{18-7}{14-p}\right)=-1$ OR Scalar product leading to $(14-p)(6-p)-33=0$	*M1	Their gradients must both come from $\frac{\text{Difference in the } ys}{\text{Difference in the } xs}$.
	$p^2-20p+84=33$ leading to $p^2-20p+51[=0]$ or $p^2-20p=-51$	A1	Clearing of fractions and collecting terms to arrive at the three-term quadratic. Allow integer multiples.
	Alternative method for first 2 marks of Question 11(a)		
	$(p-6)^2+(7-4)^2+(14-p)^2+(18-7)^2=(14-6)^2+(18-4)^2$ OR E.g. $(10-p)^2+4^2=4^2+7^2$	*M1	For correct use of Pythagoras with *A*,*B* and *C*. OR For correct use of Pythagoras with the centre, B and one of the other two points.
	$2p^2-40p+102[=0]$	A1	OE Collecting terms to arrive at the three-term quadratic.
	$[2](p-3)(p-17)$ or $\frac{20\pm\sqrt{20^2-4\times51}}{2}$	DM1	OE Solving their three-term quadratic.
	$p=3$	A1	If M1A1DM0 scored then **SC B1** is available for final answer.
		4	

Question	Answer	Marks	Guidance
11(b)	[Midpoint or Centre is] (10, 11)	**B1**	SOI by final answer.
	$\frac{1}{2}\sqrt{(14-6)^2+(18-4)^2}$ or $(18-their11)^2+(14-their10)^2$ or $(their11-4)^2+(their10-6)^2$ $\left[r^2=65 \text{ or } r=\sqrt{65}\right]$	**M1**	Finding half of the length of *AC* or using their centre, which cannot be A, B or C, to find r^2 or r. Note: $r=65$ is M0.
	$(x-10)^2+(y-11)^2=65$ or $x^2+y^2-20x-22y+156=0$	**A1**	$(x-6)(x-14)+(y-4)(y-18)=0$ scores 3/3.
		3	
11(c)	$\frac{18-their11}{14-their10}$ or $\frac{their11-4}{their10-6}$ or $\frac{18-4}{14-6}$ $\left[=\frac{7}{4}\right]$	***M1**	Gradient of *their* centre, which cannot be A, B or C, from part **(b)**, to A or C or the gradient of AC but working needed if incorrect centre. OR by clearly differentiating and substitution of (14,18).
	$y-18=-\frac{1}{their\frac{7}{4}}(x-14)$	**DM1**	OE Using (14,18) and $-\frac{1}{their\frac{7}{4}}$ to form the equation of a straight line.
	$4x+7y-182=0$	**A1**	All terms on one side in any order. Allow multiples of this format by an integer only.
		3	

Cambridge International AS & A Level

MATHEMATICS **9709/13**

Paper 1 Pure Mathematics 1 **October/November 2023**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2023 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles

1. Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.

2. Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.

3. Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.

4. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).

5. Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.

6. Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate’s intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M	Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
A	Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
B	Mark for a correct result or statement independent of method marks.
DM or **DB**	When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
FT	Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	$[y]=\left\{\frac{x^2}{2}\right\}\ \left\{\frac{-3x^{\frac{1}{2}}}{\frac{1}{2}}\right\}[+c]$	**B1 B1**	Any unsimplified correct form, ISW for extra terms, allow lists.
	$1 = 8 - 12 + c$	**M1**	Substitute (into an integrated expression) $x = 4$, $y = 1$. c must be present. Expect $c = 5$.
	$y=\frac{1}{2}x^2-6x^{\frac{1}{2}}+5$, allow f($x$) =	**A1**	Condone $c = 5$ as the final line so long as '$y =$' present.
		4	

Question	Answer	Marks	Guidance
2(a)	$(0-3)^2+(y-5)^2=40$	**M1**	OE. Substitute $x = 0$, may use $y^2-10y-6=0$.
	$y=5\pm\sqrt{31}$	**A1**	OE. Must be surd form.
		2	
2(b)	$\left\{x^2+(y-5)^2\right\}=\{31\}$ Allow $(x-0)^2$	**B1FT B1FT**	B1 FT for *their* 5 and B1 FT for *their* 31. Don't allow surd form.
		2	

Question	Answer	Marks	Guidance
3(a)	$5\cos^2\theta - \sin^2\theta + \cos\theta\ [=0]$	**M1**	Multiply by $\cos\theta$ and replace $\tan\theta$ by $\dfrac{\sin\theta}{\cos\theta}$.
	$5\cos^2\theta - \left(1-\cos^2\theta\right) + \cos\theta\ [=0]$	**M1**	
	$6\cos^2\theta + \cos\theta - 1 = 0$	**A1**	Missing '= 0' can be condoned if '= 0' appears earlier.
		3	
3(b)	$(3\cos\theta - 1)(2\cos\theta + 1) = 0$	**M1**	Must have 3 term quadratic, expect $\cos\theta = \frac{1}{3}, -\frac{1}{2}$. Factors (OE) must be shown.
	$\theta = \{1.23\}$; {2.09 or $\frac{2\pi}{3}$ } ; {5.05 and 4.19 (allow $\frac{4\pi}{3}$) }	**A1 A1 A1 FT**	For A1 FT is for <u>both</u> 2π – 1st solutions.
		4	

Question	Answer	Marks	Guidance
4(a)(i)	$1 + 10x + 40x^2$	**B1**	Ignore any additional terms (ISW). Allow x^0 or $1\,x^0$ for the first term, allow lists.
		1	
4(a)(ii)	$\{1\}\{-6ax\}\left\{+15a^2x^2\right\}$	**B2, 1, 0**	Ignore any additional terms (ISW). Allow x^0 or $1\,x^0$ for the first term, allow lists.
		2	

Question	Answer	Marks	Guidance
4(b)	$15a^2 - 60a + 40 = -5$	**M1 A1**	Correct 3 products from *their* expansions for M1. Condone inclusion of x^2 for M1.
	$[15](a-1)(a-3)[=0]$ OE	**DM1**	OE. Rearranging and solving a quadratic.
	$a = 1$ and 3	**A1**	**Special case:** If M1 A1 DM0 scored then **SC B1** can be awarded for correct answers.
		4	

Question	Answer	Marks	Guidance
5(a)	$\frac{5p}{2p+6} = \frac{8p+2}{5p}$	**M1**	OE. Setting up a valid relationship in terms of *p*.
	$9p^2 - 52p - 12$ [= 0]	**DM1**	OE. Simplifying to a 3 term quadratic equation, only condone sign errors.
	$\left[(9p+2)(p-6) = 0\right]$ leading to $p = \frac{-2}{9}$ and 6	**A1**	
		3	
5(b)	$a = 2\left(-\frac{2}{9}\right) + 6\left[= \frac{50}{9}\right]$	***M1**	FT *their* $-\frac{2}{9}$, allow any negative non-integer.
	$r = -\frac{10}{9} \div \frac{50}{9}\left[= -\frac{1}{5}\right]$	***M1**	Ft *their* $-\frac{2}{9}$, allow any negative non-integer.
	$S_\infty = \frac{50}{9} \div \left(1 - -\frac{1}{5}\right) = \frac{125}{27}$	**DM1 A1**	Can only get DM1 if |r| < 1. Accept AWRT 4.63 .
		4	

Question	Answer	Marks	Guidance
6	$cx^2+2x-3=6x-c$ leading to $cx^2-4x+(c-3)$ $[=0]$	B1	3-term quadratic.
	$16-4c(c-3)=0$	*M1	Apply $b^2-4ac=0$ ('$=0$' may be implied in subsequent work). *Their* coefficients must be substituted correctly
	$4c^2-12c-16[=0]$ leading to $[4](c-4)(c+1)$ $[=0]$ leading to $c=4$ and -1	A1	Dependent on factorisation oe.
	When c = 4, $4x^2-4x+1[=0]$ $\left[(2x-1)^2=0\right]$	DM1	OE. Substituting *their c* = 4 into *their* quadratic equation.
	$x=\frac{1}{2}$, $y=-1$	A1	Both required.
	When $c=-1$, $x^2+4x+4[=0]$ $\left[(x+2)^2=0\right]$	DM1	OE. Substituting *their c* = -1 into *their* quadratic equation.
	$x=-2$, $y=-11$	A1	Both required.
	Alternative method for Question 6		
	$\frac{dy}{dx}=2cx+2$	B1	
	$2cx+2=6$	M1	Equating *the*ir curve gradient and 6.
	$c=\frac{2}{x}$	A1	SOI
	$2x^2+3x-2$ $[=0]$	DM1	Substitute $c=\frac{2}{x}$ into $cx^2+2x-3=6x-c$. Simplify to 3-term quadratic.

Question	Answer	Marks	Guidance
6	$(2x-1)(x+2)[=0] \rightarrow x=\frac{1}{2}$ or -2	**A1**	Dependent on factorisation. Both required.
	$c=4$ and -1	**A1**	Both required, if DM0 given **SC B1** for both.
	$y=-1$ and -11	**A1**	Both required, if DM0 given **SC B1** for both. **SC** one correct (x, y). A1 only
		7	

Question	Answer	Marks	Guidance
7(a)	Range is [y] > 1	**B1**	Allow f, f(x), (1,∞), etc.
		1	
7(b)	$y=\frac{3}{x-2}+1$ leading to $y-1=\frac{3}{x-2}$ leading to $(x-2)(y-1)=3$	**M1**	Clearing the fraction.
	$x-2=\frac{3}{y-1}$	**DM1**	Reaching a stage which only requires one further operation.
	$x=\frac{3}{y-1}+2$ leading to $f^{-1}(x)=\frac{3}{x-1}+2$	**A1**	OE. Slightly longer routes lead to $f^{-1}(x)=\frac{2x+1}{x-1}$.
	[Domain is] $x>1$	**B1FT**	Must use x FT *their* **(a)**, must be a range.
		4	

Question	Answer	Marks	Guidance
7(c)	$\text{gf}(x) = 2\left(\dfrac{3}{x-2}+1\right)-2$ or $2\left(\dfrac{x+1}{x-2}\right)-2$	**M1**	Substitute f(x) into g(x).
	$\dfrac{6}{x-2}$	**A1**	
		2	

Question	Answer	Marks	Guidance
8(a)	$a = \dfrac{1}{2}$	**B1**	
	$b = \dfrac{\pi}{3}$	**B1**	$b = \dfrac{\pi}{3} + 4n\pi, n \geqslant 0$.
		2	
8(b)	x-coordinate = {$4p$} {–8}	**B1 B1**	OE, e.g. $4(p-2)$.
	y-coordinate = $-3q$	**B1**	
		3	

Question	Answer	Marks	Guidance
9(a)	$\frac{\mathrm{d}y}{\mathrm{d}x} = x^{-\frac{1}{2}} \to m = \frac{1}{2}$ at $x = 4$	B1	
	Equation of normal is $y - 3 = -2(x-4)$	M1	Through (4, 3) with gradient $-\frac{1}{their\ m}$. (Dependent on differentiation used).
	$y = -2x + 11$	A1	Only acceptable answer.
		3	
9(b)	$\frac{\mathrm{d}y}{\mathrm{d}t} = their\ \frac{1}{2} \times 3$	M1	Correct use of the chain rule with a numerical gradient.
	$\frac{3}{2}$	A1	
		2	
9(c)	Required gradient $\left[= \frac{\mathrm{d}y}{\mathrm{d}x}\right] = -2$	B1FT	SOI. FT from *their* part **(a)** if a normal gradient has been found from $m_1 m_2 = -1$ and differentiation.
	$\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{1}{their\ normal\ gradient} \times -5$	M1	Correct use of the chain rule . Allow method mark also for +5, must be numerical. *Their* normal gradient must come from $m_1 m_2 = -1$ and differentiation in part(a) unless 'restarted' here.
	$\frac{5}{2}$	A1	
		3	

Question	Answer	Marks	Guidance
10(a)	Angle $ACO = 0.7$	**B1**	Don't allow AWRT 0.7 .
		1	
10(b)	$[R =]\ 1.53\, r$	**B1**	Allow AWRT 1.53r.
		1	
10(c)	Sector $OAB = \frac{1}{2} r^2 \times 2.8 \quad \left[= 1.4r^2\right]$	**B1**	
	Sector $CAB = \frac{1}{2}(\mathit{their}\ R)^2 \times 2 \times \mathit{their}\ 0.7$	***M1**	
	$1.638\, r^2$	**A1**	Allow AWRT $1.64\, r^2$.
	$[2] \times \frac{1}{2} r^2 \sin(\pi - 1.4)$ OR $[2] \times \frac{1}{2} r \times \mathit{their} R \sin 0.7$	***M1**	
	$2 \times 0.4927 r^2$	**A1**	Allow AWRT $0.98\, r^2$ to $0.99\, r^2$.
	$1.4r^2 - \left(\mathit{their}\, 1.638 r^2 - \mathit{their}\, 0.985 r^2\right)$	**DM1**	
	$0.747r^2$ to $0.748r^2$	**A1**	
		7	

Question	Answer	Marks	Guidance
10(c)	**General guidance for alternative methods**		
	Finding any useful sector area of the circle radius, *r*	**B1**	May be 'nested' in a segment.
	Finding the area of sector CAB	***M1A1**	May be 'nested' in a segment.
	Finding the area of one useful triangle	***M1**	May be 'nested' in a segment.
	Finding the total area of useful triangles	**A1**	May be 'nested' in a segment.
	A correct plan for the shaded area	**DM1**	
	$0.747r^2$ to $0.748r^2$	**A1**	
		7	

Question	Answer	Marks	Guidance
11(a)	$\frac{dy}{dx}=\left\{-2\times 2\times(2x-1)^{-3}\times 2\right\}+\{1\}$	**B1B1**	Expect $\frac{-8}{(2x-1)^3}+1$.
	Substitute $x=\frac{3}{2}$ leading to $\frac{dy}{dx}=\frac{-8}{8}+1=0$. Hence *x*-coordinate of *R* is $\frac{3}{2}$	**DB1**	AG. Or correct solution of $\frac{dy}{dx}=0$.
	When $x=\frac{3}{2}, y=\frac{2}{4}+\frac{3}{2}=2$	**B1**	Answer only is acceptable.
		4	

Question	Answer	Marks	Guidance
11(b)	y-coordinate of $P = 3$, y-coordinate of $Q = \frac{20}{9}$	B1	Both required.
	$\left\{\frac{2(2x-1)^{-1}}{-1\times 2}\right\} + \left\{\frac{1}{2}x^2\right\}$	B1 B1	Area below curve.
	$\left[\left(-\frac{1}{3}+2\right)-\left(-1+\frac{1}{2}\right)\right] = \frac{5}{3}-\left(-\frac{1}{2}\right)$	M1	Apply limits 1→2 to an integral. Expect $\frac{13}{6}$.
	$\frac{1}{2}\left(3+\frac{20}{9}\right) = \frac{47}{18}$	M1	Area of trapezium, only allow errors in y-coordinate of Q.
	$\frac{47}{18} - \frac{13}{6} = \frac{4}{9}$	A1	Shaded region.
		6	
	Alternative method 1: Changes the award of the first M1		
	Their equation of line PQ:[$y = \frac{-7}{9}x + \frac{34}{9}$]. Integrating between 1 and 2.	M1	Must be some evidence of use of limits.
	Alternative method 2: Changes the award of the first M1, a B1 and the second M1		
	Combining line and curve: $\int\left(\frac{-16}{9}x + \frac{34}{9} - \frac{2}{(2x-1)^2}\right)\mathrm{d}x$	M1	For area under the line if *their* $\frac{34}{9}$ is seen integrated correctly and limits used. Correct first and 3rd terms.
	$= \frac{-8}{9}x^2 + \frac{34}{9}x + \frac{1}{(2x-1)}$	B1 B1	
	Use of limits on the whole integral	M1	

Cambridge Assessment International Education

Cambridge International AS & A Level

MATHEMATICS **9709/11**

Paper 1 Pure Mathematics 1 **May/June 2023**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2023 series for most Cambridge IGCSE, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

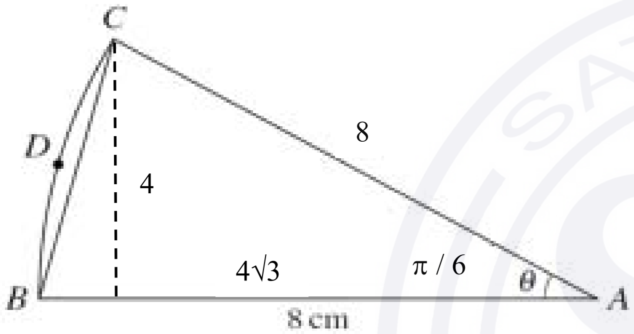
AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	$4\sin\theta + \tan\theta = 0 \Rightarrow 4\sin\theta + \dfrac{\sin\theta}{\cos\theta}[=0]$	M1	For use of $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$. BOD if θ missing.
	$\Rightarrow \sin\theta(4\cos\theta + 1)[=0 \Rightarrow \sin\theta = 0 \text{ or}]\ \cos\theta = -\dfrac{1}{4}$	M1	WWW Factorise, not divide by $\sin\theta$ or $\tan\theta$. May see $\tan\theta(4\cos\theta+1)[=0]$ or $\sin\theta(4+\sec\theta)[=0]$.
	$\theta = 104.5°$	A1	AWRT 1.82 rads A0. Ignore answers outside (0, 180°). If M1 M0, **SC B1** for $\theta = 104.5°$ max 2/3.
		3	

Question	Answer	Marks	Guidance
2(a)	$16 + 96x + 216x^2$	B2, 1, 0	ISW (higher powers of x). Terms may be in any order or presented as a list.
		2	
2(b)	$1 - 10x + 40x^2$	B2, 1, 0	ISW (higher powers of x). Terms may be in any order or presented as a list.
		2	
2(c)	$(16\times40) - (10\times96) + (1\times216)$	M1	*Their* 3 products which would give the term in x^2 (FT *their* values). Look for $640 - 960 + 216$.
	-104	A1	Condone $-104x^2$.
		2	

Question	Answer	Marks	Guidance
3	{Stretch} {factor 2} {in y-direction}	**B2, 1, 0**	**B2** for fully correct, **B1** with two elements correct. {} indicates different elements.
	{Translation} $\begin{pmatrix} \{-6\} \\ \{0\} \end{pmatrix}$	**B2, 1, 0**	**B2** for fully correct, **B1** with two elements correct. {} indicates different elements.
		4	Transformations may be in either order.

Question	Answer	Marks	Guidance
4	$\frac{1}{2}\times 8^2 \times \theta = \frac{16\pi}{3} \Rightarrow \theta = \frac{\pi}{6}$	**B1**	SOI OE e.g. $\frac{2\pi}{12}$, $0.524(3\text{s.f.})$ Use of degrees acceptable throughout provided conversion used in formulae for sector area and arc length.
	Arc length = $8 \times \textit{their}\frac{\pi}{6}$ [= 4.1887…]	**M1**	OE FT *their* θ. Look for $\frac{4\pi}{3}$.
	[BC =] $2\times 8\sin\left(\frac{1}{2}\times \textit{their}\frac{\pi}{6}\right)$ [= 4.1411…]	**M1**	Attempt to find BC or BC^2 (see alt. methods below) FT *their* θ. Look for $16\sin\frac{\pi}{12}$ or $4\sqrt{6}-4\sqrt{2}$.
	Perimeter = 8.33	**A1**	AWRT Must be combined into one term.

Question	Answer	Marks	Guidance
4	**Alternative methods for Question 4: 2nd M1 mark (use normal scheme for the other marks)**		
	ALT 1 $BC^2 = 8^2 + 8^2 - 2\times 8\times 8\cos\left(their\,\frac{\pi}{6}\right)\left[\Rightarrow BC = 4.14\ldots\right]$ **ALT 2** $BC^2 = \left(8-4\sqrt{3}\right)^2 + 4^2\left[\Rightarrow BC = 4.14\ldots\right]$  **ALT 3** $\dfrac{BC}{\sin\left(\frac{\pi}{6}\right)} = \dfrac{8}{\sin\left(\frac{5\pi}{12}\right)}\left[\Rightarrow BC = 4.14\ldots\right]$		ALT 1 Substitute into correct cosine rule. FT *their* θ Look for $128-64\sqrt{3}$ ALT 2 Find lengths 4 and $4\sqrt{3}$ then use Pythagoras in the left hand triangle. ALT 3 Substitute into correct sine rule.
		4	

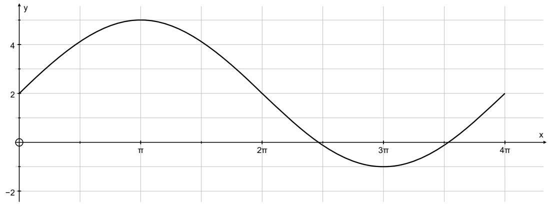
Question	Answer	Marks	Guidance
5	$kx-k=-\frac{1}{2x} \Rightarrow 2kx^2-2kx+1[=0]$ OR quadratic in y: $x=\frac{y+k}{k} \Rightarrow y=-\frac{1}{2\left(\frac{y+k}{k}\right)} \Rightarrow 2y^2+2ky+k=0$	*M1	OE e.g. $kx^2-kx+\frac{1}{2}[=0]$, $x^2-x+\frac{1}{2k}[=0]$ Equate line and curve to form 3-term quadratic (all terms on one side).
	$b^2-4ac[=0] \Rightarrow ([-]2k)^2-4(2k)(1)[=0]$ or $4k^2-8k$ $[=0 \Rightarrow 4k(k-2)=0]$ OR using equation in y: $(2k)^2-4(2)(k)=0$	DM1	Use discriminant correctly with their a,b,c not in quadratic formula. DM0 if x still present. May see $k^2-4(k)\left(\frac{1}{2}\right)=0$ or $1-4\left(\frac{1}{2k}\right)=0$.
	$k=2$ only	A1	If DM0 then $k=2$, award A0 XP then B0 B0 Allow A1 even if divides by k to solve. If $k=0$ also present but uses $k=2$, award A1.
	$4x^2-4x+1=0\left[\Rightarrow(2x-1)^2=0\right] \Rightarrow x=\frac{1}{2}$	B1	
	$y=2\times\frac{1}{2}-2=-1$	B1	

Question	Answer	Marks	Guidance
5	**Alternative method for Q5**		
	$\frac{dy}{dx} = \frac{1}{2x^2}$ or $\frac{1}{2}x^{-2}$	*M1	Differentiate $-\frac{1}{2x}$ M0 for $2x^{-2}$. No errors.
	$[y =]\frac{1}{2x^2}x - \frac{1}{2x^2} = -\frac{1}{2x}$ or $\frac{1}{x} = \frac{1}{2x^2}$ $[\Rightarrow 2x^2 - x = 0]$	DM1	Sub *their* $\frac{dy}{dx}$ into equation of line or set gradient = k to form equation in x.
	$x = \frac{1}{2}$ only	A1	If DM0 then $x = \frac{1}{2}$, award A0XP then B0 B0.
	$y = \left[2 \times \frac{1}{2} - 2\right] = -1$	B1	
	$k = 2$	B1	
		5	

Question	Answer	Marks	Guidance
6(a)	$2(2p-6)=p+\frac{p^2}{6} \Rightarrow \frac{p^2}{6}-3p+12[=0]$ OR $(2p-6)-\frac{p^2}{6}=p-(2p-6) \Rightarrow \frac{p^2}{6}-3p+12[=0]$ OR $\frac{1}{6}d^2+d[=0]$	*M1	Correct method leading to formation of a 3-term quadratic in p (all terms on one side) or 2-term quadratic in d. OE e.g. $p^2-18p+72[=0]$, $\frac{1}{2}p^2-9p+36[=0]$.
	$p^2-18p+72[=0] \Rightarrow (p-6)(p-12)[=0]$ **or** $\frac{18 \pm \sqrt{(-18)^2-4(1)(72)}}{2}$ OR $d\left(\frac{1}{6}d+1\right)[=0] \Rightarrow d=-6$	DM1	Solve a 3-term quadratic in p by factorisation, formula or completing the square or solve a 2-term quadratic in d by factorisation.
	$p = 12$ only	A1	Since $p = 6$ gives $d = 0$. If *M1 DM0 then $p = 12$ only, award **SC B1**, max 2/3 marks. A0 XP if error in either factor and $p=12$ only. $p=12$ only by trial and improvement 3/3.
		3	

Question	Answer	Marks	Guidance
6(b)	For GP $r = \left[\dfrac{2p-6}{\frac{p^2}{6}}\right] = \dfrac{18}{24}\left[=\dfrac{3}{4}\right]$	**B1**	OE SOI.
	Sum to infinity $= \dfrac{24}{1-\frac{3}{4}} = 96$	**B1 FT**	FT *their* value of p if used correctly to find r (B0 if 'p' used) provided $\lvert r\rvert < 1$. e.g. $p = 18 \Rightarrow \left[S_\infty =\right]\dfrac{54}{1-\frac{5}{9}} = 121.5$.
		2	

Question	Answer	Marks	Guidance
7(a)	[Greatest =] 5	**B1**	No inequality required.
	[Least =] –1	**B1**	No inequality required.
			Condone $(-1,5)$ or equivalent.
		2	

Question	Answer	Marks	Guidance
7(b)		**B1**	One complete cycle starting and finishing at $y = 2$. Maximum and minimum in correct quadrants. Shape and curvature approximately correct.
		B1 FT	Maximum and minimum (indicated on y-axis with numbers or lines, or labelled on graph). FT *their* greatest and least values. Award **B1** for 5 and -1 even if *their* values were incorrect in **(a)**.
		2	
7(c)	1	**B1**	WWW
		1	

Question	Answer	Marks	Guidance
8(a)	$1+\frac{2a}{7-a}=\frac{5}{2}\left[\Rightarrow\ \frac{2a}{7-a}=\frac{3}{2}\ \Rightarrow 7a=21\right]\Rightarrow a=\ldots$ OR $1+\frac{2a}{7-a}=\frac{5}{2}\left[\Rightarrow(7-a)+2a=\frac{5}{2}(7-a)\ [\Rightarrow 7a=21\right]\Rightarrow a=\ldots$	M1	OE Substitute $x=7$ then solve for a via legitimate mathematical steps. Condone sign errors only.
	$a=3$	A1	If M0, **SC B1** for $a=3$ with no working.
	$\text{f}(5)=1+\frac{2(\textit{their}3)}{5-\textit{their}3}=4\,[\Rightarrow 4b-2=4]\Rightarrow b=\ldots$ OR $\text{gf}(5)=b\left(1+\frac{2(\textit{their}3)}{5-\textit{their}3}\right)-2\ [\Rightarrow 4b-2=4]\Rightarrow b=\ldots$	M1	Evaluate f(5), either separately or within gf then solve for b via legitimate mathematical steps. Condone sign errors only. FT *their* a value.
	$b=\frac{3}{2}$	A1	OE e.g. $\frac{6}{4}$, 1.5 .
		4	
8(b)	$x>1$	B1	Accept $(1,\infty)$ or $\{*:\ *>1\}$ where * is any variable. B0 for $\text{f}^{-1}(x)>1$ or $\text{f}(x)>1$ or $y>1$.
		1	

Question	Answer	Marks	Guidance
8(c)	EITHER $x-1=\frac{6}{y-3}$ $\left[\Rightarrow (y-3)(x-1)=6\right]$ OR $x=1+\frac{6}{y-3} \Rightarrow x(y-3)=(y-3)+6$	*M1	OE $y-1=\frac{6}{x-3} \Rightarrow (x-3)(y-1)=6$. OE $y=1+\frac{6}{x-3} \Rightarrow y(x-3)=(x-3)+6$. Allow *M1 for use of *their* 3 from **(a)**.
	$y-3=\frac{6}{x-1}$ or $y(x-1)=3x+3$	DM1	OE $x-3=\frac{6}{y-1}$ or $x(y-1)=3y+3$. Allow DM1 for use of *their* 3 from **(a)**.
	$\left[f^{-1}(x)\right]=3+\frac{6}{x-1}$	A1	OE Correct answer e.g. $\frac{3x+3}{x-1}$ ISW. Must be in terms of x.
			*M1 DM1 possible for ‘ a ’ used, but A0 so max 2/3.
		3	

Question	Answer	Marks	Guidance
9(a)	$\frac{dV}{dh} = \frac{4}{3} \times 3(25+h)^2$ [= 4900 when h = 10]	**B1**	Correct expression for $\frac{dV}{dh}$.
	$\frac{dV}{dh} \times \frac{dh}{dt} = \frac{dV}{dt} \Rightarrow$ *their* "$4(25+10)^2$"$\times \frac{dh}{dt} = 500 \Rightarrow \frac{dh}{dt} = \left[\frac{500}{4900}\right]$	**M1**	Use chain rule correctly to find a numerical expression for $\frac{dh}{dt}$. Accept e.g. $\frac{500}{2500+2000+400}$.
	$\frac{dh}{dt} = 0.102\left[\text{cms}^{-1}\right]$	**A1**	AWRT OE e.g. $\frac{5}{49}$ ISW.
		3	
9(b)	$\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt} \Rightarrow 500 =$ *their* "$4(25+h)^2$"$\times 0.075$	***M1**	SOI Use chain rule correctly to form equation in h.
	$\left[(25+h)^2 = \frac{5000}{3}\right] \Rightarrow h = [15.8\,248\ldots]$	**DM1**	Solve quadratic to find h. Exact value of h is $\sqrt{\frac{5000}{3}} - 25$ or $\frac{50\sqrt{6}}{3} - 25$ $h + 25 = 40.82\ldots$
	$V = 69900 \text{ cm}^3$	**A1**	AWRT ISW Look for 698(88.5) .
		3	

Question	Answer	Marks	Guidance
10(a)	$[\pi]\int \frac{16}{(2x-1)^4}[\mathrm{d}x] = [\pi]\int 16(2x-1)^{-4}[\mathrm{d}x] = [\pi]\left(-\frac{16}{3\times 2\times(2x-1)^3}\right)$	***M1**	Integrate $\boldsymbol{y}^2$ (power incr. by 1 or div by *their* new power). M0 if more than 1 error or $-\frac{16}{6}\boldsymbol{x}(2x-1)^{-3}$.
	$[\pi]\left(-\frac{16}{3\times 2\times(2x-1)^3}\right)$	**A1**	OE e.g. $\left(-\frac{8}{3}(2x-1)^{-3}\right)$.
	$[\pi]\left(-\frac{16}{6\times 8}+\frac{16}{6\times 1}\right)$ $\left[=[\pi]\frac{112}{48}=[\pi]\frac{7}{3}\right]$	**DM1**	Sub correct limits into *their* integral: $\mathrm{F}\left(\frac{3}{2}\right)-$ F(1). Must see at least $\left(-\frac{1}{3}+\frac{8}{3}\right)$. Allow 1 sign error. Decimal: 2.33π or 7.33.
	Volume of cylinder $\left[=\pi\times 1^2\times\frac{1}{2}\right]=\frac{1}{2}\pi$ OR $[\pi]\int_1^{1.5} 1[\mathrm{d}x]=\frac{1}{2}\pi$	**B1**	$\frac{1}{2}\pi$ or $\pm\pi\left(\frac{3}{2}-1\right)$ seen.
	Volume of revolution $\left[=\frac{7}{3}\pi-\frac{1}{2}\pi\right] = \frac{11}{6}\pi$	**A1**	A0 for 5.76 (not exact). If DM0 for insufficient substitution, or B0, **SC B1** for $\frac{11}{6}\pi$.
		5	

Question	Answer	Marks	Guidance
10(b)	$\left[\dfrac{dy}{dx}=\right]\left\{-8(2x-1)^{-3}\right\}\{\times 2\}$	**B2, 1, 0**	OE B1 for each correct element in {}.
	At B gradient = -2	**B1**	
	Eqn of tangent $y-1=\textit{their}\ "-2"\left(x-\dfrac{3}{2}\right)$ OR Eqn of normal $y-1=\textit{their}\ "\dfrac{1}{2}"\left(x-\dfrac{3}{2}\right)$	**M1**	SOI Following differentiation OE e.g. $y=-2x+4$ or $y=\dfrac{1}{2}x+\dfrac{1}{4}$. (Must have $m_N=-\dfrac{1}{m_T}$ for M1).
	Tangent crosses x-axis at 2 **or** normal crosses x-axis at $-\dfrac{1}{2}$	**A1**	SOI For at least one intercept correct or correct integration.
	Area = $\dfrac{5}{4}$	**A1**	From intercepts: $\dfrac{1}{2}\times\dfrac{5}{2}\times 1=\dfrac{5}{4}$ or $1+\dfrac{1}{4}=\dfrac{5}{4}$, from lengths: $\dfrac{1}{2}\times\sqrt{5}\times\dfrac{\sqrt{5}}{2}=\dfrac{5}{4}$ or by integration.
		6	

Question	Answer	Marks	Guidance
11(a)	$6a^2-30a+6a=0$ $[\Rightarrow 6a(a-4)=0]$	**B1**	Sub $x=a$ into $\dfrac{dy}{dx}=0$. May see $a^2-5a+a=0$.
	$a=4$ only	**B1**	
		2	

Question	Answer	Marks	Guidance
11(b)	$\frac{d^2y}{dx^2}=12x-30$ or correct values of $\frac{dy}{dx}$ either side of $x=4$	M1	Differentiate $\frac{dy}{dx}$ (mult. by power or dec. power by 1) M0 if no values of $\frac{dy}{dx}$, only signs.
	At $x=4, \frac{d^2y}{dx^2}>0 \therefore$ minimum or $\frac{d^2y}{dx^2}=18 \therefore$ minimum or concludes minimum from $\frac{dy}{dx}$ values	A1	WWW A0 XP if $a=4$ obtained incorrectly in (a) Must see 'minimum'. If M0, **SC B1** for 'minimum' from $\frac{dy}{dx}$ sign diagram.
		2	
11(c)	$[y=]\ \frac{6}{3}x^3-\frac{30}{2}x^2+6(their\ a)x[+c]$	B1 FT	Expect $2x^3-15x^2+24x[+c]$. B1 poss. even if uses 'a' – no value in (a) – max 1/3.
	$-15=2(their\ "4")^3-15(their\ "4")^2+6(their\ "4")^2+c$	M1	Sub $x=their"4"$, $y=-15$ into integral (must incl $+c$) Look for –15 = 128 – 240 + 96 + c [$\Rightarrow c=1$].
	$y=2x^3-15x^2+24x+1$	A1	Coefficients must be correct and simplified. Need to see '$y=$' or '$f(x)=$' in the working.
		3	
11(d)	$\frac{dy}{dx}=6x^2-30x+6(their\ "4")[=0]$ If correct, $[6](x-1)(x-4)[=0]$ or $\frac{30\pm\sqrt{(-30)^2-4(6)(24)}}{12}$	M1	OE Forming a 3-term quadratic using the given $\frac{dy}{dx}$ and solving by factorisation, formula or completing the square. Check for working in **(b)**.
	Coordinates $(1,12)$	A1	Allow $x=1, y=12$ (ignore $x=4$ if present). If M0, award **SC B1** for $(1,12)$.
		2	

Question	Answer	Marks	Guidance
12(a)	$x^2+(y-2)^2=100$	**B1**	OE e.g. $(x-0)^2+(y-2)^2=10^2$ ISW.
		1	
12(b)	Gradient of radius $=\left[\dfrac{10-2}{6-0}=\right]\dfrac{4}{3}$ or gradient of tangent $=\dfrac{-3}{4}$	**M1**	OE SOI Use coordinates to find gradient of radius or differentiate to find m_T e.g. $2x+2(y-2)\dfrac{dy}{dx}=0 \Rightarrow \dfrac{dy}{dx}=\dfrac{-3}{4}$ at (6, 10) $y=2+\sqrt{100-x^2} \Rightarrow \dfrac{dy}{dx}=\dfrac{1}{2}(100-x^2)^{-\frac{1}{2}}(-2x)=-\dfrac{3}{4}$.
	Equation of tangent is $y-10=-\dfrac{3}{4}(x-6)$ $\left[\Rightarrow y=-\dfrac{3}{4}x+\dfrac{29}{2}\right]$	**A1**	OE ISW Allow e.g. $\dfrac{58}{4}$.
		2	
12(c)	Coordinates of centre of circle Q are $\left(0, their\,\dfrac{29}{2}\right)$	**M1**	SOI From a linear equation in **(b)**.
	Equation of circle Q is $x^2+\left(y-their\,\dfrac{29}{2}\right)^2=\left(\dfrac{5\sqrt{5}}{2}\right)^2\left[=\dfrac{125}{4}\right]$	**A1FT**	OE e.g. $(x-0)^2+(y-14.5)^2=31.25$ ISW.
	$x^2+(11-2)^2=100 \Rightarrow x^2=19$ and $x^2+\left(11-\dfrac{29}{2}\right)^2=\dfrac{125}{4} \Rightarrow x^2=19$ OR e.g. $\dfrac{125}{4}-\left(y-\dfrac{29}{2}\right)^2+(y-2)^2=100 \Rightarrow 25y=275 \Rightarrow y=11$	**B1**	OE e.g. $x=[\pm]\sqrt{19}$, $x^2-19=x^2-19$ Correct argument to verify both y-coords are 11 ISW.
		3	

Question	Answer	Marks	Guidance
12(d)	$x^2+\left(-\frac{3}{4}x+\frac{29}{2}-\frac{29}{2}\right)^2=\frac{125}{4}$ $\left[\Rightarrow \frac{25}{16}x^2=\frac{125}{4}\Rightarrow x^2=20\right]$ or $y^2-29y+199[=0]$	M1	Substitute equation of *their* tangent into equation of *their* circle. May see $y=\sqrt{31.25-x^2}+14.5$.
	$x=\pm 2\sqrt{5}$ or $y=\frac{29\mp 3\sqrt{5}}{2}$	A1	OE e.g. $x=\pm\sqrt{20}$ For 2 x-values or 2 y-values or correct (x,y) pair.
	$y\left[=\left(-\frac{3}{4}\times\pm\sqrt{20}\right)+\frac{29}{2}\right]=\frac{29\mp 3\sqrt{5}}{2}$	A1	OE e.g. $\frac{58}{4}+\frac{3\sqrt{20}}{4}$, $\frac{58}{4}-\frac{3\sqrt{20}}{4}$ Correct (x,y) pairs.
		3	

Cambridge International AS & A Level

MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **May/June 2023**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2023 series for most Cambridge IGCSE, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	$[y] = \frac{4}{-2}(x-3)^{-3+1}$ or $\frac{4}{-2(x-3)^2}[+c]$	**B1**	OE Allow $\frac{4}{-3+1}$ and $-3+1$ for the power.
	$5 = \frac{4}{-2}(4-3)^{-2} + c$ or $5 = \frac{4}{-2(4-3)^2} + c$ leading to $c =$	**M1**	Correct use of $(4, 5)$ to find c in an integrated expression (defined by the correct power and no extra x's or terms).
	$y = \frac{-2}{(x-3)^2} + 7$ or $y = -2(x-3)^{-2} + 7$	**A1**	OE $-\frac{4}{2}$ must be simplified to **–**2. Condone c = 7 as their final line as long as either y or f(x) = is seen elsewhere. Do not ISW if the result is of the form y = mx+c.
		3	

Question	Answer	Marks	Guidance
2	[Coefficient of $x^4 = p =$] $15a^2$	**B1**	May be seen in an expansion or with x^4.
	[Coefficient of $x^2 = q =$] $54\,a^2$	**B1**	May be seen in an expansion or with x^2.
	Equating *their* p + *their* q to 276 leading to an equation in a^2 only	**M1**	No x terms and no extra terms. If p and q are not identified then it needs to be clear from the expansion that the appropriate coefficients are being used. $69\,a^2 = 276$ implies the first 3 marks.
	$a = \pm 2$	**A1**	CAO
		4	

Question	Answer	Marks	Guidance
3(a)	$4(x-3)^2$ seen or $a=4$ and $b=-3$	**B1**	OE Award marks for the correct expression or their values a, b and c. Condone $4(x-3)+p-36=0$ and $4\left(\frac{p}{4}-9\right)$.
	$-36+p$ or $p-36$ seen or $c=p-36$	**B1**	
		2	
3(b)	$p-36>0$ leading to $p>36$ or $24^2-4\times4p\langle 0\Rightarrow p\rangle 36$ or $36<p$	**B1**	Allow $(36,\infty)$ or $36<p<\infty$. Consider final answer only.
		1	

Question	Answer	Marks	Guidance
4	$[8x^6+215x^3-27=0]$ leading to $(8x^3-1)(x^3+27)[=0]$ **OR** $\frac{-215\pm\sqrt{215^2-4.8.-27}}{2.8}$ or $\frac{-215\pm\sqrt{47089}}{2.8}$	**M1**	OE If a substitution is used then the correct coefficients must be retained. Condone substitution of $x=x^3$.
	$\frac{1}{8}$, -27	**A1**	Both correct values seen. **SC**: if M0 scored **SC B1** is available for sight of $\frac{1}{8}$ and -27 OE
	$\frac{1}{2}$ or $0.5, -3$	**A1**	**SC**: if M0SCB1 scored then **SCB1** is available for the correct answers and no others. Do not ISW if answers given as a range.
		3	

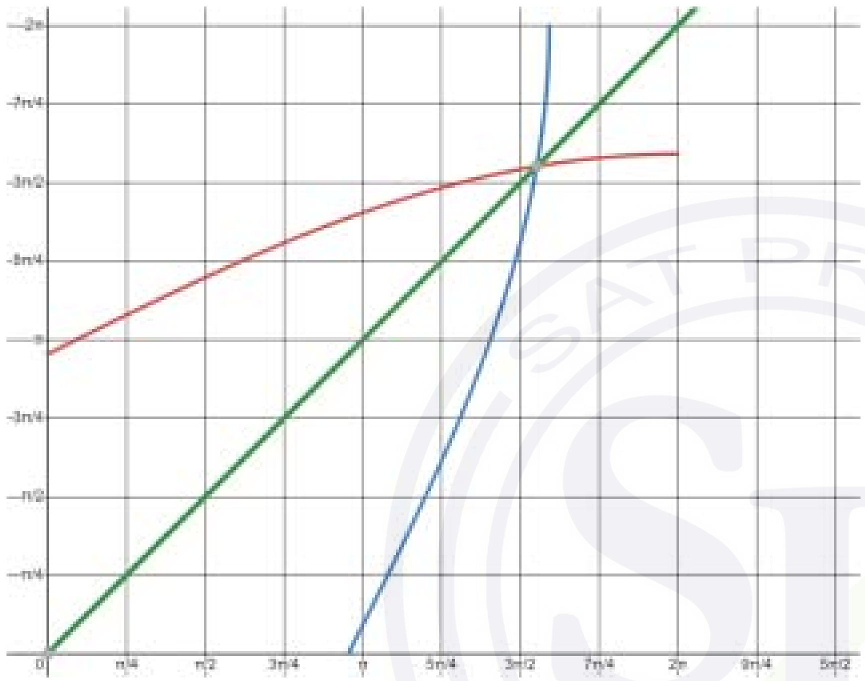
Question	Answer	Marks	Guidance
5	$\left[\int\left(10x^{\frac{1}{2}}-\frac{5}{2}x^{\frac{3}{2}}\right)=\right]\left\{\frac{10}{\frac{3}{2}}x^{\frac{3}{2}}\right\}\left\{-\frac{5}{2\times\frac{5}{2}}x^{\frac{5}{2}}\right\}\ \left[=\frac{20}{3}x^{\frac{3}{2}}-x^{\frac{5}{2}}\right]$	B1 B1	B1 for contents of each { } then ISW.
	$=\left(\textit{their}\,\frac{20}{3}\times 8-32\right)[-0]$	M1	Using limit(s) correctly in an integrated expression (defined by one correct power). Minimum acceptable working is their ($\frac{160}{3}-32$).
	[Area of shaded region =] $\frac{64}{3}$, $21\frac{1}{3}$ or 21.3[333…]	A1	Condone the presence of π for the first 3 marks. Condone using the limits the wrong way around for the M mark and if -21.3 is corrected to 21.3 allow the A mark. **SC**: if M0 scored **SCB1** is available for correct final answer If $\int\left(10x^{\frac{1}{2}}-\frac{5}{2}x^{\frac{3}{2}}\right)=21.3$ and no integration seen B1 only.
		4	

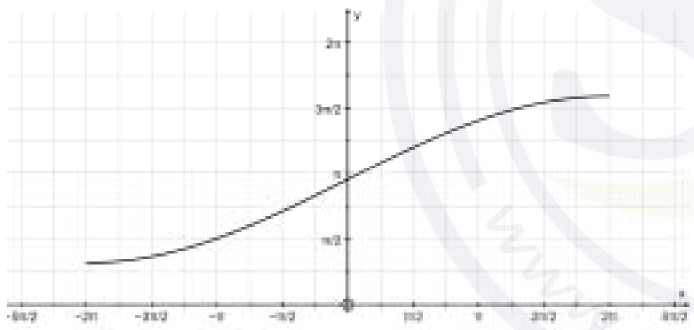
Question	Answer	Marks	Guidance
6(a)	$\frac{1}{2}OA = x\cos\theta$ or $\frac{OA}{\sin(\pi - 2\theta)} = \frac{x}{\sin\theta}$ or $OA^2 = x^2 + x^2 - 2x^2\cos(\pi - 2\theta)$ or $x^2 = r^2 + x^2 - 2rx\cos\theta$ or other valid method.	***B1**	Correct expression containing $\frac{1}{2}OA$, OA or OA^2 (allow *p*, *a* or *r* for *OA*) containing only terms with *x* and θ but not just $OA = 2x\cos\theta$. Do not condone $\sin\pi - 2\theta$ until missing brackets recovered or cos(180 -2θ) until it becomes $-cos2\theta$ etc.
	$OA =$ $2x\cos\theta$ leading to Arc length = $2x\theta\cos\theta$	**DB1**	AG Complete correct method showing all necessary working. Condone $2x\cos\theta\times\theta$.
		2	If B0 but www then **SCB1** for $OA =$ $2x\cos\theta$ leading to Arc length = $2x\theta\cos\theta$.
6(b)	Sector area = $\frac{1}{2}(2x\cos\theta)^2 \times \theta$	**M1**	OE Using sector formula with a correct OA. Condone $cos\theta^2$ for $cos^2\theta$ and missing brackets.
	Triangle area = $\frac{1}{2}\times 2x\cos\theta x\sin\theta$ OR $\frac{1}{2}x^2\sin(\pi - 2\theta)$	**M1**	Using a correct triangle formula for the correct triangle. Condone missing brackets and 180 for π.
	[Area *APB* =] *Their* sector area – *their* triangle area	**M1**	Both expressions must be areas involving terms with x^2 and θ only. Condone missing brackets and 180 for π for the triangle. Condone calling the sector a segment.
	[Area *APB* =] $\frac{1}{2}(2x\cos\theta)^2 \times \theta - \frac{1}{2}x^2\sin(\pi - 2\theta)$ [$= x^2(2\theta\cos^2\theta - \frac{1}{2}\sin 2\theta)$ or $x^2\cos\theta(2\theta\cos\theta - \sin\theta)$]	**A1**	OE A correct expression. Mark the first unsimplified result of subtraction and ISW any incorrect 'simplifications'.
		4	

Question	Answer	Marks	Guidance
7(a)(i)	$\cos^2\theta + 2\sin\theta\cos\theta + \sin^2\theta = 1$ leading to $2\sin\theta\cos\theta = 0$ or $\sin 2\theta = 0$	*B1	Or arriving at $\cos\theta = 0$ or $\sin\theta = 0$ or $\tan\theta = 0$ after first expanding and www.
	$[\theta =]\ 0, \frac{\pi}{2}, \pi$	DB 2,1,0	B2 for three correct answers only. B1 for two correct answers and one incorrect or 3 correct answers plus other values in the range. **SC DB1** for correct 3 answers in degrees and no others. Ignore extras outside of the range and allow decimal equivalents.
		3	Verifying 3 answers rather than expanding and solving 0/3.
7(a)(ii)	$\cos 0 + \sin 0 = [1 + 0 =]1$ and $\cos\frac{\pi}{2} + \sin\frac{\pi}{2}[= 0 + 1] = 1$	B1	Checking both correct values. Do not allow solving an equation. Condone use of 90 degrees.
	$\cos\pi + \sin\pi[= -1 + 0] = -1$ or $\neq 1$	B1	www
		2	

Question	Answer	Marks	Guidance
7(b)	$\dfrac{(\cos\theta-\sin\theta)\sin\theta+(\cos\theta+\sin\theta)(1-\cos\theta)}{(\cos\theta+\sin\theta)(\cos\theta-\sin\theta)}$	**M1**	Correct common denominator and correct products in the numerator and no missing terms. Correct factors in the denominator can be implied by $\cos^2\theta-\sin^2\theta$. Condone brackets missing if recovered.
	$=\dfrac{\cos\theta\sin\theta-\sin^2\theta+\cos\theta-\cos^2\theta+\sin\theta\ -\sin\theta\cos\theta}{\cos^2\theta-\sin^2\theta}$	**A1**	
	$=\dfrac{\sin\theta+\cos\theta-\cos^2\theta-\sin^2\theta}{\cos^2\theta-\sin^2\theta}=\dfrac{\cos\theta+\sin\theta-1}{1-2\sin^2\theta}$	**A1**	AG Clear evidence of using $sin^2\theta+cos^2\theta=1$ in either the numerator or denominator. Condone c, s and/or omission of θ. Working from both sides of the identity and correctly arriving at the same expression can score M1A1. A final statement is then required for the A1.
		3	

Question	Answer	Marks	Guidance
7(c)	$\dfrac{\cos\theta+\sin\theta-1}{1-2\sin^2\theta}=2(\cos\theta+\sin\theta-1)$ leading to $1=2(1-2\sin^2\theta)$	*M1	Replacing LHS with the expression from **(b)** and attempting to simplify i.e. condone omission of $(\cos\theta+\sin\theta-1)=0$ at this stage. M0 for $0=2(1-2\sin^2\theta)$
	$k\sin^2\theta=1$ or 3 leading to $\sin\theta=[\pm]\sqrt{\dfrac{1\text{ or }3}{k}}$ $\left[4sin^2\theta=1 \text{ leading to } \sin\theta=\pm\dfrac{1}{2}\right]$	DM1	Dividing by k and taking the square root of a positive value < 1. This mark can be implied by the solutions $\dfrac{1}{6}\pi,\dfrac{5}{6}\pi$.
	Solutions $0,\dfrac{1}{6}\pi,\dfrac{1}{2}\pi,\dfrac{5}{6}\pi$	A1	Allow 0, 0.524, 1.57, 2.62 AWRT. If M0 **SCB1** for $(\cos\theta+\sin\theta-1)=0 \Rightarrow 0,\dfrac{1}{2}\pi$. If M0 **SCB1** for all four correct answers and no others. Ignore answers outside of the range. Answers in degrees A0.
		3	

Question	Answer	Marks	Guidance
8(a)		*B1	The line $y = x$ correctly drawn. Can be implied by reasonably correct graph of $f^{-1}(x)$.
		DB1	Fully correct (needs to reach $y = 2\pi$ and x-axis and cross the line $y = x$ in the correct squares).
		2	

Question	Answer	Marks	Guidance
8(b)	$y = 3 + 2\sin\frac{1}{4}x$ leading to $\sin\frac{1}{4}x = \frac{y \pm 3}{2}$	**M1**	Attempting to arrive at an expression for $\sin\frac{1}{4}x$; condone $\pm$ sign errors. Variables may be interchanged initially. M1 not implied by $x = \frac{y \pm 3}{2\sin\frac{1}{4}}$.
	$x = 4\sin^{-1}\left(\frac{y-3}{2}\right)$ leading to [$\mathrm{f}^{-1}(x)$or y =] $4\sin^{-1}\left(\frac{x-3}{2}\right)$	**A1**	ISW Must clearly be $\sin^{-1}\left(\frac{x-3}{2}\right)$ NOT $\frac{\sin^{-1}(x-3)}{2}$. Allow $\left(\frac{3-x}{-2}\right)$ but not $\div\frac{1}{4}$.
		2	
8(c)		**B1**	Continuing given graph from y intercept to -2π. The correct shape needed between 0 and -2π, including starting to level off (gradient in the final two squares needs to be reducing) as -2π is approached. The y co-ordinate at -2π must be in the correct square.
	Yes it does have an inverse, because the graph is always increasing OR because it is one-one OR because it passes the horizontal line test OR it is not a many to one [function].	**B1 FT**	If there is no graph to the left of the y axis, no mark is available. FT an incorrect graph and if the answer is now 'No' provide an appropriate reason.
		2	

Question	Answer	Marks	Guidance
8(d)	**{ } indicates different elements throughout.**		
	{Stretch} {factor 4} {in *x*-direction}	**B2, 1, 0**	B2 for fully correct, B1 with two elements correct. Condone use of 'sf' instead of factor and 'co-ordinates' stretched instead of graph stretched. Allow any mention of *x*-axis, horizontally or y-axis invariant. Wavelength or period increased by a factor of 4 for B2 or by 4 for B1.
	{Stretch} {factor 2} {in *y*-direction}	**B2, 1, 0**	B2 for fully correct, B1 with two elements correct. Condone use of 'sf' instead of factor and 'co-ordinates' stretched instead of graph stretched. Allow any mention of *y*-axis, vertically or *x*-axis invariant. Allow y 'co-ordinates' doubled or amplitude doubled for B2.
	{Translation} $\begin{pmatrix}\{0\}\\ \{3\}\end{pmatrix}$	**B2, 1, 0**	B2 for fully correct, B1 with two elements correct. Allow shift. Any mention of *y* axis, *y*-direction or vertically implies $\{0\}$, so shift by 3 vertically is B2, but shift by a factor of 3 vertically or a translation of 3 'up' is B1.
		6	After scoring B2, B2 the final transformation can only be awarded B2 if the order is fully correct i.e. the translation must not be applied before the y stretch. If all correct except the order award B2B2B1.

Question	Answer	Marks	Guidance
9(a)	$\left[ar = 16, \dfrac{a}{1-r} = 100\right]$ leading to $a = \dfrac{16}{r}$ and $a = 100(1-r)$	B1	Rearranging two algebraic statements to give $a =$. These can be implied by a correct equation in one variable.
	$100(1-r)r = 16$ leading to $100r^2 - 100r + 16[=0]$	*M1	Using their two expressions and rearranging to get a 3-term quadratic expression with all of the terms on one side. Condone sign errors only.
	$(5r-4)(5r-1)=0$ OR $\dfrac{25 \pm \sqrt{25^2 - 4.25.4}}{2.25}$ leading to $r = \left[\dfrac{4}{5} \text{ or } \dfrac{1}{5}\right]$	DM1	Condone $(5r-4)(5r-1)$ following $100r^2 - 100r + 16$.
	$a = 20$, $a = 80$	A1	**SC**: if DM0 scored **SCB1** is available for sight of 20 and 80.
	Alternative Method for Question 9(a)		
	$\left[ar = 16, \dfrac{a}{1-r} = 100\right]$ leading to $r = \dfrac{16}{a}$ and $r = \dfrac{100-a}{100}$	B1	Rearranging two algebraic statements to give $r =$. These can be implied by a correct equation in one variable.
	$1600 = 100a - a^2$ leading to $a^2 - 100a + 1600[=0]$	*M1	Using their two expressions and rearranging to get a 3-term quadratic expression with all of the terms on one side. Condone sign errors and 160 instead of 1600 only.
	$(a-20)(a-80)=0$ OR $\dfrac{100 \pm \sqrt{100^2 - 4.1600}}{2}$	DM1	
	$a = 20$, $a = 80$	A1	**SC**: if DM0 scored **SCB1** is available for sight of 20 and 80.
		4	

Question	Answer	Marks	Guidance
9(b)	$r = \frac{4}{5}\ ,\ \frac{1}{5}$	**B1**	OE SOI
	$[u_n =]\mathit{their}\, 20 \times \mathit{their}\left(\frac{4}{5}\right)^{n-1}\quad [v_n =]\mathit{their} 80 \times \mathit{their}\left(\frac{1}{5}\right)^{n-1}$	**B1FT**	2 expressions for the nth term FT *their* values from part **(a)** if $\lvert r \rvert$ less than 1.
	Method 1 for final 2 marks		
	$20 \times \left(\frac{1}{5}\right)^{n-1} \times 4^{n-1}$	**M1**	Correctly separating the numerator and denominator of *their* $\left(\frac{4}{5}\right)^{n-1}$ or one correct step towards the solution eg $u_n = 80 \times \frac{4^{n-2}}{5^{n-1}}$.
	$u_n = \frac{1}{4} \times 80 \times \left(\frac{1}{5}\right)^{n-1} \times 4^{n-1} = 4^{n-2} \times 80 \times \left(\frac{1}{5}\right)^{n-1} = 4^{n-2} \times v_n$	**A1**	AG Given result clearly shown
	Method 2 for final 2 marks		
	$\frac{20 \times 0.8^{n-1}}{80 \times 0.2^{n-1}} = \frac{1}{4} \times 4^{n-1}$	**M1**	Dividing two nth terms of the correct format and simplifying their terms in r.
	$= 4^{-1} \times 4^{n-1} = 4^{n-2}$	**A1**	AG
		4	

Question	Answer	Marks	Guidance
10(a)	$(x-a)^2+\left(\frac{1}{2}x+6-3\right)^2=20$ or using $x=2y-12$	*M1	Obtaining an unsimplified equation in x or y only.
	$\frac{5}{4}x^2+(3-2a)x+a^2-11[=0]$	A1	OE e.g. $5x^2+4(3-2a)x+4a^2-44$ Rearranging to get a correct 3-term quadratic on one side. Condone terms not grouped together. $5y^2-y(54+4a)+133+a^2+24$.
	$(3-2a)^2-4\times\frac{5}{4}(a^2-11)[=0]$	DM1	OE Using b^2-4ac on *their* 3 term quadratic $[=0]$.
	Method 1 for final 2 marks		
	Using $a=4$: $(3-8)^2-5(5)=0$	A1	Clearly substituting $a=4$.
	$a=-16$	B1	Condone no method shown for this value.
	Method 2 for final 2 marks		
	$-a^2-12a+64=0 \Rightarrow (a-4)(a+16)=0 \Rightarrow a=4$	A1	AG Full method clearly shown.
	$a=-16$	B1	Condone no method shown for this value.
		5	If M0, **SCB1** available for substituting $a=4$, finding P(2, 7) and verifying that $CP^2=20$.

Question	Answer	Marks	Guidance
10(b)	Centre (4, 3) identified or used or the point P is (2, 7)	**B1**	
	$\therefore$ gradient of normal $= -2$	**B1**	SOI
	Forming normal equation using their gradient (not 0.5) and their centre or P	**M1**	Condone use of $(\pm 4, \pm 3)$.
	$\dfrac{y-3}{(x-4)} = -2$ or $y - 7 = -2(x-2)$	**A1**	OE Condone $\mathrm{f}(x) =$.
		4	

Question	Answer	Marks	Guidance
10(c)	**Method 1 for Question 10(c)**		
	Diameter: $y-3=\frac{1}{2}(x-4)$ $\left[\text{leading to } y=\frac{1}{2}x+1\right]$ Or $2(x-4)+2(y-3)\frac{dy}{dx}=0$ $\left[\text{leading to } y=\frac{1}{2}x+1\right]$	*M1	Using gradient $\frac{1}{2}$ with their centre. By implicit differentiation.
	$(x-4)^2+\left(\frac{1}{2}x+1-3\right)^2=20$ $\left[\frac{5}{4}x^2-10x=0\right]$	DM1	Obtaining an unsimplified equation in x or y only. $[y^2-6y+5=0]$.
	$x=0$ or 8, $y=1$ or 5 [(0, 1) and (8, 5)]	A1	Correct co-ordinates for both points. Condone no method shown for solution.
	Equations are $y-1=-2x$ and $y-5=-2(x-8)$	A1	$2x+y=1$ and $2x+y=21$.
	Method 2 for Question 10(c)		
	Coordinates of points at which tangents meet curve are (4+4, 3+2) = (8, 5) and (4 – 4, 3 – 2) = (0, 1)	*M1 A1	Vector approach using their centre and gradient = 0.5 . Condone answers only with no working.
	Equations are $y-5=-2(x-8)$ and $y-1=-2x$	DM1 A1	Forming equations of tangents using *their* (0, 1) and (8, 5).
	Method 3 for Question 10(c)		
	$(x-4)^2+(-2x+c-3)^2=20$ $\left[5x^2+(4-4c)x+(c-3)^2-4=0\right]$	*M1	Obtaining an unsimplified equation in x only using equation of circle with $y=-2x+c$.
	$(4-4c)^2-20\left((c-3)^2-4\right)[=0]$ [leading to $-4c^2-32c+120c+16-100=0$]	DM1	Using $b^2-4ac[=0]$.

Question	Answer	Marks	Guidance
10(c)	$4c^2 - 88c + 84[=0]$ [leading to $c^2 - 22c + 21 = 0$]	**A1**	
	$c = 21$ and $c = 1$ or $y = -2x + 21$ **and** $y = -2x + 1$	**A1**	Condone no method shown for solution.
		4	

Question	Answer	Marks	Guidance
11(a)	$\frac{dy}{dx} = \left\{k\frac{1}{2}(4x+1)^{-\frac{1}{2}}\right\}\{\times 4\}\{-1\}$	**B 2,1,0**	OE e.g. $2k(4x+1)^{-\frac{1}{2}} - 1$ B2 Three correct unsimplified **{ }** and no others. B1 Two correct **{ }** or three correct **{ }** and an additional term e.g. +5. B0 More than one error.
		2	
11(b)	$2k(4x+1)^{-\frac{1}{2}} - 1 = 0$ leading to $(4x+1)^{\frac{1}{2}} = 2k$ or $\frac{2k}{(4x+1)^{\frac{1}{2}}} = 1$	**M1**	OE Equating their $\frac{dy}{dx}$ of the form $ak(4x+1)^{-\frac{1}{2}} - 1$ where a = 2 or 0.5, to 0 and dealing with the negative power correctly including k not multiplied by $(4x+1)^{\frac{1}{2}}$.
	$x = \frac{4k^2 - 1}{4}$	**A1**	CAO OE simplified expression ISW.
		2	

Question	Answer	Marks	Guidance
11(c)	$2 \times 10.5(4x+1)^{-\frac{1}{2}} - 1 = 2$	M1	Putting k= 10.5 into their $\frac{dy}{dx}$ and equating to 2.
	$7 = (4x+1)^{\frac{1}{2}}$ leading to $4x+1=49$ leading to $x=12$	A1	If M1 earned **SCB1** available for $x = \frac{33}{64}$ from $a = \frac{1}{2}$.
	$y = \left[10.5\sqrt{4x+1} - x + 5 = \right]66.5$ [leading to (12, 66.5)]	A1	
	$y - 66.5 = -\frac{1}{2}(x-12)$	A1	OE
		4	

Cambridge Assessment International Education

Cambridge International AS & A Level

MATHEMATICS	**9709/13**
Paper 1 Pure Mathematics 1	**May/June 2023**
MARK SCHEME	
Maximum Mark: 75	

Published

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3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	{Translation} $\begin{pmatrix}\{0\}\\\{-2\}\end{pmatrix}$	**B2, 1, 0**	B2 for fully correct, B1 with two elements correct. {} indicates different elements.
	{Stretch} {[scale] factor 2} {parallel to *x*-axis}	**B2, 1, 0**	B2 for fully correct, B1 with two elements correct.
		4	Transformations can be in either order.

Question	Answer	Marks	Guidance
2	$x^2-6x+c>2$ leading to $(x-3)^2-9+c>2$	**M1 A1**	M1 for completion of the square with an equation or in equality with the '2'.
	$c>11-(x-3)^2$ and $(x-3)^2\geq 0$	**M1**	SOI
	$c>11$	**A1**	
	Alternative Method 1		
	$\frac{\mathrm{d}y}{\mathrm{d}x}=2x-6=0$	**M1**	M1 for differentiating and setting $\frac{\mathrm{d}y}{\mathrm{d}x}=0$.
	$x=3$	**A1**	
	When $x=3$, $y=9-18+c$	**M1**	
	$[-9+c>2]$ $c>11$	**A1**	
	Alternative Method 2		
	$x^2-6x+c>2$ leading to $x^2-6x+c-2[>0]$ then use of 'b^2-4ac'	**M1**	
	$36-4(1)(c-2)<0$	**M1 A1**	OE Must be correct inequality for M1.
	$c>11$	**A1**	
		4	

Question	Answer	Marks	Guidance
3(a)	$x^5+10x^3+40x+\frac{80}{x}+\frac{80}{x^3}+\frac{32}{x^5}$ or $x^5+10x^3+40x+80x^{-1}+80x^{-3}+32x^{-5}$	**B2, 1, 0**	B2, all terms correct, B1 5 terms correct. Terms must be simplified. Lists of terms allowed.
		2	
3(b)	*their* $40\times a+\left(\textit{their}\text{ coefficient of }x^{-1}\right)\times b=0$	**M1**	Coefficients of a and b must be non-zero, allow x's so long as they are dealt with correctly.
	$\left(\textit{their}\text{ coefficient of }x^{-1}\right)\times a+\left(\textit{their}\text{ coefficient of }x^{-3}\right)\times b=80$	**M1**	Coefficients of a and b must be non-zero, allow x's as long as they are dealt with correctly.
	$a=2\quad b=-1$	**A1 A1**	Dependent on both M marks, may be seen without working.
		4	

Question	Answer	Marks	Guidance
4(a)	$3\sin^2 x-3\sin^2 x\cos^2 x-4\cos^2 x\ [=0]$	**M1**	Replace $\tan^2 x$ with $\frac{\sin^2 x}{\cos^2 x}$ and multiply by $\cos^2 x$.
	$3\left(1-\cos^2 x\right)-3\left(1-\cos^2 x\right)\cos^2 x-4\cos^2 x\ [=0]$	**M1**	Replace $\sin^2 x$ by $1-\cos^2 x$ twice.
	$3\cos^4 x-10\cos^2 x+3=0$ or $-3\cos^4 x+10\cos^2 x-3=0$	**A1**	Or multiple of these equations.
		3	

Question	Answer	Marks	Guidance
4(b)	$(3\cos^2 x - 1)(\cos^2 x - 3)$ $[=0]$	**M1**	OE, using *their* equation in the given form. Allow unusual notation if meaning is clear.
	$\cos x = [\pm]\frac{1}{\sqrt{3}}$	**A1**	SOI Answer only **SC B1**.
	54.7°,	**A1**	
	125.3°	**A1 FT**	Only other answer and must be from correct factorisation for A1. FT for $180° - \textit{their}$ first answer . Answers only **SC B1**, **SC B1 FT**.
		4	

Question	Answer	Marks	Guidance
5(a)	$(x-1)^2 + (x-9+4)^2 = 40$	**M1**	Substitute line into circle.
	$x^2 - 6x - 7$ $[=0]$ leading to $(x+1)(x-7)$ $[=0]$	**M1**	Simplify to 3-term quadratic and factorise OE.
	(–1, –10), (7, –2) or $x = -1$ and 7, $y = -10$ and –2	**A1 A1**	Answers only **SC B1**, **SC B1** but must see a correct quadratic equation.
		4	

Question	Answer	Marks	Guidance
5(b)	[C is mid-point =] $\left(\frac{their\ x_1 + their\ x_2}{2}, \frac{their\ y_1 + their\ y_2}{2}\right)$	M1	Expect (3, –6).
	Radius = $\sqrt{(their\ x - their\ 3)^2 + (their\ y - their\ (-6))^2}$ OR $their\sqrt{\left((7-(-1))^2 + (-2-(-10))^2\right)}/2$	M1	Expect $\sqrt{32}$.
	$(x-3)^2 + (y+6)^2 = 32$	A1	OE
		3	

Question	Answer	Marks	Guidance
6(a)	$\frac{½r^2\theta}{r\theta} = \frac{76.8}{9.6}$ or $\frac{1}{2}\left(\frac{9.6^2}{\theta^2}\right)\theta = 76.8$	M1	Eliminate $\theta\ or\ r$ using correct formulae SOI.
	$r = 16$	A1	
	$\theta = 0.6$	A1	Accept 34.4°
	ΔOAB = ½ × *their* 16^2 × sin *their* 0.6	M1	Allow Segment = 76.8 –½ × *their* 16^2 × sin *their* 0.6. Expect 72.27 .
	[Area = 76.8 – 72.27 =] 4.53	A1	AWRT
		5	
6(b)	$AB = 2 \times 16 \times \sin 0.3$ OR $AB^2 = 16^2 + 16^2 - 2 \times 16^2 \cos 0.6$	M1	Any valid method with *their* r, θ. Expect AB = 9.46.
	Perimeter = 9.6 + 9.46 = 19.1	A1	AWRT
		2	

Question	Answer	Marks	Guidance
7(a)	$[y] < 2$ OR [f(x)] < 2	B1	OE e.g. $f < 2, (-\infty, 2), -\infty < f[x] < 2$. Do not accept $x < 2$ or f(x) $\leqslant$ 2.
		1	
7(b)	$y = 2 - \frac{5}{x+2}$ leading to $y(x+2) = 2(x+2) - 5$ leading to $xy + 2y = 2x - 1$	M1	or $\frac{5}{x+2} = 2 - y$ (allow sign errors).
	$2y + 1 = 2x - xy$ leading to $2y + 1 = x(2-y)$	DM1	or $\frac{5}{2-y} = x + 2$ (allow sign errors).
	$x = \frac{2y+1}{2-y} \rightarrow \mathrm{f}^{-1}(x) = \frac{2x+1}{2-x}$	A1	OE or $y = \frac{5}{2-x} - 2$.
	Domain is $x < 2$	B1 FT	FT on the numerical part of *their* range from part **(a),** including $x \neq 2$ not penalized. No FT for $x \in \mathcal{R}, x = k, x \neq k$.
		4	
7(c)	$\mathrm{fg}(x) = 2 - \frac{5}{x+3+2}$	B1	
	$= \frac{2(x+5)-5}{x+5}$ or $\frac{2(x+5)}{x+5} - \frac{5}{x+5}$	M1	Use of *their* common denominator.
	$= \frac{2x+5}{x+5}$	A1	
		3	

Question	Answer	Marks	Guidance
8(a)	$r = \dfrac{a}{a+2}$	**B1**	OE SOI
	$\dfrac{a}{1-\dfrac{a}{a+2}} = 264$	**M1**	Use of S∞ formula.
	$\dfrac{a(a+2)}{a+2-a} = 264$ leading to $\dfrac{a(a+2)}{2} = 264$ leading to $a^2 + 2a - 528\ [=0]$	**M1***	Process to a 3 term quadratic or a 3 term cubic. May contain terms on LHS and RHS.
	$(a-22)(a+24)\ [=0]$	**DM1**	Attempt to solve.
	$a = 22$ (only)	**A1**	22 without working **SC DB1** (dep on 2nd M1).
		5	
8(b)	$d = \dfrac{6^2}{6+2} - 6 = -\dfrac{3}{2}$	**B1**	
	$\dfrac{n}{2}\left\{12 + (n-1)\left(\dfrac{-3}{2}\right)\right\}[<] - 480$	**M1***	Forming an inequation with *their* numerical d. May use an equality.
	$[3]\left(n^2 - 9n - 640\right)\ [> 0]$	**A1**	OE May contain terms on LHS and RHS.
	$[n =]\ \dfrac{9 \pm \sqrt{81 + 2560}}{2}$	**DM1**	OE. Expect 30.19 . Working for solution must be shown.
	31 only	**A1**	Must come from a correct first inequality (or an equality). 31 no working **SC DB1** (dep on correct quadratic and correct inequality/equality).
		5	

Question	Answer	Marks	Guidance
9(a)	$[y=]\ \{x\}\ \{+(x-1)^{-2}\ \}\ \ [+c]$	B1 B1	May be unsimplified.
	Sub $x = 0$, $y = 3$ leading to $3 = 0 + 1 + c$	M1	Substitution into an integral, expect $c = 2$.
	$y = x + (x-1)^{-2} + 2$ or $f(x) = x + (x-1)^{-2} + 2$	A1	$\frac{-2}{(-2)(x-1)^2}$ or $\frac{-2(x-1)^{-2}}{-2}$ must be simplified.
		4	
9(b)	[Gradient of tangent =] $f'(0) = 3$	B1	
	Equation of tangent is $y - 3 = their$ gradient at $x = 0(x-0)$	M1*	Expect $y = 3x + 3$, normal gets M0.
	Intersection given by $3x + 3 = x + (x-1)^{-2} + 2$	DM1	FT *their* equation from part **(a)**.
	$2x+1=\frac{1}{(x-1)^2} \rightarrow (2x+1)(x-1)^2 - 1 = 0$ or solve equation before given form reached and show solution ($x = 3/2$) satisfies given result	A1	WWW AG
		4	
9(c)	Substitute $x=\frac{3}{2}$ leading to $(2x+1)(x-1)^2 - 1$ leading to $4\times¼-1=0$. Hence $x=\frac{3}{2}$ If shown in **(b)** must be referenced here (in part **(c)**)	B1	Evaluation of each bracket must be shown. Allow $\left(\frac{1}{2}\right)^2$ for second bracket. Solution of $(2x+1)(x-1)^2 - 1 = 0$ is acceptable.
	When $x=\frac{3}{2}$ $y = 7½$	B1	
		2	

Question	Answer	Marks	Guidance
10(a)	$\left[\dfrac{dy}{dx}=\right]\{9\}+\left\{-\dfrac{3}{2}(2x+1)^{1/2}\times 2\right\}$	B1, B1	Including '+c' makes the second term B0.
	$9-3(2x+1)^{1/2}=0$ leading to $2x+1=9$	M1	Set differential to zero and solve by squaring SOI. Beware $9^2-3^2(2x+1)=0$ M0A0. $2\boldsymbol{x}+1=\sqrt{3}\ or\ 2x+1=\pm 9$ get M0.
	Max point = (4, 9)	A1	WWW y = 9 must come from original equation.
		4	
10(b)	When x = 1½, shows substitution or $\dfrac{dy}{dx}=3$	M1	Substituting x = 1½ into their $\dfrac{dy}{dx}$.
	Gradient of AB is $\dfrac{5½-3½}{1½-7½}\left[=\dfrac{-1}{3}\right]$	M1	Substituting into a correct expression for m_{AB}.
	$-\dfrac{1}{3}\text{x}3=-1$. [Hence AB is the normal]	A1	
	Alternative method for Question 10(b)		
	When x = 1½ $\dfrac{dy}{dx}=3$,[perpendicular gradient is -1/3]	M1	
	Perpendicular through A has equation $y=\dfrac{-x}{3}+6$ which contains B(7.5,3.5) leading to AB is a normal to the curve at A	M1 A1	
		3	

Question	Answer	Marks	Guidance
10(c)	$\left\{\frac{9x^2}{2}\right\}+\left\{\frac{-(2x+1)^{\frac{5}{2}}}{\frac{5}{2}\times 2}\right\}$	B1 B1	Integrating y with respect to x.
	$\left\{\frac{9}{2}7.5^2-\frac{1}{5}(2\times 7.5+1)^{2.5}\right\}-\left\{\frac{9}{2}1.5^2-\frac{1}{5}(2\times 1.5+1)^{2.5}\right\}$ or $\left(\frac{9}{2}\times\frac{225}{4}-\frac{1024}{5}\right)-\left(\frac{81}{8}-\frac{32}{5}\right)$ or $\frac{1933}{40}-\frac{149}{40}$ or 48.325 – 3.725	M1	OE Apply limits 1½ to 7½ to an integral. Working must be seen. Expect 44.6 .
	$\frac{1}{2}\left(5\frac{1}{2}+3\frac{1}{2}\right)\times 6$ or $\int_{\frac{3}{2}}^{\frac{15}{2}}(\frac{-1}{3}x+6)\mathrm{d}x=$ $\left(\frac{-1}{6}\times\left(\frac{15}{2}\right)^2+6\times\frac{15}{2}\right)-\left(\frac{-1}{6}\times\left(\frac{3}{2}\right)^2+6\times\frac{3}{2}\right)$ or $\frac{285}{8}-\frac{69}{8}$ [= 27]	B1	SOI Area of trapezium. May be seen combined with the area under the curve integral.
	[Shaded area = 44.6 – 27 =] 17.6	A1	**SC B1** if no substitution of the limits seen.
		5	

Question	Answer	Marks	Guidance
10(c)	**Alternative method for Question 10(c)**		
	$A = \int_{\frac{3}{2}}^{\frac{15}{2}} ((9x-(2x+1)^{\frac{3}{2}})-\left(\frac{-1}{3}x+6\right))dx = \int_{\frac{3}{2}}^{\frac{15}{2}} ((\frac{28}{3}x-(2x+1)^{\frac{3}{2}}-6)dx$	**M1**	Finding the equation of AB and subtracting from the equation of the curve.
	$=\left\{\frac{28}{3\times 2}x^2-6x\right\}+\left\{\frac{-(2x+1)^{\frac{5}{2}}}{\frac{5}{2}\times 2}\right\}$	**A1 A1**	
	$\frac{127}{10}-\frac{-49}{10}$	**M1**	Apply limits 1½ to 7½ to an integral. Working must be seen.
	17.6	**A1**	**SC B1** if no substitution of limits seen.
		5	

Cambridge International AS & A Level

MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **February/March 2023**

MARK SCHEME

Maximum Mark: 75

Published

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M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

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B Mark for a correct result or statement independent of method marks.

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- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
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Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
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SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	$x^2 - kx + 2 = 3x - 2k$ leading to $x^2 - x(k+3) + (2+2k)$ $[=0]$	**M1**	3-term quadratic, may be implied in the discriminant.
	$b^2 - 4ac = (k+3)^2 - 8(1+k)$ (ignore '= 0' at this stage)	**DM1**	Cannot just be seen in the quadratic formula.
	$=(k-1)^2$ accept $(k-1)(k-1)$	**A1**	Or use of calculus to show minimum of zero at $k = 1$ or sketch of $\mathrm{f}(k) = k^2 - 2k + 1$.
	$\geqslant 0$ Hence will meet for all values of k	**A1**	Clear conclusion.
		4	

Question	Answer	Marks	Guidance
2	Stretch: $(2x)^2 - 2(2x) + 5$ or $(x-1)^2 + 4$ leading to $(2x-1)^2 + 4$	**M1**	Replacing x by $2x$.
	Reflection: $(-2x)^2 - 2(-2x) + 5$ or $(-2x-1)^2 + 4$	**M1**	Replacing x by $-x$. FT on *their* stretch.
	Stretch: $3\left\{(-2x)^2 - 2(-2x) + 5\right\}$ or $3\left\{(-2x-1)^2 + 4\right\}$	**M1**	Multiplying the whole function by 3. FT on *their* (stretch plus reflection).
	$12x^2 + 12x + 15$	**A1**	
		4	

Question	Answer	Marks	Guidance
3	$\frac{dy}{dx} = \left\{\frac{1}{60}(3x+1) \times 2\right\} \times \{3\}$	B1 B1	May see $\frac{1}{60}(18x+6)$.
	$\frac{1}{10}(3x+1) = 1$	M1	Equate *their* $\frac{dy}{dx}$ to 1.
	$x = 3$	A1	
		4	

Question	Answer	Marks	Guidance
4(a)	$5.00 + 20 \times 0.02$ or $5.02 + 19 \times 0.02$	M1	Allow for a = 5, n = 20 with d = 0.02 only. a = 5, n = 21(OE) with d = 0.2 gets M1 only.
	5.40	A1	
		2	
4(b)	$r = \frac{5.02}{5} = 1.004$ or $\frac{251}{250}$	B1	
	$5.00 \times (their 1.004)^{20}$ or $5.02 \times (their 1.004)^{19}$	M1	Allow a = 5, n = 20.
	5.42	A1	Any correct rounding of 5.41557108 .
		3	

Question	Answer	Marks	Guidance
5	$r^2 = (7+2)^2 + (12-5)^2$	**B1**	Expect 130, may use *AC* rather than *r*.
	Equation of circle is $(x+2)^2 + (y-5)^2 = 130$	**B1 FT**	OE FT *their* 130, may use distance *BC* rather than circle.
	$(x+2)^2 + (-2x+21)^2 = 130$	**M1**	Substitute $y = -2x + 26$ into a circle equation.
	$5x^2 - 80x + 315\ [=0]$ leading to $[5](x-9)(x-7)$	**M1**	Factorisation OE must be seen.
	$x = 9$	**A1**	With or without $x = 7$.
	$y = 8$ OR (9, 8)	**A1**	$y = 8$ or $(9, 8)$ only. Both A1's dependent on the first M1.
		6	

Question	Answer	Marks	Guidance
6	$7C1\left(\frac{x}{a}\right)^6\left(\frac{a}{x^2}\right)$ or $7C6\left(\frac{x}{a}\right)^6\left(\frac{a}{x^2}\right)$ $7C2\left(\frac{x}{a}\right)^5\left(\frac{a}{x^2}\right)^2$ or $7C5\left(\frac{x}{a}\right)^5\left(\frac{a}{x^2}\right)^2$	B1 B1	Coefficients x^4 & x. Can be seen in an expansion.
	$\frac{\left(\frac{7}{a^5}\right)}{\left(\frac{21}{a^3}\right)} = 3$	M1	OE. Allow extraneous x^4 and x at this stage; numerator and denominator must be functions of a. Allow errors in evaluation of the combinations.
		A1	Completely correct.
	$a^2 = \frac{1}{9}$	A1	SOI (implied by $a = \frac{1}{3}$).
	$a = \pm\frac{1}{3}$	A1	Allow ± 0.333 .
		6	

Question	Answer	Marks	Guidance
7(a)	$\tan\theta\sin\theta = 1$ leading to $\sin^2\theta = \cos\theta$	**M1**	Use of $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ and multiplication by $\cos\theta$.
	$1-\cos^2\theta = \cos\theta$ or $\cos^2\theta + \cos\theta - 1\ [=0]$	**M1**	Use of trig identity to form a 3-term quadratic.
	$[\cos\theta =]\ \dfrac{-1\pm\sqrt{5}}{2}$	**M1**	Use of formula or completion of the square must be seen on a 3-term quadratic. Expect 0.6180 .
	51.8º,	**A1**	Both A marks dependent on the 2nd M1.
	308.2º	**A1 FT**	FT for (360º – 1st soln), A0 if extra solutions in range. Radians 0.905 and 5.38, A1 only for both.
		5	
7(b)	$\dfrac{\tan\theta}{\sin\theta} - \dfrac{\sin\theta}{\tan\theta} = \dfrac{\sin\theta}{\sin\theta\cos\theta} - \dfrac{\sin\theta\cos\theta}{\sin\theta} = \dfrac{1}{\cos\theta} - \cos\theta$	**M1**	Use $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ twice with correct use of fractions.
	$= \dfrac{1-\cos^2\theta}{\cos\theta} = \dfrac{\sin^2\theta}{\cos\theta}$	**M1**	Use $1-\cos^2\theta = \sin^2\theta$ with correct use of fractions.
	$= \tan\theta\sin\theta$	**A1**	WWW
		3	

Question	Answer	Marks	Guidance
8(a)	$\tan BDC = \frac{4}{3}$ or $\sin BDC = \frac{4}{5}$ or $\cos BDC = \frac{3}{5}$ **used** to find ADC	**M1**	May use cosine rule or $CAD = \tan^{-1}\frac{4}{8}$.
	$BDC = 0.927[3] \rightarrow ADC = \pi - 0.927[3]$ $[= 2.214 \text{ to } 2.215]$	**A1**	Allow degrees, 126.87, and 0.7048π or 0.705π.
	$Arc\, AC = 5 \times their\, 2.214$	**M1**	Use of $r\theta$ or $\frac{\theta}{360}.2\pi r$ Expect 11.07 .
	$AC = \sqrt{8^2 + 4^2}$ *or* $2 \times 5 \times \sin 1.107$	**M1**	Expect 8.94 .
	$[\text{Perimeter} = 11.07 + 8.94 =] 20.0$	**A1**	Accept AWRT [20.01, 20.02].
		5	
8(b)	Sector $ACD = ½ \times 5^2 \times their\, 2.214$	**M1**	See use of $\frac{1}{2}r^2\theta$ or $\frac{\theta}{360}.\pi r^2$. Expect 27.7 .
	Subtracting the area of $\Delta ADC = ½ \times 5 \times 4$ or $\frac{1}{2}5^2 \sin their\, 2.214$ or $\frac{1}{2} \times 8 \times 4 - \frac{1}{2} \times 3 \times 4$	**M1**	Subtracting the area of ΔADC, expect -10.
	Shaded area $= 27.7 - 10 = 17.7$	**A1**	Accept AWRT [17.67, 17.68]. Correct answer cannot come from an angle of 2.215 .
		3	

Question	Answer	Marks	Guidance
9(a)	$[y] \leqslant -1$	B1	Accept f or $\mathrm{f}(x) \leqslant -1$, $-\infty < y \leqslant -1$, $(-\infty, -1]$. Do not accept $x \leqslant -1$.
		1	
9(b)	$y = -3x^2 + 2$ rearranged to $3x^2 = 2 - y$, leading to $x^2 = \dfrac{2-y}{3}$ or $y^2 = \dfrac{2-x}{3}$	M1	
	$x = [\pm]\sqrt{\dfrac{2-y}{3}} \rightarrow [\mathrm{f}^{-1}(x)] = \{-\}\left\{\sqrt{\dfrac{2-x}{3}}\right\}$	A1 A1	A1 for minus, A1 for $\sqrt{\dfrac{2-x}{3}}$, allow $-\sqrt{\dfrac{x-2}{-3}}$.
		3	
9(c)	$\mathrm{fg}(x) = -3\left(-x^2 - 1\right)^2 + 2$	M1	SOI expect $-3x^4 - 6x^2 - 1$.
	$\mathrm{gf}(x) = -\left(-3x^2 + 2\right)^2 - 1$	M1	SOI expect $-9x^4 + 12x^2 - 5$.
	$\mathrm{fg}(x) - \mathrm{gf}(x) + 8 = 0$ leading to $6x^4 - 18x^2 + 12\ [= 0]$	A1	OE
	$[6]\left(x^2 - 1\right)\left(x^2 - 2\right)[= 0]$ or formula or completion of the square	M1	Solving a 3-term quadratic equation in x^2 must be seen.
	$x = -1\ , -\sqrt{2}$ only these **two** solutions	A1	Allow $-\sqrt{1}$, $-1.41[4]$ Answers only **SC B1**.
		5	

Question	Answer	Marks	Guidance
10(a)	$-\frac{3}{2}=\frac{1}{2}+k$ leading to $k=-2$	B1	**AG** Need to see $4^{-\frac{1}{2}}$ evaluated as $\frac{1}{4^{\frac{1}{2}}}$ or better.
		1	
10(b)	$[y]=2x^{\frac{1}{2}}-2x \quad [+c]$	M1 A1	Allow $\frac{x^{\frac{1}{2}}}{1/2}-2x$.
	$-1=4-8+c$	M1	Substitute x = 4, y = -1 (c present) Expect c = 3.
	$y=2x^{\frac{1}{2}}-2x+3$ or $y=2\sqrt{x}-2x+3$	A1	Allow if f(x) = or y = anywhere in the solution.
		4	
10(c)	$x^{-1/2}-2=0$	M1	Set *their* $\frac{dy}{dx}$ to zero.
	$x=\frac{1}{4}$	A1	If $\left(\frac{1}{2}\right)^2=\pm\frac{1}{4}$ max of M1A1 if $\left(\frac{1}{4},3\frac{1}{2}\right)$ seen.
	(¼, 3½)	A1	
		3	

Question	Answer	Marks	Guidance
10(d)	$\frac{d^2y}{dx^2} = -\frac{1}{2}x^{-\frac{3}{2}}$	B1	
	< 0 (or -4) hence Maximum	DB1	WWW Ignore extra solutions from $x = -\frac{1}{4}$.
		2	

Question	Answer	Marks	Guidance
11(a)	Gradient of *AB* = $\frac{2-(-1)}{5-2}$	M1	Expect 1, must be from $\Delta y / \Delta x$.
	Equation of *AB* is $y-2=1(x-5)$ or $y+1=1(x-2)$	A1	OE. Expect $y = x-3$.
		2	

Question	Answer	Marks	Guidance
11(b)	$[\pi]\int x^2 dy = [\pi]\int (y^2+1)^2 dy = [\pi]\int (y^4+2y^2+1)dy$	**M1**	For curve: Attempt to square y^2+1 and attempt integration. Subtracting curve equation from line equation before squaring is M0. Integration before squaring M0.
	$[\pi]\left(\frac{y^5}{5}+\frac{2y^3}{3}+y\right)$	**A2, 1, 0**	
	$[\pi]\int (y+3)^2 dy = [\pi]\int (y^2+6y+9)dy$	**M1**	For line: Attempt to square *their* $y+3$ and attempt integration.
	$[\pi]\left(\frac{y^3}{3}+3y^2+9y\right)$ or $[\pi]\left(\frac{(y+3)^3}{3}\right)$	**A2, 1, 0**	Not available for incorrect line equations.
	$[\pi]\left\{\frac{8}{3}+12+18-\left(-\frac{1}{3}+3-9\right)\right\}$ or $[\pi]\left\{\frac{32}{5}+\frac{16}{3}+2-\left(-\frac{1}{5}-\frac{2}{3}-1\right)\right\}$	**DM1**	Apply limits $-1 \to 2$ to either integral providing they have been awarded M1. Expect $15\frac{3}{5}$ $[\pi]$ and/or $39[\pi]$. Some evidence of substitution of both -1 and 2 must be seen. Dependent on at least one of the first 2 M1 marks.
	Volume = $[\pi](39 - 15\frac{3}{5})$	**DM1**	Appropriate subtraction. Dependent on at least one of the first 2 M1 marks.
	$= 23\frac{2}{5}\pi$ or $\frac{117}{5}\pi$ or awrt 73.5[1327]	**A1**	
		9	

Cambridge Assessment International Education

Cambridge International AS & A Level

MATHEMATICS **9709/11**

Paper 1 Pure Mathematics 1 **October/November 2022**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2022 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

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Question	Answer	Marks	Guidance
1	$(3x+2)(x-1)=2 \Rightarrow 3x^2-x-4\ [=0]$	**M1**	OE Multiply by denominator and obtain a quadratic.
	$(3x-4)(x+1)[=0]$	**M1**	Solve by factorising, formula or completing the square.
	$[x=]\ -1, \frac{4}{3}$	**A1**	Allow 1.33 If M1 M0, **SC B1** possible for two correct answers.
		3	

Question	Answer	Marks	Guidance
2(a)	$12\left(\frac{1}{2}\times 6-1\right)^{-4}\left[=12(2)^{-4}=\frac{3}{4}\right]$	**M1**	Substitute $x=6$ into $\frac{dy}{dx}$ SOI by gradient $\frac{3}{4}$ used.
	$y-4=\frac{3}{4}(x-6)$ OR evaluates $c=-\frac{1}{2}$	**A1**	OE e.g. $y=\frac{3}{4}x-\frac{1}{2}$ or evaluates c in $y=\frac{3}{4}x+c$ using (6, 4) and gradient $\frac{3}{4}$. ISW
		2	

Question	Answer	Marks	Guidance
2(b)	$[y=]\left(\dfrac{12\left(\frac{1}{2}x-1\right)^{-3}}{-3}\right)\div\dfrac{1}{2}\left[=-8\left(\dfrac{1}{2}x-1\right)^{-3}\right]$	**B2, 1, 0**	
	$4=\dfrac{12\times\left(\frac{1}{2}\times6-1\right)^{-3}}{\frac{1}{2}\times-3}+c\ \left[\Rightarrow 4=-8\times2^{-3}+c\right]\ \Rightarrow c=[5]$	**M1**	Must have $+c$. Substitute $y=4,\ x=6$ and solve for c in an integrated expression. May be unsimplified.
	$[y=]-8\left(\dfrac{1}{2}x-1\right)^{-3}+5$	**A1**	OE Must see ‘ $y=$ ’ or ‘ $\mathrm{f}(x)=$ ’ in the working.
		4	

Question	Answer	Marks	Guidance
3	$\dfrac{dy}{dx}=\dfrac{1}{2}ax^{-\frac{1}{2}}-2$	**B2, 1, 0**	
	$0=\dfrac{1}{2}a(9)^{-\frac{1}{2}}-2\ \Rightarrow \dfrac{a}{6}-2=0\ \Rightarrow a=[12]$	**M1**	Substitute $x = 9$ and $\dfrac{dy}{dx}=0$ into *their* derivative and solve a linear equation for *a*.
	$[a=]12$	**A1**	
	$\left[y=\mathit{their}\ a\times(9)^{\frac{1}{2}}-18=\right]18$	**A1 FT**	FT on *their a*.
		5	

Question	Answer	Marks	Guidance
.4	Coefficient of x^2 in $\left(1+\frac{2}{p}x\right)^5$ is $10\left(\frac{2}{p}\right)^2=\frac{10\times 2^2}{p^2}\left[=\frac{40}{p^2}\right]$	**B1**	Accept with x^2 present. Must evaluate ${}^5C_2$
	Coefficient of x^2 in $(1+px)^6$ is $15(p)^2\left[=15p^2\right]$	**B1**	Accept with x^2 present. Must evaluate ${}^6C_2$
	$\frac{40}{p^2}+15p^2=70$	***M1**	Forming an equation in p with *their* coefficients, the given 70, no *x* terms and no extra terms.
	$15p^4-70p^2+40\ [=0]$ or $3p^4-14p^2+8\ [=0]$	**DM1**	Forming a 3-term equation in p (or another variable) with all terms on one side and *their* coefficients.
	$[5]\left(p^2-4\right)\left(3p^2-2\right)[=0]$ or $\frac{70\pm\sqrt{70^2-4(15)(40)}}{30}$ or $\frac{14\pm\sqrt{14^2-4(3)(8)}}{6}$	**DM1**	Attempt to solve 3-term quartic (or quadratic in another variable) by factorisation, formula or completing the square.
	$p=\pm 2,\ \pm\sqrt{\frac{2}{3}}$	**A1**	OE e.g. $\pm\frac{\sqrt{6}}{3}$ or AWRT ± 0.816 If *M1 DM1 DM0, allow **SC B1** for 4 correct values.
		6	

Question	Answer	Marks	Guidance
5(a)	$\left[2r+8=20 \Rightarrow\right]\ r=6$	**B1**	
	Angle $AOB=\dfrac{8}{their\,6}$	***M1**	Expect $\dfrac{4}{3}$ OE $(76.4°)$. M0 Assume triangle is equilateral.
	$AB=2\times 6\sin their\dfrac{2}{3}$ **or** $\sqrt{6^2+6^2-2\times 6^2\cos their\dfrac{4}{3}}$ **or** $AB=\dfrac{6}{\sin\left(\dfrac{\pi}{2}-their\dfrac{2}{3}\right)}\times\sin their\dfrac{4}{3}$	**DM1**	For 6 read $their\,6$.
	Perimeter $=\left[7.42+8=\right]15.4$	**A1**	AWRT
		4	
5(b)	Area $=\dfrac{1}{2}\times 6^2\times their\dfrac{4}{3}-\dfrac{1}{2}\times 6^2\times\sin their\dfrac{4}{3}$ **or** Area $=\dfrac{1}{2}\times 6^2\times their\dfrac{4}{3}-2\times\dfrac{1}{2}\left(6\sin their\dfrac{2}{3}\right)\left(6\cos their\dfrac{2}{3}\right)$	**M1**	Sector area – whole triangle area. For 6 read $their\,6$. Sector area – 2(half triangle area).
	$=\left[24-17.49=\right]6.51$	**A1**	AWRT
		2	

Question	Answer	Marks	Guidance
6(a)	$\dfrac{\sin\theta-\cos\theta+\sin\theta+\cos\theta}{(\sin\theta+\cos\theta)(\sin\theta-\cos\theta)}\left[=\dfrac{\sin\theta-\cos\theta+\sin\theta+\cos\theta}{\sin^2\theta-\cos^2\theta}\right]=1$	***M1**	Use common denominator and equate to 1.
	$2\sin\theta\ [=\sin^2\theta-\cos^2\theta]=\sin^2\theta-\left(1-\sin^2\theta\right)$	**DM1**	Multiply by common denominator and replace $\cos^2\theta$ by $1-\sin^2\theta$.
	$2\sin^2\theta-2\sin\theta-1=0$	**A1**	OE In the given form.
		3	
6(b)	$[\sin\theta=]\dfrac{2\pm\sqrt{(-2)^2-4(2)(-1)}}{4}\left[=\dfrac{2\pm\sqrt{4+8}}{4}=\dfrac{1\pm\sqrt{3}}{2}\right]$	**M1**	Use formula or complete the square to solve a quadratic equation of the correct form.
	201.5° or 338.5°	**A1 A1 FT**	AWRT; A1 for either solution correct. A1 FT for 540 – (first value). If M0, allow **SC B1 B1FT** similarly.
		3	

Question	Answer	Marks	Guidance
7(a)	$r = 0.8$	**B1**	SOI
	$50 \times (their\ 0.8)^7 = 10.5$	**M1**	Evaluate 8th or 9th term in GP.
	$50 \times (their\ 0.8)^8 = 8.39$. Hence 9th impact required	**A1**	**AG** Two terms correct + conclusion (mention of 9th impact or u_9 somewhere in the solution). Statement that one is <10 (and the other >10) is insufficient unless it mentions 9th impact or u_9.
	Alternative method for final two marks: Logarithm method		
	$50 \times (their\ 0.8)^n < 10 \Rightarrow (their\ 0.8)^n < 0.5$ $n \log(their\ 0.8) < \log 0.5$ $n > \dfrac{\log 0.5}{\log(their\ 0.8)} \Rightarrow [n >]7.2$	**M1**	
	$n = 8$ hence 9th impact required	**A1**	**AG** Need conclusion that mentions 9th impact or u_9.
		3	
7(b)	$\dfrac{50\left(1-(their\ 0.8)^{20}\right)}{1-their\ 0.8}$	**M1**	OE Use of formula with *their r* SOI.
	$= 247$	**A1**	Must be to the nearest mm (not 247.1).
		2	
7(c)	$\dfrac{50}{1-their\ 0.8}$	**M1**	Use of sum to infinity formula with *their r* SOI. Substituting a value of *n* into the sum formula M0.
	$= 250$	**A1**	
		2	

Question	Answer	Marks	Guidance
8(a)	$f'(x) = -3(-1)(4)(4x-p)^{-2}\left[= \dfrac{12}{(4x-p)^2}\right]$	**B2, 1, 0**	
	> 0 Hence increasing function	**B1FT**	Correct conclusion from *their* $f'(x)$.
		3	
8(b)	$y = 2 - \dfrac{3}{4x-p} \Rightarrow (y-2)(4x-p) = -3$ **or** $4xy - py = 8x - 2p - 3$	**M1**	OE Form horizontal equation. Sign errors only, no missing terms. May go directly to $4y = p - \dfrac{3}{x-2}$ OE M1 M1
	$4xy - 8x = py - 2p - 3 \Rightarrow 4x(y-2) = p(y-2) - 3$ **or** $4x = -\dfrac{3}{x-2} + p$	**M1**	OE Factorise out $[4]x$ or $[4]y$.
	$x = \dfrac{p(y-2)-3}{4(y-2)}\left[\Rightarrow x = \dfrac{p}{4} - \dfrac{3}{4y-8}\right]$ **or** $\dfrac{-\dfrac{3}{x-2}+p}{4}$	**M1**	OE Make x (or y) the subject.
	$\left[f^{-1}(x) =\right]\dfrac{p}{4} - \dfrac{3}{4x-8}$	**A1**	OE in correct form (must be in terms of *x*).
		4	
8(c)	$[p =]8$	**B1**	
		1	

Question	Answer	Marks	Guidance
9(a)	$(x-2)^2+5$	**B1**	
		1	
9(b)	$2\left(\left\{(x+1)^2\right\}+\{5\}\right)$	**B2, 1, 0**	
		2	
9(c)	$\left[g(x)=\right]\ 2f(x+3)$ or $k=2,\ h=3$	**B1**	In correct form. B0 if contradiction.
		1	
9(d)	{Translation} $\left\{\begin{pmatrix}-3\\0\end{pmatrix}\right\}$	**B2, 1, 0 FT**	FT on *their* $x+3$ or $h=3$.
	{Stretch} {y direction, factor 2}	**B2, 1, 0 FT**	FT on *their* 2 or $k=2$.
		4	

Question	Answer	Marks	Guidance
10(a)	$\pm\int\left(2x^{1/2}+1\right)-\left(\frac{1}{2}x^2-x+1\right)\,\mathrm{d}x\ [=\ \pm\int 2x^{1/2}-\frac{1}{2}x^2+x\mathrm{d}x]$	*M1	
	$\pm\left(\frac{4x^{3/2}}{3}+x-\left(\frac{x^3}{6}-\frac{x^2}{2}+x\right)\right)$ or $\pm\left(\frac{4x^{3/2}}{3}-\frac{x^3}{6}+\frac{x^2}{2}\right)$	B2, 1, 0	OE Coefficients may be unsimplified.
	$\pm\left(\frac{32}{3}-\frac{32}{3}+8\ \right)$ or $\pm\left(\frac{44}{3}-0-\frac{20}{3}+0\right)$	DM1	± (F(4) – F(0)) using *their* integral(s).
	$=8$	A1	Depends on all previous marks. If *M1 B2 DM0 **and** limits stated, **SC B1** for +8
		5	
10(b)	Upper curve: $\frac{\mathrm{d}y}{\mathrm{d}x}=x^{-\frac{1}{2}}$. Lower curve: $\frac{\mathrm{d}y}{\mathrm{d}x}=x-1$	M1 A1	Attempt at differentiating one function. A1 if both correct.
	At x = 4: gradient of upper curve $=\frac{1}{2}$, gradient of lower curve $=3$	M1	Evaluate two gradients using $x=4$.
	$\alpha=\tan^{-1}3-\tan^{-1}\frac{1}{2}\ [=71.57-26.57]$	M1	Use inverse tan to find angles then subtract. **OR** find equations of both tangents then Pythagoras using a point on each e.g. on axes. **OR** cosine rule using intercepts or proportion.
	$[\alpha=]45°$	A1	AWRT
		5	

Question	Answer	Marks	Guidance
11(a)	$x^2+(mx+10)^2=20$ **or** $y^2+\left(\frac{y-10}{m}\right)^2=20$ **or** $mx+10=\sqrt{20-x^2}$	*M1	Substitute equation of line into equation of circle.
	$x^2(1+m^2)+20mx+80\quad[=0]$ **or** $y^2(m^2+1)-20y+(100-20m^2)[=0]$	A1	Collect terms into a 3 term quadratic.
	$(20m)^2-4(1+m^2)\times 80[=0\ \Rightarrow\ 80m^2-320=0\Rightarrow[80](m^2-4)=0]$ **or** $(-20)^2-4(m^2+1)(100-20m^2)[=0\Rightarrow\ [80](m^4-4m^2)=0]$	DM1	Use $b^2-4ac[=0]$.
	$m=\pm 2$	A1	Two values for m.
		4	

Question	Answer	Marks	Guidance
11(b)	**Method 1: Use of quadratic**		
	$\left(1+2^2\right)x^2 \pm 20\left(2\right)x+80 \left[=0 \Rightarrow 5x^2 \pm 40x+80=0\right]$ **or** $y^2\left(2^2+1\right)-20y+\left(100-20\left(2^2\right)\right)\left[=0 \Rightarrow \left[5\right]\left(y^2-4y+4\right)=0\right]$	**M1**	Sub *their* m into *their* quadratic in x or y **or** restart with *their* tangent equation and equation of circle.
	$\left[5\right]\left(x \pm 4\right)^2=0 \Rightarrow x=\pm 4$ **or** $y=2$	**A1**	Correct solutions or one correct pair (x, y).
	$\left(-4,2\right)$, $\left(4,2\right)$	**A1**	Two correct points with x and y paired correctly.
	Method 2: Using equation of normal		
	$2x+10=-\frac{1}{2}x$ **or** $-2x+10=\frac{1}{2}x$	**M1**	Equate tangent and normal and solve for x.
	$x=\pm 4$	**A1**	Two correct x-values or one correct pair (x, y).
	$\left(-4,2\right)$, $\left(4,2\right)$	**A1**	Two correct points with x and y paired correctly.
		3	

Question	Answer	Marks	Guidance
11(c)	**Method 1: Using angle at circumference**		
	$\cos BOA = \frac{\sqrt{20}}{10}$ or $\sin BOA = \frac{\sqrt{80}}{10}$ or $\tan BOA = \frac{\sqrt{80}}{\sqrt{20}}[=2]$	***M1**	Use a trig function in triangle *AOB*.
	$BOA = 63.4° \Rightarrow BOC = 126.8°$ or $126.9°$	**DM1**	Strategy involving doubling
	$[BDC =]63.4°$	**A1**	AWRT
	Metho 2: Using cosine rule		
	$BC = 8$, $BD = \sqrt{\left(\sqrt{20}+4\right)^2 + 2^2}$, $CD = \sqrt{\left(\sqrt{20}-4\right)^2 + 2^2}$	***M1**	Calculate two lengths in triangle *BCD*.
	$64 = 80 - 16\sqrt{5}\cos BDC$	**DM1**	Use cosine rule with *their* lengths
	$\cos BDC = \frac{\sqrt{5}}{5} \Rightarrow [BDC =]63.4°$	**A1**	AWRT
	Method 3: Subtract angles from 90°		
	Calculate one angle at D $[=13.28]$	***M1**	*ODB* or angle between *CD* and the vertical from *D*
	Calculate a second angle at D $[=13.28]$ and subtract both from $90°$	**DM1**	
	$[BDC =]63.4°$	**A1**	AWRT
		3	

Cambridge Assessment International Education

Cambridge International AS & A Level

MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **October/November 2022**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2022 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

This document consists of **18** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1(a)	Mid-point AB is $\left(\frac{10+5}{2}, \frac{2-1}{2}\right)\left[=\left(\frac{15}{2}, \frac{1}{2}\right)\right]$	**B1**	Accept unsimplified.
	Gradient of AB = $\frac{-1-2}{10-5} = \frac{-3}{5}$ Gradient perpendicular = $\frac{5}{3}$	**M1**	For use of $\frac{\text{Change in y}}{\text{Change in } x}$, condone inconsistent order of x and y, and $m_1m_2 = -1$.
	$\frac{y-\frac{1}{2}}{x-\frac{15}{2}} = \frac{5}{3}$ $\left[y - \frac{1}{2} = \frac{5}{3}\left(x - \frac{15}{2}\right)\right]$	**A1**	OE ISW Any correct version e.g. $y = \frac{5}{3}x - 12$ or $5x - 3y = 36$.
		3	
1(b)	[Radius =] $\sqrt{34}$ or 5.8 AWRT or [(radius)^2 =] 34	**B1**	Sight of $\sqrt{34}$ or 34. Condone confusion of r and r^2.
	$(x-5)^2 + (y-2)^2$	**B1**	Sight of $(x-5)^2 + (y-2)^2$
	$(x-5)^2 + (y-2)^2 = 34$	**B1**	CAO ISW
	Alternative method for Question 1(b)		
	$x^2 + y^2 - 10x - 4y$	**B1**	
	$[c =]5$ or $[c =] - 5$	**B1**	Substitution of (10, –1) into $x^2 + y^2 - 10x - 4y + c = 0$.
	$x^2 + y^2 - 10x - 4y - 5 = 0$	**B1**	
		3	

Question	Answer	Marks	Guidance
2	$2a - a = a^2 - 2a$	**B1**	OE An unsimplified correct equation in a or d only, e.g. $a^2 + a = 4a$. Can be implied by correct values for a or d.
	$a = 3$ or $d = 3$	**B1**	Condone 'extra' solution of $a = 0$ or $d = 0$.
	$a = 3$ and $d = 3$	**B1**	SOI
	$S_{50} = \frac{50}{2}\left(2 \times \textit{their } a + 49 \times \textit{their } d\right)$	**M1**	May be done using 50th term (=150). Their a and d must be numerical.
	3825	**A1**	ISW **SC B2** for $1275a$ or $1275d$
		5	

Question	Answer	Marks	Guidance
3(a)	$k^2 - 4 \times 8 \times 2$ [< 0]	**M1**	Use of $b^2 - 4ac$ but not just in the quadratic formula.
	$-8 < k < 8$ or $-8 < k$, $k < 8$ or $\lvert \boldsymbol{k} \rvert < 8$ or $(-8, 8)$	**A1**	Condone '$-8 < k$ or $k < 8$', '$-8 < k$ and $k < 8$' but not $\sqrt{64}$.
		2	

Question	Answer	Marks	Guidance
3(b)	$2(4\cos\theta-1)(\cos\theta-1)$ or $(4\cos\theta-1)(\cos\theta-1)$	**M1**	OE Or use of formula or completing the square. Allow use of replacement variable.
	$\cos\theta=\frac{2}{8}$, $\cos\theta=1$	**A1**	OE For both answers. **SC:** If M0, **SC B1** available for sight of $\cos\theta=\frac{2}{8}$ and 1
	[θ =] 0°, 75.5°	**A1**	AWRT ISW rejection of 0°. For both answers and no others in the range $0°\leqslant\theta\leqslant180°$, must be in degrees. **SC:** If M0 B1 scored, **SC B1** available for correct answers. **SC**: If M1 A0 scored, **SC B1** available for $\cos\theta=\frac{2}{8}$ and θ = 75.5° only, WWW.
		3	

Question	Answer	Marks	Guidance
4	$a r^2 = 1764$ and $a r + a r^2 = 3444$ or $a r = 1680$ or $\frac{a(1-r^3)}{1-r}-a=3444$	**B1**	Two correct algebraic statements.
	Attempt to solve as far as r = or a =	**M1**	Any valid method, e.g. $1764\div1680$ or from $20r^2-41r+21$ OE (condone solving using a calculator).
	$r=\frac{1764}{1680}=\frac{21}{20}$ or 1.05 [a = 1600]	**A1**	Note: $r=\frac{1764}{3444-1764}$ www implies B1 and M1.
	17 500	**A1**	AWRT e.g. 17 474.1…..
		4	

Question	Answer	Marks	Guidance
5(a)	Three points at the bottom of their transformed graph plotted at $y = 2$	B1	All 5 points of the graph must be connected.
	Bottom three points of /\/\ at $x = 0$, $x = 1$ & $x = 2$	B1	Must be this shape.
	All correct	B1	Condone extra cycles outside $0 \leqslant x \leqslant 2$.
		3	**SC:** If B0 B0 scored, B1 available for /\ in one of correct positions **or** all 5 points correctly plotted and not connected **or** correctly sized shape in the wrong position.
5(b)	[g(x) =] f(2x) + 1	B1 B1	Award marks for their final answer as follows: f(2x) B1, + 1 B1. Condone $y =$ or $f(x) =$.
		2	

Question	Answer	Marks	Guidance
6(a)	$\mathbf{y} = 4\left(x + \frac{5}{2}\right)^2 - 19$		There is no requirement for the candidate to list a, b and c. Look at values in their final expression, condone omission of ^{2}, and award marks as follows:
		B1	$a = 4$
		B1	$b = \frac{5}{2}$ OE
		B1	$c = -19$
		3	

Question	Answer	Marks	Guidance
6(b)	$\left(\textit{Their } 4\left(x+\frac{5}{2}\right)^2-19\right)=45\ \left[\Rightarrow\left(x+\frac{5}{2}\right)^2=16\right]$	***M1**	Equate their quadratic completed square form from **6(a)** to 45 or re-start and use completing the square.
	Solve as far as $x=$	**DM1**	Any valid method leading to two answers.
	$[x=]\frac{3}{2},-\frac{13}{2}$	**A1**	**SC:** If M0 or M1 DM0 awarded, B1 available for correct final answers.
		3	
6(c)	Quadratic **curve** that is the right way up **(**must be seen either side of stationary point**)**	**B1**	No axes required, ignore any axes even if incorrect.
	Stationary point stated using any valid method or correctly labelled on their diagram.	**B1 FT** **B1 FT**	FT *their* values from **6(a)** as long as *their* expression is of the form $p(qx+r)^2+s$. Expect $\left(-\frac{5}{2},-19\right)$. Condone if stated correctly but plotted incorrectly.
		3	

Question	Answer	Marks	Guidance
7(a)	$\dfrac{\sin\theta(\sin\theta-\cos\theta)+\cos\theta(\sin\theta+\cos\theta)}{(\sin\theta+\cos\theta)(\sin\theta-\cos\theta)}\left[=\dfrac{\sin^2\theta+\cos^2\theta}{\sin^2\theta-\cos^2\theta}\right]$	*M1	Sight of a correct common denominator, either in one or two fractions, condone missing brackets if recovered. In the numerator condone $\pm$ sign errors only.
	$\dfrac{\dfrac{\sin^2\theta}{\cos^2\theta}+\dfrac{\cos^2\theta}{\cos^2\theta}}{\dfrac{\sin^2\theta}{\cos^2\theta}-\dfrac{\cos^2\theta}{\cos^2\theta}}$	DM1	Divide throughout by $\cos^2\theta$.
	$\dfrac{\tan^2\theta+1}{\tan^2\theta-1}$ AG	A1	
	Alternative method for Question 7(a)		
	$\dfrac{\dfrac{\sin^2\theta}{\cos^2\theta}+1}{\dfrac{\sin^2\theta}{\cos^2\theta}-1}\times\dfrac{\cos^2\theta}{\cos^2\theta}$ or the equivalent step $\left[=\dfrac{\sin^2\theta+\cos^2\theta}{\sin^2\theta-\cos^2\theta}\right]$	*M1	Replace $\tan^2\theta$ with $\dfrac{\sin^2\theta}{\cos^2\theta}$ and multiply top and bottom by $\cos^2\theta$. Condone $\pm$ sign errors.
	Sight of convincing use of partial fractions	DM1	
	$\dfrac{\boldsymbol{\sin\theta}}{\boldsymbol{\sin\theta+\cos\theta}}+\dfrac{\boldsymbol{\cos\theta}}{\boldsymbol{\sin\theta-\cos\theta}}$ AG	A1	
		3	**Note:** M1 DM1 A1 for working on both sides at the same time and finishing at the same correct expression. M1 DM1 for starting separately and finishing at the same correct expression and A1 if there is a final conclusion e.g. QED. Do not allow cross multiplication. Condone use of s, c and t and omission of θ.

Question	Answer	Marks	Guidance
7(b)	$\frac{\tan^2\theta+1}{\tan^2\theta-1}=2 \Rightarrow \tan^2\theta+1=2\left(\tan^2\theta-1\right)$	***M1**	Equate expression from **(a)** to 2 and clear fraction.
	$\tan\theta=[\pm]\sqrt{3}$	**DM1**	Simplify as far as $\tan\theta=$. May be implied by a correct final answer in degrees or radians.
	Alternative method for first two marks of Question 7(b)		
	$\frac{\sin^2\theta+\cos^2\theta}{\sin^2\theta-\cos^2\theta}=2 \Rightarrow 1=2\sin^2\theta-2\left(1-\sin^2\theta\right)$	***M1**	Equate expression to 2, clear fraction and use trig identities to form an equation in $sin\theta$ or $cos\theta$ only.
	$sin\theta=[\pm]\sqrt{\frac{3}{4}}$ or $cos\theta=[\pm]\sqrt{\frac{1}{4}}$	**DM1**	Simplify as far as $sin\theta=$, or $cos\theta=$.
	$\theta=\frac{1}{3}\boldsymbol{\pi},\frac{2}{3}\boldsymbol{\pi}$	**A1** **A1 FT**	A1 for either correct answer then A1FT For their second value being $\pi-$ (their first) and no others in range $0\leqslant\theta\leqslant\pi$, both values must be exact and in radians. **SC:** B1 for $\theta=60°,120°$ or $0.333\pi,0.667\pi$ AWRT. or 1.05, 2.09 AWRT.
		4	

Question	Answer	Marks	Guidance
8(a)	$[y=]\left\{\frac{3x^{\frac{3}{2}}}{\frac{3}{2}}\right\}+\left\{-\frac{3x^{\frac{1}{2}}}{\frac{1}{2}}\right\}[+c]\left[=2x^{\frac{3}{2}}-6x^{\frac{1}{2}}\right]$	**B1 B1**	Marks can be awarded for correct unsimplified expressions, 1 mark each for contents of { } ISW.
	$5=2\times3^{\frac{3}{2}}-6\times3^{\frac{1}{2}}+c$	**M1**	Correct use of (3,5) in an integrated expression (defined by at least one correct power) including + c.
	$\boldsymbol{y}=2\boldsymbol{x}^{\frac{3}{2}}-6\boldsymbol{x}^{\frac{1}{2}}+5$	**A1**	Condone c = 5 as their final line if either y = or f(x) = seen elsewhere in the solution, but coefficients must not contain unresolved double fractions.
		4	
8(b)	$3x^{\frac{1}{2}}-3x^{-\frac{1}{2}}=0$	**M1**	Setting given differential to 0.
	[x=] 1	**A1**	CAO WWW Condone extra solution of —1 only if it is rejected.
		2	
8(c)	x>1 or x> "*their* 8(b)"	**B1FT**	Allow ⩾
		1	

Question	Answer	Marks	Guidance
9(a)	$\boldsymbol{a}\left(\boldsymbol{x}+\frac{1}{\boldsymbol{x}}\right)+1$	**B1**	ISW
		1	

Question	Answer	Marks	Guidance
9(b)	$a\left(2+\frac{1}{2}\right)+1=11$	**M1**	Substitute $x = 2$ into *their* expression from **(a)** and equate to 11. This may be done in 2 stages: f(2)= $2.5, g(2.5)=11$.
	[a =] 4	**A1**	
		2	
9(c)	No,[because it is] not one-one	**B1**	Or other suitable explanation that may include one to many or many to one.
		1	
9(d)	$[\mathrm{g}^{-1}(x)]=\frac{x-1}{5}$ WWW	**B1**	Condone use of a instead of 5.
	$[\mathrm{g}^{-1}\mathrm{f}(x)=]\frac{x+\frac{1}{x}-1}{5}$ OE	**M1**	Correct combination of their $\mathrm{g}^{-1}(x)$ with given $\mathrm{f}(x)$ Condone use of a instead of 5.
	$\frac{\boldsymbol{x}^2-\boldsymbol{x}+1}{5\boldsymbol{x}}$ or $\frac{1}{5}\left(\boldsymbol{x}+\frac{1}{\boldsymbol{x}}-1\right)$ or $\frac{1}{5}\left(\boldsymbol{x}+\boldsymbol{x}^{-1}-1\right)$ OE ISW	**A1**	Must not contain unresolved fractions e.g. $\frac{x+x^{-1}-1}{5}$.
		3	
9(e)	The domain of f does not include the whole of the range of g. **Or** The range of g does not lie in the domain of f.	**B1**	Accept an answer that includes an example outside the domain of f, e.g. g(—1) = — 4 but for f, $x>0$.
		1	

Question	Answer	Marks	Guidance
10(a)	$2.5\times\frac{4\pi}{3}+2.24\times\frac{5\pi}{6}$ [= 10.47[2] + 5.86[4] or $\frac{10\pi}{3}+\frac{28\pi}{15}$]	**B1**	For either arc correct. Arc ARB could be AR+RB.
		M1	For adding two (or three) arc lengths using different radii and angles and nothing else. SOI
	16.34 or $\frac{26\boldsymbol{\pi}}{5}$	**A1**	AWRT Condone 16.33 only.
		3	
10(b)	Area AOB = $\frac{1}{2}\times 2.5^2\sin\frac{2\pi}{3}$ [=2.706] Area APB = $\frac{1}{2}\times 2.24^2\sin\frac{5\pi}{6}$ [=1.254]	**M1**	For either ΔAOB or ΔAPB (AB = 4.33, h= 1.25, 0.58) or any other valid method.
	[Difference =] 1.45	**A1**	AWRT Condone 1.46 only.
		2	
10(c)	Area AOB = $\frac{1}{2}\times 2.5^2\times\frac{4\pi}{3}$ [=13.09] Area APB = $\frac{1}{2}\times 2.24^2\times\frac{5\pi}{6}$ [=6.57]	**B1**	For either sector area correct
	[Area of cross section =] $\frac{1}{2}\times 2.5^2\times\frac{4\pi}{3}+\frac{1}{2}\times 2.24^2\times\frac{5\pi}{6}+$ "$their\,10(b)$" $\left[=13.09+6.57+\text{"}their\,10(b)\text{"}\right]$	**M1**	Adding two **sector areas** from different sectors and '$their\,10$**(b)**' and nothing else. SOI
	21.1	**A1**	CAO Condone slight inaccuracies in intermediate working if the correct answer is arrived at.
		3	

Question	Answer	Marks	Guidance
11(a)	$\left[\frac{dy}{dx}=\right]\frac{9}{2}x-12$ [= 0] or [y =] $\frac{9}{4}\left\{\left(x-\frac{8}{3}\right)^2+\frac{8}{9}\right\}$ or $\frac{9}{4}\left(x-\frac{8}{3}\right)^2+2$	**B1**	OE Either $\frac{dy}{dx}$ or a correct expression in completed square form. Allow unsimplified.
	$\boldsymbol{x}=\frac{24}{9}$	**B1**	OE Condone 2.67 AWRT.
	$y = 2$	**B1**	CAO Note: $x=\frac{-b}{2a}=\frac{8}{3}$ B1; substitute $\frac{8}{3}$ for x in y = B1; y =2 B1.
		3	

Question	Answer	Marks	Guidance
For 11(b) look for working to be marked on page 19 or annotate it as BP or SEEN			
11(b)	[Area =] $\int\left(18-\frac{3}{8}x^{\frac{5}{2}}-\left(\frac{9}{4}x^2-12x+18\right)\right)dx$	**M1**	Intention to integrate **and** subtract areas (either way around). Can be two separate functions or combined. Using y^2 scores 0/5 but condone inclusion of π except for the final mark.
	Note: Subtraction not required for these marks. Either separately $\left([18x]-\frac{3x^{\frac{7}{2}}}{8\times\frac{7}{2}}\right)$, $\left(\frac{9x^3}{4\times3}-\frac{12x^2}{2}[+18x]\right)$ Or combined $[18x]-\frac{3x^{\frac{7}{2}}}{8\times\frac{7}{2}}-\frac{9x^3}{4\times3}+\frac{12x^2}{2}[-18x]$	**B1,B1**	One mark for correct integration of each curve, allow unsimplified. $\left([18x]-\frac{3}{28}x^{\frac{7}{2}}\right)$ $\left(\frac{3}{4}x^3-6x^2[+18x]\right)$ or $[18x]-\frac{3}{28}x^{\frac{7}{2}}-\frac{3}{4}x^3+6x^2[-18x]$ **BUT** condone sign errors that are only due to missing brackets.
	$=\left(-\frac{3}{28}\times4^{\frac{7}{2}}-\frac{3}{4}\times4^3+6\times4^2\right)$ $[-(0)]$	**M1**	Clear substitution of 4 into at least one integrated expression (defined by at least one correct power) which can be unsimplified.
	$=\frac{240}{7}$ or **34.3** AWRT	**A1**	**SC:** If all marks awarded except the final M1, SCB1 is available for the correct final answer.
		5	

Question	Answer	Marks	Guidance
11(c)	$\left[\frac{dy}{dx}=\right]\frac{-5\times3}{2\times8}x^{\frac{3}{2}}\left[=-\frac{15}{16}x^{\frac{3}{2}}\right]$	**B1**	Allow unsimplified.
	$\frac{dy}{dt}=\frac{dy}{dx}\times\frac{dx}{dt}\Rightarrow\frac{dy}{dt}=-\frac{15}{16}\times8\times2$	**M1**	Substitute $x=4$ into their $\frac{dy}{dx}$ and multiply by 2.
	–15	**A1**	Accept decreasing [at/by] 15
		3	**Note:** If incorrect curve used, this is not a MR and only M1 mark is available. Expect $(\frac{9(4)}{2}-12)\times2[=12]$

Cambridge International AS & A Level

MATHEMATICS	**9709/13**
Paper 1 Pure Mathematics 1	**October/November 2022**
MARK SCHEME	
Maximum Mark: 75	

Published

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Generic Marking Principles

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GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
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GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

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2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

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M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	$8\left(1-\cos^2\theta\right)+6\cos\theta+1 \quad [=0]$	M1	Expect $8\cos^2\theta-6\cos\theta-9=0$.
	$(4\cos\theta+3)(2\cos\theta-3) \quad [=0]$	A1	Factors or formula or completing square must be shown.
	$[\rightarrow \cos\theta=-0.75 \rightarrow \theta=]$ $138.6°$ only,	A1	AWRT, ignore solutions outside the given range, answer in radians A0.
		3	

Question	Answer	Marks	Guidance
2(a)	$\left[\mathrm{f}(x)\right]=\left\{-2(x+2)^2\right\}-\{5\}$	B1 B1	
		2	
2(b)	$\left[\mathrm{f}(x)\right]<-7$	B1	Allow $y<-7$, <-7, $(-\infty,-7)$ or less than -7 $-\infty\langle f(x)\langle -7, -7\rangle f(x)\rangle -\infty$, $f<-7$
		1	
2(c)	$y=-2(x+2)^2-5 \quad \rightarrow \quad (x+2)^2=\dfrac{-(y+5)}{2}$	M1	Operations correct. Allow sign errors. FT *their* quadratic from **(a)**.
	$x=[\pm]\sqrt{\dfrac{-(y+5)}{2}} \ -2$	M1	Operations correct. Allow sign errors. FT *their* quadratic from **(a)**.
	$[\mathrm{f}^{-1}(x)]=-2-\sqrt{\dfrac{-(x+5)}{2}}$ or $-2-\sqrt{-\dfrac{(x+5)}{2}}$	A1	Allow $[\mathrm{f}^{-1}(x)]=-2-\sqrt{\dfrac{x+5}{-2}}$.
		3	

Question	Answer	Marks	Guidance
3(a)	$1+10x+40x^2$ May be part of a complete expansion	**B2, 1, 0**	1^5 must be simplified to 1, allow if the '1' is seen in a more complete expansion but not the final answer. Mis-reads not condoned in this question.
		2	
3(b)	$1-12x+54x^2$ May be part of a complete expansion	**B2, 1, 0**	1^4 must be simplified to 1, allow if the '1' is seen in a more complete expansion but not the final answer. Mis-reads not condoned in this question.
		2	
3(c)	$54-120+40$	**M1**	Forming exactly 3 products correctly using their terms.
	-26	**A1**	Allow $-26x^2$ If in a list with other terms it must be clear this is the required term otherwise A0.
		2	

Question	Answer	Marks	Guidance
4	$\left[\frac{dv}{dx}\right] = (9-x)^2$	**B1**	Allow unsimplified forms. Allow any or no notation
	Substitute $x = 4$ into *their* differentiated V,	***M1**	Expect 25.
	$\frac{dx}{dt} = \frac{1}{\textit{their}\text{ derivative}} \times 3.6$ (accept $\frac{dt}{dx} = \frac{\textit{their}\text{ derivative}}{3.6}$)	**M1**	Correct use of the chain rule, ignore incorrect conversions at this point. Expect 0.144
	$= \frac{1}{\textit{their numerical}\text{ derivative}} \times 3.6 \times \frac{100}{60}$	**DM1**	Correct use of the conversion factors.
	$= \frac{1}{25} \times 3.6 \times \frac{100}{60} = 0.24$	**A1**	
		5	

Question	Answer	Marks	Guidance
5(a)	3	**B1**	Ignore any description.
		1	
5(b)	2	**B1**	Ignore any description.
		1	
5(c)	(8, 2)	**B1 B1**	Ignore any description. Allow vector notation and absence of brackets.
		2	

Question	Answer	Marks	Guidance
5(d)	(1, 5)	**B1 FT**	FT each coordinate, (*their*8 – 7, *their*2 + 3) Allow vector notation and absence of brackets.
		B1 FT	
		2	

Question	Answer	Marks	Guidance
6	Use of $\sin^2\alpha + \cos^2\alpha = 1$ eg $\sin\alpha = [\pm]\sqrt{1-\left(\frac{8}{17}\right)^2}$	***M1**	Or Pythagoras seen (may quote 8, 15, 17 triple).
	$\sin\alpha = \frac{15}{17}$	**A1**	
	$\tan\alpha = \frac{15}{8}$	**A1**	
	$\frac{1}{\sin\alpha} + \frac{1}{\tan\alpha} = \frac{17}{15} + \frac{8}{15}$	**DM1**	Dealing with reciprocals and addition of fractions correctly.
	$= \frac{5}{3}$ oe	**A1**	Correct answer with no working shown scores 0. Extra answers from $\sin\alpha = -\frac{15}{17}$ are allowed.
		5	

Question	Answer	Marks	Guidance
7(a)	$\frac{-3}{(a+2)^4}=-\frac{16}{27} \rightarrow$ e.g. $16(a+2)^4=81$	M1	Equate first derivative and $-\frac{16}{27}$ and move term in a (or x) into the numerator.
	$\rightarrow(a+2)^2=\frac{9}{4}\rightarrow a+2=[\pm]\frac{3}{2}$	M1	Solve for $(a+2)$ or $(x+2)$
	$a=-\frac{1}{2}$ or $-\frac{7}{2}$	A1 A1	Allow 'x ='
		4	
7(b)	$[\mathrm{f}(x)]=\frac{1}{(x+2)^3}\ [+c]$	B1	Allow unsimplified form and 'y ='
	5 = 1 + c	M1	Sub $x=-1,\ y=5$ into an integral.
	$[\mathrm{f}(x)]=\frac{1}{(x+2)^3}+4$	A1	Allow 'y ='
		3	

Question	Answer	Marks	Guidance
8(a)	$APQ = \cos^{-1}\dfrac{\frac{5}{6}r}{r}$ $\left[= \cos^{-1}\dfrac{5}{6}\right]$	*M1	May use cosine rule to find APB. Stating APQ or APB as an incorrect multiple of π is M0.
	$= 0.5857$	A1	Accept 0.586 or 33.6° or APB (1.171 or 67.1°).
	Perimeter $= 4 \times r \times their\,0.5857 = 2.34r$ or $0.745\pi\, r$ or $(293/125)r$	DM1 A1	Must use a numerical value of *their* angle.
		4	
8(b)	Use of sector formula: Sector APB $= ½r^2 \times (2 \times their\,0.5857)$ or Sector APC (C is on PQ so PC = r) $= ½r^2 \times (their\,0.5857)$	M1	Any sector with *their* appropriate angle. It must be clear the appropriate numerical angle is being used.
	Use of appropriate formula for area of triangle and correct combination with the sector to find the area of a half segment, one segment or both segments	M1	e.g. Area APB = $½r^2 \times \sin(2 \times their\,0.5857)$.
	Shaded area [$= 2 \times 0.1250r^2$] $= 0.250r^2$	A1	or $0.0796\pi\, r^2$, allow $\dfrac{1}{4}r^2$ or $0.25\,r^2$.
		3	

Question	Answer	Marks	Guidance
9(a)	$216r^3 = 64 \rightarrow r = 2/3$	B1	Allow decimal to 3sf (AWRT).
	$S_\infty = \dfrac{216}{1 - their\,⅔} = 648$ cao	M1 A1	M1 depends on *their* |r| < 1.
		3	

Question	Answer	Marks	Guidance
9(b)	$216\left(\frac{2}{3}\right)=144 \rightarrow 144=a+d$	**B1 FT**	SOI, may be implied in the use of $96=144+3d$ and finding a. Mis-reads not condoned in **9(b)**.
	$216\left(\frac{2}{3}\right)^2=96 \rightarrow 96=a+4d$	**B1 FT**	SOI, may be implied in the use of $96=144+3d$ and finding a.
	Solve simultaneously	***M1**	No working may be seen.
	$d=-16, \quad a=160$	**A1**	Both required.
	$S_{21}=\frac{21}{2}\{320+20(-16)\}=0$	**DM1 A1**	Or use of $\frac{21}{2}(a+u_{21})$.
		6	

Question	Answer	Marks	Guidance
10(a)	$x^2+(2x-1)^2-2[=0] \rightarrow 5x^2-4x-1\ [=0]$	***M1 A1**	Or $5y^2+2y-7\ [=0]$.
	$(5x+1)(x-1)[=0]$ or $(5y+7)(y-1)[=0]$	**DM1**	May see factors or formula or completing square.
	$x=1,\ y=1$ or (1, 1) only	**A1**	May be implied on the diagram.
		4	

Question	Answer	Marks	Guidance
10(b)	$(\pi)\int\left(2-x^2\right)\mathrm{d}x = (\pi)\left(2x-\frac{x^3}{3}\right)$	*M1 A1	Attempt integration of y^2, allow $\int\left(2-y^2\right)\mathrm{d}y$.
	$(\pi)\left(\left(2\sqrt{2}-\frac{(\sqrt{2})^3}{3}\right)-\left(2-\frac{1}{3}\right)\right)$	DM1	Apply limits $1 \to \sqrt{2}$.
	$\frac{\pi}{3}\left(4\sqrt{2}-5\right)$	A1	CAO, allow $\frac{\pi}{3}\left(2\sqrt{8}-5\right)$, must be in given form.
		4	
10(c)	Arc length = $\frac{1}{8}(2\pi\sqrt{2})$ or $\frac{\pi\sqrt{2}}{4}$ oe	B1	Must be exact.
	Perimeter = $\sqrt{2}+$ *their* arc length	B1 FT	Must be exact, do not allow inverse trig functions.
		2	

Question	Answer	Marks	Guidance
11(a)	$(5-2p)^2+(p+2)^2=(10-2p)^2+(3-p)^2$	M1 A1	Allow one sign error for M mark only.
	$25-20p+4p^2+p^2+4p+4=100-40p+4p^2+9-6p+p^2$ $30p=80 \;\rightarrow\; p=\frac{8}{3}$ oe	A1	Allow 2.67 AWRT.
		3	

Question	Answer	Marks	Guidance
11(b)(i)	$m_{AC} = \frac{p+2}{2p-5}$ $m_{BC} = \frac{p-3}{2p-10}$	**M1**	Allow a sign error.
	$\frac{p+2}{2p-5} \times \frac{p-3}{2p-10} = -1$	**M1**	Use of $m_1m_2 = -1$ with their m_{AC} and m_{BC}.
	$p^2 - p - 6 = -\left(4p^2 - 30p + 50\right) \rightarrow 5p^2 - 31p + 44 \ (=0)$	**A1**	
	$p = 4$ (Ignore $p = \frac{11}{5}$)	**A1**	Factors $(p-4)(5p-11)$, or formula or completing square must be seen.
		4	
11(b)(ii)	Mid-point of *AB* = (7½, ½)	**B1**	SOI
	$r^2 = 2½^2 + 2½^2 \ \left[= \frac{50}{4}\right]$ or $r = \sqrt{(2½^2 + 2½^2)} \ \left[= \frac{5\sqrt{2}}{2}\right]$	***M1**	Or $r^2 = \frac{1}{4}\left(5^2 + 5^2\right) \ \left[= \frac{50}{4}\right]$ etc.
	Equation of circle is $\left(x - their\,7½\right)^2 + \left(y - their\,½\right)^2 = their\,\frac{50}{4}$	**DM1**	Must use r^2 not r or d or d^2
	$x^2 + y^2 - 15x - y + 44 = 0$	**A1**	CAO
		4	

Cambridge International AS & A Level

MATHEMATICS	**9709/11**
Paper 1 Pure Mathematics 1	**May/June 2022**
MARK SCHEME	
Maximum Mark: 75	

Published

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Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
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SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1(a)	$x^2 - 8x + 11 = (x-4)^2$ … **or** $p = -4$	**B1**	If p and q-values given after *their* completed square expression, mark the expression and ISW.
	… -5 **or** $q = -5$	**B1**	
		2	
1(b)	$(x-4)^2 - 5 = 1$ so $(x-4)^2 = 6$ so $x-4=[\pm]\sqrt{6}$	**M1**	Using *their* p and q values or by quadratic formula
	$x = 4 \pm \sqrt{6}$ **or** $\frac{8 \pm \sqrt{24}}{2}$	**A1**	Or **exact** equivalent. No FT; must have ± for this mark. ISW decimals 1.55, 6.45 if exact answers seen. If M0, **SC B1** possible for correct answers.
		2	

Question	Answer	Marks	Guidance
2	$a + 12d = 12$	**B1**	For correct equation.
	$\frac{30}{2}\left(2a+(30-1)d\right) = -15$	**B1**	For correct equation in a and d. If using $\frac{n}{2}(a+l)$, must replace l with an expression involving a and d.
	$a = 72$, $d = -5$	**B1**	Both values correct SOI.
	$S_{50} = \frac{50}{2}\left(2(\textit{their } a) + 49(\textit{their } d)\right)$	**M1**	Using sum formula with *their* a and d values obtained via a valid method.
	$S_{50} = -2525$	**A1**	
		5	

Question	Answer	Marks	Guidance
3(a)	x^4 term is $[10\times](2x^2)^3\left(\frac{k^2}{x}\right)^2$	**M1**	For selecting the term in x^4.
	$80k^4x^4 \Rightarrow a = 80k^4$	**A1**	For correct value of *a*. Allow $80k^4x^4$.
	[x^2 term is $[6\times](2kx)^2 \times 1 = 24k^2x^2 \Rightarrow$] $b = 24k^2$	**B1**	For correct value of *b*. Allow $24k^2x^2$.
		3	
3(b)	$80k^4 + 24k^2 - 216[=0]$ $\quad [\Rightarrow 10k^4 + 3k^2 - 27 = 0]$	**M1**	Forming a 3-term equation in *k* (all terms on one side) with *their a* and *b* and no *x*'s.
	$(2k^2-3)(5k^2+9)\ [=0]$ [$\Rightarrow k^2 = \frac{3}{2}$ or $-\frac{9}{5}$]	**M1**	Attempt to solve 3-term quartic (or quadratic in another variable) by factorisation, formula or completing the square – see guidance.
	$[k] = \pm\sqrt{\frac{3}{2}}$	**A1**	OE e.g. $\pm\frac{\sqrt{6}}{2}$, $\pm\sqrt{1.5}$, AWRT ± 1.22 Omission of ± A0. Additional answers A0. If M1 M0, **SC B1** can be awarded for correct final answer, max 2/3.
		3	

Question	Answer	Marks	Guidance
4(a)	$\frac{\sin^3\theta}{\sin\theta-1}-\frac{\sin^2\theta}{1+\sin\theta}=\frac{\sin^3\theta(1+\sin\theta)}{(\sin\theta-1)(1+\sin\theta)}-\frac{\sin^2\theta(\sin\theta-1)}{(\sin\theta-1)(1+\sin\theta)}$ $\left[=\frac{\sin^3\theta(1+\sin\theta)-\sin^2\theta(\sin\theta-1)}{(\sin\theta-1)(1+\sin\theta)}\right]$	*M1	Using a common denominator.
	$-\frac{\sin^2\theta+\sin^4\theta}{1-\sin^2\theta}$	DM1	Reaching $\pm\left(1-\sin^2\theta\right)$ in denominator. SOI by $\pm\cos^2\theta$.
	$-\frac{\sin^2\theta\left(1+\sin^2\theta\right)}{\cos^2\theta}$	DM1	Using $\sin^2\theta+\cos^2\theta=1$ in denominator and isolating $\sin^2\theta$ in numerator.
	$-\tan^2\theta\left(1+\sin^2\theta\right)$	A1	AG **-** Using/stating $\tan\theta=\frac{\sin\theta}{\cos\theta}$ is sufficient for A1. May be working from both sides provided the argument is complete. A0 if θ or brackets missing throughout, or sign errors. Allow recovery if AG follows from *their* working.
	Alternative method for Q4(a)		
	$-\tan^2\theta(1+\sin^2\theta)=-\frac{\sin^2\theta\left(1+\sin^2\theta\right)}{1-\sin^2\theta}$	*M1	Using $\tan\theta=\frac{\sin\theta}{\cos\theta}$ and $\sin^2\theta+\cos^2\theta=1$.
	$\frac{-\sin^2\theta-\sin^4\theta}{(1-\sin\theta)(1+\sin\theta)}$	DM1	Factorising denominator.
	$\frac{\sin^2\theta+\sin^3\theta-\sin^3\theta+\sin^4\theta}{(\sin\theta-1)(1+\sin\theta)}=\frac{\sin^3\theta(1+\sin\theta)-\sin^2\theta(\sin\theta-1)}{(\sin\theta-1)(1+\sin\theta)}$	DM1	Factorising numerator.

Question	Answer	Marks	Guidance
4(a)	$\dfrac{\sin^3\theta}{\sin\theta-1}-\dfrac{\sin^2\theta}{1+\sin\theta}$	**A1**	AG A0 if θ or brackets missing throughout, or sign errors. Allow recovery if AG follows from *their* working.
		4	
4(b)	$-\tan^2\theta\left(1+\sin^2\theta\right)=\tan^2\theta\left(1-\sin^2\theta\right)$ leading to $[2]\tan^2\theta=0$	**M1**	Obtaining a (trig function)$^2 = 0$ WWW.
	$\tan\theta=0$ leading to $[\theta=]\pi$	**A1**	Ignore extra solutions outside the interval $(0, 2\pi)$.
	Alternative method for Q4(b)		
	$-\dfrac{\sin^2\theta}{\cos^2\theta}(1+\sin^2\theta)=\dfrac{\sin^2\theta}{\cos^2\theta}(1-\sin^2\theta)$ leading to $-\sin^2\theta-\sin^4\theta=\sin^2\theta-\sin^4\theta$ leading to $[2]\sin^2\theta=0$	**M1**	Obtaining a (trig function)$^2 = 0$ WWW.
	$\sin\theta=0$ leading to $[\theta=]\pi$	**A1**	Ignore extra solutions outside the interval $(0, 2\pi)$.
		2	

Question	Answer	Marks	Guidance
5(a)	Sector area = $\frac{1}{2}r^2\left(\frac{\pi}{6}\right)\left[=\frac{\pi}{12}r^2\right]$	**B1**	Using $\frac{1}{2}r^2\theta$ with θ in radians SOI. B0 if using a value for r.
	$BD = \sin\frac{\pi}{6}r\left[=\frac{1}{2}r\right]$ and $AD = \cos\frac{\pi}{6}\mathrm{r}\left[=\frac{\sqrt{3}}{2}r\right]$ so triangle area = $\frac{1}{2}\left(\sin\frac{\pi}{6}r\right)\left(\cos\frac{\pi}{6}\mathrm{r}\right)\left[=\frac{1}{2}\times\frac{1}{2}r\times\frac{\sqrt{3}}{2}r\right]$ **or** $\frac{1}{2}r\left(\cos\frac{\pi}{6}r\right)\left(\sin\frac{\pi}{6}\right)\left[=\frac{1}{2}r\times\frac{\sqrt{3}}{2}r\times\frac{1}{2}\right]$	**B1**	SOI Finding triangle area. Decimals B0 unless exact values seen in working.
	Area of $BCD = \frac{1}{12}\pi r^2 - \frac{\sqrt{3}}{8}r^2$	**B1**	OE e.g. $\frac{r^2}{4}\left(\frac{\pi}{3}-\frac{\sqrt{3}}{2}\right)$ with $\cos\frac{\pi}{6}$ and $\sin\frac{\pi}{6}$ evaluated. Must be exact, in terms of r^2. ISW
		3	

Question	Answer	Marks	Guidance
5(b)	Angle $BAC = \sin^{-1}\left(\dfrac{\frac{\sqrt{3}}{2}r}{r}\right)\left[=\dfrac{\pi}{3}\right]$	**B1**	SOI by length of *AD*, *CD* or arc, or by perimeter.
	Length $AD = \cos\dfrac{\pi}{3}r\left[=\dfrac{1}{2}r\right]$ [so length $CD = \dfrac{1}{2}r$]	**M1**	SOI Finding length by Pythagoras, or by trigonometry with *their* angle *BAC*, provided $BAC \neq \dfrac{\pi}{6}$.
	Length of arc $BC = r \times \dfrac{\pi}{3}$	**M1**	SOI Using $r\theta$ with θ in radians. Condone $\theta = \dfrac{\pi}{6}$.
	Perimeter of $BCD = \dfrac{\sqrt{3}}{2}r + \dfrac{1}{2}r + \dfrac{\pi}{3}r$	**A1**	OE e.g. $r\left(\dfrac{\sqrt{3}+1}{2} + \dfrac{\pi}{3}\right)$ with e.g. $\cos\dfrac{\pi}{3}$ evaluated. Must be exact, in terms of *r*. ISW
		4	

Question	Answer	Marks	Guidance
6(a)	$y=\dfrac{x^2-4}{x^2+4}$ leading to $\left(x^2+4\right)y=\left(x^2-4\right)$ leading to $x^2y+4y=x^2-4$	*M1	For clearing denominator and expanding brackets. If swap variables first, look for $y^2x+4x=y^2-4$.
	$x^2y-x^2=-4y-4$ leading to $x^2(1-y)=4y+4$ leading to $x^2=\ldots$	DM1	For making x^2 the subject. If swap variables first, look for $y^2(1-x)=4x+4\Rightarrow y^2=\ldots$
	$x^2=\dfrac{4y+4}{1-y}$ leading to $x=\sqrt{\dfrac{4y+4}{1-y}}$ leading to $[\mathrm{f}^{-1}(x)]=\sqrt{\dfrac{4x+4}{1-x}}$	A1	OE e.g. $\sqrt{\dfrac{-4x-4}{x-1}}$ without ± in final answer.
	Alternative method for Q6(a)		
	$x=\dfrac{y^2-4}{y^2+4}$ leading to $x=1-\dfrac{8}{y^2+4}$ leading to $x-1=\dfrac{-8}{y^2+4}$	*M1	For division and reaching $x-1=\ldots$ (or $y-1=\ldots$)
	$y^2+4=\dfrac{-8}{x-1}$ leading to $y^2=\dfrac{-8}{x-1}-4$	DM1	For making y^2 (or x^2) the subject.
	$[y=]\left[\mathrm{f}^{-1}(x)\right]=\sqrt{\dfrac{-8}{x-1}-4}$	A1	OE without ± in final answer.
		3	

Question	Answer	Marks	Guidance
6(b)	$1-\dfrac{8}{x^2+4}=\dfrac{x^2+4}{x^2+4}-\dfrac{8}{x^2+4}\left[=\dfrac{x^2+4-8}{x^2+4}\right]=\dfrac{x^2-4}{x^2+4}$	**M1 A1**	Using common denominator or division to reach 1. Remainder -8. WWW
	$0<\mathrm{f}(x)<1$	**B1 B1**	B1 for each correct inequality. B0 if contradictory statement seen. Accept $\mathrm{f}(x)>0,\ \mathrm{f}(x)<1;\ 1>\mathrm{f}(x)>0;\ (0,1)$ **SC B1** for $0\leqslant \mathrm{f}(x)\leqslant 1$.
		4	
6(c)	Because the range of f does not include the whole of the domain of f (or any of it)	**B1**	Accept an answer that includes an example outside the domain of f, e.g. $\mathrm{f}(4)=\dfrac{12}{20}$. Must refer to the domain or > 2. Need not explicitly use the term 'domain' but must not refer just to the range.
		1	

Question	Answer	Marks	Guidance
7(a)	$(3x-2)^{\frac{1}{2}}=\dfrac{1}{2}x+1\Rightarrow 3x-2=\left(\dfrac{1}{2}x+1\right)^2=\dfrac{1}{4}x^2+x+1$	**M1**	Equating curve and line, attempt to square; $\dfrac{1}{4}x^2+1$ M0
	$\Rightarrow\dfrac{1}{4}x^2-2x+3[=0]\left[\Rightarrow x^2-8x+12=0\right]\Rightarrow(x-6)(x-2)[=0]$	**M1**	Forming and solving a 3TQ by factorisation, formula or completing the square – see guidance.
	(2, 2) and (6, 4)	**A1 A1**	A1 for each point, or A1 A0 for two correct *x*-values. If M0 for solving, **SC B2** possible: B1 for each point or B1 B0 for two correct *x*-values.
		4	

Question	Answer	Marks	Guidance
7(b)	Area $= \pm\int_{[2]}^{[6]}\left((3x-2)^{\frac{1}{2}}-\left(\frac{1}{2}x+1\right)\right)[\mathrm{d}x]$	***M1**	For intention to integrate and subtract (M0 if squared).
	$\pm\left[\frac{2}{9}(3x-2)^{\frac{3}{2}}-\left(\frac{1}{4}x^2+x\right)\right]_2^6$	**B1 B1**	B1 for each bracket integrated correctly (in any form).
	$\pm\left(\left[\frac{2}{9}(16)^{\frac{3}{2}}-\left(\frac{1}{4}\times 36+6\right)\right]-\left[\frac{2}{9}(4)^{\frac{3}{2}}-\left(\frac{1}{4}\times 4+2\right)\right]\right)$	**DM1**	$\pm$(F(*their* 6) – F(*their* 2)) with *their* integral. Allow 1 sign error.
	$\frac{4}{9}$	**A1**	AWRT 0.444. **SC1 B1** for $\frac{4}{9}$ if *M1 B1 B1 DM0. **SC2 B1** for $\frac{4}{9}$ if *M1 B0 B0 DM0, provided limits stated.
	Alternative method for question 7(b)		
	Area $= \pm\int_{[2]}^{[6]}(3x-2)^{\frac{1}{2}}\ [\mathrm{d}x]$ – area of trapezium (or triangle + rectangle)	***M1**	For intention to integrate and subtract (M0 if squared).
	$\pm\left[\frac{2}{9}(3x-2)^{\frac{3}{2}}\right]_2^6-4\left(\frac{2+4}{2}\right)$ **or** $\pm\left[\frac{2}{9}(3x-2)^{\frac{3}{2}}\right]_2^6-\left(\frac{2+4}{2}+(2\times 4)\right)$	**B1** **B1 FT**	B1 for bracket integrated correctly (in any form). B1 FT for using correct formula with *their* values.
	$\pm\left(\left(\frac{2}{9}(16)^{\frac{3}{2}}-\frac{2}{9}(4)^{\frac{3}{2}}\right)-12\right)$	**DM1**	$\pm$(F(*their* 6) – F(*their* 2)) using *their* integral. Allow 1 sign error.

Question	Answer	Marks	Guidance
7(b)	$\frac{4}{9}$	**A1**	AWRT 0.444. **SC1 B1** for $\frac{4}{9}$ if *M1 B1 B1 DM0. **SC2 B1** for $\frac{4}{9}$ if *M1 B0 B0 DM0, provided limits stated.
		5	

Question	Answer	Marks	Guidance
8(a)	**EITHER** **(1)**{Translation} $\begin{pmatrix}\{30°\}\\\{0\}\end{pmatrix}$ **OR** **(2)**{Translation} $\begin{pmatrix}\{60°\}\\\{0\}\end{pmatrix}$	**B2,1,0**	B2 for fully correct, B1 with two elements correct. { } indicates different elements. Accept angle in radians.
	(3){Stretch} {factor 2} {in x-direction}	**B2,1,0**	B2 for fully correct, B1 with two elements correct. { } indicates different elements.
	(4) Stretch factor 4 in y-direction **and** correct order	**B1**	Stretch, y-direction and factor **and** correct order. Correct order is **either** (1) then (3) **or** (3) then (2). (4) can be anywhere in the sequence.
		5	
8(b)	$4\sin\left(\frac{1}{2}x-30°\right)=2\sqrt{2}\Rightarrow \sin^{-1}\left(\frac{\sqrt{2}}{2}\right)[=45]$	**M1**	SOI
	$\frac{1}{2}x-30=45$ or 135 $\Rightarrow x=2(45+30)$ or $x=2(135+30)$	**M1**	SOI. The M marks are independent.
	$x = 150°$, $x = 330°$	**A1**	Both exact values, condone $\frac{5\pi}{6},\frac{11\pi}{6}$. A0 if extra solutions in the interval. Ignore other solutions outside $[0°, 360°]$.
		3	

Question	Answer	Marks	Guidance
9(a)	Express as $(x+3)^2+(y-1)^2=26+9+1[=36]$	M1	Completing the square on x and y or using the form $x^2+y^2+2gx+2fy+c=0$, centre $(-g,-f)$ and radius $\sqrt{g^2+f^2-c}$. SOI by correct answer.
	Centre (−3, 1)	B1	
	Radius 6	B1	
	So lowest point is (−3, −5)	A1 FT	FT on *their* centre and *their* radius.
		4	
9(b)	Intersects when $x^2+(kx-5)^2+6x-2(kx-5)-26=0$ **or** $(x+3)^2+(kx-5-1)^2=36$	*M1	Substituting $y=kx-5$ into *their* circle equation or rearranging and equating y.
	$x^2+k^2x^2-10kx+25+6x-2kx+10-26=0$ **or** $x^2+6x+9+k^2x^2-12kx+36=36$ leading to $k^2x^2+x^2+6x-12kx+9[=0]$ **or** $(k^2+1)x^2+(6-12k)x+9[=0]$	DM1 A1	Rearranging to 3-term quadratic (terms grouped, all on one side). Allow 1 error. Correct quadratic (need to see 9 as constant term).
	$(6-12k)^2-4(k^2+1)\times 9\ [>0]$ $\left[\text{leading to } 144k^2-144k+36-36k^2-36>0\right]$	DM1	Using discriminant $b^2-4ac[>0]$ with *their* values. Allow if in square root.
	$[108k^2-144k=0$ leading to$]$ $k=0$ or $k=\frac{4}{3}$	A1	Need not see method for solving.
	$k<0, k>\frac{4}{3}$	A1	Do not accept $\frac{4}{3}<k<0$.
		6	

Question	Answer	Marks	Guidance
10(a)	$\frac{d^2y}{dx^2} = 6(-1)^2 - \frac{4}{(-1)^3} > 0 \therefore$ minimum **or** $\frac{d^2y}{dx^2} = 10 \;\therefore$ minimum	**B1**	Sub $x = -1$ into $\frac{d^2y}{dx^2}$, correct conclusion. WWW
		1	
10(b)	$\frac{dy}{dx} = 2x^3 + \frac{2}{x^2}[+c]$	***M1**	Integrating $\frac{d^2y}{dx^2}$ (at least one term correct).
	$0 = -2 + 2 + c$ leading to $c = [0]$	**DM1**	Substituting $x = -1, \frac{dy}{dx} = 0$ (need to see) to evaluate c. DM0 if simply state $c = 0$ or omit $+c$.
	$y = \frac{1}{2}x^4 - \frac{2}{x} + (\textit{their}\ c)x + k$	**A1 FT**	Integrated. FT *their* non-zero value of c if DM1 awarded.
	$\frac{9}{2} = \frac{1}{2} + 2 + k$ leading to $k = [2]$	**DM1**	Substituting $x = -1, y = \frac{9}{2}$ to evaluate k (dep on *M1).
	$y = \frac{1}{2}x^4 - \frac{2}{x} + 2$	**A1**	OE e.g. $2x^{-1}$ or $\frac{4}{2}$. A0 (wrong process) if c not evaluated but correct answer obtained.
		5	
10(c)	$\frac{dy}{dx} = 2x^3 + \frac{2}{x^2} = 0$	**M1**	*Their* $\frac{dy}{dx} = 0$.
	Leading to $x^5 = -1$	**M1**	Reaching equation of the form $x^5 = a$.
	So only stationary point is when $x = -1$	**A1**	$x = -1$ and stating e.g. 'only' or 'no other solutions.
		3	

Question	Answer	Marks	Guidance
10(d)	At $x = 1$, $\frac{dy}{dx} = [4]$	*M1	Substituting $x = 1$ into *their* $\frac{dy}{dx}$.
	$\frac{dx}{dt} = \frac{dx}{dy} \times \frac{dy}{dt} = \frac{1}{4} \times 5$	DM1	OE Using chain rule correctly SOI.
	$\frac{5}{4}$	A1	OE e.g. 1.25.
		3	

Cambridge Assessment International Education

Cambridge International AS & A Level

MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **May/June 2022**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2022 series for most Cambridge IGCSE, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

This document consists of **21** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

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AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	Coefficient of $x^4 = 15$	**B1**	Condone inclusion of x^4. Can be seen as part of an expansion.
	Coefficient of $x^2 = 240a^2$	**B1**	Condone inclusion of x^2. Can be seen as part of an expansion.
	'*Their* 240'a^2 – '*their* 15'	**M1**	Forming an equation of the form $pa^2 = q$, where p and q are constants. Condone inclusion of powers of x as long as they then disappear.
	$a = \frac{1}{4}$ or 0.25	**A1**	OE Do not condone extra 'answer' of $-\frac{1}{4}$,or allow $\sqrt{\frac{1}{16}}$ or similar.
		4	

Question	Answer	Marks	Guidance
2	$r = 0.8$	**B1**	OE
	$a = 12.5$	**B1**	OE
	$S_\infty = 12.5 \div (1 - 0.8)$	**M1**	Using $\frac{a}{1-r}$ with '*their a*' and '*their r*' but $\lvert r \rvert$ must be < 1.
	$S_\infty = \frac{125}{2}, 62\frac{1}{2}$ or 62.5	**A1**	$\frac{12\frac{1}{2}}{\frac{1}{5}}$ or similar **does not** get A1.
		4	

Question	Answer	Marks	Guidance
3	$[y=]\left\{\dfrac{3(4x-7)^{\frac{3}{2}}}{\frac{3}{2}\times 4}\right\}+\left\{-\dfrac{4}{\frac{1}{2}}x^{\frac{1}{2}}\right\}\left[\Rightarrow \dfrac{1}{2}(4x-7)^{\frac{3}{2}}-8x^{\frac{1}{2}}\right][+c]$	B1 B1	Marks can be awarded for correct unsimplified expressions ISW.
	$\dfrac{5}{2}=\dfrac{1}{2}(9)^{\frac{3}{2}}-8\times 4^{\frac{1}{2}}+c \quad [\Rightarrow c=5]$	M1	Using $(4, \frac{5}{2})$ in an integrated expression (defined by at least one correct power) including + c.
	$y=\dfrac{3}{6}(4x-7)^{\frac{3}{2}}-8x^{\frac{1}{2}}+5$.	A1	Condone $c = 5$ as their final line if either $y =$ or $f(x) =$ seen elsewhere in the solution. Coefficients must not contain unresolved double fractions.
		4	

Question	Answer	Marks	Guidance
4(a)	$2\times 6k=k+k+6$ or $6k-k=k+6-6k$ or $2d=6$ leading to $d=3$, $\therefore 6k-3=k$	B1	OE A correct equation in k only. Can be implied by correct final answer.
	$k=\dfrac{6}{10}$ or 0.6	B1	OE
		2	

Question	Answer	Marks	Guidance
4(b)	$d = 3$	**B1**	Correct value of d can be implied by a correct final answer. Working may be seen in part **(a)** but must be used in **(b)**.
	$S_{30} = \frac{30}{2}\left(2 \times \text{‘}their\ k\text{’} + 29 \times \text{‘}their\ d\text{’}\right)$	**M1**	It needs to be clear that the candidate is using a correct sum formula. There is no requirement to check the candidates working for d but it must be clearly identified.
	$S_{30} = 1323$	**A1**	ISW if corrected to 1320.
		3	

Question	Answer	Marks	Guidance
5(a)	$4\times 0^2 - 0 + \frac{1}{2}k^2 = 0 - a$	M1	Equating the equations of curve and line and substituting $x = 0$. Condone slight errors e.g. ± sign errors.
	$4\times\left(\frac{3}{4}\right)^2 - \frac{3}{4}k + \frac{1}{2}k^2 = \frac{3}{4} - a$	M1	Equating the equations of curve and line and substituting $x = \frac{3}{4}$. Condone slight errors e.g. ± sign errors.
	$k = 2,\ a = -2$	A1 A1	WWW
	Alternative method for question 5(a)		
	$(x-0)\left(x-\frac{3}{4}\right)=0$ or $x(4x-3)=0$ $[\Longrightarrow 4x^2-3x=0]$	*M1	Use 0, $\frac{3}{4}$ to form a quadratic equation. **Do not allow** $(x+0)\left(x+\frac{3}{4}\right)=0$.
	$4x^2 - kx + \frac{1}{2}k^2 = x - a$ leading to $4x^2-(k+1)x+\frac{1}{2}k^2+a[=0]$	DM1	Equating the equations of curve and line and rearranging so that terms are all on same side. Condone slight errors e.g. ± sign errors.
	$k = 2,\ a = -2$	A1 A1	WWW
	Alternative method for question 5(a)		
	$-\frac{b}{a}=\frac{3}{4}+0$ and $\frac{c}{a}=0\times\frac{3}{4}$	*M1	Using sum and product of roots. Condone ± sign errors.
	$\frac{k+1}{4}=\frac{3}{4}$ and $\frac{\frac{1}{2}k^2+a}{4}=0$	DM1	Equating the equations of curve and line and equating to $\frac{3}{4}$ and 0.
	$k = 2,\ a = -2$	A1 A1	WWW
		4	

Question	Answer	Marks	Guidance
5(b)	$4x^2 - kx + \frac{1}{2}k^2 = x + \frac{7}{2} \Rightarrow 4x^2 - kx - x + \frac{1}{2}k^2 - \frac{7}{2}[=0]$	*M1	OE Substitute $a = -\frac{7}{2}$ and rearrange so that terms are all on same side, condone ± sign errors. Watch for multiples.
	$(k+1)^2 - 4 \times 4\left(\frac{1}{2}k^2 - \frac{7}{2}\right)$	*DM1	Use of $b^2 - 4ac$ with the coefficients from *their* 3-term quadratic. Both coefficients 'b' and 'c' must consist of two components.
	$\Rightarrow 7k^2 - 2k - 57$	A1	OE
	$(k-3)(7k+19)$ or other valid method	DM1	Factorising or use of the formula or completing the square. Must be evidence of an attempt to solve for this mark. Dependent upon both previous method marks.
	$k = 3,\ k = -\frac{19}{7}$	A1	OE e.g. AWRT -2.71. No ISW if inequalities used. **SC:** If second DM1 not scored, **SC B1** available for correct final answers.
	Alternative method for question 5(b)		
	$8x - k = 1$ **and** $4x^2 - kx + \frac{1}{2}k^2 = x + \frac{7}{2}$	*M1	Equating gradients and equating line and curve.
	$4x^2 - (8x-1)x + \frac{1}{2}(8x-1)^2 = x + \frac{7}{2}$ **or** $4(\frac{k+1}{8})^2 - k\left(\frac{k+1}{8}\right) + \frac{1}{2}k^2 = \frac{k+1}{8} + \frac{7}{2}$	*DM1	Forming an equation in x or k only.
	$28x^2 - 8x - 3$ or $7k^2 - 2k - 57$	A1	OE A correct 3 term quadratic in x or k only.
	$(14x+3)(2x-1)$ or $(k-3)(7k+19)$ or other valid method	DM1	OE Factorising or use of the formula or completing the square. Must be evidence of an attempt to solve for this mark. Dependent upon both previous method marks.

Question	Answer	Marks	Guidance
5(b)	$k=3,\ k=-\frac{19}{7}$	A1	OE e.g. AWRT -2.71. No ISW if inequalities used. **SC:** If second DM1 not scored, **SC B1** available for correct final answers.
		5	

Question	Answer	Marks	Guidance
6	Line meets curve when: $2x+2=5x^{\frac{1}{2}}$ leading to $2x - 5x^{\frac{1}{2}}+2[=0]$ **or** $4x^2+8x+4=25x$ leading to $4x^2-17x+4[=0]$ **or** $x=\frac{y^2}{25}$ leading to $2y^2-25y+50[=0]$	M1	Equating line and curve and rearranging so that terms are all on same side, condone sign errors, and making a valid attempt to solve by factorising, using the formula or completing the square. Factors are:$(2x^{\frac{1}{2}}-1)(x^{\frac{1}{2}}-2)$, $(4x-1)(x-4)$ and $(2y-5)(y-10)$.
	$x=\frac{1}{4},x=4$	A1	**SC:** If M1 not scored, **SC B1** available for correct answers, could just be seen as limits.
	Area = $\int 5x^{\frac{1}{2}}-(2x+2)\mathrm{d}x = \int 5x^{\frac{1}{2}}-2x-2\ \mathrm{d}x$	*M1	Intention to integrate and subtract areas. Condone missing brackets and/or subtraction wrong way around.
	$=\left[\frac{10}{3}x^{\frac{3}{2}}-x^2-2x\right]_{\frac{1}{4}}^{4} = \left(\left(\frac{10}{3}\times 8-16-8\right)-\left(\frac{10}{3}\times\frac{1}{8}-\frac{1}{16}-\frac{1}{2}\right)\right)$	DM1	Integrating($kx^{\frac{3}{2}}$ seen) and substituting '*their* points of intersection' (but limits need to be found, not assumed to be 0 and something else).
	$\frac{45}{16}$ or $2\frac{13}{16}$ or 2.8125	A1	OE exact answer. Condone $-\frac{45}{16}$ if corrected to $\frac{45}{16}$. A0 for inclusion of π. **SC:** If *M1 DM0 scored, **SC B1** available for correct answer.

Question	Answer	Marks	Guidance
6	**Alternative method for question 6**		
	Line meets curve when: $2x+2=5x^{\frac{1}{2}} \Rightarrow 2x - 5x^{\frac{1}{2}}+2[=0]$ **or** $4x^2+8x+4=25x \Rightarrow 4x^2-17x+4[=0]$ **or** $x=\frac{y^2}{25} \Rightarrow 2y^2-25y+50[=0]$	**M1**	Equating line and curve and rearranging so that terms are all on same side, condone sign errors, and making a valid attempt to solve by factorising, using the formula or completing the square. Factors are:$(2x^{\frac{1}{2}}-1)(x^{\frac{1}{2}}-2)$, $(4x-1)(x-4)$ and $(2y-5)(y-10)$.
	$x=\frac{1}{4}, x=4$	**A1**	**SC:** If M1 not scored, **SC B1** available for correct answers, could just be seen as limits.
	Area = $\int 5x^{\frac{1}{2}}\text{d}x - \{\int(2x+2)dx$ or area of trapezium$\}$	***M1**	Intention to integrate and subtract areas. Or integrate curve and subtract area of trapezium.
	$\left[\frac{10}{3}x^{\frac{3}{2}}\right]_{\frac{1}{4}}^{4} - \left\{\left[x^2+2x\right]_{\frac{1}{4}}^{4} \text{ or } \frac{1}{2}(\text{sum of 'their } y \text{ values'})\text{'their } \frac{15}{4}\text{'}\right\}$ $=\left(\left(\frac{10}{3}\times 8\right)-\left(\frac{10}{3}\times\frac{1}{8}\right)\right)-\left\{\left((16+8)-\left(\frac{1}{16}+\frac{1}{2}\right)\right) \text{or} \frac{1}{2}\left(\frac{5}{2}+10\right)\frac{15}{4}\right\}$	**DM1**	Integrating ($kx^{\frac{3}{2}}$ seen) and substituting '*their* points of intersection' (but limits need to be found, not assumed to be 0 and something) or a trapezium using the correct formula ('*their* $\frac{15}{4}$' must be '*their* 4' – '*their* $\frac{1}{4}$' **but not 0**).
	$\frac{45}{16}$ or $2\frac{13}{16}$ or 2.8125	**A1**	OE exact answer. Condone $-\frac{45}{16}$ if corrected to $\frac{45}{16}$. A0 for inclusion of π. **SC:** If *M1 DM0 scored, **SC B1** available for correct answer.
		5	

Question	Answer	Marks	Guidance
7(a)	$\left[A\hat{O}B=\right]\ \frac{2}{10}$	B1	OE Sight of 0.2 from $s = r\theta$ but $10\theta = 2$ is not enough. ISW if $\frac{2}{10}=\frac{\pi}{5}$.
	$[B\hat{O}C =]\frac{5\pi+6}{30}$ or $\frac{1}{6}\pi+0.2$	B1	OE e.g. 0.724^c AWRT or 41.5 degrees AWRT. But not $\frac{2+\frac{5\pi}{3}}{10}$ – fraction within a fraction. ISW incorrect simplifications.
	Alternative method for question 7(a)		
	OR [Arc AC =] $\frac{10\pi}{6}$ or $[\text{Arc } BC =]\frac{10\pi}{6}+2$ or 7.2	B1	AWRT. Sight of $\frac{10\pi}{6}$ or 5.2 or 7.2.
	$[B\hat{O}C =]\frac{5\pi+6}{30}$ or $\frac{1}{6}\pi+0.2$	B1	OE e.g. 0.724^c AWRT or 41.5 degrees AWRT. But not $\frac{2+\frac{5\pi}{3}}{10}$ – fraction within a fraction. ISW incorrect simplifications.
		2	

Question	Answer	Marks	Guidance
7(b)	$[BP] = 10\sin\left(\frac{5\pi+6}{30}\right)$ and $[OP] = 10\cos\left(\frac{5\pi+6}{30}\right)$ [= 6.6208…] and [= 7.494…] **OR** $[BP] = 10\sin\left(\frac{5\pi+6}{30}\right)$ and $[O\hat{B}P] = \left(\frac{5\pi-3}{15}\right)$ [= 6.6208…] and [= 0.84719…]	**M1**	OE Any correct method for **both** lengths, for *their* angle BOC (which may have been incorrectly 'simplified' but not 0.2) or length BP and $O\hat{B}P$. May be seen as part of $\frac{1}{2}ab\sin C$. Sight of correct method enough. Can be implied by the next A1.
	Area of $\Delta OBP = \frac{1}{2}\times\ 10\sin\left(\frac{5\pi+6}{30}\right)\ \times 10\cos\left(\frac{5\pi+6}{30}\right)$ or $\frac{1}{2}\times 10\times 10\sin\left(\frac{5\pi+6}{30}\right)\times\sin\left(\left(\frac{5\pi-3}{15}\right)\right)$ [=24.809]	**A1**	OE Can be implied by any answer in range (24.7, 24.9) or a final answer in the range (11.3, 11.5) WWW.
	$[\text{Sector } BOC] = \frac{1}{2}\times 10^2\times \textit{their}\ \left(\frac{5\pi+6}{30}\right)$ $\left[=50\left(\frac{5\pi+6}{30}\right)=36.1799...\right]$	**M1**	Use of $\frac{1}{2}r^2\theta$ with *their* angle BOC (may have been incorrectly 'simplified' but not 0.2).
	Area of region $BPC = 11.4$	**A1**	CAO
		4	

Question	Answer	Marks	Guidance
8(a)	$1+1+a+b-12=0\left[\Rightarrow a+b=10\right]$ $4+36+2a-6b-12=0\left[\Rightarrow 2a-6b=-28\right]$	**B1 B1**	B1 for each equation. Allow unsimplified. Can be implied by correct values for a and b.
	$a=4, b=6$	**B1**	
	Centre is $\left(-\frac{their\,a}{2}, -\frac{their\,b}{2}\right)$ $\left[-2,-3\right]$	**B1 FT**	Or $x=-2, y=-3$
		4	

Question	Answer	Marks	Guidance
8(b)	Gradient of AC is $\dfrac{1-their\ y}{1-their\ x}$ $\left[=\ \dfrac{1--3}{1--2}=\ \dfrac{1+3}{1+2}=\dfrac{4}{3}\ \right]$	*M1	Using *their* centre correctly.
	Gradient of tangent is $=\dfrac{-1}{their\dfrac{4}{3}}\left[=-\dfrac{3}{4}\right]$	A1 FT	Use of $m_1m_2=-1$ to obtain the gradient of the tangent.
	Equation: $y-1=$'$their-\dfrac{3}{4}$'$(x-1)$ or $y=-\dfrac{3}{4}x+\dfrac{7}{4}$	DM1	Using $(1,1)$ with *their* gradient of the tangent at A.
	$3x+4y=7$ or $4y+3x=7$. or integer multiples of these	A1	
	Alternative method for question 8(b)		
	$2x+2y\dfrac{dy}{dx}+4+6\dfrac{dy}{dx}=0$	*M1	Implicit differentiation with at least one y term differentiated correctly.
	$8\dfrac{dy}{dx}=-6\Rightarrow\dfrac{dy}{dx}=-\dfrac{6}{8}$	A1	
	Equation: $y-1=$'$their-\dfrac{3}{4}$'$(x-1)$ or $y=-\dfrac{3}{4}x+\dfrac{7}{4}$	DM1	Using $(1,1)$ with *their* gradient of the tangent at A.
	$3x+4y=7$ or $4y+3x=7$. or integer multiples of these	A1	
	Alternative method for question 8(b)		
	$\dfrac{dy}{dx}=\dfrac{1}{2}\{25-(x+2)^2\}^{-\frac{1}{2}}(-2x-4)$	*M1	Rearranging to form $y=$ and differentiating using the chain rule.
	$\dfrac{dy}{dx}=\dfrac{1}{2}(25-9)^{-\frac{1}{2}}(-6)=-\dfrac{6}{8}$	A1	

Question	Answer	Marks	Guidance
8(b)	Equation: $y-1=$ '*their* $-\frac{3}{4}$'$(x-1)$ or $y=-\frac{3}{4}x+\frac{7}{4}$	DM1	Using $(1,1)$ with *their* gradient of the tangent at *A*.
	$3x+4y=7$ or $4y+3x=7$. or integer multiples of these	A1	
		4	

Question	Answer	Marks	Guidance
9(a)	$\frac{dy}{dx}=\{3\}+\left\{-4\times\frac{1}{2}(3x+1)^{-\frac{1}{2}}\times 3\right\}\left[=3-6(3x+1)^{-\frac{1}{2}}\right]$	B1 B1	Correct differentiation of $3x+1$ and no other terms and correct differentiation of $-4(3x+1)^{\frac{1}{2}}$. Accept unsimplified.
	$\left[\frac{d^2y}{dx^2}=\right]-\frac{1}{2}\times -6(3x+1)^{-\frac{3}{2}}\times 3$ $[=9(3x+1)^{-\frac{3}{2}}]$	B1	WWW. Accept unsimplified. Do not award if $\frac{dy}{dx}$ is incorrect.
		3	
9(b)	$\frac{dy}{dx}=0$ leading to $3-6(3x+1)^{-\frac{1}{2}}=0$	M1	Setting *their* $\frac{dy}{dx}=0$.
	$(3x+1)^{\frac{1}{2}}=2 \Rightarrow 3x+1=4$ leading to $x=1$	A1	CAO – do not ISW for a second answer.
	$y=-4$ [coordinates $(1, -4)$]	A1	Condone inclusion of second value from a second answer.
	$\frac{d^2y}{dx^2}=9(3\times 1+1)^{-\frac{3}{2}}=\frac{9}{8}$ or >0 so minimum	A1	Some evidence of substitution needed but $\frac{d^2y}{dx^2}$. Do not award if $\frac{d^2y}{dx^2}$ is incorrect or wrongly evaluated. Accept correct consideration of gradients either side of $x=1$.
		4	

Question	Answer	Marks	Guidance
10(a)	$x \neq 1$ or $x < 1,\ x > 1$ or $(-\infty, 1), (1, \infty)$ $[x \in \mathbb{R}]$	**B1**	Must be x not $\mathrm{f}^{-1}(x)$ or y. Do not accept $1 < x < 1$.
		1	
10(b)	$y = \dfrac{2x+1}{2x-1}$ leading to $(2x-1)y = 2x+1$ leading to $2xy - y = 2x+1$	***M1**	Setting $y =$, removing fraction and expanding brackets.
	$2xy - 2x = y+1$ leading to $2x(y-1) = y+1$ leading to $x = \dfrac{y+1}{2(y-1)}$	**DM1**	Reorganising to get $x =$. Condone ± sign errors only.
	$[\mathrm{f}^{-1}(x)] = \dfrac{x+1}{2(x-1)}, \dfrac{x+1}{x-1} \times \dfrac{1}{2}$ or $\dfrac{1}{x-1} + \dfrac{1}{2}$	**A1**	OE. Must be in terms of x. Do not allow $\dfrac{x+1}{x-1} \div 2$.
		3	
10(c)	$\left(\textit{their } f^{-1}(3)\right)$ leading to $\left(\textit{their } f^{-1}(3)\right)^2 + 4$ $\left[f^{-1}(3) = 1, 1+4 =\right]$	**M1**	Correct order of operations and substitution of $x = 3$ needed.
	5	**A1**	
		2	
10(d)	Sight of 'not one to one' or 'many to one' or 'one to many'	**B1**	Any reason mentioning 2 values, or + and — , such as: square root gives 2 values or horizontal line test crosses curve twice or 2 values because of turning point or 2 values because it is a quadratic.
		1	

Question	Answer	Marks	Guidance
10(e)	$f(x)=1+\dfrac{2}{2x-1}=\dfrac{2x-1}{2x-1}+\dfrac{2}{2x-1}=\dfrac{2x+1}{2x-1}$	**B1**	AG Do not condone equating expressions and verification.
	$f'(x)=-4(2x-1)^{-2}$ or $2(2x-1)^{-1}+\left\{-(2x+1)2(2x-1)^{-2}\right\}$ or $\dfrac{(2\boldsymbol{x}-1)2-2(2\boldsymbol{x}+1)}{(2\boldsymbol{x}-1)^2}$	***M1**	For $k(2x-1)^{-2}$ and no other terms or correct use of the product or quotient rule then ISW.
	Gradient $m=-4$	**A1**	Differentiation must have clearly taken place.
	Equation of tangent is $y-3=-4(x-1)$ $[\Rightarrow y=-4x+7]$	**DM1**	Using (1, 3) in the equation of a line with *their* gradient.
	Crosses axes at $\left(\dfrac{7}{4},0\right)$ and $(0,7)$	**A1 FT**	SOI from *their* straight line or by integration from 0 to '*their* 7/4'.
	[Area =] $\dfrac{49}{8}$	**A1**	OE e.g. 6.13 AWRT. If M0 A0 DM0, **SC B2** available for correct answer.
		6	

Question	Answer	Marks	Guidance
11(a)	$4\cos^4 x + \cos^2 x - 3 = 0 \Rightarrow (4\cos^2 x - 3)(\cos^2 x + 1) = 0$	**M1**	Attempt to solve 3 term quartic (or quadratic in another variable).
	$\Rightarrow [\cos^2 x =]\frac{3}{4}\ [\cos^2 x = -1]$	**A1**	If M0 scored then **SC B1** is available for sight of $\frac{3}{4}$ [and -1].
	$\Rightarrow \cos x = [\pm]\sqrt{their\frac{3}{4}}$ **OE** $\left[= \pm\frac{\sqrt{3}}{2}\right]$	**M1**	Square rooting '*their* $\cos^2 x$'. Allow without ±. May be implied by correct final answer(s). Ignore $\sqrt{-1}$.
	$[x =]\ \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$	**A1** **A1 FT**	Dependent on preceding M1 only. Exact answers needed. A1 for any 2 correct answers A1 A1 for 4 correct answers and no others inside the range $0 \leqslant x \leqslant 2\pi$ A0 A1 FT can be awarded for two exact answers that are $2\pi -$ '*their* $\frac{\pi}{6}$ and $\frac{5\pi}{6}$' within the range $0 \leqslant x \leqslant 2\pi$.
			SC: If all 4 answers given in degrees (30, 150, 210, 330) or non-exact (AWRT 0.524, 2.62, 3.67, 5.76 or 0.167π, 0.833π, 1.17π, 1.83) and no others then **SC B1**.
		5	

Question	Answer	Marks	Guidance
11(b)	$\cos^2 x = \dfrac{-1-\sqrt{1+16k}}{8} < 0$ [∴ no solutions].	**B1**	State that this root is less than 0, needs to be linked to $\cos^2 x$. Can be achieved by substituting a value for $k \geqslant 0$.
	$[\cos^2 x] = \dfrac{-1 \pm \sqrt{1+16k}}{8}$	***M1**	Must use quadratic formula. Allow any value of k **but not** ±3. Condone + rather than ±.
	Substituting $k = 5$ and obtain 1 from the formula	**DM1**	Or argue logically if $k > 5 \Rightarrow 1+16k > 81 \Rightarrow >1$.
	$\cos^2 x = 1$ or $\cos^2 x >$ or $\geqslant 1$	**A1**	Needs to be linked to $\cos^2 x$.
	Concluding statement having considered both ± cases. ∴ no solutions	**A1**	Dependent upon all previous marks having been scored.
	Alternative method for question 11(b)		
	$\cos^2 x = \dfrac{-1-\sqrt{1+16k}}{8} < 0$ [∴ no solutions].	**B1**	State that this root is less than 0, needs to be linked to $\cos^2 x$. Can be achieved by substituting a value for $k \geqslant 0$.
	$[\cos^2 x] = \dfrac{-1 \pm \sqrt{1+16k}}{8}$	***M1**	Must use quadratic formula. Allow any value of k **but not** ±3. Condone + rather than ±.
	$\dfrac{-1+\sqrt{1+16k}}{8} * 1 \Rightarrow -1+\sqrt{1+16k} * 8 \Rightarrow 1+16k * 81$	**DM1**	$*$ represents any inequality or =.
	$k * 5$	**A1**	$*$ represents any inequality or =.
	Concluding statement having considered both ± cases. ∴ no solutions	**A1**	Dependent upon all previous marks having been scored.
		5	

Cambridge International AS & A Level

MATHEMATICS **9709/13**

Paper 1 Pure Mathematics 1 **May/June 2022**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2022 series for most Cambridge IGCSE, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	$4C1\times p\times\frac{1}{p^3}x^3$	**B1**	OE soi Can be seen in an expansion.
	$\frac{4}{p^2}=144$	**B1**	OE Correct with correct power of p and only one p term.
	$p=\pm\frac{1}{6}$	**B1 B1**	OE $\pm\frac{2}{12}$ etc. Allow ±0.167 for B1 B1. **SC B1** for $\pm\sqrt{\frac{1}{36}}$ B1 only,
		4	

Question	Answer	Marks	Guidance
2(a)	[p =] 3	**B1**	
		1	
2(b)	[q =] $\frac{1}{2}$	**B1**	
		1	
2(c)	[r =] –2	**B1**	
		1	

Question	Answer	Marks	Guidance
3(a)	$\frac{n}{2}\left[8+(n-1)d\right]=5863$ leading to $n\left[8+(n-1)d\right]=11726$ leading to $(n-1)d=\frac{11726}{n}-8$	**B1**	Must show a useful intermediate step. WWW AG.
		1	
3(b)	$4+(n-1)d=139$ leading to $\frac{11726}{n}-8=135$	***M1**	OE Use of correct u_n formula with expression from **(a)** or S_n formula to eliminate *d*.
	$n=\frac{11726}{143}=82$	**A1**	
	$81d=\frac{11726}{82}-8$	**DM1**	Substitute *their n* into a correct u_n or S_n formula
	$d=\frac{5}{3}$	**A1**	Accept $\frac{138}{81}$ OE fraction only If M0 DM0 scored them **SC B1 B1** for correct *n* and *d* values only.
		4	

Question	Answer	Marks	Guidance
4(a)	$\left\{(x+1)^2+2(x+1)-5\right\}+\{3\}$, or $\left\{(x+1+1)^2\right\}+\{-6+3\}$	**M1 M1**	M1 for dealing with $\begin{pmatrix}-1\\0\end{pmatrix}$ and M1 for dealing with $\begin{pmatrix}0\\3\end{pmatrix}$.
	$[y=]x^2+4x+1$	**A1**	Answer only given full marks.
		3	

Question	Answer	Marks	Guidance
4(b)	{Stretch} {x direction or horizontally or y-axis invariant} { factor ½}	B2, 1, 0	Additional transformation B0.
		2	

Question	Answer	Marks	Guidance
5(a)	$6y+2-7y^{1/2}$ [= 0]	*M1	OE Rearrange to a 3-term quadratic.
	$\left(2y^{\frac{1}{2}}-1\right)\left(3y^{\frac{1}{2}}-2\right)$ [= 0] or e.g. $(2u-1)(3u-2)$ [= 0]	DM1	Or use of formula or completing the square.
	$[y^{1/2}=]\frac{1}{2},\frac{2}{3}$	A1	Answers only **SC B1** if DM1 not scored.
	$[y=]\frac{1}{4},\frac{4}{9}$	A1	Answers only **SC B1** if DM1 not scored.
		4	
5(b)	Use of $\tan x$ = *their* y values	M1	Must have at least 2 values of y from part **(a).**
	$x=$ 14[.0], 24[.0], $x=$ 194[.0], 204[.0]	A1 A1 FT	FT for 180 + angle (twice). AWRT
		3	

Question	Answer	Marks	Guidance
6(a)	$\left\{2(x-4)^2\right\}$ $\{-9\}$	**B1 B1**	OE When a and b stated give priority to marking algebraic expression.
		2	
6(b)	$y > -7$	**B1**	Allow $f(x) > -7$ or $(-7, \infty)$ Don't allow $x > -7$.
		1	
6(c)	$(x-4)^2 = \dfrac{y+9}{2}$	**M1**	2 operations correct. Allow a sign error.
	$x = 4\ [\pm]\sqrt{\dfrac{y+9}{2}}$	**M1**	2 operations correct. Allow a sign error.
	$[f^{-1}(x) =]4 - \sqrt{\dfrac{x+9}{2}}$	**A1 FT**	OE FT on *their* answer to **(a)** i.e. $-a - \sqrt{\left(\dfrac{x-b}{2}\right)}$.
		3	
6(d)	$fg(x) = f(2x+4) = 2(2x+4-4)^2 - 9$	**M1**	Allow $2(2x+4)^2 - 16(2x+4) + 23$.
	$8x^2 - 9$ only	**A1**	
		2	

Question	Answer	Marks	Guidance
7(a)	Equation of BC is $\{y=\}\{2\}\{-3x\}$	**B2, 1, 0**	OE forms $y+4=-3(x-2)$ or $y-2=-3(x-0)$.
		2	
7(b)	$(x-2)^2+(2-3x+4)^2=20$	***M1**	OE Sub line equation into equation of circle to eliminate y.
	$10(x-2)^2=20$ or $[10](x^2-4x+2)[=0]$	**A1**	OE Accept $(10x^2-40x+20)$.
	$x-2=[\pm]\sqrt{2}$ or $x=\frac{4[\pm]\sqrt{16-8}}{2}$	**DM1**	Correctly solving *their* quadratic.
	$x=2-\sqrt{2}$	**A1**	OE only solution. Answer only **SC B1** If DM1 not scored.
	$y=3\sqrt{2}-4$	**A1**	OE only solution. Answer only **SC B1** If DM1 not scored.
		5	

Question	Answer	Marks	Guidance
8(a)	$\left[\frac{dy}{dx}=\right]½x^{-1/2}-2x^{-3/2}$	**B1 B1**	Allow unsimplified versions.
	At $x=1$, $\frac{dy}{dx}=\frac{1}{2}-2=-\frac{3}{2}$	**M1**	Substitute $x=1$ into a differentiated y.
	Equation of tangent is $y-5=-\frac{3}{2}(x-1)$	**A1**	WWW Or $y=-\frac{3}{2}x+\frac{13}{2}$.
		4	

Question	Answer	Marks	Guidance
8(b)	$\dfrac{x^{3/2}}{3/2}+8x^{1/2}$	**B1**	OE Integrate to find area under curve, allow unsimplified versions.
	$\left[\left(\dfrac{128}{3}+32\right)-\left(\dfrac{2}{3}+8\right)\right]$	**M1**	Apply limits 1 → 16 to an integrated expression.
	Area under line = 15 × 5 = 75	**B1**	Or by $\int_1^{16} 5\mathrm{d}x$.
	Required area = 75 – 66 = 9	**A1**	
		4	

Question	Answer	Marks	Guidance
9(a)	$6\sin 0.9=\dfrac{AC}{2}$ or $AC^2=6^2+6^2-2\times6\times6\cos1.8$	**M1**	OE Correct working in degrees is acceptable throughout.
	$AC = 9.40$	**A1**	SOI Accept 9.39 – 9.41, may be used but not seen for A1.
	Angle CAB = ½(π – 1.8)	**M1**	SOI Expect 0.6708 (or 0.671).
	Arc CD = *their* 9.40 × *their* 0.6708	**M1**	Expect 6.306 (or 6.31), do not accept 6 for *their* AC or 1.8 for CAB.
	[Perimeter = 6 + 3.40 + 6.306 =] 15.7	**A1**	Accept 15.69 – 15.72.
		5	

Question	Answer	Marks	Guidance
9(b)	Sector $ADC - \Delta ABC = \frac{1}{2} \times$ *their* $9.40^2 \times$ *their* $0.6708 - \frac{1}{2} \times 6^2 \times \sin 1.8$	**M1 M1**	Accept correct use of their answers from part **(a).**
	[29.64 – 17.53 =] 12.1	**A1**	AWRT
		3	

Question	Answer	Marks	Guidance
10(a)	$\left\{\frac{(4x+2)^{-1}}{-1}\right\}\{\div 4\}$ or eg $\left\{\frac{1}{16}\right\}\left\{-(x+0.5)^{-1}\right\}$ or $\frac{-1}{(16x+8)}$	**B1 B1**	OE If more than one function of x present then B0 B0.
	$0-(-1/24)$	**M1**	Apply limits to an integral, ∞ must be used correctly.
	1/24	**A1**	Allow 0.0417 AWRT.
		4	
10(b)	$\frac{dy}{dx} = \left\{-2(4x+2)^{-3}\right\}$ $\{\times 4\}$	**B1 B1**	Allow unsimplified forms.
	Recognise $\frac{dy}{dx} = -1$	**B1**	SOI
	$\textit{their}\frac{-8}{(4x+2)^3} = \textit{their} -1$	**M1**	Must be numerical. Must be some attempt to solve *their* equation and $\frac{dy}{dx} \neq 0$.
	(0, ¼)	**A1 A1**	Accept $x = 0$, $y = ¼$. $y = ¼$ must be from $x = 0$ not $x = -1$.
		6	

Question	Answer	Marks	Guidance
11(a)	$mx + c = -\frac{m}{x} \Rightarrow mx^2 + cx + m = 0$	**M1**	All x terms in the numerator. OE e.g. $mx^2 + cx = -m$.
	$b^2 - 4ac = 0 \Rightarrow c^2 - 4m^2 = 0$	**M1**	OE $b^2 - 4ac = 0$ is implied by $c^2 - 4m^2 = 0$.
	$c = [\pm]2m$	**A1**	SOI. Allow $\pm$ at this stage.
	$mx^2\ [\pm]2mx + m = 0 \Rightarrow x^2 [\pm]2x + 1 = 0$	**M1**	Sub $c = +2m$ Ignore substitution of -2m.
	$(x+1)^2 = 0 \Rightarrow x = -1$ only	**A1**	
	$y = m$ only or $(-1, m)$ only	**A1**	
	Alternative method to question 11(a)		
	$\frac{dy}{dx} = \frac{m}{x^2}$	**M1**	As this is a method mark a sign error is allowed.
	$\frac{m}{x^2} = m \Rightarrow x^2 = 1$	**M1 A1**	Equating *their* $\frac{dy}{dx}$ and m and attempt to solve.
	$x = \pm 1$ or $x = -1$	**A1**	If $x = -1$ and $y = m$ are the only answers offered here award the final M1 A1.
	Selecting $x = -1$ as the only answer and attempt to find y	**M1**	
	$y = m$ or $(-1, m)$	**A1**	
		6	

Question	Answer	Marks	Guidance
11(b)	Equation of normal is $y - m = \frac{-1}{m}(x+1)$	*M1	Through *their P* with gradient $\frac{-1}{m}$, OE e.g. $y = \frac{-1}{m}x + \frac{m^2-1}{m}$. Allow use of the gradient of the curve as $-\frac{1}{\left[\frac{m}{(their\ x)^2}\right]}$ with *their* P. Coordinates of P must be in terms of *m* only.
	$\frac{-x}{m} - \frac{1}{m} + m = \frac{-m}{x} \quad \Rightarrow \quad x^2 + x(1-m^2) - m^2\ [=0]$	DM1	OE Equating *their* normal equation to the equation of the curve and removing *x* from the denominator.
	$(x+1)(x-m^2)\ [=0] \Rightarrow x = m^2$	A1	or $x = \frac{m^2 - 1 \pm \sqrt{1 - 2m^2 + m^4 + 4m^2}}{2} = \frac{m^2 - 1 \pm (m^2+1)}{2} = m^2$
	$y = \frac{-m}{m^2} = \frac{-1}{m}$	A1	or $\left(m^2, \frac{-1}{m}\right)$, ignore the coordinates of P.
		4	

Cambridge Assessment International Education

Cambridge International AS & A Level

MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **February/March 2022**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the February/March 2022 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

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WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	$\left[\mathrm{f}(x)=\right]\dfrac{2x^{\frac{2}{3}}}{\frac{2}{3}}-\dfrac{x^{\frac{4}{3}}}{\frac{4}{3}}\ [+c]$	**B1 B1**	$\frac{2}{3}$ and $\frac{4}{3}$ may be seen as sums of 1 and a fraction.
	$5 = 12 - 12 + c$	**M1**	Substituting (8,5) into an integral.
	$\left[\mathrm{f}(x)=\right]3x^{\frac{2}{3}}-\frac{3}{4}x^{\frac{4}{3}}+5$	**A1**	Fractions in the denominators scores A0.
		4	

Question	Answer	Marks	Guidance
2	$x^2+2cx+4=4x+c$ leading to $x^2+2cx-4x+4-c\ [=0]$	***M1**	Equate *y*s and move terms to one side of equation.
	$b^2-4ac=(2c-4)^2-4(4-c)$	**DM1**	Use of discriminant with *their* correct coefficients.
	$\left[4c^2-16c+16-16+4c=\right]4c^2-12c$	**A1**	
	$b^2-4ac>0$ leading to $(4)c(c-3)>0$	**M1**	Correctly apply '> 0' considering both regions.
	$c<0,\ c>3$	**A1**	Must be in terms of *c*. **SC B1** instead of M1A1 for $c \leqslant 0$, $c \geqslant 3$
		5	

Question	Answer	Marks	Guidance
3(a)	${}^6C_2 \times (3x)^4 \left(\frac{2}{x^2}\right)^2$	**B1**	Can be seen within an expansion.
	$15 \times 3^4 \times 2^2$	**B1**	Identified. Powers must be correct.
	4860	**B1**	Without any power of x
		3	
3(b)	*Their* 4860 and one other relevant term	**M1**	Using *their* 4860 and an attempt to find a term in x^{-3}
	Other term = $6C3(3x)^3\left(\frac{2}{x^2}\right)^3$ or $6C3 \times 3^3 \times 2^3$ or 4320	**A1**	Must be identified. If M0 scored then **SC B1** for 4320 as the only answer.
	[4860 – 4320 =] 540	**A1**	
		3	

Question	Answer	Marks	Guidance
4	$ar^2 = a + d$	B1	
	$ar^4 = a + 5d$	B1	
	$a^2r^4 = a(a+5d)$ leading to $a^2 + 5ad = (a+d)^2$	*M1	Eliminating r or complete elimination of a and d.
	$[3ad - d^2 = 0$ leading to$]$ $d = 3a$ OR $[r = 2$ leading to$]$ $d = 3a$	A1	
	$S_{20} = \frac{20}{2}[2a + 19 \times 3a]$	DM1	Use of formula with *their* d in terms of a.
	$590a$	A1	
		6	

Question	Answer	Marks	Guidance
5(a)	$2[\{(x-2)^2\}\ \{+3\}]$	B1 B1	B1 for $a = 2$, B1 for $b = 3$. $2(x-2)^2 + 6$ gains B1B0
		2	
5(b)	{Translation} $\begin{pmatrix} \{2\} \\ \{3\} \end{pmatrix}$ OR {Stretch} {y direction} {factor 2}	B2,1,0	B2 for fully correct, B1 with two elements correct. {} indicates different elements.
	{Stretch} {y direction} {factor 2} OR {Translation} $\begin{pmatrix} \{2\} \\ \{6\} \end{pmatrix}$	B2,1,0	B2 for fully correct, B1 with two elements correct. {} indicates different elements.
		4	

Question	Answer	Marks	Guidance
6(a)	$(x+1)^2 + (3x-22)^2 = 85$	**M1**	OE. Substitute equation of line into equation of circle.
	$10x^2 - 130x + 400\ [=0]$	**A1**	Correct 3-term quadratic
	$[10](x-8)(x-5)$ leading to $x = 8$ or 5	**A1**	Dependent on factors or formula or completing of square seen.
	(8, 4), (5, –5)	**A1**	If M1A1A0A0 scored, then **SC B1** for correct final answer only.
		4	
6(b)	Mid-point of $AB = \left(6\tfrac{1}{2}, -\tfrac{1}{2}\right)$	**M1**	Any valid method
	Use of $C = (-1, 2)$	**B1**	SOI
	$r^2 = \left(-1-6\tfrac{1}{2}\right)^2 + \left(2+\tfrac{1}{2}\right)^2$	**M1**	Attempt to find r^2. Expect $r^2 = 62\tfrac{1}{2}$.
	Equation of circle is $(x+1)^2 + (y-2)^2 = 62\tfrac{1}{2}$	**A1**	OE.
		4	

Question	Answer	Marks	Guidance
7(a)	$\dfrac{(\sin\theta+2\cos\theta)(\cos\theta+2\sin\theta)-(\sin\theta-2\cos\theta)(\cos\theta-2\sin\theta)}{(\cos\theta-2\sin\theta)(\cos\theta+2\sin\theta)}$	*M1	Obtain an expression with a common denominator
	$\dfrac{5\sin\theta\cos\theta+2\cos^2\theta+2\sin^2\theta-\left(5\sin\theta\cos\theta-2\sin^2\theta-2\cos^2\theta\right)}{\cos^2\theta-4\sin^2\theta}$ $=\dfrac{4\left(\cos^2\theta+\sin^2\theta\right)}{\cos^2\theta-4\sin^2\theta}$	A1	
	$\dfrac{4}{\cos^2\theta-4\left(1-\cos^2\theta\right)}$	DM1	Use $\cos^2\theta+\sin^2\theta=1$ twice
	$\dfrac{4}{5\cos^2\theta-4}$	A1	**AG**
		4	
7(b)	$\dfrac{4}{5\cos^2\theta-4}=5$ leading to $25\cos^2\theta=24$ leading to $\cos\theta=\sqrt{\dfrac{24}{25}}\left[=(\pm)0.9798\right]$	M1	Make $\cos\theta$ the subject
	$\theta=11.5°$ or $168.5°$	A1 A1 FT	 FT on 180º – 1st solution
		3	

Question	Answer	Marks	Guidance
8(a)	$(-2)^2 + y^2 = 8$ leading to $y = 2$ leading to $A = (0,2)$	**B1**	
	Substitute $y = their\,2$ into circle leading to $(x-2)^2 + 4 = 8$	**M1**	Expect $x = 4$.
	$B = (4, 2)$	**A1**	
		3	
8(b)	Attempt to find $[\pi]\int\left(8-(x-2)^2\right)\mathrm{d}x$	***M1**	
	$[\pi]\left[8x-\frac{(x-2)^3}{3}\right]$ or $[\pi]\left[8x-\left(\frac{x^3}{3}-2x^2+4x\right)\right]$	**A1**	
	$[\pi]\left(32-\frac{16}{3}\right)$ or $[\pi]\left[32-\left(\frac{64}{3}-32+16\right)\right]$	**DM1**	Apply limits $0 \to their\ 4$.
	Volume of cylinder = $\pi \times 2^2 \times 4 = 16\pi$	**B1 FT**	OR from $\pi\int 2^2\,\mathrm{d}x$ with *their* limits from **(a)**. FT on *their* A and B
	$\left[\text{Volume of revolution} = 26\tfrac{2}{3}\pi - 16\pi =\right] 10\tfrac{2}{3}\pi$	**A1**	Accept 33.5
		5	

Question	Answer	Marks	Guidance
9(a)	$\left[x^{\frac{1}{2}}=\right]\frac{4\pm\sqrt{16-4}}{2}=2\pm\sqrt{3}$	**M1 A1**	OE. Answer must come from formula or completing square. If M0A0 scored then **SC B1** for $2\pm\sqrt{3}$ only.
	$[x=]\left(2\pm\sqrt{3}\right)^2$	**M1**	Attempt to square *their* $2\pm\sqrt{3}$
	$7+4\sqrt{3}$, $7-4\sqrt{3}$	**A1**	Accept $7\pm4\sqrt{3}$ or $a=7, b=\pm4, c=3$ **SC B1** instead of second M1A1 for correct final answer only.
	Alternative method for question 9(a)		
	$-4x^{\frac{1}{2}}+1=0$ leading to $(x+1)^2=16x$ leading to $x^2-14x+1=0$	***M1 A1**	OE
	$x=\frac{14\pm\sqrt{196-4}}{2}$	**DM1**	Attempt to solve for x
	$7+4\sqrt{3}$, $7-4\sqrt{3}$	**A1**	**SC B1** instead of second M1A1 for correct final answer only.
		4	
9(b)	$[\mathrm{gh}(x)=]\ m\left(x^{\frac{1}{2}}-2\right)^2+n$	**M1**	SOI
	$[\mathrm{gh}(x)=]\ m\left(x-4x^{\frac{1}{2}}+4\right)+n\ \equiv\ x-4x^{\frac{1}{2}}+1$	**A1**	SOI
	$m=1,\ n=-3$	**A1 A1**	WWW
		4	

Question	Answer	Marks	Guidance
10(a)	$\tan A = \frac{12}{5}$ or $\cos A = \frac{5}{13}$ or $\sin A = \frac{12}{13}$	M1	OR $\tan B = \frac{5}{12}$ or $\cos B = \frac{12}{13}$ or $\sin B = \frac{5}{13}$
	$A = 1.176$ $B = 0.3948$	A1	Allow 1.18 or 67.4°, Allow 0.395 or 22.6°. May be implied by $\frac{\pi}{2} - 1.176$
	$DE = 4$	B1	If trigonometry used accept AWRT 4.00
	Arcs = $5 \times their 1.176$ *and* $8 \times their 0.3948$	M1	Or corresponding calculations in degrees.
	[Perimeter = 5.880 + 3.158 + 4 =] 13.0	A1	Accept 13. If *DE* is outside the given range this mark cannot be awarded.
		5	
10(b)	Area of triangle = $\frac{1}{2} \times 5 \times their 12$ [= 30]	B1 FT	
	Area of sectors = $\frac{1}{2} \times 5^2 \times their\ 1.176 + \frac{1}{2} \times 8^2 \times their\ 0.3948$	M1	Or corresponding calculations in degrees
	[Area = 30 – 14.70 –12.63 =] 2.67	A1	Allow 2.66 to 2.67
		3	

Question	Answer	Marks	Guidance
11(a)	$\frac{dy}{dx} = \left\{-k(3x-k)^{-2}\right\} \{\times 3\} \{+3\}$	B2, 1, 0	
	$\frac{-3k}{(3x-k)^2} + 3 = 0$ leading to $(3)(3x-k)^2 = (3)k$ leading to $3x - k = [\pm]\sqrt{k}$	M1	Set $\frac{dy}{dx} = 0$ and remove the denominator
	$x = \frac{k \pm \sqrt{k}}{3}$	A1	OE
		4	
11(b)	$a = \frac{4 \pm \sqrt{4}}{3}$ leading to $a = 2$	B1	Substitute $x = a$ when $k = 4$. Allow $x = 2$.
	$f''(x) = f'\left[-12(3x-4)^{-2} + 3\right] = 72(3x-4)^{-3}$	B1	Allow $18k(3x-k)^{-3}$
	> 0 (or 9) when $x = 2 \rightarrow$ minimum	B1 FT	FT on *their* $x = 2$, providing their $x \geqslant \frac{3}{2}$ and $f''(x)$ is correct
		3	

Question	Answer	Marks	Guidance
11(c)	Substitute $k=-1$ leading to $g'(x)=\dfrac{3}{(3x+1)^2}+3$	**M1**	Condone one error.
	$g'(x)>0$ or $g'(x)$ always positive, hence g is an increasing function	**A1**	WWW. A0 if the conclusion depends on substitution of values into $g'(x)$.
	Alternative method for question 11(c)		
	$x=\dfrac{k\pm\sqrt{k}}{3}$ when $k=-1$ has no solutions, so g is increasing or decreasing	**M1**	Allow the statement 'no turning points' for increasing or decreasing
	Show g'(x) is positive for any value of x, hence g is an increasing function	**A1**	Or show g(b) > g(a) for $b>a\rightarrow$ g, hence g is an increasing function
		2	

Cambridge International AS & A Level

MATHEMATICS	**9709/11**
Paper 1 Pure Mathematics 1	**October/November 2021**
MARK SCHEME	
Maximum Mark: 75	

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2021 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

This document consists of **19** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1(a)	$1-\frac{1}{x}+\frac{1}{4x^2}$	**B1**	OE. Multiply or use binomial expansion. Allow unsimplified.
		1	
1(b)	$1+12x+60x^2+160x^3$	**B2, 1, 0**	Withhold 1 mark for each error; B2, 1, 0. ISW if more than 4 terms in the expansion.
		2	
1(c)	$\textit{their}(1\times 12)+\textit{their}(-1\times 60)+\textit{their}\left(\frac{1}{4}\times 160\right)$	**M1**	Attempts at least 2 products where each product contains one term from each expansion.
	$[12-60+40=]\ -8$	**A1**	Allow $-8x$.
		2	

Question	Answer	Marks	Guidance
2	$kx^2 + 2x - k = kx - 2$ leading to $kx^2 + (-k+2)x - k + 2\ [=0]$	*M1	Eliminate y and form 3-term quadratic. Allow 1 error.
	$(-k+2)^2 - 4k(-k+2)$	DM1	Apply $b^2 - 4ac$; allow 1 error but a, b and c must be correct for *their* quadratic.
	$5k^2 - 12k + 4$ or $(-k+2)(-k+2-4k)$	A1	May be shown in quadratic formula.
	$(-k+2)(-5k+2)$	DM1	Solving a 3-term quadratic in k (all terms on one side) by factorising, use of formula or completing the square. Factors must expand to give *their* coefficient of k^2.
	$\frac{2}{5} < k < 2$	A1	WWW, accept two separate correct inequalities. If M0 for solving quadratic, **SC B1** can be awarded for correct final answer.
		5	

Question	Answer	Marks	Guidance
3	$3\cos\theta(2\tan\theta-1)+2(2\tan\theta-1)\ [=0]$	**M1**	Or similar partial factorisation; condone sign errors.
	$(2\tan\theta-1)\ (3\cos\theta+2)\ [=0]$ [leading to $\tan\theta=\frac{1}{2}$, $\cos\theta=-\frac{2}{3}$]	**M1**	OE. At least 2 out of 4 products correct.
	26.6º, 131.8º	**A1 A1**	WWW. Must be 1 d.p. or better. Final A0 if extra solution within the interval. **SC B1** No factorisation: Division by $2\tan\theta-1$ leading to 131.8° or division by $3\cos\theta+2$ or similar leading to 26.6°.
	Alternative method for question 3		
	$6\cos\theta\left(\frac{\sin\theta}{\cos\theta}\right)-3\cos\theta+4\left(\frac{\sin\theta}{\cos\theta}\right)-2\ [=0]$ $6\cos\theta\sin\theta-3\cos^2\theta+4\sin\theta-2\cos\theta\ [=0]$ $2\sin\theta(3\cos\theta+2)-\cos\theta(3\cos\theta+2)\ \ [=0]$	**M1**	Using $\tan\theta=\frac{\sin\theta}{\cos\theta}$ and reaching a partial factorisation; condone sign errors.
	$(2\sin\theta-\cos\theta)(3\cos\theta+2)\ \ [=0]$ [leading to $\tan\theta=\frac{1}{2}$, $\cos\theta=-\frac{2}{3}$]	**M1**	At least 2 out of 4 products correct.
	26.6º, 131.8º	**A1 A1**	WWW. Must be 1 d.p. or better. Final A0 if extra solution within the interval. **SC B1** No factorisation: Division by $2\tan\theta-1$ leading to 131.8° or division by $3\cos\theta+2$ or similar leading to 26.6°.
		4	

Question	Answer	Marks	Guidance
4(a)	$\dfrac{5a}{1-\left(\pm\frac{1}{4}\right)}$	**B1**	Use of correct formula for sum to infinity.
	$\dfrac{8}{2}\left[2a+7(-4)\right]$	***M1**	Use of correct formula for sum of 8 terms and form equation; allow 1 error.
	$4a=8a-112$ leading to $a=[28]$	**DM1**	Solve equation to reach a value of a.
	$a=28$	**A1**	Correct value.
		4	
4(b)	*their* $28+(k-1)(-4)=0$	**M1**	Use of correct method with *their a*.
	$[k=]8$	**A1**	
		2	

Question	Answer	Marks	Guidance
5(a)	$a=5$	**B1**	
	$b=2$	**B1**	
	$c=3$	**B1**	
		3	

Question	Answer	Marks	Guidance
5(b)(i)	3	**B1**	
		1	
5(b)(ii)	2	**B1**	
		1	

Question	Answer	Marks	Guidance
6(a)	Recognise that at least one of angles A, B, C is $\frac{\pi}{3}$	**B1**	SOI; allow 60°.
	One arc $6 \times \textit{their}\ \frac{\pi}{3}$ leading to two arcs $2 \times 6 \times \textit{their}\ \frac{\pi}{3}$	**M1**	SOI e.g. may see 2π or 4π. Use of correct formula for length of arc and multiply by 2.
	Perimeter $= 6 + 4\pi$	**A1**	Must be exact value.
	Alternative method for question 6(a)		
	Calculate circumference of whole circle $= 12\pi$	**B1**	
	One arc $\frac{1}{6}\times 12\pi$ leading to two arcs $2\times\frac{1}{6}\times 12\pi$	**M1**	SOI e.g. may see 2π or 4π.
	Perimeter $= 6 + 4\pi$	**A1**	Must be exact value.
		3	

Question	Answer	Marks	Guidance
6(b)	Sector = $\frac{1}{2}\times 6^2 \times their\left(\frac{\pi}{3}\right)$	**M1**	Use of correct formula for area of sector. SOI e.g. may see 6π or 12π.
	$\frac{1}{2}\times\left(6^2\right)\times their\left(\frac{\pi}{3}\right)-\frac{1}{2}\times\left(6^2\right)\times\sin\left(their\left(\frac{\pi}{3}\right)\right)+6\pi\ \left[=6\pi-9\sqrt{3}+6\pi\right]$	**M1 A1**	M1 for attempt at strategy with values substituted: **area of segment + area of sector** A1 if correct (unsimplified).
	Area = $12\pi-9\sqrt{3}$	**A1**	Must be simplified exact value.
	Alternative method for question 6(b)		
	Sector = $\frac{1}{2}\times 6^2 \times their\left(\frac{\pi}{3}\right)$	**M1**	Use of correct formula for area of sector. SOI e.g. may see 6π or 12π.
	$2\times\left(\frac{1}{2}\times 6^2\times their\left(\frac{\pi}{3}\right)\right)-\frac{1}{2}\times\left(6^2\right)\times\sin\left(their\left(\frac{\pi}{3}\right)\right)$	**M1 A1**	M1 for attempt at strategy with values substituted: **2 × sector – triangle** A1 if correct (unsimplified).
	Area = $12\pi-9\sqrt{3}$	**A1**	Must be simplified exact value.
	Alternative method for question 6(b)		
	Sector = $\frac{1}{2}\times 6^2 \times their\left(\frac{\pi}{3}\right)$	**M1**	Use of correct formula for area of sector. SOI e.g. may see 6π or 12π.
	$2\times\left(\frac{1}{2}\times\left(6^2\right)\times their\left(\frac{\pi}{3}\right)-\frac{1}{2}\times\left(6^2\right)\times\sin\left(their\left(\frac{\pi}{3}\right)\right)\right)+$ $\frac{1}{2}\times\left(6^2\right)\times\sin\left(their\left(\frac{\pi}{3}\right)\right)\ \left[=12\pi-18\sqrt{3}+9\sqrt{3}\right]$	**M1 A1**	M1 for attempt at strategy with values substituted: **2 × segment + triangle** A1 if correct (unsimplified).
	Area $\left[=6\pi-9\sqrt{3}+6\pi\right]$ = $12\pi-9\sqrt{3}$	**A1**	Must be simplified exact value.
		4	

Question	Answer	Marks	Guidance
7(a)	$r^2\left[=(5-2)^2+(7-5)^2\right]=13$	**B1**	$r^2=13$ or $r=\sqrt{13}$
	Equation of circle is $(x-5)^2+(y-2)^2=13$	**B1 FT**	OE. FT on *their* 13 but LHS must be correct.
		2	
7(b)	$(x-5)^2+\ (5x-10-2)^2=13$	**M1**	Substitute $y=5x-10$ into *their* equation.
	$26x^2-130x+156\ \ [=0]$	**A1 FT**	OE 3-term quadratic with all terms on one side. FT on *their* circle equation.
	$[26](x-2)(x-3)\ \ [=0]$	**M1**	Solve 3-term quadratic in x by factorising, using formula or completing the square. Factors must expand to give *their* coefficient of x^2.
	(2, 0), (3, 5)	**A1 A1**	Coordinates must be clearly paired; A1 for each correct point. A1 A0 available if two x or y values only. If M0 for solving quadratic, **SC B2** can be awarded for correct coordinates, **SC B1** if two x or y values only.
	$(AB)^2=(3-2)^2+(5-0)^2$	**M1**	SOI. Using *their* points to find length of AB.
	$AB=\sqrt{26}$	**A1**	ISW. Dependent on final M1 only.

Question	Answer	Marks	Guidance
7(b)	**Alternative method for question 7(b)**		
	$\left(\frac{y+10}{5}-5\right)^2+(y-2)^2=13$	**M1**	Substitute $x=\frac{y+10}{5}$ into *their* equation.
	$\frac{26y^2}{25}-\frac{26y}{5}$ $[=0]$	**A1 FT**	OE 2-term quadratic with all terms on one side. FT on *their* circle equation.
	$[26]y(y-5)$ $[=0]$	**M1**	Solve 2-term quadratic in y by factorising, using formula or completing the square. Factors must expand to give *their* coefficient of y^2.
	(2, 0), (3, 5)	**A1 A1**	Coordinates must be clearly paired; A1 for each correct point. A1 A0 available if two x or y values only. If M0 for solving quadratic, **SC B2** can be awarded for correct coordinates, **SC B1** if two x or y values only.
	$(AB)^2=(3-2)^2+(5-0)^2$	**M1**	SOI. Using *their* points to find length of AB.
	$AB=\sqrt{26}$	**A1**	ISW. Dependent on final M1 only.
		7	

Question	Answer	Marks	Guidance
8(a)	$\left\{-3(x-2)^2\right\}$ $\{+14\}$	**B1 B1**	B1 for each correct term; condone a = 2, b = 14.
		2	
8(b)	[k =] 2	**B1**	Allow [x] $\leqslant$ 2.
		1	

Question	Answer	Marks	Guidance
8(c)	[Range is] [y] $\leqslant -13$	**B1**	Allow [f(x)] $\leqslant -13$, [f] $\leqslant -13$ but NOT $x \leqslant -13$.
		1	
8(d)	$y = -3(x-2)^2 + 14$ leading to $(x-2)^2 = \frac{14-y}{3}$	**M1**	Allow $\frac{y-14}{-3}$. Allow 1 error in rearrangement if x, y on opposite sides.
	$x = 2(\pm)\sqrt{\frac{14-y}{3}}$	**A1**	Allow $\frac{y-14}{-3}$.
	$[\mathrm{f}^{-1}(x)] = 2 - \sqrt{\frac{14-x}{3}}$	**A1**	OE. Allow $\frac{x-14}{-3}$. Must be x on RHS; must be negative square root <u>only.</u>
	Alternative method for question 8(d)		
	$x = -3(y-2)^2 + 14$ leading to $(y-2)^2 = \frac{14-x}{3}$	**M1**	Allow $\frac{x-14}{-3}$. Allow 1 error in rearrangement if x, y on opposite sides.
	$= 2(\pm)\sqrt{\frac{14-x}{3}}$	**A1**	Allow $\frac{x-14}{-3}$.
	$[\mathrm{f}^{-1}(x)] = 2 - \sqrt{\frac{14-x}{3}}$	**A1**	OE. Allow $\frac{x-14}{-3}$. Must be x on RHS; must be negative square root <u>only.</u>
		3	

Question	Answer	Marks	Guidance
8(e)	[g(x) =] $\left\{-3(x+3-2)^2\right\}+\{14+1\}$	B2, 1, 0	OR $\left\{-3(x+3)^2\right\}+\{12(x+3)\}+\{3\}$
	$g(x)=-3x^2-6x+12$	B1	
		3	

Question	Answer	Marks	Guidance
9(a)	$f(x)=\frac{2}{3}x^3-7x+4x^{-1}\ [+c]$	B2, 1, 0	Allow terms on different lines; allow unsimplified.
	$-\frac{1}{3}=\frac{2}{3}-7+4+c$ leading to c = [2]	M1	Substitute f(1) = $-\frac{1}{3}$ into an integrated expression and evaluate c.
	$f(x)=\frac{2}{3}x^3-7x+4x^{-1}+2$	A1	OE.
		4	

Question	Answer	Marks	Guidance
9(b)	$2x^4 - 7x^2 - 4\ [= 0]$	**M1**	Forms 3-term quadratic in x^2 with all terms on one side. Accept use of substitution e.g. $2y^2 - 7y - 4[= 0]$.
	$(2x^2 + 1)(x^2 - 4)\ [= 0]$	**M1**	Attempt factors or use formula or complete the square. Allow ± sign errors. Factors must expand to give *their* coefficient of x^2 or e.g. y. Must be quartic equation. Accept use of substitution e.g. $(2y+1)(y-4)$.
	$x = [\pm]2$	**A1**	If M0 for solving quadratic, **SC B1** can be awarded for $[\pm]2$.
	$\left[\frac{2}{3}(2)^3 - 7(2) + \frac{4}{2} + 2 \quad \text{leading to}\right]\left(2, -\frac{14}{3}\right)$ $\left[\frac{2}{3}(-2)^3 - 7(-2) + \frac{4}{-2} + 2 \quad \text{leading to}\right]\ \left(-2, \frac{26}{3}\right)$	**B1 B1**	B1 B1 for correct coordinates clearly paired; B1 for each correct point; B1 B0 if additional point.
		5	
9(c)	$f''(x) = 4x + 8x^{-3}$	**B1**	OE
		1	

Question	Answer	Marks	Guidance
9(d)	$f''(2) = 9 > 0$ MINIMUM at $x = \textit{their}\ 2$	**B1 FT**	FT on *their* $x = [\pm]2$ provided $f''(x)$ is correct. Must have correct value of $f''(x)$ if $x = 2$.
	$f''(-2) = -9 < 0$ MAXIMUM at $x = \textit{their} - 2$	**B1 FT**	FT on *their* $x = [\pm]2$ provided $f''(x)$ is correct. Must have correct value of $f''(x)$ if $x = -2$. **Special case:** If values not shown and B0B0 scored, **SC B1** for $f''(2) > 0$ MIN and $f''(-2) < 0$ MAX
	Alternative method for question 9(d)		
	Evaluate $f'(x)$ for x-values either side of 2 and –2	**M1**	FT on *their* $x = [\pm]2$
	MINIMUM at $x = \textit{their}\ 2$, MAXIMUM at $x = \textit{their}\ 2$	**A1 FT**	FT on *their* $x = [\pm]2$. Must have correct values of $f'(x)$ if shown. **Special case:** If values not shown and M0A0 scored **SC B1** $f'(2)$ $-/0/+$ MIN and $f'(-2)$ $+/0/-$ MAX
	Alternative method for question 9(d)		
	Justify maximum and minimum using correct sketch graph	**B1 B1**	Need correct coordinates in **(b)** for this method.
		2	

Question	Answer	Marks	Guidance
10(a)	$\left\{\dfrac{(3x-2)^{-\frac{1}{2}}}{-1/2}\right\} \div \{3\}$	**B2, 1, 0**	Attempt to integrate
	$-\frac{2}{3}[0-1]$	**M1**	M1 for applying limits $1 \to \infty$ to an integrated expression (either correct power or dividing by their power).
	$\dfrac{2}{3}$	**A1**	
		4	
10(b)	$[\pi]\int y^2 \mathrm{d}x = [\pi]\int (3x-2)^{-3}\,\mathrm{d}x = [\pi]\dfrac{(3x-2)^{-2}}{-2\times 3}$	***M1 A1**	M1 for attempt to integrate y^2 (power increases); allow 1 error. A1 for correct result in any form.
	$[\pi]\left[-\dfrac{1}{6}\right]\left[\dfrac{1}{16}-1\right]$	**DM1**	Apply limits 1 and 2 to an integrated expression and subtract correctly; allow 1 error.
	$\dfrac{5\pi}{32}$	**A1**	OE
		4	

Question	Answer	Marks	Guidance
10(c)	$\frac{dy}{dx} = -\frac{3}{2} \times 3(3x-2)^{-\frac{5}{2}}$	**M1**	M1 for attempt to differentiate (power decreases); allow 1 error.
	At $x = 1$, $\frac{dy}{dx} = -\frac{9}{2}$	***M1**	Substitute $x = 1$ into *their* differentiated expression; allow 1 error.
	[Equation of normal is] $y - 1 = \frac{2}{9}(x-1)$ OR evaluates c	**DM1**	Forms equation of line or evaluates c using (1, 1) and gradient $\frac{-1}{their\ \frac{dy}{dx}}$.
	At A, $y = \frac{7}{9}$	**A1**	OE e.g. AWRT 0.778; must clearly identify y-intercept
		4	

Cambridge Assessment International Education

Cambridge International AS & A Level

MATHEMATICS	**9709/12**
Paper 1 Pure Mathematics 1	**October/November 2021**
MARK SCHEME	
Maximum Mark: 75	

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2021 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

This document consists of **21** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	$2\cos^2\theta - 7\cos\theta + 3[=0]$	**M1**	Forming a 3-term quadratic expression with all terms on the same side or correctly set up prior to completing the square. Allow ± sign errors.
	$(2\cos\theta - 1)(\cos\theta - 3) = 0$	**DM1**	Solving *their* 3-term quadratic using factorisation, formula or completing the square.
	$[\cos\theta = \frac{1}{2}$ or $\cos\theta = 3$ leading to$]$ $\theta = -60°$ or $\theta = 60°$	**A1**	
	$\theta = -60°$ **and** $\theta = 60°$	**A1 FT**	FT for ± same answer between 0° and 90° or 0 and $\frac{\pi}{2}$. $\pm\frac{\pi}{3}$ or ± 1.05 AWRT scores maximum M1M1A0A1FT. **Special case:** If M1 DM0 scored then SC B1 for $\theta = -60°$ **or** $\theta = 60°$, and SC B1 FT can be awarded for $\pm($ *their* $60°)$.
		4	

Question	Answer	Marks	Guidance
2(a)	Stretch with [scale factor] either ± 2 or $\pm\frac{1}{2}$	**B1**	
	Scale factor $\frac{1}{2}$ in the x-direction	**B1**	
	Translation $\begin{pmatrix} 0 \\ -3 \end{pmatrix}$ or translation of 3 units in negative y-direction	**B1**	
		3	
2(b)	$(10, 9)$	**B1 B1**	B1 for each correct co-ordinate.
		2	

Question	Answer	Marks	Guidance
3(a)	$f(5)=[2]$ and $f(\textit{their}\ 2)=[5]$ OR $ff(5)=\left[\frac{2+3}{2-1}\right]$ OR $\dfrac{\frac{x+3}{x-1}+3}{\frac{x+3}{x-1}-1}$ and an attempt to substitute $x=5$.	**M1**	Clear evidence of applying f twice with $x = 5$.
	5	**A1**	
		2	

Question	Answer	Marks	Guidance
3(b)	$\frac{x+3}{x-1}=y\Rightarrow x+3=xy-y$ OR $\frac{y+3}{y-1}=x\Rightarrow y+3=xy-x$	*M1	Setting f(*x*) = *y* or swapping *x* and *y*, clearing of fractions and expanding brackets. Allow ± sign errors.
	$xy-x=y+3\Rightarrow x=\frac{y+3}{y-1}$ OE OR $y+3=xy-x\Rightarrow y=\left[\frac{x+3}{x-1}\right]$ OE	DM1	Finding *x* or *y* = . Allow ± sign errors.
	$[\mathrm{f}^{-1}(x)\text{ or }y]=\ \frac{\boldsymbol{x}+3}{\boldsymbol{x}-1}$	A1	OE e.g. $1+\frac{4}{x-1}$ etc. Must be a function of *x*, cannot be *x* = .
		3	

Question	Answer	Marks	Guidance
4	$y=-\frac{\frac{8}{3}}{(3x+2)}[+c]$	*B1	For $(3x+2)^{-1}$
		DB1	For $-\frac{8}{3}$
	$5\frac{2}{3}=-\frac{\frac{8}{3}}{(3\times2+2)}+c$	M1	Substituting $\left(2,\ 5\frac{2}{3}\right)$ into *their* integrated expression – defined by power = -1, or dividing by their power. + *c* needed
	$y=-\frac{8}{3(3x+2)}+6$	A1	OE e.g. $y=-\frac{8}{3}(3x+2)^{-1}+6$
		4	

Question	Answer	Marks	Guidance
5(a)	[(3rd term – 1st term) = (5th term – 3rd term) leading to…] $-6\sqrt{3}\sin x - 2\cos x = 10\cos x + 6\sqrt{3}\sin x$ $\left[\text{leading to } -12\sqrt{3}\sin x = 12\cos x\right]$ OR [(1st term + 5th term) = 2 × 3rd term leading to…] $12\cos x = -12\sqrt{3}\sin x$	***M1**	OE. From the given terms, obtain 2 expressions relating to the common difference of the arithmetic progression, attempt to solve them simultaneously and achieve an equation just involving sin$x$ and cos$x$.
	Elimination of sin$x$ and cos$x$ to give an expression in tan$x$ $\left[\tan x = -\frac{1}{\sqrt{3}}\right]$	**DM1**	For use of $\frac{\sin x}{\cos x} = \tan x$
	$[x =]\frac{5\pi}{6}$ only	**A1**	CAO. Must be exact.
		3	
5(b)	$d = 2\cos x$ or $d = 2\cos(\textit{their } x)$	**B1 FT**	Or an equivalent expression involving sin$x$ and cos$x$ e.g. $-3\sqrt{3}\sin(\textit{their } x) - \cos(\textit{their } x)$ $\left[= -\sqrt{3}\right]$ FT for *their x* from **(a)** only. If not $\pm\sqrt{3}$, must see unevaluated form.
	$S_{25} = \frac{25}{2}\left(2\times\left(2\cos(\textit{their } x)\right) + (25-1)\times(\textit{their } d)\right)$ $\left[= 12.5\left(2\times\left(-\sqrt{3}\right) + 24\left(-\sqrt{3}\right)\right)\right]$	**M1**	Using the correct sum formula with $\frac{25}{2}$, (25 – 1) and with *a* replaced by either 2(cos(*their x*)) or $\pm\sqrt{3}$ and *d* replaced by either 2(cos(*their x*)) or $\pm\sqrt{3}$.
	$-325\sqrt{3}$	**A1**	Must be exact.
		3	

Question	Answer	Marks	Guidance
6	$ar = 54$ and $\dfrac{a \text{ or } their\ a}{1-r} = 243$	B1	SOI
	$\dfrac{54}{r} = 243(1-r)$ leading to $243r^2 - 243r + 54[=0]$ $[9r^2 - 9r + 2 = 0]$ OR $a^2 - 243a + 13122[=0]$	*M1	Forming a 3-term quadratic expression in r or a using *their* 2nd term and S_∞. Allow ± sign errors.
	$k(3r-2)(3r-1)[=0]$ OR $(a-81)(a-162)[=0]$	DM1	Solving *their* 3-term quadratic using factorisation, formula or completing the square. If factorising, factors must expand to give ±*their* coefficient of r^2.
	$54 \div \left(their\ \dfrac{2}{3}\right) = a$ OR $54 \div (their\ 81) = r$	DM1	May be implied by final answer.
	Tenth term $= \dfrac{512}{243}$ $\left[\text{OR } 81\times\left(\dfrac{2}{3}\right)^9 \text{ OR } 54\times\left(\dfrac{2}{3}\right)^8\right]$	A1	OE. Must be exact. **Special case:** If B1M1DM0DM1 scored then SC B1 can be awarded for the correct final answer.
		5	

Question	Answer	Marks	Guidance
7(a)	**EITHER** By using trigonometry: $B\hat{A}C = 0.6435\ldots$ and $A\hat{B}C = \dfrac{\pi - 0.6435}{2}$ **OR** By Pythagoras: $AP = 12 \Rightarrow BP = 3$ so $\tan A\hat{B}C = \dfrac{9}{3}$ **OR** Using ΔPBC and either the sine or cosine rule $\sin A\hat{B}C = \dfrac{3}{\sqrt{10}}$ or $\cos A\hat{B}C = \dfrac{\sqrt{10}}{10}$	**M1**	$\dfrac{3}{\sqrt{10}} = 0.9486\ldots$ $\dfrac{\sqrt{10}}{10} = 0.3162\ldots$
	$A\hat{B}C = \dfrac{\pi - 0.6435}{2}$ or $\tan^{-1}\dfrac{9}{3}$ or $\sin^{-1}\dfrac{3}{\sqrt{10}}$ or $\cos^{-1}\dfrac{\sqrt{10}}{10}$ or $1.249(04\ldots)$ or $71.56°$ $= 1.25$ radians (3 sf)	**A1**	AG. Final answer must be 1.25, more accurate value 1.24904… with no rounding to 3sf seen as the final answer gets M1A0. If decimals are used all values must be given to at least 4sf for A1.
		2	
7(b)	$BC = \sqrt{(their\,3)^2 + 9^2}$ or $\dfrac{9}{\sin 1.25}$ $[= \sqrt{90}$, $3\sqrt{10}$ or $9.48697\ldots]$	**M1**	Using correct method(s) to find *BC*.
	Area of sector $= \dfrac{1}{2} \times (their\ BC)^2 \times \tan^{-1} 3$ $[= 56.207\ or\ 56.25]$	**M1**	Using $\tan^{-1} 3$ or 1.25 and *their BC*, but not 9 or 15, in correct area of sector formula.
	Area of triangle PBC $= 13.4$ to 13.6 or $\dfrac{1}{2} \times 9 \times 3$	**B1**	
	[Area = (56.207 or 56.25) – *their* 13.5 =] 42.7 or 42.8	**A1**	AWRT
		4	

Question	Answer	Marks	Guidance
8(a)	Terms required for x^2: $-5\times 2^4 \times ax + 10\times 2^3 \times a^2x^2 \left[= -80ax + 80a^2x^2\right]$	**B1**	Can be seen as part of an expansion or in correct products.
	$2\times$ ($\pm$*their* coefficient of x) + $4\times$ ($\pm$*their* coefficient of x^2)	***M1**	
	x^2 coefficient is $320a^2 - 160a = -15$ $\Rightarrow 64a^2 - 32a + 3 \Rightarrow (8a-3)(8a-1)$	**DM1**	Forming a 3-term quadratic in a, with all terms on the same side or correctly setting up prior to completing the square and solving using factorisation, formula or completing the square. If factorising, factors must expand to give *their* coefficient of a^2.
	$a = \dfrac{1}{8}$ or $a = \dfrac{3}{8}$	**A1**	OE. **Special case:** If DM0 for solving quadratic, SC B1 can be awarded for correct final answers.
		4	

Question	Answer	Marks	Guidance
8(b)	$320a^2 - 160a = k \Rightarrow 320a^2 - 160a - k[=0]$	M1	Forming a 3-term quadratic in *a* with all terms on the same side. Allow ± sign errors.
	Their $b^2 - 4ac$ [= 0], $[160^2 - 4\times320\times(-k) = 0]$	M1	Any use of discriminant on a 3-term quadratic.
	$k = -20$	A1	
	$a = \frac{1}{4}$	B1	Condone $a = \frac{1}{4}$ from $k = 20$.
	Alternative method for question 8(b)		
	$320a^2 - 160a = k$ and divide by 320 $\left[a^2 - \frac{a}{2} = \frac{k}{320}\right]$	M1	Allow ± sign errors.
	Attempt to complete the square $\left[\left(a - \frac{1}{4}\right)^2 - \frac{1}{16} = \frac{k}{320}\right]$	M1	Must have $\left(a - \frac{1}{4}\right)^2$
	$a = \frac{1}{4}$	A1	
	$k = -20$	B1	

Question	Answer	Marks	Guidance
8(b) cont'd	**Alternative method for question 8(b)**		
	$320a^2 - 160a = k$ and attempt to differentiate LHS $\left[640a - 160\right]$	**M1**	Allow ± sign errors.
	Setting *their* $(640a - 160) = 0$ and attempt to solve.	**M1**	
	$a = \dfrac{1}{4}$	**A1**	
	$k = -20$	**B1**	
		4	

Question	Answer	Marks	Guidance
9(a)	$\left[\frac{dV}{dr}=\right]\ \frac{9}{2}\left(r-\frac{1}{2}\right)^2$	B1	OE. Accept unsimplified.
	$\frac{dr}{dt}=\frac{dr}{dV}\times\frac{dV}{dt}=\frac{1.5}{their\frac{dV}{dr}}\quad\left[=\frac{1.5}{\frac{9}{2}\left(5.5-\frac{1}{2}\right)^2}=\frac{1.5}{112.5}\right]$	M1	Correct use of chain rule with 1.5, *their* differentiated expression for $\frac{dV}{dr}$ and using $r=5.5$.
	0.0133 or $\frac{3}{225}$ or $\frac{1}{75}$ [metres per second]	A1	
		3	
9(b)	$\frac{dV}{dr}$ or $their\frac{dV}{dr}=\frac{1.5}{0.1}$ or 15 OR $0.1=\frac{1.5}{their\frac{dV}{dr}}\quad\left[=\frac{2\times1.5}{9\left(r-\frac{1}{2}\right)^2}\text{OE}\right]$	B1 FT	Correct statement involving $\frac{dV}{dr}$ or *their* $\frac{dV}{dr}$, 1.5 and 0.1.
	$\left[\frac{9}{2}\left(r-\frac{1}{2}\right)^2=15\Rightarrow\right]\ r=\frac{1}{2}+\sqrt{\frac{10}{3}}$	B1	OE e.g. AWRT 2.3 Can be implied by correct volume.
	[Volume =] 8.13 AWRT	B1	OE e.g. $\frac{-3+5\sqrt{30}}{3}$. CAO.
		3	

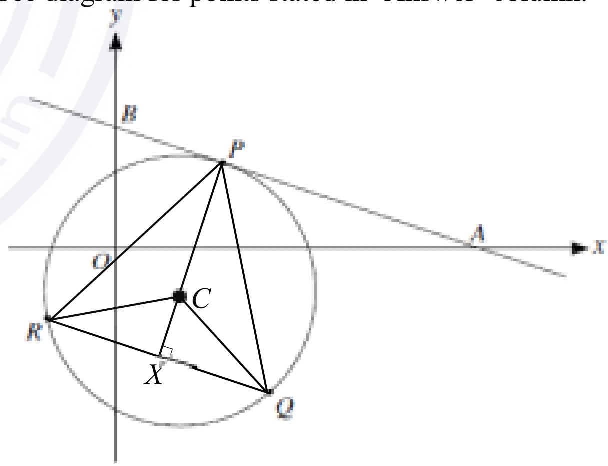
Question	Answer	Marks	Guidance
10(a)	$[f'(x) =]\ 2x - \dfrac{k}{x^2}$	**B1**	
	$f'(2) = 0\ \left[2\times 2 - \dfrac{k}{2^2} = 0\right] \Rightarrow k = \ldots$	**M1**	Setting *their* 2-term f'(2) = 0, at least one term correct and attempting to solve as far as $k =$.
	$k = 16$	**A1**	
		3	
10(b)	$f''(2) =$ e.g. $2 + \dfrac{2k}{2^3}$	**M1**	Evaluate a two term $f''(2)$ with at least one term correct. Or other valid method.
	$\left[2 + \dfrac{2k}{2^3}\right] > 0 \Rightarrow$ minimum or $= 6 \Rightarrow$ minimum	**A1 FT**	WWW. FT on positive k value.
		2	
10(c)	When $x = 2$, $f(x) = 14$	**B1**	SOI
	[Range is or y or $f(x)$] $\geqslant$ *their* f(2)	**B1 FT**	Not $x \geqslant$ *their* f(2)
		2	

Question	Answer	Marks	Guidance
11(a)	$\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{1}{2}+\dfrac{1}{3(x-2)^{\frac{4}{3}}}$	**B1**	OE. Allow unsimplified.
	Attempt at evaluating *their* $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ at $x = 3$ $\left[\dfrac{1}{2}+\dfrac{1}{3(3-2)^{\frac{4}{3}}}=\dfrac{5}{6}\right]$	***M1**	Substituting $x = 3$ into *their* differentiated expression – defined by one of 3 original terms with correct power of x.
	Gradient of normal = $\dfrac{-1}{their\dfrac{dy}{dx}}$ $\left[=-\dfrac{6}{5}\right]$	***DM1**	Negative reciprocal of *their* evaluated $\dfrac{\mathrm{d}y}{\mathrm{d}x}$.
	Equation of normal $y-\dfrac{6}{5}=(\textit{their}\text{ normal gradient})(x-3)$ $\left[y=-\dfrac{6}{5}x+4.8\Rightarrow 5y=-6x+24\right]$	**DM1**	Using *their* normal gradient and A in the equation of a straight line. Dependent on *M1 and *DM1.
	[When $y = 0$,] $x = 4$	**A1**	or (4, 0)
		5	

Question	Answer	Marks	Guidance
11(b)	Area under curve = $\int \left(\frac{1}{2}x + \frac{7}{10} - \frac{1}{(x-2)^{\frac{1}{3}}} \right) [dx]$	**M1**	For intention to integrate the curve (no need for limits). Condone inclusion of π for this mark.
	$\frac{1}{4}x^2 + \frac{7}{10}x - \frac{3(x-2)^{\frac{2}{3}}}{2}$	**A1**	For correct integral. Allow unsimplified. Condone inclusion of π for this mark.
	$\left(\frac{9}{4} + 2.1 - \frac{3}{2} \right) - \left(\frac{6.25}{4} + 1.75 - \frac{3 \times 0.5^{\frac{2}{3}}}{2} \right)$	**M1**	Clear substitution of 3 and 2.5 into *their* integrated expression (with at least one correct term) and subtracting.
	0.48[24]	**A1**	If M1A1M0 scored then SC B1 can be awarded for correct answer.
	[Area of triangle =] 0.6	**B1**	OE
	[Total area =] 1.08	**A1**	Dependent on the first M1 and WWW.
		6	

Question	Answer	Marks	Guidance
12(a)	Centre is (3, – 2)	**B1**	
	Gradient of radius = $\frac{(\textit{their}\,-2)-4}{(\textit{their}\,3)-5}$[= 3]	***M1**	Finding gradient using *their* centre (not (0, 0)) and P (5,4).
	Equation of tangent $y-4=-\frac{1}{3}(x-5)$	**DM1**	Using P and the negative reciprocal of *their* gradient to find the equation of AB.
	Sight of [x =]17 and [y =] $\frac{17}{3}$	**A1**	
	$\left[\text{Area}=\frac{1}{2}\times\frac{17}{3}\times 17=\right]\ \frac{289}{6}$	**A1**	Or $48\frac{1}{6}$ or AWRT 48.2.
	Alternative method for question 12(a)		
	$2x+2y\frac{dy}{dx}-6+4\frac{dy}{dx}=0$	**B1**	
	At P: $10+8\frac{dy}{dx}-6+4\frac{dy}{dx}=0\left[\Rightarrow\frac{dy}{dx}=-\frac{1}{3}\right]$	***M1**	Find the gradient using P (5,4) in *their* implicit differential (with at least one correctly differentiated y term).
	Equation of tangent $y-4=-\frac{1}{3}(x-5)$	**DM1**	Using P and *their* value for the gradient to find the equation of AB.
	Sight of [x =]17 and [y =] $\frac{17}{3}$	**A1**	
	$\left[\text{Area}=\frac{1}{2}\times\frac{17}{3}\times 17=\right]\ \frac{289}{6}$	**A1**	Or $48\frac{1}{6}$ or AWRT 48.2.

Question	Answer	Marks	Guidance
12(a) cont'd	**Alternative method for question 12(a)**		
	$\left[y = -2 \pm \left(40-(x-3)^2\right)^{\frac{1}{2}} \text{ OE leading to}\right] \dfrac{dy}{dx} = (3-x)\left(31+6x-x^2\right)^{-\frac{1}{2}}$	**B1**	OE. Correct differentiation of rearranged equation.
	$\dfrac{dy}{dx} = (3-5)\left(31+6(5)-(5)^2\right)^{-\frac{1}{2}} \left[\Rightarrow \dfrac{dy}{dx} = -\dfrac{1}{3}\right]$	***M1**	Find the gradient using $x = 5$ in *their* differential (with clear use of chain rule).
	Equation of tangent $y-4 = -\dfrac{1}{3}(x-5)$	**DM1**	Using P and *their* value for the gradient to find the equation of AB.
	Sight of [x =]17 and [y =] $\dfrac{17}{3}$	**A1**	
	$\left[\text{Area} = \dfrac{1}{2} \times \dfrac{17}{3} \times 17 =\right] \dfrac{289}{6}$	**A1**	Or $48\dfrac{1}{6}$ or AWRT 48.2.
		5	

Question	Answer	Marks	Guidance
12(b)	Radius of circle = $\sqrt{40}$,	**B1**	Or $2\sqrt{10}$ or 6.32 AWRT or $r^2 = 40$.
	Area of $\Delta CRQ = \frac{1}{2}\times(\mathit{their}\ r)^2 \sin 120\left[=\frac{1}{2}\times 40\times\frac{\sqrt{3}}{2}\right]$ OR Area of $\Delta CQX = \frac{1}{2}\times\sqrt{40}\cos 30\times\sqrt{40}\cos 60$ OE $\left[=\frac{1}{2}\times\sqrt{30}\times\sqrt{10}\right]$ OR Area of circle – 3× Area of segment = $40\pi - 3\times(40\frac{\pi}{3} - 10\sqrt{3})$ OR $QR = \sqrt{120}$ or $2\sqrt{30}$ and area = $\frac{1}{2}QR^2\sin 60$	**M1**	Using $\frac{1}{2}r^2\sin\theta$ with *their r* and 120 or 60 [×3] Using $\frac{1}{2}$ ×base×height in a correct right-angled triangle [×6]. Use of cosine rule and area of large triangle
	$30\sqrt{3}$	**A1**	AWRT 52[.0] implies B1M1A0.
		3	See diagram for points stated in 'Answer' column. 

Cambridge International AS & A Level

MATHEMATICS	**9709/13**
Paper 1 Pure Mathematics 1	**October/November 2021**
MARK SCHEME	
Maximum Mark: 75	

Published

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M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

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B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

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- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

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AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
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SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

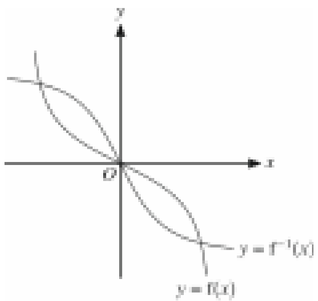
Question	Answer	Marks	Guidance
1	{Reflection} {[in the] x-axis} or {Stretch of scale factor -1} {parallel to y-axis}	***B1 DB1**	{} indicate how the B1 marks should be awarded throughout.
	Then {Translation} $\left\{\begin{pmatrix}0\\3\end{pmatrix}\right\}$	**B1 B1**	Or Translation 3 units in the positive y-direction. **N.B.** If order reversed a maximum of 3 out of 4 marks awarded.
	Alternative method for question 1		
	{Translation} $\left\{\begin{pmatrix}0\\-3\end{pmatrix}\right\}$	**B1 B1**	Or Translation 3 units in the negative y-direction.
	Then {Reflection} {in the x-axis} or {Stretch of scale factor -1} {parallel to y-axis}	***B1 DB1**	**N.B.** If order reversed a maximum of 3 out of 4 marks awarded.
		4	

Question	Answer	Marks	Guidance
2(a)	$1+6ax+15a^2x^2$	**B1**	Terms must be evaluated.
		1	
2(b)	*their* $15a^2 \pm (3\times$ *their* $6a)$	***M1**	Expect $15a^2-18a$.
	$15a^2-18a=-3$	**A1**	
	$(3)(a-1)(5a-1)\ [=0]$	**DM1**	Dependent on 3-term quadratic. Or solve using formula or completing the square.
	$a=1,\ \frac{1}{5}$	**A1**	WWW. If DM0 awarded **SC B1** if both answers correct.
		4	

Question	Answer	Marks	Guidance
3(a)	$\left\{5(y-3)^2\right\} \quad \{+5\}$	**B1 B1**	Accept $a = -3$, $b = 5$
		2	
3(b)	$\left[f'(x)=\right]5x^4-30x^2+50$	**B1**	
	$5\left(x^2-3\right)^2+5$ or $b^2<4ac$ and at least one value of $f'(x)>0$	**M1**	
	>0 and increasing	**A1**	WWW
		3	

Question	Answer	Marks	Guidance
4(a)	$84-3(n-1)=0$	**M1**	OE, SOI. Allow either = 0 or < 0 (to -3).
	Smallest n is 30	**A1**	**SC B2** for answer only n = 30 WWW.
		2	
4(b)	$\left(\frac{2k}{2}\right)\left[168+(2k-1)(-3)\right]=\left(\frac{k}{2}\right)\left[168+(k-1)(-3)\right]$	**M1 A1**	M1 for forming an equation using correct formula. A1 for at least one side correct.
	$k=19$	**A1**	
		3	

Question	Answer	Marks	Guidance
5(a)	Angle $XYC = \sin^{-1}\left(\frac{9}{11}\right) = 0.9582$ or $\sin XYC = \frac{9}{11}$ leading to $XYC = 0.9582$	**B1**	AG. OE using cosine rule.
		1	
5(b)	$XY = \sqrt{11^2 - 9^2} = \sqrt{40}$ or using 0.9582 and trigonometry	***M1 A1**	
	$AB = 9 + 11 - theirXY$	**B1 FT**	OE e.g. $20 - 2\sqrt{10}$, $2 + 9 - 2\sqrt{10} + 11 - 2\sqrt{10}$
	Arc AC = 11 × 0.9582	**M1**	
	Arc $BC = 9 \times \frac{\pi}{2}$	**M1**	
	Perimeter = [13.6(8) + 10.5(4) +14.1(4) =] 38.4	**A1**	AWRT. Answer must be evaluated as a single decimal.
		6	

Question	Answer	Marks	Guidance
6(a)		**B1**	A reflection of the given curve in $y = x$ (the line $y = x$ can be implied by position of curve).
		1	

Question	Answer	Marks	Guidance
6(b)	$y = \frac{-x}{\sqrt{4-x^2}}$ leading to $x^2 = y^2\left(4-x^2\right)$	*M1	Squaring and clearing the fraction. Condone one error in squaring $-x$ or y
	$x^2\left(1+y^2\right) = 4y^2$	DM1	OE. Factorisation of the new subject with order of operations correct. Condone sign errors.
	$x = (\pm)\frac{2y}{\sqrt{1+y^2}}$	DM1	$x = (\pm)\sqrt{\left(\frac{4y^2}{(1+y^2)}\right)}$ OE is acceptable for this mark. Isolating the new subject. Order of operations correct. Condone sign errors.
	$\mathrm{f}^{-1}(x) = \frac{-2x}{\sqrt{1+x^2}}$	A1	Selecting the correct square root. Must not have fractions in numerator or denominator.
		4	
6(c)	1 or $a = 1$	B1	Do not allow $x = 1$ or $-1 < x < 1$
		1	
6(d)	$[\mathrm{fg}(x) = \mathrm{f}(2x) =]\ \frac{-2x}{\sqrt{4-4x^2}}$	B1	Allow $\frac{-2x}{\sqrt{4-(2x)^2}}$ or any correct unsimplified form.
	$\mathrm{fg}(x) = \frac{-x}{\sqrt{1-x^2}}$ or $\frac{-x}{1-x^2}\sqrt{1-x^2}$ or $\frac{x}{x^2-1}\sqrt{1-x^2}$	B1	Result of cancelling 2 in numerator and denominator.
		2	

Question	Answer	Marks	Guidance
7(a)	$\tan x + \cos x = k(\tan x - \cos x)$ leading to $\sin x + \cos^2 x = k(\sin x - \cos^2 x)$	M1	Use $\tan x = \dfrac{\sin x}{\cos x}$ and clear fraction.
	$\sin x + 1 - \sin^2 x = k\sin x - k + k\sin^2 x$	*M1	Use $\cos^2 x = 1 - \sin^2 x$ twice to obtain an equation in sine.
	$k\sin^2 x + \sin^2 x + k\sin x - \sin x - k - 1 = 0$	DM1	Gather like terms on one side of the equation.
	$(k+1)\sin^2 x + (k-1)\sin x - (k+1) = 0$	A1	AG. Factorise to obtain answer.
		4	
7(b)	$5\sin^2 x + 3\sin x - 5 = 0$	B1	
	$\sin x = \dfrac{-3 \pm \sqrt{9+100}}{10}$	M1	Use formula or complete the square.
	$x = 48.1°, 131.9°$	A1 A1 FT	AWRT. Maximum A1 if extra solutions in range. FT for 180 – *their* answer or 540 – *their* answer if sin$x$ is negative If M0 given and correct answers only **SCB1B1** available. If answers in radians; 0.839, 2.30 can score **SCB1** for both.
		4	

Question	Answer	Marks	Guidance
8(a)	$\int\left(\frac{5}{2}-x^{\frac{1}{2}}-x^{-\frac{1}{2}}\right)\mathrm{d}x$	**M1**	OR as 2 separate integrals $\int\left(\frac{5}{2}-x^{1/2}\right)\mathrm{d}x-\int\left(x^{-1/2}\right)\mathrm{d}x$
	$\left\{\frac{5}{2}x-\frac{2}{3}x^{\frac{3}{2}}\right\}\{-\}\left\{2x^{\frac{1}{2}}\right\}$	**A1 A1 A1**	If two separate integrals with no subtraction **SC B1** for each correct integral.
	$\left(10-\frac{16}{3}-4\right)-\left(\frac{5}{8}-\frac{1}{12}-1\right)$	**DM1**	Substitute limits $\frac{1}{4}\to 4$ at least once, must be seen.
	$\frac{9}{8}$ or 1.125	**A1**	WWW. Cannot be awarded if π appears in any integral.
		6	
8(b)	$\left[\frac{\mathrm{d}y}{\mathrm{d}x}=\right]-\frac{1}{2}x^{-\frac{3}{2}}$	**B1**	
	When $x = 1$, $m = -\frac{1}{2}$	**M1**	Substitute $x = 1$ into a differential.
	[Equation of normal is] $y-1=2(x-1)$	**M1**	Through (1, 1) with gradient $-\frac{1}{m}$ or $\frac{1-p}{1}=2$
	[When $x = 0$,] $p = -1$	**A1**	WWW
		4	

Question	Answer	Marks	Guidance
9(a)	$x^2+(2x+5)^2=20$ leading to $x^2+4x^2+20x+25=20$	**M1**	Substitute $y=2x+5$ and expand bracket.
	$(5)(x^2+4x+1)[=0]$	**A1**	3-term quadratic.
	$x=\dfrac{-4\pm\sqrt{16-4}}{2}$	**M1**	OE. Apply formula or complete the square.
	$A=\left(-2+\sqrt{3},1+2\sqrt{3}\right)$	**A1**	Or 2 correct x values.
	$B=\left(-2-\sqrt{3},1-2\sqrt{3}\right)$	**A1**	Or all values correct. **SC B1** all 4 values correct in surd form without working. **SC B1** all 4 values correct in decimal form from correct formula or completion of the square
	$AB^2 = their\left(x_2-x_1\right)^2+their\left(y_2-y_1\right)^2$	**M1**	Using *their* coordinates in a correct distance formula. Condone one sign error in x_2-x_1 or y_2-y_1
	$\left[AB^2=48+12 \text{ leading to}\right]AB=\sqrt{60}$	**A1**	OE. CAO. Do not accept decimal answer. Answer must come from use of surd form in distance formula.
		7	

Question	Answer	Marks	Guidance
9(b)	$x^2 + m^2(x-10)^2 = 20$	*M1	Finding equation of tangent and substituting into circle equation.
	$x^2(m^2+1) - 20m^2x + 20(5m^2-1)$ [=0]	DM1	OE. Brackets expanded and all terms collected on one side of the equation.
	$[b^2 - 4ac =]400m^4 - 80(m^2+1)(5m^2-1)$	M1	Using correct coefficients from *their* quadratic equation.
	$400m^4 - 80(5m^4 + 4m^2 - 1) = 0 \quad \to (-80)(4m^2-1) = 0$	A1	OE. Must have '=0' for A1.
	$m = \pm\frac{1}{2}$	A1	
	Alternative method for question 9(b)		
	Length, l of tangent, is given by $l^2 = 10^2 - 20$	M1	
	$l = \sqrt{80}$	A1	
	$\tan\alpha = \frac{\sqrt{20}}{\sqrt{80}} = \frac{1}{2}$	M1 A1	Where α is the angle between the tangent and the x-axis.
	$m = \pm\frac{1}{2}$	A1	
		5	

Question	Answer	Marks	Guidance
10(a)	$f''(x) = -\left(\frac{1}{2}x + k\right)^{-3}$	B1	
	$f''(2) > 0 \Rightarrow -(1+k)^{-3} > 0$	M1	Allow for solving *their* $f''(2) > 0$
	$k < -1$	A1	WWW
		3	
10(b)	$\left[f(x) = \int\left(\left(\frac{1}{2}x-3\right)^{-2} - (-2)^{-2}\right)dx = \right] \left\{\frac{\left(\frac{1}{2}x-3\right)^{-1}}{-1\times\frac{1}{2}}\right\}\left\{-\frac{x}{4}\right\}$	B1 B1	Allow $-2\left(\frac{1}{2}x+k\right)^{-1}$ OE for 1st B1 and $-(1+k)^{-2}x$ OE for 2nd B1
	$3\frac{1}{2} = 1 - \frac{1}{2} + c$	M1	Substitute $x = 2$, $y = 3\frac{1}{2}$ into *their* integral with c present.
	$f(x) = \frac{-2}{\left(\frac{1}{2}x-3\right)} - \frac{x}{4} + 3$	A1	OE
		4	
10(c)	$\left(\frac{1}{2}x-3\right)^{-2} - (-2)^{-2} = 0$	M1	Substitute $k = -3$ and set to zero.
	leading to $\left(\frac{1}{2}x-3\right)^{2} = 4$ $\left[\frac{1}{2}x - 3 = (\pm)2\right]$ leading to $x = 10$	A1	
	$(10, -\frac{1}{2})$	A1	Or when $x = 10$, $y = -1 - 2\frac{1}{2} + 3 = -\frac{1}{2}$
	$f''(10)\left[= -(5-3)^{-3} \rightarrow\right] < 0 \rightarrow$ MAXIMUM	A1	WWW
		4	

Cambridge International AS & A Level

MATHEMATICS **9709/11**

Paper 1 Pure Mathematics 1 **May/June 2021**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2021 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

Generic Marking Principles

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Question	Answer	Marks	Guidance
1	$[y =] -\dfrac{1}{x^3} + 8x^4 \; [+ c]$	**B1 B1**	OE. Accept unsimplified.
	$4 = -8 + \dfrac{1}{2} + c$	**M1**	Substituting $\left(\dfrac{1}{2}, 4\right)$ into an integrated expression
	$y = -\dfrac{1}{x^3} + 8x^4 + \dfrac{23}{2}$	**A1**	OE. Accept $-x^{-3}$; must be 8; $y =$ must be seen in working.
		4	

Question	Answer	Marks	Guidance
2	$10(2a + 19d) = 405$	**B1**	
	$20(2a + 39d) = 1410$	**B1**	
	Solving simultaneously two equations obtained from using the correct sum formulae [$a = 6$, $d = 1.5$]	**M1**	Reach $a =$ or $d =$
	Using the correct formula for 60th term with their a and d	**M1**	
	60th term = 94.5	**A1**	OE, e.g. $\dfrac{189}{2}$
		5	

Question	Answer	Marks	Guidance
3(a)	243	**B1**	
	$-810x$	**B1**	
	$+1080x^2$	**B1**	
		3	
3(b)	$(4 + x)^2 = 16 + 8x + x^2$	**B1**	
	Coefficient of x^2 is $16 \times 1080 + 8 \times (-810) + 243$	**M1**	Allow if at least 2 pairs used correctly
	11043	**A1**	Allow $11043x^2$
		3	

Question	Answer	Marks	Guidance
4	$a = 2$	**B1**	
	$b = \frac{\pi}{4}$	**B1**	or $\frac{2\pi}{8}$
	$c = 1$	**B1**	
		3	

Question	Answer	Marks	Guidance
5	$(-12)^2 = 8k \times 2k$	**M1**	Forming an equation in k
	$k = -3$	**A1**	
	Using correct formula for S_∞ [$r = 0.5$, $a = -384$]	**M1**	With $-1 < r < 1$
	$S_\infty = -768$	**A1**	
	Alternative method for Question 5		
	$r^2 = \frac{2k}{8k}$	**M1**	
	$r = [\pm]0.5$	**A1**	
	Using correct formula for S_∞ [$r = 0.5$, $a = -384$]	**M1**	$-1 < r < 1$
	$S_\infty = -768$	**A1**	
		4	

Question	Answer	Marks	Guidance
6	$(2k-3)x^2 - kx - (k-2) = 3x - 4$	*M1	Equating curve and line
	$(2k-3)x^2 - (k+3)x - (k-6)[=0]$	DM1	Forming a 3-term quadratic
	$(k+3)^2 + 4(2k-3)(k-6)[=0]$	DM1	Use of discriminant (dependent on **both** previous M marks)
	$9k^2 - 54k + 81[=0]$ [leading to $k^2 - 6k + 9 = 0$]	M1	Simplifying and solving *their* 3-term quadratic in k
	$k = 3$	A1	
	Alternative method for Question 6		
	$(2k-3)x^2 - kx - (k-2) = 3x - 4$	*M1	Equating curve and line
	$2(2k-3)x - k = 3 \Rightarrow x = \frac{k+3}{4k-6}$ or $k = \frac{3+6x}{4x-1}$	DM1	Differentiating and solving for x or k
	Either $(2k-3)\left(\frac{k+3}{4k-6}\right)^2 - k\left(\frac{k+3}{4k-6}\right) - (k-2) = 3\left(\frac{k+3}{4k-6}\right) - 4$ **Or** $4x\left(\frac{3x^2+3x-6}{2x^2-x-1}\right) - 6x - \left(\frac{3x^2+3x-6}{2x^2-x-1}\right) = 3$	DM1	Substituting *their* x into equation or *their* $k = \frac{3x^2+3x-6}{2x^2-x-1}$ or $k = \frac{3x+6}{2x+1}$ into derivative equation (dependent on **both** previous M marks)
	$9k^2 - 54k + 81[=0]$ [leading to $k^2 - 6k + 9 = 0$]	M1	Simplifying and solving *their* 3-term quadratic in k (or solving for x)
	$k = 3$	A1	
			SC If M0, B1 for differentiating, equating to 3 and solving for x or k
		5	

Question	Answer	Marks	Guidance
7(a)	Reach $\dfrac{\cos^2\theta-\sin^2\theta}{\cos^2\theta}$ **or** $\dfrac{1-\sin^2\theta}{1-\sin^2\theta}-\dfrac{\sin^2\theta}{\cos^2\theta}$ **or** $\dfrac{\sin^2\theta+\cos^2\theta}{\cos^2\theta}-2\tan^2\theta$ **or** $\sec^2\theta-\dfrac{2\sin^2\theta}{\cos^2\theta}$ **or** $2-\sec^2\theta$ **or** $\dfrac{\cos 2\theta}{\cos^2\theta}$	**M1**	May start with $1-\tan^2\theta$
	$1-\tan^2\theta$	**A1**	AG, must show sufficient stages
		2	
7(b)	$1-\tan^2\theta=2\tan^4\theta \Rightarrow 2\tan^4\theta+\tan^2\theta-1$ [= 0]	**M1**	Forming a 3-term quadratic in $\tan^2\theta$ or e.g. u
	$\tan^2\theta=0.5$ or -1 leading to $\tan\theta=[\pm]\sqrt{0.5}$	**M1**	
	$\theta=35.3°$ and $144.7°$ (AWRT)	**A1**	Both correct. Radians 0.615, 2.53 scores A0.
		3	

Question	Answer	Marks	Guidance
8(a)	**Either** Let midpoint of *PQ* be *H:* $\sin HCP = \frac{2}{4} \Rightarrow$ Angle $HCP = \frac{\pi}{6}$ **Or** $\sin PSQ = \frac{4}{8} \Rightarrow$ Angle $PSQ = \frac{\pi}{6}$ **Or** using cosine rule: angle $PCQ = \frac{\pi}{3}$ **Or** by inspection: triangle *PCQ* or *PCT* is equilateral so angle $PCQ = \frac{\pi}{3}$	**M1**	
	Angle $PCS = \pi - \frac{\pi}{6} - \frac{\pi}{6} = \frac{2}{3}\pi$	**A1**	AG
		2	
8(b)	Perimeter $= 2\times4\times\frac{2\pi}{3}$ or $8\pi - \frac{8\pi}{3}$	**M1**	Length of two arcs *PS* and *QR*
	$+2\pi\times2$	**M1**	Adding circumference of two semicircles
	$\frac{28\pi}{3}$	**A1**	Must be a single term
		3	

Question	Answer	Marks	Guidance
8(c)	Area sector $CPQ = \frac{1}{2}\times 4^2\times\frac{\pi}{3}=\frac{8\pi}{3}$	**M1**	Uses correct formula for sector
	Area of segment of large circle beyond CPQ $= \frac{8\pi}{3}-\frac{1}{2}\times 4^2\times\sin\left(\frac{\pi}{3}\right)=\frac{8\pi}{3}-4\sqrt{3}$	**M1**	Attempts to find area of segment
	Area of small semicircle = $\pi\times 2$ or area of small circle = $\pi\times 2^2$	**M1**	
	Area of plate = Large circle – [2 ×] small semicircle – [2 ×] segment area	**M1**	
	$\pi\times 4^2-\pi\times 2^2-2\times\left(\frac{8\pi}{3}-4\sqrt{3}\right)=\frac{20\pi}{3}+8\sqrt{3}$	**A1**	AG
	Alternative method for Question 8(c)		
	Area of sector $PCS = \frac{1}{2}\times 4^2\times\frac{2\pi}{3}=\frac{16\pi}{3}$	**M1**	Uses correct formula for sector
	Area of triangle $PCQ = \frac{1}{2}\times 4^2\times\sin\frac{\pi}{3}=4\sqrt{3}$	**M1**	Uses correct formula for triangle
	Area of small semicircle = $\pi\times 2$ or area of circle = $\pi\times 2^2$	**M1**	
	Area of plate = [2 ×] large sector + [2 ×] triangle – [2 ×] small semicircle	**M1**	
	$2\left(\frac{16\pi}{3}\right)+2\left(4\sqrt{3}\right)-\pi\times 2^2=\frac{20\pi}{3}+8\sqrt{3}$	**A1**	AG
		5	

Question	Answer	Marks	Guidance
9(a)	Range of f is $\mathrm{f}(x) \geqslant -4$	**B1**	Allow y, f or 'range' or $[-4,\infty)$
		1	
9(b)	$y = (x-2)^2 - 4 \Rightarrow (x-2)^2 = y+4 \Rightarrow x-2 = +\sqrt{(y+4)}$ or $\pm\sqrt{(y+4)}$	**M1**	May swap variables here
	$\left[\mathrm{f}^{-1}(x)\right] = \sqrt{(x+4)} + 2$	**A1**	
		2	
9(c)	$(x-2)^2 - 4 = -\frac{5}{3}x + 2 \Rightarrow x^2 - 4x + 4 - 4 = -\frac{5}{3}x + 2$ $[\Rightarrow x^2 - \frac{7}{3}x - 2 = 0]$	**M1**	Equating and simplifying to a 3-term quadratic
	$(3x+2)(x-3)[=0]$ or $\dfrac{7 \pm \sqrt{7^2 - 4(3)(-6)}}{6}$ OE	**M1**	Solving quadratic
	$x = 3$ only	**A1**	
		3	

Question	Answer	Marks	Guidance
9(d)	$f^{-1}(12) = 6$	**M1**	Substitute 12 into *their* $f^{-1}(x)$ and evaluate
	$g(f^{-1}(12)) = 6a + 2$	**M1**	Substitute *their* '6' into $g(x)$
	$g(g(f^{-1}(12))) = a(6a + 2) + 2 = 62$	**M1**	Substitute the result into $g(x)$ and = 62
	$6a^2 + 2a - 60$ [= 0]	**M1**	Forming and solving a 3-term quadratic
	$a = -\dfrac{10}{3}$ or 3	**A1**	
	Alternative method for Question 9(d)		
	$g(f^{-1}(x)) = a\left(\sqrt{x+4}+2\right)+2$ or $gg(x) = a(ax+2)+2$	**M1**	Substitute *their* $f^{-1}(x)$ or $g(x)$ into $g(x)$
	$g(g(f^{-1}(x))) = a\left(a\left(\sqrt{x+4}+2\right)+2\right)+2$	**M1**	Substitute the result into $g(x)$
	$g(g(f^{-1}(12))) = a(6a + 2) + 2 = 62$	**M1**	Substitute 12 and = 62
	$6a^2 + 2a - 60$ [= 0]	**M1**	Forming and solving a 3-term quadratic
	$a = -\dfrac{10}{3}$ or 3	**A1**	
		5	

Question	Answer	Marks	Guidance
10(a)	When $y = 0$ $x^2 - 4x - 77 = 0$ [$\Rightarrow (x+7)(x-11) = 0$ or $(x-2)^2 = 81$]	**M1**	Substituting $y = 0$
	So x-coordinates are −7 and 11	**A1**	
		2	

Question	Answer	Marks	Guidance
10(b)	Centre of circle C is $(2, -3)$	**B1**	
	Gradient of AC is $-\frac{1}{3}$ or Gradient of BC is $\frac{1}{3}$	**M1**	For either gradient (M1 sign error, M0 if x-coordinate(s) in numerator)
	Gradient of tangent at A is 3 or Gradient of tangent at B is -3	**M1**	For either perpendicular gradient
	Equations of tangents are $y = 3x + 21$, $y = -3x + 33$	**A1**	For either equation
	Meet when $3x + 21 = -3x + 33$	**M1**	OR: (centre of circle has x coordinate 2) so x coordinate of point of intersection is 2
	Coordinates of point of intersection $(2, 27)$	**A1**	
	Alternative method for Question 10(b)		
	Implicit differentiation: $2y\frac{dy}{dx}$ seen	**B1**	
	$2x - 4 + 2y\frac{dy}{dx} + 6\frac{dy}{dx} = 0$	**M1**	Fully differentiated $= 0$ with at least one term involving y differentiated correctly
	Gradient of tangent at A is 3 or Gradient of tangent at B is -3	**M1**	For either gradient
	Equations of tangents are $y = 3x + 21$, $y = -3x + 33$	**A1**	For either equation
	Meet when $3x + 21 = -3x + 33$	**M1**	OR: (centre of circle has x coordinate 2) so x coordinate of point of intersection is 2
	Coordinates of point of intersection $(2, 27)$	**A1**	
		6	

Question	Answer	Marks	Guidance
11(a)	$\dfrac{dy}{dx} = 3(3x+4)^{-0.5} - 1$	B1 B1	B1 All correct with 1 error, B2 if all correct
	Gradient of tangent = $-\dfrac{1}{4}$ and Gradient of normal = 4	*M1	Substituting $x = 4$ into a differentiated expression and using $m_1 m_2 = -1$
	Equation of line is $(y-4) = 4(x-4)$ **or** evaluate c	DM1	With (4, 4) and *their* gradient of normal
	So $y = 4x - 12$	A1	
		5	
11(b)	$3(3x+4)^{-0.5} - 1 = 0$	M1	Setting *their* $\dfrac{dy}{dx} = 0$
	Solving as far as $x =$	M1	Where $\dfrac{dy}{dx}$ contains $a(bx+c)^{-0.5}$ a, b, c any values
	$x = \dfrac{5}{3}$, $y = 2\left(3\times\dfrac{5}{3}+4\right)^{0.5} - \dfrac{5}{3} = \dfrac{13}{3}$	A1	
		3	
11(c)	$\dfrac{d^2y}{dx^2} = -\dfrac{9}{2}(3x+4)^{-1.5}$	M1	Differentiating *their* $\dfrac{dy}{dx}$ OR checking $\dfrac{dy}{dx}$ to find +ve and -ve either side of their $x = \dfrac{5}{3}$
	At $x = \dfrac{5}{3}$ $\dfrac{d^2y}{dx^2}$ is negative so the point is a maximum	A1	
		2	

Question	Answer	Marks	Guidance
11(d)	Area $= \left[\int 2(3x+4)^{0.5} - x\,dx = \right] \frac{4}{9}(3x+4)^{1.5} - \frac{1}{2}x^2$	B1 B1	B1 for each correct term (unsimplified)
	$\left(\frac{4}{9}(16)^{1.5} - \frac{1}{2}(4)^2\right) - \frac{4}{9}(4)^{1.5} = \frac{256}{9} - 8 - \frac{32}{9}$	M1	Substituting limits 0 and 4 into an expression obtained by integrating y
	$16\frac{8}{9}$	A1	Or $\frac{152}{9}$
		4	

Cambridge International AS & A Level

MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **May/June 2021**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2021 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1(a)	$(4x-3)^2$ or $(4x+(-3))^2$ or $a=-3$	B1	$k(4x-3)^2$ where $k \neq 1$ scores B0 but mark final answer, allow recovery.
	$+1$ or $b=1$	B1	
		2	
1(b)	[For one root] $k = 1$ or '*their b*'	B1 FT	Either by inspection or solving or from $24^2 - 4 \times 16 \times (10-k) = 0$ WWW
	$\left[\text{Root or } x = \right]\frac{3}{4}$ or 0.75	B1	**SC B2** for correct final answer WWW.
		2	

Question	Answer	Marks	Guidance
2(a)	Translation $\begin{pmatrix}1\\0\end{pmatrix}$	B1	Allow shift and allow by 1 in x-direction or [parallel to/on/in/along/against] the x-axis or horizontally. 'Translation by 1 to the right' only, scores B0
	Stretch	B1	Stretch. **SC B2** for amplitude doubled.
	Factor 2 in y-direction	B1	With/by **factor** 2 in y-direction or [parallel to/on/in/along/against] the y-axis or vertically or with x axis invariant 'With/by factor 2 upwards' only, scores B0. Accept SF as an abbreviation for scale factor.
		3	**Note:** Transformations can be in either order
2(b)	[$-\sin 6x$][$+ 15x$] or [$\sin(-6x)$][$+ 15x$] OE	B1 B1	Accept an unsimplified version. ISW. B1 for each correct component – square brackets indicate each required component.
			If B0, **SC B1** for either $\sin(-2x) + 5x$ or $-\sin(2x) + 5x$ or $\sin 6x - 15x$ or $\sin\left(-\frac{2}{3}x\right)+\frac{5}{3}x$
		2	

Question	Answer	Marks	Guidance
3(a)	1.2679	B1	AWRT. ISW if correct answer seen. $3-\sqrt{3}$ scores B0
		1	
3(b)	1.7321	B1	AWRT. ISW if correct answer seen.
		1	
3(c)	Sight of 2 or 2.0000 or two in reference to the gradient	*B1	
	This is because the gradient at E is the limit of the gradients of the chords as the x-value tends to 3 or δx tends to 0.	DB1	Allow it gets nearer/approaches/tends/almost/approximately 2
		2	

Question	Answer	Marks	Guidance
4	[Coefficient of x or p =] 480	B1	SOI. Allow $480x$ even in an expansion.
	$\left[\text{Term in } \frac{1}{x} \text{ or } q=\right][10\times](2x)^3\left(\frac{k}{x^2}\right)^2$	M1	Appropriate term identified and selected.
	$[10\times 2^3k^2=]\ 80k^2$	A1	Allow $\frac{80k^2}{x}$
	$p=6q$ used $(480=6\times 80k^2$ or $80=80k^2)$	M1	Correct link used for *their* coefficient of x and $\frac{1}{x}$ (p and q) with no x's.
	$[k^2=1\Rightarrow]\ k=\pm 1$	A1	A0 if a range of values given. Do not allow $\pm\sqrt{1}$.
		5	

Question	Answer	Marks	Guidance
5(a)	$ff(x) = 2(2x^2 + 3)^2 + 3$	M1	Condone = 0.
	$8x^4 + 24x^2 + 21$	A1	ISW if correct answer seen. Condone = 0.
		2	
5(b)	$8x^4 + 24x^2 + 21 = 34x^2 + 19 \Rightarrow 8x^4 + 24x^2 - 34x^2 + 21 - 19$ [= 0]	M1	Equating $34x^3 + 19$ to *their* 3-term ff(x) and collect all terms on one side condone $\pm$ sign errors.
	$8x^4 - 10x^2 + 2$[= 0]	A1	
	[2]$(x^2 - 1)(4x^2 - 1)$	M1	Attempt to solve 3-term quartic or 3-term quadratic by factorisation, formula or completing the square or factor theorem.
	$\left[x^2 = 1 \text{ or } \frac{1}{4} \text{ leading to}\right] x = 1 \text{ or } x = \frac{1}{2}$	A1	If factorising, factors must expand to give $8x^4$ or $4x^4$ 4 or *their* ax^4 otherwise M0A0 due to calculator use. Condone ± 1, $\pm\frac{1}{2}$ but not $\sqrt{\frac{1}{4}}$ or $\sqrt{1}$.
		4	

Question	Answer	Marks	Guidance
6	Gradient AB = $\frac{1}{2}$	B1	SOI
	Lines meet when $-2x+4=\frac{1}{2}(x-8)+3$ Solving as far as $x =$	*M1	Equating given perpendicular bisector with the line through (8, 3) using *their* gradient of *AB* (but not -2) and solving. Expect $x = 2$, $y = 0$.
	Using mid-point to get as far as $p =$ or $q =$	DM1	Expect $\frac{8+p}{2}=2$ or $\frac{3+q}{2}=0$
	$p=-4$, $q=-3$	A1	Allow coordinates of *B* are (−4, −3).
	Alternative method for Question 6		
	Gradient AB = $\frac{1}{2}$	B1	SOI
	$\frac{q-3}{p-8}=\frac{1}{2}$ $\left[\text{leading to } 2q=p-2\right]$, $\frac{q+3}{2}=-2\left(\frac{8+p}{2}\right)+4$ $\left[\text{leading to } q=-11-2p\right]$	*M1	Equating gradient of *AB* with *their* gradient of *AB* (but not -2) and using mid-point in equation of perpendicular bisector.
	Solving simultaneously *their* 2 linear equations	DM1	Equating and solving 2 correct equations as far as $p =$ or $q =$.
	$p=-4$, $q=-3$	A1	Allow coordinates of *B* are (−4, −3).

Question	Answer	Marks	Guidance
6	**Alternative method for Question 6**		
	Gradient AB = $\frac{1}{2}$	B1	
	$\frac{q-3}{p-8}=\frac{1}{2}$ $\left[\text{leading to } p=2q+2\right]$, $y-\frac{q+3}{2}=-2\left(x-(q+5)\right)$ $\left[\text{leading to } y=-2x+\frac{5q+23}{2}\right]$	*M1	Equating gradient of AB with *their* gradient of AB (but not -2) and using mid-point in equation of perpendicular bisector.
	their $\frac{5q+23}{2}=4 \Rightarrow q=$	DM1	Equating and solving as far as q or p =
	$p=-4$, $q=-3$	A1	Allow coordinates of B are $(-4, -3)$.
		4	

Question	Answer	Marks	Guidance
7(a)	$(5-1)^2 + (11-5)^2 = 52$ or $\frac{11-5}{5-1}$	M1	For substituting (1,5) into circle equation or showing gradient $= \frac{3}{2}$.
	For both circle equation and gradient, and proving line is perpendicular and stating that *A* lies on the circle	A1	Clear reasoning.
	Alternative method for Question 7(a)		
	$(x-5)^2 + (y-11)^2 = 52$ and $y-5 = -\frac{2}{3}(x-1)$	M1	Both equations seen and attempt to solve. May see $y = -\frac{2}{3}x + \frac{17}{3}$
	Solving simultaneously to obtain $(y-5)^2 = 0$ or $(x-1)^2 = 0 \Rightarrow 1$ root or tangent or discriminant $= 0 \Rightarrow 1$ root or tangent	A1	Clear reasoning.
	Alternative method for Question 7(a)		
	$\frac{dy}{dx} = \frac{10-2x}{2y-22} = \frac{10-2}{10-22}$	M1	Attempting implicit differentiation of circle equation and substitute $x = 1$ and $y = 5$.
	Showing gradient of circle at A is $-\frac{2}{3}$	A1	Clear reasoning.
		2	
7(b)	Centre is $(-3, -1)$	B1 B1	B1 for each correct co-ordinate.
	Equation is $(x+3)^2 + (y+1)^2 = 52$	B1 FT	FT *their* centre, but not if either (1, 5) or (5, 11). Do not accept $\sqrt{52^2}$.
		3	

Question	Answer	Marks	Guidance
8(a)	$\left(a+b=2\times\frac{3}{2}a\right)\Rightarrow b=2a$	B1	SOI
	$18^2 = a(b+3)$ OE or 2 correct statements about r from the GP, e.g. $r=\frac{18}{a}$ and b + 3 = 18r or $r^2=\frac{b+3}{a}$	B1	SOI
	$324 = a(2a+3) \Rightarrow 2a^2 + 3a - 324[=0]$ or $b^2 + 3b - 648[=0]$ or $6r^2 - r - 12[=0]$ or $4d^2 + 3d - 162[=0]$	M1	Using the correct connection between AP and GP to form a 3-term quadratic with all terms on one side.
	$(a-12)(2a+27)[=0]$ or $(b-24)(b+27)[=0]$ or $(2r-3)(3r+4)[=0]$ or $(d-6)(4d+27)[=0]$	M1	Solving *their* 3-term quadratic by factorisation, formula or completing the square to obtain answers for *a, b, r* or *d*.
	$a = 12$, $b = 24$	A1	WWW. Condone extra 'solution' $a=-13.5, b=-27$ only.
		5	

Question	Answer	Marks	Guidance
8(b)	Common difference $d = 6$	B1 FT	SOI. FT *their* $\frac{a}{2}$
	$S_{20} = \frac{20}{2}(2\times12+19\times6)$	M1	Using correct sum formula with *their a, their* calculated d and 20.
	1380	A1	
		3	

Question	Answer	Marks	Guidance
9	Curve intersects $y = 1$ at (3, 1)	B1	**Throughout Question 9: 1 < *their* 3 < 5** Sight of $x = 3$
	Volume $= [\pi]\int(x-2)[\mathrm{d}x]$	M1	M1 for showing the intention to integrate $(x-2)$. Condone missing π or using 2π.
	$[\pi]\left[\frac{1}{2}x^2-2x\right]$ or $[\pi]\left[\frac{1}{2}(x-2)^2\right]$	A1	Correct integral. Condone missing π or using 2π.
	$=[\pi]\left[\left(\frac{5^2}{2}-2\times5\right)-\left(\frac{their\,3^2}{2}-2\times their\,3\right)\right]$ $=[\pi]\left[\frac{5}{2}+\frac{3}{2}\right]$ as a minimum requirement for *their* values	M1	Correct use of '*their* 3' and 5 in an integrated expression. Condone missing π or using 2π. Condone +c. Can be obtained by integrating and substituting between 5 and 2 and then 3 and 2 then subtracting.
	Volume of cylinder = $\pi\times1^2\times(5-their\,3)[=2\pi]$	B1 FT	Or by integrating 1 to obtain x (condone y if 5 and *their* 3 used).
	[Volume of solid $= 4\pi-2\pi=$]2π or 6.28	A1	AWRT

Question	Answer	Marks	Guidance
9	**Alternative method for Question 9**		
	Curve intersects $y = 1$ at (3, 1)	B1	Sight of $x = 3$
	Volume of solid = $\pi\int(x-2)-1[\mathrm{d}x]$	M1 B1	M1 for showing the intention to integrate $(x-2)$ B1 for correct integration of –1. Condone missing π or 2π for M1 but not for B1.
	$[\pi]\left[\frac{1}{2}x^2-3x\right]$ or $[\pi]\left[\frac{1}{2}(x-3)^2\right]$	A1	Correct integral, allow as two integrals. Condone missing π or using 2π.
	$=[\pi]\left[\left(\frac{5^2}{2}-3\times5\right)-\left(\frac{their\,3^2}{2}-3\times their\,3\right)\right]$	M1	Correct use of '*their* 3' and 5 in an integrated expression. Condone missing π or using 2π. Condone +c. Can be obtained by integrating and substituting between 5 and 2 and then 3 and 2 then subtracting.
	[Volume of solid = $4\pi-2\pi=$]2π or 6.28	A1	AWRT
		6	

Question	Answer	Marks	Guidance
10(a)	$\frac{1+\sin x}{1-\sin x}-\frac{1-\sin x}{1+\sin x}\equiv\frac{(1+\sin x)^2-(1-\sin x)^2}{(1-\sin x)(1+\sin x)}$	*M1	For using a common denominator of $(1-\sin x)(1+\sin x)$ and reasonable attempt at the numerator(s).
	$\equiv\frac{1+2\sin x+\sin^2 x-\left(1-2\sin x+\sin^2 x\right)}{(1-\sin x)(1+\sin x)}$	DM1	For multiplying out the numerators correctly. Condone sign errors for this mark.
	$\equiv\frac{4\sin x}{1-\sin^2 x}\equiv\frac{4\sin x}{\cos^2 x}$	DM1	For simplifying denominator to $\cos^2 x$.
	$\equiv\frac{4\sin x}{\cos x\cos x}\equiv\frac{4\tan x}{\cos x}$	A1	AG. Do not award A1 if undefined notation such as s, c, t or missing x's used throughout or brackets are missing.
	Alternative method for Question 10(a)		
	$\frac{4\tan x}{\cos x}\equiv\frac{4\sin x}{\cos^2 x}\equiv\frac{4\sin x}{1-\sin^2 x}$	*M1	Using $\tan x=\frac{\sin x}{\cos x}$ and $\cos^2 x = 1-\sin^2 x$
	$\equiv\frac{-2}{1+\sin x}+\frac{2}{1-\sin x}$	DM1	Separating into partial fractions.
	$\equiv 1+\frac{-2}{1+\sin x}+\frac{2}{1-\sin x}-1$	DM1	Use of 1-1 or similar
	$\equiv-\frac{1-\sin x}{1+\sin x}+\frac{1+\sin x}{1-\sin x}$	A1	
		4	

Question	Answer	Marks	Guidance
10(b)	$\cos x = \frac{1}{2}$	*B1	OE. WWW.
	$x = \frac{\pi}{3}$	DB1	Or AWRT 1.05
	$x = 0$ from $\tan x = 0$ or $\sin x = 0$	B1	WWW. Condone extra solutions outside the domain 0 to $\frac{\pi}{2}$ but B0 if any inside.
		3	

Question	Answer	Marks	Guidance
11(a)	At stationary point $\frac{dy}{dx} = 0$ so $6(3\times 2-5)^3 - k\times 2^2 \ = 0$	M1	Setting given $\frac{dy}{dx} = 0$ and substituting $x = 2$ into it.
	$[k =]\frac{3}{2}$	A1	OE
		2	
11(b)	$[y =]\frac{6}{4\times 3}(3x-5)^4 - \frac{1}{3}kx^3\ [+c]$.	*M1 A1FT	Integrating (increase of power by 1 in at least one term) given $\frac{dy}{dx}$. Expect $\frac{1}{2}(3x-5)^4 - \frac{1}{2}x^3$. FT *their* non zero k.
	$-\frac{7}{2} = \frac{1}{2}(3\times 2-5)^4 - \frac{1}{3}\times\frac{3}{2}\times 2^3 + c$ [leading to $-3.5 + c = -3.5$]	DM1	Using (2,-3.5) in an integrated expression. + c needed. Substitution needs to be seen, simply stating c = 0 is DM0.
	$y = \frac{1}{2}(3x-5)^4 - \frac{1}{2}x^3$	A1	y = or f(x)= must be seen somewhere in solution.

Question	Answer	Marks	Guidance
11(b)	**Alternative method for Question 11(b)**		
	$[y=]\frac{81}{2}x^4-\frac{541}{2}x^3+675x^2-750x(+c)$ or $-270x^3-k\frac{x^3}{3}$	*M1 A1 FT	From $\frac{dy}{dx}=162x^3-810x^2-kx^2-1350x-750$. FT *their k*
	$-\frac{7}{2}=\frac{81}{2}\times2^4-\frac{541}{2}\times2^3+675\times2^2-750\times2+c$	DM1	Using (2, -3.5) in an integrated expression. + c needed
	$y=\frac{81}{2}x^4-\frac{541}{2}x^3+675x^2-750x+\frac{625}{2}$	A1	$y=$ or f(x)= must be seen somewhere in solution.
		4	
11(c)	$[3\times]\left[18(3x-5)^2\right][-2kx]$	B2,1,0 FT	FT *their k*. Square brackets indicate each required component. B2 for fully correct, B1 for one error or one missing component, B0 for 2 or more errors.
	Alternative method for Question 11(c)		
	$486x^2-1623x+1350$ or $-1620x-2kx$	B2,1,0 FT	FT *their k*. B2 for fully correct, B1 for one error, B0 for 2 or more errors.
		2	
11(d)	[At $x=2$] $\left[\frac{d^2y}{dx^2}=\right]54(3\times2-5)^2-4k$ or 48	M1	OE. Substituting $x=2$ into *their* second differential or other valid method.
	[> 0] Minimum	A1	WWW
		2	

Question	Answer	Marks	Guidance
12(a)	[By symmetry] [$6 \times P\hat{A}Q = 2\pi$], [$P\hat{A}Q =$] $2\pi \div 6$,	M1	
	Explaining that there are six sectors around the diagram that make up a complete circle.	A1	AG
	Alternative method for Question 12(a)		
	Using area or circumference of circle centre $A \div 6$	M1	$\frac{400\pi}{6}$ or $\frac{40\pi}{6}$
	Justification for dividing by 6 followed by comparison with the sector area or arc length.	A1	AG
	Alternative method for Question 12(a)		
	Explain why ΔPAQ is an equilateral triangle	M1	Assumption of this scores M0
	Using ΔPAQ is an equilateral triangle $\therefore$ $P\hat{A}Q = \frac{\pi}{3}$	A1	AG
	Alternative method for Question 12(a)		
	Using the internal angle of a regular hexagon = $\frac{2\pi}{3}$ Or $F\hat{A}O + O\hat{A}B = \frac{2\pi}{3}$, equilateral triangles	M1	
	$P\hat{A}Q = 2\pi - \left(\frac{\pi}{2} + \frac{2\pi}{3} + \frac{\pi}{2}\right) = \frac{\pi}{3}$	A1	AG

Question	Answer	Marks	Guidance
12(a)	**Alternative method for Question 12(a)**		
	$Sin\theta = \frac{20}{40}$, with θ clearly identified	M1	
	$\theta = \frac{\pi}{6}, 2\theta = \frac{\pi}{3} = F\hat{A}O$ and by similar triangles $= P\hat{A}Q$	A1	AG
		2	
12(b)	Each straight section of rope has length 40 cm	B1	SOI
	Each curved section round each pipe has length $r\theta = 20 \times \frac{\pi}{3}$	*M1	Use of $r\theta$ with $r = 20$ and θ in radians
	Total length = $6 \times ((their\,40) + k\pi)$	DM1	$6\times$(*their* straight section + *their* curved section). *Their* curved section must be from acceptable use of $r\theta$ – this could now be numeric.
	$240 + 40\pi$ or 366 (AWRT) (cm)	A1	Or directly: ($6\times$ diameter) + circumference
		4	

Question	Answer	Marks	Guidance
12(c)	[Triangle area =] $\frac{1}{2}\times40\times40\times\sin\left(\frac{\pi}{3}\right)$ or $\frac{1}{2}\times40\times20\sqrt{3}$ or $400\sqrt{3}$ or 693(AWRT)	B1	
	[Total area of hexagon = $6\times 400\sqrt{3}$ =] $2400\sqrt{3}$	B1	Condone $4800\frac{\sqrt{3}}{2}$
	Alternative method for Question 12(c)		
	[Trapezium area =] $\frac{1}{2}\times(40+80)\times40\sin\left(\frac{\pi}{3}\right)$ or $1200\sqrt{3}$ or 2080 (AWRT)	B1	
	[Total area of hexagon = $2\times 1200\sqrt{3}$ =] $2400\sqrt{3}$	B1	Condone $4800\frac{\sqrt{3}}{2}$
	Alternative method for Question 12(c)		
	Area of triangle ABC = $400\sqrt{3}$ or 693 (AWRT) or 4 × Area of half of triangle ABC = $4\times200\sqrt{3}$ or 1390 (AWRT) or Area of rectangle $ABDE$ = $1600\sqrt{3}$ or 2770 (AWRT)	B1	
	[Total area of hexagon = $2\times400\sqrt{3}+1600\sqrt{3}$ =] $2400\sqrt{3}$ Or [= $4\times200\sqrt{3}+1600$ =] $2400\sqrt{3}$	B1	Condone $4800\frac{\sqrt{3}}{2}$
			If B0B0, **SC B1** can be scored for sight of 4160 (AWRT) as final answer.
		2	

Question	Answer	Marks	Guidance
12(d)	Each rectangle area = 40×20 (= 800)	B1	SOI, e.g. by sight of 4800
	Each sector area = $\frac{1}{2}r^2\theta = \frac{1}{2} \times 20^2 \times \frac{\pi}{3}\left[= \frac{200\pi}{3}\right]$	B1	SOI.
	Total area = $2400\sqrt{3} + 4800 + 400\pi$ or 10 200 (cm^2) (AWRT)	B1	Or directly: part **(c)** + 6800 + area circle radius 20.
		3	

Cambridge Assessment International Education

Cambridge International AS & A Level

MATHEMATICS **9709/13**

Paper 1 Pure Mathematics 1 **May/June 2021**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

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This document consists of **17** printed pages.

 [Turn over

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate’s intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	$\left[f(x)=\right]\ 2x^3+\frac{8}{x}\ [+c]$	**B1**	Allow any correct form
	$7 = 16 + 4 + c$	**M1**	Substitute f(2) = 7 into an integral. c must be present. Expect $c = -13$
	$f(x)=\ 2x^3+\frac{8}{x}-13$	**A1**	Allow $y =$, $f(x)$ or y can appear earlier in answer
		3	

Question	Answer	Marks	Guidance
2	$\left[f^{-1}(x)=\right]\left((2x-1)^{1/2}\right)\times\left(\frac{1}{3}\times2\times\frac{3}{2}\right)(-2)$	**B2, 1, 0**	Expect $(2x-1)^{1/2}-2$
	$(2x-1)^{1/2}-2\leqslant 0\ \rightarrow\ 2x-1\leqslant 4$ or $2x-1<4$	**M1**	SOI. Rearranging and then squaring, must have power of ½ not present Allow '=0'at this stage but do not allow '≥ 0' or '>0' If '–2' missed then must see $\leqslant$ or $<$ for the M1
	Value [of a] is 2½ or $a = 2½$	**A1**	WWW, OE e.g. $\frac{5}{2}$, 2.5 Do not allow from '=0' unless some reference to negative gradient.
		4	

Question	Answer	Marks	Guidance
3	$x^2-4x+3=mx-6$ leading to $x^2-x(4+m)+9$	*M1	Equating and gathering terms. May be implied on the next line.
	b^2-4ac leading to $(4+m)^2-4\times 9$	DM1	SOI. Use of the discriminant with *their* a, b and c
	$4+m=\pm 6$ or $(m-2)(m+10)=0$ leading to $m=2$ or -10	A1	Must come from $b^2-4ac=0$ SOI
	Substitute both *their* m values into *their* equation in line 1	DM1	
	$m = 2$ leading to $x = 3$; $m = -10$ leading to $x = -3$	A1	
	(3, 0), (–3, 24)	A1	Accept 'when $x = 3$, $y = 0$; when $x = -3$, $y = 24$' If final A0A0 scored, **SC B1** for one point correct WWW
	Alternative method for Question 3		
	$\frac{dy}{dx}=2x-4 \to 2x-4=m$	*M1	
	$x^2-4x+3=(2x-4)x-6$	DM1	
	$x^2-4x+3=2x^2-4x-6 \to 9=x^2 \to x=\pm 3$	A1	
	$y=0$, 24 or (3, 0), (–3, 24)	A1	
	Substitute both *their* x values into *their* equation in line 1	DM1	Or substitute both *their* (x, y) into $y=mx-6$
	When $x = 3$, $m = 2$; when $x = -3$, $m = -10$	A1	If A0, DM1, A0 scored, **SC B1** for one point correct WWW
		6	

Question	Answer	Marks	Guidance
4(a)	$\dfrac{\tan x+\sin x}{\tan x-\sin x}[=k]$ leading to $\dfrac{\sin x+\sin x\cos x}{\sin x-\sin x\cos x}[=k]$ or $\dfrac{\frac{1}{\cos x}+1}{\frac{1}{\cos x}-1}$ [=k] or $\dfrac{\tan x+\tan x\cos x}{\tan x-\tan x\cos x}$ [=k]	**M1**	Multiply numerator and denominator by $\cos x$, or divide numerator and denominator by tan x or sin x
	$\dfrac{\sin x(1+\cos x)}{\sin x(1-\cos x)}$ or $\dfrac{\frac{1}{\cos x}+1}{\frac{1}{\cos x}-1}.\dfrac{\cos x}{\cos x}$ or $\dfrac{\tan x(1+\cos x)}{\tan x(1-\cos x)}$ leading to $\dfrac{1+\cos x}{1-\cos x}[=k]$	**A1**	AG, WWW
		2	
4(b)	$k-k\cos x=1+\cos x$ leading to $k-1=k\cos x+\cos x$	**M1**	Gather like terms on LHS and RHS
	$k-1=(k+1)\cos x$ leading to $\cos x=\dfrac{k-1}{k+1}$	**A1**	WWW, OE
		2	
4(c)	Obtaining $\cos x$ from *their* **(b)** or **(a)**	**M1**	Expect $\cos x=\dfrac{3}{5}$
	±0.927 (only solutions in the given range)	**A1**	AWRT. Accept ±0.295π
		2	

Question	Answer	Marks	Guidance
5(a)	$\frac{1}{2} \times 4^2 \times \text{angle BAD} = 10$	**M1**	Use of sector area formula
	$\text{Angle BAD} = 1.25$	**A1**	OE. Accept 0.398π, 71.6^o for **SC B1** only
		2	
5(b)	$\text{Arc } BD = 4 \times \textit{their} 1.25$	**M1**	Use of arc length formula. Expect 5.
	$BC = 4\tan(\textit{their} 1.25)$	**M1**	Expect 12.0(4). May use ACB=0.321 or 18.4°
	$CD = \dfrac{4}{\cos(\textit{their} 1.25)} - 4$ or $\sqrt{4^2 + (\textit{their } BC)^2} - 4$	**M1**	Expect 12.69 – 4 = 8.69. May use ACB.
	Perimeter = 5 + 12.0(4) + 8.69 = 25.7 (cm)	**A1**	AWRT
		4	

Question	Answer	Marks	Guidance
6(a)	$f(x) = (x-1)^2 + 4$	**B1**	
	$g(x) = (x+2)^2 + 9$	**B1**	
	$g(x) = f(x+3) + 5$	**B1 B1**	B1 for each correct element. Accept $p = 3, q = 5$
		4	

Question	Answer	Marks	Guidance
6(b)	Translation or Shift	**B1**	
	$\begin{pmatrix}-3\\5\end{pmatrix}$ or acceptable explanation	**B1 FT**	If given as 2 single translations both must be described correctly e.g. $\begin{pmatrix}-3\\0\end{pmatrix}$ & $\begin{pmatrix}0\\5\end{pmatrix}$ FT from *their* $\mathrm{f}(x+p)+q$ *or their* $\mathrm{f}(x)\to\mathrm{g}(x)$ Do not accept $\begin{pmatrix}1\\4\end{pmatrix}$ or $\begin{pmatrix}-2\\9\end{pmatrix}$
		2	

Question	Answer	Marks	Guidance
7(a)	$(a-x)^6 = a^6 - 6a^5x + 15a^4x^2 - 20a^3x^3 + \ldots$	**B2, 1, 0**	Allow extra terms. Terms may be listed. Allow a^6x^0.
		2	
7(b)	$\left(1+\frac{2}{ax}\right)\left(\ldots 15a^4x^2 - 20a^3x^3 + \ldots\right)$ leading to $\left[x^2\right]\left(15a^4 - 40a^2\right)$	**M1**	Attempting to find 2 terms in x^2
	$15a^4 - 40a^2 = -20$ leading to $15a^4 - 40a^2 + 20[=0]$	**A1**	Terms on one side of the equation
	$\left(5a^2-10\right)\left(3a^2-2\right)$ $[=0]$	**M1**	OE. M1 for attempted factorisation or solving for a^2 or u (=a^2) using e.g. formula or completing the square
	$a = \pm\sqrt{2},\ \pm\sqrt{\frac{2}{3}}$	**B1 B1**	OE exact form only If B0B0 scored then **SC B1** for $\sqrt{2}, \sqrt{\frac{2}{3}}$ WWW or ±1.41,±0,816 WWW
		5	

Question	Answer	Marks	Guidance
8(a)	$\left[\mathrm{fg}(x)=\right]1/(2x+1)^2-1$	B1	SOI
	$1/(2x+1)^2-1=3$ leading to $4(2x+1)^2=1$ or $\dfrac{1}{(2x+1)}=[\pm]2$ or $16x^2+16x+3=0$	M1	Setting $\mathrm{fg}(x)=3$ and reaching a stage before $2x+1=\pm½$ or reaching a 3 term quadratic in x
	$2x+1=\pm½$ or $2x+1=-½$ or $(4x+1)(4x+3)[=0]$	A1	Or formula or completing square on quadratic
	$x=-\dfrac{3}{4}$ only	A1	
	Alternative method for Question 8(a)		
	$x^2-1=3$	M1	
	$\mathrm{g}(x)=-2$	A1	
	$\dfrac{1}{(2x+1)}=-2$	M1	
	$x=-\dfrac{3}{4}$ only	A1	
		4	

Question	Answer	Marks	Guidance
8(b)	$y=\frac{1}{(2x+1)^2}-1$ leading to $(2x+1)^2=\frac{1}{y+1}$ leading to $2x+1=[\pm]\frac{1}{\sqrt{y+1}}$	*M1	Obtain $2x+1$ or $2y+1$ as the subject
	$x=[\pm]\frac{1}{2\sqrt{y+1}}-\frac{1}{2}$	DM1	Make $x(\text{or } y)$ the subject
	$-\frac{1}{2\sqrt{x+1}}-\frac{1}{2}$	A1	OE e.g. $-\frac{\sqrt{x+1}}{2x+2}-\frac{1}{2}$, $-\left(\sqrt{\frac{-x}{4x+4}+\frac{1}{4}}+\frac{1}{2}\right)$
		3	

Question	Answer	Marks	Guidance
9(a)	$ar=\frac{24}{100}\times\frac{a}{1-r}$	M1	Form an equation using a numerical form of the percentage and correct formula for u_2 and S_∞
	$100r^2-100r+24[=0]$	A1	OE. All 3 terms on one side of an equation.
	$(20r-8)(5r-3)[=0]\to\ r=\frac{2}{5},\frac{3}{5}$	A1	Dependent on factors or formula seen from their quadratic.
		3	

Question	Answer	Marks	Guidance
9(b)	$3\times\{(a+4d)\}=\{(2(a+1)+11(d+1))\}$	***M1**	SOI Attempt to cross multiply with contents of at least one { } correct
	Simplifies to $a+d=13$	**A1**	
	$\left[\frac{5}{2}\right]\times3\{(2a+4d)\}=\left[\frac{5}{2}\right]\times2\{(4(a+1)+4(d+1))\}$	***M1**	SOI Attempt to cross multiply with contents of at least one { } correct
	Simplifies to $-a+2d=8$	**A1**	
	Solve 2 linear equations simultaneously	**DM1**	Elimination or substitution expected
	$d=7,\ a=6$	**A1**	**SC B1** for a=6, d=7 without complete working
		6	

Question	Answer	Marks	Guidance
10(a)	Gradient of $AB = -\frac{3}{5}$, gradient of $BC = \frac{5}{3}$ or lengths of all 3 sides or vectors	**M1**	Attempting to find required gradients, sides or vectors
	$m_{ab}m_{bc}=-1$ or Pythagoras or $\overrightarrow{AB}.\overrightarrow{BC}=0$ or $\cos ABC=0$ from cosine rule	**A1**	WWW
		2	
10(b)	Centre = mid-point of AC = (2,4)	**B1**	
		1	

Question	Answer	Marks	Guidance
10(c)	$(x - \textit{their}\ x_c)^2 + (y - \textit{their}\ y_c)^2 \left[= r^2\right] or \left(\textit{their}\ x_c - x\right)^2 + \left(\textit{their}\ y_c - y\right)^2 = \left[r^2\right]$	**M1**	Use of circle equation with *their* centre
	$(x-2)^2 + (y-4)^2 = 17$	**A1**	Accept $x^2 - 4x + y^2 - 8y + 3 = 0$ OE
		2	
10(d)	$\left(\frac{x+3}{2}, \frac{y+0}{2}\right) = (2,4)$ or **BE** = 2**BD** = $2\begin{pmatrix}-1\\4\end{pmatrix}$ Or Equation of *BE* is $y = -4(x-3)$ or $y - 4 = -4(x-2)$ leading to $y = -4x + 12$ Substitute equation of *BE* into circle and form a 3-term quadratic.	**M1**	Use of mid-point formula, vectors, steps on a diagram May be seen to find *x* coordinate at *E*
	$(x, y) = (1,8)$ or **OE** = $\begin{pmatrix}3\\0\end{pmatrix} + \begin{pmatrix}-2\\8\end{pmatrix} = \begin{pmatrix}1\\8\end{pmatrix}$	**A1**	*E* = (1, 8) Accept without working for both marks **SC B2**
	Gradient of *BD*, *m*, = –4 or gradient $AC = \frac{1}{4}$ = gradient of tangent	**B1**	Or gradient of *BE* = -4
	Equation of tangent is $y - 8 = ¼(x - 1)$ OE	**M1 A1**	For M1, equation through *their* E or (1, 8) (not, *A*, *B* or *C*) and with gradient $\frac{-1}{\textit{their} - 4}$
		5	

Question	Answer	Marks	Guidance
11(a)	$\dfrac{dy}{dx} = \dfrac{1}{2}x^{-1/2} - \dfrac{1}{2}k^2x^{-3/2}$	B1 B1	Allow any correct unsimplified form
	$\dfrac{1}{2}x^{-1/2} - \dfrac{1}{2}k^2x^{-3/2} = 0$ leading to $\dfrac{1}{2}x^{-1/2} = \dfrac{1}{2}k^2x^{-3/2}$	M1	OE. Set to zero and one correct algebraic step towards the solutions. $\dfrac{dy}{dx}$ must only have 2 terms.
	$(k^2, 2k)$	A1	
		4	
11(b)	When $x = 4k^2$, $\dfrac{dy}{dx} = \left[\dfrac{1}{4k} - \dfrac{1}{16k} = \right]\dfrac{3}{16k}$	B1	OE
	$y = \left[2k + k^2 \times \dfrac{1}{2k}\right] = \dfrac{5k}{2}$	B1	OE. Accept $2k + \dfrac{k}{2}$
	Equation of tangent is $y - \dfrac{5k}{2} = \dfrac{3}{16k}(x - 4k^2)$ or $y = mx + c \rightarrow \dfrac{5k}{2} = \dfrac{3}{16k}(4k^2) + c$	M1	Use of line equation with *their* gradient and $(4k^2, \textit{their } y)$,
	When $x = 0$, $y = \left[\dfrac{5k}{2} - \dfrac{3k}{4} = \right]\dfrac{7k}{4}$ or from $y = mx + c$, $c = \dfrac{7k}{4}$	A1	OE
		4	

Question	Answer	Marks	Guidance
11(c)	$\int\left(x^{\frac{1}{2}}+k^2x^{-\frac{1}{2}}\right)\mathrm{d}x=\dfrac{2x^{\frac{3}{2}}}{3}+2k^2x^{\frac{1}{2}}$	**B1**	Any unsimplified form
	$\left(\dfrac{16k^3}{3}+4k^3\right)-\left(\dfrac{9k^3}{4}+3k^3\right)$	**M1**	Apply limits $\dfrac{9}{4}k^2 \to 4k^2$ to an integration of y. M0 if volume attempted.
	$\dfrac{49k^3}{12}$	**A1**	OE. Accept $4.08\,k^3$
		3	

Cambridge Assessment International Education

Cambridge International AS & A Level

MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **March 2021**

MARK SCHEME

Maximum Mark: 75

Published

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- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
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SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1(a)	$1+5x+10x^2$	**B1**	
		1	
1(b)	$1-12x+60x^2$	**B2, 1, 0**	B2 all correct, B1 for two correct components.
		2	
1(c)	$(1+5x+10x^2)(1-12x+60x^2)$ leading to $60-60+10$	**M1**	3 products required
	10	**A1**	Allow $10x^2$
		2	

Question	Answer	Marks	Guidance
2	$u=2x-3$ leading to $u^4-3u^2-4\ [=0]$	**M1**	Or $u=(2x-3)^2$ leading to $u^2-3u-4\ [=0]$
	$(u^2-4)(u^2+1)\ [=0]$	**M1**	Or $(u-4)(u+1)\ [=0]$
	$2x-3=[\pm]2$	**A1**	
	$x=\frac{1}{2}, \frac{5}{2}$ **only**	**A1**	
		4	

Question	Answer	Marks	Guidance
3	$\tan\theta + 2\sin\theta = 3\tan\theta - 6\sin\theta$ leading to $2\tan\theta - 8\sin\theta\ [=0]$	**M1**	OE
	$2\sin\theta - 8\sin\theta\cos\theta\ (=0)$ leading to $[2]\sin\theta(1-4\cos\theta)\ [=0]$	**M1**	
	$\cos\theta = \frac{1}{4}$	**A1**	Ignore $\sin\theta = 0$
	$\theta = 75.5°$ **only**	**A1**	
		4	

Question	Answer	Marks	Guidance
4	$x^2 + kx + 6 = 3x + k$ leading to $x^2 + x(k-3) + (6-k)\ [=0]$	**M1**	Eliminate y and form 3-term quadratic.
	$(k-3)^2 - 4(6-k)[>0]$	**M1**	OE. Apply $b^2 - 4ac$.
	$k^2 - 2k - 15[>0]$	**A1**	Form 3-term quadratic.
	$(k+3)(k-5)[>0]$	**A1**	Or $k = -3,\ 5$ from use of formula or completing square.
	$k < -3,\quad k > 5$	**A1 FT**	Or any correct alternative notation, do not allow $\leqslant, \geqslant$. FT for *their* outside regions.
		5	

Question	Answer	Marks	Guidance
5(a)	(Stretch) (factor 3 in y direction **or** parallel to the y-axis)	**B1 B1**	
	(Translation) $\begin{pmatrix} 4 \\ 0 \end{pmatrix}$	**B1 B1**	Allow Translation 4 (units) in x direction. N.B. Transformations can be given in either order.
		4	
5(b)	$[y =]$ $3f(x-4)$	**B1 B1**	B1 for 3 , B1 for $(x-4)$ with no extra terms.
		2	

Question	Answer	Marks	Guidance
6(a)	At $x = 1$, $\frac{dy}{dx} = 6$	**B1**	
	$\frac{dx}{dt} = \left(\frac{dx}{dy} \times \frac{dy}{dt}\right) = \frac{1}{6} \times 3 = \frac{1}{2}$	**M1 A1**	Chain rule used correctly. Allow alternative and minimal notation.
		3	

Question	Answer	Marks	Guidance
6(b)	$[y=]\left(\dfrac{6(3x-2)^{-2}}{-2}\right)\div(3)\ [+c]$	B1 B1	
	$-3=-1+c$	M1	Substitute $x=1,\ y=-3$. c must be present.
	$y=-(3x-2)^{-2}-2$	A1	OE. Allow f(x)=
		4	

Question	Answer	Marks	Guidance
7(a)	$[\mathrm{f}(x)=](x+1)^2+2$	B1 B1	Accept $a=1, b=2$.
	Range [of f is (y)] $\geqslant 2$	B1FT	OE. Do not allow $x \geqslant 2$, FT on *their b*.
		3	
7(b)	$y=(x+1)^2+2$ leading to $x=[\pm]\sqrt{y-2}-1$	M1	Or by using the formula. Allow one sign error.
	$\mathrm{f}^{-1}(x)=-\sqrt{x-2}-1$	A1	
		2	

Question	Answer	Marks	Guidance
7(c)	$2(x^2+2x+3)+1=13$	B1	Or using a correct completed square form of f(x).
	$2x^2+4x-6[=0]$ leading to $(2)(x-1)(x+3)[=0]$	B1	Or $x=1, x=-3$ using formula or completing square. Must reach 2 solutions.
	$x=-3$ **only**	B1	
		3	

Question	Answer	Marks	Guidance
8(a)	Centre of circle is (4, 5)	B1 B1	
	$r^2=(7-4)^2+(1-5)^2$	M1	OE. Either using *their* centre and A or C **or** using A and C and dividing by 2.
	$r=5$	A1 FT	FT on *their* (4, 5) if used.
	Equation is $(x-4)^2+(y-5)^2=25$	A1	OE. Allow 5^2 for 25.
		5	
8(b)	Gradient of radius = $\dfrac{9-5}{7-4}=\dfrac{4}{3}$	B1 FT	FT for use of *their* centre.
	Equation of tangent is $y-9=-\dfrac{3}{4}(x-7)$	B1	or $y=\dfrac{-3x}{4}+\dfrac{57}{4}$
		2	

Question	Answer	Marks	Guidance
9(a)(i)	$\dfrac{\cos\theta}{1-r}=\dfrac{1}{\cos\theta}$	**B1**	
	$1-r=\cos^2\theta$ leading to $r=1-\cos^2\theta$	**M1**	Eliminate fractions
	$r=\sin^2\theta$ leading to 2nd term = $\cos\theta\sin^2\theta$	**A1**	AG
		3	
9(a)(ii)	$S_{12}=\dfrac{\cos\left(\frac{\pi}{3}\right)\left[1-\left(\sin^2\left(\frac{\pi}{3}\right)\right)^{12}\right]}{1-\sin^2\left(\frac{\pi}{3}\right)}=\dfrac{0.5\left[1-(0.75)^{12}\right]}{1-0.75}$	**M1**	Evidence of correct substitution, use of S_n formula and attempt to evaluate
	1.937	**A1**	
		2	
9(b)	$[d=]\cos\theta\sin^2\theta-\cos\theta$	**M1**	Use of $d=u_2-u_1$
	$-\dfrac{1}{8}$	**A1**	
	[85th term =] $\dfrac{1}{2}+84\times-\dfrac{1}{8}$	**M1**	Use of $a+84d$ with a calculated value of d
	-10	**A1**	
		4	

Question	Answer	Marks	Guidance
10(a)	$\Delta ADE = \frac{1}{2}(ka)^2 \sin\frac{\pi}{6}$	**M1**	Attempt to find the area of ΔADE.
	$\frac{1}{4}k^2a^2$	**A1**	OE.
	Sector $ABC = \frac{1}{2}a^2\frac{\pi}{6}$	**B1**	
	$2\times\frac{1}{4}k^2a^2 = \frac{1}{2}a^2\frac{\pi}{6}$	**M1**	OE. For $2\times\Delta ADE = \text{sector}ABC$ with at least one correct area.
	$k = \left(\sqrt{\frac{\pi}{6}}\right) = 0.7236$	**A1**	
		5	
10(b)	$2\times\frac{1}{2}(ka)^2\sin\theta = \frac{1}{2}a^2\theta$	**M1**	Condone omission of '2' or '1/2' on LHS for M1 only.
	$k^2 = \frac{\theta}{2\sin\theta}$	**A1**	
	$k^2 > \frac{1}{2}$ leading to $\frac{1}{\sqrt{2}} < k < 1$	**A1**	OE. Accept $k > \frac{1}{\sqrt{2}}$ or $k > 0.707$ (AWRT) or $0.707\text{(AWRT)} < k < 1$ or $k > \sqrt{\frac{1}{2}}$ OE
		3	

Question	Answer	Marks	Guidance
11(a)	$9\left(x^{-\frac{1}{2}}-4x^{-\frac{3}{2}}\right)=0$ leading to $9x^{-\frac{3}{2}}(x-4)=0$	M1	OE. Set y to zero and attempt to solve.
	$x=4$ **only**	A1	From use of a correct method.
		2	
11(b)	$\frac{dy}{dx}=9\left(-\frac{1}{2}x^{-\frac{3}{2}}+6x^{-\frac{5}{2}}\right)$	B2, 1, 0	B2; all 3 terms correct: 9, $-\frac{1}{2}x^{-\frac{3}{2}}$ and $6x^{-\frac{5}{2}}$ B1; 2 of the 3 terms correct
	At $x = 4$ gradient = $9\left(-\frac{1}{16}+\frac{6}{32}\right)=\frac{9}{8}$	M1	Using *their* $x = 4$ in *their* differentiated expression and attempt to find equation of the tangent.
	Equation is $y=\frac{9}{8}(x-4)$	A1	or $y=\frac{9x}{8}-\frac{9}{2}$ OE
		4	
11(c)	$9x^{-\frac{5}{2}}\left(-\frac{1}{2}x+6\right)=0$	M1	Set *their* $\frac{dy}{dx}$ to zero and an attempt to solve.
	$x=12$	A1	Condone $(\pm)12$ from use of a correct method.
		2	

Question	Answer	Marks	Guidance
11(d)	$\int 9\left(x^{-\frac{1}{2}} - 4x^{-\frac{3}{2}}\right)dx = 9\left(\frac{x^{\frac{1}{2}}}{\frac{1}{2}} - \frac{4x^{-\frac{1}{2}}}{-\frac{1}{2}}\right)$	**B2, 1, 0**	B2; all 3 terms correct: 9, $\frac{x^{\frac{1}{2}}}{\frac{1}{2}}$, $\frac{-4x^{-\frac{1}{2}}}{-\frac{1}{2}}$ B1; 2 of the 3 terms correct
	$9\left[\left(6+\frac{8}{3}\right)-(4+4)\right]$	**M1**	Apply limits *their* 4 → 9 to an integrated expression with no consideration of other areas.
	6	**A1**	Use of π scores A0
		4	

Cambridge International AS & A Level

MATHEMATICS **9709/11**

Paper 1 Pure Mathematics 1 **October/November 2020**

MARK SCHEME

Maximum Mark: 75

Published

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AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	$2x^2 + 5 = mx - 3 \rightarrow 2x^2 - mx + 8\ (= 0)$	**B1**	Form 3-term quadratic
	$m^2 - 64$	**M1**	Find $b^2 - 4ac$.
	$-8 < m < 8$	**A1**	Accept (-8, 8) and equality included
		3	

Question	Answer	Marks	Guidance
2	$(y =)\left[-(x-3)^{-1}\right]\left[+\frac{1}{2}x^2\right]\ (+c)$	**B1 B1**	
	$7 = 1 + 2 + c$	**M1**	Substitute $x = 2$, $y = 7$ into an integrated expansion (c present). Expect $c = 4$
	$y = -(x-3)^{-1} + \frac{1}{2}x^2\ +4$	**A1**	OE
		4	

Question	Answer	Marks	Guidance
3	(Derivative =) $4\pi r^2$ ($\rightarrow$ 400π)	**B1**	SOI Award this mark for $\frac{\mathrm{d}r}{\mathrm{d}V}$
	$\frac{50}{\textit{their}\text{ derivative}}$	**M1**	Can be in terms of r
	$\frac{1}{8\pi}$ or 0.0398	**A1**	AWRT
		3	

Question	Answer	Marks	Guidance
4	$(y=)[3]+[2]\left[\cos\frac{1}{2}\theta\right]$	**B1 B1 B1**	
		3	

Question	Answer	Marks	Guidance
5(a)	$6C2\times\left[2\left(x^2\right)\right]^4\times\left[\frac{a}{(x)}\right]^2$, $6C3\times\left[2\left(x^2\right)\right]^3\times\left[\frac{a}{(x)}\right]^3$	**B1 B1**	SOI Can be seen in an expansion
	$15\times2^4\times a^2=20\times2^3\times a^3$	**M1**	SOI Terms must be from a correct series
	$a=\frac{15\times2^4}{20\times2^3}=\frac{3}{2}$	**A1**	OE
		4	

Question	Answer	Marks	Guidance
5(b)	0	**B1**	
		1	

Question	Answer	Marks	Guidance
6	$\frac{dy}{dx} = \left[\frac{1}{2}\left(25 - x^2\right)^{-1/2}\right] \times [-2x]$	**B1 B1**	
	$\frac{-x}{\left(25 - x^2\right)^{1/2}} = \frac{4}{3} \to \frac{x^2}{25 - x^2} = \frac{16}{9}$	**M1**	Set $= \frac{4}{3}$ and square both sides
	$16\left(25 - x^2\right) = 9x^2 \to 25x^2 = 400 \to x = (\pm)4$	**A1**	
	When $x = -4$, $y = 5 \to (-4, 5)$	**A1**	
		5	

Question	Answer	Marks	Guidance
7(a)	$\left(\frac{\sin\theta}{1-\sin\theta}-\frac{\sin\theta}{1+\sin\theta}=\right)\frac{\sin\theta(1+\sin\theta)-\sin\theta(1-\sin\theta)}{1-\sin^2\theta}$	*M1	Put over a single common denominator
	$\frac{2\sin^2\theta}{\cos^2\theta}$	DM1	Replace $1-\sin^2\theta$ by $\cos^2\theta$ and simplify numerator
	$2\tan^2\theta$	A1	AG
		3	
7(b)	$2\tan^2\theta=8 \rightarrow \tan\theta=(\pm)2$	B1	SOI
	$(\theta=)$ 63.4°, 116.6°	B1 B1 FT	FT on 180 – 1st solution (with justification)
		3	

Question	Answer	Marks	Guidance
8(a)	$S=\frac{a}{1-r}$, $2S=\frac{a}{1-R}$	B1	SOI at least one correct
	$\frac{2a}{1-r}=\frac{a}{1-R}$	M1	SOI
	$2-2R=1-r \rightarrow r=2R-1$	A1	AG
		3	

Question	Answer	Marks	Guidance
8(b)	$ar^2 = aR \rightarrow (a)(2R-1)^2 = R(a)$	*M1	
	$4R^2 - 5R + 1\ (=0) \rightarrow (4R-1)(R-1)\ (=0)$	DM1	Allow use of formula or completing square.
	$R = \frac{1}{4}$	A1	Allow $R = 1$ in addition
	$S = \frac{2a}{3}$	A1	
	Alternative method for question 8(b)		
	$ar^2 = aR \rightarrow (a)r^2 = ½(r+1)(a)$	*M1	Eliminating 1 variable
	$2r^2 - r - 1\ (=0) \rightarrow (2r+1)(r-1)\ (=0)$	DM1	Allow use of formula or completing square. Must solve a quadratic.
	$r = -\frac{1}{2}$	A1	Allow $r = 1$ in addition
	$S = \frac{2a}{3}$	A1	
		4	

Question	Answer	Marks	Guidance
9(a)	$m_{AB} = \dfrac{4-2}{-1-3} = -\dfrac{1}{2}$	**B1**	
	Equation of tangent is $y-2=2(x-3)$	**B1 FT**	(3, 2) with *their* gradient $-\dfrac{1}{m_{AB}}$
		2	
9(b)	$AB^2 = 4^2 + 2^2 = 20$ or $r^2 = 20$ or $r = \sqrt{20}$ or $AB = \sqrt{20}$	**B1**	
	Equation of circle centre B is $(x-3)^2 + (y-2)^2 = 20$	**M1 A1**	FT *their* 20 for M1
		3	
9(c)	$(x-3)^2 + (2x-6)^2 = \textit{their } 20$	**M1**	Substitute *their* $y-2=2x-6$ into *their* circle, centre B
	$5x^2 - 30x + 25 = 0$ or $5(x-3)^2 = 20$	**A1**	
	$[(5)(x-5)(x-1)$ or $x-3=\pm 2]$ $x = 5, 1$	**A1**	
		3	

Question	Answer	Marks	Guidance
10(a)	$\left(\sin\theta = \frac{r}{OC} \to\right) OC = \frac{r}{\sin\theta}$	**M1 A1**	
	$CD = r + \frac{r}{\sin\theta}$	**A1**	
		3	
10(b)	Radius of arc $AB = 4 + \frac{4}{\sin\frac{\pi}{6}} = 4 + 8 = 12$	**B1**	SOI
	(Arc $AB =$) *their* $12 \times \frac{2\pi}{6}$ or $\left(\frac{1}{2}AB =\right)\left(\textit{their } 12 \times \frac{\pi}{6}\right)$	**M1**	Expect 4π, must use *their* CD, not 4
	Perimeter $= 24 + 4\pi$	**A1**	
		3	

Question	Answer	Marks	Guidance
10(c)	Area $FOC = \frac{1}{2} \times 4 \times \textit{their } OC \times \sin\frac{\pi}{3}$	**M1**	
	$8\sqrt{3}$	**A1**	
	Area sector $FOE = \frac{1}{2} \times \frac{2\pi}{3} \times 4^2 = \frac{16\pi}{3}$	**B1**	
	Shaded area $= 16\sqrt{3} - \frac{16\pi}{3}$	**A1**	
	Alternative method for question 10(c)		
	$FC = \sqrt{(\textit{their } OC)^2 - 4^2}$	**M1**	$\sqrt{48}$ or $4\sqrt{3}$
	Area $FOC = \frac{1}{2} \times 4 \times 4\sqrt{3} = 8\sqrt{3}$	**A1**	
	Area of half sector $FOE = \frac{1}{2} \times \frac{\pi}{3} \times 4^2 = \frac{8\pi}{3}$	**B1**	
	Shaded area $= 16\sqrt{3} - \frac{16\pi}{3}$	**A1**	
		4	

Question	Answer	Marks	Guidance
11(a)	$\text{fg}(x) = (2x+1)^2 + 3$	B1	OE
		1	
11(b)	$y = (2x+1)^2 + 3 \rightarrow 2x + 1 = (\pm)\sqrt{y-3}$	M1	1st two operations. Allow one sign error or x/y interchanged
	$x = (\pm)\frac{1}{2}\left(\sqrt{y-3} - 1\right)$	M1	OE 2nd two operations. Allow one sign error or x/y interchanged
	$\left(\text{fg}^{-1}(x) =\right) \frac{1}{2}\left(\sqrt{x-3} - 1\right)$ for $(x) > 3$	A1 B1	Allow $(3, \infty)$
		4	
11(c)	$\text{gf}(x) = 2\left(x^2+3\right)+1$	B1	SOI
	$(2x+1)^2 + 3 - 3 = 2\left(x^2+3\right)+1 \rightarrow 2x^2 + 4x - 6\ (= 0)$	*M1	Express as 3-term quadratic
	$(2)(x+3)(x-1)\ (= 0)$	DM1	Or quadratic formula or completing the square
	$x = 1$	A1	
		4	

Question	Answer	Marks	Guidance
12(a)	$4x^{\frac{1}{2}} - 2x = 3 - x \rightarrow x - 4x^{\frac{1}{2}} + 3 (= 0)$	*M1	3-term quadratic. Can be expressed as e.g. $u^2 - 4u + 3$ (=0)
	$\left(x^{\frac{1}{2}} - 1\right)\left(x^{\frac{1}{2}} - 3\right) (= 0)$ or $(u-1)(u-3)(= 0)$	DM1	Or quadratic formula or completing square
	$x^{\frac{1}{2}} = 1, 3$	A1	SOI
	$x = 1, 9$	A1	
	Alternative method for question 12(a)		
	$\left(4x^{\frac{1}{2}}\right)^2 = (3 + x)^2$	*M1	Isolate $x^{\frac{1}{2}}$
	$16x = 9 + 6x + x^2 \rightarrow x^2 - 10x + 9 \ (= 0)$	A1	3-term quadratic
	$(x-1)(x-9) \ (= 0)$	DM1	Or formula or completing square on a quadratic obtained by a correct method
	$x = 1, 9$	A1	
		4	
12(b)	$\frac{dy}{dx} = 2x^{1/2} - 2$	*B1	
	$\frac{dy}{dx}$ or $2x^{1/2} - 2 = 0$ when $x = 1$ hence B is a stationary point	DB1	
		2	

Question	Answer	Marks	Guidance
12(c)	Area of correct triangle = $\frac{1}{2}$ (9 – 3) × 6	**M1**	or $\int_3^9 (3-x)(\mathrm{d}x) = \left[3x - \frac{1}{2}x^2\right] \rightarrow\ -18$
	$\int (4x^{\frac{1}{2}} - 2x)(\mathrm{d}x) = \left[\frac{4x^{\frac{3}{2}}}{\frac{3}{2}} - x^2\right]$	**B1 B1**	
	$(72-81) - \left(\frac{64}{3} - 16\right)$	**M1**	Apply limits 4 → *their* 9 to an integrated expression
	$-14\frac{1}{3}$	**A1**	OE
	Shaded region = $18 - 14\frac{1}{3} = 3\frac{2}{3}$	**A1**	OE
		6	

Cambridge International AS & A Level

MATHEMATICS	**9709/12**
Paper 1 Pure Mathematics 1	**October/November 2020**
MARK SCHEME	
Maximum Mark: 75	

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2020 series for most Cambridge IGCSE™, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	Coefficient of x^3 in $(1-2x)^5$ is -80	**B1**	Can be seen in an expansion but must be simplified correctly.
	Coefficient of x^2 in $(1-2x)^5$ is 40	**B1**	
	Coefficient of x^3 in $(1+kx)(1-2x)^5$ is $40k - 80 = 20$	**M1**	Uses the relevant two terms to form an equation = 20 and solves to find k. Condone x^3 appearing in some terms if recovered.
	$(k =)\dfrac{5}{2}$	**A1**	
		4	

Question	Answer	Marks	Guidance
2	$(-2p)^2 = (2p+6) \times (p+2)$ or $\dfrac{-2p}{2p+6} = \dfrac{p+2}{-2p}$	M1	OE. Using "a, b, c then $b^2 = ac$" or $a = 2p+6$, $ar = -2p$ and $ar^2 = p+2$ to form a correct relationship in terms of p only
	$(2p^2 - 10p - 12 = 0)$ $p = 6$	A1	
	$a = 18$ and $r = -⅔$	A1	
	$(s_\infty) =$ *their* $a \div (1 -$ *their* $r)$ $\left(=18 \div \dfrac{5}{3}\right)$	M1	Correct formula used with their values for a and r , $\lvert r \rvert < 1$ Both a & r from the same value of p.
	$(s_\infty =)10.8$	A1	OE. A0 if an extra solution given
			SC B2 for $s_\infty = \dfrac{2p+6}{1-\dfrac{-2p}{2p+6}}$ *or* $\dfrac{2p+6}{1-\dfrac{p+2}{-2p}}$ ignore any subsequent algebraic simplification.
		5	

Question	Answer	Marks	Guidance
3	$2x^2 + m(2x+1) - 6x - 4(=0)$	*M1	y eliminated and all terms on one side with correct algebraic steps. Condone ± errors
	Using $b^2 - 4ac$ on $2x^2 + x(2m-6) + m - 4 \ (=0)$	DM1	Any use of discriminant with their *a*, *b* and *c* identified correctly.
	$4m^2 - 32m + 68$ **or** $2m^2 - 16m + 34$ **or** $m^2 - 8m + 17$	A1	
	$(2m-8)^2 + k$ **or** $(m-4)^2 + k$ **or** minimum point $(4, k)$ **or** finds $b^2 - 4ac \ (= -4, -16, -64)$	DM1	OE. Any valid method attempted on their 3-term quadratic
	$(m-4)^2 + 1$ oe + always > 0 → 2 solutions for all values of *m* **or** Minimum point (4,1) + (fn) always > 0 → 2 solutions for all values of *m* **or** $b^2 - 4ac < 0$ + no solutions → 2 solutions for the original equation for all values of *m*	A1	Clear and correct reasoning and conclusion without wrong working.
		5	

Question	Answer	Marks	Guidance
4	S_x *and* S_{x+1}	**M1**	Using two values of n in the given formula
	$a = 5$, $d = 2$	**A1 A1**	
	$a + (n - 1)\,d > 200 \rightarrow 5 + 2(k - 1) > 200$	**M1**	Correct formula used with their a and d to form an equation or inequality with 200, condone use of n
	$(k =)$ 99	**A1**	Condone $\geqslant 99$
	Alternative method for question 4		
	$\frac{n}{2}(\,2a + (n-1)\,d\,) \equiv n^2 + 4n \rightarrow \left(\frac{d}{2} = 1\,,\; a - \frac{1}{2}d = 4\right)$	**M1**	Equating two correct expressions of S_n and equating coefficients of n and n^2
	$d = 2$, $a = 5$	**A1 A1**	
	$a + (n - 1)\,d > 200 \rightarrow 5 + 2(k - 1) > 200$	**M1**	Correct formula used with their a and d to form an equation or inequality with 200, condone use of n
	$(k =)$ 99	**A1**	Condone $\geqslant 99$
	Alternative method for question 4		
	$sum_k - sum_{k-1} \rightarrow k^2 + 4k - (k-1)^2 - 4(k-1)$	**M1 A1**	Using given formula with consecutive expressions subtracted. Allow k+1 and k.
	$2k + 3 > 200$ or $= 200$	**M1 A1**	Simplifying to a linear equation or inequality
	$(k =)$ 99	**A1**	Condone $\geqslant 99$
		5	

Question	Answer	Marks	Guidance
5(a)	0	**B1**	
		1	
5(b)	$(\mathrm{f}^{-1}(x)) = \dfrac{x+2}{4}$, $(\mathrm{g}^{-1}(x)) = \dfrac{4-x}{x}$ or $\dfrac{4}{x}-1$	**B1 B1**	OE. Sight of correct inverses.
	$x^2 + 6x - 16$ $(= 0)$	**B1**	Equating inverses and simplifying.
	$(x+8)$ and $(x-2)$	**M1**	Correct attempt at solution of *their* 3-term quadratic-factorising, completing the square or use of formula.
	$(x =)$ 2 or -8	**A1**	Do not accept answers obtained with no method shown.
		5	

Question	Answer	Marks	Guidance
6(a)	$\left(\frac{1}{\cos x}-\frac{\sin x}{\cos x}\right)\left(\frac{1}{\sin x}+1\right)$	**B1**	Uses "tan$x$ = sin$x$ ÷ cos$x$" throughout
	$\left(\frac{1-\sin x}{\cos x}\right)\left(\frac{1+\sin x}{\sin x}\right)$ or $\left(\frac{1-\sin^2 x}{\cos x\sin x}\right)$	**M1**	Correct algebra leading to two or four terms
	$\left(\frac{\cos^2 x}{\cos x\sin x}\right)$	**A1**	OE. A correct expression which can be cancelled directly to $\frac{\cos x}{\sin x}$ e.g. $\frac{\cos x\left(1-\sin^2 x\right)}{\sin x\left(1-\sin^2 x\right)}$
	$\left(\frac{\cos^2 x}{\cos x\sin x}\right)=\left(\frac{\cos x}{\sin x}\right)=\frac{1}{\tan x}$	**A1**	AG. Must show cancelling. If x is missing throughout their working withhold this mark.
		4	
6(b)	Uses **(a)** → $\frac{1}{\tan x}=2\tan^2 x$ $\tan^3 x=\frac{1}{2}$	**M1**	Reducing to $\tan^3 x = k$.
	(x =) 38.4°	**A1**	AWRT. Ignore extra answers outside the range 0 to 180° but A0 if within.
		2	

Question	Answer	Marks	Guidance
7(a)	$\text{f}'(4)\left(=\frac{5}{2}\right)$	*M1	Substituting 4 into $\text{f}'(x)$
	$\left(\frac{\text{d}y}{\text{d}t}=\frac{\text{d}y}{\text{d}x}\times\frac{\text{d}x}{\text{d}t}\right)\to\left(\frac{\text{d}y}{\text{d}t}\right)=\frac{5}{2}\times 0.12$	DM1	Multiplies *their* $\text{f}'(4)$ by 0.12
	$\left(\frac{\text{d}y}{\text{d}t}=\right)0.3$	A1	OE
		3	
7(b)	$\frac{6x^{\frac{1}{2}}}{\frac{1}{2}}-\frac{4x^{-\frac{1}{2}}}{-\frac{1}{2}}(+c)$	B1 B1	B1 for each unsimplified integral.
	Uses (4, 7) leading to c = (-21)	M1	Uses (4, 7) to find a c value
	y or $\text{f}(x)=12x^{\frac{1}{2}}+8x^{-\frac{1}{2}}-21$ or $12\sqrt{x}+\frac{8}{\sqrt{x}}-21$	A1	Need to see y or f(x) = somewhere in *their* solution and 12 and 8
		4	

Question	Answer	Marks	Guidance
8(a)	Use of correct formula for the area of triangle ABC	**M1**	Use of 180–2θ scores M0. Condone 2π–2θ
	$\frac{1}{2}r^2\sin(\pi-2\theta)$ **or** $\frac{1}{2}r^2\sin 2\theta$ **or** $2\times\frac{1}{2}r\times r\cos\theta\times\sin\theta$ **or** $2\times\frac{1}{2}r\cos\theta\times r\sin\theta$	**A1**	OE
	[Shaded area = triangle – sector] = *their* triangle area $-\frac{1}{2}r^2\theta$	**B1 FT**	FT for *their* triangle area $-\frac{1}{2}r^2\theta$ (Condone use of 180 degrees for triangle area for B1)
		3	
8(b)	Arc $BD = r\theta = 6$ cm	**B1**	SOI
	$AC = 2r\cos\theta = (2\times10\cos0.6 = 20\cos0.6 = 16.506)$ or $\sqrt{(2r^2-2r^2\cos(\pi-2\theta))}$ or $\frac{r\times\sin(\pi-2\theta)}{\sin\theta}$	***M1**	Finding AC or $\frac{1}{2}AC$ (= 8.25)
	$DC = 2r\cos\theta - r$ or $\sqrt{(2r^2-2r^2\cos(\pi-2\theta))}-r$ (= 6.506)	**DM1**	Subtracting r from *their* AC or $r-r\cos\theta$ from *their* half AC (8.25-1.75)
	(Perimeter = 10 + 6 + 6.506 =) 22.5	**A1**	AWRT
		4	

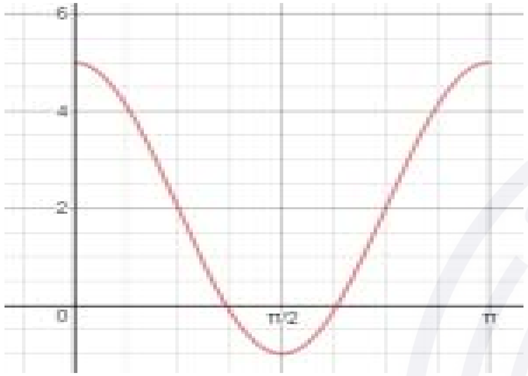
Question	Answer	Marks	Guidance
9(a)	$r = \sqrt{(6^2+3^2)}$ or $r^2 = 45$	**B1**	Sight of r = 6.7 implies B1
	$(x-5)^2+(y-1)^2=r^2$ or $x^2-10x+y^2-2y=r^2-26$	**M1**	Using centre given and *their* radius or r in correct formula
	$(x-5)^2+(y-1)^2=45$ or $x^2-10x+y^2-2y=19$	**A1**	Do not allow $\left(\sqrt{45}\right)^2$ for r^2
		3	
9(b)	C has coordinates (11, 4)	**B1**	
	0.5	**B1**	OE, Gradient of AB, BC or AC.
	Grad of CD $=-2$	**M1**	Calculation of gradient needs to be shown for this M1.
	$(\frac{1}{2}\times-2=-1)$ then states + perpendicular → hence shown or tangent	**A1**	Clear reasoning needed.
	Alternative method for question 9(b)		
	C has coordinates (11, 4)	**B1**	
	0.5	**B1**	OE, Gradient of AB, BC or AC.
	Gradient of the perpendicular is −2 → Equation of the perpendicular is $y-4=-2(x-11)$	**M1**	Use of $m_1m_2=-1$ with *their* gradient of AB, BC or AC and correct method for the equation of the perpendicular. Could use D(5, 16) instead of C(11,4).
	Checks D(5, 16) **or** checks gradient of CD and then states D lies on the line **or** CD has gradient −2→ hence shown or tangent	**A1**	Clear check and reasoning needed. Checks that the other point lies on the line or checks gradient.

Question	Answer	Marks	Guidance
9(b)	**Alternative method for question 9(b)**		
	C has coordinates (11, 4) **or** Gradient of AB, BC or AC = 0.5	**B1**	Only one of AB, BC or AC needed.
	Equation of the perpendicular is $y-4=-2(x-11)$	**B1**	Finding equation of CD.
	$(x-5)^2+(-2x+26-1)^2=45 \rightarrow (x^2-22x+121=0)$	**M1**	Solving simultaneously with the equation of the circle.
	$(x-11)^2=0$ or $b^2-4ac=0$ → repeated root → hence shown or tangent	**A1**	Must state repeated root.
	Alternative method for question 9(b)		
	C has coordinates (11, 4)	**B1**	
	Finding $CD=\sqrt{180}$ and $BD=\sqrt{225}$	**B1**	OE. Calculated from the co-ordinates of B, C & D without using r.
	Checking (their BD)^2 – (their CD)^2 is the same as (their r)^2	**M1**	
	∴ Pythagoras valid ∴ perpendicular → hence shown or tangent	**A1**	Triangle ACD could be used instead.
	Alternative method for question 9(b)		
	C has coordinates (11, 4)	**B1**	
	Finding vectors $\overrightarrow{AC}$ and $\overrightarrow{CD}$ **or** $\overrightarrow{BC}$ and $\overrightarrow{CD}$ $\left(= \begin{pmatrix}6\\3\end{pmatrix} and \begin{pmatrix}-6\\12\end{pmatrix}\right.$ **or** $\left.\begin{pmatrix}12\\6\end{pmatrix} and \begin{pmatrix}-6\\12\end{pmatrix}\right)$	**B1**	Must be correct pairing.
	Applying the scalar product to one of these pairs of vectors	**M1**	Accept *their* $\overrightarrow{AC}$ and $\overrightarrow{CD}$ or *their* $\overrightarrow{BC}$ and $\overrightarrow{CD}$
	Scalar product = 0 then states ∴ perpendicular → hence shown or tangent	**A1**	
		4	

Question	Answer	Marks	Guidance
9(c)	$E\,(-1, 4)$	**B1 B1**	WWW B1 for each coordinate Note: Equation of DE which is $y = 2x + 6$ may be used to find E
		2	

Question	Answer	Marks	Guidance
10(a)	$\left(\frac{dy}{dx}\right) = [8] \times \left[(3-2x)^{-3}\right] + [-1]$ $\left(= \frac{8}{(3-2x)^3} - 1\right)$	**B2, 1, 0**	B2 for all three elements correct, B1 for two elements correct, B0 for only one or no elements correct.
	$\frac{d^2y}{dx^2} = -3 \times 8 \times (3-2x)^{-4} \times (-2)$ $\left(= \frac{48}{(3-2x)^4}\right)$	**B1 FT**	FT providing *their* bracket is to a negative power
	$\int y\,dx = [(3-2x)^{-1}]\ [2 \div (-1 \times -2)]\ [-\tfrac{1}{2}x^2]\ (+c)$ $\left(= \frac{1}{3-2x} - \frac{1}{2}x^2 + c\right)$	**B1 B1 B1**	Simplification not needed, B1 for each correct element
		6	

Question	Answer	Marks	Guidance
10(b)	$\frac{dy}{dx} = 0 \rightarrow (3-2x)^3 = 8 \rightarrow 3-2x = k \rightarrow x =$	M1	Setting *their* 2-term differential to 0 and attempts to solve as far as $x =$
	$\frac{1}{2}$	A1	
	Alternative method for question 10(b)		
	$y = 0 \rightarrow \frac{2}{(3-2x)^2} - x = 0 \rightarrow (x-2)(2x-1)^2 = 0 \rightarrow x =$	M1	Setting y to 0 and attempts to solve a cubic as far as $x =$ (3 factors needed)
	$\frac{1}{2}$	A1	
		2	
10(c)	Area under curve = *their* $\left[\frac{1}{3-2\times\left(\frac{1}{2}\right)} - \frac{\left(\frac{1}{2}\right)^2}{2}\right] - \left[\frac{1}{3-2\times 0} - 0\right]$	M1	Using *their* integral, *their* positive x limit from **part (b)** and 0 correctly.
	$\frac{1}{24}$	A1	
		2	

Question	Answer	Marks	Guidance
11(a)	5, –1	**B1 B1**	Sight of each value
		2	
11(b)		***B1**	Needs to be a curve, not straight lines. One complete cycle starting and finishing at *their* largest value.
		DB1	One complete cycle starting and finishing at $y = 5$ and going down to $y = -1$ and starting to level off at least one end.
		2	
11(c)(i)	0 solution	**B1**	
		1	
11(c)(ii)	2 solutions	**B1**	
		1	
11(c)(iii)	1 solution	**B1**	
		1	

Question	Answer	Marks	Guidance
11(d)	Stretch by (scale factor) $\frac{1}{2}$, parallel to x-axis or in x direction (or horizontally)	**B1**	
	Translation of $\begin{pmatrix} 0 \\ 4 \end{pmatrix}$	**B1**	Accept translation/shift Accept translation 4 units in positive y-direction.
		2	
11(e)	Translation of $\begin{pmatrix} -\frac{\pi}{2} \\ 0 \end{pmatrix}$	**B1**	Accept translation/shift Accept translation $-\frac{\pi}{2}$ units in x-direction.
	Stretch by (scale factor) 2 parallel to y-axis (or vertically).	**B1**	
		2	

Cambridge Assessment International Education

Cambridge International AS & A Level

MATHEMATICS **9709/13**

Paper 1 Pure Mathematics 1 **October/November 2020**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2020 series for most Cambridge IGCSE™, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

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GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
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2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1(a)	$\left[(x+3)^2\right]$ $[-4]$	B1 B1	
		2	
1(b)	[Translation or shift] $\begin{pmatrix}-3\\-4\end{pmatrix}$	B1 B1 FT	Accept [translation/shift] $\begin{pmatrix}-their\ a\\ their\ b\end{pmatrix}$ OR translation –3 units in x-direction and (translation) –4 units in y-direction.
		2	

Question	Answer	Marks	Guidance
2(a)	$\dfrac{-2}{x+2}$	B1	Integrate f(x). Accept $-2(x+2)^{-1}$. Can be unsimplified.
	$0-\left(-\dfrac{2}{3}\right) = \dfrac{2}{3}$	M1 A1	Apply limit(s) to an integrated expansion. CAO for A1
		3	
2(b)	$-1=-2+c$	M1	Substitute $x=-1, y=-1$ into *their* integrated expression (c present)
	$y=\dfrac{-2}{x+2}+1$	A1	Accept $y=-2(x+2)^{-1}+1$. –2 must be resolved.
		2	

Question	Answer	Marks	Guidance
3	$3\tan^4\theta + \tan^2\theta - 2\ (=0)$	**M1**	SOI 3-term quartic, condone sign errors for this mark only
	$(3\tan^2\theta - 2)(\tan^2\theta + 1)\ (=0)$	**M1**	Attempt to factorise or solve 3-term quadratic in $\tan^2\theta$.
	$\tan\theta = (\pm)\sqrt{\frac{2}{3}}$ or $(\pm)0.816$ or $(\pm)0.817$	**A1**	SOI Implied by final answer = 39.2° after 1st M1 scored
	39.2°, 140.8°	**A1** **A1 FT**	FT for 2nd solution =180° – 1st solution
		5	

Question	Answer	Marks	Guidance
4	$3x^2 - 4x + 4 = mx + m - 1 \rightarrow 3x^2 - (4+m)x + (5-m)\ (=0)$	**M1**	3-term quadratic
	$b^2 - 4ac = (4+m)^2 - 4\times3\times(5-m)$	**M1**	Find $b^2 - 4ac$ for *their* quadratic
	$m^2 + 20m - 44$	**A1**	
	$(m + 22)(m - 2)$	**A1**	Or use of formula or completing square. This step must be seen
	$m > 2\ ,\ m < -22$	**A1**	Allow $x > 2\ ,\ x < -22$
		5	

Question	Answer	Marks	Guidance
5	$\left[7C1a^6b(x)\right]$, $\left[7C2a^5b^2\left(x^2\right)\right]$, $\left[7C4a^3b^4\left(x^4\right)\right]$	**B2, 1, 0**	SOI, can be seen in an expansion.
	$\dfrac{7C2a^5b^2\left(x^2\right)}{7C1a^6b(x)}=\dfrac{7C4a^3b^4\left(x^4\right)}{7C2a^5b^2\left(x^2\right)} \rightarrow \dfrac{21a^5b^2}{7a^6b}=\dfrac{35a^3b^4}{21a^5b^2}$	**M1 A1**	M1 for a correct relationship OE (Ft from *their* 3 terms). For A1 binomial coefficients must be correct & evaluated.
	$\dfrac{a}{b}=\dfrac{5}{9}$	**A1**	OE
		5	

Question	Answer	Marks	Guidance
6(a)	$y=\dfrac{2x}{3x-1} \rightarrow 3xy-y=2x \rightarrow 3xy-2x=y$ (or $-y=2x-3xy$)	***M1**	For 1st two operations. Condone a sign error
	$x(3y-2)=y \rightarrow x=\dfrac{y}{3y-2}$ (or $x=\dfrac{-y}{2-3y}$)	**DM1**	For 2nd two operations. Condone a sign error
	$\left(\mathrm{f}^{-1}(x)\right)=\dfrac{x}{3x-2}$	**A1**	Allow $\left(\mathrm{f}^{-1}(x)\right)=\dfrac{-x}{2-3x}$
		3	
6(b)	$\left[\dfrac{2(3x-1)+2}{3(3x-1)}\right] = \left[\dfrac{6x}{3(3x-1)} = \dfrac{2x}{3x-1}\right]$	**B1 B1**	AG, WWW First B1 is for a correct single unsimplified fraction. An intermediate step needs to be shown. Equivalent methods accepted.
		2	

Question	Answer	Marks	Guidance
6(c)	$(\mathrm{f}(x)) > \frac{2}{3}$	B1	Allow $(y) > \frac{2}{3}$. Do not allow $x > \frac{2}{3}$
		1	

Question	Answer	Marks	Guidance
7(a)	$(d =) - \frac{\tan^2\theta}{\cos^2\theta} - \frac{1}{\cos^2\theta}$	B1	Allow sign error(s). Award only at form $(d =)$... stage
	$-\frac{\sin^2\theta}{\cos^4\theta} - \frac{1}{\cos^2\theta}$ or $\frac{-\sec^2\theta}{\cos^2\theta}$	M1	Allow sign error(s). Can imply B1
	$\frac{-\sin^2\theta - \cos^2\theta}{\cos^4\theta}$ or $\frac{-\frac{1}{\cos^2\theta}}{\cos^2\theta}$	M1	
	$-\frac{1}{\cos^4\theta}$	A1	AG, WWW
		4	
7(b)	$a = \frac{4}{3}$, $d = -\frac{16}{9}$	B1	SOI, both required. Allow $a = \frac{1}{\frac{3}{4}}$, $d = -\frac{1}{\frac{9}{16}}$
	$u_{13} = \frac{1}{\cos^2\theta} - \frac{12}{\cos^4\theta} = \frac{4}{3} + 12\left(\frac{-16}{9}\right)$	M1	Use of correct formula with *their a* and *their d*. The first 2 steps could be reversed
	–20	A1	WWW
		3	

Question	Answer	Marks	Guidance
8(a)	$\frac{dy}{dx} = [2] \quad [-2(2x+1)^{-2}]$	B1 B1	
	$\frac{d^2y}{dx^2} = 8(2x+1)^{-3}$	B1	
		3	
8(b)	Set *their* $\frac{dy}{dx} = 0$ and attempt solution	M1	
	$(2x+1)^2 = 1 \rightarrow 2x+1 = (\pm)\,1$ or $4x^2 + 4x = 0 \rightarrow (4)x(x+1) = 0$	M1	Solving as far as $x = ...$
	$x = 0$	A1	WWW. Ignore other solution.
	(0, 2)	A1	One solution only. Accept $x = 0$, $y = 2$ only.
	$\frac{d^2y}{dx^2} > 0$ from a solution $x > -\frac{1}{2}$ hence minimum	B1	Ignore other solution. Condone arithmetic slip in value of $\frac{d^2y}{dx^2}$. *Their* $\frac{d^2y}{dx^2}$ must be of the form $k(2x+1)^{-3}$
		5	

Question	Answer	Marks	Guidance
9(a)	$\cos BAO = \frac{6}{8}$ or $\frac{8^2 + 12^2 - 8^2}{2 \times 8 \times 12}$	M1	Or other correct method
	$BAO = 0.723$	A1	
		2	

Question	Answer	Marks	Guidance
9(b)	Sector $ABC = ½ \times 12^2 \times \textit{their}\ 0.7227$	*M1	Accept 52.1
	Triangle $AOB = ½ \times 8 \times 12 \sin(\textit{their}\ 0.7227)$ or $½ \times 12 \times \sqrt{28}$	*M1	or $½ \times 8 \times 8 \sin(\pi - 2 \times \textit{their}\ 0.7227)$. Expect 31.7 or 31.8
	Shaded area = *their* 52.0 − *their* 31.7 = 20.3	DM1 A1	M1 dependent on both previous M marks
		4	
9(c)	Arc $BC = 12 \times \textit{their}\ 0.7227$	*M1	Expect 8.67
	Perimeter = 8 + 4 + *their* 8.67 = 20.7	DM1 A1	
		3	

Question	Answer	Marks	Guidance
10(a)	$\frac{dy}{dx} = \left[\frac{x^{-1/2}}{2k}\right] - \left[\frac{x^{-3/2}}{2}\right] + ([0])$	B2, 1, 0	$([0])$ implies that more than 2 terms counts as an error
	Sub $\frac{dy}{dx} = 3$ when $x = \frac{1}{4}$ Expect $3 = \frac{1}{k} - 4$	M1	
	$k = \frac{1}{7}$ (or 0.143)	A1	
		4	

Question	Answer	Marks	Guidance
10(b)	$\int \frac{1}{k}x^{1/2} + x^{-1/2} + \frac{1}{k^2} = \left[\frac{2x^{3/2}}{3k}\right] + \left[2x^{1/2}\right] + \left[\frac{x}{k^2}\right]$	**B2, 1, 0**	OE
	$\left(\frac{2k^2}{3} + 2k + 1\right) - \left(\frac{k^2}{12} + k + \frac{1}{4}\right)$	**M1**	Apply limits $\frac{k^2}{4} \to k^2$ to an integrated expression. Expect $\frac{7}{12}k^2 + k + \frac{3}{4}$
	$\frac{7}{12}k^2 + k + \frac{3}{4} = \frac{13}{12}$	**M1**	Equate to $\frac{13}{12}$ and simplify to quadratic. OE, expect $7k^2 + 12k - 4\ (=0)$
	$k = \frac{2}{7}$ only (or 0.286)	**A1**	Dependent on $(7k-2)(k+2)\ (=0)$ or formula or completing square.
		5	

Question	Answer	Marks	Guidance
11(a)	$(-6-8)^2+(6-4)^2$	**M1**	OE
	$=200$	**A1**	
	$\sqrt{200}>10$, hence outside circle	**A1**	AG ('Shown' not sufficient). Accept equivalents of $\sqrt{200}>10$
	Alternative method for question 11(a)		
	Radius = 10 and $C=(8, 4)$	**B1**	
	Min(x) on circle = 8 – 10 = –2	**M1**	
	Hence outside circle	**A1**	AG
		3	
11(b)	angle $=\sin^{-1}\left(\frac{their\,10}{their\,10\sqrt{2}}\right)$	**M1**	Allow decimals for $10\sqrt{2}$ at this stage. If cosine used, angle ACT or BCT must be identified, or implied by use of 90°– 45°.
	angle = $\sin^{-1}(\frac{1}{\sqrt{2}}$ or $\frac{\sqrt{2}}{2}$ or $\frac{10}{10\sqrt{2}}$ or $\frac{10}{\sqrt{200}}) = 45°$	**A1**	AG Do not allow decimals
	Alternative method for question 11(b)		
	$(10\sqrt{2})^2=10^2+TA^2$	**M1**	
	$TA=10\rightarrow 45°$	**A1**	AG
		2	

Question	Answer	Marks	Guidance
11(c)	Gradient, m, of $CT = -\frac{1}{7}$	**B1**	OE
	Attempt to find mid-point (M) of CT	***M1**	Expect (1, 5)
	Equation of AB is $y-5=7(x-1)$	**DM1**	Through *their* (1, 5) with gradient $-\frac{1}{m}$
	$y=7x-2$	**A1**	
		4	
11(d)	$(x-8)^2+(7x-2-4)^2=100$ or equivalent in terms of y	**M1**	Substitute *their* equation of AB into equation of circle.
	$50x^2-100x\ (=0)$	**A1**	
	$x = 0$ and 2	**A1**	WWW
	Alternative method for question 11(d)		
	MC $=\begin{pmatrix}7\\-1\end{pmatrix}$	**M1**	
	$\begin{pmatrix}1\\5\end{pmatrix}+\begin{pmatrix}-1\\-7\end{pmatrix}=\begin{pmatrix}0\\-2\end{pmatrix}$, $\begin{pmatrix}1\\5\end{pmatrix}+\begin{pmatrix}1\\7\end{pmatrix}=\begin{pmatrix}2\\12\end{pmatrix}$	**A1**	
	$x = 0$ and 2	**A1**	
		3	

Cambridge International AS & A Level

MATHEMATICS | **9709/11**

Paper 1 Pure Mathematics 1 | **May/June 2020**

MARK SCHEME

Maximum Mark: 75

Published

Students did not sit exam papers in the June 2020 series due to the Covid-19 global pandemic.

This mark scheme is published to support teachers and students and should be read together with the question paper. It shows the requirements of the exam. The answer column of the mark scheme shows the proposed basis on which Examiners would award marks for this exam. Where appropriate, this column also provides the most likely acceptable alternative responses expected from students. Examiners usually review the mark scheme after they have seen student responses and update the mark scheme if appropriate. In the June series, Examiners were unable to consider the acceptability of alternative responses, as there were no student responses to consider.

Mark schemes should usually be read together with the Principal Examiner Report for Teachers. However, because students did not sit exam papers, there is no Principal Examiner Report for Teachers for the June 2020 series.

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A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

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DM or **DB** When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

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WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks
1	$117 = \frac{9}{2}(2a + 8d)$	B1
	Either $91 = S_4$ with 'a' as $a + 4d$ **or** $117 + 91 = S_{13}$ (**M1** for overall approach. **M1** for S_n)	M1M1
	Simultaneous Equations → $a = 7$, $d = 1.5$	A1
		4

Question	Answer	Marks
2	$\left(kx + \frac{1}{x}\right)^5 + \left(1 - \frac{2}{x}\right)^8$ Coefficient in $\left(kx + \frac{1}{x}\right)^5 = 10 \times k^2$ (**B1** for 10. **B1** for k^2)	B1B1
	Coefficient in $\left(1 - \frac{2}{x}\right)^8 = 8 \times -2$	B2,1,0
	$10k^2 - 16 = 74 \rightarrow k = 3$	B1
		5

Question	Answer	Marks
3(a)	$\$36\,000 \times (1.05)^n$ (**B1** for $r = 1.05$. **M1** method for rth term)	**B1M1**
	\$53 200 after 8 years.	**A1**
		3
3(b)	$S_{10} = 36000\dfrac{(1.05^{10} - 1)}{(1.05 - 1)}$	**M1**
	\$453 000	**A1**
		2

Question	Answer	Marks
4(a)	$-1 \leqslant \mathrm{f}(x) \leqslant 2$	**B1 B1**
		2
4(b)	$k = 1$	**B1**
	Translation by 1 unit upwards parallel to the y-axis	**B1**
		2
4(c)	$y = -\dfrac{3}{2}\cos 2x - \dfrac{1}{2}$	**B1**
		1

Question	Answer	Marks
5(a)	$x(mx+c) = 16 \rightarrow mx^2 + cx - 16 = 0$	B1
	Use of $b^2 - 4ac = c^2 + 64m$	M1
	Sets to 0 $\rightarrow m = \dfrac{-c^2}{64}$	A1
		3
5(b)	$x(-4x+c) = 16$ Use of $b^2 - 4ac \rightarrow c^2 - 256$	M1
	$c > 16$ and $c < -16$	A1 A1
		3

Question	Answer	Marks
6(a)	$3(3x+b)+b=9x+4b \rightarrow 10 = 18 + 4b$	**M1**
	$b = -2$	**A1**
	Either $f(14) = 2$ **or** $f^{-1}(x) = 2(x + a)$ etc.	**M1**
	$a = 5$	**A1**
		4
6(b)	$gf(x) = 3\left(\frac{1}{2}x-5\right)-2$	**M1**
	$gf(x) = \frac{3}{2}x-17$	**A1**
		2

Question	Answer	Marks
7(a)	$\dfrac{(1+\sin\theta)^2+\cos^2\theta}{\cos\theta(1+\sin\theta)}$	**M1**
	Use of $\sin^2\theta+\cos^2\theta=1 \rightarrow \dfrac{2+2\sin\theta}{\cos\theta(1+\sin\theta)} \rightarrow \dfrac{2}{\cos\theta}$.	**M1A1**
		3
7(b)	$\dfrac{2}{\cos\theta}=\dfrac{3}{\sin\theta} \rightarrow \tan\theta=1.5$	**M1**
	$\theta = 0.983$ or 4.12 (**FT** on second value for 1st value + π)	**A1** **A1FT**
		3

Question	Answer	Marks
8	Angle $AOB = 15 \div 6 = 2.5$ radians	**B1**
	Angle $BOC = \pi - 2.5$ (**FT** on angle AOB)	**B1FT**
	$BC = 6(\pi - 2.5)$ $(BC = 3.850)$	**M1**
	$\sin(\pi - 2.5) = BX \div 6$ $(BX = 3.59)$	**M1**
	Either $OX = 6\cos(\pi - 2.5)$ **or** Pythagoras $(OX = 4.807)$	**M1**
	$XC = 6 - OX$ $(XC = 1.193) \rightarrow P = 8.63$	**A1**
		6

Question	Answer	Marks
9(a)	$\frac{dy}{dx} = 3(3-2x)^2 \times -2 + 24 = -6(3-2x)^2 + 24$ (**B1** without ×−2. **B1** for ×−2)	**B1B1**
	$\frac{d^2y}{dx^2} = -12(3-2x)\times-2 = 24(3-2x)$ (**B1FT** from $\frac{dy}{dx}$ without – 2)	**B1FT** **B1**
		4
9(b)	$\frac{dy}{dx} = 0$ when $6(3-2x)^2 = 24 \rightarrow 3-2x = \pm 2$	**M1**
	$x = ½, y = 20$ or $x = 2½, y = 52$ (**A1** for both x values or a correct pair)	**A1A1**
		3
9(c)	If $x = ½$, $\frac{d^2y}{dx^2} = 48$ Minimum	**B1FT**
	If $x = 2½$, $\frac{d^2y}{dx^2} = -48$ Maximum	**B1FT**
		2

Question	Answer	Marks
10(a)	Centre is (3, 1)	**B1**
	Radius = 5 (Pythagoras)	**B1**
	Equation of C is $(x-3)^2+(y-1)^2=25$ (**FT** on *their* centre)	**M1** **A1FT**
		4
10(b)	Gradient from (3, 1) to (7, 4) = ¾ (this is the normal)	**B1**
	Gradient of tangent $=-\frac{4}{3}$	**M1**
	Equation is $y-4=-\frac{4}{3}(x-7)$ or $3y+4x=40$	**M1A1**
		4
10(c)	*B* is centre of line joining centres → (11, 7)	**B1**
	Radius = 5 New equation is $(x-11)^2+(y-7)^2=25$ (**FT** on coordinates of B)	**M1** **A1FT**
		3

Question	Answer	Marks
11(a)	Simultaneous equations $\dfrac{8}{x+2} = 4 - ½x$	**M1**
	$x = 0$ or $x = 6 \rightarrow A\ (0, 4)$ and $B\ (6, 1)$	**B1A1**
	At C $\dfrac{-8}{(x+2)^2} = -\dfrac{1}{2} \rightarrow C\ (2, 2)$ (**B1** for the differentiation. **M1** for equating and solving)	**B1** **M1A1**
		6
11(b)	Volume under line $= \pi\int\left(-\frac{1}{2}x+4\right)^2 dx = \pi\left[\dfrac{x^3}{12} - 2x^2 + 16x\right] = (42\pi)$ (**M1** for volume formula. **A2,1** for integration)	**M1** **A2,1**
	Volume under curve $= \pi\int\left(\dfrac{8}{x+2}\right)^2 dx = \pi\left[\dfrac{-64}{x+2}\right] = (24\pi)$	**A1**
	Subtracts and uses 0 to 6 $\rightarrow 18\pi$	**M1A1**
		6

Cambridge Assessment International Education

Cambridge International AS & A Level

MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **May/June 2020**

MARK SCHEME

Maximum Mark: 75

Published

Students did not sit exam papers in the June 2020 series due to the Covid-19 global pandemic.

This mark scheme is published to support teachers and students and should be read together with the question paper. It shows the requirements of the exam. The answer column of the mark scheme shows the proposed basis on which Examiners would award marks for this exam. Where appropriate, this column also provides the most likely acceptable alternative responses expected from students. Examiners usually review the mark scheme after they have seen student responses and update the mark scheme if appropriate. In the June series, Examiners were unable to consider the acceptability of alternative responses, as there were no student responses to consider.

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Generic Marking Principles

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5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

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M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

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B Mark for a correct result or statement independent of method marks.

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Abbreviations

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CAO	Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks
1(a)	$(2+3x)\left(x-\frac{2}{x}\right)^6$ Term in x^2 in $\left(x-\frac{2}{x}\right)^6 = 15x^4 \times \left(\frac{-2}{x}\right)^2$	B1
	Coefficient = 60	B1
		2
1(b)	Constant term in $\left(x-\frac{2}{x}\right)^6 = 20x^3 \times \left(\frac{-2}{x}\right)^3 (-160)$	B2, 1
	Coefficient of x^2 in $(2+3x)\left(x-\frac{2}{x}\right)^6 = 120 - 480 = -360$	B1FT
		3

Question	Answer	Marks
2(a)	$3\cos\theta = 8\tan\theta \rightarrow 3\cos\theta = \frac{8\sin\theta}{\cos\theta}$	M1
	$3(1 - \sin^2\theta) = 8\sin\theta$	M1
	$3\sin^2\theta + 8\sin\theta - 3 = 0$	A1
		3
2(b)	$(3\sin\theta - 1)(\sin\theta + 3) = 0 \rightarrow \sin\theta = ⅓$	M1
	$\theta = 19.5°$	A1
		2

Question	Answer	Marks
3(a)	Volume after 30 s = 18000 $\frac{4}{3}\pi r^3 = 18000$	**M1**
	r = 16.3 cm	**A1**
		2
3(b)	$\frac{dV}{dr} = 4\pi r^2$	**B1**
	$\frac{dr}{dt} = \frac{dr}{dV} \times \frac{dV}{dt} = \frac{600}{4\pi r^2}$	**M1**
	$\frac{dr}{dt}$ = 0.181 cm per second	**A1**
		3

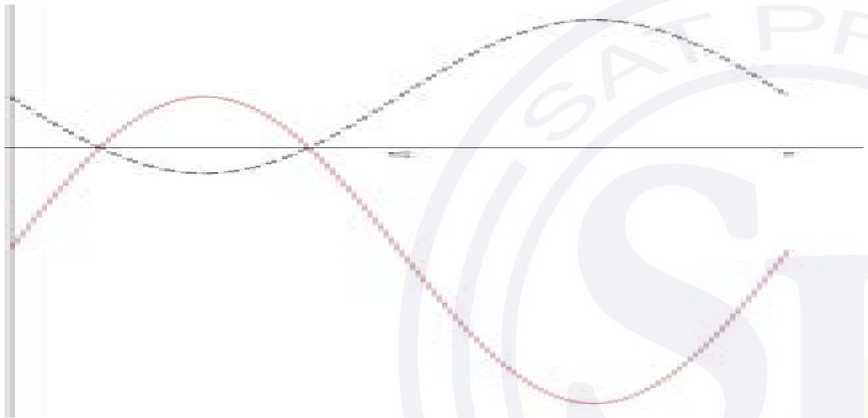
Question	Answer	Marks
4	1st term is −6, 2nd term is −4.5 (**M1** for using *k*th terms to find both *a* and *d*)	**M1**
	→ $a = -6$, $d = 1.5$	**A1 A1**
	$S_n = 84 \rightarrow 3n^2 - 27n - 336 = 0$	**M1**
	Solution $n = 16$	**A1**
		5

Question	Answer	Marks
5(a)	$\text{ff}(x) = a - 2(a - 2x)$	M1
	$\text{ff}(x) = 4x - a$	A1
	$\text{f}^{-1}(x) = \dfrac{a-x}{2}$	M1 A1
		4
5(b)	$4x - a = \dfrac{a-x}{2} \rightarrow 9x = 3a$	M1
	$x = \dfrac{a}{3}$	A1
		2

Question	Answer	Marks
6(a)	$2x^2 + kx + k - 1 = 2x + 3 \rightarrow 2x^2 + (k-2)x + k - 4 = 0$	**M1**
	Use of $b^2 - 4ac = 0 \rightarrow (k-2)^2 = 8(k-4)$	**M1**
	$k = 6$	**A1**
		3
6(b)	$2x^2 + 2x + 1 = 2\left(x + \frac{1}{2}\right)^2 + 1 - \frac{1}{2}$ $a = \frac{1}{2}$, $b = \frac{1}{2}$	**B1 B1**
	vertex $\left(-\frac{1}{2}, \frac{1}{2}\right)$ (**FT** on *a* and *b* values)	**B1FT**
		3

Question	Answer	Marks
7(a)	$BC^2 = r^2 + 4r^2 - 2r.2r \times \cos\left(\frac{\pi}{6}\right) = 5r^2 - 2r^2\sqrt{3}$	M1
	$BC = r\sqrt{\left(5-2\sqrt{3}\right)}$	A1
		2
7(b)	Perimeter = $\frac{2\pi r}{6} + r + r\sqrt{\left(5-2\sqrt{3}\right)}$	M1 A1
		2
7(c)	Area = sector – triangle	
	Sector area = $\frac{1}{2}4r^2\frac{\pi}{6}$	M1
	Triangle area = ½ r. $2r$ $\sin\frac{\pi}{6}$	M1
	Shaded area = $r^2\left(\frac{\pi}{3} - \frac{1}{2}\right)$	A1
		3

Question	Answer	Marks
8(a)	Volume = $\pi\int x^2 dy = \pi\int \frac{36}{y^2}dy$	*M1
	$= \pi\left[\frac{-36}{y}\right]$	A1
	Uses limits 2 to 6 correctly → (12π)	DM1
	Vol of cylinder = $\pi.1^2.4$ or $\int 1^2.dy$ = [y] from 2 to 6	M1
	Vol = $12\pi - 4\pi = 8\pi$	A1
		5
8(b)	$\frac{dy}{dx} = \frac{-6}{x^2}$	B1
	$\frac{-6}{x^2} = -2 \rightarrow x = \sqrt{3}$	M1
	$y = \frac{6}{\sqrt{3}} = 2\sqrt{3}$ Lies on $y = 2x$	A1
		3

Question	Answer	Marks
9(a)	f(x) from −1 to 5	**B1B1**
	g(x) from −10 to 2 (**FT** from part (**a**))	**B1FT**
		3
9(b)		**B2, 1**
		2
9(c)	Reflect in x-axis	**B1**
	Stretch by factor 2 in the y direction	**B1**
	Translation by –π in the x direction OR translation by $\begin{pmatrix} 0 \\ -\pi \end{pmatrix}$.	**B1**
		3

Question	Answer	Marks
10(a)	$\frac{dy}{dx} = 54 - 6(2x-7)^2$	B2,1
	$\frac{d^2y}{dx^2} = -24(2x-7)$ (**FT** only for omission of '×2 ' from the bracket)	B2,1 FT
		4
10(b)	$\frac{dy}{dx} = 0 \to (2x-7)^2 = 9$	M1
	$x = 5$, $y = 243$ or $x = 2$, $y = 135$	A1 A1
		3
10(c)	$x = 5$ $\frac{d^2y}{dx^2} = -72 \to$ Maximum (**FT** only for omission of '×2 ' from the bracket)	B1FT
	$x = 2$ $\frac{d^2y}{dx^2} = 72 \to$ Minimum (**FT** only for omission of '×2 ' from the bracket)	B1FT
		2

Question	Answer	Marks
11(a)	Express as $(x-4)^2+(y+2)^2=16+4+5$	M1
	Centre $C(4, -2)$	A1
	Radius = $\sqrt{25}$ = 5	A1
		3
11(b)	$P(1,2)$ to $C(4, -2)$ has gradient $-\dfrac{4}{3}$ (**FT** on coordinates of C)	B1FT
	Tangent at P has gradient = $\dfrac{3}{4}$	M1
	Equation is $y-2=\dfrac{3}{4}(x-1)$ or $4y = 3x + 5$	A1
		3
11(c)	Q has the same coordinate as P $y = 2$	B1
	Q is as far to the right of C as P $x = 3 + 3 + 1 = 7$ Q (7, 2)	B1
		2

Question	Answer	Marks
11(d)	Gradient of tangent at $Q = -\frac{3}{4}$ by symmetry (**FT** from part (**b**))	**B1FT**
	Eqn of tangent at Q is $y-2=-\frac{3}{4}(x-7)$ or $4y+3x=29$	**M1**
	$T\left(4, \frac{17}{4}\right)$	**A1**
		3

Cambridge International AS & A Level

MATHEMATICS **9709/13**

Paper 1 Pure Mathematics 1 **May/June 2020**

MARK SCHEME

Maximum Mark: 75

Published

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Question	Answer	Marks
1	$3x^2+2x+4=mx+1 \to 3x^2+x(2-m)+3\ (=0)$	B1
	$(2-m)^2-36$ SOI	M1
	$(m+4)(m-8)$ (>/= 0) **or** $2-m$ >/= 6 and $2-m$ </= –6 OE	A1
	$m<-4$, $m>8$ WWW	A1
	Alternative method for question 1	
	$\frac{dy}{dx}=6x+2 \to m=6x+2 \to 3x^2+2x+4=(6x+2)x+1$	M1
	$x=\pm1$	A1
	$m=\pm6+2 \to m=8$ or -4	A1
	$m<-4$, $m>8$ WWW	A1
		4

Question	Answer	Marks
2	$(y)=\frac{3x^{\frac{3}{2}}}{\frac{3}{2}}-\frac{3x^{\frac{1}{2}}}{\frac{1}{2}}\ (+c)$	B1 B1
	$7=16-12+c$ (**M1** for subsituting $x=4$, $y=7$ into *their* integrated expansion)	M1
	$y=2x^{\frac{3}{2}}-6x^{\frac{1}{2}}+3$	A1
		4

Question	Answer	Marks
3(a)	$(y) = \mathrm{f}(-x)$	**B1**
		1
3(b)	$(y) = 2\mathrm{f}(x)$	**B1**
		1
3(c)	$(y) = \mathrm{f}(x+4) - 3$	**B1 B1**
		2

Question	Answer	Marks
4(a)	$1 + 5a + 10a^2 + 10a^3 + \ldots$	**B1**
		1
4(b)	$1 + 5(x + x^2) + 10(x + x^2)^2 + 10(x + x^2)^3 + \ldots$ SOI	**M1**
	$1 + 5(x + x^2) + 10(x^2 + 2x^3 + \ldots) + 10(x^3 + \ldots) + \ldots$ SOI	**A1**
	$1 + 5x + 15x^2 + 30x^3 + \ldots$	**A1**
		3

Question	Answer	Marks
5	$\cos POA = \frac{5}{13} \rightarrow POA = 1.17(6)$ Allow 67.4° **or** $\sin = \frac{12}{13}$ or $\tan = \frac{12}{5}$	M1 A1
	Reflex $AOB = 2\pi - 2\times$ *their* $1.17(6)$ OE in degrees **or** minor arc AB = 5×2×*their*1.17(6)	M1
	Major arc = 5 × *their* 3.93(1) **or** $2\pi\times5$ - *their* 11.7(6)	M1
	AP (or BP) = $\sqrt{13^2 - 5^2}$ = 12	B1
	Cord length = 43.7	A1
		6

Question	Answer	Marks
6(a)	$\frac{\mathrm{d}y}{\mathrm{d}x} = \left[\frac{1}{2}(5x-1)^{-1/2}\right]\times[5]$	B1 B1
	Use $\frac{\mathrm{d}y}{\mathrm{d}t} = 2\times\left(\textit{their}\frac{\mathrm{d}y}{\mathrm{d}x}\text{ when } x=1\right)$	M1
	$\frac{5}{2}$	A1
		4

Question	Answer	Marks
6(b)	$2 \times their \frac{5}{2}(5x-1)^{-1/2} = \frac{5}{8}$ oe	M1
	$(5x-1)^{1/2} = 8$	A1
	$x = 13$	A1
		3

Question	Answer	Marks
7(a)	$\frac{\tan\theta}{1+\cos\theta} + \frac{\tan\theta}{1-\cos\theta} = \frac{\tan\theta(1-\cos\theta) + \tan\theta(1+\cos\theta)}{1-\cos^2\theta}$	M1
	$= \frac{2\tan\theta}{\sin^2\theta}$	M1
	$= \frac{2\sin\theta}{\cos\theta\sin^2\theta}$	M1
	$= \frac{2}{\sin\theta\cos\theta}$ **AG**	A1
		4

Question	Answer	Marks
7(b)	$\dfrac{2}{\sin\theta\cos\theta}=\dfrac{6\cos\theta}{\sin\theta}$	**M1**
	$\cos^2\theta=\dfrac{1}{3} \rightarrow \cos\theta=(\pm)0.5774$	**A1**
	54.7º, 125.3º (**FT** for 180º – 1st solution)	**A1** **A1FT**
		4

Question	Answer	Marks
8(a)	$r=\cos^2\theta$ SOI	**M1**
	$S_\infty=\dfrac{\sin^2\theta}{1-\cos^2\theta}$	**M1**
	1	**A1**
		3
8(b)(i)	$d=\sin^2\theta\cos^2\theta-\sin^2\theta$	**M1**
	$\sin^2\theta\left(\cos^2\theta-1\right)$	**M1**
	$-\sin^4\theta$	**A1**
		3

Question	Answer	Marks
8(b)(ii)	Use of $S_{16} = \frac{16}{2}[2a + 15d]$	M1
	With <u>both</u> $a = \frac{3}{4}$ and $d = -\frac{9}{16}$	A1
	$S_{16} = -55\frac{1}{2}$	A1
		3

Question	Answer	Marks
9(a)	$\left[(x-2)^2\right]$ $[-1]$	B1 B1
		2
9(b)	Smallest $c = 2$ (**FT** on *their* part **(a)**)	B1FT
		1
9(c)	$y = (x-2)^2 - 1 \rightarrow (x-2)^2 = y + 1$	*M1
	$x = 2(\pm)\sqrt{y+1}$	DM1
	$(\mathrm{f}^{-1}(x)) = 2 + \sqrt{x+1}$ for $x > 8$	A1
		3

Question	Answer	Marks
9(d)	$\text{gf}(x) = \dfrac{1}{(x-2)^2 - 1 + 1} = \dfrac{1}{(x-2)^2}$ OE	**B1**
	Range of gf is $0 < \text{gf}(x) < \dfrac{1}{9}$	**B1 B1**
		3

Question	Answer	Marks
10(a)	Mid-point is (–1, 7)	**B1**
	Gradient, m, of AB is 8/12 OE	**B1**
	$y - 7 = -\dfrac{12}{8}(x+1)$	**M1**
	$3x + 2y = 11$ **AG**	**A1**
		4
10(b)	Solve simultaneously $12x - 5y = 70$ and *their* $3x + 2y = 11$	**M1**
	$x = 5,\ y = -2$	**A1**
	Attempt to find distance between *their* (5, –2) and either (–7,3) or (5, 11)	**M1**
	$(r) = \sqrt{12^2 + 5^2}$ or $\sqrt{13^2 + 0} = 13$	**A1**
	Equation of circle is $(x-5)^2 + (y+2)^2 = 169$	**A1**
		5

Question	Answer	Marks
11(a)	$\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2 - 4bx + b^2$	**B1**
	$3x^2 - 4bx + b^2 = 0 \to (3x-b)(x-b)\ (=0)$	**M1**
	$x = \frac{b}{3}$ or b	**A1**
	$a = \frac{b}{3} \to b = 3a$ **AG**	**A1**
	Alternative method for question 11(a)	
	$\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2 - 4bx + b^2$	**B1**
	Sub $b = 3a$ & obtain $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ when $x = a$ and when $x = 3a$	**M1**
	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 6x - 12a$	**A1**
	< 0 Max at $x = a$ and > 0 Min at $x = 3a$. Hence $b = 3a$ **AG**	**A1**
		4

Question	Answer	Marks
11(b)	Area under curve = $\int\left(x^3-6ax^2+9a^2x\right)dx$	**M1**
	$\frac{x^4}{4}-2ax^3+\frac{9a^2x^2}{2}$	**B2,1,0**
	$\frac{a^4}{4}-2a^4+\frac{9a^4}{2}\quad\left(=\frac{11a^4}{4}\right)$ (**M1** for applying limits $0\ \to a$)	**M1**
	When $x=a,\ y=a^3-6a^3+9a^3=4a^3$	**B1**
	Area under line = $\frac{1}{2}a\times \textit{their}\ 4a^3$	**M1**
	Shaded area = $\frac{11a^4}{4}-2a^4\ =\ \frac{3}{4}a^4$	**A1**
		7

Cambridge International AS & A Level

MATHEMATICS **9709/12**

Paper 1 Pure Mathematics **March 2020**

MARK SCHEME

Maximum Mark: 75

Published

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Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
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GENERIC MARKING PRINCIPLE 5:
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GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics-Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

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Types of mark

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B Mark for a correct result or statement independent of method marks.

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WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	$f'(x) = \left[-(3x+2)^{-2}\right] \times [3] + [2x]$	**B2, 1, 0**	
	< 0 hence decreasing	**B1**	Dependent on at least B1 for $f'(x)$ and must include < 0 or '(always) neg'
		3	

Question	Answer	Marks	Guidance
2	[Stretch] [factor 2, x direction (or y-axis invariant)]	***B1** **DB1**	
	[Translation or Shift] [1 unit in y direction] **or** [Translation/Shift] $\left[\begin{pmatrix} 0 \\ 1 \end{pmatrix}\right]$	**B1B1**	Accept transformations in either order. Allow (0, 1) for the vector
		4	

Question	Answer	Marks	Guidance
3	$(\pi)\int(y-1)\,dy$	*M1	SOI Attempt to integrate x^2 or $(y-1)$
	$(\pi)\left[\frac{y^2}{2}-y\right]$	A1	
	$(\pi)\left[\left(\frac{25}{2}-5\right)-\left(\frac{1}{2}-1\right)\right]$	DM1	Apply limits 1 → 5 to an integrated expression
	8π or AWRT 25.1	A1	
		4	

Question	Answer	Marks	Guidance
4	$\frac{dy}{dx}=2x-2$	B1	
	$\frac{dy}{dx}=\frac{4}{6}$	B1	OE, SOI
	$their\,(2x-2)=their\,\frac{4}{6}$	M1	LHS and RHS must be *their* $\frac{dy}{dx}$ expression and value
	$x=\frac{4}{3}$ oe	A1	
		4	

Question	Answer	Marks	Guidance
5	$2\tan\theta - 6\sin\theta + 2 = \tan\theta + 3\sin\theta + 2 \rightarrow \tan\theta - 9\sin\theta\ (=0)$	**M1**	Multiply by denominator and simplify
	$\sin\theta - 9\sin\theta\cos\theta\ (=0)$	**M1**	Multiply by $\cos\theta$
	$\sin\theta(1-9\cos\theta)\ (=0) \rightarrow \sin\theta = 0, \quad \cos\theta = \frac{1}{9}$	**M1**	Factorise and attempt to solve at least one of the factors = 0
	$\theta = 0$ or $83.6°$ (only answers in the given range)	**A1A1**	
		5	

Question	Answer	Marks	Guidance
6(a)	$5C2\ \left[2(x)\right]^3\left[\frac{a}{(x^2)}\right]^2$	**B1**	SOI Can include correct x's
	$10\times8\times a^2\left(\frac{x^3}{x^4}\right) = 720\left(\frac{1}{x}\right)$	**B1**	SOI Can include correct x's
	$a = \pm3$	**B1**	
		3	
6(b)	$5C4\ \left[2(x)\right]\left[\frac{\textit{their } a}{(x^2)}\right]^4$	**B1**	SOI *Their a* can be just <u>one</u> of their values (e.g. just 3). Can gain mark from within an expansion but must use *their* value of *a*
	810 identified	**B1**	Allow with x^{-7}
		2	

Question	Answer	Marks	Guidance
7	$OC = 6\cos 0.8 = 4.18(0)$	M1A1	SOI
	Area sector $OCD = \frac{1}{2}(\textit{their}\ 4.18)^2 \times 0.8$	*M1	OE
	$\Delta OCA = \frac{1}{2} \times 6 \times \textit{their}\ 4.18 \times \sin 0.8$	M1	OE
	Required area = *their* ΔOCA – *their* sectorOCD	DM1	SOI. If not seen *their* areas of sector and triangle must be seen
	2.01	A1	CWO. Allow or better e.g. 2.0064
		6	

Question	Answer	Marks	Guidance
8(a)	2%	B1	
		1	
8(b)	Bonus = 600 + 23× 100 = 2900	B1	
	Salary = 30000×1.03^{23}	M1	Allow 30000×1.03^{24} (60984)
	= 59207.60	A1	Allow answers of 3significant figure accuracy or better
	$\frac{\textit{their}\ 2900}{\textit{their}\ 59200}$	M1	SOI
	4.9(0)%	A1	
		5	

Question	Answer	Marks	Guidance
9(a)	$\left[2(x+3)^2\right]\,[-7]$	B1B1	Stating $a=3, b=-7$ gets B1B1
		2	
9(b)	$y=2(x+3)^2-7 \to 2(x+3)^2=y+7 \to (x+3)^2=\frac{y+7}{2}$	M1	First 2 operations correct. Condone sign error or with *x*/*y* interchange
	$x+3=(\pm)\sqrt{\frac{y+7}{2}} \to x=(\pm)\sqrt{\frac{y+7}{2}}-3 \to \ \mathrm{f}^{-1}(x)=-\sqrt{\frac{x+7}{2}}-3$	A1FT	FT on *their a* and *b*. Allow $y=$ ...
	Domain: $x \geqslant -5$ **or** $\geqslant -5$ **or** $[-5, \infty)$	B1	Do not accept $y=\ldots,\ f(x)=\ldots, f^{-1}(x)=\ldots$
		3	
9(c)	$\mathrm{fg}(x) = 8x^2-7$	B1FT	SOI. FT on *their* –7 from part **(a)**
	$8x^2-7=193 \to x^2=25 \to x=-5$ only	B1	
	Alternative method for question 9(c)		
	$\mathrm{g}(x) = \mathrm{f}^{-1}(193) \to 2x-3=-\sqrt{100}-3$	M1	FT on *their* $\mathrm{f}^{-1}(x)$
	$x=-5$ only	A1	
		2	
9(d)	(Largest k is) $-\frac{1}{2}$	B1	Accept $-\frac{1}{2}$ or $k \leqslant -\frac{1}{2}$
		1	

Question	Answer	Marks	Guidance
10(a)	$2(a+3)^{\frac{1}{2}}-a=0$	**M1**	SOI. Set $\frac{dy}{dx}=0$ when $x=a$. Can be implied by an answer in terms of a
	$4(a+3)=a^2 \rightarrow a^2-4a-12=0$	**M1**	Take a to RHS and square. Form 3-term quadratic
	$(a-6)(a+2) \rightarrow a=6$	**A1**	Must show factors, or formula or completing square. Ignore $a=-2$ **SC** If a is never used maximum of M1A1 for $x=6$,with visible solution
		3	
10(b)	$\frac{d^2y}{dx^2}=(x+3)^{-\frac{1}{2}}-1$	**B1**	
	Sub *their a* $\rightarrow \frac{d^2y}{dx^2}=\frac{1}{3}-1=-\frac{2}{3}$ $(or<0) \rightarrow$ MAX	**M1A1**	A mark only if completely correct If the second differential is not $-\frac{2}{3}$ correct conclusion must be drawn to award the M1
		3	
10(c)	$(y=)\frac{2(x+3)^{\frac{3}{2}}}{\frac{3}{2}}-\frac{1}{2}x^2 \quad (+c)$	**B1B1**	
	Sub x = *their a* and y = 14 $\rightarrow 14=\frac{4}{3}(9)^{\frac{3}{2}}-18+c$	**M1**	Substitute into an integrated expression. c must be present. Expect $c=-4$
	$y=\frac{4}{3}(x+3)^{\frac{3}{2}}-\frac{1}{2}x^2-4$	**A1**	Allow $f(x)=\ldots.$
		4	

Question	Answer	Marks	Guidance
11(a)	$(\tan x - 2)(3\tan x + 1)\ (= 0)$. **or** formula **or** completing square	**M1**	Allow reversal of signs in the factors. Must see a method
	$\tan x = 2$ or $-\frac{1}{3}$	**A1**	
	$x = 63.4°$(only value in range) or $161.6°$(only value in range)	**B1FT** **B1FT**	
		4	
11(b)	Apply $b^2 - 4ac < 0$	**M1**	SOI. Expect $25 - 4(3)(k) < 0$, $\tan x$ must not be in coefficients
	$k > \frac{25}{12}$	**A1**	Allow $b^2 - 4ac = 0$ leading to correct $k > \frac{25}{12}$ for M1A1
		2	
11(c)	$k = 0$	**M1**	SOI
	$\tan x = 0$ or $\frac{5}{3}$	**A1**	
	$x = 0°$ or $180°$ or $59.0°$	**A1**	All three required
		3	

Question	Answer	Marks	Guidance
12(a)	Centre = (2, –1)	**B1**	
	$r^2 = \left[2-(-3)\right]^2 + \left[-1-(-5)\right]^2$ or $\left[2-7\right]^2 + \left[-1-3\right]^2$ OE	**M1**	OR $\frac{1}{2}\left[(-3-7)^2 + (-5-3)^2\right]$ OE
	$(x-2)^2 + (y+1)^2 = 41$	**A1**	Must not involve surd form **SCB3** $(x+3)(x-7)+(y+5)(y-3)=0$
		3	
12(b)	Centre = *their* (2, –1) + $\begin{pmatrix} 8 \\ 4 \end{pmatrix}$ = (10, 3)	**B1FT**	SOI FT on *their* (2, –1)
	$(x-10)^2 + (y-3)^2 = \textit{their}\, 41$	**B1FT**	FT on *their* 41 even if in surd form **SCB2** $(x-5)(x-15)+(y+1)(y-7)=0$
		2	

Question	Answer	Marks	Guidance
12(c)	Gradient m of line joining centres = $\frac{4}{8}$ OE	**B1**	
	Attempt to find mid-point of line.	**M1**	Expect (6, 1)
	Equation of RS is $y-1=-2(x-6)$	**M1**	Through *their* (6, 1) with gradient $\frac{-1}{m}$
	$y=-2x+13$	**A1**	AG
	Alternative method for question 12(c)		
	$(x-2)^2+(y+1)^2-41=(x-10)^2+(y-3)^2-41$ OE	**M1**	
	$x^2-4x+4+y^2+2y+1=x^2-20x+100+y^2-6y+9$ OE	**A1**	Condone 1 error **or** errors caused by 1 error in the first line
	$16x+8y=104$	**A1**	
	$y=-2x+13$	**A1**	AG
		4	
12(d)	$(x-10)^2+(-2x+13-3)^2=41$	**M1**	Or eliminate y between C_1 and C_2
	$x^2-20x+100+4x^2-40x+100=41\rightarrow 5x^2-60x+159=0$	**A1**	AG
		2	

Cambridge Assessment International Education
Cambridge International Advanced Subsidiary and Advanced Level

MATHEMATICS **9709/11**

Paper 1 **October/November 2019**

MARK SCHEME

Maximum Mark: 75

Published

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Question	Answer	Marks	Guidance
1	$6C2 \times (2x)^4 \times \frac{1}{(4x^2)^2}$	B1	SOI **SC:** Condone errors in $(4^{-1})^2$ evaluation or interpretation for B1 only
	$15 \times 2^4 \times \frac{1}{4^2}$	B1	Identified as required term.
	15	B1	
		3	

Question	Answer	Marks	Guidance
2	Attempt to solve $f'(x) = 0$ or $f'(x) > 0$ or $f'(x) \geqslant 0$	M1	SOI
	$(x-2)(x-4)$	A1	2 and 4 seen
	(Least possible value of n is) 4	A1	Accept $n = 4$ or $n \geqslant 4$
		3	

Question	Answer	Marks	Guidance
3	$\frac{dy}{dx} = 6x^2 - 10x - 3$	B1	
	At $x = 2$, $\frac{dy}{dx} = 24 - 20 - 3 = 1 \to a = 1$	M1 A1	
	$6 = 2 + b \to b = 4$	B1FT	Substitute $x = 2$, $y = 6$ in $y = (\textit{their } a)x + b$
	$6 = 16 - 20 - 6 + c \to c = 16$	B1	Substitute $x = 2$, $y = 6$ into equation of curve
		5	

Question	Answer	Marks	Guidance
4(i)	Identifies common ratio as 1.1	B1	
	Use of $x(1.1)^{20} = 20$	M1	SOI
	$x\left(= \frac{20}{(1.1)^{20}}\right) = 3.0$	A1	Accept 2.97
		3	
4(ii)	$\textit{their}\,3.0 \times \frac{\left[(1.1)^{21} - 1\right]}{1.1 - 1} \to 192$	M1 A1	Correct formula used for M mark. Allow 2.97 used from **(i)** Accept 190 from $x = 2.97\ldots$
		2	

Question	Answer	Marks	Guidance
5(i)	$4\tan x + 3\cos x + \dfrac{1}{\cos x} = 0 \rightarrow 4\sin x + 3\cos^2 x + 1 = 0$	**M1**	Multiply by $\cos x$ or common denominator of $\cos x$
	$4\sin x + 3\left(1-\sin^2 x\right) + 1 = 0 \rightarrow 3\sin^2 x - 4\sin x - 4 = 0$	**M1**	Use $\cos^2 x = 1-\sin^2 x$ and simplify to 3-term quadratic in $\sin x$
	$\sin x = -\dfrac{2}{3}$	**A1**	AG
		3	
5(ii)	$2x - 20^\circ = 221.8^\circ, 318.2^\circ$	**M1A1**	Attempt to solve $\sin\left(2x-20\right) = -2/3\left(\text{M1}\right)$. At least 1 correct (A1)
	$x = 120.9^\circ, 169.1^\circ$	**A1** **A1FT**	FT for 290º – other solution. **SC** A1 both answers in radians
		4	

Question	Answer	Marks	Guidance
6	Equation of line is $y = mx - 2$	**B1**	**OR**
	$x^2 - 2x + 7 = mx - 2 \rightarrow x^2 - x(2 + m) + 9 = 0$	**M1**	
	Apply $b^2 - 4ac(= 0) \rightarrow (2 + m)^2 - 4 \times 9\ (= 0)$	***M1**	
	$m = 4$ or -8	**A1**	
	$m = 4 \rightarrow x^2 - 6x + 9 = 0 \rightarrow x = 3$ $m = -8 \rightarrow x^2 + 6x + 9 = 0 \rightarrow x = -3$	**DM1**	
	(3, 10), (–3, 22)	**A1A1**	
	Alternative method for question 6		
	$\frac{dy}{dx} = 2x - 2$	**B1**	
	$2x - 2 = m$	**M1**	
	$x^2 - 2x + 7 = (2x - 2)x - 2 = 2x^2 - 2x - 2$	**M1**	
	$x^2 - 9 = 0 \rightarrow x = \pm 3$	**A1**	
	(3, 10), (–3, 22)	**A1A1**	
	When $x = 3$, $m = 4$; when $x = -3$, $m = -8$	**A1**	
		7	

Question	Answer	Marks	Guidance
7(i)	Range of f is $0 < f(x) < 3$	B1B1	OE. Range cannot be defined using x
	Range of g is $g(x) > 2$	B1	OE
		3	
7(ii)	$\left(fg(x)=\right)\dfrac{3}{2\left(\frac{1}{x}+2\right)+1}=\dfrac{3x}{2+5x}$	B1B1	Second B mark implies first B mark
		2	
7(iii)	$y=\dfrac{3x}{2+5x} \rightarrow 2y+5xy=3x \rightarrow 3x-5xy=2y$	M1	Correct order of operations
	$x(3-5y)=2y \quad \rightarrow \quad x=\dfrac{2y}{3-5y}$	M1	Correct order of operations
	$\left((fg)^{-1}(x)\right)=\dfrac{2x}{3-5x}$	A1	
		3	

Question	Answer	Marks	Guidance
8(i)	$OA \times \frac{3}{8}\pi = 6$	M1	
	$OA = \frac{16}{\pi} = 5.093(0)$	A1	
8(ii)	$AB = their5.0930 \times \tan\frac{3}{16}\pi$	M1	
	Perimeter $= 2 \times 3.4030 + 6 = 12.8$	A1	
8(iii)	Area $OABC = (2 \times ½) \times their5.0930 \times their3.4030$	M1	
	Area sector $= ½ \times (their5.0930)^2 \times \frac{3}{8}\pi$	M1	
	Shaded area $= their17.331 - their15.279 = 2.05$	M1A1	

Question	Answer	Marks	Guidance
9(i)	$y = [(5x-1)^{1/2} \div \tfrac{3}{2} \ \div 5]\ [-2x]$	B1 B1	
	$3 = \dfrac{27}{(3/2)\times 5} - 4 + c$	M1	Substitute $x = 2$, $y = 3$
	$c = 7 - \dfrac{18}{5} = \dfrac{17}{5} \rightarrow \left(y = \dfrac{2(5x-1)^{\frac{3}{2}}}{15} - 2x + \dfrac{17}{5} \right)$	A1	
9(ii)	$\mathrm{d}^2 y / \mathrm{d}x^2 = \left[½(5x-1)^{-1/2} \right]\ [\times 5]$	B1 B1	
9(iii)	$(5x-1)^{1/2} - 2 = 0 \ \rightarrow \ 5x - 1 = 4$ $x = 1$	M1A1	Set $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 0$ and attempt solution (M1)
	$y = \dfrac{16}{25} - 2 + \dfrac{17}{5} = \dfrac{37}{15}$	A1	Or 2.47 or $\left(1, \dfrac{37}{15}\right)$
	$\dfrac{\mathrm{d}^2 y}{\mathrm{d}x^x} = \dfrac{5}{2} \times \dfrac{1}{2} = \dfrac{5}{4}$ (> 0) hence minimum	A1	OE

Question	Answer	Marks	Guidance
10(i)	**AB** $= \begin{pmatrix} 2 \\ -3 \\ 5 \end{pmatrix} - \begin{pmatrix} -1 \\ 3 \\ -4 \end{pmatrix} = \begin{pmatrix} 3 \\ -6 \\ 9 \end{pmatrix}$, **BC** $= \begin{pmatrix} 4 \\ -2 \\ 5 \end{pmatrix} - \begin{pmatrix} 2 \\ -3 \\ 5 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$	**B1B1**	Condone reversal of labels
	AB.BC = 6 – 6 → = 0 (hence perpendicular)	**B1**	AG
10(ii)	**DC** $= \begin{pmatrix} 4 \\ -2 \\ 5 \end{pmatrix} - \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix} = \begin{pmatrix} 2 \\ -4 \\ 6 \end{pmatrix}$	**B1**	Or: **CD** $= \begin{pmatrix} -2 \\ 4 \\ -6 \end{pmatrix}$
	AB = k**DC**	**M1**	OE Expect $k = \frac{3}{2}$ Or: **DC.BC** = 4 – 4 = 0 hence *BC* is also perpendicular to *DC* Or: **AB.DC** = 1 or **AB.CD** = –1, angle between lines is 0 or 180
	AB is parallel to *DC*, hence *ABCD* is a trapezium	**A1**	
10(iii)	|**AB**| = $\sqrt{9+36+81} = \sqrt{126} = 11.22$ |**DC**| = $\sqrt{4+16+36} = \sqrt{56} = 7.483$ |**BC**| = $\sqrt{4+1+0} = \sqrt{5} = 2.236$	**M1**	Method for finding at least 2 magnitudes
	Area = $\frac{1}{2}\left(theirAB + theirDC\right) \times theirBC = 20.92$	**M1A1**	OE

Question	Answer	Marks	Guidance
11(i)	$(y=)\ (x+2)^2-1$	**B1** **DB1**	2nd B1 dependent on 2 in bracket
	$x+2=(\pm)(y+1)^{1/2}$	**M1**	
	$x=-2+(y+1)^{1/2}$	**A1**	
11(ii)	$x^2=4+(y+1)-/+4(y+1)^{\frac{1}{2}}$	***M1A1**	SOI. Attempt to find x^2 .The last term can be – or + at this stage
	$(\pi)\int x^2(\mathrm{d}y)=(\pi)\left[5y+\dfrac{y^2}{2}-\dfrac{4(y+1)^{\frac{3}{2}}}{\frac{3}{2}}\right]$	**A2,1,0**	
	$(\pi)\left[15+\dfrac{9}{2}-\dfrac{64}{3}-\left(-5+\dfrac{1}{2}\right)\right]$	**DM1**	Apply y limits
	$\dfrac{8\pi}{3}$ or 8.38	**A1**	

Cambridge International AS & A Level

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MATHEMATICS	**9709/12**
Paper 1	**October/November 2019**
MARK SCHEME	
Maximum Mark: 75	

Published

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SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	$\frac{6x}{2}$, $15 \times \frac{x^2}{4}$	B1 B1	OE In or from a correct expansion. Can be implied by correct equation.
	$\times (4 + ax) \to 3a + 15 = 3$	M1	2 terms in x^2 equated to 3 or $3x^2$. Condone x^2 on one side only.
	$a = -4$	A1	CAO
		4	

Question	Answer	Marks	Guidance
2	Attempt to find the midpoint *M*	M1	
	(1, 4)	A1	
	Use a gradient of $\pm\frac{2}{3}$ and *their M* to find the equation of the line.	M1	
	Equation is $y - 4 = -\frac{2}{3}(x - 1)$	A1	AEF
	Alternative method for question 2		
	Attempt to find the midpoint *M*	M1	
	(1, 4)	A1	
	Replace 1 in the given equation by c and substitute *their M*	M1	
	Equation is $y - 4 = -\frac{2}{3}(x - 1)$	A1	AEF
		4	

Question	Answer	Marks	Guidance
3	$(y =)\ \dfrac{kx^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}\left(=\dfrac{k\sqrt{x}}{\frac{1}{2}}\right)\ (+c)$	**B1**	OE
	Substitutes both points into an integrated expression with a '+c' and solve as far as a value for one variable.	**M1**	Expect to see $-1 = 2k + c$ and $4 = 4k + c$
	$k = 2½$ **and** $c = -6$	**A1**	WWW
	$y = 5\sqrt{x} - 6$	**A1**	OE From correct values of both k & c and correct integral.
		4	

Question	Answer	Marks	Guidance
4(i)	Arc length $AB = 2r\theta$	**B1**	
	Tan $\theta = \dfrac{AT}{r}$ or $\dfrac{BT}{r}$ $\rightarrow AT\ or\ BT = r\tan\theta$	**B1**	Accept or $\sqrt{\left(\left(\dfrac{r}{\cos\theta}\right)^2 - r^2\right)}$ or $\dfrac{r\sin\theta}{\sin\left(\dfrac{\pi}{2}-\theta\right)}$ NOT $(90-\theta)$
	$P = 2r\theta + 2r\tan\theta$	**B1FT**	OE, FT for *their* arc length + 2 × *their* AT
		3	

Question	Answer	Marks	Guidance
4(ii)	Area $\Delta AOT = ½ \times 5 \times 5 \tan 1.2$ **or** Area $AOBT = 2 \times ½ \times 5 \times 5 \tan 1.2$	**B1**	
	Sector area = $½ \times 25 \times 2.4$ (or 1.2)	***M1**	Use of $½r^2\theta$ with $\theta = 1.2$ or 2.4.
	Shaded area = 2 triangles – sector	**DM1**	Subtraction of sector, using 2.4 where appropriate, from 2 triangles
	Area = 34.3 (cm^2)	**A1**	AWRT
	Alternative method for question 4(ii)		
	Area of $\Delta\ ABT = ½ \times (5 \times \tan 1.2)^2 \times \sin(\pi - 2.4)$ (= 55.86)	**B1**	
	Segment area = $½ \times 25 \times (2.4 - \sin 2.4)$ (= 21.56)	***M1**	Use of $½r^2(\theta - \sin\theta)$ with $\theta = 1.2$ or 2.4
	Shaded area = triangle – segment	**DM1**	Subtraction of segment from $\Delta\ ABT$, using 2.4 where appropriate.
	Area = 34.3 (cm^2)	**A1**	AWRT
		4	

Question	Answer	Marks	Guidance
5(i)	Use of Pythagoras $\rightarrow r^2 = 15^2 - h^2$	**M1**	
	$V = ⅓\pi(225 - h^2) \times h \rightarrow ⅓\pi(225h - h^3)$	**A1**	AG WWW e.g. sight of $r = 15 - h$ gets A0.
		2	

Question	Answer	Marks	Guidance
5(ii)	$\left(\frac{\mathrm{d}v}{\mathrm{d}h}=\right)\frac{\pi}{3}(225-3h^2)$	**B1**	
	Their $\frac{\mathrm{d}v}{\mathrm{d}h}=0$	**M1**	Differentiates, sets *their* differential to 0 and attempts to solve at least as far as $h^2 \neq 0$.
	$(h=)\ \sqrt{75}$, $5\sqrt{3}$ or AWRT 8.66	**A1**	Ignore $-\sqrt{75}$ OE and ISW for both A marks
	$\frac{\mathrm{d}^2h}{\mathrm{d}h^2}=\frac{\pi}{3}\ (-6h)\ (\to -\text{ve})$	**M1**	Differentiates for a second time and considers the sign of the second differential or any other valid complete method.
	$\to$ Maximum	**A1FT**	Correct conclusion from correct 2nd differential, value for h not required, or any other valid complete method. FT for *their* h, if used, as long as it is positive.
			SC Omission of π or $\frac{\pi}{3}$ throughout can score B0M1A1M1A0
		5	

Question	Answer	Marks	Guidance
6(a)	$(2x + 1) = \tan^{-1}(⅓)$ (= 0.322 or 18.4 OR −0.339 rad or 8.7°)	*M1	Correct order of operations. Allow degrees.
	Either *their* 0.322 + π or 2π Or *their* −0.339 + $\frac{\pi}{2}$ or π	DM1	Must be in radians
	$x = 1.23$ **or** $x = 2.80$	A1	AWRT for either correct answer, accept 0.39π or 0.89π
		A1	For the second answer with no other answers between 0 and 2.8 **SC1** For both 1.2 and 2.8
		4	
6(b)(i)	$5\cos^2 x - 2$	B1	Allow $a = 5$, $b = -2$
		1	
6(b)(ii)	−2	B1FT	FT for sight of *their b*
	3	B1FT	FT for sight of *their a + b*
		2	

Question	Answer	Marks	Guidance
7(i)	$\left(\overrightarrow{PB}\right) = 5\mathbf{i} + 8\mathbf{j} - 5\mathbf{k}$	**B2,1,0**	B2 all correct, B1 for two correct components.
	$\left(\overrightarrow{PQ}\right) = 4\mathbf{i} + 8\mathbf{j} + 5\mathbf{k}$	**B2,1,0**	B2 all correct, B1 for two correct components.
			Accept column vectors. **SC** B1 for each vector if all components multiplied by –1.
		4	
7(ii)	(Length of PB =) $\sqrt{\left(5^2+8^2+5^2\right)}$ = ($\sqrt{114} \approx 10.7$) (Length of PQ =) $\sqrt{\left(4^2+8^2+5^2\right)}$ = ($\sqrt{105} \approx 10.2$)	**M1**	Evaluation of both lengths. Other valid complete comparisons can be accepted.
	P is nearer to Q.	**A1**	WWW
		2	
7(iii)	$\left(\overrightarrow{PB}.\overrightarrow{PQ}\right) = 20 + 64 - 25$	**M1**	Use of $x_1x_2 + y_1y_2 + z_1z_2$ on *their* $\overrightarrow{PB}$ and $\overrightarrow{PQ}$
	$(\mathit{Their}\sqrt{114})(\mathit{their}\sqrt{105})\cos BPQ = (\mathit{their}\ 59)$	**M1**	All elements present and in correct places.
	BPQ = 57.4(°) or 1.00 (rad)	**A1**	AWRT Calculating the obtuse angle and then subtracting gets A0.
		3	

Question	Answer	Marks	Guidance
8(a)(i)	21st term = 13 + 20 × 1.2 = 37 (km)	**B1**	
		1	

Question	Answer	Marks	Guidance
8(a)(ii)	$S_{21} = \frac{1}{2}\times 21 \times (26 + 20 \times 1.2)$ or $\frac{1}{2} \times 21 \times (13 + \textit{their } 37)$	**M1**	A correct sum formula used with correct values for a, d and n.
	525 (km)	**A1**	
		2	
8(b)(i)	$\frac{x-3}{x} = \frac{x-5}{x-3}$ oe (or use of a, ar and ar^2)	**M1**	Any valid method to obtain an equation in one variable.
	(a = *or* x =) 9	**A1**	
		2	
8(b)(ii)	$r = \left(\frac{x-3}{x}\right)$ or $\left(\frac{x-5}{x-3}\right)$ or $\sqrt{\frac{x-5}{x}} = ⅔$. Fourth term $= 9 \times (⅔)^3$	**M1**	Any valid method to find r and the fourth term with *their* a & r.
	2⅔ **or** 2.67	**A1**	OE, AWRT
		2	
8(b)(iii)	$S_\infty = \frac{a}{1-r} = \frac{9}{1-\frac{2}{3}}$	**M1**	Correct formula and using *their* 'r' and 'a', with $\lvert r \rvert < 1$, to obtain a numerical answer.
	27 **or** 27.0	**A1**	AWRT
		2	

Question	Answer	Marks	Guidance
9(i)	$f(x) = g(x) \rightarrow 2x^2 + 6x + 1 + k\ \ (= 0)$	*M1	Forms a quadratic with all terms on same side.
	Use of $b^2 = 4ac$	DM1	Uses the discriminant = 0.
	$(k =)\ 3\frac{1}{2}$	A1	OE, WWW
	Alternative method for question 9(i)		
	$4x + 8 = 2\ (\rightarrow x = -1\frac{1}{2})$	*M1	Differentiating, equating gradients and solving to give $x =$
	Substitutes *their* x value into either $2x^2 + 6x + 1 + k = 0$ OR into the curve to find $y\left(= \frac{-13}{2}\right)$ then both values into the line.	DM1	Substituting appropriately for *their* x and proceeding to find a value of k.
	$(k =)\ 3\frac{1}{2}$	A1	OE, WWW
		3	
9(ii)	$2x^2 + 6x - 8\ (< 0)$	M1	Forms a quadratic with all terms on same side
	-4 and 1	A1	
	$-4 < x < 1$	A1	CAO
		3	
9(iii)	$(g^{-1}(x)) = \frac{x-1}{2}$	B1	Needs to be in terms of x.
	$(g^{-1}f(x)) = \frac{2x^2 + 8x + 1 - 1}{2} = 0 \rightarrow (2x^2 + 8x = 0) \rightarrow x =$	M1	Substitutes f into g^{-1} and attempts to solve it = 0 as far as $x =$
	$0, -4$	A1	CAO
		3	

Question	Answer	Marks	Guidance
9(iv)	$2(x+2)^2 - 7$	B1 B1	or $a = +2$, $b = -7$
	(Least value of f(x) or $y =$) -7 or $\geqslant -7$	B1FT	FT for *their* b from a correct form of the expression.
		3	

Question	Answer	Marks	Guidance
10(i)	$\frac{dy}{dx} = [0] + \left[(2x+1)^{-3}\right] \times [+16]$	B2,1,0	OE. Full marks for 3 correct components. Withhold one mark for each error or omission.
	$\int y dx = [x] + \left[(2x+1)^{-1}\right] \times [+2]$ (+c)	B2,1,0	OE. Full marks for 3 correct components. Withhold one mark for each error or omission.
		4	
10(ii)	At A, $x = ½$.	B1	Ignore extra answer $x = -1.5$
	$\frac{dy}{dx} = 2 \rightarrow$ Gradient of normal $(= -½)$	*M1	With *their* positive value of x at A and *their* $\frac{dy}{dx}$, uses $m_1 m_2 = -1$
	Equation of normal: $y - 0 = -½(x - ½)$ or $y - 0 = -½\ (0 - ½)$ or $0 = -½ \times ½ + c$	DM1	Use of *their* x at A and *their* normal gradient.
	B (0, ¼)	A1	
		4	

Question	Answer	Marks	Guidance
10(iii)	$\int_0^{\frac{1}{2}} 1-\frac{4}{(2x+1)^2}(\mathrm{d}x)$	*M1	$\int y\,\mathrm{d}x$ SOI with 0 and *their* positive x coordinate of A.
	[½ + 1] – [0 + 2] = (–½)	DM1	Substitutes both 0 and *their* ½ into *their* $\int y\mathrm{d}x$ and subtracts.
	Area of triangle above x-axis = ½ × ½ × ¼ $\left(=\frac{1}{16}\right)$	B1	
	Total area of shaded region = $\frac{9}{16}$	A1	OE (including AWRT 0.563)
	Alternative method for question 10(iii)		
	$\int_{-3}^{0} \frac{1}{(1-y)^{\frac{1}{2}}}-\frac{1}{2}(\mathrm{d}y)$	*M1	$\int x\,\mathrm{d}y$ SOI. Where x is of the form $k\left(1-y)^{-\frac{1}{2}}+c\right)$ with 0 and *their* negative y intercept of curve.
	$[-2]-\left[-4+\frac{3}{2}\right]=(½)$	DM1	Substitutes both 0 and *their* –3 into *their* $\int x\mathrm{d}y$ and subtracts.
	Area of triangle above x-axis = ½ × ½ × ¼ $\left(=\frac{1}{16}\right)$	B1	
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	Alternative method for question 10(iii)		
	$\int_0^{\frac{1}{2}} -\frac{1}{2}x+\frac{1}{4}-y\,\mathrm{d}x$	***M1**	$\int$(*their* normal curve) with 0 and *their* positive x coordinate of A.
	Curve [½ + 1] – [0 + 2] = (−½)	**DM1**	Substitutes both 0 and *their* ½ into *their* $\int y\mathrm{d}x$ and subtracts.
	$\int_0^{\frac{1}{2}} -\frac{1}{2}x+\frac{1}{4}\mathrm{d}x = \frac{-x^2}{4}+\frac{x}{4} = \left[\frac{-1}{16}+\frac{1}{8}\right]-[0]\ \left(=\frac{1}{16}\right)$	**B1**	Substitutes both 0 and ½ into the correct integral and subtracts.
	Total area of shaded region = $\frac{9}{16}$	**A1**	OE (including AWRT 0.563)
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MATHEMATICS **9709/13**

Paper 1 **October/November 2019**

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AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1(i)	$1 + 6y + 15y^2$	**B1**	CAO
		1	
1(ii)	$1+6\left(px-2x^2\right)+15\left(px-2x^2\right)^2$	**M1**	SOI. Allow $6C1\times1^5\left(px-2x^2\right)$, $6C2\times1^4\left(px-2x^2\right)^2$
	$\left(15p^2-12\right)(x^2)=48\left(x^2\right)$	**A1**	1 term from each bracket and equate to 48
	$p=2$	**A1**	**SC:** A1 $p = 4$ from $15p - 12 = 48$
		3	

Question	Answer	Marks	Guidance
2	$(y=)\left[(x-3)^2\right][-2]$	***B1** **DB1**	DB1 dependent on 3 in 1st bracket
	$x-3=(\pm)\sqrt{y+2}$ **or** $y-3=(\pm)\sqrt{x+2}$	**M1**	Correct order of operations
	$\left(g^{-1}(x)\right)=3+\sqrt{x+2}$	**A1**	Must be in terms of x
	Domain (of g^{-1}) is $(x)>-1$	**B1**	Allow $(-1, \infty)$. Do not allow $y>-1$ or $g(x)>-1$ or $g^{-1}(x)>-1$
		5	

Question	Answer	Marks	Guidance
3	$\frac{dy}{dx} = 3x^2 + 2x - 8$	**B1**	
	Set to zero (SOI) and solve	**M1**	
	(Min) $a = -2$, (Max) $b = 4/3$. – in terms of a and b.	**A1** **A1**	Accept $a \geqslant -2$, $b \leqslant \frac{4}{3}$ **SC**: A1 for $a > -2$, $b < \frac{4}{3}$ or for $-2 < x < \frac{4}{3}$
		4	

Question	Answer	Marks	Guidance
4(i)	Angle $CAO = \frac{\pi}{3}$	**B1**	
		1	
4(ii)	(Sector AOC) $= \frac{1}{2}r^2 \times \mathit{their}\frac{\pi}{3}$	**M1**	SOI
	$(\Delta ABC) = \frac{1}{2}(r)(2r)\sin\left(\mathit{their}\frac{\pi}{3}\right)$ **or** $\frac{1}{2}(2r)(r)\frac{\sqrt{3}}{2}$ **or** $\frac{1}{2}(r)(r)\sqrt{3}$	**M1**	For M1M1, $\mathit{their}\frac{\pi}{3}$ must be of the form $k\pi$ where $0 < k < ½$
	$(\Delta ABC) = \frac{1}{2}(r)(2r)\sin\left(\frac{\pi}{3}\right)$ **or** $\frac{1}{2}(2r)(r)\frac{\sqrt{3}}{2}$ **or** $\frac{1}{2}(r)(r)\sqrt{3}$	**A1**	All correct
	$r^2\left(\frac{\sqrt{3}}{2}\right) - \frac{1}{2}r^2\left(\frac{\pi}{3}\right)$	**A1**	
		4	

Question	Answer	Marks	Guidance
5(i)	$S = 28x^2,\ V = 8x^3$	B1B1	SOI
	$7V^{\frac{2}{3}} = 7 \times 4x^2 = S$	B1	AG, WWW
		3	
5(ii)	$\left(\frac{\mathrm{d}S}{\mathrm{d}V}\right) = \frac{14V^{-\frac{1}{3}}}{3} = \frac{14}{30}$ SOI when $V = 1000$	*M1 A1	Attempt to differentiate For M mark $\left(\frac{\mathrm{d}S}{\mathrm{d}V}\right)$ to be of form $kV^{-\frac{1}{3}}$
	$\left(\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{\mathrm{d}S}{\mathrm{d}t} \times \frac{\mathrm{d}V}{\mathrm{d}S}\right)$ OE used with $\frac{\mathrm{d}S}{\mathrm{d}t} = 2$ **and** $\frac{1}{their\ \frac{14}{30}}$	DM1	
	$\frac{30}{7}$ or 4.29	A1	OE
	Alternative method for question 5(ii)		
	$V = \frac{S^{\frac{3}{2}}}{7\sqrt{7}} \to \left(\frac{\mathrm{d}V}{\mathrm{d}S}\right) = \frac{3}{2} \times S^{\frac{1}{2}} \times \frac{1}{7\sqrt{7}} = \frac{30}{14}$ SOI when $S = 700$	*M1 A1	Attempt to differentiate For M mark $\left(\frac{\mathrm{d}V}{\mathrm{d}S}\right)$ to be of form $kS^{\frac{1}{2}}$
	$\left(\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{\mathrm{d}S}{\mathrm{d}t} \times \frac{\mathrm{d}V}{\mathrm{d}S}\right)$ OE used with $\frac{\mathrm{d}S}{\mathrm{d}t} = 2$ **and** $\frac{1}{their\ \frac{14}{30}}$	DM1	
	$\frac{30}{7}$ or 4.29	A1	OE

Question	Answer	Marks	Guidance
5(ii)	**Alternative method for question 5(ii)**		
	Attempt to find either $\frac{dV}{dx}$ **or** $\left(\frac{dS}{dx} \text{and} \frac{dV}{dS}\right)$ together with either $\frac{dx}{dt}$ **or** x	***M1**	
	$\frac{dV}{dx} = 24x^2$ **or** $\left(\frac{dS}{dx} = 56x \text{ and } \frac{dV}{dS} = \frac{3x}{7}\right)$, $\frac{dx}{dt} = \frac{1}{140}$ **or** $x = 5$ (A1)	**A1**	
	Correct method for $\frac{dV}{dt}$	**DM1**	
	$\frac{30}{7}$ or 4.29	**A1**	OE
		4	

Question	Answer	Marks	Guidance
6(i)	$3kx - 2k = x^2 - kx + 2 \rightarrow \ x^2 - 4kx + 2k + 2 \ (= 0)$	**B1**	kx terms combined correctly-implied by correct $b^2 - 4ac$
	Attempt to find $b^2 - 4ac$	**M1**	Form a quadratic equation in k
	1 and $-\frac{1}{2}$	**A1**	SOI
	$k > 1, \ k < -\frac{1}{2}$	**A1**	Allow $x > 1, \ x < -1/2$
		4	
6(ii)	$y = 3x - 2, \quad y = -\dfrac{3}{2}x + 1$	**M1**	Use of *their* k values (twice) in $y = 3kx - 2k$
	$3x - 2 = -\dfrac{3}{2}x + 1$ OR $y + 2 = 2 - 2y$	**M1**	Equate *their* tangent equations OR substitute $y = 0$ into both lines
	$x = \dfrac{2}{3}, \rightarrow y = 0$ in one or both lines	**A1**	Substitute $x = \dfrac{2}{3}$ in one or both lines
		3	

Question	Answer	Marks	Guidance
7(i)	$3\cos^4\theta + 4\left(1-\cos^2\theta\right) - 3(=0)$	M1	Use $s^2 = 1 - c^2$
	$3x^2 + 4(1-x) - 3(=0) \rightarrow 3x^2 - 4x + 1(=0)$	A1	AG
		2	
7(ii)	Attempt to solve for x	M1	Expect x = 1, 1/3
	$\cos\theta = (\pm)1,\ (\pm)0.5774$	A1	Accept $(\pm)\left(\frac{1}{\sqrt{3}}\right)$ SOI
	($\theta =$) 0º, 180º, 54.7º, 125.3º	A3,2,1,0	A2,1,0 if more than 4 solutions in range
		5	

Question	Answer	Marks	Guidance
8(i)	$(2x-1)^{\frac{1}{2}} < 2$ or $3(2x-1)^{\frac{1}{2}} < 6$	**M1**	SOI
	$2x-1<4$	**A1**	SOI
	$\frac{1}{2} < x < \frac{5}{2}$	**A1 A1**	Allow 2 separate statements
		4	
8(ii)	f(x) = $[3(2x-1)^{3/2} \div \left(\frac{3}{2}\right) \div (2)]$ $[-6x]$ (+c)	**B1 B1**	
	Subsitute x = 1, y = –3 into an integrated expression.	**M1**	Dependent on c being present (c = 2)
	f(x) = $(2x-1)^{\frac{3}{2}} - 6x + 2$	**A1**	
		4	

Question	Answer	Marks	Guidance
9(i)	$\dfrac{5k-6}{3k}=\dfrac{6k-4}{5k-6} \quad \to \quad (5k-6)^2=3k(6k-4)$	M1	OR any valid relationship
	$25k^2-60k+36=18k^2-12k \quad \to \quad 7k^2-48k+36$	A1	AG
		2	
9(ii)	$k=\dfrac{6}{7}$, 6	B1B1	Allow 0.857(1) for $\dfrac{6}{7}$
	When $k=\dfrac{6}{7}$, $r=-\dfrac{2}{3}$	B1	Must be exact
	When $k=6$, $r=\dfrac{4}{3}$	B1	
		4	
9(iii)	Use of $S_\infty=\dfrac{a}{1-r}$ with $r=their-\dfrac{2}{3}$ and $a=3\times their\dfrac{6}{7}$	M1	Provided $0<\lvert their\ {-2/3}\rvert<1$
	$\dfrac{18}{7}\div\left(1+\dfrac{2}{3}\right)=\dfrac{54}{35}$ or 1.54	A1	FT if 0.857(1) has been used in part (ii).
		2	

Question	Answer	Marks	Guidance
10(i)	$\mathbf{AX} = \begin{pmatrix} 6 \\ 2 \\ 3 \end{pmatrix}$, and one of $\mathbf{AB} = \begin{pmatrix} 18 \\ 6 \\ 9 \end{pmatrix}$, $\mathbf{XB} = \begin{pmatrix} 12 \\ 4 \\ 6 \end{pmatrix}$, $\mathbf{BX} = \begin{pmatrix} -12 \\ -4 \\ -6 \end{pmatrix}$	**B1B1**	
	State $\mathbf{AB} = 3\mathbf{AX}$ (or $\mathbf{XB} = 2\mathbf{AX}$ or $\mathbf{AB} = \frac{3}{2}\mathbf{XB}$ etc) hence straight line **OR** $\frac{\mathbf{AX.AB}}{\lvert\mathbf{AX}\rvert\lvert\mathbf{AB}\rvert} = 1\ (\rightarrow \theta = 0)$ or $\frac{\mathbf{AX.BX}}{\lvert\mathbf{AX}\rvert\lvert\mathbf{BX}\rvert} = -1\ (\rightarrow \theta = 180)$ hence straight line	**B1**	WWW A conclusion (i.e. a straight line) is required.
		3	
10(ii)	$\mathbf{CX} = \begin{pmatrix} -3 \\ 6 \\ 2 \end{pmatrix}$	**B1**	
	$\mathbf{CX.AX} = -18 + 12 + 6$	**M1**	
	= 0 (hence CX is perpendicular to AX)	**A1**	
		3	
10(iii)	$\lvert\mathbf{CX}\rvert = \sqrt{3^2 + 6^2 + 2^2}$, $\lvert\mathbf{AB}\rvert = \sqrt{18^2 + 6^2 + 9^2}$ Both attempted	**M1**	
	Area $\Delta ABC = \frac{1}{2} \times$ *their* 21 $\times$ *their* 7 = $73\frac{1}{2}$	**M1A1**	Accept answers which round to 73.5
		3	

Question	Answer	Marks	Guidance
11(i)	$\frac{dy}{dx} = -2(x-1)^{-3}$	**B1**	
	When $x = 2$, $m = -2 \to$ gradient of normal $= -\frac{1}{m}$	**M1**	m must come from differentiation
	Equation of normal is $y - 3 = ½(x - 2) \to \; y = ½x + 2$	**A1**	AG Through (2, 3) with gradient $-\frac{1}{m}$. Simplify to AG
		3	

Question	Answer	Marks	Guidance
11(ii)	$(\pi)\int y_1^2(\mathrm{d}x), (\pi)\int y_2^2(\mathrm{d}x)$	*M1	Attempt to integrate y^2 for at least one of the functions
	$(\pi)\int\left(\frac{1}{2}x+2\right)^2$ or $\left(\frac{1}{4}x^2+2x+4\right)$ $(\pi)\int\left((x-1)^{-4}+4(x-1)^{-2}+4\right)$	A1A1	A1 for $\left(\frac{1}{2}x+2\right)^2$ depends on an attempt to integrate this form later
	$(\pi)\left[\frac{2}{3}\left(\frac{1}{2}x+2\right)^3\right.$ or $\left.\frac{1}{12}x^3+x^2+4x\right]$ $(\pi)\left[\frac{(x-1)^{-3}}{-3}+\frac{4(x-1)^{-1}}{-1}+4x\right]$	A1A1	Must have at least 2 terms correct for each integral
	$(\pi)\left\{18-\frac{125}{12} or \frac{2}{3}+4+8-\left(\frac{1}{12}+1+4\right)\right\}$ $\left\{\frac{-1}{24}-2+12-\left(\frac{-1}{3}-4+8\right)\right\}$	DM1	Apply limits to at least 1 integrated expansion
	Attempt to add 2 volume integrals (or 1 volume integral + frustum) $\pi\left\{7\frac{7}{12}+6\frac{7}{24}\right\}$	DM1	
	$13\frac{7}{8}\pi$ or $\frac{111}{8}\pi$ or 13.9π or 43.6	A1	$\frac{2}{3}+4+8-\left(\frac{1}{12}+1+4\right)$ $\frac{-1}{24}-2+12-\left(\frac{-1}{3}-4+8\right)$
		8	

Cambridge Assessment International Education
Cambridge International Advanced Subsidiary and Advanced Level

MATHEMATICS **9709/11**

Paper 1 **May/June 2019**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2019 series for most Cambridge IGCSE™, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

  - Note: B2 or A2 means that the candidate can earn 2 or 0.
    B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
CWO	Correct Working Only – often written by a 'fortuitous' answer
ISW	Ignore Subsequent Working
SOI	Seen or implied
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Question	Answer	Marks	Guidance
1(i)	Ind term $= (2x)^3 \times \left(\frac{k}{x}\right)^3 \times {}_6C_3$	B2,1,0	Term must be isolated
	$= 540 \rightarrow k = 1\frac{1}{2}$	B1	
		3	
1(ii)	Term, in x^2 is $(2x)^4 \times \left(\frac{k}{x}\right)^2 \times {}_6C_2$	B1	All correct – even if k incorrect.
	$15 \times 16 \times k^2 = 540$ (or $540x^2$)	B1	**FT** For $240k^2$ or $240k^2x^2$
		2	

Question	Answer	Marks	Guidance
2(i)	Eliminates x or $y \to$ $y^2 - 4y + c - 3 = 0$ or $x^2 + (2c - 16)x + c^2 - 48 = 0$	M1	Eliminates x or y completely to a quadratic
	Uses $b^2 = 4ac$ $\to 4c - 28 = 0$	M1	Uses discriminant = 0. (c the only variable) Any valid method (may be seen in part **(i)**)
	$c = 7$	A1	
	Alternative method for question 2(i)		
	$\frac{dy}{dx} = \frac{1}{2\sqrt{(x+3)}} = \frac{1}{4}$	M1	
	Solving	M1	
	$c = 7$	A1	
		3	
2(ii)	Uses $c = 7$, $y^2 - 4y + 4 = 0$	M1	Ignore (1,–2), c=-9
	(1, 2)	A1	
		2	

Question	Answer	Marks	Guidance
3	Uses $A = \frac{1}{2}r^2\theta$	M1	Uses area formula.
	$\theta = \dfrac{2A}{r^2}$	A1	
	$P = r + r + r\theta$	B1	
	$P = 2r + \dfrac{2A}{r}$	A1	Correct simplified expression for P.
		4	

Question	Answer	Marks	Guidance
4(i)	Gradient of $AB = -½ \rightarrow$ Gradient of $BC = 2$	**M1**	Use of $m_1.m_2 = -1$ for correct lines
	Forms equation in h $\frac{3h-2}{h} = 2$	**M1**	Uses normal line equation or gradients for h.
	$h = 2$	**A1**	
	Alternative method for question 4(i)		
	Vectors AB.BC=0	**M1**	Use of vectors AB and BC
	Solving	**M1**	
	$h = 2$	**A1**	
	Alternative method for question 4(i)		
	Use of Pythagoras to find 3 lengths	**M1**	
	Solving	**M1**	
	$h = 2$	**A1**	
		3	
4(ii)	y coordinate of D is 6, (3 × 'their' h) $\frac{6-0}{x-4} = 2 \rightarrow x = 7 \rightarrow D\ (7, 6)$	**B1**	**FT**
	Vectors: AD.AB=0	**M1 A1**	Must use y = 6 Realises the y values of C and D are equal. Uses gradient or line equation to find x.
		3	

Question	Answer	Marks	Guidance
5(i)	$-2(x-3)^2+15 \ (a=-3,\ b=15)$	B1 B1	Or seen as $a=-3$, $b=15$ B1 for each value
		2	
5(ii)	$(\mathrm{f}(x) \leqslant)\ 15$	B1	**FT** for ($\leqslant$) their "b" Don't accept (3,15) alone
		1	
5(iii)	$\mathrm{gf}(x) = 2(-2x^2+12x-3) + 5 = -4x^2 + 24x - 6 + 5$	B1	
	$\mathrm{gf}(x) + 1 = 0 \rightarrow -4x^2 + 24x = 0$	M1	
	$x = 0$ or 6	A1	Forms and attempts to solve a quadratic Both answers given.
		3	

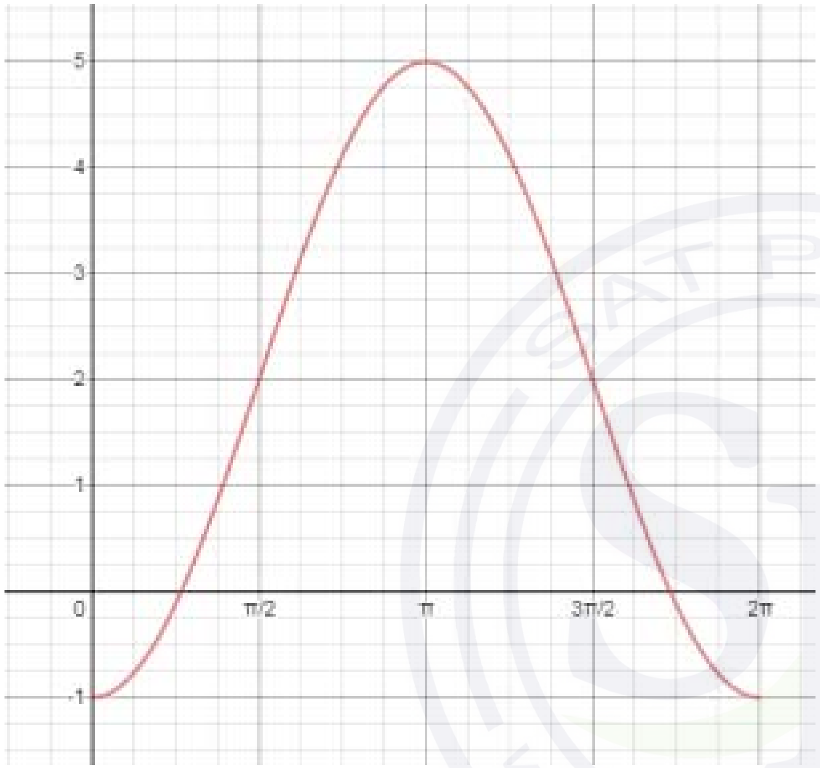
Question	Answer	Marks	Guidance
6(i)	$\text{LHS} = \left(\frac{1}{c}-\frac{s}{c}\right)^2 = \frac{(1-s)(1-s)}{c^2} = \frac{(1-s)(1-s)}{1-s^2}$	B1	Expresses tan in terms of sin and cos
		B1	correctly $1-s^2$ as the denominator
	$=\frac{(1-s)(1-s)}{(1-s)(1+s)}$	M1	Factors and correct cancelling www
	$\frac{1-\sin x}{1+\sin x}$ AG	A1	
		4	

Question	Answer	Marks	Guidance
6(ii)	Uses part **(i)** to obtain $\frac{1-\sin 2x}{1+\sin 2x} = \frac{1}{3} \rightarrow \sin 2x = ½$	**M1**	Realises use of $2x$ and makes $\sin 2x$ the subject
	$x = \frac{\pi}{12}$	**A1**	Allow decimal (0.262)
	(or) $x = \frac{5\pi}{12}$	**A1**	**FT** for ½π – 1st answer. Allow decimal (1.31) $\frac{\pi}{12}$ and $\frac{5\pi}{12}$ only, and no others in range. SC $\sin x = ½ \rightarrow \frac{\pi}{6}\ \frac{5\pi}{6}$ B1
		3	

Question	Answer	Marks	Guidance
7(i)	$\overrightarrow{AM} = 1.5\mathbf{i} + 4\mathbf{j} + 5\mathbf{k}$ $\overrightarrow{GM} = 6.5\mathbf{i} - 4\mathbf{j} - 5\mathbf{k}$	**B3,2,1**	Loses 1 mark for each error.
		3	
7(ii)	$\overrightarrow{AM} . \overrightarrow{GM} = 9.75 - 16 - 25 = -31.25$	**M1**	Use of $x_1x_2 + y_1y_2 + z_1z_2$ on AM and GM
	$\overrightarrow{AM} . \overrightarrow{GM} = \sqrt{(1.5^2+4^2+5^2)} \times \sqrt{(6.5^2+4^2+5^2)} \cos GMA$	**M1 M1**	M1 for product of 2 modulii M1 all correctly connected
	Equating → Angle GMA = 121°	**A1**	
		4	

Question	Answer	Marks	Guidance
8(a)	$ar^2 = 48$, $ar^3 = 32$, r = ⅔ **or** $a = 108$	**M1**	Solution of the 2 eqns to give r (or a). A1 (both)
	r = ⅔ **and** $a = 108$	**A1**	
	$S\infty = \dfrac{108}{\frac{1}{3}} = 324$	**A1**	**FT** Needs correct formula and r between −1 and 1.
		3	
8(b)	Scheme A $a = 2.50$, $d = 0.16$ $S_n = 12(5 + 23\times0.16)$	**M1**	Correct use of either AP S_n formula.
	$S_n = 104$ tonnes.	**A1**	
	Scheme B $a = 2.50$, $r = 1.06$	**B1**	Correct value of r used in GP.
	$= \dfrac{2.5\left(1.06^{24}-1\right)}{1.06-1}$	**M1**	Correct use of either S_n formula.
	$S_n = 127$ tonnes.	**A1**	
		5	

Question	Answer	Marks	Guidance
9(i)	$-1 \leqslant f(x) \leqslant 5$ or [–1, 5] (may use y or f instead of f(x))	**B1 B1**	$-1 < f(x) \leqslant 5$ or $-1 \leqslant x \leqslant 5$ or (–1,5) or [5,–1] B1 only
		2	

Question	Answer	Marks	Guidance
9(ii)		*B1	Start and end at –ve y, symmetrical, centre +ve.
	$g(x) = 2 - 3\cos x$ for $0 \leqslant x \leqslant p$	DB1	Shape all ok. Curves not lines. One cycle $[0,2\pi]$ Flattens at each end.
		2	

Question	Answer	Marks	Guidance
9(iii)	(greatest value of p =) π	**B1**	
		1	
9(iv)	$x = 2 - 3\cos x \rightarrow \cos x = ⅓(2 - x)$	**M1**	Attempt at $\cos x$ the subject. Use of cos^{-1}
	$g^{-1}(x) = \cos^{-1}\dfrac{2-x}{3}$ (may use 'y =')	**A1**	Must be a function of x,
		2	

Question	Answer	Marks	Guidance
10(i)	integrating $\rightarrow \dfrac{dy}{dx} = x^2 - 5x\ (+c)$	**B1**	
	$= 0$ when $x = 3$	**M1**	Uses the point to find c after $\int = 0$.
	$c = 6$	**A1**	
	integrating again $\rightarrow\ y = \dfrac{x^3}{3} - \dfrac{5x^2}{2} + 6x\ \ (+d)$	**B1**	**FT** Integration again FT if a numerical constant term is present.
	use of (3, 6)	**M1**	Uses the point to find d after $\int = 0$.
	$d = 1½$	**A1**	
		6	

Question	Answer	Marks	Guidance
10(ii)	$\frac{dy}{dx} = x^2 - 5x + 6 = 0 \rightarrow x = 2$	**B1**	
		1	
10(iii)	$x = 3$, $\frac{d^2y}{dx^2} = 1$ and/or +ve Minimum. $x = 2$, $\frac{d^2y}{dx^2} = -1$ and/or −ve Maximum	**B1**	www
	May use shape of '$+x^3$' curve or change in sign of $\frac{dy}{dx}$	**B1**	www SC: $x = 3$, minimum, $x = 2$, maximum, B1
		2	

Question	Answer	Marks	Guidance
11(i)	$3 \times -½ \times (1+4x)^{-\frac{3}{2}}$	**B1**	
	$\frac{dy}{dx} = 3 \times -½ \times (1+4x)^{-\frac{3}{2}} \times 4$	**B1**	Must have '× 4'
	If $x = 2$, $m = -\frac{2}{9}$, Perpendicular gradient = $\frac{9}{2}$	**M1**	Use of $m_1.m_2 = -1$
	Equation of normal is $y - 1 = \frac{9}{2}(x-2)$	**M1**	Correct use of line eqn (could use y=0 here)
	Put $y = 0$ or on the line before $\rightarrow \frac{16}{9}$	**A1**	AG
		5	

Question	Answer	Marks	Guidance
11(ii)	Area under the curve = $\int_0^2 \frac{3}{\sqrt{1+4x}}\,dx = \frac{3\sqrt{1+4x}}{\frac{1}{2}} \div 4$	B1 B1	Correct without '÷4'. For 2nd B1, ÷4'.
	Use of limits 0 to 2 → 4½ − 1½	M1	Use of correct limits in an integral.
	3	A1	
	Area of the triangle = ½ × 1 × $\frac{2}{9} = \frac{1}{9}$ or attempt to find $\int_{16/9}^{2} \left(\frac{9}{2}x - 8\right) dx$	M1	Any correct method.
	Shaded area = $3 - \frac{1}{9} = 2\frac{8}{9}$	A1	
		6	

Cambridge International AS & A Level

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Cambridge International Advanced Subsidiary and Advanced Level

MATHEMATICS | **9709/12**

Paper 1 | **May/June 2019**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

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Cambridge Assessment International Education

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

  - Note: B2 or A2 means that the candidate can earn 2 or 0.
    B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no “follow through” from a previous error is allowed)
CWO	Correct Working Only – often written by a ‘fortuitous’ answer
ISW	Ignore Subsequent Working
SOI	Seen or implied
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become “follow through” marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Question	Answer	Marks	Guidance
1	For $\left(\frac{2}{x}-3x\right)^5$ term in x is 10 or $5C_3$ or $5C2 \times \left(\frac{2}{x}\right)^2 \times (-3x)^3$ or $\left(\frac{2}{x}\right)^5 \frac{5.4.3}{3!}\left(-\frac{3}{2}x^2\right)^3$ or $(-3x)^5 \frac{5.4}{2!}\left(\frac{2}{3x^2}\right)^2$	**B2,1**	3 elements required. –1 for each error with or without x's. Can be seen in an expansion.
	−1080 identified	**B1**	Allow $-1080x$ Allow if expansion stops at this term. Allow from expanding brackets.
		3	

Question	Answer	Marks	Guidance
2	Midpoint of AB is (5, 1)	**B1**	Can be seen in working, accept $\left(\frac{10}{2},\frac{2}{2}\right)$.
	$m_{AB} = -½$ oe	**B1**	
	C to (5, 1) has gradient 2	***M1**	Use of $m_1 \times m_2 = -1$.
	Forming equation of line ($y = 2x - 9$)	**DM1**	Using their perpendicular gradient and their midpoint to form the equation.
	C (0, −9) **or** $y = -9$	**A1**	
		5	

Question	Answer	Marks	Guidance
3(i)	$\frac{dy}{dt}=\frac{dy}{dx}\times\frac{dx}{dt}=7\times-0.05$	**M1**	Multiply numerical gradient at $x = 2$ by ± 0.05.
	-0.35 (units/s) **or** Decreasing at a rate of (+) 0.35	**A1**	Ignore notation and omission of units
		2	
3(ii)	$(y)=\frac{x^4}{4}+\frac{4}{x}$ $(+c)$ oe	**B1**	Accept unsimplified
	Uses (2, 9) in an integral to find c.	**M1**	The power of at least one term increase by 1.
	$c = 3$ **or** $(y=)\frac{x^4}{4}+\frac{4}{x}+3$ oe	**A1**	A0 if candidate continues to a final equation that is a straight line.
		3	

Question	Answer	Marks	Guidance
4(i)	$a^2+2ab+b^2, a^2-2ab+b^2$	B1	Correct expansions.
	$\sin^2 x + \cos^2 x = 1$ used → $(a+b)^2+(a-b)^2=1$	M1	Appropriate use of $\sin^2 x + \cos^2 x = 1$ with $(a+b)^2$ and $(a-b)^2$
	$a^2 + b^2 = ½$	A1	No evidence of $\pm 2ab$, scores 2/3
	Alternative method for question 4(i)		
	$2a$ = (s+c) & $2b$ = (s−c) or a = ½(s+c) & b = ½(s−c)	B1	
	$a^2+b^2 = \frac{1}{4}(s+c)^2+\frac{1}{4}(s-c)^2$ = ½(s^2+c^2)	M1	Appropriate use of $\sin^2 x + \cos^2 x = 1$
	$a^2 + b^2 = ½$	A1	Method using only $(\sin x-b)^2$ and $(a-\cos x)^2$ scores 0/3.
		3	SC B1 for assuming θ is acute giving $a=\frac{1}{\sqrt{5}}+b$ or $2\sqrt{5}-b$

Question	Answer	Marks	Guidance
4(ii)	$\tan x = \dfrac{\sin x}{\cos x} \to \dfrac{a+b}{a-b} = 2$	M1	Use of $\tan x = \dfrac{\sin x}{\cos x}$ to form an equation in a and b only
	$a = 3b$	A1	
		2	

Question	Answer	Marks	Guidance
5	Perimeter of $AOC = 2r + r\theta$	B1	
	Angle $COB = \pi - \theta$	B1	Could be on the diagram. Condone $180 - \theta$.
	Perimeter of $BOC = 2r + r(\pi - \theta)$	B1	**FT** on angle COB if of form $(k\pi - \theta)$, $k > 0$.
	$(2r +)\ \pi r - r\theta = 2((2r) + r\theta)$ $(2 + \pi - \theta = 4 + 2\theta \to \theta = \dfrac{\pi - 2}{3})$	M1	Sets up equation using $r(k\pi - \theta)$ and ×2 on correct side. Condone any omissions of OA, OB and/or OC.
	$\theta = 0.38$	A1	Equivalent answer in degrees scores A0.
		5	

Question	Answer	Marks	Guidance
6(i)	3, –3	**B1**	Accept ± **3**
	–½	**B1**	
	2½	**B1**	
		3	Condone misuse of inequality signs.
6(ii)			Only mark the curve from 0 → 2π. If the *x* axis is not labelled assume that 0 → 2π is the range shown. Labels on axes are not required.
	2 complete oscillations of a cosine curve starting with a maximum at (0,a), a>0	**B1**	
	Fully correct curve which must appear to level off at 0 and/or 2π.	**B1**	
	Line starting on positive *y* axis and finishing below the *x* axis at 2π. Must be straight.	**B1**	
		3	
6(iii)	4	**B1**	
		1	

Question	Answer	Marks	Guidance
7(i)	$(\mathrm{f}^{-1}(x)) = \dfrac{x+2}{3}$ oe	**B1**	
	$y = \dfrac{2x+3}{x-1} \to (x-1)y = 2x+3 \to x(y-2) = y+3$	**M1**	Correct method to obtain $x =$, (or $y =$, if interchanged) but condone $+/-$ sign errors
	$(\mathrm{g}^{-1}(x)$ or $y) = \dfrac{x+3}{x-2}$ oe $\left(eg\ \dfrac{5}{x-2}+1\right)$	**A1**	Must be in terms of x
	$x \neq 2$ only	**B1**	**FT** for value of x from their denominator = 0
		4	
7(ii)	$\left(fg(x)=\right) \dfrac{3(2x+3)}{x-1} - 2\ (=\dfrac{7}{3})$	**B1**	
	$18x + 27 = 13x - 13$ or $3(4x + 11) = 7(x - 1)$ $(5x = -40)$	**M1**	Correct method from their $fg = \dfrac{7}{3}$ leading to a linear equation and collect like terms. Condone omission of $2(x-1)$.
	Alternative method for question 7(ii)		
	$(\mathrm{f}^{-1}(\dfrac{7}{3})) = \dfrac{13}{9}$	**B1**	
	$\dfrac{2x+3}{x-1} = \dfrac{13}{9} \to 9(2x+3) = 13(x-1)\ (\to 5x = -40)$	**M1**	Correct method from $g(x) =$ their $\dfrac{13}{9}$ leading to a linear equation and collect like terms.
	$x = -8$	**A1**	
		3	

Question	Answer	Marks	Guidance
8(i)	$6\times3+-2\times k+-6\times-3=0$ $(18-2k+18=0)$	M1	Use of scalar product = 0. Could be $\overrightarrow{AO}.\overrightarrow{OB}$, $\overrightarrow{AO}.\overrightarrow{BO}$ or $\overrightarrow{OA}.\overrightarrow{BO}$
	$k=18$	A1	
	Alternative method for question 8(i)		
	$76+18+k^2=18+(k+2)^2$	M1	Use of Pythagoras with appropriate lengths.
	$k=18$	A1	
		2	
8(ii)	$36+4+36=9+k^2+9$	M1	Use of modulus leading to an equation and solve to $k=$ or $k^2=$
	$k=\pm\sqrt{58}$ or ±7.62	A1	Accept exact or decimal answers. Allow decimals to greater accuracy.
		2	

Question	Answer	Marks	Guidance
8(iii)	$\overrightarrow{AB} = \begin{pmatrix} -3 \\ 6 \\ 3 \end{pmatrix} \rightarrow \overrightarrow{AC} = \begin{pmatrix} -2 \\ 4 \\ 2 \end{pmatrix}$ then $\overrightarrow{OA} + \overrightarrow{AC}$	M1	Complete method using $\overrightarrow{AC} = \pm \frac{2}{3} \overrightarrow{AB}$ And then $\overrightarrow{OA} + their\ \overrightarrow{AC}$
	$\overrightarrow{OC} = \begin{pmatrix} 4 \\ 2 \\ -4 \end{pmatrix}$	A1	
	$\div \sqrt{(their\,4)^2 + (their\,2)^2 + (their\,-4)^2}$	M1	Divides by modulus of their $\overrightarrow{OC}$
	$= \frac{1}{6}\begin{pmatrix} 4 \\ 2 \\ -4 \end{pmatrix}$ **or** $\frac{1}{6}\ (4i + 2j - 4k)$	A1	
	Alternative method for question 8(iii)		
	Let $\overrightarrow{OC} = \begin{pmatrix} p \\ q \\ r \end{pmatrix} \rightarrow \overrightarrow{AC} = \begin{pmatrix} p-6 \\ q+2 \\ r+6 \end{pmatrix} \& \overrightarrow{CB} = \begin{pmatrix} 3-p \\ 4-q \\ -3-r \end{pmatrix}$	M1	Correct method. Equates coefficients leading to values for p, q, r
	$p - 6 = 2(3 - p)$; $q+2 = 2(4 - q)$; $r + 6 = 2\ (-3 - r)$ $\rightarrow p=4,\ q=2\ \&\ r= -4$	A1	
	$\div \sqrt{(their\,4)^2 + (their\,2)^2 + (their\,-4)^2}$	M1	Divides by modulus of their $\overrightarrow{OC}$
	$= \frac{1}{6}\begin{pmatrix} 4 \\ 2 \\ -4 \end{pmatrix}$ **or** $\frac{1}{6}\ (4i + 2j - 4k)$	A1	

Question	Answer	Marks	Guidance
8(iii)	**Alternative method for question 8(iii)**		
	$\overrightarrow{CB} = \overrightarrow{OB} - \overrightarrow{OC}\ \therefore 2\left(\overrightarrow{OB} - \overrightarrow{OC}\right) = \overrightarrow{OC} - \overrightarrow{OA}$ $\rightarrow 2\overrightarrow{OB} + \overrightarrow{OA} = 3\overrightarrow{OC} \therefore\ 3\overrightarrow{OC} = \begin{pmatrix} 12 \\ 6 \\ -12 \end{pmatrix}$	M1	Correct method. Gets to a numerical expression for k$\overrightarrow{OC}$ from $\overrightarrow{OA}$ & $\overrightarrow{OB}$.
	$\overrightarrow{OC} = \begin{pmatrix} 4 \\ 2 \\ -4 \end{pmatrix}$	A1	
	$\div \sqrt{(their\,4)^2 + (their\,2)^2 + (their\,-4)^2}$	M1	Divides by modulus of their $\overrightarrow{OC}$
	$= \frac{1}{6}\begin{pmatrix} 4 \\ 2 \\ -4 \end{pmatrix}$ **or** $\frac{1}{6}\ (4i + 2j - 4k)$	A1	
		4	

Question	Answer	Marks	Guidance
9	For C_1: $\dfrac{dy}{dx} = 2x - 4 \rightarrow m = 2$	B1	
	$y -$ '*their 4*' = '*their* m' $(x - 3)$ or using $y = mx + c$	M1	Use of : $\dfrac{dy}{dx}$ and (3, their 4) to find the tangent equation.
	$y - 4 = 2(x - 3)$ or $y = 2x - 2$	A1	If using $= mx + c$, getting $c = -2$ is enough.
	$2x - 2 = \sqrt{4x + k}$ $(\rightarrow 4x^2 - 12x + 4 - k = 0)$	*M1	Forms an equation in one variable using tangent & C_2
	Use of $b^2 - 4ac = 0$ on a 3 term quadratic set to 0.	*DM1	Uses 'discriminant = 0'
	$144 = 16(4 - k) \rightarrow k = -5$	A1	
	$4x^2 - 12x + 4 - k = 0 \rightarrow 4x^2 - 12x + 9 = 0$	DM1	Uses k to form a 3 term quadratic in x
	$x = \dfrac{3}{2} \left(or \dfrac{1}{2}\right), y = 1(\text{or} - 1).$	A1	Condone 'correct' extra solution.
	Alternative method for question 9		
	For C_1: $\dfrac{dy}{dx} = 2x - 4 \rightarrow m = 2$	B1	
	$y -$ '*their 4*' = '*their* m' $(x - 3)$ or using $y = mx + c$	M1	Use of : $\dfrac{dy}{dx}$ and (3, their 4) to find the tangent equation.
	$y - 4 = 2(x - 3)$ or $y = 2x - 2$	A1	If using $= mx + c$, getting $c = -2$ is enough.
	For C_2: $\dfrac{dy}{dx} = A(4x + k)^{-\frac{1}{2}}$	*M1	Finds $\dfrac{dy}{dx}$ for C_2 in the form $A(4x + k)^{-\frac{1}{2}}$

Question	Answer	Marks	Guidance
9	At P: '*their* 2' = $A(4x+k)^{-\frac{1}{2}}$" $\to$ ($x=\frac{1-k}{4}$ *or* $4x+k=1$)	*DM1	Equating '*their* 2' to '*their* $\frac{dy}{dx}$' and simplify to form a linear equation linking $4x+k$ and a constant.
	$(2x-2)^2=4x+k \to (2x-2)^2=1 \to (4x^2-8x+3=0)$	DM1	Using *their* $y=2x-2$, $y^2=4x+k$ and *their* $4x+k=1$ (but not =0) to form a 3 term quadratic in x.
	$x=\frac{3}{2}\left(or\ \frac{1}{2}\right)$ and from $k=-5(or\ -1)$	A1	Needs correct values for x and k.
	from $y^2=4x+k$, $y=1$(or – 1).	A1	Condone 'correct' extra solution.
	Alternative method for question 9		
	For C_1: $\frac{dy}{dx}=2x-4 \to m=2$	B1	
	y – '*their 4*' = '*their* m' $(x-3)$ or using $y=mx+c$	M1	Use of : $\frac{dy}{dx}$ and (3, their 4) to find the tangent equation.
	$y-4=2(x-3)$ or $y=2x-2$	A1	If using $=mx+c$, getting $c=-2$ is enough.
	For C_2: $\frac{dy}{dx}=A(4x+k)^{-\frac{1}{2}}$	*M1	Finds $\frac{dy}{dx}$ for C_2 in the form $A(4x+k)^{-\frac{1}{2}}$
	At P: '*their* 2' = $A(4x+k)^{-\frac{1}{2}}$" $\to$ ($x=\frac{1-k}{4}$ *or* $4x+k=1$)	*DM1	Equating '*their* 2' to '*their* $\frac{dy}{dx}$' and simplify to form a linear equation linking $4x+k$ and a constant.
	From $4x+k=1$ and $y^2=4x+k \to y^2=1$	DM1	Using *their* $4x+k=1$ (but not =0) and C_2 to form y^2 = a constant

Question	Answer	Marks	Guidance
9	$y = 1(\text{or} -1)$ and $x = \frac{3}{2}\left(or \frac{1}{2}\right)$	**A1**	Needs correct values for y and x.
	From $4x + k = 1$, $k = -5$ (or -1)	**A1**	Condone 'correct' extra solution
		8	

Question	Answer	Marks	Guidance
10(a)(i)	$S_{10} = S_{15} - S_{10}$ or $S_{10} = S_{(11 \text{ to } 15)}$	**M1**	Either statement seen or implied.
	$5(2a + 9d)$ oe	**B1**	
	$7.5(2a + 14d) - 5(2a + 9d)$ or $\frac{5}{2}[(a + 10d) + (a+14d)]$ oe	**A1**	
	$d = \frac{a}{3}$ **AG**	**A1**	Correct answer from convincing working
		4	Condone starting with $d = \frac{a}{3}$ and evaluating both summations as 25a.
10(a)(ii)	$(a+9d) = 36 + (a+3d)$	**M1**	Correct use of $a+(n-1)d$ twice and addition of ± 36
	$a = 18$	**A1**	
		2	Correct answer www scores 2/2

Question	Answer	Marks	Guidance
10(b)	$S_\infty = 9 \times S_4$; $\dfrac{a}{1-r} = 9\dfrac{a\left(1-r^4\right)}{1-r}$ or $9(a + ar + ar^2 + ar^3)$	**B1**	May have 12 in place of *a*.
	$9(1 - r^n) = 1$ where $n = 3,4$ or 5	**M1**	Correctly deals with *a* and correctly eliminates '$1 - r$'
	$r^4 = \dfrac{8}{9}$ oe	**A1**	
	(5^{th} term =) 10⅔ or 10.7	**A1**	
		4	Final answer of 10.6 suggests premature approximation – award 3/4 www.

Question	Answer	Marks	Guidance
11(i)	$\frac{dy}{dx}=\left[\frac{1}{2}(4x+1)^{-\frac{1}{2}}\right]$ [× 4] $\left[-\frac{9}{2}(4x+1)^{-\frac{3}{2}}\right]$ [× 4]	B1B1B1	B1 B1 for each, without × 4. B1 for ×4 twice.
	$\left(\frac{2}{\sqrt{4x+1}}-\frac{18}{\left(\sqrt{4x+1}\right)^3}\; or \;\frac{8x-16}{(4x+1)^{\frac{3}{2}}}\right)$		SC If no other marks awarded award B1 for both powers of (4x +1) correct.
	$\int y\mathrm{d}x=\left[\frac{(4x+1)^{\frac{3}{2}}}{\frac{3}{2}}\right]$ [÷ 4] + $\left[\frac{9(4x+1)^{\frac{1}{2}}}{\frac{1}{2}}\right]$ [÷ 4] (+C)	B1B1B1	B1 B1 for each, without ÷ 4. B1 for ÷4 twice. + C not required.
	$\left(\frac{\left(\sqrt{4x+1}\right)^3}{6}+\frac{9}{2}\left(\sqrt{4x+1}\right)(+C)\right)$		SC If no other marks awarded , B1 for both powers of (4x +1) correct.
		6	
11(ii)	$\frac{dy}{dx}=0\rightarrow\frac{2}{\sqrt{4x+1}}-\frac{18}{(4x+1)^{\frac{3}{2}}}=0$	M1	Sets their $\frac{dy}{dx}$ to 0 (and attempts to solve
	$4x + 1 = 9$ or $(4x + 1)^2 = 81$	A1	Must be from correct differential.
	$x = 2$, $y = 6$ **or** M is (2, 6) only.	A1	Both values required. Must be from correct differential.
		3	

Question	Answer	Marks	Guidance
11(iii)	Realises area is $\int y \, dx$ **and** attempt to use their 2 and sight of 0.	*M1	Needs to use their integral and to see '*their 2*' substituted.
	Uses limits 0 to 2 correctly → $[4.5 + 13.5] - [\frac{1}{6} + 4.5]$ (= 13⅓)	DM1	Uses both 0 and '*their 2*' and subtracts. Condone wrong way round.
	(Area =) 1⅓ **or** 1.33	A1	Must be from a correct differential and integral.
		3	13⅓ or 1⅓ with little or no working scores M1DM0A0.

Cambridge International AS & A Level

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Cambridge International Advanced Subsidiary and Advanced Level

MATHEMATICS **9709/13**

Paper 1 **May/June 2019**

MARK SCHEME

Maximum Mark: 75

Published

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Cambridge Assessment International Education

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Marks are of the following three types:

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- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

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  - Note: B2 or A2 means that the candidate can earn 2 or 0.
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Question	Answer	Marks	Guidance
1(i)	$\left[(x-2)^2\right]$ [+4]	B1 DB1	2nd B1 dependent on 2 inside bracket
		2	
1(ii)	$(x-2)^2 < 5 \ \rightarrow\ -\sqrt{5} < x-2$ and/or $x-2 < \sqrt{5}$	M1	Allow e.g. $x-2 < \pm\sqrt{5}$, $x-2 = \pm\sqrt{5}$ and decimal equivalents for √5 For M1, ft from *their*(i). Also allow √13 instead of √5 for clear slip
	$2-\sqrt{5} < x < 2+\sqrt{5}$	A1A1	A1 for each inequality – allow two separate statements but there must be 2 inequalities for x. Non-hence methods, if completely correct, score SC 1/3. Condone $\leqslant$
		[3]	

Question	Answer	Marks	Guidance
2(i)	$\frac{-5}{x}+\frac{5}{8x^3}-\frac{1}{32x^5}$ (or $-5x^{-1}+\frac{5}{8}x^{-3}-\frac{1}{32}x^{-5}$)	B1B1B1	B1 for each correct term SCB1 for both $\frac{+5}{x}$ & $\frac{+1}{32x^5}$
		3	
2(ii)	$1\times 20+4\times their(-5)=0$	M1A1	Must be from exactly 2 terms SCB1 for $20+20=40$
		2	

Question	Answer	Marks	Guidance
3(i)	Angle EAD = Angle $ACD = \frac{3\pi}{10}$ or 54° or 0.942 soi **or** Angle $DAC = \frac{\pi}{5}$ or 36° or 0.628 soi	**B1**	
	$AD = 8\sin(\frac{3\pi}{10})$ **or** $8\cos(\frac{\pi}{5})$	**M1**	Angles used must be correct
	(AD =) 6.47	**A1**	
	Alternative method for question 3(i)		
	$AB = \frac{8}{\tan\left(\frac{\pi}{5}\right)}$ or $AB = \frac{8\sin\left(\frac{3\pi}{10}\right)}{\sin\left(\frac{\pi}{5}\right)}$ or $11.(01)$	**B1**	Angles used must be correct
	$AD = 11.0(1)\sin\frac{\pi}{5}$ **oe**	**M1**	
	(AD =) 6.47	**A1**	
		3	
3(ii)	Area sector $= \frac{1}{2}(their AD)^2 \times their\left(\frac{\pi}{2} - \frac{\pi}{5}\right)$	**M1**	19.7(4)
	Area $\Delta ADC = \frac{1}{2} \times 8 \times their AD \times \sin\frac{\pi}{5}$ or $\frac{1}{2} \times 8\cos\left(\frac{3\pi}{10}\right) \times 8\sin\left(\frac{3\pi}{10}\right)$	**M1**	Or e.g. $\frac{1}{2}\, their AD \times \sqrt{8^2 - their AD^2}$. 15.2(2)
	(Shaded area =) 35.0 or 34.9	**A1**	
		3	

Question	Answer	Marks	Guidance
4(i)	Max(***a***) is 8	**B1**	Allow $a = 8$ or $a \leqslant 8$
	Min(***b***) is 24	**B1**	Allow $b = 24$ or $b \geqslant 24$
		2	SCB1 for 8 and 24 seen
4(ii)	$\text{gf}(x) = \dfrac{96}{x-1} - 4$ **or** $\text{gf}(x) = \dfrac{100-4x}{x-1}$	**B1**	$2\left(\dfrac{48}{x-1}\right) - 4$ is insufficient Apply ISW
		1	
4(iii)	$y = \dfrac{96}{x-1} - 4 \to y + 4 = \dfrac{96}{x-1} \to x - 1 = \dfrac{96}{y+4}$	**M1**	**FT** from *their*(**ii**) provided (**ii**) involves algebraic fraction. Allow sign errors
	$(\text{gf})^{-1}(x) = \dfrac{96}{x+4} + 1$	**A1**	OR $\dfrac{100+x}{x+4}$. Must be a function of x. Apply ISW
		2	

Question	Answer	Marks	Guidance
5(i)	$\dfrac{x}{2}\left[2 + (x-1)(-/+0.02)\right]$ or $1.01x - 0.01x^2$ or $0.99x + 0.01x^2$ **oe**	**B1**	Allow – or + 0.02. Allow n used
		1	

Question	Answer	Marks	Guidance
5(ii)	Equate to 13 **then** *either* simplify to a 3-term quadratic equation **or** find at least 1 solution (need not be correct) to an unsimplified quadratic	**M1**	Expect $n^2 - 101n + 1300$ (=0) or $0.99x + 0.01x^2 = 13$. Allow x used
	16	**A1**	Ignore 85.8 or 86
		2	
5(iii)	Use of $\frac{a(1-r^{n})}{1-r}$ with $a = 1$, $r = 0.92$, $n = 20$ soi	**M1**	
	(=) 10.1	**A1**	
	Use of $\left(S_\infty =\right)\frac{a}{1-r}$ with $a = 1$, $r = 0.92$	**M1**	OR $\frac{(1)(1-0.92^{n})}{1-0.92} = 13 \rightarrow 0.92^n = -0.04$ oe
	$S_\infty = 12.5$ so never reaches target or < 13	**A1**	Conclusion required – 'Shown' is insufficient No solution so never reaches target or < 13
		4	

Question	Answer	Marks	Guidance
6(i)	**MF** = –4**i** + 2**j** + 7**k**	**B1**	
		1	
6(ii)	**FN** = 2**i** – **j**	**B1**	
		1	
6(iii)	**MN** = –2**i** + **j** + 7**k**	**B1**	**FT** on *their* (**MF** + **FN**)
		1	

Question	Answer	Marks	Guidance
6(iv)	**MF.MN** $= 8 + 2 + 49 = 59$	*M1	**MF.MN** or **FM.NM** but allow if one is reversed (implied by –59)
	$\lvert\mathbf{MF}\rvert \times \lvert\mathbf{MN}\rvert = \sqrt{4^2+2^2+7^2} \times \sqrt{2^2+1^2+7^2}$	*DM1	Product of modulus. At least one methodically correct
	$\cos FMN = \dfrac{+/-59}{\sqrt{69}\times\sqrt{54}}$	DM1	All linked correctly. Note $\sqrt{69}\times\sqrt{54} = 9\sqrt{46}$
	$FMN = 14.9°$ or 0.259	A1	Do not allow if exactly 1 vector is reversed – even if adjusted finally
		4	

Question	Answer	Marks	Guidance
7(i)	$D = (5, 1)$	B1	
		1	
7(ii)	$(x-5)^2 + (y-1)^2 = 20$ **oe**	B1	**FT** on *their D*. Apply ISW, oe but not to contain square roots
		1	

Question	Answer	Marks	Guidance
7(iii)	$(x-1)^2+(y-3)^2=(9-x)^2+(y+1)^2$ **soi**	**M1**	Allow 1 sign slip For M1 allow with √ signs round both sides but sides must be equated
	$x^2-2x+1+y^2-6y+9=x^2-18x+81+y^2+2y+1$	**A1**	
	$y=2x-9$ www **AG**	**A1**	
	Alternative method for question 7(iii)		
	grad. of $AB=-½ \rightarrow$ grad of perp bisector $=\dfrac{-1}{-½}$	**M1**	
	Equation of perp. bisector is $y-1=2(x-5)$	**A1**	
	$y=2x-9$ www **AG**	**A1**	
		3	
7(iv)	Eliminate y (or x) using equations in **(ii)** and **(iii)**	***M1**	To give an (unsimplified) quadratic equation
	$5x^2-50x+105$ $(=0)$ **or** $5(x-5)^2=20$ **or** $5y^2-10y-75$ $(=0)$ **or** $5(y-1)^2=80$	**DM1**	Simplify to one of the forms shown on the right (allow arithmetic slips)
	$x=3$ and 7, **or** $y=-3$ and 5	**A1**	
	(3, –3), (7, 5)	**A1**	Both pairs of x & y correct implies A1A1. SC B2 for no working
		4	

Question	Answer	Marks	Guidance
8	$f'(-1)=0 \to 3-a+b=0$ $f'(3)=0 \to 27+3a+b=0$	M1	Stationary points at $x=-1$ & $x=3$ gives sim. equations in a & b
	$a=-6$	A1	Solve simultaneous equation
	$b=-9$	A1	
	Hence $f'(x)=3x^2-6x-9 \to f(x)=x^3-3x^2-9x(+c)$	B1	**FT** correct integration for *their a,b* (numerical a, b)
	$2=-1-3+9+c$	M1	Sub $x=-1$, $y=2$ into *their* integrated f(x). c must be present
	$c=-3$	A1	**FT** from *their* f(x)
	$f(3)=k \to k=27-27-27-3$	M1	Sub $x=3$, $y=k$ into *their* integrated f(x) (Allow c omitted)
	$k=-30$	A1	
		8	

Question	Answer	Marks	Guidance
9(i)	$q \leqslant f(x) \leqslant p+q$	B1B1	B1 each inequality – allow two separate statements Accept < , $(q, p+q)$, $[q, p+q]$ Condone y or x or f in place of f(x)
		2	

Question	Answer	Marks	Guidance
9(ii)	**(a)** 2	B1	Allow $\frac{\pi}{4}, \frac{3\pi}{4}$
	(b) 3	B1	Allow $0, \frac{\pi}{2}, \pi$
	(c) 4	B1	Allow $\frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}$
		3	
9(iii)	$3\sin^2 2x + 2 = 4 \rightarrow \sin^2 2x = \frac{2}{3}$ soi	M1	
	Sin$2x = (\pm)0.816(5)$. Allow $\sin 2x = (\pm)\sqrt{\frac{2}{3}}$ or $2x = \sin^{-1}(\pm)\sqrt{\frac{2}{3}}$	A1	OR Implied by at least one correct value for x. Allow $\sin^{-1}$ form
	$(2x =)$ **at least two** of 0.955(3), 2.18(6), 4.09(7) , 5.32(8)	A1	Can be implied by corresponding values of x below Allow for at least two of 0.304π, 0.696π, 1.30(4)π, 1.69(6)π **OR** at least <u>two</u> of 54.7(4)°, 125.2(6)°, 234.7(4)°, 305.2(6)°
	$(x =)$ 0.478, 1.09, 2.05, 2.66.	A1A1	Allow 0.152π, 0.348π, 0.652π, 0.848π SC A1 for 2 or 3 correct. SC A1 for all of 27.4º, 62.6º, 117.4º, 152.6º Sin$2x = \pm\frac{2}{3} \rightarrow x = 0.365, 1.21, 1.94, 2.78$ scores SC M1A0A0A1
		5	

Question	Answer	Marks	Guidance
10(i)	$\left[\frac{1}{2}(3x+4)^{-\frac{1}{2}}\right]$	**B1**	oe
	$\frac{dy}{dx}=\left[\frac{1}{2}(3x+4)^{-\frac{1}{2}}\right]\times 3$	**B1**	Must have '×3'
	At $x=4$, $\frac{dy}{dx}=\frac{3}{8}$ soi	**B1**	
	Line through (4, *their*4) with gradient *their* $\frac{3}{8}$	**M1**	If $y\neq 4$ is used then clear evidence of substitution of $x=4$ is needed
	Equation of tangent is $y-4=\frac{3}{8}(x-4)$ or $y=\frac{3}{8}x+\frac{5}{2}$	**A1**	oe
		5	

Question	Answer	Marks	Guidance
10(ii)	Area under line $=\frac{1}{2}\left(4+\frac{5}{2}\right)\times 4=13$	**B1**	OR $\int_0^4 \frac{3}{8}x+\frac{5}{2}=\left[\frac{3}{16}x^2+\frac{5}{2}x\right]=\left[3+10\right]=13$
	Area under curve: $\int(3x+4)^{\frac{1}{2}}=\left[\frac{(3x+4)^{3/2}}{3/2}\right]\left[\div 3\right]$	**B1B1**	Allow if seen as part of the difference of 2 integrals First B1 for integral without $\left[\div 3\right]$ Second B1 must have $\left[\div 3\right]$
	$\frac{128}{9}-\frac{16}{9}=\frac{112}{9}=12\frac{4}{9}$	**M1**	Apply limits $0 \to 4$ to an integrated expression
	Area $= 13 - 12\frac{4}{9} = \frac{5}{9}$ (or 0.556)	**A1**	
	Alternative method for question 10(ii)		
	Area for line = 1/2 × 4 × 3/2 = 3	**B1**	OR $\int_{5/2}^{4}\frac{1}{3}(8y-20)=\frac{1}{3}\left[4y^2-20\right]=\frac{1}{3}\left[-16+25\right]=3$
	Area for curve = $\int \frac{1}{3}(y^2-4) = \left[\frac{y^3}{9}\right]-\left[\frac{4y}{3}\right]$	**B1B1**	
	$\left(\frac{64}{9}-\frac{16}{3}\right)-\left(\frac{8}{9}-\frac{8}{3}\right)=\frac{32}{9}$	**M1**	Apply limits $2 \to 4$ to an integrated expression for curve
	Area $=\frac{32}{9}-3=\frac{5}{9}$ (or 0.556)	**A1**	
		5	

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10(iii)	$\frac{dy}{dx}=\frac{1}{2}$	B1	
	$\frac{3}{2}(3x+4)^{-\frac{1}{2}}=\frac{1}{2}$	M1	Allow M1 for $\frac{3}{2}(3x+4)^{-\frac{1}{2}}=2$.
	$(3x+4)^{\frac{1}{2}}=3 \rightarrow 3x+4=9 \rightarrow x=\mathbf{\frac{5}{3}}$ oe	A1	
		3	

Cambridge International AS & A Level

Cambridge Assessment International Education
Cambridge International Advanced Subsidiary and Advanced Level

MATHEMATICS **9709/12**

Paper 1 Pure Mathematics **March 2019**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the March 2019 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

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1	$5C3\left[(-)(px)^3\right]$ soi	**B1**	Can be part of expansion. Condone omission of – sign
	$(-1)10p^3 = -2160$ then ÷ and cube root	**M1**	Condone omission of – sign.
	$p = 6$	**A1**	
		3	

Question	Answer	Marks	Guidance
2	$y = \frac{1}{3}kx^3 - x^2\ (+c)$	**M1A1**	Attempt integration for M mark
	Sub (0, 2)	**DM1**	Dep on c present. Expect $c = 2$
	Sub (3, –1) → $-1 = 9k - 9 + their\ c$	**DM1**	
	$k = 2/3$	**A1**	
		5	

Question	Answer	Marks	Guidance
3	Angle $CBA = \sin^{-1}\left(\frac{7}{8}\right) = 1.0654$ or $CBD = \cos^{-1}\left(\frac{-17}{32}\right) = 2.13$	**B1**	Accept 61.0°, 66° or 122°
	Sector $BCYD = ½ \times 8^2 \times 2 \times their 1.0654(\text{rad})$ soi or sector $CBY = ½ \times 8^2 \times their 1.0654(\text{rad})$	**M1**	Expect 68.1(9). Angle must be in radians (or *their* 61/360 × 2 × 8^2) Or sector DBY
	$\Delta BCD = 7 \times \sqrt{8^2 - 7^2}$ or $½ \times 8^2 \times \sin(2 \times their 1.0654)$ soi	**M1**	Expect 27.1(1). Award M1 for ABC or ABD
	Semi-circle $CXD = ½\pi \times 7^2 = 76.9(7)$	**M1**	M1M1 for segment area formula used correctly
	Total area = *their*68.19 – *their*27.11 + *their*76.97 = 118.0–118.1	**M1A1**	Cannot gain M1 without attempt to find angle CBA or CBD
		6	

Question	Answer	Marks	Guidance
4(i)	$dy / dx = -2(2x-1)^{-2} + 2$	**B2,1,0**	Unsimplified form ok (–1 for each error in '–2', '$(2x-1)^{-2}$' and '2')
	$d^2y / dx^2 = 8(2x-1)^{-3}$	**B1**	Unsimplified form ok
		3	

Question	Answer	Marks	Guidance
4(ii)	Set $\mathrm{d}y/\mathrm{d}x$ to zero and attempt to solve – at least one correct step	M1	
	$x = 0, 1$	A1	Expect $(2x-1)^2 = 1$
	When $x = 0$, $\mathrm{d}^2y/\mathrm{d}x^2 = -8$ (or < 0). Hence MAX	B1	
	When $x = 1$, $\mathrm{d}^2y/\mathrm{d}x^2 = 8$ (or > 0). Hence MIN	B1	Both final marks dependent on correct x and correct $\mathrm{d}^2y/\mathrm{d}x^2$ and no errors May use change of sign of $\mathrm{dy/dx}$ but not at $x = 1/2$
		4	

Question	Answer	Marks	Guidance
5(i)	**u.v** = $8q + 2q - 2 + 6q^2 - 42$	B1	May be unsimplified
	$6q^2 + 10q - 44 = 0$ oe	M1	Simplify, set to zero and attempt to solve
	$q = 2, -11/3$	A1	Both required. Accept –3.67
		3	

Question	Answer	Marks	Guidance
5(ii)	$\mathbf{u} = \begin{pmatrix} 0 \\ 2 \\ 6 \end{pmatrix}$ $\mathbf{v} = \begin{pmatrix} 8 \\ -1 \\ -7 \end{pmatrix}$ $\mathbf{u.v} = -2 - 42$	M1	Correct method for scalar product
	$\lvert\mathbf{u}\rvert \times \lvert\mathbf{v}\rvert = \sqrt{2^2+6^2} \times \sqrt{8^2+1^2+7^2}$	M1	Prod of mods. At least one methodically correct.
	$\cos\theta = \dfrac{-44}{\sqrt{40}\times\sqrt{114}} = \dfrac{-44}{4\sqrt{285}} = \dfrac{-4}{\sqrt{11}}$	M1	All linked correctly and inverse cos used correctly
	$\theta = 130.7°$ or 2.28(05) rads	A1	No other angles between 0° and 180°
		4	

Question	Answer	Marks	Guidance
6(i)	$S_n = \dfrac{p\left(2^n - 1\right)}{2-1}$ soi	M1	
	$p\left(2^n - 1\right) > 1000p \rightarrow\ 2^n > 1001$ **AG**	A1	
		2	

Question	Answer	Marks	Guidance
6(ii)	$p+(n-1)p=336$	**B1**	Expect $np = 336$
	$\frac{n}{2}\left[2p+(n-1)p\right]=7224$	**B1**	Expect $\frac{n}{2}(p+np)=7224$
	Eliminate n or p to an equation in one variable	**M1**	Expect e.g. $168(1 + n) = 7224$ or $1 + 336/p = 43$ etc
	$n = 42$, $p = 8$	**A1A1**	
		5	

Question	Answer	Marks	Guidance
7(a)	$3\left(1-\cos^2 2\theta\right)+8\cos 2\theta=0 \ \rightarrow\ 3\cos^2 2\theta-8\cos 2\theta-3\ (=0)$	**M1**	Use $s^2=1-c^2$ and simplify to 3-term quadratic in 2θ
	$\cos 2\theta=-\frac{1}{3}$ soi	**A1**	Ignore other solution
	$2\theta = 109.(47)^\circ$ or $250.(53)^\circ$	**A1**	One solution is sufficient, may be implied by either of the next solns
	$\theta = 54.7^\circ$ or 125.3°	**A1A1ft**	Ft for 180° – other solution Use of double angles leads to $3c^4-7c^2+2=0 \Rightarrow c=\pm 1/\sqrt{3}$ for M1A1A1 then A1A1 for each angle Similar marking if $3\sin^2 2\theta=-8\cos 2\theta$ is squared leading to $9\sin^4 2\theta+64\sin^2 2\theta-64=0$
		5	

Question	Answer	Marks	Guidance
7(b)	$\sqrt{3} = a + \tan 0 \rightarrow a = \sqrt{3}$	**B1**	$b = 8$ or -4 (or -10, 14 etc) scores M1A0
	$0 = \tan(-b\pi / 6) + \sqrt{3}$ taken as far as $\tan^{-1}$, angle units consistent	**M1**	A0 if $\tan^{-1}\left(-\sqrt{3}\right)$ is not exact; (b=2 no working scores B2)
	$b = 2$	**A1**	
		3	

Question	Answer	Marks	Guidance
8(i)	$\left[\left(x-2\right)^2\right]+[3]$	**B1 DB1**	2nd B1 dependent on ±2 in 1st bracket
		2	
8(ii)	Largest k is 2 Accept $k \leqslant 2$	**B1**	Must be in terms of k
		1	
8(iii)	$y = \left(x-2\right)^2 + 3 \;\Rightarrow\; x-2 = (\pm)\sqrt{y-3}$	**M1**	
	$\Rightarrow \mathrm{f}^{-1}\left(x\right) = 2 - \sqrt{x-3}$ for $x > 4$	**A1B1**	
		3	

Question	Answer	Marks	Guidance
8(iv)	$\text{gf}(x) = \dfrac{2}{x^2-4x+7-1} = \dfrac{2}{(x-2)^2+2}$	B1	Either form
	Since $\text{f}(\text{x}) > 4 \Rightarrow \text{gf}(\text{x}) < 2/3$ (or since $x < 1$etc)	M1A1	2/3 in answer implies M1 www
	range of gf(x) is 0 < gf(x)(< 2/3)	B1	Accept 0 < y < 2/3, (0, 2/3) but 0 < x < 2/3 is SCM1A1B0
		4	

Question	Answer	Marks	Guidance
9(i)	$V = (\pi)\int(x^3+x^2)(\text{d}x)$	M1	Attempt $\int y^2 \text{d}x$
	$(\pi)\left[\dfrac{x^4}{4}+\dfrac{x^3}{3}\right]_0^3$	A1	
	$(\pi)\left[\dfrac{81}{4}+9 \quad (-0)\right]$	DM1	May be implied by a correct answer
	$\dfrac{117\pi}{4}$ oe	A1	Accept 91.9 If additional areas rotated about x-axis, maximum of M1A0DM1A0
		4	

Question	Answer	Marks	Guidance
9(ii)	$\frac{dy}{dx} = \frac{1}{2}\left(x^3 + x^2\right)^{-1/2} \times \left(3x^2 + 2x\right)$	B2,1,0	Omission of $3x^2 + 2x$ is one error
	(At $x = 3$,) $y = 6$	B1	
	At $x = 3$, $m = \frac{1}{2} \times \frac{1}{6} \times 33 = \frac{11}{4}$ soi	DB1ft	Ft on *their* dy / dx providing differentiation attempted
	Equation of normal is $y - 6 = -\frac{4}{11}(x - 3)$	DM1	Equation through (3, *their* 6) and with gradient –1/*their* *m*
	When $x = 0$, $y = 7\frac{1}{11}$ oe	A1	
		6	

Question	Answer	Marks	Guidance
10(i)	$4x^{1/2}=x+3\rightarrow$ $(x^{1/2})^2-4x^{1/2}+3\ (=0)$ **OR** $16x=x^2+6x+9$	M1	Eliminate y from the 2 equations and then: **Either** treat as quad in $x^{1/2}$ **OR** square both sides and RHS is 3-term
	$x^{1/2}=1$ or 3 $\quad x^2-10x+9\ (=0)$	A1	If in 1st method $x^{1/2}$ becomes x, allow only M1 unless subsequently squared
	$x=1$ or 9	A1	
	$y=4$ or 12	A1ft	Ft from *their* x values If the 2 solutions are found by trial substitution B1 for the first coordinate and B3 for the second coordinate
	$AB^2=(9-1)^2+(12-4)^2$	M1	
	$AB=\sqrt{128}$ or $8\sqrt{2}$ oe or 11.3	A1	
		6	
10(ii)	$dy/dx=2x^{-1/2}$	B1	
	$2x^{-1/2}=1$	M1	Set *their* derivative = *their* gradient of AB and attempt to solve
	(4, 8)	A1	Alternative method without calculus: $M_{AB}=1$, tangent is $y=mx+c$ where m = 1 and meets $y=4x^{1/2}$ when $4x^{1/2}=x+c$. This is a quadratic with $b^2=4ac$, so $16-4\times 1\times c=0$ so c = 4 B1 Solving $4x^{1/2}=x+4$ gives $x=4$ and $y=8$ M1A1
		3	

Question	Answer	Marks	Guidance
10(iii)	Equation of normal is $y-8=-1(x-4)$	**M1**	Equation through *their T* and with gradient –1/*their* gradient of AB. Expect $y=-x+12$,
	Eliminate y (or x) $\rightarrow$ $-x+12=x+3$ or $y-3=12-y$	**M1**	May use *their* equation of AB
	(4½, 7½)	**A1**	
		3	

MATHEMATICS **9709/11**

Paper 1 **October/November 2018**

MARK SCHEME

Maximum Mark: 75

Published

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A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more “method” steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously “correct” answers or results obtained from incorrect working.

  - Note: B2 or A2 means that the candidate can earn 2 or 0.
    B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no “follow through” from a previous error is allowed)
CWO	Correct Working Only – often written by a ‘fortuitous’ answer
ISW	Ignore Subsequent Working
SOI	Seen or implied
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become “follow through” marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Question	Answer	Marks	Guidance
1	$\left(4x^{½}-3\right)\left(x^{½}-2\right)$ oe soi Alt: $4x+6=11\sqrt{x} \Rightarrow 16x^2-73x+36$	**M1**	Attempt solution for $x^{½}$ or sub $u=x^{½}$
	$x^{½}=3/4$ or 2 $(16x-9)(x-4)$	**A1**	Reasonable solutions for $x^{½}$ implies M1 ($x=2, 3/4$, M1A0)
	$x = 9/16$ oe or 4	**A1**	Little or no working shown scores SCB3, spotting one solution, B0
		3	

Question	Answer	Marks	Guidance
2	$x^2+bx+5=x+1 \to x^2+x(b-1)+4\ (=0)$	**M1**	Eliminate x or y with all terms on side of an equation
	$(b^2-4ac=)\ \ (b-1)^2-16$	**M1**	
	b associated with –3 & +5 or $b-1$ associated with ±4	**A1**	$(x-2)^2=0\, or\, (x+2)^2=0, x=\pm2, b-1=\pm4$ (M1A1) Association can be an equality or an inequality
	$b \geqslant 5, b \leqslant -3$	**A1**	
		4	

Question	Answer	Marks	Guidance
3(i)	Gradient of $AB = -3/4$	**B1**	Accept $-3a/4a$
	$y = -\frac{3}{4}x$ oe	**B1FT**	Answer must not include a. Ft on *their* <u>numerical</u> gradient
		2	
3(ii)	$(4a)^2 + (3a)^2 = (10/3)^2$ soi	**M1**	May be unsimplified
	$25a^2 = 100/9$ oe	**A1**	
	$a = 2/3$	**A1**	
		3	

Question	Answer	Marks	Guidance
4(i)	$S_{80} = \frac{80}{2}\left[12 + 79 \times (-4)\right]$ or $\frac{80}{2}[6 + l], l = -310$	**M1A1**	Correct formula (M1). Correct a, d and n (A1) .
	–12 160	**A1**	
		3	
4(ii)	$S_\infty = \dfrac{6}{1 - \frac{1}{3}} = 9$	**M1A1**	Correct formula with $\lvert r \rvert < 1$ for M1
		2	

Question	Answer	Marks	Guidance
5(i)	$\dfrac{(\cos\theta-4)(5\cos\theta-2)-4\sin^2\theta}{\sin\theta(5\cos\theta-2)}$ $(=0)$	**M1**	Accept numerator only
	$\dfrac{5\cos^2\theta-22\cos\theta+8-4\left(1-\cos^2\theta\right)}{\sin\theta(5\cos\theta-2)}$ $(=0)$	**M1**	Simplify numerator and use $s^2=1-c^2$. Accept numerator only
	$9\cos^2\theta-22\cos\theta+4=0$ www **AG**	**A1**	
		3	
5(ii)	Attempt to solve for $\cos\theta$, (formula, completing square expected)	**M1**	Expect $\cos\theta=0.1978$. Allow 2.247 in addition
	$\theta=78.6°,\ 281.4°$ (only, second solution in the range)	**A1A1FT**	Ft for (360º – 1st solution)
		3	

Question	Answer	Marks	Guidance
6(i)	$0 = 9a + 3a^2$	**M1**	Sub $\frac{dy}{dx} = 0$ and $x = 3$
	$a = -3$ only	**A1**	
		2	
6(ii)	$\frac{dy}{dx} = -3x^2 + 9x \to y = -x^3 + \frac{9x^2}{2}$ $(+c)$	**M1A1FT**	Attempt integration. $⅓ax^3 + ½a^2x^2$ scores M1. Ft on *their a*.
	$9½ = -27 + 40½ + c$	**DM1**	Sub $x = 3, y = 9½$. Dependent on c present
	$c = -4$	**A1**	Expect $y = -x^3 + \frac{9x^2}{2} - 4$
		4	
6(iii)	$\frac{d^2y}{dx^2} = -6x + 9$	**M1**	$2ax + a^2$ scores M1
	At $x = 3$, $\frac{d^2y}{dx^2} = -9 < 0$ MAX www	**A1**	Requires at least one of –9 or < 0. Other methods possible.
		2	

Question	Answer	Marks	Guidance
7(i)	$2 = k(8 - 28 + 24) \rightarrow k = 1/2$	**B1**	
		1	
7(ii)	When $x = 5$, $y = [½](125 - 175 + 60) = 5$	**M1**	Or solve $[½]\left(x^3 - 7x^2 + 12x\right) = x \Rightarrow x = 5\ [x = 0, 2]$
	Which lies on $y = x$, oe	**A1**	
		2	
7(iii)	$\int [\frac{1}{2}(x^3 - 7x^2 + 12x) - x]dx$.	**M1**	Expect $\int \frac{1}{2}x^3 - \frac{7}{2}x^2 + 5x$
	$\frac{1}{8}x^4 - \frac{7}{6}x^3 + \frac{5}{2}x^2$	**B2,1,0FT**	Ft on their k
	$2 - 28/3 + 10$	**DM1**	Apply limits $0 \rightarrow 2$
	$8/3$	**A1**	
	OR $\frac{1}{8}x^4 - \frac{7}{6}x^3 + 3x^2$	**B2,1,0FT**	Integrate to find area under curve, Ft on their k
	$2 - 28/3 + 12$	**M1**	Apply limits $0 \rightarrow 2$. Dep on integration attempted
	Area $\Delta = ½ \times 2 \times 2$ or $\int_0^2 x\mathrm{d}x = \left[½x^2\right] = 2$	**M1**	
	$8/3$	**A1**	
		5	

Question	Answer	Marks	Guidance
8(i)	$\overrightarrow{DF} = -6\mathbf{i} + 2\mathbf{k}$	B1	
		1	
8(ii)	$\overrightarrow{EF} = -6\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$	B1	
	$\lvert\overrightarrow{EF}\rvert = \sqrt{(-6)^2 + (-3)^2 + 2^2}$	M1	Must use *their* $\overrightarrow{EF}$
	Unit vector = $\frac{1}{7}(-6\mathbf{i} - 3\mathbf{j} + 2\mathbf{k})$	A1	
		3	
8(iii)	$\overrightarrow{DF}.\overrightarrow{EF} = (-6\mathbf{i} + 2\mathbf{k}).(-6\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}) = 36 + 4 = 40$	M1	
	$\lvert\overrightarrow{DF}\rvert = \sqrt{40}$, $\lvert\overrightarrow{EF}\rvert = 7$	M1	
	$\cos EFD = \frac{40}{7\sqrt{40}}$ oe	M1	
	$EFD = 25.4°$	A1	Special case: use of cosine rule M1(must evaluate lengths using correct method) A1 only
		4	

Question	Answer	Marks	Guidance
9	Angle $OAB = \pi/2 - \pi/5 = 3\pi/10$ soi	**B1**	Allow 54° *or* 0.9425 *rads*
	Sector $CAB = \frac{1}{2}\times\left(their\frac{3\pi}{10}\right)\times 5^2$	**M1**	Expect 11.78
	$OA = \dfrac{5}{\sin\frac{\pi}{5}} = 8.507$	**M1A1**	May be implied by $OC = 3.507$
	Sector $COD = \frac{1}{2}\times(their\,3.507)^2\times\frac{\pi}{5}$	**M1**	Expect 3.86
	$\Delta OAB = \frac{1}{2}\times 5\times(their\,8.507)\sin\frac{3\pi}{10}$	**M1**	Or $\frac{1}{2}\times 5\times\dfrac{5}{\tan\frac{\pi}{5}}$ or $2.5\times\sqrt{(their\,8.507)^2-25}$
	= 17.20 or 17.21	**A1**	
	Shaded area $17.20(or\,17.21) - 11.78 - 3.86 = 1.56$ or 1.57	**A1**	
		8	

Question	Answer	Marks	Guidance
10(i)(a)	$\frac{dy}{dx} = \left[-½(4x-3)^{-2}\right] \times [4]$	B1B1	Can gain this in part **(b)(ii)**
	When $x = 1$, $m = -2$	B1FT	Ft from *their* $\frac{dy}{dx}$
	Normal is $y - ½ = ½(x-1)$	M1	Line with gradient $-1/m$ and through A
	$y = ½x$ soi	A1	Can score in part **(b)**
		5	
10(i)(b)	$\frac{1}{2(4x-3)} = \frac{x}{2} \rightarrow 2x(4x-3) = 2 \rightarrow (2)(4x^2 - 3x - 1)\ (=0)$	M1A1	$x/2$ seen on RHS of equation can score *previous* A1
	$x = -1/4$	A1	Ignore $x = 1$ seen in addition
		3	
10(ii)	*Use* of chain rule: $\frac{dy}{dt} = (their\ -2) \times (\pm)0.3 = 0.6$	M1A1	Allow +0.3 or –0.3 for M1
		2	

Question	Answer	Marks	Guidance
11(a)(i)	[Greatest value of a is] 3	**B1**	Must be in terms of a. Allow $a < 3$. Allow $a \leqslant 3$
		1	
11(a)(ii)	Range is $y > -1$	**B1**	Ft on *their a*. Accept any equivalent notation
	$y = (x-3)^2 - 1 \to (x-3)^2 = 1 + y \to x = 3(\pm)\sqrt{1+y}$	**M1**	Order of operations correct. Allow sign errors
	$f^{-1}(x) = 3 - \sqrt{1+x}$ cao	**A1**	
		3	
11(b)(i)	gg(2x) $= \left[(2x-3)^2 - 3\right]^2$	**B1**	
	$(2x-3)^4 - 6(2x-3)^2 + 9$	**B1**	
		2	
11(b)(ii)	$\left[16x^4 - 96x^3 + 216x^2 - 216x + 81\right] + \left[\left(-24x^2 + 72x - 54\right) + 9\right]$ $16x^4 - 96x^3 + 192x^2 - 144x + 36$	**B4,3,2,1,0**	
		4	

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MATHEMATICS **9709/12**

Paper 1 **October/November 2018**

MARK SCHEME

Maximum Mark: 75

Published

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Question	Answer	Marks	Guidance
1	For a correctly selected term in $\frac{1}{x^2}$: $(3x)^4$ or 3^4	**B1**	Components of coefficient added together 0/4 B1 expect 81
	$\times \left(\frac{2}{3x^2}\right)^3$ or $(2/3)^3$	**B1**	B1 expect 8/27
	$\times\ {}_7C_3$ or ${}_7C_4$	**B1**	B1 expect 35
	$\rightarrow$ **840 or** $\boldsymbol{\frac{840}{x^2}}$	**B1**	All of the first three marks can be scored if the correct term is seen in an expansion **and it is selected** but then wrongly simplified.
			SC: A completely correct unsimplified term seen in an expansion but not correctly selected can be awarded B2.
		4	

Question	Answer	Marks	Guidance
2	Integrate $\rightarrow \dfrac{x^{\frac{3}{2}}}{\frac{3}{2}} + 2\dfrac{x^{\frac{1}{2}}}{\frac{1}{2}}$ (+C)	**B1 B1**	B1 for each term correct – allow unsimplified. C not required.
	$\left[\dfrac{x^{\frac{3}{2}}}{\frac{3}{2}} + 2\dfrac{x^{\frac{1}{2}}}{\frac{1}{2}}\right]_1^4 \rightarrow \dfrac{40}{3} - \dfrac{14}{3}$	**M1**	Evidence of 4 and 1 used correctly in their integrand ie at least one power increased by 1.
	$= \dfrac{26}{3}$ **oe**	**A1**	Allow 8.67 awrt. No integrand implies use of integration function on calculator 0/4. Beware a correct answer from wrong working.
		4	

Question	Answer	Marks	Guidance
3(i)	P is $(\boldsymbol{t}, \mathbf{5}\boldsymbol{t})$ Q is $(\boldsymbol{t}, \boldsymbol{t}(\mathbf{9} - \boldsymbol{t}^2)) \rightarrow \mathbf{4}\boldsymbol{t} - \boldsymbol{t}^3$	**B1 B1**	B1 for both y coordinates which can be implied by subsequent working. B1 for PQ allow $\lvert 4\boldsymbol{t} - \boldsymbol{t}^3\rvert$ or $\lvert \boldsymbol{t}^3 - 4\boldsymbol{t}\rvert$. **Note:** $4x - x^3$ from equating line and curve 0/2 even if x then replaced by t.
		[2]	

Question	Answer	Marks	Guidance
3(ii)	$\frac{d(PQ)}{dt} = 4 - 3t^2$	B1FT	B1FT for differentiation of their *PQ*, which MUST be a cubic expression, but can be $\frac{d}{dx}f(x)$ from **(i)** but not the equation of the curve.
	$= 0 \to t = + \frac{2}{\sqrt{3}}$	M1	Setting their differential of *PQ* to 0 and attempt to solve for t or *x*.
	$\to$ **Maximum** $\boldsymbol{PQ} = \frac{16}{3\sqrt{3}}$ **or** $\frac{16\sqrt{3}}{9}$	A1	Allow 3.08 awrt. If answer comes from wrong method in **(i)** award A0. Correct answer from correct expression by T&I scores 3/3.
		3	

Question	Answer	Marks	Guidance
4(i)	$fg(x) = 2 - 3\cos(\frac{1}{2}x)$	B1	Correct *fg*
	$2 - 3\cos(\frac{1}{2}x) = 1 \to \cos(\frac{1}{2}x) = \frac{1}{3} \to \left(\frac{1}{2}x\right) = \cos^{-1}\left(their\,\frac{1}{3}\right)$	M1	M1 for correct order of operations to solve their *fg*(x) = 1 as far as using inverse cos expect 1.23, (or 70.5°) condone *x* =.
	$\boldsymbol{x}$ **= 2.46 awrt or** $\frac{4.7\pi}{6}(0.784\pi$ **awrt**$)$	A1	One solution only in the given range, ignore answers outside the range. Answer in degrees A0.
			Alternative: Solve $f(y) = 1 \to y = 1.23 \to \frac{1}{2}x = 1.23$ **B1M1** $\to x = 2.46$ **A1**
		3	

Question	Answer	Marks	Guidance
4(ii)		B1	One cycle of ± cos curve, evidence of turning at the ends not required at this stage. Can be a poor curve but not an inverted “V”. If horizontal axis is not labelled mark everything to the right of the vertical axis. If axis is clearly labelled mark 0 → 2π.
		B1	Start and finish at roughly the same negative y value. Significantly more above the x axis than below or correct range implied by labels .
		B1	Fully correct. Curves not lines. Must be a reasonable curve clearly turning at both ends. Labels not required but must be appropriate if present.
		3	

Question	Answer	Marks	Guidance
5(i)	From the AP: $x-4=y-x$	B1	Or equivalent statement e.g. $y=2x-4$ or $x=\frac{y+4}{2}$.
	From the GP: $\frac{y}{x}=\frac{18}{y}$	B1	Or equivalent statement e.g. $y^2=18x$ or $x=\frac{y^2}{18}$.
	Simultaneous equations: $y^2-9y-36=0$ or $2x^2-17x+8=0$	M1	Elimination of either x or y to give a three term quadratic (= 0)
	OR		
	$4+d=x$, $4+2d=y \rightarrow \frac{4+2d}{4+d}=r$ oe	B1	
	$(4+d)\left(\frac{4+2d}{4+d}\right)^2=18 \rightarrow 2d^2-d-28=0$	M1	Uses $ar^2=18$ to give a three term quadratic (= 0)
	$d=4$	B1	Condone inclusion of $d=\frac{-7}{2}$ oe

Question	Answer	Marks	Guidance
5(i)	**OR**		
	From the GP $\frac{y}{x}=\frac{18}{y}$	**B1**	
	$\to x=\frac{y^2}{18} \to 4+d=\frac{y^2}{18} \to d=\frac{y^2}{18}-4$	**B1**	
	$4+2\left(\frac{y^2}{18}-4\right)=y \to y^2-9y-36=0$	**M1**	
	$\boldsymbol{x=8, y=12.}$	**A1**	Needs both x and y. Condone $\left(\frac{1}{2},-3\right)$ included in final answer. Fully correct answer www 4/4.
		4	
5(ii)	AP 4th term = **16**	**B1**	Condone inclusion of $\frac{-13}{2}$ oe
	GP 4th term = $8\times\left(\frac{12}{8}\right)^3$	**M1**	A valid method using their x and y from **(i)**.
	= **27**	**A1**	Condone inclusion of −108
			Note: Answers from fortuitous $x=8$, $y=12$ in **(i)** can only score M1. Unidentified correct answer(s) with no working seen after valid $x=8$, $y=12$ to be credited with appropriate marks.
		3	

Question	Answer	Marks	Guidance
6(i)	In ΔABD, $\tan\theta = \frac{9}{BD} \to BD = \frac{9}{\tan\theta}$ or $9\tan(90-\theta)$ or $9\cot\theta$ or $\sqrt{\left[(20\,\boldsymbol{\tan}\theta)^2 - 9^2\right]}$ (Pythag) or $\frac{9\sin(90-\theta)}{\sin\theta}$ (Sine rule)	B1	Both marks can be gained for correct equated expressions.
	In ΔDBC, $\sin\theta = \frac{BD}{20} \to BD = 20\sin\theta$	B1	
	$20\sin\theta = \frac{9}{\tan\theta}$	M1	Equates their expressions for BD and uses $\sin\theta/\cos\theta = \tan\theta$ or $\cos\theta/\sin\theta = \cot\theta$ if necessary.
	$\to$ **$20\sin^2\theta = 9\cos\theta$ AG**	A1	Correct manipulation of their expression to arrive at given answer.
			SC: In ΔDBC, $\sin\theta = \frac{BD}{20} \to BD = 20\sin\theta$ B1 In ΔABD, $BA = \frac{9}{\sin\theta}$ and $\cos\theta = \frac{BD}{BA}$ $\cos\theta = \frac{20\sin\theta}{9/\sin\theta} \to \cos\theta = \frac{20\boldsymbol{\sin^2\theta}}{9}$ M1 $\to$ **$20\sin^2\theta = 9\cos\theta$** A1 Scores 3/4
		4	
6(ii)	Uses $s^2 + c^2 = 1 \to 20\cos^2\theta + 9\cos\theta - 20\ (= 0)$	M1	Uses $s^2 + c^2 = 1$ to form a three term quadratic in $\cos\theta$
	$\to \cos\theta = 0.8$	A1	www
	$\to$ **$\theta = 36.9°$ awrt**	A1	www. Allow 0.644^c awrt. Ignore $323.1°$ or 2.50^c. Note: correct answer without working scores 0/3.
		3	

Question	Answer	Marks	Guidance
7	$\overrightarrow{PN} = 8\mathbf{i} - 8\mathbf{k}$	**B1**	
	$\overrightarrow{PM} = 4\mathbf{i} + 4\mathbf{j} - 6\mathbf{k}$	**B2,1,0**	Loses 1 mark for each component incorrect
			SC: $\overrightarrow{PN} = -8\mathbf{i} + 8\mathbf{k}$ **and** $\overrightarrow{PM} = -4\mathbf{i} - 4\mathbf{j} + 6\mathbf{k}$ scores 2/3.
	$\overrightarrow{PN}.\overrightarrow{PM} = 32 + 0 + 48 = 80$	**M1**	Evaluates $x_1x_2 + y_1y_2 + z_1z_2$ for correct vectors or one or both reversed.
	$\lvert PN\rvert \times \lvert PM\rvert = \sqrt{128} \times \sqrt{68}$ $(= 16\sqrt{34})$	**M1**	Product of their moduli – may be seen in cosine rule
	$\sqrt{128} \times \sqrt{68} \cos M\hat{P}N = 80$	**M1**	All linked correctly.
	Angle $M\hat{P}N =$ **31.0°** **awrt**	**A1**	Answer must come directly from +ve cosine ratio. Cosine rule not accepted as a complete method. Allow 0.540^c awrt. **Note:** Correct answer from incorrect vectors scores A0 (XP)
		7	

Question	Answer	Marks	Guidance
8(i)	$A\hat{B}C$ using cosine rule giving $\cos^{-1}(\frac{-1}{8})$ or $2\sin^{-1}(¾)$ or $2\cos^{-1}\left(\frac{\sqrt{7}}{2}\right)$ **or** $B\hat{A}C = \cos^{-1}(¾)$ or $B\hat{A}C = \sin^{-1}\frac{\sqrt{7}}{4}$ or $B\hat{A}C = \tan^{-1}\frac{\sqrt{7}}{3}$	M1	Correct method for $A\hat{B}C$, expect 1.696^cawrt Or for $B\hat{A}C$, expect 0.723^cawrt
	$C\hat{B}Y = \pi - A\hat{B}C$ or $2\times C\hat{A}B$	M1	For attempt at $C\hat{B}Y = \pi - A\hat{B}C$ or $C\hat{B}Y = 2\times C\hat{A}B$
	OR		
	Find CY from ΔACY using Pythagoras or similar Δs	M1	Expect $4\sqrt{7}$
	$C\hat{B}Y = \cos^{-1}\left(\frac{8^2+8^2-(their\ CY)^2}{2\times8\times8}\right)$	M1	Correct use of cosine rule
	$\boldsymbol{C\hat{B}Y = 1.445^c}$ AG	A1	Numerical values for angles in radians, if given, need to be correct to 3 decimal places. Method marks can be awarded for working in degrees. Need 82.8° awrt converted to radians for A1. Identification of angles must be consistent for A1.
		3	
8(ii)	Arc $CY = 8\times1.445$	B1	Use of $s=8\theta$ for arc CY, Expect 11.56
	$B\hat{A}C = ½(\pi - A\hat{B}C)$ or $\cos^{-1}(¾)$	*M1	For a valid attempt at $B\hat{A}C$, may be from **(i)**. Expect 0.7227^c
	Arc $XC = 12\times$ (their $B\hat{A}C$)	DM1	Expect 8.673
	Perimeter = 11.56 + 8.673 + 4 = **24.2 cm awrt www**	A1	Omission of '+4' only penalised here.
		4	

Question	Answer	Marks	Guidance
9(i)	$2x^2 - 12x + 7 = 2(\boldsymbol{x}-3)^2 - 11$	**B1 B1**	Mark full expression if present: B1 for $2(x-3)^2$ and B1 for -11. If no clear expression award $a = -3$ and $b = -11$.
		2	
9(ii)	Range (of f or y) $\geqslant$ **'their – 11'**	**B1FT**	FT for their 'b' or start again. Condone **>.** Do **NOT** accept $x >$ or $\geqslant$
		1	
9(iii)	(k =) **–**"their a" also allow $\boldsymbol{x}$ **or** $\boldsymbol{k} \leqslant$ **3**	**B1FT**	FT for their "a" or start again using $\frac{dy}{dx} = 0$. Do **NOT accept $\boldsymbol{x}$ = 3.**
		1	
9(iv)	$y = 2(x-3)^2 - 11 \rightarrow y + 11 = 2(x-3)^2$ $\frac{y+11}{2} = (x-3)^2$	***M1**	Isolating their $(x-3)^2$, condone -11.
	$x = 3 + \sqrt{\left(\frac{y+11}{2}\right)}$ or $3 - \sqrt{\left(\frac{y+11}{2}\right)}$	**DM1**	Other operations in correct order, allow ± at this stage. Condone – 3.
	($g^{-1}(x)$ or y) $= 3 - \sqrt{\left(\frac{x+11}{2}\right)}$	**A1**	needs '–'. x and y could be interchanged at the start.
		3	

Question	Answer	Marks	Guidance
10(i)	$2x+\frac{12}{x}=k-x$ or $y=2(k-y)+\frac{12}{k-y}$ → 3 term quadratic.	*M1	Attempt to eliminate y (or x) to form a 3 term quadratic. Expect $3x^2-kx+12$ or $3y^2-5ky+(2k^2+12)$ $(=0)$
	Use of $b^2-4ac \rightarrow k^2-144<0$	DM1	Using the discriminant, allow $\leqslant$, = 0; expect 12 and −12
	$\mathbf{-12<k<12}$	A1	Do **NOT accept $\leqslant$.** Separate statements OK.
		3	
10(ii)	Using $k = 15$ in their 3 term quadratic	M1	From (i) or restart. Expect $3x^2-15x+12$ or $3y^2-75y+462$ $(= 0)$
	$x = 1,4$ or $y = 11, 14$	A1	Either pair of x or y values correct..
	(1, 14) and (4, 11)	A1	Both pairs of coordinates
		3	
10(iii)	Gradient of $AB = -1$ → Perpendicular gradient = +1	B1FT	Use of $m_1m_2=-1$ to give +1 or ft from their A and B.
	Finding their midpoint using their (1, 14) and (4, 11)	M1	Expect (2½, 12½)
	Equation: $\mathbf{y-12\tfrac{1}{2}=(x-2\tfrac{1}{2})}$ $[y=x+10]$	A1	Accept correct unsimplified and isw
		3	

Question	Answer	Marks	Guidance
11(i)	$\frac{dy}{dx} = \left[\frac{3}{2}\times(4x+1)^{-\frac{1}{2}}\right]$ [×4] [− 2] $\left(\frac{6}{\sqrt{4x+1}}-2\right)$	B2,1,0	Looking for 3 components
	$\int y\,dx = \left[3(4x+1)^{\frac{3}{2}} \div \frac{3}{2}\right]$ [÷ 4] [$-\frac{2x^2}{2}$] (+ C) $\left(=\frac{(4x+1)^{\frac{3}{2}}}{2}-x^2\right)$	B1 B1 B1	B1 for $3(4x+1)^{\frac{3}{2}} \div \frac{3}{2}$ B1 for '÷4'. B1 for '$-\frac{2x^2}{2}$'. Ignore omission of + C. If included isw any attempt at evaluating.
		5	
11(ii)	At M, $\frac{dy}{dx}=0 \rightarrow \frac{6}{\sqrt{4x+1}}=2$	M1	Sets their 2 term $\frac{dy}{dx}$ to 0 and attempts to solve (as far as x = k)
	$\boldsymbol{x = 2, y = 5}$	A1 A1	
		3	

Question	Answer	Marks	Guidance
11(iii)	Area under the curve = $\left[\frac{1}{2}(4x+1)^{\frac{3}{2}} - x^2\right]_0^2$	**M1**	Uses their integral and their '2' and 0 correctly
	$(13.5 – 4) – 0.5$ or $9.5 – 0.5 = 9$	**A1**	No working implies use of integration function on calculator M0A0.
	Area under the chord = trapezium = ½ × 2 × (3 + 5) = 8 Or $\left[\frac{x^2}{2}+3x\right]_0^2 = 8$	**M1**	Either using the area of a trapezium with their 2, 3 and 5 or $\int(\textit{their}\ x+3)\,dx$ using their '2' and 0 correctly.
	(Shaded area = 9 – 8) **= 1**	**A1**	Dependent on both method marks,
	OR Area between the chord and the curve is:		
	$\int_0^2 3\sqrt{4x+1}-2x-(x+3)\,dx$ $=\int_0^2 3\sqrt{4x+1}-3x-3dx$	**M1**	Subtracts their line from given curve and uses their '2' and 0 correctly.
	$=3\left[\frac{1}{6}(4x+1)^{\frac{3}{2}} - \frac{x^2}{2}-x\right]_0^2$	**A1**	All integration correct and limits 2 and 0.
	$=3\left\{\left(\frac{27}{6}-2-2\right)-\left(\frac{1}{6}\right)\right\}$	**M1**	Evidence of substituting their '2' and 0 into their integral.
	$=3\left\{\frac{1}{2}-\frac{1}{6}\right\}=3\left\{\frac{1}{3}\right\}=1$	**A1**	No working implies use of a calculator M0A0.
		[4]	

Cambridge Assessment International Education
Cambridge International Advanced Subsidiary and Advanced Level

MATHEMATICS **9709/13**

Paper 1 **October/November 2018**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2018 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

  - Note: B2 or A2 means that the candidate can earn 2 or 0.
    B2/1/0 means that the candidate can earn anything from 0 to 2.

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1	$7C5\ x^2(-2/x)^5$ soi	**B1**	Can appear in an expansion. Allow 7C2
	21×-32 soi	**B1**	Identified. Allow $(21x^2)\times(-32\ x^{-5})$. Implied by correct answer
	–672	**B1**	Allow $\frac{-672}{x^3}$. If 0/3 scored, 672 scores SCB1
		3	

Question	Answer	Marks	Guidance
2	$f'(x)=3x^2+4x-4$	**B1**	
	Factors or crit. values or sub any 2 values $(x\neq -2)$ into $f'(x)$ soi	**M1**	Expect $(x+2)(3x-2)$ or –2, ⅔ or any 2 subs (excluding x = –2).
	For $-2<x<\frac{2}{3}$, $f'(x)<0$; for $x>\frac{2}{3}$, $f'(x)>0$ soi Allow $\leqslant, \geqslant$	**M1**	Or at least 1 specific value $(\neq -2)$ in each interval giving opp signs Or $f'(\frac{2}{3})=0$ and $f''(\frac{2}{3})\neq 0$ (i.e. gradient changes sign at $x=\frac{2}{3}$)
	Neither www	**A1**	Must have 'Neither'
	ALT 1 At least 3 values of f(x)	**M1**	e.g. f(0) = 7, f(1) = 6, f(2) = 15
	At least 3 <u>correct</u> values of f(x)	**A1**	
	At least 3 <u>correct</u> values of f(x) spanning x = ⅔	**A1**	
	Shows a decreasing and then increasing pattern. Neither www	**A1**	Or similar wording. Must have 'Neither'
	ALT 2 $f'(x)=3x^2+4x-4 = 3(x+\frac{2}{3})^2-\frac{16}{3}$	**B1B1**	Do not condone sign errors
	$f'(x)\geqslant -\frac{16}{3}$	**M1**	
	$f'(x)<0$ for some values and >0 for other values. Neither www	**A1**	Or similar wording. Must have 'Neither'
		4	

Question	Answer	Marks	Guidance
3(i)	0.8 oe	**B1**	
		1	
3(ii)	$BD = 5\sin \textit{their}\, 0.8$	**M1**	Expect 3.58(7). Methods using degrees are acceptable
	DC = 5 – 5cos *their* 0.8	**M1**	Expect 1.51(6)
	Sector = ½ × 5^2 × *their*0.8 OR Seg = ½ × 5^2 × [*their* 0.8 – sin*their* 0.8]	**M1**	Expect 10 for sector. Expect 1.03(3) for segment
	Trap = ½(5 + *theirDC*) × *theirBD* oe OR Δ*BDC* = ½*theirBD* × *theirCD*	**M1**	OR (for last 2 marks) if *X* is on *AB* and *XC* is parallel to *BD*:
	Shaded area = 11.69 – 10 OR 2.71(9) – 1.03(3) = 1.69 cao	**A1**	*BDCX* –(sector – Δ*AXC*) = 5.43(8) – [10 – 6.24(9)] = 1.69 cao M1A1
		5	

Question	Answer	Marks	Guidance
4(i)	Gradient, *m*, of *AB* = 3/4	**B1**	
	Equation of *BC* is $y-4=\frac{-4}{3}(x-3)$	**M1A1**	Line through (3, 4) with gradient $\frac{-1}{m}$ (M1). (Expect $y=\frac{-4}{3}x+8$)
	$x=6$	**A1**	Ignore any *y* coordinate given.
		4	

Question	Answer	Marks	Guidance
4(ii)	$(AC)^2 = 7^2 + 1^2 \;\rightarrow\; AC = 7.071$	M1A1	M mark for $\sqrt{(their 6 + / -1)^2 + 1}$.
		2	

Question	Answer	Marks	Guidance
5	$a + (n-1)3 = 94$	B1	
	$\frac{n}{2}\left[2a + (n-1)3\right] = 1420$ OR $\frac{n}{2}\left[a + 94\right] = 1420$	B1	
	Attempt elimination of *a* or *n*	M1	
	$3n^2 - 191n + 2840\ (=0)$ OR $a^2 - 3a - 598\ (=0)$	A1	3-term quadratic (not necessarily all on the same side)
	$n = 40$ (only)	A1	
	$a = -23$ (only)	A1	Award 5/6 if a 2nd pair of solutions (71/3, 26) is given in addition or if given as the only answer.
		6	

Question	Answer	Marks	Guidance
6	$(\mathbf{BO}) = -8\mathbf{i} - 6\mathbf{j}$	B1	OR **(OB)** = 8**i** + 6**j**
	$(\mathbf{BF}) = -6\mathbf{j} - 8\mathbf{i} + 7\mathbf{k} + 4\mathbf{i} + 2\mathbf{j} = -4\mathbf{i} - 4\mathbf{j} + 7\mathbf{k}$	B1	OR **(FB)** = 4**i** + 4**j** – 7**k**
	$(\mathbf{BF.BO}) = (-4)(-8) + (-4)(-6)$	M1	OR **(FB.OB)** Expect 56. Accept one reversed but award final A0
	$\lvert\mathbf{BF}\rvert \times \lvert\mathbf{BO}\rvert = \sqrt{4^2+4^2+7^2} \times \sqrt{8^2+6^2}$	M1	Expect 90. At least one magnitude <u>methodically</u> correct
	Angle $OBF = \cos^{-1}\left(\dfrac{their\,56}{their\,90}\right) = \cos^{-1}\left(\dfrac{56}{90}\right)$ or $\cos^{-1}\left(\dfrac{28}{45}\right)$	DM1A1	Or equivalent 'integer' fractions. All M marks dependent on use of (±)**BO** and (±)**BF**. 3rd M mark dep on both preceding M marks
		6	

Question	Answer	Marks	Guidance
7(i)	$\dfrac{(\tan\theta+1)(1-\cos\theta)+(\tan\theta-1)(1+\cos\theta)}{(1+\cos\theta)(1-\cos\theta)}$ soi	M1	
	$\dfrac{\tan\theta-\tan\theta\cos\theta+1-\cos\theta+\tan\theta-1+\tan\theta\cos\theta-\cos\theta}{1-\cos^2\theta}$ www	A1	
	$\dfrac{2(\tan\theta-\cos\theta)}{\sin^2\theta}$ www **AG**	A1	
		3	

Question	Answer	Marks	Guidance
7(ii)	$(2)(\tan\theta-\cos\theta)\ (=0)\to(2)\left(\dfrac{\sin\theta}{\cos\theta}-\cos\theta\right)\ (=0)$ soi	**M1**	Equate numerator to zero and replace $\tan\theta$ by $\sin\theta/\cos\theta$
	$(2)\left(\sin\theta-\left(1-\sin^2\theta\right)\right)\ (=0)$	**DM1**	Multiply by $\cos\theta$ and replace $\cos^2\theta$ by $1-\sin^2\theta$
	$\sin\theta=0.618(0)$ soi	**A1**	Allow $(\sqrt{5}-1)/2$
	$\theta=38.2°$	**A1**	Apply penalty –1 for extra solutions in range
		4	

Question	Answer	Marks	Guidance
8(i)	$y=\frac{1}{3}ax^3+\frac{1}{2}bx^2-4x\ (+c)$	**B1**	
	$11=0+0+0+c$	**M1**	Sub $x=0$, $y=11$ into an integrated expression. c must be present
	$y=\frac{1}{3}ax^3+\frac{1}{2}bx^2-4x+11$	**A1**	
		3	
8(ii)	$4a+2b-4=0$	**M1**	Sub $x=2$, $\mathrm{d}y/\mathrm{d}x=0$
	$\frac{1}{3}(8a)+2b-8+11=3$	**M1**	Sub $x=2$, $y=3$ into an integrated expression. Allow if 11 missing
	Solve simultaneous equations	**DM1**	Dep. on both M marks
	$a=3,\ b=-4$	**A1A1**	Allow if no working seen for simultaneous equations
		5	

Question	Answer	Marks	Guidance
9(i)	For *their* 3-term quad a recognisable application of b^2-4ac	**M1**	Expect $2x^2-x(3+k)+1-k^2\quad(=0)$ oe for the 3-term quad.
	$(b^2-4ac=)\ (3+k)^2-4(2)(1-k^2)$ oe	**A1**	Must be correct. Ignore any RHS
	$9k^2+6k+1$	**A1**	Ignore any RHS
	$(3k+1)^2\geqslant 0$ Do not allow > 0. Hence curve and line meet. **AG**	**A1**	Allow $(9)\left(k+\frac{1}{3}\right)^2\geqslant 0$. Conclusion required.
	ALT Attempt solution of 3-term quadratic	**M1**	
	Solutions $x=k+1,\quad ½(1-k)$	**A1A1**	
	Which exist for all values of *k*. Hence curve and line meet. **AG**	**A1**	
		4	

Question	Answer	Marks	Guidance
9(ii)	$k = -1/3$	B1	**ALT** $dy/dx = 4x - 3 \Rightarrow 4x - 3 = k$
	Sub (one of) *their* $k = -\frac{1}{3}$ into either line 1 $\rightarrow$ $2x^2 - \frac{8}{3}x + \frac{8}{9}\ (=0)$ Or into the derivative of line 1 $\rightarrow 4x\ -(3+k)(=0)$	M1	Sub $k = 4x - 3$ into line 1 $\rightarrow$ $2x^2 - x(4x) + 1 - (4x-3)^2\ (=0)$
	x = 2/3 Do not allow unsubstantiated $\left(\frac{2}{3}, -\frac{1}{9}\right)$ following $k = -\frac{1}{3}$	A1	x = 2/3, y = –1/9 (both required) [from $-18x^2 + 24x - 8$ (=0) oe]
	$y = -1/9$ Do not allow unsubstantiated $\left(\frac{2}{3}, -\frac{1}{9}\right)$ following $k = -\frac{1}{3}$	A1	$k = -1/3$
		4	

Question	Answer	Marks	Guidance
10(i)	$V = 4(\pi)\int(3x-1)^{-2/3}\,dx \quad = 4(\pi)\left[\frac{(3x-1)^{1/3}}{1/3}\right][\div 3]$	M1A1A1	Recognisable integration of y^2 (M1) Independent A1, A1 for [ ] [ ]
	$4(\pi)[2-1]$	DM1	Expect $4(\pi)(3x-1)^{\frac{1}{3}}$
	4π or 12.6	A1	Apply limits ⅔ → 3. Some working must be shown.
		5	

Question	Answer	Marks	Guidance
10(ii)	$\mathrm{d}y/\mathrm{d}x = (-2/3)(3x-1)^{-4/3} \times 3$	B1	Expect $-2(3x-1)^{-4/3}$
	When $x = 2/3$, $y = 2$ soi $\mathrm{d}y/\mathrm{d}x = -2$	B1B1	2nd B1 dep. on correct expression for $\mathrm{d}y/\mathrm{d}x$
	Equation of normal is $y - 2 = \frac{1}{2}\left(x - \frac{2}{3}\right)$	M1	Line through ($\frac{2}{3}$, *their* 2) and with grad $-1/m$. Dep on m from diffn
	$y = \dfrac{1}{2}x + \dfrac{5}{3}$	A1	
		5	

Question	Answer	Marks	Guidance
11(i)	$[2]\left[(x-3)^2\right][-7]$	B1B1B1	
		3	
11(ii)	Largest value of k is 3. Allow ($k =$) 3.	B1	Allow $k \leqslant 3$ but not $x \leqslant 3$ as final answer.
		1	

Question	Answer	Marks	Guidance
11(iii)	$y = 2(x-3)^2 - 7 \rightarrow (x-3)^2 = ½(y+7)$ or with *x/y* transposed	**M1**	Ft *their a*, *b*, *c*. Order of operations correct. Allow sign errors
	$x = 3 \pm \sqrt{½(y+7)}$ Allow 3 + √ or 3 –√ or with *x/y* transposed	**DM1**	Ft *their a*, *b*, *c*. Order of operations correct. Allow sign errors
	$f^{-1}(x) = 3 - \sqrt{½(x+7)}$	**A1**	
	(Domain is *x*) $\geqslant$ *their* -7	**B1FT**	Allow other forms for interval but if variable appears must be *x*
		4	
11(iv)	$x+3 \leqslant 1$. Allow *x* + 3 = 1	**M1**	Allow $x+3 \leqslant k$
	largest p is -2 . Allow (p =) –2	**A1**	Allow $p \leqslant -2$ but not $x \leqslant -2$ as final answer.
	$fg(x) = f(x+3) = 2x^2 - 7$ cao	**B1**	
		3	

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MATHEMATICS	**9709/11**
Paper 1	**May/June 2018**
MARK SCHEME	
Maximum Mark: 75	

Published

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Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously 'correct' answers or results obtained from incorrect working.

  - Note: B2 or A2 means that the candidate can earn 2 or 0.
    B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only – often written by a 'fortuitous' answer
ISW	Ignore Subsequent Working
SOI	Seen or implied
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become 'follow through' marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Question	Answer	Marks	Guidance
1(i)	$(1-2x)^5 = 1 - 10x + 40x^2$ (no penalty for extra terms)	**B2,1**	Loses a mark for each incorrect term. Treat $-32x^5 + 80x^4 - 80x^3$ as MR −1
		2	
1(ii)	$\rightarrow (1 + ax + 2x^2)(1 - 10x + 40x^2)$		
	3 terms in x^2 $\rightarrow$ $40 - 10a + 2$	**M1 A1FT**	Selects 3 terms in x^2. FT from **(i)**
	Equate with 12 $\rightarrow$ a = 3	**A1**	CAO
		3	

Question	Answer	Marks	Guidance
2	$y = 2x + \frac{5}{x} \rightarrow \frac{dy}{dx} = 2 - \frac{5}{x^2} = -3$ (may be implied) when $x = 1$.	**M1 A1**	Reasonable attempt at differentiation CAO (−3)
	$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt} \rightarrow -0.06$	**M1 A1**	Ignore notation, but needs to multiply $\frac{dy}{dx}$ by 0.02.
		4	

Question	Answer	Marks	Guidance
3	$\frac{dy}{dx}=\frac{12}{(2x+1)^2} \rightarrow y=\frac{-12}{2x+1} \div 2 \ (+c)$	**B1 B1**	Correct without " ÷ 2". For " ÷ 2". Ignore "c".
	Uses (1, 1) $\rightarrow c=3$ $(\rightarrow y=\frac{-6}{2x+1}+3)$	**M1 A1**	Finding "c" following integration. CAO
	Sets y to 0 and attempts to solve for $x \rightarrow x=\frac{1}{2} \rightarrow ((\frac{1}{2}, 0))$	**DM1 A1**	Sets y to 0. $x=\frac{1}{2}$ is sufficient for A1.
		6	

Question	Answer	Marks	Guidance
4(i)	$(\sin\theta + \cos\theta)(1 - \sin\theta\cos\theta) \equiv \sin^3\theta + \cos^3\theta$.		Accept abbreviations s and c
	LHS = $\sin\theta + \cos\theta - \sin^2\theta\cos\theta - \sin\theta\cos^2\theta$	**M1**	Expansion
	$= \sin\theta(1 - \cos^2\theta) + \cos\theta(1 - \sin^2\theta)$ or $(s + c - c(1 - c^2) - s(1 - s^2))$	**M1A1**	Uses identity twice. Everything correct. AG
	Uses $\sin^2\theta + \cos^2\theta = 1 \rightarrow \sin^3\theta + \cos^3\theta$ (RHS)		or from RHS: M1 for use of trig ID twice
	Or		
	LHS = $(\sin\theta+\cos\theta)(\sin^2\theta+\cos^2\theta-\sin\theta\cos\theta)$	**M1**	M1 for factorisation
	$= \sin^3\theta+\sin\theta\cos^2\theta-\sin^2\theta\cos\theta+\cos\theta\sin^2\theta+\cos^3\theta-\sin\theta\cos^2\theta=\sin^3\theta+\cos^3\theta$	**M1A1**	
		3	

Question	Answer	Marks	Guidance
4(ii)	$(\sin\theta + \cos\theta)(1 - \sin\theta\cos\theta) = 3\cos^3\theta \rightarrow \sin^3\theta = 2\cos^3\theta$	**M1**	
	$\rightarrow \tan^3\theta = 2 \rightarrow \theta = 51.6°$ or $231.6°$ (only)	**A1A1FT**	Uses $\tan^3 = \sin^3 \div \cos^3$. A1 CAO. A1FT, 180 + their acute angle. $tan^3\theta = 0$ gets M0
		3	

Question	Answer	Marks	Guidance
5(i)	Eqn of AC $y = -\frac{1}{2}x + 4$ (gradient must be $\Delta y / \Delta x$)	**M1A1**	Uses gradient and a given point for equa. CAO
	Gradient of $OB = 2 \rightarrow y = 2x$ (If y missing only penalise once)	**M1 A1**	Use of $m_1m_2 = -1$, answers only ok.
		4	

Question	Answer	Marks	Guidance
5(ii)	Simultaneous equations $\to$ ((1.6, 3.2))	**M1**	Equate and solve for M1 and reach $\geqslant 1$ solution
	This is mid-point of OB. $\to$ B (3.2, 6.4)	**M1 A1**	Uses mid-point. CAO
	or		
	Let coordinates of B (h, k) $OA = AB \to h^2 = 8k - k^2$ $OC = BC \to k^2 = 16h - h^2 \to (3.2, 6.4)$		M1 for both equations, M1 for solving with $y = 2x$
	or		
	gradients ($\frac{k-4}{h} \times \frac{k}{h-8} = -1$)		M1 for gradient product as –1, M1 solving with $y = 2x$
	or		
	Pythagoras: $h^2 + (k-4)^2 + (h-8)^2 + k^2 = 4^2 + 8^2$		M1 for complete equation, M1 solving with $y = 2x$
		3	

Question	Answer	Marks	Guidance
6(i)	$(\tan\theta = \frac{AT}{r}) \to AT = r\tan\theta$ or $OT = \frac{r}{\cos\theta}$ SOI	**B1**	CAO
	$\to A = \frac{1}{2}r^2\tan\theta \quad - \frac{1}{2}r^2\theta$	**B1 B1**	B1 for $\frac{1}{2}r^2\tan\theta$. B1 for "$-\frac{1}{2}r^2\theta$" If Pythagoras used may see area of triangle as $\frac{1}{2}r\sqrt{r^2 + r^2 tan^2\theta}$ or $\frac{1}{2}r\left(\frac{r}{\cos\theta}\right)sin\theta$
		3	

Question	Answer	Marks	Guidance
6(ii)	$\tan\theta = \dfrac{AT}{3} \rightarrow AT = 7.716$	**M1**	Correct use of trigonometry and radians in rt angle triangle
	Arc length = $r\theta$ = 3.6	**B1**	Accept 3×1.2
	OT by Pythagoras or $\cos 1.2 = \dfrac{3}{OT}$ $(= 8.279)$	**M1**	Correct method for OT
	Perimeter = AT + arc + OT – radius = 16.6	**A1**	CAO, www
		4	

Question	Answer	Marks	Guidance
7	$\overrightarrow{OA} = \begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix}$, $\overrightarrow{OB} = \begin{pmatrix} -1 \\ 3 \\ 5 \end{pmatrix}$ and $\overrightarrow{OC} = \begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix}$		
7(i)	$\overrightarrow{AC} = \begin{pmatrix} 2 \\ 4 \\ -4 \end{pmatrix}$	**B1**	B1 for $\overrightarrow{AC}$.
		1	

Question	Answer	Marks	Guidance
7(ii)	$\overrightarrow{OM} = \overrightarrow{OA} + \overrightarrow{AM} = \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix}$ or $\frac{1}{2}[\begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix} + \begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix}]$	M1	M1 for their $\overrightarrow{OM} = \overrightarrow{OA} + \overrightarrow{AM}$ oe
	Unit vector in direction of $\overrightarrow{OM} = \frac{1}{\sqrt{5}}(\overrightarrow{OM})$	M1 A1	M1 for dividing their $\overrightarrow{OM}$ by their modulus
		3	
7(iii)	$\overrightarrow{AB} = \begin{pmatrix} -2 \\ 6 \\ 3 \end{pmatrix}$, Allow ±	B1	
	$\lvert\overrightarrow{AB}\rvert=7$, $\lvert\overrightarrow{AC}\rvert=6$ $\begin{pmatrix} -2 \\ 6 \\ 3 \end{pmatrix} . \begin{pmatrix} 2 \\ 4 \\ -4 \end{pmatrix} = -4 + 24 - 12 = 8$	M1 M1	Product of both moduli, Scalar product of ± their AB and AC
	$7 \times 6 \cos\theta = 8 \rightarrow \theta = 79.(0)^{\circ}$	A1	1.38 radians ok
		4	

Question	Answer	Marks	Guidance
8(a)	$ar = 12$ and $\dfrac{a}{1-r} = 54$	**B1 B1**	CAO, OE CAO, OE
	Eliminates a or r $\rightarrow$ $9r^2 - 9r + 2 = 0$ or $a^2 - 54a + 648 = 0$	**M1**	Elimination leading to a 3-term quadratic in a or r
	$\rightarrow r = \dfrac{2}{3}$ or $\dfrac{1}{3}$ hence to a $\rightarrow$ $a = 18$ or 36	**A1**	Needs both values.
		4	
8(b)	nth term of a progression is $p + qn$		
8(b)(i)	first term = $p + q$. Difference = q or last term = $p + qn$	**B1**	Need first term and, last term or common difference
	$S_n = \dfrac{n}{2}\left(2(p+q)+(n-1)q\right)$ or $\dfrac{n}{2}(2p+q+nq)$	**M1A1**	Use of S_n formula with their a and d. ok unsimplified for A1.
		3	
8(b)(ii)	Hence $2(2p+q+4q)=40$ and $3(2p+q+6q)=72$	**DM1**	Uses their S_n formula from **(i)**
	Solution $\rightarrow$ $p = 5$ and $q = 2$ [Could use S_n with a and d $\rightarrow$ $a = 7, d = 2$ $\rightarrow$ $p = 5, q = 2$.]	**A1**	Note: answers 7, 2 instead of 5, 2 gets M1A0 – must attempt to solve for M1
		2	

Question	Answer	Marks	Guidance
9	$\mathrm{f} : x \mapsto \frac{x}{2} - 2, \quad \mathrm{g} : x \mapsto 4 + x - \frac{x^2}{2}$		
9(i)	$4 + x - \frac{x^2}{2} = \frac{x}{2} - 2 \ \rightarrow \ x^2 - x - 12 = 0$	**M1**	Equates and forms 3 term quadratic
	$\rightarrow$ (4, 0) and (−3, −3.5) Trial and improvement, B3 all correct or B0	**A1 A1**	A1 For both x values or a correct pair. A1 all.
		3	
9(ii)	$\mathrm{f}(x) > \mathrm{g}(x)$ for $x > 4$, $x < -3$	**B1, B1**	B1 for each part. Loses a mark for $\leqslant$ or $\geqslant$.
		2	
9(iii)	$\mathrm{fg}(x) = 2 + \frac{x}{2} - \frac{x^2}{4} - 2 \ (= \frac{x}{2} - \frac{x^2}{4})$	**B1**	CAO, any correct form
	i.e. $-\frac{1}{4}((x-1)^2 - 1)$ or $\frac{dy}{dx} = \frac{1}{2} - \frac{2x}{4} = 0 \ \rightarrow \ x = 1$	**M1 A1**	Completes the square or uses calculus. First A1 is for $x = 1$ or completed square form
	$\rightarrow y = \frac{1}{4} \ \rightarrow$ Range of fg $\leqslant \frac{1}{4}$,	**A1**	CAO, OE e.g. $y \leqslant \frac{1}{4}$, $[-\infty, \frac{1}{4})$ etc.
		4	
9(iv)	Calculus or completing square on 'h' $\rightarrow x = 1$	**M1**	May use a sketch or $-\frac{b}{2a}$
	$k = 1$ (accept $k \geqslant 1$)	**A1**	Complete method. CAO
		2	

Question	Answer	Marks	Guidance
10	$y = x^3 - 2x^2 + 5x$		
10(i)	$\frac{dy}{dx} = 3x^2 - 4x + 5$	**B1**	CAO
	Using $b^2 - 4ac \rightarrow 16 - 60 \rightarrow$ negative $\rightarrow$ some explanation or completed square and explanation	**M1 A1**	Uses discriminant on equation (set to 0). CAO
		3	
10(ii)	$m = 3x^2 - 4x + 5$ $\frac{dm}{dx} = 6x - 4\ (= 0)$ (must identify as $\frac{dm}{dx}$)	**B1FT**	FT providing differentiation is equivalent
	$\rightarrow x = \frac{2}{3},\ m = \frac{11}{3}$ or $\frac{dy}{dx} = \frac{11}{3}$ Alt1: $m = 3\left(x - \frac{2}{3}\right)^2 + \frac{11}{3},\ m = \frac{11}{3}$ Alt2: $3x^2 - 4x + 5 - m = 0,\ b^2 - 4ac = 0,\ m = \frac{11}{3}$	**M1 A1**	Sets to 0 and solves. A1 for correct *m*. Alt1: B1 for completing square, M1A1 for ans Alt2: B1 for coefficients, M1A1 for ans
	$\frac{d^2m}{dx^2} = 6$ +ve $\rightarrow$ Minimum value or refer to sketch of curve or check values of m either side of $x = \frac{2}{3}$,	**M1 A1**	M1 correct method. A1 (no errors anywhere)
		5	

Question	Answer	Marks	Guidance
10(iii)	Integrate $\rightarrow \frac{x^4}{4} - \frac{2x^3}{3} + \frac{5x^2}{2}$	**B2,1**	Loses a mark for each incorrect term
	Uses limits 0 to 6 $\rightarrow$ 270 (may not see use of lower limit)	**M1 A1**	Use of limits on an integral. CAO Answer only 0/4
		4	

Cambridge International AS & A Level

Cambridge Assessment International Education
Cambridge International Advanced Subsidiary and Advanced Level

MATHEMATICS **9709/12**

Paper 1 **May/June 2018**

MARK SCHEME

Maximum Mark: 75

Published

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CWO	Correct Working Only – often written by a 'fortuitous' answer
ISW	Ignore Subsequent Working
SOI	Seen or implied
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become 'follow through' marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Question	Answer	Marks	Guidance
1	Coefficient of x^2 in $\left(2+\frac{x}{2}\right)^6$ is ${}_6C_2\times2^4\times(½)^2\ (x^2)$ (= 60)	B2,1,0	3 things wanted –1 each incorrect component, must be multiplied together. Allow ${}_6C_4$, $\begin{pmatrix}6\\4\end{pmatrix}$ and factorial equivalents. Marks can be awarded for correct term in an expansion.
	Coefficient of x^2 in $(a+x)^5$ is ${}_5C_2\times a^3\ (x^2)$ (= $10a^3$)	B1	Marks can be awarded for correct term in an expansion.
	$\rightarrow 60 + 10a^3 = 330$	M1	Forms an equation '*their 60*' + '*their $10a^3$*' = 330, OK with x^2 in all three terms initially. This can be recovered by a correct answer.
	$a = 3$	A1	Condone ±3 as long as +3 is selected.
		5	

Question	Answer	Marks	Guidance
2(i)			A complete method as far as finding a set of values for k by:
	Either $(x-3)^2 + k - 9 > 0$, $k - 9 > 0$		Either completing the square and using '*their $k-9$*' > or $\geqslant$ 0 OR
	or $2x - 6 = 0 \rightarrow (3, k - 9)$, $k - 9 > 0$	M1	Differentiating and setting to 0, using '*their $x=3$*' to find y and using '*their $k-9$*' > or $\geqslant$0 OR
	or $b^2 < 4ac$ oe $\rightarrow 36 < 4k$		Use of discriminant < or $\leqslant$ 0. Beware use of > and incorrect algebra.
	$\rightarrow k > 9$ Note: not $\geqslant$	A1	T&I leading to (or no working) correct answer 2/2 otherwise 0/2.
		2	

Question	Answer	Marks	Guidance
2(ii)	**EITHER**		
	$x^2 - 6x + k = 7 - 2x \rightarrow x^2 - 4x + k - 7 \ (= 0)$	***M1**	Equates and collects terms.
	Use of $b^2 - 4ac = 0$ $(16 - 4(k - 7) = 0)$	**DM1**	Correct use of discriminant = 0, involving k from a 3 term quadratic.
	OR		
	$2x - 6 = -2 \rightarrow x = 2$ $(y = 3)$	***M1**	Equates their $\frac{dy}{dx}$ to ± 2, finds a value for x.
	(*their* 3) or 7– 2(*their* 2) = (*their* 2)^2 – 6(*their* 2) + k	**DM1**	Substitutes their value(s) into the appropriate equation.
	$\rightarrow k = 11$	**A1**	
		3	

Question	Answer	Marks	Guidance
3(i)	$r = 1.02$ or $\frac{102}{100}$ used in a GP in some way.	**B1**	Can be awarded here for use in S_n formula.
	Amount in 12th week = 8000 (*their r*)^{11} or (*their a from* $\frac{8000}{their\ r.}$) (*their r*)^{12}	**M1**	Use of ar^{n-1} with a = 8000 & n = 12 or with a = $\frac{8000}{1.02}$ and n = 13.
	= 9950 (kg) awrt	**A1**	Note: Final answer of either 9943 or 9940 implies M1. Full marks can be awarded for a correct answer from a list of terms.
		3	

Question	Answer	Marks	Guidance
3(ii)	In 12 weeks, total is $\dfrac{8000\left(\left(their\ r\right)^{12}-1\right)}{\left(\left(their\ r\right)-1\right)}$	M1	Use of S_n with a = 8000 and n = 12 or addition of 12 terms.
	= 107000 (kg) awrt	A1	Correct answer but no working 2/2
		2	

Question	Answer	Marks	Guidance
4(i)	$a + ½b = 5$	B1	Alternatively these marks can be awarded when ½ and –1 appear after a or b has been eliminated.
	$a – b = 11$	B1	
	$\rightarrow a = 7$ and $b = – 4$	B1	
		[3]	
4(ii)	$a + b$ or *their a* + *their b* (3)	B1	Not enough to be seen in a table of values – must be selected. Graph from their values can get both marks. **Note: Use of b^2 – 4*ac* scores 0/3**
	$a – b$ or *their a* – *their b* (11).	B1	
	$\rightarrow k < 3$, $k > 11$	B1	Both inequalities correct. Allow combined statement as long as correct inequalities if taken separately. Both answers correct from T & I or guesswork 3/3 otherwise 0/3
		3	

Question	Answer	Marks	Guidance
5(i)	$\overrightarrow{DA} = 6\mathbf{i} - 4\mathbf{k}$	B1	
	$\overrightarrow{CA} = 6\mathbf{i} - 5\mathbf{j} - 4\mathbf{k}$	B1	
		2	
5(ii)	Method marks awarded only for ***their*** vectors $\pm\overrightarrow{CA}$ & $\pm\overrightarrow{DA}$		Full marks can be obtained using $\overrightarrow{AC}$ & $\overrightarrow{AD}$
	$\overrightarrow{CA}.\overrightarrow{DA} = 36 + 16\ (= 52)$	M1	Using $x_1x_2+y_1y_2+z_1z_2$
	$\lvert\overrightarrow{DA}\rvert = \sqrt{52}$, $\lvert\overrightarrow{CA}\rvert = \sqrt{77}$	M1	Uses modulus twice
	$52 = \sqrt{77}\sqrt{52}\cos C\hat{A}D$ oe	M1	All linked correctly
	$\text{Cos } C\hat{A}D = 0.82178.. \rightarrow C\hat{A}D = 34.7^{\circ}$ or 0.606^{c} awrt	A1	Answer must come from +ve cosine ratio
		4	

Question	Answer	Marks	Guidance
6(i)	AT or $BT = r\tan\theta$ or $OT = \dfrac{r}{\cos\vartheta}$	B1	May be seen on diagram.
	$\frac{1}{2}r^2 2\theta$, & $\frac{1}{2}\times r\times(r\tan\theta$ or $AT)$ or $\frac{1}{2}\times r\times(\dfrac{r}{\cos\vartheta}$ or $OT)\sin\theta$	M1	Both formulae, ($\frac{1}{2}r^2\theta$, $\frac{1}{2}bh$ or $\frac{1}{2}ab\sin\theta$), seen with 2θ used when needed.
	$\frac{1}{2}r^2 2\theta = 2\times\frac{1}{2}\times r\times r\tan\theta - \frac{1}{2}r^2 2\theta$ oe $\rightarrow 2\theta = \tan\theta$ **AG**	A1	Fully correct working from a correct statement. Note: $\frac{1}{2}r^2 2\theta = \frac{1}{2}r^2\tan\theta$ is a valid statement.
		3	

Question	Answer	Marks	Guidance
6(ii)	$\theta = 1.2$ or sector area = 76.8	**B1**	
	Area of kite = 165 awrt	**B1**	
	164.6 – 76.8 = 87.8 awrt	**B1**	awrt 87.8 with little or no working can be awarded 3/3. SC Final answers that round to 88 with little or no working can be awarded 2/3.
		3	

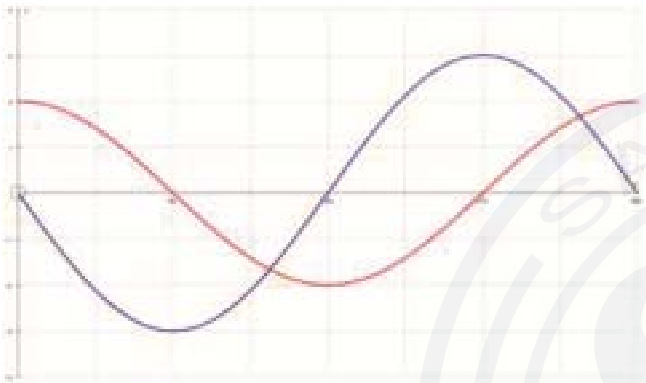
Question	Answer	Marks	Guidance
7(i)	$25 - 2(x + 3)^2$	**B1 B1**	Mark expression if present: B1 for 25 and B1 for $-2(x + 3)^2$. If no expression award $a = 25$ B1 and $b = 3$ B1.
		2	
7(ii)	$(-3, 25)$	**B1FT**	FT from answers to (i) or by calculus
		1	
7(iii)	$(k) = -3$ also allow x or $k \geqslant -3$	**B1FT**	FT from answer to (i) or (ii) **NOT** $x = -3$
		1	

Question	Answer	Marks	Guidance
7(iv)	**EITHER**		
	y = 25 – 2(x + 3)² → 2(x + 3)² = 25 – y	*M1	Makes their squared term containing x the subject or equivalent with x/y interchanged first. Condone errors with +/- signs.
	$x+3=(\pm)\sqrt{½(25-y)}$	DM1	Divide by ±2 and then square root allow ±.
	OR		
	y = 7 – 2x^2 – 12x → 2x^2 + 12x + y – 7 (= 0)	*M1	Rearranging equation of the curve.
	$x = \dfrac{-12\pm\sqrt{12^2-8(y-7)}}{4}$	DM1	Correct use of their 'a, b and c' in quadratic formula. Allow just + in place of ±.
	$g^{-1}(x) = \sqrt{\left(\dfrac{25-x}{2}\right)} - 3$ oe isw if substituting x = – 3	A1	± gets A0. Must now be a function of x. Allow y =
		3	

Question	Answer	Marks	Guidance
8	**EITHER**		
	Gradient of bisector $= -\frac{3}{2}$	B1	
	gradient $AB = \frac{5h-h}{4h+6-h}$	*M1	Attempt at $\frac{y-step}{x-step}$
	Either $\frac{5h-h}{4h+6-h} = \frac{2}{3}$ or $-\frac{4h+6-h}{5h-h} = -\frac{3}{2}$	*M1	Using $m_1m_2 = -1$ appropriately to form an equation.
	OR		
	Gradient of bisector $= -\frac{3}{2}$	B1	
	Using gradient of AB and A, B or midpoint $\rightarrow \frac{2}{3}x + \frac{h}{3} = y$ oe	*M1	Obtain equation of AB using gradient from $m_1m_2 = -1$ and a point.
	Substitute co-ordinates of one of the other points	*M1	Arrive at an equation in h.
	h = 2	A1	
	Midpoint is $\left(\frac{5h+6}{2}, 3h\right)$ or (8, 6)	B1FT	Algebraic expression or FT for numerical answer from '*their h*'
	Uses midpoint and '*their h*' with $3x + 2y = k$	DM1	Substitutes '*their midpoint*' into $3x + 2y = k$. If $y = -\frac{3}{2}x + c$ is used (expect $c = 18$) the method mark should be withheld until they ×2.
	$\rightarrow k = 36$ soi	A1	
		7	

Question	Answer	Marks	Guidance
9(i)	$y = \frac{2}{3}\left(4x+1\right)^{\frac{3}{2}} \div 4\ (+\text{C})\ \left(=\frac{\left(4x+1\right)^{\frac{3}{2}}}{6}\right)$	B1 B1	B1 without ÷ 4. B1 for ÷ 4 oe. Unsimplified OK
	Uses $x = 2$, $y = 5$	M1	Uses (2, 5) in an integral (indicated by an increase in power by 1).
	→ ***c* = ½** oe isw	A1	No isw if candidate now goes on to produce a straight line equation
		4	
9(ii)	$\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$		
	$\frac{dx}{dt} = 0.06 \div 3$	M1	Ignore notation. Must be 0.06÷3 for M1.
	= 0.02 oe	A1	Correct answer with no working scores 2/2
		2	
9(iii)	$\frac{d^2y}{dx^2} = ½\left(4x+1\right)^{-½} \times 4$	B1	
	$\frac{d^2y}{dx^2} \times \frac{dy}{dx} = \frac{2}{\sqrt{4x+1}} \times \sqrt{4x+1} \quad (= 2)$	B1FT	Must either show the algebraic product and state that it results in a constant or evaluate it as '= 2'. Must not evaluate at x =2. ft to apply only if $\frac{d^2y}{dx^2}$ is of the form $k\left(4x+1\right)^{-½}$
		2	

Question	Answer	Marks	Guidance
10(i)	$2\cos x = -3\sin x \rightarrow \tan x = -⅔$	**M1**	Use of tan=sin/cos to get tan =, or other valid method to find sin or cos =. M0 for $\tan x = +/-\frac{3}{2}$
	$\rightarrow x = 146.3°$ or $326.3°$awrt	**A1 A1FT**	FT for 180 added to an incorrect first answer in the given range. The second A1 is withheld if any further values in the range $0° \leqslant x \leqslant 360°$ are given. Answers in radians score A0, A0.
		3	

Question	Answer	Marks	Guidance
10(ii)	No labels required on either axis. Assume that the diagram is 0º to 360º unless labelled otherwise. Ignore any part of the diagram outside this range.		
		B1	Sketch of $y = 2\cos x$. One complete cycle; start and finish at <u>top of curve</u> at roughly the same positive y value and go below the x axis by roughly the same distance. (Can be a poor curve but not straight lines.)
		B1	Sketch of $y = -3\sin x$ One complete cycle; start and finish on the x axis, must be inverted and go below and then above the x axis by roughly the same distance. (Can be a poor curve but not straight lines.)
		B1	Fully correct answer including the sine curve with clearly larger amplitude than cosine curve. Must now be reasonable curves.
			Note: Separate diagrams can score 2/3
		3	
10(iii)	$x < 146.3^\circ$, $x > 326.3^\circ$	**B1FT B1FT**	Does not need to include 0º, 360º. √ from their answers in (i) Allow combined statement as long as correct inequalities if taken separately. SC For two correct values including ft but with $\leqslant$ and $\geqslant$ B1
		2	

Question	Answer	Marks	Guidance
11(i)	$y=\frac{x}{2}+\frac{6}{x}=4 \rightarrow x=2$ or 6	**B1 B1**	Inspection or guesswork OK
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{2}-\frac{6}{x^2}$	**B1**	Unsimplified OK
	When $x=2$, $m=-1 \rightarrow x+y=6$ When $x=6$, $m=\frac{1}{3} \rightarrow y=\frac{1}{3}x+2$	***M1**	Correct method for either tangent
	Attempt to solve simultaneous equations	**DM1**	Could solve BOTH equations separately with $y=x$ and get $x=3$ both times.
	(3,3)	**A1**	Statement about $y=x$ not required.
		6	

Question	Answer	Marks	Guidance
11(ii)	$V = (\pi)\int\left(\frac{x^2}{4}+6+\frac{36}{x^2}\right)(dx)$	*M1	Integrate using $\pi\int y^2dx$ (doesn't need π or dx). Allow incorrect squaring. Not awarded for $\pi\int\left\{4-\left(\frac{x}{2}+\frac{6}{x}\right)\right\}^2 dx$. Integration indicated by increase in any power by 1.
	Integration $\rightarrow \frac{x^3}{12}+6x-\frac{36}{x}$	A2,1	3 things wanted —1 each error, allow + C. (Doesn't need π)
	Using limits '*their 2*' to '*their 6*' ($53\frac{1}{3}\pi$, $\frac{160}{3}\pi$, 168 awrt)	DM1	Evidence of their values 6 and 2 from **(i)** substituted into their integrand and then subtracted. $48-\left(-\frac{16}{3}\right)$ is enough.
	Vol for line: integration or cylinder ($\rightarrow 64\pi$)	M1	Use of πr^2h or integration of 4^2 (could be from $\left\{4-\left(\frac{x}{2}+\frac{6}{x}\right)\right\}^2$)
	Subtracts $\rightarrow 10\frac{2}{3}\boldsymbol{\pi}$ oe $\left(\text{e.g.}\frac{32}{3}\pi, 33.5\text{ awrt}\right)$	A1	

Question	Answer	Marks	Guidance
11(ii)	**OR**		
	V = $(\pi)\int 4^2 - \left(\frac{x}{2}+\frac{6}{x}\right)^2 \,(\mathrm{d}x)$	M1 *M1	Integrate using $\pi\int y^2 \mathrm{d}x$ (doesn't need π or dx) Integration indicated by increase in any power by 1.
	$= (\pi)\int 16 - \left(\frac{x^2}{4}+6+\frac{36}{x^2}\right)\,(\mathrm{d}x)$		
	$= (\pi)\left[16x - \left(\frac{x^3}{12}+6x-\frac{36}{x}\right)\right]\,(\mathrm{d}x)$	A2,1	Or $\left[10x - \frac{x^3}{12}+\frac{36}{x}\right]$
	$= (\pi)\,(48 - 37\frac{1}{3})$	DM1	Evidence of their values 6 and 2 from (i) substituted
	$= 10\frac{2}{3}\boldsymbol{\pi}$ oe $\left(\text{eg}\,\frac{32}{3}\pi, 33.5\,\text{awrt}\right)$	A1	
		6	

Cambridge Assessment International Education
Cambridge International Advanced Subsidiary and Advanced Level

MATHEMATICS **9709/13**

Paper 1 **May/June 2018**

MARK SCHEME

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CWO Correct Working Only – often written by a ‘fortuitous’ answer

ISW Ignore Subsequent Working

SOI Seen or implied

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become ‘follow through’ marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Question	Answer	Marks	Guidance
1	[3] $\left[(x-2)^2\right]$ [–5]	**B1B1B1**	OR $a = 3$, $b = -2$, $c = -5$. 1st mark is dependent on the form $(x+a)^2$ following 3
		3	

Question	Answer	Marks	Guidance
2	${}_5C_3\ x^2\left(\frac{-2}{x}\right)^3$ SOI	**B2,1,0**	–80 www scores B3. Accept ${}_5C_2$.
	–80 Accept $\frac{-80}{x}$	**B1**	+80 without clear working scores SCB1
		3	

Question	Answer	Marks	Guidance
3	$\left[\dfrac{a\left(1-r^n\right)}{1-r}\right][\div]\left[\dfrac{a}{1-r}\right]$	M1M1	Correct formulae **used** with/without r = 0.99 or n = 100.
		DM1	Allow numerical a (M1M1). 3rd M1 is for division $\dfrac{S_n}{S_\infty}$ (or ratio) SOI
	$1-0.99^{100}$ SOI OR $\dfrac{63(a)}{100(a)}$ SOI	A1	Could be shown multiplied by 100(%). Dep. on DM1
	63(%) Allow 63.4 or 0.63 but not 2 infringements (e.g. 0.634, 0.63%)	A1	n = 99 used scores Max M3. Condone a = 0.99 throughout $S_n = S_\infty$ (without division shown) scores 2 / 5
		5	

Question	Answer	Marks	Guidance
4	$\mathrm{f}(x)=\left[\dfrac{(3x-1)^{\frac{2}{3}}}{\frac{2}{3}}\right][\div 3]\quad(+c)$	**B1B1**	
	$1=\dfrac{8^{\frac{2}{3}}}{2}+c$	**M1**	Sub $y=1, x=3$ Dep. on attempt to integrate and c present
	$c=-1 \rightarrow y=\dfrac{1}{2}(3x-1)^{\frac{2}{3}}-1$ SOI	**A1**	
	When $x = 0$, $y=\dfrac{1}{2}(-1)^{\frac{2}{3}}-1 \quad =-\dfrac{1}{2}$	**DM1A1**	Dep. on previous M1
		6	

Question	Answer	Marks	Guidance
5	Angle $AOC = \frac{6}{5}$ or 1.2	M1	Allow 68.8°. Allow $\frac{5}{6}$
	AB = 5 × tan(*their* 1.2) OR by e.g. Sine Rule Expect 12.86	DM1	OR $OB = \frac{5}{\cos their 1.2}$. Expect 13.80
	Area $\Delta OAB = \frac{1}{2} \times 5 \times their\, 12.86$ Expect 32.15	DM1	OR $\frac{1}{2} \times 5 \times$ *their OB* × sin *their* 1.2
	Area sector $\frac{1}{2} \times 5^2 \times their\, 1.2$ Expect 15	DM1	All DM marks are dependent on the first M1
	Shaded region = 32.15 – 15 = 17.2	A1	Allow degrees used appropriately throughout. 17.25 scores A0
		5	

Question	Answer	Marks	Guidance
6(i)	Gradient, m, of $AB = \dfrac{3k+5-(k+3)}{k+3-(-3k-1)}$ OE $\left(=\dfrac{2k+2}{4k+4}\right) = \dfrac{1}{2}$	**M1A1**	Condone omission of brackets for M mark
		2	
6(ii)	Mid-pt = $[\frac{1}{2}(-3k-1+k+3), \frac{1}{2}(3k+5+k+3)] = \left(\dfrac{-2k+2}{2}, \dfrac{4k+8}{2}\right)$ SOI	**B1B1**	B1 for $\dfrac{-2k+2}{2}$, B1 for $\dfrac{4k+8}{2}$ (ISW) or better, i.e. $(-k+1, 2k+4)$
	Gradient of perpendicular bisector is $\dfrac{-1}{their\ m}$ SOI Expect –2	**M1**	Could appear in subsequent equation and/or could be in terms of k
	Equation: $y-(2k+4) = -2[x-(-k+1)]$ OE	**DM1**	Through *their* mid-point and with *their* $\dfrac{-1}{m}$ (now numerical)
	$y+2x=6$	**A1**	Use of numerical k in **(ii)** throughout scores SC2/5 for correct answer
		5	

Question	Answer	Marks	Guidance
7(a)(i)	$\dfrac{\tan^2\theta-1}{\tan^2\theta+1} = \dfrac{\dfrac{\sin\theta^2}{\cos\theta^2}-1}{\dfrac{\sin\theta^2}{\cos\theta^2}+1}$	M1	
	$= \dfrac{\sin\theta^2-\cos\theta^2}{\sin\theta^2+\cos\theta^2}$	A1	multiplying by $\cos\theta^2$ Intermediate stage can be omitted by multiplying directly by $\cos\theta^2$
	$= \sin\theta^2-\cos\theta^2 = \sin\theta^2-\left(1-\sin\theta^2\right) = 2\sin^2\theta-1$	A1	Using $\sin\theta^2+\cos\theta^2=1$ twice. Accept $a=2,\ b=-1$
	ALT 1 $\dfrac{\sec^2\theta-2}{\sec^2\theta}$	M1	ALT 2 $\dfrac{\tan^2\theta-1}{\sec^2\theta}$
	$1-\dfrac{2}{\sec^2\theta} = 1-2\cos^2\theta$	A1	$(\tan^2\theta-1)\cos^2\theta$
	$1-2\left(1-\sin^2\theta\right)=2\sin^2\theta-1$	A1	$\sin^2\theta-\cos^2\theta=\sin^2\theta-\left(1-\sin^2\theta\right)=2\sin^2\theta-1$
		3	
7(a)(ii)	$2\sin^2\theta-1=\dfrac{1}{4} \rightarrow \sin\theta=(\pm)\sqrt{\dfrac{5}{8}}$ or $(\pm)0.7906$	M1	OR $\dfrac{t^2-1}{t^2+1}=\dfrac{1}{4}\rightarrow 3t^2=5\rightarrow t=(\pm)\sqrt{\dfrac{5}{3}}$ or $t=(\pm)1.2910$
	$\theta=-52.2$	A1	
		2	

Question	Answer	Marks	Guidance
7(b)(i)	$\sin x = 2\cos x \rightarrow \tan x = 2$	**M1**	Or $\sin x = \sqrt{\frac{4}{5}}$ or $\cos x = \sqrt{\frac{1}{5}}$
	$x = 1.11$ with no additional solutions	**A1**	Accept 0.352π or 0.353π. Accept in co-ord form ignoring y co-ord
		2	
7(b)(ii)	Negative answer in range $-1 < y < -0.8$	**B1**	
	-0.894 or -0.895 or -0.896	**B1**	
		2	

Question	Answer	Marks	Guidance
8(i)	$\frac{dy}{dx} = 3x^2 - 18x + 24$	**M1A1**	Attempt to differentiate. All correct for A mark
	$3x^2 - 18x + 24 = -3$	**M1**	Equate *their* $\frac{dy}{dx}$ to –3
	$x = 3$	**A1**	
	$y = 6$	**A1**	
	$y - 6 = -3(x - 3)$	**A1FT**	FT on *their A*. Expect $y = -3x + 15$
		6	
8(ii)	$(3)(x-2)(x-4)$ SOI or $x = 2, 4$ Allow $(3)(x+2)(x+4)$	**M1**	Attempt to factorise or solve. Ignore a RHS, e.g. = 0 or > 0, etc.
	Smallest value of k is 4	**A1**	Allow $k \geqslant 4$. Allow $k = 4$. Must be in terms of k
		2	

Question	Answer	Marks	Guidance
9(i)	**OE** = $\frac{2}{10}(8\mathbf{i} + 6\mathbf{j}) = 1.6\mathbf{i} + 1.2\mathbf{j}$ **AG**	**M1A1**	Evidence of OB = 10 or other valid method (e.g. trigonometry) is required
		2	
9(ii)	**OD =** 1.6i + 1.2**j** + 7k	**B1**	Allow reversal of one or both of **OD**, **BD.**
	BD = –8**i** – 6**j** + 1.6i + 1.2**j** + 7k OE = –6.4**i** –4.8**j** + 7**k**	**M1A1**	For M mark allow sign errors. Also if 2 out of 3 components correct
	Correct method for ±**OD.**±**BD** (using *their* answers)	**M1**	Expect 1.6 × –6.4 + 1.2 × –4.8 + 49 = 33 or $\frac{825}{25}$ 825 / 25.
	Correct method for |**OD**| or |**BD**| (using *their* answers)	**M1**	Expect $\sqrt{1.6^2+1.2^2+7^2}$ or $\sqrt{6.4^2+4.8^2+7^2}$ = √53 or √113
	Cos BDO = *their* $\frac{\mathbf{OD}.\mathbf{BD}}{\lvert\mathbf{OD}\rvert\times\lvert\mathbf{BD}\rvert}$	**DM1**	Expect $\frac{33}{77.4}$. Dep. on all previous M marks and either B1 or A1
	64.8º Allow 1.13(rad)	**A1**	Can’t score A1 if 1 vector only is reversed unless explained well
		7	

Question	Answer	Marks	Guidance
10(i)	Smallest value of c is 2. Accept 2, $c = 2$, $c \geqslant 2$. Not in terms of x	B1	Ignore superfluous working, e.g. $\frac{d^2y}{dx^2} = 2$
		1	
10(ii)	$y = (x-2)^2 + 2 \to x - 2 = (\pm)\sqrt{y-2} \to x = (\pm)\sqrt{y-2} + 2$	M1	Order of operations correct. Allow sign errors
	$f^{-1}(x) = \sqrt{x-2} + 2$	A1	Accept $y = \sqrt{x-2} + 2$
	Domain of f^{-1} is $x \geqslant 6$. Allow $\geqslant 6$.	B1	Not $f^{-1}(x) \geqslant 6$. Not $f(x) \geqslant 6$. Not $y \geqslant 6$
		3	
10(iii)	$\left[(x-2)^2 + 2 - 2\right]^2 + 2 = 51$ SOI Allow 1 term missing for M mark Or $\left(x^2 - 4x + 6\right)^2 - 4\left(x^2 - 4x + 6\right) + 6 = 51$	M1A1	ALT. $f(x) = f^{-1}(51)$ (M1)$= \sqrt{51-2} + 2$ (A1)
	$(x-2)^4 = 49$ or $(x^2 - 4x + 4)^2 = 49$ OR $x^4 - 8x^3 + 24x^2 - 32x - 33 = 0$ often implied by next line	A1	$(x-2)^2 + 2 = \sqrt{49} + 2$ OR $f(x) = 9$
	$(x-2)^2 = (\pm)7$ OR $x^2 - 4x - 3 = 0$. Ignore $x^2 - 4x + 11 = 0$	A1	$(x-2)^2 = 7$ OR $x = f^{-1}(9)$
	$x = 2 + \sqrt{7}$ only CAO $x = 2 + \sqrt[4]{49}$ scores 3/5	A1	$x = 2 + \sqrt{7}$
		5	

Question	Answer	Marks	Guidance
11(i)	$\frac{dy}{dx} = 2(x+1) - (x+1)^{-2}$	**B1**	
	Set = 0 and obtain $2(x+1)^3 = 1$ convincingly www **AG**	**B1**	
	$\frac{d^2y}{dx^2} = 2 + 2(x+1)^{-3}$ www	**B1**	
	Sub, e.g., $(x+1)^{-3} = 2$ OE or $x = \left(\frac{1}{2}\right)^{\frac{1}{3}} - 1$	**M1**	Requires exact method – otherwise scores M0
	$\frac{d^2y}{dx^2} = 6$ CAO www	**A1**	and exact answer – otherwise scores A0
		5	

Question	Answer	Marks	Guidance
11(ii)	$y^2=(x+1)^4+(x+1)^{-2}+2(x+1)$ SOI	B1	OR $y^2=(x^4+4x^3+6x^2+4x+1)+(2x+2)+(x+1)^{-2}$
	$(\pi)\int y^2dx=(\pi)\left[\frac{(x+1)^5}{5}\right]+\left[\frac{(x+1)^{-1}}{-1}\right]+\left[\frac{2(x+1)^2}{2}\right]$ OR $(\pi)\left[\frac{x^5}{5}+x^4+2x^3+2x^2+x\right]+\left[x^2+2x\right]+\left[-\frac{1}{x+1}\right]$	B1B1B1	Attempt to integrate y^2. Last term might appear as (x^2+2x)
	$(\pi)\left[\frac{32}{5}-\frac{1}{2}+4-\left(\frac{1}{5}-1+1\right)\right]$	M1	Substitute limits $0\rightarrow1$ into an attempted integration of y^2. Do not condone omission of value when $x=0$
	9.7π or 30.5	A1	Note: omission of $2(x+1)$ in first line $\rightarrow 6.7\pi$ scores 3/6 Ignore initially an extra volume, e.g. $(\pi)\int(4½)^2$. Only take into account for the final answer
		6	

Cambridge International AS & A Level

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MATHEMATICS **9709/12**

Paper 1 Pure Mathematics **March 2018**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the March 2018 series for most Cambridge IGCSE®, Cambridge International A and AS Level components and some Cambridge O Level components.

Cambridge Assessment International Education

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

  - Note: B2 or A2 means that the candidate can earn 2 or 0.
    B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
CWO	Correct Working Only – often written by a 'fortuitous' answer
ISW	Ignore Subsequent Working
SOI	Seen or implied
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

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PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Question	Answer	Marks	Guidance
1	$(y) = \dfrac{x^{½}}{½} - 3x \ (+c)$	**B1B1**	
	Sub (4, –6) $-6 = 4 - 12 + c \rightarrow \quad \boldsymbol{c} = 2$	**M1A1**	Expect $(y) = 2x^{½} - 3x + 2$
		4	

Question	Answer	Marks	Guidance
2(i)	${}^7C_2(+/-2x)^2$ or ${}^7C_3(-2x)^3$	**M1**	SOI, Allow for either term correct. Allow + or – inside first bracket.
	$84(x^2)$, $-280(x^3)$	**A1A1**	
		3	
2(ii)	$2\times(\textit{their}-280)+5\times(\textit{their}\,84)$ only	**M1**	
	–140	**A1**	
		2	

Question	Answer	Marks	Guidance
3(i)	$40+60\times1.2 = 112$	**M1A1**	Allow 1.12 m. Allow **M1** for 40 + 59 × 1.2 OE
		2	

Question	Answer	Marks	Guidance
3(ii)	Find rate of growth e.g. 41.2/40 or 1.2/40	*M1	SOI, Also implied by 3% , 0.03 or 1.03 seen
	$40 \times (1 + their\, 0.03)^{60\, or\, 59}$	DM1	
	236	A1	Allow 2.36 m
		3	

Question	Answer	Marks	Guidance
4(i)	$\frac{1}{\sqrt{3}} = \frac{2}{x}$ or $y - 2 = \frac{-1}{\sqrt{3}}x$	M1	OE, Allow $y - 2 = \frac{+1}{\sqrt{3}}x$. Attempt to express $\tan\frac{\pi}{6}\, or\, \tan\frac{\pi}{3}$ exactly is required or the use of $1/\sqrt{3}\, or \sqrt{3}$
	$(x =) 2\sqrt{3}$	A1	OE
		2	
4(ii)	Mid-point (a, b) = (½ *their* **(i)**, 1)	B1FT	Expect (√3, 1)
	Gradient of AB leading to gradient of bisector, m	M1	Expect $-1/\sqrt{3}$ leading to $m = \sqrt{3}$
	Equation is $y - their\, b = m(x - their\, a)$ OE	DM1	Expect $y - 1 = \sqrt{3}(x - \sqrt{3})$
	$y = \sqrt{3}x - 2$ OE	A1	
		4	

Question	Answer	Marks	Guidance
5(a)	$2\tan x+5=2\tan^2 x+5\tan x+3 \rightarrow 2\tan^2 x+3\tan x-2(=0)$	M1A1	Multiply by denom., collect like terms to produce 3-term quad. in tan$x$
	0.464 (accept 0.148π), 2.03 (accept 0.648π)	A1A1	**SCA1** for both in degrees 26.6º, 116.6º only
		4	
5(b)	$\alpha=30°$ $k=4$	B1B1	Accept $\alpha=\pi/6$
		2	

Question	Answer	Marks	Guidance
6(i)	$\frac{PQ}{2}=10\times\sin 1.1$	M1	Correct use of sin/cos rule
	(PQ =) 17.8 (17.82…implies **M1**, **A1**) AG	A1	OR $PQ=\frac{10\sin 2.2}{\sin\left(\frac{\pi}{2}-1.1\right)}$ *or* $\frac{10\sin 2.2}{\sin 0.4708}$ *or* $\sqrt{200-200\cos 2.2}=17.8$
		2	
6(ii)	Angle OPQ = (π/2 – 1.1) [accept 27°]	B1	OE Expect 0.4708 or 0.471. Can be scored in part **(i)**
	Arc QR =17.8 × *their* (π/2 – 1.1)	M1	Expect 8.39. (or 8.38).
	Perimeter $=17.8-10+10+\textit{their}\,\text{arc}\,QR$	M1	
	26.2	A1	For both parts allow correct methods in degrees
		4	

Question	Answer	Marks	Guidance
7(i)	$\overrightarrow{CE} = -4\mathbf{i} - \mathbf{j} + 8\mathbf{k}$	**B1**	
	$\lvert\overrightarrow{CE}\rvert = \sqrt{((their -4)^2 + (their -1)^2 + (their\, 8)^2} = 9$	**M1A1**	Could use Pythagoras' theorem on triangle *CDE*
		3	
7(ii)	$\overrightarrow{CA} = 3\mathbf{i} - 3\mathbf{j}$ or $\overrightarrow{AC} = -3\mathbf{i} + 3\mathbf{j}$	**B1**	
	$\overrightarrow{CE} . \overrightarrow{CA} = (-4\mathbf{i} - \mathbf{j} + 8\mathbf{k}).(3\mathbf{i} - 3\mathbf{j}) = -12 + 3$ (Both vectors reversed ok)	**M1**	Scalar product of *their* $\overrightarrow{CE}$, $\overrightarrow{CA}$. One vector reversed ok for all **M** marks
	$\lvert\overrightarrow{CE}\rvert \times \lvert\overrightarrow{CA}\rvert = \sqrt{16+1+64} \times \sqrt{9+9}$	**M1**	Product of moduli of *their* $\overrightarrow{CE}$, $\overrightarrow{CA}$
	$\cos^{-1}\left(\frac{-12+3}{9\sqrt{18}}\right) = \cos^{-1}\left(\frac{-1}{\sqrt{18}}\right)$ $\left[\text{or e.g. } \cos^{-1}\left(\frac{-3}{\sqrt{162}}\right), \cos^{-1}\left(\frac{-9}{\sqrt{1458}}\right)\right]$ etc.	**A1A1**	**A1** for any correct expression, **A1** for required form Equivalent answers must be in required form $m/\sqrt{n}$ (m, n integers)
		5	

Question	Answer	Marks	Guidance
8(i)	$\mathrm{d}y / \mathrm{d}x = x - 6x^{\frac{1}{2}} + 8$	**B2,1,0**	
	Set to zero and attempt to solve a quadratic for $x^{\frac{1}{2}}$	**M1**	Could use a substitution for $x^{\frac{1}{2}}$ or rearrange and square correctly*
	$x^{\frac{1}{2}} = 4$ or $x^{\frac{1}{2}} = 2$ [$x = 2$ *and* $x = 4$ gets **M1 A0**]	**A1**	Implies **M1**. 'Correct' roots for *their* $\mathrm{d}y / \mathrm{d}x$ also implies **M1**
	$x = 16$ or 4	**A1FT**	Squares of their solutions *Then **A1,A1** for each answer
		5	

Question	Answer	Marks	Guidance
8(ii)	$\mathrm{d}^2y/\mathrm{d}x^2 = 1 - 3x^{-½}$	B1FT	FT on *their* dy/dx, providing a fractional power of x is present
		1	
8(iii)	(When $x = 16$) $\mathrm{d}^2y/\mathrm{d}x^2 = 1/4 > 0$ hence MIN	M1	Checking both of their values in their $\mathrm{d}^2y/\mathrm{d}x^2$
	(When $x = 4$) $\mathrm{d}^2y/\mathrm{d}x^2 = -1/2 < 0$ hence MAX	A1	All correct Alternative methods ok but must be explicit about values of x being considered
		2	

Question	Answer	Marks	Guidance
9(i)	$1 + cx = cx^2 - 3x \rightarrow cx^2 - x(c+3) - 1\ (=0)$	M1	Multiply throughout by x and rearrange terms on one side of equality
	Use $b^2 - 4ac\left[= (c+3)^2 + 4c = c^2 + 10c + 9 \text{ or } (c+5)^2 - 16\right]$	M1	Select their correct coefficients which must contain 'c' twice Ignore = 0, < 0, >0 etc. at this stage
	(Critical values) –1, –9	A1	SOI
	$c \leqslant -9, \quad c \geqslant -1$	A1	
		4	

Question	Answer	Marks	Guidance
9(ii)	Sub their c to obtain a quadratic $\left[c=-1\to -x^2-2x-1(=0)\right]$	M1	
	$x=-1$	A1	
	Sub their c to obtain a quadratic $\left[c=(-9 \to -9x^2+6x-1(=0)\right]$	M1	
	$x=1/3$	A1	[Alt 1: $dy/dx=-1/x^2=c$, when $c=-1, x=\pm 1, c=-9, x=\pm\frac{1}{3}$ Give M1 for equating the gradients, A1 for all four answers and M1A1 for checking and eliminating] [Alt 2: $dy/dx=-1/x^2=c$ leading to $1/x-1/x^2=(-1/x^2)(x)-3$ Give M1 A1 at this stage and M1A1 for solving]
		4	

Question	Answer	Marks	Guidance
10(i)(a)	$f(x)>2$	B1	Accept $y>2$, $(2,\infty)$, $(2,\infty]$, $range>2$
		1	
10(i)(b)	$g(x)>6$	B1	Accept $y>6$, $(6,\infty)$, $(6,\infty]$, $range>6$
		1	
10(i)(c)	$2<fg(x)<4$	B1	Accept $2<y<4$, $(2,4)$, $2<range<4$
		1	

Question	Answer	Marks	Guidance
10(ii)	The range of f is (partly) outside the domain of g	**B1**	
		1	
10(iii)	$f'(x) = \dfrac{-8}{(x-2)^2}$	**B1**	SOI
	$y = \dfrac{8}{x-2} + 2 \to y - 2 = \dfrac{8}{x-2} \to x - 2 = \dfrac{8}{y-2}$	**M1**	Order of operations correct. Accept sign errors
	$f^{-1}(x) = \dfrac{8}{x-2} + 2$	**A1**	SOI
	$\dfrac{-48}{(x-2)^2} + \dfrac{16}{x-2} + 4 - 5 \ (< 0) \ \to \ x^2 - 20x + 84 \ (< 0)$	**M1**	Formation of 3-term quadratic in $x, (x-2)$ or $1/(x-2)$
	$(x-6)(x-14)$ or 6, 14	**A1**	SOI
	$2 < x < 6$, $x > 14$	**A1**	CAO
		6	

Question	Answer	Marks	Guidance
11(i)	$dy/dx = [-2] - \left[3(1-2x)^2\right] \times [-2]$ $(= 4 - 24x + 24x^2)$	**B2,1,0**	Award for the accuracy within each set of square brackets
	At $x = ½$ $dy/dx = -2$	**B1**	
	Gradient of line $y = 1 - 2x$ is –2 (hence AB is a tangent) **AG**	**B1**	
		4	

Question	Answer	Marks	Guidance
11(ii)	Shaded region = $\int_0^{1/2}(1-2x) - \int_0^{1/2}[1-2x-(1-2x)^3]$ oe	M1	Note: If area triangle OAB – area under the curve is used the first part of the integral for the area under the curve must be evaluated
	$= \int_0^{1/2}(1-2x)^3\, dx$ **AG**	A1	
		2	
11(iii)	Area = $\left[\frac{(1-2x)^4}{4}\right][\div -2]$	*B1B1	
	0 – (–1/8) = 1/8	DB1	OR $\int 1-6x+12x^2-8x^3 = x-3x^2+4x^3-2x^4$ (**B2,1,0**) Applying limits 0 → ½
		3	

Cambridge International AS & A Level

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Cambridge International Advanced Subsidiary and Advanced Level

MATHEMATICS **9709/11**

Paper 1 **October/November 2017**

MARK SCHEME

Maximum Mark: 75

Published

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 Cambridge Assessment International Education

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B Mark for a correct result or statement independent of method marks.

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- The symbol FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously “correct” answers or results obtained from incorrect working.

  - Note: B2 or A2 means that the candidate can earn 2 or 0.
    B2/1/0 means that the candidate can earn anything from 0 to 2.

The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

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The following abbreviations may be used in a mark scheme or used on the scripts:

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
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ISW	Ignore Subsequent Working
SOI	Seen or implied
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become “follow through” marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Question	Answer	Marks	Guidance
1	$\dfrac{dy}{dx} = 3x^{1/2} - 3 - 2x^{-1/2}$	**B2,1,0**	
	at $x = 4$, $\dfrac{dy}{dx} = 6 - 3 - 1 = 2$	**M1**	
	Equation of tangent is $y = 2(x-4)$ OE	**A1FT**	Equation through (4, 0) with *their* gradient
		4	

Question	Answer	Marks	Guidance
2	$f'(x) = 3x^2 - 2x - 8$	**M1**	Attempt differentiation
	$-\dfrac{4}{3}$, 2 SOI	**A1**	
	$f'(x) > 0 \Rightarrow x < -\dfrac{4}{3}$ SOI	**M1**	Accept $x > 2$ in addition. FT *their* solutions
	Largest value of a is $-\dfrac{4}{3}$	**A1**	Statement in terms of a. Accept $a \leqslant -\dfrac{4}{3}$ or $a < -\dfrac{4}{3}$. Penalise extra solutions
		4	

Question	Answer	Marks	Guidance
3(i)	$\frac{3a}{1-r}=\frac{a}{1+2r}$	M1	Attempt to equate 2 sums to infinity. At least one correct
	$3+6r=1-r$	DM1	Elimination of 1 variable (a) at any stage and multiplication
	$r=-\frac{2}{7}$	A1	
		3	
3(ii)	$½n\left[2\times15+(n-1)4\right]=½n\left[2\times420+(n-1)(-5)\right]$	M1A1	Attempt to equate 2 sum to n terms, at least one correct (**M1**). Both correct (**A1**)
	$n=91$	A1	
		3	

Question	Answer	Marks	Guidance
4(i)	$V = \frac{1}{3}\pi r^2(18-r) = 6\pi r^2 - \frac{1}{3}\pi r^3$	**B1**	AG
		1	
4(ii)	$\frac{dV}{dr} = 12\pi r - \pi r^2 = 0$	**M1**	Differentiate and set = 0
	$\pi r(12-r) = 0 \to r = 12$	**A1**	
	$\frac{d^2V}{dr^2} = 12\pi - 2\pi r$	**M1**	
	Sub $r = 12 \to 12\pi - 24\pi = -12\pi \to$ MAX	**A1**	AG
		4	
4(iii)	Sub $r = 12,\ h = 6 \to \text{Max}\,V = 288\pi$ or 905	**B1**	
		1	

Question	Answer	Marks	Guidance
5(i)	$\cos A = 8/10 \rightarrow A = 0.6435$	B1	AG Allow other valid methods e.g. $\sin A = 6/10$
		1	
5(ii)	*EITHER:* Area $\Delta ABC = ½ \times 16 \times 6$ or $½ \times 10 \times 16 \sin 0.6435 = 48$	(M1A1	
	Area 1 sector $½ \times 10^2 \times 0.6435$	M1	
	Shaded area $= 2 \times$ *their* sector $-$ *their* ΔABC	M1)	
	OR: $\Delta BDE = 12$, $\Delta BDC = 30$	(B1 B1	
	Sector = 32.18	M1	
	2×segment + ΔBDE	M1)	
	=16.4	A1	
		5	

Question	Answer	Marks	Guidance
6(i)	Mid-point of $AB = (3, 5)$	**B1**	Answers may be derived from simultaneous equations
	Gradient of $AB = 2$	**B1**	
	Eqn of perp. bisector is $y - 5 = -½(x - 3) \rightarrow 2y = 13 - x$	**M1A1**	AG For **M1** FT from mid-point and gradient of AB
		4	
6(ii)	$-3x + 39 = 5x^2 - 18x + 19 \rightarrow (5)(x^2 - 3x - 4)(= 0)$	**M1**	Equate equations and form 3-term quadratic
	$x = 4$ or -1	**A1**	
	$y = 4½$ or 7	**A1**	
	$CD^2 = 5^2 + 2½^2 \rightarrow CD = \sqrt{\frac{125}{4}}$	**M1A1**	Or equivalent integer fractions ISW
		5	

Question	Answer	Marks	Guidance
7(a)	$a=-2, \quad b=3$	**B1B1**	
		2	
7(b)(i)	$s+s^2-sc+2c+2sc-2c^2=s+sc \rightarrow s^2-2c^2+2c=0$	**B1**	Expansion of brackets must be correct
	$1-\cos^2\theta-2\cos^2\theta+2\cos\theta=0$	**M1**	Uses $s^2=1-c^2$
	$3\cos^2\theta-2\cos\theta-1=0$	**A1**	AG
		3	
7(b)(ii)	$\cos\theta=1$ or $-\frac{1}{3}$	**B1**	
	$\theta=0°$ or $109.5°$ or $-109.5°$	**B1B1B1 FT**	FT for – *their* $109.5°$
		4	

Question	Answer	Marks	Guidance
8(a)	*EITHER:* $\overrightarrow{PR} = 2\overrightarrow{PQ} = 2(\mathbf{q} - \mathbf{p})$	(B1	
	$\overrightarrow{OR} = \mathbf{p} + 2\mathbf{q} - 2\mathbf{p} = 2\mathbf{q} - \mathbf{p}$	M1A1)	
	OR: $\overrightarrow{QR} = \overrightarrow{PQ} = \mathbf{q} - \mathbf{p}$	(B1	
	$\overrightarrow{OR} = \overrightarrow{OQ} + \overrightarrow{QR} = \mathbf{q} + \mathbf{q} - \mathbf{p} = 2\mathbf{q} - \mathbf{p}$	M1A1)	Or other valid method
		3	
8(b)	$6^2 + a^2 + b^2 = 21^2$ SOI	B1	
	$18 + 2a + 2b = 0$	B1	
	$a^2 + (-a - 9)^2 = 405$	M1	Correct method for elimination of a variable. (Or same equation in b)
	$(2)(a^2 + 9a - 162)(= 0)$	A1	Or same equation in b
	$a = 9$ or -18	A1	
	$b = -18$ or 9	A1	
		6	

Question	Answer	Marks	Guidance
9(i)	$gg(x) = g(2x - 3) = 2(2x - 3) - 3 = 4x - 9$	**M1A1**	
		2	
9(ii)	$y = \dfrac{1}{x^2 - 9} \rightarrow x^2 = \dfrac{1}{y} + 9$ OE	**M1**	Invert; add 9 to both sides or with *x/y* interchanged
	$f^{-1}(x) = \sqrt{\dfrac{1}{x} + 9}$	**A1**	
	Attempt soln of $\sqrt{\dfrac{1}{x} + 9} > 3$ or attempt to find range of f. $(y > 0)$	**M1**	
	Domain is $x > 0$ CAO	**A1**	May simply be stated for **B2**
		4	

Question	Answer	Marks	Guidance
9(iii)	*EITHER:* $\frac{1}{(2x-3)^2-9}=\frac{1}{7}$	**(M1**	
	$(2x-3)^2=16$ or $4x^2-12x-7=0$	**A1**	
	x = 7/2 or –1/2	**A1**	
	x = 7/2 only	**A1)**	
	OR: g(x) = $f^{-1}\left(\frac{1}{7}\right)$	**(M1**	
	g(x) = 4	**A1**	
	2x – 3 = 4	**A1**	
	x = 7/2	**A1)**	
		4	

Question	Answer	Marks	Guidance
10(i)	Area $=\int ½\left(x^4-1\right)\mathrm{d}x = ½\left[\frac{x^5}{5}-x\right]$	*B1	
	$½\left[\frac{1}{5}-1\right]-0 \quad = \quad (-)\frac{2}{5}$	DM1A1	Apply limits 0→1
		3	
10(ii)	Vol $=\pi\int y^2\mathrm{d}x \quad = \quad ¼(\pi)\int\left(x^8-2x^4+1\right)\mathrm{d}x$	M1	(If middle term missed out can only gain the M marks)
	$¼(\pi)\left[\frac{x^9}{9}-\frac{2x^5}{5}+x\right]$	*A1	
	$¼(\pi)[\left(\frac{1}{9}-\frac{2}{5}+1\right]-0$	DM1	
	$\frac{8\pi}{45}$ or 0.559	A1	
		4	

Question	Answer	Marks	Guidance
10(iii)	Vol $= \pi \int x^2 \mathrm{d}y \;=\; (\pi)\int (2y+1)^{1/2}\,\mathrm{d}y$	**M1**	Condone use of x if integral is correct
	$(\pi)\left[\dfrac{(2y+1)^{3/2}}{3/2}\right][\div 2]$	***A1A1**	Expect $(\pi)\left[\dfrac{(2y+1)^{3/2}}{3}\right]$
	$(\pi)\left[\dfrac{1}{3}-0\right]$	**DM1**	
	$\dfrac{\pi}{3}$ or 1.05	**A1**	Apply $-\dfrac{1}{2} \to 0$
		5	

Cambridge Assessment International Education
Cambridge International Advanced Subsidiary and Advanced Level

MATHEMATICS **9709/12**

Paper 1 **October/November 2017**

MARK SCHEME

Maximum Mark: 75

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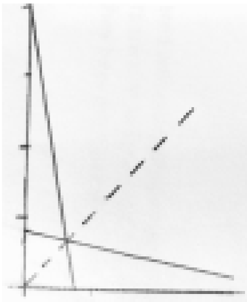
Question	Answer	Marks	Guidance
1	*EITHER:* Term is $^9C_3 \times 2^6 \times (-\frac{1}{4})^3$	**(B1, B1, B1)**	OE
	OR1: $\left(\frac{8x^3-1}{4x^2}\right)^9 = \left(\frac{1}{4x^2}\right)^9 \left(8x^3-1\right)^9 \; or -\left(\frac{1}{4x^2}\right)^9 \left(1-8x^3\right)^9$		
	Term is $-\frac{1}{4^9} \times {}^9C_3 \times 8^6$	**(B1, B1, B1)**	OE
	OR2: $(2x)^9 \left(1-\frac{1}{8x^3}\right)^9$		
	Term is $2^9 \times {}^9C_3 \times \left(-\frac{1}{8}\right)^3$	**(B1, B1, B1)**	OE
	Selected term, which must be independent of x $= -84$	**B1**	
		4	

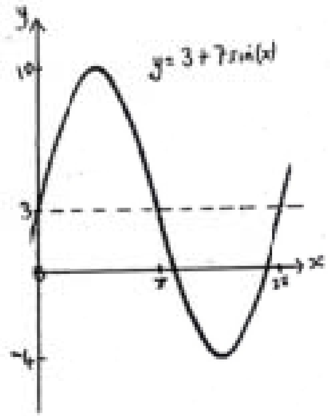
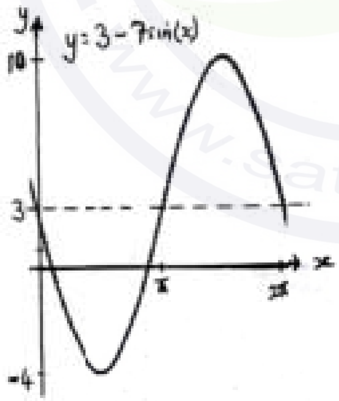
Question	Answer	Marks	Guidance
2(i)	$\frac{4-x}{5}$	**B1**	OE
	Equate a valid attempt at f^{-1} with f, or with x, or f with x $\rightarrow \left(\frac{2}{3}, \frac{2}{3}\right)$ **or** (0.667, 0.667)	**M1, A1**	Equating and an attempt to solve as far $x =$. Both coordinates.
		3	
2(ii)		**B1**	Line $y = 4 - 5x$ – must be straight, through approximately (0,4) and intersecting the positive x axis near (1,0) as shown.
		B1	Line $y = \frac{4-x}{5}$ – must be straight and through approximately (0, 0.8). No need to see intersection with x axis.
		B1	A line through (0,0) and the point of intersection of a pair of straight lines with negative gradients. This line must be at 45° unless scales are different in which case the line must be labelled $y=x$.
		3	

Question	Answer	Marks	Guidance
3(a)	Uses $r = (1.05 \text{ or } 105\%)^{9,\ 10 \text{ or } 11}$	**B1**	Used to multiply repeatedly or in any GP formula.
	New value $= 10000 \times 1.05^{10}$ = ($)16 300	**B1**	
		2	

Question	Answer	Marks	Guidance
3(b)	*EITHER:* $n = 1 \to 5 \qquad a = 5$	**(B1**	Uses $n = 1$ to find a
	$n = 2 \to 13$	**B1**	Correct S_n for any other value of n (e.g. $n = 2$)
	$a + (a + d) = 13 \quad \to d = 3$	**M1 A1)**	Correct method leading to $d =$
	OR: $\left(\frac{n}{2}\right)\left(2a+(n-1)d\right)=\left(\frac{n}{2}\right)(3n+7)$		$\left(\frac{n}{2}\right)$ maybe be ignored
	$\therefore dn + 2a - d = 3n + 7 \to dn = 3n \to d = 3$	**(*M1A1**	Method mark awarded for equating terms in n from correct S_n formula.
	$2a - (their\,3) = 7, \quad a = 5$	**DM1 A1)**	
		4	

Question	Answer	Marks	Guidance
4(i)	Pythagoras → $r = \sqrt{72}$ OE or $\cos 45 = \frac{6}{r} \rightarrow r = \frac{6}{\cos 45} = 6\sqrt{2}$	**M1**	Correct method leading to $r =$
	Arc DC = $\sqrt{72} \times \frac{1}{4}\pi = \frac{3\sqrt{2}}{2}\pi$, 2.12π, 6.66	**M1 A1**	Use of $s=r\theta$ with their r (NOT 6) and $\frac{1}{4}\pi$
		3	
4(ii)	Area of sector-BDC is ½ × 72 × ¼π (= 9π or 28.274…)	***M1**	Use of $\frac{1}{2}r^2\theta$ with their r (NOT 6) and $\frac{1}{4}\pi$
	Area Q = 9π – 18 (10.274…)	**DM1**	Subtracts their ½ × 6 × 6 from their $\frac{1}{2}r^2\theta$
	Area P is ($\frac{1}{4}\pi 6^2$ – area Q) = 18	**M1**	Uses {¼ $\pi 6^2$ – (their area Q using $\sqrt{72}$)}
	Ratio is $\frac{18}{9\pi - 18}$ $\left(\frac{18}{10.274}\right) \rightarrow 1.75$	**A1**	
		4	

Question	Answer	Marks	Guidance
5(i)	*EITHER:* Uses $\tan^2 2x = \frac{\sin^2 2x}{\cos^2 2x}$	(M1	Replaces $\tan^2 2x$ by $\frac{\sin^2 2x}{\cos^2 2x}$ not $\frac{\sin^2}{\cos^2}2x$
	Uses $\sin^2 2x = (1 - \cos^2 2x)$	M1	Replaces $\sin^2 2x$ by $(1 - \cos^2 2x)$
	$\rightarrow 2\cos^2 2x + 3\cos 2x + 1 = 0$	A1)	AG. All correct
	OR: $\tan^2 2x = \sec^2 2x - 1$	(M1	Replaces $\tan^2 2x$ by $\sec^2 2x - 1$
	$\sec^2 2x = \frac{1}{\cos^2 2x}$ Multiply through by $\cos^2 2x$ and rearrange	M1	Replaces $\sec^2 2x$ by $\frac{1}{\cos^2 2x}$
	$\rightarrow 2\cos^2 2x + 3\cos 2x + 1 = 0$	A1)	AG. All correct
		3	
5(ii)	$\cos 2x = -½, -1$	M1	Uses **(i)** to get values for $\cos 2x$. Allow incorrect sign(s).
	$2x = 120°, 240°$ or $2x = 180°1$ $x = 60°$ or $120°$	A1 A1 FT	**A1** for 60° or 120° FT for 180−1st answer
	or $x = 90°$	A1	Any extra answer(s) in given range only penalise fourth mark so max 3/4.
		4	

Question	Answer	Marks	Guidance
6(a)(i)	$4 = a + \frac{1}{2}b$ $3 = a + b$	**M1**	Forming simultaneous equations and eliminating one of the variables – probably a. May still include $\sin\frac{\pi}{2}$ and / or $\sin\frac{\pi}{6}$
	$\rightarrow a = 5,\ b = -2$	**A1 A1**	
		3	
6(a)(ii)	$ff(x) = a + b\sin(a + b\sin x)$	**M1**	Valid method for ff. Could be f(0) = N followed by f(N) = M.
	$ff(0) = 5 – 2\sin 5\ = 6.92$	**A1**	
6(b)	*EITHER:* $10 = c + d$ *and* $-4 = c – d$ $10 = c – d$ *and* $-4 = c + d$	**(M1**	Either pair of equations stated.
	$c = 3$, $d = 7$, -7 or ±7	**A1 A1)**	Either pair solved ISW **Alternately** c=3 **B1**, range = 14 **M1**→ d = 7, –7 or ±7 **A1**
	OR: 	**(M1 A1 A1)**	Either of these diagrams can be awarded M1.Correct values of c and/or d can be awarded the A1, A1
		3	

Question	Answer	Marks	Guidance
7(i)	$\frac{dy}{dx} = 2x - 4 = 0$		Can use completing the square.
	$\rightarrow x = 2,\ y = 3$	**B1 B1**	
	Midpoint of *AB* is (3, 5)	**B1 FT**	**FT** on (*their* 2, *their* 3) with (4,7)
	$\rightarrow m = \frac{7}{3}$ (or 2.33)	**B1**	
		4	
7(ii)	Simultaneous equations $\rightarrow$ $x^2 - 4x - mx + 9 (= 0)$	***M1**	Equates and sets to 0 must contain m
	Use of $b^2 - 4ac \rightarrow (m + 4)^2 - 36$	**DM1**	Any use of $b^2 - 4ac$ on equation set to 0 must contain m
	Solves = 0 → −10 or 2	**A1**	Correct end-points.
	$-10 < m < 2$	**A1**	Don't condone $\leqslant$ at either or both end(s). Accept $-10 < m$, $m < 2$.
		4	

Question	Answer	Marks	Guidance
8(i)	$\frac{dy}{dx} = 0$	**M1**	Sets $\frac{dy}{dx}$ to 0 and attempts to solve leading to two values for x.
	$x = 1, x = 4$	**A1**	Both values needed
		2	

Question	Answer	Marks	Guidance
8(ii)	$\frac{d^2y}{dx^2} = -2x + 5$	**B1**	
	Using both of their x values in their $\frac{d^2y}{dx^2}$	**M1**	Evidence of any valid method for both points.
	$x = 1 \rightarrow (3) \rightarrow$ Minimum, $x = 4 \rightarrow (-3) \rightarrow$ Maximum	**A1**	
		3	
8(iii)	$y = -\frac{x^3}{3} + \frac{5x^2}{2} - 4x \quad (+c)$	**B2, 1, 0**	$+c$ not needed. –1 each error or omission.
	Uses $x = 6$, $y = 2$ in an integrand to find c $\rightarrow c = 8$	**M1 A1**	Statement of the final equation not required.
		4	

Question	Answer	Marks	Guidance
9(i)	$\overrightarrow{AB} = \begin{pmatrix}4\\3\\2\end{pmatrix}$ or $\overrightarrow{BA} = \begin{pmatrix}-4\\-3\\-2\end{pmatrix}$	M1	Use of **b** – **a** or **a** – **b**
	e.g. $\overrightarrow{AO}\,.\,\overrightarrow{AB} = -8+6+2=0 \rightarrow O\hat{A}B = 90°$ AG **OR** $\lvert\overrightarrow{OA}\rvert = 3, \lvert\overrightarrow{OB}\rvert = \sqrt{38}, \lvert\overrightarrow{AB}\rvert = \sqrt{29}$ $OA^2 + AB^2 = OB^2 \rightarrow O\hat{A}B = 90°$ AG	M1 A1	Use of dot product with either $\overrightarrow{AO}$ *or* $\overrightarrow{OA}$ & *either* $\overrightarrow{AB}$ *or* $\overrightarrow{BA}$. Must see 3 component products OR Correct use of Pythagoras. In both methods must state angle or $\Theta = 90^{\circ}$ or similar for **A1**
		3	
9(ii)	$\overrightarrow{CB} = \begin{pmatrix}6\\-6\\-3\end{pmatrix}$ or $\overrightarrow{BC} = \begin{pmatrix}-6\\6\\3\end{pmatrix}$	B1	Must correctly identify the vector.
	$\overrightarrow{OC} = \overrightarrow{OB} + \overrightarrow{BC}$ (or $-\overrightarrow{CB}$) $= \begin{pmatrix}0\\7\\4\end{pmatrix}$	M1 A1	Correct link leading to $\overrightarrow{OC}$
		3	

Question	Answer	Marks	Guidance
9(iii)	$\left\|\overrightarrow{OA}\right\| = 3,\ \left\|\overrightarrow{BC}\right\| = 9, \left\|\overrightarrow{AB}\right\| = \sqrt{29}\ \ (5.39\ )$	**B1**	For any one of these
	Area = ½(3 + 9) $\sqrt{29}$ or 3 $\sqrt{29}$ + 3 $\sqrt{29}$	**M1**	Correct formula(e) used for trapezium or (rectangle + triangle) or two triangles using their lengths.
	= 6 $\sqrt{29}$ $(1\sqrt{1044}, 2\sqrt{261}$ ***or*** $3\sqrt{116})$	**A1**	Exact answer in correct form.
		3	

Question	Answer	Marks	Guidance
10(i)	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{2} \times (5x-1)^{-\frac{1}{2}} \times 5 \quad (= \frac{5}{6})$	**B1 B1**	**B1** Without × 5 **B1** × 5 of an attempt at differentiation
	m of normal = $-\frac{6}{5}$	**M1**	Uses $m_1m_2 = -1$ with their numeric value from their dy/dx
	Equation of normal $\boldsymbol{y} - 3 = -\frac{6}{5}(\boldsymbol{x} - 2)$ OE or $5y + 6x = 27$ or $\boldsymbol{y} = \frac{-6}{5}\boldsymbol{x} + \frac{27}{5}$	**A1**	Unsimplified. Can use $y = mx + c$ to get $c = 5.4$ ISW

Question	Answer	Marks	Guidance
10(ii)	*EITHER:*	(B1	Correct expression without ÷5
	For the curve $(\int)\sqrt{5x-1}\mathrm{d}x = \dfrac{(5x-1)^{\frac{3}{2}}}{\frac{3}{2}} \div 5$	B1	For dividing an attempt at integration of y by 5
	Limits from $\frac{1}{5}$ to 2 used → 3.6 or $\frac{18}{5}$ OE	M1 A1	Using $\frac{1}{5}$ and 2 to evaluate an integrand $\left(\text{may be } \int y^2\right)$
	Normal crosses x-axis when $y = 0$, → x= (4½)	M1	Uses their equation of normal, NOT tangent
	Area of triangle = 3.75 or $\frac{15}{4}$ OE	A1	This can be obtained by integration
	Total area=3.6 + 3.75 = 7.35, $\frac{147}{20}$ OE	A1)	
	OR: For the curve: $(\int)\frac{1}{5}\left(y^2+1\right)\mathrm{d}y = \frac{1}{5}\left(\frac{y^3}{3}+y\right)$	(B2, 1, 0	–1 each error or omission.
	Limits from 0 to 3 used → 2.4 or $\frac{12}{5}$ OE	M1 A1	Using 0 and 3 to evaluate an integrand
	Uses their equation of normal, NOT tangent.	M1	Either to find side length for trapezium or attempt at integrating between 0 and 3
	Area of trapezium = $\frac{1}{2}(2+4½)\times 3 = \frac{39}{4}\, or\, 9\frac{3}{4}$	A1	This can be obtained by integration
	Shaded area = $\frac{39}{4}-\frac{12}{5}=7.35, \frac{147}{20}$ OE	A1)	

Question	Answer	Marks	Guidance
		7	

Cambridge Assessment International Education
Cambridge International Advanced Subsidiary and Advanced Level

MATHEMATICS **9709/13**

Paper 1 **October/November 2017**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

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This document consists of **15** printed pages.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

  - Note: B2 or A2 means that the candidate can earn 2 or 0.
    B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
CWO	Correct Working Only – often written by a 'fortuitous' answer
ISW	Ignore Subsequent Working
SOI	Seen or implied
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Question	Answer	Marks	Guidance
1	$\frac{1}{2}n\left[-24+(n-1)6\right]\ \sim 3000$ Note: ~ denotes <u>any</u> inequality or equality	M1	Use correct formula with RHS ≈ 3000 (e.g. 3010).
	$(3)\left(n^2-5n-1000\right)\ (\sim 0)$	A1	Rearrange into a 3-term quadratic.
	$n \sim 34.2\ (\& -29.2)$	A1	
	35. Allow $n \geqslant 35$	A1	
		4	
2	$ax+3a=-\frac{2}{x} \to ax^2+3ax+2\ (=0)$	*M1	Rearrange into a 3-term quadratic.
	Apply $b^2-4ac>0$ SOI	DM1	Allow ⩾. If no inequalities seen, **M1** is implied by 2 correct final answers in *a* or *x*.
	$a<0,\ a>\frac{8}{9}$ (or 0.889) OE	A1 A1	For final answers accept $0 > a > \frac{8}{9}$ but not ⩽, ⩾.
		4	

Question	Answer	Marks	Guidance
3(i)	$6C3\left(\frac{2}{x}\right)^3(-3x)^3$ SOI also allowed if seen in an expansion	M1	Both x's can be missing.
	–4320 Identified as answer	A1	Cannot be earned retrospectively in **(ii)**.
		2	
3(ii)	$6C2\left(\frac{2}{x}\right)^4\left[(-)3x\right]^2$ SOI clearly identified as critical term	M1	Both x's and minus sign can be missing.
	$15a\times16\times9-\textit{their}4320(=0)$	A1 FT	FT on *their* 4320.
	$a=2$	A1	
		3	

Question	Answer	Marks	Guidance
4	$f'(x)=\left[\left(\frac{3}{2}\right)(2x-1)^{1/2}\right]\times[2]-[6]$	B2, 1, 0	Deduct 1 mark for each [...] incorrect.
	$f'(x)<0$ or $\leqslant 0$ or $=0$ SOI	M1	
	$(2x-1)^{1/2}<2$ or $\leqslant 2$ or $=2$ OE	A1	Allow with k used instead of x
	Largest value of k is $\frac{5}{2}$	A1	Allow $k\leqslant\frac{5}{2}$ or $k=\frac{5}{2}$ Answer must be in terms of k (not x)
		5	

Question	Answer	Marks	Guidance
5(i)	$\cos\theta + 4 + 5\sin^2\theta + 5\sin\theta - 5\sin\theta - 5\ (=0)$	M1	Multiply throughout by $\sin\theta + 1$. Accept if $5\sin\theta - 5\sin\theta$ is not seen
	$5\left(1-\cos^2\theta\right) + \cos\theta - 1\ (=0)$	M1	Use $s^2 = 1 - c^2$
	$5\cos^2\theta - \cos\theta - 4 = 0$ AG	A1	Rearrange to AG
		3	
5(ii)	$\cos\theta = 1$ and -0.8	B1	Both required
	$\theta = [0°, 360°]$, $[143.1°]$, $[216.9°]$	B1 B1 B1 FT	Both solutions required for 1st mark. For 3rd mark FT for $(360° - \mathit{their}\,143.1°)$ Extra solution(s) in range (e.g. 180º) among 4 correct solutions scores $\frac{3}{4}$
		4	

Question	Answer	Marks	Guidance
6(i)	$y = \dfrac{2}{x^2 - 1} \Rightarrow x^2 = \dfrac{2}{y} + 1$ OE	M1	
	$x = (\pm)\sqrt{\dfrac{2}{y} + 1}$ OE	A1	With or without x/y interchanged.
	$f^{-1}(x) = -\sqrt{\dfrac{2}{x} + 1}$ OE	A1	Minus sign obligatory. Must be a function of x.
		3	

Question	Answer	Marks	Guidance
6(ii)	$\left(\frac{2}{x^2-1}\right)^2+1=5$	**B1**	
	$\frac{2}{x^2-1}=(\pm)2$ OE OR $x^4-2x^2=0$ OE $x^2-1=(\pm)1 \Rightarrow x^2=2\ (\text{or}\,0)$ $x=-\sqrt{2}$ or -1.41 only	**B1**	Condone $x^2=0$ as an additional solution
		4	

Question	Answer	Marks	Guidance
7(i)	$\sin^{-1}\left(\frac{3}{5}\right)=0.6435$ AG	**M1**	OR $(PBC=)\cos^{-1}\left(\frac{3}{5}\right)=0.9273 \Rightarrow (ABP=)\frac{\pi}{2}-0.9273=0.6435$ Or other valid method. Check working and diagram for evidence of incorrect method
7(ii)	Use (once) of sector area $=½r^2\theta$	**M1**	
	Area sector $BAP=½\times5^2\times0.6435$ = 8.04	**A1**	
	Area sector $DAQ=½\times½\pi\times3^2$ = 7.07 , Allow $\frac{9\pi}{4}$	**A1**	
		3	

Question	Answer	Marks	Guidance
7(iii)	*EITHER:* Region = sect + sect – (rect – Δ) or sect – [rect – (sect + Δ)]	**(M1**	Use of correct strategy
	(Area Δ BPC =) ½ × 3 × 4 = 6 Seen	**A1**	
	8.04 + 7.07 – (15 – 6) = 6.11	**A1)**	
	OR1: Region = sector ADQ – (trap $ABPD$ – sector ABP).	**(M1**	Use of correct strategy
	(Area trap $ABPD$ =) ½ (5 + 1) × 3 = 9 Seen	**A1**	
	7.07 – (9 – 8.04) = 7.07 – 0.96 = 6.11	**A1)**	
	OR2: Area segment AP= 2.5686 Area segment AQ = 0.5438 Region = segment AP + segment AQ + ΔAPQ.	**(M1**	Use of correct strategy
	(Area ΔAPQ =) ½ × 2 × 3 = 3 Seen	**A1**	
	2.57 + 0.54 + 3 = 6.11	**A1)**	
		3	

Question	Answer	Marks	Guidance
8(i)	*EITHER:* $4 - 3\sqrt{x} = 3 - 2x \rightarrow 2x - 3\sqrt{x} + 1\ (=0)$ or e.g. $2k^2 - 3k + 1\ (=0)$	**(M1**	Form 3-term quad & attempt to solve for $\sqrt{x}$.
	$\sqrt{x} = \frac{1}{2}, 1$	**A1**	Or $k = \frac{1}{2}$ or 1 (where $k = \sqrt{x}$).
	$x = \frac{1}{4}, 1$	**A1)**	
	OR1: $(3\sqrt{x})^2 = (1+2x)^2$	**(M1**	
	$4x^2 - 5x + 1\ (=0)$	**A1**	
	$x = \frac{1}{4}, 1$	**A1)**	
	OR2: $\frac{3-y}{2} = \left(\frac{4-y}{3}\right)^2\ \left(\rightarrow 2y^2 - 7y + 5(=0)\right)$	**(M1**	Eliminate x
	$y = \frac{5}{2}, 1$	**A1**	
	$x = \frac{1}{4}, 1$	**A1)**	
		3	

Question	Answer	Marks	Guidance
8(ii)	*EITHER:* Area under line = $\int(3-2x)\mathrm{d}x = 3x - x^2$	**(B1**	
	$=\left[(3-1)-\left(\frac{3}{4}-\frac{1}{16}\right)\right]$	**M1**	Apply *their* limits (e.g. ¼ → 1) after integn.
	Area under curve $=\int\left(4-3x^{1/2}\right)\mathrm{d}x = 4x - 2x^{3/2}$	**B1**	
	$\left[(4-2)-(1-¼)\right]$	**M1**	Apply *their* limits (e.g. ¼ → 1) after integration.
	Required area $= \frac{21}{16} - \frac{5}{4} = \frac{1}{16}$ (or 0.0625)	**A1)**	
	OR: $+/-\int(3-2x)-\left(4-3x^{\frac{1}{2}}\right) = +/-\int(-1-2x+3x^{\frac{1}{2}})$	**(*M1**	Subtract functions and then attempt integration
	$+/-\left[-x-x^2+\frac{3x^{3/2}}{3/2}\right]$	**A2, 1, 0 FT**	FT on *their* subtraction. Deduct 1 mark for each term incorrect
	$+/-\left[-1-1+2-\left(-\frac{1}{4}+\frac{1}{16}+\frac{1}{8}\right)\right]=\frac{1}{16}$ (or 0.0625)	**DM1 A1)**	Apply *their* limits ¼ → 1
		5	

Question	Answer	Marks	Guidance
9(i)	$\overrightarrow{AB} = +/-\begin{pmatrix}-18\\9\\-18\end{pmatrix}, \quad \overrightarrow{BC} = +/-\begin{pmatrix}12\\-6\\12\end{pmatrix},$	**B1 B1**	Allow **i**, **j**, **k** form throughout.
	$\lvert\overrightarrow{AB}\rvert = 27, \quad \lvert\overrightarrow{BC}\rvert = 18$	**B1 FT** **B1 FT**	FT on *their* $\overrightarrow{AB}$, *their* $\overrightarrow{OD}$.
	$\lvert\overrightarrow{CD}\rvert = \left(\frac{18}{27}\right)\times 18$ OR $\left(\frac{18}{27}\right)^2 \times 27 = 12$	**B1**	
		5	
9(ii)	$\overrightarrow{CD} = (\pm)\,their\,\frac{18}{27}\times their\ \overrightarrow{BC}$ SOI	**M1**	Expect $(\pm)\begin{pmatrix}8\\-4\\8\end{pmatrix}$.
	$\overrightarrow{OD} = \begin{pmatrix}2\\-3\\-1\end{pmatrix}(\pm)\ their\ \frac{18}{27}\begin{pmatrix}12\\-6\\12\end{pmatrix} = \begin{pmatrix}10\\-7\\7\end{pmatrix}, \begin{pmatrix}-6\\1\\-9\end{pmatrix}$	**M1 A1 A1**	Other methods possible for $\overrightarrow{OD}$, e.g. $\overrightarrow{OB} + \frac{5}{2}\overrightarrow{CD}$, $\overrightarrow{OB} + \frac{1}{2}\overrightarrow{CD}$ (One soln **M2A1**, 2nd soln **A1**) OR $\overrightarrow{OB} + \frac{5}{3}\overrightarrow{BC}$, $\overrightarrow{OB} + \frac{1}{3}\overrightarrow{BC}$ (One soln **M2A1**, 2nd soln **A1**)
		4	

Question	Answer	Marks	Guidance
10(i)	$ax^2 + bx = 0 \to x(ax+b) = 0 \to x = \frac{-b}{a}$	**B1**	
	Find $f''(x)$ and attempt sub *their* $\frac{-b}{a}$ into *their* $f''(x)$	**M1**	
	When $x = \frac{-b}{a}$, $f''(x) = 2a\left(\frac{-b}{a}\right) + b = -b$ MAX	**A1**	
		3	
10(ii)	Sub $f'(-2) = 0$	**M1**	
	Sub $f'(1) = 9$	**M1**	
	$a = 3 \qquad b = 6$	***A1**	Solve simultaneously to give both results.
	$f'(x) = 3x^2 + 6x \to f(x) = x^3 + 3x^2 \ (+c)$	***M1**	Sub *their a, b* into $f'(x)$ and integrate 'correctly'. Allow $\frac{ax^3}{3} + \frac{bx^2}{2}(+c)$
	$-3 = -8 + 12 + c$	**DM1**	Sub $x = -2,\ y = -3$. Dependent on c present. Dependent also on a, b substituted.
	$f(x) = x^3 + 3x^2 - 7$	**A1**	
		6	

Question	Answer	Marks	Guidance
11(i)	Gradient of $AB = \frac{1}{2}$	**B1**	
	Equation of AB is $y = \frac{1}{2}x - \frac{1}{2}$	**B1**	
		2	
11(ii)	$\frac{dy}{dx} = ½(x-1)^{-\frac{1}{2}}$	**B1**	
	$½(x-1)^{-\frac{1}{2}} = ½$. Equate *their* $\frac{dy}{dx}$ to *their* ½	***M1**	
	$x = 2,\ \ y = 1$	**A1**	
	$y - 1 = ½(x - 2)$ (thro' *their*(2,1) & *their* ½) → $y = ½x$	**DM1 A1**	
		5	

Question	Answer	Marks	Guidance
11(iii)	*EITHER:* $\sin\theta = \frac{d}{1} \rightarrow d = \sin\theta$	(M1	Where θ is angle between AB and the x-axis
	gradient of $AB = ½ \Rightarrow \tan\theta = ½ \Rightarrow \theta = 26.5(7)°$	B1	
	$d = \sin 26.5(7)° = 0.45$ (or $\frac{1}{\sqrt{5}}$)	A1)	
	OR1: Perpendicular through O has equation $y = -2x$	(M1	
	Intersection with AB: $-2x = ½x - ½ \rightarrow \left(\frac{1}{5}, \frac{-2}{5}\right)$	A1	
	$d = \sqrt{\left(\frac{1}{5}\right)^2 + \left(\frac{2}{5}\right)^2} = 0.45$ (or $\frac{1}{\sqrt{5}}$)	A1)	
	OR2: Perpendicular through (2, 1) has equation $y = -2x + 5$	(M1	
	Intersection with AB: $-2x + 5 = ½x - ½ \rightarrow \left(\frac{11}{5}, \frac{3}{5}\right)$	A1	
	$d = \sqrt{\left(\frac{1}{5}\right)^2 + \left(\frac{2}{5}\right)^2} = 0.45$ (or $1/\sqrt{5}$)	A1)	

Question	Answer	Marks	Guidance
11(iii)	*OR3:* ΔOAC has area $\frac{1}{4}$ [where $C = (0, -\frac{1}{2})$]	**(B1**	
	$\frac{1}{2} \times \frac{\sqrt{5}}{2} \times d = \frac{1}{4} \rightarrow d = \frac{1}{\sqrt{5}}$	**M1 A1)**	
		3	

Cambridge International AS & A Level

Cambridge International Examinations
Cambridge International Advanced Subsidiary and Advanced Level

MATHEMATICS **9709/11**

Paper 1 **May/June 2017**

MARK SCHEME

Maximum Mark: 75

Published

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  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

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The following abbreviations may be used in a mark scheme or used on the scripts:

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
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ISW	Ignore Subsequent Working
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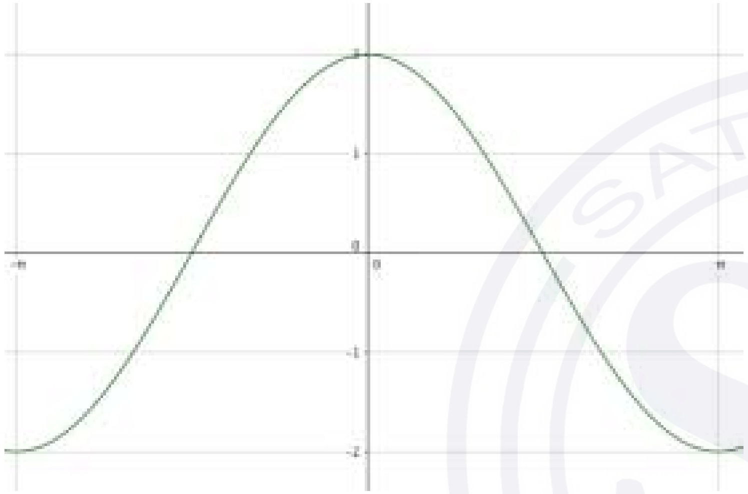
Penalties

MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Question	Answer	Marks	Guidance
1	$(3-2x)^6$		
	Coeff of $x^2 = 3^4 \times (-2)^2 \times {}_6C_2 = a$ Coeff of $x^3 = 3^3 \times (-2)^3 \times {}_6C_3 = b$	**B3,2,1**	Mark unsimplified forms. –1 each independent error but powers must be correct. Ignore any 'x' present.
	$\frac{a}{b} = -\frac{9}{8}$	**B1**	OE. Negative sign must appear before or in the numerator
	Total:	**4**	
2	$\overrightarrow{OA} = \begin{pmatrix} 3 \\ -6 \\ p \end{pmatrix}$ and $\overrightarrow{OB} = \begin{pmatrix} 2 \\ -6 \\ -7 \end{pmatrix}$		
2(i)	Angle $AOB = 90° \to 6 + 36 - 7p = 0$	**M1**	Use of $x_1x_2 + y_1y_2 + z_1z_2 = 0$ or Pythagoras
	$\to p = 6$	**A1**	
	Total:	**2**	

Question	Answer	Marks	Guidance
2(ii)	$\overrightarrow{OC} = \frac{2}{3}\begin{pmatrix}3\\-6\\p\end{pmatrix} = \begin{pmatrix}2\\-4\\4\end{pmatrix}$	**B1 FT**	CAO FT on their value of p
	$\overrightarrow{BC} = \mathbf{c} - \mathbf{b} = \begin{pmatrix}0\\2\\11\end{pmatrix}$; magnitude = $\sqrt{125}$	**M1 M1**	Use of **c** – **b**. Allow magnitude of **b** + **c** or **b – c** Allow first **M1** in terms of p
	Unit vector = $\frac{1}{\sqrt{125}}\begin{pmatrix}0\\2\\11\end{pmatrix}$	**A1**	OE Allow ± and decimal equivalent
3(i)	$\frac{1+\cos\theta}{\sin\theta}+\frac{\sin\theta}{1+\cos\theta}\equiv\frac{2}{\sin\theta}$.		
	$\frac{(1+c)^2+s^2}{s(1+c)} = \frac{1+2c+c^2+s^2}{s(1+c)}$	**M1**	Correct use of fractions
	$= \frac{2+2c}{s(1+c)} = \frac{2(1+c)}{s(1+c)} \to \frac{2}{s}$	**M1 A1**	Use of trig identity, **A1** needs evidence of cancelling
	Total:	**3**	
3(ii)	$\frac{2}{\boldsymbol{s}} = \frac{3}{\boldsymbol{c}} \to t = \frac{2}{3}$	**M1**	Use part **(i)** and $t = s \div c$, may restart from given equation
	$\to \theta = 33.7°$ or $213.7°$	**A1 A1FT**	FT for 180° + 1st answer. 2nd **A1** lost for extra solns in range
	Total:	**3**	

Question	Answer	Marks	Guidance
4(a)	$a = 32$, $a + 4d = 22$, $\rightarrow d = -2.5$	**B1**	
	$a + (n-1)d = -28 \rightarrow n = 25$	**B1**	
	$S_{25} = \dfrac{25}{2}\left(64 - 2.5 \times 24\right) = 50$	**M1 A1**	**M1** for correct formula with $n = 24$ or $n = 25$
	Total:	**4**	
4(b)	$a = 2000$, $r = 1.025$	**B1**	$r = 1 + 2.5\%$ ok if used correctly in S_n formula
	$S_{10} = 2000\left(\dfrac{1.025^{10} - 1}{1.025 - 1}\right) = 22400$ or a value which rounds to this	**M1 A1**	**M1** for correct formula with $n = 9$ or $n = 10$ and their a and r
			SR: correct answer only for $n = 10$ **B3**, for $n = 9$, **B1** (£19 900)
	Total:	**3**	

Question	Answer	Marks	Guidance
5	$y = 2\cos x$		
5(i)		**B1**	One whole cycle – starts and finishes at –ve value
		**DB1**	Smooth curve, flattens at ends and middle. Shows (0, 2).
	Total:	**2**	
5(ii)	$P(\frac{\pi}{3}, 1)$ $Q(\pi, -2)$		
	$\rightarrow PQ^2 = \left(\frac{2\pi}{3}\right)^2 + 3^2 \rightarrow PQ = 3.7$	**M1 A1**	Pythagoras (on their coordinates) must be correct, OE.
	Total:	**2**	

Question	Answer	Marks	Guidance
5(iii)	Eqn of PQ $y-1=-\frac{9}{2\pi}\left(x-\frac{\pi}{3}\right)$	**M1**	Correct form of line equation or sim equations from their P & Q
	If $y=0 \to h=\frac{5\pi}{9}$	**A1**	AG, condone $x=\frac{5\pi}{9}$
	If $x=0 \to k=\frac{5}{2}$,	**A1**	SR: non-exact solutions **A1** for both
	Total:	**3**	
6(i)	Volume = $\left(\frac{1}{2}\right)x^2\frac{\sqrt{3}}{2}h = 2000 \to h=\frac{8000}{\sqrt{3}x^2}$	**M1**	Use of (area of triangle, with attempt at ht) × h =2000, $h=\mathrm{f}(x)$
	$A = 3xh+(2)\times\left(\frac{1}{2}\right)\times x^2\times\frac{\sqrt{3}}{2}$	**M1**	Uses 3 rectangles and at least one triangle
	Sub for $h \to A=\frac{\sqrt{3}}{2}x^2+\frac{24000}{\sqrt{3}}x^{-1}$	**A1**	AG
	Total:	**3**	
6(ii)	$\frac{dA}{dx}=\frac{\sqrt{3}}{2}2x-\frac{24000}{\sqrt{3}}x^{-2}$	**B1**	CAO, allow decimal equivalent
	$= 0$ when $x^3 = 8000 \to x = 20$	**M1 A1**	Sets their $\frac{dA}{dx}$ to 0 and attempt to solve for x
	Total:	**3**	

Question	Answer	Marks	Guidance
6(iii)	$\frac{d^2A}{dx^2}=\frac{\sqrt{3}}{2}2+\frac{48000}{\sqrt{3}}x^{-3}>0$	**M1**	Any valid method, ignore value of $\frac{d^2A}{dx^2}$ providing it is positive
	$\rightarrow$ Minimum	**A1 FT**	FT on their x providing it is positive
	Total:	**2**	
7	$\frac{dy}{dx}=7-x^2-6x$		
7(i)	$y=7x-\frac{x^3}{3}-\frac{6x^2}{2}\ (+c)$	**B1**	CAO
	Uses $(3,-10)\rightarrow c=5$	**M1 A1**	Uses the given point to find c
	Total:	**3**	
7(ii)	$7-x^2-6x=16-(x+3)^2$	**B1 B1**	**B1** $a=16$, **B1** $b=3$.
	Total:	**2**	
7(iii)	$16-(x+3)^2>0\rightarrow(x+3)^2<16$, and solve	**M1**	or factors $(x+7)(x-1)$
	End-points $x=1$ or -7	**A1**	
	$\rightarrow -7<x<1$	**A1**	needs $<$, not $\leqslant$. (SR $x<1$ only, or $x>-7$ only **B1** i.e. 1/3)
	Total:	**3**	

Question	Answer	Marks	Guidance
8(i)	Letting M be midpoint of AB		
	$OM = 8$ (Pythagoras) $\rightarrow XM = 2$	**B1**	(could find $\sqrt{40}$ and use $\sin^{-1}$ or $\cos^{-1}$)
	$\tan AXM = \frac{6}{2}$ $AXB = 2\tan^{-1}3 = 2.498$	**M1 A1**	AG Needs × 2 and correct trig for **M1**
	(Alternative 1: $\sin AOM = \frac{6}{10}, AOM = 0.6435, AXB = \pi - 0.6435$)		(Alternative 1: Use of isosceles triangles, **B1** for AOM, **M1**,**A1** for completion) (Alternative 2: Use of circle theorem, **B1** for AOB, **M1**,**A1**for completion)
	Total:	**3**	
8(ii)	$AX = \sqrt{(6^2 + 2^2)} = \sqrt{40}$	**B1**	CAO, could be gained in part (i) or part (iii)
	Arc $AYB = r\theta = \sqrt{40} \times 2.498$	**M1**	Allow for incorrect $\sqrt{40}$ (not $r = 6\, or\, 12\, or\, 10$)
	Perimeter = 12 + arc = 27.8 cm	**A1**	
	Total:	**3**	
8(iii)	area of sector $AXBY = ½ \times (\sqrt{40})^2 \times 2.498$	**M1**	Use of $½r^2\theta$ with their r, (not $r = 6\, or\, r = 10$)
	Area of triangle $AXB = ½ \times 12 \times 2$, Subtract these $\rightarrow 38.0$ cm²	**M1 A1**	Use of $½bh$ and subtraction. Could gain **M1** with $r = 10$.
	Total:	**3**	

Question	Answer	Marks	Guidance
9	$\mathrm{f}: x \mapsto \dfrac{2}{3-2x}$ $\mathrm{g}: x \mapsto 4x + a$,		
9(i)	$y = \dfrac{2}{3-2x} \to y(3-2x) = 2 \to 3-2x = \dfrac{2}{y}$	M1	Correct first 2 steps
	$\to 2x = 3 - \dfrac{2}{y} \to \mathrm{f}^{-1}(x) = \dfrac{3}{2} - \dfrac{1}{x}$	M1 A1	Correct order of operations, any correct form with $\mathrm{f}(x)$ *or* $y =$
	Total:	3	
9(ii)	$\mathrm{gf}(-1) = 3$ $\mathrm{f}(-1) = \dfrac{2}{5}$	M1	Correct first step
	$\dfrac{8}{5} + a = 3 \to a = \dfrac{7}{5}$	M1 A1	Forms an equation in a and finds a, OE
			(or $\dfrac{8}{3-2x} + a = 3$, **M1** Sub and solves **M1**, **A1**)
	Total:	3	
9(iii)	$\mathrm{g}^{-1}(x) = \dfrac{x-a}{4} = \mathrm{f}^{-1}(x)$	M1	Finding $\mathrm{g}^{-1}(x)$ and equating to their $\mathrm{f}^{-1}(x)$ even if $a = 7/5$
	$\to x^2 - x(a+6) + 4 (= 0)$	M1	Use of $b^2 - 4ac$ on a quadratic with a in a coefficient
	Solving $(a+6)^2 = 16$ *or* $a^2 + 12a + 20 (= 0)$	M1	Solution of a 3 term quadratic
	$\to a = -2$ or -10	A1	
	Total:	4	

Question	Answer	Marks	Guidance
10(i)	$\frac{dy}{dx} = \frac{-4}{(5-3x)^2} \times (-3)$	**B1 B1**	**B1** without $\times(-3)$ **B1** For $\times(-3)$
	Gradient of tangent = 3, Gradient of normal – ⅓	***M1**	Use of $m_1m_2 = -1$ after calculus
	$\to$ eqn: $y-2=-\frac{1}{3}(x-1)$	**DM1**	Correct form of equation, with (1, their y), not (1,0)
	$\to\ y=-\frac{1}{3}x+\frac{7}{3}$	**A1**	This mark needs to have come from $y = 2$, y must be subject
	Total:	**5**	
10(ii)	$\text{Vol} = \pi\int_0^1 \frac{16}{(5-3x)^2}dx$	**M1**	Use of $V = \pi\int y^2 dx$ with an attempt at integration
	$\pi\left[\frac{-16}{(5-3x)} \quad \div -3\right]$	**A1 A1**	**A1** without($\div -3$), **A1** for ($\div -3$)
	$= (\ \pi\left(\frac{16}{6}-\frac{16}{15}\right)\ ) = \frac{8\pi}{5}$ (if limits switched must show – to +)	**M1 A1**	Use of both correct limits **M1**
	Total:	**5**	

Cambridge International Examinations
Cambridge International Advanced Subsidiary and Advanced Level

MATHEMATICS	**9709/12**
Paper 1	**May/June 2017**
MARK SCHEME
Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the May/June 2017 series for most Cambridge IGCSE®, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

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1(i)	Coefficient of $x = 80(x)$	**B2**	Correct value must be selected for both marks. SR +80 seen in an expansion gets **B1** or −80 gets **B1** if selected.
	Total:	**2**	
1(ii)	Coefficient of $\frac{1}{x} = -40\left(\frac{1}{x}\right)$	**B2**	Correct value soi in (ii), if powers unsimplified only allow if selected. SR +40 soi in (ii) gets **B1**.
	Coefficient of $x = (1 \times \text{their } 80) + (3 \times \text{their} - 40) = -40(x)$	**M1 A1**	Links the appropriate 2 terms only for **M1**.
	Total:	**4**	
2(i)	Gradient = 1.5 Gradient of perpendicular = $-⅔$	**B1**	
	Equation of AB is $y - 6 = -⅔(x+2)$ Or $3y + 2x = 14$ oe	**M1 A1**	Correct use of straight line equation with a changed gradient and (− 2, 6), the (− (− 2)) must be resolved for the **A1** ISW.
			Using $y = mx + c$ gets **A1** as soon as c is evaluated.
	Total:	**3**	
2(ii)	Simultaneous equations → Midpoint (1, 4)	**M1**	Attempt at solution of simultaneous equations as far as x =, or y =.
	Use of midpoint or vectors → B (4, 2)	**M1A1**	Any valid method leading to x, or to y.
	Total:	**3**	

Question	Answer	Marks	Guidance
3(i)	LHS = $\left(\frac{1}{c}-\frac{s}{c}\right)^2$	M1	Eliminates tan by replacing with $\frac{\sin}{\cos}$ leading to a function of sin and/or cos only.
	$=\frac{(1-s)^2}{1-s^2}$	M1	Uses $s^2+c^2=1$ leading to a function of sin only.
	$=\frac{(1-s)(1-s)}{(1-s)(1+s)}=\frac{1-\sin\theta}{1+\sin\theta}$	A1	AG. Must show use of factors for **A1**.
	Total:	3	
3(ii)	Uses part (i) → $2-2s=1+s$		
	→ $s=⅓$	M1	Uses part (i) to obtain $s=k$
	$\theta=19.5°$ or $160.5°$	A1A1 FT	FT from error in 19.5° Allow 0.340^c (0.3398^c) & 2.80(2) or $0.108\pi^c$ & $0.892\pi^c$ for **A1** only. Extra answers in the range lose the second **A1** if gained for 160.5°.
	Total:	3	
4(i)	$(AB)=2r\sin\theta$ (or $r\sqrt{2-2\cos2\theta}$ or $\frac{r\sin2\theta}{\sin\left(\frac{\pi}{2}-\theta\right)}$)	B1	Allow unsimplified throughout eg r + r, $\frac{2\theta}{2}$ etc
	(Arc AB) = $2r\theta$	B1	
	(P =) $2r+2r\theta+2r\sin\theta$ (or $r\sqrt{2-2\cos2\theta}$ or $\frac{r\sin2\theta}{\sin\left(\frac{\pi}{2}-\theta\right)}$)	B1	
	Total:	3	

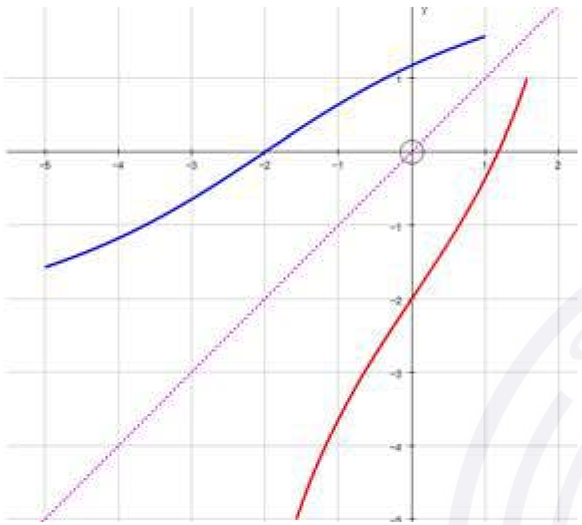
Question	Answer	Marks	Guidance
4(ii)	Area sector AOB = (½ r^2 2θ) $\frac{25\pi}{6}$ or13.1	**B1**	Use of segment formula gives 2.26 **B1B1**
	Area triangle AOB = (½×2$r$sinθ×$r$cosθ or ½ × r^2 sin2θ) $\frac{25\sqrt{3}}{4}$ or10.8	**B1**	
	Area rectangle $ABCD$ = (r × 2$r$sinθ) 25	**B1**	
	(Area =) Either 25 – (25π/6 – 25√3/4) or 22.7	**B1**	Correct final answer gets **B4**.
	Total:	**4**	
5(i)	Crosses x-axis at (6, 0)	**B1**	$x = 6$ is sufficient.
	$\frac{dy}{dx} = (0 +) -12\,(2 - x)^{-2} \times (-1)$	**B2,1,0**	−1 for each incorrect term of the three or addition of + C.
	Tangent $y = ¾(x-6)$ or $4y = 3x - 18$	**M1 A1**	Must use dy/dx, x= their 6 but not x = 0 (which gives m = 3), and correct form of line equation.
			Using $y = mx + c$ gets **A1** as soon as c is evaluated.
	Total:	**5**	
5(ii)	If x = 4, dy/dx = 3		
	$\frac{dy}{dt} = 3 \times 0.04 = 0.12$	**M1 A1FT**	**M1** for (“their m” from $\frac{dy}{dx}$ and x = 4) × 0.04. Be aware: use of x = 0 gives the correct answer but gets **M0**.
	Total:	**2**	

Question	Answer	Marks	Guidance
6	$\text{Vol} = \pi \int (5-x)^2 \,\mathrm{d}x - \pi \int \frac{16}{x^2}\mathrm{d}x$	**M1***	Use of volume formula at least once, condone omission of π and limits and $\mathrm{d}x$.
		DM1	Subtracting volumes somewhere must be <u>after</u> squaring.
	$\int (5-x)^2 \mathrm{d}x = \frac{(5-x)^3}{3} \div -1$	**B1 B1**	**B1** Without ÷ (−1). **B1** for ÷(−1)
	(or $25x - 10x^2/2 + \frac{1}{3}x^3$)	**(B2,1,0)**	−1 for each incorrect term
	$\int \frac{16}{x^2}\mathrm{d}x = -\frac{16}{x}$	**B1**	
	Use of limits 1 and 4 in an integrated expression and subtracted.	**DM1**	Must have used "y^2" at least once. Need to see values substituted.
	→ 9π or 28.3	**A1**	
	Total:	**7**	
7(a)	$(S_n =)\ \frac{n}{2}\left[32 + (n-1)8\right]$ and 20000	**M1**	**M1** correct formula used with d from $16 + d = 24$
		A1	**A1** for correct expression linked to 20000.
	→ $n^2 + 3n - 5000$ (<,=,> 0)	**DM1**	Simplification to a three term quadratic.
	→ (n = 69.2) → 70 terms needed.	**A1**	Condone use of 20001 throughout. Correct answer from trial and improvement gets 4/4.
	Total:	**4**	

Question	Answer	Marks	Guidance
7(b)	$a = 6,\ \frac{a}{1-r} = 18 \rightarrow r = ⅔$	M1A1	Correct $S\infty$ formula used to find r.
	New progression $a = 36$, $r = \frac{4}{9}$ oe	M1	Obtain new values for a and r by any valid method.
	New $S\infty = \frac{36}{1-\frac{4}{9}} \rightarrow 64.8$ or $\frac{324}{5}$ oe	A1	(Be aware that $r = -⅔$ leads to 64.8 but can only score M marks)
	Total:	**4**	
8(i)	Uses scalar product correctly: $3 \times 6 + 2 \times 6 + (-4) \times 3 = 18$	M1	Use of dot product with $\overrightarrow{OA}$ or $\overrightarrow{AO}$ & $\overrightarrow{OB}$ or $\overrightarrow{BO}$ only.
	$\lvert\overrightarrow{OA}\rvert = \sqrt{29}$, $\lvert\overrightarrow{OB}\rvert = 9$	M1	Correct method for any one of $\lvert\overrightarrow{OA}\rvert$, $\lvert\overrightarrow{AO}\rvert$, $\lvert\overrightarrow{OB}\rvert$ or $\lvert\overrightarrow{BO}\rvert$.
	$\sqrt{29} \times 9 \times \cos AOB = 18$	M1	All linked correctly.
	$\rightarrow AOB = 68.2°$ or 1.19^c	A1	Multiples of π are acceptable (e.g. $0.379\pi^c$)
	Total:	**4**	
8(ii)	$\overrightarrow{AB} = 3\mathbf{i} + 4\mathbf{j} + (3+2p)\mathbf{k}$	*M1	For use of $\overrightarrow{OB} - \overrightarrow{OA}$ **,** allow with p = 2
	Comparing "**j**"	DM1	For comparing, $\overrightarrow{OC}$ must contain p & q. Can be implied by $\overrightarrow{AB} = 2\overrightarrow{OC}$.
	$\rightarrow p = 2½$ and $q = 4$	A1 A1	Accuracy marks only available if $\overrightarrow{AB}$ is correct.
	Total:	**4**	

Question	Answer	Marks	Guidance
9(i)	$\frac{dy}{dx} = 4x^{-\frac{1}{2}} - 2$	B1	Accept unsimplified.
	$= 0$ when $\sqrt{x} = 2$		
	$x = 4, y = 8$	B1B1	
	Total:	**3**	
9(ii)	$\frac{d^2y}{dx^2} = -2x^{-\frac{3}{2}}$	B1FT	FT providing –ve power of x
	$\left(\frac{d^2y}{dx^2} = -\frac{1}{4}\right) \to$ Maximum	B1	Correct $\frac{d^2y}{dx^2}$ and x=4 in (i) are required. Followed by"< 0 or negative" is sufficient" but $\frac{d^2y}{dx^2}$ must be correct if evaluated.
	Total:	**2**	
9(iii)	*EITHER:* Recognises a quadratic in $\sqrt{x}$	(M1	Eg $\sqrt{x} = u \to 2u^2 - 8u + 6 = 0$
	1 and 3 as solutions to this equation	A1	
	$\to x = 9, x = 1.$	A1)	

Question	Answer	Marks	Guidance
	OR: Rearranges then squares	(M1	$\sqrt{x}$ needs to be isolated before squaring both sides.
	$\rightarrow x^2 - 10x + 9 = 0$ oe	A1	
	$\rightarrow x = 9, x = 1.$	A1)	Both correct by trial and improvement gets 3/3
	Total:	3	
9(iv)	$k > 8$	B1	
	Total:	1	
10(i)	$3\tan\left(\frac{1}{2}x\right) = -2 \rightarrow \tan\left(\frac{1}{2}x\right) = -⅔$	M1	Attempt to obtain $\tan\left(\frac{1}{2}x\right) = k$ from $3\tan\left(\frac{1}{2}x\right) + 2 = 0$
	$½x = -0.6\ (-0.588) \rightarrow x = -1.2$	M1 A1	$\tan^{-1} k$. Seeing $½x = -33.69°$ or $x = -67.4°$ implies **M1M1**.
			Extra answers between -1.57 & 1.57 lose the **A1**. Multiples of π are acceptable (eg -0.374π)
	Total:	3	
10(ii)	$\frac{y+2}{3} = \tan\left(\frac{1}{2}x\right)$	M1	Attempt at isolating tan(½x)
	$\rightarrow f^{-1}(x) = 2\tan^{-1}\left(\frac{x+2}{3}\right)$	M1 A1	Inverse tan followed by × 2. Must be function of x for **A1**.
	$-5, 1$	B1 B1	Values stated **B1** for -5, **B1** for 1.
	Total:	5	

Question	Answer	Marks	Guidance
10(iii)		**B1 B1 B1**	A tan graph through the first, third and fourth quadrants. (**B1**) An invtan graph through the first, second and third quadrants.(**B1**) Two curves clearly symmetrical about $y = x$ either by sight or by exact end points. Line not required. Approximately in correct domain and range. (Not intersecting.) (**B1**) Labels on axes not required.
	Total:	**3**	

Cambridge International AS & A Level

Cambridge International Examinations
Cambridge International Advanced Subsidiary and Advanced Level

MATHEMATICS **9709/13**

Paper 1 **May/June 2017**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

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Cambridge is publishing the mark schemes for the May/June 2017 series for most Cambridge IGCSE®, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

CAMBRIDGE
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Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

  - Note: B2 or A2 means that the candidate can earn 2 or 0.
    B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF/OE Any Equivalent Form (of answer is equally acceptable) / Or Equivalent

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

SOI Seen or implied

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Question	Answer	Marks	Guidance
1	$7C1 \times 2^6 \times a(x)$, $7C2 \times 2^5 \times [a(x)]^2$	B1 B1	SOI Can be part of expansion. Condone ax^2 only if followed by a^2. ALT $2^7[1+ax/2]^7 \to 7C1[a(x)/2] = 7C2[a(x)/2]^2$
	$a = \frac{7 \times 2^6}{21 \times 2^5} = \frac{2}{3}$	B1	Ignore extra soln $a = 0$. Allow $a = 0.667$. Do not allow an extra x in the answer
	Total:	**3**	

Question	Answer	Marks	Guidance
2(i)	$S = \frac{r^2 - 3r + 2}{1 - r}$	M1	
	$S = \frac{(r-1)(r-2)}{1-r} = \frac{-(1-r)(r-2)}{1-r} = 2 - r$ OR $\frac{(1-r)(2-r)}{1-r} = 2 - r$ OE	A1	**AG** Factors must be shown. Expressions requiring minus sign taken out must be shown
	Total:	**2**	
2(ii)	Single range $1 < S < 3$ or (1, 3)	B2	Accept $1 < 2 - r < 3$. Correct range but with $S = 2$ omitted scores SR **B1** $1 \leqslant S \leqslant 3$ scores SR **B1**. [$S > 1$ and $S < 3$] scores SR **B1**.
	Total:	**2**	

Question	Answer	Marks	Guidance
3	**EITHER** Elim y to form 3-term quad eqn in $x^{1/3}$ (or u or y or even x)	**(M1**	Expect $x^{2/3}-x^{1/3}-2(=0)$ or $u^2-u-2\ (=0)$ etc.
	$x^{1/3}$ (or u or y or x) = 2, −1	***A1**	Both required. But $\underline{x}$ = 2,–1 and not then cubed or cube rooted scores **A0**
	Cube solution(s)	**DM1**	Expect x = 8, – 1. Both required
	(8, 3), (–1,0)	**A1)**	
	OR Elim x to form quadratic equation in y	**(M1**	Expect $y+1=(y-1)^2$
	$y^2-3y=0$	***A1**	
	Attempt solution	**DM1**	Expect y = 3, 0
	(8, 3), (–1,0)	**A1)**	
	Total:	**4**	

Question	Answer	Marks	Guidance
4(i)	$\overrightarrow{OB}-\overrightarrow{OA}\left(=\overrightarrow{AB}\right)=\begin{pmatrix}5\\4\\-3\end{pmatrix}-\begin{pmatrix}5\\1\\3\end{pmatrix}=\begin{pmatrix}0\\3\\-6\end{pmatrix}$	**B1**	
	$\overrightarrow{OP}=\begin{pmatrix}5\\1\\3\end{pmatrix}+\frac{1}{3}\begin{pmatrix}0\\3\\-6\end{pmatrix}=\begin{pmatrix}5\\2\\1\end{pmatrix}$	**M1 A1**	If $\overrightarrow{OP}$ not scored in (i) can score SR **B1** if seen correct in (ii). Other equivalent methods possible
	Total:	**3**	
4(ii)	Distance $OP=\sqrt{5^2+2^2+1^2}$ = $\sqrt{30}$ or 5.48	**B1 FT**	FT on *their* $\overrightarrow{OP}$ from (i)
	Total:	**1**	
4(iii)	Attempt $\overrightarrow{AB}.\overrightarrow{OP}$. Can score as part of $\overrightarrow{AB}.\overrightarrow{OP}=(AB)(OP)\cos\theta$ Rare ALT: Pythagoras $\left\|\overrightarrow{OP}\right\|^2+\left\|\overrightarrow{AP}\right\|^2=5+30=\left\|\overrightarrow{OA}\right\|^2$	**M1**	Allow any combination of $\overrightarrow{AB}.\overrightarrow{PO}$ etc. and also if $\overrightarrow{AP}$ or $\overrightarrow{PB}$ used instead of $\overrightarrow{AB}$ giving 2–2 = 0 & 4–4 = 0 respectively. Allow notation × instead of **.** .
	(0 + 6 – 6) = 0 hence perpendicular. (Accept 90º)	**A1 FT**	If result not zero then 'Not perpendicular' can score **A1**FT if value is 'correct' for *their* values of $\overrightarrow{AB},\overrightarrow{OP}$ etc. from (i).
	Total:	**2**	

Question	Answer	Marks	Guidance
5(i)	$\dfrac{2\sin\theta+\cos\theta}{\sin\theta+\cos\theta}=\dfrac{2\sin\theta}{\cos\theta}$	M1	Replace $\tan\theta$ by $\sin\theta/\cos\theta$
	$2\sin\theta\cos\theta+\cos^2\theta=2\sin^2\theta+2\sin\theta\cos\theta \Rightarrow c^2=2s^2$	M1 A1	Mult by c(s + c) or making this a common denom.. For **A1** simplification to **AG** without error or omission must be seen.
	Total:	**3**	
5(ii)	$\tan^2\theta=1/2$ or $\cos^2\theta=2/3$ or $\sin^2\theta=1/3$	B1	Use $\tan\theta=s/\text{c}$ or $\text{c}^2+\text{s}^2=1$ and simplify to one of these results
	$\theta=35.3°$ or $144.7°$	B1 B1 FT	FT for 180 – other solution. SR **B1** for radians 0.615, 2.53 (0.196π, 0.804π) Extra solutions in range amongst solutions of which 2 are correct gets **B1B0**
	Total:	**3**	

Question	Answer	Marks	Guidance
6	Gradient of normal is – 1/3 → gradient of tangent is 3 SOI	**B1 B1 FT**	FT from *their* gradient of normal.
	$dy/dx = 2x - 5 = 3$	**M1**	Differentiate and set = *their* 3 (numerical).
	$x = 4$	***A1**	
	Sub $x = 4$ into line → $y = 7$ & sub *their* (4, 7) into curve	**DM1**	OR sub $x = 4$ into curve → $y = k - 4$ and sub *their*$(4, k - 4)$ into line OR other valid methods deriving a linear equation in k (e.g. equating curve with either normal or tangent and sub $x = 4$).
	$k = 11$	**A1**	
	Total:	**6**	

Question	Answer	Marks	Guidance
7(i)	$\sin ABC = 8/10 \rightarrow ABC = 0.927(3)$	B1	Or cos = 6/10 or tan = 8/6. Accept 0.295π.
	Total:	**1**	
7(ii)	$AB = 6(\text{Pythagoras}) \quad \rightarrow \quad \Delta BCD = 8\times 6 = 48.0$	M1A1	**OR** 8×10sin0.6435 or ½×10×10sin((2)×0.927)=48. 24or 40or80 gets **M1A0**
	Area sector $BCD = ½\times 10^2 \times (2)\times \textit{their}\, 0.9273$	*M1	Expect 92.7(3). 46.4 gets **M1**
	Area segment = 92.7(3) – 48	*A1	Expect 44.7(3). Might not appear until final calculation.
	Area semi-circle – segment = $½\times \pi \times 8^2 - \textit{their}(92.7 - 48)$	DM1	Dep. on previous **M1A1** OR $\pi \times 8^2 - \left(½\times \pi \times 8^2 + \textit{their}\, 44.7\right)$.
	Shaded area = 55.8 – 56.0	A1	
	Total:	**6**	

Question	Answer	Marks	Guidance
8(i)	$(b-1)/(a+1)=2$	M1	OR Equation of *AP* is $y-1=2(x+1) \to y=2x+3$
	$b=2a+3$ CAO	A1	Sub $x=a$, $y=b \to b=2a+3$
	Total:	2	
8(ii)	$AB^2=11^2+2^2=125$ oe	B1	Accept $AB=\sqrt{125}$
	$(a+1)^2+(b-1)^2=125$	B1 FT	FT on *their* 125.
	$(a+1)^2+(2a+2)^2=125$	M1	Sub from part (i) → quadratic eqn in *a* (or possibly in *b* → $b^2-2b-99=0$)
	$(5)(a^2+2a-24)=0 \to \text{eg}(a-4)(a+6)=0$	M1	Simplify and attempt to solve
	$a=4$ or -6	A1	
	$b=11$ or -9	A1	OR (4, 11), (–6, –9) If **A0A0**, SR1 for either (4, 11) or (–6, –9)
	Total:	6	

Question	Answer	Marks	Guidance
9(i)	$(3x-1)^2+5$	B1B1B1	First 2 marks dependent on correct $(ax+b)^2$ form. OR $a=3,\ b=-1,\ c=5$ e.g. from equating coefs
	Total:	3	
9(ii)	Smallest value of p is 1/3 seen. (Independent of (i))	B1	Allow $p \geqslant 1/3$ or $p=1/3$ or 1/3 seen. But not in terms of x.
	Total:	1	
9(iii)	$y=(3x-1)^2+5 \Rightarrow 3x-1=(\pm)\sqrt{y-5}$	B1 FT	OR $y=9\left(x-\frac{1}{3}\right)^2+5 \Rightarrow (y-5)/9=\left(x-\frac{1}{3}\right)^2$ (Fresh start)
	$x=(\pm)\frac{1}{3}\sqrt{y-5}+\frac{1}{3}$ OE	B1 FT	Both starts require 2 operations for each mark. FT for *their* values from part (i)
	$\mathrm{f}^{-1}(x)=\frac{1}{3}\sqrt{x-5}+\frac{1}{3}$ OE domain is $x \geq their5$	B1B1 FT	Must be a function of x and $\pm$ removed. Domain must be in terms of x. Note: $\sqrt{y-5}$ expressed as $\sqrt{y}-\sqrt{5}$ scores Max **B0B0B0B1** [See below for general instructions for different starts]
	Total:	4	
9(iv)	$q<5$ CAO	B1	
	Total:	1	

Alt 9(iii)
For start $(ax-b)^2+c$ or $a(x-b)^2+c$ ($a\neq 0$) ft for their a, b, c
For start $(x-b)^2+c$ ft but award only **B1** for 3 correct operations
For start $a(bx-c)^2+d$ ft but award **B1** for first2 operations correct and **B1** for the next 3 operations correct

Question	Answer	Marks	Guidance
10(a)(i)	Attempt to integrate $V = (\pi)\int (y+1)\mathrm{d}y$	**M1**	Use of h in integral e.g. $\int (h+1) = ½h^2 + h$ is **M0**. Use of $\int y^2 \mathrm{d}x$ is **M0**
	$= (\pi)\left[\frac{y^2}{2} + y\right]$	**A1**	
	$= \pi\left[\frac{h^2}{2} + h\right]$	**A1**	**AG**. Must be from clear use of limits 0→ h somewhere.
	Total:	**3**	
10(ii)	$\int (y+1)^{1/2}\mathrm{d}y$ **ALT** $6 - \int (x^2 - 1)\mathrm{d}x$	**M1**	Correct variable and attempt to integrate
	$⅔(y+1)^{3/2}$ oe **ALT** $6 - (⅓x^3 - x)$ CAO	***A1**	Result of integration must be shown
	$⅔[8-1]$ **ALT** $6 - [\left(\frac{8}{3} - 1\right) - \left(\frac{1}{3} - 1\right)]$	**DM1**	Calculation seen with limits 0→3 for y. For ALT, limits are 1→2 and rectangle.
	14/3 **ALT** 6 – 4/3 = 14/3	**A1**	16/3 from $⅔ \times 8$ gets **DM1A0** provided work is correct up to applying limits.
	Total:	**4**	

Question	Answer	Marks	Guidance
10(b)	Clear attempt to differentiate wrt h	**M1**	Expect $\frac{dV}{dh} = \pi\left(h+1\right)$. Allow $h + 1$. Allow h.
	Derivative = 4π SOI	***A1**	
	$\frac{2}{\textit{their}\ \text{derivative}}$. Can be in terms of h	**DM1**	
	$\frac{2}{4\pi}$ or $\frac{1}{2\pi}$ or 0.159	**A1**	
	Total:	**4**	

Question	Answer	Marks	Guidance
11(i)	$\text{f}'(x)=[(4x+1)^{1/2}\div ½]\ [\div 4]\ (+c)$	B1 B1	Expect $½(4x+1)^{1/2}\ (+c)$
	$\text{f}'(2)=0 \Rightarrow \frac{3}{2}+c=0 \Rightarrow c=-\frac{3}{2}$ (Sufficient)	B1 FT	Expect $½(4x+1)^{1/2}-\frac{3}{2}$. FT on *their* $\text{f}'(x)=k(4x+1)^{1/2}+c$. (i.e. $c=-3k$)
	Total:	**3**	
11(ii)	$\text{f}''(0)=1$ SOI	B1	
	$\text{f}'(0)=1/2-1½=-1$ SOI	B1 FT	Substitute $x=0$ into *their* f ′(x) but must not involve c otherwise **B0B0**
	f(0) = – 3	B1 FT	FT for 3 terms in AP. FT for 3rd **B1** dep on 1st **B1**. Award marks for the AP method only.
	Total:	**3**	
11(iii)	$\text{f}(x)=\left[½(4x+1)^{3/2}\div 3/2\div 4\right]-\left[1½x\right]\ (+k)$	B1 FT B1 FT	Expect $(1/12)(4x+1)^{3/2}-1½x\ (+k)$. FT from *their* f ′($x$) but c numerical.
	$-3=1/12-0+k \Rightarrow k=-37/12$ CAO	M1A1	Sub $x=0, y=$ *their* $\text{f}(0)$ into *their* f(x). Dep on cx & k present (c numerical)
	Minimum value = f(2) = $\frac{27}{12}-3-\frac{37}{12}=-\frac{23}{6}$ or -3.83	A1	
	Total:	**5**	

Cambridge International Examinations
Cambridge International Advanced Subsidiary and Advanced Level

MATHEMATICS **9709/12**

Paper 1 Pure Mathematics **March 2017**

MARK SCHEME

Maximum Mark: 75

Published

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Question	Answer	Marks	Guidance
1	$(3k)^2 - 4 \times 2 \times k$	**M1**	Attempt $b^2 - 4ac$
	$9k^2 - 8k > 0$ soi Allow $9k^2 - 8k \geqslant 0$	**A1**	Must involve correct inequality. Can be implied by correct answers
	0, 8/9 soi	**A1**	
	$k < 0$, $k > 8/9$ (or 0.889)	**A1**	Allow $(-\infty, 0)$, $(8/9, \infty)$
	Total:	**4**	

Question	Answer	Marks	Guidance
2	$5C2 \left(\frac{1}{ax}\right)^3 \left(2ax^2\right)^2$ soi	**B1**	Seen or implied. Can be part of an expansion.
	$10 \times \frac{1}{a^3} \times 4a^2 = 5$ soi	**M1A1**	M1 for identifying relevant term and equating to 5, all correct. Ignore extra x
	$a = 8$ cao	**A1**	
	Total:	**4**	

Question	Answer	Marks	Guidance
3(i)	$V = \frac{1}{12}h^3$ oe	B1	
	Total:	1	
3(ii)	$\frac{dV}{dh} = \frac{1}{4}h^2$ or $\frac{dh}{dV} = 4(12v)^{-2/3}$	M1A1	Attempt differentiation. Allow incorrect notation for M. For A mark accept *their* letter for volume - but otherwise correct notation. Allow V'
	$\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt} \quad = \frac{4}{h^2} \times 20$ soi	DM1	Use chain rule correctly with $\frac{d(V)}{dt} = 20$. Any equivalent formulation. Accept non-explicit chain rule (or nothing at all)
	$\left(\frac{dh}{dt}\right) = \frac{4}{10^2} \times 20 = 0.8$ or equivalent fraction	A1	
	Total:	4	

Question	Answer	Marks	Guidance
4(i)	$ABC = \pi/2 - \pi/7 = 5\pi/14$. $CBD = \pi - 5\pi/14 = 9\pi/14$	**B1**	**AG** Or other valid exact method.
	Total:	**1**	
4(ii)	$\sin\frac{\pi}{7} = \frac{\frac{1}{2}BC}{8}$ **or** $\frac{BC}{\sin\frac{2\pi}{7}} = \frac{8}{\sin\frac{5\pi}{14}}$ **or** $BC^2 = 8^2 + 8^2 - 2(8)(8)\cos\frac{2\pi}{7}$	**M1**	
	$BC = 6.94(2)$	**A1**	
	arc CD = *their* $6.94 \times 9\pi/14$	**M1**	Expect 14.02(0)
	arc $CB = 8 \times 2\pi/7$	**M1**	Expect 7.18(1)
	perimeter = 6.94 + 14.02 + 7.18 = 28.1	**A1**	
	Total:	**5**	

Question	Answer	Marks	Guidance
5(i)	$\tan x = \cos x \rightarrow \sin x = \cos^2 x$	M1	Use tan = sin/cos and multiply by cos
	$\sin x = 1 - \sin^2 x$	M1	Use $\cos^2 x = 1 - \sin^2 x$
	$\sin x = 0.6180$. Allow $(-1 + \sqrt{5})/2$	M1	Attempt soln of quadratic in $\sin x$. Ignore solution –1.618. Allow x = 0.618
	x-coord of A = $\sin^{-1} 0.618 = 0.666$ cao	A1	Must be radians. Accept 0.212π
	Total:	**4**	
5(ii)	***EITHER*** x-coord of B is $\pi - \textit{their}\, 0.666$	**(M1**	Expect 2.475(3). Must be radians throughout
	y-coord of B is $\tan(\textit{their}\, 2.475)$ or $\cos(\textit{their}\, 2.475)$	M1	
	$x = 2.48$, $y = -0.786$ or -0.787 cao	**A1)**	Accept $x = 0.788\pi$
	OR y-coord of B is – (cos or tan (*their* 0.666))	**(M1**	
	x-coord of B is $\cos^{-1}(\textit{their}\ y)$ or $\pi + \tan^{-1}(\textit{their}\ y)$	M1	
	$x = 2.48$, $y = -0.786$ or -0.787	**A1)**	Accept $x = 0.788\pi$
	Total:	**3**	

Question	Answer	Marks	Guidance
6(i)	**BA** = **OA** – **OB** = –5**i** – **j** + 2**k**	B1	Allow vector reversed. Ignore label **BA** or **AB**
	OA.BA = –10 – 3 + 10 = –3	M1	soi by ±3
	|**OA**|×|**BA**| = $\sqrt{2^2+3^2+5^2}\times\sqrt{5^2+1^2+2^2}$	M1	Prod. of mods for at least 1 correct vector or reverse.
	$\cos OAB = \dfrac{+/-3}{\sqrt{38}\times\sqrt{30}}$	M1	
	$OAB = 95.1^{\circ}$ (or 1.66^{c})	A1	
	Total:	**5**	
6(ii)	$\Delta\, OAB = \dfrac{1}{2}\sqrt{38}\times\sqrt{30}\sin 95.1$. Allow $½\sqrt{38}\times\sqrt{74}\sin 39.4$	M1	Allow their moduli product from **(i)**
	= 16.8	A1	cao but <u>NOT</u> from sin 84.9 (1.482^{c})
	Total:	**2**	

Question	Answer	Marks	Guidance
7(i)	$f'(x) = \left[\frac{3}{2}(4x+1)^{1/2}\right]\ [4]$	**B1B1**	Expect $6(4x+1)^{1/2}$ but can be unsimplified.
	$f''(x) = 6\times 1/2\times(4x+1)^{-1/2}\times 4$	**B1√**	Expect $12(4x+1)^{-1/2}$ but can be unsimplified. Ft from *their* $f'(x)$.
	Total:	**3**	
7(ii)	f(2), f ′(2), *k*f ″(2) = 27, 18, 4*k* OR 12	**B1B1√B1√**	Ft dependent on attempt at differentiation
	27/18 = 18/4*k* oe OR *k*f ″(2) = 12 $\Rightarrow k = 3$	**M1A1**	
	Total:	**5**	

Question	Answer	Marks	Guidance
8(i)	$\text{gf}(x) = 3\left(2x^2+3\right)+2 = 6x^2+11$	B1	**AG**
	$\text{fg}(x) = 2\left(3x+2\right)^2+3$ Allow $18x^2+24x+11$	B1	ISW if simplified incorrectly. Not retrospectively from **(ii)**
	Total:	2	
8(ii)	$y = 2\left(3x+2\right)^2+3 \quad \Rightarrow 3x+2 = (\pm)\sqrt{(y-3)/2}$ oe	M1	Subtract 3;divide by 2;square root. Or *x/y* interchanged. Allow $\frac{\sqrt{y-3}}{2}$ for 1st M
	$\Rightarrow x = (\pm)\frac{1}{3}\sqrt{(y-3)/2}-\frac{2}{3}$ oe	M1	Subtract 2; divide by 3; Indep. of 1st M1. Or *x/y* interchanged.
	$\Rightarrow (\text{fg})^{-1}(x) = \frac{1}{3}\sqrt{(x-3)/2}-\frac{2}{3}$ oe	A1	Must be a function of x. Allow alt. method $\text{g}^{-1}\text{f}^{-1}(x)$ OR $18\left(x+\frac{2}{3}\right)^2+3 \Rightarrow (\text{fg})^{-1}(x) = \sqrt{\frac{x-3}{18}}-\frac{2}{3}$
	Solve *their* $(\text{fg})^{-1}(x) \geqslant 0$ or attempt range of fg	M1	Allow <u>range</u> $\geqslant 3$ for M only. Can be implied by correct answer or $x > 11$
	Domain is $x \geqslant 11$	A1	
	Total:	5	

Question	Answer	Marks	Guidance
8(iii)	$6(2x)^2 + 11 = 2(3x+2)^2 + 3$	M1	Replace x with $2x$ in gf and equate to *their* fg(x) from (i). Allow $\underline{12}\,x^2 + 11 =$
	$6x^2 - 24x = 0$ oe	A1	Collect terms to obtain correct quadratic expression.
	$x = 0$, 4	A1	Both required
	Total:	**3**	

Question	Answer	Marks	Guidance
9(i)	$\frac{dy}{dx} = 2x - 2$. At $x = 2$, $m = 2$	B1B1	Numerical m
	Equation of tangent is $y - 2 = 2(x-2)$	B1	Expect y = $2x - 2$
	Total:	**3**	
9(ii)	Equation of normal $y - 2 = -½(x-2)$	M1	Through (2, 2) with gradient = $-1/m$. Expect $y = -½x + 3$
	$x^2 - 2x + 2 = -½x + 3 \rightarrow 2x^2 - 3x - 2 = 0$	M1	Equate and simplify to 3-term quadratic
	$x = -½$, $y = 3¼$	A1A1	Ignore answer of (2, 2)
	Total:	**4**	

Question	Answer	Marks	Guidance
9(iii)	At $x = -½$, grad $= 2(-½) - 2 = -3$	B1√	Ft *their* –½.
	Equation of tangent is $y - 3¼ = -3(x + ½)$	*M1	Through *their B* with grad *their* –3 (not m_1 or m_2). Expect $y = -3x + 7/4$
	$2x - 2 = -3x + 7/4$	DM1	Equate *their* tangents or attempt to solve simultaneous equations
	$x = 3/4,\ \ y = -½$	A1	Both required.
	Total:	4	

Question	Answer	Marks	Guidance
10(i)	$2x - 2/x^3 = 0$	M1	Set = 0.
	$x^4 = 1 \Rightarrow x = 1$ at A cao	A1	Allow 'spotted' $x = 1$
	Total:	2	
10(ii)	$f(x) = x^2 + 1/x^2\ (+c)$ cao	B1	
	$\frac{189}{16} = 16 + 1/16 + c$	M1	Sub (4, $\frac{189}{16}$). c must be present. Dep. on integration
	$c = -17/4$	A1	
	Total:	3	

Question	Answer	Marks	Guidance
10(iii)	$x^2+1/x^2-17/4=0 \Rightarrow 4x^4-17x^2+4\ (=0)$	M1	Multiply by $4x^2$ (or similar) to transform into 3-term quartic.
	$(4x^2-1)(x^2-4)\ (=0)$	M1	Treat as quadratic in x^2 and attempt solution or factorisation.
	$x=½,\ 2$	A1A1	Not necessary to distinguish. Ignore negative values. No working scores 0/4
	Total:	**4**	
10(iv)	$\int(x^2+x^{-2}-17/4)\mathrm{d}x\ =\ \frac{x^3}{3}-\frac{1}{x}-\frac{17x}{4}$	B2,1,0√	Mark final integral
	$(8/3-1/2-17/2)-(1/24-2-17/8)$	M1	Apply *their* limits from **(iii)** (Seen). Dep. on integration of at least 1 term of y
	Area $=9/4$	A1	Mark final answer. $\int y^2$ scores 0/4
	Total:	**4**	

Cambridge International AS & A Level

Cambridge International Examinations
Cambridge International Advanced Subsidiary and Advanced Level

MATHEMATICS **9709/11**

Paper 1 **October/November 2016**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the October/November 2016 series for most Cambridge IGCSE®, Cambridge International A and AS Level components and some Cambridge O Level components.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol $\sqrt{}$ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

  - Note: B2 or A2 means that the candidate can earn 2 or 0.
    B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF/OE Any Equivalent Form (of answer is equally acceptable) / Or Equivalent

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

SOI Seen or implied

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

1 (i)	$(x+3)^2-7$	**B1B1**	[2]	For $a = 3$, $b = -7$
(ii)	$1,-7$ seen $x>1,\quad x<-7$ oe	**B1** **B1**	[2]	$x>1$ or $x<-7$ Allow $x\leqslant -7, x\geqslant 1$ oe
2	$8C6(2x)^6\left(\frac{1}{2x^3}\right)^2$ soi $28\times 64\times\frac{1}{4}$ oe (powers and factorials evaluated) 448	**B1** **B2,1,0** **B1**	[4]	May be seen within a number of terms May be seen within a number of terms Identified as answer
3 (i)	$2r\alpha + r\alpha + 2r = 4.4r$ $\alpha = 0.8$	**M1** **A1**	[2]	At least 3 of the 4 terms required
(ii)	$½(2r)^2\,0.8 - ½(r^2)0.8 = 30$ $(3/2)r^2\times 0.8 = 30 \rightarrow r=5$	**M1A1**√ **A1**	[3]	Ft through on *their* α
4 (i)	$C = (4, -2)$ $m_{AB} = -1/2 \rightarrow m_{CD} = 2$ Equation of CD is $y+2=2(x-4)$ oe $y = 2x-10$	**B1** **M1** **M1** **A1**	[4]	Use of $m_1m_2=-1$ on their m_{AB} Use of *their C* and m_{CD} in a line equation
(ii)	$AD^2=(14-0)^2+(-7-(-10))^2$ AD = 14.3 or √205	**M1** **A1**	[2]	Use *their D* in a correct method
5	$a(1+r)=50$ or $\frac{a(1-r^2)}{1-r}=50$ $ar(1+r)=30$ or $\frac{a(1-r^3)}{1-r}=30+a$ Eliminating a or r $r=3/5$ $a=125/4$ oe $S=625/8$ oe	**B1** **B1** **M1** **A1** **A1** **A1**√	[6]	 Or otherwise attempt to solve for r Any correct method Ft through on *their* r and a $(-1<r<1)$

6	**(i)**	$\cos^4 x = \left(1-\sin^2 x\right)^2 \quad = 1-2\sin^2 x+\sin^4 x$ AG	**B1**	[1]	Could be LHS to RHS or vice versa
	(ii)	$8\sin^4 x+1-2\sin^2 x+\sin^4 x = 2\left(1-\sin^2 x\right)$ $9\sin^4 x = 1$ $x = 35.3^o$ (or any correct solution) Any correct second solution from 144.7°, 215.3°, 324.7° The remaining 2 solutions	**M1** **A1** **A1** **A1**√ **A1**	[5]	Substitute for $\cos^4 x$ and $\cos^2 x$ or OR sub for $\sin^4 x \to 3\cos^2 x = 2$ $\to \cos x = (\pm)\sqrt{2/3}$ Allow the first 2 **A1** marks for radians (0.616, 2.53, 3.76, 5.67)
7	**(i)**	A = (½, 0)	**B1**	[1]	Accept $x = 0$ *at* $y = 0$
	(ii)	$\int(1-2x)^{\frac{1}{2}}\,dx = \left[\dfrac{(1-2x)^{3/2}}{3/2}\right]\left[\div(-2)\right]$ $\int(2x-1)^2\,dx = \left[\dfrac{(2x-1)^3}{3}\right]\left[\div 2\right]$ $\left[0-(-1/3)\right]-\left[0-(-1/6)\right]$ 1/6	**B1B1** **B1B1** **M1** **A1**	[6]	May be seen in a single expression May use $\int_a^1 x\,dy$, may expand $(2x-1)^2$ Correct use of *their* limits
8	**(i)**	fg(x) = $5x$ Range of fg is $y \geqslant 0$ oe	**M1A1** **B1**	[3]	only Accept $y > 0$
	(ii)	$y = 4/(5x+2) \Rightarrow x = (4-2y)/5y$ oe $g^{-1}(x) = (4-2x)/5x$ oe 0, 2 with no incorrect inequality $0 < x \leqslant 2$ oe, c.a.o.	**M1** **A1** **B1,B1** **B1**	[5]	Must be a function of x
9	**(i)**	**XP** = −4**i** + (p – 5)**j** + 2**k** [−4**i** + (p – 5)**j** + 2**k**].(p**j** + 2**k**) = 0 $p^2-5p+4=0$ $p = 1$ or 4	**B1** **M1** **A1** **A1**	[4]	Or **PX** Attempt scalar prod with **OP/PO** and set = 0 (= 0 could be implied)
	(ii)	**XP** = −4**i** + 4**j** + 2**k** → $\lvert \mathbf{XP} \rvert = \sqrt{16+16+4}$ Unit vector = 1 / 6 (−4**i** + 4**j** + 2**k**) oe	**M1** **A1**	[2]	Expect 6
	(iii)	**AG** = −4**i** + 15**j** + 2**k** **XQ** = λ**AG** soi λ = 2/3 → **XQ** $= -\dfrac{8}{3}\mathbf{i} + 10\mathbf{j} + \dfrac{4}{3}\mathbf{k}$	**B1** **M1** **A1**	[3]	

10 (i)	$3z-\frac{2}{z}=-1 \Rightarrow 3z^2+z-2=0$	M1		Express as 3-term quad. Accept $x^{1/2}$ for z
	$x^{1/2}(\text{or } z)=2/3 \text{ or } -1$	A1		(OR
	$x=4/9$ only	A1	[3]	$3x-1=-\sqrt{x},\ 9x^2-13x+4=0$ **M1, A1,A1** $x=4/9$)
(ii)	$\mathrm{f}(x)=\frac{3x^{3/2}}{3/2}-\frac{2x^{1/2}}{1/2} \quad (+c)$	B1B1		
	Sub $x=4, y=10 \quad 10=16-8+c \Rightarrow c=2$	M1A1		c must be present
	When $x=\frac{4}{9},\ y=2\left(\frac{4}{9}\right)^{3/2}-4\left(\frac{4}{9}\right)^{1/2}+2$	M1		Substituting x value from part **(i)**
	$-2/27$	A1	[6]	
11 (i)	$\frac{\mathrm{d}y}{\mathrm{d}x}=-(x-1)^{-2}+9(x-5)^{-2}$	M1A1		May be seen in part **(ii)**
	$m_{\text{tangent}}=-\frac{1}{4}+\frac{9}{4}=2$	B1		
	Equation of normal is $y-5=-½(x-3)$	M1		Through (3, 5) and with $m=-1/m_{tangent}$
	$x=13$	A1	[5]	
(ii)	$(x-5)^2=9(x-1)^2$	B1		Set $\frac{\mathrm{d}y}{\mathrm{d}x}=0$ and simplify
	$x-5=(\pm)3(x-1)$ or $(8)(x^2-x-2)=0$	M1		Simplify further and attempt solution
	$x=-1$ or 2	A1		
	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=2(x-1)^{-3}-18(x-5)^{-3}$	B1		If change of sign used, x values close to the roots must be used and all must be correct
	When $x=-1,\ \frac{\mathrm{d}^2y}{\mathrm{d}x^2}=-\frac{1}{6}<0$ MAX	B1		
	When $x=2,\ \frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\frac{8}{3}>0$ MIN	B1	[6]	

Cambridge International AS & A Level

Cambridge International Examinations
Cambridge International Advanced Subsidiary and Advanced Level

MATHEMATICS	**9709/12**
Paper 1	**October/November 2016**
MARK SCHEME	
Maximum Mark: 75	

Published

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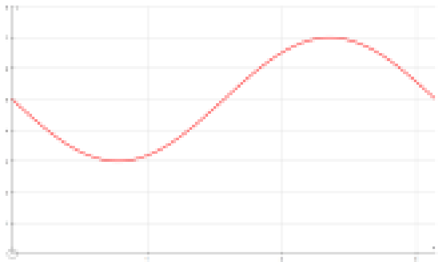
1	$(y)=8(4x+1)^{\frac{1}{2}}$ ÷ ½ ÷ 4 (+*c*) Uses x = 2 and y = 5 $c=-7$	**B1** **B1** **M1** **A1**	[4]	Correct integrand (unsimplified) without ÷4 ÷4. Ignore *c*. Substitution of correct values into an integrand to find c. $y=4\sqrt{4x+1}-7$
2 (i)	2sin2x = 6cos2x tan2x = k → tan2x = 3 or k =3	 **M1** **A1**	[2]	 Expand and collect as far as tan2x = a constant from sin ÷ cos soi cwo
(ii)	$x=\ (\tan^{-1}(their\ k))\div 2$ (71.6° or −108.4°) ÷2 x = 35.8°, −54.2° x = 0.624^c, – 0.946^c x = 0.198π^c, – 0.301π^c	**M1** **A1 A1**√	[3]	Inverse then ÷2. soi. √ on 1st answer +/ − 90° if in given range but no extra solutions in the given range. Both SR A1A0
3 (i)	$2x^2-6x+5>13$ $2x^2-6x-8(>0)$ (x =) −1 and 4. $x>4$, $x<-1$	 **M1** **A1** **A1**	[3]	 Sets to 0 + attempts to solve Both values required Allow all recognisable notation.
(ii)	$2x^2-6x+5=2x+k$ $\rightarrow\ 2x^2-8x+5-k(=0)$ Use of b^2-4ac → −3 **OR** $\frac{dy}{dx}=4x-6$ $4x-6=2$ $x=2$ $x=2\rightarrow y=1$ Using *their* (2,1) in $y=2x+k$ or $y=2x^2-6x+5$ $\rightarrow k=-3$	 **M1*** **DM1** **A1** **M1*** **DM1** **A1**	[3] [3]	 Equates and sets to 0. Use of discriminant Sets (their $\frac{dy}{dx}$) = 2 Uses *their* x = 2 and *their* y = 1

4	Term in $x = \frac{nx}{2}$ $(3-2x)(1+\frac{nx}{2}+\ldots) \rightarrow 7 = \frac{3n}{2} - 2$ $\rightarrow n = 6$ Term in $x^2 = \frac{n(n-1)}{2}\left(\frac{x}{2}\right)^2$ Coefficient of $x^2 = \frac{3n(n-1)}{8} - \frac{2n}{2}$ $= \frac{21}{4}$	**B1** **M1** **A1** **B1** **M1** **A1**	[6]	Could be implied by use of a numerical *n*. (Their 2 terms in *x*) = 7 May be implied by (their *n*) × (their *n-1*) ÷ 8. Considers 2 terms in x^2. aef
5	$A(a, 0)$ and $B(0, b)$ $a^2+b^2=100$ M has coordinates $\left(\frac{a}{2}, \frac{b}{2}\right)$ M lies on $2x+y=10$ $\rightarrow a + \frac{b}{2} = 10$ Sub $\rightarrow a^2+(20-2a)^2=100$ or $\left(10-\frac{b}{2}\right)^2+b^2=100$ $\rightarrow a = 6, b = 8$.	**B1** **M1*** **B1**√ **M1*** **DM1** **A1**	[6]	soi Uses Pythagoras with their *A* & *B*. √ on their *A* and *B*. Subs into given line, using their M, to link *a* and *b*. Forms quadratic in *a* or in *b*. cao

6 (i)	$\frac{r}{10} = \sin 0.6$ or $\frac{r}{10} = \cos 0.97$ or $BD = \sqrt{200 - 200\cos 1.2}\,(=11.3)$ $r = 10 \times 0.5646$, $r = 10 \times \sin 0.6$, $r = 10 \times \cos 0.971$ or $r = ½\,BD$ $\rightarrow\ r = 5.646$ AG	**M1** **A1**	[2]	Or other valid alternative.
(ii)	Major arc $= 10(\theta)$ $(= 50.832)$ $\theta = 2\pi - 1.2$ $(= 5.083)$ or C $= 2\pi \times 10$, Minor arc $= 1.2 \times 10$ Semicircle $= 5.646\pi$ $(= 17.737)$ Major arc + semicircle $= 68.6$	**M1** **B1** **A1**	[3]	$\theta = 2\pi - 1.2$ or $\pi - 1.2$ Implied by 5.1
(iii)	Area of major sector $= ½10^2(\theta)$ $(= 254.159)$ Area of triangle OBD $= ½10^2\sin 1.2$ $(= 46.602)$ Area = semicircle + sector + triangle $(= 50.1 + 254.2 + 46.6)$ $= 351$	**M1** **M1** **A1**	[3]	$\theta = 2\pi - 1.2$ or $\pi - 1.2$ Use of $½ab\sin C$ or other complete method
7 (i)	$\frac{dy}{dx} = \frac{-3}{(2x-1)^2} \times 2$	**B1** **B1**	[2]	B1for a single correct term (unsimplified) without ×2.
(ii)	e.g. Solve for $\frac{dy}{dx} = 0$ is impossible.	**B1**√	[1]	Satisfactory explanation.
(iii)	If $x = 2$, $\frac{dy}{dx} = \frac{-6}{9}$ and $y = 3$ Perpendicular has $m = \frac{9}{6}$ $\rightarrow\ y - 3 = \frac{3}{2}(x-2)$ Shows when x=0 then y=0 AG	**M1*** **M1*** **DM1** **A1**	[4]	Attempt at both needed. Use of $m_1 m_2 = -1$ numerically. Line equation using (2, their 3) and their m.
(iv)	$\frac{dx}{dt} = -0.06$ $\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt} \rightarrow -\frac{2}{3} \times -0.06 = 0.04$	**M1 A1**	[2]	

8 (a) (i)	$200+(15-1)(+/-5)$ $= 130$	**M1** **A1**	 [2]	Use of nth term with $a = 200$, $n = 14$ or 15and $d = +/- 5$.
(ii)	$\frac{n}{2}\left[400+(n-1)(+/-5)\right] = (3050)$ $\rightarrow\ 5n^2-405n+6100\ (=0)$ $\rightarrow\ 20$	**M1** **A1** **A1**	 [3]	Use of S_n a=200 and $d = +/- 5$.
(b) (i)	ar^2, ar^5 $\rightarrow$ $r = ½$ $\frac{63}{2} = \frac{a\left(1-½^6\right)}{½}$ $\rightarrow$ $a = 16$	**M1 A1** **M1 A1**	 [4]	Both terms correct. Use of $S_n = 31.5$ with a numeric r.
(ii)	Sum to infinity = $\frac{16}{½}$ = 32	**B1**$\sqrt{}$	[1]	$\sqrt{}$ for their a and r with $\lvert r \rvert < 1$.
9 (i)	$-4-6-6=-16$ $\sqrt{x_1^2+y_1^2+z_1^2}$ or $\sqrt{x_2^2+y_2^2+z_2^2}$ $3 \times 7 \times \cos\theta = -16$ $\rightarrow$ $\theta = 139.6°$ or 2.44^c or 0.776π	**M1** **M1** **M1** **A1**	 [4]	Use of $x_1x_2 + y_1y_2 + z_1z_2$ on their $\overrightarrow{OA}$ & $\overrightarrow{OB}$ Modulus once on either their $\overrightarrow{OA}$ or $\overrightarrow{OB}$ All linked using their $\overrightarrow{OA}$ & $\overrightarrow{OB}$
(ii)	$\overrightarrow{AC} = \text{c} - \text{a} = \begin{pmatrix}0\\8\\6\end{pmatrix}$ Magnitude = 10 Scaling $\rightarrow$ $\frac{15}{\textit{their}10} \times \begin{pmatrix}0\\8\\6\end{pmatrix} = \begin{pmatrix}0\\12\\9\end{pmatrix}$	**B1** **M1** **A1**	 [3]	 For 15 × *their* unit vector.
(iii)	$\begin{pmatrix}2+2p\\6-2p\\5-p\end{pmatrix}$ $\rightarrow -2(2+2p)+3(6-2p)+6(5-p)=0$ $\rightarrow$ $p = 2¾$	**B1** **M1** **A1**	 [3]	Single vector soi by scalar product. Dot product of ($p\,\overrightarrow{OA} + \overrightarrow{OC}$) and $\overrightarrow{OB} = 0$.

10 (i)	$3 \leqslant f(x) \leqslant 7$	**B1** **B1**	[2]	Identifying both 3 and 7 or correctly stating one inequality. Completely correct statement. **NB** $3 \leqslant x \leqslant 7$ scores B1B0
(ii)		**B1*** **DB1**	[2]	One complete oscillation of a sinusoidal curve between 0 and π. All correct, initially going downwards, all above $f(x)=0$
(iii)	$5\text{-}2\sin 2x = 6 \ \rightarrow\ \sin 2x = -\frac{1}{2}$ $\rightarrow\ 2x = \frac{7\pi}{6}$ or $\frac{11\pi}{6}$ $\rightarrow\ x = \frac{7\pi}{12}$ or $\frac{11\pi}{12}$ 0.583π or 0.917π $\frac{\pi + 0.524}{2}$ or $\frac{2\pi - 0.524}{2}$ 1.83^c or 2.88^c	**M1** **A1 A1**$\checkmark$	[3]	Make $\sin 2x$ the subject. $\checkmark$ for $\frac{3\pi}{2}$ – 1st answer from $\sin 2x = -\frac{1}{2}$ only, if in given range SR A1A0 for both.
(iv)	$k = \frac{\pi}{4}$	**B1**	[1]	
(v)	$2\sin 2x = 5 - y \ \rightarrow\ \sin 2x = \frac{1}{2}(5 - y)$ $(g^{-1}(x)) = \frac{1}{2}\sin^{-1}\frac{(5-x)}{2}$	**M1** **M1** **A1**	[3]	Makes $\pm\sin 2x$ the subject soi by final answer. Correct order of operations including correctly dealing with " – ". Must be a function of x

Cambridge International Examinations
Cambridge International Advanced Subsidiary and Advanced Level

MATHEMATICS **9709/13**

Paper 1 **October/November 2016**

MARK SCHEME

Maximum Mark: 75

Published

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Mark Scheme Notes

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- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

  - Note: B2 or A2 means that the candidate can earn 2 or 0.
    B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

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CWO Correct Working Only – often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

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PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

1	$kx^2 - 3x = x - k \quad \Rightarrow \quad kx^2 - 4x + k(=0)$	M1		Eliminate y and rearrange into 3-term quad
	$(-4)^2 - 4(k)(k)$ soi	M1		$b^2 - 4ac$.
	$k > 2$, $k < -2$ cao Allow $(2, \infty)$ etc. Allow $2<k<-k$	A1	[3]	
2	$(+/-)20 \times 3^3 (x^3)$, $10a^3 (x^3)$ soi	B1B1		Each term can include x^3
	$-540 + 10a^3 = 100$ oe	M1		Must have 3 terms and include a^3 and 100
	$a = 4$	A1	[4]	
3	$4\sin^2 x = 6\cos^2 x \Rightarrow \tan^2 x = \frac{6}{4}$ or $4\sin^2 x = 6(1 - \sin^2 x)$	M1		Or $4(1 - \cos^2 x) = 6\cos^2 x$
	[tan x = (±)1.225 or sin x = (±)0.7746 or cos x = (±)0.6325]			
	x= 50.8 (Allow 0.886 (rad))	A1		Or any other angle correct
	Another angle correct	A1√		Ft from 1st angle (Allow radians)
	$x = 50.8°, 129.2°, 230.8°, 309.2°$ [0.886, 2.25/6, 4.03, 5.40 (rad)]	A1	[4]	All 4 angles correct in degrees
4	$f'(x) = 3x^2 - 6x - 9$ soi	B1		
	Attempt to solve $f'(x) = 0$ or $f'(x) > 0$ or $f'(x) \geqslant 0$ soi	M1		
	$(3)(x-3)(x+1)$ or 3,–1 seen or 3 only seen	A1		With or without equality/inequality signs
	Least possible value of n is 3. Accept n = 3. Accept $n \geqslant 3$	A1	[4]	Must be in terms of n
5 (i)	$\cos 0.9 = OE / 6$ or $= \sin\left(\frac{\pi}{2} - 0.9\right)$ oe	M1		Other methods possible
	$OE = 6\cos 0.9 = 3.73$ oe AG	A1	[2]	
(ii)	Use of $(2\pi - 1.8)$ or equivalent method	M1		Expect 4.48
	Area of large sector $= ½ \times 6^2 \times (2\pi - 1.8)$ oe	M1		Or $\pi 6^2 - ½6^2 1.8$. Expect 80.70
				Expect 12.52
	Area of small sector $½ \times 3.73^2 \times 1.8$	M1		Other methods possible
	Total area = 80.7(0) + 12.5(2) = 93.2	A1	[4]	
6 (i)	$\frac{2+x}{2} = n \Rightarrow x = 2n - 2$	B1		No MR for (½(2+n), ½(m – 6))
	$\frac{m+y}{2} = -6 \Rightarrow y = -12 - m$	B1	[2]	Expect $(2n-2, -12-m)$

(ii)	Sub *their* x, y into $y = x+1 \rightarrow -12-m = 2n-2+1$ $\frac{m+6}{2-n} = -1$ oe Not nested in an equation Eliminate a variable $m=-9,\ n=-1$	**M1*** **B1** **DM1** **A1A1**	[5]	Expect $m+2n=-11$ Expect $m-n=-8$ Note: other methods possible
7 (i)	**AB.AC** $=3-2-1=0$ hence perpendicular or 90° **AB.AD** $=3+4-7=0$ hence perpendicular or 90° **AC.AD** $=1-8+7=0$ hence perpendicular or 90° AG	**B1** **B1** **B1**	[3]	3 – 2 – 1 or sum of prods etc must be seen Or single statement: mutually perpendicular or 90˚ seen at least once .
(ii)	Area $ABC = (½)\sqrt{3^2+1^2+1^2} \times \sqrt{1^2+(-2)^2+(-1)^2}$ $= ½\sqrt{11} \times \sqrt{6}$ Vol. $= ⅓\times$ *their* $\Delta ABC \times \sqrt{1^2+4^2+(-7)^2}$ $= \frac{1}{6}\sqrt{66}\times\sqrt{66} = 11$	**M1** **A1** **M1** **A1**	[4]	 Expect $½\sqrt{66}$ Not 11.0
8 (i)	$(2x+3)^2+1$ Cannot score retrospectively in **(iii)**	**B1B1B1**	[3]	For $a=2,\ b=3,\ c=1$
(ii)	$g(x) = 2x+3$ cao	**B1**	[1]	In **(ii)**,**(iii)** Allow if from $4\left(x+\frac{3}{2}\right)^2+1$
(iii)	$y=(2x+3)^2+1 \Rightarrow 2x+3 = (\pm)\sqrt{y-1}$ or ft from **(i)** $x = (\pm)\frac{1}{2}\sqrt{y-1}-\frac{3}{2}$ or ft from **(i)** $(fg)^{-1}(x) = \frac{1}{2}\sqrt{x-1}-\frac{3}{2}$ cao Note alt. method $g^{-1}f^{-1}$ Domain is $(x) > 10$ ALT. method for first 3 marks: Trying to obtain $g^{-1}\left[f^{-1}(x)\right]$ $g^{-1} = ½(x-3),\ f^{-1} = \sqrt{x-1}$ A1 for $\frac{1}{2}\sqrt{x-1}-\frac{3}{2}$	**M1** **M1** **A1** **B1** ***M1** **DM1** **A1**	[4]	Or with *x/y* transposed. Or with *x/y* transposed Allow sign errors. Must be a function of *x*. Allow *y* = ... Allow (10, ∞), 10 < *x* < ∞ etc. but not with *y* or f or g involved. Not ⩾10 Both required

9 (a)	$\frac{6}{1-r}=\frac{12}{1+r}$ $r=\frac{1}{3}$ $S=9$	**M1** **A1** **A1**	[3]	
(b)	$\frac{13}{2}\left[2\cos\theta+12\sin^2\theta\right]=52$ $2\cos\theta+12(1-\cos^2\theta)=8\rightarrow 6\cos^2\theta-\cos\theta-2(=0)$ $\cos\theta=2/3$ or $-1/2$ soi $\theta=0.841$, 2.09 Dep on previous A1	**M1*** **DM1** **A1** **A1A1**	[5]	Use of correct formula for sum of AP Use $s^2=1-c^2$ & simplify to 3-term quad Accept 0.268π, 2π/3. SRA1 for 48.2˚, 120˚ Extra solutions in range –1
10 (i)	at $x=a^2$, $\frac{dy}{dx}=\frac{2}{a^2}+\frac{1}{a^2}$ or $2a^{-2}+a^{-2}\left(=\frac{3}{a^2}\text{ or }3a^{-2}\right)$ $y-3=\frac{3}{a^2}\left(x-a^2\right)$ or $y=\frac{3}{a^2}x+c\rightarrow 3=\frac{3}{a^2}a^2+c$ $y=\frac{3}{a^2}x$ or $3a^{-2}x$ cao	**B1** **M1** **A1**	[3]	$\frac{2}{a^2}+\frac{1}{a^2}$ or $2a^{-2}+a^{-2}$ seen anywhere in **(i)** Through $(a^2,3)$ & with *their* grad as f(a)
(ii)	$(y)=\frac{2}{a}\frac{x^{1/2}}{1/2}+\frac{ax^{-1/2}}{-1/2}$ $(+c)$ sub $x=a^2$, $y=3$ into $\int dy/dx$ $c=1$ $\left(y=\frac{4x^{1/2}}{a}-2ax^{-1/2}+1\right)$	**B1B1** **M1** **A1**	[4]	*c* must be present. Expect $3=4-2+c$
(iii)	sub $x=16$, $y=8\rightarrow$ $8=\frac{4}{a}\times4-2a\times\frac{1}{4}+1$ $a^2+14a-32(=0)$ $a=2$ $A=(4, 3)$, $B=(16, 8)$ $AB^2=12^2+5^2$ $\rightarrow AB=13$	***M1** **A1** **A1** **DM1A1**	[5]	Sub into *their y* Allow –16 in addition

11 (i)	Attempt diffn. and equate to 0 $\frac{dy}{dx} = -k(kx-3)^{-2} + k = 0$	*M1		Must contain $(kx-3)^{-2}$ + other term(s)
	$(kx-3)^2 = 1$ or $k^3x^2 - 6k^2x + 8k(=0)$	DM1		Simplify to a quadratic
	$x = \frac{2}{k}$ or $\frac{4}{k}$	*A1*A1		Legitimately obtained
	$\frac{d^2y}{dx^2} = 2k^2(kx-3)^{-3}$	B1√		Ft must contain $Ak^2(kx-3)^{-3}$ where $A>0$
	When $x = \frac{2}{k}$, $\frac{d^2y}{dx^2} = (-2k^2) < 0$ MAX All previous	DB1		Convincing alt. methods (values either side) must show which values used & cannot use $x = 3/k$
	When $x = \frac{4}{k}$, $\frac{d^2y}{dx^2} = (2k^2) > 0$ MIN working correct	DB1	[7]	
(ii)	$V = (\pi)\int\left[(x-3)^{-1} + (x-3)\right]^2 dx$	*M1		Attempt to expand y^2 and then integrate
	$= (\pi)\int[(x-3)^{-2} + (x-3)^2 + 2\,]dx$	A1		
	$= (\pi)\left[-(x-3)^{-1} + \frac{(x-3)^3}{3}(+2x)\right]$ Condone missing $2x$	A1		Or $\left[-(x-3)^{-1} + \frac{x^3}{3} - 3x^2 + 9x + 2x\right]$
	$= (\pi)\left[1 - \frac{1}{3} + 4 - \left(\frac{1}{3} - 9 + 0\right)\right]$	DM1		Apply limits 0→2
	$= 40\pi/3$ oe or 41.9	A1	[5]	2 missing → $28\pi/3$ scores M1A0A1M1A0

Cambridge International Examinations
Cambridge International Advanced Subsidiary and Advanced Level

MATHEMATICS **9709/11**

Paper 1 **May/June 2016**

MARK SCHEME

Maximum Mark: 75

Published

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CWO Correct Working Only – often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

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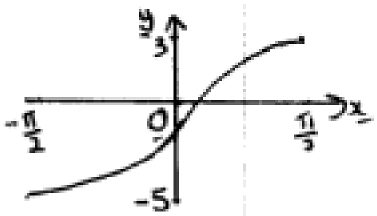
PA–1 This is deducted from A or B marks in the case of premature approximation. The PA–1 penalty is usually discussed at the meeting.

1	$\left(x-\frac{3}{2x}\right)^6$ Term is ${}^6C_3 \times x^3 \times \left(\frac{-3}{2x}\right)^3$ $\rightarrow -67.5$ oe	 **B1 B1** **B1** [3]	 B1 for Bin coeff. B1 for rest.
2	$3\sin^2\theta = 4\cos\theta - 1$ Uses $s^2 + c^2 = 1$ $\rightarrow 3c^2 + 4c - 4$ (= 0) ($\rightarrow c = \frac{2}{3}$ or -2) $\rightarrow \theta = 48.2°$ or $311.8°$ 0.841, 5.44 rads, **A1** only (0.268π, 1.73π)	 **M1 A1** **A1 A1**√ [4]	Equation in $\cos\theta$ only. All terms on one side of (=) For 360° − 1st answer.
3	$x = \frac{12}{y^2} - 2.$ Vol = $(\pi) \times \int x^2 \, dy$ $\rightarrow \left[\frac{-144}{3y^3} + 4y + \frac{48}{y}\right]$ Limits 1 to 2 used $\rightarrow 22\pi$	 **M1** **3 × A1** **A1** [5]	Ignore omission of π at this stage Attempt at integration Un-simplified only from correct integration
4 (i)	$\frac{dy}{dx} = 2 - 8(3x+4)^{-½}$ ($x = 0$, $\rightarrow \frac{dy}{dx} = -2$) $\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt} \rightarrow -0.6$	 **M1A1** [2]	Ignore notation. Must be $\frac{dy}{dx} \times 0.3$
(ii)	$y = \{2x\}\left\{-\frac{8\sqrt{3x+4}}{\frac{1}{2}} \div 3\right\} \quad (+c)$ $x = 0$, $y = \frac{4}{3} \rightarrow c = 12.$	**B1 B1** **M1 A1** [4]	No need for $+c$. Uses x, y values after ∫ with c
5 (i)	$A = 2y \times 4x (= 8xy)$ $10y + 12x = 480$ $\rightarrow A = 384x - 9.6x^2$	**B1** **B1** **B1** [3]	 answer given

(ii)	$\frac{dA}{dx} = 384 - 19.2x$ $= 0$ when $x = 20$ $\rightarrow x = 20, y = 24.$ Uses $x = -\frac{b}{2a} = \frac{-384}{-19.2} = 20$, **M1, A1** $y = 24$, A1 From graph: **B1** for $x = 20$, **M1**, **A1** for $y = 24$	**B1** **M1** **A1** [3]	 Sets to 0 and attempt to solve oe Might see completion of square Needs both x and y Trial and improvement **B3**.
6 (a)	$y = 2x^2 - 4x + 8$ Equates with $y = mx$ and selects a, b, c Uses $b^2 = 4ac$ $\rightarrow m = 4$ or -12.	**M1** **M1** **A1** [3]	 Equate + solution or use of dy/dx Use of discriminant for both.
(b) (i)	$f(x) = x^2 + ax + b$ Eqn of form $(x-1)(x-9)$ $\rightarrow a = -10, b = 9$ (or using 2 sim eqns **M1 A1**)	**M1** **A1** [2]	Any valid method allow $(x+1)(x+9)$ for **M1** must be stated
(ii)	Calculus or $x = \frac{1}{2}(1 + 9)$ by symmetry $\rightarrow (5, -16)$	**M1** **A1** [2]	Any valid method
7 (i)	$CD = r\cos\theta$, $BD = r - r\sin\theta$ oe Arc $CB = r(\frac{1}{2}\pi - \theta)$ oe $\rightarrow P = r\cos\theta + r - r\sin\theta + r(\frac{1}{2}\pi - \theta)$ oe	**B1 B1** **B1** **B1**$\checkmark$ [4]	allow degrees but not for last B1 $\checkmark$ sum – assuming trig used
(ii)	Sector $= \frac{1}{2}.5^2.(\frac{1}{2}\pi - 0.6)$ (12.135) Triangle $= \frac{1}{2}.5\cos 0.6.5\sin 0.6$ (5.825) $\rightarrow$ Area = 6.31 (or $\frac{1}{4}$ circle − triangle – sector)	**M1** **M1** **A1** [3]	Uses $\frac{1}{2}r^2\theta$ Uses $\frac{1}{2}bh$ with some use of trig.

8	$y = 3x - \frac{4}{x}$		
	$\frac{dy}{dx} = 3 + \frac{4}{x^2}$	**B1**	
	m of $AB = 4$	**B1**	
	Equate $\rightarrow x = \pm 2$		
	$\rightarrow C\ (2, 4)$ and $D\ (-2, -4)$	**M1 A1**	Equating + solution.
	$\rightarrow M\ (0, 0)$ or stating M is the origin	**B1**$\checkmark$	$\checkmark$ on their C and D
	m of $CD = 2$		
	Perpendicular gradient ($= -\frac{1}{2}$)	**M1**	Use of $m_1 m_2 = -1$, must use m_{CD}
		A1	(not m = 4)
	$\rightarrow y = -\frac{1}{2}x$	[7]	
9 (a)	$a = 50$, $ar^2 = 32$	**B1**	seen or implied
	$\rightarrow r = \frac{4}{5}$ (allow $-\frac{4}{5}$ for M mark)	**M1**	Finding r and use of correct S_∞ formula
	$\rightarrow S_\infty = 250$	**A1**	Only if $\lvert r \rvert < 1$
		[3]	
(b) (i)	$2\sin x$, $3\cos x$, $(\sin x + 2\cos x)$.		
	$3c - 2s = (s + 2c) - 3c$	**M1**	Links terms up with AP, needs one expression for d.
	(or uses a, $a + d$, $a + 2d$)		
	$\rightarrow 4c = 3s \rightarrow t = \frac{4}{3}$	**M1 A1**	Arrives at $t = k$. ag
		[3]	
	SC uses $t = \frac{4}{3}$ to show		
	$u_1 = \frac{8}{5}, u_2 = \frac{9}{5}, u_3 = \frac{10}{5}$, **B1** only		
(ii)	$\rightarrow c = \frac{3}{5}$, $s = \frac{4}{5}$ or calculator $x = 53.1°$	**M1**	
	$\rightarrow a = 1.6$, $d = 0.2$	**M1**	Correct method for both a and d.
	$\rightarrow S_{20} = 70$	**A1**	(Uses S_n formula)
		[3]	
10 (i)	$\overrightarrow{OA} = \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$, $\overrightarrow{OB} = \begin{pmatrix} 5 \\ -1 \\ k \end{pmatrix}$, $\overrightarrow{OC} = \begin{pmatrix} 2 \\ 6 \\ -3 \end{pmatrix}$		
	$10 - 1 - 2k = 0 \rightarrow k = 4\frac{1}{2}$	**M1 A1**	Use of scalar product = 0.
		[2]	

(ii)	$\overrightarrow{AB} = \begin{pmatrix} 3 \\ -2 \\ k+2 \end{pmatrix}$,	B1	
	$\lvert\overrightarrow{OC}\rvert = 7$ (seen or implied)	B1	
	$3^2 + (-2)^2 + (k+2)^2 = 49$ $\rightarrow k = 4$ or -8	M1 A1 [4]	Correct method. Both correct. Condone sign error in $\overrightarrow{AB}$
(iii)	$\lvert\overrightarrow{OA}\rvert = 3$		
	$\overrightarrow{OD} = 3\overrightarrow{OA} = \begin{pmatrix} 6 \\ 3 \\ -6 \end{pmatrix}$ and $\overrightarrow{OE} = 2$	M1 A1	Scaling from magnitudes / unit vector – oe.
	$\overrightarrow{OC} = \begin{pmatrix} 4 \\ 12 \\ -6 \end{pmatrix}$		
	$\overrightarrow{DE} = \overrightarrow{OE} - \overrightarrow{OD} = \begin{pmatrix} -2 \\ 9 \\ 0 \end{pmatrix}$,	M1	Correct vector subtraction.
	$\rightarrow$ Magnitude of $\sqrt{85}$.	A1 [4]	
11 (i)	$f : x \rightarrow 4\sin x - 1$ for $-\frac{\pi}{2} \leqslant x \leqslant \frac{\pi}{2}$	B1	–5 and 3
	Range $-5 \leqslant f(x) \leqslant 3$	B1 [2]	Correct range
(ii)	$4s - 1 = 0 \rightarrow s = \frac{1}{4} \rightarrow x = 0.253$	M1 A1	Makes $\sin x$ subject. Degrees **M1 A0**, (14.5°)
	$x = 0 \rightarrow y = -1$	B1 [3]	
(iii)		B1√ B1 [2]	Shape from their range in (i) Flattens, curve.
(iv)	range $-\frac{1}{2}\pi \leqslant f^{-1}(x) \leqslant \frac{1}{2}\pi$	B1	
	domain $-5 \leqslant x \leqslant 3$	B1√	√ on part (i) (only for 2 numerical values)
	Inverse $f^{-1}(x) = \sin^{-1}\left(\frac{x+1}{4}\right)$	M1 A1 [4]	Correct order of operations

Cambridge International AS & A Level

Cambridge International Examinations
Cambridge International Advanced Subsidiary and Advanced Level

MATHEMATICS **9709/12**

Paper 1 **May/June 2016**

MARK SCHEME

Maximum Mark: 75

Published

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- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

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PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

1	$f : x \mapsto 10 - 3x,\ g : x \mapsto \dfrac{10}{3-2x}$,		
	$ff(x) = 10 - 3(10 - 3x)$	**B1**	Correct unsimplified expression
	$gf(2) = \dfrac{10}{3 - 2(10 - 3(2))}$ $(= -2)$	**B1**	Correct unsimplified expression with 2 in for x
	$x = 2$	**B1** [3]	
2	$f'(x) = \dfrac{8}{(5-2x)^2}$		
	$f(x) = \dfrac{8(5-2x)^{-1}}{-1}$ $\div -2$ $(+c)$	**B1** **B1**	Correct without (÷ by −2) An attempt at integration (÷ by−2)
	Uses $x = 2$, $y = 7$,	**M1**	Substitution of correct values into an integral to find c
	$c = 3$	**A1** [4]	
3	$\overrightarrow{OA} = 2\mathbf{i} - 5\mathbf{j} - 2\mathbf{k}$ and $\overrightarrow{OB} = 4\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}$.		
	$\overrightarrow{AB} = 2\mathbf{i} + \mathbf{j} + 4\mathbf{k}$ **or** $\overrightarrow{AC} = 4\mathbf{i} + 2\mathbf{j} + 8\mathbf{k}$	**B1**	
	$\overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{AC} = 6\mathbf{i} - 3\mathbf{j} + 6\mathbf{k}$	**M1**	correct method for $\overrightarrow{OC}$
	OR		
	$\begin{pmatrix}2\\1\\4\end{pmatrix} = \begin{pmatrix}x-4\\y+4\\z-2\end{pmatrix}$,	**B1**	
	$\overrightarrow{OC} = \begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}6\\-3\\6\end{pmatrix}$	**M1**	
	OR		
	$\overrightarrow{OB} - \overrightarrow{OA} = \overrightarrow{OC} - \overrightarrow{OB}$ $\therefore \overrightarrow{OC} = 2\overrightarrow{OB} - \overrightarrow{OA}$	**B1**	
	$= \begin{pmatrix}8\\-8\\4\end{pmatrix} - \begin{pmatrix}2\\-5\\-2\end{pmatrix} = \begin{pmatrix}6\\-3\\6\end{pmatrix}$	**M1**	
	Unit vector = (Their $\overrightarrow{OC}$) ÷ (Mod their $\overrightarrow{OC}$)	**M1**	Divides by their mod of their $\overrightarrow{OC}$
	= **(**6**i** −3**j** + 6**k)** ÷ 9	**A1** [4]	Correct unsimplified expression

4 (i)	$\left(x-\frac{2}{x}\right)^6$ Term is ${}_6C_3 \times (-2)^3 = (-)160$ -160	 B1 B1 [2]	 ±160 seen anywhere
(ii)	$\left(2+\frac{3}{x^2}\right)\left(x-\frac{2}{x}\right)^6$ Term in $x^2 = {}_6C_2(-2)^2 x^2$ $= 60\ (x^2)$ Term independent of x: $= 2 \times (\text{their}-160) + 3 \times (\text{their } 60)$ -140	 B1 B1 M1 A1 [4]	 ±60 seen anywhere Using 2 products correctly
5 (i)	$\tan\left(\frac{\pi}{3}\right) = \frac{AC}{2x}$ or $\cos\left(\frac{\pi}{3}\right)\left(=\sin\frac{\pi}{6}\right) = \frac{2x}{AB}$ $\to AC = 2\sqrt{3}x$ or $AB = 4x$ $AM = \sqrt{13\boldsymbol{x}^2}, \sqrt{13}\boldsymbol{x}, 3.61\boldsymbol{x}$	B1 M1A1 [3]	Either trig ratio Complete method.
(ii)	$\tan(M\hat{A}C) = \frac{x}{\text{Their } AC}$ $\theta = \frac{1}{6}\pi - \tan^{-1}\frac{1}{2\sqrt{3}}$ **AG**	M1 A1 [2]	"Their AC" must be f(x), $\left(M\hat{A}C\right) \neq \theta$. Justifies $\frac{\pi}{6}$ and links MAC & θ
6 (i)	$PT = r\tan\alpha$ $QT = OT - OQ = \frac{r}{\cos\alpha} - r$ or $\sqrt{r^2 + r^2\tan^2\alpha} - r$ Perimeter = sum of the 3 parts including $r\alpha$	B1 B1 B1 [3]	
(ii)	Area of triangle = $\frac{1}{2}\times 10\times 10\tan\frac{\pi}{3}$ Area of sector = $\frac{1}{2} \times 10^2 \times \frac{1}{3}\pi$ Shaded region has area 34 (2sf)	M1 M1 A1 [3]	Correct formula used, $50\sqrt{3}, 86.6$ Correct formula used, $\frac{50\pi}{3}$, 52.36

7 **(i)**	$\frac{1+\cos\theta}{1-\cos\theta}-\frac{1-\cos\theta}{1+\cos\theta}\equiv\frac{4}{\sin\theta\tan\theta}$		
	LHS = $\frac{1+2c+c^2-\left(1-2c+c^2\right)}{(1-c)(1+c)}$	**M1**	Attempt at combining fractions.
	$=\frac{4c}{1-c^2}$	**A1 A1**	A1 for numerator. A1 denominator
	$=\frac{4c}{s^2}$		Essential step for award of A1
	$=\frac{4}{ts}$ **AG**	**A1** [4]	
(ii)	$\sin\theta\left(\frac{1+\cos\theta}{1-\cos\theta}-\frac{1-\cos\theta}{1+\cos\theta}\right)=3.$		Uses part (i) to eliminate "s" correctly.
	$\to s\times\frac{4}{ts}=3$ ($\to t=\frac{4}{3}$)	**M1**	
	$\theta=53.1°$ and $233.1°$	**A1 A1$\sqrt{}$** [3]	$\sqrt{}$ for 180° + 1st answer.
8	A (0, 7), B (8, 3) and C ($3k$, k)		
(i)	m of AB is −½ oe.	**B1**	Using A,B or C to get an equation
	Eqn of AB is $y=-½x+7$	**M1**	Using C or A,B in the equation
	Let $x=3k$, $y=k$	**M1**	
	$\boldsymbol{k}$ **= 2.8 oe**	**A1**	
	OR		
	$\frac{7-k}{0-3k}=\frac{3-k}{8-3k}$	**M1A1**	Using A,B & C to equate gradients
	$\to 20k=56\to k=2.8$	**DM1A1**	Simplifies to a linear or 3 term quadratic = 0.
	OR		
	$\frac{7-k}{0-3k}=\frac{7-3}{0-8}$	**M1A1**	Using A,B and C to equate gradients
	$\to 20k=56\to k=2.8$	**DM1A1** [4]	Simplifies to a linear or 3 term quadratic = 0.

(ii)	M(4, 5) Perpendicular gradient = 2. Perp bisector has eqn $y-5=2(x-4)$ Let $x = 3k$, $y = k$ $k=\frac{3}{5}$ oe **OR** $(0-3k)^2+(7-k)^2=(8-3k)^2+(3-k)^2$ $-14k+49=73-54k \rightarrow 40k=24 \rightarrow k=0.6$	**B1** **M1** **M1** **A1** **M1A1** **DM1A1** [4]	anywhere in (ii) Use of $m_1m_2=-1$ soi Forming eqn using their M and their "*perpendicular m*" Use of Pythagoras. Simplifies to a linear or 3 term quadratic = 0.
9 (i) (a)	$a+(n-1)d=10+29\times2$ $=68$	**M1** **A1** [2]	Use of nth term of an AP with a=±10, d=±2, n=30 or 29 Condone – 68 → 68
(b)	$\frac{1}{2}n(20+2(n-1))=2000$ or 0 $\rightarrow 2n^2+18n-4000=0$ oe (n=) 41	**M1** **A1** **A1** [3]	Use of S_n formula for an AP with a=±10, d=±2 and equated to either 0 or 2000. Correct 3 term quadratic = 0.
(ii)	$r = 1.1$, oe Uses $S_{30}=\frac{10(1.1^{30}-1)}{1.1-1}$ (= 1645) Percentage lost = $\frac{2000-1645}{2000}\times100$ = 17.75	**B1** **M1** **DM1** **A1** [4]	e.g. $\frac{11}{10}$, 110% Use of S_n formula for a GP, a=±10, n=30. Fully correct method for % left with "their 1645" allow 17.7 or 17.8.
10	$y=\frac{8}{x}+2x.$		
(i)	$\frac{dy}{dx}=-8x^{-2}+2$ $\frac{d^2y}{dx^2}=16x^{-3}$ $\int y^2dx=-64x^{-1}$ oe$+32x$ oe $+\frac{4x^3}{3}$ **oe** $(+c)$	**B1** **B1** **3 × B1** [5]	unsimplified ok unsimplified ok B1 for each term – unsimplified ok

(ii)	sets $\frac{dy}{dx}$ to 0 $\rightarrow x = \pm 2$ $\rightarrow M(2, 8)$ Other turning point is $(-2, -8)$ If $x = -2$, $\frac{d^2y}{dx^2} < 0$ ∴Maximum	**M1** **A1** **A1** **M1** **A1** [5]	Sets to 0 and attempts to solve Any pair of correct values A1 Second pair of values A1 Using their $\frac{d^2y}{dx^2}$ if kx^{-3} and $x < 0$
(iii)	Vol = π × [part (i)] from 1 to 2 $\frac{220\pi}{3}, 73.3\pi, 230$	**M1** **A1** [2]	Evidence of using limits 1&2 in their integral of y^2 (ignore π)
11 **(i)**	$f : x \mapsto 6x - x^2 - 5$ $6x - x^2 - 5 \leqslant 3$ $\rightarrow x^2 - 6x + 8 \geqslant 0$ $\rightarrow x = 2, x = 4$ $x \leqslant 2, x \geqslant 4$ condone < and/or >	 **M1** **A1** **A1** [3]	 $\pm(6x - x^2 - 8) =, \leqslant, \geqslant 0$ and attempts to solve Needs both values whether =2, <2, >2 Accept all recognisable notation.
(ii)	Equate $mx + c$ and $6x - x^2 - 5$ Use of "$b^2 - 4ac$" $4c = m^2 - 12m + 16$. **AG** OR $\frac{dy}{dx} = 6 - 2x = m \rightarrow x = \left(\frac{6-m}{2}\right)$ $m\left(\frac{6-m}{2}\right) + c = 6\left(\frac{6-m}{2}\right) - \left(\frac{6-m}{2}\right)^2 - 5$ $4c = m^2 - 12m + 16$. **AG**	**M1** **DM1** **A1** **M1** **M1** **A1** [3]	Equates, sets to 0. Use of discriminant with values of $a.b.c$ independent of x. = (0) must appear before last line. Equates $\frac{dy}{dx}$ to m and rearrange Equates $mx + c$ and $6x - x^2 - 5$ and substitutes for x
(iii)	$6x - x^2 - 5 = 4 - (x - 3)^2$	**B1 B1** [2]	4 B1 – $(x - 3)^2$ B1
(iv)	$k = 3$.	**B1**√ [1]	√ for "b".
(v)	$g^{-1}(x) = \sqrt{4 - x} + 3$	**M1 A1** [2]	Correct order of operations. $\pm\sqrt{4-x} + 3$ M1A0 $\sqrt{x-4} + 3$ M1A0 $\sqrt{4-y} + 3$ M1A0

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MATHEMATICS **9709/13**

Paper 1 **May/June 2016**

MARK SCHEME

Maximum Mark: 75

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1	$5C2\left(\frac{1}{x}\right)^3\left(3x^2\right)^2$	**B1**	Can be seen in expansion
	$10(\times 1)\times 3^2$	**B1**	Identified as leading to answer
	$90\ (x)$	**B1** [3]	
2	$(\pi)\int\left(x^3+1\right)\mathrm{d}x$	**M1**	Attempt to resolve y^2 and attempt to integrate
	$(\pi)\left[\frac{x^4}{4}+x\right]$	**A1**	
	6π or 18.8	**DM1A1** [4]	Applying limits 0 and 2. (Limits reversed: Allow M mark and allow A mark if final answer is 6π)
3 (i)	$6+k=2 \rightarrow k=-4$	**B1** [1]	
(ii)	$(y)=\frac{6x^3}{3}-\frac{4}{-2}x^{-2}\quad(+c)$	**B1B1**√	ft on *their k*. Accept $+\frac{k}{-2}x^{-2}$
	$9=2+2+c$ *c* must be present	**M1**	Sub (1,9) with numerical *k*. Dep on attempt $\int$
	$(y)=2x^3+2x^{-2}+5$	**A1** [4]	Equation needs to be seen. Sub (2, 3) $\rightarrow c=-13\frac{1}{2}$ scores M1A0
4	$r=\frac{3+2d}{3}$ or $\frac{3+12d}{3+2d}$ or $r^2=\frac{3+12d}{3}$	**B1**	1 correct equation in *r* and *d* only is sufficient
	$(3+2d)^2=3(3+12d)$ oe OR sub $2d=3r-3$	**M1**	Eliminate *r* or *d* using valid method
	$(4)d(d-6)=0$ OR $3r^2=18r-15\rightarrow(r-1)(r-5)$	**DM1**	Attempt to simplify and solve quadratic
	$d=6$	**A1**	Ignore $d=0$ or $r=1$
	$r=5$	**A1** [5]	Do not allow -5 or ± 5

5	$\frac{dy}{dx} = [8] + [-2]\,[(2x-1)^{-2}]$	B2,1,0	
	$= 0 \to 4(2x-1)^2 = 1$ oe eg $16x^2 - 16x + 3 = 0$	M1	Set to zero, simplify and attempt to solve soi
	$x = \frac{1}{4}$ and $\frac{3}{4}$	A1	Needs both x values. Ignore y values
	$\frac{d^2y}{dx^2} = 8(2x-1)^{-3}$	B1√*	ft to $k(2x-1)^{-3}$ where $k > 0$
	When $x = \frac{1}{4}$, $\frac{d^2y}{dx^2}(=-64)$ and/or < 0 MAX	DB1	Alt. methods for last 3 marks (values either side of 1/4 & 3/4) must indicate <u>which</u> x-values and cannot use $x = 1/2$. (M1A1A1)
	When $x = \frac{3}{4}$, $\frac{d^2y}{dx^2}(=64)$ and/or > 0 MIN	DB1 [7]	
6	$BAC = \sin^{-1}(3/5)$ or $\cos^{-1}(4/5)$ or $\tan^{-1}(3/4)$	B1	Accept 36.8(7)º
	$ABC = \sin^{-1}(4/5)$ or $\cos^{-1}(3/5)$ or $\tan^{-1}(4/3)$	B1	Accept 53.1(3)º
	$ACB = \pi/2$ (Allow 90º)	B1	
	Shaded area = ΔABC – sectors ($AEF + BEG + CFG$)	M1	
	$\Delta ABC = \frac{1}{2} \times 4 \times 3$ oe	B1	
	Sum sectors $= \frac{1}{2}\left[3^2 0.6435\right) + 2^2 0.9273 + 1^2 1.5708]$ **OR** $\frac{\pi}{360}\left[3^2 36.8(7) + 2^2 53.1(3) + 1^2 90\right]$	M1	
	6 – 5.536 = 0.464	A1 [7]	
7	$\frac{dy}{dx} = 2x - 5x^{1/2} + 5$	B1	
	$\frac{dy}{dx} = 2$	B1	
	$2x - 5x^{1/2} + 5 = 2$	M1	Equate their dy/dx to *their* 2 or ½.
	$2x - 5x^{1/2} + 3(=0)$ or equivalent 3-term quadratic	A1	
	Attempt to solve for $x^{1/2}$ e.g. $(2x^{1/2} - 3)(x^{1/2} - 1) = 0$	DM1	Dep. on 3-term quadratic
	$x^{1/2} = 3/2$ and 1	A1	ALT
	$x = 9/4$ and 1	A1 [7]	$5x^{½} = 2x + 3 \to 25x = (2x+3)^2$ $4x^2 - 13x + 9(=0)$ $x = 9/4$ and 1

8 **(i)**	$3\sin^2 x - \cos^2 x + \cos x = 0$	**M1**	Multiply by $\cos x$
	Use $s^2 = 1 - c^2$ and simplify to 3-term quad	**M1**	Expect $4c^2 - c - 3 = 0$
	$\cos x = -3/4$ and 1	**A1**	
	$x = 2.42$ (allow 0.77π) or 0 (extra in range max 1)	**A1A1** [5]	SC1 for 0.723 (or 0.23π), π following $4c^2 + c - 3 = 0$
(ii)	$2x = 2\pi - their 2.42$ or 360 – 138.6	**B1**√	Expect $2x = 3.86$
	$x = 1.21$ (0.385π), 1.93 (0.614/5π), 0, π (3.14) (extra max 1)	**B1B1** [3]	Any 2 correct B1. Remaining 2 correct B1. SCB1for all 69.3, 110.7, 0, 180 (degrees) SCB1 for .361, π/2, 2.78 after $4c^2 + c - 3 = 0$
9 **(i)**	**AB** = **OB** – **OA** = $\begin{pmatrix} -1 \\ 2 \\ p+4 \end{pmatrix}$	**B1**	Ignore labels. Allow **BA** or **BC**
	CB = **OB** – **OC** = $\begin{pmatrix} -4 \\ 5 \\ p-2 \end{pmatrix}$	**B1**	
	$1 + 4 + (p+4)^2 = 16 + 25 + (p-2)^2$	**M1**	
	$p = 2$	**A1** [4]	
(ii)	**AB.CB** = 4+10−5 = 9	**M1**	Use of $x_1x_2 + y_1y_2 + z_1z_2$
	$\lvert \mathbf{AB} \rvert = \sqrt{1+4+25} = \sqrt{30}$, $\lvert \mathbf{CB} \rvert = \sqrt{16+25+1} = \sqrt{42}$	**M1**	Product of moduli
	$\cos ABC = \dfrac{9}{\sqrt{30}\sqrt{42}}$ or $\dfrac{9}{6\sqrt{35}}$	**M1**	Allow one of **AB**, **CB** reversed - but award **A0**
	$ABC = 75.3°$ or 1.31rads (ignore reflex angle 285°)	**A1** [4]	
10 **(i)**	$2(ax^2 + b) + 3 = 6x^2 - 21$	**M1**	
	$a = 3,\ b = -12$	**A1A1** [3]	
(ii)	$3x^2 - 12 \geqslant 0$ or $6x^2 - 21 \geqslant 3$	**M1**	Allow = or ⩽ or > or <. Ft from *their a, b*
	$x \leqslant -2$ i.e. (max) $q = -2$	**A1** [2]	Must be in terms of q (eg $q \leqslant -2$)
(iii)	$y \geqslant 6(-3)^2 - 21 \Rightarrow$ range is $(y) \geqslant 33$	**B1** [1]	Do not allow $y > 33$.Accept all other notations e.g. [33, ∞) or [33, ∞]

(iv)	$y = 6x^2 - 21 \Rightarrow x = (\pm)\sqrt{\frac{y+21}{6}}$	**M1**	
	$(\mathrm{fg})^{-1}(x) = -\sqrt{\frac{x+21}{6}}$	**A1**	Allow y =..... . Must be a function of x
	Domain is $x \geqslant 33$	**B1**$\sqrt{}$ [3]	ft from *their* part **(iii)** but x essential
11 (i)	$AB^2 = 6^2 + 7^2 = 85$, $BC^2 = 2^2 + 9^2 = 85$ (→ isosceles)	**B1B1**	Or $AB = BC = \sqrt{85}$ etc
	$AC^2 = 8^2 + 2^2 = 68$	**B1**	
	$M = (2, -2)$ or $BM^2 = (\sqrt{85})^2 - (½\sqrt{68})^2$	**B1**	Where M is mid-point of AC
	$BM = \sqrt{2^2 + 8^2} = \sqrt{68}$ or $\sqrt{85-17} = \sqrt{68}$	**B1**	
	Area $\Delta ABC = \frac{1}{2}\sqrt{68}\sqrt{68} = 34$	**B1** [6]	
(ii)	Gradient of $AB = 7/6$	**B1**	
	Equation of AB is $y + 1 = \frac{7}{6}(x+2)$	**M1**	Or $y - 6 = \frac{7}{6}(x-4)$
	Gradient of $CD = -6/7$	**M1**	
	Equation of CD is $y + 3 = \frac{-6}{7}(x-6)$	**M1**	
	Sim Eqns $2 = \frac{-6}{7}x + \frac{36}{7} - \frac{7}{6}x - \frac{14}{6}$	**M1**	
	$x = \frac{34}{85} = \frac{2}{5}$ oe	**A1** [6]	

CAMBRIDGE INTERNATIONAL EXAMINATIONS

Cambridge International Advanced Subsidiary and Advanced Level

MARK SCHEME for the March 2016 series

9709 MATHEMATICS

9709/12 Paper 1 (Pure Mathematics), maximum raw mark 75

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A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

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- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

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AEF Any Equivalent Form (of answer is equally acceptable)

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CWO Correct Working Only – often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

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MR−1 A penalty of MR−1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR−2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA−1 This is deducted from A or B marks in the case of premature approximation. The PA−1 penalty is usually discussed at the meeting.

1 (i)	$80(x^4)$, $-32(x^5)$	**B1B1** **[2]**	Fully simplified
(ii)	$(-32+80p)(x^5)=0$ $p=2/5$ or 32/80 oe	**M1** **A1**√ **[2]**	Attempt to mult. relevant terms & put $=0$
2	$y=\frac{3x^3}{3}-\frac{2x^{-2}}{-2}$ $(+c)$ $3=-1+1+c$ $y=x^3+x^{-2}+3$	**B1B1** **M1** **A1** **[4]**	 Sub $x=-1, y=3$. c must be present Accept $c=3$ www
3	$a+11d=17$ $\frac{31}{2}(2a+30d)=1023$ Solve simultaneous equations $d=4$, $a=-27$ 31st term = 93	**B1** **B1** **M1** **A1** **A1** **[5]**	 At least one correct
4 (a)	$3x=-\sqrt{3}/2$ $x=\frac{-\sqrt{3}}{6}$ oe	**M1** **A1** **[2]**	Accept −0.866 at this stage Or $\frac{-3}{6\sqrt{3}}$ or $\frac{-1}{2\sqrt{3}}$
(b)	$(2\cos\theta-1)(\sin\theta-1)=0$ $\cos\theta=1/2$ or $\sin\theta=1$ $\theta=\pi/3$ or $\pi/2$	**M1** **A1** **A1A1** **[4]**	Reasonable attempt to factorise and solve Award B1B1 www Allow 1.05, 1.57. SCA1for both 60°, 90°
5 (i)	Mid-point of AB = (7, 3) soi Grad. of AB = −2 →grad of perp. bisector = 1/2 soi Eqn of perp. bisector is $y-3=\frac{1}{2}(x-7)$	**B1** **M1** **A1** **[3]**	 Use of $m_1m_2=-1$
(ii)	Eqn of CX is $y-2=-2(x-1)$ $\frac{1}{2}x-\frac{1}{2}=-2x+4$ x = 9/5, y = 2/5 $BX^2=7.2^2+1.4^2$ soi BX = 7.33	**M1** **DM1** **A1** **M1** **A1** **[5]**	Using their original gradient and (1,2) Solve simultaneously dependent on both previous M's

6 (i)	$A = 2\pi r^2 + 2\pi rh$	**B1**	
	$\pi r^2 h = 1000 \rightarrow h = \dfrac{1000}{\pi r^2}$	**M1**	
	Sub for h into $A \rightarrow A = 2\pi r^2 + \dfrac{2000}{r}$ **AG**	**A1** **[3]**	
(ii)	$\dfrac{dA}{dr} = 0 \Rightarrow 4\pi r - \dfrac{2000}{r^2} = 0$	**M1A1**	Attempt differentiation & set = 0
	$r ==$ 5.4	**DM1 A1**	Reasonable attempt to solve to $r^3 =$
	$\dfrac{d^2A}{dr^2} = 4\pi + \dfrac{4000}{r^3}$		
	> 0 hence MIN hence MOST EFFICIENT **AG**	**B1** **[5]**	Or convincing alternative method
7 (i)	$CP = \dfrac{3}{5}CA$ soi	**M1**	
	$CP = \dfrac{3}{5}(4\mathbf{i} - 3\mathbf{k})$ = $2.4\mathbf{i} - 1.8\mathbf{k}$ **AG**	**A1** **[2]**	
(ii)	OP = $2.4\mathbf{i} + 1.2\mathbf{k}$	**B1**	
	BP = $2.4\mathbf{i} - 2.4\mathbf{j} + 1.2\mathbf{k}$	**B1** **[2]**	
(iii)	$BP.CP = 5.76 - 2.16 = 3.6$	**M1**	Use of $x_1x_2 + y_1y_2 + z_1z_2$
	$\lvert BP \rvert \, \lvert CP \rvert = \sqrt{2.4^2 + 2.4^2 + 1.2^2}\sqrt{2.4^2 + 1.8^2}$	**M1**	Product of moduli
	$\cos BPC = \dfrac{3.6}{\sqrt{12.96}\sqrt{9}} \quad \left(= \dfrac{1}{3}\right)$	**M1**	All linked correctly
	Angle BPC = 70.5° (or 1.23 rads) cao	**A1** **[4]**	
8 (i)	$2a + 4b = 8$	**M1**	Substitute in –2 and –3
	$2a^2 + 3a + 4b = 14$	**A1**	
	$2a^2 + 3a + (8 - 2a) = 14 \rightarrow (a + 2)(2a - 3) = 0$	**M1**	Sub linear into quadratic & attempt solution
	$a = -2$ or $3/2$	**A1**	If A0A0 scored allow SCA1 for either
	$b = 3$ or $5/4$	**A1** **[5]**	$(-2, 3)$ or (3/2, 5/4)
(ii)	$y = \left(x - \dfrac{1}{2}\right)^2 - \dfrac{13}{4}$ Attempt completing of square	**M1A1**	Allow with x/y transposed
	$x - \dfrac{1}{2} = (\pm)\sqrt{y + \dfrac{13}{4}}$ oe	**DM1**	Allow with x/y transposed
	$f^{-1}(x) = \dfrac{1}{2} - \sqrt{x + \dfrac{13}{4}}$ oe	**A1**	Allow y =..... Must be a function of x
	Domain of f^{-1} is $(x) \geqslant -13/4$	**B1**√ **[5]**	Allow >, $-13/4 \leqslant x \leqslant \infty$, $\left[-\dfrac{13}{4}, \infty\right]$ etc

9 (a) (i)	$BAO = OBA = \frac{\pi}{2} - \alpha$ $AOB = \pi - \left(\frac{\pi}{2} - \alpha\right) - \left(\frac{\pi}{2} - \alpha\right) = 2\alpha$ **AG**	M1A1 [2]	Allow use of 90º or 180º Or other valid reasoning
(ii)	$\frac{1}{2}r^2(2\alpha) - \frac{1}{2}r^2\sin 2\alpha$ oe	B2,1,0 [2]	SCB1 for reversed subtraction
(b)	Use of $\alpha = \frac{\pi}{6}$, $r = 4$ 1 segment $S = \left(\frac{1}{2}\right)4^2\left(\frac{\pi}{3}\right) - \left(\frac{1}{2}\right)4^2\sin\frac{\pi}{3}$ $= \left(\frac{8\pi}{3} - 4\sqrt{3}\right)$ Area ABC $T = \left(\frac{1}{2}\right)4^2\sin\frac{\pi}{3}$ $\left(= 4\sqrt{3}\right)$ $T - 3S = \left(\frac{1}{2}\right)4^2\sin\frac{\pi}{3} - 3$ $\left[\left(\frac{1}{2}\right)4^2\left(\frac{\pi}{3}\right) - \left(\frac{1}{2}\right)4^2\sin\frac{\pi}{3}\right]$ $16\sqrt{3} - 8\pi$ cao	B1B1 M1 B1 M1 A1 [6]	Ft *their* **(ii)**, α, r OR $AXB = \frac{T}{3} = 4\tan\frac{\pi}{6}$ or $\frac{1}{2}\left(\frac{4}{\sqrt{3}}\right)^2\sin\frac{2\pi}{3}$ $\left(= \frac{4\sqrt{3}}{3}\right)$ OR $3\left[\frac{T}{3} - S\right] = 3\left[\frac{4\sqrt{3}}{3} - \left(\frac{8\pi}{3} - 4\sqrt{3}\right)\right]$
10 (i)	$x = 1/3$	B1 [1]	
(ii)	$\frac{dy}{dx} = \left[\frac{2}{16}(3x-1)\right][3]$ When $x = 3$ $\frac{dy}{dx} = 3$ soi Equation of QR is $y - 4 = 3(x - 3)$ When $y = 0$ $x = 5/3$	B1B1 M1 M1 A1 [5]	
(iii)	Area under curve $= \left[\frac{1}{16 \times 3}(3x-1)^3\right]\left[\times\frac{1}{3}\right]$ $\frac{1}{16 \times 9}\left[8^3 - 0\right] = \frac{32}{9}$ Area of $\Delta = 8/3$ Shaded area $= \frac{32}{9} - \frac{8}{3} = \frac{8}{9}$ (or 0.889)	B1B1 M1A1 B1 A1 [6]	Apply limits: *their* $\frac{1}{3}$ and 3

CAMBRIDGE INTERNATIONAL EXAMINATIONS

Cambridge International Advanced Subsidiary and Advanced Level

MARK SCHEME for the October/November 2015 series

9709 MATHEMATICS

9709/11 Paper 1, maximum raw mark 75

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International Examinations

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1	$(a+x)^5 = a^5 + {}^5C_1 a^4 x + {}^5C_2 a^3 x^2 + \ldots$ soi $\left(-\frac{2}{a} \times (their\ 5a^4) + (their\ 10a^3)\right)(x^2)$ 0	**M1** **M1** **A1** [3]	Ignore subsequent terms **AG**
2	$f(x) = x^3 - 7x\,(+c)$ $5 = 27 - 21 + c$ $c = -1 \rightarrow f(x) = x^3 - 7x - 1$	**B1** **M1** **A1** [3]	Sub $x = 3$, $y = 5$. Dep. on c present
3	$4x^2 + x^2 = 1/2$ soi Solve as quadratic in x^2 $x^2 = 1/4$ $x = \pm 1/2$	**B1** **M1** **A1** **A1** [4]	E.g. $(4x^2 - 1)(2x^2 + 1)$ or $x^2 =$ formula Ignore other solution
4 (i)	$4\cos^2\theta + 15\sin\theta = 0$ $4(1 - s^2) + 15s = 0 \rightarrow 4\sin^2\theta - 15\sin\theta - 4 = 0$	**M1** **M1A1** **[3]**	Replace $\tan\theta$ by $\frac{\sin\theta}{\cos\theta}$ and multiply by $\sin\theta$ or equivalent Use $c^2 = 1 - s^2$ and rearrange to **AG** **(www)**
(ii)	$\sin\theta = -1/4$ $\theta = 194.5$ or 345.5	**B1** **B1B1**$\checkmark$ [3]	Ignore other solution Ft from 1st solution, SC B1 both angles in rads (3.39 and 6.03)
5 (i)	$\frac{dy}{dx} = -\frac{8}{x^2} + 2$ cao $\frac{d^2y}{dx^2} = \frac{16}{x^3}$ cao	**B1B1** **B1** [3]	
(ii)	$-\frac{8}{x^2} + 2 = 0 \rightarrow 2x^2 - 8 = 0$ $x = \pm 2$ $y = \pm 8$ $\frac{d^2y}{dx^2} > 0$ when $x = 2$ hence MINIMUM $\frac{d^2y}{dx^2} < 0$ when $x = -2$ hence MAXIMUM	**M1** **A1** **A1** **B1**$\checkmark$ **B1**$\checkmark$ [5]	Set = 0 and rearrange to quadratic form If A0A0 scored, SCA1 for just (2, 8) $\left\{\text{Ft for "correct" conclusion if } \frac{d^2y}{dx^2} \text{ incorrect or any valid method inc. a good sketch}\right\}$

6 (i)	$x^2 - x + 3 = 3x + a \to x^2 - 4x + (3-a) = 0$	**B1** [1]	**AG**
(ii)	$5 + (3-a) = 0 \to a = 8$ $x^2 - 4x - 5 = 0 \to x = 5$	**B1** **B1** [2]	Sub $x = -1$ into **(i)** **OR B2** for $x = 5$ www
(iii)	$16 - 4(3-a) = 0$ (applying $b^2 - 4ac = 0$) $a = -1$ $(x-2)^2 = 0 \to x = 2$ $y = 5$	**M1** **A1** **A1** **A1** [4]	**OR** $dy/dx = 2x - 1 \to 2x - 1 = 3$ $x = 2$ $y = 2^2 - 2 + 3 \to y = 5$ $5 = 6 + a \to a = -1$
7 (i)	$BC^2 = r^2 + r^2 = 2r^2 \to BC = r\sqrt{2}$	**B1** [1]	**AG**
(ii)	Area sector $BCFD = \frac{1}{4}\pi(r\sqrt{2})^2$ soi Area $\Delta\,BCAD = \frac{1}{2}(2r)r$ Area segment $CFDA = \frac{1}{2}\pi r^2 - r^2$ .oe Area semi-circle $CADE = \frac{1}{2}\pi r^2$ Shaded area $\frac{1}{2}\pi r^2 - \left(\frac{1}{2}\pi r^2 - r^2\right)$ or $\pi r^2 - \left(\frac{1}{2}\pi r^2 + \left(\frac{1}{2}\pi r^2 - r^2\right)\right)$ $= r^2$	**M1** **M1** **A1** **B1** **DM1** **A1** [6]	Expect $\frac{1}{2}\pi r^2$ Expect r^2 (could be embedded) Depends on the area $\Delta\,BCD$

8	**(i)**	$x^2 - 4x = 12$	**M1**	$4x - x^2 = 12$ scores M1A0
		$x = -2$ or 6	**A1**	
		3^{rd} term $= (-2)^2 + 12 = 16$ or $6^2 + 12 = 48$	**A1A1** [4]	SC1 for 16, 48 after $x = 2, -6$
	(ii)	$r^2 = \frac{x^2}{4x}\left(= \frac{x}{4}\right)$ soi	**M1**	
		$\frac{4x}{1 - \frac{x}{4}} = 8$	**M1**	Accept use of unsimplified $\frac{x^2}{4x}$ or $\frac{4x}{x^2}$ or $\frac{4}{x}$
		$x = \frac{4}{3}$ or $r = \frac{1}{3}$	**A1**	
		3^{rd} term $= \frac{16}{27}$ (or 0.593)	**A1** [4]	
		ALT		
		$\frac{4x}{1-r} = 8 \rightarrow r = 1 - \frac{1}{2}x$ **or** $\frac{4x}{1-r} = 8 \rightarrow x = 2(1-r)$	**M1**	
		$x^2 = 4x\left(1 - \frac{1}{2}x\right)$ $r = \frac{2(1-r)}{4}$	**M1**	
		$x = \frac{4}{3}$ $r = \frac{1}{3}$	**A1**	
9	**(i)**	$-(1)(x-3)^2 + 4$	**B1B1B1** [3]	
	(ii)	Smallest (m) is 3	**B1**$\sqrt{}$ [1]	Accept $m \geqslant 3$, $m = 3$. **Not** $x \geqslant 3$. Ft *their* b
	(iii)	$(x-3)^2 = 4 - y$	**M1**	Or x/y transposed. Ft *their* a, b, c
		Correct order of operations	**M1**	
		$\text{f}^{-1}(x) = 3 + \sqrt{4 - x}$ cao	**A1**	Accept $y =$ if clear
		Domain is $x \leqslant 0$	**B1** [4]	

10	**(i)**	PM = 2**i** – 10**k** + $\frac{1}{2}$(6**j** + 8**k**) oe	**M1**	Any valid method
		PM = 2**i** + 3**j** – 6**k**	**A1**	
		$\div\sqrt{4+9+36}$	**M1**	
		Unit vector = $\frac{1}{7}(2\mathbf{i}+3\mathbf{j}-6\mathbf{k})$	**A1** [4]	
	(ii)	AT = 6**j** + 8**k**, PT = a**i** + 6**j** – 2**k** soi (or TA and TP)	**B1**	Allow 1 vector reversed at this stage. (**AM** or **MT** could be used for **AT**)
		$(\cos ATP)=\dfrac{(6\mathbf{j}+8\mathbf{k}).(a\mathbf{i}+6\mathbf{j}-2\mathbf{k})}{\sqrt{36+64}\sqrt{a^2+36+4}}$	**M1**	
		$=\dfrac{36-16}{\sqrt{36+64}\sqrt{a^2+36+4}}$		
		$\dfrac{20}{10\sqrt{a^2+40}}$	**A1√**	Ft from their **AT** and **PT**
		$\dfrac{2}{\sqrt{a^2+40}}=\dfrac{2}{7}$ oe and attempt to solve	**M1**	
		$a=3$	**A1** [5]	Withheld if only 1 vector reversed
		ALT Alt (Cosine Rule) Vectors (AT, PT etc.)	**B1**	
		$\cos ATP=\dfrac{a^2+36+4+36+64-(100+a^2)}{2\sqrt{(a^2+40)}\sqrt{100}}$	**M1A1**	
		then as above		

11 (i)	$\frac{dy}{dx} = \left[\frac{1}{2}(1+4x)^{-1/2}\right] \times [4]$	**B1B1**	
	At $x = 6, \frac{dy}{dx} = \frac{2}{5}$	**B1**	
	Gradient of normal at $P = -\frac{1}{2}$	**B1**√	**OR** eqn of norm $y - 5 = their - \frac{5}{2}(x-6)$ When $y = 0, x = 8$ hence result
	Gradient of $PQ = -\frac{5}{2}$ hence PQ is a normal, or $m_1 m_2 = -1$	**B1** [5]	
(ii)	Vol for curve $= (\pi)\int(1+4x)$ and attempt to integrate y^2	**M1**	
	$= (\pi)\left[x + 2x^2\right]$ ignore '+ c'	**A1**	
	$= (\pi)[6 + 72 - 0]$	**DM1**	Apply limits $0 \to 6$ (allow reversed if corrected later)
	$= 78(\pi)$	**A1**	
	Vol for line $= \frac{1}{3} \times (\pi) \times 5^2 \times 2$	**M1**	**OR** $(\pi)\left[\frac{\left(-\frac{5}{2}x + 20\right)^3}{3 \times -\frac{5}{2}}\right]_6^8$
	$= \frac{50}{3}(\pi)$	**A1**	
	Total Vol $= 78\pi + 50\pi/3 = 94\frac{2}{3}\pi$ (or $284\pi/3$)	**A1** [7]	

CAMBRIDGE INTERNATIONAL EXAMINATIONS

Cambridge International Advanced Subsidiary and Advanced Level

MARK SCHEME for the October/November 2015 series

9709 MATHEMATICS

9709/12 Paper 1, maximum raw mark 75

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International Examinations

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A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

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- The symbol $\sqrt{}$ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

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- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

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CWO Correct Working Only – often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR−1 A penalty of MR−1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures − this is regarded as an error in accuracy. An MR−2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA−1 This is deducted from A or B marks in the case of premature approximation. The PA−1 penalty is usually discussed at the meeting.

1	f : $x \mapsto 3x + 2$, g : $x \mapsto 4x - 12$ $f^{-1}(x) = \dfrac{x-2}{3}$ $gf(x) = 4(3x + 2) - 12$ Equate $\rightarrow$ $x = \dfrac{2}{7}$	**B1** **B1** **M1** **A1** [4]	 Equates, collects terms, +soln
2	$(x + 2k)^7$ Term in $x^5 = 21 \times 4k^2 = 84k^2$ Term in $x^4 = 35 \times 8k^3 = 280k^3$ Equate and solve $\rightarrow$ $k = 0.3$ or $\dfrac{3}{10}$	 **B1** **B1** **M1 A1** [4]	 Correct method to obtain k.
3 (i)	$\tan 60 = \dfrac{x}{h}$ $\rightarrow$ $x = h\tan 60$ $A = h \times x$ $V = 40\sqrt{(3h^2)}$	**B1** **M1** **A1** [3]	Any correct unsimplified length Correct method for area ag
(ii)	$\dfrac{dV}{dh} = 80\sqrt{(3h)}$ If $h = 5$, $\dfrac{dh}{dt} = \dfrac{1}{2\sqrt{(3)}}$ or 0.289	**B1** **M1A1** [3]	 B1 M1 (must be ÷, not ×).
4 (i)	$\left(\dfrac{1}{\sin x} - \dfrac{1}{\tan x}\right)^2 = \left(\dfrac{1}{s} - \dfrac{c}{s}\right)^2$ $\dfrac{(1-c)^2}{s^2} = \dfrac{(1-c)^2}{1-c^2}$ $= \dfrac{(1-c)(1-c)}{(1-c)(1+c)}$ or $\dfrac{(1-c)^2}{(1-c)(1+c)}$ $\equiv \dfrac{1-\cos x}{1+\cos x}$	**M1** **M1** **A1** **A1** [4]	Use of tan = sin/cos Use of $s^2 = 1 - c^2$ ag
(ii)	$\left(\dfrac{1}{\sin x} - \dfrac{1}{\tan x}\right)^2 = \dfrac{2}{5}$ $\dfrac{1-\cos x}{1+\cos x} = \dfrac{2}{5} \rightarrow \cos x \dfrac{3}{7}$ $\rightarrow$ $x = 1.13$ or 5.16	 **M1** **A1 A1**√ [3]	 Making cos$x$ the subject $2\pi - 1^{\text{st}}$ answer.

5	(i)	Length of $OB = \dfrac{6}{\cos 0.6} = 7.270$	M1 [1]	ag Any valid method
	(ii)	$AB = 6\tan 0.6$ or 4.1 Arc length = $7.27 \times (\frac{1}{2}\pi - 0.6) = (7.06)$ Perimeter = $6 + 7.27 + 7.06 + 6\tan 0.6 = 24.4$	B1 M1 A1 [3]	Sight of in **(ii)** Use of $s = r\theta$ with sector angle
	(iii)	Area of $AOB = \frac{1}{2} \times 6 \times 7.27 \times \sin 0.6$ Area of $OBC = \frac{1}{2} \times 7.27^2 \times (\frac{1}{2}\pi - 0.6)$ $\rightarrow$ area = $12.31 + 25.65 = 38.0$	M1 M1 A1 [3]	Use of any correct area method Use of $\frac{1}{2}r^2\theta$.
6		$A(-3, 7)$, $B(5, 1)$ and $C(-1, k)$		
	(i)	$AB = 10$ $6^2 + (k-1)^2 = 10^2$ $k = -7$ and 9	B1 M1 A1 [3]	 Use of Pythagoras
	(ii)	m of $AB = -\frac{3}{4}$ m perp $= \dfrac{4}{3}$ $M = (1, 4)$ Eqn $y - 4 = \dfrac{4}{3}(x-1)$ Set y to 0, $\rightarrow$ $x = -2$	B1 M1 B1 M1 A1 [5]	B1 M1 Use of $m_1m_2 = -1$ Complete method leading to D.
7		$\overrightarrow{OA} = \begin{pmatrix} 0 \\ 2 \\ -3 \end{pmatrix}$, $\overrightarrow{OB} = \begin{pmatrix} 2 \\ 5 \\ -2 \end{pmatrix}$, $\overrightarrow{OC} = \begin{pmatrix} 3 \\ p \\ q \end{pmatrix}$.		
	(i)	$\overrightarrow{AB} = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$ $\overrightarrow{AC}\begin{pmatrix} 3 \\ p-2 \\ q+3 \end{pmatrix}$ $\overrightarrow{BC}\begin{pmatrix} 1 \\ p-5 \\ q+2 \end{pmatrix}$ $\rightarrow$ $p = 6\frac{1}{2}$ and $q = -1\frac{1}{2}$	B1B1 B1 B1 [4]	Any 2 of 3 relevant vectors
	(ii)	$6 + 3p - 6 + q + 3 = 0$ $\rightarrow$ $q = -3p - 3$	M1 A1 [2]	Use of $x_1x_2 + y_1y_2 + z_1z_2 = 0$
	(iii)	$AB^2 = 4 + 9 + 1$ $AC^2 = 9 + 1 + (q+3)^2$ $\rightarrow$ $(q+3)^2 = 4$ $\rightarrow q = -1$ or -5	M1 A1 A1 [3]	For attempt at either

8	$f : x \to x^2 + ax + b$,		
(i)	$x^2 + 6x - 8 = (x+3)^2 - 17$ or $2x + 6 = 0 \;\to\; x = -3 \to y = -17$ $\to$ Range $f(x) \geqslant -17$	**B1 B1** **B1**√ [3]	B1 for $(x + 3)^2$. B1 for -17 or B1 for $x = -3$, B1 $y = -17$ Following through visible method.
(ii)	$(x - k)(x + 2k) = 0$ $\equiv x^2 + 5x + b = 0$ $\to\; k = 5$ $\to\; b = -2k^2 = -50$	**M1** **A1** **A1** [3]	Realises the link between roots and the equation comparing coefficients of x
(iii)	$(x+a)^2 + a(x+a) + b = a$ Uses $b^2 - 4ac \;\to\; 9a^2 - 4(2a^2 + b - a)$ $\to\; a^2 < 4(b - a)$	**M1** **DM1** **A1** [3]	Replaces "x" by "$x + a$" in 2 terms Any use of discriminant
9	$f''(x) = \dfrac{12}{x^3}$		
(i)	$f'(x) = -\dfrac{6}{x^2}$ $(+c)$ $= 0$ when $x = 2 \;\to\; c = \dfrac{3}{2}$ $f(x) = \dfrac{6}{x} + \dfrac{3x}{2}$ $(+A)$ $= 10$ when $x = 2 \;\to\; A = 4$	**B1** **M1 A1** **B1**√**B1**√ **A1** [6]	Correct integration Uses $x = 2$, $f'(x = 0)$ For each integral
(ii)	$-\dfrac{6}{x^2} + \dfrac{3}{2} = 0 \;\to\; x = \pm 2$ Other point is $(-2, -2)$	**M1** **A1** [2]	Sets their 2 term $f'(x)$ to 0.
(iii)	At $x = 2$, $f''(x) = 1.5$ Min At $x = -2$, $f''(x) = -1.5$ Max	**B1** **B1** [2]	

10	$y=\sqrt{(9-2x^2)}$ $P\,(2, 1)$		
(i)	$\dfrac{dy}{dx}=\dfrac{1}{2\sqrt{(9-2x^2)}}\times -4x$	**B1** **B1**	Without " $\times -4x$" Allow even if B0 above.
	At P, $x = 2$, $m = -4$ Normal grad = ¼ Eqn AP $y-1=\frac{1}{4}(x-2)$	**M1** **M1**	For $m_1m_2 = -1$ calculus needed Normal, not tangent
	→ $A\,(-2, 0)$ or $B\,(0, ½)$ Midpoint AP also (0, ½)	**A1** **A1** [6]	 Full justification.
(ii)	$\int x^2 dy = \int\left(\dfrac{9}{2}-\dfrac{y^2}{2}\right)dy$	**M1**	Attempt to integrate x^2
	$=\dfrac{9y}{2}-\dfrac{y^3}{6}$	**A1**	Correct integration
	Upper limit = 3 Uses limits 1 to 3 → volume = $4\frac{2}{3}\pi$	**B1** **DM1** **A1** [5]	Evaluates upper limit Uses both limits correctly

CAMBRIDGE INTERNATIONAL EXAMINATIONS

Cambridge International Advanced Subsidiary and Advanced Level

MARK SCHEME for the October/November 2015 series

9709 MATHEMATICS

9709/13 Paper 1, maximum raw mark 75

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1	$x^2 - 4x + c = 2x - 7 \to x^2 - 6x + c + 7 (= 0)$ $36 - 4(c+7) < 0$ $c > 2$	**M1** **DM1** **A1** [3]	All terms on one side Apply $b^2 - 4ac < 0$. Allow $\leqslant$.
2	$[7C2] \times \left[\left(\frac{x}{3}\right)^5\right] \times \left[\left(\frac{9}{x^2}\right)^2\right]$ soi $21 \times \frac{1}{3^5}(x^5) \times 81\left(\frac{1}{x^4}\right)$ soi 7	**B2,1,0** **B1** **B1** [4]	Seen Identified as required term Accept $7x$
3 (i)	$[3]\left[(x-1)^2\right][-1]$	**B1B1B1** [3]	
(ii)	$f'(x) = 3x^2 - 6x + 7$ $= 3(x-1)^2 + 4$ > 0 hence increasing	**B1** **B1**√ **DB1** [3]	Ft *their* **(i)** + 5 Dep B1√ unless other valid reason
4 (i)	Sector $OCD = \frac{1}{2}(2r)^2\theta \quad (= 2r^2\theta)$ Sector(s) $OAB/OEF = (2)\frac{1}{2}r^2(\pi - \theta)$ Total $= r^2(\pi + \theta)$	**B1** **B1** **B1** [3]	$2r^2\theta$ seen somewhere Accept with/without factor (2) **AG** **www**
(ii)	Arc $CD = 2r\theta$ Arc(s) $AB/EF \; (2)r(\pi - \theta)$ Straight edges $= 4r$ Total $2\pi r + 4r$ (which is independent of θ)	**B1** **B1** **B1** **B1** [4]	Accept with/without factor (2) Must be simplified

5 (i)	$-2p^2+16p-24+2p^2-6p+2$	**M1**	Good attempt at scalar product
	Set scalar product = 0 and attempt solution	**DM1**	
	$p=2.2$	**A1** [3]	
(ii)	$4-2p=2(p-6)$ *or* $p=2(2p-6)$	**M1**	
	$p=4\rightarrow\overrightarrow{OA}=\begin{pmatrix}-2\\2\\1\end{pmatrix}\quad\overrightarrow{OB}=\begin{pmatrix}-4\\4\\2\end{pmatrix}$	**A1**	At least one of **OA** and **OB** correct
	$\lvert\overrightarrow{OA}\rvert=\sqrt{(-2)^2+2^2+1^2}=3$	**M1A1** [4]	For M1 accept a numerical p
	ALT 1 Compare *AB* with *OA* $\rightarrow 10-3p=p-6$ or $6-p=2p-6$. Similarly cf *AB* with *OB*	**M1**	
	ALT 2 (*OA.OB*)/(|*OA*|×|*OB*|) = 1 or –1 → $10p-22=\sqrt{5p^2-36p+}$ $73\sqrt{5p^2-16p+20}$ $\rightarrow 125p^4-260p^3+941p^2-1448p+$ $976=0\rightarrow p=4$. Similarly with *OA.AB* or *OB.AB*.	**M1**	
	ALT 3 *OA* & *OB* have equal unit vectors. (Similarly with *OA* & *AB* or *OB* & *AB*.) Hence $\frac{1}{\sqrt{5p^2-36p+73}}\begin{pmatrix}p-6\\2p-6\\1\end{pmatrix}$ $=\frac{1}{\sqrt{5p^2-16p+20}}\begin{pmatrix}4-2p\\p\\2\end{pmatrix}$ $\rightarrow\frac{1}{\sqrt{5p^2-36p+73}}=\frac{2}{\sqrt{5p^2-16p+20}}$ $\rightarrow 15p^2-128p+272=0$ $\rightarrow(p-4)(15p-68)=0$ $\rightarrow p=4(or68/15)$	**M1**	

6 **(i)** **(a)**	$1.92 + 1.84 + 1.76 + \ldots$ oe $\frac{20}{2}\left[2\times 1.92 + 19\times(-0.08)\right]$ oe 23.2 cao	**B1** **M1** **A1** [3]	OR a=0.96, d= –.04 & ans doubled/adjusted Corr formula used with corr d & *their* a, n $a = 1$, $n = 21 \to 12.6$ (25.2), $a = 0.96$, $n = 21 \to 11.76$ (23.52)
(b)	$1.92 + 1.92(.96) + 1.92(.96)^2 + \ldots$ $\frac{1.92(1-.96^{20}}{1-.96}$ 26.8 cao	**B1** **M1** **A1** [3]	OR a=.96, r =.96 & ans /doubled/adjusted Corr formula used with r =.96 & *their* a, n $a = .96$, $n = 21 \to 13.82$ (27.63) $a = 1$, $n = 21 \to 14.39$ (28.78)
(ii)	$\frac{1.92}{1-.96} = 48$ or $\frac{0.96}{1-0.96} = 24$ & then Double **AG**	**M1A1** [2]	$a = 1 \to 25$ (50) but must be doubled for M1 $1.92\frac{(1-0.96^n)}{1-0.96} < 48 \to 0.96^n > 0$ (www) 'which is true' scores SCB1
7 **(a)**	$1 + 3\sin^2\theta + 4\cos\theta = 0$ $1 + 3(1-\cos^2\theta) + 4\cos\theta + 0$ $3\cos^2\theta - 4\cos\theta - 4 = 0$ **AG** $\cos\theta = -2/3$ $\theta = 131.8$ *or* 228.2	**M1** **M1** **A1** **B1** **B1B1**√ [6]	Attempt to multiply by $\cos\theta$ Use $c^2 + s^2 = 1$ Ignore other solution Ft for 360 – 1st soln. –1 extra solns in range Radians 2.30 & 3.98 scores SCB1
(b)	$c = b/a$ cao $d = a - b$	**B1** **B1** [2]	Allow $D = (0, a - b)$
8 **(i)**	$3x+1 \leqslant -1$ (Accept $3x+1=-1, 3a+1=-1$) $x \leqslant -2/3 \Rightarrow$ largest value of a is $-2/3$ (in terms of a)	**M1** **A1** [2]	Do not allow gf in (i) to score in (iii) Accept $a \leqslant -2/3$ and $a = -2/3$
(ii)	$\mathrm{fg}(x) = 3(-1-x^2)+1$ $\mathrm{fg}(x) + 14 = 0 \Rightarrow 3x^2 = 12$ oe (2 terms) $x = -2$ only	**B1** **B1** **B1** [3]	No marks in this part for gf used
(iii)	$\mathrm{gf}(x) = -1-(3x+1)^2$ oe $\mathrm{gf}(x) \leqslant -50 \Rightarrow (3x+1)^2 \geqslant 49$ (Allow $\leqslant$ *or* = $3x+1 \geqslant 7$ *or* $3x+1 \leq -7$ (one sufficient) www $x \leqslant -8/3$ only www	**B1** **M1** **A1** **A1** [4]	No marks in this part for fg used OR attempt soln of $9x^2 + 6x - 48 + / \leqslant / \geqslant 0$ OR $x - 2 \geqslant$ *or* $3x + 8 \leqslant 0$(one suffic)

9 (i)	At $x=4$, $\frac{dy}{dx}=2$	**B1**	
	$\frac{dy}{dt}=\frac{dy}{dx}\times\frac{dx}{dt}=2\times3=6$	**M1A1** [3]	Use of Chain rule
(ii)	$(y)=x+4x^{\frac{1}{2}}(+c)$	**B1**	
	Sub $x=4,\ y=6\rightarrow 6=4+(4\times4^{\frac{1}{2}})+c$	**M1**	Must include c
	$c=-6\rightarrow (y=x+4x^{\frac{1}{2}}-6$	**A1** [3]	
(iii)	Eqn of tangent is $y-6=2(x-4)$ or $(6-0)/(4-x)=2$	**M1** **A1**	Correct eqn thru (4, 6) & with $m=$ *their* 2
	$B=(1,0)$ (Allow $x=1$) Gradient of normal $=-1/2$ $C=(16,0)$ (Allow $x=16$)	**M1** **A1** **A1**	[Expect eqn of normal: $y=-½x+8$]
	Area of triangle $=\frac{1}{2}\times15\times6=45$	[5]	Or $AB=\sqrt{45}$, $AC=\sqrt{180}\rightarrow$ Area = 45.0
10 (i)	$f'(x)=2-2(x+1)^{-3}$	**B1**	
	$f''(x)=6(x+1)^{-4}$	**B1**	
	$f'0=0$ hence stationary at $x=0$	**B1**	**AG**
	$f''0=6>0$ hence minimum	**B1** [4]	www. Dependent on correct f $''(x)$ except $-6(x+1)^{-4}\rightarrow<0$ MAX scores SC1
(ii)	$AB^2=(3/2)^2+(3/4)^2$	**M1**	
	$AB=1.68$ or $\sqrt{45/4}$ oe	**A1** [2]	
(iii)	Area under curve = $\int f(x)=x^2-(x+1)^{-1}$	**B1**	Ignore +c even if evaluated Do not penalise reversed limits
	$=\left(1-\frac{1}{2}\right)-\left(\frac{1}{4}-2\right)=9/4$ (Apply limits $-½\rightarrow1$)	**M1A1**	Allow reversed subtn if final ans positive
	Area trap. $=\frac{1}{2}(3+\frac{9}{4})\times\frac{3}{2}$	**M1**	
	$=63/16$ or 3.94	**A1**	
	Shaded area $63/16-9/4+27/16$ or 1.69	**A1** [6]	
	ALT eqn AB is $y=-½x+11/4$	**B1**	
	Area $=\int-½x+11/4-\int 2x+(x+1)^{-2}$	**M1**	Attempt integration of at least one
	$=\left[-\frac{1}{4}x^2+\frac{11}{4}x\right]-\left[x^2-(x+1)^{-1}\right]$	**A1A1**	Ignore +c even if evaluated Dep. on integration having taken place
	Apply limits $-½\rightarrow1$ to both integrals	**M1**	Allow reversed subtn if final ans positive
	27/16 or 1.69	**A1**	

CAMBRIDGE INTERNATIONAL EXAMINATIONS

Cambridge International Advanced Subsidiary and Advanced Level

MARK SCHEME for the May/June 2015 series

9709 MATHEMATICS

9709/11 Paper 1, maximum raw mark 75

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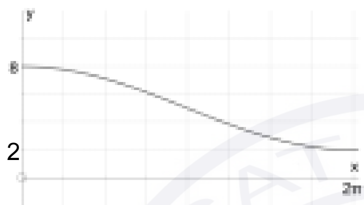
PA–1 This is deducted from A or B marks in the case of premature approximation. The PA–1 penalty is usually discussed at the meeting.

1	θ is obtuse , $\sin\theta = k$		
(i)	$\cos\theta = -\sqrt{(1-k^2)}$	B1 [1]	cao
(ii)	$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ used	M1	Used, attempt at cosine seen in (i)
	$\rightarrow\ \tan\theta = -\dfrac{k}{\sqrt{(1-k^2)}}$ aef	A1$\checkmark$ [2]	Ft for their cosine as a function of k only, from part (i)
(iii)	$\sin(\theta+\pi) = -k$	B1 [1]	cao
2	$y = 2x^2$, $X(-2, 0)$ and $P(p, 0)$		
(i)	$A = \dfrac{1}{2}\times(2+p)\times 2p^2\ (= 2p^2 + p^3)$	M1 A1 [2]	Attempt at base and height in terms of p and use of $\dfrac{bh}{2}$
(ii)	$\dfrac{dA}{dp} = 4p + 3p^2$	B1	cao
	$\dfrac{dA}{dt} = \dfrac{dA}{dp}\times\dfrac{dp}{dt} = 0.02\times 20 = 0.4$ or $\dfrac{dA}{dt} = 4p\dfrac{dp}{dt} + 3p^2\dfrac{dp}{dt}$	M1 A1 [3]	any correct method, cao
3	$(1-x)^2(1+2x)^6$.		
(i) (a)	$(1-x)^6 = 1 - 6x + 15x^2$	B2,1 [2]	−1 each error
(b)	$(1+2x)^6 = 1 + 12x + 60x^2$	B2,1 [2]	−1 each error **SC** B1 only, in each part, for all 3 correct descending powers **SC** only one penalty for omission of the '1' in each expansion
(ii)	Product of **(a)** and **(b)** with >1 term $\rightarrow\ 60 - 72 + 15 = 3$	M1 DM1A1 [3]	Must be 2 or more products M1 exactly 3 products. cao, condone $3x^2$

4	$\overrightarrow{OA}=\begin{pmatrix}3\\0\\-4\end{pmatrix}, \overrightarrow{OB}=\begin{pmatrix}6\\-3\\2\end{pmatrix}, \overrightarrow{OC}=\begin{pmatrix}k\\-2k\\2k-3\end{pmatrix}$		
(i)	$OA \cdot OB = 18 - 8 = 10$ Modulus of $OA = 5$, of $OB = 7$ Angle $AOB = \cos^{-1}\left(\frac{10}{35}\right)$ aef $\rightarrow \frac{10}{35}$ or $\frac{2}{7}$	M1 M1 A1 [3]	Use of $x_1x_2 + y_1y_2 + z_1z_2$ All linked with modulus cao, (if angle given, no penalty), correct angle implies correct cosine
(ii)	$\overrightarrow{AB} = \mathbf{b} - \mathbf{a} = \begin{pmatrix}3\\-3\\6\end{pmatrix}$ $k^2 + 4k^2 + (2k-3)^2 = 9 + 9 + 36$ $\rightarrow 9k^2 - 12k - 45 (= 0)$ $\rightarrow k = 3$ or $k = -\frac{5}{3}$	B1 M1 DM1 A1 [4]	allow for $\mathbf{a} - \mathbf{b}$ Correct use of moduli using their AB obtains 3 term quadratic. cao
5 (i)	$24 = r + r + r\theta$ $\rightarrow \theta = \frac{24-2r}{r}$ $A = \frac{1}{2}r^2\theta = \frac{24r}{2} - r^2 = 12r - r^2$. aef, ag	 M1 M1A1 [3]	(May not use θ) Attempt at $s = r\theta$ linked with 24 and r Uses A formula with θ as f(r). cao
(ii)	$(A =) 36 - (r-6)^2$	B1 B1 [2]	cao
(iii)	Greatest value of $A = 36$ $(r = 6) \rightarrow \theta = 2$	B1√ B1 [2]	Ft on **(ii).** cao, may use calculus or the discriminant on $12r - r^2$

6 (i)	$y-2t=-2(x-3t)(y+2x=8t)$ Set x to 0 $\rightarrow$ $B(0, 8t)$ Set y to 0 $\rightarrow A(4t, 0)$ $\rightarrow$ Area = $16t^2$	M1 M1 A1 [3]	Unsimplified or equivalent forms Attempt at both A and B, then using cao
(ii)	$m=\frac{1}{2}$ $\rightarrow y-2t=\frac{1}{2}(x-3t)(2y=x+t)$ Set y to 0 $\rightarrow$ $C(-t, 0)$ Midpoint of CP is (t, t) This lies on the line $y = x$.	B1 M1 A1 A1 [4]	cao Unsimplified or equivalent forms co correctly shown.
7 (a)	$ar^2=\frac{1}{3}$, $ar^3=\frac{2}{9}$ $\rightarrow r=\frac{2}{3}$ aef Substituting $\rightarrow$ $a=\frac{3}{4}$ $\rightarrow S_\infty=\frac{\frac{3}{4}}{\frac{1}{3}}=2\frac{1}{4}$ aef	M1 A1 M1 A1 [4]	Any valid method, seen or implied. Could be answers only. Both a and r Correct formula with $\lvert r\rvert<1$, cao
(b)	$4a=a+4d$ $\rightarrow$ $3a=4d$ $360=S_5=\frac{5}{2}(2a+4d)$ or $12.5a$ $\rightarrow a = 28.8°$ aef Largest = $a + 4d$ or $4a = 115.2°$ aef	B1 M1 A1 B1 [4]	May be implied in $360=5/2(a+4a)$ Correct S_n formula or sum of 5 terms cao, may be implied (may use degrees or radians)

8	f : $x \mapsto 5 + 3\cos\left(\frac{1}{2}x\right)$ for $0 \leqslant x \leqslant 2\pi$.		
(i)	$5 + 3\cos\left(\frac{1}{2}x\right) = 7$ $\cos\left(\frac{1}{2}x\right) = \frac{2}{3}$	B1	Makes $\cos\left(\frac{1}{2}x\right) = \frac{2}{3}$
	$\frac{1}{2}x = 0.84$ $x = 1.68$ only, aef (in given range)	M1A1 [3]	Looks up $\cos^{-1}$ first, then ×2
(ii)		B1 B1 [2]	y always +ve, m always –ve. from (0, 8) to (2π, 2) (may be implied)
(iii)	No turning point on graph or 1:1	B1 [1]	cao, independent of graph in (ii)
(iv)	$y = 5 + 3\cos\left(\frac{1}{2}x\right)$	M1	Tries to make x subject.
	Order; −5, ÷3, $\cos^{-1}$, ×2	M1	Correct order of operations
	$x = 2\cos^{-1}\left(\frac{x-5}{3}\right)$	A1 [3]	cao

9	$y = x^3 + px^2$		
(i)	$\frac{dy}{dx} = 3x^2 + 2px$	B1	cao
	Sets to 0 $\rightarrow$ $x = 0$ or $-\frac{2p}{3}$	M1	Sets differential to 0
	$\rightarrow$ (0, 0) or $\left(-\frac{2p}{3}, \frac{4p^3}{27}\right)$	A1 A1 [4]	cao cao, first A1 for any correct turning point or any correct pair of x values. 2nd A1 for 2 complete TPs
(ii)	$\frac{d^2y}{dx^2} = 6x + 2p$	M1	Other methods include; clear demonstration of sign change of gradient, clear reference to the shape of the curve
	At (0, 0) $\rightarrow$ $2p$ +ve Minimum	A1	www
	At $\left(-\frac{2p}{3}, \frac{4p^3}{27}\right) \rightarrow -2p$ –ve Maximum	A1 [3]	
(iii)	$y = x^3 + px^2 + px \rightarrow 3x^2 + 2px + p \ (= 0)$	B1	
	Uses $b^2 - 4ac$	M1	Any correct use of discriminant
	$\rightarrow 4p^2 - 12p < 0$		
	$\rightarrow$ $0 < p < 3$ aef	A1 [3]	cao (condone $\leqslant$)

10	$y = \dfrac{8}{\sqrt{3x+4}}$		
(i)	$\dfrac{dy}{dx} = \dfrac{-4}{(3x+4)^{\frac{3}{2}}} \times 3$ aef	B1 B1	Without the "×3" For "×3" even if 1st B mark lost.
	$\rightarrow m_{(x=0)} = -\dfrac{3}{2}$ Perpendicular $m_{(x=0)} = \dfrac{2}{3}$	M1	Use of $m_1 m_2 = -1$ after attempting to find $\dfrac{dy}{dx}_{(x=0)}$
	Eqn of normal $y - 4 = \dfrac{2}{3}(x-0)$	M1	Unsimplified line equation
	Meets $x = 4$ at $B\left(4, \dfrac{20}{3}\right)$	A1 [5]	cao
(ii)	$\displaystyle\int \frac{8}{\sqrt{(3x+4)}}\,dx = \frac{8\sqrt{(3x+4))}}{\frac{1}{2}} \div 3$	B1 B1	Without "÷3". For "÷3"
	Limits from 0 to 4 $\rightarrow$ Area $P = \dfrac{32}{3}$	M1 A1	Correct use of correct limits. cao
	Area Q = Trapezium – P Area of Trapezium = $\dfrac{1}{2}\left(4 + \dfrac{20}{3}\right) \times 4 = \dfrac{64}{3}$	M1	Correct method for area of trapezium
	$\rightarrow$ Areas of P and Q are both $\dfrac{32}{3}$	A1 [6]	All correct.

CAMBRIDGE INTERNATIONAL EXAMINATIONS

Cambridge International Advanced Subsidiary and Advanced Level

MARK SCHEME for the May/June 2015 series

9709 MATHEMATICS

9709/12 Paper 1, maximum raw mark 75

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1	$f'(x) = 5 - 2x^2$ and (3, 5) $f(x) = 5x - \frac{2x^3}{3}$ $(+c)$ Uses (3, 5) $\rightarrow c = 8$	 B1 M1 A1 [3]	 For integral Uses the point in an integral co
2	Radius of semicircle = $\frac{1}{2}AB = r\sin\theta$ Area of semicircle = $\frac{1}{2}\pi r^2\sin^2\theta = A_1$ Shaded area = semicircle – segment $= A_1 - \frac{1}{2}r^2 2\theta + \frac{1}{2}r^2\sin 2\theta$	B1 B1√ B1B1 [4]	aef Uses $\frac{1}{2}\pi r^2$ with $r = f(\theta)$ B1 (–sector), B1 for + (triangle)
3 (i)	$(2-x)^6$ Coeff of x^2 is 240 Coeff of x^3 is − 20 × 8 = −160	 B1 B2,1 [3]	 co B1 for +160
(ii)	$(3x+1)(2-x)^6$ Product needs exactly 2 terms → 720 – 160 = 560	 M1 A1√ [2]	 3 × their 240 + their -160 √ for candidate's answers.
4	$u = 2x(y - x)$ and $x + 3y = 12$, $u = 2x\left(\frac{12-x}{3} - x\right)$ $= 8x - \frac{8x^2}{3}$ $\frac{du}{dx} = 8 - \frac{16x}{3}$ $= 0$ when $x = 1\frac{1}{2}$ $\rightarrow (y = 3\frac{1}{2})$ $\rightarrow u = 6$	 M1 A1 M1 A1 A1 [5]	 Expresses u in terms of x Differentiate candidate's quadratic, sets to 0 + attempt to find x, or other valid method Complete method that leads to u Co
5 (i)	$\frac{\sin\theta - \cos\theta}{\sin\theta + \cos\theta}$. Divides top and bottom by cos θ $\rightarrow \frac{t-1}{t+1}$	 B1 [1]	 Answer given.
(ii)	$\frac{\sin\theta - \cos\theta}{\sin\theta + \cos\theta} = \frac{1}{6}\tan\theta$ $\rightarrow \frac{t-1}{t+1} = \frac{t}{6}$ $\rightarrow t^2 - 5t + 6 = 0$ $\rightarrow t = 2$ or $t = 3$ $\rightarrow \theta = 63.4°$ or $71.6°$	 B1 M1 A1 A1 [4]	 Using the identity. Forms a 3 term quadratic with terms all on same side. co co

6	$h = 60(1 - \cos kt)$		
(i)	Max h when cos $= -1 \rightarrow 120$	B1 [1]	Co
(ii)	$h = 0$ and $t = 30$, or $h = 120$ and $t = 15$ $\rightarrow \cos 30k = 1$ or $\cos 15k = -1$ $\rightarrow 30k = 2\pi$ or $15k = \pi$ $\rightarrow k = \frac{2\pi}{30} = \frac{\pi}{15}$	M1 A1 [2]	Substituting a correct pair of values into the equation. co ag
(iii)	$90 = 60(1 - \cos kt)$ $\rightarrow \cos kt = \frac{-30}{60} = -0.5$ $\rightarrow kt = \frac{2\pi}{3}$ or $\rightarrow kt = \frac{4\pi}{3}$ $\rightarrow$ Either $t = 10$ or 20 or both $\rightarrow t = 10$ minutes	 B1 B1 B1 [3]	 co – but there must be evidence of correct subtraction.
7	$A(4, 6)$, $B(10, 2)$.		
(i)	$M = (7, 4)$ m of $AB = -\frac{2}{3}$ m of perpendicular $= \frac{3}{2}$ $\rightarrow y - 4 = \frac{3}{2}(x - 7)$	B1 B1 M1 A1 [4]	co co Use of $m_1m_2 = -1$ & their midpoint in the equation of a line. co
(ii)	Eqn of line parallel to AB through (3, 11) $\rightarrow y - 11 = -\frac{2}{3}(x - 3)$ Sim eqns $\rightarrow C(9, 7)$	 M1 DM1A1 [3]	 Needs to use m of AB Must be using their correct lines. Co
8 (a)	1st, 2nd, nth are 56, 53 and -22 $a = 56$, $d = -3$ $-22 = 56 + (n - 1)(-3)$ $\rightarrow n = 27$ $S_{27} = \frac{27}{2}(112 + 26(-3))$ $\rightarrow 459$	 M1 A1 M1 A1 [4]	 Uses correct u_n formula. co Needs positive integer n Co
(b)	1^{st}, 2^{nd}, 3^{rd} are $2k + 6$, $2k$ and $k + 2$.		
(i)	Either $\frac{2k}{2k+6} = \frac{k+2}{2k}$ or uses a, r and eliminates $\rightarrow 2k^2 - 10k - 12 = 0$ $\rightarrow k = 6$	 M1 DM1 A1 [3]	 Correct method for equation in k. Forms quad. or cubic equation with no brackets or fractions. Co

(ii)	$S_\infty = \frac{a}{1-r}$ with $r = \frac{2k}{2k+6}$ or $\frac{k+2}{2k}$ $(=\frac{2}{3})$ $\rightarrow$ 54	M1 A1 [2]	Needs attempt at a and r and S_∞ Co
9	$\overrightarrow{OA} = 2\mathbf{i} + 4\mathbf{j} + 4\mathbf{k}$ and $\overrightarrow{OB} = 3\mathbf{i} + \mathbf{j} + 4\mathbf{k}$		
(i)	$\overrightarrow{OA}.\overrightarrow{OB} = 6 + 4 + 16 = 26$ $\lvert\overrightarrow{OA}\rvert = \sqrt{36}$, $\lvert\overrightarrow{OB}\rvert = \sqrt{26}$ Cos $AOB = \frac{26}{6\sqrt{26}}$ $\rightarrow$ 31.8°	M1 M1 M1 A1 [4]	Must be numerical at some stage Product of 2 moduli All linked correctly co
(ii)	$\overrightarrow{AB} = \mathbf{b} - \mathbf{a} = \begin{pmatrix}1\\-3\\0\end{pmatrix}$	B1	
	$\overrightarrow{OC} = \begin{pmatrix}2\\4\\4\end{pmatrix} + 2\overrightarrow{AB}$ *or* $\begin{pmatrix}3\\1\\4\end{pmatrix} + \overrightarrow{AB}$	M1	Correct link
	$\overrightarrow{OC} = \begin{pmatrix}4\\-2\\4\end{pmatrix}$		
	Unit vector ÷ modulus $\rightarrow \frac{1}{6}\begin{pmatrix}4\\-2\\4\end{pmatrix}$	M1 A1 [4]	÷ by modulus. co
(iii)	$\lvert\overrightarrow{OC}\rvert = 6$, $\lvert\overrightarrow{OA}\rvert = 6$	B1 [1]	co

10	$y = \dfrac{4}{2x-1}$.	B1	Correct without the ÷2
(i)	$\displaystyle\int \frac{16}{(2x-1)^2}\,dx = \frac{-16}{2x-1} \div 2$	B1	For the ÷2 even if first B1 is lost
	Vol $= \pi\left[\dfrac{-8}{2x-1}\right]$ with limits 1 and 2	M1	Use of limits in a changed expression.
	$\rightarrow \dfrac{16\pi}{3}$	A1 [4]	co
(ii)	$m = \frac{1}{2}m$ of tangent $= -2$	M1	Use of $m_1m_2 = -1$
	$\dfrac{dy}{dx} = \dfrac{-4}{(2x-1)^2} \times 2$	B1 B1	Correct without the ×2 For the ×2 even if first B1 is lost
	Equating their $\frac{dy}{dx}$ to -2	DM1	
	$\rightarrow x = \frac{3}{2}$ or $-\frac{1}{2}$ ($y = 2$ or -2)	A1	co
	$\rightarrow c = \dfrac{5}{2}$ or $-\dfrac{7}{2}$	A1 [6]	co
11	$f : x \mapsto 2x^2 - 6x + 5$		
(i)	$2x^2 - 6x + 5 - p = 0$ has no real roots	M1	Sets to 0 with p on LHS.
	Uses $b^2 - 4ac \rightarrow 36 - 8(5 - p)$	DM1	Uses discriminant.
	Sets to 0 $\rightarrow p < \frac{1}{2}$	A1 [3]	co – must be "$<$", not "$\leqslant$".
(ii)	$2x^2 - 6x + 5 = 2\left(x - \frac{3}{2}\right)^2 + \frac{1}{2}$	3 × B1 [3]	co
(iii)	Range of g $\frac{1}{2} \leqslant g(x) \leqslant 13$	B1√ B1 [2]	√ on **(ii)** co from sub of $x = 4$
	$h : x \mapsto 2x^2 - 6x + 5$ for $k \leqslant x \leqslant 4$		
(iv)	Smallest $k = \frac{3}{2}$	B1√ [1]	√ on **(ii)**
(v)	$h(x) = 2\left(x - \frac{3}{2}\right)^2 + \frac{1}{2}$	M1	Using comp square form to try and get x as subject or y if transposed.
	Order of operations $\pm\frac{1}{2}$, ÷2, √, $\pm\frac{3}{2}$	DM1	Order must be correct
	$\rightarrow$ Inverse $= \frac{3}{2} + \sqrt{\left(\frac{x}{2} - \frac{1}{4}\right)}$	A1 [3]	co (without ±)

CAMBRIDGE INTERNATIONAL EXAMINATIONS

Cambridge International Advanced Subsidiary and Advanced Level

MARK SCHEME for the May/June 2015 series

9709 MATHEMATICS

9709/13 Paper 1 (Paper 1), maximum raw mark 75

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PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

1	$2(x-3)^2-11$	**B1B1B1** **[3]**	For 2, $(x-3)^2$, -11. Or a=2, b= 3, c= 11
2	$\left[\dfrac{(2x+1)^{\frac{3}{2}}}{\frac{3}{2}}\right]$ [÷2] (+c)	**B1B1**	
	$7=9+c$	**M1**	Attempt subst $x=4, y=7$. c must be there. Dep. on attempt at integration.
	$y=\dfrac{(2x+1)^{\frac{3}{2}}}{3}-2$ or unsimplified	**A1** **[4]**	$c=-2$ sufficient
3 (i)	$a^5-5a^4x+10a^3x^2-10a^2x^3$ + ...	**B2,1,0** **[2]**	Ok full expansion (ignore extra terms) Descending: Ok if full expansion but max B1 for 4 terms
(ii)	$(1-ax)(..10a^3x^2-10a^2x^3..)=(x^3)(-10a^4-10a^2)$	**M1**	Attempt to find coeff. of x^3 from 2 terms
	$-10a^4-10a^2=-200$	**A1**√	Ft from *their* $10a^3, -10a^2$ from part (i)
	$a^2=4$ ignore $a^2=-5$	**M1**	Attempt soln. for a^2 from 3-term quad. in a^2
	$a=\pm 2$ cao	**A1** **[4]**	Ignore any imaginary solutions
4 (i)	$\tan\theta=1/3$	**M1**	
	$\theta=18.4°$ only	**A1** **[2]**	Ignore solns. outside range 0→180
(ii)	$\mathbf{tan}2x=(\pm)1/\sqrt{3}$ Must be sq. root soi	**M1**	$\sin 2x=(\pm)1/2$ **or** $\cos 2x=(\pm)\sqrt{\mathbf{3/2}}$ using $c^2+s^2=1$. Not $\tan x=(\pm)\dfrac{1}{\sqrt{3}}$ etc.
	$(x)=15$	**A1**	ft for (90 ± their 15) or (180 – their 15)
	$(x)=$ any correct second value (75, 105, 165)	**A1**√	All four correct. Extra solns in range 1
	$(x)=$ cao	**A1** **[4]**	
5 (i)	$\overrightarrow{AB}=\begin{pmatrix}5\\-1\\-2\end{pmatrix}-\begin{pmatrix}3\\2\\-3\end{pmatrix}=\begin{pmatrix}2\\-3\\1\end{pmatrix}$	**B1**	Or $\overrightarrow{BA}, \overrightarrow{CB}$. Allow any combination. Ignore labels.
	$\overrightarrow{BC}=\begin{pmatrix}6\\1\\2\end{pmatrix}-\begin{pmatrix}5\\-1\\-2\end{pmatrix}=\begin{pmatrix}1\\2\\4\end{pmatrix}$	**B1**	
	$\overrightarrow{AB}.\overrightarrow{BC}=2-6+4$ oe must be seen = 0	**M1**	Could be part of calculation for angle ABC
	hence $ABC=90°$	**A1** **[4]**	AG Alt methods Pythag, Cosine Rule

(ii)	$\left\|\overrightarrow{AB}\right\| = \sqrt{14}$, $\left\|\overrightarrow{BC}\right\| = \sqrt{21}$ oe Area $= \frac{1}{2}\sqrt{14}\sqrt{21}$ 8.6 oe	**B1** **M1** **A1** **[3]**	At least one correct Reasonable attempt at vectors and their magnitudes Allow $\frac{7\sqrt{6}}{2}$
6 (i)	Attempt to find $(\mathrm{f}^{-1})^{-1}$ $2xy = 1 - 5x$ or $\frac{1}{2x} = y + \frac{5}{2}$ Allow 1 sign error $x = \frac{1}{2y+5}$ oe Allow 1 sign error (total) $(\mathrm{f}(x)) = \frac{1}{2x+5}$ for $x \geqslant -\frac{9}{4}$ **(Allow** $-\frac{9}{4} \leqslant x \leqslant \infty$)	**M1** **A1** **A1** **A1 B1** **[5]**	Or with *x*/*y* transposed. Or with *x*/*y* transposed. Allow $x = \frac{\frac{1}{2}}{y + \frac{5}{2}}$. Allow $\frac{\frac{1}{2}}{x + \frac{5}{2}}$. Condone $x > \frac{-9}{4}$, $(\frac{-9}{4}, \infty)$ (etc.)
(ii)	$\mathbf{f}^{-1}\left(\frac{1}{x}\right) = \frac{1 - \frac{5}{x}}{\frac{2}{x}}$ $\frac{x-5}{2}$ or $\frac{1}{2}x - \frac{5}{2}$	**M1** **A1** **[2]**	Reasonable attempt to find $\mathbf{f}^{-1}\left(\frac{1}{x}\right)$.
7 (i)	$(9-p)^2 + (3p)^2 = 169$ $10p^2 - 18p - 88 (= 0)$ oe $p = 4$ ***or*** $-11/5$ oe	**M1** **A1** **A1** **[3]**	Or $\sqrt{\ } = 13$ 3-term quad
(ii)	Gradient of given line $= -\frac{2}{3}$ Hence gradient of $AB = \frac{3}{2}$ $\frac{3}{2} = \frac{3p}{9-p}$ oe eg $\left(\frac{-2}{3}\right)\left(\frac{3p}{9-p}\right) = 1$ (includes previous M1) $p = 3$	**B1** **M1** **M1** **A1** **[4]**	Attempt using $m_1 m_2 = -1$ Or vectors $\begin{pmatrix} 9-p \\ 3p \end{pmatrix} . \begin{pmatrix} 3 \\ -2 \end{pmatrix}$
8 (i)	$-(x+1)^{-2} - 2(x+1)^{-3}$	**M1A1** **A1** **[3]**	M1 for recognisable attempt at differentn. Allow $\frac{-x^2 - 4x - 3}{(x+1)^4}$ from Q rule. (A2,1,0)

(ii)	$f'(x) < 0$ hence decreasing	B1 [1]	Dep. on *their* (i) < 0 for $x >$ 1
(iii)	$\frac{-1}{(x+1)^2} - \frac{2}{(x+1)^3} = 0$ **or** $\frac{-x^2-4x-3}{(x+1)^4} = 0$ $\frac{-(x+1)-2}{(x+1)^3} = 0 \to -x-1-2=0$ or $-x^2-4x-3=0$ $x=-3,\ y=-1/4$	M1* M1 Dep* A1A1 [4]	Set $\frac{dy}{dx}$ to 0 OR mult by $(x+1)^3$ or $(x+1)^5$ (i.e.×mult) × multn $\to -(x+1)^3 - 2(x+1)^2 = 0$ (−3, −1/4) www scores 4/4
9 (a)	2222/17 (=131 or 130.7) 131 × 17 (=2227) −2222 + 2227 = 5	M1 M1 A1 [3]	Ignore signs. Allow 2239/17→131.7 or 132 Ignore signs. Use 131. 5 www gets 3/3
(b)	$r = \frac{2\cos\theta}{\sqrt{3}}$ soi oe $(-1<)\frac{2\cos\theta}{\sqrt{3}} < 1$ or $(0<)\frac{2\cos\theta}{\sqrt{3}} < 1$ soi $\pi/6,\ 5\pi/6$ soi (but dep. on M1) $\pi/6 < \theta < 5\pi/6$ cao	B1 M1√ A1A1 A1 [5]	Ft on *their* r. Ignore a 2nd inequality on LHS Allow 30º, 150º. Accept ⩽
10 (i)	$\frac{dy}{dx} = 6-6x$ At $x=2$, gradient $=-6$ soi $y-9=-6(x-2)$ oe Expect $y=-6x+21$ When $y=0,\ x=3\frac{1}{2}$ cao	B1 B1√ M1 A1 [4]	Line through (2, 9) and with gradient *their* −6
(ii)	Area under curve: $\int 9+6x-3x^2 dx = 9x+3x^2-x^3$ $(27+27-27)-(18+12-8)$ Area under tangent: $\frac{1}{2}\times\frac{3}{2}\times 9\ (=\frac{27}{4})$ Area required $\frac{27}{4} - 5 = \frac{7}{4}$	B2,1,0 M1 B1√ A1 [5]	Allow unsimplified terms Apply limits 2,3. Expect 5 OR $\int_2^{7/2}(-6x+21)\,dx\ (\to \frac{27}{4})$. Ft on *their* $-6x+21$ and/or *their* 7/2.

11 (i)	$OC = r\cos\alpha$ or $AC = r\sin\alpha$ or oe soi (Area $\Delta OAC =$) $\frac{1}{2}r^2\sin\alpha\cos\alpha$ $\frac{1}{2}r^2\sin\alpha\cos\alpha = \frac{1}{2}\times\frac{1}{2}r^2\alpha$ oe $\boldsymbol{\sin\alpha\cos\alpha = \frac{1}{2}\alpha}$	**M1** **A1** **M1** **A1** **[4]**	 Or e.g. $\frac{1}{2}r^2\alpha - \frac{1}{2}r^2\cos\alpha\sin\alpha = \frac{1}{4}r^2\alpha$ $\frac{1}{2}r^2\alpha - \frac{1}{2}r^2\cos\alpha\sin\alpha = \frac{1}{2}r^2\cos\alpha\sin\alpha$ AG
(ii)	Perimeter $\Delta OAC = r + r\sin\alpha + r\cos\alpha = 2.4(0)r$ Perim. $ACB = r\alpha + r\sin\alpha + r - r\cos\alpha = 2.18r$ or $2.17r$ Ratio $= \dfrac{2.4(0)}{2.18 \text{ or } 2.17} : 1 = 1.1 : 1$	**M1A1** **M1A1** **A1** **[5]**	Allow with r a number. 2.0164 gets M1A0 Allow with r a number. 0.9644 gets M1A0 Allow 2.2 www. Use of cos = 0.6, sin = 0.8, $\alpha = 0.9$ is PA 1
(iii)	54.3° cao	**B1** **[1]**	

CAMBRIDGE INTERNATIONAL EXAMINATIONS

Cambridge International Advanced Subsidiary and Advanced Level

MARK SCHEME for the October/November 2014 series

9709 MATHEMATICS

9709/11 Paper 1, maximum raw mark 75

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1	${}^7C_1 \times 2^6 \times a$ (=) ${}^7C_2 \times 2^5 \times a^2$ soi $a = \left(\dfrac{7\times 2^6}{21\times 2^5}\right) = \dfrac{2}{3}$ oe	**B2, 1, 0** **B1** **[3]**	Treat the same error in each expression as a single error
2	$\tan^{-1}(3) =$ 1.249 or 71.565° sin 1.25 or sin71.6 or 0.949 soi (x =) 1.95 cao, accept $1 + \dfrac{3}{\sqrt{10}}$ oe	**M1** **M1** **A1** **[3]**	Attempt at $\tan^{-1}3$ or right angle triangle with attempt at hypotenuse = √10 Attempt at sin $\tan^{-1}3$ Answer only **B3**
3	$13\sin^2\theta + 2\cos\theta + \cos^2\theta = 4 + 2\cos\theta$ $13\sin^2\theta + 1 - \sin^2\theta = 4 \to \sin^2\theta = \frac{1}{4}$ or $13 - 13\cos^2\theta + \cos^2\theta = 4 \to \cos^2\theta = \frac{3}{4}$ 30°, 150°	**M1** **M1** **A1A1**√ **[4]**	Attempt to multiply by $2+\cos\theta$ Use of $s^2 + c^2$ appropriately **SC** both answers correct in radians, **A1** only Ft on 180 – their first value of θ
4 (i)	$32 - 4k = 20 \Rightarrow k = 3$ $4b + 3\times 2b = 20$ $b = 2$	**M1A1** **M1** **A1** **[4]**	Sub (8, −4) [alt: $(2b+4)/(b-8) = -4/k$ Sub (b, 2b), $4b + 2bk = 20$ **M1** both **M1** solving **A1**, **A1**]
(ii)	Mid-point = (5, 0)	**B1**√ **[1]**	Ft on *their b*
5	$x^2 + x(k-2) + (k-2)(=0)$ $(k-2)^2 - 4(k-2)(>0)$ soi $(k-2)(k-6)(>0)$ $k < 2$ or $k > 6$ (condone ⩽, ⩾) Allow {–∞, 2}U{6, ∞} etc.	**M1** **M1** **DM1** **A2** **[5]**	Equate and move terms to one side of equ. Apply $b^2 - 4ac$ (>0). Allow ⩾ at this stage. Attempt to factorise or solve or find 2 solns. **SCA1** for 2, 6 seen with wrong inequalities
6 (i)	***AB*** or ***BA*** = ±[(7**i** − 3**j** + **k**) – (3**i** + 2**j** – **k**)]= ±(4**i** − 5**j** + 2**k**) (***AO.AB***) = ±(12 – 10 – 2) [allow as column if total given] = 0 hence *OAB* = 90°	**M1A1** **DM1** **A1** **[4]**	May be seen in part **(ii)** **OR** $AB^2 = 45, AO^2 = 14, OB^2 = 59$ Hence $AB^2 + AO^2 = OB^2$ Hence *OAB* = 90°
(ii)	$\lvert \boldsymbol{OA}\rvert = \sqrt{9+4+1} = \sqrt{14}$, $\lvert \boldsymbol{AB}\rvert = \sqrt{16+25+4} = \sqrt{45}$ Area $\Delta = \frac{1}{2}\sqrt{14}\left(\sqrt{45}\right) = 12.5$	**B1** **M1A1** **[3]**	At least one magnitude correct in **(i)** or **(ii)** Accept 12.6, $\dfrac{\left(3\sqrt{70}\right)}{2}$ oe

7 **(i)**	$S=\frac{a}{1-r}$, $3S=\frac{a}{1-2r}$	**B1**	At least $3S=\frac{a}{1-2r}$
	$1-r=3-6r$	**M1**	Eliminate S
	$r=\frac{2}{5}$	**A1** **[3]**	
(ii)	$7+(n-1)d=84$ and/or $7+(3n-1)d=245$	**B1**	At least one of these equations seen
	$[(n-1)d=77,\ (3n-1)d=238,\ 2nd=161]$	**B1**	Two different seen – unsimplified ok
	$\frac{n-1}{3n-1}=\frac{77}{238}$ (must be from the correct u_n formula)	**M1**	Or other attempt to elim d. E.g. sub $d=\frac{161}{2n}$ (if n is eliminated d must be found)
	$n=23$ $(d=\frac{77}{22}=3.5)$	**A1** **[4]**	
8 **(i)**	Arc $AB=4\alpha$	**B1**	
	Arc $DC=(4\cos\alpha)\alpha$	**B1**	
	AC (or DB) $=4-4\cos\alpha$	**B1**	
	Perimeter $=4\alpha\cos\alpha+4\alpha+8-8\cos\alpha$	**B1** **[4]**	
(ii)	$OD=4\cos\frac{\pi}{6}\left(=2\sqrt{3}\right)$	**B1**	
	Shaded area = $\left[\frac{1}{2}\times4^2\times\frac{\pi}{6}\right]\left[-\frac{1}{2}\left(2\sqrt{3}\right)^2\times\frac{\pi}{6}\right]$	**B1B1**	
	$\frac{\pi}{3}$	**B1** **[4]**	Or $k=\frac{1}{3}$
9 **(i)**	$f'(2)=4-\frac{1}{2}=\frac{7}{2}\rightarrow$ gradient of normal $=-\frac{2}{7}$	**B1M1**	
	$y-6=-\frac{2}{7}(x-2)$ AEF	**A1√** **[3]**	Ft from their $f'(2)$
(ii)	$f(x)=x^2+\frac{2}{x}(+c)$	**B1B1**	
	$6=4+1+c\Rightarrow c=1$	**M1A1** **[4]**	Sub (2, 6) – dependent on c being present
(iii)	$2x-\frac{2}{x^2}=0\Rightarrow 2x^3-2=0$	**M1**	Put $f'(x)=0$ and attempt to solve
	$x=1$	**A1**	Not necessary for last A mark as $x>0$ given
	$f''(x)=2+\frac{4}{x^3}$ or any valid method	**M1**	
	$f''(1)=6$ OR >0 hence minimum	**A1** **[4]**	Dependent on everything correct

10 (i)	$(x-1)^2-16$	**B1B1** **[2]**	
(ii)	-16	**B1**√ **[1]**	Ft from **(i)**
(iii)	$9 \leqslant (x-1)^2-16 \leqslant 65$ **OR** $x^2-2x-15=9 \to 6,-4$ $25 \leqslant (x-1)^2 \leqslant 81$ $\quad x^2-2x-15=65 \to 10,-8$ $5 \leqslant x-1 \leqslant 9$ $\quad p=6$ $6 \leqslant x \leqslant 10$ $\quad q=10$	**M1** **M1** **A1** **A1** **[4]**	**OR** $x^2-2x-24 \geqslant 0$, $x^2-2x-80 \leqslant 0$, $(x-6)(x+4) \geqslant 0$ $(x-10)(x+8) \leqslant 0$ $x \geqslant 6$ $x \leqslant 10$ **SC B2, B2** for trial/improvement
(iv)	$x=(y-1)^2-16$ $\quad$ [interchange x/y] $y-1=(\pm)\sqrt{x+16}$ $\mathrm{f}^{-1}(x)=1+\sqrt{x+16}$	**M1** **M1** **A1** **[3]**	**OR** $(x-1)^2=y+16$ $x=1+(\pm)\sqrt{y+16}$ $\mathrm{f}^{-1}(x)=1+\sqrt{x+16}$
11 (i)	For $y=(4x+1)^{\frac{1}{2}}$, $\frac{\mathrm{d}y}{\mathrm{d}x}=\left[\frac{1}{2}(4x+1)^{-\frac{1}{2}}\right]\times[4]$ When $x=2$, gradient $m_1=\frac{2}{3}$ For $y=\frac{1}{2}x^2+1$, $\frac{\mathrm{d}y}{\mathrm{d}x}=x \to$ gradient $m_2=2$ $\alpha=\tan^{-1}m_2-\tan^{-1}m_1$ $\alpha=63.43-33.69=29.7$ $\quad$ cao	**B1B1** **B1**√ **B1** **M1** **A1** **[6]**	 Ft from *their* derivative above
(ii)	$\int(4x+1)^{\frac{1}{2}}\,\mathrm{d}x=\left[\frac{(4x+1)^{\frac{3}{2}}}{2/3}\right]\div[4]$ $\int\left(\frac{1}{2}x^2+1\right)\mathrm{d}x=\frac{1}{6}x^3+x$ $\int_0^2(4x+1)^{\frac{1}{2}}\,\mathrm{d}x=\frac{1}{6}[27-1]$, $\int_0^2\left(\frac{1}{2}x^2+1\right)\mathrm{d}x=\left[\frac{8}{6}+2\right]$ $\frac{13}{3}-\frac{10}{3}$ 1	**B1B1** **B1** **M1** **M1** **A1** **[6]**	 Apply limits $0 \to 2$ to at least the 1st integral Subtract the integrals (at some stage)

CAMBRIDGE INTERNATIONAL EXAMINATIONS

Cambridge International Advanced Subsidiary and Advanced Level

MARK SCHEME for the October/November 2014 series

9709 MATHEMATICS

9709/12 Paper 1, maximum raw mark 75

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PA−1 This is deducted from A or B marks in the case of premature approximation. The PA−1 penalty is usually discussed at the meeting.

1 Vol = (π) $\int x^2 dy = (\pi) \int (y-1)\,dy$	M1	Use of $\int x^2$ – not $\int y^2$ – ignore π
	A1	co
Integral is $\frac{1}{2}y^2 - y$ or $\frac{(y-1)^2}{2}$	B1	Sight of an integral sign with 1 and 5
Limits for y are 1 to 5		
$\rightarrow 8\pi$ or 25.1(AWRT)	A1 [4]	co (no π max 3/4)
2 (i) $\tan\theta = \frac{5}{12}$ $\rightarrow (\theta = 0.3948)$	M1 [1]	Any valid trig method ag
(ii) Other angle in triangle $= \frac{1}{2}\pi - 0.3948$	B1	Unsimplified OK
Area of triangle $AOB = \frac{1}{2}\times12\times5$ (= 30)	B1	co
Use of $\frac{1}{2}r^2\theta$ once	M1	With θ in radians and r = 5 or 12
Shaded area = sector + sector – triangle $= \frac{1}{2}\times12^2\times0.3948 + \frac{1}{2}5^2\theta - 30$	DM1	Sum of 2 sectors – triangle or any other valid method using the given angle and a different one.
= 28.43 + 14.70 – 30 = 13.1	A1 [5]	co
3 (i) $(1+x)^5 = 1 + 5x + 10x^2$	B2,1 [2]	Loses 1 for each error
(ii) $(1+px+x^2)^5$ $(1+)\ 5(px+x^2) + 10(px+x^2)^2$	M1	Replace x by $(px + x^2)$ in their expansion
Coeff of $x^2 = 5 + 10p^2$	DM1	Considers 2 terms
$= 95 \rightarrow p = 3$	A1 [3]	co – no penalty for ±3
4 $y = \frac{12}{3-2x}$		
(i) Differential $= -12(3-2x)^{-2} \times -2$	B1 B1 [2]	co co (even if 1st B mark lost)
(ii) $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt} = 0.4 \div 0.15$	M1	Chain rule used correctly (AEF)
$\rightarrow \frac{24}{(3-2x)^2} = \frac{8}{3}$	M1	Equates their $\frac{dy}{dx}$ with their $\frac{8}{3}$ or $\frac{3}{8}$
$\rightarrow x = 0$ or 3	A1 A1 [4]	co co

5 $1 + \sin x \tan x = 5\cos x$		
(i) Replaces t by s/c	M1	Correct formula
$1 + \dfrac{s^2}{c} = 5c$		
Replace s^2 by $1 - c^2$	M1	Correct formula used in appropriate place
$\rightarrow\ 6c^2 - c - 1\ (= 0)$	A1 [3]	AG
(ii) Soln of quadratic $\rightarrow$ ($c = -⅓$ or ½)	M1	Correct method
$\rightarrow\ x = 60°$ or $109.5°$	A1 A1 [3]	co co
6 $y = x^3 + ax^2 + bx$		
(i) $\dfrac{dy}{dx} = 3x^2 + 2ax + b$	B1	co
(ii) $b^2 - 4ac = 4a^2 - 12b\ (< 0)$	M1	Use of discriminant on their quadratic $\dfrac{dy}{dx}$ or other valid method
$\rightarrow\ a^2 < 3b$	A1 [3]	co – answer given
(iii) $y = x^3 - 6x^2 + 9x$		
$\dfrac{dy}{dx} = 3x^2 - 12x + 9 < 0$	M1	Attempt at differentiation
$= 0$ when $x = 1$ and 3	A1	co
$\rightarrow\ 1 < x < 3$	A1 [3]	condone $\leqslant$
7 **(i)** $\mathbf{AM} = -6\mathbf{i} + 2\mathbf{j} + 5\mathbf{k}$	B2,1	co −1 each error
$\mathbf{AC} = -8\mathbf{i} + 8\mathbf{j}$	B1 [3]	co
(ii) $\mathbf{AM.AC} = 48 + 16 = 64$	M1	Use of x_1y_1 + etc. with suitable vectors
$64 = \sqrt{128}\sqrt{65}\cos\theta$	M1 M1	Product of moduli. Correct link.
$\rightarrow\ \theta = 45.4°$	A1 [4]	co

8 **(a)** $S_n = 32n - n^2$. Set n to 1, a or $S_1 = 31$ Set n to 2 or other value $S_2 = 60$ $\rightarrow$ 2nd term = 29 $\rightarrow$ $d = -2$ (or equates formulae – compares coeffs n^2, n) [M1 comparing, A1 d A1 a]	B1 M1 A1 [3]	co Correct method. co [M1 only when coeffs compared]
(b) $\frac{a}{1-r} = 20$, $\frac{a(1-r)^2}{1-r}$, or $a + ar = 12.8$	B1 B1	co co
Elimination of $\frac{a}{1-r}$ or a or r	M1	'Correct' elimination to form equation in a or r
$\rightarrow$ $(r = 0.6)$ $\rightarrow$ $a = 8$	DM1 A1 [5]	Complete method leading to $a =$ Condone $a = 8$ and 32
9 **(i)** $m_{AB} = -3$ or $\frac{-9}{3}$	B1	oe
$m_{AD} = \frac{1}{3}$	M1	use of $m_1m_2 = -1$ with grad AB
Eqn AD $y - 6 = \frac{1}{3}(x - 2)$ or $3y = x + 16$	A1 [3]	co – OK unsimplified
(ii) Eqn CD $y - 3 = -3(x - 8)$ or $y = -3x + 27$ Sim Eqns $\rightarrow$ D (6½, 7½)	B1√ M1 A1 [3]	OK unsimplified. √on m of AB. Reasonable algebra leading to $x =$ or $y =$ with AD and CD
(iii) Use of vectors or mid-point $\rightarrow$ E (5, 12) or mid-point (5,4.5) Length of $BE = 15$	 B1 B1 [2]	 May be implied co
10 $\frac{d^2y}{dx^2} = \frac{24}{x^3} - 4$ **(i)** (If $x = 2$) it's negative $\rightarrow$ Max	 B1 [1]	 www
(ii) $\left(\frac{dy}{dx} =\right) -12x^{-2} - 4x + (A)$	B2,1,0	oe one per term
$= 0$ when $x = 2$ $\rightarrow$ $A = 11$	M1 A1 [4]	Attempt at the constant A after ∫n co
(iii) $(y =)\ 12x^{-1} - 2x^2 + Ax + (c)$	B2,1,0 √	oe Doesn't need $+c$, but does need a term A to give "Ax".
$y = 13$ when $x = 1$ $\rightarrow$ $c = -8$	M1	Attempt at c after ∫n
(If $x = 2$) $y = 12$	A1 [4]	co

11 $f : x \mapsto 6 - 4\cos\left(\frac{1}{2}x\right)$		
(i) $6 - 4\cos\left(\frac{1}{2}x\right) = 4 \rightarrow 4\cos\left(\frac{1}{2}x\right) = 2$	M1	Makes $\cos\left(\frac{1}{2}x\right)$ the subject.
$\frac{1}{2}x = \frac{1}{3}\pi \quad x = \frac{2}{3}\pi$	M1	Looks up "$\frac{1}{2}x$" before ×2
	A1 [3]	co (120° gets A0 − decimals A0)
(ii) Range is $2 \leqslant f(x) \leqslant 10$	B1 B1 [2]	condone <
(iii)	B1 B1 [2]	Point of inflexion at π Fully correct
(iv) $\cos\left(\frac{1}{2}x\right) = \frac{1}{4}(6 - y)$	M1	Makes $\cos\left(\frac{1}{2}x\right)$ the subject
$\frac{1}{2}x = \cos^{-1}\left(\frac{1}{4}(6 - y)\right)$	M1	Order of operations correct (M marks allowed if + for −)
$f^{-1}(x) = 2\cos^{-1}\left(\frac{6 - x}{4}\right)$	A1 [3]	oe – needs to be a function of x not y

CAMBRIDGE INTERNATIONAL EXAMINATIONS

Cambridge International Advanced Subsidiary and Advanced Level

MARK SCHEME for the October/November 2014 series

9709 MATHEMATICS

9709/13 Paper 1, maximum raw mark 75

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1	$\left(15 \text{ or } {}^{16}C_2\right)\times 2^4 \times (ax)^2$, $\left(20 \text{ or } {}^{6}C_3\right)\times 2^3 \times (ax)^3$ $a = \dfrac{15\times 2^4}{20\times 2^3} = \dfrac{3}{2}$	**B1B1** **M1A1** **[4]**	$240a = 160a$ is M0
2 (i)	CB or $AB = \dfrac{3}{\tan\frac{\pi}{6}}$ or $3\tan\dfrac{\pi}{3}$ Arc or $AC = 3\times\left[\dfrac{2\pi}{3} \text{ or } \dfrac{\pi}{3}\right]$ $(= 2\pi \text{ or } \pi)$ Perimeter $= 6\sqrt{3} + 2\pi$ oe	**B1** **B1** **B1** **[3]**	Allow throughout for e.g. $3\sqrt{3}$, $\sqrt{27}, \sqrt{3^3}, \left(\sqrt{3}\right)^3$, $\dfrac{9}{\sqrt{3}}$ After B0B0 SCB1 for 16.7
(ii)	Area $OABC$ $(2)\times\dfrac{1}{2}\times 3\times$ *their AB* $\left(= 9\sqrt{3} \text{ or } \dfrac{9\sqrt{3}}{2}\right)$ Area $OADC$ $\dfrac{1}{2}\times 3^2\times\left(\dfrac{2\pi}{2} \text{ or } \dfrac{\pi}{3}\right)$ $\left(= 3\pi \text{ or } \dfrac{3\pi}{2}\right)$ Shaded area $9\sqrt{3} - 3\pi$ oe	**B1** **B1** **B1**√ **[3]**	*Their AB* in form $k\sqrt{3}$ After B0B0 SCB1 for 6.16 or 6.17. Allow $\left(\sqrt{3}\right)^5 - 3\pi$
3 (i)	$(3x-2)^2 + 1$	**B1B1B1** **[3]**	For either of 1st 2 marks bracket must be in the form $(ax+b)^2$ except for SCB2 for $9\left(x-\dfrac{2}{3}\right)^2 + 1$
(ii)	$f'(x) = 9x^2 - 12x + 5$ $=$ their $(3x-2)^2 + 1$ > 0 (or $\geqslant 1$) hence an increasing function	**B1** **M1** **A1** **[3]**	Ft from (i). Some reference/recognition Allow > 1. Allow *their* 1 provided positive. Allow a complete alt method (2/2 or 0/2)
4 (i)	$S_P = \dfrac{2}{1-\frac{1}{2}}, S_P = \dfrac{3}{1-\frac{1}{3}}$ $S_P = 4, S_Q = \dfrac{9}{2}$ $S_R = 5$ cao	**M1** **A1** **A1** **[3]**	At least one correct At least one correct
(ii)	$\dfrac{4}{1-r} =$ *their* S_R $r = \dfrac{1}{5}$	**M1** **A1**	

$R = 4 + \frac{4}{5} + \frac{4}{25} = 4\frac{24}{25}$ or 4.96 cao	**A1** **[3]**	
5 (i) $\left(s^2 - c^2\right)\left(s^2 + c^2\right)$ OR $s^2\left(1-c^2\right) - c^2\left(1-s^2\right)$ $\sin^2\theta - \cos^2\theta$ $2\sin^2\theta - 1$ www **AG**	**M1** **A1** **A1** **[3]**	**OR** $\sin^4\theta - (1-\sin^2\theta)^2$ $\sin^4\theta - (1 - 2\sin^2\theta + \sin^4\theta)$ $= 2\sin^2\theta - 1$ **AG**
(ii) $2\sin^2\theta - 1 = \frac{1}{2} \Rightarrow \sin\theta = (\pm)\frac{\sqrt{3}}{2}$ or $(\pm)0.866$	**B1**	**OR** $\cos 2\theta = -\frac{1}{2} \to 2\theta = 120, 240$ etc.
$\theta = 60°$ $\theta = 120°$	**B1** **B1**√	Ft for 180 – *their* 60 Ft for 180 + *their* 60, 360 – *their* 60
$\theta = 240°, 300°$	**B1**√ **[4]**	Allow $\frac{\pi}{3}, \frac{2\pi}{3}$ etc. Extra sols in range −1
6 (i) $m = \frac{3a + 9 - (2a-1)}{2a + 4 - a} = \frac{a+10}{a+4}$ oe e.g. $\frac{-a-10}{-a-4}$	**M1A1**	cao Allow omission of brackets for M1
Gradient of perpendicular $= \frac{-(a+4)}{a+10}$ oe but not $\frac{-1}{\left(\frac{a+10}{a+4}\right)}$	**A1**√ **[3]**	Do not ISW. Max penalty for erroneous cancellation 1 mark
(ii) $(\sqrt{})\left[(a+4)^2 + (a+10)^2\right] = (\sqrt{})260$	**M1**	Allow *their* $(a + 4)$, $(a + 10)$ from (i). Allow $(-a-4)^2$ etc. Allow omission of brackets
$(\sqrt{})\left[(a+4)^2 + (a+10)^2\right]$ cao $(2)\left(a^2 + 14a - 72\right)(= 0)$ $a = 4$ or -18 cao	**A1** **A1** **A1** **[4]**	

7	**(i)** $\boldsymbol{OA.OB} = -7+3-3p+p^2$ $(p+1)(p-4)=0$ $p=-1$ or 4	**M1** **DM1** **A1** **[3]**	Correct method for scalar product Equate to zero & attempt to factorise/solve '= 0' implied by answers
	(ii) $49+(1-p^2)+p^2=2(1+9+p^2)$ $p=15$	**M1** **A1** **[2]**	Scalar result required
	(iii) $\boldsymbol{AB} = -8\mathbf{i}+6\mathbf{j}$ Divide $\boldsymbol{AB}$ by $\lvert\boldsymbol{AB}\rvert$ $=\sqrt{(-8)^2+6^2}=10$ soi Unit vector $=\frac{1}{10}(-8\mathbf{i}+6\mathbf{j})$ oe cao	**B1** **M1** **A1** **[3]**	p = 15 used – treat as MR $\to \frac{1}{\sqrt{353}}\begin{pmatrix}-8\\-17\\0\end{pmatrix}$
8	**(i)** Minimum since f ″(3) (= 4/3) > 0 www	**B1** **[1]**	
	(ii) $f'(x)=-18x^{-2}\ (+c)$ $0=-2+c$ c = 2 $(\to f'(x)=-18x^{-2}+2)$ $f(x)=18x^{-1}+2x\ (+k)$ $7=6+6+k$ $k=-5\to(f(x)=18x^{-1}+2x-5)$ cao	**B1** **M1** **A1** **B1√B1√** **M1** **A1** **[7]**	 Sub $f'(3)=0$. (dep c present) c = 2 sufficient at this stage Allow cx at this stage Sub f(3) = 3 (k present & numeric (or no) c)
9	**(i)** $x-3\sqrt{x}+2$ or k^2-3k+2 or $(3\sqrt{x})^2=(x+2)^2$ $\sqrt{x}=1$ or 2 or $k=1$ or 2 or $x^2-5x+4\,(=0)$ $x=1$ or 4 $y=3$ or 6	**M1** **A1** **A1** **A1** **[4]**	**OR** attempt to eliminate x eg sub $x=\frac{y^2}{9}$ $y^2-9y+18=0$ $y=3$ or 6 $x=1$ or 4

(ii) $\int 3x^{\frac{1}{2}}\,dx - \left[\int (x+2)\,dx \text{ or attempt at trapezium}\right]$	**M1DM1**	Attempt to integrate. Subtract at some stage
$2x^{\frac{3}{2}} - \left[\left(\frac{1}{2}x^2 + 2x\right) \text{ or } \frac{1}{2}(y_2+y_1)(x_2-x_1)\right]$	**A1A1**	Where $(x_1, y_1), (x_2, y_2)$ is *their* (1, 3), (4, 6)
$(16-2) - \left[\left[(8+8) - \left(\frac{1}{2}+2\right)\right] \text{ or } \mathit{their}\ \frac{1}{2}\times 9\times 3\right]$	**DM1**	Apply *their* 1→4 limits correctly to curve
$\frac{1}{2}$	**A1** **[6]**	For A mark allow reverse subtn→ $-\frac{1}{2} \to \frac{1}{2}$ but not reversed limits
OR		
$\left[\int (y-2)\,dy \text{ or attempt at trap}\right] - \int \frac{y^2}{9}\,dy$	**M1DM1**	
$\left[\frac{1}{2}y^2 - 2y \text{ or } \frac{1}{2}(x_1+x_2)(y_2-y_1)\right] - \frac{y^3}{27}$	**A1A1**	
$\left[(18-12) - \left(4\frac{1}{2} - 6\right) \text{ or } \frac{1}{2}\times 5\times 3\right] - [8-1]$	**DM1**	Apply *their* 3→6 limits correctly to curve
$\frac{1}{2}$	**A1**	
10 (a) (i) $(a+b)^{\frac{1}{3}} = 2,\ (9a+b)^{\frac{2}{3}} = 16$	**B1B1**	Ignore 2nd soln (–9, 17) throughout
$a+b=8,\ 9a+b=64$	**M1**	Cube etc. & attempt to solve
$a=7,\ b=1$	**A1** **[4]**	Correct answers without any working 0/4
(ii) $x = (7y+1)^{\frac{1}{3}}$ (x/y interchange as first or last step)	**B1√**	ft on from *their a, b* or in terms of *a, b*
$x^3 = 7y+1$ or $y^3 = 7x+1$	**B1√**	ft on from *their a*, *b* or in terms of *a*, *b*
$f^{-1}(x) = \frac{1}{7}(x^3-1)$ cao	**B1**	A function of x required
Domain of f^{-1} is $x \geqslant 1$ cao	**B1** **[4]**	Accept >. Must be x
(b) $\frac{dy}{dx} = \left[\frac{1}{3}(7x^2+1)^{-\frac{2}{3}}\right] \times [14x]$	**B1B1**	
When $x=3$, $\frac{dy}{dx} = \frac{1}{3}\times(64)^{-\frac{2}{3}}\times 42 \quad \left(=\frac{7}{8}\right)$	**M1**	
$\frac{dy}{dt} = \frac{dy}{dx}\times\frac{dx}{dt} = \frac{7}{8}\times 8$	**DM1**	Use chain rule
7	**A1** **[5]**	

CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level and GCE Advanced Level

MARK SCHEME for the May/June 2014 series

9709 MATHEMATICS

9709/11 Paper 1, maximum raw mark 75

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PA–1 This is deducted from A or B marks in the case of premature approximation. The PA–1 penalty is usually discussed at the meeting.

1 $a = 1, \quad b = 2$	**B1B1** **[2]**	Or $1 + 2\sin x$
2 **(i)** $(2x-3)^2 - 9$	**B1B1** **[2]**	For −3 and −9
(ii) $2x-3>4 \quad 2x-3<-4$	**M1**	At least one of these statements
$x>3\frac{1}{2}$ (or) $x<-\frac{1}{2}$ cao	**A1**	Allow 'and' $3\frac{1}{2}, -\frac{1}{2}$ soi scores first M1
Allow $-\frac{1}{2}>x>3\frac{1}{2}$		
OR $4x^2 - 12x - 7 \to (2x-7)(2x+1)$	**M1**	Attempt to solve 3-term quadratic
$x>3\frac{1}{2}$ (or) $<-\frac{1}{2}$ cao	**A1** **[2]**	Allow 'and' $3\frac{1}{2}, -\frac{1}{2}$ soi scores first M1
Allow $-\frac{1}{2}>x>3\frac{1}{2}$		
3 $[{}^8C_6 \text{ or } 28]\times[16 \text{ or } 4^2](x^6)\times\left[\frac{1}{(64 \text{or } 2^6)(x^6)}\right]$	**B1B1B1**	Seen in expansion ok. Allow ${}^8C_2$
7	**B1** **[4]**	Identified as answer
4 $\frac{dy}{dx} = \left[-2\times 4(3x+1)^{-3}\right]\times[3]$	**B1B1**	$[-2 \times 4u^{-3}] \times [3]$ is B0B1 unless resolved
When $x=-1$, $\frac{dy}{dx}=3$	**B1**	
When $x=-1$, $y=1$ soi	**B1**	
$y - 1 = 3(x + 1)$ $(\to y = 3x + 4)$	**B1**√ **[5]**	Ft on *their* '3' only (not $-\frac{1}{3}$). Dep on diffn
5 **(i)** $200/2(2a + 199d) = 4 \times 100/2(2a + 99d)$	**M1A1**	Correct formula used (once) M1, correct eqn A1
$d = 2a$ cao	**A1** **[3]**	
(ii) $a + 99d = a + 99 \times 2a$	**M1**	Sub. *their* part(i) into correct formula
$199a$ cao	**A1** **[2]**	
6 **(i)** area $\Delta = \frac{1}{2}\times 4\times 4\tan\alpha$ oe soi	**B1**	$4\tan\alpha = \sqrt{16/\cos^2\alpha - 16}$. (Can also score in answer) Accept θ throughout
Area sector $= \frac{1}{2}\times 2^2\alpha$ oe soi	**B1**	
Shaded area $= 8\tan\alpha - 2\alpha$ cao	**B1** **[3]**	Little/no working – accept terms in answer
(ii) $DC = \frac{4}{\cos\alpha} - 2$ oe soi	**B1**	
Arc $DE = 2\alpha$ soi anywhere provided clear	**B1**	$\frac{4}{\cos\alpha} = \sqrt{16+16\tan^2\alpha}$. Can score in answer
Perimeter $= \frac{4}{\cos\alpha} + 4\tan\alpha + 2\alpha$ cao	**B1** **[3]**	Little/no working – accept terms in answer

7 $(a-3)^2+(2-b)^2=125$ oe $\frac{2-b}{a-3}=2$ oe $(a-3)^2+(2a-6)^2=125$ (sub for a or b) $(5)(a+2)(a-8)$ (= 0) Attempt factorise/solve $a=-2$ or 8, $b=12$ or -8	**B1** **B1** **M1** **M1** **A1A1** **[6]**	 Or $1/4(2-b)^2+(2-b)^2=125$ Or $(5)(b-12)(b+8)$ (= 0) Answers (no working) after 2 correct eqns score SCB1B1 for each correct pair (a, b)
8 (i) $\boldsymbol{OA.OB}=-3p^2-4+p^4$ soi $(p^2+1)(p^2-4)=0$ oe e.g. with substitution $p=\pm 2$ and no other real solutions	**M1** **M1** **A1** **[3]**	 Put = 0 (soi) and attempt to solve
(ii) $\overrightarrow{BA}=\begin{pmatrix}9\\4\\9\end{pmatrix}-\begin{pmatrix}-3\\-1\\9\end{pmatrix}=\begin{pmatrix}12\\5\\0\end{pmatrix}$ $\lvert\overrightarrow{BA}\rvert=\sqrt{12^2+5^2}=13$ and division by *their* 13 Unit vector $=\frac{1}{13}\begin{pmatrix}12\\5\\0\end{pmatrix}$ cao	**M1** **M1** **A1** **[3]**	Reversed subtraction can score M1M1A0
9 (i) LHS $\equiv\frac{\sin^2\theta-(1-\cos\theta)}{(1-\cos\theta)\sin\theta}$ cao $\equiv\frac{1-\cos^2\theta-1+\cos\theta}{(1-\cos\theta)\sin\theta}$ $\equiv\frac{\cos\theta(1-\cos\theta)}{(1-\cos\theta)\sin\theta}$ $\equiv\frac{1}{\tan\theta}$	**B1** **M1** **M1** **A1** **[4]**	Put over common denominator Use $s^2=1-c^2$ oe Correct factorisation from line 2 **AG**
(ii) $\tan\theta=(\pm)\frac{1}{2}$ 26.6°, 153.4°	**M1** **A1A1√** **[3]**	 Ft for 180 – 1st answer

10 **(i)** $-5 \leqslant f(x) \leqslant 4$ For f(x) allow x or y; allow <, $[-5, 4]$, $(-5,4)$	**B1** **[1]**	Allow less explicit answers (eg $-5 \to 4$)
(ii) $f^{-1}(x)$ approximately correct (independent of f)	**B1**	
Closed region between (1, 1) and (4, 4); line reaches x-axis	**DB1** **[2]**	Ignore line $y = x$
(iii) LINE: $f^{-1}(x) = \frac{1}{3}(x+2)$	**B1**	Allow $y =$ ..... but must be a function of x
for $-5 \leqslant x \leqslant 1$	**B1B1**	cao but allow <
CURVE: $5 - y = \frac{4}{x}$ OR $x = 5 - \frac{4}{y}$	**M1**	
$f^{-1}(x) = 5 - \frac{4}{x}$ oe	**A1**	cao
for $1 < x \leqslant 4$	**B1** **[6]**	cao but allow < or <
11 **(i)** $x^2 + 4x + c - 8 (= 0)$	**M1**	Attempt to simplify to 3-term quadratic
$16 - 4(c - 8) = 0$	**M1**	Apply $b^2 - 4ac = 0$. '= 0' soi
$c = 12$	**A1**	
OR		
$-2 - 2x = 2 \to x = (-2)$	**M1**	Equate derivs of curve and line. Expect $x = -2$
$-4 + c = 8 + 4 - 4$	**M1**	Sub *their* $x = -2$ into line and curve, and equate
$c = 12$	**A1** **[3]**	
(ii) $x^2 + 4x + 3 \to (x + 1)(x + 3) (= 0) \to$ $x = -1$ or -3	**B1**	
$\int(8 - 2x - x^2) - [\int(2x + 11)$ or area of trapezium$]$	**M1M1**	Attempt to integrate. At some stage subtract
$\left[8x - x^2 - \frac{x^3}{3}\right] - \left[x^2 + 11x\right]$ or $\left[8x - x^2 - \frac{x^3}{3}\right] - \frac{1}{2}(5+9)\times 2$	**A1B1**	A1 for curve, B1 for line **OR** $\left[-3x - 2x^2 - \frac{x^3}{3}\right]$ A2,1,0
Apply *their* limits to at least integral for curve	**M1**	For M marks allow reversed limits and/or subtraction of areas but then final A0
$1\frac{1}{3}$ oe	**A1** **[7]**	

12 **(i)** $y = \frac{2}{3}x^{\frac{3}{2}} - 2x^{\frac{1}{2}} + (c)$ oe	**B1B1**	Attempt to integrate
$\frac{2}{3} = \frac{16}{3} - 4 + c$	**M1**	Sub $\left(4, \frac{2}{3}\right)$. Dependent on c present
$c = -\frac{2}{3}$	**A1** **[4]**	
(ii) $\frac{1}{2}x^{-\frac{1}{2}} + \frac{1}{2}x^{-\frac{3}{2}}$ oe	**B1B1** **[2]**	
(iii) $x^{\frac{1}{2}} - x^{-\frac{1}{2}} = 0 \rightarrow \frac{x-1}{\sqrt{x}} = 0$	**M1**	Equate to zero and attempt to solve
$x = 1$	**A1**	
When $x = 1$, $y = \frac{2}{3} - 2 - \frac{2}{3} = -2$	**M1A1**	Sub. *their* '1' into *their* 'y'
When $x = 1$, $\frac{d^2y}{dx^2}(=1) > 0$ Hence minimum	**B1** **[5]**	Everything correct on final line. Also dep on correct (ii). Accept other valid methods

CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level and GCE Advanced Level

MARK SCHEME for the May/June 2014 series

9709 MATHEMATICS

9709/12 Paper 1, maximum raw mark 75

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1 (2, 7) to (10, 3) Mid-point (6, 5) Gradient = −½ Perp gradient = 2 Eqn $y-5=2(x-6)$ Sets y to 0, → (3½, 0)	 B1 B1 B1√ M1 A1 [5]	 co co co Must be correct form of Perp co x = 3½ only is ok.
2 $\left(1+x^2\right)\left(\frac{x}{2}-\frac{4}{x}\right)^6$. Term in x^2 = 15 × $\frac{1}{16}$ × $(-4)^2$ = 15 Constant term = 20 × $\frac{1}{8}$ × $(-4)^3$ = −160 Coefficient of x^2 = −145	 B1 B1 B1 B1 B1√ [5]	 B1 unsimplified. B1 15. B1 unsimplified. B1 −160 Uses 2 terms. √ on previous answers
3 reflex angle θ is such that $\cos\theta = k$, **(i)** **(a)** $\sin\theta = -\sqrt{(1-k^2)}$ **(b)** Uses $t=s/c$ → $\frac{-\sqrt{1-k^2}}{k}$ **(ii)** θ is in 4th quadrant. 2θ lies between 540° and 720° $\sin 2\theta$ is negative in both these quadrants.	 B1 B1 [2] B1√ [1] B1 B1 [2]	 (−) B1 rest B1 √ for **(i)** ÷ k. co co
4 **(i)** $\frac{1}{2}r^2\theta = \frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta$ → $2\sin\theta = \theta$ → p = 2. **(ii)** Chord length = 8sin1.2 × 2 (14.9) (or from cosine rule) Arc length = 2.4 × 8 (19.2) Perimeter = sum of these = 34.1	 B1 B1 [2] M1 B1 A1 [3]	 Correct equation. All ok – answer given. Needs ×2. Any method ok. co
5 **(i)** $\frac{1}{\cos\theta} - \frac{\cos\theta}{1+\sin\theta} \equiv \tan\theta$. LHS $= \frac{1+s-c^2}{c(1+s)} = \frac{s^2+s}{c(1+s)} = \frac{s}{c}$ $= \tan\theta$ **(ii)** → $\tan\theta + 2 = 0$ ie $\tan\theta = -2$ → θ = 116.6° or 296.6°	M1 M1M1 A1 [4] M1 A1 A1√ [3]	Correct addition of fractions Use of $s^2+c^2=1$. $(1+s)$cancelled. → answer given. Uses part **(i).** Allow $\tan\theta=\pm2$ Co. √ for 180°+ and no other solutions in the range.

6 **(i)** GP 8 $8r$ $8r^2$ AP 8 $8+8d$ $8+20d$ $8r = 8+8d$ and $8r^2 = 8+20d$ Eliminates $d \to 2r^2 - 5r + 3 = 0$ $\to r = 1.5$ (or 1)	 B1 B1 M1 A1 [4]	 B1 for each equation. Correct elimination. co (no penalty for including $r = 1$)
(ii) 4th term of GP = $ar^3 = 8 \times 27/8 = 27$ If $r = 1.5$, $d = 0.5$ 4th term of AP = $a + 3d = 9\frac{1}{2}$	B1√ M1A1 [3]	co needs $a + 3d$ and correct method for d
7 **(i)** $(\mathbf{b}-\mathbf{a}).(\mathbf{b}-\mathbf{c}) = \begin{pmatrix}-2\\-1\\2\end{pmatrix}.\begin{pmatrix}3\\2\\4\end{pmatrix}$ $\to -6 - 2 + 8 = 0 \to 90°$	M1 M1 A1 [3]	**AB** = **b** − **a** once (**a** – **b** is ok) Use of $x_1x_2...$ with **AB** and **CB** All correct
(ii) Unit vector = $\frac{1}{3}\begin{pmatrix}2\\1\\-2\end{pmatrix}$	M1	Method for unit vector.
CD = 12 × unit vector = $\pm\begin{pmatrix}8\\4\\-8\end{pmatrix}$	M1	Knows to multiply by 12 or ±4**BA**
OD = **OC** + **CD** = $\begin{pmatrix}12\\9\\-2\end{pmatrix}$	M1 A1 [4]	Correct method. co
8 $\frac{d^2y}{dx^2} = 2x - 1$ $\to \int \quad \frac{dy}{dx} = x^2 - x \quad + c$ $= 0$ when $x = 3 \quad \to c = -6$ $x^2 - x - 6 = 0$ when $x = -2$ (or 3) $\to \int \quad y = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 6x \quad (+k)$ $= -10$ when $x = 3$ $\to k = 3\frac{1}{2}$ $\to y = 10\frac{5}{6}$	 B1 M1 A1 A1 B1√B1√ M1 A1 [8]	 Correct integration (ignore $+c$) Uses a constant of integration. co Puts dy/dx to 0 √ first 2 terms, √ for cx. Correct method for k Co –r 10.8

9 $y = 8 - \sqrt{4-x}$		
(i) $\frac{dy}{dx} = -\frac{1}{2}(4-x)^{-\frac{1}{2}} \times -1$	B1 B1	Without (−1). For (×−1).
$\int y\,dx = 8x - \frac{(4-x)^{\frac{3}{2}}}{\frac{3}{2}} \div -1$	3 × B1 [5]	B1 for "$8x$" and $+c$". B1 for all except ÷(−1). B1 for ÷(−1). (n.b. these 5 marks can be gained in(ii) or (iii))
(ii) Eqn $y - 7 = \frac{1}{2}(x-3)$ → $y = \frac{1}{2}x + 5\frac{1}{2}$	M1A1 [2]	M1 unsimplified. A1 as $y=mx+c$
(iii) Area under curve = ∫ from 0 to 3 (58/3) Area under line = ½(5½ + 7)×3 Or $\left[\frac{1}{4}x^2 + \frac{11x}{2}\right]$ from 0 to 3 → $\frac{58}{3} - \frac{75}{4} = \frac{7}{12}$	M1 M1 M1 A1 [4]	Use of limits – needs use of "0" Correct method M1 Subtraction. A1 co
10 f : $x \mapsto 2x - 3,\ x \in \mathbb{R}$, g : $x \mapsto x^2 + 4x,\ x \in \mathbb{R}$.		
(i) ff = $2(2x - 3) - 3$ Solves = 11 → x = 5 (or $2x-3 = 11$, $x = 7$. $2x-3=7 \rightarrow x = 5$)	M1 A1 [2]	Either forms ff correctly, or solves 2 equations co
(ii) min at $x = -2$ → Range ⩾ −4	M1 A1 [2]	Any valid method – could be guesswork.
(iii) $x^2 + 4x - 12$ (> 0) → $x = 2$ or -6 → $x < -6$, $x > 2$.	M1 A1 A1 [3]	Makes quadratic = 0 + 2 solutions Correct limits – even if >,<,⩾,⩽,= co
(iv) gf(x) = $(2x - 3)^2 + 4(2x - 3) = p$ → $4x^2 - 4x - 3 - p = 0$ Uses "$b^2 - 4ac$" $16 = 16(-3 - p)$ → $p = -4$	B1 M1 A1 [3]	co unsimplified Use of discriminant co
(v) − 2	B1 [1]	co
(vi) $y = (x+2)^2 - 4$ $\sqrt{y+4} = x + 2$ $h^{-1}(x) = \sqrt{x+4} - 2$	B2,1 M1 A1 [4]	−1 for each error Correct order of operations co with x, not y. ± left A0.

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1 $\left(x^2-\dfrac{2}{x}\right)^5$ Term in x is $10 \times (x^2)^2 \times \left(\dfrac{-2}{x}\right)^3$ Coefficient $= -80(x)$	 B1 B1 B1 [3]	 B1 10 or $^5C_2$ or $^5C_3$, B1 $\left(\dfrac{-2}{x}\right)^3$ co Must be identified
2 36, 32, **...** **(i)** $r = \dfrac{8}{9}$ $S_\infty = (\textit{their } a) \div (1 - \textit{their } r)$ $S_\infty = 36 \div \dfrac{1}{9} = 324$	 M1 A1 [2]	 Method for r and S_∞ ok. ($\lvert r \rvert < 1$) co
(ii) $d = -4$ $0 = \dfrac{n}{2}(72 + (n-1)(-4))$ $\rightarrow n = 19$	B1 M1 A1 [3]	co S_n formula ok and a value for d $\left(\neq \dfrac{8}{9}\right)$ Condone $n = 0$ but no other soln
3 **(i)** $s = r\theta$ Angle of major arc $= 2\pi - 2.2 = (4.083)$ Perimeter $= 12 + 24.5 = 36.5$ or $12\pi - 1.2$ (or full circle − minor arc B1)	M1 B1 A1 [3]	Used with major or minor arc Could be gained in **(ii)**. co
(ii) Area of major sector $= \dfrac{1}{2}r^2\theta = (73.49)$ Area of triangle $= \dfrac{1}{2}.6^2\sin 2.2 = (14.55)$ Ratio $= 5.05 : 1$ (Allow $5.03 \rightarrow 5.06$)	M1 M1 A1 [3]	Used with major/minor sector. Correct formula or method. $(2\pi - 2.2)/\sin 2.2$ gets M1M1 co
4 $\dfrac{\tan x + 1}{\sin x \tan x + \cos x} \equiv \sin x + \cos x$ **(i)** LHS $\dfrac{\left(\frac{s}{c}\right)+1}{\left(\frac{s^2}{c}+c\right)} = \dfrac{s+c}{s^2+c^2}$ = RHS	 M1 M1 A1 [3]	 Use of t = s/c twice Correct algebra and use of $s^2 + c^2 = 1$ AG all ok
(ii) $s + c = 3s - 2c$ $\rightarrow \tan x = \dfrac{3}{2}$ Allow $\cos^2 = \dfrac{4}{13}$, $\sin^2 = \dfrac{9}{13}$ $\rightarrow x = 0.983$ and 4.12 or 4.13	 M1 A1 A1√ [3]	 Uses (i) and $t = \dfrac{s}{c}$ $t = \dfrac{2}{3}$ or 0 is M0 co. √ 1st + π, providing no excess solns in range. Allow 0.313π, 1.31π

5 $f(x) = \dfrac{15}{2x+3}$		
(i) $f'(x) = \dfrac{-15}{(2x+3)^2} \times 2$	B1 B1	Without the "×2". For "×2" (indep of 1st B1).
( )² always +ve → $f'(x) < 0$ (No turning points) – therefore an inverse	B1√ [3]	√ providing ( )² in $f'(x)$. 1–1 insuff.
(ii) $y = \dfrac{15}{2x+3} \rightarrow 2x + 3 = \dfrac{15}{y}$	M1	Order of ops – allow sign error
$\rightarrow x = \dfrac{\frac{15}{y} - 3}{2} \rightarrow \dfrac{15-3x}{2x}$	A1	co as function of x. Allow $y = \ldots$
(Range) $0 \leqslant f^{-1}(x) \leqslant 6$. Allow $0 \leqslant y \leqslant 6$, [0,6] (Domain) $1 \leqslant x \leqslant 5$. Allow [1, 5]	B1 B1 [4]	For range / domain ignore letters unless range / domain not identified
6 $\dfrac{dy}{dx} = \dfrac{12}{\sqrt{4x+a}}$ P (2, 14) Normal $3y + x = 44$		
(i) m of normal $= -\dfrac{1}{3}$	B1	co
$\dfrac{dy}{dx} = 3 = \dfrac{12}{\sqrt{4x+a}} \rightarrow a = 8$	M1 A1 [3]	Use of $m_1m_2 = -1$. AG.
(ii) $\int y = 12(4x + a)^{\frac{1}{2}} \div \dfrac{1}{2}$ $\div 4$ $(+c)$	B1 B1	Correct without "÷4". for "÷4".
Uses (2, 14) $c = -10$	M1 A1 [4]	Uses in an integral only. Dep 'c'. co All 4 marks can be given in (i)

7 **(i)** Angle BAC needs sides AB,AC or BA,CA **AB.AC** = (**b** − **a**).(**c** − **a**)		Ignore *their* labels:
$= \begin{pmatrix}4\\-2\\4\end{pmatrix}.\begin{pmatrix}0\\3\\4\end{pmatrix} = 10$	B1 M1	One of **AB, BA, AC, CA** correct Use of $x_1x_2 + y_1y_2$, etc.
$= \sqrt{36} \times \sqrt{25} \cos BAC$	M1M1	M1 prod of moduli. M1 all linked
$\rightarrow \quad BAC = \cos^{-1}\frac{1}{3}$ AG	A1 [5]	If e.g. **BA.OC** max B1M1M1. If both vectors wrong 0/5. If e.g. **BA.AC** used $\rightarrow \cos^{-1}\left(-\frac{1}{3}\right)$ final mark A0
(ii) $\sin BAC = \sqrt{1-\frac{1}{9}}$	B1	Use of $s^2 + c^2 = 1$ − not decimals
Area = $\frac{1}{2} \times 6 \times 5 \times \sqrt{\frac{8}{9}} = 5\sqrt{8}$ oe	M1 A1 [3]	Correct formula for area. Decimals seen A0
8 $2x^2 - 10x + 8 \rightarrow a(x+b)^2 + c$		
(i) $a = 2,\ b = -2\frac{1}{2},\ c = -4\frac{1}{2}$	3 × B1	Or $2\left(x-2\frac{1}{2}\right)^2 - 4\frac{1}{2}$
$\rightarrow$ min value is $-4\frac{1}{2}$ Allow $(2\frac{1}{2}, -4\frac{1}{2})$	B1√ [4]	Can score by sub $x = 2\frac{1}{2}$ into original but not by differentiation
(ii) $2x^2 - 10x + 8 - kx = 0$ Use of "$b^2 - 4ac$" $(-10-k)^2 - 64 < 0$ or $k^2 + 20k + 36 < 0$ $\rightarrow\ k = -18$ or -2 $-18 < k < -2$	 M1 M1 A1 A1 [4]	Sets equation to 0 and uses discriminant correctly Realises discriminant < 0. Allow ⩽ co Dep on 1st M1 only co
9 **(i)** $3x^2y = 288$ y is the height	B1	co
$A = 2(3x^2 + xy + 3xy)$	M1	Considers at least 5 faces ($y \neq x$)
Sub for $y \rightarrow A = 6x^2 + \frac{768}{x}$	A1 [3]	co answer given
(ii) $\frac{dA}{dx} = 12x - \frac{768}{x^2}$	B1	co
= 0 when $x = 4 \rightarrow A = 288$. Allow (4, 288)	M1 A1	Sets differential to 0 + solution. co
$\frac{d^2A}{dx^2} = 12 + \frac{1536}{x^3}$	M1	Any valid method
(= 36) > 0 Minimum	A1 [5]	co www dep on correct f″ and $x = 4$

10	pts of intersection $2x + 1 = -x^2 + 12x - 20$ $\rightarrow\ x = 3, 7$	M1A1	Attempt at soln of sim eqns. co
	Area of trapezium = $\frac{1}{2}(4)(7 + 15) = 44$ (or $\int (2x+1)\,dx$ from 3 to 7 = 44)	M1A1	Either method ok. co
	Area under curve = $-\frac{1}{3}x^3 + 6x^2 - 20x$	B2,1	−1 each term incorrect
	Uses 3 to 7 $\rightarrow$ $(54\frac{2}{3})$	DM1	Correct use of limits (Dep 1st M1)
	Shaded area = $10\frac{2}{3}$	A1 [8]	co
	OR		
	$\int_3^7 \left(-x^2 + 10x - 21\right) = -\frac{x^3}{3} + 5x^2 - 21x\right)$		Functions subtracted before integration
	M1 subtraction, A1A1A1 for integrated terms, DM1 correct use of limits, A1		Subtraction reversed allow A3A0. Limits reversed allow DM1A0
11	Sim eqns $\rightarrow$ $A\,(1, 3)$	M1 A1	co Allow answer only B2
	Vectors or mid-point $\rightarrow$ $C\,(12, 14)$	M1 A1$\checkmark$	Allow answer only B2$\checkmark$
	Eqn of BC $4y = x + 44$ or CD $y = 3x - 22$	M1	equation ok – unsimplified
	Sim eqns $\rightarrow$ $B(4, 12)$ or $D\,(9, 5)$	DM1A1	Sim eqns. co
	Vectors or mid-point $\rightarrow$ $B(4, 12)$ or $D\,(9, 5)$	DM1A1 [9]	Valid method (or sim eqns) co

CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level and GCE Advanced Level

MARK SCHEME for the October/November 2013 series

9709 MATHEMATICS

9709/11 Paper 1, maximum raw mark 75

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1 **(i)** $64+576x+2160x^2$	**B1B1B1** **[3]**	Can score in (ii)
(ii) $576a(x^2)+2160(x^2)=0$	**M1**	
$a=-\dfrac{2160}{576}$ oe (eg $-\dfrac{15}{4}$) or -3.75	**A1** **[2]**	
2 Attempt integration	**M1**	
$f(x)=2(x+6)^{\frac{1}{2}}-\dfrac{6}{x}(+c)$	**A1A1**	Accept unsimplified terms
$2(3)-\dfrac{6}{3}+c=1$	**M1**	Sub.x = 3, y = 1. c must be present
$c=-3$	**A1** **[5]**	
3 **(i)** **DB** = 6**i** + 4**j** – 3**k** cao	**B1**	
DE = 3**i** +2**j** – 3**k** cao	**B1** **[2]**	
(ii) **DB.DE** = 18 + 8 + 9 = 35	**M1**	Use of $x_1x_2+y_1y_2+z_1z_2$
$\lvert \mathbf{DB} \rvert = \sqrt{61}$ or $\lvert \mathbf{DE} \rvert = \sqrt{22}$	**M1**	Correct method for moduli
$35=\sqrt{61}\times\sqrt{22}\times\cos\theta$ oe	**M1**	All connected correctly
$\theta=17.2°$ (0.300 rad) cao	**A1** **[4]**	Use of e.g. **BD. DE** can score M marks (leads to obtuse angle)
4 **(i)** $4(1-\cos^2 x)+8\cos x-7=0$	**M1**	Use $c^2+s^2=1$
$4c^2-8c+3=0 \rightarrow (2\cos x-1)(2\cos x-3)=0$	**M1**	Attempt to solve
x = 60° or 300°	**A1A1** **[4]**	
(ii) $\frac{1}{2}\theta$ = 60° (or 300°)	**M1**	Allow 300° in addition
$\theta=120°$ only	**A1** **[2]**	
5 **(i)** $x=(\pm)\sqrt{y-1}$	**B1**	**OR** $y^2=x-1$ (x/y interchange 1st)
$f^{-1}: x \mapsto \sqrt{x-1}$ for $x > 1$	**B1B1** **[3]**	
(ii) $ff(x)=(x^2+1)^2+1$	**B1**	Or $x^4+2x^2-(153/16)=0$
$x^2+1=(\pm)13/4$	**M1**	Or $x^2=9/4, (-17/4)$
x = 3/2	**A1** **[3]**	www. Condone ± 3/2
Alt. (ii) $f(x)=f^{-1}(185/16)=13/4$ **M1**		**Alt.**(ii) f(3/2) = 13/4 **B1**
$x=f^{-1}(13/4)$ **M1**		f(13/4) = 185/16 **B1**
$x=3/2$ **A1**		x = 3/2 **B1**
		SC.B2 answer 1.5 with no working

6	(i)	$r(2\pi-\alpha)$ $\quad + 2r\alpha + 2r$ $2\pi r + r\alpha + 2r$	B1B1 B1√ [3]	ft for $r\alpha$ instead of $2r\alpha$ or omission $2r$ SC1 for $2r\alpha + 4r$. (Plate = shaded part)
	(ii)	$\frac{1}{2}(2r)^2\alpha + \pi r^2 - \frac{1}{2}r^2\alpha$ $\frac{3r^2\alpha}{2} + \pi r^2$	B1B1 B1 [3]	Either B1 can be scored in (iii)
	(iii)	$\pi r^2 - \frac{1}{2}r^2\alpha = 2r^2\alpha$ $\alpha = \frac{2}{5}\pi$	M1 A1 [2]	For equating *their* 2 parts from (ii)
7	(i)	mid-point = (3, 4) Grad. $AB = -½ \rightarrow$ grad. of perp., = 2 $y-4=2(x-3)$ $y=2x-2$	B1 M1 M1 A1 [4]	soi For use of $-1/m$ soi ft on *their* (3, 4) and 2
	(ii)	$q=2p-2$ √ $\quad p^2+q^2=4$ oe $p^2+(2p-2)^2=4 \rightarrow 5p^2-8p=0$ $\{\text{OR } ¼(q+2)^2+q^2=4 \rightarrow 5q^2+4q-12=0\}$ $(0,-2)$ and $\left(\frac{8}{5},\frac{6}{5}\right)$	B1√ B1 M1 A1A1 [5]	ft for 1st eqn. Attempt substn (linear into quadratic) & simplify
8	(i)	$A=2xr+\pi r^2$ $2x+2\pi r=400$ $(\Rightarrow x=200-\pi r)$ $A=400r-\pi r^2$	B1 B1 M1A1 [4]	Subst & simplify to **AG** (www)
	(ii)	$\frac{dA}{dr}=400-2\pi r$ $=0$ $r=\frac{200}{\pi}$ oe $x=0 \Rightarrow$ no straight sections $\quad$ **AG** $\frac{d^2A}{dr^2}=-2\pi$ $\quad(<0)$ $\quad$ Max	B1 M1 A1 A1 B1 [5]	Differentiate Set to zero and attempt to find r Dep on -2π, or use of other valid reason

9 **(a)**	$\frac{10}{2}(2a+9d)=400$ oe	**B1**	$\rightarrow 2a+9d=80$
	$\frac{20}{2}(2a+19d)=1400$ OR		
	$\frac{10}{2}[2(a+10d)+9d]=1000$	**B1**	$\rightarrow 2a+19d=140$ or $2a+29d=200$
	$d=6 \quad a=13$	**M1A1A1** **[5]**	Solve sim. eqns both from S_n formulae
(b)	$\frac{a}{1-r}=6 \qquad \frac{2a}{1-r^2}=7$	**B1B1**	
	$\frac{12(1-r)}{1-r^2}=7$ or $\frac{1-r^2}{1-r}=\frac{12}{7}$	**M1**	Substitute or divide
	$r=\frac{5}{7}$ or 0.714	**A1**	
	$a=\frac{12}{7}$ or 1.71(4)	**A1√** **[5]**	Ignore any other solns for r and a
10 **(i)**	$\frac{dy}{dx}=\left[3(3-2x)^2\right]\times[-2]$	**B1B1**	OR $-54+72x-24x^2$ B2,1,0
	At $x=\frac{1}{2}, \frac{dy}{dx}=-24$	**M1**	
	$y-8=-24\left(x-\frac{1}{2}\right)$	**DM1**	
	$y=-24x+20$	**A1** **[5]**	
(ii)	Area under curve = $\left[\frac{(3-2x)^4}{4}\right]\times\left[-\frac{1}{2}\right]$	**B1B1**	OR $27x-27x^2+12x^3-2x^4$ B2,1,0
	$-2-\left(-\frac{81}{8}\right)$	**M1**	Limits 0→ ½ applied to integral with intention of subtraction shown
	Area under tangent = $\int(-24x+20)$	**M1**	or area trap =½(20 + 8) × ½
	$=\left\| -12x^2+20x \right\|$ or 7 (from trap)	**A1**	Could be implied
	$\frac{9}{8}$ or 1.125	**A1** **[6]**	Dep on both M marks

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9709/12 Paper 1, maximum raw mark 75

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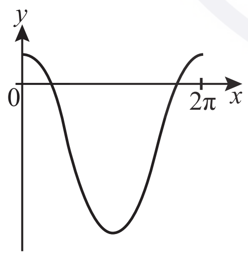
1 **(i)** $\sin x = \sqrt{(1-p^2)}$	B1 [1]	Allow $1-p$ if following $\sqrt{(1-p^2)}$ ± is B0.
(ii) $\tan x = \dfrac{\sin x}{\cos x} = \dfrac{\sqrt{1-p^2}}{p}$	B1√ [1]	√ for answer to **(i)** used.
(iii) $\tan(90-x) = \dfrac{p}{\sqrt{1-p^2}}$	B1√ [1]	√ for reciprocal of **(ii)**
2 **(i)** slant length = 10 cm. circumference of base = 12π arc length = 10θ ($= 12\pi$) $\rightarrow \theta = 1.2\pi$ or 3.77 radians.	B1 B1 B1√ B1 [4]	 Use of $r\theta$, θ calculated, not 6 or 8.
(ii) $\tfrac{1}{2}r^2\theta = 188.5$ cm ^{2} or 60π.	M1 A1√ [2]	Use of $\tfrac{1}{2}r^2\theta$ with radians and r = calculated ‘10’, not 6 or 8.
3 $y = \dfrac{2}{\sqrt{5x-6}}$		
(i) $\dfrac{dy}{dx} = 2\times -\tfrac{1}{2}\times(5x-6)^{-\frac{3}{2}}\times 5$ $\rightarrow -\frac{5}{8}$	B1 B1 B1 [3]	B1 without ‘×5’. B1 For ‘×5’ Use of ‘uv’ or ‘u/v’ ok.
(ii) integral = $\dfrac{2\sqrt{5x-6}}{\frac{1}{2}} \div 5$ Uses 2 to 3 → 2.4 − 1.6 = 0.8	B1 B1 M1 A1 [4]	B1 without ‘÷5’. B1 for ‘÷ 5’ Use of limits in an integral.
4 $\overrightarrow{OA} = \mathbf{i}+2\mathbf{j}$ and $\overrightarrow{OB} = 4\mathbf{i}+p\mathbf{k}$,		
(i) $\overrightarrow{AB} = \mathbf{b}-\mathbf{a} = 3\mathrm{i}-2\mathrm{j}+6\mathrm{k}$ Unit vector = $(3\mathrm{i}-2\mathrm{j}+6\mathrm{k}) \div 7$	B1 M1 A1√ [3]	Must be $\overrightarrow{AB} = \mathbf{b}-\mathbf{a}$ Divides by modulus. √ on vector AB.
(ii) Scalar product = 4 $= \sqrt{5}\times\sqrt{(16+p^2)}\times\cos\theta$ $\rightarrow p = \pm 8$	M1 M1 M1 A1 [4]	Use of $x_1x_2+y_1y_2+z_1z_2$ For modulus. All linked correctly including correct use of $\cos\theta=1/5$.
5 A (0, 8) B (4, 0) $8y+x=33$ m of $AB = -2$ m of $BC = \tfrac{1}{2}$ Eqn BC → $y-0=\tfrac{1}{2}(x-4)$ Sim eqns → C (16, 6) Vector step method → D (12, 14) (or AD $y=\tfrac{1}{2}x+8$, CD $y=-2x+38$) (or $M=(8, 7) \rightarrow D=(12, 14)$)	 B1 M1 M1 M1 A1 M1 A1 [7]	 Use of $m_1m_2=-1$for BC or AD Correct method for equation of BC Sim Eqns for BC, AC. M1 valid method.

6		
(i) Sim triangles $\frac{y}{16-x}=\frac{12}{16}$ (or trig)	M1	Trig, similarity or eqn of line
$\rightarrow y = 12 - \frac{3}{4}x$	A1	(could also come from eqn of line)
$A = xy = 12x - \frac{3}{4}x^2$.	A1 [3]	ag – check working.
(ii) $\frac{dA}{dx} = 12 - \frac{6x}{4}$	B1	
$= 0$ when $x = 8$. $\rightarrow A = 48$.	M1 A1	Sets to 0 + solution.
This is a Maximum. From −ve quadratic or 2nd differential.	B1 [4]	Can be deduced without any working. Allow even if '48' incorrect.
7 (a) (i) $a = 300$, $d = 12$		
$\rightarrow 540 = 300 + (n-1)12 \rightarrow n = 21$	M1 A1 [2]	Use of nth term. Ans 20 gets 0. Ignore incorrect units
(ii) $S_{26} = 13\,(600 + 25\times12) = 11700$	M1	Correct use of s_n formula.
$\rightarrow$ 3 hours 15 minutes.	A1 [2]	
(b) $ar = 48$ and $ar^2 = 32 \rightarrow r = \frac{2}{3}$	M1	Needs ar and ar^2 + attempt at a and r.
$\rightarrow a = 72$.	A1	
$S_\infty = 72 \div \frac{1}{3} = 216$.	M1	Correct S_∞ formula with $\lvert r \rvert < 1$
	A1√ [4]	
8 $f : x \mapsto 3\cos x - 2$ for $0 \leqslant x \leqslant 2\pi$.		
(i) $3\cos x - 2 = 0 \rightarrow \cos x = \frac{2}{3}$	M1	Makes cos subject, then $\cos^{-1}$
$\rightarrow x = 0.841$ or 5.44	A1 A1√ [3]	√ for 2π − 1st answer.
(ii) range is $-5 \leqslant f(x) \leqslant 1$	B2,1 [2]	B1 for $\geqslant -5$. B1 for $\leqslant 1$.
(iii) 	B1,B1 [2]	B1 starts and ends at same point. Starts decreasing. One cycle only. B1 for shape, not 'V' or 'U'.
(iv) max value of $k = \pi$ or 180º.	B1 [1]	
(iv) $g^{-1}(x) = \cos^{-1}\left(\frac{x+2}{3}\right)$	M1	Make x the subject, copes with 'cos'.
	A1 [2]	Needs to be in terms of x.

9 $y = \frac{8}{x} + 2x$		
(i) $\frac{dy}{dx} = \frac{-8}{x^2} + 2$	M1	Attempt at differentiation.
(− 6 at A)	A1	algebraic – unsimplified.
$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dy}{dt}$	M1	Ignore notation – needs product of 0.04
$\rightarrow -0.24$	A1 [4]	and 'his' $\frac{dy}{dx}$.
(ii) $\int y^2 = \int \frac{64}{x^2} + 4x^2 + 32$	M1	Use of integral of y^2 (ignore π)
$= \left(\frac{-64}{x} + \frac{4x^3}{3} + 32x\right)$	A3,2,1	3 terms → −1 each error.
Limits 2 to 5 used correctly	DM1	Uses correct limits correctly.
→ 271.2π or 852	A1	
(allow 271π or 851 to 852)	[6]	(omission of π loses last mark)
10 $f : x \mapsto 2x^2 - 3x$, $g : x \mapsto 3x + k$,		
(i) $2x^2 - 3x - 9 > 0$		
→ x =3 or −1½	M1 A1	For solving quadratic. Ignore > or ⩾
Set of x $x > 3$, or $x < -1½$	A1 [3]	condone ⩾ or ⩽
(ii) $2x^2 - 3x = 2(x - \frac{3}{4})^2 - \frac{9}{8}$	B3,2,1	– x^2 in bracket is an error.
Vertex $(\frac{3}{4}, -\frac{9}{8})$	B1√ [4]	√ on 'c' and 'b'.
(iii) gf(x) = $6x^2 - 9x + k = 0$	B1	
Use of $b^2 - 4ac \rightarrow k = \frac{27}{8}$ oe.	M1 A1 [3]	Used on a quadratic (even fg).

CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level and GCE Advanced Level

MARK SCHEME for the October/November 2013 series

9709 MATHEMATICS

9709/13 Paper 1, maximum raw mark 75

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

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Mark Scheme Notes

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M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol $\sqrt{}$ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

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The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through $\sqrt{}$" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

1 $(x+1)(x-2)$ or other valid method $-1, 2$ $x < -1, x > 2$	M1 A1 A1 [3]	Attempt soln of eqn or other method Penalise $\le, \ge$
2 $f(x) = 2x^{-\frac{1}{2}} + x\ (+c)$ $5 = -2 \times \frac{1}{2} + 4 + c$ $c = 2$	M1A1 M1 A1 [4]	Attempt integ $x^{-\frac{1}{2}}$ or $+x$ needed for M Sub (4, 5). c must be present
3 **(i)** gradient of perpendicular = $-½$ soi $y - 1 = -½(x - 3)$	B1 B1 [2]	
(ii) $C = (-9, 6)$ $AC^2 = [3-(-9)]^2 + [1-6]^2$ (ft on *their C*) $AC = 13$	B1 M1 A1 [3]	soi in **(i)** or **(ii)** **OR** $AB^2 = [3-(-21)]^2 + [1-11]^2$ M1 $AB = 26$ A1 $AC = 13$ A1
4 **(i)** **OD** = 4**i** + 3**j** **CD** = 4**i** + 3**j** −10**k**	B1 B1√ [2]	√ for **OD** – **10k**
(ii) **OD.CD** = 9 + 16 = 25 $\|\mathbf{OD}\| = \sqrt{25}$ or $\|\mathbf{CD}\| = \sqrt{125}$ $25 = \sqrt{25} \times \sqrt{125} \times \cos\theta$ oe ODC = 63.4° (or 1.11 rads)	M1 M1 M1 A1 [4]	Use of $x_1x_2 + y_1y_2 + z_1z_2$ Correct method for moduli All connected correctly cao
5 **(a)** $\frac{a}{1-r} = 8a \Rightarrow 1(a) = 8(a)(1-r)$ $r = \frac{7}{8}$ oe	B1 B1 [2]	
(b) $a + 4d = 197$ $\frac{10}{2}[2a + 9d] = 2040$ $d = 14$	B1 B1 M1A1 [4]	Or $2a + 9d = 408$ Attempt to solve simultaneously
6 **(i)** sector areas are $\frac{1}{2}11^2\alpha, \frac{1}{2}5^2\alpha$ $k = \dfrac{\frac{1}{2}\times 11^2\alpha - \frac{1}{2}\times 5^2\alpha}{\frac{1}{2}\times 5^2\alpha}$ $k = \frac{96}{25}$ or 3.84	B1 M1 A1 [3]	Sight of $11^2, 5^2$ Or $\frac{11^2 - 5^2}{5^2}$

(ii) perimeter shaded region= $11\alpha + 5\alpha + 6 + 6 = 16\alpha + 12$ perimeter unshaded region = $5\alpha + 5 + 5 = 5\alpha + 10$ $16\alpha + 12 = 2\,(5\alpha + 10)$ $\alpha = 4/3$ or 1.33	B1 B1 M1 A1 [4]	
7 **(a)** $x^2 - 1 = \sin\frac{\pi}{3}$ $x = \pm\ 1.366$	M1 A1A1√ [3]	 √ for negative of 1st answer
(b) $2\theta + \frac{\pi}{3} = \frac{5\pi}{6}\left(\text{or } \frac{13\pi}{6} \text{ or } \frac{\pi}{6}\right)$ $2\theta = \frac{\pi}{2} = \left(\text{or } \frac{11\pi}{6}\right)$ $\theta = \frac{\pi}{4}, \frac{11\pi}{12}$	B1 M1 A1A1 [4]	1 correct angle on RHS is sufficient Isolating 2θ SC decimals 0.785 & 2.88 scores M1B1
8 **(i)** 81 (x^8)	B1 [1]	
(ii) 10×3^3 (x^8) soi leading to their answer 270 (x^8)	B1B1 B1 [3]	B1 for 10, 5C2 or 5C3. B1 for 3^3. But must be multiplied.
(iii) k × **(i)** 405 soi + **(ii)** 675 (x^8)	M1 A1 DM1 A1 [4]	$k \neq 1{,}0$
9 $\frac{dy}{dx} = -k^2(x+2)^{-2} + 1 = 0$ $x + 2 = \pm\ k$ $x = -2 \pm k$ $\frac{d^2y}{dx^2} = 2k^2(x+2)^{-3}$ When $x = -2 = k$, $\frac{d^2y}{dx^2} = \left(\frac{2}{k}\right)$ which is (> 0) min When $x = -2 - k$, $\frac{d^2y}{dx^2} = \left(\frac{2}{-k}\right)$ which is (< 0) max	M1A1 DM1 A1 M1 M1 A1 A1 [8]	Attempt differentiation & set to zero Attempt to solve cao Attempt to differentiate again Sub their x value with k in it into $\frac{d^2y}{dx^2}$ Only 1 of bracketed items needed for each but $\frac{d^2y}{dx^2}$ and x need to be correct.

10 (i) Range is (y) $\geq c^2 + 4c$	B1	Allow >
$x^2 + 4x = (x+2)^2 - 4$	M1	**OR** $\frac{dy}{dx} = 2x + 4 = 0$
(Smallest value of c is) -2	A1 [3]	-2 with no (wrong) working gets B2
(ii) $5a + b = 11$	B1	
$(a+b)^2 + 4(a+b) = 21$	B1	
$(11 - 5a + a)^2 + 4(11 - 5a + a) = 21$	M1	**OR** corresponding equation in b
$(8)(2a^2 - 13a + 18) = (8)(2a-9)(a-2) = 0$	M1	**OR** $(8)(2b + 23)(b - 1) = 0$
$a = \frac{9}{2}, 2$ OR $b = \left(-\frac{23}{2}\right), 1$	A1	A1 for either a or b correct. Condone 2nd
	A1 [6]	value. Spotted solution scores only B marks.
Alt. (ii) Last 5 marks $f^{-1}(x) = \sqrt{x+4} - 2$ B1 g(1) = f^{-1} = (21) used M1 $a + b = \sqrt{25} - 2 = 3$ A1 Solve $a + b = 3$, $5a + b = 11$ M1 $a = 2, b = 1$ A1		**Alt. (ii)** Last 4 marks $(a + b + 7)(a + b - 3) = 0$ M1A1 (Ignore solution involving $a + b = -7$) Solve $a + b = 3$, $5a + b = 11$ M1 $a = 2, b = 1$ A1
11 (i) $\frac{dy}{dx} = \left[\frac{1}{2}(x4 + 4x + 4)^{-\frac{1}{2}}\right] \times \left[4x^3 + 4\right]$	B1B1	
At $x = 0$, $\frac{dy}{dx} = \frac{1}{2} \times \frac{1}{2} \times 4 = (1)$	M1	Sub $x = 0$ and attempt eqn of line following differentiation.
Equation is $y - 2 = x$	A1 [4]	
(ii) $x + 2 = \sqrt{x^4 + 4x + 4} \Rightarrow (x+2)^2 = x4 + 4x + 4$	B1	**AG** www
$x^2 - x^4 = 0$ oe	B1	
$x = 0, \pm 1$	B2,1,0 [4]	
(iii) $(\pi)\left[\frac{x^5}{5} + 2x^2 + 4x\right]$	M1A1	Attempt to integrate y^2
$(\pi)\left[0 - \left(\frac{-1}{5} + 2 - 4\right)\right]$	DM1	
$\frac{11\pi}{5}$ (6.91) oe	A1 [4]	Apply limits $-1 \to 0$

CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level and GCE Advanced Level

MARK SCHEME for the May/June 2013 series

9709 MATHEMATICS

9709/11 Paper 1, maximum raw mark 75

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CAMBRIDGE
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PA -1	This is deducted from A or B marks in the case of premature approximation. The PA -1 penalty is usually discussed at the meeting.

1 $f'(x)=(2x-5)^2\times 2+1$ or $24\left(x-\frac{5}{2}\right)^2+1$ >0 (allow $\geq$)	**B1B1** **B1** √ **[3]**	**B1** for $3(2x-5)^2$, **B1** for $(\times 2+1)$ SC B1 for $24x^2-120x+151$ Dep on $k(2x-5)^2+c$ $(k>0)$, $(c\geq 0)$ Subst of particular values is B0
2 **(i)** $1-6px \quad +15p^2x^2$	**B1B1** **[2]**	Simplificn of nCr can be scored in (ii)
(ii) $15p^2\times 1-6p\times -1$ $3p(5p+2)=0$ $p=-\frac{2}{5}$ oe	**M1** **DM1** **A1** **[3]**	 Obtain & attempt to solve quadratic Allow $p=0$ in addition
3 **(i)** $(OAB)=\frac{1}{2}\times 8^2\alpha$, $(OAC)=\frac{1}{2}\times\pi\times 4^2$ $\alpha=\frac{\pi}{8}$	**B1B1** **B1** **[3]**	Accept 25.1 (for OAC)
(ii) $8+8\times$ their $\alpha+\frac{1}{2}\times 8\times\pi$ $8+5\pi$	**B1** √ **B1** **[2]**	23.7 gets B1B0 SC B1 for e.g. 5π (omitted OB)
4 **(i)** $ar^2=-108$, $ar^5=32$ $r^3=\frac{32}{-108}=\left(-\frac{8}{27}\right)$ $r=\left(-\frac{2}{3}\right)$ or -0.666 or -0.667	**B1** **M1** **A1** **[3]**	 Eliminating a $-\frac{2}{3}$ from little or no working $\rightarrow\frac{3}{3}$ www
(ii) $a=-243$	**B1** √ **[1]**	ft on *their r* $\left(-\frac{108}{r^2}\text{ or }\frac{32}{r^5}\right)$
(iii) $S_\infty=\frac{-243}{1+\frac{2}{3}}=-\frac{729}{5}$ or -145.8	**M1A1** **[2]**	Accept -146. For M1 $\lvert r\rvert$ must be <1
5 **(i)** $\frac{\sin\theta(\sin\theta-\cos\theta)+\cos\theta(\sin\theta+\cos\theta)}{(\sin\theta+\cos\theta)(\sin\theta-\cos\theta)}$ $\frac{\sin^2\theta-\sin\theta\cos\theta+\cos\theta\sin\theta=\cos^2\theta}{\sin^2\theta-\cos^2\theta}$ $\frac{1}{\sin^2\theta-\cos^2\theta}$ **AG**	**M1** **A1** **A1** **[3]**	 www

(ii) $s^2-(1-s^2)=\frac{1}{3}$ or $1-c^2-c^2=\frac{1}{3}$ or $3(s^2-c^2)=c^2+s^2$	M1	Applying $c^2+s^2=1$
$\sin\theta=(\pm)\sqrt{\frac{2}{3}}$ or $\cos\theta=(\pm)\sqrt{\frac{1}{3}}$ or $\tan\theta=(\pm)\sqrt{2}$	A1	Or $s=(\pm)\ 0.816$, $c=(\pm)\ 0.577$, $t=(\pm)\ 1.414$
$\theta=54.7°, 125.3°, 234.7°, 305.3°$	A1A1 **[4]**	<u>any</u> 2 solutions for 1st A1 >4 solutions in range max A1A0
6 (i) **OA.OC** $=-4p^2-q^2+4p^2+q^2$ $=0$	M1 A1 **[2]**	Attempt scalar product. Allow M1 even for e.g. **OA.OB** $=2pq-2pq$ etc.
(ii) **CA = OA – OC** = $(\pm)(1+4p^2+q^2)$ **(i)** $\lvert\mathbf{CA}\rvert=1+4p^2+q^2$	M1 A1 **[2]**	Ignore **CA = OC–OA** Not $\sqrt{(1+4p^2+q^2)^2}$
(iii) **BA = OA – OB** = $\mathbf{i}+6\mathbf{j}+2\mathbf{k}-(2\mathbf{j}-6\mathbf{k})$ $=(\pm)(\mathbf{i}+4\mathbf{j}+8\mathbf{k})$	M1	Allow subtn reversed for both M marks
$\frac{x\mathbf{i}+y\mathbf{j}+z\mathbf{k}}{\sqrt{x^2+y^2+z^2}}\to\frac{1}{9}(\mathbf{i}+4\mathbf{j}+8\mathbf{k})$	M1A1 **[3]**	M1 independent of 1st M1
7 (i) $x^2-4x+4=x\Rightarrow x^2-5x+4=0$ $(x-1)(x-4)(=0)$ or other valid method (1, 1), (4, 4) Mid-point = (2½, 2½)	M1 M1 A1 A1 √ **[4]**	Eliminate y to reach 3-term quadratic Attempt solution ft dependent on 1st M1
(ii) $x^2-(4+m)x+4=0\to(4+m)^2-4(4)=0$ $4+m=\pm4$ or $m(8+m)=0$ $m=-8$ $x^2+4x+4=0$ $x=-2,\ y=16$	M1 DM1 A1 M1 A1 **[5]**	Applying $b^2-4ac=0$ Attempt solution Ignore $m=0$ in addition Sub non-zero m and attempt to solve Ignore (2, 0) solution from $m=0$
Alt (ii) $2x-4=m$	M1	**OR** $2x-4=m$
$x^2-4x+4=(2x-4)x$	DM1	Sub $x=\frac{m+4}{2}$, $y=\frac{m(m+4)}{2}$ into quad
$x=-2$ (ignore +2) $m=-8$ (ignore 0) $y=16$	A1 A1 A1	$m=-8$ from resulting quad $m(m+8)=0$ $x=-2$ $y=16$
8 (i) $2(x-3)^2-5$ or $a=2$, $b=-3$, $c=-5$	B1B1B1 **[3]**	
(ii) 3	B1 √ **[1]**	ft on – *their b*. Allow $k\geq3$ or $x\geq3$

(iii) $(y) \geq 27$	B1 **[1]**	Allow >. Allow $27 \leq y \leq \infty$ etc. **OR** (x/y interchange as 1st operation)
(iv) $2(x-3)^2 = (y+5)$	M1	$x = 2(y-3)^2 - 5$
$x - 3 = (\pm)\sqrt{\frac{1}{2}(y+5)}$	M1	$(y-3)^2 = \frac{1}{2}(x+5)$
$x = 3 + /\pm\sqrt{\frac{1}{2}(y+5)}$	A1 ✓	$y - 3 = (\pm)\sqrt{\frac{1}{2}(x+5)}$
$(\mathrm{f}^{-1}(x)) = 3 + \sqrt{\frac{1}{2}(x+5)}$ for $x \geq 27$	A1B1 ✓ **[5]**	ft on *their* 27 from **(iii)**
9 (i) $3u + \frac{3}{u} - 10 = 0$	B1	Or $3x - 10\sqrt{x} + 3 = 0$ Or $(3\sqrt{x}-1)(\sqrt{x}-3)$ or apply formula etc.
$3u^2 - 10u + 3 = 0 \Rightarrow (3u-1)(u-3) = 0$	M1	
$\sqrt{x} = \frac{1}{3}$ or 3	A1	
$\sqrt{x} = \frac{1}{9}$ or 9	A1 **[4]**	
(ii) $\mathrm{f}''(x) = \frac{3}{2}x^{-\frac{1}{2}} - \frac{3}{2}x^{-\frac{3}{2}}$	B1	Allow anywhere
At $x = \frac{1}{9}$ $\mathrm{f}''(x) = \frac{3}{2}(3) - \frac{3}{2}(27)\ (= -36) < 0 \rightarrow$ Max	M1	Valid method. Allow innac subs, even 3, $\frac{1}{3}$
At $x = 9$ $\mathrm{f}''(x) = \frac{3}{2}\times\frac{1}{3} - \frac{3}{2}\times\frac{1}{27}\ (= \frac{4}{9}) > 0 \rightarrow$ Min	A1 **[3]**	Fully correct. No working, no marks.
(iii) $\mathrm{f}(x) = 2x^{\frac{3}{2}} + 6x^{\frac{1}{2}} - 10x\ (+c)$ $-7 = 16 + 12 - 40 + c$ $c = 5$	B2 M1 A1 **[4]**	B1 for 2/3 terms correct. Allow in **(i)** Sub (4, –7). c must be present.
10 (i) $\frac{\mathrm{dy}}{\mathrm{dx}} = 4(x-2)^3$	B1	Or $4x^3 - 24x^2 + 48x - 32$
Grad of tangent = –4	M1	Sub x = 1 into *their* derivative
Eq. of tangent is y – 1 = –4(x – 1)	M1	Line thru (1, 1) and with m from deriv
$\rightarrow$ B $(\frac{5}{4}, 0)$	A1	
Grad of normal = $\frac{1}{4}$	M1	Use of $m_1 m_2 = -1$
Eq. of normal is $y - 1 = \frac{1}{4}(x-1) \rightarrow \mathrm{C}(0, \frac{3}{4})$	A1 **[6]**	

(ii) $AC^2 = 1^2 + \left(\frac{1}{4}\right)^2$	**M1**	
$\frac{\sqrt{17}}{4}$	**A1** **[2]**	Allow $\sqrt{\frac{17}{16}}$
(iii) $\int (x-2)^4 \mathrm{d}x = \frac{(x-2)^5}{5}$	**B1**	Or $\frac{x^5}{5} - 2x^4 + 8x^3 - 16x^2 + 16x$
$\left[0 - (-\frac{1}{5})\right] = \frac{1}{5}$	**M1**	Apply limits $1 \to 2$ for curve
$\Delta = \frac{1}{2} \times 1 \times (\mathit{their}\,\frac{5}{4} - 1) = \frac{1}{8}$	**M1**	Or $\int_1^{\frac{5}{4}} (-4x+5)\ \mathrm{d}x = \frac{1}{8}$
$\frac{1}{5} - \frac{1}{8} = \frac{3}{40}$ or 0.075	**A1** **[4]**	

CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level and GCE Advanced Level

MARK SCHEME for the May/June 2013 series

9709 MATHEMATICS

9709/12 Paper 1, maximum raw mark 75

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1 $\frac{dy}{dx}=\frac{6}{x^2}$ $y=-6x^{-1}+c$ Uses (2, 9) → $c = 12$ $y=-6x^{-1}+12$	B1 M1 A1 [3]	Integration only – unsimplified Uses (2, 9) in an integral
2 $\left(2x-\frac{1}{2x}\right)^6$ **(i)** Coeff of $x^2 = 15\times16\times(-½)^2 = 60$ **(ii)** Constant term is $20 \times 8x^3 \times(-1 \div 8x^3)$ $\times(1 + x^2)$ needs to consider 2 terms → $60 - 20 = 40$	B1 B1 [2] B1 M1 A1 [3]	B1 for 2/3 parts. B1 B1 unsimplified Needs to consider the constant term
3 $mx+14=\frac{12}{x}+2 \rightarrow mx^2+12x-12=0$ Uses $b^2=4ac \rightarrow m=-3$ $-3x^2+12x-12=0 \rightarrow P\ (2, 8)$ [Or $m=-12x^{-2}$ M1 Sub M1 $x = 2$ A1] [→ $m = -3$ and $y = 8$ M1 A1]	M1 M1 A1 DM1 A1 [5]	Eliminates x (or y) Any use of discriminant Any valid method.
4 **(i)** $BOC = 2\tan^{-1}½ = 0.9273$ **(ii)** $OB = \sqrt{(10^2 + 5^2)}$ or $11.2 = r$ Arc $BXC = \sqrt{125} \times 0.9273$ → Perimeter = 20.4 cm **(iii)** Area = $½r^2\theta$ $- ½.10.10 \rightarrow 7.96$ cm².	M1 A1 [2] B1 M1 A1 [3] M1 A1 [2]	Correct trigonometry. (ans given) Use of trig (or Pyth) for the $OB = \sqrt{125}$. Use of $s = r\theta$ with θ in rads , $r \neq 10$ Correct formula used with rads, $r \neq 10$. Allow 7.95 or 7.96
5 $a=\sin\theta-3\cos\theta,\ b=3\sin\theta+\cos\theta$ **(i)** $a^2 + b^2 =$ $(s^2+9c^2-6sc)+(9s^2+c^2+6sc)$ $10c^2 + 10s^2 = 10$ **(ii)** $2s-6c=3s+c \rightarrow s=-7c$ → $\tan\theta = -7$ → 98.1° and 278.1°	B1 M1 A1 [3] M1 A1 A1 A1√ [4]	Correct squaring Use of $s^2 + c^2 =1$ to get constant. (can get 2/3 for missing $6sc$) Collecting and $t = s\div c$ For 180^0 + first answer, providing no extra answers in the range.

6 $\overrightarrow{OA} = \mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$, $\overrightarrow{OB} = 3\mathbf{i} + p\mathbf{j} + q\mathbf{k}$ **(i)** $p = -6$, $q = 6$	B1 B1 [2]	
(ii) dot product = 0 → $3 - 2p + 4p = 0$ → $p = -1.5$	M1 A1 [2]	Use of $x_1x_2 + y_1y_2 + z_1z_2 = 0$
(iii) $\overrightarrow{AB}$ = b − a = **2**i + **3**j + 6**k** Unit vector = (**2**i + **3**j + 6**k**) ÷ **7**	B1 M1 A1√ [3]	not for **b** – **a**. M1 for division by modulus. √ on B1.
7 $3y + 2x = 33.$ Gradient of line = $-\frac{2}{3}$ Gradient of perpendicular = 3/2 Eqn of perp $y - 3 = \frac{3}{2}(x+1)$ Sim Eqns → (3, 9)	B1 M1 M1 M1 A1	 Use of $m_1m_2 = -1$ with gradient of line Correct form of perpendicular eqn. Sim eqns.
(−1, 3) → (3, 9) → (7, 15)	M1 A1 [7]	Vectors or other method.
8 **(i)** $\pi r^2 h = 250\pi \rightarrow h = \frac{250}{r^2}$ → $S = 2\pi rh + 2\pi r^2$ → $S = 2\pi r^2 + \frac{500\pi}{r}$	M1 M1 [2]	Makes h the subject. $\pi r^2 h$ must be right Ans given – check all formulae..
(ii) $\frac{dS}{dr} = 4\pi r - \frac{500\pi}{r^2}$ = 0 when $r^3 = 125$ → $r = 5$ → $S = 150\pi$	B1 B1 M1 A1 [4]	B1 for each term Sets differential to 0 + attempt at soln
(iii) $\frac{d^2S}{dr^2} = 4\pi + \frac{1000\pi}{r^3}$ This is positive → Minimum	M1 A1 [2]	Any valid method. 2nd differential must be correct – no need for numerical answer or correct r.
9 $f(x) = \frac{5}{1-3x}$, $x \geq 1$ **(i)** $f'(x) = \frac{-5}{(1-3x)^2} \times -3$	B1 B1 [2]	B1 without × −3. B1 for ×−3, even if first B mark is incorrect
(ii) $15 > 0$ and $(1-3x)^2 > 0$, $f'(x) > 0$ → increasing	B1√ [1]	√ providing ( )² in denominator.
(iii) $y = \frac{5}{1-3x}$ → $3x = 1 - \frac{5}{y}$ → $f^{-1}(x) = \frac{x-5}{3x}$ or $\frac{1}{3} - \frac{5}{3x}$	M1 A1	Attempt to make x the subject. Must be in terms of x.
Range is ≥ 1 Domain is $-2.5 \leq x < 0$	B1 B1 B1 [5]	must be ≥ condone <

10 **(a)** $57 = 2(24 + 3d) \rightarrow d = 1.5$ $48 = 12 + (n - 1)1.5 \rightarrow n = 25$	 M1 A1 M1 A1 [4]	 Use of correct S_n formula. Use of correct T_n formula.
(b) $ar^2 = 4a \quad r = \pm 2$ $\dfrac{a(r^6 - 1)}{r - 1} = ka$ $\rightarrow k = 63$ or $k = -21$	B1 B1 B1 B1 [4]	(allow for $r = 2$)
11 $y = \sqrt{1+4x}$ **(i)** $\dfrac{dy}{dx} = \frac{1}{2}(1+4x)^{-\frac{1}{2}} \times 4$ $= 2$ at B (0, 1) Gradient of normal $= -½$ Equation $y - 1 = -½\, x$	B1 B1 M1 M1 A1 [5]	B1 Without "×4". B1 for "×4" even if first B mark lost. Use of $m_1m_2 = -1$ Correct method for eqn.
(ii) At A $x = -¼$ $\int \sqrt{1+4x}\,dx = \dfrac{(1+4x)^{\frac{3}{2}}}{\frac{3}{2}} \div 4$ Limits $-¼$ to 0 $\rightarrow \frac{1}{6}$ Area $BOC = ½ \times 2 \times 1 = 1$ $\rightarrow$ Shaded area $= \dfrac{7}{6}$	B1 B1 B1 B1 B1√ [5]	B1 Without the "÷4". For "÷4" even if first B mark lost. For 1 + his "1/6".

CAMBRIDGE INTERNATIONAL EXAMINATIONS

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9709/13 Paper 1, maximum raw mark 75

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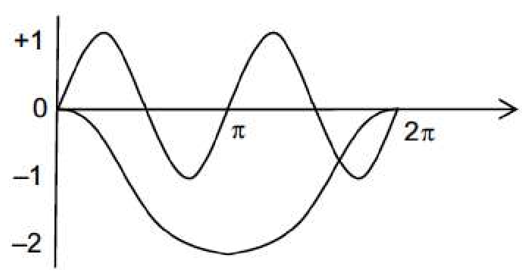
1		$\frac{dy}{dx} = \sqrt{2x+5}$			
		$\frac{(2x+5)^{\frac{3}{2}}}{\frac{3}{2}} \div 2 \quad (+c)$	B1 B1		B1 Everything without "÷2". B1 "÷2"
		Uses (2, 5) $\rightarrow c = -4$	M1 A1	[4]	Uses point in an integral.
2	(i)	$\frac{1}{2}.3^2\pi = \frac{1}{2}9^2\theta - \frac{1}{2}3^2\theta$ $\rightarrow \theta = \frac{1}{4}\pi$	M1 A1 A1	[3]	M1 needs $\frac{1}{2}r^2\theta$ once. A1 all correct. Answer given
	(ii)	$P = 6+6+3\times\frac{1}{4}\pi + 9\times\frac{1}{4}\pi = 21.4$ cm. or $12 + 3\pi$	M1 A1	[2]	M1 is for use of $s=r\theta$ once.
3		$2\cos^2\theta = \tan^2\theta$			
	(i)	$\rightarrow 2\cos^2\theta = \frac{\sin^2\theta}{\cos^2\theta}$ $\rightarrow$ Uses $c^2 + s^2 = 1 \rightarrow 2c^4 = 1 - c^2$	M1 A1	[2]	Use of $t^2 = s^2 \div c^2$ or alternative. Correct eqn.
	(ii)	$(2c^2 - 1)(c^2 + 1) = 0 \rightarrow c = \pm\frac{1}{\sqrt{2}}$ $\rightarrow \theta = \frac{1}{4}\pi$ or $\frac{3}{4}\pi$.	M1 A1 A1√	[3]	Method of solving for 3-term quadratic. (in terms of π). √ for π − 1st ans. Cannot gain A1√ if other answers given in the range.
4	(i)	$(2+ax)^5 = 32 + 80ax + 80a^2x^2$	3 × B1	[3]	B1 for each term.
	(ii)	$\times (1 + 2x)$ $240 = 80a^2 + 160a$ $\rightarrow a = 1$ or $a = -3$.	M1 DM1A1	[3]	Realises need to consider 2 terms. Solution of 3-term quadratic.
5	(i)		B1 DB1 B1 DB1	[4]	y =sin2x has 2 cycles, starts and finishes on the x-axis, max comes first. From +1 to −1. Smooth curves. y = cosx − 1 has one cycle, starts and finishes on x-axis, with a minimum pt. From 0 to −2, smooth curve, flattens.
	(ii)	**(a)** $\sin 2x = -\frac{1}{2} \rightarrow$ 4 solutions	B1√	[1]	√ for their curve.
		(b) $\sin 2x + \cos x + 1 = 0 \rightarrow$ 3 solutions.	B1√	[1]	√ for intersections of their curves.

6	$u = x^2 y \quad y + 3x = 9$ $u = x^2(9-3x)$ or $\left(\frac{9-y}{3}\right)^2 y$	M1		Expressing u in terms of 1 variable
	$\frac{du}{dx} = 18x - 9x^2$ or $\frac{du}{dy} = 27 - 12y + y^2$	DM1A1		Knowing to differentiate.
	$= 0$ when $x = 2$ or $y = 3 \rightarrow u = 12$	DM1 A1		Setting differential to 0.
	$\frac{d^2u}{dx^2} = 18 - 18x \quad -\text{ve}$	DM1 A1	[7]	Any valid method
7	A (2, 14), B (14, 6) and C (7, 2).			
(i)	m of $AB = -\frac{2}{3}$	B1		
	m of perpendicular $= \frac{3}{2}$	M1		For use of $m_1 m_2 = -1$
	eqn of $AB \quad y - 14 = -\frac{2}{3}(x-2)$	M1		Allow M1 for unsimplified eqn
	eqn of $CX \quad y - 2 = \frac{3}{2}(x-7)$	M1		Allow M1 for unsimplified eqn
	Sim Eqns $\rightarrow X$ (11, 8)	M1 A1	[6]	For solution of sim eqns.
(ii)	$AX : XB = 14-8 : 8-6 = 3 : 1$ Or $\sqrt{(9^2+6^2)} : \sqrt{(3^2+2^2)} = 3 : 1$	M1 A1	[2]	Vector steps or Pythagoras.
8	$\overrightarrow{OA} = \begin{pmatrix} 3 \\ 3 \\ -4 \end{pmatrix}$ and $\overrightarrow{OB} = \begin{pmatrix} 5 \\ 0 \\ 2 \end{pmatrix}$.			
	(i) $\mathbf{OC} = \mathbf{AB} = \mathbf{b} - \mathbf{a} = \begin{pmatrix} 2 \\ -3 \\ 6 \end{pmatrix}$	M1		Knowing how to find **OC**
	Uses **OC** and **OB**	B1		Using **OC.OB** or **CO.BO**
	OC.OB $= 22 = 7 \times \sqrt{29} \cos BOC$	M1 M1		M1 Use of $x_1x_2 + \ldots$ M1 for modulus
	$\rightarrow$ Angle $BOC = 54.3^0$ (or 0.948 rad)	M1 A1	[6]	M1 everything linked. (nb uses **BO.OC** loses B1 A1) (nb uses other vectors – max M1M1)

(ii)	Modulus of **OC** = 7 Vector = 35 ÷ 7 × **OC**	M1		Knows to scale by factor of 35 ÷ Mod
	$\rightarrow \pm \mathbf{5}\begin{pmatrix}2\\-3\\6\end{pmatrix}$	A1√	[2]	For their **OC.**
9 (a)	$S_n = 2n^2 + 8n$			
	$S_1 = 10 \;\; = a$	B1		
	$S_2 = 24 = a + (a + d)\; d = 4$	M1 A1	[3]	correct use of S_n formula.
(b)	GP $a = 64$ $ar = 48 \rightarrow r = ¾$	B1		
	→3rd term is $ar^2 = 36$	M1		ar^2 numerical – for their r
	AP $a = 64,\; a + 8d = 48 \;\rightarrow\; d = -2$	B1		
	$36 = 64 + (n-1)(-2)$	M1		correct use of $a+(n-1)d$
	$\rightarrow n = 15.$	A1	[5]	
10	$\mathrm{f} : x \mapsto 2x + k,\; \mathrm{g} : x \mapsto x^2 - 6x + 8,$			
(i)	$2(2x + 3) + 3 = 25$ $\rightarrow x = 4$ or $\{\mathrm{f}(11) = 25,\; \mathrm{f}(4) = 11\}$	M1 A1	[2]	ff(x) needs to be correctly formed
(ii)	$x^2 - 6x + 8 = 2x + k$ $x^2 - 8x + 8 - k = 0$ Uses $b^2 - 4ac < 0$ $\rightarrow k < -8$	M1 M1 A1	[3]	Eliminates y to form eqn in x. Uses the discriminant – even if =0.>0
(iii)	$x^2 - 6x + 8 = (x-3)^2 - 1$ $y = (x-3)^2 - 1$ Makes x the subject $\rightarrow \pm\sqrt{(x+1)} + 3$ Needs specifically to lose the "–".	B1 B1 M1 A1√	[4]	For "–3" and "–1" Makes x the subject, in terms of x and without − or ±.

11	$y = \dfrac{8}{\sqrt{x}} - x$			
(i)	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = -4x^{-\frac{3}{2}} - 1$	B1		needs both
	$= -\frac{3}{2}$ when $x = 4$.	M1		Subs $x = 4$ into $\mathrm{d}y/\mathrm{d}x$
	Eqn of BC $y - 0 = -\frac{3}{2}(x - 4)$	M1		Must be using differential + correct form of line at $B(4,0)$.
	$\rightarrow C\,(1, 4\frac{1}{2})$	A1	[4]	
(ii)	area under curve = $\int(\dfrac{8}{\sqrt{x}} - x)$			
	$= \dfrac{8x^{\frac{1}{2}}}{\frac{1}{2}} - \frac{1}{2}x^2$	B1 B1		(both unsimplified)
	Limits 1 to 4 $\rightarrow$ $8\frac{1}{2}$	M1		Using correct limits.
	Area under tangent = $\frac{1}{2} \times 4\frac{1}{2} \times 3 = 6\frac{3}{4}$	M1		Or could use calculus)
	Shaded area = $1\frac{3}{4}$	A1	[5]	