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CANDIDATE NAME

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8986046942

MATHEMATICS **9709/11**

Paper 1 Pure Mathematics 1 **October/November 2024**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS
- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
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- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION
- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

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1 In the expansion of $\left(kx+\frac{2}{x}\right)^4$, where k is a positive constant, the term independent of x is equal to 150.

Find the value of k and hence determine the coefficient of x^2 in the expansion. [4]

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2 The curve $y = x^2 - \frac{a}{x}$ has a stationary point at $(-3, b)$.

Find the values of the constants a and b. [4]

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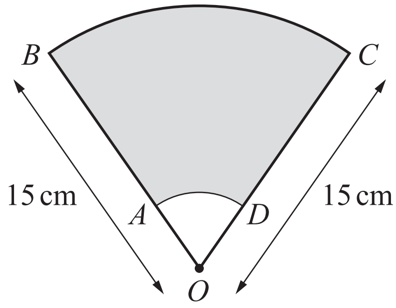

The diagram shows a sector of a circle, centre O, where $OB = OC = 15\,\text{cm}$. The size of angle BOC is $\frac{2}{5}\pi$ radians. Points A and D on the lines OB and OC respectively are joined by an arc AD of a circle with centre O. The shaded region is bounded by the arcs AD and BC and by the straight lines AB and DC. It is given that the area of the shaded region is $\frac{209}{5}\pi\,\text{cm}^2$.

Find the perimeter of the shaded region. Give your answer in terms of π. [5]

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4 Show that the curve with equation $x^2 - 3xy - 40 = 0$ and the line with equation $3x + y + k = 0$ meet for all values of the constant k. [5]

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5 The equation of a curve is such that $\frac{dy}{dx} = 4x - 3\sqrt{x} + 1$.

(a) Find the x-coordinate of the point on the curve at which the gradient is $\frac{11}{2}$. [3]

(b) Given that the curve passes through the point (4, 11), find the equation of the curve. [4]

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6 Circles C_1 and C_2 have equations

$$x^2 + y^2 + 6x - 10y + 18 = 0 \quad \text{and} \quad (x-9)^2 + (y+4)^2 - 64 = 0$$

respectively.

(a) Find the distance between the centres of the circles. [4]

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P and *Q* are points on C_1 and C_2 respectively. The distance between *P* and *Q* is denoted by *d*.

(b) Find the greatest and least possible values of *d*. [3]

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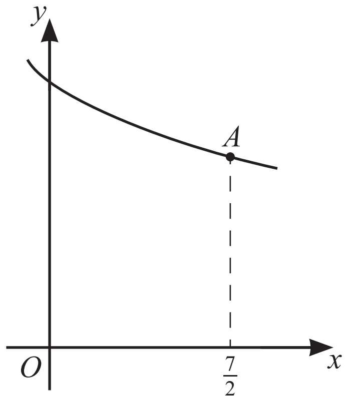

The diagram shows part of the curve with equation $y = \dfrac{12}{\sqrt[3]{2x+1}}$. The point A on the curve has coordinates $\left(\frac{7}{2}, 6\right)$.

(a) Find the equation of the tangent to the curve at A. Give your answer in the form $y = mx + c$. [4]

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(b) Find the area of the region bounded by the curve and the lines $x = 0$, $x = \frac{7}{2}$ and $y = 0$. [4]

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8 **(a)** It is given that β is an angle between 90° and 180° such that $\sin\beta = a$.

Express $\tan^2\beta - 3\sin\beta\cos\beta$ in terms of a. [3]

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(b) Solve the equation $\sin^2\theta + 2\cos^2\theta = 4\sin\theta + 3$ for $0° < \theta < 360°$. [5]

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9 The equation of a curve is $y = 4 + 5x + 6x^2 - 3x^3$.

(a) Find the set of values of x for which y decreases as x increases. [4]

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(b) It is given that $y = 9x + k$ is a tangent to the curve.

Find the value of the constant k. [4]

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10 An arithmetic progression has first term 5 and common difference d, where $d > 0$. The second, fifth and eleventh terms of the arithmetic progression, in that order, are the first three terms of a geometric progression.

(a) Find the value of d. [3]

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(b) The sum of the first 77 terms of the arithmetic progression is denoted by S_{77}. The sum of the first 10 terms of the geometric progression is denoted by G_{10}.

Find the value of $S_{77} - G_{10}$. [5]

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11 The function f is defined by $f(x) = 3 + 6x - 2x^2$ for $x \in \mathbb{R}$.

(a) Express $f(x)$ in the form $a - b(x-c)^2$, where a, b and c are constants, and state the range of f. [3]

(b) The graph of $y = f(x)$ is transformed to the graph of $y = h(x)$ by a reflection in one of the axes followed by a translation. It is given that the graph of $y = h(x)$ has a minimum point at the origin.

Give details of the reflection and translation involved. [2]

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The function g is defined by $g(x) = 3 + 6x - 2x^2$ for $x \leqslant 0$.

(c) Sketch the graph of $y = g(x)$ and explain why g is a one-one function. You are **not** required to find the coordinates of any intersections with the axes. [2]

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(d) Sketch the graph of $y = g^{-1}(x)$ on your diagram in **(c)**, and find an expression for $g^{-1}(x)$. You should label the two graphs in your diagram appropriately and show any relevant mirror line. [4]

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MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **October/November 2024**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
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INFORMATION

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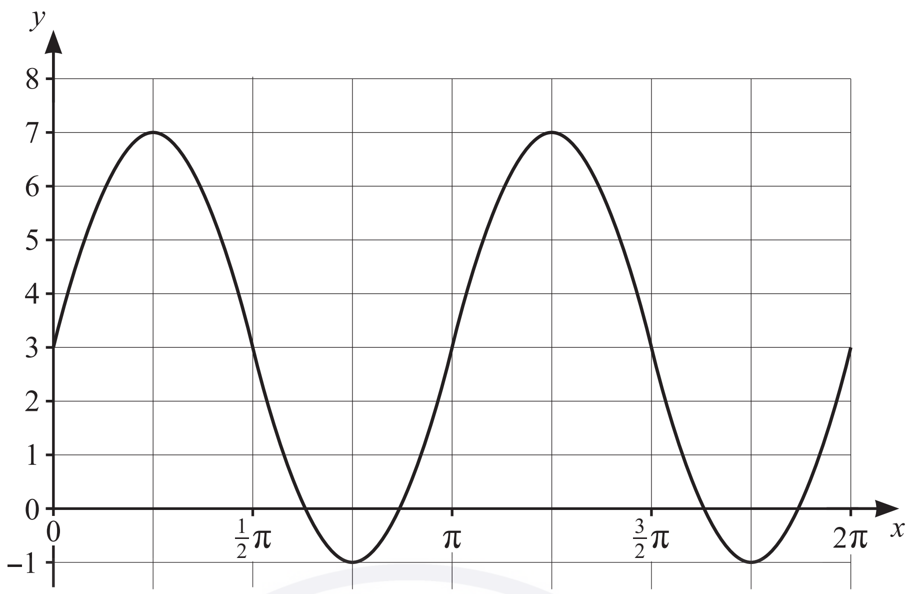

The diagram shows the curve with equation $y = a\sin(bx) + c$ for $0 \leqslant x \leqslant 2\pi$, where a, b and c are positive constants.

(a) State the values of a, b and c. [3]

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(b) For these values of a, b and c, determine the number of solutions in the interval $0 \leqslant x \leqslant 2\pi$ for each of the following equations:

(i) $a\sin(bx) + c = 7 - x$ [1]

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(ii) $a\sin(bx) + c = 2\pi(x - 1)$. [1]

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2 The first term of an arithmetic progression is −20 and the common difference is 5.

(a) Find the sum of the first 20 terms of the progression. [2]

It is given that the sum of the first $2k$ terms is 10 times the sum of the first k terms.

(b) Find the value of k. [3]

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3 The equation of a curve is $y = 2x^2 - 3$. Two points A and B with x-coordinates 2 and $(2 + h)$ respectively lie on the curve.

(a) Find and simplify an expression for the gradient of the chord AB in terms of h. [3]

(b) Explain how the gradient of the curve at the point A can be deduced from the answer to part **(a)**, and state the value of this gradient. [2]

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4 Find the term independent of x in the expansion of each of the following:

(a) $\left(x+\frac{3}{x^2}\right)^6$ [2]

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(b) $(4x^3-5)\left(x+\frac{3}{x^2}\right)^6$. [4]

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5 The function f is defined by $f(x) = \frac{2x+1}{2x-1}$ for $x < \frac{1}{2}$.

(a) **(i)** State the value of $f(-1)$. [1]

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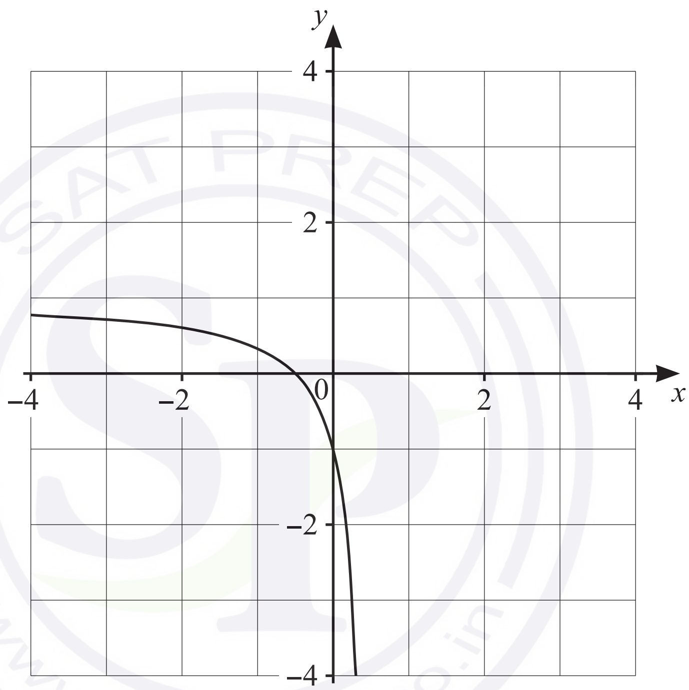
(ii)



The diagram shows the graph of $y = f(x)$. Sketch the graph of $y = f^{-1}(x)$ on this diagram. Show any relevant mirror line. [2]

(iii) Find an expression for $f^{-1}(x)$ and state the domain of the function f^{-1}. [4]

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The function g is defined by $g(x) = 3x + 2$ for $x \in \mathbb{R}$.

(b) Solve the equation $f(x) = gf\left(\frac{1}{4}\right)$. [3]

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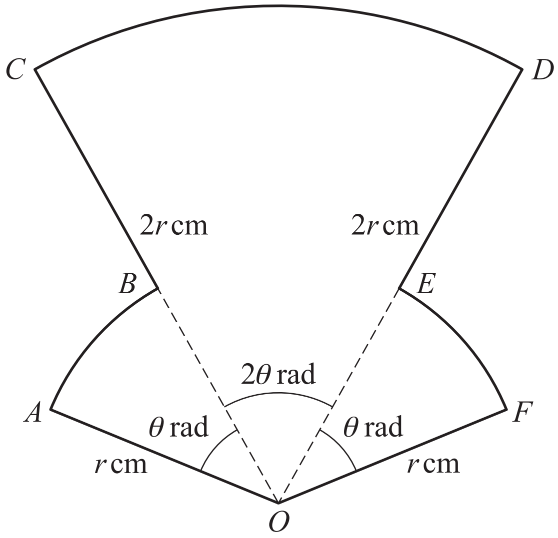

The diagram shows a metal plate *OABCDEF* consisting of sectors of two circles, each with centre *O*. The radii of sectors *AOB* and *EOF* are r cm and the radius of sector *COD* is $2r$ cm. Angle AOB = angle $EOF = \theta$ radians and angle $COD = 2\theta$ radians.

It is given that the perimeter of the plate is 14 cm and the area of the plate is $10\,\text{cm}^2$.

Given that $r > \frac{3}{2}$ and $\theta < \frac{3}{4}$, find the values of r and θ. [6]

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7 (a) By expressing $-2x^2+8x+11$ in the form $-a(x-b)^2+c$, where a, b and c are positive integers, find the coordinates of the vertex of the graph with equation $y=-2x^2+8x+11$. [3]

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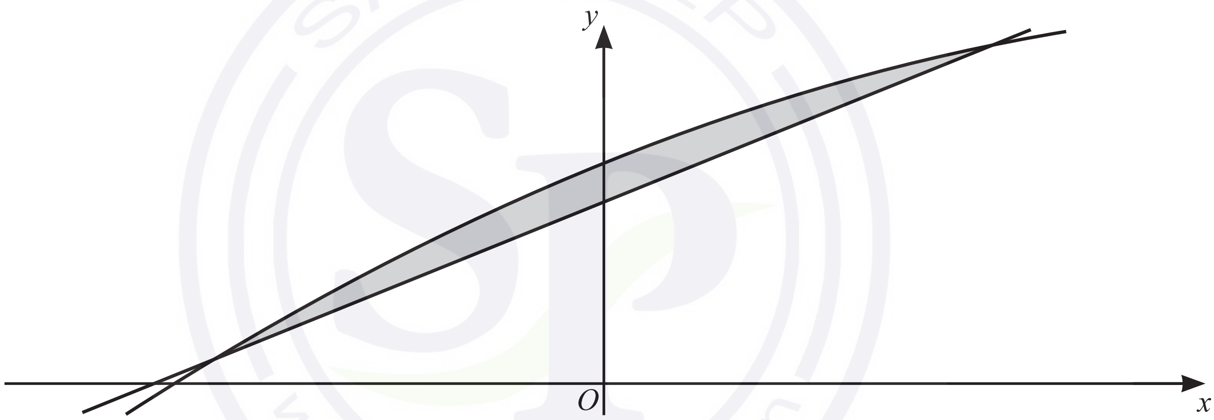
(b)



The diagram shows part of the curve with equation $y=-2x^2+8x+11$ and the line with equation $y=8x+9$.

Find the area of the shaded region. [5]

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8 The equation of a circle is $x^2 + y^2 + px + 2y + q = 0$, where p and q are constants.

(a) Express the equation in the form $(x-a)^2 + (y-b)^2 = r^2$, where a is to be given in terms of p and r^2 is to be given in terms of p and q. [2]

The line with equation $x + 2y = 10$ is the tangent to the circle at the point $A(4, 3)$.

(b) **(i)** Find the equation of the normal to the circle at the point A. [3]

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(ii) Find the values of p and q. [5]

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9 The equation of a curve is $y = \frac{1}{2}k^2x^2 - 2kx + 2$ and the equation of a line is $y = kx + p$, where k and p are constants with $0 < k < 1$.

(a) It is given that one of the points of intersection of the curve and the line has coordinates $\left(\frac{5}{2}, \frac{1}{2}\right)$.

Find the values of k and p, and find the coordinates of the other point of intersection. [7]

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(b) It is given instead that the line and the curve do **not** intersect.

Find the set of possible values of p. [3]

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10 A function f with domain $x > 0$ is such that $f'(x) = 8(2x-3)^{\frac{1}{3}} - 10x^{\frac{2}{3}}$. It is given that the curve with equation $y = f(x)$ passes through the point $(1, 0)$.

(a) Find the equation of the normal to the curve at the point $(1, 0)$. [3]

(b) Find $f(x)$. [4]

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It is given that the equation $f'(x) = 0$ can be expressed in the form

$$125x^2 - 128x + 192 = 0.$$

(c) Determine, making your reasoning clear, whether f is an increasing function, a decreasing function or neither. [3]

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MATHEMATICS **9709/13**

Paper 1 Pure Mathematics 1 **October/November 2024**

1 hour 50 minutes

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1 An arithmetic progression has fourth term 15 and eighth term 25.

Find the 30th term of the progression. [3]

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2 Find the exact solution of the equation

$$\cos\frac{1}{6}\pi + \tan 2x + \frac{\sqrt{3}}{2} = 0 \text{ for } -\frac{1}{4}\pi < x < \frac{1}{4}\pi.$$ [2]

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3 **(a)** Find the coefficients of x^3 and x^4 in the expansion of $(3-ax)^5$, where a is a constant. Give your answers in terms of a. [3]

(b) Given that the coefficient of x^4 in the expansion of $(ax+7)(3-ax)^5$ is 240, find the positive value of a. [3]

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4 Solve the equation $4\sin^4\theta + 12\sin^2\theta - 7 = 0$ for $0° \leqslant \theta \leqslant 360°$. [4]

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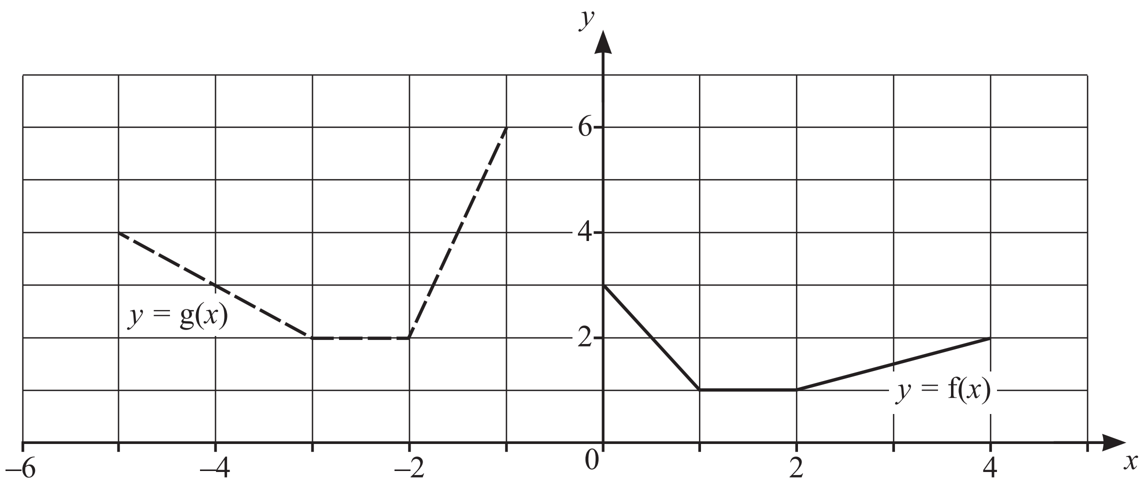

In the diagram, the graph with equation $y = \mathrm{f}(x)$ is shown with solid lines and the graph with equation $y = \mathrm{g}(x)$ is shown with broken lines.

(a) Describe fully a sequence of three transformations which transforms the graph of $y = \mathrm{f}(x)$ to the graph of $y = \mathrm{g}(x)$. [6]

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(b) Find an expression for $\mathrm{g}(x)$ in the form $a\mathrm{f}(bx+c)$, where a, b and c are integers. [2]

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6 The first term of a convergent geometric progression is 10. The sum of the first 4 terms of the progression is p and the sum of the first 8 terms of the progression is q. It is given that $\frac{q}{p} = \frac{17}{16}$.

Find the two possible values of the sum to infinity. [5]

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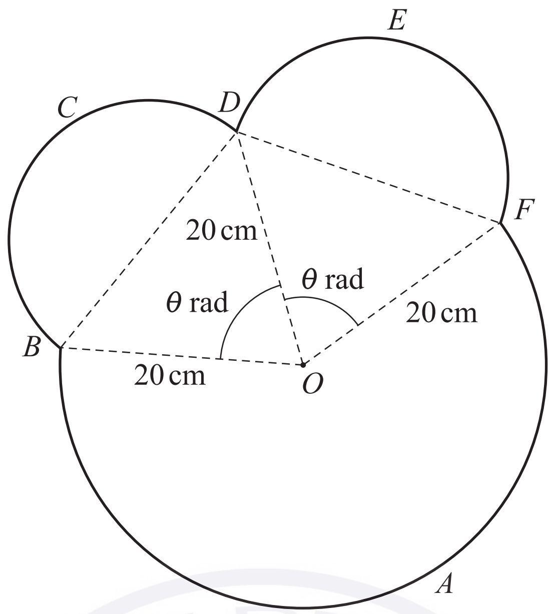

The diagram shows a metal plate *ABCDEF* consisting of five parts. The parts *BCD* and *DEF* are semicircles. The part *BAFO* is a sector of a circle with centre *O* and radius 20 cm, and *D* lies on this circle. The parts *OBD* and *ODF* are triangles. Angles *BOD* and *DOF* are both θ radians.

(a) Given that $\theta = 1.2$, find the area of the metal plate. Give your answer correct to 3 significant figures. [5]

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(b) Given instead that the area of each semicircle is $50\pi\,\text{cm}^2$, find the exact perimeter of the metal plate. [5]

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8 **(a)** Express $3x^2 - 12x + 14$ in the form $3(x+a)^2 + b$, where a and b are constants to be found. [2]

The function $f(x) = 3x^2 - 12x + 14$ is defined for $x \geqslant k$, where k is a constant.

(b) Find the least value of k for which the function f^{-1} exists. [1]

For the rest of this question, you should assume that k has the value found in part **(b)**.

(c) Find an expression for $f^{-1}(x)$. [3]

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(d) Hence or otherwise solve the equation $ff(x) = 29$. [3]

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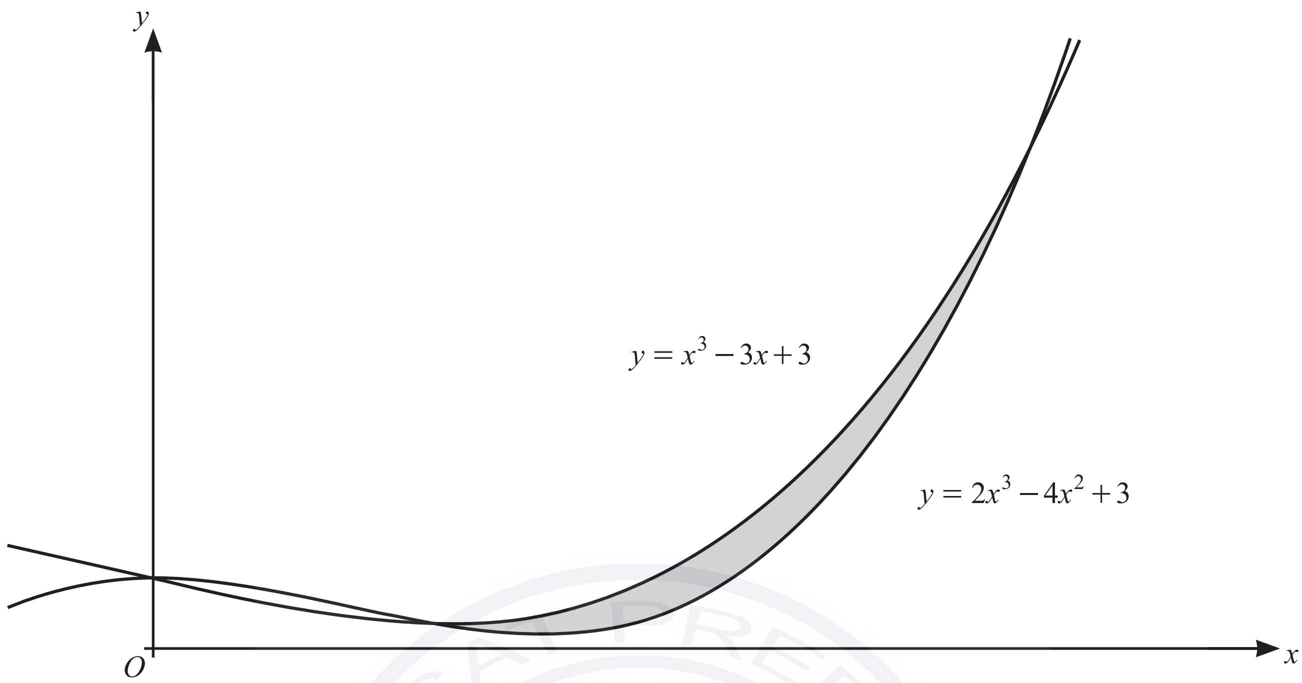

The diagram shows the curves with equations $y = x^3 - 3x + 3$ and $y = 2x^3 - 4x^2 + 3$.

(a) Find the x-coordinates of the points of intersection of the curves. [3]

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(b) Find the area of the shaded region. [4]

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10 Points A and B have coordinates (4, 3) and (8, −5) respectively. A circle with radius 10 passes through the points A and B.

(a) Show that the centre of the circle lies on the line $y = \frac{1}{2}x - 4$. [4]

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(b) Find the two possible equations of the circle. [5]

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11 The equation of a curve is $y = kx^{\frac{1}{2}} - 4x^2 + 2$, where k is a constant.

(a) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ in terms of k. [2]

(b) It is given that $k = 2$.

Find the coordinates of the stationary point and determine its nature. [4]

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(c) Points A and B on the curve have x-coordinates 0.25 and 1 respectively. For a different value of k, the tangents to the curve at the points A and B meet at a point with x-coordinate 0.6.

Find this value of k. [6]

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CENTRE NUMBER		CANDIDATE NUMBER	

8671641646

MATHEMATICS **9709/11**

Paper 1 Pure Mathematics 1 **May/June 2024**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS
- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION
- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

DC (CJ/SW) 331704/6
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1 **(a)** Express $3y^2 - 12y - 15$ in the form $3(y+a)^2 + b$, where a and b are constants. [2]

(b) Hence find the exact solutions of the equation $3x^4 - 12x^2 - 15 = 0$. [3]

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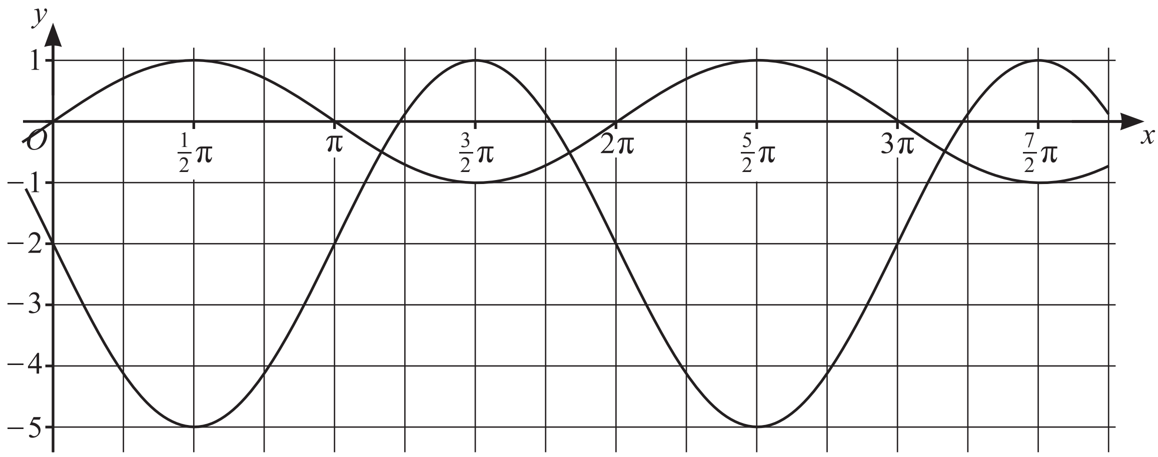

The diagram shows two curves. One curve has equation $y = \sin x$ and the other curve has equation $y = f(x)$.

(a) In order to transform the curve $y = \sin x$ to the curve $y = f(x)$, the curve $y = \sin x$ is first reflected in the x-axis.

Describe fully a sequence of two further transformations which are required. [4]

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(b) Find $f(x)$ in terms of $\sin x$. [2]

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3 The coefficient of x^3 in the expansion of $(3+ax)^6$ is 160.

(a) Find the value of the constant a. [2]

(b) Hence find the coefficient of x^3 in the expansion of $(3+ax)^6(1-2x)$. [3]

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4 The equation of a curve is $y = f(x)$, where $f(x) = (2x-1)\sqrt{3x-2} - 2$. The following points lie on the curve. Non-exact values have been given correct to 5 decimal places.

$A(2, 4)$, $B(2.0001, k)$, $C(2.001, 4.00625)$, $D(2.01, 4.06261)$, $E(2.1, 4.63566)$, $F(3, 11.22876)$

(a) Find the value of k. Give your answer correct to 5 decimal places. [1]

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The table shows the gradients of the chords AB, AC, AD and AF.

Chord	AB	AC	AD	AE	AF
Gradient of chord	6.2501	6.2511	6.2608		7.2288

(b) Find the gradient of the chord AE. Give your answer correct to 4 decimal places. [1]

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(c) Deduce the value of $f'(2)$ using the values in the table. [1]

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5 **(a)** Prove the identity $\frac{\sin^2 x - \cos x - 1}{1 + \cos x} \equiv -\cos x$. [3]

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(b) Hence solve the equation $\frac{\sin^2 x - \cos x - 1}{2 + 2\cos x} = \frac{1}{4}$ for $0° \leqslant x \leqslant 360°$. [3]

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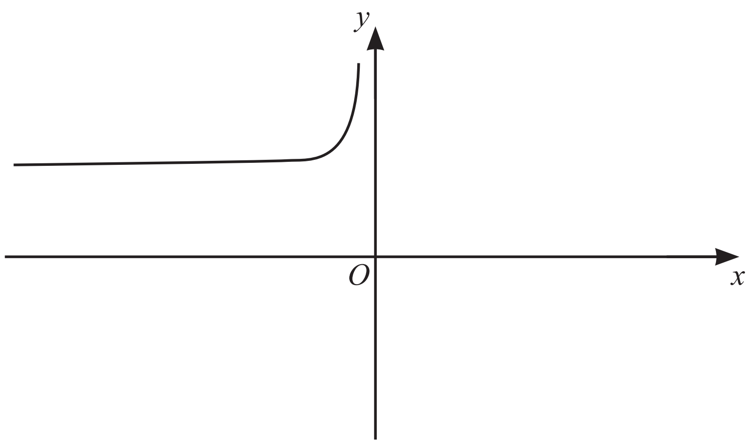

The function f is defined by $f(x) = \frac{2}{x^2} + 4$ for $x < 0$. The diagram shows the graph of $y = f(x)$.

(a) On this diagram, sketch the graph of $y = f^{-1}(x)$. Show any relevant mirror line. [2]

(b) Find an expression for $f^{-1}(x)$. [3]

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(c) Solve the equation $f(x) = 4.5$. [1]

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(d) Explain why the equation $f^{-1}(x) = f(x)$ has no solution. [1]

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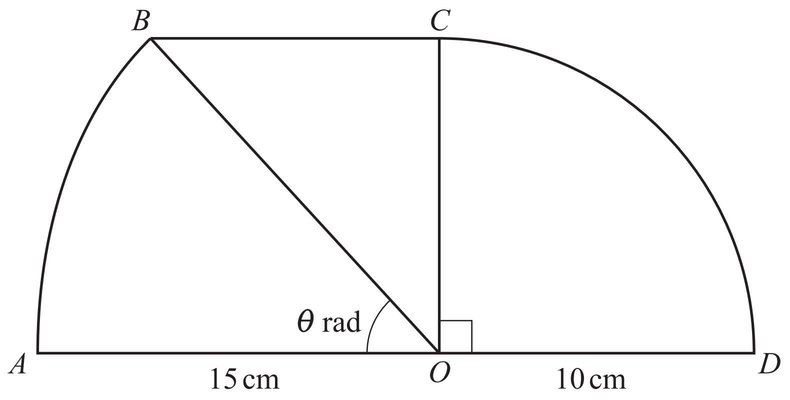

In the diagram, AOD and BC are two parallel straight lines. Arc AB is part of a circle with centre O and radius 15 cm. Angle $BOA = \theta$ radians. Arc CD is part of a circle with centre O and radius 10 cm. Angle $COD = \frac{1}{2}\pi$ radians.

(a) Show that $\theta = 0.7297$, correct to 4 decimal places. [1]

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(b) Find the perimeter and the area of the shape $ABCD$. Give your answers correct to 3 significant figures. [7]

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8 **(a)** The first three terms of an arithmetic progression are 25, $4p-1$ and $13-p$, where p is a constant.

Find the value of the tenth term of the progression. [4]

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(b) The first three terms of a geometric progression are $25, 4q-1$ and $13-q$, where q is a positive constant.

Find the sum to infinity of the progression. [4]

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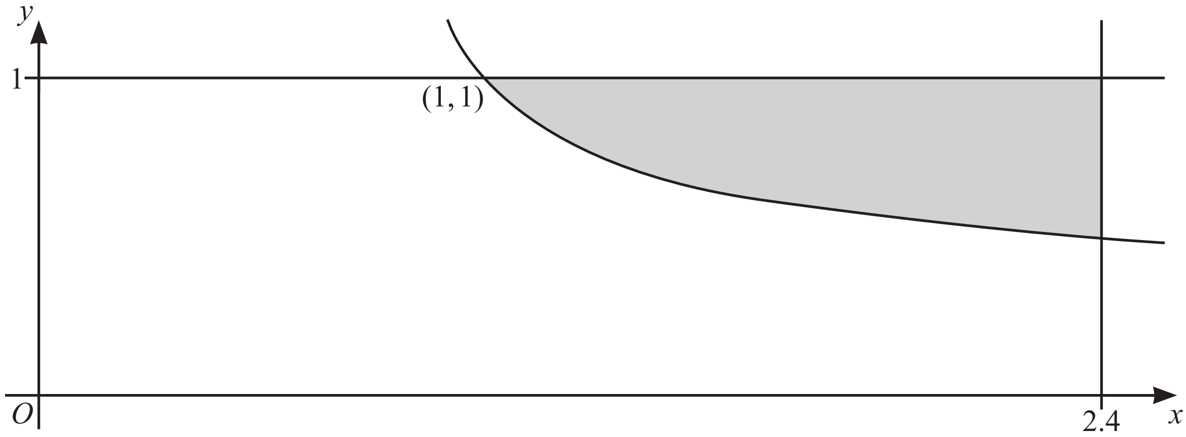

The diagram shows part of the curve with equation $y = \dfrac{1}{(5x-4)^{\frac{1}{3}}}$ and the lines $x = 2.4$ and $y = 1$. The curve intersects the line $y = 1$ at the point $(1, 1)$.

Find the exact volume of the solid generated when the shaded region is rotated through 360° about the x-axis. [6]

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10 The equation of a circle is $(x-3)^2+y^2=18$. The line with equation $y=mx+c$ passes through the point $(0,-9)$ and is a tangent to the circle.

Find the two possible values of m and, for each value of m, find the coordinates of the point at which the tangent touches the circle. [8]

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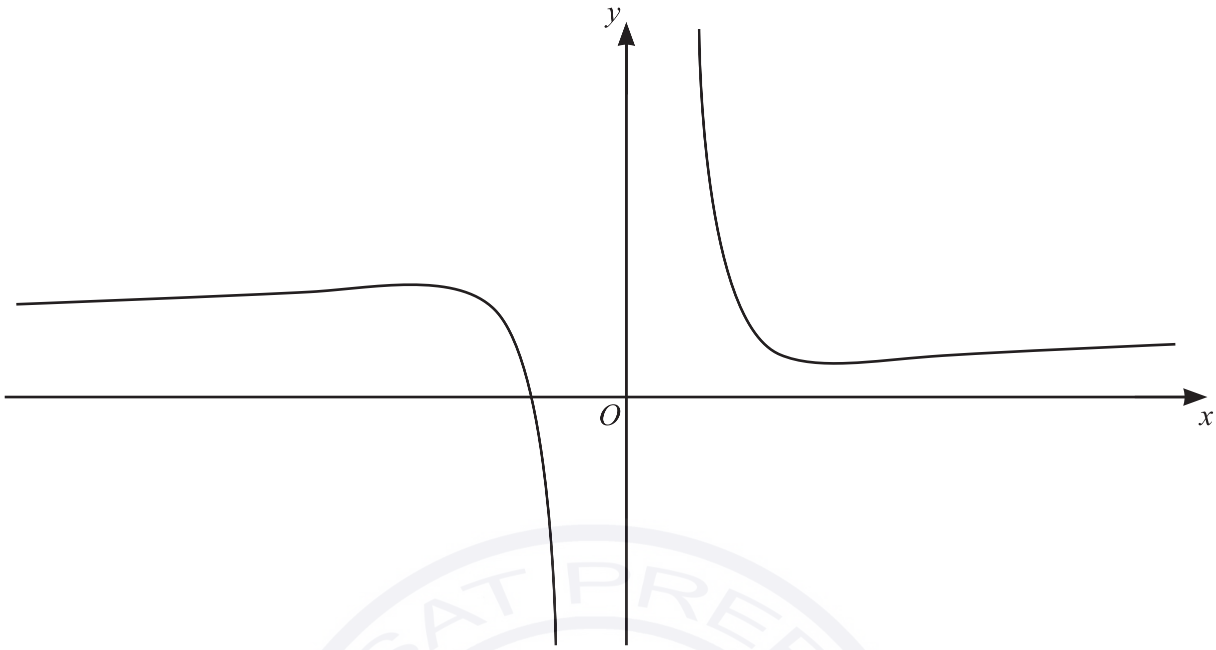

A function is defined by $f(x) = \frac{4}{x^3} - \frac{3}{x} + 2$ for $x \neq 0$. The graph of $y = f(x)$ is shown in the diagram.

(a) Find the set of values of x for which $f(x)$ is decreasing. [5]

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(b) A triangle is bounded by the y-axis, the normal to the curve at the point where $x = 1$ and the tangent to the curve at the point where $x = -1$.

Find the area of the triangle. Give your answer correct to 3 significant figures. [8]

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2894789041

MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **May/June 2024**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS
- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
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- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION
- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

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1 The coefficient of x^2 in the expansion of $(1-4x)^6$ is 12 times the coefficient of x^2 in the expansion of $(2+ax)^5$.

Find the value of the positive constant a. [3]

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2 The curve $y = x^2$ is transformed to the curve $y = 4(x-3)^2 - 8$.

Describe fully a sequence of transformations that have been combined, making clear the order in which the transformations have been applied. [5]

3 **(a)** Show that the equation $\dfrac{7\tan\theta}{\cos\theta} + 12 = 0$ can be expressed as

$$12\sin^2\theta - 7\sin\theta - 12 = 0.$$ [3]

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(b) Hence solve the equation $\dfrac{7\tan\theta}{\cos\theta} + 12 = 0$ for $0^\circ \leqslant \theta \leqslant 360^\circ$. [3]

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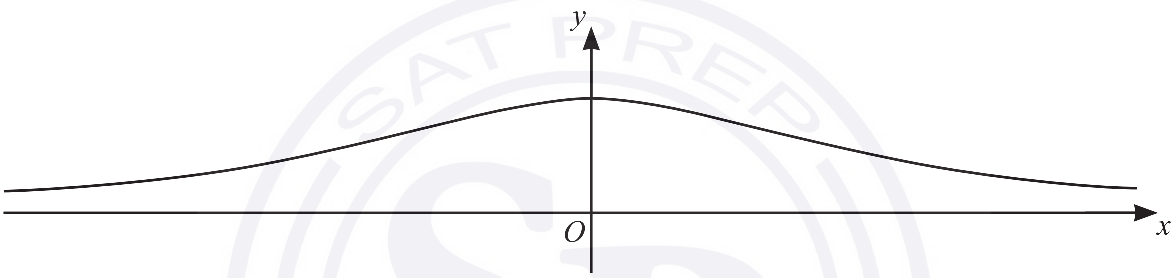
4 The function f is defined as follows:

$$f(x) = \sqrt{x} - 1 \text{ for } x > 1.$$

(a) Find an expression for $f^{-1}(x)$. [1]



The diagram shows the graph of $y = g(x)$ where $g(x) = \dfrac{1}{x^2 + 2}$ for $x \in \mathbb{R}$.

(b) State the range of g and explain whether g^{-1} exists. [2]

The function h is defined by $h(x) = \frac{1}{x^2+2}$ for $x \geqslant 0$.

(c) Solve the equation $hf(x) = f\left(\frac{25}{16}\right)$. Give your answer in the form $a + b\sqrt{c}$, where a, b and c are integers. [4]

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5 The first and second terms of an arithmetic progression are $\tan\theta$ and $\sin\theta$ respectively, where $0 < \theta < \frac{1}{2}\pi$.

(a) Given that $\theta = \frac{1}{4}\pi$, find the exact sum of the first 40 terms of the progression. [4]

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The first and second terms of a geometric progression are $\tan\theta$ and $\sin\theta$ respectively, where $0 < \theta < \frac{1}{2}\pi$.

(b) **(i)** Find the sum to infinity of the progression in terms of θ. [2]

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(ii) Given that $\theta = \frac{1}{3}\pi$, find the sum of the first 10 terms of the progression. Give your answer correct to 3 significant figures. [3]

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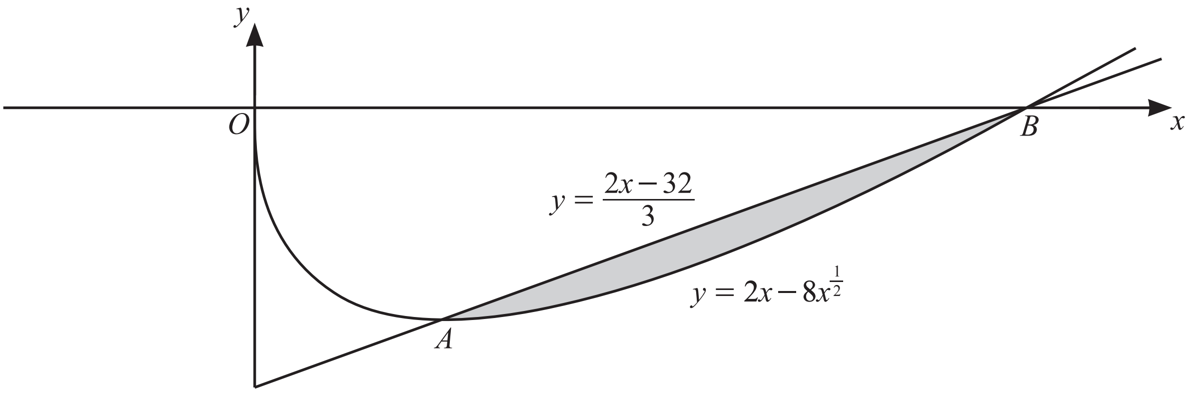
6 The curve with equation $y = 2x - 8x^{\frac{1}{2}}$ has a minimum point at A and intersects the positive x-axis at B.

(a) Find the coordinates of A and B. [4]

(b)



The diagram shows the curve with equation $y = 2x - 8x^{\frac{1}{2}}$ and the line AB. It is given that the equation of AB is $y = \frac{2x-32}{3}$.

Find the area of the shaded region between the curve and the line. [5]

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7 The equation of a circle is $(x-6)^2+(y+a)^2=18$. The line with equation $y=2a-x$ is a tangent to the circle.

(a) Find the two possible values of the constant a. [5]

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(b) For the greater value of a, find the equation of the diameter which is perpendicular to the given tangent. [3]

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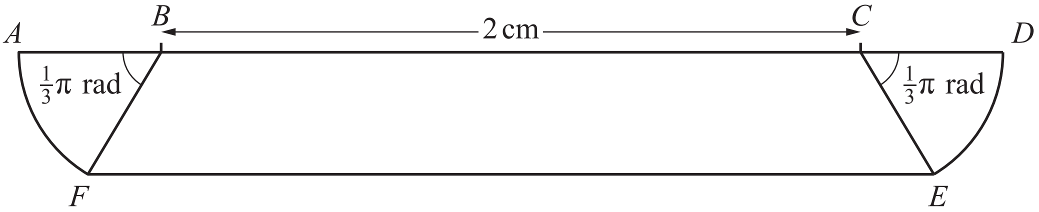

The diagram shows a symmetrical plate *ABCDEF*. The line *ABCD* is straight and the length of *BC* is 2 cm. Each of the two sectors *ABF* and *DCE* is of radius r cm and each of the angles *ABF* and *DCE* is equal to $\frac{1}{3}\pi$ radians.

(a) It is given that $r = 0.4$ cm.

(i) Show that the length $EF = 2.4$ cm. [2]

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(ii) Find the area of the plate. Give your answer correct to 3 significant figures. [4]

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(b) It is given instead that the perimeter of the plate is 6 cm.

Find the value of r. Give your answer correct to 3 significant figures. [4]

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9 A function f is such that $f'(x) = 6(2x-3)^2 - 6x$ for $x \in \mathbb{R}$.

(a) Determine the set of values of x for which $f(x)$ is decreasing. [4]

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(b) Given that $f(1) = -1$, find $f(x)$. [4]

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10 The equation of a curve is $y = (5-2x)^{\frac{3}{2}} + 5$ for $x < \frac{5}{2}$.

(a) A point P is moving along the curve in such a way that the y-coordinate of point P is decreasing at 5 units per second.

Find the rate at which the x-coordinate of point P is increasing when $y = 32$. [4]

(b) Point A on the curve has y-coordinate 32. Point B on the curve is such that the gradient of the curve at B is -3.

Find the equation of the perpendicular bisector of AB. Give your answer in the form $ax + by + c = 0$, where a, b and c are integers. [6]

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MATHEMATICS **9709/13**

Paper 1 Pure Mathematics 1 **May/June 2024**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
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- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
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1 Find the coefficient of x^2 in the expansion of

$$(2-5x)(1+3x)^{10}.$$

[4]

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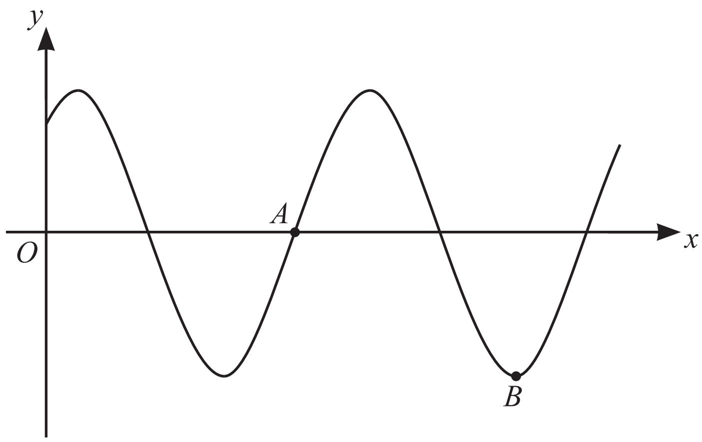
2 **(a)**



The diagram shows the curve $y = k\cos(x - \frac{1}{6}\pi)$ where k is a positive constant and x is measured in radians. The curve crosses the x-axis at point A and B is a minimum point.

Find the coordinates of A and B. [3]

(b) Find the exact value of t that satisfies the equation

$$3\sin^{-1}(3t) + 2\cos^{-1}\left(\tfrac{1}{2}\sqrt{2}\right) = \pi.$$ [2]

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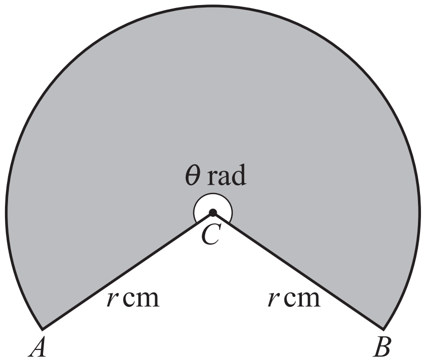

The diagram shows a sector of a circle with centre C. The radii CA and CB each have length r cm and the size of the reflex angle ACB is θ radians. The sector, shaded in the diagram, has a perimeter of 65 cm and an area of 225 cm^2.

(a) Find the values of r and θ. [4]

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(b) Find the area of triangle ACB. [2]

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4 **(a)** Show that the equation $\cos\theta(7\tan\theta - 5\cos\theta) = 1$ can be written in the form $a\sin^2\theta + b\sin\theta + c = 0$, where a, b and c are integers to be found. [3]

(b) Hence solve the equation $\cos 2x(7\tan 2x - 5\cos 2x) = 1$ for $0° < x < 180°$. [3]

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5 The equation of a curve is $y = 2x^2 - \frac{1}{2x} + 3$.

(a) Find the coordinates of the stationary point. [3]

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(b) Determine the nature of the stationary point. [2]

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(c) For positive values of x, determine whether the curve shows a function that is increasing, decreasing or neither. Give a reason for your answer. [2]

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6 A curve passes through the point $\left(\frac{4}{5}, -3\right)$ and is such that $\frac{dy}{dx} = \frac{-20}{(5x-3)^2}$.

(a) Find the equation of the curve. [4]

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(b) The curve is transformed by a stretch in the x-direction with scale factor $\frac{1}{2}$ followed by a translation of $\begin{pmatrix} 2 \\ 10 \end{pmatrix}$.

Find the equation of the new curve. [3]

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7 The first term of an arithmetic progression is 1.5 and the sum of the first ten terms is 127.5 .

(a) Find the common difference. [2]

(b) Find the sum of all the terms of the arithmetic progression whose values are between 25 and 100. [5]

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8 A circle with equation $x^2 + y^2 - 6x + 2y - 15 = 0$ meets the y-axis at the points A and B. The tangents to the circle at A and B meet at the point P.

Find the coordinates of P. [8]

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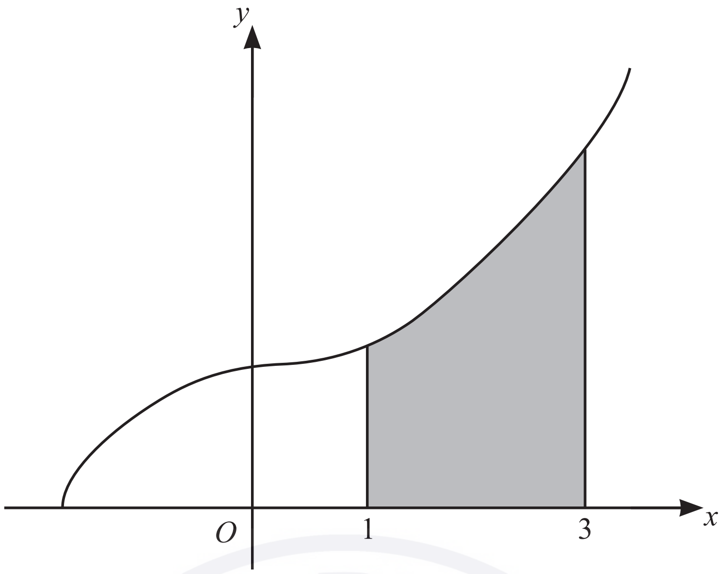

The diagram shows the curve with equation $y = \sqrt{2x^3 + 10}$.

(a) Find the equation of the tangent to the curve at the point where $x = 3$. Give your answer in the form $ax + by + c = 0$ where a, b and c are integers. [5]

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(b) The region shaded in the diagram is enclosed by the curve and the straight lines $x = 1$, $x = 3$ and $y = 0$.

Find the volume of the solid obtained when the shaded region is rotated through 360° about the x-axis. [3]

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10 The geometric progression $a_1, a_2, a_3, \ldots$ has first term 2 and common ratio r where $r > 0$.
It is given that $\frac{9}{2}a_5 + 7a_3 = 8$.

(a) Find the value of r. [3]

(b) Find the sum of the first 20 terms of the geometric progression. Give your answer correct to 4 significant figures. [2]

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(c) Find the sum to infinity of the progression $a_2, a_5, a_8, \dots$. [3]

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11 The function f is defined by $f(x) = 10 + 6x - x^2$ for $x \in \mathbb{R}$.

(a) By completing the square, find the range of f. [3]

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The function g is defined by $g(x) = 4x + k$ for $x \in \mathbb{R}$ where k is a constant.

(b) It is given that the graph of $y = g^{-1}f(x)$ meets the graph of $y = g(x)$ at a single point P.

Determine the coordinates of P. [6]

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CANDIDATE NAME

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MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **February/March 2024**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS
- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
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- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION
- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages.

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1 Find the exact value of $\int_3^\infty \frac{2}{x^2}\,dx$. [3]

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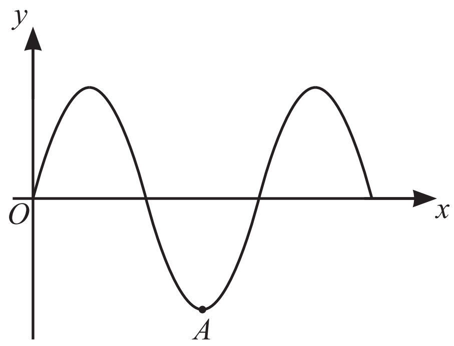
2



The diagram shows part of the curve with equation $y = k\sin\frac{1}{2}x$, where k is a positive constant and x is measured in radians. The curve has a minimum point A.

(a) State the coordinates of A. [1]

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(b) A sequence of transformations is applied to the curve in the following order.

Translation of 2 units in the negative y-direction

Reflection in the x-axis

Find the equation of the new curve and determine the coordinates of the point on the new curve corresponding to A. [3]

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3 A curve is such that $\frac{dy}{dx} = 3(4x+5)^{\frac{1}{2}}$. It is given that the points $(1, 9)$ and $(5, a)$ lie on the curve.

Find the value of a. [5]

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4 **(a)** Prove that $\dfrac{(\sin\theta + \cos\theta)^2 - 1}{\cos^2\theta} \equiv 2\tan\theta$. [3]

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(b) Hence solve the equation $\dfrac{(\sin\theta + \cos\theta)^2 - 1}{\cos^2\theta} = 5\tan^3\theta$ for $-90^\circ < \theta < 90^\circ$. [3]

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5 A curve has the equation $y = \frac{3}{2x^2 - 5}$.

Find the equation of the normal to the curve at the point $(2, 1)$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers. [6]

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6 It is given that the coefficient of x^3 in the expansion of

$$(2+ax)^4(5-ax)$$

is 432.

Find the value of the constant a. [5]

7 The straight line $y = x + 5$ meets the curve $2x^2 + 3y^2 = k$ at a single point P.

(a) Find the value of the constant k. [4]

(b) Find the coordinates of P. [2]

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8 **(a)** An arithmetic progression is such that its first term is 6 and its tenth term is 19.5 .

Find the sum of the first 100 terms of this arithmetic progression. [4]

(b) A geometric progression $a_1, a_2, a_3, \ldots$ is such that $a_1 = 24$ and the common ratio is $\frac{1}{2}$.

The sum to infinity of this geometric progression is denoted by S. The sum to infinity of the even-numbered terms (i.e. $a_2, a_4, a_6, \ldots$) is denoted by S_E.

Find the values of S and S_E. [4]

9 The functions f and g are defined for all real values of x by

$$f(x) = (3x-2)^2 + k \quad \text{and} \quad g(x) = 5x-1,$$

where k is a constant.

(a) Given that the range of the function gf is $gf(x) \geqslant 39$, find the value of k. [4]

(b) For this value of k, determine the range of the function fg. [2]

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(c) The function h is defined for all real values of x and is such that $gh(x) = 35x + 19$.

Find an expression for $g^{-1}(x)$ and hence, or otherwise, find an expression for $h(x)$. [3]

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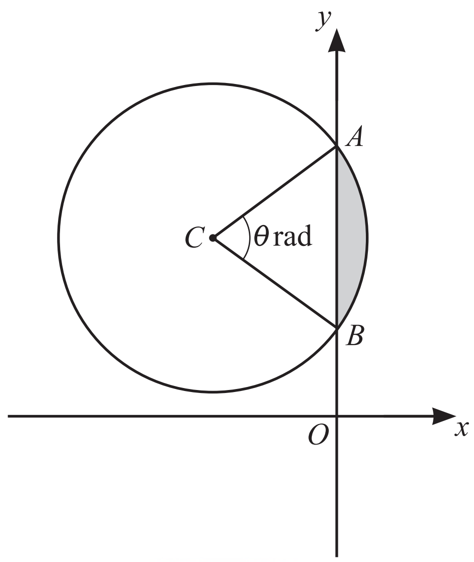

The diagram shows the circle with centre $C(-4, 5)$ and radius $\sqrt{20}$ units. The circle intersects the y-axis at the points A and B. The size of angle ACB is θ radians.

(a) Find the equation of the tangent to the circle at the point $(-6, 9)$. [3]

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(b) Find the equation of the circle in the form $x^2 + y^2 + ax + by + c = 0$. [2]

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(c) Find the value of θ correct to 4 significant figures. [3]

(d) Find the perimeter and area of the segment shaded in the diagram. [4]

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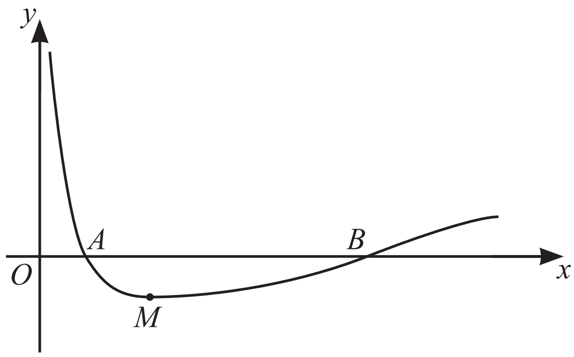

The diagram shows the curve with equation $y = 2x^{-\frac{2}{3}} - 3x^{-\frac{1}{3}} + 1$ for $x > 0$. The curve crosses the x-axis at points A and B and has a minimum point M.

(a) Find the exact coordinates of M. [4]

(b) Find the area of the region bounded by the curve and the line segment AB. [7]

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Additional page

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* 5 2 6 8 1 3 4 6 6 0 *

MATHEMATICS **9709/11**

Paper 1 Pure Mathematics 1 **October/November 2023**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
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- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
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INFORMATION

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- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

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1 **(a)** Expand $(1 + 3x)^6$ in ascending powers of x up to, and including, the term in x^2. [2]

(b) Hence find the coefficient of x^2 in the expansion of $(1 - 7x + x^2)(1 + 3x)^6$. [2]

2 A line has equation $y = 2cx + 3$ and a curve has equation $y = cx^2 + 3x - c$, where c is a constant.

Showing all necessary working, determine which of the following statements is correct.

A The line and curve intersect only for a particular set of values of c.

B The line and curve intersect for all values of c.

C The line and curve do not intersect for any values of c. [4]

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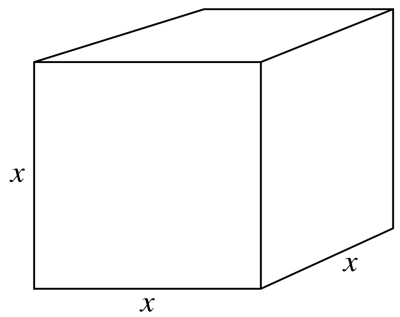

The diagram shows a cubical closed container made of a thin elastic material which is filled with water and frozen. During the freezing process the length, x cm, of each edge of the container increases at the constant rate of 0.01 cm per minute. The volume of the container at time t minutes is V cm^3.

Find the rate of increase of V when $x = 20$. [3]

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4 The transformation R denotes a reflection in the x-axis and the transformation T denotes a translation of $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$.

(a) Find the equation, $y = g(x)$, of the curve with equation $y = x^2$ after it has been transformed by the sequence of transformations R followed by T. [2]

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(b) Find the equation, $y = h(x)$, of the curve with equation $y = x^2$ after it has been transformed by the sequence of transformations T followed by R. [2]

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(c) State fully the transformation that maps the curve $y = g(x)$ onto the curve $y = h(x)$. [2]

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5 **(a)** Show that the equation

$$4\sin x + \frac{5}{\tan x} + \frac{2}{\sin x} = 0$$

may be expressed in the form $a\cos^2 x + b\cos x + c = 0$, where a, b and c are integers to be found. [3]

(b) Hence solve the equation $4\sin x + \frac{5}{\tan x} + \frac{2}{\sin x} = 0$ for $0° \leqslant x \leqslant 360°$. [3]

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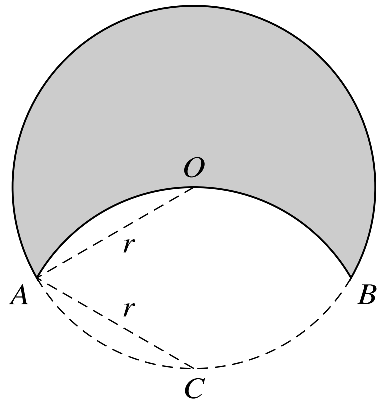

The diagram shows a motif formed by the major arc AB of a circle with radius r and centre O, and the minor arc AOB of a circle, also with radius r but with centre C. The point C lies on the circle with centre O.

(a) Given that angle $ACB = k\pi$ radians, state the value of the fraction k. [1]

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(b) State the perimeter of the shaded motif in terms of π and r. [1]

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(c) Find the area of the shaded motif, giving your answer in terms of π, r and $\sqrt{3}$. [5]

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7 The sum of the first two terms of a geometric progression is 15 and the sum to infinity is $\frac{125}{7}$. The common ratio of the progression is negative.

Find the third term of the progression. [7]

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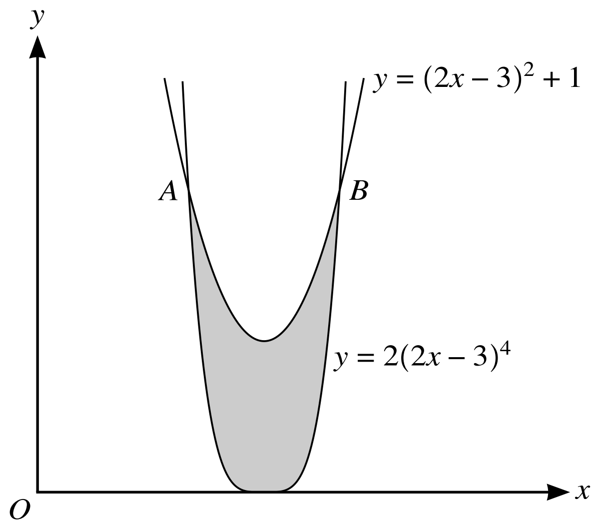

The diagram shows the curves with equations $y = 2(2x - 3)^4$ and $y = (2x - 3)^2 + 1$ meeting at points A and B.

(a) By using the substitution $u = 2x - 3$ find, by calculation, the coordinates of A and B. [4]

(b) Find the exact area of the shaded region. [5]

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9 **(a)** Express $4x^2 - 12x + 13$ in the form $(2x + a)^2 + b$, where a and b are constants. [2]

The function f is defined by $f(x) = 4x^2 - 12x + 13$ for $p < x < q$, where p and q are constants. The function g is defined by $g(x) = 3x + 1$ for $x < 8$.

(b) Given that it is possible to form the composite function gf, find the least possible value of p and the greatest possible value of q. [3]

(c) Find an expression for $gf(x)$. [1]

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The function h is defined by $h(x) = 4x^2 - 12x + 13$ for $x < 0$.

(d) Find an expression for $h^{-1}(x)$. [3]

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10 A curve has a stationary point at $(2, -10)$ and is such that $\frac{d^2y}{dx^2} = 6x$.

(a) Find $\frac{dy}{dx}$. [3]

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(b) Find the equation of the curve. [3]

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(c) Find the coordinates of the other stationary point and determine its nature. [3]

(d) Find the equation of the tangent to the curve at the point where the curve crosses the y-axis. [2]

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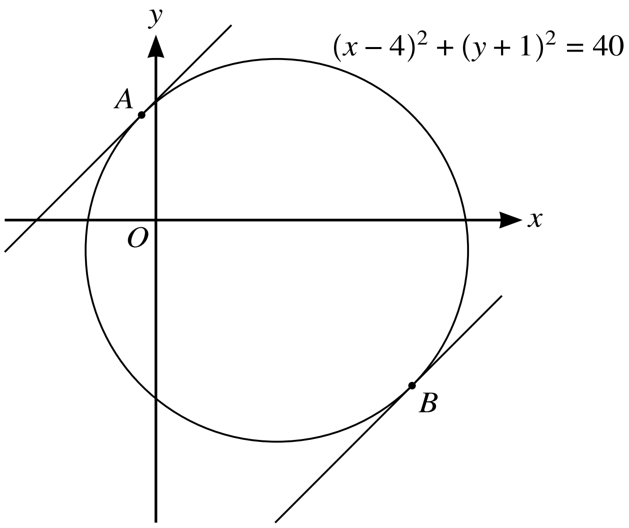

The diagram shows the circle with equation $(x-4)^2 + (y+1)^2 = 40$. Parallel tangents, each with gradient 1, touch the circle at points A and B.

(a) Find the equation of the line AB, giving the answer in the form $y = mx + c$. [3]

(b) Find the coordinates of A, giving each coordinate in surd form. [4]

(c) Find the equation of the tangent at A, giving the answer in the form $y = mx + c$, where c is in surd form. [2]

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MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **October/November 2023**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

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INFORMATION

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[Turn over

1 The coefficient of x^3 in the expansion of $(3 + 2ax)^5$ is six times the coefficient of x^2 in the expansion of $(2 + ax)^6$.

Find the value of the constant a. [4]

2 Find the exact solution of the equation

$$\tfrac{1}{6}\pi + \tan^{-1}(4x) = -\cos^{-1}\left(\tfrac{1}{2}\sqrt{3}\right).$$ [2]

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3 The equation of a curve is such that $\frac{dy}{dx} = \frac{1}{2}x + \frac{72}{x^4}$. The curve passes through the point $P(2, 8)$.

(a) Find the equation of the normal to the curve at P. [2]

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(b) Find the equation of the curve. [4]

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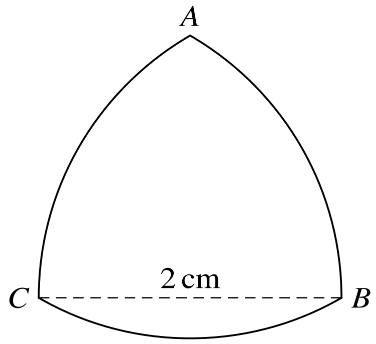

The diagram shows the shape of a coin. The three arcs AB, BC and CA are parts of circles with centres C, A and B respectively. ABC is an equilateral triangle with sides of length 2 cm.

(a) Find the perimeter of the coin. [2]

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(b) Find the area of the face ABC of the coin, giving the answer in terms of π and $\sqrt{3}$. [4]

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5 The first, second and third terms of a geometric progression are $\sin\theta$, $\cos\theta$ and $2 - \sin\theta$ respectively, where θ radians is an acute angle.

(a) Find the value of θ. [3]

(b) Using this value of θ, find the sum of the first 10 terms of the progression. Give the answer in the form $\frac{b}{\sqrt{c}-1}$, where b and c are integers to be found. [3]

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6 The equation of a curve is $y = x^2 - 8x + 5$.

(a) Find the coordinates of the minimum point of the curve. [2]

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The curve is stretched by a factor of 2 parallel to the y-axis and then translated by $\begin{pmatrix} 4 \\ 1 \end{pmatrix}$.

(b) Find the coordinates of the minimum point of the transformed curve. [2]

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(c) Find the equation of the transformed curve. Give the answer in the form $y = ax^2 + bx + c$, where a, b and c are integers to be found. [4]

7 **(a)** Verify the identity $(2x-1)(4x^2+2x-1) \equiv 8x^3-4x+1$. [1]

(b) Prove the identity $\dfrac{\tan^2\theta+1}{\tan^2\theta-1} \equiv \dfrac{1}{1-2\cos^2\theta}$. [3]

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(c) Using the results of **(a)** and **(b)**, solve the equation

$$\frac{\tan^2\theta + 1}{\tan^2\theta - 1} = 4\cos\theta,$$

for $0° \leqslant \theta \leqslant 180°$. [5]

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8 Functions f and g are defined by

$$f(x) = (x+a)^2 - a \text{ for } x \leqslant -a,$$
$$g(x) = 2x - 1 \text{ for } x \in \mathbb{R},$$

where a is a positive constant.

(a) Find an expression for $f^{-1}(x)$. [3]

(b) **(i)** State the domain of the function f^{-1}. [1]

(ii) State the range of the function f^{-1}. [1]

(c) Given that $a = \frac{7}{2}$, solve the equation $gf(x) = 0$. [3]

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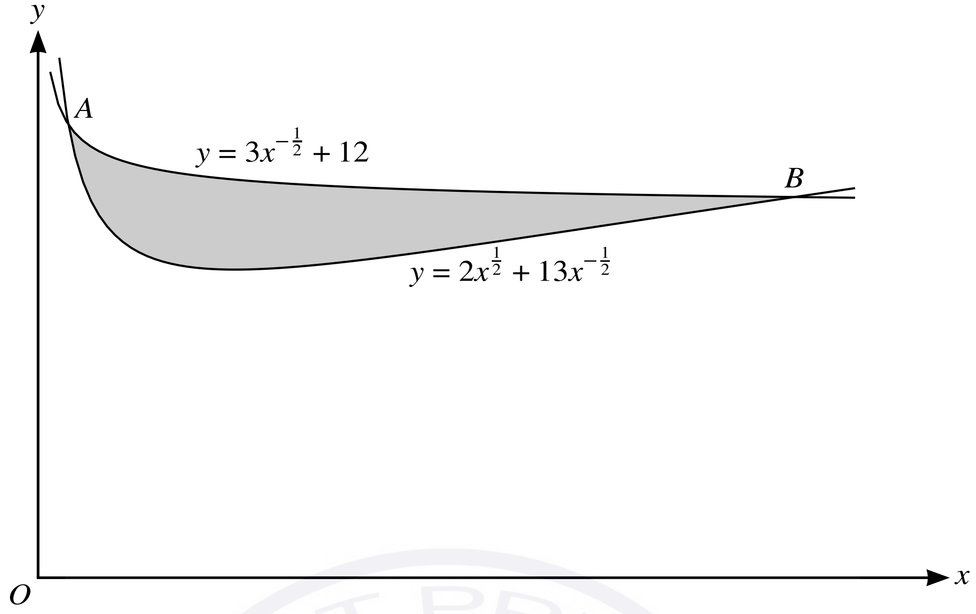

The diagram shows curves with equations $y = 2x^{\frac{1}{2}} + 13x^{-\frac{1}{2}}$ and $y = 3x^{-\frac{1}{2}} + 12$. The curves intersect at points A and B.

(a) Find the coordinates of A and B. [4]

(b) Hence find the area of the shaded region. [5]

10 The equation of a curve is $y = f(x)$, where $f(x) = (4x - 3)^{\frac{5}{3}} - \frac{20}{3}x$.

(a) Find the x-coordinates of the stationary points of the curve and determine their nature. [6]

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(b) State the set of values for which the function f is increasing. [1]

11 The coordinates of points A, B and C are $(6, 4)$, $(p, 7)$ and $(14, 18)$ respectively, where p is a constant. The line AB is perpendicular to the line BC.

(a) Given that $p < 10$, find the value of p. [4]

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A circle passes through the points A, B and C.

(b) Find the equation of the circle. [3]

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(c) Find the equation of the tangent to the circle at C, giving the answer in the form $dx + ey + f = 0$, where d, e and f are integers. [3]

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CANDIDATE NAME			
CENTRE NUMBER		CANDIDATE NUMBER	

2108969358

MATHEMATICS **9709/13**

Paper 1 Pure Mathematics 1 **October/November 2023**

1 hour 50 minutes

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1 A curve is such that its gradient at a point (x, y) is given by $\frac{dy}{dx} = x - 3x^{-\frac{1}{2}}$. It is given that the curve passes through the point $(4, 1)$.

Find the equation of the curve. [4]

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2 The circle with equation $(x-3)^2 + (y-5)^2 = 40$ intersects the y-axis at points A and B.

(a) Find the y-coordinates of A and B, expressing your answers in terms of surds. [2]

(b) Find the equation of the circle which has AB as its diameter. [2]

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3 **(a)** Show that the equation

$$5\cos\theta - \sin\theta\tan\theta + 1 = 0$$

may be expressed in the form $a\cos^2\theta + b\cos\theta + c = 0$, where a, b and c are constants to be found. [3]

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(b) Hence solve the equation $5\cos\theta - \sin\theta\tan\theta + 1 = 0$ for $0 < \theta < 2\pi$. [4]

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4 **(a)** Expand the following in ascending powers of x up to and including the term in x^2.

(i) $(1 + 2x)^5$. [1]

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(ii) $(1 - ax)^6$, where a is a constant. [2]

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In the expansion of $(1 + 2x)^5(1 - ax)^6$, the coefficient of x^2 is -5.

(b) Find the possible values of a. [4]

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5 The first, second and third terms of a geometric progression are $2p + 6$, $5p$ and $8p + 2$ respectively.

(a) Find the possible values of the constant p. [3]

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(b) One of the values of p found in **(a)** is a negative fraction.

Use this value of p to find the sum to infinity of this progression. [4]

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6 A line has equation $y = 6x - c$ and a curve has equation $y = cx^2 + 2x - 3$, where c is a constant. The line is a tangent to the curve at point P.

Find the possible values of c and the corresponding coordinates of P. [7]

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7 The function f is defined by $f(x) = 1 + \frac{3}{x-2}$ for $x > 2$.

(a) State the range of f. [1]

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(b) Obtain an expression for $f^{-1}(x)$ and state the domain of f^{-1}. [4]

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The function g is defined by $g(x) = 2x - 2$ for $x > 0$.

(c) Obtain a simplified expression for $gf(x)$. [2]

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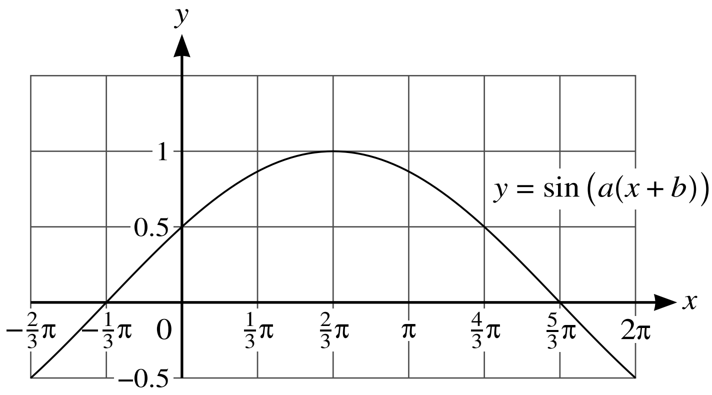

The diagram shows part of the graph of $y = \sin\big(a(x+b)\big)$, where a and b are positive constants.

(a) State the value of a and one possible value of b. [2]

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Another curve, with equation $y = \mathrm{f}(x)$, has a single stationary point at the point (p, q), where p and q are constants. This curve is transformed to a curve with equation

$$y = -3\mathrm{f}\big(\tfrac{1}{4}(x+8)\big).$$

(b) For the transformed curve, find the coordinates of the stationary point, giving your answer in terms of p and q. [3]

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9 A curve has equation $y = 2x^{\frac{1}{2}} - 1$.

(a) Find the equation of the normal to the curve at the point $A\,(4,\ 3)$, giving your answer in the form $y = mx + c$. [3]

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A point is moving along the curve $y = 2x^{\frac{1}{2}} - 1$ in such a way that at A the rate of increase of the x-coordinate is $3\,\text{cm}\,\text{s}^{-1}$.

(b) Find the rate of increase of the y-coordinate at A. [2]

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At A the moving point suddenly changes direction and speed, and moves down the normal in such a way that the rate of decrease of the y-coordinate is constant at $5\,\text{cm}\,\text{s}^{-1}$.

(c) As the point moves down the normal, find the rate of change of its x-coordinate. [3]

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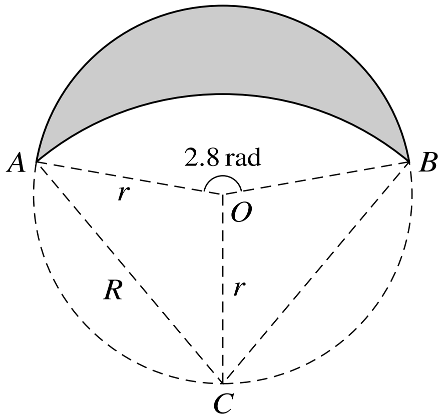

The diagram shows points A, B and C lying on a circle with centre O and radius r. Angle AOB is 2.8 radians. The shaded region is bounded by two arcs. The upper arc is part of the circle with centre O and radius r. The lower arc is part of a circle with centre C and radius R.

(a) State the size of angle ACO in radians. [1]

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(b) Find R in terms of r. [1]

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(c) Find the area of the shaded region in terms of r. [7]

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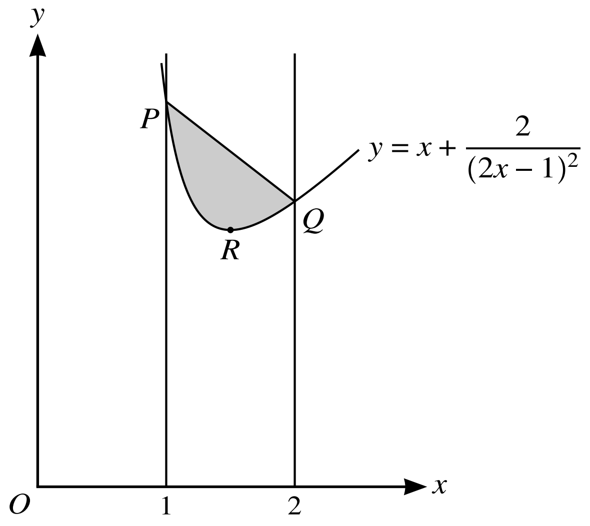

The diagram shows part of the curve with equation $y = x + \frac{2}{(2x-1)^2}$. The lines $x = 1$ and $x = 2$ intersect the curve at P and Q respectively and R is the stationary point on the curve.

(a) Verify that the x-coordinate of R is $\frac{3}{2}$ and find the y-coordinate of R. [4]

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(b) Find the exact value of the area of the shaded region. [6]

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CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

3967186601

MATHEMATICS **9709/11**

Paper 1 Pure Mathematics 1 **May/June 2023**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

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1 Solve the equation $4\sin\theta + \tan\theta = 0$ for $0° < \theta < 180°$. [3]

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2 **(a)** Find the first three terms in the expansion, in ascending powers of x, of $(2+3x)^4$. [2]

(b) Find the first three terms in the expansion, in ascending powers of x, of $(1-2x)^5$. [2]

(c) Hence find the coefficient of x^2 in the expansion of $(2+3x)^4(1-2x)^5$. [2]

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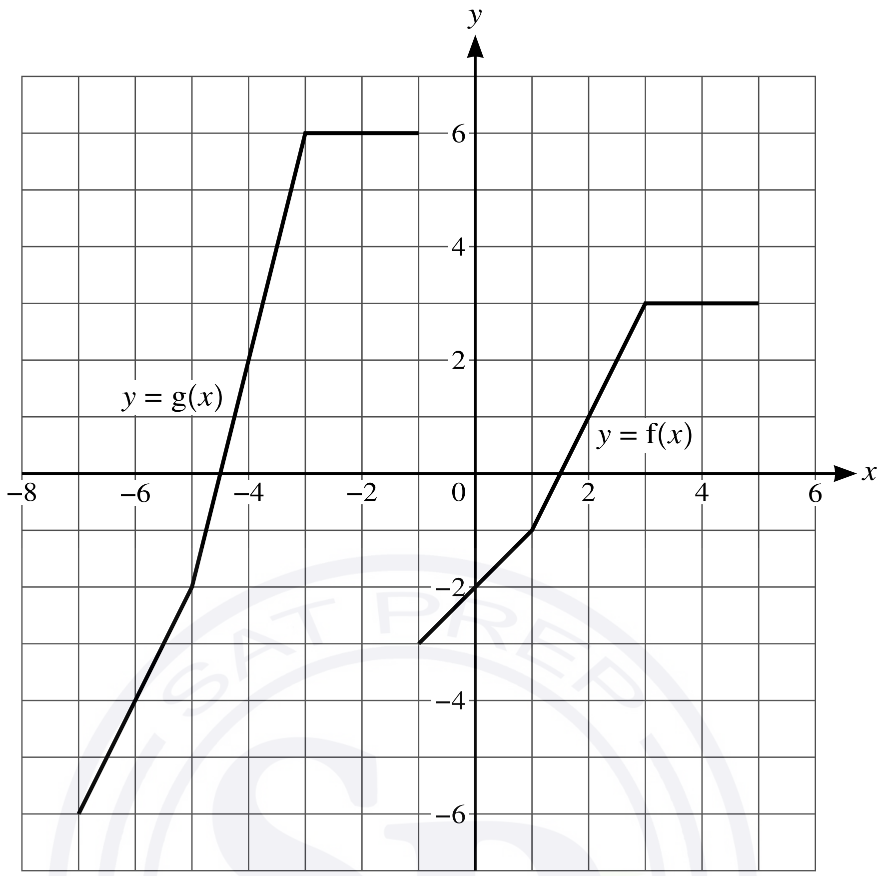

The diagram shows graphs with equations $y = \mathrm{f}(x)$ and $y = \mathrm{g}(x)$.

Describe fully a sequence of two transformations which transforms the graph of $y = \mathrm{f}(x)$ to $y = \mathrm{g}(x)$. [4]

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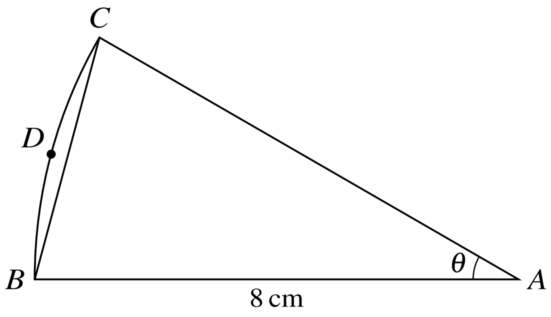

The diagram shows a sector ABC of a circle with centre A and radius 8 cm. The area of the sector is $\frac{16}{3}\pi\,\text{cm}^2$. The point D lies on the arc BC.

Find the perimeter of the segment BCD. [4]

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5 The line with equation $y = kx - k$, where k is a positive constant, is a tangent to the curve with equation $y = -\frac{1}{2x}$.

Find, in either order, the value of k and the coordinates of the point where the tangent meets the curve. [5]

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6 The first three terms of an arithmetic progression are $\frac{p^2}{6}$, $2p - 6$ and p.

(a) Given that the common difference of the progression is not zero, find the value of p. [3]

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(b) Using this value, find the sum to infinity of the geometric progression with first two terms $\frac{p^2}{6}$ and $2p - 6$. [2]

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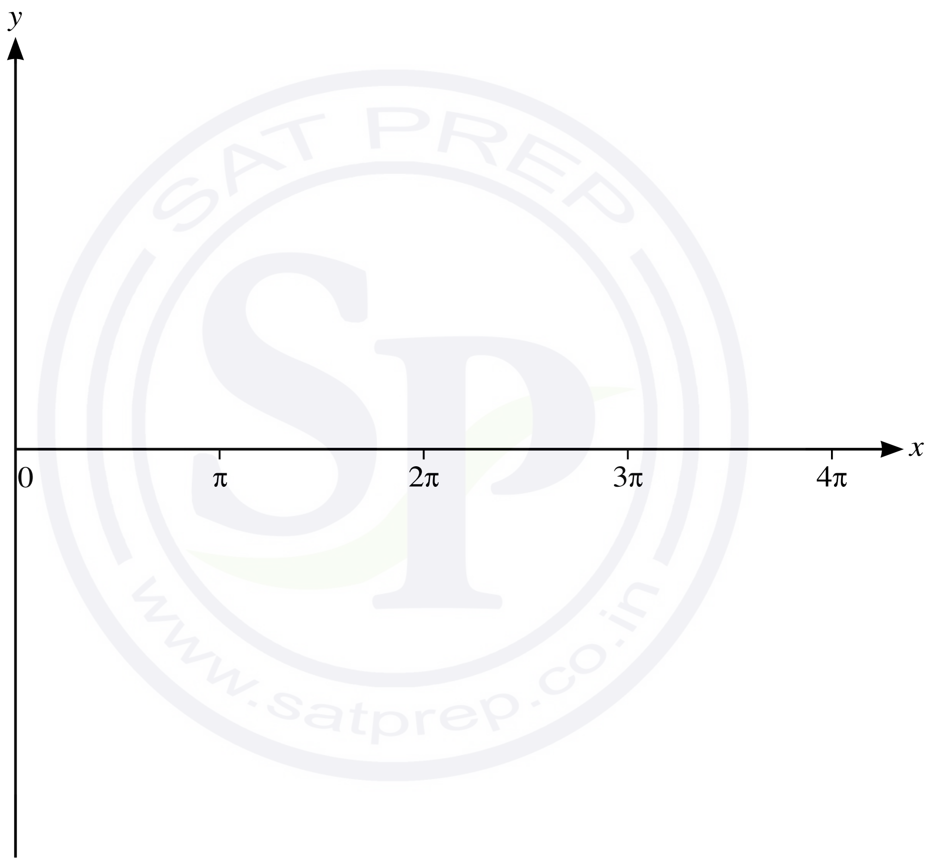
7 A curve has equation $y = 2 + 3\sin\frac{1}{2}x$ for $0 \leqslant x \leqslant 4\pi$.

(a) State greatest and least values of y. [2]

(b) Sketch the curve. [2]



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(c) State the number of solutions of the equation

$$2 + 3\sin\tfrac{1}{2}x = 5 - 2x$$

for $0 \leqslant x \leqslant 4\pi$. [1]

8 The functions f and g are defined as follows, where a and b are constants.

$$f(x) = 1 + \frac{2a}{x-a} \text{ for } x > a$$

$$g(x) = bx - 2 \text{ for } x \in \mathbb{R}$$

(a) Given that $f(7) = \frac{5}{2}$ and $gf(5) = 4$, find the values of a and b. [4]

For the rest of this question, you should use the value of a which you found in **(a)**.

(b) Find the domain of f^{-1}. [1]

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(c) Find an expression for $f^{-1}(x)$. [3]

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[Turn over

9 Water is poured into a tank at a constant rate of $500\,\text{cm}^3$ per second. The depth of water in the tank, t seconds after filling starts, is h cm. When the depth of water in the tank is h cm, the volume, $V\,\text{cm}^3$, of water in the tank is given by the formula $V = \frac{4}{3}(25+h)^3 - \frac{62500}{3}$.

(a) Find the rate at which h is increasing at the instant when $h = 10$ cm. [3]

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(b) At another instant, the rate at which h is increasing is 0.075 cm per second.

Find the value of V at this instant. [3]

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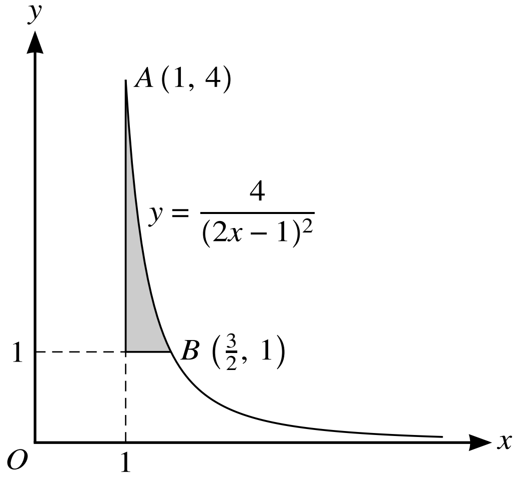

The diagram shows part of the curve with equation $y = \dfrac{4}{(2x-1)^2}$ and parts of the lines $x = 1$ and $y = 1$. The curve passes through the points $A\,(1, 4)$ and $B, \left(\frac{3}{2}, 1\right)$.

(a) Find the exact volume generated when the shaded region is rotated through 360° about the x-axis. [5]

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(b) A triangle is formed from the tangent to the curve at B, the normal to the curve at B and the x-axis.

Find the area of this triangle. [6]

11 The equation of a curve is such that $\frac{dy}{dx} = 6x^2 - 30x + 6a$, where a is a positive constant. The curve has a stationary point at $(a, -15)$.

(a) Find the value of a. [2]

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(b) Determine the nature of this stationary point. [2]

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(c) Find the equation of the curve. [3]

(d) Find the coordinates of any other stationary points on the curve. [2]

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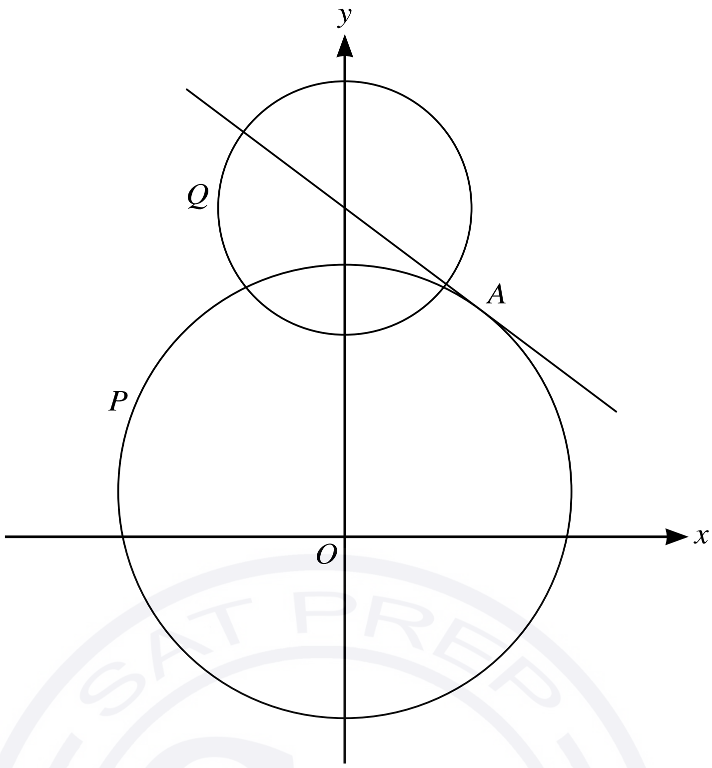

The diagram shows a circle P with centre (0, 2) and radius 10 and the tangent to the circle at the point A with coordinates (6, 10). It also shows a second circle Q with centre at the point where this tangent meets the y-axis and with radius $\frac{5}{2}\sqrt{5}$.

(a) Write down the equation of circle P. [1]

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(b) Find the equation of the tangent to the circle P at A. [2]

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(c) Find the equation of circle Q and hence verify that the y-coordinates of both of the points of intersection of the two circles are 11. [3]

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(d) Find the coordinates of the points of intersection of the tangent and circle Q, giving the answers in surd form. [3]

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CANDIDATE NAME			
CENTRE NUMBER		CANDIDATE NUMBER	

2330973605

MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **May/June 2023**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
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- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

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1 The equation of a curve is such that $\frac{dy}{dx} = \frac{4}{(x-3)^3}$ for $x > 3$. The curve passes through the point (4, 5).

Find the equation of the curve. [3]

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2 The coefficient of x^4 in the expansion of $(x + a)^6$ is p and the coefficient of x^2 in the expansion of $(ax + 3)^4$ is q. It is given that $p + q = 276$.

Find the possible values of the constant a. [4]

3 **(a)** Express $4x^2 - 24x + p$ in the form $a(x + b)^2 + c$, where a and b are integers and c is to be given in terms of the constant p. [2]

(b) Hence or otherwise find the set of values of p for which the equation $4x^2 - 24x + p = 0$ has no real roots. [1]

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4 Solve the equation $8x^6 + 215x^3 - 27 = 0$. [3]

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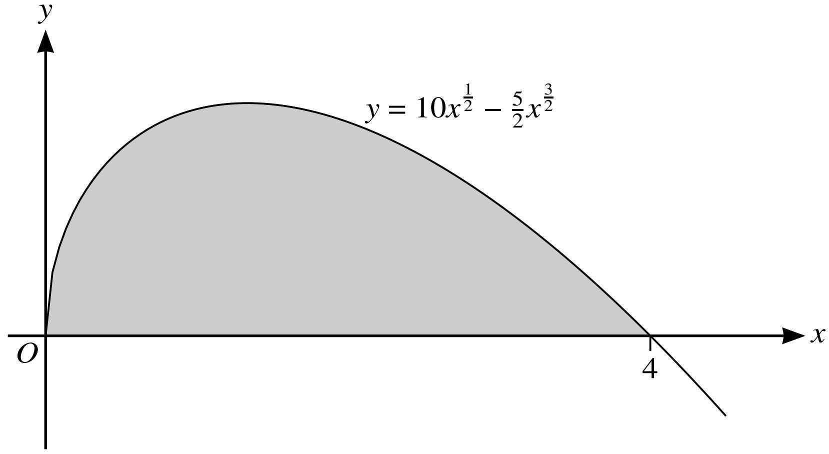

The diagram shows the curve with equation $y = 10x^{\frac{1}{2}} - \frac{5}{2}x^{\frac{3}{2}}$ for $x > 0$. The curve meets the x-axis at the points (0, 0) and (4, 0).

Find the area of the shaded region. [4]

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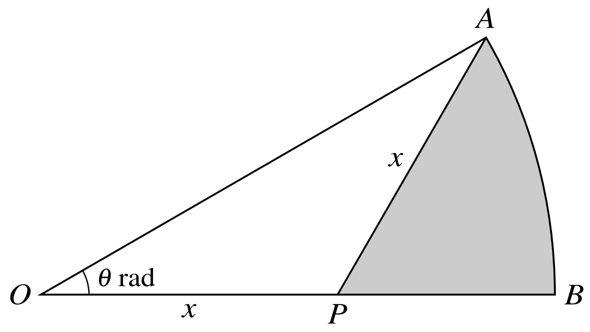

The diagram shows a sector OAB of a circle with centre O. Angle $AOB = \theta$ radians and $OP = AP = x$.

(a) Show that the arc length AB is $2x\theta\cos\theta$. [2]

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(b) Find the area of the shaded region APB in terms of x and θ. [4]

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7 **(a)** **(i)** By first expanding $(\cos\theta + \sin\theta)^2$, find the three solutions of the equation

$$(\cos\theta + \sin\theta)^2 = 1$$

for $0 \leqslant \theta \leqslant \pi$. [3]

(ii) Hence verify that the only solutions of the equation $\cos\theta + \sin\theta = 1$ for $0 \leqslant \theta \leqslant \pi$ are 0 and $\frac{1}{2}\pi$. [2]

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(b) Prove the identity $\dfrac{\sin\theta}{\cos\theta + \sin\theta} + \dfrac{1 - \cos\theta}{\cos\theta - \sin\theta} \equiv \dfrac{\cos\theta + \sin\theta - 1}{1 - 2\sin^2\theta}$. [3]

(c) Using the results of **(a)(ii)** and **(b)**, solve the equation

$$\frac{\sin\theta}{\cos\theta + \sin\theta} + \frac{1 - \cos\theta}{\cos\theta - \sin\theta} = 2(\cos\theta + \sin\theta - 1)$$

for $0 \leqslant \theta \leqslant \pi$. [3]

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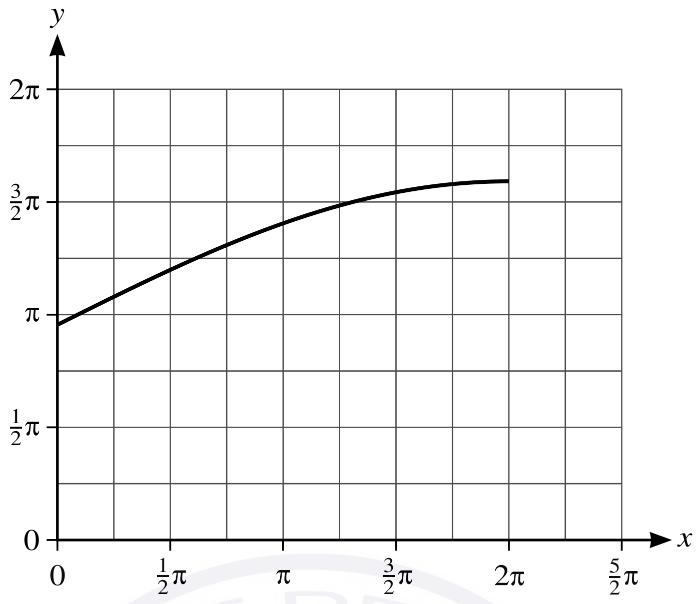

The diagram shows the graph of $y = f(x)$ where the function f is defined by

$$f(x) = 3 + 2\sin\frac{1}{4}x \text{ for } 0 \leqslant x \leqslant 2\pi.$$

(a) On the diagram above, sketch the graph of $y = f^{-1}(x)$. [2]

(b) Find an expression for $f^{-1}(x)$. [2]

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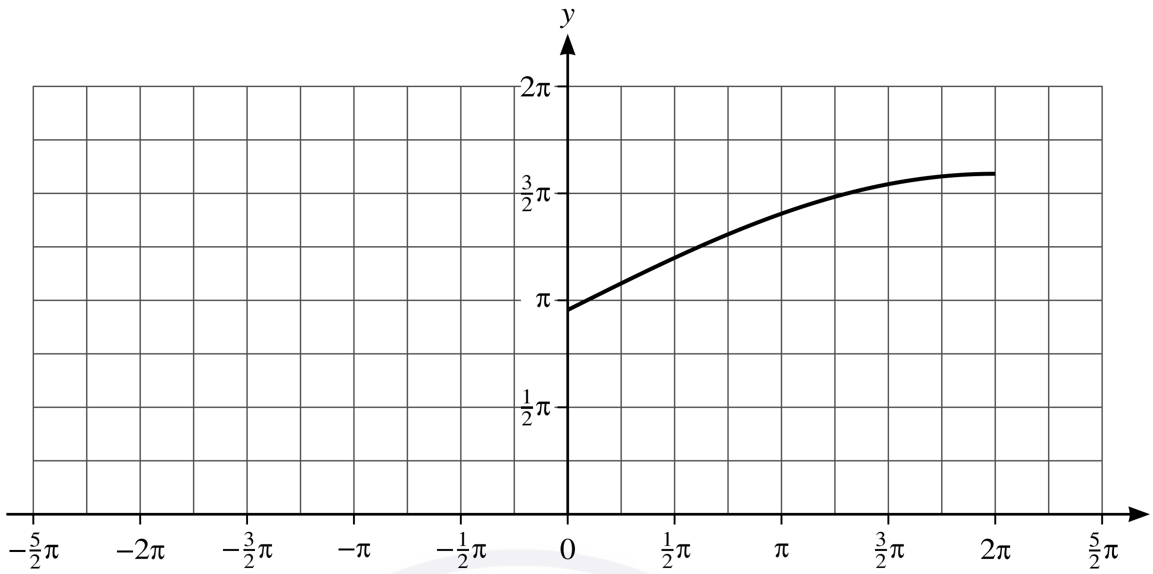

The diagram above shows part of the graph of the function $g(x) = 3 + 2\sin\frac{1}{4}x$ for $-2\pi \leqslant x \leqslant 2\pi$.

Complete the sketch of the graph of $g(x)$ on the diagram above and hence explain whether the function g has an inverse. [2]

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(d) Describe fully a sequence of three transformations which can be combined to transform the graph of $y = \sin x$ for $0 \leqslant x \leqslant \frac{1}{2}\pi$ to the graph of $y = f(x)$, making clear the order in which the transformations are applied. [6]

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9 The second term of a geometric progression is 16 and the sum to infinity is 100.

(a) Find the two possible values of the first term. [4]

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(b) Show that the nth term of one of the two possible geometric progressions is equal to 4^{n-2} multiplied by the nth term of the other geometric progression. [4]

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10 The equation of a circle is $(x - a)^2 + (y - 3)^2 = 20$. The line $y = \frac{1}{2}x + 6$ is a tangent to the circle at the point P.

(a) Show that one possible value of a is 4 and find the other possible value. [5]

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(b) For $a = 4$, find the equation of the normal to the circle at P. [4]

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(c) For $a = 4$, find the equations of the two tangents to the circle which are parallel to the normal found in **(b)**. [4]

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11 The equation of a curve is

$$y = k\sqrt{4x+1} - x + 5,$$

where k is a positive constant.

(a) Find $\dfrac{dy}{dx}$. [2]

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(b) Find the x-coordinate of the stationary point in terms of k. [2]

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(c) Given that $k = 10.5$, find the equation of the normal to the curve at the point where the tangent to the curve makes an angle of $\tan^{-1}(2)$ with the positive x-axis. [4]

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CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

2780821796

MATHEMATICS **9709/13**

Paper 1 Pure Mathematics 1 **May/June 2023**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
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- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

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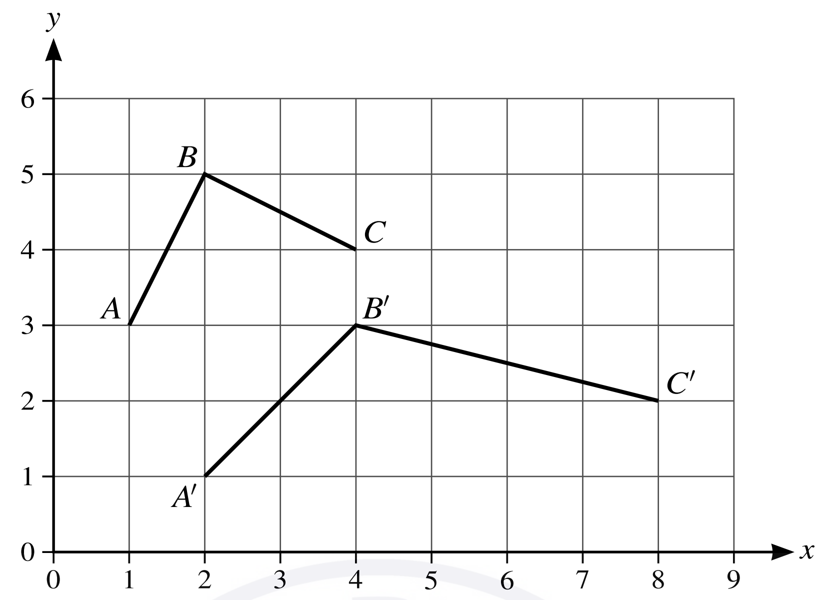

The diagram shows the graph of $y = f(x)$, which consists of the two straight lines AB and BC. The lines $A'B'$ and $B'C'$ form the graph of $y = g(x)$, which is the result of applying a sequence of two transformations, in either order, to $y = f(x)$.

State fully the two transformations. [4]

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2 The function f is defined for $x \in \mathbb{R}$ by $f(x) = x^2 - 6x + c$, where c is a constant. It is given that $f(x) > 2$ for all values of x.

Find the set of possible values of c. [4]

3 **(a)** Give the complete expansion of $\left(x + \frac{2}{x}\right)^5$. [2]

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(b) In the expansion of $(a + bx^2)\left(x + \frac{2}{x}\right)^5$, the coefficient of x is zero and the coefficient of $\frac{1}{x}$ is 80.

Find the values of the constants a and b. [4]

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4 **(a)** Show that the equation

$$3\tan^2 x - 3\sin^2 x - 4 = 0$$

may be expressed in the form $a\cos^4 x + b\cos^2 x + c = 0$, where a, b and c are constants to be found. [3]

(b) Hence solve the equation $3\tan^2 x - 3\sin^2 x - 4 = 0$ for $0° \leqslant x \leqslant 180°$. [4]

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5 A circle has equation $(x-1)^2 + (y+4)^2 = 40$. A line with equation $y = x - 9$ intersects the circle at points A and B.

(a) Find the coordinates of the two points of intersection. [4]

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(b) Find an equation of the circle with diameter AB. [3]

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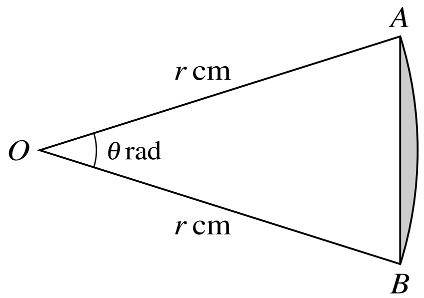

The diagram shows a sector OAB of a circle with centre O and radius r cm. Angle $AOB = \theta$ radians. It is given that the length of the arc AB is 9.6 cm and that the area of the sector OAB is 76.8 cm^2.

(a) Find the area of the shaded region. [5]

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(b) Find the perimeter of the shaded region. [2]

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7 The function f is defined by $f(x) = 2 - \dfrac{5}{x+2}$ for $x > -2$.

(a) State the range of f. [1]

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(b) Obtain an expression for $f^{-1}(x)$ and state the domain of f^{-1}. [4]

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The function g is defined by $g(x) = x + 3$ for $x > 0$.

(c) Obtain an expression for $fg(x)$ giving your answer in the form $\frac{ax + b}{cx + d}$, where a, b, c and d are integers. [3]

8 A progression has first term a and second term $\frac{a^2}{a+2}$, where a is a positive constant.

(a) For the case where the progression is geometric and the sum to infinity is 264, find the value of a. [5]

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(b) For the case where the progression is arithmetic and $a = 6$, determine the least value of n required for the sum of the first n terms to be less than -480. [5]

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9 A curve which passes through $(0, 3)$ has equation $y = \mathrm{f}(x)$. It is given that $\mathrm{f}'(x) = 1 - \dfrac{2}{(x-1)^3}$.

(a) Find the equation of the curve. [4]

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The tangent to the curve at (0, 3) intersects the curve again at one other point, P.

(b) Show that the x-coordinate of P satisfies the equation $(2x + 1)(x - 1)^2 - 1 = 0$. [4]

(c) Verify that $x = \frac{3}{2}$ satisfies this equation and hence find the y-coordinate of P. [2]

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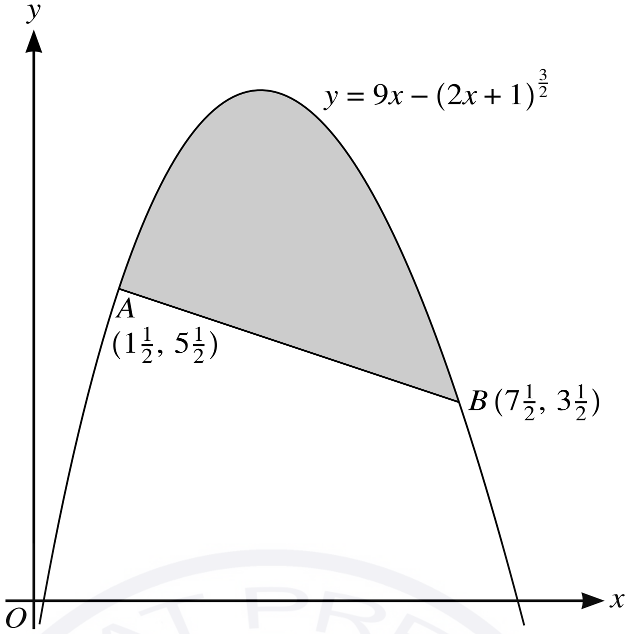

The diagram shows the points $A\left(1\frac{1}{2},\ 5\frac{1}{2}\right)$ and $B\left(7\frac{1}{2},\ 3\frac{1}{2}\right)$ lying on the curve with equation $y = 9x - (2x+1)^{\frac{3}{2}}$.

(a) Find the coordinates of the maximum point of the curve. [4]

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(b) Verify that the line AB is the normal to the curve at A. [3]

(c) Find the area of the shaded region. [5]

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MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **February/March 2023**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
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- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

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1 A line has equation $y = 3x - 2k$ and a curve has equation $y = x^2 - kx + 2$, where k is a constant.

Show that the line and the curve meet for all values of k. [4]

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2 A function f is defined by $f(x) = x^2 - 2x + 5$ for $x \in \mathbb{R}$. A sequence of transformations is applied in the following order to the graph of $y = f(x)$ to give the graph of $y = g(x)$.

Stretch parallel to the x-axis with scale factor $\frac{1}{2}$

Reflection in the y-axis

Stretch parallel to the y-axis with scale factor 3

Find $g(x)$, giving your answer in the form $ax^2 + bx + c$, where a, b and c are constants. [4]

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3 A curve has equation $y = \frac{1}{60}(3x + 1)^2$ and a point is moving along the curve.

Find the x-coordinate of the point on the curve at which the x- and y-coordinates are increasing at the same rate. [4]

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4 The circumference round the trunk of a large tree is measured and found to be 5.00 m. After one year the circumference is measured again and found to be 5.02 m.

(a) Given that the circumferences at yearly intervals form an arithmetic progression, find the circumference 20 years after the first measurement. [2]

(b) Given instead that the circumferences at yearly intervals form a geometric progression, find the circumference 20 years after the first measurement. [3]

5 Points $A\,(7,\ 12)$ and B lie on a circle with centre $(-2,\ 5)$. The line AB has equation $y = -2x + 26$.

Find the coordinates of B. [6]

6 In the expansion of $\left(\frac{x}{a} + \frac{a}{x^2}\right)^7$, it is given that

$$\frac{\text{the coefficient of } x^4}{\text{the coefficient of } x} = 3.$$

Find the possible values of the constant a. [6]

7 (a) By first obtaining a quadratic equation in $\cos\theta$, solve the equation

$$\tan\theta\sin\theta = 1$$

for $0° < \theta < 360°$. [5]

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(b) Show that $\frac{\tan\theta}{\sin\theta} - \frac{\sin\theta}{\tan\theta} \equiv \tan\theta\sin\theta$. [3]

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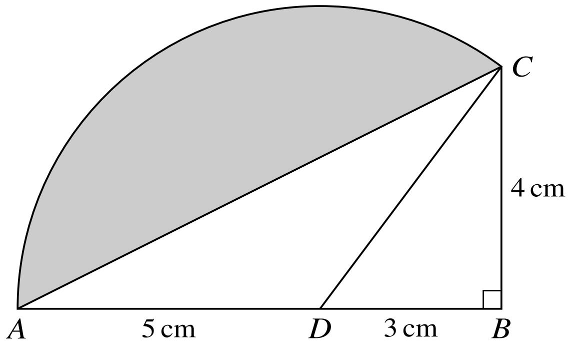

The diagram shows triangle ABC in which angle B is a right angle. The length of AB is 8 cm and the length of BC is 4 cm. The point D on AB is such that $AD = 5$ cm. The sector DAC is part of a circle with centre D.

(a) Find the perimeter of the shaded region. [5]

(b) Find the area of the shaded region. [3]

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9 The function f is defined by $f(x) = -3x^2 + 2$ for $x \leqslant -1$.

(a) State the range of f. [1]

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(b) Find an expression for $f^{-1}(x)$. [3]

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The function g is defined by $g(x) = -x^2 - 1$ for $x \leqslant -1$.

(c) Solve the equation $fg(x) - gf(x) + 8 = 0$. [5]

10 At the point $(4, -1)$ on a curve, the gradient of the curve is $-\frac{3}{2}$. It is given that $\frac{dy}{dx} = x^{-\frac{1}{2}} + k$, where k is a constant.

(a) Show that $k = -2$. [1]

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(b) Find the equation of the curve. [4]

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(c) Find the coordinates of the stationary point. [3]

(d) Determine the nature of the stationary point. [2]

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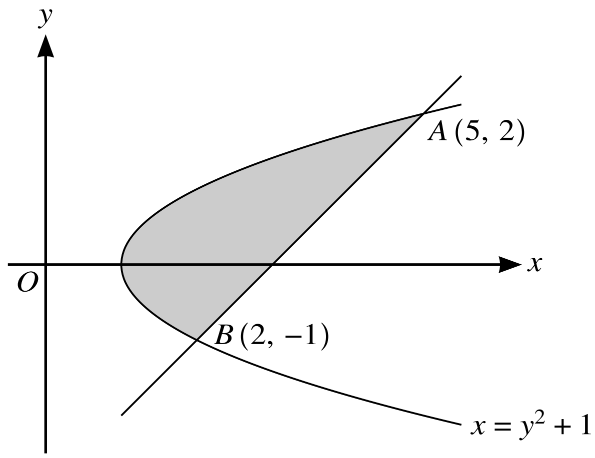

The diagram shows the curve with equation $x = y^2 + 1$. The points $A\,(5,\ 2)$ and $B\,(2,\ -1)$ lie on the curve.

(a) Find an equation of the line AB. [2]

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(b) Find the volume of revolution when the region between the curve and the line AB is rotated through 360° about the ***y*-axis**. [9]

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MATHEMATICS **9709/11**

Paper 1 Pure Mathematics 1 **October/November 2022**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
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- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages.

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1 Solve the equation $3x + 2 = \dfrac{2}{x - 1}$. [3]

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2 The equation of a curve is such that $\frac{dy}{dx} = 12\left(\frac{1}{2}x - 1\right)^{-4}$. It is given that the curve passes through the point $P(6, 4)$.

(a) Find the equation of the tangent to the curve at P. [2]

(b) Find the equation of the curve. [4]

3 A curve has equation $y = ax^{\frac{1}{2}} - 2x$, where $x > 0$ and a is a constant. The curve has a stationary point at the point P, which has x-coordinate 9.

Find the y-coordinate of P. [5]

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4 The coefficient of x^2 in the expansion of $\left(1 + \frac{2}{p}x\right)^5 + (1 + px)^6$ is 70.

Find the possible values of the constant p. [6]

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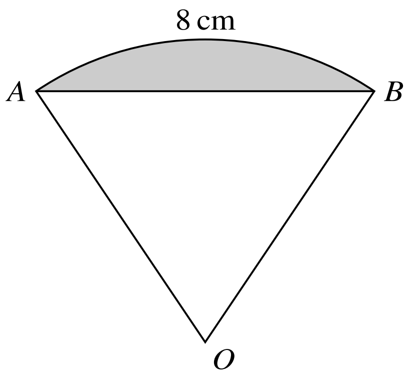

The diagram shows a sector OAB of a circle with centre O. The length of the arc AB is 8 cm. It is given that the perimeter of the sector is 20 cm.

(a) Find the perimeter of the shaded segment. [4]

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(b) Find the area of the shaded segment. [2]

6 **(a)** Show that the equation

$$\frac{1}{\sin\theta + \cos\theta} + \frac{1}{\sin\theta - \cos\theta} = 1$$

may be expressed in the form $a\sin^2\theta + b\sin\theta + c = 0$, where a, b and c are constants to be found. [3]

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(b) Hence solve the equation $\dfrac{1}{\sin\theta + \cos\theta} + \dfrac{1}{\sin\theta - \cos\theta} = 1$ for $0° \leqslant \theta \leqslant 360°$. [3]

7 A tool for putting fence posts into the ground is called a 'post-rammer'. The distances in millimetres that the post sinks into the ground on each impact of the post-rammer follow a geometric progression. The first three impacts cause the post to sink into the ground by 50 mm, 40 mm and 32 mm respectively.

(a) Verify that the 9th impact is the first in which the post sinks less than 10 mm into the ground. [3]

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(b) Find, to the nearest millimetre, the total depth of the post in the ground after 20 impacts. [2]

(c) Find the greatest total depth in the ground which could theoretically be achieved. [2]

8 The function f is defined by $f(x) = 2 - \dfrac{3}{4x - p}$ for $x > \dfrac{p}{4}$, where p is a constant.

(a) Find $f'(x)$ and hence determine whether f is an increasing function, a decreasing function or neither. [3]

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(b) Express $f^{-1}(x)$ in the form $\dfrac{p}{a} - \dfrac{b}{cx-d}$, where a, b, c and d are integers. [4]

(c) Hence state the value of p for which $f^{-1}(x) \equiv f(x)$. [1]

9 Functions f and g are both defined for $x \in \mathbb{R}$ and are given by

$$f(x) = x^2 - 4x + 9,$$
$$g(x) = 2x^2 + 4x + 12.$$

(a) Express $f(x)$ in the form $(x - a)^2 + b$. [1]

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(b) Express $g(x)$ in the form $2[(x + c)^2 + d]$. [2]

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(c) Express $g(x)$ in the form $kf(x + h)$, where k and h are integers. [1]

(d) Describe fully the two transformations that have been combined to transform the graph of $y = f(x)$ to the graph of $y = g(x)$. [4]

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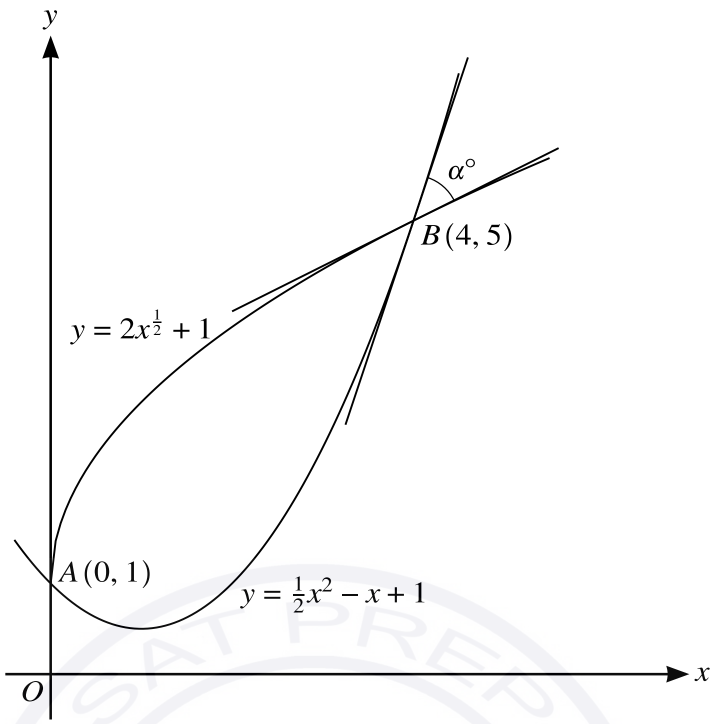

Curves with equations $y = 2x^{\frac{1}{2}} + 1$ and $y = \frac{1}{2}x^2 - x + 1$ intersect at $A\,(0,\ 1)$ and $B\,(4,\ 5)$, as shown in the diagram.

(a) Find the area of the region between the two curves. [5]

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The acute angle between the two tangents at B is denoted by $\alpha°$, and the scales on the axes are the same.

(b) Find α. [5]

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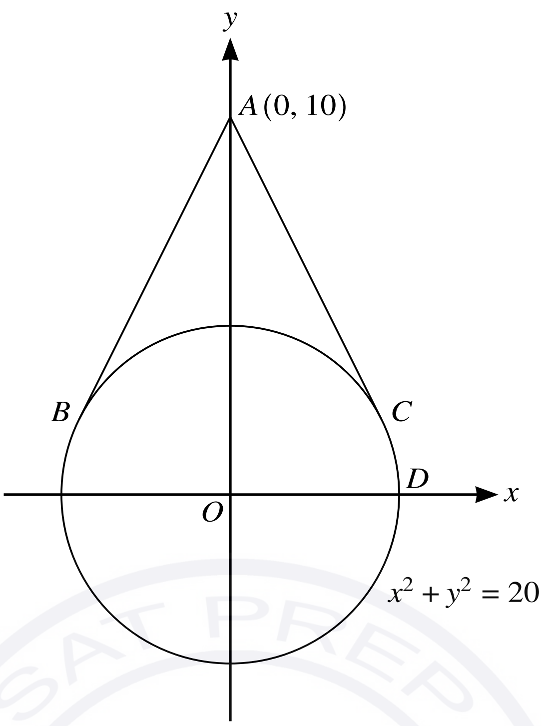

The diagram shows the circle with equation $x^2 + y^2 = 20$. Tangents touching the circle at points B and C pass through the point $A\ (0,\ 10)$.

(a) By letting the equation of a tangent be $y = mx + 10$, find the two possible values of m. [4]

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(b) Find the coordinates of B and C. [3]

The point D is where the circle crosses the positive x-axis.

(c) Find angle BDC in degrees. [3]

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MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **October/November 2022**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
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- You should use a calculator where appropriate.
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- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

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1 Points A and B have coordinates $(5, 2)$ and $(10, -1)$ respectively.

(a) Find the equation of the perpendicular bisector of AB. [3]

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(b) Find the equation of the circle with centre A which passes through B. [3]

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2 The first, second and third terms of an arithmetic progression are a, $2a$ and a^2 respectively, where a is a positive constant.

Find the sum of the first 50 terms of the progression. [5]

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3 **(a)** Find the set of values of k for which the equation $8x^2 + kx + 2 = 0$ has no real roots. [2]

(b) Solve the equation $8\cos^2\theta - 10\cos\theta + 2 = 0$ for $0° \leqslant \theta \leqslant 180°$. [3]

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4 A geometric progression is such that the third term is 1764 and the sum of the second and third terms is 3444.

Find the 50th term. [4]

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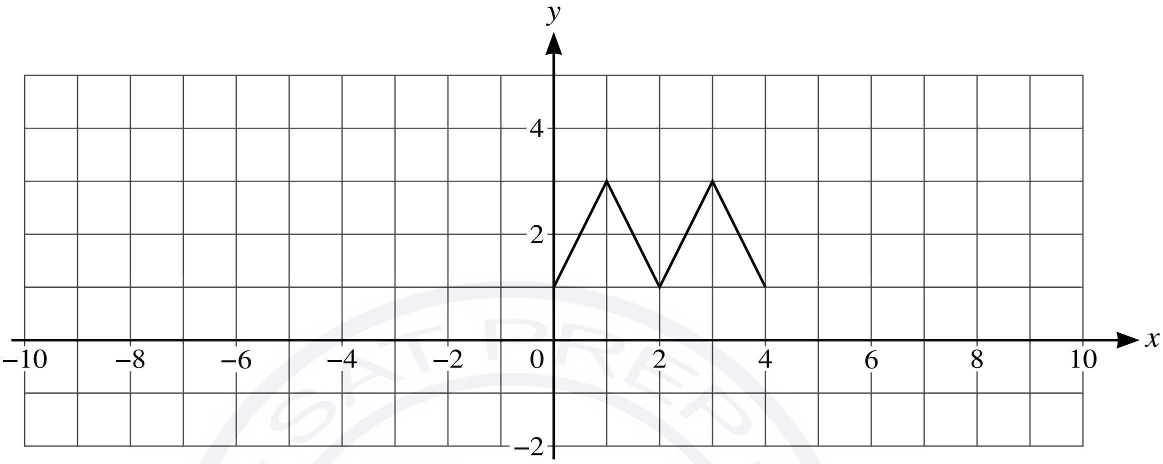
5 The graph with equation $y = \mathrm{f}(x)$ is transformed to the graph with equation $y = \mathrm{g}(x)$ by a stretch in the x-direction with factor 0.5, followed by a translation of $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$.

(a) The diagram below shows the graph of $y = \mathrm{f}(x)$.

On the diagram sketch the graph of $y = \mathrm{g}(x)$. [3]



(b) Find an expression for $\mathrm{g}(x)$ in terms of $\mathrm{f}(x)$. [2]

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6 The equation of a curve is $y = 4x^2 + 20x + 6$.

(a) Express the equation in the form $y = a(x + b)^2 + c$, where a, b and c are constants. [3]

(b) Hence solve the equation $4x^2 + 20x + 6 = 45$. [3]

(c) Sketch the graph of $y = 4x^2 + 20x + 6$ showing the coordinates of the stationary point. You are not required to indicate where the curve crosses the x- and y-axes. [3]

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7 **(a)** Prove the identity $\dfrac{\sin\theta}{\sin\theta+\cos\theta}+\dfrac{\cos\theta}{\sin\theta-\cos\theta}\equiv\dfrac{\tan^2\theta+1}{\tan^2\theta-1}$. [3]

(b) Hence find the exact solutions of the equation $\dfrac{\sin\theta}{\sin\theta + \cos\theta} + \dfrac{\cos\theta}{\sin\theta - \cos\theta} = 2$ for $0 \leqslant \theta \leqslant \pi$. [4]

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8 The equation of a curve is such that $\frac{dy}{dx} = 3x^{\frac{1}{2}} - 3x^{-\frac{1}{2}}$. The curve passes through the point (3, 5).

(a) Find the equation of the curve. [4]

(b) Find the x-coordinate of the stationary point. [2]

(c) State the set of values of x for which y increases as x increases. [1]

9 Functions f and g are defined by

$$f(x) = x + \frac{1}{x} \quad \text{for } x > 0,$$
$$g(x) = ax + 1 \quad \text{for } x \in \mathbb{R},$$

where a is a constant.

(a) Find an expression for $gf(x)$. [1]

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(b) Given that $gf(2) = 11$, find the value of a. [2]

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(c) Given that the graph of $y = f(x)$ has a minimum point when $x = 1$, explain whether or not f has an inverse. [1]

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It is given instead that $a = 5$.

(d) Find and simplify an expression for $g^{-1}f(x)$. [3]

(e) Explain why the composite function fg cannot be formed. [1]

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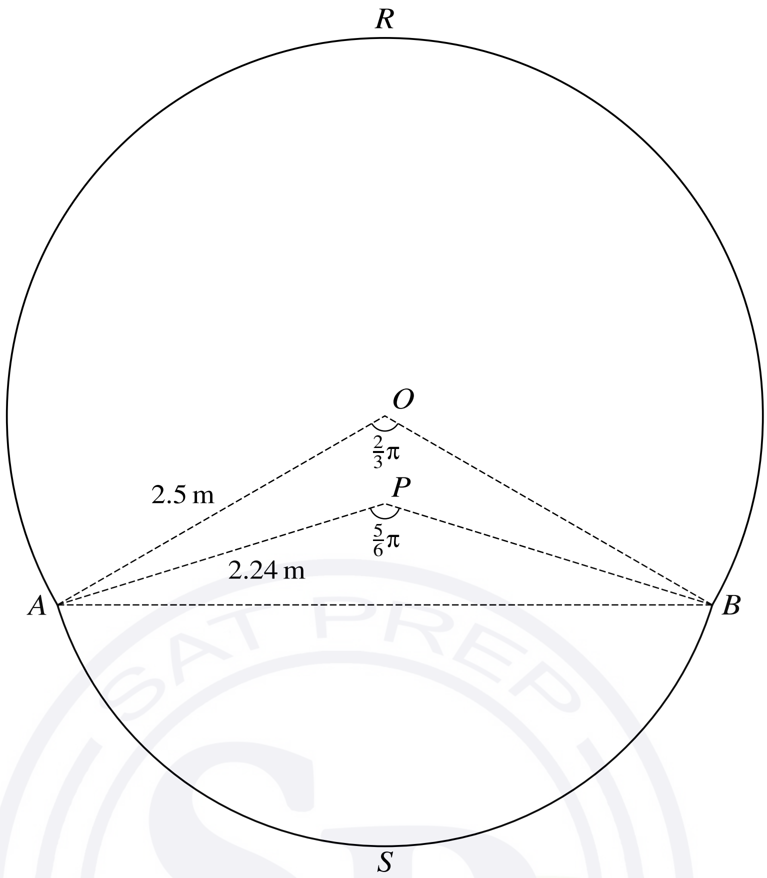

The diagram shows a cross-section *RASB* of the body of an aircraft. The cross-section consists of a sector *OARB* of a circle of radius 2.5 m, with centre *O*, a sector *PASB* of another circle of radius 2.24 m with centre *P* and a quadrilateral *OAPB*. Angle $AOB = \frac{2}{3}\pi$ and angle $APB = \frac{5}{6}\pi$.

(a) Find the perimeter of the cross-section *RASB*, giving your answer correct to 2 decimal places. [3]

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(b) Find the difference in area of the two triangles AOB and APB, giving your answer correct to 2 decimal places. [2]

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(c) Find the area of the cross-section $RASB$, giving your answer correct to 1 decimal place. [3]

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11 **(a)** Find the coordinates of the minimum point of the curve $y = \frac{9}{4}x^2 - 12x + 18$. [3]

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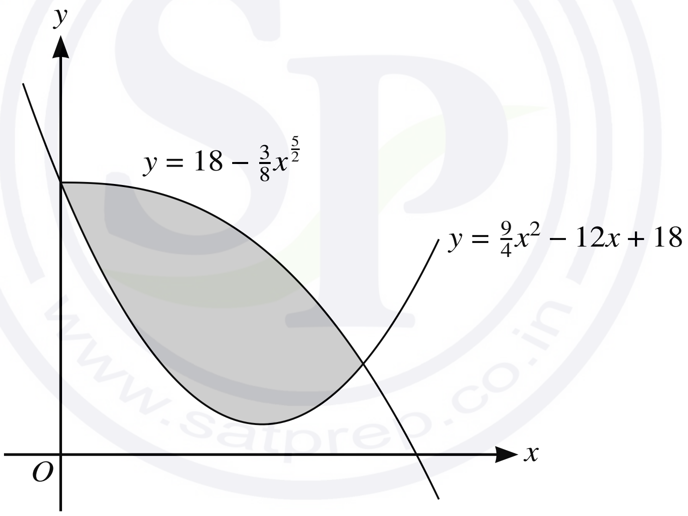

The diagram shows the curves with equations $y = \frac{9}{4}x^2 - 12x + 18$ and $y = 18 - \frac{3}{8}x^{\frac{5}{2}}$. The curves intersect at the points (0, 18) and (4, 6).

(b) Find the area of the shaded region. [5]

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(c) A point P is moving along the curve $y = 18 - \frac{3}{8}x^{\frac{5}{2}}$ in such a way that the x-coordinate of P is increasing at a constant rate of 2 units per second.

Find the rate at which the y-coordinate of P is changing when $x = 4$. [3]

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CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

6552648013

MATHEMATICS **9709/13**

Paper 1 Pure Mathematics 1 **October/November 2022**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

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1 Solve the equation $8\sin^2\theta + 6\cos\theta + 1 = 0$ for $0° < \theta < 180°$. [3]

2 The function f is defined by $f(x) = -2x^2 - 8x - 13$ for $x < -3$.

(a) Express $f(x)$ in the form $-2(x + a)^2 + b$, where a and b are integers. [2]

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(b) Find the range of f. [1]

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(c) Find an expression for $f^{-1}(x)$. [3]

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3 **(a)** Find the first three terms in ascending powers of x of the expansion of $(1 + 2x)^5$. [2]

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(b) Find the first three terms in ascending powers of x of the expansion of $(1 - 3x)^4$. [2]

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(c) Hence find the coefficient of x^2 in the expansion of $(1 + 2x)^5(1 - 3x)^4$. [2]

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4 A large industrial water tank is such that, when the depth of the water in the tank is x metres, the volume $V\,\text{m}^3$ of water in the tank is given by $V = 243 - \frac{1}{3}(9 - x)^3$. Water is being pumped into the tank at a constant rate of $3.6\,\text{m}^3$ per hour.

Find the rate of increase of the depth of the water when the depth is 4 m, giving your answer in cm per minute. [5]

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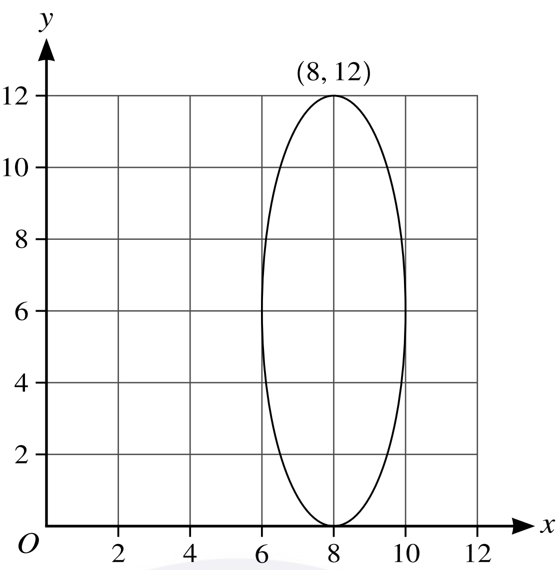

The diagram shows a curve which has a maximum point at (8, 12) and a minimum point at (8, 0). The curve is the result of applying a combination of two transformations to a circle. The first transformation applied is a translation of $\begin{pmatrix} 7 \\ -3 \end{pmatrix}$. The second transformation applied is a stretch in the y-direction.

(a) State the scale factor of the stretch. [1]

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(b) State the radius of the original circle. [1]

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(c) State the coordinates of the centre of the circle after the translation has been completed but before the stretch is applied. [2]

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(d) State the coordinates of the centre of the original circle. [2]

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6 It is given that $\alpha = \cos^{-1}\left(\frac{8}{17}\right)$.

Find, without using the trigonometric functions on your calculator, the exact value of $\dfrac{1}{\sin\alpha} + \dfrac{1}{\tan\alpha}$. [5]

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7 The curve $y = f(x)$ is such that $f'(x) = \dfrac{-3}{(x+2)^4}$.

(a) The tangent at a point on the curve where $x = a$ has gradient $-\frac{16}{27}$.

Find the possible values of a. [4]

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(b) Find $f(x)$ given that the curve passes through the point $(-1, 5)$. [3]

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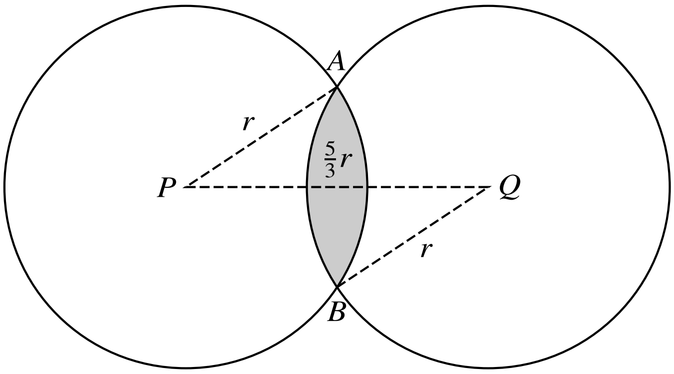

The diagram shows two identical circles intersecting at points A and B and with centres at P and Q. The radius of each circle is r and the distance PQ is $\frac{5}{3}r$.

(a) Find the perimeter of the shaded region in terms of r. [4]

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(b) Find the area of the shaded region in terms of r. [3]

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9 The first term of a geometric progression is 216 and the fourth term is 64.

(a) Find the sum to infinity of the progression. [3]

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The second term of the geometric progression is equal to the second term of an arithmetic progression.

The third term of the geometric progression is equal to the fifth term of the same arithmetic progression.

(b) Find the sum of the first 21 terms of the arithmetic progression. [6]

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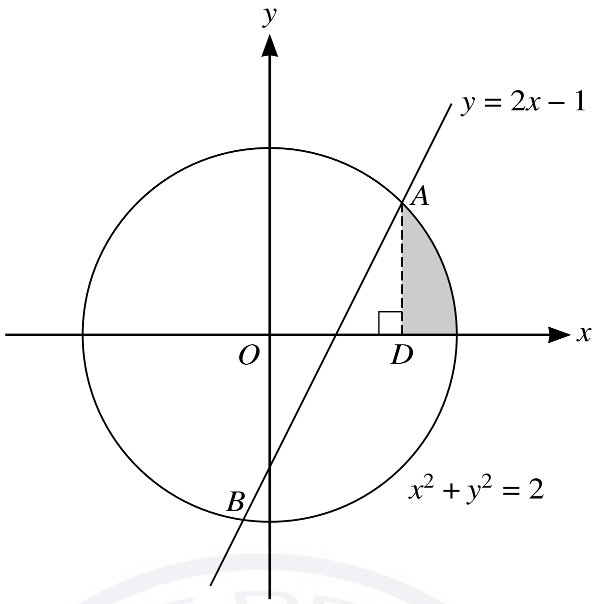

The diagram shows the circle $x^2 + y^2 = 2$ and the straight line $y = 2x - 1$ intersecting at the points A and B. The point D on the x-axis is such that AD is perpendicular to the x-axis.

(a) Find the coordinates of A. [4]

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(b) Find the volume of revolution when the shaded region is rotated through 360° about the x-axis. Give your answer in the form $\frac{\pi}{a}\left(b\sqrt{c}-d\right)$, where a, b, c and d are integers. [4]

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(c) Find an exact expression for the perimeter of the shaded region. [2]

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11 The coordinates of points A, B and C are $A\,(5, -2)$, $B\,(10, 3)$ and $C\,(2p, p)$, where p is a constant.

(a) Given that AC and BC are equal in length, find the value of the fraction p. [3]

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(b) It is now given instead that AC is perpendicular to BC and that p is an integer.

(i) Find the value of p. [4]

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(ii) Find the equation of the circle which passes through A, B and C, giving your answer in the form $x^2 + y^2 + ax + by + c = 0$, where a, b and c are constants. [4]

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CANDIDATE NAME

CENTRE NUMBER | CANDIDATE NUMBER

8494117371

MATHEMATICS **9709/11**

Paper 1 Pure Mathematics 1 **May/June 2022**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
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- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages.

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1 **(a)** Express $x^2 - 8x + 11$ in the form $(x + p)^2 + q$ where p and q are constants. [2]

(b) Hence find the exact solutions of the equation $x^2 - 8x + 11 = 1$. [2]

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2 The thirteenth term of an arithmetic progression is 12 and the sum of the first 30 terms is −15.

Find the sum of the first 50 terms of the progression. [5]

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3 The coefficient of x^4 in the expansion of $\left(2x^2 + \frac{k^2}{x}\right)^5$ is a. The coefficient of x^2 in the expansion of $(2kx - 1)^4$ is b.

(a) Find a and b in terms of the constant k. [3]

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(b) Given that $a + b = 216$, find the possible values of k. [3]

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4 **(a)** Prove the identity $\dfrac{\sin^3\theta}{\sin\theta - 1} - \dfrac{\sin^2\theta}{1 + \sin\theta} \equiv -\tan^2\theta(1 + \sin^2\theta)$. [4]

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(b) Hence solve the equation

$$\frac{\sin^3\theta}{\sin\theta - 1} - \frac{\sin^2\theta}{1 + \sin\theta} = \tan^2\theta(1 - \sin^2\theta)$$

for $0 < \theta < 2\pi$. [2]

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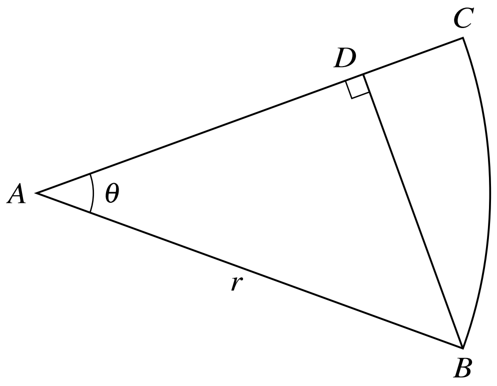

The diagram shows a sector ABC of a circle with centre A and radius r. The line BD is perpendicular to AC. Angle CAB is θ radians.

(a) Given that $\theta = \frac{1}{6}\pi$, find the exact area of BCD in terms of r. [3]

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(b) Given instead that the length of BD is $\frac{\sqrt{3}}{2}r$, find the exact perimeter of BCD in terms of r. [4]

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6 The function f is defined as follows:

$$f(x) = \frac{x^2 - 4}{x^2 + 4} \quad \text{for } x > 2.$$

(a) Find an expression for $f^{-1}(x)$. [3]

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(b) Show that $1 - \dfrac{8}{x^2 + 4}$ can be expressed as $\dfrac{x^2 - 4}{x^2 + 4}$ and hence state the range of f. [4]

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(c) Explain why the composite function ff cannot be formed. [1]

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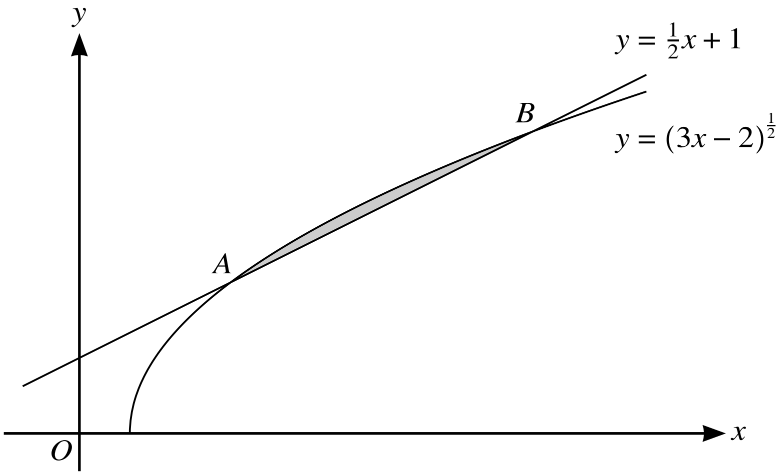

The diagram shows the curve with equation $y = (3x-2)^{\frac{1}{2}}$ and the line $y = \frac{1}{2}x + 1$. The curve and the line intersect at points A and B.

(a) Find the coordinates of A and B. [4]

(b) Hence find the area of the region enclosed between the curve and the line. [5]

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8 **(a)** The curve $y = \sin x$ is transformed to the curve $y = 4\sin\left(\frac{1}{2}x - 30°\right)$.

Describe fully a sequence of transformations that have been combined, making clear the order in which the transformations are applied. [5]

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(b) Find the exact solutions of the equation $4\sin\left(\frac{1}{2}x - 30°\right) = 2\sqrt{2}$ for $0° \leqslant x \leqslant 360°$. [3]

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9 The equation of a circle is $x^2 + y^2 + 6x - 2y - 26 = 0$.

(a) Find the coordinates of the centre of the circle and the radius. Hence find the coordinates of the lowest point on the circle. [4]

(b) Find the set of values of the constant k for which the line with equation $y = kx - 5$ intersects the circle at two distinct points. [6]

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10 The equation of a curve is such that $\frac{d^2y}{dx^2} = 6x^2 - \frac{4}{x^3}$. The curve has a stationary point at $\left(-1, \frac{9}{2}\right)$.

(a) Determine the nature of the stationary point at $\left(-1, \frac{9}{2}\right)$. [1]

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(b) Find the equation of the curve. [5]

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(c) Show that the curve has no other stationary points. [3]

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(d) A point A is moving along the curve and the y-coordinate of A is increasing at a rate of 5 units per second.

Find the rate of increase of the x-coordinate of A at the point where $x = 1$. [3]

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CANDIDATE NAME	

CENTRE NUMBER						CANDIDATE NUMBER				

* 0 3 3 2 6 5 5 4 0 7 *

MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **May/June 2022**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
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- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages.

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1 The coefficient of x^4 in the expansion of $(3 + x)^5$ is equal to the coefficient of x^2 in the expansion of $\left(2x + \frac{a}{x}\right)^6$.

Find the value of the positive constant a. [4]

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2 The second and third terms of a geometric progression are 10 and 8 respectively.

Find the sum to infinity. [4]

3 The equation of a curve is such that $\frac{dy}{dx} = 3(4x-7)^{\frac{1}{2}} - 4x^{-\frac{1}{2}}$. It is given that the curve passes through the point $\left(4, \frac{5}{2}\right)$.

Find the equation of the curve. [4]

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4 The first, second and third terms of an arithmetic progression are k, $6k$ and $k + 6$ respectively.

(a) Find the value of the constant k. [2]

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(b) Find the sum of the first 30 terms of the progression. [3]

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5 The equation of a curve is $y = 4x^2 - kx + \frac{1}{2}k^2$ and the equation of a line is $y = x - a$, where k and a are constants.

(a) Given that the curve and the line intersect at the points with x-coordinates 0 and $\frac{3}{4}$, find the values of k and a. [4]

(b) Given instead that $a = -\frac{7}{2}$, find the values of k for which the line is a tangent to the curve. [5]

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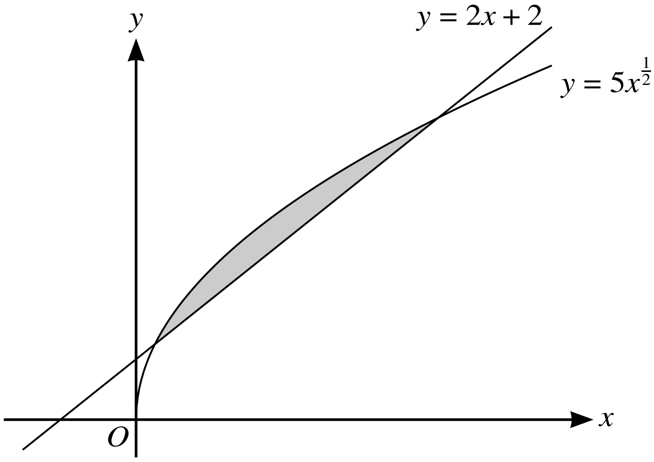

The diagram shows the curve with equation $y = 5x^{\frac{1}{2}}$ and the line with equation $y = 2x + 2$.

Find the exact area of the shaded region which is bounded by the line and the curve. [5]

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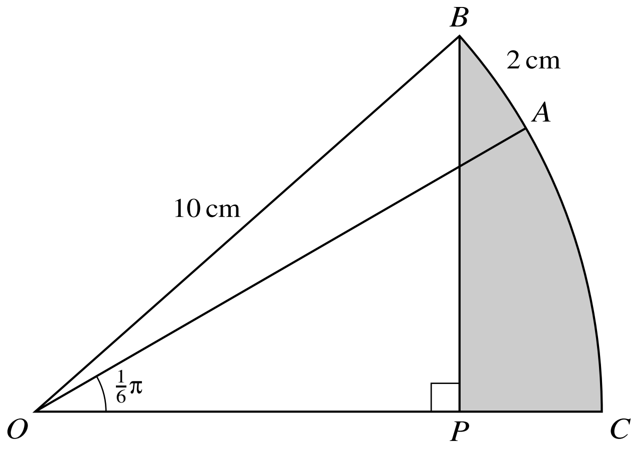

The diagram shows a sector *OBAC* of a circle with centre *O* and radius 10 cm. The point *P* lies on *OC* and *BP* is perpendicular to *OC*. Angle $AOC = \frac{1}{6}\pi$ and the length of the arc *AB* is 2 cm.

(a) Find the angle *BOC*. [2]

(b) Hence find the area of the shaded region BPC giving your answer correct to 3 significant figures. [4]

8 The equation of a circle is $x^2 + y^2 + ax + by - 12 = 0$. The points $A(1, 1)$ and $B(2, -6)$ lie on the circle.

(a) Find the values of a and b and hence find the coordinates of the centre of the circle. [4]

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(b) Find the equation of the tangent to the circle at the point A, giving your answer in the form $px + qy = k$, where p, q and k are integers. [4]

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9 The equation of a curve is $y = 3x + 1 - 4(3x + 1)^{\frac{1}{2}}$ for $x > -\frac{1}{3}$.

(a) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. [3]

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(b) Find the coordinates of the stationary point of the curve and determine its nature. [4]

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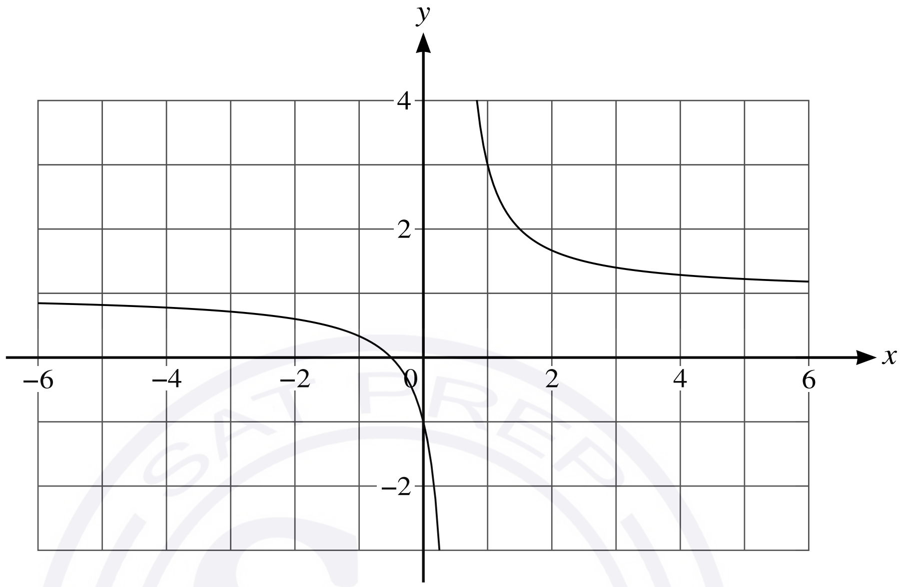
10 Functions f and g are defined as follows:

$$f(x) = \frac{2x+1}{2x-1} \quad \text{for } x \neq \tfrac{1}{2},$$

$$g(x) = x^2 + 4 \quad \text{for } x \in \mathbb{R}.$$

(a)



The diagram shows part of the graph of $y = f(x)$.

State the domain of f^{-1}. [1]

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(b) Find an expression for $f^{-1}(x)$. [3]

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(c) Find $gf^{-1}(3)$. [2]

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(d) Explain why $g^{-1}(x)$ cannot be found. [1]

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(e) Show that $1 + \frac{2}{2x - 1}$ can be expressed as $\frac{2x + 1}{2x - 1}$. Hence find the area of the triangle enclosed by the tangent to the curve $y = f(x)$ at the point where $x = 1$ and the x- and y-axes. [6]

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11 The function f is given by $f(x) = 4\cos^4 x + \cos^2 x - k$ for $0 \leqslant x \leqslant 2\pi$, where k is a constant.

(a) Given that $k = 3$, find the exact solutions of the equation $f(x) = 0$. [5]

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(b) Use the quadratic formula to show that, when $k > 5$, the equation $f(x) = 0$ has no solutions. [5]

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CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

7036013722

MATHEMATICS **9709/13**

Paper 1 Pure Mathematics 1 **May/June 2022**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
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- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

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1 The coefficient of x^3 in the expansion of $\left(p + \frac{1}{p}x\right)^4$ is 144.

Find the possible values of the constant p. [4]

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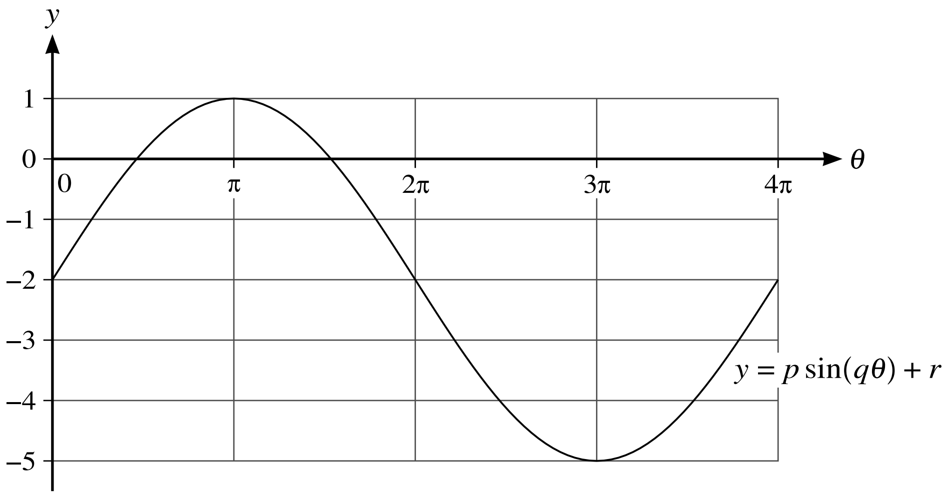

The diagram shows part of the curve with equation $y = p\sin(q\theta) + r$, where p, q and r are constants.

(a) State the value of p. [1]

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(b) State the value of q. [1]

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(c) State the value of r. [1]

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3 An arithmetic progression has first term 4 and common difference d. The sum of the first n terms of the progression is 5863.

(a) Show that $(n-1)d = \dfrac{11726}{n} - 8$. [1]

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(b) Given that the nth term is 139, find the values of n and d, giving the value of d as a fraction. [4]

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4 **(a)** The curve with equation $y = x^2 + 2x - 5$ is translated by $\begin{pmatrix} -1 \\ 3 \end{pmatrix}$.

Find the equation of the translated curve, giving your answer in the form $y = ax^2 + bx + c$. [3]

(b) The curve with equation $y = x^2 + 2x - 5$ is transformed to a curve with equation $y = 4x^2 + 4x - 5$.

Describe fully the single transformation that has been applied. [2]

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5 **(a)** Solve the equation $6\sqrt{y} + \dfrac{2}{\sqrt{y}} - 7 = 0$. [4]

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(b) Hence solve the equation $6\sqrt{\tan x} + \dfrac{2}{\sqrt{\tan x}} - 7 = 0$ for $0° \leqslant x \leqslant 360°$. [3]

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6 The function f is defined by $f(x) = 2x^2 - 16x + 23$ for $x < 3$.

(a) Express $f(x)$ in the form $2(x + a)^2 + b$. [2]

(b) Find the range of f. [1]

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(c) Find an expression for $f^{-1}(x)$. [3]

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The function g is defined by $g(x) = 2x + 4$ for $x < -1$.

(d) Find and simplify an expression for $fg(x)$. [2]

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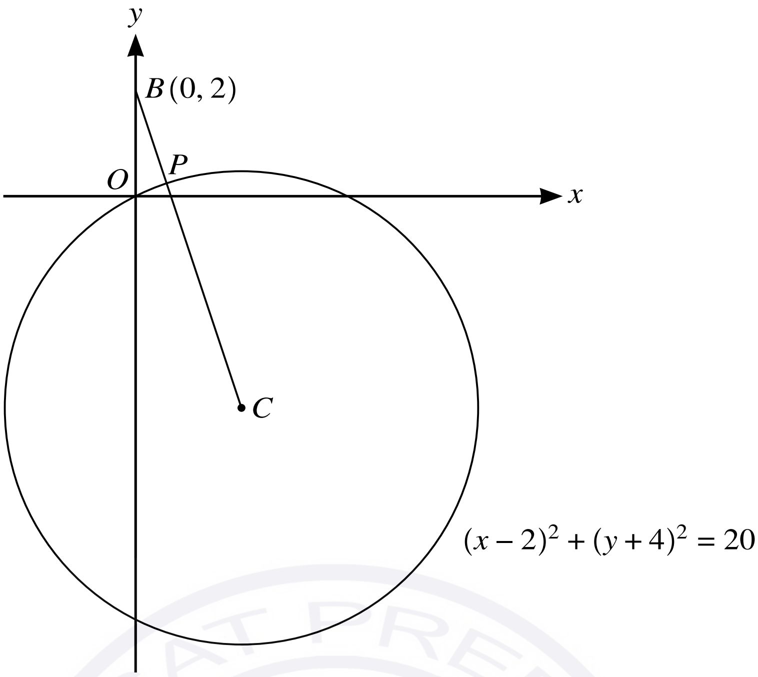

The diagram shows the circle with equation $(x-2)^2 + (y+4)^2 = 20$ and with centre C. The point B has coordinates (0, 2) and the line segment BC intersects the circle at P.

(a) Find the equation of BC. [2]

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(b) Hence find the coordinates of P, giving your answer in exact form. [5]

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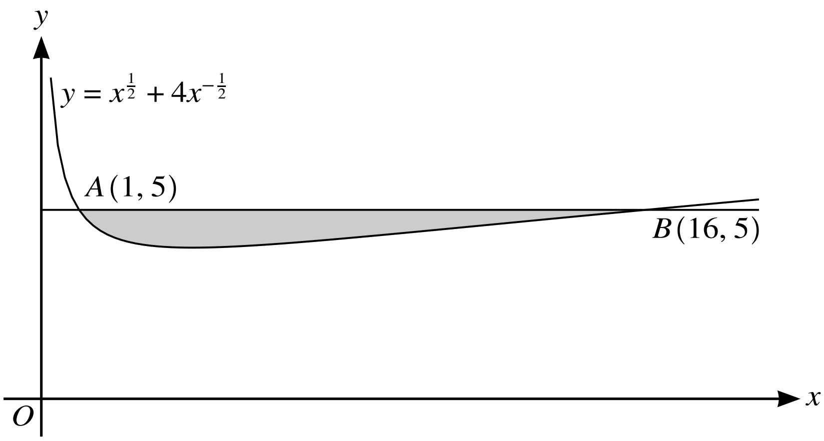

The diagram shows the curve with equation $y = x^{\frac{1}{2}} + 4x^{-\frac{1}{2}}$. The line $y = 5$ intersects the curve at the points $A\,(1,\ 5)$ and $B\,(16,\ 5)$.

(a) Find the equation of the tangent to the curve at the point A. [4]

(b) Calculate the area of the shaded region. [4]

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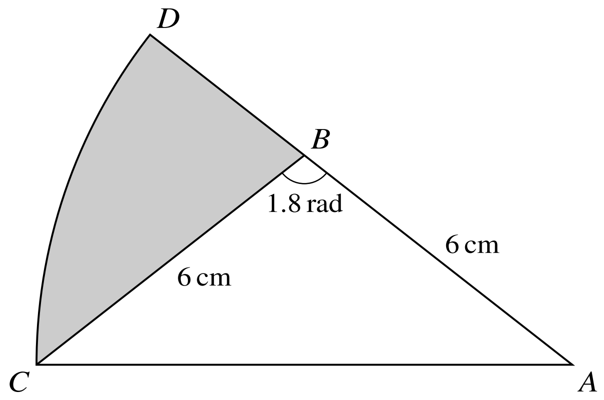

The diagram shows triangle ABC with $AB = BC = 6$ cm and angle $ABC = 1.8$ radians. The arc CD is part of a circle with centre A and ABD is a straight line.

(a) Find the perimeter of the shaded region. [5]

(b) Find the area of the shaded region. [3]

10 The function f is defined by $f(x) = (4x + 2)^{-2}$ for $x > -\frac{1}{2}$.

(a) Find $\int_1^{\infty} f(x)\,dx$. [4]

A point is moving along the curve $y = f(x)$ in such a way that, as it passes through the point A, its y-coordinate is **decreasing** at the rate of k units per second and its x-coordinate is **increasing** at the rate of k units per second.

(b) Find the coordinates of A. [6]

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11 The point P lies on the line with equation $y = mx + c$, where m and c are positive constants. A curve has equation $y = -\frac{m}{x}$. There is a single point P on the curve such that the straight line is a tangent to the curve at P.

(a) Find the coordinates of P, giving the y-coordinate in terms of m. [6]

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The normal to the curve at P intersects the curve again at the point Q.

(b) Find the coordinates of Q in terms of m. [4]

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CANDIDATE NAME

CENTRE NUMBER

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6482658238

MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **February/March 2022**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
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- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

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1 A curve with equation $y = f(x)$ is such that $f'(x) = 2x^{-\frac{1}{3}} - x^{\frac{1}{3}}$. It is given that $f(8) = 5$.

Find $f(x)$. [4]

2 A curve has equation $y = x^2 + 2cx + 4$ and a straight line has equation $y = 4x + c$, where c is a constant.

Find the set of values of c for which the curve and line intersect at two distinct points. [5]

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3 Find the term independent of x in each of the following expansions.

(a) $\left(3x + \frac{2}{x^2}\right)^6$ [3]

(b) $\left(3x + \frac{2}{x^2}\right)^6 (1 - x^3)$ [3]

4 The first term of a geometric progression and the first term of an arithmetic progression are both equal to a.

The third term of the geometric progression is equal to the second term of the arithmetic progression.

The fifth term of the geometric progression is equal to the sixth term of the arithmetic progression.

Given that the terms are all positive and not all equal, find the sum of the first twenty terms of the arithmetic progression in terms of a. [6]

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5 **(a)** Express $2x^2 - 8x + 14$ in the form $2[(x-a)^2 + b]$. [2]

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The functions f and g are defined by

$$\mathrm{f}(x) = x^2 \quad \text{for } x \in \mathbb{R},$$
$$\mathrm{g}(x) = 2x^2 - 8x + 14 \quad \text{for } x \in \mathbb{R}.$$

(b) Describe fully a sequence of transformations that maps the graph of $y = \mathrm{f}(x)$ onto the graph of $y = \mathrm{g}(x)$, making clear the order in which the transformations are applied. [4]

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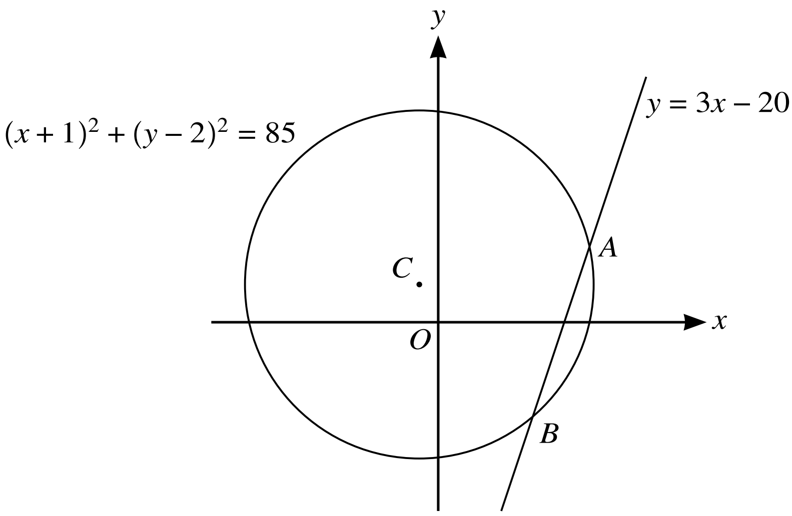

The circle with equation $(x + 1)^2 + (y - 2)^2 = 85$ and the straight line with equation $y = 3x - 20$ are shown in the diagram. The line intersects the circle at A and B, and the centre of the circle is at C.

(a) Find, by calculation, the coordinates of A and B. [4]

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(b) Find an equation of the circle which has its centre at C and for which the line with equation $y = 3x - 20$ is a tangent to the circle. [4]

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7 **(a)** Show that $\dfrac{\sin\theta + 2\cos\theta}{\cos\theta - 2\sin\theta} - \dfrac{\sin\theta - 2\cos\theta}{\cos\theta + 2\sin\theta} \equiv \dfrac{4}{5\cos^2\theta - 4}$. [4]

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(b) Hence solve the equation $\dfrac{\sin\theta + 2\cos\theta}{\cos\theta - 2\sin\theta} - \dfrac{\sin\theta - 2\cos\theta}{\cos\theta + 2\sin\theta} = 5$ for $0° < \theta < 180°$. [3]

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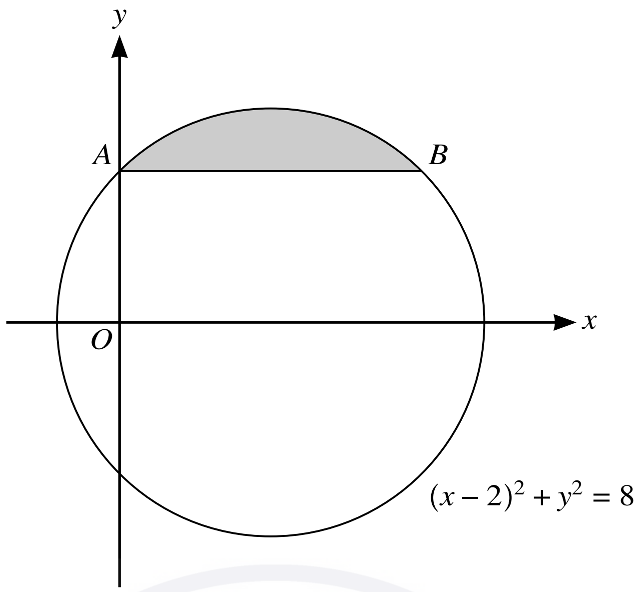

The diagram shows the circle with equation $(x-2)^2 + y^2 = 8$. The chord AB of the circle intersects the positive y-axis at A and is parallel to the x-axis.

(a) Find, by calculation, the coordinates of A and B. [3]

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(b) Find the volume of revolution when the shaded segment, bounded by the circle and the chord AB, is rotated through 360° about the x-axis. [5]

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9 Functions f, g and h are defined as follows:

$$\mathrm{f} : x \mapsto x - 4x^{\frac{1}{2}} + 1 \quad \text{for } x \geqslant 0,$$
$$\mathrm{g} : x \mapsto mx^2 + n \quad \text{for } x \geqslant -2, \text{ where } m \text{ and } n \text{ are constants,}$$
$$\mathrm{h} : x \mapsto x^{\frac{1}{2}} - 2 \quad \text{for } x \geqslant 0.$$

(a) Solve the equation $\mathrm{f}(x) = 0$, giving your solutions in the form $x = a + b\sqrt{c}$, where a, b and c are integers. [4]

(b) Given that $f(x) \equiv gh(x)$, find the values of m and n. [4]

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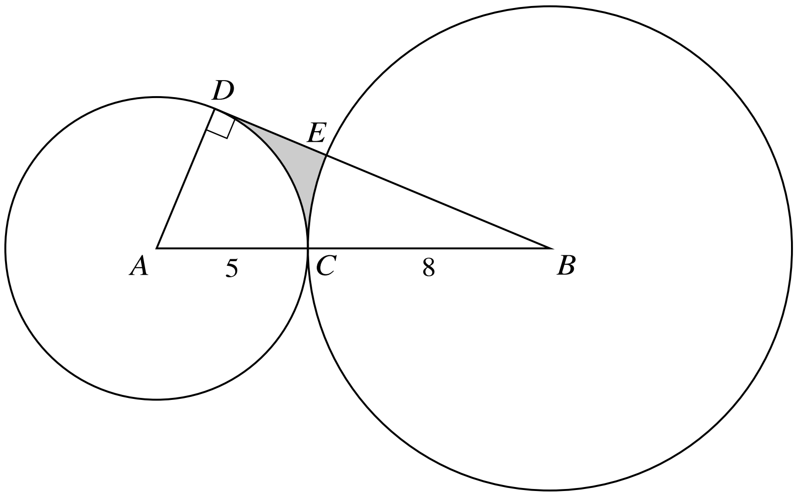

The diagram shows a circle with centre A of radius 5 cm and a circle with centre B of radius 8 cm. The circles touch at the point C so that ACB is a straight line. The tangent at the point D on the smaller circle intersects the larger circle at E and passes through B.

(a) Find the perimeter of the shaded region. [5]

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(b) Find the area of the shaded region. [3]

11 It is given that a curve has equation $y = k(3x - k)^{-1} + 3x$, where k is a constant.

(a) Find, in terms of k, the values of x at which there is a stationary point. [4]

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The function f has a stationary value at $x = a$ and is defined by

$$f(x) = 4(3x - 4)^{-1} + 3x \quad \text{for } x \geqslant \tfrac{3}{2}.$$

(b) Find the value of a and determine the nature of the stationary value. [3]

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(c) The function g is defined by $g(x) = -(3x + 1)^{-1} + 3x$ for $x \geqslant 0$.

Determine, making your reasoning clear, whether g is an increasing function, a decreasing function or neither. [2]

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CENTRE NUMBER

CANDIDATE NUMBER

2554281371

MATHEMATICS **9709/11**

Paper 1 Pure Mathematics 1 **October/November 2021**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
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- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

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1 **(a)** Expand $\left(1 - \dfrac{1}{2x}\right)^2$. [1]

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(b) Find the first four terms in the expansion, in ascending powers of x, of $(1 + 2x)^6$. [2]

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(c) Hence find the coefficient of x in the expansion of $\left(1 - \dfrac{1}{2x}\right)^2 (1 + 2x)^6$. [2]

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2 A curve has equation $y = kx^2 + 2x - k$ and a line has equation $y = kx - 2$, where k is a constant.

Find the set of values of k for which the curve and line do not intersect. [5]

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3 Solve, by factorising, the equation

$$6\cos\theta\tan\theta - 3\cos\theta + 4\tan\theta - 2 = 0,$$

for $0° \leqslant \theta \leqslant 180°$. [4]

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4 The first term of an arithmetic progression is a and the common difference is -4. The first term of a geometric progression is $5a$ and the common ratio is $-\frac{1}{4}$. The sum to infinity of the geometric progression is equal to the sum of the first eight terms of the arithmetic progression.

(a) Find the value of a. [4]

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The kth term of the arithmetic progression is zero.

(b) Find the value of k. [2]

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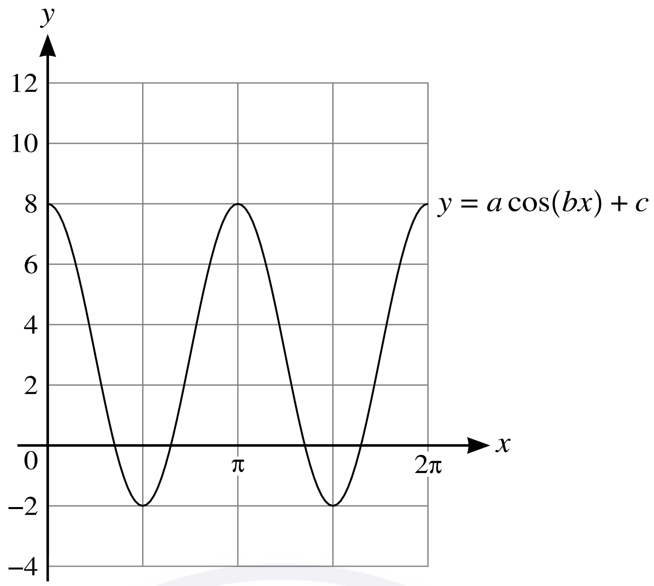

The diagram shows part of the graph of $y = a\cos(bx) + c$.

(a) Find the values of the positive integers a, b and c. [3]

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(b) For these values of a, b and c, use the given diagram to determine the number of solutions in the interval $0 \leqslant x \leqslant 2\pi$ for each of the following equations.

(i) $a\cos(bx) + c = \frac{6}{\pi}x$ [1]

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(ii) $a\cos(bx) + c = 6 - \frac{6}{\pi}x$ [1]

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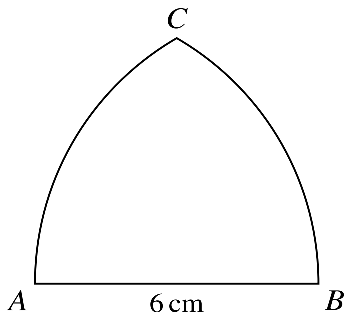

The diagram shows a metal plate *ABC* in which the sides are the straight line *AB* and the arcs *AC* and *BC*. The line *AB* has length 6 cm. The arc *AC* is part of a circle with centre *B* and radius 6 cm, and the arc *BC* is part of a circle with centre *A* and radius 6 cm.

(a) Find the perimeter of the plate, giving your answer in terms of π. [3]

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(b) Find the area of the plate, giving your answer in terms of π and $\sqrt{3}$. [4]

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7 A circle with centre (5, 2) passes through the point (7, 5).

(a) Find an equation of the circle. [2]

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The line $y = 5x - 10$ intersects the circle at A and B.

(b) Find the exact length of the chord AB. [7]

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8 **(a)** Express $-3x^2 + 12x + 2$ in the form $-3(x - a)^2 + b$, where a and b are constants. [2]

The one-one function f is defined by $\mathrm{f} : x \mapsto -3x^2 + 12x + 2$ for $x \leqslant k$.

(b) State the largest possible value of the constant k. [1]

It is now given that $k = -1$.

(c) State the range of f. [1]

(d) Find an expression for $f^{-1}(x)$. [3]

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The result of translating the graph of $y = f(x)$ by $\begin{pmatrix} -3 \\ 1 \end{pmatrix}$ is the graph of $y = g(x)$.

(e) Express $g(x)$ in the form $px^2 + qx + r$, where p, q and r are constants. [3]

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9 A curve has equation $y = f(x)$, and it is given that $f'(x) = 2x^2 - 7 - \dfrac{4}{x^2}$.

(a) Given that $f(1) = -\frac{1}{3}$, find $f(x)$. [4]

(b) Find the coordinates of the stationary points on the curve. [5]

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(c) Find $f''(x)$. [1]

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(d) Hence, or otherwise, determine the nature of each of the stationary points. [2]

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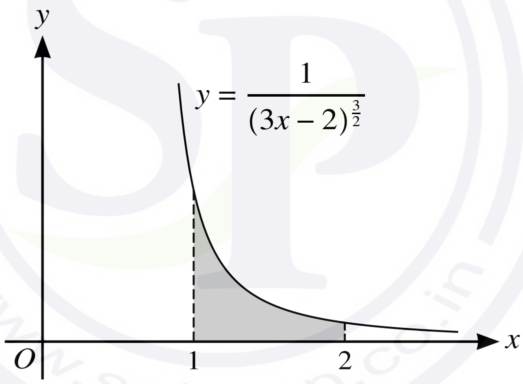
10 **(a)** Find $\displaystyle\int_1^{\infty} \frac{1}{(3x-2)^{\frac{3}{2}}}\,dx$. [4]



The diagram shows the curve with equation $y = \dfrac{1}{(3x-2)^{\frac{3}{2}}}$. The shaded region is bounded by the curve, the x-axis and the lines $x = 1$ and $x = 2$. The shaded region is rotated through 360° about the x-axis.

(b) Find the volume of revolution. [4]

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The normal to the curve at the point $(1,\ 1)$ crosses the y-axis at the point A.

(c) Find the y-coordinate of A. [4]

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Additional Page

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* 2 9 5 2 8 8 8 5 9 5 *

MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **October/November 2021**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
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- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages.

JC21 11_9709_12/2R
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[Turn over

1 Solve the equation $2\cos\theta = 7 - \dfrac{3}{\cos\theta}$ for $-90° < \theta < 90°$. [4]

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2 The graph of $y = f(x)$ is transformed to the graph of $y = f(2x) - 3$.

(a) Describe fully the two single transformations that have been combined to give the resulting transformation. [3]

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The point $P(5, 6)$ lies on the transformed curve $y = f(2x) - 3$.

(b) State the coordinates of the corresponding point on the original curve $y = f(x)$. [2]

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3 The function f is defined as follows:

$$f(x) = \frac{x+3}{x-1} \text{ for } x > 1.$$

(a) Find the value of ff(5). [2]

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(b) Find an expression for $f^{-1}(x)$. [3]

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4 A curve is such that $\frac{dy}{dx} = \frac{8}{(3x+2)^2}$. The curve passes through the point $\left(2,\ 5\frac{2}{3}\right)$.

Find the equation of the curve. [4]

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5 The first, third and fifth terms of an arithmetic progression are $2\cos x$, $-6\sqrt{3}\sin x$ and $10\cos x$ respectively, where $\frac{1}{2}\pi < x < \pi$.

(a) Find the exact value of x. [3]

(b) Hence find the exact sum of the first 25 terms of the progression. [3]

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6 The second term of a geometric progression is 54 and the sum to infinity of the progression is 243. The common ratio is greater than $\frac{1}{2}$.

Find the tenth term, giving your answer in exact form. [5]

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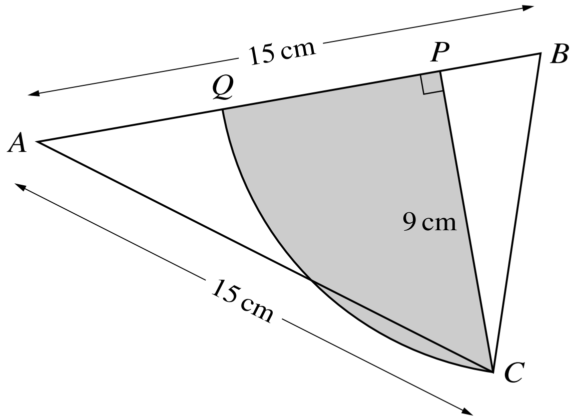

In the diagram the lengths of AB and AC are both 15 cm. The point P is the foot of the perpendicular from C to AB. The length $CP = 9$ cm. An arc of a circle with centre B passes through C and meets AB at Q.

(a) Show that angle $ABC = 1.25$ radians, correct to 3 significant figures. [2]

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(b) Calculate the area of the shaded region which is bounded by the arc CQ and the lines CP and PQ. [4]

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8 **(a)** It is given that in the expansion of $(4 + 2x)(2 - ax)^5$, the coefficient of x^2 is -15.

Find the possible values of a. [4]

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(b) It is given instead that in the expansion of $(4 + 2x)(2 - ax)^5$, the coefficient of x^2 is k. It is also given that there is only one value of a which leads to this value of k.

Find the values of k and a. [4]

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9 The volume $V\,\text{m}^3$ of a large circular mound of iron ore of radius r m is modelled by the equation $V = \frac{3}{2}\left(r - \frac{1}{2}\right)^3 - 1$ for $r \geqslant 2$. Iron ore is added to the mound at a constant rate of $1.5\,\text{m}^3$ per second.

(a) Find the rate at which the radius of the mound is increasing at the instant when the radius is 5.5 m. [3]

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(b) Find the volume of the mound at the instant when the radius is increasing at 0.1 m per second. [3]

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10 The function f is defined by $f(x) = x^2 + \frac{k}{x} + 2$ for $x > 0$.

(a) Given that the curve with equation $y = f(x)$ has a stationary point when $x = 2$, find k. [3]

(b) Determine the nature of the stationary point. [2]

(c) Given that this is the only stationary point of the curve, find the range of f. [2]

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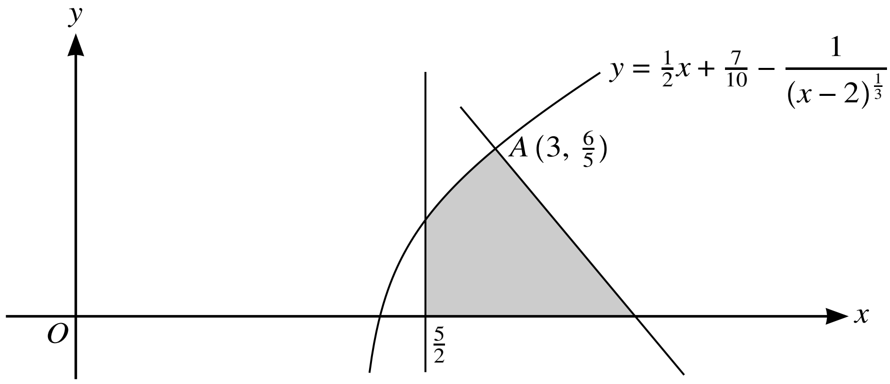

The diagram shows the line $x = \frac{5}{2}$, part of the curve $y = \frac{1}{2}x + \frac{7}{10} - \frac{1}{(x-2)^{\frac{1}{3}}}$ and the normal to the curve at the point $A\ \left(3,\ \frac{6}{5}\right)$.

(a) Find the x-coordinate of the point where the normal to the curve meets the x-axis. [5]

(b) Find the area of the shaded region, giving your answer correct to 2 decimal places. [6]

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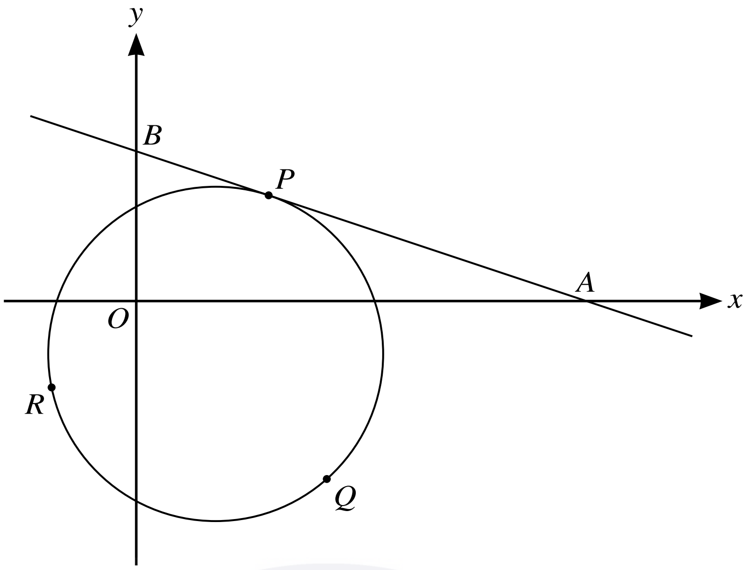

The diagram shows the circle with equation $x^2 + y^2 - 6x + 4y - 27 = 0$ and the tangent to the circle at the point $P(5, 4)$.

(a) The tangent to the circle at P meets the x-axis at A and the y-axis at B.

Find the area of triangle OAB, where O is the origin. [5]

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(b) Points Q and R also lie on the circle, such that PQR is an equilateral triangle.

Find the exact area of triangle PQR. [3]

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* 5 8 9 6 4 2 1 5 1 7 *

MATHEMATICS **9709/13**

Paper 1 Pure Mathematics 1 **October/November 2021**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS
- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
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- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION
- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

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1 The graph of $y = \mathrm{f}(x)$ is transformed to the graph of $y = 3 - \mathrm{f}(x)$.

Describe fully, in the correct order, the two transformations that have been combined. [4]

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2 **(a)** Find the first three terms, in ascending powers of x, in the expansion of $(1 + ax)^6$. [1]

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(b) Given that the coefficient of x^2 in the expansion of $(1 - 3x)(1 + ax)^6$ is -3, find the possible values of the constant a. [4]

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3 **(a)** Express $5y^2 - 30y + 50$ in the form $5(y + a)^2 + b$, where a and b are constants. [2]

(b) The function f is defined by $f(x) = x^5 - 10x^3 + 50x$ for $x \in \mathbb{R}$.

Determine whether f is an increasing function, a decreasing function or neither. [3]

4 The first term of an arithmetic progression is 84 and the common difference is -3.

(a) Find the smallest value of n for which the nth term is negative. [2]

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It is given that the sum of the first $2k$ terms of this progression is equal to the sum of the first k terms.

(b) Find the value of k. [3]

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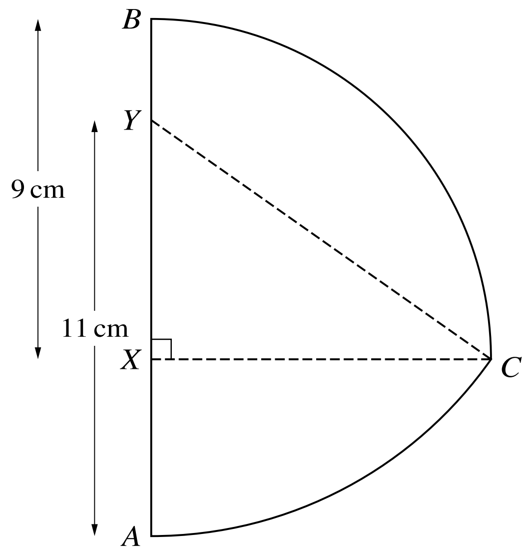

In the diagram, X and Y are points on the line AB such that $BX = 9$ cm and $AY = 11$ cm. Arc BC is part of a circle with centre X and radius 9 cm, where CX is perpendicular to AB. Arc AC is part of a circle with centre Y and radius 11 cm.

(a) Show that angle $XYC = 0.9582$ radians, correct to 4 significant figures. [1]

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(b) Find the perimeter of ABC. [6]

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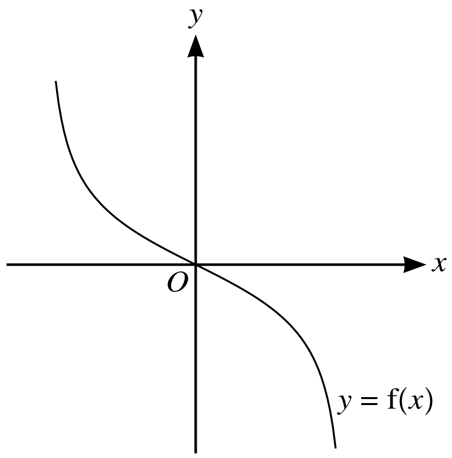

The diagram shows the graph of $y = \mathrm{f}(x)$.

(a) On this diagram sketch the graph of $y = \mathrm{f}^{-1}(x)$. [1]

It is now given that $\mathrm{f}(x) = -\dfrac{x}{\sqrt{4 - x^2}}$ where $-2 < x < 2$.

(b) Find an expression for $\mathrm{f}^{-1}(x)$. [4]

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The function g is defined by $g(x) = 2x$ for $-a < x < a$, where a is a constant.

(c) State the maximum possible value of a for which fg can be formed. [1]

(d) Assuming that fg can be formed, find and simplify an expression for $fg(x)$. [2]

7 (a) Show that the equation $\frac{\tan x + \cos x}{\tan x - \cos x} = k$, where k is a constant, can be expressed as

$$(k+1)\sin^2 x + (k-1)\sin x - (k+1) = 0.$$ [4]

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(b) Hence solve the equation $\dfrac{\tan x + \cos x}{\tan x - \cos x} = 4$ for $0° \leqslant x \leqslant 360°$. [4]

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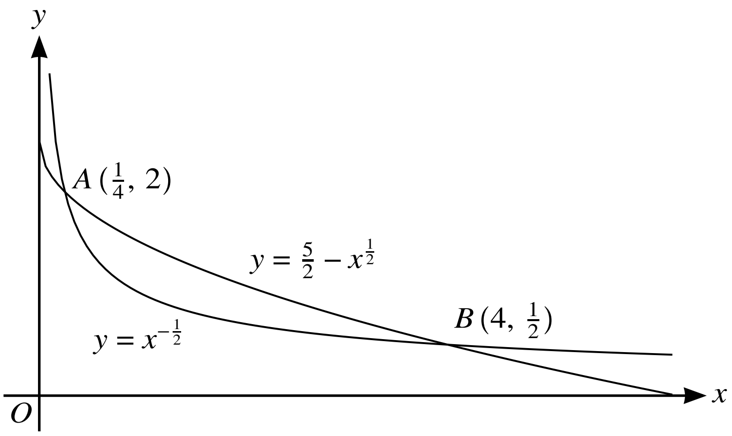

The diagram shows the curves with equations $y = x^{-\frac{1}{2}}$ and $y = \frac{5}{2} - x^{\frac{1}{2}}$. The curves intersect at the points $A\left(\frac{1}{4}, 2\right)$ and $B\left(4, \frac{1}{2}\right)$.

(a) Find the area of the region between the two curves. [6]

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(b) The normal to the curve $y = x^{-\frac{1}{2}}$ at the point $(1,\ 1)$ intersects the y-axis at the point $(0,\ p)$.

Find the value of p. [4]

9 The line $y = 2x + 5$ intersects the circle with equation $x^2 + y^2 = 20$ at A and B.

(a) Find the coordinates of A and B in surd form and hence find the exact length of the chord AB. [7]

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A straight line through the point (10, 0) with gradient m is a tangent to the circle.

(b) Find the two possible values of m. [5]

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10 A curve has equation $y = \mathrm{f}(x)$ and it is given that

$$\mathrm{f}'(x) = \left(\tfrac{1}{2}x + k\right)^{-2} - (1 + k)^{-2},$$

where k is a constant. The curve has a minimum point at $x = 2$.

(a) Find $\mathrm{f}''(x)$ in terms of k and x, and hence find the set of possible values of k. [3]

It is now given that $k = -3$ and the minimum point is at $(2, 3\tfrac{1}{2})$.

(b) Find $\mathrm{f}(x)$. [4]

(c) Find the coordinates of the other stationary point and determine its nature. [4]

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CENTRE NUMBER		CANDIDATE NUMBER	

7942312992

MATHEMATICS **9709/11**

Paper 1 Pure Mathematics 1 **May/June 2021**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
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- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

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[Turn over

1 The equation of a curve is such that $\frac{dy}{dx} = \frac{3}{x^4} + 32x^3$. It is given that the curve passes through the point $(\frac{1}{2}, 4)$.

Find the equation of the curve. [4]

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2 The sum of the first 20 terms of an arithmetic progression is 405 and the sum of the first 40 terms is 1410.

Find the 60th term of the progression. [5]

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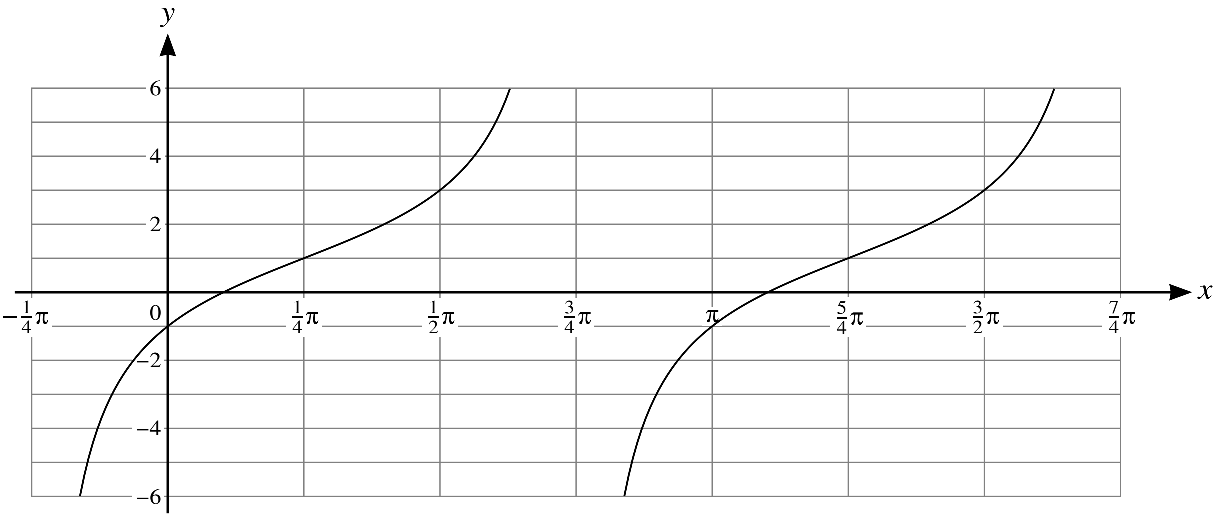
3 **(a)** Find the first three terms in the expansion of $(3-2x)^5$ in ascending powers of x. [3]

(b) Hence find the coefficient of x^2 in the expansion of $(4+x)^2(3-2x)^5$. [3]

4



The diagram shows part of the graph of $y = a\tan(x - b) + c$.

Given that $0 < b < \pi$, state the values of the constants a, b and c. [3]

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5 The fifth, sixth and seventh terms of a geometric progression are $8k$, -12 and $2k$ respectively.

Given that k is negative, find the sum to infinity of the progression. [4]

6 The equation of a curve is $y = (2k - 3)x^2 - kx - (k - 2)$, where k is a constant. The line $y = 3x - 4$ is a tangent to the curve.

Find the value of k. [5]

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7 **(a)** Prove the identity $\dfrac{1-2\sin^2\theta}{1-\sin^2\theta} \equiv 1-\tan^2\theta$. [2]

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(b) Hence solve the equation $\frac{1 - 2\sin^2\theta}{1 - \sin^2\theta} = 2\tan^4\theta$ for $0° \leqslant \theta \leqslant 180°$. [3]

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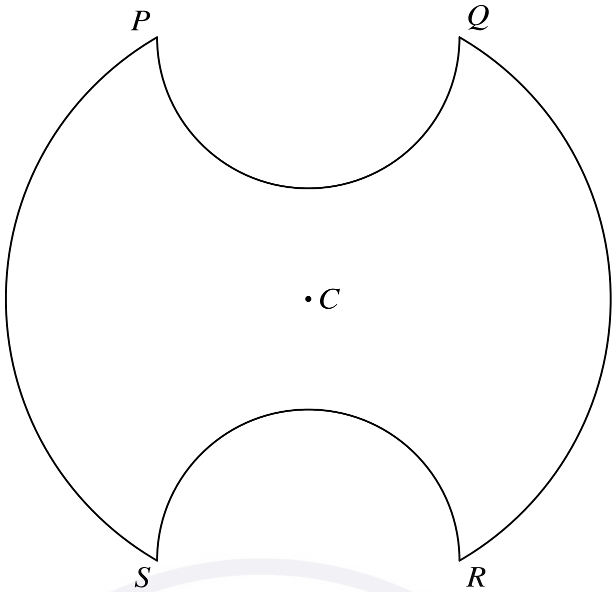

The diagram shows a symmetrical metal plate. The plate is made by removing two identical pieces from a circular disc with centre C. The boundary of the plate consists of two arcs PS and QR of the original circle and two semicircles with PQ and RS as diameters. The radius of the circle with centre C is 4 cm, and $PQ = RS = 4$ cm also.

(a) Show that angle $PCS = \frac{2}{3}\pi$ radians. [2]

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(b) Find the exact perimeter of the plate. [3]

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(c) Show that the area of the plate is $\left(\frac{20}{3}\pi + 8\sqrt{3}\right)\,\text{cm}^2$. [5]

9 Functions f and g are defined as follows:

$$f(x) = (x-2)^2 - 4 \text{ for } x \geqslant 2,$$
$$g(x) = ax + 2 \text{ for } x \in \mathbb{R},$$

where a is a constant.

(a) State the range of f. [1]

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(b) Find $f^{-1}(x)$. [2]

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(c) Given that $a = -\frac{5}{3}$, solve the equation $f(x) = g(x)$. [3]

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(d) Given instead that $gg f^{-1}(12) = 62$, find the possible values of a. [5]

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10 The equation of a circle is $x^2 + y^2 - 4x + 6y - 77 = 0$.

(a) Find the x-coordinates of the points A and B where the circle intersects the x-axis. [2]

(b) Find the point of intersection of the tangents to the circle at A and B. [6]

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11 The equation of a curve is $y = 2\sqrt{3x + 4} - x$.

(a) Find the equation of the normal to the curve at the point (4, 4), giving your answer in the form $y = mx + c$. [5]

(b) Find the coordinates of the stationary point. [3]

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(c) Determine the nature of the stationary point. [2]

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(d) Find the exact area of the region bounded by the curve, the x-axis and the lines $x = 0$ and $x = 4$. [4]

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CENTRE NUMBER

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2355526103

MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **May/June 2021**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
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INFORMATION

- The total mark for this paper is 75.
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1 **(a)** Express $16x^2 - 24x + 10$ in the form $(4x + a)^2 + b$. [2]

(b) It is given that the equation $16x^2 - 24x + 10 = k$, where k is a constant, has exactly one root.

Find the value of this root. [2]

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2 **(a)** The graph of $y = f(x)$ is transformed to the graph of $y = 2f(x - 1)$.

Describe fully the two single transformations which have been combined to give the resulting transformation. [3]

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(b) The curve $y = \sin 2x - 5x$ is reflected in the y-axis and then stretched by scale factor $\frac{1}{3}$ in the x-direction.

Write down the equation of the transformed curve. [2]

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3 The equation of a curve is $y = (x - 3)\sqrt{x + 1} + 3$. The following points lie on the curve. Non-exact values are rounded to 4 decimal places.

$A\,(2,\ k)$ $B\,(2.9,\ 2.8025)$ $C\,(2.99,\ 2.9800)$ $D\,(2.999,\ 2.9980)$ $E\,(3,\ 3)$

(a) Find k, giving your answer correct to 4 decimal places. [1]

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(b) Find the gradient of AE, giving your answer correct to 4 decimal places. [1]

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The gradients of BE, CE and DE, rounded to 4 decimal places, are 1.9748, 1.9975 and 1.9997 respectively.

(c) State, giving a reason for your answer, what the values of the four gradients suggest about the gradient of the curve at the point E. [2]

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4 The coefficient of x in the expansion of $\left(4x + \frac{10}{x}\right)^3$ is p. The coefficient of $\frac{1}{x}$ in the expansion of $\left(2x + \frac{k}{x^2}\right)^5$ is q.

Given that $p = 6q$, find the possible values of k. [5]

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5 The function f is defined by $f(x) = 2x^2 + 3$ for $x \geqslant 0$.

(a) Find and simplify an expression for $ff(x)$. [2]

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(b) Solve the equation $ff(x) = 34x^2 + 19$. [4]

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6 Points A and B have coordinates $(8, 3)$ and (p, q) respectively. The equation of the perpendicular bisector of AB is $y = -2x + 4$.

Find the values of p and q. [4]

7 The point A has coordinates $(1, 5)$ and the line l has gradient $-\frac{2}{3}$ and passes through A. A circle has centre $(5, 11)$ and radius $\sqrt{52}$.

(a) Show that l is the tangent to the circle at A. [2]

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(b) Find the equation of the other circle of radius $\sqrt{52}$ for which l is also the tangent at A. [3]

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8 The first, second and third terms of an arithmetic progression are a, $\frac{3}{2}a$ and b respectively, where a and b are positive constants. The first, second and third terms of a geometric progression are a, 18 and $b + 3$ respectively.

(a) Find the values of a and b. [5]

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(b) Find the sum of the first 20 terms of the arithmetic progression. [3]

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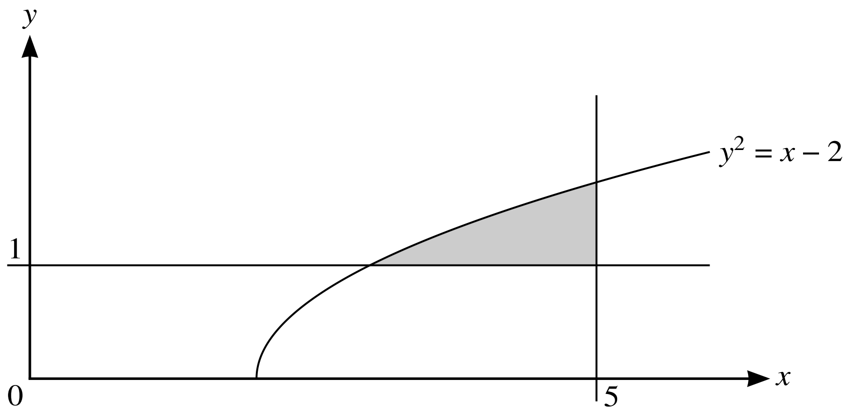

The diagram shows part of the curve with equation $y^2 = x - 2$ and the lines $x = 5$ and $y = 1$. The shaded region enclosed by the curve and the lines is rotated through 360° about the x-axis.

Find the volume obtained. [6]

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10 **(a)** Prove the identity $\dfrac{1+\sin x}{1-\sin x} - \dfrac{1-\sin x}{1+\sin x} \equiv \dfrac{4\tan x}{\cos x}$. [4]

(b) Hence solve the equation $\dfrac{1+\sin x}{1-\sin x} - \dfrac{1-\sin x}{1+\sin x} = 8\tan x$ for $0 \leqslant x \leqslant \frac{1}{2}\pi$. [3]

11 The gradient of a curve is given by $\frac{dy}{dx} = 6(3x - 5)^3 - kx^2$, where k is a constant. The curve has a stationary point at $(2, -3.5)$.

(a) Find the value of k. [2]

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(b) Find the equation of the curve. [4]

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(c) Find $\frac{d^2y}{dx^2}$. [2]

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(d) Determine the nature of the stationary point at $(2, -3.5)$. [2]

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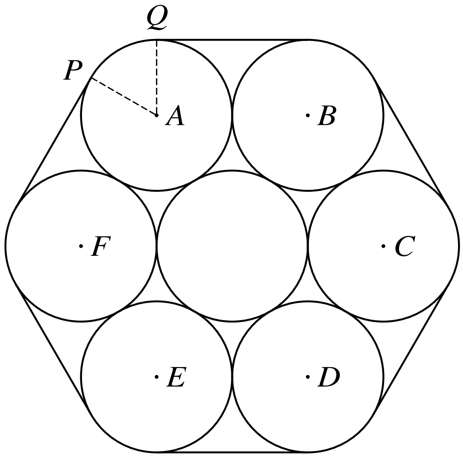

The diagram shows a cross-section of seven cylindrical pipes, each of radius 20 cm, held together by a thin rope which is wrapped tightly around the pipes. The centres of the six outer pipes are A, B, C, D, E and F. Points P and Q are situated where straight sections of the rope meet the pipe with centre A.

(a) Show that angle $PAQ = \frac{1}{3}\pi$ radians. [2]

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(b) Find the length of the rope. [4]

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(c) Find the area of the hexagon $ABCDEF$, giving your answer in terms of $\sqrt{3}$. [2]

(d) Find the area of the complete region enclosed by the rope. [3]

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CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

6412245051

MATHEMATICS **9709/13**

Paper 1 Pure Mathematics 1 **May/June 2021**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

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1 A curve with equation $y = f(x)$ is such that $f'(x) = 6x^2 - \frac{8}{x^2}$. It is given that the curve passes through the point $(2, 7)$.

Find $f(x)$. [3]

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2 The function f is defined by $f(x) = \frac{1}{3}(2x - 1)^{\frac{3}{2}} - 2x$ for $\frac{1}{2} < x < a$. It is given that f is a decreasing function.

Find the maximum possible value of the constant a. [4]

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3 A line with equation $y = mx - 6$ is a tangent to the curve with equation $y = x^2 - 4x + 3$.

Find the possible values of the constant m, and the corresponding coordinates of the points at which the line touches the curve. [6]

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4 **(a)** Show that the equation

$$\frac{\tan x + \sin x}{\tan x - \sin x} = k,$$

where k is a constant, may be expressed as

$$\frac{1 + \cos x}{1 - \cos x} = k. \qquad [2]$$

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(b) Hence express $\cos x$ in terms of k. [2]

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(c) Hence solve the equation $\dfrac{\tan x + \sin x}{\tan x - \sin x} = 4$ for $-\pi < x < \pi$. [2]

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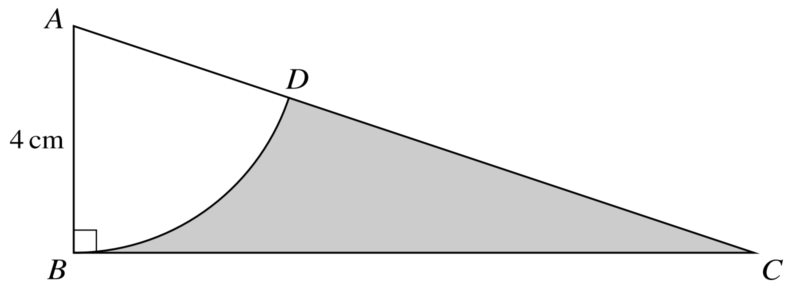

The diagram shows a triangle ABC, in which angle $ABC = 90°$ and $AB = 4$ cm. The sector ABD is part of a circle with centre A. The area of the sector is $10\,\text{cm}^2$.

(a) Find angle BAD in radians. [2]

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(b) Find the perimeter of the shaded region. [4]

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6 Functions f and g are both defined for $x \in \mathbb{R}$ and are given by

$$f(x) = x^2 - 2x + 5,$$
$$g(x) = x^2 + 4x + 13.$$

(a) By first expressing each of $f(x)$ and $g(x)$ in completed square form, express $g(x)$ in the form $f(x + p) + q$, where p and q are constants. [4]

(b) Describe fully the transformation which transforms the graph of $y = f(x)$ to the graph of $y = g(x)$. [2]

7 **(a)** Write down the first four terms of the expansion, in ascending powers of x, of $(a - x)^6$. [2]

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(b) Given that the coefficient of x^2 in the expansion of $\left(1 + \frac{2}{ax}\right)(a - x)^6$ is -20, find in exact form the possible values of the constant a. [5]

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8 Functions f and g are defined as follows:

$$f : x \mapsto x^2 - 1 \text{ for } x < 0,$$
$$g : x \mapsto \frac{1}{2x+1} \text{ for } x < -\tfrac{1}{2}.$$

(a) Solve the equation $fg(x) = 3$. [4]

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(b) Find an expression for $(\mathrm{fg})^{-1}(x)$. [3]

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9 **(a)** A geometric progression is such that the second term is equal to 24% of the sum to infinity.

Find the possible values of the common ratio. [3]

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(b) An arithmetic progression P has first term a and common difference d. An arithmetic progression Q has first term $2(a + 1)$ and common difference $(d + 1)$. It is given that

$$\frac{\text{5th term of } P}{\text{12th term of } Q} = \frac{1}{3} \quad \text{and} \quad \frac{\text{Sum of first 5 terms of } P}{\text{Sum of first 5 terms of } Q} = \frac{2}{3}.$$

Find the value of a and the value of d. [6]

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10 Points $A(-2, 3)$, $B(3, 0)$ and $C(6, 5)$ lie on the circumference of a circle with centre D.

(a) Show that angle $ABC = 90°$. [2]

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(b) Hence state the coordinates of D. [1]

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(c) Find an equation of the circle. [2]

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The point E lies on the circumference of the circle such that BE is a diameter.

(d) Find an equation of the tangent to the circle at E. [5]

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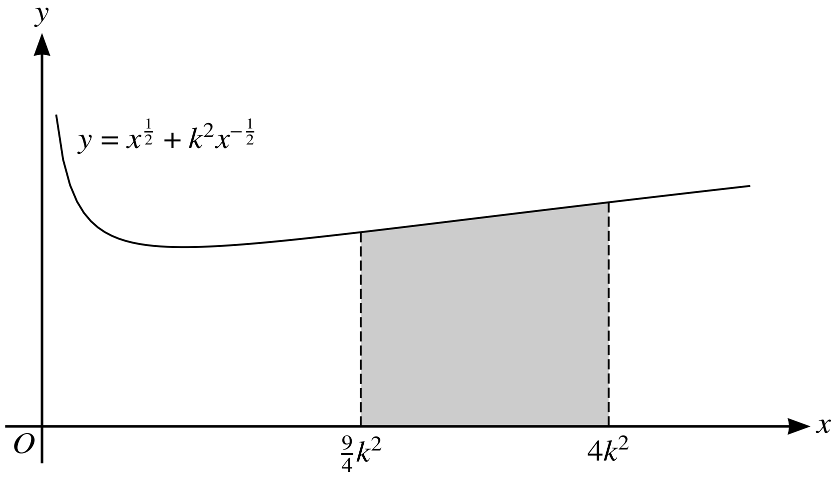

The diagram shows part of the curve with equation $y = x^{\frac{1}{2}} + k^2x^{-\frac{1}{2}}$, where k is a positive constant.

(a) Find the coordinates of the minimum point of the curve, giving your answer in terms of k. [4]

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The tangent at the point on the curve where $x = 4k^2$ intersects the y-axis at P.

(b) Find the y-coordinate of P in terms of k. [4]

The shaded region is bounded by the curve, the x-axis and the lines $x = \frac{9}{4}k^2$ and $x = 4k^2$.

(c) Find the area of the shaded region in terms of k. [3]

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CENTRE NUMBER

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5712505207

MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **February/March 2021**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
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- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

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1 **(a)** Find the first three terms in the expansion, in ascending powers of x, of $(1 + x)^5$. [1]

(b) Find the first three terms in the expansion, in ascending powers of x, of $(1 - 2x)^6$. [2]

(c) Hence find the coefficient of x^2 in the expansion of $(1 + x)^5(1 - 2x)^6$. [2]

2 By using a suitable substitution, solve the equation

$$(2x-3)^2 - \frac{4}{(2x-3)^2} - 3 = 0.$$ [4]

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3 Solve the equation $\dfrac{\tan\theta + 2\sin\theta}{\tan\theta - 2\sin\theta} = 3$ for $0° < \theta < 180°$. [4]

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4 A line has equation $y = 3x + k$ and a curve has equation $y = x^2 + kx + 6$, where k is a constant.

Find the set of values of k for which the line and curve have two distinct points of intersection. [5]

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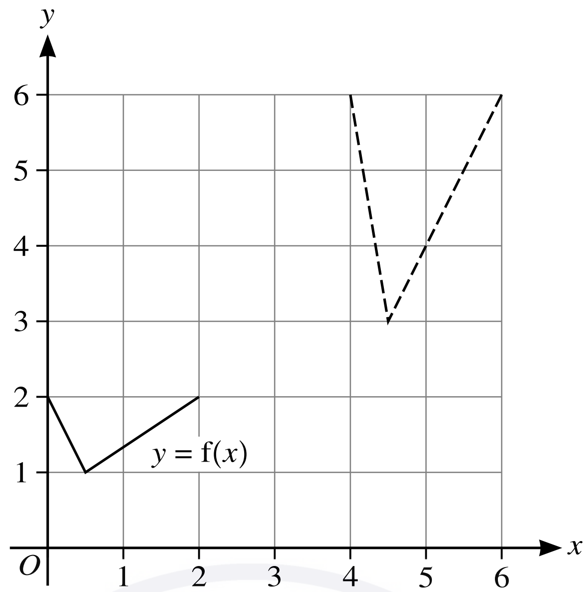

In the diagram, the graph of $y = f(x)$ is shown with solid lines. The graph shown with broken lines is a transformation of $y = f(x)$.

(a) Describe fully the two single transformations of $y = f(x)$ that have been combined to give the resulting transformation. [4]

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(b) State in terms of y, f and x, the equation of the graph shown with broken lines. [2]

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6 A curve is such that $\frac{dy}{dx} = \frac{6}{(3x-2)^3}$ and $A\,(1, -3)$ lies on the curve. A point is moving along the curve and at A the y-coordinate of the point is increasing at 3 units per second.

(a) Find the rate of increase at A of the x-coordinate of the point. [3]

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(b) Find the equation of the curve. [4]

7 Functions f and g are defined as follows:

$$\text{f} : x \mapsto x^2 + 2x + 3 \text{ for } x \leqslant -1,$$
$$\text{g} : x \mapsto 2x + 1 \text{ for } x \geqslant -1.$$

(a) Express $\text{f}(x)$ in the form $(x + a)^2 + b$ and state the range of f. [3]

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(b) Find an expression for $f^{-1}(x)$. [2]

(c) Solve the equation $gf(x) = 13$. [3]

8 The points $A(7, 1)$, $B(7, 9)$ and $C(1, 9)$ are on the circumference of a circle.

(a) Find an equation of the circle. [5]

(b) Find an equation of the tangent to the circle at B. [2]

9 The first term of a progression is $\cos\theta$, where $0 < \theta < \frac{1}{2}\pi$.

(a) For the case where the progression is geometric, the sum to infinity is $\dfrac{1}{\cos\theta}$.

(i) Show that the second term is $\cos\theta\sin^2\theta$. [3]

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(ii) Find the sum of the first 12 terms when $\theta = \frac{1}{3}\pi$, giving your answer correct to 4 significant figures. [2]

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(b) For the case where the progression is arithmetic, the first two terms are again $\cos\theta$ and $\cos\theta\sin^2\theta$ respectively.

Find the 85th term when $\theta = \frac{1}{3}\pi$. [4]

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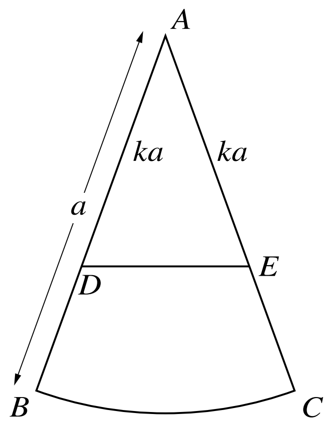

The diagram shows a sector ABC which is part of a circle of radius a. The points D and E lie on AB and AC respectively and are such that $AD = AE = ka$, where $k < 1$. The line DE divides the sector into two regions which are equal in area.

(a) For the case where angle $BAC = \frac{1}{6}\pi$ radians, find k correct to 4 significant figures. [5]

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(b) For the general case in which angle $BAC = \theta$ radians, where $0 < \theta < \frac{1}{2}\pi$, it is given that $\dfrac{\theta}{\sin\theta} > 1$.

Find the set of possible values of k. [3]

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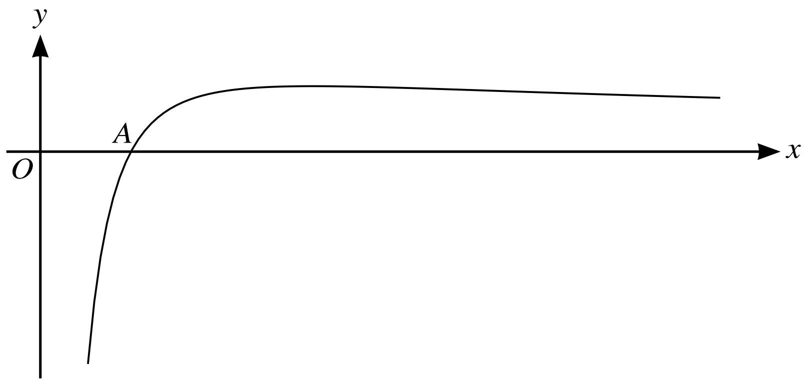

The diagram shows the curve with equation $y = 9\left(x^{-\frac{1}{2}} - 4x^{-\frac{3}{2}}\right)$. The curve crosses the x-axis at the point A.

(a) Find the x-coordinate of A. [2]

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(b) Find the equation of the tangent to the curve at A. [4]

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(c) Find the x-coordinate of the maximum point of the curve. [2]

(d) Find the area of the region bounded by the curve, the x-axis and the line $x = 9$. [4]

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CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

2028675937

MATHEMATICS **9709/11**

Paper 1 Pure Mathematics 1 **October/November 2020**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
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- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

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1 Find the set of values of m for which the line with equation $y = mx - 3$ and the curve with equation $y = 2x^2 + 5$ do not meet. [3]

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2 The equation of a curve is such that $\frac{dy}{dx} = \frac{1}{(x-3)^2} + x$. It is given that the curve passes through the point (2, 7).

Find the equation of the curve. [4]

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3 Air is being pumped into a balloon in the shape of a sphere so that its volume is increasing at a constant rate of $50\,\text{cm}^3\,\text{s}^{-1}$.

Find the rate at which the radius of the balloon is increasing when the radius is 10 cm. [3]

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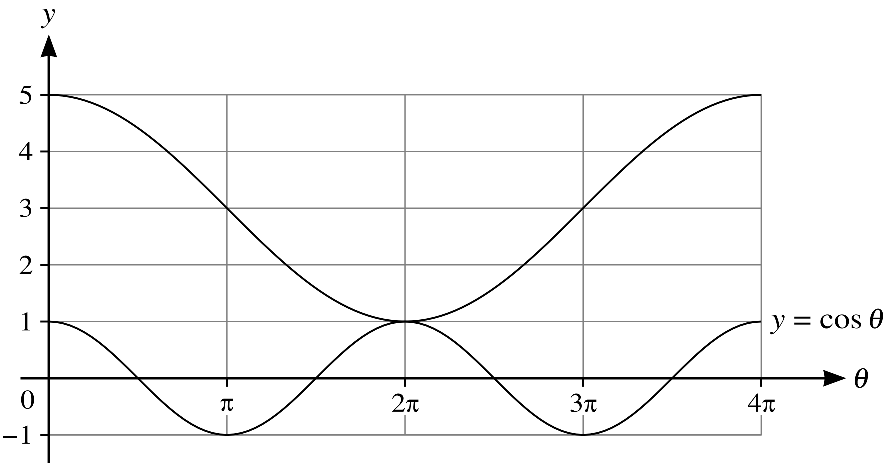

In the diagram, the lower curve has equation $y = \cos\theta$. The upper curve shows the result of applying a combination of transformations to $y = \cos\theta$.

Find, in terms of a cosine function, the equation of the upper curve. [3]

5 In the expansion of $\left(2x^2 + \frac{a}{x}\right)^6$, the coefficients of x^6 and x^3 are equal.

(a) Find the value of the non-zero constant a. [4]

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(b) Find the coefficient of x^6 in the expansion of $(1 - x^3)\left(2x^2 + \frac{a}{x}\right)^6$. [1]

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6 The equation of a curve is $y = 2 + \sqrt{25 - x^2}$.

Find the coordinates of the point on the curve at which the gradient is $\frac{4}{3}$. [5]

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7 **(a)** Show that $\dfrac{\sin\theta}{1-\sin\theta} - \dfrac{\sin\theta}{1+\sin\theta} \equiv 2\tan^2\theta$. [3]

(b) Hence solve the equation $\frac{\sin\theta}{1-\sin\theta} - \frac{\sin\theta}{1+\sin\theta} = 8$, for $0° < \theta < 180°$. [3]

8 A geometric progression has first term a, common ratio r and sum to infinity S. A second geometric progression has first term a, common ratio R and sum to infinity $2S$.

(a) Show that $r = 2R - 1$. [3]

It is now given that the 3rd term of the first progression is equal to the 2nd term of the second progression.

(b) Express S in terms of a. [4]

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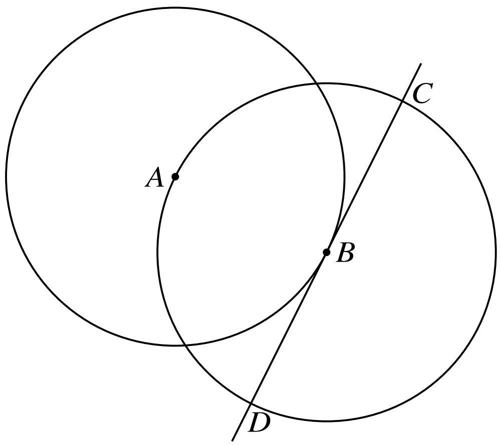

The diagram shows a circle with centre A passing through the point B. A second circle has centre B and passes through A. The tangent at B to the first circle intersects the second circle at C and D.

The coordinates of A are $(-1, 4)$ and the coordinates of B are $(3, 2)$.

(a) Find the equation of the tangent CBD. [2]

(b) Find an equation of the circle with centre B. [3]

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(c) Find, by calculation, the x-coordinates of C and D. [3]

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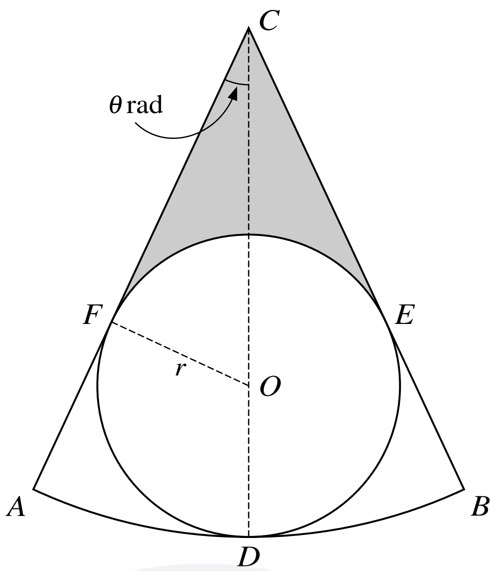

The diagram shows a sector CAB which is part of a circle with centre C. A circle with centre O and radius r lies within the sector and touches it at D, E and F, where COD is a straight line and angle ACD is θ radians.

(a) Find CD in terms of r and $\sin\theta$. [3]

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It is now given that $r = 4$ and $\theta = \frac{1}{6}\pi$.

(b) Find the perimeter of sector CAB in terms of π. [3]

(c) Find the area of the shaded region in terms of π and $\sqrt{3}$. [4]

11 The functions f and g are defined by

$$f(x) = x^2 + 3 \quad \text{for } x > 0,$$
$$g(x) = 2x + 1 \quad \text{for } x > -\tfrac{1}{2}.$$

(a) Find an expression for $fg(x)$. [1]

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(b) Find an expression for $(fg)^{-1}(x)$ and state the domain of $(fg)^{-1}$. [4]

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(c) Solve the equation $fg(x) - 3 = gf(x)$. [4]

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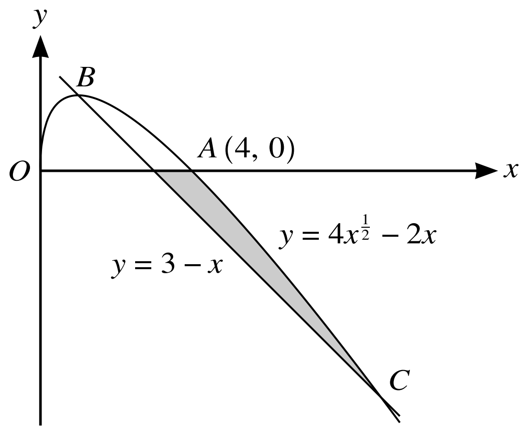
12



The diagram shows a curve with equation $y = 4x^{\frac{1}{2}} - 2x$ for $x \geqslant 0$, and a straight line with equation $y = 3 - x$. The curve crosses the x-axis at $A\,(4, 0)$ and crosses the straight line at B and C.

(a) Find, by calculation, the x-coordinates of B and C. [4]

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(b) Show that B is a stationary point on the curve. [2]

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(c) Find the area of the shaded region. [6]

Additional Page

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CANDIDATE NAME

CENTRE NUMBER

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8279021580

MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **October/November 2020**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
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- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

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1 The coefficient of x^3 in the expansion of $(1 + kx)(1 - 2x)^5$ is 20.

Find the value of the constant k. [4]

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2 The first, second and third terms of a geometric progression are $2p + 6$, $-2p$ and $p + 2$ respectively, where p is positive.

Find the sum to infinity of the progression. [5]

3 The equation of a curve is $y = 2x^2 + m(2x + 1)$, where m is a constant, and the equation of a line is $y = 6x + 4$.

Show that, for all values of m, the line intersects the curve at two distinct points. [5]

4 The sum, S_n, of the first n terms of an arithmetic progression is given by

$$S_n = n^2 + 4n.$$

The kth term in the progression is greater than 200.

Find the smallest possible value of k. [5]

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5 Functions f and g are defined by

$$f(x) = 4x - 2, \quad \text{for } x \in \mathbb{R},$$

$$g(x) = \frac{4}{x+1}, \quad \text{for } x \in \mathbb{R},\ x \neq -1.$$

(a) Find the value of fg(7). [1]

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(b) Find the values of x for which $f^{-1}(x) = g^{-1}(x)$. [5]

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6 (a) Prove the identity $\left(\dfrac{1}{\cos x} - \tan x\right)\left(\dfrac{1}{\sin x} + 1\right) \equiv \dfrac{1}{\tan x}$. [4]

(b) Hence solve the equation $\left(\dfrac{1}{\cos x} - \tan x\right)\left(\dfrac{1}{\sin x} + 1\right) = 2\tan^2 x$ for $0° \leqslant x \leqslant 180°$. [2]

7 The point (4, 7) lies on the curve $y = \mathrm{f}(x)$ and it is given that $\mathrm{f}'(x) = 6x^{-\frac{1}{2}} - 4x^{-\frac{3}{2}}$.

(a) A point moves along the curve in such a way that the x-coordinate is increasing at a constant rate of 0.12 units per second.

Find the rate of increase of the y-coordinate when $x = 4$. [3]

(b) Find the equation of the curve. [4]

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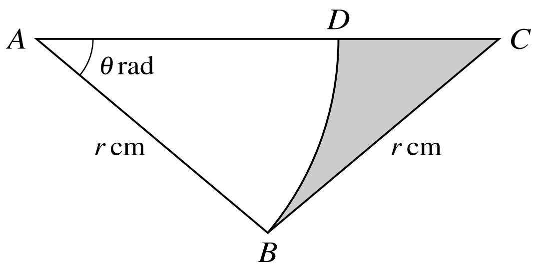

In the diagram, ABC is an isosceles triangle with $AB = BC = r$ cm and angle $BAC = \theta$ radians. The point D lies on AC and ABD is a sector of a circle with centre A.

(a) Express the area of the shaded region in terms of r and θ. [3]

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(b) In the case where $r = 10$ and $\theta = 0.6$, find the perimeter of the shaded region. [4]

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9 A circle has centre at the point $B(5, 1)$. The point $A(-1, -2)$ lies on the circle.

(a) Find the equation of the circle. [3]

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Point C is such that AC is a diameter of the circle. Point D has coordinates $(5, 16)$.

(b) Show that DC is a tangent to the circle. [4]

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The other tangent from D to the circle touches the circle at E.

(c) Find the coordinates of E. [2]

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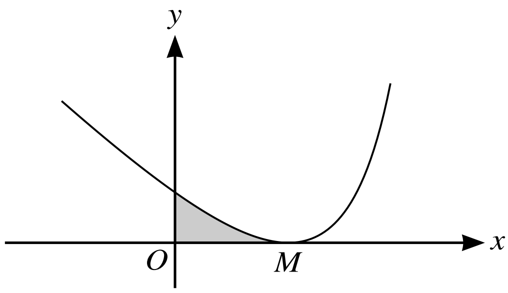

The diagram shows part of the curve $y = \dfrac{2}{(3-2x)^2} - x$ and its minimum point M, which lies on the x-axis.

(a) Find expressions for $\dfrac{dy}{dx}$, $\dfrac{d^2y}{dx^2}$ and $\int y\,dx$. [6]

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(b) Find, by calculation, the x-coordinate of M. [2]

(c) Find the area of the shaded region bounded by the curve and the coordinate axes. [2]

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11 A curve has equation $y = 3\cos 2x + 2$ for $0 \leqslant x \leqslant \pi$.

(a) State the greatest and least values of y. [2]

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(b) Sketch the graph of $y = 3\cos 2x + 2$ for $0 \leqslant x \leqslant \pi$. [2]

(c) By considering the straight line $y = kx$, where k is a constant, state the number of solutions of the equation $3\cos 2x + 2 = kx$ for $0 \leqslant x \leqslant \pi$ in each of the following cases.

(i) $k = -3$ [1]

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(ii) $k = 1$ [1]

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(iii) $k = 3$ [1]

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Functions f, g and h are defined for $x \in \mathbb{R}$ by

$$f(x) = 3\cos 2x + 2,$$
$$g(x) = f(2x) + 4,$$
$$h(x) = 2f\left(x + \tfrac{1}{2}\pi\right).$$

(d) Describe fully a sequence of transformations that maps the graph of $y = f(x)$ on to $y = g(x)$. [2]

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(e) Describe fully a sequence of transformations that maps the graph of $y = f(x)$ on to $y = h(x)$. [2]

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CANDIDATE NUMBER

8677467398

MATHEMATICS **9709/13**

Paper 1 Pure Mathematics 1 **October/November 2020**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
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- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

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1 **(a)** Express $x^2 + 6x + 5$ in the form $(x + a)^2 + b$, where a and b are constants. [2]

(b) The curve with equation $y = x^2$ is transformed to the curve with equation $y = x^2 + 6x + 5$.

Describe fully the transformation(s) involved. [2]

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2 The function f is defined by $f(x) = \dfrac{2}{(x+2)^2}$ for $x > -2$.

(a) Find $\displaystyle\int_1^{\infty} f(x)\,dx$. [3]

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(b) The equation of a curve is such that $\dfrac{dy}{dx} = f(x)$. It is given that the point $(-1, -1)$ lies on the curve.

Find the equation of the curve. [2]

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3 Solve the equation $3\tan^2\theta + 1 = \dfrac{2}{\tan^2\theta}$ for $0° < \theta < 180°$. [5]

4 A curve has equation $y = 3x^2 - 4x + 4$ and a straight line has equation $y = mx + m - 1$, where m is a constant.

Find the set of values of m for which the curve and the line have two distinct points of intersection. [5]

5 In the expansion of $(a + bx)^7$, where a and b are non-zero constants, the coefficients of x, x^2 and x^4 are the first, second and third terms respectively of a geometric progression.

Find the value of $\frac{a}{b}$. [5]

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6 The function f is defined by $f(x) = \dfrac{2x}{3x - 1}$ for $x > \frac{1}{3}$.

(a) Find an expression for $f^{-1}(x)$. [3]

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(b) Show that $\dfrac{2}{3} + \dfrac{2}{3(3x - 1)}$ can be expressed as $\dfrac{2x}{3x - 1}$. [2]

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(c) State the range of f. [1]

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7 The first and second terms of an arithmetic progression are $\frac{1}{\cos^2\theta}$ and $-\frac{\tan^2\theta}{\cos^2\theta}$, respectively, where $0 < \theta < \frac{1}{2}\pi$.

(a) Show that the common difference is $-\frac{1}{\cos^4\theta}$. [4]

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(b) Find the exact value of the 13th term when $\theta = \frac{1}{6}\pi$. [3]

8 The equation of a curve is $y = 2x + 1 + \dfrac{1}{2x+1}$ for $x > -\frac{1}{2}$.

(a) Find $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$. [3]

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(b) Find the coordinates of the stationary point and determine the nature of the stationary point. [5]

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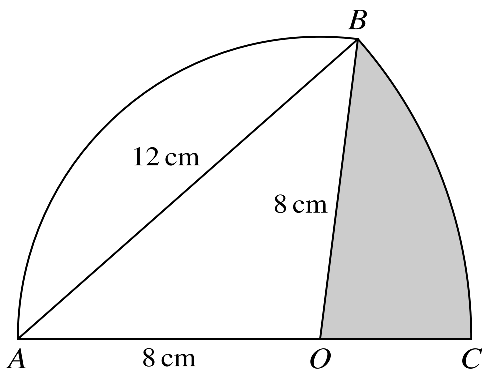

In the diagram, arc AB is part of a circle with centre O and radius 8 cm. Arc BC is part of a circle with centre A and radius 12 cm, where AOC is a straight line.

(a) Find angle BAO in radians. [2]

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(b) Find the area of the shaded region. [4]

(c) Find the perimeter of the shaded region. [3]

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10 A curve has equation $y = \frac{1}{k}x^{\frac{1}{2}} + x^{-\frac{1}{2}} + \frac{1}{k^2}$ where $x > 0$ and k is a positive constant.

(a) It is given that when $x = \frac{1}{4}$, the gradient of the curve is 3.

Find the value of k. [4]

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(b) It is given instead that $\int_{\frac{1}{4}k^2}^{k^2} \left(\frac{1}{k} x^{\frac{1}{2}} + x^{-\frac{1}{2}} + \frac{1}{k^2} \right) \mathrm{d}x = \frac{13}{12}$.

Find the value of k. [5]

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11 A circle with centre C has equation $(x-8)^2 + (y-4)^2 = 100$.

(a) Show that the point $T\,(-6, 6)$ is outside the circle. [3]

Two tangents from T to the circle are drawn.

(b) Show that the angle between one of the tangents and CT is exactly 45°. [2]

The two tangents touch the circle at A and B.

(c) Find the equation of the line AB, giving your answer in the form $y = mx + c$. [4]

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(d) Find the x-coordinates of A and B. [3]

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CENTRE NUMBER		CANDIDATE NUMBER	

* 2 8 5 1 7 9 8 5 0 9 *

MATHEMATICS **9709/11**

Paper 1 Pure Mathematics 1 **May/June 2020**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
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- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

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1 The sum of the first nine terms of an arithmetic progression is 117. The sum of the next four terms is 91.

Find the first term and the common difference of the progression. [4]

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2 The coefficient of $\frac{1}{x}$ in the expansion of $\left(kx + \frac{1}{x}\right)^5 + \left(1 - \frac{2}{x}\right)^8$ is 74.

Find the value of the positive constant k. [5]

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3 Each year the selling price of a diamond necklace increases by 5% of the price the year before. The selling price of the necklace in the year 2000 was \$36 000.

(a) Write down an expression for the selling price of the necklace n years later and hence find the selling price in 2008. [3]

(b) The company that makes the necklace only sells one each year. Find the total amount of money obtained in the ten-year period starting in the year 2000. [2]

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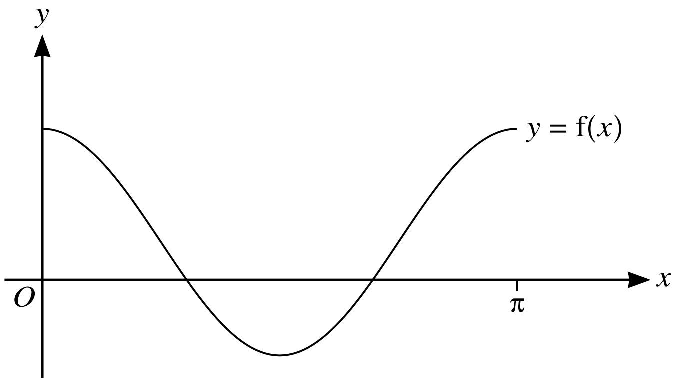

The diagram shows the graph of $y = f(x)$, where $f(x) = \frac{3}{2}\cos 2x + \frac{1}{2}$ for $0 \leqslant x \leqslant \pi$.

(a) State the range of f. [2]

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A function g is such that $g(x) = f(x) + k$, where k is a positive constant. The x-axis is a tangent to the curve $y = g(x)$.

(b) State the value of k and hence describe fully the transformation that maps the curve $y = f(x)$ on to $y = g(x)$. [2]

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(c) State the equation of the curve which is the reflection of $y = f(x)$ in the x-axis. Give your answer in the form $y = a\cos 2x + b$, where a and b are constants. [1]

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5 The equation of a line is $y = mx + c$, where m and c are constants, and the equation of a curve is $xy = 16$.

(a) Given that the line is a tangent to the curve, express m in terms of c. [3]

(b) Given instead that $m = -4$, find the set of values of c for which the line intersects the curve at two distinct points. [3]

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6 Functions f and g are defined for $x \in \mathbb{R}$ by

$$f : x \mapsto \tfrac{1}{2}x - a,$$
$$g : x \mapsto 3x + b,$$

where a and b are constants.

(a) Given that $gg(2) = 10$ and $f^{-1}(2) = 14$, find the values of a and b. [4]

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(b) Using these values of a and b, find an expression for $gf(x)$ in the form $cx + d$, where c and d are constants. [2]

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[Turn over

7 **(a)** Prove the identity $\dfrac{1 + \sin\theta}{\cos\theta} + \dfrac{\cos\theta}{1 + \sin\theta} \equiv \dfrac{2}{\cos\theta}$. [3]

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(b) Hence solve the equation $\dfrac{1+\sin\theta}{\cos\theta}+\dfrac{\cos\theta}{1+\sin\theta}=\dfrac{3}{\sin\theta}$, for $0 \leqslant \theta \leqslant 2\pi$. [3]

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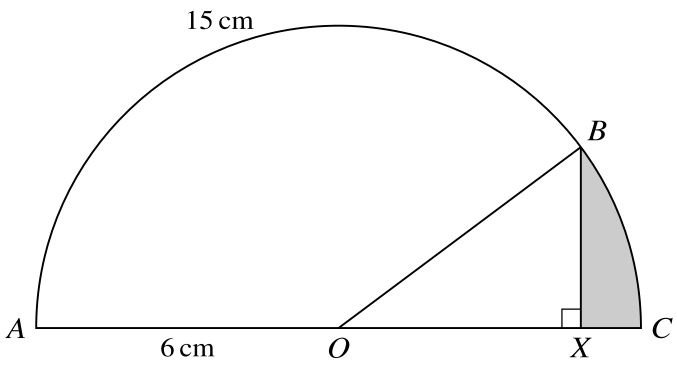

In the diagram, ABC is a semicircle with diameter AC, centre O and radius 6 cm. The length of the arc AB is 15 cm. The point X lies on AC and BX is perpendicular to AX.

Find the perimeter of the shaded region BXC. [6]

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9 The equation of a curve is $y = (3 - 2x)^3 + 24x$.

(a) Find expressions for $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. [4]

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(b) Find the coordinates of each of the stationary points on the curve. [3]

(c) Determine the nature of each stationary point. [2]

10 The coordinates of the points A and B are $(-1, -2)$ and $(7, 4)$ respectively.

(a) Find the equation of the circle, C, for which AB is a diameter. [4]

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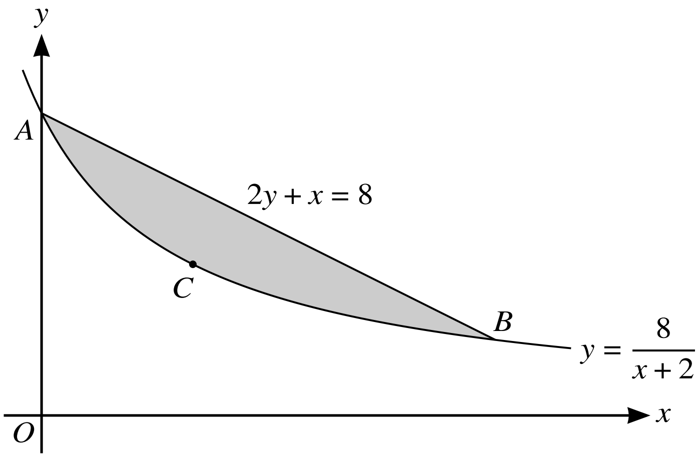
(b) Find the equation of the tangent, T, to circle C at the point B. [4]

(c) Find the equation of the circle which is the reflection of circle C in the line T. [3]

11



The diagram shows part of the curve $y = \dfrac{8}{x+2}$ and the line $2y + x = 8$, intersecting at points A and B. The point C lies on the curve and the tangent to the curve at C is parallel to AB.

(a) Find, by calculation, the coordinates of A, B and C. [6]

(b) Find the volume generated when the shaded region, bounded by the curve and the line, is rotated through 360° about the x-axis. [6]

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CANDIDATE NAME

CENTRE NUMBER | CANDIDATE NUMBER

5874701744

MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **May/June 2020**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
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- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

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1 **(a)** Find the coefficient of x^2 in the expansion of $\left(x - \frac{2}{x}\right)^6$. [2]

(b) Find the coefficient of x^2 in the expansion of $(2 + 3x^2)\left(x - \frac{2}{x}\right)^6$. [3]

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2 **(a)** Express the equation $3\cos\theta = 8\tan\theta$ as a quadratic equation in $\sin\theta$. [3]

(b) Hence find the acute angle, in degrees, for which $3\cos\theta = 8\tan\theta$. [2]

3 A weather balloon in the shape of a sphere is being inflated by a pump. The volume of the balloon is increasing at a constant rate of 600 cm^3 per second. The balloon was empty at the start of pumping.

(a) Find the radius of the balloon after 30 seconds. [2]

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(b) Find the rate of increase of the radius after 30 seconds. [3]

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4 The *n*th term of an arithmetic progression is $\frac{1}{2}(3n - 15)$.

Find the value of n for which the sum of the first n terms is 84. [5]

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5 The function f is defined for $x \in \mathbb{R}$ by

$$f : x \mapsto a - 2x,$$

where a is a constant.

(a) Express $ff(x)$ and $f^{-1}(x)$ in terms of a and x. [4]

(b) Given that $ff(x) = f^{-1}(x)$, find x in terms of a. [2]

6 The equation of a curve is $y = 2x^2 + kx + k - 1$, where k is a constant.

(a) Given that the line $y = 2x + 3$ is a tangent to the curve, find the value of k. [3]

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It is now given that $k = 2$.

(b) Express the equation of the curve in the form $y = 2(x + a)^2 + b$, where a and b are constants, and hence state the coordinates of the vertex of the curve. [3]

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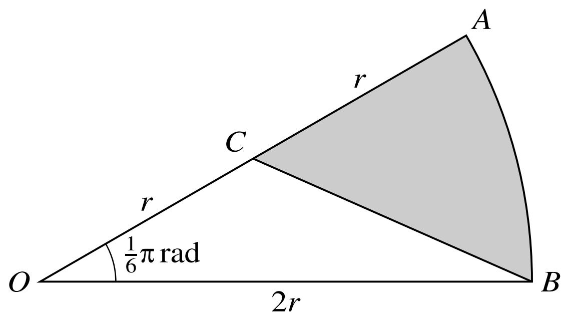

In the diagram, OAB is a sector of a circle with centre O and radius $2r$, and angle $AOB = \frac{1}{6}\pi$ radians. The point C is the midpoint of OA.

(a) Show that the exact length of BC is $r\sqrt{5 - 2\sqrt{3}}$. [2]

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(b) Find the exact perimeter of the shaded region. [2]

(c) Find the exact area of the shaded region. [3]

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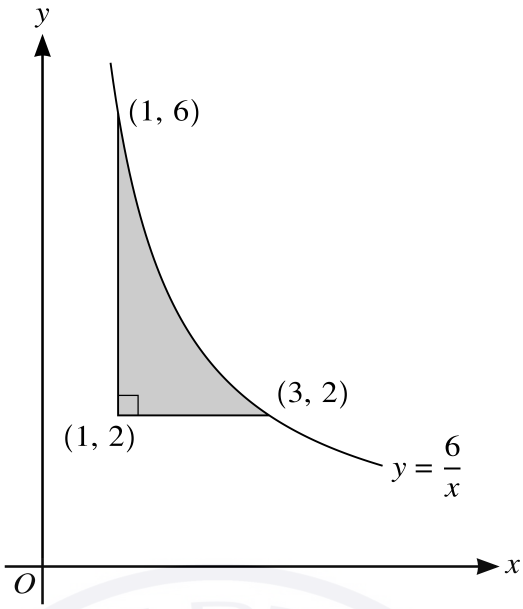

The diagram shows part of the curve $y = \frac{6}{x}$. The points (1, 6) and (3, 2) lie on the curve. The shaded region is bounded by the curve and the lines $y = 2$ and $x = 1$.

(a) Find the volume generated when the shaded region is rotated through 360° about the ***y*-axis**. [5]

(b) The tangent to the curve at a point X is parallel to the line $y + 2x = 0$. Show that X lies on the line $y = 2x$. [3]

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9 Functions f and g are such that

$$f(x) = 2 - 3\sin 2x \quad \text{for } 0 \leqslant x \leqslant \pi,$$
$$g(x) = -2f(x) \quad \text{for } 0 \leqslant x \leqslant \pi.$$

(a) State the ranges of f and g. [3]

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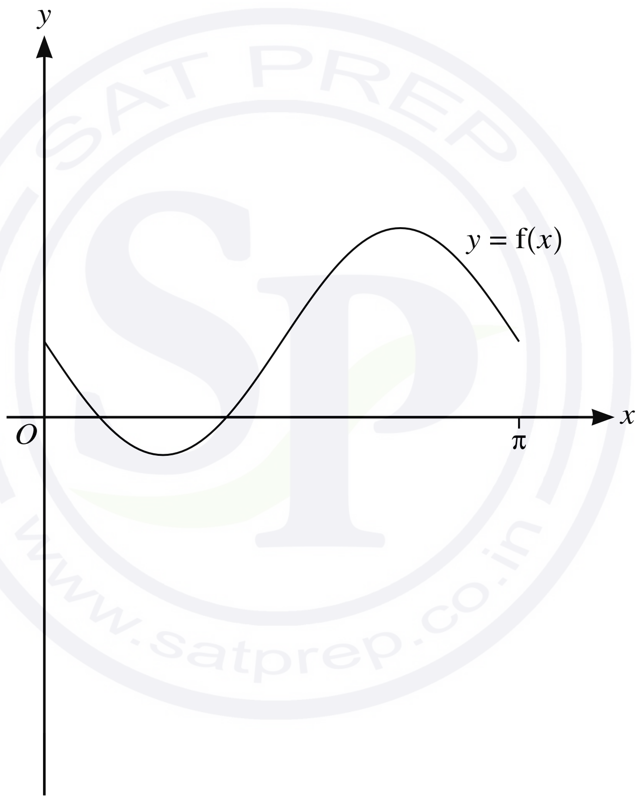
The diagram below shows the graph of $y = f(x)$.



(b) Sketch, on this diagram, the graph of $y = g(x)$. [2]

The function h is such that

$$h(x) = g(x + \pi) \quad \text{for } -\pi \leqslant x \leqslant 0.$$

(c) Describe fully a sequence of transformations that maps the curve $y = f(x)$ on to $y = h(x)$. [3]

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10 The equation of a curve is $y = 54x - (2x - 7)^3$.

(a) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. [4]

(b) Find the coordinates of each of the stationary points on the curve. [3]

(c) Determine the nature of each of the stationary points. [2]

11 The equation of a circle with centre C is $x^2 + y^2 - 8x + 4y - 5 = 0$.

(a) Find the radius of the circle and the coordinates of C. [3]

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The point $P(1, 2)$ lies on the circle.

(b) Show that the equation of the tangent to the circle at P is $4y = 3x + 5$. [3]

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The point Q also lies on the circle and PQ is parallel to the x-axis.

(c) Write down the coordinates of Q. [2]

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The tangents to the circle at P and Q meet at T.

(d) Find the coordinates of T. [3]

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CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

1360524731

MATHEMATICS **9709/13**

Paper 1 Pure Mathematics 1 **May/June 2020**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

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1 Find the set of values of m for which the line with equation $y = mx + 1$ and the curve with equation $y = 3x^2 + 2x + 4$ intersect at two distinct points. [4]

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2 The equation of a curve is such that $\frac{dy}{dx} = 3x^{\frac{1}{2}} - 3x^{-\frac{1}{2}}$. It is given that the point $(4, 7)$ lies on the curve.

Find the equation of the curve. [4]

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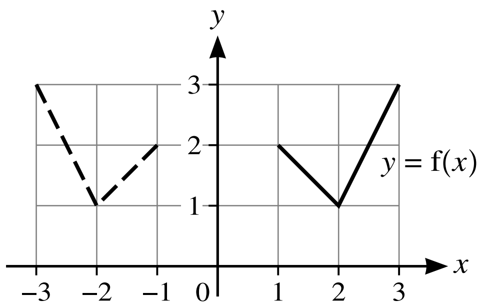
3 In each of parts **(a)**, **(b)** and **(c)**, the graph shown with solid lines has equation $y = \text{f}(x)$. The graph shown with broken lines is a transformation of $y = \text{f}(x)$.

(a)



State, in terms of f, the equation of the graph shown with broken lines. [1]

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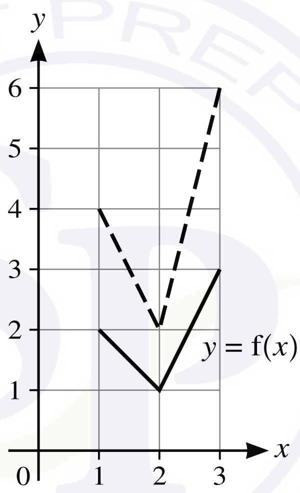
(b)



State, in terms of f, the equation of the graph shown with broken lines. [1]

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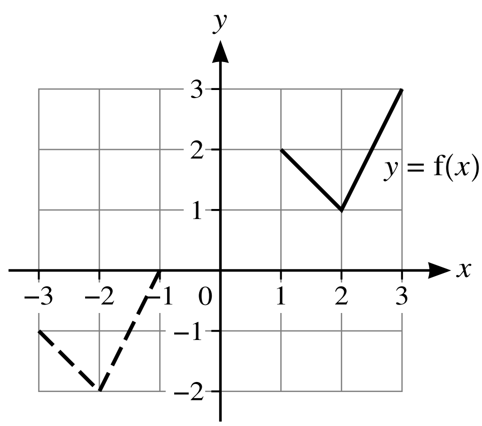
(c)



State, in terms of f, the equation of the graph shown with broken lines. [2]

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4 **(a)** Expand $(1 + a)^5$ in ascending powers of a up to and including the term in a^3. [1]

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(b) Hence expand $[1 + (x + x^2)]^5$ in ascending powers of x up to and including the term in x^3, simplifying your answer. [3]

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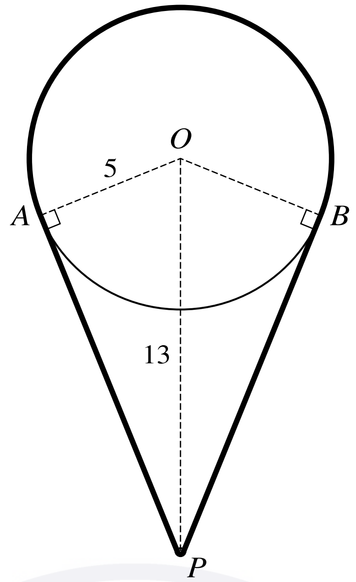

The diagram shows a cord going around a pulley and a pin. The pulley is modelled as a circle with centre O and radius 5 cm. The thickness of the cord and the size of the pin P can be neglected. The pin is situated 13 cm vertically below O. Points A and B are on the circumference of the circle such that AP and BP are tangents to the circle. The cord passes over the major arc AB of the circle and under the pin such that the cord is taut.

Calculate the length of the cord. [6]

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6 A point P is moving along a curve in such a way that the x-coordinate of P is increasing at a constant rate of 2 units per minute. The equation of the curve is $y = (5x - 1)^{\frac{1}{2}}$.

(a) Find the rate at which the y-coordinate is increasing when $x = 1$. [4]

(b) Find the value of x when the y-coordinate is increasing at $\frac{5}{8}$ units per minute. [3]

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7 **(a)** Show that $\dfrac{\tan\theta}{1+\cos\theta} + \dfrac{\tan\theta}{1-\cos\theta} \equiv \dfrac{2}{\sin\theta\cos\theta}$. [4]

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(b) Hence solve the equation $\frac{\tan\theta}{1+\cos\theta} + \frac{\tan\theta}{1-\cos\theta} = \frac{6}{\tan\theta}$ for $0° < \theta < 180°$. [4]

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8 The first term of a progression is $\sin^2\theta$, where $0 < \theta < \frac{1}{2}\pi$. The second term of the progression is $\sin^2\theta\cos^2\theta$.

(a) Given that the progression is geometric, find the sum to infinity. [3]

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It is now given instead that the progression is arithmetic.

(b) **(i)** Find the common difference of the progression in terms of $\sin\theta$. [3]

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(ii) Find the sum of the first 16 terms when $\theta = \frac{1}{3}\pi$. [3]

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9 The functions f and g are defined by

$$f(x) = x^2 - 4x + 3 \quad \text{for } x > c, \text{ where } c \text{ is a constant,}$$

$$g(x) = \frac{1}{x+1} \quad \text{for } x > -1.$$

(a) Express $f(x)$ in the form $(x - a)^2 + b$. [2]

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It is given that f is a one-one function.

(b) State the smallest possible value of c. [1]

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It is now given that $c = 5$.

(c) Find an expression for $f^{-1}(x)$ and state the domain of f^{-1}. [3]

(d) Find an expression for $gf(x)$ and state the range of gf. [3]

10 **(a)** The coordinates of two points A and B are $(-7, 3)$ and $(5, 11)$ respectively.

Show that the equation of the perpendicular bisector of AB is $3x + 2y = 11$. [4]

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(b) A circle passes through A and B and its centre lies on the line $12x - 5y = 70$.

Find an equation of the circle. [5]

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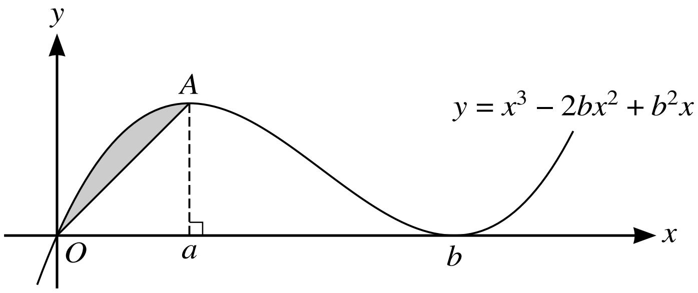

The diagram shows part of the curve with equation $y = x^3 - 2bx^2 + b^2x$ and the line OA, where A is the maximum point on the curve. The x-coordinate of A is a and the curve has a minimum point at $(b, 0)$, where a and b are positive constants.

(a) Show that $b = 3a$. [4]

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(b) Show that the area of the shaded region between the line and the curve is ka^4, where k is a fraction to be found. [7]

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* 1 9 3 9 4 7 8 2 4 0 *

MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **February/March 2020**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

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1 The function f is defined by $f(x) = \frac{1}{3x+2} + x^2$ for $x < -1$.

Determine whether f is an increasing function, a decreasing function or neither. [3]

2 The graph of $y = \mathrm{f}(x)$ is transformed to the graph of $y = 1 + \mathrm{f}\left(\frac{1}{2}x\right)$.

Describe fully the two single transformations which have been combined to give the resulting transformation. [4]

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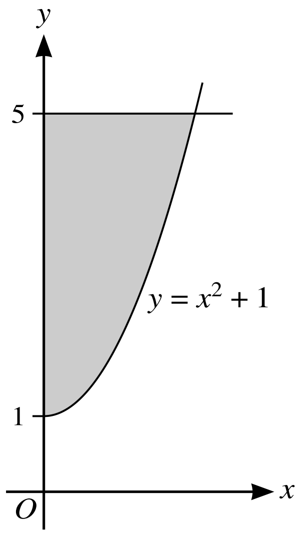

The diagram shows part of the curve with equation $y = x^2 + 1$. The shaded region enclosed by the curve, the y-axis and the line $y = 5$ is rotated through 360° about the ***y*-axis**.

Find the volume obtained. [4]

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4 A curve has equation $y = x^2 - 2x - 3$. A point is moving along the curve in such a way that at P the y-coordinate is increasing at 4 units per second and the x-coordinate is increasing at 6 units per second.

Find the x-coordinate of P. [4]

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5 Solve the equation

$$\frac{\tan\theta + 3\sin\theta + 2}{\tan\theta - 3\sin\theta + 1} = 2$$

for $0° \leqslant \theta \leqslant 90°$. [5]

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6 The coefficient of $\frac{1}{x}$ in the expansion of $\left(2x + \frac{a}{x^2}\right)^5$ is 720.

(a) Find the possible values of the constant a. [3]

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(b) Hence find the coefficient of $\frac{1}{x^7}$ in the expansion. [2]

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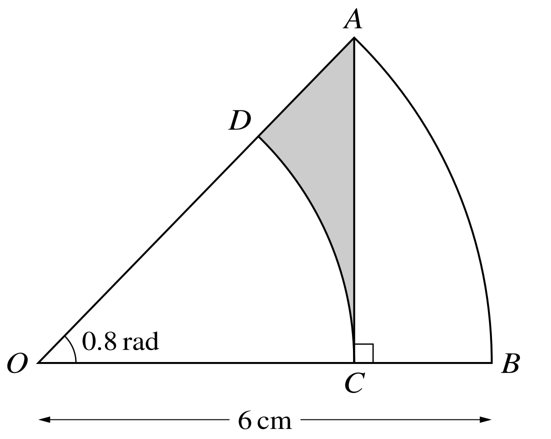

The diagram shows a sector AOB which is part of a circle with centre O and radius 6 cm and with angle $AOB = 0.8$ radians. The point C on OB is such that AC is perpendicular to OB. The arc CD is part of a circle with centre O, where D lies on OA.

Find the area of the shaded region. [6]

8 A woman's basic salary for her first year with a particular company is \$30 000 and at the end of the year she also gets a bonus of \$600.

(a) For her first year, express her bonus as a percentage of her basic salary. [1]

At the end of each complete year, the woman's basic salary will increase by 3% and her bonus will increase by \$100.

(b) Express the bonus she will be paid at the end of her 24th year as a percentage of the basic salary paid during that year. [5]

9 **(a)** Express $2x^2 + 12x + 11$ in the form $2(x + a)^2 + b$, where a and b are constants. [2]

The function f is defined by $f(x) = 2x^2 + 12x + 11$ for $x \leqslant -4$.

(b) Find an expression for $f^{-1}(x)$ and state the domain of f^{-1}. [3]

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The function g is defined by $g(x) = 2x - 3$ for $x \leqslant k$.

(c) For the case where $k = -1$, solve the equation $fg(x) = 193$. [2]

(d) State the largest value of k possible for the composition fg to be defined. [1]

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10 The gradient of a curve at the point (x, y) is given by $\frac{dy}{dx} = 2(x+3)^{\frac{1}{2}} - x$. The curve has a stationary point at $(a, 14)$, where a is a positive constant.

(a) Find the value of a. [3]

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(b) Determine the nature of the stationary point. [3]

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(c) Find the equation of the curve. [4]

11 (a) Solve the equation $3\tan^2 x - 5\tan x - 2 = 0$ for $0° \leqslant x \leqslant 180°$. [4]

(b) Find the set of values of k for which the equation $3\tan^2 x - 5\tan x + k = 0$ has no solutions. [2]

(c) For the equation $3\tan^2 x - 5\tan x + k = 0$, state the value of k for which there are three solutions in the interval $0° \leqslant x \leqslant 180°$, and find these solutions. [3]

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12 A diameter of a circle C_1 has end-points at $(-3, -5)$ and $(7, 3)$.

(a) Find an equation of the circle C_1. [3]

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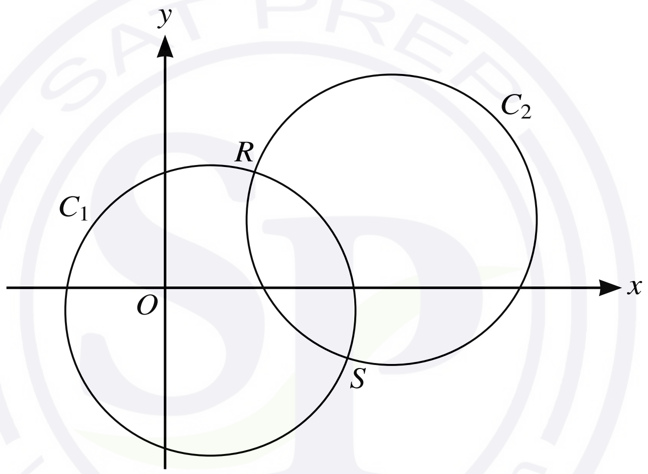

The circle C_1 is translated by $\begin{pmatrix} 8 \\ 4 \end{pmatrix}$ to give circle C_2, as shown in the diagram.

(b) Find an equation of the circle C_2. [2]

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The two circles intersect at points R and S.

(c) Show that the equation of the line RS is $y = -2x + 13$. [4]

(d) Hence show that the x-coordinates of R and S satisfy the equation $5x^2 - 60x + 159 = 0$. [2]

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CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

MATHEMATICS **9709/11**

Paper 1 Pure Mathematics 1 **(P1)** **October/November 2019**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

Write your centre number, candidate number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions in the space provided. If additional space is required, you should use the lined page at the end of this booklet. The question number(s) must be clearly shown.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.

This document consists of **20** printed pages.

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1 Find the term independent of x in the expansion of $\left(2x + \frac{1}{4x^2}\right)^6$. [3]

2 An increasing function, f, is defined for $x > n$, where n is an integer. It is given that $f'(x) = x^2 - 6x + 8$. Find the least possible value of n. [3]

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3 The line $y = ax + b$ is a tangent to the curve $y = 2x^3 - 5x^2 - 3x + c$ at the point $(2, 6)$. Find the values of the constants a, b and c. [5]

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4 A runner who is training for a long-distance race plans to run increasing distances each day for 21 days. She will run x km on day 1, and on each subsequent day she will increase the distance by 10% of the previous day's distance. On day 21 she will run 20 km.

(i) Find the distance she must run on day 1 in order to achieve this. Give your answer in km correct to 1 decimal place. [3]

(ii) Find the total distance she runs over the 21 days. [2]

5 **(i)** Given that $4\tan x + 3\cos x + \dfrac{1}{\cos x} = 0$, show, without using a calculator, that $\sin x = -\frac{2}{3}$. [3]

(ii) Hence, showing all necessary working, solve the equation

$$4\tan(2x - 20°) + 3\cos(2x - 20°) + \frac{1}{\cos(2x - 20°)} = 0$$

for $0° \leqslant x \leqslant 180°$. [4]

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6 A straight line has gradient m and passes through the point $(0, -2)$. Find the two values of m for which the line is a tangent to the curve $y = x^2 - 2x + 7$ and, for each value of m, find the coordinates of the point where the line touches the curve. [7]

7 Functions f and g are defined by

$$f : x \mapsto \frac{3}{2x+1} \quad \text{for } x > 0,$$

$$g : x \mapsto \frac{1}{x} + 2 \quad \text{for } x > 0.$$

(i) Find the range of f and the range of g. [3]

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(ii) Find an expression for $\mathrm{fg}(x)$, giving your answer in the form $\frac{ax}{bx+c}$, where a, b and c are integers. [2]

(iii) Find an expression for $(\mathrm{fg})^{-1}(x)$, giving your answer in the same form as for part **(ii)**. [3]

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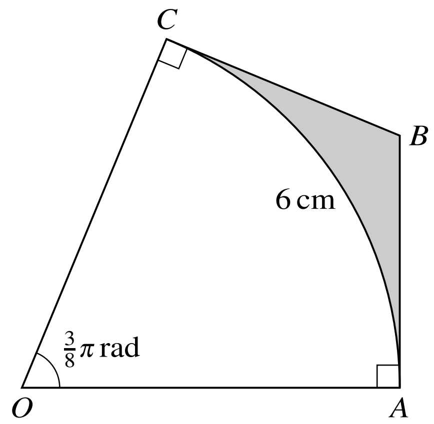

The diagram shows a sector OAC of a circle with centre O. Tangents AB and CB to the circle meet at B. The arc AC is of length 6 cm and angle $AOC = \frac{3}{8}\pi$ radians.

(i) Find the length of OA correct to 4 significant figures. [2]

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(ii) Find the perimeter of the shaded region. [2]

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(iii) Find the area of the shaded region. [4]

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9 A curve for which $\frac{dy}{dx} = (5x - 1)^{\frac{1}{2}} - 2$ passes through the point (2, 3).

(i) Find the equation of the curve. [4]

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(ii) Find $\frac{d^2y}{dx^2}$. [2]

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(iii) Find the coordinates of the stationary point on the curve and, showing all necessary working, determine the nature of this stationary point. [4]

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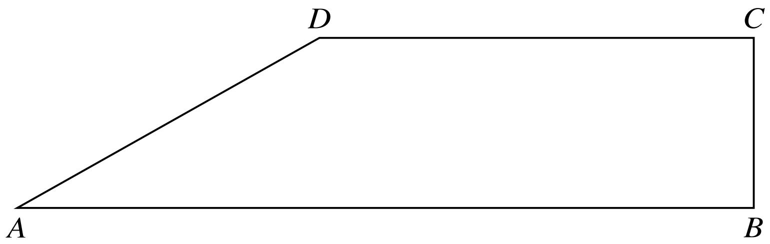

Relative to an origin O, the position vectors of the points A, B, C and D, shown in the diagram, are given by

$$\overrightarrow{OA} = \begin{pmatrix} -1 \\ 3 \\ -4 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 2 \\ -3 \\ 5 \end{pmatrix}, \quad \overrightarrow{OC} = \begin{pmatrix} 4 \\ -2 \\ 5 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OD} = \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}.$$

(i) Show that AB is perpendicular to BC. [3]

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(ii) Show that $ABCD$ is a trapezium. [3]

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(iii) Find the area of *ABCD*, giving your answer correct to 2 decimal places. [3]

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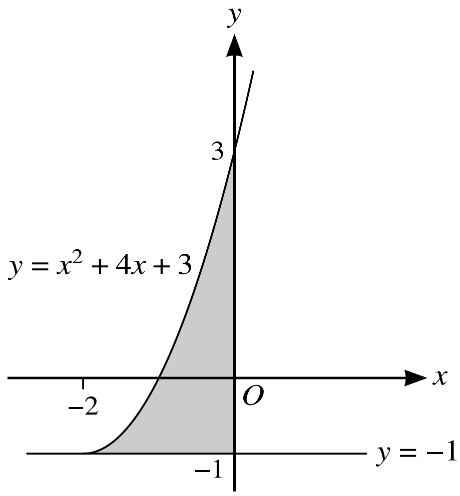

The diagram shows a shaded region bounded by the y-axis, the line $y = -1$ and the part of the curve $y = x^2 + 4x + 3$ for which $x \geqslant -2$.

(i) Express $y = x^2 + 4x + 3$ in the form $y = (x + a)^2 + b$, where a and b are constants. Hence, for $x \geqslant -2$, express x in terms of y. [4]

(ii) Hence, showing all necessary working, find the volume obtained when the shaded region is rotated through 360° about the ***y*-axis**. [6]

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CANDIDATE NUMBER

* 9 8 1 0 3 7 4 8 8 3 *

MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **(P1)** **October/November 2019**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

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Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
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[Turn over

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1 The coefficient of x^2 in the expansion of $(4 + ax)\left(1 + \frac{x}{2}\right)^6$ is 3. Find the value of the constant a. [4]

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2 The point M is the mid-point of the line joining the points $(3, 7)$ and $(-1, 1)$. Find the equation of the line through M which is parallel to the line $\frac{x}{3} + \frac{y}{2} = 1$. [4]

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3 A curve is such that $\frac{dy}{dx} = \frac{k}{\sqrt{x}}$, where k is a constant. The points $P(1, -1)$ and $Q(4, 4)$ lie on the curve. Find the equation of the curve. [4]

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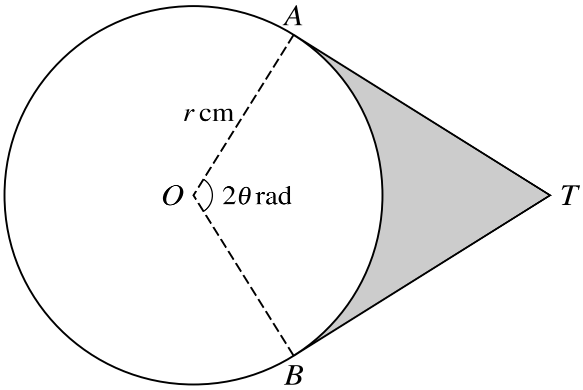

The diagram shows a circle with centre O and radius r cm. Points A and B lie on the circle and angle $AOB = 2\theta$ radians. The tangents to the circle at A and B meet at T.

(i) Express the perimeter of the shaded region in terms of r and θ. [3]

(ii) In the case where $r = 5$ and $\theta = 1.2$, find the area of the shaded region. [4]

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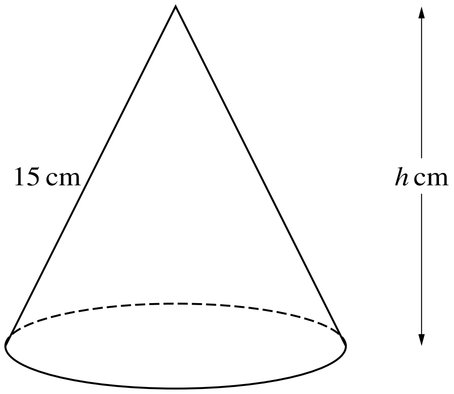

The diagram shows a solid cone which has a slant height of 15 cm and a vertical height of h cm.

(i) Show that the volume, V cm^3, of the cone is given by $V = \frac{1}{3}\pi(225h - h^3)$. [2]

[The volume of a cone of radius r and vertical height h is $\frac{1}{3}\pi r^2 h$.]

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(ii) Given that h can vary, find the value of h for which V has a stationary value. Determine, showing all necessary working, the nature of this stationary value. [5]

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6 **(a)** Given that $x > 0$, find the two smallest values of x, in radians, for which $3\tan(2x + 1) = 1$. Show all necessary working. [4]

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(b) The function $f : x \mapsto 3\cos^2 x - 2\sin^2 x$ is defined for $0 \leqslant x \leqslant \pi$.

(i) Express $f(x)$ in the form $a\cos^2 x + b$, where a and b are constants. [1]

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(ii) Find the range of f. [2]

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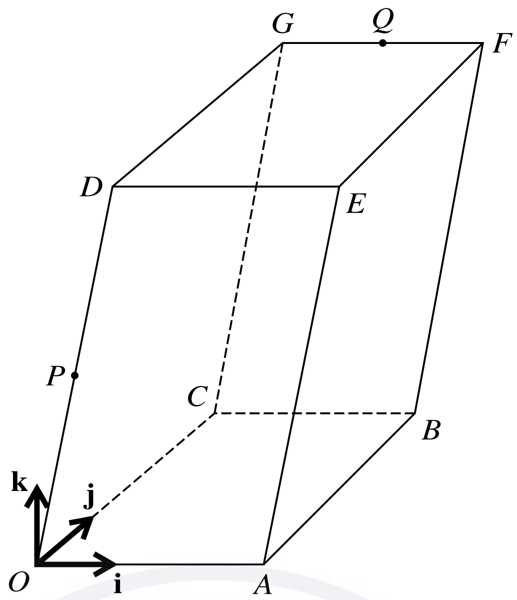

The diagram shows a three-dimensional shape $OABCDEFG$. The base $OABC$ and the upper surface $DEFG$ are identical horizontal rectangles. The parallelograms $OAED$ and $CBFG$ both lie in vertical planes. Points P and Q are the mid-points of OD and GF respectively. Unit vectors **i** and **j** are parallel to $\overrightarrow{OA}$ and $\overrightarrow{OC}$ respectively and the unit vector **k** is vertically upwards. The position vectors of A, C and D are given by $\overrightarrow{OA} = 6\mathbf{i}$, $\overrightarrow{OC} = 8\mathbf{j}$ and $\overrightarrow{OD} = 2\mathbf{i} + 10\mathbf{k}$.

(i) Express each of the vectors $\overrightarrow{PB}$ and $\overrightarrow{PQ}$ in terms of **i**, **j** and **k**. [4]

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(ii) Determine whether P is nearer to Q or to B. [2]

(iii) Use a scalar product to find angle BPQ. [3]

8 **(a)** Over a 21-day period an athlete prepares for a marathon by increasing the distance she runs each day by 1.2 km. On the first day she runs 13 km.

(i) Find the distance she runs on the last day of the 21-day period. [1]

(ii) Find the total distance she runs in the 21-day period. [2]

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(b) The first, second and third terms of a geometric progression are x, $x - 3$ and $x - 5$ respectively.

(i) Find the value of x. [2]

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(ii) Find the fourth term of the progression. [2]

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(iii) Find the sum to infinity of the progression. [2]

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9 Functions f and g are defined by

$$f(x) = 2x^2 + 8x + 1 \quad \text{for } x \in \mathbb{R},$$
$$g(x) = 2x - k \quad \text{for } x \in \mathbb{R},$$

where k is a constant.

(i) Find the value of k for which the line $y = g(x)$ is a tangent to the curve $y = f(x)$. [3]

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(ii) In the case where $k = -9$, find the set of values of x for which $f(x) < g(x)$. [3]

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(iii) In the case where $k = -1$, find $g^{-1}f(x)$ and solve the equation $g^{-1}f(x) = 0$. [3]

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(iv) Express $f(x)$ in the form $2(x + a)^2 + b$, where a and b are constants, and hence state the least value of $f(x)$. [3]

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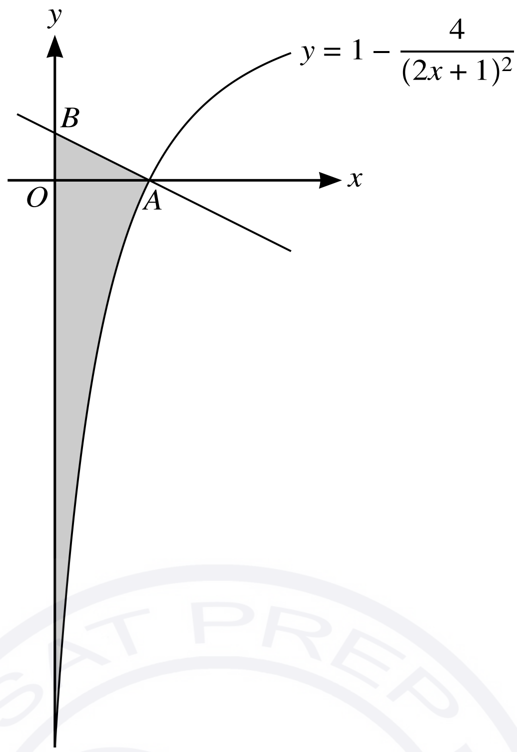

The diagram shows part of the curve $y = 1 - \frac{4}{(2x+1)^2}$. The curve intersects the x-axis at A. The normal to the curve at A intersects the y-axis at B.

(i) Obtain expressions for $\frac{dy}{dx}$ and $\int y\,dx$. [4]

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(ii) Find the coordinates of B. [4]

(iii) Find, showing all necessary working, the area of the shaded region. [4]

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CENTRE NUMBER | CANDIDATE NUMBER

3117579859

MATHEMATICS **9709/13**

Paper 1 Pure Mathematics 1 **(P1)** **October/November 2019**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

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You are reminded of the need for clear presentation in your answers.

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[Turn over

1 **(i)** Expand $(1 + y)^6$ in ascending powers of y as far as the term in y^2. [1]

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(ii) In the expansion of $\left(1 + (px - 2x^2)\right)^6$ the coefficient of x^2 is 48. Find the value of the positive constant p. [3]

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2 The function g is defined by $g(x) = x^2 - 6x + 7$ for $x > 4$. By first completing the square, find an expression for $g^{-1}(x)$ and state the domain of g^{-1}. [5]

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3 The equation of a curve is $y = x^3 + x^2 - 8x + 7$. The curve has no stationary points in the interval $a < x < b$. Find the least possible value of a and the greatest possible value of b. [4]

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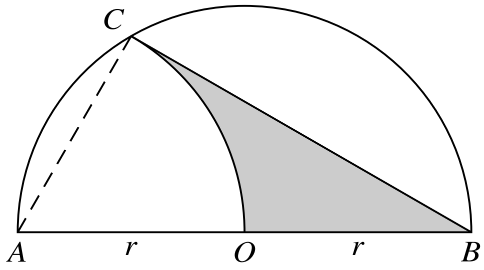

The diagram shows a semicircle ACB with centre O and radius r. Arc OC is part of a circle with centre A.

(i) Express angle CAO in radians in terms of π. [1]

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(ii) Find the area of the shaded region in terms of r, π and $\sqrt{3}$, simplifying your answer. [4]

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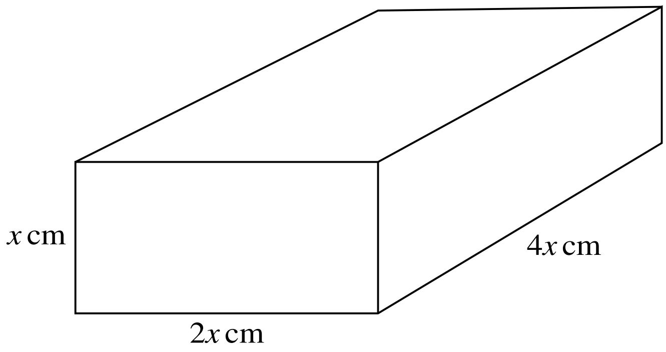

The dimensions of a cuboid are x cm, $2x$ cm and $4x$ cm, as shown in the diagram.

(i) Show that the surface area S cm^2 and the volume V cm^3 are connected by the relation

$$S = 7V^{\frac{2}{3}}.$$ [3]

(ii) When the volume of the cuboid is $1000\,\text{cm}^3$ the surface area is increasing at $2\,\text{cm}^2\,\text{s}^{-1}$. Find the rate of increase of the volume at this instant. [4]

6 A line has equation $y = 3kx - 2k$ and a curve has equation $y = x^2 - kx + 2$, where k is a constant.

(i) Find the set of values of k for which the line and curve meet at two distinct points. [4]

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(ii) For each of two particular values of k, the line is a tangent to the curve. Show that these two tangents meet on the x-axis. [3]

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7 **(i)** Show that the equation $3\cos^4\theta + 4\sin^2\theta - 3 = 0$ can be expressed as $3x^2 - 4x + 1 = 0$, where $x = \cos^2\theta$. [2]

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(ii) Hence solve the equation $3\cos^4\theta + 4\sin^2\theta - 3 = 0$ for $0° \leqslant \theta \leqslant 180°$. [5]

8 A function f is defined for $x > \frac{1}{2}$ and is such that $f'(x) = 3(2x - 1)^{\frac{1}{2}} - 6$.

(i) Find the set of values of x for which f is decreasing. [4]

(ii) It is now given that $f(1) = -3$. Find $f(x)$. [4]

9 The first, second and third terms of a geometric progression are $3k$, $5k - 6$ and $6k - 4$, respectively.

(i) Show that k satisfies the equation $7k^2 - 48k + 36 = 0$. [2]

(ii) Find, showing all necessary working, the exact values of the common ratio corresponding to each of the possible values of k. [4]

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(iii) One of these ratios gives a progression which is convergent. Find the sum to infinity. [2]

10 Relative to an origin O, the position vectors of the points A, B and X are given by

$$\overrightarrow{OA} = \begin{pmatrix} -8 \\ -4 \\ 2 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 10 \\ 2 \\ 11 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OX} = \begin{pmatrix} -2 \\ -2 \\ 5 \end{pmatrix}.$$

(i) Find $\overrightarrow{AX}$ and show that AXB is a straight line. [3]

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The position vector of a point C is given by $\overrightarrow{OC} = \begin{pmatrix} 1 \\ -8 \\ 3 \end{pmatrix}$.

(ii) Show that CX is perpendicular to AX. [3]

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(iii) Find the area of triangle ABC. [3]

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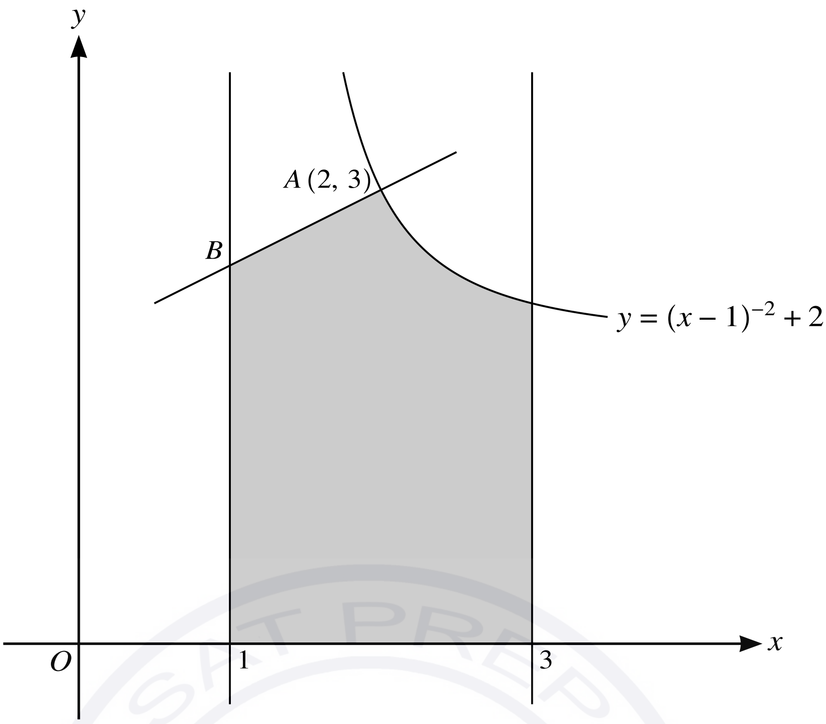

The diagram shows part of the curve $y = (x - 1)^{-2} + 2$, and the lines $x = 1$ and $x = 3$. The point A on the curve has coordinates $(2, 3)$. The normal to the curve at A crosses the line $x = 1$ at B.

(i) Show that the normal AB has equation $y = \frac{1}{2}x + 2$. [3]

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(ii) Find, showing all necessary working, the volume of revolution obtained when the shaded region is rotated through 360° about the x-axis. [8]

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CENTRE NUMBER

CANDIDATE NUMBER

2339823814

MATHEMATICS **9709/11**

Paper 1 Pure Mathematics 1 **(P1)** **May/June 2019**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

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You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
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1 The term independent of x in the expansion of $\left(2x + \frac{k}{x}\right)^6$, where k is a constant, is 540.

(i) Find the value of k. [3]

(ii) For this value of k, find the coefficient of x^2 in the expansion. [2]

2 The line $4y = x + c$, where c is a constant, is a tangent to the curve $y^2 = x + 3$ at the point P on the curve.

(i) Find the value of c. [3]

(ii) Find the coordinates of P. [2]

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3 A sector of a circle of radius r cm has an area of A cm^2. Express the perimeter of the sector in terms of r and A. [4]

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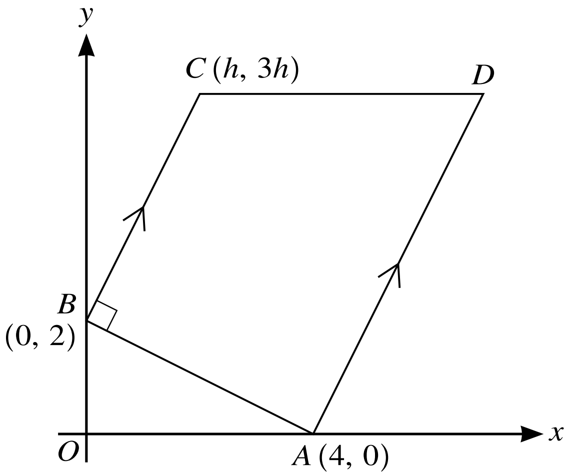
4



The diagram shows a trapezium $ABCD$ in which the coordinates of A, B and C are (4, 0), (0, 2) and $(h, 3h)$ respectively. The lines BC and AD are parallel, angle $ABC = 90°$ and CD is parallel to the x-axis.

(i) Find, by calculation, the value of h. [3]

(ii) Hence find the coordinates of D. [3]

5 The function f is defined by $f(x) = -2x^2 + 12x - 3$ for $x \in \mathbb{R}$.

(i) Express $-2x^2 + 12x - 3$ in the form $-2(x + a)^2 + b$, where a and b are constants. [2]

(ii) State the greatest value of $f(x)$. [1]

The function g is defined by $g(x) = 2x + 5$ for $x \in \mathbb{R}$.

(iii) Find the values of x for which $gf(x) + 1 = 0$. [3]

6 **(i)** Prove the identity $\left(\frac{1}{\cos x} - \tan x\right)^2 \equiv \frac{1 - \sin x}{1 + \sin x}$. [4]

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(ii) Hence solve the equation $\left(\frac{1}{\cos 2x} - \tan 2x\right)^2 = \frac{1}{3}$ for $0 \leqslant x \leqslant \pi$. [3]

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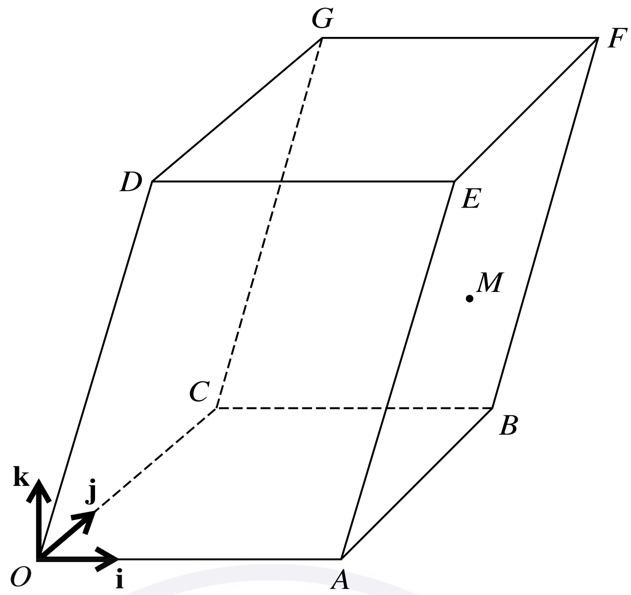

The diagram shows a three-dimensional shape in which the base $OABC$ and the upper surface $DEFG$ are identical horizontal squares. The parallelograms $OAED$ and $CBFG$ both lie in vertical planes. The point M is the mid-point of AF.

Unit vectors **i** and **j** are parallel to OA and OC respectively and the unit vector **k** is vertically upwards. The position vectors of A and D are given by $\overrightarrow{OA} = 8\mathbf{i}$ and $\overrightarrow{OD} = 3\mathbf{i} + 10\mathbf{k}$.

(i) Express each of the vectors $\overrightarrow{AM}$ and $\overrightarrow{GM}$ in terms of **i**, **j** and **k**. [3]

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(ii) Use a scalar product to find angle *GMA* correct to the nearest degree. [4]

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8 **(a)** The third and fourth terms of a geometric progression are 48 and 32 respectively. Find the sum to infinity of the progression. [3]

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(b) Two schemes are proposed for increasing the amount of household waste that is recycled each week.

Scheme A is to increase the amount of waste recycled each month by 0.16 tonnes.

Scheme B is to increase the amount of waste recycled each month by 6% of the amount recycled in the previous month.

The proposal is to operate the scheme for a period of 24 months. The amount recycled in the first month is 2.5 tonnes.

For each scheme, find the total amount of waste that would be recycled over the 24-month period. [5]

Scheme A ....

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9 The function f is defined by $f(x) = 2 - 3\cos x$ for $0 \leqslant x \leqslant 2\pi$.

(i) State the range of f. [2]

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(ii) Sketch the graph of $y = f(x)$. [2]

The function g is defined by $g(x) = 2 - 3\cos x$ for $0 \leqslant x \leqslant p$, where p is a constant.

(iii) State the largest value of p for which g has an inverse. [1]

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(iv) For this value of p, find an expression for $g^{-1}(x)$. [2]

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[Turn over

10 A curve for which $\frac{d^2y}{dx^2} = 2x - 5$ has a stationary point at (3, 6).

(i) Find the equation of the curve. [6]

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(ii) Find the x-coordinate of the other stationary point on the curve. [1]

(iii) Determine the nature of each of the stationary points. [2]

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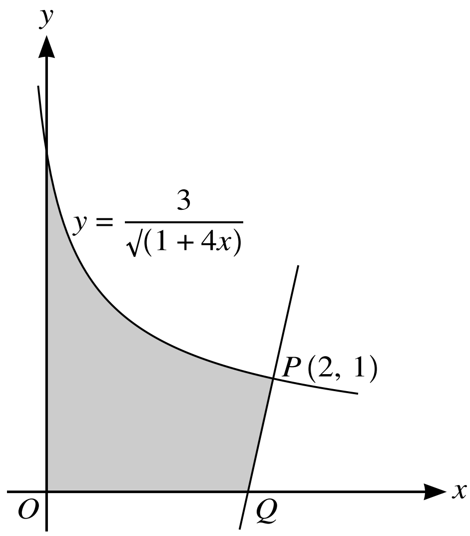
11



The diagram shows part of the curve $y = \dfrac{3}{\sqrt{(1+4x)}}$ and a point $P\,(2,\ 1)$ lying on the curve. The normal to the curve at P intersects the x-axis at Q.

(i) Show that the x-coordinate of Q is $\frac{16}{9}$. [5]

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(ii) Find, showing all necessary working, the area of the shaded region. [6]

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CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

6611123410

MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **(P1)** **May/June 2019**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

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Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.

This document consists of **18** printed pages and **2** blank pages.

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1 Find the coefficient of x in the expansion of $\left(\frac{2}{x} - 3x\right)^5$. [3]

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2 Two points A and B have coordinates $(1, 3)$ and $(9, -1)$ respectively. The perpendicular bisector of AB intersects the y-axis at the point C. Find the coordinates of C. [5]

3 A curve is such that $\frac{dy}{dx} = x^3 - \frac{4}{x^2}$. The point $P(2, 9)$ lies on the curve.

(i) A point moves on the curve in such a way that the x-coordinate is decreasing at a constant rate of 0.05 units per second. Find the rate of change of the y-coordinate when the point is at P. [2]

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(ii) Find the equation of the curve. [3]

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4 Angle x is such that $\sin x = a + b$ and $\cos x = a - b$, where a and b are constants.

(i) Show that $a^2 + b^2$ has a constant value for all values of x. [3]

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(ii) In the case where $\tan x = 2$, express a in terms of b. [2]

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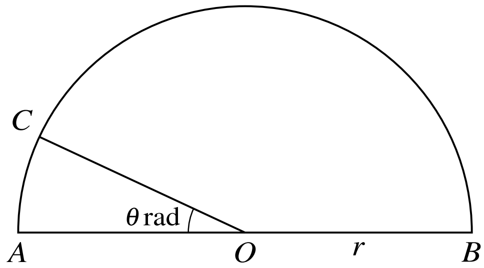

The diagram shows a semicircle with diameter AB, centre O and radius r. The point C lies on the circumference and angle $AOC = \theta$ radians. The perimeter of sector BOC is twice the perimeter of sector AOC. Find the value of θ correct to 2 significant figures. [5]

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6 The equation of a curve is $y = 3\cos 2x$ and the equation of a line is $2y + \frac{3x}{\pi} = 5$.

(i) State the smallest and largest values of y for both the curve and the line for $0 \leqslant x \leqslant 2\pi$. [3]

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(ii) Sketch, on the same diagram, the graphs of $y = 3\cos 2x$ and $2y + \frac{3x}{\pi} = 5$ for $0 \leqslant x \leqslant 2\pi$. [3]

(iii) State the number of solutions of the equation $6\cos 2x = 5 - \frac{3x}{\pi}$ for $0 \leqslant x \leqslant 2\pi$. [1]

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7 Functions f and g are defined by

$$\mathrm{f} : x \mapsto 3x - 2, \quad x \in \mathbb{R},$$
$$\mathrm{g} : x \mapsto \frac{2x+3}{x-1}, \quad x \in \mathbb{R},\ x \neq 1.$$

(i) Obtain expressions for $\mathrm{f}^{-1}(x)$ and $\mathrm{g}^{-1}(x)$, stating the value of x for which $\mathrm{g}^{-1}(x)$ is not defined. [4]

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(ii) Solve the equation $fg(x) = \frac{7}{3}$. [3]

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8 The position vectors of points A and B, relative to an origin O, are given by

$$\overrightarrow{OA} = \begin{pmatrix} 6 \\ -2 \\ -6 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OB} = \begin{pmatrix} 3 \\ k \\ -3 \end{pmatrix},$$

where k is a constant.

(i) Find the value of k for which angle AOB is 90°. [2]

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(ii) Find the values of k for which the lengths of OA and OB are equal. [2]

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The point C is such that $\overrightarrow{AC} = 2\overrightarrow{CB}$.

(iii) In the case where $k = 4$, find the unit vector in the direction of $\overrightarrow{OC}$. [4]

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9 The curve C_1 has equation $y = x^2 - 4x + 7$. The curve C_2 has equation $y^2 = 4x + k$, where k is a constant. The tangent to C_1 at the point where $x = 3$ is also the tangent to C_2 at the point P. Find the value of k and the coordinates of P. [8]

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10 **(a)** In an arithmetic progression, the sum of the first ten terms is equal to the sum of the next five terms. The first term is a.

(i) Show that the common difference of the progression is $\frac{1}{3}a$. [4]

(ii) Given that the tenth term is 36 more than the fourth term, find the value of a. [2]

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(b) The sum to infinity of a geometric progression is 9 times the sum of the first four terms. Given that the first term is 12, find the value of the fifth term. [4]

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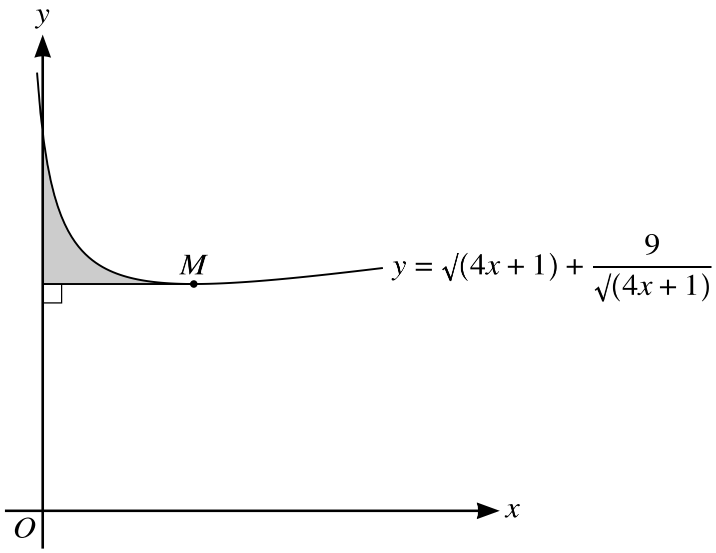

The diagram shows part of the curve $y = \sqrt{(4x+1)} + \dfrac{9}{\sqrt{(4x+1)}}$ and the minimum point M.

(i) Find expressions for $\dfrac{dy}{dx}$ and $\displaystyle\int y\,dx$. [6]

(ii) Find the coordinates of M. [3]

The shaded region is bounded by the curve, the y-axis and the line through M parallel to the x-axis.

(iii) Find, showing all necessary working, the area of the shaded region. [3]

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CANDIDATE NAME

CENTRE NUMBER

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MATHEMATICS **9709/13**

Paper 1 Pure Mathematics 1 **(P1)** **May/June 2019**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

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At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.

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1 The function f is defined by $f(x) = x^2 - 4x + 8$ for $x \in \mathbb{R}$.

(i) Express $x^2 - 4x + 8$ in the form $(x - a)^2 + b$. [2]

(ii) Hence find the set of values of x for which $f(x) < 9$, giving your answer in exact form. [3]

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2 **(i)** In the binomial expansion of $\left(2x - \frac{1}{2x}\right)^5$, the first three terms are $32x^5 - 40x^3 + 20x$. Find the remaining three terms of the expansion. [3]

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(ii) Hence find the coefficient of x in the expansion of $(1 + 4x^2)\left(2x - \frac{1}{2x}\right)^5$. [2]

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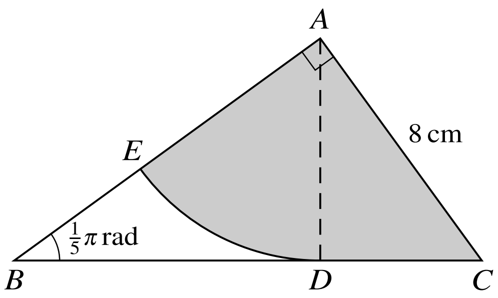

The diagram shows triangle ABC which is right-angled at A. Angle $ABC = \frac{1}{5}\pi$ radians and $AC = 8$ cm. The points D and E lie on BC and BA respectively. The sector ADE is part of a circle with centre A and is such that BDC is the tangent to the arc DE at D.

(i) Find the length of AD. [3]

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(ii) Find the area of the shaded region. [3]

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4 The function f is defined by $f(x) = \dfrac{48}{x-1}$ for $3 \leqslant x \leqslant 7$. The function g is defined by $g(x) = 2x - 4$ for $a \leqslant x \leqslant b$, where a and b are constants.

(i) Find the greatest value of a and the least value of b which will permit the formation of the composite function gf. [2]

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It is now given that the conditions for the formation of gf are satisfied.

(ii) Find an expression for $gf(x)$. [1]

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(iii) Find an expression for $(gf)^{-1}(x)$. [2]

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5 Two heavyweight boxers decide that they would be more successful if they competed in a lower weight class. For each boxer this would require a total weight loss of 13 kg. At the end of week 1 they have each recorded a weight loss of 1 kg and they both find that in each of the following weeks their weight loss is slightly less than the week before.

Boxer *A*'s weight loss in week 2 is 0.98 kg. It is given that his weekly weight loss follows an arithmetic progression.

(i) Write down an expression for his total weight loss after x weeks. [1]

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(ii) He reaches his 13 kg target during week n. Use your answer to part **(i)** to find the value of n. [2]

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Boxer B's weight loss in week 2 is 0.92 kg and it is given that his weekly weight loss follows a geometric progression.

(iii) Calculate his total weight loss after 20 weeks and show that he can never reach his target. [4]

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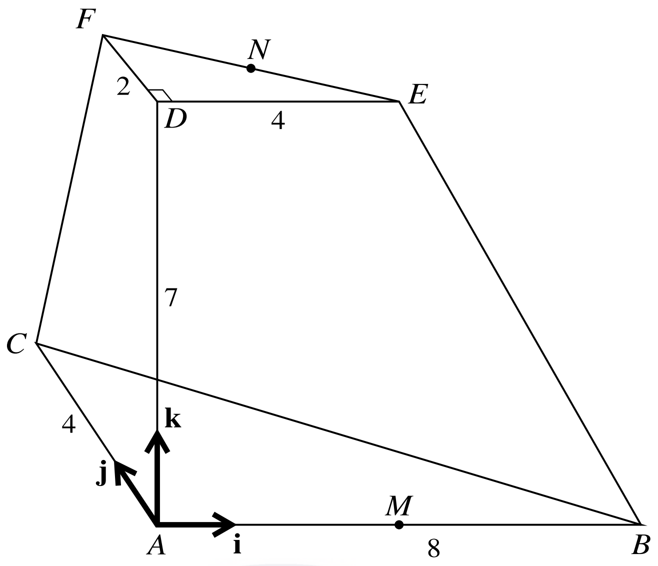

The diagram shows a solid figure $ABCDEF$ in which the horizontal base ABC is a triangle right-angled at A. The lengths of AB and AC are 8 units and 4 units respectively and M is the mid-point of AB. The point D is 7 units vertically above A. Triangle DEF lies in a horizontal plane with DE, DF and FE parallel to AB, AC and CB respectively and N is the mid-point of FE. The lengths of DE and DF are 4 units and 2 units respectively. Unit vectors **i**, **j** and **k** are parallel to $\overrightarrow{AB}$, $\overrightarrow{AC}$ and $\overrightarrow{AD}$ respectively.

(i) Find $\overrightarrow{MF}$ in terms of **i**, **j** and **k**. [1]

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(ii) Find $\overrightarrow{FN}$ in terms of **i** and **j**. [1]

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(iii) Find $\overrightarrow{MN}$ in terms of **i**, **j** and **k**. [1]

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(iv) Use a scalar product to find angle FMN. [4]

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7 The coordinates of two points A and B are $(1,\ 3)$ and $(9,\ -1)$ respectively and D is the mid-point of AB. A point C has coordinates $(x,\ y)$, where x and y are variables.

(i) State the coordinates of D. [1]

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(ii) It is given that $CD^2 = 20$. Write down an equation relating x and y. [1]

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(iii) It is given that AC and BC are equal in length. Find an equation relating x and y and show that it can be simplified to $y = 2x - 9$. [3]

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(iv) Using the results from parts **(ii)** and **(iii)**, and showing all necessary working, find the possible coordinates of C. [4]

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8 A curve is such that $\frac{dy}{dx} = 3x^2 + ax + b$. The curve has stationary points at $(-1, 2)$ and $(3, k)$. Find the values of the constants a, b and k. [8]

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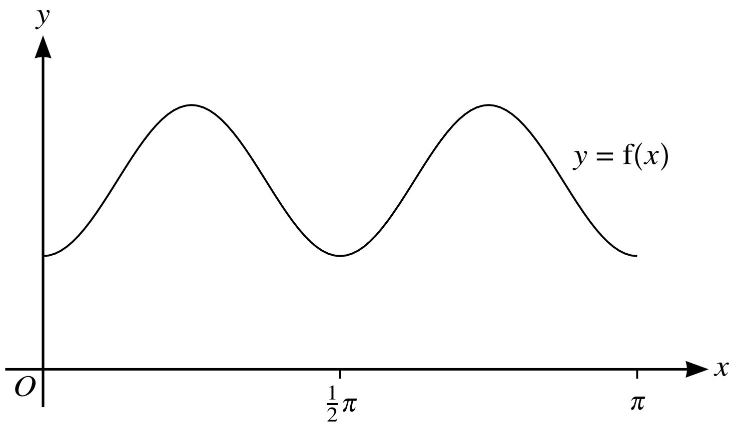

The function $f : x \mapsto p \sin^2 2x + q$ is defined for $0 \leqslant x \leqslant \pi$, where p and q are positive constants. The diagram shows the graph of $y = f(x)$.

(i) In terms of p and q, state the range of f. [2]

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(ii) State the number of solutions of the following equations.

(a) $f(x) = p + q$ [1]

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(b) $f(x) = q$ [1]

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(c) $f(x) = \frac{1}{2}p + q$ [1]

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(iii) For the case where $p = 3$ and $q = 2$, solve the equation $f(x) = 4$, showing all necessary working. [5]

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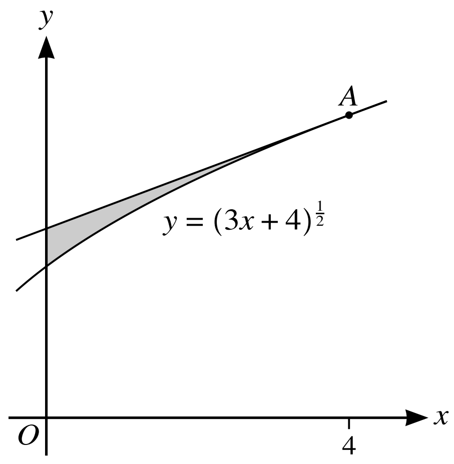

The diagram shows part of the curve with equation $y = (3x + 4)^{\frac{1}{2}}$ and the tangent to the curve at the point A. The x-coordinate of A is 4.

(i) Find the equation of the tangent to the curve at A. [5]

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(ii) Find, showing all necessary working, the area of the shaded region. [5]

[Question 10 (iii) is printed on the next page.]

(iii) A point is moving along the curve. At the point P the y-coordinate is increasing at half the rate at which the x-coordinate is increasing. Find the x-coordinate of P. [3]

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MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **(P1)** **February/March 2019**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

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1 The coefficient of x^3 in the expansion of $(1 - px)^5$ is -2160. Find the value of the constant p. [3]

2 A curve with equation $y = \mathrm{f}(x)$ passes through the points $(0, 2)$ and $(3, -1)$. It is given that $\mathrm{f}'(x) = kx^2 - 2x$, where k is a constant. Find the value of k. [5]

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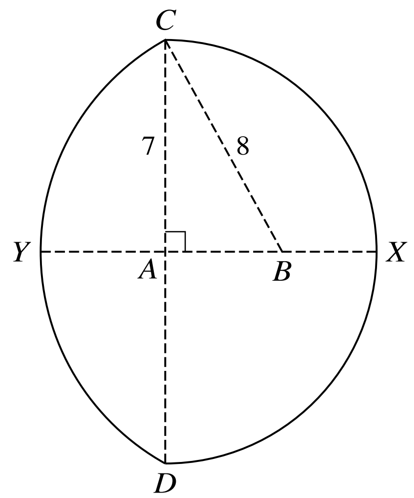

In the diagram, CXD is a semicircle of radius 7 cm with centre A and diameter CD. The straight line $YABX$ is perpendicular to CD, and the arc CYD is part of a circle with centre B and radius 8 cm. Find the total area of the region enclosed by the two arcs. [6]

4 A curve has equation $y = (2x - 1)^{-1} + 2x$.

(i) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. [3]

(ii) Find the x-coordinates of the stationary points and, showing all necessary working, determine the nature of each stationary point. [4]

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5 Two vectors, **u** and **v**, are such that

$$\mathbf{u} = \begin{pmatrix} q \\ 2 \\ 6 \end{pmatrix} \quad \text{and} \quad \mathbf{v} = \begin{pmatrix} 8 \\ q-1 \\ q^2-7 \end{pmatrix},$$

where q is a constant.

(i) Find the values of q for which **u** is perpendicular to **v**. [3]

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(ii) Find the angle between **u** and **v** when $q = 0$. [4]

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6 **(i)** The first and second terms of a geometric progression are p and $2p$ respectively, where p is a positive constant. The sum of the first n terms is greater than $1000p$. Show that $2^n > 1001$. [2]

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(ii) In another case, p and $2p$ are the first and second terms respectively of an arithmetic progression. The nth term is 336 and the sum of the first n terms is 7224. Write down two equations in n and p and hence find the values of n and p. [5]

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7 **(a)** Solve the equation $3\sin^2 2\theta + 8\cos 2\theta = 0$ for $0° \leqslant \theta \leqslant 180°$. [5]

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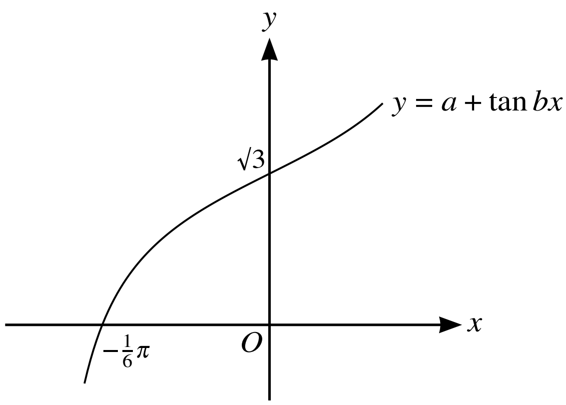
(b)



The diagram shows part of the graph of $y = a + \tan bx$, where x is measured in radians and a and b are constants. The curve intersects the x-axis at $\left(-\frac{1}{6}\pi, 0\right)$ and the y-axis at $(0, \sqrt{3})$. Find the values of a and b. [3]

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8 **(i)** Express $x^2 - 4x + 7$ in the form $(x + a)^2 + b$. [2]

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The function f is defined by $f(x) = x^2 - 4x + 7$ for $x < k$, where k is a constant.

(ii) State the largest value of k for which f is a decreasing function. [1]

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The value of k is now given to be 1.

(iii) Find an expression for $f^{-1}(x)$ and state the domain of f^{-1}. [3]

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(iv) The function g is defined by $g(x) = \dfrac{2}{x-1}$ for $x > 1$. Find an expression for $gf(x)$ and state the range of gf. [4]

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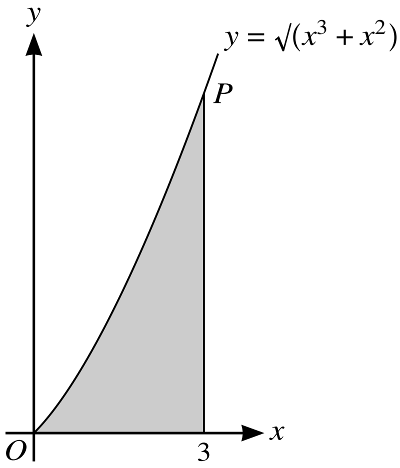

The diagram shows part of the curve with equation $y = \sqrt{(x^3 + x^2)}$. The shaded region is bounded by the curve, the x-axis and the line $x = 3$.

(i) Find, showing all necessary working, the volume obtained when the shaded region is rotated through 360° about the x-axis. [4]

(ii) P is the point on the curve with x-coordinate 3. Find the y-coordinate of the point where the normal to the curve at P crosses the y-axis. [6]

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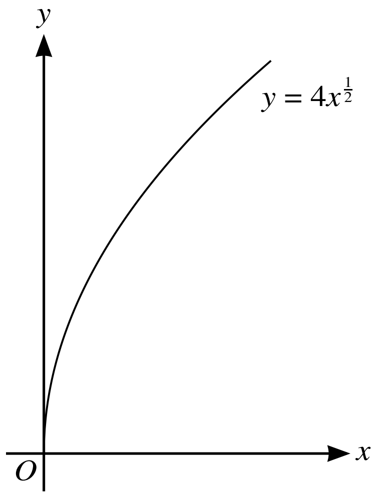

The diagram shows the curve with equation $y = 4x^{\frac{1}{2}}$.

(i) The straight line with equation $y = x + 3$ intersects the curve at points A and B. Find the length of AB. [6]

(ii) The tangent to the curve at a point T is parallel to AB. Find the coordinates of T. [3]

(iii) Find the coordinates of the point of intersection of the normal to the curve at T with the line AB. [3]

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CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

0906973490

MATHEMATICS **9709/11**

Paper 1 Pure Mathematics 1 **(P1)** **October/November 2018**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

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The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.

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1 Showing all necessary working, solve the equation $4x - 11x^{\frac{1}{2}} + 6 = 0$. [3]

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2 A line has equation $y = x + 1$ and a curve has equation $y = x^2 + bx + 5$. Find the set of values of the constant b for which the line meets the curve. [4]

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[Turn over

3 Two points A and B have coordinates $(3a, -a)$ and $(-a, 2a)$ respectively, where a is a positive constant.

(i) Find the equation of the line through the origin parallel to AB. [2]

(ii) The length of the line AB is $3\frac{1}{3}$ units. Find the value of a. [3]

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4 The first term of a series is 6 and the second term is 2.

(i) For the case where the series is an arithmetic progression, find the sum of the first 80 terms. [3]

(ii) For the case where the series is a geometric progression, find the sum to infinity. [2]

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5 **(i)** Show that the equation

$$\frac{\cos\theta - 4}{\sin\theta} - \frac{4\sin\theta}{5\cos\theta - 2} = 0$$

may be expressed as $9\cos^2\theta - 22\cos\theta + 4 = 0$. [3]

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(ii) Hence solve the equation

$$\frac{\cos\theta - 4}{\sin\theta} - \frac{4\sin\theta}{5\cos\theta - 2} = 0$$

for $0° \leqslant \theta \leqslant 360°$. [3]

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6 A curve has a stationary point at $(3, 9\frac{1}{2})$ and has an equation for which $\frac{dy}{dx} = ax^2 + a^2x$, where a is a non-zero constant.

(i) Find the value of a. [2]

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(ii) Find the equation of the curve. [4]

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(iii) Determine, showing all necessary working, the nature of the stationary point. [2]

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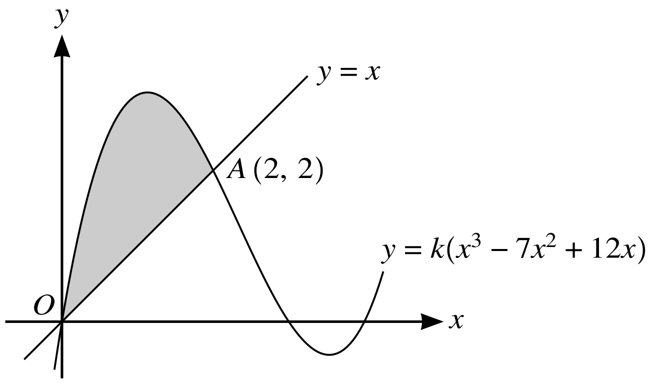

The diagram shows part of the curve with equation $y = k(x^3 - 7x^2 + 12x)$ for some constant k. The curve intersects the line $y = x$ at the origin O and at the point $A\ (2, 2)$.

(i) Find the value of k. [1]

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(ii) Verify that the curve meets the line $y = x$ again when $x = 5$. [2]

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(iii) Find, showing all necessary working, the area of the shaded region. [5]

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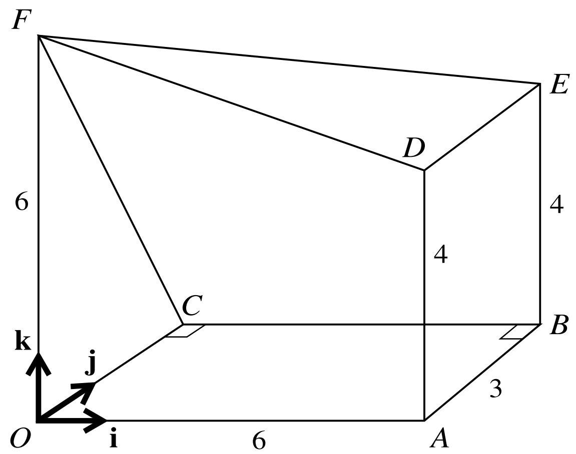

The diagram shows a solid figure $OABCDEF$ having a horizontal rectangular base $OABC$ with $OA = 6$ units and $AB = 3$ units. The vertical edges OF, AD and BE have lengths 6 units, 4 units and 4 units respectively. Unit vectors $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ are parallel to OA, OC and OF respectively.

(i) Find $\overrightarrow{DF}$. [1]

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(ii) Find the unit vector in the direction of $\overrightarrow{EF}$. [3]

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(iii) Use a scalar product to find angle EFD. [4]

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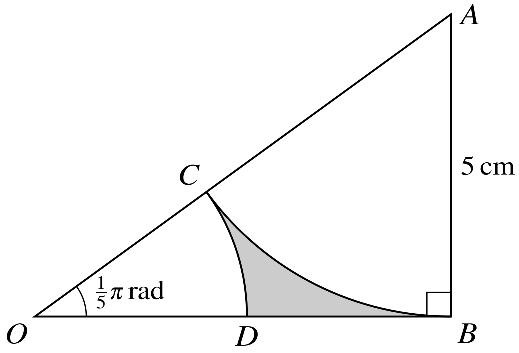

The diagram shows a triangle OAB in which angle ABO is a right angle, angle $AOB = \frac{1}{5}\pi$ radians and $AB = 5$ cm. The arc BC is part of a circle with centre A and meets OA at C. The arc CD is part of a circle with centre O and meets OB at D. Find the area of the shaded region. [8]

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10 A curve has equation $y = \frac{1}{2}(4x - 3)^{-1}$. The point A on the curve has coordinates $\left(1, \frac{1}{2}\right)$.

(i) (a) Find and simplify the equation of the normal through A. [5]

(b) Find the x-coordinate of the point where this normal meets the curve again. [3]

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(ii) A point is moving along the curve in such a way that as it passes through A its x-coordinate is decreasing at the rate of 0.3 units per second. Find the rate of change of its y-coordinate at A. [2]

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11 **(a)** The one-one function f is defined by $f(x) = (x-3)^2 - 1$ for $x < a$, where a is a constant.

(i) State the greatest possible value of a. [1]

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(ii) It is given that a takes this greatest possible value. State the range of f and find an expression for $f^{-1}(x)$. [3]

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(b) The function g is defined by $g(x) = (x-3)^2$ for $x \geqslant 0$.

(i) Show that gg$(2x)$ can be expressed in the form $(2x-3)^4 + b(2x-3)^2 + c$, where b and c are constants to be found. [2]

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(ii) Hence expand gg$(2x)$ completely, simplifying your answer. [4]

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CANDIDATE NAME	
CENTRE NUMBER	
CANDIDATE NUMBER	

9945076812

MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **(P1)** **October/November 2018**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

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You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.

This document consists of **20** printed pages.

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[Turn over

1 Find the coefficient of $\frac{1}{x^2}$ in the expansion of $\left(3x + \frac{2}{3x^2}\right)^7$. [4]

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2 Showing all necessary working, find $\int_1^4 \left(\sqrt{x} + \frac{2}{\sqrt{x}}\right) \mathrm{d}x$. [4]

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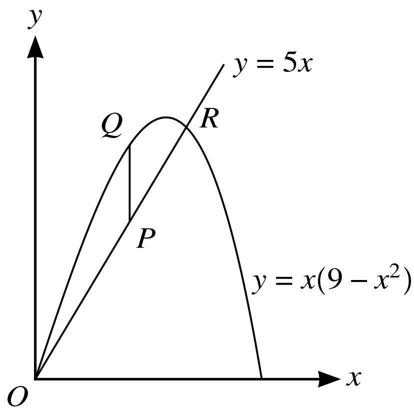
3



The diagram shows part of the curve $y = x(9 - x^2)$ and the line $y = 5x$, intersecting at the origin O and the point R. Point P lies on the line $y = 5x$ between O and R and the x-coordinate of P is t. Point Q lies on the curve and PQ is parallel to the y-axis.

(i) Express the length of PQ in terms of t, simplifying your answer. [2]

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(ii) Given that t can vary, find the maximum value of the length of PQ. [3]

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4 Functions f and g are defined by

$$\mathrm{f}: x \mapsto 2 - 3\cos x \quad \text{for } 0 \leqslant x \leqslant 2\pi,$$

$$\mathrm{g}: x \mapsto \tfrac{1}{2}x \quad \text{for } 0 \leqslant x \leqslant 2\pi.$$

(i) Solve the equation $\mathrm{fg}(x) = 1$. [3]

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(ii) Sketch the graph of $y = \mathrm{f}(x)$. [3]

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5 The first three terms of an arithmetic progression are 4, x and y respectively. The first three terms of a geometric progression are x, y and 18 respectively. It is given that both x and y are positive.

(i) Find the value of x and the value of y. [4]

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(ii) Find the fourth term of each progression. [3]

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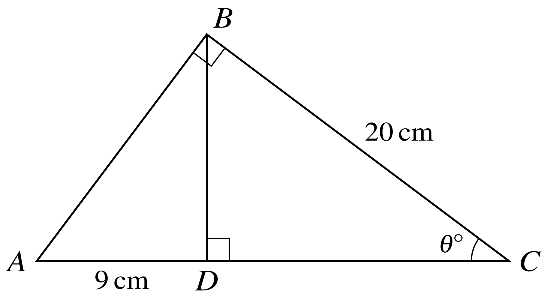
6



The diagram shows a triangle ABC in which $BC = 20$ cm and angle $ABC = 90°$. The perpendicular from B to AC meets AC at D and $AD = 9$ cm. Angle $BCA = \theta°$.

(i) By expressing the length of BD in terms of θ in each of the triangles ABD and DBC, show that $20\sin^2\theta = 9\cos\theta$. [4]

(ii) Hence, showing all necessary working, calculate θ. [3]

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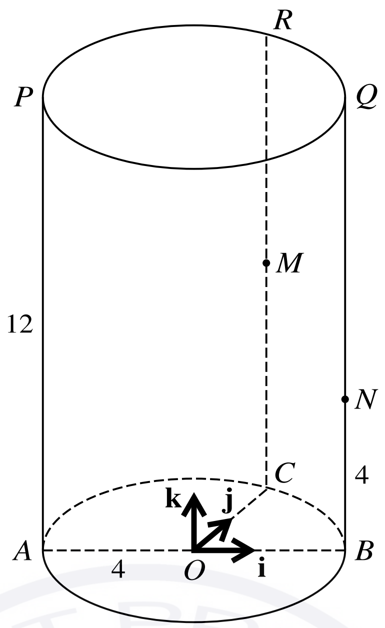

The diagram shows a solid cylinder standing on a horizontal circular base with centre O and radius 4 units. Points A, B and C lie on the circumference of the base such that AB is a diameter and angle $BOC = 90°$. Points P, Q and R lie on the upper surface of the cylinder vertically above A, B and C respectively. The height of the cylinder is 12 units. The mid-point of CR is M and N lies on BQ with $BN = 4$ units.

Unit vectors **i** and **j** are parallel to OB and OC respectively and the unit vector **k** is vertically upwards.

Evaluate $\overrightarrow{PN}.\overrightarrow{PM}$ and hence find angle MPN. [7]

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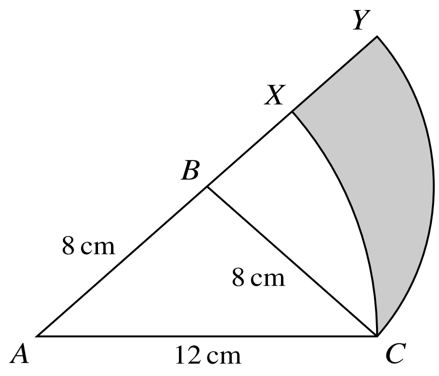

The diagram shows an isosceles triangle ACB in which $AB = BC = 8$ cm and $AC = 12$ cm. The arc XC is part of a circle with centre A and radius 12 cm, and the arc YC is part of a circle with centre B and radius 8 cm. The points A, B, X and Y lie on a straight line.

(i) Show that angle $CBY = 1.445$ radians, correct to 4 significant figures. [3]

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(ii) Find the perimeter of the shaded region. [4]

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9 The function f is defined by $\mathrm{f} : x \mapsto 2x^2 - 12x + 7$ for $x \in \mathbb{R}$.

(i) Express $2x^2 - 12x + 7$ in the form $2(x + a)^2 + b$, where a and b are constants. [2]

(ii) State the range of f. [1]

The function g is defined by $g : x \mapsto 2x^2 - 12x + 7$ for $x \leqslant k$.

(iii) State the largest value of k for which g has an inverse. [1]

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(iv) Given that g has an inverse, find an expression for $g^{-1}(x)$. [3]

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10 The equation of a curve is $y = 2x + \dfrac{12}{x}$ and the equation of a line is $y + x = k$, where k is a constant.

(i) Find the set of values of k for which the line does not meet the curve. [3]

In the case where $k = 15$, the curve intersects the line at points A and B.

(ii) Find the coordinates of A and B. [3]

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(iii) Find the equation of the perpendicular bisector of the line joining A and B. [3]

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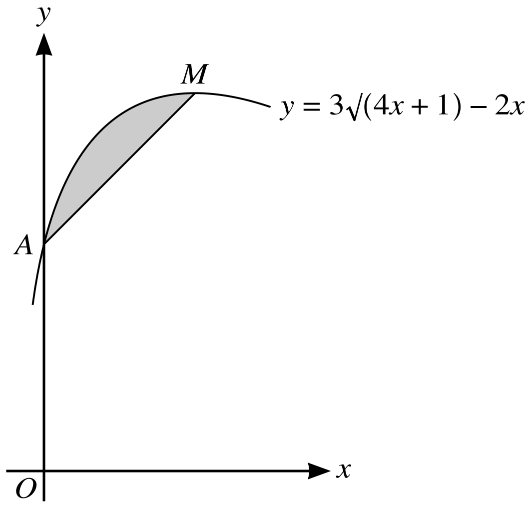
11



The diagram shows part of the curve $y = 3\sqrt{(4x + 1)} - 2x$. The curve crosses the y-axis at A and the stationary point on the curve is M.

(i) Obtain expressions for $\frac{dy}{dx}$ and $\int y \, dx$. [5]

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(ii) Find the coordinates of M. [3]

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(iii) Find, showing all necessary working, the area of the shaded region. [4]

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CANDIDATE NAME

CENTRE NUMBER | CANDIDATE NUMBER

3414314086

MATHEMATICS **9709/13**

Paper 1 Pure Mathematics 1 **(P1)** **October/November 2018**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

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[Turn over

1 Find the coefficient of $\frac{1}{x^3}$ in the expansion of $\left(x - \frac{2}{x}\right)^7$. [3]

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2 The function f is defined by $f(x) = x^3 + 2x^2 - 4x + 7$ for $x \geqslant -2$. Determine, showing all necessary working, whether f is an increasing function, a decreasing function or neither. [4]

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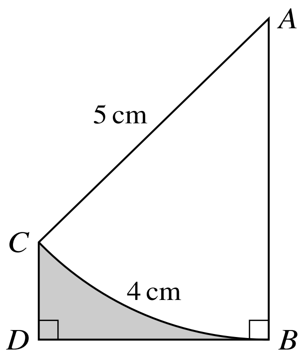

The diagram shows an arc BC of a circle with centre A and radius 5 cm. The length of the arc BC is 4 cm. The point D is such that the line BD is perpendicular to BA and DC is parallel to BA.

(i) Find angle BAC in radians. [1]

(ii) Find the area of the shaded region BDC. [5]

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4 Two points A and B have coordinates $(-1, 1)$ and $(3, 4)$ respectively. The line BC is perpendicular to AB and intersects the x-axis at C.

(i) Find the equation of BC and the x-coordinate of C. [4]

(ii) Find the distance AC, giving your answer correct to 3 decimal places. [2]

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5 In an arithmetic progression the first term is a and the common difference is 3. The nth term is 94 and the sum of the first n terms is 1420. Find n and a. [6]

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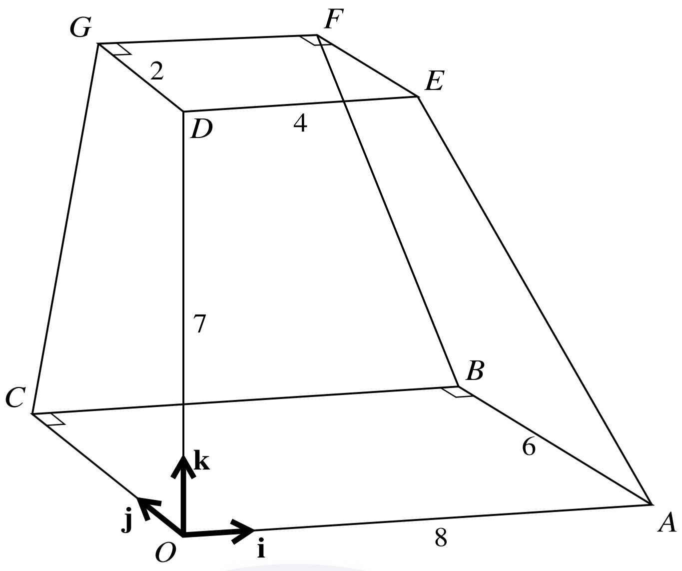

The diagram shows a solid figure $OABCDEFG$ with a horizontal rectangular base $OABC$ in which $OA = 8$ units and $AB = 6$ units. The rectangle $DEFG$ lies in a horizontal plane and is such that D is 7 units vertically above O and DE is parallel to OA. The sides DE and DG have lengths 4 units and 2 units respectively. Unit vectors **i**, **j** and **k** are parallel to OA, OC and OD respectively. Use a scalar product to find angle OBF, giving your answer in the form $\cos^{-1}\left(\frac{a}{b}\right)$, where a and b are integers.

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7 (i) Show that $\frac{\tan\theta + 1}{1 + \cos\theta} + \frac{\tan\theta - 1}{1 - \cos\theta} \equiv \frac{2(\tan\theta - \cos\theta)}{\sin^2\theta}$. [3]

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(ii) Hence, showing all necessary working, solve the equation

$$\frac{\tan\theta + 1}{1 + \cos\theta} + \frac{\tan\theta - 1}{1 - \cos\theta} = 0$$

for $0° < \theta < 90°$. [4]

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8 A curve passes through (0, 11) and has an equation for which $\frac{dy}{dx} = ax^2 + bx - 4$, where a and b are constants.

(i) Find the equation of the curve in terms of a and b. [3]

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(ii) It is now given that the curve has a stationary point at $(2, 3)$. Find the values of a and b. [5]

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9 A curve has equation $y = 2x^2 - 3x + 1$ and a line has equation $y = kx + k^2$, where k is a constant.

(i) Show that, for all values of k, the curve and the line meet. [4]

(ii) State the value of k for which the line is a tangent to the curve and find the coordinates of the point where the line touches the curve. [4]

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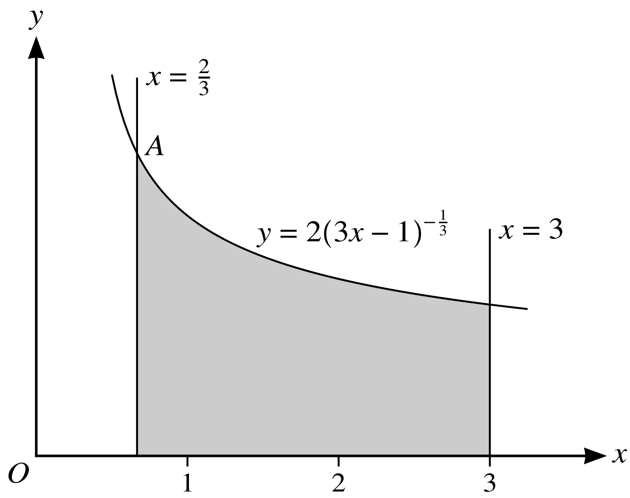

The diagram shows part of the curve $y = 2(3x-1)^{-\frac{1}{3}}$ and the lines $x = \frac{2}{3}$ and $x = 3$. The curve and the line $x = \frac{2}{3}$ intersect at the point A.

(i) Find, showing all necessary working, the volume obtained when the shaded region is rotated through 360° about the x-axis. [5]

(ii) Find the equation of the normal to the curve at A, giving your answer in the form $y = mx + c$. [5]

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11 **(i)** Express $2x^2 - 12x + 11$ in the form $a(x + b)^2 + c$, where a, b and c are constants. [3]

The function f is defined by $f(x) = 2x^2 - 12x + 11$ for $x \leqslant k$.

(ii) State the largest value of the constant k for which f is a one-one function. [1]

(iii) For this value of k find an expression for $f^{-1}(x)$ and state the domain of f^{-1}. [4]

The function g is defined by $g(x) = x + 3$ for $x \leqslant p$.

(iv) With k now taking the value 1, find the largest value of the constant p which allows the composite function fg to be formed, and find an expression for $fg(x)$ whenever this composite function exists. [3]

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CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

2262006173

MATHEMATICS **9709/11**

Paper 1 Pure Mathematics 1 **(P1)** **May/June 2018**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

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You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.

This document consists of **19** printed pages and **1** blank page.

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1 **(i)** Find the first three terms in the expansion, in ascending powers of x, of $(1 - 2x)^5$. [2]

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(ii) Given that the coefficient of x^2 in the expansion of $(1 + ax + 2x^2)(1 - 2x)^5$ is 12, find the value of the constant a. [3]

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2 A point is moving along the curve $y = 2x + \dfrac{5}{x}$ in such a way that the x-coordinate is increasing at a constant rate of 0.02 units per second. Find the rate of change of the y-coordinate when $x = 1$. [4]

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3 A curve is such that $\frac{dy}{dx} = \frac{12}{(2x+1)^2}$. The point $(1, 1)$ lies on the curve. Find the coordinates of the point at which the curve intersects the x-axis. [6]

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4 **(i)** Prove the identity $(\sin\theta + \cos\theta)(1 - \sin\theta\cos\theta) \equiv \sin^3\theta + \cos^3\theta$. [3]

(ii) Hence solve the equation $(\sin\theta + \cos\theta)(1 - \sin\theta\cos\theta) = 3\cos^3\theta$ for $0° \leqslant \theta \leqslant 360°$. [3]

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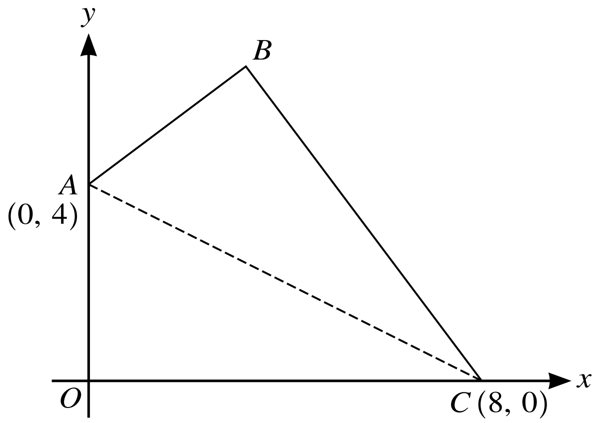

The diagram shows a kite $OABC$ in which AC is the line of symmetry. The coordinates of A and C are (0, 4) and (8, 0) respectively and O is the origin.

(i) Find the equations of AC and OB. [4]

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(ii) Find, by calculation, the coordinates of B. [3]

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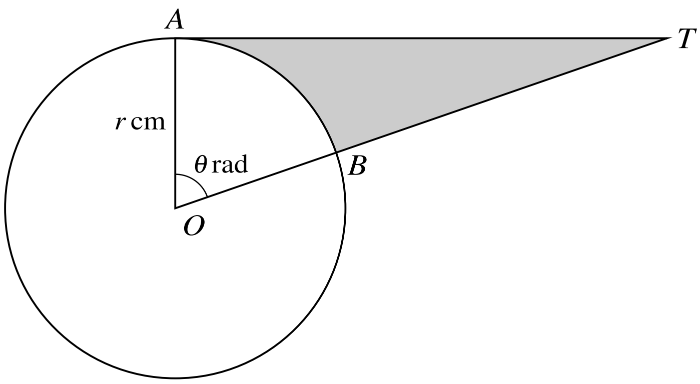

The diagram shows a circle with centre O and radius r cm. The points A and B lie on the circle and AT is a tangent to the circle. Angle $AOB = \theta$ radians and OBT is a straight line.

(i) Express the area of the shaded region in terms of r and θ. [3]

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(ii) In the case where $r = 3$ and $\theta = 1.2$, find the perimeter of the shaded region. [4]

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7 Relative to an origin O, the position vectors of the points A, B and C are given by

$$\overrightarrow{OA} = \begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} -1 \\ 3 \\ 5 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix}.$$

(i) Find $\overrightarrow{AC}$. [1]

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(ii) The point M is the mid-point of AC. Find the unit vector in the direction of $\overrightarrow{OM}$. [3]

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(iii) Evaluate $\overrightarrow{AB}.\overrightarrow{AC}$ and hence find angle BAC. [4]

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8 **(a)** A geometric progression has a second term of 12 and a sum to infinity of 54. Find the possible values of the first term of the progression. [4]

(b) The nth term of a progression is $p + qn$, where p and q are constants, and S_n is the sum of the first n terms.

(i) Find an expression, in terms of p, q and n, for S_n. [3]

(ii) Given that $S_4 = 40$ and $S_6 = 72$, find the values of p and q. [2]

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9 Functions f and g are defined for $x \in \mathbb{R}$ by

$$f : x \mapsto \tfrac{1}{2}x - 2,$$
$$g : x \mapsto 4 + x - \tfrac{1}{2}x^2.$$

(i) Find the points of intersection of the graphs of $y = f(x)$ and $y = g(x)$. [3]

(ii) Find the set of values of x for which $f(x) > g(x)$. [2]

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(iii) Find an expression for $\mathrm{fg}(x)$ and deduce the range of fg. [4]

The function h is defined by $\mathrm{h} : x \mapsto 4 + x - \frac{1}{2}x^2$ for $x \geqslant k$.

(iv) Find the smallest value of k for which h has an inverse. [2]

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10 The curve with equation $y = x^3 - 2x^2 + 5x$ passes through the origin.

(i) Show that the curve has no stationary points. [3]

(ii) Denoting the gradient of the curve by m, find the stationary value of m and determine its nature. [5]

(iii) Showing all necessary working, find the area of the region enclosed by the curve, the x-axis and the line $x = 6$. [4]

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6653000674

MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **(P1)** **May/June 2018**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

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The total number of marks for this paper is 75.

This document consists of **20** printed pages.

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[Turn over

1 The coefficient of x^2 in the expansion of $\left(2 + \frac{x}{2}\right)^6 + (a + x)^5$ is 330. Find the value of the constant a. [5]

2 The equation of a curve is $y = x^2 - 6x + k$, where k is a constant.

(i) Find the set of values of k for which the whole of the curve lies above the x-axis. [2]

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(ii) Find the value of k for which the line $y + 2x = 7$ is a tangent to the curve. [3]

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3 A company producing salt from sea water changed to a new process. The amount of salt obtained each week increased by 2% of the amount obtained in the preceding week. It is given that in the first week after the change the company obtained 8000 kg of salt.

(i) Find the amount of salt obtained in the 12th week after the change. [3]

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(ii) Find the total amount of salt obtained in the first 12 weeks after the change. [2]

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4 The function f is such that $\mathrm{f}(x) = a + b\cos x$ for $0 \leqslant x \leqslant 2\pi$. It is given that $\mathrm{f}\left(\frac{1}{3}\pi\right) = 5$ and $\mathrm{f}(\pi) = 11$.

(i) Find the values of the constants a and b. [3]

(ii) Find the set of values of k for which the equation $\mathrm{f}(x) = k$ has no solution. [3]

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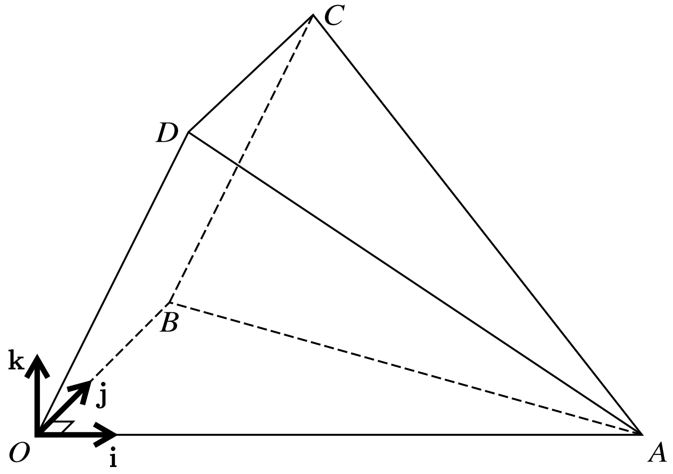

The diagram shows a three-dimensional shape. The base OAB is a horizontal triangle in which angle AOB is 90°. The side $OBCD$ is a rectangle and the side OAD lies in a vertical plane. Unit vectors **i** and **j** are parallel to OA and OB respectively and the unit vector **k** is vertical. The position vectors of A, B and D are given by $\overrightarrow{OA} = 8\mathbf{i}$, $\overrightarrow{OB} = 5\mathbf{j}$ and $\overrightarrow{OD} = 2\mathbf{i} + 4\mathbf{k}$.

(i) Express each of the vectors $\overrightarrow{DA}$ and $\overrightarrow{CA}$ in terms of **i**, **j** and **k**. [2]

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(ii) Use a scalar product to find angle *CAD*. [4]

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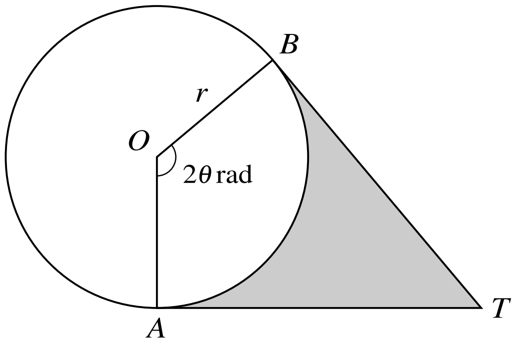

The diagram shows points A and B on a circle with centre O and radius r. The tangents to the circle at A and B meet at T. The shaded region is bounded by the minor arc AB and the lines AT and BT. Angle AOB is 2θ radians.

(i) In the case where the area of the sector AOB is the same as the area of the shaded region, show that $\tan\theta = 2\theta$. [3]

(ii) In the case where $r = 8$ cm and the length of the minor arc AB is 19.2 cm, find the area of the shaded region. [3]

7 The function f is defined by $\mathrm{f} : x \mapsto 7 - 2x^2 - 12x$ for $x \in \mathbb{R}$.

(i) Express $7 - 2x^2 - 12x$ in the form $a - 2(x + b)^2$, where a and b are constants. [2]

(ii) State the coordinates of the stationary point on the curve $y = \mathrm{f}(x)$. [1]

The function g is defined by $g : x \mapsto 7 - 2x^2 - 12x$ for $x \geqslant k$.

(iii) State the smallest value of k for which g has an inverse. [1]

(iv) For this value of k, find $g^{-1}(x)$. [3]

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8 Points A and B have coordinates $(h,\ h)$ and $(4h+6,\ 5h)$ respectively. The equation of the perpendicular bisector of AB is $3x + 2y = k$. Find the values of the constants h and k. [7]

9 A curve is such that $\frac{dy}{dx} = \sqrt{(4x + 1)}$ and $(2, 5)$ is a point on the curve.

(i) Find the equation of the curve. [4]

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(ii) A point P moves along the curve in such a way that the y-coordinate is increasing at a constant rate of 0.06 units per second. Find the rate of change of the x-coordinate when P passes through (2, 5). [2]

(iii) Show that $\frac{d^2y}{dx^2} \times \frac{dy}{dx}$ is constant. [2]

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10 **(i)** Solve the equation $2\cos x + 3\sin x = 0$, for $0° \leqslant x \leqslant 360°$. [3]

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(ii) Sketch, on the same diagram, the graphs of $y = 2\cos x$ and $y = -3\sin x$ for $0° \leqslant x \leqslant 360°$. [3]

(iii) Use your answers to parts **(i)** and **(ii)** to find the set of values of x for $0° \leqslant x \leqslant 360°$ for which $2\cos x + 3\sin x > 0$. [2]

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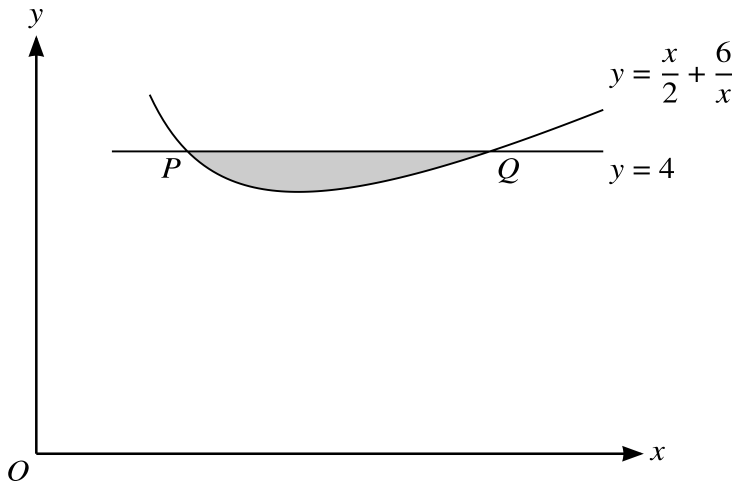

The diagram shows part of the curve $y = \frac{x}{2} + \frac{6}{x}$. The line $y = 4$ intersects the curve at the points P and Q.

(i) Show that the tangents to the curve at P and Q meet at a point on the line $y = x$. [6]

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(ii) Find, showing all necessary working, the volume obtained when the shaded region is rotated through 360° about the x-axis. Give your answer in terms of π. [6]

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MATHEMATICS **9709/13**

Paper 1 Pure Mathematics 1 **(P1)** **May/June 2018**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

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[Turn over

1 Express $3x^2 - 12x + 7$ in the form $a(x + b)^2 + c$, where a, b and c are constants. [3]

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2 Find the coefficient of $\frac{1}{x}$ in the expansion of $\left(x - \frac{2}{x}\right)^5$. [3]

3 The common ratio of a geometric progression is 0.99. Express the sum of the first 100 terms as a percentage of the sum to infinity, giving your answer correct to 2 significant figures. [5]

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4 A curve with equation $y = \mathrm{f}(x)$ passes through the point $A\ (3,\ 1)$ and crosses the y-axis at B. It is given that $\mathrm{f}'(x) = (3x - 1)^{-\frac{1}{3}}$. Find the y-coordinate of B. [6]

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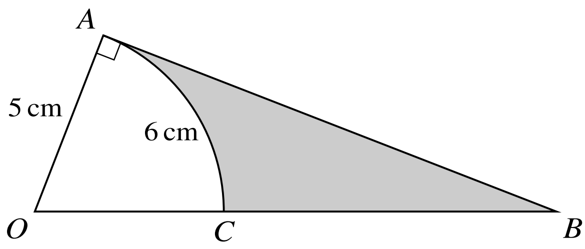

The diagram shows a triangle OAB in which angle $OAB = 90°$ and $OA = 5$ cm. The arc AC is part of a circle with centre O. The arc has length 6 cm and it meets OB at C. Find the area of the shaded region. [5]

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6 The coordinates of points A and B are $(-3k - 1,\ k + 3)$ and $(k + 3,\ 3k + 5)$ respectively, where k is a constant $(k \neq -1)$.

(i) Find and simplify the gradient of AB, showing that it is independent of k. [2]

(ii) Find and simplify the equation of the perpendicular bisector of AB. [5]

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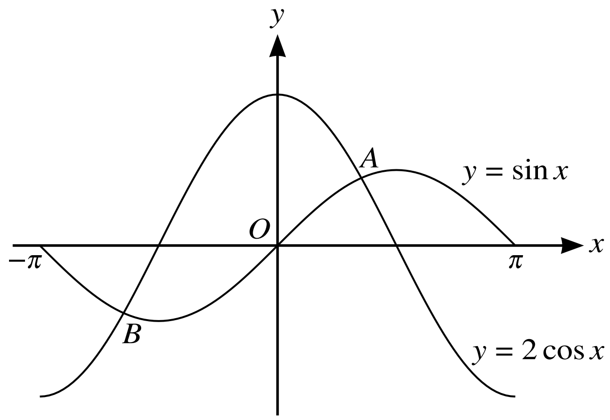
7 **(a)** **(i)** Express $\dfrac{\tan^2\theta - 1}{\tan^2\theta + 1}$ in the form $a\sin^2\theta + b$, where a and b are constants to be found. [3]

(ii) Hence, or otherwise, and showing all necessary working, solve the equation

$$\frac{\tan^2\theta - 1}{\tan^2\theta + 1} = \frac{1}{4}$$

for $-90° \leqslant \theta \leqslant 0°$. [2]

(b)



The diagram shows the graphs of $y = \sin x$ and $y = 2\cos x$ for $-\pi \leqslant x \leqslant \pi$. The graphs intersect at the points A and B.

(i) Find the x-coordinate of A. [2]

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(ii) Find the y-coordinate of B. [2]

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8 **(i)** The tangent to the curve $y = x^3 - 9x^2 + 24x - 12$ at a point A is parallel to the line $y = 2 - 3x$. Find the equation of the tangent at A. [6]

(ii) The function f is defined by $f(x) = x^3 - 9x^2 + 24x - 12$ for $x > k$, where k is a constant. Find the smallest value of k for f to be an increasing function. [2]

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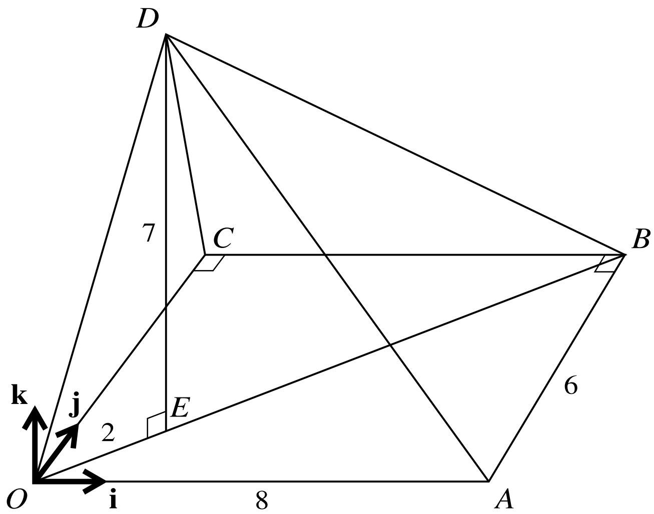

The diagram shows a pyramid $OABCD$ with a horizontal rectangular base $OABC$. The sides OA and AB have lengths of 8 units and 6 units respectively. The point E on OB is such that $OE = 2$ units. The point D of the pyramid is 7 units vertically above E. Unit vectors $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ are parallel to OA, OC and ED respectively.

(i) Show that $\overrightarrow{OE} = 1.6\mathbf{i} + 1.2\mathbf{j}$. [2]

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(ii) Use a scalar product to find angle BDO. [7]

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10 The one-one function f is defined by $f(x) = (x-2)^2 + 2$ for $x \geqslant c$, where c is a constant.

(i) State the smallest possible value of c. [1]

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In parts **(ii)** and **(iii)** the value of c is 4.

(ii) Find an expression for $f^{-1}(x)$ and state the domain of f^{-1}. [3]

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(iii) Solve the equation $ff(x) = 51$, giving your answer in the form $a + \sqrt{b}$. [5]

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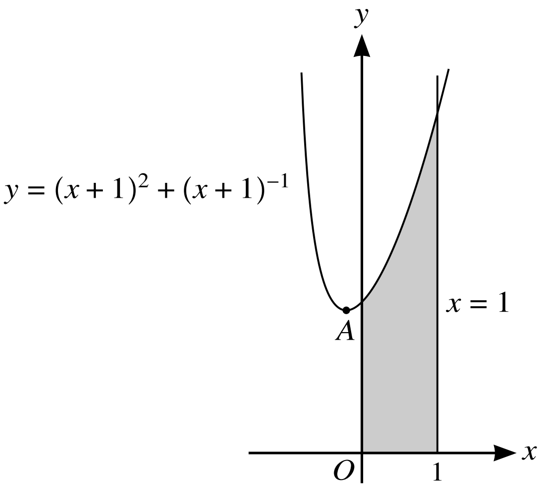
11



The diagram shows part of the curve $y = (x+1)^2 + (x+1)^{-1}$ and the line $x = 1$. The point A is the minimum point on the curve.

(i) Show that the x-coordinate of A satisfies the equation $2(x+1)^3 = 1$ and find the exact value of $\frac{d^2y}{dx^2}$ at A. [5]

(ii) Find, showing all necessary working, the volume obtained when the shaded region is rotated through 360° about the x-axis. [6]

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CENTRE NUMBER

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8348568421

MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **(P1)** **February/March 2018**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

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1 A curve passes through the point (4, −6) and has an equation for which $\frac{dy}{dx} = x^{-\frac{1}{2}} - 3$. Find the equation of the curve. [4]

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2 **(i)** Find the coefficients of x^2 and x^3 in the expansion of $(1 - 2x)^7$. [3]

(ii) Hence find the coefficient of x^3 in the expansion of $(2 + 5x)(1 - 2x)^7$. [2]

3 On a certain day, the height of a young bamboo plant was found to be 40 cm. After exactly one day its height was found to be 41.2 cm. Two different models are used to predict its height exactly 60 days after it was first measured.

- Model A assumes that the daily amount of growth continues to be constant at the amount found for the first day.
- Model B assumes that the daily percentage rate of growth continues to be constant at the percentage rate of growth found for the first day.

(i) Using model A, find the predicted height in cm of the bamboo plant exactly 60 days after it was first measured. [2]

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(ii) Using model B, find the predicted height in cm of the bamboo plant exactly 60 days after it was first measured. [3]

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4 A straight line cuts the positive x-axis at A and the positive y-axis at $B\,(0,\ 2)$. Angle $BAO = \frac{1}{6}\pi$ radians, where O is the origin.

(i) Find the exact value of the x-coordinate of A. [2]

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(ii) Find the equation of the perpendicular bisector of AB, giving your answer in the form $y = mx + c$, where m is given exactly and c is an integer. [4]

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5 **(a)** Express the equation $\frac{5 + 2\tan x}{3 + 2\tan x} = 1 + \tan x$ as a quadratic equation in $\tan x$ and hence solve the equation for $0 \leqslant x \leqslant \pi$. [4]

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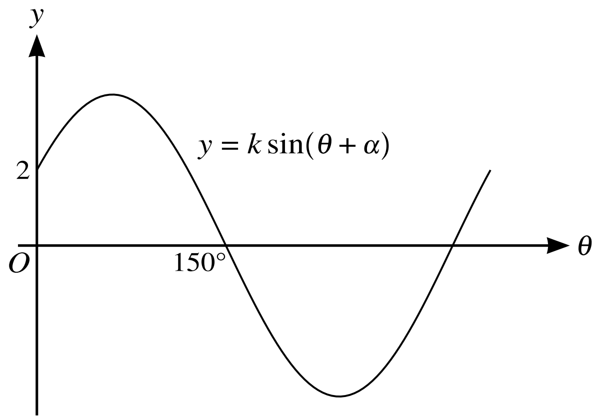
(b)



The diagram shows part of the graph of $y = k\sin(\theta + \alpha)$, where k and α are constants and $0° < \alpha < 180°$. Find the value of α and the value of k. [2]

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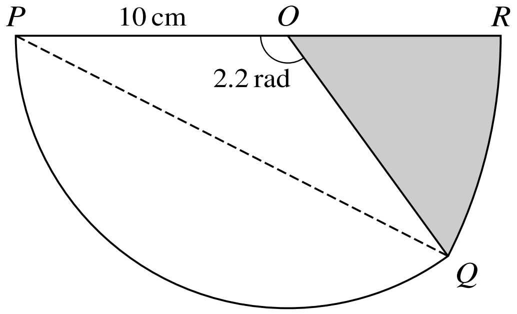

The diagram shows a sector POQ of a circle of radius 10 cm and centre O. Angle POQ is 2.2 radians. QR is an arc of a circle with centre P and POR is a straight line.

(i) Show that the length of PQ is 17.8 cm, correct to 3 significant figures. [2]

(ii) Find the perimeter of the shaded region. [4]

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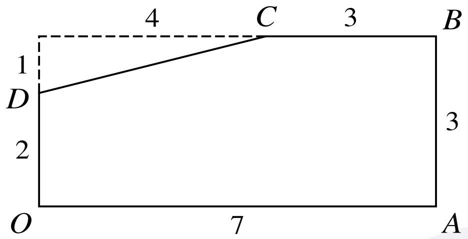

Fig. 1

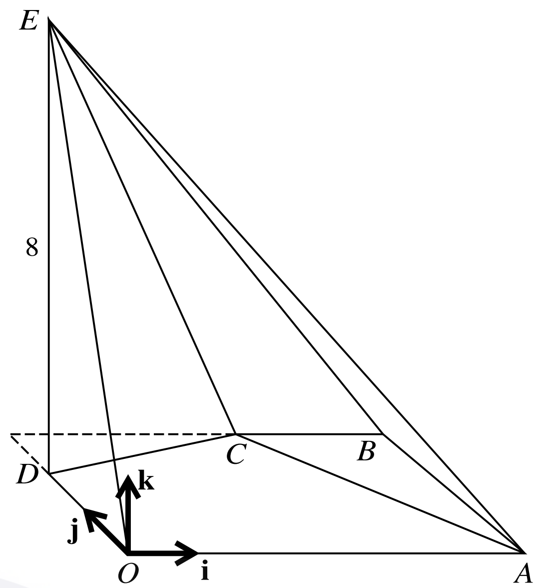

Fig. 2

Fig. 1 shows a rectangle with sides of 7 units and 3 units from which a triangular corner has been removed, leaving a 5-sided polygon *OABCD*. The sides *OA*, *AB*, *BC* and *DO* have lengths of 7 units, 3 units, 3 units and 2 units respectively. Fig. 2 shows the polygon *OABCD* forming the horizontal base of a pyramid in which the point *E* is 8 units vertically above *D*. Unit vectors **i**, **j** and **k** are parallel to *OA*, *OD* and *DE* respectively.

(i) Find $\overrightarrow{CE}$ and the length of *CE*. [3]

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(ii) Use a scalar product to find angle ECA, giving your answer in the form $\cos^{-1}\left(\frac{m}{\sqrt{n}}\right)$, where m and n are integers. [5]

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8 A curve has equation $y = \frac{1}{2}x^2 - 4x^{\frac{3}{2}} + 8x$.

(i) Find the x-coordinates of the stationary points. [5]

(ii) Find $\frac{d^2y}{dx^2}$. [1]

(iii) Find, showing all necessary working, the nature of each stationary point. [2]

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9 A curve has equation $y = \frac{1}{x} + c$ and a line has equation $y = cx - 3$, where c is a constant.

(i) Find the set of values of c for which the curve and the line meet. [4]

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(ii) The line is a tangent to the curve for two particular values of c. For each of these values find the x-coordinate of the point at which the tangent touches the curve. [4]

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10 Functions f and g are defined by

$$f(x) = \frac{8}{x-2} + 2 \quad \text{for } x > 2,$$
$$g(x) = \frac{8}{x-2} + 2 \quad \text{for } 2 < x < 4.$$

(i) (a) State the range of the function f. [1]

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(b) State the range of the function g. [1]

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(c) State the range of the function fg. [1]

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(ii) Explain why the function gf cannot be formed. [1]

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(iii) Find the set of values of x satisfying the inequality $6\mathrm{f}'(x) + 2\mathrm{f}^{-1}(x) - 5 < 0$. [6]

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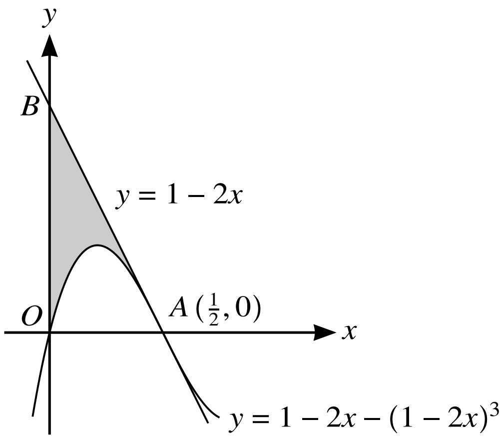
11



The diagram shows part of the curve $y = 1 - 2x - (1 - 2x)^3$ intersecting the x-axis at the origin O and at $A\left(\frac{1}{2}, 0\right)$. The line AB intersects the y-axis at B and has equation $y = 1 - 2x$.

(i) Show that AB is the tangent to the curve at A. [4]

(ii) Show that the area of the shaded region can be expressed as $\int_0^{\frac{1}{2}} (1-2x)^3 \, dx$. [2]

(iii) Hence, showing all necessary working, find the area of the shaded region. [3]

Additional Page

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CENTRE NUMBER		CANDIDATE NUMBER	

* 7 1 2 5 9 9 6 2 3 7 *

MATHEMATICS **9709/11**

Paper 1 Pure Mathematics 1 **(P1)** **October/November 2017**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

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Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.

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1 A curve has equation $y = 2x^{\frac{3}{2}} - 3x - 4x^{\frac{1}{2}} + 4$. Find the equation of the tangent to the curve at the point $(4, 0)$. [4]

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2 A function f is defined by $\mathrm{f} : x \mapsto x^3 - x^2 - 8x + 5$ for $x < a$. It is given that f is an increasing function. Find the largest possible value of the constant a. [4]

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3 **(a)** A geometric progression has first term $3a$ and common ratio r. A second geometric progression has first term a and common ratio $-2r$. The two progressions have the same sum to infinity. Find the value of r. [3]

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(b) The first two terms of an arithmetic progression are 15 and 19 respectively. The first two terms of a second arithmetic progression are 420 and 415 respectively. The two progressions have the same sum of the first n terms. Find the value of n. [3]

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4 Machines in a factory make cardboard cones of base radius r cm and vertical height h cm. The volume, V cm^3, of such a cone is given by $V = \frac{1}{3}\pi r^2 h$. The machines produce cones for which $h + r = 18$.

(i) Show that $V = 6\pi r^2 - \frac{1}{3}\pi r^3$. [1]

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(ii) Given that r can vary, find the non-zero value of r for which V has a stationary value and show that the stationary value is a maximum. [4]

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(iii) Find the maximum volume of a cone that can be made by these machines. [1]

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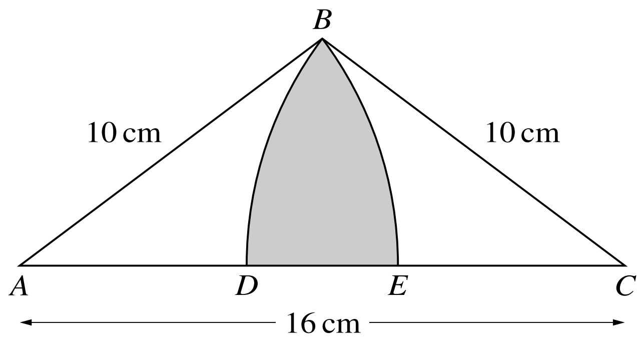

The diagram shows an isosceles triangle ABC in which $AC = 16$ cm and $AB = BC = 10$ cm. The circular arcs BE and BD have centres at A and C respectively, where D and E lie on AC.

(i) Show that angle $BAC = 0.6435$ radians, correct to 4 decimal places. [1]

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(ii) Find the area of the shaded region. [5]

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6 The points $A\,(1,\ 1)$ and $B\,(5,\ 9)$ lie on the curve $6y = 5x^2 - 18x + 19$.

(i) Show that the equation of the perpendicular bisector of AB is $2y = 13 - x$. [4]

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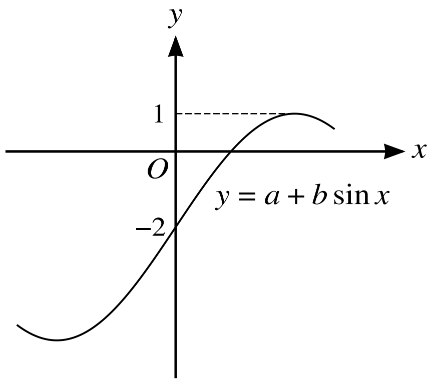
The perpendicular bisector of AB meets the curve at C and D.

(ii) Find, by calculation, the distance CD, giving your answer in the form $\sqrt{\left(\frac{p}{q}\right)}$, where p and q are integers. [5]

7 **(a)**



The diagram shows part of the graph of $y = a + b \sin x$. Find the values of the constants a and b. [2]

(b) **(i)** Show that the equation

$$(\sin\theta + 2\cos\theta)(1 + \sin\theta - \cos\theta) = \sin\theta(1 + \cos\theta)$$

may be expressed as $3\cos^2\theta - 2\cos\theta - 1 = 0$. [3]

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(ii) Hence solve the equation

$$(\sin\theta + 2\cos\theta)(1 + \sin\theta - \cos\theta) = \sin\theta(1 + \cos\theta)$$

for $-180° \leqslant \theta \leqslant 180°$. [4]

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8 **(a)** Relative to an origin O, the position vectors of two points P and Q are $\mathbf{p}$ and $\mathbf{q}$ respectively. The point R is such that PQR is a straight line with Q the mid-point of PR. Find the position vector of R in terms of $\mathbf{p}$ and $\mathbf{q}$, simplifying your answer. [3]

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(b) The vector $6\mathbf{i} + a\mathbf{j} + b\mathbf{k}$ has magnitude 21 and is perpendicular to $3\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$. Find the possible values of a and b, showing all necessary working. [6]

9 Functions f and g are defined for $x > 3$ by

$$f : x \mapsto \frac{1}{x^2 - 9},$$
$$g : x \mapsto 2x - 3.$$

(i) Find and simplify an expression for $gg(x)$. [2]

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(ii) Find an expression for $f^{-1}(x)$ and state the domain of f^{-1}. [4]

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(iii) Solve the equation $fg(x) = \frac{1}{7}$. [4]

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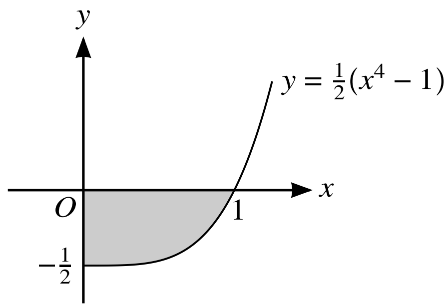

The diagram shows part of the curve $y = \frac{1}{2}(x^4 - 1)$, defined for $x \geqslant 0$.

(i) Find, showing all necessary working, the area of the shaded region. [3]

(ii) Find, showing all necessary working, the volume obtained when the shaded region is rotated through 360° about the x-axis. [4]

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(iii) Find, showing all necessary working, the volume obtained when the shaded region is rotated through 360° about the y-axis. [5]

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CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

3053484001

MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **(P1)** **October/November 2017**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

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Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.

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1 Find the term independent of x in the expansion of $\left(2x - \frac{1}{4x^2}\right)^9$. [4]

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2 A function f is defined by $\mathrm{f} : x \mapsto 4 - 5x$ for $x \in \mathbb{R}$.

(i) Find an expression for $\mathrm{f}^{-1}(x)$ and find the point of intersection of the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{f}^{-1}(x)$. [3]

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(ii) Sketch, on the same diagram, the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{f}^{-1}(x)$, making clear the relationship between the graphs. [3]

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3 **(a)** Each year, the value of a certain rare stamp increases by 5% of its value at the beginning of the year. A collector bought the stamp for \$10 000 at the beginning of 2005. Find its value at the beginning of 2015 correct to the nearest \$100. [2]

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(b) The sum of the first n terms of an arithmetic progression is $\frac{1}{2}n(3n+7)$. Find the 1st term and the common difference of the progression. [4]

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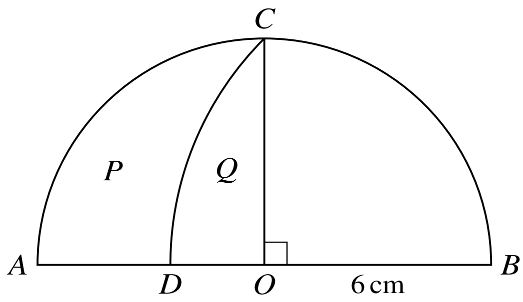

The diagram shows a semicircle with centre O and radius 6 cm. The radius OC is perpendicular to the diameter AB. The point D lies on AB, and DC is an arc of a circle with centre B.

(i) Calculate the length of the arc DC. [3]

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(ii) Find the value of

$$\frac{\text{area of region } P}{\text{area of region } Q},$$

giving your answer correct to 3 significant figures. [4]

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5 **(i)** Show that the equation $\cos 2x(\tan^2 2x + 3) + 3 = 0$ can be expressed as

$$2\cos^2 2x + 3\cos 2x + 1 = 0.$$ [3]

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(ii) Hence solve the equation $\cos 2x(\tan^2 2x + 3) + 3 = 0$ for $0° \leqslant x \leqslant 180°$. [4]

6 **(a)** The function f, defined by $f : x \mapsto a + b\sin x$ for $x \in \mathbb{R}$, is such that $f\left(\frac{1}{6}\pi\right) = 4$ and $f\left(\frac{1}{2}\pi\right) = 3$.

(i) Find the values of the constants a and b. [3]

(ii) Evaluate ff(0). [2]

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(b) The function g is defined by $g : x \mapsto c + d \sin x$ for $x \in \mathbb{R}$. The range of g is given by $-4 \leqslant g(x) \leqslant 10$. Find the values of the constants c and d. [3]

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7 Points A and B lie on the curve $y = x^2 - 4x + 7$. Point A has coordinates $(4, 7)$ and B is the stationary point of the curve. The equation of a line L is $y = mx - 2$, where m is a constant.

(i) In the case where L passes through the mid-point of AB, find the value of m. [4]

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(ii) Find the set of values of m for which L does not meet the curve. [4]

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8 A curve is such that $\frac{dy}{dx} = -x^2 + 5x - 4$.

(i) Find the x-coordinate of each of the stationary points of the curve. [2]

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(ii) Obtain an expression for $\frac{d^2y}{dx^2}$ and hence or otherwise find the nature of each of the stationary points. [3]

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(iii) Given that the curve passes through the point (6, 2), find the equation of the curve. [4]

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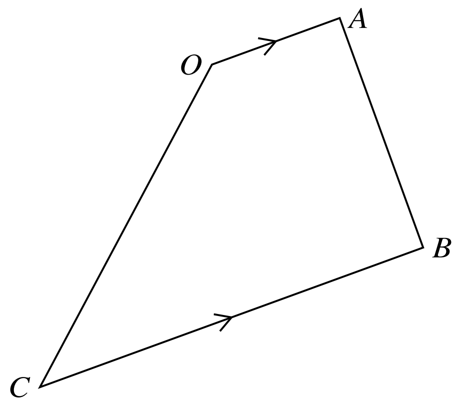

The diagram shows a trapezium $OABC$ in which OA is parallel to CB. The position vectors of A and B relative to the origin O are given by $\overrightarrow{OA} = \begin{pmatrix} 2 \\ -2 \\ -1 \end{pmatrix}$ and $\overrightarrow{OB} = \begin{pmatrix} 6 \\ 1 \\ 1 \end{pmatrix}$.

(i) Show that angle OAB is 90°. [3]

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The magnitude of $\overrightarrow{CB}$ is three times the magnitude of $\overrightarrow{OA}$.

(ii) Find the position vector of C. [3]

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(iii) Find the exact area of the trapezium $OABC$, giving your answer in the form $a\sqrt{b}$, where a and b are integers. [3]

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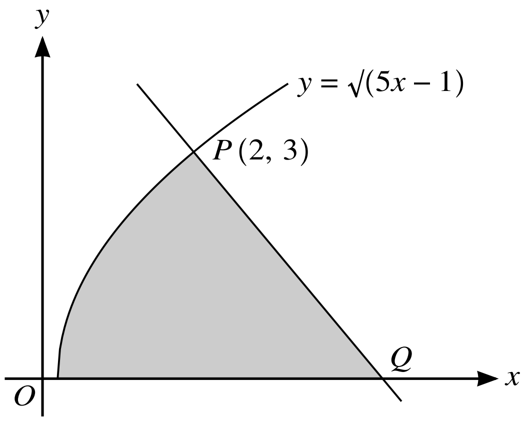

The diagram shows part of the curve $y = \sqrt{(5x - 1)}$ and the normal to the curve at the point $P(2, 3)$. This normal meets the x-axis at Q.

(i) Find the equation of the normal at P. [4]

(ii) Find, showing all necessary working, the area of the shaded region. [7]

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MATHEMATICS **9709/13**

Paper 1 Pure Mathematics 1 **(P1)** **October/November 2017**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

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[Turn over

1 An arithmetic progression has first term -12 and common difference 6. The sum of the first n terms exceeds 3000. Calculate the least possible value of n. [4]

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2 Find the set of values of a for which the curve $y = -\frac{2}{x}$ and the straight line $y = ax + 3a$ meet at two distinct points. [4]

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3 **(i)** Find the term independent of x in the expansion of $\left(\frac{2}{x} - 3x\right)^6$. [2]

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(ii) Find the value of a for which there is no term independent of x in the expansion of

$$(1 + ax^2)\left(\frac{2}{x} - 3x\right)^6.$$ [3]

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4 The function f is such that $f(x) = (2x-1)^{\frac{3}{2}} - 6x$ for $\frac{1}{2} < x < k$, where k is a constant. Find the largest value of k for which f is a decreasing function. [5]

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5 **(i)** Show that the equation $\dfrac{\cos\theta + 4}{\sin\theta + 1} + 5\sin\theta - 5 = 0$ may be expressed as $5\cos^2\theta - \cos\theta - 4 = 0$. [3]

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(ii) Hence solve the equation $\dfrac{\cos\theta + 4}{\sin\theta + 1} + 5\sin\theta - 5 = 0$ for $0° \leqslant \theta \leqslant 360°$. [4]

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6 The functions f and g are defined by

$$f(x) = \frac{2}{x^2 - 1} \text{ for } x < -1,$$
$$g(x) = x^2 + 1 \text{ for } x > 0.$$

(i) Find an expression for $f^{-1}(x)$. [3]

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(ii) Solve the equation $gf(x) = 5$. [4]

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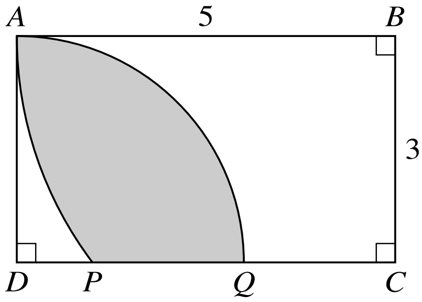

The diagram shows a rectangle $ABCD$ in which $AB = 5$ units and $BC = 3$ units. Point P lies on DC and AP is an arc of a circle with centre B. Point Q lies on DC and AQ is an arc of a circle with centre D.

(i) Show that angle $ABP = 0.6435$ radians, correct to 4 decimal places. [1]

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(ii) Calculate the areas of the sectors BAP and DAQ. [3]

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(iii) Calculate the area of the shaded region. [3]

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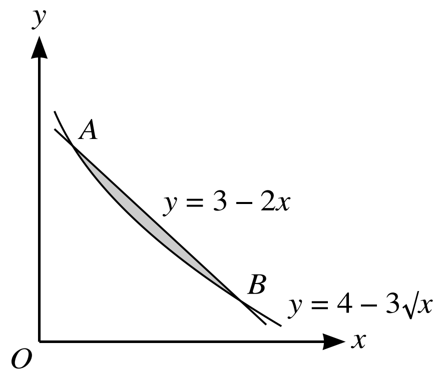

The diagram shows parts of the graphs of $y = 3 - 2x$ and $y = 4 - 3\sqrt{x}$ intersecting at points A and B.

(i) Find by calculation the x-coordinates of A and B. [3]

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(ii) Find, showing all necessary working, the area of the shaded region. [5]

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9 Relative to an origin O, the position vectors of the points A, B and C are given by

$$\overrightarrow{OA} = \begin{pmatrix} 8 \\ -6 \\ 5 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} -10 \\ 3 \\ -13 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} 2 \\ -3 \\ -1 \end{pmatrix}.$$

A fourth point, D, is such that the magnitudes $|\overrightarrow{AB}|$, $|\overrightarrow{BC}|$ and $|\overrightarrow{CD}|$ are the first, second and third terms respectively of a geometric progression.

(i) Find the magnitudes $|\overrightarrow{AB}|$, $|\overrightarrow{BC}|$ and $|\overrightarrow{CD}|$. [5]

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(ii) Given that D is a point lying on the line through B and C, find the two possible position vectors of the point D. [4]

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10 A curve has equation $y = f(x)$ and it is given that $f'(x) = ax^2 + bx$, where a and b are positive constants.

(i) Find, in terms of a and b, the non-zero value of x for which the curve has a stationary point and determine, showing all necessary working, the nature of the stationary point. [3]

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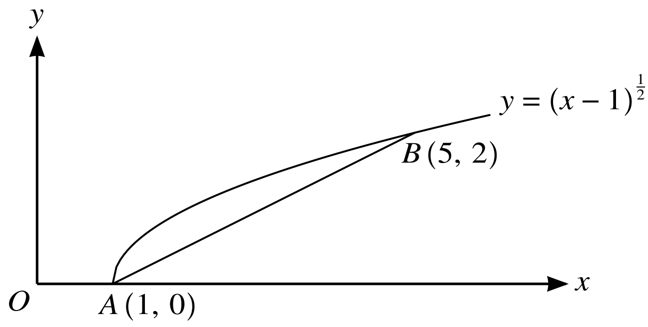
(ii) It is now given that the curve has a stationary point at $(-2, -3)$ and that the gradient of the curve at $x = 1$ is 9. Find $\mathrm{f}(x)$. [6]

11



The diagram shows the curve $y = (x-1)^{\frac{1}{2}}$ and points $A\,(1,\ 0)$ and $B\,(5,\ 2)$ lying on the curve.

(i) Find the equation of the line AB, giving your answer in the form $y = mx + c$. [2]

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(ii) Find, showing all necessary working, the equation of the tangent to the curve which is parallel to AB. [5]

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(iii) Find the perpendicular distance between the line AB and the tangent parallel to AB. Give your answer correct to 2 decimal places. [3]

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MATHEMATICS **9709/11**

Paper 1 Pure Mathematics 1 **(P1)** **May/June 2017**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

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[Turn over

1 The coefficients of x^2 and x^3 in the expansion of $(3 - 2x)^6$ are a and b respectively. Find the value of $\frac{a}{b}$. [4]

2 Relative to an origin O, the position vectors of points A and B are given by

$$\overrightarrow{OA} = \begin{pmatrix} 3 \\ -6 \\ p \end{pmatrix} \quad \text{and} \quad \overrightarrow{OB} = \begin{pmatrix} 2 \\ -6 \\ -7 \end{pmatrix},$$

and angle $AOB = 90°$.

(i) Find the value of p. [2]

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The point C is such that $\overrightarrow{OC} = \frac{2}{3}\overrightarrow{OA}$.

(ii) Find the unit vector in the direction of $\overrightarrow{BC}$. [4]

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3 (i) Prove the identity $\dfrac{1+\cos\theta}{\sin\theta}+\dfrac{\sin\theta}{1+\cos\theta}\equiv\dfrac{2}{\sin\theta}$. [3]

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(ii) Hence solve the equation $\frac{1+\cos\theta}{\sin\theta}+\frac{\sin\theta}{1+\cos\theta}=\frac{3}{\cos\theta}$ for $0° \leqslant \theta \leqslant 360°$. [3]

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4 (a) An arithmetic progression has a first term of 32, a 5th term of 22 and a last term of −28. Find the sum of all the terms in the progression. [4]

(b) Each year a school allocates a sum of money for the library. The amount allocated each year increases by 2.5% of the amount allocated the previous year. In 2005 the school allocated \$2000. Find the total amount allocated in the years 2005 to 2014 inclusive. [3]

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5 The equation of a curve is $y = 2\cos x$.

(i) Sketch the graph of $y = 2\cos x$ for $-\pi \leqslant x \leqslant \pi$, stating the coordinates of the point of intersection with the y-axis. [2]

Points P and Q lie on the curve and have x-coordinates of $\frac{1}{3}\pi$ and π respectively.

(ii) Find the length of PQ correct to 1 decimal place. [2]

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The line through P and Q meets the x-axis at $H(h, 0)$ and the y-axis at $K(0, k)$.

(iii) Show that $h = \frac{5}{9}\pi$ and find the value of k. [3]

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6 The horizontal base of a solid prism is an equilateral triangle of side x cm. The sides of the prism are vertical. The height of the prism is h cm and the volume of the prism is $2000\,\text{cm}^3$.

(i) Express h in terms of x and show that the total surface area of the prism, $A\,\text{cm}^2$, is given by

$$A = \frac{\sqrt{3}}{2}x^2 + \frac{24\,000}{\sqrt{3}}x^{-1}.$$ [3]

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(ii) Given that x can vary, find the value of x for which A has a stationary value. [3]

(iii) Determine, showing all necessary working, the nature of this stationary value. [2]

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7 A curve for which $\frac{dy}{dx} = 7 - x^2 - 6x$ passes through the point $(3, -10)$.

(i) Find the equation of the curve. [3]

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(ii) Express $7 - x^2 - 6x$ in the form $a - (x + b)^2$, where a and b are constants. [2]

(iii) Find the set of values of x for which the gradient of the curve is positive. [3]

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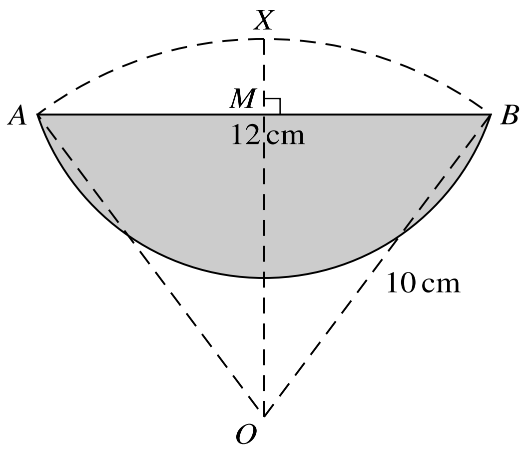

In the diagram, $OAXB$ is a sector of a circle with centre O and radius 10 cm. The length of the chord AB is 12 cm. The line OX passes through M, the mid-point of AB, and OX is perpendicular to AB. The shaded region is bounded by the chord AB and by the arc of a circle with centre X and radius XA.

(i) Show that angle AXB is 2.498 radians, correct to 3 decimal places. [3]

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(ii) Find the perimeter of the shaded region. [3]

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(iii) Find the area of the shaded region. [3]

9 The function f is defined by $\mathrm{f} : x \mapsto \dfrac{2}{3 - 2x}$ for $x \in \mathbb{R},\ x \neq \frac{3}{2}$.

(i) Find an expression for $\mathrm{f}^{-1}(x)$. [3]

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The function g is defined by $g : x \mapsto 4x + a$ for $x \in \mathbb{R}$, where a is a constant.

(ii) Find the value of a for which $gf(-1) = 3$. [3]

(iii) Find the possible values of a given that the equation $f^{-1}(x) = g^{-1}(x)$ has two equal roots. [4]

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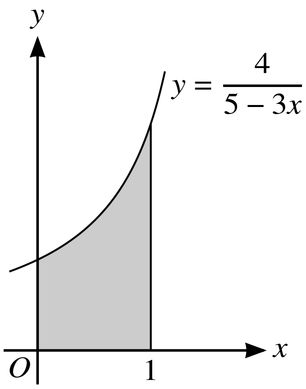
10



The diagram shows part of the curve $y = \dfrac{4}{5-3x}$.

(i) Find the equation of the normal to the curve at the point where $x = 1$ in the form $y = mx + c$, where m and c are constants. [5]

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The shaded region is bounded by the curve, the coordinate axes and the line $x = 1$.

(ii) Find, showing all necessary working, the volume obtained when this shaded region is rotated through 360° about the x-axis. [5]

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MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **(P1)** **May/June 2017**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

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[Turn over

1 **(i)** Find the coefficient of x in the expansion of $\left(2x - \frac{1}{x}\right)^5$. [2]

(ii) Hence find the coefficient of x in the expansion of $(1 + 3x^2)\left(2x - \frac{1}{x}\right)^5$. [4]

2 The point A has coordinates $(-2, 6)$. The equation of the perpendicular bisector of the line AB is $2y = 3x + 5$.

(i) Find the equation of AB. [3]

(ii) Find the coordinates of B. [3]

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3 **(i)** Prove the identity $\left(\dfrac{1}{\cos\theta} - \tan\theta\right)^2 \equiv \dfrac{1-\sin\theta}{1+\sin\theta}$. [3]

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(ii) Hence solve the equation $\left(\frac{1}{\cos\theta} - \tan\theta\right)^2 = \frac{1}{2}$, for $0° \leqslant \theta \leqslant 360°$. [3]

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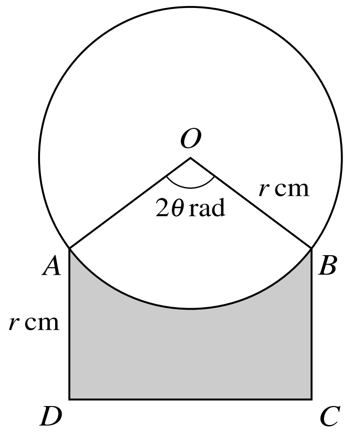

The diagram shows a circle with radius r cm and centre O. Points A and B lie on the circle and $ABCD$ is a rectangle. Angle $AOB = 2\theta$ radians and $AD = r$ cm.

(i) Express the perimeter of the shaded region in terms of r and θ. [3]

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(ii) In the case where $r = 5$ and $\theta = \frac{1}{6}\pi$, find the area of the shaded region. [4]

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5 A curve has equation $y = 3 + \dfrac{12}{2 - x}$.

(i) Find the equation of the tangent to the curve at the point where the curve crosses the x-axis. [5]

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(ii) A point moves along the curve in such a way that the x-coordinate is increasing at a constant rate of 0.04 units per second. Find the rate of change of the y-coordinate when $x = 4$. [2]

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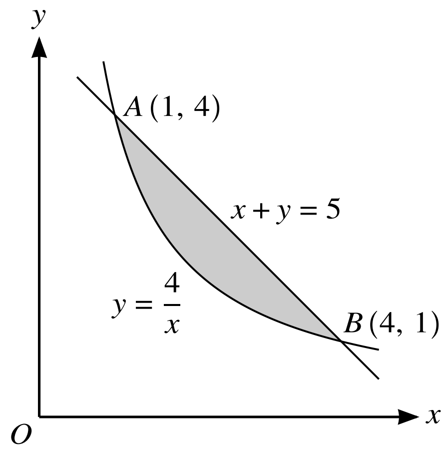

The diagram shows the straight line $x + y = 5$ intersecting the curve $y = \frac{4}{x}$ at the points $A\,(1,\ 4)$ and $B\,(4,\ 1)$. Find, showing all necessary working, the volume obtained when the shaded region is rotated through 360° about the x-axis. [7]

7 **(a)** The first two terms of an arithmetic progression are 16 and 24. Find the least number of terms of the progression which must be taken for their sum to exceed 20 000. [4]

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(b) A geometric progression has a first term of 6 and a sum to infinity of 18. A new geometric progression is formed by squaring each of the terms of the original progression. Find the sum to infinity of the new progression. [4]

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8 Relative to an origin O, the position vectors of three points A, B and C are given by

$$\overrightarrow{OA} = 3\mathbf{i} + p\mathbf{j} - 2p\mathbf{k}, \quad \overrightarrow{OB} = 6\mathbf{i} + (p+4)\mathbf{j} + 3\mathbf{k} \quad \text{and} \quad \overrightarrow{OC} = (p-1)\mathbf{i} + 2\mathbf{j} + q\mathbf{k},$$

where p and q are constants.

(i) In the case where $p = 2$, use a scalar product to find angle AOB. [4]

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(ii) In the case where $\overrightarrow{AB}$ is parallel to $\overrightarrow{OC}$, find the values of p and q. [4]

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9 The equation of a curve is $y = 8\sqrt{x} - 2x$.

(i) Find the coordinates of the stationary point of the curve. [3]

(ii) Find an expression for $\frac{d^2y}{dx^2}$ and hence, or otherwise, determine the nature of the stationary point. [2]

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(iii) Find the values of x at which the line $y = 6$ meets the curve. [3]

(iv) State the set of values of k for which the line $y = k$ does not meet the curve. [1]

10 The function f is defined by $f(x) = 3\tan\left(\frac{1}{2}x\right) - 2$, for $-\frac{1}{2}\pi \leqslant x \leqslant \frac{1}{2}\pi$.

(i) Solve the equation $f(x) + 4 = 0$, giving your answer correct to 1 decimal place. [3]

(ii) Find an expression for $f^{-1}(x)$ and find the domain of f^{-1}. [5]

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(iii) Sketch, on the same diagram, the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{f}^{-1}(x)$. [3]

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MATHEMATICS **9709/13**

Paper 1 Pure Mathematics 1 **(P1)** **May/June 2017**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

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[Turn over

1 The coefficients of x and x^2 in the expansion of $(2 + ax)^7$ are equal. Find the value of the non-zero constant a. [3]

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2 The common ratio of a geometric progression is r. The first term of the progression is $(r^2 - 3r + 2)$ and the sum to infinity is S.

(i) Show that $S = 2 - r$. [2]

(ii) Find the set of possible values that S can take. [2]

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3 Find the coordinates of the points of intersection of the curve $y = x^{\frac{2}{3}} - 1$ with the curve $y = x^{\frac{1}{3}} + 1$. [4]

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4 Relative to an origin O, the position vectors of points A and B are given by

$$\overrightarrow{OA} = \begin{pmatrix} 5 \\ 1 \\ 3 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OB} = \begin{pmatrix} 5 \\ 4 \\ -3 \end{pmatrix}.$$

The point P lies on AB and is such that $\overrightarrow{AP} = \frac{1}{3}\overrightarrow{AB}$.

(i) Find the position vector of P. [3]

(ii) Find the distance OP. [1]

(iii) Determine whether OP is perpendicular to AB. Justify your answer. [2]

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5 (i) Show that the equation $\dfrac{2\sin\theta + \cos\theta}{\sin\theta + \cos\theta} = 2\tan\theta$ may be expressed as $\cos^2\theta = 2\sin^2\theta$. [3]

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(ii) Hence solve the equation $\dfrac{2\sin\theta + \cos\theta}{\sin\theta + \cos\theta} = 2\tan\theta$ for $0° < \theta < 180°$. [3]

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6 The line $3y + x = 25$ is a normal to the curve $y = x^2 - 5x + k$. Find the value of the constant k. [6]

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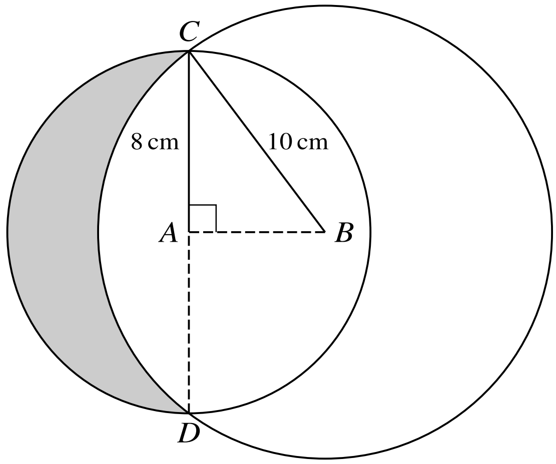

The diagram shows two circles with centres A and B having radii 8 cm and 10 cm respectively. The two circles intersect at C and D where CAD is a straight line and AB is perpendicular to CD.

(i) Find angle ABC in radians. [1]

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(ii) Find the area of the shaded region. [6]

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8 $A(-1, 1)$ and $P(a, b)$ are two points, where a and b are constants. The gradient of AP is 2.

(i) Find an expression for b in terms of a. [2]

(ii) $B(10, -1)$ is a third point such that $AP = AB$. Calculate the coordinates of the possible positions of P. [6]

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9 **(i)** Express $9x^2 - 6x + 6$ in the form $(ax + b)^2 + c$, where a, b and c are constants. [3]

The function f is defined by $f(x) = 9x^2 - 6x + 6$ for $x \geqslant p$, where p is a constant.

(ii) State the smallest value of p for which f is a one-one function. [1]

(iii) For this value of p, obtain an expression for $f^{-1}(x)$, and state the domain of f^{-1}. [4]

(iv) State the set of values of q for which the equation $f(x) = q$ has no solution. [1]

10 (a)

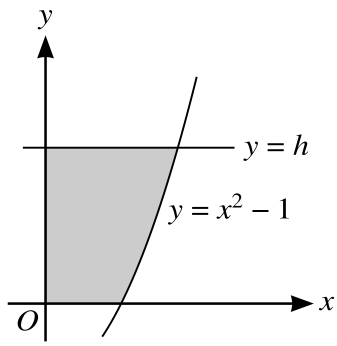

Fig. 1

Fig. 1 shows part of the curve $y = x^2 - 1$ and the line $y = h$, where h is a constant.

(i) The shaded region is rotated through 360° about the ***y*-axis**. Show that the volume of revolution, V, is given by $V = \pi\left(\frac{1}{2}h^2 + h\right)$. [3]

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(ii) Find, showing all necessary working, the area of the shaded region when $h = 3$. [4]

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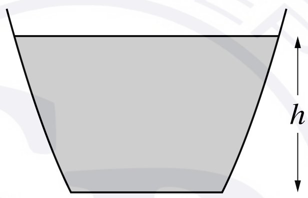

Fig. 2

Fig. 2 shows a cross-section of a bowl containing water. When the height of the water level is h cm, the volume, V cm^3, of water is given by $V = \pi\left(\frac{1}{2}h^2 + h\right)$. Water is poured into the bowl at a constant rate of 2 cm^3 s^{-1}. Find the rate, in cm s^{-1}, at which the height of the water level is increasing when the height of the water level is 3 cm. [4]

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11 The function f is defined for $x \geqslant 0$. It is given that f has a minimum value when $x = 2$ and that $f''(x) = (4x + 1)^{-\frac{1}{2}}$.

(i) Find $f'(x)$. [3]

It is now given that $f''(0)$, $f'(0)$ and $f(0)$ are the first three terms respectively of an arithmetic progression.

(ii) Find the value of $f(0)$. [3]

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(iii) Find $f(x)$, and hence find the minimum value of f. [5]

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MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **(P1)** **February/March 2017**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

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This document consists of **20** printed pages.

JC17 03_9709_12/RP
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1 Find the set of values of k for which the equation $2x^2 + 3kx + k = 0$ has distinct real roots. [4]

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2 In the expansion of $\left(\frac{1}{ax} + 2ax^2\right)^5$, the coefficient of x is 5. Find the value of the constant a. [4]

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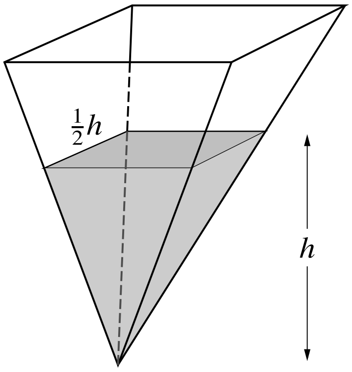

The diagram shows a water container in the form of an inverted pyramid, which is such that when the height of the water level is h cm the surface of the water is a square of side $\frac{1}{2}h$ cm.

(i) Express the volume of water in the container in terms of h. [1]

[The volume of a pyramid having a base area A and vertical height h is $\frac{1}{3}Ah$.]

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Water is steadily dripping into the container at a constant rate of $20\,\text{cm}^3$ per minute.

(ii) Find the rate, in cm per minute, at which the water level is rising when the height of the water level is 10 cm. [4]

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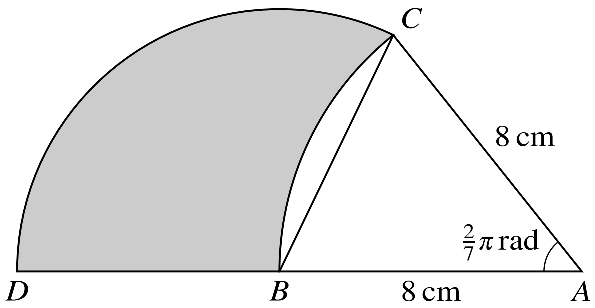
4



In the diagram, $AB = AC = 8$ cm and angle $CAB = \frac{2}{7}\pi$ radians. The circular arc BC has centre A, the circular arc CD has centre B and ABD is a straight line.

(i) Show that angle $CBD = \frac{9}{14}\pi$ radians. [1]

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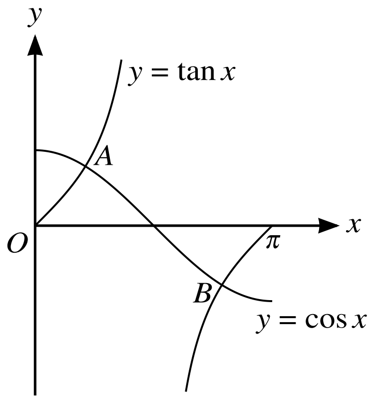
(ii) Find the perimeter of the shaded region. [5]

5



The diagram shows the graphs of $y = \tan x$ and $y = \cos x$ for $0 \leqslant x \leqslant \pi$. The graphs intersect at points A and B.

(i) Find by calculation the x-coordinate of A. [4]

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(ii) Find by calculation the coordinates of B. [3]

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6 Relative to an origin O, the position vectors of the points A and B are given by

$$\overrightarrow{OA} = 2\mathbf{i} + 3\mathbf{j} + 5\mathbf{k} \qquad \text{and} \qquad \overrightarrow{OB} = 7\mathbf{i} + 4\mathbf{j} + 3\mathbf{k}.$$

(i) Use a scalar product to find angle OAB. [5]

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(ii) Find the area of triangle OAB. [2]

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7 The function f is defined for $x \geqslant 0$ by $f(x) = (4x + 1)^{\frac{3}{2}}$.

(i) Find $f'(x)$ and $f''(x)$. [3]

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The first, second and third terms of a geometric progression are respectively $f(2)$, $f'(2)$ and $kf''(2)$.

(ii) Find the value of the constant k. [5]

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8 The functions f and g are defined for $x \geqslant 0$ by

$$f : x \mapsto 2x^2 + 3,$$
$$g : x \mapsto 3x + 2.$$

(i) Show that $gf(x) = 6x^2 + 11$ and obtain an unsimplified expression for $fg(x)$. [2]

(ii) Find an expression for $(fg)^{-1}(x)$ and determine the domain of $(fg)^{-1}$. [5]

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(iii) Solve the equation $gf(2x) = fg(x)$. [3]

9 The point $A\,(2,\ 2)$ lies on the curve $y = x^2 - 2x + 2$.

(i) Find the equation of the tangent to the curve at A. [3]

The normal to the curve at A intersects the curve again at B.

(ii) Find the coordinates of B. [4]

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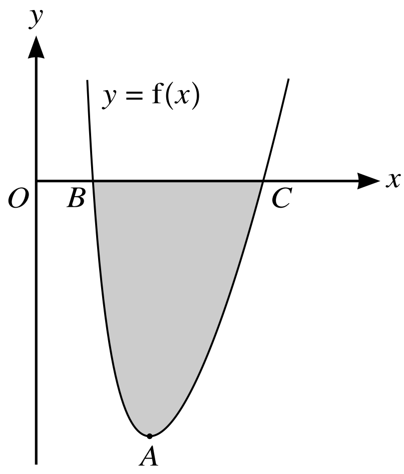
The tangents at A and B intersect each other at C.

(iii) Find the coordinates of C. [4]

10



The diagram shows the curve $y = \mathrm{f}(x)$ defined for $x > 0$. The curve has a minimum point at A and crosses the x-axis at B and C. It is given that $\frac{\mathrm{d}y}{\mathrm{d}x} = 2x - \frac{2}{x^3}$ and that the curve passes through the point $\left(4, \frac{189}{16}\right)$.

(i) Find the x-coordinate of A. [2]

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(ii) Find $\mathrm{f}(x)$. [3]

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(iii) Find the x-coordinates of B and C. [4]

[Question 10 (iv) is printed on the next page.]

(iv) Find, showing all necessary working, the area of the shaded region. [4]

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MATHEMATICS **9709/11**

Paper 1 Pure Mathematics 1 **(P1)** **October/November 2016**

1 hour 45 minutes

* 9 2 0 4 7 5 9 1 2 3 *

Additional Materials: List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

An answer booklet is provided inside this question paper. You should follow the instructions on the front cover of the answer booklet. If you need additional answer paper ask the invigilator for a continuation booklet.

Answer **all** the questions.
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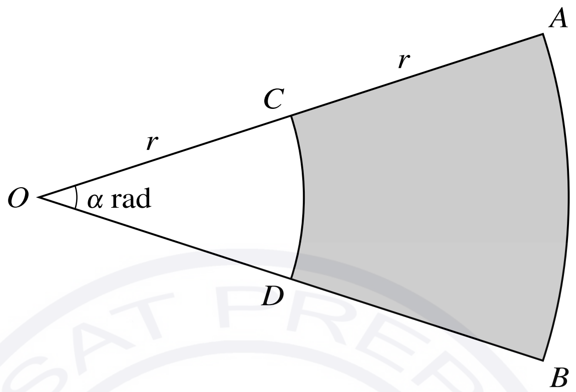
[Turn over

1 **(i)** Express $x^2 + 6x + 2$ in the form $(x + a)^2 + b$, where a and b are constants. [2]

(ii) Hence, or otherwise, find the set of values of x for which $x^2 + 6x + 2 > 9$. [2]

2 Find the term independent of x in the expansion of $\left(2x + \frac{1}{2x^3}\right)^8$. [4]

3



In the diagram OCA and ODB are radii of a circle with centre O and radius $2r$ cm. Angle $AOB = \alpha$ radians. CD and AB are arcs of circles with centre O and radii r cm and $2r$ cm respectively. The perimeter of the shaded region $ABDC$ is $4.4r$ cm.

(i) Find the value of α. [2]

(ii) It is given that the area of the shaded region is $30\,\text{cm}^2$. Find the value of r. [3]

4 C is the mid-point of the line joining $A\,(14, -7)$ to $B\,(-6, 3)$. The line through C perpendicular to AB crosses the y-axis at D.

(i) Find the equation of the line CD, giving your answer in the form $y = mx + c$. [4]

(ii) Find the distance AD. [2]

5 The sum of the 1st and 2nd terms of a geometric progression is 50 and the sum of the 2nd and 3rd terms is 30. Find the sum to infinity. [6]

6 **(i)** Show that $\cos^4 x \equiv 1 - 2\sin^2 x + \sin^4 x$. [1]

(ii) Hence, or otherwise, solve the equation $8\sin^4 x + \cos^4 x = 2\cos^2 x$ for $0° \leqslant x \leqslant 360°$. [5]

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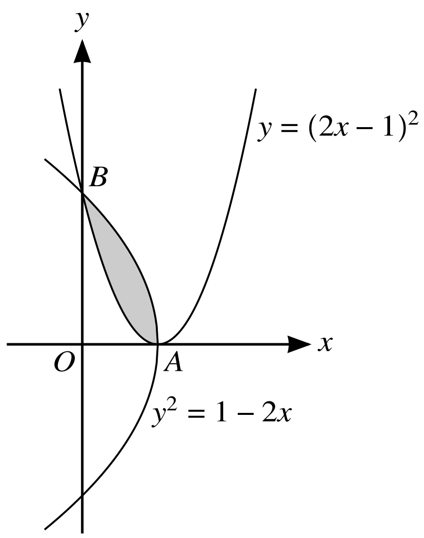
7



The diagram shows parts of the curves $y = (2x-1)^2$ and $y^2 = 1-2x$, intersecting at points A and B.

(i) State the coordinates of A. [1]

(ii) Find, showing all necessary working, the area of the shaded region. [6]

8 The functions f and g are defined by

$$f(x) = \frac{4}{x} - 2 \quad \text{for } x > 0,$$

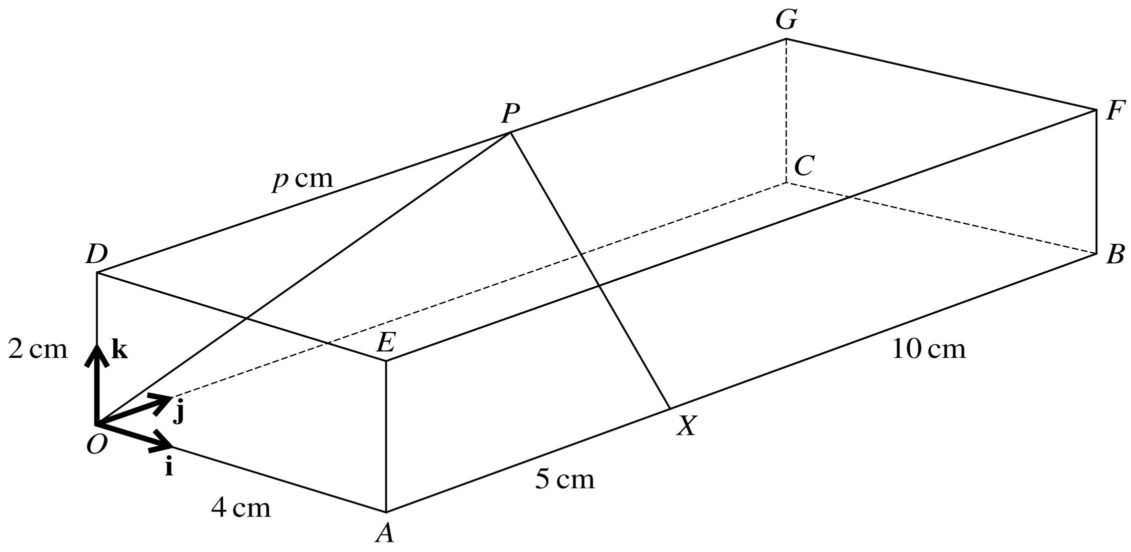
$$g(x) = \frac{4}{5x+2} \quad \text{for } x \geqslant 0.$$

(i) Find and simplify an expression for $fg(x)$ and state the range of fg. [3]

(ii) Find an expression for $g^{-1}(x)$ and find the domain of g^{-1}. [5]

[Questions 9, 10 and 11 are printed on the next page.]

9



The diagram shows a cuboid $OABCDEFG$ with a horizontal base $OABC$ in which $OA = 4$ cm and $AB = 15$ cm. The height OD of the cuboid is 2 cm. The point X on AB is such that $AX = 5$ cm and the point P on DG is such that $DP = p$ cm, where p is a constant. Unit vectors **i**, **j** and **k** are parallel to OA, OC and OD respectively.

(i) Find the possible values of p such that angle $OPX = 90°$. [4]

(ii) For the case where $p = 9$, find the unit vector in the direction of $\overrightarrow{XP}$. [2]

(iii) A point Q lies on the face $CBFG$ and is such that XQ is parallel to AG. Find $\overrightarrow{XQ}$. [3]

10 A curve has equation $y = \mathrm{f}(x)$ and it is given that $\mathrm{f}'(x) = 3x^{\frac{1}{2}} - 2x^{-\frac{1}{2}}$. The point A is the only point on the curve at which the gradient is -1.

(i) Find the x-coordinate of A. [3]

(ii) Given that the curve also passes through the point $(4, 10)$, find the y-coordinate of A, giving your answer as a fraction. [6]

11 The point $P\,(3, 5)$ lies on the curve $y = \dfrac{1}{x-1} - \dfrac{9}{x-5}$.

(i) Find the x-coordinate of the point where the normal to the curve at P intersects the x-axis. [5]

(ii) Find the x-coordinate of each of the stationary points on the curve and determine the nature of each stationary point, justifying your answers. [6]

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MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **(P1)** **October/November 2016**

1 hour 45 minutes

5199196634

Additional Materials: List of Formulae (MF9)

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1 A curve is such that $\frac{dy}{dx} = \frac{8}{\sqrt{(4x+1)}}$. The point (2, 5) lies on the curve. Find the equation of the curve. [4]

2 **(i)** Express the equation $\sin 2x + 3\cos 2x = 3(\sin 2x - \cos 2x)$ in the form $\tan 2x = k$, where k is a constant. [2]

(ii) Hence solve the equation for $-90° \leqslant x \leqslant 90°$. [3]

3 A curve has equation $y = 2x^2 - 6x + 5$.

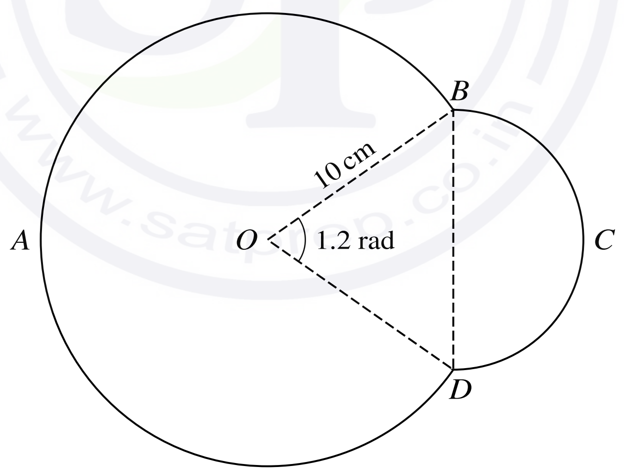
(i) Find the set of values of x for which $y > 13$. [3]

(ii) Find the value of the constant k for which the line $y = 2x + k$ is a tangent to the curve. [3]

4 In the expansion of $(3 - 2x)\left(1 + \frac{x}{2}\right)^n$, the coefficient of x is 7. Find the value of the constant n and hence find the coefficient of x^2. [6]

5 The line $\frac{x}{a} + \frac{y}{b} = 1$, where a and b are positive constants, intersects the x- and y-axes at the points A and B respectively. The mid-point of AB lies on the line $2x + y = 10$ and the distance $AB = 10$. Find the values of a and b. [6]

6



The diagram shows a metal plate $ABCD$ made from two parts. The part BCD is a semicircle. The part DAB is a segment of a circle with centre O and radius 10 cm. Angle BOD is 1.2 radians.

(i) Show that the radius of the semicircle is 5.646 cm, correct to 3 decimal places. [2]

(ii) Find the perimeter of the metal plate. [3]

(iii) Find the area of the metal plate. [3]

7 The equation of a curve is $y = 2 + \dfrac{3}{2x - 1}$.

(i) Obtain an expression for $\dfrac{dy}{dx}$. [2]

(ii) Explain why the curve has no stationary points. [1]

At the point P on the curve, $x = 2$.

(iii) Show that the normal to the curve at P passes through the origin. [4]

(iv) A point moves along the curve in such a way that its x-coordinate is decreasing at a constant rate of 0.06 units per second. Find the rate of change of the y-coordinate as the point passes through P. [2]

8 (a) A cyclist completes a long-distance charity event across Africa. The total distance is 3050 km. He starts the event on May 1st and cycles 200 km on that day. On each subsequent day he reduces the distance cycled by 5 km.

(i) How far will he travel on May 15th? [2]

(ii) On what date will he finish the event? [3]

(b) A geometric progression is such that the third term is 8 times the sixth term, and the sum of the first six terms is $31\frac{1}{2}$. Find

(i) the first term of the progression, [4]

(ii) the sum to infinity of the progression. [1]

9 Relative to an origin O, the position vectors of the points A, B and C are given by

$$\overrightarrow{OA} = \begin{pmatrix} 2 \\ -2 \\ -1 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} -2 \\ 3 \\ 6 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} 2 \\ 6 \\ 5 \end{pmatrix}.$$

(i) Use a scalar product to find angle AOB. [4]

(ii) Find the vector which is in the same direction as $\overrightarrow{AC}$ and of magnitude 15 units. [3]

(iii) Find the value of the constant p for which $p\overrightarrow{OA} + \overrightarrow{OC}$ is perpendicular to $\overrightarrow{OB}$. [3]

10 A function f is defined by $\mathrm{f} : x \mapsto 5 - 2\sin 2x$ for $0 \leqslant x \leqslant \pi$.

(i) Find the range of f. [2]

(ii) Sketch the graph of $y = \mathrm{f}(x)$. [2]

(iii) Solve the equation $\mathrm{f}(x) = 6$, giving answers in terms of π. [3]

The function g is defined by $\mathrm{g} : x \mapsto 5 - 2\sin 2x$ for $0 \leqslant x \leqslant k$, where k is a constant.

(iv) State the largest value of k for which g has an inverse. [1]

(v) For this value of k, find an expression for $\mathrm{g}^{-1}(x)$. [3]

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MATHEMATICS **9709/13**

Paper 1 Pure Mathematics 1 **(P1)** **October/November 2016**

1 hour 45 minutes

Additional Materials: List of Formulae (MF9)

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1 Find the set of values of k for which the curve $y = kx^2 - 3x$ and the line $y = x - k$ do not meet. [3]

2 The coefficient of x^3 in the expansion of $(1 - 3x)^6 + (1 + ax)^5$ is 100. Find the value of the constant a. [4]

3 Showing all necessary working, solve the equation $6\sin^2 x - 5\cos^2 x = 2\sin^2 x + \cos^2 x$ for $0° \leqslant x \leqslant 360°$. [4]

4 The function f is such that $f(x) = x^3 - 3x^2 - 9x + 2$ for $x > n$, where n is an integer. It is given that f is an increasing function. Find the least possible value of n. [4]

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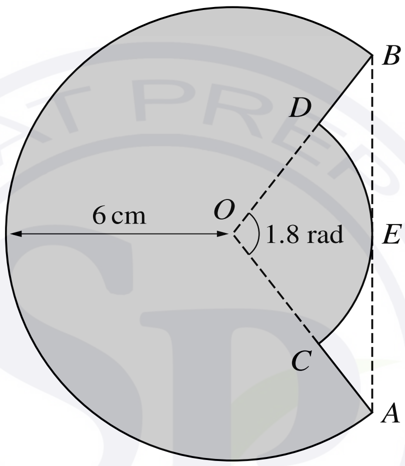


The diagram shows a major arc AB of a circle with centre O and radius 6 cm. Points C and D on OA and OB respectively are such that the line AB is a tangent at E to the arc CED of a smaller circle also with centre O. Angle $COD = 1.8$ radians.

(i) Show that the radius of the arc CED is 3.73 cm, correct to 3 significant figures. [2]

(ii) Find the area of the shaded region. [4]

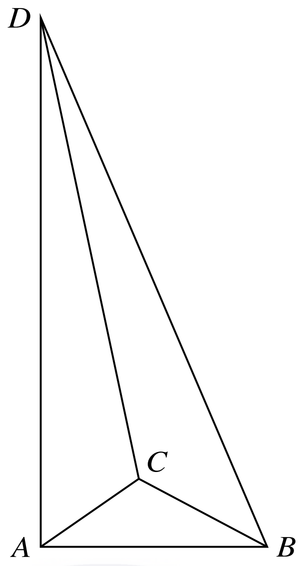
6 Three points, A, B and C, are such that B is the mid-point of AC. The coordinates of A are $(2, m)$ and the coordinates of B are $(n, -6)$, where m and n are constants.

(i) Find the coordinates of C in terms of m and n. [2]

The line $y = x + 1$ passes through C and is perpendicular to AB.

(ii) Find the values of m and n. [5]

7



The diagram shows a triangular pyramid $ABCD$. It is given that

$$\overrightarrow{AB} = 3\mathbf{i} + \mathbf{j} + \mathbf{k}, \quad \overrightarrow{AC} = \mathbf{i} - 2\mathbf{j} - \mathbf{k} \quad \text{and} \quad \overrightarrow{AD} = \mathbf{i} + 4\mathbf{j} - 7\mathbf{k}.$$

(i) Verify, showing all necessary working, that each of the angles DAB, DAC and CAB is 90°. [3]

(ii) Find the exact value of the area of the triangle ABC, and hence find the exact value of the volume of the pyramid. [4]

[The volume V of a pyramid of base area A and vertical height h is given by $V = \frac{1}{3}Ah$.]

8 **(i)** Express $4x^2 + 12x + 10$ in the form $(ax + b)^2 + c$, where a, b and c are constants. [3]

(ii) Functions f and g are both defined for $x > 0$. It is given that $\mathrm{f}(x) = x^2 + 1$ and $\mathrm{fg}(x) = 4x^2 + 12x + 10$. Find $\mathrm{g}(x)$. [1]

(iii) Find $(\mathrm{fg})^{-1}(x)$ and give the domain of $(\mathrm{fg})^{-1}$. [4]

9 **(a)** Two convergent geometric progressions, P and Q, have the same sum to infinity. The first and second terms of P are 6 and $6r$ respectively. The first and second terms of Q are 12 and $-12r$ respectively. Find the value of the common sum to infinity. [3]

(b) The first term of an arithmetic progression is $\cos\theta$ and the second term is $\cos\theta + \sin^2\theta$, where $0 \leqslant \theta \leqslant \pi$. The sum of the first 13 terms is 52. Find the possible values of θ. [5]

[Questions 10 and 11 are printed on the next page.]

10 A curve is such that $\dfrac{dy}{dx} = \dfrac{2}{a}x^{-\frac{1}{2}} + ax^{-\frac{3}{2}}$, where a is a positive constant. The point $A\,(a^2, 3)$ lies on the curve. Find, in terms of a,

(i) the equation of the tangent to the curve at A, simplifying your answer, [3]

(ii) the equation of the curve. [4]

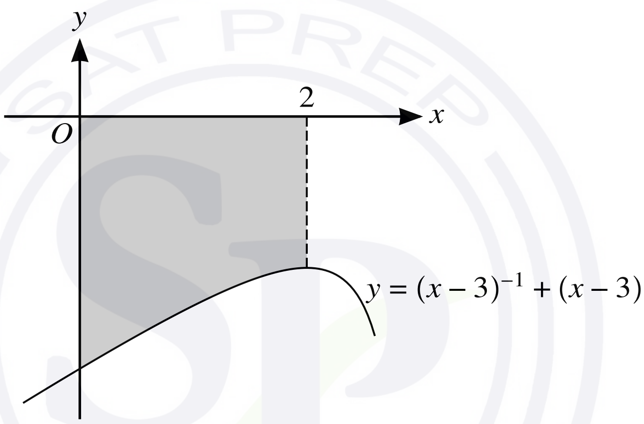
It is now given that $B\,(16, 8)$ also lies on the curve.

(iii) Find the value of a and, using this value, find the distance AB. [5]

11 A curve has equation $y = (kx - 3)^{-1} + (kx - 3)$, where k is a non-zero constant.

(i) Find the x-coordinates of the stationary points in terms of k, and determine the nature of each stationary point, justifying your answers. [7]

(ii)



The diagram shows part of the curve for the case when $k = 1$. Showing all necessary working, find the volume obtained when the region between the curve, the x-axis, the y-axis and the line $x = 2$, shown shaded in the diagram, is rotated through 360° about the x-axis. [5]

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MATHEMATICS **9709/11**

Paper 1 Pure Mathematics 1 **(P1)** **May/June 2016**

1 hour 45 minutes

1120942846

Additional Materials: List of Formulae (MF9)

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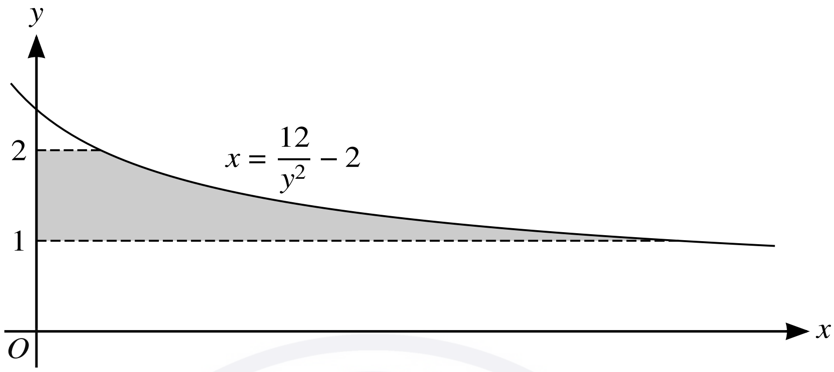
1 Find the term independent of x in the expansion of $\left(x - \frac{3}{2x}\right)^6$. [3]

2 Solve the equation $3\sin^2\theta = 4\cos\theta - 1$ for $0° \leqslant \theta \leqslant 360°$. [4]

3



The diagram shows part of the curve $x = \frac{12}{y^2} - 2$. The shaded region is bounded by the curve, the y-axis and the lines $y = 1$ and $y = 2$. Showing all necessary working, find the volume, in terms of π, when this shaded region is rotated through 360° about the y-axis. [5]

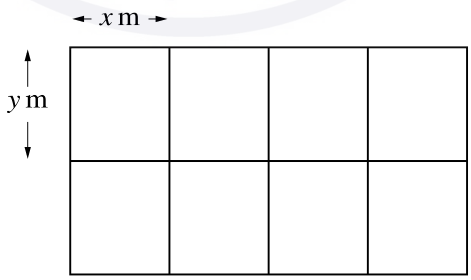
4 A curve is such that $\frac{dy}{dx} = 2 - 8(3x + 4)^{-\frac{1}{2}}$.

(i) A point P moves along the curve in such a way that the x-coordinate is increasing at a constant rate of 0.3 units per second. Find the rate of change of the y-coordinate as P crosses the y-axis. [2]

The curve intersects the y-axis where $y = \frac{4}{3}$.

(ii) Find the equation of the curve. [4]

5



A farmer divides a rectangular piece of land into 8 equal-sized rectangular sheep pens as shown in the diagram. Each sheep pen measures x m by y m and is fully enclosed by metal fencing. The farmer uses 480 m of fencing.

(i) Show that the total area of land used for the sheep pens, A m^2, is given by

$$A = 384x - 9.6x^2.$$ [3]

(ii) Given that x and y can vary, find the dimensions of each sheep pen for which the value of A is a maximum. (There is no need to verify that the value of A is a maximum.) [3]

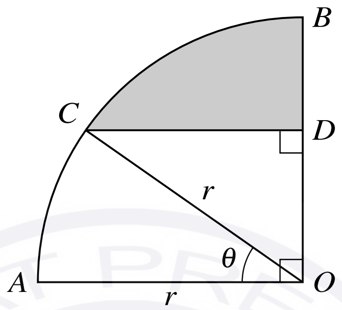
6 **(a)** Find the values of the constant m for which the line $y = mx$ is a tangent to the curve $y = 2x^2 - 4x + 8$. [3]

(b) The function f is defined for $x \in \mathbb{R}$ by $f(x) = x^2 + ax + b$, where a and b are constants. The solutions of the equation $f(x) = 0$ are $x = 1$ and $x = 9$. Find

(i) the values of a and b, [2]

(ii) the coordinates of the vertex of the curve $y = f(x)$. [2]

7



In the diagram, AOB is a quarter circle with centre O and radius r. The point C lies on the arc AB and the point D lies on OB. The line CD is parallel to AO and angle $AOC = \theta$ radians.

(i) Express the perimeter of the shaded region in terms of r, θ and π. [4]

(ii) For the case where $r = 5$ cm and $\theta = 0.6$, find the area of the shaded region. [3]

8 A curve has equation $y = 3x - \dfrac{4}{x}$ and passes through the points $A\,(1, -1)$ and $B\,(4, 11)$. At each of the points C and D on the curve, the tangent is parallel to AB. Find the equation of the perpendicular bisector of CD. [7]

9 **(a)** The first term of a geometric progression in which all the terms are positive is 50. The third term is 32. Find the sum to infinity of the progression. [3]

(b) The first three terms of an arithmetic progression are $2\sin x$, $3\cos x$ and $(\sin x + 2\cos x)$ respectively, where x is an acute angle.

(i) Show that $\tan x = \frac{4}{3}$. [3]

(ii) Find the sum of the first twenty terms of the progression. [3]

[Questions 10 and 11 are printed on the next page.]

10 Relative to an origin O, the position vectors of points A, B and C are given by

$$\overrightarrow{OA} = \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 5 \\ -1 \\ k \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} 2 \\ 6 \\ -3 \end{pmatrix}$$

respectively, where k is a constant.

(i) Find the value of k in the case where angle $AOB = 90°$. [2]

(ii) Find the possible values of k for which the lengths of AB and OC are equal. [4]

The point D is such that $\overrightarrow{OD}$ is in the same direction as $\overrightarrow{OA}$ and has magnitude 9 units. The point E is such that $\overrightarrow{OE}$ is in the same direction as $\overrightarrow{OC}$ and has magnitude 14 units.

(iii) Find the magnitude of $\overrightarrow{DE}$ in the form $\sqrt{n}$ where n is an integer. [4]

11 The function f is defined by $\mathrm{f} : x \mapsto 4\sin x - 1$ for $-\frac{1}{2}\pi \leqslant x \leqslant \frac{1}{2}\pi$.

(i) State the range of f. [2]

(ii) Find the coordinates of the points at which the curve $y = \mathrm{f}(x)$ intersects the coordinate axes. [3]

(iii) Sketch the graph of $y = \mathrm{f}(x)$. [2]

(iv) Obtain an expression for $\mathrm{f}^{-1}(x)$, stating both the domain and range of f^{-1}. [4]

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MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **(P1)** **May/June 2016**

1 hour 45 minutes

Additional Materials: List of Formulae (MF9)

9566317764

READ THESE INSTRUCTIONS FIRST

An answer booklet is provided inside this question paper. You should follow the instructions on the front cover of the answer booklet. If you need additional answer paper ask the invigilator for a continuation booklet.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.

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1 Functions f and g are defined by

$$\text{f} : x \mapsto 10 - 3x, \quad x \in \mathbb{R},$$
$$\text{g} : x \mapsto \frac{10}{3 - 2x}, \quad x \in \mathbb{R},\ x \neq \tfrac{3}{2}.$$

Solve the equation $\text{ff}(x) = \text{gf}(2)$. [3]

2 A curve is such that $\dfrac{dy}{dx} = \dfrac{8}{(5 - 2x)^2}$. Given that the curve passes through (2, 7), find the equation of the curve. [4]

3 Relative to an origin O, the position vectors of points A and B are given by

$$\overrightarrow{OA} = 2\mathbf{i} - 5\mathbf{j} - 2\mathbf{k} \quad \text{and} \quad \overrightarrow{OB} = 4\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}.$$

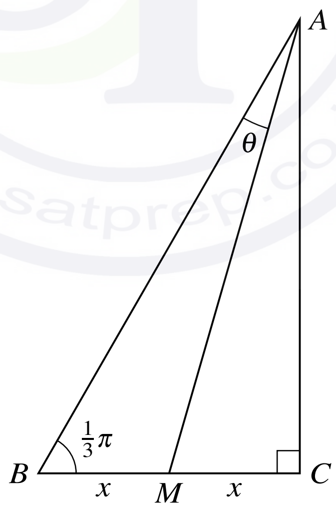
The point C is such that $\overrightarrow{AB} = \overrightarrow{BC}$. Find the unit vector in the direction of $\overrightarrow{OC}$. [4]

4 Find the term that is independent of x in the expansion of

(i) $\left(x - \dfrac{2}{x}\right)^6$, [2]

(ii) $\left(2 + \dfrac{3}{x^2}\right)\left(x - \dfrac{2}{x}\right)^6$. [4]

5

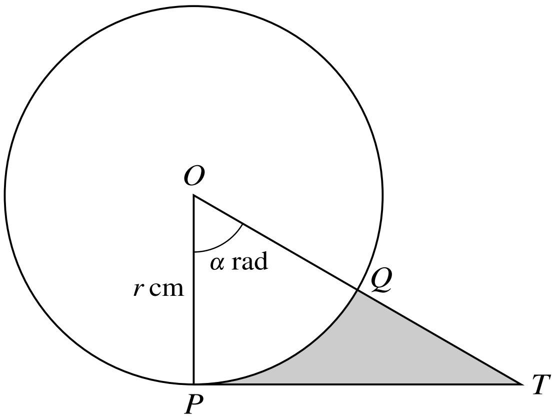


In the diagram, triangle ABC is right-angled at C and M is the mid-point of BC. It is given that angle $ABC = \frac{1}{3}\pi$ radians and angle $BAM = \theta$ radians. Denoting the lengths of BM and MC by x,

(i) find AM in terms of x, [3]

(ii) show that $\theta = \frac{1}{6}\pi - \tan^{-1}\left(\dfrac{1}{2\sqrt{3}}\right)$. [2]

6



The diagram shows a circle with radius r cm and centre O. The line PT is the tangent to the circle at P and angle $POT = \alpha$ radians. The line OT meets the circle at Q.

(i) Express the perimeter of the shaded region PQT in terms of r and α. [3]

(ii) In the case where $\alpha = \frac{1}{3}\pi$ and $r = 10$, find the area of the shaded region correct to 2 significant figures. [3]

7 **(i)** Prove the identity $\dfrac{1+\cos\theta}{1-\cos\theta} - \dfrac{1-\cos\theta}{1+\cos\theta} \equiv \dfrac{4}{\sin\theta\tan\theta}$. [4]

(ii) Hence solve, for $0° < \theta < 360°$, the equation

$$\sin\theta\left(\frac{1+\cos\theta}{1-\cos\theta} - \frac{1-\cos\theta}{1+\cos\theta}\right) = 3.$$ [3]

8 Three points have coordinates $A\,(0, 7)$, $B\,(8, 3)$ and $C\,(3k, k)$. Find the value of the constant k for which

(i) C lies on the line that passes through A and B, [4]

(ii) C lies on the perpendicular bisector of AB. [4]

9 A water tank holds 2000 litres when full. A small hole in the base is gradually getting bigger so that each day a greater amount of water is lost.

(i) On the first day after filling, 10 litres of water are lost and this increases by 2 litres each day.

(a) How many litres will be lost on the 30th day after filling? [2]

(b) The tank becomes empty during the nth day after filling. Find the value of n. [3]

(ii) Assume instead that 10 litres of water are lost on the first day and that the amount of water lost increases by 10% on each succeeding day. Find what percentage of the original 2000 litres is left in the tank at the end of the 30th day after filling. [4]

[Questions 10 and 11 are printed on the next page.]

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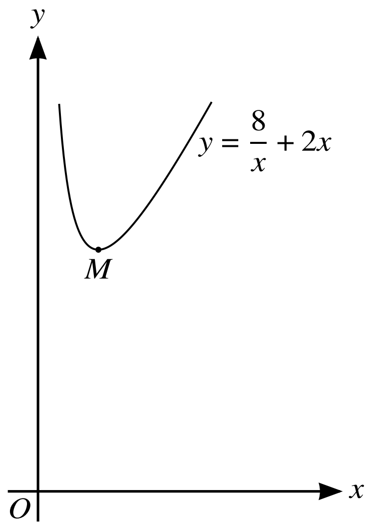
10



The diagram shows the part of the curve $y = \frac{8}{x} + 2x$ for $x > 0$, and the minimum point M.

(i) Find expressions for $\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$ and $\int y^2\,dx$. [5]

(ii) Find the coordinates of M and determine the coordinates and nature of the stationary point on the part of the curve for which $x < 0$. [5]

(iii) Find the volume obtained when the region bounded by the curve, the x-axis and the lines $x = 1$ and $x = 2$ is rotated through 360° about the x-axis. [2]

11 The function f is defined by $\mathrm{f} : x \mapsto 6x - x^2 - 5$ for $x \in \mathbb{R}$.

(i) Find the set of values of x for which $\mathrm{f}(x) \leqslant 3$. [3]

(ii) Given that the line $y = mx + c$ is a tangent to the curve $y = \mathrm{f}(x)$, show that $4c = m^2 - 12m + 16$. [3]

The function g is defined by $\mathrm{g} : x \mapsto 6x - x^2 - 5$ for $x \geqslant k$, where k is a constant.

(iii) Express $6x - x^2 - 5$ in the form $a - (x - b)^2$, where a and b are constants. [2]

(iv) State the smallest value of k for which g has an inverse. [1]

(v) For this value of k, find an expression for $\mathrm{g}^{-1}(x)$. [2]

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MATHEMATICS **9709/13**

Paper 1 Pure Mathematics 1 **(P1)** **May/June 2016**

1 hour 45 minutes

8426520173

Additional Materials: List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

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At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.

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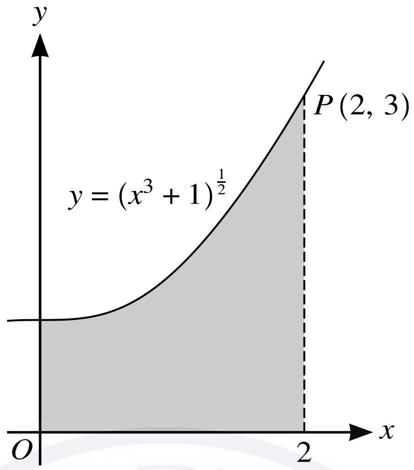
1 Find the coefficient of x in the expansion of $\left(\frac{1}{x} + 3x^2\right)^5$. [3]

2



The diagram shows part of the curve $y = (x^3 + 1)^{\frac{1}{2}}$ and the point $P\,(2, 3)$ lying on the curve. Find, showing all necessary working, the volume obtained when the shaded region is rotated through 360° about the x-axis. [4]

3 A curve is such that $\frac{dy}{dx} = 6x^2 + \frac{k}{x^3}$ and passes through the point $P\,(1, 9)$. The gradient of the curve at P is 2.

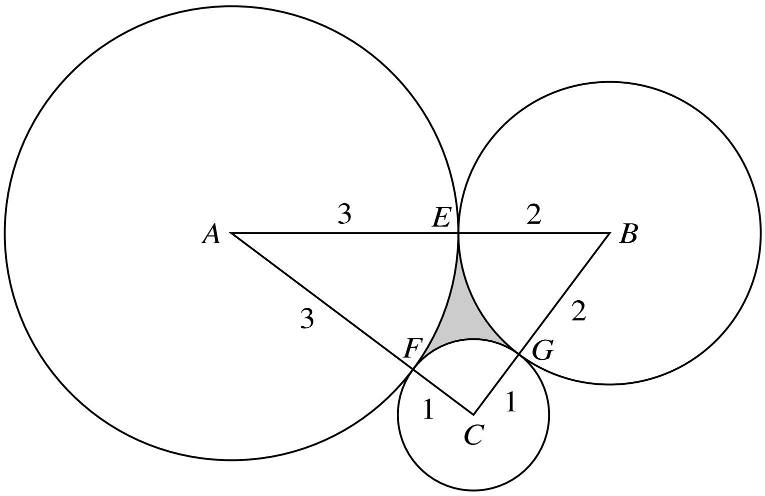
(i) Find the value of the constant k. [1]

(ii) Find the equation of the curve. [4]

4 The 1st, 3rd and 13th terms of an arithmetic progression are also the 1st, 2nd and 3rd terms respectively of a geometric progression. The first term of each progression is 3. Find the common difference of the arithmetic progression and the common ratio of the geometric progression. [5]

5 A curve has equation $y = 8x + (2x - 1)^{-1}$. Find the values of x at which the curve has a stationary point and determine the nature of each stationary point, justifying your answers. [7]

6



The diagram shows triangle ABC where $AB = 5\,\text{cm}$, $AC = 4\,\text{cm}$ and $BC = 3\,\text{cm}$. Three circles with centres at A, B and C have radii 3 cm, 2 cm and 1 cm respectively. The circles touch each other at points E, F and G, lying on AB, AC and BC respectively. Find the area of the shaded region EFG. [7]

7 The point $P(x, y)$ is moving along the curve $y = x^2 - \frac{10}{3}x^{\frac{3}{2}} + 5x$ in such a way that the rate of change of y is constant. Find the values of x at the points at which the rate of change of x is equal to half the rate of change of y. [7]

8 **(i)** Show that $3\sin x \tan x - \cos x + 1 = 0$ can be written as a quadratic equation in $\cos x$ and hence solve the equation $3\sin x \tan x - \cos x + 1 = 0$ for $0 \leqslant x \leqslant \pi$. [5]

(ii) Find the solutions to the equation $3\sin 2x \tan 2x - \cos 2x + 1 = 0$ for $0 \leqslant x \leqslant \pi$. [3]

9 The position vectors of A, B and C relative to an origin O are given by

$$\overrightarrow{OA} = \begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 1 \\ 5 \\ p \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} 5 \\ 0 \\ 2 \end{pmatrix},$$

where p is a constant.

(i) Find the value of p for which the lengths of AB and CB are equal. [4]

(ii) For the case where $p = 1$, use a scalar product to find angle ABC. [4]

[Questions 10 and 11 are printed on the next page.]

10 The function f is such that $f(x) = 2x + 3$ for $x \geqslant 0$. The function g is such that $g(x) = ax^2 + b$ for $x \leqslant q$, where a, b and q are constants. The function fg is such that $fg(x) = 6x^2 - 21$ for $x \leqslant q$.

(i) Find the values of a and b. [3]

(ii) Find the greatest possible value of q. [2]

It is now given that $q = -3$.

(iii) Find the range of fg. [1]

(iv) Find an expression for $(fg)^{-1}(x)$ and state the domain of $(fg)^{-1}$. [3]

11 Triangle ABC has vertices at $A\,(-2, -1)$, $B\,(4, 6)$ and $C\,(6, -3)$.

(i) Show that triangle ABC is isosceles and find the exact area of this triangle. [6]

(ii) The point D is the point on AB such that CD is perpendicular to AB. Calculate the x-coordinate of D. [6]

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MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **(P1)** **February/March 2016**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

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Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
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You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

This document consists of **5** printed pages and **3** blank pages.

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1 **(i)** Find the coefficients of x^4 and x^5 in the expansion of $(1 - 2x)^5$. [2]

(ii) It is given that, when $(1 + px)(1 - 2x)^5$ is expanded, there is no term in x^5. Find the value of the constant p. [2]

2 A curve for which $\frac{dy}{dx} = 3x^2 - \frac{2}{x^3}$ passes through $(-1, 3)$. Find the equation of the curve. [4]

3 The 12th term of an arithmetic progression is 17 and the sum of the first 31 terms is 1023. Find the 31st term. [5]

4 **(a)** Solve the equation $\sin^{-1}(3x) = -\frac{1}{3}\pi$, giving the solution in an exact form. [2]

(b) Solve, by factorising, the equation $2\cos\theta\sin\theta - 2\cos\theta - \sin\theta + 1 = 0$ for $0 \leqslant \theta \leqslant \pi$. [4]

5 Two points have coordinates $A(5, 7)$ and $B(9, -1)$.

(i) Find the equation of the perpendicular bisector of AB. [3]

The line through $C(1, 2)$ parallel to AB meets the perpendicular bisector of AB at the point X.

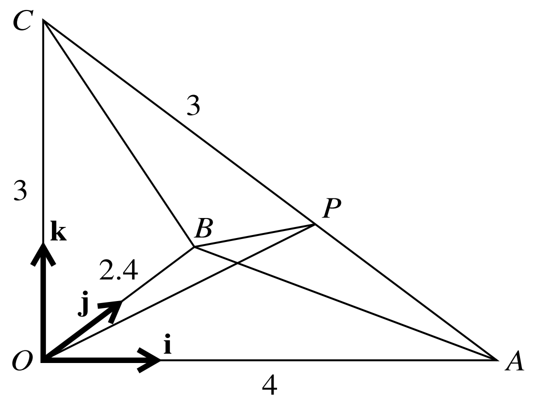
(ii) Find, by calculation, the distance BX. [5]

6 A vacuum flask (for keeping drinks hot) is modelled as a closed cylinder in which the internal radius is r cm and the internal height is h cm. The volume of the flask is $1000\,\text{cm}^3$. A flask is most efficient when the total internal surface area, $A\,\text{cm}^2$, is a minimum.

(i) Show that $A = 2\pi r^2 + \frac{2000}{r}$. [3]

(ii) Given that r can vary, find the value of r, correct to 1 decimal place, for which A has a stationary value and verify that the flask is most efficient when r takes this value. [5]

7



The diagram shows a pyramid $OABC$ with a horizontal triangular base OAB and vertical height OC. Angles AOB, BOC and AOC are each right angles. Unit vectors **i**, **j** and **k** are parallel to OA, OB and OC respectively, with $OA = 4$ units, $OB = 2.4$ units and $OC = 3$ units. The point P on CA is such that $CP = 3$ units.

(i) Show that $\overrightarrow{CP} = 2.4\mathbf{i} - 1.8\mathbf{k}$. [2]

(ii) Express $\overrightarrow{OP}$ and $\overrightarrow{BP}$ in terms of **i**, **j** and **k**. [2]

(iii) Use a scalar product to find angle BPC. [4]

8 The function f is such that $\mathrm{f}(x) = a^2x^2 - ax + 3b$ for $x \leqslant \dfrac{1}{2a}$, where a and b are constants.

(i) For the case where $\mathrm{f}(-2) = 4a^2 - b + 8$ and $\mathrm{f}(-3) = 7a^2 - b + 14$, find the possible values of a and b. [5]

(ii) For the case where $a = 1$ and $b = -1$, find an expression for $\mathrm{f}^{-1}(x)$ and give the domain of f^{-1}. [5]

9 **(a)**

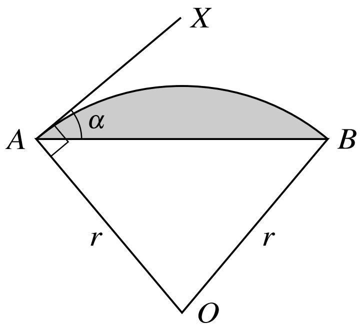


Fig. 1

In Fig. 1, OAB is a sector of a circle with centre O and radius r. AX is the tangent at A to the arc AB and angle $BAX = \alpha$.

(i) Show that angle $AOB = 2\alpha$. [2]

(ii) Find the area of the shaded segment in terms of r and α. [2]

(b)

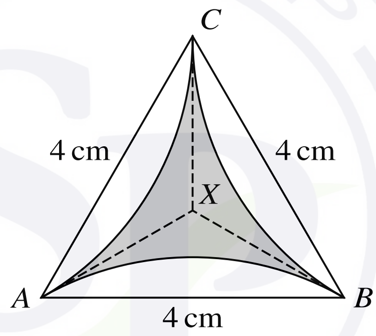
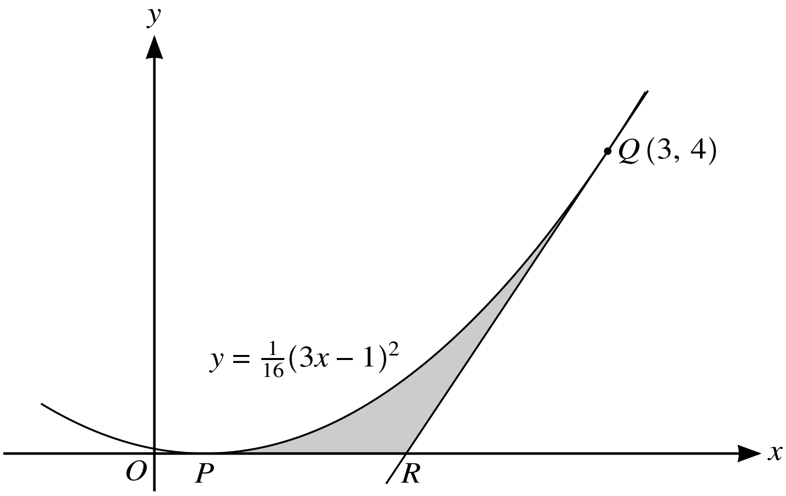


Fig. 2

In Fig. 2, ABC is an equilateral triangle of side 4 cm. The lines AX, BX and CX are tangents to the equal circular arcs AB, BC and CA. Use the results in part **(a)** to find the area of the shaded region, giving your answer in terms of π and $\sqrt{3}$. [6]

10



The diagram shows part of the curve $y = \frac{1}{16}(3x-1)^2$, which touches the x-axis at the point P. The point $Q\,(3, 4)$ lies on the curve and the tangent to the curve at Q crosses the x-axis at R.

(i) State the x-coordinate of P. [1]

Showing all necessary working, find by calculation

(ii) the x-coordinate of R, [5]

(iii) the area of the shaded region PQR. [6]

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MATHEMATICS **9709/11**

Paper 1 Pure Mathematics 1 **(P1)** **October/November 2015**

1 hour 45 minutes

* 1 4 1 3 8 0 2 6 4 5 *

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

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The total number of marks for this paper is 75.
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JC15 11_9709_11/RP
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[Turn over

1 In the expansion of $\left(1 - \frac{2x}{a}\right)(a + x)^5$, where a is a non-zero constant, show that the coefficient of x^2 is zero. [3]

2 The function f is such that $f'(x) = 3x^2 - 7$ and $f(3) = 5$. Find $f(x)$. [3]

3 Solve the equation $\sin^{-1}(4x^4 + x^2) = \frac{1}{6}\pi$. [4]

4 **(i)** Show that the equation $\frac{4\cos\theta}{\tan\theta} + 15 = 0$ can be expressed as

$$4\sin^2\theta - 15\sin\theta - 4 = 0.$$ [3]

(ii) Hence solve the equation $\frac{4\cos\theta}{\tan\theta} + 15 = 0$ for $0° \leqslant \theta \leqslant 360°$. [3]

5 A curve has equation $y = \frac{8}{x} + 2x$.

(i) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. [3]

(ii) Find the coordinates of the stationary points and state, with a reason, the nature of each stationary point. [5]

6 A curve has equation $y = x^2 - x + 3$ and a line has equation $y = 3x + a$, where a is a constant.

(i) Show that the x-coordinates of the points of intersection of the line and the curve are given by the equation $x^2 - 4x + (3 - a) = 0$. [1]

(ii) For the case where the line intersects the curve at two points, it is given that the x-coordinate of one of the points of intersection is -1. Find the x-coordinate of the other point of intersection. [2]

(iii) For the case where the line is a tangent to the curve at a point P, find the value of a and the coordinates of P. [4]

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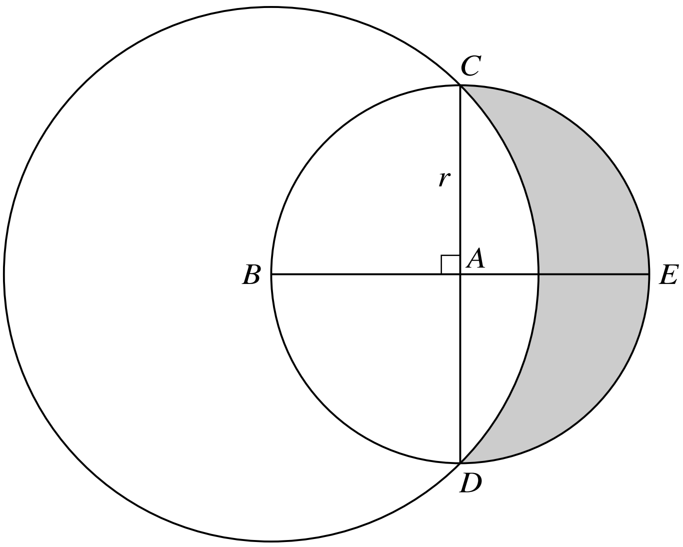

The diagram shows a circle with centre A and radius r. Diameters CAD and BAE are perpendicular to each other. A larger circle has centre B and passes through C and D.

(i) Show that the radius of the larger circle is $r\sqrt{2}$. [1]

(ii) Find the area of the shaded region in terms of r. [6]

8 The first term of a progression is $4x$ and the second term is x^2.

(i) For the case where the progression is arithmetic with a common difference of 12, find the possible values of x and the corresponding values of the third term. [4]

(ii) For the case where the progression is geometric with a sum to infinity of 8, find the third term. [4]

9 **(i)** Express $-x^2 + 6x - 5$ in the form $a(x+b)^2 + c$, where a, b and c are constants. [3]

The function $\mathrm{f} : x \mapsto -x^2 + 6x - 5$ is defined for $x \geqslant m$, where m is a constant.

(ii) State the smallest value of m for which f is one-one. [1]

(iii) For the case where $m = 5$, find an expression for $\mathrm{f}^{-1}(x)$ and state the domain of f^{-1}. [4]

[Questions 10 and 11 are printed on the next page.]

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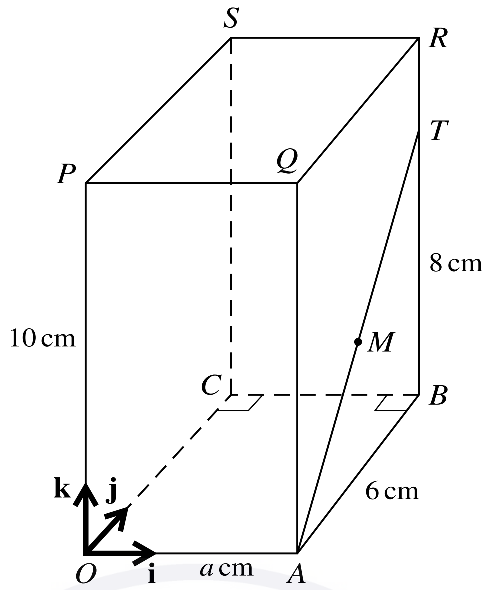
10

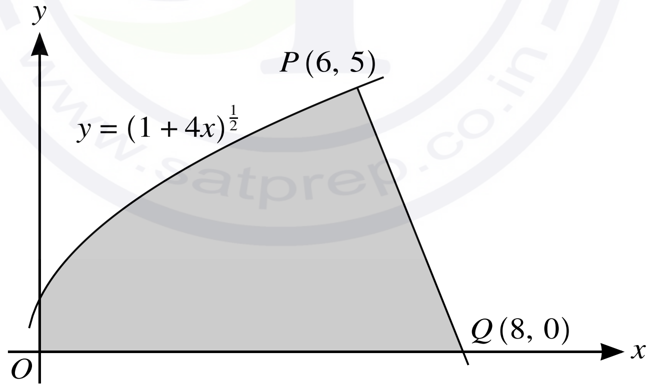


The diagram shows a cuboid $OABCPQRS$ with a horizontal base $OABC$ in which $AB = 6$ cm and $OA = a$ cm, where a is a constant. The height OP of the cuboid is 10 cm. The point T on BR is such that $BT = 8$ cm, and M is the mid-point of AT. Unit vectors **i**, **j** and **k** are parallel to OA, OC and OP respectively.

(i) For the case where $a = 2$, find the unit vector in the direction of $\overrightarrow{PM}$. [4]

(ii) For the case where angle $ATP = \cos^{-1}\left(\frac{2}{7}\right)$, find the value of a. [5]

11



The diagram shows part of the curve $y = (1 + 4x)^{\frac{1}{2}}$ and a point $P\,(6, 5)$ lying on the curve. The line PQ intersects the x-axis at $Q\,(8, 0)$.

(i) Show that PQ is a normal to the curve. [5]

(ii) Find, showing all necessary working, the exact volume of revolution obtained when the shaded region is rotated through 360° about the x-axis. [7]

[In part **(ii)** you may find it useful to apply the fact that the volume, V, of a cone of base radius r and vertical height h, is given by $V = \frac{1}{3}\pi r^2 h$.]

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MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **(P1)** **October/November 2015**

1 hour 45 minutes

0446695683

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

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DO **NOT** WRITE IN ANY BARCODES.

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Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
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1 Functions f and g are defined by

$$\mathrm{f} : x \mapsto 3x + 2, \quad x \in \mathbb{R},$$
$$\mathrm{g} : x \mapsto 4x - 12, \quad x \in \mathbb{R}.$$

Solve the equation $\mathrm{f}^{-1}(x) = \mathrm{gf}(x)$. [4]

2 In the expansion of $(x + 2k)^7$, where k is a non-zero constant, the coefficients of x^4 and x^5 are equal. Find the value of k. [4]

3

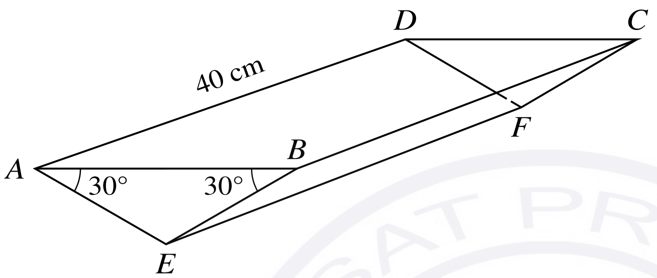


Fig. 1

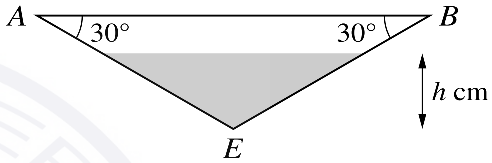


Fig. 2

Fig. 1 shows an open tank in the shape of a triangular prism. The vertical ends ABE and DCF are identical isosceles triangles. Angle ABE = angle BAE = 30°. The length of AD is 40 cm. The tank is fixed in position with the open top $ABCD$ horizontal. Water is poured into the tank at a constant rate of $200\,\text{cm}^3\,\text{s}^{-1}$. The depth of water, t seconds after filling starts, is h cm (see Fig. 2).

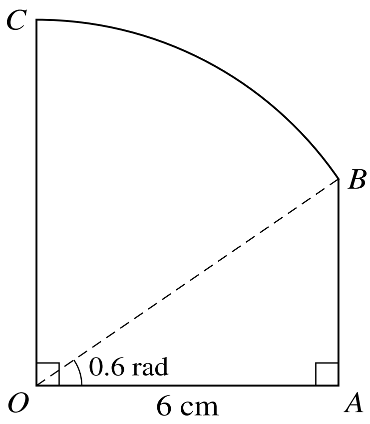
(i) Show that, when the depth of water in the tank is h cm, the volume, $V\,\text{cm}^3$, of water in the tank is given by $V = (40\sqrt{3})h^2$. [3]

(ii) Find the rate at which h is increasing when $h = 5$. [3]

4 **(i)** Prove the identity $\left(\dfrac{1}{\sin x} - \dfrac{1}{\tan x}\right)^2 \equiv \dfrac{1 - \cos x}{1 + \cos x}$. [4]

(ii) Hence solve the equation $\left(\dfrac{1}{\sin x} - \dfrac{1}{\tan x}\right)^2 = \frac{2}{5}$ for $0 \leqslant x \leqslant 2\pi$. [3]

5



The diagram shows a metal plate $OABC$, consisting of a right-angled triangle OAB and a sector OBC of a circle with centre O. Angle $AOB = 0.6$ radians, $OA = 6$ cm and OA is perpendicular to OC.

(i) Show that the length of OB is 7.270 cm, correct to 3 decimal places. [1]

(ii) Find the perimeter of the metal plate. [3]

(iii) Find the area of the metal plate. [3]

6 Points A, B and C have coordinates $A(-3, 7)$, $B(5, 1)$ and $C(-1, k)$, where k is a constant.

(i) Given that $AB = BC$, calculate the possible values of k. [3]

The perpendicular bisector of AB intersects the x-axis at D.

(ii) Calculate the coordinates of D. [5]

7 Relative to an origin O, the position vectors of points A, B and C are given by

$$\overrightarrow{OA} = \begin{pmatrix} 0 \\ 2 \\ -3 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 2 \\ 5 \\ -2 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} 3 \\ p \\ q \end{pmatrix}.$$

(i) In the case where ABC is a straight line, find the values of p and q. [4]

(ii) In the case where angle BAC is $90°$, express q in terms of p. [2]

(iii) In the case where $p = 3$ and the lengths of AB and AC are equal, find the possible values of q. [3]

8 The function f is defined, for $x \in \mathbb{R}$, by $\mathrm{f} : x \mapsto x^2 + ax + b$, where a and b are constants.

(i) In the case where $a = 6$ and $b = -8$, find the range of f. [3]

(ii) In the case where $a = 5$, the roots of the equation $\mathrm{f}(x) = 0$ are k and $-2k$, where k is a constant. Find the values of b and k. [3]

(iii) Show that if the equation $\mathrm{f}(x + a) = a$ has no real roots, then $a^2 < 4(b - a)$. [3]

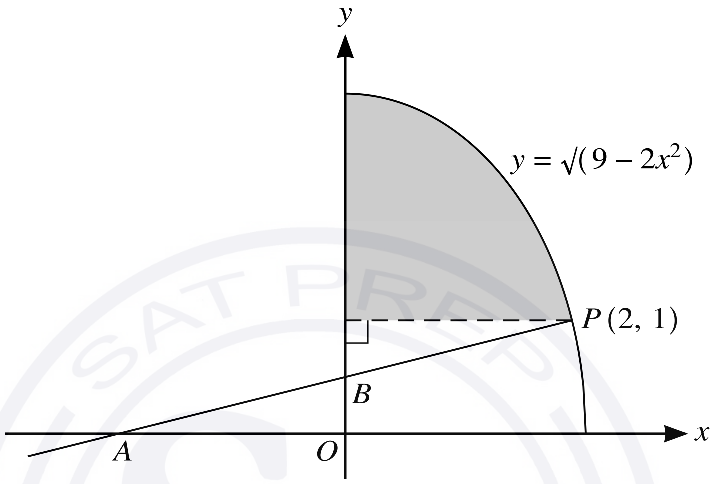
9 The curve $y = \mathrm{f}(x)$ has a stationary point at $(2, 10)$ and it is given that $\mathrm{f}''(x) = \dfrac{12}{x^3}$.

(i) Find $\mathrm{f}(x)$. [6]

(ii) Find the coordinates of the other stationary point. [2]

(iii) Find the nature of each of the stationary points. [2]

10



The diagram shows part of the curve $y = \sqrt{(9 - 2x^2)}$. The point $P\,(2, 1)$ lies on the curve and the normal to the curve at P intersects the x-axis at A and the y-axis at B.

(i) Show that B is the mid-point of AP. [6]

The shaded region is bounded by the curve, the y-axis and the line $y = 1$.

(ii) Find, showing all necessary working, the exact volume obtained when the shaded region is rotated through 360° about the ***y*-axis**. [5]

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MATHEMATICS **9709/13**

Paper 1 Pure Mathematics 1 **(P1)** **October/November 2015**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

4520717899

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You are reminded of the need for clear presentation in your answers.

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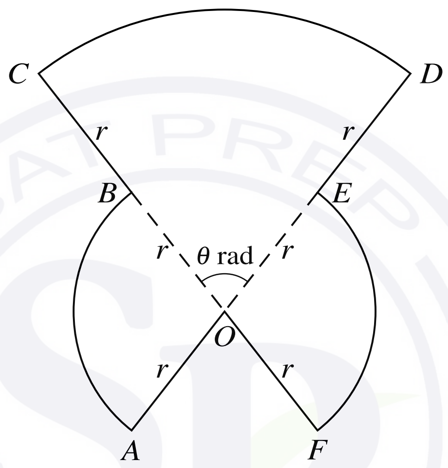
1 A line has equation $y = 2x - 7$ and a curve has equation $y = x^2 - 4x + c$, where c is a constant. Find the set of possible values of c for which the line does not intersect the curve. [3]

2 Find the coefficient of x in the expansion of $\left(\frac{x}{3} + \frac{9}{x^2}\right)^7$. [4]

3 **(i)** Express $3x^2 - 6x + 2$ in the form $a(x + b)^2 + c$, where a, b and c are constants. [3]

(ii) The function f, where $f(x) = x^3 - 3x^2 + 7x - 8$, is defined for $x \in \mathbb{R}$. Find $f'(x)$ and state, with a reason, whether f is an increasing function, a decreasing function or neither. [3]

4



The diagram shows a metal plate $OABCDEF$ consisting of 3 sectors, each with centre O. The radius of sector COD is $2r$ and angle COD is θ radians. The radius of each of the sectors BOA and FOE is r, and $AOED$ and $CBOF$ are straight lines.

(i) Show that the area of the metal plate is $r^2(\pi + \theta)$. [3]

(ii) Show that the perimeter of the metal plate is independent of θ. [4]

5 Relative to an origin O, the position vectors of the points A and B are given by

$$\overrightarrow{OA} = \begin{pmatrix} p - 6 \\ 2p - 6 \\ 1 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OB} = \begin{pmatrix} 4 - 2p \\ p \\ 2 \end{pmatrix},$$

where p is a constant.

(i) For the case where OA is perpendicular to OB, find the value of p. [3]

(ii) For the case where OAB is a straight line, find the vectors $\overrightarrow{OA}$ and $\overrightarrow{OB}$. Find also the length of the line OA. [4]

6 A ball is such that when it is dropped from a height of 1 metre it bounces vertically from the ground to a height of 0.96 metres. It continues to bounce on the ground and each time the height the ball reaches is reduced. Two different models, A and B, describe this.

Model A : The height reached is reduced by 0.04 metres each time the ball bounces.

Model B : The height reached is reduced by 4% each time the ball bounces.

(i) Find the total distance travelled vertically (up and down) by the ball from the 1st time it hits the ground until it hits the ground for the 21st time,

(a) using model A, [3]

(b) using model B. [3]

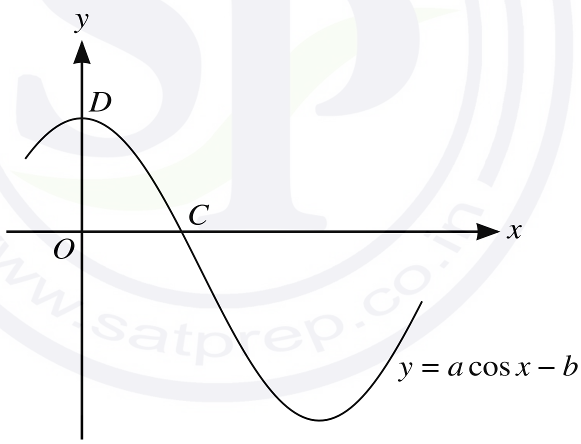
(ii) Show that, under model B, even if there is no limit to the number of times the ball bounces, the total vertical distance travelled after the first time it hits the ground cannot exceed 48 metres. [2]

7 **(a)** Show that the equation $\frac{1}{\cos\theta} + 3\sin\theta\tan\theta + 4 = 0$ can be expressed as

$$3\cos^2\theta - 4\cos\theta - 4 = 0,$$

and hence solve the equation $\frac{1}{\cos\theta} + 3\sin\theta\tan\theta + 4 = 0$ for $0° \leqslant \theta \leqslant 360°$. [6]

(b)



The diagram shows part of the graph of $y = a\cos x - b$, where a and b are constants. The graph crosses the x-axis at the point $C\,(\cos^{-1} c,\ 0)$ and the y-axis at the point $D\,(0,\ d)$. Find c and d in terms of a and b. [2]

8 The function f is defined by $f(x) = 3x + 1$ for $x \leqslant a$, where a is a constant. The function g is defined by $g(x) = -1 - x^2$ for $x \leqslant -1$.

(i) Find the largest value of a for which the composite function gf can be formed. [2]

For the case where $a = -1$,

(ii) solve the equation $fg(x) + 14 = 0$, [3]

(iii) find the set of values of x which satisfy the inequality $gf(x) \leqslant -50$. [4]

9 A curve passes through the point $A\ (4, 6)$ and is such that $\dfrac{dy}{dx} = 1 + 2x^{-\frac{1}{2}}$. A point P is moving along the curve in such a way that the x-coordinate of P is increasing at a constant rate of 3 units per minute.

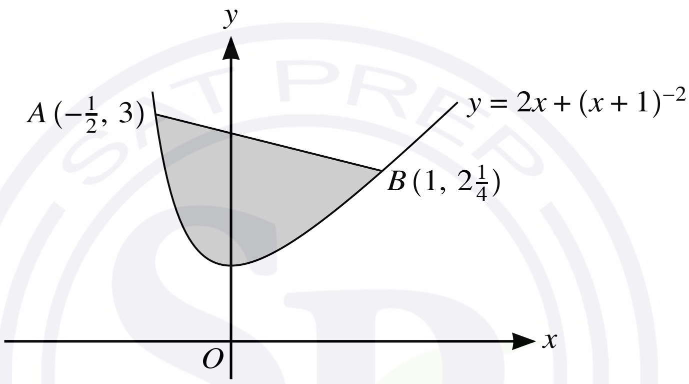
(i) Find the rate at which the y-coordinate of P is increasing when P is at A. [3]

(ii) Find the equation of the curve. [3]

(iii) The tangent to the curve at A crosses the x-axis at B and the normal to the curve at A crosses the x-axis at C. Find the area of triangle ABC. [5]

10 The function f is defined by $f(x) = 2x + (x + 1)^{-2}$ for $x > -1$.

(i) Find $f'(x)$ and $f''(x)$ and hence verify that the function f has a minimum value at $x = 0$. [4]



The points $A\ (-\frac{1}{2}, 3)$ and $B\ (1, 2\frac{1}{4})$ lie on the curve $y = 2x + (x + 1)^{-2}$, as shown in the diagram.

(ii) Find the distance AB. [2]

(iii) Find, showing all necessary working, the area of the shaded region. [6]

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MATHEMATICS **9709/11**

Paper 1 Pure Mathematics 1 **(P1)** **May/June 2015**

1 hour 45 minutes

* 8 4 6 1 1 9 7 1 7 4 *

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

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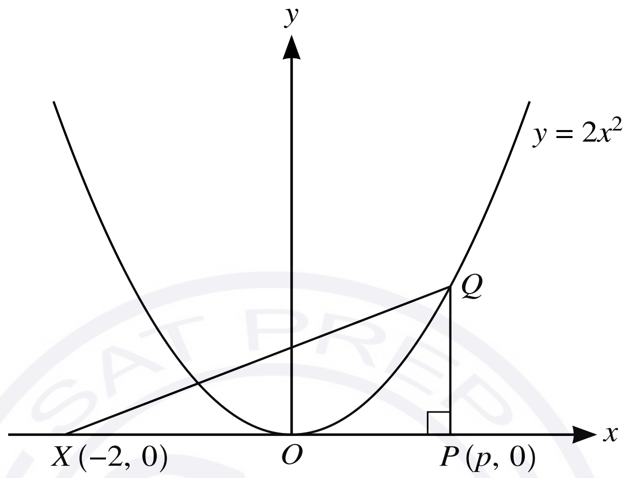
1 Given that θ is an obtuse angle measured in radians and that $\sin\theta = k$, find, in terms of k, an expression for

(i) $\cos\theta$, [1]

(ii) $\tan\theta$, [2]

(iii) $\sin(\theta + \pi)$. [1]

2



The diagram shows the curve $y = 2x^2$ and the points $X(-2, 0)$ and $P(p, 0)$. The point Q lies on the curve and PQ is parallel to the y-axis.

(i) Express the area, A, of triangle XPQ in terms of p. [2]

The point P moves along the x-axis at a constant rate of 0.02 units per second and Q moves along the curve so that PQ remains parallel to the y-axis.

(ii) Find the rate at which A is increasing when $p = 2$. [3]

3 **(i)** Find the first three terms, in ascending powers of x, in the expansion of

(a) $(1 - x)^6$, [2]

(b) $(1 + 2x)^6$. [2]

(ii) Hence find the coefficient of x^2 in the expansion of $[(1 - x)(1 + 2x)]^6$. [3]

4 Relative to the origin O, the position vectors of points A and B are given by

$$\overrightarrow{OA} = \begin{pmatrix} 3 \\ 0 \\ -4 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OB} = \begin{pmatrix} 6 \\ -3 \\ 2 \end{pmatrix}.$$

(i) Find the cosine of angle AOB. [3]

The position vector of C is given by $\overrightarrow{OC} = \begin{pmatrix} k \\ -2k \\ 2k - 3 \end{pmatrix}$.

(ii) Given that AB and OC have the same length, find the possible values of k. [4]

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5 A piece of wire of length 24 cm is bent to form the perimeter of a sector of a circle of radius r cm.

(i) Show that the area of the sector, A cm^2, is given by $A = 12r - r^2$. [3]

(ii) Express A in the form $a - (r - b)^2$, where a and b are constants. [2]

(iii) Given that r can vary, state the greatest value of A and find the corresponding angle of the sector. [2]

6 The line with gradient -2 passing through the point $P(3t, 2t)$ intersects the x-axis at A and the y-axis at B.

(i) Find the area of triangle AOB in terms of t. [3]

The line through P perpendicular to AB intersects the x-axis at C.

(ii) Show that the mid-point of PC lies on the line $y = x$. [4]

7 **(a)** The third and fourth terms of a geometric progression are $\frac{1}{3}$ and $\frac{2}{9}$ respectively. Find the sum to infinity of the progression. [4]

(b) A circle is divided into 5 sectors in such a way that the angles of the sectors are in arithmetic progression. Given that the angle of the largest sector is 4 times the angle of the smallest sector, find the angle of the largest sector. [4]

8 The function $\mathrm{f} : x \mapsto 5 + 3\cos\left(\frac{1}{2}x\right)$ is defined for $0 \leqslant x \leqslant 2\pi$.

(i) Solve the equation $\mathrm{f}(x) = 7$, giving your answer correct to 2 decimal places. [3]

(ii) Sketch the graph of $y = \mathrm{f}(x)$. [2]

(iii) Explain why f has an inverse. [1]

(iv) Obtain an expression for $\mathrm{f}^{-1}(x)$. [3]

9 The equation of a curve is $y = x^3 + px^2$, where p is a positive constant.

(i) Show that the origin is a stationary point on the curve and find the coordinates of the other stationary point in terms of p. [4]

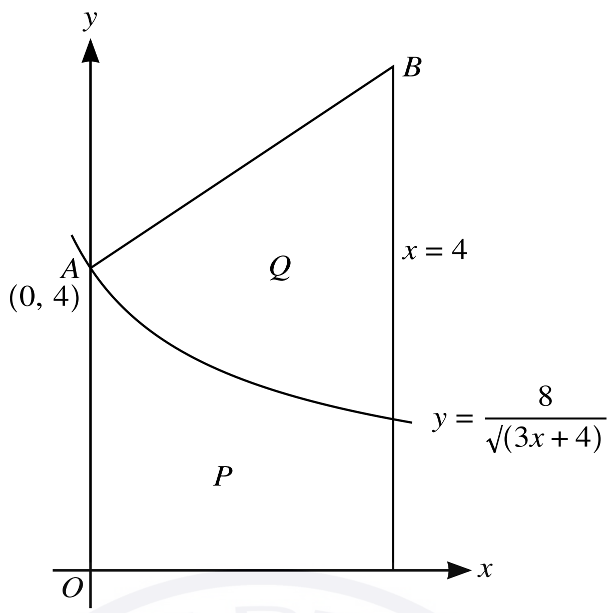
(ii) Find the nature of each of the stationary points. [3]

Another curve has equation $y = x^3 + px^2 + px$.

(iii) Find the set of values of p for which this curve has no stationary points. [3]

[Question 10 is printed on the next page.]

10



The diagram shows part of the curve $y = \frac{8}{\sqrt{(3x+4)}}$. The curve intersects the y-axis at A (0, 4). The normal to the curve at A intersects the line $x = 4$ at the point B.

(i) Find the coordinates of B. [5]

(ii) Show, with all necessary working, that the areas of the regions marked P and Q are equal. [6]

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MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **(P1)** **May/June 2015**

1 hour 45 minutes

2057450266

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

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The total number of marks for this paper is 75.
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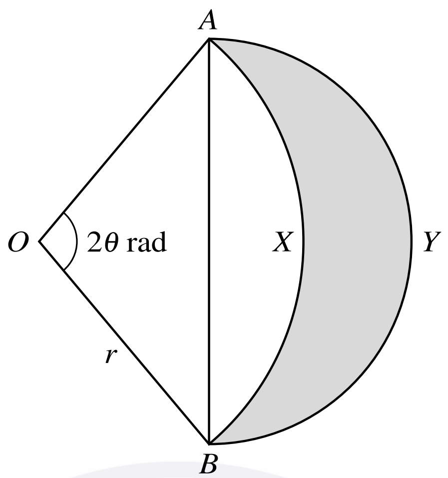
[Turn over

1 The function f is such that $f'(x) = 5 - 2x^2$ and $(3, 5)$ is a point on the curve $y = f(x)$. Find $f(x)$. [3]

2



In the diagram, AYB is a semicircle with AB as diameter and $OAXB$ is a sector of a circle with centre O and radius r. Angle $AOB = 2\theta$ radians. Find an expression, in terms of r and θ, for the area of the shaded region. [4]

3 **(i)** Find the coefficients of x^2 and x^3 in the expansion of $(2 - x)^6$. [3]

(ii) Find the coefficient of x^3 in the expansion of $(3x + 1)(2 - x)^6$. [2]

4 Variables u, x and y are such that $u = 2x(y - x)$ and $x + 3y = 12$. Express u in terms of x and hence find the stationary value of u. [5]

5 **(i)** Prove the identity $\dfrac{\sin\theta - \cos\theta}{\sin\theta + \cos\theta} \equiv \dfrac{\tan\theta - 1}{\tan\theta + 1}$. [1]

(ii) Hence solve the equation $\dfrac{\sin\theta - \cos\theta}{\sin\theta + \cos\theta} = \dfrac{\tan\theta}{6}$, for $0° \leqslant \theta \leqslant 180°$. [4]

6 A tourist attraction in a city centre is a big vertical wheel on which passengers can ride. The wheel turns in such a way that the height, h m, of a passenger above the ground is given by the formula $h = 60(1 - \cos kt)$. In this formula, k is a constant, t is the time in minutes that has elapsed since the passenger started the ride at ground level and kt is measured in radians.

(i) Find the greatest height of the passenger above the ground. [1]

One complete revolution of the wheel takes 30 minutes.

(ii) Show that $k = \frac{1}{15}\pi$. [2]

(iii) Find the time for which the passenger is above a height of 90 m. [3]

7 The point C lies on the perpendicular bisector of the line joining the points $A\,(4, 6)$ and $B\,(10, 2)$. C also lies on the line parallel to AB through $(3, 11)$.

(i) Find the equation of the perpendicular bisector of AB. [4]

(ii) Calculate the coordinates of C. [3]

8 **(a)** The first, second and last terms in an arithmetic progression are 56, 53 and −22 respectively. Find the sum of all the terms in the progression. [4]

(b) The first, second and third terms of a geometric progression are $2k + 6$, $2k$ and $k + 2$ respectively, where k is a positive constant.

(i) Find the value of k. [3]

(ii) Find the sum to infinity of the progression. [2]

9 Relative to an origin O, the position vectors of points A and B are given by

$$\overrightarrow{OA} = 2\mathbf{i} + 4\mathbf{j} + 4\mathbf{k} \quad \text{and} \quad \overrightarrow{OB} = 3\mathbf{i} + \mathbf{j} + 4\mathbf{k}.$$

(i) Use a vector method to find angle AOB. [4]

The point C is such that $\overrightarrow{AB} = \overrightarrow{BC}$.

(ii) Find the unit vector in the direction of $\overrightarrow{OC}$. [4]

(iii) Show that triangle OAC is isosceles. [1]

10 The equation of a curve is $y = \dfrac{4}{2x - 1}$.

(i) Find, showing all necessary working, the volume obtained when the region bounded by the curve, the x-axis and the lines $x = 1$ and $x = 2$ is rotated through 360° about the x-axis. [4]

(ii) Given that the line $2y = x + c$ is a normal to the curve, find the possible values of the constant c. [6]

11 The function f is defined by $\mathrm{f} : x \mapsto 2x^2 - 6x + 5$ for $x \in \mathbb{R}$.

(i) Find the set of values of p for which the equation $\mathrm{f}(x) = p$ has no real roots. [3]

The function g is defined by $\mathrm{g} : x \mapsto 2x^2 - 6x + 5$ for $0 \leqslant x \leqslant 4$.

(ii) Express $\mathrm{g}(x)$ in the form $a(x + b)^2 + c$, where a, b and c are constants. [3]

(iii) Find the range of g. [2]

The function h is defined by $\mathrm{h} : x \mapsto 2x^2 - 6x + 5$ for $k \leqslant x \leqslant 4$, where k is a constant.

(iv) State the smallest value of k for which h has an inverse. [1]

(v) For this value of k, find an expression for $\mathrm{h}^{-1}(x)$. [3]

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Cambridge International AS & A Level

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Cambridge International Advanced Subsidiary and Advanced Level

MATHEMATICS **9709/13**

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[Turn over

1 Express $2x^2 - 12x + 7$ in the form $a(x+b)^2 + c$, where a, b and c are constants. [3]

2 A curve is such that $\frac{dy}{dx} = (2x+1)^{\frac{1}{2}}$ and the point (4, 7) lies on the curve. Find the equation of the curve. [4]

3 **(i)** Write down the first 4 terms, in ascending powers of x, of the expansion of $(a-x)^5$. [2]

(ii) The coefficient of x^3 in the expansion of $(1-ax)(a-x)^5$ is -200. Find the possible values of the constant a. [4]

4 **(i)** Express the equation $3\sin\theta = \cos\theta$ in the form $\tan\theta = k$ and solve the equation for $0° < \theta < 180°$. [2]

(ii) Solve the equation $3\sin^2 2x = \cos^2 2x$ for $0° < x < 180°$. [4]

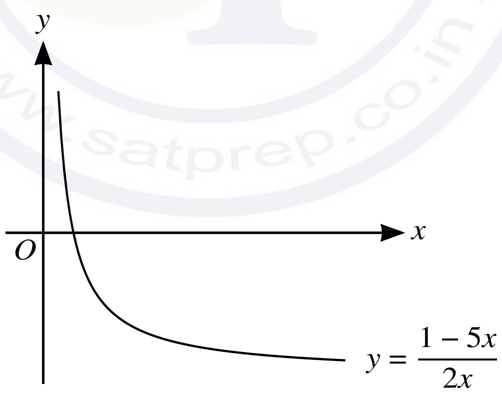
5 Relative to an origin O, the position vectors of the points A, B and C are given by

$$\overrightarrow{OA} = \begin{pmatrix} 3 \\ 2 \\ -3 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 5 \\ -1 \\ -2 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} 6 \\ 1 \\ 2 \end{pmatrix}.$$

(i) Show that angle ABC is 90°. [4]

(ii) Find the area of triangle ABC, giving your answer correct to 1 decimal place. [3]

6



The diagram shows the graph of $y = f^{-1}(x)$, where f^{-1} is defined by $f^{-1}(x) = \frac{1-5x}{2x}$ for $0 < x \leqslant 2$.

(i) Find an expression for $f(x)$ and state the domain of f. [5]

(ii) The function g is defined by $g(x) = \frac{1}{x}$ for $x \geqslant 1$. Find an expression for $f^{-1}g(x)$, giving your answer in the form $ax + b$, where a and b are constants to be found. [2]

7 The point A has coordinates $(p, 1)$ and the point B has coordinates $(9, 3p + 1)$, where p is a constant.

(i) For the case where the distance AB is 13 units, find the possible values of p. [3]

(ii) For the case in which the line with equation $2x + 3y = 9$ is perpendicular to AB, find the value of p. [4]

8 The function f is defined by $\mathrm{f}(x) = \dfrac{1}{x+1} + \dfrac{1}{(x+1)^2}$ for $x > -1$.

(i) Find $\mathrm{f}'(x)$. [3]

(ii) State, with a reason, whether f is an increasing function, a decreasing function or neither. [1]

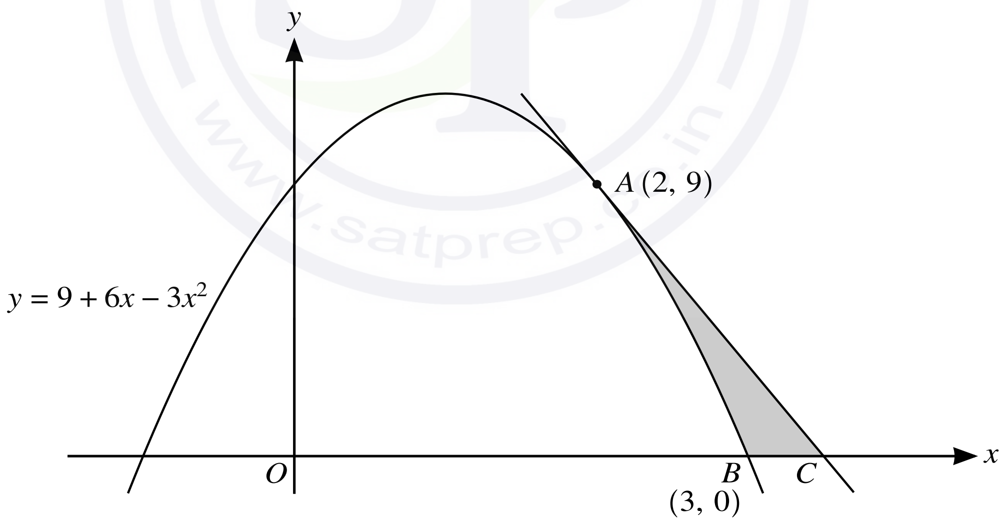
The function g is defined by $\mathrm{g}(x) = \dfrac{1}{x+1} + \dfrac{1}{(x+1)^2}$ for $x < -1$.

(iii) Find the coordinates of the stationary point on the curve $y = \mathrm{g}(x)$. [4]

9 **(a)** The first term of an arithmetic progression is -2222 and the common difference is 17. Find the value of the first positive term. [3]

(b) The first term of a geometric progression is $\sqrt{3}$ and the second term is $2\cos\theta$, where $0 < \theta < \pi$. Find the set of values of θ for which the progression is convergent. [5]

10



Points $A\,(2, 9)$ and $B\,(3, 0)$ lie on the curve $y = 9 + 6x - 3x^2$, as shown in the diagram. The tangent at A intersects the x-axis at C. Showing all necessary working,

(i) find the equation of the tangent AC and hence find the x-coordinate of C, [4]

(ii) find the area of the shaded region ABC. [5]

[Question 11 is printed on the next page.]

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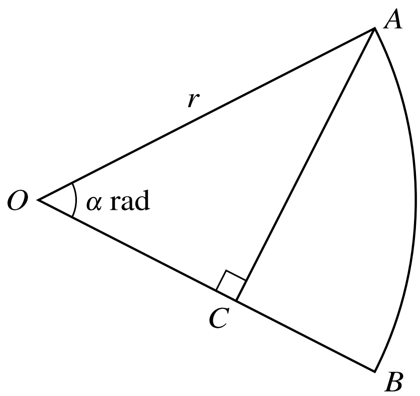
11



In the diagram, OAB is a sector of a circle with centre O and radius r. The point C on OB is such that angle ACO is a right angle. Angle AOB is α radians and is such that AC divides the sector into two regions of equal area.

(i) Show that $\sin\alpha\cos\alpha = \frac{1}{2}\alpha$. [4]

It is given that the solution of the equation in part **(i)** is $\alpha = 0.9477$, correct to 4 decimal places.

(ii) Find the ratio

perimeter of region OAC : perimeter of region ACB,

giving your answer in the form $k : 1$, where k is given correct to 1 decimal place. [5]

(iii) Find angle AOB in degrees. [1]

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MATHEMATICS **9709/11**

Paper 1 Pure Mathematics 1 **(P1)** **October/November 2014**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

1395233496

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[Turn over

1 In the expansion of $(2 + ax)^7$, the coefficient of x is equal to the coefficient of x^2. Find the value of the non-zero constant a. [3]

2 Find the value of x satisfying the equation $\sin^{-1}(x - 1) = \tan^{-1}(3)$. [3]

3 Solve the equation $\dfrac{13\sin^2\theta}{2 + \cos\theta} + \cos\theta = 2$ for $0° \leqslant \theta \leqslant 180°$. [4]

4 The line $4x + ky = 20$ passes through the points $A\,(8, -4)$ and $B\,(b, 2b)$, where k and b are constants.

(i) Find the values of k and b. [4]

(ii) Find the coordinates of the mid-point of AB. [1]

5 Find the set of values of k for which the line $y = 2x - k$ meets the curve $y = x^2 + kx - 2$ at two distinct points. [5]

6 Relative to an origin O, the position vector of A is $3\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ and the position vector of B is $7\mathbf{i} - 3\mathbf{j} + \mathbf{k}$.

(i) Show that angle OAB is a right angle. [4]

(ii) Find the area of triangle OAB. [3]

7 **(i)** A geometric progression has first term a $(a \neq 0)$, common ratio r and sum to infinity S. A second geometric progression has first term a, common ratio $2r$ and sum to infinity $3S$. Find the value of r. [3]

(ii) An arithmetic progression has first term 7. The nth term is 84 and the $(3n)$th term is 245. Find the value of n. [4]

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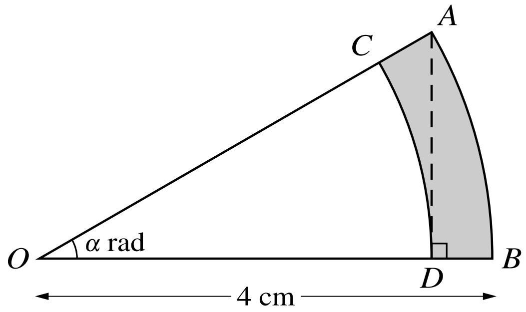
8



In the diagram, AB is an arc of a circle with centre O and radius 4 cm. Angle AOB is α radians. The point D on OB is such that AD is perpendicular to OB. The arc DC, with centre O, meets OA at C.

(i) Find an expression in terms of α for the perimeter of the shaded region $ABDC$. [4]

(ii) For the case where $\alpha = \frac{1}{6}\pi$, find the area of the shaded region $ABDC$, giving your answer in the form $k\pi$, where k is a constant to be determined. [4]

9 The function f is defined for $x > 0$ and is such that $\mathrm{f}'(x) = 2x - \dfrac{2}{x^2}$. The curve $y = \mathrm{f}(x)$ passes through the point $P(2, 6)$.

(i) Find the equation of the normal to the curve at P. [3]

(ii) Find the equation of the curve. [4]

(iii) Find the x-coordinate of the stationary point and state with a reason whether this point is a maximum or a minimum. [4]

10 **(i)** Express $x^2 - 2x - 15$ in the form $(x + a)^2 + b$. [2]

The function f is defined for $p \leqslant x \leqslant q$, where p and q are positive constants, by

$$\mathrm{f} : x \mapsto x^2 - 2x - 15.$$

The range of f is given by $c \leqslant \mathrm{f}(x) \leqslant d$, where c and d are constants.

(ii) State the smallest possible value of c. [1]

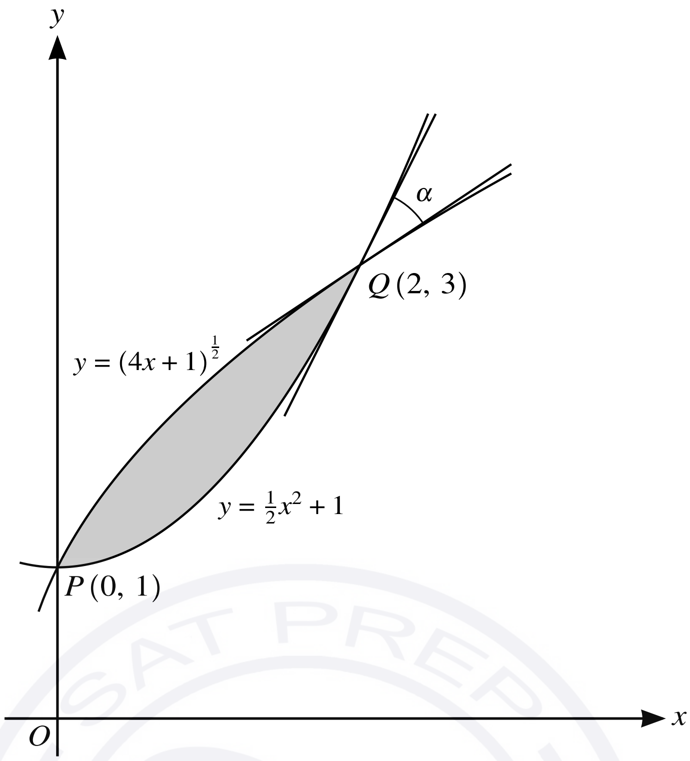
For the case where $c = 9$ and $d = 65$,

(iii) find p and q, [4]

(iv) find an expression for $\mathrm{f}^{-1}(x)$. [3]

[Question 11 is printed on the next page.]

11



The diagram shows parts of the curves $y = (4x + 1)^{\frac{1}{2}}$ and $y = \frac{1}{2}x^2 + 1$ intersecting at points $P\,(0,\ 1)$ and $Q\,(2,\ 3)$. The angle between the tangents to the two curves at Q is α.

(i) Find α, giving your answer in degrees correct to 3 significant figures. [6]

(ii) Find by integration the area of the shaded region. [6]

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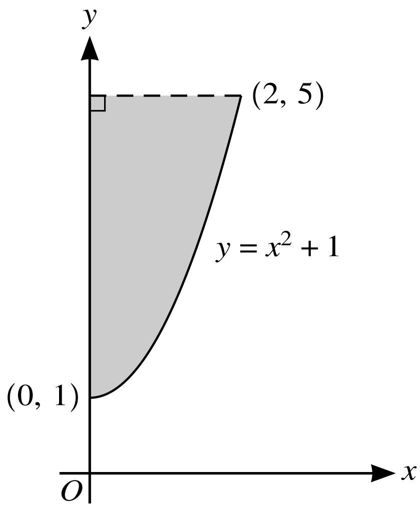
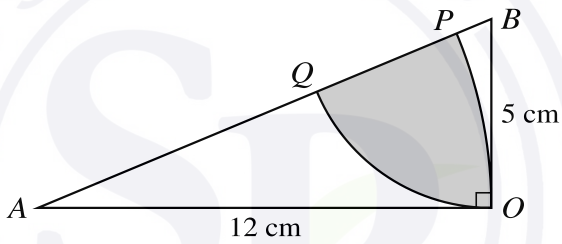


The diagram shows part of the curve $y = x^2 + 1$. Find the volume obtained when the shaded region is rotated through 360° about the ***y*-axis**. [4]

2



The diagram shows a triangle AOB in which OA is 12 cm, OB is 5 cm and angle AOB is a right angle. Point P lies on AB and OP is an arc of a circle with centre A. Point Q lies on AB and OQ is an arc of a circle with centre B.

(i) Show that angle BAO is 0.3948 radians, correct to 4 decimal places. [1]

(ii) Calculate the area of the shaded region. [5]

3 **(i)** Find the first 3 terms, in ascending powers of x, in the expansion of $(1 + x)^5$. [2]

The coefficient of x^2 in the expansion of $\left(1 + (px + x^2)\right)^5$ is 95.

(ii) Use the answer to part **(i)** to find the value of the positive constant p. [3]

4 A curve has equation $y = \dfrac{12}{3 - 2x}$.

(i) Find $\dfrac{dy}{dx}$. [2]

A point moves along this curve. As the point passes through A, the x-coordinate is increasing at a rate of 0.15 units per second and the y-coordinate is increasing at a rate of 0.4 units per second.

(ii) Find the possible x-coordinates of A. [4]

5 **(i)** Show that the equation $1 + \sin x \tan x = 5 \cos x$ can be expressed as

$$6 \cos^2 x - \cos x - 1 = 0.$$ [3]

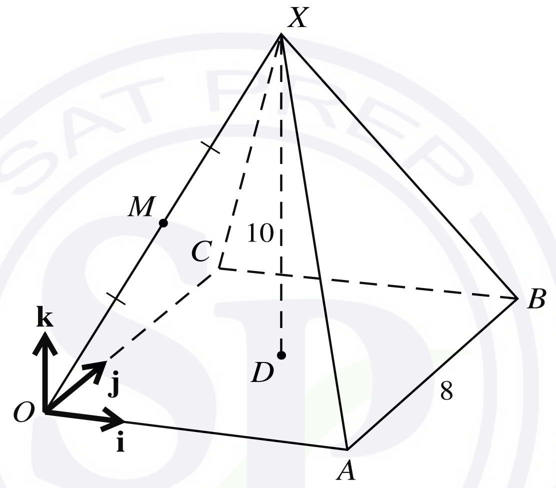
(ii) Hence solve the equation $1 + \sin x \tan x = 5 \cos x$ for $0° \leqslant x \leqslant 180°$. [3]

6 The equation of a curve is $y = x^3 + ax^2 + bx$, where a and b are constants.

(i) In the case where the curve has no stationary point, show that $a^2 < 3b$. [3]

(ii) In the case where $a = -6$ and $b = 9$, find the set of values of x for which y is a decreasing function of x. [3]

7



The diagram shows a pyramid $OABCX$. The horizontal square base $OABC$ has side 8 units and the centre of the base is D. The top of the pyramid, X, is vertically above D and $XD = 10$ units. The mid-point of OX is M. The unit vectors **i** and **j** are parallel to $\overrightarrow{OA}$ and $\overrightarrow{OC}$ respectively and the unit vector **k** is vertically upwards.

(i) Express the vectors $\overrightarrow{AM}$ and $\overrightarrow{AC}$ in terms of **i**, **j** and **k**. [3]

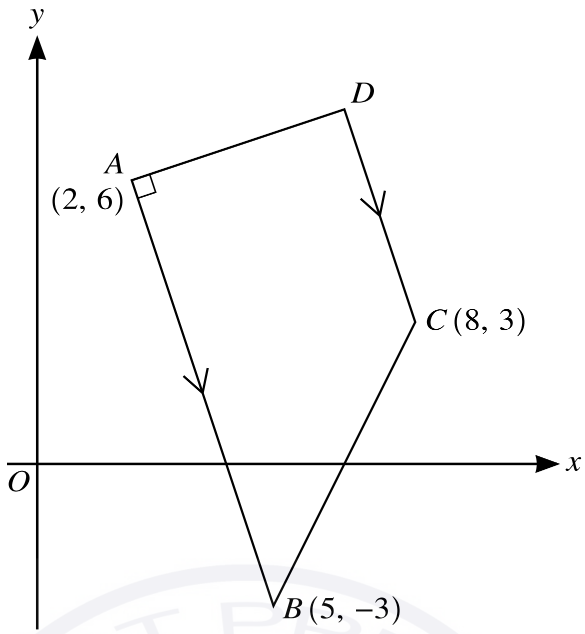
(ii) Use a scalar product to find angle MAC. [4]

8 **(a)** The sum, S_n, of the first n terms of an arithmetic progression is given by $S_n = 32n - n^2$. Find the first term and the common difference. [3]

(b) A geometric progression in which all the terms are positive has sum to infinity 20. The sum of the first two terms is 12.8. Find the first term of the progression. [5]

[Questions 9, 10 and 11 are printed on the next page.]

9



The diagram shows a trapezium $ABCD$ in which AB is parallel to DC and angle BAD is 90°. The coordinates of A, B and C are (2, 6), (5, −3) and (8, 3) respectively.

(i) Find the equation of AD. [3]

(ii) Find, by calculation, the coordinates of D. [3]

The point E is such that $ABCE$ is a parallelogram.

(iii) Find the length of BE. [2]

10 A curve is such that $\frac{d^2y}{dx^2} = \frac{24}{x^3} - 4$. The curve has a stationary point at P where $x = 2$.

(i) State, with a reason, the nature of this stationary point. [1]

(ii) Find an expression for $\frac{dy}{dx}$. [4]

(iii) Given that the curve passes through the point (1, 13), find the coordinates of the stationary point P. [4]

11 The function $f : x \mapsto 6 - 4\cos\left(\frac{1}{2}x\right)$ is defined for $0 \leqslant x \leqslant 2\pi$.

(i) Find the exact value of x for which $f(x) = 4$. [3]

(ii) State the range of f. [2]

(iii) Sketch the graph of $y = f(x)$. [2]

(iv) Find an expression for $f^{-1}(x)$. [3]

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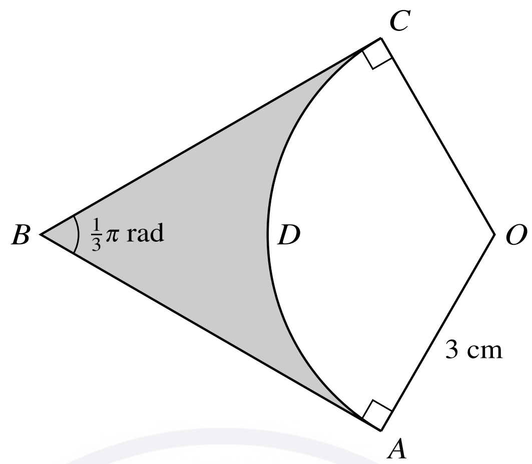
1 In the expansion of $(2 + ax)^6$, the coefficient of x^2 is equal to the coefficient of x^3. Find the value of the non-zero constant a. [4]

2



In the diagram, $OADC$ is a sector of a circle with centre O and radius 3 cm. AB and CB are tangents to the circle and angle $ABC = \frac{1}{3}\pi$ radians. Find, giving your answer in terms of $\sqrt{3}$ and π,

(i) the perimeter of the shaded region, [3]

(ii) the area of the shaded region. [3]

3 **(i)** Express $9x^2 - 12x + 5$ in the form $(ax + b)^2 + c$. [3]

(ii) Determine whether $3x^3 - 6x^2 + 5x - 12$ is an increasing function, a decreasing function or neither. [3]

4 Three geometric progressions, P, Q and R, are such that their sums to infinity are the first three terms respectively of an arithmetic progression.

Progression P is $2,\ 1,\ \frac{1}{2},\ \frac{1}{4},\ \dots$.

Progression Q is $3,\ 1,\ \frac{1}{3},\ \frac{1}{9},\ \dots$.

(i) Find the sum to infinity of progression R. [3]

(ii) Given that the first term of R is 4, find the sum of the first three terms of R. [3]

5 **(i)** Show that $\sin^4\theta - \cos^4\theta \equiv 2\sin^2\theta - 1$. [3]

(ii) Hence solve the equation $\sin^4\theta - \cos^4\theta = \frac{1}{2}$ for $0° \leqslant \theta \leqslant 360°$. [4]

6 A is the point $(a,\ 2a - 1)$ and B is the point $(2a + 4,\ 3a + 9)$, where a is a constant.

(i) Find, in terms of a, the gradient of a line perpendicular to AB. [3]

(ii) Given that the distance AB is $\sqrt{(260)}$, find the possible values of a. [4]

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7 Three points, O, A and B, are such that $\overrightarrow{OA} = \mathbf{i} + 3\mathbf{j} + p\mathbf{k}$ and $\overrightarrow{OB} = -7\mathbf{i} + (1-p)\mathbf{j} + p\mathbf{k}$, where p is a constant.

(i) Find the values of p for which $\overrightarrow{OA}$ is perpendicular to $\overrightarrow{OB}$. [3]

(ii) The magnitudes of $\overrightarrow{OA}$ and $\overrightarrow{OB}$ are a and b respectively. Find the value of p for which $b^2 = 2a^2$. [2]

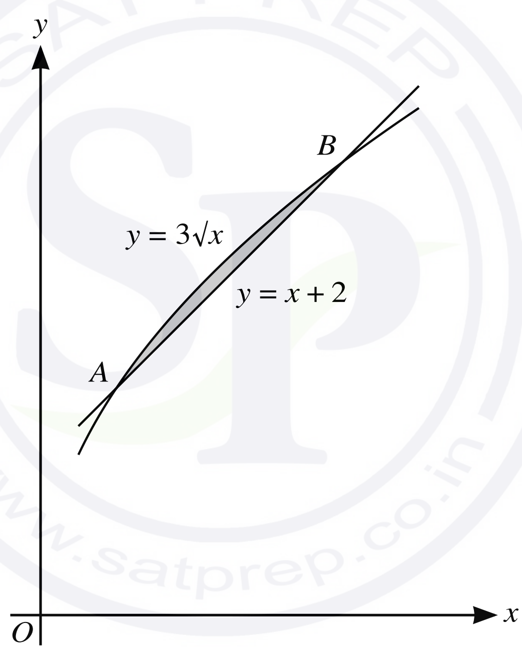
(iii) Find the unit vector in the direction of $\overrightarrow{AB}$ when $p = -8$. [3]

8 A curve $y = f(x)$ has a stationary point at $(3, 7)$ and is such that $f''(x) = 36x^{-3}$.

(i) State, with a reason, whether this stationary point is a maximum or a minimum. [1]

(ii) Find $f'(x)$ and $f(x)$. [7]

9



The diagram shows parts of the graphs of $y = x + 2$ and $y = 3\sqrt{x}$ intersecting at points A and B.

(i) Write down an equation satisfied by the x-coordinates of A and B. Solve this equation and hence find the coordinates of A and B. [4]

(ii) Find by integration the area of the shaded region. [6]

[Question 10 is printed on the next page.]

10 **(a)** The functions f and g are defined for $x \geqslant 0$ by

$$f : x \mapsto (ax + b)^{\frac{1}{3}}, \text{ where } a \text{ and } b \text{ are positive constants,}$$
$$g : x \mapsto x^2.$$

Given that $fg(1) = 2$ and $gf(9) = 16$,

(i) calculate the values of a and b, [4]

(ii) obtain an expression for $f^{-1}(x)$ and state the domain of f^{-1}. [4]

(b) A point P travels along the curve $y = (7x^2 + 1)^{\frac{1}{3}}$ in such a way that the x-coordinate of P at time t minutes is increasing at a constant rate of 8 units per minute. Find the rate of increase of the y-coordinate of P at the instant when P is at the point (3, 4). [5]

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MATHEMATICS **9709/11**

Paper 1 Pure Mathematics 1 **(P1)** **May/June 2014**

1 hour 45 minutes

* 3 4 0 1 6 3 7 2 2 8 *

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

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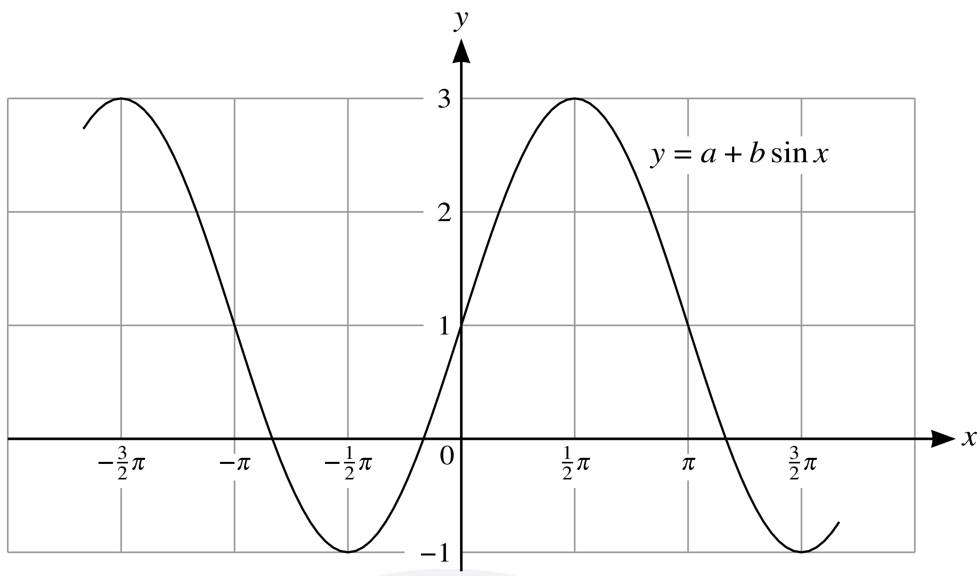
[Turn over

1



The diagram shows part of the graph of $y = a + b\sin x$. State the values of the constants a and b. [2]

2 **(i)** Express $4x^2 - 12x$ in the form $(2x + a)^2 + b$. [2]

(ii) Hence, or otherwise, find the set of values of x satisfying $4x^2 - 12x > 7$. [2]

3 Find the term independent of x in the expansion of $\left(4x^3 + \dfrac{1}{2x}\right)^8$. [4]

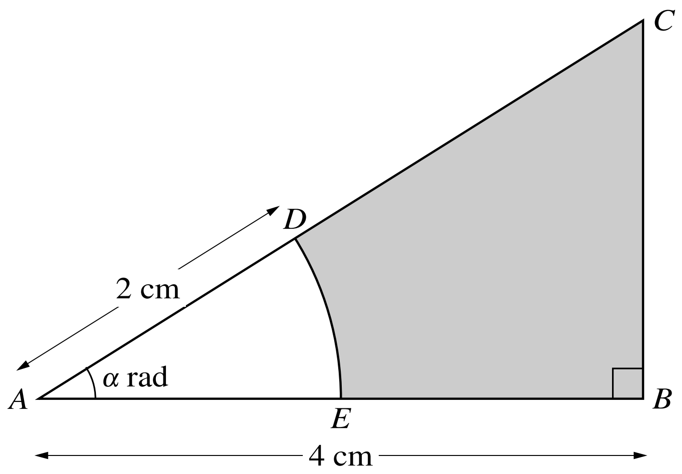
4 A curve has equation $y = \dfrac{4}{(3x+1)^2}$. Find the equation of the tangent to the curve at the point where the line $x = -1$ intersects the curve. [5]

5 An arithmetic progression has first term a and common difference d. It is given that the sum of the first 200 terms is 4 times the sum of the first 100 terms.

(i) Find d in terms of a. [3]

(ii) Find the 100th term in terms of a. [2]

6



The diagram shows triangle ABC in which AB is perpendicular to BC. The length of AB is 4 cm and angle CAB is α radians. The arc DE with centre A and radius 2 cm meets AC at D and AB at E. Find, in terms of α,

(i) the area of the shaded region, [3]

(ii) the perimeter of the shaded region. [3]

7 The coordinates of points A and B are $(a, 2)$ and $(3, b)$ respectively, where a and b are constants. The distance AB is $\sqrt{(125)}$ units and the gradient of the line AB is 2. Find the possible values of a and of b. [6]

8 Relative to an origin O, the position vectors of points A and B are given by

$$\overrightarrow{OA} = \begin{pmatrix} 3p \\ 4 \\ p^2 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OB} = \begin{pmatrix} -p \\ -1 \\ p^2 \end{pmatrix}.$$

(i) Find the values of p for which angle AOB is 90°. [3]

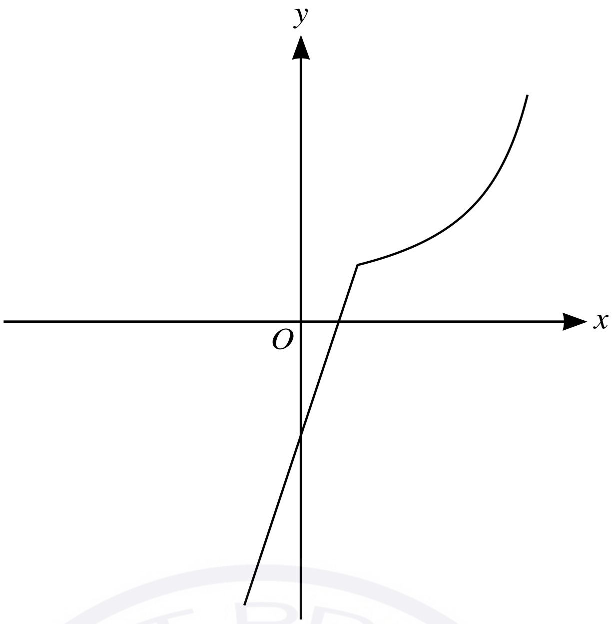
(ii) For the case where $p = 3$, find the unit vector in the direction of $\overrightarrow{BA}$. [3]

9 **(i)** Prove the identity $\dfrac{\sin\theta}{1-\cos\theta} - \dfrac{1}{\sin\theta} \equiv \dfrac{1}{\tan\theta}$. [4]

(ii) Hence solve the equation $\dfrac{\sin\theta}{1-\cos\theta} - \dfrac{1}{\sin\theta} = 4\tan\theta$ for $0° < \theta < 180°$. [3]

[Questions 10, 11 and 12 are printed on the next page.]

10



The diagram shows the function f defined for $-1 \leqslant x \leqslant 4$, where

$$f(x) = \begin{cases} 3x - 2 & \text{for } -1 \leqslant x \leqslant 1, \\ \dfrac{4}{5-x} & \text{for } 1 < x \leqslant 4. \end{cases}$$

(i) State the range of f. [1]

(ii) Copy the diagram and on your copy sketch the graph of $y = f^{-1}(x)$. [2]

(iii) Obtain expressions to define the function f^{-1}, giving also the set of values for which each expression is valid. [6]

11 A line has equation $y = 2x + c$ and a curve has equation $y = 8 - 2x - x^2$.

(i) For the case where the line is a tangent to the curve, find the value of the constant c. [3]

(ii) For the case where $c = 11$, find the x-coordinates of the points of intersection of the line and the curve. Find also, by integration, the area of the region between the line and the curve. [7]

12 A curve is such that $\dfrac{dy}{dx} = x^{\frac{1}{2}} - x^{-\frac{1}{2}}$. The curve passes through the point $\left(4, \frac{2}{3}\right)$.

(i) Find the equation of the curve. [4]

(ii) Find $\dfrac{d^2y}{dx^2}$. [2]

(iii) Find the coordinates of the stationary point and determine its nature. [5]

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MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **(P1)** **May/June 2014**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

6140629685

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1 Find the coordinates of the point at which the perpendicular bisector of the line joining (2, 7) to (10, 3) meets the x-axis. [5]

2 Find the coefficient of x^2 in the expansion of $(1 + x^2)\left(\frac{x}{2} - \frac{4}{x}\right)^6$. [5]

3 The reflex angle θ is such that $\cos\theta = k$, where $0 < k < 1$.

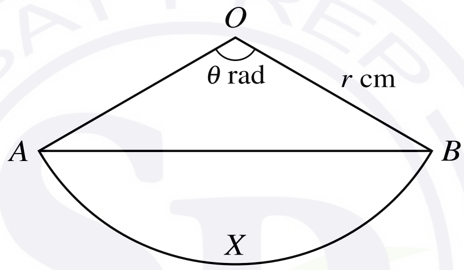
(i) Find an expression, in terms of k, for

(a) $\sin\theta$, [2]

(b) $\tan\theta$. [1]

(ii) Explain why $\sin 2\theta$ is negative for $0 < k < 1$. [2]

4



The diagram shows a sector of a circle with radius r cm and centre O. The chord AB divides the sector into a triangle AOB and a segment AXB. Angle AOB is θ radians.

(i) In the case where the areas of the triangle AOB and the segment AXB are equal, find the value of the constant p for which $\theta = p\sin\theta$. [2]

(ii) In the case where $r = 8$ and $\theta = 2.4$, find the perimeter of the segment AXB. [3]

5 **(i)** Prove the identity $\frac{1}{\cos\theta} - \frac{\cos\theta}{1 + \sin\theta} \equiv \tan\theta$. [4]

(ii) Solve the equation $\frac{1}{\cos\theta} - \frac{\cos\theta}{1 + \sin\theta} + 2 = 0$ for $0° \leqslant \theta \leqslant 360°$. [3]

6 The 1st, 2nd and 3rd terms of a geometric progression are the 1st, 9th and 21st terms respectively of an arithmetic progression. The 1st term of each progression is 8 and the common ratio of the geometric progression is r, where $r \neq 1$. Find

(i) the value of r, [4]

(ii) the 4th term of each progression. [3]

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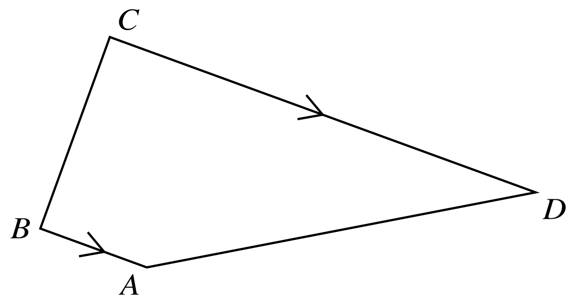
7



The diagram shows a trapezium $ABCD$ in which BA is parallel to CD. The position vectors of A, B and C relative to an origin O are given by

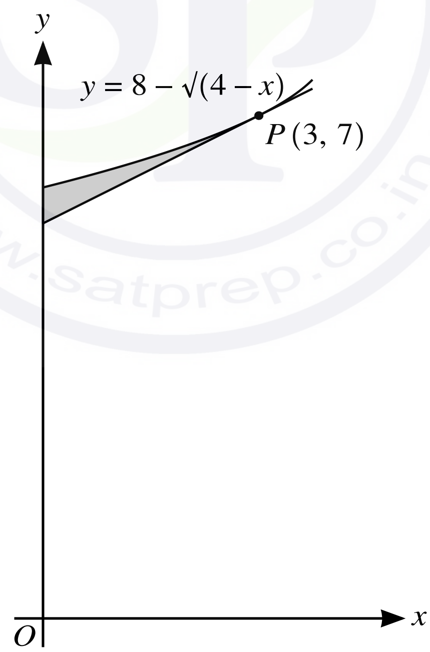
$$\overrightarrow{OA} = \begin{pmatrix} 3 \\ 4 \\ 0 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix}.$$

(i) Use a scalar product to show that AB is perpendicular to BC. [3]

(ii) Given that the length of CD is 12 units, find the position vector of D. [4]

8 The equation of a curve is such that $\dfrac{d^2y}{dx^2} = 2x - 1$. Given that the curve has a minimum point at $(3, -10)$, find the coordinates of the maximum point. [8]

9



The diagram shows part of the curve $y = 8 - \sqrt{(4 - x)}$ and the tangent to the curve at $P\,(3, 7)$.

(i) Find expressions for $\dfrac{dy}{dx}$ and $\displaystyle\int y\,dx$. [5]

(ii) Find the equation of the tangent to the curve at P in the form $y = mx + c$. [2]

(iii) Find, showing all necessary working, the area of the shaded region. [4]

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10 Functions f and g are defined by

$$\mathrm{f} : x \mapsto 2x - 3, \quad x \in \mathbb{R},$$
$$\mathrm{g} : x \mapsto x^2 + 4x, \quad x \in \mathbb{R}.$$

(i) Solve the equation $\mathrm{ff}(x) = 11$. [2]

(ii) Find the range of g. [2]

(iii) Find the set of values of x for which $\mathrm{g}(x) > 12$. [3]

(iv) Find the value of the constant p for which the equation $\mathrm{gf}(x) = p$ has two equal roots. [3]

Function h is defined by $\mathrm{h} : x \mapsto x^2 + 4x$ for $x \geqslant k$, and it is given that h has an inverse.

(v) State the smallest possible value of k. [1]

(vi) Find an expression for $\mathrm{h}^{-1}(x)$. [4]

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MATHEMATICS **9709/13**

Paper 1 Pure Mathematics 1 **(P1)** **May/June 2014**

1 hour 45 minutes

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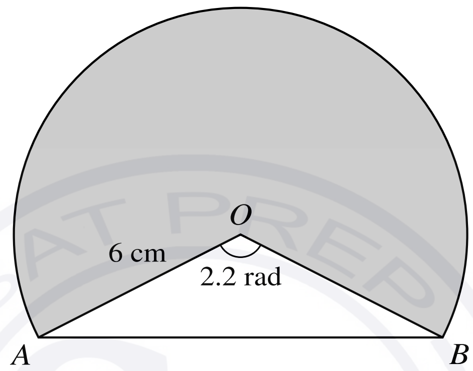
1 Find the coefficient of x in the expansion of $\left(x^2 - \frac{2}{x}\right)^5$. [3]

2 The first term in a progression is 36 and the second term is 32.

(i) Given that the progression is geometric, find the sum to infinity. [2]

(ii) Given instead that the progression is arithmetic, find the number of terms in the progression if the sum of all the terms is 0. [3]

3



The diagram shows part of a circle with centre O and radius 6 cm. The chord AB is such that angle $AOB = 2.2$ radians. Calculate

(i) the perimeter of the shaded region, [3]

(ii) the ratio of the area of the shaded region to the area of the triangle AOB, giving your answer in the form $k : 1$. [3]

4 **(i)** Prove the identity $\frac{\tan x + 1}{\sin x \tan x + \cos x} \equiv \sin x + \cos x$. [3]

(ii) Hence solve the equation $\frac{\tan x + 1}{\sin x \tan x + \cos x} = 3 \sin x - 2 \cos x$ for $0 \leqslant x \leqslant 2\pi$. [3]

5 A function f is such that $\mathrm{f}(x) = \frac{15}{2x + 3}$ for $0 \leqslant x \leqslant 6$.

(i) Find an expression for $\mathrm{f}'(x)$ and use your result to explain why f has an inverse. [3]

(ii) Find an expression for $\mathrm{f}^{-1}(x)$, and state the domain and range of f^{-1}. [4]

6 A curve is such that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{12}{\sqrt{(4x + a)}}$, where a is a constant. The point $P\,(2, 14)$ lies on the curve and the normal to the curve at P is $3y + x = 5$.

(i) Show that $a = 8$. [3]

(ii) Find the equation of the curve. [4]

7 The position vectors of points A, B and C relative to an origin O are given by

$$\overrightarrow{OA} = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 6 \\ -1 \\ 7 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} 2 \\ 4 \\ 7 \end{pmatrix}.$$

(i) Show that angle $BAC = \cos^{-1}\left(\frac{1}{3}\right)$. [5]

(ii) Use the result in part **(i)** to find the exact value of the area of triangle ABC. [3]

8 **(i)** Express $2x^2 - 10x + 8$ in the form $a(x+b)^2 + c$, where a, b and c are constants, and use your answer to state the minimum value of $2x^2 - 10x + 8$. [4]

(ii) Find the set of values of k for which the equation $2x^2 - 10x + 8 = kx$ has no real roots. [4]

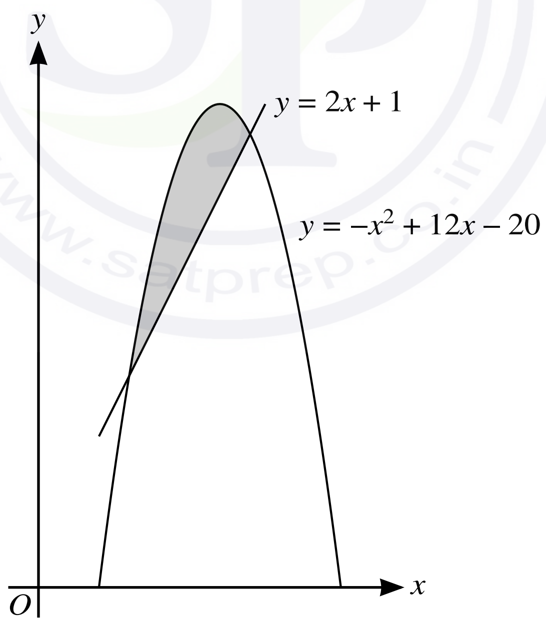
9 The base of a cuboid has sides of length x cm and $3x$ cm. The volume of the cuboid is 288 cm^3.

(i) Show that the total surface area of the cuboid, A cm^2, is given by

$$A = 6x^2 + \frac{768}{x}.$$ [3]

(ii) Given that x can vary, find the stationary value of A and determine its nature. [5]

10



The diagram shows the curve $y = -x^2 + 12x - 20$ and the line $y = 2x + 1$. Find, showing all necessary working, the area of the shaded region. [8]

[Question 11 is printed on the next page.]

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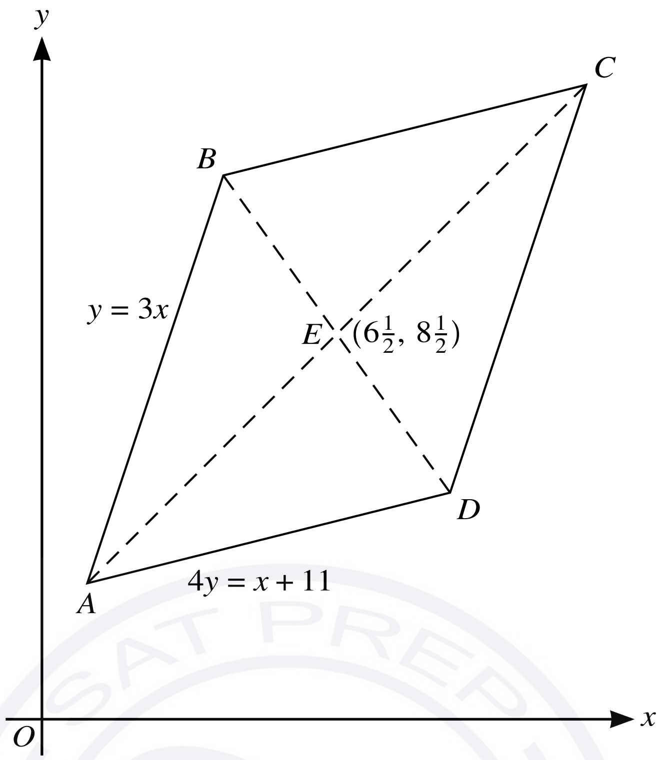
11



The diagram shows a parallelogram $ABCD$, in which the equation of AB is $y = 3x$ and the equation of AD is $4y = x + 11$. The diagonals AC and BD meet at the point $E\left(6\frac{1}{2}, 8\frac{1}{2}\right)$. Find, by calculation, the coordinates of A, B, C and D. [9]

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General Certificate of Education
Advanced Subsidiary Level and Advanced Level

MATHEMATICS **9709/11**

Paper 1 Pure Mathematics 1 **(P1)** **October/November 2013**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

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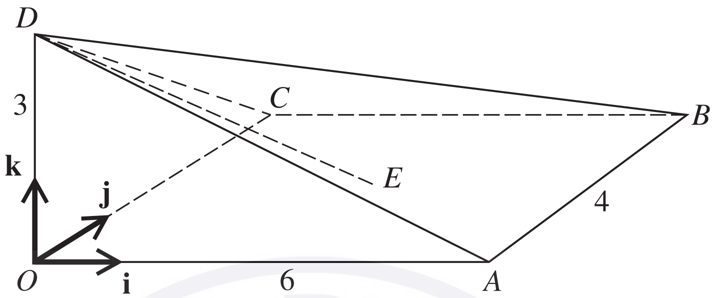
1 **(i)** Find the first three terms when $(2 + 3x)^6$ is expanded in ascending powers of x. [3]

(ii) In the expansion of $(1 + ax)(2 + 3x)^6$, the coefficient of x^2 is zero. Find the value of a. [2]

2 A curve has equation $y = \mathrm{f}(x)$. It is given that $\mathrm{f}'(x) = \dfrac{1}{\sqrt{(x+6)}} + \dfrac{6}{x^2}$ and that $\mathrm{f}(3) = 1$. Find $\mathrm{f}(x)$. [5]

3



The diagram shows a pyramid $OABCD$ in which the vertical edge OD is 3 units in length. The point E is the centre of the horizontal rectangular base $OABC$. The sides OA and AB have lengths of 6 units and 4 units respectively. The unit vectors $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ are parallel to $\overrightarrow{OA}$, $\overrightarrow{OC}$ and $\overrightarrow{OD}$ respectively.

(i) Express each of the vectors $\overrightarrow{DB}$ and $\overrightarrow{DE}$ in terms of $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$. [2]

(ii) Use a scalar product to find angle BDE. [4]

4 **(i)** Solve the equation $4\sin^2 x + 8\cos x - 7 = 0$ for $0° \leqslant x \leqslant 360°$. [4]

(ii) Hence find the solution of the equation $4\sin^2\left(\frac{1}{2}\theta\right) + 8\cos\left(\frac{1}{2}\theta\right) - 7 = 0$ for $0° \leqslant \theta \leqslant 360°$. [2]

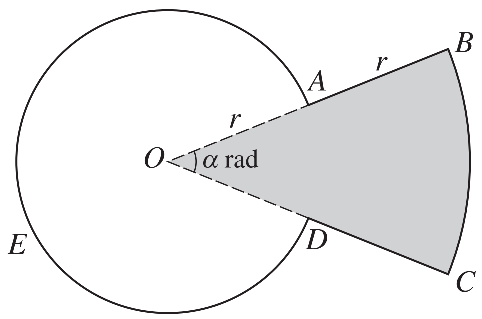
5 The function f is defined by

$$\mathrm{f} : x \mapsto x^2 + 1 \quad \text{for } x \geqslant 0.$$

(i) Define in a similar way the inverse function f^{-1}. [3]

(ii) Solve the equation $\mathrm{ff}(x) = \frac{185}{16}$. [3]

6



The diagram shows a metal plate made by fixing together two pieces, $OABCD$ (shaded) and $OAED$ (unshaded). The piece $OABCD$ is a minor sector of a circle with centre O and radius $2r$. The piece $OAED$ is a major sector of a circle with centre O and radius r. Angle AOD is α radians. Simplifying your answers where possible, find, in terms of α, π and r,

(i) the perimeter of the metal plate, [3]

(ii) the area of the metal plate. [3]

It is now given that the shaded and unshaded pieces are equal in area.

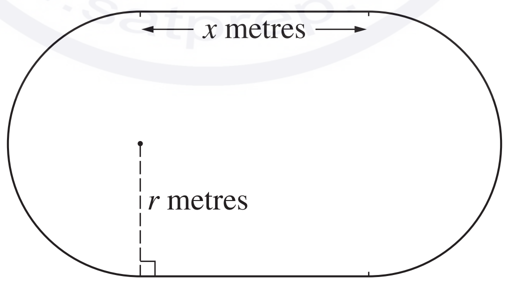
(iii) Find α in terms of π. [2]

7 The point A has coordinates $(-1, 6)$ and the point B has coordinates $(7, 2)$.

(i) Find the equation of the perpendicular bisector of AB, giving your answer in the form $y = mx + c$. [4]

(ii) A point C on the perpendicular bisector has coordinates (p, q). The distance OC is 2 units, where O is the origin. Write down two equations involving p and q and hence find the coordinates of the possible positions of C. [5]

8



The inside lane of a school running track consists of two straight sections each of length x metres, and two semicircular sections each of radius r metres, as shown in the diagram. The straight sections are perpendicular to the diameters of the semicircular sections. The perimeter of the inside lane is 400 metres.

(i) Show that the area, $A\ \text{m}^2$, of the region enclosed by the inside lane is given by $A = 400r - \pi r^2$. [4]

(ii) Given that x and r can vary, show that, when A has a stationary value, there are no straight sections in the track. Determine whether the stationary value is a maximum or a minimum. [5]

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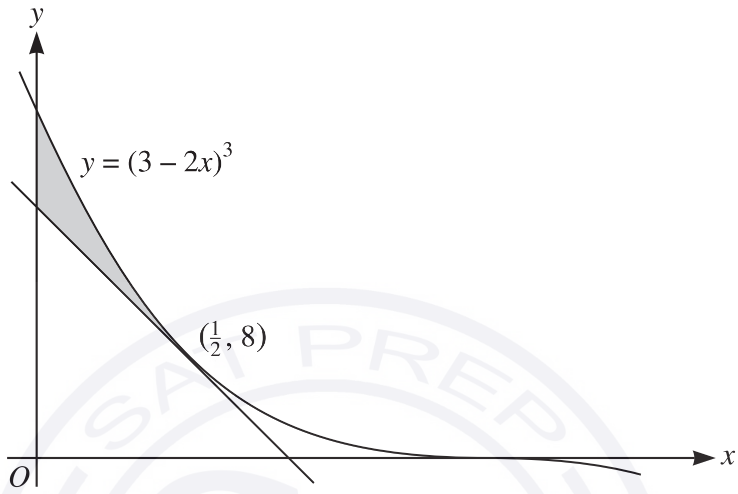
9 **(a)** In an arithmetic progression the sum of the first ten terms is 400 and the sum of the next ten terms is 1000. Find the common difference and the first term. [5]

(b) A geometric progression has first term a, common ratio r and sum to infinity 6. A second geometric progression has first term $2a$, common ratio r^2 and sum to infinity 7. Find the values of a and r. [5]

10



The diagram shows the curve $y = (3 - 2x)^3$ and the tangent to the curve at the point $\left(\frac{1}{2}, 8\right)$.

(i) Find the equation of this tangent, giving your answer in the form $y = mx + c$. [5]

(ii) Find the area of the shaded region. [6]

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Advanced Subsidiary Level and Advanced Level

MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **(P1)** **October/November 2013**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

4579480351

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[Turn over

1 Given that $\cos x = p$, where x is an acute angle in degrees, find, in terms of p,

(i) $\sin x$, [1]

(ii) $\tan x$, [1]

(iii) $\tan(90° - x)$. [1]

2

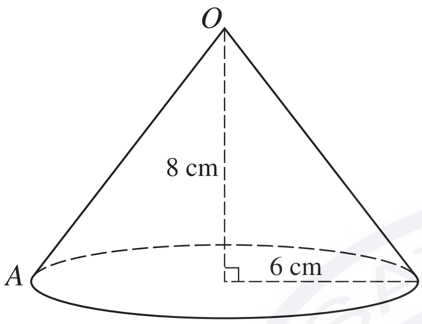

Fig. 1

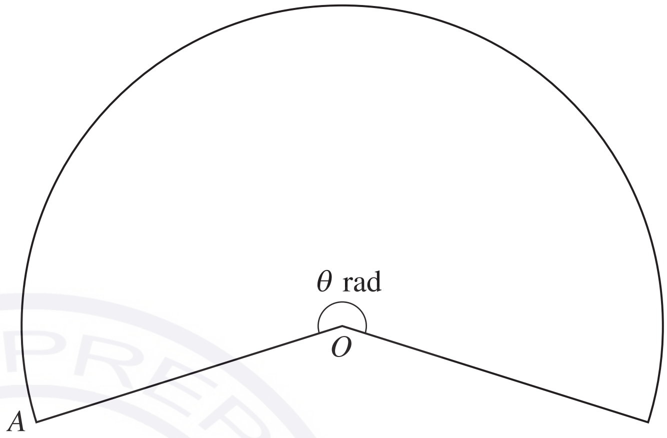

Fig. 2

Fig. 1 shows a hollow cone with no base, made of paper. The radius of the cone is 6 cm and the height is 8 cm. The paper is cut from A to O and opened out to form the sector shown in Fig. 2. The circular bottom edge of the cone in Fig. 1 becomes the arc of the sector in Fig. 2. The angle of the sector is θ radians. Calculate

(i) the value of θ, [4]

(ii) the area of paper needed to make the cone. [2]

3 The equation of a curve is $y = \dfrac{2}{\sqrt{(5x - 6)}}$.

(i) Find the gradient of the curve at the point where $x = 2$. [3]

(ii) Find $\displaystyle\int \frac{2}{\sqrt{(5x-6)}}\,dx$ and hence evaluate $\displaystyle\int_2^3 \frac{2}{\sqrt{(5x-6)}}\,dx$. [4]

4 Relative to an origin O, the position vectors of points A and B are given by

$$\overrightarrow{OA} = \mathbf{i} + 2\mathbf{j} \quad \text{and} \quad \overrightarrow{OB} = 4\mathbf{i} + p\mathbf{k}.$$

(i) In the case where $p = 6$, find the unit vector in the direction of $\overrightarrow{AB}$. [3]

(ii) Find the values of p for which angle $AOB = \cos^{-1}\left(\frac{1}{5}\right)$. [4]

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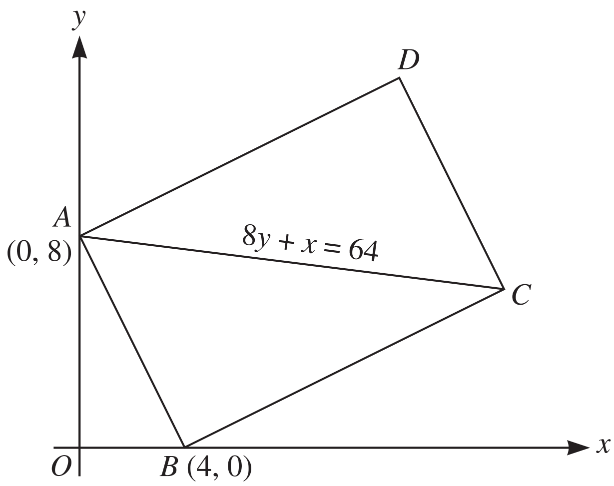
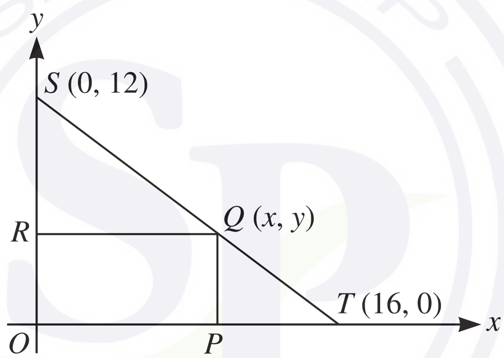
5



The diagram shows a rectangle $ABCD$ in which point A is (0, 8) and point B is (4, 0). The diagonal AC has equation $8y + x = 64$. Find, by calculation, the coordinates of C and D. [7]

6



In the diagram, S is the point (0, 12) and T is the point (16, 0). The point Q lies on ST, between S and T, and has coordinates (x, y). The points P and R lie on the x-axis and y-axis respectively and $OPQR$ is a rectangle.

(i) Show that the area, A, of the rectangle $OPQR$ is given by $A = 12x - \frac{3}{4}x^2$. [3]

(ii) Given that x can vary, find the stationary value of A and determine its nature. [4]

7 **(a)** An athlete runs the first mile of a marathon in 5 minutes. His speed reduces in such a way that each mile takes 12 seconds longer than the preceding mile.

(i) Given that the nth mile takes 9 minutes, find the value of n. [2]

(ii) Assuming that the length of the marathon is 26 miles, find the total time, in hours and minutes, to complete the marathon. [2]

(b) The second and third terms of a geometric progression are 48 and 32 respectively. Find the sum to infinity of the progression. [4]

[Questions 8, 9 and 10 are printed on the next page.]

8 A function f is defined by $\mathrm{f} : x \mapsto 3\cos x - 2$ for $0 \leqslant x \leqslant 2\pi$.

(i) Solve the equation $\mathrm{f}(x) = 0$. [3]

(ii) Find the range of f. [2]

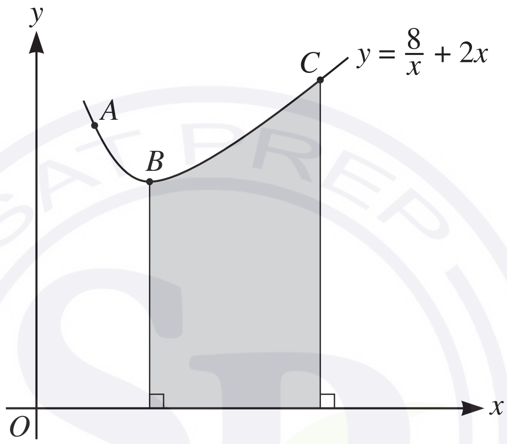
(iii) Sketch the graph of $y = \mathrm{f}(x)$. [2]

A function g is defined by $\mathrm{g} : x \mapsto 3\cos x - 2$ for $0 \leqslant x \leqslant k$.

(iv) State the maximum value of k for which g has an inverse. [1]

(v) Obtain an expression for $\mathrm{g}^{-1}(x)$. [2]

9



The diagram shows part of the curve $y = \frac{8}{x} + 2x$ and three points A, B and C on the curve with x-coordinates 1, 2 and 5 respectively.

(i) A point P moves along the curve in such a way that its x-coordinate increases at a constant rate of 0.04 units per second. Find the rate at which the y-coordinate of P is changing as P passes through A. [4]

(ii) Find the volume obtained when the shaded region is rotated through 360° about the x-axis. [6]

10 A curve has equation $y = 2x^2 - 3x$.

(i) Find the set of values of x for which $y > 9$. [3]

(ii) Express $2x^2 - 3x$ in the form $a(x + b)^2 + c$, where a, b and c are constants, and state the coordinates of the vertex of the curve. [4]

The functions f and g are defined for all real values of x by

$$\mathrm{f}(x) = 2x^2 - 3x \quad \text{and} \quad \mathrm{g}(x) = 3x + k,$$

where k is a constant.

(iii) Find the value of k for which the equation $\mathrm{gf}(x) = 0$ has equal roots. [3]

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MATHEMATICS **9709/13**

Paper 1 Pure Mathematics 1 **(P1)** **October/November 2013**

1 hour 45 minutes

7355250755

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

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1 Solve the inequality $x^2 - x - 2 > 0$. [3]

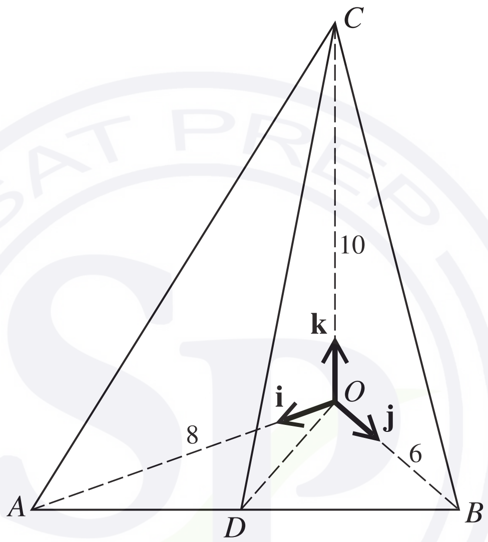
2 A curve has equation $y = \mathrm{f}(x)$. It is given that $\mathrm{f}'(x) = x^{-\frac{3}{2}} + 1$ and that $\mathrm{f}(4) = 5$. Find $\mathrm{f}(x)$. [4]

3 The point A has coordinates $(3, 1)$ and the point B has coordinates $(-21, 11)$. The point C is the mid-point of AB.

(i) Find the equation of the line through A that is perpendicular to $y = 2x - 7$. [2]

(ii) Find the distance AC. [3]

4



The diagram shows a pyramid $OABC$ in which the edge OC is vertical. The horizontal base OAB is a triangle, right-angled at O, and D is the mid-point of AB. The edges OA, OB and OC have lengths of 8 units, 6 units and 10 units respectively. The unit vectors $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ are parallel to $\overrightarrow{OA}$, $\overrightarrow{OB}$ and $\overrightarrow{OC}$ respectively.

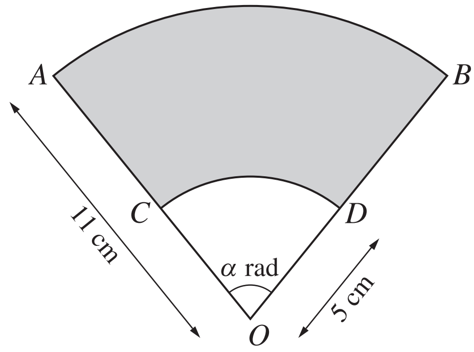
(i) Express each of the vectors $\overrightarrow{OD}$ and $\overrightarrow{CD}$ in terms of $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$. [2]

(ii) Use a scalar product to find angle ODC. [4]

5 **(a)** In a geometric progression, the sum to infinity is equal to eight times the first term. Find the common ratio. [2]

(b) In an arithmetic progression, the fifth term is 197 and the sum of the first ten terms is 2040. Find the common difference. [4]

6



The diagram shows sector OAB with centre O and radius 11 cm. Angle $AOB = \alpha$ radians. Points C and D lie on OA and OB respectively. Arc CD has centre O and radius 5 cm.

(i) The area of the shaded region $ABDC$ is equal to k times the area of the unshaded region OCD. Find k. [3]

(ii) The perimeter of the shaded region $ABDC$ is equal to twice the perimeter of the unshaded region OCD. Find the exact value of α. [4]

7 **(a)** Find the possible values of x for which $\sin^{-1}(x^2 - 1) = \frac{1}{3}\pi$, giving your answers correct to 3 decimal places. [3]

(b) Solve the equation $\sin(2\theta + \frac{1}{3}\pi) = \frac{1}{2}$ for $0 \leqslant \theta \leqslant \pi$, giving θ in terms of π in your answers. [4]

8 **(i)** Find the coefficient of x^8 in the expansion of $(x + 3x^2)^4$. [1]

(ii) Find the coefficient of x^8 in the expansion of $(x + 3x^2)^5$. [3]

(iii) Hence find the coefficient of x^8 in the expansion of $\left[1 + (x + 3x^2)\right]^5$. [4]

9 A curve has equation $y = \dfrac{k^2}{x+2} + x$, where k is a positive constant. Find, in terms of k, the values of x for which the curve has stationary points and determine the nature of each stationary point. [8]

10 The function f is defined by $\mathrm{f} : x \mapsto x^2 + 4x$ for $x \geqslant c$, where c is a constant. It is given that f is a one-one function.

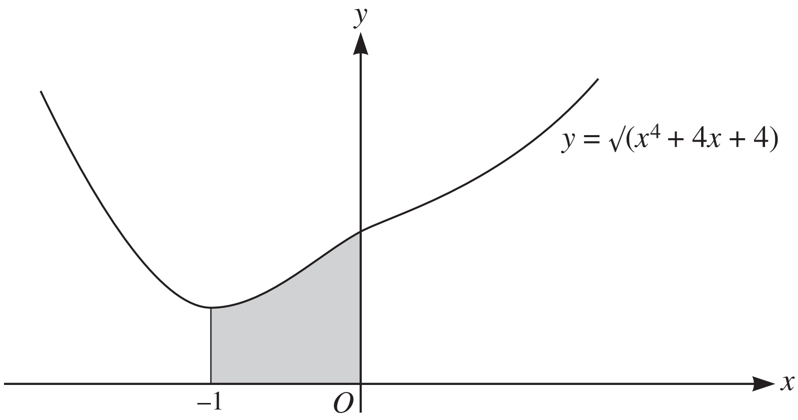
(i) State the range of f in terms of c and find the smallest possible value of c. [3]

The function g is defined by $\mathrm{g} : x \mapsto ax + b$ for $x \geqslant 0$, where a and b are positive constants. It is given that, when $c = 0$, $\mathrm{gf}(1) = 11$ and $\mathrm{fg}(1) = 21$.

(ii) Write down two equations in a and b and solve them to find the values of a and b. [6]

[Question 11 is printed on the next page.]

11



The diagram shows the curve $y = \sqrt{(x^4 + 4x + 4)}$.

(i) Find the equation of the tangent to the curve at the point (0, 2). [4]

(ii) Show that the x-coordinates of the points of intersection of the line $y = x + 2$ and the curve are given by the equation $(x + 2)^2 = x^4 + 4x + 4$. Hence find these x-coordinates. [4]

(iii) The region shaded in the diagram is rotated through 360° about the x-axis. Find the volume of revolution. [4]

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MATHEMATICS **9709/11**

Paper 1 Pure Mathematics 1 **(P1)** **May/June 2013**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

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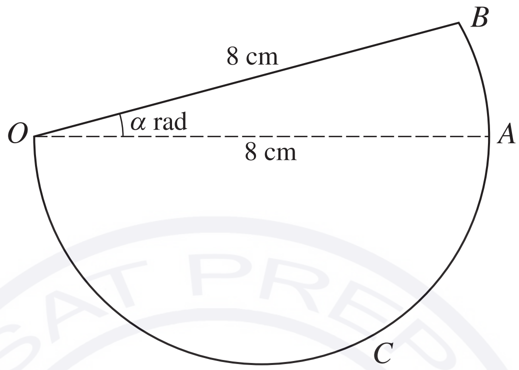
1 It is given that $f(x) = (2x - 5)^3 + x$, for $x \in \mathbb{R}$. Show that f is an increasing function. [3]

2 **(i)** In the expression $(1 - px)^6$, p is a non-zero constant. Find the first three terms when $(1 - px)^6$ is expanded in ascending powers of x. [2]

(ii) It is given that the coefficient of x^2 in the expansion of $(1 - x)(1 - px)^6$ is zero. Find the value of p. [3]

3



In the diagram, OAB is a sector of a circle with centre O and radius 8 cm. Angle BOA is α radians. OAC is a semicircle with diameter OA. The area of the semicircle OAC is twice the area of the sector OAB.

(i) Find α in terms of π. [3]

(ii) Find the perimeter of the complete figure in terms of π. [2]

4 The third term of a geometric progression is -108 and the sixth term is 32. Find

(i) the common ratio, [3]

(ii) the first term, [1]

(iii) the sum to infinity. [2]

5 **(i)** Show that $\dfrac{\sin\theta}{\sin\theta + \cos\theta} + \dfrac{\cos\theta}{\sin\theta - \cos\theta} \equiv \dfrac{1}{\sin^2\theta - \cos^2\theta}$. [3]

(ii) Hence solve the equation $\dfrac{\sin\theta}{\sin\theta + \cos\theta} + \dfrac{\cos\theta}{\sin\theta - \cos\theta} = 3$, for $0° \leqslant \theta \leqslant 360°$. [4]

6 Relative to an origin O, the position vectors of three points, A, B and C, are given by

$$\overrightarrow{OA} = \mathbf{i} + 2p\mathbf{j} + q\mathbf{k}, \quad \overrightarrow{OB} = q\mathbf{j} - 2p\mathbf{k} \quad \text{and} \quad \overrightarrow{OC} = -(4p^2 + q^2)\mathbf{i} + 2p\mathbf{j} + q\mathbf{k},$$

where p and q are constants.

(i) Show that $\overrightarrow{OA}$ is perpendicular to $\overrightarrow{OC}$ for all non-zero values of p and q. [2]

(ii) Find the magnitude of $\overrightarrow{CA}$ in terms of p and q. [2]

(iii) For the case where $p = 3$ and $q = 2$, find the unit vector parallel to $\overrightarrow{BA}$. [3]

7 A curve has equation $y = x^2 - 4x + 4$ and a line has equation $y = mx$, where m is a constant.

(i) For the case where $m = 1$, the curve and the line intersect at the points A and B. Find the coordinates of the mid-point of AB. [4]

(ii) Find the non-zero value of m for which the line is a tangent to the curve, and find the coordinates of the point where the tangent touches the curve. [5]

8 **(i)** Express $2x^2 - 12x + 13$ in the form $a(x + b)^2 + c$, where a, b and c are constants. [3]

(ii) The function f is defined by $f(x) = 2x^2 - 12x + 13$ for $x \geqslant k$, where k is a constant. It is given that f is a one-one function. State the smallest possible value of k. [1]

The value of k is now given to be 7.

(iii) Find the range of f. [1]

(iv) Find an expression for $f^{-1}(x)$ and state the domain of f^{-1}. [5]

9 A curve has equation $y = f(x)$ and is such that $f'(x) = 3x^{\frac{1}{2}} + 3x^{-\frac{1}{2}} - 10$.

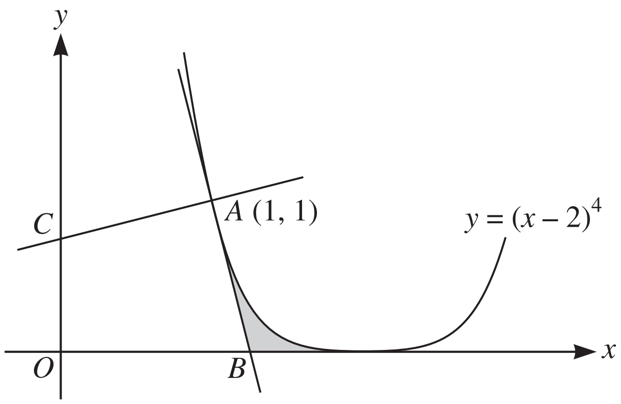
(i) By using the substitution $u = x^{\frac{1}{2}}$, or otherwise, find the values of x for which the curve $y = f(x)$ has stationary points. [4]

(ii) Find $f''(x)$ and hence, or otherwise, determine the nature of each stationary point. [3]

(iii) It is given that the curve $y = f(x)$ passes through the point $(4, -7)$. Find $f(x)$. [4]

[Question 10 is printed on the next page.]

10



The diagram shows part of the curve $y = (x - 2)^4$ and the point $A\,(1,\ 1)$ on the curve. The tangent at A cuts the x-axis at B and the normal at A cuts the y-axis at C.

(i) Find the coordinates of B and C. [6]

(ii) Find the distance AC, giving your answer in the form $\dfrac{\sqrt{a}}{b}$, where a and b are integers. [2]

(iii) Find the area of the shaded region. [4]

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* 2 7 4 0 8 5 2 1 2 8 *

MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **(P1)** **May/June 2013**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

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1 A curve is such that $\frac{dy}{dx} = \frac{6}{x^2}$ and (2, 9) is a point on the curve. Find the equation of the curve. [3]

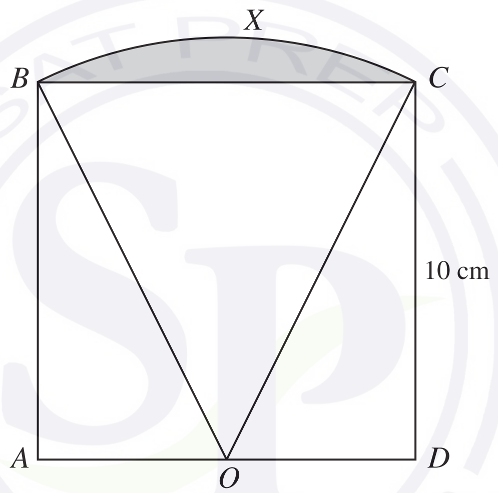
2 Find the coefficient of x^2 in the expansion of

(i) $\left(2x - \frac{1}{2x}\right)^6$, [2]

(ii) $(1 + x^2)\left(2x - \frac{1}{2x}\right)^6$. [3]

3 The straight line $y = mx + 14$ is a tangent to the curve $y = \frac{12}{x} + 2$ at the point P. Find the value of the constant m and the coordinates of P. [5]

4



The diagram shows a square $ABCD$ of side 10 cm. The mid-point of AD is O and BXC is an arc of a circle with centre O.

(i) Show that angle BOC is 0.9273 radians, correct to 4 decimal places. [2]

(ii) Find the perimeter of the shaded region. [3]

(iii) Find the area of the shaded region. [2]

5 It is given that $a = \sin\theta - 3\cos\theta$ and $b = 3\sin\theta + \cos\theta$, where $0° \leqslant \theta \leqslant 360°$.

(i) Show that $a^2 + b^2$ has a constant value for all values of θ. [3]

(ii) Find the values of θ for which $2a = b$. [4]

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6 Relative to an origin O, the position vectors of points A and B are given by

$$\overrightarrow{OA} = \mathbf{i} - 2\mathbf{j} + 2\mathbf{k} \quad \text{and} \quad \overrightarrow{OB} = 3\mathbf{i} + p\mathbf{j} + q\mathbf{k},$$

where p and q are constants.

(i) State the values of p and q for which $\overrightarrow{OA}$ is parallel to $\overrightarrow{OB}$. [2]

(ii) In the case where $q = 2p$, find the value of p for which angle BOA is $90°$. [2]

(iii) In the case where $p = 1$ and $q = 8$, find the unit vector in the direction of $\overrightarrow{AB}$. [3]

7 The point R is the reflection of the point $(-1, 3)$ in the line $3y + 2x = 33$. Find by calculation the coordinates of R. [7]

8 The volume of a solid circular cylinder of radius r cm is 250π cm^3.

(i) Show that the total surface area, S cm^2, of the cylinder is given by

$$S = 2\pi r^2 + \frac{500\pi}{r}.$$ [2]

(ii) Given that r can vary, find the stationary value of S. [4]

(iii) Determine the nature of this stationary value. [2]

9 A function f is defined by $\mathrm{f}(x) = \dfrac{5}{1 - 3x}$, for $x \geqslant 1$.

(i) Find an expression for $\mathrm{f}'(x)$. [2]

(ii) Determine, with a reason, whether f is an increasing function, a decreasing function or neither. [1]

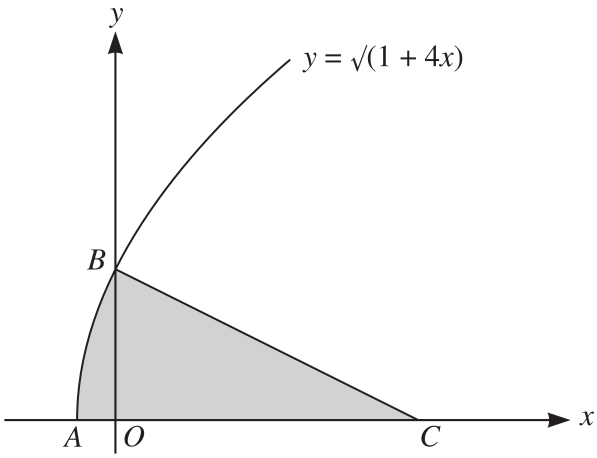
(iii) Find an expression for $\mathrm{f}^{-1}(x)$, and state the domain and range of f^{-1}. [5]

10 **(a)** The first and last terms of an arithmetic progression are 12 and 48 respectively. The sum of the first four terms is 57. Find the number of terms in the progression. [4]

(b) The third term of a geometric progression is four times the first term. The sum of the first six terms is k times the first term. Find the possible values of k. [4]

[Question 11 is printed on the next page.]

11



The diagram shows the curve $y = \sqrt{(1 + 4x)}$, which intersects the x-axis at A and the y-axis at B. The normal to the curve at B meets the x-axis at C. Find

(i) the equation of BC, [5]

(ii) the area of the shaded region. [5]

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UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education
Advanced Subsidiary Level and Advanced Level

MATHEMATICS **9709/13**

Paper 1 Pure Mathematics 1 **(P1)** **May/June 2013**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

9396345367

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

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JC13 06_9709_13/RP
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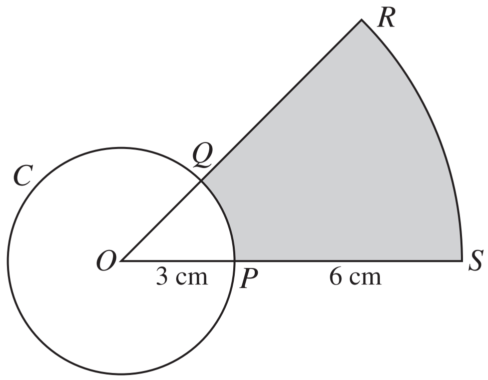
1 A curve is such that $\frac{dy}{dx} = \sqrt{(2x + 5)}$ and (2, 5) is a point on the curve. Find the equation of the curve. [4]

2



The diagram shows a circle C with centre O and radius 3 cm. The radii OP and OQ are extended to S and R respectively so that ORS is a sector of a circle with centre O. Given that $PS = 6$ cm and that the area of the shaded region is equal to the area of circle C,

(i) show that angle $POQ = \frac{1}{4}\pi$ radians, [3]

(ii) find the perimeter of the shaded region. [2]

3 **(i)** Express the equation $2\cos^2\theta = \tan^2\theta$ as a quadratic equation in $\cos^2\theta$. [2]

(ii) Solve the equation $2\cos^2\theta = \tan^2\theta$ for $0 \leqslant \theta \leqslant \pi$, giving solutions in terms of π. [3]

4 **(i)** Find the first three terms in the expansion of $(2 + ax)^5$ in ascending powers of x. [3]

(ii) Given that the coefficient of x^2 in the expansion of $(1 + 2x)(2 + ax)^5$ is 240, find the possible values of a. [3]

5 **(i)** Sketch, on the same diagram, the curves $y = \sin 2x$ and $y = \cos x - 1$ for $0 \leqslant x \leqslant 2\pi$. [4]

(ii) Hence state the number of solutions, in the interval $0 \leqslant x \leqslant 2\pi$, of the equations

(a) $2\sin 2x + 1 = 0$, [1]

(b) $\sin 2x - \cos x + 1 = 0$. [1]

6 The non-zero variables x, y and u are such that $u = x^2y$. Given that $y + 3x = 9$, find the stationary value of u and determine whether this is a maximum or a minimum value. [7]

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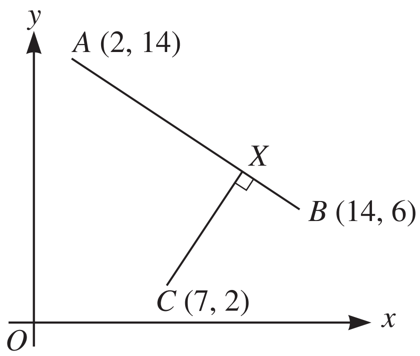
7

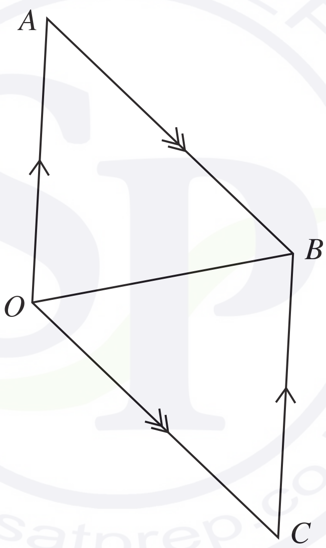


The diagram shows three points $A\,(2,\ 14)$, $B\,(14,\ 6)$ and $C\,(7,\ 2)$. The point X lies on AB, and CX is perpendicular to AB. Find, by calculation,

(i) the coordinates of X, [6]

(ii) the ratio $AX : XB$. [2]

8



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The diagram shows a parallelogram $OABC$ in which

$$\overrightarrow{OA} = \begin{pmatrix} 3 \\ 3 \\ -4 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OB} = \begin{pmatrix} 5 \\ 0 \\ 2 \end{pmatrix}.$$

(i) Use a scalar product to find angle BOC. [6]

(ii) Find a vector which has magnitude 35 and is parallel to the vector $\overrightarrow{OC}$. [2]

9 **(a)** In an arithmetic progression, the sum, S_n, of the first n terms is given by $S_n = 2n^2 + 8n$. Find the first term and the common difference of the progression. [3]

(b) The first 2 terms of a geometric progression are 64 and 48 respectively. The first 3 terms of the geometric progression are also the 1st term, the 9th term and the nth term respectively of an arithmetic progression. Find the value of n. [5]

10 The function f is defined by $f : x \mapsto 2x + k,\ x \in \mathbb{R}$, where k is a constant.

(i) In the case where $k = 3$, solve the equation $ff(x) = 25$. [2]

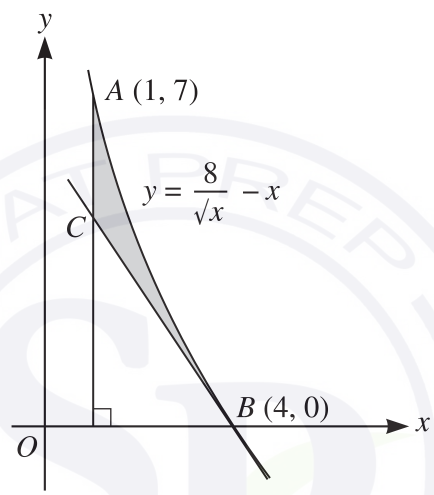
The function g is defined by $g : x \mapsto x^2 - 6x + 8,\ x \in \mathbb{R}$.

(ii) Find the set of values of k for which the equation $f(x) = g(x)$ has no real solutions. [3]

The function h is defined by $h : x \mapsto x^2 - 6x + 8,\ x > 3$.

(iii) Find an expression for $h^{-1}(x)$. [4]

11



The diagram shows part of the curve $y = \frac{8}{\sqrt{x}} - x$ and points $A\,(1, 7)$ and $B\,(4, 0)$ which lie on the curve. The tangent to the curve at B intersects the line $x = 1$ at the point C.

(i) Find the coordinates of C. [4]

(ii) Find the area of the shaded region. [5]

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