CAMBRIDGE
International Education

Cambridge International AS & A Level

MATHEMATICS **9709/51**

Paper 5 Probability & Statistics 1 **October/November 2024**

MARK SCHEME

Maximum Mark: 50

Published

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WWW	Without Wrong Working
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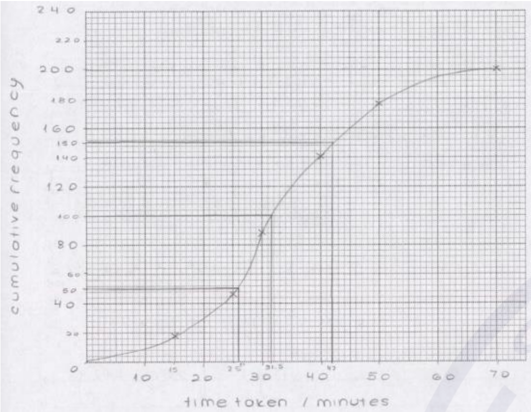
Question	Answer	Marks	Guidance
1(a)	**Method 1**		
	[P(X < 8) =] $1-\left(\frac{5}{6}\right)^7$	**M1**	$1 - b^d$, $b = \frac{5}{6}, \frac{1}{6}$ d = 7, 8. $1 - c^e - (1 - c) \times c^{e-1}$, $c = \frac{5}{6}, \frac{1}{6}$ e = 8,9.
	= 0.721	**A1**	0.720918… $\frac{201811}{279936}$. If M0 scored, **SC B1** for 0.7209… or $\frac{201811}{279936}$ only.
	Method 2		
	[P(X < 8) =] $\frac{1}{6}+\left(\frac{5}{6}\right)\left(\frac{1}{6}\right)+\left(\frac{5}{6}\right)^2\left(\frac{1}{6}\right)+\left(\frac{5}{6}\right)^3\left(\frac{1}{6}\right)+\left(\frac{5}{6}\right)^4\left(\frac{1}{6}\right)+\left(\frac{5}{6}\right)^5\left(\frac{1}{6}\right)+\left(\frac{5}{6}\right)^6\left(\frac{1}{6}\right)$	**M1**	$a+ba+b^2a+b^3a+b^4a+b^5a+b^6a\left[+b^7a\right]$. $a=\frac{1}{6},\frac{5}{6}$ $a+b=1$.
	= 0.721	**A1**	0.720918… $\frac{201811}{279936}$. If M0 scored, **SC B1** for 0.7209… or $\frac{201811}{279936}$ only.
		2	

Question	Answer	Marks	Guidance
1(b)	$\left(\frac{5}{6}\right)^6 \times \left(\frac{1}{6}\right)^2 \times 7$	**M1**	$\left(\frac{5}{6}\right)^6 \times \left(\frac{1}{6}\right)^2 \times d$ d an integer ≥ 1, no inappropriate addition.
	0.0651	**A1**	$0.0651 \leqslant p < 0.06512$.
		2	

Question	Answer	Marks	Guidance
2(a)	[Probs $6k, 3k, 2k, 6k, 11k$ so $28k = 1$,] $k = \frac{1}{28}$	**B1**	k must be identified
	<table><tr><td>x</td><td>–2</td><td>–1</td><td>0</td><td>2</td><td>3</td></tr><tr><td>P($X = x$)</td><td>$\frac{6}{28}$ 0.2143</td><td>$\frac{3}{28}$ 0.1071</td><td>$\frac{2}{28}$ 0.07143</td><td>$\frac{6}{28}$ 0.2143</td><td>$\frac{11}{28}$ 0.3929</td></tr></table>	**M1**	Table with correct outcomes and 2 correct probabilities. FT substituting *their* k correctly into formula, with $0 < p < 1$. No additional x values unless probability 0. Condone in terms of k of the form $\frac{6k}{28}$ or $6k$.
		A1	Fully correct. Decimal answers to at least 3 sig figures, condone not summing exactly to 1.
		3	

Question	Answer	Marks	Guidance
2(b)	$E(X) = \left[-2\times\frac{6}{28}+-1\times\frac{3}{28}+\left[0\times\frac{2}{28}\right]+2\times\frac{6}{28}+3\times\frac{11}{28}\right]$ $\frac{1}{28}(-12-3+12+33)\left[=\frac{15}{14}\right]$	**M1**	Accept unsimplified expression. May be calculated in the variance. FT *their* table with 5 probabilities $0 < p < 1$ that sum to 1.
	$\mathrm{Var(X)} = \frac{6\times(-2)^2+3\times(-1)^2+6\times2^2+11\times3^2}{28}$ $-\textit{their}\left(\frac{15}{14}\right)^2$	**M1**	Appropriate variance formula using *their* $(E(X))^2$ value. FT *their* table with at least 4 probabilities $0 < p < 1$, that may not sum to 1.
	$= 4.21,\ 4\frac{41}{196}$	**A1**	Condone $\frac{825}{196}$. If one or both M marks not awarded, **SC B1** for correct answer WWW.
		3	

Question	Answer	Marks	Guidance
3(a)		**M1**	At least 4 points plotted within tolerance at upper bounds. Linear cf scale $0 \leqslant \text{cf} \leqslant 200$ and linear time scale $0 \leqslant t \leqslant 70$, with at least 3 values identified on each. Minimum scale uses at least ½ the grid.
		A1	All points plotted correctly. Curve drawn and joined to (0, 0). Axes labelled cumulative frequency (cf), time (t) and minutes (min) – or a suitable title.
		2	
3(b)	Median = 33	**B1 FT**	Must be identified. Evidence of use of graph must be seen. Strict FT ± ½ square on time axis.
	[IQR =] 42 – 26	**M1**	$41 \leqslant \text{UQ} \leqslant 43 - 25 < \text{LQ} \leqslant 27$. If outside of range FT ± ½ square on time axis.
	16	**A1 FT**	
		3	

Question	Answer	Marks	Guidance
3(c)	Midpoint: 7.5, 20, 27.5, 35, 45, 60 Frequency: 18, 28, 42, 52, 36, 24	**B1**	At least 5 correct midpoints or 5 correct frequencies seen.
	Mean = $\dfrac{18\times7.5+28\times20+42\times27.5+52\times35+36\times45+24\times60}{200}$	**M1**	Correct mean formula using *their* 6 midpoints (must be within class, not upper bound, not lower bound) condone 1 error and *their* 6 frequencies (not cumulative frequencies).
	$= 33.65,\ 33\dfrac{13}{20}$	**A1**	Accept 33.7, not $\dfrac{673}{20}$.
		3	

Question	Answer	Marks	Guidance
4(a)	P(HRR) = $\dfrac{1}{4}\times\dfrac{4}{6}\times\dfrac{4}{6}=\dfrac{16}{144},\dfrac{4}{36}$ P(TRR) = $\dfrac{3}{4}\times\dfrac{3}{7}\times\dfrac{2}{6}=\dfrac{18}{168},\dfrac{3}{28}$ P(HBB) = $\dfrac{1}{4}\times\dfrac{2}{6}\times\dfrac{2}{6}=\dfrac{4}{144},\dfrac{1}{36}$ P(TBB) = $\dfrac{3}{4}\times\dfrac{4}{7}\times\dfrac{3}{6}=\dfrac{36}{168},\dfrac{6}{28},\dfrac{3}{14}$	**B1**	2 clearly identified unsimplified probabilities from P(HRR), P(TRR), P(HBB) and P(TBB) correct.
	$\dfrac{4}{36}+\dfrac{3}{28}+\dfrac{1}{36}+\dfrac{6}{28}$	**M1**	Sum of 4 correct scenarios, may be identified by the unsimplified probability calculations.
	$=\dfrac{29}{63}$ or 0.460	**A1**	(0.460317… to at least 3SF).
		3	

Question	Answer	Marks	Guidance
4(b)	$\left[\mathrm{P}(\mathrm{T}\mid\mathrm{BB}) = \dfrac{\mathrm{P}(\mathrm{T}\cap\mathrm{BB})}{\mathrm{P}(\mathrm{BB})}\right] = \left[\dfrac{\frac{3}{4}\times\frac{4}{7}\times\frac{3}{6}}{\frac{1}{36}+\frac{6}{28}}\right]$	M1	$\frac{3}{4}\times\frac{4}{7}\times\frac{3}{6}, \frac{36}{168}$ oe, $\frac{3}{14}$, 0.2142857… seen as numerator of a fraction, accept unsimplified, FT *their* P(TBB) from **4(a)**
	$\frac{3}{14}\div\frac{61}{252}$ or $\dfrac{\frac{3}{14}}{\frac{61}{252}}$	M1	*their* $\frac{1}{36}$ + *their* $\frac{6}{28}$ FT from **4(a)** or correct, 0.24206…, seen as denominator of a fraction, accept unsimplified.
	$\frac{54}{61}$ or 0.885	A1	Accept 0.8852589… rounded to at least 3SF. If one or both Ms not awarded, **SC B1** for correct final answer WWW.
		3	

Question	Answer	Marks	Guidance
5(a)	$\mathrm{P}(83 < X < 95) = \mathrm{P}(\frac{83-90}{8} < Z < \frac{95-90}{8})$	M1	Using ± standardisation formula with 90, 8 and *either* 83 *or* 95. Not σ^2, not σ, no continuity correction.
	$= \mathrm{P}(-0.875 < Z < 0.625)$	A1	Both ±0.875 OE and ±0.625 OE seen. If M0 scored, **SC B1** for both ±0.875 and ±0.625 seen
	$[\ \Phi(0.625) + \Phi(0.875) - 1\ ]$ = 0.7340 + 0.8092 – 1	M1	Calculating the appropriate probability area, leading to their final probability. Expect final answer > 0.5.
	= 0.543	A1	0.5432, $0.543 \leqslant p < 0.5435$. Only dependent on the 2nd M mark.
		4	

Question	Answer	Marks	Guidance
5(b)	[Mean $=160\times0.6=$]96 [Var $=160\times0.6\times0.4=$]38.4	B1	96 and 38.4 seen, allow unsimplified. May be seen in the standardisation formula. $\left(\frac{8\sqrt{15}}{5}, 6.19677\ldots \text{to at least } 4 \text{ SF}\right.$ implies correct variance$\left.\right)$ Withold mark if variance clearly identified as standard deviation, condone N(96, $\sqrt{38.4}$) if standardisation formula correct or variance/standard deviation correctly stated as well.
	$P(X < 105) = P(Z < \frac{104.5-96}{\sqrt{38.4}})$ [$P(Z < 1.372) = \Phi(1.372)$]	M1	Substituting *their* 96 and *their* 38.4 into the ±standardising formula (any number for 104.5), condone σ^2 or $\sqrt{\sigma}$.
		M1	Use continuity correction 104.5 or 105.5 in *their* standardisation formula. Note: $\frac{\pm8.5}{\sqrt{38.4}}$ or $\frac{\pm8.5}{6.197}$ seen gains M2 BOD.
		M1	Appropriate area Φ, from final process, must be a probability. Expect final answer > 0.5.
	= 0.915[0]	A1	$0.9149 \leqslant p \leqslant 0.915$. If one or more M marks not scored, **SC B1** for $0.9149 \leqslant p \leqslant 0.915$.

Question	Answer	Marks	Guidance
6(a)	$\frac{182.7-\mu}{\sigma}=1.282$	B1	1.282 or – 1.282 seen, CAO (critical value).
	$\frac{162.5-\mu}{\sigma}=-0.253$	B1	$-0.2535 < z \leqslant -0.253$ or $0.253 \leqslant z < 0.2535$ seen.
		M1	One standardisation formula, not σ^2, or $\sqrt{\sigma}$, with 182.7 or 162.5 substituted correctly equated to a z value (not 0.9, 0.1, 0.8159, 0.5398, 0.4, 0.6, 0.6554, 0.7257, …).
	Solve, obtaining values for μ and σ	M1	Either a single expression with one variable eliminated formed or two expressions with both variables on the same side seen with at least one variable value stated.
	$\mu=165.8$, $\sigma=13.2$	A1	Answers must be to at least 1 DP (context).
		5	

Question	Answer	Marks	Guidance
6(b)	**Method 1**		
	[P(X < 8) = 1 – P(8, 9, 10) =] 1 – ($^{10}C_8\,(0.6)^8(0.4)^2 + {}^{10}C_9\,(0.6)^9(0.4)^1 + (0.6)^{10}$) [= 1 – (0.12093 + 0.040311 + 0.0060466)]	**M1**	One term $^{10}C_x\,(p)^x(1-p)^{10-x}$. With $0 < p < 1, x \neq 0$ or 10.
		A1	Correct unsimplified expression. Allow 10 for $^{10}C_9$. Condone omission of last bracket only. If both brackets omitted in unsimplified expression allow recovery for final stated calculation of 1 – 0.1673 or final answer WRT 0.8327.
	= 0.833	**B1**	0.8327 < $p \leqslant$ 0.833.
	Method 2		
	$(0.4)^{10} + {}^{10}C_1\,(0.6)^1(0.4)^9 + {}^{10}C_2\,(0.6)^2(0.4)^8 + {}^{10}C_3\,(0.6)^3(0.4)^7 + {}^{10}C_4\,(0.6)^4(0.4)^6 + {}^{10}C_5\,(0.6)^5(0.4)^5 + {}^{10}C_6\,(0.6)^6(0.4)^4 + {}^{10}C_7\,(0.6)^7(0.4)^3$ $\left[1.0486\times10^{-4} + 1.5729\times10^{-3} + \ldots + 0.21499\right]$	**M1**	One term $^{10}C_x\,(p)^x(1-p)^{10-x}$. With $0 < p < 1, x \neq 0$ or 10.
		A1	Correct unsimplified expression.
	= 0.833	**B1**	0.8327 < $p \leqslant$ 0.833.
		3	

Question	Answer	Marks	Guidance
7(a)	$\dfrac{8!}{2!2!}$	**M1**	$\dfrac{k!}{2!m!}$ $k = 7$ or 8, $m = 1, 2$.
	$= 10080$	**A1**	
7(b)	**Method 1** Number of ways with no restriction on Es – ways with Es together		
	$\dfrac{7!}{2!2!} - \dfrac{6!}{2!}$	**M1**	$\dfrac{7!}{2!2!} - r$, r integer > 1.
	[= 1260 – 360]	**M1**	$s - \dfrac{6!}{2!}$, s integer > 360.
	$= 900$	**A1**	
	Method 2 T ^ ^ ^ ^ ^ T with Es inserted in gaps		
	$\dfrac{5!}{2!} \times \dfrac{6 \times 5}{2}$ or $\dfrac{5!}{2!} \times {}^6C_2$	**M1**	$t \times \dfrac{6 \times 5}{2}$ or $t \times {}^6C_2$, t an integer > 1.
	[= 60 × 15]	**M1**	$\dfrac{5!}{2!} \times u$, u an integer > 1.
	$=900$	**A1**	
		3	

Question	Answer	Marks	Guidance
7(c)	**Method 1 – addition**		
	T E _ _ = $^2C_1 \times {}^2C_1 \times {}^5C_2 = 40$	**B1**	Either identified or correct unsimplified expression, either alone or in an addition.
	T E E _ = $^2C_1 \times {}^2C_2 \times {}^5C_1 = 10$	**B1**	Either identified or correct unsimplified expression, either alone or in an addition.
	Probability $\dfrac{(40+10)}{^9C_4}$	**M1**	$\dfrac{a}{^9C_4}$, a an integer < 126. Denominator value must be seen as $^9C_4$ somewhere.
	$\left[\text{Percentage} = \dfrac{50}{126} \times 100\right] = 39.7\%$	**A1**	$39.68 \leqslant$ percentage $\leqslant 39.7$.
	Method 2 – subtraction (total arrangements with 1 T – number of arrangements with 1T 0 E)		
	T ^ ^ ^ = $^2C_1 \times {}^7C_3 = 70$	**B1**	Either identified or correct unsimplified expression, either alone or in a subtraction.
	T * * * = $^2C_1 \times {}^5C_3 = 20$	**B1**	Either identified or correct unsimplified expression, either alone or in a subtraction.
	Probability $\dfrac{(70-20)}{^9C_4}$	**M1**	$\dfrac{a}{^9C_4}$, a an integer < 126. Denominator value must be seen as $^9C_4$ somewhere.
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		1	
1(b)	$[\mathrm{P}(N\|X) =]\ \frac{12}{50}$	**B1**	0.24, $\frac{6}{25}$ OE.
		1	
1(c)	$\mathrm{P}(N \cap X) = \frac{12}{120},\ P(N) = \frac{40}{120},\ P(X) = \frac{50}{120}$ $\frac{40}{120} \times \frac{50}{120} = \frac{5}{36}, 0.138[8\ldots] \neq \frac{12}{120}, 0.1$ Not independent	**B1**	$P(N), P(X)$ and $P(N \cap X)$ or $P(N \text{ and } X)$ notation seen and equated to the values for $P(N), P(X)$ and $P(N \cap X)$ or $P(N \text{ and } X)$. Calculation stated and evaluated. Not independent clearly stated. $\frac{5}{36} \neq \frac{12}{120}$ does not need to be stated. All values OE. Condone consistent use of *A*, *B* etc. If values for P(*N*), P(*X*) stated, accept $\mathrm{P}(N) \times \mathrm{P}(X) = \frac{5}{36}$.
		1	

Question	Answer	Marks	Guidance
1(d)	**Method 1**		
	[P(0, 1, 2) =] $(0.85)^8 + {}^8C_1(0.85)^7(0.15) + {}^8C_2(0.85)^6(0.15)^2$ [= 0.27249 + 0.38469 + 0.23760]	**M1**	One term of form ${}^8C_x\,(p)^x(1-p)^{8-x}$. With $0 < p < 1,\ x \neq 0$ or 8.
		A1	Correct unsimplified expression, no terms omitted leading to final answer.
	= 0.895	**B1**	$0.8945 \leqslant p \leqslant 0.895$.
	Method 2		
	[P(0, 1, 2) =] $1 - \{{}^8C_3(0.85)^5(0.15)^3 + {}^8C_4(0.85)^4(0.15)^4 + {}^8C_5(0.85)^3(0.15)^5$ $+ {}^8C_6(0.85)^2(0.15)^6 + {}^8C_7(0.85)(0.15)^7 + (0.15)^8\}$	**M1**	One term of form ${}^8C_x\,(p)^x(1-p)^{8-x}$ With $0 < p < 1, x \neq 0$ or 8.
		A1	Correct unsimplified expression. Condone omission of final bracket '}'. If other brackets omitted, allow recovery if 1 – 0.1052[…] seen.
	= 0.895	**B1**	$0.8945 \leqslant p \leqslant 0.895$.
		3	

Question	Answer	Marks	Guidance
2(a)	$\left[\frac{9!}{2!}=\right]181440$	**B1**	Exact value must be seen. CAO.
		1	

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td>2(b)</td><td colspan="3">Method 1</td></tr>
<tr><td></td><td>
<table>
<tr><td>Scenario number of letters between As</td><td></td><td></td></tr>
<tr><td>A A [^ ^ ^ ^ ^ ^ ^]</td><td>7! × 8 or 8!</td><td>[= 40320]</td></tr>
<tr><td>A ^ A [^ ^ ^ ^ ^ ^]</td><td>7! × 7</td><td>[= 35280]</td></tr>
<tr><td>A ^ ^ A [^ ^ ^ ^ ^]</td><td>7! × 6</td><td>[= 30240]</td></tr>
</table>
Total: 7! × (8 + 7 + 6)</td><td>B1

M1</td><td>Correct outcome/value for 1 identified scenario, accept unsimplified.

Add values of 3 correct scenarios, no incorrect/repeated scenarios.</td></tr>
<tr><td></td><td>= 105 840</td><td>A1</td><td>If M1 not awarded, SC B1 for 105840 WWW.</td></tr>
<tr><td></td><td colspan="3">Method 2</td></tr>
<tr><td></td><td>
<table>
<tr><td>Scenario number of letters between As</td><td></td><td></td></tr>
<tr><td>A A [^ ^ ^ ^ ^ ^ ^]</td><td>8!</td><td>[= 40320]</td></tr>
<tr><td>A ^ A [^ ^ ^ ^ ^ ^]</td><td>$^7P_1 \times 7!$ or $^7C_1 \times 7!$</td><td>[= 35280]</td></tr>
<tr><td>A ^ ^ A [^ ^ ^ ^ ^]</td><td>$^7P_2 \times 6!$ or $^7C_2 \times 2 \times 6!$</td><td>[= 30240]</td></tr>
</table>
Total: $8! + {^7P_1} \times 7! + {^7P_2} \times 6!$
or $8! + {^7C_1} \times 7! + {^7C_2} \times 2 \times 6!$</td><td>B1

M1</td><td>Correct outcome/value for 1 identified scenario, accept unsimplified.

Add values of 3 correct scenarios, no incorrect/repeated scenarios.</td></tr>
<tr><td></td><td>=105 840</td><td>A1</td><td>If M1 not awarded, SC B1 for 105840 WWW.</td></tr>
</table>

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td rowspan="5">2(b)</td><td colspan="3">Method 3</td></tr>
<tr><td rowspan="2">
<table>
<tr><td>Scenario number of letters between As</td><td></td><td></td></tr>
<tr><td>A ^ ^ ^ A [^ ^ ^ ^]</td><td>7! × 5</td><td>[= 25200]</td></tr>
<tr><td>A ^ ^ ^ ^ A [^ ^ ^]</td><td>7! × 4</td><td>[= 20160]</td></tr>
<tr><td>A ^ ^ ^ ^ ^ A [^ ^]</td><td>7! × 3</td><td>[= 15120]</td></tr>
<tr><td>A ^ ^ ^ ^ ^ ^ A [^]</td><td>7! × 2</td><td>[= 10080]</td></tr>
<tr><td>A ^ ^ ^ ^ ^ ^ ^ A</td><td>7! [× 1]</td><td>[= 5040]</td></tr>
</table>
Total = $\frac{9!}{2!} - 7! \times (5+4+3+2+1)$
</td><td>B1</td><td>Correct outcome/value for 1 identified scenario, accept unsimplified.</td></tr>
<tr><td>M1</td><td>their 2(a), or correct, subtract values of 5 correct scenarios, no incorrect/repeated scenarios.</td></tr>
<tr><td>=105 840</td><td>A1</td><td>If M1 not awarded, SC B1 for 105840 WWW.</td></tr>
<tr><td></td><td>3</td><td></td></tr>
</table>

Question	Answer	Marks	Guidance
3(a)	<table><tr><td>x</td><td>0</td><td>2</td><td>4</td><td>6</td><td>8</td><td>10</td><td>12</td></tr><tr><td>P($X = x$)</td><td>$\frac{3}{12}$, $\frac{1}{4}$ 0.25</td><td>$\frac{2}{12}$, $\frac{1}{6}$ 0.167</td><td>$\frac{2}{12}$</td><td>$\frac{2}{12}$</td><td>$\frac{1}{12}$ 0.0833</td><td>$\frac{1}{12}$</td><td>$\frac{1}{12}$</td></tr></table>	**B1**	Table with correct x values and at least 2 correct probabilities. Condone any additional x values if probability stated as 0.
		B1	Four more probabilities correctly linked to the correct x value, need not be in table, accept unsimplified.
		B1	7 correct probabilities linked with correct outcomes, may not be in table. Decimals correct to at least 3 SF. **SC B1** 7 or more probabilities summing to 1 placed in a probability distribution table.
		3	

Question	Answer	Marks	Guidance
3(b)	[E(X) =] $[0]+2\times\frac{2}{12}+4\times\frac{2}{12}+6\times\frac{2}{12}+8\times\frac{1}{12}+10\times\frac{1}{12}+12\times\frac{1}{12}$ $\left[[0]+\frac{4}{12}+\frac{8}{12}+\frac{12}{12}+\frac{8}{12}+\frac{10}{12}+\frac{12}{12}=\frac{54}{12}=4.5\right]$	**M1**	Accept unsimplified expression. May be calculated in variance. Accept $\frac{1}{3}+\frac{2}{3}+1+\frac{2}{3}+\frac{5}{6}+1$ OE for the M mark FT *their* table with 7 or 8 probabilities summing to $0.999 \leqslant total \leqslant 1$ $(0 < p < 1)$. FT acceptable at the bold partially evaluated stage.
	[Var(X) =] $[0]+2^2\times\frac{2}{12}+4^2\times\frac{2}{12}+6^2\times\frac{2}{12}+8^2\times\frac{1}{12}+10^2\times\frac{1}{12}+12^2\times\frac{1}{12}-(\textit{their}\ 4.5)^2$ $\left[\frac{[0]+2\times4+2\times16+2\times36+[1\times]64+[1\times]100+[1\times]144}{12}-\frac{81}{4}\right]$ $\left[=35-4.5^2\right]$	**M1**	Appropriate variance formula using *their* $(\mathrm{E}(X))^2$ value. FT *their* table with 6 or more probabilities. $(0 < p < 1)$ which need not sum to 1 or with an expression no more evaluated than shown. FT acceptable at the bold partially evaluated stage with *their* probabilities.
	$= 14.75,\ 14\frac{3}{4}$	**A1**	CAO Accept $\frac{59}{4}$. If either or both M marks not awarded, **SC B1** for correct answer WWW
		3	

Question	Answer	Marks	Guidance
4(a)	[P(X < 18.2) = P(Z <] $\frac{18.2-19.8}{2.4}$)	**M1**	Use of ± standardisation formula with 18.2, 19.8 and 2.4 substituted appropriately, no continuity correction. Condone $\sigma^2\left(2.4^2\right)$ or $\sqrt{\sigma}\left(\sqrt{2.4}\right)$.
	$=\left[\Phi(-0.6667)=1-\Phi(0.6667)\right]$ = 1 – 0.7477	**M1**	Calculating the appropriate probability areas (leading to their final answer, expect < 0.5). Note: 0.432 is z-value of 0.667 so is not an appropriate probability area (M0).
	= 0.252(3)	**A1**	AWRT 0.252 SOI, accept 0.2525. If one or both M marks not awarded, **SC B1** for AWRT 0.252 SOI, accept 0.2525.
	[Expected number = 0.2523 $\times 450 = 113.5$,] = 113 or 114	**B1FT**	Strict FT *their* at least 4 figure probability × 450. (Check with calculator). One integer answer only. No indication of 'approximation', e.g. $\approx, \cong, about$.
		4	
4(b)	[P(X >25.5) = 0.26 so P(Z > $\frac{25.5-23.4}{\sigma}$) = 0.26] $\frac{25.5-23.4}{\sigma}=0.643$	**B1**	$0.643 \leqslant z \leqslant 0.6435$ or $-0.6435 \leqslant z \leqslant -0.643$ seen.
		M1	± standardisation formula with 25.5, 23.4, σ equating to a z-value, (not 1 – *their* z-value…). Condone continuity correction ±0.05, not σ^2, not $\sqrt{\sigma}$.
	$\sigma = 3.27$	**A1**	$3.26 \leqslant \sigma \leqslant 3.27$. Do not award for improper fractions.
		3	

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td rowspan="3">5(a)</td><td>$^{10}C_4 \times {}^6C_2 \times {}^5C_1$</td><td>**M1**</td><td>$^{10}C_a \times {}^6C_b \times {}^5C_c$, $a + b + c = 7$, a,b,c integers.
No other terms present but condone × 6 or × 3!.</td></tr>
<tr><td>[= 210 ×15×5] = 15750</td><td>**A1**</td><td></td></tr>
<tr><td></td><td>**2**</td><td></td></tr>
<tr><td rowspan="5">5(b)</td><td rowspan="3">
<table>
<tr><td>Scenario</td><td>V G P</td><td></td><td></td></tr>
<tr><td>VVVVG</td><td>4 1 0</td><td>$^{10}C_4 \times {}^6C_1$ [$\times {}^5C_0$]</td><td>[1260]</td></tr>
<tr><td>VVVGG</td><td>3 2 0</td><td>$^{10}C_3 \times {}^6C_2$ [$\times {}^5C_0$]</td><td>[1800]</td></tr>
<tr><td>VVGGG</td><td>2 3 0</td><td>$^{10}C_2 \times {}^6C_3$ [$\times {}^5C_0$]</td><td>[900]</td></tr>
<tr><td>VVVGP</td><td>3 1 1</td><td>$^{10}C_3 \times {}^6C_1 \times {}^5C_1$</td><td>[3600]</td></tr>
<tr><td>VVGGP</td><td>2 2 1</td><td>$^{10}C_2 \times {}^6C_2 \times {}^5C_1$</td><td>[3375]</td></tr>
</table>
</td><td>**M1**</td><td>One product using 2 or 3 combinations with upper numbers correct and lower numbers summing to 5 and linked to a correct identified scenario.
Condone the consistent use of permutations.</td></tr>
<tr><td>**B1**</td><td>2 identified outcomes evaluated accurately, accept unsimplified.</td></tr>
<tr><td>**M1**</td><td>Add values of 5 correct scenarios, no incorrect/repeated scenarios.</td></tr>
<tr><td>Total = 10935</td><td>**A1**</td><td>If either or both Ms not awarded,
SC B1 for 10935 WWW</td></tr>
<tr><td></td><td>**4**</td><td></td></tr>
</table>

Question	Answer	Marks	Guidance
6(a)	Falcons | | Kites 9 8 | 3 | 2 8 6 4 2 | 4 | 0 0 5 7 8 9 8 6 2 1 0 | 5 | 2 4 8 9 9 9 4 | 6 | 0 1 3 5 6 | 7 | Key: 1|5|4 means 51 minutes for Falcons and 54 minutes for Kites	**B1**	Correct stem, ignore extra values (not in reverse, not split). If a split stem-and-leaf plot is used (i.e. stem values are repeated) the remaining B marks are available.
		B1	Correct Falcons labelled on left, leaves in order from right to left and lined up vertically, no commas or other punctuation.
		B1	Correct Kites labelled on same diagram, leaves in order and lined up vertically, no commas or other punctuation. Penalise each error only once in question. E.g. commas in both sets of data.
		B1	Correct key, for their diagram, need both teams names and ‘mins’ at least once here, or in leaf headings or title. <u>If 2 separate diagrams drawn </u>max marks **B1** if both stems correct. **B1** if Falcons correct to the left of the stem. **B1** if both keys correct including ‘mins’ and team name.
		4	
6(b)	Median = 51 [minutes]	**B1**	Accept **Q2**, **must** be identified.
	[IQR =] 59 – 44	**M1**	$58 \leqslant UQ \leqslant 64 - 42 \leqslant LQ \leqslant 46$. Implied if both quartile values are stated and an appropriate IQR calculated accurately.
	= 15 [minutes]	**A1**	WWW
		3	

Question	Answer	Marks	Guidance
6(c)	$\left[\text{mean} = \frac{792+783}{30} = \frac{1575}{30} = \right]$ 52.5, $52\frac{1}{2}$, $\frac{105}{2}$	**B1**	$\frac{1575}{30}$ is not acceptable for this mark.
	$\sum 'x^2' = 85727$ $sd^2 = [\text{Variance} =]\left[\frac{(43504+42223)}{30} - \left(\frac{792+783}{30}\right)^2\right]$ $\frac{85727}{30} - \left(\frac{1575}{30}\right)^2$ [= 101.3167]	**M1**	Accept unsimplified variance formula. FT *their* mean. Ignore any square root leading to sd for this mark.
	$\sigma = \left(\sqrt{101.3167} = \right) 10.1$	**A1**	AWRT. Must be identified, e.g. sd, s, std d, σ. Condone 'short' square root signs. If M1 not awarded, **SC B1** for, $\sigma = \sqrt{101.3167}$ *or* $\sqrt{\frac{6079}{60}}$ oe $= 10.1$.
		3	

Question	Answer	Marks	Guidance
7(a)	$[(0.6)^4 \times 0.4 =]$ 0. 0518[4], $\frac{162}{3125}$	**B1**	
		1	

Question	Answer	Marks	Guidance
7(b)	**Method 1**		
	[P($X \le 7$) – P($X \le 2$) =] $(1 – 0.6^7) – (1 – 0.6^2)$	**M1**	$\left(1-p^7\right)-\left(1-p^2\right) or\ p^2-p^7$ seen, $0 < p < 1$.
	[= 0.36 – 0.02799] = 0.332[0…], $\frac{25938}{78125}$	**A1**	If M0 awarded **SC B1** 0.3320064 or $\frac{25938}{78125}$ CAO.
	Method 2		
	[P(X = 3,4,5,6,7) =] $0.4\times0.6^2+0.4\times0.6^3+0.4\times0.6^4+0.4\times0.6^5+0.4\times0.6^6$	**M1**	$(1-p)\times p^2+(1-p)\times p^3+(1-p)\times p^4+(1-p)\times p^5+(1-p)\times p^6$ seen, $0 < p < 1$.
	[= 0.144 + 0.0864 + 0.05184 + 0.031104 + 0.0186624] = 0.332[0…], $\frac{25938}{78125}$	**A1**	If M0 awarded **SC B1** 0.3320064 or $\frac{25938}{78125}$ CAO.
	Method 3 – geometric series		
	[P(X = 3,4,5,6,7) =] $\frac{0.144\left(1-0.6^5\right)}{1-0.6\ or\ 0.4}$	**M1**	$\frac{0.144\left(1-p^5\right)}{1-p}$ seen $0 < p < 1$.
	=0.332[0…], $\frac{25938}{78125}$	**A1**	If M0 awarded **SC B1** 0.3320064 or $\frac{25938}{78125}$ CAO.
		2	

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td rowspan="9">7(c)</td><td colspan="3">Method 1</td></tr>
<tr><td rowspan="2"><table>
<tr><td>2nd goal scored on:</td><td></td><td></td></tr>
<tr><td>2nd attempt</td><td>$(0.4)^2$</td><td>[= 0.16]</td></tr>
<tr><td>3rd attempt</td><td>$(0.4)^2\ (0.6) \times (2 \text{ or } {}^2C_1)$</td><td>[= 0.192]</td></tr>
<tr><td>4th attempt</td><td>$(0.4)^2\ (0.6)^2 \times (3 \text{ or } {}^3C_1)$</td><td>[= 0.1728]</td></tr>
<tr><td>5th attempt</td><td>$(0.4)^2\ (0.6)^3 \times (4 \text{ or } {}^4C_1)$</td><td>[=0.13824]</td></tr>
</table></td><td>M1</td><td>2 correct unsimplified outcomes.
Condone not identified but not incorrectly identified.</td></tr>
<tr><td>M1</td><td>Add values for 4 identified correct scenarios.
Condone adding values of 2nd, 3rd and 4th attempts only.
No incorrect scenarios.</td></tr>
<tr><td>$= 0.663,\ \dfrac{2072}{3125}$</td><td>A1</td><td>If either or both M marks not awarded, SC B1 for 0.663, $\dfrac{2072}{3125}$ WWW condone 1 index error.</td></tr>
<tr><td colspan="3">Method 2</td></tr>
<tr><td rowspan="2">${}^5C_2(0.4)^2(0.6)^3 + {}^5C_3(0.4)^3(0.6)^2 + {}^5C_4(0.4)^4(0.6)^1 + {}^5C_5(0.4)^5$
[0.3456 + 0.2304 + 0.0768 + 0.01024]
or
$1 - ({}^5C_0\ (0.6)^5 + {}^5C_1(0.4)^1(0.6)^4)$</td><td>M1</td><td>At least 2 correct unsimplified terms.</td></tr>
<tr><td>M1</td><td>Add values for 4 terms of the form ${}^5C_a(0.4)^a(0.6)^{5-a}$
or
1 – sum of 2 terms of the form ${}^5C_a(0.4)^a(0.6)^{5-a}$.</td></tr>
<tr><td>$=0.663,\ \dfrac{2072}{3125}$</td><td>A1</td><td>If either or both M marks not awarded, SC B1 for 0.663 www condone 1 index error.</td></tr>
<tr><td></td><td>3</td><td></td></tr>
</table>

Question	Answer	Marks	Guidance
7(d)	[Mean = 75 $\times 0.4 =$] 30 [Variance = 75 $\times 0.4 \times 0.6 =$] 18	**B1**	30 and 18 seen, allow unsimplified. May be seen in standardisation formula. ($\sigma = \sqrt{18}, 3\sqrt{2}, 4.2426 \leqslant \sigma \leqslant 4.243$ implies correct variance) Withold mark if variance clearly identified as standard deviation.
	P(28 < X < 35) = P($\frac{28.5-30}{\sqrt{18}} < Z < \frac{34.5-30}{\sqrt{18}}$)	**M1**	Substituting *their* μ and positive σ into one ±standardising formula (any number for 28.5 or 34.5), not σ^2, not $\sqrt{\sigma}$.
		M1	Using continuity corrections 27.5 *or* 28.5 and 34.5 *or* 35.5 in *their* 2 separate standardisation formula.
	$\left[= \Phi(1.0607) + \Phi(0.3536) - 1\right]$ = 0.8556 + 0.6383 – 1 Or 0.8556 – (1 – 0.6383) Or 0.8556 – 0.3617 Or (0.8556 – 0.5) + (0.6383 – 0.5) Or 0.3556 + 0.1383	**M1**	Appropriate area Φ, from final process. Must be a probability.
	= 0.494	**A1**	AWRT.
		5	

CAMBRIDGE
International Education

Cambridge International AS & A Level

MATHEMATICS **9709/53**

Paper 5 Probability & Statistics 1 **October/November 2024**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2024 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics-Specific Marking Principles

1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.

2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.

3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.

4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).

5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.

6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

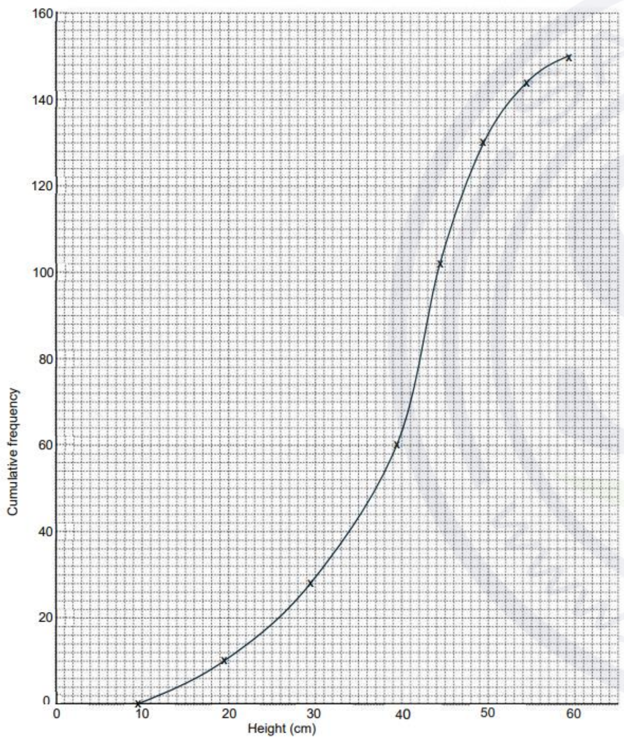
AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1(a)	$0.3^3 + 0.7^3$ or $1 - (3\times0.3^2\times0.7 + 3\times0.3\times0.7^2) = [1-0.63]$	M1	$p^3 + q^3,\ p+q=1,\ p,q>0$ or $1-(3\times p^2\times q + 3\times p\times q^2),$ $p+q=1, p,q>0$.
	0.37[0]	A1	$\frac{37}{100}$.
		2	

Question	Answer	Marks	Guidance
1(b)	[Mean = $125 \times 0.3 =$] 37.5 [Variance = $125 \times 0.3 \times 0.7 =$] 26.25	**B1**	37.5 or 37½ and 26.25, 26¼ seen, allow unsimplified. May be seen in standardisation formula. ($[\sigma =]5.12, \frac{\sqrt{105}}{2}$ implies correct variance).
	$P(X > 45) = P(Z > \frac{45.5 - 37.5}{\sqrt{26.25}})$	**M1**	Substituting their *mean* and *their* positive standard deviation into the ±standardising formula (any number for 45.5), not *their* σ^2, not *their* $\sqrt{\sigma}$.
		M1	Use continuity corrections 44.5 or 45.5 in *their* standardisation formula Note: $\frac{\pm 8}{\sqrt{26.25}}$ or $\frac{\pm 8}{5.123}$ seen gains M2 BOD
	$[1 - \Phi(\textit{their}\, 1.5614)] = 1 - \textit{their}\ 0.9407$	**M1**	Appropriate area Φ, from final process, must be a probability. Expect final answer < 0.5. Note: appropriate final answer implies this M1.
	0.0593	**A1**	$0.0592 \leqslant p \leqslant 0.0593$.
		5	

Question	Answer	Marks	Guidance
2(a)	<table><tr><td>x</td><td>1</td><td>2</td><td>3</td><td>4</td><td>6</td></tr><tr><td>$P(X = x)$</td><td>$\frac{6}{36}$</td><td>$\frac{12}{36}$</td><td>$\frac{6}{36}$</td><td>$\frac{6}{36}$</td><td>$\frac{6}{36}$</td></tr><tr><td></td><td>$\frac{1}{6}$</td><td>$\frac{1}{3}$</td><td>$\frac{1}{6}$</td><td>$\frac{1}{6}$</td><td>$\frac{1}{6}$</td></tr><tr><td></td><td>0.167</td><td>0.333</td><td>0.167</td><td>0.167</td><td>0.167</td></tr></table>	**B1**	Table with correct x values and at least one correct probability linked with the correct x-value. Values need not be in order, lines may not be drawn, may be vertical, x and $P(X)$ may be omitted. Condone any additional x values if probability stated as 0.
		B1	4 correct probabilities linked with the correct x-values, need not be in table, accept unsimplified.
		B1	5 correct probabilities linked with correct x-values, may not be in table. Decimals correct to at least 3 SF. **SC B1** 4 or 5 probabilities summing to 1 placed in a probability distribution table with 4 or 5 x-values between 1 and 6 inclusive.
		3	
2(b)	$[E(X) = \frac{1}{36}(6+24+18+24+36) =]\ 3$	**B1 FT**	FT *their* table with 4 or 5 probabilities ($0 < p < 1$) summing to 1.
		1	

Question	Answer	Marks	Guidance
3(a)	[P(X > 170) = P(Z >] $\frac{170-176}{4.8}$)	**M1**	Using ± standardisation formula with 170, 176 and 4.8 substituted appropriately. Condone σ^2 and $\sqrt{\sigma}$. No continuity correction.
	$\left[\Phi(1.25)\right]$ = 0.894	**M1**	Appropriate area Φ, from final process, must be a probability. Expect final answer > 0.5.
		A1	0.894 or 0.89435 ⩽ p ⩽ 0.8944. If A0 scored, **SC B1** for 0.894 or 0.89435 ⩽ p ⩽ 0.8944, WWW.
		3	
3(b)	P(h < 170) = 1 – 0.8944 = 0.1056	**M1**	1 – *their* **3(a)** seen or implied by 0.7056 or 0.2944
	$\frac{k-176}{4.8}\left[=\Phi^{-1}(0.1056+0.6)\right]=0.541$	**B1**	0.540 < z ⩽ 0.541 or −0.541 ⩽ z < 0.540 seen.
		M1	Use of ±standardisation formula with k, 176, 4.8 equated to a z-value (**not** 1.25, 0.7601, 0.2399, 0.7056, 0.7257, 0.8313, 0.253 ±0.894, 0.6, 0.4), not 4.8^2, not $\sqrt{4.8}$, no continuity correction.
	k = 178.6	**A1**	CAO (answer required to 1 dp).
		4	

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td rowspan="5">4(a)</td><td><table><tr><td>H(cm)</td><td>10–19</td><td>20–29</td><td>30–39</td><td>40–44</td><td>45–49</td><td>50–54</td><td>55–59</td></tr><tr><td>UB</td><td>19.5</td><td>29.5</td><td>39.5</td><td>44.5</td><td>49.5</td><td>54.5</td><td>59.5</td></tr><tr><td>cf</td><td>[10]</td><td>28</td><td>60</td><td>102</td><td>130</td><td>144</td><td>150</td></tr></table></td><td>B1</td><td>Cf values 28, 60, 102, 130, 144, 150 seen, Condone omission of 10.
May be implied by accurate plotting (scale no less than 1cm = 10).
May be by data table.</td></tr>
<tr><td rowspan="3"></td><td>B1</td><td>Linearly scaled axes correctly labelled cumulative frequency (cf) (from 0 to 150) and height (h) and centimetres (cm) (from 9.5 to 59.5) with at least 3 values identified on each.
Axes can be the other way round.</td></tr>
<tr><td>M1</td><td>At least 4 points plotted at upper boundary ± 0.5, (e.g. allow (19, 19.5 or 20, 10) etc.) on correctly scaled axes.
(9.5,0), (19.5,10), (29.5, 28), (39.5, 60), (44.5,102),
(49.5, 130), (54.5, 144), (59.5,150).</td></tr>
<tr><td>A1</td><td>All points plotted correctly, curve drawn (within tolerance), joined to (9.5, 0) and not going beyond above 150 vertically.
A0 if straight line segments used.</td></tr>
<tr><td></td><td>4</td><td></td></tr>
</table>

Question	Answer	Marks	Guidance
4(b)	[$150 \times 0.3 = 45$]	**M1**	Use of graph must be seen.
	Line drawn from 45 on cf axis to meet graph at $h = 36$	**A1 FT**	Must be an increasing cf graph. Expect an answer in range $35 \leqslant h \leqslant 37$ for a correct graph.
		2	
4(c)	Midpoints 14.5, 24.5, 34.5, 42, 47, 52, 57	**B1**	At least 6 correct midpoints seen, may be unsimplified, may be in calculation, may be by data table.
	Mean = $\dfrac{10\times14.5+18\times24.5+32\times34.5+42\times42+28\times47+14\times52+6\times57}{150}$ $\left[=\dfrac{145+441+1104+1764+1316+728+342}{150}\right]$	**M1**	Correct unsimplified mean formula with *their* midpoints (not ub, lb, upper limits, lower limits, cw, fd, f or cf and must be within class). If midpoints correct, accept partially evaluated.
	$=\dfrac{5840}{150},\ \dfrac{584}{15},\ 38\dfrac{14}{15}$, 38.9	**A1**	Accept answers wrt 38.9 WWW If M1 withheld, **SC B1** for $\dfrac{5840}{150},\dfrac{584}{15},38\dfrac{14}{15}$, 38.9.
	$sd^2 =$ $\dfrac{10\times14.5^2+18\times24.5^2+32\times34.5^2+42\times42^2+28\times47^2+14\times52^2+6\times57^2}{150}-\left(their\dfrac{5840}{150}\right)^2$ $[=\dfrac{244285}{150}-\left(their\dfrac{5840}{150}\right)^2]$	**M1**	Correct unsimplified variance formula with *their* midpoint (not ub, lb, upper limits, lower limits, cw, fd, f or cf and must be within class). If midpoints correct, accept partially evaluated
	[= 112.76] standard deviation $\left[\sqrt{112.76}\right] = 10.6$	**A1**	AWRT 10.6 WWW. If second M1 withheld, **SC B1** for 10.6 WWW.
		5	

Question	Answer	Marks	Guidance
5(a)	**Method 1**		
	[P(0, 1, 2) =] ${}^8C_2(0.75)^6(0.25)^2 + {}^8C_1(0.75)^7(0.25)^1 + (0.75)^8$ [= 0.31146 + 0.26697 + 0.10011] =	**M1**	One term ${}^8C_x$. $(p)^x(1-p)^{8-x}, 0 < p < 1, 0 < x < 8$.
		A1	Correct expression, accept unsimplified, no terms omitted leading to final answer.
	= 0.679	**B1**	AWRT.
		3	
	Method 2		
	[P(0, 1, 2) = 1 – P(3, 4, 5, 6, 7, 8) =] $1 - \{{}^8C_3(0.75)^5(0.25)^3 + {}^8C_4(0.75)^4(0.25)^4 + {}^8C_5(0.75)^3(0.25)^5 + {}^8C_6(0.75)^2(0.25)^6 + {}^8C_7(0.75)(0.25)^7 + (0.25)^8\}$	**M1**	One term ${}^8C_x\ (p)^x(1-p)^{8-x}$, $0 < p < 1, \quad 0 < x < 8$.
		A1	Correct expression, accept unsimplified, condone omission of up to 3 'middle' terms, leading to final answer.
	= 0.679	**B1**	AWRT.
		3	

Question	Answer	Marks	Guidance
5(b)	**Method 1**		
	$1-0.75^6$	**M1**	$1-0.75^n, n=6,7$.
	$= 0.822, \frac{3367}{4096}$	**A1**	0.82202148… to at least 3SF.
	Method 2		
	$0.25+0.25\times0.75+0.25\times0.75^2+0.25\times0.75^3+0.25\times0.75^4+0.25\times0.75^5$	**M1**	Summing 6 or 7 terms – condone extra term 0.25×0.75^6 .
	=0.822	**A1**	
	Method 3		
	$1 - 0.75^7 - 0.25\times0.75^6$	**M1**	Correct expression.
	=0.822	**A1**	
		2	

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td>5(c)</td><td colspan="3">Method 1 P(2nd gold $\cap$ 5th is first unwrapped).
R G RorG RorG U</td></tr>
<tr><td></td><td rowspan="2">$0.25\times0.3\times0.55\times0.55\times0.45[=0.01021]$
on its own or as a numerator</td><td>M1</td><td>$a\times0.3\times b\times c\times0.45$ $0 < a, b, c < 1$.
$a \neq 0.3, 0.45$ $b, c \neq 0.45$.
multiplied in that order, or correct.</td></tr>
<tr><td></td><td>A1</td><td>5 correct probabilities multiplied.</td></tr>
<tr><td></td><td colspan="3">Method 2 P(2nd gold $\cap$ 5th is first unwrapped).4 possible scenarios</td></tr>
<tr><td></td><td rowspan="2">
<table>
<tr><td>R G R G U</td><td>$0.25\times0.3\times0.25\times0.3\times0.45$</td><td>[= 0.00253125]</td></tr>
<tr><td>R G R R U</td><td>$0.25\times0.3\times0.25\times0.25\times0.45$</td><td>[= 0.002109375]</td></tr>
<tr><td>R G G R U</td><td>$0.25\times0.3\times0.3\times0.25\times0.45$</td><td>[= 0.00253125]</td></tr>
<tr><td>R G G G U</td><td>$0.25\times0.3\times0.3\times0.3\times0.45$</td><td>[= 0.0030375]</td></tr>
<tr><td></td><td>[Total</td><td>0.010209375]</td></tr>
</table>
</td><td>M1</td><td>$a\times0.3\times b\times c\times0.45$ $0 < a, b, c < 1$.
$a \neq 0.3, 0.45$ $b, c \neq 0.45$.
4 terms in this form seen added on their own or as a numerator.</td></tr>
<tr><td></td><td>A1</td><td>All probabilities correct and attempt to sum the 4 scenarios.</td></tr>
<tr><td></td><td colspan="3">For either approach</td></tr>
<tr><td></td><td>[P(5th is first unwrapped) =] $(0.55)^4(0.45)[=0.041178]$</td><td>B1</td><td></td></tr>
<tr><td></td><td>[P(2nd is first gold | 5th is first unwrapped) =] $\dfrac{0.25\times0.3\times0.55\times0.55\times0.45}{(0.55)^4(0.45)}$
$\left[=\dfrac{0.010209375}{0.0411778125}\right]$</td><td>M1</td><td>$\dfrac{\textit{their}\ \mathrm{P}(\text{2nd gold} \cap \text{5th is first unwrapped})}{\textit{their}\ \mathrm{P}(\text{5th is first unwrapped})}$.
Their probabilities must be clearly identified if incorrect.</td></tr>
<tr><td></td><td>$= 0.248, \dfrac{30}{121}$</td><td>A1</td><td>0.24793…</td></tr>
</table>

Question	Answer	Marks	Guidance
5(c)	**Method 3** First chocolate is red and second gold P(red given that it is wrapped) $\times$ P(gold given that it is wrapped) = $\frac{0.25}{0.55} \times \frac{0.3}{0.55}$	**M1**	Either $\frac{0.25}{0.55\, or\, 0.45}$ or $\frac{0.3}{0.55\, or\, 0.45}$.
		A1	Either $\frac{0.25}{0.55}$ or $\frac{0.3}{0.55}$.
		B1	Both probs correct, can be unsimplified.
		M1	Multiplying their identified P(red given wrapped) by their identified P(gold given wrapped) or correct.
	$= 0.248,\ \frac{30}{121}$	**A1**	
		5	

Question	Answer	Marks	Guidance
6(a)	$\left[\frac{9!}{2!2!} =\right]$ 90720	**B1**	
		1	

Question	Answer	Marks	Guidance
6(b)	**Method 1** Total arrangements – arrangements with repeated letters at ends		
	$\frac{9!}{2!2!}-\frac{7!}{2!}\times 2$	**M1**	$a-\frac{7!}{2!}\times b$ a = *their* 6(a) or correct, b = 1,2.
		M1	$a-\frac{7!}{c!}\times 2$ a = *their* 6(a) or correct, c = 1,2.
	85680	**A1 FT**	ft *their* **6(a)** – 5040.
	Method 2 Adding no of different ways		
	P and S at ends $2\times 7!=$ 10080 P or S at one end only $4\times 5\times\frac{7!}{2!}=$ 50400 Neither P nor S at an end $5\times 4\times\frac{7!}{2!2!}=$ 25200	**M1**	Finding correct number of ways for one of these correctly identified scenarios.
		M1	Adding no of ways for 3 correctly identified scenarios.
	Total 85680	**A1**	
	Method 3		
	P at beginning $7\times\frac{7!}{2!}$ = 17640 S at beginning $7\times\frac{7!}{2!}$ = 17640 Neither P nor S at beginning $5\times\frac{8!}{2!2!}$ = 50400	**M1**	Finding correct number of ways for one of these correctly identified scenarios.
		M1	Adding no of ways for 3 correctly identified scenarios.
	Total 85680	**A1**	
		3	

Question	Answer	Marks	Guidance
6(c)	**Method 1** arrangements with PP between Ss { S P P S ^ ^ ^ ^ ^ } add arrangements with PP not between Ss { (S ^ ^ S) ^ P P ^ }		
	$6! + 5! \times 5 \times 4$	M1	$6! + d$, d an integer ≥ 1, may be implied.
	$6! + 5! \times 5 \times 4$	M1	$e + 5! \times f$, e, f integers ≥ 1, may be implied.
		M1	$e + g! \times \left(5 \times 4 \; or \; {}^5P_2\right)$, e an integer ≥ 1, g = 4,5,6.
	[Total]= 3120	A1	
	Method 2 - considers the 6 positions for S ^^S		
	Positions 1 and 6 there are $5 \times 5!$ ways	M1	Identifying no of ways if S^^S is in position 1 or 6.
	Positions 2, 3, 4 and 5 there are $4 \times 5!$ ways	M1	Identifying no of ways if S^^S is in position 2, 3, 4 or 5.
	$2 \times 5 \times 5! + 4 \times 4 \times 5!$	M1	Adding no of ways for 6 scenarios (or 26 × 5!).
	[Total] = 3120	A1	SC B1 for 3120 if any method marks are withheld.
		4	

Question	Answer	Marks	Guidance
6(d)	**Method 1** Either PP in the group of 5 or PP in the group of 4		
	$\frac{^5C_3}{^9C_5}+\frac{^5C_2}{^9C_5}$, $\frac{^5C_3+{^5C_2}}{^9C_5}$	M1	$a\times\ ^5C_2,\ a\times\ ^5C_3$, or $^5C_2+{^5C_3}$ seen as a numerator of one or two fractions where a is 1 or 2, no extra terms.
		M1	$^9C_5$ *or* $^9C_4$ seen (no addition, multiplication) as a denominator of one or two fractions.
	Probability = $\frac{20}{126},\frac{10}{63}$, 0.159	A1	
	Method 2 Considering the positions of P and then S		
	$\left(\frac{5}{9}\times\frac{4}{8}\times\frac{4}{7}\times\frac{3}{6}\right)+\left(\frac{5}{7}\times\frac{4}{6}\times\frac{4}{9}\times\frac{3}{8}\right)$	M1	$a \times 5 \times 4 \times 4 \times 3$ seen as a numerator of a fraction. where a = 1 or 2.
	$\left(\frac{5}{9}\times\frac{4}{8}\times\frac{4}{7}\times\frac{3}{6}\right)+\left(\frac{5}{7}\times\frac{4}{6}\times\frac{4}{9}\times\frac{3}{8}\right)$ $=\frac{10}{63}$	M1	$9 \times 8 \times 7 \times 6$ seen as a denominator of a fraction.
		A1	
		3	

Cambridge International AS & A Level

MATHEMATICS **9709/51**

Paper 5 Probability & Statistics 1 **May/June 2024**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2024 series for most Cambridge IGCSE, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

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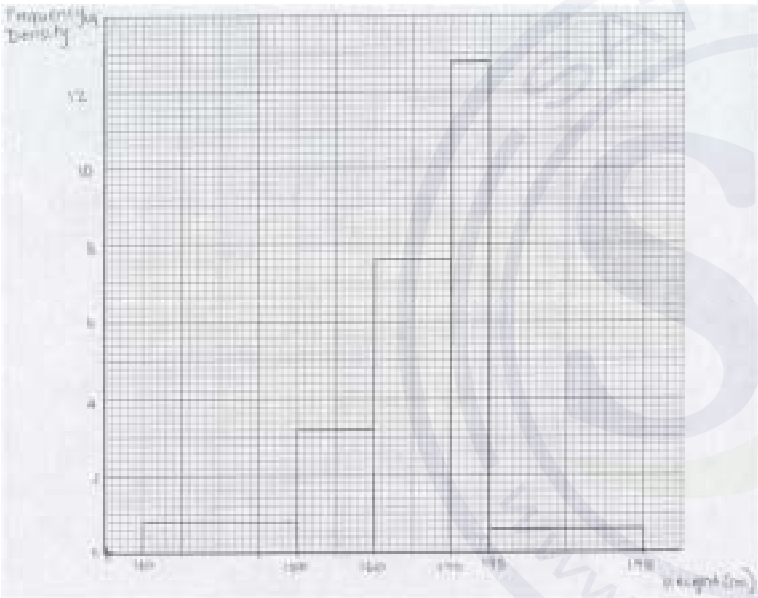
Question	Answer	Marks	Guidance
1(a)	[For all 45 values Mean =] $\frac{439+470}{45}+30$	M1	$\frac{439+470}{45}$ or $\frac{909}{45}$ seen.
	$=50.2$	A1	If M0 awarded, **SC B1** 50.2 WWW.
	Alternative Method for Question 1(a)		
	[For all 45 values Mean =] $\frac{25\times30+470+20\times30+439}{45}$	(M1)	$\frac{1220+1039}{45}$ or $\frac{2259}{45}$ seen.
	$=50.2$	(A1)	If M0 awarded, **SC B1** 50.2 WWW.
		2	
1(b)	For all 45 values $\mathrm{Sd}^2=\frac{12405+11346}{45}-\left(\frac{909}{45}\right)^2$	M1	$\frac{their\,(12405+11346)\text{ or }23751}{45}-\left(their\,\frac{909}{45}\right)^2$
	sd [= $\sqrt{119.76}$] $=10.9$	A1	If M0 awarded, **SC B1** 10.9 WWW.
		2	

Question	Answer	Marks	Guidance
2(a)	$P(23 < X < 35) = P(\frac{23-28}{3.3} < Z < \frac{35-28}{3.3})$ $[= P(-1.515 < Z < 2.121)]$	**M1**	Using ±standardisation formula once with 23 or 35, 28 and 3.3, allow σ^2, allow $\sqrt{\sigma}$, no continuity correction.
		A1	One fully correct ± standardisation formula.
	$[= \Phi(2.121) + \Phi(1.5151) - 1]$ $= 0.9830 + 0.9351 - 1$	**M1**	Appropriate area Φ, from final process, must be a probability.
	= 0.918	**A1**	AWRT
		4	
2(b)	$[P(X > 7.6) = P(Z > \frac{7.6-8.5}{\sigma}) = 0.75]$ $\frac{7.6-8.5}{\sigma} = -0.674$	**B1**	0.674 or – 0.674 seen. CAO as critical value.
		M1	Use of the ±standardisation formula with 7.6, 8.5, σ and a z-value (not 0.75, 0.25, 0.7734, 0.2266, 0.5987 nor 1 – z-value: 0.326, 0.5987). Condone use of $\frac{\pm 0.9}{\sigma}$.
	$\sigma = 1.34$	**A1**	$1.33 \leqslant \sigma \leqslant 1.34$
		3	

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td rowspan="5">3(a)</td><td><table><tr><td>cw</td><td>20</td><td>10</td><td>10</td><td>5</td><td>20</td></tr><tr><td>fd</td><td>0.8</td><td>3.2</td><td>7.6</td><td>12.8</td><td>0.6</td></tr></table></td><td>M1</td><td>At least four frequency densities calculated $\frac{f}{cw}$ $\left(\text{e.g. } \frac{16}{20}\right)$.
Condone $\frac{f}{cw \pm 0.5}$ if unsimplified.
Accept unsimplified, may be read from graph using their scale no lower than 1 cm = fd 2.</td></tr>
<tr><td rowspan="3"></td><td>A1</td><td>All bar heights correct on graph, not FT.
Using their suitable linear scale with at least three values indicated, no lower than 1 cm = fd 2.</td></tr>
<tr><td>B1</td><td>Bar ends at 150, 160, 170, 175, 195. Five bars drawn with a horizontal linear scale no lower than 1 cm = 10 cm, with at least three values indicated, 130 ⩽ horizontal scale ⩽ 195.</td></tr>
<tr><td>B1</td><td>Axes labelled frequency density (fd) height (h) and cm, OE, or an appropriate title.
(Axes may be reversed)</td></tr>
<tr><td></td><td>4</td><td></td></tr>
<tr><td rowspan="4">3(b)</td><td rowspan="3">[LQ:] $160 \leqslant h < 170$ [UQ:] $170 \leqslant h < 175$
175 – 160 = 15</td><td>M1</td><td>$170 \leqslant h < 175 - 160 \leqslant h < 170$
UQ and LQ classes seen.</td></tr>
<tr><td>A1</td><td>175 – 160 = 15</td></tr>
<tr><td></td><td>If M0 scored, SC B1 for 175 – 160 = 15.</td></tr>
<tr><td></td><td>2</td><td></td></tr>
</table>

Question	Answer	Marks	Guidance
4(a)	**Method 1**		
	[Probability of 4 in 3 throws is] $1-\left(\frac{3}{4}\right)^3 = \frac{37}{64}$	**M1**	$1-(s)^3,\ s=\frac{3}{4}$ or $\frac{1}{4}$.
		A1	AG
	Method 2		
	[Probability of 4 in 3 throws is] $\frac{1}{4}+\frac{1}{4}\times\frac{3}{4}+\frac{1}{4}\times\left(\frac{3}{4}\right)^2 = \frac{37}{64}$	**(M1)**	$t+t(1-t)+t(1-t)^2,\ t=\frac{1}{4}$ or $\frac{3}{4}$.
		(A1)	AG
	Method 3		
	${}^3C_1\times\frac{1}{4}\times\left(\frac{3}{4}\right)^2+{}^3C_2\times\left(\frac{1}{4}\right)^2\times\frac{3}{4}+{}^3C_3\times\left(\frac{1}{4}\right)^3 = \frac{37}{64}$	**(M1)**	
		(A1)	AG
		2	

Question	Answer	Marks	Guidance
4(b)	**Method 1**		
	$\left(1-\frac{37}{64}\right)^4 = 0.0317$	**B1FT**	$\left(1 - \textit{their}\,\textbf{(a)}\right)^4$, accept unsimplified.
		1	
	Method 2		
	[Probability no 4s is] $\left(\frac{3}{4}\right)^6 \times \left(\frac{3}{4}\right)^6 = 0.0317$	**(B1FT)**	Accept unsimplified.
		1	
4(c)	X3 Y1 $\left(\frac{37}{64}\right)^3 \times \frac{37}{64} \times \left(\frac{27}{64}\right)^2 \times 3$ [= 0.059645] X2 Y0 $\left(\frac{37}{64}\right)^2 \times \frac{27}{64} \times \left(\frac{27}{64}\right)^3 \times 3$ [= 0.03176]	**B1**	Correct probability for 1 identified scenario. Accept unsimplified.
		M1	Add values of 2 correct scenarios. Identification may be implied by correct unsimplified expressions (condone omission of × 3). Values may not be probabilities.
	Probability = 0.0914	**A1**	If A0 scored, **SC B1** for 0.0914 WWW.
		3	

Question	Answer	Marks	Guidance
5(a)	**Method 1**		
	[P(5, 6, 7) =]	M1	One term ${}^7C_x\,(p)^x(1-p)^{7-x}$, with $0 < p < 1$, $x \neq 0$ or 7.
	${}^7C_5\,0.7^5\,0.3^2 + {}^7C_6\,0.7^6\,0.3^1 + 0.7^7$ [= 0.31765 + 0.24706 + 0.08235]	A1	Correct expression, accept unsimplified, no terms omitted leading to final answer.
	= 0.647	B1	$0.647 \leqslant p < 0.6475$
	Method 2		
	[P(5, 6, 7) = 1 – P(0, 1, 2, 3, 4) =]	(M1)	One term ${}^7C_x\,(p)^x(1-p)^{7-x}$, with $0 < p < 1$, $x \neq 0$ or 7.
	$1 - \{0.3^7 + {}^7C_1\,0.7^1\,0.3^6 + {}^7C_2\,0.7^2\,0.3^5 + {}^7C_3\,0.7^3\,0.3^4 + {}^7C_4\,0.7^4\,0.3^3\}$	(A1)	Correct expression, accept unsimplified, no terms omitted leading to final answer. Condone omission of final bracket '}'. If other brackets omitted, allow recovery if 1 – 0.35294 seen.
	= 0.647	(B1)	$0.647 \leqslant p < 0.6475$
		3	

Question	Answer	Marks	Guidance
5(b)	**Method 1**		
	[1 – P(0 white weeks) =] $1-(1-0.647)^3$	**M1**	$1-p^3$, $0<p<1$, $p=1-$ *their* **(a)**, or correct.
	0.956	**A1**	
	Method 2		
	[P(1, 2, 3 white weeks) =] $3\times0.647\times0.353^2+3\times0.647^2\times0.353+0.647^3$	**(M1)**	$3\times q\times(1-q)^2+3\times q^2\times(1-q)+q^3$, $q=$ *their* **(a)**, or correct.
	0.956	**(A1)**	
		2	

Question	Answer	Marks	Guidance
5(c)	[Mean = $60 \times 0.8 =$] 48 [Variance = $60 \times 0.8 \times 0.2 =$] 9.6	**B1**	48 and 9.6, $9\frac{3}{5}, \frac{48}{5}$ seen, allow unsimplified. May be seen in the standardisation formula ([$\sigma =$] $3.098 \leqslant \sigma \leqslant 3.1[0]$ implies correct variance). Incorrect notation penalised but values can be used as anticipated in remainder of question.
	$\text{P}(X < 47) = \text{P}\left(Z < \frac{46.5 - 48}{\sqrt{9.6}}\right)$	**M1**	Substituting *their* μ and σ into $\pm$standardising formula (any number for 46.5), not *their* σ^2 or $\sqrt{their\,\sigma}$.
		M1	Use continuity correction 46.5 or 47.5 in *their* standardised formula. Note: $\frac{\pm 1.5}{\sqrt{9.6}}$ or $\frac{\pm 1.5}{3.098}$ seen gains M2 BOD.
	[$\text{P}(Z < -0.4841) = 1 - \Phi(0.4841)$] 1 – 0.6858	**M1**	Appropriate area Φ, from final process, must be a probability. Expect final answer < 0.5. Note: appropriate final answer implies this M1.
	= 0.314	**A1**	$0.314 \leqslant p < 0.3145$
		5	

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td rowspan="5">6(a)</td><td rowspan="4">

x	0	1	2	3
$P(X = x)$	$\frac{24}{60}$	$\frac{26}{60}$	$\frac{9}{60}$	$\frac{1}{60}$
	$\frac{2}{5}$	$\frac{13}{30}$	$\frac{3}{20}$	$\frac{1}{60}$
	0.4	0.433	0.15	0.0167

</td><td>B1</td><td>Table with correct X values and at least one probability.
Values need not be in order, lines may not be drawn, may be vertical, X and $P(X)$ may be omitted.
Condone any additional X values if probability stated as 0.</td></tr>
<tr><td>B1</td><td>$P(X = 1)$ or $P(X = 2)$ correct and identified, need not be in table, accept unsimplified.</td></tr>
<tr><td>B1</td><td>Two more correct and identified probabilities, need not be in table, accept unsimplified.</td></tr>
<tr><td>B1</td><td>4 correct probabilities linked with correct outcomes, may not be in table.
Decimals correct to at least 3sf.
SC B1 for four probabilities summing to 1 placed in a probability distribution table with the correct x values.</td></tr>
<tr><td></td><td>4</td><td></td></tr>
</table>

Question	Answer	Marks	Guidance
6(b)	[P($Y = 0$) =] $\frac{2}{3}\times\frac{3}{4}\times\frac{4}{5}\times(1-p)^2$; [P($Y = 5$) =] $\frac{1}{3}\times\frac{1}{4}\times\frac{1}{5}\times p^2$	B1	Either $\frac{2}{3}\times\frac{3}{4}\times\frac{4}{5}\times(1-p)^2$, not $\frac{2}{5}\times(1-p)^2$; or $\frac{1}{3}\times\frac{1}{4}\times\frac{1}{5}\times p^2$, not $\frac{1}{60}\times p^2$.
	$\frac{2}{3}\times\frac{3}{4}\times\frac{4}{5}\times(1-p)^2 = 6\times\frac{1}{3}\times\frac{1}{4}\times\frac{1}{5}\times p^2$ $\left[24(1-p)^2 = 6\times p^2\right]$ $3p^2 - 8p + 4 = 0$	M1	Equating and forming a 3 term quadratic equation. *Their* P($Y = 0$) = 6 × *their* P($Y = 5$).
	$p = \frac{2}{3}$	A1	Not dependent on B1. A0 if $p = 2$ seen and not clearly rejected.
			SC B1 if $p = \frac{2}{3}$ obtained from a correct quadratic with more than three terms. If $p = 2$ seen and not clearly rejected, **SC B0**.
		3	

Question	Answer	Marks	Guidance
7(a)	$\left[\frac{8!}{2!3!}=\right]$ 3360	**B1**	
		1	
7(b)	Number of arrangements with 2s at the end – number of arrangements with 2s at the end and the 4s together 2 _ _ _ _ _ _ 2 – 2 _ (<u>444</u>) _ _ 2		
	$\frac{6!}{3!}-4!$	**M1**	$\frac{6!}{3!}\times r-s$, r = 1, 2 and s a positive integer (including 0).
		B1	4! Seen either alone or in t – 4!, t an integer value > 24.
		M1	$\frac{6!}{3!}\times r-4!\times u,\ r=1,\ 2$ and $u=1,\ 2$.
	= 96	**A1**	
		4	

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td rowspan="6">7(c)</td><td colspan="3">Method 1</td></tr>
<tr><td rowspan="3"><table><tr><td>2s</td><td>4s</td><td>1,3,5</td><td></td><td></td></tr><tr><td>0</td><td>0</td><td>3</td><td>$^3C_3$</td><td>1</td></tr><tr><td>0</td><td>1</td><td>2</td><td>$^3C_1 \times {}^3C_2$</td><td>9</td></tr><tr><td>1</td><td>0</td><td>2</td><td>$^2C_1 \times {}^3C_2$</td><td>6</td></tr><tr><td>1</td><td>1</td><td>1</td><td>$^2C_1 \times {}^3C_1 \times {}^3C_1$</td><td>18</td></tr></table>[Total 34 ways]</td><td>M1</td><td>One correct calculation, unsimplified for an identified scenario containing 1, 2 and/or 1, 4.</td></tr>
<tr><td>A1</td><td>Two correct outcomes evaluated, accept unsimplified.</td></tr>
<tr><td>M1</td><td>Four correct scenarios added.</td></tr>
<tr><td>[Total number of selections =] $^8C_3$ [= 56]</td><td>B1</td><td>Used as denominator of probability expression.</td></tr>
<tr><td>[Probability =] $\left[\frac{34}{^8C_3}\right] = \frac{17}{28}$, 0.607</td><td>A1</td><td></td></tr>
</table>

Question	Answer	Marks	Guidance
7(c)	**Method 2**		
	Combinations of 3 numbers	(M1)	One correct calculation, unsimplified for an identified scenario containing 1, 2 and/or 1, 4.
		(A1)	Five correct outcomes evaluated, accept unsimplified.
		(M1)	Ten correct scenarios added.

1,2,3	${}^1C_1 \times {}^2C_1 \times {}^1C_1$	2
1,2,4	${}^1C_1 \times {}^2C_1 \times {}^3C_1$	6
1,2,5	${}^1C_1 \times {}^2C_1 \times {}^1C_1$	2
1,3,4	${}^1C_1 \times {}^1C_1 \times {}^3C_1$	3
1,3,5	${}^1C_1 \times {}^1C_1 \times {}^1C_1$	1
1,4,5	${}^1C_1 \times {}^3C_1 \times {}^1C_1$	3
2,3,4	${}^2C_1 \times {}^1C_1 \times {}^3C_1$	6
2,3,5	${}^2C_1 \times {}^1C_1 \times {}^1C_1$	2
2,4,5	${}^2C_1 \times {}^3C_1 \times {}^1C_1$	6
3,4,5	${}^1C_1 \times {}^3C_1 \times {}^1C_1$	3

Question	Answer	Marks	Guidance
	[Total 34 ways] [Total number of selections =] ${}^8C_3$ [= 56]	(B1)	${}^8C_3$ or 56 as denominator of probability expression.
	[Probability =] $\left[\dfrac{34}{{}^8C_3}\right] = \dfrac{17}{28},\ 0.607$	(A1)	

Question	Answer	Marks	Guidance
7(c)	**Method 3**		
		(M1)	One correct calculation, unsimplified for an identified scenario containing 1, 2 and/or 1, 4.
		(A1)	Five correct outcomes evaluated, accept unsimplified.
		(M1)	Ten correct scenarios added.

1,2,3	$\frac{1}{8}\times\frac{2}{7}\times\frac{1}{6}\times 3!$	$\frac{12}{336}$
1,2,4	$\frac{1}{8}\times\frac{2}{7}\times\frac{3}{6}\times 3!$	$\frac{36}{336}$
1,2,5	$\frac{1}{8}\times\frac{2}{7}\times\frac{1}{6}\times 3!$	$\frac{12}{336}$
1,3,4	$\frac{1}{8}\times\frac{1}{7}\times\frac{3}{6}\times 3!$	$\frac{18}{336}$
1,3,5	$\frac{1}{8}\times\frac{1}{7}\times\frac{1}{6}\times 3!$	$\frac{6}{336}$
1,4,5	$\frac{1}{8}\times\frac{3}{7}\times\frac{1}{6}\times 3!$	$\frac{18}{336}$
2,3,4	$\frac{2}{8}\times\frac{1}{7}\times\frac{3}{6}\times 3!$	$\frac{36}{336}$
2,3,5	$\frac{2}{8}\times\frac{1}{7}\times\frac{1}{6}\times 3!$	$\frac{12}{336}$
2,4,5	$\frac{2}{8}\times\frac{3}{7}\times\frac{1}{6}\times 3!$	$\frac{36}{336}$
3,4,5	$\frac{2}{8}\times\frac{3}{7}\times\frac{1}{6}\times 3!$	$\frac{18}{336}$

Question	Answer	Marks	Guidance
7(c)		(B1)	336 or 8×7×6 seen as a denominator.
	[Probability] = $\frac{17}{28}$, 0.607	(A1)	
	Method 4		
	444 — $\frac{3}{8}\times\frac{2}{7}\times\frac{1}{6}$ — $\frac{6}{336}$	(M1)	1-1 correct calculation, unsimplified for an identified scenario not containing three 4s.
		(A1)	Five correct probabilities evaluated, accept unsimplified.
7(c)		(M1)	Nine correct scenarios subtracted.

444	$\frac{3}{8}\times\frac{2}{7}\times\frac{1}{6}$	$\frac{6}{336}$
445	$\frac{3}{8}\times\frac{2}{7}\times\frac{1}{6}\times 3$	$\frac{18}{336}$
443	$\frac{2}{8}\times\frac{3}{7}\times\frac{1}{6}\times 3$	$\frac{18}{336}$
442	$\frac{2}{8}\times\frac{3}{7}\times\frac{1}{6}\times 3$	$\frac{36}{336}$
441	$\frac{2}{8}\times\frac{3}{7}\times\frac{1}{6}\times 3$	$\frac{18}{336}$
225	$\frac{2}{8}\times\frac{3}{7}\times\frac{1}{6}\times 3$	$\frac{6}{336}$
224	$\frac{2}{8}\times\frac{3}{7}\times\frac{1}{6}\times 3$	$\frac{18}{336}$
223	$\frac{2}{8}\times\frac{3}{7}\times\frac{1}{6}\times 3$	$\frac{6}{336}$
221	$\frac{2}{8}\times\frac{3}{7}\times\frac{1}{6}\times 3$	$\frac{6}{336}$

Question	Answer	Marks	Guidance
		(B1)	336 or 8 × 7 × 6 seen as a denominator.
	[Probability] $=1-\frac{132}{336},\ \frac{204}{336},\ 0.607$	**(A1)**	
		5	

Cambridge International AS & A Level

MATHEMATICS **9709/52**

Paper 5 Probability & Statistics 1 **May/June 2024**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

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This document consists of **17** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics-Specific Marking Principles

1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.

2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.

3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.

4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).

5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.

6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Question	Answer	Marks	Guidance
1(a)	$[(0.7)^6 \times 0.3]\ = 0.0353$	**B1**	$\dfrac{352947}{10000000}$ or 0.03529… to at least 3sf.
		1	
1(b)	**Method 1**		
	[P(X < 6) =] $1 - 0.7^5$	**M1**	$1 - 0.7^d$, d = 5, 6.
	= 0.832	**A1**	Accept 0.83193 to at least 3sf. If M0 scored, **SC B1** for 0.8319[3].
	Method 2		
	[P(X < 6) =] $0.3+(0.3)(0.7)+(0.3)(0.7)^2+(0.3)(0.7)^3+(0.3)(0.7)^4$	**(M1)**	$0.3+(0.3)(0.7)+(0.3)(0.7)^2+(0.3)(0.7)^3+(0.3)(0.7)^4\Big[+(0.3)(0.7$
	= 0.832	**(A1)**	Accept 0.83193 to at least 3sf. If M0 scored, **SC B1** for 0.8319[3].
		2	
1(c)	$(0.7)^8 \times (0.3)^2 \times {}^9C_1$ or $(0.7)^8 \times (0.3) \times {}^9C_1 \times (0.3)$	**M1**	$(0.7)^8 \times (0.3)^2 \times k$, k a positive integer, 1 may be implied. No addition/subtraction/additional terms.
	= 0.0467	**A1**	
		2	

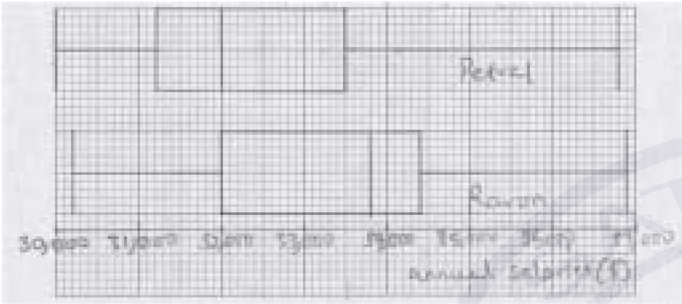
Question	Answer	Marks	Guidance
2(a)	**Method 1**		
	P(HR) + P(TR) + P(TBR) $\frac{1}{3}\times\frac{4}{9}+\frac{2}{3}\times\frac{4}{9}+\frac{2}{3}\times\frac{5}{9}\times\frac{4}{8}$ or P(HR) + (P(TRR) + P(TRB)) + P(TBR) $\frac{1}{3}\times\frac{4}{9}+\left(\frac{2}{3}\times\frac{4}{9}\times\frac{3}{8}+\frac{2}{3}\times\frac{4}{9}\times\frac{5}{8}\right)+\frac{2}{3}\times\frac{5}{9}\times\frac{4}{8}$	**B1**	Two of the calculations for P(HR), P(TBR), either P(TR) or P(TRR) + P(TRB) unsimplified, ignore any identification. Condone $\frac{4}{8}=\frac{1}{2}$ in the unsimplified calculation. Condone use of tree diagram to show calculation if values correct at end.
		M1	Values of all correct identified scenarios added. Correct branches may be identified on the tree diagram.
	$\left[=\frac{4}{27}+\frac{8}{27}+\frac{5}{27}\right]=\frac{17}{27}$	**A1**	0.6296…, 0.630 If M0 scored **SC B1** for acceptable answers, WWW.
	Method 2		
	$1-P(HB)-P(TBB)=1-\left(\frac{1}{3}\times\frac{5}{9}+\frac{2}{3}\times\frac{5}{9}\times\frac{4}{8}\right)=\left[1-\frac{5}{27}-\frac{5}{27}\right]$	**(B1)**	One calculation of P(HB), P(TBB), unsimplified, ignore any identification. 1 – probability must be seen. Condone use of tree diagram to show calculation if values correct at end.
		(M1)	1 – values of two correct identified scenarios subtracted. Correct branches may be identified on the tree diagram.
	$=\frac{17}{27}$	**(A1)**	0.6296…, 0.630 If M0 scored **SC** B1 for acceptable answers, WWW.

Question	Answer	Marks	Guidance
2(a)	**Method 3**		
	P(HR) + P(T, (1 – no R)) = $\frac{1}{3}\times\frac{4}{9}+\frac{2}{3}\left(1-\left(\frac{5}{9}\times\frac{4}{8}\right)\right)=$	**(B1)**	Calculation for P(T, (1 – no R)) seen unsimplified. Condone use of tree diagram to show calculation if values correct at end.
	$\left[\frac{4}{27}+\frac{2}{3}\left(1-\frac{20}{27}\right)\right]$	**(M1)**	Values of two correct identified scenarios added. Correct branches may be identified on the tree diagram.
	$=\frac{17}{27}$	**(A1)**	0.6296…, 0.630 If M0 scored **SC B1** for acceptable answers, WWW.
		3	

Question	Answer	Marks	Guidance
2(b)	**Method 1**		
	$\left[\text{P(head}\mid\text{no reds)} = \dfrac{\text{P}(\text{head} \cap \text{no reds})}{\text{P}(\text{no reds})}\right]$ $= \dfrac{\frac{1}{3}\times\frac{5}{9}}{1-\frac{17}{27}} = \dfrac{\frac{5}{27}}{\frac{10}{27}} =$	**M1**	$\dfrac{d}{1-their(\mathbf{a})}$ or $\dfrac{d}{1-\frac{17}{27}}$ or $\dfrac{d}{\frac{10}{27}}$, $0 < d < 1$. Condone $\dfrac{10}{27} = 0.3704$ or more accurate.
	$= \dfrac{1}{2}$	**A1**	OE Condone $0.499[9\ldots]$.
	Method 2		
	$\left[\text{P(head}\mid\text{no reds)} = \dfrac{\text{P}(\text{head} \cap \text{blue})}{\text{P(HB)}+\text{P(TBB)}}\right]$ $= \dfrac{\frac{1}{3}\times\frac{5}{9}}{\frac{1}{3}\times\frac{5}{9}+\frac{2}{3}\times\frac{5}{9}\times\frac{4}{8}} = \dfrac{\frac{5}{27}}{\frac{10}{27}} =$	**(M1)**	$\dfrac{d}{\frac{1}{3}\times\frac{5}{9}+\frac{2}{3}\times\frac{5}{9}\times\frac{4}{8}}$ or $\dfrac{d}{\frac{10}{27}}$, $0 < d < 1$. Condone $\dfrac{10}{27} = 0.3704$ or more accurate.
	$= \dfrac{1}{2}$	**(A1)**	OE Condone $0.499[9\ldots]$.
		2	

Question	Answer	Marks	Guidance
3(a)	$P(X > 184) = P\left(Z > \frac{184-131}{54}\right)$ [= P($Z > 0.9815$)]	**M1**	Use of ±standardisation formula with 184, 131 and 54, no continuity correction. Condone use of σ^2, $\sqrt{\sigma}$.
	1 – 0.837 [= 0.163]	**M1**	Calculating the appropriate probability area (leading to their final answer).
	Percentage [= 0.163 ×100] = 16.3	**A1**	AWRT
		3	
3(b)	[$P(X < w) = P(Z < \frac{w-131}{54}) = 0.2$] $\frac{w-131}{54} = -0.842$	**B1**	$-0.842 \leqslant z < -0.8415$ or $0.8415 < z \leqslant 0.842$ seen.
		M1	Use of the ±standardisation formula with 131, 54, w and a z-value (not 0.2, 0.8, 0.158, 0.508[0], 0.492[0], 0.7881, 0.2119, 0.5593, 0.4407).
	$w = 85.5$	**A1**	$85.5 \leqslant p \leqslant 85.6$ Signs must be consistent to create a positive answer.
		3	

Question	Answer	Marks	Guidance
4(a)	Median = 32000	**B1**	Clearly identified, e.g. Q2, med. Accept 32k.
	[UQ = 33500, LQ = 31200] [IQR =] 33500 – 31200	**M1**	$33300 \leqslant UQ \leqslant 33700 - 31100 \leqslant LQ \leqslant 31200$ Implied if both quartile values are stated and an appropriate IQR is calculated accurately.
	= 2300	**A1**	WWW Ignore $ signs. If M0 scored, **SC B1** for 2300 WWW. If key ignored consistently: B0 Median = 320 **SC M1** $325 \leqslant UQ \leqslant 335 - 311 \leqslant LQ \leqslant 312$ **SC A1** 23.
		3	

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td rowspan="4">4(b)</td><td rowspan="3">Box-and-whisker plot on provided grid

<table><tr><td>R</td><td>30 200</td><td>32 000</td><td>33 800</td><td>34 400</td><td>36 900</td></tr><tr><td>P</td><td>30 000</td><td>31 200</td><td>32 000</td><td>33 500</td><td>36 800</td></tr></table></td><td>B1</td><td>All five key values for R plotted accurately in standard format using a linear scale with at least three linear values. Labelled R.
Condone whiskers through box or at corners of boxes or extending $\frac{1}{2}$ square beyond limit.
Scale no less than 1 cm = $1000.
Daylight rule applied to vertical lines of box.</td></tr>
<tr><td>B1FT</td><td>All five key values for P, FT from (a), plotted accurately in standard format using a linear scale with at least three linear values. Labelled P.
Condone whiskers through box or at corners of boxes or extending $\frac{1}{2}$ square beyond limit.
Scale no less than 1 cm = $1000.
Daylight rule applied to vertical lines of box.</td></tr>
<tr><td>B1</td><td>Whiskers not through box (condone $\frac{1}{2}$ square in box) for either, not drawn at corners of boxes. single linear scale for the diagram and labelled ‘salaries’ (OE) and $.
If only one plot attempted, SC B1 for meeting all the requirements above.</td></tr>
<tr><td></td><td>3</td><td></td></tr>
<tr><td rowspan="2">4(c)</td><td>Median because there is an extreme value ($36 800)</td><td>B1</td><td>Do not accept ‘values’.
Must identify median and reference either the extreme value (anomaly, outlier, 36 800) or the skew in context (e.g. concentrated in lower values, positive skew).</td></tr>
<tr><td></td><td>1</td><td></td></tr>
</table>

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td>5(a)</td><td>[$7 =] [$]5 + [$]2
[Probability =] $\frac{1}{5}\times\frac{2}{4}\times 2=\frac{1}{5}=0.2$
Or
[Probability =] $0.2 \times 0.5 \times 2 = 0.2$</td><td>B1</td><td>AG
Must include [$7], 5, 2 and link the probabilities to the appropriate value
$\frac{\left[{}^{1}C_1\right]\times {}^{2}C_1}{{}^{5}C_2}=0.2.$
$\frac{1}{5}\times\frac{2}{4}+\frac{2}{5}\times\frac{1}{4}$, not $\frac{1}{5}\times\frac{2}{4}+\frac{1}{5}\times\frac{2}{4}$ unless 5 and 2 and 2 and 5 seen in solution.
If all possibilities identified (e.g. outcome table), must be clearly labelled and terms fulfilling the condition identified.</td></tr>
<tr><td></td><td></td><td>1</td><td></td></tr>
<tr><td>5(b)</td><td><table><tr><td>x</td><td>2</td><td>3</td><td>4</td><td>6</td><td>7</td></tr><tr><td>$P(X=x)$</td><td>0.1</td><td>0.4</td><td>0.1</td><td>0.2</td><td>0.2</td></tr></table></td><td>B1</td><td>Table with correct x values and at least one further non-zero probability correct.
Condone extra x values if probability stated as 0.</td></tr>
<tr><td></td><td></td><td>B1</td><td>Two more correct non-zero probabilities linked with correct outcomes. Accept probabilities not in table if clearly identified.</td></tr>
<tr><td></td><td></td><td>B1</td><td>All five probabilities correct. Accept probabilities not in table if clearly identified.
SC B1 for four further non-zero probabilities adding to 0.8 if B1 max scored.</td></tr>
<tr><td></td><td></td><td>3</td><td></td></tr>
</table>

Question	Answer	Marks	Guidance
5(c)	[E(X) = 0.1 × 2 + 0.4 × 3 + 0.1 × 4 + 0.2 × 6 + 0.2 × 7] **0.2 + 1.2 + 0.4 + 1.2 + 1.4** [= 4.4]	**M1**	Accept unsimplified expression. May be calculated in the variance, FT *their* table with **at least 5** probabilities, $0 < p < 1$, that sum to 1. FT acceptable at the bold partially evaluated stage.
	$[\text{Var}(X) = 0.1\times2^2 + 0.4\times3^2 + 0.1\times4^2 + 0.2\times6^2$ $+0.2\times7^2 - 4.4^2$ $0.1\times4 + 0.4\times9 + 0.1\times16 + 0.2\times36 + 0.2\times49 - 4.4^2$	**M1**	Appropriate variance formula using *their* $(E(X))^2$ value. FT *their* table with **at least 4** probabilities, $0 < p < 1$, that may not sum to 1. FT acceptable at the bold partially evaluated stage. Note: if table is correct, 22.6 – (4.4^2 or 19.36) implies this M1.
	= 3.24	**A1**	CAO $\frac{81}{25}$, $3\frac{6}{25}$ scores A0. Only dependent upon previous M1 (M0 M1 A1 possible). If M0 M0 scored, **SC B1** for 3.24 WWW.
		3	

Question	Answer	Marks	Guidance
6(a)	Mean $[=110\times 0.25]=27.5$ Variance $[=110\times 0.25\times 0.75]=20.625,\ \frac{165}{8}$	**B1**	27.5 and 20.625 (CAO) seen, allow unsimplified. May be in standardisation formula (4.541475… to at least 4sf or $\sqrt{\frac{165}{8}}$ or $\frac{\sqrt{330}}{4}$ implies correct variance). Penalise incorrect identification, condone no identification.
	$P(X<22)=P\left(Z<\frac{21.5-27.5}{\sqrt{20.625}}\right)$	**M1**	Substituting *their* 27.5 and *their* 20.625 into the ±standardising formula (any number for 21.5), not σ^2, not $\sqrt{\sigma}$.
		M1	Using continuity correction 21.5 or 22.5 in *their* standardisation formula.
	$[P(Z<-1.3212)\ =1-\Phi(1.3212)]$ $1-0.9068=$	**M1**	Appropriate probability area, from final process, must be a probability. May be implied by a sketch of the required probability area. Expect final answer < 0.5.
	0.0932	**A1**	$0.0932 \leqslant p < 0.09325$ If either M1 M1 not awarded for standardisation and/or M1 not awarded for finding probability area, **SC B1** $0.0932 \leqslant p < 0.09325$ WWW.
		5	

Question	Answer	Marks	Guidance
6(b)	**Method 1**		
	[1 – P(8, 9, 10) =]	M1	One term ${}^{10}C_x\,(p)^x(1-p)^{10-x}$, $0 < p < 1$, $x \neq 0$ or 10.
	$1 - ({}^{10}C_8\,0.85^8\,0.15^2 + {}^{10}C_9\,0.85^9 0.15^1 + 0.85^{10})$ [= 1 – (0.275897 + 0.347425 + 0.196874)]	A1	Correct unsimplified expression. Condone omission of last bracket only.
	= 0.180	B1	$0.1795 < p \leqslant 0.180$
	Method 2		
	[P(0, 1, 2, 3, 4, 5, 6, 7) =] $0.15^{10} + {}^{10}C_1\,0.85 \times 0.15^9 + \ldots + {}^{10}C_7\,0.85^7 0.15^3$	(M1)	One term ${}^{10}C_x\,(p)^x(1-p)^{10-x}$, $0 < p < 1$, $x \neq 0$ or 10.
		(A1)	Correct unsimplified expression.
	= 0.180	(B1)	$0.1795 < p \leqslant 0.180$
		3	
6(c)	$0.25 \times 0.6 \times 0.15 \times 6$	M1	$0.25 \times 0.6 \times 0.15 \times k$, k an integer > 1.
	0.135, $\frac{27}{200}$	A1	
		2	

Question	Answer	Marks	Guidance
7(a)	$\left[\dfrac{10!}{2!4!}=\right]$ 75600	**B1**	
		1	
7(b)	$4! \times 3!$	**M1**	4! SOI in all terms leading to final answer. Allow 24 if 4! = 24 is seen.
		M1	Ignoring any values used to justify 4!. Either 3! SOI in expression leading to final answer, or at least 6 distinct scenarios identified and added in expression leading to final answer. Condone 3 distinct scenarios × 2. Ignore repeated scenarios.
		A1	$4! \times 3!$ Fully correct unsimplified expression leading to final answer.
	144	**B1**	WWW
		4	

Question	Answer	Marks	Guidance
7(c)	**Method 1: If denominator is from 7(a), no denominator or incorrect denominator**		
	[Numerator = Number of required arrangements =] $\frac{5!}{2!} \times \frac{5!}{4!} \times 2$ [= 600]	B1	$\frac{5!}{2!}$ seen (arrangements of consonants).
		B1	$\frac{5!}{4!}$ seen (arrangements of vowels).
		M1	$\frac{5!}{r} \times \frac{5!}{s} \times 2$, $r = 1$ or 2, $s = 1, 4, 4!$ or 24.
	[Probability =] $\frac{\textit{their } 600}{\textit{their } 75600}$	M1	$\frac{\textit{their } 600}{\textit{their } (\mathbf{a})}$ or $\frac{\textit{their } 600}{75600}$.
	$= \frac{1}{126}$, 0.00794	A1	Accept $\frac{600}{75600}$ OE.
	Method 2: If denominator 10!		
	[Numerator = Number of required arrangements =] $5! \times 5! \times 2$ [= 28800]	(B1)	5! seen (arrangements of consonants).
		(B1)	A second 5! seen (arrangements of vowels).
		(M1)	$5! \times 5! \times k$, $k = 1$ or 2.
	[Probability =] $\frac{\textit{their } 28800}{10!}$	(M1)	
	$= \frac{1}{126}$, 0.00794	(A1)	Accept $\frac{600}{75600}$ OE.

Question	Answer	Marks	Guidance
7(c)	**Method 3: Using probabilities**		
	$\frac{5}{10}\times\frac{5}{9}\times\frac{4}{8}\times\frac{4}{7}\times\frac{3}{6}\times\frac{3}{5}\times\frac{2}{4}\times\frac{2}{3}\times\frac{1}{2}\times\frac{1}{1}\times 2$	**(B1)**	$\frac{5}{a}\times\frac{4}{b}\times\frac{3}{c}\times\frac{2}{d}\times\frac{1}{e}$ seen. $10 \geq a > b > c > d > e \geq 1$ (arrangements of consonants).
		(B1)	A second $\frac{5}{f}\times\frac{4}{g}\times\frac{3}{h}\times\frac{2}{i}\times\frac{1}{j}$ seen. $10 \geq f > g > h > i > j \geq 1$ (arrangements of vowels).
		(M1)	$\frac{5}{a}\times\frac{4}{b}\times\frac{3}{c}\times\frac{2}{d}\times\frac{1}{e}\times\frac{5}{f}\times\frac{4}{g}\times\frac{3}{h}\times\frac{2}{i}\times\frac{1}{j}\times k$ $k = 1$ or 2.
		(M1)	$\frac{\textit{their}\ 5\times4\times3\times2\times1\times5\times4\times3\times2\times1}{10\times9\times8\times7\times6\times5\times4\times3\times2\times1}$.
	$= \frac{1}{126}$, 0.00794	**(A1)**	Accept $\frac{28800}{362800}$ OE.
		5	

Cambridge International AS & A Level

MATHEMATICS **9709/53**

Paper 5 Probability & Statistics 1 **May/June 2024**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2024 series for most Cambridge IGCSE, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics-Specific Marking Principles

1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.

2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.

3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.

4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).

5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.

6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

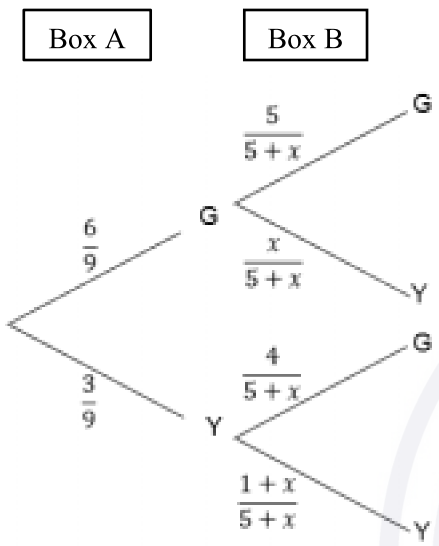
AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td rowspan="4">1(a)</td><td rowspan="3"><table>
<tr><td>x</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr>
<tr><td>$\mathrm{P}(X = x)$</td><td>$\frac{1}{36}$</td><td>$\frac{4}{36}$</td><td>$\frac{10}{36}$</td><td>$\frac{12}{36}$</td><td>$\frac{9}{36}$</td></tr>
<tr><td></td><td>$\frac{1}{36}$</td><td>$\frac{1}{9}$</td><td>$\frac{5}{18}$</td><td>$\frac{1}{3}$</td><td>$\frac{1}{4}$</td></tr>
</table>

Decimal equivalent 3sf:
0.0278, 0.111, 0.278, 0.333, 0.25</td><td>B1</td><td>Table with correct X values and at least one correct probability associated with the correct X value.
Values need not be in order, lines may not be drawn, may be vertical, X and $\mathrm{P}(X)$ may be omitted.
Condone any additional X values if probability stated as 0.</td></tr>
<tr><td>B1</td><td>Three other probabilities associated with correct x values, need not be in table, accept unsimplified.</td></tr>
<tr><td>B1</td><td>Five correct probabilities linked with correct outcomes, may not be in table.
Decimals correct to at least 3sf.
SC B1 for five probabilities summing to 1 placed in a probability distribution table with the correct x values.</td></tr>
<tr><td></td><td>3</td><td></td></tr>
</table>

Question	Answer	Marks	Guidance
1(b)	$E(X) = \dfrac{1\times2+4\times3+10\times4+12\times5+9\times6}{36} =$ $\dfrac{2}{36}+\dfrac{12}{36}+\dfrac{40}{36}+\dfrac{60}{36}+\dfrac{54}{36}\left[=\dfrac{14}{3} \text{ or } 4.67\right]$	M1	Accept unsimplified expression or sum of fractions seen. May be calculated in variance. FT *their* table with five probabilities summing to $0.999 \leqslant total \leqslant 1$ $(0 < p < 1)$.
	$\text{Var}(X) = \dfrac{1\times2^2+4\times3^2+10\times4^2+12\times5^2+9\times6^2}{36} - \left(\dfrac{14}{3}\right)^2$	M1	Appropriate variance formula using *their* $(E(X))^2$ value. FT *their* table with 4 or more probabilities. $(0 < p < 1)$ which need not sum to 1. Note: If table is correct, then $\left(\dfrac{824}{36} \text{ or } \dfrac{206}{9} \text{ or } 22.89\right) - \left(\dfrac{196}{9} \text{ or } 21.78 \text{ or } \left(\dfrac{14}{3}\right)^2\right)$ implies M1.
	$\left[= \dfrac{824}{36} - \dfrac{196}{9} = 22.89 - 21.78\right] = \dfrac{10}{9}$	A1	$1\dfrac{1}{9}$, $1.11[1]$, $1.\dot{1}$
		3	
1(c)	$P(X \text{ even} \mid X > 3) = \dfrac{\dfrac{10}{36}+\dfrac{9}{36}}{\dfrac{31}{36}}$	M1	$\dfrac{their\ P(4) + their\ P(6)}{their\ P(4) + P(5) + P(6)}$, all probabilities $(0 < p < 1)$. If sample space seen in any part of the question, then M1 $\dfrac{their\,19}{their\,31}$.
	$= \dfrac{19}{31}$	A1	0.613
		2	

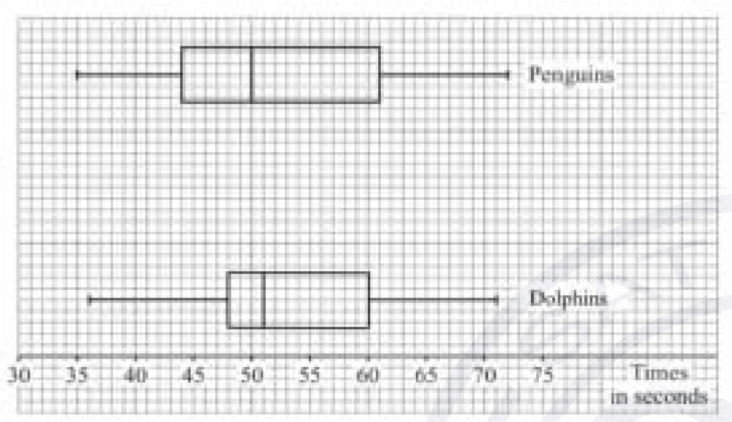
Question	Answer	Marks	Guidance
2(a)	$\mathrm{P}\left(Z > \frac{1.93-1.64}{0.25}\right) = \mathrm{P}(Z > 1.16)$	**M1**	Using ± standardisation formula with 1.93, 1.64 and 0.25 substituted, not σ^2, not $\sqrt{\sigma}$, no continuity correction.
	1 – 0.8770	**M1**	Appropriate area Φ resulting from a standardisation, from final process, must be probability. Note: the appropriate probability answer implies this M1.
	0.123	**A1**	$0.123 \leqslant p < 0.12303$ If M0 M0, **SC B1** if no standardisation shown.
		3	
2(b)	$\frac{1.56-\mu}{\sigma} = -0.44$ $\frac{1.86-\mu}{\sigma} = 0.674$	**B1**	$-0.441 < z_1 < -0.439$ or $0.439 < z_1 < 0.441$ seen.
		B1	$z_2 = 0.674$ or $z_2 = -0.674$ seen, CAO, critical value.
		M1	Use of the ±standardisation formula once with μ, σ equating to a z-value (not 0.33, 0.67. 0.25, 0.75, 0.6293, 0.5987, 0.7486, 0.7734, (1 – 0.44), (1 – 0.674)). Condone continuity correct ±0.005, not $\sigma^2, \sqrt{\sigma}$.
	Solve, obtaining values for μ and σ $\mu = 1.68, \quad \sigma = 0.269$	**M1**	Solve two equations in μ and σ using the elimination method, substitution method or other appropriate approach to obtain values for both μ and σ.
		A1	AWRT $\mu = 1.68$, $\sigma = 0.269$. If one or both of the M marks have not been awarded, **SC B1** for both correct.
		5	

Question	Answer	Marks	Guidance
3(a)		**B1**	Correct structure and probabilities for Box A branches.
		B1	Completely correct structure and one correct probability for a Box B branch including label for G or Y.
		B1	Completely correct structure and second correct probability on a Box B branch including label G or Y.
		B1	Completely correct structure and remaining two probabilities correct on Box B branches, including labels for G or Y. **SC B1** if correct shape diagram but only four correct algebraic probs for GG, GY, YG and YY.
		4	
3(b)	P(same colour) = $\frac{6}{9}\times\frac{5}{5+x}+\frac{3}{9}\times\frac{1+x}{5+x}$	**M1**	P(GG) + P(YY) = $\left(\frac{6}{9}\text{ or }\frac{3}{9}\right)\times their\left(\frac{5}{5+x}\right)+\left(\frac{3}{9}\text{ or }\frac{6}{9}\right)\times their\left(\frac{1+x}{5+x}\right)$
	$\frac{6}{9}\times\frac{5}{5+x}+\frac{3}{9}\times\frac{1+x}{5+x}=\frac{8}{15}$ and arrange as a linear equation	**M1**	$15(x+11)=24(x+5)$ OE Accept sum of their products equated to $\frac{8}{15}$ and rearranged to form a linear equation.
	Solve: $x=5$	**A1**	
		3	

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td rowspan="5">4(a)</td><td rowspan="4">
<table>
<tr><th>Penguins</th><th></th><th>Dolphins</th></tr>
<tr><td>9 5</td><td>3</td><td>6</td></tr>
<tr><td>8 5 5 4 2</td><td>4</td><td>1 3 8 9 9</td></tr>
<tr><td>9 8 6 0</td><td>5</td><td>0 1 4 6 6</td></tr>
<tr><td>8 6 1</td><td>6</td><td>0 1 4</td></tr>
<tr><td>2</td><td>7</td><td>1</td></tr>
</table>
Key: 2|4|1 means 42 seconds for Penguins and 41 seconds for Dolphins</td><td>B1</td><td>Correct stem, ignore extra values (not in reverse, not split).
If a split stem-and-leaf plot is used (i.e. stem values are repeated), the remaining B marks are available.</td></tr>
<tr><td>B1</td><td>Correct Penguins labelled on left, leaves in order from right to left and lined up vertically (less than halfway to next column), no commas or other punctuation.</td></tr>
<tr><td>B1</td><td>Correct Dolphins labelled on same diagram, leaves in order and lined up vertically (less than halfway to next column), no commas or other punctuation.

If the correct data for Penguins & Dolphins is transposed, treat as a single error in Penguins and condone in Dolphins.</td></tr>
<tr><td>B1</td><td>Correct key for their diagram, need both clubs labelled and ‘sec’ or ‘s’ stated at least once here, or in leaf headings or title.

If two separate diagrams drawn: SC B1 if both keys meet these criteria.</td></tr>
<tr><td></td><td>4</td><td></td></tr>
</table>

Question	Answer	Marks	Guidance
4(b)			
	For Dolphins, median is 51	**B1**	Plotted on box.
	LQ = 48, UQ = 60	**B1**	Plotted on box.
	Correct end points for whiskers and diagram labelled Dolphins	**B1**	Correct end points of whiskers (36 and 71). Whiskers not through box, not drawn at corners of boxes, diagram labelled.
		3	
4(c)	Dolphins have more consistent times than Penguins or Penguins are faster (have faster times) than Dolphins	**B1**	Reason given in context. Can be reference to either the central tendency or spread.
		1	

Question	Answer	Marks	Guidance
5(a)	$\left[(0.35)^4 \times 0.65 =\right]$ 0.00975	B1	AWRT
		1	
5(b)	$(0.35)^3 \times (0.65)^2 \times 4$	M1	$(0.35)^3 \times (0.65)^2 \times k$, where k is an integer. $1 \leqslant k \leqslant 5$ no + or −
	= 0.0725	A1	
		2	
5(c)	**Method 1**		
	[1 – P(5, 6, 7) =] $1 - ({}^7C_5\, 0.65^5\, 0.35^2 + {}^7C_6\, 0.65^6 0.35^1 + 0.65^7)$ [= 1 – (0.29848 + 0.18478 + 0.049022)]	M1	One term ${}^7C_x (p)^x (1-p)^{7-x}$, $0 < p < 1, 0 < x < 7$.
		A1	Correct expression, accept unsimplified, no terms omitted leading to final answer
	= 0.468	B1	AWRT
	Method 2		
	[P(0, 1, 2, 3, 4) =] $0.35^7 + {}^7C_1\, 0.65^1\, 0.35^6 + {}^7C_2\, 0.65^2\, 0.35^5 + {}^7C_3\, 0.65^3\, 0.35^4 + {}^7C_4\, 0.65^4\, 0.35^3$ [0.00064 + 0.00836 + 0.04660 + 0.14424 + 0.26787]	(M1)	One term ${}^7C_x (p)^x (1-p)^{7-x}$, $0 < p < 1, 0 < x < 7$.
		(A1)	Correct expression, accept unsimplified, no terms omitted leading to final answer.
	= 0.468	(B1)	AWRT
		3	

Question	Answer	Marks	Guidance
5(d)	[Mean =] $84 \times 0.65 = 54.6$ [Var =] $84 \times 0.65 \times 0.35 = 19.11$	**B1**	54.6 and 19.11 seen, allow unsimplified. May be seen in the standardisation formula ([σ =] 4.371 or $\frac{7\sqrt{39}}{10}$ implies correct variance). Incorrect notation is penalised, but condone use of values in standardisation formula.
	$P(X > 50) = P\left(Z > \frac{50.5 - 54.6}{\sqrt{19.11}}\right)$	**M1**	Substituting *their* μ and *their* positive σ into the ±standardising formula (any number), not *their* σ^2, or $\sqrt{their\,\sigma}$.
		M1	Use continuity correction 49.5 or 50.5 in *their* standardisation formula. Note: $\frac{\pm 4.1}{\sqrt{19.11}}$ or $\pm\frac{4.1}{4.371}$ seen gains M2 BOD.
	$P(Z > -0.9379) = \Phi(0.9379)$	**M1**	Appropriate area Φ, from final process, must be a probability.
	0.826	**A1**	$0.8255 < p \leq 0.826$
		5	

Question	Answer	Marks	Guidance
6(a)	$\left[\dfrac{9!}{2!3!}=\right]$ 30240	B1	
		1	
6(b)	**Method 1: Number of arrangements with E at each end – Number of arrangements with E at each end and the three Rs together**		
	$\dfrac{7!}{3!}-5!=$	B1	$\dfrac{7!}{3!}-e$, ${}^7P_4-e$, e a positive integer.
		M1	$f-\dfrac{5!}{r!}$, $f>120$, $r=1, 2$
	720	A1	If no marks scored **SC B1** for 840 – 120 = 720.
	Method 2: Number of arrangements with E at each end and no Rs together + No of arrangements with E at each end and two Rs together		
	${}^5C_3\times 4!+{}^4C_1\times 5!$ or $\dfrac{{}^5P_3}{3!}\times 4!+{}^5P_2\times 4!$	(B1)	One of ${}^5C_3\times 4!$, $\dfrac{{}^5P_3}{3!}\times 4!$, ${}^4C_1\times 5!$ or ${}^5P_2\times 4!$ seen.
		(M1)	$a\times 4!+b\times 5!$ where a and b are integers between 1 and 10 inclusive, or $c\times 4!+d\times 4!$ where c and d are integers between 1 and 20 inclusive.
	240 + 480 = 720	(A1)	
		3	

Question	Answer	Marks	Guidance
6(c)	**Method 1**		
	Group of 5 3 Rs 2 Es = 1 3 Rs 1 E = ${}^2C_1 \times {}^4C_1$ = 8 3 Rs 0 Es = ${}^4C_2$ = 6 Group of 4 3 Rs 1 E = ${}^2C_1$ = 2 3 Rs 0 Es = ${}^4C_1$ = 4	**B1**	Correct no of ways for two correct identified scenarios other than three Rs two Es.
	[Total =] 21	**M1**	No of ways for five correct identified scenarios added or correct.
	[Number of ways of splitting into the two groups =] ${}^9C_5$ (= 126) seen as a denominator	**M1**	Accept evaluated, accept ${}^9C_4$.
	Probability = $\frac{21}{126} = \frac{1}{6}$ (0.167)	**A1**	

Question	Answer	Marks	Guidance
6(c)	**Method 2**		
	3Rs in Group of 5 = ${}^6C_2$ = 15 3Rs in Group of 4 = ${}^6C_1$ = 6	**(B1)**	One correct case evaluated accurately and linked with correct scenario.
	[Total =] 21	**(M1)**	No of ways for two correct scenarios added or correct.
	[Number of ways of splitting into the two groups =] ${}^9C_5$ (= 126) seen as a denominator	**(M1)**	Accept evaluated, accept ${}^9C_4$.
	Probability = $\frac{21}{126} = \frac{1}{6}$ (0.167)	**(A1)**	
	Method 3: Considering the possible positions of R within the groups		
	3Rs in Group of 5 $\frac{5}{9} \times \frac{4}{8} \times \frac{3}{7} = \frac{15}{126}$	**(B1)**	For one correct product unsimplified and linked with correct scenario.
	3Rs in Group of 4 $\frac{4}{9} \times \frac{3}{8} \times \frac{2}{7} = \frac{6}{126}$	**(M1)**	For second correct product.
	$\frac{15}{126} + \frac{6}{126}$	**(M1)**	For adding probabilities of two correct scenarios or correct.
	Probability = $\frac{21}{126} = \frac{1}{6}$ (0.167)	**(A1)**	

Question	Answer	Marks	Guidance
6(c)	**Method 4: Probability method**		
	Group of 5 2Es $\frac{3}{9}\times\frac{2}{8}\times\frac{1}{7}\times\frac{2}{6}\times\frac{1}{5}\times\frac{5!}{3!2!}=\frac{1}{126}$	**(B1)**	Two correct probabilities linked with correct scenarios, accept unsimplified.
	1E $\frac{4}{9}\times\frac{3}{8}\times\frac{2}{7}\times\frac{1}{6}\times\frac{2}{5}\times\frac{5!}{3!}=\frac{8}{126}$ 0E $\frac{3}{9}\times\frac{2}{8}\times\frac{1}{7}\times\frac{4}{6}\times\frac{3}{5}\times\frac{5!}{3!2!}=\frac{6}{126}$ Group of 4 $\frac{3}{9}\times\frac{2}{8}\times\frac{1}{7}\times\frac{6}{6}\times\frac{4!}{3!}=\frac{6}{126}$	**(M1)**	Four probabilities with denominators including a factor of $9 \times 8 \times 7 \times 6 \times n$, where n is 1 or 5.
	$\frac{1+8+6+6}{126}$	**(M1)**	Probabilities of four correct scenarios added or correct.
	Probability $=\frac{21}{126}=\frac{1}{6}$ (0.167)	**(A1)**	
		4	

Cambridge International AS & A Level

MATHEMATICS **9709/51**

Paper 5 Probability & Statistics 1 **October/November 2023**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2023 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics-Specific Marking Principles

1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.

2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.

3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.

4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).

5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.

6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate’s intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1(a)	[IQR =] 31 – 23.7	**M1**	30.5 < UQ < 31.25 – 23.25 < LQ ⩽ 24 Evidence of graph use must be seen at least once.
	7.3	**A1**	7.0 ⩽ IQR ⩽ 7.5 If M0 scored, **SC B1** for 7.0 ⩽ IQR ⩽ 7.5 www.
		2	
1(b)	[65% of 120 =]78	**B1**	Seen or implied by use on graph.
	28.5	**B1**	28 < ans < 29
		2	

Question	Answer	Marks	Guidance
2(a)	$\left(\frac{21}{36}\right)^4\left(\frac{15}{36}\right)$	**M1**	$(1-p)^4 \times p$, $0 < p < 1$
	$= \frac{12005}{248832}$, 0.0482	**A1**	0.0482454… to at least 3SF.
		2	

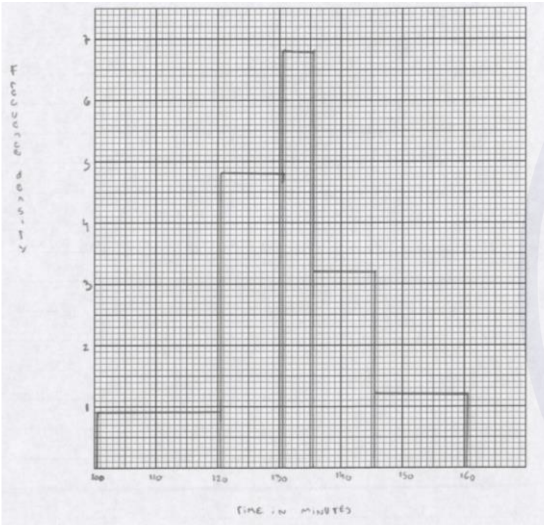
Question	Answer	Marks	Guidance
2(b)	**Method 1**		
	$[\mathrm{P}(X \leqslant 4) =]\ 1-\left(\frac{21}{36}\right)^4$	**M1**	$1-b^r$, b = *their* $(1-p)$ in **2(a)** or correct; r = 4, 5.
	$= \frac{18335}{20736}$, 0.884	**A1**	0.884211… to at least 3SF.
		2	
	Method 2		
	$[\mathrm{P}(X \leqslant 4) =]\ \frac{15}{36}+\frac{15}{36}\times\frac{21}{36}+\frac{15}{36}\times\left(\frac{21}{36}\right)^2+\frac{15}{36}\times\left(\frac{21}{36}\right)^3$	**M1**	$p + p(1-p) + p(1-p)^2 + p(1-p)^3$ $\left[+p\left(1-p\right)^4\right]$ FT from **2(a)** or correct.
	$= \frac{18335}{20736}$, 0.884	**A1**	0.884211… to at least 3SF.
		2	

Question	Answer	Marks	Guidance
2(c)	**Method 1**		
	[P(0,1,2) =] ${}^8C_0\left(\frac{5}{12}\right)^0\left(\frac{7}{12}\right)^8 + {}^8C_1\left(\frac{5}{12}\right)^1\left(\frac{7}{12}\right)^7 + {}^8C_2\left(\frac{5}{12}\right)^2\left(\frac{7}{12}\right)^6$	**M1**	One term ${}^8C_x(q)^x(1-q)^{8-x}$, $0 < q < 1,\ x \neq 0, 8$.
	0.01341 + 0.07661 + 0.1915	**A1 FT**	Correct expression, accept unsimplified, no terms omitted leading to final answer. FT only with unsimplified expression.
	= 0.282	**B1**	$0.2815 \leqslant q \leqslant 0.282$
	Method 2		
	[1 – P(3,4,5,6,7,8) =] $1 - \left({}^8C_3\left(\frac{5}{12}\right)^3\left(\frac{7}{12}\right)^5 + {}^8C_4\left(\frac{5}{12}\right)^4\left(\frac{7}{12}\right)^4 + \ldots + {}^8C_7\left(\frac{5}{12}\right)^7\left(\frac{7}{12}\right)^1 + {}^8C_8\left(\frac{5}{12}\right)^8\left(\frac{7}{12}\right)^0\right)$	**M1**	One term ${}^8C_x(q)^x(1-q)^{8-x}$, $0 < q < 1,\ x \neq 0, 8$.
	$= 1 - (0.2736 + 0.2443 + \ldots + 0.01017 + 9.084\times10^{-4})$	**A1 FT**	Correct expression, accept unsimplified, no terms omitted leading to final answer. FT only with unsimplified expression.
	= 0.282	**B1**	$0.2815 \leqslant q \leqslant 0.282$
		3	

Question	Answer	Marks	Guidance
3(a)	[P(X < 76) =] P($Z < \frac{76-80.5}{6.6}$)	**M1**	Use of ± standardisation formula with 76, 80.5 and 6.6, condone 6.6^2 or $\sqrt{6.6}$, no continuity correction.
	[= Φ(−0.6818) = 1 −Φ(0.6818) =] 1 – 0.7524 = 0.2476	**M1**	Calculating the appropriate probability area (leading to their final answer).
	24.8%	**A1**	24.75% < ans ⩽ 24.8% (percentage value required). If A0 scored, **SC B1** for 24.75% < ans ⩽ 24.8% www.
		3	
3(b)	[% of large eggs = 100 – 40 – 24.76 = 35.24] [P(Z > $\frac{x-80.5}{6.6}$) $= 0.40 + 0.2476 = 0.6476$] $\frac{x-80.5}{6.6} = 0.378$	**B1**	0.378 ⩽ z < 0.3791 or −0.3791 < z ⩽ −0.378 seen.
		M1	Use of ± standardisation formula with x, 80.5, 6.6 and a z-value (not 0.6476, 0.3524, 0.4, 0.2476) (treat ±0.38 as a z-value), not 6.6^2, not $\sqrt{6.6}$, no continuity correction.
	$x = 83[.0]$	**A1**	awrt 83.0
		3	

Question	Answer	Marks	Guidance
3(c)	Mean $= 150 \times 0.4 = 60$ Var $= 150 \times 0.4 \times 0.6 = 36$	**B1**	60 and 36 seen, allow unsimplified.
	$P(X > 68) = P\left(Z > \dfrac{68.5 - 60}{\sqrt{36}}\right)$	**M1**	Substituting *their* 60 and *their* 6 into ±standardisation formula (any number for 68.5), condone *their* σ^2 and *their* $\sqrt{\sigma}$.
		M1	Using continuity correction 67.5 or 68.5 in *their* standardisation formula.
	$P(Z > 1.417) = 1 - \Phi(1.417)$ [= 1 – 0.9217]	**M1**	Appropriate area Φ, from final process, must be a probability.
	0.0783	**A1**	$0.07825 < p \leqslant 0.0783$ If A0 scored, **SC B1** for $0.07825 < p \leqslant 0.0783$.
		5	

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td rowspan="5">4(a)</td><td><table><tr><td>Class width</td><td>20</td><td>10</td><td>5</td><td>10</td><td>15</td></tr><tr><td>Frequency density</td><td>0.9</td><td>4.8</td><td>6.8</td><td>3.2</td><td>1.2</td></tr></table></td><td>M1</td><td>At least 4 frequency densities calculated by $\frac{f}{cw}$ e.g. $\frac{18}{20}$ (condone $\frac{f}{cw \pm 0.5}$ if unsimplified).

Accept unsimplified, may be read from graph using their scale, no lower than 1cm = 1 fd.</td></tr>
<tr><td rowspan="3"></td><td>A1</td><td>All bar heights correct on graph (no FT), using their suitable linear scale with at least 3 values indicated, no lower than 1cm = 1 fd.</td></tr>
<tr><td>B1</td><td>Bar ends at 120.5, 130.5, 135.5, 145.5, 160.5. 5 bars drawn with a horizontal linear scale, no lower than 1 cm = 10 min, with at least 3 values indicated. 100 $\leqslant$ horizontal scale $\leqslant$ 160.</td></tr>
<tr><td>B1</td><td>Axes labelled frequency density (fd), time (t) and minutes (min, m) oe, or an appropriate title.
(Axes may be reversed).</td></tr>
<tr><td></td><td>4</td><td></td></tr>
</table>

Question	Answer	Marks	Guidance
4(b)	[Midpoints] 110.5 125.5 133 140.5 153	**B1**	At least 4 correct mid-points seen, may be by data table or used in formula.
	Mean = $\frac{18\times110.5+48\times125.5+34\times133+32\times140.5+18\times153}{150}$ $=\frac{1989+6024+4522+4496+2754}{150}$	**M1**	Correct formula for mean using midpoints ±0.5, condone 1 midpoint error within class.
	= 131.9	**A1**	Accept 132, $131\frac{9}{10}$, or $\frac{1319}{10}$. Must be identified.
	Variance = $\frac{18\times110.5^2+48\times125.5^2+34\times133^2+32\times140.5^2+18\times153^2}{150}-(their\,131.9)^2$	**M1**	Appropriate variance formula with *their* 5 midpoints within class (not upper bound, lower bound, class width, frequency density, frequency or cumulative frequency). Condone 1 error. If correct midpoints seen, accept $\left\{\frac{3200+41400+194400+157300+153600}{150}\, or\, \frac{2630272.5}{150}\right\}$ $-\{131.9^2\, or\, 17397.61\}$.
	[= 137.54] [Standard deviation =] 11.7	**A1**	11.7277448… to at least 3SF. Accept 11.6 ⩽ σ < 11.95 www. If M0 awarded, **SC B1** 11.6 ⩽ σ < 11.95 www.
		5	

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td rowspan="2">5(a)</td><td>$0.28+6p=1,\quad p=0.12$</td><td>B1</td><td>Using sum of probabilities = 1 to form an equation.
Accept 0.28 + p + 2p + 3p = 1, p = 0.12.
Substitution of 0.12 into the expression scores B0.</td></tr>
<tr><td></td><td>1</td><td></td></tr>
<tr><td rowspan="4">5(b)</td><td rowspan="2">[For fair spinners (blue and green), probability of any score is 0.25
Scenarios to give total 4 or less:]
<table>
<tr><td>R</td><td>B</td><td>G</td><td></td><td></td></tr>
<tr><td>1</td><td>1</td><td>2</td><td>$0.28\times(0.25)^2$</td><td>= 0.0175</td></tr>
<tr><td>1</td><td>2</td><td>1</td><td>$0.28\times(0.25)^2$</td><td>= 0.0175</td></tr>
<tr><td>1</td><td>1</td><td>1</td><td>$0.28\times(0.25)^2$</td><td>= 0.0175</td></tr>
<tr><td>2</td><td>1</td><td>1</td><td>$0.12\times(0.25)^2$</td><td>= 0.0075</td></tr>
</table></td><td>B1</td><td>Correct probability for 1 identified scenario, accept unsimplified, www.</td></tr>
<tr><td>M1</td><td>Add values of 4 correct scenarios, may be implied by correct unsimplified expressions. No incorrect/repeated scenarios.</td></tr>
<tr><td>0.06</td><td>A1</td><td>If A0 scored, SC B1 for 0.06 www.</td></tr>
<tr><td></td><td>3</td><td></td></tr>
</table>

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td>5(c)</td><td>[P(X is odd) = 0.28 + 2×0.12 or 0.24]= 0.52[0]</td><td>B1</td><td>Seen alone or as the denominator of a conditional probability fraction. Accept unsimplified.</td></tr>
<tr><td></td><td>
<table>
<tr><td>R</td><td>B</td><td>G</td><td></td><td></td></tr>
<tr><td>1</td><td>1</td><td>1</td><td>$0.28\times(0.25)^2$</td><td>= 0.0175</td></tr>
<tr><td>1</td><td>1</td><td>2</td><td>$0.28\times(0.25)^2$</td><td>= 0.0175</td></tr>
<tr><td>1</td><td>1</td><td>3</td><td>$0.28\times(0.25)^2$</td><td>= 0.0175</td></tr>
<tr><td>1</td><td>1</td><td>4</td><td>$0.28\times(0.25)^2$</td><td>= 0.0175</td></tr>
<tr><td>1</td><td>2</td><td>1</td><td>$0.28\times(0.25)^2$</td><td>= 0.0175</td></tr>
<tr><td>1</td><td>2</td><td>2</td><td>$0.28\times(0.25)^2$</td><td>= 0.0175</td></tr>
<tr><td>1</td><td>3</td><td>1</td><td>$0.28\times(0.25)^2$</td><td>= 0.0175</td></tr>
<tr><td>1</td><td>4</td><td>1</td><td>$0.28\times(0.25)^2$</td><td>= 0.0175</td></tr>
<tr><td>3</td><td>1</td><td>1</td><td>$0.24\times(0.25)^2$</td><td>= 0.015</td></tr>
</table>
</td><td>M1</td><td>Values of at least 5 identified correct scenarios added, accept unsimplified, condone incorrect scenarios in calculation.</td></tr>
<tr><td></td><td>[P(product of 3 scores $\leqslant$ 4 $\cap$ X is odd) =] $0.28\times(0.25)^2\times 8+0.24\times(0.25)^2$</td><td>M1</td><td>$0.28\times(0.25)^2\times x+0.24\times(0.25)^2$,
or 0.0175 × x + 0.015 where x = 4, 5, 6, 7, or 8.
Seen alone or as numerator/denominator of a conditional probability fraction.</td></tr>
</table>

Question	Answer	Marks	Guidance
5(c)	$\left[\mathrm{P}(\text{product of 3 scores} \leqslant 4 \mid X \text{ is odd}) = \dfrac{\mathrm{P}(\text{product of 3 scores} \leqslant 4 \cap X \text{ is odd})}{\mathrm{P}(X \text{ is odd})} = \right]$ $\dfrac{0.155}{0.52}$	**M1**	$\dfrac{0.28 \times (0.25)^2 \times x + 0.24 \times (0.25)^2}{0.28 + 0.24}$ $x = 4, 5, 6, 7, 8$ or $\dfrac{\textit{their} \text{ identified P(product of 3 scores is 4 or less and } X \text{ is odd)}}{\textit{their} \text{ identified P(odd)}}$.
	$= 0.298, \dfrac{155}{520}, \dfrac{31}{104}$	**A1**	0.2980769… to at least 3SF.
		5	

Question	Answer	Marks	Guidance
6(a)	${}^5C_2 \times 2$	**M1**	${}^5C_2 \times r$, r = positive integer, 1 implied, no addition.
		M1	$s \times 2$, $s = {}^5C_2$ or ${}^5P_2$ or if ${}^5C_2$ or ${}^5P_2$ not present, s = a single integer > 1 or $t! \times 2$, $2 \leqslant t \leqslant 8$, no other terms.
	20	**A1**	
		3	

Question	Answer	Marks	Guidance
6(b)	**Method 1**		
	${}^6C_2 \times 2 \times 2 \times 2 \times 4!$	**M1**	${}^6C_2 \times 2 \times 2 \times 2 \times t$, t = positive integer $\geqslant 1$. ${}^6P_2 \times 2 \times 2 \times t$, t = positive integer $\geqslant 1$.
		M1	$u \times 4!$, u = positive integer > 1.
	2880	**A1**	If A0 scored, **SC B1** for 2880 nfww.
	Method 2		
	$6! \times 2 \times 2$	**M1**	$6! \times v$, v = positive integer $\geqslant 1$.
		M1	$w \times 2 \times 2$, w = positive integer > 1. condone $w \times 4$, w = positive integer > 1.
	2880	**A1**	If A0 scored, **SC B1** for 2880 nfww.
		3	

Question	Answer	Marks	Guidance
6(c)	**Method 1:** Number of arrangements with Rajid and Sue together – Number of arrangements with Rajid and Sue together and at end of line		
	$7!\times 2 - 6!\times 4$	**M1**	$7! \times 2 - a$, a = positive integer > 1.
		M1	$b - 6! \times 4$, b = positive integer > 2880.
		M1	$7! \times c - 6! \times d$, c = 1,2 and d = 1, 4.
	= 7200	**A1**	If A0 scored, **SC B1** for 7200 nfww.
	Method 2: Arrangements of 6 people and then place Rajid and Sue		
	$6!\times 2\times 5$	**M1**	$6! \times e \times f$, e, f = positive integers $\geqslant$ 1.
		M1	$6! \times 2 \times f$, f = positive integer $\geqslant$ 1. If 5! Used, **SC B1** $5! \times 2 \times f$, f = positive integer > 1.
		M1	$6! \times e \times 5$, e = positive integer $\geqslant$ 1.
	7200	**A1**	If A0, scored **SC B1** for 7200 nfww.
	Method 3: Friends at ends picked first F ^ RS ^ ^ ^ F		
	${}^6P_2 \times 5! \times 2$	**M1**	${}^6P_2 \times e \times f$, e, f = positive integers $\geqslant$ 1.
		M1	${}^6P_2 \times 5! \times f$, f = positive integer $\geqslant$ 1. Condone ${}^6C_2 \times 5! \times f$, f = positive integer $\geqslant$ 1.
		M1	${}^6P_2 \times e \times 2$, e = positive integer $\geqslant$ 1. Condone ${}^6C_2 \times e \times 2$, e = positive integer $\geqslant$ 1.
	7200	**A1**	If A0 scored, **SC B1** for 7200 nfww.

Question	Answer	Marks	Guidance
6(c)	**Method 4:** RS placed in different possible positions		
	^ RS ^ ^ ^ ^ ^ $^6P_1 \times 2 \times 5! = 1440$ ^ ^ RS ^ ^ ^ ^ $^6P_2 \times 2 \times 4! = 1440$ ^ ^ ^ RS ^ ^ ^ $^6P_3 \times 2 \times 3! = 1440$ ^ ^ ^ ^ RS ^ ^ $^6P_4 \times 2 \times 2! = 1440$ ^ ^ ^ ^ ^ RS ^ $^6P_5 \times 2 \times 1! = 1440$	**M1**	$^6P_n \times a \times (6-n)!$, a = positive integer, $1 \leqslant n \leqslant 5$ seen once.
		M1	$^6P_n \times 2 \times (6-n)!$, a = positive integer, $1 \leqslant n \leqslant 5$ seen at least 3 times in identified scenarios.
		M1	Add 5 values of appropriate scenarios only. No additional, incorrect or repeated scenarios. Accept unsimplified.
	7200	**A1**	If A0 scored, **SC B1** for 7200 nfww.
		4	

Cambridge Assessment International Education

Cambridge International AS & A Level

MATHEMATICS	**9709/52**
Paper 5 Probability & Statistics 1	**October/November 2023**
MARK SCHEME	
Maximum Mark: 50	

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2023 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

This document consists of **21** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics-Specific Marking Principles

1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.

2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.

3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.

4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).

5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.

6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1(a)	$p+r+0.55=1$	**M1**	Using sum of probabilities = 1 to form an equation. Accept $p + r = 0.45$ oe.
	$p+2r+0.45=1.1$	**M1**	Use $E(X) = 1.1$ to form an equation. Accept $p + 2r = 0.65$ oe. NB: These marks can be gained in either order; the second M may have an algebraic substitution.
	$p=0.25,\ r=0.2$	**A1**	If both Ms not awarded, **SC B1** for $p=0.25,\ r=0.2$ stated.
		3	
1(b)	[Var(X) =] $\left[0.4\times 0^2+\right] \textit{their}\ 0.25\left[\times 1^2\right]+\left(\textit{their}\ 0.2\right)\times 2^2+0.15\times 3^2-1.1^2$ [=[0+] 0.25 + 0.8 + 1.35 – 1.21]	**M1**	Correct formula for variance method using their probability distribution table, 0 < their P(x) < 1.
	= 1.19, $1\frac{19}{100}$	**A1**	If M0 awarded, **SC B1** for 1.19 www. $\frac{119}{100}$ is A0.
		2	

Question	Answer	Marks	Guidance
2(a)	**Method 1:**		
	[P(5) = 0.2] [P(X < 7) =] $1-0.8^6$	**M1**	$1 – 0.8^n$, n = 6, 7.
	= 0.738, $\frac{11529}{15625}$	**A1**	0.737856 to at least 3SF.
	Method 2:		
	[P(X < 7) =] $0.2+0.2\times0.8+0.2\times0.8^2+0.2\times0.8^3+0.2\times0.8^4+0.2\times0.8^5$	**M1**	$0.2+0.2\times0.8+0.2\times0.8^2+0.2\times0.8^3+$ $0.2\times0.8^4+0.2\times0.8^5\left(+0.2\times0.8^6\right)$
	= 0.738, $\frac{11529}{15625}$	**A1**	0.737856 to at least 3SF.
		2	

Question	Answer	Marks	Guidance
2(b)	**Method 1:**		
	[P(5, 6, 7) =] $^{10}C_5\,(0.2)^5(0.8)^5 + {}^{10}C_6\,(0.2)^6(0.8)^4 + {}^{10}C_7\,(0.2)^7(0.8)^3$	**M1**	One term: $^{10}C_x\,(p)^x(1-p)^{10-x},\ 0<p<1,\ x\neq 0, 10.$
	[$0.02642 + 5.505\times10^{-3} + 7.864\times10^{-4}$]	**A1**	Correct expression, accept unsimplified, no terms omitted leading to final answer.
	= 0.0327	**B1**	awrt
	Method 2:		
	[P(X < 8) – P($X \leqslant 4$) = 1 – P($X \geqslant 8$) – P($X \leqslant 4$) =] $1 - \{{}^{10}C_8(0.2)^8(0.8)^2 + {}^{10}C_9(0.2)^9 0.8 + (0.2)^{10}\}$ $- \{(0.8)^{10} + {}^{10}C_1(0.2)(0.8)^9 + {}^{10}C_2(0.2)^2(0.8)^8 + {}^{10}C_3(0.2)^3(0.8)^7 + {}^{10}C_4(0.2)^4(0.8)^6\}$	**M1**	One term: $^{10}C_x\,(p)^x(1-p)^{10-x},\ 0<p<1,\ x\neq 0, 10.$
	[$1-\{7.373\times10^{-5}+4.096\times10^{-6}+1.024\times10^{-7}\}-$ $\{0.1074+0.2684+0.3020+0.2013+0.08808\}$]	**A1**	Correct expression, accept unsimplified, no terms omitted leading to final answer.
	= 0.0327	**B1**	awrt
		3	

Question	Answer	Marks	Guidance
3	[Mean = $200 \times 0.15 =$] 30 [Var = $200 \times 0.15 \times 0.85 =$] 25.5	B1	30 and 25.5, $25\frac{1}{2}, \frac{51}{2}$ seen, allow unsimplified. May be seen in standardisation formula. [σ =] $5.049 \leqslant \sigma \leqslant 5.05[0]$, $\frac{\sqrt{102}}{2}$ implies correct variance. Correct notation is required.
	[P(X > 40) =] P($Z > \frac{40.5 - 30}{\sqrt{25.5}}$)	M1	Substituting *their* mean and *their* positive 5.04975 into ±standardisation formula (any number for 40.5), not *their* σ^2 or $\sqrt{their\ \sigma}$.
		M1	Using continuity correction 39.5 or 40.5 in *their* standardisation formula.
	[1 – Φ (2.079)] 1 – 0.9812	M1	Appropriate area Φ, from final process, must be a probability.
	= 0.0188	A1	$0.01875 < p \leqslant 0.0188$
		5	

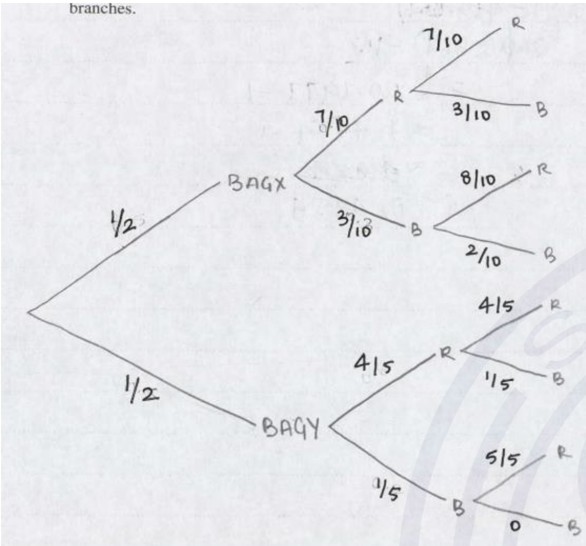
<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td rowspan="5">4(a)</td><td rowspan="2"><table><tr><th>Aces</th><th></th><th>Jets</th></tr><tr><td>9 8 6 4</td><td>16</td><td>6 8</td></tr><tr><td>4 3 1</td><td>17</td><td>0 4 4 5</td></tr><tr><td>2 2 1 0</td><td>18</td><td>1 1 8 8</td></tr><tr><td></td><td>19</td><td>0</td></tr></table></td><td>B1</td><td>Correct stem, ignore extra values (not in reverse, not split).

If a split stem-and-leaf plot is used (i.e., stem values are repeated), the remaining B marks are available.</td></tr>
<tr><td>B1</td><td>Correct Aces labelled on left, leaves in order from right to left and lined up vertically, no commas or other punctuation.</td></tr>
<tr><td rowspan="2">Key: 1 | 17 | 0 means 171 cm for the Aces and 170 cm for the Jets</td><td>B1</td><td>Correct Jets labelled on same diagram, leaves in order and lined up vertically, no commas or other punctuation.</td></tr>
<tr><td>B1</td><td>Correct key for their diagram, need both teams labelled and ‘cm’ stated at least once here, or in leaf headings or title.</td></tr>
<tr><td></td><td>4</td><td></td></tr>
</table>

Question	Answer	Marks	Guidance
4(b)	Median = 173 [cm]	**B1**	Accept Q_2; must be identified.
	[IQR =] 181 – 168	**M1**	$180 \leqslant UQ \leqslant 182 - 166 \leqslant LQ \leqslant 169$ Implied if both quartile values are stated and an appropriate IQR calculated accurately.
	13 [cm]	**A1**	www If M0 scored **SC B1** for 13 www.
		3	
4(c)	Jets have a greater variety of heights. Jets have a wider range of height. Jets have a greater/larger/bigger/wider/'more' spread of heights. Aces have a smaller variety of height etc…	**B1**	[Jets IQR = 18 cm, Range = 24 cm Aces IQR = *their* **4(b)**, Range = 18 cm] Comment about spread in context, must include height. Comparison of values does not score until a comment in context is made. If values for range or IQR are stated, they must be correct or FT from **4(b)**. If more than one comment about spread, mark the final comment. Additional comments about central tendency score B0.
		1	

Question	Answer	Marks	Guidance
5(a)(i)	[P(X < 170) =] P(Z < $\frac{170-166}{10}$)	**M1**	Use of ± standardisation formula with 170, 166 and 10 substituted appropriately, condone $10^2, \sqrt{10}$, condone continuity correction ±0.5.
	[= P(Z < 0.4) =] 0.655	**A1**	$0.655 \leqslant p < 0.6555$ If M0 awarded, **SC B1** for correct answer www.
		2	
5(a)(ii)	$\left[P\left(Z > \frac{h-166}{10}\right) = 0.4\right]$ $\frac{h-166}{10} = 0.253$	**B1**	$0.253 \leqslant z \leqslant 0.2535$ or $-0.2535 \leqslant z \leqslant -0.253$ seen.
		M1	Use of the ±standardisation formula with h, 166, 10 and a z-value (not 1 – z-value), not $10^2, \sqrt{10}$, no continuity correction.
	$h = 168.53$	**A1**	If M0 scored, **SC B1** for $168.53 \leqslant h \leqslant 168.535$, 168.5. **SC B1** for 168.54 from z = 0.254.
		3	

Question	Answer	Marks	Guidance
5(b)	$\left[P(X>0)=P\left(Z>\frac{0-\mu}{\sigma}\right)=\right]P\left(Z>\frac{[0]-\mu}{\frac{2}{3}\mu}\right)$ Or $P\left(Z>\frac{[0]-\frac{3}{2}\sigma}{\sigma}\right)$	**M1**	Use of the ±standardisation formula with 0, μ and $\frac{2}{3}\mu$ substituted for σ. Or use of the ±standardisation formula with 0, σ and $\frac{3}{2}\sigma$ substituted for μ.
	= P($Z > -1.5$)	**A1**	-1.5 seen, no additional terms (e.g. $x - 1.5$ A0). Condone $Z < 1.5$. If M0 scored, **SC B1** $Z > -1.5$ or $Z < 1.5$ seen www.
	= 0.933 final answer	**A1**	$0.933 \leqslant p < 0.9333$. If M0 scored, **SC B1** $0.933 \leqslant p < 0.9333$ seen www.
		3	

Question	Answer	Marks	Guidance
6(a)	 .	**B1**	1st column, 2 branches identified X, Y with probabilities ½, ½ indicated.
		B1	2nd column (1st marble pick) of 4 branches identified R B R B (oe) and probabilities $\frac{7}{10},\frac{3}{10},\frac{4}{5},\frac{1}{5}$ indicated appropriately.
		B1	3rd column (2nd marble pick) of 8 branches identified R B R B R B R [B] (oe) and probabilities $\frac{7}{10},\frac{3}{10},\frac{8}{10},\frac{2}{10},\frac{4}{5},\frac{1}{5},1,[0]$. Condone omission of YBB branch if YBR branch is fully correct. Ignore any additional columns of branches. If separate tree diagrams for bags X and Y, B0B1B1 max if bags clearly identified.
		3	

Question	Answer	Marks	Guidance
6(b)	[P(both same colour) = P(BB) + P(RR) = P(XBB) + P(XRR) + P(YRR) =] $\frac{1}{2}\times\frac{3}{10}\times\frac{2}{10}+\frac{1}{2}\times\frac{7}{10}\times\frac{7}{10}+\frac{1}{2}\times\frac{4}{5}\times\frac{4}{5}$ $\left[=\frac{6}{200}+\frac{49}{200}+\frac{16}{50},\ 0.03+0.245+0.32\right]$	**B1 FT**	$\left[\mathrm{P(BB)}=\right]\left[\frac{1}{2}\times\frac{3}{10}\times\frac{2}{10}\left[+\frac{1}{2}\times\frac{1}{5}\times 0\right]=\right]\frac{6}{200}$ seen. Accept unsimplified. FT from **6(a)** unsimplified only with 3 term probabilities.
		B1 FT	Either [P(XRR) =] $\frac{1}{2}\times\frac{7}{10}\times\frac{7}{10}$ or [P(YRR) =] $\frac{1}{2}\times\frac{4}{5}\times\frac{4}{5}$ seen. FT from **6(a)** unsimplified only with 3 term probabilities.
		M1	[P(BB) + P(XRR) + P(YRR) =] $\mathit{their}\,\frac{6}{200}+\mathit{their}\,\frac{49}{200}+\mathit{their}\,\frac{16}{50}$ Accept unsimplified, consistent with tree diagram if not clearly identified by notation.
	$=\frac{119}{200}$, 0.595	**A1**	
		4	Special case: if ½ omitted consistently in the tree diagram and the calculation (i.e., no probability for picking the bags), no FT. **SC B1** [P(BB) =] $\frac{3}{10}\times\frac{2}{10}\left[+\frac{1}{5}\times 0\right]$ **SC B1** [P(RR) =] $\frac{7}{10}\times\frac{7}{10}+\frac{4}{5}\times\frac{4}{5}$ **SC B1** $\frac{3}{10}\times\frac{2}{10}\left[+\frac{1}{5}\times 0\right]+\frac{7}{10}\times\frac{7}{10}+\frac{4}{5}\times\frac{4}{5}$

Question	Answer	Marks	Guidance
6(c)	$\left[P(\text{ bag } Y \mid \text{different colours}) = \left(\dfrac{\text{P}(\text{bag } Y \cap \text{different colours})}{\text{P}(\text{different colours})}\right)\right]$ $\dfrac{\frac{1}{2}\times\frac{4}{5}\times\frac{1}{5}+\frac{1}{2}\times\frac{1}{5}[\times 1]}{1-\mathit{their}\left(\frac{119}{200}\right)}$ or $\dfrac{\frac{1}{2}\times\frac{4}{5}\times\frac{1}{5}+\frac{1}{2}\times\frac{1}{5}[\times 1]}{\frac{1}{2}\times\frac{7}{10}\times\frac{3}{10}+\frac{1}{2}\times\frac{3}{10}\times\frac{8}{10}+\frac{1}{2}\times\frac{4}{5}\times\frac{1}{5}+\frac{1}{2}\times\frac{1}{5}[\times 1]}$	**M1**	FT from *their* **6(a)** and *their* **6(b)** with 3 term probabilities unsimplified only or correct. Accept $\dfrac{\frac{4}{50}+\frac{1}{10}}{\frac{81}{200}}$, $\dfrac{\frac{2}{25}+\frac{1}{10}}{\frac{81}{200}}$, $\dfrac{0.08+0.1}{0.405}$.
	$=\left[\dfrac{\frac{9}{50}}{\frac{81}{200}}\right] = \dfrac{4}{9}$, 0.444	**A1**	Accept $\dfrac{36}{81}$, $0.\dot{4}$.
		2	Special case: if ½ omitted consistently in the tree diagram and the calculation (ie no probability for picking the bags), no FT. **SC B1** $\dfrac{\frac{4}{5}\times\frac{1}{5}+\frac{1}{5}[\times 1]}{1-\mathit{their}\ \mathbf{6(b)}}$, or $\dfrac{\frac{4}{5}\times\frac{1}{5}+\frac{1}{5}[\times 1]}{\frac{7}{10}\times\frac{3}{10}+\frac{3}{10}\times\frac{8}{10}+\frac{4}{5}\times\frac{1}{5}+\frac{1}{5}[\times 1]}$.

Question	Answer	Marks	Guidance
7(a)	$\dfrac{5!\times 4!}{2!\times 2!}$	**M1**	$\dfrac{5!\times 4!}{e}$, e a positive integer, 1 can be implied. No other terms on numerator. No addition etc.
		M1	$\dfrac{f}{2!\times g!}$, f a positive integer, g = 1, 2. No other terms on denominator.
	720	**A1**	
		3	

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td rowspan="13">7(b)</td><td colspan="3">Method 1 Number of arrangements with A at each end – Number of arrangements with A at each end and 2 Ds together.</td></tr>
<tr><td rowspan="2">$\frac{7!}{2!} - 6!$</td><td>B1</td><td>$\frac{7!}{2!} - e$, ${}^7P_5 - e$, e a positive integer or 0.</td></tr>
<tr><td>M1</td><td>$d - \frac{6!}{r!}$, $d > 720$, $r = 1, 2$.</td></tr>
<tr><td>= 1800</td><td>A1</td><td></td></tr>
<tr><td colspan="3">Method 2 A ^ ^ ^ ^ ^ A and Ds inserted separately</td></tr>
<tr><td rowspan="2">$5! \times \frac{{}^6P_2}{2!}$ or $5! \times \frac{6 \times 5}{2}$ or $5! \times {}^6C_2$</td><td>B1</td><td>$5! \times s$, s a positive integer, 1 may be implied.</td></tr>
<tr><td>M1</td><td>$t \times \frac{6 \times 5}{u}$, t a positive integer > 1, u =1, 2.</td></tr>
<tr><td>= 1800</td><td>A1</td><td></td></tr>
<tr><td colspan="3">Method 3 Number of arrangements with As at each end and Ds placed in different scenarios.</td></tr>
<tr><td rowspan="2"><table>
<tr><td>Scenario position of first D</td><td></td><td></td></tr>
<tr><td>A D ^ ^ ^ ^ ^ ^ A</td><td>5! × 5</td><td>600</td></tr>
<tr><td>A ^ D ^ ^ ^ ^ ^ A</td><td>5! × 4</td><td>480</td></tr>
<tr><td>A ^ ^ D ^ ^ ^ ^ A</td><td>5! × 3</td><td>360</td></tr>
<tr><td>A ^ ^ ^ D ^ ^ ^ A</td><td>5! × 2</td><td>240</td></tr>
<tr><td>A ^ ^ ^ ^ D ^ ^ A</td><td>5! × 1</td><td>120</td></tr>
</table></td><td>B1</td><td>Correct outcome/value for 1 identified scenario, accept unsimplified, www.</td></tr>
<tr><td>M1</td><td>Add values of 5 correct scenarios, no incorrect/repeated scenarios.</td></tr>
<tr><td>[Total =] 1800</td><td>A1</td><td></td></tr>
<tr><td></td><td>3</td><td></td></tr>
</table>

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td>7(c)</td><td colspan="3">Method 1:</td></tr>
<tr><td></td><td><table>
<tr><td>Scenarios</td><td></td><td></td></tr>
<tr><td>A D ^ ^</td><td>$^{2}C_{1} \times {}^{2}C_{1} \times {}^{5}C_{2}$</td><td>= 40</td></tr>
<tr><td>A D D ^</td><td>$^{2}C_{1} \times [\,{}^{2}C_{2} \times]\ {}^{5}C_{1}$</td><td>= 10</td></tr>
</table></td><td>M1</td><td>At least one correct unsimplified expression for an identified scenario.</td></tr>
<tr><td></td><td>[Total =] 40 + 10 or 50 soi</td><td>A1</td><td>www
If M0 scored, SC B1 [total =]50 www.</td></tr>
<tr><td></td><td>[Total number of selections =] $^{9}C_{4}$ [= 126]</td><td>B1</td><td>Accept evaluated, accept as denominator of probability expression.
Do not condone $^{9}C_{5}$ unless there is a clear explanation for selecting the letters not in the group.</td></tr>
<tr><td></td><td>[Probability =] $\frac{50}{126}$, $\frac{25}{63}$</td><td>B1 FT</td><td>0.396825… to at least 3SF.

FT $\frac{\textit{their}\text{ attempted } 40+10}{126}$. Numerator must be from an attempt to find the 2 appropriate scenarios and must be evaluated.</td></tr>
</table>

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td rowspan="5">7(c)</td><td colspan="3">Method 2:</td></tr>
<tr><td rowspan="3">
<table>
<tr><td>Scenarios</td><td></td><td></td></tr>
<tr><td>A D ^ ^</td><td>$\frac{2}{9}\times\frac{2}{8}\times\frac{5}{7}\times\frac{4}{6}\times{}^4P_2$</td><td>$=\frac{960}{3024}, \frac{20}{63}$</td></tr>
<tr><td>A D D ^</td><td>$\frac{2}{9}\times\frac{2}{8}\times\frac{1}{7}\times\frac{5}{6}\times\frac{{}^4P_3}{2!}$</td><td>$=\frac{240}{3024}, \frac{5}{63}$</td></tr>
</table>

[Total Probability =] $\frac{20}{63}+\frac{5}{63}$
</td><td>M1</td><td>Numerator for at least one correct unsimplified expression for an identified scenario.
$either \frac{2\times2\times5\times4\times12}{a\times b\times c\times d}$ or $\frac{2\times2\times1\times5\times12}{a\times b\times c\times d}$
seen, $6 \leqslant a,b,c,d \leqslant 9$.</td></tr>
<tr><td>A1</td><td>$\frac{2\times2\times5\times4\times12}{a\times b\times c\times d}+\frac{2\times2\times1\times5\times12}{a\times b\times c\times d}$,
$6 \leqslant a,b,c,d \leqslant 9$.
If M0 scored, SC B1 $\frac{1200}{g}$, $g > 1200$, or $\frac{25}{63}$ seen.</td></tr>
<tr><td>B1</td><td>$\frac{p}{9}\times\frac{q}{8}\times\frac{r}{7}\times\frac{s}{6}$ present in all scenarios attempted, accept $\frac{t}{3024}$, $t < 3024$.</td></tr>
<tr><td>$\frac{1200}{3024}, \frac{25}{63}$ oe</td><td>B1 FT</td><td>0.396825… to at least 3SF.

FT $\frac{their \text{ attempted } 960+240}{3024}$. Numerator must be from an attempt to find the 2 appropriate scenarios.</td></tr>
</table>

Question	Answer	Marks	Guidance
7(c)	**Method 3: selecting the A and then selecting 3 any letters and removing selections without Ds.**		
	${}^2C_1 \times ({}^7C_3 - {}^5C_3)$ [$= 2 \times (35 - 10)$]	**M1**	$a \times ({}^7C_3 - {}^5C_3)$, $a = 1, 2$.
	[Total =] 50	**A1**	www If M0 scored, **SC B1** [total =]50 www.
	[Total number of selections =] ${}^9C_4$ [= 126]	**B1**	Accept evaluated, accept as denominator of probability expression. Do not condone ${}^9C_5$ unless there is a clear explanation for selecting the letters not in the group.
	[Probability =] $\frac{50}{126}, \frac{25}{63}$	**B1 FT**	0.396825… to at least 3SF. FT $\frac{\textit{their}\text{ attempted } 40+10}{126}$. Numerator must be from an attempt to find the 2 appropriate scenarios.
	Method 4: Listing outcomes.		
	Either 10 correct outcomes for ADD^ listed or 40 correct outcomes for AD^^ listed	**M1**	
	50 stated	**A1**	www If M0 scored, **SC B1** [total =]50 www.
	126 stated or correct outcomes listed	**B1**	
	[Probability =] $\frac{50}{126}, \frac{25}{63}$	**B1**	0.396825… to at least 3SF. FT $\frac{\textit{their}\text{ attempted } 40+10}{126}$. Numerator must be from an attempt to find the 2 appropriate scenarios.
		4	

Cambridge Assessment International Education

Cambridge International AS & A Level

MATHEMATICS **9709/53**

Paper 5 Probability & Statistics 1 **October/November 2023**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Due to a series-specific issue during the live exam series, all candidates were awarded full marks for questions 1 and 4. This published mark scheme for these questions was created alongside the question paper, but has not been used by examiners.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2023 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics-Specific Marking Principles

1. Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.

2. Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.

3. Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.

4. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).

5. Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.

6. Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

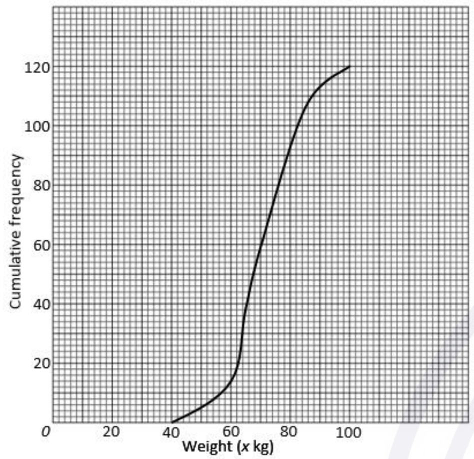
AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

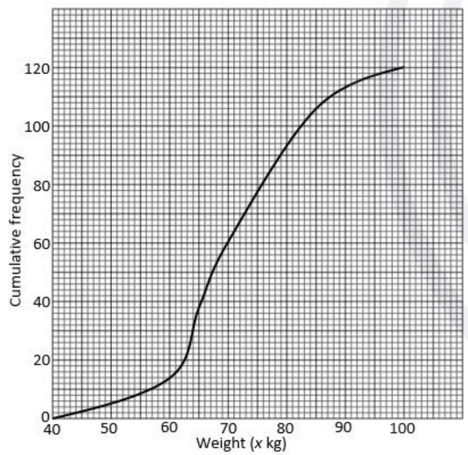
<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td>1(a)</td><td>$2k+6k+12k+20k=1,\ \left[k=\frac{1}{40}\right]$</td><td>M1</td><td>Using sum of probabilities = 1 to form an equation in k.
Accept $1\times2\times k + 2\times3\times k + 3\times4\times k + 4\times5\times k = 1$.</td></tr>
<tr><td></td><td>
<table><tr><td>X</td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>P(X)</td><td>$\frac{2}{40}$</td><td>$\frac{6}{40}$</td><td>$\frac{12}{40}$</td><td>$\frac{20}{40}$</td></tr><tr><td></td><td>0.05</td><td>0.15</td><td>0.3</td><td>0.5</td></tr></table>
</td><td>M1</td><td>
<table><tr><td>X</td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>P(X)</td><td>$2k$</td><td>$6k$</td><td>$12k$</td><td>$20k$</td></tr></table>
Two correctly linked, accurate probabilities. May be in terms of k.
May not be in a table.</td></tr>
<tr><td></td><td></td><td>A1</td><td>Table with correct X values and correct probabilities.</td></tr>
<tr><td></td><td></td><td>3</td><td></td></tr>
<tr><td>1(b)</td><td>$[\mathrm{E}(X)=]\ [\mathrm{E}(X)=\frac{1\times2+2\times6+3\times12+4\times20}{40}]\ \frac{2+12+36+80}{40}$</td><td>M1</td><td>$[\mathrm{E}(X)=1\times2k+2\times6k+3\times12k+4\times20k=130k]$
Accept unsimplified expression. May be calculated in variance. FT *their* table with 3 or more probabilities summing to 1 ($0 < p < 1$).
If there are outcomes in the table without probabilities, condone and treat as $p = 0$.</td></tr>
<tr><td></td><td>$\left[\mathrm{Var}(X)=\frac{1^2\times2+2^2\times6+3^2\times12+4^2\times20}{40}-(\textit{their}\ \mathrm{E}(X))^2=\right]$
$\frac{1\times2+4\times6+9\times12+16\times20}{40}-\left(\textit{their}\ \frac{13}{4}\right)^2$
$\left[\frac{2+24+108+320}{40}-\left(\textit{their}\ \frac{13}{4}\right)^2\right]$</td><td>M1</td><td>$[\mathrm{Var}(X)=1^2\times2k+2^2\times6k+3^2\times12k+4^2\times20k-(130k)^2]$
Appropriate variance formula using *their* $(\mathrm{E}(X))^2$ value. FT *their* table with 3 or more probabilities ($0 < p < 1$) which need not sum to 1 and the highlighted calculation (or less simplified) seen,
Note: if table is correct,
$\frac{454}{40}\left(\text{or } \frac{227}{20} \text{ or any calculation}\right)-(\textit{their}\ \mathrm{E}(X))^2$ implies M1.</td></tr>
<tr><td></td><td>$\mathrm{E}(X) = \frac{13}{4}, 3\frac{1}{4}, 3.05\ \ \mathrm{Var}(X) = \frac{63}{80}, 0.7875$</td><td>A1</td><td>Answers for E(X) and Var(X) must be identified. E(X) may be identified by correct use in variance. Condone E, V, μ, σ etc.
If A0 earned, **SC B1** for identified correct final solutions.</td></tr>
<tr><td></td><td></td><td>3</td><td></td></tr>
</table>

Question	Answer	Marks	Guidance
2(a)	[P(1.42 < X <1.52) =] P($\frac{1.42-1.5}{0.05} < Z < \frac{1.52-1.5}{0.05}$)	**M1**	Use of ± standardisation formula once with 1.5, 0.05 and either 1.42 or 1.52, allow σ^2 or $\sqrt{\sigma}$, no continuity correction.
	[= P($-1.6 < Z < 0.4$) = $\Phi(0.4)+\Phi(1.6)-1$] = 0.6554 + 0.9452 – 1 or 0.6554 – 0.0548	**M1**	Calculating the appropriate probability area (leading to their final answer, expect > 0.5).
	= 0.601	**A1**	$0.6005 < p \leqslant 0.601$ **SC B1** for 0.601 with no standardisation seen.
		3	
2(b)	$\left[P(X<0.9)=P\left(Z<\frac{0.9-0.75}{\sigma}\right)=0.68\right]$ $\frac{0.9-0.75}{\sigma}=0.468$	**B1**	$0.467 < z \leqslant 0.468$ or $-0.468 \leqslant z < -0.467$ seen
		M1	±standardisation formula with 0.9, 0.75, σ equating to a z-value (not 0.32, 0.68, 0.532, 0.7517, 0.2483, 0.6255,) . Condone continuity correct ±0.05, not $\sigma^2, \sqrt{\sigma}$. Condone $\pm\frac{0.15}{\sigma}=0.468$.
	$\sigma=0.321$, $\frac{25}{78}$	**A1**	$0.3205 \leqslant \sigma < 0.3215$ **SC B1** if M0 www.
		3	

Question	Answer	Marks	Guidance
3(a)	[P(WW) = P(AWW) + P(BWW) =] $\frac{2}{6}\times\frac{8}{15}\times\frac{7}{14}+\frac{4}{6}\times\frac{6}{15}\times\frac{5}{14}$	**M1**	Either $\frac{2}{6}\times\frac{8}{15}\times\frac{7}{14}$ or $\frac{4}{6}\times\frac{6}{15}\times\frac{5}{14}$ seen, accept unsimplified.
		M1	$\frac{q}{6}\times\frac{r}{15}\times\frac{r-1}{14}+\frac{6-q}{6}\times\frac{s}{15}\times\frac{s-1}{14}$ seen, no additional terms, accept unsimplified. Condone $\frac{q}{6}\times\frac{r}{15}\times\frac{r}{15}+\frac{6-q}{6}\times\frac{s}{15}\times\frac{s}{15}$, $1 \leqslant q \leqslant 5$, $1 < r, s < 9$.
	$\left[=\frac{56}{630}+\frac{60}{630}=\frac{4}{45}+\frac{2}{21}\right]=\frac{58}{315}$ or 0.184	**A1**	**SC B1** for 58/315 if either M mark withheld.
		3	

Question	Answer	Marks	Guidance
3(b)	$\left[\text{P(B}\mid\text{WR or RW)} = \dfrac{\text{P(W \& R from bag B)}}{\text{P(W and R)}} = \right]$ $\dfrac{\frac{4}{6}\times\frac{6}{15}\times\frac{7}{14}+\frac{4}{6}\times\frac{7}{15}\times\frac{6}{14}}{\frac{2}{6}\times\frac{8}{15}\times\frac{4}{14}+\frac{2}{6}\times\frac{4}{15}\times\frac{8}{14}+\frac{4}{6}\times\frac{6}{15}\times\frac{7}{14}+\frac{4}{6}\times\frac{7}{15}\times\frac{6}{14}}$ or $\dfrac{2\times\frac{4}{6}\times\frac{6}{15}\times\frac{7}{14}}{2\times\frac{2}{6}\times\frac{8}{15}\times\frac{4}{14}+2\times\frac{4}{6}\times\frac{6}{15}\times\frac{7}{14}}$	B1	P(W & R from bag B) $= \frac{2}{3}\times\frac{6}{15}\times\frac{7}{14}+\frac{2}{3}\times\frac{7}{15}\times\frac{6}{14}$ or $2\times\frac{2}{3}\times\frac{6}{15}\times\frac{7}{14}$ $[= \frac{4}{15}$ *or* $0.267]$ Seen alone or as numerator/denominator of conditional probability.
		M1	P(WR or RW) = P(W & R from bag A) + P(W & R from bag B) $= a\times\frac{2}{6}\times\frac{8}{15}\times\frac{4}{14}+a\times\frac{4}{6}\times\frac{6}{15}\times\frac{7}{14}$ or $= a\times\frac{2}{6}\times\frac{8}{15}\times\frac{4}{14}+$ *their* P(W & R from bag B). a = 1 or 2. [expect $\frac{116}{315}$ or 0.368] Seen alone or as numerator/denominator of conditional probability.
	$\dfrac{168}{630}\Big/\dfrac{232}{630} = \dfrac{4}{15}\Big/\dfrac{116}{315}$	M1	$\dfrac{\textit{their}\text{ identified P}(W\ \&R\ from\,bag\ B)}{\textit{their}\text{ identified P(WR or RW)}}$ Accept unsimplified.
	$= \dfrac{168}{232}, \dfrac{21}{29}$ or 0.724	A1	0.7241379 to at least 3SF.
		4	

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td rowspan="3">4(a)</td><td rowspan="2">


</td><td>M1</td><td>At least 3 points plotted accurately at class upper end points (40,0) (60,14) (65,38) (70,60) (85,106) (100,120).
Linear cumulative frequency scale $0 \leqslant$ cf $\leqslant 120$ and linear weight scale $40 \leqslant$ weight(kg) $\leqslant 100$ with at least 3 values identified on each axis.
Condone scale reversed.</td></tr>
<tr><td>A1</td><td>All points plotted correctly, curve drawn (within tolerance) and joined to (40,0).
Axes labelled cumulative frequency (cf), weight (w) and kg (kilograms) – or a suitable title.</td></tr>
<tr><td></td><td>2</td><td></td></tr>
<tr><td rowspan="3">4(b)</td><td>[120 × 0.65 =] 78 seen</td><td>M1</td><td>May be implied by use on graph.</td></tr>
<tr><td>76 [kg]</td><td>A1</td><td>75 < hours < 79. Indication of use of graph required.</td></tr>
<tr><td></td><td>2</td><td></td></tr>
</table>

Question	Answer	Marks	Guidance
4(c)	Frequencies: [0] 14 24 22 46 14	**B1**	At least 5 correct frequencies seen (condone omission of 0).
	Midpoints: 20 50 62.5 67.5 77.5 92.5	**B1**	At least 5 correct midpoints seen (condone omission of 20).
	Mean = $\frac{0\times20+14\times50+24\times62.5+22\times67.5+46\times77.5+14\times92.5}{120}$ $=\frac{[0]+700+1500+1485+3565+12950}{120}\left[=\frac{8545}{120}\right]$	**M1**	Correct formula for mean using *their* midpoints and *their* frequencies, implied by $\frac{8545}{120}$ if correct midpoints & frequencies seen. May be gained in variance calculation. If midpoints not clearly identified, condone midpoints ±0.5.
	= 71.2	**A1**	Accept $\frac{1709}{24}$, $71\frac{5}{24}$ or 71.208333 to at least 3SF. If M0 scored, **SC B1** for $\frac{1709}{24}$, $71\frac{5}{24}$ or 71.208333 to at least 3SF www.
	Variance = $\frac{0\times20^2+14\times50^2+24\times62.5^2+22\times67.5^2+46\times77.5^2+14\times92.5^2}{120}-71.2^2$ $\frac{[0]+35000+93750+100237.5+276287.5+119787.5}{120}-\left(\frac{8545}{120}\right)^2$ [=138.23]	**M1**	Correct formula for variance using *their* midpoints, *their* frequencies and *their* mean. Implied by $\frac{625062.5}{120}-\left(\frac{8545}{120}\right)^2$ if correct midpoints & frequencies seen.
	Standard deviation = 11.8	**A1**	11.757016 to at least 3SF.
		6	

Question	Answer	Marks	Guidance
5(a)(i)	**Method 1**		
	$[\mathrm{P}(2 \leqslant X \leqslant 6) = \mathrm{P}(X \leqslant 6) - \mathrm{P}(X \leqslant 1) =]\ 1-(0.7)^6-(1-0.7)$	**M1**	$1 - 0.7^n$ seen, n = 5, 6.
	= 0.582	**A1**	www 0.582351 to at least 3SF.
	Method 2		
	P(X= 2, 3, 4, 5, 6) $= 0.7 \times 0.3 + 0.7^2 \times 0.3 + 0.7^3 \times 0.3 + 0.7^4 \times 0.3 + 0.7^5 \times 0.3$ = 0.21 + 0.147 + 0.1029 + 0.07203 + 0.050421	**M1**	Sum of first 4 or 5 correct terms – no incorrect terms.
	= 0.582	**A1**	www 0.582351 to at least 3SF.
		2	
5(a)(ii)	$3\frac{1}{3}$	**B1**	Condone 3.33, $3.\dot{3}$ or $\frac{10}{3}$ – NOT $\frac{1}{0.3}$.
		1	

Question	Answer	Marks	Guidance
5(b)	**Method 1**		
	[P(3, 4, 5) =] $^5C_3(0.3)^3(0.7)^2 + {}^5C_4(0.3)^4(0.7)^1 + {}^5C_5(0.3)^5(0.7)^0$	**M1**	One term seen $^5C_x\ (p)^x(1-p)^{5-x}$, $0<p<1$, $x\neq 0,5$.
	= 0.1323 + 0.02835 + 0.00243	**A1**	Correct expression, accept unsimplified, no terms omitted leading to final answer.
	= 0.163, $\frac{4077}{25000}$	**B1**	0.16308 to at least 3SF.
	Method 2		
	[1 – P(0, 1, 2) = $1-({}^5C_0(0.3)^0(0.7)^5 + {}^5C_1(0.3)^1(0.7)^4 + {}^5C_2(0.3)^2(0.7)^3)$	**M1**	One term $^5C_x\ (p)^x(1-p)^{5-x}$, $0<p<1$, $x\neq 0,5$.
	= 1 – (0.16807 + 0.36015 + 0.3087)	**A1**	Correct expression, accept unsimplified, no terms omitted leading to final answer.
	= 0.163, $\frac{4077}{25000}$	**B1**	0.16308 to at least 3SF.
		3	

Question	Answer	Marks	Guidance
5(c)	[Mean = $75 \times 0.3 =$] 22.5 [Var = $75 \times 0.3 \times 0.7 =$] 15.75	B1	22.5, 22½ and 15.75, $15\frac{3}{4}$ seen, allow unsimplified. ($\sigma = \frac{3\sqrt{7}}{2}$ or 3.9686269… to at least 3SF implies correct variance)
	$[P(X > 20) =]\ P\left(Z > \frac{20.5 - 22.5}{\sqrt{15.75}}\right)$	M1	Substituting their μ and σ into ±standardisation formula (any number for 20.5), not σ^2 not $\sqrt{\sigma}$.
		M1	Using continuity correction 19.5 or 20.5 in *their* standardisation formula.
	$[P(Z > -0.504) = \Phi(0.504)]$ = 0.693	M1	Appropriate area Φ, from final process, must be a probability. Expect final answer > 0.5. Note: correct final answer implies this M1.
		A1	$0.6925 < p \leqslant 0.693$
		5	

Question	Answer	Marks	Guidance
6(a)(i)	**Method 1**		
	$6! \times 2^6$	**M1**	$6! \times a$, a integer > 1.
		M1	$b \times 2^6$, b integer $\geqslant 1$.
	= 46080	**A1**	Accurate answer required. **SC B1** for 46080 if M0 M0 www.
	Alternative method for question 6(a)(i)		
	$12 \times 10 \times 8 \times 6 \times 4 \times 2$	**M1**	$c \times d \times e \times f \times g \times h$ $2 \leqslant c,d,e,f,g,h$ (different integers) $\leqslant 12$
		M1	Correct unsimplified.
	= 46080	**A1**	Accurate answer required. **SC B1** for 46080 if M0 M0 www.
		3	
6(a)(ii)	$5! \times 5! \times 2 \times 2$	**M1**	$5! \times 5! \times k$, k positive integer, 1 may be implied (no adding/subtracting).
	= 57600	**A1**	
		2	

Question	Answer	Marks	Guidance
6(b)	**Method 1 probabilities of J & K being placed:**		
	In the group of 5: $\frac{5}{12}\times\frac{4}{11}$ $\left[=\frac{20}{132},\frac{5}{33}\right]$ In the group of 4: $\frac{4}{12}\times\frac{3}{11}$ $\left[=\frac{12}{132},\frac{1}{11}\right]$ In the group of 3: $\frac{3}{12}\times\frac{2}{11}$ $\left[=\frac{6}{132},\frac{1}{22}\right]$	**B1**	Correct probability for one identified scenario.
		M1	Denominator 12×11 for all probabilities, (1, 2 or 3 scenarios).
		A1	3 correct probabilities, accept unsimplified.
	$\frac{5}{12}\times\frac{4}{11}+\frac{4}{12}\times\frac{3}{11}+\frac{3}{12}\times\frac{2}{11}$	**M1**	Adding probabilities for 3 correct scenarios.
	$\frac{19}{66}$, 0.288	**A1**	0.2878787 to at least 3SF.

Question	Answer	Marks	Guidance
6(b)	**Method 2 number of arrangements of J & K being placed:**		
	In the group of 5: ${}^{10}C_3 \times {}^{7}C_4$ [= 120 × 35 = 4200] In the group of 4: ${}^{10}C_2 \times {}^{8}C_5$ [= 45 × 56 = 2520] In the group of 3: ${}^{10}C_1 \times {}^{9}C_5$ [= 10 × 126 = 1260]	**B1**	Correct value of one identified scenario seen, accept unsimplified.
		M1	${}^{12}C_a \times {}^{12-a}C_b$, a = 3, 4, 5; b = 3, 4, 5 ($a \neq b$)
	[Total number of ways of arranging the 3 groups =] ${}^{12}C_5 \times {}^{7}C_4 = 792 \times 35 = 27720$ or ${}^{12}C_3 \times {}^{9}C_4$ or ${}^{12}C_4 \times {}^{8}C_5$	**A1**	27720 Seen alone or as denominator of probability –accept unsimplified. **SC B1** if M0.
	4200 + 2520 + 1260 = 7980	**M1**	Values of 3 correct scenarios added, accept unsimplified – or correct.
	[Probability =] $\frac{7980}{27720}$, $\frac{19}{66}$, 0.288	**A1**	0.2878787 to at least 3SF.
		5	Note, alternative arrangement calculations possible e.g. In the group of 5: ${}^{10}C_3 \times {}^{7}C_4$ [= 120 × 35 = 4200] In the group of 4: ${}^{10}C_5 \times {}^{5}C_2$ [= 252× 10 = 2520] In the group of 3: ${}^{10}C_5 \times {}^{5}C_4$ [= 252 × 5 = 1260]

Cambridge International AS & A Level

MATHEMATICS **9709/51**

Paper 5 Probability & Statistics 1 **May/June 2023**

MARK SCHEME

Maximum Mark: 50

Published

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GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

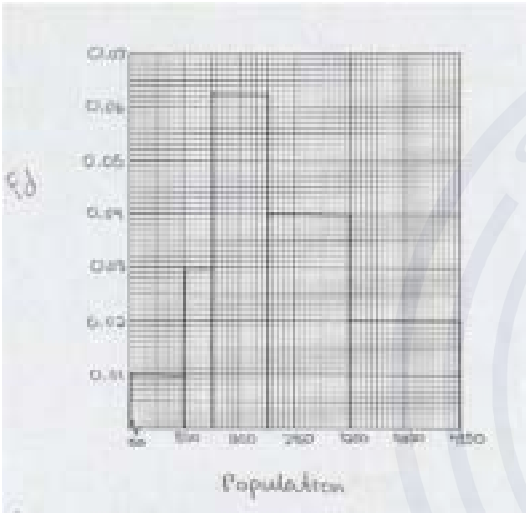
AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1(a)	Var = $\left[\frac{\Sigma(x-q)^2}{50} - \left(\frac{\Sigma(x-q)}{50}\right)^2 = \right]\frac{14235}{50} - \left(\frac{700}{50}\right)^2$ $[= 284.7 - 196 = 88.7]$	**M1**	$\frac{14235}{a} - \left(\frac{700}{a}\right)^2$; where a = 49, 50, 51.
	[sd = $\sqrt{88.7}$ =] 9.42	**A1**	9.4180677 rounded to at least 3SF.
		2	
1(b)	$\sum x - 50q = 700$ [2865 – 50q = 700]	**M1**	Forming equation with Σx, 50q and 700.
	$q = 43.3, 43\frac{3}{10}$	**A1**	If M0 scored, **SC B1** for 43.3 WWW.
		2	

Question	Answer	Marks	Guidance
2(a)	${}^{6}C_{3} \times {}^{8}C_{3}$	M1	${}^{6}C_{3} \times b$ or $c \times {}^{8}C_{3}$ seen. b, c integers $\geqslant 1$ (1 may be implied).
	1120	A1	
		2	
2(b)	**Method 1**		
	0 brothers $[{}^{3}C_{0}] \times {}^{11}C_{6}$ 462 1 brother ${}^{3}C_{1} \times {}^{11}C_{5}$ 1386 2 brothers ${}^{3}C_{2} \times {}^{11}C_{4}$ 990	B1	${}^{3}C_{x} \times {}^{11}C_{6-x}$, with $x = 1$ or 2 seen.
		M1	Add values of 3 correct scenarios, (may be identified by the appropriate calculations) no incorrect/repeated scenarios, condone use of permutations.
	2838	A1	Only dependent on the M mark. **SC B1** for the correct calculation or 2838 seen WWW.
	Method 2		
	${}^{14}C_{6} - {}^{11}C_{3}$ 3003 – 165	B1	${}^{14}C_{6} - d$, where d a positive integer.
		M1	$e - {}^{11}C_{3}$, where e is a positive integer >165.
	= 2838	A1	
		3	

Question	Answer	Marks	Guidance
3(a)	$\left[\dfrac{8!}{2!3!}=\right]$ 3360	**B1**	
		1	
3(b)	$\dfrac{6!}{2!2!}$	**M1**	$\dfrac{6!}{2!f!}$; f = 1, 2, 3.
	180	**A1**	
		2	
3(c)	$\left[P(OOO\|CC)=\dfrac{P(OOO\cap CC)}{P(CC)}=\right]$ $\dfrac{5!}{\frac{7!}{3!}}$	**M1**	$\dfrac{5!}{g}$ g a positive integer, g ≠ 3360, 1. Condone numerator of $\dfrac{5!}{3360g}$.
		M1	$\dfrac{h}{\frac{7!}{3!}}$ or $\dfrac{h}{\frac{8!}{3!}}$, where h is a positive integer. Condone division by 3360 in denominator.
	$=\dfrac{120}{840},\dfrac{1}{7},0.143$	**A1**	0.1428571… to at least 3SF. If M0 scored **SC B1** for $\dfrac{1}{7}$ WWW.
		3	

Question	Answer	Marks	Guidance
4(a)	$P(Z > \frac{20-14.6}{5.2}) = P(Z > 1.03846)$	**M1**	Use of ± standardisation formula with 20, 14.6 and 5.2 not σ^2, not $\sqrt{\sigma}$, no continuity correction.
	1 – 0.8504	**M1**	Calculating the appropriate probability area (leading to their final answer).
	0.150	**A1**	0.1496, $0.149 < p \leqslant 0.15[0]$. Only dependent on the 2nd M mark so M0M1A1 possible. **SC B1** for $0.149 < p \leqslant 0.15[0]$ if M0M0A0 awarded.
	[250 × *their* 0.1496 =] 37, 38	**B1 FT**	Strict FT *their* at least 4-figure probability seen anywhere (give BOD if they go on to use 0.150). Final answer must be positive integer, no approximation or rounding stated.
		4	
4(b)	$z_1 = \frac{14.5-\mu}{\sigma} = -0.842$ $z_2 = \frac{18.5-\mu}{\sigma} = -0.44$	**B1**	$-0.843 < z_1 < -0.841$ or $0.841 < z_1 < 0.843$.
		B1	$-0.441 < z_2 < -0.439$ or $0.439 < z_2 < 0.441$.
		M1	Use of the ±standardisation formula once with μ, σ and a z-value (not 0.20, 0.80, 0.67, 0.23, 0.5793, 0.7881, 0.7486, 0.591 or $1 - z$ i.e. 0.158 etc.). Condone continuity correction ±0.05, not $\sigma^2, \sqrt{\sigma}$.
	Solve, obtaining values for μ and σ. $\mu = 22.9$, $\sigma = 9.95$	**M1**	Solve using the elimination method, substitution method or other appropriate approach to obtain values for both μ and σ.
		A1	AWRT 22.9, 9.95 .
		5	

Question	Answer	Marks	Guidance
5(a)	<table><tr><td>cw</td><td>800</td><td>400</td><td>800</td><td>1200</td><td>1600</td></tr><tr><td>fd</td><td>0.01</td><td>0.03</td><td>0.0625</td><td>0.04</td><td>0.02</td></tr></table>	M1	At least 4 frequency densities calculated (F/cw, e.g. $\frac{8}{800}\left(\text{condone}\frac{8}{n}, 799 \leqslant n \leqslant 801\right)$) Accept unsimplified, may be read from graph using *their* scale.
		A1	All heights correct on graph.
		B1	Bar ends at 50, 850, 1250, 2050, 3250, 4850 read at the axis with a horizontal linear scale with at least 3 values indicated. $50 \leqslant$ horizontal scale $\leqslant 4850$.
		B1	Axes labelled frequency density (fd) and population (pop) OE, or in a title. Linear vertical scale, with at least 3 values indicated. Vertical axis must cover at least the range $0 \leqslant$ vertical axis $\leqslant 0.0625$. Axes may be reversed.
		4	
5(b)	2100 – 3200	B1	Accept 2050 – 3250 OE. Condone '4th interval'.
		1	
5(c)	3249 – 1250	M1	$2050 \leqslant \text{UQ} \leqslant 3250 - 1250 \leqslant \text{LQ} \leqslant 2050$.
	1999	A1	Condone 3250 – 1250 = 2000.
		2	

Question	Answer	Marks	Guidance
6(a)	[P($X=3$) =] $\frac{3}{4}\times\left(\frac{1}{4}\right)^3\times 4$	**M1**	$\frac{3}{4}\times\left(\frac{1}{4}\right)^3\times q$; q a positive integer (1 may be implied).
	$=\frac{3}{64}$	**A1**	AG.
		2	
6(b)	<table><tr><td>x</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>P($X=x$)</td><td>$\frac{81}{256}$</td><td>$\frac{27}{64}$</td><td>$\frac{27}{128}$</td><td>$\frac{3}{64}$</td><td>$\frac{1}{256}$</td></tr></table>	**B1**	Either P(1) = $\frac{27}{64}$, 0.421875 or P(2) = $\frac{27}{128}$, 0.2109375 correct to at least 3SF. Condone not in table.
		B1 FT	Both values in table. FT P(1) + P(2) = $\frac{81}{128}$, 0.6328125 .
		2	
6(c)	[E(X) =] $[0\times\frac{81}{256}]+1\times their\frac{27}{64}+2\times their\frac{27}{128}+3\times\frac{12}{256}+4\times\frac{1}{256}$	**M1**	Correct method from *their* probability distribution table with at least 4 terms, $0 < their$ P(x) < 1, accept partially evaluated. $= 0+\frac{27}{64}+\frac{54}{128}+\frac{36}{256}+\frac{4}{256}$
	= 1	**A1**	
		2	

Question	Answer	Marks	Guidance
6(d)	Mean = $96 \times \frac{67}{256} = 25.125$ Var = $96 \times \frac{67}{256} \times \frac{189}{256} = 18.549$	**B1**	25.125, $25\frac{1}{8}$ and 18.5493… to at least 3SF seen, allow unsimplified ($4.3068 \leqslant \sigma \leqslant 4.307$ implies correct variance).
	$P(X < 20) = P(Z < \frac{19.5 - 25.125}{\sqrt{18.549}})$	**M1**	Substituting *their* μ and σ into ± standardisation formula (any number for 19.5). Condone σ^2 and $\sqrt{\sigma}$.
		M1	Using continuity correction 19.5 or 20.5 in *their* standardisation formula. Note: $\frac{\pm 5.625}{\sqrt{18.549}}$ seen gains M2 BOD.
	[= $P(Z < -1.306)$ = 1 – Φ (1.306)=] 1 – 0.9042 =	**M1**	Appropriate area Φ, from final process. Must be a probability.
	0.0958	**A1**	$0.0957 \leqslant p \leqslant 0.0958$. **SC B1** for $0.0957 \leqslant p \leqslant 0.0958$ if B1M0M0M1 scored.
		5	

Question	Answer	Marks	Guidance
7(a)	**Method 1**		
	$[\mathrm{P}(X < 6) = \mathrm{P}(X \leqslant 5) =]\ 1 - 0.8^5$	**M1**	$1 - 0.8^r$, $r = 5, 6$.
	$= 0.672$	**A1**	
	Method 2		
	$[\mathrm{P}(X < 6) = \mathrm{P}(X = 1) + \mathrm{P}(X = 2) + \mathrm{P}(X = 3) + \mathrm{P}(X = 4) + \mathrm{P}(X = 5) =]$ $\frac{1}{5} + \frac{4}{5} \times \frac{1}{5} + \left(\frac{4}{5}\right)^2 \times \frac{1}{5} + \left(\frac{4}{5}\right)^3 \times \frac{1}{5} + \left(\frac{4}{5}\right)^4 \times \frac{1}{5}$	**M1**	Condone an extra term $(\frac{4}{5})^5 \times \frac{1}{5}$. First, last and one of the 3 middle terms implies M1.
	$= 0.672$	**A1**	
		2	

Question	Answer	Marks	Guidance
7(b)	**Method 1**		
	$[1 - P(0, 1, 2)]$ $= 1 - ({}^{12}C_0\,(0.8)^{12} + {}^{12}C_1\,(0.2)(0.8)^{11} + {}^{12}C_2\,(0.2)^2\,(0.8)^{10})$ $[= 1 - (0.06872 + 0.20615 + 0.28347)]$	M1	One term ${}^{12}C_x\,(p)^x\,(1-p)^{12-x}$, $0 < p < 1$, $x \neq 0, 1, 2$.
		A1	Correct expression, accept unsimplified, no terms omitted, leading to final answer. Correct unsimplified expression or better.
	$= 0.442$	B1	$0.411 < p \leqslant 0.442$ WWW.
	Method 2		
	$[P(3,4,5,6,7,8,9,10,11,12) =]$ ${}^{12}C_3\,(0.2)^3\,(0.8)^9 + {}^{12}C_4\,(0.2)^4\,(0.8)^8 + \ldots + {}^{12}C_{11}\,(0.2)^{11}\,(0.8)^1 + {}^{12}C_{12}\,(0.2)^{12}$ $[= 0.23622 + 0.13288 + \ldots + 1.966 \times 10^{-7} + 4.096 \times 10^{-9}]$	M1	One term ${}^{12}C_x\,(p)^x\,(1-p)^{12-x}$, $0 < p < 1$, $x \neq 0, 1, 2$.
		A1	Correct expression, accept unsimplified, leading to final answer. Accept first, last and 8 of the middle terms.
	$=0.442$	B1	$0.411 < p \leqslant 0.442$.
		3	

Question	Answer	Marks	Guidance
7(c)	$(0.2)^5 \times 5!$	M1	$(0.2)^5 \times s$, s a positive integer. 1 may be implied.
		M1	$t \times 5!$ where $0 < t < 1$.
	$= 0.0384, \frac{24}{625}$	A1	
	Alternative Method for Question 7(c)		
	$\frac{{}^5C_1 \times {}^4C_1 \times {}^3C_1 \times {}^2C_1 \times [{}^1C_1]}{({}^5C_1)^5}$	M1	$({}^5C_1)^5$ or 5^5 as denominator.
		M1	${}^5C_1 \times {}^4C_1 \times {}^3C_1 \times {}^2C_1 \times [{}^1C_1]$ or 5! as numerator.
	$= 0.0384, \frac{24}{625}$	A1	
		3	

Cambridge Assessment International Education

Cambridge International AS & A Level

MATHEMATICS **9709/52**

Paper 5 Probability & Statistics 1 **May/June 2023**

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GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

	Mathematics Specific Marking Principles
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

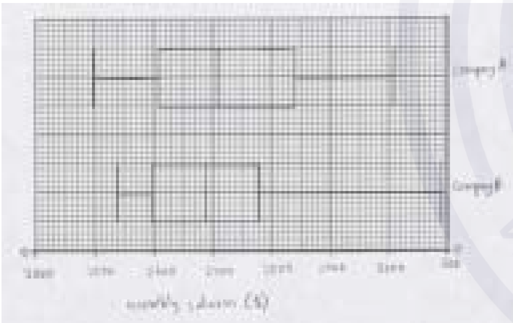
AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td rowspan="5">1(a)</td><td>$\left[3k+3k+8k=1, so\right] k=\frac{1}{14}$</td><td>B1</td><td></td></tr>
<tr><td rowspan="2"><table><tr><td>x</td><td>–2</td><td>2</td><td>3</td></tr><tr><td>P(x)</td><td>$\frac{3}{14}$, 0.214</td><td>$\frac{3}{14}$, 0.214</td><td>$\frac{8}{14}$, 0.571</td></tr></table></td><td>B1 FT</td><td>Table with correct values of x, and at least one correct probability linked with outcome. FT their k.
Condone any additional X values if probability stated as 0.</td></tr>
<tr><td>B1 FT</td><td>The outcomes in the table must be –2, 2 and 3.
2 further correct probabilities in table
or 3 correct probabilities not in table linked to outcomes,
or 3 correct FT probabilities in table using their k,
or 3 incorrect probabilities summing to 1 in table if k not stated.</td></tr>
<tr><td></td><td></td><td>If k not calculated, SC B1 for the below.<table><tr><td>x</td><td>–2</td><td>2</td><td>3</td></tr><tr><td>P(x)</td><td>$3k$</td><td>$3k$</td><td>$8k$</td></tr></table></td></tr>
<tr><td></td><td>3</td><td></td></tr>
</table>

Question	Answer	Marks	Guidance
1(b)	$\left[E(X) = -2\times\frac{3}{14} + 2\times\frac{3}{14} + 3\times\frac{8}{14} = \right]$ $-\frac{6}{14} + \frac{6}{14} + \frac{24}{14}$	M1	Accept unsimplified expression. May be calculated in variance. FT *their* table with 3 probabilities summing to $0.999 \leqslant$ *total* $\leqslant 1$ ($0 < p < 1$) or in terms of k.
	$\left[\text{Var}(X) = (-2)^2\times\frac{3}{14} + 2^2\times\frac{3}{14} + 3^2\times\frac{8}{14} - (\textit{their}\ E(X))^2 = \right]$ $4\times\frac{3}{14} + 4\times\frac{3}{14} + 9\times\frac{8}{14} - \left(\textit{their}\ \frac{12}{7}\right)^2$ $\left[\frac{12+12+72}{14} - \left(\textit{their}\ \frac{12}{7}\right)^2\right]$	M1	Appropriate variance formula using *their* $(E(X))^2$ value. FT *their* table with 3 or more probabilities ($0 < p < 1$) which need not sum to 1, or in terms of k with an expression no more evaluated than shown.
	$E(X) = \frac{12}{7}$, 1.71, $1\frac{5}{7}$ $\text{Var}(X) = \frac{192}{49}$, 3.92, $3\frac{45}{49}$	A1	Answers for $E(X)$ and $\text{Var}(X)$ must be identified. $E(X)$ may be identified by correct use in Variance (condone E, V, μ, σ^2, etc.). If A0 earned, **SC B1** for identified correct final answers.
		3	

Question	Answer	Marks	Guidance
2(a)	[P(no rain) = $0.6\times(0.8)^3$ =] 0.3072, $\frac{192}{625}$	B1	Exact value required
		1	

Question	Answer	Marks	Guidance
2(b)	$0.6 \times 0.8 \times 0.2$	**M1**	$a \times b \times c$ where $a,\ b = 0.6,\ 0.8,\ c = 0.2,\ 0.4,\ 0.7$. Condone including Wednesday with both 0.3 and 0.7 used.
	$= 0.096[0],\ \frac{12}{125}$	**A1**	
		2	
2(c)	$P(RDDD) = 0.4 \times 0.3 \times 0.8 \times 0.8 = 0.0768,\ \frac{48}{625}$ $P(DRDD) = 0.6 \times 0.2 \times 0.3 \times 0.8 = 0.0288,\ \frac{18}{625}$ $P(DDRD) = 0.6 \times 0.8 \times 0.2 \times 0.3 = 0.0288,\ \frac{18}{625}$ $P(DDDR) = 0.6 \times 0.8 \times 0.8 \times 0.2 = 0.0768,\ \frac{48}{625}$	**B1**	Correct probability for one clearly identified outcome evaluated accept unsimplified. A correct unsimplified expression is not sufficient.
		M1	Add 4 probability values, $0 < p < 1$, for appropriate identified scenarios. Accept unsimplified. Ways of identifying scenarios for this mark**:** Stating the days. All the unsimplified probability calculations exactly as stated in the mark scheme. Identifying the correct branches on a tree diagram and linking with the values. No repeated scenarios. No incorrect scenarios.
	$0.2112,\ \frac{132}{625}$	**A1**	Accept 0.211 If 0/3 scored **SC B1** for $0.2112,\ \frac{132}{625}$.
		3	

Question	Answer	Marks	Guidance
3(a)	Median = 2710	**B1**	Must be identified, condone Q2. Ignore units throughout.
	2840 – 2610	**M1**	$2820 \leqslant \text{UQ} \leqslant 2850 - 2600 \leqslant \text{LQ} \leqslant 2620$.
	230	**A1**	www If M0 scored **SC B1** for 230 www. If key ignored consistently: B0 Median = 271 **SC M1** $282 \leqslant \text{UQ} \leqslant 285 - 260 \leqslant \text{LQ} \leqslant 262$ **SC A1** 23.
		3	
3(b)	Box-and-whisker plot on provided grid.  B: 2540 2600 2690 2780 3090 A: 2500 *2610* *2710* *2840* 3010	**B1**	All 5 key values for *B* plotted accurately in standard format using a linear scale with 3 identified values. Labelled *B*. Scale at least 1 cm = \$100.
		B1FT	All 5 key values for *A*, **FT** from **(a)**, plotted accurately in standard format using a linear scale with 3 identified values. Labelled *A*. Scale at least 1cm = \$100
		B1	Whiskers not through box for both, not drawn at corners of boxes, single linear scale for the diagram and labelled ‘salaries’ (oe) and \$.
		3	

Question	Answer	Marks	Guidance
3(c)	Examples: Mean less appropriate than median because of extreme value for company *B* [at \$3090]. No, extreme value in company B. No, \$3090 is an anomaly.	**B1**	Must refer to company B, may be implied by appropriate use of \$3090. Must include an indication that the mean is not appropriate. No contradictory statements can be present, e.g. acceptable comment with 'but mean could be used for company A'. Condone reference to \$309.
		1	

Question	Answer	Marks	Guidance
4(a)	$[\mathrm{P}(X=4) = (0.8)^3(0.2) =]\ 0.1024, \frac{64}{625}$	**B1**	Condone 0.102 .
		1	

Question	Answer	Marks	Guidance
4(b)	$[P(X < 6) =] 1 - 0.8^5$	**M1**	$1 - 0.8^d$, $d = 5, 6$.
	$= 0.672, \frac{2101}{3125}$	**A1**	0.67232 to at least 3SF. If M0 awarded, **SC B1** for $\frac{2101}{3125}$ or 0.67232 only.
	Alternative Method for Question 4(b)		
	$[P(X < 6) =]\left(\frac{1}{5}\right)+\left(\frac{4}{5}\right)\left(\frac{1}{5}\right)+\left(\frac{4}{5}\right)^2\left(\frac{1}{5}\right)+\left(\frac{4}{5}\right)^3\left(\frac{1}{5}\right)+\left(\frac{4}{5}\right)^4\left(\frac{1}{5}\right)$	**M1**	If answer correct, condone omission of 2 from 3 middle terms. Allow M1 for $\left(\frac{1}{5}\right)+\left(\frac{4}{5}\right)\left(\frac{1}{5}\right)+\left(\frac{4}{5}\right)^2\left(\frac{1}{5}\right)+\left(\frac{4}{5}\right)^3\left(\frac{1}{5}\right)+\left(\frac{4}{5}\right)^4\left(\frac{1}{5}\right)+\left(\frac{4}{5}\right)^5\left(\frac{1}{5}\right)$
	$= 0.672, \frac{2101}{3125}$	**A1**	0.67232 to at least 3SF. If M0 awarded, **SC B1** for $\frac{2101}{3125}$ or 0.67232 only.
		2	

Question	Answer	Marks	Guidance
4(c)	$\left[P(X>0 \mid X \neq 2) = \dfrac{P(X>0 \cap X \neq 2)}{P(X \neq 2)} = \right]$ $= \dfrac{14}{25} \Big/ \dfrac{19}{25}$ $= \dfrac{14}{19}$, 0.737	M1	[P($X > 0 \cap X \neq 2$) =] $\frac{14}{25}$, 0.56[0] seen as numerator or denominator of conditional probability fraction.
		M1	[P($X \neq 2$) =] $\frac{19}{25}$, 0.76[0] seen as denominator of conditional probability fraction.
		A1	Final answer = $\frac{14}{19}$, 0.7368421… to at least 3SF. If A0, **SC B1** for correct final answer www.
	Alternative Method for Question 4(c)		
	$\begin{array}{c\|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 0 & 1 & 2 & 3 & 4 \\ 2 & 1 & 0 & 1 & 2 & 3 \\ 3 & 2 & 1 & 0 & 1 & 2 \\ 4 & 3 & 2 & 1 & 0 & 1 \\ 5 & 4 & 3 & 2 & 1 & 0 \end{array}$ $\left[P(X>0 \mid X \neq 2) = \dfrac{\text{Number of outcome}(X>0 \cap X \neq 2)}{\text{Number of outcomes } X \neq 2} = \right]$ $\dfrac{14}{19}$, 0.737	M1	$\left[\text{Number of outcome}(X>0 \cap X \neq 2) = \right]$14 seen as numerator or denominator of conditional probability fraction.
		M1	$\left[\text{Number of outcome}(X \neq 2) = \right]$19 seen as denominator of conditional probability fraction.
		A1	Final answer = $\frac{14}{19}$, 0.7368421… to at least 3SF.
		3	

Question	Answer	Marks	Guidance
4(d)	[P(X >2) = 1 – P(0, 1, 2) with $p = \dfrac{6}{25}$] $1 - \left({}^9C_0\left(\frac{19}{25}\right)^9 + {}^9C_1\left(\frac{6}{25}\right)^1\left(\frac{19}{25}\right)^8 + {}^9C_2\left(\frac{6}{25}\right)^2\left(\frac{19}{25}\right)^7\right)$ [1 – (0.08459 + 0.2404 + 0.3037)]	M1	One term ${}^9C_x\,(p)^x(1-p)^{9-x}$, $0 < p < 1$, $0 < x < 9$.
		A1	$1 - ({}^9C_0\,(1-p)^9 + {}^9C_1\,(p)^1(1-p)^8 + {}^9C_2\,(p)^2(1-p)^7)$, $0 < p < 1$. Correct expression from *their p*, accept unsimplified, no terms omitted leading to final answer. Condone omission of last bracket only.
	0.371	B1	$0.371 \leqslant p < 0.3715$.
	Alternative Method for Question 4(d)		
	[P(X>2) = P(3,4,5,6,7,8,9) with $p = \dfrac{6}{25}$] ${}^9C_3\left(\frac{6}{25}\right)^3\left(\frac{19}{25}\right)^6 + {}^9C_4\left(\frac{6}{25}\right)^4\left(\frac{19}{25}\right)^5 + \ldots + {}^9C_8\left(\frac{6}{25}\right)^8\left(\frac{19}{25}\right)^1 + {}^9C_9\left(\frac{6}{25}\right)^9$ [$0.2238 + 0.1060 + \ldots + 7.529 \times 10^{-5} + 2.642 \times 10^{-6}$]	M1	One term ${}^9C_x\,(p)^x(1-p)^{9-x}$, $0 < p < 1$, $0 < x < 9$.
		A1	${}^9C_3\,(p)^3(1-p)^6 + {}^9C_4\,(p)^4(1-p)^5 + \ldots + {}^9C_8\,(p)^8(1-p)^1 + {}^9C_9\,(p)^9$, $0 < p < 1$. Correct expression from *their p*, accept unsimplified, no terms omitted leading to final answer.
	0.371	B1	$0.371 \leqslant p < 0.3715$.
		3	

Question	Answer	Marks	Guidance
5(a)	[P(15.4 < X < 16.8) =] $P(\frac{15.4-16.5}{0.6} < Z < \frac{16.8-16.5}{0.6})$ [= $P(-1.833 < Z < 0.5)$]	**M1**	Use of ± standardisation formula once with 16.5, 0.6 and either 15.4 or 16.8 substituted.
	[= $\Phi(0.5)+\Phi(1.833)-1=$] 0.6915 + 0.9666 – 1	**M1**	Calculating the appropriate probability area (leading to their final answer, expect > 0.5). 0.6915 – (1 – 0.9666) or (0.6915 – 0.5) + (0.9666 – 0.5) OE are alternatives.
	= 0.658	**A1**	$0.658 \le p < 0.6585$. If A0 scored, **SC B1** for $0.658 \leqslant p < 0.6585$.
	[Expected number =] 0.6581 × 150 = 98, 99	**B1 FT**	FT *their* 4SF (or better) probability from a normal calculation. Must be a positive single integer answer. No approximation notation.
		4	
5(b)	$\left[P\left(Z > \frac{17.1-18.4}{\sigma}\right) = 0.72\right]$ $\frac{17.1-18.4}{\sigma} = -0.583$	**B1**	$0.5825 < z \leqslant 0.583$ or $-0.583 \leqslant z < -0.5825$ seen.
		M1	Use of the ± standardisation formula with 17.1, 18.4, σ and a z-value (not 0.28, 0.72, 0.4175, 0.2358, 0.7642, 0.6103, 0.3897, …). Condone continuity correct ± 0.05, not $\sigma^2, \sqrt{\sigma}$.
	$\sigma = 2.23$	**A1**	AWRT
		3	

Question	Answer	Marks	Guidance
5(c)	[Mean = 120 × 0.72 =] 86.4 [Var = 120 × 0.72 × 0.28 =] 24.192	B1	86.4, $84\frac{2}{5}$ and $24\frac{24}{125}$, 24.192 to at least 3SF seen, allow unsimplified. May be seen in standardisation formula. ($4.918 \leqslant \sigma \leqslant 4.919$ implies correct variance) Incorrect notation is penalised.
	$P(X < 80) = P(Z < \frac{79.5 - 86.4}{\sqrt{24.192}})$	M1	Substituting *their* mean (not 18.4) and *their positive* 4.9185 into ± standardisation formula (any number for 79.5), condone *their* 4.918^2 and $\sqrt{their\ 4.918}$.
		M1	Using continuity correction 79.5 or 80.5 in *their* standardisation formula.
	$[P(Z < -1.4029) = 1 - \Phi(1.403)\ ]$ $1 - 0.9196$	M1	Appropriate area Φ, from final process, must be a probability. Expect final answer < 0.5 . Note: correct final answer implies this M1.
	0.0804	A1	$0.0803 \leqslant p \leqslant 0.0804$
		5	

Question	Answer	Marks	Guidance
6(a)	S + 4C + 2R $^6C_1 \times {}^8C_4 \times {}^{11}C_2$ [$= 6 \times 70 \times 55$] $= 23\,100$ S + 5C + 1R $^6C_1 \times {}^8C_5 \times {}^{11}C_1$ [$= 6 \times 56 \times 11$] $= 3696$ S + 6C [+ 0R] $^6C_1 \times {}^8C_6$ [$\times {}^{11}C_0$][$= 6 \times 28$] $= 168$	**M1**	$^6C_e \times {}^8C_f \times {}^{11}C_g$, with $e + f + g = 7$ seen.
		B1	Correct outcome/value for 1 identified scenario, accept unsimplified, www.
		M1	Add values of 3 correct scenarios. No incorrect scenarios, no repeated scenarios. Condone $^6C_e \times {}^8C_f \times {}^{11}C_g$, with $e + f + g = 7$ to identify S, C, R.
	[Total =] 26964	**A1**	cao
		4	
6(b)	$2! \times 3! \times 4! \times 6$	**M1**	$2! \times 3! \times 4! \times k$, k an integer > 0. 1 can be implied.
	=1728	**A1**	If A0 scored **SC B1** for 1728 www.
		2	

Question	Answer	Marks	Guidance
6(c)	**Method 1**		
	$6! \times 7 \times 6 \times 5$	**M1**	$6! \times k$, k an integer > 0. 1 can be implied.
		M1	$\dfrac{m!}{a! \times b!} \times 7 \times n \times r$; $6 \leqslant m \leqslant 9$; $a = 1, 2$; $b = 1, 4$; $1 \leqslant n, r \leqslant 6, n \neq r$.
		M1	$\dfrac{m!}{a! \times b!} \times 7 \times 6 \times 5$; $6 \leqslant m \leqslant 9$; $a = 1, 2$; $b = 1, 4$.
	151 200	**A1**	Condone 151 000. If A0 scored **SC B1** for 151 200 www.
	Method 2		
	$6! \times {}^7P_3$	**M1**	$6! \times k$, k an integer > 0. 1 can be implied.
		M1	$\dfrac{m!}{a! \times b!} \times {}^7P_q$, or $\dfrac{m!}{a! \times b!} \times {}^7C_q \times q!$; $6 \le m \le 9$; $a = 1, 2$; $b = 1, 4$; $1 \le q \le 6$.
		M1	$\dfrac{m!}{a! \times b!} \times {}^7P_3$, or $\dfrac{m!}{a! \times b!} \times {}^7C_3 \times 3!$; $6 \le m \le 9$; $a = 1, 2$; $b = 1, 4$.
	151 200	**A1**	Condone 151 000. If A0 scored **SC B1** for 151 200 www.

Question	Answer	Marks	Guidance
6(c)	**Method 3**		
	$6! \times 35 \times 3!$	**M1**	$6! \times k$, k an integer > 0. 1 can be implied.
		M1	$\frac{m!}{a! \times b!} \times 35 \times q!$; $6 \leqslant m \leqslant 9$; $a = 1, 2$; $b = 1, 4$; $1 \leqslant q \leqslant 3$.
		M1	$\frac{m!}{a! \times b!} \times 35 \times 6$; $6 \leqslant m \leqslant 9$; $a = 1, 2$; $b = 1, 4$.
	151 200	**A1**	Condone 151 000. If A0 scored **SC B1** for 151 200 www.
	Method 4		
	$9! - 7!3! - {}^3P_2 \times 6! \times 7 \times 6$ Or $9! - 7!3! - 3! \times 7! \times 6$ [= 362 880 – 30 240 – 181 440]	**M1**	$9! - 7!r! - q$, r an integer > 1, q an integer $\leqslant 0$. 0 and 1 may be implied.
		M1	$\frac{s!}{a! \times b! \times c!} - 7!3! - q$; $s = 8, 9$; $a = 1, 2$; $b = 1, 3$; $c = 1, 4$; q an integer $\geqslant 0$. 0 and 1 may be implied.
		M1	$\frac{s!}{a! \times b! \times c!} - 7!3! - {}^3P_2 \times 6! \times 6 \times 7$, $6 \leqslant s \leqslant 9$, **or** $\frac{s!}{a! \times b! \times c!} - 7!3! - 3! \times 7! \times 6$, $6 \leqslant s \leqslant 9$. $a = 1, 2$ $b = 1, 3$ $c = 1, 4$. 1 may be implied.
	151 200	**A1**	Condone 151 000. If A0 scored **SC B1** for 151 200 www.
		4	

Cambridge Assessment International Education

Cambridge International AS & A Level

MATHEMATICS	**9709/53**
Paper 5 Probability & Statistics 1	**May/June 2023**
MARK SCHEME	
Maximum Mark: 50	

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2023 series for most Cambridge IGCSE, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
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FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

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- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1(a)	$\left[\mathrm{P}(\mathrm{HH}) = \frac{1}{4}\right]$ [E(X) =] 4	**B1**	
		1	
1(b)	$\left[\mathrm{P}(X = 5) = \left(\frac{3}{4}\right)^4\left(\frac{1}{4}\right) = \right]$ 0.0791	**B1**	$\frac{81}{1024}$
		1	
1(c)	$[\mathrm{P}(X < 7) =]\, 1 - \left(\frac{3}{4}\right)^6$ or $\frac{1}{4} + \frac{3}{4} \times \frac{1}{4} + \frac{3}{4}^2 \times \frac{1}{4} + \ldots + \frac{3}{4}^5 \times \frac{1}{4}$	**M1**	$1 - p^n$, $0 < p < 1$, $n = 6, 7$ or $p + p(1-p) + p(1-p)^2 + \ldots + p(1-p)^n$, where $n = 4, 5$.
	$= \frac{3367}{4096}$, 0.822	**A1**	Accept 0.82202148… to at least 3SF.
		2	

Question	Answer	Marks	Guidance
2	Mean $=120\times 0.4=48$ Var $=120\times 0.4\times 0.6=28.8$	**B1**	48 and $28\frac{4}{5}$, 28.8 seen, allow unsimplified. ($5.366 \leqslant \sigma \leqslant 5.367$ or $\frac{12\sqrt{5}}{5}$ implies correct variance).
	$P(36 \leqslant X \leqslant 54) = P(\frac{35.5-48}{\sqrt{28.8}} < Z < \frac{54.5-48}{\sqrt{28.8}})$	**M1**	Substituting *their* μ and σ into one ±standardisation formula (any number for 35.5 or 54.5), condone σ^2 and $\sqrt{\sigma}$.
		M1	Using continuity correction 35.5, 36.5 or 53.5, 54.5 once in *their* standardisation formula. Note: $\frac{\pm 12.5}{\sqrt{28.8}}$ or $\frac{\pm 6.5}{\sqrt{28.8}}$ seen gains M2 BOD.
	[= $P(-2.3292 < Z < 1.211)$ =] 0.8871 + 0.9900 – 1	**M1**	Appropriate area Φ, from final process. Must be a probability. Expect final answer > 0.5 . Note: correct final answer implies this M1.
	= 0.877	**A1**	$0.877 \leq p < 0.8772$.
		5	

Question	Answer	Marks	Guidance
3(a)	[P(X = 4) = 3P(X = 2)] $4k(4+a)=3\times 2k(2+a)$ $16k + 4ak = 12k + 6ak$	**M1**	Using P(X = 4) = 3P(X = 2) to form an equation in a and k.
	$a = 2$	**A1**	If M0 scored, **SC B1** for a = 2 www.
	$3k + 8k + 15k + 24k = 1$	**M1**	Using sum of probabilities = 1 to form an equation in k: $k(1 + a) + 2k(2 + a) + 3k(3 + a) + 4k(4 + a) = 1$.
	$k=\dfrac{1}{50}$	**A1**	If M0 scored, **SC B1** for $k=\dfrac{1}{50}$ www.
		4	
3(b)	$\begin{array}{\|c\|c\|c\|c\|c\|}\hline X & 1 & 2 & 3 & 4 \\ \hline \mathrm{P}(X) & \frac{3}{50}, 0.06 & \frac{8}{50}, 0.16 & \frac{15}{50}, 0.3 & \frac{24}{50}, 0.48 \\ \hline\end{array}$	**B1 FT**	$\begin{array}{\|c\|c\|c\|c\|c\|}\hline X & 1 & 2 & 3 & 4 \\ \hline \mathrm{P}(X) & k(1+a) & 2k(2+a) & 3k(3+a) & 4k(4+a) \\ \hline\end{array}$ $0 < p < 1$ for all outcomes, must be numerical.
		1	
3(c)	Var(X) = $\dfrac{3}{50}\times 1+\dfrac{8}{50}\times 2^2+\dfrac{15}{50}\times 3^2+\dfrac{24}{50}\times 4^2-3.2^2$	**M1**	Correct formula for variance method from their probability distribution table, $0 \leqslant$ *their* P(x) $\leqslant 1$. Accept $\dfrac{3+32+135+384}{50}-\dfrac{256}{25}$.
	[= 11.08 – 3.2^2 =] 0.84[0], $\dfrac{21}{25}$	**A1**	If M0 score **SC B1** for 0.84 www.
		2	

Question	Answer	Marks	Guidance
4(a)	Median = 99 [minutes]	**B1**	
	[IQR =] 106 – 83	**M1**	$105 \leqslant UQ \leqslant 112 - 82 \leqslant LQ \leqslant 87$.
	23 [minutes]	**A1**	www. If M0 scored **SC B1** for 23 www.
		3	
4(b)	The times for the Cheetahs are faster than the times for the Panthers	**B1**	Correct statement comparing central tendency in context.
	The times for the Cheetahs are more spread than the times for the Panthers	**B1**	Correct statement comparing range/IQR in context.
		2	
4(c)	[Total time including Kenny = 99 × 20 =]1980	**B1**	Accept unsimplified.
	[Kenny's time =] 1980 – 1862	**M1**	For *their* 1980 – *their* 1862.
	= 118 [minutes]	**A1**	Accept 1 hour 58 mins.
	Alternative Method for Question 4(c)		
	$\frac{1862 + \textit{their}\ \text{Kenny's time}}{20} = 99$ [Kenny's time = $99 \times 20 - 1862$]	**B1**	$\frac{1862 + \textit{their}\ \text{Kenny's time}}{20} = 99$ seen.
		M1	For *their* 99 ×20 – *their* 1862.
	= 118 [minutes]	**A1**	Accept 1 hour 58 mins.
		3	

Question	Answer	Marks	Guidance
5(a)	$P(A) = \frac{10}{36}$ $P(B) = \frac{24}{36}$	**B1**	Accept $P(A) = \frac{10}{36}, \frac{5}{18}, 0.278$ and $P(B) = \frac{24}{36}, \frac{2}{3}, 0.667$.
	$P(A \cap B) = \frac{8}{36}$	**B1**	
	$\frac{10}{36} \times \frac{24}{36}$	**M1**	*Their* P(A) × *their* P(B) seen numerically, $0 \leqslant$ their P(A), P(B) $\leqslant 1$.
	$= \frac{5}{27}, 0.185 \left[\neq \frac{8}{36}\right]$ Events are not independent	**A1 FT**	Multiplication evaluated correctly and compared with intersection that is not a product of multiplication, conclusion stated, notation P(A), P(B) and P($A \cap B$) used.
		4	

Question	Answer	Marks	Guidance
5(b)	$\left[\mathrm{P}(B\|A') = \dfrac{\mathrm{P}(B \cap A')}{\mathrm{P}(A')} = \right]$ $\dfrac{\frac{16}{36}}{\left(1-\frac{10}{36}\right)}$	M1	$\left[\mathrm{P}(B \cap A') = \right]\dfrac{16}{36}, 0.4444$ or *their* P(B) – *their* P($A \cap B$) seen as numerator or denominator of conditional probability fraction.
		M1	$\left[\mathrm{P}(A') = \right]\left(1-\dfrac{10}{36}\right), \dfrac{26}{36}, 0.7222$ or 1 – *their* P(A) seen as denominator of conditional probability fraction.
	$= \dfrac{8}{13}$	A1	Final answer $\dfrac{16}{26}, \dfrac{8}{13}, 0.6153846$ to at least 3SF.
	Alternative Method for Question 5(b): Direct from outcome tables		
	$\left[\mathrm{P}(B\|A') = \dfrac{\text{Number of outcomes}(B \cap A')}{\text{Number of outcomes}(A')} = \right]$ $\dfrac{16}{26}$	M1	$\left[\text{Number of outcomes}(B \cap A') = \right]16$ seen as numerator or denominator of conditional probability fraction.
		M1	$\left[\text{Number of outcomes}(A') = \right]26$ seen as denominator of conditional probability fraction.
		A1	Final answer $\dfrac{16}{26}, \dfrac{8}{13}, 0.6153846$ to at least 3SF.
		3	

Question	Answer	Marks	Guidance
6(a)	$\left[\mathrm{P}(X<16)=\mathrm{P}\left(Z<\frac{16-28}{\sigma}\right)=0.1\right]$ $\frac{16-28}{\sigma}=-1.282$	**B1**	±1.282 seen, cao – critical value.
		M1	Use of the ±standardisation formula with 16, 28, σ and a z-value (not 0.1, 0.9, 0.282, 0.5398, 0.8159) equated to a z-value. Condone continuity correct ±0.5, not $\sigma^2, \sqrt{\sigma}$. Condone $\pm\frac{12}{\sigma}=-1.282$.
	$\sigma=9.36$	**A1**	
		3	
6(b)	[1 − P(0, 1, 2) =] $1-({}^{12}C_0(0.1)^0(0.9)^{12}+{}^{12}C_1(0.1)^1(0.9)^{11}+{}^{12}C_2(0.1)^2(0.9)^{10})$ [1 – (0.2824 + 0.3766 + 0.2301)]	**M1**	One term ${}^{12}C_x\,(p)^x(1-p)^{12-x}$, $0<p<1$. $x\neq 0,1,2$.
		A1	Correct expression, accept unsimplified, no terms omitted leading to final answer.
	0.111	**B1**	0.1108699… rounded to at least 3SF.
	Alternative Method for Question 6(b)		
	$\mathrm{P}(3,4,5,6,7,8,9,10,11,12)={}^{12}C_3(0.1)^3(0.9)^9+{}^{12}C_4(0.1)^4(0.9)^8+\ldots+{}^{12}C_{11}(0.1)^{11}(0.9)^1+{}^{12}C_{12}(0.1)^{12}(0.9)^0$ $[0.08523+0.02131+\ldots+1.08\times10^{-10}+1\times10^{-12}]$	**M1**	One term ${}^{12}C_x\,(p)^x(1-p)^{12-x}$, $0<p<1$. $x\neq 0,1,2$.
		A1	Correct expression, accept unsimplified, no terms omitted leading to final answer.
	0.111	**B1**	0.1108699… rounded to at least 3SF.
		3	

Question	Answer	Marks	Guidance
6(c)	[P($-1.3 < Z < 1.3$) = 2 $\Phi(1.3) - 1$] = 2 × 0.9032 – 1	B1	Identifying at least one of −1.3 or 1.3 as the appropriate z-values.
		M1	Calculating the appropriate probability area from 2 symmetrical z-values (leading to their final answer, expect > 0.5).
	= 0.806, $\frac{504}{625}$	A1	0.8064, $0.806 \leqslant p < 0.8065$.
	[In 365 days 0.8064×365] = 294 or 295	B1 FT	Strict FT *their* at least 4-figure probability (not z-value). Final answer must be positive integer, no approximation or rounding stated.
		4	

Question	Answer	Marks	Guidance
7(a)	**Method 1: Total number of arrangements – number of arrangements with Cs together**		
	$\frac{10!}{2!4!}-\frac{9!}{4!}$ [75600-15120]	**M1**	$\frac{10!}{a!b!}-c$, $a \neq b$, a = 1, 2, b = 1, 4, with c being a positive integer.
		M1	$d-\frac{e!}{4!}$, e = 8, 9, 10, with d being a positive integer.
	= 60480	**A1**	Exact value only. **SC B1** for final answer 60480 www.
	Method 2: Arrangements ^ ^ C ^ C ^ ^ ^ ^ ^		
	$\frac{8!}{4!}\times\frac{9\times8}{2}$	**M1**	$\frac{8!}{4!}\times f$ seen, with f being a positive integer.
		M1	$g\times\frac{9\times8}{h}$, with g being a positive integer, h = 1, 2. $g\times{}^9C_2$ and $g\times{}^9P_2$ are acceptable.
	= 60480	**A1**	Exact value only. **SC B1** for final answer 60480 www.
		3	

Question	Answer	Marks	Guidance
7(b)	AC^^^C^^^A $\frac{6!}{2!} \times 4$	**M1**	$\frac{6!}{2!} \times s$, with s being a positive integer.
		M1	$\frac{t!}{r!} \times 4$, $r = 1, 2, 3$ and $t = 8, 7, 6$.
	1440	**A1**	
	Alternative Method for Question 7(b)		
	$\frac{4 \times {}^6P_3 \times 3!}{2!}$	**M1**	$\frac{{}^6P_3}{2!} \times k$, with k being a positive integer.
		M1	$4 \times 3! \times \frac{{}^6P_m}{n!}$, $m = 2, 3$ and $n = 1, 2, 3$.
	1440	**A1**	
		3	

Question	Answer	Marks	Guidance
7(c)	Scenarios AA _ _ _ $^5C_3 = 10$ AAA _ _ $^5C_2 = 10$ AAAA _ $^5C_1 = 5$	**B1**	Correct number of ways for identified scenarios of 2 or 3 As, accept unsimplified, www.
		M1	Add 3 values for 2, 3 and 4 As, no additional, incorrect or repeated scenarios. Accept unsimplified.
	25	**A1**	
	Alternative Method 2 for Question 7(c)		
	Scenarios: AAC _ _ $^4C_2 = 6$ AA _ _ _ $^4C_3 = 4$ AAAC _ $^4C_1 = 4$ AAA _ _ $^4C_2 = 6$ AAAAC 1 AAAA _ 4	**B1**	Correct total number of ways for identified scenarios of 2 or 3 As, accept unsimplified, www (e.g., both values for AAC^^ and AA^^^ shown would be fine for 2As).
		M1	Add 6 values of appropriate scenarios only, no additional, incorrect or repeated scenarios. Accept unsimplified.
	25	**A1**	
		3	

Cambridge Assessment International Education

Cambridge International AS & A Level

MATHEMATICS **9709/52**

Paper 5 Probability and Statistics 1 **February/March 2023**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the February/March 2023 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

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DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

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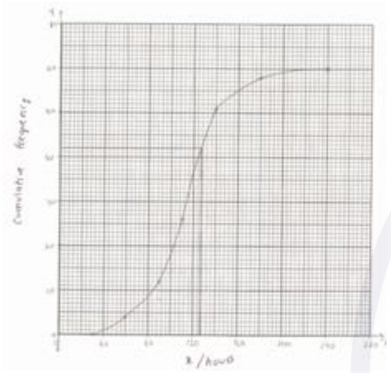
<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td rowspan="4">1(a)</td><td><table><tr><td>Upper value</td><td>60</td><td>90</td><td>110</td><td>140</td><td>180</td><td>240</td></tr><tr><td>cf</td><td>4</td><td>12</td><td>26</td><td>51</td><td>58</td><td>60</td></tr></table></td><td>B1</td><td>All cumulative frequencies stated.
May be under data table, condone omission of 4.
May be read accurately from graph, must include 4.</td></tr>
<tr><td rowspan="2"></td><td>M1</td><td>At least 5 points plotted at class upper end points, daylight rule tolerance.
Linear cf scale $0 \leqslant \text{cf} \leqslant 60$, linear time scale $30 \leqslant \text{time} \leqslant 240$ with at least 3 values identified on each axis.</td></tr>
<tr><td>A1</td><td>All points plotted correctly. Curve drawn (within tolerance), no ruled segments, and joined to $(30, 0)$.
Axes labelled ‘cumulative frequency’ and ‘hours [of sunshine]’ (OE including appropriate title).</td></tr>
<tr><td></td><td>3</td><td></td></tr>
<tr><td rowspan="3">1(b)</td><td>[60 × 0.7 =] 42</td><td>M1</td><td>42 may be implied by clear use on graph.</td></tr>
<tr><td>126</td><td>A1 FT</td><td>Must be clear evidence on graph of use of 42, e.g. an appropriate mark on either axis, appropriate mark on curve.
FT from increasing cf graph only read at 42 only.</td></tr>
<tr><td></td><td>2</td><td></td></tr>
</table>

Question	Answer	Marks	Guidance
1(c)	Midpoints: 45, 75, 100, 125, 160, 210	**B1**	At least 5 correct mid-points seen, check by data table or used in formula.
	[Mean =] $\dfrac{4\times45+8\times75+14\times100+25\times125+7\times160+2\times210}{60}$ $\left[=\dfrac{6845}{60}\right]$	**M1**	Correct mean formula using their 6 midpoints (must be within class, not upper bound, lower bound), condone 1 data error If correct midpoints seen accept $\dfrac{180+600+1400+3125+1120+420}{60}$.
	$= 114, 114\dfrac{1}{12}$	**A1**	Accept 114.1, 114.08[3…] If A1 not awarded, **SC B1** for 114, $114\dfrac{1}{12}$, 114.1 or 114.08[3…].
		3	

Question	Answer	Marks	Guidance
2(a)	$0.6(0.5)^3+0.4(0.5)^3\times3$	**B1**	Either $0.6(0.5)^3+a$ or $b+0.4(0.5)^3\times(3 or {}^3C_1)$, $0 < a, b < 1$ seen.
		M1	$0.6(0.5)^3+0.4(0.5)^3\times d$ seen, d = 1, 3. Condone 0.075 + 0.05 × d, d = 1, 3.
	= 0.225	**A1**	AG full supporting working required. Scenarios identified and linked to calculations.
		3	

Question	Answer	Marks	Guidance
2(b)	<table><tr><td>x</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>$P(X = x)$</td><td>0.05</td><td>0.225</td><td>**0.375**</td><td>**0.275**</td><td>0.075</td></tr></table>	**B1**	Either [P(2) =] 0.375, $\frac{3}{8}$ or [P(3) =] 0.275, $\frac{11}{40}$ seen. Condone not in table if identified.
		B1 FT	Both values in table. FT P(2) + P(3) = 0.650 .
		2	
2(c)	Var(X) $= [1^2\times]0.225 + 2^2 \times \textit{their}\ 0.375 + 3^2 \times \textit{their}\ 0.275 + 4^2 \times 0.075 - 2.1^2$	**M1**	Appropriate variance formula from their probability distribution table with at least 4 terms, $0 < \textit{their}\ P(x) < 1$. Condone 4.41 for 2.1^2 . Condone mean clearly recalculated inaccurately. Or $0.225 + 4 \times \textit{their}\ 0.375 + 9 \times \textit{their}\ 0.275 + 16 \times 0.075 - 2.1^2$ Condone 2.1^2 for 4.41.
	$\left[5.4 - 2.1^2\right] = 0.99[0]$	**A1**	If M0 awarded **SC B1** for 0.99[0] WWW.
		2	

Question	Answer	Marks	Guidance
3(a)	**Method 1 for Question 3(a)**		
	[P(X > 17) = P(18, 19, 20) =] $^{20}C_{18}\,(0.8)^{18}(0.2)^{2} + {}^{20}C_{19}\,(0.8)^{19}(0.2)^{1}$	**M1**	One term $^{20}C_x\,(p)^x(1-p)^{20-x},\ 0<p<1, 0<x<20$.
	$+\ {}^{20}C_{20}\,(0.8)^{20}$ = 0.13691 + 0.05765 + 0.01153	**A1**	Correct expression, accept unsimplified, no terms omitted leading to final answer.
	0.206	**B1**	Mark the final answer at the most accurate value $0.206 \leqslant p \leqslant 0.2061$.
	Method 2 for Question 3(a)		
	[P(X > 17) = 1 – P(0,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17) =] $1-({}^{20}C_0\,(0.8)^0(0.2)^{20} + {}^{20}C_1\,(0.8)^1(0.2)^{19}$	**M1**	One term $^{20}C_x\,(p)^x(1-p)^{20-x},\ 0<p<1, 0<x<20$.
	$+\ {}^{20}C_2\,(0.8)^2(0.2)^{18} + \ldots + {}^{20}C_{16}\,(0.8)^{16}(0.2)^4$ $+\ {}^{20}C_{17}\,(0.8)^{17}(0.2)^3)$ $= 1-(1.048\times10^{-14} + 8.389\times10^{-13}$ $+3.188\times10^{11} + \ldots + 0.2182 + 0.2054)$	**A1**	Correct expression, accept unsimplified, no terms omitted leading to final answer. If answer correct, condone omission of any 15 of the 16 middle terms.
	0.206	**B1**	Mark the final answer at the most accurate value $0.206 \leqslant p \leqslant 0.2061$. Condone omission of brackets.
		3	
3(b)	$\left[(0.8)^4(0.2)=\right]0.08192, \dfrac{256}{3125}$	**B1**	Accept $\dfrac{8192}{100000}$ OE.
		1	

Question	Answer	Marks	Guidance
3(c)	$(0.8)^5(0.2)^2 \times 6$	**M1**	$(0.8)^5(0.2)^2 \times k$ *or* $(0.8)^5(0.2) \times k \times 0.2$, $2 \leqslant k \leqslant 7$.
	$= 0.0786, \dfrac{8144}{78125}$	**A1**	$0.0786 \leqslant p < 0.07865$, $\dfrac{786432}{10000000}$. If A0 awarded, **SC B1** for correct answer WWW.
		2	

Question	Answer	Marks	Guidance
4	$(1 - x) \times 0.7 \times 0.9 = 0.36$	**M1**	$(1-x)\times a \times b = 0.36$, $a = 0.7$ or 0.3, $b = 0.9$ or 0.1
		B1	$(1-x)\times0.7\times0.9=0.36$, $(1-x)\times0.63=0.36$, $0.63 - 0.63x = 0.36$ or $1 - x = \dfrac{0.36}{0.63}$ seen. Condone recovery from omission of brackets.
	$x = \dfrac{3}{7}$	**A1**	Accept 0.428571 to at least 3 sf. Condone 0.4285 rounding to 0.429 . If M0 awarded, **SC B1** for $x = \dfrac{3}{7}$ or 0.428571 to at least 3 sf.
		3	

Question	Answer	Marks	Guidance
5	$\text{P}(A) = \frac{1}{2}$, $\text{P}(B) = \frac{8}{24} = \frac{1}{3}$,	**B1**	Both stated, accept unsimplified.
	$\text{P}(A \cap B) = \frac{1}{6}$	**M1**	Evidence that independence properties not used.
	$\text{P}(A) \times \text{P}(B) = \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$ so events are independent	**A1**	Evaluated and conclusion stated. $\text{P}(A) \times \text{P}(B)$ and $\text{P}(A \cap B)$ seen.
		3	

Question	Answer	Marks	Guidance
6(a)	$[\text{P}(X < 74) =]\ \text{P}\left(Z < \frac{74 - 62.3}{8.4}\right)\left[= \text{P}(Z < 1.393)\right]$	**M1**	Use of ± standardisation formula with 74, 62.3 and 8.4 substituted appropriately, not 8.4^2, not $\sqrt{8.4}$, no continuity correction.
	= 0.918	**A1**	$0.918 \leqslant p \leqslant 0.9185$.
		2	

Question	Answer	Marks	Guidance
6(b)	$[\text{P}(50 < X < 74) = \text{P}]\left(\frac{50-62.3}{8.4} < Z < \frac{74-62.3}{8.4}\right)$ $[\text{P}(-1.464 < Z < 1.393)]$	**M1**	Use of ± standardisation formula with both 74 (may be seen in 6(a) if *their* value seen) & 50, 62.3 and 8.4 substituted appropriately. Condone use of $8.4^2, \sqrt{8.4}$ and continuity correction ±0.5 (73.5 or 74.5 and 49.5 or 50.5).
	$[\Phi(1.464) + \Phi(1.393) - 1\ ]$ 0.9285 + 0.9182 – 1	**M1**	Calculating the appropriate probability area from stated Φ of z-values (leading to *their* final answer > 0.5) but not symmetrical values.
	= 0.847	**A1**	$0.8465 \leqslant p < 0.8475$. **SC B1** for $0.8465 \leqslant p < 0.8475$ if M0A0 awarded.
	$(0.8467)^4 = 0.514$	**B1 FT**	Accept $0.513 \leqslant p \leqslant 0.514$. FT (*their* 4-figure p)^4, $0 < p < 1$.
		4	
6(c)	$z_1 = \frac{36-\mu}{\sigma} = -0.739$ $z_2 = \frac{54-\mu}{\sigma} = 1.282$	**B1**	$-0.740 < z_1 < -0.738$ or $0.738 < z_1 < 0.740$.
		B1	$z_2 = \pm 1.282$ (critical value).
		M1	Use of the ±standardisation formula once with μ, σ and a z-value (not 0.23, 0.77, 0.90, 0.10, ±0.261, ±0.282…). Condone continuity correction ±0.5, not $\sigma^2, \sqrt{\sigma}$.
	Solve, obtaining values for μ and σ $\mu = 42.6, \quad \sigma = 8.91$	**M1**	Solve using the elimination method, substitution method or other appropriate approach to obtain values for both μ and σ.
		A1	$42.58 \leqslant \mu \leqslant 42.6$, $8.90 \leqslant \sigma \leqslant 8.91$.
		5	

Question	Answer	Marks	Guidance
7(a)	**Method 1: Arrangements with 3 Es together – arrangements with 3 Es together and 2 Ds together**		
	$\frac{7!}{2!}-6!$	**B1**	$\frac{7!}{2!}-e$, e a positive integer (including 0).
		M1	$f-6!$, $f>6!$
		M1	$\frac{7!}{a!b!}-\frac{6!}{c!d!}$, a,c = 1, 2 and b,d = 1, 3.
	1800	**A1**	
	Method 2: Identified scenarios ^ EEE ^ ^ ^		
	$5!\times\frac{6\times5}{2}$	**B1**	$5!\times j$, j a positive integer (j = 1 may be implied).
		M1	$\frac{k!}{m!}\times\frac{6\times5}{2}$, $\frac{k!}{m!}\times{}^6C_2$, $\frac{k!}{m!}\times\frac{{}^6P_2}{2}$ or $k!\times\frac{7\times6}{n}$, k a positive integer (k = 1 may be implied), m = 1, 2 n = 1, 2, 3.
		M1	$k!\times\frac{m\times(m-1)}{n}$ k a positive integer > 1, m = 10, 9, 8, 7, 6 and n = 1, 2.
	1800	**A1**	
		4	

Question	Answer	Marks	Guidance
7(b)	**First 2 marks: Method 1 – Number of arrangements with 2 Ds in one position with 4 letters in between – repeats allowed**		
	$7! \times 4 \times 2$	**M1**	$7! \times s$, s = positive integer > 1.
		M1	$t! \times 4 \times 2$, t = 8, 7, 6. Condone $t! \times 8$.
	First 2 marks: Method 2 – Picking 2Ds, arranging 4 letters from remaining letters between and then arranging terms		
	${}^7P_4 \times 4! \times 2!$	**M1**	${}^7P_4 \times a! \times b!$, $1 \leqslant a \leqslant 6$ and b = 1, 2, 3.
		M1	${}^7P_c \times 4! \times 2!$, c = 3, 4, 5.
	First 2 marks: Method 3 – Identified scenarios involving Es between Ds		
	D ^ ^ ^ ^ D E E E = ${}^4C_4 \times 4! \times 4! \times 2! = 1152$ D E ^ ^ ^ D E E ^ = ${}^4C_3 \times 4! \times 4! \times 3 \times 2! = 13824$ D E E ^ ^ D E ^ ^ = ${}^4C_2 \times 4! \times 4! \times 3 \times 2! = 20736$ D E E E ^ D ^ ^ ^ = ${}^4C_1 \times 4! \times 4! \times 2! = 4608$	**M1**	1 identified scenario value correct.
		M1	4 appropriate scenarios added, no incorrect.

Question	Answer	Marks	Guidance
7(b)	**Final 3 marks for Methods 1, 2 and 3**		
	40320	**A1**	If A0 scored, **SC B1** for 40320 WWW.
	[Total number of arrangements =] $[9! =]$ 362880	**B1**	Accept unsimplified. May be seen as denominator of probability.
	Probability = $\dfrac{40320}{362880} = \dfrac{1}{9}$	**B1FT**	$\dfrac{\textit{their}\ 40320}{\textit{their}\ 362880}$, accept unsimplified. B1FT if *their* 40320 and *their* 362880 supported by work in this part. Condone *their* 362880 supported by calculation in **7(a)**.
		5	
7(c)	Scenarios D E _ _ _ $\quad {}^4C_3 \quad 4$ D E E _ _ $\quad {}^4C_2 \quad 6$ D E E E _ $\quad {}^4C_1 \quad 4$ D D E _ _ $\quad {}^4C_2 \quad 6$ D D E E _ $\quad {}^4C_1 \quad 4$ D D E E E $\quad [{}^4C_0] \quad 1$	**B1**	1 correct unsimplified outcome/value for one identified scenario excluding DDEEE. Note: ${}^4C_1$ cannot be used for ${}^4C_3$.
		M1	Add values of 6 appropriate scenarios, no additional, incorrect or repeated scenarios. Accept unsimplified.
	[Total =] 25	**A1**	
		3	

Cambridge Assessment International Education

Cambridge International AS & A Level

MATHEMATICS **9709/51**

Paper 5 Probability & Statistics 1 **October/November 2022**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2022 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:
- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:
- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	$0.12 + p + q + 0.16 + 0.3 = 1$	**B1**	Sum of probabilities = 1 $p + q = 0.42$ OE.
	$-0.24 - p + 0.5q + 0.16 + 0.6 = 0.28$	**B1**	Form equation using $E(X) = 0.28$ $-p + 0.5q = -0.24$ OE. Accept unsimplified.
	Attempt to solve *their* two equations in p and q	**M1**	**Either** Substitution method to form a single equation in either p or q and finding values for both unknowns. **Or** Elimination method by writing both equations in the same form (usually $ap + bq = c$) and + or – to find an equation in one unknown and finding values for both unknowns.
	$q = 0.12,\ p = 0.3$	**A1**	CAO, both WWW. If M0 awarded **SC B1** for both correct WWW.
		4	

Question	Answer	Marks	Guidance
2(a)	[P(3, 4, …7) = 1 – P(0, 1, 2, 8)] $= 1-({}^8C_0\ 0.48^0\,0.52^8 + {}^8C_1\ 0.48^1\,0.52^7$ $+ {}^8C_2\ 0.48^2\,0.52^6 + {}^8C_8\ 0.48^8\,0.52^0)$	**M1**	One term ${}^8C_x\ p^x(1-p)^{8-x}$, for $0 < x < 8$, $0 < p < 1$
	= 1 – (0.00534597 + 0.039478 + 0.127544 + 0.0028179)	**A1**	Correct expression, accept unsimplified, no terms omitted, leading to final answer.
	0.825	**B1**	Mark the final answer at the most accurate value. $0.8248 < p \leqslant 0.825$ WWW.
	Alternative method for Question 2(a)		
	[P(3, 4, 5, 6, 7) =] ${}^8C_3\ 0.48^3\,0.52^5 + {}^8C_4\ 0.48^4\,0.52^4 + {}^8C_5\ 0.48^5\,0.52^3 + {}^8C_6$ $0.48^6\,0.52^2 + {}^8C_7\ 0.48^7\,0.52^1$	**M1**	One term ${}^8C_x\ p^x(1-p)^{8-x}$, for $0<x<8$, $0<p<1$
		A1	Correct expression, accept unsimplified, no terms omitted, leading to final answer.
	0.825	**B1**	Final answer $0.8248 < p \leqslant 0.825$ WWW.
		3	

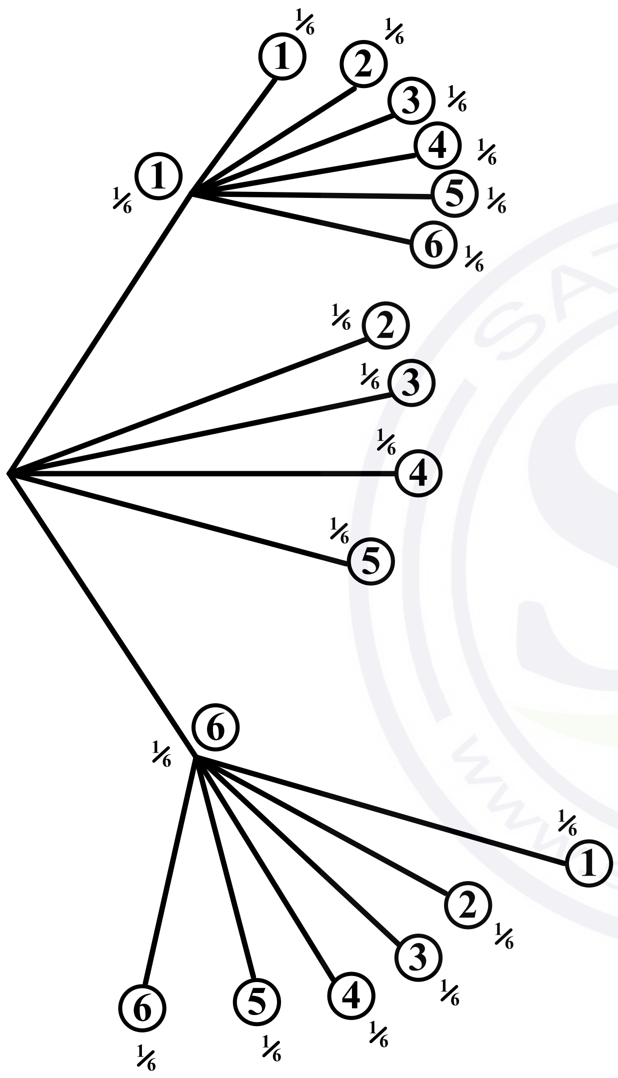
Question	Answer	Marks	Guidance
2(b)	[Mean = $0.52 \times 125 =$]65, $\left[\text{var} = 0.52 \times 0.48 \times 125 =\right]$31.2	**B1**	65 and 31.2 seen, allow unsimplified. May be seen in standardisation formula. (5.585 < $\sigma \leqslant$ 5.586 imply correct variance).
	[P(X > 72) =]P($Z > \frac{72.5-65}{\sqrt{31.2}}$) [= P($Z > 1.343$)]	**M1**	Substituting *their* 65 and $\sqrt{their\,31.2}$ into ±standardisation formula (any number for 72·5), not *their* 31.2, √ *their* 5.586 .
		M1	Using continuity correction 72·5 or 71·5 in *their* standardisation formula . Note $\frac{\pm 7.5}{\sqrt{31.2}}$ or $\frac{\pm 7.5}{5.586}$ seen gains **M2** BOD
	= 1 – 0.9104	**M1**	Appropriate area Φ, from final process, must be probability.
	0.0896	**A1**	0.0896 $\leqslant p \leqslant$ 0.0897 WWW.
		5	

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td rowspan="5">3(a)</td><td rowspan="3">
<table>
<tr><th>Lions</th><th></th><th>Tigers</th></tr>
<tr><td>9</td><td>16</td><td></td></tr>
<tr><td>9 8</td><td>17</td><td>9</td></tr>
<tr><td>9 7 6 1 0</td><td>18</td><td>0 3 4 7</td></tr>
<tr><td>6 0 0</td><td>19</td><td>0 1 4 5 7</td></tr>
<tr><td></td><td>20</td><td>1</td></tr>
</table>
</td><td>B1</td><td>Correct stem can be upside down, ignore extra values (not in reverse).</td></tr>
<tr><td>B1</td><td>Correct Lions labelled on left, leaves in order from right to left and lined up vertically, no commas or other punctuation.</td></tr>
<tr><td>B1</td><td>Correct Tigers labelled on same diagram, leaves in order and lined up vertically, no commas or other punctuation.

If the correct data for Lions and Tigers is transposed, treat as a single error in Lions and condone in Tigers.</td></tr>
<tr><td>Key 1|18|3 means 181 cm for Lions and 183 cm for Tigers</td><td>B1</td><td>Correct single key for their diagram, need both teams identified and ‘cm’ stated at least once here or in leaf headings or title.

SC If 2 separate diagrams drawn, SC B1 if both keys meet these criteria (Max B1, B0, B0, B1).</td></tr>
<tr><td></td><td>4</td><td></td></tr>
<tr><td rowspan="4">3(b)</td><td>Median = 186 cm</td><td>B1</td><td></td></tr>
<tr><td>[UQ = 190 cm, LQ = 179 cm]
IQR = 190 – 179</td><td>M1</td><td>$189 \leqslant UQ \leqslant 190 - 178 \leqslant LQ \leqslant 180$</td></tr>
<tr><td>11[cm]</td><td>A1</td><td>WWW</td></tr>
<tr><td></td><td>3</td><td></td></tr>
<tr><td rowspan="3">3(c)</td><td>Tigers are (generally) taller</td><td>B1</td><td>Comparison about central tendency in context.</td></tr>
<tr><td>Heights of Tigers are slightly less consistent than heights of Lions</td><td>B1</td><td>Comparison about spread in context. (Condone ‘similar spread’ in context.)</td></tr>
<tr><td></td><td>2</td><td></td></tr>
</table>

Question	Answer	Marks	Guidance
4(a)	P(X<132) = P($Z < \frac{132-125.4}{18.6}$) = P($Z < 0.3548$)	**M1**	Use of ±standardisation formula with 132 and 125.4 substituted, condone continuity correction 132±0.5 and use of 18.6^2, $\sqrt{18.6}$
	0.639	**A1**	$0.6385 < p \leqslant 0.639$ If M0 scored, **SC B1** for $0.6385 < p \leqslant 0.639$
		2	
4(b)	$\frac{108-117}{\sigma} = -1.175$	**B1**	$1.1749 < z \leqslant 1.175$ or $-1.175 \leqslant z < -1.1749$
		M1	108 and 117 substituted in ±standardisation formula, no continuity correction, not σ^2, $\sqrt{\sigma}$, equated to a z-value.
	$\sigma = 7.66$	**A1**	$7.659 \leqslant \sigma \leqslant 7.66$ If M0 scored, **SC B1** for $7.659 \leqslant \sigma \leqslant 7.66$
		3	
4(c)	$P(-1.5 < Z < 1.5)$ $[\Phi(1.5) - \Phi(-1.5)]$ $[= 2\Phi(1.5) - 1]$ $= 2 \times \textit{their}\ 0.9332 - 1$ **or** $\textit{their}\ 0.9332 - (1 - \textit{their}\ 0.9332)$ **or** $2 \times (\textit{their}\ 0.9332 - 0.5)$	**M1**	{Both 1.5 and –1.5 seen as z-values **or** appropriate use of 1.5 or −1.5} **and** {no other z-values in part}.
		M1	Calculating the appropriate area from stated phis of z-values which must be ± the same number. Condone *their* 0.0668 as (1 – *their* 0.9332).
	0.8664	**A1**	Accept answers wrt 0.866 If A0 scored **SC B1** for answers wrt 0.866
	$0.8664^3 = 0.650[36\ldots]$	**B1 FT**	FT *their* 4SF (or better) probability, accept final answers to 3SF.
		4	

Question	Answer	Marks	Guidance
5(a)		**B1**	1st throw fully correct with probabilities and outcomes identified. (Probabilities $\left(\text{all}\,\frac{1}{6}\right)$ and outcomes (1,2,3,4,5,6) on branches).
		B1	2nd throw fully correct with probabilities and outcomes identified. (Probabilities $\left(\text{all}\,\frac{1}{6}\right)$ and outcomes (1,2,3,4,5,6) on branches).
		2	

Question	Answer	Marks	Guidance
5(b)	5 comes from 1+4 or 5: P(5) = $\frac{1}{6}\times\frac{1}{6}+\frac{1}{6}=\frac{7}{36}$ 6 comes from 1+5 P(6) = $\frac{1}{6}\times\frac{1}{6}=\frac{1}{36}$ 7 comes from 1+6 or 6+1 P(7) = $\frac{1}{6}\times\frac{1}{6}+\frac{1}{6}\times\frac{1}{6}=\frac{2}{36}$ 8 comes from 6+2 P(8) = $\frac{1}{6}\times\frac{1}{6}=\frac{1}{36}$ 9 comes from 6+3 P(9) = $\frac{1}{6}\times\frac{1}{6}=\frac{1}{36}$	**B1**	P(5) or P(7) identified and correct unsimplified, accept if supported by correct scenarios shown or from tree diagram .
	P(A) = $\frac{7}{36}+\frac{1}{36}+\frac{2}{36}+\frac{1}{36}+\frac{1}{36}$	**M1**	Adding only the values from 5 correct scenarios.
	$=\frac{12}{36}=\frac{1}{3}$	**A1**	Scenarios identified (may be on tree diagram in **5(a)**), all probabilities seen, WWW AG.
		3	
5(c)	P(B) = $\frac{1}{3}$, P(A∩B)= $\frac{6}{36}$	**M1**	Both identified and evaluated, consistent with *their* tree diagram or correct.
	P(A)P(B) = $\frac{1}{3}\times\frac{1}{3}=\frac{1}{9}$ $\frac{6}{36}\neq\frac{1}{9}$, so not independent	**A1**	P(A) × P(B) seen and evaluated, all notation present and correct. Correct conclusion WWW.
		2	

Question	Answer	Marks	Guidance
5(d)	$\text{P}(B \mid A') = \dfrac{P(B \cap A')}{P(A')} = \dfrac{\textit{their}\,\frac{6}{36}}{\frac{2}{3}}$	B1	$\dfrac{6}{36}$ oe as numerator of a fraction.
		M1	$\dfrac{\textit{their}\,\frac{6}{36}\ \textbf{or}\ \textit{correct}}{\textit{their}\,1-\frac{1}{3}\ \textbf{or}\ \textit{correct}}$ seen, consistent with *their* tree diagram.
	$\dfrac{1}{4}$, 0.25	A1	
		3	

Question	Answer	Marks	Guidance
6(a)	5M0W $^8C_5$ [$\times {}^7C_0$] = 56 4M1W $^8C_4 \times {}^7C_1$ = 490 3M2W $^8C_3 \times {}^7C_2$ = 1176	**M1**	$^8C_x \times {}^7C_{5-x}$ for x = 1, 2, 3, 4,or 5
		B1	Outcome for 4M1W **or** 3M2W correct and identified, accept unsimplified.
		M1	Add 3 values of appropriate scenarios, no incorrect scenarios, no repeated scenarios, accept unsimplified. Addition may be implied by final answer.
	[Total =] 1722	**A1**	Value stated WWW.
	Alternative method for Question 6(a)		
	2M3W $^8C_2 \times {}^7C_3$ = 980 1M4W $^8C_1 \times {}^7C_4$ = 280 0M5W $^8C_0 \times {}^7C_5$ = 21	**M1**	$^8C_x \times {}^7C_{5-x}$ for x = 1, 2, 3, 4,or 5
		B1	Outcome for 2M3W **or** 1M4W correct and identified, accept unsimplified.
	[Total = $^{15}C_5$ – (980 + 280 + 21)] 3003 – (980 + 280 + 21)	**M1**	Subtract 3 values of appropriate scenarios from *their* identified total or correct, no incorrect scenarios, no repeated scenarios, accept unsimplified.
	[Total =] 1722	**A1**	Value stated WWW.
		4	
6(b)	$^{15}C_3 \times {}^{12}C_5$ [$\times {}^7C_7$] [= 455 × 792]	**M1**	$^{15}C_r \times q$, r = 3, 5, 7; q a positive integer >1
		M1	$^{15}C_s \times {}^{15-s}C_t$ [$\times {}^{15-s-t}C_u$] s = 3,5,7; t = 3,5,7 $\neq s$; u = 3,5,7 $\neq s,t$
	360360	**A1**	Final answer. If A0 awarded **SC B1** for final answer 360360.
		3	

Question	Answer	Marks	Guidance
6(c)	**Method 1: Total number of arrangements with AB together – Arrangements with AB and FG together**		
	$6! \times 2 - 5! \times 2 \times 2$ [= 1440 – 480]	**M1**	$a! \times 2! \times b$, $a = 5, 6$; $b = 1,2$ seen.
		M1	Either $6! \times 2 - c$,$1 < c < 1440$ or $d - 5! \times 2 \times 2$, $1440 < d$
	960	**A1**	
	Method 2: arrangements with AB together with F and G not together.		
	$2 \times 4! \times 5 \times 4$	**M1**	$2 \times 4! \times e$, e positive integer >1
		M1	$f \times 5 \times 4$, f positive integer >1 condone $f \times 20$, $f \times {}^5C_2$, f positive integer >1
	960	**A1**	
		3	

Cambridge Assessment International Education

Cambridge International AS & A Level

MATHEMATICS **9709/52**

Paper 5 Probability & Statistics 1 **October/November 2022**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

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Generic Marking Principles

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GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

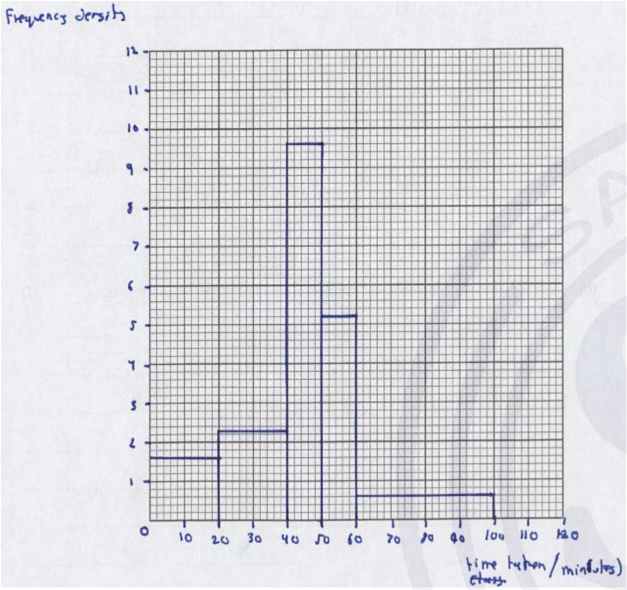
Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1(a)	$0.2\times x+0.1\times 2x+0.7\times 0.25=0.235$	**M1**	$0.2\times x+0.1\times 2x+0.7\times 0.25$ or $0.2x + 0.2x + 0.175$ seen.
		M1	Equating *their* 3 term expression (2 terms involving x) to 0.235
	$x=0.15$	**A1**	
		3	
1(b)	$\left[P(car\mid not\,late)=\dfrac{P(car\text{ and }not\,late)}{P(not\,late)}\right]$ $\dfrac{0.1\times(1-0.3)}{1-0.235}$	**M1**	$0.1\times(1-2\times their\ x)$ or 0.1×0.7 as numerator **and** $0.2 \times (1 - their\ x) + 0.1 \times (1 - 2 \times their\ x) + 0.7 \times 0.75$ with values substituted or $1 - 0.235$ or 0.765 as denominator of fraction. Condone $0.2 \times (1 - their\ x) + 0.1 \times (1 - \times\ their\ x) + 0.7 \times 0.75$ as denominator consistent with **1(a)**.
	$\left[\dfrac{0.07}{0.765}=\right]0.0915, \dfrac{70}{765}, \dfrac{14}{153}$	**A1**	0.091503267 to at least 3SF. If M0 scored **SC B1** for 0.091503267 to at least 3SF.
		2	

Question	Answer	Marks	Guidance
2(a)	[P(X<54.8)] = P($Z < \frac{54.8-55.6}{1.2}$)	**M1**	Use of ± standardisation formula, with 54.8, 55.6 and 1.2 substituted. condone $1.2^2, \sqrt{1.2}$ or continuity correction of 54.75 or 54.85
	[= P($Z < -0.6667$)] $= 1 - 0.7477$	**M1**	Appropriate area Φ, from final process, must be probability.
	= 0.2523	**A1**	$0.252 \leqslant p \leqslant 0.2525$ If A0 scored **S CB1** for $0.252 \leqslant p \leqslant 0.2525$
	[Expected number =] $400 \times 0.2523 = 100.92$ 100 or 101	**B1 FT**	FT *their* 4SF (or better) probability from a normal calculation. Must be a single integer answer.
		4	
2(b)	[P($-\frac{1}{2} < Z < \frac{1}{2}$) = Φ(½) – Φ(−½) =] $2\Phi\left(\frac{1}{2}\right)-1$ = $2 \times \textit{their}\, 0.6915 - 1$ or $\textit{their}\, 0.6915 - (1 - \textit{their}\, 0.6915)$ or $2 \times (0.6915 - 0.5)$	**M1**	{Both ½ and –½ seen as z-values **or** appropriate use of +½ or −½} **and** {no other z-values in part}. Condone $\frac{56.2-55.6}{1.2}$ and $\frac{55[.0]-55.6}{1.2}$ seen as z-values.
		M1	Calculating the appropriate area from stated phis of z-values which must be ± the same number.
	0.383	**A1**	$0.3829 \leqslant z \leqslant 0.383$ If A0 scored **SC B1** for $0.3829 \leqslant z \leqslant 0.383$
		3	

Question	Answer	Marks	Guidance
3(a)	[P(17 or 18) =] $\frac{4}{216}=\frac{1}{54}$,0.0185(185…)	**B1**	May be seen used in calculation.
	P(X = 6) = $\left(\frac{53}{54}\right)^5 . \frac{1}{54}$	**M1**	$p(1-p)^5$, $0 < p < 1$
	0.0169	**A1**	$0.01686 < p \leqslant 0.0169$ If A0 scored **SC B1** for $0.01686 < p \leqslant 0.0169$
		3	
3(b)	[P(X < 8) =] $1-\left(\frac{53}{54}\right)^7$	**M1**	$1-\left(their\left(\frac{53}{54}\text{or}\,0.98148\right)\text{or correct}\right)^r$, $r = 7,8$ $0 < their\ p < 1$
	0.123	**A1**	$0.1225 \leqslant p \leqslant 0.123$
	Alternative method for Question 3(b)		
	[P(X < 8) =] $\left(\frac{1}{54}\right)+\left(\frac{53}{54}\right)\left(\frac{1}{54}\right)+\left(\frac{53}{54}\right)^2\left(\frac{1}{54}\right)+\left(\frac{53}{54}\right)^3\left(\frac{1}{54}\right)+\left(\frac{53}{54}\right)^4\left(\frac{1}{54}\right)+$ $\left(\frac{53}{54}\right)^5\left(\frac{1}{54}\right)+\left(\frac{53}{54}\right)^6\left(\frac{1}{54}\right)$	**M1**	$q+pq+p^2q+p^3q+p^4q+p^5q\left[+p^6q\right],\ p+q=1, 0<p,q<1, q$ $=their\,\frac{53}{54}$
	0.123	**A1**	$0.1225 \leqslant p \leqslant 0.123$
		2	

Question	Answer	Marks	Guidance
4(a)	Cw 20 20 10 10 40 Fd 1.6 2.3 9.6 5.2 0.6 	**M1**	At least 4 frequency densities calculated $\frac{f}{cw}$ eg $\frac{32}{20}\left(\textit{condone}\ \frac{f}{cw \pm 0.5}\ \textit{if unsimplified}\right)$, accept unsimplified, may be read from graph using *their* scale no lower than 1 cm = fd 1
		A1	All bar heights correct on graph, using *their* suitable linear scale with at least 3 values indicated, no lower than 1 cm = fd 2.
		B1	Bar ends at [0,] 20, 40, 50, 60, 100 (at axis), 5 bars drawn 0 ⩽ time axis ⩽ 100, linear scale with at least 3 values indicated.
		B1	Axes labelled frequency density (fd), time (t) and minutes (mins, m) or appropriate title. (Axes may be reversed).
		4	

Question	Answer	Marks	Guidance
4(b)	Midpoints 10 30 45 55 80	**B1**	At least 4 correct midpoints seen (check data table).
	[Mean = 43.2 given] [Var =] $\frac{32\times10^2+46\times30^2+96\times45^2+52\times55^2+24\times80^2}{250}-43.2^2$ **Or** $\frac{32(10-43.2)^2+46(30-43.2)^2+96(45-43.2)^2+52(55-43.2)^2+24(80-43.2)^2}{250}$	**M1**	Appropriate variance formula with *their* 5 midpoints (not upper bound, lower bound, class width, frequency density, frequency or cumulative frequency). Condone 1 frequency error. If correct midpoints seen accept $\left\{\frac{3200+41400+194400+157300+153600}{250}\ or\ \frac{549900}{250}\right\}$ $-\{43.2^2\ or\ 1866.24\}$.
	$=\left[\frac{549900}{250}-43.2^2=333.36\right]$ Sd = 18.3	**A1**	www, final answer 18.25814887 to at least 3SF. If M0 earned **SC B1** for final answer 18.25814887 to at least 3SF.
		3	

Question	Answer	Marks	Guidance
5(a)	**Method 1: Scenarios identified ignoring unbiased coin**		
	P(BH_1 BT_2) = $\frac{1}{4}\times\frac{3}{4}=\frac{3}{16}$ P(BT_1 BH_2) = $\frac{3}{4}\times\frac{1}{4}=\frac{3}{16}$ P(BH_1 BH_2) = $\frac{1}{4}\times\frac{1}{4}=\frac{1}{16}$	**M1**	All 3 different calculations seen unsimplified.
	$\frac{3}{16}+\frac{3}{16}+\frac{1}{16}=\frac{7}{16}$	**A1**	Clear identification of **all scenarios**, linked probabilities and sum. AG

Question	Answer	Marks	Guidance
5(a)	**Method 2: Scenarios identified with all 3 coins**		
	$P(H\ BH_1\ BT_2) = \frac{1}{2}\times\frac{1}{4}\times\frac{3}{4}=\frac{3}{32}$ $P(T\ BH_1\ BT_2) = \frac{1}{2}\times\frac{1}{4}\times\frac{3}{4}=\frac{3}{32}$ $P(H\ BT_1\ BH_2) = \frac{1}{2}\times\frac{3}{4}\times\frac{1}{4}=\frac{3}{32}$ $P(T\ BT_1\ BH_2) = \frac{1}{2}\times\frac{3}{4}\times\frac{1}{4}=\frac{3}{32}$ $P(H\ BH_1\ BH_2) = \frac{1}{2}\times\frac{1}{4}\times\frac{1}{4}=\frac{1}{32}$ $P(T\ BH_1\ BH_2) = \frac{1}{2}\times\frac{1}{4}\times\frac{1}{4}=\frac{1}{32}$	**M1**	All 6 different calculations seen unsimplified.
	$P(B) = \frac{1+3+3+1+3+3}{32}=\frac{14}{32}=\frac{7}{16}$	**A1**	Clear identification of **all scenarios**, linked probabilities and sum. AG
	Method 3: 1- P(BT₁ BT₂) ignoring unbiased coin		
	$1 - P(BT_1\ BT_2) = 1-\left(\frac{3}{4}\right)^2$	**M1**	Calculation seen unsimplified **and** 1 – probability seen.
	$=\frac{7}{16}$	**A1**	Clear identification of scenario used, linked probability and calculation. AG

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td rowspan="4">5(a)</td><td colspan="3">Method 4: 1- P(BT₁ BT₂) with all 3 coins</td></tr>
<tr><td>$1 - P(H\ BT_1\ BT_2) - P(T\ BT_1\ BT_2) = 1 - \left(\frac{1}{2}\times\frac{3}{4}\times\frac{3}{4}\right) - \left(\frac{1}{2}\times\frac{3}{4}\times\frac{3}{4}\right)$</td><td>M1</td><td>Both calculations seen unsimplified
and 1 – 2 probabilities seen.</td></tr>
<tr><td>$= 1 - \frac{9}{32} - \frac{9}{32} = \frac{7}{16}$</td><td>A1</td><td>Clear identification of all scenarios used, linked probabilities and calculation. AG</td></tr>
<tr><td></td><td>2</td><td></td></tr>
<tr><td rowspan="3">5(b)</td><td>$\left[P(A|B) = \frac{P(A\cap B)}{P(B)} = \right] \frac{\frac{1}{2}\times\frac{1}{4}\times\frac{1}{4}}{\frac{7}{16}} = \frac{\frac{1}{32}}{\frac{7}{16}}$</td><td>M1</td><td>Their identified P(HHH) or correct as numerator
and their identified P(B) or correct as denominator.
Either numerical expression acceptable.</td></tr>
<tr><td>$= \frac{1}{14}$, 0.0714</td><td>A1</td><td>Accept 0.071428… rounded to at least 3SF.</td></tr>
<tr><td></td><td>2</td><td></td></tr>
<tr><td rowspan="5">5(c)</td><td rowspan="4">$P(1H) = \frac{1}{2}\times\frac{3}{4}\times\frac{1}{4}+\frac{1}{2}\times\frac{1}{4}\times\frac{3}{4}+\frac{1}{2}\times\frac{3}{4}\times\frac{3}{4}=\frac{15}{32}$
$P(2H) = \frac{1}{2}\times\frac{1}{4}\times\frac{1}{4}+\frac{1}{2}\times\frac{3}{4}\times\frac{1}{4}+\frac{1}{2}\times\frac{1}{4}\times\frac{3}{4}=\frac{7}{32}$

<table>
<tr><td>X</td><td>0</td><td>1</td><td>2</td><td>3</td></tr>
<tr><td>$p(X)$</td><td>$\frac{9}{32}$
0.28125</td><td>$\frac{15}{32}$
0.46875</td><td>$\frac{7}{32}$
0.21875</td><td>$\frac{1}{32}$
0.03125</td></tr>
</table></td><td>B1</td><td>Table with correct X values and at least one probability.
Condone any additional X values if probability stated as 0.</td></tr>
<tr><td>B1</td><td>P(1) or P(2) correct, need not be in table, accept unsimplified.</td></tr>
<tr><td>B1</td><td>4 correct probabilities linked with correct outcomes, may not be in table.
Decimals correct to at least 3 SF.</td></tr>
<tr><td></td><td>SC B1 for 4 probabilities ($0 < p < 1$) sum to 1 ± 0.005 with P(1) and P(2) incorrect.</td></tr>
<tr><td></td><td>3</td><td></td></tr>
</table>

Question	Answer	Marks	Guidance
6(a)	[1 – P(10, 11, 12) =] $1-({}^{12}C_{10}\ 0.9^{10}\,0.1^{2} + {}^{12}C_{11}\ 0.9^{11}\,0.1^{1} + {}^{12}C_{12}\ 0.9^{12}\,0.1^{0})$ = 1 – (0.230128 + 0.376573 + 0.282430)	**M1**	One term ${}^{12}C_x\ p^x(1-p)^{12-x}$, for $0 < x < 12$, $0 < p < 1$
		A1	Correct expression, accept unsimplified, no terms omitted, leading to final answer.
	0.111	**B1**	Mark the final answer at the most accurate value, $0.1108 < p \leqslant 0.111$ WWW.
	Alternative method for Question 6(a)		
	[P(0,1,2,3,4,5,6,7,8,9) =] ${}^{12}C_0\ 0.9^0\,0.1^{12} + {}^{12}C_1\ 0.9^1\,0.1^{11} + {}^{12}C_2\ 0.9^2\,0.1^{10} + {}^{12}C_3\ 0.9^3\,0.1^9 + {}^{12}C_4\,0.9^4\,0.1^8 + {}^{12}C_5\ 0.9^5\,0.1^7 + {}^{12}C_6\ 0.9^6\,0.1^6 + {}^{12}C_7\ 0.9^7\,0.1^5 + {}^{12}C_8\ 0.9^8\,0.1^4 + {}^{12}C_9\ 0.9^9\,0.1^3)$	**M1**	One term ${}^{12}C_x\ p^x(1-p)^{12-x}$, for $0 < x < 12$, $0 < p < 1$
		A1	Correct expression, accept unsimplified, no terms omitted, leading to final answer. If answer correct condone omission of any 7 of the 8 middle terms.
	0.111	**B1**	Final answer $0.1108 < p \leqslant 0.111$ WWW.
		3	

Question	Answer	Marks	Guidance
6(b)	[Mean = 80 × 0.9 =] 72, $\left[\text{Variance} = 80 \times 0.9 \times 0.1\right] = 7.2$	**B1**	72 and 7.2 seen, allow unsimplified. May be seen in standardisation formula. ($2.683 \leqslant \sigma < 2.684$ imply correct variance).
	$\text{P}(X > 69) = \text{P}(Z > \frac{69.5 - 72}{\sqrt{7.2}})$	**M1**	Substituting *their* mean and $\sqrt{\textit{their}\ \text{variance}}$ into ±standardisation formula (any number for 69·5), not *their* 7.2, not √*their* 2.683
		M1	Using continuity correction 69·5 or 68·5 in *their* standardisation formula.
	[= P($Z > -0.9317$) =] $\Phi(0.9317)$	**M1**	Appropriate area Φ, from final process, must be probability.
	0.824	**A1**	$0.8239 \leqslant p \leqslant 0.8243$ WWW.
		5	
6(c)	$np = 72,\ nq = 8$ Both greater than 5, [so approximation is valid]	**B1**	*np*, *nq* evaluated accurately. **both** *np* & *nq* referenced correctly. > 5 or greater than 5 seen.
		1	

Question	Answer	Marks	Guidance
7(a)	7!	**M1**	$\frac{7!}{b! \times c!}$ $b,c = 1,2$ $7! \times \frac{2!}{2!} \times \frac{2!}{2!}$ oe, no further terms present.
	5040	**A1**	
		2	

Question	Answer	Marks	Guidance
7(b)	**Method 1 for first 3 marks: Arrangements of 6 letters including Ls between As**		
	$5! \times 5 \times 2$	**M1**	$5! \times d$, d integer > 1
		M1	$e! \times f \times g$, e = 5, 6, 7; f = 1, 5; g = 1, 2; $f \neq g$, 1 can be implicit.
	1200	**A1**	
	Method 2 for first 3 marks: Number of arrangements of LL^^^^^ – number of arrangements with the Ls split by an A		
	$6! \times 2 - 5! \times 2$	**M1**	$6! \times 2 - h$ h an integer $1 < h < 1440$
		M1	$k - 5! \times 2$ k an integer $k > 240$
	1200	**A1**	
	Method 3 for first 3 marks: Alternative approaches to Method 1		
	^A ^ ^ ^ ^ ^ A $\quad {}^5P_1 \times {}^1P_1 \times {}^5P_5 \times {}^1P_1 = 600$	**M1**	LL treated as a single unit.
		M1	
	1200	**A1**	

Question	Answer	Marks	Guidance
7(b)	**Final 2 marks of Question 7(b)**		
	[Total number of arrangements =] $\left[\frac{9!}{2!2!}=\right]90\,720$	**B1**	Accept unsimplified. May be seen as denominator of probability.
	Probability = $\frac{1200}{90\,720}, \frac{5}{378}$, 0.0132	**B1 FT**	$\frac{\textit{their}\,1200}{\textit{their}\,90\,720}$ unsimplified B1 FT if *their* 1200 and *their* 90 720 supported by work in this part.
		5	

Question	Answer	Marks	Guidance
7(c)	**Method 1: Scenarios identified Both As and Ls removed**		
	A _ _ _ _ $^5C_4 = 5$ AA _ _ _ $^5C_3 = 10$ AL _ _ _ $^5C_3 = 10$ AAL _ _ $^5C_2 = 10$	**B1**	1 correct, identified outcome/value for A, AL or AAL scenario, accept unsimplified $^5C_{5-x}$ cannot be used in place of ^5C_x
		M1	Add 4 values of appropriate scenarios, no incorrect scenarios, no repeated scenarios, accept unsimplified, condone use of permutations.
	[Total =] 35	**A1**	Value stated WWW.
	Method 2: 1 A fixed, 1 L removed No other scenarios can be present anywhere in solution		
	A ^ ^ ^ ^ $^7C_4$	**M1**	^7C_h, $3 \leqslant h \leqslant 5$
		B1	$^7C_4$ oe, no other terms, scenario identified.
	[Total =] 35	**A1**	Value stated.
	Method 3: 1 A fixed, both Ls removed		
	A ^ ^ ^ ^ = $^6C_4 = 15$ A L ^ ^ ^ = $^6C_3 = 20$	**B1**	Correct outcome/value for 1 identified scenario, accept unsimplified. WWW
		M1	Add 2 values of appropriate scenarios, no incorrect scenarios, no repeated scenarios, accept unsimplified, condone use of permutations.
	[Total =] 35	**A1**	Value stated.
		3	

Cambridge Assessment International Education

Cambridge International AS & A Level

MATHEMATICS **9709/53**

Paper 5 Probability & Statistics 1 **October/November 2022**

MARK SCHEME

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The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

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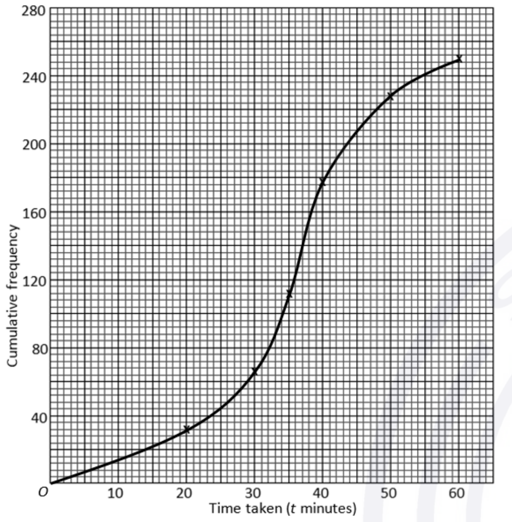
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	$\sum x - 50\times 20 = 35$; $\sum x = 1035$ **or** $\bar{x} = \dfrac{35}{50} + 20 = \dfrac{1035}{50}$ [= 20.7]	**B1**	Correct value for $\sum x$ or $\bar{x}$.
	$\dfrac{25036}{50} - \left(\dfrac{\sum x}{50}\right)^2 = \dfrac{25036}{50} - \left(\dfrac{1035}{50}\right)^2$	**M1**	$\dfrac{25036}{50} - \left(their \left(\dfrac{\sum x}{50}\right)^2 \right)$
	72.23	**A1**	Exact answer only **SC B1** for 72.23 with no substitution in formula.
		3	

Question	Answer	Marks	Guidance
2	Mean = $80\times 0.32 = 25.6$, $\text{var} = 80\times 0.32\times 0.68 = 17.408$	**B1**	25.6 and 17.4[08] seen, allow unsimplified. 4.172… implies correct variance.
	P(X < 20) = P($Z < \dfrac{19.5-25.6}{\sqrt{17.408}}$) = P($Z < -1.462$)	**M1**	Substituting *their* 25.6 and 17.408 into ±standardisation formula (any number for 19·5), not σ^2, $\sqrt{\sigma}$.
		M1	Using continuity correction 19·5 or 20·5 in *their* standardisation formula.
	= [1 – Φ(1.462)] = 1 – 0.9282	**M1**	Appropriate area Φ, from final process, must be probability. (Expect final ans < 0·5). Note: the correct final answer may imply M1 from use of calculator.
	0.0718	**A1**	$0.0718 \leqslant p \leqslant 0.0719$
		5	

Question	Answer	Marks	Guidance
3(a)	Cumulative frequency graph 	**M1**	At least 3 points plotted accurately at class upper end points: (20,32), (30, 66), (35, 112), (40, 178), (50, 228), (60, 250). Linear cf scale $0 \leqslant$ cf $\leqslant 250$ and linear time scale $0 \leqslant$ time $\leqslant 60$ with at least 3 values identified on each.
		A1	All points plotted correct, curve drawn (within tolerance) and joined to (0,0). Axes labelled cumulative frequency (cf), time (t) and minutes (min or m) – or a suitable title. Axes can be the other way round.
		2	
3(b)	Line drawn from 150 on cf axis to meet graph at about t =38 minutes	**B1 FT**	Must be an increasing cf graph with correct upper bounds. Use of graph must be seen. Expect an answer in range $37 \leqslant t \leqslant 39$ for a correct graph
		1	

Question	Answer	Marks	Guidance
3(c)	[Frequencies] [32] 34 46 66 50 22	**B1**	May be unsimplified and/or in variance calculation.
	[Midpoints] 10 25 32.5 37.5 45 55	**M1**	At least 5 correct midpoints seen , may be unsimplified.
	[Variance] = $\frac{32\times10^2+34\times25^2+46\times32.5^2+66\times37.5^2+50\times45^2+22\times55^2}{250}-34.4^2$ [= $\frac{333650}{250}-34.4^2=151.24$]	**M1**	Correct unsimplified Variance formula with *their* midpoints and *their* frequencies for var or sd. (− mean^2 included)
	[Sd =] 12.3	**A1**	Awrt WWW **SC B1** for 12.3 if second M1 not awarded.
		4	

Question	Answer	Marks	Guidance
4(a)	**Method 1: Scenarios identified**		
	[no of ways for score of 2 are] 222, 211, 212, 221, 122, 112, 121 [Total options = 64]	**B1**	7 correct scenarios identified, no incorrect.
	[So P(X = 2) =] $\frac{7}{4\times4\times4} = \frac{7}{64}$	**M1**	$\frac{a}{4\times4\times4}$, a = *their* number of correct identified scenarios > 4
		A1	Approach identified, WWW.
	Method 2: P(2 on all spinners) + P(2 on two spinners and 1 on one spinner) + P(2 on one spinner and 1 on two spinners)		
	$\left(\frac{1}{4}\right)^3 + {}^3C_2\left(\frac{1}{4}\times\frac{1}{4}\times\frac{1}{4}\right) + {}^3C_1\left(\frac{1}{4}\times\frac{1}{4}\times\frac{1}{4}\right)$	**B1**	$\left(\frac{1}{4}\right)^3 + {}^3C_2\left(or\ {}^3C_1\right)\left(\frac{1}{4}\times\frac{1}{4}\times\frac{1}{4}\right) + d$, $0 < d < 1$
		M1	$\left(\frac{1}{4}\right)^3 + e\left(\frac{1}{4}\right)^3 + f\left(\frac{1}{4}\right)^3$ $1 < e < 5$ and $1 < f < 5$
	[So P(X = 2) =] = $\frac{7}{64}$	**A1**	Approach identified, WWW.

Question	Answer	Marks	Guidance
	Method 3: P(1 or 2 on each spinner) – P(1 on all spinners)		
	$\left(\frac{1}{2}\right)^3 - \left(\frac{1}{4}\right)^3$	**B1**	$\left(\frac{1}{2}\right)^3 - b$ seen, $0 < b < 1$
		M1	$\left(\frac{1}{2}\right)^3 - c^3$, $0 < c < ½$
	[So P(X = 2) =] $\frac{7}{64}$	**A1**	Approach identified, WWW.
		3	
4(b)	$\mathrm{P}(X=1)=\frac{1}{64}$	**B1**	P(X = 1) or P(X = 4) correct. Condone answers not in probability distribution table if clearly identified.
	P(X = 4) = $\left[1-\frac{1}{64}-\frac{7}{64}-\frac{19}{64}=\right]\frac{37}{64}$	**B1 FT**	All 4 probabilities summing to 1.
		2	
4(c)	P(Y = 6) = $\left[\left(\frac{3}{4}\right)^5\times\frac{1}{4}=\right]0.0593, \frac{243}{4096}$	**B1**	Accept 0.059326… to 4 or more SF.
		1	

Question	Answer	Marks	Guidance
4(d)	$\left(\frac{3}{4}\right)^4$	M1	$\left(\frac{3}{4}\right)^g$, $g = 4, 5$ **or** p^4 where $0 < p < 1$
	$= \frac{81}{256}, 0.316$	A1	Accept 0.316406…to 4 or more SF.
	Alternative method for Question 4(d)		
	$P(Y > 4) = 1 - P(Y \leqslant 4) = 1-\left(\frac{1}{4}+\frac{3}{4}\times\frac{1}{4}+\left(\frac{3}{4}\right)^2\times\frac{1}{4}+\left(\frac{3}{4}\right)^3\times\frac{1}{4}\right)$ $\left[=1-\frac{175}{256}\right]$	M1	Correct or $1-\left(\frac{1}{4}+\frac{3}{4}\times\frac{1}{4}+\left(\frac{3}{4}\right)^2\times\frac{1}{4}+\left(\frac{3}{4}\right)^3\times\frac{1}{4}+\left(\frac{3}{4}\right)^4\right)$ **or** $1-\left(p+qp+q^2p+q^3p\right)$ where $0 < p < 1$ and $q = 1-p$
	$= \frac{81}{256}, 0.316$	A1	Accept 0.316406…to 4 or more SF.
		2	

Question	Answer	Marks	Guidance
5(a)	[P(0, 1, 2) =] $^{10}C_0\ 0.1^0\, 0.9^{10} + {}^{10}C_1\ 0.1^1\, 0.9^9 + {}^{10}C_2\ 0.1^2\, 0.9^8$	**M1**	One term $^{10}C_x\ p^x\left(1-p\right)^{10-x}$, $0<p<1, x\neq 0$
	= 0.348678+0.38742+0.19371	**A1**	Correct expression, accept unsimplified.
	0.930	**B1**	$0.9298 \leqslant p \leqslant 0.9303$
	Alternative method for Question 5(a)		
	[1 – P(3, 4, 5, 6, 7, 8, 9, 10) = 1 – ($^{10}C_3\ 0.9^7\, 0.1^3 + {}^{10}C_4\ 0.9^6\, 0.1^4 + {}^{10}C_5\ 0.9^5\, 0.1^5 + {}^{10}C_6\ 0.9^4\, 0.1^6 + {}^{10}C_7\, 0.9^3\, 0.1^7 + {}^{10}C_8\ 0.9^2\, 0.1^8 + {}^{10}C_9\ 0.9^1\, 0.1^9 + {}^{10}C_{10}\ 0.9^0\, 0.1^{10}$)	**M1**	One term $^{10}C_x\ p^x\left(1-p\right)^{10-x}$, $0<p<1, x\neq 0$
		A1	Correct expression, accept unsimplified.
	0.930	**B1**	$0.9298 \leqslant p \leqslant 0.9303$
		3	
5(b)	[P(X > 1.11) =]P($Z > \dfrac{1.11-1.04}{0.06}$) = P($Z > 1.167$)	**M1**	1.11, 1.04 and 0.06 substituted into ±Standardisation formula, no continuity correction not 0.06^2 or $\sqrt{0.06}$
	= 1 – 0.8784	**M1**	1 – *their* 0.8784 as final answer, must be probability. (Expect final ans < 0·5).
	0.122	**A1**	$0.1216 \leqslant p \leqslant 0.122$ **SC M0 M1 B1** for 0.122 with no standardisation formula.
		3	

Question	Answer	Marks	Guidance
5(c)	$[\mathrm{P}(X < w) = \mathrm{P}(Z < \frac{w-1.04}{0.06}) = 0.81]$	B1	$0.8775 < z \leqslant 0.878$ or $-0.878 \leqslant z < -0.8775$ seen.
	$\frac{w-1.04}{0.06} = 0.878$	M1	1.04 and 0.06 substituted in ±standardisation formula, no continuity correction, not σ^2, $\sqrt{\sigma}$, equated to a z-value.
	$w = 1.09$	A1	$1.09 \leqslant w \leqslant 1.093$
		3	

Question	Answer	Marks	Guidance
6(a)	$\frac{9!}{2!2!}$	M1	$\frac{h!}{2!\times j!}$, $h = 7, 8, 9$; $j = 1, 2$
	90720	A1	
		2	
6(b)	**Arrangements with 5 letters between As + Arrangements with 6 letters between As + Arrangements with 7 letters between As**		
	With gap of 5: $\frac{7!}{2!}\times 3$ [= 7560] With gap of 6: $\frac{7!}{2!}\times 2$ [= 5040] With gap of 7: $\frac{7!}{2!}\times 1$ [= 2520]	M1	$\frac{7!}{2!}\times k$, k positive integer $1 < k < 7$
		M1	Add their no of ways for 3 identified correct scenarios, no additional incorrect scenarios, accept unsimplified.
	[Total no $= \frac{7!}{2!}\times 6 =$] 15120	A1	
		3	

Question	Answer	Marks	Guidance
6(c)	**Method 1: Summing number of ways**		
	AT _ _ _ $2\times2\times{}^5C_3$ 40 A _ _ _ _ $2\times{}^5C_4$ 10 AATT _ ${}^5C_1$ 5 AAT _ _ $2\times{}^5C_2$ 20 AA _ _ _ ${}^5C_3$ 10 _ _ _ _ _ ${}^5C_5$ 1	**B1**	Correct no of ways for 4 correctly identified scenarios, accept unsimplified.
		M1	Add no of ways for 5 or 6 identified correct scenarios, no additional incorrect scenarios, no repeated scenarios, accept unsimplified.
	[Total no of ways not containing more Ts than As =] = 40+10+5+20+10+1 [=86]	**A1**	All correct and added
	Probability = $\frac{86}{{}^9C_5}$	**M1**	$\frac{\textit{their}\ 86}{9C5\ \textit{or their identified total}}$ accept numerator unevaluated
	$\frac{86}{126}, \frac{43}{63}$, 0.683	**A1**	
	Method 2: Subtracting no of ways with more Ts from total		
	T _ _ _ _ $2\times{}^5C_4$ 10 TTA _ _ $2\times{}^5C_2$ 20 TT _ _ _ ${}^5C_3$ 10	**B1**	Correct no of ways for 2 correctly identified scenarios, no additional incorrect scenarios, no repeated scenarios, accept unsimplified, condone use of permutations
		M1	Add no of ways for 2 or 3 correct scenarios and subtract from their total no of ways All correct and subtracted
	Total no of ways with more Ts than As =40 ${}^9C_5 - 40 = 86$	**A1**	
	Probability = $\frac{86}{{}^9C_5}$	**M1**	$\frac{\textit{their}\ 86}{9C5\ \textit{or their identified total}}$ accept numerator unevaluated

Question	Answer	Marks	Guidance
6(c)	$\frac{43}{63}$, 0.683	**A1**	
		5	

Question	Answer	Marks	Guidance
7(a)	[P(SR TR) + P(SW TR) =] $\frac{3}{8}\times\frac{2}{7}+\frac{5}{8}\times\frac{3}{7}$	**M1**	$\frac{3}{8}\times\frac{2}{7}+k$ or $l+\frac{5}{8}\times\frac{3}{7}$ $0 < k,l < 1$
	$= \frac{21}{56}, \frac{3}{8}$, 0.375	**A1**	**SC B1** for $\frac{3}{8}$ with no explanation.
		2	
7(b)	[RRWR, WRRR, WRWR] $\frac{3}{8}\times\frac{2}{7}\times\frac{5}{6}\times\frac{1}{5}+\frac{5}{8}\times\frac{3}{7}\times\frac{2}{6}\times\frac{1}{5}+\frac{5}{8}\times\frac{3}{7}\times\frac{4}{6}\times\frac{2}{5}$ $[=\frac{1}{56}+\frac{1}{56}+\frac{1}{14}]$	**M1**	$\frac{m}{8}\times\frac{n}{7}\times\frac{o}{6}\times\frac{q}{5}$ $1 \leqslant m,n,o,q \leqslant 5$, $m \neq n \neq o \neq q$
		A1	Probability for one scenario correct, accept unsimplified.
		M1	Adding probabilities for 3 correct scenarios and no incorrect.
	$= \frac{180}{1680}, \frac{3}{28}$, 0.107	**A1**	Or 0.1071428… to 4SF or better. **SC B1** for 3/28 with inadequate explanation.
		4	

Question	Answer	Marks	Guidance
7(c)	[P(S first disc R |T2) =] $\dfrac{\frac{30}{1680}}{\frac{3}{28}} = \dfrac{\frac{1}{56}}{\frac{3}{28}}$	**M1**	$\dfrac{their\ P(RRWR)\ or\ \frac{3}{8}\times\frac{2}{7}\times\frac{5}{6}\times\frac{1}{5}}{their\ 7(\mathbf{b}) - must\ be\ a\ prob\ or\ \frac{3}{28}}$
	$\frac{1}{6}$, 0.167	**A1**	
		2	

Cambridge International AS & A Level

MATHEMATICS **9709/51**

Paper 5 Probability & Statistics 1 **May/June 2022**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2022 series for most Cambridge IGCSE, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

This document consists of **15** printed pages.

Generic Marking Principles

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GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
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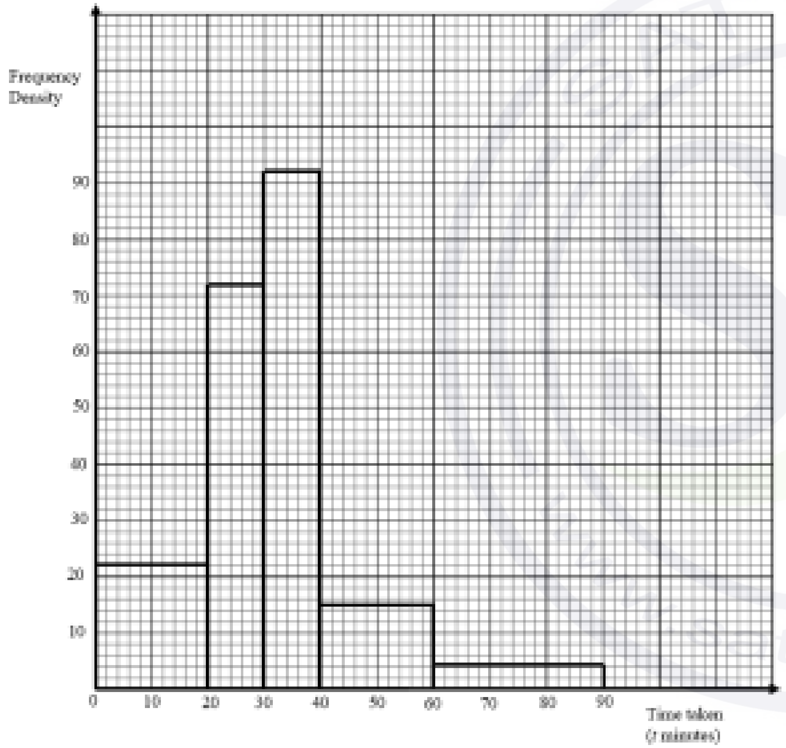
Question	Answer	Marks	Guidance
1(a)	5!	**M1**	$k!$ where $k = 5$, 6 or 7 Condone × 1 OE
	120	**A1**	
		2	
1(b)	[Total no of ways =] $\frac{8!}{2!3!}$ [= 3360]	**M1**	$\frac{8!}{a!b!}$, $a = 1,2$ $b = 1,3$ $a \neq b$
	[With 3Es together =] $\frac{6!}{2!}$ [= 360]	**M1**	$\frac{6!}{c!}$, $c = 1,2$ seen in an addition/subtraction
	[With 3Es not together] = 3360 – 360	**M1**	$\frac{8!}{d!e!} - \frac{6!}{f!}$ where $d, f = 1,2$ & $e = 1,3$
	3000	**A1**	
		4	

Question	Answer	Marks	Guidance
2(a)	${}^{12}C_4 \times 2$	M1	${}^{g}C_4 \times h$ $g = 12, 13, h = 1,2$
	990	A1	
	Alternative method for question 2(a)		
	[total – both on – neither on] ${}^{14}C_5 - ({}^{12}C_3 + {}^{12}C_5) = [2002 - 220 - 792]$	M1	${}^{k}C_5 - ({}^{a}C_3 + {}^{a}C_5)$ $a = 12, 13$ and $k = 13, 14$
	990	A1	
		2	
2(b)	[Mrs Lan plus] 2W 2M ${}^{7}C_2 \times {}^{6}C_2 = 315$ 3W 1M ${}^{7}C_3 \times {}^{6}C_1 = 210$ 4W ${}^{7}C_4 = 35$	M1	${}^{7}C_r \times {}^{6}C_{4-r}$ for $r = 2, 3$ or 4
		B1	Outcome for one identifiable scenario correct, accept unevaluated
		M1	Add outcomes for 3 identifiable correct scenarios Note: if scenarios not labelled, they may be identified by seeing ${}^{7}C_r \times {}^{6}C_s$ $r + s = 4$ to imply r women and s men for both **B & M** marks only
	[Total =] 560	A1	
		4	

Question	Answer	Marks	Guidance
3(a)	See table and graph below	M1	At least 4 frequency densities calculated (Frequency ÷ class width, e.g. $\frac{440}{20}\left(condone \frac{440}{19.5}, \frac{440}{20.5}\right)$ Accept unsimplified, may be read from graph using *their* scale
		A1	All heights correct on graph **NOT FT**
		B1	Bar ends at [0,] 20, 30, 40, 60, 90 at axis with a horizontal linear scale with at least 3 values indicated. $0 \leqslant$ horizontal scale $\leqslant 90$
		B1	Axes labelled frequency density (fd), time (t) and minutes (mins) or in a title. Linear vertical scale, with at least 3 values indicated $0 \leqslant$ vertical axes $\leqslant 92$ (condone 90 used).
		4	

Answer 3(a):

Class width	20	10	10	20	30
Frequency density	22	72	92	15	4



Question	Answer	Marks	Guidance
3(b)	Midpoints: 10, 25, 35, 50, 75	**B1**	At least 4 correct midpoints seen
	[Mean = 31.44 given] [Variance = $\frac{440\times10^2+720\times25^2+920\times35^2+300\times50^2+120\times75^2}{2500}-31.44^2$] $=\frac{44000+450000+1127000+750000+675000}{2500}-31.44^2$ [= $\frac{3046000}{2500}-31.44^2$ = 229.9264] Or Variance = $\frac{440(10-31.44)^2+720(25-31.44)^2+920(35-31.44)^2+300(50-31.44)^2+120(75-31.44)^2}{2500}$ $=\frac{202256+29860+11659+103342+227697}{2500}=\frac{574814}{2500}=229.9264$	**M1**	Correct formula for variance or standard deviation (− mean^2 included with *their* midpoints (not upper bound, lower bound, class width, frequency density, frequency or cumulative frequency) and *their* $\sum$f if calculated. Condone 1 data error.
	Standard deviation = 15.2	**A1**	WWW, allow 15.16[3…]
		3	
3(c)	30–40	**B1**	
		1	
3(d)	Stays the same, data still in same intervals	**B1**	Frequencies unchanged
		1	

Question	Answer	Marks	Guidance
4(a)	$a = \text{P(1 head)} = 0.7\times(0.5)^3 + 0.3\times(0.5)^3\times 3 = \dfrac{1}{5}$	**B1**	Clear statement of unevaluated correct calculation $=\dfrac{1}{5}$. AG
	$b = 0.7\times0.5^3\times3+0.3\times0.5^3\times3=\dfrac{3}{8}$	**M1**	Clear statement of unevaluated calculation for either b or c
	$c = 0.7\times0.5^3\times3+0.3\times0.5^3=\dfrac{3}{10}$	**A1**	For either b or c correct
	$\left[or\, c=\dfrac{27}{40}-b\right]$	**B1 FT**	$their\, b + their\, c = \dfrac{27}{40}$
		4	
4(b)	$\left[\text{E}(X)=\dfrac{3\times0+16\times1+30\times2+24\times3+7\times4}{80}=\right]\dfrac{176}{80}$ or 2.2	**B1 FT**	Correct or accept unsimplified calculation using *their* values for b and c seen (sum of probabilities = 1)
		1	

Question	Answer	Marks	Guidance
4(c)	$[\mathrm{P}(0, 1, 2) =]\,{}^{10}C_0\ 0.2^0\,0.8^{10} + {}^{10}C_1\ 0.2^1\,0.8^9 + {}^{10}C_2\ 0.2^2\,0.8^8$	**M1**	One term ${}^{10}C_x\ p^x(1-p)^{10-x}$, for $0 < x < 10$, $0 < p < 1$
	0.107374 + 0.268435 + 0.301989	**A1**	Correct expression, accept unsimplified leading to final answer
	0.678	**B1**	$0.677 < p \leqslant 0.678$
	Alternative method for question 4(c)		
	$1 - [{}^{10}C_{10}\ 0{\cdot}2^{10}0.8^0 + {}^{10}C_9\ 0{\cdot}2^9 0.8^1 + {}^{10}C_8\ 0{\cdot}2^8 0.8^2 + {}^{10}C_7\ 0{\cdot}2^7 0.8^3 + {}^{10}C_6\ 0{\cdot}2^6 0.8^4 + {}^{10}C_5\ 0{\cdot}2^5 0.8^5 + {}^{10}C_4\ 0{\cdot}2^4 0.8^6 + {}^{10}C_3\ 0{\cdot}2^3 0.8^7]$	**M1**	One term ${}^{10}C_x\ p^x(1-p)^{10-x}$, for $0 < x < 10$, $0 < p < 1$
		A1	Correct expression, accept unsimplified
	0.678	**B1**	$0.677 < p \leq 0.678$
		4	
4(d)	$0.8^6 \times 0.2 + 0.8^7 \times 0.2 = 0.0524288 + 0.041943$	**M1**	$p^l \times (1-p) + p^m \times (1-p)$, $l = 6, 7$ $m = l + 1$, $0 < p < 1$
	0.0944	**A1**	$0.09437 \leqslant p \leqslant 0.0944$
		2	

Question	Answer	Marks	Guidance
5(a)	$P(X < 6) = P(Z < \frac{6-5.2}{1.5}) = P(Z < 0.5333)$	**M1**	6, 5.2, 1.5 substituted into ± standardisation formula, condone 1.5^2, continuity correction ±0.5
	0.703	**A1**	
		2	
5(b)	$z_1 = \frac{3-\mu}{\sigma} = -1.329$ $z_2 = \frac{8-\mu}{\sigma} = 0.878$	**B1**	$1.328 < z_1 \leqslant 1.329$ or $-1.329 \leqslant z_1 < -1.328$
		B1	$0.877 < z_2 \leqslant 0.878$ or $-0.878 \leqslant z_2 < -0.877$
	Solve to find at least one unknown: $\frac{3-\mu}{\sigma} = -1.329$ $\frac{8-\mu}{\sigma} = 0.878$	**M1**	Use of the ± standardisation formula once with μ, σ, a z-value (not 0.8179, 0.7910, 0.5367, 0.5753, 0.19, 0.092 etc.) and 3 or 8, condone continuity correction but not σ^2 or $\sqrt{\sigma}$
		M1	Use either the elimination method or the substitution method to solve their two equations in μ and σ
	$\sigma = 2.27, \quad \mu = 6.01$	**A1**	$2.26 \leqslant \sigma \leqslant 2.27$, $6.01 \leqslant \mu \leqslant 6.02$
		5	

Question	Answer	Marks	Guidance
5(c)	$[P(Z < -1) + P(Z > 1)]$ $\Phi(1) - \Phi(-1) =$ $= 2 - 2\ \Phi(1)$ $= 2 - 2 \times 0.8413$	**M1**	Identify 1 and –1 as the appropriate z-values.
		M1	Calculating the appropriate area from stated phis of z-values which must be ± the same number
	0.3174	**A1**	Accept AWRT 0.317
	Number of leaves: $2000 \times 0.3174 = 634.8$ so 634 or 635	**B1 FT**	FT *their* 4 s.f. (or better) probability, final answer must be positive integer no approximation or rounding stated
		4	

Question	Answer	Marks	Guidance
6(a)	$0.6+0.4\times 0.3=0.72$ or $1-0.4 \times 0.7=0.72$	**B1**	Clear identified calculation AG
		1	
6(b)	$0.72\times\left(0.4+0.6\times 0.2\right)$	**M1**	$0.72\times u$, $0<u<1$
		M1	$v\times(0.4+0.6\times 0.2)$, or $v\times(1-0.6\times 0.8)$ $0<v\leqslant 1$ no additional terms **SC B1** for $0.72\times(0.4+0.12)$ or $0.72\times(1-0.48)$
	0.3744	**A1**	WWW. Condone 0.374. **SC B1** for 0.3744 only
		3	
	Alternative method for question 6(b)		
	[p(P1P2) + p(F1P1P2) + p(P1F2P2) + p(F1P1F2P2)] = $0.6\times 0.4+0.4\times 0.3\times 0.4+0.6\times 0.6\times 0.2+0.4\times 0.3\times 0.6\times 0.2$	**M1**	Any two terms unsimplified and correct
		M1	Summing 4 appropriate scenarios by listing or on a tree diagram **SC B1** for $0.24+0.048+0.072+0.0144$
	0.3744	**A1**	WWW. Condone 0.374. **SC B1** for 0.3744 only
		3	

Question	Answer	Marks	Guidance
6(c)	$P(\text{fails first or second level} \mid \text{finishes game}) = \dfrac{P(\text{fails first or second level} \cap \text{finishes game})}{\textit{their}\ \textbf{(b)}}$	**M1**	Either $0.6 \times 0.6 \times 0.2$ or $0.4 \times 0.3 \times 0.4$ seen Condone 0.072 or 0.048 if seen in **(b)**
	Numerator = P(S SF) + P(FS S) = $0.6 \times 0.6 \times 0.2 + 0.4 \times 0.3 \times 0.4 = 0.072 + 0.048 = 0.12$	**A1**	Both correct accept unsimplified expression. No additional terms
	Required probability = $\dfrac{0.12}{\textit{their}\ \textbf{(b)}}$	**M1**	$\dfrac{\textit{Their}\ \text{sum of two 3-term probabilities as numerator}}{\textit{their}\ \textbf{(b)}\ \text{or correct}}$
	0.321 or $\dfrac{25}{78}$	**A1**	$0.3205 < p \leqslant 0.321$
		4	

Cambridge Assessment International Education

Cambridge International AS & A Level

MATHEMATICS **9709/52**

Paper 5 Probability & Statistics 1 **May/June 2022**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2022 series for most Cambridge IGCSE, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (ISW).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

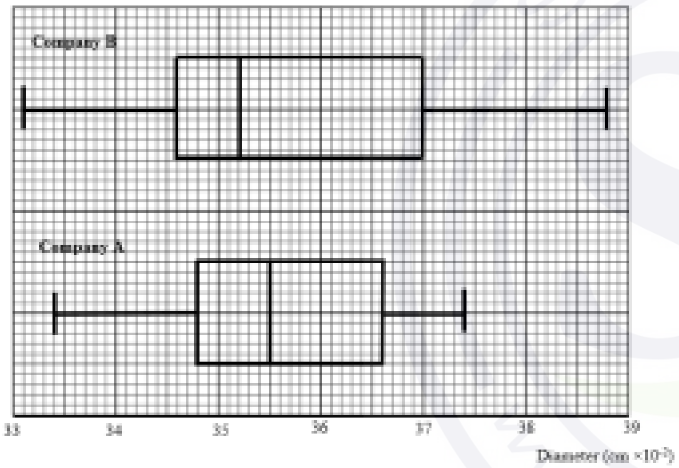
AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	$\sum x - \sum 200 = \sum(x-200)$	**B1**	Forming a correct 3-term (linear) equation from $\sum x,\ \sum 200$ and $\sum(x-200)$. Accept 6846 – 200n = 446 OE. Condone 1 sign error.
	$\sum 200 = 200n$	**B1**	SOI
	$[200n = 6846 - 446 = 6400]\ \ n = 32$	**B1**	WWW
		3	

Question	Answer	Marks	Guidance
2(a)	(table below)	**B1**	Table with correct X values and at least one probability. Condone any additional X values if probability stated as 0.
		B1	3 correct probabilities linked with correct outcomes. Accept 3 sf decimals.
		B1	2 further correct probabilities linked with correct outcomes. Accept 3 sf decimals.
		3	**SC B1** for 5 probabilities (0 < p <1) that sum to 1 with less than 3 correct probabilities.

x	2	3	4	5	6
p	$\frac{1}{36}$	$\frac{4}{36}$	$\frac{10}{36}$	$\frac{12}{36}$	$\frac{9}{36}$
	0.02778	0.1111	0.2778	0.3333	0.25

Question	Answer	Marks	Guidance
2(b)	If method FT from *their* incorrect **(a)**, expressions for E(*X*) and Var(*X*) must be seen at the stage shown in **bold** (or less simplified) in the scheme with all probabilities < 1.		
	$\left[\mathrm{E}(X) = \frac{1\times2+4\times3+10\times4+12\times5+9\times6}{36} =\right]\ \boldsymbol{\frac{2+12+40+60+54}{36}}$	**M1**	Accept unsimplified expression. May be calculated in variance. FT *their* table with 4 or more probabilities summing to 0.999 ⩽ total ⩽ 1 ($0 < p < 1$).
	$\left[\mathrm{Var}(X) = \frac{1\times2^2+4\times3^2+10\times4^2+12\times5^2+9\times6^2}{36} - \left(\mathit{their}\ \mathrm{E}(X)\right)^2 =\right]$ $\boldsymbol{\frac{1\times4+4\times9+10\times16+12\times25+9\times36}{36} - \left(\mathit{their}\ \frac{14}{3}\right)^2}$ $\left[\frac{4+36+160+300+324}{36} - \left(\mathit{their}\ \frac{14}{3}\right)^2\right]$	**M1**	Appropriate variance formula using *their* $(\mathrm{E}(X))^2$ value. FT *their* table with 3 or more probabilities ($0 < p < 1$) which need not sum to 1 and the calculation in **bold** (or less simplified) seen.
	$\mathrm{E}(X) = \frac{168}{36},\ \frac{14}{3},\ 4.67$ $\mathrm{Var}(X) = \frac{10}{9},\ 1\frac{1}{9},\ 1.11,\ \frac{1440}{1296}$	**A1**	Answers for E(*X*) and Var(*X*) must be identified. E(*X*) may be identified by correct use in Variance. Condone E, V, μ, σ^2 etc. If M0 earned **SC B1** for identified correct final answers.
		3	

Question	Answer	Marks	Guidance
3(a)	Median = 0.355	**B1**	Identified condone Q2.
	[IQR =] 0.366 – 0.348	**M1**	0.365 ⩽ UQ ⩽ 0.369 – 0.343 ⩽ LQ ⩽ 0.349. Subtraction may be implied by answer.
	0.018	**A1**	If 0/3 scored **SC B1** for figs Median = 355 IQR = 18.
		3	
3(b)	Box-and-whisker plot on provided grid 	**B1**	All 5 key values for *B* plotted accurately in standard format using *their* scale. Labelled *B*. Check accuracy in the middle of vertical line.
		B1 FT	All 5 key values for *A*, FT from part **3(a),** plotted in standard format accurately using *their* scale. Labelled *A*. Check accuracy in the middle of vertical line.
		B1	Whiskers not through box for both, not drawn at corners of boxes, single linear scale with at least 3 values stated, covering at least 0.34 to 0.38 and labelled diameter (*d* etc) and cm. Accept as a title.
		3	If both plots attempted and plot(s) not labelled, **SC B1** for at least 1 fully correct set of values plotted.
3(c)	A comparison in context	**B1**	Single comment comparing spread or central tendency in context. Must reference either diameter or pipes. Not a simple numerical comparison of statistical values such as median, range, IQR or min/max.
		1	

Question	Answer	Marks	Guidance
4(a)	[P(1.98 < X < 2.03) =]P($\frac{1.98-2.02}{0.03} < z < \frac{2.03-2.02}{0.03}$) [= P(−1.333 < z < 0.333)]	**M1**	Use of ±standardisation formula once with 2.02, 0.03 and either 1.98 or 2.03 substituted appropriately. Condone 0.03^2 and continuity correction ±0.005, not √0.03.
	[= $\Phi(0.333)-(1-\Phi(1.333))$] = 0.6304 + 0.9087 – 1	**M1**	Calculating the appropriate probability area from *their* z-values. (or 0.6304 – 0.09121 or (0.9087 – 0.5) + (0.6304 – 0.5) etc)
	0.539	**A1**	0.539 ⩽ z < 0.5395 Only dependent upon 2nd M mark. If M0 scored **SC B1** for 0.539 ⩽ z < 0.5395.
		3	
4(b)	[P(X > 2.6) = $\frac{134}{5000}$ = 0.0268] [P(X<2.6) = 1 – 0.0268 =] 0.9732	**B1**	0.9732 or $\frac{4866}{5000}$ or $\frac{2433}{2500}$ seen.
	$\frac{2.6-2.55}{\sigma}=1.93$	**M1**	Use of ±standardisation formula with 2.6 and 2.55 substituted, no $\sigma^2, \sqrt{\sigma}$ or continuity correction.
		M1	*Their* standardisation formula with values substituted equated to z-value which rounds to ±1.93.
	$\sigma = 0.0259$	**A1**	AWRT 0.0259 or $\frac{5}{193}$. If M0 earned, **SC B1** for correct final answer.
		4	

Question	Answer	Marks	Guidance
5(a)	[P(10, 11, 12) =] ${}^{12}C_{10}\,0.72^{10}\,0.28^{2} + {}^{12}C_{11}\,0.72^{11}\,0.28^{1} + {}^{12}C_{12}\,0.72^{12}\,0.28^{0}$	**M1**	One term ${}^{12}C_x\,p^x\left(1-p\right)^{12-x}$, for $0 < x < 12$, $0 < p < 1$.
	= 0.193725 + 0.0905726 + 0.0194084	**A1**	Correct expression, accept unsimplified, no terms omitted, leading to final answer.
	0.304	**B1**	Final answer $0.3036 < p \leqslant 0.304$.
	Alternative method for question 5(a)		
	[1 – P(0,1,2,3,4,5,6,7,8,9) =] $1 - ({}^{12}C_0\,0.72^0\,0.28^{12} + {}^{12}C_1\,0.72^1\,0.28^{11} + {}^{12}C_2\,0.72^2\,0.28^{10} +$ ${}^{12}C_3\,0.72^3\,0.28^9 + {}^{12}C_4\,0.72^4\,0.28^8 + {}^{12}C_5\,0.72^5\,0.28^7 +$ ${}^{12}C_6\,0.72^6\,0.28^6 + {}^{12}C_7\,0.72^7\,0.28^5 + {}^{12}C_8\,0.72^8\,0.28^4 +$ ${}^{12}C_9\,0.72^9\,0.28^3)$	**M1**	One term ${}^{12}C_x\,p^x\left(1-p\right)^{12-x}$, for $0 < x < 12$, $0 < p < 1$.
		A1	Correct expression, accept unsimplified, no terms omitted, leading to final answer.
	0.304	**B1**	Final answer $0.3036 < p \leqslant 0.304$.
		3	
5(b)	Mean = $[\,0.52 \times 90 =\,]46.8$, var $=[\,0.52 \times 0.48 \times 90\,] = 22.464$	**B1**	46.8 and 22.464 or 22.46 seen, allow unsimplified, ($4.739 < \sigma \leqslant 4.740$ imply correct variance).
	[P(X < 40) =] $\mathrm{P}\left(z < \dfrac{39.5 - 46.8}{\sqrt{22.464}}\right)$	**M1**	Substituting *their* mean and *their* variance into ±standardisation formula (any number for 39.5), not σ^2, $\sqrt{\sigma}$.
		M1	Using continuity correction 39.5 or 40.5 in *their* standardisation formula.
	$= [\mathrm{P}(\,Z < -1.540)] = 1 - 0.9382$	**M1**	Appropriate area Φ, from final process, must be probability.
	0.0618	**A1**	$0.06175 \leqslant p \leqslant 0.0618$
		5	

Question	Answer	Marks	Guidance
6(a)	$\left[\frac{9!}{2!2!}=\right]$ 90 720	**B1**	
		1	
6(b)	**Method 1** Arrangements Cs at ends – Arrangements Cs at ends and Os together		
	[Os not together =] $\frac{7!}{2!}-6!$ [= 2520 – 720]	**M1**	$\frac{w!}{2!}-y$, w = 6, 7 y an integer. Condone $2\times\left(\frac{w!}{2!}\right)-y$.
		M1	a – 6! or a – 720, a an integer resulting in a positive answer.
	1800	**A1**	
	Method 2 identified scenarios R ^ ^ ^ R		
	[Os not together =] $5!\times\frac{6\times5}{2!}$ =	**M1**	5! × b, b integer >1.
		M1	$c\times\left(\frac{6\times5}{2!}\text{or}\,{}^6C_2\,\text{or}\,\frac{{}^6P_2}{2!}\text{or}\,15\right)$, c integer > 1.
	1800	**A1**	
		3	

Question	Answer	Marks	Guidance
6(c)	CCO _ $^5C_1 = 5$ CC _ _ $^5C_2 = 10$ OOC _ $^5C_1 = 5$ OO _ _ $^5C_2 = 10$ C _ _ _ $^5C_3 = 10$ O _ _ _ $^5C_3 = 10$	B1	Correct outcome/value for 1 identified scenario. Accept unsimplified. WWW
		M1	Add 5 or 6 values of appropriate scenarios only, no additional incorrect scenarios, no repeated scenarios. Accept unsimplified. Condone use of permutations.
	[Total =] 50	A1	
		3	
6(d)	Both Os in group with a C $^5C_2 = 10$ Both Os in group without a C $^5C_2 \times {}^3C_2 = 30$ One O in a C group, one not $^5C_1 \times {}^4C_2 = 30$ One O with each C $({}^5C_1 \times {}^4C_1) \div 2! = 10$	B1	A correct scenario calculated accurately. Accept unsimplified.
		M1	Add 3 or 4 correct scenario values, no incorrect scenarios, accept repeated scenarios. Accept unsimplified.
	[Total =] 80	A1	
	Alternative method for question 6(d)		
	CCO O^^ ^^^ = $^5C_2 = 10$ CC^ O^^ O^^ = $^5C_1 \times {}^4C_2 = 30$ CC^ OO^ ^^^ = $^5C_1 \times {}^4C_1 = 20$	B1	A correct scenario calculated accurately. Accept unsimplified.
	Total ways of making three groups $\frac{^9C_6 \times {}^6C_3}{2 \times 2 \times 3} = 140$ 140 – (*their* 10+ *their* 30+ *their* 20)	M1	Total subtract 2 or 3 correct scenario values, no incorrect scenarios. Accept unsimplified.
	80	A1	
		3	

Question	Answer	Marks	Guidance
7(a)	YYY: $\frac{5}{12}\times\frac{4}{11}\times\frac{3}{10} = \frac{60}{1320},\frac{1}{22}$	M1	**Either** $12 \times 11 \times 10$ in denominator **or** $a \times (a-1) \times (a-2)$, $a = 5, 4, 3$ in numerator seen in at least one expression.
	OOO: $\frac{4}{12}\times\frac{3}{11}\times\frac{2}{10} = \frac{24}{1320},\frac{1}{55}$ RRR: $\frac{3}{12}\times\frac{2}{11}\times\frac{1}{10} = \frac{6}{1320},\frac{1}{220}$	A1	One expression $\frac{a}{12}\times\frac{a-1}{11}\times\frac{a-2}{10}$, $a = 5, 4, 3$ (consistent in expression). Correct order of values in the numerator is essential.
		M1	$\frac{5}{12}\times\frac{4}{d}\times\frac{3}{e}+\frac{4}{12}\times\frac{3}{d}\times\frac{2}{e}+\frac{3}{12}\times\frac{2}{d}\times\frac{1}{e}$, either $d = 11$, $e = 10$ or $d = 12$, $e = 12$. Condone $\frac{1}{22}+\frac{1}{55}+\frac{1}{220}$ OE
	[Total =] $\frac{90}{1320},\frac{3}{44},0.0682$	A1	0.06818. Dependent only upon the second M mark.

Question	Answer	Marks	Guidance
7(a)	**Alternative method for question 7(a)**		
	YYY: $\frac{^5C_3}{^{12}C_3} = \frac{10}{220}, \frac{1}{22}$ OOO: $\frac{^4C_3}{^{12}C_3} = \frac{4}{220}, \frac{1}{55}$ RRR: $\frac{^3C_3}{^{12}C_3} = \frac{1}{220}$	M1	Either $^{12}C_3$ in denominator or $^{a}C_3$ in numerator seen in at least one expression.
		A1	One expression $\frac{^aC_3}{^{12}C_3}$ $a = 5, 4, 3$
		M1	$\frac{^5C_3}{^{12}C_3} + \frac{^4C_3}{^{12}C_3} + \frac{^3C_3}{^{12}C_3}$ Condone $\frac{1}{22} + \frac{1}{55} + \frac{1}{220}$ OE
	[Total =] $\frac{90}{1320}, \frac{3}{44}, 0.0682$	A1	0.06818. Dependent only upon the second M mark.
		4	
7(b)	[P(YYY | all same colour) =] $\frac{60}{1320} \div \frac{90}{1320}$	M1	$\frac{\textit{their}\ \text{P(YYY)} \text{ or } \frac{60}{1320} \text{ or } \frac{1}{22}}{\textit{their}\ 7\text{(a)} \text{ or } \frac{90}{1320} \text{ or } \frac{3}{44}}$
	$\frac{2}{3}$, 0.667	A1	OE
		2	

Question	Answer	Marks	Guidance
7(c)	In each method, the M mark requires the scenarios to be identifiable. This may be implied by a list of scenarios and then the calculations which will be assumed to be in the same order. A correct value/expression will be condoned as identifying the connected scenario.		
	Method 1		
	[1 – no orange =] $1-\frac{8}{12}\times\frac{7}{11}\times\frac{6}{10}$ or $1-\frac{^8C_3}{^{12}C_3}$ $=1-\frac{14}{55}$	B1	$\frac{8}{12}\times\frac{7}{11}\times\frac{6}{10}$ or $\frac{^8C_3}{^{12}C_3}$ seen, condone $\frac{336}{1320}$ or $\frac{56}{220}$ only, not OE.
		M1	$1-\frac{f}{12}\times\frac{g}{d}\times\frac{h}{e}$ Either $d = 11$, $e = 10$ or $d = 12$, $e = 12$ or $1-\frac{^8C_3}{^{12}C_3}$. Condone $1-\frac{14}{55}$ OE (not $\frac{41}{55}$).
	$\frac{41}{55}$	A1	$0.745 \leqslant p \leqslant 0.74545$ If M0 scored **SC B1** $0.745 \leqslant p \leqslant 0.74545$.

Question	Answer	Marks	Guidance
7(c)	**Method 2**		
	P(1 O)= $\left(\frac{4}{12}\times\frac{3}{11}\times\frac{2}{10}+\frac{4}{12}\times\frac{5}{11}\times\frac{4}{10}+2\times\frac{4}{12}\times\frac{5}{11}\times\frac{3}{10}\right)\times 3=\frac{672}{1320}$ P(2O) = $\frac{4}{12}\times\frac{3}{11}\times\frac{8}{10}\times 3=\frac{288}{1320}$ P(3O) = $\frac{24}{1320}$	B1	P(1 O)or P(2 O) correct, accept unsimplified.
		M1	3 correct scenarios added, with at least one 3-term product of form $\frac{f}{12}\times\frac{g}{d}\times\frac{h}{e}$ seen, either $d = 11$, $e = 10$ or $d = 12$, $e = 12$.
	[Total =] $\frac{984}{1320}=\frac{41}{55}$, 0.745	A1	$0.745 \leqslant p \leqslant 0.74545$ If M0 scored **SC B1** $0.745 \leqslant p \leqslant 0.74545$.
	Method 3		
	O Y R = ${}^4C_1\times{}^5C_1\times{}^3C_1$ = 60 O R R = ${}^4C_1\times{}^3C_2$ = 12 O Y Y = ${}^4C_1\times{}^5C_2$ = 40 O O Y = ${}^4C_2\times{}^5C_1$ = 30 O O R = ${}^4C_2\times{}^3C_1$ = 18 O O O = ${}^4C_3$ = 4 Total = 164 Prob = $\frac{164}{{}^{12}C_3}$	B1	Number of ways either 1 or 2 orange sweets obtained correctly (112 or 48). Accept unsimplified Note ${}^4C_1\times{}^8C_2 = 112$ or ${}^4C_2\times{}^8C_1 = 48$ are correct alternatives.
		M1	3 correct scenarios (1, 2 or 3 orange sweets) added on numerator, denominator ${}^{12}C_3$
	$\frac{984}{1320}=\frac{41}{55}$, 0.745	A1	$0.745 \leqslant p \leqslant 0.74545$ If M0 scored **SC B1** $0.745 \leqslant p \leqslant 0.74545$.

Question	Answer	Marks	Guidance
7(c)	**Method 4**		
	$P(R\ R\ O) = \frac{3}{12}\times\frac{2}{11}\times\frac{4}{10}=\frac{1}{55}$ $P(R\ O\ \ \ ) = \frac{3}{12}\times\frac{4}{11}=\frac{1}{11}$ $P(R\ Y\ O) = \frac{3}{12}\times\frac{5}{11}\times\frac{4}{10}=\frac{1}{22}$ $P(O\ \ \ \ \ ) = \frac{4}{12}=\frac{1}{3}$ $P(Y\ R\ O) = \frac{5}{12}\times\frac{3}{11}\times\frac{4}{10}=\frac{1}{22}$ $P(Y\ O\ \ \ ) = \frac{5}{12}\times\frac{4}{11}=\frac{5}{33}$ $P(Y\ Y\ O) = \frac{5}{12}\times\frac{4}{11}\times\frac{4}{10}=\frac{2}{33}$	**B1**	$P(R\ \hat{}\ \hat{}) = \frac{17}{110}$ or $P(Y\ \hat{}\ \hat{}) = \frac{17}{66}$. Accept unsimplified.
		M1	3 correct scenarios added, with at least one 3-term product of form $\frac{f}{12}\times\frac{g}{d}\times\frac{h}{e}$ seen, either $d = 11$, $e = 10$ or $d = 12$, $e = 12$.
	$\frac{984}{1320}=\frac{41}{55}$, 0.745	**A1**	$0.745 \leqslant p \leqslant 0.74545$ If M0 scored **SC B1** $0.745 \leqslant p \leqslant 0.74545$.

Question	Answer	Marks	Guidance
7(c)	**Method 5**		
	$\text{P}(\text{O}\quad) = \frac{4}{12} = \frac{1}{3}$ $\text{P}(\hat{}\ \text{O}\quad) = \frac{8}{12} \times \frac{4}{11} = \frac{8}{33}$ $\text{P}(\hat{}\ \hat{}\ \text{O}) = \frac{8}{12} \times \frac{7}{11} \times \frac{4}{10} = \frac{28}{165}$	**B1**	$\text{P}(\hat{}\ \text{O}\quad) = \frac{8}{33}$ or $\text{P}(\hat{}\ \hat{}\ \text{O}) = \frac{28}{165}$. Accept unsimplified.
		M1	3 correct scenarios added, with at least one 3-term product of form $\frac{f}{12} \times \frac{g}{d} \times \frac{h}{e}$ seen, either $d = 11$, $e = 10$ or $d = 12$, $e = 12$ with correct numerator.
	$\frac{984}{1320} = \frac{41}{55}$, 0.745	**A1**	$0.745 \leqslant p \leqslant 0.74545$ If M0 scored **SC B1** $0.745 \leqslant p \leqslant 0.74545$.
		3	

Cambridge International AS & A Level

MATHEMATICS	**9709/53**
Paper 5 Probability & Statistics 1	**May/June 2022**
MARK SCHEME	
Maximum Mark: 50	

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2022 series for most Cambridge IGCSE, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathsematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (ISW).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

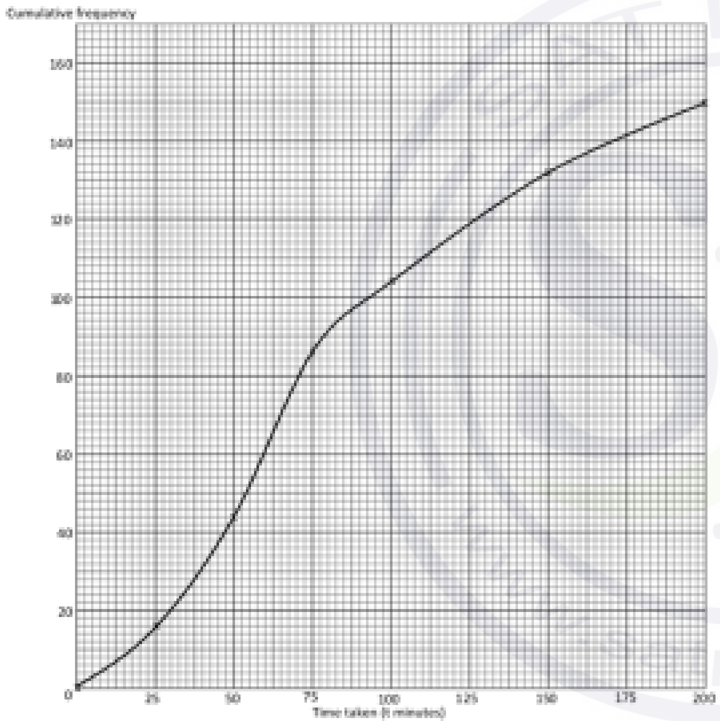
DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1(a)	Cumulative frequency (cf) graph	**M1**	At least 3 points plotted accurately at class upper end points (25,16) (50,44) (75,86) (100,104) (150, 132) (200, 150). Linear cf scale $0 \leqslant$ cf $\leqslant 150$ and linear time scale $0 \leqslant$ time(mins) $\leqslant 200$ with at least 3 values identified on each axis.
		**A1**	All points plotted correctly, curve drawn (within tolerance) and joined to (0,0). Axes labelled cumulative frequency (cf), time (t) and minutes (min), or a suitable title.
		2	
1(b)	Line from cumulative frequency = 30 to meet graph at t is between 37.5 and 42	**B1 FT**	Not from wrong working. Must be an increasing cumulative frequency graph.
		1	

Question	Answer	Marks	Guidance
2(a)	$\left[\frac{123.4}{20}=\right]6.17$	**B1**	Accept 6 m 17 cm, $\frac{1234}{200}$.
		1	
2(b)	$\frac{10\text{th}+11\text{th}}{2}=\frac{5.4+5.5}{2}=5.45$ (m)	**B1**	Accept 5 m 45 cm.
		1	
2(c)	The mean is unduly influenced by an extreme value, 19.4.	**B1**	Comment must be within context.
		1	

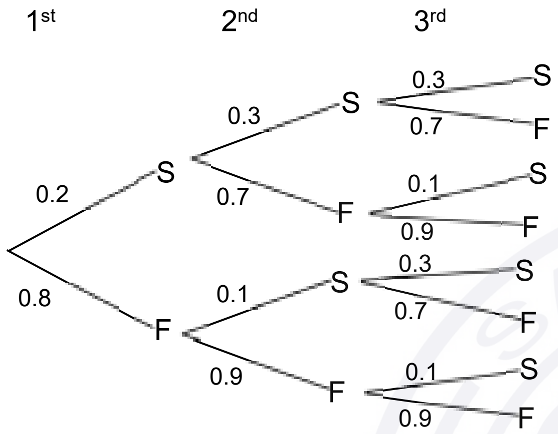
Question	Answer	Marks	Guidance
3(a)	$k=\frac{1}{18}$ $(4k+k+4k+9k=18k=1)$	**B1**	SOI
	<table><tr><td>x</td><td>–2</td><td>1</td><td>2</td><td>3</td></tr><tr><td>P(X=x)</td><td>$\frac{4}{18}$</td><td>$\frac{1}{18}$</td><td>$\frac{4}{18}$</td><td>$\frac{9}{18}$</td></tr></table>	**M1**	Table with correct x values and at least one probability accurate using *their k*. Values need not be in order, lines may not be drawn, may be vertical, x and P(X=x) may be omitted. Condone any additional X values if probability stated as 0.
		A1	Remaining probabilities correct.
		3	

Question	Answer	Marks	Guidance
3(b)	$\left[E(X) = \frac{4\times -2 + 1\times 1 + 4\times 2 + 9\times 3}{18} =\right]$ $\frac{-8+1+8+27}{18}$	**M1**	$-8k + k + 8k + 27k$ May be implied by use in Variance. Accept unsimplified expression. FT *their* table if probabilities sum to 1 or 0.999. **SC B1** $28k$.
	$\left[\mathrm{Var}(X) = \frac{4\times(-2)^2 + 1\times 1^2 + 4\times 2^2 + 9\times 3^2}{18} - (\textit{their}\,E(X))^2 =\right]$ $= \frac{16+1+16+81}{18} - \left(\textit{their}\,\frac{28}{18}\right)^2$	**M1**	$16k + k + 16k + 81k - (\textit{their}\text{ mean})^2$ FT *their* table even if probabilities not summing to 1. Note: If table is correct, $\frac{114}{18} - (\textit{their}\,E(X))^2$ M1. **SC B1** $114k - (\textit{their}\text{ mean})^2$.
	$E(X) = \frac{14}{9},\ 1\frac{5}{9},\ 1.56$, $\mathrm{Var}(X) = \frac{317}{81},\ 3\frac{74}{81},\ 3.91$	**A1**	Answers for $E(X)$ and $\mathrm{Var}(X)$ must be identified. $3.91 \leqslant \mathrm{Var}(X) \leqslant 3.914$
		3	

Question	Answer	Marks	Guidance
4(a)	$\left[\left(\frac{5}{6}\right)^7 \times \frac{1}{6} =\right] 0.0465, \frac{78125}{1679616}$	**B1**	$0.0465 \leqslant p < 0.04652$
		1	

Question	Answer	Marks	Guidance
4(b)	$P(X<6)=1-\left(\frac{5}{6}\right)^5$ or $\frac{1}{6}+\left(\frac{5}{6}\right)\left(\frac{1}{6}\right)+\left(\frac{5}{6}\right)^2\left(\frac{1}{6}\right)+\left(\frac{5}{6}\right)^3\left(\frac{1}{6}\right)+\left(\frac{5}{6}\right)^4\left(\frac{1}{6}\right)$	**M1**	$1-p^n$, $0<p<1$, $n=4, 5, 6$ or sum of 4, 5 or 6 terms $p\times(1-p)^n$ for $n=0,1,2,3,4(5)$.
	0.598, $\frac{4651}{7776}$	**A1**	
		2	
4(c)	[Probability of total less than 4 is] $\frac{3}{36}$ or $\frac{1}{12}$	**B1**	SOI
	[1 – P(0, 1, 2)] $=1-\left({}^{10}C_0\left(\frac{1}{12}\right)^0\left(\frac{11}{12}\right)^{10}+{}^{10}C_1\left(\frac{1}{12}\right)^1\left(\frac{11}{12}\right)^9+{}^{10}C_2\left(\frac{1}{12}\right)^2\left(\frac{11}{12}\right)^8\right)$	**M1**	One term ${}^{10}C_x\ p^x(1-p)^{10-x}$, for $0<x<10$, $0<p<1$.
	1 – (0.418904 + 0.380822 + 0.155791)	**A1 FT**	Correct expression. Accept unsimplified.
	0.0445	**A1**	$0.04448 \leqslant p \leqslant 0.0445$
		4	

Question	Answer	Marks	Guidance
5(a)	$[\text{P}(142 < X < 205)] = \text{P}\left(\frac{142-170}{25} < z < \frac{205-170}{25}\right)$	M1	Use of ± standardisation formula once substituting 170, 25 and either 142 or 205 appropriately.. Condone 25^2 and continuity correction ±0.5.
	$\text{P}(-1.12 < z < 1.4)$	A1	Both correct. Accept unsimplified.
	$\Phi(1.4) - (1 - \Phi(1.12)) = 0.9192 + 0.8686 - 1$	M1	Calculating the appropriate area from stated phis of z-values.
	0.788	A1	AWRT, not from wrong working
		4	
5(b)	$\text{P}(X > 205) = 1 - 0.9192 = 0.0808$	B1 FT	Correct or FT from part **5(a).**
	$(0.0808 \times 0.30 + \textit{their}\ 0.788 \times 0.24) \times 20000$	M1	Correct or *their* 0.0808 × 0.30 × k + *their* 0.788 × 0.24 × k, k positive integer.
	[$]4266.24	A1	$4265 <$ income $\leqslant 4270$, not from wrong working
		3	
5(c)	$[\text{P}(Z > \frac{w-182}{20}) = 0.72]$ $\frac{w-182}{20} = -0.583$	B1	$0.5828 \leqslant z \leqslant 0.583$ or $-0.583 \leqslant z \leqslant -0.5828$ seen.
		M1	182 and 20 substituted in ± standardisation formula, no continuity correction, not σ^2, $\sqrt{\sigma}$, equated to a z-value.
	$w = 170$	A1	$170 \leqslant w < 170.35$
		3	

Question	Answer	Marks	Guidance
6(a)		**B1**	First and second jumps correct with probabilities and outcomes identified.
		B1	Third jump correct with probabilities and outcomes identified.
		2	
6(b)	SFF $0.2\times0.7\times0.9 = 0.126$ FSF $0.8\times0.1\times0.7 = 0.056$ FFS $0.8\times0.9\times0.1 = 0.072$	**M1**	Two or three correct 3 factor probabilities added, correct or FT from part **6(a)**. Accept unsimplified.
	[Total = probability of 1 success =] 0.254 $\left(\frac{127}{500}\right)$	**A1**	Accept unsimplified.
	[Probability of at least 1 success = $1-0.8\times0.9\times0.9=$]0.352 $\left(\frac{44}{125}\right)$	**B1 FT**	Accept unsimplified.
	P(exactly 1 success | at least 1 success)= $\frac{\textit{their}\,0.254}{\textit{their}\,0.352}$	**M1**	Accept unsimplified.
	0.722, $\frac{127}{176}$	**A1**	$0.7215 < p \leqslant 0.722$
		5	

Question	Answer	Marks	Guidance
6(c)	$0.8\times0.9\times0.9\times0.1\times0.3\times0.3=0.005832$ [FFFSSS] $0.2\times0.3\times0.3\times0.7\times0.9\times0.9=0.010206$ [SSSFFF]	**M1**	$a \times b \times c \times d \times e \times f$ FT from *their* tree diagram. Either a, b and c all = 0.8 or 0.9 (at least one of each) and d, e and f all = 0.1 or 0.3 (at least one of each). Or a, b, c = 0.2 or 0.3 (at least one of each) and d, e, f = 0.7 or 0.9 (at least one of each).
		A1	Either correct. Accept unsimplified.
	[Total =] 0.0160[38]	**A1**	
		3	

Question	Answer	Marks	Guidance
7(a)	${}^{12}C_5 \times {}^{7}C_4$ [$\times {}^{3}C_3$]	**M1**	${}^{12}C_r \times q$, $r = 3, 4, 5$ q a positive integer > 1, no + or − .
		M1	${}^{12}C_s \times {}^{12-s}C_t$ [$\times {}^{12-s-t}C_u$] $s = 3, 4, 5$; $t = 3, 4, 5 \neq s$; $u = 3, 4, 5 \neq s, t$
	Alternative method for question 7(a)		
	$\dfrac{12!}{5!\times 3!\times 4!}$	**M1**	$12! \div$ by a product of three factorials.
		M1	$\dfrac{n!}{5!\times 3!\times 4!}$
	[792 × 35 =] 27 720	**A1**	CAO
		3	

Question	Answer	Marks	Guidance
7(b)	4! (Lizo) × 6! (Kenny) × 2! (Martin) × 2! (Nantes)	**M1**	Product involving at least 3 of 4!, 6!, 2!, 2!
	× 3! (orders of K, M and N)	**M1**	w × 3! , *w* integer > 1.
	414 720	**A1**	WWW CAO
		3	
7(c)	$^7C_4$ (adults) × $^4C_1$ × $^3C_1$	**M1**	$^7C_4$ × *b*, *b* integer > 1 no + or – .
	420	**A1**	
		2	
7(d)	K not L $^5C_3 \times {}^8C_3 = 560$ L not K $^5C_3 \times {}^8C_3 = 560$ L and K $^5C_2 \times {}^8C_3 = 560$	**M1**	$^8C_3$(or $^8P_3$) × *c* for one of the products or $^5C_3$ (or $^5P_3$)× *c*, positive integer >1 for first 2 products only.
		M1	Add 2 or 3 correct scenarios only values, no additional incorrect scenarios, no repeated scenarios. Accept unsimplified.
	[Total or Difference=] 1680	**A1**	
	Alternative method for question 7(d)		
	Total no of ways – neither L nor K Total = $^7C_4 \times {}^8C_3 = 1960$ Neither K nor L = $^5C_4 \times {}^8C_3 = 280$	**M1**	$^8C_3$ × *c, c* a positive integer >1.
		M1	Subtracting the number of ways with neither from their total number of ways.
	[Total or Difference=] 1680	**A1**	

Question	Answer	Marks	Guidance
7(d)	**Alternative method for question 7(d)**		
	Subtracting K and L from sum of K and L K $^6C_3 \times {}^8C_3 = 1120$ L $^6C_3 \times {}^8C_3 = 1120$ L and K $^5C_2 \times {}^8C_3 = 560$ $1120 + 1120 - 560 = 1680$	**M1**	$^8C_3 \times c$, c a positive integer >1.
		M1	Subtracting number of ways with both from sum of number of ways with K and number of ways with L.
	[Total or Difference=] 1680	**A1**	
		3	

Cambridge International AS & A Level

MATHEMATICS **9709/52**

Paper 5 Probability & Statistics 1 **February/March 2022**

MARK SCHEME

Maximum Mark: 50

Published

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Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
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SOI	Seen Or Implied
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WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1(a)	<table><tr><td>X</td><td>–2</td><td>–1</td><td>0</td><td>1</td><td>2</td></tr><tr><td rowspan="2">P(X)</td><td>$\frac{1}{16}$</td><td>$\frac{3}{16}$</td><td>$\frac{5}{16}$</td><td>$\frac{5}{16}$</td><td>$\frac{2}{16}$</td></tr><tr><td>0.0625</td><td>0.1875</td><td>0.3125</td><td>0.3125</td><td>0.125</td></tr></table>	**B1**	Table with correct X values and at least one probability $0 < p < 1$. Condone any additional X values if probability stated as 0. No repeated X values.
		B1	3 correct probabilities linked with correct outcomes, may not be in table.
		B1	2 further correct probabilities linked with correct outcomes, may not be in table No repeated X values. **SC** if less than 3 correct probabilities seen, award **SCB1** Sum of *their* probabilities, $0 < p < 1$, of 4,5 or 6 X values = 1 (condone summing to 1±0·01 or better).
		3	
1(b)	$\left[\frac{1}{16}\times -2^2 + \frac{3}{16}\times -1^2\left(+\frac{5}{16}\times 0^2\right)+\frac{5}{16}\times 1^2+\frac{2}{16}\times 2^2-\left(\frac{1}{4}\right)^2\right]$ $\frac{1\times 4+3\times 1+5\times 0+5\times 1+2\times 4}{16}-0.25^2$	**M1**	Appropriate variance formula using $(E(X))^2$ value, accept unsimplified. FT *their* table with at least 3 different X values even if probabilities not summing to 1, $0 < p < 1$. Condone 1 error providing all probabilities <1 and 0.25^2 used
	$\left[=\frac{5}{4}-\frac{1}{16}=\right]\frac{19}{16}$, 1.1875	**A1**	Condone 1.188 or 1.19 WWW
		2	

Question	Answer	Marks	Guidance
2(a)	[P(>2) = 1 – P(0,1,2) =] $1 - ({}^7C_0\ 0.18^0\ 0.82^7 + {}^7C_1\ 0.18^1\ 0.82^6 + {}^7C_2\ 0.18^2\ 0.82^5)$	**M1**	One term ${}^7C_x\ p^x(1-p)^{7-x}$, $0 < p < 1, 0 < x < 7$
	= 1 – (0.249285 + 0.383048 + 0.252251) = 1 – 0.88458	**A1**	Correct unsimplified expression or better Condone omission of brackets if recovered
	0.115	**B1**	WWW. $0{\cdot}115 \leqslant p < 0{\cdot}1155$ not from wrong working
		3	
2(b)	[P(at least 1 day of rain) = 1 – P(0) = $1-(0.82)^7$ =] 0.7507	**B1**	AWRT 0.751 seen
	[P(exactly 2 periods) =] $0.7507^2 \times (1-0.7507) \times 3$	**M1**	FT *their* $1-p^7$ or *their* 0.7507 if identified, not 0.18, 0.82 Accept $\times{}^3C_r$, r=1,2 or $\times{}^3P_1$ for ×3 Condone ×2
	0.421	**A1**	Accept $0.421 \leqslant p \leqslant 0.4215$ **SC B1** if 0/3 scored for final answer only $0.421 \leqslant p \leqslant 0.4215$
		3	

Question	Answer	Marks	Guidance
3(a)	<table><tr><td>Class Width</td><td>30</td><td>15</td><td>20</td><td>10</td><td>25</td></tr><tr><td>Frequency Density</td><td>0.7</td><td>2</td><td>3.4</td><td>8.6</td><td>1.8</td></tr></table>	**M1**	At least 4 frequency densities calculated
		A1	All heights correct on graph
		B1	Bar ends at 0·5, 30·5, 45·5, 65.5, 75.5, 100.5 (at axis), 5 bars drawn, condone 0 in first bar $0.5 \leqslant$ time axis $\leqslant 100.5$, linear scale with at least 3 values indicated.
		B1	Axes labelled: Frequency density (fd), time (t) and mins (or appropriate title). Linear fd scale, with at least 3 values indicated $0 \leqslant$ fd axis $\leqslant 8{\cdot}6$
		4	
3(b)	66 – 75	**B1**	Condone 65.5 – 75.5
		1	
3(c)	Distribution is not symmetrical	**B1**	Or skewed, ignore nature of skew
		1	

Question	Answer	Marks	Guidance
4(a)	$\text{P}(46 < X < 62) = \text{P}\left(\frac{46-55}{6} < Z < \frac{62-55}{6}\right)$	**M1**	46 or 62, 55 and 6 substituted into ±standardisation formula once. Condone 6^2 and continuity correction ±0.5
	$= \text{P}\left(-1.5 < Z < \frac{7}{6}\right)$	**B1**	Both standardisation values correct, accept unsimplified
	$\left[=\Phi\left(\frac{7}{6}\right) - \left(1 - \Phi(1.5)\right)\right]$ $= 0.8784 + (0.9332 - 1)$	**M1**	Calculating the appropriate area from stated Φs of z-values, must be probabilities.
	0.812	**A1**	$0{\cdot}8115 < p \leqslant 0{\cdot}812$
		4	
4(b)	$z = \pm 0.674$	**B1**	CAO, critical z-value
	$\frac{36-42}{\sigma} = -0.674$	**M1**	36 and 42 substituted in ±standardisation formula, no continuity correction, not σ^2, $\sqrt{\sigma}$, equated to a z-value
	$\sigma = 8.9[0]$	**A1**	WWW. Only dependent on M.
		3	

Question	Answer	Marks	Guidance
4(c)	P(male < 46) = 1−*their* 0.9332 = 0.0668	M1	FT value from part **(a)** or Correct: $1-\Phi\left(\frac{46-55}{6}\right)$,condone continuity correction, σ^2, $\sqrt{\sigma}$, **and** probability found. Condone unsupported correct value stated.
	P(female < 46) = $P(Z < \frac{46-42}{their\,8.90})\left[=\Phi(0.449)\right]$ $= 0.6732$	M1	46, 42 and *their* 4(b) σ (or correct σ) substituted in ±standardisation formula, condone continuity correction, σ^2, $\sqrt{\sigma}$, **and** probability found Condone $\frac{4}{their\,8.90}$.
	P(both) = 0.0668 ×0.6732	M1	Product of *their* 2 probabilities (0 < both < 1) Not 0.25 or *their* final answer to **4(a)** used.
	0.0450 or 0.0449	A1	$0{\cdot}0449 \leqslant p \leqslant 0{\cdot}0450$
		4	

Question	Answer	Marks	Guidance
5(a)	${}^5C_1 \times {}^7C_4$	M1	${}^7C_4 \times k$, k integer $\geqslant 1$ Condone ${}^5P_1$ for M1 only
	175	A1	
		2	

Question	Answer	Marks	Guidance
5(b)	2B 1G 2A $\quad {}^3C_2 \times {}^4C_1 \times {}^5C_2 = 120$ 2B 2G 1A $\quad {}^3C_2 \times {}^4C_2 \times {}^5C_1 = 90$ 2B 3G $\quad {}^3C_2 \times {}^4C_3 = 12$ 3B 1G 1A $\quad {}^3C_3 \times {}^4C_1 \times {}^5C_1 = 20$ 3B 2G $\quad {}^3C_3 \times {}^4C_2 = 6$	**M1**	${}^3C_x \times {}^4C_y \times {}^5C_z$, $x + y + z = 5$, x,y,z integers $\geqslant 1$ Condone use of permutations for this mark
		B1	2 appropriate identified outcomes correct, allow unsimplified
		M1	Summing *their* values for 4 or 5 correct identified scenarios only (no repeats or additional scenarios), condone identification by unsimplified expressions
	[Total =] 248	**A1**	Note: Only dependent upon M marks
		4	
5(c)	$8! \times 3! \times {}^5P_2$	**M1**	$8! \times m$, m an integer $\geqslant 1$ Accept $8 \times 7!$ for $8!$
		M1	$3! \times n$, n an integer > 1
		M1	$p \times {}^5P_2$, $p \times {}^5C_2 \times 2$, $p \times 20$, p an integer > 1 If extra terms present, maximum 2/3 M marks available
	4 838 400	**A1**	Exact value required
		4	

Question	Answer	Marks	Guidance
6(a)	[Probability of lemon = $\frac{3}{15}=\frac{1}{5}$] $\left[\left(\frac{4}{5}\right)^6 \times \frac{1}{5} =\right]\frac{4096}{78125}, 0.0524$	B1	0.0524288 rounded to more than 3SF if final answer
		1	
6(b)	$\left(1-\frac{1}{5}\right)^6$	M1	or $\left(\frac{4}{5}\right)^6$. FT *their* $\frac{1}{5}$ or correct. From final answer Condone $\left(\frac{4}{5}\right)^5$ or $\left(\frac{1}{5}\right)\times\left(\frac{4}{5}\right)^5+\left(\frac{4}{5}\right)^6$
	$\frac{4096}{15625}$, 0.262	A1	0.262144 rounded to more than 3SF
	Alternative method for question 6(b)		
	[1 – P(1,2,3,4,5,[6]) =] $1-\left(\frac{1}{5}+\frac{4}{5}\times\frac{1}{5}+\left(\frac{4}{5}\right)^2\times\frac{1}{5}+\left(\frac{4}{5}\right)^3\times\frac{1}{5}+\left(\frac{4}{5}\right)^4\times\frac{1}{5}+\left(\frac{4}{5}\right)^5\times\frac{1}{5}\right)$	M1	From final answer Condone omission of $\left(\frac{4}{5}\right)^5\times\frac{1}{5}$
	$\frac{4096}{15625}$, 0.262	A1	0.262144 rounded to more than 3SF
		2	

Question	Answer	Marks	Guidance
6(c)	$\frac{10}{15}\times\frac{9}{14}\times\frac{8}{13}$	M1	$\frac{a}{15}\times\frac{a-1}{14}\times\frac{a-2}{13}$, no additional terms
	$\frac{24}{91}$, 0·264	A1	0.263736 rounded to more than 3SF
	Alternative method for question 6(c)		
	$\frac{3}{15}\times\frac{2}{14}\times\frac{1}{13}+3\times\frac{3}{15}\times\frac{2}{14}\times\frac{7}{13}+3\times\frac{3}{15}\times\frac{7}{14}\times\frac{6}{13}+\frac{7}{15}\times\frac{6}{14}\times\frac{5}{13}$	M1	[3Ls + 2Ls1S + 1L2Ss + 3Ss] Condone one numerator error. Condone no multiplications seen if tree diagram complete with probabilities on each branch, scenarios listed and attempt at evaluation
	$\frac{24}{91}$, 0·264	A1	0.263736 rounded to more than 3SF
	Alternative method for question 6(c)		
	$1-\left(\frac{5}{15}\times\frac{4}{14}\times\frac{3}{13}+3\times\frac{5}{15}\times\frac{4}{14}\times\frac{10}{13}+3\times\frac{5}{15}\times\frac{10}{14}\times\frac{9}{13}\right)$	M1	1 – P(3,2,1 oranges) Condone one numerator error.
	$\frac{24}{91}$, 0·264	A1	0.263736 rounded to more than 3SF
	Alternative method for question 6(c)		
	$\frac{{}^{10}C_3}{{}^{15}c_3}$	M1	
	$\frac{24}{91}$, 0·264	A1	0.263736 rounded to more than 3SF
		2	

Question	Answer	Marks	Guidance
6(d)	$\frac{7}{15}\times\frac{5}{14}\times\frac{3}{13}\times 3!$	M1	All probabilities of the form: $\frac{7}{a}\times\frac{5}{b}\times\frac{3}{c}$, $13 \leqslant a,b,c \leqslant \underline{15}$
		M1	$\frac{e}{f}\times\frac{g}{h}\times\frac{i}{j}\times 3!$ $e,f,g,h,i.j$ positive integers forming probabilities or 6 identical probability calculations or values added, no additional terms
	$\frac{3}{13}$, 0·231	A1	0·230769 rounded (not truncated) to more than 3SF
	Alternative method for question 6(d)		
	$\frac{{}^{3}C_{1}\times{}^{5}C_{1}\times{}^{7}C_{1}}{{}^{15}C_{3}}$	M1	$\frac{{}^{3}C_{1}\times{}^{5}C_{1}\times{}^{7}C_{1}}{k}$, k integer > 1 Condone use of permutations
		M1	$\frac{{}^{3}C_{a}\times{}^{5}C_{b}\times{}^{7}C_{c}}{{}^{15}C_{3}}$, $0<a<3$, $0<b<5$, $0<c<7$, Condone use of permutations
	$\frac{3}{13}$, 0·231	A1	0·230769 rounded (not truncated) to more than 3SF
		3	

Question	Answer	Marks	Guidance
6(e)	$\dfrac{\frac{7}{15}\times\frac{6}{14}\times\frac{5}{13}+\frac{3}{15}\times\frac{7}{14}\times\frac{6}{13}\times 3}{their\,(\boldsymbol{c})}\ \left[=\frac{14}{65}\div\frac{24}{91}\right]$	B1	$\frac{3}{15}\times\frac{7}{14}\times\frac{6}{13}\times 3$ seen (SSL, SLS, LSS) **SC B1** $\frac{3}{65}\times 3, \frac{126}{2730}\times 3$ seen
		B1	$\frac{7}{15}\times\frac{6}{14}\times\frac{5}{13}$ seen in numerator (SSS) **SCB1** $\frac{210}{2730}, \frac{1}{13}$ seen in numerator
		M1	Fraction with *their* **(c)** or correct in denominator $\left(\frac{720}{2730}, \frac{24}{91}, 0.263736\right)$
	$=\frac{49}{60}$, 0·817	A1	Accept $0.81\dot{6}$
	Alternative method for question 6(e)		
	$\dfrac{{}^{7}C_{2}\times{}^{3}C_{1}+{}^{7}C_{3}}{{}^{10}C_{3}}$	B1	${}^{7}C_{2}\times{}^{3}C_{1}$ seen (SSL, SLS, LSS) **SCB1** 21 × 3 seen or use of permutations
		B1	${}^{7}C_{3}$ seen in numerator (SSS) **SCB1** 35 seen in numerator or use of permutations
		M1	Fraction with ${}^{10}C_{3}$ or consistent with *their* numerator of **6(c)** in denominator
	$=\frac{49}{60}$, 0·817	A1	Accept $0.81\dot{6}$
		4	

Cambridge Assessment International Education

Cambridge International AS & A Level

MATHEMATICS **9709/51**

Paper 5 Probability & Statistics 1 **October/November 2021**

MARK SCHEME

Maximum Mark: 50

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1(a)	$\left(\frac{3}{4}\right)^6 \frac{1}{4}$	M1	$(1-p)^6\ p,\ 0<p<1$
	$0.0445,\ \frac{729}{16384}$	A1	
		2	
1(b)	$\left(\frac{3}{4}\right)^9$	M1	$\left(\frac{3}{4}\right)^n$ *or* p^n, $0<p<1, n=8,9,10$
	$0.0751,\ \frac{19683}{262144}$	A1	
		2	

Question	Answer	Marks	Guidance
2(a)	$\left[\frac{\sum x}{40}-k=\frac{\sum(x-k)}{40}\right]$ $\frac{40\times 34}{40}-k=\frac{520}{40}$	M1	Forms an equation involving Σx, $\Sigma(x\text{-}k)$ and k. Accept at a numeric stage with k.
	$k[=34-13]=21$	A1	Evaluated.
		2	

Question	Answer	Marks	Guidance
2(b)	$\text{Var} = \left[\frac{\sum(x-k)^2}{40} - \left(\frac{\sum(x-k)}{40}\right)^2\right] = \frac{9640}{40} - \left(\frac{520}{40}\right)^2 = [241 - 13^2 =]$	**M1**	Values substituted into an appropriate variance formula, accept unsimplified.
	72	**A1**	
		2	

Question	Answer	Marks	Guidance
3	$\left[\text{P}(T\|B') = \frac{\text{P}(T \cap B')}{\text{P}(B')}\right]$ $\text{P}(B') = 0.45\times0.7 + 0.35\times0.4 + 0.2\times1$ $\left[= 0.655, \frac{131}{200}\right]$	**M1**	$0.45\times a + 0.35\times b + 0.2[\times1], a = 0.7, 0.3b = 0.4, 0.6$, seen anywhere.
		A1	Correct, accept unsimplified.
	$\text{P}(T \cap B') = 0.35\times0.4$ $[= 0.14, \frac{7}{50}]$	**M1**	Seen as numerator or denominator of a fraction.
	$\text{P}(T \mid B') = \frac{\textit{their}\ 0.14}{\textit{their}\ 0.655}$	**M1**	Values substituted into conditional probability formula correctly. Accept unsimplified. Denominator sum of 3 two-factor probabilities (condone omission of 1 from final factor). If clearly identified, condone from incomplete denominator.
	$0.214, \frac{28}{131}$	**A1**	If 0 marks awarded, **SC B1** 0.214 WWW.
		5	

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td rowspan="4">4(a)</td><td rowspan="3"><table><tr><td>x</td><td>-1</td><td>0</td><td>1</td><td>2</td><td>3</td></tr><tr><td>p</td><td>$\frac{1}{12}=0.0833$</td><td>$\frac{2}{12}=0.167$</td><td>$\frac{4}{12}=0.333$</td><td>$\frac{3}{12}=0.25$</td><td>$\frac{2}{12}=0.167$</td></tr></table></td><td>B1</td><td><table><tr><td></td><td>0</td><td>1</td><td>2</td><td>2</td></tr><tr><td>– 1</td><td>–1</td><td>0</td><td>1</td><td>1</td></tr><tr><td>0</td><td>0</td><td>1</td><td>2</td><td>2</td></tr><tr><td>1</td><td>1</td><td>2</td><td>3</td><td>3</td></tr></table>Table with x values and at least one probability substituted, $0 < p < 1$.
Condone any additional x values if probability stated as 0.</td></tr>
<tr><td>B1</td><td>2 correct identified probabilities.</td></tr>
<tr><td>B1</td><td>All probabilities correct (accept to 3sf).

SC if less than 2 correct probabilities:
SC B1 4 or 5 probabilities summing to one.</td></tr>
<tr><td></td><td>3</td><td></td></tr>
<tr><td rowspan="4">4(b)</td><td>$\mathrm{E}(X) = -\frac{1}{12}+\frac{4}{12}+\frac{6}{12}+\frac{6}{12}\left[=\frac{15}{12}\right]$</td><td>M1</td><td>May be implied by use in Variance, accept unsimplified expression.
Probabilities must sum to 1 ± 0.001.</td></tr>
<tr><td>$\mathrm{Var}(X) = \frac{1}{12}+0+\frac{4}{12}+\frac{12}{12}+\frac{18}{12}-\left(\frac{15}{12}\right)^2$</td><td>M1</td><td>Appropriate variance formula using *their* $(\mathrm{E}(X))^2$. **FT** accept probabilities not summing to 1.
Condone $\frac{35}{12}-\left(\frac{15}{12}\right)^2$ *or* $\frac{35}{12}-\frac{25}{9}$ from correct table.</td></tr>
<tr><td>$\left[\frac{35}{12}-\frac{25}{16}=\right]\frac{65}{48}$, 1.35</td><td>A1</td><td>WWW</td></tr>
<tr><td></td><td>3</td><td></td></tr>
</table>

Question	Answer	Marks	Guidance
5(a)	[8! =] 40 320	**B1**	Evaluated, exact value only.
		1	
5(b)	**Method 1** [^ ^ ^ R ^ ^ S ^ ^]		
	$7! \times {}^{8}C_{2} \times 2$	**M1**	$7! \times k$ seen, k an integer > 1.
		M1	$m \times n(n-1)$ or $m \times {}^{n}C_{2}$ or $m \times {}^{n}P_{2}$, n = 7, 8 or 9, m an integer > 1.
	282 240	**A1**	Exact value only. **SC B1** for final answer 282 240 WWW.
	Method 2 [Total number of arrangements – Arrangements with R & S together]		
	$9! - 8! \times 2$	**M1**	$9! - k$, k an integer < 362 880 .
		M1	$m - 8! \times n$, m an integer > 40 320, n = 1,2.
	282 240	**A1**	Exact value only. **SC B1** for final answer 282 240 WWW.
		3	
5(c)	${}^{9}C_{5}$ [$\times {}^{4}C_{4}$]	**M1**	${}^{9}C_{x}$ [$\times {}^{9-x}C_{9-x}$,] x = 4, 5. Condone × 1 for ${}^{9-x}C_{9-x}$. Condone use of P.
	126	**A1**	WWW
		2	

Question	Answer	Marks	Guidance
5(d)	[Number of ways with Raman and Sanjay together on back row =] $^7C_3$ [Number of ways with Raman and Sanjay together on front row =] $^7C_2$	**M1**	^7C_x seen, $x = 3$ or 2.
	[Total =] 35 + 21	**M1**	Summing two correct scenarios.
	56	**A1**	Evaluated – may be seen used in probability. If **M0** scored, **SC B1** for 56 WWW.
	Probability = $\frac{their\,56}{their\,(c)} = \frac{56}{126}, \frac{4}{9}$, 0.444	**B1 FT**	FT *their* 56 from adding 2 or more scenarios in numerator and *their* **(c)** or correct as denominator.
		4	

Question	Answer	Marks	Guidance
6(a)	Rebels | | Sharks 6 | 6 8 9 8 5 | 7 | 1 2 4 5 5 6 8 9 6 5 4 3 2 2 0 | 8 | 3 3 4 5 6 9 5 3 | 9 | 2 2 | 10 | Key: 8 | 7 | 2 means 78 kg for Rebels and 72 kg for Sharks	**B1**	Correct stem, ignore extra values (not in reverse).
		B1	Correct Rebels labelled on left, leaves in order from right to left and lined up vertically, no commas.
		B1	Correct Sharks labelled on same diagram, leaves in order and lined up vertically, no commas.
		B1	Correct key for their diagram, need both teams identified and 'kg' stated at least once here or in leaf headings or title. **SC** If 2 separate diagrams drawn, **SC B1** if both keys meet these criteria.
		4	

Question	Answer	Marks	Guidance
6(b)	Median = 84 (kg)	**B1**	
	[UQ = 93, LQ = 80] 93 – 80	**M1**	$95 \leqslant UQ \leqslant 89 - 79 \leqslant LQ \leqslant 82$
	[IQR =] 13 (kg)	**A1**	WWW
		3	
6(c)	Box and whisker with end points 75 and 102	**B1**	Whiskers drawn to correct end points not through box, not joining at top or bottom of box.
	Median and quartiles plotted as found in **(b)**	**B1 FT**	Quartiles and median plotted as box graph.
		2	
6(d)	e.g. Average weight of Rebels is higher than average weight of Sharks	**B1**	Acceptable answers refer to: Range, skew, central tendency within context. E.g. range of Rebels is greater **B0**. Range of weights of the rebels is greater **B1**. Simple value comparison insufficient.
		1	

Question	Answer	Marks	Guidance
7(a)(i)	$\text{P}(X > 142) = \text{P}\left(Z > \frac{142-125}{24}\right)$	M1	Substitution of correct values into the ±Standardisation formula, allow continuity correction, not σ^2, $\sqrt{\sigma}$.
	$[= \text{P}(Z > 0.7083) =]1 - 0.7604$	M1	Appropriate numerical area Φ, from final process, must be probability, expect $p < 0.5$.
	0.2396	A1	$0.239 \leqslant p \leqslant 0.240$ to at least 3sf.
	Their 0.2396×365 [= 87.454]	M1	**FT** *their* 4sf (or better) probability.
	87 or 88	A1 FT	Final answer must be positive integer, no indication of approximation/rounding, only dependent on previous **M** mark. **SC B1 FT** for *their* 3sf probability × 365 = integer value, condone 0.24 used.
		5	
7(a)(ii)	$\text{P}(0, 1) = 0.7604^{10} + {}^{10}\text{C}_1 \times 0.2396^1 \times 0.7604^9$ [= 0.064628 + 0.20364]	M1	One term: ${}^{10}\text{C}_x\, p^x (1-p)^{10-x}$ for $0 < x < 10$, any p.
		A1 FT	Correct unsimplified expression using *their* probability to at least 3sf from **(a)(i)** or correct.
	0.268	A1	AWRT, WWW.
		3	
7(b)	$z = \pm 1.282$	B1	Correct value only, critical value.
	$\frac{t-125}{24} = -1.282$	M1	Use of ± Standardisation formula with correct values substituted, allow continuity correction, σ^2, $\sqrt{\sigma}$, to form an equation with a z-value and not probability.
	$t = 94.2$	A1	AWRT, condone AWRT 94.3. Not dependent on **B** mark.
		3	

Cambridge Assessment International Education

Cambridge International AS & A Level

MATHEMATICS **9709/52**

Paper 5 Probability & Statistics 1 **October/November 2021**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2021 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

This document consists of **16** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1(a)	$\frac{82}{180}, \frac{41}{90}$, 0.456	B1	
		1	
1(b)	$\left[\mathrm{P}(\mathrm{M}\vert\mathrm{D}) = \frac{P(M \cap D)}{P(D)}\right] = \frac{\frac{11}{180}}{\frac{20}{180}+\frac{11}{180}}$ or $\frac{0.6011}{0.1722}$	M1	*Their* identified $\frac{P(M \cap D)}{P(D)}$ *or* from data table $\frac{11}{20+11}$, accept unsimplified, condone $\times$ 180.
	$\frac{11}{31}$, 0.355	A1	Final answer.
		2	

Question	Answer	Marks	Guidance
1(c)	$P(F) = \frac{100}{180}, \frac{5}{9}, 0.5556$ OE $P(G) = \frac{82}{180}, \frac{41}{90} 0.4556$ OE	M1	*Their* identified P(F) × *their* identified P(G) or correct seen, can be unsimplified.
	$P(F \cap G) = \frac{38}{180}, \frac{19}{90}, 0.2111$ OE $P(F) \times P(G) = \frac{100}{180} \times \frac{82}{180} = \frac{41}{162}, 0.2531$ OE $\left[\neq \frac{38}{180}\right]$ Not independent	A1	$\frac{41}{162}, \frac{38}{180}, P(F \cap G)$ and $P(F) \times P(G)$ seen with correct conclusion, WWW. Values and labels must be seen.
	Alternative method for question 1(c)		
	$P(F \cap G) = \frac{38}{180}, \frac{19}{90}, 0.2111$ OE $P(G) = \frac{82}{180}, \frac{41}{90}, 0.4556$ OE	M1	P(F|G) (OE) unsimplified with *their* identified probs or correct
	$P(F\|G) = \frac{\frac{38}{180}}{\frac{82}{180}} = \frac{19}{41}, 0.4634$ OE $\neq P(F) = \frac{100}{180}, \frac{5}{9}, 0.5556$ OE Not independent	A1	$\frac{19}{41}, \frac{100}{180}, P(F \cap G)$ and $P(F\|G)$ seen with correct conclusion WWW. Values and labels must be seen.
		2	

Question	Answer	Marks	Guidance
2(a)	${}^{11}C_5 \times {}^{4}C_1$	**M1**	${}^{11}C_5 \times {}^{4}C_1$ condone ${}^{11}P_5 \times {}^{4}P_1$ no +, –, × or ÷.
	1848	**A1**	CAO as exact.
		2	
2(b)	**Method 1 [**Identifying scenarios]		
	[Neither selected =] ${}^{13}C_6$ [= 1716] [Only Jane selected =] ${}^{13}C_5$ [= 1287] [Only Kate selected =] ${}^{13}C_5$ [= 1287]	**M1**	Either ${}^{13}C_6$ seen alone or ${}^{13}C_5$ seen alone or × 2 (condone ${}^{13}P_n$, n = 5,6).
	[Total =] 1716 + 1287 + 1287	**M1**	Three correct scenarios only added, accept unsimplified (values may be incorrect).
	4290	**A1**	
	Method 2 [Total number of selections – selections with Jane and Kate both picked]		
	${}^{15}C_6$ - ${}^{13}C_4$ [= 5005 – 715]	**M1**	${}^{15}C_6 - k$, k a positive integer < 5005, condone ${}^{15}P_6$.
		M1	$m - {}^{13}C_4$, m integer > 715, condone $n - {}^{13}P_4$, n > 17 160.
	4290	**A1**	
		3	
			SC Where the condition of **2(a)** is also applied in **2(b)**, the final answer is 1512 **SC M1 M1 A0** max. The method marks can be earned for the equivalent stages in each method. Method 1 ${}^{4}C_1 \times {}^{9}C_5 + {}^{4}C_1 \times {}^{9}C_4 \times 2$ Method 2 ${}^{4}C_1 \times {}^{11}C_5 - {}^{4}C_1 \times {}^{9}C_3$

Question	Answer	Marks	Guidance
3(a)	For one yellow: YGG + GYG +GGY $\frac{5}{9}\times\frac{4}{8}\times\frac{3}{7}\times 3$	**M1**	$\frac{a}{9}\times\frac{b}{8}\times\frac{c}{7}$, $0 < a,b,c$ integers $\leqslant 5$, for one arrangement.
		M1	*Their* three-factor probability $\times$ 3, ${}^3C_1$, ${}^3C_2$ or ${}^3P_1$, (or repeated adding) no additional terms.
	$\left[\frac{180}{504}=\right]\frac{5}{14}$	**A1**	AG. Convincingly shown, including identifying possible scenarios, may be on tree diagram WWW.
		3	
	Alternative method for question 3(a)		
	$\frac{{}^5C_1\times{}^4C_2}{{}^9C_3}$	**M1**	$\frac{{}^5C_1\times{}^4C_2}{{}^9C_r}$, $r = 2, 3, 4$
		M1	$\frac{{}^5C_s\times{}^4C_t}{{}^9C_3}$, $s + t = 3$
	$\left[\frac{30}{84}=\right]\frac{5}{14}$	**A1**	AG. Convincingly shown, WWW.
		3	

Question	Answer	Marks	Guidance
3(b)	X: 0, 1, 2, 3 $P(X)$: $\frac{24}{504}$ $\left[=\frac{1}{21}, 0.0476\right]$, $\frac{180}{504}$ $\left[=\frac{5}{14}, 0.357\right]$, $\frac{240}{504}$ $\left[=\frac{10}{21}, 0.476\right]$, $\frac{60}{504}$ $\left[=\frac{5}{42}, 0.119\right]$	B1	Table with correct X values and one correct probability inserted appropriately. Condone any additional X values if probability stated as 0.
		B1	Second identified correct probability, may not be in table.
		B1	All probabilities identified and correct . **SC** if less than 2 correct probabilities or X value(s) omitted: **SC B1** 3 or 4 probabilities summing to one.
		3	
3(c)	[E(X) =] $\frac{840}{504}, \frac{5}{3}$, 1.67	B1	OE Must be evaluated. **SC B1 FT** correct unsimplified expression from incorrect **3(b)** using at least 3 probabilities, $0 < p < 1$.
		1	

Question	Answer	Marks	Guidance
4(a)	$\frac{9!}{3!}$	M1	$\frac{9!}{e!}$, $e = 2, 3$
	60 480	A1	
		2	

Question	Answer	Marks	Guidance
4(b)	$\frac{7!}{3!}\times 2\times 6$	M1	$\frac{7!}{3!}\times k$ seen, k an integer > 0.
		M1	$\frac{m!}{n!}\times 2\times q$ $7 \leqslant m \leqslant 9$, $1 \leqslant n \leqslant 3$, $1 \leqslant q \leqslant 8$ all integers.
		M1	$\frac{m!}{n!}\times p\times 6$ $7 \leqslant m \leqslant 9$, $1 \leqslant n \leqslant 3$, $1 \leqslant p \leqslant 2$ all integers. (Accept 3P2 for 6) If **M0 M0 M0** awarded, **SC M1** for $t\times 12$, t an integer $\geqslant 20$, $\frac{5!}{3!}$.
	10 080	A1	Exact value.
	Alternative method for question 4(b)		
	$\frac{{}^7P_2\times 6!\times 2}{3!}$	M1	$\frac{6!}{3!}\times k$ seen, k an integer > 0.
		M1	$\frac{m!}{n!}\times {}^7P_2\times q$ $m = 6,9$, $1 \leqslant n \leqslant 3$, $1 \leqslant q \leqslant 2$ all integers.
		M1	$\frac{m!}{n!}\times {}^7P_r\times 2$ $m = 6, 9$, $1 \leqslant n \leqslant 3$, $1 \leqslant r \leqslant 5$ all integers. If **M0 M0 M0** awarded, **SC M1** for $t\times 84$, t an integer $\geqslant 20$, $\frac{5!}{3!}$.
	10 080	A1	Exact value.

Question	Answer	Marks	Guidance
4(b)	**Alternative method for question 4(b)**		
	$\frac{7!}{3!}\times 4P2$	**M1**	$\frac{7!}{3!}\times k$ seen, k an integer > 0.
		M1	$t \times {}^4P_2$ or 12, t an integer $\geqslant 20$, $\frac{5!}{3!}$.
		M1	$\frac{m!}{n!}\times 4P2$ $7 \leqslant m \leqslant 9$, $1 \leqslant n \leqslant 3$ all integers.
	10 008	**A1**	Exact value.
		4	

Question	Answer	Marks	Guidance
5(a)	[P(0, 1, 2) =] ${}^{10}C_0\, 0.16^0\, 0.84^{10} + {}^{10}C_1\, 0.16^1\, 0.84^9 + {}^{10}C_2\, 0.16^2\, 0.84^8$ [= 0.17490 + 0.333145 + 0.28555]	**M1**	One term: ${}^{10}C_x\, p^x\, (1-p)^{10-x}$ for $0 < x < 10$, any p.
		A1	Correct unsimplified expression, or better.
	0.794	**A1**	$0.7935 < p \leqslant 0.794$, mark at most accurate. If **M0** scored, **SC B1** for final answer 0.794.
		3	
5(b)	$(0.84)^7\, 0.16$	**M1**	$(1-p)^7 p$, $0 < p < 1$
	0.0472	**A1**	0.0472144 to at least 3sf.
		2	

Question	Answer	Marks	Guidance
5(c)	$4 \times 0.0472 \times (1 - 0.0472)^3$	**M1**	$4 \times q(1-q)^3$, q = *their* **(b)** or correct.
	0.163	**A1**	$0.163 \leqslant p \leqslant 0.1634$, mark at most accurate from *their* probability to at least 3sf.
		2	

Question	Answer	Marks	Guidance
6(a)	$\left[\mathrm{P}(X > 28.6) =\right] P\left(Z > \frac{28.6 - 32.2}{9.6}\right)$ $\left[= P(Z > -0.375)\right]$	**M1**	28.6, 32.2 and 9.6 substituted appropriately in ± Standardisation formula once, allow continuity correction of ± 0.05, no σ^2, $\sqrt{\sigma}$.
	$[\Phi(\textit{their}\ 0.375) =]$ *their* 0.6462	**M1**	Appropriate numerical area, from final process, must be probability, expect > 0.5.
	0.646	**A1**	AWRT
		3	
6(b)	$z = \pm 0.842$	**B1**	$0.841 < z \leqslant 0.842$ or $-0.842 \leqslant z < -0.841$ seen.
	$\frac{t - 32.2}{9.6} = 0.842$	**M1**	Substituting 32.2 and 9.6 into ± standardisation formula, no continuity correction, allow σ^2, $\sqrt{\sigma}$, must be equated to a z-value.
	$t = 40.3$	**A1**	$40.28 \leqslant t \leqslant 40.3$ WWW
		3	

Question	Answer	Marks	Guidance
6(c)	$P\left(-\frac{15}{9.6} < Z < \frac{15}{9.6}\right)$ $P(-1.5625 < Z < 1.5625)$	M1	Identifying at least one of $\frac{15}{9.6}$ and $-\frac{15}{9.6}$ as the appropriate z-values or substituting *their* (32.2 ± 15) into ± Standardisation formula once, no continuity correction, σ^2 nor $\sqrt{\sigma}$. Condone ±1.563 for **M1**.
	$[2\ \Phi(\frac{15}{9.6}) - 1\ ]$ $= 2 \times 0.9409 - 1$	A1	$p = 0.941$ AWRT SOI
		M1	Appropriate area $2\Phi - 1$ oe, (eg $1 - 2 \times 0.0591$, $2 \times (0.9409 - 0.5)$ or $0.9409 - 0.0591$), from final process, must be probability > 0.5.
	0.882	A1	
		4	

Question	Answer	Marks	Guidance
7(a)	Cumulative frequency graph drawn	B1	Axes labelled ‘cumulative frequency’ (or cf) from 0 to at least 140 and ‘distance (or d) [in] m’ from 0 to at least 1600, linear scales with at least 3 values stated.
		B1	All plotted correctly at correct upper end points (200 etc.) **curve** drawn accurately joined to (0, 0) (straight line segments B0) but no daylight above 140. Cf scale no less than 2 cm = 20 children .
		2	

Question	Answer	Marks	Guidance
7(b)	[UQ at 75% of 140 = 105, LQ at 25% of 140 = 35] [IQR:] 700 – 260	**M1**	Accept 660 $\leqslant$ UQ $\leqslant$ 720 – 240 $\leqslant$ LQ $\leqslant$ 290. If values are outside our range, FT providing scales linear and increasing cf drawn.
	440	**A1**	Accept correct evaluation of 660 $\leqslant$ *their* UQ $\leqslant$ 720 – 240 $\leqslant$ *their* LQ $\leqslant$ 290 with clear indication that graph has been used for at least one of 105 or 35.
		2	

Question	Answer	Marks	Guidance
7(c)	[Mean =] $\dfrac{16\times100+30\times250+42\times400+34\times700+12\times1050+6\times1400}{140}$	**B1**	Frequencies: 16, 30, 42, 34, 12, 6; Mid-points: 100, 250, 400, 700, 1050, 1400 5 or 6 correct frequency values seen.
		B1	5 or 6 correct midpoint values seen.
		M1	Values substituted into mean formula using *their* midpoints which must be in the class – condone 1 data error. Accept $\dfrac{1600+7500+16\,800+23\,800+12\,600+8400}{140}$ or $\dfrac{70\,700}{140}$. Condone $\dfrac{70\,770}{140}$ for **M1**.
	505	**A1**	WWW
	Variance = $\dfrac{16\times100^2+30\times250^2+42\times400^2+34\times700^2+12\times1050^2+6\times1400^2}{140}$ -505^2	**M1**	Values substituted into variance formula using (*their* mean)^2 and *their* midpoints and *their* frequencies (including for denominator). Accept unsimplified. Condone 1 data error. Accept: $[\dfrac{160\,000+1\,875\,000+6\,720\,000+16\,660\,000+13\,230\,000+11\,760\,000}{140}$ or $\dfrac{50\,405\,000}{140}$ or $360\,035.7143]-\left[505^2 \text{ or } 255\,025\right]$ If formula stated accept 105 010 or 105 011 WWW.
	S.d. = $\left[\sqrt{105\,010.7}=\right]324$	**A1**	WWW
		6	

Cambridge Assessment International Education

Cambridge International AS & A Level

MATHEMATICS **9709/53**

Paper 5 Probability & Statistics 1 **October/November 2021**

MARK SCHEME

Maximum Mark: 50

Published

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GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
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2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
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Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

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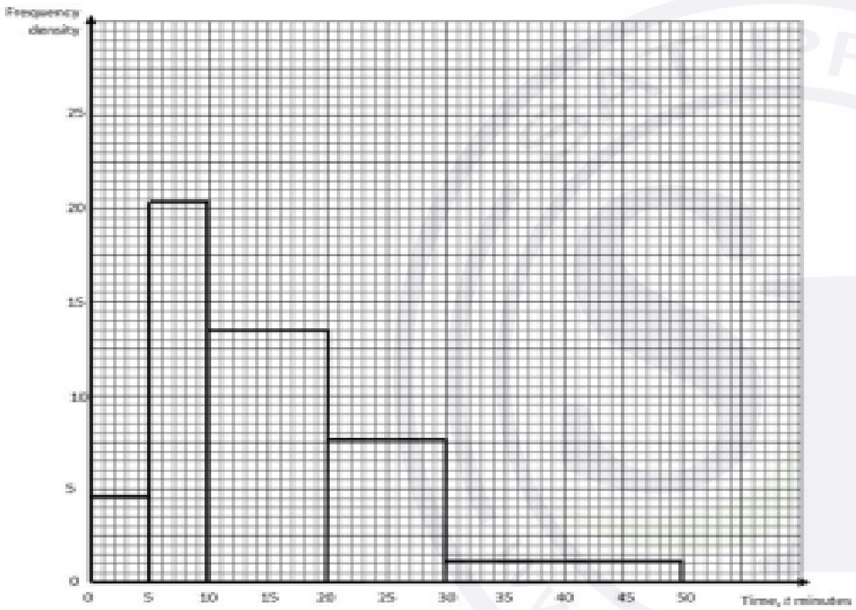
Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	${}^{23}C_{17}$	**M1**	${}^{23}C_x$ or ${}^{y}C_{17}$ or ${}^{z}C_6$, *x, y* or *z* are integers no +, –, × or ÷.
	100947	**A1**	CAO
		2	

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td rowspan="5">2(a)</td><td rowspan="4"><table>
<tr><th>Lakeview</th><th></th><th>Riverside</th></tr>
<tr><td>9 4 0</td><td>1</td><td>8 8</td></tr>
<tr><td>8 7 6 2</td><td>2</td><td>0 1 3 4 5 5</td></tr>
<tr><td>3 2 0</td><td>3</td><td>0 6 7</td></tr>
<tr><td>1</td><td>4</td><td></td></tr>
</table>
Key: 6|2|3 means 26m for Lakeview and 23m for Riverside</td><td>B1</td><td>Correct stem, ignore extra values.</td></tr>
<tr><td>B1</td><td>Correct Lakeview labelled on left, leaves in order from right to left and lined up vertically, no commas.</td></tr>
<tr><td>B1</td><td>Correct Riverside labelled on same diagram, leaves in order and lined up vertically, no commas.</td></tr>
<tr><td>B1</td><td>Correct key for their diagram, need both teams identified and ‘m’ stated at least once here or in leaf headings or title.

SC If 2 separate diagrams drawn: SC B1 if both keys meet these criteria.</td></tr>
<tr><td></td><td>4</td><td></td></tr>
<tr><td rowspan="3">2(b)</td><td>UQ = 32, LQ = 19</td><td>M1</td><td>(30 ⩽ UQ ⩽ 33) – (14 ⩽ LQ ⩽ 22)</td></tr>
<tr><td>IQR = 32 – 19 = 13</td><td>A1</td><td>WWW</td></tr>
<tr><td></td><td>2</td><td></td></tr>
</table>

Question	Answer	Marks	Guidance
3(a)	Cw: 5 5 10 10 20	**M1**	At least 4 frequency densities calculated (f/cw), accept unsimplified and class widths ± 1 of true values. May be implied by graph.
	Fd: 4.6 20.4 13.5 7.6 1.2	**A1**	All heights correct on graph **NOT FT**
		**B1**	Bar ends at 0, 5, 10, 20, 30, 50 clear intention not to draw at 4.5 or 5.5 etc.
		B1	Axes labelled: Frequency density (fd), time (t) and mins (or appropriate title). Linear scales between 0 and 20.4 or above on vertical axis, and 0 and 50 or above on the horizontal axis. (Axes may be reversed.)
		4	
3(b)	$\dfrac{2.5\times 23+7.5\times 102+15\times 135+25\times 76+40\times 24}{360}$	**M1**	Uses at least 4 midpoint attempts (e.g. 2.5 ± 0.5) in correct formula, accept unsimplified expression, denominator either correct or *their* Σfrequencies .
	$\left[\dfrac{5707.5}{360}=\right]15.9,\ 15\dfrac{41}{48}$	**A1**	Evaluated.
		2	

Question	Answer	Marks	Guidance
4(a)	$P(X > 43.2) = P\left(Z > \frac{43.2-41.2}{3.6}\right) = P(Z > 0.5556)$	**M1**	Use of ±Standardisation formula once, allow continuity correction, not σ^2, $\sqrt{\sigma}$.
	$1 - \Phi(0.5556) = 1 - 0.7108$	**M1**	Appropriate area Φ, from final process, must be probability.
	0.289	**A1**	AWRT
		3	
4(b)	Probability = 1 – *their* **(a)** = 1 – 0.2892 = 0.7108	**B1FT**	1 – *their* **(a)** or correct.
	$0.7108 \times 365 = 259.4$ 259, 260	**B1FT**	FT *their* 4SF (or better) probability, final answer must be positive integer.
		2	
4(c)	$z = \pm 1.645$	**B1**	CAO, critical z value.
	$\frac{t-41.2}{3.6} = -1.645$	**M1**	Use of ±standardisation formula with μ, σ equated to a z-value, no continuity correction, allow σ^2, $\sqrt{\sigma}$.
	$t = 35.3$	**A1**	
		3	

Question	Answer	Marks	Guidance
5(a)	${}^5P_2 \times {}^7P_4$ **or** $5 \times 4 \times 7 \times 6 \times 5 \times 4$	**M1**	${}^5P_x \times {}^7P_y$, $1 \leqslant x \leqslant 4$, $1 \leqslant y \leqslant 6$
	16 800	**A1**	
		2	

Question	Answer	Marks	Guidance
5(b)	**Method 1 [**Identify scenarios]		
	With A and no 5: $8 \times {}^6P_4$ or $(1 \times 4 \times 6 \times 5 \times 4 \times 3) \times 2$ or $4C1 \times 2! \times 6P4 = 2880$ With 5 and no A: ${}^4P_2 \times 4 \times {}^6P_3$ or $(4 \times 3 \times 1 \times 6 \times 5 \times 4) \times 4$ or $4P2 \times 6C3 \times 4! = 5760$ With A and 5: $8 \times 4 \times {}^6P_3$ or $(4 \times 1 \times 1 \times 6 \times 5 \times 4) \times 8$ or $4C1 \times 2! \times 6C3 \times 4! = 3840$	**M1**	One number of ways correct, accept unsimplified.
		M1	Add 2 or 3 identified correct scenarios only, accept unsimplified.
	[Total =] 12 480	**A1**	CAO
	Method 2 [total number of codes – number of codes with no A or 5]		
	No A or 5 : $(4 \times 3) \times (6 \times 5 \times 4 \times 3) = 4320$	**M1**	${}^4P_2 \times {}^6P_4$ or ${}^4C_2 \times {}^6C_4$ seen, accept unsimplified.
	Required number = *their* **(a)** – *their* 4320	**M1**	*Their* **5(a)** (or correct) – *their* (No A or 5) value.
	12 480	**A1**	
	Method 3 [subtracting **double counting**]		
	With A ${}^4P_1 \times {}^7P_4 \times 2$ or ${}^4C_1 \times 2 \times {}^7C_4 \times 4! = 6720$ With 5 ${}^5P_2 \times {}^6P_3 \times 4$ or ${}^5C_2 \times 2 \times {}^6C_3 \times 4! = 9600$ With A and 5 = ${}^4P_1 \times {}^6P_3 \times 8$or $4C1 \times 2! \times 6C3 \times 4! \times 8 = 3840$	**M1**	One outcome correct, accept unsimplified.
	Required number = 6720 + 9600 – 3840	**M1**	Adding 'with a' to 'with 5' and subtracting 'A and 5'.
	12 480	**A1**	CAO
		3	

Question	Answer	Marks	Guidance
5(c)	**Method 1 –** number of successful codes divided by total		
	$(1 \times)\ 3 \times {}^5P_2$	M1	$3 \times {}^5P_n$, $n = 2, 3$. Condone $3 \times {}^5C_2$, no + or –.
	Probability $= \dfrac{their\,3\times 5P2}{their\,16\,800}$	M1	Probability $= \dfrac{their\,60}{their\,16\,800}$.
	$\dfrac{1}{280}$, 0.00357	A1	
	Method 2 – product of probabilities of each part of code		
	$\dfrac{1}{5}\times\dfrac{1}{4}\times\dfrac{1}{7}\times\dfrac{3}{6}\left(\times\dfrac{5}{5}\times\dfrac{4}{4}\right)$ or $\dfrac{1}{5}\times\dfrac{1}{4}\times\dfrac{3\times 5P2}{7P4}$	M1	$\dfrac{1}{5}\times\dfrac{1}{4}\times k$ where $0 < k < 1$ for considering letters.
		M1	$t\times\dfrac{1}{7}\times\dfrac{3}{6}$ or $t\times\dfrac{3\times 5P2}{7P4}$ where $0 < t < 1$.
	$\dfrac{1}{280}$	A1	CAO
		3	

Question	Answer	Marks	Guidance
6(a)	$p+q+0.65=1$	B1	Sum of probabilities = 1.
	$p+2q+0.15=0.55$	B1	Use given information.
	Solve 2 linear equations	M1	Either a single expression with one variable eliminated formed or two expressions with both variables on the same side seen with at least one variable value stated.
	$p=0.3, \frac{3}{10}, \quad q=0.05, \frac{1}{20}$	A1	CAO, both WWW If **M0** with correct answers **SC B1**.
		4	
6(b)	Var (X) = $\textit{their}\, 0.3+4\times \textit{their}\, 0.05+9\times 0.05-0.55^2$	M1	Appropriate variance formula including $(E(X))^2$, accept unsimplified.
	0.6475 $\left[\frac{259}{400}\right]$	A1	CAO (must be exact).
		2	
6(c)	1 – P(0, 1, 2) = 1 – ($^{12}C_0\ 0.3^0\ 0.7^{12}$ + $^{12}C_1\ 0.3^1\ 0.7^{11}$ + $^{12}C_2\ 0.3^2\ 0.7^{10}$)	M1	One correct term: $^{12}C_x\, p^x\, (1-p)^{12-x}$ for $0 < x < 12$, $0 < p < 1$.
	1 – (0.01384 + 0.07118 + 0.16779)	A1FT	Correct unsimplified expression, or better in final answer. Unsimplified expression must be seen to **FT** *their p* from **6(a)** or correct.
	0.747	A1	
		3	
6(d)	$(0.95)^8\times 0.05 = 0.0332$ or $0.95^8-0.95^9=0.0332$	B1	Evaluated.
		1	

Question	Answer	Marks	Guidance
7(a)	Probabilities: $\frac{x+1}{x+10}$, $\frac{9}{x+10}$, $\frac{x}{x+10}$, $\frac{10}{x+10}$	**B1**	One probability correct in correct position.
		B1	Another probability correct in correct position.
		B1	Other two probabilities correct in correct positions.
		3	
7(b)	$\frac{4}{10} \times \textit{their}\, \frac{10}{x+10}$	**M1**	Method consistent with *their* tree diagram.
	$\frac{4}{x+10}$	**A1**	AG
		2	

Question	Answer	Marks	Guidance
7(c)	$\frac{4}{x+10}=\frac{1}{6}$ $x+10=24, \quad x=14$	**B1**	Find value of x. Can be implied by correct probabilities in calculation.
	P(ARed|BRed) = P(ARed ∩ BRed) ÷ P(BRed) $\dfrac{\frac{6}{10}\times their\,\frac{x+1}{x+10}}{\frac{6}{10}\times their\,\frac{x+1}{x+10}+\frac{4}{10}\times their\,\frac{x}{x+10}} = \dfrac{\frac{6}{10}\times\frac{15}{24}}{\frac{6}{10}\times\frac{15}{24}+\frac{4}{10}\times\frac{14}{24}} = \dfrac{\frac{3}{8}}{\frac{73}{120}}$	**B1 FT**	$\frac{6}{10}\times their\,\frac{x+1}{x+10}$ as numerator or denominator of fraction.
		M1	$\frac{6}{10}\times their\,\frac{x+1}{x+10}+\frac{4}{10}\times their\,\frac{x}{x+10}$ seen anywhere.
		A1 FT	Seen as denominator of fraction.
	$\frac{45}{73}$, 0.616[4…]	**A1**	If B0 M0: **SC B1** for $\dfrac{\frac{3}{8}}{\frac{73}{120}}$ or $\frac{0.375}{0.6083}$ **SC B1** $\frac{45}{73}$ or 0.616.
		5	

Cambridge Assessment International Education

Cambridge International AS & A Level

MATHEMATICS **9709/51**

Paper 5 Probability & Statistics 1 **May/June 2021**

MARK SCHEME

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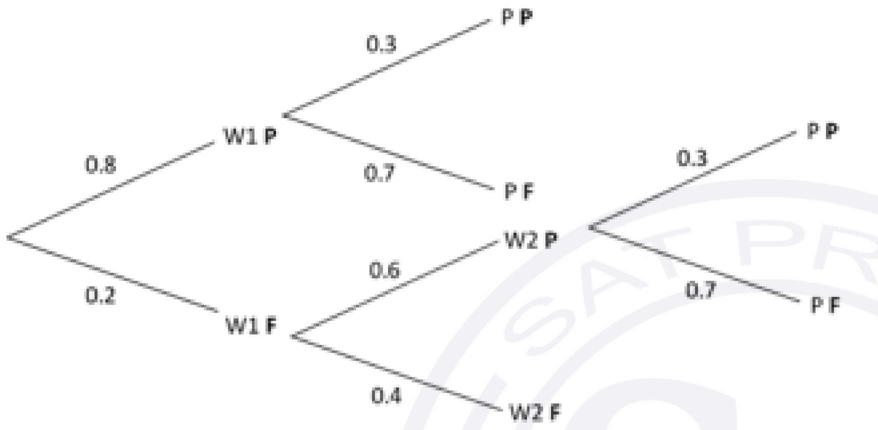
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	RRRRB $^8C_4 \times {}^4C_1 = 280$ BBBBR $^8C_1 \times {}^4C_4 = 8$ RRRRR $^8C_5 = 56$	M1	$^8C_x \times {}^4C_y$ with $x + y = 5$. x, y both integers, $1 \leqslant x \leqslant 5$, $0 \leqslant y \leqslant 4$ condone $^8C_1 \times 1$
		A1	Two correct outcomes evaluated
		M1	Add 2 or 3 identified correct scenarios only (no additional terms, not probabilities)
	[Total =] 344	A1	WWW, only dependent on 2nd M mark
		4	**SC** not all (or no) scenarios identified **B1** 280 + 8 + 56 **DB1** 344

Question	Answer	Marks	Guidance
2	$\left[P\left(\left(\frac{25.2-(25.5+0.50)}{0.4}\right)<z<\left(\frac{25.2-(25.2-0.50)}{0.4}\right)\right)\right]$ $=P\left(-\frac{0.5}{0.4}<z<\frac{0.5}{0.4}\right)$	M1	Use of $\pm$ Standardisation formula once; no continuity correction, σ^2, $\sqrt{\sigma}$
	$[=2\Phi(1.25)-1]$ $=2\times0.8944-1$	A1	For AWRT 0.8944 SOI
		M1	Appropriate area $2\Phi - 1$ OE, from final process, must be probability
	0.7888	A1	Accept AWRT 0.789
	Number of rods = 0.7888×500 = 394 or 395	B1FT	Correct or FT *their* 4SF (or better) probability, final answer must be positive integer, not 394.0 or 395.0, no approximation/rounding stated, only 1 answer
		5	

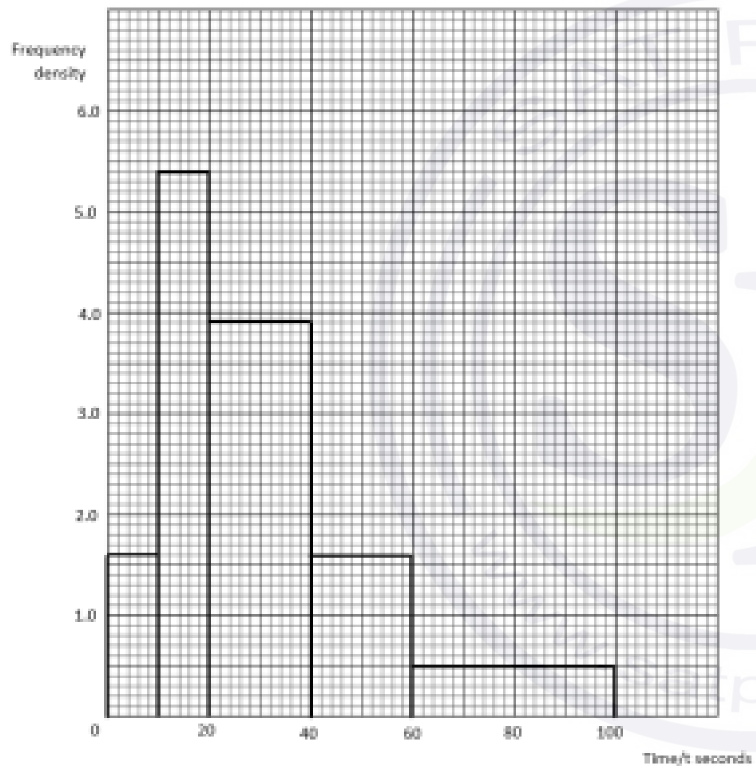
Question	Answer	Marks	Guidance
3(a)	$\left[\dfrac{8!}{3!}\right] = 6720$	B1	NFWW, must be evaluated
		1	
3(b)	_ _ _ L E D _ _ : With LED together: $\dfrac{6!}{2!}$	M1	$\dfrac{6!}{k}$ or $\dfrac{5!x6}{k}$ $k \geqslant 1$ and no other terms
		M1	$\dfrac{m}{2!}$, m an integer, $m \geqslant 5$
	360	A1	CAO
		3	
3(c)	Method using _ _ _ A _ D _ _ : Arrange the 6 letters RELESE = $\dfrac{6!}{3!}$ [= 120]	*M1	$\dfrac{6!}{3!}\times k$ seen, k an integer > 0
	Multiply by number of ways of placing AD in non-adjacent places = *their* 120 × $^7P_2$ [= 5040]	*M1	$m\times n(n-1)$ or $m\times {}^nC_2$ or $m\times {}^nP_2$, n = 6, 7 or 8, m an integer > 0
	[Probability =] $\dfrac{\mathit{their}\ 5040}{\mathit{their}\ 6720}$	DM1	Denominator = *their* **(a)** or correct, dependent on at least one M mark already gained.
	$\dfrac{5040}{6720}$ or $\dfrac{3}{4}$ or 0.75	A1	
	Alternative method for Question 3(c)		
	Method using 'Total arrangements – Arrangements with A and D together': *Their* 6720 – $\dfrac{7!\times 2}{3!}$ [= 5040]	*M1	*Their* 6720 – k, k a positive integer
		*M1	$(m-)\dfrac{7!\times k}{3!}, k=1,2$

Question	Answer	Marks	Guidance
	[Probability =] $\frac{their\,5040}{their\,6720}$	DM1	With denominator = *their* **(a)** or correct, dependent on at least one M mark already gained.
	$\frac{5040}{6720}$ or $\frac{3}{4}$ or 0.75	A1	
	Alternative method for Question 3(c)		
	Method using '1 – Probability of arrangements with A and D together': $\frac{7!\times 2}{3!}$ [= 1680]	*M1	$\frac{7!\times k}{3!}, k = 1, 2$
	[Probability =] $\frac{their\,1680}{their\,6720}$	*M1	With denominator = *their* **(a)** or correct
	$1 - \frac{their\,1680}{their\,6720}$	DM1	$1 - m$, $0 < m < 1$, dependent on at least one M mark already gained
	$\frac{5040}{6720}$ or $\frac{3}{4}$ or 0.75	A1	
		4	

Question	Answer	Marks	Guidance
4(a)		B1	Fully correct labelled tree diagram for each pair of branches clearly identifying written and practical, pass and fail for each intersection (no additional branches)
		B1	‘One written test’ branch all probabilities (or %) correct
		B1	‘Two written tests’ branch all probabilities (or %) correct, condone additional branches after W2F with probabilities 1 for PF and 0 for PP
		3	
4(b)	[P(W1P) × P(PP) + P(W1F) × P(W2P) × P(PP)] $0.8\times0.3+0.2\times0.6\times0.3$	M1	Consistent with *their* tree diagram or correct
	0.276 or $\dfrac{69}{250}$	A1	
		2	
4(c)	$P(W1\|P)=\dfrac{P(W1\cap \text{Practical})}{P(\text{getting place})}=\dfrac{0.8\times0.3}{their(b)}\left[=\dfrac{0.24}{0.276}\right]$	M1	Correct expression or FT *their* **(b)**
	$\dfrac{20}{23}$ or $0.87[0]$	A1	
		2	

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td rowspan="4">5(a)</td><td rowspan="4">

Class width	10	10	20	20	40
Frequency Density	1.6	5.4	3.9	1.6	0.5



</td><td>M1</td><td>At least 4 frequency densities calculated, accept unsimplified.
May be read from graph using *their* scale, 3SF or correct</td></tr>
<tr><td>A1</td><td>All heights correct on graph</td></tr>
<tr><td>B1</td><td>Bar ends at 0, 10, 20 …, etc. with a horizontal linear scale with at least 3 values indicated,
$0 \leqslant$ horizontal axis $\leqslant 100$</td></tr>
<tr><td>B1</td><td>Axes labelled: Frequency density (fd), time (t) and seconds. Linear vertical scale, with at least 3 values indicated $0 \leqslant$ vertical axis $\leqslant 5.4$</td></tr>
<tr><td></td><td></td><td>**4**</td><td></td></tr>
</table>

Question	Answer	Marks	Guidance
5(b)	Mean $= \left[\dfrac{16\times5+54\times15+78\times30+32\times50+20\times80}{200}\right]$ $= \dfrac{80+810+2340+1600+1600}{200}$	M1	Uses at least 4 midpoint attempts (e.g. 5 ± 0.5). Accept unsimplified expression, denominator either correct or *their* Σfrequencies
	$\left[\dfrac{6430}{200}=\right]$ $32\dfrac{3}{20}$ or 32.15	A1	Accept 32.2
		2	
5(c)	A value in correct UQ (40–60) – a value in correct LQ (10–20)	M1	
	Greatest possible value is 60 – 10 = 50	A1	Condone $49.\dot{9}$
		2	

Question	Answer	Marks	Guidance
6(a)	1 – P(10, 11, 12) = $1 - ({}^{12}C_{10}\,0.6^{10}\,0.4^{2} + {}^{12}C_{11}\,0.6^{11}\,0.4^{1} + {}^{12}C_{12}\,0.6^{12}\,0.4^{0})$ [= 1 – (0.063852 + 0.017414 + 0.0021768)]	M1	One term: ${}^{12}C_x\,p^x\,(1-p)^{12-x}$ for $0 < x < 12$, any p allowed.
		A1	Correct unsimplified expression, or better.
	[1 – 0.083443] = 0.917	A1	AWRT
	Alternative method for Question 6(a)		
	P (0,1,2,3,4,5,6,7,8,9) = ${}^{12}C_0 0.6^0\,0.4^{12} + {}^{12}C_1\,0.6^1\,0.4^{11} + \ldots\ldots\ldots\ldots {}^{12}C_9\,0.6^9\,0.4^3$ [= 0.000016777 + 0.00030199 + 0.0024914 + 0.012457 + 0.042043 + 0.10090 + 0.17658 + 0.22703 + 0.21284 + 0.14189]	M1	One term: ${}^{12}C_x\,p^x\,(1-p)^{12-x}$ for $0 < x < 12$, any p allowed.
		A1	Correct unsimplified expression with at least the first two and last terms
	0.917	A1	WWW, AWRT
		3	

Question	Answer	Marks	Guidance
6(b)	[Mean =] 0.6×150 [= 90]; [Variance =] $0.6 \times 150 \times 0.4$ [= 36]	B1	Correct mean and variance. Accept evaluated or unsimplified
	$P(X < 81) = P\left(Z < \frac{80.5 - 90}{6}\right)$	M1	Substituting *their* mean and variance into ±standardisation formula (with a numerical value for 80.5), allow σ^2, $\sqrt{\sigma}$, but not $\mu \pm 0.5$
		M1	Using continuity correction 80.5 or 81.5
	$\Phi(-1.5833) = 1 - 0.9433$	M1	Appropriate area Φ, from final process, must be probability
	0.0567	A1	AWRT
		5	
6(c)	$np = 90$, $nq = 60$ both greater than 5	B1	At least nq evaluated and statement >5 required
		1	

Question	Answer	Marks	Guidance
7(a)	$P(X = 3) = \frac{4}{7} \times \frac{3}{6} \times \frac{3}{5}$	M1	$\frac{m}{7} \times \frac{n}{6} \times \frac{o}{5}$ used throughout. condone use of $\frac{1}{2}$
	$\frac{6}{35}$	A1	AG. The fractions must be identified, e.g. P(NC, NC, C), may be seen in a tree diagram.
		2	

Question	Answer	Marks	Guidance
7(b)	$\begin{array}{c\|c\|c\|c\|c\|c} x & 1 & 2 & 3 & 4 & 5 \\ \hline p & \frac{15}{35} & \frac{10}{35} & \frac{6}{35} & \frac{3}{35} & \frac{1}{35} \end{array}$	B1	Table with x values and at least one probability Condone any additional x values if probability stated as 0.
		B1	One correct probability other than $X = 3$ linked to the correct outcome
		B1	Two further correct probabilities other than $X = 3$ seen linked to the correct outcome
		B1FT	All probabilities correct, or at least 4 probabilities summing to 1
		4	
7(c)	$[E(X) = 1\times\frac{15}{35} + 2\times\frac{10}{35} + 3\times\frac{6}{35} + 4\times\frac{3}{35} + 5\times\frac{1}{35}]$ $E(X) = \frac{15+20+18+12+5}{35}\left[=\frac{70}{35}=2\right]$	M1	At least 4 correct terms FT *their* values in **(a)** with probabilities summing to 1 May be implied by use in Variance, accept unsimplified expression.
	$\text{Var}(X) = \left[\frac{1^2\times15+2^2\times10+3^2\times6+4^2\times3+5^2\times1}{35} - 2^2 =\right]$ $\frac{15+40+54+48+25}{35} - 2^2$	M1	Appropriate variance formula using *their* $(E(X))^2$. FT *their* table accept probabilities not summing to 1.
	$\left[=\frac{182}{35} - 4\right] = \frac{6}{5}$	A1	**N.B.** If method FT for M marks from *their* incorrect **(b)**, expressions for E(X) and Var(X) must be seen unsimplified with all probabilities <1
		3	

Cambridge International AS & A Level

MATHEMATICS **9709/52**

Paper 5 Probability & Statistics 1 **May/June 2021**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2021 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1(a)	6	B1	WWW
		1	
1(b)	$\left(\frac{5}{6}\right)^3\frac{1}{6}+\left(\frac{5}{6}\right)^4\frac{1}{6}+\left(\frac{5}{6}\right)^5\frac{1}{6}+\left(\frac{5}{6}\right)^6\frac{1}{6}$	M1	$p^3(1-p)+p^4(1-p)+p^5(1-p)+p^6(1-p)$, $0<p<1$
	0.300 (0.2996…)	A1	At least 3s.f. Award at most accurate value.
	Alternative method for Question 1(b)		
	$\left(\frac{5}{6}\right)^3-\left(\frac{5}{6}\right)^7$	M1	p^3-p^7, $0<p<1$
	0.300 (0.2996…)	A1	At least 3s.f. Award at most accurate value.
		2	
1(c)	$1-\left(\frac{5}{6}\right)^9$	M1	$1-p^n$, $0<p<1$, $n=9, 10$
	0.806	A1	
	Alternative method for Question 1(c)		
	$\frac{1}{6}+\frac{1}{6}\left(\frac{5}{6}\right)+\frac{1}{6}\left(\frac{5}{6}\right)^2+\cdots+\frac{1}{6}\left(\frac{5}{6}\right)^8$	M1	$p+p(1-p)+p(1-p)^2+p(1-p)^3+p(1-p)^4+p(1-p)^5+p(1-p)^6+p(1-p)^7+p(1-p)^8(+p(1-p)^9)$, $0<p<1$ As per answer for minimum terms shown
	0.806	A1	
		2	

Question	Answer	Marks	Guidance
2	$\left[P(X>1.1)=\frac{72}{2000}(=0.036)\right]$ $z=\pm 1.798$	B1	$1.79 < z \leqslant 1.80$, $-1.80 \leqslant z < -1.79$ seen
	$\frac{1.1-1.04}{\sigma}=1.798$	B1	1.1 and 1.04 substituted in ±standardisation formula, allow continuity correction, not σ^2 or $\sqrt{\sigma}$
	$\left[\frac{0.06}{\sigma}=1.798\right]$	M1	Equate *their* ±standardisation formula to a z-value and to solve for the appropriate area leading to final answer (expect $\sigma < 0.5$). $\left(\text{Accept} \pm\frac{0.06}{\sigma}=z-\text{value}\right)$
	$\sigma = 0.0334$	A1	$0.03335 \leq \sigma \leq 0.0334$. At least 3 3s.f.
		4	

Question	Answer	Marks	Guidance
3(a)	P(not late) = 0.4 × 0.45 + 0.35 × 0.3 + 0.25 × (1 – x) or P(late) = 0.4 × 0.55 + 0.35 × 0.7 + 0.25x	M1	$0.4\times p+0.35\times q+0.25\times r$, p = 0.45, 0.55, q = 0.3, 0.7 and r = (1 – x), x
	0.18 + 0.105 + 0.25 (1 – x) = 0.48 or 0.22 + 0.245 + 0.25x = 0.52	A1	Linear equation formed using sum of 3 probabilities and 0.48 or 0.52 as appropriate. Accept unsimplified.
	x = 0.22	A1	Final answer
		3	
3(b)	$\left[P\left(train\middle\|late\right)=\dfrac{P\left(train\cap late\right)}{P\left(late\right)}\right]$ $=\dfrac{0.35\times0.7}{1-0.48}$ or $\dfrac{0.35\times0.7}{0.4\times0.55+0.35\times0.7+0.25\times \mathit{their}\,0.22}$	B1	0.35×0.7 or 0.245 seen as numerator of fraction
		M1	P(late) seen as a denominator with *their* probability as numerator (Accept $\dfrac{\mathit{their}\ p}{0.52}$ or $\dfrac{\mathit{their}\ p}{0.22+0.245+0.25\times \mathit{their}\,0.22}$)
	= 0.471 or $\dfrac{49}{104}$	A1	
		3	

Question	Answer	Marks	Guidance
4(a)	$\begin{array}{c\|c\|c\|c\|c\|c} X & -1 & 0 & 1 & 2 & 3 \\ \hline P(X) & \frac{1}{9} & \frac{2}{9} & \frac{1}{9} & \frac{3}{9} & \frac{2}{9} \end{array}$	B1	Table with correct X values and at least one probability Condone any additional X values if probability stated as 0.
		B1	2 correct probabilities linked with correct outcomes, may not be in table.
		B1	3 further correct probabilities linked with correct outcomes, may not be in table. **SC** if less than 2 correct probabilities seen, award **SCB1** for sum of *their* 4 or 5 probabilities in table = 1
		3	
4(b)	$\left[E(X) = \frac{-1\times1+(0\times2)+1\times1+2\times3+3\times2}{9} = \right]$ $\frac{-1+1+6+6}{9}$	M1	May be implied by use in variance, accept unsimplified expression. FT *their* table if *their* 3 or more probabilities sum to 1 or 0.999
	[Var (X) =] $\left[\frac{-1^2\times1+(0^2\times2)+1^2\times1+2^2\times3+3^2\times2}{9} - (their\,E(X))^2\right]$ $\frac{1+0+1+12+18}{9} - (their\,E(X))^2$	M1	Appropriate variance formula using *their* $(E(X))^2$ value. FT *their* table even if *their* 3 or more probabilities not summing to 1.
	$E(X) = \frac{4}{3}$ or 1.33 and $Var(X) = \frac{16}{9}$ or 1.78	A1	Answers for E(X) and Var(X) must be identified
		3	**N.B.** If method FT for M marks from *their* incorrect **(b)**, expressions for E(X) and Var(X) must be seen unsimplified with all probabilities <1

Question	Answer	Marks	Guidance
5(a)	$[(0.7)^3 =]0.343$	B1	Evaluated WWW
	Alternative method for Question 5(a)		
	$[(0.15)^3 + {}^3C_1(0.15)^2(0.55) + {}^3C_2(0.15)(0.55)^2 + (0.55)^3 =]\ 0.343$	B1	Evaluated WWW
		1	
5(b)	$1 - (0.85^9 + {}^9C_1\,0.15^1\,0.85^8 + {}^9C_2\,0.15^2\,0.85^7)$ $[1 - (0.231617 + 0.367862 + 0.259667)]$	M1	One term: ${}^9C_x\,p^x\,(1-p)^{9-x}$ for $0 < x < 9$, any $0 < p < 1$
		A1	Correct expression, accept unsimplified.
	0.141	A1	$0.1408 \leqslant$ ans $\leqslant 0.141$, award at most accurate value.
	Alternative method for Question 5(b)		
	${}^9C_3\,0.15^3\,0.85^6 + {}^9C_4\,0.15^4\,0.85^5 + {}^9C_5\,0.15^5\,0.85^4 + {}^9C_6\,0.15^6\,0.85^3 +$ ${}^9C_7\,0.15^7\,0.85^2 + {}^9C_8\,0.15^8\,0.85 + 0.15^9$	M1	One term: ${}^9C_x\,p^x\,(1-p)^{9-x}$ for $0 < x < 9$, any $0 < p < 1$
		A1	Correct expression, accept unsimplified.
	0.141	A1	$0.1408 \leqslant$ ans $\leqslant 0.141$, award at most accurate value.
		3	

Question	Answer	Marks	Guidance
5(c)	Mean $=[60\times0.15=]9$ Variance $=[60\times0.15\times0.85=]7.65$	B1	Correct mean and variance, allow unsimplified. ($2.765 \le \sigma \le 2.77$ imply correct variance)
	$\left[(X\ge12)=\right]P\left(Z>\frac{11.5-9}{\sqrt{7.65}}\right)$	M1	Substituting *their* mean and variance into ±standardisation formula (any number for 11.5), not σ^2 or $\sqrt{\sigma}$
		M1	Using continuity correction 11.5 or 12.5 in *their* standardisation formula.
	$1-\Phi(0.9039)=1-0.8169$	M1	Appropriate area Φ, from final process, must be probability.
	0.183	A1	Final AWRT
		5	

Question	Answer	Marks	Guidance
6(a)	$\frac{8!}{2!3!}$	M1	$\frac{8!}{k!\times m!}$ $k=1$ or 2, $m=1$ or 3, not $k=m=1$ no additional terms
	3360	A1	
		2	

Question	Answer	Marks	Guidance
6(b)	**Method 1** Arrangements Rs at ends – Arrangements Rs at ends and Os together		
	[Os not together $=$] $\frac{6!}{3!}-4!$	M1	$\frac{6!}{k!}-m$, $1 \leqslant k \leqslant 3$, m an integer, condone $2\times\left(\frac{6!}{k!}\right)-m$.
		M1	$w-4!$ or $w-24$, w an integer Condone $w-2\times 4!$
	96	A1	
	Method 2 identified scenarios R _ _ _ R, Arrangement No Os together + 2Os and a single O		
	${}^4C_3\times 3!+{}^4C_2\times 2\times 3!$	M1	${}^4C_3\times 3!+r$ or $4\times 3!+r$ or ${}^4P_3\times 3!+r$, r an integer. Condone $2\times{}^4C_3\times 3!+r$. $2\times 4\times 3!+r$ or $2\times{}^4P_3\times 3!+r$.
		M1	$q+{}^4C_2\times 3!\times k$ or $q+{}^4P_2\times 3!\times k$, $k=1,2$, q an integer
	[24 + 72 =] 96	A1	
		3	
6(c)	**Method 1** Identified scenarios		
	OORR ${}^3C_2\times{}^2C_2\times\left[{}^3C_0\right]=3\times1=3$ ORR_ ${}^3C_1\times{}^2C_2\times{}^3C_1=3\times1\times3=9$ OOR_ ${}^3C_2\times{}^2C_1\times{}^3C_1=3\times2\times3=18$ OR_ _ ${}^3C_1\times{}^2C_1\times{}^3C_2=3\times2\times3=18$ OOOR ${}^3C_3\times{}^2C_1\times\left[{}^3C_0\right]=1\times2=2$	B1	Outcomes for 2 identifiable scenarios correct, accept unsimplified.
		M1	Add 4 or 5 identified correct scenarios only values, no additional incorrect scenarios, no repeated scenarios, accept unsimplified, condone use of permutations.
	Total 50	A1	All correct and added
	Probability $=\frac{50}{{}^8C_4}$	M1	$\frac{their\text{'}50\text{'}}{{}^8C_4}$, accept numerator unevaluated

Question	Answer	Marks	Guidance
6(c) cont'd	$\frac{50}{70}$ or 0.714	A1	
	Method 2 Identified outcomes		
	ORTM $^3C_1 \times {}^2C_1 = 6$ ORTW $^3C_1 \times {}^2C_1 = 6$ ORMW $^3C_1 \times {}^2C_1 = 6$ ORRM $^3C_1 \times {}^2C_2 = 3$ ORRW $^3C_1 \times {}^2C_2 = 3$ ORRT $^3C_1 \times {}^2C_2 = 3$ OROR $^3C_2 \times {}^2C_2 = 3$ OROT $^3C_2 \times {}^2C_1 = 6$ OROM $^3C_2 \times {}^2C_1 = 6$ OROW $^3C_2 \times {}^2C_1 = 6$ OROO $^3C_3 \times {}^2C_1 = 2$	B1 M1	Outcomes for 5 identifiable scenarios correct, accept unsimplified. Add 9, 10 or 11 identified correct scenarios only values, no additional incorrect scenarios, no repeated scenarios, accept unsimplified, condone use of permutations.
	Total 50	A1	All correct and added
	Probability = $\frac{50}{^8C_4}$	M1	$\frac{their\text{'50'}}{^8C_4}$, accept numerator unevaluated.
	$\frac{50}{70}$ or 0.714	A1	
		5	

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td rowspan="2">7(a)</td><td>Includes all data</td><td>B1</td><td>Reference to either including all/raw data
or further statistical processes are possible that cannot be found using data from box-and-whisker, eg frequency, mean, mode or standard deviation not only median, IQR, range or spread which can be found from both.</td></tr>
<tr><td></td><td>1</td><td></td></tr>
<tr><td rowspan="5">7(b)</td><td rowspan="4">
<table>
<tr><th colspan="3">Amazons</th><th></th><th colspan="5">Giants</th></tr>
<tr><td></td><td></td><td>8</td><td>17</td><td>5</td><td></td><td></td><td></td><td></td></tr>
<tr><td>4</td><td>2</td><td>1</td><td>18</td><td>2</td><td>4</td><td>7</td><td>9</td><td></td></tr>
<tr><td>8</td><td>6</td><td>0</td><td>19</td><td>2</td><td>3</td><td>5</td><td>5</td><td>5</td></tr>
<tr><td>5</td><td>2</td><td>1</td><td>20</td><td>4</td><td></td><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td>5</td><td>21</td><td></td><td></td><td></td><td></td><td></td></tr>
</table>
Key: 1|18|2 means 181 cm for Amazons and 182 cm for Giants</td><td>B1</td><td>Correct stem can be upside down, ignore extra values</td></tr>
<tr><td>B1</td><td>Correct Amazons labelled on left, leaves in order from right to left and lined up vertically (less than halfway to next column), no commas or other punctuation.</td></tr>
<tr><td>B1</td><td>Correct Giants labelled on same diagram, leaves in order and lined up vertically (less than halfway to next column), no commas or other punctuation.</td></tr>
<tr><td>B1</td><td>Correct single key for their diagram, need both teams identified and ‘cm’ stated at least once here or in leaf headings or title.

SC for if 2 separate diagrams drawn, award SCB1 if both keys meet these criteria (Max B1, B0, B0, B1)</td></tr>
<tr><td></td><td>4</td><td></td></tr>
<tr><td rowspan="3">7(c)</td><td rowspan="2">[UQ = 202 (cm), LQ = 182 (cm)]
[IQR =] 202 – 182 = 20 (cm)</td><td>M1</td><td>$201 \leqslant UQ \leqslant 205 - 181 \leqslant LQ \leqslant 184$</td></tr>
<tr><td>A1</td><td>WWW</td></tr>
<tr><td></td><td>2</td><td></td></tr>
</table>

Question	Answer	Marks	Guidance
7(d)	[$\Sigma_{11} = 2132$ $\Sigma_{15} = 191.2 \times 15 = 2868$]	B1	Both Σ_{11} and Σ_{15} found. Accept unevaluated.
	their 2868 = *their* 2132 + (180 + 185 + 190) + *h*	M1	Forming an equation for the height using *their* Σ_{11} and Σ_{15}.
	181 (cm)	A1	
	Alternative method for Question 7(d)		
	[$\Sigma_{15} = 191.2 \times 15 = 2868$ $\Sigma_{15} = 2687 + h$]	B1	Σ_{15} found using the mean and raw data methods. Accept unevaluated.
	their 2868 = *their* 2687 + *h*	M1	Forming an equation for the height using *their* Σ_{15} expressions.
	181 (cm)	A1	
	Alternative method for Question 7(d)		
	[$\Sigma_{15} = 2687 + h$ $\frac{\Sigma_{15}}{15} = 191.2$]	B1	Σ_{15} found using raw data method and statement on calculating new mean. Accept unevaluated.
	$\frac{\textit{their}\ 2687 + h}{15} = 191.2$	M1	Forming an equation for the height using *their* Σ_{15} expressions
	181 (cm)	A1	
		3	**N.B.** All methods can be presented as a logical numerical argument which can be condoned if clear.

Cambridge International AS & A Level

MATHEMATICS **9709/53**

Paper 5 Probability & Statistics 1 **May/June 2021**

MARK SCHEME

Maximum Mark: 50

Published

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Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
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- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
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GENERIC MARKING PRINCIPLE 5:
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GENERIC MARKING PRINCIPLE 6:
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3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
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A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
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- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
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SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1(a)	60	**B1**	Accept 60 or 61. No decimals
		1	
1(b)	65% of 160 = 104	**M1**	0.65 × 160 (=104) seen unsimplified or implied by use on graph
	136 (cm)	**A1**	Use of graph must be seen. **SCB1** correct value (136 only) if neither 104 nor use of graph are evident
		2	
1(c)	UQ: 150 LQ: 76 IQR = 150 – 76 = 74 [cm]	**M1**	UQ – LQ ; $148 \leqslant UQ \leqslant 152$; $74 \leqslant LQ \leqslant 78$.
		A1	Must be from 150 - 76
		2	

Question	Answer	Marks	Guidance
2	$p + p + 0.1 + q + q = 1$	**B1**	Sum of probabilities = 1
	$0.1 + 2q = 3(2p)$	**B1**	Use given information
	Attempt to solve two correct equations in p and q	**M1**	**Either** use of Substitution method to form a single equation in either p or q and finding values for both unknowns. **Or** use of Elimination method by writing both equations in same form (usually $ap + bq = c$) and + or – to find an equation in one unknown and finding values for both unknowns.
	$p = \frac{1}{8}$ or 0.125 and $q = \frac{13}{40}$ or 0.325	**A1**	CAO, both WWW
		4	

Question	Answer	Marks	Guidance
3(a)	Mean height $= \frac{\Sigma x + \Sigma y}{6+11} = \frac{1050+1991}{6+11} = \frac{3041}{17}$	**M1**	Use of appropriate formula with values substituted, accept unsimplified.
	178.9	**A1**	Allow 178.88, $178\frac{15}{17}$, 179
		2	

Question	Answer	Marks	Guidance
3(b)	$\frac{\Sigma x^2 + \Sigma y^2}{6+11} = \frac{193700+366400}{6+11}$	**M1**	Use of appropriate formula with values substituted, accept unsimplified.
	$\text{Sd}^2 = \frac{560100}{17} - \textit{their}\,178.88^2\,[=948.289]$	**M1**	Appropriate variance formula using *their* mean^2, accept unsimplified expression.
	Standard deviation = 30.8	**A1**	Accept 30.7
		3	

Question	Answer	Marks	Guidance
4(a)	[Possible cases: 1 1 2, 1 2 1, 2 1 1] Probability = $\left(\frac{1}{6}\right)^3 \times 3$	**M1**	$\left(\frac{1}{6}\right)^3 \times k$, where k is an integer.
		M1	Multiply a probability by 3, not +, – or ÷
	$\frac{1}{72}$	**A1**	Accept $\frac{3}{216}$ or $0.013\dot{8}$ or 0.0139
		3	
4(b)	$\text{P}(18) = \left(\frac{1}{6}\right)^3 \left[= \frac{1}{216}\right]$	**B1**	
	$\text{P(18 on 5th throw)} = \left(\frac{215}{216}\right)^4 \times \frac{1}{216}$	**M1**	$(1-p)^4 p$, $0 < \textit{their}\ p < 1$
	0.00454	**A1**	
		3	

Question	Answer	Marks	Guidance
5(a)	$z_1 = \dfrac{4-\mu}{\delta} = -1.378$	**B1**	$1.378 \leqslant z_1 \leq 1.379$ or $-1.379 \leqslant z_1 \leqslant -1.378$
	$z_2 = \dfrac{10-\mu}{\sigma} = 0.842$	**B1**	$0.841 \leqslant z_2 \leqslant 0.842$ or $-0.842 \leqslant z_2 \leqslant -0.841$
	Solve to find at least one unknown: $\dfrac{4-\mu}{\sigma} = -1.378$ $\dfrac{10-\mu}{\sigma} = 0.842$	**M1**	Use of ±standardisation formula once with μ, σ, a z-value and 4 or 10, allow continuity correction, not σ^2 or $\sqrt{\sigma}$
		M1	Use either the elimination method or the substitution method to solve two equations in μ and σ.
	$\sigma = 2.70$ $\mu = 7.72$	**A1**	$2.70 \leqslant \sigma \leqslant 2.71$ $7.72 \leqslant \mu \leqslant 7.73$
		5	
5(b)	$\Phi(2) - \Phi(-2) = 2\Phi(2) - 1$	**M1**	Identifying 2 and –2 as the appropriate z-values
	$2 \times their\ 0.9772 - 1$	**B1**	Calculating the appropriate area from stated phis of z-values which must be ± the same number
	0.9544 or 0.9545	**A1**	Accept AWRT 0.954
	$0.9544 \times 800 = 763.52$ 763 or 764	**B1 FT**	FT *their* 4SF (or better) probability, final answer must be positive integer
		4	

Question	Answer	Marks	Guidance
6(a)	$\dfrac{11!}{2!3!}$	**M1**	11! alone on numerator – must be a fraction. $k! \times m!$ on denominator, $k = 1, 2$, $m = 1, 3$, 1 can be implied but cannot both = 1. No additional terms
	3326400	**A1**	Exact value only
		2	
6(b)	8! = 40320	**B1**	Evaluate, exact value only
		1	
6(c)	$\dfrac{9!}{3!}\times 7$	**M1**	$\dfrac{9!}{3!}\times k$ seen, k an integer > 0, no +, – or ÷
		M1	7 × an integer seen in final answer, no +, – or ÷
	423360	**A1**	Exact value only
	Alternative method for Question 6(c)		
	${}^9C_3 \times 7!\,(\times\dfrac{3!}{3!})$	**M1**	$9C3\times k$ seen, k an integer > 0, no + or –
		M1	$7! \times k$ seen, , k an integer > 0, no + or –
	423360	**A1**	Exact value only but there must be evidence of $\times\dfrac{3!}{3!}$

Question	Answer	Marks	Guidance
6(c) cont'd	**Alternative method for Question 6(c)**		
	$3\times7\times\dfrac{8!}{2!}$	M1	$3\times\dfrac{8!}{2!}\times k$ seen, k an integer > 0, no + or –
		M1	7 × an integer seen in final answer, no +, – or ÷
	423360	A1	Exact value only
	Alternative method for Question 6(c)		
	$7\times\dfrac{2}{11}\times\dfrac{9}{10}\times\dfrac{8}{9}\times\dfrac{7}{8}\times\dfrac{1}{7}\times$total no. of arrangements	M1	Product of correct five fractions × k seen, k an integer > 0, no + or –
		M1	7×'total no of arrangements' ×k seen, k an integer > 0, no + or –
	423360	A1	Exact value only
	Alternative method for Question 6(c)		
	No E between the Rs – $\dfrac{^6C_3\times3!\times7!}{3!}=100800$ 1E between the Rs – $\dfrac{^6C_2\times3!\times7!}{2!}=226800$ 2Es between the Rs $-{}^6C_1\times3!\times7!=90720$ 3Es between the Rs – 7! = 5040	M1	Finding the correct number of ways for no, 1 or 2 Es between the Rs, accept unsimplified.
		M1	Adding the number of ways for 3 or 4 correct scenarios
	$\left[\text{Total}=7!\times(20+45+18+1)=7!\times84=\right]423360$	A1	CAO
		3	

Question	Answer	Marks	Guidance
6(d)	E E R _ _ $^6C_2 = 15$ E E R R _ $^6C_1 = 6$ E E E R _ $^6C_1 = 6$ E E E R R $^6C_0 = 1$	**M1**	Identifying four correct scenarios only.
		B1	Correct number of selections unsimplified for 2 or more scenario.
		M1	Adding the number of selections for 3 or 4 identified correct scenarios only, accept unsimplified. $^3C_x \times {}^2C_y \times {}^6C_z$, $x+y+z=5$ correctly identifies x Es and y Rs
	[Total =] 28	**A1**	WWW, only dependent upon 2nd M mark.
	Alternative method for Question 6(d) – Fixing EER first. No other scenarios can be present anywhere in solution.		
	E E R ^ ^ = $^8C_2$	**M1**	^8C_x seen alone or $^8C_x \times k$, , $k = 1$ or 2, $0<x<8$ Condone ^8P_x or $^8P_x \times k$, $k = 1$ or 2, $0<x<8$
		B1	$^8C_2 \times k$, $k = 1$ or 2 OE
		M1	$^8C_2 \times k$, $k = 1$ OE and no other terms
	[Total =] 28	**A1**	Value stated
		4	

Question	Answer	Marks	Guidance
7(a)(i)	$\frac{40}{800}$ or $\frac{1}{20}$ or 0.05	**B1**	
		1	
7(a)(ii)	$\frac{177}{223+177+40}$	**M1**	*Their* 223 + 177 + 40 seen as denominator of fraction in the final answer, accept unsimplified
	$\frac{177}{440}$ or 0.402	**A1**	CAO
	Alternative method for Question 7(a)(ii)		
	$P(G \mid S) = \frac{P(G \cap S)}{P(S)} = \frac{\frac{177}{800}}{\frac{223+177+40}{800}} = \frac{\frac{177}{800}}{\frac{440}{800}} = \frac{\frac{177}{800}}{\frac{11}{20} \text{ or } 0.55}$	**M1**	*Their* P(S) seen as denominator of fraction in the final answer, accept unsimplified
	$\frac{177}{440}$ or 0.402	**A1**	CAO
		2	
7(b)(i)	P(0, 1, 2) = ${}^{10}C_0 (0.35)^0 (0.65)^{10} + {}^{10}C_1 (0.35)^1 (0.65)^9 + {}^{10}C_2 (0.35)^2 (0.65)^8$	**M1**	One term: ${}^{10}C_x p^x (1-p)^{10-x}$ for $0 < x < 10$, any $0<p<1$
	0.013463 + 0.072492 + 0.17565	**A1**	Correct unsimplified expression, or better
	0.262	**A1**	
		3	

Question	Answer	Marks	Guidance
7(b)(ii)	Mean = $120 \times 0.35 [= 42]$ Variance = $120 \times 0.35 \times 0.65 [= 27.3]$	**B1**	Correct mean and variance seen, allow unsimplified
	$P(X > 32) = P(Z > \frac{32.5 - 42}{\sqrt{27.3}}) = P(Z > -1.818)$	**M1**	Substituting *their* mean and variance into ±standardisation formula (any number), condone σ^2 or $\sqrt{\sigma}$
		M1	Using continuity correction 31.5 or 32.5
	$\Phi(1.818)$	**M1**	Appropriate area Φ, from final process, must be probability
	0.966	**A1**	$0.965 \leqslant p \leqslant 0.966$
		5	

Cambridge Assessment International Education

Cambridge International AS & A Level

MATHEMATICS	**9709/52**
Paper 5 Probability and Statistics 1	**March 2021**
MARK SCHEME	
Maximum Mark: 50	

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WWW	Without Wrong Working
AWRT	Answer Which Rounds To

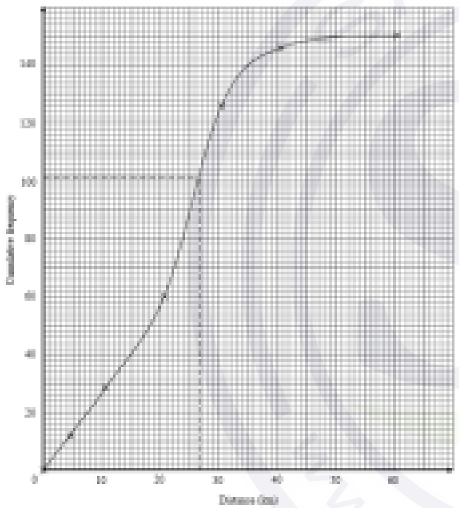
Question	Answer	Marks	Guidance
1(a)	$\left[\left(\frac{4}{5}\right)^7 \frac{1}{5} =\right]$ $\frac{16384}{390625}$ or 0·0419[43…]	**B1**	Evaluated, final answer.
		1	
1(b)	$1-\left(\frac{4}{5}\right)^5$ or $\frac{1}{5}+\frac{4}{5}\times\frac{1}{5}\left(\frac{4}{5}\right)^2\times\frac{1}{5}+\left(\frac{4}{5}\right)^3\times\frac{1}{5}+\left(\frac{4}{5}\right)^4\times\frac{1}{5}$	**M1**	$1 - p^n$ $n = 5,6$ or $p + pq + pq^2 + pq^3 + pq^4$ $(+ pq^5)$ $0 < p < 1$, $p + q = 1$, Sum of a geometric series may be used.
	$\frac{2101}{3125}$ or 0·672[32]	**A1**	Final answer.
	Alternative method for question 1(b)		
	[P(at least 1 three scored in 5 throws) =] $\left(\frac{1}{5}\right)^5 + {}^5C_4\left(\frac{1}{5}\right)^4\left(\frac{4}{5}\right) + {}^5C_3\left(\frac{1}{5}\right)^3\left(\frac{4}{5}\right)^2 + {}^5C_2\left(\frac{1}{5}\right)^2\left(\frac{4}{5}\right)^3 + {}^5C_4\left(\frac{1}{5}\right)\left(\frac{4}{5}\right)^4$	**M1**	$(p)^5 + {}^5C_4(p)^4(q) + {}^5C_3(p)^3(q)^2 + {}^5C_2(p)^2(q)^3 + {}^5C_1(p)(q)^4$ or $(p)^6 + {}^6C_5(p)^5(q) + {}^6C_4(p)^4(q)^2 + {}^6C_3(p)^3(q)^3$ $+{}^6C_2(p)^2(q)^4 + {}^6C_1(p)(q)^5$, $0 < p < 1, p + q = 1$ At least first, last and one intermediate term is required to show pattern of terms if not all terms stated.
	$\frac{2101}{3125}$ or 0·672[32]	**A1**	Final answer.
		2	

Question	Answer	Marks	Guidance
2(a)	$0.2[\times 1]+0.45\times 0.4+0.35\times 0.3$	**M1**	0·2 [× 1] + 0·45 × b + 0·35 × c, b = 0·4, 0·6 c = 0·3, 0·7
	0·485 or $\frac{97}{200}$	**A1**	
		2	
2(b)	$P(Y\mid\overline{H})=\frac{P(Y\cap\overline{H})}{P(\overline{H})}=\frac{0.35\times 0.7}{1-\textit{their}\,\mathbf{(a)}}=\frac{0.245}{0.515}$	**B1**	0·35 × 0·7 or 0·245 seen as numerator or denominator of fraction.
		M1	0·515 or 1 – *their* **(a)** or [0·3 × 0 +] 0·45 × d + 0·35 × e, where d = *their* b', e = *their* c' seen as denominator of fraction.
	0·476 or $\frac{49}{103}$	**A1**	$0{\cdot}4757 \leqslant p \leqslant 0{\cdot}476$
		3	

Question	Answer	Marks	Guidance
3(a)	$\mathrm{P}\left(\left(\frac{85-96}{18}\right)<z<\left(\frac{100-96}{18}\right)\right)$	**M1**	Use of ±standardisation formula once with appropriate values substituted, no continuity correction, not σ^2 or $\sqrt{\sigma}$.
	$\mathrm{P}(-0.6111<z<0.2222)$ $=\Phi(0.2222)+\Phi(0.6111)-1$ $=0.5879+0.7294-1$	**M1**	Appropriate area Φ, from final process, must be probability. Use of (1 – z) implies M0.
	0·317	**A1**	Final answer which rounds to 0·317.
		3	

Question	Answer	Marks	Guidance
3(b)	$z = \pm 1{\cdot}175$	**B1**	$1{\cdot}17 \leqslant z \leqslant 1{\cdot}18$ or $-1{\cdot}18 \leqslant z \leqslant -1{\cdot}17$
	$-1.175 = \dfrac{t-96}{18}$	**M1**	An equation using ±standardisation formula with a z-value, condone σ^2, $\sqrt{\sigma}$ or continuity correction. E.g. equating to 0·88, 0·12, 0·8106, 0·1894, 0·5478, 0·4522, ±0·175 or ±2·175 implies M0.
	74·85 or 74·9	**A1**	$74{\cdot}85 \leqslant t \leqslant 74{\cdot}9$
		3	

Question	Answer	Marks	Guidance
4(a)	$\begin{array}{\|c\|c\|c\|c\|c\|} \hline x & 1 & 2 & 3 & 4 \\ \hline \text{prob} & 4k & 6k & 6k & 4k \\ \hline \end{array}$	**B1**	Table with × values and one correct probability expressed in terms of *k*. Condone any additional × values if probability stated as 0.
		B1	Remaining 3 probabilities correct expressed in terms of *k* – condone if the first correct probability is not in table.
		2	
4(b)	$[4k+6k+6k+4k=1]\ k=\frac{1}{20}\quad(=0.05)$	**B1**	Correct value for *k* SOI. May be calculated in **4(a)**. **SC B1** If denominator 20*k* used throughout.
	$E(X)=1\times\frac{4}{20}+2\times\frac{6}{20}+3\times\frac{6}{20}+4\times\frac{4}{20}\ =\frac{4}{20}+\frac{12}{20}+\frac{18}{20}+\frac{16}{20}$ (= 2.5)	**M1**	Accept unsimplified expression. Condone 4*k* + 12*k* + 18*k* + 16*k*. May be implied by use in Variance expression. **Special ruling:** Allow use of denominator 20*k*.
	$\text{Var}(X)=1^2\times\frac{4}{20}+2^2\times\frac{6}{20}+3^2\times\frac{6}{20}+4^2\times\frac{4}{20}-\left(their\,2\frac{1}{2}\right)^2$ $=(4+24+54+64)\times their\,0.05-(their\,2.5)^2$ **Or** $(1-2.5)^2\times\frac{4}{20}+(2-2.5)^2\times\frac{6}{20}+(3-2.5)^2\times\frac{6}{20}+(4-2.5)^2\times\frac{4}{20}$	**M1**	Appropriate variance formula with *their* numerical probabilities using *their* $(E(X))^2$, accept unsimplified, with *their k* substituted. **Special ruling:** If denominator 20*k* used throughout, accept appropriate variance formula in terms of *k*.
	1·05	**A1**	AG, NFWW.
		4	

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td>5(a)</td><td><table><tr><td>Distance</td><td>0-4</td><td>5-10</td><td>11-20</td><td>21-30</td><td>31-40</td><td>41-60</td></tr><tr><td>Upper boundary</td><td>4·5</td><td>10·5</td><td>20·5</td><td>30·5</td><td>40·5</td><td>60·5</td></tr><tr><td>Cumulative frequency</td><td>12</td><td>28</td><td>60</td><td>126</td><td>146</td><td>150</td></tr></table></td><td>B1</td><td>Correct cumulative frequencies seen (may be by table or plotted accurately on graph), condone 12 not stated.</td></tr>
<tr><td></td><td rowspan="3"></td><td>B1</td><td>Axes labelled ‘distance (or d) [in] km’ from 0 to 60 and ‘cumulative frequency’ (or cf) from 0 to 150.</td></tr>
<tr><td></td><td>M1</td><td>At least 5 points plotted at upper end points for d (allow upper boundary ±0·5) with a linear scale for distance, condone 0 – 4 interval inaccurate, no scale break on axis.
Not bar graph/histogram unless clear indication of upper end point only of each bar.</td></tr>
<tr><td></td><td>A1</td><td>All plotted correctly at correct upper end points (4.5 etc.) with both scales linear ($0 \leqslant d \leqslant 60$, $0 \leqslant \text{cf} \leqslant 150$), curve drawn accurately joined to (0,0), cf line>150, no daylight if >150.</td></tr>
<tr><td></td><td></td><td>4</td><td></td></tr>
<tr><td>5(b)</td><td>70% of 150 = 105</td><td>M1</td><td>105 seen or implied by indication on grid.</td></tr>
<tr><td></td><td>Approx. 27</td><td>A1 FT</td><td>Strict FT their increasing cumulative frequency graph, use of graph must be seen.
If no clear evidence of use of graph:
SC B1 FT correct value from their increasing cumulative frequency graph.</td></tr>
<tr><td></td><td></td><td>2</td><td></td></tr>
</table>

Question	Answer	Marks	Guidance
5(c)	Midpoints: 2.25, 7.5, 15.5, 25.5, 35.5, 50.5	**B1**	At least 5 correct midpoints seen.
	$\text{Mean} = \frac{2.25\times12+7.5\times16+15.5\times32+25.5\times66+35.5\times20+50.5\times4}{150}$ $= \frac{27+120+496+1683+710+202}{150}$	**M1**	Using 6 midpoint attempts (e.g. 2·25 ±0·5), condone one error not omission, multiplied by frequency, accept unevaluated, denominator either correct or *their* Σ frequencies.
	$\left[=\frac{3238}{150}\right] = 21.6,\ 21\frac{44}{75}$	**A1**	Evaluated, WWW, accept 21·5[866…].
		3	

Question	Answer	Marks	Guidance
6(a)	$\frac{11!}{2!2!2!}$	**M1**	11! alone as numerator. 2! × *m*! × *n*! on denominator, *m* = 1, 2, *n* = 1, 2. no additional terms, no additional operations.
	4989600	**A1**	Exact answer only.
		2	

Question	Answer	Marks	Guidance
6(b)	**Method 1** R ^ ^ ^ ^ ^ ^ ^ R		
	Arrange the 7 letters CTEPILL = $\frac{7!}{2!}$	**B1**	$\frac{7!}{2!}\times k$ seen, k an integer > 1.
	Number of ways of placing As in non-adjacent places = ${}^{8}C_{2}$	**M1**	$m\times n(n-1)$ or $m\times {}^{n}C_{2}$ or $m\times {}^{n}P_{2}$, n = 7, 8 or 9, m an integer > 1.
	$\frac{7!}{2!}\times {}^{8}C_{2}$	**M1**	$\frac{7!}{p!}\times {}^{8}C_{2}$ or $\frac{7!}{p!}\times {}^{8}P_{2}$, p integer $\geqslant$ 1, condone 2520×28.
	= 70560	**A1**	Exact answer only. **SC B1** 70560 from M0, M1 only.
	Method 2 [Arrangements Rs at ends – Arrangements Rs at ends and As together]		
	Total arrangements with R at beg. and end $=\frac{9!}{2!2!}$	**M1**	$\frac{9!}{2!m!}-k$, 90720 > k integer > 1, m = 1, 2.
	Arrangements with R at ends and As together $=\frac{8!}{2!}$	**B1**	$s-\frac{8!}{2!}$, s an integer >1
	With As not together $=\frac{9!}{2!2!}-\frac{8!}{2!}$	**M1**	$\frac{9!}{p}-\frac{8!}{q}$, p, q integers $\geqslant$ 1, condone 90720 – 20160.
	[90720 – 20160] = 70560	**A1**	Exact answer only. **SC B1** 70560 from M0, M1 only.
		4	

Question	Answer	Marks	Guidance
6(c)	**Method 1**		
	R R A L _ _ $^5C_2$ = 10 R R A L L _ $^5C_1$ = 5 R R A A L _ $^5C_1$ = 5 R R A A L L = 1	**M1**	^5C_x seen alone or $^5C_x \times k$, $2 \geqslant k \geqslant 1$, k an integer, $0 < x < 5$ linked to an appropriate scenario.
		A1	$^5C_2 \times k$, $k = 1$ oe or $^5C_1 \times m$, $m = 1,2$ oe alone. **SC** if ^5C_x not seen. **B2** for 5 or 10 linked to the appropriate scenario WWW.
		M1	Add outcomes from 3 or 4 identified correct scenarios only, accept unsimplified. $^2C_w \times {}^2C_x \times {}^2C_y \times {}^5C_z$, $w+x+y+z=6$ identifies w Rs, × As and y Ls.
	[Total =] 21	**A1**	WWW, only dependent on 2nd M mark. Note: $^5C_2 + {}^5C_1 + {}^5C_1 + 1 = 21$ is sufficient for 4/4.
			SC not all (or no) scenarios identified. **B1** 10 + 5 + 5 + 1 **DB1** = 21
	Method 2 – Fixing RRAL first. N.B. No other scenarios can be present anywhere in solution.		
	R R A L ^ ^ = $^7C_2$	**M1**	^7C_x seen alone or $^7C_x \times k$, $2 \geqslant k \geqslant 1$, k an integer, $0<x<7$. Condone ^7P_x or $^7P_x \times k$, $2 \geqslant k \geqslant 1$, k an integer, $0<x<7$.
		M1	$^7C_2 \times k$, $2 \geqslant k \geqslant 1$oe
		A1	$^7C_2 \times k$, $k = 1$oe no other terms.
	[Total =] 21	**A1**	Value stated.
		4	

Question	Answer	Marks	Guidance
7(a)(i)	$\left[\dfrac{104+31}{400}=\right]\dfrac{135}{400}, \dfrac{27}{80}, 0\cdot3375$	**B1**	Evaluated, exact value.
		1	
7(a)(ii)	**Method 1**		
	$P(M)=\dfrac{180}{400}, 0\cdot45$ $P(S)=\dfrac{135}{400}, 0\cdot3375$ $P(M\cap S)=\dfrac{31}{400}, 0\cdot0775$ $\dfrac{180}{400}\times\dfrac{135}{400}=\dfrac{243}{1600}, 0.151875\neq\dfrac{31}{400}$ so NOT independent	**M1**	*Their* P(*M*) × *their* P(*S*) seen, accept unsimplified.
		A1	P(*M*), P(*S*) and P(*M* ∩ *S*) notation seen, numerical comparison and correct conclusion, WWW.
	Method 2		
	$P(M\cap S)=\dfrac{31}{400}$ $P(S)=\dfrac{135}{400}$ $P(M)=\dfrac{180}{400}$ $P(M\mid S)=\dfrac{\frac{31}{400}}{\frac{135}{400}}=\dfrac{31}{135}, 0.2296\ldots\neq\dfrac{180}{400}$ so NOT independent	**M1**	$[P(M\mid S)=]\dfrac{\textit{their}\ P(M\cap S)}{\textit{their}\ P(S)}$ (oe) seen, accept unsimplified.
		A1	P(*M*), P(*S*) and P(*M* ∩ *S*) notation seen, numerical comparison and correct conclusion, WWW.
		2	

Question	Answer	Marks	Guidance
7(b)(i)	**Method 1** [1 – P(0,1,2)]		
	$= 1 - ({}^{10}C_0\, 0{\cdot}3^0\, 0{\cdot}7^{10} + {}^{10}C_1\, 0{\cdot}3^1\, 0{\cdot}7^9$ $+ {}^{10}C_2\, 0{\cdot}3^2\, 0{\cdot}7^8)$	**M1**	${}^{10}C_x\, p^x\, (1-p)^{10-x}$ for $0 < x < 10$, $0 < p < 1$, any p.
	$= 1 - (0{\cdot}028248 + 0{\cdot}121061 + 0{\cdot}233474)$	**A1**	Correct expression, accept unsimplified, condone omission of final bracket, condone recovery from poor notation.
	$= 0{\cdot}617$	**A1**	Accept $0{\cdot}61715 \leqslant p \leqslant 0{\cdot}61722$, WWW.
	Method 2 [P(3,4,5,6,7,8,9,10) =]		
	${}^{10}C_3\, 0{\cdot}3^3\, 0{\cdot}7^7 + {}^{10}C_4\, 0{\cdot}3^4\, 0{\cdot}7^6 + {}^{10}C_5\, 0{\cdot}3^5\, 0{\cdot}7^5$ $+ {}^{10}C_6\, 0{\cdot}3^6\, 0{\cdot}7^4 + {}^{10}C_7\, 0{\cdot}3^7\, 0{\cdot}7^3 + {}^{10}C_8\, 0{\cdot}3^8\, 0{\cdot}7^2$ $+ {}^{10}C_9\, 0{\cdot}3^9\, 0{\cdot}7^1 + {}^{10}C_{10}\, 0{\cdot}3^{10}\, 0{\cdot}7^0$	**M1**	${}^{10}C_x\, p^x\, (1-p)^{10-x}$ for $0 < x < 10$, $0 < p < 1$, any p.
		A1	Correct unsimplified expression.
	$= 0{\cdot}617$	**A1**	Accept $0{\cdot}61715 \leqslant p \leqslant 0{\cdot}61722$, WWW.
		3	

Question	Answer	Marks	Guidance
7(b)(ii)	[p = 0·3] Mean = 0·3 × 90 = 27; variance = 0·3 × 90 × 0·7 = 18·9	**B1**	Correct mean and variance, allow unsimplified. Condone $\sigma = 4{\cdot}347$ evaluated.
	$P(X < 32) = P\left(z < \frac{31.5 - 27}{\sqrt{18.9}}\right)$	**M1**	Substituting *their* μ and σ (not σ^2, $\sqrt{\sigma}$) into the ±standardising formula with a numerical value for '31·5'.
		M1	Using either 31·5 or 32·5 within a ±standardising formula with numerical values for *their* μ and σ (condone σ^2, $\sqrt{\sigma}$).
	$= \Phi(1.035)$	**M1**	Appropriate area Φ, from standardisation formula P(z<…) in final solution, must be probability.
	= 0·850	**A1**	Allow $0{\cdot}8495 < p \leqslant 0{\cdot}85(0)$, final answer WWW.
		5	

Cambridge International A Level

MATHEMATICS **9709/51**

Paper 5 Probability & Statistics 1 **October/November 2020**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2020 series for most Cambridge IGCSE™, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

This document consists of **16** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td rowspan="3">1(a)</td><td>
<table>
<tr><td></td><td></td><td colspan="6">Red</td></tr>
<tr><td></td><td></td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr>
<tr><td rowspan="6">Blue</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td></tr>
<tr><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td></tr>
<tr><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td></tr>
<tr><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td></tr>
<tr><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td><td>11</td></tr>
<tr><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td><td>11</td><td>12</td></tr>
</table>
</td><td>M1</td><td>Complete outcome space or
or listing A and B outcomes
or listing A∩B outcomes</td></tr>
<tr><td>P(A∩B) = $\frac{5}{36}$</td><td>A1</td><td>With evidence</td></tr>
<tr><td></td><td>2</td><td></td></tr>
</table>

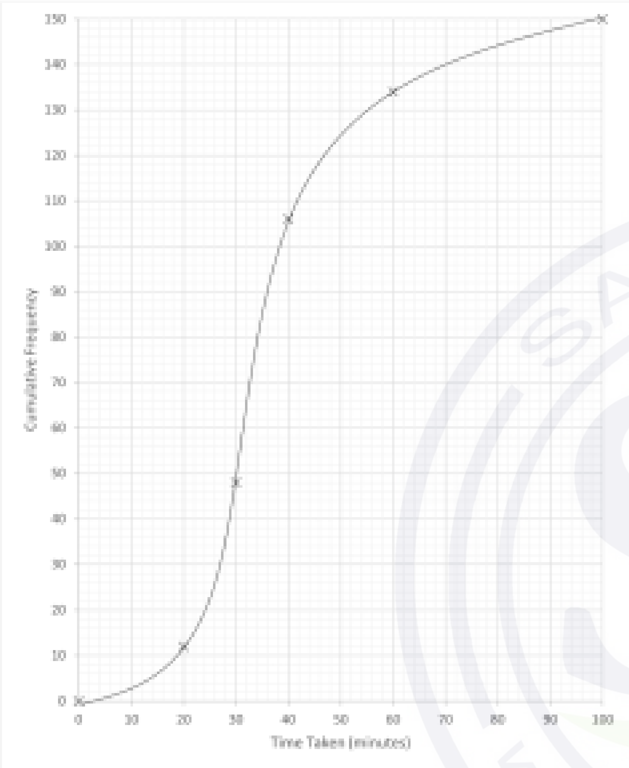
Question	Answer	Marks	Guidance
1(b)	$\text{P(A)} \times \text{P(B)} = \frac{1}{3} \times \frac{10}{36}$	M1	$\textit{Their}\,\frac{1}{3} \times \textit{their}\,\frac{10}{36}$ seen
	$\frac{5}{54} \neq \frac{5}{36}$ so not independent	A1	$\frac{5}{54}, \frac{5}{36}$, P(A) × P(B) and P(A∩*B*) seen in workings and correct conclusion stated Condone $\frac{5}{36}$ being stated in **(a)**
	Alternative method for question 1(b)		
	P(B|A) = P(B) $\text{P(B\|A)} = \frac{\text{P}(\text{A} \cap \text{B})}{\text{P}(\text{A})} = \frac{\frac{5}{36}}{\frac{1}{3}}$	M1	OE, $\frac{\textit{their}\,1(\text{a})}{\textit{their}\,\text{P}(\text{A})}$ seen
	$\frac{5}{12} \neq \frac{5}{18}$ so not independent	A1	P(A|B), P(B), $\frac{5}{12}, \frac{5}{18}$ seen in workings and correct conclusion stated Condone $\frac{5}{18} \equiv \frac{10}{36}$ being identified in **(a)**
		2	

Question	Answer	Marks	Guidance
2(a)	$0{\cdot}6 \times 0{\cdot}7 + 0{\cdot}4(1 - x) = 0{\cdot}58$ $\equiv 0{\cdot}42 + 0{\cdot}4(1 - x) = 0{\cdot}58$	**M1**	Equation of form $0{\cdot}6 \times a + 0{\cdot}4 \times \text{b} = 0{\cdot}58$; $a = 0{\cdot}3, 0{\cdot}7$, $\text{b} = x, (1 - x)$
		B1	Single correct product seen, condone 0·42, in an equation of appropriate form
	$x = 0{\cdot}6$	**A1**	
	Alternative method for question 2(a)		
	$0{\cdot}6 \times 0{\cdot}3 + 0{\cdot}4x = 0{\cdot}42$ $\equiv 0{\cdot}18 + 0{\cdot}4x = 0{\cdot}42$	**M1**	Equation of form $0{\cdot}6$ x $a + 0{\cdot}4$ x $\text{b} = 0{\cdot}42$; $a = 0{\cdot}3, 0{\cdot}7$, $\text{b} = x, (1 - x)$
		B1	Single correct product seen, condone 0·18, in an equation of appropriate form
	$x = 0{\cdot}6$	**A1**	
		3	
2(b)	$(0.6\times0.3)^2$	**M1**	$(a \times b)^2$, $a = 0{\cdot}6, 0{\cdot}4$ and $b = 0{\cdot}7, 0{\cdot}3, x, (1–x)$ or $0{\cdot}18^2$, alone.
	0.0324	**A1**	
		2	
3(a)	$\text{P}(X > 6) = 0.75^6$	**M1**	p^n, $n = 6, 7$ $0 < p < 1$
	$0.178, \dfrac{729}{4096}$	**A1**	0·17797…
		2	

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td rowspan="4">3(b)</td><td>$1 - P(0, 1, 2) = 1 - (0.75^{10} + {}^{10}C_1\ 0.25^1\, 0.75^9 + {}^{10}C_2\, 0.25^2\, 0.75^8)$</td><td>M1</td><td>Binomial term of form ${}^{10}C_x\ p^x(1-p)^{10-x}$, $0 < p < 1$, any p, $x \neq 0, 10$</td></tr>
<tr><td>1 – (0·0563135 + 0·1877117 + 0·2815676)</td><td>A1</td><td>Correct unsimplified expression</td></tr>
<tr><td>0·474</td><td>A1</td><td>$0.474 \leqslant p \leqslant 0.4744$</td></tr>
<tr><td></td><td>3</td><td></td></tr>
<tr><td rowspan="5">4(a)</td><td rowspan="4"><table><tr><td>y</td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>prob</td><td>$\frac{7}{16}$</td><td>$\frac{5}{16}$</td><td>$\frac{3}{16}$</td><td>$\frac{1}{16}$</td></tr></table></td><td>B1</td><td><table><tr><td></td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>1</td><td>1</td><td>1</td><td>2</td><td>3</td></tr><tr><td>2</td><td>1</td><td>2</td><td>1</td><td>2</td></tr><tr><td>3</td><td>2</td><td>1</td><td>3</td><td>1</td></tr><tr><td>4</td><td>3</td><td>2</td><td>1</td><td>4</td></tr></table>Probability distribution table with correct scores with at least one probability, allow extra score values if probability of zero stated’</td></tr>
<tr><td>B1</td><td>One probability (linked with correct score) correct</td></tr>
<tr><td>B1</td><td>2 more probs (linked with correct scores) correct</td></tr>
<tr><td>B1 FT</td><td>4th prob correct, FT sum of 3 or 4 terms = 1</td></tr>
<tr><td></td><td>4</td><td></td></tr>
</table>

Question	Answer	Marks	Guidance
4(b)	P(2|even) = $\dfrac{\frac{5}{16}}{\frac{6}{16}}$	**M1**	$\dfrac{their\,\mathrm{P}(2)}{their\ \mathrm{P}(2)+their\,\mathrm{P}(4)}$ seen or correct outcome space.
	$\frac{5}{6}$ or 0·833	**A1**	
		2	
5(a)	$\mathrm{P}(X > 4.2) = \mathrm{P}(z > \dfrac{4.2-3.5}{0.9})$ $= \mathrm{P}(z > 0.7778)$	**M1**	Using ± standardisation formula, no $\sqrt{\sigma}$ or σ^2, continuity correction
	1 – 0·7818	**M1**	Appropriate area Φ, from standardisation formula P(z>…) in final solution
	0·218	**A1**	
		3	
5(b)	$z = -1.282$	**B1**	±1.282 seen (critical value)
	$\dfrac{t-3.5}{0.9} = -1.282$	**M1**	An equation using ±standardisation formula with a z-value, condone $\sqrt{\sigma}$, σ^2 and continuity correction
	$t = 2.35$	**A1**	AWRT, only dependent on M mark
		3	

Question	Answer	Marks	Guidance
5(c)	$P(2.8 < X < 4.2) = 1 - 2 \times$ *their* **5(a)** $\equiv 2(1 -$ *their* **5(a)**$) - 1$ $\equiv 2(0{\cdot}5 -$ *their* **5(a)**$)$ $= 0{\cdot}5636$	**B1 FT**	FT from *their* **5(a)** < 0.5 or correct Accept unevaluated probability OE Accept 0·564
	Number of days = $365 \times 0{\cdot}5636 = 205{\cdot}7$	**M1**	$365 \times$ *their* p
	So, 205 (days)	**A1 FT**	Accept 205 or 206, not 205·0 or 206·0 no approximation/ rounding stated FT must be an integer value
	Alternative method for question 5(c)		
	$P\left(\frac{2.8-3.5}{0.9} < z < \frac{4.2-3.5}{0.9}\right)$ $= \Phi(0.7778) - (1 - \Phi 0.7778)$ $= 0{\cdot}7818 - (1 - 0{\cdot}7818)$ $= 0{\cdot}5636$	**B1**	$0{\cdot}5635 < p \leqslant 0{\cdot}564$ OE
	Number of days = $365 \times 0{\cdot}5636 = 205{\cdot}7$	**M1**	$365 \times$ *their* p
	So, 205 (days)	**A1 FT**	Accept 205 or 206, not 205·0 or 206·0 no approximation/ rounding stated FT must be an integer value
		3	

Question	Answer	Marks	Guidance
6(a)		M1	At least 4 points plotted at upper end points, with both scales linear with at least 3 values indicated
	Correct cumulative frequency curve	A1	All plotted correctly with curve drawn joined to (0, 0), axes labelled cumulative frequency, time, minutes
		2	
6(b)	$150 \times 0{\cdot}76 = 114$	M1	114 SOI, may be on graph
	$k = 45$ (mins)	A1 FT	Clear indication that *their* graph has been used, tolerance ±1mm
		2	

Question	Answer	Marks	Guidance
6(c)	Frequencies: 12 36 58 28 16	**B1**	Correct frequencies seen
	Mean = $\dfrac{10\times12+25\times36+35\times58+50\times28+80\times16}{150}$	**B1**	At least 4 correct midpoints seen and used
	$\dfrac{120+900+2030+1400+1280}{150}$	**M1**	Correct formula with *their* midpoints (not upper boundary, lower boundary, class width or frequency density).
	38.2, $38\dfrac{1}{5}$	**A1**	
	Variance = $\dfrac{12\times10^2+36\times25^2+58\times35^2+28\times50^2+16\times80^2}{150}-mean^2$ $=\dfrac{1200+22500+71050+70000+102400}{150}-mean^2$	**M1**	Substitute *their* midpoints and frequencies (condone use of cumulative frequency) in correct variance formula, must have '– *their* mean^2'
	(Standard deviation = $\sqrt{321.76}$) = 17.9	**A1**	
		6	
7(a)	$\dfrac{8!}{2!}$	**M1**	$\dfrac{8!}{k}\equiv\dfrac{7!\times8}{k}$,where $k\in\mathbb{N}$, $\dfrac{a!}{2(!)}$, where $a\in\mathbb{N}$
	20160	**A1**	
		2	

Question	Answer	Marks	Guidance
7(b)	Total number of ways: $\frac{10!}{2!3!}$ $(= 302\,400)$ (A)	**B1**	Accept unsimplified
	With Ps together: $\frac{9!}{3!}$ (= 60 480) (B)	**B1**	Accept unsimplified
	With Ps not together: 302 400 – 60 480	**M1**	$\frac{10!}{m} - \frac{9!}{n}$, *m, n* integers or (A) – (B) if clearly identified
	241 920	**A1**	
	Alternative method for question 7(b)		
	$\frac{8!}{3!}$	**B1**	$k \times 8!$ in numerator, k a positive integer, no ±
		B1	$m \times 3!$ in denominator, m a positive integer, no ±
	$\times \frac{9 \times 8}{2}$	**M1**	*Their* $\frac{8!}{3!}$ multiplied by ${}^9C_2$ or ${}^9P_2$ no additional terms
	241 920	**A1**	Exact value, WWW
		4	

Question	Answer	Marks	Guidance
7(c)	Probability = $\dfrac{\text{Number of ways Es at beginning and end}}{\text{Total number of ways}}$ Probability = $\dfrac{\frac{8!}{2!}}{\frac{10!}{2!\times 3!}} = \dfrac{20160}{302400}$	**M1**	$\dfrac{\left(\frac{8!}{k!}\right)}{\frac{10!}{k!l!}}$ $1 \leqslant k, l \in \mathbb{N} \leqslant 3$, FT denominator from **7(b)** or correct
	$\dfrac{1}{15}$, 0·0667	**A1**	
	Alternative method for question 7(c)		
	Probability = $\dfrac{3}{10}\times\dfrac{2}{9}$	**M1**	$\dfrac{a}{10}\times\dfrac{a-1}{9}$ $a = 3, 2$
	$\dfrac{1}{15}$, 0·0667	**A1**	
	Alternative method for question 7(c)		
	Probability = $\dfrac{1}{10}\times\dfrac{1}{9}\times 3!$	**M1**	$\dfrac{1}{10}\times\dfrac{1}{9}\times m!, m = 3, 2$
	$\dfrac{1}{15}$, 0.0667	**A1**	
		2	

Question	Answer	Marks	Guidance
7(d)	Scenarios: P E E E $\quad {}^5C_0 = 1$ P E E _ $\quad {}^5C_1 = 5$ P E _ _ $\quad {}^5C_2 = 10$ P _ _ _ $\quad {}^5C_3 = 10$	**M1**	${}^5C_x$ seen alone, $1 \leqslant x \leqslant 4$
		M1	Summing the number of ways for 3 or 4 correct scenarios (can be unsimplified), no incorrect scenarios
	Total = 26	**A1**	
		3	

Cambridge Assessment International Education

Cambridge International A Level

MATHEMATICS **9709/52**

Paper 5 Probability & Statistics 1 **October/November 2020**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2020 series for most Cambridge IGCSE™, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

This document consists of **14** printed pages.

[Turn over

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:
- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:
- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate’s intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

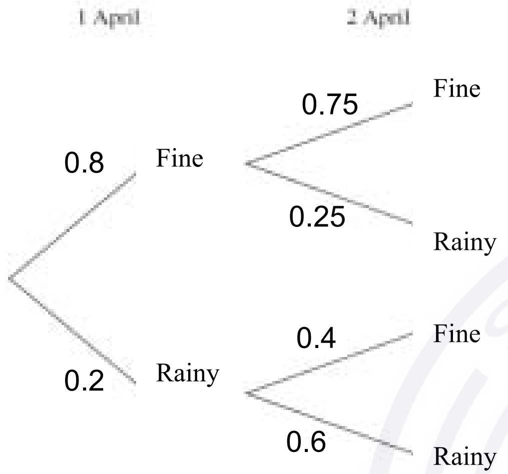
Question	Answer	Marks	Guidance
1(a)	$1-\left(\frac{5}{6}\right)^5$ or $\frac{1}{6}+\frac{5}{6}\times\frac{1}{6}+\left(\frac{5}{6}\right)^2\times\frac{1}{6}+\left(\frac{5}{6}\right)^3\times\frac{1}{6}+\left(\frac{5}{6}\right)^4\times\frac{1}{6}$	**M1**	$1-p^n$ $n = 5,6$ or $p + pq + pq^2+pq^3+ pq^4$ $(+ pq^5)$ $0 < p < 1, p + q = 1,$
	$0{\cdot}598$, $\frac{4651}{7776}$	**A1**	
		2	
1(b)	$(1 - P(0, 1, 2))$ $1-\left(\left(\frac{5}{6}\right)^{10}+{}^{10}C_1\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)^9+{}^{10}C_2\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^8\right)$	**M1**	${}^{10}C_x\ p^x(1-p)^{10-x}$, $0 < p < 1$, any p, $x \neq 0,10$
	$1 - (0{\cdot}1615056 + 0{\cdot}3230111 + 0{\cdot}290710)$	**A1**	Correct expression, accept unsimplified, condone omission of final bracket
	$0{\cdot}225$	**A1**	$0{\cdot}2247 < p \leq 0{\cdot}225$, WWW
		3	

Question	Answer	Marks	Guidance
2(a)	P(1 red) = $\frac{5}{8}\times\frac{3}{7}\times\frac{2}{6}\times 3$	M1	$\frac{a}{8}\times\frac{b}{7}\times\frac{c}{6}\times k$ or $\frac{5}{d}\times\frac{3}{e}\times\frac{2}{f}\times 3$, $1\leqslant a,b,c \leqslant 5$, $d, e, f \leqslant 8$, a, b, c, d, e, f, k all integers. $1 < k \leqslant 3$,
	$\frac{15}{56}$	A1	AG, WWW
	Alternative method for question 2(a)		
	$\frac{{}^5C_1\times{}^3C_2}{{}^8C_3}$	M1	$\frac{{}^aC_1\times{}^bC_2}{{}^8C_3}$ or $\frac{{}^5C_d\times{}^3C_e}{{}^8C_3}$ or $\frac{{}^5C_d\times{}^3C_e\left(or\ {}^aC_1\times{}^bC_2\right)}{{}^5C_3\times{}^3C_0+{}^5C_2\times{}^3C_1+{}^5C_1\times{}^3C_2+{}^5C_0\times{}^3C_3}$, $a + b = 8$, $d + e = 3$
	$\frac{15}{56}$	A1	AG, WWW, $\frac{15}{56}$ must be seen
		2	
2(b)	<table><tr><td>x</td><td>0</td><td>1</td><td>2</td><td>3</td></tr><tr><td>Prob.</td><td>$\frac{1}{56}$</td><td>$\frac{15}{56}$</td><td>$\frac{30}{56}=\frac{15}{28}$</td><td>$\frac{10}{56}=\frac{5}{28}$</td></tr><tr><td></td><td>0·0179</td><td>0·268</td><td>0·536</td><td>0·179</td></tr></table>	B1	Probability distribution table with correct outcomes with at least one probability less than 1, allow extra outcome values if probability of zero stated.
		B1	2 of P(0), P(2) and P(3) correct
		B1 FT	4th probability correct or FT sum of 3 or more probabilities = 1, with P(1) correct
		3	

Question	Answer	Marks	Guidance
2(c)	$\text{Var}(X) = \dfrac{(0^2 \times 1) + 1^2 \times 15 + 2^2 \times 30 + 3^2 \times 10}{56} - \left(\dfrac{15}{8}\right)^2$ $= \dfrac{15}{56} + \dfrac{120}{56} + \dfrac{90}{56} - \left(\dfrac{15}{8}\right)^2$	M1	Substitute *their* attempts at scores in correct variance formula, must have ‘ – mean^2’ (FT if mean calculated) (condone probabilities not summing to 1 for this mark)
	$\dfrac{225}{448}$, 0·502	A1	
		2	

Question	Answer	Marks	Guidance
3(a)	$P(X > 11.3) = P(z > \dfrac{11.3 - 10.1}{1.3}) = P(z > 0 \cdot 9231)$	M1	Using ± standardisation formula, no $\sqrt{\sigma}$ or σ^2, continuity correction
	1 – 0.822	M1	Appropriate area Φ, from standardisation formula P(z>…) in final solution
	0·178	A1	0.1779…
		3	
3(b)	$z = -0 \cdot 674$	B1	±0.674 seen (critical value)
	$\dfrac{t - 10.1}{1.3} = -0 \cdot 674$	M1	An equation using ±standardisation formula with a z-value, condone $\sqrt{\sigma}$ or σ^2, continuity correction.
	$t = 9 \cdot 22$	A1	AWRT. Only dependent on M1
		3	

Question	Answer	Marks	Guidance
3(c)	$P(8.9 < X < 11.3) = 1 - 2 \times$ *their* **3(a)** $\equiv 2(1 -$ *their* **3(a)**$) - 1$ $\equiv 2(0{\cdot}5 -$ *their* **3(a)**$)$ $=0.644$	**B1 FT**	FT from *their* **3(a)** $< 0{\cdot}5$ or correct, accept unevaluated probability OE
	Number of days $= 90 \times 0{\cdot}644$ $= 57{\cdot}96$	**M1**	$90 \times$ *their* p seen, $0 < p < 1$
	So 57 (days)	**A1 FT**	Accept 57 or 58, not 57·0 or 58·0, no approximation/rounding stated FT must be an integer value
	Alternative method for question 3(c)		
	$P\left(\frac{8\cdot9-10\cdot1}{1\cdot3} < z < \frac{11.3-10.1}{1\cdot3}\right)$ $= \Phi(0\cdot9231) - (1-\Phi(0\cdot9231))$ oe $= 0{\cdot}822 - (1 - 0{\cdot}822)$ $= 0{\cdot}644$	**B1**	Accept unevaluated probability
	Number of days $= 90 \times 0{\cdot}644$ $= 57{\cdot}96$	**M1**	$90 \times$ *their* p seen, $0 < p < 1$
	So 57 (days)	**A1 FT**	Accept 57 or 58, not 57·0 or 58·0, no approximation/rounding stated FT must be an integer value
		3	

Question	Answer	Marks	Guidance
4(a)		**B1**	All probabilities correct, may be on branch or next to 'Fine/Rainy' Ignore additional branches.
		1	
4(b)	$0{\cdot}8 \times 0{\cdot}75 + 0{\cdot}2 \times 0{\cdot}4 \quad (= 0{\cdot}6 + 0{\cdot}08)$	**M1**	Correct or FT from *their* diagram unsimplified, all probabilities $0 < p < 1$. Partial evaluation only sufficient when correct. Accept working in **4(b)** or by the tree diagram.
	$0.68, \dfrac{17}{25}$	**A1**	From supporting working
		2	

Question	Answer	Marks	Guidance
4(c)	$0{\cdot}8 \times 0{\cdot}75 \times 0{\cdot}25 + 0{\cdot}8 \times 0{\cdot}25 \times 0{\cdot}6$	**M1**	$a \times b \times c + a \times 1\text{-}b \times d$, $0 < c, d \leqslant 1$, *a, b* consistent with *their* tree diagram or correct, no additional terms
	$0{\cdot}15 + 0{\cdot}12$	**A1**	At least one term correct, accept unsimplified
	0.27	**A1**	Final answer
		3	
4(d)	P(Y) = *their* **(c)** + $0{\cdot}2 \times 0{\cdot}4 \times 0{\cdot}25 + 0{\cdot}2 \times 0{\cdot}6 \times 0{\cdot}6$ (= 0·362)	**B1 FT**	*their* **(c)** + $e \times f \times g + e \times (1\text{-}f) \times h$, $0 < g, h \leqslant 1$, *e, f* consistent with *their* tree diagram, or correct
	$\text{P(X\|Y)} = \dfrac{\textit{their}\,(\text{c})}{\textit{their}\,\text{P}(\text{Y})} = \dfrac{0.27}{0.362}$	**M1**	*their* **4(c)** (or correct)/*their* previously calculated and identified P(Y) or a denominator involving 3 or 4 3-factor probability terms consistent with *their* tree diagram & third factor $0 < p < 1$
	$0{\cdot}746$, $\dfrac{373}{500}$ or $\dfrac{135}{181}$	**A1**	(0.7458…)
		3	

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td rowspan="5">5(a)</td><td rowspan="4"><table><tr><th>Dados</th><th></th><th>Linva</th></tr><tr><td>8 6</td><td>0</td><td>0 2 9</td></tr><tr><td>6 5 2 0 0</td><td>1</td><td>0 1 2 5 6</td></tr><tr><td>8 2</td><td>2</td><td></td></tr><tr><td>6</td><td>3</td><td>2 6</td></tr><tr><td>2</td><td>4</td><td>0</td></tr></table>KEY 6| 3| 2 means 36 cm (snow) in Dados
and 32 cm (snow) in Linva</td><td>B1</td><td>Correct stem can be upside down, ignore extra values</td></tr>
<tr><td>B1</td><td>Correct Dados labelled, leaves in order and lined up vertically (less than midway to next column), no commas etc, no extra terms</td></tr>
<tr><td>B1</td><td>Correct Linva on opposite side of stem labelled, leaves in order and lined up vertically (less than midway to next column), no commas etc, no extra terms</td></tr>
<tr><td>B1</td><td>Correct single key for their diagram, need both resorts identified and ‘cm’ stated at least once here or in leaf headings or title.

SC If 2 separate diagrams drawn, SCB1 if both keys meet these criteria B0B1B0SCB1 max.</td></tr>
<tr><td></td><td>4</td><td></td></tr>
<tr><td rowspan="4">5(b)</td><td>Median or Q2 = 15 (cm)</td><td>B1</td><td>Correct</td></tr>
<tr><td>UQ or Q3 = 28 cm, LQ or Q1 = 10 cm
IQR = 28 – 10</td><td>M1</td><td>$22 \leqslant UQ \leqslant 36 - 8 \leqslant LQ \leqslant 10$</td></tr>
<tr><td>18 (cm)</td><td>A1</td><td>WWW</td></tr>
<tr><td></td><td>3</td><td></td></tr>
<tr><td rowspan="3">5(c)</td><td>On average the snowfall in Davos is higher</td><td>B1 FT</td><td>FT from their 5(b) values for Dados.
Statement comparing central tendency in context</td></tr>
<tr><td>The amount of snowfall in Linva varies more than in Davos</td><td>B1 FT</td><td>Statement comparing spread in context
Note: simply stating and comparing the values is not sufficient.</td></tr>
<tr><td></td><td>2</td><td></td></tr>
</table>

Question	Answer	Marks	Guidance
6(a)	${}^9C_6$ ($\times\,{}^3C_3$)	**M1**	${}^9C_k \times n$, $k = 6, 3$, $n = 1,2$ oe Condone ${}^9C_6 + {}^3C_3$, ${}^9P_6 \times {}^3P_3$
	84	**A1**	Accept unevaluated.
		2	
6(b)	Number with 3 Baker children = ${}^6C_2$ or 15	**B1**	Correct seen anywhere, not multiplied or added
	Total no of selections = ${}^9C_5$ or 126 Probability = $\dfrac{\text{number of selections with3 Baker children}}{\text{total number of selections}}$	**M1**	Seen as denominator of fraction
	$\dfrac{15}{126}$, 0·119	**A1**	OE, e.g. $\dfrac{5}{42}$
	Alternative method for question 6(b)		
	$\dfrac{3}{9}\times\dfrac{2}{8}\times\dfrac{1}{7}\left(\times\dfrac{6}{6}\right)\left(\times\dfrac{5}{5}\right)\times{}^5C_3$	**B1**	${}^5C_3$ (OE) or 10 seen anywhere, multiplied by fractions only, not added
		M1	$\dfrac{3}{9}\times\dfrac{2}{8}\times\dfrac{1}{7}\left(\times\dfrac{6}{6}\right)\left(\times\dfrac{5}{5}\right)\times k$, $1 \leqslant k$, k integer
	$\dfrac{15}{126}$, 0·119	**A1**	OE, e.g. $\dfrac{5}{42}$
		3	

Question	Answer	Marks	Guidance
6(c)	[Total no of arrangements = 9!] [Arrangements with men together = $8! \times 2$] Not together: 9! –	**M1**	$9! - k$ or $362880 - k$, k an integer<362 880
	$8! \times 2$	**B1**	$8! \times 2(!)$ or 80 640 seen anywhere
	282 240	**A1**	Exact value
	Alternative method for question 6(c)		
	$7! \times 8 \times 7$	**B1**	$7! \times k$, k positive integer > 1
		M1	$m \times 8 \times 7$, $m \times {}^8P_2$, $m \times {}^8C_2$ m positive integer > 1
	282 240	**A1**	Exact value
		3	
6(d)	$7! \times 2 \times 7$	**M1**	$7! \times k$, k positive integer > 1 If 7! not seen, condone $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times (1) \times k$ *or* $7 \times 6! \times k$ only
		M1	$m \times 2 \times 7$, m positive integer > 1
	70 560	**A1**	
		3	

Cambridge Assessment International Education

Cambridge International A Level

MATHEMATICS **9709/53**

Paper 5 Probability & Statistics 1 **October/November 2020**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2020 series for most Cambridge IGCSE™, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
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2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1(a)	$\mathrm{P}(56 < X < 66) = \mathrm{P}\left(\frac{56-62}{5} < z < \frac{66-62}{5}\right)$ $= \mathrm{P}(-1.2 < z < 0.8)$	**M1**	Using ± standardisation formula at least once, no $\sqrt{\sigma}$ or σ^2, allow continuity correction
	$\Phi(0.8)+\Phi(1.2)-1$ $= 0.7881 + 0.8849 - 1$	**M1**	Appropriate area Φ, from standardisation formula in final solution
	0.673	**A1**	
		3	
1(b)	$z = 1.127$	**B1**	±(1.126 – 1.127) seen, 4 sf or more
	$\frac{60t-62}{5} = 1.127$ $60t = 5.635+62=67.635$	**M1**	z-value = $\pm\frac{(60t-62)}{5}$ condone z-value = $\pm\frac{(t-62)}{5}$ no continuity correction, condone $\sqrt{\sigma}$ or σ^2
	$t = 1.13$	**A1**	CAO
		3	

Question	Answer	Marks	Guidance
2(a)	$\left(\frac{5}{6}\right)^8$	**M1**	p^8, $0 < p < 1$, no x, + or -
	0.233	**A1**	
		2	
2(b)	36	**B1**	
		1	
2(c)	P(X =10) + P(X=11) = $\left(\frac{35}{36}\right)^9 \frac{1}{36} + \left(\frac{35}{36}\right)^{10} \frac{1}{36}$	**M1**	OE, unsimplified expression in form $p^9 q + p^{10} q$, $p + q = 1$, no ×
	0.0425	**A1**	
		2	

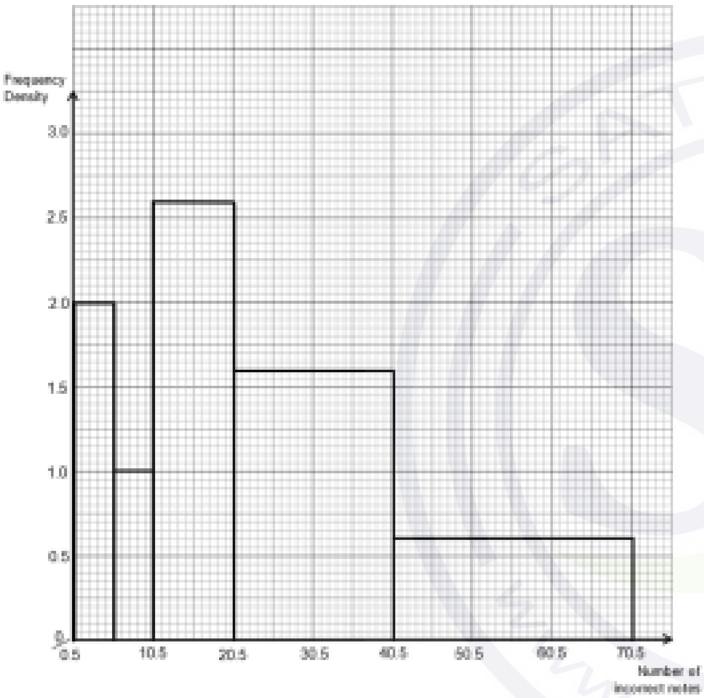
Question	Answer	Marks	Guidance
3(a)	Scenarios: 6W 0M ${}^9C_6 = 84$ 5W 1M ${}^9C_5 \times {}^5C_1 = 126 \times 5 = 630$ 4W 2M ${}^9C_4 \times {}^5C_2 = 126 \times 10 = 1260$	**M1**	Correct number of ways for either 5 or 4 women, accept unsimplified
		M1	Summing the number of ways for 2 or 3 correct scenarios (can be unsimplified), no incorrect scenarios.
	Total = 1974	**A1**	
		3	
3(b)	Total number of ways = ${}^{14}C_6$ (3003) Number with sister and brother = ${}^{12}C_4$ (495) Number required = ${}^{14}C_6 -$	**M1**	${}^{14}C_6$ – a value
	${}^{12}C_4 = 3003 - 495$	**M1**	${}^{12}C_x$ or ${}^nC_4$ seen on its own or subtracted from *their* total, $x \leqslant 6$, $n \leqslant 13$
	2508	**A1**	
	Alternative method for question 3(b)		
	Number of ways with neither = ${}^{12}C_6 = 924$	**M1**	${}^{12}C_6$ + a value
	Number of ways with either brother or sister (not both) $= {}^{12}C_5 \times 2$ $(= 792 \times 2) = 1584$	**M1**	${}^{12}C_x \times 2$ or ${}^nC_5 \times 2$ seen on its own or added to *their* number of ways with neither, $x \leqslant 5$, $n \leqslant 12$
	Number required = 924 + 1584 = 2508	**A1**	
		3	

Question	Answer	Marks	Guidance
4(a)	$0.65^7 + {}^7C_1\ 0.65^6\, 0.35^1 + {}^7C_2\ 0.65^5\, 0.35^2$	**M1**	Binomial term of form ${}^7C_x\ p^x(1-p)^{7-x}$, $0 < p < 1$, any p, $x \neq 0, 7$
	0.049022 + 0.184776 + 0.29848	**A1**	Correct unsimplified answer
	0.532	**A1**	
		3	
4(b)	Mean = $142 \times 0.35 = 49.7$ Variance = $142 \times 0.35 \times 0.65 = 32.305$	**B1**	Correct unsimplified *np* and *npq* (condone $\sigma = 5.684$ evaluated)
	$P(X > 40) = P(z > \frac{40.5 - 49.7}{\sqrt{32.305}})$	**M1**	Substituting *their* μ *and* σ (no $\sqrt{\sigma}$ or σ^2) into ±standardisation formula with a numerical value for '40.5'
	$P(z > -1.619)$	**M1**	Using either 40.5 or 39.5 within a ±standardisation formula
		M1	Appropriate area Φ, from standardisation formula $P(z > \ldots)$ in final solution, must be probability
	0.947	**A1**	Correct final answer
		5	

Question	Answer	Marks	Guidance
5(a)	Total number of ways = $\frac{8!}{3!2!}$ (= 3360)	**B1**	Correct unsimplified expression for total number of ways
	Number of ways with V and E in correct positions = $\frac{6!}{2!\times 2!}$ (= 180)	**B1**	$\frac{6!}{2!\times 2!}$ alone or as numerator in an attempt to find the number of ways with V and E in correct positions. No $\times, \pm$
	Probability = $\frac{180}{3360}$ $\left(=\frac{3}{56}\right)$ or 0.0536	**B1 FT**	Final answer from *their* $\frac{6!}{2!\times 2!}$ divided by *their* total number of ways
	Alternative method for question 5(a)		
	$\frac{1}{8}\times\frac{3}{7}$	**M1**	$\frac{a}{8}\times\frac{b}{7}$ seen, no other terms (correct denominators)
		M1	$\frac{1}{c}\times\frac{3}{d}$ seen, no other terms (correct numerators)
	$\frac{3}{56}$ or 0.0536	**A1**	
		3	

Question	Answer	Marks	Guidance
5(b)	Rs together and Es together: 5! (120)	B1	Alone or as numerator of probability to represent the number of ways with Rs and Es together, no ×, +, –
	Es together: $\frac{6!}{2!}(=360)$	B1	Alone or as denominator of probability to represent the number of ways with Es together, no ×, + or –
	Probability = $\frac{5!}{\frac{6!}{2!}}$	M1	$\frac{\textit{their}\ 5!}{\textit{their}\ \frac{6!}{2!}}$ seen
	$\frac{1}{3}$	A1	OE
	Alternative method for question 5(b)		
	P(Rs together and Es together): $\frac{5!}{\textit{their}\text{ total number of ways}}\left(=\frac{1}{28}\right)$	B1	
	P(Es together): $\frac{\frac{6!}{2!}}{\textit{their}\text{ total number of ways}}\left(=\frac{3}{28}\right)$	B1	Alone or as numerator of probability to represent the P(Rs and Es together), no ×, +, –
	Probability = $\frac{\frac{1}{28}}{\frac{3}{28}}$	M1	Alone or as denominator of probability to represent the P(Es together), no ×, + or –
	$\frac{1}{3}$	A1	OE, $\frac{\textit{their}\ \frac{1}{28}}{\textit{their}\ \frac{3}{28}}$ seen
		4	

Question	Answer	Marks	Guidance
6(a)	Scenarios: HHT: $\frac{2}{3}\times\frac{2}{3}\times\frac{1}{5}=\frac{4}{45}$ HTH: $\frac{2}{3}\times\frac{1}{3}\times\frac{4}{5}=\frac{8}{45}$ THH: $\frac{1}{3}\times\frac{2}{3}\times\frac{4}{5}=\frac{8}{45}$	**M1**	One 3 factor probability with 3, 3, 5 as denominators
		M1	3 factor probabilities for 2 or 3 correct scenarios added, no incorrect scenarios
	Total = $\frac{20}{45}=\frac{4}{9}$	**A1**	AG, Total of 3 products with clear context
		3	
6(b)	<table><tr><td>x</td><td>0</td><td>1</td><td>2</td><td>3</td></tr><tr><td>Prob.</td><td>$\frac{1}{45}$</td><td>$\frac{8}{45}$</td><td>$\frac{20}{45}$</td><td>$\frac{16}{45}$</td></tr></table>	**B1**	Probability distribution table with correct outcomes with at least one probability, allow extra outcome values if probability of zero stated'
		B1	2 of P(0), P(1) and P(3) correct
		B1 FT	3 or 4 probabilities sum to 1 with P(2) correct
		3	
6(c)	$\text{Var}(X)=\frac{0^2\times1+1^2\times8+2^2\times20+3^2\times16}{45}-\left(\frac{32}{15}\right)^2$ $=\frac{8}{45}+\frac{80}{45}+\frac{144}{45}-\left(\frac{32}{15}\right)^2$	**M1**	Substitute *their* attempts at scores in correct variance formula, must have '– mean^2' (FT if calculated) (condone probs not summing to 1); must be at least 2 non-zero values
	$\frac{136}{225}$ or 0.604	**A1**	
		2	

Question	Answer	Marks	Guidance
7(a)	Class widths: 5, 5, 10, 20, 30 Frequency density: 2, 1, 2.6, 1.6, 0.6	**M1**	At least 3 class widths correct and used in a calculation
		M1	At least 3 correct frequency densities unsimplified – FT *their* class widths
		**A1**	All correct heights on a histogram using a linear vertical scale from zero – no FT
		B1	Correct upper bar ends (5.5, 10.5, 20.5, 40.5, 70.5) and 4 correct lower bar ends of 5.5, 10.5, 20.5, 40.5. Condone 0 or 1.
		B1	Linear scales with at least 3 values indicated on each axis, vertical scale from 0, axes labelled 'fd' and 'no. of (incorrect) notes', or better.
		5	
7(b)	LQ: 11 – 20 UQ: 21 – 40	**B1**	Both UQ and LQ correct
	Greatest IQR = 40 – 11 = 29	**B1 FT**	Subtract lower end of *their* LQ interval from upper end of *their* UQ interval
		2	

Question	Answer	Marks	Guidance
7(c)	Midpoints: 3 8 15.5 30.5 55.5	**M1**	At least 4 midpoints correct and used
	Mean = $\frac{3\times10+8\times5+15.5\times26+30.5\times32+55.5\times18}{91}$ $=\frac{30+40+403+976+999}{91}$ $=\frac{2448}{91}$	**M1**	Correct formula with *their* midpoints (not upper boundary, lower boundary, class width, frequency density, frequency or cumulative frequency)
	26.9, $26\frac{82}{91}$	**A1**	Accept 26 or 27
		3	

Cambridge Assessment International Education

Cambridge International AS & A Level

MATHEMATICS	**9709/51**
Paper 5 Probability & Statistics 1	**May/June 2020**
MARK SCHEME	
Maximum Mark: 50	

Published

Students did not sit exam papers in the June 2020 series due to the Covid-19 global pandemic.

This mark scheme is published to support teachers and students and should be read together with the question paper. It shows the requirements of the exam. The answer column of the mark scheme shows the proposed basis on which Examiners would award marks for this exam. Where appropriate, this column also provides the most likely acceptable alternative responses expected from students. Examiners usually review the mark scheme after they have seen student responses and update the mark scheme if appropriate. In the June series, Examiners were unable to consider the acceptability of alternative responses, as there were no student responses to consider.

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This document consists of **13** printed pages.

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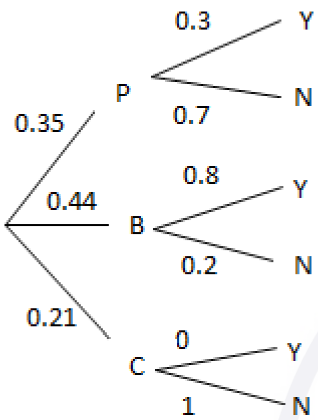
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks
1(a)	Prob of 4 (from 1,3, 3,1 or 2,2) = $\frac{3}{36} = \frac{1}{12}$ **AG**	**B1**
		1
1(b)	Mean = $\frac{1}{\frac{1}{12}} = 12$	**B1**
		1
1(c)	$\left(\frac{11}{12}\right)^5 \times \frac{1}{12} = 0.0539$ or $\frac{161051}{2985984}$	**B1**
		1
1(d)	$1 - \left(\frac{11}{12}\right)^7$	**M1**
	0.456 or $\frac{16344637}{35831808}$	**A1**
		2

Question	Answer	Marks
2(a)	6!	**M1**
	720	**A1**
		2
2(b)	Total number: $\frac{9!}{3!2!}(30240)$	**M1**
	Number with Ls together $= \frac{8!}{3!}(6720)$	**M1**
	Number with Ls not together $= \frac{9!}{3!2!} - \frac{8!}{3!}$ $= 30\,240 - 6720$	**M1**
	23 520	**A1**
	Alternative method for question 2(b)	
	$\frac{7!}{3!} \times \frac{8 \times 7}{2}$	
	$7! \times k$ in numerator, k integer ≥ 1	**M1**
	$8 \times 7 \times m$ in numerator or 8C2 $\times m$, m integer ≥ 1	**M1**
	3! in denominator	**M1**
	23 520	**A1**
		4

Question	Answer	Marks
3(a)	<table><tr><td>x</td><td>0</td><td>1</td><td>2</td><td>3</td></tr><tr><td>Probability</td><td>$\frac{1}{56}$</td><td>$\frac{15}{56}$</td><td>$\frac{30}{56}$</td><td>$\frac{10}{56}$</td></tr></table> (**B1** for probability distribution table with correct outcome values)	**B1**
	$P(0) = \frac{3}{8}\times\frac{2}{7}\times\frac{1}{6} = \frac{1}{56}$ $P(1) = \frac{5}{8}\times\frac{3}{7}\times\frac{2}{6}\times 3 = \frac{15}{56}$ $P(2) = \frac{5}{8}\times\frac{4}{7}\times\frac{3}{6}\times 3 = \frac{30}{56}$ $P(3) = \frac{5}{8}\times\frac{4}{7}\times\frac{3}{6} = \frac{10}{56}$ (**M1** for denominator 8×7×6)	**M1**
	Any one probability correct (with correct outcome)	**A1**
	All probabilities correct	**A1**
		4
3(b)	$1 - P(8, 9, 10) = 1 - \left[{}^{10}C_8\, 0.64^8 0.36^2 + {}^{10}C_9\, 0.64^9 0.36^1 + 0.64^{10}\right]$	**M1**
	$1 - (0.164156 + 0.064852 + 0.11529)$	**M1**
	0.759	**A1**
		3

Question	Answer	Marks
4	Scenarios: 2P 3V 2G $\quad {}^8C_2 \times {}^4C_2 \times {}^6C_3 = 28 \times 6 \times 20 = 3360$ 2P 4V 1G $\quad {}^8C_2 \times {}^4C_1 \times {}^6C_4 = 28 \times 4 \times 15 = 1680$ 3P 3V 1G $\quad {}^8C_3 \times {}^4C_1 \times {}^6C_3 = 56 \times 4 \times 20 = 4480$ 4P 2V 1G $\quad {}^8C_4 \times {}^4C_1 \times {}^6C_2 = 70 \times 4 \times 15 = 4200$ (**M1** for ${}^8C_r \times {}^4C_r \times {}^6C_r$ with $\sum r = 7$)	**M1**
	Two unsimplified products correct	**B1**
	Summing the number of ways for 3 or 4 correct scenarios	**M1**
	Total: 13 720	**A1**
		4

Question	Answer	Marks
5(a)		
	Fully correct labelled tree for method of transport with correct probabilities.	**B1**
	Fully correct labelled branches with correct probabilities for lateness with either 1 branch after W or 2 branches with the prob 0	**B1**
		2
5(b)	$0.35 \times 0.3 + 0.44 \times 0.8\ (+\ 0)$	**M1**
	0.457	**A1**
		2

Question	Answer	Marks
5(c)	P(not B|not fruit) = $\dfrac{P(B' \cap F')}{P(F')}$	M1
	$\dfrac{0.35 \times 0.7 + 0.21 \times 1}{1 - \textit{their}\,(\mathbf{b})}$	M1
	$\dfrac{0.455}{0.543}$ (**M1** for 1 – *their* **(b)** or summing three appropriate 2-factor probabilities, correct or consistent with *their* tree diagram as denominator)	M1
	0.838 or $\dfrac{455}{543}$	A1
		4

Question	Answer	Marks
6(a)	$P\left(\dfrac{50-54}{6.1} < z < \dfrac{60-54}{6.1}\right) = P(-0.6557 < Z < 0.9836)$	M1
	Both values correct	A1
	$\Phi(0.9836) - \Phi(-0.6557) = \Phi(0.9836) + \Phi(0.6557) - 1$ $= 0.8375 + 0.7441 - 1$ (Correct area)	M1
	0.582	A1
		4

Question	Answer	Marks
6(b)	$\frac{45-\mu}{\sigma}=-0.994$	B1
	$\frac{56-\mu}{\sigma}=1.372$	B1
	One appropriate standardisation equation with μ, σ, z-value (not probability) and 45 or 56.	M1
	$11 = 2.366\ \sigma$ (**M1** for correct algebraic elimination of μ or σ from *their* two simultaneous equations to form an equation in one variable)	M1
	$\sigma = 4.65$, $\mu = 49.6$	A1
		5

Question	Answer	Marks
7(a)	Class widths: 10, 5, 15, 20, 10	**M1**
	Frequency density = frequency/*their* class width: 1.8, 4.8, 2, 1, 0.8	**M1**
	All heights correct on diagram (using a linear scale)	**A1**
	Correct bar ends	**B1**
	Bar ends: 10.5, 15.5, 30.5, 50.5, 60.5	**B1**
		5
7(b)	11 – 15 and 31 – 50	**B1**
	Greatest IQR = 50 – 11 = 39	**B1**
		2
7(c)	Mean = $\dfrac{18\times5.5+24\times13+30\times23+20\times40.5+8\times55.5}{100}=\dfrac{2355}{100}=23.6$	**B1**
	Var = $\dfrac{18\times5.5^2+24\times13^2+30\times23^2+20\times40.5^2+8\times55.5^2}{100}-\text{mean}^2$	**M1**
	$\dfrac{77917.5}{100}-\text{mean}^2=224.57$	**A1**
	Standard deviation = 15.0 (**FT** *their* variance)	**A1 FT**
		4

Cambridge International AS & A Level

MATHEMATICS **9709/52**

Paper 5 Probability & Statistics 1 **May/June 2020**

MARK SCHEME

Maximum Mark: 50

Published

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This mark scheme is published to support teachers and students and should be read together with the question paper. It shows the requirements of the exam. The answer column of the mark scheme shows the proposed basis on which Examiners would award marks for this exam. Where appropriate, this column also provides the most likely acceptable alternative responses expected from students. Examiners usually review the mark scheme after they have seen student responses and update the mark scheme if appropriate. In the June series, Examiners were unable to consider the acceptability of alternative responses, as there were no student responses to consider.

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Cambridge International will not enter into discussions about these mark schemes.

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Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
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- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

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- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
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5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
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CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks
1	$\sum x - 50n = 144$	**B1**
	$50n + 144 = 944$	**M1**
	$n = 16$	**A1**
		3

Question	Answer	Marks
2(a)	$\frac{56}{500}$ or $\frac{14}{125}$ or 0.112	**B1**
		1
2(b)	$\mathrm{P}(\mathrm{D}\mid\mathrm{S}) = \frac{\mathrm{P}(\mathrm{D}\cap\mathrm{S})}{\mathrm{P}(\mathrm{S})} = \frac{120}{280}$	**M1**
	$\frac{120}{280}$ or $\frac{3}{7}$	**A1**
		2

Question	Answer	Marks
2(c)	P(hockey) = $\frac{220}{500} = 0.44$ P(Amos or Benn) = $\frac{242}{500} = 0.484$ P(hockey ∩ A or B) = $\frac{104}{500} = 0.208$ P(H) × P(A U B) = P(H ∩ (A U B)) if independent	M1
	$\frac{220}{500} \times \frac{242}{500} = \frac{1331}{6250}$ so not independent	A1
		2

Question	*Answer*	Marks
3(a)	Median = 0.238	B1
	UQ = 0.245, LQ = 0.231, So IQR = 0.245 – 0.231	M1
	0.014	A1
		3

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th></tr>
<tr><td rowspan="5">3(b)</td><td><table><tr><td></td><td></td><td>LQ</td><td>M</td><td>UQ</td><td></td></tr><tr><td>A</td><td>0.220</td><td>0.231
FT</td><td>0.238
FT</td><td>0.245
FT</td><td>0.254</td></tr><tr><td>B</td><td>0.211</td><td>0.224</td><td>0.232</td><td>0.243</td><td>0.256</td></tr></table></td><td></td></tr>
<tr><td>Medians and quartiles correctly plotted for A or B</td><td>B1</td></tr>
<tr><td>End points correct for A or B</td><td>B1</td></tr>
<tr><td>Completely correct, including scale</td><td>B1</td></tr>
<tr><td></td><td>3</td></tr>
<tr><td rowspan="3">3(c)</td><td>Lengths of rods produced by machine A are longer.
(B1 for comparison of central tendency)</td><td>B1</td></tr>
<tr><td>Lengths of rods produced by machine A are less spread out
(B1 for comparison of spread)</td><td>B1</td></tr>
<tr><td></td><td>2</td></tr>
</table>

Question	Answer	Marks
4(a)	$\mathrm{P}(X < 25) = \mathrm{P}\left(z < \frac{25-40}{12}\right) = \mathrm{P}(z < -1.25)\mathrm{P}(X < 25) = \mathrm{P}(z <)$	**M1**
	$1 - 0.8944$	**M1**
	0.106	**A1**
		3
4(b)	0.8944 divided by 3 (**M1** for 1 - *their* **(a)** divided by 3)	**M1**
	0.298 **AG**	**A1**
		2
4(c)	0.2981 gives $z = 0.53$	**B1**
	$\frac{h-40}{12} = 0.53$	**M1**
	$h = 46.4$	**A1**
		3

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th></tr>
<tr><td>5(a)</td><td>
<table>
<tr><td></td><td>1</td><td>1</td><td>2</td><td>2</td><td>3</td></tr>
<tr><td>1</td><td>1</td><td>1</td><td>2</td><td>2</td><td>3</td></tr>
<tr><td>2</td><td>2</td><td>2</td><td>2</td><td>2</td><td>3</td></tr>
<tr><td>3</td><td>3</td><td>3</td><td>3</td><td>3</td><td>3</td></tr>
</table>
</td><td>M1</td></tr>
<tr><td></td><td>$\frac{7}{15}$ AG</td><td>A1</td></tr>
<tr><td></td><td></td><td>2</td></tr>
<tr><td>5(b)</td><td>
<table>
<tr><td>x</td><td>1</td><td>2</td><td>3</td></tr>
<tr><td>Probability</td><td>$\frac{2}{15}$</td><td>$\frac{6}{15}$</td><td>$\frac{7}{15}$</td></tr>
</table>
</td><td>B1</td></tr>
<tr><td></td><td>P(1) or P(2) correct</td><td>B1</td></tr>
<tr><td></td><td>3rd probability correct, FT sum to 1</td><td>B1</td></tr>
<tr><td></td><td></td><td>3</td></tr>
</table>

Question	Answer	Marks
5(c)	$\text{E}(X) = \dfrac{2+12+21}{15} = \dfrac{35}{15} = \dfrac{7}{3}$	B1
	$\text{Var}(X) = \dfrac{1^2 \times 2 + 2^2 \times 6 + 3^2 \times 7}{15} - \left(\dfrac{7}{3}\right)^2$	M1
	$\dfrac{22}{45}(0.489)$	A1
		3

Question	*Answer*	Marks
6(a)	$\dfrac{8!}{3!}$	M1
	6720	A1
		2

Question	Answer	Marks
6(b)	Total number $= \dfrac{10!}{2!3!}$ (302400) (A)	B1
	With Es together $= \dfrac{9!}{3!}$ (60480) (B)	B1
	Es not together = *their* (A) – *their* (B)	M1
	241920	A1
	Alternative method for question 6(b)	
	_^_^_^_^_^_^_^_^_ $\dfrac{8!}{3!}\times\dfrac{9\times8}{2}$	
	8! × *k* in numerator, *k* integer ≥ 1, denominator ≥ 1	B1
	3! × *m* in denominator, *m* integer ≥ 1	B1
	Their $\dfrac{8!}{3!}$ Multiplied by $^9C_2$ (OE) only (no additional terms)	M1
	241920	A1
		4

Question	Answer	Marks
6(c)	Scenarios: E M M M $\quad {}^5C_0 = 1$ E M M _ $\quad {}^5C_1 = 5$ E M _ _ $\quad {}^5C_2 = 10$	M1
	Summing the number of ways for 2 or 3 correct scenarios	M1
	Total = 16	A1
		3

Question	Answer	Marks
7(a)	1 – P(10, 11, 12) $= 1 - [{}^{12}C_{10}\, 0.72^{10} 0.28^2 + {}^{12}C_{11}\, 0.72^{11} 0.28^1 + 0.72^{12}]$	M1
	$1 - (0.19372 + 0.09057 + 0.01941)$	A1
	0.696	A1
		3
7(b)	$0.28^3 \times 0.72 = 0.0158$	B1
		1

Question	Answer	Marks
7(c)	Mean = $100 \times 0.72 = 72$ Var = $100 \times 0.72 \times 0.28 = 20.16$	**M1**
	P(less than 64) = $P\left(z < \frac{63.5 - 72}{\sqrt{20.16}}\right)$ (**M1** for substituting *their* μ *and* σ into ±standardisation formula with a numerical value for '*63.5*')	**M1**
	Using either 63.5 or 64.5 within a ±standardisation formula	**M1**
	Appropriate area Φ, from standardisation formula P(z<…) in final solution $= P(z < -1.893)$	**M1**
	0.0292	**A1**
		5

Cambridge Assessment International Education

Cambridge International AS & A Level

MATHEMATICS	**9709/53**
Paper 5 Probability & Statistics 1	**May/June 2020**
MARK SCHEME	
Maximum Mark: 50	

Published

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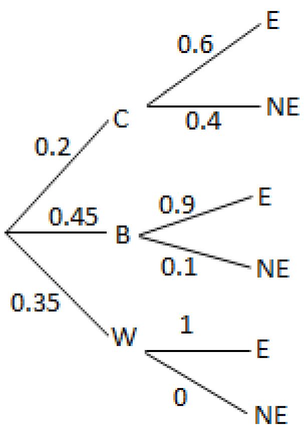
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks
1(a)		
	Fully correct labelled tree for method of transport with correct probabilities.	B1
	Fully correct labelled branches with correct probabilities for lateness with either 1 branch after W or 2 branches with the probability 0.	B1
		2
1(b)	$P(C\|E) = \dfrac{P(C \cap E)}{P(E)} = \dfrac{0.2 \times 0.6}{0.2 \times 0.6 + 0.45 \times 0.1 + 0.35 \times 1}$	M1
	Summing three appropriate 2-factor probabilities	M1
	$\dfrac{0.12}{0.515}$	A1
	0.233 or $\dfrac{12}{515}$	A1
		4

Question	Answer	Marks
2(a)	$0.22^3 = 0.0106$	B1
		1
2(b)	$P(2, 3, 4) = {}^{16}C_2\ 0.22^2 0.78^{14} + {}^{16}C_3 0.22^3 0.78^{13} + {}^{16}C_4 0.22^4 0.78^{12}$	M1
	$0.179205 + 0.235877 + 0.216221$	A1
	0.631	A1
		3

Question	Answer	Marks
3(a)	$P(X < 21) = P\left(z < \dfrac{21-15.8}{4.2}\right) = \Phi(1.238)$	M1
	0.892	A1
		2
3(b)	$z = \pm 0.674$	B1
	$\dfrac{k-15.8}{4.2} = 0.674$	M1
	18.6	A1
		3

Question	Answer	Marks
4(a)	<table><tr><td>–1</td><td>0</td><td>0</td><td>1</td></tr><tr><td>0</td><td>1</td><td>1</td><td>2</td></tr><tr><td>2</td><td>3</td><td>3</td><td>4</td></tr></table> <table><tr><td>x</td><td>–1</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>Probability</td><td>$\frac{1}{12}$</td><td>$\frac{3}{12}$</td><td>$\frac{3}{12}$</td><td>$\frac{2}{12}$</td><td>$\frac{2}{12}$</td><td>$\frac{1}{12}$</td></tr></table> Probability distribution table with correct scores with at least one probability	**B1**
	At least 4 probabilities correct	**B1**
	All probabilities correct	**B1**
		3
4(b)	$E(X) = \frac{-1+0+3+4+6+4}{12} = \frac{16}{12} = \frac{4}{3}$	**B1**
	$\text{Var}(X) = \frac{1+0+3+8+18+16}{12} - \left(\frac{4}{3}\right)^2$	**M1**
	$\frac{37}{18}$ (= 2.06)	**A1**
		3

Question	Answer	Marks
5(a)	$\frac{1}{\frac{1}{4}} = 4$	B1
		1
5(b)	$\frac{9}{64}$ $(= 0.141)$	B1
		1
5(c)	$P(X < 6) = 1 - \left(\frac{3}{4}\right)^5$ (**FT** *their* probability/mean from part **(a)**)	M1
	0.763	A1
		2
5(d)	Mean $= 80 \times 0.25 = 20$ Var $= 80 \times 0.25 \times 0.75 = 15$	M1
	P(more than 25) $= P\left(z > \frac{25.5 - 20}{\sqrt{15}}\right)$	M1
	$P(z > 1.42)$	M1
	$1 - 0.9222$	M1
	0.0778	A1
		5

Question	Answer	Marks
6(a)	<table><tr><th>A</th><th></th><th>B</th></tr><tr><td></td><td>2</td><td>6</td></tr><tr><td>5 2 0</td><td>3</td><td>0 1 5 8</td></tr><tr><td>9 7 2 1 1</td><td>4</td><td>1 2 2 7 9</td></tr><tr><td>3 2</td><td>5</td><td>2</td></tr><tr><td>4</td><td>6</td><td></td></tr></table> KEY 1 | 4 | 2 means $41 000 for A and $42 000 for B Correct stem	B1
	Correct A on LHS	B1
	Correct B on same diagram	B1
	Correct key for *their* diagram, both companies identified and correct units	B1
		4
6(b)	Median = [$]42 000	B1
	LQ = [$]35 000 UQ = [$]52 000	B1
	IQR = [$]17 000 (**FT** if $49000 \leqslant UQ \leqslant 53000 - 32000 \leqslant LQ \leqslant 41000$)	B1 FT
		3

Question	Answer	Marks
6(c)	Sum of given 11 numbers is 433 000	M1
	Sum of 12 numbers, including new = 38 500 × 12 = 462 000	M1
	Difference = new salary = [$]29 000	A1
		3

Question	Answer	Marks
7(a)	$\frac{9!}{2!2!} = 90\ 720$	B1
		1
7(b)	$\frac{6!}{2!}$	M1
	360	A1
		2

Question	Answer	Marks
7(c)	2 Es together $= \frac{8!}{2!}(=20160)$	M1
	Es not together $= 90720 - 20160 = 70560$	M1
	Probability $= \frac{70560}{90720}$	M1
	$\frac{7}{9}$ or 0.778	A1
	Alternative method for question 7(c)	
	_^_^_^_^_^_^_^_ $\frac{7!}{2!}\times\frac{8\times 7}{2} = 70560$	
	$7! \times k$ in numerator, k integer $\geqslant 1$, denominator $\geqslant 1$	M1
	Multiplying by ${}^8C_2$ OE	M1
	Probability $= \frac{70560}{90720}$	M1
	$\frac{7}{9}$ or 0.778	A1
		4

Question	Answer	Marks
7(d)	Scenarios are: E L _ _ _ $^5C_3$ 10 E E L _ _ $^5C_2$ 10 E _ _ _ _ $^5C_4$ 5 E E _ _ _ $^5C_3$ 10	**M1**
	Summing the number of ways for 3 or 4 correct scenarios	**M1**
	Total = 35	**A1**
		3

Cambridge International AS & A Level

MATHEMATICS	**9709/52**
Paper 5 Probability and Statistics	**March 2020**
MARK SCHEME	
Maximum Mark: 50	

Published

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- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
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Marks must be awarded **positively**:

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- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics-Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

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Types of mark

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Abbreviations

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SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	${}^{38}C_r$ or ${}^{n}C_{34}$	**M1**	Either expression seen OE, no other terms, condone x1
	${}^{38}C_{34}$	**A1**	Correct unsimplified OE
	73815	**A1**	If M0, **SCB1** ${}^{38}C_{34}$ x *k*, *k* an integer
		3	

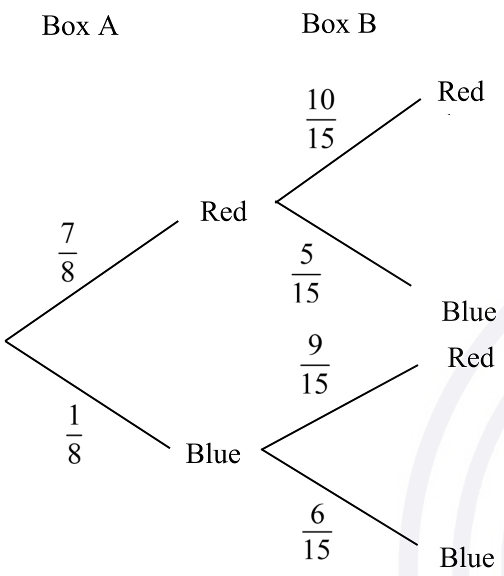
Question	Answer	Marks	Guidance
2(a)	$\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^2+\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^3+\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^4$	**M1**	One correct term with $0<p<1$
	$=\frac{4}{27}+\frac{8}{81}+\frac{16}{243}\left(=\frac{2432}{7776}\right)$	**A1**	Correct expression, accept unsimplified
	$=\frac{76}{243}$ or 0.313	**A1**	
		3	

Question	Answer	Marks	Guidance
2(b)	<table><tr><td>x</td><td>0</td><td>1</td><td>2</td><td>3</td></tr><tr><td>P(x)</td><td>$\frac{8}{27}$</td><td>$\frac{12}{27}$</td><td>$\frac{6}{27}$</td><td>$\frac{1}{27}$</td></tr></table>	**B1**	Probability distribution table with correct values of x, no additional values unless with probability of 0 stated, at least one non-zero probability included
	$\text{P}(0) = \left(\frac{2}{3}\right)^3$ $\text{P}(1) = \left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^2 \times 3$ $\text{P}(2) = \left(\frac{2}{3}\right)\left(\frac{1}{3}\right)^2 \times 3$ $\text{P}(3) = \left(\frac{1}{3}\right)^3$	**B1**	1 correct probability seen (may not be in table) **or** 3 or 4 non-zero probabilities summing to 1
		B1	All probabilities correct
		3	
2(c)	$\text{E}(X) = \left[0 \times \frac{8}{27}\right] + 1 \times \frac{12}{27} + 2 \times \frac{6}{27} + 3 \times \frac{1}{27}$ $= \left[\frac{0}{27}\right] + \frac{12}{27} + \frac{12}{27} + \frac{3}{27}$	**M1**	Correct method from *their* probability distribution table with at least 3 terms, $0 \leqslant$ *their* P(x) $\leqslant 1$, accept unsimplified
	$= 1$	**A1**	
		2	

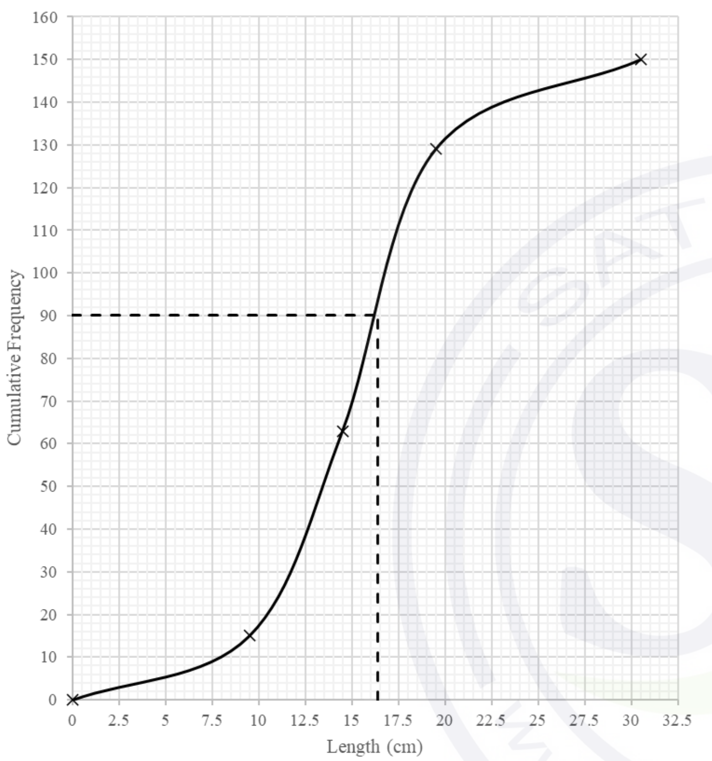
Question	Answer	Marks	Guidance
3(a)	$P(X > 87) = P\left(Z > \frac{87-82}{\sigma}\right) = 0.22$	M1	Using ± standardisation formula, not σ^2, not $\sqrt{\sigma}$, no continuity correction
	$P\left(Z < \frac{5}{\sigma}\right) = 0.78$ $\left(\frac{5}{\sigma} =\right) 0.772$	B1	AWRT ±0.772 seen B0 for ±0.228
	$\sigma = 6.48$	A1	
		3	
3(b)	$P\left(-\frac{4}{\sigma} < Z < \frac{4}{\sigma}\right) = P(-0.6176 < Z < 0.6176)$	M1	Using ±4 used within a standardisation formula (SOI), allow σ^2, $\sqrt{\sigma}$ and continuity correction
		M1	Standardisation formula applied to **both** *their* ±4
	$\Phi = 0.7317$ Prob = $2\Phi - 1 = 2(0.7317) - 1$	M1	Correct area $2\Phi - 1$ oe linked to final solution
	= 0.463	A1	
		4	

Question	Answer	Marks	Guidance
4(a)	R ^ ^ ^ ^ ^ ^ ^ ^ ^ ^ R $\frac{9!}{3!6!}$	**M1**	9! Alone on numerator, $3! \times k$ or $6! \times k$ on denominator
	= 84	**A1**	
		2	
4(b)	^ (B B B) ^ ^ ^ ^ ^	**M1**	$\frac{7!}{6!}\times k$ or $7k$ seen, k an integer > 0
	$\frac{7!}{6!}\times\frac{8\times7}{2}$	**M1**	$m\times n(n-1)$ or $m\times {}^nC_2$ or $m\times {}^nP_2$, n=7, 8 or 9, m an integer > 0
		M1	n = 8 used in above expression
	= 196	**A1**	
	Alternative for question 4(b)		
	[Arrangements, blues together – Arrangements with blues together and reds together =] $\frac{9!}{2!6!}-\frac{8!}{6!}$	**M1**	9! Seen alone or as numerator with subtraction
	= [252 – 56]	**M1**	8! Seen alone or as numerator in a second term and no other terms
		M1	All terms divided by 6! x k, k an integer
	= 196	**A1**	
		4	

Question	Answer	Marks	Guidance
5(a)	1 – P(6, 7, 8) $= 1 - ({}^8C_6\ 0.7^6 0.3^2 + {}^8C_7\ 0.7^7 0.3^1 + 0.7^8)$	M1	One term ${}^8C_x\ p^x(1-p)^{8-x}$, $0 < p < 1$, $x \neq 0$
	= 1 – 0.55177	A1	Correct unsimplified expression, or better
	= 0.448	A1	
	Alternative method for question 5(a)		
	P(0, 1, 2, 3, 4, 5) $= 0.3^8 + {}^8C_1\,0.7^1 0.3^7 + {}^8C_2\,0.7^2 0.3^6 + {}^8C_3\,0.7^3 0.3^5 + {}^8C_4 0.7^4 0.3^4 + {}^8C_5\,0.7^5 0.3^3$	M1	One term ${}^8C_x\ p^x(1-p)^{8-x}$, $0 < p < 1$, $x \neq 0$
		A1	Correct unsimplified expression, or better
	= 0.448	A1	
		3	
5(b)	Mean = $120 \times 0.7 = 84$ Var = $120 \times 0.7 \times 0.3 = 25.2$	B1	Correct mean and variance, allow unsimplified
	P(more than 75) = $P\left(z > \frac{75.5 - 84}{\sqrt{25.2}}\right)$	M1	Substituting *their* μ and σ into the ±standardising formula (any number), not σ^2, not $\sqrt{\sigma}$
		M1	Using continuity correction 75.5 or 74.5
	P($z > -1.693$)	M1	Appropriate area Φ, from final process, must be a probability
	= 0.955	A1	Allow $0.9545 < p \leqslant 0.955$
		5	

Question	Answer	Marks	Guidance
6(a)		**B1**	Both correct probs, box A
		B1	2 probs correct for box B
		B1	All correct probs for box B
		3	
6(b)	$\frac{7}{8}\times\frac{5}{15}+\frac{1}{8}\times\frac{9}{15}$	**M1**	Two 2 factor terms added, correct or FT *their* **6(a)**.
	$=\frac{44}{120}\left[\frac{11}{30}\text{ or }0.367\right]$	**A1**	OE
		2	

Question	Answer	Marks	Guidance
6(c)	$\text{P(A blue} \mid \text{B blue)} = \dfrac{\text{P}(\text{A blue} \cap \text{B blue})}{\text{P}(\text{B blue})}$ $= \dfrac{\frac{1}{8}\times\frac{6}{15}}{\frac{7}{8}\times\frac{5}{15}+\frac{1}{8}\times\frac{6}{15}} = \dfrac{\frac{1}{20}}{\frac{41}{120}}$	M1	*their* $\frac{1}{8}\times\frac{6}{15}$ seen as numerator or denom of fraction
		M1	*their* $\frac{7}{8}\times\frac{5}{15}+\frac{1}{8}\times\frac{6}{15}$ seen
		M1	*their* $\frac{7}{8}\times\frac{5}{15}+\frac{1}{8}\times\frac{6}{15}$ seen as denominator
	$= \dfrac{6}{41}$ or 0.146	A1	
		4	

Question	Answer	Marks	Guidance
7(a)	15, 63, 129, 150	**B1**	Correct cumulative frequencies seen (may be on graph)
		**B1**	$0 \leqslant$ Horizontal axis $\leqslant 30$, $0 \leqslant$ vertical axis $\leqslant 150$ Labels correct: length cm, cf
		M1	At least 3 points plotted at upper end points (e.g. allow 9, 9.5, 10) with a linear horizontal scale.
		A1	Linear vertical scale, all points at correct upper end points (9.5 etc.), curve drawn accurately, joined to (0,0) (condone (–0.5, 0))
		4	
7(b)	60% of 150 = 90	**M1**	90 seen or implied by use on graph
	Approx. 16.5 [cm]	**A1FT**	FT *their* increasing cumulative frequency graph, Use of graph must be seen. If no clear evidence of use of graph **SCB1FT** correct value from *their* graph
		2	

Question	Answer	Marks	Guidance
7(c)	Midpoints: 4.75, 12, 17, 25	**M1**	At least 3 correct midpoints used (39449.4375 implies M1)
	Var = $\dfrac{4.75^2 \times 15 + 12^2 \times 48 + 17^2 \times 66 + 25^2 \times 21}{150} - 15.295^2$	**M1**	Using midpoints ±0.5 in correct var formula, including subtraction of *their* μ^2.
	= 29.1	**A1**	
		3	

Cambridge International AS & A Level

Cambridge Assessment International Education
Cambridge International Advanced Subsidiary and Advanced Level

MATHEMATICS	**9709/61**
Paper 6	**October/November 2019**
MARK SCHEME	
Maximum Mark: 50	

Published

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Cambridge Assessment International Education

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WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	$0.8 \times 0.6 + 0.2(1-x) = 0.63$	**M1**	Equation of form $0.8 \times \text{A} + 0.2 \times \text{B} = \text{C}$, A,B involving $1-x$ and 0.6 or 0.4 and $\text{C} = 0.63$ or 0.37
	$0.2x = 0.05$	**M1**	Correct unsimplified equation
	$x = 0.25$	**A1**	
	Alternative method for question 1		
	$0.8 \times 0.4 + 0.2x = 1 - 0.63$	**M1**	Equation of form $0.8 \times \text{A} + 0.2 \times \text{B} = \text{C}$, A,B involving x and 0.6 or 0.4 and $\text{C} = 0.63$ or 0.37
	$0.2x = 0.05$	**M1**	Correct unsimplified equation
	$x = 0.25$	**A1**	
		3	

Question	Answer	Marks	Guidance
2(i)	$1-({}^{10}C_2\, 0.42^8\, 0.58^2 + {}^{10}C_9\, 0.42^9\, 0.58^1 + 0.42^{10})$	**M1**	Binomial term of form ${}^{10}C_a p^a(1-p)^b$ $0 < p < 1$ any p, $0 \leqslant a,b \leqslant 10$
		A1	Correct unsimplified expression
	0.983	**A1**	
		3	
2(ii)	$1 - P(0) > 0.995$ $0.58^n < 0.005$	**M1**	Equation or inequality involving 0.58^n or 0.42^n and 0.995 or 0.005
	$n > \dfrac{\log 0.005}{\log 0.58}$ $n > 9.727$	**M1**	Attempt to solve using logs or Trial and Error. May be implied by their answer (rounded or truncated)
	$n = 10$	**A1**	CAO
		3	

Question	Answer	Marks	Guidance
3(i)	$\sum x = 60 \times 20$ $\quad = 1200$	**B1**	
	$\dfrac{\sum x^2}{20} - 60^2 = 4^2$	**M1**	Correct variance formula used, condone $= 4$
	$\sum x^2 = 3616 \times 20$ $\quad = 72320$	**A1**	Exact value
		3	

Question	Answer	Marks	Guidance
3(ii)	$\sum x = 1200 + 550 = 1750$ $\sum x^2 = 72320 + 40500 = 112800$	**M1**	Summing both values of $\sum x$ and $\sum x^2$
	Mean = $\dfrac{their\,1750}{30} = 58.3$	**B1FT**	FT *their* 1750 (not 550 or 1200)/*their*(20+10), accept unsimplified
	Variance = $\dfrac{their\,112820}{30} - \left(\dfrac{their\,1750}{30}\right)^2 \quad (= 357.89)$	**M1**	substitute *their* Σx and Σx^2 into correct variance formula
	s.d. = 18.9	**A1**	
		4	

Question	Answer	Marks	Guidance
4(i)	$\dfrac{1}{4} + + p + p + \dfrac{3}{8} + 4p = 1$	**M1**	Unsimplified sum of probabilities equated to 1
	$p = \dfrac{1}{16}$	**A1**	If method FT from *their* incorrect **(i)**, expressions for E(X) and Var(X) must be seen unsimplified with all probabilities <1, condone not adding to 1
		2	

Question	Answer	Marks	Guidance
4(ii)	$[\text{E}(X)] = -\frac{1}{4}+\frac{1}{16}+\frac{6}{8}+1=\frac{25}{16}$	**M1**	May be implied by use in Variance, accept unsimplified
	$[\text{Var}(X)] = \frac{1}{4}+\frac{1}{16}+\frac{12}{8}+\frac{16}{4}-\left(\textit{their}\,\frac{25}{16}\right)^2$	**M1**	Substitute into correct variance formula, must have '– their mean^2'
	$\frac{863}{256}$ or 3.37	**A1**	OE
		3	
4(iii)	$\text{P}(X=2 \mid X>0) = \frac{\text{P}(X=2)}{\text{P}(X>0)} = \frac{\frac{3}{8}}{\frac{11}{16}}$	**M1**	Conditional probability formula used consistent with their probabilities
	$\frac{6}{11}$ or 0.545	**A1**	
		2	

Question	Answer	Marks	Guidance
5(i)	$156 - 55 = 99$	B1	$98 \leqslant$ answer < 100
		1	
5(ii)	90% of $160 = 144$	M1	144 seen, may be marked on graph
	(L =) 22	A1	
		2	
5(iii)	Median = 15.6 UQ = 18.8, LQ = 12.7	B1	$15.5 <$ median < 15.8
	IQR = $18.8 - 12.7$	M1	$18.5 <$ UQ $< 19 - 12.5 <$ LQ < 13
	6.1	A1	$6.0 \leqslant$ IQR $\leqslant 6.2$
		3	
5(iv)	The Median higher for Ransha (1st set of data)	B1	Any correct comparison of central tendency, must mention median
	IQR lower for Ransha (1st set of data)	B1	Any correct comparison of spread, must refer to IQR
		2	

Question	Answer	Marks	Guidance
6(i)	$\frac{9!}{2!} = 181\,440$	**B1**	Exact value
		1	
6(ii)	Total no of ways = $\frac{12!}{2!4!} = 9\,979\,200$ (A)	**B1**	Accept unevaluated
	With Ss together = $\frac{11!}{4!} = 1\,663\,200$ (B)	**B1**	Accept unevaluated
	With Ss not together = (B) – (A)	**M1**	Correct or $\frac{12!}{m} - \frac{8!}{n}, m, n$ integers > 1 or *their* identified total – *their* identified Ss together
	8 316 000	**A1**	Exact value
	Alternative method for question 6(ii)		
	_ T _ E _ E _ P _ L _ E _ C _ H _ A _ E _	**B1**	$10! \times k$ in numerator k integer $\geqslant 1$
	$\frac{10!}{4!} \times \frac{11 \times 10}{2!}$	**B1**	$4! \times k$ in numerator k integer $\geqslant 1$
	$\frac{their\,10!}{their\,4!} \times {}^{11}C_2$ or ${}^{11}P_2$	**M1**	OE
	8 316 000	**A1**	Exact value
		4	

Question	Answer	Marks	Guidance
6(iii)	S E E E : 1	**M1**	${}^6C_x$ seen alone or times $K > 1$
	S E E _ : ${}^6C_1 = 6$ S E _ _ : ${}^6C_2 = 15$ S _ _ _ : ${}^6C_3 = 20$	**B1**	${}^6C_3$ or ${}^6C_2$ or ${}^6C_1$ alone
	Add 3 or 4 correct scenarios	**M1**	No extras
	Total = 42	**A1**	
		4	

Question	Answer	Marks	Guidance
7(i)	$P(46 < X < 53) = P\left(\frac{46-49.2}{2.8} < Z < \frac{53-49.2}{2.8}\right)$	**M1**	Using $\pm$ standardisation formula for either 46 or 53, no continuity correction, σ^2 or $\sqrt{\sigma}$
	$P(-1.143 < Z < 1.357)$	**A1**	Both standardisations correct unsimplified
	$\Phi(1.357) + \Phi(1.143) - 1$ $= 0.9126 + 0.8735 - 1$	**M1**	Correct final area
	0.786	**A1**	Final answer
		4	

Question	Answer	Marks	Guidance
7(ii)	$\frac{t-49.2}{2.8}=-1.406$	**B1**	$\pm$1.406 seen
		M1	An equation using $\pm$ standardisation formula with a z-value, condone σ^2 or $\sqrt{\sigma}$
	45.3	**A1**	
		3	
7(iii)	$P(X<46)=0.1265$	**M1**	Calculated or ft from **(i)**
	$P(2PB<46)=3(1-0.1265)0.1265^2$	**M1**	$3(1-p)p^2$, $0<p<1$
	0.0419	**A1**	
		3	

Cambridge International AS & A Level

Cambridge Assessment International Education
Cambridge International Advanced Subsidiary and Advanced Level

MATHEMATICS **9709/62**

Paper 6 **October/November 2019**

MARK SCHEME

Maximum Mark: 50

Published

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The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

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Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
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SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To
NFWW	Not From Wrong Working

Question	Answer	Marks	Guidance
1(i)	Median = 51 UQ = 57.5, LQ = 40	**B1**	
	IQR = UQ – LQ	**M1**	$55 \leqslant \text{UQ} \leqslant 62 - 38 \leqslant \text{LQ} \leqslant 45$
	17.5	**A1**	NFWW
		3	
1(ii)	Result will be disproportionately affected by 110	**B1**	Affected by an extreme/large value There is a large outlier …contains outliers such as 110… Not 'mean affected by extreme values'
		1	

Question	Answer	Marks	Guidance
2(i)	$0.4x + 0.6 \times 2x = 0.36$ or $0.4(1-x) + 0.6(1-2x) = 0.64$	**M1**	$0.4a + (1 - 0.4)b = 0.36$ or 0.64, a,b terms involving x
	$1.6x = 0.36$ $x = 0.225$	**A1**	Fully justified by algebra AG
		2	

Question	Answer	Marks	Guidance
2(ii)	$P(H\|L') =$ $\frac{0.4(1-x)}{1-0.36} = \frac{0.4\times(1-0.225)}{0.64} = \frac{0.4\times 0.775}{0.4\times 0.775+0.6\times 0.55}$	**M1**	Correct numerical numerator of a fraction. Allow unsimplified.
		M1	Denominator 0.36 or 0.64. Allow unsimplified.
	$\frac{31}{64}$ or 0.484	**A1**	
		3	

Question	Answer	Marks	Guidance
3(i)	0.5 2.4 3 1.4 0.4	**M1**	At least 3 frequency densities calculated (frequency ÷ class width) e.g. $\left(\frac{10}{20}, \frac{10}{19} \text{or} \frac{10}{19.5}\right)$ may be read from graph using *their* scale, 3SF or exact
	All heights correct on graph.	**A1**	
	Bar ends of 9.5, 29.5, 39.5, 59.5, 89.5	**B1**	
	Axes labelled: Frequency density (fd) and speed/km h^{-1} (or appropriate title). Linear scales $9.5 \leqslant$ horizontal axis $\leqslant 89.5$, $0 \leqslant$ vertical axis $\leqslant 3$, 5 bars with no gaps	**B1**	
		4	

Question	Answer	Marks	Guidance
3(ii)	$\frac{19.5\times10+34.5\times24+44.5\times30+54.5\times14+74.5\times12}{\textit{their}\ 90}$ $=\frac{195+828+1335+763+894}{90}$ $=\frac{4015}{90}$ or $\frac{803}{18}$	M1	Uses at least 4 midpoint attempts (e.g. 19.5 $\pm$ 0.5). Allow unsimplified expression.
	$44\frac{11}{18}$ or 44.6 (km h^{-1})	A1	Final answer not an improper fraction NFWW
		2	

Question	Answer	Marks	Guidance
4(i)	P(8, 9, 10) = ${}^{10}C_8\ 0.66^8\ 0.34^2 + {}^{10}C_9\ 0.66^9\ 0.34^1 + 0.66^{10}$	M1	Correct binomial term, ${}^{10}C_a\ 0.66^a(1–0.66)^b$ $a+b = 10$, $0 < a,b < 10$
		A1	Correct unsimplified expression
	0.284	B1	CAO
		3	

Question	Answer	Marks	Guidance
4(ii)	$np = 0.66 \times 150 = 99$ $npq = 0.66 \times (1-0.66) \times 150 = 33.66$	**B1**	Accept evaluated or unsimplified μ, σ^2 numerical expressions, condone $\sigma = \sqrt{33.66} = 5.8017 \text{ or } 5.802$ CAO
	$P(X > 84) = P\left(Z > \frac{84.5 - 99}{\sqrt{33.66}}\right)$	**M1**	$\pm$ Standardise, $\frac{x - \textit{their}\ 99}{\sqrt{\textit{their}\ 33.66}}$, condone σ^2, x a value
		M1	84.5 or 83.5 used in *their* standardisation formula
	$(= P(Z > -2.499))$	**M1**	Correct final area
	0.994	**A1**	Final answer (accept 0.9938) SC if no standardisation formula seen, B2 P(Z > -2.499) = 0.994
		5	

Question	Answer	Marks	Guidance
5(i)	<table><tr><td>x</td><td>–1</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>p</td><td>$\frac{1}{12}$</td><td>$\frac{1}{12}$</td><td>$\frac{3}{12}$</td><td>$\frac{2}{12}$</td><td>$\frac{3}{12}$</td><td>$\frac{2}{12}$</td></tr></table>	**B1**	Table with correct values of x, at least 1 probability, all probabilities $\leqslant 1$
		B1	2 probabilities correct, may not be in table
		B1	2 more probabilities correct, may not be in table
		B1	All correct, values in table SC1 No more than 1 correct probability and at least 5 probabilities summing to 1 in table
		4	
5(ii)	$[\mathrm{E}(X)] = \left(\frac{-1+0+3+4+9+8}{12}\right) = \frac{23}{12}$	**M1**	May be implied by use in variance. Allow unsimplified expression
	$[\mathrm{Var}(X)] = \frac{1+0+3+8+27+32(=71)}{12} - \left(\frac{23}{12}\right)^2$	**M1**	Appropriate variance formula using *their* $\mathrm{E}(X)^2$
	2.24 or $\frac{323}{144}$ or $2\frac{35}{144}$	**A1**	CAO
		3	

Question	Answer	Marks	Guidance
6(i)	$P(X<45)=P\left(Z<\frac{45-40}{8}\right)$ $=P(Z<0.625)$	M1	$\pm$ Standardise, no continuity correction, σ^2 or $\sqrt{\sigma}$, formula must be seen
	0.734(0)	A1	CAO
		2	
6(ii)	$1-2(1-(i))=2(i)-1=2((i)-0.5)$	M1	Use result of **part (i)** or recalculated to find area OE
	0.468	A1ft	$0<$ FT from **(i)** <1 or correct.
		2	
6(iii)	$P(X<10)=48/500=0.096$ $z=-1.305$	B1	$z=\pm 1.305$
	$P(X>24)=76/500=0.152$ $z=1.028$	B1	$z=\pm 1.028$
	$10-\mu=-1.305\sigma$ $24-\mu=1.028\sigma$	M1	Form 1 equation using 10 or 24 with μ,σ,z–value. Allow continuity correction, not $\sigma^2,\sqrt{\sigma}$
	$14=2.333\sigma$	M1	OE Solve two equations in σ and μ to form equation in one variable
	$\sigma=6.[00],\quad \mu=17.8[3]$	A1	CAO, WWW
		5	

Question	Answer	Marks	Guidance
7(i)	$6! = 720$	**B1**	Evaluated
		1	
7(ii)	Total no of arrangements: $\frac{9!}{2!3!} = 30240$	**B1**	Accept unevaluated
	No with Ts together $= \frac{8!}{3!} = 6720$	**B1**	Accept unevaluated
	With Ts not together: 30 240 – 6720	**M1**	correct or $\frac{9!}{m} - \frac{8!}{n}, m, n$ integers > 1 or *their* identified total – *their* identified Ts together
	23 520	**A1**	CAO
	Alternative method for question 7(ii)		
	$\frac{7!}{3!} \times \frac{8 \times 7}{2}$	**B1**	$7! \times (k > 0)$ in numerator, cannot be implied by $^7P_2$, etc.
		B1	$3! \times (k > 0)$ in denominator
		M1	$\frac{\textit{their } 7!}{\textit{their } 3!} \times {}^8C_2$ or ${}^8P_2$
	23 520	**A1**	CAO
		4	

Question	Answer	Marks	Guidance
7(iii)	Number of arrangements = $\frac{7!}{3!}$ Probability = $\dfrac{their\ \frac{7!}{3!}}{their\ \frac{9!}{3!2!}} = \frac{840}{30240}$	M1	$\dfrac{their \text{ identified number of arrangements with T at ends}}{their \text{ identified total number of arrangements}}$ $or\ \dfrac{\frac{7!}{m}}{\frac{9!}{n}}\ m, n$ integers > 1
	$\frac{1}{36}$ or 0.0278	A1	Final answer
		2	
7(iv)	OOT_ _ $\quad {}^4C_2 = 6$ OOTT_ $\quad {}^4C_1 = 4$ OOOT_ $\quad {}^4C_1 = 4$ OOOTT $\quad = 1$	M1	${}^4C_x$ seen alone or ${}^4C_x$ x $k \geq 1$, k an integer, $0 < x < 4$
		A1	${}^4C_2$ x k, $k = 1$ oe or ${}^4C_1$ x m, $m = 1$ oe alone
		M1	Add 3 or 4 identified correct scenarios only, accept unsimplified
	(Total) = 15	A1	CAO, WWW Only dependent on 2nd M mark
		4	

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MATHEMATICS	**9709/63**
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Question	Answer	Marks	Guidance
1(i)	$\frac{120}{300} = 0.4$	**B1**	OE
		1	
1(ii)	P(male) × P(not piano) = $\frac{160}{300} \times \frac{225}{300}\left(\frac{8}{15} \times \frac{3}{4}\right) = \frac{2}{5}$	**M1**	P(M) × P(P') seen Can be unsimplified but the events must be named in a product
	As P(male ∩ not piano) also = $\frac{120}{300} = \frac{2}{5}$ The events are **Independent**	**A1**	Numerical comparison and correct conclusion
	Alternative method for question 1(ii)		
	P(male ∩ not piano) = $\frac{120}{300}$; P(not piano) = $\frac{225}{300}$	**M1**	P(M|P') or P(P'|M) unsimplified seen with *their* probs with correctly named events
	P(M | not piano) = $\frac{\frac{120}{300}}{\frac{225}{300}} = \frac{120}{225} = \frac{8}{15}$ = P(male) **or** P(not piano | M) = $\frac{\frac{120}{300}}{\frac{160}{300}} = \frac{120}{160} = \frac{3}{4}$ = P(not piano) Therefore the events are **Independent**	**A1**	Numerical comparison with P(M) or P(P') and correct conclusion
		2	

Question	Answer	Marks	Guidance
2(i)	$\frac{9!}{2!3!} = 30240$	**B1**	9! Divided by at least one of 2! or 3!
		B1	Exact value
		2	
2(ii)	D _ _ _ _ _ _ _ R: $\frac{7!}{2!2!} = 1260$ D _ _ _ _ _ _ _ O: $\frac{7!}{3!} = 840$	**B1**	7! Seen alone or as numerator in a term, can be multiplied not + or –
		B1	One term correct, unsimplified
	Total = 2100	**B1**	Final answer
		3	

Question	Answer	Marks	Guidance
3(i)	$3A\ 2D\ 2M$: ${}^6C_3 \times {}^5C_2 \times {}^4C_2$ (= 1200) $4A\ 2D\ 1M$: ${}^6C_4 \times {}^5C_2 \times {}^4C_1$ (= 600) $3A\ 3D\ 1M$: ${}^6C_3 \times {}^5C_3 \times {}^4C_1$ (= 800)	**M1**	${}^6C_x \times {}^5C_y \times {}^4C_z$, $x + y + z = 7$
		A1	2 correct products, allow unsimplified
		M1	Summing their totals for 3 correct scenarios only
	Total = 2600	**A1**	Correct answer **SC1** ${}^6C_3 \times {}^5C_2 \times {}^4C_1 \times {}^9C_1 = 7200$
		4	

Question	Answer	Marks	Guidance
3(ii)	${}^7C_4 \times 1$	**B1**	${}^7C_3$ or ${}^7C_4$ seen anywhere
	35	**B1**	
		2	

Question	Answer	Marks	Guidance
4(i)	$P(h < 148) = 0.67$	**B1**	$z = \pm 0.44$ seen
	$\frac{h-148}{8} = 0.44$	**M1**	z-value $= \pm\frac{(h-148)}{8}$
	$151.52 \approx 152$	**A1**	CAO
		3	
4(ii)	$P(144 < X < 152) = P\left(\frac{144-148}{8} < Z < \frac{152-148}{8}\right)$	**M1**	Using ± standardisation formula for either 144 or 152, $\mu = 148$, $\sigma = 8$ and no continuity correction, allow σ^2 or $\sqrt{\sigma}$
	$= P\left(-\frac{1}{2} < Z < \frac{1}{2}\right) = 0.6915 - (1 - 0.6915) = 2 \times 0.6915 - 1$	**M1**	Correct final area legitimately obtained from phi(*their* z_2) – phi(*their* z_1)
	$= 0.383$	**A1**	Final probability answer
	$0.383 \times 120 = 45.96$ Accept 45 or 46 only	**B1FT**	*Their* prob (to 3 or 4 sf) × 120, rounded to a whole number or truncated
		4	

Question	Answer	Marks	Guidance
5(i)	Correct labels and scales	**B1**	Axes labelled 'cumulative frequency' (or cf) and 'time (or t) [in] min(utes)', linear scales from 0 to 90 and 0 to 200 with at least 3 values marked on each axis.
	7 correctly plotted points above upper boundaries joined in a curve or line segments	**B1**	(0, 0); (10, 16); (20, 50); (30, 106); (50, 146); (70,176); (90,200)
		2	
5(ii)	29	**B1**	28 $\leqslant$ median $\leqslant$ 30
		1	
5(iii)	120 seen	**M1**	For seeing 120 in a calculation or marked on the graph
	37	**A1FT**	36 $\leqslant$ Ans $\leqslant$ 39 or FT from *their* graph **SC1** unsupported answer in range
		2	
5(iv)	Frequencies 16 34 56 40 30 24	**B1**	Seen. Allow unsimplified
	Est. Mean = $\frac{5\times16+15\times34+25\times56+40\times40+60\times30+80\times24}{200}$	**M1**	At least 4 correct midpoints (5, 15, 25, 40, 60, 80) used in a calculation
	$\frac{7310}{200}$	**M1**	Summing products of *their* 6 mid-points (not lower or upper bound or class width) × *their* frequencies / 200 (or *their* $\sum$f), unsimplified
	36.55	**A1**	Accept 36.6
		4	

Question	Answer	Marks	Guidance
6(i)	P(RR) = $\frac{3}{8}\times\frac{2}{7}=\frac{3}{28}$	**B1**	OE
		1	
6(ii)	P(RW) + P(WR) $\frac{3}{8}\times\frac{5}{7}+\frac{5}{8}\times\frac{3}{7}$	**M1**	Method shown, numerical calculations identified, may include replacements
	$=\frac{15}{28}$	**A1**	AG, Fully correct calculations
	Alternative method for question 6(ii)		
	1 – (P(RR) + P(WW) $1-\left(\frac{3}{28}+\frac{5}{8}\times\frac{4}{7}\right)$	**M1**	Method shown, numerical calculations identified, may include replacements
	$=\frac{15}{28}$	**A1**	AG, Fully correct calculations
		2	
6(iii)	P(first red|second red) = $\frac{\textit{their}\ \textbf{(i)}}{\textit{their}\ \textbf{(i)}+\frac{5}{8}\times\frac{3}{7}}=\frac{\frac{3}{8}\times\frac{2}{7}}{\frac{3}{8}\times\frac{2}{7}+\frac{5}{8}\times\frac{3}{7}}=\frac{\frac{3}{28}}{\frac{21}{56}}$	**M1**	Conditional probability formula used consistent with *their* probabilities or correct
	$=\frac{2}{7}$	**A1**	OE
		2	

Question	Answer	Marks	Guidance
6(iv)	<table><tr><td>x</td><td>0</td><td>1</td><td>2</td></tr><tr><td>p</td><td>$\frac{10}{28}$</td><td>$\frac{15}{28}$</td><td>$\frac{3}{28}$</td></tr></table>	**B1**	Probability distribution table with correct values of x and at least one correct probability placed. Extra x values allowed with probability of zero stated.
		B1FT	Fully correct FT P(2) = *their* **(i)**, P(1) = *their* **(ii)**, $\Sigma(p) = 1$.
		2	
6(v)	$E(X) = \frac{30}{56} + \frac{12}{56} = \frac{42}{46} \quad \left(= \frac{3}{4}\right)$	**B1**	May be implied by use in variance formula
	$\text{Var}(X) = \frac{30}{56} + \frac{24}{56} - \left(\textit{their } \frac{3}{4}\right)^2$	**M1**	Substitute into correct variance formula, must have '$-$ *their* mean^2' Must be for 2 or more non-zero x-values
	$\frac{45}{112}$ or 0.402	**A1**	Correct final answer
		3	

Question	Answer	Marks	Guidance
7(i)(a)	$P(0, 1, 2) = {}^6C_0\, 0.3^0\, 0.7^6 + {}^6C_1\, 0.3^1\, 0.7^5 + {}^6C_2\, 0.3^2\, 0.7^4$	**M1**	Binomial term of form ${}^6C_x p^x (1-p)^{6-x}$ $\quad 0 < p < 1$ any p, $x \neq 6,0$
	0.1176 ... + 0.3025 ... + 0.3241 ...	**A1**	Correct unsimplified answer
	0.744	**A1**	Correct final answer
		3	

Question	Answer	Marks	Guidance
7(i)(b)	P(support neither choir) = 1 – (0.3 + 0.45) = 0.25	**M1**	0.25^n seen alone, $1 < n \leqslant 6$
	P(6 support neither choir) = 0.25^6 $= 0.000244$ or $\frac{1}{4096}$	**A1**	Correct final answer
		2	
7(ii)	Mean = 240 × 0.25 = 60 Variance = 240 × 0.25 × 0.75 = 45	**B1FT**	Correct unsimplified *240p* and *240pq* where p =*their* P(support neither choir) or 0.25
	$P(X < 50) = P\left(Z < \frac{49.5 - 60}{\sqrt{45}}\right) = P(Z < -1.565)$	**M1**	Substituting *their* μ and σ (condone σ^2) into the ±Standardisation Formula with a numerical value for *'49.5'*.
		M1	Using continuity correction 49.5 or 50.5 within a standardisation expression
	1 – 0.9412	**M1**	Appropriate area Φ from standardisation formula P(z< …) in final solution, (< 0.5 if z is –ve, > 0.5 if z is +ve)
	0.0588	**A1**	Correct final answer
		5	

Cambridge International AS & A Level

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MATHEMATICS **9709/61**

Paper 6 **May/June 2019**

MARK SCHEME

Maximum Mark: 50

Published

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Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

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Cambridge International is publishing the mark schemes for the May/June 2019 series for most Cambridge IGCSE™, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

Cambridge Assessment International Education

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more “method” steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously “correct” answers or results obtained from incorrect working.

  - Note: B2 or A2 means that the candidate can earn 2 or 0.
    B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF/OE Any Equivalent Form (of answer is equally acceptable) / Or Equivalent

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

CAO Correct Answer Only (emphasising that no “follow through” from a previous error is allowed)

CWO Correct Working Only – often written by a ‘fortuitous’ answer

ISW Ignore Subsequent Working

SOI Seen or implied

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become “follow through” marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Question	Answer	Marks	Guidance
1(i)	$\Sigma(t-120) = -25 + 6 - 3 + 15 + 0 + 5 - 6 - 1 + 16 = 7$	**M1**	Attempt to sum both $(t-120)$ and $(t-120)^2$ Correct ans using $\Sigma t - 9 \times 120$ and $\Sigma(t-120)^2$ M1A1
	$\Sigma(t-120)^2 = 25^2 + 6^2 + 3^2 + 15^2 + 0^2 + 5^2 + 6^2 + 1^2 + 16^2 = 1213$	**A1**	Both correct, www, **SC** correct ans no working **B1B1**
		2	
1(ii)	$\text{Var} = \frac{\Sigma(t-120)^2}{9} - \left(\frac{\Sigma(t-120)}{9}\right)^2 = \frac{their\,1213}{9} - \left(\frac{their\,7}{9}\right)^2$	**M1**	Using two coded values in correct formula including finding Σt from 7 etc
	$= 134(.2)$	**A1**	Correct answer **SC** if correct variance obtained by another method from raw data give **SCB1**
		2	

Question	Answer	Marks	Guidance
2	Jameel: P(plum) = $\frac{5}{8}$, Rosa: P(plum) = $\frac{x}{x+6}$	**M1**	*Their* 2 probabilities for P(plum) multiplied and equated to 1/4
	$\frac{5}{8} \times \frac{x}{x+6} = \frac{1}{4}$	**A1**	Correct equation oe
	$(x =)\ 4$	**A1**	**SC** correct answer with no appropriate equations i.e. common sense **B1**
		3	

Question	Answer	Marks	Guidance
3	P(X) = $\frac{3}{36}$ $\left(\frac{1}{12}oe\right)$	**B1**	
	P(Y) = $\frac{12}{36}$ $\left(\frac{1}{3}oe\right)$	**B1**	
	P(X∩Y) = $\frac{1}{36}$	**M1**	Independent method to find P($X \cap Y$) without multiplication, either stated or by listing or circling numbers on a probability space diagram. OR condititional prob with a single fraction numerator
	P(X) × P(Y) = P(X∩Y), independent	**A1**	Numerical comparison and conclusion, www
		4	

Question	Answer	Marks	Guidance
4	Median Maths = 40	**M1**	Indication of finding medians, such as mark on graph or reference marks to 700 pupils, condone poor terminology such as 'mean'
	Median English = 55	**A1**	Both values correct, condone 54<English<56 but 54, 56 get A0
	Median of English is larger than median of Maths	**B1**	Correct statement, median must be referenced within answer. No credit if statement references 'means'
	Range Maths is 100 or IQ range Maths = 80 – 12 = 68	**M1**	Evidence of finding either both ranges or both IQ ranges i.e. see a minus
	Range English is 60 or IQ range English = 62 – 42 = 20	**A1**	Both ranges or IQR correct
	Maths marks have more spread then English marks	**B1**	Correct conclusion. Accept standard deviation but must see some figures
		6	

Question	Answer	Marks	Guidance
5(i)	(P > 12) = P(13, 14, 15)	**M1**	Binomial term of form ${}^{15}C_x p^x(1-p)^{15-x}$ $0 < p < 1$ any p, $x \neq 15,0$
	$= {}^{15}C_{13}(0.65)^{13}(0.35)^2 + {}^{15}C_{14}(0.65)^{14}(0.35)^1 + (0.65)^{15}$	**A1**	Correct unsimplified answer
	= 0.0617	**A1**	SC if use np and npq with justification give (12.5 – 9.75)/√3.41 M1 1–F(1.489) A1 0.0681 A0
		3	
5(ii)	mean = 250 × 0.65 = 162.5 variance = 250 × 0.65 × 0.35 = 56.875	**B1**	Correct unsimplified *np* and *npq*
	$P(< 179) = P(z < \frac{178.5 - 162.5}{\sqrt{56.875}}) = P(z < 2.122)$	**M1**	Substituting *their* μ and σ (condone σ^2) into the Standardisation Formula with a numerical value for *'178.5'*. Continuity correct not required for this M1. Condone ± standardisation formula
	Using continuity correction 178.5 or 179.5	**M1**	
	= 0.983	**A1**	Correct final answer
		4	

Question	Answer	Marks	Guidance
6(i)	P(loses \$1) = P(F and F) = 0.8 × 0.8	**M1**	0.8 x 0.8 or (1 – 0.2)(1-0.2) or P(F) × P(F) or P(F)+P(F) seen or implied
	= 0.64 AG	**A1**	Must see probabilities multiplied together with final answer and a clear probability statement or implied by labelled tree diagram
		2	

Question	Answer	Marks	Guidance
6(ii)	Amount gained ($): –1, 0.50, 2; Prob: , 0.16, 0.2	B1	–1 linked with 0.64 in table
		B1	0.5 seen in table
		B1	0.16 seen in table linked to their 0.5
		B1	**FT** P(2.00 gained) = 0.36 – P(0.50 gained) or correct, and all amount gained linked correctly in table
		4	
6(iii)	E(winnings) = –1 × 0.64 + 0.5 × 0.16 + 2 × 0.2 = –($)0.16, –16 cents	B1	**FT** Accept ($)0.16 or 16 cents **loss.** FT unsimplified E(winnings) from their table provided $\Sigma p = 1$
		1	

Amount gained ($)	–1	0.50	2
Prob		0.16	0.2

Question	Answer	Marks	Guidance
7(i)	$\text{P}(<700) = \text{P}\left(z < \frac{700-830}{120}\right) = \text{P}(z < -1.083)$	M1	Using ± standardisation formula, no continuity correction, not σ^2 or $\sqrt{\sigma}$
	= 1 – 0.8606	M1	Appropriate area Φ from standardisation formula P(z<….) in final probability solution, (<0.5 if z is –ve, >0.5 if z is +ve)
	= 0.1394	A1	Correct final probability rounding to 0.139
	Expected number of female adults = 430 × *their* 0.1394 = 59.9 So 59 or 60	B1	**FT** *their* 3 or 4 SF probability, rounded or truncated to integer
		4	

Question	Answer	Marks	Guidance
7(ii)	P(giraffe < 830+w) = 95% so z = 1.645	**B1**	±1.645 seen (critical value)
	$\frac{(830+w)-830}{120} = \frac{w}{120} = 1.645$	**M1**	An equation using the standardisation formula with a z-value (not 1 – z), condone σ^2 or $\sqrt{\sigma}$ not 0.8519, 0.8289
	w = 197	**A1**	Correct answer
		3	
7(iii)	P(male > 950) = 0.834, so z = –0.97	**B1**	± 0.97 seen
	$\frac{950-1190}{\sigma} = -0.97$	**M1**	Using ± standardisation formula, condone continuity correction, σ^2 or $\sqrt{\sigma}$, condone equating with non z-value not 0.834, 0.166
	σ = 247	**A1**	Condone $-\sigma = -247$. www.
		3	

Question	Answer	Marks	Guidance
8(i)	($^9C_4$ =) 126	**B1**	
		1	
8(ii)	$^7C_2$	**B1**	^7C_x or $^yC_2$ (implied by correct answer) or ^7P_x or ^7P_y, seen alone
	= 21	**B1**	correct answer
		2	

Question	Answer	Marks	Guidance
8(iii)	_ C_1 (B_1 B_2 B_3) C_2 _ C_3 _ C_4 _ C_5 _ C_6	**B1**	3! or 6! seen alone or multiplied by k > 1 need not be an integer
	3! × 6! × 7	**B1**	3! and 6! seen multiplied by k > 1, integer, no division
	= 30240	**B1**	Exact value
	Alternative method for question 8(iii)		
	C_1 (B_1 B_2 B_3) C_2 C_3 C_4 C_5 C_6	**B1**	3! or 7! seen alone or multiplied by k > 1 need not be an integer
	3! × 7!	**B1**	3! and 7! seen multiplied by k > or = 1, no division
	= 30240	**B1**	Exact value
		3	
8(iv)	C_1 _ C_2 _ C_3 _ C_4 _ C_5 _C_6	**B1**	6! or 4! X 6P2 seen alone or multiplied by k > 1, no division (arrangements of cars)
	6! × 5P3 *or* 6! × 5 × 4 × 3 or 6! x 3! x10	**B1**	Multiply by 5P3 oe i.e. putting Bs in between 4 of the Cs OR multiply by 3! x n where n = 7, 8, 9, 10 (number of options)
	= 43200	**B1**	Correct answer
		3	

Cambridge International AS & A Level

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MATHEMATICS	**9709/62**
Paper 6	**May/June 2019**
MARK SCHEME	
Maximum Mark: 50	

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CWO	Correct Working Only – often written by a 'fortuitous' answer
ISW	Ignore Subsequent Working
SOI	Seen or implied
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

<u>Penalties</u>

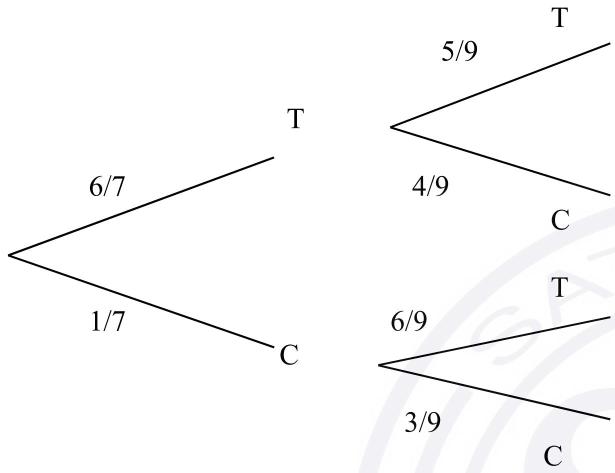
MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Question	Answer	Marks	Guidance
1	$P(S) = \frac{1}{2}$	**B1**	
	$P(T) = \frac{16}{36}\left(\frac{4}{9}\right)$	**B1**	
	$P(S \cap T) = \frac{10}{36}\left(\frac{5}{18}\right)$	**M1**	$P(S \cap T)$ found by multiplication scores M0 M1 awarded if *their* value is identifiable in their sample space diagram **or** Venn diagram **or** list of terms **or** probability distribution table (oe)
	$P(S)\,P(T) \neq P(S \cap T)$ so not independent	**A1**	8/36, 10/36 P(S) × P(T) and $P(S \cap T)$ seen in workings and correct conclusion stated, www
	Alternative method for question 1		
	$P(S) = \frac{1}{2}$	**B1**	
	$P(T) = \frac{16}{36}\left(\frac{4}{9}\right)$	**B1**	
	$P(S \cap T) = \frac{10}{36}\left(\frac{5}{18}\right)$	**M1**	$P(S \cap T)$ found by multiplication scores M0 M1 awarded if *their* value is identifiable in their sample space diagram **or** Venn diagram **or** list of terms **or** probability distribution table (oe)
	$P(S\|T) = \frac{10}{16}$ or $P(T\|S) = \frac{10}{18}$ $P(S\|T) \neq P(S)$ or $P(T\|S) \neq P(T)$ so not independent	**A1**	**Either** 18/36, 10/16,P(S) and $P(S\|T)$ seen in workings and correct conclusion stated, www **Or** 16/36, 10/18, P(T) and $P(T\|S)$ seen in workings and correct conclusion stated, www
		4	

Question	Answer	Marks	Guidance
2	$\text{P}(<28.9) = \text{P}\left(z < \frac{28.9-30}{1.5}\right)$	**B1**	Using ± standardising formula, no continuity correction, not σ^2 or $\sqrt{\sigma}$,
	$= \text{P}(z < -0.733)$ $= 1 - 0.7682$	**M1**	Appropriate area Φ from standardisation formula P(z <….) in final probability solution, Must be a probability, e.g. 1 – 0.622 is M0
	$= 0.2318$	**A1**	Correct final probability rounding to 0.232. (Only requires M1 not B1 to be awarded
	Number of cartridges is *their* 0.2318 × 8 = 1.85, so 2 (Also accept 1 but not both)	**B1**	**FT** using *their* 4 SF (or better) value, ans. rounded or truncated to integer, no approximation indicated.
		4	

Question	Answer	Marks	Guidance
3(i)	P(at most 7) = 1 – P(8, 9, 10) $= 1 - {}^{10}C8(0.35)^8(0.65)^2 - {}^{10}C_9(0.35)^9(0.65)^1 - (0.35)^{10}$	**M1**	Use of normal approximation M0 Binomial term of form ${}^{10}C_x p^x(1-p)^{10-x}$ $0 < p < 1$ any p, $x \neq 10{,}0$
	[= 1 – 0.004281 – 0.0005123 – 0.00002759]	**A1**	Correct unsimplified (or individual terms evaluated) answer seen Condone 1 – A + B + C leading to correct solution
	= 0.995	**B1**	B1 **not** dependent on previous marks.
	Alternative method for question 3(i)		
	P(at most 7) = P(0,1,2,3,4,5,6,7)	**M1**	Binomial term of form ${}^{10}C_x p^x(1-p)^{10-x}$ $0 < p < 1$ any p, $x \neq 10{,}0$
	$= (0.65)^{10} + {}^{10}C1(0.35)^1(0.65)^9 + \ldots + {}^{10}C_7(0.35)^7(0.65)^3$	**A1**	Correct unsimplified answer or individual terms evaluated seen
	= 0.995	**B1**	
		3	
3(ii)	$1 - (0.65)^n > 0.99$ $0.01 > (0.65)^n$	**M1**	Equation or inequality with $(0.65)^n$ **and** 0.01 or $(0.35)^n$ **and** 0.99 only (Note 1 – 0.99 is equivalent to 0.01 etc.)
	$n > 10.69$	**M1**	Solving their $a^n = c$, $0 < a,c < 1$ using logs **or** Trial and Error If answer inappropriate, at least 2 trials are required for Trial and Error M mark
	smallest $n = 11$	**A1**	CAO
		3	

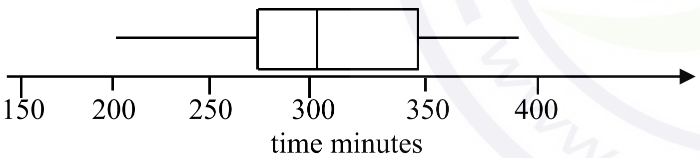
Question	Answer	Marks	Guidance
4	$z = 0.842 = \left(\dfrac{121-\mu}{\sigma}\right)$ so $0.842\sigma = 121 - \mu$	**B1**	± 0.842 seen but B0 if 1 ± 0.842 oe seen
		M1	One appropriate standardisation equation with a z-value, μ, σ and 121 **or** 102, condone continuity correction. Not 0.158, 0.42,…
	$z = -0.58 = \left(\dfrac{102-\mu}{\sigma}\right)$ so $-0.58\sigma = 102 - \mu$	**B1**	± 0.58(0) seen but B0 if 1 ± 0.58 oe seen
	Solving	**M1**	Correct algebraic elimination of μ or σ from *their* two simultaneous equations to form an equation in one variable, condone 1 numerical slip
	$\sigma = 13.4$ $\mu = 110$	**A1**	If M0A0 scored (i.e. no algebraic elimination seen), **SC B1** can be awarded for both answers correct Consistent use of σ^2 or $\sqrt{\sigma}$ throughout apply **MR** penalty to A mark or SC B mark.
		5	

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td rowspan="4">5(i)</td><td rowspan="3">
</td><td>B1</td><td>First pair of branches labels and probs correct (6/7 and 1/7 or rounding to 0.857 and 0.143)

(Labelling must be logically…e.g. (T and T) or (T and Not T) would be acceptable)</td></tr>
<tr><td>B1</td><td>Either of second top pair or bottom of branches labels and probs correct</td></tr>
<tr><td>B1</td><td>Both second pairs of branches labels and probs correct. No additional / further branches.</td></tr>
<tr><td></td><td>3</td><td></td></tr>
<tr><td rowspan="4">5(ii)</td><td rowspan="3"><table><tr><td>No of toffees taken (T)</td><td>0</td><td>1</td><td>2</td></tr><tr><td>prob</td><td>$\frac{3}{63}$, 0.0476(2)</td><td>$\frac{30}{63}$, 0.476(2)</td><td>$\frac{30}{63}$, 0.476(2)</td></tr></table></td><td>B1</td><td>P(1) correct</td></tr>
<tr><td>B1</td><td>P(0) or P(2) correct</td></tr>
<tr><td>B1</td><td>FT Correct values in table, any additional values of T have stated probability of zero. For FT $\Sigma p = 1$,</td></tr>
<tr><td></td><td>3</td><td></td></tr>
<tr><td rowspan="2">5(iii)</td><td>$\mathrm{E}(X) = \frac{90}{63}$ $\left(\frac{10}{7}\right)$ (1.43)</td><td>B1</td><td>Not FT</td></tr>
<tr><td></td><td>1</td><td></td></tr>
</table>

Question	Answer	Marks	Guidance
5(iv)	$P(1^{st}\ C \mid 2^{nd}\ T) = \dfrac{P(C \cap T)}{P(T)} = \dfrac{\frac{1}{7} \times \frac{6}{9}}{\frac{1}{7} \times \frac{6}{9} + \frac{6}{7} \times \frac{5}{9}} = \dfrac{\frac{6}{63}}{\frac{36}{63}}$	B1	$P(C \cap T)$ attempt seen as numerator of a fraction, consistent with *their* tree diagram or correct
		M1	Summing 2 appropriate two-factor probabilities, consistent with *their* tree diagram or correct seen anywhere
		A1	$\frac{36}{63}$ oe or correct unsimplifed expression seen as numerator or denominator of a fraction
	$\frac{1}{6}$ oe	A1	Final answer
		4	

Question	Answer	Marks	Guidance
6(i)	Advantage: comment referring to spread or range or shape	**B1**	Comments referring to quartiles, IQR, Range, median, shape, skewness, data distribution, spread score B1 Any comments with reference to mean or standard deviation or any other 'disadvantage' will score B0 Comments referring to '5-value plot', comparison with another data set, overview or ease of drawing/plotting/reading require an appropriate advantage statement.
	Disadvantage: comment referring to limited data information provided	**B1**	Comments referring to no individual data, no information about the number of values, unable to calculate mean, standard deviation, variance and mode score B1 Any comments with reference to median, shape or any other 'advantage' will score B0 Comments referring to 'size of data set' or 'average' require an appropriate disadvantage statement. Comments referring to outliers are ignored in all cases (as outliers are not in the syllabus content) unless supported by an appropriate advantage / disadvantage statement. If comments not clearly identified, assume first comment is the advantage.
		2	

Question	Answer	Marks	Guidance
6(ii)	Not mean as data skewed by one large value	**B1**	Comment which identifies 768 (or 'a very large number') as the problem. Condone the use of 'outlier'
	Not mode as frequencies all the same	**B1**	Comment which indicates that no mode exists (e.g. all the data is different, there is no repeated number, all the values are different)
	Median	**B1**	Median identified as choice, dependent upon statements for mean and mode being given, even if incorrect or very general.
	SC: Mean is identified as most suitable		
	Not mode as frequencies all the same	**SCB1**	Comment which indicates that no mode exists
	Not median as not all values used	**SCB1**	Comment which indicates limitation of median e.g. median is not in middle of range.
		3	
6(iii)(a)	LQ = 256 or 256.5 Med = 280 UQ = 329 Min 190 max 375 	**B1**	Median, UQ and LQ values seen, may not be identified or identified correctly. (Not read from box plot unless value stated)
		B1	**FT** Median and quartiles plotted in box on graph, linear scale
		B1	Correct end points, whiskers from ends of box but not through box, not at top or bottom of box
		B1	Uniform scale from 190 to 375 (need at least 3 linear identified points min) and labelled 'time' and 'minutes' (can be in title) **No time axis or time axis with no scale attempt,** Max B1B0B0B0
		4	

Question	Answer	Marks	Guidance
6(iii)(b)	IQR = *their* 329 – *their* 256 = 73 or 72.5	B1	**FT Must** follow through only from ***their*** stated values (condone if correct quartiles stated here), not reading from graph.
		1	

Question	Answer	Marks	Guidance
7(a)	${}^6C_3 \times {}^3C_2 \times {}^1C_1$	M1	${}^6C_a \times {}^{6-a}C_b \times {}^{6-a-b}C_{6-a-b}$ seen oe ${}^{6-a-b}C_{6-a-b}$ can be implied by 1 or omission, condone use of permutations,
	= 20 × 3	A1	Any correct method seen no addition/additional scenarios
	= 60	A1	Correct answer
	Alternative method for question 7(a)		
	$\frac{{}^6P_6}{{}^3P_3 \times {}^2P_2 \times {}^1P_1} = \frac{6!}{3! \times 2!}$	M1	${}^6P_6$ / (${}^nP_n$ x k) with $3 \geqslant n > 1$and $6 \geqslant$ k an integer $\geqslant 1$, not 6!/1
		A1	Correct method with no additional terms
	= 60	A1	Correct answer
		3	
7(b)(i)	$\frac{4!}{3!} \times \frac{3!}{2!} \times 2$	M1	A single expression with either 4!/3! × k or 3!/2! × k, k a positive integer seen oe (condone 2 identical expressions being added)
		M1	Correctly multiplying *their* single expression by 2 or 2 identical expressions being added.
	= 24	A1	Correct answer
		3	

Question	Answer	Marks	Guidance
7(b)(ii)	Total no of arrangements = $\frac{7!}{2!3!}$ = 420 (A)	**B1**	Accept unsimplified
	No with 2s together = $\frac{6!}{3!}$ = 120 (B)	**B1**	Accept unsimplified
	With 2s not together: *their* (A) – *their* (B)	**M1**	Subtraction indicated, possibly by *their* answer, no additional terms present
	= 300 ways	**A1**	Exact value www
	Alternative method for question 7(b)(ii)		
	3 _ 7 _ 7 _ 7 _ 8 _		
	$\frac{5!}{3!}\times\frac{6\times5}{2}$	**B1**	*k* x 5! in numerator, *k* a positive integer
		B1	*m* x *3*! In denominator, *m* a positive integer
		M1	*Their* 5!/3! multiplied by $^6C_2$ only (no additional terms)
	= 300 ways	**A1**	Exact value www
		4	

Cambridge Assessment International Education
Cambridge International Advanced Subsidiary and Advanced Level

MATHEMATICS **9709/63**

Paper 6 **May/June 2019**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2019 series for most Cambridge IGCSE™, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

  - Note: B2 or A2 means that the candidate can earn 2 or 0.
    B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

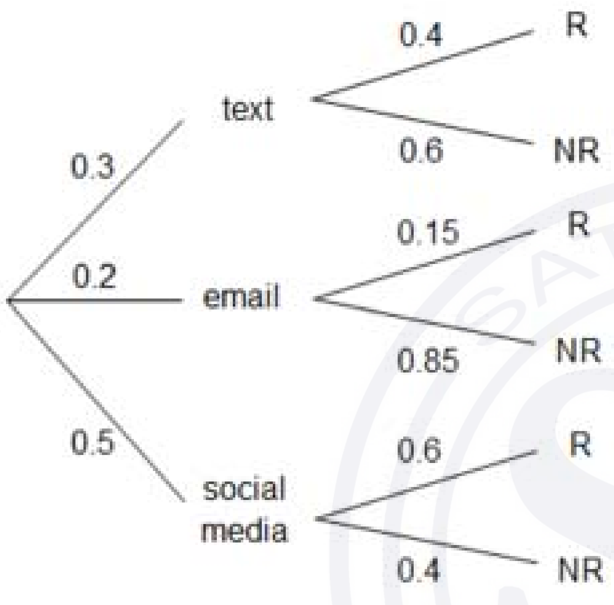
The following abbreviations may be used in a mark scheme or used on the scripts:

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
CWO	Correct Working Only – often written by a 'fortuitous' answer
ISW	Ignore Subsequent Working
SOI	Seen or implied
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Question	Answer	Marks	Guidance
1(i)	$P(79 < X < 91) = P\left(\frac{79-85}{6.8} < Z < \frac{91-85}{6.8}\right)$ $= P(-0.8824 < Z < 0.8824)$	**M1**	Using ± standardisation formula for either 79 or 91, no continuity correction
	$= \Phi(0.8824) - \Phi(-0.8824)$ $= 0.8111 - (1 - 0.8111)$	**M1**	Correct area ($\Phi - \Phi$) with one +ve and one –ve z-value or $2\Phi - 1$ or $2(\Phi - 0.5)$
	$= 0.622$	**A1**	Correct answer
		3	
1(ii)	$z = -1.751$	**B1**	± 1.751 seen
	$-1.751 = \frac{t-85}{6.8}$	**M1**	An equation using ± standardisation formula with a z-value, condone σ^2 or $\sqrt{\sigma}$
	$t = 73.1$	**A1**	Correct answer
		3	

Question	Answer	Marks	Guidance
2(i)		B1	Fully correct labelled tree with correct probabilities for 'Send'
		B1	Fully correct labelled branches with correct probabilities for the 'reply'
		2	

Question	Answer	Marks	Guidance
2(ii)	$P(email \mid NR) = \frac{P(email \cap NR)}{P(NR)} = \frac{0.2 \times 0.85}{0.3 \times 0.6 + 0.2 \times 0.85 + 0.5 \times 0.4}$	**M1**	P(email) × P(NR) seen as numerator of a fraction, consistent with *their* tree diagram
	$= \frac{0.17}{0.18 + 0.17 + 0.2} = \frac{0.17}{0.55}$	**M1**	Summing three appropriate 2-factor probabilities, consistent with *their* tree diagram, seen anywhere 0.55 oe (can be unsimplified) seen as denom of a fraction
	$= 0.309, \frac{17}{55}$	**A1**	
		A1	Correct answer
		4	

Question	Answer	Marks	Guidance
3(i)	9! × 2	**B1**	9! seen multiplied by $k \geqslant 1$, no addition
	= 725760	**B1**	Exact value
		2	
3(ii)	Eg $(K_1K_2K_3K_4K_5)$ A A A (U_1U_2) A	**B1**	2! or 5! seen mult by k > 1, no addition (arranging Us or Ks)
	= 5! × 2! × 6!	**B1**	6! Seen mult by k > 1, no addition (arranging AAAAKU)
	= 172800	**B1**	Exact value
		3	

Question	Answer	Marks	Guidance
4(i)	M(8) W(4) 4 2 in $^8C_4 \times {}^4C_2 = 420$ ways 5 1 in $^8C_5 \times {}^4C_1 = 224$ ways 6 0 in $^8C_6 \times {}^4C_0 = 28$ ways	**B1**	One unsimplified product correct
		M1	Summing the number of ways for 2 or 3 correct scenarios (can be unsimplified), no incorrect scenarios
	Total 672 ways	**A1**	Correct answer
		3	

Question	Answer	Marks	Guidance
4(ii)	Total number of selections = ${}^{12}C_6$ = 924 (A)	M1	${}^{12}C_x$ – (subtraction seen), accept unsimplified
	Selections with males together = ${}^{10}C_4$ = 210 (B)	A1	Correct unsimplified expression
	Total = (A) – (B) = 714	A1	Correct answer
	Alternative method for question 4(ii)		
	No males + Only male 1 + Only male 2 = ${}^{10}C_6 + {}^{10}C_5 + {}^{10}C_5$	M1	${}^{10}C_x + 2 \times {}^{10}C_y$, $x \neq y$ seen, accept unsimplified
	= 210 + 252 + 252	A1	Correct unsimplified expression
	= 714	A1	Correct answer
	Alternative method for question 4(ii)		
	Pool without male 1 + Pool without male 2 – Pool without either male	M1	$2 \times {}^{11}C_x - {}^{10}C_x$
	= ${}^{11}C_6 + {}^{11}C_6 - {}^{10}C_6$ = 462 + 462 – 210	A1	Correct unsimplified expression
	= 714	A1	Correct answer
		3	

Question	Answer	Marks	Guidance
5(i)	$P(0, 1, 2) = (0.66)^{14} + {}^{14}C_1(0.34)(0.66)^{13} + {}^{14}C_2(0.34)^2(0.66)^{12}$	**M1**	Binomial term of form ${}^{14}C_x p^x(1-p)^{14-x}$ $0 < p < 1$ any p, $x \neq 14,0$
	$= 0.0029758 + 0.02146239 + 0.071866$	**A1**	Correct unsimplified answer
	$= 0.0963$	**A1**	Correct answer
		3	
5(ii)	Mean $=600 \times 0.34 = 204$, Var $= 600 \times 0.34 \times 0.66 = 134.64$	**B1**	Correct unsimplified *np* and *npq* (or sd = 11.603 or Variance = 3366/25)
	$P(< 190) = P\left(z < \frac{189.5 - 204}{\sqrt{134.64}}\right) = P(z < -1.2496)$	**M1**	Substituting *their* μ and σ, (no σ^2 or $\sqrt{\sigma}$) into the Standardisation Formula with a numerical value for *'189.5'*. Condone ± standardisation formula
		M1	Using continuity correction 189.5 or 190.5 within a Standardisation formula
	$= 1 - \Phi(1.2496)$	**M1**	Appropriate area Φ from standardisation formula P(z<….) in final solution, (<0.5 if z is –ve, >0.5 if z is +ve)
	$= 1 - 0.8944 = 0.106$	**A1**	Correct final answer
		5	

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td rowspan="5">6(i)</td><td rowspan="4">

score	1	2	3	4	6	9
prob	$\frac{3}{15}$	$\frac{4}{15}$	$\frac{4}{15}$	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{1}{15}$

</td><td>B1</td><td>Probability distribution table with correct scores, allow extra score values if probability of zero stated</td></tr>
<tr><td>B1</td><td>2 probabilities (with correct score) correct</td></tr>
<tr><td>B1</td><td>3 or more correct probabilities with correct scores</td></tr>
<tr><td>B1</td><td>FT $\Sigma p = 1$, at least 4 probabilities</td></tr>
<tr><td></td><td>4</td><td></td></tr>
<tr><td rowspan="4">6(ii)</td><td>mean = $\frac{(3+8+12+4+12+9)}{15} = \frac{48}{15}$ (3.2)</td><td>B1</td><td></td></tr>
<tr><td>Var = $\frac{(3+16+36+16+72+81)}{15} - (\textit{their}\, 3.2)^2$</td><td>M1</td><td>FT Substitute their attempts at scores in correct var formula, must have "– mean^2" (condone probabilities not summing to 1)</td></tr>
<tr><td>$= \frac{224}{15} - 3.2^2 = 4.69 \left(\frac{352}{75}\right)$</td><td>A1</td><td></td></tr>
<tr><td></td><td>3</td><td></td></tr>
<tr><td rowspan="3">6(iii)</td><td>Score of 4, 6, 9</td><td>M1</td><td>Identifying relevant scores from their mean and their table</td></tr>
<tr><td>Prob $\frac{4}{15}$ (0.267)</td><td>A1</td><td>Correct answer

SC B1 for 4/15 with no working</td></tr>
<tr><td></td><td>2</td><td></td></tr>
</table>

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td rowspan="5">7(i)</td><td rowspan="4"><table><tr><th>Thaters School</th><th></th><th>Whitefay Park School</th></tr><tr><td>8</td><td>3</td><td></td></tr><tr><td>8 3</td><td>4</td><td>5 7</td></tr><tr><td>8 8 7 6 4 2</td><td>5</td><td>3 6 6</td></tr><tr><td>6 2 1</td><td>6</td><td>1 4 6 9</td></tr><tr><td>5</td><td>7</td><td>3 5 8</td></tr><tr><td></td><td>8</td><td>3</td></tr></table>
Key 8 | 4 | 5 represents 48 minutes for Thaters School and 45 minutes for Whitefay Park School.</td><td>B1</td><td>Correct stem can be upside down, ignore extra values,</td></tr>
<tr><td>B1</td><td>Correct Thaters School labelled on left, leaves in order from right to left and lined up vertically, no commas</td></tr>
<tr><td>B1</td><td>Correct Whitefay Park School labelled on same diagram on right hand side in order from left to right and lined up vertically, no commas</td></tr>
<tr><td>B1</td><td>FT Correct key for their diagram, need both teams identified and ‘minutes’ stated at least once here or in leaf headings or title.

SC If 2 separate diagrams drawn, SCB1 if both keys meet these criteria</td></tr>
<tr><td></td><td>4</td><td></td></tr>
<tr><td rowspan="3">7(ii)</td><td>LQ = 50
UQ = 61.5</td><td>B1</td><td>Both quartiles correct</td></tr>
<tr><td>IQ range = 61.5 – 50 = 11.5</td><td>B1</td><td>FT $61 \leqslant UQ \leqslant 62 - 48 \leqslant LQ \leqslant 52$</td></tr>
<tr><td></td><td>2</td><td></td></tr>
<tr><td rowspan="3">7(iii)</td><td>$\Sigma(x-60)^2 = (-15)^2 + (-13)^2 + (-7)^2 + (-4)^2 + (-4)^2 + 1^2 + 4^2 + 6^2 + 9^2 + 13^2 + 23^2 + 15^2 + 18^2$</td><td>M1</td><td>Summing squares with at least 5 correct unsimplified terms</td></tr>
<tr><td>= 1856</td><td>A1</td><td>Exact value</td></tr>
<tr><td></td><td>2</td><td></td></tr>
</table>

Question	Answer	Marks	Guidance
7(iv)	Var = mean of coded squares – (coded mean)^2 $= \frac{\sum(x-60)^2}{13} - \left(\frac{\sum(x-60)}{13}\right)^2$	**M1**	Using two coded values in correct formula (variance or sd)
	Var $= \frac{\textit{their}\,1856}{13} - \left(\frac{46}{13}\right)^2$ $= 130$	**A1**	Correct answer **SC** if correct variance obtained by another method give **SCB1**
		2	

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MATHEMATICS **9709/62**

Paper 6 Probability and Statistics **March 2019**

MARK SCHEME

Maximum Mark: 50

Published

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Cambridge Assessment International Education

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ISW	Ignore Subsequent Working
SOI	Seen or implied
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Question	Answer	Marks	Guidance
1(i)	$0.6 \times 0.2 + 0.4 \times 0.32$	**M1**	Addition of 2 two-factor terms $0.6 \times a + 0.4 \times b$
	$= 0.248, \ \frac{31}{125}$	**A1**	CAO
		2	
1(ii)	**Method 1**		
	P(GS|Not Red socks) = $\frac{0.4 \times 0.68}{1 - \boldsymbol{(i)}}$	**B1**	Correct [unsimplified] numerator seen in fraction
		M1	1 – their **(i)** as denominator in fraction
	$= 0.362, \ \frac{17}{47}$	**A1**	
	Method 2		
	P(GS|Not Red socks) = $\frac{0.4 \times 0.68}{0.6 \times 0.8 + 0.4 \times 0.68}$	**B1**	Correct [unsimplified] numerator seen in fraction
		M1	Correct or (their **(i)**)' as denominator in fraction
	$= 0.362, \ \frac{17}{47}$	**A1**	
		3	

Question	Answer	Marks	Guidance
2(i)	$\sigma^2 = \frac{\Sigma(x-c)^2}{n} - \left(\frac{\Sigma(x-c)}{n}\right)^2$ $3.2^2 = \frac{3099.2}{40} - \left(\frac{\Sigma(x-c)}{40}\right)^2$	**M1**	Use correct formula with values substituted
	$\left(\frac{\Sigma(x-c)}{40}\right)^2 = 67.24$: $\Sigma(x-c) = 40 \times \sqrt{67.24}$	**M1**	Rearrange to make *their* $\left(\frac{\Sigma(x-c)}{40}\right)^2$ the subject, unsimplified.
	= 328	**A1**	Exact value, cao
		3	
2(ii)	$\Sigma x - 40c =$ *their* **(i)** Mean = $\frac{\textit{their}\ \textbf{(i)}}{40} + 50$ = 58.2	**B1FT**	FT *their* **(i)**
		1	

Question	Answer	Marks	Guidance
3(i)	$P(X< 132) = P\left(Z < \frac{132-140}{12}\right) = P(Z < -0.6667)$	M1	Using ± standardisation formula, no continuity correction, not σ^2 or $\sqrt{\sigma}$
	$= 1 - 0.7477$	M1	Appropriate area Φ from standardisation formula P(z<….) in final solution
	= 0.252 awrt	A1	Condone linear interpolation = 0.25243
		3	
3(ii)	P(time>k) = 0.675, $z = -0.454$	B1	±0.454 seen
	$\frac{k-140}{12} = -0.454$	M1	An equation using the standardisation formula with a z-value (not $1 - z$), condone σ^2 or $\sqrt{\sigma}$
	k = 135, 134.6, 134.55	A1	B0M1A1 max from –0.45
		3	

Question	Answer	Marks	Guidance
4(i)	<table><tr><td>x</td><td>−1</td><td>1</td><td>2</td><td>3</td></tr><tr><td>p</td><td>k</td><td>k</td><td>$4k$</td><td>$9k$</td></tr></table>	B1	Probability distribution table with correct values of x, no additional values unless with probability 0 stated, at least one correct probability including k
	$15k = 1$,	M1	Equating $\Sigma p = 1$, may be implied by answer
	$k = \frac{1}{15}$	A1	If 0 scored, SCB2 for probability distribution table with correct numerical probabilities.
		3	

Question	Answer	Marks	Guidance
4(ii)	**Method 1**		
	$E(X) = 8k + 27k = 35k = \frac{35}{15} = \frac{7}{3}$	**B1FT**	FT if 0< *their* k<1
	$\text{Var}(X) = (k + k + 16k + 81k) - (35k)^2$	**M1**	Correct formula for variance, in terms of k at least – must have '– mean^2'(ft).
	$= 1.16,\ \frac{52}{45}$	**A1**	
	Method 2		
	$E(X) = \frac{8}{15} + \frac{27}{15} = \frac{35}{15} = \frac{7}{3}$	**B1FT**	FT if 0< *their* k<1
	$\text{Var}(X) = \frac{1}{15} + \frac{1}{15} + \frac{16}{15} + \frac{81}{15} - \left(\frac{7}{3}\right)^2$	**M1**	Subst *their* values in correct var formula – must have '– mean^2'(ft) (condone probs not summing to exactly 1)
	= 1.16 (= 52/45)	**A1**	Using their values from **(i)**
		3	

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td rowspan="5">5(i)</td><td rowspan="4"><table><tr><th>Dolphins</th><th></th><th>Sharks</th></tr><tr><td></td><td>5</td><td>9</td></tr><tr><td>9 5 5 3 2</td><td>6</td><td>4 6 8</td></tr><tr><td>5 3 2</td><td>7</td><td>0 1 2 4 7</td></tr><tr><td>2 2 0</td><td>8</td><td>0 4</td></tr><tr><td></td><td></td><td>Key: 3|6|4 means 63 kg for Dolphins and 64 kg for Sharks</td></tr></table></td><td>B1</td><td>Correct stem can be upside down, ignore extra values,</td></tr>
<tr><td>B1</td><td>Correct Dolphin must be on LHS,</td></tr>
<tr><td>B1</td><td>Correct Sharks on either LHS or RHS of back-to-back. Alignment ± half a space, no late entries squeezed in, no crossing out if shape is changed. Condone a separate RHS stem-and-leaf diagram</td></tr>
<tr><td>B1FT</td><td>Correct single key for their single diagram, need both teams identified and ‘kg’ stated at least once here or in leaf headings or title.</td></tr>
<tr><td></td><td>4</td><td></td></tr>
<tr><td rowspan="4">5(ii)</td><td>Median = 72
LQ = 65, UQ = 80,</td><td>B1</td><td>72<UQ<82 – 62<LQ<72</td></tr>
<tr><td>IQR = 80 – 65</td><td>M1</td><td>nfww</td></tr>
<tr><td>= 15</td><td>A1</td><td>SCB1 if M0 scored for LQ = 65 and UQ = 80</td></tr>
<tr><td></td><td>3</td><td></td></tr>
</table>

Question	Answer	Marks	Guidance
6(i)	P(4, 5, 6) = ${}^6C_4\,0.35^4 0.65^2 + {}^6C_5\,0.35^5 0.65^1 + 0.35^6$	M1	Binomial term of form ${}^6C_x p^x (1-p)^{6-x}$ $0 < p < 1$ any p, $x \neq 6,0$
		A1	Correct unsimplified answer
	= 0.117	A1	
		3	
6(ii)	$1 - 0.65^n > 0.95$ $0.65^n < 0.05$	M1	Equation or inequality involving '0.65^n or 0.35^n' and '0.95 or 0.05'
	$n > \dfrac{\log 0.05}{\log 0.65} = 6.95$	M1	Attempt to solve *their* exponential equation using logs or Trial and Error.
	$n = 7$	A1	CAO
		3	
6(iii)	Mean = $0.35 \times 100 = 35$ Variance = $0.35 \times 0.65 \times 100 = 22.75$	B1	Correct unsimplified np and npq,
	$\text{P}\left(z > \dfrac{39.5 - 35}{\sqrt{22.75}}\right) = P(z > 0.943)$	M1	Substituting *their* μ and σ (condone σ^2) into the ±Standardisation Formula with a numerical value for *'39.5'*.
		M1	Using continuity correction 39.5 or 40.5
	$= 1 - 0.8272$	M1	Appropriate area Φ from standardisation formula P(z>….) in final solution, (>0.5 if z is -ve, <0.5 if z is +ve)
	= 0.173	A1	Final answer
		5	

Question	Answer	Marks	Guidance
7(i)	$\dfrac{9!}{2!3!}$	M1	9! alone on numerator, 2! and/or 3! on denominator
	= 30240	A1	Exact value, final answer
		2	
7(ii)	A ^ ^ ^ A ^ ^ ^ A Arrangements = $\dfrac{6!}{2!} = 360$	B1	Final answer
		1	
7(iii)	M ^ M ^ ^ ^ ^ ^ ^ $= \dfrac{7!}{3!} \times 7$	M1	7! in numerator, (considering letters not M)
		M1	Division by 3! only (removing repeated As)
		M1	Multiply by 7 (positions of M-M)
	= 5880	A1	Exact value, final answer
	Method 2 (choosing letter between Ms)		
	$1 \times \dfrac{6!}{2!} \times 7 + 4 \times \dfrac{6!}{3!} \times 7$	M1	6! in sum of 2 expressions $a6! + b6!$
		M1	Multiply by 7 in both expressions (positions of M-M)
	= 2520 + 3360	M1	$\dfrac{c}{2!} + \dfrac{d}{3!}$ seen (removing repeated As)
	= 5880	A1	Exact value

Question	Answer	Marks	Guidance
7(iii)	**Method 3**		
	(MAM) ^ ^ ^ ^ ^ ^ = 7!/2! = 2520	**M1**	7! in numerator (considering 6 letters + block)
	(MA'M) ^ ^ ^ ^ ^ ^ = 7!/3! × 4 = 840 × 4 = 3360	**M1**	Division by 2! and 3! seen in different terms
	Total = 2520 + 3360	**M1**	Summing 5 correct scenarios only
	= 5880	**A1**	Exact value
		4	
7(iv)	M A ^ = $^4C_1$ = 4	**B1**	Final answer
		1	
7(v)	M ^ ^ : $^4C_2$ = 6 M M ^ : $^4C_1$ = 4	**M1**	Either option M M ^ or M ^ ^ correct, accept unsimplified
	M M A : = 1 M A A : = 1 (M A _ :$^4C_1$ = 4)	**M1**	Add 4 or 5 correct scenarios only
	Total = 16	**A1**	Value must be clearly stated
	Method 2		
	M M ^ = $^5C_1$ = 5	**M1**	Either option M M ^ or M ^ ^ correct, accept unsimplified
	M ^ ^ = $^5C_2$ = 10	**M1**	Adding 2 or 3 correct scenarios only
	M A A = = 1 Total = 16	**A1**	Value must be clearly stated
		3	

Cambridge International AS & A Level

Cambridge Assessment International Education
Cambridge International Advanced Subsidiary and Advanced Level

MATHEMATICS **9709/61**

Paper 6 **October/November 2018**

MARK SCHEME

Maximum Mark: 50

Published

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GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

  - Note: B2 or A2 means that the candidate can earn 2 or 0.
    B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no “follow through” from a previous error is allowed)
CWO	Correct Working Only – often written by a ‘fortuitous’ answer
ISW	Ignore Subsequent Working
SOI	Seen or implied
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become “follow through” marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Question	Answer	Marks	Guidance
1	${}^9C_4 \times {}^5C_3 \times {}^2C_2$	**B1**	${}^9C_4$ or ${}^9C_3$ or ${}^9C_2$ seen *(1st group)*
	$=126 \times 10 \times 1$	**B1**	${}^{5\text{ or }7}C_3$ or ${}^{6\text{ or }7}C_4$ or ${}^{6\text{ or }5}C_2$ times an integer *(2nd group)*
	$=1260$	**B1**	Correct answer
		3	

Question	Answer	Marks	Guidance
2(i)	$6p + 0.1 = 1$ $p = 0.15$	**B1**	Correct answer
		1	
2(ii)	$\text{Var}(X) = 1 \times p + 1 \times 2p + 4 \times 2p + 16 \times 0.1 - 1.15^2$	**M1**	Correct unsimplified formula, *their p* substituted (allow 1 error)
	$0.15 + 0 + 0.3 + 1.2 + 1.6 - 1.15^2$ $= 1.9275 = 1.93$ (3sf)	**A1**	Correct answer
		2	

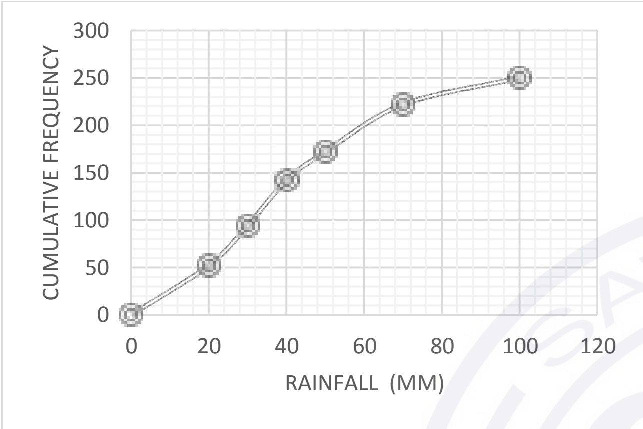
Question	Answer	Marks	Guidance
3(i)	Scenarios are: 4V + 1C + 1DB: ${}^{11}C_4 \times {}^5C_1 \times {}^4C_1$	**M1**	${}^{11}C_a \times {}^5C_b \times {}^4C_c$, $a+b+c=6$,
	4V + 2C: ${}^{11}C_4 \times {}^5C_2$ 5V + 1C: ${}^{11}C_5 \times {}^5C_1$	**B1**	2 correct unsimplified options
	6600 + 3300 + 2310	**M1**	Add 2 or 3 correct scenarios only
	= 12210	**A1**	Correct answer
		4	
3(ii)	$4! \times 3!$	**M1**	k multiplied by 3! or 4!, k an integer $\geqslant 1$
		A1	Correct unsimplified expression
	= 144	**A1**	Correct answer
		3	

Question	Answer	Marks	Guidance
4(a)	$P(X < 29.4) = P\left(Z < \frac{29.4 - 31.4}{\sqrt{3.6}}\right)$ $= P(Z < -1.0541)$	**M1**	Standardise, no cc, must have sq rt.
	$= 1 - 0.8540$	**M1**	Obtain 1 − prob
	= 0.146	**A1**	Correct final answer
		3	

Question	Answer	Marks	Guidance
4(b)	$P(X < 12) = \dfrac{42}{400} = 0.105$ and $P(X > 19) = \dfrac{58}{400} = 0.145$	M1	Eqn with μ, σ and a z-value. Allow cc, wrong sign, but not $\sqrt{\sigma}$ or σ^2
	$\dfrac{12-\mu}{\sigma} = -1.253$	B1	Any form with z value rounding to ± 1.25
	$\dfrac{19-\mu}{\sigma} = 1.058$	B1	Any form with z value rounding to ± 1.06
	$12 - \mu = -1.253\sigma$ $19 - \mu = 1.058\sigma$ $7 = 2.307\sigma$ or $36.455 + 2.307\mu = 0$ oe	M1	Solve 2 equations in μ, σ eliminating to 1 unknown
	$\mu = 15.8$, $\sigma = 3.03$	A1	Correct answers
		5	

Question	Answer	Marks	Guidance
5(i)	$1 - (P(7) + P(8) + P(9))$ $= 1 - ({}^9C_7\ 0.8^7 \times 0.2^2 + {}^9C_8\ 0.8^8 \times 0.2^1 + {}^9C_9\ 0.8^9 \times 0.2^0)$	M1	Any binomial term of form ${}^9C_x p^x (1-p)^{9-x}$, $x \neq 0$
		M1	Correct unsimplified expression
	$= 1 - (0.3019899 + 0.3019899 + 0.1342177)$ $= 0.262$	A1	Correct answer
		3	

Question	Answer	Marks	Guidance
5(ii)	Mean = 200 × 0.8 = 160: var = 200 × 0.8 × 0.2 = 32	**B1**	Both unsimplified
	$P(X > 166) = P(Z > \frac{166.5-160}{\sqrt{32}})$	**M1**	Standardise, $z = \pm\frac{x - their\,160}{\sqrt{their\,32}}$ with square root
		M1	166.5 or 165.5 seen in attempted standardisation expression
	$= P(Z > 1.149) = 1 - 0.8747$	**M1**	1 – a Φ -value, correct area expression, linked to final answer
	= 0.125	**A1**	Correct final answer
		5	
5(iii)	$np = 160$, $nq = 40$: both > 5 (so normal approx. holds)	**B1**	Both parts required
		1	

Question	Answer	Marks	Guidance
6(i)		**B1**	Appropriate linear scales starting at (0,0), axes labelled cf and Rainfall, mm
		B1	Correct graph, points plotted at ucb, allow straight lines or curve
		2	
6(ii)		**M1**	Read off from increasing graph at cf = 150
	42	**A1**	Correct answer ($41 \leqslant r \leqslant 43$)
		2	

Question	Answer	Marks	Guidance
6(iii)	Frequencies 52, 42, 48, 30, 50, 28	B1	Correct frequencies
	Mean age = $(10\times52+25\times42+35\times48+45\times30+60\times50+85\times28)/250$	B1	Correct midpoints (allow one error)
	=9980/250	M1	Using Σfx/250 with mid-points attempt, not cf, cw, lb, ub
	= 39.9(2) oe	A1	Correct answer
	Variance = $10^2\times52+25^2\times42+35^2\times48+45^2\times30+60^2\times50+85^2\times28)/250-$ mean ^{2} = 539.59	M1	Attempt at variance using their midpoints and their mean
	$\sigma = 23.2$	A1	Correct answer for sd
		6	

Question	Answer	Marks	Guidance
7(i)	52/160 = 13/40, 0.325	B1	oe
		1	
7(ii)	P(boy) = 96/160: P(Music) = 52/160 P(boy and Music) = 40/160	M1	Use of P(B) × P(M) = P(B∩M), appropriate probabilities used
	96/160 × 52/160 ≠ 40/160: Not independent	A1	Numerical comparison and conclusion stated
		2	

Question	Answer	Marks	Guidance
7(iii)	**Method 1**		
	P(not Music/girl) = P(not Music and girl)/P(girl) (27/160) / (64/160)	**M1**	Appropriate probabilities in a fraction
	$= \frac{27}{64}$	**A1**	Correct answer www implies method
	Method 2		
	Direct from table	**M1**	*27/a or b/64, a ≠ 160*
	$\frac{27}{64}$	**A1**	Correct answer www implies method
		2	
7(iv)	P(B M) × P(B NM) × P(G NM) **or** P(G M) × P(B NM) ×P(B NM)	**M1**	One scenario identified with 3 probs multiplied
	40/160 × 56/159 × 52/158 **or** 12/160 × 56/159 × 55/158	**A1**	One scenario correct (ignore multiplying factor)
	× 3! × 3!/2!	**B1**	Both multiplying factors correct
	0.17387 0.02759 P = 0.17387 + 0.02759	**M1**	Both cases attempted and added (multiplying factor not required), accept unsimplified
	= 0.201 Note: If score in this part is 0, award SCB1 for $\frac{1}{160}\times\frac{1}{159}\times\frac{1}{158}\times k$, for positive integer *k*, seen	**A1**	Correct answer, oe

Question	Answer	Marks	Guidance
7(iv)	**Method 2**		
	$\dfrac{\binom{40}{1}\times\binom{56}{1}\times\binom{52}{1}+\binom{12}{1}\times\binom{56}{2}}{\binom{160}{3}}$	**M1**	One scenario identified with 2 or 3 combination multiplied
		A1	One scenario correct
		B1	Denominator correct
	$\dfrac{116480+18480}{669920}$	**M1**	Both scenarios attempted, and added, seen as a numerator of a fraction
	$\dfrac{1687}{8374}$	**A1**	Correct answer, oe
		5	

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MATHEMATICS **9709/62**

Paper 6 **October/November 2018**

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M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

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The following abbreviations may be used in a mark scheme or used on the scripts:

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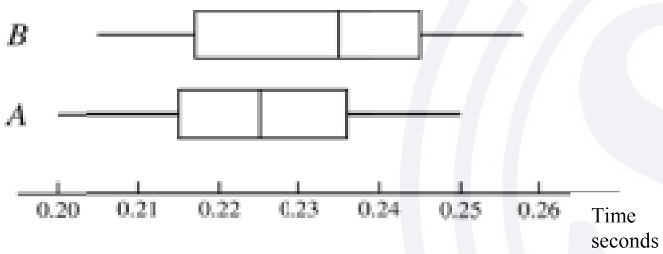
Question	Answer	Marks	Guidance
1(i)	$\dfrac{11!}{4!4!2!}$	**M1**	$\dfrac{11!}{4!\times k}$ *or* $\dfrac{11!}{2!\times k}$, k a positive integer
	$= 34650$	**A1**	Correct final answer
		2	
1(ii)	**Method 1**		
	P(SS) = $\dfrac{4}{11}\times\dfrac{3}{10}=\dfrac{12}{110}$ (= 0.10911)	**B1**	One of P(SS), P(PP) or P(II) correct, allow unsimplified
	P(PP) = $\dfrac{2}{11}\times\dfrac{1}{10}=\dfrac{2}{110}$ (= 0.01818) P(II) = $\dfrac{4}{11}\times\dfrac{3}{10}=\dfrac{12}{110}$ (= 0.10911) $\dfrac{4}{11}\times\dfrac{3}{10}$	**M1**	Sum of probabilities from 3 appropriate identifiable scenarios (either by labelling or of form $\dfrac{4}{11}\times\dfrac{a}{b}+\dfrac{2}{11}\times\dfrac{c}{b}+\dfrac{4}{11}\times\dfrac{a}{b}$ where a = 4 *or* 3, b = 11 *or* 10, c = 2 *or* 1)
	Total = $\dfrac{26}{110}=\dfrac{13}{55}$ oe (0.236)	**A1**	Correct final answer
	Method 2		
	Total number of selections = $^{11}C_2 = 55$ Selections with 2 Ps = 1	**B1**	Seen as the denominator of fraction (no extra terms) allow unsimplified
	Selections with 2 Ss = $^{4}C_2 = 6$ Selections with 2 Is = $^{4}C_2 = 6$,	**M1**	Sum of 3 appropriate identifiable scenarios (either by labelling or values, condone use of permutations. May be implied by 2,12,12)
	Total selections with 2 letters the same = 13 Probability of 2 letters the same = $\dfrac{13}{55}$ oe (0.236)	**A1**	Correct final answer, without use of permutations
		3	

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td>2(i)</td><td>median = 0.225;
LQ = 0.215: UQ = 0.236</td><td>B1</td><td>Correct median (Q_2)</td></tr>
<tr><td></td><td>IQR = 0.236 – 0.215</td><td>M1</td><td>0.232 < UQ (Q_3) < 0.238 – 0.204 < LQ (Q_1) < 0.219</td></tr>
<tr><td></td><td>= 0.021</td><td>A1</td><td>www
Omission of all decimal points MR-1
<u>If M0 awarded</u>
SCB1 for both LQ = 0.215: UQ = 0.236 seen</td></tr>
<tr><td></td><td></td><td>3</td><td></td></tr>
<tr><td>2(ii)</td><td>
</td><td>B1</td><td>Linear scale between 0.20 to 0.26 (condone omission of 0.26) axis labelled (time and seconds), at least one box plot attempted, no lines through boxes, whiskers not at corner of boxes</td></tr>
<tr><td></td><td></td><td>B1 ft</td><td>Labelled correct graph for A, (ft their median/quartiles), condone lines through boxes, whiskers at corner of boxes</td></tr>
<tr><td></td><td><table>
<tr><td>A</td><td>0.200</td><td>0.215</td><td>0.225</td><td>0.236</td><td>0.250</td></tr>
<tr><td>B</td><td>0.205</td><td>0.217</td><td>0.235</td><td>0.245</td><td>0.258</td></tr>
</table></td><td>B1</td><td>Labelled correct graph for B, condone lines through boxes, whiskers at corner of boxes

SC If B0B0 scored because graphs not labelled/labels reversed SCB1 if both ‘correct’

Penalty MR-1 if graphs plotted on separate axes unless both scales align exactly.</td></tr>
<tr><td></td><td></td><td>3</td><td></td></tr>
</table>

Question	Answer	Marks	Guidance
3(i)	**Method 1**		
	$P(3) + P(4) + P(5) = {}^5C_3\ 0.75^3 \times 0.25^2\ +$	**M1**	One binomial term ${}^5C_x p^x(1-p)^{5-x}$, $x \neq 0$ or 5, any p
	${}^5C_4\ 0.75^4 \times 0.25^1\ + {}^5C_5\ 0.75^5 \times 0.25^0$	**M1**	Correct unsimplified expression
	$= 0.26367 + 0.39551 + 0.23730$ $= 0.896\ (459/512)$	**A1**	Correct final answer, allow 0.8965 (isw) but not 0.897 alone
	Method 2		
	$1 - P(0) - P(1) - P(2) = 1 - {}^5C_0\ 0.75^0 \times 0.25^5$	**M1**	One binomial term ${}^5C_x p^x(1-p)^{5-x}$, $x \neq 0$ or 5, any p
	$-\ {}^5C_1\ 0.75^1 \times 0.25^4 -\ {}^5C_2\ 0.75^2 \times 0.25^3$	**M1**	Correct simplified expression
	$= 1 - 0.00097656 - 0.014648 - 0.087891$ $= 0.896\ (459/512)$	**A1**	Correct final answer, allow 0.8965 (isw) but not 0.897 alone
		3	

Question	Answer	Marks	Guidance
3(ii)	**Method 1**		
	$P(C,C) + P(C,C') + P(C',C)$ 0.8×0.9	**B1**	Unsimplified prob completed on both days
	$0.8 \times 0.1 + 0.2 \times 0.6$	**M1**	Unsimplified prob $0.8 \times a + 0.2 \times b$, $a = 0.1$or 0.4, $b = 0.6$ or 0.9
	$= 0.92$ oe	**A1**	Correct final answer
	Method 2		
	$1 - P(C',C') = 1 - 0.2 \times 0.4$	**B1**	Unsimplified prob completed on no days
		M1	$1 - 0.2 \times a$, a=0.1or 0.4 allow unsimplified
	$= 0.92$	**A1**	Correct final answer
		3	

Question	Answer	Marks	Guidance
4(i)	$5! \times 6! \times 2$	**B1**	$k \times 5!$ or $m \times 6!$ (k,m integer, $k,m \geqslant 1$), no inappropriate addition
		B1	$n \times 5! \times 6!$ (n integer, $n \geqslant 1$), no inappropriate addition
	$= 172800$	**B1**	Correct final answer, isw rounding (www scores B3) All marks based on their **final** answer
		3	

Question	Answer	Marks	Guidance
4(ii)	… G … G … G … G … G … G … No. ways girls placed × No. ways boys placed in gaps =	**M1**	$k \times 6!$ or $k \times {}^7P_5$ (k is an integer, $k \geqslant 1$) no inappropriate add. (${}^7P_5 \equiv 7 \times 6 \times 5 \times 4 \times 3$ or ${}^7C_5 \times 5!$)
	$6! \times {}^7P_5$	**M1**	Correct unsimplified expression
	= 1814400	**A1**	Correct exact final answer (ignore subsequent rounding)
		3	

Question	Answer	Marks	Guidance
5(i)	$\dfrac{15.5 \times 12 + 910}{12 + 20}$	**M1**	Unsimplified total age divided by *their* total members (not 12, 20 or 2)
	=34.25 or 34¼ (years)	**A1**	Correct exact answer (isw rounding), oe (34 years 3 months)
		2	
5(ii)	Considering Juniors: variance = $\dfrac{\sum x^2}{12} - 15.5^2 = 1.2^2$	**M1**	$\dfrac{\sum x^2}{k} - 15.5^2 = 1.2^2$, k = 12 or 20
	$\sum x^2 = 2900.28$	**A1**	Answer wrt 2900
	Considering whole group: $\sum z^2 = \sum x^2 + \sum y^2 = 2900.28 + 42850 = 45750$ Variance = $\dfrac{\sum z^2}{32} - \mu^2 = \dfrac{\textit{their}\ 45750}{12+20} - (\textit{their}\ 34.25)^2$ (= 256.63)	**M1**	*Their* 45750 > 42850 (not 85700 or rounding to 1.8×10^9) in correct variance or std deviation formula (Σx^2 and addition may not be seen)
	s d = 16.0(2)	**A1**	Correct final answer, condone 16.03
		4	

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td>6(i)</td><td>

x	–2	–1	0	1	2	3
p	$\frac{1}{12}$	$\frac{2}{12}$	$\frac{3}{12}$	$\frac{3}{12}$	$\frac{2}{12}$	$\frac{1}{12}$

</td><td>B1</td><td>–2, –1, 0, 1, 2, 3 seen as top line of a pdf table with at least 1 probability OR attempting to evaluate P(–2), P(–1), P(0), P(1), P(2), P(3) (condone additional values with p=0 stated)</td></tr>
<tr><td></td><td></td><td>B1</td><td>At least 4 probs correct (need not be in table)</td></tr>
<tr><td></td><td></td><td>B1</td><td>All probs correct in a table</td></tr>
<tr><td></td><td></td><td>3</td><td></td></tr>
<tr><td>6(ii)</td><td>$\mathrm{E}(X) = \frac{-2\times1-1\times2+0+1\times3+2\times2+1\times3}{12} = 0.5$</td><td>M1</td><td>Unsimplified expression for mean using their pdf table (or correct) with at least 2 non-zero values (may be seen in variance). Numerator terms may be implied by values.</td></tr>
<tr><td></td><td>$\mathrm{Var}(X) = \frac{(-2)^2\times1+(-1)^2\times2+1^2\times3+2^2\times2+3^2\times1}{12} - (\mathit{their}\ 0.5)^2$</td><td>M1</td><td>Unsimplified expression for variance using their pdf table (or correct) with at least 2 non-zero values and their E(X). Numerator terms may be implied by values. If $-k^2$ is seen for $(-k)^2$, the method must be confirmed by seeing value used correctly</td></tr>
<tr><td></td><td>26/12 −1/4 = 23/12</td><td>A1</td><td>Correct final answer</td></tr>
<tr><td></td><td></td><td>3</td><td></td></tr>
</table>

Question	Answer	Marks	Guidance
6(iii)	**Method 1**		
	P(X non-zero) = 9/12	**B1ft**	<u>If Binomial distribution used 0/3</u> P(X non-zero) ft from *their* pdf table, Σp=1 oe
	P(X = 1 | X non-zero) = $\dfrac{P(X=1 \cap X\ non-zero)}{P(X\ non-zero)} = \dfrac{3/12}{9/12}$	**M1**	*Their* P(X = 1)/*their* P(X non-zero) from *their* pdf table oe
	= 1/3 oe	**A1**	Correct final answer www
	Method 2		
	P(X = 1 | X non-zero) = $\dfrac{Number\ of\ outcomes = 1}{Number\ of\ non-zero\ outcomes}$	**B1ft**	Number of non-zero outcomes (expect 9) ft from *their* outcome table or pdf table numerators oe
		M1	*a/b, a = their* 3 from *their* outcome table or pdf table numerators, *b = their* 9 (not 12)
	$= \dfrac{3}{9} = \dfrac{1}{3}$ oe	**A1**	Correct final answer www
		3	

Question	Answer	Marks	Guidance
7(a)(i)	$P(X<4)=P\left(Z<\frac{4-3.24}{0.96}\right)$	M1	±Standardisation formula, no cc, no sq rt, no square
	$= P\left(Z<0.7917\right) = 0.7858$	A1	$0.7855 < p \leqslant 0.7858$ or $p = 0.786$ Cao (implies M1A1 awarded), may be seen used in calculation
	their 0.7858 × 365 = 286 (or 287)	B1ft	*Their* probability × 365 provided 4sf probability **seen**. FT answer rounded or truncated to nearest integer. No approximation notation used.
		3	
7(a)(ii)	$P(X<k)=P(Z<\frac{k-3.24}{0.96})=0.8$	B1	(z=) ± 0.842 seen
	$\frac{k-3.24}{0.96}=0.842$	M1	$z=\pm\frac{k-3.24}{0.96}$, allow cc, sq rt or square equated to a z-value (0.7881, 0.2119, 0.158, 0.8, 0.2 etc. are not acceptable)
	$k = 4.05$	A1	Correct final answer, www
		3	
7(a)(iii)	$P(-1.5<Z<1.5)=$	M1	$\Phi(z = 1.5)$ or $\Phi(z = -1.5)$ seen used or $p = 0.9332$ seen
	$\Phi(1.5)-\Phi(-1.5)=2\Phi(1.5)-1$ = 2 × 0.9332 – 1 oe	M1	Correct final area expression using *their* probabilities
	= 0.866	A1	Correct final answer
		3	

Question	Answer	Marks	Guidance
7(b)	$\mathrm{P}(Y>0)=\mathrm{P}\left(Z>\dfrac{0-\mu}{\sigma}\right)\equiv \mathrm{P}\left(Z>\dfrac{0-\mu}{3\mu/4}\right)$ or $\mathrm{P}\left(Z>\dfrac{0-\left(\frac{4\sigma}{3}\right)}{\sigma}\right)$	**M1**	±Standardisation attempt in terms of one variable no sq rt or square, condone ±0.5 as cc
	$=\mathrm{P}(Z>-4/3)$	**A1**	Correct unsimplified standardisation, no variables
	$=0.909$	**A1**	Correct final answer
		3	

Alternative methods for Question 1(ii)

Method 3

$P(S,S') = \frac{4}{11} \times \frac{7}{10} = \frac{28}{110}$

$P(P,P') = \frac{2}{11} \times \frac{9}{10} = \frac{18}{110}$

$P(I,I') = \frac{4}{11} \times \frac{7}{10} = \frac{28}{110}$

$P(M,M') = \frac{1}{11} \times \frac{10}{10} = \frac{10}{110}$

$Total = \frac{84}{110}$

$P(Same) = 1 - \frac{84}{110} = \frac{26}{110}$

B1 one of products correct
M1 1 – sum of probabilities from 4 appropriate scenarios
A1 Correct final answer

Method 4

PP' = $\dfrac{2\times 9}{2} = 9$

SS' = $\dfrac{4\times 7}{2} = 14$

II' = $\dfrac{4\times 7}{2} = 14$

MM' = $\dfrac{1\times 10}{2} = 5$

Total number of ways = $\dfrac{10\times 11}{2} = 55$

Number of ways of letters repeating = 55 – (9 + 14 + 14 + 5) = 13

P(Same) = $\dfrac{13}{55}$

B1 $^{11}C_2$ seen as the denominator of fraction (no extra terms) allow unsimplified
M1 1 – sum of 4 appropriate scenarios
A1 Correct final answer

Cambridge Assessment International Education
Cambridge International Advanced Subsidiary and Advanced Level

MATHEMATICS **9709/63**

Paper 6 **October/November 2018**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2018 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

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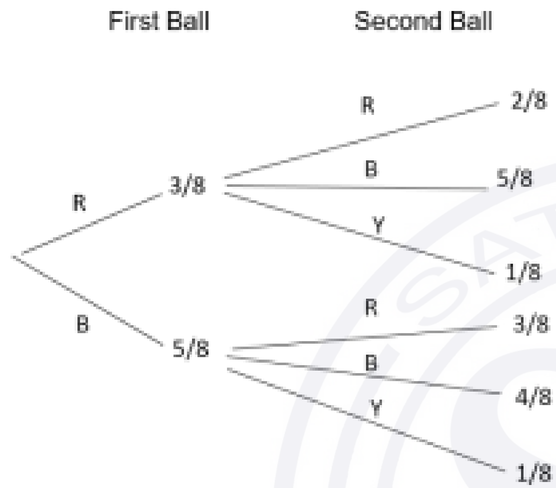
1	**Method 1**		
	... M … M … M … M … M …	**M1**	k × 5! (120) or k × 6P2 (30), k is an integer ⩾ 1,
	No. ways men placed × No. ways women placed in gaps = $5! \times {}^6P_2$	**M1**	Correct unsimplified expression
	= 3600	**A1**	Correct answer
	Method 2		
	Number with women together = 6! × 2 (1440) Total number of arrangements = 7! (5040)	**M1**	6! × 2 or 7! – k seen, k is an integer ⩾ 1
	Number with women not together = 7! – 6! × 2	**M1**	Correct unsimplified expression
	= 3600	**A1**	Correct answer
		3	

Question	Answer	Marks	Guidance
2(i)	<table><tr><td>x</td><td>–2</td><td>–1</td><td>0</td><td>1</td><td>2</td><td>3</td></tr><tr><td>P($X = x$)</td><td>$\frac{2}{18}$</td><td>$\frac{4}{18}$</td><td>$\frac{5}{18}$</td><td>$\frac{4}{18}$</td><td>$\frac{2}{18}$</td><td>$\frac{1}{18}$</td></tr></table>	**B1**	–2, –1, 0, 1, 2, 3 seen as top line of a pdf table OR attempting to evaluate P(–2), P(–1), P(0), P(1), P(2), P(3),
		B1	At least 4 probs correct (need not be in table)
		B1	All probs correct in a table
		3	

Question	Answer	Marks	Guidance
2(ii)	$E(X) = \dfrac{-4-4+0+4+4+3}{18} = \dfrac{1}{6}$	**M1**	Correct unsimplified expression for the mean using their table, $\Sigma p = 1$, may be implied
	$\text{Var}(X) = \dfrac{8+4+0+4+8+9}{18} - \left(\dfrac{1}{6}\right)^2$ $= 11/6 - 1/36$ (1.8333 – 0.02778)	**M1**	Correct, unsimplified expression for the variance using their table, and their mean^2 subtracted. Allow $\Sigma p \neq 1$
	$= 65/36$, (1.81)	**A1**	Correct answer
		3	

Question	Answer	Marks	Guidance
3(i)		B1	Fully correct labelled tree and correct probabilities for 'First Ball'
		B1	Correct probabilities (with corresponding labels) for 'Second Ball'
		2	
3(ii)	P(RR) + P(BB) = $3/8 \times 2/8 + 5/8 \times 4/8 = 3/32 + 5/16$	M1	Correct unsimplified expression from their tree diagram, $\Sigma p = 1$ on each branch
	$= 13/32$ (0.406)	A1	Correct answer
		2	

Question	Answer	Marks	Guidance
3(iii)	P(RB) = $3/8 \times 5/8 = 15/64$	**M1**	$P(\text{1st ball red}) \times P(\text{2nd ball blue})$ from their tree diagram seen unsimplified as numerator or denominator of a fraction Allow $\Sigma p \neq 1$ on each branch
	P(B) = 3/8 × 5/8 + 5/8 × 4/8 = 35/64	**M1**	Correct unsimplified expression for P(B) from their tree diagram seen as denominator of a fraction. Allow $\Sigma p \neq 1$ on each branch
	P(R|B) = P(RB) / P(B) = (15/64) ÷ (35/64) = 3/7 (0.429)	**A1**	Correct answer
		3	

Question	Answer	Marks	Guidance
4(i)	Total number of selections = ${}^{12}C_7 = 792$	**B1**	Seen as denominator of fraction
	Selections with boy included = ${}^{11}C_6$ or ${}^{12}C_7 - {}^{11}C_7 = 462$	**M1**	Correct unsimplified expression for selections with boy included seen as numerator of fraction
	Probability = 462/792 = 7/12 (0.583)	**A1**	Correct answer
	OR		
	prob of boy not included = 11/12 × 10/11 × …. × 5/6= 5/12	**B1**	Correct unsimplified prob
	1 – 5/12	**M1**	Subtracting prob from 1
	= 7/12	**A1**	Correct answer
		3	

Question	Answer	Marks	Guidance
4(ii)	**Method 1**		
	Scenarios are: 2G + 5B: $^4C_2 \times {}^8C_5 = 336$	**B1**	One unsimplified product correct
	3G + 4B: $^4C_3 \times {}^8C_4 = 280$ 4G + 3B: $^4C_4 \times {}^8C_3 = 56$	**M1**	No of selections (products of ^nC_r and ^nP_r) added for 2, 3 and 4 girls with no of girls and no of boys summing to 7
	Total = 672	**A1**	Correct total
	Probability = 672/792 (28/33) (0.848)	**A1ft**	Correct answer – ‘total’/(‘total no of selections’ from i)
	Method 2		
	0G + 7B $^4C_0 \times {}^8C_7 = 8$	**B1**	One unsimplified no of selections correct
	1G + 6B $^4C_1 \times {}^8C_6 = 112$ Total = 8 + 112 = 120	**M1**	No of selections (products of ^nC_r and ^nP_r) added for 0 and 1 girls with no of girls and no of boys summing to 7
	$(^{12}C_7 - 120)/792$ or $1 - 120/792$	**A1**	792 – 120 = 672 or 1 – 120/792
	Probability = 672/792 (28/33) (0.848)	**A1ft**	‘672’ over ‘792’ from i
	Method 3 (probability)		
	1 – P(0) – P(1) $= 1 - (8/12 \times 7/11 \times \ldots\ldots \times 2/6) - (8/12 \times \ldots \times 3/7 \times 4/6 \times 7)$	**B1**	One correct unsimplified prob for 0 or 1
	= 1 – 1/99 –14/99	**M1**	Subtracting ‘P(0)’ and ‘P(1)’ (using products of 7 fractions with denominators from 12 to 6) from 1
		A1	Both probs correct unsimplified
	= 84/99 = 28/33	**A1ft**	1 – ‘P(0)’ – ‘P(1)’

Question	Answer	Marks	Guidance
4(ii)	**Method 4 (probability)**		
	P(2) + P(3) + P(4) =	**B1**	One correct unsimplified prob for 2, 3 or 4
	42/99 + 35/99 + 7/99	**M1**	Adding ‘P(2)’, ‘P(3)’ and P(4)’ (using products of 7 fractions with denominators from 12 to 6)
		A1	Three probs correct unsimplified
	= 84/99 = 28/33	**A1ft**	‘P(2)’+ ‘P(3)’ + ‘P(4)’
		4	

Question	Answer	Marks	Guidance
5(i)	$z_1 = \pm\frac{90-120}{24} = -\frac{5}{4}$, $z_2 = \pm\frac{140-120}{24} = \frac{5}{6}$	**M1**	At least one standardisation, no cc, no sq rt, no sq using 120 and 24 and either 90 or 140
	$= \Phi\left(\frac{20}{24}\right) - \Phi\left(-\frac{30}{24}\right)$	**A1**	–5/4 and 5/6 unsimplified
	$= \Phi(0.8333) - (1 - \Phi(1.25))$ = 0.7975 – (1 – 0.8944) or 0.8944 – 0.2025 = 0.6919	**M1**	Correct area $\Phi - \Phi$ legitimately obtained and evaluated from phi(their z_2) – phi (their z_1)
	= 0.692 AG	**A1**	Correct answer obtained from 0.7975 and 0.1056 oe to 4sf or 0.6919 seen www
		4	

Question	Answer	Marks	Guidance
5(ii)	**Method 1**		
	Probability = P(2, 3, 4) $= 0.692^2(1 – 0.692)^2 \times {}^4C_2 + 0.692^3(1 – 0.692) \times {}^4C_3 + 0.692^4$	**M1**	Any binomial term of form $4C_x p^x (1-p)^{4-x}$, $x \neq 0$ or 4
		B1	One correct bin term with $n = 4$ and $p = 0.692$,
	= 0.27256 + 0.40825 + 0.22931	**M1**	Correct unsimplified expression using 0.692 or better
	= 0.910	**A1**	Correct answer
	Method 2:		
	1 – P(0, 1) =	**M1**	Any binomial term of form $4C_x p^x (1-p)^{4-x}$, $x\neq0$ or 4
	$1 - 0.692^0(1 – 0.692)^4 \times {}^4C_0 - 0.692^1(1 – 0.692)^3 \times {}^4C_1$	**B1**	One correct bin term with $n = 4$ and $p = 0.692$
	= 1 – 0.00899 – 0.0808757	**M1**	Correct unsimplified expression using 0.692 or better
	= 0.910	**A1**	Correct answer
		4	

Question	Answer	Marks	Guidance
6(i)	P(X >1800) = 0.96, so P($Z > \dfrac{1800-2000}{\sigma}$) = 0.96	**B1**	± 1.75 seen
	$\Phi(\dfrac{200}{\sigma}) = 0.96$ $\dfrac{200}{\sigma} = 1.751$	**M1**	$z = \pm\dfrac{1800-2000}{\sigma}$, allow cc, allow sq rt, allow sq equated to a z-value
	$\sigma = 114$	**A1**	Correct final answer www
		3	
6(ii)	Mean = 300 × 0.2 = 60 and variance = 300 × 0.2 × 0.8 = 48	**B1**	Correct unsimplified mean and variance
	P(X < 70) = P($Z > \dfrac{69.5-60}{\sqrt{48}}$)	**M1**	$Z = \pm\dfrac{x - their\,60}{\sqrt{their\,48}}$
	$= \Phi(1.371)$	**M1**	69.5 or 70.5 seen in an attempted standardisation expression as cc
	=0.915	**A1**	Correct final answer
		4	
6(iii)	np = 60, nq = 240: both > 5, (so normal approximation holds)	**B1**	Both parts evaluated are required
		1	

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td rowspan="5">7(i)</td><td rowspan="1"><table>
<tr><th>Anvils</th><th></th><th>Brecons</th></tr>
<tr><td>8</td><td>15</td><td></td></tr>
<tr><td>9 5</td><td>16</td><td>6</td></tr>
<tr><td>5 3 2 0</td><td>17</td><td>0 1 2 2 8</td></tr>
<tr><td>4 1 0</td><td>18</td><td>1 2 3 3</td></tr>
<tr><td>6</td><td>19</td><td>2</td></tr>
<tr><td></td><td></td><td>Key: 5|16|6 means 165 cm for Anvils and 166 cm for Brecons</td></tr>
</table></td><td>B1</td><td>Correct stem, up or down</td></tr>
<tr><td></td><td>B1</td><td>Correct Anvils labelled on left, leaves in order from right to left and lined up vertically, no commas</td></tr>
<tr><td></td><td>B1</td><td>Correct Brecons labelled on same diagram on right hand side in order from left to right and lined up vertically, no commas</td></tr>
<tr><td></td><td>B1</td><td>Correct key, not split, both teams, at least one with cm</td></tr>
<tr><td></td><td>4</td><td></td></tr>
<tr><td rowspan="4">7(ii)</td><td>Median = 173</td><td>B1</td><td>Correct median (or Q2)</td></tr>
<tr><td>LQ = 169; UQ = 181
IQR = 181 – 169</td><td>M1</td><td>Either UQ = 181 ± 4, or LQ = 169 ± 4 and evaluating UQ – LQ</td></tr>
<tr><td>= 12</td><td>A1</td><td>Correct answer from 181 and 169 only</td></tr>
<tr><td></td><td>3</td><td></td></tr>
</table>

Question	Answer	Marks	Guidance
7(iii)	$\Sigma x = 1923 + 166 + 172 + 182\ (= 2443)$ $\sum x^2 = 337221 + 166^2 + 172^2 + 182^2\ (= 427485)$	M1	Correct unsimplified expression for $\sum x$ and $\sum x^2$, may be implied
	Mean $= \frac{\sum x}{14} = \frac{2443}{14} = 174.5$	M1	Correct unsimplified mean
	Variance $= \frac{\sum x^2}{14} - \left(\frac{\sum x}{14}\right)^2 = \frac{427485}{14} - \left(\frac{2443}{14}\right)^2$	M1	Correct unsimplified variance using 14, their Σx and their Σx^2, not using 1923 and/or 337221
	S d = 9.19	A1	Correct answer
		4	

Cambridge International AS & A Level

Cambridge Assessment International Education
Cambridge International Advanced Subsidiary and Advanced Level

MATHEMATICS	**9709/61**
Paper 6	**May/June 2018**
MARK SCHEME	
Maximum Mark: 50	

Published

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Cambridge Assessment International Education

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GENERIC MARKING PRINCIPLE 4:

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GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously 'correct' answers or results obtained from incorrect working.

  - Note: B2 or A2 means that the candidate can earn 2 or 0.
    B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

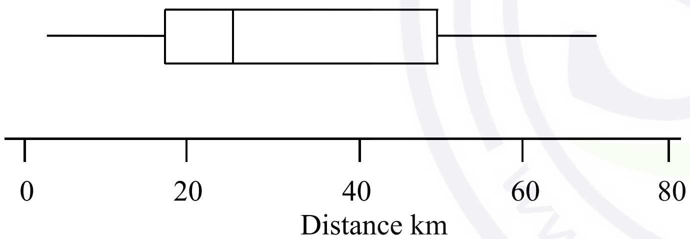
The following abbreviations may be used in a mark scheme or used on the scripts:

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only – often written by a 'fortuitous' answer
ISW	Ignore Subsequent Working
SOI	Seen or implied
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become 'follow through' marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Question	Answer	Marks	Guidance
1	$\Sigma(x - 10) = 186 - 12 \times 10 = 66$	B1	Correct answer
	$\dfrac{\Sigma(x-10)^2}{12} - \left(\dfrac{\Sigma(x-10)}{12}\right)^2 = 4.5^2$	M1	Consistent substituting in the correct coded variance formula OR Valid method for Σx^2 then expanding $\Sigma(x - 10)^2$, 3 terms with at least 2 correct
	$\Sigma(x - 10)^2 = 606$	B1	Correct answer
		3	

Question	Answer	Marks	Guidance
2(i)	LQ = 18, Median = 25, UQ = 50 	B1	median correct
		B1	LQ and UQ correct
		B1	Quartiles and median plotted as box graph with linear scale min 3 values
		B1ft	Whiskers drawn to correct end points with linear scale, not thr' box, not joining at top or bottom of box. Ft their UQ and LQ. Whiskers must be with ruler If scale non-linear or non-existent SCB1if all 5 data values (quartiles and end points) have values shown and all are correct numerically and fulfil the 'box' and 'whiskers ruled line' requirements
		B1	Label to include 'distance or travelled' and 'km,' allow 'total km', linear scale, numbered at least 5 – 70.
		5	

Question	Answer	Marks	Guidance
2(ii)	$1.5 \times \text{IQR} = 48$ **Method 1** LQ – 48 = –ve, (i.e. < 0) UQ + 48 = 98 (i.e. > 70)	M1	Attempt to find $1.5 \times$ their IQR and add to UQ **or** subt from LQ
	hence no outliers	A1	Correct conclusion from correct working, need both ends. No need to state comparisons.
	Method 2 LQ – 5 = 13 (< 48) 70 – UQ = 20 (< 48)	M1	Compare their $1.5 \times$ IQR (= 48) > gap (20) between UQ and max 70 **or** LQ and min 5
	Hence no outliers	A1	Correct conclusion from correct working, need both ends. No need to state comparisons
		2	

Question	Answer	Marks	Guidance
3(i)	P(RB) + P(BR) = $\frac{4}{12} \times \frac{8}{11} + \frac{8}{12} \times \frac{4}{11}$ oe	M1	Multiply 2 probs together and summing two 2-factor probs, unsimplified, condone replacement
	P(diff colours) = $\frac{64}{132}$ ($\frac{16}{33}$) (0.485) oe	A1	Correct answer
	Method 2 1 – P(BB) – P(RR) = $1 - \frac{4}{12} \times \frac{3}{11} - \frac{8}{12} \times \frac{7}{11}$	M1	Multiply 2 probs together and subtracting two 2-factor probs from 1, unsimplified, condone replacement
	P(diff colours) = $\frac{64}{132}$ ($\frac{16}{33}$) oe	A1	Correct answer
	Method 3 P(diff colours) = $\frac{({}^4C_1 \times {}^8C_1)}{{}^{12}C_2}$	M1	Multiply 2 combs together and dividing by a combination
	$= \frac{16}{33}$	A1	Correct answer
		2	
3(ii)	Number of red socks: 0, 1, 2; Prob: $\frac{14}{33}$, $\frac{16}{33}$, $\frac{3}{33}$	B1	Prob distribution table drawn, top row correct, condone additional values with $p = 0$ stated
		B1	P(0) or P(2) correct to 3sf (need not be in table)
		B1	All probs correct to 3sf, condone P(0) and P(2) swapped if correct
		3	

Question	Answer	Marks	Guidance
3(iii)	$E(X) = 1 \times \frac{16}{33} + 2 \times \frac{3}{33} = \frac{16}{33} + \frac{6}{33} = \frac{22}{33}$ $(\frac{2}{3})$	B1ft	ft their table if 0, 1, 2 only, $0 < p < 1$
		1	

Question	Answer	Marks	Guidance
4(a)	$z_1 = 2.4$	B1	± 2.4 seen accept 2.396
	$z_2 = -0.5$	B1	± 0.5 seen
	$2.4 = \frac{36800 - \mu}{\sigma}$	M1	Either standardisation eqn with z value, not 0.5082, 0.7565, 0.0082, 0.6915, 0.3085, 0.6209, 0.0032 or any other probability
	$-0.5 = \frac{31000 - \mu}{\sigma}$	M1	Sensible attempt to eliminate μ or σ by substitution or subtraction from their 2 equations (z-value not required), need at least 1 value stated
	σ = 2000 μ = 32000	A1	Both correct answers
		5	
4(b)	$P(X < 3\mu) = P\left(z < \frac{3\mu - \mu}{(4\mu/3)}\right)$ or $P = \left(z < \frac{(9\sigma/4) - (3\sigma/4)}{\sigma}\right)$	M1	Standardise, in terms of one variable, accept σ^2 or $\sqrt{\sigma}$
	$P(z < \frac{6}{4})$	M1	$\frac{6}{4}$ or $\frac{6}{4\sigma}$ seen
	= 0.933	A1	Correct final answer
		3	

Question	Answer	Marks	Guidance
5(i)	$P(4, 5, 6) = {}^{15}C_4(0.22)^4(0.78)^{11} + {}^{15}C_5(0.22)^5(0.78)^{10} +$	M1	One binomial term ${}^{15}C_x p^x(1-p)^{15-x}$ $0 < p < 1$
	${}^{15}C_6(0.22)^6(0.78)^9$	A1	Correct unsimplified expression
	$= 0.398$	A1	Correct answer
		3	
5(ii)	$\mu = 145 \times 0.22 = 31.9$ $\sigma^2 = 145 \times 0.22 \times 0.78 = 24.882$	B1	Correct unsimplified mean and variance
	$P(x > 26) = P\left(z > \frac{26.5 - 31.9}{\sqrt{24.882}}\right) = P(z > -1.08255)$	M1	Standardising must have sq rt
		M1	25.5 or 26.5 seen as a cc
	$= \Phi(1.08255)$	M1	Correct area Φ, must agree with their μ
	$= 0.861$	A1	Correct final answer accept 0.861, or 0.860 from 0.8604 not from 0.8599
		5	

Question	Answer	Marks	Guidance
6(i)	P(SLL) = (0.3)(0.55)(0.55) = 0.09075 ($\frac{363}{4000}$)	M1	P(SLL), P(SRR), P(SSL) or P(SSR) seen
	P(SRR) = (0.3)(0.15)(0.15) = 0.00675 ($\frac{27}{4000}$)	A1	Two correct options 0.09075 or 0.00675 can be unsimplified
	Total = ${}^3C_1 \times$ P(SLL) + ${}^3C_1 \times$ P(SRR) = 0.27225 + 0.02025	M1	Summing 6 prob options not all identical
	Prob = 0.293 accept 0.2925 ($\frac{117}{400}$)	A1	Correct answer
		4	
6(ii)	$\text{P(SSS} \mid \text{all same dir}^n) = \dfrac{P(SSS\ \ and\ same\ dir^n)}{P(same\ direction)}$	B1	$(0.3)^3$ oe seen on its own as num or denom of a fraction
		M1	Attempt at P(*SSS*+*LLL*+*RRR*) seen anywhere
	$= \dfrac{0.3 \times 0.3 \times 0.3}{(0.15)^3 + (0.55)^3 + (0.3)^3}$	A1	$(0.15)^3 + (0.55)^3 + (0.3)^3$ oe seen as denom of a fraction
	= 0.137 ($\frac{108}{787}$)	A1	Correct answer
		4	

Question	Answer	Marks	Guidance
7(i)	$\frac{9!}{2!2!} = 90720$	**B1**	Must see 90720
		1	
7(ii)	**Method 1** ↑ * * * * * A	**B1**	5! seen multiplied (arrangement of consonants allowing repeats)
	No. arrangements of consonants × ways of inserting vowels =	**B1**	${}^6P_4$ oe (i.e. 6 × 5 × 4 × 3, ${}^6C_4$ × 4!) seen mult (allowing repeats) no extra terms
	$\frac{5!}{2!}$ $\times \frac{{}^6P_4}{2!}$	**B1**	Dividing by at least one 2! (removing at least one set of repeats)
	Answer $\frac{{}^6P_4}{2!} \times \frac{5}{2} = 10\,800$	**B1**	Correct final answer
		4	
7(iii)	${}^5C_3 = 10$	**M1**	${}^5C_x$ or ${}^5P_x$ seen alone, x = 2 or 3
		A1	Correct final answer not from ${}^5C_2$
		2	

Question	Answer	Marks	Guidance
7(iv)	**Method 1** Considering separate groups	**M1**	Considering two scenarios of MME or EEM or MMEE with attempt, may be probs or perms
	MME** = ${}^5C_2$ = 10 MEE** = ${}^5C_2$ = 10 MMEE* = ${}^5C_1$ = 5	**M1**	Summing three appropriate scenarios from the four need ${}^5C_x$ seen in all of them
	ME*** = ${}^5C_3$ = 10 see **(iii)** Total = 35	**A1**	Correct final answer
	Method 2 Considering criteria are met if ME are chosen	**M1**	${}^7C_x$ only seen, no other terms
		M1	${}^xC_3$ only seen, no other terms
	ME *** = ${}^7C_3$ = 35	**A1**	Correct final answer
		3	

Cambridge International AS & A Level

Cambridge Assessment International Education
Cambridge International Advanced Subsidiary and Advanced Level

MATHEMATICS	**9709/62**
Paper 6	**May/June 2018**
MARK SCHEME	
Maximum Mark: 50	

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M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously 'correct' answers or results obtained from incorrect working.

  - Note: B2 or A2 means that the candidate can earn 2 or 0.
    B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

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Penalties

MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become ‘follow through’ marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Question	Answer	Marks	Guidance
1(i)	38	**B1**	
		1	
1(ii)	Median = 38.5	**B1**	CAO
	IQR = 40 – 38	**M1**	39 < UQ < 45 – 36 < LQ $\leqslant$ 38
	= 2	**A1**	If M0 awarded **SCB1** for **both** UQ = 40 or 40.5 **and** LQ = 38 or 37.75 seen
		3	

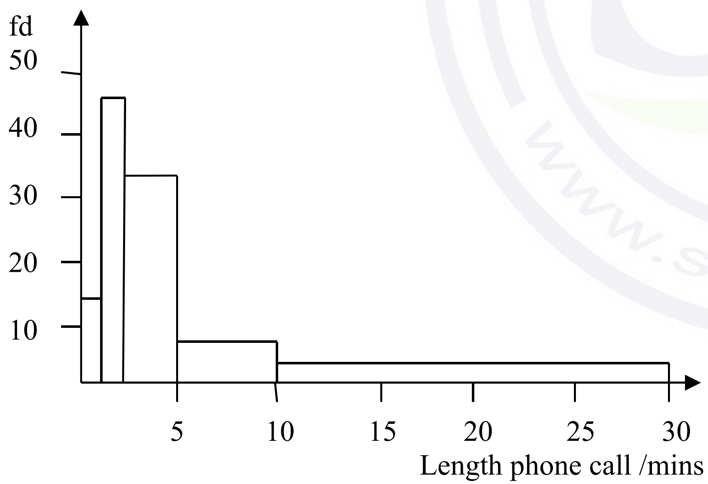
Question	Answer	Marks	Guidance
2(i)	**Method 1** $P(M \cap H) = \frac{3}{4} \times \frac{3}{5} = \frac{9}{20}$ (0.45)	**B1**	Seen, accept unsimplified
	$P(F \text{ or } M \cap H) = \frac{1}{4} + \frac{9}{20} = \frac{14}{20}$	**M1**	Numerical attempt at $P(F) + P(M \cap H)$
		A1	Correct unsimplified expression
	$= \frac{7}{10}$ (0.7) OE	**A1**	Correct final answer
	Method 2 $P(M \cap H') = \frac{3}{4} \times \frac{2}{5} = \frac{6}{20}$ (0.3)	**B1**	Seen, accept unsimplified
	$P(F \text{ or } M \cap H) = 1 - P(M \cap H')$	**M1**	Numerical attempt at $1 - P(M \cap H')$
	$= 1 - \frac{3}{4} \times \frac{2}{5}$	**A1**	Correct unsimplified expression
	$= \frac{7}{10}$ (0.7) OE	**A1**	Correct final answer

Question	Answer	Marks	Guidance
2(i)	**Method 3** $P(F \cap H' \text{ or } H) = \frac{1}{4} \times \frac{1}{5} + \frac{1}{4} \times \frac{4}{5} + \frac{3}{4} \times \frac{3}{5}$	**B1**	$\frac{3}{4} \times \frac{3}{5}$ $(\frac{9}{20})$ **or** $\frac{1}{4} \times \frac{4}{5}$ $(\frac{4}{20})$ **or** $\frac{3}{4} \times \frac{3}{5} + \frac{1}{4} \times \frac{4}{5}$ $(\frac{13}{20})$ seen
	$= \frac{1}{20} + \frac{4}{20} + \frac{9}{20}$	**M1**	Numerical attempt at $P(F \cap H') + P(F \cap H) + P(M \cap H)$
		A1	Correct unsimplified expression
	$= \frac{7}{10}$ (0.7) oe	**A1**	Correct final answer
	Method 4 – Venn diagram style approach $P(F \cup H) = P(F) + P(H) - P(F \cap H)$	**B1**	$\frac{3}{4} \times \frac{3}{5}$ $(\frac{9}{20})$ **or** $\frac{1}{4} \times \frac{4}{5}$ $(\frac{4}{20})$ **or** $\frac{3}{4} \times \frac{3}{5} + \frac{1}{4} \times \frac{4}{5}$ $(\frac{13}{20})$ seen
	$= \frac{1}{4} + \frac{1}{4} \times \frac{4}{5} + \frac{3}{4} \times \frac{3}{5} - \frac{1}{4} \times \frac{4}{5}$	**M1**	Numerical attempt at $P(F) + P(H) - P(F \cap H)$
	$= \frac{1}{4} + \frac{4}{20} + \frac{9}{20} - \frac{4}{20}$	**A1**	Correct unsimplified expression
	$= \frac{7}{10}$ (0.7) oe	**A1**	Correct final answer
		4	

Question	Answer	Marks	Guidance
2(ii)	**Method 1** (P(M) × P(H) =) $\frac{3}{4} \times \textit{their}\frac{13}{20} = \frac{39}{80}$ (P($M \cap H$) =) $\frac{3}{4} \times \frac{3}{5} = 0.45$	**M1**	Unsimplified, or better, legitimate numerical attempt at P(M) × P(H) **and** P($M \cap H$) Descriptors P($M \cap H$) **and** P(M) × P(H) seen, correct numerical evaluation and comparison, conclusion stated
	$\frac{39}{80}$ (0.4875) ≠ 0.45, not independent	**A1**	
	Method 2 $\mathrm{P}(M \mid H) = \frac{\mathrm{P}(M \cap H)}{\mathrm{P}(H)} = \frac{\frac{9}{20}}{\text{their } \frac{13}{20}} = \frac{9}{13}$ $\mathrm{P}(M) = \frac{3}{4}$	**M1**	Unsimplified, or better, numerical attempt at P(H) and P($M \cap H$), P(M)
	$\frac{9}{13} \neq \frac{3}{4}$, not independent	**A1**	Descriptors P($M \cap H$), P(H) **and** P(M) **OR** P($M\|H$) **and** P(M) seen, numerical evaluation **and** comparison, conclusion stated Any appropriate relationship can be used, the M is awarded for an unsimplified, or better, numerical attempt at the terms required, the A mark requires the correct descriptors, numerical evaluation and comparison and the conclusion
		2	

Question	Answer	Marks	Guidance
3(i)	$z = -1.282$	**B1**	± 1.282 seen
	$-1.282 = \dfrac{440-\mu}{9}$	**M1**	±Standardisation equation with 440, 9 and μ, equated to a z-value, (not $1 - z$-value or probability e.g. 0.1841, 0.5398, 0.6202, 0.8159)
	$\mu = 452$	**A1**	Correct answer rounding to 452, not dependent on B1
		3	
3(ii)	$P(z > 1.8) = 1 - 0.9641 = 0.0359$	**B1**	
	Number $= 0.0359 \times 150$ $= 5.385$	**M1**	$p \times 150$, $0 < p < 1$
	(Number of cartons =) 5	**A1FT**	Accept either 5 or 6, not indicated as an approximation, e.g. ~, about **FT** *their* $p \times 150$, answer as an integer
		3	

Question	Answer	Marks	Guidance
4(i)	<table><tr><td>X</td><td>0</td><td>1</td><td>2</td></tr><tr><td>Prob</td><td>$\frac{2}{7}$</td><td>$\frac{4}{7}$</td><td>$\frac{1}{7}$</td></tr></table>	**B1**	Prob distribution table drawn, top row correct with at least one probability $0 < p < 1$ entered, condone additional values with $p = 0$ stated
	$\text{P}(0) = \frac{5}{7}\times\frac{4}{6}\times\frac{3}{5} = \frac{2}{7}$ (0.2857)	**B1**	One probability correct (need not be in table)
	$\text{P}(1) = \frac{2}{7}\times\frac{5}{6}\times\frac{4}{5}\times {}^3C_1 = \frac{4}{7}$ (0.5713)	**B1**	Another probability correct (need not be in table).
	$\text{P}(2) = \frac{2}{7}\times\frac{1}{6}\times\frac{5}{5}\times {}^3C_2 = \frac{1}{7}$ (0.1429)	**B1**	Values in table, all probs correct (to 3SF) or 3 probabilities summing to 1
		4	
4(ii)	$\text{Var}(X) = 1\times\frac{4}{7} + 4\times\frac{1}{7} - (\frac{6}{7})^2$ $= \frac{8}{7} - (\frac{6}{7})^2$	**M1**	Unsimplified correct numerical expression for variance **or** *their* probabilities from **(i)** $0 < p < 1$ in **unsimplified** variance expression
	$= \frac{20}{49}$ or 0.408	**A1**	Correct answer (0.40816…) nfww Final answer does **not** imply the method mark
		2	

Question	Answer	Marks	Guidance
5(i)	$a = 40$	**B1**	
		1	
5(ii)	Mean $= \dfrac{0.5\times14+1.5\times46+3.5\times102+7.5\times \mathit{their}\,40+20\times40}{242}$ $= \dfrac{1533}{242}$	**M1**	Numerator: 5 products with at least 3 acceptable mid-points × appropriate frequency FT **(i).** Denominator: 242 CAO $\dfrac{1533}{242}$ implies M1, but if **FT** an unsimplified expression required
	$= 6\dfrac{81}{242}$ or 6.33	**A1**	CAO (6.3347… rounded to 3 or more SF)
		2	
5(iii)	fd = 14, 46, 34, ($\dfrac{\mathit{their}\ (i)}{5}$ =) 8, 2	**M1**	Attempt at fd [f/(attempt at cw)] or scaled freq
		**A1FT**	Correct heights seen on diagram with linear vertical scale from (x, 0) **FT** their $\dfrac{a}{5}$ only
		B1	Correct bar widths (1:1:3:5:20) at axis, visually no gaps, with linear horizontal scale from (0, y), first bar starting at (0,0)
		B1	Labels (time, mins, and fd(OE) seen, some may be as a title) and a linear scale with at least 3 values marked on each axis. (Interval notation not acceptable)
		4	

Question	Answer	Marks	Guidance
6(a)(i)	(AAAIU) * * * * Arrangements of vowels/repeats × arrangements of (consonants & vowel group) =	**M1**	$k \times 5!$ (k is an integer, $k \geqslant 1$)
	$\frac{5! \times 5!}{3!}$	**M1**	$\frac{m}{3}!$ (m is an integer, $m \geqslant 1$) **Both** Ms can **only** be awarded if expression is **fully correct**
	= 2400	**A1**	Correct answer
		3	
6(a)(ii)	E.g. R * * * T * * * L . Arrangements of consonants RL, RS, SL = $^3P_2 = 6$ Arrangements of remaining letters = $\frac{6!}{3!} = 120$	**M1**	$k \times \frac{6!}{3!}$ **or** $k \times {}^3P_2$ **or** $k \times {}^3C_2$ **or** $k \times 3!$ **or** $k \times 3 \times 2$ (k is an integer, $k \geqslant 1$), no irrelevant addition
	Total 120 × 6	**M1**	Correct unsimplified expression or $\frac{6!}{3!} \times {}^3C_2$
	= 720 ways	**A1**	Correct answer
		3	

Question	Answer	Marks	Guidance
6(b)	**Method 1** N(2) R(8) Br(4) 1 2 1 $= 2\times{}^8C_2\times4 = 224$	**M1**	Multiply 3 combinations, ${}^2C_x\times{}^8C_y\times{}^4C_z$. Accept ${}^2C_1 = 2$ etc.
	2 1 1 $= 1\times{}^8C_1\times4 = 32$ 1 1 2 $= 2\times8\times{}^4C_2 = 96$	**A1**	3 or more options correct unsimplified
	2 0 2 $= 1\times1\times{}^4C_2 = 6$ 1 0 3 $= 2\times1\times4 = 8$	**M1**	Summing *their* values of 4 or 5 legitimate scenarios (no extra scenarios)
	Total = 366 ways	**A1**	Correct answer
	Method 2 ${}^{14}C_4$ – (2N2R or 1N3R or 4R or 3R1B or 2R2B or 1R3B or 4B)	**M1**	'${}^{14}C_4 - k$' seen, k an integer from an expression containing ${}^8C_x$
	$1001 - (1\times{}^8C_2 + 2\times{}^8C_3 + {}^8C_4 + {}^8C_3\times4 + {}^8C_2\times{}^4C_2 + 8\times4 + 1)$	**A1**	4 or more 'subtraction' options correct unsimplified, may be in a list
	$1001 - (28 + 112 + 70 + 224 + 168 + 32 + 1)$	**M1**	*Their* ${}^{14}C_4$ – [*their* values of 6 or more legitimate scenarios] (no extra scenarios, condone omission of final bracket)
	= 366	**A1**	Correct answer
		4	

Question	Answer	Marks	Guidance
7(i)	**Method 1** P(< 11) = 1 – P(11, 12, 13)	M1	Binomial expression of form $^{13}C_x\ (p)^x(1–p)^{13–x}$, $0 < x < 13,\ 0 < p < 1$
	$= 1 - {}^{13}C_{11}(0.6)^{11}(0.4)^2 - {}^{13}C_{12}(0.6)^{12}(0.4) - (0.6)^{13}$	M1	Correct unsimplified answer
	= 0.942	A1	CAO
	Method 2 P(< 11) = P(0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10)	M1	Binomial expression of form $^{13}C_x\ (p)^x(1–p)^{13–x}$ $0 < x < 13,\ 0 < p < 1$
	$= (0.4)^{13} + {}^{13}C_1(0.4)^{12}(0.6) + \ldots + {}^{13}C_{10}(0.4)^3(0.6)^{10}$	M1	Correct unsimplified answer
	= 0.942	A1	CAO
		3	
7(ii)	$\mu = 130 \times 0.35 = 45.5$ var $= 130 \times 0.35 \times 0.65 = 29.575$	B1	Correct unsimplified mean and var (condone $\sigma^2 = 29.6$, $\sigma = 5.438$)
	$P(\geqslant 50) = P\left(z > \dfrac{49.5 - 45.5}{\sqrt{29.575}}\right) = P\ (z > 0.7355)$	M1	Standardising, using $\pm\left(\dfrac{x - \textit{their}\text{ mean}}{\textit{their}\ \sigma}\right)$, x = value to standardise 49.5 or 50.5 seen in ± standardisation equation
	$= 1 - \Phi(0.7355)$	M1	Correct final area
	= 1 – 0.7691	M1	
	= 0.231	A1	Correct final answer
		5	

Question	Answer	Marks	Guidance
7(iii)	$1 - (0.65)^n > 0.98$ or $0.02 > (0.65)^n$	**M1**	Eqn or inequality involving, 0.65^n and 0.02 **or** 0.35^n and 0.98
	$n > 9.08$	**M1**	Attempt to solve their eqn or inequality by logs **or** trial and error
	$n = 10$	**A1**	CAO
		3	

Cambridge Assessment International Education
Cambridge International Advanced Subsidiary and Advanced Level

MATHEMATICS **9709/63**

Paper 6 **May/June 2018**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2018 series for most Cambridge IGCSE™, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

This document consists of **14** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

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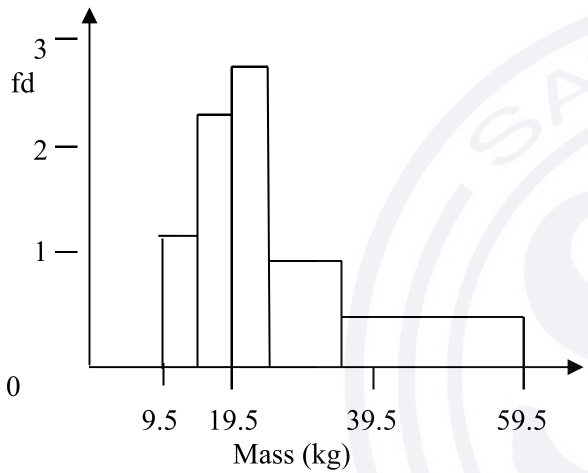
1(i)	15–19 (kg) cao	**B1**	kg not necessary; condone 14.5 – 19.5
	Total:	**1**	
1(ii)	fd = 1.2, 2.4, 2.8, 1, 0.32 	**M1**	Attempt at fd [f/(attempt at cw)] or scaled freq (may be implied by 4 correct)
		A1	Correct heights seen on diagram with linear vertical scale from (x, 0)
		B1	Correct bar widths (1:1:1:2:5) visually no gaps with linear horizontal scale from (9.5,y) and first bar starting at (9.5, y)
		B1	Histogram, using attempted fds, with labels (mass, kg and fd seen) and at least 3 linearly spaced values on each axis. Horizontal axis must range from at least 9.5 to 59.5 If horizontal axis clearly starts from zero, either a break in the scale must be indicated or the scale must be linear from zero.

Question	Answer	Marks	Guidance
2(i)	$z = 0.674$	**B1**	z value ± 0.674
	$0.674 = \dfrac{0 - -3}{\sigma}$	**M1**	$\pm$Standardising with 0 and equating to a z-value
	$\sigma = 4.45$	**A1**	Correct answer www ie not ignoring a minus sign
	Total:	**3**	
2(ii)	P(0, 1) $= (0.75)^8 + {}^8C_1(0.25)(0.75)^7$	**M1**	Any bin of form ${}^8C_x(0.75)^x(0.25)^{8-x}$ any x
		M1	Correct unsimplified answer, may be implied by numerical values
	$0.1001 + 0.2670 \quad = 0.367$	**A1**	Correct answer
	Method 2 $1 - P(8,7,6,5,4,3,2) = 1 - (0.25)^8 - {}^8C_1(0.75)(0.25)^7 - \ldots$ $- {}^8C_2(0.75)^6(0.25)^2$ $= 0.367$	**M1**	Any bin of form ${}^8C_x(0.75)^x(0.25)^{8-x}$ any x
		M1	Correct unsimplified answer
		A1	Correct answer
	Total:	**3**	

Question	Answer	Marks	Guidance
3(i)	$(1-x)$ and 0.45 (or 0.3)	**B1**	Seen, either on tree diagram or elsewhere
	Beginners: $0.7 \times x$ + '0.45' × '$(1-x)$' = 0.5 Or Advanced: '0.3' × x + 0.55 × '$(1-x)$' = 0.5 Or $0.7 \times x$ + '0.45'× '$(1-x)$' = '0.3' × x + 0.55× '$(1-x)$'	**M1**	One of the three correct probability equations
	$x = 0.2$ oe	**A1**	Correct answer
	Total:	**3**	
3(ii)	$P(M \mid A) = \dfrac{P(M \cap A)}{P(A)} = \dfrac{0.2 \times 0.3}{0.5}$	**M1**	'i' × 0.3 as num or denom of a fraction
		M1	0.5 (or (1 – 'i') × 0.55 + 'i' × 0.3 unsimplified) seen as denom of a fraction
	$= 0.12 \left(\dfrac{3}{25}\right)$	**A1**	Correct answer
	Total:	**3**	

Question	Answer	Marks	Guidance
4(i)	Mean = $(30 \times 1500 + 21 \times 2400)/51$	**M1**	Multiply by 30 and 21, summing and dividing total by 51 $\left(\frac{45000+50400}{51}\right)$
	= 1870 (1870.59)	**A1**	correct answer (to 3sf)
	Total:	**2**	
4(ii)	$230^2 = \frac{\Sigma x_F^{\ 2}}{30} - 1500^2$ so $\Sigma x_F^{\ 2} = 69\,087\,000$	**M1**	One correct substitution into a correct variance formula
		A1	Correct $\Sigma x_F^{\ 2}$ (rounding to 69 000 000 2sf)
	$160^2 = \frac{\Sigma x_L^{\ 2}}{21} - 2400^2$ so $\Sigma x_L^{\ 2} = 121\,497\,600$	**A1**	Correct $\Sigma x_L^{\ 2}$ (rounding to 121 000 000 3sf)
	New var = $\frac{69087000+121497600}{51} - 1870.588^2 = 237\,853$	**M1**	using '$\Sigma x_F^{\ 2}$'+ '' $\Sigma x_L^{\ 2}$ dividing by 51 and subtracting 'i' squared. (Correct '$\Sigma x_F^{\ 2}$' + '' $\Sigma x_L^{\ 2}$ = 190 584 600)
	New sd = 488	**A1**	Correct answer accept anything between 486 and 490
	Total:	**5**	

Question	Answer	Marks	Guidance
5(i)	$P(0) = 0.6 \times 0.25 \times 0.5 = 0.075$ $P(1) = 0.4 \times 0.25 \times 0.5 + 0.6 \times 0.75 \times 0.5 + 0.6 \times 0.25 \times 0.5 = 0.35$ $P(2) = 0.4 \times 0.75 \times 0.5 + 0.4 \times 0.25 \times 0.5 + 0.6 \times 0.75 \times 0.5 = 0.425$ $P(3) = 0.4 \times 0.75 \times 0.5 = 0.15$	B1	0, 1, 2, 3 seen as top line of a pdf table OR attempting to evaluate P(0), P(1), P(2) and P(3)
		M1	Multiply 3 probabilities together from 0.4 or 0.6, 0.25 or 0.75, 0.5 with or without a table
	No of heads: 0, 1, 2, 3 Prob: 0.075 $\left(\frac{3}{40}\right)$, 0.35 $\left(\frac{7}{20}\right)$, 0.425 $\left(\frac{17}{40}\right)$, 0.15 $\left(\frac{3}{20}\right)$	M1	Summing 3 probabilities for P(1) or P(2) with or without a table
		B1	One correct probability seen.
		A1	All correct in a table
	Total:	**5**	
5(ii)	$E(X) = 0.35 + 2 \times 0.425 + 3 \times 0.15 = 1.65 \left(\frac{33}{20} \text{ oe}\right)$	M1	Correct unsimplified expression for the mean using their table, $\sum p = 1$; can be implied by correct answer
5(ii)	$\text{Var}(X) = 0.35 + 4 \times 0.425 + 9 \times 0.15 - 1.65^2$	M1	Correct unsimplified expression for the variance using their table and their mean^2 subtracted, $\sum p = 1$
	$= 0.678\ (0.6775) \left(\frac{271}{400} \text{ oe}\right)$	A1	Correct answer
	Total:	**3**	

Inner table in 5(i):

No of heads	0	1	2	3
Prob	0.075 $\left(\frac{3}{40}\right)$	0.35 $\left(\frac{7}{20}\right)$	0.425 $\left(\frac{17}{40}\right)$	0.15 $\left(\frac{3}{20}\right)$

Question	Answer	Marks	Guidance
6(i)	$z_1 = \pm\dfrac{4.1-5.7}{0.8} = -2$ $\quad z_2 = \pm\dfrac{5-5.7}{0.8} = -0.875$	**M1**	At least one standardising no cc no sq rt no sq using 5.7 and 0.8 and either 4.1 or 5
	P(Toffee Apple) $= \text{P}(d < 5.0) - \text{P}(d < 4.1)$ $= \text{P}(z < -0.875) - \text{P}(z < -2)$ $= \Phi(-0.875) - \Phi(-2)$ $= \Phi(2) - \Phi(0.875)$	**M1**	Correct area $\Phi - \Phi$ legitimately obtained – need 2 negative z-values or 2 positives – not one of each
	$= 0.9772 - 0.8092 = 0.168$ (or $0.1908 - 0.0228$)	**A1**	Correct final answer
	Total:	**3**	
6(ii)	$np = 250 \times 0.168 = 42$, $\quad npq = 34.944$	**B1ft**	Correct unsimplified mean and var – ft their prob for **(i)** providing ($0 < p < 1$) Implied by $\sigma = \sqrt{34.944} = 5.911$
	$\text{P}(< 50) = \text{P}\left(z < \dfrac{49.5-42}{\sqrt{34.944}}\right) = \text{P}(z < 1.2687)$	**M1**	$\pm$ Standardising using 50, their mean and sd; must have sq rt.
		M1	49.5 or 50.5 seen as a cc
	$= \Phi(1.2687)$	**M1**	Correct area Φ(> 0.5 for $+z$ and < 0.5 for $-z$)in their final answer
	$= 0.898$	**A1**	Correct final answer
	Total:	**5**	

Question	Answer	Marks	Guidance
7(i)	****E**** Other letters arranged in $\frac{8!}{2!3!}$	M1	Mult by 8! or $^8P_8$ oe (arrangements ignoring repeats)
	= 3360 ways	A1	Correct final answer www
	OR $\frac{8\times7\times6\times5\times4\times4\times3\times2\times1}{4!2!}$ = 3360 ways	M1	Correct numerator (161 280)
		A1	Correct final answer www
	Total:	**2**	
7(ii)	* * * * * ↑ Arrangements other letters × ways Es inserted	M1	k mult by $^6C_4$ or $^6P_4$ oe (ways to insert Es ignoring repeats), k can = 1 or k mult by $\frac{5!}{2!}$
	$= \frac{5!}{2!} \times {}^6C_4 \left(\frac{5!}{2!}\times\frac{^6P_4}{4!}\right)$	M1	Correct unsimplified expression or $\frac{5!}{2!} \times {}^6P_4$
	= 900 ways	A1	Correct answer
	OR Total no of ways – no of ways with Es touching 9!/(4! × 2!) – … or 7 560 – …	M1	7560 unsimplified – k
	$\frac{6!}{2!} + {}^6P_2\times\frac{5!}{2!} + \frac{^6P_2}{2!}\times\frac{5!}{2!} + \frac{^6P_3}{2!\times\frac{5!}{2!}}$ = 360 + 1800 + 900 + 3600 = 6660	M1	Attempting to find four ways of Es touching (4 Es, 3Es and a single, 2 lots of 2 Es, 2 Es and 2 singles)
	7 560 – 6 660 = 900	A1	Correct answer

Question	Answer	Marks	Guidance
7(ii)	OR Adding the number of ways with the first E in the 1st (E_1), 2nd (E_2) or 3rd (E_3) position. $\frac{5!}{2!}(E_1 + E_2 + E_3)$ where $E_1 = 10$, $E_2 = 4$, $E_3 = 1$	**M1**	For any values for E_1, E_2 and E_3
	$\frac{5!}{2!}(E_1 + E_2 + E_3)$	**M1**	For any two correct values of E_1, E_2 and E_3
	600 + 240 + 60 = 900	**A1**	Correct answer
	Total:	**3**	
7(iii)	EENN* in 3 ways	**B1**	Numerical value must be stated
	Total:	**1**	

Question	Answer	Marks	Guidance
7(iv)	EE *** with no N: 1 way EEN** 3C2 or listing 3 ways EENN* 3 ways from **(iii)**	**M1**	Identifying the three different scenarios of EE, EEE or EEEE
		A1	Total no of ways with two Es (7 or 3 + 3 + 1)
	EEE** with no N: 3 ways EEEN* 3 ways EEENN 1 way	**A1**	Total no. of ways with 3 Es (7)
	EEEE* no N 3 ways EEEEN 1 way Total 18 ways	**A1**	Correct answer stated
	Method List containing ways with 2Es, 3Es and 4Es List containing at least 8 correct different ways List of all 18 correct ways Total 18	**M1**	At least 1 option listed for each of EE^^^, EEE^^, EEEE^
		A1	Ignore repeated options
		A1	Ignore repeated/incorrect options
		A1	Correct answer stated
	Total:	**4**	

Cambridge Assessment International Education
Cambridge International Advanced Subsidiary and Advanced Level

MATHEMATICS **9709/62**

Paper 6 Probability and Statistics **March 2018**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the March 2018 series for most Cambridge IGCSE®, Cambridge International A and AS Level components and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

  - Note: B2 or A2 means that the candidate can earn 2 or 0.
    B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

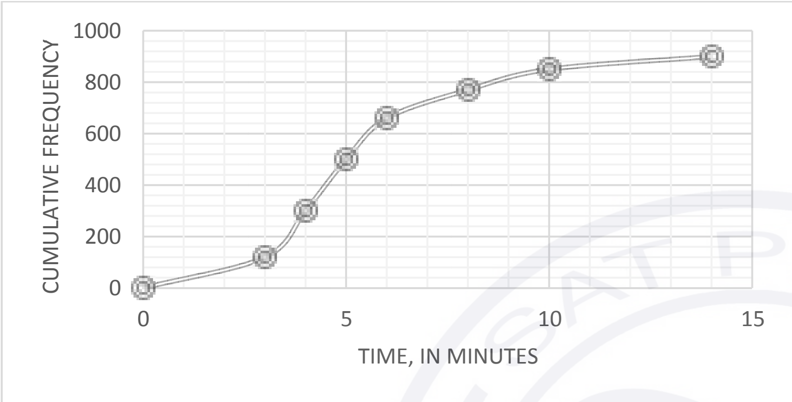
- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no “follow through” from a previous error is allowed)
CWO	Correct Working Only – often written by a ‘fortuitous’ answer
ISW	Ignore Subsequent Working
SOI	Seen or implied
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become “follow through” marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td rowspan="4">1</td><td rowspan="2">

<table>
<tr><td>t</td><td>0</td><td>3</td><td>4</td><td>5</td><td>6</td><td>8</td><td>10</td><td>14</td></tr>
<tr><td>cf</td><td>0</td><td>120</td><td>300</td><td>500</td><td>660</td><td>770</td><td>850</td><td>900</td></tr>
</table></td><td>M1 A1</td><td>Attempt to plot cumulative frequencies at ucb and all points joined between $(3,y_1)$ and $(14,y_2)$. Cf table not required.

Linear scales starting at (0,0) and axes labelled cf and time in mins, all points correct; (allow straight lines or curves)</td></tr>
<tr><td>M1</td><td>450 seen in median attempt on increasing CF graph (independent);</td></tr>
<tr><td>Median value: 4.8 (minutes)</td><td>A1 FT</td><td>Correct ($4.7 \leqslant m < 4.9$) or FT from reading their increasing graph at cf = 450</td></tr>
<tr><td></td><td>4</td><td></td></tr>
</table>

Question	Answer	Marks	Guidance
2(i)	1 L: ${}^6C_2 = 15$	**B1**	
		1	
2(ii)	No L: ${}^6C_3 = 20$ (1 L: ${}^6C_2 = 15$)	**M1**	Either 0L or 2L correct unsimplified
	2 L: ${}^6C_1 = 6$	**M1**	Summing the 3 correct scenarios
	Total = 41	**A1**	
		3	

Question	Answer	Marks	Guidance
3(i)	(10/160 =) 1/16, 0.0625	**B1**	OE
		1	
3(ii)	(90/160) = 9/16, 0.5625	**B1**	OE
		1	
3(iii)	P(red/hatchback) = P(red hatchback) / P(hatchback) = 40/160 / 90/160	**M1**	Appropriate probabilities in a fraction
	= 4/9	**A1**	OE *Altn method: Direct from table* *M1 for 40/*a *or* b*/90,* a $\neq$ *160* *A1 for 40/90 oe*
		2	

Question	Answer	Marks	Guidance
3(iv)	*EITHER:* P(red) × P(hatchback) = $\frac{72}{160}\times\frac{90}{160} \neq \frac{40}{160}$	**(M1**	Use correct approach with appropriate probabilities substituted
	Not independent	**A1)**	Numerical comparison and conclusion stated
	OR: P(red/hatchback) = 40/90 and $\frac{40}{90}\neq\frac{72}{160}$	**(M1**	Use correct approach with appropriate probabilities substituted
	Not independent	**A1)**	Numerical comparison and conclusion stated
		2	

Question	Answer	Marks	Guidance
4(i)	$\Sigma p = 1$: $0.2 + 0.1 + p + 0.1 + q = 1$: $p + q = 0.6$	**M1**	Unsimplified sum of probabilities equated to 1
	$\Sigma px = 1.7$: $-0.4 + 0 + p + 0.3 + 4q = 1.7$:	**M1**	Unsimplified Sum of px equated to 1.7
	$p + 4q = 1.8$	**M1**	Solve simult. equations to find expression in p or q
	$p = 0.2$, $q = 0.4$	**A1**	
		4	
4(ii)	$\text{Var}(X) = \Sigma px^2 - 1.7^2 = 4\text{x}0.2 + 1p + 9\text{x}0.1 + 16q - 1.7^2$ $= 8.3 - 2.89$	**M1**	Use correct unsimplified expression for variance
	$= 5.41$	**A1**	
		2	

Question	Answer	Marks	Guidance
5(i)	$24.25n - 20n = 136$ **Or** $\frac{136}{n} + 20 = 24.25$	**M1**	Unsimplified correct equation
	$n = 32$	**A1**	
		2	
5(ii)	Using coded information: Variance = $\frac{2888}{32} - \left(\frac{136}{32}\right)^2$	**M1**	unsimplified expression for variance
	$= 72.1875 = 72.19$	**A1**	accept answers 72.2 SOI
	Using uncoded information: Variance = $\frac{\sum x^2}{32} - 24.25^2$ Equate with 72.1875 to give	**M1**	Equate two expressions for variance and solve
	$\sum x^2 = 21128$	**A1**	
		4	

Question	Answer	Marks	Guidance
6(i)	$3! \times \dfrac{4!}{3!} \times 2$	**M1**	3! oe seen multiplied by integer $\geqslant$ 1, no addition
		M1	4!/3! oe seen multiplied by integer > 1, no addition
	= 48	**A1**	
		3	
6(ii)	*EITHER:* Even = Total number of arrangements – Odd numbers	**B1**	7!/3! –
	$= 7!/3! - 3\times\dfrac{6\times5\times4\times3\times2\times1}{3!} = (7!/3! - 6!/2!)$ $= 840 - 360$	**B1**	6!/2! OE
	= 480	**B1**	
	OR: No of arrangements ending in 8: $\dfrac{6!}{3!}$	**B1**	No. ending in 8 or no. ending in 6 correct unsimplified
	No ending in 6: 6!/2!	**B1**	Both correct and added unsimplified
	Total: $\dfrac{6!}{3!} + 6!/2 = 120 + 360 = 480$	**B1**	
		3	

Question	Answer	Marks	Guidance
7(i)	$P(X > 410) = 225/6000 = 0.0375$ $P\left(Z > \frac{410-400}{\sigma}\right) = 0.0375$: 0.9625	**M1**	Use 1 −225/6000 = 0.9625 to find z value
	z value = ±1.78	**A1**	z value: ± 1.78
	$\frac{10}{\sigma} = 1.78$	**M1**	(410-400)/σ = *their z* (must be a z value)
	$\sigma = 5.62$	**A1**	
		4	
7(ii)	We need $P(Z < -1.5)$ and $P(Z > 1.5)$	**M1**	Attempt at $P(Z < -1.5)$ or $P(Z > 1.5)$ $1 - \Phi(1.5)$ seen
	$\Phi(-1.5) + 1 - \Phi(1.5)$ $= 2 - 2\Phi(1.5)$	**M1**	Or equivalent expression with values
	$= 2 - 2 \times 0.9332 = 0.1336$ (0.134)	**A1**	Correct to 3sf
	Number expected = 500 × 0.1336 = 66.8: 66 or 67 packets	**B1ft**	0.1336 used or FT their 4sf probability times 500, (not 0.9625 or 0.0375) rounded or truncated
		4	

Question	Answer	Marks	Guidance
8(i)	$P(4) + P(5) = {}^5C_4\left(\frac{1}{4}\right)^4\left(\frac{3}{4}\right)^1 + {}^5C_5\left(\frac{1}{4}\right)^5\left(\frac{3}{4}\right)^0$	**M1**	One binomial term, with $p < 1$, n=5, $p + q$=1
	= 0.014648.. + 0.00097656..	**M1**	Add 2 correct unsimplified binomial terms
	= 0.0156 or $\frac{1}{64}$	**A1**	
		3	
8(ii)	$1 - P(0) > 0.995$: $0.75^n < 0.005$	**M1**	Equation or inequality involving 0.75^n and 0.005 or 0.25^n and 0.995
	$n\log 0.75 < \log 0.005$ $n > 18.4$:	**M1**	Attempt to solve *their* exponential equation using logs, or trial and error May be implied by their answer
	$n = 19$	**A1**	
		3	
8(iii)	$p = 0.25$, $n = 160$: mean = 160 x 0.25 (= 40) variance = 160 x 0.25 x 0.75 (=30)	**B1**	Correct unsimplified mean and variance
	$P(X < 50) = P\left(Z < \frac{49.5-40}{\sqrt{30}}\right)$	**M1**	Use standardisation formulae must include square root.
		M1	Use continuity correction ±0.5 (49.5 or 50.5)
	= $P(Z < 1.734) = 0.959$	**A1**	Correct final answer
		4	

Cambridge Assessment International Education
Cambridge International Advanced Subsidiary and Advanced Level

MATHEMATICS **9709/61**

Paper 6 **October/November 2017**

MARK SCHEME

Maximum Mark: 50

Published

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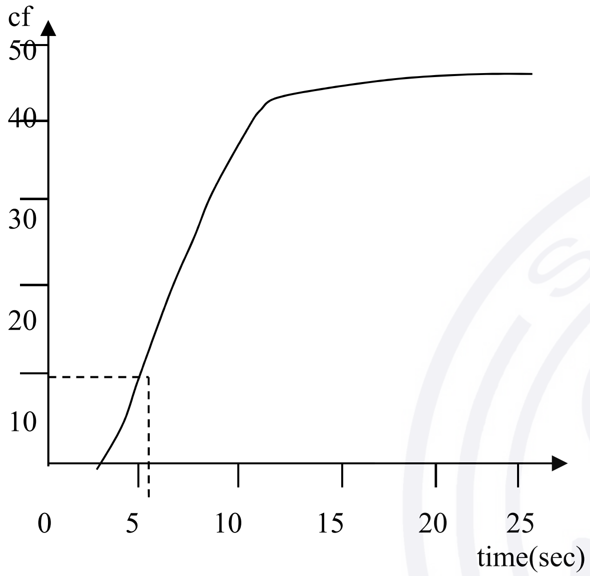
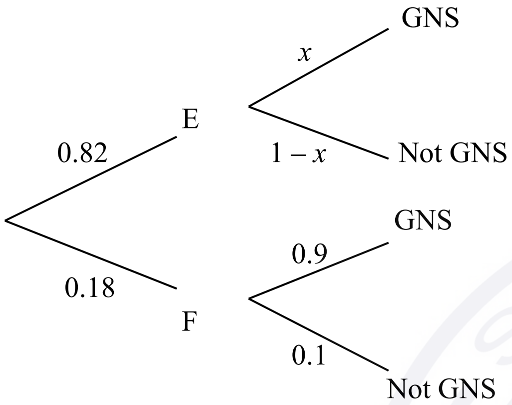
Question	Answer	Marks	Guidance
1	$p + q = 0.45$	**M1**	Equation involving $\Sigma P(x) = 1$
	$0.15 + 2p + 1.2 + 6q = 3.05$	**M1**	Equation using $E(X) = 3.05$
	$q = 0.2$	**M1**	Solving simultaneous equations to one variable
	$p = 0.25$	**A1**	Both answers correct
		4	

Question	Answer	Marks	Guidance
2(i)	Points (5.5,10), (8.5,25), (11.5,42), (16.5,46), (25.5,48)	**B1**	Correct cfs values seen listed, in or by table or on graph, 0 not required
		**B1**	Axes labelled "cumulative frequency" (or cf) and "time [or t etc.] (in) seconds (or sec etc.)". Linear scales – cf 0–48, time 2.5 – 25.5 (ignore <2.5 on time.) At least 3 values stated on each axis, but (0,0) can be implied without stating.
		B1	All points plotted accurately, (5, 10) etc. scores **B0**. Curve or line segments drawn starting at (5.5,10) and passing within '1 scale unit' vertically and horizontally of plotted points
		3	

Question	Answer	Marks	Guidance
2(ii)	48 – 35 = 13 $t =$ 6.5 sec	M1	Subt 35 (checked ±1 mm on graph) from 48 or 50,
		A1	6 ⩽ Ans ⩽ 7
		2	

Question	Answer	Marks	Guidance
3(i)	$p = 0.207$	B1	
		1	
3(ii)	Var = 30 × 0.207 × 0.793 = 4.92	B1	
		1	
3(iii)	P(⩾ 2) = 1 – P(0, 1)	M1	
	$= 1 - (0.793)15 - \binom{15}{1}(0.207)(0.793)14$	M1	1 – P(0, 1) seen n =15 p = any prob
	= 0.848	A1	
		3	

Question	Answer	Marks	Guidance
4(i)	$\dfrac{(48.7\times 12+38.1\times 7)}{19}$	**M1**	Accept unsimplified (may be separate calculations)
	$= 44.8$	**A1**	
		2	
4(ii)	$7.65^2 = \dfrac{\Sigma x^2}{12} - 48.7^2 \quad \Sigma x^2 = 29162.55$	**M1**	Substitution in one correct variance formula
	$4.2^2 = \dfrac{\Sigma y^2}{7} - 38.1^2 \quad \Sigma y^2 = 10284.75$	**A1**	One Σx^2 or Σy^2 correct (can be rounded to 4sf))
	Combined var = $\dfrac{(29162.55+10284..75)}{19} - 44.79^2$ $= \dfrac{39447.3}{19} - 44.79^2$	**M1**	Using their Σx^2 and Σy^2 and their **4(i)** in the variance formula
	Combined $\sigma = 8.37$ or 8.36	**A1**	
		4	

Question	Answer	Marks	Guidance
5(i)		**B1**	Must see at least 4 probs correct including one with an x in, correct shape
		B1	Shape, clear labels/annotation and all probs correct
		2	
5(ii)	$0.82x + 0.18 \times 0.9 = 0.285$	**M1**	Eqn with x in , two 2-factors on one side
	$x = 0.15$	**A1**	
		2	
5(iii)	$P(E \mid notGNS) = \dfrac{\mathrm{P}(\mathrm{E} \cap \mathrm{notGNS})}{\mathrm{P}(\mathrm{notGNS})}$	**M1**	Attempt at P(E∩not GNS) seen as num or denom of fraction
		M1	Attempt at P(not GNS) seen anywhere
	$= \dfrac{0.82 \times 0.85}{1 - 0.285} = 0.975$	**A1**	Correct answer
		3	

Question	Answer	Marks	Guidance
6(a)(i)	$^{40}P_5$	**M1**	$^{40}P_x$ or $^{y}P_5$ oe seen, can be mult by $k \geqslant 1$
	= 78 960 960	**A1**	
		2	
6(a)(ii)	not front row e.g. WEJ** in 3× 3! = 18 ways	**B1**	3! seen mult by $k \geqslant 1$
	7 rows in 7 × 18= 126 ways	**B1**	mult by 7
	front row: e.g. *MA** in 4 × 2 = 8 ways	**M1**	attempt at front row arrangements and multiplying by the 7 other rows arrangements, need not be correct
	Total 126×8 = 1008	**A1**	
		4	
6(b)	*EITHER:* e.g. *R** in $^8C_3$ ways = 56 ways *L** in $^8C_3$ = 56 ways	**(M1**	Considering either R or L only in team
	**** in $^8C_4$ = 70 ways	**M1***	Considering neither in team
		DM1	summing 3 scenarios
	Total 182 ways	**A1)**	
	OR1: No restrictions $^{10}C_4$ = 210 ways	**(M1**	$^{10}C_4$ – , Considering no restrictions with subtraction
	RL = $^8C_2$ = 28	**M1***	Considering both in team
	210 – 28	**DM1**	subt
	= 182 ways	**A1)**	

Question	Answer	Marks	Guidance
6(b)	*OR2:* R out in ${}^9C_4 = 126$ ways L out in ${}^9C_4 = 126$ ways	**(M1**	Considering either R out or L out
	Both out in ${}^8C_4 = 70$	**M1***	Considering both out
		DM1	Summing 2 scenarios and subtracting 1 scenario
	126 + 126 – 70 = 182 ways.	**A1)**	
		4	

Question	Answer	Marks	Guidance
7(i)	$\text{P}(< 570) = \text{P}\left(z < \dfrac{570-500}{91.5}\right) = \text{P}(z < 0.7650)$ $= 0.7779$	**M1**	Standardising for either 570 or 390, no cc, no sq, no √
	$\text{P}(< 390) = \text{P}\left(z < \dfrac{390-500}{91.5}\right) = \text{P}(z < -1.202)$	**A1**	One correct z value
	= 1 – 0.8853 = 0.1147	**A1**	One correct Φ, final solution
	Large:0.222 (0.2221) Small: 0.115 (0.1147)	**A1**	Correct small and large
	Medium: 0.663 (0.6632)	**A1FT**	Correct Medium rounding to 0.66 or ft 1 – (their small + their large)
		5	

Question	Answer	Marks	Guidance
7(ii)	$1.645 = \left(\dfrac{x-500}{91.5}\right)$	**B1**	$\pm$ 1.645 seen (critical value)
		M1	Standardising accept cc, sq, sq rt
	$x = 651$	**A1**	$650 \leqslant \text{Ans} \leqslant 651$
		3	
7(iii)	$P(x > 610) = 0.1147$ (symmetry)	**M1**	Attempt to find upper end prob $x > 610$ or $\Phi(x)$, ft their $P(< 390)$ from **(i)**
	$0.3 + 0.1147 = 0.4147 \Rightarrow \Phi(x) = 0.5853$	**M1**	Adding 0.3 to *their* $P(x > 610)$ or subt 0.5 from $\Phi(x)$ or $0.8853 - 0.3$
	$z = 0.215$ or 0.216	**M1**	Finding $z = \Phi^{-1}(0.5853)$
	$0.215 = \dfrac{k-500}{91.5}$	**M1**	Standardising and solving, accept cc, sq, sq rt
	$k = 520$	**A1**	
		5	

Cambridge Assessment International Education
Cambridge International Advanced Subsidiary and Advanced Level

MATHEMATICS	**9709/62**
Paper 6	**October/November 2017**
MARK SCHEME	
Maximum Mark: 50	

Published

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AEF/OE Any Equivalent Form (of answer is equally acceptable) / Or Equivalent

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CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

SOI Seen or implied

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

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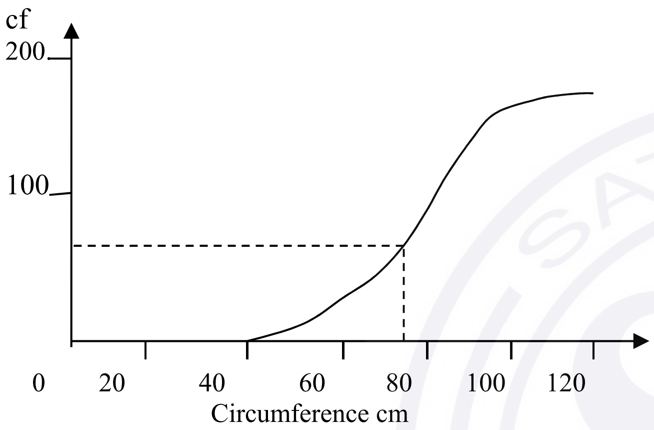
Question	Answer	Marks	Guidance
1	*EITHER:* ($\Sigma x =$) $11.5n = 27 + 10n$	**(M1**	Expanding brackets and forming a three term equation involving 27 and at least one term in n, without x
		M1	$10n$ or $11.5n$ seen in expression without x ($1.5n = 27$ implies **M2**)
	$n = 18$	**A1)**	
	OR: $11.5 = \dfrac{27}{n} + 10$	**(M1**	Dividing coded sum by n and forming a three term equation involving 11.5 and at least one term in n, without x
		M1	$27/n$ seen in expression without x ($1.5 = \dfrac{27}{n}$ implies **M2**)
	$n = 18$	**A1)**	
		3	

Question	Answer	Marks	Guidance
2(i)	points (50, 14), (80, 62), (100, 132), (120, 140)	**B1**	Correct cfs values seen listed, in or by table or on graph, 0 not required
		**B1**	Axes labelled 'cumulative frequency' (or cf) and 'circumference [or cir or c etc.] (in) cm'. Linear scales – c.f. 0–140 circumference 40–120 (ignore <40 on circ.) At least 3 values stated on each axis, but (0,0) can be implied without stating.
		B1	All points plotted accurately
		3	
2(ii)	140 – 54 = 86	**M1**	Finding correct value from graph (checked ±1 mm) or linear interpolation. Subtraction from 140 can be implied
	Percentage = 61.4%	**A1**	60.5% ⩽ Ans ⩽ 64.5%
		2	

Question	Answer	Marks	Guidance
3(i)	*EITHER:* P(X = 3) = P(RRB) = $\frac{2}{6} \times \frac{1}{5} \times \frac{4}{4}$	(M1	probabilities in order $\frac{2}{p} \times \frac{1}{q} \times \frac{4}{r}$, $p, q, r \leqslant 6$ and $p \geqslant q \geqslant r$, $r \geqslant 4$, accept $\times$ 1 as $\frac{4}{r}$.
	= $\frac{1}{15}$ AG	A1)	Needs either P(RRB) OE stated or identified on tree diagram.
	OR1: P(X = 3) = P(RRB) = $\frac{{}^2C_2}{{}^6C_2} \times \frac{{}^4C_1}{{}^4C_1}$	(M1	probabilities stated clearly, $\times \frac{{}^4C_1}{{}^4C_1}$ or $\times$ 1 or $\times \frac{4}{4}$ included
	= $\frac{1}{15}$ AG	A1)	Needs either P(RRB) OE stated or identified on tree diagram.
	OR2: P(X = 3) = P(RRB) = $\frac{{}^2C_1}{{}^6C_1} \times \frac{{}^1C_1}{{}^5C_1} \times \frac{{}^4C_1}{{}^4C_1}$	(M1	probabilities in order $\frac{{}^2C_1}{{}^pC_1} \times \frac{{}^1C_1}{{}^qC_1} \times \frac{{}^4C_1}{{}^rC_1}$ $p, q, r \leqslant 6$ and $p \geqslant q \geqslant r$, $r \geqslant 4$ ($\times \frac{{}^4C_1}{{}^4C_1}$ or $\times$ 1 or $\times \frac{4}{4}$ acceptable)
	= 1/15 AG	A1)	Needs either P(RRB) OE stated or identified on tree diagram.
		2	

Question	Answer	Marks	Guidance
3(ii)	$P(1) = P(B) = \frac{4}{6}\ (\frac{2}{3} = 0.667)$	**B1**	Probability distribution table drawn with at least 2 correct x values and at least 1 probability. All probabilities $0 \leqslant p < 1$.
	$P(2) = P(RB) = \frac{2}{6} \times \frac{4}{5} = \frac{4}{15}$ (= 0.267)	**B1**	P(1) or P(2) correct unsimplified, or better, and identified.
	$P(3) = P(RRB) = \frac{2}{6} \times \frac{1}{5} \times \frac{4}{4} = \frac{1}{15}$ (= 0.0667) x: 1, 2, 3 P: $\frac{10}{15}$, $\frac{4}{15}$, $\frac{1}{15}$	**B1**	All probabilities in table, evaluated correctly OE. Additional x values must have a stated probability of 0
		3	

Question	Answer	Marks	Guidance
4(i)	$P(4, 2H) = \frac{1}{4} \times {}^4C_2 \times (\frac{1}{3})^2(\frac{2}{3})^2$	M1	Multiplying their 2H expression by ¼ [P(4)]
		M1	Remaining factor is $(\frac{1}{3})^2(\frac{2}{3})^2$ [or $\frac{4}{81}$] multiplied by integer value $k \geqslant 1$ OE
	$= \frac{2}{27}$ (0.0741)	A1	
		3	
4(ii)	$P(3, 3H) = \frac{1}{4} \times (\frac{1}{3})^3 = \frac{1}{108}$ (0.00926)	B1	
		1	
4(iii)	$P(1, 1H) = \frac{1}{4} \times \frac{1}{3} = \frac{1}{12}$ (0.08333)	M1	Correct expression for 1 of P(1, 1H), P(2, 2H), P(4, 4H) Unsimplified (or better)
	$P(2, 2H) = \frac{1}{4} \times (\frac{1}{3})^2 = \frac{1}{36}$ (0.02778) $P(3, 3H) = \frac{1}{4} \times (\frac{1}{3})^3 = \frac{1}{108}$ (0.009259) $P(4, 4H) = \frac{1}{4} \times (\frac{1}{3})^4 = \frac{1}{324}$ (0.003086)	M1	Summing their values for 3 or 4 appropriate outcomes for the 'game' with no additional outcomes.
	Prob $= \frac{10}{81}$ (0.123)	A1	
		3	

Question	Answer	Marks	Guidance
5(i)	*EITHER:* P(> 2) = 1 – P(0, 1, 2)	(M1	Binomial term of form ${}^{30}C_x p^x(1-p)^{30-x}$, $0 < p < 1$ any p
	$= 1 - (0.96)^{30} - {}^{30}C_1(0.04)(0.96)^{29} - {}^{30}C_2(0.04)^2(0.96)^{28}$ (= 1 – 0.2938… – 0.3673… – 0.2219…)	A1	Correct unsimplified answer
	= 1-0.883103 = 0.117 (0.116896)	A1)	
	OR: P(> 2) = P(3,4,5,6,….30)	(M1	Binomial term of form ${}^{30}C_x p^x(1-p)^{30-x}$, $0 < p < 1$ any p
	$= {}^{30}C_3(0.04)^3(0.96)^{27} + {}^{30}C_4(0.04)^4(0.96)^{26} + \ldots + (0.04)^{30}$	A1	Correct unsimplified answer
	= 0.117	A1)	
		3	

Question	Answer	Marks	Guidance
5(ii)	$np = 280 \times 0.1169 = 32.73$, $npq = 280 \times 0.1169 \times 0.8831 = 28.9$	**M1 FT**	Correct unsimplified *np* and *npq*, FT their *p from* **(i)**,
	$P(\geqslant 30) = P\left(z > \frac{29.5 - 32.73}{\sqrt{28.9}}\right) = P(z > -0.6008)$	**M1**	Substituting *their* μ and σ ($\sqrt{npq}$ only) into the Standardisation Formula
		M1	Using continuity correction of 29.5 or 30.5
		M1	Appropriate area Φ from standardisation formula P(z >….) in final solution
	= 0.726	**A1**	
		5	

Question	Answer	Marks	Guidance
6(a)(i)	*EITHER:* 3**, 4**, 6**, 8**	**(M1**	${}^5P_2$ or ${}^5C_2 \times 2!$ or 5×4 OE (considering final 2 digits)
	options $4 \times 5 \times 4 = 80$	**M1**	Mult by 4 or summing 4 options (considering first digit)
		A1)	Correct final answer
	OR: Total number of values: $6 \times 5 \times 4 = 120$	**(M1**	Calculating total number of values (with subtraction seen)
	Number of values less than 300: $2 \times 5 \times 4 = 40$	**M1**	Calculating number of unwanted values
	Number of evens = $120 - 40 = 80$	**A1)**	Correct final answer
		3	

Question	Answer	Marks	Guidance
6(a)(ii)	3**, 4**, 6**, 8** *EITHER:* options $4 \times 6 \times 4$ (last)	(M1	6 linked to considering middle digit e.g. multiplied or in list
		M1	Multiply an integer by 4×4 (condone $\times$ 16) (No additional figures present for both M's to be awarded)
	= 96	A1)	
	OR: Total number of values $4 \times 6 \times 6 = 144$	(M1	Calculating total number of values (with subtraction seen)
	Number of odd values $4 \times 6 \times 2 = 48$	M1	Calculating number of unwanted values
	Number of evens = 144 – 48 = 96	A1)	
		3	
6(b)(i)	252	B1	
		1	

Question	Answer	Marks	Guidance
6(b)(ii)	B (6)G(4)		
	5 0 in $^6C_5$ ($\times{}^4C_0$) $= 6 \times 1 = 6$ 4 1 in $^6C_4 \times {}^4C_1 = 15 \times 4 = 60$ 3 2 in $^6C_3 \times {}^4C_2 = 20 \times 6 = 120$	M1	Multiplying 2 combinations $^6C_q \times {}^4C_r$, $q + r = 5$, or $^6C_5$ seen alone
		M1	Summing 2 or 3 appropriate outcomes, involving perm/comb, no extra outcomes.
	Total = 186 ways	A1	
		3	

Question	Answer	Marks	Guidance
7(i)	$P(> 65) = P\left(z > \frac{65 - 61.4}{12.3}\right) = P\ (z > 0.2927)$	M1	Standardising no continuity correction, no square or square root, condone ± standardisation formula
		M1	Correct area (< 0.5)
	$= 1 - 0.6153 = 0.385$	A1	
		3	

Question	Answer	Marks	Guidance
7(ii)	P (< 65) = 0.6153 so P(< k) = 0.25 + 0.6153 = 0.8653	**B1**	
	$z = 1.105$	**B1**	$z = \pm$ 1.105 seen or rounding to 1.1
	$1.105 = \dfrac{k-61.4}{12.3}$	**M1**	standardising allow ±, cc, sq rt, sq. Need to see use of tables backwards so must be a z-value, not 1 – z value.
	$k = 75.0$	**A1**	Answers which round to 75.0. Condone 75 if supported.
		4	
7(iii)	$2.326 = \dfrac{97.2-\mu}{\sigma}$	**B1**	± 2.326 seen (Use of critical value)
	$-0.44 = \dfrac{55.2-\mu}{\sigma}$	**B1**	± 0.44 seen
		M1	An equation with a z-value, μ, σ and 97.2 or 55.2, allow $\sqrt{\sigma}$ or σ^2
		M1	Algebraic elimination μ or σ from *their* two simultaneous equations
	$\mu = 61.9$ $\sigma = 15.2$	**A1**	both correct answers
		5	

Cambridge International AS & A Level

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MATHEMATICS **9709/63**

Paper 6 **October/November 2017**

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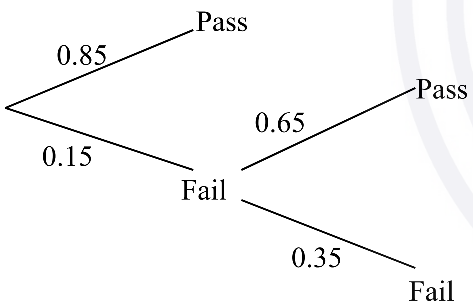
1	*EITHER:* P(at least 1 completes) = 1 – P(0 people complete) $= 1 - (0.8)^3$	**(M1**	Fully correct unsimplified expression $1 - (0.8)^3$ OE
	$= 0.488 \left(\frac{61}{125}\right)$	**A1)**	
	OR1: $P(1, 2, 3) = {}^3C_1(0.2)(0.8)^2 + {}^3C_2(0.2)^2(0.8) + (0.2)^3$	**(M1**	Unsimplified correct 3 term expression
	$= 0.488 \left(\frac{61}{125}\right)$	**A1)**	
	OR2: $0.2 + 0.8 \times 0.2 + 0.8 \times 0.8 \times 0.2$	**(M1**	Unsimplified sum of 3 correct terms
	$= 0.488 \left(\frac{61}{125}\right)$	**A1)**	
		2	

Question	Answer	Marks	Guidance
2	$\Sigma(x-45) = 1218 - 20 \times 45 = 318$	**B1**	
	$\dfrac{\Sigma(x-45)^2}{20} - \left(\dfrac{\Sigma(x-45)}{20}\right)^2 = 4.2^2$	**M1**	Fully correct substitution in the correct coded variance formula with their $\Sigma(x-45)$ **OR** valid method for $\Sigma x^2 = 74\,529$ ($4.2^2 = \dfrac{\Sigma x^2}{20} - \left(\dfrac{1218}{20}\right)^2$) and expanding $\Sigma(x–45)^2$ correctly $= \Sigma x^2 - 90\Sigma x + 20 \times 45^2 = '74\,529' - 90 \times 1218 + 40\,500 = 5409$
	$\Sigma(x-45)^2 = 5409$	**A1**	
		3	

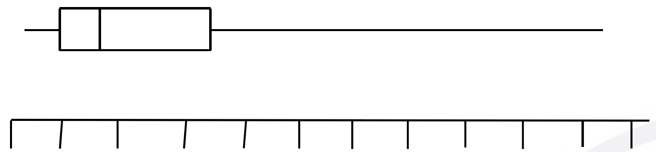
Question	Answer	Marks	Guidance
3(i)		**M1**	Correct shape
		A1	All correct labels and probabilities
		2	

Question	Answer	Marks	Guidance
3(ii)	$\text{P}(F \mid P) = \dfrac{\text{P}(F \cap P)}{\text{P}(P)}$	**M1**	P(P) consistent with their tree diagram seen anywhere
	$= \dfrac{0.15 \times 0.65}{0.85 + 0.15 \times 0.65}$ or $\dfrac{0.15 \times 0.65}{1 - 0.15 \times 0.35}$	**A1**	Correct unsimplified P(P) seen as num or denom of a fraction
	$= \dfrac{0.0975}{0.9475}$	**M1**	P($F \cap P$) found as correct product or consistent with their tree diagram seen as num or denom of a fraction
	$= \dfrac{39}{379} = 0.103$	**A1**	
		4	

Question	Answer	Marks	Guidance
4(i)	<table><tr><td>x</td><td>–3</td><td>0</td><td>5</td><td>32</td></tr><tr><td>Prob</td><td>1/6</td><td>1/2</td><td>1/6</td><td>1/6</td></tr></table>	**B1**	At least 3 different correct values of X (can be unsimplified)
		B1	Four correct probabilities in a Probability Distribution table
		B1	Correct probs with correct values of X
		3	

Question	Answer	Marks	Guidance
4(ii)	$E(X) = –3/6 + 5/6 + 32/6 = 34/6 = 17/3$ (5.67)	**M1**	Subst their attempts at scores in correct formula as long as 'probs' sum to 1
	$\text{Var}(X) = 9/6 + 25/6 + 1024/6 – (34/6)^2$	**M1**	Subst their attempts at scores in correct var formula
	$= 144 \left(\frac{1298}{9}\right)$	**A1**	Both answers correct
		3	

Question	Answer	Marks	Guidance
5(i)	0 | 2 2 5 6 9 1 | 0 0 0 2 2 3 3 4 7 7 8 8 2 | 8 8 3 | 4 5 8 4 | 4 5 | 6 | 5 7 | 8 | 2 8 9 | 10 | 4 key 2 | 8 means 28 medals	B1	Stem, digits 5, 7, 9 can be missing here, can be upside down
		B1	All leaves in correct order increasing from stem, (5, 7 and 9 can be missing), condone commas
		B1	Reasonable shape, requires all values of the stem, only one line for each stem and leaves must be lined up. Can be upside down or sideways. No commas. Condone one 'leaf' error.
		B1	Correct key must state 'medals' or have 'medals' in leaf heading or title
		4	

Question	Answer	Marks	Guidance
5(ii)	Med = 17 LQ = 10 UQ = 35	B1	Median correct
	 	B1	LQ and UQ correct
		B1	Uniform scale from 2 to 104 (need 3 identified points min) and label including medals (can be in title)
		B1 FT	Correct box med and quartiles on diagram, FT their values
		B1	Correct end-whiskers from ends of box but not through box
		5	

Question	Answer	Marks	Guidance
6(i)	$^{18}P_5$	M1	$^{18}P_x$ or $^{y}P_5$ OE seen, $0 < x < 18$ and $5 < y < 18$, can be mult by $k \geqslant 1$
	$= 1\ 028\ 160$	A1	
		2	

Question	Answer	Marks	Guidance
6(ii)	*EITHER:* e.g. ***(CCCCC)********** in 5!×14 ways	(B1	5! OE mult by k ⩾ 1, considering the arrangements of cars next to each other
	= 1680	B1	Mult by 14 OE, (or 14 on its own) considering positions within the line
	P (next to each other) = 1680/1 028 160	M1	Dividing by **(i)** for probability
	P(not next to each other) = 1 – 1680/1 028 160	M1	Subtracting prob from 1 (or their ‘ 5! × 14 ’ from **(i)**)
	$= 0.998 \left(\frac{611}{612}\right)$ OE	A1)	
	OR1: $\frac{5! \times 14!}{18!} = 0.001634$	(B1	5! OE mult by k ⩾ 1 (on its own or in numerator of fraction) considering the arrangements of cars next to each other
		B1	Multiply by 14!, (or 14! on its own) considering all ways of arranging spaces with 5 cars together
		M1	Dividing by 18!, total number of ways of arranging spaces
	1 – 0.001634	M1	Subtracting prob from 1 (or ‘5! × 14!’ from 18!)
	= 0.998(366)	A1)	
	OR2: 4 together – $2\times5!\times14C12 = 21\,840$ 3, 1, 1 – $3\times5!\times14C11 = 131\,040$ 3, 2 – $2\times5!\times14C12 = 21\,840$ 2,2,1 – $3\times5!\times14C11 = 131\,040$ 2,1,1,1 – $4\times5!\times14C10 = 480\,480$ 1,1,1,1,1 – $5!\times14C9\; or\; 14P5 = 240\,240$	(M1	Listing the six correct scenarios (only): 4 together; 3 together and 2 separate; 3 together and 2 together; two sets of 2 together and 1 separate; 2 together and 3 separate; 5 separate.
		M1	Summing total of the six scenarios, at least 2 correct unsimplified

Question	Answer	Marks	Guidance
	Total = 1 026 480	A1	Total of 1 026 480
		M1	Dividing their 1 026 480 by their **6(i)**
	$1\,026\,480 \div 1\,028\,160 = 0.998(366)$	A1)	
		5	

Question	Answer	Marks	Guidance
6(iii)	R(5) W(4) B(3) Scenarios No. of ways 1 1 1 $= 5 \times 4 \times 3 = 60$ 0 1 2 $= 4 \times {}^3C_2 = 12$ 0 2 1 $= {}^4C_2 \times 3 = 18$ 1 0 2 $= 5 \times {}^3C_2 = 15$ 2 0 1 $= {}^5C_2 \times 3 = 30$ 1 2 0 $= 5 \times {}^4C_2 = 30$ 2 1 0 $= {}^5C_2 \times 4 = 40$	B1	$5C1 \times 4C1 \times 3C1$ or better seen i.e. no. of ways with 3 different colours
		M1	Any of ${}^5C_2$ or ${}^4C_2$ or ${}^3C_2$ seen multiplied by $k > 1$ (can be implied)
		A1	2 correct unsimplified 'no. of ways' other than 5C1 × 4C1 × 3C1
		M1	Summing no more than 7 scenario totals containing at least 6 correct scenarios
	Total = 205	A1	
	OR		
	${}^{12}C_3 -$	M1	Seeing '${}^{12}C_3 -$', considering all selections of 3 cars
	$- {}^5C_3$	M1	Subt ${}^5C_3$ OE, removing only red selections
	$- {}^4C_3$	M1	Subt ${}^4C_3$ OE, removing only white selections
	$- {}^3C_3$	M1	Subt ${}^3C_3$ OE, removing only black selections
	$= 205$	A1	Correct answer
		5	

Question	Answer	Marks	Guidance
7(i)	$P(t > 6) = P\left(z > \frac{6-5.3}{2.1}\right) = P(z > 0.333)$	M1	Standardising, no continuity correction, no sq, no sq rt
	$= 1 - 0.6304$	M1	Correct area $1 - \Phi$ (< 0.5), final solution
	$= 0.370$ or 0.369	A1	
		3	
7(ii)	$z = 1.645$	B1	± 1.645
	$1.645 = \frac{x-5.3}{2.1}$	M1	Standardising, no continuity correction, allow sq, sq rt. Must be equated to a z-value
	$x = 8.75$ or 8.755 or 8.7545	A1	
		3	
7(iii)	$n = 10, p = 0.05$	M1	Bin term ${}^{10}C_x\, p^x(1-p)^{10-x}$
	$P(0, 1, 2) = (0.95)^{10} + {}^{10}C_1(0.05)(0.95)^9 + {}^{10}C_2(0.05)^2(0.95)^8$	M1	Correct unsimplified answer
	$= 0.988$ (0.9885 to 4 sf)	A1	
		3	
7(iv)	P(misses bus) $= P(t < 0)$	*M1	Seeing t linked to zero
	$= P\left(z < \frac{0-5.3}{2.1}\right) = P(z < -2.524) = 1 - \Phi(2.524)$ $= 1 - 0.9942$	DM1	Standardising with $t = 0$, no continuity correction, no sq, no sq rt
	$= 0.0058$	A1	
		3	

Cambridge International Examinations
Cambridge International Advanced Subsidiary and Advanced Level

MATHEMATICS **9709/61**

Paper 6 **May/June 2017**

MARK SCHEME

Maximum Mark: 50

Published

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    B2/1/0 means that the candidate can earn anything from 0 to 2.

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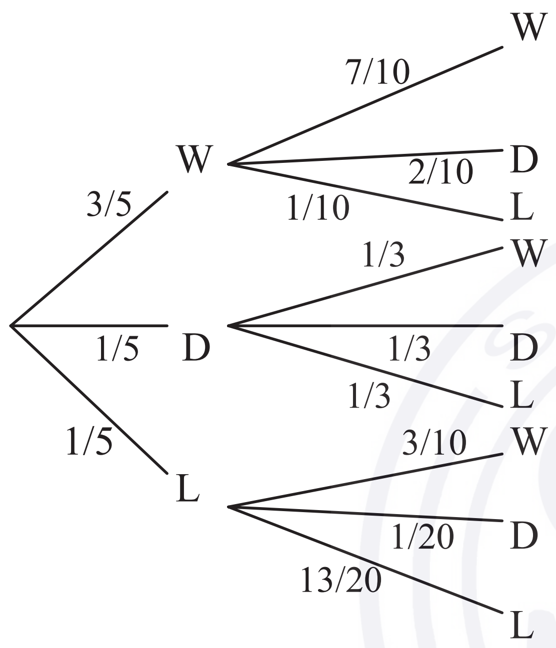
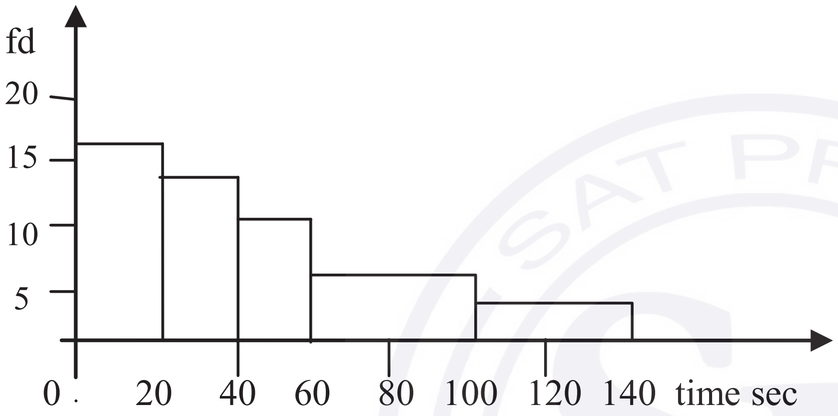
PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Question	Answer	Marks	Guidance
1(i)	*EITHER:* $\frac{\sum x}{30} - k = \frac{315}{30} = 10.5$	**(M1**	Dividing 315 by ±30 and + or – from 50.5 need both and no more
	$k = 5.5 - 10.5 = 40$	**A1)**	Correct answer from correct working
	OR: $\sum x = 50.5 \times 30 = 1515$, $1515 - 30k = 315$	**(M1**	Mult by 50.5 by 30 and + or – 315 and dividing by ±30 need all these
	$k = 40$	**A1)**	Correct answer from correct working. 1200 gets **M0**
	Total:	**2**	
1(ii)	*EITHER:* var = 4022/30–10.5^2(=23.817)	**(M1**	Subst in correct coded variance formula
	sd = 4.88	**A1)**	
	OR: $\sum x^2 - 2(40)\sum x + 30(40)^2 = 4022$, $\sum x^2 = 77222$ Var = 77222/30 – 50.5^2 (= 23.817)	**(M1**	Expanding with ± 40Σx and ± 30(40)^2 seen
	sd = 4.88	**A1)**	
	Total:	**2**	

Question	Answer	Marks	Guidance
2	$P(R) = 4/36 = 1/9$	**M1**	Attempt at $P(R)$ by probability space diag or listing more than half the options, must see a prob, just a list is not enough
	$P(T) = P(O, E) + P(E, O) = 1/4 + 1/4 = 1/2$ OR $P(R \mid T) = 1/9$	**M1**	Attempt at $P(T)$ or $P(R\mid T)$ involving more than half the options
	$P(R \cap T) = P(3, 4) + P(4, 3) = 2/36 = 1/18$ OR $P(R \mid T) = 1/9$	**B1**	Value stated, not from P(R) × P(T) e.g. from probability space diagram
	As $P(R) \times P(T) = P(R \cap T)$ OR as $P(R \mid T) = P(R)$	**M1**	Comparing product values with $P(R \cap T)$, or comparing P $(R\mid T)$ with $P(R)$
	The events are independent.	**A1**	Correct conclusion must have all probs correct
	Total:	**5**	

Question	Answer	Marks	Guidance
3(i)		**M1**	Correct shape i.e. 3 branches then 3 by 3 branches, labelled and clear annotation Condone omission of lines for first match result providing the probabilities are there.
		A1	All correct probs with fully correct shape and probs either fractions or decimals not 1.5/5 etc.
	Total:	**2**	

Question	Answer	Marks	Guidance
3(ii)	$P(L_1 \text{ given } W_2) = \dfrac{P(L_1 \cap W_2)}{P(W_2)}$	M1	Attempt at P(L1∩W2) as a two-factor prod only as num or denom of a fraction
	$= \dfrac{1/5 \times 3/10}{3/5 \times 7/10 + 1/5 \times 1/3 + 1/5 \times 3/10}$	M1	Attempt at P(W2) as sum of appropriate 3 two-factor probs OE seen anywhere
		A1	Unsimplified correct P(W2) num or denom of a fraction
	$= \dfrac{3/50}{41/75} = 9/82\,(0.110)$	A1	
	Total:	4	

Question	Answer	Marks	Guidance
4(i)	fd 16, 14, 11, 505, 2.5	M1	Attempt at fd (must be at least 3 freq/cw) – may be implied by graph
		A1	Correct heights seen on graph i.e. must see a gap for fd = 2.5 etc.
		B1	Correct end points of bars and correct widths
		B1	labels fd, sec. Time can be optional. Linear axes, condone $0 \leqslant t < 20$ etc.
	Total:	**4**	

Question	Answer	Marks	Guidance
4(ii)	(10 × 320 + 30 × 280 + 50 × 220 + 80 × 220 + 120 × 100) / 1140	**M1**	using $\Sigma fx / n$ with mid-point attempt ±0.5, not ends not class widths
	= 45.8	**A1**	
	Total:	**2**	
5(i)	$p = 0.07$	**B1**	
	$\mathrm{P}(2) = {}^{20}\mathrm{C}_2 (0.07)^2 (0.93)^{18}$	**M1**	Bin term ${}^{20}\mathrm{C}_x p^x (1-p)^{20-x}$ their p
	= 0.252	**A1**	
	Total:	**3**	
5(ii)	P(at least 1 cracked egg)=1–$(0.93)^{20}$=1–0.2342	**M1**	Attempt to find P(at least1 cracked egg) with their p from (i) allow 1 – P(0, 1) OE
	= 0.766	**A1**	Rounding to 0.766
	Total:	**2**	
5(iii)	$(0.7658)^n < 0.01$	**M1**	Eqn or inequal containing (their $0.766)^n$ or (their $0.234)^n$, together with 0.01 or 0.99
	$n = 18$	**A1**	
	Total:	**2**	

Question	Answer	Marks	Guidance
6(a)(i)	$z = 0.674$	**B1**	rounding to ±0.674 or 0.675
	$0.674 = \dfrac{6.8-\mu}{0.25\mu}$	**M1**	standardising, no cc, no sq rt, no sq, σ may still be present on RHS
		M1	subst and sensible solving for μ must collect terms, no z-value needed can be 0.75 or 0.7734 need a value for μ
	$\mu = 5.82$	**A1**	
	Total:	**4**	
6(a)(ii)	$\text{P}(X < 4.7) = \text{P}\left(z < \dfrac{4.7-5.819}{1.4548}\right)$	**M1**	± standardising no cc, no sq rt, no sq unless penalised in (a)(i)
	= ϕ(–0.769) = 1 – 0.7791	**M1**	correct side for their mean i.e. 1–ϕ (final solution)
	= 0.221	**A1**	
	Total:	**3**	
6(b)	$\text{P}(<15.75) = \text{P}\left(z < \dfrac{15.75-16}{0.2}\right) = 1-\text{P}(z<1.25)$ =1 – 0.8944 = 0.1056 and P(>16.25) = 0.1056 by sym	***M1**	Standardising for 15.75 or 16.25 no cc no sq no sq rt unless penalised in (a)(i) or (a)(ii)
	P(usable) = 1 – 0.2112 = 0.7888	**B1**	2ϕ– 1 OE for required prob, (final solution)
	Usable rods=1000 × 0.7888 =	**DM1**	Mult their prob by 1000 dep on recognisable attempt to standardise
	788 or 789	**A1**	
	Total:	**4**	

Question	Answer	Marks	Guidance
7(a)	*EITHER:* e.g. xxxxx =5! for the other children	(B1	5! OE seen alone or mult by integer $k \geqslant 1$, no addition
	Put y in 6 ways, then 5 then 4 for the youngest children	B1	Mult by 6P3 OE
	Answer $5! \times 6P3 = 14400$	B1)	Correct answer
	OR: total – 3 tog – 2 tog = $8! - 6!3! - 6! \times 2 \times 5 \times 3 = 14400$	(B1	$8! - 6! \times k \geqslant 1$seen
		B1	$6!3!$ or $6! \times 2 \times 5 \times 3$ seen subtracted
		B1)	Correct answer
	Total:	**3**	
7(b)	D W M 2 2 1 = $6C2 \times 4C2 \times 1$ = 90	B1	One correct unsimplified option
	3 1 1 = $6C3 \times 4 \times 1$ = 80	M1	Summing 2 or more 3-factor options which can contain perms or 3 factors added. The 1 can be implied
	1 3 1 = $6 \times 4C3 \times 1$ = 24	M1	Summing the correct 3 unsimplified outcomes only
	Total=194 ways	A1	
	Total:	**4**	

Question	Answer	Marks	Guidance
7(c)	C D S 2 1 1 = ${}^{26}C_2 \times 9 \times 5 \times 4!$ = 351 000	**M1**	summing 2 or more options of the form (2 1 1), (1 2 1), (1 1 2), can have perms, can be added
	1 2 1 = $26 \times {}^{9}C_2 \times 5 \times 4!$ = 112 320	**M1**	4 relevant products seen excluding 4! e.g. $26 \times 9 \times 8 \times 5$ or $26 \times {}^{9}P_2 \times 5$ for 2nd outcome, condone $26 \times 9 \times 5 \times 37$ as being relevant
	1 1 2 = $26 \times 9 \times {}^{5}C_2 \times 4!$ = 56 160	**M1**	mult all terms by 4! or 4!/2!
	Total = 519 480	**A1**	
	Total:	**4**	

Cambridge International AS & A Level

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MATHEMATICS **9709/62**

Paper 6 **May/June 2017**

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1(i)	$4 \times 5.5 + 3x + 90 = 8 \times 29$	M1	An expression to work out total cost of individual items = 8 × mean, x may be implied.
	$112 + 3x = 232$ $x = 40$	A1	Correct complete unsimplified expression / calculation
	(Cost = $)40	A1	Units not required
	Total:	**3**	
1(ii)	sd = 0 so all cost the same	M1	Must see comment interpreting sd = 0, OE
	shirts cost 4 × $26 = $104 **AG**	A1	See 4 × $26, $130 – $26 OE. Must have a final value of $104 stated
	Total:	**2**	
2(i)	med = 3.2	B1	Accept 3.2 ± 0.05
	UQ = $3.65 \leqslant uq \leqslant 3.7$ LQ = $2.55 \leqslant lq \leqslant 2.6$	M1	UQ – LQ, UQ greater than *their* 'median', LQ less than *their* 'median'
	IQR = $1.05 \leqslant iqr \leqslant 1.15$	A1	Correct answer from both LQ and UQ in given ranges
	Total:	**3**	
2(ii)	134 – 24 = 110	B1	Accept $108 \leqslant n \leqslant 112$, n an integer
	Total:	**1**	

Question	Answer	Marks	Guidance
2(iii)	200 – 12 = 188 less than length l	M1	188 seen, can be implied by answer in range, mark on graph.
	$l = 4.5$ cm	A1	Correct answer accept $4.4 \leqslant l \leqslant 4.5$
	Total:	2	
3(i)	$k(-2)^2$ is the same as $k(2)^2 = 4k$	B1	need to see $-2^2\,k$, 2^2k and $4k$, algebraically correct expressions OE
	Total:	1	
3(ii)	<table><tr><td>x</td><td>–2</td><td>–1</td><td>2</td><td>4</td></tr><tr><td>Prob</td><td>$4k$</td><td>k</td><td>$4k$</td><td>$16k$</td></tr></table>	B1	–2, -1, 2, 4 only seen in a table, together with at least one attempted probability involving k
	$4k + k + 4k + 16k = 1$	M1	Summing 4 probs equating to 1. Must all be positive (table not required)
	$k = 1/25$ (0.04)	A1	CWO
	Total:	3	
3(iii)	$\mathrm{E}(X) = -8k + -k + 8k + 64k = 63k$	M1	using Σpx unsimplified. FT their k substituted before this stage, no inappropriate dividing
	= 63/25 (2.52)	A1	
	Total:	2	

Question	Answer	Marks	Guidance
4	P(score is 6) = P(3, 3)	**M1**	Realising that score 6 is only P(3, 3)
	$= r^2 = 1/36$ $r = 1/6$	**A1**	Correct ans [SR **B2** $r = 1/6$ without workings]
	P(2, 3) + P(3, 2) = 1/9 $qr + rq = 1/9$	**M1**	Eqn involving qr (OE) equated to 1/9 (r may be replaced by *their* 'r value')
	$q/6 + q/6 = 1/9$	**M1**	Correct equation with *their* 'r value' substituted
	$q = 1/3$	**A1**	Correct answer seen, does **not** imply previous M's
	$p = 1 – 1/6 – 1/3 = 1/2$	**B1 FT**	FT their p + their r + their q =1 , $0 < p < 1$
	Total:	**6**	
5(i)	$(z =)\dfrac{4.2 - 3.9}{\sigma}$	**M1**	Standardising, not square root of σ, not σ^2
	$z = 0.916$ or 0.915	**B1**	Accept $0.915 \leqslant \pm z \leqslant 0.916$ seen
	$\sigma = 0.328$	**A1**	Correct final answer (allow 20/61 or 75/229)
	Total:	**3**	

Question	Answer	Marks	Guidance
5(ii)	$z = 4.4 - 3.9$/their 0.328 or $z = 3.4 - 3.9$/their 0.328 = 1.5267 = –1.5267	**M1**	Standardising attempt with 3.4 or 4.4 only, allow square root of σ, or σ^2
	$\Phi = 0.9364$	**A1**	$0.936 \leqslant \Phi \leqslant 0.937$ or $0.063 \leqslant \Phi \leqslant 0.064$ seen
	Prob = $2\Phi - 1 = 2(0.9364) - 1$	**M1**	Correct area $2\Phi - 1$OE i.e. $\Phi = -(1 - \Phi)$, linked to final solution
	= 0.873	**A1**	Correct final answer from $0.9363 \leqslant \Phi \leqslant 0.9365$
	Total:	**4**	
5(iii)	dividing (0.5) by a larger number gives a smaller z-value or more spread out as sd larger or use of diagrams	***B1**	No calculations or calculated values present e.g. ($\sigma =$)0.656 seen Reference to spread or z value required
	Prob is less than that in (ii)	**DB1**	Dependent upon first B1
	Total:	**2**	
6(i)	*EITHER*: Route 1 A*********A in 9! / 2!2!5! = 756 ways	**(*M1**	*Considering AA and BB options* with values
	B*********B in 9! / 4!5! = 126 ways	**A1**	Any one option correct
	756 + 126	**DM1**	*Summing their AA and BB outcomes* only
	Total = 882 ways	**A1)**	

Question	Answer	Marks	Guidance
	OR1: Route 2 *A***********A* in $^9C_5 \times {}^4C_2 = 756$ ways	**(M1**	*Considering AA and BB options* with values
	*B***********B* in $^9C_4 \times {}^5C_5 = 126$ ways	**A1**	Any one option correct
	756 + 126	**DM1**	*Summing their AA and BB outcomes* only
	Total = 882	**A1)**	
	Total:	**4**	

Question	Answer	Marks	Guidance
6(ii)	*EITHER:* (The subtraction method) *A*s together, no restrictions 8! / 2!5! = 168	(*M1	*Considering all As together* – 8! seen alone or as numerator – condone × 4! for thinking A's not identical
	*A*s together and *B*s together 7! / 5! = 42	M1	*Considering all As together and all Bs together* – 7! seen alone or numerator
		M1	*Removing repeated Bs or Cs* – Dividing by 5! either expression or 2! 1st expression only – OE
	Total 168 – 42	DM1	Subt their 42 from their 168 (dependent upon **first M** being awarded)
	= 126	A1)	
	OR1: *A*s together, no restrictions ${}^8C_5$ x ${}^3C_1$ = 168	(*M1	${}^8C_5$ seen alone or multiplied
		M1	${}^7C_5$ seen alone or multiplied
	*A*s together and *B*s together ${}^7C_5$ x ${}^2C_1$ = 42	M1	First expression x ${}^3C_1$ or second expression x ${}^2C_1$
	Total 168 – 42	DM1	Subt their 42 from their 168 (dependent upon **first M** being awarded)
	= 126	A1)	
	OR2: (The intersperse method)	(M1	*Considering all "As together" with Cs* – Mult by 6!
	(AAAA)CCCCC then intersperse *B* and another *B*	M1	*Removing repeated Cs* – Dividing by 5!– [Mult by 6 implies M2]
		*M1	*Considering positions for Bs* – Mult by 7P2 oe –

Question	Answer	Marks	Guidance
	$\frac{6!}{5!}\times 7\times 6\div 2$	**DM1**	Dividing by 2! Oe – removing repeated *B*s (dependent upon **3rd M** being awarded)
	= 126	**A1)**	
	Total:	**5**	
7(i)	P(H) = P(BH) + P(SH) = 0.6×0.05 + 0.4×0.75	**M1**	Summing two 2-factor probs using 0.6 with 0.05 or 0.95, and 0.4 with 0.75 or 0.25
	= 0.330 or $\frac{33}{100}$	**A1**	Correct final answer accept 0.33
	Total:	**2**	
7(ii)	$\mathrm{P}(S\|H) = \frac{P(S\cap H)}{P(H)} = \frac{0.4\times 0.75}{0.33} = \frac{0.3}{0.33}$	**M1 FT**	Their $\frac{P(S\cap H)}{P(H)}$ unsimplified, FT from (i)
	$= \frac{10}{11}$ or 0.909	**A1**	
	Total:	**2**	
7(iii)	Var (B) = 45×0.6×0.4 Var (S)= 45×0.4×0.6	**B1**	One variance stated unsimplified
	Variances same	**B1**	Second variance stated unsimplified **and** at least one variance clearly identified, **and** both evaluated *or* showing equal *or* conclusion made SR **B1** – Standard Deviation calculated Fulfil all the criteria for the variance method but calculated to Standard Deviation
	Total:	**2**	

Question	Answer	Marks	Guidance
7(iv)	1 – P(0, 1) $= 1 - [(0.6)^{10} + {}^{10}C_1(0.4)(0.6)^9] = 1 - 0.0464$ OR P(2,3,4,5,6,7,8,9,10) $= {}^{10}C_2(0.4)^2(0.6)^8 + \ldots + {}^{10}C_9(0.4)^9(0.6) + (0.4)^{10}$	**M1** **M1**	Bin term ${}^{10}C_x p^x(1-p)^{10-x}$ $0 < p < 1$ Correct unsimplified answer
	= 0.954	**A1**	
	Total:	**3**	

Cambridge International Examinations
Cambridge International Advanced Subsidiary and Advanced Level

MATHEMATICS **9709/63**

Paper 6 **May/June 2017**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the May/June 2017 series for most Cambridge IGCSE®, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

  - Note: B2 or A2 means that the candidate can earn 2 or 0.
    B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
CWO	Correct Working Only – often written by a 'fortuitous' answer
ISW	Ignore Subsequent Working
SOI	Seen or implied
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

<u>Penalties</u>

MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

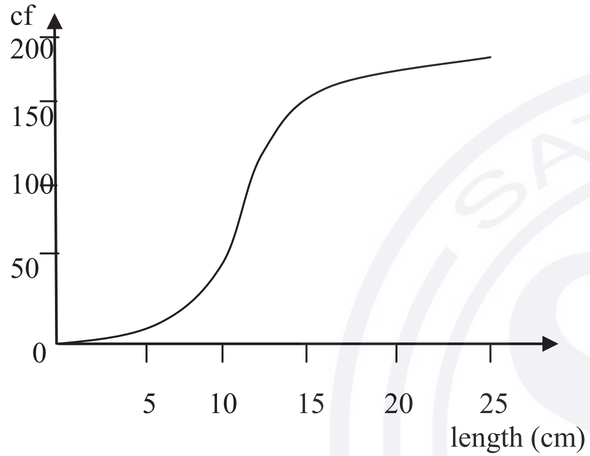
Question	Answer	Marks	Guidance
1	P(6) = 0.3	**B1**	SOI
	P(sum is 9) = P(3, 6) + P(4, 5) + P(5, 4) + P(6, 3)	**M1**	Identifying the four ways of summing to 9 (3,6), (6,3) (4,5) and (5,4)
	= (0.03 + 0.02) × 2	**M1**	Mult 2 probs together to find one correct prob of (3,6), (6,3) (4,5) or (5,4) unsimplified
	= 0.1	**A1**	OE
	Total:	**4**	
2	$np = 270 \times 1/3 = 90$, $npq = 270 \times 1/3 \times 2/3 = 60$	**B1**	Correct unsimplified *np* and *npq*, SOI
	$\text{P}(x > 100) = \text{P}\left(z > \frac{99.5 - 90}{\sqrt{60}}\right) = \text{P}(z > 1.2264)$	**M1** **M1**	±Standardising using 100 need sq rt Continuity correction, 99.5 or 100.5 used
	= 1 – 0.8899	**M1**	Correct area 1 – Φ implied by final prob. < 0.5
	= 0.110	**A1**	
	Total:	**5**	
3(i)	P(*S*) = 0.65 × 0.6 + 0.35 × 0.75	**M1**	Summing two 2-factor probs or 1 – (sum of two 2-factor probs)
	= 0.653 (261/400)	**A1**	
	Total:	**2**	

Question	Answer	Marks	Guidance
3(ii)	$\mathrm{P}\left(Std \mid L\right) = \dfrac{P(Std \cap L)}{P(L)} = \dfrac{0.35 \times 0.25}{1 - 0.6525} = 0.0875/0.3475$	M1	'P(Std)' × 'P(L/Std)'as num of a fraction. Could be from tree diagram in 3(i).
		M1	Denominator (1 - their (i)) or their (i) **or** 0.65 × 0.4(or 0.6) + 0.35 × 0.25(or 0.75) = 0.26+0.0875 **or** P(L) from their tree diagram
	= 0.252 (35/139)	A1	
	Total:	3	
4(a)	$\mathrm{P}(x > 0) = \mathrm{P}\left(z > \pm\dfrac{0 - \mu}{\sigma}\right)$ $= \mathrm{P}\left(z > \dfrac{-\mu}{\mu/1.5}\right)$ or $\mathrm{P}\left(z > \dfrac{-1.5\sigma}{\sigma}\right)$	M1	±Standardising, in terms of μ and/or σ with 0 - …. in numerator, no continuity correction, no √
	= P(z > -1.5)	A1	Obtaining z value of ±1.5 by eliminating μ and σ, SOI
	= 0.933	A1	
	Total:	3	
4(b)	$z = -1.151$	B1	± z value rounding to 1.1 or 1.2
	$-1.151 = \dfrac{70 - 120}{s}$	M1	± Standardising (using 70) equated to a z-value, no cc, no squaring, no √
	σ = 43.4 or 43.5	A1	
	Totals:	3	

Question	Answer	Marks	Guidance
5(i)	constant probability (of completing)	**B1**	Any one condition of these two
	independent trials/events	**B1**	The other condition
	Totals:	**2**	
5(ii)	P(5, 6, 7) = ${}^7C_5(0.7)^5(0.3)^2 + {}^7C_6(0.7)^6(0.3)^1 + (0.7)^7$	**M1** **A1**	Bin term ${}^7C_x(0.7)^x(0.3)^{7-x}$, $x \neq 0, 7$ Correct unsimplified answer (sum) OE
	= 0.647	**A1**	
	Total:	**3**	
5(iii)	P(0, 1, 2, 3, 4) = 1 – their '0.6471' = 0.3529	**M1**	Find P($\leqslant 4$) either by subtracting their (ii) from 1 or from adding Probs of 0,1,2,3,4 with n=7 (or 10) and p = 0.7
	P(3) = ${}^{10}C_3(0.3529)^3(0.6471)^7$	**M1**	${}^{10}C_3$ (their $0.353)^3(1$ – their $0.353)^7$ on its own
	= 0.251	**A1**	
6(a)(i)	First digit in 2 ways. 2 × 4 × 3 × 2 or 2 × 4P3	**M1**	1, 2 or 3 × 4P3 OE as final answer
	Total = 48 ways	**A1**	
	Total:	**2**	
6(a)(ii)	2 × 5 × 5 × 3	**M1** **M1**	Seeing 5^2 mult; this mark is for correctly considering the middle two digits with replacement Mult by 6; this mark is for correctly considering the first and last digits
	= 150 ways	**A1**	
	Totals:	**3**	

Question	Answer	Marks	Guidance
6(b)(i)	OO**** in $^{18}C_4$ ways	**M1**	$^{18}C_x$ or the sum of five 2-factor products with n = 14 and 4, may be × by 2C2: 4C0 × 14C4 + 4C1 × 14C3 + 4C2 × 14C2 + 4C3 × 14C1 + 4C4 (× 14C0)
	= 3060	**A1**	
	Totals:	**2**	

Question	Answer	Marks	Guidance
6(b)(ii)	Choc / Not Choc 0 / $6 = 1 \times {}^{16}C_6 = 8008$ 0.2066 1 / $5 = {}^{4}C_1 \times {}^{16}C_5 = 17472$ 0.4508 2 / $4 = {}^{4}C_2 \times {}^{16}C_4 = 10920$ 0.2817 OR Choc / Oats / Ginger 0 / 0 / 6 0 / 1 / 5 0 / 2 / 4 1 / 0 / 5 1 / 1 / 4 1 / 2 / 3 2 / 0 / 4 2 / 1 / 3 2 / 2 / 2	**B1**	The correct number of ways with one of 0, 1 or 2 chocs , unsimplified **or** any three correct number of ways of combining choc/oat/ginger, unsimplified
	Total = 36400 ways	**M1**	sum the number of ways with 0, 1 and 2 chocs and two must be totally correct, unsimplified OR sum the nine combinations of choc, ginger, oats, six must be totally correct, unsimplified
	Probability = $36400/{}^{20}C_6$	**M1**	dividing by ${}^{20}C_6$ (38760) oe
	= 0.939 (910/969)	**A1**	
	Totals:	**4**	
7(i)	freq = fd × cw 10, 40, 120, 30	**M1** **A1**	Attempt to multiply at least 3 fds by their 'class widths'
	Totals:	**2**	

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td rowspan="2">7(ii)</td><td><table><tr><td>length</td><td>< 5</td><td>< 10</td><td>< 20</td><td>< 25</td></tr><tr><td>cf</td><td>10</td><td>50</td><td>170</td><td>200</td></tr></table></td><td>B1

B1

M1

A1</td><td>3 or more correct cfs heights on graph 10, 50, 170, 200

Labels correct cf and length(cm), linear scales from zero (allow 0.5 on horizontal axis)

Attempt (at least three) at plotting at upper end points (either 5 or 5.5, 10 or 10.5 etc.)

Starting at (0, 0) polygon or smooth curve increasing with plotted points at lengths 5, 10, 20 and 25</td></tr>
<tr><td>Totals:</td><td>4</td><td></td></tr>
<tr><td rowspan="4">7(iii)</td><td>median = 14.2</td><td>B1</td><td>Median (accept 13.2 – 15.2)</td></tr>
<tr><td>‘18.5’ – ‘10’</td><td>M1</td><td>Subt their LQ from their UQ if reasonable from their graph</td></tr>
<tr><td>IQ range = 8.5</td><td>A1FT</td><td>Correct FT using LQ = 10 and UQ between 17.5 and 19.5</td></tr>
<tr><td>Totals:</td><td>3</td><td></td></tr>
<tr><td rowspan="3">7(iv)</td><td>mean = (2.5×10 + 7.5×40 + 15×120 + 22.5×30) / 200</td><td>M1</td><td>Using mid points (± 0.5) and their frequencies from 7(i) in correct formula</td></tr>
<tr><td>= 14</td><td>A1</td><td></td></tr>
<tr><td>Totals:</td><td>2</td><td></td></tr>
</table>

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MATHEMATICS **9709/62**

Paper 6 Probability and Statistics **March 2017**

MARK SCHEME

Maximum Mark: 50

Published

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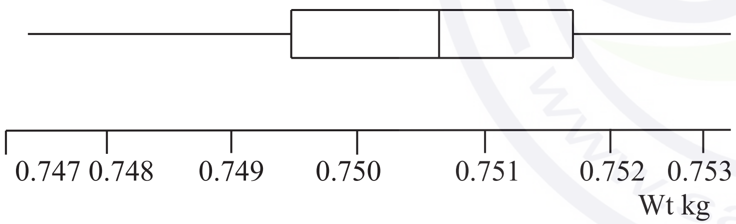
Question	Answer	Marks	Guidance
1	1.6 –1.5 2.3 1.4 –0.6 –0.9 2.5 1.9 2.4 1.9 2.8 1.0	**M1**	Subtracting 1760, allow max 2 slips
	Mean = 1.23	**A1**	
	sd = 1.39	**A1**	
	Mean of x = 1761.23 , sd of x = 1.39	**A1**$\sqrt{}$	ft their coded mean and sd.
			SR B1 correct mean and sd without use of coded process
	Total:	**4**	

Question	Answer	Marks	Guidance
2	$\dfrac{{}^{12}C_3 \times {}^{28}C_4}{{}^{40}C_7}$	**M1**	Using combinations with attempt to evaluate 2 terms in num. and 1 in denom.
		M1	Correct numerator or denominator unsimplified
	= 0.242	**A1**	
	OR		
	P(GGG) $= \dfrac{12}{40}\times\dfrac{11}{39}\times\dfrac{10}{38}\times\dfrac{28}{37}\times\dfrac{27}{36}\times\dfrac{26}{35}\times\dfrac{25}{34}\times {}^{7}C_3$	**M1**	Multiplying 3 green probs with 4 non-green probs, without replacement
		M1	Multiplying by ${}^{7}C_3$
	= 0.242	**A1**	
	Total:	**3**	

Question	Answer	Marks	Guidance
3	$np = 160\times0.1$ (16) $npq = 160\times0.1\times0.9$ (14.4)	**B1**	Correct unsimplified np and npq
	$P(>17) = P\left(z > \dfrac{17.5-16}{\sqrt{14.4}}\right) = P(z > 0.3953)$	**M1**	Standardising need √
		M1	16.5 or 17.5 seen in standardised eqn for continuity correction
	$= 1 - 0.6536$	**M1**	Correct area from their mean $(1 - \Phi)$, final solution
	$= 0.346$	**A1**	
	Total:	**5**	

Question	Answer	Marks	Guidance
4(i)	LQ = 0.7495 Med = 0.7507 UQ = 0.7517	**M1**	Attempt to find all 3 quartiles can be implied, Condone LQ=0.7496, Med=0.7506, UQ=0.7515
		**B1**	Correct median line in box using their scale
		A1	Correct quartiles in box
		B1	Correct end whiskers(not dots or boxes), lines not through box,
		B1	Correct uniform scale from at least 0.7473 to 0.7532, and label (wt) kg oe can be seen in title or scale
	Total:	**5**	

Question	Answer	Marks	Guidance
4(ii)	Normal	**B1**	
	Symmetrical/peaks in middle or tails off quickly	**B1**	Need symm + another reason
	Total:	**2**	

Question	Answer	Marks	Guidance
5(i)	${}^{12}C_1 + {}^{12}C_3 + {}^{12}C_5 + {}^{12}C_7 + {}^{12}C_9 + {}^{12}C_{11}$	**M1**	Summing at least 4 ${}^{12}C_x$ combinations with x = odd numbers
		A1	Correct unsimplified answer (can be implied by final answer)
	= 2048	**A1**	Correct answer
	Total:	**3**	
5(ii)	$7! \times {}^8P_4$	**B1**	7! seen alone or multiplied only (cupcakes ordered)
		M1	multiplying by ${}^8P_4$ o.e (placing brownies)
	= 8467200	**A1**	correct answer
	Total:	**3**	
5(iii)	$9! / (6! \times 2!)$	**B1**	9! oe seen alone or as numerator
		M1	dividing by at least one of 6!,2! (removing repeated shortbread or gingerbread biscuits) ignore 4! if present
	= 252	**A1**	correct answer
	Total:	**3**	

Question	Answer	Marks	Guidance
6(i)	$P(2) = P(0,2) = 2/10 \times 4/6$	**M1**	Mult 2 probs seen (or complete listing of all options)
	= 2/15 **AG**	**A1**	Correct answer legit obtained
	Total:	**2**	
6(ii)	x: 0, 1, 2, 3, 5 $P(X=x)$: 2/30, 5/30, 4/30, 13/30, 6/30	**B1**	Correct values for x in table. Any additional values must have $P(x)$=0 stated
		B1	One correct prob other than P(2) or P(3)
		B1	Correct P(3)
		B1	All correct
	Total:	**4**	
6(iii)	$P(A1 \mid \text{Sum } 3) = \dfrac{P(A1 \cap \text{Sum3})}{P(\text{Sum3})} = \dfrac{5/10 \times 4/6}{13/30}$	**M1**	Attempt at $P(A1 \cap \text{Sum } 3)$ as num or denom of a fraction, can be by counting
		M1	Their P(3) from **(ii)** as num or denom of a fraction
	= 10/13(0.769)	**A1**	
	Total:	**3**	

Question	Answer	Marks	Guidance
7(a)(i)	$0.674 = \dfrac{8.8-\mu}{\sigma} \Rightarrow 0.674\sigma = 8.8 - \mu$	B1	±0.674 seen
	$-0.935 = \dfrac{7.7-\mu}{\sigma} \Rightarrow -0.935\sigma = 7.7 - \mu$	B1	±0.935 seen (condone ±0.934)
		M1	An eqn with a z-value, μ and σ allow sq rt, sq cc
		M1	sensible attempt to eliminate μ or σ by substitution or subtraction
	$\sigma = 0.684$ $\mu = 8.34$	A1	correct answers (from –0.935)
	Total:	**5**	
7(a)(ii)	$P(< 8.2) = P\left(z < \dfrac{8.2-7.9}{0.44}\right)$	M1	Standardising no cc no sq rt no sq
		M1	Correct area ie Φ, final solution
	$= P(z < 0.6818) = 0.7524$	A1	Correct prob rounding to 0.752
	$P(3) = {}^5C_3\,(0.7524)^3(0.2476)^2$	M1	Binomial ${}^5C_x$ powers summing to 5, any p, $\Sigma p = 1$
	$= 0.261$	A1	
	Total:	**5**	

Question	Answer	Marks	Guidance
7(b)	$P(< 1.5\mu) = P\left(z < \dfrac{1.5\mu - \mu}{\mu}\right) = P\ (z < 0.5)$	*M1	standardising with μ and σ (σ may be replaced by μ)
		DM1	just one variable
	= 0.692	A1	
	Total:	**3**	

Cambridge International Examinations
Cambridge International Advanced Subsidiary and Advanced Level

MATHEMATICS **9709/61**

Paper 6 **October/November 2016**

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1	$z = 0.674$ $0.674 = \dfrac{k-20}{7}$ $k = 24.7$	**M1** **M1** **A1**	[3]	±0.674 seen Standardising no cc, no sq, no sq rt
2	diff: 0, 1, 2, 3, 4, 5 prob: 6/36, 10/36, 8/36, 6/36, 4/36, 2/36 Expectation = (0+10+16+18+16+10)/36 = 70/36 = 1.94	**B1** **M1** **A1** **M1** **A1**	[5]	0, 1, 2, 3, 4, 5 seen in table heading or considering all different differences Attempt at finding prob of any difference 1 correct prob Probs summing to 1
3 (i)	$0.9 \times 0.95 \times 0.85 \times 0.1 = 0.0727$	**B1**	[1]	
(ii)	P(0, 1, 2) $= (0.9)^{12} + {}^{12}C_1\,(0.1)(0.9)^{11} + {}^{12}C_2\,(0.1)^2(0.9)^{10}$ = 0.889	**M1** **M1** **A1**	[3]	Bin term ${}^{12}C_x\,(p)^x(1-p)^{12-x}$ $p < 1$, $x \neq 0$ Bin expression $p = 0.1$ or 0.9, $n = 12$, 2 or 3 terms
(iii)	$X \sim B(50, 0.85)$ Expectation = 50 × 0.85 (= 42.5) Var = 50 × 0.85 × 0.15 (= 6.375)	**M1** **A1**	[2]	50 × 0.85 seen oe can be implied Correct unsimplified mean and var
4 (i)	$P(<1) = P\left(z < \dfrac{1-1.04}{0.017}\right) = P(z < -2.353)$ $= 1 - 0.9907$ $= 0.0093$	**M1** **M1** **A1**	[3]	Standardising no cc, no √ or sq 1 – Φ (final process)
(ii)	expected number 1000 ÷ 1.04 = 961 or 962	**B1**	[1]	Or anything in between
(iii)	$z = -1.765$ $-1.765 = \dfrac{1-\mu}{0.017}$ $= 1.03$	**B1** **M1** **A1**	[3]	± 1.76 to 1.77 Standardising must have a z-value, allow √ or sq
(iv)	expected number = 1000 ÷ 1.03 = 971 or 970	**B1**√	[1]	Or anything in between, ft their (iii)

5 (a)	e.g. P*N*P*P*L $= \frac{5!}{3!} \times \frac{^6P_4}{2!}$ = 3600	**M1** **M1** **M1** **A1**	 [4]	Mult by 5! in num Dividing by 3! or 2! Mult by $^6P_4$ oe
(b) (i)	$^7C_5 \times {}^5C_4 \times {}^2C_1 \times {}^2C_1$ = 420	**M1** **A1**	 [2]	Mult 4 combs of which three are correct
(ii)	both in team $^6C_4 \times {}^4C_3 \times 2 \times 2 = 240$ 420 – 240 = 180 ways **OR** Bat in bowl out + bowl in bat out + both out $= {}^6C_4 \times {}^4C_3 \times 2 \times 2 + {}^6C_5 \times {}^4C_3 \times 2 \times 2 + {}^6C_5 \times {}^4C_4 \times 2 \times 2$ = 60 + 96 + 24 = 180 ways **OR** Bat in bowl out + bat out $= 60 + {}^6C_5 \times {}^5C_4 \times 2 \times 2 = 60 + 120 = 180$ ways	**M1** **M1** **A1** **M1** **A1** **A1** **M1** **A1 A1**	 [3]	Evaluating both in team and subtracting from (i) 240 seen can be unsimplified ft their 420, their 240 summing 2 or 3 options not both in team 2 or 3 options correct unsimplified Correct ans from correct working As above, or bowl in bat out + bowl out
6 (i)	P(B, B) = 1/4 ×2/5 = 1/10	**M1** **A1**	 [2]	Multiplying two different probs
(ii)	P(X = 1) = P(R,R) + P(B,B) = 3/4 × 4/5 + 1/10 = 14/20 (7/10)	**M1** **M1** **A1**	 [3]	Finding P(R, R) (=3/5) Summing two options
(iii)	P(B B) $= \frac{P(B \cap B)}{P(B)} = \frac{1/10}{3/4 \times 1/5 + 1/4 \times 2/5}$ = 2/5	**M1** **M1** **A1** **A1**	 [4]	their (i) seen as num or denom of a fraction ¾ × p_1 + ¼ × p_2 seen anywhere 1/4 (unsimplified) seen as num or denom of a fraction, www

7	(i)	Factory A | | Factory B | 3 | 1 5 8 9 | 4 | 2 4 7 8 9 9 8 8 7 4 3 0 | 5 | 1 4 6 8 5 3 1 1 1 | 6 | 4 Key: 9 | 4 | 2 represents 0.049g for factory A and 0.042 g for factory B	**M1** **B1** **B1** **B1** **B1**	[5]	Attempt at ordering factory B Correct stem Correct leaves factory A Correct leaves factory B Correct key need factory A and factory B and units
	(ii)	median factory B = 0.048 g IQR = UQ – LQ = 0.055 – 0.04 = 0.015	**B1** **M1** **A1**	[3]	using their key i.e. 48, 0.48 etc or correct Subt their LQ from their UQ for factory B
	(iii)	generally heavier in factory A Masses more spread out in factory B	**B1** **B1**	[2]	oe must refer to context, e.g. mass

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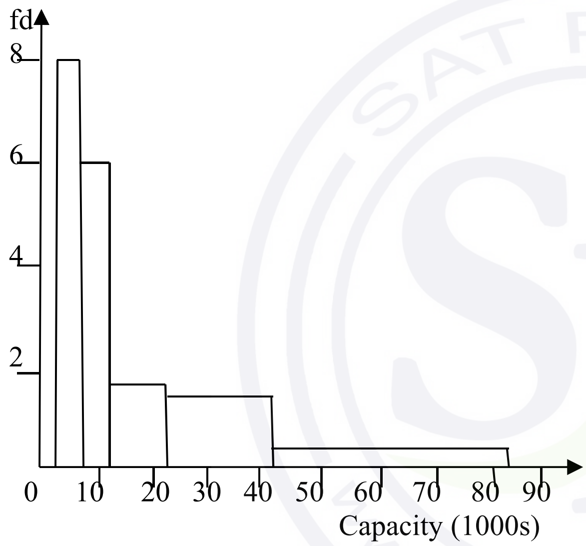
PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

1	P(C given L) = $\frac{P(C \cap L)}{P(L)}$	**M1**		$P(C\cap L)$ seen as num or denom of a fraction
	$= \frac{0.65 \times 0.1}{0.65 \times 0.1 + 0.3 \times 0.15 + 0.05 \times 0.6}$	**A1**		Correct unsimplified $P(C\cap L)$ as numerator
		M1		Summing three 2-factor products seen anywhere
	$= \frac{0.065}{0.14}$	**A1**		0.14 (unsimplified) seen as num or denom of a fraction
	$= 0.464, \frac{13}{28}$	**A1**	[5]	oe
2 (i)	P(1 T-shirt) = $\frac{{}^3C_1 \times {}^9C_2}{{}^{12}C_3}$	**B1**		Correct num unsimplified
		B1		Correct denom unsimplified
	= 27/55 AG	**B1**	[3]	Answer given, so process needs to be convincing
	OR $3/12\times9/11\times8/10\times{}^3C_1$oe	**M1**		Mult 3 probs diff denoms (not a/3 x b/4 x c/5)
		M1		Mult by ${}^3C_1$ oe
	= 27/55 AG	**A1**		Answer given, so process needs to be convincing
(ii)	*X*: 0, 1, 2, 3; Prob: 84/220, 27/55, 27/220, 1/220	**B1**		0, 1, 2, 3 only seen in top line (condone additional values if Prob stated as 0)
		B1		One correct prob, correctly placed in table
		B1		One other correct prob, correctly placed in table
		B1√	[4]	One other correct prob ft $\Sigma p = 1$, 4 values in table
3 (i)	Bin (7, 0.8)	**M1**		${}^7C_n\, p^n(1-p)^{7-n}$ seen
	$P(6, 7) = {}^7C_6\,(0.8)^6(0.2)^1 + (0.8)^7$	**M1**		Correct unsimplified expression for P(6,7)
	= 0.577	**A1**	[3]	
(ii)	mean = 100×0.2 = 20 Var = 100×0.2×0.8 = 16	**B1**		Correct unsimplified mean and var
	P(at most 30) = $P\left(z < \frac{30.5 - 20}{\sqrt{16}}\right)$	**M1**		Standardising must have sq rt, their μ, variance
		M1		cc either 29.5 or 30.5
		M1		Correct area Φ , from final process
	= P(z < 2.625) = 0.996	**A1**	[5]	
4 (i)	$P(< 4.5) = P\left(z < \frac{4.5 - 4.2}{0.6}\right) = P(z < 0.5)$	**M1**		Standardising once no cc no sq no sq rt
	= 0.6915			
	$P(< 3.5) = P\left(z < \frac{3.5 - 4.2}{0.6}\right) = P(z < -1.167)$			
	= 1 – 0.8784 = 0.1216	**M1**		$\Phi_1 - (1 - \Phi_2)$ [$P_1 - P_2$, $1>P_1>0.5$, $0.5>P_2>0$] oe
	0.6915 – 0.1216 = 0.570	**A1**	[3]	

(ii)	$z = 1.175$ $1.175 = \dfrac{t-4.2}{0.6}$ $t = 4.91$	B1 M1 A1	 [3]	±1.17 to 1.18 seen Standardising no cc, allow sq, sq rt with z – value (not ±0.8106, 0.5478, 0.4522, 0.1894, 0.175 etc.) Correct answer from z = 1.175 seen (4sf)
(iii)	$(0.88)^n < 0.003$ $n > \lg(0.003)/\lg(0.88)$ $n > 45.4$ $n = 46$	M1 M1 A1	 [3]	Inequality or eqn in 0.88, power correctly placed using n or (n±1), 0.003 or (1 – 0.003) oe Attempt to solve by logs or trial and error (may be implied by answer) Correct integer answer
5 (i)	cw 5, 5, 10, 20, 40 fd 8, 6, 1.8, 1.7, 0.2 	M1 M1 A1 B1 B1	 [5]	cw either 4 or 5 etc fd or scaled freq [f/their cw attempt] fd may be ÷ 1000 Correct heights seen accurately on diagram Correct bar ends, accurately plotted on axis Labels fd and capacity (thousands) Correct horizontal scale required. Vertical scale linear from 0
(ii)	(5×40+10×30+17.5×18+32.5×34+62.5×8)/130 = 2420/130 = 18.6 thousand	M1 A1	 [2]	Σfx/130 where x is mid point attempt (value within class, not end pt or cw)
(iii)	median group = 8 – 12 thousand LQ group = 3 – 7 thousand	B1 B1	 [2]	Thousands not needed

6	**(i)**	e.g. (OAEE)(CPNHGN) or cv $\frac{4!}{2!}\times\frac{6!}{2!}\times 2 = 8640$	**M1** **M1** **A1**	[3]	4!/2! or 6!/2! seen anywhere All multiplied by 2 oe
	(ii)	First Method Total ways = 10!/2!2! = 907200 EE together in 9!/2! ways = 181440 EE not together = 907200 – 181440 = 725760	**B1** **M1** **M1** **A1**	[4]	Total ways together correct EE together attempt alone Considering total – EE together
		OR Second Method C P N H G N O A in 8!/2! ways Insert E in 9 ways Insert 2nd E in 8 ways, ÷ 2 Total = 8!/2!×9×8 ÷ 2 = 725760	**B1** **M1** **M1** **A1**		8!/2! Seen Interspersing an E, x n where n=7,8,9. Condone additional factors. Mult by 9×8(÷2), ${}^9C_2$ or ${}^9P_2$ only oe
	(iii)	First Method EN** in ${}^6C_2$ ways = 15 different ways EENN in 1 way Total 16 ways	**M1** **M1** **A1** **B1** **A1**	[5]	${}^6C_x$ or ${}^yC_2$ seen alone or mult by $k > 1$, x<6, y>2 (1x1x) ${}^6C_2$ seen strictly alone or added to their EENN only
		OR Second Method Listing with at least 8 different correct options Listing all correct options Total = 15 different ways EENN in 1 way Total 16 ways	**M1** **M1** **A1** **B1** **A1**		Value stated or implied by final answer correct value stated Award 16 SRB2 if no method is present

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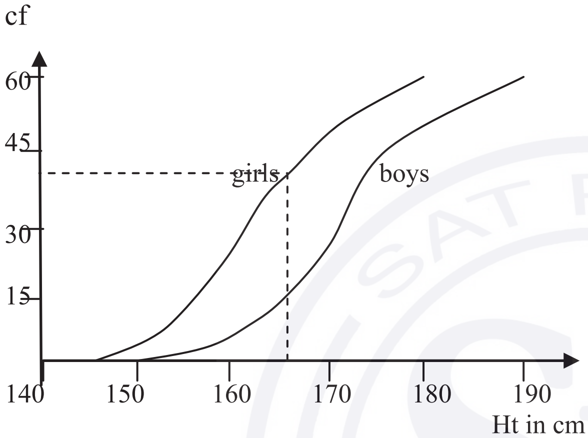
1	total ways $^{10}C_5 = 252$ MW together e.g. (MW)*** in $^8C_3$ ways = 56 MW not together = 252 – 56 = 196 ways **OR 1** $2\ ^8C_4 + {}^8C_5$ $2\ ^8C_4 = 2 \times 70 = 140$; $^8C_5 = 56$ $2\ ^8C_4 + {}^8C_5 = 196$ **OR 2** $2\ ^9C_5 - {}^8C_5$ $2\ ^9C_5 = 2 \times 126 = 252$; $^8C_5 = 56$ $2\ ^9C_5 - {}^8C_5 = 196$	**M1** **B1** **A1** **M1** **B1** **A1** **M1** **B1** **A1**	 [3]	$^{10}C_5 - \ldots$ or $252 - \ldots$ 252 and 56 seen, may be unsimplified $2\ ^nC_4 + {}^nC_5$ 140 and 56 seen may be unsimplified $2\ ^9C_5 - ..$ 252 and 56 seen, may be unsimplified
2 (i)	$p = 1/3$ $P(\geqslant 2) = 1 - P(0, 1) = 1 - (2/3)^4 - {}^4C_1(1/3)(2/3)^3$ or $P(2,3,4) = {}^4C_2(1/3)^2(2/3)^2 + {}^4C_3(1/3)^3(2/3) + (1/3)^4$ $= \frac{11}{27}$, 0.407	 **M1** **M1** **A1**	 [3]	 Bin term $^4C_x p^x(1-p)^{4-x}$ $0 < p < 1$ Correct unsimplified answer
(ii)	P(sum is 5) = P(1, 1, 1, 2) ×4 $= (1/3)^4 \times 4$ $= \frac{4}{81}$, 0.0494	**M1** **M1** **A1**	 [3]	1, 1, 1, 2 seen or 4 options Mult by $(1/3)^4$
3 (i)	e.g. **5 in $^3P_2$ ways = 6 **7 in $^3P_2 = 6$ Total 12 AG **OR** listing 457, 547, 467, 647, 567, 657, 475, 745 465, 645, 675, 765 Total 12 AG	**M1** **M1** **A1** **M1** **M1** **A1**	 [3]	Recognising ends in 5 or 7, can be implied Summing ends in 5 + ends in 7 oe Correct answer following legit working Listing at least 5 different numbers ending in 5 Listing at least 5 different numbers ending in 7
(ii)	1 digit in 2 ways 2 digits in *5 or *7 = $^3P_1 \times 2 = 6$ 4 digits in ***5 or ***7 = $^3P_3 \times 2 = 12$ Total ways = 32	**M1** **A1** **A1**	 [3]	Consider at least 3 options with different number of digits. If no working, must be 3 or 4 from 2, 6, 12, 12 One option correct from 1, 2 or 4 digits
4 (i)	64/250, 0.256	**B1**	[1]	oe
(ii)	190/250, 0.76(0)	**B1**	[1]	oe

(iii)	$P(X) = 80/250 = 8/25$	M1		attempt at $P(X)$
	$P(Y) = 100/250 = 2/5$	M1		attempt at $P(Y)$
	$P(X \cap Y) = 32/250 = 16/125$	B1		oe
	$P(X) \times P(Y) = \dfrac{8}{25} \times \dfrac{2}{5} = \dfrac{16}{125}$	M1		comparing $P(X) \times P(Y)$ and $P(X \cap Y)$ so long as independence has not been assumed
	Since $P(X) \times P(Y) = P(X \cap Y)$ therefore independent	A1	[5]	correct answer with all working correct
5 (i)		B1		Horizontal axis from min of 140 to 190 and vertical axis from 0 to minimum of 60 and two CF graphs on the same set of axes.
		B1		Labels: CF; height (ht) in cm; girls; boys in correct places
		B1		CF graph going through (150, 0) , (160, 20), (170, 43), (180, 55) and (190, 60)
		B1	[4]	CF graph going through (140, 0), (150, 12), (160,33), (170,50), (180, 60) [and (190, 60)]
(ii)	42 (± 1) shorter than 165.	M1		Line or reading from 165 on their cf graph oe subtracting from 60
	$(18(\pm 1))/60 \times 100$ $= 30\% \ (\pm 1.7\%)$	M1 A1	[3]	
(iii)	can see which is taller; see which of boys or girls is more spread out	B1	[1]	any sensible comment in context
6 (i)	$P(\text{small}) = P\left(z < \dfrac{95-150}{50}\right)$	M1		± standardising using 95, no cc, no sq, no sq rt
	$= P(z < -1.1)$ $= 1 - 0.8643$ $= 0.136$	M1 A1	[3]	$1 - \Phi$ (in final answer)
(ii)	$z = 1.282$	B1		± rounding to 1.28
	$1.282 = \dfrac{x-150}{50}$	M1		Standardised eqn in their z allow cc
	$x = 214$ g	A1	[3]	
(iii)	P(small) = 0.1357, P(large) = 0.1357 symmetry P(medium) = 1 – 0.1357×2 = 0.7286 **AG**	B1	[1]	Correct answer legit obtained
(b)	Expected cost per banana = 0.1357×10 + 0.1357×25 + 0.7286×20 = 19.3215 cents	*M1		Attempt at multiplying each ‘prob’ by a price and summing
	Total cost of 100 bananas = 1930 (cents) ($19.30)	DM1 A1	[3]	Mult by 100

7 (i)	$P(2) = {}^7C_2(0.1)^2(0.9)^5$ $= 0.124$	**M1** **A1**	 [2]	Bin term ${}^7C_2p^2(1-p)^5 \quad 0 < p < 1$
(ii)	$(0.15)^1(0.1)^2(0.75)^2 \times 5!/2!2!$ $= 0.0253$ or $81/3200$	**M1** **M1** **A1**	 [3]	Mult probs for options, $(0.15)^a(0.1)^b(0.75)^c$ where a + b + c sum to 5 Mult by 5!/2!2! oe
(iii)	mean = 365×0.15 (= 54.75 or 219/4) Var = $365\times 0.15\times0.85$ (= 46.5375 or 3723/80) $P(x > 44) = P\left(z > \frac{44.5-54.75}{\sqrt{46.5375}}\right)$ $= P(z > -1.5025)$ $= 0.933$	**B1** **M1** **M1** **M1** **A1**	 [5]	Correct unsimplified mean **and** var, oe ± Standardising need sq rt cc either 44.5 (or 43.5) Φ Correct answer accept 0.934

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MATHEMATICS **9709/61**

Paper 6 **May/June 2016**

MARK SCHEME

Maximum Mark: 50

Published

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CAMBRIDGE International Examinations

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MR	Misread
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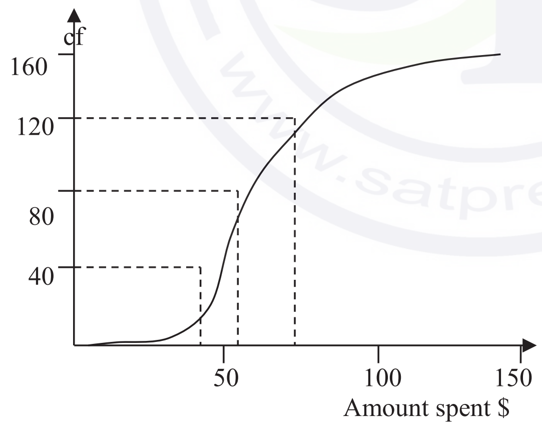
Question	Answer	Marks	Guidance
1	$z = 1.037$ $1.037 = \dfrac{1.8-1.62}{\sigma}$ $\sigma = 0.18/1.037 = 0.174$	B1 M1 A1 [3]	Rounding to 1.04 Standardising attempt allow cc no sq rt must have a z-value i.e. not 0.8023 or 0.5596.
2	P (throwing a 4) = (1 – 0.4) / 4 = 0.15 P(at most 1) = P(0, 1) or 1 – P(2, 3) $= (0.85)^3 + {}^3C_1\,(0.15)\,(0.85)^2$ = 0.939	M1 A1 M1 M1 A1 [5]	Sensible attempt to find P(1) Correct answer A binomial term with ${}^3C_n$ oe any p Binomial expression with ${}^3C_n$ P(0, 1) or 1 – P(2, 3) p = 0.15 or 0.85
3 (i)	P (cup of coffee) = 0.6×0.9 + 0.4× 0.3 = 0.66	M1 A1 [2]	Summing two 2-factor probabilities Correct answer accept 0.660
(ii)	P(Not on time | no cup of coffee) $= \dfrac{P(\text{not on time} \cap \text{no cup})}{P(\text{no cup})} = \dfrac{0.4\times 0.7}{1-0.66}$ $= \dfrac{0.28}{0.34} = 0.824$	M1 M1 A1 [3]	0.4×0.7 seen as num or denom of a fraction Attempt at P(no cup) as $0.1\times p_1 + 0.7\times p_2$ or as 1 – (i) seen anywhere
4	[P(X = 0)] = P(B, B) = 5/7×4/6 = 10/21 [P(X = 1)] = P(G,B) + P(B,G) = 2/7×5/6 ×2 = 10/21 [P(X = 2] = P(G, G) = 2/7×1/6 = 1/21 E(X) = 0 + 10/21 + 2/21 = 4/7 (0.571) Var(X) = 0 + 10/21 + 4/21 – $(4/7)^2$ = 50/147 (0.340)	M1 A1 A1 B1√ M1 A1 [6]	Attempt to find P(0) or P(1) or P(2) can be seen as P(BB) etc. or table unsimplified P(1) or P(BG)+P(GB) correct P(0) or P(2) correct must see X value Correct answer ft their probs P(1) and P(2) Attempt at $\Sigma x^2p - [E(X)]^2$
5 (i)	$P(x < 3.0) = P\left(z < \dfrac{3.0-2.6}{0.25}\right)$ + P(z < 1.6) = 0.945	M1 M1 A1 [3]	Standardising no sq rt no cc Correct area i.e. prob > 0.5 legit
(ii)	X~B(500, 0.9452) ~ N(472.6, 25.898) $P\left(z > \dfrac{479.5-472.6}{\sqrt{25.89848}}\right) = P\,(z > 1.3558)$ = 1 – 0.9125 = 0.0875	M1 M1 M1 M1 A1 [5]	500 ×'0.9452' and 500×'0.9452'×('1 – 0.9452') seen oe Standardising must have sq rt. All M marks indep cc either 479.5 or 480.5 seen correct area i.e. < 0.5

Question	Answer	Marks	Guidance
(iii)	500× 0.9452 and 500× (1–0.9452) are both > 5	**B1**√ [1]	must see at least 500× 0.0548 > 5oe ft their (i) accept $np > 5$, $nq > 5$ if both not $npq > 5$
6 (a) (i)	9 × 9 × 8 = 648 OR 900 – 28 × 9 = 648	**M1 M1** **A1** [3]	Logical listing attempt
(ii)	(7….in 1 × 8 × 4 = 32 ways 8 …in 1 × 8 × 5 = 40 9… in 1 × 8 × 4 = 32 Total 104 ways	**M1** **M1** **M1** **A1** [4]	Listing #s starting with 7 or 9 and ending odd
(b)	R(6) T(5) D(4) 2 2 3 = ${}^6C_2 \times {}^5C_2 \times {}^4C_3 = 600$ 2 3 2 = ${}^6C_2 \times {}^5C_3 \times {}^4C_2 = 900$ 3 2 2 = ${}^6C_3 \times {}^5C_2 \times {}^4C_2 = 1200$ Total = 2700	 **M1** **M1** **A1** **A1** [4]	 Mult 3 combs, ${}^6C_x \times {}^5C_y \times {}^4C_z$ Summing 2 or 3 three-factor outcomes can be perms, + instead of × 2 options correct unsimplified
7 (i)	cf 16, 56, 104, 130, 160  	**M1** **B1** **M1** **A1** [4]	Attempt at cf table (up to 160) no graph needed accept %cf but give final linear scale minimum 0 to 160 and 0 to 120 Attempt to plot points at (30, 16), (50, 56), (70,104), (90, 130), (140, 160) up to 2 errors can have a polygon All points correct from their scale and joined up, with (0,0) as well
(ii)	median \$59 IQR = 82 – 43 = \$39	**B1**√ **M1** **A1**√ [3]	accept 57–60 or ft their graph if used lb, midpts instead of ub or assume linear interpolation. Subt a (sensible) LQ from a sensible UQ (generous) Ans ft need a cf graph and UQ 80–84, LQ

Question	Answer	Marks	Guidance
(iii)	160 – 149 = 11 OR 115 is mid pt of last interval so # of shoppers is 30/2 = 15 (can be implied)	M1 A1 [2]	41–46 Subtracting from 160 can be implied Correct answer accept 9–16
(iv)	mean = (15×16+ 40×40 +60×48+ 80×26 + 115×30)/160 = 10250/160 = \$64.1= \$64.1	M1 A1 [2]	Using $\Sigma xf/160$ with mid-points

Cambridge International AS & A Level

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MATHEMATICS	**9709/62**
Paper 6	**May/June 2016**
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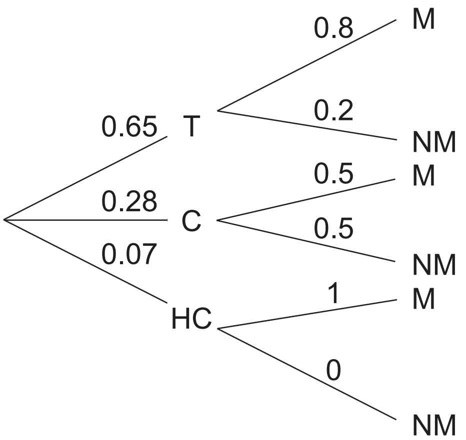
Qu		Answer	Marks	Notes
1	(i)		M1 A1 [2]	Correct shape with either one branch after HC or 2 branches with 0 prob seen correct Labelled and clear annotation All probs correct
	(ii)	$\text{P}(C \mid \text{milk}) = \dfrac{P(coffee \cap milk)}{P(milk)}$ $= \dfrac{0.28 \times 0.5}{0.65 \times 0.8 + 0.28 \times 0.5 + 0.07(\times 1)}$ $= \dfrac{0.14}{0.73}$ $=0.192$	M1 M1 A1 [3]	Attempt at P(coffee∩ milk)as a two-factor prod only seen as num or denom of a fraction Summing appropriate three 2-factor products seen anywhere (can omit the 1) Correct answer oe
2	(i)	0.72	B1 [1]	
	(ii)	$np = 180 \times 0.72$, $npq = 180 \times 0.72 \times 0.28$ $X \sim \text{N}(129.6, 36.288)$ $\text{P}(x > 115) = P\left(z > \dfrac{115.5 - 129.6}{\sqrt{36.288}}\right)$ $= \text{P}(z > -2.341)$ $= 0.990$	B1√ M1 M1 M1 A1 [5]	180 × 0.72, 180 × 0.72 × 0.28 seen, their values or correct Standardising (±) must have sq rt cc either 115.5 or 114.5 seen Correct area, Φ from final answer attempt fully correct method
3	(i)	<table><tr><td>x</td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>P(x)</td><td>k</td><td>$2k$</td><td>$3k$</td><td>$4k$</td></tr></table>$10k = 1$ $k = 1/10$	B1 M1 A1 [3]	Probability Distribution Table, either k or correct numerical values Summing probs involving k to = 1, 3 or 4 terms
	(ii)	$\text{E}(X) = 1/10 + 4/10 + 9/10 + 16/10 = 3$ $\text{Var}(X) = 1/10 + 8/10 + 27/10 + 64/10 - 3^2$ $= 1$	B1 M1 A1 [3]	Correct mean Correct method seen for var, their k and μ

4 **(i)**	$p = 0.66$ $X \sim B(15, 0.66)$ P(at least 14) = P(14, 15) = $^{15}C_{14}\,(0.66)^{14}(0.34) + (0.66)^{15}$ = 0.0171	**M1** **M1** **A1** [3]	Bin term $^{15}C_x p^x(1-p)^{15-x}$ seen any p Unsimplified correct expression for P(14,15)
(ii)	$(0.87)^n < 0.04$ $n = 24$	**M1** **M1** **A1** [3]	Eqn involving 0.87, power of n, 0.04 only Solving by logs or trial and error(can be implied). Must be exponential equation
5 **(i)**	Bronlea Rogate 6 3 | 0 | 4 5 7 7 7 4 3 | 1 | 0 1 3 5 6 8 8 7 5 4 2 1 | 2 | 3 3 6 3 2 | 3 | 4 5 | 4 | Key 3| 1 | 5 represents 13 kph for Bronlea and 15 kph for Rogate	**B1** **B1** **B1** **B1** **B1** [5]	Correct single stem Correct ordered leaves Bronlea Correct ordered leaves Rogate Correct overall shape Single key must have both towns and units consistent with their values
(ii)	median Bronlea = 23 km per hour IQ range Rogate = 23 – 7 = 16	**B1** **M1** **A1** [3]	Units not necessary Subt their LQ <14 from their UQ>14 from Rogate leaf
(iii)	Rogate is less windy than Bronlea	**B1** [1]	Not a comparison of a statistic but interpretation of information
6 **(i)**	$P(x > 10.2) = P\left(z > \frac{10.2 - 9.5}{1.3}\right)$ $= P(z > 0.53846)$ $= 1 - 0.7046$ $= 0.295$	**M1** **M1** **A1** [3]	Standardising allow cc, sq rt, sq 1 – Φ final solution attempt
(ii)	$z = -1.282$ $-1.282 = \frac{t - 9.5}{1.3}$ $t = 7.83$	**B1** **M1** **A1** [3]	± rounding to 1.28 seen Standardising correctly can be ± z value here Correct answer from $z = -1.282$ only
(iii)	$P(x < 8.8) = 0.2954$ by symmetry Days = 365×0.2954 = 107 or 108	**B1** **M1** **A1** [3]	oe method, FT *their 0.2954 from (i)* Mult a probability <1 by 365 Correct answer (no decimals)
7 **(a)** **(i)**	$\frac{10!}{2!3!} = 302400$	**B1** [1]	Exact value only, isw rounding

(ii)	e.g. *W******W*, **W******W, W******W**	**M1**	8! Seen mult or alone. Cannot be embedded (arrangements of other 8 letters).
	$\frac{8!}{3!} \times 3$(for the Ws)	**M1**	Dividing by 3! (removing repeated L's)
		M1	Mult by 3 (different W positions) may be sum of 3 terms
	= 20160	**A1** [4]	
(b)	S(5) A(7) C(4) 1 3 2 : $5\times{}^7C_3\times{}^4C_2 = 1050$ 1 4 1 : $5\times{}^7C_4\times 4 = 700$ 2 3 1 : ${}^5C_2\times{}^7C_3\times 4 = 1400$ 3 2 1 : ${}^5C_3\times{}^7C_2\times 4 = 840$ (Outcomes : Options)	**M1**	Mult 3 combinations, ${}^5C_x$, ${}^7C_y$, ${}^4C_z$ (not 5 x 7 x 4)
		A1	2 correct options unsimplified
		M1	Summing only 3 or 4 correct outcomes involving combs or perms
	Total = 3990	**A1** [4]	

Cambridge International AS & A Level

Cambridge International Examinations
Cambridge International Advanced Subsidiary and Advanced Level

MATHEMATICS **9709/63**

Paper 6 **May/June 2016**

MARK SCHEME

Maximum Mark: 50

Published

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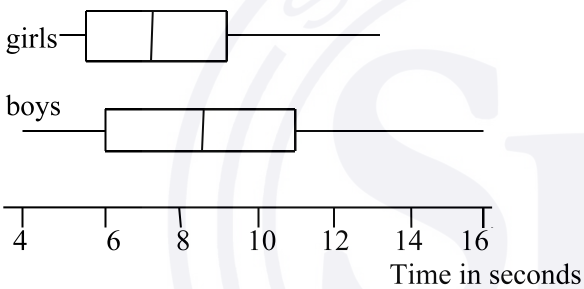
<table>
<tr><th>Qu</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td>1 (i)</td><td>
<table>
<tr><th></th><th>Wears specs</th><th>Not wears specs</th><th>Total</th></tr>
<tr><td>RH</td><td>6</td><td>19</td><td>25</td></tr>
<tr><td>Not RH</td><td>2</td><td>3</td><td>5</td></tr>
<tr><td>Total</td><td>8</td><td>22</td><td></td></tr>
</table>
</td><td>B1
B1 [2]</td><td>One correct row or col including total other than the Total row/column
All correct</td></tr>
<tr><td>(ii)</td><td>P(X) = 25/30, P(Y) = 8/30
P(X) × P(Y) = 25/30 × 8/30 = 200/900 = 2/9
P(X∩Y) = 6/30 = 1/5 ≠ P(X) × P(Y)
Not independent</td><td>M1
M1
A1 [3]</td><td>P(X) or P(Y) from their table or correct from question (denom 30) oe
Comparing their P(X) × P(Y) (values substituted) with their evaluated P(X∩Y) – not P(X)×P(Y)</td></tr>
<tr><td>2 (i)</td><td></td><td>B1
B1
B1 [3]</td><td>Labels ‘time’ and ‘seconds’, ‘boys’ and ‘girls’ on correct plots and scaled line
One box and whisker all correct on graph paper – ignore boy or girl label
Second box and whisker all correct (on graph paper and ignore boy/girl label) on SAME scaled line.</td></tr>
<tr><td>(ii)</td><td>girls smaller range or IQ range than boys /girls less spread out oe
girls generally quicker than boys or girls median<boys median (not mean) oe
boys almost symmetrical, girls +vely skewed oe</td><td>B1
B1 [2]</td><td>Any 2 comments – MUST be a comparison</td></tr>
<tr><td>3 (i)</td><td>P(0) = 6/36, P(1) = 10/36, P(2) = 8/36
P(3) = 6/36, P(4) = 4/36, P(5) = 2/36</td><td>B1
B1
M1
A1 [4]</td><td>Table oe seen with 0, 1, 2, 3, 4, 5 (6 if P(6) = 0)
Any three probs correct
Σ p = 1 and at least 3 outcomes
All probs correct</td></tr>
<tr><td>(ii)</td><td>mean score = (0×6+1×10 +16 +18 +16+10)/36
= 70/36 (35/18, 1.94)</td><td>M1
A1 [2]</td><td>Using Σxp (unsimplified) on its own – condone
Σ p not =1</td></tr>
</table>

Qu		Answer	Marks		Guidance
4	(i)	1845/9 (= 205) c = 2205 - 205 = 2000	M1 A1		Accept (1845± anything)/ 9
		OR $\Sigma x = 2205 \times 9$ (= 19845) $\Sigma x - \Sigma c = 1845$ Σc = 19845 -1845 = 18000 c = 2000	M1 A1	 [2]	For 2205×9 seen
	(ii)	var = $\dfrac{477450}{9} - 205^2$ = 11025	M1 A1		For $\dfrac{477450}{9} - (\text{their coded mean})^2$
		OR var = $\dfrac{43857450}{9} - 2205^2$ = 11025	M1 A1	 [2]	For their $\Sigma x^2/9 - 2205^2$ where Σx^2 is obtained from expanding $\Sigma(x-c)^2$ with $2c\Sigma x$ seen
	(iii)	new total = 2120.5×10 = 21205 new price = 21205 – 19845 = 1360	M1 A1	 [2]	Attempt at new total
5	(i)	z = 1.015 $1.015 = \dfrac{70-69}{\sigma}$ σ = 0.985 (200/203)	B1 M1 A1	 [3]	Accept z between ±1.01 and 1.02 Standardising
	(ii)	58 + 9 = 67 P (> 67) = $\text{P}\left(z > \dfrac{67-69}{0.9852}\right)$	M1 M1		58 + 9 seen or implied (or 69-58 or 69-9) Standardising ± z no cc allow their sd (must be +ve) Alt. 1 69-58 =11, P(>9)=$\text{P}\left(z > \dfrac{9-11}{0.9852}\right)$ Alt.2 69-9 =60, P(>58) =$\text{P}\left(z > \dfrac{58-60}{0.9852}\right)$
		= P(z > – 2.03) = 0.9788	M1		Correct prob area
		300 × 0.9788	M1		Multiply their prob (from use of tables) by 300
		= 293.6 so 293	A1	[5]	– accept 293 or 294 from fully correct working

Qu		Answer	Marks		Guidance
6	(i)	7560 ways	B1	[1]	
	(ii)	RxxxxxxxG in $\frac{7!}{4!}$	B1		7! alone seen in num or 4! alone in denom Must be in a fraction. $\frac{7!\times 2}{4!\times 2}$ gets full marks
		= 210 ways	B1	[2]	
	(iii)	eg EEEExxxxx in $\frac{6!}{2!}$	B1		6! or 5! × 6 seen in numerator or on own Can be 6! × k but not 6! ± k
		= 360 ways	B1	[2]	
	(iv)	1 R eg RVG or RVN or RGN = 3	B1	[1]	
	(v)	no Rs eg VGN or 3C3 ways = 1 2 Rs eg RRV or 3C1 ways = 3	M1		Summing at least 2 options for R
			A1		Correct outcome for no Rs or 2 Rs – evaluated
		Total = 7	A1	[3]	
7	(i)	${}^{12}C_8\,(0.65)^8(0.35)^4 + {}^{12}C_9\,(0.65)^9(0.35)^3 + {}^{12}C_{10}\,(0.65)^{10}(0.35)^2$	M1		Bin term with ${}^{12}C_r p^r (1-p)^{12-r}$ seen $r\neq 0$ any $p<1$
			M1		Summing 2 or 3 bin probs p = 0.65 or 0.35, n = 12
		= 0.541	A1	[3]	
	(ii)	$P\left(\bar{R}\bar{R}\bar{R}R\right) = 0.35\times 0.35\times 0.35\times 0.65$	M1		Mult 4 probs either $(0.35)^3(0.65)$ or $(0.65)^3(0.35)$
		= 0.0279	A1	[2]	
	(iii)	P(7) = 0.2039 (unsimplified)	B1		${}^{12}C_7\,(0.65)^7(0.35)^5$
		Mean = 250×’0.2039’ (= 50.9798) Var = 250×’0.2039’ × ‘(1 – 0.2039)’ (= 40.5851)	B1		Correct unsimplified np and npq using ‘their 0.2039’ but not 0.65 or 0.35
		$P(>54) = P\left(\frac{54.5-50.9798}{\sqrt{40.5851}}\right)$	M1		Standardising need sq rt – must be from working with 54
		= P(z > 0.5526) = 1 – Φ(0.5526) = 1 – 0.7098	M1		cc either 53.5 or 54.5
			M1		correct area < 0.5 i.e. 1 – Φ - must be from working with 54
		= 0.290	A1	[6]	

CAMBRIDGE INTERNATIONAL EXAMINATIONS

Cambridge International Advanced Subsidiary and Advanced Level

MARK SCHEME for the March 2016 series

9709 MATHEMATICS

9709/62 Paper 6 (Probability and Statistics), maximum raw mark 50

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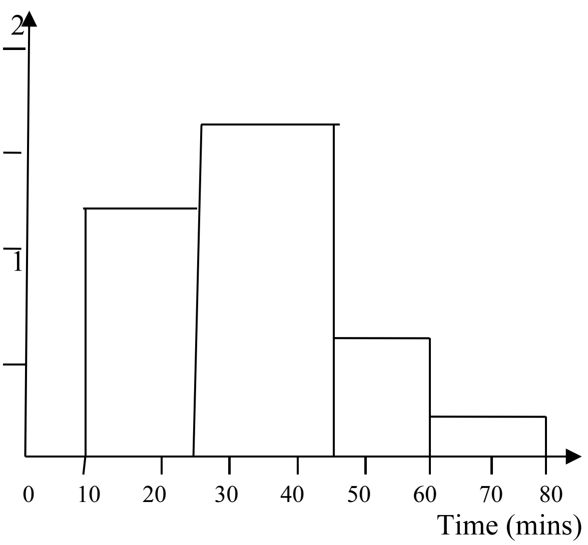
1 (i)	$\Sigma x = 862$	B1	1	Must be stated or replaced in (ii) Can see **(i)** and **(ii)** in any order
(ii)	$362/10 + a = 86.2$ $a = 50$	M1 A1	 2	86.2 ± 36.2 seen oe Correct answer, nfww
2	<table><tr><td>No of W</td><td>0</td><td>1</td><td>2</td></tr><tr><td>Prob</td><td>42/90</td><td>42/90</td><td>6/90</td></tr></table>	B1		0, 1, 2, seen in table with attempt at prob.
	$P(0) = 8/10 \times 7/9 \times 6/8 = 42/90$ $P(1W) = P(W,NW, NW) \times 3 = 2/10 \times 8/9 \times 7/8 \times 3$ $= 42/90$ $P(2W) = P(W, W, NW) \times 3 = 2/10 \times 1/9 \times 8/8 \times 3$ $= 6/90$	M1 M1 A1	 4	3-factor prob seen with different denoms. Mult by 3 All correct
3 (i)	$P(R)$ [(1, 4),(2,5), (3,6),(4,7),(5,8)] $\times 2/64$ $= 10/64$	M1 A1	 2	List of at least 4 different options or possibility space diagram Correct answer
(ii)	$P(S)$ = [(3,8)(3,7)(4,8)(4,7)(4,6)(4,5)(5,8)(5,7)(5,6)(6,8)(6,7)(7,8)] $\times 2$ + (5,5)(6,6)(7,7)(8,8) $= 28/64$	M1 A1	 2	List of at least 14 different options or ticks **oe** from possibility space Correct answer
(iii)	$P(R \cap S) = 4/64$ $4/64 \neq 10/64 \times 28/64$ Events are not independent	B1 M1 A1	 3	 Comparing their $P(R \cap S)$ with (i) ×(ii) with values Correct answer
4 (i)	32	B1	1	
(ii)	freqs 0 18 32 9 4 fd 0 1.2 1.6 0.6 0.2 cf 	M1 A1 B1 B1	 4	attempt at fd or scaled freq (at least 3 f/cw attempt) correct heights seen on diagram Correct bar ends Labels fd and time (mins) and linear axes or squiggle

(iii)	$(17.5 \times 18 + 35 \times 32 + 52.5 \times 9 + 70 \times 4)/63$ $= 2187.5/63 = 34.7$	**M1** **A1**	 2	$\Sigma fx/63$ where x is midpoint attempt not end pt or cw Correct answer
5 (i)	P(Abroad given camping) $= \dfrac{P(A\cap C)}{P(A\cap C)+P(H\cap C)}$ $= \dfrac{0.35\times 0.15}{0.35\times 0.15+0.65\times 0.4}$ $= \dfrac{0.0525}{0.3125}$ $= 0.168$	**M1** **A1** **M1** **A1** **A1**	 5	Attempt at P($A\cap C$) seen alone anywhere Correct answer seen as num or denom of a fraction Attempt at P(C) seen anywhere Correct unsimplified answer seen as num or denom of a fraction Correct answer
(ii)	$(0.65)^n < 0.002$ $n > \lg(0.002)/\lg(0.65)$ $n = 15$	**M1** **M1** **A1**	 3	Eqn with 0.65 or 0.35, power n, 0.002 or 0.998 Attempt to solve their eqn by logs or trial and error need a power Correct answer
6 (i)	$^{15}P_5$ $= 360360$	**M1** **A1**	 2	oe, can be implied Not $^{15}C_5$ Correct answer
(ii)	$5 \times 10 \times 4 \times 9 \times 3$ $= 5400$	**M1** **A1**	 2	Mult 5 numbers Correct answer
(iii)	M(5) F(10) 3 2 $= {}^5C_3 \times {}^{10}C_2 = 450$ ways 4 1 $= {}^5C_4 \times {}^{10}C_1 = 50$ 5 0 $= {}^5C_5 \times {}^{10}C_0 = 1$ Total = 501 ways	 **M1** **M1** **A1**	 3	 Mult 2 combs, $^5C_x \times {}^{10}C_y$ Summing 2 or 3 two-factor options, $x + y = 5$ Correct answer
(iv)	(Couple) M(4) F(9) ManWife + 3 0 $= {}^4C_3 \times {}^9C_0 = 4$ ManWife + 2 1 $= {}^4C_2 \times {}^9C_1 = 54$ Total = 58	 **M1** **M1** **A1**	 3	 Mult 2 combs ^4C_x and ^9C_y Summing both options $x + y = 3$, gender correct Correct answer
7 (i)	$z = -1.645$ $-1.645 = \dfrac{0.9 - m}{0.35}$ $m = 1.48$	**B1** **M1** **A1**	 3	± 1.64 to 1.65 seen Standardising with a z-value accept $(0.35)^2$ Correct answer
(ii)	$P(<2) = P\left(z < \dfrac{2-1.476}{0.35}\right)$ $= P(z < 1.50)$ $= 0.933$ Prob $= (0.9332)^4$ $= 0.758$	**M1** **M1** **A1** **M1** **A1**	 5	Standardising no sq , FT *their m*, no cc Correct area i.e. F Accept correct to 2sf here Power of 4, from attempt at P(z) Correct answer

(iii)	$P(t > 0.6\mu) = P\left(z > \dfrac{0.6\mu - \mu}{\mu / 3}\right)$ $= P(z > -1.2)$ $= 0.885$	**M1** **M1** **A1**	 3	Standardising attempt with 1 or 2 variables Eliminating μ or σ Correct final answer

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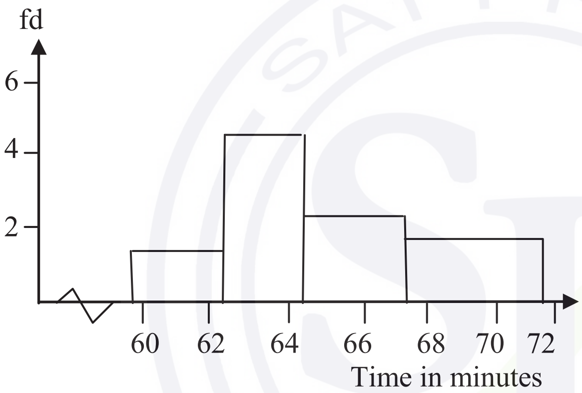
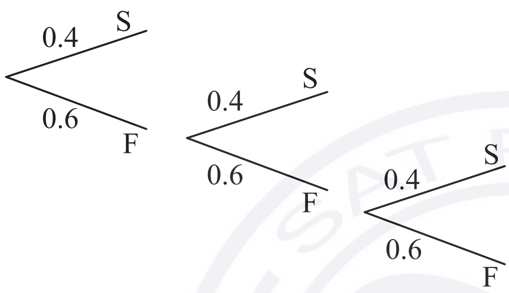
1	$p = 0.76$ P(fewer than 10) = 1 – P(10, 11) $= 1 - (0.76)^{10}(0.24)^{11}C_{10} - (0.76)^{11}$ = 1 – 0.219 = 0.781	 **M1** **M1** **M1** **A1** [4]	 Any binomial term ${}^{11}C_x p^x(1\text{-}p)^{11-x}$, $0 < p < 1$ Any binomial term ${}^{n}C_x(0.76)^x(0.24)^{n-x}$ 1 – P(10, 11) oe binomial expression Correct answer
2	$\mu = 54.1$ $z = -1.11$ $-1.11 = \dfrac{50.9 - 54.1}{\sigma}$ $\sigma = 2.88$	**B1** **B1** **M1** **A1** [4]	Stated or evaluated Accept rounding to ± 1.1 Standardising no cc no sq rt Correct answer
3 (i)	$a = 9$/cw $= 9/2 = 4.5$ $1.5 = b/4$ so $b = 6$	**M1** **A1** **A1** [3]	Using fd = f/cw Correct a Correct b
(ii)		**B1**√ **B1** **B1** [3]	Correct heights ft their b Correct widths, ie 3, 2, 3, 4 starting either 60 or 59.5 Labels fd, time or minutes and squiggle and bars from 59.5 to 71.5
4 (i)	$\bar{x} = 80 - 147/30 = 80 - 4.9$ $= 75.1$ $\text{sd} = \sqrt{\left(\dfrac{952}{30} - \left(\dfrac{147}{30}\right)^2\right)} = \sqrt{7.72\ldots}$ sd = 2.78	**M1** **A1** **M1** **A1** [4]	For –147/30 oe seen Correct answer 952/30 – (± their coded mean)^2 Correct answer
(ii)	$P(x > 160) = P\left(z > \dfrac{160 - 148.6}{18.5}\right)$ $= P(z > 0.616)$ $= 1 - 0.7310$ $= 0.269$	**M1** **M1** **A1** [3]	Standardising no cc no sq rt $1 - \Phi$ Correct answer

5 (i)	**5 (i)** eg **(EEEE)*** Number of ways = $\frac{6!}{2!2!}$ = 180	**M1** **M1** **A1** [3]	Mult by 6! oe Dividing by 2!2! oe Correct answer
(ii)	S*******T or T*******S Number of ways = $\frac{7!}{4!2!} \times 2$ = 210	**M1** **M1** **A1** [3]	Mult by 7! Or dividing by one of 2! or 4! Mult by 2 Correct answer
(iii)	exactly one E in ${}^6C_3$ ways = 20	**M1** **M1** **A1** [3]	${}^6C_x$ as a single answer ${}^xC_3$ as a single answer correct answer
6 (i)		**M1** **A1** **A1** [3]	3 pairs S (bank, log in, success oe) and F oe seen no extra bits. Exactly 3 pairs, must be labelled Correct diagram with all probs correct
(ii)	<table><tr><td>x</td><td>0</td><td>1</td><td>2</td><td>3</td></tr><tr><td>Prob</td><td>0.4</td><td></td><td>0.144</td><td>0.216</td></tr></table>	**B1** **M1** **A1** **B1** [4]	P(0) correct Multiplying two of more factors of 0.4 and 0.6 One more correct prob One more correct prob
(iii)	E(X) = 0.24+ 2× 0.144 + 3× 0.216 = 1.176 (1.18)	**M1** **A1** [2]	Using $\Sigma p_i x_i$ Correct answer
7 (i)	let P(2, 4, 6) all = p then P(1, 3, 5) all = 2p $3p + 6p = 1$ p = 1/9 so prob (3) = 2/9 (0.222)	**M1** **M1** **A1** [3]	Using P(even) = 2P(odd) or vice versa oe Summing P(odd+ even) or P(1, 2, 3, 4, 5, 6) = 1 Correct answer
(ii)	P(5, 5, 6) = 2/9 × 2/9 × 1/9 × ${}^3C_2$ = 4/243 (0.0165)	**M1** **M1** **A1** [3]	Mult three probs together Mult by 3 oe ie summing 3 options Correct answer
(iii)	μ = 100×1/3 = 33.3, σ = 100×1/3×2/3 = 22.2 $P(x \leqslant 37) = P\left(z \leqslant \frac{37.5 - \frac{100}{3}}{\sqrt{\frac{200}{9}}}\right) = P(z \leqslant 0.8839)$ = 0.812	**B1** **M1** **M1** **M1** **A1** [5]	Unsimplified 100/3 and 200/9 seen Standardising need sq rt 36.5 or 37.5 seen correct area using their mean Correct answer

CAMBRIDGE INTERNATIONAL EXAMINATIONS

Cambridge International Advanced Subsidiary and Advanced Level

MARK SCHEME for the October/November 2015 series

9709 MATHEMATICS

9709/62 Paper 6, maximum raw mark 50

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Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the October/November 2015 series for most Cambridge IGCSE®, Cambridge International A and AS Level components and some Cambridge O Level components.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol $\sqrt{}$ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)
AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no “follow through” from a previous error is allowed)

CWO Correct Working Only – often written by a ‘fortuitous’ answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become “follow through √” marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

1	$\Sigma x - 100n = 216$ $2416 - 100n = 216$ $n = 22$ OR $\frac{2416}{n} = \frac{216}{n} + 100$ $n = 22$	**B1** **B1** **B1** **B1** **B1** **B1**	 3	$\Sigma x - 100n$ seen Subst 2416 for their Σx Correct answer $2416/n$ seen or $216/n + 100$ oe eg $\Sigma x/n - 100 = 216/n$ correct equation Correct answer
2	P(no men) $\frac{{}^{9}C_{6}}{{}^{16}C_{6}} = \frac{84}{8008} = \frac{21}{2002} = \frac{3}{286}$ $= 0.0105$ OR $\frac{9}{16} \times \frac{8}{15} \times \frac{7}{14} \times \frac{6}{13} \times \frac{5}{12} \times \frac{4}{11} = 0.0105$	**B1** **B1** **B1** **B1** **B1** **B1**	 3	${}^{9}C_{6}$ seen anywhere ${}^{16}C_{6}$ seen as denom of fraction oe Correct final answer $(9 \times 8 \times 7 \times 6 \times 5 \times 4)$ seen anywhere Correct unsimplified denom Correct final answer
3 (i)	$\frac{1}{4}$	**B1**	1	
(ii)	$\left(\frac{3}{4}\right)^4\left(\frac{1}{4}\right) = \frac{81}{1024} = 0.0791$	**M1** **A1**	 2	Expression of form $p^4(1-p)$ only, $p = 1/4$ or $3/4$ Correct answer
(iii)	P(all diff) $= \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} \times 4!$ $= \frac{3}{32}$ (0.0938) OR $1 \times \frac{3}{4} \times \frac{2}{4} \times \frac{1}{4} = \frac{3}{32}$	**M1** **M1** **A1**	 3	4! on numerator seen mult by k $\geqslant$ 1 or $3\times2\times1$ on num oe, must be in a fraction. 4^4 on denom or 4^3 on denom with the $3\times 2\times 1$ Correct answer
4 (i)	Two in same taxi: ${}^{6}C_{2} \times {}^{4}C_{4} \times 2$ or ${}^{6}C_{2} + {}^{6}C_{4}$ $= 30$	**M1** **M1** **A1**	 3	${}^{6}C_{4}$ or ${}^{6}C_{2}$ oe seen anywhere 'something' ×2 only or adding 2 equal terms Correct final answer
(ii)	MJS in taxi $({}^{5}C_{1}\times2\times2)\times {}^{4}P_{4}$ $= 480$	**M1** **M1** **M1** **A1**	 4	${}^{5}P_{1}$, ${}^{5}C_{1}$ or 5 seen anywhere Mult by 2 or 4 oe Mult by ${}^{4}P_{4}$ oe eg 4! or $4\times{}^{3}P_{3}$ or can be part of 5! Correct final answer

5 (i)

team A		team B
	7	5 7 9
4 4 2	8	2 3 4 6
9 8 7 6 1	9	4 5 6
9 7 4 0	10	1 8
6 5	11	1 3 5
2	12	

key 1 | 9 | 4 means 91 kg for team A and 94 kg for B

Mark	Total	Guidance
B1		Correct stem can be upside down, ignore extra values, allow 70, 80 etc with suitable numerical key
B1		Correct team A must be on LHS, alignment ± half a space, no late entries squeezed in, no crossing out if shape is changed
B1		Correct team B in single diagram can be either LHS or RHS
B1	4	Correct key or keys for their diagram/s, need both teams, at least one kg.

(ii)

LQ = 91 UQ = 109
IQ range = 18

Mark	Total	Guidance
B1		Both quartiles correct
B1√	2	Correct IQR ft wrong quartiles, LQ < UQ, not 12 – 4 etc

(iii)

$\Sigma x_{15} = 1399$
$\Sigma x_{16} = 16 \times 93.9 = 1502.4$
New wt = $1502.4 - 1399 = 103$ (103.4)

Mark	Total	Guidance
M1		Attempt at Σx_{15} for either team
M1		Mult 93.9 by 16 attempt
A1	3	Correct answer

6 (i)

Spinner A

Spinner B	1	2	3	3
–3	(–2)	–1	0	0
–2	–1	0	(1)	1
–1	0	1	2	2
1	2	3	4	4

B1 1

(ii)

x	–2	–1	0	1	2	3	4
prob	$\frac{1}{16}$	$\frac{2}{16}$	$\frac{4}{16}$	$\frac{3}{16}$	$\frac{3}{16}$	$\frac{1}{16}$	$\frac{2}{16}$

Mark	Total	Guidance
M1		Their values in (i) as the top line, seen listed in (ii) or used in part (iii)
M1		Attempt at probs seen evaluated, need at least 4 correct from their table
A1	3	Correct table seen

(iii)

$E(X) = 1$

$\mathrm{Var}(X) = ((-2)^2 + 2 + 3 + 12 + 9 + 32)/16 - 1^2$

$= \frac{62}{16} - 1$

$= \left(\frac{23}{8}\right)$ (2.875)

OR using $\Sigma p(x - \bar{x})^2 = (9 + 8 + 4 + 0 + 3 + 4 + 18)/16$

$= \frac{46}{16} = 2.875$

Mark	Total	Guidance
M1		Attempt at $E(X)$ from their table if $\Sigma p = 1$
M1		Evaluating $\Sigma x^2 p$ – [their $E(X)$]^2 allow $\Sigma p \neq 1$ but all p's <1
A1	3	Correct answer
M1		
M1		
A1		

(iv)	P(even given +ve) $= \frac{5}{9}$	**M1** **A1** 2	Counting their even numbers and dividing by their positive numbers Correct answer
	OR P(even given +ve) = $\frac{\left(\frac{5}{16}\right)}{\left(\frac{9}{16}\right)}$	**M1**	Using cond prob formula not P(E) × P(+ve) need fraction over fraction accept any of $\frac{5/16 or 6/16 or 9/16}{9/16 or 10/16 or 13/16}$
	$= \frac{5}{9}$ (0.556)	**A1**	Correct answer
7 (a) (i)	P (x > 3900) = P$\left(z > \frac{3900-4520}{560}\right)$ = P(z > –1.107) = Φ(1.107) = 0.8657 Number of days = 365 × 0.0.8657 = 315 or 316 (315.98)	**M1** **M1** **A1** **B1√** 4	Standardising no cc no sq rt no sq Correct area Φ ie > 0.5 Prob rounding to 0.866 Correct answer ft their wrong prob if previous A0, p < 1 , ft must be accurate to 3sf
(ii)	z = 1.165 $1.165 = \frac{8000-m}{560}$ m = 7350 (7347.6)	**B1** **M1** **A1** 3	± 1.165 seen Standardising eqn allow sq, sq rt, cc, must have z-value eg not 0.122, 0.878, 0.549, 0.810. Correct answer rounding to 7350
(iii)	P(0, 1) = $(0.878)^6 + {}^6C_1(0.122)^1(0.878)^5$ = 0.840 accept 0.84 Normal approx. to Binomial. M0, M0, A0	**M1** **M1** **A1** 3	Binomial term ${}^6C_x\, p^x(1-p)^{6-x}$ $0 < p < 1$ seen Correct unsimplified expression Correct answer
(b)	P(< 2μ) = P$\left(z > \frac{2\mu-\mu}{\sigma}\right)$ = P (z < 1.5) = 0.933	**M1** **M1** **A1** 3	Standardising with μ and σ Attempt at one variable and cancel Correct answer

CAMBRIDGE INTERNATIONAL EXAMINATIONS

Cambridge International Advanced Subsidiary and Advanced Level

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9709 MATHEMATICS

9709/63 Paper 6, maximum raw mark 50

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International Examinations

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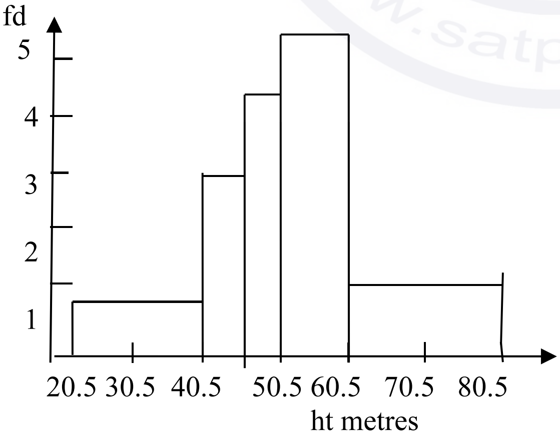
PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

1	coded mean = 0.3 oe sd $= \sqrt{\frac{96.1}{250} - (0.3)^2}$ $= 0.543$	**B1** **M1** **A1**	 3	$\Sigma(t - 2.5) = 75$ B0 until ÷ 250 Subst in variance formula both terms coded Correct answer
	Alt: $\Sigma(t-2.5)^2$ expanded $\Sigma t^2 = 2033.6$ sd $= \sqrt{\frac{2033.6}{250} - 2.8^2}$ $= 0.543$	**Or** **B1** **M1** **A1**	 3	 Substituting their Σt^2 from expanded 3-term expression, 250 and 2.8 in variance formula
2 (i)	$P(X) = \frac{20}{28}\left(\frac{5}{7}\right)(0.714), 71.4\%$	**B1**	1	oe
(ii)	$P(F) = \frac{20}{28} \times \frac{1}{4} \times \frac{8}{28} \times \frac{6}{10} = \frac{7}{20}$	**M1** **A1**	 2	Summing two 2-factor probs created by One of ¼ or ¾ multiplied by 20/28 or 8/28 Added to 4/10 or 6/10 × altn population prob Correct answer
(iii)	$P(X \mid F) = \frac{5/28}{7/20} = \frac{25}{49}(0.510)$	**M1** **A1**	 2	Their unsimplified country X probability (5/28) as num or denom of a fraction Or (their fair hair population) ÷ (total fair hair pop) Correct answer
3 (i)	$P(S) = \frac{3}{16}$ $P(T) = \frac{4}{16}$ $P(S \cap T) = \frac{2}{16}$ $P(S) \times P(T) = \frac{3}{64} \neq \frac{2}{16}$ Not independent	**M1** **M1** **B1** **M1** **A1**	 5	Sensible attempt at $P(S)$ Sensible attempt at $P(T)$ Correct $P(S \cap T)$ comp $P(S) \times P(T)$ with $P(S \cap T)$ (their values), evaluated Correct conclusion following all correct working
(ii)	not exclusive since $P(S \cap T) \neq 0$ Or counter example e.g. 1 and 3 Or $P(S \cup T) \neq P(S) + P(T)$ with values	**B1**√	1	FT their $P(S \cap T)$, not obtained from $P(S) \times P(T)$, with value and statement.
4 (i)	$z = 1.127$ $1.127 = \frac{136 - 125}{\sigma}$ $\sigma = 9.76$	**B1** **M1** **A1**	 3	± 1.127 seen accept rounding to ±1.13 Standardising no cc no sq rt, with attempt at z (not ±0.8078, ±0.5517, ±0.13, ±0.87) Correct ans

(ii)	P(131<x<141)= P ($\frac{131-125}{9.76}$ <z< $\frac{141-125}{9.76}$) = Φ(1.639) – Φ(0.6147) = 0.9493 – 0.7307 = 0.2186 Number = 0.2186 × 170 = 37 or 38 or awrt 37.2	M1 M1 M1 A1 4	Standardising once with their sd, no √,², allow cc Correct area Φ2 – Φ1 Mult by 170, P<1 Correct answer, nfww
5 (a)	e.g. **(AAOOOI)***** $\frac{8!}{2!2!} \times \frac{6!}{2!3!} = 604800$	B1 M1 A1 3	8! (8 × 7!) or 6! seen anywhere, either alone or in numerator) Dividing by at least 3 of 2!2!2!3! (may be fractions added) Correct answer
(b)	C(7) E(6) A(4) 1 1 2 = 7 × 6 × $^4C_2$ = 252 1 2 1 = 7 × $^6C_2$ × 4 = 420 1 3 0 = 7 × $^6C_3$ × 1 = 140 2 1 1 = $^7C_2$ × 6 × 4 = 504 2 2 0 = $^7C_2$ × $^6C_2$ × 1 = 315 3 1 0 = $^7C_3$ × 6 × 1 = 210 Total = 1841	M1 A1 M1* DM1 A1 5	Mult 3 appropriate combinations together assume 6=$^6C_1$, 1=$^4C_0$ etc., $\sum r$=4, C&E both present At least 3 correct unsimplified products Listing at least 4 different correct options Summing at least 4 outcomes, involving 3 combs or perms, $\sum r$=4 Correct answer SC if CE removed, M1 available for listing at least 4 different correct options for remaining 2. DM1 for $^7C_1$×$^6C_1$×(sum of at least 4 outcomes)
6 (i)	fd 0.9, 3, 4.2, 5.2, 1.4 	M1 A1 B1 B1 4	Attempt at scaled freq [f/(attempt at cw)] Correct heights seen on diagram Scale no less than 1cm to 1 unit Correct bar widths visually no gaps Labels (ht/metres and fd or freq per 20 m etc.) and end points at 20.5 etc. condone 2 end point errors, scale no less than 1cm to 5m for 20,30… unless clearly accurate, linear scale between 20.5 and 80

(ii)	(30.5 × 18 + 43 × 15 + 48 × 21 + 55.5 × 52 + 70.5 × 28)/134	**M1**		Attempt at unsimplified, mid points (at least 4 within 0.5)
	$= \dfrac{7062}{134} = 52.701$	**M1** **A1**		Attempt at Σfx their mid points ÷ 134 Correct mean rounding to 53
	Var = $(30.5^2 \times 18 + 43^2 \times 15 + 48^2 \times 21 + 55.5^2 \times 52 + 70.5^2 \times 28)/134 - 52.701^2$ $= 392203.5/134 - 52.701^2 = 149.496$ sd = 12.2	**M1** **A1**	 5	Attempts at Σfx^2 their mid points ÷ their Σf – mean^2 Correct answer, nfww
7 (i)	P(0, 1, 2) = $(0.92)^{19} + {}^{19}C_1(0.08)(0.92)^{18} + {}^{19}C_2(0.08)^2(0.92)^{17}$ = 0.809	**M1** **M1** **A1**	 3	Binomial term ${}^{19}C_x p^x(1-p)^{19-x}$ seen $0<p<1$ Correct unsimplified expression Correct answer (no working SC B2)
(ii)	P(at least 1) = 1 – P(0) $= 1 - P(0.92)^n > 0.90$ $0.1 > (0.92)^n$ $n > 27.6$ Ans 28	 **M1** **M1** **A1**	 3	 Eqn with their 0.92^n, 0.9 or 0.1, 1 not nec Solving attempt by logs or trial and error, power eqn with one unknown power Correct answer, not approx., ≈, ⩾, >, ⩽, <
(iii)	np = 1800 × 0.08 = 144 npq = 132.48	**B1**		correct unsimplified np and npq seen accept 132.5, 132, 11.5, awrt 11.51
	P(at least 152) = $P\left(z > \left(\dfrac{151.5-144}{\sqrt{132.48}}\right)\right)$	**M1** **M1**		standardising, with √ cont correction 151.5 or 152.5 seen
	$= P(z > 0.6516)$ $= 1 - 0.7429$ $= 0.257$	**M1** **A1**	 5	correct area 1 – Φ (probability) correct answer
(iv)	Use because 1800 ×0.08 (and 1800 × 0.92 are both) > 5	**B1**	1	1800 ×0.08 > 5 is sufficient np>5 is sufficient if clearly evaluated in (iii) If npq>5 stated then award B0

CAMBRIDGE INTERNATIONAL EXAMINATIONS

Cambridge International Advanced Subsidiary and Advanced Level

MARK SCHEME for the May/June 2015 series

9709 MATHEMATICS

9709/61 Paper 6, maximum raw mark 50

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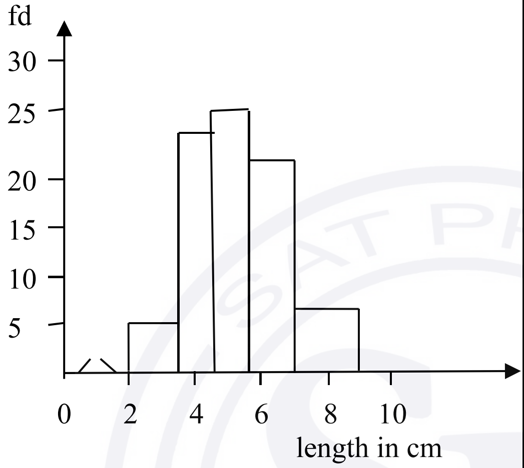
1	$P(x < 3.273) = 0.5 - 0.475 = 0.025$ $z = -1.96$ $\frac{3.2 - \mu}{0.714} = -1.96$ $\mu = 4.60$s	**M1** **A1** **M1** **A1 [4]**	Attempt to find z-value using tables in reverse ±1.96 seen Solving their standardised equation z-value not nec Correct ans accept 4.6
2 (i)	UQ 5.5 – 7.0 cm	**B1 [1]**	
(ii)	fd 5.33, 25, 28, 20.7, 6, 	**M1** **A1** **B1** **B1 [4]**	Attempt at fd or scaled freq [fr/cw] Correct heights seen on graph Correct bar widths no gaps Labels (fd and length/cm) and correct bar ends
3 (i)	$P(A) = \frac{1}{3} \times \frac{2}{3} + \frac{2}{3} \times \frac{1}{3} = \frac{4}{9}$ $P(B) = \frac{27}{36} = \frac{3}{4}$ $P(A \cap B) = \frac{12}{36} = \frac{1}{3}$ $P(A) \times P(B) = \frac{4}{9} \times \frac{3}{4} = \frac{1}{3}$ Independent as $P(A \cap B) = P(A) \times P(B)$	**M1** **M1** **B1** **M1** **A1 [5]**	Sensible attempt at P(A) Sensible attempt at P(B) correct P($A \cap B$) Cf P($A \cap B$) with P(A)×P(B) need at least 1 correct Correct conclusion following all correct working
(ii)	Not mutually exclusive because $P(A \cap B) \neq 0$ Or give counter example e.g. 1 and 6	**B1**√ **[1]**	ft their P($A \cap B$)
4 (i)	$(1 - x)0.9 + x \times 0.24 = 0.801$ $x = 0.15$	**M1** **A1** **A1 [3]**	Eqn with sum of two 2-factor probs = 0.801 Correct equation Correct answer

(ii)	P(⩾100 times given ⩽ 3 views) $\frac{P(\geqslant 100\text{ times} \cap \geqslant 3\text{ views})}{P(\geqslant 3\text{ views})} =$ $\frac{0.85\times 0.1}{0.85\times 0.1 + 0.15\times 0.76 \text{ or } 1-0.801}$ = 0.427	**B1** **M1** **A1** **A1 [4]**	0.85×0.1 seen on its own as num or denom of a fraction Attempt at P(⩾ 3 views) either ($0.85\times p_1 + 0.15\times p_2$) or 1 – 0.801 seen anywhere Correct unsimplified P(⩾ 3 views) as num or denom of a fraction Correct answer
5 (i)	new mean = $\frac{9\times 7.1 + 18\times 5.2}{27}$ = 5.83	**M1** **A1 [2]**	Mult by 9 and 18 and dividing by 27 correct answer
(ii)	$1.45^2 =$ so $\frac{\sum x_t^2}{9} = 472.6125\text{ mm}$ $0.96^2 = \frac{\sum x_g^2}{18} - 5.2^2$ so $\Sigma x_g^2 = 503.3088$ New sd^2 $\frac{472.6...^2 + 503.3...^2}{27} - 5.83...^2 = 2.117$ New sd = 1.46	**M1** **A1** **A1** **M1** **A1 [5]**	subst in a correct variance formula sq rt or not correct Σx_t^2 (rounding to 470) correct Σx_g^2 (rounding to 500) using $\Sigma x_t^2 + \Sigma x_g^2$, dividing by 27 and subt comb mean^2 correct answer
6 (i)	P(5, 6, 7) = $^8C_5(0.68)^5(0.32)^3 + {}^8C_6(0.68)^6(0.32)^2 + {}^8C_7(0.68)^7(0.32)$ = 0.722	M1 M1 A1 A1 **[4]**	Binomial term $^8C_x\, p^x(1-p)^{8-x}$ seen $0 < p < 1$ Summing 3 binomial terms Correct unsimplified answer Correct answer
(ii)	$np = 340$, $npq = 108.8$ $P(x > 337) = P\left(z > \frac{337.5 - 340}{\sqrt{108.8}}\right)$ $= P(z > -0.2396)$ = 0.595	B1 M1 M1 M1 A1 **[5]**	Correct (unsimplified) mean and var standardising with sq rt must have used 500 cc either 337.5 or 336.5 correct area (> 0.5) must have used 500 correct answer
(iii)	np (340) > 5 and nq(160) > 5	B1 **[1]**	must have both or at least the smaller, need numerical justification
7 (a) (i)	$\frac{9!}{2!2!3!}$ = 15120 ways	B1 B1 **[2]**	Dividing by 2!2!3! Correct answer

(ii)	********3 in $\frac{8!}{2!2!3!}$ = 1680 ways	B1	Correct ways end in 3
	********7 in $\frac{8!}{2!3!}$ = 3360 ways	B1	Correct ways end in 7
	Total even = 15120 – 1680 – 3360	M1	Finding odd and subt from 15120 or their **(i)**
	= 10080 ways	A1 **[4]**	Correct answer
	OR		
	********2 in 8!/2!3! = 3360 ways	B1	One correct way end in even
	********6 in 8!/2!2!3! = 1680 ways	B1	correct way end in another even
	********8 in 8!/2!2!2! = 5040ways	M1	Summing 2 or 3 ways
	Total = 10080 ways	A1	Correct answer
	OR		
	"15120" ×6/9 = 10080	M2 A2	Mult their **(i)** by 2/3 oe Correct answer
(b)	T(3) S(6) G(14)		
	1 1 3 in $3\times6\times{}^{14}C_3 = 6552$	M1	Mult 3 (combinations) together assume 6 = ${}^6C_1$etc
	1 3 1 in $3\times{}^6C_3\times14 = 840$		
	3 1 1 in $1\times6\times14 = 84$	M1	Listing at least 4 different options
	2 2 1 in ${}^3C_2\times{}^6C_2\times14 = 630$	M1	Summing at least 4 different options
	2 1 2 in ${}^3C_2\times6\times{}^{14}C_2 = 1638$		
	1 2 2 in $3\times{}^6C_2\times{}^{14}C_2 = 4095$	B1	At least 3 correct numerical options
	Total ways = 13839 (13800)	A1 **[5]**	Correct answer

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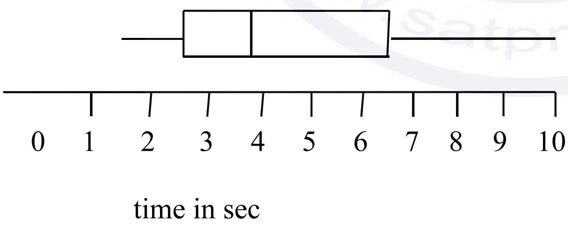
2	mid points 13, 30.5, 40.5, 50.5, 73 Mean = $\frac{4\times13 + 24\times30.5 + 38\times40.5 + 34\times50.5 + 20\times73}{120}$ $= \frac{5500}{120} = 45.8$ var = $\frac{4\times13^2 + 24\times30.5^2 + 38\times40.5^2 + 34\times50.5^2 + 20\times73^2}{120}$ $-(45.8\ldots)^2$ $= \frac{278620}{120} - 45.8\ldots^2$ $= 2321.8333 - 45.8\ldots^2$ sd = 14.9	M1 M1 A1 M1 A1 **5**	Attempt at midpoints at least 3 correct Using their midpoints i.e. cw, ucb, 1/2 cw and freqs into correct formula must be divided by 120 Correct answer from correct working Evaluating $\frac{\Sigma fx^2}{120}$ – their $\bar{x}^2$ must see their 45.8^2 subtracted allow cw etc Correct answer
3 (i)		B1 B1 √ B1 B1 **4**	LQ = 2.6 med = 3.8– 3.85, UQ = 6.4– 6.6 Correct quartiles and median on graph ft linear from 2–10 End whiskers correct not through box Label need seconds and linear 2–10 axis or can have 5 values on boxplot no line provided correct
(ii)	1.5 × IQR = 1.5 × 3.8 = 5.7 LQ – 5.7 = –ve, UQ + 5.7 = 12.1 i.e. > 10 So no outliers AG	M1 A1 **2**	Attempt to find 1.5 × IQR and add to UQ or subt from LQ OR compare 1.5 × IQR with gap 3.6 between UQ and max 10 Correct conclusion from correct working need both
4 (i)	$0.3\times0.72 + 0.7\times x = 0.783$ $x = 0.81$	M1 A1 A1 **3**	Eqn with sum of two 2-factor probs =0.783 Correct equation Correct answer

(ii)	P(S given not like) = $\dfrac{P(S \cap NL)}{P(NL)}$ $= \dfrac{0.3 \times 0.28}{0.3 \times 0.28 + 0.7 \times 0.19 \textit{ or } 1 - 0.783}$ = 0.387 (12/31)	B1 M1 A1 A1 **4**	0.3×0.28 seen on its own as num or denom of a fraction Attempt at P(NL) either (0.3 × p_1) + (0.7 × p_2) or 1 – 0.783 seen anywhere Correct unsimplified P(NL) as num or denom of a fraction Correct answer
5 (i)	P(2Es 1O) = $\frac{3}{5} \times \frac{2}{4} \times \frac{2}{3} \times {}^3C_2 = \frac{3}{5}$ (0.6) OR P(2Es 1O) = $\dfrac{{}^3C_2 \times {}^2C_1}{{}^5C_3} = \dfrac{6}{10}$ = 0.6 OR 241, 247, 261, 267, 461, 467 = 6 options 124 126 127 146 147 167 246 247 267 467 Prob = 6/10	M1 M1 A1 **3** M1 M1 A1 M1 M1 A1	5×4×3 seen in denom Mult a prob by ${}^3C_2$ oe Correct answer ${}^3C_x$ or ${}^yC_2$ or ${}^2C_1$ oe seen mult by $k \geqslant 1$ in num ${}^5C_3$ seen in denom Correct answer List at least 3 of 241, 247, 261, 267, 461, 467 ${}^5C_3$ or list to get all 10 options in denom see below Correct answer
(ii)	124 126 127 146 147 167 246 247 267 467 s: 1, 2, 4 P($S = s$): 6/10, 3/10, 1/10	M1 A1 B1 B1 B1 **5**	Attempt at listing with at least 7 correct All correct and no others or all 60 1, 2, 4 only seen in top row Any two correct All correct
6 (a) (i)	N*****B Number of ways = $\dfrac{5!}{3!}$ = 20	B1 B1 B1 **3**	5! Seen in num oe or alone mult by $k \geqslant 1$ 3! Seen in denom can be mult by $k \geqslant 1$ Correct final answer
(ii)	B(AAA)NNS Number of ways = $\dfrac{5!}{2!}$ or ${}^5P_3$ = 60	M1 M1 A1 **3**	5! seen as a num can be mult by $k \geqslant 1$ Dividing by 2! Correct final answer
(b)	${}^{14}C_9$ total options = 2002 T and M both in ${}^{12}C_7$ = 792 Ans 2002 – 792 = 1210 OR Neither in ${}^{12}C_9$ = 220 One in ${}^{12}C_8$ = 495 Other in ${}^{12}C_8$ = 495	M1 B1 A1 **3** M1 B1	${}^{14}C_9$ or ${}^{14}P_9$ in subtraction attempt ${}^{12}C_7$ (792) seen Correct final answer Summing 2 or 3 options at least 1 correct condone ${}^{12}P_9 + {}^{12}P_8 + {}^{12}P_8$ here only Second correct option seen accept another 495 or if M1 not awarded, any correct option

	total = 1210	A1	Correct final answer
7 (a) (i)	prob = $p\left(z < \frac{30-35.2}{4.7}\right)$ $= \text{P}(z < -1.106)$ $= 1 - 0.8655 = 0.1345$ $0.1345 \times 52 = 6.99$	M1 M1 A1 A1 **4**	Standardising no sq rt no cc no sq $1 - \Phi$ Correct ans rounding to 0.13 Correct final answer accept 6 or 7 if 6.99 not seen but previous prob 0,1345 correct
(ii)	$\Phi(t) = 0.648 \quad z = 0.380$ $0.380 = \frac{t-35.2}{4.7}$ $t = 37.0$	B1 M1 A1 **3**	0.648 seen standardising allow cc, sq rt,sq, need use of tables not 0.148, 0.648, 0.352, 0.852 correct answer rounding to 37.0
(b)	$\frac{7-\mu=-0.8\sigma}{\sigma}$ so $7-\mu=-0.8\sigma$ $\frac{10-\mu}{\sigma}=0.44$ so $10-\mu=0.44\sigma$ $\mu = 8.94 \qquad \sigma = 2.42$	B1 B1 M1 M1 A1 **5**	± 0.8 seen ± 0.44 seen An eqn with z-value, μ and σ no sq rt no cc no sq Sensible attempt to eliminate μ or σ by subst or subtraction, need at least one value Correct answers

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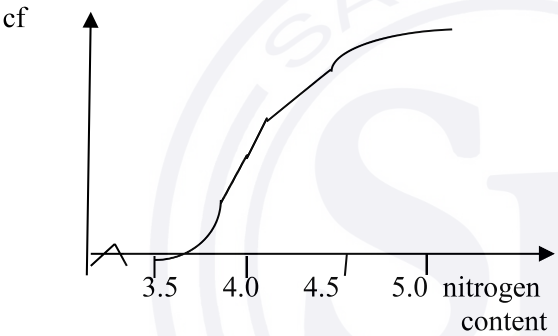
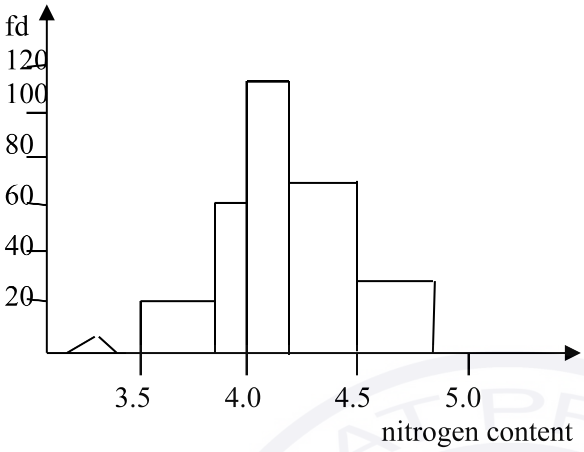
<table>
<tr><td>1</td><td>$z = 1.136$
$1.136 = \dfrac{195-\mu}{22}$
$\mu = 170$</td><td>B1
M1
A1 [3]</td><td>± 1.136 seen, not ±1.14,
Standardising, no cc no sq rt, equated to their z not 0.128 or 0.872
Correct answer, nfww</td></tr>
<tr><td>2 (i)</td><td><table><tr><td></td><td>Kitchen mess</td><td>Kitchen not mess</td><td>Total</td></tr><tr><td>On time</td><td>1/10</td><td>1/10</td><td></td></tr><tr><td>Not on time</td><td>1/2</td><td></td><td>4/5</td></tr><tr><td>Total</td><td>3/5</td><td>4/10</td><td></td></tr></table></td><td>B1
B1
B1 [3]</td><td>All values may be decimals or %
2 probabilities correct
2 further probabilities correct
2 further probabilities correct</td></tr>
<tr><td>(ii)</td><td>P(not on time given kitchen mess) = $\dfrac{1/2}{3/5}$
= 5/6 o.e.</td><td>M1
A1 [2]</td><td>A cond prob fraction seen (using corresponding combined outcomes and total)
FT from their values, 3sf or better, <1, 3/5ft<1</td></tr>
<tr><td>3</td><td>$\mu = 300\times0.072 = 21.6,\ \sigma^2 = 20.0448$
$P(x < 18) = P\left(z < \dfrac{17.5-21.6}{\sqrt{20.0448}}\right)$
$= P(z < -0.9157)$
$= 1 - 0.8201$
$= 0.180$</td><td>B1
M1
M1
M1
A1 [5]</td><td>300×0.072 seen and 300×0.072×0.928 seen or implied ($\sigma = 4.4771, \sigma^2 = 20(.0)$) oe
±Standardising, their mean/var, with sq root
Cont corr 17.5 or 18.5
Correct area 1 - Φ
Answer wrt 0.180, nfww</td></tr>
<tr><td>4 (i)</td><td>P(1 W) = 6/9×3/8 + 3/9×6/8
= ½ AG
OR $\dfrac{{}^6C_1 \times {}^3C_1}{{}^9C_2}$
= ½ AG</td><td>M1
A1 [2]
M1
A1</td><td>summing 2 two-factor probs (condone replacement) not ½×½ + ½×½
Correct answer, fully justified
Using combinations consistent, correct format
Correct answer, fully justified</td></tr>
<tr><td>(ii)</td><td>$P(\overline{W},\overline{W})$ = 3/9×2/8 = 6/72 (1/12)
$P(W,W)$ = 6/9×5/8 = 30/72 (5/12)
<table><tr><td>x</td><td>0</td><td>1</td><td>2</td></tr><tr><td>Prob</td><td>1/12</td><td>1/2</td><td>5/12</td></tr></table></td><td>B1
B1
B1√ [3]</td><td>Distribution table with 0,1,2 only
$P(W,W)$ or $P(\overline{W},\overline{W})$ correct
$P(W,W) + P(\overline{W},\overline{W}) = 0.5$</td></tr>
<tr><td>(iii)</td><td>E(X) = 16/12 (4/3) (1.33) isw</td><td>B1 [1]</td><td>Condone 1(.3) if correct working seen, nfww</td></tr>
</table>

Question	Answer	Marks	Guidance
5 (i)	P(large) = $1 - \Phi\left(\frac{29-21.7}{6.5}\right)$ $= 1 - \Phi(1.123) = 1 - 0.8692$ $= 0.1308$ $P(0,1) = (0.8692)^{8} + {}^8C_1(0.1308)(0.8692)^7$ $= 0.718$	M1 M1 A1 M1 M1 A1 [6]	Standardising no cc no sq rt Correct area $1 - \Phi$ Rounding to 0.13 Any bin term with ${}^8C_x p^x(1-p)^{8-x}$ $0 < p < 1$ Summing bin P(0) + P(1) only with $n = 8$, oe Correct ans
(ii)	$= 1 - (0.8692)^n > 0.98$ $(0.8692)^n < 0.02$ Least number = 28	M1 M1 A1 [3]	eq/ineq involving their $(0.8692)^n$ or $(0.1308)^n$, 0.02 or 0.98 oe with or without a 1 solving attempt (could be trial and error) – may be implied by their answer correct answer
6 (i)		B1 M1 A1 [3]	Uniform axes cf and nitrogen content labelled, at least 0 to 70 and 3.5 to 4.8 seen 5 points plotted correctly on graph paper 3.5, 3.8, 4.0, 4.2, 4.5, 4.8 0, 6, 18, 41, 62, 70 All points correct and a reasonable curve (condone 1 missed point) or line segments.
(ii)	70 – their 55 = 15 = 21.4%	M1 A1 [2]	Subt a value > 41 from 70 (or n/70, n<29) Correct ans, accept 18.5 – 22
(iii)	median = 4.15	B1 [1]	Accept 4.1< median < 4.2, nfww

<table>
<tr><td>(iv)</td><td>
<table>
<tr><td>nit cont</td><td>3.5–3.8</td><td>3.8–4.0</td><td>4.0–4.2</td><td>4.2–4.5</td><td>4.5–4.8</td></tr>
<tr><td>fr</td><td>6</td><td>12</td><td>23</td><td>21</td><td>8</td></tr>
<tr><td>fd</td><td>20</td><td>60</td><td>115</td><td>70</td><td>26.7</td></tr>
</table>


</td><td>M1

M1

A1

B1

B1 [5]</td><td>Attempt at freqs, at least 3 correct, ignore labelling

Attempt at fd as f/cw only at least 3 correct FT
(Accept f/cw × k)

Correct heights seen on graph (plot at 4.8,27 A0)
Graph paper must be used
(3 correct relative heights implies M1M1)

Correct bar ends seen on graph – graph paper used

Correct linear scale and labels.</td></tr>
<tr><td>7 (i)</td><td>W S D
1 1 $3 = 6\times4\times{}^3C_3 = 24$
1 3 $1 = 6\times{}^4C_3\times3 = 72$

3 1 $1 = {}^6C_3\times4\times3 = 240$
1 2 $2 = 6\times{}^4C_2\times{}^3C_2 = 108$
2 1 $2 = {}^6C_2\times4\times{}^3C_2 = 180$

2 2 $1 = {}^6C_2\times{}^4C_2\times3 = 270$

Total = 894</td><td>M1
M1

M1

B1

A1 [5]</td><td>Listing at least 4 different options
Mult 3 (combs) together assume $6 = {}^6C_1$, $\Sigma r=5$
Summing at least 4 different evaluated/unsimplified options >1

At least 3 correct unsimplified options
Correct answer</td></tr>
<tr><td>(ii)</td><td>${}^3P_2 \times {}^{10}P_8$

$= 10886400$</td><td>B1

B1

B1 [3]</td><td>${}^3P_2$ oe seen multiplied either here or in (iii)
$k\,{}^{10}P_x$ seen or $k\,{}^yP_8$ with no addition, $k\geqslant1$, $y>8$, $x<10$
Correct answer, nfww</td></tr>
<tr><td>(iii)</td><td>DSWSWSWSWD or DWSWSWSWSD
D in ${}^3P_2$ ways = 6

S in ${}^4P_4$ ways = 24
W in ${}^6P_4 = 360$
Swap SW in 2 ways
Total = 103680 ways</td><td>

B1

B1
B1 [3]</td><td>If ${}^3P_2$ has not gained credit in (ii) may be awarded
${}^4P_4$ or ${}^6P_4$ oe seen multiplied or common in all terms (no division)

Mult by 2 (condone 2!)
Correct answer, 3sf or better, nfww</td></tr>
</table>

CAMBRIDGE INTERNATIONAL EXAMINATIONS

Cambridge International Advanced Subsidiary and Advanced Level

MARK SCHEME for the October/November 2014 series

9709 MATHEMATICS

9709/61 Paper 6, maximum raw mark 50

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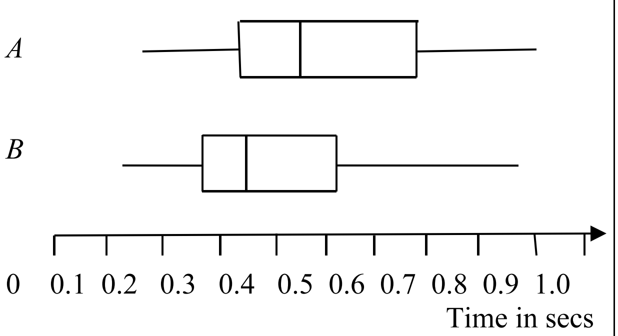
1	mean = $(5 + (-2) + 12 + 7 + (-3) + 2 + (-6) + 4 + 0 + 8) / 10$ $= 2.7$ var $= (5^2 + (-2)^2 + \ldots + 8^2) / 10 - 2.7^2 = 35.1 - 2.7^2$ $= 27.8$	B1 M1 A1	 **3**	 Subst in correct var formula must have $-$ mean^2 Correct answer
2	**(i)** $0.24 + 0.35 + 2k + k + 0.05 = 1$ $k = 0.12$	M1 A1	 **2**	Summing probs = 1 Correct answer
	(ii) model number is 1	B1	**1**	
	(iii) mean $= 1 \times 0.35 + 2 \times 0.24 + 3 \times 0.12 + 4 \times 0.05$ $P(>1.39) = P(2, 3, 4) = 0.41$	B1 M1 B1	 **3**	1.39 seen Finding $P(X >$ their mean) Correct ans following mean or mode only
3	$P(8) = P(H\ 4\ 4) + P(T\ 2\ 4) + P(T\ 4\ 2)$ $= \frac{1}{3}\times\frac{1}{16}+\frac{2}{3}\times\frac{1}{16}+\frac{2}{3}\times\frac{1}{16}$ $= \frac{5}{48}$ $P(H \mid 8) = \frac{P(H \cap 8)}{P(8)}$ $= \frac{\frac{1}{48}}{\frac{5}{48}} = \frac{1}{5}$	M1 M1 A1 B1 A1	 **5**	$\frac{1}{3}$ or $\frac{2}{3}$ mult by dice related prob, seen anywhere Summing two or three 2-factor probs involving $\frac{1}{3}$ and $\frac{2}{3}$ $\frac{5}{48}$ oe seen as num or denom of a fraction $\frac{1}{48}$ oe seen as num or denom of a fraction Correct ans
4	**(i)** median A = 0.52 LQ = 0.41 UQ = 0.79	B1 B1 B1ft	 **3**	 ft wrong units
	(ii) 	B1 B1 B1	 **3**	2 correct boxes ft (i) OK if superimposed 2 pairs correct whiskers lines up to box not inside Correct uniform scale need at least 4 values on it. No scale no marks unless perfect A and B with all 10 values shown, in which case score B1B1B0

	(iii)	Smartphone B is quicker, slightly less variable, etc.	B1	**1**	oe sensible answer
5	**(i)**	$1.2 = 15p \quad p = 0.08$ Var $= npq = 15 \times 0.08 \times 0.92 = 1.104$ **AG**	M1 A1	 **2**	Attempt to find p using $1.2 = 15p$ Correct answer
	(ii)	$P(0, 1, 2) = (0.92)^{15} + {}^{15}C_1(0.08)(0.92)^{14}$ $+ {}^{15}C_2(0.08)^2(0.92)^{13}$ $= 0.887$	M1 M1 A1	 **3**	Binomial expression ${}^{15}C_x p^x(1-p)^{15-x} \quad 0 < p < 1$ Correct unsimplified expression for P(0, 1, 2) Correct answer
	(iii)	P(at least 1 faulty screw) $= 1 - P(0) = 1 - (0.92)^{15}$ $= 0.7137\ldots$ P(at least 1 faulty screw in 7 packets) $=$ ${}^8C_7(0.713\ldots)^7(0.2863\ldots)$ $= 0.216$	M1 A1 M1 A1	 **4**	Attempt at P(0) or 1 – P(0) Rounding to 0.71 Binomial expression ${}^8C_7p^7(1-p) \quad 0 < p < 1$ Correct answer
6	**(i)**	$z_1 = \dfrac{70 - 66.4}{5.6} = 0.6429$ $z_2 = \dfrac{72.5 - 66.4}{5.6} = 1.089$ $\Phi(1.089) - \Phi(0.643) = 0.8620 - 0.7399$ $= 0.1221$ $0.1221 \times 250 = 30.5$ 30 or 31 sheep	M1 M1 A1 M1 A1ft	 **5**	Standardising one variable, no cc, no sq rt Correct area $\Phi_2 - \Phi_1$ Correct answer rounding to 0.12 Mult by 250 Correct answer ft their 0.1221
	(ii)	$66.4 - 59.2 = 7.2$ $66.4 + 7.2 = 73.6$	M1 A1	 **2**	Subt from 66.4 Correct answer
	(iii)	$z = 0.674$ $\dfrac{67.5 - \mu}{4.92} = 0.674$ $\mu = 64.2$	B1 M1 A1	 **3**	$\pm$ 0.674 or 0.675 seen Standardising with a z-value no cc no sq rt Correct answer
7	**(i)**	W(8) M(5) 4 2 $= {}^8C_4 \times {}^5C_2 = 700$ 5 1 $= {}^8C_5 \times {}^5C_1 = 280$ 6 0 $= {}^8C_6 \times {}^5C_0 = 28$ Total = 1008	 M1 M1 A1 A1	 **4**	 Mult 2 combs, ${}^8C_x \times {}^5C_y$ Summing 2 or 3 options 2 correct options unsimplified Correct answer
	(ii)	M1 and MMWWW $= {}^3C_2 \times {}^8C_3 = 168$ M2 and MMWWW $= {}^3C_2 \times {}^8C_3 = 168$ Neither and MMMWWW $= {}^3C_1 \times {}^8C_3 = 56$ Total = 392 OR total, no restrictions $= {}^5C_3 \times {}^8C_3 = 560$ M1M2 and MWWW $= {}^3C_1 \times {}^8C_3 = 168$ $560 - 168 = 392$	M1 B1 A1 M1 B1 A1	 **3**	Summing 3 options One correct option Correct answer Subt 2 men together from no restrictions One correct of 560 or 168 Correct answer

(iii)	e.g. WWMWWW = 5! (women) × 4 = 480	M1 M1 A1	 **3**	5! Seen mult by integer ⩾ 1 Mult by 4 Correct answer
	OR 6! – MWWWWW – WWWWWM = 6! – 5! – 5! = 480	M1 M1 A1		6! seen with a subtraction 5! or 2 × 5! Seen subtracted Correct answer

CAMBRIDGE INTERNATIONAL EXAMINATIONS

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9709/62 Paper 6, maximum raw mark 50

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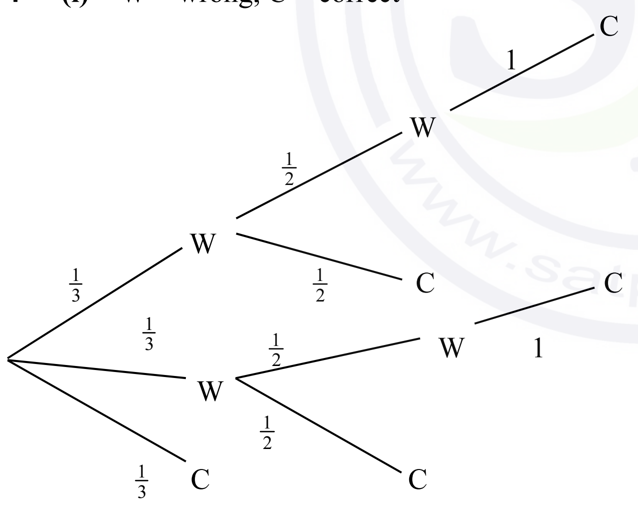
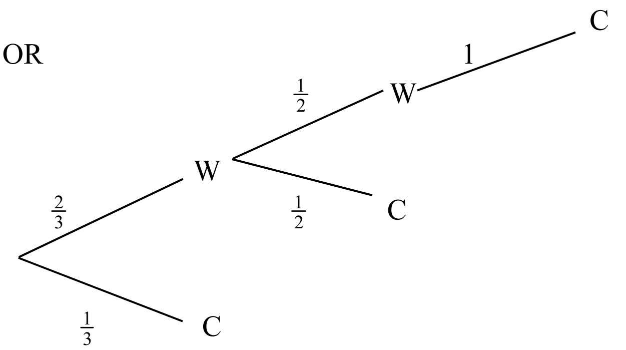
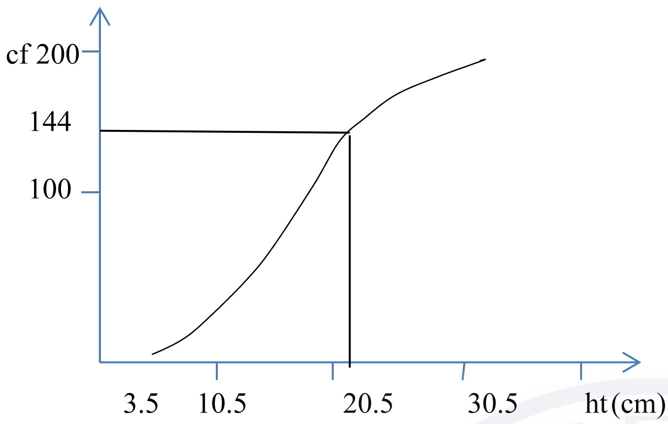
1 $^{48}C_{43}$ = 1712304 (1710000)	B1 B1 B1 **3**	48 seen in a single term combination oe 43 or 5 seen in a single term combination oe Both can be mult by integer $k \geqslant 1$ Correct final answer
2 **(i)** 6! ×5! = 86400 **(ii)** 6! × 7 × 6 × 5 × 4 = 604800	B1 B1 B1 **3** B1 B1 B1 **3**	6! oe seen multiplied by integer $k \geqslant 1$ 5! oe seen multiplied by integer $k \geqslant 1$ Correct final answer 6! seen mult by integer $k \geqslant 1$ Mult by $^7P_4$ oe Correct final answer
3 **(i)** 1 1 1 2 or 1 1 2 1 or 1 2 1 1 or 2 1 1 1 Prob = $\frac{1}{6}\times\frac{1}{6}\times\frac{1}{6}\times\frac{1}{6}\times 4$ $= \frac{1}{324}$ (0.00309) **(ii)** P(1,2) = $^7C_1 \times (1/324)\ (323/324)^6 +$ $^7C_2(1/324)^2(323/324)^5$ = 0.0214	M1 M1 A1 **3** M1 M1 M1 A1 **4**	One of 1 1 1 2 seen Mult a prob by 4 or $(\frac{1}{6})^4$ × integer $k \geqslant 1$ seen Correct answer Bin term $^7C_x p^x (q)^{7-x}$, $0.99 \leqslant p + q \leqslant 1$ Using their p from **(i)** in a bin term Correct unsimplified answer Correct answer
4 **(i)** W = wrong, C = correct  OR 	M1 M1 B1 A1 **4** M1 M1 B1 A1	3 branches first qn and 2 by 2 for second qn only One branch twice for third qn or two branches twice with 0 and 1 seen on branches Any two of $\frac{1}{3}$, $\frac{1}{2}$ and 1 seen as probs Probs all correct and sensible labels NB SR for 4 outcomes instead of 3, M1 B1 only 2 branches first qn and 1 by 2 for second qn only One branch once for third qn or two branches with 0 and 1 seen on branches Any two of $\frac{1}{3}$ or $\frac{2}{3}$, $\frac{1}{2}$ and 1 seen as probs Probs all correct and sensible labels

(ii)		
<table><tr><td>x</td><td>1</td><td>2</td><td>3</td></tr><tr><td>Prob</td><td>$\frac{1}{3}$</td><td>$\frac{1}{3}$</td><td>$\frac{1}{3}$</td></tr></table>	B1 B1	1, 2, 3 seen only oe 2 correct probs
P(1) = P(C) say $= \frac{1}{3}$ P(2) = P(WC) = $\frac{1}{6}$ P(WC) = $\frac{1}{6}$ total P (2) $= \frac{1}{3}$ P(3) = P(WWC) = $\frac{1}{6}$ P(WWC) = $\frac{1}{6}$ total P(3) = $\frac{1}{3}$	B1	3 correct probs
$E(X) = 1 \times \frac{1}{3} + 2 \times \frac{1}{3} + 3 \times \frac{1}{3} = 2$	B1√ **4**	Correct answer ft their probs provided $0.999 \leqslant \Sigma p \leqslant 1$
5 (a) (i) $P(x < 8) = P\left(z < \frac{8 - 7.15}{0.88}\right)$ $= \Phi(0.9659)$ $= 0.833$	M1 A1 **2**	Standardising ± , no cc no sq rt no sq Correct answer
(ii) $z = 0.674$ $\frac{q - 7.15}{0.88} = 0.674$ $q = 7.74$	B1 M1 A1 **3**	Accept ± 0.674 or 0.675 only Standardised eqn = ± their z-value, allow sq or sq rt if already penalised in **(i)** Correct answer
(b) $P(Y > 4\mu) = P\left(z > \left(\frac{4\mu - \mu}{(3\mu/2)}\right)\right) = P(z > 2)$ $= 1 - 0.9772$ $= 0.0228$	M1 A1 A1 **3**	Standardising no sq rt, no cc, no sq, one variable $z = \pm 2$ seen correct ans SR B1 if made-up values used and 0.0228 obtained

6 (i) | ht | <10.5 | <15.5 | <20.5 | <25.5 | <30.5 | | CF | 22 | 54 | 132 | 172 | 200 |	B1	At least 4 CFs correct seen on graph
	B1	Labels correct, i.e. all of ht, cm, cf
	M1	Attempt at upper end points either 10 or 10.5 or 11 at least 4 upper end points
	A1 **4**	All correct, i.e. points joined up from (3.5, 0) to (10.5, 22)….to (30.5, 200) Straight lines or curve
(ii) 72% less, i.e. 144 less than ht *h*. $h = 22.5$ cm	M1 A1 **2**	144 **used** can be implied single value in range 21 to 23 inclusive
(iii) var = $(7^2 \times 22 + 13^2 \times 32 + 18^2 \times 78 + 23^2 \times 40 + 28^2 \times 28)/200 - 18.39^2$ $= 74870/200 - 18.39^2$ $= 374.35 - 18.39^2$ $= 36.1579$	M1	Using mid points attempt 7 ± 0.5in correct var formula incl – mean^2
	B1	At least 4 correct midpoints
sd = 6.01	A1 **3**	Correct ans
7 (i) P (4, 5, 6) = $(0.75)^4(0.25)^4 \times {}^8C_4 + (0.75)^5(0.25)^3 \times {}^8C_5 + (0.75)^6(0.25)^2 \times {}^8C_6$	M1	Bin term $p^r(1-p)^{8-r} \times {}^8C_r$ seen any p
	M1	Correct unsimplified answer
= 0.606	A1 **3**	Correct ans
(ii) $np = 160 \times 0.75 = 120$ $\quad npq = 30$	B1	Unsimplified mean and var correct
$P(>114) = P\left(z > \left(\dfrac{114.5-120}{\sqrt{30}}\right)\right)$ $= P(z > -1.004)$	M1 M1 M1	Standardising, need sq rt Cont correction either 114.5 or 113.5 Correct area consistent with their working
$= \Phi(1.004) = 0.842$	A1 **5**	Correct ans
(iii) np and nq both > 5	B1 **1**	Need both

CAMBRIDGE INTERNATIONAL EXAMINATIONS

Cambridge International Advanced Subsidiary and Advanced Level

MARK SCHEME for the October/November 2014 series

9709 MATHEMATICS

9709/63 Paper 6, maximum raw mark 50

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International Examinations

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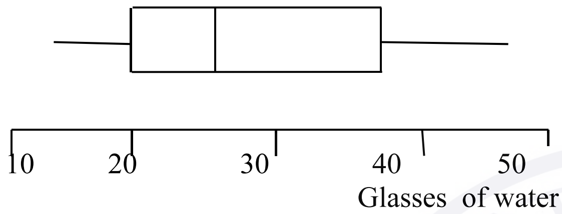
1	$z = -2.326$ $\dfrac{250-260}{\sigma} = -2.326$ $\sigma = 4.30$	B1 M1 A1 **3**	± 2.325 to ± 2.33 seen Standardising and = or < their z, no cc, sq, sq rt Correct ans
2	**(i)** $0.7 - 2.4 + 2.2 - 0.5 + 6.3 + 4.9 + 0 + 0.3$ $= 11.5$	 B1 **1**	
	(ii) $(0.7^2 + 2.4^2 + 2.2^2 + 0.5^2 + 6.3^2 + 4.9^2 + 0.3^2)$ $= 75.13\ (75.1)$	 B1 **1**	
	(iii) mean = 63.4375 Variance $= 75.13/8 - (11.5/8)^2$ $= 7.32$	B1√ M1 A1 **3**	ft 62 + their (i)/8 their(ii)/8 – $((\text{i})/8)^2$ correct answer
	OR mean = 507.5/8 = 63.4375 Var $= 32253/8 - 63.4375^2 = 7.32$	B1 M1 A1	 subst in correct variance or standard deviation formula correct answer – allow 6.62, 6.93–7.04, 7.260–7.325 Marks can be awarded in (i) or (ii) if not 'contradicted' by further working
3	**(i)** max = 12 $P(12) = (0.7)^{12} = 0.0138$	B1 B1 **2**	(Implied by P(12) with power 12) Accept 0.014
	(ii) P(fewer than 10) = 1– P (10, 11, 12) $= 1 - {}^{12}C_{10} \times (0.7)^{10}(0.3)^2 - 12 \times (0.7)^{11}(0.3) - (0.7)^{12}$ $= 1 - 0.2528$ $= 0.747$	M1 A1 A1 **3**	Binomial term ${}^{12}C_r(0.7)^r(0.3)^{12-r}$ or ${}^{12}C_r(p)^r(q)^{12-r}$, $0.99 \leqslant p + q \leqslant 1.00$ Correct unsimplified expression oe Correct answer

4 **(i)** Stem | leaf 1 | 4 5 7 8 9 9 2 | 1 2 2 3 4 5 6 6 8 8 3 | 0 2 6 8 4 | 1 2 5 6 7	B1 B1		Correct stem (or reversed order) Correct leaves, ordered in numerical sequence, with ½ 'column' tolerance
Key 1|4 represents 14 glasses (of water)	B1	3	Key must include 'glasses' or similar drinking item
(ii) LQ = 20 Med = 26 UQ = 37  	B1 B1 B1√ B1 B1	 5	Correct median Correct quartiles Correct on diagram ft any wrong med or quartiles. Linear scale based upon 3 quartiles plotted Correct end points of attached whiskers not through box Linear axis, label, both must be seen
SC No values stated 3 quartiles on diagram in correct relative positions End points of attached whiskers not through box correct relative to quartiles	B2 B1		
5 **(i)** $\mathrm{P}(<1.2)=\mathrm{P}\left(z<\frac{1.2-1.9}{0.55}\right)=\mathrm{P}(z<-1.2727)$ $= 1-\Phi(1.273) = 1-0.8986$ $= 0.1014$	M1 A1		Standardising for wt 1.2 or 2.5, no cc, sq, sq rt May be awarded in (ii) if not attempted in (i) Accept 0.102 First correct proportion seen
$\mathrm{P}(>2.5)=\mathrm{P}\left(z<\frac{2.5-1.9}{0.55}\right)=\mathrm{P}(z>1.0909)$ $= 1-\Phi(1.0909) = 1-0.8623$ $= 0.138$	A1		Second correct proportion seen
P (1.2 < *wt* < 2.5) = 1 – 0.101 – 0.138	M1		Third proportion 1 – their previous 2 proportions or correct attempt for remaining proportion
= 0.761	A1√	5	Correct answer or 1 – *their* 2 previous correct proportions
(ii) $\mathrm{P}(x > k) = 0.8 + 0.1377 = 0.9377$ $z = -1.536$	M1 A1		Valid method to obtain $\mathrm{P}(x > k)$ or $\mathrm{P}(x < k)$ ± 1.536 seen accept 3sf rounding to 1.53 or 1.54
$-1.536 = \frac{k-1.9}{0.55}$	M1		Attempt to solve equation with their 'correct' area z value, *k,* 1.9 and 0.55
$k = 1.06$	A1	4	Correct answer or rounding to 1.05

6 **(a)** 1*****3 or 3*****1 or 2*****2 $= 6^5 \times 3$ = 23328	M1 M1 A1	 **3**	Mult by 6^5 (for middle 5 dice outcomes) Mult by 3 or summing 3 different combinations (for end dice outcomes) Correct answer accept 23 300
(b) W J H 1 1 $7 = {}^9C_1 \times {}^8C_1 \times 1 = 72$ 1 7 $1 = {}^9C_1 \times {}^8C_7 \times 1 = 72$ 7 1 $1 = {}^9C_7 \times {}^2C_1 \times 1 = 72$ 1 3 $5 = {}^9C_1 \times {}^8C_3 \times 1 = 504$ mult by 3! 3 3 $3 = {}^9C_3 \times {}^6C_3 \times 1 = 1680$ Total 4920 If no marks gained Listing all 10 different outcomes	M1 A1 A1 M1 M1 A1 SCM1	 **6**	Multiplying 3 combinations (may be implied) 1 unsimplified correct answer (72, 504, 1680, 216 or 3024) A 2nd unsimplified different correct answer Summing options for 1,1.7 or 1,3,5 oe (mult by 3 or 3!) Summing at least 2 different options of the 3 Correct ans If games replaced M1M1M1 max available If factorials used M0M1M1 max available
7 **(a)** **(i)** $P(X = 3) = P(GRR) + P(RGR)$ $\frac{2}{4} \times \frac{2}{3} \times \frac{1}{2} + \frac{2}{4} \times \frac{2}{3} \times \frac{1}{2}$ $\frac{1}{3}$ **AG**	M1 M1 A1	 **3**	Mult 3 probs Summing 2 options Correct working with appropriate justification and fraction sequencing
(ii) <table><tr><td>X</td><td>2</td><td>3</td><td>4</td></tr><tr><td>Prob</td><td>$\frac{1}{6}$</td><td>$\frac{1}{3}$</td><td>$\frac{1}{2}$</td></tr></table>	B1		Values 2, 3, 4 only in table Condone X=0,1 if P(X)=0 stated
$P(X = 2) = P(RR) = \frac{2}{4} \times \frac{1}{3} = \frac{1}{6}$	B1		One correct prob other than (i)
$P(X = 4) = 1 - \left(\frac{1}{6} + \frac{1}{3}\right) = \frac{1}{2}$ Or P(GGRR) + P(RGGR) + P(GRGR) $= \left(\frac{2}{4} \times \frac{1}{3} \times \frac{2}{2} \times \frac{1}{1}\right) \times 3 = \frac{1}{2}$	B1√	**3**	Second correct prob ft 1 – their previous 2 probs

(iii) P(3 orange | at least 2 O) = $\dfrac{P(3O)}{P(at\ least\ 2O)}$			
P(3 orange) = P(OOO)	M1		Atttempt at P(OOO) one three-factor option, not added
$= \frac{5}{7}\times\frac{4}{6}\times\frac{3}{5}=\frac{2}{7}$	A1		Correct unsimplified num of a fraction
P(at least 2O) = P(YOO) + P(OYO) + P(OOY) + $\frac{2}{7}$			
$=\frac{2}{7}\times\frac{5}{6}\times\frac{4}{5}+\frac{5}{7}\times\frac{2}{6}\times\frac{4}{5}+\frac{5}{7}\times\frac{4}{6}\times\frac{2}{5}+\frac{2}{7}$	M1		Attempt at P(at least 2O) sum 3 or 4 three-factor options
$=\frac{6}{7}$	A1		Correct unsimplified answer seen anywhere
P(30 | at least 2O) = $\frac{2}{7}\div\frac{6}{7}=\frac{1}{3}(0.333)$	A1	5	Correct answer evaluated
<u>Alternative 1</u> 3 Orange = ${}^5C_3$	M1		Attempt at combinations for 3 orange oe, not added
	A1		Correct unsimplified num of a fraction
At least 2 Orange = ${}^5C_2 \times {}^2C_1 + {}^5C_3$	M1		Attempt at combinations for at least 2 orange condone omission of $+{}^5C_3$
	A1		Correct unsimplified answer seen anywhere
P(3O | at least 2O) = $\dfrac{{}^5C_3}{{}^5C_2\times{}^2C_1+{}^5C_3}=\dfrac{1}{3}$	A1	5	Correct answer evaluated
<u>Alternative 2</u> No Yellow = ${}^2C_0$	M1		Attempt at combinations for 0 yellow oe, not added
	A1		Correct unsimplified num of a fraction
No more than 1 Yellow = ${}^2C_1 + {}^2C_0$	M1		Attempt at combinations for no more than 1 yellow. Condone omission of +2C0
	A1		Correct unsimplified answer seen anywhere
P(3O | at least 2O) = $\dfrac{{}^2C_0}{{}^2C_1+{}^2C_0}=\dfrac{1}{3}$	A1	5	Correct answer evaluated
<u>Misread – with replacement</u> MR–1 applied to first Accuracy Mark earned	M1		Attempt at P(OOO) one three factor option oe not added
P(3O) = $\frac{5}{7}\times\frac{5}{7}\times\frac{5}{7}=\frac{125}{343}$	A1		Correct unsimplified num of a fraction
P(at least 2O) = $\frac{5}{7}\times\frac{5}{7}\times\frac{2}{7}\times{}^3C_2+\left(\frac{5}{7}\right)^3$	M1		Attempt at P(at least 2O) sum of 3 or 4 three factor options
	A1		Correct unsimplified seen anywhere
P(3O | at least 2O) = $\frac{5}{11}$	A1	4 max	Answer evaluated

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GCE Advanced Subsidiary Level and GCE Advanced Level

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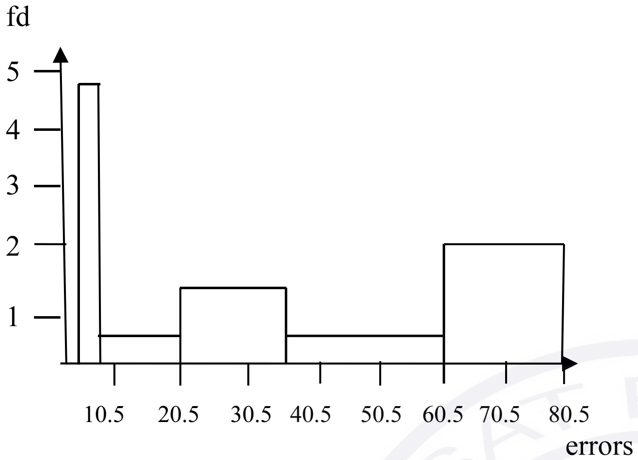
1 $P(21.6 < x < 28.7)$			
$= P\left(\left(\frac{21.6-24}{4.7}\right) < z < \left(\frac{28.7-24}{4.7}\right)\right)$	M1 A1		Standardising; no cc, no sq rt One rounding to Φ(0.841 or 0.695)
$= P(-0.5106 < z < 1) = \Phi(1) - \Phi(-0.5106)$	M1		$\Phi_1 + \Phi_2 - 1$
$= 0.8413 - (1 - 0.6953)$			
$= 0.537$ (0.5366)	A1	**4**	Correct answer
2 $1.751 = \frac{12-\mu}{\sigma}$	B1		Rounding to ±1.75 seen
$0.468 = \frac{9-\mu}{\sigma}$	B1		±0.468 seen
	M1		An eqn with a z-value, μ and σ no $\sqrt{\sigma}$, no σ^2
$\sigma = 2.34$	M1		Sensible attempt to eliminate μ or σ by substitution or subtraction, need a value
$\mu = 7.91$	A1	**5**	correct answers
3 **(i)** constant / given p, independent trials, fixed / given no. of trials, only two outcomes	B1 B1	**2**	Any one correct Any 3 correct
(ii) $P(x \geqslant 3) = 1 - P(0, 1, 2)$	M1		Any binomial expression $p^r(1-p)^{18-r}\,{}^{18}C_r$ seen
$= 1 - [(0.85)^{18} + (0.85)^{17}(0.15) \times 18 + (0.85)^{16}(0.15)^2 \times {}^{18}C_2]$	M1		$1 - P(0, 1, 2)$, any n,p,q
$= 0.520$	A1	**3**	Correct answer
4 **(i)** $P(\text{exactly } 2) = \frac{{}^6C_2}{{}^8C_4} = \frac{15}{70} = \frac{3}{14}$ AG	M1		${}^6Cx / {}^8Cx$ seen or ${}^4C_2$ mult by 4 fractions (last 2 can be implied)
OR $P(2) = \frac{6}{8} \times \frac{5}{7} \times \frac{2}{6} \times \frac{1}{5} \times {}^4C_2 = \frac{3}{14}$ AG	A1	**2**	Answer legit obtained
(ii) <table><tr><td>x</td><td>2</td><td>3</td><td>4</td></tr><tr><td>Prob</td><td>3/14</td><td>8/14</td><td>3/14</td></tr></table>	B1 B1 B1√	**3**	2, 3, 4 only in top line one correct prob other than P(2) third correct prob ft $\Sigma = 1$
(iii) $\text{Var}(X) = \frac{12}{14} + \frac{72}{14} + \frac{48}{14} - 3^2$	M1		using $\Sigma x^2 p - 3^2$ (or their $\{E(X)\}^2$) must be evaluated
$= \frac{3}{7}$ (0.429)	A1	**2**	correct answer

5	**(i)**	P(X and P) = $\frac{1}{4} \times \frac{4}{9} = \frac{1}{9}$	M1		Mult a playground prob with a P prob
		P(Y and P) = $\frac{1}{4} \times \frac{2}{12} = \frac{1}{24}$	A1		One correct prob
		P(Z and P) = $\frac{1}{2} \times \frac{1}{16} = \frac{1}{32}$	M1		Summing at least two 2-factor probs
		P(P) = $\frac{53}{288}$ = 0.184	A1	**4**	Correct answer
	(ii)	$\mathrm{P}(Y \mid C) = \dfrac{\mathrm{P}(Y \cap C)}{\mathrm{P}(C)}$	M1		Attempt at $\mathrm{P}(Y \cap C)$ as numerator of a fraction
		$\dfrac{\frac{1}{4} \times \frac{1}{12}}{\frac{1}{4} \times \frac{1}{12} + \frac{1}{2} \times \frac{4}{16}}$	M1		Attempt at P(C) in form of summing two 2-factor products, seen anywhere
			A1		Correct unsimplified P(C) seen anywhere
		$= \dfrac{\frac{1}{48}}{\frac{7}{48}} = \dfrac{1}{7}$	A1	**4**	Correct answer
6	**(i)**	$\frac{6!}{2!} = 360$	B1		6! Seen alone
			B1	**2**	Dividing by 2! only
	(ii)	$\frac{4!}{2!} \times \frac{4!}{3!}$	B1		4! seen mult
			B1		Dividing by 2! or 3! (Mult by 4 implied B1B1)
		= 48	B1	**3**	Correct answer
	(iii)	1N and 1A: N A xx in ${}^3C_2$	M1		${}^3C_x$ or ${}^xC_2$ seen alone
		= 3 ways	A1	**2**	Correct answer
	(iv)	0 A : Nxxx = 1 way	M1		Finding ways with 0 or 2 or 3 As
		2 As: NAAx in ${}^3C_1$ = 3 ways	M1		Summing 3 or 4 options
		3 As: NAAA in 1 way			
		Total = 8 ways	A1	**3**	Correct answer

7 **(i)** class widths 5, 15, 15, 25, 20	M1		Attempt at class widths
fd = $\frac{24}{5}, \frac{9}{15}, \frac{21}{15}, \frac{15}{25}, \frac{42}{20}$	B1		Correct widths of bars, with or without halves, seen on diagram
= 4.8, 0.6, 1.4, 0.6, 2.1			
	M1		Attempt at fd or scaled freq
	A1		Correct heights seen on graph ft their fd
	B1	5	Correct labels, scales and halves
(ii) mean = $\frac{(3\times24+13\times9+28\times21+48\times15+70.5\times42)}{111}$	M1 M1		Using mid points using (Σ their fx) / their 111
= 40.2 errors	A1	3	correct answer
(iii) LQ in 6 – 20 UQ in 61 – 80 Least value of IQ range is 61 – 20 = 41	B1 B1 B1√	3	ft any or both wrong quartile ranges if sensible

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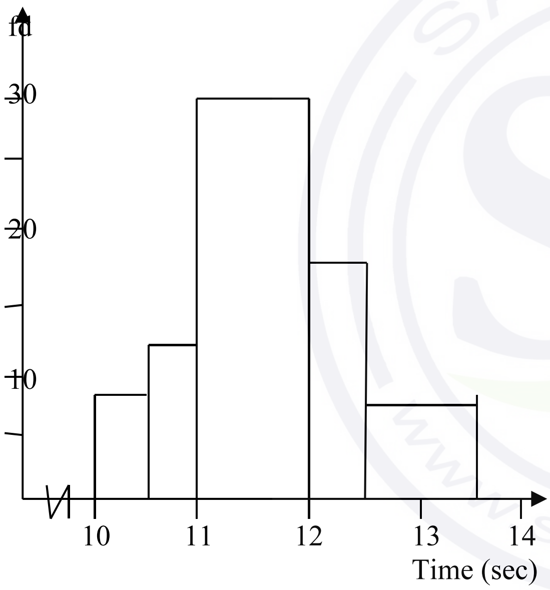
Question	Answer	Marks	Total	Guidance
1	$X \sim B(19, 0.12)$ $P(X < 4) = P(0, 1, 2, 3)$ $= (0.88)^{19} + {}^{19}C_1(0.12)^1(0.88)^{18} + {}^{19}C_2(0.12)^2(0.88)^{17} + {}^{19}C_3(0.12)^3(0.88)^{16}$	M1 M1 M1		Any binomial term ${}^{19}C_x\, p^x(1-p)^{19-x}$, $0<p<1$ Any binomial term ${}^nCx(0.12 \text{ or } 0.88)^x(0.88 \text{ or } 0.12)^{n-x}$ P(0, 1, 2, 3) binomial expr with at least 2 consistent terms
	$= 0.813$	A1	4	Correct answer
2	Y1(7) Y2(2) Y3(2) 1 2 2 $= 7 \times 1 \times 1 = 7$ 2 1 2 $= {}^7C_2 \times {}^2C_1 \times 1 = 42$ 2 2 1 $= {}^7C_2 \times 1 \times {}^2C_1 = 42$ 3 1 1 $= {}^7C_3 \times {}^2C_1 \times {}^2C_1 = 140$	B1 B1 M1		One unsimplified correct 3-factor product of combinations A second unsimplified correct 3-factor product of combinations Summing 3 or 4 options allow perms, wrong combs but second numbers must sum to 5 etc.
	Total = 231	A1	4	Correct answer
3 (i)	P(RR) = 0.6 × 0.7 = 0.42 P(AA) = 0.4 × 0.75 = 0.3 P(2 sets in match) = 0.72	B1 B1 B1√	3	Only 2 factors Only 2 factors ft previous answers
(ii)	$\dfrac{P(A\text{ wins and 2 sets})}{P(2\text{ sets})} = \dfrac{P(AA)}{P(2\text{ sets})}$	B1√		Correct num or correct denom of a fraction ft their **(i)**
	$= \dfrac{0.3}{0.72} = \dfrac{5}{12}$ (0.417)	B1√	2	Correct answer ft their or recovered AA/their or recovered **(i)**
4 (i)	A:P(H) = 2/3, P(T) = 1/3 B: P(H) = ¼, P(T) = 3/4	M1		Using some of 2/3, 1/3, ¼ or 3/4 in a calculation involving prod of 3 probs
	P(1H) = P(HTT) + P(THT) + P(TTH) = (2/3 × 1/3 × 3/4) + (1/3 × 2/3 × 3/4)	M1		Summing 3 options not all the same
	+ (1/3 × 1/3 × 1/4) = 13/36 **AG**	A1	3	Correct answer
(ii)	Table: x: 0, 1, 2, 3; P: 3/36, 13/36, 16/36, 4/36	B1		0, 1, 2, 3 seen for table no probs needed, table not absolutely necessary if calcs shown
	P(0H) = P(TTT) = 1/3 × 1/3 × 3/4 = 1/12	B1		One prob correct other than (i) condone 0.083 for 0.0833
	P(2H) = P(HHT) + P(HTH) + P(THH) = (2/3 × 2/3 × 3/4) + (2/3 × 1/3 × 1/4) + (1/3 × 2/3 × 1/4) = 4/9 not 2/3 × 2/3	B1		A second prob correct need 3 factors can be implied
	P(3H) = P(HHH) = 2/3 × 2/3 × 1/4 = 1/9	B1√	4	A third prob correct ft 23/36 – Σ their 2 probs
(iii)	$E(X)$ = 13/36 + 32/36 + 12/36	M1		Attempt to evaluate Σxp at least 3 vals of x in table
	= 57/36 (19/12) (1.58)	A1	2	Correct answer

Table for 4 (ii):

x	0	1	2	3
P	3/36	13/36	16/36	4/36

5 (i)	$5! \times 3!$ or $6!$	B1		5! or 3! or 6! oe seen mult or alone
	= 720	B1	**2**	Correct final answer
(ii)	3**4, 3**8, 4**8	M1		considering at least 2 types of 4-figure options ending with 4 or 8 and starting with 3 or 4
		B1		One option correct unsimplified can be implied
	$= 5 \times 4 + 5 \times 4 + 5 \times 4 = 60$	A1	**3**	Correct final answer
(iii)	5, *5, **5,	M1		Appreciating that the number must end in 5 (can be implied)
	$= 1 + 7 + 7^2$	M1		summing numbers ending in 5 with at least 2 different numbers of digits
	= 57	A1	**3**	Correct final answer
6 (i)	6	B1	**1**	Must see in **(i)**
(ii)	freqs 4 6 30 9 8 fd 8 12 30 18 8	M1		Attempt at scaled freq or fd (must be f/cw) at least three f/cw
		A1		Correct heights seen on graph
		B1		Correct-looking widths from 10, 10.5 etc. no gaps no extra lines
		B1	**4**	Labels and linear axes or squiggle need time or secs, fd,
(iii)	$E(X) = (10.25 \times 4 + 10.75 \times 6 + 11.5 \times 30 + 12.25 \times 9 + 13 \times 8)/57$	M1		Using mid-point attempt (not end points) with their freq or cf at least 2 sensible ones
	= 11.7(11.662)	A1		Correct mean
	$\text{Var}(X) = (10.25^2 \times 4 + 10.75^2 \times 6 + 11.5^2 \times 30 + 12.25^2 \times 9 + 13^2 \times 8) / 57 - (11.662\ldots)^2$	M1		numerical attempt at correct variance formula with mean^2 subt ft their "midpoints" i.e. ucb, cw, etc.
	= 0.547	A1	**4**	accept answers between 0.547 and 0.610 condone 0.6, 0.60

7	(i)	$z = -0.842$ $\text{P}(x > 1.35) = \text{P}\left(z > \dfrac{1.35-1.9}{\sigma}\right)$ $-0.842 = -0.55/\sigma$ $\sigma = 0.653$	B1 M1 A1	 3	± rounding to 0.84 seen $\pm\dfrac{1.35-1.9}{\sigma}$ = a prob or a z-value NOT 0.8 or 0.2 allow a 1–... Correct answer from correct working
	(ii)	$\text{P}(x < 2) = \text{P}\left(z < \dfrac{2-1.9}{0.6532}\right)$ $= \text{P}(z < 0.1531)$ $= 0.561$	M1 A1	 2	± standardising no continuity correction their σ Correct answer
	(iii)	$X \sim \text{N}(160, 32)$ $\text{P}(162.5 < x < 173.5) =$ $\text{P}\left(\dfrac{162.5-160}{\sqrt{32}} < z < \dfrac{173.5-160}{\sqrt{32}}\right)$ $\text{P}(0.442 < z < 2.386)$ $= \Phi(2.386) - \Phi(0.442)$ $= 0.9915 - 0.6707$ $= 0.321$	B1 M1 M1 M1 A1 A1	 6	Unsimplified 160 and 32 seen Standardising need sq rt Any of 162.5, 163.5, 172.5, 173.5 seen $\Phi_2 - \Phi_1$ oe One correct Φ to 3sf Correct answer accept 0.320

CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level and GCE Advanced Level

MARK SCHEME for the May/June 2014 series

9709 MATHEMATICS

9709/63 Paper 6, maximum raw mark 50

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- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

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<table>
<tr><td>1 (i)</td><td>
<table>
<tr><td>Adults</td><td></td><td>Children</td></tr>
<tr><td></td><td>4</td><td>3</td></tr>
<tr><td>8 6 5 4 3</td><td>5</td><td>4</td></tr>
<tr><td>7 4 3 3 2 1</td><td>6</td><td>1 2 7 8</td></tr>
<tr><td>8 4 3 1</td><td>7</td><td>2 7</td></tr>
<tr><td></td><td>8</td><td>1 3 4 6 9</td></tr>
<tr><td></td><td>9</td><td>2 5</td></tr>
</table>
key 3 │ 5 │ 4 represents 53 seconds for adults and 54 seconds for children</td><td>B1

B1

B1</td><td>

3</td><td>Single stem and key correct – including adults, children and seconds

Right hand leaves correct shape

Left hand leaves correct shape</td></tr>
<tr><td>(ii)</td><td>Two from:
Children's estimates more spread out
Adults estimates lower
Adults are symmetrical whereas children are skewed</td><td>

B1
B1</td><td>

2</td><td>
oe
oe
oe</td></tr>
<tr><td>2 (i)</td><td>$np = 252 \times 1/7 = 36$,
$npq = 252 \times 1/7 \times 6/7 = 30.857$

$\mathrm{P}\left(z < \left(\frac{29.5-36}{\sqrt{30.857}}\right)\right) + \mathrm{P}\left(z > \left(\frac{44.5-36}{\sqrt{30.857}}\right)\right)$

$= \mathrm{P}(z < -1.170) + \mathrm{P}(z > 1.530)$

$= 1 - 0.8790 + 1 - 0.9370$

$= 0.184$</td><td>
B1

M1
M1

M1

A1</td><td>

5</td><td>
Unsimplified 36 and 30.857 seen, oe

any standardising, sq rt needed
any continuity correction either 29.5, 30.5, 43.5, 44.5

correct area $2 - (\Phi_1 + \Phi_2)$

correct answer</td></tr>
<tr><td>(ii)</td><td>np and nq are both > 5</td><td>B1</td><td>1</td><td>must have both</td></tr>
<tr><td>3 (i)</td><td>$\mathrm{P}(2) = {}^6C_3 \times {}^3C_2/{}^9C_5$
OR
$\frac{{}^6C_3 \times {}^3C_2}{{}^6C_5 + {}^6C_4 \times {}^3C_1 + {}^6C_3 \times {}^3C_2 + {}^6C_2 \times {}^3C_3}$
OR
$3/9 \times 2/8 \times 6/7 \times 5/6 \times 4/5 \times {}^5C_2 = 10/21$

$= 60/126$ AG</td><td>M1
OR
M1
OR
M1

A1</td><td>

2</td><td>Using combinations ${}^aC_b \times {}^cC_d/{}^eC_f$

Mult 5 probs with a ${}^pC_q$
If ${}^5C_2$ replace by 10, oe must be justified
Legit method, as answer given</td></tr>
<tr><td>(ii)</td><td>
<table>
<tr><td>x</td><td>0</td><td>1</td><td>2</td><td>3</td></tr>
<tr><td>Prob</td><td>2/42</td><td>15/42</td><td>20/42</td><td>5/42</td></tr>
</table>
$\mathrm{P}(0) = {}^6C_5/{}^9C_5 = 6/126$
$\mathrm{P}(1) = {}^6C_4 \times {}^3C_1/{}^9C_5 = 45/126$
$\mathrm{P}(3) = {}^6C_2 \times {}^3C_3/126 = 15/126$</td><td>B1

B1
B1
B1√</td><td>

4</td><td>0, 1, 2, 3 only seen in table.
Condone $x = 4,5$ in table if $\mathrm{P}(x) = 0$ or blank and values in table for $x = 0,1,2,3$

Any correct prob other than P(2)
Any other correct prob
$\Sigma\mathrm{P}(x) = 1$, $3 < \mathrm{n}(x) < 6$</td></tr>
</table>

4 **(i)**	new mean $\dfrac{172.6\times 28-161.8}{27}=173$	M1 A1	 **2**	Mult by 28, subt 161.8 and dividing by 27 or 28 Correct ans
(ii)	original $\Sigma x^2 = (4.58^2 + 172.6^2) \times 28$ $= 834728.6$ (835000) Remaining $\Sigma x^2 = 834728.6 - 161.8^2$ $= 808549.36$ sd of remaining $= \sqrt{\dfrac{808549.36}{27}-173^2}$ $= 4.16$	M1 A1 M1 A1	 **4**	Subst in formula to find Σx^2 and attempt to make Σx^2subject, with 2 terms both squared Correct answer Subtract 161.8^2 from their original Σx^2 Correct ans, accept 4.15 or 3.93
5 **(i)**	$z = -1.282$ $-1.282 = \dfrac{t-6.5}{1.76}$ $t = 4.24$	B1 M1 A1	 **3**	Rounding to ± 1.28 seen Standardising, no cc, no sq or sq rt, $z \neq \pm0.9, \pm0.1$ Correct answer, accept 4.25
(ii)	$P(z < 1) = 0.8413$ P(within 1sd of mean) $= 2\Phi - 1$ $= 0.6826$ P(8, 9) $= {}^9C_8(0.6826)^8(0.3174) + (0.6826)^9$ $= 0.167$	M1 B1 M1 M1 A1	 **5**	$z = 1$ used to find a probability correct prob, accept answer rounding to 0.66, 0.67, 0.68, not from wrong working. If quoted, then implies first M1. Binomial term $p^r(1-p)^{9-r}\,{}^9C_r$, ${}^9C_r$ must be seen Binomial expression for P(8)+P(9), any p Correct ans
6 **(i)**	P(B champ) $= 0.7\times 0.7 = 0.49$	B1	**1**	
(ii)	P (B champ) = P(WW) + P(WLW) + P(LWW) $= (0.7\times0.7) + (0.7\times0.3\times0.7) + (0.3\times0.7\times0.7)$ $= 0.49 + 0.147 + 0.147$ $= 0.784$	 M1 B1 A1	 **3**	 Summing at least 2 options, at least one of which is 3-factor 0.147 seen, unsimplified Correct answer
(iii)	$P(T2 \mid T) = \dfrac{P(T2\cap T)}{P(T)}$ $= \dfrac{0.3\times0.3+0.7\times0.3\times0.3}{0.216}$ $= 0.708$	M1 A1 M1 A1	 **4**	Attempt P(T2∩T) seen anywhere sum of 2 terms Correct unsimplified num of a fraction Dividing by their (1 – **(ii)** √) oe Correct answer

7	**(i)**	**(a)**	$6!$ $(\times)\ 4!$ **OR** $(\times)\ 4 \times 3$ $\div\ 2!2!3!$ **OR** $\div\ 2!3!$ Total 720 ways	M1 M1 M1 A1	 **4**	Seen in a single term expression as numerator Seen in a single term expression as numerator (denominator may be 1) Seen in a single term expression as denominator Correct ans
	(i)	**(b)**	1*******3 = $\frac{7!}{3!2!}$ = 420 3*******1= 420 3*******3= 420 Total = 1260 ways	B1 M1 A1	 **3**	$\frac{7!}{3!2!}$ seen oe Attempting to evaluate and sum at least 2 of 1***3, 3***1, 3***3 Correct ans
	(ii)	**(a)**	$5 \times 4 \times 3 = 60$ ways (${}^5P_3$)	M1 A1	 **2**	${}^5P_3$ or ${}^5C_3 \times 3!$ (can be implied) Correct ans
	(ii)	**(b)**	2** in 212, 213, 214, 216, 221, 223, 224, 226, 231, 232, 233, 234, 236, 241, 242, 243, 246 261, 262, 263, 264, 266 Total = 22 ways **Alternative Methods:** $3 \times {}^4C_1 + 2 \times {}^5C_1$ **OR** ${}^5P_2 + {}^2C_1$ **OR** ${}^4P_2 + 2 \times {}^4P_1 + {}^2C_1$	 M1 A1 M1 **OR** M1 **OR** M1	 **2**	 Listing attempt starting with 2, at least 10 correct entries Correct ans $p \times {}^4C_1 + q \times {}^5C_1$, oe $p + q > 2$ ${}^5P_2$ seen Any 2 terms added

CAMBRIDGE INTERNATIONAL EXAMINATIONS

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9709/61 Paper 6, maximum raw mark 50

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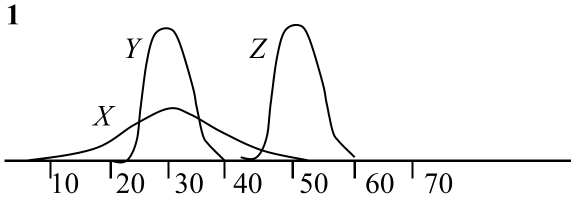
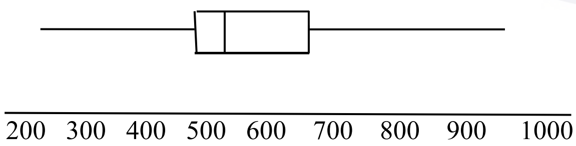
1 	B1 B1 B1ft	 3	X mean at 30, roughly from 10 to 50 or 15 – 45 Y same mean as X but higher and thinner Z same shape as Y but mean at 50 ft wrong Y
2 either 55/90 (11/18) or 95/160 (19/32) seen P(M and 18 – 60) = 0.6 × 55/90 = 0.367 (11 / 30) P(18 – 60) = 0.6 × 55/90 + 0.4 × 95/160 (= 29/48 or 0.604) P(M | 18 – 60) = $\frac{P(M \cap 18-60)}{P(18-60)}$ = 88/145 (0.607)	B1 M1 M1 A1 A1	 5	oe 0.6 mult by 55/90 seen as num / denom of a fraction Summing 2 two-factor products seen anywhere Correct unsimplified answer seen as num/denom of a fraction Correct answer
3 $\Sigma(x-5) = 116 - 18 \times 5 = 26$ $\frac{\Sigma(x-5)^2}{18} - \left(\frac{26}{18}\right)^2 = \frac{967}{18} - \left(\frac{58}{9}\right)^2$ $\Sigma(x-5)^2 = 257$ OR coded mean = 58/9 – 5 = 1.444 $\Sigma(x-5) = 1.444 \times 18 = 26$ $\Sigma(x-5)^2 = \Sigma x^2 - 10\Sigma x + 25 \times 18$ $= 967 - 1160 + 450 = 257$	M1 A1 M1 M1 A1 M1 A1 M1 A1 A1	 5	Obtaining Σx and subtracting 18 × 5 Correct answer Subst in correct var formula all coded vals Subst in correct var formula all uncoded Correct answer Subtracting 5 from true mean and mult by 18 Correct answer Expanding $\Sigma(x–5)^2$ 3 terms needed Any 2 terms correct Correct answer
4 (i) 	B1 B1 B1 B1	 4	Linear scale or 5 values shown and labels or in heading, need thousands of dollars, Correct median Correct quartiles Correct end points of whiskers not through box
(ii) 1.5 × 170 = 255 Expensive houses above 690 + 170 × 1.5 = 945 i.e. 957 and 986 thousands of dollars	M1 A1	 2	Mult their IQ range by 1.5 Correct answers from correct wkg need thousands of dollars
(iii) doesn't show all the data items	B1	1	Need to see 'individual items' oe

5	**(i)**	$z = -1.406$ $\dfrac{c-14.2}{3.6} = -1.406$ $c = 9.14$	B1 M1 A1	 **3**	Rounding to ± 1.41 seen Standardising allow sq rt no cc Correct answer
	(ii)	$P\left(\dfrac{15-14.2}{3.6}\right) < z < \left(\dfrac{16-14.2}{3.6}\right)$ $= \Phi(0.5) - \Phi(0.222)$ $= 0.6915 - 0.5879$ $= 0.1036$ P(at least 2) = 1 – P(0, 1) $= 1 - (0.8964)^7 - (0.8964)^6(0.1036)_7C_1$ $= 1 - 0.8413$ $= 0.159$	M1 M1 A1 M1 M1 A1	 **6**	2 attempts at standardising no cc no sq rt Subt two Φs (indep mark) Needn't be entirely accurate, rounding to 0.10 Binomial term with ${}_7C_r p^r(1-p)^{7-r}$ seen $r \neq 0$ any $p < 1$ 1 – P(0), 1 – P(1), 1 – P(0, 1) seen their p Correct answer accept 3sf rounding to 0.16
6	**(i)**	M R O 3 1 2 = 7C3 × 5C1 × 8C2 = 4900 3 2 1 = 7C3 × 5C2 × 8C1 = 2800 2 2 2 = 7C2 × 5C2 × 8C2 = 5880 Total = 13580	 M1 M1 A1 A1	 **4**	 Summing more than one 3term option involving combs (can be added) Mult 3 combs only (indep) 1 option correct unsimplified Correct answer
	(ii)	4 groups in 4! ways 3 mountain in 3! ways 2 ordinary in 2! ways 4! × 3! × 2 = 288	M1 M1 A1	 **3**	4! seen mult by something Mult by 3! for racing or 2! for ordinary Correct answer
	(iii)	e.g. s O x x x x O s s s Ordinary in 2! Rest of bikes in 4! Bikes and spaces 5 groups in 5 ways 2! × 4! × 5 = 240	M1 M1 A1	 **3**	2! or 4! seen mult Mult by 5 (ssssb) Correct answer

7	**(i)** if throw H then smallest score is 2 P(T, 1) = 1/2 × 1/4 = 1/8 **AG**	B1 B1 **2**	Or equivalent
	(ii) P(3) from two dice = 2/16 seen P(H, 3) = 1/2 × 2/16 = 2/32 P(T, 3) = 1/2 × 1/4 = 1/8 So P(3) = 6/32 = 3/16 **AG**	B1 M1 A1 A1 **4**	From (1, 2) and (2, 1) Summing P(H, 3) and P(T, 3) One correct Correct answer must see clear reasoning
	(iii)	B1 B1 B1 **3**	One correct prob A second correct prob A third correct prob
	(iv) P($Q \cap R$) = 0 or 'if you throw a tail you can't get a 7' Yes they are exclusive	M1 A1dep **2**	Stating P($Q \cap R$) = 0 or implying by words Dep on previous M

X	1	2	3	4	5	6	7	8
Prob		5/32		7/32		3/32		

CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level and GCE Advanced Level

MARK SCHEME for the October/November 2013 series

9709 MATHEMATICS

9709/62 Paper 6, maximum raw mark 50

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1		$P(x < -2.4) = P\left(z < \frac{-2.4-1.5}{3.2}\right)$	M1		Standardising no cc can have sq
		$= P(z < -1.219)$	M1		Correct area, i.e. < 0.5
		$= 1 - 0.8886$ $= 0.111$	A1	**[3]**	Correct answer rounding to 0.111
2	**(i)**	$P(C \cap < 50) = 0.35 \times 0.2 = 0.07$	B1	**[1]**	
	(ii)	$P(C \mid < 50) = \frac{P(C \cap < 50)}{P(< 50)}$	M1	**[4]**	Summing three 2-factor products seen anywhere (can omit the 1)
		$= \frac{0.35 \times 0.2}{0.25 \times 0.3 + 0.35 \times 0.2 + 0.4(\times 1)}$	A1		0.545 (unsimplified) seen as num or denom of a fraction
		$= \frac{0.07}{0.545}$	M1		Attempt at $P(C \cap < 50)$ as 2-factor prod only seen as num or denom of a fraction
		$= 0.128$ (14/109)	A1		Correct answer
3	**(i)**	$z = 0.878$	B1		± 0.878, 0.88, rounding to 0.88 seen
		$\frac{190-160}{\sigma} = 0.878$	M1		$(190 - 160)/\sigma$ = something
		$\sigma = 34.2$	A1	**[3]**	Correct answer
	(ii)	P(at least 1) = 1 – P(0)	M1		Using 1 – P(0), 1 – P(0, 1), P(1,2 … 12) or P(2, … 12) with p = 0.19 or 0.81, terms must be evaluated to get the M1
		$= 1 - (0.81)^{12} = 0.920$	A1	**[2]**	Correct answer accept 0.92
4	**(i)**	number = 1.5 × 50 = 75 (AG)	B1	**[1]**	Must see 1.5 × 50
	(ii)	freqs are 10, 25, 50, 75, 30 (15, 15)	M1 A1		Attempt at freqs not fd Correct freqs
		Mean = (10 × 125 + 25 × 162.5 + 50 × 187.5 + 75 × 225 + 30 × 300)/190	M1		attempt at mid points not cw or ucb or lcb
		= 40562.5/190 = 213 (213.48 …)	A1		correct mean
		$sd^2 = 10 \times 125^2 + 25 \times 162.5^2 + 50 \times 187.5^2 + 75 \times 225^2 + 30 \times 300^2)/190 - (213.48\ldots)^2$	M1		subst their Σfx^2 in correct variance formula
		sd = 46.5 or 46.6	A1	**[6]**	
	(iii)	have used the mid-point of each interval and not the raw data	B1	**[1]**	

5	(i)	$P(4, 5, 6) = (0.22)^4(0.78)^4 8C4 + (0.22)^5(0.78)^3 8C5 + (0.22)^6(0.78)^2 8C6$	M1 M1		Bin term with ${}_8C_r\, p^r\,(1-p)^{8-r}$ seen $r \neq 0$ any $p < 1$ Summing 2 or 3 bin probs $p = 0.22$, $n = 8$
		$= 0.0763$	A1	[3]	Correct answer
	(ii)	prob = 0.13 mean = 300 × 0.13 = 39 var = 300 × 0.13 × 0.87 = 33.93	B1 B1ft		Correct prob can be implied Correct unsimplified np and npq ft wrong 0.13
		$P(30 < x < 50) = P\left(\frac{30.5-39}{\sqrt{33.93}} < z < \frac{49.5-39}{\sqrt{33.93}}\right)$	M1 M1		Standardising a value need sq rt Cont correction 30.5 / 31.5 or 48.5/49.5 only
		$= P(-1.4592 < z < 1.8026)$ $= \Phi(1.8026) + \Phi(1.4592) - 1$ $= 0.9643 + 0.9278 - 1 = 0.892$	M1 A1	 [6]	Correct area $\Phi_1 + \Phi_2 - 1$ oe Rounding to correct answer SC $P(31,..49) = 300C31(0.13)^{31}(0.87)^{269} + \ldots + 300C49$ etc.) B1B1
6	(i)	1663200	B1	[1]	
	(ii)	M xxxxxxxxx M Number of ways = $\frac{9!}{3!2!}$ = 30240	M1 A1	 [2]	9! or 9P9 seen Correct answer
	(iii)	4 vowels together = 8! × 4/2!2! = 40320 1663200 – 40320 = 1622880	M1 M1 B1	 [3]	8!/2!2! seen mult by something 4 oe 4!/3! or 4C1 etc. seen mult by something Correct answer SC 7!/2!2! × 8P4 or 7! × 8P4/3! Or 7!/2!2! × 8P4/3! M1
	(iv)	Exactly 2 Es 4C2 = 6 Exactly 3 Es 4C1 = 4 Total = 10 ways	M1 B1 A1	 [3]	Summing 2 options One option correct Correct answer
		OR 5C2 = 10	M2 A1		M1 for k5C2 Correct ans

7	(i)	options (3, 4, 4,) or (4, 3, 4) or (4, 4, 3) Probs (4/10 × 6/9 × 5/8) ×3C1 = 360/720 = ½ AG	M1 M1 A1	 [3]	Summing three 3-factor options oe 10 × 9 × 8 seen in denom Correct answer
		OR $\frac{{}_6C_2 \times {}_4C_1}{{}_{10}C_3} = \frac{1}{2}$ AG	M1 M1 A1		One of 6C2 or 4C1 seen in num 10C3 in denom Correct answer
	(ii)		B1	[4]	9, 10, 11, 12 only seen
		sum: 9, 10, 11, 12 Prob: 24/720, 216/720, 360/720, 120/720	B1		One correct prob other than P(11), with or without replacement
		P(3, 3, 3) = 4/10 × 3/9 × 2/8 = 24/720 (1/30) P(3, 3, 4) = 4/10 × 3/9 × 6/8 × 3C1 = 216/720 (3/10) P(4, 4, 4) = 6/10 × 5/9 × 4/8 = 120/720(1/6)	B1 B1		Another correct prob Σ all 4 probs = 1
	(iii)	P(*R*) = 0.5 P(*S*) = 0.4 P(*R*∩*S*) = 120/720 P(*R*∩*S*) = 120/720 ≠ P(*R*) × P(*S*) Not indep	B1 M1 A1ft	[3]	P(*R*∩*S*) = 120/720 (1/6) Numerical attempt to compare P(*R* and *S*) with P(*R*) × P(*S*) provided P(*R*∩*S*) ≠ 1/5 Correct conclusion ft wrong P(*R*∩*S*) ≠ 1/5, P(*S*) correct
	(iv)	P(*R*∩*S*) ≠ 0 or there is an overlap between *R* and *S* (34,4) Not exclusive Σ*x*f/Σf	B1ft	[1]	Correct answer following correct reasoning ft wrong non zero P(*R*∩*S*)

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1	bars are not touching oe	B1		Sensible reason involving not touching, no gaps, class boundaries, group data not continuous (may be the negative)
	Area not rep by frequency, not used fd, not labelled fd	B1	**2**	Must be frequency density oe. Wrong height not sufficient. (Best 2 reasons awarded)
2	$P(13.6 < X < 14.8) = P\left(\frac{13.6-14}{0.52} < z < \frac{14.8-14}{0.52}\right)$	M1		Standardising 1 expression, no cc, no sq rt, no sq, ±, mean on num.
	$= P(-0.7692 < z < 1.538)$ $= \Phi(1.538) - [1 - \Phi(0.7692)]$	M1		$\Phi1 + \Phi2 - 1$ (indep) oe ($\Phi2 - \Phi1$ if cc used)
	$= 0.9380 - [1 - 0.7791]$ $= 0.7171$	A1		Correct probability rounding to 0.72 here
	$P(8) = (0.7171)^8(0.2829)^2\,{}_{10}C_8$	M1		Binomial expression 10C8 p^8q^2 , $\Sigma p + q = 1$, any p
	$= 0.252$	A1	**5**	Correct answer (rounding to 0.252)
3 (i)	$(p =)0.85$	B1		$(p =)0.85$ oe seen anywhere
	$P(< 12) = 1 - P(12, 13, 14)$ $= 1 -[(0.85)^{12}(0.15)^2\,{}_{14}C_{12} + (0.85)^{13}(0.15)\,{}_{14}C_{13} + (0.85)^{14}]$	M1		Summing 2 or 3 consistent bin probs, any $p < 1$, $n = 14$ (or summing 12 or 13 consistent bin probs)
	$= 1 - 0.6479$ $= 0.352$	A1	**3**	Correct answer
(ii)	$(0.85)^n \geqslant 0.1$	M1		Eqn or inequality in 0.85(or 0.15), n, 0.1, n as a power
	$n \leqslant 14.2$	M1		Attempt to solve (can be implied) if n a power
	$n = 14$	A1	**3**	Correct answer – must be equals, not approx. MR allowed for 0.01, M1M1A0 max.
4 (i)	$(220\times20 + 118\times25)/45$	M1		Mult by 20 and 25 and dividing their sum by 45
	$= 163$	A1	**2**	Correct answer, 163.3 or 490/3 oe acceptable
(ii)	$\Sigma x_o^2/20 - 220^2 = 32^2$	M1		Subst in correct variance formula
	$\Sigma x_o^2 = 988480$	A1		Correct Σx_o^2
	$\Sigma x_l^2/25 - 118^2 = 12^2$ $\Sigma x_l^2 = 351700$	A1		correct Σx_l^2
	$\Sigma x_o^2 + \Sigma x_l^2 = 1340180$	M1		Subst their combined results in correct var formula
	New var $= 1340180/45 - (7350/45)^2$ $= 3100 - 3120$	A1	**5**	Correct answer

5	**(a)** $P(X < q + 82) = 0.72$ $z = 0.583$ $\frac{\pm q}{7.4}$ *or* $\frac{\pm 2q}{7.4} = z$ *or probabilty* (o.e.) $q = 4.31$	M1 M1 A1	 3	Rounding to ± 0.58 or ± 0.15 seen Standardising, no cc, no sq, no sq rt correct answer
	(b) $\frac{0.5\mu - \mu}{\sigma} = \frac{\pm 0.5\mu}{\sigma}$ $\frac{0.2\sigma^2}{\sigma} = -0.2\sigma = -0.580$ $\sigma = 2.90$ $\mu = 3.36$	M1 B1 M1 A1	 4	Standardising attempt some μ/σ allow cc, sq rt, sq Can be implied ± 0.580 seen (accept ± 0.58) substituting to eliminate μ or σ , arriving at numerical solution, any z value or probability – not dependent both answers correct , accept 2.9
6	**(i)** $\frac{8!}{3!2!2!}$ $= 1680$	M1 A1	 2	8! Divided by at least one of 3!2!2! oe Correct answer
	(ii) 5! = 120	M1 A1	 2	5! Seen (not added, may be divided/multipled) Correct answer
	(iii) $\frac{5!4!}{3!2!2!}$ $= 120$	B1 M1 A1	 3	5! Or 4! Seen in sum or product in numerator (denominator may by 1) $\frac{k5!4!}{3!2!2!}$ in a numerical expression Correct final answer
	(iv) GG with AA, AE, EE, RA, RE, RT, TA, TE, = 8 ways GGG with A, E, R, T = 4 ways Total = 12 ways	M1 A1 A1	 3	Summing 2 options (could be lists) 1 correct option Correct answer

7 (i)	P(same) = P(1, 1)+ P(3, 3) + P(5, 5)	M1		Summing 3 two-factor options
	$= \frac{2}{9}\times\frac{1}{8}+\frac{4}{9}\times\frac{3}{8}+\frac{3}{9}\times\frac{2}{8}$	M1		Multiplying terms by one less in the numerator or denominator
	= 5/18 (0.278)	A1	3	Correct answer
	Alt. method: $\frac{2C2+4C2+3C2}{9C2}$ or $\frac{2\times1+3\times4+2\times3}{9C2\times2}$ *oe*			M1 for numerator, M1 for denominator, A1 correct answer
(ii)	$P(5,\overline{5})+P(\overline{5},5)$	M1 M1		Mult 2 probs whose numerators sum to 9 o.e. Summing 2 options or mult by 2 (may be 4 options)
	$= \frac{3}{9}\times\frac{6}{8}+\frac{6}{9}\times\frac{3}{8}=\frac{36}{72}$ = ½ or 0.5	A1	3	Correct answer
	Alt. method: $\frac{6C1\times3C1\,(\times2)}{9C2\,(\times2)}$ *oe*			M1 for numerator, M1 for denominator, A1 correct answer
(iii)	$P(5\cap\overline{5})=\frac{3}{9}\times\frac{6}{8}=\frac{1}{4}$	M1		Attempt at P(5 and not 5) seen as numerator or denominator of a fraction
	$P(\overline{5})=\frac{1}{4}+\frac{6}{9}\times\frac{5}{8}$ = 48/72 = 0.6666	M1		Attempt at P(not 5) sum of 2 two-factor terms seen anywhere
		A1		Correct $P(\overline{5})$ as numerator or denominator in fraction
	$P(5_1\mid\overline{5}_2)=\frac{1/4}{48/72}$ = 3/8 = 0.375	A1	4	Correct answer
(iv)	<table><tr><td>x</td><td>0</td><td>1</td><td>2</td></tr><tr><td>$P(X=x)$</td><td>5/12</td><td>1/2</td><td>1/12</td></tr></table>	B1		Values 0, 1, 2 seen in table with at least 1 prob
	$P(0)=P(\overline{5},\overline{5})=\frac{6}{9}\times\frac{5}{8}$ = 30/72 (5/12) (0.4166)	B1		Correct P(0) unsimplified
	P(1) = 0.5 from part (ii)			
	P(2) = 6/72 (1/12) (0.0833) from part (i)	B1ft	3	If x=0,1,2(,3) ft Σp = 1, no –ve values, all probabilities <1

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9709 MATHEMATICS

9709/61 Paper 6, maximum raw mark 50

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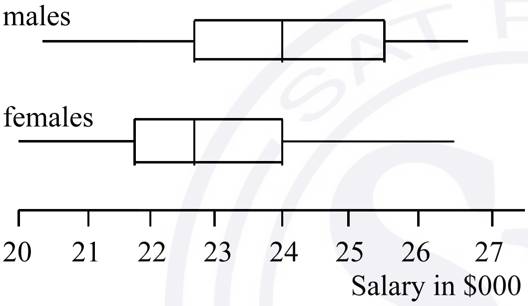
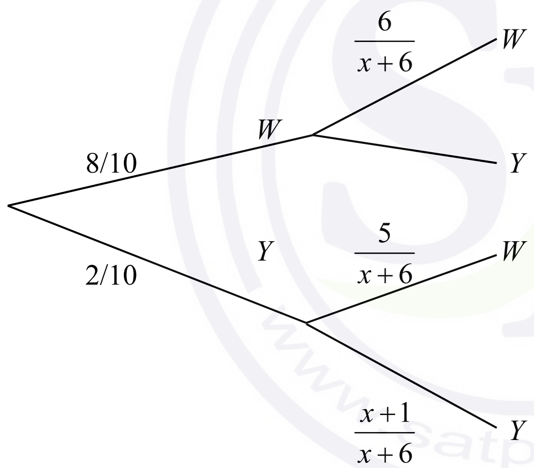
1	**(i)**	$sd^2 = 1957.5/30 - (234/30)^2$ sd = 2.1	M1 A1	 **[2]**	Subst in formula or expand Accept 2.10
	(ii)	86 = 234/30 + *c* *c* = 78.2	M1 A1	 **[2]**	234/30 seen
2		$np = 350 \times 1/7$ (= 50) $npq = 350 \times 1/7 \times 6/7$ (= 42.857) $P(x \quad 47) = P\left(z > \frac{46.5-50}{\sqrt{42.857}}\right) =$ $P(z > -0.5346)$ = 0.704	B1 M1 M1 M1 A1	 **[5]**	Correct unsimplified *np* and *npq* standardising, with or without cc, must have sq rt continuity correction 46.5 or 47.5 correct area ie > 0.5 must be a Φ correct answer
3	**(i)**	females: med $22 700 LQ $21700 UQ $24 000	B1 B1	 **[2]**	Any 2 correct All correct
	(ii)		B1 B1 B1	 **[3]**	Uniform scale and labels must see Salary, $000 Correct graph for females ft their quartiles. Line not through box Correct graph for males
4	**(a)**	$P(y < 0) = P\left(z < \frac{0-\mu}{\mu/2}\right)$ $= P(z < -2)$ = 1 – 0.9772 = 0.0228	M1 A1 A1	 **[3]**	Standardising containing 0 (can be implied) and μ only $z < -2$ seen Correct answer
(b)		$P(x > 2.1) = 253/8000 = 0.031625$ $P(x < 2.1) = 0.968375 = \Phi(z)$ $z = 1.857$ or 1.858 or $1.859 = \frac{2.1-2.04}{\sigma}$ σ = 0.0323	M1 A1 M1 A1	 **[4]**	1 – their 253/8000 used to obtain a *z*-value Rounded to 1.86 seen Solving for σ using their *z* val must be a *z* val Correct answer
5	**(i)**	*X* ~ Bin (12, 0.2)	B1 B1 B1	 **[3]**	Bin or B 12 0.2 or 1/5
	(ii)	$P(X = 3, 4, 5) = 0.2^3 0.8^9 {}_{12}C_3 + 0.2^4 0.8^8 {}_{12}C_4 + 0.2^5 0.8^7 {}_{12}C_5$ = 0.23622 + 0.13287 + 0.05315 = 0.422	M1 A1ft A1	 **[3]**	Bin exprerssion with any p Correct unsimplified expression, their p Correct answer

	(iii)	P (X = 0) < 0.01 $0.8^n < 0.01$ $n = 21$	M1 M1 A1	**[3]**	Statement involving P(X = 0) and 0.01 can be implied Equn involving ‘0.8’, 0.01 or 0.99 Correct answer
6	**(i)**	4! × 3! × 5! × 2! × 4! = 829440	B1 B1 B1	 **[3]**	4!, 3!, 5!, 2 seen multiplied 1, not in denominator Mult by 4! Correct answer
	(ii)	8! × 9 × 8 × 7 × 6 × 5 × 4 = 2438553600 (2.44×10^9)	B1 B1 B1	 **[3]**	8! seen multiplied 1 Mult by $_9P_6$ Correct answer
	(iii)	8C3 × 5C3 × 2C2 = 560	B1 B1 B1	 **[3]**	8C3 seen mult 5C3 seen mult Correct answer
7	**(i)**	number of balls in B is 5+ x + 1 = x + 6 P(Y) = $x/(x + 6)$ **AG**	B1	**[1]**	Sensible reason
	(ii)		B1 B1 B1 B1	 **[4]**	both correct for box A 1 correct 1 correct 1 correct
	(iii)	P(W_B) = $\frac{6}{x+6} = \frac{1}{3}$ x = 12 **AG**	M1 A1	 **[2]**	their $\frac{6}{x+6}$ = 1/3 or x/x+6 = 2/3 Verification or solving legit

(iv)	$P(Y) = \frac{8}{10}\times\frac{12}{18}+\frac{2}{10}\times\frac{13}{18}$	M1		Attempt at P(,Y) involving 2 two-factor fractions, seen anywhere.
	$= \frac{61}{90}$	A1		Correct P(Y) seen as num or denom of a fraction
	$P(= (AY \mid BY) = \frac{P(AY \cap BY)}{P(Y)}$ $= \frac{2}{10}\times\frac{13}{18} / \frac{61}{90}$	B1		(2/10) × (13/18) seen as num or denom of a fraction
	$= \frac{13}{61}$ (0.213)	A1	**[4]**	Correct answer

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1	$z = 1.452$	B1		Rounding to ± 1.45
	$1.452 = \dfrac{20-\mu}{\mu/5}$	B1		$\dfrac{20-\mu}{\mu/5}$ or $\dfrac{20-5\sigma}{\sigma}$ seen oe
	$\mu = 15.5$	B1	**[3]**	rounding to correct answer
2	$\bar{x} = 50 + 81.4/22 = 53.7$	M1		Attempt to find variance using coding in both, correct formula
	var $= 671/22 - 3.7^2 = 16.81(16.8)$	A1		Correct answer
	$16.81 = \Sigma x^2/22 - 53.7^2$	M1		using their var and their mean with uncoded formula for both
	$= 63811(63800)$	A1	**[4]**	correct answer
	OR			
	$\Sigma x - 22\times50 = 81.4$ ($\Sigma x = 1181.4$)	M1		expanded eqn with 22×50 seen
	$\Sigma x^2 - 100\Sigma x + 22\times50^2 = 671$	M1		expanded eqn with 2 or 3 terms correct
	$\Sigma x^2 = 671 + 118140 - 55000 = 63811$	A1		correct answer
	Var $= \Sigma x^2/22 - (\Sigma x/22)^2 = 16.81$	A1		correct answer
3 (i)	P($x < 440$)			
	$= \mathrm{P}\left(z < \dfrac{440-445}{3.6}\right) = 1 - \Phi(1.389)$	M1		Standardising no cc no sq or sq rt
	$= 1 - 0.9176$	M1		Correct area $(1 - \Phi)$ oe (indep)
	Ans = 0.0824	A1	**[3]**	Rounding to correct answer accept 0.0825
(ii)	$z = 1.881$	M1		±1.88 or 1.881 or 1.882 or 1.555 seen±
	$\dfrac{c}{3.6} = 1.881$	M1		Equation with ± $c/3.6$ or $2c/3.6$ only = z or prob (can be implied)
	$c = 6.77$	A1	**[3]**	Correct answer accept 6.78
4 (i)	$p = 4/9$ or $5/9$	B1		Binomial term ${}_5C_x p^x(1-p)^{5-x}$ seen
	P(at least 2) = 1 – P(0, 1) $= 1 - (5/9)^5 - (4/9)(5/9)^4\ {}_5C_1$	M1		
	$= 0.735$	A1	**[3]**	Correct answer
(ii)	$np = 96$ $npq = 32$ $p = \mathrm{P}(\le k)$	M1		Using $np = 96$ $npq = 32$ to obtain eqn in 1 variable
	$p = 2/3$ $q = 1/3$ $n = 144$	A1		1/3 or 2/3 seen or implied
	$k = 6$	A1ft		Correct k ft $k = 9p$
	$n = 144$	A1	**[4]**	correct n

5 (i)	Stem | leaf 0 | 1 4 6 8 1 | 0 3 4 4 4 5 5 5 6 6 6 6 7 8 8 2 | 0 1 5 7 8 3 | 1 4 | 5 5 | 7	B1 B1		Correct stem condone a space under the 1 Correct leaves must be single digits and one line for each stem value or 2 lines each stem value
	Key 1 | 4 represents \$140	B1ft	**[3]**	Correct key must have \$, ft 2 special cases
(ii)	Median = 160 LQ = 140 UQ = 210 IQ range = UQ - LQ = 70	B1 M1 A1	**[3]**	Subt their LQ from their UQ Correct answer cwo
(iii)	1.5 × IQ range = 105 Lower outlier is below 35 Upper outlier is above 315 Outliers 10, 450, 570	M1 A1ft A1	**[3]**	Mult their IQ range by 1.5 can be implied Correct limits ft their IQ range and quartiles Correct outliers
6 (i)	H J O 1. 28 2 = 4C2×9C8×2C2 = 54 3 7 2 = 4C3×9C7×2C2 = 144 4 6 2 = 4C4×9C6×2C2 = 84 Total = 282 ways	M1 M1 A1 A1	**[4]**	Mult 3 combs, 2C2 may be implied $4Cx\times 9Cy\times 2Cz$ Summing 2 or 3 three-factor options 2 options correct unsimplified Correct answer
(ii)	4! × 6! × 2! × 3! = 207360 (207000)	M1 M1 A1	**[3]**	4! × 6! × 2! oe seen multiplied by int ≥ 1 3! seen mult by int ≥ 1 Correct answer
(iii)	8 J and O trees in 8! = 40320 ways 9 gaps × 8 × 7 × 6 = 121,927,680 (122,000,000)	B1 M1 A1	**[3]**	8! seen mult by int ≥ 1 no division 9P4 oe or 7P4 or 8P4 seen mult by int ≥ 1 no division Correct answer
(i)	SR 4C2×9C2×2C2×9C6	M1		
(ii)	SR $\frac{4!\times6!\times2!}{4!\times6!\times2!}$ or 3! or both M1	M1		

(iii)	SR1 12! – 9! 4!	M1		
	SR2 $\frac{9P4}{4!}$ or $\frac{8!}{6!\,2!}$ or both	M1		
7 (i)	$P(T,B) = \frac{5}{12} \times \frac{2}{10} = \frac{1}{12}$ (0.0833)	M1 A1	[2]	Mult their P(T) by 2/9 or 2/10 only Correct answer
(ii)	$P(C_S \cap C_A) = \frac{7}{12} \times \frac{4}{10} = \frac{28}{120}$ (0.2333)	M1		Mult their P(C_S) by 3/9 or 4/10 seen as num or denom of a fraction
	$P(C_A) = \frac{7}{12} \times \frac{4}{10} + \frac{5}{12} \times \frac{3}{10} = \frac{43}{120}$ (0.3583)	M1		Summing 2 two-factor products to find P(C_A) seen anywhere
	$P(C_S \mid C_A) = \frac{P(C \cap C)}{P(C_A)} = \frac{28/120}{43/120}$	A1		Correct unsimplified P(C_A) seen as num or denom of a fraction
	$= \frac{28}{43}$ (0.651)	A1	[4]	Correct answer
(iii)	<table><tr><td>x</td><td>0</td><td>1</td><td>2</td></tr><tr><td>Prob</td><td>7/24</td><td>19/40</td><td>7/30</td></tr></table>	B1		x = 0, 1, 2, can be implied from table or working
	$P(X = 0) = P(T, B) + P(T, T)$	M1		1 or 2 two-factor products, denoms 12 and 10 or 12 and 9, implied if ans is correct
	$= \frac{5}{12} \times \frac{2}{10} + \frac{5}{12} \times \frac{5}{10} = \frac{7}{24}$ (0.292)	A1		One correct unsimplified
	$P(X = 2) = P(C, C) = \frac{7}{12} \times \frac{4}{10} = \frac{28}{120}$ (0.233)	B1		One other correct unsimplified
	$P(X = 1) = 1 - 7/24 - 28/120 = \frac{19}{40}$ (0.475)	B1ft	[5]	Third correct ft 1 – P(2 of their probs))

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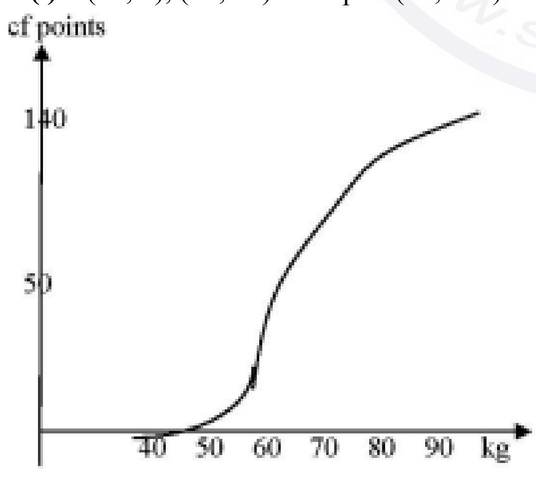
1 $P(Q) = \frac{4}{36}$ or $P(S) = \frac{1}{2}$	B1		oe
$P(Q \cap S) = \frac{2}{36}$ or $P(S\|Q) = \frac{1}{2}$ or $P(Q\|S) = \frac{2}{18}$	B1		oe
$P(Q \cap S) = P(Q) \times P(S)$ or $P(S\|Q) = P(S)$ or $P(Q\|S) = P(Q)$	M1		Comparing correct pair of terms $0 \le$ all probabilities < 1
Independent	A1	**[4]**	Correct conclusion must have all probs correct
2 P(at least 2) = P(2, 3) or 1 – P(0, 1)	M1		Summing, or 1–, two different three-factor prob expressions, ${}_3C_2$ not needed
$= \frac{5}{12} \times \frac{4}{11} \times \frac{7}{10} \times {}_3C_2 + \frac{5}{12} \times \frac{4}{11} \times \frac{3}{10}$	M1		12, 11, 10 seen or implied in denominator
	M1		Mult a prob by ${}_3C_2$ or ${}_3C_1$ oe
$= \frac{4}{11}$ (0.364)	A1	**[4]**	Correct answer
OR $\frac{({}_5C_3) + ({}_5C_2 \times {}_7C_1)}{{}_{12}C_3}$	M1		${}_5C_3$ seen added in numerator
	M1		${}_5C_2$ seen mult alone or in numerator
	M1		${}_{12}C_3$ seen in denom
	A1		Correct answer
3 (i) $P(\text{tall}) = P\left(z > \frac{70-50}{16}\right) = P(z > 1.25)$	M1		+ve/-ve Standardising no cc no sq rt no sq
$= 1 - 0.8944$ $= 0.106$	A1	**[2]**	Correct answer
(ii) P(short) = (1 – 0.1056)/3	M1		Subt their **(i)** from 1 or their **(i)** and multiplying by $\frac{1}{3}$ or $\frac{2}{3}$
$= 0.2981$	A1 ft		Rounding to 0.298, only ft for $\frac{(1-\textbf{(i)})}{3}$
$z = -0.53$	A1		± z-value rounding to 0.53, condone ±0.24
$-0.53 = \frac{x-50}{16}$	M1		Standardising with their z value (not a probability), no cc sq rt etc.
$x = 41.5$	A1	**[5]**	Correct answer

4 **(i)** $(0.8)^n < 0.001$ $n > 30.9$ $n = 31$	M1 M1 A1	 **[3]**	Eqn or inequ involving 0.8^n or 0.2^n and 0.001 or 0.999 Trial and error or logs (can be implied) Correct answer **MR** 0.01, max available M1M1A0
(ii) $\mu = 120 \times 0.2 = 24$ $\sigma^2 = 120 \times 0.2 \times 0.8 = 19.2$ $P(x < 33) = P \times \left(z < \frac{32.5 - 24}{\sqrt{19.2}}\right)$ $= P(z < 1.9398)$ $= 0.974$	B1 M1 M1 A1	 **[4]**	24 and 19.2 or $\sqrt{19.2}$ seen Standardising with or without cc, must have sq rt in denom Continuity correction 32.5 or 33.5 Correct answer
5 **(a)** $P(W_2) = P(W_1W_2) + P(L_1W_2)$ $= 0.3 \times 0.6 + 0.7 \times 0.15$ $= 0.285$ $P(W_1 \mid W_2) = \frac{P(W_1 \cap W_2)}{P(W_2)} = \frac{0.18}{0.285}$ $= 0.632, \frac{12}{19}$	B1 M1 A1 A1	 **[4]**	0.3 × 0.6 alone as num or denom of a fraction Attempt at $P(W_2)$ as sum of two 2-factor options seen anywhere Correct unsimplified $P(W_2)$ as num or denom of a fraction Correct answer
(b) $x + 4$ oe seen $\frac{10}{15} \times \frac{7}{x+4} = \frac{7}{18}$ $x = 8$	B1 M1 A1 A1	 **[4]**	Seen anywhere Mult two probabilities, one containing x and equating to $\frac{7}{18}$ Correct unsimplified equation Correct answer
6 **(i)** (40, 0), (50, 12) etc. up to (90, 144) 	B1 B1	 **[2]**	Axes, (cf) and labels (kg), uniform scales from at least 0–140 and 40.5–69.5 either way round All points correct, sensible scale (not 12), polygon or smooth curve
(ii) 80 weigh less than 67.2 kg $c = 67.2$	M1 A1 ft	 **[2]**	Subt 64 from 144 Accept anything between 67 and 68 ft from incorrect graph

(iii) freqs 12, 22, 30, 28, 52	M1 A1		frequencies attempt not cf Correct freqs
mean wt = (45 × 12 + 55 × 22 + 62.5 × 30 + 67.5 × 28 + 80 × 52) / 144	M1		Using mid points attempt, i.e. 44.5, 45, 45.5, in correct mean formula, unsimplified, no cfs, condone 1 error.
= 9675 / 144 = 67.2 kg	A1		Correct mean
Var $(45^2 \times 12 + 55^2 \times 22 + 62.5^2 \times 30 + 67.5^2 \times 28 + 80^2 \times 52) / 144 - (9675/144)^2 = 127.59$	M1		Substituting their mid-pts squared (may be class widths, lower or upper bound) in correct var formula even with cfs with their mean^2
sd = 11.3, allow 11.2	A1	**[6]**	Correct answer
7 (i)			
S(10) R(14) P(6)	M1		Summing 2 or more 3-factor options perms or combs
1 2 4 = 10C1×14C2×6C4= 13650 1 3 3 = 10C1×14C3×6C3= 72800 2 2 3 = 10C2×14C2×6C3= 81900	M1 B1		Mult 3 combs or 4 combs with $\Sigma r=7$ 2 options correct, unsimplified
Total = 168350 or 168000	A1	**[4]**	Correct answer
(ii) 2! × 2! × 5!	M1		2! × 2! oe, seen mult by an integer ≥1, no division
	M1		Mult by 5!, or 5! alone, seen mult by an integer ≥ 1 no division
= 480	A1	**[3]**	Correct answer
If M0 earned $\frac{2!\times2!}{2!\times2!}$ or $\frac{5!}{3!}$ or both, seen mult by an integer ≥ 1 Or 2!×2!×5! divided by a value	SCM1		
(iii) spaniels and retrievers in 4! ways gaps in 5P3 or 5 × 4 × 3 ways = 1440	M1 M1 A1	**[3]**	4! seen multiplied by an integer >1 Mult by 5P3 oe Correct answer
If M0 earned $\frac{4!}{2!\times2!}$ or $\frac{{}_5P_3}{3!}$ or both, seen multiplied by an integer > 1	SCM1		$_5C_3$ oe
or 7! – 5! × 3!	M1		oe
– {(4! × 2 × 4 × 3!) +	M1		oe, e.g. 6 × 5 × 4 × 4!
(4! × 3 × 4 × 3!)} = 1440	A1		
If M0 earned 3! × 2! × 2! used as a denominator in all 4 terms	SCM1		Marks cannot be earned from both methods.