Cambridge International A Level

MATHEMATICS **9709/61**

Paper 6 Probability & Statistics 2 **May/June 2023**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

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This document consists of **12** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M	Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
A	Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
B	Mark for a correct result or statement independent of method marks.
DM or **DB**	When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
FT	Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1(a)	20540/6012300 = 0.0034163	**B1**	
	[1000 × 0.0034163 = 3.4163]		
	Po(3.4163)	**B1**	Could be implied by expression seen.
	$e^{-their\ '3.4163'}(1 + 3.4163 + \frac{3.4163^2}{2!} + \frac{3.4163^3}{3!})$ OR $e^{-their\ '3.4163'}(1 + 3.4163 + 5.8356 + 6.6453)$ or 0.03283 + 0.1122 +0.1916 + 0.21819)	**M1**	Allow any λ. Allow with one end error. Must see expression.
	= 0.555 (3sf)	**A1**	CAO **SC** No working: B1 B1 (Po must be stated) B1 correct answer (max 3/4). **SC** Binomial: B1 B0 B1 correct answer (max 2/4).
		4	
1(b)	$n = 1000 > 50$	**B1**	Must show comparison with 50.
	$np = 3.4163 < 5$	**B1**	Must show comparison with 5.
		2	**SC B1**: $n > 50$ (or n large), $np < 5$. **SC B1**: n large, p small.

Question	Answer	Marks	Guidance
2(a)	$\frac{1}{2} \times 2 \times 1$ or $\int_0^2 \frac{1}{2}x\,dx = 1$, which is the correct area under a pdf.	**B1**	Calculation and result.
	$f(x) \geqslant 0$	**B1**	Condone $f(x) > 0$ or 'Line is above x-axis' OE.
		2	

Question	Answer	Marks	Guidance
2(b)	$\frac{1}{2}\pi r^2 = 1$	**M1**	Area of semi-circle equated to 1 OE. Missing factor of ½ gets M1A0.
	$r = \sqrt{\frac{2}{\pi}}$ or 0.798 (3sf)	**A1**	
		2	
2(c)(i)	Area to the left of 15 is greater than 0.5	**B1**	OE, e.g. 'The distribution of X is skewed to the right / positively skewed, suggesting the median will be less than the mid-point of the interval.' or 'The distribution of X is skewed to the right / positively skewed' or 'It is a decreasing function suggesting the median will be less than the mid-point of the interval'.
		1	
2(c)(ii)	$\int_{10}^{20} (\frac{40}{x} - \frac{x}{10})\mathrm{d}x$	**M1**	Integration of $x\mathrm{h}(x)$ attempted. Ignore limits.
	$\left[40\ln x - \frac{x^2}{20}\right]_{10}^{20}$	**A1**	Correct integration and limits (can be implied by final answer).
	= 40ln 2 – 15 or 12.7 (3sf)	**A1**	
		3	

Question	Answer	Marks	Guidance
3	Assume SD still = 5.2	**B1**	OE i.e. 'Assume the SD remains unchanged'.
	H$_0$: $\mu = 24.0$ H$_1$: $\mu > 24.0$	**B1**	Or population mean; not just mean.
	$\frac{25.8-24.0}{\frac{5.2}{\sqrt{50}}}$	**M1**	For standardising (could be implied). Must have $\sqrt{50}$.
	= 2.448	**A1**	Or $\mathrm{P}(\overline{X} > 25.8) = 0.0071$.
	'2.448' > 2.326	**M1**	Or 0.0071 < 0.01 . For valid comparison.
	[Reject H$_0$] There is evidence that (mean) amount of wheat is greater.	**A1FT**	OE. FT their z_{calc}. In context, not definite, eg not 'Mean amount of wheat is greater' No contradictions CV method: CV= 25.71 M1A1 25.71<25.8 M1 A1FT or CV=24.09 M1 A1 24.09>24 M1 A1FT.
		6	

Question	Answer	Marks	Guidance
4(a)	$z \times \sqrt{\frac{11.2}{n}} = 1.4076 \div 2$	**M1**	Any z, but must be a z.
	$z = 1.881$ or 1.882	**B1**	
	$[n = \left(\frac{1.881}{0.7038}\right)^2 \times 11.2]$ $n = 80$	**A1**	Must be a whole number.
		3	

Question	Answer	Marks	Guidance
4(b)	Jan, Feb and March not typical of whole year.	**B1**	Or, e.g., weather is different at different times of year.
		1	
4(c)	$0.94^3 \times 0.06 \times 4$	**M1**	
	= 0.199 (3 sf)	**A1**	
		2	

Question	Answer	Marks	Guidance
5(a)	$2.0^2 + 20 \times 3.4^2$	**M1**	
	= 235.2	**A1**	
		2	
5(b)	E(C – 3B) = 50 + 20×1010 – 3×6730 or 60	**B1**	
	Var(C – 3B) = ‘235.2’ + 9×15^2 or 2260.2	**M1**	FT *their* values from **(a)**.
	[C – 3B ~ N(‘60’, ‘2260.2’)] $= \frac{0-60}{\sqrt{2260.2}}$ [= –1.262]	**M1**	Standardising with their values (could be implied).
	1 – Φ(‘–1.262’) = Φ(‘1.262’)	**M1**	Probability area consistent with their values.
	= 0.897 (3 sf)	**A1**	
		5	

Question	**Answer**	**Marks**	**Guidance**
6	$\frac{5}{4}\left(\frac{1+2^2+6^2+1+a^2}{5}-\left(\frac{1+2+6+1+a}{5}\right)^2\right)=\frac{11}{2}$ or $\frac{1}{4}\left((42+a^2)-\frac{(10+a^2)}{5}\right)=\frac{11}{2}$	**M1***	OE attempted or e.g., $\frac{42+a^2}{5}-\left(\frac{10+a}{5}\right)^2=\frac{22}{5}$. Allow use of biased i.e., without $\frac{5}{4}$.
	$4a^2 - 20a + 0 = 0$ or $a^2 - 5a + 0 = 0$	**DM1**	Two- or three-term quadratic equation in a, with at least two terms correct.
	$a = 5$	**A1**	Ignore $a = 0$, if seen.
		3	

Question	**Answer**	**Marks**	**Guidance**
7(a)	H_0: $\lambda = 7.6$ [or 1.9] H_1: $\lambda < 7.6$ [or 1.9]	**B1**	Or Population mean = 7.6 or μ (not just ‘mean’). Or Population mean < 7.6 or μ.
		1	

Question	Answer	Marks	Guidance
7(b)	Mean = 7.6	**B1**	Seen.
	$P(X \leqslant 2) = e^{-7.6}\left(1 + 7.6 + \frac{7.6^2}{2}\right)$ [= 0.0188 or 0.0187]	**M1**	OE.
	$P(X \leqslant 3) = e^{-7.6}\left(1 + 7.6 + \frac{7.6^2}{2} + \frac{7.6^3}{3!}\right)$ [= 0.0554 or 0.0553]	**M1**	OE. Expression must be seen in at least one probability calculation.
	0.0188 or 0.0187 and 0.0554 or 0.0553	**A1**	A1 for both values.
	Critical region is $X \leqslant 2$	**A1**	Dep on both M marks. **SC** No Poisson expression seen in either prob scores B1 for 0.0188 or 0.0187 and B1 for 0.0554 or 0.0553 and B1 for CR.
	P(Type I error) = $P(X \leqslant 2)$ = 0.0188 or 0.0187 (3 sf)	**B1FT**	FT *their* $P(X \leqslant 2)$ or *their* CR.
		6	
7(c)	Concluding that the (mean) no. of accidents has reduced when it has not.	**B1**	OE. Must be in context. Accept: ‘It is believed that the booklet has helped to improve safety when actually it has not’.
		1	
7(d)	3 not in critical region.	**M1**	FT their CR or $P(X \leq 3) = 0.0554 > 0.05$.
	No evidence mean number of accidents has decreased.	**A1FT**	In context. Cannot be a definite statement, e.g., ‘mean number accidents has not decreased’.
		2	

Question	Answer	Marks	Guidance
7(e)	N(98.8, 98.8)	**B1**	May be implied.
	$\frac{100.5-98.8}{\sqrt{98.8}}$ [= 0.171]	**M1**	For standardising (could be implied by correct answer). Allow with wrong or no continuity correction.
	1 – Φ(‘0.171’)	**M1**	For probability area consistent with their working.
	= 0.432 (3 sf)	**A1**	
		4	

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WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	$0.23 \pm z \times \sqrt{\frac{0.23\times(1-0.23)}{200}}$	**M1**	Expression of correct form. Any z, but $z = 0.8328$ scores B0M0.
	$z = 1.811$ or 1.812	**B1**	
	0.176 to 0.284 (3 sf)	**A1**	Must be an interval.
		3	

Question	Answer	Marks	Guidance
2(a)	E(W) = Var(W).	**B1**	Allow 'they are the same' OE. Must be = not ≈ (and not both = and ≈). Condone E(W) =λ and Var(W)= λ.
		1	
2(b)	$np \approx np(1-p)$, hence $1-p$ must be close to 1	**B1**	OE. Must see formulae and $q = 1 - p$ must be seen or implied and conclusion made.
		1	

Question	Answer	Marks	Guidance
2(c)	$\lambda = 1.4$	**B1**	Seen.
	$1 - e^{-1.4}(1 + 1.4 + \frac{1.4^2}{2})$ or $1 - e^{-1.4}(1 + 1.4 + 0.98)$ or 1-(0.2466+0.3452+0.2417)	**M1**	Allow any λ; allow one end error. Expression must be seen (accept correct sigma notation).
	= 0.167 (3 sf) or 0.166	**A1**	Use of Binomial scores SCB1 for 0.167 or 0.166 . No working: 0.167 [or 0.166] **SC B1**. Note: λ=1.4 and 0.167 with no working seen scores **SC B1B1**. Use of Normal scores B0M0.
		3	

Question	Answer	Marks	Guidance
3(a)	Est $(\mu) = 3.25 = 13/4$ or 1625/500	**B1**	
	Est$(\sigma^2) = \frac{500}{499}(\frac{5663.5}{500} - "3.25"^2)$ or $\frac{1}{499}\left(5663.5 - \frac{1625^2}{500}\right)$	**M1**	Expression of correct form.
	= 0.766 (3 sf) or 1529/1996	**A1**	Biased variance of 0.7645 scores M0A0.
		3	

Question	Answer	Marks	Guidance
3(b)	H_0: Pop mean (or μ) = '3.25' H_1: Pop mean (or μ) ≠ '3.25'	**B1FT**	Not just 'mean'. FT their 3.25 .
	$\dfrac{2.95-\text{"}3.25\text{"}}{\sqrt{\text{"}0.766\text{"}\div 60}}$	**M1**	Standardising with their values. Must have √60.
	= –2.655	**A1**	Or $P(\overline{X} < 2.95) = 0.0039$ or 0.00396 or 0.00397 . **SC** FT their biased est(σ^2), i.e. 0.7645 to give $z = 2.658$ A1.
	'2.655' > 2.576 or '–2.655' < –2.576	**M1**	For valid comparison, e.g. 0.0039 or 0.00396 or 0.00397 < 0.005, or 0.0078 < 0.01, or 0.00792 < 0.01 .
	[Reject H_0] There is evidence that (mean) mass in (country B) is different (from country A).	**A1FT**	OE. Must be in context and not definite, e.g., not 'Mean mass is not different', No contradictions. Context needs either 'mass' or 'countries' OE.
			SC, Use of one-tail test. '2.655' > 2.326 or 0.0039 < 0.01 M1A0 (Max B0M1A1M1A0 3/5).
			Accept critical value method. Either: Xcrit=2.959 M1A1 2.95<2.959 M1A1FT with correct conclusion, or Xcrit=3.241 M1A1 3.25>3,241 M1A1FT with correct conclusion.
		5	

Question	Answer	Marks	Guidance
4(a)	Books received independently or singly or randomly.	**B1**	OE. Must be in context. If more than one condition given, ignore extras.
		1	

Question	Answer	Marks	Guidance
4(b)	$e^{-15.3} \times \frac{15.3^{10}}{10!}$	**M1**	Allow incorrect λ.
	= 0.0439 (3sf)	**A1**	**SC** No working shown but correct answer seen scores B1.
		2	
4(c)	N(153, 153)	**B1**	Seen or implied.
	$\frac{180.5-153}{\sqrt{153}}$ [= 2.223]	**M1**	For standardising with their values (can be implied). Allow with wrong or missing continuity correction.
	1 −ϕ(‘2.223’)	**M1**	For correct probability area consistent with their values.
	= 0.0131 (3sf)	**A1**	
		4	
4(d)	(λ =) 5.1 + 2.5 [= 7.6]	**B1**	Give at early stage (seen or implied).
	$1 - e^{-7.6}(1+7.6+\frac{7.6^2}{2}+\frac{7.6^3}{3!})= 1- e^{-7.6}(1+7.6+28.88+73.16)$ = 1 – (0.0005005+0.003803+0.01445+0.03661)	**M1**	Allow incorrect λ. Allow one end error. Must see an expression (accept correct sigma notation).
	= 0.945 (3sf)	**A1**	**SC** No working, 0.945 B1(could be implied) **SC B1**.
		3	

Question	Answer	Marks	Guidance
5(a)	E($X-Y$) = 1 Var($X-Y$) = 5	**B1**	Seen or implied, OE e.g. $X-Y-2$.
	$\frac{2-1}{\sqrt{5}}$ [= 0.447] $\frac{-2-1}{\sqrt{5}}$ [= –1.342]	**M1**	Standardising with their values must come from a combination.
	1 – Φ('0.447') Φ('–1.342') = 1 – Φ('1.342)	**M1**	Correct probability area consistent with their values.
	= 0.327 or 0.328 = 0.0898 or 0.0899	**A1**	Seen or implied.
	Probability that difference is more than 2 = 0.417 (3 sf) or 0.418	**A1**	
		5	
5(b)(i)	E(X) = 62 + 1.5(42) [= 125]	**B1**	OE.
	Var(X) = 158 + 1.5^2 × 108 [= 401]	**B1**	Correct expression OE.
	$\frac{90-\text{"125"}}{\sqrt{\text{"401"}}}$ [= – 1.748]	**M1**	Correct standardisation using their E(X) and Var(X). Must both be from a combination attempt. Ignore any attempted continuity correction.
	Φ('1.748')	**M1**	Correct probability area consistent with their stated values.
	= 0.960 or 96.0% (3 sf)	**A1**	Allow 0.96 or 96%.
		5	
5(b)(ii)	Unlikely. A candidate who does well in Theory is likely to do well in Practical.	**B1**	Need both. Accept 'unlikely', 'not independent', 'dependent', 'not realistic', or similar; and accept 'both testing knowledge from the same syllabus', 'theory and practical share same content' or similar statement.
		1	

Question	Answer	Marks	Guidance
6(a)	$(1-\frac{1}{3})^{10}$	**M1**	
	$= 0.0173$ (3 sf)	**A1**	No working scores **SC B1**.
		2	
6(b)	$1-(1-p)^{10} = 0.8926$	**M1**	Accept $1-q^{10} = 0.8926$. Equation must be in p or in q but not both.
	$1-p = 0.1074^{0.1}$ [= 0.800]	**M1**	For valid attempt to solve their (binomial) equation in p^{10} or q^{10}.
	$p = 0.200$ (3 sf) or 0.2	**A1**	
		3	

Question	Answer	Marks	Guidance
7(a)(i)	$\frac{1}{2}\times 4\times a = 1$	**M1**	For use of area = 1 or let $f(x) = kx$ and attempt $\int_0^4 kx \, dx = 1$.
	$[a = \frac{1}{2}]$ $f(x) = \frac{1}{8}x$	**A1**	$k\left[\frac{x^2}{2}\right]_0^4 = 1;\ 8k = 1;\ k = \frac{1}{8}$. $f(x) = \frac{1}{8}x$ or $k = \frac{1}{8}$.
		2	

Question	Answer	Marks	Guidance
7(a)(ii)	$\int_0^4 x \times \frac{1}{8} x \text{ d}x$	**M1**	Attempt to integrate $x \times$ *their* f(x). Ignore limits accept in terms of k.
	$\left[\frac{x^3}{24}\right]_0^4$	**A1ft**	Their integral and correct limits accept in terms of k.
	$= \frac{8}{3}$ or 2.67 (3 sf)	**A1**	Note: Final answer of 64k/3 scores 2/3.
		3	
7(b)	$\frac{a-1}{a} = \frac{1}{\sqrt{2}}$	**M1**	Or attempt $\int_0^1 \text{g}(w)\text{d}w = \frac{1}{2}$ i.e. $\int_0^1 (\frac{2}{a} - \frac{2}{a^2} w)\text{d}w = \frac{1}{2}$, or integral from 1 to a. g(w) must be linear of form g(w) = mw (+c). Or area attempt: attempt to calculate heights using their linear equation (h_1=2/a and h_2= –2/a^2 + 2/a) and use in either area trapezium = 0.5, or area trapezium =area small triangle or area small triangle = 0.5 . Area trapezium =1/2 × 1 (2/a + –2/a^2 + 2/a) Area triangle = 1/2(a – 1)(–2/a^2 + 2/a)) Note: alternative expression for h_1 = (a – 2)/(a – 1).
	$a\sqrt{2} - \sqrt{2} = a$	**A1**	Or $a^2 - 4a + 2 = 0$. Any correct equation in a, a not in denominator.
	$a = 2 + \sqrt{2} = 3.41$	**A1**	
		3	

Cambridge International A Level

MATHEMATICS **9709/63**

Paper 6 Probability & Statistics 2 **May/June 2023**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2023 series for most Cambridge IGCSE, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

This document consists of **14** printed pages.

Generic Marking Principles

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GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
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Mathematics Specific Marking Principles	
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2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M	Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
A	Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
B	Mark for a correct result or statement independent of method marks.
DM or **DB**	When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
FT	Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	$\frac{3}{2}\int_0^1 (x - x^3)\mathrm{d}x$	**M1**	Attempt to integrate $x\mathrm{f}(x)$; ignore limits.
	$= \frac{3}{2}\left[\frac{x^2}{2} - \frac{x^4}{4}\right]_0^1$	**A1**	Correct integration and limits.
	$= \frac{3}{8}$	**A1**	
		3	

Question	Answer	Marks	Guidance
2(a)	180, 227	**B1**	One correct. Ignore incorrect numbers.
		B1	Both correct and no extra numbers seen. (Allow other correct use of list of digits).
		2	
2(b)	These numbers are not independent of the previous numbers OR Only a finite number of digits used	**B1**	Already used these numbers, so therefore not random. Does not include numbers not in the list, therefore not random (not random or biased needs a reason).
		1	

Question	Answer	Marks	Guidance
3(a)	$z = 1.645$	**B1**	
	$z \times \dfrac{\sqrt{\frac{x}{100} \times (1 - \frac{x}{100})}}{100} = 0.07896$	**M1**	OE. Equation of correct form. Accept $p = x/100$. Any z. Allow missing factor of 2.
	$[x(100 - x) = 100^3 \times 0.07896^2 \div 1.645^2]$ $x^2 - 100x + 2304 = 0$	**A1**	Any correct (likely scalar multiple) three-term quadratic equation in x or p with simplified coefficients. Accept $p^2 - p + 0.2304 = 0$ or $p(1-p) = 0.2304$.
	$x = 36$ or 64	**A1**	
		4	
3(b)	$0.1^2 = 0.01$	**B1**	Accept either.
		1	

Question	Answer	Marks	Guidance
4	**Method 1: Based on mass**		
	Mean = 7 × 65.2 = 456.4	**B1**	
	Var = 7 × 3.6^2 [= 90.72]	**M1**	
	22 000/50 = 440 used in standardising equation	**M1**	
	$\frac{'440' - '456.4'}{\sqrt{'90.72'}}$ [= –1.722] no mixed methods	**M1**	For standardising with their values. No mixed methods.
	ϕ(–‘1.722’) = 1 – ϕ(‘1.722’)	**M1**	For correct probability area consistent with their values.
	= 0.0425 or 0.0426	**A1**	Note: accept alt method using per day. N(65.2, $\frac{3.6^2}{7}$). No mixed methods.
	Method 2: Based on profit		
	Mean = 7 × 65.2 × 50 = 22820	**B1**	
	Var = 7 × 3.6^2	**M1**	
	Var = 50^2 × ‘90.72’ [= 226800]	**M1**	
	$\frac{22000 - '22820'}{\sqrt{'226800'}}$ [= –1.722] no mixed methods	**M1**	For standardising with their values. No mixed methods.
	ϕ(–‘1.722’) = 1 – ϕ(‘1.722’)	**M1**	For correct probability area consistent with their values.
	= 0.0425 or 0.0426	**A1**	
		6	

Question	Answer	Marks	Guidance
5(a)	$\overline{x} = 1700/50 = 34$	**B1**	
	$\text{Est}(\sigma^2) = \frac{50}{49}\left(\frac{59050}{50} - 34^2\right)$ or $\frac{1}{49}\left(59050 - \frac{1700^2}{50}\right)$	**M1**	$\text{Est}(\sigma^2) = \frac{59050}{50} - 34^2$ biased scores M0.
	$= 25.5$ (3 sf) or $\frac{1250}{49}$	**A1**	= 25 scores A0.
		3	
5(b)	H$_0$: Population mean time = 32.4 H$_1$: Population mean time ≠ 32.4	**B1**	Not just ‘mean’ but allow just ‘μ’.
	$\frac{34 - 32.4}{\frac{\sqrt{'25.5'}}{\sqrt{50}}}$	**M1**	Must have $\sqrt{50}$ and not 50. FT *their* mean and var. Can be implied.
	= 2.24 (3 sf)	**A1**	or $P(\overline{T} > 34) = 0.0125$. **SC** use of biased var (25) $z = 2.26$ or $p = 0.0119$, allow M1A1.
	‘2.24’ < 2.326	**M1**	Or 0.0125 > 0.01 for a valid comparison.
	[Not reject H$_0$] Insufficient evidence that (mean) time has changed	**A1FT**	In context, not definite, e.g. not ‘Time not changed’. No contradictions. Note: accept CV method $x_{cri} = 34.06$ for M1A1. Compares 34 < 34.06 for M1, conclusion for A1. Condone $x = 32.34$ M1A1: compares 32.4 > 32.34 for M1, conclusion for A1.
		5	**SC** for using a one-tail method. Award max 3/5 (B0 M1 A1 M1 A0).

Question	Answer	Marks	Guidance
5(c)	Distribution of times in the population is normal	**B1**	Accept answers with no context here. Accept underlying distribution for population.
		1	

Question	Answer	Marks	Guidance
6(a)	$X \sim \text{Po}(2.5)$	**B1**	SOI.
	$e^{-2.5}(1 + 2.5 + \frac{2.5^2}{2} + \frac{2.5^3}{3!})$	**M1**	Any λ. Allow one end error.
	= 0.758 (3 sf)	**A1**	**SC** use of binomial B1 for 0.758. **SC** when no working is shown, $X \sim \text{Po}(2.5)$ seen scores B1, 0.758 seen also scores B1.
		3	
6(b)	$\text{E}(X) = \frac{5}{2}$ or 2.5 , $\text{Var}(X) = \frac{4999}{2000}$ or 2.4995	***B1**	Just an answer of 2.5 for the variance is not sufficient. However, 2.4995 is sufficient.
	These are almost equal	**DB1**	Condone ‘equal’.
		2	

Question	Answer	Marks	Guidance
7(a)	$\frac{1}{2}\pi\left(\sqrt{\frac{2}{\pi}}\right)^2$	**M1**	
	= 1, which is the area under a PDF [and f(x) ≥ 0]	**A1**	Result and statement are both needed.
		2	
7(b)	$\cos^{-1}\left(\frac{\sqrt{\frac{1}{\pi}}}{\sqrt{\frac{2}{\pi}}}\right)=\frac{\pi}{4}$	**B1**	AG. Accept alternative approaches, e.g. using Pythagoras, tangent, or isosceles right-angle triangles. Answer should be convincingly obtained and all correct.
	Area of sector = $\frac{1}{4}$	**B1**	
	Area of triangle AOB = $\frac{1}{2}OA\times OB=\frac{1}{2}\times\sqrt{\frac{1}{\pi}}\times\sqrt{\frac{2}{\pi}-\frac{1}{\pi}}$ or Area of triangle AOB = $\frac{1}{2}OA\times OB\times\sin(AOB)=\frac{1}{2}\times\sqrt{\frac{1}{\pi}}\times\sqrt{\frac{2}{\pi}}\sin\frac{\pi}{4}$	**M1**	Accept alternative approaches. Note: AB = $\sqrt{0.7979^2-0.5642^2}$ [= 0.5642] Allow values to 3sf.
	$\frac{1}{2\pi}$ or 0.1592	**A1**	
	'$\frac{1}{4}$' – '$\frac{1}{2\pi}$' or '0.25' – '0.1592'	**M1**	Attempt area of sector – area of triangle AOB.
	= $\frac{1}{4}-\frac{1}{2\pi}$ or 0.0908 (3sf)	**A1**	

Question	Answer	Marks	Guidance
7(b)	**Alternative Method for Question Q7(b): Using integration**		
	Find equation of curve $x^2 + y^2 = \frac{2}{\pi}$	**M1**	
	$y = \sqrt{\frac{2}{\pi} - x^2}$	**A1**	
	Attempt to integrate (any limits)	**M1**	
	Use of correct limits $\sqrt{\frac{1}{\pi}}$ to $\sqrt{\frac{2}{\pi}}$	**B1**	
	Correct integration with correct limits	**A1**	
	$= \frac{1}{4} - \frac{1}{2\pi}$ or 0.0908 (3sf)	**A1**	Correct final answer.
		6	

Question	Answer	Marks	Guidance
8(a)	H_0: Pop mean no. people = 3.03 or 1.01 (per 20 min) H_1: Pop mean no. people > 3.03 or 1.01 (per 20 min)	**B1**	These must not just be 'mean', but allow just 'λ' or 'μ'.
	Use of Po(3.03)	**M1**	
	$= 1 - e^{-3.03}(1 + 3.03 + \frac{3.03^2}{2} + \frac{3.03^3}{3!} + \frac{3.03^4}{4!} + \frac{3.03^5}{5!})$ $= 1 - e^{-3.03}(1 + 3.03 + 4.5905 + 4.6364 + 3.5120 + 2.128)$ $= 1 - (0.04832 + 0.1464 + 0.2218 + 0.2240 + 0.1697 + 0.1028)$	**M1**	Allow incorrect λ. Allow one end error. Must see Poisson expression used.
	= 0.0870 (3sf) [0.0869727]	**A1**	Allow 0.087 .
	0.0870 > 0.05	**M1**	For a valid comparison.
	(Do not reject H_0) Insufficient evidence to believe (mean) number of people has increased	**A1FT**	Conclusion stated must be in context, not definite and include no contradictions (e.g. not 'mean number people has not increased').
		6	If only $P(x = 6)$ award max 2/6 (single term not valid). **SC** No working B1 B2 M1 A1. Award maximum 5/6.

Question	Answer	Marks	Guidance
8(b)	$"0.0869727" - e^{-3.03} \times \frac{3.03^6}{6!}$ or $0.869727 - e^{-3.03}(1.0748)$ or 0.869727 – 0.05193 or $1 - e^{-3.03}(1 + 3.03 + \frac{3.03^2}{2} + \frac{3.03^3}{3!} + \frac{3.03^4}{4!} + \frac{3.03^5}{5!} + \frac{3.03^6}{6!})$	**M1**	OE. Must see Poisson expression (may be in part (a)).
	0.0350 or 0.0351	**A1**	Accept 0.035. **SC** no working seen, award B1 for 0.0350, 0.0351 or 0.035.
		2	
8(c)	Concluding that the (mean) number of people (using the path per 20 mins in the evening) has increased when it has not	**B1**	OE. Conclusion must be in context.
		1	
8(d)	A value for the true mean	**B1**	Allow without context for this mark.
	Number of people using the path per 20 mins in the evening.	**B1**	Condone equivalent comment on three randomly chosen 20-minute periods.
		2	

Cambridge International A Level

MATHEMATICS **9709/62**

Paper 6 Probability and Statistics 2 **February/March 2023**

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SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1(a)	$\left[\frac{49}{140} = 0.35\right]$		
	$0.35 \pm z\sqrt{\frac{0.35(1-0.35)}{140}}$	**M1**	Use of formula of correct form, ft *their* $\frac{49}{140}$, any z (not a probability).
	$z = 2.326$	**B1**	Accept 2.326 to 2.329 .
	Confidence interval = 0.256 to 0.444 (3 sf)	**A1**	Must be an interval.
		3	
1(b)	Find a smaller **percentage** confidence interval/ lower **level** of confidence	**B1**	ISW if 2 reasons given. Just saying 'use smaller z' oe B0. Accept a correct example e.g. 90% (even if not qualified with statement).
		1	

Question	Answer	Marks	Guidance
2(a)	Orders arrive at constant mean rate (must say mean or rate) Orders arrive at random Orders arrive independently Orders arrive singly		Must be in context (accept 25.2 as context).
		B1	Any one reason correctly stated.
		B1	A second reason correctly stated.
			SC B1: both correct, not in context.
		2	

Question	Answer	Marks	Guidance
2(b)(i)	$\lambda = \frac{3}{8} \times 25.2$ [= 9.45]	**B1**	
	$e^{-\text{'}9.45\text{'}}(\frac{9.45^3}{3!} + \frac{9.45^4}{4!} + \frac{9.45^5}{5!})$ **or** $e^{-\text{'}9.45\text{'}}(140.65 + 332.29 + 628.03)$ **or** 0.01107 + 0.02615 + 0.04942	**M1**	Allow any λ. Allow end errors. Expression must be seen.
	= 0.0866 (3 sf)	**A1**	If M0 allow **SC B1** for 0.0866 no working seen.
		3	
2(b)(ii)	$e^{-3.15} \times 3.15$ **or** $(1 - e^{-3.15}(1 + 3.15))$ **or** 0.135 **or** 0.822 (3 sf)	**B1**	
	$e^{-3.15} \times 3.15 \times (1 - e^{-3.15}(1 + 3.15))$	**M1**	M1 for product of two Poisson probabilities P(1) × (1– P(0,1)) (no end errors accepted). Accept any λ.
	× 2 or 0.111 × 2	**M1**	M1 for *their* product of two Poisson probabilities (accept end errors) × 2. Accept any λ
	0.222 (3 sf)	**A1**	
		4	
2(c)	N(113.4, 113.4)	**B1**	SOI
	$\frac{120.5-113.4}{\sqrt{113.4}}$ [= 0.667]	**M1**	Standardise with *their* values. Allow wrong or no cc. Must have √.
	1 – ϕ(*their* ‘0.667’)	**M1**	For probability area consistent with *their* values.
	= 0.252 (3 sf)	**A1**	
		4	

Question	Answer	Marks	Guidance
3(a)	$1 - 2(a + b)$ **or** $1 - 2a$ **or** $0.5 - a - b$ **or** 1-(a+b) **or** a+a+b	**M1**	OE. Seen or implied – may be on the diagram (or for correct un-simplified final expression).
	$P(0.6 \le X \le 1.8) = 1 - 2a - b$	**A1**	Accept 1 – (2a + b).
		2	
3(b)(i)	$k\int_0^3 (9x^2 - 6x^3 + x^4)dx = 1$	**M1**	Attempt integrate f(x) ignore limits and '= 1'.
	$k\left[\frac{9x^3}{3} - \frac{6x^4}{4} + \frac{x^5}{5}\right]_0^3 = 1$	**A1**	Correct integration seen, correct limits.
	$k \times \frac{81}{10} = 1, k = \frac{10}{81}$	**A1**	AG. Convincingly obtained. No errors seen. (Must see integration).
		3	
3(b)(ii)	$\frac{10}{81}\int_0^3 (9x^4 - 6x^5 + x^6)dx$ $\left[\frac{10}{81}\left[\frac{9x^5}{5} - x^6 + \frac{x^7}{7}\right]_0^3\right]$ [= $\frac{18}{7}$ or 2.57...]	**M1**	Attempt integrate $x^2f(x)$ between 0 and 3 condone missing k. Must see integration or correct answer of 18/7 seen or implied.
	$\frac{18}{7}$ – '1.5'^{2}	**M1**	Their integral of $x^2f(x) - 1.5^2$ (or their mean^2).
	= $\frac{9}{28}$ or 0.321	**A1**	
		3	

Question	Answer	Marks	Guidance
4(a)	$e^{-5.7}(1 + 5.7 + \frac{5.7^2}{2!})$ **or** $e^{-5.7}(1 + 5.7 + 16.245)$ **or** 0.003346 + 0.01907 + 0.05436	**M1**	Allow one end error. Must see this expression.
	= 0.0768 (3 sf)	**A1**	**SC B1** for unsupported answer of 0.0768 .
		2	
4(b)	$e^{-0.9}(1 + 0.9 + \frac{0.9^2}{2!})$	**M1**	Attempted; allow one end error (must see expression).
	$= 1 - e^{-0.9}(1 + 0.9 + \frac{0.9^2}{2!}) = 1 - e^{-0.9}(1 + 0.9 + 0.405) = 1 - (0.4066 + 3659 + 0.1647)$	**A1**	Correct expression P($X \geqslant 3$) no end errors (must see expression).
	= 0.0629 (3 sf)	**A1**	**SC B2** for unsupported answer of 0.0629 .
		3	

Question	Answer	Marks	Guidance
5(a)	D = L – 2S E(D) = 410 – 2(206) = –2	**B1**	SOI. OE using 2S–L.
	$\text{Var}(D) = 3.6^2 + 4 \times 3.7^2$ [= 67.72]	**B1**	SOI
	$\frac{0-(-2)}{\sqrt{'67.72'}}$ [= 0.243]	**M1**	For standardising using *their* values.
	1 – ϕ(*their* '0.243')	**M1**	For probability area consistent with *their* values.
	= 0.404 (3 sf)	**A1**	As final answer.
		5	

Question	Answer	Marks	Guidance
5(b)	$T_L \sim N(4100, 10\times3.6^2)$ $T_S \sim N(4120, 20\times3.7^2)$	**B1**	One of N(4100, 129.6) or N(4120, 273.8) USED (unchanged) in a standardising equation.
	$\frac{4080-4100}{\sqrt{'129.6'}}$ (= –1.757) $\frac{4080-4120}{\sqrt{'273.8'}}$ (= –2.417)	**M1**	Standardising with either *their* N(4100, 129.6) or N(4120, 273.8) or their N(…,…) (could be from a combination).
	$1 - \phi$(‘–1.757’) = ϕ(1.757) $1 - \phi$(‘–2.417’) = ϕ(2.417)	**M1**	One area consistent with *their* working (could be from a combination). Do not ISW.
	= 0.9605 **or** 0.961 = 0.9921 **or** 0.9922 **or** 0.992	**A1**	Both of these correct. Do not ISW.
	0.6 × ‘*their* 0.9605’ + 0.4 × ‘*their* 0.9921’	**M1**	Must be using probabilities.
	= 0.973 (3 sf)	**A1**	
		6	

Question	Answer	Marks	Guidance
6(a)	He is expecting a **decrease** (in μ)	**B1**	OE
		1	
6(b)	$-2.02 < -1.96$	**M1**	For valid comparison. Allow 2.02 > 1.96 **or** 0.0217 < 0.025 **or** 0.9783 > 0.975
	(Reject H_0) There is evidence to suggest that this year’s (mean) time is less than 25	**A1**	OE (such as evidence to support Akash’s belief), in context, not definite. No contradictions.
		2	

Question	Answer	Marks	Guidance
6(c)	1 – ϕ(2.14) [= 0.0162]	**M1**	
	1.62	**A1**	Allow 1.62% or 1.6 or 1.6%.
	$\alpha \geqslant 1.62$ (3 sf)	**A1ft**	FT *their* 1.62 . Allow $\alpha \geqslant 1.62\%$ or 1.6 or 1.6%. Condone >.
		3	
6(d)	$\frac{24.8-m}{3.9\div\sqrt{10}}$	**M1**	For standardising.
	$\frac{24.8-m}{3.9\div\sqrt{10}} = -1.645$	**M1**	Equate *their* standardised value to −1.645 (signs must be consistent).
	$m = 25.4$ (3 sf)	**A1**	
		3	

Cambridge International AS & A Level

MATHEMATICS **9709/61**

Paper 6 Probability & Statistics 2 **May/June 2022**

MARK SCHEME

Maximum Mark: 50

Published

This document consists of **11** printed pages.

Generic Marking Principles

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- the standard of response required by a candidate as exemplified by the standardisation scripts.

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Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
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- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
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2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
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4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

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- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
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Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1(a)	Est(μ) = $\frac{2520}{200}$ [= 12.6]	**B1**	OE
	Est(σ^2) = $\frac{200}{199}\left(\frac{31582}{200} - \text{‘}12.6\text{’}^2\right)$ or $\frac{1}{199}\left(31852 - \frac{2520^2}{200}\right)$	**M1**	Allow M1 if $\frac{200}{199}$ omitted
	= 0.5025 or 0.503 or $\frac{100}{199}$	**A1**	CWO or σ = 0.7088 or 0.709
	z = 1.96	**B1**	
	‘12.6’ ± z × $\sqrt{\text{‘}0.5025\text{’} \div 200}$	**M1**	For expression of correct form Any z but must be z
	CI = 12.5 to 12.7 (3 sf)	**A1**	CWO Must be an interval Note: Use of biased can score maximum B1 M1 A0 B1 M1 A0
		6	
1(b)	0.95 × 40 [= 38]	**B1**	Give at early stage
		1	

Question	Answer	Marks	Guidance
2	H_0: P(correct) = $\frac{1}{6}$ H_1: P(correct) > $\frac{1}{6}$	**B1**	Allow $p = \frac{1}{6}$ Allow $p > \frac{1}{6}$
	$1 - ({}^{15}C_4 \times (\frac{5}{6})^{11} \times (\frac{1}{6})^4 + {}^{15}C_3 \times (\frac{5}{6})^{12} \times (\frac{1}{6})^3 + {}^{15}C_2 \times (\frac{5}{6})^{13} \times (\frac{1}{6})^2 + 15 \times (\frac{5}{6})^{14} \times \frac{1}{6} + (\frac{5}{6})^{15})$	**M1**	Expression must be seen Allow one end error
	0.0898 or 0.0897 (3 sf)	**A1**	SC if M0 scored allow SCB1 for 0.0898 or 0.0897
	$0.0898 < 0.1$	**M1**	Valid comparison For valid comparison with 0.9 ($0.9102 > 0.9$ seen the previous M1and A1 can be recovered
	[Reject H_0] There is evidence (at the 10% level) that Arvind can predict scores	**FTA1**	Not definite, e.g. not 'He can predict' or 'Claim true' In context and no contradictions
		5	

Question	Answer	Marks	Guidance
3	$D = X - (Y_1 + Y_2 + Y_3)$ OE $E(D) = 6.2 - 2.4 \times 3$ [= –1] OE	**B1**	Give at early stage
	$\text{Var}(D) = 0.36 + 3 \times 0.25$ [= 1.11]	**B1**	Give at early stage
	$\frac{0-(-1)}{\sqrt{\text{‘}1.11\text{’}}}$ [= 0.949]	**M1**	No standard deviation/variance mixes Var must come from a combination attempt
	$1 - \Phi(\text{‘}0.949\text{’})$	**M1**	Area consistent with *their* values
	= 0.171 (3 s.f.)	**A1**	
		5	

Question	Answer	Marks	Guidance
4(a)	$E(Y) = \frac{20}{4}\left[=5\right]$, $\text{Var}(Y) = 20 \times \frac{1}{4} \times \frac{3}{4}\left[=\frac{15}{4}\right]$	**B1**	Both OE, SOI
	$\text{Var}(X) = 2$	**B1**	SOI or standard deviation = $\sqrt{2}$
	$E(X - 3Y) = -13$	**B1**	
	$\text{Var}(X - 3Y) = 2 + 9 \times \text{‘}\frac{15}{4}\text{’}$ [= 35.75]	**M1**	Correct formula using *their* values
	Standard deviation of $(X - 3Y) = 5.98$ (3 s.f.) or $\frac{1}{2}\sqrt{143}$	**A1**	CWO
		5	

Question	Answer	Marks	Guidance
4(b)	(0, 0) and (1, 15)	**M1**	
	$e^{-2} \times (\frac{3}{4})^{20} + e^{-2} \times 2 \times {}^{20}C_{15}\,(\frac{3}{4})^5\,(\frac{1}{4})^{15}$	**M1**	
	0.000430 (3 sf)	**A1**	CWO (must have evidence of addition) Allow 0.00043 or $4.3(0) \times 10^{-4}$
		3	

Question	Answer	Marks	Guidance
5(a)	$\lambda = 4.5$	**B1**	
	$1 - e^{-4.5}\,(1 + 4.5 + \frac{4.5^2}{2!} + \frac{4.5^3}{3!} + \frac{4.5^4}{4!})$	**M1**	Allow one end error Allow any λ. Poisson expressions must be seen
	= 0.468 (3 sf)	**A1**	If M0 awarded allow SC B1 for 0.468
		3	
5(b)	$\lambda = 162$ $(X \sim \text{Po}(162) \Rightarrow X \sim \text{N}(162, 162))$	**B1**	
	$\frac{149.5 - \text{‘}162\text{’}}{\sqrt{\text{‘}162\text{’}}}$ and $\frac{160.5 - \text{‘}162\text{’}}{\sqrt{\text{‘}162\text{’}}}$ (= –0.982 and –0.118)	**M1**	One of these; allow with incorrect or no continuity correction
	$\Phi(\text{‘}0.982\text{’}) - \phi(\text{‘}0.118\text{’})$ oe	**M1**	Area consistent with *their* values (both standardisations must be seen)
	= 0.290 (3 sf)	**A1**	Allow 0.29
		4	

Question	Answer	Marks	Guidance
5(c)	$\lambda = \frac{13.5}{6} + 3.6 \times \frac{2}{3}$ OE or 4.65	**M1**	Attempt to find λ
	$e^{-4.65}(\frac{4.65^4}{4!} + \frac{4.65^5}{5!} + \frac{4.65^6}{6!})$	**M1**	Allow any λ Allow one end error Poisson terms not be seen
	0.494 (3 sf)	**A1**	If M0 allow SC B1 for 0.494
		3	
6(a)	$\frac{a}{2}$	**B1**	
		1	
6(b)	$\frac{1}{4}$	**B1**	
		1	
6(c)	$\mathrm{f}(x) = \frac{1}{a}$	**B1**	SOI (may be seen in part **(a)** or part **(b)**)
	$\mathrm{E}(X) = \frac{a}{2}$	**B1**	SOI
	$\int_0^a \frac{1}{a} x^2 \mathrm{d}x$	**M1**	Attempt integrate *their* f(*x*) × x^2 with correct limits
	$= \left[\frac{x^3}{3a}\right]_0^a = \frac{a^2}{3}$	**A1**	
	$\frac{a^2}{3} - (\frac{a}{2})^2$ or $\frac{a^2}{3} - \frac{a^2}{4}$ [= $\frac{a^2}{12}$ **AG**]	**A1**	Must see previous line and answer No errors seen
		5	

Question	Answer	Marks	Guidance
6(d)	$\text{P}(X < \frac{b}{3}) = \frac{p}{3}$	**M1**	SOI (could be on a diagram) OR by integration: prob = 1-(2/3)(b/a)
	$\text{P}(\frac{b}{3} < X < a - .\frac{b}{3}) = 1 - \frac{2p}{3}$	**A1**	
		2	

Question	Answer	Marks	Guidance
7(a)	H_0: pop mean run time = 28.2 mins H_1: pop mean run time < 28.2 mins	**B1**	Allow 'μ'. Not 'mean journey time'
		1	
7(b)	$\frac{27-28.2}{4/\sqrt{40}}$ [= –1.897]	**M1**	For standardising Must have √40
	Φ(< '–1.897') = 1 – Φ('1.897')	**M1**	For correct area consistent with these values
	0.0289 (3 sf)	**A1**	
		3	
7(c)	H_0 is not rejected so…	**M1**	
	Type II error can be made and Type I error cannot be made	**A1**	Both needed (accept 'only a Type II error could be made')
		2	

Cambridge International AS & A Level

MATHEMATICS **9709/62**

Paper 6 Probability & Statistics 2 **May/June 2022**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2022 series for most Cambridge IGCSE, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

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WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1(a)	$72.3 \pm z\sqrt{\frac{64.3}{50}}$	**M1**	Expression of correct form (allow only one side for M1). Must be a *z* value
	$z = 1.751$	**B1**	Accept 1.75 if nothing better seen
	CI is 70.3 to 74.3 metres (3 s.f.)	**A1**	Allow without units Must be an interval
		3	
1(b)	Not random sample	**B1**	Need 'random' or 'not representative/biased because…' OE
		1	

Question	Answer	Marks	Guidance
2	H_0: Pop mean height = 2.3 H_1: Pop mean height > 2.3	**B1**	Not just 'mean' Allow μ
	$\dfrac{2.4-2.3}{\frac{0.4}{\sqrt{60}}}$	**M1**	For standardising, must have $\sqrt{60}$
	1.936 or 1.937 or 1.94	**A1**	
	'1.936' < 1.96	**M1**	Valid comparison with 1.96 Or 2.64% > 2.5% OE Accept 1.936 < 2.24 or 2.64% > 1.25% OE if $H_1\mu \neq 2.3$
	[Do not reject H_0] No evidence that (mean) height (with fertiliser) is more than without	**A1 FT**	FT *their z* In context, not definite. E.g. not 'Mean height is not greater' with no contradictions No FT for 2 tail test (max B0 M1 A1 M1 A0 3/5) Accept critical values method 2.401 (M1 A1) 2.4 < 2.401 (M1) Condone 2.299 (M1 A1) < 2.3 (M1) A1 conclusion
		5	

Question	Answer	Marks	Guidance
3(a)	Poisson	**B1**	SOI
	Mean = 3.6	**B1**	Can be awarded for N(3.6, …)
	$e^{-3.6}(1 + 3.6 + \frac{3.6^2}{2})$	**M1**	Allow any λ Allow one end error Expression must be seen
	0.303 (3 s.f.)	**A1**	If M0 awarded allow SC B1 for 0.303 SC Use of binomial: B1 for answer 0.300 (3 sf)
		4	
3(b)	[Binomial with] 200 > 50	**B1**	
	[200 × 0.018 =] 3.6 < 5 or [p =] 0.018 < 0.1	**B1**	If B0 B0 then SC n large, p small: B1 or n large $np < 5$: B1 or $n > 50$ and either $np < 5$ or $p < 0.1$: B1
		2	

Question	Answer	Marks	Guidance
4(a)	H_0: Pop mean = 4.6 [or 9.2] H_1: Pop mean < 4.6 [or 9.2]	**B1**	or $\lambda = 4.6$ or μ (Not just 'mean') or $\lambda < 4.6$
		1	

Question	Answer	Marks	Guidance
4(b)	Use of Poisson with $\lambda = 9.2$	**B1**	SOI
	$P(X \leqslant 3) = e^{-9.2}(1 + 9.2 + \frac{9.2^2}{2} + \frac{9.2^3}{3!}) = 0.0184$ or 0.018 [< 0.02] $P(X \leqslant 4) = 0.0184 + e^{-9.2} \times \frac{9.2^4}{4!} = 0.0486$ or 0.049 [> 0.02]	**M1**	At least one of these attempted correct λ (with Poisson expression seen not implied)
		***A1**	Both correct SC Use of $\lambda = 4.6$ scores B1 for $P(X = 0) = 0.01[0][1]$ and $P(X \leqslant 1) = 0.056[3]$only
	CR is $X \leqslant 3$	**DA1**	From CWO and at least one comparison seen SC If M0 awarded allow *B1 for both 0.018 and 0.049 or better and DB1 for correct critical region from CWO and at least one comparison seen.
		4	
4(c)	5 is not in critical region OR $P(X \leqslant 5) = 0.104 > 0.02$ so [not reject H_0] no evidence that number of cars arriving is now fewer	**M1** **A1 FT**	For a comparison (i.e. $5 > 3$) OE In context, not definite No contradictions e.g. not 'No. of cars arriving is not fewer' ft *their* critical region if used (but must be from Poisson and integers)
		2	
4(d)	No, because H_0 was not rejected	**B1 FT**	OE, FT *their* **(c)**
		1	

Question	Answer	Marks	Guidance
4(e)	N(276, 276)	**B1**	SOI
	$\frac{300.5-276}{\sqrt{276}}$ [= 1.475]	**M1**	Standardising with *their* values Allow with wrong or no continuity correction
	1 – ϕ('1.475') = 0.0701 (3 s.f.)	**A1**	SC Use of Poisson: B1 for answer 0.0727 (3 sf)
		3	

Question	Answer	Marks	Guidance
5(a)	$\frac{3}{16}\int_2^4 (4x^2 - x^3)\mathrm{d}x$	**M1**	Attempt to integrate xf(x) Ignore limits (must see a power increase for attempted integration)
	$= \frac{3}{16}\left[\frac{4x^3}{3} - \frac{x^4}{4}\right]_2^4$	**M1**	Attempt integrate xf(x) with correct limits (must see a power increase for attempted integration)
	$= \frac{3}{16}\ (\frac{256}{3} - 64 - (\frac{32}{3} - 4)\ ) = \frac{11}{4}$ **(AG)**	**A1**	Correct substitution of correct limits (at least 2 terms seen) and answer seen. No errors seen i.e. NO recovery of errors and no non-exact decimals (e.g. 21.33) seen
		3	

Question	Answer	Marks	Guidance
5(b)	$\frac{3}{16}\int_{2}^{4}(4x^3 - x^4)\mathrm{d}x$	***M1**	Attempt to integrate x^2 f(x) with correct limits (integration must be seen not implied. Must see a power increase for attempted integration)
	$=\frac{3}{16}\left[x^4 - \frac{x^5}{5}\right]_2^4$ [$=\frac{39}{5}$ or 7.8] Var(X) = '$\frac{39}{5}$' $-(\frac{11}{4})^2$	**DM1**	*their* $\int x^2\mathrm{f}(x)\mathrm{d}x - (\frac{11}{4})^2$, with $\int x^2\mathrm{f}(x)\mathrm{d}x$ evaluated, not necessarily simplified
	$=\frac{19}{80}$ or 0.2375 (or 0.238 (3 sf))	**A1**	SC If M0 then score B1 for $\frac{39}{5}$ and B1 for $\frac{19}{80}$
		3	
5(c)	$\frac{3}{16}\int_{2}^{3}(4x - x^2)\mathrm{d}x$	**M1**	Attempt to integrate correct integral and limits must see a power increase for attempted integration Oe (Integrate 3 to 4) OR ALTERNATIVE METHOD integrate from m to 4 or 2 to m and equate to 0.5 to obtain cubic (m^3 -6m^2 + 24 = 0 oe) (NB Integrating from m to 3 and equating to 0.5M0)
	$=\frac{3}{16}\left[2x^2 - \frac{x^3}{3}\right]_2^3$ $\left[=\frac{3}{16}\left(18-9-\left(8-\frac{8}{3}\right)\right)\right]$ $\left[=\frac{11}{16}\right]$ '$\frac{11}{16}$' $-\frac{1}{2}$	**M1**	*Their* $\int \mathrm{f}(x)\mathrm{d}x - \frac{1}{2}$ oe (1/2 – 5/16) OR ALTERNATIVE METHOD m obtained from cubic (m = 2.69459) and attempt to integrate f(x) from '*their m*' (2 < m < 4) to 3 must see a power increase for attempted integration and limits substituted
	$\frac{3}{16}$ or 0.1875	**A1**	Condone 0.187 or 0.188
		3	

Question	Answer	Marks	Guidance
6(a)	$E(D) = 53 - (4 \times 14) = -3$	**B1**	OE Give at early stage
	$\text{Var}(D) = 11 + 4^2 \times 3$ [= 59]	**B1**	or $\sqrt{(11+4^2\times3)}$ (= 7.68 (3 s.f.)) Give at early stage
	$\dfrac{0-(-3)}{\sqrt{59}}$ [= 0.391]	**M1**	For standardising with *their* values (var must be from a combination attempt) Ignore continuity correction attempts
	$1 - \Phi$(‘0.391’)	**M1**	For area consistent with *their* values
	0.348 (3 s.f.)	**A1**	As final answer
		5	
6(b)	$E(T) = 12 \times 53 + 25 \times 14$ [= 986]	**B1**	Give at early stage (N.B. accept $E(T - 1000) = -14$)
	$\text{Var}(T) = 12 \times 11 + 25 \times 3$ [= 207]	**B1**	Or $\sqrt{(12\times11+25\times3)}$ (= 14.4 (3sf)) Give at early stage
	$\dfrac{1000-986}{\sqrt{207}}$ [= 0.973]	**M1**	For standardising with *their* values (var must be from a combination attempt) Ignore continuity correction attempts
	Φ(‘0.973’)	**M1**	For area consistent with *their* values
	0.835 (3 sf)	**A1**	As final answer
		5	

Question	Answer	Marks	Guidance
7	$\bar{X} \sim \text{N}(2.9, \frac{2.9}{100})$ OR Totals method N(290, 290)	**B1**	B1 for N(2.9, …) OR N(290, …)
		B1	B1 for Var = $\frac{2.9}{100}$ OR for var = 290 SOI
	$\frac{2.88-2.90}{\sqrt{\frac{2.9}{100}}}$ [= –0.1174] OR $\frac{288-290}{\sqrt{290}}$	**M1**	Standardising with *their* values Allow without –ve sign AND/OR with incorrect continuity correction No mixed methods
	1– Φ(‘0.1174’)	**M1**	For area consistent with *their* values
	0.453 (3 sf)	**A1**	As final answer
	Alternative method for question 7		
	$\bar{X} \sim \text{N}(2.9, \frac{2.9}{100})$ OR Totals method N(290, 290)	**B1**	B1 for N(2.9, …) OR N(290, …)
		B1	B1 for Var = $\frac{2.9}{100}$ OR Var = 290 stated or implied
	$\frac{(2.88-\frac{1}{200})-2.90}{\sqrt{\frac{2.9}{100}}}$ [= –0.1468] OR (287.5 – 290)/√290	**M1**	Standardising with *their* values Allow without –ve sign AND/OR with incorrect continuity correction No mixed methods
	1 – ϕ(‘0.1468’)	**M1**	For area consistent with *their* values
	0.442 (3 sf)	**A1**	As final answer
		5	

Cambridge International A Level

MATHEMATICS **9709/63**

Paper 6 Probability & Statistics 2 **May/June 2022**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2022 series for most Cambridge IGCSE, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

This document consists of **11** printed pages.

Generic Marking Principles

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GENERIC MARKING PRINCIPLE 1:
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- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M	Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
A	Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
B	Mark for a correct result or statement independent of method marks.
DM or **DB**	When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
FT	Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	$\dfrac{1300+\frac{1}{200}-1250}{\frac{480}{10}}$ or $\dfrac{1300-1250}{\frac{480}{10}}$ [= 1.042]	**M1**	Allow with incorrect or omitted continuity correction Must have 10 Accept totals method
	1 – Φ('1.042')	**M1**	For area consistent with *their* values
	0.149 (3 s.f.)	**A1**	
		3	

Question	Answer	Marks	Guidance
2(a)	Conclude more than 10% of the students are left handed when this is not true	**B1**	OE. Must be in context (accept use of p). Need the context of one tail test.
		1	
2(b)	$1 - (0.9^{20} + 20 \times 0.9^{19} \times 0.1 + {}^{20}C_2 \times 0.9^{18} \times 0.1^2 + {}^{20}C_3 \times 0.9^{17} \times 0.1^3 + {}^{20}C_4 \times 0.9^{16} \times 0.1^4)$	**M2**	M2: fully correct M1: attempt 1 – P(X = 0, 1, 2, 3, 4); allow 1 – P(X = 0,1,2,3,4,5) or 1 – P(X = 0,1,2,3) need 1 – … the method mark cannot be implied
	0.0432 (3 s.f.)	**A1**	If M0 awarded allow SC B2 for 0.0432
		3	

Question	Answer	Marks	Guidance
2(c)	$0.7^{20} + 20 \times 0.7^{19} \times 0.3 + {}^{20}C_2 \times 0.7^{18} \times 0.3^2 + {}^{20}C_3 \times 0.7^{17} \times 0.3^3 + {}^{20}C_4 \times 0.7^{16} \times 0.3^4$	**M1**	Attempt to find $P(\leqslant 4)$ using B(20,0.3) Allow one end error The method mark cannot be implied
	0.238 or 0.237 (3 s.f.)	**A1**	If M0 awarded allow SC B1for 0.238 or 0.237
		2	

Question	Answer	Marks	Guidance
3(a)	Batteries unusable after testing or Population too big or too costly or too time consuming to use the whole population oe	**B1**	
		1	
3(b)	$H_0: \mu = 150$ $H_1: \mu < 150$	**B1**	Or population mean = 150; not just ‘mean’ = 150
	$\frac{147-150}{\sqrt{225} \div \sqrt{120}}$	**M1**	Allow with continuity correction Need $\sqrt{120}$
	–2.191	**A1**	Condone – 2.19
	–2.191 < –2.054 [or –2.055]	**M1**	OE. For valid comparison with 2.054 or 2.055 Or 0.0143 (or 0.0142) < 0.02 For two tail test allow comp –2.326 OE if $H_1: \mu \neq 150$ (can score B0M1A1M1A0 max 3/5)
	[Reject Ho] There is evidence that the (mean) life of type *B* is less than type *A* (or less than 150)	**A1 FT**	In context, not definite with no contradictions Accept critical value method 147.19 M1A1 147 < 147.19 M1 conclusion A1 Or 150 > 149.81
		5	

Question	Answer	Marks	Guidance
3(c)	$147 \pm z \times \frac{15}{\sqrt{120}}$	**M1**	Expression of correct form must be a *z* value
	$z = 1.881$ [or 1.882]	**B1**	
	144 to 150 (3 s.f.)	**A1**	Must be an interval Incorrect *z* value can only score M1B0A0
		3	

Question	Answer	Marks	Guidance
4(a)	T~N(515, 74)	**B1**	B1 for N(515, ..) give at early stage
		B1	B1 for Var = 45 + 25 + 4 = 74 give at early stage
	$\frac{500-'515'}{\sqrt{'74'}}$ [= –1.744]	**M1**	Standardise with *their* values. No standard deviation/variance mix Need combination for variance. Allow continuity correction.
	Φ(’1.744’)	**M1**	Area consistent with *their* working
	0.959 or 0.96[0] (3 s.f.)	**A1**	
		5	

Question	Answer	Marks	Guidance
4(b)	$E(S - 1.4R) = 300\text{-}1.4\text{x}200 = 20$	**B1**	Give at early stage
	$\text{Var}(S - 1.4R) = 45 + 1.4^2 \times 25 = 94$	**B1**	Give at early stage SC: if B0B0 awarded allow SC B1 for 14 and 105.84
	$\frac{0-(20)}{\sqrt{'94'}}$ [= –2.063]	**M1**	Standardise with *their* values. No standard deviation/variance mix. Need combination for variance.
	$1 - \Phi(\text{‘}2.063\text{’})$	**M1**	Area consistent with *their* working
	0.0196 (3 s.f.)	**A1**	
		5	

Question	Answer	Marks	Guidance
5(a)	$\lambda = 6.6$	**B1**	
	$e^{-6.6} \times \frac{6.6^6}{6!}$	**M1**	Any λ
	0.156 (3 s.f.)	**A1**	If M0 awarded SC B1 for 0.156
		3	
5(b)	$1 - e^{-2.2}(1 + 2.2 + \frac{2.2^2}{2} + \frac{2.2^3}{3!} + \frac{2.2^4}{4!})$	**M1**	Allow one end error. Need 1 – … Any λ
	0.0725 (3 s.f.)	**A1**	If M0 awarded SC B1 for 0.0725
		2	

Question	Answer	Marks	Guidance
5(c)	N(26.4, 26.4)	**B1**	Give at early stage 2.2 × 12
	$\frac{19.5-'26.4'}{\sqrt{'26.4'}}$ [= –1.343]	**M1**	Standardising with *their* values. Allow wrong or no continuity correction
	$\phi('-1.343') = 1 - \phi('1.343')$	**M1**	Area consistent with *their* working
	0.0897 or 0.0896 (3 s.f.)	**A1**	
		4	

Question	Answer	Marks	Guidance
6(a)	$\frac{13+a}{5}$	**B1**	Accept $\frac{2+3+3+5+a}{5}$. Do not ignore subsequent working
		1	
6(b)	$\frac{5}{4}\left(\frac{47+a^2}{5}-\left(\frac{13+a}{5}\right)^2\right)=4$ or $\frac{1}{4}\left(47+a^2-\frac{(13+a)^2}{5}\right)=4$	**M1**	Use of correct formula using *their* value from **(a)**, in terms of a, and equate to 4
	$2a^2 - 13a - 7 = 0$	**A1**	Any correct three-term quadratic equation rearranged to a form ready to solve
	$a = 7$	**A1**	Condone the other value of a $(-\frac{1}{2})$
		3	

Question	Answer	Marks	Guidance
7(a)(i)	1	**B1**	no ambiguity
		1	
7(a)(ii)	$\frac{1}{2}$	**B1**	No ambiguity
		1	
7(a)(iii)	$[q =]\ \frac{1}{2}p$	**B1**	Accept $2q = p$
		1	
7(b)	$p\int_0^a (a^2 - x^2)\mathrm{d}x\ = 1$	**M1**	Attempt to integrate $\mathrm{f}(x)$ and equated to 1
	$\frac{2}{3}a^3 p\ = 1$	**A1**	OE, simplified
	$"\frac{3}{2a^3}"\int_0^a (a^2x - x^3)\mathrm{d}x = 3$ or $"\frac{3}{2a^3}"\int_0^a (a^2x - x^3)\mathrm{d}x\ = 3$	**M1**	Attempt to integrate $x\mathrm{f}(x)$, with multiplier p or $\frac{3}{2a^3}$ or *their* p, and equate to 3
	$p \times \frac{a^4}{4}\ = 3$	**A1**	May be implied by next line
	$"\frac{3}{2a^3}" \times \frac{a^4}{4}\ = 3$	**M1**	OE. Substitute from one equation into the other. FT *their* equations
	$a = 8$	**A1**	
		6	

Cambridge Assessment
International Education

Cambridge International A Level

MATHEMATICS **9709/62**

Paper 6 Probability and Statistics 2 **February/March 2022**

MARK SCHEME

Maximum Mark: 50

Published

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5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Question	Answer	Marks	Guidance
1(a)	Est (μ) = $\frac{1199}{6}$ or 199.833 or 200 or $\frac{2398}{12}$ [mm]	**B1**	Accept in any form
	Est (σ^2) = $\frac{12}{11}\left(\frac{479226}{12}-\frac{'1199'^2}{6}\right)$ or $\frac{1}{11}\left('479226'-\frac{'2398'^2}{6}\right)$	**M1**	Use of their values in correct formula (may be implied)
	= 2.33 (3 sf) [mm^2]	**A1**	Accept $\frac{7}{3}$
		3	
1(b)	Small sample	**B1**	Accept not ‘not representative’ unless qualified.
		1	

Question	Answer	Marks	Guidance
2	B(300, $\frac{1}{5}$) → N(60,48)	**B1**	SOI
	$\frac{45.5-60}{\sqrt{48}}$	**M1**	Condone with wrong or no continuity correction
	= –2.093	**A1**	
	‘2.093’ > 1.96	**M1**	Valid comparison Note: ϕ('–2.093') (= 0.0182), 0.0182< 0.025
	[Evidence to reject H_0] There is evidence that P(landing on blue) ≠ $\frac{1}{5}$	**A1 FT**	Allow ‘There is evidence that the spinner is biased.’ In context, not definite, no contradictions Condone critical values method (critical value 46.42 M1 A1 and 45.5 < ‘46.42’ M1 for valid comparison A1 for correct conclusion)
			SC: 0.0182 unsupported: 0.0182 < 0.025 And there is evidence that the spinner is biased. In context, not definite B1 only
		5	

Question	Answer	Marks	Guidance
3	est(p) = 0.4	**B1**	
	$'0.4' + z\sqrt{\frac{'0.4'\times(1-'0.4')}{500}}\left[=0.445\right]$	**M1**	OE Use of *their* 0.4 in a correct expression
	$z\left[=0.045\div\sqrt{\frac{'0.4'\times(1-'0.4')}{500}}\right]=2.054$	**A1**	Condone 2.053 and 2.05
	0.98 – (1 – 0.98)	**M1**	
	96% confidence	**A1**	CWO, must be integer
		5	

Question	Answer	Marks	Guidance
4(a)	H_0: $\mu = 25.5$ H_1: $\mu < 25.5$	**B1**	
	$\frac{23.7 - 25.5}{5.2 \div \sqrt{40}}$	**M1**	Must have √40
	= –2.189	**A1**	
	'2.189' < 2.326	**M1**	For valid comparison For two-tailed test: allow compare 2.576 if H_1: $\mu \neq 25.5$
	[Accept H_0] No evidence that mean time has decreased	**A1 FT**	In context, not definite, no contradictions FT *their* 2.189 but no FT for two-tailed test **N.B.** Use of two-tailed test can score max B0 M1 A1 M1 A0 Condone use of critical value method (23.59 M1 A1 and 23.7 > 23.59 M1 A1 correct conclusion or 25.612 M1 A1 and 25.5 < 25.612 M1 A1 with correct conclusion)
		5	
4(b)	No, because H_0 was not rejected	**B1 FT**	FT *their* conclusion in **(a)**
		1	

Question	Answer	Marks	Guidance
5(a)	Mean = 5×18.3 and Variance = 5×2.5^2 [= N(91.5, 31.25)]	**B1**	SOI
	$\frac{95-'91.5}{\sqrt{'31.25'}}$ [= 0.626]	**M1**	FT *their* mean and variance
	1 – Φ(‘0.626’)	**M1**	For finding area consistent with *their* values
	0.266 (3 sf)	**A1**	
		4	
5(b)	E(*D*) = 0	**B1**	Or E(D – 1) = –1
	Var(*D*) = 2.5^2 × 2 [= 12.5]	**B1**	
	$\frac{1-0}{\sqrt{'12.5'}}$ [= 0.283] or $\frac{-1-0}{\sqrt{'12.5'}}$ [= –0.283]	**M1**	FT *their* E and Var
	Φ(‘0.283’) – (1 – ϕ(0.283)) [= 0.6115 – 0.3885]	**M1**	For finding area consistent with *their* values
	0.223 (3 sf)	**A1**	
		5	

Question	Answer	Marks	Guidance
6(a)	Quadratic curve, hence symmetrical	**B1**	OE. Allow sketch and 'symmetrical' or just 'curve symmetrical'
		1	
6(b)	$-k\int_1^3 (x^2 - 4x + 3)\mathrm{d}x = 1$	**M1**	Attempt to integrate f(x) and '= 1'. Ignore limits at this stage
	$-k\left[\frac{x^3}{3} - 2x^2 + 3x\right]_1^3$	**A1**	Fully correct expression (correct integration and limits)
	$-k\times\left[0-\frac{4}{3}\right]=1$ or $k\times\frac{4}{3}=1$ $\left[k=\frac{3}{4}\right]$	**A1**	AG, OE. Correctly substitute limits and '= 1' and correctly obtain result with no errors seen.
		3	
6(c)	$-\frac{3}{4}\int_1^3\left(x^4-4x^3+3x^2\right)\mathrm{d}x$	**M1**	Attempt to integrate x^2 f(x) from 1 to 3
	$-\frac{3}{4}\times\left[\frac{x^5}{5}-x^4+x^3\right]_1^3$ $\left[=\frac{3}{4}\times\frac{28}{5}=\frac{21}{5}\right]$	**A1**	Correct integration and limits
	$\left[\frac{21}{5}-2^2\right]=0.2$	**A1**	
		3	

Question	Answer	Marks	Guidance
6(d)	$-\frac{3}{4}\int_{2.5}^{3}\left(x^2-4x+3\right)\mathrm{d}x$	**M1**	OE. Attempt to integrate f(x), from 2.5 to 3 (or 1 to 2.5)
	$=-\frac{3}{4}\times\left[\frac{x^3}{3}-2x^2+3x\right]_{2.5}^{3}=\frac{5}{32}$ or 0.15625	**A1**	
	$1-\left(1-'\frac{5}{32}'\right)^3$	**M1**	OE. FT *their* $\frac{5}{32}$.
	= 0.399 (3 sf)	**A1**	
		4	

Question	Answer	Marks	Guidance
7(a)(i)	0.024×50 [= 1.2] and 0.018×60 [= 1.08]	**B1**	
	$(1 - e^{-1.2}(1 + 1.2)) \times (1 - e^{-1.08}(1 + 1.08))$	**M1**	For $(1 - e^{-\lambda}(1+\lambda)) \times (1 - e^{-\mu}(1+\mu))$ any λ, μ ($\lambda \neq \mu$) Allow one end error on either or both terms
	= 0.0991 (3 sf)	**A1**	Unsupported answer scores maximum **SC** B1 B1 **SC** Use of binomial 0.0994 scores B1 only
		3	
7(a)(ii)	$\lambda = 0.024 \times 50 + 0.018 \times 60$	**M1**	or *their* 1.2 + 1.08 (NB 0.024+0.018 is M0)
	$1 - e^{-2.28} \times \left(1 + 2.28 + \frac{2.28^2}{2!} + \frac{2.28^3}{3!}\right)$	**M1**	any λ and allow one end error
	= 0.197 (3 sf)	**A1**	Unsupported answer scores maximum **SC** B2
		3	
7(b)	$e^{-\lambda} = \left[e^{-\mu}\right]^2 = e^{-2\mu}$	**M1**	
	$e^{-\lambda} \times \frac{\lambda^2}{2} = k\left[e^{-\mu} \times \mu\right]^2$	**M1**	
	$e^{-2\mu} \times 2\mu^2 = k \times e^{-2\mu} \times \mu^2$	**M1**	OE. Use of $\lambda = 2\mu$ to find equation in μ and k only (or λ and k only)
	$k = 2$	**A1**	
		4	

Cambridge Assessment
International Education

Cambridge International AS & A Level

MATHEMATICS **9709/61**

Paper 6 Probability & Statistics 2 **October/November 2021**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2021 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

This document consists of **11** printed pages.

Generic Marking Principles

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Marks awarded are always **whole marks** (not half marks, or other fractions).

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- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M	Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
A	Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
B	Mark for a correct result or statement independent of method marks.
DM or **DB**	When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
FT	Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1(a)	N(12.5, ….)	**B1**	
	Variance = 0.4096	**B1**	Accept 0.410 (3sf), condone $\frac{10.24}{25}$
		2	
1(b)	$\frac{13-'12.5'}{\sqrt{'0.4096'}}$ [= 0.781]	**M1**	For standardising with *their* values. Accept standardising with 12.
	ϕ(‘0.781’) – (1 – Φ(‘0.781’))	**M1**	For attempting to find *their* central area.
	0.565 (3sf)	**A1**	
		3	

Question	Answer	Marks	Guidance
2	N(45.2, 45.2)	**B1**	SOI
	$\frac{60.5-45.2}{\sqrt{45.2}}$ [= 2.276]	**M1**	Allow with wrong or no continuity correction.
	1 –ϕ(‘2.276’)	**M1**	
	0.0114	**A1**	
		4	

Question	Answer	Marks	Guidance
3	est(p) = 0.2 accept $\frac{15}{75}$	**B1**	SOI
	$2 \times z \times \sqrt{\frac{0.2 \times 0.8}{75}} = 0.162$	**M1**	Expression of the correct form. Condone missing 2*x*.
	$z\left[= 0.081 \times \sqrt{\frac{75}{0.2 \times 0.8}}\right] = 1.754$	**A1**	Correct *z*. Condone 3sf accuracy.
	Φ(‘1.754’) = 0.96[03] '0.96' – (1 – '0.96')	**M1**	OE. Using *their z* to find alpha.
	$\alpha = 92$	**A1**	Following correct working.
		5	

Question	Answer	Marks	Guidance
4(a)	$\frac{1}{18}\int_0^{1.2}(9 - x^2)\mathrm{d}x$	**M1**	Attempt to integrate f(*x*), ignore limits. Must see an increase of power.
	$\frac{1}{18}\left[9x - \frac{x^3}{3}\right]_0^{1.2}$	**A1**	Correct integration and correct limits.
	$\frac{71}{125}$ or 0.568	**A1**	SC unsupported answer scores **B2** only.
		3	

Question	Answer	Marks	Guidance
4(b)	$\frac{1}{18}\int_0^3 (9x - x^3)\mathrm{d}x$	**M1**	Attempt to integrate $x\mathrm{f}(x)$, ignore limits. Must see an increase of power.
	$\frac{1}{18}\left[\frac{9x^2}{2} - \frac{x^4}{4}\right]_0^3$	**A1**	Correct integration and correct limits.
	$\frac{9}{8}$ or 1.125	**A1**	SC unsupported answer scores **B2** only.
		3	
4(c)	$\frac{1}{18}\left[9x - \frac{x^3}{3}\right]_0^m = 0.5$	**M1**	Attempt to integrate f(x) with correct limits and = 0.5. OE. Accept limits m to 3. Allow x instead of m.
	$\frac{1}{18}\left[9m - \frac{m^3}{3}\right] - 0.5 = 0$	**A1**	Any correct cubic equation in m or x.
	$m^3 - 27m + 27 = 0$	**A1**	AG. Correctly obtain this equation. No errors seen.
		3	

Question	Answer	Marks	Guidance
5(a)(i)	Po(0.025)	**B1**	For Poisson and correct parameter.
	$n = 2500 > 50$, $np = 0.025 < 5$	**B1**	Must show 2500 and 0.025. Accept p = $\frac{1}{100000} < 0.1$ in place of $np = 0.025 < 5$.
		2	

Question	Answer	Marks	Guidance
5(a)(ii)	$1 - e^{-0.025}$	**M1**	Allow any λ. FT *their* **(a)(i)** if normal; must have continuity correction.
	0.0247 (3sf)	**A1**	Must be from Poisson. Unsupported correct answer scores **B1** instead of **M1 A1**.
		2	
5(b)	H_0: $p = 0.3$ H_1: $p < 0.3$	**B1**	
	$0.7^{28} + 28 \times 0.7^{27} \times 0.3 + {}^{28}C_2 \times 0.7^{26} \times 0.3^2 + {}^{28}C_3 \times 0.7^{25} \times 0.3^3 + {}^{28}C_4 \times 0.7^{24} \times 0.3^4$	**M1**	Use of B(28, 0.3). Addition of terms must be intended. Allow one term wrong or omitted or extra.
	0.0474	**A1**	Unsupported correct answer scores **B1** instead of **M1 A1**.
	$0.0474 > 0.02$ [Not reject H_0]	**M1**	Valid comparison.
	No evidence that suspicion is true.	**A1 ft**	Not definite e.g. not 'Suspicion is not true', in context, no contradictions. SC use of N(8.4, 5.88) leading to 0.054 > 0.2 OE can score **B1** only for comparison and correct conclusion. Correct hypotheses with *p* will also score B1.
		5	

Question	Answer	Marks	Guidance
6(a)	est $\mu = 14$ accept $\frac{560}{40}$	**B1**	
	est $\sigma^2 = \frac{40}{39}\left(\frac{7850}{40} - 14^2\right)$ or $\frac{1}{39}\left(7850 - \frac{560^2}{40}\right)$	**M1**	
	0.25641 or 0.256 (3sf)	**A1**	Accept $\frac{10}{39}$ Without $\frac{40}{39}$ i.e. biased: est $\sigma^2 = 0.25$ **M0 A0**.
		3	
6(b)	E($S - T$) = 14.2 – '14' [= 0.2]	**B1 FT**	FT *their* 14.
	Var($S - T$) = 0.3 + \` 0.256\` [= 0.55641]	**B1 FT**	Accept $\frac{217}{390}$ FT *their* 0.256 including FT biased. Var($S - T$) = 0.55.
	$\frac{0.1 - '0.2'}{\sqrt{'0.55641'}}$ [= –0.134]	**M1**	Standardising with *their* values (note biased gives –0.135). FT *their* E & Var.
	P(S – T > 0.1) = 1 – Φ('–0.134') = Φ('0.134')	**M1**	Finding correct area consistent with *their* values.
	0.553 (3sf)	**A1**	Use of biased gives 0.554 (3sf) can score the **A1**. Similar scheme for P($T - S$) < –0.1. Similar scheme for S – T – 0.1 > 0. And T – S + 0.1 < 0.
		5	

Question	Answer	Marks	Guidance
7(a)	H$_0$: $\mu = 64.6$ H$_1$: $\mu < 64.6$	B1	Allow population mean, not just 'mean'.
	$[\pm]\dfrac{63.5-64.6}{5.2\div\sqrt{100}}$	M1	Standardising. Must have $\sqrt{100}$.
	$[\pm]-2.115$	A1	Accept -2.12 (3sf)
	'2.115' > 1.96 or '–2.115' < –1.96 [do not accept H$_0$]	M1	Valid comparison (0.0172 < 0.025 for area comparison).
	There is evidence that $\mu < 64.6$	A1 FT	Not definite, e.g. not '$\mu < 64.6$'. in context. No contradictions. Accept critical value method leading to 63.5 < 63.58 or 64.6 > 64.52.
		5	
7(b)	$\dfrac{m-64.6}{5.2\div\sqrt{100}} = -1.96$	M1	Finding the critical value using $N\left(64.6, \dfrac{5.2}{\sqrt{100}}\right)$ and a z value.
	$m = 63.5808$	A1	
	$\dfrac{63.5808-62.7}{5.2\div\sqrt{100}}$ [= 1.694]	M1	Standardising using $N\left(62.7, \dfrac{5.2}{\sqrt{100}}\right)$ and a critical value.
	$1-\Phi(\text{'}1.694\text{'})$	M1	For area consistent with *their* values.
	0.0451	A1	Accept answers that round to 0.045.
		5	

Cambridge International AS & A Level

MATHEMATICS **9709/62**

Paper 6 Probability & Statistics 2 **October/November 2021**

MARK SCHEME

Maximum Mark: 50

Published

This document consists of **12** printed pages.

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WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1(a)	$\frac{20.5}{40} = 0.5125$	**B1**	Accept 0.513 or $\frac{41}{80}$. Condone $\frac{20.5}{40}$.
	$\frac{40}{39}\left(\frac{10.728}{40} - \left('0.5125'^2\right)\right)$ or $\frac{1}{39}\left(10.728 - \frac{20.50^2}{40}\right)$	**M1**	Biased variance (0.005544 or $\frac{887}{160\,000}$) scores **M0 A0.**
	0.0056859 or 0.00569 (3 sf) or $\frac{887}{156\,000}$	**A1**	CAO
		3	
1(b)	[11 × ‘0.5125’ + 0.5]) = 6.1375 or $\frac{491}{80}$ or 6.14 (3sf)	**B1 FT**	FT *their* 0.5125
	11^2 × ‘0.0056859’	**M1**	With nothing added. Using *their* variance in **(a)** (no sd/var confusion)
	0.688 (3sf)	**A1**	CAO
		3	

Question	Answer	Marks	Guidance
2(a)	E.g. Bias towards students who play instruments or only music students or e.g. the six will possibly be friends/have similar music preferences	**B1**	OE Or any reason that some are excluded e.g. because it is lunchtime or because the music building is chosen or any suggestion that opinions may not be independent. Note: ‘not representative of all students’ needs qualifying
		1	

Question	Answer	Marks	Guidance
2(b)	28, 119, 207	**B1**	**B1** for 28, 119 (condone 028).
		B1	**B1** for 207 and only 3 values stated.
		2	

Question	Answer	Marks	Guidance
3(a)	$0.25 \pm z\sqrt{\frac{0.25\times0.75}{140}}$	**M1**	Expression of correct form (allow **M1** for just one side stated). Must be a z-value.
	$z = 2.054$ or 2.055	**B1**	
	0.175 to 0.325 (3sf)	**A1**	Must be an interval.
		3	
3(b)	$0.90 \times 0.95 \times 0.01$ $+ 0.90 \times 0.05 \times 0.99$ $+ 0.10 \times 0.95 \times 0.99$	**M1 M1**	**M1** for one correct triple product. **M1** for all correct and added.
	0.147	**A1**	SC If zero scored award **B1** for a 2 or 3 term expression of the form 0.90×0.95 [$\times c$] OE. ($0 < c \leqslant 1$)
		3	

Question	Answer	Marks	Guidance
4(a)	Fireworks are destroyed when tested.	**B1**	
		1	
4(b)	H_0: Pop mean time lasted (or μ) = 30 H_1: Pop mean time lasted (or μ) < 30	**B1**	Not just 'mean'.
	$\pm \dfrac{29-30}{\frac{5}{\sqrt{100}}}$	**M1**	For standardising. Must have $\sqrt{100}$. Use of totals N(3000,2500) giving $\dfrac{(2900-3000)}{\sqrt{2500}}$ scores **M1**. No mixed methods.
	± –2	**A1**	
	–2 > –2.326 [Do not reject H_0]	**M1**	Accept –2.326 to –2.329. Valid comparison or area comparison 0.0228>0.01 or 0.9772<0.99. Accept CR method 28.837<29 or 30.163>30.
	There is not enough evidence that mean time lasted is less than 30 seconds OR Not enough evidence to support the inspector's suspicion	**A1 FT**	In context (if used need mean or time / condone average instead of mean), not definite, e.g. not 'mean time lasted is not less than 30 seconds'. No contradictions. **Note** 2 tailed test can score **B0 M1 A1 M1** (comparison with 2.574–2.579) **A0** (no FT).
		5	
4(c)	Yes. Because population distribution is unknown [condone not Normal].	**B1**	Both needed. Condone *X* for parent population.
		1	

Question	Answer	Marks	Guidance
5(a)	$e^{-2}(1 + 2 + \frac{2^2}{2!})$	**M1**	P(X < 3) any λ. Allow one end error.
	0.677 (3sf)	**A1**	Unsupported correct answer scores SC **B1** only.
		2	
5(b)	N(40, 40)	**M1**	SOI
	$\frac{50.5 - 40}{\sqrt{40}}$ [= 1.660]	**M1**	For standardising with *their* values. Allow with wrong or no cc must have square root.
	P(z > '1.660') = 1 – Φ('1.660')	**M1**	Correct area consistent with *their* working.
	0.0485 or 0.0484 (3sf)	**A1**	
		4	
5(c)	$\lambda = 10$	**B1**	Condone mean = 10.
	$e^{-10}\left(\frac{10^8}{8!} + \frac{10^9}{9!} + \frac{10^{10}}{10!} + \frac{10^{11}}{11!}\right)$	**M1**	Allow any λ (allow one end error).
	0.477 (3sf)	**A1**	Unsupported correct answer scores SC **B2** only.
		3	

Question	Answer	Marks	Guidance
6(a)	H_0: P(0) = $\frac{1}{10}$ H_1: P(0) < $\frac{1}{10}$	**B1**	Accept p.
		1	
6(b)	For B(30,0.1)	**M1**	Used not just stated.
	$P(X = 0) = 0.9^{30}$ [= 0.0424] [<0.1]	**M1**	
	$P(X = 0 \text{ or } 1) = 0.9^{30} + 30\times0.9^{29}\times0.1=0.184$ [>0.1]	**B1**	Accept 0.184 or 0.183.
	Rejection region is 0 zeros	**A1**	Dependent on **M1 M1** and at least one comparison, no errors seen. **SC** One unsupported correct answer 0.0424/0.184(or 0.183) and correct rejection region scores **B1**; with comparison with 0.1 scores **B2**. Two unsupported correct answers 0.0424 and 0.184(or 0.183) and correct rejection region scores **B2** or if with one comparison with 0.1 scores **B3**.
		4	
6(c)	0.0424	**B1**	FT *their* **(b)** must have a critical region (only follow though Binomial), dependent on answer < 0.1.
		1	

Question	Answer	Marks	Guidance
6(d)	Bin(30, $\frac{1}{40}$)	**B1**	SOI
	$1 - 0.975^{30}$	**M1**	FT *their* rr and with Bin(30, 1/40)).
	0.532 (3dp)	**A1**	SC Unsupported correct answer scores **B2** only.
		3	
6(e)	Not concluding that the probability is less than $\frac{1}{10}$, when in fact it is.	**B1**	In context.
		1	

Question	Answer	Marks	Guidance
7(a)(i)	$k\int_0^2 (4x - x^2)\mathrm{d}x = 1$	**M1**	Attempt integral f(*x*) and = 1. Ignore limits (must see a power increase for attempted integration).
	$k\left[\frac{4x^2}{2} - \frac{x^3}{3}\right]_0^2 = 1$	**A1**	Correct integration and correct limits.
	$k \times \frac{16}{3} = 1 \left[k = \frac{3}{16}\right]$	**A1**	OE AG Convincingly obtained. At least one interim step. No errors seen.
		3	

Question	Answer	Marks	Guidance
7(a)(ii)	$\frac{3}{16}\int_0^2 (4x^2 - x^3)\mathrm{d}x$	**M1**	Attempt integral *x*f(*x*). Ignore limits. (must see a power increase for attempted integration). Condone missing *k*.
	$\frac{3}{16}\left[\frac{4x^2}{2} - \frac{x^3}{3}\right]_0^2$	**A1**	Correct integration and correct limits. Condone missing *k*.
	$\frac{5}{4}$	**A1**	Unsupported correct answer scores **SC B2** only.
		3	
7(b)	Symmetrical frequency density graph, 0 to 5, showing area 0.2 to left of *a*	**B1**	With *a* to the left of centre.
	Either 0.2 between 5–*a* and 5-or 0.8 between 0 and 5–*a*	**B1**	Shown on graph or stated (5–*a* seen or implied). *a* must be non-numerical.
	[P(2.5<Y<5–*a*)] = 0.3	**B1**	Must be clearly final answer. *a* must be non-numerical.
		3	

Cambridge International AS & A Level

MATHEMATICS **9709/63**

Paper 6 Probability & Statistics 2 **October/November 2021**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2021 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

This document consists of **11** printed pages.

Generic Marking Principles

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- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

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- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
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Mathematics Specific Marking Principles	
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2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	**Answer**	**Marks**	**Guidance**
1(a)	N(12.5, ….)	**B1**	
	Variance = 0.4096	**B1**	Accept 0.410 (3sf), condone $\frac{10.24}{25}$
		2	
1(b)	$\frac{13-\text{'}12.5\text{'}}{\sqrt{\text{'}0.4096\text{'}}}$ [= 0.781]	**M1**	For standardising with *their* values. Accept standardising with 12.
	ϕ(‘0.781’) – (1 – Φ(‘0.781’))	**M1**	For attempting to find *their* central area.
	0.565 (3sf)	**A1**	
		3	

Question	**Answer**	**Marks**	**Guidance**
2	N(45.2, 45.2)	**B1**	SOI
	$\frac{60.5-45.2}{\sqrt{45.2}}$ [= 2.276]	**M1**	Allow with wrong or no continuity correction.
	1 –ϕ(‘2.276’)	**M1**	
	0.0114	**A1**	
		4	

Question	Answer	Marks	Guidance
3	est(p) = 0.2 accept $\frac{15}{75}$	**B1**	SOI
	$2 \times z \times \sqrt{\frac{0.2 \times 0.8}{75}} = 0.162$	**M1**	Expression of the correct form. Condone missing 2*x*.
	$z\left[= 0.081 \times \sqrt{\frac{75}{0.2 \times 0.8}}\right] = 1.754$	**A1**	Correct *z*. Condone 3sf accuracy.
	Φ('1.754') = 0.96[03] '0.96' – (1 – '0.96')	**M1**	OE. Using *their z* to find alpha.
	$\alpha = 92$	**A1**	Following correct working.
		5	

Question	Answer	Marks	Guidance
4(a)	$\frac{1}{18}\int_{0}^{1.2}(9 - x^2)\mathrm{d}x$	**M1**	Attempt to integrate f(*x*), ignore limits. Must see an increase of power.
	$\frac{1}{18}\left[9x - \frac{x^3}{3}\right]_0^{1.2}$	**A1**	Correct integration and correct limits.
	$\frac{71}{125}$ or 0.568	**A1**	SC unsupported answer scores **B2** only.
		3	

Question	Answer	Marks	Guidance
4(b)	$\frac{1}{18}\int_0^3 (9x - x^3)\mathrm{d}x$	**M1**	Attempt to integrate $x\mathrm{f}(x)$, ignore limits. Must see an increase of power.
	$\frac{1}{18}\left[\frac{9x^2}{2} - \frac{x^4}{4}\right]_0^3$	**A1**	Correct integration and correct limits.
	$\frac{9}{8}$ or 1.125	**A1**	SC unsupported answer scores **B2** only.
		3	
4(c)	$\frac{1}{18}\left[9x - \frac{x^3}{3}\right]_0^m = 0.5$	**M1**	Attempt to integrate f(x) with correct limits and = 0.5. OE. Accept limits m to 3. Allow x instead of m.
	$\frac{1}{18}\left[9m - \frac{m^3}{3}\right] - 0.5 = 0$	**A1**	Any correct cubic equation in m or x.
	$m^3 - 27m + 27 = 0$	**A1**	AG. Correctly obtain this equation. No errors seen.
		3	

Question	Answer	Marks	Guidance
5(a)(i)	Po(0.025)	**B1**	For Poisson and correct parameter.
	$n = 2500 > 50$, $np = 0.025 < 5$	**B1**	Must show 2500 and 0.025. Accept $p = \frac{1}{100000} < 0.1$ in place of $np = 0.025 < 5$.
		2	

Question	Answer	Marks	Guidance
5(a)(ii)	$1 - e^{-0.025}$	**M1**	Allow any λ. FT *their* **(a)(i)** if normal; must have continuity correction.
	0.0247 (3sf)	**A1**	Must be from Poisson. Unsupported correct answer scores **B1** instead of **M1 A1**.
		2	
5(b)	H_0: $p = 0.3$ H_1: $p < 0.3$	**B1**	
	$0.7^{28} + 28 \times 0.7^{27} \times 0.3 + {}^{28}C_2 \times 0.7^{26} \times 0.3^2 + {}^{28}C_3 \times 0.7^{25} \times 0.3^3 + {}^{28}C_4 \times 0.7^{24} \times 0.3^4$	**M1**	Use of B(28, 0.3). Addition of terms must be intended. Allow one term wrong or omitted or extra.
	0.0474	**A1**	Unsupported correct answer scores **B1** instead of **M1 A1**.
	$0.0474 > 0.02$ [Not reject H_0]	**M1**	Valid comparison.
	No evidence that suspicion is true.	**A1 ft**	Not definite e.g. not 'Suspicion is not true', in context, no contradictions. SC use of N(8.4, 5.88) leading to $0.054 > 0.2$ OE can score **B1** only for comparison and correct conclusion. Correct hypotheses with p will also score B1.
		5	

Question	**Answer**	**Marks**	**Guidance**
6(a)	est $\mu = 14$ accept $\frac{560}{40}$	**B1**	
	est $\sigma^2 = \frac{40}{39}\left(\frac{7850}{40} - 14^2\right)$ or $\frac{1}{39}\left(7850 - \frac{560^2}{40}\right)$	**M1**	
	0.25641 or 0.256 (3sf)	**A1**	Accept $\frac{10}{39}$ Without $\frac{40}{39}$ i.e. biased: est $\sigma^2 = 0.25$ **M0 A0**.
		3	
6(b)	E(*S* – *T*) = 14.2 – ‘14’ [= 0.2]	**B1 FT**	FT *their* 14.
	Var(*S* – *T*) = 0.3 + ` 0.256` [= 0.55641]	**B1 FT**	Accept $\frac{217}{390}$ FT *their* 0.256 including FT biased. Var(*S* – *T*) = 0.55.
	$\frac{0.1-'0.2'}{\sqrt{'0.55641'}}$ [= –0.134]	**M1**	Standardising with *their* values (note biased gives –0.135). FT *their* E & Var.
	P(*S* – T > 0.1) = 1 – Φ(‘–0.134’) = Φ(‘0.134’)	**M1**	Finding correct area consistent with *their* values.
	0.553 (3sf)	**A1**	Use of biased gives 0.554 (3sf) can score the **A1**. Similar scheme for P(*T* – *S*) < –0.1. Similar scheme for S – T – 0.1 > 0. And T – S + 0.1 < 0.
		5	

Question	Answer	Marks	Guidance
7(a)	H$_0$: $\mu = 64.6$ H$_1$: $\mu < 64.6$	**B1**	Allow population mean, not just 'mean'.
	$[\pm]\frac{63.5-64.6}{5.2 \div \sqrt{100}}$	**M1**	Standardising. Must have $\sqrt{100}$.
	$[\pm]-2.115$	**A1**	Accept -2.12 (3sf)
	'2.115' > 1.96 or '–2.115' < –1.96 [do not accept H$_0$]	**M1**	Valid comparison (0.0172 < 0.025 for area comparison).
	There is evidence that $\mu < 64.6$	**A1 FT**	Not definite, e.g. not '$\mu < 64.6$'. in context. No contradictions. Accept critical value method leading to 63.5 < 63.58 or 64.6 > 64.52.
		5	
7(b)	$\frac{m-64.6}{5.2 \div \sqrt{100}} = -1.96$	**M1**	Finding the critical value using $N\left(64.6, \frac{5.2}{\sqrt{100}}\right)$ and a *z* value.
	$m = 63.5808$	**A1**	
	$\frac{63.5808-62.7}{5.2 \div \sqrt{100}}$ [= 1.694]	**M1**	Standardising using $N\left(62.7, \frac{5.2}{\sqrt{100}}\right)$ and a critical value.
	1 – Φ('1.694')	**M1**	For area consistent with *their* values.
	0.0451	**A1**	Accept answers that round to 0.045.
		5	

Cambridge Assessment
International Education

Cambridge International A Level

MATHEMATICS **9709/61**

Paper 6 Probability & Statistics 2 **May/June 2021**

MARK SCHEME

Maximum Mark: 50

Published

This document consists of **13** printed pages.

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AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	$\lambda = (3.1 + 1.7) \times 2$	M1	Attempt combined mean. Allow 3.1 + 1.7 for M1
	= 9.6	A1	Correct mean
	$1 - e^{-9.6}\left(1 + 9.6 + \frac{9.6^2}{2} + \frac{9.6^3}{3!}\right)$	M1	Allow incorrect mean. Allow one end error.
	= 0.986 (3 sf)	A1	**SC** If 9.6 seen and unsupported 0.986 M1A1B1. **SC** Unsupported correct answer of 0.986 only if 9.6 also not seen scores B2 only.
		4	

Question	Answer	Marks	Guidance
2(a)	$\pm\dfrac{123-125}{\frac{6}{\sqrt{40}}}$ [= –2.108...]	M1	Must have √40 No standard deviation/variance mix. Ignore any continuity correction attempts for this mark.
	$P(z <$ '–2.108'$) = 1 - \Phi($'2.108'$)$	M1	For correct probability area consistent with *their* working.
	= 0.0175 or 0.0176 (3 sf)	A1	
		3	
2(b)	No, population is normal	B1	Need both.
		1	

Question	Answer	Marks	Guidance
3	$1-\frac{20}{27}$ or $\frac{20}{27}-\frac{1}{2}$ $\frac{20}{27}-\left(1-\frac{20}{27}\right)$ or $\left(\frac{20}{27}-\frac{1}{2}\right)$	M1	For either expression seen.
	$\frac{13}{27}$	A1	OE. Accept 0.481 or 0.482.
		2	

Question	Answer	Marks	Guidance
4	$\frac{3820}{100}$ [= 38.2]	B1	
	$\frac{100}{99}\left(\frac{182200}{100}-\text{'}38.2\text{'}^2\right)$ or $\frac{1}{99}\left(182200-\frac{3820^2}{100}\right)$	M1	Use of biased (362.76) scores M0
	$=\frac{12092}{33}$ or 366.424 or 366 (3 sf)	A1	Accept SD=19.1422 or 19.1(3sf)
	'38.2' ± $z\times\sqrt{\frac{\text{'}366.424\text{'}}{100}}$	M1	Expression of the correct form must be a *z*-value.
	z = 1.881 or 1.882	B1	Seen.
	34.6 to 41.8 (3 sf)	A1	Allow use of biased giving (34.6,41.8) Must be an interval.
		6	

Question	Answer	Marks	Guidance
5(a)	$\text{Po}\left(\frac{2}{15}\right)$	M1	SOI. Allow Po(0.133).
	$\text{P}(X \geqslant 1) = 1 - \text{e}^{-\frac{2}{15}}$	M1	Allow incorrect λ allow one end error
	= 0.125 (3 sf)	A1	**SC** Partially unsupported final answer: $\text{Po}\left(\frac{2}{15}\right)$ stated B1 then unsupported 0.125 B1 **SC** Use of Binomial (0.1248) B1 only Use of Normal scores M0
		3	

Question	Answer	Marks	Guidance
5(b)	$\lambda = \dfrac{n}{75000}$	B1	
	$e^{-\frac{n}{75000}} > 0.9$	M1	Allow '=' Allow incorrect λ
	$-\dfrac{n}{75000} > \ln 0.9 \quad [n < 7902.04]$	M1	Attempt ln both sides
	Largest value of n is 7902	A1	CWO. Must be an integer.
	Alternative method for Question 5(b)		
	$e^{-\mu} > 0.9$	M1	Allow '='
	$-\mu > \ln 0.9 \quad [\mu < 0.10536]$	M1	Attempt ln both sides
	$n = \mu \times 75000$	B1	
	Largest value of n is 7902	A1	CWO. Must be an integer.
	Alternative method for Question 5(b)		
	$\dfrac{74999}{75000}$	B1	
	$\left(\dfrac{74999}{75000}\right)^n > 0.9$	M1	
	$n\ln\dfrac{74999}{75000} > \ln 0.9$	M1	Attempt ln or log both sides
	Largest value of n is 7901	A1	CWO Must be an integer
		4	

Question	Answer	Marks	Guidance
6	E(X) = 3	B1	N.B. E(X)=108k is B0 until correct k substituted in.
	$k\int_0^6 (6x - x^2)\mathrm{d}x = 1$ $k\left[3x^2 - \frac{x^3}{3}\right]_0^6$ [= 1]	M1	Attempt integration of f(x) and =1. Ignore limits at this stage.
	$k\left(108 - \frac{216}{3}\right) = 1$ $k = \frac{3}{108}$ or $\frac{1}{36}$	A1	
	$'\frac{3}{108}'\int_0^6 (6x^3 - x^4)\mathrm{d}x$ $= \frac{3}{108}\left[\frac{3x^4}{2} - \frac{x^5}{5}\right]_0^6 = 10.8$	*M1	Attempt integration of *their* $k \times x^2\mathrm{f}(x)$. Ignore limits at this stage. Accept in terms of k.
	$'10.8' - '3'^2$	DM1	*Their* 10.8 (from use of limits 0 and 6) minus *their* $(\mathrm{E}(X))^2$. Accept in terms of k: $388.8k-(108k)^2$
	$\frac{9}{5}$ or 1.8	A1	CWO. Must be convincingly obtained as AG.
		6	

Question	Answer	Marks	Guidance
7(a)	$\mathrm{E}(T) = 3\times55+6\times27[=327]$	B1	OE. Accept unsimplified.
	$\mathrm{Var}(T) = 3\times3^2 + 6\times2.5^2$ [= 64.5]	B1	Accept unsimplified.
	$\frac{340-'327'}{\sqrt{'64.5'}}$ [= 1.619]	M1	Must have √
	$\mathrm{P}(z < '1.619') = \Phi('1.619')$	M1	Correct probability area consistent with *their* working.
	0.947 (3 sf)	A1	
		5	
7(b)	$\mathrm{E}(L–S_1–S_2) = 55 – 2 \times 27$ [=1]	B1	OE e.g. $\mathrm{E}(S_1+S_2 – L)= –1$. Accept unsimplified.
	$\mathrm{Var}(L– S_1–S_2) = 3^2 + 2 \times 2.5^2$ [= 21.5]	B1	Accept unsimplified.
	$\frac{0-'1'}{\sqrt{'21.5'}}$ [= –0.216]	M1	Standardising with *their* values. Must come from a combination attempt.
	$\mathrm{P}(L–S_1–S_2 > 0) = \Phi(‘0.216’)$	M1	Correct probability area consistent with *their* working.
	0.586 or 0.585 (3 sf)	A1	
		5	

Question	Answer	Marks	Guidance
8(a)	Not representative (of all students in the school)	**B1**	OE idea of 'not being representative' e.g. different grades in the school have different characteristics/proportions.... Don't accept 'not random' or 'biased' without further explanation.
		1	
8(b)	H_0: P(not correct uniform) = 0.15 H_1: P(not correct uniform) < 0.15	**B1**	Allow "*p*"
		1	
8(c)	Any two probs attempted using B(50,0.15)	**M1**	
	$P(X \leqslant 3) = 0.85^{50} + 50 \times 0.85^{49} \times 0.15 + {}^{50}C_2 \times 0.85^{48} \times 0.15^2 + {}^{50}C_3 \times 0.85^{47} \times 0.15^3$	**M1**	Attempt the tail probability P(0,1,2,3) with B(50,0.15) must be added.
	$P(X \leqslant 4) = 0.04605 + {}^{50}C_4 \times 0.85^{46} \times 0.15^4$	**M1**	OE. *Their* $P(X \leqslant 3) + P(X = 4)$ or P(0,1,2,3,4) with B(50,0.15) must be added.
	$P(X \leqslant 3) = 0.0460$ or 0.0461 [<0.05] $P(X \leqslant 4) = 0.112$ or [>0.05]	**A1**	Both correct. OR if $P(X \leqslant 4)$ not seen; P(4)=0.06606 **and** 0.06606>0.05 **and** $P(X \leqslant 3)$=0.0460 scores M1 A1
	P(Type I) = 0.0460 or 0.0461 (3 sf)	**A1**	Dependent on second M1. **SC** If M1M1M1A0 scored allow A1FT for incorrect $P(X \leqslant 3)$ as long as <0.05
		5	

Question	Answer	Marks	Guidance
8(d)	4 is outside critical region ($\leqslant 3$) OE or $P(X \leqslant 4) = 0.112$ which is > 0.05	**M1**	FT working from **(c)**.
	No evidence that proportion not wearing the correct uniform has decreased (Accept Ho)	**A1**	In context not definite, e.g. not 'Proportion has not decreased'. No contradiction.
		2	
8(e)	Not rejected H_0	***B1 FT**	FT If Reject H_0 in **(d)**
	Type II	**DB1 FT**	FT Type I
		2	

Cambridge International AS & A Level

MATHEMATICS **9709/62**

Paper 6 Probability & Statistics 2 **May/June 2021**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

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This document consists of **12** printed pages.

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- the specific skills defined in the mark scheme or in the generic level descriptors for the question
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GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
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- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M	Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
A	Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
B	Mark for a correct result or statement independent of method marks.
DM or **DB**	When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
FT	Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1(a)	H$_0$: $p = \frac{1}{4}$ H$_1$: $p \neq \frac{1}{4}$	B1	or H$_0$: $\mu = 25$ or H$_1$: $\mu \neq 25$
		1	
1(b)	$\mathrm{N}\left(25, \frac{75}{4}\right)$	B1	SOI. Allow B1 for $\mathrm{N}\left(25, \frac{75}{4}\right)$ or N(0.25,0.001875) SOI.
	$\pm \frac{15.5-25}{\sqrt{\frac{75}{4}}}$ or $\frac{\frac{15.5}{100}-0.25}{\sqrt{\frac{0.25\times0.75}{100}}}$	M1	Standardise with *their* N(25,…) Allow with no or wrong continuity correction.
	$\pm$ –2.194 (2.19)	A1	
	–2.326 < –2.194 or 0.0141 > 0.01 or 0.9859 < 0.99	M1	For valid comparison (accept 2.326 to 2.329)
	No evidence to reject that the probability is $\frac{1}{4}$	A1 FT	OE must be in context and not definite, e.g. not 'Claim untrue'. No contradictions. FT *their z* ; dependent on two–tailed test (one-tailed test can score B1 M1 A1 M1 A0) **SC** for use of Binomial B(100,0.25) P = 0.0111 for B1 and then comparison with 0.01 and correct conclusion for B1, maximum 2 out of 5 marks.
		5	

Question	Answer	Marks	Guidance
2(a)	$\text{Var}(X) = 400 \times 0.01 \times 0.99\ \ (= 3.96)$	M1	
	$\text{Var}(4X + 2) = 16 \times \text{Var}(X)$	M1	For 16 × *their* Var(X)
	63.36	A1	Accept 63.4
		3	
2(b)(i)	Po(4)	B1	
	$n = 400 > 50$ and either $np = 4 < 5$ or p=0.01 < 0.1	B1	Must quote values 400 and 4 or clearly see n=400 and np=4 (or p=0.01) in working
		2	
2(b)(ii)	$e^{-4}\left(\frac{4^2}{2!}+\frac{4^3}{3!}+\frac{4^4}{4!}+\frac{4^5}{5!}\right)$	M1	FT *their* '4' Allow one end error FT from **(b)(i)** Use of Normal allow M1 for attempt at standardising (with correct continuity correction) using *their* N(4,3.96) and attempt at probability. FT from **(b)(i)** Use of Binomial allow M1 for attempt at P(2,3,4,5) Binomial terms clearly seen and added
	0.694 (3 sf)	A1	CWO **SCB1** only for unsupported answer of 0.694
		2	

Question	Answer	Marks	Guidance
3(a)	$\frac{1}{2}p(p-1)=1$	M1	For area =1 For verification methods accept $\frac{1}{2}\times 2\times 1=1$ or $\frac{1}{2}\times 2\times(p-1)=1$ or $\frac{1}{2}\times 1\times p=1$ as indication that area=1
	$p=2$	A1	AG - Convincing method and answer. Must see quadratic rearranged to =0 and no errors seen. N.B. Accept convincing verification methods (e.g. statement such as 'assume $p = 2$' or 'if $p = 2$' or 'using $p = 2$' or showing by clear substitution that $p = 2$ fits $\frac{1}{2}p(p-1)=1$ with clear conclusion)
		2	
3(b)	Gradient = 2 equation of line is $y = 2x + c$ line passes through (1, 0), hence $c = -2$	M1	Award for attempting equation of line $y=mx+c$ with $m = 2, -2, \frac{1}{2}$ or $-\frac{1}{2}$ and numerical c ($c\neq 0$)
	$y = 2x - 2$	A1	May be seen in **(a)** M1 can be implied by correct answer
	$2\int_1^2 (x^2 - x)\mathrm{d}x$	M1	For attempting $\int x\mathrm{f}(x)\mathrm{d}x$. Ignore limits, FT *their* equation.
	$2\left[\frac{x^3}{3}-\frac{x^2}{2}\right]_1^2$	A1 FT	Correct integration FT *their* f(x)and correct limits
	$\frac{5}{3}$ or 1.67 (3 sf)	A1	
		5	

Question	Answer	Marks	Guidance
4(a)	Mean = 15.0+32.0+8.6 [= 55.6]	B1	Allow unsimplified
	Var = $1.1^2+3.5^2+1.2^2$ [= 14.9]	B1	Allow unsimplified
		2	
4(b)	$\dfrac{60-\text{"55.6"}}{\sqrt{\text{"14.9"}}}$ [= 1.140]	M1	FT *their* 55.6 and 14.9 Ignore continuity correction
	1 – ϕ("1.140")	M1	For correct probability area consistent with *their* working
	0.127 (3 sf)	A1	CWO
		3	
4(c)	$\dfrac{54.5-\text{"55.6"}}{\sqrt{\dfrac{\text{"14.9"}}{15}}}$ or $\dfrac{817.5-834}{\sqrt{223.5}}$ [= –1.104]	**M1**	FT *their* 55.6 and 14.9 No mixed methods
	1 – ϕ("1.104")	**M1**	For correct probability area consistent with their working
	0.135 (3 sf)	**A1**	As final answer
		3	

Question	Answer	Marks	Guidance
5(a)	Conclude that (population) mean time has changed (or is not 42.4) although μ has not changed (or is still 42.4)	B1	OE. In context.
		1	
5(b)	H_0: population mean (or μ) = 42.4 H_1: population mean (or μ) $\neq$ 42.4	B1	Not just 'mean'. (could be seen in **(a)**)
	$\pm \dfrac{45.6-42.4}{\sqrt{38.2 \div 20}}$	M1	For standardising (must have $\sqrt{20}$)
	$\pm$ 2.315	A1	
	2.240 < '2.315'	M1	For valid comparison (accept 2.241) or P(z > 2.315) = 0.0103 < 0.0125 oe
	There is evidence that μ or mean time has changed	A1 FT	FT *their z* In context, not definite. No contradictions. **Note:** Accept correct alternative methods **SC**: One tail test no FT. Can score B0 M1 A1 M1 (comparison with 1.96) A0 (maximum 3 out of 5)
		5	

Question	Answer	Marks	Guidance
6(a)	est (μ) = $\frac{7570}{100}$ (= 75.7)	B1	
	est(σ^2) = $\frac{100}{99}\left(\frac{\sum h^2}{100} - \text{'}75.7\text{'}^2\right)$ or $\frac{1}{99}\left(588050 - \frac{7570^2}{100}\right)$ = $\frac{100}{99}\left(\frac{588050}{100} - \text{'}75.7\text{'}^2\right)$ [= 151.525]	M1	Attempted (**Note:** Biased variance (150.01) scores M0)
	= 152 (3 sf)	A1	Or $\frac{15001}{99}$
		3	
6(b)	‘75.7’ ± $z\sqrt{\frac{\text{'}151.525\text{'}}{100}}$	M1	For expression of correct form. Must be a *z* value. Condone just + or just -.
	$z = 2.576$	B1	Accept 2.574 to 2.579
	72.5 to 78.9	A1 FT	FT biased variance only Must be an interval
		3	
6(c)	0.99^4	B1	
	0.961 (3 sf)	B1	
		2	

Question	Answer	Marks	Guidance
7(a)	$e^{-4.2} \times \frac{4.2^4}{4!}$	M1	P(4), allow any λ
	0.194 (3 sf)	A1	As final answer. **SC** Unsupported correct answer scores B1 only.
		2	
7(b)	$1-e^{-8.4}\left(1+8.4+\frac{8.4^2}{2}+\frac{8.4^3}{3!}\right)$	M1	Allow M1 with incorrect λ. Accept one end error.
	0.968 (3 sf)	A1	As final answer. **SC** Unsupported correct answer scores B1 only.
		2	
7(c)	N(50.4, 50.4)	M1	SOI
	$\frac{39.5-50.4}{\sqrt{50.4}}$ $[=-1.535]$	M1	Allow wrong or no continuity correction. Must have √
	Φ('–1.535') = 1 – Φ('1.535')	M1	For correct probability area consistent with *their* working.
	0.0624 (3 sf) or 0.0623	A1	
		4	

Cambridge Assessment
International Education

Cambridge International A Level

MATHEMATICS **9709/63**

Paper 6 Probability & Statistics 2 **May/June 2021**

MARK SCHEME

Maximum Mark: 50

Published

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SOI	Seen Or Implied
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WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	$\lambda = 10 \times 1.36$ [= 13.6]	**M1**	
	E(amount) = 5×13.6 = [\$]68	**A1**	
	Var(amount) = $5^2 \times 13.6$ [= 340]	**M1**	$5^2 \times \ldots$
		M1	$\ldots \times$ *their* λ
	Standard Deviation = [\$]18.4(4) (3 s.f.)	**A1**	CAO condone $2\sqrt{85}$
		5	

Question	Answer	Marks	Guidance
2(a)	Conclude (mean) (journey) time has not decreased when in fact it has.	**B1**	OE in context
		1	
2(b)	H_0: Pop mean (or μ) = 1.4 H_1: Pop mean (or μ) < 1.4	**B1**	May be seen in **(a)**
	$\dfrac{1.36-1.4}{\frac{0.12}{\sqrt{50}}}$	**M1**	Accept totals method $\dfrac{68-70}{\sqrt{50}\times 0.12}$ No mixed methods or no standard deviation/variance mixes
	–2.357 or – 2.36	**A1**	Correct z or correct area if used
	$-2.357 < -1.96$ or $0.0092 < 0.025$ or $0.9908 > 0.975$ Or CV method $1.36 < 1.367$	**M1**	valid comparison
	There is evidence that (mean) (journey) times have decreased	**A1 FT**	in context not definite no contradictions NB use of two tail test scores max B0M1A1M1A0 no ft for two tail test
		5	
2(c)	H_0 was rejected OE	***B1 FT**	FT H_0 was accepted OE
	Type I	**DB1 FT**	FT Type II
		2	

Question	Answer	Marks	Guidance
3(a)(i)	H_0: $\lambda = 2.4$ H_1: $\lambda > 2.4$	**B1**	Accept λ or μ Accept 2.4 or 0.8 (per year)
		1	
3(a)(ii)	$1 - e^{-2.4}(1 + 2.4 + \frac{2.4^2}{2} + \frac{2.4^3}{3!} + \frac{2.4^4}{4!})$	**M1**	Any λ; allow one end error
	0.0959 (3 sf)	**A1**	**SC** unsupported answer 0.0959 scores B1 only not M1A1
	$0.0959 > 0.05$	**M1**	Valid comparison Use of $0.9041 < 0.95$ can recover either M1A1 or B1
	There is evidence that Jane's claim not justified or There is insufficient evidence to support Jane's claim	**A1 FT**	OE. In context, not definite, e.g. not 'Jane is wrong', no contradictions. Condone omission of Jane.
		4	
3(b)	Mean not constant so Poisson model not valid	**B1**	
		1	

Question	Answer	Marks	Guidance
4(a)	$\frac{4509}{90}$ [= 50.1]	**B1**	
	$\frac{90}{89}(\frac{225950}{90} - \text{'}50.1\text{'}^2)$ or $\frac{1}{89}\left(225950 - \frac{4509^2}{90}\right)$	**M1**	Attempted. Use of biased = 0.5455 scores M0A0
	$\frac{491}{890}$ or 0.552 (3 sf)	**A1**	
		3	
4(b)	'50.1' $\pm z\sqrt{\frac{\text{'}\frac{491}{890}\text{'}}{90}}$	**M1**	Expression of the correct form, allow any *z*-value but must be a *z*-value
	$z = 2.326$	**B1**	Accept 2.326 to 2.329
	49.9 to 50.3 (3 sf)	**A1**	FT from biased variance. Must be an interval.
		3	
4(c)	Population of masses is unknown	**B1**	Accept population of masses is not normal
		1	
4(d)	1 – 0.98	**M1**	0.02 seen
	0.02 ÷ 2 = 0.01	**A1**	As final answer
		2	

Question	Answer	Marks	Guidance
5(a)	Po(2.5)	**B1**	Accept Poisson with mean = 2.5 not just np = 2.5
	$n = 25\,000 > 50$ and np (or λ) = 2.5 which is < 5 or $n = 25\,000 > 50$ and p = 0.0001 < 0.1	**B1**	Must see 2.5 (or 0.0001) and 25000 OE, not just $np < 5$ (or p < 0.1) and $n > 50$
		2	
5(b)	$e^{-2.5}(1 + 2.5 + \frac{2.5^2}{2} + \frac{2.5^3}{3!})$	**M1**	Any λ, accept one end error. FT binomial from part **(a)** scores M1 only for equivalent binomial expressions FT normal from part **(a)** must use correct continuity correction and tables scores M1 only for complete method
	0.758 (3 sf)	**A1**	Unsupported answer of 0.758 scores B1 instead of M1A1
		2	
5(c)	$e^{-2.5} \times \frac{2.5^k}{(k)!} = 2e^{-2.5} \times \frac{2.5^{k+1}}{(k+1)!}$	**M1**	Any λ FT binomial from **(a)** scores M1 only for equivalent binomial expression FT from **(a)** normal for equivalent expressions continuity correction must be included
	$k = 4$	**A1**	No errors seen **SC** $k = 4$ unsupported scores B1 only, but see full Poisson expressions for P(4) and P(5) and 0.134 scores M1A1
		2	

Question	Answer	Marks	Guidance
5(d)	1 - $e^{-\lambda}$ = 0.963	**M1**	Accept *their* attempt at λ
	$\lambda = -\ln 0.037$ (= 3.2968 or 3.30 or 3.3)	**M1**	Correct use of lns
	n = 33 000 (3 sf)	**A1**	Allow n = 32 950 to 33 050 (must be an integer) **SC** use of binomial leading to 32 967 scores B1 for $(0.9999)^n = 0.037$ B1 for 33 000 to 3sf (32 967)
		3	
6(a)	$P(X > 10) = \int_{10}^{20} \frac{3}{8000}(x-20)^2 \, dx$	**M1**	Attempt integration of f(x), ignore limits.
	$= \left[\frac{3}{8000} \times \frac{(x-20)^3}{3}\right]_{10}^{20}$ or $\frac{3}{8000}\left[\frac{x^3}{3} - \frac{40x^2}{2} + 400x\right]_{10}^{20}$ $= \frac{1}{8000}\left[0-(-10)^3\right]$	**M1**	Substitute correct limits 10 to 20 or 1 – … limits 0 to 10 in *their* integral
	$\frac{1}{8}$ or 0.125	**A1**	**SC** Unsupported answer of $\frac{1}{8}$ scores B1 only
	$(\frac{1}{8})^2 = \frac{1}{64}$ or 0.0156 (3 sf)	**B1 FT**	FT *their* $P(X > 10)$ dependent on first M1 gained
		4	

Question	Answer	Marks	Guidance
6(b)	$\int_0^{20} \frac{3}{8000}(x^3 - 40x^2 + 400x)\mathrm{d}x$	**M1**	Attempt integration of $x\mathrm{f}(x)$. Ignore limits.
	$\frac{3}{8000}\left[\frac{x^4}{4} - \frac{40x^3}{3} + \frac{400x^2}{2}\right]_0^{20}$ or $\left(\frac{3x}{8000} \times \frac{(x-20)^3}{3}\right) - \frac{1}{8000}\left(\frac{(x-20)^4}{4}\right)$	**A1**	Correct integral (by expanding or by parts)
	$\frac{3}{8000}\left[\frac{160000}{4} - \frac{40 \times 8000}{3} + 200 \times 400\right]$	**M1**	Subst correct limits in their (4th degree) integral
	5	**A1**	
		4	
6(c)	$\int_0^{m} \frac{3}{8000}(x-20)^2\mathrm{d}x \ = 0.5$	**M1**	Attempt to integrate f(x) and equate to 0.5. Ignore limits.
	$\left[\frac{3}{8000} \times \frac{(x-20)^3}{3}\right]_0^m = 0.5$ or $\frac{3}{8000}\left[\frac{x^3}{3} - \frac{40x^2}{2} + 400x\right]_0^m = 0.5$ $\frac{1}{8000}\left[(m-20)^3 - (-20)^3\right] = 0.5$	**M1**	Attempt integral and substitute limits 0 and m or m and 20 and = 0.5
	$(m-20)^3 = -4000$	**A1**	AG. Found convincingly.
	$(m = \ 20 + \sqrt[3]{-4000}\ )$ $m = 4.13$ (3 sf)	**B1**	
		4	

Question	Answer	Marks	Guidance
6(d)	Doesn't allow for trains > 20 mins late or Doesn't allow for trains being early	**B1**	or any relevant comment e.g. trains on Sun may be different to trains on Mon
		1	

Cambridge Assessment
International Education

Cambridge International AS & A Level

MATHEMATICS **9709/62**

Paper 6 Probability and Statistics 2 **March 2021**

MARK SCHEME

Maximum Mark: 50

Published

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- the standard of response required by a candidate as exemplified by the standardisation scripts.

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- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
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Mathematics Specific Marking Principles	
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2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
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Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

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A	Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
B	Mark for a correct result or statement independent of method marks.
DM or **DB**	When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
FT	Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
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WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1(a)	Est(μ) = $\frac{4820}{60}$ or $\frac{241}{3}$ or 80.3 (3 sf)	**B1**	
	Est(σ^2) = $\frac{60}{59}(\frac{392\,050}{60}-(\frac{4820}{60})^2)$	**M1**	Use of biased (80.72) score M0 A0.
	82.0904 $\left(\frac{14530}{177}\right)$ to 82.635 or SD = 9.0604 to 9.0904 (3sf)	**A1**	
	$z = 2.326$	**B1**	
	$\frac{4820}{60} \pm z\times\sqrt{\frac{\text{'}82.0904\text{'}}{60}}$	**M1**	Expression of the correct form – must be z value.
	77.6 to 83.1 (3 sf)	**A1**	CWO Use of biased 77.6 to 83.0(3) can score B1M1A1 (max 4/6).
		6	
1(b)	Population distribution of times unknown	**B1**	Accept 'not normal'.
		1	

Question	Answer	Marks	Guidance
2(a)	$\frac{1}{2}\times\frac{1}{2}k\times k\ =1$	**M1**	Or use of $\int_0^k\left(-\frac{1}{2}x+\frac{1}{2}k\right)\mathrm{d}x=1$ and attempt at integral.
	$k=2$	**A1**	Unsupported answers M0 A0. Do not accept ± 2.
		2	
2(b)	$\mathrm{f}(x)=-\frac{1}{2}x+1$	**B1 FT**	FT *their* k from $y=-\frac{1}{2}x+\frac{1}{2}k$.
	$\int_0^2(-\frac{1}{2}x^2+x)\mathrm{d}x\ =\left[-\frac{x^3}{6}+\frac{x^2}{2}\right]_0^2$	**M1**	Attempt integration of $x\mathrm{f}(x)$ limits 0 to k. FT *their* f(x). Could be in terms of k.
	$\frac{2}{3}$ or 0.667 (3 sf)	**A1**	
		3	

Question	Answer	Marks	Guidance
2(c)	$\int_p^1 (-\frac{1}{2}x+1)\mathrm{d}x$ [= 0.25]	**M1**	FT their equation of **line**; correct integral and limits (could be reversed) stated or $\frac{1}{2}(1-p)(1-\frac{1}{2}p+\frac{1}{2})$ [= 0.25].
	$\left[-\frac{x^2}{4}+x\right]_p^1 = 0.25$ $-\frac{1}{4} + 1 + \frac{p^2}{4} - p = 0.25$	**M1**	Attempt substitution of correct limits (not reversed) into their integral or attempt expand must equal 0.25. OE
	$p^2 - 4p + 2 = 0$	**M1**	Obtain 3-term quadratic set equal to 0, obtain at least 1 solution.
	$p = 2 - \sqrt{2}$ or 0.586	**A1**	CAO
		4	

Question	Answer	Marks	Guidance
3(a)	One-tail because investigating whether "higher"	**B1**	OE. Must have both parts.
		1	
3(b)	H_0: Population mean (or μ) in city same as for others H_1: Population mean (or μ) in city greater than for others	**B1 FT**	If **(a)** two-tail: H_0: Pop mean (or μ) in city same as for others. H_1: Pop mean (or μ) in region different from others.
	2.41 > 2.326 or 0.008 < 0.01 or 0.992 > 0.99	**M1**	If **(a)** two-tail: 2.41 < 2.576 or 0.992 < 0.995.
	There is evidence that buildings are higher [on average].	**A1 FT**	In context, not definite. No contradictions. If **(a)** two-tail: There is no evidence that the [average] height of buildings is different.
		3	

Question	Answer	Marks	Guidance
4(a)	B(1000, $\frac{1}{400}$)	**B1**	Accept Bin and $n = 1000$, $p = \frac{1}{400}$.
		1	
4(b)	Po(2.5)	**B2**	B1 for Po. B1 for $\lambda = 2.5$.
		2	
4(c)(i)	$e^{-2.5} \times \frac{2.5^4}{4!}$	**M1**	FT *their* **(b)** for Normal must have a continuity correction. Allow any λ
	0.134 (3 sf)	**A1**	CWO
		2	
4(c)(ii)	$e^{-2.5}(\frac{2.5^2}{2!} + \frac{2.5^3}{3!} + \frac{2.5^4}{4!})$	**M1**	FT *their* **(b)** for Normal must have a continuity correction. Allow with one term extra or omitted or wrong. Allow any λ.
	0.604 (3 sf)	**A1**	CWO
		2	
4(d)	$\lambda = 2.5 \times 0.7$ or $\lambda = 700 \times \frac{1}{400}$ [= 1.75]	**M1**	Must see λ or use of Poisson.
	$1 - e^{-1.75}$	**M1**	Allow any λ. Allow 1 – P(0,1).
	0.826	**A1**	**SC B1** Use of B(700,0.0025) leading to 0.826.
		3	

Question	Answer	Marks	Guidance
5(a)	$\mathrm{E}(L_1+L_2+L_3+S_1+S_2+S_3+S_4) = 3\times5.10+4\times2.51$ [= 25.34]	**B1**	OE (E(3L + 4S – 25.5) = -0.16)
	$\mathrm{Var}(L_1+L_2+L_3+S_1+S_2+S_3+S_4) = 3\times0.0102+4\times0.0036$ [= 0.045]	**B1**	or $\mathrm{SD} = \frac{3\sqrt{2}}{20} = 0.2121$.
	$\frac{25.5-'25.34'}{\sqrt{'0.045'}}$ [= 0.754]	**M1**	No SD/variance mix. Standardising with *their* values (must be from a combination attempt).
	Φ('0.754')	**M1**	For the correct area consistent with *their* working.
	0.775 (3 sf)	**A1**	
		5	
5(b)	E(L – 2S) = 5.10 – 2×2.51 [= 0.08]	**B1**	OE
	Var(L – 2S) = $0.0102+2^2\times0.0036$ [= 0.0246]	**B1**	Or SD = 0.1568.
	$\frac{0-'0.08'}{\sqrt{'0.0246'}}$ [= –0.510]	**M1**	No SD/variance mix. Standardising with *their* values (must be from a combination attempt).
	$P(Z > '-0.510') = \phi('0.510')$	**M1**	For the correct area consistent with *their* working.
	0.695 (3 sf)	**A1**	
		5	

Question	Answer	Marks	Guidance
6(a)	H_0: population proportion = 0.08 OE H_1: population proportion > 0.08 OE	**B1**	Allow ‘$p = 0.08$’ etc.
	$P(X \geq 4) = 1 - P(X \leq 3) =$ $1 - (0.92^{25} + 25\times0.92^{24}\times0.08 + {}^{25}C_2\times0.92^{23}\times0.08^2 + {}^{25}C_3\times0.92^{22}\times0.08^3)$	**M1**	Allow 1 – (one term omitted or extra or wrong).
	0.135 (3 sf)	**A1**	
	$0.135 > 0.05$	**M1**	Valid comparison. Note: ‘0.865'<0.95 can score M1 A1 and can recover previous M1 A1 for 0.865.
	There is no evidence that proportion owning Chantor has increased	**A1 FT**	In context. Not definite, e.g. not ‘Proportion not increased’. No contradictions.
		5	
6(b)	H_0 was not rejected.	***B1 FT**	H_0 was rejected (consistent with **(a)**).
	Hence Type II might have been made.	**DB1 FT**	Type I error.
		2	
6(c)	$P(X \geq 5) = 1 - P(X \leq 4)$ $= 1-\left((1-0.1351)+{}^{25}C_4\times0.92^{21}\times0.08^4\right)$ [= 0.0451]	***M1**	Attempted. Note: If critical region method used in **(a)** marks can be awarded here.
	$0.0451 < 0.05$	**A1**	Comparison of 0.045[1] with 0.05. Note: If critical region method used in **(a)** marks can be awarded here.
	P(Type I error) = 0.0451 or 0.0452	**A1**	Dependent on M1* only. **SC** Unsupported answers score: **B1** for 0.0451<0.05 and **B1** for final answer 0.0451 only.
		3	

Cambridge Assessment
International Education

Cambridge International AS & A Level

MATHEMATICS **9709/61**

Paper 6 Probability & Statistics 2 **October/November 2020**

MARK SCHEME

Maximum Mark: 50

Published

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WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1(a)	$\text{Po}\left(\frac{2}{3}\right)$	**B1**	Poisson with correct mean stated (to at least 3 sf) or implied in working.
	$1-e^{-\frac{2}{3}}\left(1+\frac{2}{3}\right)$	**M1**	1 – P(X = 0 or 1); allow incorrect λ; allow one end error
	= 0.144 (3 sf)	**A1**	**SC** B1 for use of binomial or no working shown leading to correct final answer.
		3	
1(b)	$n > 50$ and $np = \frac{2}{3} < 5$ **or** $n > 50$ and $p = \frac{1}{300} < 0.1$	**B1**	Accept p or np clearly stated in part **(a)**. Do not accept n is large and p is small.
		3	
1(c)	$\text{Po}\left(\frac{11}{3}\right)$	**B1**	Poisson with correct mean stated (to at least 3sf) or implied in working.
	$e^{-\frac{11}{3}}\left(1+\frac{11}{3}+\frac{\left(\frac{11}{3}\right)^2}{2!}+\frac{\left(\frac{11}{3}\right)^3}{3!}\right)$	**M1**	P(X = 0, 1, 2, 3); allow incorrect λ; allow one end error. Must not be multiplied by any additional values.
	= 0.501 (3 sf)	**A1**	As final answer.
		3	

Question	Answer	Marks	Guidance
2(a)	$\dfrac{\frac{102}{250}\times\frac{250-102}{250}}{250}$ (= 0.000966144) $\frac{102}{250}\pm z\sqrt{'0.00096614'}$	**M1**	Any *z* but must be a *z* value. One side of the interval scores M1.
	$z = 1.645$	**B1**	
	Confident Interval is 0.357 to 0.459 (3 sf)	**A1**	Must be an interval.
		3	
2(b)	Estimate of mean $\left(\frac{50460}{250}\right)$ = \$201.84	**B1**	Allow without units. Allow 3s.f. \$202.
	$\frac{250}{249}\left(\frac{19854200}{250}-\left(\frac{50460}{250}\right)^2\right)$ or $\frac{1}{249}\left(19854200-\frac{50460^2}{250}\right)$	**M1**	
	Estimate of variance = 38 832.75 dollars^2 or 38 800 (3 sf)	**A1**	Allow with missing units. (Calculation of biased gives 38 700 scores M0A0)
		3	
2(c)	e.g. Every house doesn't have an equal chance of being selected or most houses have no chance of being selected.	**B1**	Or other similar e.g. Houses in streets with few houses are more likely to be selected. Not just 'biased', OE, without explanation
		1	

Question	Answer	Marks	Guidance
3	F – 0.5M	**M1**	SOI
	$\sim N(17, 27^2 + 0.25 \times 55^2)$	**B1**	for 102 – 0.5(170) (= 17) or 34
		B1	for $27^2 + 0.25 \times 55^2$ (= 1485.25) or $2^2 \times 27^2 + 55^2$ (= 5941)
	$\frac{0-'17'}{\sqrt{'1485.25'}}$ (= –0.4411)	**M1**	Must have an attempt at combining F and M. No standard deviation/variance errors.
	P(F – 0.5M < 0) = ϕ('-0.4411') = 1 – ϕ('0.4411')	**M1**	Correct area consistent with *their* figures.
	= 0.330 (3 sf)	**A1**	Allow 0.33 if no greater accuracy given
	Alternative method for question 3		
	2F – M		
	$\sim N(34, \ 2^2 \times 27^2 + 55^2)$	**B1**	for 102 – 0.5(170) (= 17) or 34
		B1	for $27^2 + 0.25 \times 55^2$ (= 1485.25) or $2^2 \times 27^2 + 55^2$ (= 5941)
	$\frac{0-'34'}{\sqrt{'5941'}}$ (= –0.4411)	**M1**	Must have an attempt at combining F and M. No standard deviation/variance errors.
	P(2F – M < 0) = ϕ('-0.4411') = 1 – ϕ('0.4411')	**M1**	Correct area consistent with *their* figures.
	= 0.330 (3 sf)	**A1**	Allow 0.33 if no greater accuracy given
		6	

Question	Answer	Marks	Guidance
4(a)	$(k =)\ \frac{1}{a}$	**B1**	
		1	
4(b)	(Mean =) *their* $k \times \frac{a^2}{2}\ \left(= \frac{a}{2}\right)$	**B1 FT**	OE seen. FT *their k*
	$\frac{1}{a}\int_0^a x^2 \mathrm{d}x \left(= \frac{a^2}{3} \right)$	**M1**	Attempt at correct integral and use of limits. Accept in terms of *k* or incorrect *k*.
	$-\left(\frac{a}{2}\right)^2 \left(= \frac{a^2}{12}\right)$	**M1**	For subtracting mean^2, allow if integration not complete. FT incorrect values of *k*.
	$\left(\frac{a^2}{12} = 3\right)\ a = 6$	**A1**	Can be in terms of *k*.
		4	

Question	Answer	Marks	Guidance
5(a)	$\sqrt{2.1}$ or 1.45 (3 sf)	**B1**	
		1	
5(b)	$\lambda = 4.2$	**B1**	
	$1 - e^{-4.2}(1 + 4.2)$	**M1**	$1 - P(X \leqslant 1)$ any λ, allow one end error.
	= 0.922 (3 sf)	**A1**	
		3	
5(c)	$\lambda = 6.3$ $e^{-6.3}\left(\frac{6.3^5}{5!} + \frac{6.3^6}{6!} + \frac{6.3^7}{7!}\right)$	**M1**	$P(X = 5, 6, 7)$ any λ, allow one end error.
	= 0.455 (3 sf)	**A1**	
		2	
5(d)	H_0: $\lambda = 6.3$ H_1: $\lambda < 6.3$	**B1**	Accept μ, accept 2.1 (per week)
	$P(X \leqslant 2) = e^{-6.3}\left(1 + 6.3 + \frac{6.3^2}{2!}\right)$	**M1**	
	= 0.0498 or 0.0499	**A1**	Accept 0.0499
	‘0.0498’ < 0.1	**M1**	For valid comparison. For CV method the comparison can be ‘2 lies in CR of $X \leqslant 2$’
	There is evidence that mean number of absences has decreased.	**A1 FT**	In context, not definite, e.g. not ‘Mean number of absences has decreased.’ No contradictions.
		5	

Question	Answer	Marks	Guidance
5(e)	H_0 rejected	***B1 FT**	OE
	Hence Type I error possible	**DB1 FT**	
		2	
6(a)	$\frac{40-38.4}{\frac{6.9}{\sqrt{30}}} = 1.270 \quad \frac{38-38.4}{\frac{6.9}{\sqrt{30}}} = -0.3175$	**M1**	M1 for either correct expression must have √30 (condone continuity correction)
		A1	A1 for ±1.270 or for 1.27 or AWRT
		A1	A1 for ± (–0.3175) must be opposite sign or for 0.317 or 0.318 or AWRT
	Φ('1.270') – (1 – ϕ('0.3175')	**M1**	For correct method consistent with *their* values
	= 0.523 (3 sf) or 0.522	**A1**	
		5	
6(b)(i)	2-tail because looking for 'change', not decrease or increase	**B1**	OE
		1	

Question	Answer	Marks	Guidance
6(b)(ii)	H_0: Population mean journey time (or μ) = 38.4 H_1: Population mean journey time (or μ) ≠ 38.4	**B1**	Not just 'mean journey time'
	$\dfrac{40.2-38.4}{\frac{6.9}{\sqrt{30}}}$	**M1**	For standardising (must have $\sqrt{30}$)
	= 1.429	**A1**	
	'1.429' < 1.645	**M1**	For valid comparison (area comparison 0.0765 > 0.05)
	There is no evidence that mean journey time has changed.	**A1 FT**	In context. Not definite (e.g. not 'mean journey time has not changed'). No contradictions. FT *their* '1.429' (Note use of 1-tail test scores B0 M1A1M1(comparison with 1.282) A0 max)
	Alternative method for question 6(b)(ii) – critical values method		
	H_0: Population mean journey time (or μ) = 38.4 H_1: Population mean journey time (or μ) ≠ 38.4	**B1**	Not just 'mean journey time'
	$38+1.645\left(\dfrac{6.9}{\sqrt{30}}\right)$	**M1**	
	= 40.47	**A1**	
	40.2 < 40.47	**M1**	For valid comparison
	There is no evidence that mean journey time has changed.	**A1 FT**	In context. Not definite (e.g. not 'mean journey time has not changed'). No contradictions.
		5	

Question	Answer	Marks	Guidance
6(b)(iii)	Yes, because population distribution unknown.	**B1**	Allow: Yes, because population distribution not normal.
		1	

Cambridge International AS & A Level

MATHEMATICS **9709/62**

Paper 6 Probability & Statistics 2 **October/November 2020**

MARK SCHEME

Maximum Mark: 50

Published

This document consists of **13** printed pages.

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- marks are awarded when candidates clearly demonstrate what they know and can do
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- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

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GENERIC MARKING PRINCIPLE 5:

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GENERIC MARKING PRINCIPLE 6:

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The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

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M	Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
A	Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
B	Mark for a correct result or statement independent of method marks.
DM or **DB**	When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
FT	Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

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Abbreviations

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WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	Poisson, any λ	**M1**	Used
	$1 - e^{-3}\left(1+3+\frac{3^2}{2}\right)$	**M1**	Allow one end error
	= 0.577 (3sf)	**A1**	**SC** Use of Binomial (or unsupported correct answer) scores B1 only
		3	

Question	Answer	Marks	Guidance
2(a)	$\frac{56}{300} \pm z \times \sqrt{\frac{\frac{56}{300} \times \frac{244}{300}}{300}}$	**M1**	For expression of the correct form. Must be a z value
	z = 2.054 or 2.055	**B1**	
	0.14(0) to 0.233 (3sf) or 0.141 to 0.233 (3sf)	**A1**	Must be an interval
		3	
2(b)	$\frac{1}{6}$ (= 0.167) This is within confidence interval, so no reason to believe die is biased.	**B1 FT**	Note if confidence interval set up with $\frac{1}{6}, \frac{56}{300}$ it should be the value used here. FT *their* confidence interval. Not definite, e.g. not 'Die not biased'.
		1	

Question	Answer	Marks	Guidance
3(a)	$\frac{1}{2} \times 3 \times c = 1$ ($c = \frac{2}{3}$ **AG**)	**B1**	Must see this line, oe, and result (Alternative method involving equation of line $\left(y = \left(\frac{-c}{3}\right)x + c\right)$ must have all relevant working shown)
		1	
3(b)	$\left(\frac{1}{3}\right)^2$	**M1**	Allow M1 for $\frac{1}{3}$ seen as a linear scale factor or Attempt to find equation of line (of form y = mx + c where c ≠ 0) and interval from 2 to 3 OE or Attempt to find the point (2, $\frac{2}{9}$) using the equation of the line (of form y = mx +c where c ≠ 0) and then use area of triangle
	$= \frac{1}{9}$ or 0.111(3sf)	**A1**	
		2	

Question	Answer	Marks	Guidance
3(c)	Equation of line is $y = \frac{2}{3} - \left(\frac{2}{3} \div 3\right)x$	***M1**	OE Must be of form y = mx + c (c ≠ 0).
	$\text{E}(X) = \int_0^3 \left(\frac{2}{3}x - \frac{2}{9}x^2\right)\text{d}x$	**DM1**	Attempt integration *x* × *their* f(*x*), ignore limits
	$= \left[\frac{x^2}{3} - \frac{2x^3}{27}\right]_0^3$	**A1 FT**	Correct integration and limits. FT *their* equation of line
	= 1	**A1**	
		4	

Question	Answer	Marks	Guidance
4	est(μ) = $\frac{1850}{200}$ or 9.25	**B1**	
	est(σ^2) = $\frac{200}{199}\left(\frac{17850}{200}-\left(\frac{1850}{200}\right)^2\right)$ or $\frac{1}{199}\left(17850-\frac{1850^2}{200}\right)$	**M1**	
	= 3.71 or 3.7060 or $\frac{1475}{398}$	**A1**	
	H_0: $\mu = 8.9$ H_1: $\mu \neq 8.9$	**B1**	Accept Population mean (not just mean)
	$\dfrac{\frac{1850}{200}-8.9}{\sqrt{\frac{"3.706"}{200}}}$	**M1**	Use of biased variance (3.6875) still scores M1
	= 2.57(3sf) (or using areas 0.00507 – 0.0051)	**A1**	Accept 2.58 (3sf) or using areas 0.0049–0.005 where biased variance used.
	2.24 < 2.57 or 0.00507 < 0.0125	**M1**	For valid comparison with 2.240 or 2.241 or valid comparison with 0.0125 Accept 2.24 < 2.58 or 0.00496 < 0.0125 where biased variance used
	(Reject H_0) There is evidence that μ is not 8.9	**A1 FT**	Not definite, e.g. NOT '$\mu \neq 8.9$' Must be in context. No contradictions. (Accept cv method) (**Note:** Use of 1 tail test scores Max B1M1A1B0M1A1M1A0, max 6 out of 8)
		8	

Question	Answer	Marks	Guidance
5(a)	Customers arrive independently or singly or at random	**B1**	Any one of these in context
		1	
5(b)	$e^{-2.3} \times \frac{2.3^3}{3!}$	**M1**	Attempt correct expression seen
	= 0.203 (3sf)	**A1**	
		2	
5(c)	Po(4.6)	**B1**	SOI
	$1 - e^{-4.6}\left(1 + 4.6 + \frac{4.6^2}{2!} + \frac{4.6^3}{3!}\right)$	**M1**	Correct expression, with any λ, allow one end error
	= 0.674 (3sf)	**A1**	As final answer.
		3	
5(d)	P(none arrive) = $e^{-2.3}$ (= 0.10026)	**M1**	Must be clearly *their* P(none arrive)
	${}^5C_2(e^{-2.3})^2(1 - e^{-2.3})^3$	**M1**	FT *their* $e^{-2.3}$
	= 0.0732 or 0.0733 (3sf)	**A1**	
		3	

Question	Answer	Marks	Guidance
6(a)	H_0: P(contains offer) = $\frac{1}{3}$ H_1: P(contains offer) < $\frac{1}{3}$	**B1**	Allow *p* for P(contains offer) but not just proportion
	P(0,1 or 2 offers in 20 | H_0) $= \left(\frac{2}{3}\right)^{20} + 20\left(\frac{2}{3}\right)^{19}\left(\frac{1}{3}\right) + {}^{20}C_2\left(\frac{2}{3}\right)^{18}\left(\frac{1}{3}\right)^{2}$	**M1**	
	= 0.0176 (3sf)	**A1**	
	'0.0176' < 0.1	**M1**	For valid comparison. **SC** comparison of 0.982(4) > 0.9 scores M1 and recovers the previous M1 A1
	(Reject H_0) No evidence (at 10% level) to support manufacturers claim	**A1 FT**	In context. Not definite. No contradictions. (Note 2 tail test scores max B0M1A1M1A0, max 3 out of 5) Accept critical region method: M1 A1 for correctly finding critical region of < 4 ; 2 in critical region M1; A1 conclusion **SC** Use of Normal approximation $N\left(\frac{20}{3},\frac{40}{9}\right)$ scores B1 M1 A0 M1 A1 max; the first M1 for $\frac{\left(2.5-\frac{20}{3}\right)}{\sqrt{\left(\frac{40}{9}\right)}}$ requires use of correct continuity correction and the comparison 0.024 < 0.1 OE must be a valid comparison
		5	

Question	Answer	Marks	Guidance
6(b)	$1 - P(X \leqslant 3)$	**M1**	M1 for 1 – (one term omitted or extra or incorrect) or omit '$1-$'
	$=1-\left[\left(\frac{6}{7}\right)^{20}+20\left(\frac{6}{7}\right)^{19}\left(\frac{1}{7}\right)+{}^{20}C_2\left(\frac{6}{7}\right)^{18}\left(\frac{1}{7}\right)^2+{}^{20}C_3\left(\frac{6}{7}\right)^{17}\left(\frac{1}{7}\right)^3\right]$	**A1**	for all correct expression
	= 0.318 (3sf)	**A1**	As final answer.
		3	
6(c)	Concluding that prop is 1 in 3 when it is actually less(1 in 7)	**B1**	OE, in context.
		1	

Question	Answer	Marks	Guidance
7(a)	$P(S > L + 200) = P(S – L > 200)$ $E(S – L) = 380–210$ (=170) or $E(S–L–200)= 380–210–200$ (=–30)	**B1**	These may be implied by next line
	$Var(S – L) = 140+80$ (=220) or $Var(S–L–200)= 140+80$ (=220)	**B1**	
	$\frac{200-\text{"170"}}{\sqrt{\text{"220"}}}$ or $\frac{0-\text{"}-30\text{"}}{\sqrt{\text{"220"}}}$ (= 2.023)	**M1**	Standardising with *their* values (must be from a combination attempt) Allow with attempted continuity correction.
	$1 – \phi(\text{"2.023"})$	**M1**	Area consistent with their values
	= 0.0216 (3sf)	**A1**	(0.0234 with continuity correction)
		5	

Question	**Answer**	**Marks**	**Guidance**
7(b)	E(total cost) = 380 × 20 + 210 × 50 (= 18 100)	**B1**	or \$181
	Var(total cost) = 140 × 20^2 + 80 × 50^2 (= 256 000)	**B1**	or 25.6 (dollar^2) These may be implied by next line
	$\frac{19000-"18100"}{\sqrt{"256000"}}$ or $\frac{190-"181"}{\sqrt{"25.6"}}$ (= 1.778)	**M1**	Standardising with *their* values (must be from a combination attempt). No mixed methods. Allow with attempted continuity correction.
	$\phi("1.778")$	**M1**	Area consistent with *their* values
	= 0.962 or 0.963 (3 sf)	**A1**	(0.953 or 0.954 with continuity correction)
		5	

Cambridge International AS & A Level

MATHEMATICS **9709/63**

Paper 6 Probability & Statistics 2 **October/November 2020**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2020 series for most Cambridge IGCSE™, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

This document consists of **13** printed pages.

[Turn over

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WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1(a)	$\text{Po}\left(\frac{2}{3}\right)$	**B1**	Poisson with correct mean stated (to at least 3 sf) or implied in working.
	$1-e^{-\frac{2}{3}}\left(1+\frac{2}{3}\right)$	**M1**	1 – P(X = 0 or 1); allow incorrect λ; allow one end error
	= 0.144 (3 sf)	**A1**	**SC** B1 for use of binomial or no working shown leading to correct final answer.
		3	
1(b)	$n > 50$ and $np = \frac{2}{3} < 5$ **or** $n > 50$ and $p = \frac{1}{300} < 0.1$	**B1**	Accept p or np clearly stated in part **(a)**. Do not accept n is large and p is small.
		3	
1(c)	$\text{Po}\left(\frac{11}{3}\right)$	**B1**	Poisson with correct mean stated (to at least 3sf) or implied in working.
	$e^{-\frac{11}{3}}\left(1+\frac{11}{3}+\frac{\left(\frac{11}{3}\right)^2}{2!}+\frac{\left(\frac{11}{3}\right)^3}{3!}\right)$	**M1**	P(X = 0, 1, 2, 3); allow incorrect λ; allow one end error. Must not be multiplied by any additional values.
	= 0.501 (3 sf)	**A1**	As final answer.
		3	

Question	Answer	Marks	Guidance
2(a)	$\dfrac{\frac{102}{250}\times\frac{250-102}{250}}{250}$ (= 0.000966144) $\dfrac{102}{250}\pm z\sqrt{'0.00096614'}$	**M1**	Any *z* but must be a *z* value. One side of the interval scores M1.
	$z = 1.645$	**B1**	
	Confident Interval is 0.357 to 0.459 (3 sf)	**A1**	Must be an interval.
		3	
2(b)	Estimate of mean $\left(\dfrac{50460}{250}\right)$ = \$201.84	**B1**	Allow without units. Allow 3s.f. \$202.
	$\dfrac{250}{249}\left(\dfrac{19854200}{250}-\left(\dfrac{50460}{250}\right)^2\right)$ or $\dfrac{1}{249}\left(19854200-\dfrac{50460^2}{250}\right)$	**M1**	
	Estimate of variance = 38 832.75 dollars^2 or 38 800 (3 sf)	**A1**	Allow with missing units. (Calculation of biased gives 38 700 scores M0A0)
		3	
2(c)	e.g. Every house doesn't have an equal chance of being selected or most houses have no chance of being selected.	**B1**	Or other similar e.g. Houses in streets with few houses are more likely to be selected. Not just 'biased', OE, without explanation
		1	

Question	Answer	Marks	Guidance
3	F – 0.5M	**M1**	SOI
	$\sim \text{N}(17, 27^2 + 0.25 \times 55^2)$	**B1**	for 102 – 0.5(170) (= 17) or 34
		B1	for $27^2 + 0.25 \times 55^2$ (= 1485.25) or $2^2 \times 27^2 + 55^2$ (= 5941)
	$\frac{0 - '17'}{\sqrt{'1485.25'}}$ (= –0.4411)	**M1**	Must have an attempt at combining F and M. No standard deviation/variance errors.
	P(F – 0.5M < 0) = ϕ('-0.4411') = 1 – ϕ('0.4411')	**M1**	Correct area consistent with *their* figures.
	= 0.330 (3 sf)	**A1**	Allow 0.33 if no greater accuracy given
	Alternative method for question 3		
	2F – M		
	$\sim \text{N}(34,\ \ 2^2 \times 27^2 + 55^2)$	**B1**	for 102 – 0.5(170) (= 17) or 34
		B1	for $27^2 + 0.25 \times 55^2$ (= 1485.25) or $2^2 \times 27^2 + 55^2$ (= 5941)
	$\frac{0 - '34'}{\sqrt{'5941'}}$ (= –0.4411)	**M1**	Must have an attempt at combining F and M. No standard deviation/variance errors.
	P(2F – M < 0) = ϕ('-0.4411') = 1 – ϕ('0.4411')	**M1**	Correct area consistent with *their* figures.
	= 0.330 (3 sf)	**A1**	Allow 0.33 if no greater accuracy given
		6	

Question	Answer	Marks	Guidance
4(a)	$(k =)\ \frac{1}{a}$	**B1**	
		1	
4(b)	(Mean =) $\text{their } k \times \frac{a^2}{2}\ \left(= \frac{a}{2}\right)$	**B1 FT**	OE seen. FT *their k*
	$\frac{1}{a}\int_0^a x^2 \mathrm{d}x \left(= \frac{a^2}{3}\right)$	**M1**	Attempt at correct integral and use of limits. Accept in terms of *k* or incorrect *k*.
	$-\left(\frac{a}{2}\right)^2 \left(= \frac{a^2}{12}\right)$	**M1**	For subtracting mean^2, allow if integration not complete. FT incorrect values of *k*.
	$\left(\frac{a^2}{12} = 3\right)\ a = 6$	**A1**	Can be in terms of *k*.
		4	

Question	Answer	Marks	Guidance
5(a)	$\sqrt{2.1}$ or 1.45 (3 sf)	**B1**	
		1	
5(b)	$\lambda = 4.2$	**B1**	
	$1 - e^{-4.2}(1 + 4.2)$	**M1**	$1 - P(X \leqslant 1)$ any λ, allow one end error.
	= 0.922 (3 sf)	**A1**	
		3	
5(c)	$\lambda = 6.3$ $e^{-6.3}\left(\frac{6.3^5}{5!}+\frac{6.3^6}{6!}+\frac{6.3^7}{7!}\right)$	**M1**	$P(X = 5, 6, 7)$ any λ, allow one end error.
	= 0.455 (3 sf)	**A1**	
		2	
5(d)	H_0: $\lambda = 6.3$ H_1: $\lambda < 6.3$	**B1**	Accept μ, accept 2.1 (per week)
	$P(X \leqslant 2) = e^{-6.3}\left(1+6.3+\frac{6.3^2}{2!}\right)$	**M1**	
	= 0.0498 or 0.0499	**A1**	Accept 0.0499
	'0.0498' < 0.1	**M1**	For valid comparison. For CV method the comparison can be '2 lies in CR of $X \leqslant 2$'
	There is evidence that mean number of absences has decreased.	**A1 FT**	In context, not definite, e.g. not 'Mean number of absences has decreased.' No contradictions.
		5	

Question	Answer	Marks	Guidance
5(e)	H_0 rejected	***B1 FT**	OE
	Hence Type I error possible	**DB1 FT**	
		2	
6(a)	$\frac{40-38.4}{\frac{6.9}{\sqrt{30}}} = 1.270 \quad \frac{38-38.4}{\frac{6.9}{\sqrt{30}}} = -0.3175$	**M1**	M1 for either correct expression must have √30 (condone continuity correction)
		A1	A1 for ±1.270 or for 1.27 or AWRT
		A1	A1 for ± (–0.3175) must be opposite sign or for 0.317 or 0.318 or AWRT
	Φ('1.270') – (1 – ϕ('0.3175')	**M1**	For correct method consistent with *their* values
	= 0.523 (3 sf) or 0.522	**A1**	
		5	
6(b)(i)	2-tail because looking for 'change', not decrease or increase	**B1**	OE
		1	

Question	Answer	Marks	Guidance
6(b)(ii)	H_0: Population mean journey time (or μ) = 38.4 H_1: Population mean journey time (or μ) ≠ 38.4	**B1**	Not just 'mean journey time'
	$\dfrac{40.2-38.4}{\frac{6.9}{\sqrt{30}}}$	**M1**	For standardising (must have √30)
	= 1.429	**A1**	
	'1.429' < 1.645	**M1**	For valid comparison (area comparison 0.0765 > 0.05)
	There is no evidence that mean journey time has changed.	**A1 FT**	In context. Not definite (e.g. not 'mean journey time has not changed'). No contradictions. FT *their* '1.429' (Note use of 1-tail test scores B0 M1A1M1(comparison with 1.282) A0 max)
	Alternative method for question 6(b)(ii) – critical values method		
	H_0: Population mean journey time (or μ) = 38.4 H_1: Population mean journey time (or μ) ≠ 38.4	**B1**	Not just 'mean journey time'
	$38+1.645\left(\dfrac{6.9}{\sqrt{30}}\right)$	**M1**	
	= 40.47	**A1**	
	40.2 < 40.47	**M1**	For valid comparison
	There is no evidence that mean journey time has changed.	**A1 FT**	In context. Not definite (e.g. not 'mean journey time has not changed'). No contradictions.
		5	

Question	Answer	Marks	Guidance
6(b)(iii)	Yes, because population distribution unknown.	**B1**	Allow: Yes, because population distribution not normal.
		1	

Cambridge Assessment
International Education

Cambridge International AS & A Level

MATHEMATICS **9709/51**

Paper 5 Probability & Statistics 1 **May/June 2020**

MARK SCHEME

Maximum Mark: 50

Published

Students did not sit exam papers in the June 2020 series due to the Covid-19 global pandemic.

This mark scheme is published to support teachers and students and should be read together with the question paper. It shows the requirements of the exam. The answer column of the mark scheme shows the proposed basis on which Examiners would award marks for this exam. Where appropriate, this column also provides the most likely acceptable alternative responses expected from students. Examiners usually review the mark scheme after they have seen student responses and update the mark scheme if appropriate. In the June series, Examiners were unable to consider the acceptability of alternative responses, as there were no student responses to consider.

Mark schemes should usually be read together with the Principal Examiner Report for Teachers. However, because students did not sit exam papers, there is no Principal Examiner Report for Teachers for the June 2020 series.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the June 2020 series for most Cambridge IGCSE™ and Cambridge International A & AS Level components, and some Cambridge O Level components.

This document consists of **13** printed pages.

Generic Marking Principles

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GENERIC MARKING PRINCIPLE 1:
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- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
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2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M	Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
A	Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
B	Mark for a correct result or statement independent of method marks.
DM or **DB**	When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
FT	Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

Abbreviations

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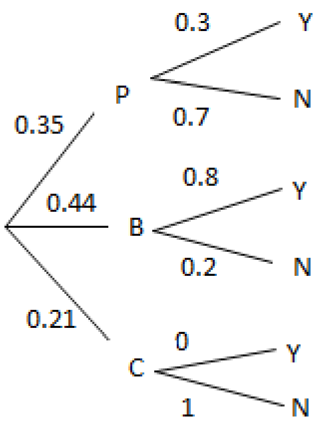
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no “follow through” from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks
1(a)	Prob of 4 (from 1,3, 3,1 or 2,2) = $\frac{3}{36}=\frac{1}{12}$ **AG**	**B1**
		1
1(b)	Mean = $\frac{1}{\frac{1}{12}}$ = 12	**B1**
		1
1(c)	$\left(\frac{11}{12}\right)^5\times\frac{1}{12}=0.0539$ or $\frac{161051}{2985984}$	**B1**
		1
1(d)	$1-\left(\frac{11}{12}\right)^7$	**M1**
	0.456 or $\frac{16344637}{35831808}$	**A1**
		2

Question	Answer	Marks
2(a)	6!	**M1**
	720	**A1**
		2
2(b)	Total number: $\frac{9!}{3!2!}(30240)$	**M1**
	Number with Ls together $= \frac{8!}{3!}(6720)$	**M1**
	Number with Ls not together $= \frac{9!}{3!2!} - \frac{8!}{3!}$ $= 30\,240 - 6720$	**M1**
	23 520	**A1**
	Alternative method for question 2(b)	
	$\frac{7!}{3!} \times \frac{8 \times 7}{2}$	
	7! × k in numerator, k integer ≥ 1	**M1**
	8 × 7 × m in numerator or 8C2 × m, m integer ≥ 1	**M1**
	3! in denominator	**M1**
	23 520	**A1**
		4

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th></tr>
<tr><td>3(a)</td><td><table><tr><td>x</td><td>0</td><td>1</td><td>2</td><td>3</td></tr><tr><td>Probability</td><td>$\frac{1}{56}$</td><td>$\frac{15}{56}$</td><td>$\frac{30}{56}$</td><td>$\frac{10}{56}$</td></tr></table>
(B1 for probability distribution table with correct outcome values)</td><td>B1</td></tr>
<tr><td></td><td>$\mathrm{P}(0) = \frac{3}{8}\times\frac{2}{7}\times\frac{1}{6}=\frac{1}{56}$
$\mathrm{P}(1) = \frac{5}{8}\times\frac{3}{7}\times\frac{2}{6}\times3=\frac{15}{56}$
$\mathrm{P}(2) = \frac{5}{8}\times\frac{4}{7}\times\frac{3}{6}\times3=\frac{30}{56}$
$\mathrm{P}(3) = \frac{5}{8}\times\frac{4}{7}\times\frac{3}{6}=\frac{10}{56}$
(M1 for denominator 8×7×6)</td><td>M1</td></tr>
<tr><td></td><td>Any one probability correct (with correct outcome)</td><td>A1</td></tr>
<tr><td></td><td>All probabilities correct</td><td>A1</td></tr>
<tr><td></td><td></td><td>4</td></tr>
<tr><td>3(b)</td><td>$1-\mathrm{P}(8, 9, 10) = 1-\left[{}^{10}\mathrm{C}_8\, 0.64^8 0.36^2 + {}^{10}\mathrm{C}_9\, 0.64^9 0.36^1 + 0.64^{10}\right]$</td><td>M1</td></tr>
<tr><td></td><td>$1-(0.164156+0.064852+0.11529)$</td><td>M1</td></tr>
<tr><td></td><td>0.759</td><td>A1</td></tr>
<tr><td></td><td></td><td>3</td></tr>
</table>

Question	Answer	Marks
4	Scenarios: 2P 3V 2G $^8C_2 \times {}^4C_2 \times {}^6C_3 = 28 \times 6 \times 20 = 3360$ 2P 4V 1G $^8C_2 \times {}^4C_1 \times {}^6C_4 = 28 \times 4 \times 15 = 1680$ 3P 3V 1G $^8C_3 \times {}^4C_1 \times {}^6C_3 = 56 \times 4 \times 20 = 4480$ 4P 2V 1G $^8C_4 \times {}^4C_1 \times {}^6C_2 = 70 \times 4 \times 15 = 4200$ (**M1** for $^8C_r \times {}^4C_r \times {}^6C_r$ with $\sum r = 7$)	**M1**
	Two unsimplified products correct	**B1**
	Summing the number of ways for 3 or 4 correct scenarios	**M1**
	Total: 13 720	**A1**
		4

Question	Answer	Marks
5(a)	 Fully correct labelled tree for method of transport with correct probabilities.	**B1**
	Fully correct labelled branches with correct probabilities for lateness with either 1 branch after W or 2 branches with the prob 0	**B1**
		2
5(b)	$0.35 \times 0.3 + 0.44 \times 0.8 \ (+ 0)$	**M1**
	0.457	**A1**
		2

Question	Answer	Marks
5(c)	P(not B|not fruit) = $\frac{P(B'\cap F')}{P(F')}$	M1
	$\frac{0.35\times0.7+0.21\times1}{1-\textit{their}(\mathbf{b})}$	M1
	$\frac{0.455}{0.543}$ (**M1** for 1 – *their* **(b)** or summing three appropriate 2-factor probabilities, correct or consistent with *their* tree diagram as denominator)	M1
	0.838 or $\frac{455}{543}$	A1
		4

Question	Answer	Marks
6(a)	$P\left(\frac{50-54}{6.1}<z<\frac{60-54}{6.1}\right)=P(-0.6557<Z<0.9836)$	M1
	Both values correct	A1
	Φ (0.9836) – Φ (–0.6557) = Φ (0.9836) + Φ (0.6557) – 1 = 0.8375 + 0.7441 – 1 (Correct area)	M1
	0.582	A1
		4

Question	Answer	Marks
6(b)	$\frac{45-\mu}{\sigma}=-0.994$	**B1**
	$\frac{56-\mu}{\sigma}=1.372$	**B1**
	One appropriate standardisation equation with μ, σ, z-value (not probability) and 45 or 56.	**M1**
	$11 = 2.366\,\sigma$ (**M1** for correct algebraic elimination of μ or σ from *their* two simultaneous equations to form an equation in one variable)	**M1**
	$\sigma = 4.65$, $\mu = 49.6$	**A1**
		5

Question	Answer	Marks
7(a)	Class widths: 10, 5, 15, 20, 10	**M1**
	Frequency density = frequency/*their* class width: 1.8, 4.8, 2, 1, 0.8	**M1**
	All heights correct on diagram (using a linear scale)	**A1**
	Correct bar ends	**B1**
	Bar ends: 10.5, 15.5, 30.5, 50.5, 60.5	**B1**
		5
7(b)	11 – 15 and 31 – 50	**B1**
	Greatest IQR = 50 – 11 = 39	**B1**
		2
7(c)	Mean = $\frac{18\times5.5+24\times13+30\times23+20\times40.5+8\times55.5}{100}=\frac{2355}{100}=23.6$	**B1**
	Var = $\frac{18\times5.5^2+24\times13^2+30\times23^2+20\times40.5^2+8\times55.5^2}{100}-\text{mean}^2$	**M1**
	$\frac{77917.5}{100}-\text{mean}^2=224.57$	**A1**
	Standard deviation = 15.0 (**FT** *their* variance)	**A1 FT**
		4

Cambridge International AS & A Level

MATHEMATICS **9709/52**

Paper 5 Probability & Statistics 1 **May/June 2020**

MARK SCHEME

Maximum Mark: 50

Published

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Question	Answer	Marks
1	$\sum x - 50n = 144$	**B1**
	$50n + 144 = 944$	**M1**
	$n = 16$	**A1**
		3

Question	Answer	Marks
2(a)	$\frac{56}{500}$ or $\frac{14}{125}$ or 0.112	**B1**
		1
2(b)	$\mathrm{P}(\mathrm{D}\mid\mathrm{S}) = \frac{\mathrm{P}(\mathrm{D}\cap\mathrm{S})}{\mathrm{P}(\mathrm{S})} = \frac{120}{280}$	**M1**
	$\frac{120}{280}$ or $\frac{3}{7}$	**A1**
		2

Question	Answer	Marks
2(c)	P(hockey) = $\frac{220}{500} = 0.44$ P(Amos or Benn) = $\frac{242}{500} = 0.484$ P(hockey ∩ A or B) = $\frac{104}{500} = 0.208$ P(H) × P(A U B) = P(H ∩ (A U B)) if independent	**M1**
	$\frac{220}{500} \times \frac{242}{500} = \frac{1331}{6250}$ so not independent	**A1**
		2

Question	*Answer*	Marks
3(a)	Median = 0.238	**B1**
	UQ = 0.245, LQ = 0.231, So IQR = 0.245 – 0.231	**M1**
	0.014	**A1**
		3

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th></tr>
<tr><td>3(b)</td><td><table>
<tr><td></td><td></td><td>LQ</td><td>M</td><td>UQ</td><td></td></tr>
<tr><td>A</td><td>0.220</td><td>0.231
FT</td><td>0.238
FT</td><td>0.245
FT</td><td>0.254</td></tr>
<tr><td>B</td><td>0.211</td><td>0.224</td><td>0.232</td><td>0.243</td><td>0.256</td></tr>
</table></td><td></td></tr>
<tr><td></td><td>Medians and quartiles correctly plotted for <i>A</i> or <i>B</i></td><td>B1</td></tr>
<tr><td></td><td>End points correct for <i>A</i> or <i>B</i></td><td>B1</td></tr>
<tr><td></td><td>Completely correct, including scale</td><td>B1</td></tr>
<tr><td></td><td></td><td>3</td></tr>
<tr><td>3(c)</td><td>Lengths of rods produced by machine <i>A</i> are longer.
(B1 for comparison of central tendency)</td><td>B1</td></tr>
<tr><td></td><td>Lengths of rods produced by machine <i>A</i> are less spread out
(B1 for comparison of spread)</td><td>B1</td></tr>
<tr><td></td><td></td><td>2</td></tr>
</table>

Question	Answer	Marks
4(a)	$\mathrm{P}(X < 25) = \mathrm{P}\left(z < \frac{25-40}{12}\right) = \mathrm{P}(z < -1.25)\mathrm{P}(X < 25) = \mathrm{P}(z <)$	**M1**
	$1 - 0.8944$	**M1**
	0.106	**A1**
		3
4(b)	0.8944 divided by 3 (**M1** for 1 - *their* **(a)** divided by 3)	**M1**
	0.298 **AG**	**A1**
		2
4(c)	0.2981 gives $z = 0.53$	**B1**
	$\frac{h-40}{12} = 0.53$	**M1**
	$h = 46.4$	**A1**
		3

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th></tr>
<tr><td rowspan="3">5(a)</td><td><table>
<tr><td></td><td>1</td><td>1</td><td>2</td><td>2</td><td>3</td></tr>
<tr><td>1</td><td>1</td><td>1</td><td>2</td><td>2</td><td>3</td></tr>
<tr><td>2</td><td>2</td><td>2</td><td>2</td><td>2</td><td>3</td></tr>
<tr><td>3</td><td>3</td><td>3</td><td>3</td><td>3</td><td>3</td></tr>
</table></td><td>M1</td></tr>
<tr><td>$\frac{7}{15}$ AG</td><td>A1</td></tr>
<tr><td></td><td>2</td></tr>
<tr><td rowspan="4">5(b)</td><td><table>
<tr><td>x</td><td>1</td><td>2</td><td>3</td></tr>
<tr><td>Probability</td><td>$\frac{2}{15}$</td><td>$\frac{6}{15}$</td><td>$\frac{7}{15}$</td></tr>
</table></td><td>B1</td></tr>
<tr><td>P(1) or P(2) correct</td><td>B1</td></tr>
<tr><td>3rd probability correct, FT sum to 1</td><td>B1</td></tr>
<tr><td></td><td>3</td></tr>
</table>

Question	Answer	Marks
5(c)	$\mathrm{E}(X) = \frac{2+12+21}{15} = \frac{35}{15} = \frac{7}{3}$	**B1**
	$\mathrm{Var}(X) = \frac{1^2 \times 2 + 2^2 \times 6 + 3^2 \times 7}{15} - \left(\frac{7}{3}\right)^2$	**M1**
	$\frac{22}{45}(0.489)$	**A1**
		3

Question	*Answer*	Marks
6(a)	$\frac{8!}{3!}$	**M1**
	6720	**A1**
		2

Question	Answer	Marks
6(b)	Total number $= \frac{10!}{2!3!}$ (302400) (A)	**B1**
	With Es together $= \frac{9!}{3!}$ (60480) (B)	**B1**
	Es not together = *their* (A) – *their* (B)	**M1**
	241920	**A1**
	Alternative method for question 6(b)	
	_^_^_^_^_^_^_^_^_ $\frac{8!}{3!} \times \frac{9 \times 8}{2}$	
	8! × *k* in numerator, *k* integer ≥ 1, denominator ≥ 1	**B1**
	3! × *m* in denominator, *m* integer ≥ 1	**B1**
	Their $\frac{8!}{3!}$ Multiplied by ${}^9C_2$ (OE) only (no additional terms)	**M1**
	241920	**A1**
		4

Question	Answer	Marks
6(c)	Scenarios: E M M M $^5C_0 = 1$ E M M _ $^5C_1 = 5$ E M _ _ $^5C_2 = 10$	**M1**
	Summing the number of ways for 2 or 3 correct scenarios	**M1**
	Total = 16	**A1**
		3

Question	Answer	Marks
7(a)	1 – P(10, 11, 12) $= 1 - [{}^{12}C_{10}\, 0.72^{10} 0.28^2 + {}^{12}C_{11}\, 0.72^{11} 0.28^1 + 0.72^{12}]$	**M1**
	1 – (0.19372 + 0.09057 + 0.01941)	**A1**
	0.696	**A1**
		3
7(b)	$0.28^3 \times 0.72 = 0.0158$	**B1**
		1

Question	Answer	Marks
7(c)	Mean = $100 \times 0.72 = 72$ Var = $100 \times 0.72 \times 0.28 = 20.16$	**M1**
	P(less than 64) = $\mathrm{P}\left(z < \frac{63.5-72}{\sqrt{20.16}}\right)$ (**M1** for substituting *their μ and σ* into ±standardisation formula with a numerical value for '*63.5*')	**M1**
	Using either 63.5 or 64.5 within a ±standardisation formula	**M1**
	Appropriate area Φ, from standardisation formula P(*z*<…) in final solution $= \mathrm{P}(z < -1.893)$	**M1**
	0.0292	**A1**
		5

Cambridge International AS & A Level

MATHEMATICS **9709/53**

Paper 5 Probability & Statistics 1 **May/June 2020**

MARK SCHEME

Maximum Mark: 50

Published

Students did not sit exam papers in the June 2020 series due to the Covid-19 global pandemic.

This mark scheme is published to support teachers and students and should be read together with the question paper. It shows the requirements of the exam. The answer column of the mark scheme shows the proposed basis on which Examiners would award marks for this exam. Where appropriate, this column also provides the most likely acceptable alternative responses expected from students. Examiners usually review the mark scheme after they have seen student responses and update the mark scheme if appropriate. In the June series, Examiners were unable to consider the acceptability of alternative responses, as there were no student responses to consider.

Mark schemes should usually be read together with the Principal Examiner Report for Teachers. However, because students did not sit exam papers, there is no Principal Examiner Report for Teachers for the June 2020 series.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the June 2020 series for most Cambridge IGCSE™ and Cambridge International A & AS Level components, and some Cambridge O Level components.

This document consists of **13** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

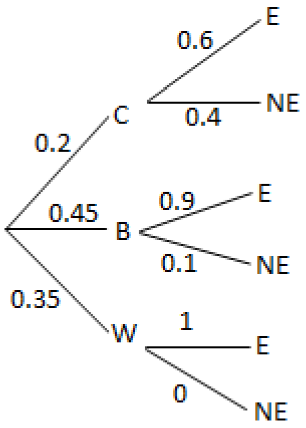
The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M	Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
A	Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
B	Mark for a correct result or statement independent of method marks.
DM or **DB**	When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
FT	Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no “follow through” from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks
1(a)	 Fully correct labelled tree for method of transport with correct probabilities.	**B1**
	Fully correct labelled branches with correct probabilities for lateness with either 1 branch after W or 2 branches with the probability 0.	**B1**
		2
1(b)	$\mathrm{P(C\|E)} = \dfrac{\mathrm{P}(\mathrm{C}\cap\mathrm{E})}{\mathrm{P}(\mathrm{E})} = \dfrac{0.2\times0.6}{0.2\times0.6+0.45\times0.1+0.35\times1}$	**M1**
	Summing three appropriate 2-factor probabilities	**M1**
	$\dfrac{0.12}{0.515}$	**A1**
	0.233 or $\dfrac{12}{515}$	**A1**
		4

Question	Answer	Marks
2(a)	$0.22^3 = 0.0106$	**B1**
		1
2(b)	$\text{P}(2, 3, 4) = {}^{16}\text{C}_2\ 0.22^2 0.78^{14} + {}^{16}C_3 0.22^3 0.78^{13} + {}^{16}C_4 0.22^4 0.78^{12}$	**M1**
	$0.179205 + 0.235877 + 0.216221$	**A1**
	0.631	**A1**
		3

Question	Answer	Marks
3(a)	$\text{P}(\text{X} < 21) = \text{P}\left(z < \frac{21-15.8}{4.2}\right) = \Phi(1.238)$	**M1**
	0.892	**A1**
		2
3(b)	$z = \pm 0.674$	**B1**
	$\frac{k-15.8}{4.2} = 0.674$	**M1**
	18.6	**A1**
		3

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th></tr>
<tr><td>4(a)</td><td>
<table>
<tr><td>-1</td><td>0</td><td>0</td><td>1</td></tr>
<tr><td>0</td><td>1</td><td>1</td><td>2</td></tr>
<tr><td>2</td><td>3</td><td>3</td><td>4</td></tr>
</table>
<table>
<tr><td>x</td><td>-1</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td></tr>
<tr><td>Probability</td><td>$\frac{1}{12}$</td><td>$\frac{3}{12}$</td><td>$\frac{3}{12}$</td><td>$\frac{2}{12}$</td><td>$\frac{2}{12}$</td><td>$\frac{1}{12}$</td></tr>
</table>
Probability distribution table with correct scores with at least one probability</td><td>B1</td></tr>
<tr><td></td><td>At least 4 probabilities correct</td><td>B1</td></tr>
<tr><td></td><td>All probabilities correct</td><td>B1</td></tr>
<tr><td></td><td></td><td>3</td></tr>
<tr><td>4(b)</td><td>$\text{E(X)} = \frac{-1+0+3+4+6+4}{12} = \frac{16}{12} = \frac{4}{3}$</td><td>B1</td></tr>
<tr><td></td><td>$\text{Var(X)} = \frac{1+0+3+8+18+16}{12} - \left(\frac{4}{3}\right)^2$</td><td>M1</td></tr>
<tr><td></td><td>$\frac{37}{18}$ $(= 2.06)$</td><td>A1</td></tr>
<tr><td></td><td></td><td>3</td></tr>
</table>

Question	Answer	Marks
5(a)	$\frac{1}{\frac{1}{4}} = 4$	**B1**
		1
5(b)	$\frac{9}{64} \quad (= 0.141)$	**B1**
		1
5(c)	$P(X < 6) = 1 - \left(\frac{3}{4}\right)^5$ (**FT** *their* probability/mean from part **(a)**)	**M1**
	0.763	**A1**
		2
5(d)	$\text{Mean} = 80 \times 0.25 = 20$ $\text{Var} = 80 \times 0.25 \times 0.75 = 15$	**M1**
	$P(\text{more than } 25) = P\left(z > \frac{25.5 - 20}{\sqrt{15}}\right)$	**M1**
	$P(z > 1.42)$	**M1**
	$1 - 0.9222$	**M1**
	0.0778	**A1**
		5

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th></tr>
<tr><td>6(a)</td><td><table><tr><th>A</th><th></th><th>B</th></tr><tr><td></td><td>2</td><td>6</td></tr><tr><td>5 2 0</td><td>3</td><td>0 1 5 8</td></tr><tr><td>9 7 2 1 1</td><td>4</td><td>1 2 2 7 9</td></tr><tr><td>3 2</td><td>5</td><td>2</td></tr><tr><td>4</td><td>6</td><td></td></tr></table>KEY 1 | 4 | 2 means $41 000 for A and $42 000 for B
Correct stem</td><td>B1</td></tr>
<tr><td></td><td>Correct A on LHS</td><td>B1</td></tr>
<tr><td></td><td>Correct B on same diagram</td><td>B1</td></tr>
<tr><td></td><td>Correct key for <i>their</i> diagram, both companies identified and correct units</td><td>B1</td></tr>
<tr><td></td><td></td><td>4</td></tr>
<tr><td>6(b)</td><td>Median = [$]42 000</td><td>B1</td></tr>
<tr><td></td><td>LQ = [$]35 000
UQ = [$]52 000</td><td>B1</td></tr>
<tr><td></td><td>IQR = [$]17 000
(FT if $49000 \leqslant \text{UQ} \leqslant 53000 - 32000 \leqslant \text{LQ} \leqslant 41000$)</td><td>B1 FT</td></tr>
<tr><td></td><td></td><td>3</td></tr>
</table>

Question	Answer	Marks
6(c)	Sum of given 11 numbers is 433 000	**M1**
	Sum of 12 numbers, including new = 38 500 × 12 = 462 000	**M1**
	Difference = new salary = [$]29 000	**A1**
		3

Question	Answer	Marks
7(a)	$\frac{9!}{2!2!} = 90\,720$	**B1**
		1
7(b)	$\frac{6!}{2!}$	**M1**
	360	**A1**
		2

Question	Answer	Marks
7(c)	2 Es together $= \frac{8!}{2!}(=20160)$	**M1**
	Es not together $= 90720 - 20160 = 70560$	**M1**
	Probability $= \frac{70560}{90720}$	**M1**
	$\frac{7}{9}$ or 0.778	**A1**
	Alternative method for question 7(c)	
	_^_^_^_^_^_^_^_ $\frac{7!}{2!}\times\frac{8\times7}{2} = 70560$	
	7! × k in numerator, k integer $\geqslant 1$, denominator $\geqslant 1$	**M1**
	Multiplying by $^8C_2$ OE	**M1**
	Probability $= \frac{70560}{90720}$	**M1**
	$\frac{7}{9}$ or 0.778	**A1**
		4

Question	Answer	Marks
7(d)	Scenarios are: E L _ _ _ $^5C_3$ 10 E E L _ _ $^5C_2$ 10 E _ _ _ _ $^5C_4$ 5 E E _ _ _ $^5C_3$ 10	**M1**
	Summing the number of ways for 3 or 4 correct scenarios	**M1**
	Total = 35	**A1**
		3

Cambridge International A Level

MATHEMATICS **9709/62**

Paper 6 Probability and Statistics **March 2020**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the March 2020 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

This document consists of **12** printed pages.

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Mathematics-Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
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3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
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5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
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Abbreviations

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WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	$(\lambda =) \frac{5}{12} = 0.417$ or better	**B1**	
	$1 - e^{-\frac{5}{12}}(1 + \frac{5}{12})$	**M1**	1 – P(X = 0 or 1), by Poisson, using any λ, allow 1 – P(X = 0 or 1 or 2) for M1
	= 0.0661 or 0.0662 (3 sf)	**A1**	Final answer **SC** use of Binomial (from 0.06607…) B1 only
		3	

Question	Answer	Marks	Guidance
2	$2 \times z \times \frac{3.2}{10} = 1.25$	**M1**	OE Allow without '2 ×'
	$z = 1.953$	**A1**	SOI
	ϕ('*their* 1.953') (= 0.9746)	**M1**	
	= 1 – 2(1 – '0.9746') = 0.9492	**M1**	OE
	α = 94.9 or 95	**A1**	CWO
		5	

Question	Answer	Marks	Guidance
3(a)	est (μ) = 37.6 or $\frac{1504}{40}$ or $\frac{188}{5}$	**B1**	
	est (σ^2) = $\frac{40}{39}\left[\frac{57760}{40} - 37.6^2\right] = 31.0154 = \frac{2016}{65}$	**M1**	Correct substitution in any correct formula $\frac{1}{39}\left[57760 - \frac{1504^2}{40}\right]$
	= 31.(0) (3 sf)	**A1**	Accept $\frac{2016}{65}$ or $31\frac{1}{65}$
		3	
3(b)	H_0: Pop mean (or μ) = 39.2 H_1: Pop mean (or μ) < 39.2	**B1**	Both. Not just 'mean'
	$\frac{'37.6'-39.2}{\frac{\sqrt{'31.0154'}}{\sqrt{40}}}$	**M1**	Allow use of biased variance (30.2), must have √40
	= –1.817	**A1**	**SC** FT use of biased = –1.840 for A1
	'1.817' > 1.645 OE	**M1**	Valid comparison '*their* 1.817' with 1.645 or valid area comparison 0.0346 < 0.05 OE
	There is evidence that mean time has decreased	**A1FT**	FT *their* 1.817; in context, not definite, no contradictions **SC** For 2 tail test: H_1: $\mu \neq 39.2$ and comp 1.96, max B0M1A1M1A0 (no FT for final mark)
		5	

Question	Answer	Marks	Guidance
4(a)	λ (= 0.4 × 365 ÷ 50) = 2.92	**B1**	
	$e^{-2.92}(1 + 2.92 + \frac{2.92^2}{2})$	**M1**	Any λ. Allow one end error
	= 0.441 (3 sf)	**A1**	
		3	
4(b)	$e^{-\lambda} > 0.95$	**M1**	Allow '=' throughout
	$-\lambda > \ln 0.95$ or $\lambda < 0.051293$ OE	**M1**	Attempt ln both sides
	'0.051293' × 50 ÷ 0.4 (= 6.411)	**M1**	
	Largest n is 6 (3 sf) Allow $n = 6$ or $n \leqslant 6$ (NOT $n < 6$ or $n \geqslant 6$ as final answer)	**A1**	**SC** Trial and Improvement M1 for $e^{-\lambda} > 0.95$ SOI; M1 for $\lambda = n \times \frac{0.4}{50}$; M1 for use of both $n = 6$ giving 0.9531 and $n = 7$ giving 0.9455; A1 $n = 6$
		4	

Question	Answer	Marks	Guidance
5(a)	$\frac{3}{4000}\int_5^{10}(100-x^2)\mathrm{d}x$ $=\frac{3}{4000}\left[100x - \frac{x^3}{3}\right]_5^{10}$	**M1**	Attempt integration of f(x), ignore limits. Condone omission of $\frac{3}{4000}$
	$= \frac{3}{4000}(1000 - \frac{1000}{3} - 500 + \frac{125}{3})$	**M1**	Correct limits 5 and 10. OE SOI
	$= 0.156$ (3 sf) or $\frac{5}{32}$	**A1**	For fully correct working seen including substitution of limits
		3	
5(b)	$\frac{3}{4000}\int_p^{10}(100-x^2)\mathrm{d}x = \frac{1}{4}$	**M1**	Attempt integration of f(x) with any limits and $=\frac{1}{4}$ or $=\frac{3}{4}$ seen. Condone omission of $\frac{3}{4000}$
	$\frac{3}{4000}\left[100x - \frac{x^3}{3}\right]_p^{10} = \frac{1}{4}$	**A1**	Correct integration with correct limits seen (or implied for limits p and 10) and $= \frac{1}{4}$ OE Condone omission of $\frac{3}{4000}$
	$\frac{3}{4000}(1000 - \frac{1000}{3} - 100p + \frac{p^3}{3}) = \frac{1}{4}$	**M1**	Attempt substitution correct limits in their integration of f(x). Accept limits 0 to p if clearly seen, accept limits –10 and p. Substitution must be seen.
	e.g. $\frac{2000}{3} - 100p + \frac{p^3}{3} = \frac{1000}{3}$ $p^3 - 300p + 1000 = 0$	**A1**	AG No errors seen
		4	

Question	Answer	Marks	Guidance
5(c)	Curve is symmetrical about $x = 0$	**B1**	May be implied by sketch. No contradictions or integrate f(x) between $-q$ and $+q$ and equate to 0.5 leading to q^3 -300q + 1000 = 0 oe
	$q = 3.47$	**B1**	
		2	

Question	Answer	Marks	Guidance
6(a)	N(310, 50)	**B1**	SOI
	$\frac{300-'310'}{\sqrt{'50'}}$ (= –1.414)	**M1**	Standardise using their values
	Φ(‘–1.414’) = 1 – ϕ(‘1.414’)	**M1**	Area consistent with their values
	= 0.0786 or 0.0787 (3 sf)	**A1**	As final answer
		4	

Question	Answer	Marks	Guidance
6(b)	$P(L - 2S > 0)$	**M1**	OE SOI
	$E(X) = 200\text{-}2\text{x}110$ or $= -20$	**B1**	OE seen
	$\text{Var} = 30 + 2^2 \times 20$ or $= 110$	**B1**	Seen
	$N(-20, 110)$ $\frac{0-('-20')}{\sqrt{'110'}}$ (= 1.907)	**M1**	Standardising with their values. Mean and variance must come from a combination attempt.
	$1 - \Phi('1.907')$	**M1**	Correct area consistent with their working
	= 0.0283 (3 sf)	**A1**	Final answer
		6	

Question	Answer	Marks	Guidance
7(a)	$P(X \leqslant n)$ $(n \leqslant 20)$ attempted, using B(20, 0.95)	**M1**	OE
	$P(X \leqslant 17)$ <u>or</u> $P(X \leqslant 16)$ attempted, using B(20, 0.95)	**M1**	OE
	$(P(X \leqslant 17)) = 0.0755$ <u>and</u> $(P(X \leqslant 16)) = 0.0159$	**A1**	OE (0.925 and 0.984) both correct
	Rej region is $X \leqslant 16$ or $X < 17$	**A1**	Dependent on M1M1 and previous answers correct to at least 0.075/0.076 and 0.016 **or** 0.92/0.93 and 0.98 Correct unsupported answers of 0.0755 and 0.0159 OE scores M1 M1 A0
		4	

Question	Answer	Marks	Guidance
7(b)	0.0159	**B1**	FT *their* rejection region, from Binomial in a, if P(X in rejection region) < 0.025
		1	
7(c)	Use of B(20, 0.7)	**M1**	
	$P(X > 16 \mid p = 0.7)$	**M1**	Correct method using B(20, 0.7)
	= 0.107	**A1**	
		3	

Cambridge Assessment International Education
Cambridge International Advanced Level

MATHEMATICS **9709/72**
Paper 7 **October/November 2019**
MARK SCHEME
Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

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This document consists of **11** printed pages.

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- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

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Mark Scheme Notes

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Types of mark

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A	Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
B	Mark for a correct result or statement independent of method marks.
DM or **DB**	When a part of a question has two or more “method” steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
FT	Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

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WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1(i)	Binomial	**B1**	
	$n = 500$ **and** $p = \frac{1}{150}$ or 0.00667	**B1**	Or $\mathrm{B}\left(500, \frac{1}{150}\right)$ for B1B1
		2	
1(ii)	Poisson	**B1**	
	n large and mean = $\frac{10}{3}$ or 3.3 or better, which is < 5	**B1**	Accept $n > 50$
		2	
1(iii)	$1 - e^{-\frac{10}{3}} \times \left(1 + \frac{10}{3} + \frac{\left(\frac{10}{3}\right)^2}{2}\right)$	**M1**	1-P(X = 0, 1, 2)
	= 1 – 0.353	**A1**	Correct expression with λ=3.3 or better
	= 0.647 (3 sf)	**A1**	**SC** Use of Binomial scores B1 for 0.648. Use of Normal scores B1 for 0.67(0) to 0.677
		3	

Question	Answer	Marks	Guidance
2(i)(a)	Assume standard deviation for the region is 7.1	**B1**	Or standard deviation is same as for whole population OE
	$\frac{63.2-65.2}{\frac{7.1}{\sqrt{n}}}=-2.182$	**M1**	Attempt to find correct equation (accept +2.182)
	$n = \{-2.182 \times 7.1 \div (-2)\}^2$	**A1**	Any correct expression for n or $\sqrt{n}$. SOI
	$n = 60$	**A1**	CWO. Must be an integer
		4	
2(i)(b)	H_0: population mean (or μ) = 65.2 H_1: population mean (or μ) < 65.2	**B1**	Not just 'mean'
	2.182 > 1.751	**M1**	Or valid area comparison.
	There is evidence that animals are shorter in this region	**A1**	CWO. No contradictions
		3	
2(ii)	Population unknown or population not given as normal	**B1**	Allow population not normal. Accept distribution of X unknown.
		1	

Question	Answer	Marks	Guidance
3(i)	$\text{est}(\mu) = \dfrac{25110}{50}$ (= 502.2)	**B1**	
	$\text{est}\left(\sigma^2\right) = \dfrac{50}{49}\left(\dfrac{12610300}{50} - \left(\dfrac{25110}{50}\right)^2\right) \left(= \dfrac{50}{49} \times \dfrac{58}{50} = 1.1836\right)$	**M1**	OE
	1.18 (3 sf) or $\dfrac{58}{49}$	**A1**	Accept SD = 1.0879
	z = 2.054 or 2.055	**B1**	
	$502.2 \pm z \times \dfrac{\sqrt{'1.1836'}}{\sqrt{50}}$	**M1**	Must be of correct form.
	501.9 to 502.5 (1dp)	**A1**	CWO. Must be in interval. **SC** accept use of biased variance (1.16) for M1 A1
		6	
3(ii)	More confident **or** z would be greater, Hence wider.	**B1**	OE Reason needed
		1	

Question	Answer	Marks	Guidance
4(i)	$\frac{1}{2}\times a\times\frac{a}{2}=1$ **or** $\frac{1}{2}\int_0^a x\,\mathrm{d}x=1$ $\frac{a^2}{4}=1$ OE	**M1**	Attempt at triangle area or integral f(x) **and** = 1,
	$a = 2$	**A1**	
		2	
4(ii)	$\frac{1}{2}\int_0^2 x^2\,\mathrm{d}x$	**M1**	Attempt integral xf(x)
	$=\left[\frac{x^3}{6}\right]_0^2$	**M1**	Correct integral and limits 0 to their 'a'
	$\left(=\frac{8}{6}\right)=\frac{4}{3}$	**A1**	AG CWO
		3	

Question	Answer	Marks	Guidance
4(iii)	$P\left(X<\frac{4}{3}\right)=\frac{1}{2}\int_0^{\frac{4}{3}} x\mathrm{d}x$	**M1**	Attempt integral f(*x*) between correct limits
	$=\frac{4}{9}$	**A1**	or $\frac{5}{9}$
	$P\left(E\left(X\right)<X<m\right)=\frac{1}{2}-\,'\frac{4}{9}'$	**M1**	or $\frac{5}{9}-\frac{1}{2}$
	$\frac{1}{18}$	**A1**	
	Alternative method for question 4(iii)		
	Attempt to find *m*	**M1**	
	$m=\sqrt{2}$	**A1**	
	Integrate f(*x*) between $\frac{4}{3}$ and $'\sqrt{2}'$	**M1**	
	$\frac{1}{18}$	**A1**	
		4	

Question	Answer	Marks	Guidance
5(i)	mean = 3250 var. = 61	**B1**	Or mean = 325 var. $= \frac{6.1}{10}$
	$\frac{3240-3250}{\sqrt{61}}(=-1.280)$	**M1**	Standardise with their values (no mixed methods)
	$\phi('-1.280') = 1 - \phi('1.280)$	**M1**	Area consistent with their figures
	0.100	**A1**	Allow 0.1
		4	
5(ii)	E(D) = 325 – 2 × 167 = –9	**B1**	Accept ±9
	Var(D) = $6.1 + 2^2 \times 5.6$ (= 28.5)	**B1**	
	$\frac{0-(-9)}{\sqrt{28.5}}(=1.686)$	**M1**	Standardising with *their* values. Must have a combination attempt on denominator and $\sqrt{\ }$
	$1 - \phi('1.686')$	**M1**	Area consistent with their figures
	0.0459	**A1**	
		5	

Question	Answer	Marks	Guidance
6(i)	H_0: Pop mean (or λ or μ) is 1.1 H_1: Pop mean (or λ or μ) is more than 1.1	**B1**	
	$P(X \geqslant 4) = 1 - e^{-1.1}\left(1 + 1.1 + \frac{1.1^2}{2} + \frac{1.1^3}{3!}\right)$	**M1**	Correct expression for either $P(X \geqslant 4)$ or $P(X \geqslant 5)$
	0.0257	**A1**	Correct value of either $P(X \geqslant 4)$ or $P(X \geqslant 5)$
	$P(X \geqslant 5) = 0.0257 - e^{-1.1} \times \frac{1.1^4}{4!} = 0.00544$	**B1**	B1 for the other value (Note use of $P(X < 4) = 0.9743$ and $P(X < 5) = 0.99456$ can score only if comparison with 0.99 seen)
	$0.00544 < 0.01 < 0.0257$	**M1**	OE stated (valid comparison)
	There is evidence mean has increased	**B1**	**SC** $P(X \geqslant 6) = 0.000968$ M1A1 Conclusion B1
		6	
6(ii)	Concluding mean has increased when it has not	**B1**	In context
	'0.00544'	**B1FT**	FT *their* $P(X \geqslant 5)$, dep < 0.01
		2	
6(iii)	$e^{-7.0}\left(1 + 7 + \frac{7^2}{2} + \frac{7^3}{3!} + \frac{7^4}{4!}\right)$	**M1**	Correct expression for $P(X \leqslant 4 \mid \lambda = 7.0)$
	0.173 (3 sf)	**A1**	
		2	

Cambridge International A Level

Cambridge Assessment International Education
Cambridge International Advanced Level

MATHEMATICS **9709/73**

Paper 7 **October/November 2019**

MARK SCHEME

Maximum Mark: 50

Published

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Question	Answer	Marks	Guidance
1(i)	9.6, 12.4	**B1 B1**	
		2	
1(ii)	6.6, 49.6	**B1 B1**	
		2	

Question	Answer	Marks	Guidance
2(i)	$\left(\lambda\left(=2\times2.4\right)=4.8\right)$ $e^{-4.8}\left(1+4+\frac{4.8^2}{2}+\frac{4.8^3}{3!}\right)$	**M1**	Any λ
	0.294 (3 sf)	**A1**	
		2	
2(ii)	$\left(\lambda\left(=60\times2.4\right)=144\right)$ N('144', '144')	**M1**	N and $\sigma^2=\mu$ SOI
	$\frac{139.5-'144'}{\sqrt{'144'}}\left(=-0.375\right)$	**M1**	Allow with no continuity correction
	ϕ('0.375')	**M1**	Correct area consistent with their working
	0.646 (3 sf)	**A1**	
		4	

Question	Answer	Marks	Guidance
3(i)	Assume population is normally distributed	**B1**	
	$\overline{x} = 25.9$	**B1**	Allow $\frac{259}{10}$
	$z = 2.17$	**B1**	
	$'25.9' \pm z \times \frac{3}{\sqrt{10}}$	**M1**	Must have correct form and *z*.
	23.8 to 28.0 (3 sf)	**A1**	CWO
		5	
3(ii)	0.03^2 (=0.0009)	**B1**	
		1	

Question	Answer	Marks	Guidance
4	Assume trains are independent OR probability of being on time is constant	**B1**	Must be in context
	H_0: P(on time)=0.92 H_1: P(on time)<0.92	**B1**	Both. Allow '*p*' or π
	$1-$ $\left({}^{20}C_{17}\times 0.92^{17}\times 0.08^{3}+{}^{20}C_{18}\times 0.92^{18}\times 0.08^{2}+20\times 0.92^{19}\times 0.08+0.92^{20}\right)$	**M1**	Allow one end error Must have 1 – …
	=0.0706 (3 sf)	**A1**	
	Compare with 0.05	**M1**	Valid comparison needed
	No evidence that percentage less than 92%	**A1FT**	OE No contradictions. <u>Method using normal approximation:</u> If the first B1B1 is earned then: $\text{CV}-1.566\left(\text{from}\frac{16.5-20\times 0.92}{\sqrt{20\times 0.92\times 0.08}},\text{with continuity correction}\right)$ **or** CV=1.978 (without continuity correction) comp z=1.645 No evidence that % decreased (1.566) or evidence that % decreased (1.978) is awarded **SC2** after B marks
		6	

Question	Answer	Marks	Guidance
5(i)	Po(3)	**B1**	SOI
	$\mathrm{e}^{-3}\left(\frac{3^3}{3!}+\frac{3^4}{4!}+\frac{3^5}{5!}\right)$	**M1**	Allow one or two extra terms (2 or 6 or both)
	0.493 (3 sf)	**A1**	
		3	
5(ii)	A correct equation from $\mathrm{P}(0)=\mathrm{P}(2)$ $\left(\text{leading to } 1=\frac{\lambda^2}{2}\right)$	**M1**	
	$\lambda=\sqrt{2}$ or 1.41 (3 sf)	**A1**	CWO
		2	

Question	Answer	Marks	Guidance
5(iii)(a)	Correct inequality $\left(\text{leading to } \frac{5.2^n}{n!} < \frac{5.2^{n+1}}{(n+1)!}\right)$	**B1**	
		1	
5(iii)(b)	$n + 1 < 5.2$ or $1 < \frac{5.2}{n+1}$	**M1**	Simplify to a stage without exponentials, powers or factorials.
	Largest n is 4	**A1**	
		2	

Question	Answer	Marks	Guidance
6(i)	$k\int_0^3 (3x - x^2)\,\mathrm{d}x = 1$	**M1**	Attempt to integrate f(x) and = 1
	$k\left[\frac{3}{2}x^2 - \frac{x^3}{3}\right]_0^3$ $k\left(\frac{27}{2} - \frac{27}{3}\right) = 1$	**A1**	Correct integral and limits
	$k = \frac{2}{9}$	**A1**	AG No errors seen
		3	

Question	Answer	Marks	Guidance
6(ii)	$\frac{2}{9}\int_1^2 (3x - x^2)\,dx \ = \frac{2}{9}\left[\frac{3}{2}x^2 - \frac{x^3}{3}\right]_1^2 \ = \frac{2}{9} \times \left(6 - \frac{8}{3} - \frac{3}{2} + \frac{1}{3}\right)$	M1	Attempt to integrate f(x) dx with limits 1 and 2 OE
	$\frac{13}{27}$ or 0.481 (3 sf)	A1	
		2	
6(iii)	$y = 3x - x^2$ symmetrical about $x = \frac{3}{2}$	M1	Attempt $\frac{2}{9}\int_0^3 (3x^2 - x^3)\,dx$
	E(X)= $\frac{3}{2}$	A1	
	$\frac{2}{9}\int_0^3 (3x^3 - x^4)\,dx$	M1	Attempt to integrate x^2f(x)
	$= \frac{2}{9}\left[\frac{3x^4}{4} - \frac{x^5}{5}\right]_0^3 (= \frac{2}{9} \times \frac{243}{20} = \frac{27}{10})$ $\frac{'27'}{10} - \left(\frac{'3'}{2}\right)^2$	M1	Subtract their (E(X))^2 from their integral x^2f(x) with correct limits substituted
	$\frac{9}{20}$ or 0.45	A1	
		5	

Question	Answer	Marks	Guidance
7(i)	H_0: Pop mean=546 H_1: Pop mean>546	**B1**	Both. Allow just μ, but not just 'mean'
	$\frac{581-546}{\frac{120}{\sqrt{40}}}$	**M1**	Standardising. Need $\frac{120}{\sqrt{40}}$
	=1.845 allow 1.844	**A1**	Allow 1.84 or 1.85 AWRT
	1.845<1.96	**M1**	OE. Or area comparison 0.0325>0.025 or large probabilities
	No evidence that mean weekly income has increased	**A1FT**	No contradictions. If H_1: $\neq$, and 2.241 used, max B0M1A1M1A0
		5	
7(ii)	$\frac{a-546}{\frac{120}{\sqrt{40}}}=1.96$	**M1**	Standardise to find a. Need $\frac{120}{\sqrt{40}}$ **and** 546 **and** a value of z
	a = 583.19	**A1**	Allow 583 to 3sf
	$\frac{'583.19'-595}{\frac{120}{\sqrt{40}}}(=-0.622)$	**M1**	Standardise. Need $\frac{120}{\sqrt{40}}$ and 595
	ϕ('–0.622')=1 – ϕ('0.622')	**M1**	Consistent area
	0.267	**A1**	
		5	

Cambridge Assessment International Education
Cambridge International Advanced Level

MATHEMATICS **9709/71**

Paper 7 **May/June 2019**

MARK SCHEME

Maximum Mark: 50

Published

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This document consists of **10** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

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- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

  - Note: B2 or A2 means that the candidate can earn 2 or 0.
    B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no “follow through” from a previous error is allowed)
CWO	Correct Working Only – often written by a ‘fortuitous’ answer
ISW	Ignore Subsequent Working
SOI	Seen or implied
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become “follow through” marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Question	Answer	Marks	Guidance
1(i)	Mean = 115	**B1**	
	SD = 40	**B1**	
		2	
1(ii)	Mean = 15 × ‘115’ = 1725	**B1ft**	
	15 × $‘40’^2$ (= 24000)	**M1**	or SD = $\sqrt{15}$ × ‘40’. ft their **(i)**
	SD = $\sqrt{24000}$ SD = 155 (cents) (3 sf)	**A1**	Accept $\sqrt{24000}$ SC: Allow correct answers in dollars
		3	

Question	Answer	Marks	Guidance
2(i)	Assume sd still 4.8 or is unchanged	**B1**	or Assume the 150 times can be treated as a random sample / are independent
	H_0: Pop mean = 26.5 H_1: Pop mean > 26.5	**B1**	Allow ‘μ’ but not just ‘mean’
	$\frac{27.5-26.5}{\frac{4.8}{\sqrt{150}}}$	**M1**	Standardise, with √ Accept CV method
	= 2.552	**A1**	
	Comp with *z*-value ‘2.552’ > 2.326	**M1**	or comp 1 – Φ(‘2.552’) with 0.01 1 – 0.9946 = 0.0054 < 0.01
	There is evidence time has increased	**A1ft**	oe No contradictions (2 tail test scores max. B1 B0 M1 A1 M1 (for comparison with 2.576) A0 no ft)
		6	

Question	Answer	Marks	Guidance
2(ii)	No because pop is normal so distr of $\overline{X}$ is normal	**B1**	Condone just 'No because pop is normal'
		1	

Question	Answer	Marks	Guidance
3(i)	H_0: $P(6) = \frac{1}{6}$ H_1: $P(6) < \frac{1}{6}$	**B1**	
	$(\frac{5}{6})^{30} + 30(\frac{1}{6}) \times (\frac{5}{6})^{29} + {}^{30}C_2(\frac{1}{6})^2 \times (\frac{5}{6})^{28}$	**M1**	Allow one term incorrect, omitted or extra
	= 0.103	**A1**	
	'0.103' > 0.05	**M1**	
	No evidence (at 5% level) that die biased	**A1ft**	oe No contradictions
		5	
3(ii)	$(\frac{5}{6})^{30} + 30(\frac{1}{6}) \times (\frac{5}{6})^{29}$	**M1**	
	P(Type I) = 0.0295	**A1**	
		2	

Question	Answer	Marks	Guidance
4(a)(i)	$0.5 \times 1/\text{a} = (\frac{0.5}{a})$	**M1**	Or attempt to integrate f(x) (=1/a) between 0 and 0.5
	$= \frac{1}{2a}$ oe	**A1**	Accept 0.5/a for A1
		2	
4(a)(ii)	$\frac{a}{2}$	**B1**	
		1	
4(a)(iii)	$\int_0^a \frac{x^2}{a}\mathrm{d}x - (‘\frac{a}{2}’)^2$	**M1**	Integ their x^2f(x) from 0 to a and sub their mean^2
	$\text{Var}(X) = \frac{a^2}{3} - \frac{a^2}{4}$ $(\text{Var}(X) = \frac{a^2}{12}$ **AG**)	**A1**	Must see this line oe
		2	
4(b)	$\int_2^b \frac{3}{2(t-1)^2}\mathrm{dt}$	**M1**	Attempt integ g(t) ignore limits
	$\left[-\frac{3}{2(t-1)}\right]_2^b$	**A1**	Correct integral
	$-\frac{3}{2}(\frac{1}{(b-1)}-1) = \frac{3}{4}$ $(1 - \frac{1}{(b-1)} = \frac{1}{2})$	**M1**	Attempt subst correct limits in their integ and $= \frac{3}{4}$
	$b = 3$	**A1**	
		4	

Question	Answer	Marks	Guidance
5(a)(i)	$e^{-2.3}(\frac{2.3^2}{2}+\frac{2.3^3}{3!}+\frac{2.3^4}{4!}$	**M1**	Allow one end error
	= 0.585	**A1**	
		2	
5(a)(ii)	(λ) = 4.6	**B1**	
	$1-e^{-4.6}(1+4.6+\frac{4.6^2}{2})$	**M1**	any λ, Allow one end error
	= 0.837 (3 sf)	**A1**	
		3	
5(a)(iii)	$S \sim N(115, 115)$	**B1**	May be implied
	$\frac{110.5-115}{\sqrt{115}}$ (= –0.420)	**M1**	Allow with wrong or no cc OR no √
	1 – Φ(‘0.420’) (= 1 – 0.663)	**M1**	
	= 0.337	**A1**	Accept alternative method using N(2.3, 2.3) no mixed methods.
		4	
5(b)	$e^{-\lambda}\times\frac{\lambda^3}{3!}=e^{-\lambda}\times\frac{\lambda^5}{5!}$	**M1**	
	$\lambda^3=\frac{\lambda^5}{4\times5}$ or $\lambda^2=20$ oe	**A1**	any correct simplification without $e^{-\lambda}$ or !
	$\lambda=\sqrt{20}$ or $2\sqrt{5}$ or 4.47 (3 sf)	**A1**	
		3	

Question	Answer	Marks	Guidance
6(i)	Biased towards people who like tennis Excludes people who don't like tennis	**B1**	or other sensible
		1	
6(ii)	Obtain a list of all people in the town	**B1**	
	Use random numbers	**B1**	or, e.g. pick numbers from a hat or other sensible
		2	
6(iii)	Var(*p*) = $\frac{\frac{47}{350}(1-\frac{47}{350})}{350}$ (= 0.000332152)	**M1**	
	$z = 1.645$	**B1**	
	$\frac{47}{350} \pm z\sqrt{\frac{\frac{47}{350}(1-\frac{47}{350})}{350}}$	**M1**	Must be a z value
	0.104 to 0.164 (3 sf)	**A1**	Must be an interval
		4	
6(iv)	1.25 × 1.645 (= 2.056)	**M1**	or 1.25 × their width ÷ 2 ÷ their $\sqrt{\frac{\frac{47}{350}(1-\frac{47}{350})}{350}}$ (Complete method)
	Φ(‘2.056’) (= 0.980)	**M1**	Attempt Φ(their z)
	$x = 96$ (2 sf)	**A1**	Allow 0.96 (2 sf) CWO
		3	

Cambridge Assessment International Education
Cambridge International Advanced Level

MATHEMATICS **9709/72**

Paper 7 **May/June 2019**

MARK SCHEME

Maximum Mark: 50

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PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Question	Answer	Marks	Guidance
1(i)	0.0842 (3 sf)	**B1**	
		1	
1(ii)	e^{-5} x $\frac{5^n}{n!}$ = e^{-5} x $\frac{5^{n+1}}{(n+1)!}$	**B1**	or $\frac{5^n}{n!} = \frac{5^{n+1}}{(n+1)!}$ or better ISW
		1	
1(iii)	$1 = \frac{5}{n+1}$ $n = 4$	**B1**	
		1	

Question	Answer	Marks	Guidance
2(i)	Normal with mean 372	**B1**	
	sd = $\frac{54}{\sqrt{36}}$	**M1**	or variance = $\frac{54^2}{36}$ M1
	(= 9)	**A1**	(= 81) A1
		3	
2(ii)	Pop normal	**B1**	Allow X is normal
		1	

Question	Answer	Marks	Guidance
3(i)	Est(μ) = 1.85	**B1**	
	Est(σ^2) = $\frac{50}{49}\left(\frac{175.25}{50} - '1.85'^2\right)$	**M1**	Allow $\sqrt{\frac{50}{49}\left(\frac{175.25}{150} - '1.85'^2\right)}$ or 0.0290 for M1
	= 0.0842 (3 sf) or $\frac{33}{392}$	**A1**	Cao If $\frac{50}{49}$ omitted (giving var = 0.0825 or sd = 0.287) M0A0
		3	
3(ii)	H_0: Pop mean time = 1.9 (h) H_1: Pop mean time < 1.9 (h)	**B1**	Allow 'μ' but not just 'mean'
	$\pm\frac{1.85-1.9}{\sqrt{\frac{'0.0842'}{50}}}$	**M1**	$\pm\frac{1.85-1.9}{\frac{'0.290'}{\sqrt{50}}}$ Accept totals method (92.5–95) / $\sqrt{4.21}$
	= –1.22	**A1**	= –1.22
	comp z = –1.645	**M1**	Or other valid comparison 0.888 or 0.889 < 0.95 OR 0.111 or 0.112>0.05
	No evidence that mean time < 1.9 h	**A1**	**FT** their z. Correct conclusion. No contradictions If $\frac{50}{49}$ not used in (1): var = 0.8225, sd = 0.907, cr = 1.17 can score all marks in **(ii)** Note- 2 tail test can score B0 M1 A1 M1 (comparison with 1.96) A0 (no ft) max3/5
		5	

Question	Answer	Marks	Guidance
4	Use of $1.5X_1 - X_2$ or similar	**B1**	
	$E(1.5X_1–X_2) = 1.5(110) – 110$ (= 55)	**B1**	or $E(X_1–1.5X_2) = 110 – 1.5(110)$ (= –55)
	$Var(1.5X_1–X_2) = 1.5^2×1050+1050$ (or 3412.5)	**M1**	Correct expression or result
	$\frac{0–55}{\sqrt{'3412.5'}}$ or $\frac{0–(–55)}{\sqrt{'3412.5'}}$ (= ± 0.942)	**M1**	Their ‘55’. Allow incorrect var (dep > 0 and ≠ 1050)
	1 – Φ(‘0.942’)	**M1**	Area consistent with their working
	= 0.173	**A1**	
	Ans 0.346 (3 sf)	**B1**	**FT** double their prob (must be <1)
		7	

Question	Answer	Marks	Guidance
5(i)	H_0: $p = 0.1$ H_1: $p < 0.1$	**B1**	
		1	
5(ii)	B(40, 0.1) stated or implied by use of	**B1**	e.g. by ${}^{40}C_x$ or $0.9^p×0.1^q$ ($p + q = 40$)
	$0.9^{40} + 40×0.9^{39}×0.1$	**M1**	Correct working (if seen). If working not seen, M1 may be implied by 0.0805
	= 0.0805	**A1**	
		3	

Question	Answer	Marks	Guidance
5(iii)	$z = 1.645$	**B1**	seen
	$\frac{6}{80} \pm z\sqrt{\frac{\frac{6}{80}\times\frac{(80-6)}{80}}{80}}$	**M1**	Formula of correct form. Must be a 'z'
	= 0.0266 to 0.123 (3 sfs)	**A1**	Allow 0.03 to 0.12 or better Must be an interval
		3	
5(iv)	10% (or manufacturer's claim) is within CI Hence no reason to question claim	**B1**	**FT** Allow '10% is within CI, accept claim' oe Must include both parts. No contradictions. **FT** their CI Note if CI is centred on 0.1 allow ft 0.075 is within CI, accept claim
		1	

Question	Answer	Marks	Guidance
6(i)	$a\int_1^b \frac{1}{x^2}\,dx = 1$	**M1**	Attempt int f(x) and = 1, ignore limits
	$a[-\frac{1}{x}]_1^b = 1$	**A1**	correct integ and limits = 1
	$a[1 - \frac{1}{b}] = 1$ or $a\times\frac{b-1}{b} = 1$ $b = \frac{a}{a-1}$ **AG**	**A1**	No errors seen
		3	

Question	Answer	Marks	Guidance
6(ii)	$a\int_{1}^{\frac{3}{2}} \frac{1}{x^2}dx = \frac{1}{2}$ $a\left[-\frac{1}{x}\right]_{1}^{\frac{3}{2}} = \frac{1}{2}$	**M1**	Attempt int f(x) with limits 1 to $\frac{3}{2}$ and = $\frac{1}{2}$
	$a\left[1 - \frac{2}{3}\right] = \frac{1}{2}$	**A1**	oe correct equn in *a*
	$a = \frac{3}{2}$, $b = 3$	**A1**	Both
		3	
6(iii)	$\frac{3}{2}\int_{1}^{3} \frac{1}{x}dx$	**M1**	Attempt int xf(x), ignore limits – condone missing a
	$= \frac{3}{2}\ [\ln x]_{1}^{3}$	**A1**	**FT** Correct integ and *their* limits 1 to b – condone missing a
	$= \frac{3}{2}\ln 3$ or 1.65 (3 sf)	**A1**	**FT** *their a* and *b* (valid *b* i.e. >1)
		3	

Question	Answer	Marks	Guidance
7(i)	Max no. of passengers plane can take oe	**B1**	oe e.g. No of passengers who bought tickets
		1	

Question	Answer	Marks	Guidance
7(ii)	$\lambda = 3.2$	**B1**	
	$e^{-3.2}(\frac{3.2^3}{3!}+\frac{3.2^4}{4!}+\frac{3.2^5}{5!})$	**M1**	Any λ. Allow one end error
	= 0.5146 = 0.515 (3 sfs)	**A1**	SR Use of Bin(640,0.005) scores B1 (only) for 0.516
		3	
7(iii)	$n > 50$	**B1**	Accept n is large
	$np = 1.6$, which is < 5 or p=0.005 which is <0.1	**B1**	Allow $np = 3.2$
		2	
7(iv)	H_0: Pop mean (for 5 days) = 8 H_1: Pop mean (for 5 days) < 8	**B1**	or Pop mean (for 1 day) = 1.6 Pop mean (for 1 day) < 1.6 Allow λ or μ but not just 'mean'
	$e^{-8}(1 + 8 + \frac{8^2}{2!})$	**M1**	Any λ ($\neq$ 1.6) No end errors. Accept use of Bin(1600,0.005) P(0,1,2)=0.0136
	= 0.0138	**A1**	
	Comp 0.025	**M1**	Valid comparison
	Evidence that mean no. failing to arrive has decreased	**A1**	**FT** their '0.0138' or '0.0136'. No contradictions
		5	

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- marks are not deducted for errors
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- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

  - Note: B2 or A2 means that the candidate can earn 2 or 0.
    B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

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The following abbreviations may be used in a mark scheme or used on the scripts:

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
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ISW	Ignore Subsequent Working
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SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Question	Answer	Marks	Guidance
1	$0.6 \pm z\sqrt{\frac{0.4\times0.6}{100}}$	**M1**	Recognisable value of z
	$z = 2.326$	**B1**	2.326 to 2.329
	0.486 to 0.714 (3 sf)	**A1**	Must be an interval
		3	

Question	Answer	Marks	Guidance
2	$\frac{50}{49}\left(\frac{4361}{50}-\bar{x}^2\right) = 9.62$	**M1**	or $\left(\frac{4361}{49}-\frac{(\Sigma x)^2}{50\times49}\right) = 9.62$ BOD regarding symbols used
	$\bar{x}^2 = \frac{4361}{50}$ - $9.62\times\frac{49}{50} = 77.7924$	**A1**	$(\Sigma x)^2 = 4361 \times 50 - 9.62 \times 50 \times 49 = 194481$or $\Sigma x = 441$ (Σx) or $(\bar{x})$ must be correctly identified
	$\bar{x}$ = 8.82 (3 sf)	**A1**	SC use of 'biased' leading to 8.81 B1
		3	

Question	Answer	Marks	Guidance
3(i)	*D* more likely to be chosen	**B1**	oe, e.g. P(*D*) > P(*A*) e.g. P(A)=P(B)=P(C)=1/6 P(D)=1/2 no contradictions
		1	
3(ii)	Reject scores of 5 or 6	**B1**	or other correct: choose D when the score is 4
		1	

Question	Answer	Marks	Guidance
3(iii)	AB AC AD BC BD CD	**B1**	
	Allocate as follows: 1: AB; 2: AC; 3: AD; 4: BC; 5: BD 6: CD	**B1**	or similar
		2	

Question	Answer	Marks	Guidance
4	Total ~ N(1208, ...)	**B1**	
	Var(total) (= 10×1.2 + 20×0.7 (+ 0)) = 26	**B1**	May be implied by next line
	$\pm\frac{1200-\text{"}1208\text{"}}{\sqrt{\text{"}26\text{"}}}$ (= –1.569)	**M1**	**FT** their mean and var of total mass, e.g. allow 1200 and 11.24 (from $10 \times 1.2^2 + 20 \times 0.7^2$)
	1 – Φ(“1.569”)	**M1**	Correct area consistent with their working
	= 0.0583 (3 sf)	**A1**	
		5	

Question	Answer	Marks	Guidance
5	H_0: Pop mean = 20 H_1: Pop mean ≠ 20	**B1**	Accept μ
	$\frac{\Sigma x}{6}$ $(= \frac{126.9}{6} = 21.15)$	**M1**	Attempted or 126.9 and 11.64 attempted
	$\frac{'21.15'-20}{\sqrt{\frac{1.94}{6}}}$	**M1**	Must have $\sqrt{6}$ or $\frac{120-126.9}{\sqrt{11.64}}$ no mixed method
	= 2.022	**A1**	
	2(1 - ϕ('2.022')) 2 (1 – '0.9784)' = 0.0432)	**M1**	**FT** $2\times\left(1-'.9784'\right)$
	α = 4.32 (3 sf)	**A1**	**FT** Allow 4.3 or 4, if correct working seen, or clearly implied, as far as 0.0216 **FT** their z, no error seen One-tail test scores maximum 3/6
		6	

Question	Answer	Marks	Guidance
6(i)	$\frac{3}{a^3}\int_0^a x^2 dx$ (= $\frac{3}{a^3}\left[\frac{x^3}{3}\right]_0^a$)	**M1**	Attempt to integrate f(x) with limits 0 and a (condone missing $\frac{3}{a^3}$)
	= $\frac{3a^3}{3a^3}$	**A1**	$\frac{3a^3}{3a^3} - 0$ or better seen
	= 1 Hence f is pdf for all *a*	**A1**	Answer = 1 and comment
		3	
6(ii)	$\frac{3}{a^3}\int_0^2 x^2 dx = 0.5$ $\frac{3}{a^3}\left[\frac{x^3}{3}\right]_0^2 = 0.5$	**M1**	Attempt to integrate f(x)=0.5, limits 0 and 2 oe, condone missing $\frac{3}{a^3}$
	$\frac{3}{a^3} \times \frac{8}{3} = 0.5$ oe	**A1**	$\frac{2^3}{3} - 0$ or better, condone missing $\frac{3}{a^3}$
	$a^3 = 16$ or $a = \sqrt[3]{16}$ (= 2.52 **AG**)	**A1**	Convincingly obtained Note: Attempt to verify 2.52, M1 as stated except not equated to 0.5.A1 as stated, A1 for evaluation to 0.499..apprx 0.5
		3	

Question	Answer	Marks	Guidance
6(iii)	$\frac{3}{16}\int_0^{2.52} x^3 dx$ or $\frac{3}{16}\int_0^{a} x^3 dx$ $= \frac{3}{16}\left[\frac{x^4}{4}\right]_0^{2.52}$ or $\frac{3}{16}\left[\frac{x^4}{4}\right]_0^{a}$	M1	Attempt integ xf(x), correct limits, condone missing $\frac{3}{a^3}$
	$= \frac{3}{16}\times\frac{40.317}{4}$	A1	$\frac{2.52^4}{4} - 0$ or better, condone missing $\frac{3}{a^3}$
	= 1.89 (3 sf)	A1	
		3	

Question	Answer	Marks	Guidance
7(i)	Use of Po(2.8)	M1	May be implied
	$1 - e^{-2.8}(1 + 2.8 + \frac{2.8^2}{2}))$	M1	Any λ allowing one end error
	= 0.531or 0.53(0) (3 sf)	A1	SC Binomial 0.534 B1
		3	
7(ii)	Use of Po(5.8)	M1	May be implied
	$e^{-5.8} \times \frac{5.8^6}{6!}$	M1	Any λ
	= 0.16(0) (3 sf)	A1	
		3	

Question	Answer	Marks	Guidance
7(iii)	Use of N(58, 58)	**M1**	May be implied or N(58, 55.38)
	$\frac{50.5-'58'}{\sqrt{'58'}}$ (= -0.985)	**M1**	Standardised with their values, allow wrong or incorrect cc
	$\Phi('0.985')$	**M1**	Correct area consistent with their working or $\Phi(''1.008)$
	= 0.838 (3 sf)	**A1**	or 0.843
		4	

Question	Answer	Marks	Guidance
8(i)	H$_0$: $p = \frac{1}{4}$ H$_1$: $p > \frac{1}{4}$	**B1**	
	$^{10}C_6(\frac{1}{4})^6(\frac{3}{4})^4 + {}^{10}C_7(\frac{1}{4})^7(\frac{3}{4})^3 + {}^{10}C_8(\frac{1}{4})^8(\frac{3}{4})^2 + 10(\frac{1}{4})^9(\frac{3}{4}) + (\frac{1}{4})^{10}$	**M1**	Correct terms, allow one term incorrect or omitted or extra or summing all correct terms from 0 to 5 allow one term incorrect or omitted or extra
	= 0.0197	**A1**	or 0.9803
	comp '0.0197' with 0.01	**M1**	Valid comparison with 0.01 or valid comparison with 0.99
	No evidence to conclude $p > \frac{1}{4}$	**A1**	**FT** No contradictions Use of two-tail test can score BOM1A1M1(comparison with 0.005) A0
		5	
8(ii)	$^{10}C_7(\frac{1}{4})^7(\frac{3}{4})^3 + {}^{10}C_8(\frac{1}{4})^8(\frac{3}{4})^2 + 10(\frac{1}{4})^9(\frac{3}{4}) + (\frac{1}{4})^{10}$	**M1**	Their P(X $\geqslant$ 6) – $^{10}C_6(0.25)^6(0.75)^4$
	P(Type I) = 0.00351 (3 sf)	**A1**	Accept 0.00348 to 0.00351
		2	
8(iii)	C.R is $X \geqslant 7$ P(Type II) = 1 – P($X \geqslant 7 \mid p = \frac{3}{5}$) =	**M1**	May be implied
	$1-({}^{10}C_7(\frac{3}{5})^7(\frac{2}{5})^3 + {}^{10}C_8(\frac{3}{5})^8(\frac{2}{5})^2 + 10(\frac{3}{5})^9(\frac{2}{5}) + (\frac{3}{5})^{10})$	**M1**	Accept 1 – P($X \geqslant 8 \mid p = \frac{3}{5}$) or 1 – P($X \geqslant 6 \mid p = \frac{3}{5}$)
	= 0.618	**A1**	
		3	

Cambridge Assessment International Education
Cambridge International Advanced Level

MATHEMATICS **9709/72**

Paper 7 Probability and Statistics **March 2019**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the March 2019 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

This document consists of **12** printed pages.

[Turn over

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Question	Answer	Marks	Guidance
1(i)	$z = 2.326$	**B1**	
	$62.3 \pm z\frac{13.2}{\sqrt{200}}$	**M1**	Any z. Expression of correct form. Must be a ‘z’
	60.1 to 64.5 (3 sfs)	**A1**	Must be an interval
		3	
1(ii)	Yes, because pop not (given to be) normal, or pop distribution unknown	**B1**	No contradictions
		1	

Question	Answer	Marks	Guidance
2	$E(X - 3Y) = 0.2$	**B1**	oe
	$Var(X - 3Y) = 12.1 + 9 \times 8.6$ (= 89.5)	**B1**	
	$\frac{0 - 0.2}{\sqrt{"89.5"}}$ (= –0.021)	**M1**	For area consistent with their working
	Φ(‘0.021’)	**M1**	
	= 0.508 (3 sfs)	**A1**	
		5	

Question	Answer	Marks	Guidance
3	$H_0: \lambda = 32$ $H_1: \lambda < 32$	B1	Accept 'population mean' (μ)
	$X \sim N(32, 32)$	B1	seen or implied
	$\frac{21.5-32}{\sqrt{32}}$	M1	Standardise with their values. Allow with no or wrong cc
	$= -1.856$ cv of $z = -2.054$ (or -2.055 or -2.053)	A1	
	'1.856' < 2.054	M1	Valid comparison or comp ϕ("1.856") with 0.98 i.e. 0.9682 < 0.98 oe
	No evidence that fewer accidents at B than at A	A1f	No contradictions Note Use of CV method x = 20.38 M1 A1 comparison 21.5 > 20.38 M1 conc A1
		6	

Question	Answer	Marks	Guidance
4(i)	$\bar{x} = \frac{420}{50} = 8.4$	B1	
	$s^2 = \frac{50}{49}(\frac{27530}{50} - (\frac{420}{50})^2)$	M1	Or $1/49(27530 - (420)^2/50)$
	= 489.8(36....)	A1	Must see ⩾ 4 sf
		3	

Question	Answer	Marks	Guidance
4(ii)	$\Phi^{-1}(0.9377) = 1.536$	**B1**	
	$\frac{5-'8.4'}{\sqrt{\frac{490}{n}}} = -1.536$	**M1**	Attempting to standardise – must have correct form
	$n = (\frac{1.536}{3.4})^2 \times 490$ (= 100.0048)	**M1**	Attempting numerical expression for n or √n (must have used a 'z' value) may be implied by answer
	$n = 100$	**A1**	No errors seen. Must be whole number
		4	

Question	Answer	Marks	Guidance
5(i)	$1-e^{-1.8}(1 + 1.8)$	**M1**	Accept any λ. Accept 1–P(0,1,2)
	= 0.537 (3 sf)	**A1**	
		2	
5(ii)	$\lambda = 2.2$	**B1**	
	$e^{-2.2}(1 + 2.2 + \frac{2.2^2}{2!} + \frac{2.2^3}{3!} + \frac{2.2^4}{4!})$	**M1**	Attempt expr'n for $P(X \leqslant 4)$,allow one end error, allow any λ
	= 0.928 (3 sf) or 0.927	**A1**	
		3	

Question	Answer	Marks	Guidance
5(iii)	$1 - e^{-1.8t} \geqslant 0.99$ or $1 - e^{-\lambda} \geqslant 0.99$	**M1**	Condone = signs/incorrect inequality signs
	$e^{-1.8t} \leqslant 0.01$ or $e^{-\lambda} \leqslant 0.01$ $-1.8t \leqslant \ln 0.01$	**M1**	Valid attempt take logs (must have single term on each side)
	$t \geqslant 2.56$ She must watch for at least 2.56 (hours)	**A1**	or 2 hours, 34 mins or better. No errors seen
		3	

Question	Answer	Marks	Guidance
6(i)	Test is for “difference” oe	**B1**	Test is not for ‘increase’ or ‘decrease’ oe No contradictions
		1	
6(ii)	0.05	**B1**	
	Conclude mean time is different when it is not	**B1**	oe, in context
		2	

Question	Answer	Marks	Guidance
6(iii)	Assume $\sigma = 6.4$	**B1**	
	H_0: pop mean = 91.4 H_1: pop mean $\neq$ 91.4	**B1**	Allow μ, but not 'mean'
	$\overline{x} = \frac{568.5}{6}$ (= 94.75)	**B1**	
	$\frac{'94.75'-91.4}{\frac{6.4}{\sqrt{6}}}$	**M1**	Must have √6
	= 1.282 cv of z = 1.96	**A1**	
	'1.282' < 1.96	**M1**	Valid comparison or comp ϕ("1.282") with 0.975 0.9(001) < 0.975 or 0.0999 (or 0.1) > 0.025 consistent use of one tail test can score M1 for comparison with 1.645oe but not A1ft oe. No contradictions. ft their z.
	No evidence mean time different	**A1 ft**	CV method x = 96.52 M1 A1 94.75 < 96.52 M1 Conc A1
		7	

Question	Answer	Marks	Guidance
7(i)	$\sqrt{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \cos x\,dx$ $= \sqrt{2}\,[\sin x]_{\frac{\pi}{6}}^{\frac{\pi}{4}}$	**M1**	Attempt integ f(*x*) with correct limits
	$= \frac{2-\sqrt{2}}{2}$ oe or 0.293 (3 sf)	**A1**	SC Final answer of 0.707 scores B1sc
		2	
7(ii)	$\sqrt{2}\int_0^m \cos x\,dx = 0.5$	**M1**	Attempt to integ f(x) & = 0.5. Ignore limits. Condone missing √2
	$\sqrt{2}\,[\sin x]_0^m = 0.5$ $\sqrt{2}\,\sin m = 0.5$	**A1**	Correct integral and limits 0 to unknown & = 0.5 Condone missing √2
	$\sin m = \frac{1}{2\sqrt{2}}$ oe	**M1**	For rearranging their expression to the form sin*m*=… (sin*m* = 0.35355.... or 0.354) seen or implied
	m = 0.361 (3 sfs)	**A1**	No errors seen (Note 20.705 can score M1 A1 M1 A0)
		4	

Question	Answer	Marks	Guidance
7(iii)	$\sqrt{2}\int_0^{\frac{\pi}{4}} x\cos x\mathrm{d}x$	**M1**	Attempt to integ $x\mathrm{f}(x)$. Ignore limits. Condone missing $\sqrt{2}$
	$= \sqrt{2}\ \{[x(\sin x)]_0^{\frac{\pi}{4}} - \int_0^{\frac{\pi}{4}} \sin x\mathrm{d}x\ \}$	**M1**	Attempt to integ by parts leading to expression of form $\pm x\sin x \pm \cos x$ with correct limits
	$= \sqrt{2}\ \{\frac{\pi}{4\sqrt{2}} - 0 - [-\cos x]_0^{\frac{\pi}{4}}\}$	**A1**	For $\sqrt{2}(x\sin x - (-\cos x))$ with correct limits
	$= \sqrt{2}\ \{\frac{\pi}{4\sqrt{2}} + \cos\frac{\pi}{4} - 1\}$ $= \frac{\pi}{4} + 1 - \sqrt{2}$ oe or 0.371 (3 sf)	**A1**	
		4	

Cambridge Assessment International Education
Cambridge International Advanced Level

MATHEMATICS **9709/71**

Paper 7 **October/November 2018**

MARK SCHEME

Maximum Mark: 50

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Question	Answer	Marks	Guidance
1(i)	$176 \pm z \times \frac{7.2}{\sqrt{200}}$	**M1**	need correct form must be z
	$z = 2.24$	**B1**	allow 2.241 and 2.242
	175 to 177	**A1**	cwo
		3	
1(ii)	Sample random	**B1**	oe. both words essential
		1	

Question	Answer	Marks	Guidance
2(i)	H_0: $p = \frac{1}{3}$ H_1: $p < \frac{1}{3}$	**B1**	
		1	
2(ii)	0.0084 < 0.01	**B1**	Allow P(N ⩽ 36) < 0.01 or 1%
	There is evidence that *p* has decreased	**B1 dep**	Allow '*p* has decreased' or $p < \frac{1}{3}$
		2	
2(iii)	150	**B1**	
		1	

Question	Answer	Marks	Guidance
3	$\frac{12.2-12}{2.5/\sqrt{n}}$	**M1**	Standardisation. Allow cc. need correct form incl sqrt
	(=) 1.96	**B1**	Correct *z*
	$\sqrt{n} = 1.96 \times 2.5 \div 0.2$	**M1**	Rearrange equation in n or sqrt n with numerical z to the stage n= or sqrt n = allow arithmetical slips only
	$n = 600$	**A1**	accept 601 SR whole number ans from 595 to 605 can score full marks if fully justified
		4	

Question	Answer	Marks	Guidance
4(i)	$\lambda = 10\times0.25 + 10\times0.36 \quad (= 6.1)$	**B1**	
	$1 - e^{-6.1}\,(1 + 6.1 + \frac{6.1^2}{2} + \frac{6.1^3}{3!})$	**M1**	1 – P($X \leqslant 3$), any λ Allow one end error
	= 0.857	**A1**	Allow 0.858
		3	
4(ii)	$\lambda = 61$	**B1 ft**	Ft from (i)
	N('61', '61')	**M1**	N with $\mu = \lambda$, any λ. May be implied
	$\frac{59.5-61}{\sqrt{'61'}} \quad (= -0.192)$	**M1**	Standardise with their mean and variance Allow no or wrong cc. not 61/100
	Φ('–0.192') = 1 – Φ('0.192')	**M1**	Correct area consistent with their working
	= 0.424	**A1**	
		5	

Question	Answer	Marks	Guidance
5(i)	$T_1 + T_2 \sim N(5, 0.4^2 + 0.5^2)$	**B1**	or N(5, 0.41)
	$\frac{6-5}{\sqrt{'0.41'}} \quad (= 1.562)$	**M1**	Allow cc
	Φ('1.562')	**M1**	Correct area consistent with their working
	= 0.941	**A1**	
		4	
5(ii)	$\text{Var}(T_2 - 1.2T_1) = 0.5^2 + 1.2^2 \times 0.4^2$ (= 0.4804)	**B1**	Or similar using $1.2T_1 - T_2$
	$T_2 - 1.2T_1 - N(0.16, 0.4804)$	**B1 ft**	Only ft attempt at combination. no ft for neg var.
	$\frac{0-'0.16'}{\sqrt{'0.4804'}} \quad (= -0.231)$	**M1**	Standardise with their mean and variance. Allow cc
	$P(T_2 - 1.2T_1) > 0$		
	= Φ('0.231')	**M1**	Correct area consistent with their working
	= 0.591 (3 sfs)	**A1**	
		5	

Question	Answer	Marks	Guidance
6(i)	$k\int_2^6 x^{-1}\mathrm{d}x = 1$	**M1**	Attempt integrate f(*x*) & = 1. Ignore limits
	$k\left[\ln x\right]_2^6 = 1$ $k(\ln 6 - \ln 2) = 1$ or $k\ln 3 = 1$ $k = \frac{1}{\ln 3}$ **AG**	**A1**	correct sub of correct limits in correct integral leading to correct ans. No errors seen.
		2	
6(ii)	$\frac{1}{\ln 3}\int_2^6 1\ \mathrm{d}x$	**M1**	Attempt integ *x*f(*x*). Ignore limits
	$= \frac{1}{\ln 3}\left[x\right]_2^6 \quad (=\frac{1}{\ln 3}(6-2))$	**A1**	Correct integral and limits
	$= \frac{4}{\ln 3} = 3.64$ **AG**	**A1**	No errors seen
		3	
6(iii)	$\mathrm{P}(X < \mathrm{E}(X)) = \frac{1}{\ln 3}\int_2^{3.64} x^{-1}\mathrm{d}x$	**M1**	Attempt integ f(*x*) from 2 to $\frac{4}{\ln 3}$ or 3.64 oe
	$= \frac{1}{\ln 3}\left[\ln x\right]_2^{3.64}$ $= \frac{1}{\ln 3}(\ln 3.64 - \ln 2) \quad (= 0.545)$	**A1**	Correct sub correct limits into correct integral
	$\mathrm{P}(m < \mathrm{X} < \mathrm{E}(X)) =$ "0.545" – 0.5	**M1**	Subt 0.5 from their P(*X* < E(*X*)) art 0.045 . ft their P(*X* < E(*X*) (> 0.5)
	= 0.045 (2 sfs)	**A1**	equivalent method M1 method for median-need 0.5 and limits 2 to m or m to 6 A1 sqrt 12 or 3.464 M1 calc area from "3.464" to 3.64 A1 0.045 or better, not 0.046
		4	

Question	Answer	Marks	Guidance
7(i)	$H_0: \mu = 51 \quad H_1: \mu < 51$	**B1**	Or popn mean …
	$\bar{x} = \frac{7480}{150} = 49.8667 = 49.9$	**B1**	
	$s^2 = \frac{150}{149}(\frac{380000}{150} - (\frac{748}{15})^2)$ $= 46.9620 = 47.0$ or $s = 6.85$	**M1**	Correct subst in s^2 or $\sqrt{s^2}$ formula Biased var scores M0
	$\frac{49.8667-51}{\sqrt{\frac{'46.962'}{150}}}$ allow $\frac{49.9-51}{\sqrt{\frac{'47'}{150}}}$	**M1**	Allow 49.8667 to 49.9 in numerator Need sqrt 150
	$= (-)\ 2.025 \ = (-)\ 1.965$	**A1**	Accept 2.02 or 2.03 Accept –2.0264 –1.9651 provided correct working
	comp $z = 1.96$	**M1**	or comp $1 - \phi(2.025)$ with 0.025
	There is evidence that $\mu < 51$	**A1 ft**	no contradictions biased var B1B1M0M1A0M1A1ft (max 5/7) accept cv method $x_{crit} = 49.9028$ M1A1 49 867 < 49.9… M1A1
		7	
7(ii)	$\frac{\bar{x}-51}{\frac{6.856}{\sqrt{150}}} = -1.96$	**M1**	Need 51 and sqrt 150 and correct form
	$\bar{x} = 51 - 1.097 = 49.9$ Rejection region is $\bar{x} < 49.9$	**A1**	This may have been found in part (i)
	$\frac{49.9-49}{\frac{6.856}{\sqrt{150}}}$ (= 1.608 to 1.614)	**M1**	Need 49 and sqrt 150 and correct form
	$P(\bar{x} > 49.9 \mid \mu = 49) = 1 - \Phi('1.608')$	**M1**	
	P(Type II error) = 0.0539	**A1**	Allow 0.0533 to 0.0539
		5	

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Cambridge International Advanced Level

MATHEMATICS **9709/72**

Paper 7 **October/November 2018**

MARK SCHEME

Maximum Mark: 50

Published

This document consists of **12** printed pages.

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1	$e^{-2.3}(\frac{2.3^2}{2}+\frac{2.3^3}{3!}+\frac{2.3^4}{4!})$	**M2**	M1 for one term wrong or one end error or 1 – P(2, 3, 4)
	= 0.585 (3 sf)	**A1**	
		3	

Question	Answer	Marks	Guidance
2(i)	$z = 1.96$	**B1**	seen
	$330.1 \pm z \times \frac{4.8}{\sqrt{180}}$	**M1**	Must be of correct form. Any z
	= 329.4 to 330.8 (1 dp)	**A1**	Must be to 1 dp. Must be an interval.
		3	
2(ii)	Yes, because vol of all cans not stated to be normal	**B1**	Or Yes, population not stated to be normal
		1	

Question	Answer	Marks	Guidance
3	E(T) = 2 × 250 + 5 × 160 (= 1300)	**B1**	
	Var(T) = 2 × 10 + 5 × 9 (= 65)	**B1**	
	$\frac{1310-'1300'}{\sqrt{'65'}}$ (= 1.240)	**M1**	Standardise using their values (must come from a combination attempt). Ignore cc
	1 – ϕ(‘1.240’)	**M1**	Correct area consistent with their working
	= 0.1075	**A1**	Allow 0.107 to 0.108 (no errors seen)
		5	

Question	Answer	Marks	Guidance
4(i)	$\int_0^a \frac{k}{(x+1)^2}\mathrm{d}x = 1$	**M1**	Any attempt integ f(x) and = 1. Ignore limits
	$-\left[\frac{k}{(x+1)}\right]_0^a = 1$ $-k(\frac{1}{a+1}-1) = 1$	**M1**	Attempt subst correct limits into correct integral
	$k \times \frac{a}{a+1} = 1$ and $k = \frac{a+1}{a}$ **AG**	**A1**	No errors seen
		3	

Question	Answer	Marks	Guidance
4(ii)	Max time allowed by model (for runners to finish)	**B1**	Allow: All runners finish in time *a* or less or Longest time (taken by any runner) oe
		1	
4(iii)	$\frac{a+1}{a}\int_0^{0.5}\frac{1}{(x+1)^2}\,dx = \frac{3}{4}$	**M1**	Attempt integ f(x) and $= \frac{3}{4}$; ignore limits oe. Condone missing / incorrect k
	$-\frac{a+1}{a}\left[\frac{1}{(x+1)}\right]_0^{0.5} = \frac{3}{4}$ $-\frac{a+1}{a}(\frac{2}{3}-1) = \frac{3}{4}$	**M1**	Attempt subst correct limits into correct integral. Condone missing / incorrect k
	$a = 0.8$ oe	**A1**	
		3	

Question	Answer	Marks	Guidance
5(i)	$\hat{\mu} = \frac{126}{70}$ or $\frac{9}{5}$ or 1.8 oe	**B1**	
	$\Sigma x^2 f = 286$	**B1**	Seen or implied
	$\text{Est}(\sigma^2) = \frac{70}{69}(\frac{\Sigma x^2 f}{70} - '1.8'^2)$	**M1**	oe attempted
	= 0.858 or 296 / 345	**A1**	Note: Final answer for var 0.846 (biased) and no working implies B1 for 286
		4	

Question	Answer	Marks	Guidance
5(ii)	H$_0$: $\mu = 1.9$ H$_1$: $\mu < 1.9$	**B1**	Or 'pop mean'; not just 'mean'
	$\frac{1.8-1.9}{\sqrt{\frac{\text{'0.858'}}{70}}}$	**M1**	Standardise with their values from **(i)**. Must have sqr 70. No SD / Var mix
	$= -0.903$	**A1**	Accept ±
	0.903 < 1.645	**M1**	comp 1.645 allow comp 1.96 if H$_1$: $\mu \neq 1.9$ or comp 1 – ϕ('0.903')=0.182 or 0.183 with 0.05 (or 0.025 if H$_1$: $\mu \neq 1.9$)
	No evidence that mean no courts in S is less than in N	**A1ft**	No contradictions. ft their 0.903, but not comp 1.96 i.e. no ft for a 2 tail test Accept cv method: cv = 1.718 M1A1 1.718 < 1.8 M1 conclusion A1 (cv centred on 1.8 gives 1.982 M1A1 and M1 for 1.982 > 1.9 A1 conclusion)
		5	
5(iii)	Type II because H$_0$ was not rejected	**B1ft**	ft their conclusion, i.e. if H$_0$ rejected, 'Type I because H$_0$ rejected' B1 Answer must be consistent with their conclusion. No conclusion in **(ii)** will score B0
		1	

Question	Answer	Marks	Guidance
6(i)	H_o: $p = 0.15$ H_1: $p < 0.15$ $(N(60 \times 0.15, 60 \times 0.15 \times 0.85))$ $= N(9, 7.65)$	**B1**	Accept H_0: $\mu = 9$ H_1: $\mu < 9$ Use of Normal approximation: $(N(0.15, \frac{0.15\times0.85}{60}))$ $= N(0.15, 0.002125)$
	$\frac{6.5-'9'}{\sqrt{'7.65'}}$	**M1**	For standardising (or $\frac{\frac{6}{60}+\frac{0.5}{60}-'0.15'}{\sqrt{'0.002125'}}$ $= –0.904$) Allow wrong or no cc
	$= –0.904$	**A1**	Accept ±
	‘0.904’ < 1.282	**M1**	Valid comparison of z values or $\phi('–0.904') = 0.183 > 0.1$ ft their 0.904
	No evidence train late less often	**A1ft**	Use of Bin (60,0.15) to give Pr (< = 6) = 0.1848 M1A1 Valid comparison with 0.1 M1 Conclusion A1ft
		5	
6(ii)	$0.1 + z \times \sqrt{\frac{0.1\times0.9}{60}} = 0.150$	**M1**	For $\sqrt{(0.1 \times 0.9 / 60)}$ seen
		M1	for $0.1 + z \times ... = 0.150$ or $2z… = 0.1$
	$z = 1.291$	**A1**	
	ϕ(‘1.291’) (= 0.90(16))	**M1**	for correct method to find α
	$\alpha = 80$	**A1ft**	ft their z. Must be a +ve non-zero integer < 100
		5	

Question	Answer	Marks	Guidance
7(i)	$e^{-5.6} \times \frac{5.6^3}{3!}$	M1	Allow any λ
	$= 0.108$ (3 sf)	A1	
		2	
7(ii)	$P(X=2\ \&\ Y=1) = e^{-2.1} \times \frac{2.1^2}{2} \times e^{-3.5} \times 3.5$ $(0.2700 \times 0.10569 = 0.028538)$	M1	
	$\frac{P(X\ =2\ \&\ Y\ =1)}{P(X+Y=3)}$ attempted $= \frac{'0.028538'}{'0.108234}$	M1	For attempt at fraction with their **(i)** as denominator or $\frac{2.1^2}{2} \times 3.5 \div \frac{5.6^3}{3}$ M2
	$= 0.264$ (3 sf)	A1	
		3	

Question	Answer	Marks	Guidance
7(iii)	Var(X) = 2.1	**B1**	soi
	$\bar{X} \sim \text{N}(2.1, \frac{2.1}{100})$ or N (210,210)	**B1**	soi B1 for N(2.1, ...)
		B1	B1 for $\frac{2.1}{100}$ oe Standardise with their values. Allow with or without cc or with incorrect cc
	$\frac{2.2-2.1}{\frac{\sqrt{2.1}}{\sqrt{100}}}$ oe $(220 - 210) / \sqrt{210}$ (= 0.690)	**M1**	or $\frac{2.2+0.5\div100-2.1}{\frac{\sqrt{2.1}}{\sqrt{100}}}$ or $(220.5 - 210) / \sqrt{210})$ (= 0.725) no mixed methods
	$1 - \phi$(‘0.690’)	**M1**	Correct area consistent with their working or $1 - \phi$(‘0.725’)
	= 0.245 (3 sf)	**A1**	= 0.234 (3 sf)
		6	

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Cambridge International Advanced Level

MATHEMATICS **9709/73**
Paper 7 **October/November 2018**
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Maximum Mark: 50

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MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become ‘follow through’ marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.
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Question	Answer	Marks	Guidance
1(i)	$176 \pm z \times \frac{7.2}{\sqrt{200}}$	**M1**	need correct form must be z
	$z = 2.24$	**B1**	allow 2.241 and 2.242
	175 to 177	**A1**	cwo
		3	
1(ii)	Sample random	**B1**	oe. both words essential
		1	

Question	Answer	Marks	Guidance
2(i)	$H_0: p = \frac{1}{3}$ $H_1: p < \frac{1}{3}$	**B1**	
		1	
2(ii)	$0.0084 < 0.01$	**B1**	Allow $P(N \leqslant 36) < 0.01$ or 1%
	There is evidence that p has decreased	**B1 dep**	Allow 'p has decreased' or $p < \frac{1}{3}$
		2	
2(iii)	150	**B1**	
		1	

Question	Answer	Marks	Guidance
3	$\frac{12.2-12}{2.5/\sqrt{n}}$	**M1**	Standardisation. Allow cc. need correct form incl sqrt
	(=) 1.96	**B1**	Correct z
	$\sqrt{n} = 1.96 \times 2.5 \div 0.2$	**M1**	Rearrange equation in n or sqrt n with numerical z to the stage n= or sqrt n = allow arithmetical slips only
	$n = 600$	**A1**	accept 601 SR whole number ans from 595 to 605 can score full marks if fully justified
		4	

Question	Answer	Marks	Guidance
4(i)	$\lambda = 10\times0.25 + 10\times0.36 \quad (= 6.1)$	**B1**	
	$1 - e^{-6.1}\,(1 + 6.1 + \frac{6.1^2}{2} + \frac{6.1^3}{3!})$	**M1**	1 – P($X \leqslant 3$), any λ Allow one end error
	= 0.857	**A1**	Allow 0.858
		3	
4(ii)	$\lambda = 61$	**B1 ft**	Ft from (i)
	N('61', '61')	**M1**	N with $\mu = \lambda$, any λ. May be implied
	$\frac{59.5-61}{\sqrt{'61'}} \quad (= -0.192)$	**M1**	Standardise with their mean and variance Allow no or wrong cc. not 61/100
	Φ('–0.192') = 1 – Φ('0.192')	**M1**	Correct area consistent with their working
	= 0.424	**A1**	
		5	

Question	Answer	Marks	Guidance
5(i)	$T_1 + T_2 \sim N(5, 0.4^2 + 0.5^2)$	**B1**	or N(5, 0.41)
	$\frac{6-5}{\sqrt{'0.41'}} \quad (= 1.562)$	**M1**	Allow cc
	Φ('1.562')	**M1**	Correct area consistent with their working
	= 0.941	**A1**	
		4	
5(ii)	$\text{Var}(T_2 - 1.2T_1) = 0.5^2 + 1.2^2 \times 0.4^2$ (= 0.4804)	**B1**	Or similar using $1.2T_1 - T_2$
	$T_2 - 1.2T_1 - N(0.16, 0.4804)$	**B1 ft**	Only ft attempt at combination. no ft for neg var.
	$\frac{0-'0.16'}{\sqrt{'0.4804'}} \quad (= -0.231)$	**M1**	Standardise with their mean and variance. Allow cc
	$P(T_2 - 1.2T_1) > 0$		
	= Φ('0.231')	**M1**	Correct area consistent with their working
	= 0.591 (3 sfs)	**A1**	
		5	

Question	Answer	Marks	Guidance
6(i)	$k\int_2^6 x^{-1}\mathrm{d}x = 1$	**M1**	Attempt integrate f(*x*) & = 1. Ignore limits
	$k[\ln x]_2^6 = 1$ $k(\ln 6 - \ln 2) = 1$ or $k\ln 3 = 1$ $k = \frac{1}{\ln 3}$ **AG**	**A1**	correct sub of correct limits in correct integral leading to correct ans. No errors seen.
		2	
6(ii)	$\frac{1}{\ln 3}\int_2^6 1\ \mathrm{d}x$	**M1**	Attempt integ *x*f(*x*). Ignore limits
	$= \frac{1}{\ln 3}[x]_2^6 \quad (= \frac{1}{\ln 3}(6-2))$	**A1**	Correct integral and limits
	$= \frac{4}{\ln 3} = 3.64$ **AG**	**A1**	No errors seen
		3	
6(iii)	$P(X < E(X)) = \frac{1}{\ln 3}\int_2^{3.64} x^{-1}\mathrm{d}x$	**M1**	Attempt integ f(*x*) from 2 to $\frac{4}{\ln 3}$ or 3.64 oe
	$= \frac{1}{\ln 3}[\ln x]_2^{3.64}$ $= \frac{1}{\ln 3}(\ln 3.64 - \ln 2) \quad (= 0.545)$	**A1**	Correct sub correct limits into correct integral
	$P(m < X < E(X)) =$ “0.545” – 0.5	**M1**	Subt 0.5 from their $P(X < E(X))$ art 0.045 . ft their $P(X < E(X)$ (> 0.5)
	= 0.045 (2 sfs)	**A1**	equivalent method M1 method for median-need 0.5 and limits 2 to m or m to 6 A1 sqrt 12 or 3.464 M1 calc area from “3.464” to 3.64 A1 0.045 or better, not 0.046
		4	

Question	Answer	Marks	Guidance
7(i)	H_0: μ = 51 H_1: μ < 51	**B1**	Or popn mean …
	$\bar{x} = \frac{7480}{150} = 49.8667 = 49.9$	**B1**	
	$s^2 = \frac{150}{149}(\frac{380000}{150} - (\frac{748}{15})^2)$ = 46.9620 = 47.0 or s = 6.85	**M1**	Correct subst in s^2 or $\sqrt{s^2}$ formula Biased var scores M0
	$\frac{49.8667-51}{\sqrt{\frac{'46.962'}{150}}}$ allow $\frac{49.9-51}{\sqrt{\frac{'47'}{150}}}$	**M1**	Allow 49.8667 to 49.9 in numerator Need sqrt 150
	= (–) 2.025 = (–) 1.965	**A1**	Accept 2.02 or 2.03 Accept –2.0264 –1.9651 provided correct working
	comp z = 1.96	**M1**	or comp 1 – ϕ(2.025) with 0.025
	There is evidence that μ < 51	**A1 ft**	no contradictions biased var B1B1M0M1A0M1A1ft (max 5/7) accept cv method x_{crit} = 49.9028 M1A1 49 867 < 49.9… M1A1
		7	
7(ii)	$\frac{\bar{x}-51}{\frac{6.856}{\sqrt{150}}} = -1.96$	**M1**	Need 51 and sqrt 150 and correct form
	$\bar{x}$ = 51 – 1.097 = 49.9 Rejection region is $\bar{x}$ < 49.9	**A1**	This may have been found in part (i)
	$\frac{49.9-49}{\frac{6.856}{\sqrt{150}}}$ (= 1.608 to 1.614)	**M1**	Need 49 and sqrt 150 and correct form
	P($\bar{x}$ > 49.9 | μ = 49) = 1 – Φ('1.608')	**M1**	
	P(Type II error) = 0.0539	**A1**	Allow 0.0533 to 0.0539
		5	

Cambridge Assessment International Education
Cambridge International Advanced Subsidiary and Advanced Level

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MATHEMATICS **9709/71**

Paper 7 **May/June 2018**

MARK SCHEME

Maximum Mark: 50

Published

This document consists of **8** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

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- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

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Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

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- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

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GENERIC MARKING PRINCIPLE 6:

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Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously 'correct' answers or results obtained from incorrect working.

  - Note: B2 or A2 means that the candidate can earn 2 or 0.
    B2/1/0 means that the candidate can earn anything from 0 to 2.

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PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Question	Answer	Marks	Guidance
1	est(μ) (= 153.2 ÷ 75) = 2.04 (3 sf)	**B1**	
	est(σ^2) = $\frac{75}{74}(\frac{340.24}{75} - "2.04267"^2)$ oe	**M1**	
	= 0.369 (3 sf)	**A1**	Accept 0.368
		3	

Question	Answer	Marks	Guidance
2(i)	$\frac{20}{100} \pm z \times \sqrt{\frac{0.2\times(1-0.2)}{100}}$	**M1**	Any z
	z = 1.881 or 1.882	**B1**	
	= 0.125 to 0.275	**A1**	
		3	
2(ii)	$\frac{1}{6}$ is within this range No evidence of bias concerning 2	**B1ft**	Both statements needed
		1	

Question	Answer	Marks	Guidance
3	N(153, 153)	**B1**	Seen or implied
	$\frac{139.5-153}{\sqrt{"153"}}$ (= –1.091)	**M1**	Allow with wrong or no cc
	ϕ("–1.091") = 1 – ϕ("1.091")	**M1**	For area consistent with their working
	= 0.138 (3 sf)	**A1**	
		4	

Question	Answer	Marks	Guidance
4(i)	mean= 155.1	**B1**	
	var = $1.5^2 \times 10.2$ (= 22.95) sd = $\sqrt{"22.95"}$	**M1**	or $1.5 \times \sqrt{10.2}$
	= 4.79	**A1**	
		3	

Question	Answer	Marks	Guidance
4(ii)	mean = 103.4 + "155.1" (= 258.5) var = 10.2 + "22.95" (=33.15)	**B1ft**	Both. ft their 155.1 and 22.95. Accept sd.
	$\frac{250-"258.5"}{\sqrt{"33.15"}}$ (= –1.476)	**M1**	Standardising – no sd/var mix. Their mean/sd must be from an attempt at combination
	1– ϕ(–1.476) = ϕ(1.476)	**M1**	For area consistent with their working
	= 0.930 (3 sf)	**A1**	Allow 0.93
		4	

Question	Answer	Marks	Guidance
5(i)	$\frac{14-14.2}{\frac{3.1}{\sqrt{50}}}$ (= – 0.456)	**M1**	For stand'n; must have √50
	1 – Φ("0.456")	**M1**	for area consistent with their working
	= 0.324 (3 sfs)	**A1**	
		3	
5(ii)	No because *n* large	**B1**	Accept n > 30
		1	
5(iii)	H_0: $\mu = 14.2$ H_1: $\mu < 14.2$	**B1**	or 'pop mean', but not just 'mean'
	$\frac{13.5-14.2}{\frac{3.1}{\sqrt{100}}}$	**M1**	For stand'n; must have √100
	= –2.258	**A1**	
	comp –2.054 (or –2.055)	**M1**	Valid comparison of z values or areas (0.0119 < 0.02)
	There is evidence (at 2% level) that mean mass in this area < 14.2	**A1ft**	Ft their z. Correct conclusion no contradictions
		5	

Question	Answer	Marks	Guidance
6(i)	$\int_5^{10} \frac{k}{x^2}\mathrm{d}x = 1$	**M1**	Attempt integration f(*x*) and '= 1'; ignore limits
	$\left[-\frac{k}{x}\right]_5^{10} = 1$ oe $(\frac{k}{5} - \frac{k}{10} = 1)$	**A1**	Correct integration and limits and '= 1'
	$k = 10$ **AG**	**A1**	No errors seen
		3	
6(ii)	$10\int_5^{10} \frac{1}{x}\mathrm{d}x$ $10\left[\ln x\right]_5^{10}$	**M1**	Attempt integ *x*f(*x*); ignore limits. or 10(ln 10 – ln 5)
	= 10ln 2 **AG**	**A1**	No errors seen
		2	
6(iii)	$10\int_9^{10} \frac{1}{x^2}\mathrm{d}x$ $(10\left[-\frac{1}{x}\right]_9^{10})$	**M1**	Attempt integ f(x) with correct limits
	$10\left[-\frac{1}{10} + \frac{1}{9}\right]$	**A1**	Substitute correct limits in correct integration
	$= \frac{1}{9}$ or 0.111 (3 sf)	**A1**	
		3	
6(iv)	$\int_5^{a} \frac{k}{x^2}\mathrm{d}x = 0.6$ $10\left[-\frac{1}{x}\right]_5^{a} = 0.6$	**M1**	Attempt integration of f(*x*) with correct limits and = 0.6
	$10[\frac{1}{5} - \frac{1}{a}] = 0.6$	**A1**	Substitute correct limits in correct integration
	$a = \frac{50}{7}$ or 7.14 (3 sf)	**A1**	
		3	

Question	Answer	Marks	Guidance
7(i)	Po(1.0)	**B1**	Seen or implied
	e^{-1} $(1 + 1 + \frac{1^2}{2})$	**M1**	Allow any λ. Allow one end error.
	= 0.920 (3 sfs)	**A1**	
		3	
7(ii)	$P(X > 3) = 1 - e^{-1.5}(1+1.5+ \frac{1.5^2}{2} + \frac{1.5^3}{3!})$	**M1**	Allow any λ. Allow one end error
	= 0.0656	**A1**	
		2	
7(iii)(a)	Incorrectly concluding that more absences than usual when there are not oe	**B1**	In context
		1	
7(iii)(b)	H_0: $\lambda = 1.5$ (or 0.3) H_1: $\lambda > 1.5$ (or 0.3)	**B1**	Or μ Both
	$P(X > 4) = \text{“0.0656”} - e^{-1.5} \times \frac{1.5^4}{4!}$ = 0.0186 (3 sf)	**M1**	or $1 - e^{-1.5}(1+1.5+ \frac{1.5^2}{2} + \frac{1.5^3}{3!} + \frac{1.5^4}{4!})$
	P(Type I) = 0.0186 or 0.0185	**A1ft**	Ft their $P(X > 4)$ if less than 0.05
		3	
7(iii)(c)	$P(X > 3) = \text{"0.0656"}$	**B1ft**	Ft their **(ii)**
	0.0656 > 0.05	**M1**	
	No evidence of more than usual male absences	**A1ft**	Ft their P(X>3). Correct conclusion. No contradictions.
		3	

Cambridge Assessment International Education
Cambridge International Advanced Level

MATHEMATICS **9709/72**

Paper 7 **May/June 2018**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2018 series for most Cambridge IGCSE™, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

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Question	Answer	Marks	Guidance
1	$\lambda = 4.4$	**B1**	
	$P(X<4) = e^{-4.4}(1 + 4.4 + \frac{4.4^2}{2} + \frac{4.4^3}{3!})$	**M1**	Allow any λ allow one end error
	= 0.359	**A1**	
		3	

Question	Answer	Marks	Guidance
2	A: N(6, 4.8)	**B1 B1**	B1 for N(6, ..) for either A or B. B1 for 4.8 (or 2.19^2) (or SD=2.19)
	B: N(6, 2.4)	**B1**	B1 For 2.4 (or 1.55^2) (or SD=1.55) (SR 3/3 but error seen withhold B1 so 2/3 scored)
		3	

Question	Answer	Marks	Guidance
3(i)	$52 \pm z \times \frac{6.5}{\sqrt{15}}$	**M1**	Expression of the correct form. Any z
	$z = 1.96$	**B1**	Seen or used
	48.7 to 55.3 (3 sf)	**A1**	Must be an interval
		3	

Question	Answer	Marks	Guidance
3(ii)	Narrower because more information or because $\frac{\sigma}{\sqrt{n}}$ smaller	**B1**	oe Accept 'sample size is larger' 'more employees' 'width inversely proportional to sq root of n' 'if n increases width decreases' '95% CI is 49.7 to 54.3' or similar. No contradictions
		1	

Question	Answer	Marks	Guidance
4(i)	Est(μ) = 495.9	**B1**	Accept 496
	Est(σ^2) = $\frac{10}{9}(\frac{2459283}{10} - \text{"}495.9\text{"}^2)$	**M1**	Attempt Σx^2 and subst in correct formula (1/9("2459283" – "4959"^{2}/10)). May be implied by correct answer
	= 12.8 (3 sf) or 383/30	**A1**	(Note: Biased var "11.49" scores M0 A0)
		3	
4(ii)	H_0: μ = 505 H_1: μ < 505 $\frac{\frac{75660}{150} - 505}{3.6 \div \sqrt{150}}$	**B1**	Allow 'Pop mean' but not just 'mean'
	= –2.04	**M1**	Correct stand'n; must have √150. No sd/var mixes. Condone sample SD (3.58/3.39) Accept standardisation of totals ((75660-75750)/44.091) Accept CV method
		A1	Accept +2.04 (Note: if valid area comparison done 0.0207/0.0206 or 0.979 needed for A1)
	comp z = –2.054	**M1**	Valid comparison of z's or area (0.0207/6>0.02; 0.979(3)<0.98)
	No evidence (at 2%) that machine pkts mean mass < 505	**A1ft**	oe No contradictions. SC Two tail test can score B0 M1 A1 M1 for comparison with 2.326 A0 (max 3/5)
		5	

Question	Answer	Marks	Guidance
4(iii)	Large sample, so sample mean approx normally distr'd	**B1**	Allow just 'Sample is large' or '*n* is large' n>30
		1	

Question	Answer	Marks	Guidance
5(i)	$\frac{1}{2}\times a\times b\ = 1$	**M1**	Attempt Δ area = 1 or $\int(b - bx/\text{a})\,\text{d}x = 1$ with correct limits
	$b = \dfrac{2}{a}$	**A1**	
		2	
5(ii)	grad = $-\frac{2}{a^2}$ or $-\frac{b}{a}$	**B1**	allow without '–' sign (could be implied or seen in **(i)**)
	y - ($\frac{2}{a}$) = grad × x or y = grad × $(x - a)$	**M1**	correct use of $y = mx + c$ or $y - y_1 = m(x - x_1)$ with (0,b) or (a,0) including attempt at substitution of their b
	y - ($\frac{2}{a}$) = $-\frac{2}{a^2}x$ or $y = -\frac{2}{a^2}(x - a)$ and $y = \dfrac{2}{a} - \dfrac{2}{a^2}x$ **AG**	**A1**	No errors seen
		3	

Question	Answer	Marks	Guidance
5(iii)	$\int_0^a (\frac{2}{a}x - \frac{2}{a^2}x^2)\mathrm{d}x$	**M1**	Attempt int xf(x) ignore limits
	$= [\frac{1}{a}x^2 - \frac{2}{3a^2}x^3]_0^a$	**A1**	Correct integration ignore limits
	a - $\frac{2}{3}a = 0.5$	**M1**	Sub correct limits into their integral and = 0.5
	$a = 1.5$	**A1**	
		4	

Question	Answer	Marks	Guidance
6(i)	Accidents occur independently or randomly	**B1**	In context. Allow 'singly'.
		1	
6(ii)	$\mathrm{e}^{-2.5} \times \frac{2.5^4}{4!}$	**M1**	Poisson P(4), allow any λ
	= 0.134 (3 sfs)	**A1**	
		2	

Question	Answer	Marks	Guidance
6(iii)	$\lambda = \frac{25}{12}$ or 2.08(333)	**B1**	
	$1 - e^{\frac{-25}{12}}\left(1 + \frac{25}{12} + \frac{\frac{25}{12}^2}{2!} + \frac{\frac{25}{12}^3}{3!}\right)$	**M1**	1 – Poisson P(0, 1, 2, 3), allow any λ allow one end error
	= 0.158 (3 sfs)	**A1**	As final answer
		3	
6(iv)	$N(\frac{1825}{84}, \frac{1825}{84})$ or N(21.7(26), 21.7(26))	**B1**	Stated or implied
	$\dfrac{29.5 - \frac{1825}{84}}{\sqrt{\frac{1825}{84}}}$	**M1**	Allow with wrong or no cc with their mean/sd
	Φ("1.668")	**M1**	Correct area consistent with their working
	= 0.952 (3 sfs)	**A1**	
		4	

Question	Answer	Marks	Guidance
7(i)	H_0: P(10) = 0.1 H_1: P(10) > 0.1	**B1**	Both. Allow ‘*p*’ for P(10)
	B(9,0.1) $P(X \geqslant 3) =$ $1 - (0.9^9 + 9\times0.9^8 \times 0.1 + {}^9C_2 \times 0.9^7 \times 0.1^2)$	**M1**	Allow one extra term in bracket
	= 0.05297... or 0.053(0)	**A1**	
	comp 0.01	**M1**	Valid comparison. (comparison with 0.99 can recover previous M1 A1 for 0.9470)
	No evidence (at 1% level) to reject H_0 Claim not justified	**A1ft**	No contradictions
		5	
7(ii)	H_0 not rejected oe	**B1**	
		1	
7(iii)	$P(X \geqslant 4)$ $= \text{"0.05297"} - {}^9C_3\times0.9^6\times0.1^3$	**M1**	or $1-(0.9^9 + 9 \times 0.9^8 \times 0.1 + {}^9C_2 \times 0.9^7 \times 0.1^2 + {}^9C_3 \times 0.9^6 \times 0.1^3)$
	= 0.00833	**A1**	Note: 0.05297 and 0.00833 both needed in **(i)** or **(iii)** to justify CV
	Hence crit value is 4	**B1**	Allow without working. Or in **(i)** May be implied by attempt at $P(X < 4)$ below
	B(9,0.5) $P(X < 4)$	**M1**	stated or implied
	$= 0.5^9 + 9 \times 0.5^8 \times 0.5 + {}^9C_2 \times 0.5^7 \times 0.5^2 + {}^9C_3\times0.5^6\times0.5^3$	**M1**	Attempt $P(X < 4)$ with $p = 0.5$
	P(Type II) = 0.254 (3 sf)	**A1**	
		6	

Cambridge Assessment International Education
Cambridge International Advanced Level

MATHEMATICS **9709/73**

Paper 7 **May/June 2018**

MARK SCHEME

Maximum Mark: 50

Published

This document consists of **13** printed pages.

Generic Marking Principles

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GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously 'correct' answers or results obtained from incorrect working.

  - Note: B2 or A2 means that the candidate can earn 2 or 0.
    B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only – often written by a 'fortuitous' answer
ISW	Ignore Subsequent Working
SOI	Seen or implied
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become 'follow through' marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Question	Answer	Marks	Guidance
1(i)	Po(2.25)	**B1**	Stated or implied
	$e^{-2.25}(1 + 2.25 + \frac{2.25^2}{2})$	**M1**	Allow any λ, one end error
	= 0.609 (3 sf)	**A1**	SC B1 Use of B(75,0.03) leading to 0.608
		3	
1(ii)	μ = 2.25, which is less than 5; n large	**B1**	Allow np<5 and n large or p<0.1 and n>50, no contradictions
		1	

Question	Answer	Marks	Guidance
2(i)	213, 165, 73, 196 Allow 073	**B1**	For 3-digit no, < 265, consisting of three consecutive integers from given digits, backwards or forward. (73 or 073 counts as a 3-digit no.)
		B1	For another three such. Other answers may be valid. If other method used, method must be clear
		2	

Question	Answer	Marks	Guidance
2(ii)	$\frac{510}{25} = \frac{102}{5}$ or 20.4	**B1**	
	$\frac{25}{24}\left[\frac{13225}{25}-\left(\frac{102}{5}\right)^2\right]$	**M1**	$\frac{1}{24}\left(13225-\frac{510^2}{25}\right)$
	118 (3 sf) or $\frac{2821}{24}$	**A1**	
		3	
2(iii)	(Average) weekly earnings of all students in Amy's year	**B1**	Not 'All students in Amy's year'
		1	

Question	Answer	Marks	Guidance
3	$\dfrac{\frac{8}{64}\times(1-\frac{8}{64})}{64}$ (= $\dfrac{7}{4096}$ or 0.00171)	**M1**	OE, e.g. $\dfrac{\frac{1}{8}\times\frac{7}{8}}{64}$
	$2\times z\sqrt{\text{"}\dfrac{7}{4096}\text{"}} = 0.130$	**M1**	Correct equation using their variance
	$z = 1.572$	**A1**	
	ϕ("1.572") (= 0.942) (0.942 – (1 – 0.942) = 0.884)	**M1**	2ϕ(their z) -1
	$\alpha = 88$	**A1**	CAO
		5	

Question	Answer	Marks	Guidance
4(i)	No of males leaving (to do eng) each yr has const mean or Males leave (to do eng) indep of other males leaving (to do eng) or Males leave (to do eng) at random	**B1**	One of these or any equiv statement in context.
		1	
4(ii)	$\lambda = 3.9$	**B1**	
	$1 - e^{-3.9}(1 + 3.9 + \frac{3.9^2}{2!} + \frac{3.9^3}{3!})$	**M1**	Any λ. Allow one end error or extra term.
	0.546753 or 0.547 (3 sf)	**A1**	
		3	
4(iii)	P(*F* = 0 and *M* > 3) = $e^{-0.8} \times \left[1 - e^{-3.1}\left(1 + 3.1 + \frac{3.1^2}{2!} + \frac{3.1^3}{3!}\right)\right]$ (= 0.16857)	**M1**	Attempt P(*F* = 0) × P(*M* > 3) allow one end error for P(*M* > 3) provided $\lambda = 3.1$
	$\frac{\text{P(F=0 and M>3)}}{\text{P(M+F>3)}}$ $\frac{\text{"0.16857"}}{\text{"0.54675"}}$	**M1**	Attempted, allow any probability/their **(ii)** provided the answer is <1
	= 0.308 (3 sf)	**A1**	
		3	

Question	Answer	Marks	Guidance
5(i)	Assume (pop) sd same (0.3) H_0: Pop mean = 2.4	**B1**	
	H_1: Pop mean ≠ 2.4	**B1**	Allow 'μ' but not just 'mean'
	$\pm\dfrac{2.3-2.4}{\frac{0.3}{\sqrt{30}}}$	**M1**	Must have $\sqrt{30}$, Critical region approach (2.293, 2.507) or (2.193, 2.407)
	= ±1.826	**A1**	
	comp $z =$ ±1.96	**M1**	Valid comparison (e.g. compare 0.034 with 0.025)
	No evidence that mean time changed	**A1f**	In context, allow accept H_0 if correctly defined, no contradictions. One-tail test can score B1, B0, M1, A1, M1, A0 Max 4/6
		6	
5(ii)(a)	0.05	**B1**	
		1	
5(ii)(b)	Concluding mean time has not changed when it has.	**B1**	OE, must have e.g. conclude/accept SR Allow mean has decreased if a one tailed test in Part **(i)**
		1	

Question	Answer	Marks	Guidance
6(i)	$E(T) = 4.5 + 2.3$ (= 6.8) $\text{Var}(T) = 1.1^2 + 0.7^2$ (= 1.7)	**M1**	Both methods seen or implied
	$\frac{8.5 - "6.8"}{\sqrt{"1.7"}}$ (= 1.304)	**M1**	Correct stand'n using their μ and σ^2 must be a combination of the two variables
	$\phi("1.304")$	**M1**	Area consistent with their working
	= 0.904 (3 sf)	**A1**	
		4	
6(ii)	$E(D) = 4.5 - 2 \times 2.3$ or -0.1	**M1**	
	$\text{Var}(D) = 1.1^2 + 2^2 \times 0.7^2$ or 3.17	**M1**	Both can seen or implied
	$\frac{0 - ('-0.1')}{\sqrt{'3.17'}}$ (= 0.056)	**M1**	Correct stand'n using their μ and σ^2 must be a Combination of the two variables
	$1 - \phi("0.056")$	**M1**	Area consistent with their working
	= 0.478 (3 sf)	**A1**	
		5	

Question	Answer	Marks	Guidance
7(i)	$k\int_1^2 (\frac{1}{x^2}+\frac{1}{x^3})\,dx = 1$	**M1**	Attempt integ f(*x*) & '= 1'; ignore limits
	$k\left[-\frac{1}{x}-\frac{1}{2x^2}\right]_1^2 = 1$	**A1**	Correct integral & limits & '= 1'
	$k\left[-\frac{1}{2}-\frac{1}{8}+1+\frac{1}{2}\right] = 1$ $k = \frac{8}{7}$ **AG**	**A1**	Sufficient working must be shown, no errors seen
		3	
7(ii)	$\frac{8}{7}\int_1^2 (\frac{1}{x}+\frac{1}{x^2})\,dx$	**M1**	Attempt integ *x*f(*x*), ignore limits
	$= \frac{8}{7}\left[\ln x-\frac{1}{x}\right]_1^2$	**A1**	Correct integral & limits, condone missing k
	$= \frac{8}{7}(\ln 2+\frac{1}{2})$ or 1.36 (3 sf)	**A1**	
		3	

Question	Answer	Marks	Guidance
7(iii)	$\frac{8}{7}\int_{1}^{1.5}(\frac{1}{x^2}+\frac{1}{x^3})\,dx$ $=\frac{8}{7}\left[-\frac{1}{x}-\frac{1}{2x^2}\right]_{1}^{1.5}$	**M1**	Attempt integration f(x) between 1 and 1.5 or between 1.5 and 2
	$=\frac{44}{63}$ or 0.698.......	**A1**	Or $\frac{19}{63}$ or 0.302
	$'\frac{44}{63}'(1-'\frac{44}{63}')^2$	**M1**	FT their $\frac{44}{63}$
	× 3	**M1**	Independent provided answer is <1
	= 0.191	**A1**	
		5	

Cambridge Assessment International Education
Cambridge International Advanced Level

MATHEMATICS **9709/72**

Paper 7 Probability and Statistics **March 2018**

MARK SCHEME

Maximum Mark: 50

Published

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Question	Answer	Marks	Guidance
1	$\dfrac{5-4.9}{\frac{2.21}{\sqrt{75}}}$ (= 0.392)	**M1**	Correct stand'n. Must have √75
	1 – Φ("0.392")	**M1**	Correct area consistent with working
	= 0.348 (3 sfs)	**A1**	
		3	

Question	Answer	Marks	Guidance
2	$\lambda = 98.4$	**B1**	
	N(98.4, 98.4) seen or implied	**B1**	
	$\frac{90.5-\text{"98.4"}}{\sqrt{\text{"98.4"}}}$ (= –0.796)	**M1**	allow with wrong or no cc. No sd/var mix.
	ϕ("0.796")	**M1**	Correct area consistent with working
	= 0.787 (3 sf)	**A1**	
		5	

Question	Answer	Marks	Guidance
3(i)	$E(H_A) = 6$	**B1**	
	$\text{Var}(H_A) = 5 \times 0.03^2$	**M1**	
	= 0.0045 or 9/2000	**A1**	
		3	
3(ii)	$E(H_A - 2H_B) = 0$	**B1**	From 6–6
	$\text{Var}(H_A - 2H_B)$ = '0.0045' + $4\times5\times0.02^2$	**M2**	Allow **M1** for '0.0045' – $4\times5\times0.02^2$ *or* '0.0045' + $2\times5\times0.02^2$ *or* '0.0045' + 4×0.02^2 *or* '0.0045' + $4\times5^2\times0.02^2$
	= 0.0125 (3 sf) or 1/80	**A1**	
		4	

Question	Answer	Marks	Guidance
4(i)	(Po)(2.4)	**B1**	seen or implied
	$e^{-2.4}(1 + 2.4 + \frac{2.4^2}{2} + \frac{2.4^3}{3!})$	**M1**	allow + P(4)/one end error. Allow wrong λ
	= 0.779 (3 sfs)	**A1**	Final answer (Note: accept combination method)
		3	
4(ii)	H_0: λ (or mean) = 3.6 (or 0.9) H_1: λ (or mean) < 3.6 (or 0.9)	**B1**	Accept μ for both
	$e^{-3.6}(1 + 3.6)$	**M1**	Allow any λ
	= 0.126	**A1**	
	0.126 > 0.1	**M1**	Valid comparison. (Comparison with 0.9 could recover previous **M1A1**)
	No evidence that fewer than usual sold	**A1FT**	Correct conclusion. No contradictions
		5	

Question	Answer	Marks	Guidance
5(i)	H_0: P(Orange) = 0.17 H_1: P(Orange) < 0.17	**B1**	or H_0: $p = 0.17$ H_1:$p < 0.17$
5(ii)	Wrongly concluding that % age is less than 17%	**B1**	OE in context allow "fewer than 3 orange in packet even though average 17% is correct"
		1	
5(iii)	B(30, 0.17) stated or implied	**M1**	eg by $0.17^p \times 0.83^q$ $(p + q = 30)$ or ${}^{30}C_r$ $(r < 30)$
	$(1 – 0.17)^{30} + 30(1 – 0.17)^{29} \times 0.17 + {}^{30}C_2(1– 0.17)^{28} \times 0.17^2$	**M1**	correct, but allow + ${}^{30}C_3(1–0.17)^{27}\times 0.17^3$
	= 0.0949 (3 sf)	**A1**	(SR: use of N(5.1,4.233) **M1** standardising (with or without cc) **M1** max 2/3)
		3	

Question	Answer	Marks	Guidance
5(iv)	P($\geqslant$ 3 orange | p = 0.05)	**M1**	stated or attempted; can be implied
	= $1-[(0.95)^{30}+30(0.95)^{29}\times 0.05+{}^{30}C_2(0.95)^{28}\times 0.05^2]$	**M1**	allow + ${}^{30}C_3(0.95)^{27}\times 0.05^3$ in bracket, or ans 0.0608
	= 0.188 (3 sfs)	**A1**	
		3	

Question	Answer	Marks	Guidance
6(i)	$1-6\int_{0.3}^{0.7}\left(x-x^2\right)\mathrm{d}x$	**M1**	or $2\times 6\int_{0}^{0.3}\left(x-x^2\right)\mathrm{d}x$ or similar correct expression before integration
	$1-\left[6(\frac{x^2}{2}-\frac{x^3}{3})\right]_{0.3}^{0.7}$	**A1**	or similar correct expression after integration
	$1-6\left[\frac{0.7^2}{2}-\frac{0.7^3}{3}-\frac{0.3^2}{2}+\frac{0.3^3}{3}\right]$	**M1**	Attempt subst correct limits in this or other correct expression
	= 0.432 (or 54/125)	**A1**	(SR1 Omission of '1–' scores **B2** for 0.568 or 71/125) (SR2 Omission of '2x' scores **B2** for 0.216 or 27/125)
		4	
6(ii)	Correct shape between x = 0 and 1	**B1**	No curve outside this range.
	E(X) = 0.5	**B1**	
		2	
6(iii)	$6\int_0^1\left(x^3-x^4\right)\mathrm{d}x$ $=\left[6\left(\frac{x^4}{4}-\frac{x^5}{5}\right)\right]_0^1$	**M1**	attempt int x^2f(x), ignore limits
	$6[\frac{1^4}{4}-\frac{1^5}{5}]$ (= 0.3)	**M1**	attempt subst correct limits in correct integ
	Var(X) = '0.3' – '0.5'^{2} = 0.05	**A1FT**	FT their mean, dep their Var(X) > 0
		3	

Question	Answer	Marks	Guidance
7(i)	$\bar{x} = 11.83$	**B1**	
	$11.83 \pm z\,\dfrac{0.1}{\sqrt{10}}$	**M1**	any z
	$z = 2.576$	**B1**	accept 2.574 to 2.579
	[11.75 to 11.91]	**A1**	or equiv. Accept 11.7 to 11.9
		4	
7(ii)	No because pop normal (so $\bar{X}$ normally distr)	**B1**	
		1	
7(iii)	11.7 not within CI	**B1FT**	
		1	
7(iv)	No because 95% CI is narrower than 99% CI	**B1**	OE
		1	
7(v)	Σx^2 (= 1399.67)	**M1**	attempted
	$\text{Est}(\sigma^2) = \dfrac{10}{9}\left(\dfrac{\text{"1399.67"}}{10} - \left(\dfrac{\text{"118.3"}}{10}\right)^2\right)$ OE	**M1**	correct sub of their Σs into correct formula
	= 0.0201 (3 sf) or 181/9000	**A1**	
		3	

Cambridge Assessment International Education
Cambridge International Advanced Level

MATHEMATICS **9709/71**

Paper 7 **October/November 2017**

MARK SCHEME

Maximum Mark: 50

Published

This document consists of **8** printed pages.

Mark Scheme Notes

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Question	Answer	Marks	Guidance
1	$\frac{40.5-31}{\sqrt{31}}$ (= 1.706)	**M1**	standn correct but allow with no or incorrect cc
	1 – ϕ(“1.706”)	**M1**	indep correct area consistent with working
	= 0.0441 (3 sf) or 0.0440	**A1**	not 0.044
		3	

Question	Answer	Marks	Guidance
2	Poisson	**B1**	seen or implied
	$\lambda = 4.03$	**B1**	seen or implied
	$e^{-4.03}(1 + 4.03 + \frac{4.03^2}{2!})$	**M1**	any λ; e.g. allow $\lambda = 4$ no extra or missing terms
	= 0.234 (3 sf)	**A1**	
		4	

Question	Answer	Marks	Guidance
3	$\frac{153}{200} + z \times \sqrt{\frac{\frac{153}{200} \times \frac{200-153}{200}}{200}} = 0.835$ (Var(P_s) = 0.000898875) (s.d. 0.02998)	**M1**	
	$z = 2.335$	**A1**	allow 2.33 or 2.34
	$2\Phi(z)-1$	**M1**	or equivalent method indep
	$\alpha = 98$	**A1**	allow 98.0 but not e.g. 98.04
		4	

Question	Answer	Marks	Guidance
4(i)	$300.1 \pm z \times \frac{0.9}{\sqrt{75}}$	**M1**	allow any value of z
	$z = 2.576$	**B1**	allow 2.574 to 2.579
	299.83 to 300.37 (2 dps)	**A1**	answer must be seen to 2 dps need an interval
		3	
4(ii)	CI includes 300 so claim supported or justified or probably true	**B1 FT**	or equivalent FT from CI in **(i)**
		1	

Question	Answer	Marks	Guidance
5(i)	$\frac{1}{4}\int_0^2 (x^2 + x)\,dx \qquad (= \frac{1}{4}\left[\frac{x^3}{3} + \frac{x^2}{2}\right]_0^2)$	**M1**	Attempt integ xf(x), ignore limits
	$= \frac{1}{4}(\frac{8}{3} + 2) \quad (-\ 0)$	**A1**	Subst correct limits in correct integration
	$= \frac{7}{6}$ OE or 1.17 (3 sf)	**A1**	
		3	
5(ii)	$\frac{1}{4}\int_0^m (x+1)\,dx = 0.5 \quad (= \frac{1}{4}\left[\frac{x^2}{2} + x\right]_0^m = 0.5)$	**M1**	attempt integ f(x), limits 0 to unknown (or unknown to 2) and = 0.5
	$\frac{1}{4}(\frac{m^2}{2} + m) = 0.5$ $m^2 + 2m - 4 = 0$ $m = \frac{-2 \pm \sqrt{4+16}}{2}$ OE	**A1**	a correct equation in m (any form) or $\sqrt{5} - 1$
	$m = 1.24$	**A1**	must reject the negative value if there
		3	

Question	Answer	Marks	Guidance
6(i)	Mean = 3.2 × 90 = 288	**B1**	
	Variance = $0.4^2 \times 90^2$	**M1**	
	= 1296	**A1**	
		3	
6(ii)	Mean = '288' + 4.3 × 95 = 696.5	**B1 FT**	
	Variance = '1296' + $0.6^2 \times 95^2$ = 4545	**B1 FT**	FT their **(i)**
	$\frac{670-696.5}{\sqrt{4545}}$ (= -0.393)	**M1**	FT Var provided both given Vars used standardising (ignore cc) no sd / Var mix
	1 – ϕ('–0.393') = ϕ('0.393')	**M1**	correct area consistent with their working (i.e. their mean)
	= 0.653 (3 sf)	**A1**	
		5	

Question	Answer	Marks	Guidance
7(i)	H_0: mean no. sales = 3.5	**B1**	or " ... = 0.7 (per day)"
	H_1: mean no. sales > 3.5	**M1**	allow 'λ' or 'μ' but not just 'mean'
	$P(X \geqslant 5) = 1 - e^{-3.5}(1 + 3.5 + \frac{3.5^2}{2!} + \frac{3.5^3}{3!} + \frac{3.5^4}{4!})$	**M1**	
	= 0.275	**A1**	allow 0.274
	Comp with 0.10	**M1**	valid comparison using Poisson
	No evidence (at 10%) to believe that sales per day have increased	**A1 FT**	correct conclusion FT no contradictions
		6	

Question	Answer	Marks	Guidance
7(ii)	$\lambda = 3.9$	**B1**	
	$e^{-3.9} \times \frac{3.9^2}{2!}$	**M1**	any λ ($\neq$ 0.7 or 0.6), single term
	= 0.154 (3 sf)	**A1**	
		3	

Question	Answer	Marks	Guidance
8(i)	$\bar{x} = 27/150$ (= 0.18)	**B1**	
	$s = \sqrt{\frac{150}{149}} \times \sqrt{\frac{5.01}{150} - 0.18^2}$ or variance (= 0.031729) (var = 3/2980 = 0.0010067)	**M1**	or var = 1/149(5.01 – 27.0^2/150)
	H_0: Pop mean = 0.185 H_1: Pop mean < 0.185	**B1**	allow just 'μ'
	$\frac{0.18 - 0.185}{\frac{'0.031729'}{\sqrt{150}}}$	**M1**	standardising, need $\sqrt{150}$
	= (–) 1.930 (3 sfs) or 1.93	**A1**	
	Comp with z = (–) 2.326	**M1**	consistent signs or using probs 0.0268 > 0.01 or 0.9732 < 0.99 or using x_{crit} 0.18 > 0.17897
	There is no evidence (at 1% level) that concentration with drug is less than without drug	**A1 FT**	conclusion FT no contradictions
		7	

Question	Answer	Marks	Guidance
8(ii)	$\frac{cv-0.185}{\frac{'0.031729'}{\sqrt{150}}}$ (= – 2.326)	**M1**	must use 0.185 and $\sqrt{150}$
	= 0.17897 or 0.179	**A1**	acceptance region (for H_0) is > 0.179
	$\frac{"0.17897"-0.175}{\frac{'0.031729'}{\sqrt{150}}}$ (=1.534)	**M1**	must use 0.175 and $\sqrt{150}$
	1 – ϕ(“1.534”)	**M1**	indep mark
	= 0.0625 (3 sf)	**A1**	Accept 0.0610 to 0.0628
		5	

Cambridge International AS & A Level

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Cambridge International Advanced Subsidiary and Advanced Level

MATHEMATICS **9709/72**

Paper 7 Paper 7 **October/November 2017**

MARK SCHEME

Maximum Mark: 50

Published

This document consists of **9** printed pages.

Cambridge Assessment International Education

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Question	Answer	Marks	Guidance
1(a)(i)	Po(2.54)	**M1**	seen or implied Po(2540 × 0.001)
	$1 - e^{-2.54}(1 + 2.54)$	**M1**	any λ Allow 1 end error
	= 0.721 (3 sf)	**A1**	
		3	
1(a)(ii)	n large and p small (or np (= 2.54) < 5)	**B1**	$n > 50$, $p < 0.1$
		1	
1(b)	$\mu = 5.6$	**B1**	
	$\sigma = 2.37$ (3 sf)	**B1**	Accept $\sqrt{5.6}$
		2	

Question	Answer	Marks	Guidance
2(i)	$4820 \pm z \times \frac{1420}{\sqrt{125}}$	**M1**	Must be a z value
	$z = 2.326$	**B1**	Accept 2.326 - 2.329
	4524/4525 to 5115/5116 or 4520 to 5120 (3 sf)	**A1**	Must be an interval
		3	

Question	Answer	Marks	Guidance
2(ii)	$\bar{x} = 4840$	**B1**	or width = 280 or half width = 140
	$4840 + 1.96 \times \frac{1420}{\sqrt{n}} = 4980$ OE	**M1**	or $140 = 1.96 \times \frac{1420}{\sqrt{n}}$ OE
	$n = 395$	**A1**	CAO must be an integer
		3	

Question	Answer	Marks	Guidance
3(i)	$\overline{m} = \frac{98.2}{100} = 0.982$	**B1**	Accept either
	$s = \sqrt{\frac{100}{99}} \times \sqrt{\frac{104.52}{100} - 0.982^2}$ (= 0.28582) or var = 0.08169	**M1**	
	H_0: Pop mean mass = 1.01 H_1: Pop mean mass < 1.01	**B1**	not just 'mean', but allow just 'μ'
	$\pm \frac{0.982 - 1.01}{\frac{0.28582}{\sqrt{100}}}$	**M1**	$\pm \frac{0.982 - 1.01}{\frac{0.284387}{\sqrt{100}}}$ **M1**
	= −0.980 (3 sf) accept ±	**A1**	= –0.985 (3 sfs) accept ± **A1**
	Comp with $z = -1.645$ (or areas 0.1635 > 0.05)	**M1**	Valid comparison of z's or area's
	No evidence that (mean) mass is less than 1.01	**A1 FT**	Correct conclusion FT their z
		7	

Question	Answer	Marks	Guidance
3(ii)	Distr of X normal (so distr of $\overline{X}$ normal) Must state or imply No	**B1**	X/parent population
		1	

Question	Answer	Marks	Guidance
4(i)	$k\int_0^a \frac{1}{\sqrt{x}}\mathrm{d}x = 1$	**M1**	Attempt int f(x) and = 1 ignore limits
	$(2k[x^{0.5}]_0^a = 1)$ $2ka^{0.5} = 1$ or $a = \frac{1}{4k^2}$	**A1**	OE; a correct eqn in k & a after sub limits
	$k\int_0^a \frac{x}{\sqrt{x}}\mathrm{d}x = 3$	**M1**	Attempt int xf(x) and = 3
	e.g. $\frac{2}{3}ka^{1.5} = 3$ or $a^3 = \frac{81}{4k^2}$	**A1**	OE; a correct eqn in k and a after sub limits
	e.g. $a^2 = 81$ or e.g. $k^2 = \frac{81}{4\times9^3}$	**M1**	Attempt eliminate one letter
	$a = 9$	**A1**	Convincingly obtained
	e.g. $k = \frac{9}{54}$ $k = \frac{1}{6}$ AG	**A1**	
		7	

Question	Answer	Marks	Guidance
4(ii)	$\frac{1}{6}\int_0^m \frac{1}{\sqrt{x}}\mathrm{d}x = 0.5$ OE	**M1**	Attempt int f(x), unknown limit and = 0.5
	$\frac{1}{3}m^{0.5} = 0.5$	**A1**	a correct equn in m after sub limits
	$m = 2.25$	**A1**	
		3	

Question	Answer	Marks	Guidance
5(i)	$\text{E}(X - Y) = 56\text{-}43$ (= 13)	**B1**	
	$\text{Var}(X - Y) = 6^2 + 5^2$ (= 61)	**M1**	
	$\frac{0-13}{\sqrt{'61'}}$ (= −1.664)	**M1**	Ignore any attempted cc/no SD/var mixes. var must be attempt at a combination
	$1 - \phi('-1.664') = \phi('1.664')$	**M1**	For area consistent with their working
	= 0.952 (3 sf)	**A1**	Similar scheme for use of $Y - X$
		5	

Question	Answer	Marks	Guidance
5(ii)	$E(M) = 56 + 1.5(43)$ (= 120.5)	**B1**	
	$\text{Var}(M) = 6^2 + 1.5^2 \times 5^2$ (= 92.25)	**M1**	
	$\frac{135-120.5}{\sqrt{'92.25}}$ (= 1.510)	**M1**	Ignore any attempted cc/no SD/var mixes. var must be attempt at a combination
	$1 - \phi('1.510')$	**M1**	For area consistent with their working
	= 0.0655 or 0.0656 or 6.55% or 6.56% (3 sf) As final answer	**A1**	Allow 6.6% or 6.5% or 7% if correct working seen
		5	

Question	Answer	Marks	Guidance
6(i)	H_0: Pop mean no. defectives = 5.15 H_1: Pop mean no. defectives < 5.15	**B1**	or '= 1.03 (per day)' not just 'mean', but allow just 'λ' or 'μ'
	$P(X \leqslant 2)$	**M1**	Attempted. Any one term error/end error/incorrect λ/expression 1–…
	$= e^{-5.15}\left(1 + 5.15 + \frac{5.15^2}{2}\right)$	**M1**	Correct expression attempted
	= 0.113	**A1**	
	Comp with 0.1	**M1**	Valid comparison
	No evidence to believe mean no. of defectives has decreased	**A1 FT**	Correct conclusion (FT their value) No contradictions
		6	

Question	Answer	Marks	Guidance
6(ii)	BOTH P($X \leqslant 1$) = $e^{-5.15}$ (1 + 5.15) (= 0.0357) AND P($X \leqslant 2$) = = $e^{-5.15}$ $(1 + 5.15 + \frac{5.15^2}{2})$= (0.113)	**B1***	(Could be seen in **(i)**)
	Comp either with 0.1	**DB1**	One comparison with 0.01 (could be seen in **(i)**)
	P(Type I error) = 0.0357 (3 sf)	**B1**	
		3	
6(iii)	Actually mean = 1.03 but conclude that mean < 1.03	**B1**	Mean no. of defectives not reduced, but conclude that it is reduced.
		1	

Cambridge International A Level

Cambridge Assessment International Education
Cambridge International Advanced Level

MATHEMATICS **9709/73**

Paper 7 **October/November 2017**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2017 series for most Cambridge IGCSE®, Cambridge International A and AS Level components and some Cambridge O Level components.

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Question	Answer	Marks	Guidance
1	$\frac{40.5-31}{\sqrt{31}}$ (= 1.706)	**M1**	standn correct but allow with no or incorrect cc
	1 – ϕ(“1.706”)	**M1**	indep correct area consistent with working
	= 0.0441 (3 sf) or 0.0440	**A1**	not 0.044
		3	

Question	Answer	Marks	Guidance
2	Poisson	**B1**	seen or implied
	$\lambda = 4.03$	**B1**	seen or implied
	$e^{-4.03}(1 + 4.03 + \frac{4.03^2}{2!})$	**M1**	any λ; e.g. allow $\lambda = 4$ no extra or missing terms
	= 0.234 (3 sf)	**A1**	
		4	

Question	Answer	Marks	Guidance
3	$\frac{153}{200} + z \times \sqrt{\frac{\frac{153}{200} \times \frac{200-153}{200}}{200}} = 0.835$ ($\text{Var}(P_s) = 0.000898875$) (s.d. 0.02998)	**M1**	
	$z = 2.335$	**A1**	allow 2.33 or 2.34
	$2\Phi(z) - 1$	**M1**	or equivalent method indep
	$\alpha = 98$	**A1**	allow 98.0 but not e.g. 98.04
		4	

Question	Answer	Marks	Guidance
4(i)	$300.1 \pm z \times \frac{0.9}{\sqrt{75}}$	**M1**	allow any value of z
	$z = 2.576$	**B1**	allow 2.574 to 2.579
	299.83 to 300.37 (2 dps)	**A1**	answer must be seen to 2 dps need an interval
		3	
4(ii)	CI includes 300 so claim supported or justified or probably true	**B1 FT**	or equivalent FT from CI in **(i)**
		1	

Question	Answer	Marks	Guidance
5(i)	$\frac{1}{4}\int_0^2 (x^2 + x)\,dx \quad (= \frac{1}{4}\left[\frac{x^3}{3} + \frac{x^2}{2}\right]_0^2)$	**M1**	Attempt integ $x\mathrm{f}(x)$, ignore limits
	$= \frac{1}{4}(\frac{8}{3} + 2) \quad (-\ 0)$	**A1**	Subst correct limits in correct integration
	$= \frac{7}{6}$ OE or 1.17 (3 sf)	**A1**	
		3	
5(ii)	$\frac{1}{4}\int_0^m (x+1)\,dx = 0.5 \quad (= \frac{1}{4}\left[\frac{x^2}{2} + x\right]_0^m = 0.5)$	**M1**	attempt integ f(x), limits 0 to unknown (or unknown to 2) and = 0.5
	$\frac{1}{4}(\frac{m^2}{2} + m) = 0.5$ $m^2 + 2m - 4 = 0$ $m = \frac{-2 \pm \sqrt{4+16}}{2}$ OE	**A1**	a correct equation in m (any form) or $\sqrt{5} - 1$
	$m = 1.24$	**A1**	must reject the negative value if there
		3	

Question	Answer	Marks	Guidance
6(i)	Mean = 3.2 × 90 = 288	**B1**	
	Variance = $0.4^2 \times 90^2$	**M1**	
	= 1296	**A1**	
		3	
6(ii)	Mean = ‘288’ + 4.3 × 95 = 696.5	**B1 FT**	
	Variance = ‘1296’ + $0.6^2 \times 95^2$ = 4545	**B1 FT**	FT their **(i)**
	$\frac{670-696.5}{\sqrt{4545}}$ (= -0.393)	**M1**	FT Var provided both given Vars used standardising (ignore cc) no sd / Var mix
	1 – ϕ(‘–0.393’) = ϕ(‘0.393)	**M1**	correct area consistent with their working (i.e. their mean)
	= 0.653 (3 sf)	**A1**	
		5	

Question	Answer	Marks	Guidance
7(i)	H_0: mean no. sales = 3.5	**B1**	or “ ... = 0.7 (per day)”
	H_1: mean no. sales > 3.5	**M1**	allow ‘λ’ or ‘μ’ but not just ‘mean’
	$P(X \geqslant 5) = 1 - e^{-3.5}(1 + 3.5 + \frac{3.5^2}{2!} + \frac{3.5^3}{3!} + \frac{3.5^4}{4!})$	**M1**	
	= 0.275	**A1**	allow 0.274
	Comp with 0.10	**M1**	valid comparison using Poisson
	No evidence (at 10%) to believe that sales per day have increased	**A1 FT**	correct conclusion FT no contradictions
		6	

Question	Answer	Marks	Guidance
7(ii)	$\lambda = 3.9$	**B1**	
	$e^{-3.9} \times \frac{3.9^2}{2!}$	**M1**	any λ ($\neq 0.7$ or 0.6), single term
	= 0.154 (3 sf)	**A1**	
		3	

Question	Answer	Marks	Guidance
8(i)	$\overline{x} = 27/150$ (= 0.18)	**B1**	
	$s = \sqrt{\frac{150}{149}} \times \sqrt{\frac{5.01}{150} - 0.18^2}$ or variance (= 0.031729) (var = 3/2980 = 0.0010067)	**M1**	or var = $1/149(5.01 - 27.0^2/150)$
	H_0: Pop mean = 0.185 H_1: Pop mean < 0.185	**B1**	allow just 'μ'
	$\frac{0.18 - 0.185}{\frac{'0.031729'}{\sqrt{150}}}$	**M1**	standardising, need $\sqrt{150}$
	= (–) 1.930 (3 sfs) or 1.93	**A1**	
	Comp with z = (–) 2.326	**M1**	consistent signs or using probs 0.0268 > 0.01 or 0.9732 < 0.99 or using x_{crit} 0.18 > 0.17897
	There is no evidence (at 1% level) that concentration with drug is less than without drug	**A1 FT**	conclusion FT no contradictions
		7	

Question	Answer	Marks	Guidance
8(ii)	$\frac{cv - 0.185}{\frac{'0.031729'}{\sqrt{150}}}$ (= – 2.326)	**M1**	must use 0.185 and $\sqrt{150}$
	= 0.17897 or 0.179	**A1**	acceptance region (for H_0) is > 0.179
	$\frac{"0.17897" - 0.175}{\frac{'0.031729'}{\sqrt{150}}}$ (=1.534)	**M1**	must use 0.175 and $\sqrt{150}$
	1 – ϕ("1.534")	**M1**	indep mark
	= 0.0625 (3 sf)	**A1**	Accept 0.0610 to 0.0628
		5	

Cambridge International A Level

Cambridge International Examinations
Cambridge International Advanced Level

MATHEMATICS **9709/71**

Paper 7 **May/June 2017**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

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Cambridge is publishing the mark schemes for the May/June 2017 series for most Cambridge IGCSE®, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

This document consists of **8** printed pages.

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Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

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    B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

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CWO	Correct Working Only – often written by a 'fortuitous' answer
ISW	Ignore Subsequent Working
SOI	Seen or implied
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR – 1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA – 1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Question	Answer	Marks	Guidance
1(i)	Poisson with $\lambda = 0.2$	**B1**	
	$1-e^{-0.2}\left(1+0.2+\frac{0.2^2}{2}\right)$	**M1**	1 – Poisson P(0, 1, 2, 3) attempted, any λ, allow one end error
	= 0.00115 (3 sf)	**A1**	SR: using Bin, ans 0.00115: **B1**
	Total:	**3**	
1(ii)	*n* large (n > 50)	**B1**	
	$np = 0.2 < 5$ or *p* small	**B1**	
	Total:	**2**	
2	Assume sd still = 3.8	**B1**	or sd unchanged
	H_0: $\mu = 64.0$ H_1: $\mu < 64.0$	**B1**	
	$\frac{63.3-64.0}{\frac{3.8}{\sqrt{100}}}$	**M1**	Standardising with their values (no sd / var mixes) Must have √100
	= –1.842	**A1**	
	comp "1.842" with *z*-value "1.842" < 1.96	**M1**	comp +ve with +ve or –ve with –ve or comp Φ ("1.842") with 0.975 0.9672 < 0.975 OE
	No evidence that heights are shorter	**A1FT**	OE FT their z_{calc}
	Total:	**6**	

Question	Answer	Marks	Guidance
3(a)	$7.1 \pm z \times \sqrt{\frac{2.6}{75}}$	**M1**	Expression of correct form must be z (note MR var = 2.6^2 can score **M1**) seen
	$z = 1.751$	**B1**	
	6.77 to 7.43 (3 sfs)	**A1**	Must be an interval
	Total:	**3**	
3(b)	0.04^3	**M1**	Allow 0.08^3 for **M1**
	= 0.000064	**A1**	
	Total:	**2**	
3(c)	e.g. Particular day or time of day	**B1**	Allow "Not random"
	Total:	**1**	
4(i)	Greater area where $x < 7.5$ than $x > 7.5$	**B1**	Allow Graph higher for $x < 7.5$ than for $x > 7.5$ or Graph decreasing or equiv expl'n
	Total:	**1**	
4(ii)	$\int_5^{10} \frac{k}{x^2}\,dx = 1$	**M1**	Attempt Integ f(x) = 1 ignore limits
	$k\left[-\frac{1}{x}\right]_5^{10} = 1$ $k \times \frac{1}{10} = 1$	**A1**	Correct integration and limits
	$k = 10$ AG	**A1**	No errors seen
	Total:	**3**	

Question	Answer	Marks	Guidance
4(iii)	$10\int_5^{10} \frac{1}{x}\ \mathrm{d}x$	**M1**	Attempt Integ $x\mathrm{f}(x)$ ignore limits
	$= 10\left[\ln x\right]_5^{10}$ $= 10(\ln 10 - \ln 5)$	**M1**	Correct integration and limits
	$= 10\ln 2$ or 6.93 (3 sf)	**A1**	OE
	Total:	**3**	
4(iv)	$10\int_5^{10} 1\ \mathrm{d}x - "6.93"^2$	**M1**	Attempt (Integ $x^2\mathrm{f}(x)$) – $(\mathrm{E}(x))^2$. No limits **M0**
	= 1.95 (accept 1.96)	**A1**	*Use of 6.93 gives 1.97* **A0**
	Total:	**2**	
5(i)	$W \sim \mathrm{N}(6210, 171.88)$	**B2**	seen or implied. **B1** each parameter
	$\frac{6200 - "6210"}{\sqrt{"171.88"}}$ (= – 0.763)	**M1**	Standardising with their values. No sd / var mix
	$1 - \Phi(“0.763”)$	**M1**	For area consistent with their mean
	= 0.223 (3 sfs)	**A1**	
	Total:	**5**	

Question	Answer	Marks	Guidance
5(ii)	E(C – 2B) = −50	**M1**	"6210"–2(3130) (or E(2B–C)=50
	Var(C – 2B) = "171.88" + $2^2 \times 12.1^2$ (= 757.52)	**M1**	
	$\frac{0-(-50)}{\sqrt{"757.52"}}$ (= 1.817)	**M1**	Standardising with their values
	Φ("1.817")	**M1**	For area consistent with their mean
	= 0.965 (3 sfs)	**A1**	
	Total:	**5**	
6(i)	mean = 6.6	**B1**	**B1** for 6.6 (could be scored in iii)
	$P(X \leqslant 1) = e^{-6.6}(1 + 6.6) = 0.0103$	**M1**	Allow incorrect λ in both probs
	$P(X \leqslant 2) = e^{-6.6}(1 + 6.6 + \frac{6.6^2}{2}) = 0.0400$	**M1A1**	**A1** for both values
	CR is $X \leqslant 1$	**DA1**	Dep on at least one **M**
	P(Type I error) = $P(X \leqslant 1) = 0.0103$	**B1FT**	FT their $P(X \leqslant 1)$
	Total:	**6**	
6(ii)	Wrongly concluding that (mean) no of (sports) injuries has decreased	**B1**	Must be in context
	Total:	**1**	

Question	Answer	Marks	Guidance
6(iii)	H_0: $\lambda = 6.6$ H_1: $\lambda < 6.6$	**B1**	Can be scored in (i). Allow μ or λ / 1.1 or 6.6 or $P(X \leqslant 2) = 0.0400 > 0.02$
	2 not in CR	**M1**	
	No evidence mean no. of injuries has decreased	**A1FT**	
	Total:	**3**	
6(iv)	N(39.6, 39.6)	**B1**	May be implied
	$\frac{29.5-39.6}{\sqrt{39.6}}$ (= −1.605)	**M1**	Allow with wrong or no cc
	Φ("–1.605") = 1 – Φ("1.605")	**M1**	For area consistent with their mean
	= 0.0543 (3 sfs)	**A1**	
	Total:	**4**	

Cambridge International Examinations
Cambridge International Advanced Level

MATHEMATICS **9709/72**

Paper 7 **May/June 2017**

MARK SCHEME

Maximum Mark: 50

Published

This document consists of **9** printed pages.

Mark Scheme Notes

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Question	Answer	Marks	Guidance
1	$\frac{0.801\times(1-0.801)}{2000}$ (= 0.0000797)	**M1**	
	$0.801\pm z\times\sqrt{"0.0000797"}$	**M1**	Allow any z-value
	$z = 1.96$	**B1**	
	0.784 to 0.818 (3 sf)	**A1**	As final answer. Must be an interval Allow 0.783 to 0.819
	Total:	**4**	
2(i)	E(X) = 4.197	**B1**	
	Var (X) = 4.196	**B1**	Both to 3dp or better
	Total:	**4**	
2(ii)	$\mathrm{E}(X)\approx\mathrm{Var}(X)$	**B1**	Condone =
2(iii)	$e^{-4.1968}\left(1+4.1968+\frac{4.1968^2}{2}+\frac{4.1968^3}{3!}+\frac{4.1968^4}{4!}\right)$	**M1**	Any λ. Allow with one end error
	= 0.59(0) (3 sfs)	**A1**	Allow 0.591
	Total:	**2**	

Question	Answer	Marks	Guidance
3(i)	Est (μ) = 923/400 or 2.3075 or 2.31 (3 sf)	**B1**	
	Est(σ^2) = $\frac{400}{399}\left(\frac{3170}{400}-"2.3075"^2\right)$ OE	**M1**	
	= 2.60696 or 2.61 (3 sf)	**A1**	(Note**:** Biased Var= 2.600 scores **M0**)
	Total:	**3**	
3(ii)	H_0: Pop mean (or μ) = "2.31" or "2310" H_1: Pop mean (or μ) > "2.31" or "2310"	**B1 FT**	
	$\pm\ \frac{2.6-"2.310"}{\sqrt{2.60696\div 50}} = 1.27$	**M1 A1**	Standardising using their values, Accept 1.28
	Comp 1.645 (OE)	**M1**	Valid comparison z values or areas
	No evidence that incomes in the region greater	**A1 FT**	OE FT their z. No contradictions (No FT for 2 tail test – max score **B0 M1 A1 M1** for comp 1.96 **A0**) **Note:** Accept alternative CV method
	Total:	**5**	

Question	Answer	Marks	Guidance
4(i)	$0.75^{20} + 20 \times 0.75^{19} \times 0.25 + {}^{20}C_2 \times 0.75^{18} \times 0.25^2$	**M1**	No end errors
	= 0.0913	**A1**	As final answer
	Total:	**2**	
4(ii)	H_0: Pop proportion=0.25 H_1: Pop proportion<0.25	**B1**	Allow *p* or π, not "proportion" (Accept anywhere in the question)
	$0.75^{25} + 25 \times 0.75^{24} \times 0.25$	**M1**	Must be B(25,0,25) No end errors
	= 0.00702	**A1**	
	comp 0.01	**M1**	Valid comparison
	There is evidence that the claim is not justified	**A1 FT**	OE. No contradictions
	Total:	**5**	

Question	Answer	Marks	Guidance
5(i)	$0.5\times1\times h=0.25$ $h = 0.5$ grad = 0.5	**M1**	$P(X < 2) = 4 \times P(X < 1)$ **M1**
	$f(x) = 0.5x$	**A1**	$P(X < 2) = 1$ **A1** $a = 2$ **A1**
	$0.5 \times a \times 0.5a = 1$	**M1**	$0.5 \times 2 \times h' = 1$ **M1** $h' = 1$
	$a = 2$	**A1**	grad = 0.5
	$P(X < 2) = 1$	**A1**	$f(x) = 0.5x$ **A1**
	Total:	**5**	
5(ii)	$\int_0^m 0.5x dx = 0.5$	**M1**	Attempt $\int f(x)dx = 0.5$ Ignore limits
	$=\left[\frac{x^2}{4}\right]_0^m = 0.5$	**A1FT**	Correct integration (ft f(x)) & limits = 0.5
	$m=\sqrt{2}$ or 1.41 (3 sf)	**A1**	or by similarity $m=\frac{1}{\sqrt{2}}\times 2$ **M2** $=\sqrt{2}$ **A1**
	Total:	**3**	

Question	Answer	Marks	Guidance
6(i)	$e^{-2.4} \times \frac{2.4^2}{2!}$	**M1**	Allow incorrect λ
	= 0.261 (3 sfs)	**A1**	
	Total:	**2**	
6(ii)	N(60, 60)	**B1**	seen or implied
	$\frac{54.5-60}{\sqrt{60}}$ (= –0.710)	**M1**	allow with wrong or missing cc
	1– ϕ("–0.710") = ϕ("0.710")	**M1**	For area consistent with their working
	= 0.761 (3 sf)	**A1**	
	Total:	**4**	
6(iii)	$\lambda = 3.6 + 12 \div 7$ (= 186/35) (= 5.314)	**M1**	
	$e^{-5.314}\left(1+5.314+\frac{5.314^2}{2}+\frac{5.314^3}{3!}\right)$	**M1**	Allow incorrect λ. Allow one end error.
	= 0.224 (3 sfs)	**A1**	
	Total:	**3**	

Question	Answer	Marks	Guidance
7(a)	$E(X_1+X_2) = 2 \times 4.2 = 8.4$ $Var(X_1+X_2) = 2 \times 1.1^2 = 2.42$	**B1**	Both. Seen or implied (or sd = 1.56)
	$\frac{10-8.4}{\sqrt{2.42}}$ (= 1.029)	**M1**	Standardising with their mean and var (no sd / var mix)
	$1 - \phi(\text{"1.029"})$	**M1**	For area consistent with their working
	= 0.152 (3 sf)	**A1**	
	Total:	**4**	
7(b)	$E(X) = 20.5$	**B1**	
	$Var(X) = 105 + 0.5^2 \times 15$ (= 108.75)	**M1**	correct expression oe
	$\frac{0-\text{"20.5"}}{\sqrt{\text{"108.75"}}}$ (= −1.966)	**M1**	correct standardisation using their E & V (no sd/var mix) ignore any attempted cc
	$\phi(\text{"−1.966"}) = 1 - \phi(\text{"1.966"})$ (= (1 − 0.9754))	**M1**	For area consistent with their working
	= 0.0246 or 2.46% (3 sf)	**A1**	Accept 0.0247
	Total:	**5**	

Cambridge International A Level

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Cambridge International Advanced Level

MATHEMATICS **9709/73**

Paper 7 **May/June 2017**

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Cambridge is publishing the mark schemes for the May/June 2017 series for most Cambridge IGCSE®, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

This document consists of **8** printed pages.

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International Examinations

Mark Scheme Notes

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The following abbreviations may be used in a mark scheme or used on the scripts:

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Question	Answer	Marks	Guidance
1	573, 43 (or 043), 289	**B1B1B1**	Ignore incorrect numbers. But allow other correct use of table (i.e. 573, 650, 431)
	Total:	**3**	
2(i)	$z = 1.751$	**B1**	
	$\frac{103}{200} \pm z\sqrt{\frac{\frac{103}{200}\times(1-\frac{103}{200})}{200}}$ oe	**M1**	all correct except for recognisable value of z, allow for one side only
	= 0.453 to 0.577 (3 sf) as final answer	**A1**	must be an interval
	Total:	**3**	
2(ii)	0.08 oe 8%, 8/100	**B1**	
3	10×0.46^2 (= 2.116) or $\frac{0.46}{\sqrt{10}}$	**B1**	SOI
	Total mass of ore ~ N(70, 2.116) or $\sim\text{N}\left(7, \left(\frac{0.46}{\sqrt{10}}\right)^2\right)$	**B1**	
	$\pm\frac{71-"70"}{\sqrt{"2.116"}}$ or $\pm\frac{7.1-"7.0"}{0.46/\sqrt{10}}$ (= 0.687)	**M1**	correct, using their sd or √(their var) e.g. allow $\frac{71-"70"}{4.6}$ for **M1**
	$1 - \phi("0.687")$	**M1**	for correct area consistent with their working
	= 0.246 (3 sf)	**A1**	
	Total:	**5**	

Question	Answer	Marks	Guidance
4(i)	$\overline{x} = 6.7/200\ (= 67/2000 = 0.0335)$	B1	
	$s^2 = \frac{200}{199} \times \left(\frac{0.2312}{200} - \text{"}0.0335\text{"}^2\right)$	M1	$s^2 = \frac{0.2312}{200} - 0.0335^2$ **M0**
	$= 0.0000339(2) = 27/796000$	A1	$= 0.00003375$ **A0**
	Total:	3	
4(ii)	H_0: Pop mean level = 0.034 H_1: Pop mean level ≠ 0.034	B1	not just "mean", but allow just "μ"
	$\frac{\text{"}030335\text{"} - 0.034}{\frac{\sqrt{\text{"}0.00003392\text{"}}}{\sqrt{200}}}$	M1	must have $\sqrt{200}$ $\frac{0.0335 - 0.034}{\frac{\sqrt{\text{"}0.00003375\text{"}}}{\sqrt{200}}}$ **M1**
	$= -1.21(4)$ (3 sfs) $(-1.22 \leftrightarrow -1.21)$	A1	$= -1.217$ (3 sfs) **A1**
	Comp with $z = -1.645$ (or $0.1124 > 0.05$)	M1	$0.112 > 0.05$ valid comparison z or areas
	No evidence that (mean) pollutant level has changed, accept H_0 (if correctly defined)	A1FT	correct conclusion no contradictions SR: One tail test: **B0**, **M1A1** as normal, **M1** (comparison with 1.282 consistent signs) **A0**
	Total:	5	

Question	Answer	Marks	Guidance
5(i)(a)	$X \sim \mathrm{N}(42, 42)$	**B1**	stated or implied
	$\frac{39.5 - "42"}{\sqrt{"42"}}$ (= −0.386)	**M1**	allow with wrong or no cc
	1 − ϕ ("−0.386") = ϕ ("0.386")	**M1**	correct area consistent with their working
	= 0.65(0) (3 sf)	**A1**	
	Total:	**4**	
5(i)(b)	42 > (e.g. 15) or mean is large	**B1**	$\lambda > 15$ or higher, λ = large ignore subsequent work if not undermining what already written
	Total:	**1**	
5(ii)(a)	$Y \sim \mathrm{Po}(1.2)$	**B1**	stated or implied
	$1 - e^{-1.2}(1 + 1.2 + \frac{1.2^2}{2})$	**M1**	allow any λ allow one end error
	= 0.121 (3 sf)	**A1**	Using binomial: 0.119 SR **B1**
	Total:	**3**	
5(ii)(b)	60 × 0.02 = 1.2 < 5 or mean is small	**B1FT**	or large *n* small *p* FT Poisson only
	Total:	**1**	

Question	Answer	Marks	Guidance
6(i)	$k\int_0^1 (x-x^2)\,\mathrm{d}x = 1$	**M1**	Attempt integ f(x) and "= 1", ignore limits
	$= k\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1 = 1$	**A1**	correct integration, limits 0 and 1
	$= k\left[\frac{1}{2} - \frac{1}{3}\right] = 1$ or $\frac{k}{6} = 1$	**A1**	correctly obtained, no errors seen
	Total:	**3**	
6(ii)	E(X) = 0.5	**B1**	
	$6\int_0^1 (x^3-x^4)\,\mathrm{d}x$	**M1**	Attempt integ x^2f(x), limits 0 to 1
	$(= 6\left[\frac{1}{4} - \frac{1}{5}\right] = 0.3)$ "0.3" – "0.5"^{2}	**M1**	their int x^2f(x) – their (E(X))^2 dep +ve result
	= 0.05(= 1/20)	**A1**	
	Total:	**4**	
6(iii)	$6\int_{0.4}^{1} (x-x^2)\,\mathrm{d}x$	**M1**	ignore limits, eg **M1**for $6\int_{0.4}^{2} (x-x^2)\ \mathrm{d}x$
	$= 6\{\frac{1}{2}-\frac{1}{3}-(\frac{0.4^2}{2}-\frac{0.4^3}{3})\}$	**A1FT**	subst correct limits into correct integration
	= 0.648(= 81/125)	**A1**	condone incorrect “k” for **A1**
	Total:	**3**	

Question	Answer	Marks	Guidance
7(i)	H_0: Pop mean no. accidents = 5.64 H_1: Pop mean no. accidents < 5.64	**B1**	or "= 0.47 (per month)" not just "mean", but allow just "λ" or "μ"
	Use of $\lambda = 5.64$	**B1**	used in a Poisson calculation
	$= e^{-5.64}\left(1 + 5.64 + \frac{5.64^2}{2}\right)$	**M1**	Allow incorrect λ in otherwise correct
	= 0.08(0)	**A1**	
	Comp with 0.05	**M1**	Valid comparison (Poisson only), no contradictions.
	No evidence to believe mean no. of accidents has decreased; accept H_0 (if correctly defined)	**A1FT**	Normal distribution: **M0M0**
	Total:	**6**	
7(ii)	Mean < 0.47 but conclude that this is not so	**B1**	(Mean) no. of accidents **reduced**, but conclude not reduced. Must be in context.
	Total:	**1**	
7(iii)	(Need greatest x such that $P(X \leqslant x) < 0.05$) $P(X \leqslant 1) = e^{-5.64}(1 + 5.64) = 0.024$ $P(X \leqslant 2) = 0.08$	**B1**	Both, could be seen in (i)
	Hence rejection region is $X \leqslant 1$	**B1**	Can be implied
	With $\lambda = 12\times 0.05 = 0.6$, $1 - P(X \leqslant 1) = 1 - e^{-0.6}(1+ 0.6)$	**M1**	λ=0.6 and $1 - P(X \leqslant 1)$
	= 0.122 (3 sf)	**A1**	Normal scores 0
	Total:	**4**	

Cambridge International Examinations
Cambridge International Advanced Level

MATHEMATICS **9709/72**

Paper 7 Probability and Statistics **March 2017**

MARK SCHEME

Maximum Mark: 50

Published

This document consists of **7** printed pages.

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Question	Answer	Marks	Guidance
1	Var(Ps) = $\frac{0.3(1-0.3)}{120}$ (= 0.00175)	**M1**	Attempt correct values in correct formula
	$0.3 \pm z\sqrt{\text{"}0.00175\text{"}}$	**M1**	must be a z-value, not a prob
	$z = 1.645$	**B1**	
	CI = 0.231 to 0.369 (3 sf)	**A1**	
	Total:	**4**	

Question	Answer	Marks	Guidance
2(i)	(H_1): $\mu \neq 6.4$	**B1**	
	Total:	**1**	
2(ii)	comp 2.43 with a z-value $z = 2.576$ AND	**M1**	oe valid comparison
	No evidence that μ is not 6.4 or do not reject $\mu = 6.4$	**A1**	Allow "Accept $\mu = 6.4$" Must mention μ, not just "H_0" or "H_1"
	Total:	**2**	
2(iii)	Testing for an increase in μ, or for a decrease in μ, rather than a change	**B1**	Any equiv statement
	Total:	**1**	

Question	Answer	Marks	Guidance
3(i)	$\frac{53-52}{6.1\div\sqrt{75}}$ (= 1.420)	**M1**	
	$\frac{51-52}{6.1\div\sqrt{75}}$ (= –1.420)	**M1**	or –"1.420" seen
	Φ("1.420") – Φ("–1.420")	**M1**	
	= 0.844 (3 sfs)	**A1**	
	Total:	**4**	
3(ii)	Need to assume $\bar{X}$ (approx.) normally distributed	**B1**	or X not stated to be normally distributed
	Total:	**1**	

Question	Answer	Marks	Guidance
4(i)	$(\lambda =)$ 4.5	**B1**	
	$e^{-4.5}(1 + 4.5 + \frac{4.5^2}{2!})$	**M1**	Allow any λ. Allow one end error
	= 0.174	**A1**	
	Total:	**3**	
4(ii)	Accept reduction in mean no. of missed appts although untrue	**B1**	or Mean is 0.9 (or 4.5) but < 3 missed appts. In context
	Total:	**1**	
4(iii)	$P(X \geqslant 3)$	**M1**	Attempted
	$= 1 - e^{-1}(1 + 1 + \frac{1^2}{2!})$	**M1**	Allow any λ except 4.5 or 0.9, Allow one end error
	= 0.0803 (3 sfs)	**A1**	
	Total:	**3**	

Question	Answer	Marks	Guidance
5(a)(i)	$k = 1$	**B1**	
	Total:	**1**	
5(a)(ii)	f_2: area > 1 (area ≠ 1)	**B1**	oe
	f_3: includes negative values of f_3	**B1**	oe
	Total:	**2**	
5(b)(i)	$6\int_{-a}^{a}(a^2 - x^2)dx = 1$	**M1**	Integ f(x) = 1, ignore limits
	$6[a^2x - \frac{x^3}{3}]_{-a}^{a} = 1$	**A1**	Correct integral and limits
	$6(2a^3 - \frac{2a^3}{3}) = 1$ $\frac{24a^3}{3} = 1$ or $8a^3 = 1$ $a = 1/2$ **AG**	**A1**	Correctly obtained. No errors seen. (SR Verification scores M1A1 only max 2/3)
	Total:	**3**	

Question	Answer	Marks	Guidance
5(b)(ii)	0	**B1**	
	Total:	**1**	
5(b)(iii)	$6\int_{-0.5}^{0.5}(\frac{x^2}{4}-x^4)\mathrm{d}x$ $(= 6\left[\frac{x^3}{12}-\frac{x^5}{5}\right]_{-0.5}^{0.5} = 0.05)$ Var = $0.05 - 0^2$	**M1**	attempt int x^2f(x) & correct limits
	= 0.05 oe	**A1**	cao; allow omission of -0^2
	Total:	**2**	

Question	Answer	Marks	Guidance
6(i)	Assume cartons are random sample(s)	**B1**	or masses of cartons are independent of each other oe
	E(T) = 816.4 Var(T) = 1570.08	**B1**	Both
	$z = \frac{900-\text{"816.4"}}{\sqrt{\text{"1570.08"}}}$ (= 2.110)	**M1**	
	1 – Φ(“2.110”)	**M1**	
	= 0.0174 = 1.74% (3 sfs)	**A1**	% only (accept 1.7% if 0.0174 seen)
	Total:	**5**	
6(ii)	P($F - S > 0$) stated or implied	**M1**	P($S - F < 0$)
	62.0 – 78.8 (= –16.8) & $10.0^2 + 12.6^2$ (= 258.76)	**B1**	78.8 – 62.0 (= 16.8) & $12.6^2 + 10.0^2$ (= 258.76)
	$z = \frac{0-(\text{"}-16.8\text{"})}{\sqrt{\text{"258.76"}}}$ (= 1.044)	**M1**	$z = \frac{0-\text{"16.8"}}{\sqrt{\text{"258.76"}}}$ (= –1.044)
	1 – Φ(“1.044”)	**M1**	Φ(“–1.044”) = 1 – Φ(“1.044”)
	(= 1 – 0.8517) = 0.148 (3 sfs)	**A1**	
	Total:	**5**	

Question	Answer	Marks	Guidance
7(i)	Planes arrive at constant mean rate	**B1**	
	Planes arrive at random	**B1**	or Planes arrive independently Must be in context
	Total:	**2**	
7(ii)(a)	(λ =) 5.2 ÷ 4	**M1**	
	$e^{-1.3}(\frac{1.3^2}{2}+\frac{1.3^3}{3!})$	**M1**	Allow any λ , allow one end error
	= 0.330 (3 sfs)	**A1**	Accept 0.33
	Total:	**3**	
7(ii)(b)	$1-e^{-3.467}\times(1+3.467+\frac{3.467^2}{2!}+\frac{3.467^3}{3!})$	**M1**	Allow any λ except 5.2 or 1.3, allow one end error
	= 0.456 (3 sfs)	**A1**	
	Total:	**2**	
7(iii)	N(52, 52) stated or implied	**B1**	
	$\frac{60.5-52}{\sqrt{52}}$ (= 1.179)	**M1**	ft their mean and var. Allow wrong or no cc or no √
	Φ("1.179")	**M1**	
	= 0.881 (3 sf)	**A1**	
	Total:	**4**	

Cambridge International Examinations
Cambridge International Advanced Level

MATHEMATICS **9709/71**

Paper 7 **October/November 2016**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

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PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

1	$\left(\frac{508}{8}\right) = 63.5$	**B1**		
	$(\Sigma x^2 = 32360.12)$			
	$\frac{8}{7}\left(\frac{'32360.12'}{8} - '63.5'^2\right)$	**M1**		oe
	= 14.6 (3 sf) or 2553/175	**A1**	[3]	From correct working
2 (i)	$H_0: P(6) = {}^1/_6 \quad H_1: P(6) < {}^1/_6$	**B1**	[1]	Allow $H_0: p = {}^1/_6 \quad H_1: p < {}^1/_6$
(ii)	$\left(\frac{5}{6}\right)^{15}$	**M1**		
	$= 0.065 > 0.05$	**A1**	[2]	Correct result and comparison needed for A1 SR if 2 tail test followed allow A1 for $0.065 > 0.025$
(iii)	$\left(\frac{5}{6}\right)^{16} = 0.054$ and $\left(\frac{5}{6}\right)^{17} = 0.045$	**M1**		both
	Smallest n is 17	**A1**	[2]	No errors seen
	OR $\left(\frac{5}{6}\right)^n < 0.05$ and attempt to solve $n\ln\left(\frac{5}{6}\right) < \ln 0.05$	**M1**		
	smallest n is 17	**A1**		
3 (i)	$(\lambda) = 3.6 \div 3 = 1.2$	**B1**		1.2 seen
	$1 - e^{-1.2}\left(1 + 1.2 + \frac{1.2^2}{2} + \frac{1.2^3}{3!}\right)$	**M1**		Allow any λ
	= 0.0338 (3 sf)	**A1**	[3]	As final answer
(ii)	N(60 × 3.6, 60 × 3.6)	**M1**		Stated or implied
	$\frac{240.5 - '216'}{\sqrt{'216'}}$ (= 1.667)	**M1**		Allow with no or wrong cc (no sd/var mixes)
	1– Φ(‘1.667’)	**M1**		Area consistent with their working
	= 0.0478 (3 sf)	**A1**	[4]	SR use of Poisson 0.0497 scores 4/4
4 (i)	6080 (litres)	**B1**		
	106 (litres)	**B1**	[2]	
(ii)	$E(21Y - 2X) = 635$	**B1**		
	$Var(21Y - 2X) =$ $21^2 \times 12^2 + 2^2 \times 53^2$ (= 74740)	**B1**		correct expression or result or sd = 273 seen
	$\frac{0 - 635}{\sqrt{'74740'}}$ (= –2.323)	**M1**		no sd/var mixes
	1–Φ(‘–2.323’) = Φ(‘2.323’)	**M1**		Area consistent with their working
	= 0.99(0) (3 sf)	**A1**	[5]	No errors seen
5 (a)	$63 \pm z \times \frac{9}{\sqrt{100}}$	**M1**	B1	Expression of correct form, any z
	$z = 1.645$	**B1**		Seen
	61.5 to 64.5 (3 sf)	**A1**	[3]	Must be an interval

(b) (i)	$z = \frac{1.96}{2}$ (= 0.98) Φ("0.98") (= 0.8365) "0.8365" – (1 – "0.8365") (= 0.673) $\alpha = 67.3$ (3 sf)	**M1** **M1** **A1**	[3]	Allow $\frac{\text{any } z}{2}$ Allow 67 from correct working
(ii)	4=(2x'z'x'σ')/√n $n = 200$	**M1** **A1**	[2]	Attempt to solve equ of correct form SR B1 for $n = 100$
6 (i)	m_X, m_Y, m_Z, m_W or *X, Y, Z, W*	**B2**	[2]	B1 if two adjacent means interchanged, i.e. m_Y, m_X, m_Z, m_W or m_X, m_Z, m_Y, m_W or m_X, m_Y, m_W, m_Z B1 for correct order reversed.
(ii) (a)	$\int_0^3 \frac{4}{81}x^4\mathrm{d}x$ $= \left[\frac{4}{81}\frac{x^5}{5}\right]_0^3$ $= \frac{4}{81}\times\frac{3^5}{5}$ or $\frac{4}{81}\times\frac{243}{5}$ or $\frac{972}{405}$ oe $= \frac{12}{5}$ or 2.4 AG	**M1** **A1** **A1**	[3]	Attempt int xf(x). Ignore limits Correct integration and limits (condone missing 4/81) Must see correct expression as well as $\frac{12}{5}$ or 2.4 No errors seen
(b)	$\int_{2.4}^3 \frac{4}{81}x^3\mathrm{d}x$ or $1 - \int_0^{2.4} \frac{4}{81}x^3\mathrm{d}x$ $= \left[\frac{4}{81}\frac{x^4}{4}\right]_{2.4}^3$ or $1 - \left[\frac{4}{81}\frac{x^4}{4}\right]_0^{2.4}$ $= 1 - \frac{4}{81}\times\frac{2.4^4}{4}$ oe $= \frac{369}{625}$ or 0.59(0) (3 sf)	**M1** **A1** **A1**	[3]	Attempt int f(x) ignore limits Correct integration and limits (condone missing 4/81) As final answer
(c)	1	**B1**	[1]	

7 (i)	H_0: Pop mean time (or μ) = 20.5 H_1: Pop mean time (or μ) < 20.5	**B1**		Not just "mean"
	$\frac{20.3-20.5}{1.2\div\sqrt{100}}$	**M1**		Allow without √ sign (accept ±1.667/1.67)
	= –1.667 or 0.0478/0.952 if areas compared	**A1**		
	'1.667' < 1.751 (or '–1.667' > –1.751)	**M1**		Correct comparison of their z_{calc} with 1.751/1.75 oe valid comparison of areas (0.0478 > 0.04)
	No evidence that (pop) mean time has decreased	**A1ft**	[5]	No contradictions (ft their z)
(ii)	$\frac{cv-20.5}{1.2\div\sqrt{100}} = -1.751$	**M1***		
	cv = 20.29 or 20.3	**A1**		
	$\frac{'20.29'-20.1}{1.2\div\sqrt{100}}$ (= 1.583 or 1.582)	**DM1**		Allow $\frac{20.3-20.1}{1.2\div\sqrt{100}}$ (= 1.667) M1
	1 – Φ('1.583')	**M1**		1 – Φ('1.667') M1
	= 0.0567 – 0.0569 (3 sf)	**A1**	[5]	= 0.0478 (3 sf) A1
(iii)	Concluding (mean) time not decreased when in fact it has.	**B1**	[1]	Must be in context oe

Cambridge International Examinations
Cambridge International Advanced Level

MATHEMATICS **9709/72**

Paper 7 **October/November 2016**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the October/November 2016 series for most Cambridge IGCSE®, Cambridge International A and AS Level components and some Cambridge O Level components.

This document consists of **6** printed pages.

[Turn over

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

  - Note: B2 or A2 means that the candidate can earn 2 or 0.
    B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
CWO	Correct Working Only – often written by a 'fortuitous' answer
ISW	Ignore Subsequent Working
SOI	Seen or implied
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

1	$\left(\frac{508}{8}\right) = 63.5$	**B1**		
	($\Sigma x^2 = 32360.12$)			
	$\frac{8}{7}\left(\frac{'32360.12'}{8} - '63.5'^2\right)$	**M1**		oe
	= 14.6 (3 sf) or 2553/175	**A1**	[3]	From correct working
2 (i)	H_0: P(6) = $^1/_6$ H_1: P(6) < $^1/_6$	**B1**	[1]	Allow H_0: $p = {}^1/_6$ H_1: $p < {}^1/_6$
(ii)	$\left(\frac{5}{6}\right)^{15}$	**M1**		
	= 0.065 > 0.05	**A1**	[2]	Correct result and comparison needed for A1 SR if 2 tail test followed allow A1 for 0.065 > 0.025
(iii)	$\left(\frac{5}{6}\right)^{16} = 0.054$ and $\left(\frac{5}{6}\right)^{17} = 0.045$	**M1**		both
	Smallest *n* is 17	**A1**	[2]	No errors seen
	OR			
	$\left(\frac{5}{6}\right)^{n} < 0.05$ and attempt to solve	**M1**		
	$n\ln\left(\frac{5}{6}\right) < \ln 0.05$			
	smallest *n* is 17	**A1**		
3 (i)	(λ) = 3.6 ÷ 3 = 1.2	**B1**		1.2 seen
	$1 - e^{-1.2}\left(1 + 1.2 + \frac{1.2^2}{2} + \frac{1.2^3}{3!}\right)$	**M1**		Allow any λ
	= 0.0338 (3 sf)	**A1**	[3]	As final answer
(ii)	N(60 × 3.6, 60 × 3.6)	**M1**		Stated or implied
	$\frac{240.5 - '216'}{\sqrt{'216'}}$ (= 1.667)	**M1**		Allow with no or wrong cc (no sd/var mixes)
	1– Φ(‘1.667’)	**M1**		Area consistent with their working
	= 0.0478 (3 sf)	**A1**	[4]	
				SR use of Poisson 0.0497 scores 4/4
4 (i)	6080 (litres)	**B1**		
	106 (litres)	**B1**	[2]	
(ii)	E(21*Y* – 2*X*) = 635	**B1**		
	Var(21*Y* – 2*X*) = $21^2 \times 12^2 + 2^2 \times 53^2$ (= 74740)	**B1**		correct expression or result or sd = 273 seen
	$\frac{0 - 635}{\sqrt{'74740'}}$ (= –2.323)	**M1**		no sd/var mixes
	1–Φ(‘–2.323’) = Φ(‘2.323’)	**M1**		Area consistent with their working
	= 0.99(0) (3 sf)	**A1**	[5]	No errors seen
5 (a)	$63 \pm z \times \frac{9}{\sqrt{100}}$	**M1**	B1	Expression of correct form, any *z*
	z = 1.645	**B1**		Seen
	61.5 to 64.5 (3 sf)	**A1**	[3]	Must be an interval

(b) (i)	$z = \frac{1.96}{2}$ (= 0.98) Φ("0.98") (= 0.8365) "0.8365" – (1 – "0.8365") (= 0.673) $\alpha = 67.3$ (3 sf)	**M1** **M1** **A1**	[3]	Allow $\frac{\text{any } z}{2}$ Allow 67 from correct working
(ii)	4=(2x'z'x'σ')/√n $n = 200$	**M1** **A1**	[2]	Attempt to solve equ of correct form SR B1 for $n = 100$
6 (i)	m_X, m_Y, m_Z, m_W or X, Y, Z, W	**B2**	[2]	B1 if two adjacent means interchanged, i.e. m_Y, m_X, m_Z, m_W or m_X, m_Z, m_Y, m_W or m_X, m_Y, m_W, m_Z B1 for correct order reversed.
(ii) (a)	$\int_0^3 \frac{4}{81}x^4\mathrm{d}x$ $= \left[\frac{4}{81}\frac{x^5}{5}\right]_0^3$ $= \frac{4}{81}\times\frac{3^5}{5}$ or $\frac{4}{81}\times\frac{243}{5}$ or $\frac{972}{405}$ oe $= \frac{12}{5}$ or 2.4 AG	**M1** **A1** **A1**	[3]	Attempt int xf(x). Ignore limits Correct integration and limits (condone missing 4/81) Must see correct expression as well as $\frac{12}{5}$ or 2.4 No errors seen
(b)	$\int_{2.4}^3 \frac{4}{81}x^3\mathrm{d}x$ or $1 - \int_0^{2.4} \frac{4}{81}x^3\mathrm{d}x$ $= \left[\frac{4}{81}\frac{x^4}{4}\right]_{2.4}^3$ or $1 - \left[\frac{4}{81}\frac{x^4}{4}\right]_0^{2.4}$ $= 1 - \frac{4}{81}\times\frac{2.4^4}{4}$ oe $= \frac{369}{625}$ or 0.59(0) (3 sf)	**M1** **A1** **A1**	[3]	Attempt int f(x) ignore limits Correct integration and limits (condone missing 4/81) As final answer
(c)	1	**B1**	[1]	

7 (i)	H_0: Pop mean time (or μ) = 20.5 H_1: Pop mean time (or μ) < 20.5	**B1**		Not just "mean"
	$\frac{20.3-20.5}{1.2\div\sqrt{100}}$	**M1**		Allow without √ sign (accept ±1.667/1.67)
	= –1.667 or 0.0478/0.952 if areas compared	**A1**		
	'1.667' < 1.751 (or '–1.667' > –1.751)	**M1**		Correct comparison of their z_{calc} with 1.751/1.75 oe valid comparison of areas (0.0478 > 0.04)
	No evidence that (pop) mean time has decreased	**A1ft**	[5]	No contradictions (ft their z)
(ii)	$\frac{cv-20.5}{1.2\div\sqrt{100}} = -1.751$	**M1***		
	cv = 20.29 or 20.3	**A1**		
	$\frac{'20.29'-20.1}{1.2\div\sqrt{100}}$ (= 1.583 or 1.582)	**DM1**		Allow $\frac{20.3-20.1}{1.2\div\sqrt{100}}$ (= 1.667) M1
	1 – Φ('1.583') = 0.0567 – 0.0569 (3 sf)	**M1** **A1**	[5]	1 – Φ('1.667') M1 = 0.0478 (3 sf) A1
(iii)	Concluding (mean) time not decreased when in fact it has.	**B1**	[1]	Must be in context oe

Cambridge International Examinations
Cambridge International Advanced Level

MATHEMATICS **9709/73**

Paper 7 **October/November 2016**

MARK SCHEME

Maximum Mark: 50

Published

This document consists of **6** printed pages.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

  - Note: B2 or A2 means that the candidate can earn 2 or 0.
    B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

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SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become “follow through √ ” marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

1	$e^{-3.5}(1 + 3.5 + \frac{3.5^2}{2!})$ $= 0.321$ (3 sf)	**M2** **A1**	[3]	Allow M1 if extra term $e^{-3.5} \times \frac{3.5^3}{3!}$ or " 1 - . " or omit P(0)
2 (i)	59	**B1**	[1]	
(ii)	Any x such that $0.687 \leqslant x \leqslant 0.693$ (3 sf)	**B1**	[1]	or 0.69 or " ... 0.686.. < 0.693 rec "
(iii)	Possible repeats	**B1**	[1]	
3	N(178, …) Var = $3.2^2 + 4.1^2 + 3.8^2$ or 41.49 $\frac{175-'178'}{\sqrt{'41.49'} \div \sqrt{15}}$ (= '–1.804') Φ('–1.804') = 1 – Φ ('1.804') = 0.0356 (3 sf)	**B1** **B1** **M1** **M1** **A1**	[5]	stated or implied or sd = 6.44 stated or implied need √15 but allow var / sd mix for "41.49" allow cc for method marks independent M1 for area / prob consistent with working
4	$\frac{11.8-11}{1.6 \div \sqrt{n}} = 1.645$ $\frac{11.8-11}{1.6 \div \sqrt{n}} = 1.96$ $n = 10.8$ (allow 11) $n = 15.4$ (allow 15) Possible values are 11, 12, 13, 14, 15	**M1** **B1** **B1** **A1** **A1**	[5]	M1 for $\frac{11.8-11}{1.6 \div \sqrt{n}}$ = any z allow var / sd mix for 1.6 but need $\sqrt{n}$ B1 for each correct z for both not for just $11 \leqslant n \leqslant 15$ oe

5 (i)	H$_0$: P(free gift) = 0.3 or $p = 0.3$ H$_1$: P(free gift) < 0.3 or $p < 0.3$	**B1**	[1]	
(ii)	$P(X \leqslant 2) =$ $0.7^{20} + 20 \times 0.7^{19} \times 0.3 + {}^{20}C_2 \times 0.7^{18} \times 0.3^2$	**M1***		$P(X \leqslant 2)$ attempted
	$= 0.03548$ or 0.0355	**A1**		
	$P(X \leqslant 3) =$ '0.03548' + ${}^{20}C_3 \times 0.7^{17} \times 0.3^3$ (= 0.107)	**M1***		$P(X \leqslant 3)$ attempted
	One comparison with 0.05 seen	**M1***		or implied by fully correct methods for $P(X \leqslant 2)$ and $P(X \leqslant 3)$
	P(Type I error) = 0.0355 (3 sf)	**DA1** √	[5]	dep on all 3 Ms
(iii)	$P(X \leqslant 3)$ = '0.107' '0.107' > 0.05 **or** cv = 2 and compare 3 >2	**M1**		Compare their $P(X \leqslant 3)$ with 0.05
	No evidence to reject claim oe	**A1** √	[2]	No evidence that 30% is not correct oe ft their 0.107
6 (i)	est(μ) = 3.4	**B1**		
	est(σ^2)$= \frac{100}{99}\left(\frac{1356}{100} - '3.4'^2\right)$	**M1**		$1 / 99\ (1356 - 340^2/100\ )$ or 200/99
	= 2.02(0202)	**A1**		
	$z = 1.96$	**B1**		
	$3.4 \pm z \times \sqrt{\frac{'2.020202'}{100}}$	**M1**		correct working only
	= 3.12 to 3.68 (3 sf)	**A1**	[6]	allow from unbiased or biased variance
(ii)	Mean should be 3	**B1***		stated or implied
	CI does not include 3 Machine probably not working properly	**DB1**√	[2]	√ their CI or evidence that….
7 (i)	$1 - e^{-1}\ (1 + 1)$ (= 0.26424)	**B1**		B1 for either λ correct.
	$1 - e^{-1.5}\,(1 + 1.5 + \frac{1.5^2}{2!})$ (= 0.19115)	**B1**		B1 for either correct expression with correct λ
	'0.26424' × '0.19115'	**M1**		product of their values for $\leqslant 2$ and $\leqslant 3$ from Poisson, need correct form " 1 - .. " , but allow incorrect λ values and end errors
	= 0.0505 (3 sf)	**A1**	[4]	accept 0.0504

(ii)	λ = 30	**B1**		
	N(30, 30)	**B1√**		seen or implied, need N(λ,λ)
	$\frac{35.5-30}{\sqrt{30}}$ (= 1.004)	**M1**		allow with wrong or no cc or no √
	Φ ('1.004')	**M1**		consistent with their working
	= 0.842 (3 sf)	**A1**	[5]	
8 (i)	σ_X, σ_Z, σ_Y, σ_W **or** X, Z, Y, W	**B2**	[2]	B1 if two adjacent sds interchanged, ie σ_Z, σ_X, σ_Y, σ_W **or** σ_X, σ_Y, σ_Z, σ_W **or** σ_X, σ_Z, σ_W, σ_Y B1 for correct order reversed
(ii) (a)	Mean = 0 stated or found or "– 0" seen	**B1**		
	$\frac{1}{18}\int_{-3}^{3} x^4 \mathrm{d}x - 0$ $= \frac{1}{18}\left[\frac{x^5}{5}\right]_{-3}^{3}$ $= \frac{1}{18}\left[\frac{3^5}{5}+\frac{3^5}{5}\right]$ oe = 5.4	**M1**		Attempt integral ^{2} f(x). Ignore limits Allow without "– 0"
	sd = √5.4 or $\sqrt{\frac{1}{18}\left[\frac{3^5}{5}+\frac{3^5}{5}\right]}$ or 2.324 sd = 2.32 (3 sf) AG	**A1**	[3]	Must see $\sqrt{\text{correct expression or 5.4}}$ or 2.324 or better
(b)	$\frac{1}{18}\int_{'2.324'}^{3} x^2 \mathrm{d}x$	**M1**		Attempt to integrate f(x), ignore limits
	$\frac{1}{18}\left[\frac{x^3}{3}\right]_{'2.324'}^{3} = \frac{1}{18}\left[\frac{3^3}{3}-\frac{'2.324'^3}{3}\right]$	**A1**		Sub correct limits into correct integral
	= 0.268 (3 sf)	**A1**	[3]	Allow 0.269
(c)	0	**B1**	[1]	

Cambridge International Examinations
Cambridge International Advanced Level

MATHEMATICS **9709/71**

Paper 7 **May/June 2016**

MARK SCHEME

Maximum Mark: 50

Published

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B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

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  B2 / 1 / 0 means that the candidate can earn anything from 0 to 2.

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AEF	Any Equivalent Form (of answer is equally acceptable)
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BOD	Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only – often written by a 'fortuitous' answer
ISW	Ignore Subsequent Working
MR	Misread
PA	Premature Approximation (resulting in basically correct work that is insufficiently accurate)
SOS	See Other Solution (the candidate makes a better attempt at the same question)
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become 'follow through √' marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Note: '(3 sfs)' means 'answer which rounds to … to 3 sfs'. If correct ans seen to > 3sfs, ISW for later rounding. Penalise < 3 sfs only once in paper.

1	$B(200, \frac{1}{6}) \rightarrow N(\frac{100}{3}, \frac{250}{9})$	**B1**	seen or implied
	$\frac{25.5-\frac{100}{3}}{\sqrt{\frac{250}{9}}}$	**M1**	allow with wrong or no cc
	$= -1.486$	**A1**	(Accept alternative correct methods)
	comp '1.486' with 1.282	**M1**	or comp ('1.486') with 0.1
	Evidence to reject H_0 There is some evidence that $p < \frac{1}{6}$ or, e.g. It is likely that $p < \frac{1}{6}$ oe	**A1 ft** [5]	No contradictions
2 **(i)**	Each employee has an equal chance of being chosen	**B1** [1]	oe
(ii)	Est $(\mu) = 4$	**B1**	
	Est $(\sigma^2) = \frac{10}{9}(\frac{199.22}{10} - '4'^2)$	**M1**	sub in correct formula attempted
	= 4.36 (3 sf)	**A1** [3]	working may not be seen
(iii)	Distances travelled by all employees at the firm	**B1** [1]	oe
3 **(i)**	((0.5672 + 0.6528) ÷ 2) = 0.61	**B1** [1]	
(ii)	$'0.61' + z\sqrt{\frac{'0.61'\times(1-'0.61')}{350}} = 0.6528$	**M1**	oe
	$z = 0.0428 \times \sqrt{\frac{700}{'0.61'\times(1-'0.61')}}$ oe	**M1**	correct rearrangement of correct equn, ft '0.61'
	= 2.321	**A1**	
	98% confidence	**A1 ft** [4]	ft their *z* (dep on both Ms)

4	**(i)**	H_0: $\mu = 12.5$ H_1: $\mu \neq 12.5$	**B1**		
		$\frac{13.5-12.5}{4.2\div\sqrt{50}}$	**M1**		allow 4.2 ÷ 50
		= 1.68(4)	**A1**		
		‘1.684’ < 1.96	**M1**		comp 1.96 allow comp 1.645 if H1: μ > 12.5 or comp 1 – (‘1.684’) with 0.025
		No evidence that mean time has changed	**A1 ft**	[5]	No contradictions ft their 1.684, but not comp 1.645
	(ii)	0.05	**B1**	[1]	
5	**(i)**	T ~ N(6×2.4, 6×0.3^2) (= N(14.4, 0.54)	**M1**		seen or implied
		$\frac{16-'14.4'}{\sqrt{'0.54'}}$ (= 2.177)	**M1**		ft their E and Var; allow without √ (Accept alternative method N(2.4,(0.3²)/6))
		1 – (‘2.177’)	**M1**		correct area consistent with their working
		= 0.0147 (3 sf)	**A1**	[4]	
	(ii)	$D = X_1 - 1.1X_2$ E(D) = –0.24	**B1**		
		Var(D) = $0.3^2 + 1.1^2 \times 0.3^2$ (= 0.1989)	**M1**		
		$\frac{0-(-0.24)}{\sqrt{'0.1989'}}$ (= 0.538)	**M1**		ft their E and Var; allow without √
		(‘0.538’)	**M1**		correct area consistent with their working
		= 0.705 (3 sf)	**A1**	[5]	
6	**(i)**	2 m	**B1**	[1]	allow without units
	(ii)	$k\int_0^2 x^2(2-x)\mathrm{d}x = 1$	**M1**		attempt integ f(x) and ‘= 1’. Ignore limits
		$k\left[\frac{2x^3}{3} - \frac{x^4}{4}\right]_0^2$	**A1**		correct integration and limits
		$k \times \left[\frac{16}{3} - 4\right] = 1$ or $k \times \frac{4}{3} = 1$ oe			
		$k = \frac{3}{4}$ **AG**	**A1**	[3]	No errors seen
	(iii)	$\frac{3}{4}\int_0^2 x^3(2-x)\mathrm{d}x$	**M1**		attempt integ xf(x), condone missing k
		$= \frac{3}{4} \times \left[\frac{2x^4}{4} - \frac{x^5}{5}\right]_0^2$	**A1**		correct integration and limits, condone missing k
		1.2 m oe	**A1**	[3]	allow without units

(iv)	$\frac{3}{4}\int_0^1 x^2(2-x)\mathrm{d}x$	**M1**	attempt integ f(x), 0 to 1, condone missing k
	(= $\frac{3}{4}$ × ($\frac{2}{3}$ – $\frac{1}{4}$)) = $\frac{5}{16}$ or 0.3125 oe	**A1**	
	400 × $\frac{5}{16}$ = 125	**A1 ft** [3]	ft their $\frac{5}{16}$
7 (a) (i)	0.01 × 80 and 0.015 × 60	**M1**	$(1 - e^{-\lambda}) \times (1 - e^{-\mu})$ any λ, μ ($\lambda \neq \mu$)
	$(1 - e^{-0.8}) \times (1 - e^{-0.9})$	**M1**	allow one end error
	= 0.327 (3 sf)	**A1** [3]	
(ii)	λ = 0.02 × 40 + 0.015 × 60	**M1**	or their 0.8 + 0.9
	$e^{-1.7} \times (1 + 1.7 + \frac{1.7^2}{2})$	**M1**	
	= 0.757 (3 sf)	**A1** [3]	
(b)	$e^{-\lambda} \times \lambda = p$ and $e^{-\lambda} \times \frac{\lambda^2}{2} = 1.5p$	**M1**	or $e^{-\lambda} \times \frac{\lambda^2}{2} = 1.5 \times e^{-\lambda} \times \lambda$ seen or implied
	$\lambda = 3$	**A1**	
	$p = e^{-3} \times 3$	**M1**	their λ
	= 0.149 (3 sf)	**A1** [4]	

[Total for paper 50]

Cambridge International Examinations
Cambridge International Advanced Level

MATHEMATICS **9709/72**

Paper 7 **May/June 2016**

MARK SCHEME

Maximum Mark: 50

Published

This document consists of **6** printed pages.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously 'correct' answers or results obtained from incorrect working.

  Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2 / 1 / 0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF	Any Equivalent Form (of answer is equally acceptable)
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
BOD	Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only – often written by a 'fortuitous' answer
ISW	Ignore Subsequent Working
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PA	Premature Approximation (resulting in basically correct work that is insufficiently accurate)
SOS	See Other Solution (the candidate makes a better attempt at the same question)
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Penalties

MR – 1	A penalty of MR – 1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become 'follow through ✓' marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR – 2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA – 1	This is deducted from A or B marks in the case of premature approximation. The PA – 1 penalty is usually discussed at the meeting.

1	$\frac{6.2}{\sqrt{50}}$ or $\frac{6.2^2}{50}$ $\frac{51-53}{6.2\div\sqrt{50}}$ (= –2.281) P(z > '–2.281') = ϕ('2.281') = 0.989 (3 sf)	**B1** **M1** **M1** **A1** [4]	seen or implied allow without ÷√50 for finding correct area consistent with working as final answer
2 (i)	Conclude less than 90% satisfied when this is not true oe	**B11**	In context
(ii)	$1-(0.9^{15}+15\times0.9^{14}\times0.1$ $+{}^{15}C_2\times0.9^{13}\times0.1^2+{}^{15}C_3\times0.9^{12}\times0.1^3)$ = 0.0556 (3 sf) or 0.0555	**M1** **M1** **A1** [3]	Attempt (1–)P(X = 15,14,13,12) allow 1 end error Attempt fully correct expression
3 (i)	Pop too big or takes too long oe or testing destroys articles oe	**B1** [1]	or too expensive oe or pop inaccessible oe
(ii) (a)	z = 1.96 $65.7\pm z\times\frac{\sqrt{15}}{10}$ = 64.9 to 66.5 (3 sf)	**B1** **M1** **A1** [3]	seen Expression of correct form (must be 'z' must be 65.7) Must be an interval
(b)	CI does not include 64.7 Probably has affected (or increased) mean bounce ht.	**B1**√ [1]	allow 64.7 not within CI both needed. ft their CI ft 65.7/64.7 mix
4	H_0: λ (or μ) = 42 H_1: λ (or μ) ≠ 42 Po(42) ~ N(42, 42) stated or implied $\frac{53.5-42}{\sqrt{42}}$ = 1.77(4) (or 0.038 for area comparison) comp 1.96 No evidence that mean has changed	**B1** **B1**√ **M1** **A1** **M1** **A1**√ [6]	Or pop weekly mean = 2.1 etc. allow 'population mean' not just 'mean' ft their '42' (Accept alt method N(2.1,2.1/20) allow with wrong or no cc. Accept alt method using N(2.1,2.1/20) with or without cc Valid comp z or 1 – ('1.774') with 0.025 seen allow comp 1.645 if H_1: λ (or μ) > 42 No contradictions. No ft for H_1: λ (or μ) > 42 Note – accept other valid methods(e.g. cv method)

5	(i)	$T \sim \mathrm{N}(520, 70)$	B1	for N(520,..) or N(500,..) if standardising with 510
		$\frac{530-520}{\sqrt{'70'}}$ (= 1.195)	B1	for Var = 70 seen or implied
		(‘1.195’)	M1	ft their E and Var; allow without √
		= 0.884 (3 sf)	M1	finding correct area consistent with working
			A1 [5]	CWO
	(ii)	$\mathrm{E}(T) = -10$	B1	or +10 for T < 0
		$\mathrm{Var}(T) = 50 + 4.1^2 \times 20$ (= 386.2)	B1	Seen or implied
		$\frac{0-(-10)}{\sqrt{'386.2'}}$ (= 0.509)	M1	ft their E and Var; allow without √
		1 – (‘0.509’)	M1	finding correct area consistent with working
		= 0.305 (3 sf)	A1 [5]	CWO
6	(i)	$\lambda = 6.8$	B1	
		$\mathrm{e}^{-6.8} \times \frac{6.8^5}{5!}$	M1	any λ
		= 0.135 (3 sf)	A1 [3]	
	(ii) (a)	$\mathrm{e}^{-3.4}(1 + 3.4 + \frac{3.4^2}{2} + \frac{3.4^3}{3!} + \frac{3.4^4}{4!})$	M1	any λ, allow one end-error
		= 0.744 (3 sf)	A1 [2]	
	(b)	‘0.744’ + $\mathrm{e}^{-3.4} \times \frac{3.4^5}{5!}$	M1	or complete method, any λ, allow one end-error
		= 0.87(0) (3 sf) or 0.871	A1 [2]	
	(iii)	$\mathrm{P}(X \leqslant 6)$ = ‘0.870’ + $\mathrm{e}^{-3.4} \times \frac{3.4^6}{6!}$	M1	or complete method, any λ
		= 0.94	A1	fully correct un-simplified expression or better
		Need 6 hair driers	A1 [3]	dep M1A1 with numerical justification (0.94 or better)
7	(a)	0.3 or 1 – 0.6 or 0.4 or 0.2 seen	M1	
		0.8	A1 [2]	
	(b) (i)	$k\int_0^{1.5}(2.25 - x^2)\mathrm{d}x = 1$	M1	attempt integ f(x) and ‘= 1’. Ignore limits
		$k\left[2.25x - \frac{x^3}{3}\right]_0^{1.5} = 1$	A1	correct integration and limits
		$k \times [3.375 - 1.125] = 1$ or $k \times \frac{9}{4} = 1$ oe		
		$k = \frac{4}{9}$ **AG**	A1 [3]	No errors seen

(ii)	$\frac{4}{9}\int_0^{1.5}(2.25x - x^3)\mathrm{d}x$ $= \frac{4}{9}\left[2.25\frac{x^2}{2} - \frac{x^4}{4}\right]_0^{1.5}$ = 0.5625 or 0.563 Mean no. of hours = 56.25 or 56.3 56 hrs 15 mins	**M1** **A1** **A1** **A1**√ [4]	attempt integ xf(x), ignore limits, condone missing k correct integration and limits, condone missing k ft their 0.5625
(iii)	Max x is 1.5, less than 2.9 or 150 < 290	**B1** [1]	Needs numerical justification
(iv)	any a such that $2.9 \leqslant a \leqslant 5$	**B1** [1]	
	Total for paper	**50**	

Cambridge International Examinations
Cambridge International Advanced Level

MATHEMATICS **9709/73**

Paper 7 **May/June 2016**

MARK SCHEME

Maximum Mark: 50

Published

This document consists of **6** printed pages.

Mark Scheme Notes

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A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more “method” steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously “correct” answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

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Penalties

MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become “follow through √” marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Qu	Answer	Marks	Notes
1	$192.4 \pm z\sqrt{\frac{43.6}{150}}$	M1	Allow $\frac{43.6}{\sqrt{150}}$ Allow one side for M1
	$z = 2.326$ to 2.329	B1	Condone √(43.6/149) oe
	191 to 194 (3 sf)	A1 [3]	CWO
2	H_o: Pop mean yield = 8.2 H_1: Pop mean yield > 8.2	B1	or $\mu = 8.2$(not just "mean") $\mu > 8.2$
	$(\pm)\frac{8.7-8.2}{1.2/\sqrt{16}}$	M1	Allow without √ sign (Allow cc)
	= (±)1.667	A1	
	Comp $z = 1.645$ Or Area comparison 0.0475-0.0478)	M1	Or comp 1 - Φ('1.667') with 0.05 Valid Comparison z-values (same sign) or areas
	Reject H_0 Evidence that mean yield has increased	A1√ [5]	No Contradictions No follow through for 2 tail test
3 (i)	Use of Poisson	B1	
	Mean = 2.4	B1	
	$1 - e^{-2.4}(1 + 2.4 + \frac{2.4^2}{2})$	M1	Allow any λ *(Allow one end error)*
	= 0.43(0) (3 sf)	A1 [4]	Final answer SR Use of binomial: B1 for ans 0.431 (3 sf)
(ii)	240 > 50 or n>50	B1	
	240 × 0.01 = 2.4 < 5 or np<5 or p<0.1	B1 [2]	SR *n* large, *p* small: B1
4 (i)	H_0: Pop mean = 2.5 (or 7.5) H_0: Pop mean < 2.5 (or 7.5)	B1	or $\lambda = 2.5$(Not just "mean") Allow μ or $\lambda < 2.5$
	$\lambda = 7.5$ $P(X \leqslant 2) = e^{-7.5}(1+7.5+\frac{7.5^2}{2}) = 0.0203$ $P(X \leqslant 3) = 0.0203 + e^{-7.5} \times \frac{7.5^3}{3!} = 0.0591$	M1 A1	Either $P(X \leqslant 2)$ or $P(X \leqslant 3)$, allow any λ *Both Correct*
	CR is $X \leqslant 2$	A1	*Clear statement*
	Reject H_0 Evidence that no of sightings fewer	A1√ [5]	*Follow through their CR/their P(X⩽2)*
(ii)	P(Type I) = 0.0203 (3 sf)	B1√ [1]	ft their $P(X \leqslant 2)$
(iii)	H_0 was rejected oe	B1 [1]	or Type II is P(not reject H_0)oe
5 (i)	$k\int_{5}^{10}(10t - t^2)\,dt = 1$	M1	Attempt to integrate, ignore limits
	$k\left[5t^2 - \frac{t^3}{3}\right]_5^{10} = 1$	A1	Correct integral and limits
	$k(500 - \frac{1000}{3} - (125 - \frac{125}{3})) = 1$ $k \times \frac{250}{3} = 1$ ($k = \frac{3}{250}$ **AG**)	A1 [3]	No errors seen; No inexact decimals seen

Qu	Answer	Marks	Notes
(ii)	$\frac{3}{250}\int_{5}^{10}(10t^2 - t^3)\mathrm{d}t$	M1	Attempt to integrate, ignore limits
	$= \frac{3}{250}\left[\frac{10t^3}{3} - \frac{t^4}{4}\right]_{5}^{10}$	A1	Correct integral and limit. Condone missing k
	$= \frac{3}{250}(\frac{10000}{3} - \frac{10000}{4} - (\frac{1250}{3} - \frac{625}{4})$		
	= 6.875 or 55/8	A1 [3]	Allow 6.88
(iii)	$P(T < E(T) = \frac{3}{250}\left[5t^2 - \frac{t^3}{3}\right]_{5}^{"6.875"}$	M1*	ft their E(*T*)
	= 0.5361		
	"0.5361" – 0.5	DM1*	allow 0.036
	P(*T* between E(*T*) & median = 0.0361	A1 [3]	**Alternative Method** Integrate f(t)limits 5 and m equated to 0.5 M1* Integrate f(t)limits their 6.736 (provided between 5 and 10) and their 6.875DM1
(iv)	10 (minutes)	B1 [1]	Allow without "minutes"
6 (i)	λ= 3.9	B1	
	$e^{-3.9} \times \frac{3.9^4}{4!}$	M1	M1 allow any λ
	= 0.195	A1 [3]	SR Combination method B1 for λ =1.6 AND λ =2.3 used in combination method (at least 3 combinations) M1 All correctly combined and added
(ii)	$\bar{X} \sim N(1.6, \frac{1.6}{75})$	B1	B1 for N(1.6, ….)stated
		B1 [2]	B1 for Var = $\frac{1.6}{75}$ stated SR, not stated but all implied in (iii): B1
(iii)	$\frac{1.7-1.6}{\sqrt{\frac{1.6}{75}}}$ (= 0.685)	M1	For standardising (using their values or correct values .Ignore cc
	1– Φ("0.685")	M1	Correct area consistent with their working
	= 0.247 (3 sf)	A1 [3]	Accept use of 1/2n correction leading to 0.233. NB Use of Poisson sum Po(120) and N(120,120) with μ=127.5 leads to 0.247, or 0.233 with cc
(iv)	*X* not normally distr. So CLT needed	B1 [1]	Not "it"

Qu	Answer	Marks	Notes
7 (i)	E(T) =20.8	**B1**	
	Var(T)= 20 × $0.03^2 + 0.01^2$(= 0.0181)	**B1**	or √(20 × $0.03^2 + 0.01^2$) = 0.135 (3sf)
	$\frac{20.6-20.8}{\sqrt{"0.0181"}}$ (= –1.487)	**M1**	For standardising (σ must come from combination)
	1 – Φ(“1.487”)	**M1**	Area consistent with their working
	= 0.0684 to 0.686	**A1** [5]	Any answer within range
(ii)	E(D) = 0 Var(D) = 2 × 0.0181(= 0.0362)	**B1**√	Both (Seen or implied)
	$\frac{0.02-0}{\sqrt{"0.0362)}}$ (= 0.105)	**M1**	Allow without √
	Φ(“0.105”) = 0.5418 or 1-Φ(0.015) =0.4582	**A1**	Allow to 3sf
	Φ(“0.105”) – (1 – Φ(“0.105”) (= 0.5418 – 0.4582)	**M1**	or 1 – 2(1 – Φ(“0.105”)) (= 1 – 2 × 0.4582)
	= 0.0836/0.0837	**A1** [5]	

CAMBRIDGE INTERNATIONAL EXAMINATIONS

Cambridge International Advanced Subsidiary and Advanced Level

MARK SCHEME for the March 2016 series

9709 MATHEMATICS

9709/72 Paper 7 (Probability and Statistics), maximum raw mark 50

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PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

1	$E(X) = \frac{10}{3}$ oe $Var(X) = \frac{25}{9}$ oe	B1	For E(X) and Var(X)
	$E(Y) = 10$ $Var(Y) = 5$	B1	For E(Y) and Var(Y) OR For E(X) and E(Y) For Var(X) and Var(Y)
	$E(X + Y) = \frac{40}{3}$ oe or 13.3 (3 sf)	B1	
	$Var(X + Y) =$ "$\frac{25}{9}$" + "5"	M1	For adding 2 (appropriate) variances
	sd = $\frac{\sqrt{70}}{3}$ oe or 2.79 (3 sf)	A1 [5]	or sd = or $\sqrt{2} \times \frac{5}{3}$
2	H_0: P(hit target) = 0.65 H_1: P(hit target) > 0.65	B1	Allow $p = 0.65$ Allow $p > 0.65$
	$^{20}C_2 \times 0.35^2 \times 0.65^{18} + 19 \times 0.35 \times 0.65^{19} + 0.65^{20}$	M1	Allow one end error. Allow p/q mix. Allow (1–) for **M** mark
	= 0.0121 (3 sf)	A1	**A** mark recovered following valid comparison
	Comp 0.01 There is no evidence (at the 1% level) that she has improved	M1 A1√ [5]	For valid comparison She has probably not improved. No contradictions. (SR Use of Normal **M0**, but **M1A1** for valid comparison could be awarded)
3 (i)	H_0: pop mean journey time = 35.2 mins H_1: pop mean journey time < 35.2 mins	B1	Allow "μ". Not "mean journey time"
	$\frac{34.7-35.2}{5.6/\sqrt{25}}$ (= –0.446)	M1	For standardising (√25 needed)
	Φ(< "–0.446") = 1 – Φ("0.446") = 0.328 (3 sf)	M1 A1 [4]	For correct area consistent with their working As final answer
(ii)	H_0 *is* rejected but Type II error can only be made if H_0 is *not* rejected	B1 [1]	Allow just "H_0 is rejected." oe
4	$X - 2Y \sim N(0.1, 0.2^2 + 4 \times 0.1^2)$ soi (= N(0.1, 0.08))	B1 B1	**B1** for ± 0.1 **B1** for $0.2^2 + 4 \times 0.1^2$
	$\frac{0-0.1}{\sqrt{"0.08"}}$ (= –0.354)	M1	For standardising. Allow without √ sign
	Φ("–0.354 ") = 1 – Φ("0.354") = 0. 362 (3 sf)	M1 A1 [5]	For correct area consistent with their working
5 (i)	Est(μ) = $\frac{14\,910}{150}$ (= 99.4)	B1	
	Est(σ^2) = $\frac{150}{149}(\frac{1525000}{150}$ – "99.4"^{2}) = 288.228	M1 A1	Allow **M1** if $\frac{150}{149}$ omitted
	$z = 2.576$	B1	Accept 2.574–2.579
	"99.4" ± $z \times \sqrt{288.228 \div 150}$	M1	Any z
	CI = 95.8 to 103 (3 sf)	A1 [6]	(NB Use of biased Var can score 5/6 max)
(ii)	100 lies within this CI Hence yes	B1√ [1]	Both needed, ft their CI

(iii)	To avoid bias or Necessary to enable statistical inference	**B1** [1]	Or any equivalent
6 (i)	$\lambda = 3.3 \times \frac{25}{30} = 2.75$	**B1**	
	$e^{-2.75}(1 + 2.75 + \frac{2.75^2}{2})$	**M1**	Allow any λ Allow one end error
	$= 0.481$ (3 sf)	**A1** [3]	As final answer. Accept 0.482
(ii) (a)	$\lambda\ (= 3.3 \times \frac{365}{30}) = 40.15$	**B1**	Accept 40.1 or 40.2
	$(X \sim \text{Po}(40.15) \Rightarrow X \sim \text{N}(40.15, 40.15))$		
	$\frac{50.5-"40.15"}{\sqrt{"40.15"}}$ $(= 1.633)$	**M1**	Allow with incorrect or no cc OR no √ sign
	$1 - \Phi("1.633")$	**M1**	For correct area consistent with their working
	$= 0.0513$ (3 sf)	**A1** [4]	Accept 0.0512
(b)	$\lambda > 15$	**B1** [1]	or similar
(iii)	$\lambda = \frac{73}{30}$ oe or 1.1+1.33= 2.43 (3 sf)	**B1**	
	$1 - e^{-2.43}(1 + 2.43 + \frac{2.43^2}{2} + \frac{2.43^3}{3!})$	**M1**	Allow any λ. Allow one end error
	$= 0.228$ (3 sf)	**A1** [3]	
7 (a) (i)	$\text{E}(X) = 1.5$	**B1**	
	$\frac{2}{9}\int_0^3 (3x^3 - x^4)\text{d}x$	**M1**	Attempt integ $x^2\text{f}(x)$ ignore limits
	$= \frac{2}{9}\left[\frac{3x^4}{4} - \frac{x^5}{5}\right]_0^3$		
	$= \frac{2}{9}\left[\frac{243}{4} - \frac{243}{5}\right]$ $(= 2.7)$	**M1**	Sub correct limits into correct integral
	$\text{Var}(X)\ (= 2.7 - 1.5^2) = 0.45$ oe	**A1**√ [4]	Ft their E(X), but no ft for –ve Var.
(ii)	0.5	**B1** [1]	
(iii)	$(1 - \frac{13}{27}) \div 2$	**M1**	or $\frac{2}{9}\int_2^3 (3x - x^2)\ \text{d}x$ oe
	$= \frac{7}{27}$ or 0.259	**A1** [2]	As final answer
(b)	$\frac{1}{2} \times 2 \times 2a = \frac{1}{2}$ or $\int_0^2 ax\text{d}x = \frac{1}{2}$	**M1**	Attempt correct equation in ‘a’
	$a = \frac{1}{4}$	**A1**	
	$\frac{1}{2} \times b \times \frac{1}{4}b = 1$ or $\int_0^b \frac{1}{4}x\text{d}x = 1$ or $b = 2 \times \sqrt{2}$	**M1**	or $\frac{1}{2} \times b \times ab = 1$ or $\int_0^b ax\text{d}x = 1$ attempt correct equation in (a and) b
	$b = 2\sqrt{2}$	**A1**√ [4]	Allow $b = \sqrt{8}$ or 2.83 (3 sf) Ft incorrect a, both **M**s needed
		Total for paper 50	

CAMBRIDGE INTERNATIONAL EXAMINATIONS

Cambridge International Advanced Level

MARK SCHEME for the October/November 2015 series

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1	λ= (1.2 + 2.3) ÷ 2	**M1**	Attempt combined mean, allow 1.2 + 2.3
	= 1.75	**A1**	Correct mean
	$e^{-1.75}\left(\frac{1.75^2}{2}+\frac{1.75^3}{3!}\right)$	**M1**	Allow incorrect mean. Allow end errors (1 and/or 4)
	= 0.421 (3 sf)	**A1** [4]	
		Total: 4	
2 (i)	$\frac{6}{\sqrt{120}}$ oe seen	**B1**	Or $6^2/120$ oe seen
	$\frac{30-29}{\left(\frac{6}{\sqrt{120}}\right)}$ (= 1.826)	**M1**	± Allow without √120. No sd/var mix
	P(z > '1.826') = 1 – Φ('1.826')	**M1**	Correct tail consistent with their working
	= 0.034 (2 sf)	**A1** [4]	0.0339
(ii)	No n is large (⩾30)	B1	1st B1 for either comment
	Sample mean is (appr) normally distrib or The CLT applies oe	B1 [2]	2nd B1 for'No'with 2nd comment (No mark for 'No' alone)
		Total: 6	
3 (i)	$\frac{3420}{60}(=57)$	**B1**	
	$\frac{60}{59}\left(\frac{195200}{60}-'57'^2\right)$ (= 4.40678)	**M1**	Oe
	= 4.41 (3 sf)	**A1** [3]	As final answer
(ii)	$'57'\pm z\sqrt{\frac{'4.40678'}{60}}$	**M1**	
	z = 2.326	**B1**	2.326 – 2.329 (accept 2.33 if no better seen)
	[56.4 to 57.6] (3 sf)	**A1** [3]	NB: use of biased variance in (ii) can score in full
		Total: 6	

4 **(i)**	$k\int_1^2 (3-x)dx = 1$	**M1**	Attempt ∫f(x) = 1, ignore limits or $\frac{k}{2}(h_1 + h_2) = 1$
	$k\left[3x - \frac{x^2}{2}\right]_1^2 = 1$	**A1**	Correct integration & limits or $\frac{k}{2}(2 + 1) = 1$
	(k(6 – 2 – (3 – 0.5)) = 1)		
	$k \times 1.5 = 1$ or $k \times \frac{3}{2} = 1$ or $k = \frac{1}{1.5}$ oe		
	$k = \frac{2}{3}$ **AG**	**A1** [3]	No errors seen
(ii)	$\frac{2}{3}\int_1^m (3-x)dx = 0.5$ oe ∫ from m to 2	**M1***	Attempt Int f(x) = 0.5, ignore limits oe Or use of area of trapezium
	$\left(\frac{2}{3}\left[3x - \frac{x^2}{2}\right]_1^m = 0.5\right)$		
	$\frac{2}{3}\left[3m - \frac{m^2}{2} - 2.5\right] = 0.5$	**dep M1***	Sub of correct limits into their integral. Or trapezium using 1 and m/m and 2 Any correct 3-term QE = 0 or $(m–3)^2$ =2.5
	$m^2 - 6m + 6.5 = 0$ oe	**A1**	
	$\left(m = \frac{6 \pm \sqrt{36 - 4\times 6.5}}{2} = 1.42 \text{ or } 4.58\right)$		
	m = 1.42 (3 sf)	**A1** [4]	or $\frac{6-\sqrt{10}}{2}$ oe; single correct ans
		Total: 7	

5 **(i)**	Po(1.6) stated or implied	**M1**		
	$P(X > 3) = 1 - e^{-1.6}\left(1 + 1.6 + \frac{1.6^2}{2} + \frac{1.6^3}{3!}\right)$	**M1**	Allow M1 for 1 – P($X \leqslant 3$), incorrect λ and allow one end error	
	= 0.0788 (3 sf)	**A1** [3]	SR Use of Bin scores B1 only for 0.0788	
(ii)	$\lambda = \frac{n}{2500}$	**B1**	or $e^{-\mu} < 0.05$ M1	or $\frac{2499}{2500}$ B1
	$e^{-\frac{n}{2500}} < 0.05$ Allow = Allow incorrect λ	**M1**		$\left(\frac{2499}{2500}\right)^n < 0.05$ M1
	$-\frac{n}{2500} < \ln 0.05$ Attempt ln bs	**M1**	$-\mu < \ln 0.05$ M1 ($\mu > 2.9957$)	$n\ln\frac{2499}{2500} < \ln 0.05$ M1
	$n > 7489.3$ (1 dp) Smallest $n = 7490$	**A1** [4]	$n = \mu \times 2500$ B1 Smallest $n = 7490$ A1	Smallest $n = 7488$ A1
		Total: 7		
6 **(i)**	E(T) = 9 × 78 + 7 × 66 (= 1164)	**B1**	Or 9 × 78 + 7 × 66 – 1200	
	Var(T) = $9 \times 7^2 + 7 \times 5^2$ (= 616)	**B1**		
	$\frac{1200 - '1164'}{\sqrt{'616'}}$ (= 1.450)	**M1**	± Allow without √	
	P(z < 1.450) = Φ (1.450) = 0.927 (3 sf)	**M1** **A1** [5]	Correct tail consistent with their mean	
(ii)	E(D) = 66 – 78 (= –12)	**B1**	Both needed	
	Var(D) = $7^2 + 5^2$ (= 74)			
	$\frac{0 - ('-12')}{\sqrt{74}}$ (= 1.395)	**M1**	± Allow without √	
	P(D > 0) = 1 – Φ (‘1.395’) 0.0815 (3 sf)	**M1** **A1** [4]	Correct tail consistent with their mean Similar scheme for P(M – W) < 0	
		Total: 9		

7 (i)	Prob could be different later in day or on a different day oe	**B1** [1]	or any explanation why not random or "Not random" or "Not representative"
(ii)	Looking for decrease (or improvement) H_0: P(not arrive) = 0.2 H_1: P(not arrive) < 0.2	**B1** **B1** [2]	oe Allow "$p = 0.2$"
(iii)	Concluding that prob has decreased (or publicity has worked) when it hasn't oe	**B1** [1]	In context
(iv)	$P(X = 0)$ and $P(X = 1)$ attempted $P(X \leqslant 2) = 0.8^{30} + 30 \times 0.8^{29} \times 0.2 + {}^{30}C_2 \times 0.8^{28} \times 0.2^2$ $(= 0.0442)$ $P(X \leqslant 3) = 0.8^{30} + 30 \times 0.8^{29} \times 0.2 + {}^{30}C_2 \times 0.8^{28} \times 0.2^2 + {}^{30}C_3 \times 0.8^{27} \times 0.2^3 = 0.123$ cr is $X \leqslant 2$ P(Type I) = 0.0442 (3 sf)	**M1** **M1** **B1** **A1** **A1** [5]	B(30, 0.2) Not nec'y added May be implied by calc $P(X \leqslant 2)$ or $P(X \leqslant 3)$ Attempt $P(X \leqslant 2)$ Or '0.0442' + ${}^{30}C_3 \times 0.8^{27} \times 0.2^3 = 0.123$
(v)	3 is outside cr No evidence that p has decreased (or that publicity has worked)	**M1** **A1** ✓ [2]	Comparison of 3 with their cr or $P(X \leqslant 3) = 0.123$ which is > 0.05 Correct conclusion. No contradictions
		Total: 11	
		Total for paper: 50	

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		Total: 4	
2 (i)	$\frac{6}{\sqrt{120}}$ oe seen $\frac{30-29}{\left(\frac{6}{\sqrt{120}}\right)}$ (= 1.826) P(z > '1.826') = 1 – Φ('1.826') = 0.034 (2 sf)	**B1** **M1** **M1** **A1** [4]	Or $6^2/120$ oe seen ± Allow without √120. No sd/var mix Correct tail consistent with their working 0.0339
(ii)	No n is large (⩾30) Sample mean is (appr) normally distrib or The CLT applies oe	B1 B1 [2]	1st B1 for either comment 2nd B1 for'No'with 2nd comment (No mark for 'No' alone)
		Total: 6	
3 (i)	$\frac{3420}{60}(=57)$ $\frac{60}{59}\left(\frac{195200}{60}-{'57'}^2\right)$ (= 4.40678) = 4.41 (3 sf)	**B1** **M1** **A1** [3]	Oe As final answer
(ii)	$'57'\pm z\sqrt{\frac{'4.40678'}{60}}$ z = 2.326 [56.4 to 57.6] (3 sf)	**M1** **B1** **A1** [3]	2.326 – 2.329 (accept 2.33 if no better seen) NB: use of biased variance in (ii) can score in full
		Total: 6	

4 (i)	$k\int_1^2 (3-x)dx = 1$	**M1**	Attempt ∫f(x) = 1, ignore limits or $\frac{k}{2}(h_1 + h_2) = 1$
	$k\left[3x-\frac{x^2}{2}\right]_1^2 = 1$	**A1**	Correct integration & limits or $\frac{k}{2}(2 + 1) = 1$
	($k(6 - 2 - (3 - 0.5)) = 1$) $k \times 1.5 = 1$ or $k \times \frac{3}{2} = 1$ or $k = \frac{1}{1.5}$ oe $k = \frac{2}{3}$ **AG**	**A1** [3]	No errors seen
(ii)	$\frac{2}{3}\int_1^m (3-x)dx = 0.5$ oe ∫ from m to 2 $\left(\frac{2}{3}\left[3x-\frac{x^2}{2}\right]_1^m = 0.5\right)$	**M1***	Attempt Int f(x) = 0.5, ignore limits oe Or use of area of trapezium
	$\frac{2}{3}\left[3m-\frac{m^2}{2}-2.5\right] = 0.5$	**dep M1***	Sub of correct limits into their integral. Or trapezium using 1 and m/m and 2 Any correct 3-term QE = 0 or $(m-3)^2 = 2.5$
	$m^2 - 6m + 6.5 = 0$ oe	**A1**	
	$\left(m = \frac{6 \pm \sqrt{36 - 4 \times 6.5}}{2} = 1.42 \text{ or } 4.58\right)$ $m = 1.42$ (3 sf)	**A1** [4]	or $\frac{6-\sqrt{10}}{2}$ oe; single correct ans
		Total: 7	

5 (i)	Po(1.6) stated or implied $\mathrm{P}(X > 3) = 1 - \mathrm{e}^{-1.6}\left(1 + 1.6 + \frac{1.6^2}{2} + \frac{1.6^3}{3!}\right)$ = 0.0788 (3 sf)	**M1** **M1** **A1** [3]	Allow M1 for 1 – P($X \leqslant 3$), incorrect λ and allow one end error SR Use of Bin scores B1 only for 0.0788	
(ii)	$\lambda = \frac{n}{2500}$ $\mathrm{e}^{-\frac{n}{2500}} < 0.05$ Allow = Allow incorrect λ $-\frac{n}{2500} < \ln 0.05$ Attempt ln bs $n > 7489.3$ (1 dp) Smallest $n = 7490$	**B1** **M1** **M1** **A1** [4]	or $e^{-\mu} < 0.05$ M1 $-\mu < \ln 0.05$ M1 ($\mu > 2.9957$) $n = \mu \times 2500$ B1 Smallest $n = 7490$ A1	or $\frac{2499}{2500}$ B1 $\left(\frac{2499}{2500}\right)^n < 0.05$ M1 $n\ln\frac{2499}{2500} < \ln 0.05$ M1 Smallest $n = 7488$ A1
		Total: 7		
6 (i)	E(T) = 9 × 78 + 7 × 66 (= 1164) Var(T) = 9 × 7^2 + 7 × 5^2 (= 616) $\frac{1200-\text{'1164'}}{\sqrt{\text{'616'}}}$ (= 1.450) P(z < 1.450) = Φ (1.450) = 0.927 (3 sf)	**B1** **B1** **M1** **M1** **A1** [5]	Or 9 × 78 + 7 × 66 – 1200 ± Allow without √ Correct tail consistent with their mean	
(ii)	E(D) = 66 – 78 (= –12) Var(D) = 7^2 + 5^2 (= 74) $\frac{0-(\text{'}-12\text{'})}{\sqrt{74}}$ (= 1.395) P(D > 0) = 1 – Φ (‘1.395’) 0.0815 (3 sf)	**B1** **M1** **M1** **A1** [4]	Both needed ± Allow without √ Correct tail consistent with their mean Similar scheme for P(M – W) < 0	
		Total: 9		

7 (i)	Prob could be different later in day or on a different day oe	**B1** [1]	or any explanation why not random or "Not random" or "Not representative"
(ii)	Looking for decrease (or improvement) H_0: P(not arrive) = 0.2 H_1: P(not arrive) < 0.2	**B1** **B1** [2]	oe Allow "$p = 0.2$"
(iii)	Concluding that prob has <u>decreased</u> (or publicity has worked) when it hasn't oe	**B1** [1]	In context
(iv)	$P(X = 0)$ and $P(X = 1)$ attempted $P(X \leqslant 2) = 0.8^{30} + 30 \times 0.8^{29} \times 0.2 + {}^{30}C_2 \times 0.8^{28} \times 0.2^2$ $(= 0.0442)$ $P(X \leqslant 3) = 0.8^{30} + 30 \times 0.8^{29} \times 0.2 + {}^{30}C_2\times0.8^{28}\times0.2^2 + {}^{30}C_3\times0.8^{27}\times0.2^3 = 0.123$ cr is $X \leqslant 2$ P(Type I) = 0.0442 (3 sf)	**M1** **M1** **B1** **A1** **A1** [5]	B(30, 0.2) Not nec'y added May be implied by calc $P(X \leqslant 2)$ or $P(X \leqslant 3)$ Attempt $P(X \leqslant 2)$ Or '0.0442' + ${}^{30}C_3 \times 0.8^{27} \times 0.2^3 = 0.123$
(v)	3 is outside cr No evidence that p has decreased (or that publicity has worked)	**M1** **A1** √ [2]	Comparison of 3 with their cr or $P(X \leqslant 3) = 0.123$ which is > 0.05 Correct conclusion. No contradictions
		Total: 11	
		Total for paper: 50	

CAMBRIDGE INTERNATIONAL EXAMINATIONS

Cambridge International Advanced Level

MARK SCHEME for the October/November 2015 series

9709 MATHEMATICS

9709/73 Paper 7, maximum raw mark 50

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the October/November 2015 series for most Cambridge IGCSE®, Cambridge International A and AS Level components and some Cambridge O Level components.

CAMBRIDGE
International Examinations

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
BOD	Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)
CAO	Correct Answer Only (emphasising that no “follow through” from a previous error is allowed)
CWO	Correct Working Only – often written by a ‘fortuitous’ answer
ISW	Ignore Subsequent Working
MR	Misread
PA	Premature Approximation (resulting in basically correct work that is insufficiently accurate)
SOS	See Other Solution (the candidate makes a better attempt at the same question)
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become “follow through √” marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

1 (i)	N(352, …) Variance = 2.9	**B1** **B1**	[2]	no recovery in (ii) for each B mark accept sd = √ 2.9 = 1.70(29) stated
(ii)	$\frac{354-352}{\sqrt{2.9}}$ (= 1.174) 1 – Φ('1.174') = 0.120 (3 sf)	**M1** **M1** **A1**	[3]	with their mean and var Or $\frac{354.05-352}{\sqrt{2.9}}$ or correct restart (= 1.204) (accept sd/var mix)1 – Φ('1.204') = 0.114 (3 sf) Incorrect cc can score M1M1A0
Total			**[5]**	
2	($\Phi^{-1}(0.99) =$) 2.326 seen N(*λ*, *λ*) seen or implied $\frac{55.5-\lambda}{\sqrt{\lambda}}$ = + "2.326" λ + "2.326" $\sqrt{\lambda}$ – 55.5 = 0 $\sqrt{\lambda} = \frac{-\text{"2.326"} \pm \sqrt{\text{"2.326"}^2 + 4\times 55.5}}{2}$ (= 6.377.. or – 8.703..)) λ = 40.7 (3 sf)	**B1** **M1** **M1** **M1** **A1**	[5]	must be Φ^{-1}, not Φ allow with wrong or no cc & Φ(0.99) (= 0.8389) must = " z " or attempt at z (0.99 / 0.01 M0) for correct method of solving their quad in √λ and squaring to find λ cao, one ans only Without cc, λ = 40.2: lose final A1
Total			**[5]**	
3 (i)	0.4 or 2/5 or 26/65	**B1**	[1]	no recovery in (ii) for the B mark
(ii)	" 0.4 " + $z \times \sqrt{\frac{0.4\times 0.6}{65}}$ = 0.516 oe $z = \left(0.116 \times \sqrt{\frac{65}{0.4\times 0.6}}\right)$ = 1.909 (Φ('1.909') = 0.97(18)) 2 ('0.97' – 1) α = 94	**M1** **A1** **M1** **A1**	[4]	or " 0.4 " – $z \times \sqrt{\frac{0.4\times 0.6}{65}}$ = 0.284 or $z \times \sqrt{\frac{0.4\times 0.6}{65}}$ = 0.116 oe for fully correct method to find α from their z allow 94.36 or 94.4 or 94.374
Total			**[5]**	

4 (i)	$k\int_{-2}^{2}(4-x^2)\mathrm{d}x = 1$	**M1**		attempt Integral f(*x*) = 1, ignore limits
	$k[4x-\frac{x^3}{3}]_{-2}^{2} = 1$	**A1**		correct integration & limits
	$\left(k\left(8-\frac{8}{3}-\left(-8-\left(-\frac{8}{3}\right)=1\right)\right)\right)$			
	$k\times\frac{32}{3} = 1$ oe Not e.g. $k\times10.7 = k$			
	$k=\frac{3}{32}$ **AG**	**A1**	[3]	exact answer correctly found
(ii)	Inverted parabola, vertex on *y* axis	**B1**		parabola must finish on x axis at ± 2, labelled (ignore markings on y axis)
	E(*X*) = 0	**B1**	[2]	
(iii)	$\frac{3}{32}\int_{-2}^{1}(4-x^2)\mathrm{d}x$	**M1**		or $1-\frac{3}{32}\int_{1}^{2}(4-x^2)\mathrm{d}x$ ignore limits
	$\frac{3}{32}[4x-\frac{x^3}{3}]_{-2}^{1}$	**A1**		or $1-\frac{3}{32}[4x-\frac{x^3}{3}]_{1}^{2}$
	$\frac{3}{32}(4-\frac{1}{3}-(-8-(-\frac{8}{3})))$			correct integration and correct limits $= 1-\frac{3}{32}(8-\frac{8}{3}-(4-\frac{1}{3})$
	$=\frac{27}{32}$ or 0.844 (3 sf)	**A1**	[3]	
Total			**[8]**	

5 (a)	$\lambda = 4.5$	**B1**		
	$e^{-4.5}$ (= 0.011109)	**M1**		alone allow any λ
	$\left(\frac{99}{100}\right)^{450}$ (= 0.010860)	**M1**		
	$\left(\frac{'0.011109'-'0.010860'}{'0.010860}\times 100\right)$			
	= 2.29% (3 sf)	**A1**	[4]	
(b)	H_0: P(6) = $\frac{1}{6}$ or $p = \frac{1}{6}$ H_1: P(6) < $\frac{1}{6}$ or $p < \frac{1}{6}$	**B1**		Both needed
	$\left(\frac{5}{6}\right)^{25} + 25\left(\frac{5}{6}\right)^{24} \times \frac{1}{6} + {}^{25}C_2\left(\frac{5}{6}\right)^{23} \times \left(\frac{1}{6}\right)^{2}$	**M1**		allow one error (extra term/missing term / incorrect term) CR method: attempt at least P(0) and P(0 and 1) (0.010... and 0.06... < 0.1)
	= 0.189 (3 sf)	**A1**		CR is 0,1 and must see 0.189 for A1
	comp 0.1	**M1**		valid comp ‘0.189’ with 0.1 oe valid comparison of 2 with CR
	No reason to believe die biased	**A1**	[5]	correct conclusion, √ their 0.189 no contradictions
Total			**[9]**	
6 (i)	Ho: $\mu = 2.60$ H_1: $\mu > 2.60$	**B1**		allow pop mean, not just ‘mean’
	$\pm\ \frac{2.64 - 2.6}{0.2 \div \sqrt{75}}$	**M1**		
	= ± 1.732	**A1**		accept ± 1.73 (3 sf)
	‘1.732’ > 1.645 Reject Ho. There is evidence that μ has increased	**B1** √	[4]	valid comparison with 1.645 (or 0.0416 < 0.05) and correct conclusion √ their 1.732 no contradictions (or CV method x_{crit} = 2.638 M1A1 comp 2.64 > 2.638 and concln B1 √) SR two tail test, using 1.96 (or using 0.025) can score B0M1A1B1ft max 3/4

(ii)	$\frac{x-2.6}{0.2 \div \sqrt{75}} = 1.645$ ($x = 2.638$)	**M1**		
	$\pm\ \frac{'2.638'-2.68}{0.2 \div \sqrt{75}}$	**M1**		for standardising with their " 2.638 " using 2.68
	= ± 1.819	**A1**		accept 1.82 (3 sf)
	Φ('–1.819') = 1 – Φ('1.819')	**M1**		indep M mark, calculate correct area/prob consistent with their working
	= 0.0345 or 0.0344	**A1**	[5]	
Total			**[9]**	
7 (i)	est $\mu = 2.087$	**B1**		allow 2.09
	est $\sigma^2 = \frac{100}{99}\left(\frac{435.57}{100} - 2.087^2\right)$	**M1**		1/99 (435.57 – 208.7^2/100)
	= 0.000132(3232) or 131/990000	**A1**	[3]	without $\frac{100}{99}$: 0.000131 M0A0
(ii)	E($Y-X$) = 2.12 – 2.087 (= 0.033)	**B1**		or 2.12 – 2.087 – 0.01 for Y – X – 0.01 < 0 allow 2.09 for 2.087
	Var($Y-X$) = 0.000144 + '0.00013232'	**M1**		or √(0.012^2 + '0.00013232') M1
	= 0.000276(32)	**A1**		= 0.016623 A1
	$\frac{0.01-'0.033'}{\sqrt{'0.00027632'}}$ (= –1.384)	**M1**		√their E($Y-X$) & Var($Y-X$) var must be a combination of the two vars
	Φ('–1.384') = 1 – Φ('1.384')	**M1**		correct area/prob consistent with their working
	= 0.0832	**A1**	[6]	SR use of biased var (0.000131) in (i) and (ii) scores in (ii) B1M1 A1 for 0.000275 and M1M1 A1 for 0.0827 (6/6 available)
Total			**[9]**	
	Total for paper		**[50]**	

CAMBRIDGE INTERNATIONAL EXAMINATIONS

Cambridge International Advanced Level

MARK SCHEME for the May/June 2015 series

9709 MATHEMATICS

9709/71 Paper 7, maximum raw mark 50

Mark Scheme Notes

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A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more “method” steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

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  B2/1/0 means that the candidate can earn anything from 0 to 2.

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- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

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The following abbreviations may be used in a mark scheme or used on the scripts:

AEF	Any Equivalent Form (of answer is equally acceptable)
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PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Note: "(3 sfs)" means "answer which rounds to ... to 3 sfs". If correct ans seen to ≥ 3sfs, ISW for later rounding. Penalise < 3 sfs only once in paper.

1	$\frac{1}{2}a^2 = 1$	M1	or $\int_0^a x\,dx = 1$
	$a = \sqrt{2}$	A1	Allow 1.41 or better
	$\int_0^{\sqrt{2}} x^2 dx$	M1	ignore limits
	$= \left[\frac{x^3}{3}\right]_0^{\sqrt{2}}$	A1f	correct integral and limits, but ft their a
	$= \frac{(\sqrt{2})^3}{3}$ = or $\frac{2^{1.5}}{3}$ or $\frac{2.83}{3}$ or 0.9428 (= 0.943 **AG**)	A1 [5]	must see this numerical expression, or equiv SR Equating $\int x$ f(x) to 0.943 scores M1 Solving to find a = 1.41 scores A1
		[Total 5]	
2 (i)	H_0: $p = 0.2$ or $\mu = 10$ H_1: $p > 0.2$ or $\mu > 10$	B1 [1]	
(ii)	N(10, 8) seen or implied	B1	or $N\left(0.2, \frac{0.2 \times 0.8}{50}\right)$
	$\frac{125 - 10}{\sqrt{8}}$ or $\frac{\frac{125}{50} - 02}{\sqrt{\frac{0.2\times0.8}{50}}}$	M1	For standardising allow with no or wrong cc
	= 0.884	A1	
	comp 1.282	M1f	Allow area comparison with 0.188 or comp 1.645 if H_1 $p \neq 0.2$
	Claim not justified or No evidence to support claim	A1f [5]	Allow accept H_0 provided correctly defined. Follow through their test statistic ;dep 1–tail test No Contradictions SR; Use of B(50,0.2) scores B1 provided at least two probabilities calculated. M1 For finding P($X \geqslant$ 13) allow one end error. A1 for 0.186
		[Total: 6]	

3 **(i)**	34 $2.2^2 + 1.3^2 + 2.6^2$ (=13.29)	B1 B1 [2]	Accept 13.3 or 3.65^2 Allow at early stage
(ii)	$\frac{33-'34'}{\sqrt{\frac{'13.29'}{70}}}$ (= –2.295) $\frac{35-'34'}{\sqrt{\frac{'13.29'}{70}}}$ (= 2.295)	M1	correct standardisation method for either
	Φ(‘2.295’) – Φ(‘–2.295’)	M1	For attempt to use tables to find the probability between two z values ,may be implied by next line
	= Φ(‘2.295’) – (1 – Φ(‘2.295’)) oe	M1	For a correct method to find the area between their two z values
	= 0.978 (3 sf)	A1 [4]	
		[Total: 6]	
4 **(i)**	H_0: pop mean (or μ) = 12.4 H_1: pop mean (or μ) > 12.4	B1	not just “mean”
	$\frac{12.9-12.4}{2.1+\sqrt{50}}$	M1	Allow with 50 instead of √50
	1.684	A1	
	comp cv z = 1.96 No evidence that pop mean time has increased	B1f [4]	or P(z > 1.684) = 0.0461 > 0.025 Allow accept H_0 if correctly defined. Ft their test statistic. No contradictions
(ii)	Not reject (or accept) that mean time is unchanged (or is 12.4) oe	B1	
	although mean time has increased (or is more than 12.4) oe	B1 [2]	
(iii)	True (or new) mean	B1 [1]	
		[Total: 7]	

5 **(i)**	4200/80 (=52.5)	B1	
	$= \frac{80}{79}\left(\frac{229\,000}{80} - {'52.5'}^{2}\right)$ (= 107.595)	M1	
	= 108 (3 sf)	A1 [3]	
(ii)	$'52.5' \pm z\sqrt{\frac{'107.595'}{80}}$	M1	Correct form – must be *z*-value – allow one side only
	$z = 2.326$	B1	Seen
	49.8 to 55.2	A1f [3]	ft their 52.5 and 107.595. Must be an interval
(iii)	49	B1 [1]	
		[Total: 7]	
6 **(i)**	$e^{-\frac{10}{3}} \times \frac{\left(\frac{10}{3}\right)^{2}}{2}$	M1	P(2), allow any *λ*
	= 0.198 (3 sf)	A1 [2]	
(ii)	$1 - e^{-2}\left(1 + 2 + \frac{2^{2}}{2}\right)$	M1 M1	M1 allow any *λ* and/or 1end error Correct expression , correct λ
	= 0.323 (3 sf)	A1 [3]	
(iii)	$N\left(\frac{200}{3}, \frac{200}{3}\right)$	M1	seen or implied
	$\frac{49.5 - \frac{200}{3}}{\sqrt{\frac{200}{3}}}$ (= –2.102)	M1	For standardising allow either wrong or no cc No sd/var mix
	Φ('–2.102') = 1 – Φ('2.102') = 0.0178 (3 sf)	M1 A1 [4]	For finding area consistent with their working
		[Total: 9]	

7 (i)	7E(X) + 5E(Y) – 2 (= 7 × 8 + 5 × 3) – 2 = 69	M1 A1 [2]	allow incorrect means
(ii)	Var(X) = 1.6, Var(Y) = 3 16Var(X) + 9Var(Y) (= 16×1.6 + 9 × 3) = 52.6	B1 M1 M1 A1 [4]	both M1 for mult by 16 and 9; allow with '+ 3' M1 for add without '+ 3'; allow incorrect multipliers
(iii)	X = 10, Y = 2 and X = 9, Y = 0 $0.8^{10} \times e^{-3} \times \frac{3^2}{2}$ or $10 \times 0.8^9 \times 0.2 \times e^{-3}$ $0.8^{10} \times e^{-3} \times \frac{3^2}{2} + 10 \times 0.8^9 \times 0.2 \times e^{-3}$ = 0.0374/5	B1 M1 M1 A1 [4]	both pairs seen or implied or 0.0241 or 0.0134 (3sf) one correct product all correct
		[Total: 10]	

[Total for paper 50]

CAMBRIDGE INTERNATIONAL EXAMINATIONS

Cambridge International Advanced Level

MARK SCHEME for the May/June 2015 series

9709 MATHEMATICS

9709/72 Paper 7, maximum raw mark 50

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International Examinations

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PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

1	Var = 16×9 + 25×36 (= 1044) sd = 32.3 or 6√29 or √1044	B1 M1 A1	M1 for 16 (or 4^2) & 25 (or 5^2) used M1 for add any multiples of 9 and 36 only
	Total	**3**	
2 (i)	H_0: $\lambda = 0.5$ H_1: $\lambda > 0.5$	 B1 1	or Pop mean = 0.5, not just Mean = 0.5 or Pop mean (per m^2)= 0.1 Accept μ instead of λ
(ii)	$1 - e^{-0.5}(1 + 0.5)$ = 0.0902 (3 sf) comp 0.1 Claim justified or there is evidence to support claim	M1 A1 M1 A1√ 4	1 – P(X = 0,1) attempted, any λ. Allow 1 end error Allow 0.09 Valid comparison NB 0.9098>0.9 recovers M1A1 M1 oe Accept 'Reject H_0' if correctly defined No contradictions.
	Total	**5**	
3	$\lambda = 5×0.15$ (= 0.75) E(amount) = 200×0.75 = 150 Var(weekly no of hole–in–ones) = 0.75 Var(amount) = 200^2 × 0.75 = 30,000	M1 A1 B1√ M1 A1 5	 Allow 200^2 × their variance (with nothing added/subtracted at any stage) (SR probability table can score M1A0 srB1 if var rounds to 30,000 (2sf))
	Total	**5**	
4 (i)	Conclude flight times affected when in fact they have not been.	B1 B1 2	Or accept pop mean changed from 6.2 although pop mean has not changed from 6.2
(ii)	H_0: Pop mean (or μ) = 6.2 H_0: Pop mean (or μ) ≠ 6.2 $\frac{5.98-6.2}{\frac{0.8}{\sqrt{40}}}$ = –1.739 (±) Accept (±)1.74 comp z = 1.96 No evidence that flight times affected	 B1 M1 A1 B1√ 4	 Allow with 40 instead of √40 Allow SD/Var mix (CV method 5.952 or 6.2279 M1 A1) For valid comparison or P(z < –1.739) = 0.041 > 0.025 or 5.98 > 5.952 or 6.2 < 6.228 and correct conclusion
(iii)	H_0 was not rejected oe Type II	B1* B1*dep 2	If in (ii) H_0 was rejected, then: H_0 rejected B1; Type I B1dep
	Total	**8**	

5 (i)	14800/50 or 296 $\frac{50}{49}\left(\frac{4390000}{50}-'296'^2\right)$ (= 187.755) = 188 (3 sf)	B1 M1 A1 3	 Oe
(ii)	$2 \times z \times \sqrt{\frac{'187.755'}{50}} = 5.45$ oe $z = 1.406$ or 1.405 Φ('1.406') (= 0.92 or 0.9199) $\alpha = 84$ (2 sf) allow 83.98	M1 A1 M1 A1 4	If '2 ×' omitted: $z \times \sqrt{\frac{'187.755'}{50}} = 5.45$ M1 $z = 2.812$ or 2.810 A0 Φ('2.812') (= 0.9975) $\alpha = 99.5$ or 99 or 100 M1 A0 For complete method to find α SR use of biased var(184) scores M1A1(1.4205) A=84.5 M1A1
(iii)	0.96^4 = 0.849 (3 sf)	M1 A1 2	
	Total	**9**	
6 (i)	$k\int_0^{15}(225-t^2)\mathrm{d}t = 1$ $k\left[225t-\frac{t^3}{3}\right]_0^{15} = 1$ $k\times[3375 - 1125\} = 1$ or $k\times2250 = 1$ ($k = \frac{1}{2250}$ **AG**)	M1 A1 A1 3	Attempt integ f(x) and = 1. Ignore limits Correct integration and limits No errors seen
(ii)	$\frac{1}{2250}\int_{10}^{15}(225-t^2)\mathrm{d}t$ (= $\frac{1}{2250}\left[225t-\frac{t^3}{3}\right]_{10}^{15}$) = $\frac{1}{2250}$ [2250 – (2250 – $\frac{1000}{3}$)] = $\frac{4}{27}$ or 0.148 (3 sf)	M1 A1 A1 3	Attempt integ, ignore limits Or $1-\int_0^{10}$ Correct integration and limits. Condone missing k
(iii)	$\frac{1}{2250}\int_0^{15}(225t-t^3)\mathrm{d}t$ = $\frac{1}{2250}\left[\frac{225t^2}{2}-\frac{t^4}{4}\right]_0^{15}$ = $\frac{1}{2250}\left[\frac{50625}{2}-\frac{50625}{4}\right]$ = $\frac{45}{8}$ or 5.625 or 5.63 (3 sf)	M1* A1 M1*dep A1 4	Attempt integ xf(x) , ignore limits Correct integration and limits. Condone missing k Sub correct limits into their integral Accept 5 mins 37 or 38 secs
	Total	**10**	

7 (i)	Poisson (Actually binomial with) $n > 50$ and np (or λ) (= 2.1) which is < 5	B1 B1 B1 3	Allow without "binomial" Accept n large Accept p small (p < 0.1)
(ii)	$\lambda = 2.1$ $e^{-2.1}\left(1+2.1+\frac{2.1^2}{2}+\frac{2.1^3}{3!}\right)$ = 0.839 (3 sf)	B1 M1 A1 3	Attempt P(0,1,2,3) any λ allow 1 end error SR$_1$ Ft Normal N(2.1,2.1) B1 standardising M1 0.833 A1 SR$_2$ Ft Binomial B(10500,0.0002) B1 calculating binomial prob P(0,1,2,3) M1 = 0.8386 A1
(iii)	$P(X \geqslant 1) = 1 - e^{-2.1}$ (= 0. 87754) $P(X = 1,2,3) = e^{-2.1}\left(2.1+\frac{2.1^2}{2}+\frac{2.1^3}{3!}\right)$ (= 0.71619) $\frac{P(X=1,2,3)}{P(X>1)}$ $\left(=\frac{0.71619}{0.87754}\right)$ = 0.816 (3 sf)	M1 M1 M1 A1 4	Any λ Or '0.839' – $e^{-2.1}$ Any λ Allow any attempted $\frac{P(X=1,2,3)}{P(X>1)}$ Any λ SR$_1$ Ft Normal P(>0.5)=0.86523 M1 P(1,2,3)=0.698 M1 0.698/0.86523 = 0.807 M1A1 SR$_2$ FT Binomial M1 M1 M1 A1
	Total	**10**	
	Total for paper	**50**	

CAMBRIDGE INTERNATIONAL EXAMINATIONS

Cambridge International Advanced Level

MARK SCHEME for the May/June 2015 series

9709 MATHEMATICS

9709/73 Paper 7, maximum raw mark 50

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PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

1 (i)	Eg: Only students who use canteen The five will probably be friends	B1 B1 [2]	or any reason that some are excluded B1 each sensible reason must be in context
(ii)	2–digits ignore > 82 (anything too big) Ignore repeats	B1 B1 B1 [3]	
		[Total 5]	
2 (i)	H_0: P(correct) = $\frac{1}{8}$ H_1: P(correct) > $\frac{1}{8}$	B1 [1]	Or H_0 p = 1/8 H_1p > 1/8
(ii)	$1-\left(\left(\frac{1}{8}\right)^{10}+10\left(\frac{1}{8}\right)^{9}\left(\frac{7}{8}\right)+{}^{10}C_2\left(\frac{1}{8}\right)^{8}\left(\frac{7}{8}\right)^{2}\right)$ = 0.120 (3 sf) or 0.119	M1 A1 A1 [3]	M1 for attempt at correct expression accept 1 error only, e.g. 1 term extra, omitted or wrong, or omit “1–” or incorrect p/q Correct expression Note Use of Poisson in (ii) could score M1 only for expression 1 –P(0,1,2) λ =1.25
(iii)	12%	B1f [1]	ft their (ii) Must be a probability
		Total 5	
3 (i)	Var(p_s) = $\frac{0.22\times(1-0.22)}{100}$ $\left(=\frac{429}{250\,000}\text{ or }0.001716\right)$ $0.22\pm z\sqrt{'\frac{429}{250\,000}'}$ z = 2.17 or 2.168/9 or 2.171 0.13(0) to 0.31(0) (2 sf)	M1 M1 B1 A1 [4]	pq/100 Expression of correct form with their variance Any z (must be a z value) accept one side only Seen Must be an interval
(ii)	‘2’ × (1 – 0.97) × 0.97 = 0.0582	M1 A1 [2]	
		Total 6	

4 (i)	$\left(\frac{1508}{50}\right) = 30.16\ (30.2)$ $\frac{50}{49}\left(\frac{51\,825}{50} - ('30.16'^2)\right)$ = 129 (3 sf) Or 130	B1 M1 A1 [3]	Allow any form (129.46367)
(ii)	(1.5 × ‘30.16’ + 10) = 55.24 (1.5^2 × ‘129….’) = 291 (3 sf)	B1ft M1 A1ft [3]	ft their 30.16 1.5^2 × their(129) with nothing added at any stage Allow 290
		Total 6	
5 (i)	Cables broken or not all cables can be accessed oe or Too many cables oe or too time consuming oe	B1 [1]	e.g. previous days’ stocks may have gone
(ii)	H_0: Pop mean brk str (or μ) = 5 H_1: Pop mean brk str (or μ) < 5 $(\pm)\frac{4.95-5}{\frac{0.15}{\sqrt{60}}}$ (= ±2.582) comp ±2.326 There is evidence that mean breaking strength is less than it should be Or reject H_0 (H_0 correctly defined)	B1 M1 A1 B1 ft [4]	Not just “mean” Allow 60 instead of √60 Ft their –2.582 (No ft 2 tailed test) Correct comparison shown, no errors seen. Accept area comparison 0.0049 with 0.01 [CR method (x – 5)/(0.15/√60) = –2.326 M1 A1 leading to x = 4.955 compared to 4.95and correct conclusion B1ft OR ((x – 4.95)/0.15/√60) leading to 4.995 M1 A1 compared to 5and correct conclusion B1ft]
(iii)	Population not necessarily normal so yes	B1 B1dep [2]	SR B1 For “it” is not necc normal (no mention of population) AND Yes
		Total 7	

6 (i)	$e^{-3.5} \times \frac{3.5^3}{3!}$	M1	P(*X* = 3) any *λ*
	= 0.216 (3 sf)	A1 [2]	
(ii)	N(42, 42) stated or implied	B1	
	$\frac{29.5 - 42}{\sqrt{42}}$ (= –1.929)	M1	Allow with wrong or no cc <u>OR</u> without √
	P(*z* > '–1.929') = Φ('1.929') = 0.973 (3 sf)	M1 A1 [4]	For correct area consistent with their working
(iii)	(*λ*) = 2.4	B1	
	$1 - e^{-2.4}\left(1 + 2.4 + \frac{2.4^2}{2} + \frac{2.4^3}{3!}\right)$	M1 M1	for 1 – P(*X* ⩽ 3), any *λ* allow one end error Correct expression any λ
	= 0.221 (3 sf)	A1 4	NB For combination method B1 attempting 10 combinations with λ=1, λ=1.4 M1 6 expressions M1 10 expressions 0.221 A1
		Total 10	
7 (i)	$\frac{3}{4}\int_0^c (cx - x^2)dx = 1$	M1	Attempt integ f(*x*) and = 1. Ignore limits
	$\frac{3}{4}\left[\frac{cx^2}{2} - \frac{x^3}{3}\right]_0^c = 1$	A1	Correct integration and limits (condone c = 2
	$\frac{3}{4}\left(\frac{c^3}{2} - \frac{c^3}{3}\right) = 1$ or $\frac{3}{4} \times \frac{c^3}{6} = 1$ or $\frac{c^3}{8} = 1$	A1 [3]	No errors seen
	(*c* = 2 **AG**)		
(ii)	Inverted parabola Through (0, 0) and (2, 0) and zero elsewhere Median = 1	B1 B1 B1 [3]	 Must not extend beyond [0,2]
(iii)	$\frac{3}{4}\int_0^{1.5} (2x - x^2)dx$	M1	Attempt integ f(*x*) ignore limits
	$= \frac{3}{4}\left[x^2 - \frac{x^3}{3}\right]_0^{1.5}$	A1	Correct integration ignore limits
	$\frac{3}{4}\left(1.5^2 - \frac{1.5^3}{3}\right)$	B1	Use of correct limits [0,1.5] or 1–[1.5,2]
	$= \frac{27}{32}$ or 0.844 (3 sf)	A1 [4]	

(iv)	$\left(\frac{27}{32}-\frac{1}{2}\text{ or }0.844-0.5\right)$ $=\frac{11}{32}$ or 0.344 (3 sf)	B1f [1]	ft their (iii) For use of symmetry Note If do not use "hence" and start again B1 for cwo
		Total 11	

Total for paper 50

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Cambridge International Advanced Level

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1	$N(-35, 60^2 + 4 \times 28^2)$ ⋮ $N(35, 60^2 + 4 \times 28^2)$ $\frac{0-(-35)}{\sqrt{'6736'}}$ (= 0.426) ⋮ $\frac{0-35}{\sqrt{'6736'}}$ (= –0.426) 1 – Φ("0.426") = 0.335 (3 sf)	B1 B1 M1 M1 A1 5	for ±(175 – 2 × 105) or ±35 for $60^2 + 4 \times 28^2$ or 6736 For standardising with their mean and variance. Allow without √ For use of tables and finding area consistent with working
		Total: 5	
2 (i)	(Bin) with $n > 50$ and mean (or np) < 5 Po(1.5) $1 - e^{-1.5}$ = 0.777 (3 sf)	B1 B1 M1 A1 4	Accept n 'large', p 'small' Poisson with correct mean stated or implied Poisson 1 – $P(X = 0)$; allow incorrect λ; allow 1 end error SR If zero scored use of Bin leading to 0.778 / 0.779 scores B1
(ii)	3.5 $e^{-3.5}\left(\frac{3.5^4}{4!} + \frac{3.5^5}{5!} + \frac{3.5^6}{6!}\right)$ = 0.398 (3 sf)	B1 M1 A1 3	Correct mean stated or implied Poisson $P(X = 4, 5, 6)$; allow incorrect λ; allow 1 end error
		Total: 7	
3 (a)	$\int_0^{0.5} \left(1.5t - 0.75t^2\right)dt$ o.e. $= \left[0.75t^2 - 0.25t^3\right]_0^{0.5}$ o.e. $= \frac{5}{32}$ or 0.156 (3 sf)	M1 A1 A1 3	Attempt int f(t) Correct integration and limits
(b) (i)	$\frac{1}{2}\pi a^2 = 1$ or $\pi a^2 = 2$ oe $a = \sqrt{\frac{2}{\pi}}$ or 0.798 (3 sf)	M1 A1 2	Attempt to find the area and equate to 1
(ii)	0	B1 1	
(iii)	Symmetry stated, seen or implied 0.8	M1 A1 2	Could be a diagram As final answer
		Total: 8	
4 (i)	$\text{Var}(P_s) = \frac{\frac{33}{150} \times \frac{150-33}{150}}{150}$ (= 0.001144) $z = 2.576$ $\frac{33}{150} \pm z\sqrt{'0.001144'}$ = 0.133 to 0.307 (3 sf)	M1 B1 M1 A1 4	 Seen. Accept 2.574 to 2.579 Expression of correct form. Any z Must be an interval

(ii)	$\frac{19035}{150}$ (= 126.9 =127(3sf))	B1	
	$\frac{150}{149}\left(\frac{4054716}{150}-\left(\frac{19035}{150}\right)^2\right)$ o.e.	M1	For use of a correct formula
	= 11001.17 or 11000(3 sf)	A1 3	
(iii)	4-digit nos. each digit 0-9 Ignore nos > 9526 Ignore repeats	B1 B1 B1 3	Some valid way of generating 4 digit random nos from valid method from valid method SR If zero score, full explanation of method for drawing numbers out of a hat can score B1. NB Systematic sampling follows the scheme with first B1 for some way of generating a random starting point.
		Total: 10	
5 (i)	$\frac{4.8}{\sqrt{40}}$	B1	or $\frac{4.8^2}{40}$. Accept $4.8\sqrt{40}$ or $4.8^2 \times 40$ for totals method
	$\frac{50.3-49.5}{\frac{4.8}{\sqrt{40}}}$ (= 1.054)	M1	For standardising with their SD Accept ± Accept totals method. No mixed methods
	1 – Φ(‘1.054’)	M1	For use of tables and finding area consistent with their working
	= 0.146 (3 sf)	A1 4	
(ii) (a)	Looking for decrease	B1 1	
(b)	H_0: Pop mean time spent (or μ) = 49.5 H_1: Pop mean time spent (or μ) < 49.5	B1	Not just “mean time spent”
	$\frac{\frac{1920}{40}-49.5}{\frac{4.8}{\sqrt{40}}}$ (= –1.976)	M1	For standardising. Allow $\div\frac{4.8}{40}$ Accept totals method; CV method. No mixed methods
	‘1.976’ > 1.555 (or ‘–1.976’ < –1.555)	M1	For valid comparison (area comparison 0.024 < 0.06)
	There is evidence that mean time has decreased.	A1 4	CWO. No contradictions in conclusions
(c)	Population normally distr so No	B1 1	Both needed
		Total: 10	

6 (i)	$\lambda = 4.65$ $e^{-4.65} \times \frac{4.65^4}{4!}$ $= 0.186$ (3 sf)	B1 M1 A1	 3	 Poisson P(X = 4) with any λ
(ii)	$\lambda = 3.875$ $= e^{-3.875}\left(1 + 3.875 + \frac{3.875^2}{2!}\right) = 0.257$ (3 sf)	B1 M1 A1	 3	P(X = 0, 1, 2) Attempted, any λ As final answer
(iii)	$\lambda = 1.5$ $1 - e^{-1.5}\left(1 + 1.5 + \frac{1.5^2}{2!}\right)$ $= 0.191$ (3 sf)	B1 M1 A1	 3	1 – P(X = 0, 1, 2) Attempted, any λ As final answer
(iv)	He will reject H_0.	B1	1	
		Total: 10		

CAMBRIDGE INTERNATIONAL EXAMINATIONS

Cambridge International Advanced Level

MARK SCHEME for the October/November 2014 series

9709 MATHEMATICS

9709/72 Paper 7, maximum raw mark 50

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

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CAMBRIDGE
International Examinations

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A	Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
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CWO	Correct Working Only – often written by a 'fortuitous' answer
ISW	Ignore Subsequent Working
MR	Misread
PA	Premature Approximation (resulting in basically correct work that is insufficiently accurate)
SOS	See Other Solution (the candidate makes a better attempt at the same question)
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

1	N(–35, $60^2 + 4 \times 28^2$) ⋮ N(35, $60^2 + 4 \times 28^2$) $\frac{0-(-35)}{\sqrt{'6736'}}$ (= 0.426) ⋮ $\frac{0-35}{\sqrt{'6736'}}$ (= –0.426) 1 – Φ("0.426") = 0.335 (3 sf)	B1 B1 M1 M1 A1 5	for ±(175 – 2 × 105) or ±35 for $60^2 + 4 \times 28^2$ or 6736 For standardising with their mean and variance. Allow without √ For use of tables and finding area consistent with working
		Total: 5	
2 (i)	(Bin) with $n > 50$ and mean (or np) < 5 Po(1.5) $1 - e^{-1.5}$ = 0.777 (3 sf)	B1 B1 M1 A1 4	Accept n 'large', p 'small' Poisson with correct mean stated or implied Poisson 1 – P($X = 0$); allow incorrect λ; allow 1 end error SR If zero scored use of Bin leading to 0.778 / 0.779 scores B1
(ii)	3.5 $e^{-3.5}\left(\frac{3.5^4}{4!}+\frac{3.5^5}{5!}+\frac{3.5^6}{6!}\right)$ = 0.398 (3 sf)	B1 M1 A1 3	Correct mean stated or implied Poisson P(X = 4, 5, 6); allow incorrect λ; allow 1 end error
		Total: 7	
3 (a)	$\int_0^{0.5}\left(1.5t - 0.75t^2\right)\mathrm{d}t$ o.e. $= \left[0.75t^2 - 0.25t^3\right]_0^{0.5}$ o.e. $= \frac{5}{32}$ or 0.156 (3 sf)	M1 A1 A1 3	Attempt int f(t) Correct integration and limits
(b) (i)	$\frac{1}{2}\pi a^2 = 1$ or $\pi a^2 = 2$ oe $a = \sqrt{\frac{2}{\pi}}$ or 0.798 (3 sf)	M1 A1 2	Attempt to find the area and equate to 1
(ii)	0	B1 1	
(iii)	Symmetry stated, seen or implied 0.8	M1 A1 2	Could be a diagram As final answer
		Total: 8	
4 (i)	Var(P_s) = $\frac{\frac{33}{150}\times\frac{150-33}{150}}{150}$ (= 0.001144) $z = 2.576$ $\frac{33}{150} \pm z\sqrt{'0.001144'}$ = 0.133 to 0.307 (3 sf)	M1 B1 M1 A1 4	Seen. Accept 2.574 to 2.579 Expression of correct form. Any z Must be an interval

(ii)	$\frac{19035}{150}$ (= 126.9 =127(3sf)) $\frac{150}{149}\left(\frac{4054716}{150}-\left(\frac{19035}{150}\right)^2\right)$ o.e. = 11001.17 or 11000(3 sf)	B1 M1 A1 3	 For use of a correct formula
(iii)	4-digit nos. each digit 0-9 Ignore nos > 9526 Ignore repeats	B1 B1 B1 3	Some valid way of generating 4 digit random nos from valid method from valid method SR If zero score, full explanation of method for drawing numbers out of a hat can score B1. NB Systematic sampling follows the scheme with first B1 for some way of generating a random starting point.
		Total: 10	
5 (i)	$\frac{4.8}{\sqrt{40}}$ $\frac{50.3-49.5}{\frac{4.8}{\sqrt{40}}}$ (= 1.054) 1 – Φ('1.054') = 0.146 (3 sf)	B1 M1 M1 A1 4	or $\frac{4.8^2}{40}$. Accept $4.8\sqrt{40}$ or $4.8^2 \times 40$ for totals method For standardising with their SD Accept ± Accept totals method. No mixed methods For use of tables and finding area consistent with their working
(ii) (a)	Looking for decrease	B1 1	
(b)	H_0: Pop mean time spent (or μ) = 49.5 H_1: Pop mean time spent (or μ) < 49.5 $\frac{\frac{1920}{40}-49.5}{\frac{4.8}{\sqrt{40}}}$ (= –1.976) '1.976' > 1.555 (or '–1.976' < –1.555) There is evidence that mean time has decreased.	B1 M1 M1 A1 4	Not just "mean time spent" For standardising. Allow ÷ $\frac{4.8}{40}$ Accept totals method; CV method. No mixed methods For valid comparison (area comparison 0.024 < 0.06) CWO. No contradictions in conclusions
(c)	Population normally distr so No	B1 1	Both needed
		Total: 10	

6 (i)	$\lambda = 4.65$ $e^{-4.65} \times \frac{4.65^4}{4!}$ $= 0.186$ (3 sf)	B1 M1 A1 3	 Poisson P($X = 4$) with any λ
(ii)	$\lambda = 3.875$ $= e^{-3.875}\left(1 + 3.875 + \frac{3.875^2}{2!}\right) = 0.257$ (3 sf)	B1 M1 A1 3	P($X = 0, 1, 2$) Attempted, any λ As final answer
(iii)	$\lambda = 1.5$ $1 - e^{-1.5}\left(1 + 1.5 + \frac{1.5^2}{2!}\right)$ $= 0.191$ (3 sf)	B1 M1 A1 3	 1 – P($X = 0, 1, 2$) Attempted, any λ As final answer
(iv)	He will reject H_0.	B1 1	
		Total: 10	

CAMBRIDGE INTERNATIONAL EXAMINATIONS
Cambridge International Advanced Level

MARK SCHEME for the October/November 2014 series

9709 MATHEMATICS

9709/73 Paper 7, maximum raw mark 50

CAMBRIDGE
International Examinations

Mark Scheme Notes

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Penalties

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PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

1 (i)	"Different" being investigated	**B1**	**[1]**	Oe ("changed", "not equal to")
(ii)	H_0: Pop mean (or μ) in region same as elsewhere			Must be "pop mean", not just "mean" Can be awarded in (i)
	H_1: Pop mean (or μ) in region diff from elsewhere	**B1**		oe
	$1.91 < 2.054$ (or 2.055) or $-1.91 > -2.054$	**M1**		or $P(z > 1.91) = 0.0281 > 0.02$ or $0.0562 > 0.04$ or $0.972 < 0.98$ Accept 2.05 if nothing better seen.
	No evidence that mean is different	**A1**	**[3]**	inequality sign incorrect M1A0 no contradictions "accept H_0" provided H_0 reasonably well defined
Total			**[4]**	
2 (i)	$\frac{1}{2}c^2 = 1$	**M1**		Area of triangle = 1 or integral of kx with limits 0 and c and equated to 1
	$c = \sqrt{2}$ or 1.41 (3 sf)	**A1**	**[2]**	
(ii)	$f(x) = x$ or $y = x$	**B1**		Seen or implied, e.g. by next line. Can be awarded anywhere in the question. Implied by $(a + 1)$ in area of trapezium.
	$\int_a^1 x\,dx = 0.1$	**M1**		Ignore limits. Must be integral of kx and equated to 0.1. Or trapezium area.
	$\left[\frac{x^2}{2}\right]_a^1 = 0.1$	**A1√**		Correct limits, ft incorrect kx.
	$1 - a^2 = 0.2$ $a = 0.894$ (3 sf)	**A1**	**[4]**	$\sqrt{\left(\frac{4}{5}\right)}$ oe
(iii)	$\int_0^{\sqrt{2}} x^2\,dx$ $\left[\frac{x^3}{3}\right]_0^{\sqrt{2}}$	**M1**		Ignore limits; ft their f(x) but not $\int x\,dx$
	$= \frac{2}{3}\sqrt{2}$ **or 0.943 or** $\sqrt{\left(\frac{8}{3}\right)}$	**A1√**	**[2]**	ft their c, dep $0 <$ ans $<$ their c. Not ft their f(x)
Total			**[8]**	

3 (i)	$\text{Est}(\mu)=\frac{7220}{80}$ or 90.25	B1		Accept 90.3
	$\text{Est}(\sigma^2)=\frac{80}{79}\left(\frac{656060}{80}-\left(\frac{7220}{80}\right)^2\right)$	M1		$\frac{1}{79}\left(656060-\frac{7220^2}{80}\right)$
	$= 56.3924$ or $\frac{4455}{79}$	A1		
	$z = 2.17$	B1		Accept 56.4
	$\frac{7220}{80}\pm z\times\sqrt{\frac{'56.3924'}{80}}$	M1		Expression of correct form
	= 88.4 to 92.1 (3 sf)	A1	[6]	Must be an interval (N.B. biased var gives 88.4 to 92.1 scores possible B1M0A0B1M1A1)
(ii)	Pop normal No	B1 B1dep	[2]	X normal or full definition of pop normal SR B1 for “no” and relevant reference to normal
Total			[8]	
4 (i)	$4 \times 125 + 6 \times 130$ (= 1280) $4 \times 30^2 + 6 \times 32^2$ (= 9744)	B1 B1		Give at early stage. Could be implied by 220. (If B0B0 then 1.28 and 0.009744 can score B1B1).
	$(\pm)\frac{1500-1280}{\sqrt{9744}}(= 2.229)$	M1		Standardising. Accept sd/var mix. Must be from combination attempt.
	$\Phi(\text{"}2.229\text{"})$	M1		Use of tables and correct area consistent with their working
	= 0.987 (3 sf)	A1	[5]	cwo
(ii)	$125 - 0.9(130)$ (= 8) (or –8) $30^2 + 0.9^2(32^2)$ (= 1729.44)	B1 B1		Give at early stage. (If B0B0 scored then accept 0.008 and 0.0017944 for B1B1)
	$(\pm)\frac{0-'8'}{\sqrt{'1729.44'}}$ (= –0.192)	M1		Accept sd/var mix. Must come from a linear combination.
	$\Phi('0.192')$	M1		Use of tables and correct area consistent with their working (unclear M0)
	= 0.576 (3 sf)	A1	[5]	
Total			[10]	

5 (i)	H_0: population proportion = 0.1 oe H_1: population proportion > 0.1 oe	**B1**		Allow "p = 0.1" and "p > 0.1"
	$P(X \geqslant 4) = 1 - P(X \leqslant 3) =$ $1 - \left(\begin{matrix} 0.9^{18} + 18 \times 0.9^{17} \times 0.1 + \\ {}^{18}C_2 \times 0.9^{16} \times 0.1^2 + {}^{18}C_3 \times 0.9^{15} \times 0.1^3 \end{matrix}\right)$	**M1**		Allow 1 – (one term omitted or extra or wrong)
	= 0.0982 (3 sf)	**A1**		(note CR method 0.0982 and CR ⩾ 5 for A1)
	Comp 0.08	**M1**		Valid comparison (0.9018 < 0.92 also recovered previous A1). Or 4 is not in CR
	No evidence that more reach 1m	**A1√**	**[5]**	Dep M1M1 no contraditions "Accept H_0" provided H_0 defined
(ii)	Not rejected H_0 Type II	**B1√** **B1dep √**	**[2]**	Ft their (i) If (i) "reject H_0" then ft gives Type I error
(iii)	$P(X \geqslant 5)$ $(= 0.0282)$ 0.0282 < 0.08	**M1** **B1√**		Attempt $P(X \geqslant 5)$ e.g. '0.0982' – ${}^{18}C_4 \times 0.9^{14} \times 0.1^4$ oe. Valid comp of their ⩾ 5 (if CR method used, could be awarded in (i))
	P(Type I error) = 0.0282 (3 sf)	**A1**	**[3]**	
Total			**[10]**	
6 (i)	$e^{-3.84} \times \frac{3.84^4}{4!}$	**M1**		Poisson P(X = 4), any λ
	= 0.195 (3 sf)	**A1**	**[2]**	
(ii)	1.44	**B1**		Seen
	$1 - e^{-1.44}\left(1 + 1.44 + \frac{1.44^2}{2}\right)$	**M1**		Any λ, allow one end error, need "1 –…"
	= 0.176	**A1**	**[3]**	
(iii)	X~N(41, 41)	**B1**		Seen or implied
	$\frac{40.5 - 41}{\sqrt{41}}$ (= –0.078) $\frac{59.5 - 41}{\sqrt{41}}$ (= 2.889)	**M1M1**		M1M0 if no cc or incorrect cc OR no √ in both
	Φ('2.889') – Φ('–0.078') = Φ('2.889') – (1 – Φ('0.078')) = 0.9981 – (1 – 0.5311)	**M1**		Use of tables and correct area consistent with their working.
	= 0.529 (3sf)	**A1**	**[5]**	cwo
Total			**[10]**	

CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Level

MARK SCHEME for the May/June 2014 series

9709 MATHEMATICS

9709/71 Paper 7, maximum raw mark 50

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CAMBRIDGE
International Examinations

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PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Note: “(3 sfs)” means “answer which rounds to ... to 3 sfs”. If correct ans seen to ≥ 3sfs, ISW for later rounding. Penalise < 3 sfs only once in <u>paper.</u>

1	N(483.2, 537.92) or N(483.2, 23.2^2) $\frac{436-483.2}{\sqrt{537.92}}$ or $\frac{436-483.2}{23.2}$ (= –2.035) Φ(“–2.035”) = 1 – Φ(“2.035”) = 0.021 or 2.1%	B1 M1 M1 A1 [4]	or $\frac{8.2}{\sqrt{8}}$ or $\frac{8.2^2}{8}$ seen or implied or $\frac{\frac{436}{8}-60.4}{8.2/\sqrt{8}}$ standardising (no mixed methods) Correct area consistent with their working
		[Total: 4]	
2	$\frac{70}{69}\times 2.70$ = 2.73913 $3.61 \pm z\sqrt{\frac{\text{"2.73913"}}{70}}$ z = 1.96 3.22 to 4.00 (3 sf)	M1A1 M1 B1 A1 [5]	or $3.61 \pm z\sqrt{\frac{2.70}{69}}$ M2A1(implied) without $\frac{70}{69}$: $3.61 \pm z\sqrt{\frac{2.70}{70}}$ M0A0M1 z = 1.96 B1 3.23 to 3.99(4.00) (3 sf) A1 Answer must be an interval
		[Total: 5]	
3	H_0: μ = 250 H_1: μ > 250 $\frac{250.06-250}{0.2\div\sqrt{40}}$ = 1.90 comp with z = 1.645 Claim is justified or There is evidence that claim is true	B1 M1 A1 M1 A1√ [5]	Both hypotheses M1 for standardising, must have √40. Accept cv method For valid comparison “1.90” with 1.645 or area comparison or CVs Correct conclusion. No contradictions NB 2-tail test scores B0 M1 A1 M1 (use 1.96) A0
		[Total: 5]	
4 (i)	B(3500, 0.001) Poisson with mean = 3.5 n > 50 and np < 5	B1 B1 B1 [3]	or Po(3.5) Both. Or n > 50 and λ < 5 or 3.5 < 5
(ii)	$e^{-3.5}(1 + 3.5 + \frac{3.5^2}{2}+\frac{3.5^3}{3!})$ = 0.537 (3 dp)	M1 A1 [2]	Allow any λ
		[Total: 5)	

5 (i)	$0.25(1 + 4 + 9) - 1.5^2$ (=1.25 **AG**)	B1 [1]	
(ii)	$\frac{1.4-1.5}{\sqrt{\frac{5}{4}\div 300}}$ (= –1.549) Φ(“–1.549”) = 1 – Φ(“1.549”) = 0.0607 (3 sf)	M1 M1 A1 [3]	$\frac{1.4-\frac{1}{600}-1.5}{\sqrt{\frac{5}{4}\div 300}}$ (= –1.523) Φ(“–1.523”) = 1 – Φ(“1.523”) = 0.0639 (3 sf)
(iii)	Large sample or large *n* ($\overline{X}$ (approx) normally distr) or Central Limit Theorem	B1 [1]	
		[Total: 5]	
6 (i)	H_0: Rate = 0.9 H_1: Rate < 0.9 1 – P(17, 18, 19, 20) $1 - ({}^{20}C_{17} \times 0.1^3 \times 0.9^{17} + {}^{20}C_{18} \times 0.1^2 \times 0.9^{18} + 20 \times 0.1 \times 0.9^{19} + 0.9^{20})$ = 0.133 (3 sf)	B1 M1 M1 A1 [4]	p = 0.9 p < 0.9 Use of B(20,0.1) Allow 1–P(18,19,20) or 1–P(16,17,18,19,20)
(ii)	Type II H_0 will not be rejected	B1 B1 [2]	or Stephan will conclude standard not fallen No contradictions
		[Total: 6]	

7 (i)	$\int_1^a \frac{k}{x}\mathrm{d}x = 1$	M1	Int f(x) & equate to 1. Ignore limits
	$k[\ln x]_1^a = 1$	A1	Correct integration and limits and = 1
	$k\ln a = 1 \quad k = 1/\ln a$	A1 [3]	AG
(ii)	$\frac{1}{\ln a}\int_1^a 1\mathrm{d}x$ or $k\int_1^a 1\mathrm{d}x$	M1	Int xf(x). Ignore limits
	$= \frac{1}{\ln a}[x]_1^a$ or $k[x]_1^a$	A1	Correct integration and limits (condone missing k)
	$= \frac{1}{\ln a}(a - 1)$	A1 [3]	
(iii)	$\frac{1}{\ln a}\int_1^m \frac{1}{x}\mathrm{d}x = 0.5$	M1	Int f(x) and equate to 0.5. Ignore limits
	$\frac{1}{\ln a}[\ln x]_1^m = 0.5$	A1	Correct integration and limits (1 to m or m to a) (condone missing k)
	$\frac{1}{\ln a}\ln m = 0.5$		
	$\ln m = 0.5\ln a$	A1	or $\ln m = \ln a^{0.5}$
	$m = \sqrt{a}$	A1 [4]	
		[Total: 10]	
8 (i)	V: cannot have neg value W: cannot have non-integer value	B1 B1 [2]	
(ii)	(a) $e^{-\lambda} = p$ and $\lambda e^{-\lambda} = 2.5p$ (Hence $\lambda = 2.5$ **AG**)	B1 [1]	or equiv explanation
(ii)	(b) $1 - e^{-2.5}(1 + 2.5 + \frac{2.5^2}{2})$	M1	Allow one end error
	$= 0.456$ (3 sf)	A1 [2]	
(iii)	$\Phi^{-1}(0.5793) = -0.2$	B1	
	N(μ, μ) seen or implied	M1	
	$\frac{40.5-\mu}{\sqrt{\mu}} =$ "–0.2"	M1	Allow no cc or incorrect cc
	$\mu +$ "–0.2" $\sqrt{\mu} - 40.5 = 0$		
	$\sqrt{\mu} = \frac{"0.2" \pm \sqrt{"0.2"^2 + 4\times 40.5}}{2}$ (= 6.4647..)	M1	For solving quadratic in $\sqrt{\mu}$ (or μ) Ignore other answer for $\sqrt{\mu}$, but not for μ
	$\mu = 41.8$ (3 sf)	A1 [5]	
		[Total: 10]	

[Total for paper 50]

CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Level

MARK SCHEME for the May/June 2014 series

9709 MATHEMATICS

9709/72 Paper 7, maximum raw mark 50

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the May/June 2014 series for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level components and some Ordinary Level components.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF	Any Equivalent Form (of answer is equally acceptable)
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BOD	Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)
CAO	Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
CWO	Correct Working Only – often written by a 'fortuitous' answer
ISW	Ignore Subsequent Working
MR	Misread
PA	Premature Approximation (resulting in basically correct work that is insufficiently accurate)
SOS	See Other Solution (the candidate makes a better attempt at the same question)
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through ✓" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

1	$\frac{\Sigma x}{8} = \frac{2006}{8}$ = 250.75 or 251 (3 s.f.)	B1	Any equivalent form
	($\Sigma x^2 = 503274$)		
	$\frac{8}{7}\left(\frac{"503274"}{8} - "250.75"^2\right)$	M1	For use of formula of correct form
	= 38.5 o.e. (accept 6.204^2)	A1 [3]	cao (as final answer)
2	$(X + Y - Z) \sim \text{N}(8, \ldots)$		
	μ=8 (or –8)	B1	seen or implied
	$\text{Var}(X + Y - Z) = 2^2 + 1.5^2 + 1.8^2$ (= 9.49)	B1	– award at early stage
	$\frac{0-8}{\sqrt{'9.49'}}$ (= –2.597)	M1	For standardising (accept sd/var mixes, but variance must be a combination of at least 2 of X, Y, Z)
	Φ('–2.597') = 1 – Φ('2.597')	M1	For area consistent with their working
	= 0.0047	A1 [5]	
3	H_0: Pop mean (or μ or λ) = 50 (or 5)		
	H_1: Pop mean (or μ or λ) $\neq$ 50 (or 5)	B1	Not just "mean"
	$\frac{60.5-50}{\sqrt{50}}$ (±)	M1	For standardising with N(50,50) or N(5,5/√10)
	= (±)1.485 OR 0.0687 OR C.V	A1	Allow M1 with wrong or no continuity correction OR no √ (accept c.v method M1, A1 for 61.63 or 48.868)
	1.485 < 1.645 or 0.0687 > 0.05	M1	
	No evidence that mean changed	A1√ [5]	For valid comparison (zs or areas or cv) (S.R For cv comparison 61.63 only award final A1 if cc used) or if H_1: $\lambda > 50$, 1.485 < 1.96 M1 No evid mean changed A0 (i.e. if one-tail test, max B0 M1 A1 M1 A0)

4 **(i)**	$\lambda = 4.5$	B1	seen
	$1 - e^{-4.5}\left(1+4.5+\frac{4.5^2}{2}\right)$	M1	any λ. Allow one end error
	$= 0.826$ (3 s.f.)	A1 [3]	
(ii)	$e^{-\lambda} = 0.523$	B1	
	$(-\lambda = \ln 0.523)$		
	$\lambda = 0.648$ (3 s.f.)	B1 [2]	
(iii)	$e^{-\mu} \times \frac{\mu 3}{3!} = 24 \times e^{-\mu} \times \mu$	B1	
	$\frac{\mu 2}{6} = 24$	M1	For a simplified expression in μ^2 with $e^{-\mu}$ and μ cancelled and no factorials.
	$\mu = 12$	A1 [3]	
5 **(i)**	$p = \frac{184}{400}$ or 0.46	B1	Used
	$z = 1.96$	B1	Seen
	$"0.46" \pm z \times \sqrt{\frac{"0.46"(1-"0.46")}{400}}$	M1	Using expression of correct form
	$= 0.411$ to 0.509	A1 [4]	Must be an interval
(ii)	0.5 within CI		
	Claim not supported or not justified	B1√ [1]	Both needed. No contradictions. ft their **(i)**
(iii)	$z \times \sqrt{\frac{"0.46"(1-"0.46")}{400}} = 0.05$	M1	Allow M1 for $z \times \sqrt{\frac{"0.46"(1-"0.46")}{400}} = 0.1$
	$z = 2.006$	A1	
	$\Phi('2.006') = 0.9775$		
	$\alpha = '0.9775' - (1 - '0.9775')$	M1	or $1 - 2(1 - '0.9775')$
	$= 95.5\%$	A1 [4]	

6 (i)	$k\int_0^4 (16t - t^3)\mathrm{d}t = 1$	M1	Int f(t) = 1 ignore limits
	$k\left[8t^2 - \frac{t^4}{4}\right]_0^4 = 1$	A1	correct integration with correct limits
	$k(128 - 64) = 1$ o.e.		
	$k \times 64 = 1$		
	$\left(k = \frac{1}{64}\right)$ **AG**	A1 [3]	must be convinced (AG)
(ii)	$\frac{1}{64}\int_0^1 (16t - t^3)\mathrm{d}t$	M1	Int f(t) between 0 and 1 (accept 0 and a value < 1 , 1 and 4)
	$= \frac{1}{64}\left[8t^2 - \frac{t^4}{4}\right]_0^1$	A1	correct integration and correct limits (ignore “k”)
	$= \frac{1}{64}\left[8 - \frac{1}{4}\right]$		
	$= \frac{31}{256}$ or 0.121094	A1	
	$\left(\frac{31}{256}\right)^2 = 0.0147$ (3 s.f.) o.e.	B1√ [4]	ft their “$\frac{31}{256}$”
iii	$\frac{1}{64}\int_0^4 (16t^2 - t^4)\mathrm{d}t$	M1	Int tf(t) ignore limits
	$= \frac{1}{64}\left[\frac{16t^3}{3} - \frac{t^5}{5}\right]_0^4$	A1	correct integration and correct limits (ignore “k”)
	$= \frac{1}{64}\left(\frac{1024}{3} - \frac{1024}{5}\right)$		
	$= \frac{32}{15}$ or 2.13 (3 s.f.) o.e.	A1 [3]	

7 (i)	2nd More representative of all appointments or Lengths may vary during the day or 1st does not include later appts so not representative	B1 B1 [2]	Any implication that times or conditions vary throughout day, e.g. doctors get tired	
(ii)	0.01 o.e. Concluding that times spent are too long when they are not.	B1 B1 [2]	Concluding that the mean time spent is more than 10 mins when it is not. Must be in context.	
(iii)	H_0: Pop mean appt time (or μ) = 10 H_1: Pop mean appt time (or μ) > 10	B1	Both correct. Allow μ, but not just "mean"	
	$\dfrac{\frac{147}{12}-10}{\frac{3.4}{\sqrt{12}}}$ (±)	M1	Allow incorrect $\frac{147}{12}$ Must have √12 (accept totals method)	$10 + 2.326 \times \frac{3.4}{\sqrt{12}}$ M1
	= (±)2.292 or (0.0109 if area comparison done)	A1		= 12.28 A1
	"2.292" < 2.326 o.e.	M1	For valid comparison Comp "2.292" with 2.326 Or 0.0109 with 0.01 Or 147/12 with 12.28	$\frac{147}{12}$ < 12.28 M1
	(No evidence to reject H_0.) No reason to believe appts are too long	A1√ [5]	Dep 2.326, ft their "2.292" No contradictions	
(iv)	Normal population	B1 [1]	Must have "population" or equiv	

CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Level

MARK SCHEME for the May/June 2014 series

9709 MATHEMATICS

9709/73 Paper 7, maximum raw mark 50

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CAMBRIDGE
International Examinations

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PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

1	$e^{-4}(1 + 4)$	M1	M1 for P(0 or 1) using Poisson, any λ
		M1	Expression of correct form correct λ (allow 1 end error)
	$= 0.0916$ (3 s.f.)	A1 [3]	SR Use of Bin(100000, 1/25000) scores M1 for P(0,1) allow one end error. A1 0.0916
2	ht = $\frac{1}{2}$ seen	B1	or $y = \frac{1}{8}x$
	$\frac{1}{2} \times m \times \left(\frac{m}{4} \times "\frac{1}{2}"\right) = \frac{1}{2}$	M1	$\frac{1}{2} \times m \times ("\frac{1}{8}"m) = \frac{1}{2}$ or $\frac{m^2}{16} = \frac{1}{2}$ o.e.
	N.B. B1 M1 must be consistent		Or Integrating linear function of form $y = kx$ with limits 0 and m or m and 4 and equated to 0.5
	$m = \sqrt{8}$ or $2\sqrt{2}$ or 2.83 (3 s.f.)	A1 [3]	
3	$p = 0.56$	B1	Used
	'0.56' $\pm z \times \sqrt{\frac{0.56 \times 0.44}{100}}$	M1	Equation of correct form condone just +ve or –ve Must be z
	$z = 2.17$, or 2.169 or 2.171	B1	
	0.452 to 0.668 (3 s.f.)	A1 [4]	Seen Must be an interval
4	$\bar{x} = 1.65$	B1	
	$\text{est}(\sigma^2) = \frac{100}{99}\left(\frac{276.25}{100} - 1.65^2\right)$	B1	
	$= 0.040404\ldots = 4/99$		
	$(\pm)\frac{1.65 - 1.6}{\sqrt{\frac{"0.040404"}{100}}}$	M1	Without $\frac{100}{99}$: $\frac{1.65 - 1.6}{\sqrt{\frac{"0.04"}{100}}}$ B1 B0 M1
	= (±) 2.487/2.488 accept 2.49 Or 0.0065/0.0064 if area comparison done	A1	= 2.50 A1 CV Method M1 must use 1.96 A1 for 1.639 or 1.6106
	comp with 1.96	M1	For valid comparison (z/z Signs consistent or area/area cv)
	There is evidence that μ is not 1.6	A1√ [6]	Accept Reject H_0 No contradictions

5 (i)	Longest lifetime	B1 [1]	Must be in context
(ii)	$\int_1^a \frac{k}{x^2}\mathrm{d}x = 1$	M1	Int f(x) and equate to 1. Ignore limits
	$k\left[-\frac{1}{x}\right]_1^a = 1$	A1	Correct integral and limits
	$\left(k\left[-\frac{1}{a}+1\right]=1\right)$		
	$k\left[\frac{-1+a}{a}\right] = 1$ or $k(-1+a) = a$		
	$k = \frac{a}{a-1}$ **AG**	A1 [3]	Must be convinced (**AG**)
(iii)	$\frac{5}{3}\int_1^{2.5}\frac{1}{x}\mathrm{d}x$ or $k\int_1^{2.5}\frac{1}{x}\mathrm{d}x$	M1	Int xf(x). Ignore limits
	$= \frac{5}{3}[\ln x]_1^{2.5}$ or $k[\ln x]_1^{2.5}$	A1	Correct integral and limits (Accept "k" or "their k")
	$= \frac{5}{3}\ln 2.5$ or 1.53 (3 s.f.)	A1 [3]	
6 (i)	H_0: $p = 0.2$ H_1: $p < 0.2$	B1	(Allow π)
	P(0 or 1 5s in 25 $\mid H_0$)	M1	$0.8^{25} + 25 \times 0.8^{24} \times 0.2$ Use of B(25,1/5) and P(0) or P(1) or both – may be implied by "0.0274"
	= 0.0274 (3 s.f.)	A1	
	Comp with 0.025	M1	Valid comparison
	No evidence (at 2.5% level) to support claim	A1√ [5]	No contradictions **SR** Use of Normal N(5,4) leading to $z = 1.75$ or 0.0401 **B1*** H_0 $\mu = 5$ H_1 $\mu < 5$ B1. Comparison $1.75 < 1.96$ or $0.0401 > 0.025$ **B1* dep**
(ii)	Normal	B1	
	$\mu = 200$, $\sigma^2 = 160$ or $\sigma = \sqrt{160}$	B1 [2]	
(iii)	Concluding that the machine produces the right proportion of 5s, although it doesn't.	B1 [1]	Not concluding that the machine produces too few 5s although it does. Must be in context o.e. No contradictions

7 (i)	Constant mean (or average) rate	B1 [1]	Constant mean per day (or week, etc.) o.e.
(ii)	$e^{-\frac{4}{7}} \times \frac{{\frac{4}{7}}^{2}}{2!}$ or $e^{-0.571} \times \frac{0.571^2}{2!}$	M1	Expression for P(2) allow any λ
	= 0.0922 or 0.0921 (3 s.f.)	A1 [2]	
(iii)	$\lambda = \frac{40}{7}$ or 5.71…	B1	
	$1 - e^{-\frac{40}{7}}\left(1+\frac{40}{7}+\frac{{\frac{40}{7}}^{2}}{2!}+\frac{{\frac{40}{7}}^{3}}{3!}\right)$	M1	Allow any λ allow one end error
	= 0.821 (3 s.f.)	A1 [3]	
(iv)	$\frac{24}{7}$ o.e. 3 s.f. or better seen	B1	
	$e^{-\frac{4}{7}} \times e^{-\frac{24}{7}} \times \frac{{\frac{24}{7}}^{5}}{5!}$	M1	M1 for P(0) × P(5) any consistent λ
	= 0.0723 (3 s.f.)	A1 [3]	
8 (i)	$X + 2.5Y \sim \text{N}(127, 44.25)$	B1 B1	B1 for 127 Allow at early stage (57 + 2.5 × 28) B1 for 44.25 or 6.65 Allow at early stage ($13 + 2.5^2 \times 5$) May be implied by next line
	$(\pm)\frac{140-\text{"127"}}{\sqrt{\text{"44.25"}}}$	M1	For standardising
	= ±(1.954)	M1	For area consistent with their working
	1 – Φ("1.954")		
	= 0.0254/0.0253 (3 s.f.)	A1 [5]	
(ii)	$X - Y \sim \text{N}(29, 18)$	B1 B1	B1 for 29 Give at early stage (57 – 28) B1 for 18 Give at early stage (13 + 5) May be implied by next line
	$\frac{20-\text{"29"}}{\sqrt{\text{"18"}}}$ (= –2.121)	M1	For Standardising
	1 – Φ("–2.121") = Φ("2.121")	M1	For area consistent with their working
	= 0.983 (3 s.f.)	A1 [5]	

CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Level

MARK SCHEME for the October/November 2013 series

9709 MATHEMATICS

9709/71 Paper 7, maximum raw mark 50

Mark Scheme Notes

Marks are of the following three types:

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A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

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- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF	Any Equivalent Form (of answer is equally acceptable)
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
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CAO	Correct Answer Only (emphasising that no “follow through” from a previous error is allowed)
CWO	Correct Working Only – often written by a ‘fortuitous’ answer
ISW	Ignore Subsequent Working
MR	Misread
PA	Premature Approximation (resulting in basically correct work that is insufficiently accurate)
SOS	See Other Solution (the candidate makes a better attempt at the same question)
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

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1 $\lambda = \frac{1}{30}$ $1 - e^{-\frac{1}{30}}$ $= 0.0328$ (3 s.f.)	B1 M1 M1 A1 [4]	o.e 1 – P(X = 0) by Poisson, any λ allow 1 end error 1 – P(X = 0) by Poisson, correct λ no end errors S.R. Binomial with final answer 0.0328 B2 Correct answer, no working scores B2
2 $z = 2.576$ $2 \times z \times \frac{0.17}{\sqrt{n}} = 0.2$ oe $n = (\frac{2 \times 0.17 \times 2.576}{0.2})^2$ oe (= 19.2) Smallest n is 20	B1 M1 M1 A1 [4]	Seen (accept 2.574 to 2.579) Allow without '2 ×' OR with incorrect z Attempt to arrange equ of correct form (with correct z and '2×' into the form n= or $\sqrt{n}$ =
3 (i) est (μ) = 2866 or 2870 (3 s.f.) est (σ^2) = $\frac{1}{49}(410900000 - \frac{143300^2}{50})$ (= 4126.53) = 4130 (3 sf)	B1 M1 A1 [3]	Accept 143300/50 o.e. Correct subst in correct formula
(ii) H_0: Pop mean (or μ) = 2850 H_1: Pop mean (or μ) ≠ 2850 $\dfrac{\frac{143300}{50} - 2850}{\frac{\sqrt{'4126.53'}}{\sqrt{50}}}$ = 1.761 '1.761' < 1.96 No evidence mean distance changed	B1 M1 A1 M1 A1f [5]	Both. Not just 'mean' Allow '4126.53' without $\sqrt{\ }$, but must have all $\sqrt{50}$ Or correct c.v. (2867.81) for alt method For valid comparison of z values, areas or c.v. Dep 1.96; ft their 1.761 If H_1: $\mu > 2850$ and c.f. 1.645, max B0M1A1M1A0 (c.v. for 1 tail test 2864.94)
4 (i) $\lambda = 2.8$ $e^{-2.8}(1 + 2.8 + \frac{2.8^2}{2})$ = 0.469 (3 s.f.) or 0.47(0)	B1 M1 A1 [3]	seen any λ allow one end error As final answer
(ii) $e^{-0.7n} \geqslant 0.99$ or $e^{-\lambda} \geqslant 0.99$ $-0.7n \geqslant \ln 0.99$ or $-\lambda \geqslant \ln 0.99$ $n \leqslant 0.01436$ or $\lambda \leqslant 0.01005$ '0.01436' × 150 or '0.01005' × 150 ÷ 0.7 Max period is 2.15 mins (3 sf)	M1 M1 A1 M1 A1 [5]	Allow '=' throughout Attempt ln both sides Can be implied. Accept 3 s.f. Note $e^{-(0.7/150)n} \geqslant 0.99$ scores 1st and 3rd M1 T & I leading to ans 2.2 mins, SC: B2

5 (i)	$\int_0^2 k(x-2)^2\,dx = 1$ $(\left[\frac{k(x-2)^3}{3}\right]_0^2 = 1)$ $k[0-(-\frac{8}{3})] = 1$ $k = \frac{3}{8}$ **AG**	M1 A1 [2]	Attempt to integrate f(x) with correct limits and = 1 Must see this line or better, e.g. $k \times \frac{8}{3} = 1$
(ii)	$\frac{3}{8}\int_d^2 (x-2)^2\,dx = 0.2$ $(\frac{3}{8}\left[\frac{(x-2)^3}{3}\right]_d^2 = 0.2)$ $\frac{3}{8}\left[0-\frac{(d-2)^3}{3}\right] = 0.2$ oe $((d-2)^3 = -1.6)$ $d = 0.83(0)$ (3 s.f.)	M1 M1 A1 [3]	∫f(x)dx with limits d and 2 or 0 and d, and = 0.2 or =0.8 Condone missing 'k' Reasonable attempt to integrate from a correct expression, with limits substituted to give expression in d³. Condone missing 'k'
(iii)	$\frac{3}{8}\int_0^2 x(x-2)^2\,dx$ $(= \frac{3}{8}\int_0^2 x^3 - 4x^2 + 4x\,dx)$ $= \frac{3}{8}\left[\frac{x^4}{4} - \frac{4x^3}{3} + 2x^2\right]_0^2$ $= \frac{1}{2}$	M1 A1 A1 [3]	Attempt integ xf(x); ignore limits, condone missing k $(\frac{3}{8}\left[x\times\frac{(x-2)^3}{3} - \int\frac{(x-2)^3}{3}\,dx\right]_0^2)$ $=\frac{3}{8}\left[x\times\frac{(x-2)^3}{3} - \frac{(x-2)^4}{12}\right]_0^2$ Correct integration & limits, condone missing k
6 (i)	P(Type I) = 1 - P($\geq$ 4 assuming p = 0.7) $1-({}^6C_4 \times 0.7^4\times0.3^2 + {}^6C_5\times0.7^5\times0.3 + 0.7^6)$ (= 1 – 0.744) = 0.256 (3 s.f.)	M1 M1 A1 [3]	or P($\leq$ 3 assuming p = 0.7) May be implied ${}^6C_3\times0.7^3\times0.3^3+{}^6C_2\times0.7^2\times0.3^4+{}^6C_1\times0.7\times0.3^5 + 0.3^6$ Allow one end error = 0.256 (3 s.f.) SR if zero scored allow B1 for use of B(6, 0.7) in any two or more terms
(ii)	P(Type II) = P($\geqslant$ 4 assuming p = 0.35) $= {}^6C_4\times0.35^4\times0.65^2 + {}^6C_5\times0.35^5\times0.65+0.35^6$ = 0.117	M1 M1 A1 [3]	May be implied Allow one end error SR if zero scored allow B1 for use of B(6, 0.35) in any two or more terms
(iii)	Type 1 They will reject Luigi's belief, although it might be true.	B1 B1 [2]	In context

7 (i)	N(10.61, 0.1017) $\frac{11-'10.61'}{\sqrt{'0.1017'}}$ (= 1.223) Φ('1.223') = 0.889 (3 s.f.)	B1 M1 M1 A1 [4]	o.e. Stated or implied (accept in un-simplified form) Allow without √ For attempt to find correct area consistent with their working
(ii)	P($K - 1.2A > 0$) Var = $0.0576 + 1.2^2 \times 0.0441$ (= 0.121104) N(–0.324, 0.121104) $\frac{0-(-0.324)}{\sqrt{'0.121104'}}$ (= 0.931) 1 – Φ('0.931') = 0.176 (3 s.f.)	M1 B1 B1 M1 M1 A1 [6]	Or similar stated or implied o.e. May be implied (accept in un-simplified form) Allow without √ For attempt to find correct area consistent with their working

CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Level

MARK SCHEME for the October/November 2013 series

9709 MATHEMATICS

9709/72 Paper 7, maximum raw mark 50

CAMBRIDGE
International Examinations

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CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Level

MARK SCHEME for the October/November 2013 series

9709 MATHEMATICS

9709/73 Paper 7, maximum raw mark 50

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

CAMBRIDGE
International Examinations

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PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

1	Est(μ) = 1.8775 or 1.88 (3 sf)	B1		Accept 751/400 (not 150.2/80)
	Est(σ^2) = $\frac{80}{79}\left(\frac{820.24}{80} - "1.8775"^2\right)$	M1		Correct subt'n in correct formula 1/79 (820.24 – 150.2^2/80)
	= 6.81316 or 6.81 (3 sf)	A1		
	z = 1.96	B1		Seen
	"1.8775" ± z × $\sqrt{\frac{"6.81316"}{80}}$	M1		
	= 1.31 to 2.45 (3 sf)	A1	6	Must be an interval. NB use of biased var can still score A1.
Total			[6]	
2 (i)	Assume sd unchanged or sd = 10.4	B1		Oe e.g. var unchanged
	H_0: Pop mean speed (or μ) = 62.3 H_1: Pop mean speed (or μ) < 62.3	B1		Both. Not just "Mean . . "
	$\frac{59.9-62.3}{\frac{10.4}{\sqrt{75}}}$	M1		Accept sd/var mixes, but must have √ 75
	= – 1.999 or – 2.00 (allow + or –)	A1		Correct z value (or correct critical value)
	Compare – 2.054 or –2.055	M1		Valid comparison of z`s/areas/critical values
	No evidence that mean speed decreased	A1 ft	6	No contradictions. Do not ft 2-tail test.
(ii)	Pop distribution unknown	B1		
	Yes	B1	2	
Total			[8]	
3 (i)	$\int_0^{10} \frac{1}{2500}\left(100t^3 - t^5\right)dt$	M1		Attempt integ t^2f(t)
	(= $\frac{1}{2500}\left[25t^4 - \frac{t^6}{6}\right]_0^{10}$ = $\frac{100}{3}$)			
	"$\frac{100}{3}$"– $(\frac{16}{3})^2$	M1		For E (T^2) – (E (T) $)^2$
	= $\frac{44}{9}$ or 4.89 (3 sf)	A1	3	
(ii)	$\int_n^{10} \frac{1}{2500}\left(100t - t^3\right)dt$	M1		Attempt integ f(t), ignore limits
	$\frac{1}{2500}\left[50t^2 - \frac{t_4}{4}\right]$ = 0.1	M1		Attempt integ f(t), limits n to 10 or 0 to n Equated to 0.1 or 0.9. Not need to be matched
	$\frac{1}{2500}\left[2500 - \left(50n^2 - \frac{n^4}{4}\right)\right]$ = 0.1	M1		0.1/0.9 matched to correct limits and used
	(n^4 – 200n^2 + 9000 = 0)	M1		Correct method of solution of a QE in n^2
	(n^2 = 68.3772, n = 8.27) n = 8	A1	5	Must be single ans only
Total			[8]	

4 (i) (a)	$e^{-2.1} \times \frac{2.1^3}{3!}$ alone	**M1**		Allow any λ. Allow sum of 3 or 4 rel products, e.g. P (3, 0)
	$= 0.189$	**A1**	**2**	
(b)	$e^{-1.2} \times \frac{1.2^3}{3!} \times e^{-0.9}$	**M1**		P (Fem = 3) × P (Opp = 0) or P (Fem = 2) × P (Opp = 1)
	$+ e^{-1.2} \times \frac{1.2^2}{2!} \times e^{-0.9} \times 0.9$	**M1**		P (3, 0) + P (2, 1)
	$= 0.115$	**A1**	**3**	As final answer
(ii)	N (30, 30)	**B1**		seen or implied
	$\frac{34.5-30}{\sqrt{30}}$ (= 0.8216)	**M1**		standardising with their N (λ, λ)
	1 – Φ(“0.822”)	**M1**		Allow with no or incorrect cc or no √
	= 0.206 (3sf)	**A1**	**4**	Area consistent with their working
Total				
5 (i)	E (X)= 3.5	**B1**		21/6
	$(1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2) \div 6 - “3.5”^2$ (= $\frac{35}{12}$ **AG**)	**B1**	**2**	oe, must see correct expression and no incorrect working
(ii)	Attempt P (X < 3) or 1 – P($X \geq 3$)	**M1**		seen or implied
	N (3.5, $\frac{35}{12}$/50)	**M1**		seen or implied
	$\frac{3-"3.5"}{\sqrt{\frac{35}{12}/50}}$ (= –2.070)	**M1**		or $\frac{2.99-"3.5"}{\sqrt{\frac{35}{12}/50}}$ (= –2.111)
	Φ (“–2.070”) = 1 – Φ(“2.070”) = 0.0192	**M1**		Φ (‘–2.111’) = 1 – Φ (‘2.111’) = 0.0174 or 0.0173 Consistent area
	as final answer	**A1**	**5**	As final answer or valid total method Allow with incorrect cc (e.g. 2.5) OR no √.Must have ÷50
(iii)	Die is biased (towards lower numbers)	**B1 indep**		Comment implying die is biased
	Mean of 50 throws ≥ 3 (Allow > 3) or Equal nos of high and low scores or More high scores	**B1 indep**	**2**	Comment implying results of exp’t do not indicate bias (or indicate bias towards higher numbers) Both must be in context
Total			**[9]**	

6 **(i)**	N(5100, 5×45^2) or N(5100, 10125) $\frac{5200-"5100"}{\sqrt{"10125"}}$ (= 0.994) Φ ("0.994") = 0.840 (3 sf)	**B1** **M1** **M1** **A1**	**4**	seen or implied standardising with their new mean and new var area consistent with their working with normal
(ii)	Use of $E-3L$ or similar E $(E-3L)=-260$ Var $(E-3L)=52^2+9\times45^2$ or 20929 $\frac{0-("-260")}{\sqrt{"20929"}}$ (= 1.797) 1 – Φ ("1.797") = 0.0361 (3 sf) or 0.0362	**M1** **B1** **B1** **M1** **M1** **A1**	**6**	2800 – 3 x 1020 with a pos var with 45^2 and 52^2 combined consistent area, must clearly be φ P $(3L-E<0)$: similar scheme SR: use of 3E – L, M1, 7380 B1, 26361 B1 stand 0 with these values M1, M0A0 max 4/6
Total			**[10]**	

CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level and GCE Advanced Level

MARK SCHEME for the May/June 2013 series

9709 MATHEMATICS

9709/71 Paper 7, maximum raw mark 50

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Note: "(3 sfs)" means "answer which rounds to ... to 3 sfs". If correct ans seen to $\geq$ 3sfs, ISW for later rounding. Penalise < 3 sfs only once in <u>paper.</u>

1	**(i)**	One of each is more likely	B1		
		P(one of each = 0.5), P(HH) = 0.25	B1 [2]	or P(TT) = 0.25	
	(ii)	Choose Charlie only if H then T	B1	or similar e.g. HH for A, HT for B, TT for C	
		Throw again if T then H	B1 [2]	or vice versa	
2		H_0: Pop mean = 17 H_1: Pop mean $\neq$ 17	B1	Both correct. Allow μ, but not just "mean"	
		$\frac{18.2-17}{\frac{2.4}{\sqrt{5}}}$	M1	Allow incorrect 18.2. Must have √5	$17 \pm 1.96\frac{2.4}{\sqrt{5}}$ M1
		= 1.12 (3 sf)	A1		= (14.9, 19.1) A1
		'1.12' < 1.96 oe	M1	Comp '1.12' with 1.96 or area '0.132' with 0.025	'14.9'<18.2<'19.1' M1
		Claim can be accepted	A1ft	ft their '1.12'	
			[5]	If H_1: μ > 17 and cf 1.645: can score max B0M1A1M1A1ft	
3		Var(total) = $6(3.2^2 + 2.6^2)$ (+ 0)) (= 102)			
		Total ~ N(1528, 102))	B1 B1	For mean (1528)oe and for variance (102) May be implied by use of N(1528, 10.1^2)	
		$\frac{1550-"1528"}{\sqrt{"102"}}$ (= 2.178)	M1	For standardising. No SD/Var mix	
		1 – Φ("2.178")	M1	For correct area consistent with working	
		= 0.0147 (3 sf)	A1 [5]		
4	**(i)**	est(μ) = 2005/200 = (10.025)	B1		
		est(σ^2) = $\frac{1}{99}20175 - \frac{2005^2}{200}$)	M1	Correct subst in correct formula	
		= 0.376 (3 sf)	A1 [3]		
	(ii)	$\frac{10-'10.025'}{\sqrt{\frac{'0.376256'}{50}}}$ (= –0.288)	M1	Allow without √, but ÷√50 essential	
		1 – Φ('0.288')	M1		
		= 0.387 (3 sf)	A1 [3]	(Use of 'biased' variance can still score fully in (ii))	

(iii)	Yes; (assumed distr of $\overline{X}$ normal) although distr of X unknown	B1 B1 [2]	
5 (i)	B(520, 0.008) Po(4.16) n = 500 which is large, np = 4.16 which is < 5 or p small <0.1	B1 B1B1 B1 [4]	Po: B1, λ = 4.16: B1 Both needed
(ii)	**(a)** $1 - e^{-4.16}(1 + 4.16 + \frac{4.16^2}{2} + \frac{4.16^3}{3!})$ = 0.597 (3 sf)	M1 A1 [2]	1–P(0,1,2,3) any λ allow one end error
	(b) $e^{-4.16} \times \frac{4.16^n}{n!} > e^{-4.16} \times \frac{4.16^{n+1}}{(n+1)!}$ $1 > \frac{4.16}{n+1}$ $n > 3.16$ Smallest n is 4	M1 A1 A1 [3]	any λ or equiv equn without e and without factorials (Calculation of P(0), P(1),....P(5) scores M1for at least 3 attempted, A1 all correct, A1 for n = 4)
6 (i)	$\frac{1}{2}\int_4^t \frac{1}{\sqrt{t}}dt = 0.9$ or $\frac{1}{2}\int_t^9 \frac{1}{\sqrt{t}}dt = 0.1$ $\left[\sqrt{t}\right]_4^t = 0.9$ or $\left[\sqrt{t}\right]_t^9 = 0.1$ $((\sqrt{t} - 2) = 0.9$ or $(3 - \sqrt{t}) = 0.1)$ t = 8.41 (mins) (3 sf)	M1 A1 A1 [3]	Attempt integ f(t) with unknown limit and 0.9/0.1. Correct integration & limits = 0.9 or 0.1.
(ii)	$\frac{1}{2}\int_4^9 \frac{t}{\sqrt{t}}dt$ oe $\frac{1}{2}\left[\frac{t1.5}{1.5}\right]_4^9$ oe $= \frac{19}{3}$ $\frac{1}{2}\int_4^9 \frac{t2}{\sqrt{t}}dt$ oe $(=\frac{1}{2}\left[\frac{t2.5}{2.5}\right]_4^9 = \frac{211}{5})$ $= \frac{'211'}{5} - (\frac{'19'}{3})^2$ $= \frac{94}{45}$ or 2.09 (3 sf)	M1 A1 A1 M1 M1 A1 [6]	Attempt integ tf(t). Ignore limits Correct integration & limits Attempt integ t^2f(t). Ignore limits integ t^2f(t) – (integ tf(t))^2 attempted

7 **(i)**	Conclude die is biased when it isn't oe $^5C_3\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^2+5\left(\frac{1}{6}\right)^4\left(\frac{5}{6}\right)+\left(\frac{1}{6}\right)^5$ + 5 $=\frac{23}{648}$ or 0.0355 (3 sf)	B1 M1 A1 [3]	In context or 1 – $\left(^5C_2\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)3+5\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)^4+\left(\frac{5}{6}\right)^5\right)$ allow 1 end error
(ii)	State or attempt P(0, 1, 2) with $p=\frac{2}{3}$ $^5C_2\left(\frac{2}{3}\right)^2\left(\frac{1}{3}\right)^3+5\left(\frac{2}{3}\right)\left(\frac{1}{3}\right)^4+\left(\frac{1}{3}\right)^5$ $=\frac{17}{81}$ or 0.210 (3 sf)	M1 M1 A1 [3]	Or 1– P(3,4,5) Attempt at correct expression Allow 0.21
(iii)	Est Var(P_s) = $\frac{0.625\times(1-0.625)}{80}$ $(=\frac{3}{1024})$ z = 2.054 (or 2.055) $0.625 \pm z\times\sqrt{\frac{'3'}{1024}}$ = 0.514 to 0.736 (3 sf)	M1 B1 M1 A1 [4]	 Any z

CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level and GCE Advanced Level

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1	**(i)**	Binomial	B1	
		$n = 400, \quad p = 0.012$	B1 [2]	Both. Not $p = 1.2\%$ Or B(400, 0.012): B1B1
	(ii)	Poisson	B1	
		n large and mean = 4.8, which is < 5	B1 [2]	n large, p small
	(iii)	$1 - e^{-4.8}(1 + 4.8 + \frac{4.8^2}{2})$	M1	$P(X = 0, 1, 2)$; allow any λ; allow one end error
		$= 0.857/0.858$	A1 [2]	(Normal/Binomial in (ii) can score M1 only)
		[Total: 6]		
2	**(i)**	$\frac{2}{3}\int_1^2 x^2 dx$	M1	Attempt integ. xf(x); ignore limits
		$= \frac{2}{3}\left[\frac{x^3}{3}\right]_1^2$	A1	Correct integration and limits
		$= \frac{14}{9}$ or 1.56 o.e.	A1 [3]	
	(ii)	$\frac{2}{3}\int_1^{\frac{14}{9}} x\, dx$	M1	Attempt integ. f(x); with limits
		$(= \frac{2}{3}\left[\frac{x^3}{3}\right]_1^2)$		
		$= \frac{115}{243}$ or 0.473 (3 s.f.)	A1 [2]	
	(iii)	$\frac{115}{243} < \frac{1}{2}$ o.e.	M1	Comparison of prob. or values
		Hence mean < median	A1ft[2]	ft (i) or (ii)
		[Total: 7]		

3 (i)	$\frac{73.1-75.2}{\frac{5.7}{\sqrt{n}}} = -1.563$	M1	For standardising (with $\sqrt{n}$)
	$n = \{-1.563\times5.7\div(-2.1)\}^2$	A1	Any correct expression for n or $\sqrt{n}$. May be implied by ans.
	$n = 18$ Assume s.d. for the region is 5.7	A1 B1 [4]	
(ii)	H_0: pop mean (or μ) = 75.2 H_0: pop mean (or μ) < 75.2 1.563 comp 1.555 Evidence that plants shorter	 B1 M1 A1 [3]	 Both (could be stated in (i)) For comparison of z values / areas / x values CWO. No contradictions
	[Total: 7]		
4 (i)	est(μ) = 9750/150 = (65)	B1	
	est(σ^2) = $\frac{1}{149}(647500 - \frac{9750^2}{150})$	M1	Correct subst. in correct formula
	= 92.3 (3 s.f.)	A1 [3]	
(ii)	$z = 2.326$	B1	
	'65' $\pm z\times\frac{\sqrt{'92.28188'}}{\sqrt{150}}$	M1	Any z
	= 63.2 to 66.8 (3 s.f.)	A1 [3]	(Use of 'biased' can still score here)
(iii)	0.02^2	M1	Allow M1 for 0.02 seen
	= 0.0004 o.e.	A1 [2]	
	[Total: 8]		

5 (i)	$6 \times 510 + 70 = (3130)$ $6 \times 12^2 + 4^2 = (880)$ N(3130, 880)	B1 B1	
	$\frac{3050-3130}{\sqrt{880}}$ (= – 2.697) $\frac{3150-3130}{\sqrt{880}}$ (= 0.674)	M1	Both. With their mean and variance(≥0) Allow without √
	Φ(‘0.674’) – (1 – Φ(‘2.697’)) (= 0.7499 – 0.0035)	M1	Use of tables and attempt to find area consistent with their working
	= 0.746 (3 sf)	A1 [5]	
(ii)	$510 - 8 \times 70 =(- 50)$ $12^2 + 8^2 \times 4^2 = (1168)$ P – 8C ~ N(– 50, 1168)	B1 B1	o.e. +50; 510/8 -70; – (510/8 – 70) o.e. $(12/8)^2 + 4^2$
	$\frac{0-(-50)}{\sqrt{1168}}$ (= 1.463)	M1	For standardising with attempt “P-8C” oe with their mean and variance(≥0).Allow without √
	1 – Φ(‘1.463’)	M1	Use of tables and attempt to find area consistent with their working
	= 0.0717 (3 s.f.)	A1 [5]	
	[Total: 10]		

6 (i)	H_0: Pop mean (or λ or μ) is 5.3 H_1: Pop mean (or λ or μ) is less than 5.3	B1	Both
	$P(X \le 1) = e^{-5.3}(1 + 5.3)$ $P(X \le 2) = e^{-5.3}(1 + 5.3 + \frac{5.3^2}{2})$/P(X=2)	M1	Both attempted
	$P(X \le 1) = 0.0314$ or 0.0315 & $P(X \le 2) = 0.102$/ P(X=2)=0.7071	A1	Both correct
	CR is 0 or 1 cases	A1	Dep. M1 and $P(X \le 1) < 0.05 < P(X \le 2)$
	No evidence mean has decreased	B1f [5]	ft their CR
(ii)	Concluding mean has decreased when it hasn't	B1	In context
	'0.0314 or 0.0315'	B1ft[2]	ft their $P(X \le 1)$, dep. < 0.05
(iii)	(Po(18.4)) N(18.4, 18.4)	B1 B1ft	Stated or implied B1 for N(18.4, ..); B1f for var. = 18.4
	$\frac{20.5-18.4}{\sqrt{18.4}}$ (= 0.490)	M1	For standardising with or without cc.Allow without √
	1 – Φ('0.490')	M1	Use of tables and attempt to find area consistent with their working
	= 0.312 (3 s.f.)	A1 [5]	
	[Total: 12]		

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SOS	See Other Solution (the candidate makes a better attempt at the same question)
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

1 (i)	9.3	B1 1	
(ii)	27.9	B1 1	
(iii)	E (S) = 17.4, E(T) = 19.4 E ($S – T$) = – 2.0, Var ($S – T$) = 37.2	M1 A1 B1ft 3	For subtracting their E[S] – E[T] can be non-numerical ft (i) & (ii) Adding (i) and (ii) ft non-negative answers only
		[Total: 5]	
2	Assume shots independent OR prob of scoring constant H_0: P(score) = 0.82 H_1: P(score) > 0.82 $20 \times 0.82^{19} \times 0.18 + 0.82^{20}$ = 0.102 (3 sf) No evidence that improved	B1 B1 M1 A1 B1f 5	In context Both. Allow 'p' For use of Bin(20,0.82)and either P(19) and/or P(20) attempted Valid comparison seen (with 0.05 if H_1 $p \neq 0.82$) and correct conclusion ft numerical errors in 0.102 only Normal approx'n: B1 B1 (μ= 16.4 acceptable here) if earned, then: CR = 1.222 (from $\frac{18.5-20\times0.82}{\sqrt{20\times0.82\times(1-0.82)}}$, need cc) comp z = 1.282 No evidence that improved SC 1 Same scheme for proportions
		[Total: 5]	
3 (i)	$\bar{x}$ = 930/15 =(62) z = 1.751 '62' ± $z \times \frac{12}{\sqrt{15}}$ = 56.6 to 67.4 (3 sf)	B1 B1 M1 A1 4	Any z Must be an interval
(ii)	92% of such intervals will contain μ	B1 1	Accept P(This interval contains μ) = 0.92
(iii)	Each possible sample of this size is equally likely	B1 1	Each member of pop equally likely to be chosen
		[Total: 6]	

4	(i)	$e^{-2} \times 2\ (\times)\ e^{-3} \times \frac{3^4}{4!}$	M1		Correct exp'n for P(1) with λ=2 OR P(4) with λ=3
		$e^{-5} \times \frac{5^4}{5!}$	B1		Correct exp'n
		÷	M1		dep M1B1
		$\frac{162}{625}$ or 0.259 (3 sf)	A1	4	
	(ii)	$(e^{-2} \times \frac{2^r}{r!} = \frac{2}{3}e^{-2} \Rightarrow)$			
		$3 \times 2^r = 2 \times r!$ OR $2^{r-1} = \frac{1}{3} \times r!$ $(\Rightarrow 3 \times 2^{r-1} = r!)$	B1		Legitimately shown
		$3 \times 2^3 = 24$ OR $3! = 24$ seen	B1	2	Legitimately shown on either equation
			[Total: 6]		
5	(i)	$\int_1^\infty \frac{k}{x^3}\,dx = 1$	M1		All correct, including limits and an attempt to integrate
		$\left[-\frac{k}{2x^2}\right]_1^\infty = 1$			
		$0 - (-\frac{k}{2}) = 1$	A1	2	or $0 + \frac{k}{2} = 1$ or $\frac{k}{2} = 1$ AG must be convincing
	(ii)	$\int_1^2 \frac{2}{x^3}\,dx$	M1		Attempt integ f(x); ignore limits
		$= \left[-\frac{1}{x^2}\right]_1^2$			
		$= \frac{3}{4}$	A1	2	
	(iii)	$\int_1^\infty \frac{2}{x^2}\,dx$	M1		Attempt integ xf(x); ignore limits
		$= \left[-\frac{2}{x}\right]_1^\infty$	A1		Correct & correct limits
		$= 2$	A1	3	
			[Total: 7]		

6	(i)	λ (= 1.4 × 2.5) = 3.5 $1 - e^{-3.5}(1 + 3.5 + \frac{3.5^2}{2} + \frac{3.5^3}{3!})$ = 0.463 (3 sf)	B1 M1 A1 3	 Any λ allow one end error
	(ii)	(λ = 672 × 1.4 = 940.8) N(940.8, 940.8) $\frac{999.5 - 940.8}{\sqrt{940.8}}$ (= 1.914) Φ('1.914') = 0.972 (3 sf)	 B1 M1 M1 A1 4	 Seen or implied Allow with wrong or no cc . no sd/var mixes
		[Total: 7]		
7	(i)	Assume sd unchanged or 4500 H_0: Pop mean = 34600 H_1: Pop mean > 34600 $\frac{35400 - 34600}{\frac{4500}{\sqrt{90}}}$ = 1.687/1.686 (1.69) cf 1.645 < 1.686 Evidence that mean wkly profit has increased	B1 B1 M1 A1 M1 A1 f 6	 Both. Allow just μ, but not just "mean" Allow without √90 Valid comparison (or 0.0458/0.0459 <0.05 or 35380 < 35400 or 34600 < 34620) If H_1: ≠, and 1.96 used, max B1B0M1A1M1A1f No contradictions
	(ii)	Distr'n of *X* unknown. Yes	B1* B1* dep 2	Allow not Normal
	(iii)	0.05 or 5%	B1 1	
	(iv)	$\frac{a - 34600}{\frac{4500}{\sqrt{90}}} = 1.645$ a = 35380 $\frac{35380 - 36500}{\frac{4500}{\sqrt{90}}}$ (= – 2.361) 1 – Φ('2.361') = 0.0091	M1 A1 M1 M1 A1 6	Attempt to find cv must see (+) 1.645 allow without √90. If found in (i) award when used Standardising with their " CV " must use √90 Correct tail
		[Total: 14]		