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International Education

# Cambridge International A Level

**MATHEMATICS** **9709/61**

Paper 6 Probability & Statistics 2 **October/November 2024**

MARK SCHEME

Maximum Mark: 50

**Published**

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2024 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.



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## Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

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**Mathematics-Specific Marking Principles**

1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.

2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.

3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.

4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).

5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.

6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

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## Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

### Types of mark

**M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

**A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

**B** Mark for a correct result or statement independent of method marks.

**DM** or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

**FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

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**Abbreviations**

| | |
|---|---|
| AEF/OE | Any Equivalent Form (of answer is equally acceptable) / Or Equivalent |
| AG | Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid) |
| CAO | Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed) |
| CWO | Correct Working Only |
| ISW | Ignore Subsequent Working |
| SOI | Seen Or Implied |
| SC | Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance) |
| WWW | Without Wrong Working |
| AWRT | Answer Which Rounds To |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 1(a) | $z = 2.054$ or 2.055 | **B1** | Accept 3 sf if nothing better seen (2.05 or 2.06). |
| | $1.42 \pm z \frac{0.35}{\sqrt{150}}$ | **M1** | Must be a z value. |
| | 1.36 to 1.48 [m] (3 sf) | **A1** | Correct working only. Must be an interval. |
| | | **3** | |
| 1(b) | No. CI is about mean, not individual values. | **B1** | Or similar. Need both. |
| | | **1** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 2 | $E(S_1 + S_2 + S_3 - L) = 16 \times 3 - 51$ [= −3 ] | **B1** | Oe, using $L - (S_1 + S_2 + S_3)$. |
| | $Var(S_1 + S_2 + S_3 - L) = 3 \times 0.4 + 0.9$ [= 2.1] | **M1** | |
| | $\frac{0-(-3)}{\sqrt{'2.1'}}$ [= 2.070] | **M1** | For standardising with their values. |
| | $1 - \Phi(\text{'2.070'})$ | **M1** | For area consistent with their values. |
| | = 0.0192 (3 sf) | **A1** | |
| | | **5** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 3(a) | $\bar{t} = \frac{230}{75}$ [= 3.0666… or 3.07 (3 sf)] [ 0r 46/15 ] | **B1** | |
| | $s^2 = \frac{75}{74}(\frac{930}{75} - (\frac{230}{75})^2)$ or $1/74(930 - 230^2/75)$ | **M1** | Use of correct formula. |
| | = 3.0360… or 3.04 (3 sf) or = 337/111 | **A1** | |
| | | **3** | |
| 3(b) | [ $\Phi^{-1}(1 - 0.234)$ ] = 0.726 | **B1** | |
| | $\pm \frac{a - '3.0667'}{\sqrt{'3.04'/75}} = \pm$ '0.726' | **M1** | Ft their 0.726 but must be a z value.<br>Note using 0.766 is M0.<br>Must have sqrt 75. |
| | $a$ = 3.21 (3 sf) | **A1** | CWO |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4(a) | $\int_2^3 (\frac{a}{x^2} - \frac{18}{x^3})dx = 1$ | **M1** | Attempt integrate f($x$), ignore limits and '= 1'. |
| | $\left[-\frac{a}{x} + \frac{9}{x^2}\right]_2^3 = 1$ | **A1** | OE Correct integration and limits. |
| | $\left[-\frac{a}{3} + 1 + \frac{a}{2} - \frac{9}{4}\right] = 1$ $a = \frac{27}{2}$ **(AG)** | **A1** | Must see correct substitution of limits.<br>Correct working no errors seen. |
| | | **3** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4(b) | $\int_2^3 (\frac{27}{2x} - \frac{18}{x^2})\mathrm{d}x$ | **M1** | Attempt to integrate xf(x), ignore limits. |
| | $\left[\frac{27}{2}\ln x + \frac{18}{x}\right]_2^3$ or $\left[\frac{27}{2}\ln 2x + \frac{18}{x}\right]_2^3$ | **A1** | Correct integration and limits.<br>OE e.g. using ln 2x. |
| | $= \frac{27}{2}\ln 3 + 6 - \frac{27}{2}\ln 2 - 9 = \frac{27}{2}\ln\frac{3}{2} - 3$ **AG** | **A1** | Must see correct substitution of limits.<br>Correct working no errors seen. |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5(a) | Two-tailed because looking for difference | **B1** | |
| | | **1** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5(b) | $H_0$: $\mu = 12.7$ $H_1$: $\mu \neq 12.7$ | **B1** | No ft from part **(a).** |
| | $\frac{\frac{597.1}{50} - 12.7}{\frac{2.3}{\sqrt{50}}}$ | **M1** | |
| | $= -2.330$<br>Accept – 2.336 or – 2.337 | **A1** | or 0.00989 or 0.0099.<br>or 0.0097 if area comparison used. |
| | '−2.330' > −2.576 or '2.330' < 2.576<br><br>or '0.00989' > 0.005<br>or `0.0097` > 0.005 | **M1** | Accept 2.574 to 2.579.<br><br>Or use of CV.<br>12.7- 2.576 x ( 2.3 / sqrt 50 ) = 11.862 **M1A1**.<br>11.942 > 11.862 **M1A1**. |
| | [Not reject $H_0$] There is insufficient evidence to suggest that $\mu$ is not 12.7 | **A1 FT** | OE ft their $z_{calc.}$<br>In context, not definite, e.g. not '$\mu$ =12.7'.<br>No contradictions.<br><br>SC use of 1 tailed test can score **B0M1A1M1** for comparison with 0.01 **A0** max 3/5. |
| | | **5** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(a) | [$\lambda = 2.7$] $1 - e^{-2.7}(1 + 2.7 + \frac{2.7^2}{2})$ or $1 - e^{-2.7}(1 + 2.7 + 3.645)$<br>or 1- ( 0.06721 + 0.1815 + 0.2450 ) | **M1** | Any $\lambda$. Allow one end error.<br>Must see expression. |
| | = 0.506 (3 sf) | **A1** | SC unsupported answer 0.506 scores **B1**. |
| | | **2** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(b) | $\lambda = 1.95$ | **B1** | |
| | $e^{-1.95}(1 + 1.95 + \frac{1.95^2}{2} + \frac{1.95^3}{3!})$ or $e^{-1.95}(1 + 1.95 + 1.90125 + 1.2358)$ or $0.1423 + 0.2774 + 0.2705 + 0.1758$ | **M1** | Any $\lambda$. Allow one end error.<br>Must see expression. |
| | $= 0.866$ | **A1** | SC unsupported answer 0.866 scores **B1B1**. |
| | | **3** | |
| 6(c) | $1 - e^{-2.1x} \geqslant 0.90$ or $1 - e^{-\lambda} \geqslant 0.90$ | **M1** | OE Condone use of '=' throughout. |
| | | | |
| | $[e^{-2.1x} \leq 0.1]$ or $e^{-\lambda} \leq 0.1$<br>$-2.1x \leq \ln 0.1$ or $-\lambda \leq \ln 0.1$ [ $\lambda \geq 2.3026$, $2.3026/2.1$ ] | **M1** | Rearrange and attempt take logs of relevant form. |
| | 1.096 or 10.96 accept 1.097 or 10.97 | ***A1** | Seen. |
| | She must wait for at least 11 minutes | **A1 dep** | |
| | | | SC Use of trial and improvement.<br>Use of $1-e^{-\lambda}$ any numerical $\lambda$ (not 2.1) ie one trial **M1**.<br>Use of enough trials to give an answer of 0.90 (2sf) **M1**.<br>$\lambda$=2.30 i.e. 3sf accuracy AND 1.09… or 10.9 … **A1**.<br>Then 11 **A1** dep. |
| | | **4** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 7(a) | $\lambda = 6.6$ | **B1** | |
| | $P(X \leq 2) = e^{-6.6}(1 + 6.6 + \frac{6.6^2}{2})$ [= 0.0400] [ < 0.05 ]<br>or $e^{-6.6}(1 + 6.6 + 21.78)$<br>or 0.001360 + 0.008978 + 0.02963 | **M1** | Expression must be seen. No end errors.<br>Allow use of 3.3 here. |
| | $P(X \leq 3) = e^{-6.6}(1 + 6.6 + \frac{6.6^2}{2} + \frac{6.6^3}{3!})$ or $0.0400 + e^{-6.6} \times \frac{6.6^3}{3!}$ = 0.105 [ > 0.05 ] | **B1** | Condone unsupported 0.105. |
| | P(Type I error) = 0.0400 (3 sf) | **A1** | Allow 0.040 or 0.04 AWRT<br><br>SC unsupported ans of 0.0400 can score max **B1B1B1.** |
| | | **4** | |
| 7(b) | $H_0$: $\lambda = 6.6$, $H_1$: $\lambda < 6.6$ | **B1** | May be seen in part (a) and award **B1** mark here.<br>Accept μ or $\lambda$.<br>Accept 3.3 or 6.6. |
| | [$P(X \leq 2) = 0.0400$] ` 0.04 ` < 0.05 | **M1** | For comparing their $P(X \leq 2)$ any $\lambda$ with 0.05. |
| | [Reject $H_0$] There is evidence to suggest that mean number of accidents has decreased | **A1** | In context, not definite.<br>No contradictions.<br>CWO. |
| | | **3** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 7(c) | P($X > 2$) attempted, with any $\lambda$ | **M1** | |
| | $P(X > 2) = 1 - e^{-1.2}(1 + 1.2 + \frac{1.2^2}{2})$<br>or $= 1 - e^{-1.2}(1 + 1.2 + 0.72)$<br>or $= 1 - ( 0.3012 + 0.3614 + 0.2169 )$ | **M1** | Expression must be seen.<br>Correct $\lambda$.<br>No end errors. |
| | 0.121 (3 sf) or 0.120 | **A1** | SC unsupported answer scores **B2**. |
| | | **3** | |
| 7(d) | N(18, 18) seen or implied | **B1** | |
| | $\frac{10.5-18}{\sqrt{18}}$ [$= -1.768$] | **M1** | Allow with no or incorrect continuity correction.<br>Their 18. |
| | P($X >$ '$-1.768$') = $\Phi$('1.768') | **M1** | ft *their* standardised value.<br>Area consistent with their values. |
| | = 0.961 or 0.962 (3 sf) | **A1** | |
| | | **4** | |

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|---|---|
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| SOI | Seen Or Implied |
| SC | Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance) |
| WWW | Without Wrong Working |
| AWRT | Answer Which Rounds To |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 1 | $\lambda = 4.5$ | **B1** | |
| | $1 - e^{-4.5}(1 + 4.5 + \frac{4.5^2}{2} + \frac{4.5^3}{3!}) = 1 - e^{-4.5}(1 + 4.5 + 10.125 + 15.1875)$<br>$= 1 - (0.011109 + 0.049999 + 0.11248 + 0.16872)$ | **M1** | Expression must be seen or implied by correct figures.<br>Any $\lambda$. Allow one end error.<br>Accept fully correct $\Sigma$ notation. |
| | 0.658 (3 sf) | **A1** | SC unsupported 0.658 scores **B1 B1**. |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 2(a) | 254 [m] | **B1** | |
| | | **1** | |
| 2(b) | $263 =$ '254' $+ 1.96 \times \frac{\sigma}{\sqrt{50}}$ oe or $2 \times 1.96 \times \frac{\sigma}{\sqrt{50}} = 18$ | **M1** | ft their '254' accept 1.96 or 1.645 for **M1**. |
| | $[\sigma = \frac{9\sqrt{50}}{1.96} =]$. s.d. = 32.5 [m] (3 sf) | **A1** | |
| | | **2** | |
| 2(c) | No<br>Because the sample mean is approximately normally distributed [for large $n$] | **B1** | Both needed.<br>Or because of the Central Limit theorem.<br>Or because n is large [accept $\geqslant 30$ condone $\geqslant 50$]. |
| | | **1** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 3(a) | $H_0$: $p = 0.8$ $H_1$: $p > 0.8$ | **B1** | |
| | [Assuming $H_0$, P($X \geqslant 23$) =] ${}^{25}C_{23}\times 0.2^2\times 0.8^{23} + {}^{25}C_{24}\times 0.2\times 0.8^{24} + 0.8^{25}$ =0.070835 + 0.0236118 + 0.0037779 | **M1** | No end errors. Expression must be seen or supported by enough figures to be convinced B(25,0.8) used.<br>Accept correct Σ notation. |
| | = 0.0982 | **A1** | **SC B1** for 0.0982 unsupported. |
| | 0.0982 < 0.1 | **M1** | Valid comparison their 0.0982 must be a tail probability. |
| | [There is evidence to reject $H_0$]<br>There is sufficient evidence to suggest that $p$ has increased | **ftA1** | No contradictions. In context, non-definite.<br>Condone 'there is sufficient evidence that the 'claim' is correct' and condone 'there is sufficient evidence that the number of employees (using the canteen) has increased'<br>Note: CR method will include P($X \geqslant 23$) so **M1 A1** as above, and P($X \geqslant 22$)=0.234>0.1 with at least one probability comparison with 0.1 needed to find CR of 23,24,25 (so 23 in CR) **M1 A1ft** as above. |
| | | **5** | |
| 3(b) | Not suitable as model does not allow for more than 25 employees to use the canteen/Not suitable as uses a sample instead of all employees/Not suitable doesn't include all employees /Not suitable as 30 is only just bigger than 25 should have used 30<br>OR<br>Suitable as owner knows that not all employees use the canteen, or similar | **B1** | Need both (i.e. suitable or not suitable plus reason). |
| | | **1** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4(a) | $\frac{36-35}{8.1\div\sqrt{140}}$ [= 1.461] | **M1** | Ignore inclusion of cc for **M1**. Must have √140. |
| | 1 − Φ('1.461') | **M1** | For area consistent with their values. |
| | = 0.0720 (3 sf) | **A1** | Allow 0.072. |
| | | **3** | |
| 4(b) | $[\Phi^{-1}(0.986)]$ = 2.197 to 2.198 | **B1** | Seen.<br>Note: 2.2 and nothing better seen scores **B0** |
| | $\pm\frac{a-35}{8.1\div\sqrt{140}}$ = ± '2.198' | **M1** | Must be a z value. |
| | $a$ = 36.5 (3 sf) | **A1** | CWO<br>Note: use of 2.2 scores **A1** so 2/3. But e.g. 2.196 gives 36.5 but scores **B0 M1 A0** so 1/3 |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5(a) | $e^{-3.7}(\frac{3.7^3}{3!}+\frac{3.7^4}{4!}+\frac{3.7^5}{5!}) = e^{-3.7}$ (8.44217 + 7.80900 + 5.77866) = 0.20872 + 0.19307 +0.14287 | **M1** | Expression must be seen or implied by correct figures.<br>Any $\lambda$. Allow one end error.<br>Accept fully correct Σ notation. |
| | = 0.545 (3 sf) | **A1** | SC 0.545 unsupported scores **B1**. |
| | | **2** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5(b) | $[\lambda] = 6.3$ | **B1** | |
| | $e^{-6.3}(1 + 6.3 + \frac{6.3^2}{2!} + \frac{6.3^3}{3!}) = e^{-6.3}(1 + 6.3 + 19.845 + 41.6745) =$ $0.0018363 + 0.011569 + 0.0364415 + 0.076527$ | **M1** | Expression must be seen or implied by correct figures.<br>Any $\lambda$. Allow one end error.<br>Accept fully correct $\Sigma$ notation. |
| | $= 0.126$ (3 sf) | **A1** | SC 0.126 unsupported scores **B1 B1** |
| | | **3** | |
| 5(c) | L~N(37, 37), O~N(26, 26) | **B1** | SOI. |
| | (O − L) ~ N(−11, 63) | **B1** | For N(±11, …..) SOI. |
| | | **M1** | For var = 37+26 SOI. |
| | $\frac{0-'-11'}{\sqrt{'63'}}$ [=1.386] or $\frac{0+0.5-'-11'}{\sqrt{'63'}}$ [=1.449] | **M1** | Standardising with their values (wrong cc scores **M1**). |
| | 1 − Φ('1.386') or 1 − Φ('1.449') | **M1** | For area consistent with their working. |
| | = 0.0828 or 0.0829 (3 sf) or = 0.0737 or 0.0736 (3 sf) | **A1** | |
| | | | $SC_1$ 10 used twice N(37,37), N(26,26) and use of 10O-10L>0 Apply MR rules max **B1 B1 M1 M1 M1 A0** (MR)<br>$SC_2$ P(11) giving N(11,11) scores **B0 B1 M0 M1 M1 A0** |
| | | **6** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(a) | Min and max times [to complete challenge] | **B1** | In context (e.g. min and max x scores **B0**). |
| | | **1** | |
| 6(b) | $\int_a^b \frac{1}{x^2}\,dx = 1$ | **M1** | Attempt to integrate f(x) and =1, ignore limits. |
| | $\left[\left[-\frac{1}{x}\right]_a^b = 1\right] -\frac{1}{b}+\frac{1}{a} = 1$ | **A1** | For correct equation using correct limits into correct integration and = 1. |
| | $-a + b = ab$ or $b = a(b + 1)$ | **A1** | Convincingly obtained. No errors seen.<br>OE $\Rightarrow a = \frac{b}{b+1}$ **AG.** |
| | | **3** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(c) | $E(X) = \int_a^b \frac{1}{x} dx$ | **M1** | Attempt to integrate xf(x). Limits a and b or b/(b+1) and b (condone a and 2 for **M1**) See SC for use of limits 2/3 and 2 |
| | $= \ln b - \ln a$ or ln b – ln (b/(b+1) | **A1** | Correct integration and limits substituted.<br>Condone ln 2 – ln a. |
| | $[= \ln b - (\ln b - \ln (b + 1)) ] = \ln b - \ln(b/(b+1) = \ln 3$<br>$= \ln (b + 1) = \ln 3$ or $b+1 = 3$ or $b^2 + b = 3b$ or $\frac{b}{\left(\frac{b}{b+1}\right)} = 3$ | **A1** | For correct equation in b only (i.e. using part **(b)**). |
| | $b = 2$ **(AG)** $a = \frac{2}{3}$ | **A1** | Both obtained correctly (Note: if b=2 not shown but used can score **M1 A1, A1/A0** depending on where b=2 is introduced, **A0**) |
| | | | SC verification: using b=2 and a=2/3 then integrating xf(x) from 2/3 to 2 scores **M1 A1** for integration and limits substituted, then **A1** for showing =ln 3 Final **A0** (as verified not shown) max ¾. |
| | | **4** | |
| 6(d) | $\int_{\frac{2}{3}}^{m} \frac{1}{x^2} dx = 0.5$ or $\int_{m}^{2} \frac{1}{x^2} dx = 0.5$ | **M1** | Attempt to integrate f(x) equated to 0.5 and correct limits stated. |
| | $\left[-\frac{1}{x}\right]_{\frac{2}{3}}^{m} = 0.5$ or $\left[-\frac{1}{x}\right]_{m}^{2} = 0.5$ | **A1FT** | Correct integration FT *their a* . |
| | $[-\frac{1}{m} + \frac{3}{2} = 0.5]$ or $[-\frac{1}{2} + \frac{1}{m} = 0.5]$<br>$m = 1$ | **A1** | |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 7(a) | 0.02 or 2% | **B1** | <0.02 **B0** |
| | | **1** | |
| 7(b) | $H_0$: $\mu = 2.3$ $H_1$: $\mu > 2.3$ | **B1** | Accept 'population mean' for μ (not just mean) If not seen here, can be awarded if correctly seen in part **(a)** |
| | $s^2 = \frac{100}{99}\left(\frac{580}{100} - (2.38)^2\right)$ ) or 1/99 (580 – $238^2$/100) | **M1** | Correct substitution in $s^2$ or $\sqrt{s^2}$ formula. |
| | = 0.137 = 113/825 or $s$ = 0.370 (3 sf) and $\bar{x}$ = 238/100 [= 2.38] | **A1** | $\bar{x}$ and $s^2$ (or $s$) correct.<br>(SC biased estimate 0.1356 and $\bar{x}$ = 2.38 scores **B1**). |
| | $\frac{2.38-2.3}{\sqrt{\frac{'0.137'}{100}}}$ [=2.161 or 2.162] | **M1** | |
| | = 2.16 (3 sf) OR 0.0153/0.0154 if area comparison used | **A1** | |
| | '2.16' > 2.054 (or 2.055) OR '0.0153 or 0.0154'<0.02 | **M1** | Valid comparison. |
| | [There is evidence to reject $H_o$.]<br>There is sufficient evidence to suggest that the [mean] height [in scientist's region] is greater than 2.3 [m] OR there is sufficient evidence to suggest that the scientist's claim is justified. | **A1FT** | No contradictions. In context, non-definite.<br>Accept CV method x = 2.376<2.38 or x = 2.304>2.3 **M1 A1** for x and M1 A1ft for comparison and conclusion<br>Two tail test can score **B0 M1 A1 M1 A1 M1** (comparison with 0.01oe) A0ft max 5/7 |
| | | **7** | |
| 7(c) | Not possible since $H_o$ was rejected. | **B1FT** | Need both. Accept No as $H_0$ was rejected.<br>Follow through their conclusion in **(b)** .<br>Condone a definite statement. |
| | | **1** | |

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CAMBRIDGE
International Education

# Cambridge International A Level

**MATHEMATICS** **9709/63**

Paper 6 Probability & Statistics 2 **October/November 2024**

MARK SCHEME

Maximum Mark: 50

**Published**

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2024 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.



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**PUBLISHED**

## Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

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**Mathematics-Specific Marking Principles**

1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.

2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.

3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.

4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).

5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.

6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

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## Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

### Types of mark

**M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

**A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

**B** Mark for a correct result or statement independent of method marks.

**DM** or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

**FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

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**Abbreviations**

| | |
|---|---|
| AEF/OE | Any Equivalent Form (of answer is equally acceptable) / Or Equivalent |
| AG | Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid) |
| CAO | Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed) |
| CWO | Correct Working Only |
| ISW | Ignore Subsequent Working |
| SOI | Seen Or Implied |
| SC | Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance) |
| WWW | Without Wrong Working |
| AWRT | Answer Which Rounds To |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 1(a) | $z = 2.054$ or 2.055 | **B1** | Accept 3 sf if nothing better seen (2.05 or 2.06). |
| | $1.42 \pm z \frac{0.35}{\sqrt{150}}$ | **M1** | Must be a z value. |
| | 1.36 to 1.48 [m] (3 sf) | **A1** | Correct working only. Must be an interval. |
| | | **3** | |
| 1(b) | No. CI is about mean, not individual values. | **B1** | Or similar. Need both. |
| | | **1** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 2 | $E(S_1 + S_2 + S_3 - L) = 16 \times 3 - 51$ [= −3 ] | **B1** | Oe, using $L - (S_1 + S_2 + S_3)$. |
| | $Var(S_1 + S_2 + S_3 - L) = 3 \times 0.4 + 0.9$ [= 2.1] | **M1** | |
| | $\frac{0-(-3)}{\sqrt{'2.1'}}$ [= 2.070] | **M1** | For standardising with their values. |
| | $1 - \Phi(\text{'2.070'})$ | **M1** | For area consistent with their values. |
| | = 0.0192 (3 sf) | **A1** | |
| | | **5** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 3(a) | $\bar{t} = \frac{230}{75}$ [= 3.0666… or 3.07 (3 sf)] [ 0r 46/15 ] | **B1** | |
| | $s^2 = \frac{75}{74}(\frac{930}{75} - (\frac{230}{75})^2)$ or $1/74(930 - 230^2/75)$ | **M1** | Use of correct formula. |
| | = 3.0360… or 3.04 (3 sf) or = 337/111 | **A1** | |
| | | **3** | |
| 3(b) | $[\ \Phi^{-1}(1 - 0.234)\ ] = 0.726$ | **B1** | |
| | $\pm \frac{a - '3.0667'}{\sqrt{'3.04'/75}} = \pm$ '0.726' | **M1** | Ft their 0.726 but must be a z value.<br>Note using 0.766 is M0.<br>Must have sqrt 75. |
| | $a$ = 3.21 (3 sf) | **A1** | CWO |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4(a) | $\int_2^3 (\frac{a}{x^2} - \frac{18}{x^3})dx = 1$ | **M1** | Attempt integrate f($x$), ignore limits and '= 1'. |
| | $\left[-\frac{a}{x} + \frac{9}{x^2}\right]_2^3 = 1$ | **A1** | OE Correct integration and limits. |
| | $\left[-\frac{a}{3} + 1 + \frac{a}{2} - \frac{9}{4}\right] = 1$ $a = \frac{27}{2}$ **(AG)** | **A1** | Must see correct substitution of limits.<br>Correct working no errors seen. |
| | | **3** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4(b) | $\int_2^3 (\frac{27}{2x} - \frac{18}{x^2})\mathrm{d}x$ | **M1** | Attempt to integrate xf(x), ignore limits. |
| | $\left[\frac{27}{2}\ln x + \frac{18}{x}\right]_2^3$ or $\left[\frac{27}{2}\ln 2x + \frac{18}{x}\right]_2^3$ | **A1** | Correct integration and limits.<br>OE e.g. using ln 2x. |
| | $= \frac{27}{2}\ln 3 + 6 - \frac{27}{2}\ln 2 - 9 = \frac{27}{2}\ln\frac{3}{2} - 3$ **AG** | **A1** | Must see correct substitution of limits.<br>Correct working no errors seen. |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5(a) | Two-tailed because looking for difference | **B1** | |
| | | **1** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5(b) | $H_0$: $\mu = 12.7$ $H_1$: $\mu \neq 12.7$ | **B1** | No ft from part **(a).** |
| | $\frac{\frac{597.1}{50} - 12.7}{\frac{2.3}{\sqrt{50}}}$ | **M1** | |
| | $= -2.330$<br>Accept – 2.336 or – 2.337 | **A1** | or 0.00989 or 0.0099.<br>or 0.0097 if area comparison used. |
| | '−2.330' > −2.576 or '2.330' < 2.576<br><br>or '0.00989' > 0.005<br>or `0.0097` > 0.005 | **M1** | Accept 2.574 to 2.579.<br><br>Or use of CV.<br>12.7- 2.576 x ( 2.3 / sqrt 50 ) = 11.862 **M1A1**.<br>11.942 > 11.862 **M1A1**. |
| | [Not reject $H_0$] There is insufficient evidence to suggest that $\mu$ is not 12.7 | **A1 FT** | OE ft their $z_{calc.}$<br>In context, not definite, e.g. not '$\mu$ =12.7'.<br>No contradictions.<br><br>SC use of 1 tailed test can score **B0M1A1M1** for comparison with 0.01 **A0** max 3/5. |
| | | **5** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(a) | [$\lambda = 2.7$] $1 - e^{-2.7}(1 + 2.7 + \frac{2.7^2}{2})$ or $1 - e^{-2.7}(1 + 2.7 + 3.645)$<br>or 1- ( 0.06721 + 0.1815 + 0.2450 ) | **M1** | Any $\lambda$. Allow one end error.<br>Must see expression. |
| | = 0.506 (3 sf) | **A1** | SC unsupported answer 0.506 scores **B1**. |
| | | **2** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(b) | $\lambda = 1.95$ | **B1** | |
| | $e^{-1.95}(1 + 1.95 + \frac{1.95^2}{2} + \frac{1.95^3}{3!})$ or $e^{-1.95}(1 + 1.95 + 1.90125 + 1.2358)$ or $0.1423 + 0.2774 + 0.2705 + 0.1758$ | **M1** | Any $\lambda$. Allow one end error.<br>Must see expression. |
| | $= 0.866$ | **A1** | SC unsupported answer 0.866 scores **B1B1**. |
| | | **3** | |
| 6(c) | $1 - e^{-2.1x} \geqslant 0.90$ or $1 - e^{-\lambda} \geqslant 0.90$ | **M1** | OE Condone use of '=' throughout. |
| | | | |
| | $[e^{-2.1x} \leq 0.1]$ or $e^{-\lambda} \leq 0.1$<br>$-2.1x \leq \ln 0.1$ or $-\lambda \leq \ln 0.1$ [ $\lambda \geq 2.3026$, $2.3026/2.1$ ] | **M1** | Rearrange and attempt take logs of relevant form. |
| | 1.096 or 10.96 accept 1.097 or 10.97 | ***A1** | Seen. |
| | She must wait for at least 11 minutes | **A1 dep** | |
| | | | SC Use of trial and improvement.<br>Use of $1-e^{-\lambda}$ any numerical $\lambda$ (not 2.1) ie one trial **M1**.<br>Use of enough trials to give an answer of 0.90 (2sf) **M1**.<br>$\lambda$=2.30 i.e. 3sf accuracy AND 1.09… or 10.9 … **A1**.<br>Then 11 **A1** dep. |
| | | **4** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 7(a) | $\lambda = 6.6$ | **B1** | |
| | $P(X \leq 2) = e^{-6.6}(1 + 6.6 + \frac{6.6^2}{2})$ [= 0.0400] [ < 0.05 ]<br>or $e^{-6.6}(1 + 6.6 + 21.78)$<br>or 0.001360 + 0.008978 + 0.02963 | **M1** | Expression must be seen. No end errors.<br>Allow use of 3.3 here. |
| | $P(X \leq 3) = e^{-6.6}(1 + 6.6 + \frac{6.6^2}{2} + \frac{6.6^3}{3!})$ or $0.0400 + e^{-6.6} \times \frac{6.6^3}{3!}$ = 0.105 [ > 0.05 ] | **B1** | Condone unsupported 0.105. |
| | P(Type I error) = 0.0400 (3 sf) | **A1** | Allow 0.040 or 0.04 AWRT<br><br>SC unsupported ans of 0.0400 can score max **B1B1B1.** |
| | | **4** | |
| 7(b) | $H_0$: $\lambda = 6.6$, $H_1$: $\lambda < 6.6$ | **B1** | May be seen in part (a) and award **B1** mark here.<br>Accept μ or $\lambda$.<br>Accept 3.3 or 6.6. |
| | [$P(X \leq 2) = 0.0400$] ` 0.04 ` < 0.05 | **M1** | For comparing their $P(X \leq 2)$ any $\lambda$ with 0.05. |
| | [Reject $H_0$] There is evidence to suggest that mean number of accidents has decreased | **A1** | In context, not definite.<br>No contradictions.<br>CWO. |
| | | **3** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 7(c) | P($X > 2$) attempted, with any $\lambda$ | **M1** | |
| | $P(X > 2) = 1 - e^{-1.2}(1 + 1.2 + \frac{1.2^2}{2})$<br>or $= 1 - e^{-1.2}(1 + 1.2 + 0.72)$<br>or $= 1 - ( 0.3012 + 0.3614 + 0.2169 )$ | **M1** | Expression must be seen.<br>Correct $\lambda$.<br>No end errors. |
| | 0.121 (3 sf) or 0.120 | **A1** | SC unsupported answer scores **B2**. |
| | | **3** | |
| 7(d) | N(18, 18) seen or implied | **B1** | |
| | $\frac{10.5-18}{\sqrt{18}}$ [$= -1.768$] | **M1** | Allow with no or incorrect continuity correction.<br>Their 18. |
| | P($X >$ '$-1.768$') = $\Phi$('1.768') | **M1** | ft *their* standardised value.<br>Area consistent with their values. |
| | = 0.961 or 0.962 (3 sf) | **A1** | |
| | | **4** | |

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International Education

# Cambridge International A Level

**MATHEMATICS** **9709/61**

Paper 6 Probability & Statistics 2 **May/June 2024**

MARK SCHEME

Maximum Mark: 50

**Published**

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**M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

**A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

**B** Mark for a correct result or statement independent of method marks.

**DM** or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

**FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

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**Abbreviations**

| | |
|---|---|
| AEF/OE | Any Equivalent Form (of answer is equally acceptable) / Or Equivalent |
| AG | Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid) |
| CAO | Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed) |
| CWO | Correct Working Only |
| ISW | Ignore Subsequent Working |
| SOI | Seen Or Implied |
| SC | Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance) |
| WWW | Without Wrong Working |
| AWRT | Answer Which Rounds To |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 1 | $\lambda = 7.2$ | **B1** | |
| | $P(X > 3) = 1 - e^{-7.2}(1 + 7.2 + \frac{7.2^2}{2!} + \frac{7.2^3}{3!})$<br>or $1 - e^{-7.2}(1 + 7.2 + 25.92 + 62.21)$<br>or $1 - (0.0007466 + 0.005375 + 0.01935 + 0.04644)$ | **M1** | Allow any $\lambda$.<br>Allow one end error.<br>Must see expression. Allow fully correct sigma notation. |
| | = 0.928 (3sf) | **A1** | **SC** 0.928 with no working seen scores **B1 B1**. |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 2 | $E(X_1 - 3X_2) = 31.2 - 3\times31.2$ [ = −62.4] | **B1** | OE $E(3X_2 - X_1) = +62.4$ |
| | $Var(X_1 - 3X_2) = 10.4^2 + 3^2 \times 10.4^2$ [= 1081.6] | **B1** | |
| | $\frac{0-('-62.4')}{\sqrt{'1081.6'}}$ [= 1.897] | **M1** | Standardising (with attempt at E and Var, not just using 31.2 and 10.4). |
| | $1 - \Phi('1.897')$ | **M1** | For area consistent with their working. |
| | = 0.0289 (3 sf) | **A1** | |
| | | **5** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 3(a) | $81.5 \pm z \times \frac{5.8}{\sqrt{20}}$ | **M1** | For a correct expression (accept if only one side of the interval calculated).<br>Any $z$ (must be a $z$). |
| | $z = 2.326$ | **B1** | |
| | 78.5 to 84.5 (3sf) | **A1** | Must be an interval. |
| | | **3** | |
| 3(b) | Not true.<br>C. I. is for **mean** time, not individual times. | **B1** | OE<br>Both comments needed. |
| | | **1** | |
| 3(c) | $\frac{0.98^2}{1-0.02^2}$ | **M1** | Attempt $\frac{\text{P(both contain } \mu)}{\text{P(at least one contains } \mu)}$ with numerator attempt $0.98^2$ and denominator attempt involving 0.02.<br>Must see their quotient. |
| | = 0.961 (3sf) | **A1** | NB: [$0.98^{2=}$] 0.9604 scores M0 A0. |
| | | **2** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4(a) | Est($\mu$) = $\frac{2044}{8}$ [=255.5] | **B1** | Accept 3sf if nothing better seen. |
| | Est($\sigma^2$) = $\frac{8}{7}\left(\frac{522348}{8} - "255.5"^2\right)$ or $\frac{1}{7}$('522348' – $\frac{'2044'^2}{8}$) | **M1** | Attempt to find $\Sigma x^2$ and substitute in correct formula. May be implied by correct answer.<br>Biased 13.25 scores M0. |
| | = 15.1 (3 sf) or $\frac{106}{7}$ | **A1** | OE |
| | | **3** | |
| 4(b)(i) | $H_0$: $\mu = 253$<br>$H_1$: $\mu > 253$ | **B1** | Allow 'Population mean' but not just 'mean'. |
| | $\frac{\frac{25360}{100} - 253}{3.5 \div \sqrt{100}}$ | **M1** | Standardising must have $\sqrt{100}$. |
| | = 1.714 | **A1** | |
| | 1.714 > 1.645 or 0.0432 < 0.05 | **M1** | OE |
| | [Reject $H_0$ ] There is sufficient evidence (at 5% level) to suggest [mean] mass is greater than 253 | **A1FT** | OE<br>FT *their* '1.714' in context, not definite, no contradictions.<br>Accept critical value method of 253.57 < 253.60 or 253.02 > 253.<br>Use of a two-tailed test scores B0 M1 A1 M1 A0 (comp with $\frac{0.025}{1.96}$ ). |
| | | **5** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4(b)(ii) | Not true. Large sample, [so sample mean is approx normally distributed]. | **B1** | OE<br>Allow 'Not true. Large sample'<br>or 'Not true. $n$ is large'<br>or 'Not true. CLT used'. |
| | | **1** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5(a) | Constant average rate | **B1** | OE<br>Accept constant rate. Allow without context. |
| | | **1** | |
| 5(b) | $\lambda = 14.4$ | **B1** | |
| | $e^{-14.4}\left(\frac{14.4^{13}}{13!} + \frac{14.4^{14}}{14!} + \frac{14.4^{15}}{15!}\right)$<br>or $e^{-14.4}(183837 + 189089 + 181526)$<br>or $(0.102469 + 0.105396 + 0.101181)$ | **M1** | Poisson P(13, 14, 15). Expression must be seen.<br>Allow one end error; allow any $\lambda$. Allow fully correct sigma notation. |
| | = 0.309 (3sf) | **A1** | **SC**: 0.309 with no working scores **B1 B1**. |
| | | **3** | |
| 5(c) | N(140×1.2, 140×1.2) or N(168, 168) | **B1** | Stated or implied. |
| | $\frac{149.5 - 168}{\sqrt{168}}$ [= −1.427] | **M1** | Standardising using their mean and variance. Allow with wrong or no continuity correction. |
| | Φ("−1.427") = 1 − Φ("1.427") | **M1** | For area consistent with their working. |
| | = 0.0768 or 0.0767 (3sf) | **A1** | |
| | | **4** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(a) | $\frac{1}{4}\pi a^2 = 1$ | **M1** | OE<br>Attempt to set area = 1. |
| | $a = \frac{2}{\sqrt{\pi}}$ | **A1** | AG<br>Correct equation and correctly rearranged to $a = \ldots$<br>No errors seen. |
| | | **2** | |
| 6(b) | $x^2 + y^2 = \left(\frac{2}{\sqrt{\pi}}\right)^2$ | **M1** | Or $x^2 + y^2 = a^2$. |
| | $[y^2 = \frac{4}{\pi} - x^2\ ]$ | | Must see at least one intermediate step |
| | $\mathrm{f}(x) = \sqrt{\frac{4}{\pi} - x^2}$ | **A1** | AG<br>Convincingly rearranged to reach given answer.<br>No errors seen. |
| | | **2** | |
| 6(c) | $\int_0^{\frac{2}{\sqrt{\pi}}} x\sqrt{\frac{4}{\pi} - x^2}\,\mathrm{d}x$ | **B1** | Correct expression for $\mathrm{E}(X) = \int x\mathrm{f}(x)\,\mathrm{d}x$ with correct limits (accept limits 0 and $a$). |
| | $-\frac{1}{3}\left[\left(\frac{4}{\pi} - x^2\right)^{\frac{3}{2}}\right]_0^{\frac{2}{\sqrt{\pi}}}$ | **M1** | Integrate $x\mathrm{f}(x)$ with any limits or none.<br>Must reach expression of form:<br>any constant $\times \left(\frac{4}{\pi} - x^2\right)^{\frac{3}{2}}$. |
| | | **A1** | Wholly correct integration and limits. |
| | $\frac{8}{3\sqrt{\pi^3}}$ | **A1** | AG<br>Correctly obtained with no errors seen. |
| | | **4** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 7(a) | $H_0$: $p = 0.3$<br>$H_1$: $p < 0.3$ | **B1** | |
| | B(31, 0.3), P($X \leqslant 4$) =<br>$0.7^{31} + 31\times0.7^{30}\times0.3 + {}^{31}C_2\times0.7^{29}\times0.3^2 + {}^{31}C_3\times0.7^{28}\times0.3^3 +$<br>${}^{31}C_4\times0.7^{27}\times0.3^4$<br>= 0.00001577 + 0.0002096 + 0.0013475 + 0.0055826 + 0.016748 | **M1** | No end errors. |
| | = 0.0239 (3sf) | **A1** | **SC** 0.0239 with no working scores **B1**. |
| | '0.0239' < 0.05 | **M1** | Valid comparison. |
| | [reject $H_0$]<br>'There is sufficient evidence (at 5% level) to support Rita's suspicion',<br>or 'There is sufficient evidence to suggest the probability of seeing this type of bird has decreased' | **A1FT** | In context.<br>Not definite. No contradictions.<br>FT *their* 0.0239. |
| | | **5** | |
| 7(b) | P($X \leq 5$) = ['0.0239' + ${}^{31}C_5\times0.7^{26}\times0.3^5$] = 0.0627 [which is > 0.05] | **B1FT** | Attempt P($X \leqslant 5$).<br>Only FT if > 0.05.<br>Only FT *their* 0.0239 if P($X \leqslant 4$) attempted in **(a)**; arithmetic error only. |
| | P(Type I error) = '0.0239' | **B1FT** | Only FT *their* 0.0239 if P($X \leqslant 4$) attempted in **(a)**; arithmetic error only and *their* 0.0239 < 0.05. |
| | | **2** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 7(c)(i) | [λ=] 3.65 | **B1** | Stated or implied. |
| | $e^{-3.65} \times \frac{3.65^4}{4!}$ | **M1** | Must see expression. Any λ. |
| | = 0.192 (3sf) | **A1** | **SC**: Use of Binomial. 0.193 scores **B1**.<br>**SC**: 0.192 with no working scores **B1 B1**. |
| | | **3** | |
| 7(c)(ii) | $n = 365 > 50$<br>$np = 3.65 < 5$ or $p = 0.01 < 0.1$ | **B1** | Explicit. Both needed.<br>Note: and '*n* large, *p* small' is insufficient. |
| | | **1** | |

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# Cambridge International A Level

**MATHEMATICS** **9709/62**

Paper 6 Probability & Statistics 2 **May/June 2024**

MARK SCHEME

Maximum Mark: 50

**Published**

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

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**PUBLISHED**

## Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

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**Mathematics-Specific Marking Principles**

1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.

2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.

3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.

4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).

5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.

6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 1(a) | N(145, 145) | **B1** | Stated or implied. |
| | $\pm\frac{150.5-145}{\sqrt{145}}$ [= ±0.457] | **M1** | Condone incorrect or omitted continuity correction. |
| | Φ('0.457') | **M1** | For area consistent with their working. |
| | = 0.676 (3sf) | **A1** | **SC**: Unsupported answer of 0.676 scores **B3**.<br>Unsupported answer of 0.646 or 0.661 scores **B2**.<br>Unsupported answer of 0.6799 scores **B1**. |
| | | **4** | |
| 1(b) | 145 > 15 | **B1** | Explicit.<br>λ > 15 B0 if λ = 145 not stated.<br>Accept ⩾<br>Accept mean for λ. |
| | | **1** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 2(a) | [567, 109], 665, 21 | **B2** | B1 for each.<br>Allow 021.<br>If more than 2 **answers** given, count first two and ISW. |
| | | **2** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 2(b) | Est($\mu$) = $\frac{610}{30}$ or $\frac{61}{3}$ | **B1** | OE or 20.3. |
| | Est($\sigma^2$) = $\frac{30}{29}(\frac{12405}{30} - (\frac{610}{30})^2)$ or $\frac{1}{29}\left(12405 - \frac{610^2}{30}\right)$ | **M1** | Use of correct formula. |
| | = 0.0575 (3sf) | **A1** | Accept $\frac{5}{87}$. |
| | | **3** | |
| 2(c) | Variance is [unrealistically] small<br>so Henri has [probably] made a mistake/claim is [probably] correct | **B1 FT** | Need both parts.<br>Need 'small' OE, not just < 0.1. FT *their* < 0.1 variance value (not –ve), e.g. 0.0556 (if omit $\frac{30}{29}$).<br>Accept 's.d. = 0.24 is small, so Henri has probably made a mistake'.<br>Note: 'mean is large/small' scores B0, but 'mean large compared to variance so Henri prob made a mistake' scores B1. |
| | | **1** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 3 | $\frac{18}{50} - z \times \sqrt{\frac{\frac{18}{50}\times(1-\frac{18}{50})}{50}} = 0.244$ | **M1** | Use of correct equation. |
| | $z$ = 1.709 or 1.708 | **A1** | Accept 1.71 if nothing better seen. |
| | $\Phi^{-1}$('1.709') = 0.956 ; 1 − 2(1 – '0.956') [= 0.912] | **M1** | Attempt area above or below their 1.709 and use correct method to find $\alpha$. |
| | $\alpha$ = 91 | **A1** | Allow $\alpha$ = 91%<br>0.91 or 91.2 score A0. |
| | | **4** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4(a) | 0.5 | **B1** | |
| | | **1** | |
| 4(b) | E($X_1 - 2X_2 + 3$) =10-20+3 [= −7]<br>or E($2X_2 - X_1 - 3$) = 20 − 10 – 3 [= 7] | **B1** | Or equivalent using $X_1 - 2X_2$ =10 – 20 [= –10]<br>or $2X_2 - X_1$ = 20 – 10 [= +10]. |
| | Var($X_1 - 2X_2 + 3$) = 12 + $2^2$×12 + 0 [= 60] | **B1** | |
| | $\frac{0-('-7')}{\sqrt{'60'}}$ [= 0.904] | **M1** | Or numerator 3–‘10’ or –3–(‘–10’), but not ‘–3 –10’ (i.e. numerator must be ‘7’ or ‘–7’). |
| | 1 − Φ(‘0.904’) | **M1** | For area consistent with their working. |
| | = 0.183 | **A1** | |
| | | **5** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5(a) | $e^{-3.1}(1 + 3.1 + \frac{3.1^2}{2!} + \frac{3.1^3}{3!})$<br>or $e^{-3.1}$(1 + 3.1 + 4.805 + 4.965)<br>or 0.0450 + 0.1397 + 0.2165 + 0.22368 | **M1** | Condone one end error. Any λ.<br>Accept fully correct Σ notation.<br>Expression must be seen. |
| | = 0.625 (3sf) | **A1** | Correct answer with no working scores **SC B1**. |
| | | **2** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5(b) | $[\lambda]= 5.5$ | **B1** | SOI |
| | $1 - e^{-5.5}(1 + 5.5 + \frac{5.5^2}{2!} + \frac{5.5^3}{3!} + \frac{5.5^4}{4!})$<br>or $1 - e^{-5.5}(1 + 5.5 + 15.125 + 27.7292 + 38.1276)$<br>or $1 - e^{-5.5}(0.004087 + 0.0224772 + 0.061812 + 0.113323 + 0.155819)$ | **M1** | Condone one end error.<br>Any λ.<br>Accept fully correct Σ notation.<br>Expression must be seen. |
| | = 0.642 or 0.643 (3sf) | **A1** | Correct answer with no working scores **SC B1 B1**. |
| | | **3** | |
| 5(c) | $[P(X = 3) \times P(Y = 2) = ] = e^{-3.1} \times \frac{3.1^3}{3!} \times e^{-2.4} \times \frac{2.4^2}{2!}$<br>or 0.223676 × 0.261267 [= 0.05844] | **M1** | Find P(3 in first half AND 2 in second half).<br>Must see expression. |
| | [P(total 5) = ] $e^{-5.5} \times \frac{5.5^5}{5!}$ or 0.17140 | **M1** | Use of 5.5 to find P(5). |
| | P(P(exactly 3 in 1st half given total 5) = $\frac{\text{P(exactly 3 in 1st half and total 5)}}{\text{P(total 5)}}$ | **M1** | Attempt at conditional probability;<br>numerator = *their* 0.05844 and<br>denominator = P(total 5)<br>Note: $(\frac{3.1^3}{3!} \times \frac{2.4^2}{2!}) \div (\frac{5.5^5}{5!})$ scores M1 M1 M1. |
| | $[= \frac{'0.05844'}{'0.17140'}] = 0.341$ (3sf) | **A1** | |
| | | **4** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(a) | $H_0$: Population mean mass = 510 g<br>$H_1$: Population mean mass < 510 g | **B1** | Allow '$\mu$' but not just 'mean'. |
| | $\pm\frac{508-510}{10\div\sqrt{120}}$ | **M1** | Standardising must have $\sqrt{120}$. |
| | $= \pm -2.191$ or $-2.190$ | **A1** | |
| | $-2.191 < -1.96$ or $2.191 > 1.96$<br>Area comparison: 0.0143 or 0.0142 < 0.025 | **M1** | OE<br>For valid comparison.<br>Inequality sign the wrong way round scores M1 A0. |
| | [Reject $H_0$] There is sufficient evidence to suggest that the [mean] mass has decreased | **A1FT** | OE<br>In context (must be 'decreased' OE, not 'changed'); not definite. No contradictions. Condone 'there is sufficient evidence to support the inspector's claim'.<br>NB: Accept alternative method using critical value (= 508.21) and comparison with 508. Condone 509.79 compared with 510.<br>Two tail test scores maximum B0 M1 A1 M1 A0; must have comparison with 0.0125 or 2.24/2.241. |
| | | **5** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(b) | $\frac{\text{cv} - 510}{10 \div \sqrt{120}} = -1.96$ | **M1** | Standardising to find critical value (must use 510 and $10 \div \sqrt{120}$). Accept ± 1.96. |
| | cv = 508.21 | **A1** | Accept 3 sf if nothing better seen.<br>Note: cv could be found in **(a)**. |
| | $z = \pm \frac{508.21 - 506}{10 \div \sqrt{120}}$ [= 2.421] | **M1** | Standardising with their 508.21 and 506 (must use $10 \div \sqrt{120}$). |
| | $\text{P}(\overline{X} > 508.21 \mid \mu = 506) = 1 - \Phi(\text{'2.421'})$ | **M1** | For area consistent with their working. |
| | = 0.0077 to 0.0080 (2sf) | **A1** | Note: $\frac{510 - 506}{10 \div \sqrt{120}}$ scores max M0 A0 M1 M1 A0. |
| | | **5** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 7(a) | $k\int_0^{\pi}(1+\cos x)\text{d}x = 1$ | **M1** | Attempt integrate f($x$) with correct limits and equate to 1. |
| | $k\left[x + \sin x\right]_0^{\pi} = 1$ | **A1** | Correct integration. |
| | [e.g. $k(\pi + \sin\pi - (0 + 0)) = 1$], $k\pi = 1$, $k = \frac{1}{\pi}$ | **A1** | AG<br>Some evidence of substitution of limits, i.e. at least one interim step (e.g. $k\pi = 1$) as minimum requirement.<br>Convincingly obtained; no errors seen. |
| | | **3** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 7(b) | $\frac{1}{\pi}\left[x+\sin x\right]_0^{0.83}$ or $\frac{1}{\pi}(0.83 + \sin 0.83)$ <br> $\frac{1}{\pi}\left[x+\sin x\right]_0^{0.84}$ or $\frac{1}{\pi}(0.84 + \sin 0.84)$ | **M1** | Substitute correct limits into their integral.<br>$OR_1$: integrate 0 to 0.83 and 0.84 to π.<br>$OR_2$: use g($m$) = $m$ + sin $m$ – (π/2) and find g(0.83) and g(0.84).<br>$OR_3$: use h($m$) = $m$ + sin $m$ and find h(0.83) and h(0.84).<br>Both attempted. |
| | = 0.499 (3 sf) <br> = 0.504 (3 sf) | **A1** | $OR_1$: 0.499 and 0.496.<br>$OR_2$: g(0.83) = –0.00286/7 and g(0.84) = 0.0138/9.<br>$OR_3$: h(0.83) = 1.57 and h(0.84) = 1.58 or 1.59.<br>Both correct. |
| | '0.499' < 0.5 < '0.504' hence 0.83 < median < 0.84<br>Equivalent to<br>–0.000912 < 0 < 0.00441 hence 0.83 < median < 0.84 | **A1FT** | FT their areas; dep 0.5 is between their areas OE.<br>$OR_1$: 0.499 < 0.5 and 0.496 < 0.5, so 0.83 < $m$ < 0.84.<br>$OR_2$: g(0.83) > 0 g(0.84) < 0 OE, so 0.83 < m < 0.84.<br>$OR_3$: h(0.83) < $\frac{\pi}{2}$ h(0.84) > $\frac{\pi}{2}$, so 0.83 < $m$ < 0.84.<br>Both statements needed.<br>Note: A score of M1 A0 A1FT is possible. |
| | | | If 0 scored, **SC:** $\frac{1}{\pi}(m + \sin m) = 0.5$ **B1**<br>and $m$ = 0.831 to 0.832, so 0.83< $m$ < 0.84 **B1**. |
| | | **3** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 7(c) | $\frac{1}{\pi}\int_0^{\pi}(x+x\cos x)\mathrm{d}x$ | **M1*** | Attempt integrate $xf(x)$.<br>Ignore limits. |
| | $=\frac{1}{\pi}\left(\left[\frac{x^2}{2}\right]_0^{\pi}+\left[x\sin x\right]_0^{\pi}-\int_0^{\pi}(\sin x\mathrm{d}x)\right)$ | **DM1** | OE<br>Attempt to integrate (using 'parts') with correct limits, reaching an expression of the form<br>$ax^2 + uv - \int v\,\mathrm{d}u$.<br><br>OR using parts to integrate $x(1+\cos x)$ reaching an expression of the form $uv - \int v\,\mathrm{d}u$<br>i.e. $\frac{1}{\pi}(x^2 + x\sin x - \int(x + \sin x)\,\mathrm{d}x))$ |
| | $=\frac{1}{\pi}\left(\frac{x^2}{2}+x\sin x+\cos x\right)$<br>or e.g. $\frac{\pi}{2}+\frac{1}{\pi}(0-[-\cos x]_0^{\pi})$<br>or e.g. $\frac{\pi}{2}+\frac{1}{\pi}((-1-1))$ | **A1** | Integration fully correct. |
| | $=\frac{\pi}{2}-\frac{2}{\pi}$ | **A1** | OE<br>ISW after correct exact value seen.<br>**SC1**: Unsupported answer of $\frac{\pi}{2}-\frac{2}{\pi}$ scores **B3**.<br>**SC2**: Unsupported answer of 0.934 scores **B2**. |
| | | **4** | |

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CAMBRIDGE
International Education

# Cambridge International A Level

**MATHEMATICS** **9709/63**

Paper 6 Probability & Statistics 2 **May/June 2024**

MARK SCHEME

Maximum Mark: 50

**Published**

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2024 series for most Cambridge IGCSE, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

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**PUBLISHED**

## Generic Marking Principles

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Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

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**Mathematics-Specific Marking Principles**

1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.

2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.

3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.

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5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.

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**PUBLISHED**

## Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

### Types of mark

**M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

**A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

**B** Mark for a correct result or statement independent of method marks.

**DM** or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

**FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

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**Abbreviations**

| | |
|---|---|
| AEF/OE | Any Equivalent Form (of answer is equally acceptable) / Or Equivalent |
| AG | Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid) |
| CAO | Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed) |
| CWO | Correct Working Only |
| ISW | Ignore Subsequent Working |
| SOI | Seen Or Implied |
| SC | Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance) |
| WWW | Without Wrong Working |
| AWRT | Answer Which Rounds To |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 1(a) | $[\lambda =] 4$ | **B1** | |
| | $e^{-4}\left(\frac{4^2}{2!}+\frac{4^3}{3!}+\frac{4^4}{4!}\right)$<br>or $e^{-4}(8 + 10.67 + 10.67)$<br>or $0.1465 + 0.19537 + 0.19537$ | **M1** | Allow one end error.<br>Any $\lambda$.<br>Expression must be seen. |
| | $= 0.537$ (3sf) | **A1** | **SC B1 B1** for unsupported correct answer.<br>**SC B2** for use of Binomial leading to 0.537.<br>Note: use of normal could score B1 only for mean = 4. |
| | | **3** | |
| 1(b) | $n = 4000 > 50$ and either $np = 4 < 5$ or $p = 0.001 < 0.1$ | **B1** | Explicit values seen. |
| | | **1** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 2(a) | $3.12 + z \times \frac{\sigma}{\sqrt{150}} = 3.23$ | **M1** | OE – correct expression.<br>Any $z$, but must be a $z$ value. |
| | $z = 1.96$ | **B1** | |
| | $\sigma = 0.687$ (3sf) [cm] | **A1** | |
| | | **3** | |
| 2(b) | Yes,<br>because population [of widths] not given to be normally distributed | **B1** | Or 'underlying distribution' instead of population.<br>Allow 'yes, because population distribution not known'.<br>Need both statements. |
| | | **1** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 3 | Diff ~ N(5×50 − 10×26, 5×2.4 + 10×1.8) [= N(−10, 30) ] | **B1** | For N and mean = ±(5×50 − 10×26) SOI |
| | | **B1** | For var = 5×2.4 + 10×1.8 SOI |
| | $\frac{0-('-10')}{\sqrt{'30'}}$ [= 1.826] | **M1** | Standardising with *their* values. |
| | 1 − Φ('1.826') | **M1** | For area consistent with *their* values. |
| | = 0.0339 or 0.034[0] (3sf) | **A1** | |
| | | **5** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4 | $H_0$: $p = 0.24$<br>$H_1$: $p < 0.24$ | **B1** | |
| | $P(X \leq 2) = 0.76^{25} + 25\times0.76^{24}\times0.24 + {}^{25}C_2\times0.76^{23}\times0.24^2$<br>or 0.0010479 + 0.0082732 + 0.0313513 | **M1** | Expression must be seen.<br>No end errors. |
| | = 0.0407 | **A1** | **SC B1** for unsupported 0.0407. |
| | 0.0407 < 0.05 | **M1** | For valid comparison. |
| | [Evidence to reject $H_0$.] There is sufficient evidence to suggest that the support for the Today Party has decreased. | **A1FT** | FT *their* probability.<br>In context, not definite, no contradictions.<br>**SC**: if $H_1$: $p \neq 0.24$ and compare with 0.025;<br>max **B0 M1 A1 M1 A0**. |
| | | **5** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5(a) | $\int_0^{\sqrt{2}} (ax - x^3)\mathrm{d}x = 1$ | **M1** | Attempted integration of f($x$) and equated to 1. |
| | $\left[a\frac{x^2}{2} - \frac{x^4}{4}\right]_0^{\sqrt{2}} = 1$<br>$a - \frac{4}{4} = 1$ | **A1** | Correct integration and substitute correct limits. |
| | $a = 2$ | **A1** | AG<br>Convincingly obtained and no errors seen. |
| | | **3** | |
| 5(b) | $\int_0^m \left(2x - x^3\right)dx =$ $= \frac{1}{2}$ | **M1** | Attempt integrate f($x$) with limits 0 to $m$ (or $m$ to $\sqrt{2}$) and equate to $\frac{1}{2}$. |
| | $m^2 - \frac{m^4}{4} = \frac{1}{2}$ | **A1** | For correct quartic in any form. |
| | $m^4 - 4m^2 + 2 = 0 \Rightarrow m^2 = \frac{4 \pm \sqrt{16-8}}{2}$ $[= 2 \pm \sqrt{2}\,]$ | **M1** | For solving their three term quartic to find $m^2$. |
| | $m = \sqrt{2-\sqrt{2}}$ or 0.765 (3sf) | **A1** | |
| | | **4** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5(c) | $\int_0^{\sqrt{2}} (2x^2 - x^4)\mathrm{d}x$ | **M1** | Attempt to integrate $xf(x)$. Ignore limits. |
| | $\left[\frac{2x^3}{3} - \frac{x^5}{5}\right]_0^{\sqrt{2}}$ | **A1** | Correct integration and correct limits. |
| | $[= \frac{4\sqrt{2}}{3} - \frac{4\sqrt{2}}{5}] = \frac{8}{15}\sqrt{2}$ | **A1** | OE<br>For single exact term. |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(a) | $\hat{\mu} = \frac{307}{200}$ or 1.535 | **B1** | Accept 3sf if nothing better seen (1.53 or 1.54). |
| | $\Sigma x^2 f = 627$, $\text{Est}(\sigma^2) = \frac{200}{199}(\frac{'627'}{200} - '1.535'^2)$ or $\frac{1}{199}\left('627' - \frac{'307'^2}{200}\right)$ | **M1** | Use of a correct formula with *their* values. |
| | = 0.783 | **A1** | AG<br>Correctly obtained with no errors seen. |
| | | **3** | |
| 6(b) | $H_0$: $\mu = 1.65$<br>$H_1$: $\mu < 1.65$ | **B1** | Accept 'population mean' but not just 'mean'. |
| | | **1** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(c) | $\frac{'1.535'-1.65}{\sqrt{0.783\div 200}}$ | **M1** | Standardising with *their* mean. |
| | $= -1.838$ or $-1.84$ | **A1*** | |
| | $\Phi(0.05)$ and $\Phi(0.01)$ attempted | **M1** | Or P($z < -$`1.838`) attempted.<br>**SC**: Condone $\Phi(0.025) = 2.807$ and $\Phi(0.005) = 3.291$ following two-tailed test in **(b)**. |
| | $-1.645 > -1.838 > -2.326$<br><br>[Hence significant at 5% but not 1% level] | **DA1** | AG<br>$= 0.033$ and $0.05 > 0.033 > 0.01$<br>**SC**: use of 1.54 or 1.53 for the mean leading to -1.645 > –1.758 > –2.326<br>or –1.645 > -1.918 > –2.326<br>or 0.95 < 0.9606 or 0.9724 < 0.99<br>scores **M1 M1 A1**.<br>Accept use of critical value method<br>1.535 < 1.547<br>or accept 1.65 > 1.638. |
| | | **4** | |
| 6(d) | At the 1% level $H_0$ is not rejected<br>Or a Type I error can only occur if $H_0$ is rejected. | **B1** | OE |
| | | **1** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 7(a) | $\lambda = 1.9 + 2.2$ [= 4.1] | **B1** | |
| | $e^{-4.1}(1 + 4.1 + \frac{4.1^2}{2!} + \frac{4.1^3}{3!})$<br>or $e^{-4.1}(1 + 4.1 + 8.405 + 11.487)$<br>or 0.01657 + 0.06795 + 0.13929 + 0.19037 | **M1** | Allow any $\lambda$.<br>Allow one end error.<br>Must see expression. |
| | = 0.414 (3sf) | **A1** | **SC**: unsupported answer 0.414 scores **B1 B1**. |
| | | **3** | |
| 7(b) | P($X + Y < 4$ and $X = 2$) = P(2, 0 or 2, 1) | **M1** | Stated or implied. |
| | $= e^{-1.9} \times \frac{1.9^2}{2}(e^{-2.2} + e^{-2.2} \times 2.2)$ [= 0.0957] | **M1** | |
| | P($X = 2 \mid X + Y < 4$) = $\frac{'0.0957'}{'0.414'}$ | **M1** | Attempt $\frac{P(X+Y<4 \text{ and } X=2)}{P(X+Y<4)}$.<br>Prob for denominator can be found in **(a)**. |
| | 0.231 (3sf) | **A1** | |
| | | **4** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 7(c) | E($X + Y$) = 4.1 Var($X + Y$) = 4.1<br><br>or Po(246) | **B1** | SOI |
| | Normal and var = $\frac{4.1}{60}$<br><br>Or normal and var = 246 | **M1** | |
| | $\frac{4.0-4.1}{\sqrt{4.1\div 60}}$<br><br>or totals method $\frac{240-246}{\sqrt{246}}$<br><br>or use of continuity correction | **M1** | No mixed methods.<br>Or continuity correction: $\frac{4.0-\frac{1}{120}-4.1}{\sqrt{(4.1\div 60)}}$<br>or $\frac{239.5-246}{\sqrt{246}}$.<br><br>Condone incorrect continuity correction for M1. |
| | = −0.383 (3sf) | **A1** | = −0.414 |
| | Φ(‘−0.383’) = 1 − Φ(‘0.383’) | **M1** | Φ(‘−0.414’) = 1 − Φ(‘0.414’) |
| | = 0.351 (3sf) | **A1** | = 0.340 or 0.339 |
| | | **6** | |

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International Education

# Cambridge International A Level

**MATHEMATICS** **9709/62**

Paper 6 Probability and Statistics 2 **February/March 2024**

MARK SCHEME

Maximum Mark: 50

**Published**

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| WWW | Without Wrong Working |
| AWRT | Answer Which Rounds To |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 1(a) | est($\mu$) = 0.368 = $\frac{46}{125}$ | **B1** | Oe. |
| | est($\sigma^2$) = $\frac{100}{99}\left(\frac{17.34}{100} - \textit{their}\,'0.368'^2\right)$ or $\frac{1}{99}\left(17.34 - \frac{36.8^2}{100}\right)$ | **M1** | For use of a correct formula (ft *their* $\mu$). |
| | = 0.0384 (3 sf) | **A1** | |
| | | **3** | |
| 1(b) | Must be a random sample | **B1** | E.g.<br>• Values must have been randomly selected.<br>• Sample should be representative of the population.<br>• All values should have equal chance of being selected.<br>• It should be an unbiased sample.<br>• Independent sample/insect lengths are independent of one another.<br>ISW |
| | | **1** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 2(a) | $\frac{78}{250} \pm z \times \sqrt{\frac{\frac{78}{250}\times(1-\frac{78}{250})}{250}}$ | **M1** | Use of a correct formula (any $z$). |
| | $z$ = 2.326 | **B1** | |
| | = 0.244 to 0.38[0] (3 sf) | **A1** | Must be an interval. |
| | | **3** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 2(b) | Unlikely to be true because confidence interval does not contain 0.4. | **B1 ft** | FT their confidence interval.<br>Must include this reason and 'unlikely', oe.<br>Allow "not true because 0.4 is not in the confidence interval."<br>But "Confidence interval only goes up to 0.38 so not true" and "'it' lies outside the confidence interval" both score B0. |
| | | **1** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 3(a) | $[\lambda =] 0.6$ | **B1** | Mean = 0.6 seen. |
| | $1 - e^{-0.6}\left(1 + 0.6 + \frac{0.6^2}{2}\right)$<br>or $1 - e^{-0.6}(1 + 0.6 + 0.18)$<br>or $1 - (0.5488 + 0.3293 + 0.09879)$ | **M1** | Any $\lambda$ Allow one end error.<br>Must see expression.<br>Accept correct $\Sigma$ notation. |
| | $= 0.0231$ | **A1** | **SC** 0.0231 and no working scores B1 (could be implied).<br>**SC** use of binomial scores M1A1 for 0.0231. |
| | | **3** | |
| 3(b) | $6000 > 50$ and either $np = 0.6 < 5$ or $\frac{1}{10000} < 0.1$ | **B1** | Must state values of $n$ and either $np$ or $p$.<br>Note: '$n$ large, $p$ small' is insufficient. |
| | | **1** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4(a) | $E(X - Y) = 10700 - 13400$ [=−2700] | **B1** | Oe, e.g. $(Y - X)$. |
| | $Var(X - Y) = 950^2 + 1210^2$ [= 2366600] | **M1** | |
| | $\frac{0-(\textit{their}\ '-2700')}{\sqrt{\textit{their}\ '2366600'}}$ [= 1.755] | **M1** | For standardising with *their* E and Var. |
| | $1 - \Phi(\textit{their}\text{`}1.755\text{'})$ | **M1** | For area consistent with *their* values. |
| | = 0.0396 or 0.0397 (3 sf) | **A1** | |
| | | **5** | |
| 4(b) | E(Total) = 10700 × 0.8 + 13400 × 0.85 [=19950] | **B1** | |
| | $Var(Total) = 950^2 \times 0.8^2 + 1210^2 \times 0.85^2$ [= 1635412.25] | **M1** | |
| | $\frac{22000-\textit{their}\ '19950'}{\sqrt{\textit{their}\ '1635412.25'}}$ [= 1.603] or $\frac{20000-\textit{their}\ '19950'}{\sqrt{\textit{their}\ '1635412.25'}}$ [= 0.0391] | **M1** | For one standardisation with *their* E and Var. |
| | $\Phi(\textit{their}\text{`}1.603\text{'}) - \Phi(\textit{their}\text{`}0.0391\text{'}) = 0.9455 - 0.5156$ | **M1** | For area consistent with *their* values. |
| | = 0.43[0] (3 sf) | **A1** | |
| | | **5** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5(a) | $[e^{-0.2}] = 0.819$ (3 sf) | **B1** | Accept $e^{-0.2}$ as final answer. |
| | | **1** | |
| 5(b) | $\lambda = 1.25$ | **B1** | |
| | $e^{-1.25}\left(1+1.25+\frac{1.25^2}{2}\right)$<br>or $e^{-1.25}(1 + 1.25 + 0.78125)$<br>or $0.2865 + 0.3581 + 0.2238$ | **M1** | Any $\lambda$ Allow one end error.<br>Must see expression (in any form).<br>Accept correct $\Sigma$ notation. |
| | = 0.868 (3 sf) | **A1** | **SC** Answer with no working seen scores B1 (could be implied). |
| | | **3** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5(c) | $e^{-0.15} \times e^{-0.1}(0.1) = 0.077879$<br>$e^{-0.15} \times e^{-0.1}\left(\frac{0.1^2}{2}\right) = 0.003894$<br>$e^{-0.15} \times 0.15 \times e^{-0.1} \times \frac{0.1^2}{2} = 0.0005841$ | **M1** | $P(B = 0) \times P(G = 1)$ $0.8607 \times 0.09048$<br>$P(B = 0) \times P(G = 2)$ $0.8607 \times 0.004524$<br>$P(B = 1) \times P(G = 2)$ $0.1291 \times 0.004524$<br><br>Note: $P(B = 0) \times P(G = 2)$ and $P(B = 1) \times P(G = 2)$ may be seen within $P(G = 2) \times P(B < 2)$.<br>For one expression seen. |
| | $e^{-0.15} \times e^{-0.1}(0.1) + e^{-0.15} \times e^{-0.1}\left(\frac{0.1^2}{2}\right) + e^{-0.15} \times 0.15 \times e^{-0.1} \times \frac{0.1^2}{2}$<br>$= 0.077879 + 0.00389036 + 0.0005841$ | **M1** | $P(B = 0) \times P(G = 1) + P(B = 0) \times P(G = 2)$ $+ P(B = 1) \times P(G = 2)$.<br>For the three Poisson terms added (must be from a complete attempt at all 3 terms). |
| | $= 0.0824$ (3 sf) | **A1** | |
| | **Alternative method for Question 5(c)** | | |
| | $P(B = 0) \times P(G > 0)$<br>$e^{-0.15} \times (1 - e^{-0.1})$<br><br>$P(B = 1) \times P(G > 1)$<br>$e^{-0.15} \times 0.15 \times (1 - e^{-0.1}(1 + 0.1))$ | **M1** | For one expression seen. |
| | $e^{-0.15} \times (1 - e^{-0.1}) + e^{-0.15} \times 0.15 \times (1 - e^{-0.1}(1 + 0.1))$ | **M1** | For adding their expressions. |
| | $= 0.0824$ (3 sf) | **A1** | |
| | | **3** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5(d) | $H_0$: $\lambda = 1.25$ or 0.25[per day]<br>$H_1$: $\lambda > 1.25$ or 0.25[per day] | **B1** | Or $\mu$ or 'population mean'. |
| | $P(\geq 4 \text{ late}) = 1 - e^{-1.25}\left(1+1.25+\frac{1.25^2}{2}+\frac{1.25^3}{3!}\right)$<br>or $1 - e^{-1.25}(1 + 1.25 + 0.7813 + 0.3255)$<br>or $1 - (0.2865 + 0.3581 + 0.2238 + 0.09326)$ | **M1** | Any $\lambda$. No end errors. Expression must be seen (in any form). Accept correct $\Sigma$ notation. |
| | $= 0.0383$ | **A1** | **SC** 0.0383 with no working scores B1. |
| | $0.0383 < 0.05$ | **M1** | For a valid comparison. |
| | [Reject $H_0$]<br>'Hence there is sufficient evidence to suggest that the teacher's claim is true' or 'There is sufficient evidence to suggest that more students are late on average'. | **A1 FT** | No contradictions. In context and not definite, e.g. not 'More students are late' or 'Claim is correct'.<br>Ft their 0.0383. |
| | | **5** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(a) | $\frac{1}{2} - \frac{117}{256}$ $\left[=\frac{11}{256}\right]$ | **M1** | For use of symmetry about $x = 2$, oe.<br>E.g. $(1 - 2 \times \frac{117}{256}) \div 2$ or $\frac{1}{2} + \frac{117}{256}$. |
| | $1 - \frac{11}{256}$ or $2 \times \frac{117}{256} + \frac{11}{256}$<br>$= \frac{245}{256}$ **AG** | **A1** | Any correct numerical expression seen leading to AG. |
| | | **2** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(b) | $k\int_{2}^{5}(12+4x-x^2)\mathrm{d}x$<br>$\left[=k\left[12x+2x^2-\frac{x^3}{3}\right]_2^5\right]$ | **M1** | Attempt to integrate f($x$) with any limits. |
| | $39k=\frac{117}{256}$ | **M1** | Use of limits 2 and 5 and equating *their* integration attempt to $\frac{117}{256}$.<br>Or limits –2 and 6 equated to 1.<br>Or limits –1 and 6 equated to $\frac{245}{256}$.<br>Or limits –1 to 5 equated to $\frac{234}{256}$.<br>Oe.<br>No mixed methods. |
| | $k=\frac{3}{256}$ or 0.0117 | **A1** | |
| | | **3** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(c) | [$2 + x - x^2 = 0$]<br>$x = -1$ and $x = 2$ seen or implied<br>[Domain is $-1 \leqslant x \leqslant 2$] | **B1** | |
| | Mean = 0.5 | **B1** | |
| | $\frac{2}{9}\int_{-1}^{2}(2x^2 + x^3 - x^4)\mathrm{d}x$<br>$\left[= \frac{2}{9}\left[\frac{2}{3}x^3 + \frac{x^4}{4} - \frac{x^5}{5}\right]_{-1}^{2}\right]$<br>[= 0.7] | ***M1** | Attempt to integrate $x^2\mathrm{g}(x)$ with any limits. |
| | *their* '0.7' – *their* '0.5'$^2$ | **DM1** | Subtract *their* mean$^2$ from *their* $\int x^2\mathrm{g}(x)\mathrm{d}x$<br>(both must be numerical). |
| | = 0.45 or $\frac{9}{20}$ | **A1** | |
| | | **5** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 7(a) | 0.01 or 1% | **B1** | Note: $x \leqslant 0.01$ scores B0. |
| | | **1** | |
| 7(b) | $2.326 = \dfrac{\bar{h} - 163.21}{6.95 \div \sqrt{300}}$ | **M1** | Accept any $z$ (±). |
| | $\bar{h} = 164.14$<br>[Rejection region is $\bar{h} > 164.14$]<br>[P(Type II) = P($\bar{h} < 164.14 \mid \mu = 164.91$)] | **A1** | Accept 3 sf accuracy here. |
| | $\dfrac{their\ '164.14' - 164.91}{6.95 \div \sqrt{300}}$ [= −1.919] | **M1** | For standardising 164.91 with their 164.14 (could be 163.21). |
| | Φ(*their*'−1.919') = 1 − Φ(*their*'1.919') | **M1** | For area attempt consistent with their values. |
| | = 0.0275 or 0.0276 or 0.028[0] (3.s.f) | **A1** | Accept anything in range 0.0275 to 0.028[0]. |
| | | **5** | |

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Cambridge Assessment International Education

# Cambridge International A Level

**MATHEMATICS** **9709/61**

Paper 6 Probability & Statistics 2 **October/November 2023**

MARK SCHEME

Maximum Mark: 50

**Published**

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2023 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.



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**[Turn over**

**PUBLISHED**

## Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

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**Mathematics-Specific Marking Principles**

1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.

2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.

3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.

4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).

5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.

6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

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## Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

### Types of mark

**M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

**A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

**B** Mark for a correct result or statement independent of method marks.

**DM** or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

**FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

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**Abbreviations**

| | |
|---|---|
| AEF/OE | Any Equivalent Form (of answer is equally acceptable) / Or Equivalent |
| AG | Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid) |
| CAO | Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed) |
| CWO | Correct Working Only |
| ISW | Ignore Subsequent Working |
| SOI | Seen Or Implied |
| SC | Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance) |
| WWW | Without Wrong Working |
| AWRT | Answer Which Rounds To |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 1 | $\frac{405-410}{\frac{20}{6}}$ [= –1.5] | **M1** | For standardising, must have $\sqrt{36}$.<br>Allow totals method $\frac{14580-14760}{\sqrt{14400}}$.<br>No mixed methods. |
| | $\Phi$('–1.5') = 1 – $\Phi$('1.5') | **M1** | For area consistent with their working. |
| | = 0.0668 | **A1** | |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 2 | $\frac{134}{300} + z\sqrt{\frac{\frac{134}{300}\times\frac{166}{300}}{300}} = 0.487$ | **M1** | For expression of the correct form. |
| | $z$ = 1.405 | **A1** | Accept 1.404, or anything that rounds to 1.39 to 1.41. |
| | $\Phi^{-1}$('1.405') = 0.9199 or 0.92; 1 − 2(1 – 0.92) | **M1** | Attempt area above or below their 1.405 and convert to a confidence level. |
| | $\alpha$ = 84 | **A1** | Allow $\alpha$ = 84%.<br>cwo<br>Note: final answer 0.84 scores A0. |
| | | **4** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 3(a)(i) | $\lambda = 3$ | **B1** | For mean = 3. |
| | $1 - e^{-3}(1 + 3 + \frac{3^2}{2} + \frac{3^3}{3!})$ or $1 - e^{-3}(1 + 3 + 4.5 + 4.5)$ or $1 - (0.04979 + 0.14936 + 0.22404 + 0.22404)$ | **M1** | Any $\lambda$. Allow one end error. |
| | = 0.353 (3 sf) | **A1** | No working scores B1. |
| | | **3** | |
| 3(a)(ii) | N(54, 54) | **M1** | soi |
| | $\frac{39.5 - 54}{\sqrt{54}}$ (= –1.973) | **M1** | Allow with wrong or no continuity correction.<br>For standardising with their mean and variance. |
| | $1 - \phi$ ('1.973') | **M1** | For area consistent with their working. |
| | = 0.0242 (3 sf) | **A1** | Special case: if no working seen, 0.0242 scores **SC B3**, 0.0284 scores **SC B2**. |
| | | **4** | |
| 3(b)(i) | 'Mean not constant' or' 'number of hits per minute not constant' or 'not a constant rate' | **B1** | |
| | | **1** | |
| 3(b)(ii) | $2p + p = 2 \times 0.3$ [$\Rightarrow p = 0.2$]<br>[where $p$ is the rate per minute for night time] | **M1** | May be implied by answer. |
| | [During day-time]: Po(0.4). [During night-time]: Po(0.2) | **A1** | Accept Po(24) [per daytime hour], Po(12) [per night time hour].<br>Accept Po(288) [per day time shift], Po(144)[ per night time shift].<br>Note: Po(432), Po(216) scores M0A0. |
| | | **2** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4(a) | E(*A* income) = [10.3 × 2.50 ] = 25.75 [$] | **B1** | Accept 3sf. |
| | Var(*A* income) = [5.76 × $2.50^2$ ] = 36 [$\$^2$] | **B1** | |
| | | **2** | |
| 4(b) | *B* income ~ N(37.05, 101.506)<br>or<br>E(*B* income) = 37.05 and Var(*B* income) = 101.51 | **B1** | Or N(37.1, 102) soi. |
| | *A* income – *B* income ~ N('25.75' – '37.05', '36' + '101.506') | **M1** | Ft their values for *A* and *B*. |
| | = N(–11.3, 137.506) | **A1** | Accept 3sf. |
| | $\frac{0-(-'11.3')}{\sqrt{'137.506'}}$ [= 0.964] | **M1** | Standardising with their values from attempt at *A* income – *B* income. |
| | 1 – ϕ('0.964') | **M1** | For area consistent with their values. |
| | = 0.168 or 0.167 (3 sf) | **A1** | cwo |
| | | **6** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5 | $H_0$: Population mean no. enquiries = 1.55<br>$H_1$: Population mean no. enquiries > 1.55 | **B1** | Or "population mean no. enquiries = 0.31 (per minute)" oe.<br>Allow '$\lambda$ = 1.55' or $\mu$ = '1.55'. |
| | $P(X \geqslant 5) = 1 - e^{-1.55}(1 + 1.55 + \frac{1.55^2}{2!} + \frac{1.55^3}{3!} + \frac{1.55^4}{4!})$<br>or $1 - e^{-1.55}(1 + 1.55 + 1.20125 + 0.62065 + 0.24050)$<br>or $1 - (0.21225 + 0.32898 + 0.25496 + 0.13173 + 0.05105)$ | **M1** | Allow one end error, e.g. extra term: $e^{-1.55} \times \frac{1.55^5}{5!}$. |
| | = 0.0210 (3 sf) | **A1** | Allow 0.021.<br>**SC B1** no working scores B1 instead of M1A1. |
| | 0.0210 < 0.025 | **M1** | For valid comparison. |
| | [Reject $H_0$] There is sufficient evidence [at 2.5% level] to suggest that mean no. of enquiries has increased. | **A1 FT** | In context, not definite,<br>e.g., not "Mean no. of enquiries has increased".<br>No contradictions. |
| | | **5** | Note: $e^{-1.55} \times \frac{1.55^5}{5!}$ = 0.0158 < 0.025: scores max B1 |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(a) | $p + \frac{13}{10}p \leqslant \frac{1}{2} \Rightarrow p \leqslant \frac{5}{23}$ AG | **B1** | Allow '=' in working but need an inequality in the answer.<br>Allow $0 < p \leqslant \frac{5}{23}$. |
| | | **1** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(b) | e.g. $0.5 - 2.3p$, 0 to $a$ / $p + 1.3p$, $a$ to 3 / $2 \times 1.3p$, $2 \times (b$ to $3)$ / $2.3p + 1.3p$, $a$ to $3 + b$ to 3<br>$2p + 2.6p$, $2 \times (a$ to $3)$ / $0.5 - 1.3p$, 0 to $b$ / $0.5 + 1.3p$, $b$ to 6 | **M1** | Any correct expression for the probability of a relevant region. |
| | $\frac{18}{5}p$ or $3.6p$ | **A1** | |
| | | **2** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(c) | $\frac{1}{36}\int_a^2 (6x - x^2)\,dx = \frac{5}{27}$ | **M1** | Attempt to integrate with correct limits and equate to $\frac{5}{27}$, oe.<br>Integrate from 2 to 6 – $a$ and equate $\frac{18}{5}p = \frac{2}{3}$.<br>Integrate from $a$ to 3 and equate to $\frac{23}{54}$.<br>Integrate from 0 to $a$ and equate to $\frac{2}{27}$. |
| | $\left[\Rightarrow \frac{1}{36}\left[3x^2 - \frac{x^3}{3}\right]_a^2 = \frac{5}{27} \Rightarrow \frac{1}{36}\left(12 - \frac{8}{3} - 3a^2 + \frac{a^3}{3}\right) = \frac{5}{27}\right]$ | **M1** | For integrating and substitution of limits to form cubic in $a$. |
| | $a^3 - 9a^2 + 8 = 0$ | **A1** | Any correct three term cubic equation in $a$. |
| | $(a - 1)(a^2 - 8a - 8) = 0$ | **M1** | Attempt to factorise their cubic equation. |
| | $a = \frac{8 \pm \sqrt{96}}{2} = 4 \pm \sqrt{24}$ or –0.899 or 8.90, [not between 0 and 6] | | |
| | $a = 1$ only [other two values rejected] | **A1** | **SC B1** for $a = 1$ only, if no method seen for solving the cubic. |
| | | **5** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 7(a) | Est ($\mu$) = 23/50 = 0.46 | **B1** | |
| | Est ($\sigma$) = $\sqrt{\frac{50}{49}} \times \sqrt{\frac{13.02}{50} - 0.46^2}$ or Est ($\sigma^2$) = $\frac{50}{49} \times \left(\frac{13.02}{50} - 0.46^2\right)$ oe<br><br>Or estimated unbiased variance = $\frac{1}{49}\left(13.02 - \frac{(23.0)^2}{50}\right)$ | **M1** | For an expression of the correct form for unbiased standard deviation or variance. |
| | Est ($\sigma$) = 0.22315 or Est ($\sigma^2$) = 0.0497959 = $\left(\frac{61}{1225}\right)$ or 0.0498 | **A1** | |
| | $\frac{0.46 - 0.5}{\frac{'0.22315'}{\sqrt{50}}}$ | **M1** | Standardising with their values. |
| | = –1.268 or -1.267 or = –1.27 (3sf) | **A1** | |
| | | | |
| | −1.268 > –1.645 or 0.102 to 0.103 > 0.05 | **M1** | For a valid comparison. |
| | [Do not reject Ho] There is insufficient evidence [at 5% level] that the mean concentration is less than 0.5. | **A1 FT** | In context, not definite. E.g., not ‘Mean concentration is not less than 0.5’. No contradictions. |
| | | **7** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 7(b) | $\frac{cv - 0.5}{\frac{'0.22315'}{\sqrt{50}}} = -1.645$ | **M1** | |
| | cv = 0.448(1) or 0.448 (3 sf) | **A1** | |
| | $\frac{'0.448' - 0.4}{\frac{'0.22315'}{\sqrt{50}}}$ [=1.521 to 1.524] | **M1** | |
| | $1 - \Phi('1.524')$ | **M1** | For area consistent with their working. |
| | = 0.0638 to 0.0642 | **A1** | |
| | | **5** | |

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Cambridge Assessment International Education

# Cambridge International A Level

**MATHEMATICS** **9709/62**

Paper 6 Probability & Statistics 2 **October/November 2023**

MARK SCHEME

Maximum Mark: 50

**Published**

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

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Cambridge International is publishing the mark schemes for the October/November 2023 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.



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**[Turn over**

**PUBLISHED**

## Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

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**Mathematics-Specific Marking Principles**

1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.

2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.

3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.

4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).

5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.

6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

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**PUBLISHED**

## Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

### Types of mark

**M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

**A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

**B** Mark for a correct result or statement independent of method marks.

**DM** or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

**FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

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**Abbreviations**

| | |
|---|---|
| AEF/OE | Any Equivalent Form (of answer is equally acceptable) / Or Equivalent |
| AG | Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid) |
| CAO | Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed) |
| CWO | Correct Working Only |
| ISW | Ignore Subsequent Working |
| SOI | Seen Or Implied |
| SC | Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance) |
| WWW | Without Wrong Working |
| AWRT | Answer Which Rounds To |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 1(a) | $X \sim N(25, 25)$ | **B1** | soi |
| | $\frac{30.5-25}{\sqrt{25}}$ [= 1.1] | **M1** | Standardising with their values.<br>Allow with missing or incorrect continuity correction. |
| | $1 - \phi('1.1')$ | **M1** | For area consistent with their working. |
| | = 0.136 (3 sf) | **A1** | |
| | | **4** | |
| 1(b) | $e^{-100p}\left(1+100p+\frac{(100p)^2}{2!}\right)$ | **M1** | For $P_o(100p)$ expression.<br>Accept un-simplified terms (e.g. $p^0/0!$ For M1).<br>Allow one end error (e.g. for correct with extra term $e^{-100p} \times \frac{(100p)^3}{3!}$ oe), or brackets omitted. |
| | $e^{-100p}\left(1+100p+\frac{(100p)^2}{2!}\right)$ or $e^{-100p} + e^{-100p} \times 100p + e^{-100p} \times \frac{(100p)^2}{2!}$<br>or $e^{-100p}(1 + 100p + 5000p^2)$ oe | **A1** | Must have brackets.<br>Allow with or without ! sign (but not 0! or $p^0$).<br>ISW once a fully correct answer seen. |
| | | **2** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 2(a) | $\frac{4590}{85} \pm z \times \frac{8.3}{\sqrt{85}}$ | **M1** | For expression of correct form. Any $z$ (but not $\phi(z)$). |
| | $z$ = 1.96 | **B1** | |
| | 52.2 to 55.8 (3 sf) | **A1** | Must be an interval. |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 2(b) | $1.96\times\frac{s}{\sqrt{100}}=1.4$ or $2\times1.96\times\frac{s}{\sqrt{100}}=2.8$ | **M1** | Equation of correct form (any $z$).<br>Allow factor of 2 error (i.e. first equation = 2.8). |
| | $s$ = 7.14 (3 sf) or $\frac{50}{7}$ | **A1** | |
| | | **2** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 3(a)(i) | Need to find P($X \geqslant 8$) | **B1** | oe (e.g. invalid because it should be a tail probability compared with 0.05). |
| | | **1** | |
| 3(a)(ii) | $H_0$: P(green) = 0.5<br>$H_1$: P(green) > 0.5 | **B1** | Allow $p$ = 0.5.<br>Allow $p$ > 0.5. |
| | P($X \geqslant 8$) = 0.0439 + $^{10}C_9 \times (0.5) \times (0.5)^9 + 0.5^{10}$ | **M1** | Attempt 0.0439 + P($X$ = 9) + P($X$ = 10).<br>Must see Binomial expressions B(10,0.5). |
| | = 0.0547 or 0.0546 (3 sf) | **A1** | **SC B1** 0.0547 or 0.0546 with no working. |
| | 0.0547 > 0.05 | **M1** | Valid comparison of tail probability with 0.05. |
| | [Do not reject $H_0$] 'There is insufficient evidence [at the 5% level] to accept the hypothesis that boys prefer green.'<br>Or 'There is sufficient evidence to support the **researcher's** claim.' | **A1FT** | In context, not definite. No contradictions.<br>Allow 'There is insufficient evidence to reject the hypothesis that boys like green and orange equally'.<br>Not definite, e.g. not 'They don't prefer green" or 'Researchers claim true' 'Magazine's claim untrue'.<br>Any mention of 'claim' must be clear which claim it is. |
| | | **5** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 3(b) | $H_0$ was not rejected | **B1** | Mark independently. |
| | | **1** | |
| 3(c) | ${}^{10}C_9\times(0.5)\times(0.5)^9 + 0.5^{10}$ or "0.0547" – 0.0439 | **M1** | Finding P(9,10) using B(10,0.5).<br>Could be seen in **(a)(ii)**. |
| | P(Type I error) = 11/1024 or 0.0107 (3 sf) or 0.0108 | **A1** | |
| | | **2** | |
| 4(a) | $H_0$: population mean [of $H$ ] = 4.23<br>$H_1$: population mean [of $H$ ] > 4.23 | **B1** | Allow $\mu$ = 4.23 or population mean of $h$ = 4.23<br>but NOT $h$ = 4.23 or $H$ = 4.23 or $\bar{h}$ = 4.23 or $\overline{H}$ = 4.23. |
| | | **1** | |
| 4(b) | $\dfrac{\bar{h}-4.23}{\frac{0.67}{\sqrt{200}}}$ = 1.645 | **M1** | For standardising and forming an equation. Must have $\sqrt{200}$ . Allow $\pm$ 1.645 or $\pm$ 1.96.<br>Accept '>' and '<'.<br>Allow $\overline{H}$ or any letter instead of $\bar{h}$ . |
| | $\bar{h}$ = 4.31 (3 sf) | **A1** | May be implied by $\bar{h}$ > 4.31. Allow $\bar{h}$ < 4.31 for this A1 only, condone 4.15 also seen. |
| | $\bar{h}$ > 4.31 or $\bar{h} \geqslant$ 4.31 (3 sf) | **A1** | Condone any letter instead of $\bar{h}$ . |
| | | **3** | |
| 4(c) | Incorrect, because the population of $H$ is given as normally distributed [with known variance]. | **B1** | Allow $h$ instead of $H$ or just 'The population is normal.'<br>Must use 'population' or 'underlying distribution'. |
| | | **1** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5(a) | $\int_a^b \frac{x}{x^2}\,\mathrm{d}x$ | **M1** | Attempt to integrate $xf(x)$ from $a$ to $b$. |
| | $=\left[\ln x\right]_a^b$ or $\ln b - \ln a$ | **A1** | |
| | $\ln\frac{b}{a} = \ln 2$ or $\ln 2a = \ln b$ oe<br><br>Hence $b = 2a$ (AG) | **A1** | Must see both statements. No errors seen. |
| | | **3** | |
| 5(b) | $\int_a^b \frac{1}{x^2}\,\mathrm{d}x = 1$ | **M1** | Attempt to integrate f($x$) and equate to 1. Ignore limits. |
| | $\left[\left[-\frac{1}{x}\right]_a^b = 1 \text{ or } \frac{1}{a}-\frac{1}{b}=1\right]\ \frac{1}{a}-\frac{1}{2a} = 1$ | **A1** | Integrate with correct limits and substitute $b = 2a$. |
| | e.g. $\frac{1}{2a}=1$ or $2 + (-1) = 2a$<br><br>Hence $a =$ $a=\frac{1}{2}$ (AG) | **A1** | Obtain convincingly (at least one step from previous answer), no errors seen (ignore $a = 0$). |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5(c) | $\int_{0.5}^{m} \frac{1}{x^2}\,\mathrm{d}x = \frac{1}{2}$ or $\int_{m}^{1} \frac{1}{x^2}\,\mathrm{d}x = \frac{1}{2}$ | **M1** | Attempt integrate f($x$), with correct limits, and equate to $\frac{1}{2}$. |
| | $\left[-\frac{1}{x}\right]_{0.5}^{m} = \frac{1}{2}$ or $\left[-\frac{1}{x}\right]_{m}^{1} = \frac{1}{2}$ | | |
| | $2 - \frac{1}{m} = \frac{1}{2}$ or $\frac{1}{m} - 1 = \frac{1}{2}$ | **A1** | oe. Correct equation after substituting limits. |
| | $m = \frac{2}{3}$ or 0.667 (3 sf) | **A1** | |
| | | **3** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6 | Cost of dried yeast and flour: \$*D* and \$*F*<br>E(*D*) = 13.5 × 0.7 = 9.45 E(*F*) = 0.9 × 100 = 90 | **B1** | One of these soi – can be given at early stage. |
| | Var(*D*) = $0.02^2 \times 13.50^2$ =0.0729 Var(*F*) = $3.0^2 \times 0.90^2$ = 7.29 | **B1** | One of these soi – can be given at early stage. |
| | Total cost: $T \sim N(99.45, (0.02^2 \times 13.50^2 + 3.0^2 \times 0.90^2))$ | **M1** | Attempt to combine their *D* and their *F* with or without 55 and 200 (but variance must not include 55 or 200).<br>Or $N(99.45 +55, (0.02^2 \times 13.50^2 + 3.0^2 \times 0.90^2))$.<br>Or $N(200-55 -99.45, (0.02^2 \times 13.50^2 + 3.0^2 \times 0.90^2))$.<br>Or $N(99.45 +c, (0.02^2 \times 13.50^2 + 3.0^2 \times 0.90^2))$. |
| | N(99.45, 7.3629) accept 99.4 or 99.5 | **A1** | Or N(154.45,7.3629) or N(45.55, 7.3629).<br>Accept 3sf (accept 45.5 or 45.6). |
| | [P(profit > \$40) = P(T < 105)]<br>$\frac{105 - '99.45'}{\sqrt{'7.3629'}}$ [= 2.045] | **M1** | $160–154.45/\sqrt{7.3629}$ or $40–45.55/\sqrt{7.3629}$.<br>No mixed methods. |
| | [P(T < 105) = P(*z* < 2.045) =]<br>ϕ(2.045) | **M1** | For area consistent with their working. |
| | = 0.9795 or 0.9796 or 0.98(0) or 0.979 (3 sf) | **A1** | |
| | | **7** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 7(a) | $(e^{-2.4}\left(\frac{2.4^2}{2!}+\frac{2.4^3}{3!}\right)=e^{-2.4}(2.88+2.304)=0.2613+0.2090$ | **M1** | Allow M1 for $e^{-2.4}\left(\frac{2.4^2}{2!}+\frac{2.4^3}{3!}+\frac{2.4^4}{4!}\right)$.<br>Expression must be seen. |
| | = 0.47(0) | **A1** | **SC B1** 0.47(0) with no working. |
| | | **2** | |
| 7(b) | $1 - e^{-2.4}(1 + 2.4)$ [= 0.691558] | **M1** | Allow one end error. Allow any λ. |
| | $(1 - e^{-2.4}(1 + 2.4))^2$ | **M1** | Squaring their probability ($\lambda \neq 4.8$). |
| | = 0.478 (3 sf) | **A1** | **SC B2** 0.478 with no working. |
| | | **3** | |
| 7(c)(i) | $e^{-2.4} \times \frac{2.4^r}{r!} < e^{-2.4} \times \frac{2.4^{r+1}}{(r+1)!}$ | **M1** | For both expressions seen. |
| | $r + 1 < 2.4$ | **A1*** | Or $r < 1.4$ (must have correct inequality). |
| | Set is $r = 0, 1$ | **DA1** | |
| | | **3** | For trial and error solutions.<br>M1 for substituting one value into correct expression (can be implied by correct values).<br>A1* for [P(0) = 0.0907], P(1) = 0.218, P(2) = 0.261, P(3) = 0.209 (accept 2sf accuracy).<br>**DA1** Set is $r$ = 0,1. |
| 7(c)(ii) | $r = 2$ | **B1** | |
| | | **1** | |

Cambridge Assessment International Education

# Cambridge International A Level

**MATHEMATICS** **9709/61**

Paper 6 Probability & Statistics 2 **May/June 2023**

MARK SCHEME

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| Mathematics Specific Marking Principles | |
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**FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
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| WWW | Without Wrong Working |
| AWRT | Answer Which Rounds To |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 1(a) | 20540/6012300 = 0.0034163 | **B1** | |
| | [1000 × 0.0034163 = 3.4163] | | |
| | Po(3.4163) | **B1** | Could be implied by expression seen. |
| | $e^{-their\ '3.4163'}(1 + 3.4163 + \frac{3.4163^2}{2!} + \frac{3.4163^3}{3!})$ OR<br>$e^{-their\ '3.4163'}$(1 + 3.4163 + 5.8356+6.6453) or 0.03283 + 0.1122 +0.1916 + 0.21819) | **M1** | Allow any $\lambda$. Allow with one end error. Must see expression. |
| | = 0.555 (3sf) | **A1** | CAO<br>**SC** No working: B1 B1 (Po must be stated) B1 correct answer (max 3/4).<br>**SC** Binomial: B1 B0 B1 correct answer (max 2/4). |
| | | **4** | |
| 1(b) | $n = 1000 > 50$ | **B1** | Must show comparison with 50. |
| | $np = 3.4163 < 5$ | **B1** | Must show comparison with 5. |
| | | **2** | **SC B1**: $n > 50$ (or n large), $np < 5$.<br>**SC B1**: n large, p small. |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 2(a) | $\frac{1}{2} \times 2 \times 1$ or $\int_0^2 \frac{1}{2}x\,dx = 1$, which is the correct area under a pdf. | **B1** | Calculation and result. |
| | $f(x) \geqslant 0$ | **B1** | Condone $f(x) > 0$ or 'Line is above $x$-axis' OE. |
| | | **2** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 2(b) | $\frac{1}{2}\pi r^2 = 1$ | **M1** | Area of semi-circle equated to 1 OE.<br>Missing factor of ½ gets M1A0. |
| | $r = \sqrt{\frac{2}{\pi}}$ or 0.798 (3sf) | **A1** | |
| | | **2** | |
| 2(c)(i) | Area to the left of 15 is greater than 0.5 | **B1** | OE, e.g. 'The distribution of X is skewed to the right / positively skewed, suggesting the median will be less than the mid-point of the interval.' or 'The distribution of X is skewed to the right / positively skewed' or 'It is a decreasing function suggesting the median will be less than the mid-point of the interval'. |
| | | **1** | |
| 2(c)(ii) | $\int_{10}^{20} (\frac{40}{x} - \frac{x}{10}) \mathrm{d}x$ | **M1** | Integration of $x$h($x$) attempted. Ignore limits. |
| | $\left[40\ln x - \frac{x^2}{20}\right]_{10}^{20}$ | **A1** | Correct integration and limits (can be implied by final answer). |
| | = 40ln 2 – 15 or 12.7 (3sf) | **A1** | |
| | | **3** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 3 | Assume SD still = 5.2 | **B1** | OE i.e. 'Assume the SD remains unchanged'. |
| | $H_0$: $\mu = 24.0$ $H_1$: $\mu > 24.0$ | **B1** | Or population mean; not just mean. |
| | $\frac{25.8-24.0}{\frac{5.2}{\sqrt{50}}}$ | **M1** | For standardising (could be implied). Must have $\sqrt{50}$. |
| | = 2.448 | **A1** | Or P($\overline{X} > 25.8$) = 0.0071 . |
| | '2.448' > 2.326 | **M1** | Or 0.0071 < 0.01 .<br>For valid comparison. |
| | [Reject $H_0$] There is evidence that (mean) amount of wheat is greater. | **A1FT** | OE. FT their $z_{calc}$.<br>In context, not definite, eg not 'Mean amount of wheat is greater' No contradictions<br><br>CV method: CV= 25.71 M1A1 25.71<25.8 M1 A1FT<br>or CV=24.09 M1 A1 24.09>24 M1 A1FT. |
| | | **6** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4(a) | $z \times \sqrt{\frac{11.2}{n}} = 1.4076 \div 2$ | **M1** | Any $z$, but must be a $z$. |
| | $z$ = 1.881 or 1.882 | **B1** | |
| | $[n = \left(\frac{1.881}{0.7038}\right)^2 \times 11.2]$<br>$n$ = 80 | **A1** | Must be a whole number. |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4(b) | Jan, Feb and March not typical of whole year. | **B1** | Or, e.g., weather is different at different times of year. |
| | | **1** | |
| 4(c) | $0.94^3 \times 0.06 \times 4$ | **M1** | |
| | = 0.199 (3 sf) | **A1** | |
| | | **2** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5(a) | $2.0^2 + 20 \times 3.4^2$ | **M1** | |
| | = 235.2 | **A1** | |
| | | **2** | |
| 5(b) | E(C – 3B) = 50 + 20×1010 – 3×6730 or 60 | **B1** | |
| | Var(C – 3B) = '235.2' + 9×$15^2$ or 2260.2 | **M1** | FT *their* values from **(a)**. |
| | [C – 3B ~ N('60', '2260.2')]<br>= $\frac{0-60}{\sqrt{2260.2}}$ [= –1.262] | **M1** | Standardising with their values (could be implied). |
| | 1 – Φ('–1.262') = Φ('1.262') | **M1** | Probability area consistent with their values. |
| | = 0.897 (3 sf) | **A1** | |
| | | **5** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6 | $\frac{5}{4}\left(\frac{1+2^2+6^2+1+a^2}{5}-\left(\frac{1+2+6+1+a}{5}\right)^2\right)=\frac{11}{2}$ or $\frac{1}{4}\left((42+a^2)-\frac{(10+a^2)}{5}\right)=\frac{11}{2}$ | **M1*** | OE attempted or e.g., $\frac{42+a^2}{5}-\left(\frac{10+a}{5}\right)^2=\frac{22}{5}$. Allow use of biased i.e., without $\frac{5}{4}$. |
| | $4a^2 - 20a + 0 = 0$ or $a^2 - 5a + 0 = 0$ | **DM1** | Two- or three-term quadratic equation in $a$, with at least two terms correct. |
| | $a = 5$ | **A1** | Ignore $a = 0$, if seen. |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 7(a) | $H_0$: $\lambda = 7.6$ [or 1.9]<br>$H_1$: $\lambda < 7.6$ [or 1.9] | **B1** | Or Population mean = 7.6 or $\mu$ (not just 'mean').<br>Or Population mean < 7.6 or $\mu$. |
| | | **1** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 7(b) | Mean = 7.6 | **B1** | Seen. |
| | $P(X \leqslant 2) = e^{-7.6}(1 + 7.6 + \frac{7.6^2}{2})$ [= 0.0188 or 0.0187] | **M1** | OE. |
| | $P(X \leqslant 3) = e^{-7.6}(1 + 7.6 + \frac{7.6^2}{2} + \frac{7.6^3}{3!})$ [= 0.0554 or 0.0553] | **M1** | OE. Expression must be seen in at least one probability calculation. |
| | 0.0188 or 0.0187 and 0.0554 or 0.0553 | **A1** | A1 for both values. |
| | Critical region is $X \leqslant 2$ | **A1** | Dep on both M marks.<br>**SC** No Poisson expression seen in either prob scores B1 for 0.0188 or 0.0187 and B1 for 0.0554 or 0.0553 and B1 for CR. |
| | P(Type I error) = $P(X \leqslant 2)$ = 0.0188 or 0.0187 (3 sf) | **B1FT** | FT *their* $P(X \leqslant 2)$ or *their* CR. |
| | | **6** | |
| 7(c) | Concluding that the (mean) no. of accidents has reduced when it has not. | **B1** | OE. Must be in context.<br>Accept: 'It is believed that the booklet has helped to improve safety when actually it has not'. |
| | | **1** | |
| 7(d) | 3 not in critical region. | **M1** | FT their CR<br>or $P(X \leq 3) = 0.0554 > 0.05$ . |
| | No evidence mean number of accidents has decreased. | **A1FT** | In context.<br>Cannot be a definite statement, e.g., 'mean number accidents has not decreased'. |
| | | **2** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 7(e) | N(98.8, 98.8) | **B1** | May be implied. |
| | $\frac{100.5 - 98.8}{\sqrt{98.8}}$ [= 0.171] | **M1** | For standardising (could be implied by correct answer). Allow with wrong or no continuity correction. |
| | 1 – Φ('0.171') | **M1** | For probability area consistent with their working. |
| | = 0.432 (3 sf) | **A1** | |
| | | **4** | |

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Cambridge Assessment International Education

# Cambridge International A Level

**MATHEMATICS** **9709/62**

Paper 6 Probability & Statistics 2 **May/June 2023**

MARK SCHEME

Maximum Mark: 50

**Published**

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

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**[Turn over**

**PUBLISHED**

## Generic Marking Principles

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- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
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GENERIC MARKING PRINCIPLE 3:
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- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

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| Mathematics Specific Marking Principles | |
|---|---|
| 1 | Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing. |
| 2 | Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected. |
| 3 | Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points. |
| 4 | Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw). |
| 5 | Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread. |
| 6 | Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear. |

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## Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

### Types of mark

**M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

**A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

**B** Mark for a correct result or statement independent of method marks.

**DM** or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

**FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

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**Abbreviations**

| | |
|---|---|
| AEF/OE | Any Equivalent Form (of answer is equally acceptable) / Or Equivalent |
| AG | Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid) |
| CAO | Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed) |
| CWO | Correct Working Only |
| ISW | Ignore Subsequent Working |
| SOI | Seen Or Implied |
| SC | Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance) |
| WWW | Without Wrong Working |
| AWRT | Answer Which Rounds To |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 1 | $0.23 \pm z \times \sqrt{\frac{0.23\times(1-0.23)}{200}}$ | **M1** | Expression of correct form. Any $z$, but $z = 0.8328$ scores B0M0. |
| | $z = 1.811$ or 1.812 | **B1** | |
| | 0.176 to 0.284 (3 sf) | **A1** | Must be an interval. |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 2(a) | $E(W) = Var(W)$. | **B1** | Allow 'they are the same' OE.<br>Must be = not ≈ (and not both = and ≈).<br>Condone $E(W) = \lambda$ and $Var(W) = \lambda$. |
| | | **1** | |
| 2(b) | $np \approx np(1-p)$, hence $1-p$ must be close to 1 | **B1** | OE. Must see formulae and $q = 1-p$ must be seen or implied and conclusion made. |
| | | **1** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 2(c) | $\lambda = 1.4$ | **B1** | Seen. |
| | $1 - e^{-1.4}(1 + 1.4 + \frac{1.4^2}{2})$ or $1 - e^{-1.4}(1 + 1.4 + 0.98)$ or 1-(0.2466+0.3452+0.2417) | **M1** | Allow any $\lambda$; allow one end error. Expression must be seen (accept correct sigma notation). |
| | = 0.167 (3 sf) or 0.166 | **A1** | Use of Binomial scores SCB1 for 0.167 or 0.166 .<br>No working: 0.167 [or 0.166] **SC B1**.<br>Note: λ=1.4 and 0.167 with no working seen scores **SC B1B1**.<br>Use of Normal scores B0M0. |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 3(a) | Est ($\mu$) = 3.25 = 13/4 or 1625/500 | **B1** | |
| | $\text{Est}(\sigma^2) = \frac{500}{499}(\frac{5663.5}{500} - \text{"3.25"}^2)$ or $\frac{1}{499}\left(5663.5 - \frac{1625^2}{500}\right)$ | **M1** | Expression of correct form. |
| | = 0.766 (3 sf) or 1529/1996 | **A1** | Biased variance of 0.7645 scores M0A0. |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 3(b) | $H_0$: Pop mean (or $\mu$) = '3.25'<br>$H_1$: Pop mean (or $\mu$) ≠ '3.25' | **B1FT** | Not just 'mean'.<br>FT their 3.25 . |
| | $\frac{2.95-"3.25"}{\sqrt{"0.766"\div 60}}$ | **M1** | Standardising with their values.<br>Must have √60. |
| | = –2.655 | **A1** | Or P($\bar{X}$ < 2.95) = 0.0039 or 0.00396 or 0.00397 .<br>**SC** FT their biased est($\sigma^2$), i.e. 0.7645 to give $z$ = 2.658 A1. |
| | '2.655' > 2.576 or '–2.655' < –2.576 | **M1** | For valid comparison, e.g. 0.0039 or 0.00396 or 0.00397 < 0.005, or 0.0078 < 0.01, or 0.00792 < 0.01 . |
| | [Reject $H_0$]<br>There is evidence that (mean) mass in (country B) is different (from country A). | **A1FT** | OE. Must be in context and not definite, e.g., not 'Mean mass is not different', No contradictions.<br>Context needs either 'mass' or 'countries' OE. |
| | | | **SC**, Use of one-tail test.<br>'2.655' > 2.326 or 0.0039 < 0.01 M1A0 (Max B0M1A1M1A0 3/5). |
| | | | Accept critical value method. Either:<br>Xcrit=2.959 M1A1 2.95<2.959 M1A1FT with correct conclusion, or<br>Xcrit=3.241 M1A1 3.25>3,241 M1A1FT with correct conclusion. |
| | | **5** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4(a) | Books received independently or singly or randomly. | **B1** | OE. Must be in context.<br>If more than one condition given, ignore extras. |
| | | **1** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4(b) | $e^{-15.3} \times \frac{15.3^{10}}{10!}$ | **M1** | Allow incorrect λ. |
| | = 0.0439 (3sf) | **A1** | **SC** No working shown but correct answer seen scores B1. |
| | | **2** | |
| 4(c) | N(153, 153) | **B1** | Seen or implied. |
| | $\frac{180.5-153}{\sqrt{153}}$ [= 2.223] | **M1** | For standardising with their values (can be implied). Allow with wrong or missing continuity correction. |
| | 1 − ϕ('2.223') | **M1** | For correct probability area consistent with their values. |
| | = 0.0131 (3sf) | **A1** | |
| | | **4** | |
| 4(d) | ($\lambda$ =) 5.1 + 2.5 [= 7.6] | **B1** | Give at early stage (seen or implied). |
| | $1 - e^{-7.6}(1+7.6+\frac{7.6^2}{2}+\frac{7.6^3}{3!}) = 1 - e^{-7.6}(1+7.6+28.88+73.16)$<br>$= 1 - (0.0005005+0.003803+0.01445+0.03661)$ | **M1** | Allow incorrect λ. Allow one end error.<br>Must see an expression (accept correct sigma notation). |
| | = 0.945 (3sf) | **A1** | **SC** No working, 0.945 B1(could be implied) **SC B1**. |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5(a) | $E(X-Y) = 1$ $Var(X-Y) = 5$ | **B1** | Seen or implied, OE e.g. $X - Y - 2$. |
| | $\frac{2-1}{\sqrt{5}}$ [= 0.447] $\frac{-2-1}{\sqrt{5}}$ [= –1.342] | **M1** | Standardising with their values must come from a combination. |
| | 1 – Φ('0.447') Φ('–1.342') = 1 – Φ('1.342) | **M1** | Correct probability area consistent with their values. |
| | = 0.327 or 0.328 = 0.0898 or 0.0899 | **A1** | Seen or implied. |
| | Probability that difference is more than 2 = 0.417 (3 sf) or 0.418 | **A1** | |
| | | **5** | |
| 5(b)(i) | $E(X) = 62 + 1.5(42)$ [= 125] | **B1** | OE. |
| | $Var(X) = 158 + 1.5^2 \times 108$ [= 401] | **B1** | Correct expression OE. |
| | $\frac{90-"125"}{\sqrt{"401"}}$ [= −1.748] | **M1** | Correct standardisation using their $E(X)$ and $Var(X)$. Must both be from a combination attempt. Ignore any attempted continuity correction. |
| | Φ('1.748') | **M1** | Correct probability area consistent with their stated values. |
| | = 0.960 or 96.0% (3 sf) | **A1** | Allow 0.96 or 96%. |
| | | **5** | |
| 5(b)(ii) | Unlikely. A candidate who does well in Theory is likely to do well in Practical. | **B1** | Need both. Accept 'unlikely', 'not independent', 'dependent', 'not realistic', or similar; and accept 'both testing knowledge from the same syllabus', 'theory and practical share same content' or similar statement. |
| | | **1** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(a) | $(1 - \frac{1}{3})^{10}$ | **M1** | |
| | = 0.0173 (3 sf) | **A1** | No working scores **SC B1**. |
| | | **2** | |
| 6(b) | $1 - (1 - p)^{10} = 0.8926$ | **M1** | Accept $1 - q^{10} = 0.8926$ .<br>Equation must be in $p$ or in $q$ but not both. |
| | $1 - p = 0.1074^{0.1}$ [= 0.800] | **M1** | For valid attempt to solve their (binomial) equation in $p^{10}$ or $q^{10}$. |
| | $p$ = 0.200 (3 sf) or 0.2 | **A1** | |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 7(a)(i) | $\frac{1}{2} \times 4 \times a = 1$ | **M1** | For use of area = 1<br>or let f($x$) = $kx$ and attempt $\int_0^4 kx \, dx = 1$. |
| | [$a = \frac{1}{2}$ ] f($x$) = $\frac{1}{8}x$ | **A1** | $k\left[\frac{x^2}{2}\right]_0^4 = 1$; $8k = 1$; $k = \frac{1}{8}$.<br>f($x$) = $\frac{1}{8}x$ or $k = \frac{1}{8}$. |
| | | **2** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 7(a)(ii) | $\int_0^4 x \times \frac{1}{8}x \, dx$ | **M1** | Attempt to integrate $x \times$ *their* f($x$). Ignore limits accept in terms of $k$. |
| | $\left[\frac{x^3}{24}\right]_0^4$ | **A1ft** | Their integral and correct limits accept in terms of $k$. |
| | $= \frac{8}{3}$ or 2.67 (3 sf) | **A1** | Note: Final answer of $64k/3$ scores 2/3. |
| | | **3** | |
| 7(b) | $\frac{a-1}{a} = \frac{1}{\sqrt{2}}$ | **M1** | Or attempt $\int_0^1 g(w)dw = \frac{1}{2}$ i.e. $\int_0^1 (\frac{2}{a} - \frac{2}{a^2}w)dw = \frac{1}{2}$, or integral from 1 to $a$.<br>g($w$) must be linear of form g($w$) = $mw$ (+$c$).<br>Or area attempt: attempt to calculate heights using their linear equation ($h_1$=2/$a$ and $h_2$= –2/$a^2$ + 2/$a$) and use in either area trapezium = 0.5, or area trapezium =area small triangle or area small triangle = 0.5 .<br>Area trapezium =1/2 × 1 (2/$a$ + –2/$a^2$ + 2/$a$)<br>Area triangle = 1/2($a$ – 1)(–2/$a^2$ + 2/$a$))<br>Note: alternative expression for $h_1$ = ($a$ – 2)/($a$ – 1). |
| | $a\sqrt{2} - \sqrt{2} = a$ | **A1** | Or $a^2 - 4a + 2 = 0$.<br>Any correct equation in $a$, $a$ not in denominator. |
| | $a = 2 + \sqrt{2} = 3.41$ | **A1** | |
| | | **3** | |

Cambridge Assessment International Education

# Cambridge International A Level

**MATHEMATICS** **9709/63**

Paper 6 Probability & Statistics 2 **May/June 2023**

MARK SCHEME

Maximum Mark: 50

**Published**

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**A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

**B** Mark for a correct result or statement independent of method marks.

**DM** or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

**FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

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**Abbreviations**

| | |
|---|---|
| AEF/OE | Any Equivalent Form (of answer is equally acceptable) / Or Equivalent |
| AG | Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid) |
| CAO | Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed) |
| CWO | Correct Working Only |
| ISW | Ignore Subsequent Working |
| SOI | Seen Or Implied |
| SC | Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance) |
| WWW | Without Wrong Working |
| AWRT | Answer Which Rounds To |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 1 | $\frac{3}{2}\int_0^1 (x - x^3)\mathrm{d}x$ | **M1** | Attempt to integrate $xf(x)$; ignore limits. |
| | $= \frac{3}{2}\left[\frac{x^2}{2} - \frac{x^4}{4}\right]_0^1$ | **A1** | Correct integration and limits. |
| | $= \frac{3}{8}$ | **A1** | |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 2(a) | 180, 227 | **B1** | One correct. Ignore incorrect numbers. |
| | | **B1** | Both correct and no extra numbers seen. (Allow other correct use of list of digits). |
| | | **2** | |
| 2(b) | These numbers are not independent of the previous numbers<br><br>OR<br><br>Only a finite number of digits used | **B1** | Already used these numbers, so therefore not random. Does not include numbers not in the list, therefore not random (not random or biased needs a reason). |
| | | **1** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 3(a) | $z = 1.645$ | **B1** | |
| | $z \times \frac{\sqrt{\frac{x}{100} \times (1 - \frac{x}{100})}}{100} = 0.07896$ | **M1** | OE. Equation of correct form. Accept $p = x/100$. Any $z$. Allow missing factor of 2. |
| | $[x(100 - x) = 100^3 \times 0.07896^2 \div 1.645^2]$<br>$x^2 - 100x + 2304 = 0$ | **A1** | Any correct (likely scalar multiple) three-term quadratic equation in $x$ or p with simplified coefficients.<br>Accept $p^2 - p + 0.2304 = 0$ or $p(1-p) = 0.2304$ . |
| | $x = 36$ or 64 | **A1** | |
| | | **4** | |
| 3(b) | $0.1^2 = 0.01$ | **B1** | Accept either. |
| | | **1** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4 | **Method 1: Based on mass** | | |
| | Mean = $7 \times 65.2 = 456.4$ | **B1** | |
| | Var = $7 \times 3.6^2$ [= 90.72] | **M1** | |
| | 22 000/50 = 440 used in standardising equation | **M1** | |
| | $\frac{'440' - '456.4'}{\sqrt{'90.72'}}$ [= –1.722] no mixed methods | **M1** | For standardising with their values. No mixed methods. |
| | $\phi(-'1.722') = 1 - \phi('1.722')$ | **M1** | For correct probability area consistent with their values. |
| | = 0.0425 or 0.0426 | **A1** | Note: accept alt method using per day.<br>N(65.2, $\frac{3.6^2}{7}$). No mixed methods. |
| | **Method 2: Based on profit** | | |
| | Mean = $7 \times 65.2 \times 50 = 22\,820$ | **B1** | |
| | Var = $7 \times 3.6^2$ | **M1** | |
| | Var = $50^2 \times$ '90.72' [= 226 800] | **M1** | |
| | $\frac{22\,000 - '22\,820'}{\sqrt{'226800'}}$ [= –1.722] no mixed methods | **M1** | For standardising with their values. No mixed methods. |
| | $\phi(-'1.722') = 1 - \phi('1.722')$ | **M1** | For correct probability area consistent with their values. |
| | = 0.0425 or 0.0426 | **A1** | |
| | | **6** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5(a) | $\bar{x} = 1700/50 = 34$ | **B1** | |
| | $\text{Est}(\sigma^2) = \frac{50}{49}\left(\frac{59050}{50} - 34^2\right)$ or $\frac{1}{49}\left(59050 - \frac{1700^2}{50}\right)$ | **M1** | $\text{Est}(\sigma^2) = \frac{59050}{50} - 34^2$ biased scores M0. |
| | $= 25.5$ (3 sf) or $\frac{1250}{49}$ | **A1** | $= 25$ scores A0. |
| | | **3** | |
| 5(b) | $H_0$: Population mean time = 32.4<br>$H_1$: Population mean time $\neq$ 32.4 | **B1** | Not just 'mean' but allow just '$\mu$'. |
| | $\frac{34 - 32.4}{\frac{\sqrt{'25.5'}}{\sqrt{50}}}$ | **M1** | Must have $\sqrt{50}$ and not 50. FT *their* mean and var. Can be implied. |
| | $= 2.24$ (3 sf) | **A1** | or $P(\bar{T} > 34) = 0.0125$.<br>**SC** use of biased var (25) $z = 2.26$ or $p = 0.0119$, allow M1A1. |
| | '2.24' < 2.326 | **M1** | Or $0.0125 > 0.01$ for a valid comparison. |
| | [Not reject $H_0$] Insufficient evidence that (mean) time has changed | **A1FT** | In context, not definite, e.g. not 'Time not changed'. No contradictions.<br>Note: accept CV method $x_{cri} = 34.06$ for M1A1. Compares $34 < 34.06$ for M1, conclusion for A1.<br>Condone $x = 32.34$ M1A1: compares $32.4 > 32.34$ for M1, conclusion for A1. |
| | | **5** | **SC** for using a one-tail method. Award max 3/5 (B0 M1 A1 M1 A0). |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5(c) | Distribution of times in the population is normal | **B1** | Accept answers with no context here.<br>Accept underlying distribution for population. |
| | | **1** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(a) | $X \sim \text{Po}(2.5)$ | **B1** | SOI. |
| | $e^{-2.5}(1 + 2.5 + \frac{2.5^2}{2} + \frac{2.5^3}{3!})$ | **M1** | Any $\lambda$. Allow one end error. |
| | = 0.758 (3 sf) | **A1** | **SC** use of binomial B1 for 0.758.<br><br>**SC** when no working is shown, $X \sim \text{Po}(2.5)$ seen scores B1, 0.758 seen also scores B1. |
| | | **3** | |
| 6(b) | $\text{E}(X) = \frac{5}{2}$ or 2.5 , $\text{Var}(X) = \frac{4999}{2000}$ or 2.4995 | ***B1** | Just an answer of 2.5 for the variance is not sufficient. However, 2.4995 is sufficient. |
| | These are almost equal | **DB1** | Condone 'equal'. |
| | | **2** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 7(a) | $\frac{1}{2}\pi\left(\sqrt{\frac{2}{\pi}}\right)^2$ | **M1** | |
| | = 1, which is the area under a PDF [and f($x$) ≥ 0] | **A1** | Result and statement are both needed. |
| | | **2** | |
| 7(b) | $\cos^{-1}\left(\frac{\sqrt{\frac{1}{\pi}}}{\sqrt{\frac{2}{\pi}}}\right)=\frac{\pi}{4}$ | **B1** | AG.<br>Accept alternative approaches, e.g. using Pythagoras, tangent, or isosceles right-angle triangles.<br>Answer should be convincingly obtained and all correct. |
| | Area of sector = $\frac{1}{4}$ | **B1** | |
| | Area of triangle $AOB = \frac{1}{2}OA\times OB = \frac{1}{2}\times\sqrt{\frac{1}{\pi}}\times\sqrt{\frac{2}{\pi}-\frac{1}{\pi}}$<br>or<br>Area of triangle $AOB = \frac{1}{2}OA\times OB\times\sin(AOB) = \frac{1}{2}\times\sqrt{\frac{1}{\pi}}\times\sqrt{\frac{2}{\pi}}\sin\frac{\pi}{4}$ | **M1** | Accept alternative approaches.<br>Note: AB = $\sqrt{0.7979^2-0.5642^2}$ [= 0.5642]<br>Allow values to 3sf. |
| | $\frac{1}{2\pi}$ or 0.1592 | **A1** | |
| | '$\frac{1}{4}$' – '$\frac{1}{2\pi}$' or '0.25' – '0.1592' | **M1** | Attempt area of sector – area of triangle $AOB$. |
| | = $\frac{1}{4}-\frac{1}{2\pi}$ or 0.0908 (3sf) | **A1** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 7(b) | **Alternative Method for Question Q7(b): Using integration** | | |
| | Find equation of curve $x^2 + y^2 = \frac{2}{\pi}$ | **M1** | |
| | $y = \sqrt{\frac{2}{\pi} - x^2}$ | **A1** | |
| | Attempt to integrate (any limits) | **M1** | |
| | Use of correct limits $\sqrt{\frac{1}{\pi}}$ to $\sqrt{\frac{2}{\pi}}$ | **B1** | |
| | Correct integration with correct limits | **A1** | |
| | $= \frac{1}{4} - \frac{1}{2\pi}$ or 0.0908 (3sf) | **A1** | Correct final answer. |
| | | **6** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 8(a) | $H_0$: Pop mean no. people = 3.03 or 1.01 (per 20 min)<br>$H_1$: Pop mean no. people > 3.03 or 1.01 (per 20 min) | **B1** | These must not just be 'mean', but allow just '$\lambda$' or '$\mu$'. |
| | Use of $P_O(3.03)$ | **M1** | |
| | $= 1 - e^{-3.03}(1 + 3.03 + \frac{3.03^2}{2} + \frac{3.03^3}{3!} + \frac{3.03^4}{4!} + \frac{3.03^5}{5!})$<br>$= 1 - e^{-3.03}(1 + 3.03 + 4.5905 + 4.6364 + 3.5120 + 2.128)$<br>$= 1 - (0.04832 + 0.1464 + 0.2218 + 0.2240 + 0.1697 + 0.1028)$ | **M1** | Allow incorrect $\lambda$. Allow one end error. Must see Poisson expression used. |
| | = 0.0870 (3sf) [0.0869727] | **A1** | Allow 0.087 . |
| | 0.0870 > 0.05 | **M1** | For a valid comparison. |
| | (Do not reject $H_0$) Insufficient evidence to believe (mean) number of people has increased | **A1FT** | Conclusion stated must be in context, not definite and include no contradictions (e.g. not 'mean number people has not increased'). |
| | | **6** | If only P($x$ = 6) award max 2/6 (single term not valid).<br><br>**SC** No working B1 B2 M1 A1. Award maximum 5/6. |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 8(b) | "0.0869727" – $e^{-3.03} \times \frac{3.03^6}{6!}$<br>or 0.869727 – $e^{-3.03}$(1.0748)<br>or 0.869727 – 0.05193<br>or $1 - e^{-3.03}(1 + 3.03 + \frac{3.03^2}{2} + \frac{3.03^3}{3!} + \frac{3.03^4}{4!} + \frac{3.03^5}{5!} + \frac{3.03^6}{6!})$ | **M1** | OE. Must see Poisson expression (may be in part (a)). |
| | 0.0350 or 0.0351 | **A1** | Accept 0.035.<br>**SC** no working seen, award B1 for 0.0350, 0.0351 or 0.035. |
| | | **2** | |
| 8(c) | Concluding that the (mean) number of people (using the path per 20 mins in the evening) has increased when it has not | **B1** | OE. Conclusion must be in context. |
| | | **1** | |
| 8(d) | A value for the true mean | **B1** | Allow without context for this mark. |
| | Number of people using the path per 20 mins in the evening. | **B1** | Condone equivalent comment on three randomly chosen 20-minute periods. |
| | | **2** | |

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Cambridge Assessment International Education

# Cambridge International A Level

**MATHEMATICS** **9709/62**

Paper 6 Probability and Statistics 2 **February/March 2023**

MARK SCHEME

Maximum Mark: 50

**Published**

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the February/March 2023 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.



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**[Turn over**

**PUBLISHED**

## Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

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| **Mathematics Specific Marking Principles** | |
|---|---|
| 1 | Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing. |
| 2 | Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected. |
| 3 | Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points. |
| 4 | Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw). |
| 5 | Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread. |
| 6 | Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear. |

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## Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

### Types of mark

**M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

**A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

**B** Mark for a correct result or statement independent of method marks.

**DM** or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

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- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
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**Abbreviations**

| | |
|---|---|
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| SOI | Seen Or Implied |
| SC | Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance) |
| WWW | Without Wrong Working |
| AWRT | Answer Which Rounds To |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 1(a) | $\left[\frac{49}{140} = 0.35\right]$ | | |
| | $0.35 \pm z\sqrt{\frac{0.35(1-0.35)}{140}}$ | **M1** | Use of formula of correct form, ft *their* $\frac{49}{140}$ , any $z$ (not a probability). |
| | $z = 2.326$ | **B1** | Accept 2.326 to 2.329 . |
| | Confidence interval = 0.256 to 0.444 (3 sf) | **A1** | Must be an interval. |
| | | **3** | |
| 1(b) | Find a smaller **percentage** confidence interval/ lower **level** of confidence | **B1** | ISW if 2 reasons given. Just saying 'use smaller z' oe B0.<br>Accept a correct example e.g. 90% (even if not qualified with statement). |
| | | **1** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 2(a) | Orders arrive at constant mean rate (must say mean or rate)<br>Orders arrive at random<br>Orders arrive independently<br>Orders arrive singly | | Must be in context (accept 25.2 as context). |
| | | **B1** | Any one reason correctly stated. |
| | | **B1** | A second reason correctly stated. |
| | | | **SC B1**: both correct, not in context. |
| | | **2** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 2(b)(i) | $\lambda = \frac{3}{8} \times 25.2$ [= 9.45] | **B1** | |
| | $e^{-'9.45'}(\frac{9.45^3}{3!} + \frac{9.45^4}{4!} + \frac{9.45^5}{5!})$ **or** $e^{-'9.45'}$ (140.65 + 332.29 + 628.03) **or** 0.01107 + 0.02615 + 0.04942 | **M1** | Allow any $\lambda$. Allow end errors.<br>Expression must be seen. |
| | = 0.0866 (3 sf) | **A1** | If M0 allow **SC B1** for 0.0866 no working seen. |
| | | **3** | |
| 2(b)(ii) | $e^{-3.15} \times 3.15$ **or** $(1 - e^{-3.15}(1 + 3.15))$ **or** 0.135 **or** 0.822 (3 sf) | **B1** | |
| | $e^{-3.15} \times 3.15 \times (1 - e^{-3.15}(1 + 3.15))$ | **M1** | M1 for product of two Poisson probabilities P(1) × (1– P(0,1)) (no end errors accepted). Accept any λ. |
| | × 2 or 0.111 × 2 | **M1** | M1 for *their* product of two Poisson probabilities (accept end errors) × 2. Accept any λ |
| | 0.222 (3 sf) | **A1** | |
| | | **4** | |
| 2(c) | N(113.4, 113.4) | **B1** | SOI |
| | $\frac{120.5-113.4}{\sqrt{113.4}}$ [= 0.667] | **M1** | Standardise with *their* values. Allow wrong or no cc. Must have √. |
| | 1 – ϕ(*their* '0.667') | **M1** | For probability area consistent with *their* values. |
| | = 0.252 (3 sf) | **A1** | |
| | | **4** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 3(a) | $1 - 2(a + b)$ **or** $1 - 2a$ **or** $0.5 - a - b$ **or** 1-(a+b) **or** a+a+b | **M1** | OE. Seen or implied – may be on the diagram (or for correct un-simplified final expression). |
| | $P(0.6 \leq X \leq 1.8) = 1 - 2a - b$ | **A1** | Accept $1 - (2a + b)$. |
| | | **2** | |
| 3(b)(i) | $k\int_0^3 (9x^2 - 6x^3 + x^4)\mathrm{d}x = 1$ | **M1** | Attempt integrate f($x$) ignore limits and '= 1'. |
| | $k\left[\frac{9x^3}{3} - \frac{6x^4}{4} + \frac{x^5}{5}\right]_0^3 = 1$ | **A1** | Correct integration seen, correct limits. |
| | $k \times \frac{81}{10} = 1$, $k = \frac{10}{81}$ | **A1** | AG. Convincingly obtained. No errors seen. (Must see integration). |
| | | **3** | |
| 3(b)(ii) | $\frac{10}{81}\int_0^3 (9x^4 - 6x^5 + x^6)\mathrm{d}x$<br>$\left[\frac{10}{81}\left[\frac{9x^5}{5} - x^6 + \frac{x^7}{7}\right]_0^3\right]$ [= $\frac{18}{7}$ or 2.57...] | **M1** | Attempt integrate $x^2$f($x$) between 0 and 3<br>condone missing k.<br>Must see integration or correct answer of 18/7 seen or implied. |
| | $\frac{18}{7}$ – '1.5'$^2$ | **M1** | Their integral of $x^2$f($x$) – $1.5^2$ (or their mean$^2$). |
| | = $\frac{9}{28}$ or 0.321 | **A1** | |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4(a) | $e^{-5.7}(1 + 5.7 + \frac{5.7^2}{2!})$ **or** $e^{-5.7}(1 + 5.7 + 16.245)$ **or** 0.003346 + 0.01907 + 0.05436 | **M1** | Allow one end error.<br>Must see this expression. |
| | = 0.0768 (3 sf) | **A1** | **SC B1** for unsupported answer of 0.0768 . |
| | | **2** | |
| 4(b) | $e^{-0.9}(1 + 0.9 + \frac{0.9^2}{2!})$ | **M1** | Attempted; allow one end error (must see expression). |
| | $= 1 - e^{-0.9}(1 + 0.9 + \frac{0.9^2}{2!}) = 1 - e^{-0.9}(1 + 0.9 + 0.405) = 1 - (0.4066 + 3659 + 0.1647)$ | **A1** | Correct expression $P(X \geqslant 3)$ no end errors (must see expression). |
| | = 0.0629 (3 sf) | **A1** | **SC B2** for unsupported answer of 0.0629 . |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5(a) | D = L – 2S E(D) = 410 – 2(206) = –2 | **B1** | SOI. OE using 2S–L. |
| | Var(D) = $3.6^2 + 4 \times 3.7^2$ [= 67.72] | **B1** | SOI |
| | $\frac{0-(-2)}{\sqrt{'67.72'}}$ [= 0.243] | **M1** | For standardising using *their* values. |
| | $1 - \Phi$(*their* '0.243') | **M1** | For probability area consistent with *their* values. |
| | = 0.404 (3 sf) | **A1** | As final answer. |
| | | **5** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5(b) | $T_L \sim N(4100, 10\times3.6^2)$ \| $T_S \sim N(4120, 20\times3.7^2)$ | **B1** | One of N(4100, 129.6) or N(4120, 273.8) USED (unchanged) in a standardising equation. |
| | $\frac{4080-4100}{\sqrt{'129.6'}}$ (= –1.757) \| $\frac{4080-4120}{\sqrt{'273.8'}}$ (= –2.417) | **M1** | Standardising with either *their* N(4100, 129.6) or N(4120, 273.8) or their N(…,…) (could be from a combination). |
| | 1 – ϕ(‘–1.757’) = ϕ(1.757) \| 1 – ϕ(‘–2.417’) = ϕ(2.417) | **M1** | One area consistent with *their* working (could be from a combination). Do not ISW. |
| | = 0.9605 **or** 0.961 \| = 0.9921 **or** 0.9922 **or** 0.992 | **A1** | Both of these correct. Do not ISW. |
| | 0.6 × ‘*their* 0.9605’ + 0.4 × ‘*their* 0.9921’ | **M1** | Must be using probabilities. |
| | = 0.973 (3 sf) | **A1** | |
| | | **6** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(a) | He is expecting a **decrease** (in $\mu$) | **B1** | OE |
| | | **1** | |
| 6(b) | −2.02 < −1.96 | **M1** | For valid comparison.<br>Allow 2.02 > 1.96 **or** 0.0217 < 0.025 **or** 0.9783 > 0.975 |
| | (Reject $H_0$)<br>There is evidence to suggest that this year’s (mean) time is less than 25 | **A1** | OE (such as evidence to support Akash’s belief), in context, not definite.<br>No contradictions. |
| | | **2** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(c) | $1 - \phi(2.14)$ [= 0.0162] | **M1** | |
| | 1.62 | **A1** | Allow 1.62% or 1.6 or 1.6%. |
| | $\alpha \geqslant 1.62$ (3 sf) | **A1ft** | FT *their* 1.62 .<br>Allow $\alpha \geqslant 1.62$% or 1.6 or 1.6%. Condone >. |
| | | **3** | |
| 6(d) | $\frac{24.8-m}{3.9 \div 10}$ | **M1** | For standardising. |
| | $\frac{24.8-m}{3.9 \div 10} = -1.645$ | **M1** | Equate *their* standardised value to −1.645 (signs must be consistent). |
| | $m = 25.4$ (3 sf) | **A1** | |
| | | **3** | |

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Cambridge Assessment International Education

# Cambridge International A Level

**MATHEMATICS** **9709/61**

Paper 6 Probability & Statistics 2 **October/November 2022**

MARK SCHEME

Maximum Mark: 50

**Published**

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

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Cambridge International is publishing the mark schemes for the October/November 2022 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

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**[Turn over**

**PUBLISHED**

## Generic Marking Principles

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GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

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| **Mathematics Specific Marking Principles** | |
|---|---|
| 1 | Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing. |
| 2 | Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected. |
| 3 | Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points. |
| 4 | Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw). |
| 5 | Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread. |
| 6 | Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear. |

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## Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

### Types of mark

**M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

**A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

**B** Mark for a correct result or statement independent of method marks.

**DM** or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

**FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

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**Abbreviations**

| | |
|---|---|
| AEF/OE | Any Equivalent Form (of answer is equally acceptable) / Or Equivalent |
| AG | Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid) |
| CAO | Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed) |
| CWO | Correct Working Only |
| ISW | Ignore Subsequent Working |
| SOI | Seen Or Implied |
| SC | Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance) |
| WWW | Without Wrong Working |
| AWRT | Answer Which Rounds To |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 1 | $\frac{62.1}{10} = 6.21$ | **B1** | OE |
| | $[\Sigma x^2 = 387.05]$<br>$\frac{10}{9}\left(\frac{\textit{their } '387.05'}{10} - \left(\textit{their } '6.21'\right)^2\right)$<br>**or** $\frac{1}{9}\left(\frac{\textit{their } '387.05'}{10} - \frac{\left(\textit{their } '6.21'\right)^2}{10}\right)$ | **M1** | Can be implied.<br>Accept alternative methods (e.g. working mean of 6).<br>Biased 0.1409 M0. |
| | = 0.157 (3 sf) or $\frac{1409}{9000}$ | **A1** | |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 2(a) | $H_0$: P(red) = 0.2 $H_1$: P(red) < 0.2 | **B1** | Allow $H_0$: $p = 0.2$ $H_1$: $p < 0.2$ . |
| | $P(X \leq 4) = 0.8^{40} + 40\times0.8^{39}\times0.2 + {}^{40}C_2\times0.8^{38}\times0.2^2$<br>$+ {}^{40}C_3\times0.8^{37}\times0.2^3 + {}^{40}C_4\times0.8^{36}\times0.2^4$ | **M1** | For full expression seen.<br>Allow one term omitted, incorrect or extra. |
| | 0.0759 | **A1** | **SC** 0.0759 without working B1. |
| | *their* '0.0759' > 0.05 | **M1** | Valid comparison (from binomial probs) of their $P(X \leqslant 4)$ with 0.05. |
| | [Do not reject $H_0$ ]. Not enough evidence that it lands on red fewer times than if it were fair or not enough evidence to suggest that the spinner is biased | **A1 FT** | FT their 0.0759.<br>In context, not definite, no contradictions. |
| | | **5** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 2(b) | P($X \leqslant 3$) = ` 0.0759 ` – $^{40}C_4 \times 0.8^{36} \times 0.2^4$ | **M1** | OE Attempted. Must be using $B(40, 0.2)$.<br>Method could be implied by correct answer here. |
| | = 0.0285 or 0.0284 | ***A1** | |
| | Largest value of $r$ is 3 | **DA1** | |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 3(a) | $\lambda = 5.2 \div 2$ [= 2.6] | **B1** | |
| | $1 - e^{-2.6}(1 + 2.6 + \frac{2.6^2}{2})$ or $1 - e^{-2.6}( 1 + 2.6 + 3.38 )$<br>**or** 1- ( 0.07427 + 0.1931 + 0.2510 ) | **M1** | Allow any $\lambda$.<br>Allow one end error.<br>Must see expression. |
| | = 0.482 (3 sf) | **B1** | |
| | | **3** | |
| 3(b) | N(120×5.2, 120×5.2) | **B1** | Stated or implied. Give at early stage. |
| | $\frac{649.5 - \textit{their } '624'}{\sqrt{\textit{their } '624'}}$ [= 1.021] | **M1** | Allow with no or wrong continuity correction. |
| | 1– Φ(*their* '1.021') | **M1** | For area consistent with *their* working. |
| | = 0.154 (3 sf) | **A1** | |
| | | **4** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4(a) | Mean = [3 (2500 + 3700)] = 18600 (kg) | **B1** | |
| | Var(Total profit) = $3(120^2 + 130^2)$ or 93900 | **M1** | or √ of this stated. |
| | sd = 306 (kg) (3 sf) | **A1** | |
| | | **3** | |
| 4(b) | E(1.5*X* – 0.2*Y*) = 1.5x2500 – 0.20x3700 = [3010] | **B1** | Give at early stage. |
| | Var(1.5*X* – 0.2*Y*) = $1.5^2 \times 120^2 + 0.2^2 \times 130^2$ [= 33076] | **B1** | Correct expression or result or sd = 182 (3 sf) seen. |
| | $\frac{3000 - 3010}{\sqrt{their\ '33076'}}$ [= –0.055] | **M1** | Ignore continuity correction attempts.<br>E(X) and Var must come from a combination attempt.<br>Can be implied. |
| | Φ(*their* '–0.055') = 1 – Φ(*their* '0.055') | **M1** | For area consistent with their values.<br>Can be implied. |
| | = 0.478 (3 sf) | **A1** | |
| | | **5** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5(a) | $45 \pm z \times \frac{6}{\sqrt{200}}$ | **M1** | For expression of correct form, any *z*.<br>Accept one side of interval for M1. |
| | *z* = 1.96 | **B1** | Must be seen. |
| | 44.2 to 45.8 (3 sf) | **A1** | Must be an interval. |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5(b) | $z \times \frac{11}{\sqrt{200}} = 2$ | **M1** | Or … = 4 for M1 |
| | $z = 2.571$ | **A1** | Accept 3sf if nothing better seen. |
| | $\Phi$(*their* '2.571') = 0.9949<br>**and** *their* '0.9949' – (1 – *their* '0.9949') [= 0.9898] | **M1** | OE For area consistent with their values.<br>Must be seen. |
| | $\alpha = 99.0$ (3 sf) | **A1** | Allow 99.<br>cwo<br>Final answer of 0.99 scores A0. |
| | | **4** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(a) | Curve of similar shape, $x = 0$ to $x = 4$, with highest point (2, 0.375) | **B1** | Not straight lines, not bell shaped. Must be correct at $x = 0$ and $x = 4$, highest point must be at $x = 2$, y value ± ¼ square.<br>Must not go below the x-axis. |
| | | **1** | |
| 6(b) | Curve of similar shape, from $x = 0$ to $x = 2$, highest point at $x = 1$ | **B1** | Not straight lines, not bell shaped. Must be correct at $x = 0$ and $x = 2$. Highest point must be at $x = 1$. |
| | Highest point (1, 0.75) | **B1** | |
| | | **2** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(c) | $\frac{3}{32}\int_{1+a}^{3}(3+2x-x^2)\mathrm{d}x=\frac{1}{4}$ **or** $\frac{3}{32}\int_{1-a}^{1+a}(3+2x-x^2)\mathrm{d}x=\frac{1}{2}$ | **M1** | OE<br>Attempt to integrate f(x) and correct limits with correct RHS. |
| | $\frac{3}{32}\left[3x+x^2-\frac{x^3}{3}\right]_{1+a}^{3}=\frac{1}{4}$ or $\frac{3}{32}\left[3x+x^2-\frac{x^3}{3}\right]_{1-a}^{1+a}=\frac{1}{2}$ | **A1** | Correct integration. |
| | $a^3 - 12a + 8 = 0$ | **A1** | AG Substitute limits and correctly obtain equation.<br>May see $3/32(6a+4a-6a/3-2a^3/3) = 0.5$<br>No errors seen.. |
| | | **3** | |
| 6(d) | $0.69^3 - 12\times0.69 + 8 = 0.049$ (2 sf) > 0<br>$0.70^3 - 12\times0.70 + 8 = -0.057$ (2 sf) < 0<br>Hence $0.69 < a < 0.70$ | **B1** | AG<br>Must state either the correct expression and > 0 and < 0 or both answers to 2 sf.<br>Both answers correct and conclusion.<br>Accept equivalent expressions.<br><br>OR: $a = 0.695$ (3 sf) which is between 0.69 & 0.70. |
| | | **1** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 7(a) | $H_0$: Population mean time (or $\mu$) = 32.5<br>$H_1$: Population mean time (or $\mu$) < 32.5 | **B1** | Not just "mean". |
| | $\pm\frac{31.8-32.5}{3.1\div\sqrt{50}}$ | **M1** | Must have $\sqrt{50}$.<br>Could be implied. |
| | = ± –1.597 | **A1** | |
| | '–1.597' < –1.406 [or '1.597' > 1.406] | **M1** | Valid comparison of their $z_{calc}$ with ±1.406.<br>or 0.0551 < 0.08 (or 0.0552 < 0.08). |
| | [ reject $H_0$ ]<br>There is evidence that [population] [mean ] **time** has **decreased** | **A1 FT** | In context, not definite, no contradictions.<br><br>Note: Accept critical value method 31.88 (31.9) **M1 A1** and 31.8 < 31.88 **M1** conclusion **A1**. |
| | | **5** | |
| 7(b) | $\frac{a-32.5}{3.1\div\sqrt{50}}=-1.406$ | **M1** | Standardise with 32.5 and $\sqrt{50}$ and $z$ value on RHS. |
| | $a$ = 31.88 or 31.9 | **A1** | May be seen in part **(a)**. Can score M1A1 here as well using a similar approach to **(a)**. |
| | $\frac{their\ '31.88-31.5}{3.1\div\sqrt{50}}$ [= 0.8668 to 0.8760] | **M1** | Standardise with *their* cv and mean = 31.5.<br>Must have $\sqrt{50}$. |
| | 1 – Φ('0.8668') | **M1** | For area consistent with their working. |
| | = 0.190 to 0.193 (3 sf) | **A1** | |
| | | **5** | |

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Cambridge Assessment International Education

# Cambridge International A Level

**MATHEMATICS** **9709/62**

Paper 6 Probability & Statistics 2 **October/November 2022**

MARK SCHEME

Maximum Mark: 50

**Published**

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|---|---|
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| SC | Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance) |
| WWW | Without Wrong Working |
| AWRT | Answer Which Rounds To |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 1(a) | Est $\mu = 25.6$ or $\frac{2048}{80}$ or $\frac{128}{5}$ | **B1** | |
| | Est $\sigma^2 = \frac{80}{79}\left(\frac{52760}{80} - \left(\frac{2048}{80}\right)^2\right)$ or $\frac{1}{79}\left(52760 - \frac{2048^2}{80}\right)$ | **M1** | Substitution into a correct formula.<br>Biased 4.14 scores M0. |
| | = 4.19 (3 sf) or $\frac{1656}{395}$ | **A1** | |
| | | **3** | |
| 1(b) | '25.6' $+ z\sqrt{\frac{'4.19'}{80}} = 26.0$ | **M1** | Use of correct equation with their values. |
| | $z = 1.748$ or 1.747 | **A1** | Accept 3sf.<br>FT Biased z = 1.758. |
| | ($\phi$('1.748') = 0.960) '0.960' – (1 – '0.960') | **M1** | Correct area using their values. |
| | $\alpha = 92.0$ or 91.9 | **A1** | Allow 92 .<br>FT Biased 92.1.<br>A final answer of 0.92 or 0.919 scores A0. |
| | | **4** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 2 | $H_0$: Population mean length = 10.3 cm<br>$H_1$: Population mean length < 10.3 cm | **B1** | or $\mu = 10.3$ (not just 'mean').<br>$\mu < 10.3$ |
| | $\pm\frac{9.8-10.3}{2.6/\sqrt{100}}$ | **M1** | If $\pm$ 1.923 (or 0.0272) seen allow M1 implied. |
| | = –1.923 | **A1** | Accept $\pm$ .<br>Accept 3sf. |
| | –1.923 > –2.054 or –2.055 | **M1** | OE For a valid comparison.<br>Or compare 1 – ϕ('1.923'') with 0.02 e.g. 0.0272 > 0.02<br>Use of CV 9.8 > 9.766 scores M1 A1 for 9.766 and M1 for comparison. |
| | [Not reject $H_0$ ] No evidence that [mean] **length** has **decreased** | **A1 FT** | FT *their* z. No contradictions, not definite, in context. |
| | | **5** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 3(a) | Use of Poisson. mean = 3.2 | **B1 B1** | |
| | $1-e^{-3.2}\left(1+3.2+\frac{3.2^2}{2}+\frac{3.2^3}{3!}\right)$ or $1 - e^{-3.2}(1 + 3.2 + 5.12 + 5.46133)$<br>or 1 – (0.04076 + 0.1304 + 0.2087 + 0.2226) | **M1** | Allow any $\lambda$ .<br>Allow one end error. |
| | = 0.397 or 0.398 | **A1** | **SC** Use of binomial: B1 for answer 0.398 (3 sf).<br>0.397 or 0.398 with no working scores **SC B1**. |
| | | **4** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 3(b) | [Binomial with] [n =] 200 > 50 | **B1** | |
| | [np =][200 × 0.016 =] 3.2 < 5 or [p =]0.016 < 0.1 | **B1** | If B0 B0<br>SC *n* large (or n > 50),<br>**and** *p* small or p < 0.1 or np <5 : B1. |
| | | **2** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4(a) | $H_0$: Population mean = 7.2 or 2.4<br>$H_1$: Population mean < 7.2 or 2.4 | **B1** | or $\lambda$ or $\mu$ = 7.2 or 2.4 (Not just 'mean').<br>or $\lambda$ or $\mu$ < 7.2 or 2.4 |
| | | **1** | |
| 4(b) | $\lambda$ = 7.2 | **B1** | SOI |
| | $\left[\mathrm{P}(X \leqslant 2)\right] = \mathrm{e}^{-7.2}\left(1 + 7.2 + \frac{7.2^2}{2}\right)$ or $\mathrm{e}^{-7.2}(1 + 7.2 + 25.92)$<br>or 0.0007465 + 0.0053754 + 0.01935 [= 0.0255]<br>$\left[\mathrm{P}(X \leqslant 3)\right] = \text{'0.0255'} + \mathrm{e}^{-7.2} \times \frac{7.2^3}{3!}$ or '0.0255' + $\mathrm{e}^{-7.2}$ (62.21)<br>or '0.0255' +0.04644 [= 0.0719] | **M1** | Both expressions needed, allow any $\lambda$<br>If $\lambda \neq 7.2$ allow P(X ⩽ n) for 2 consecutive values of n with P(X ⩽ n) < 0.05 and P(X ⩽ n + 1) > 0.05. |
| | P(Type I) = 0.02547 or 0.0255 (3 sf) | **B1** | |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4(c) | $3 > 2$ or $P(X \leqslant 3) > 0.05$ or '0.0719' > 0.05 | **M1** | For a valid comparison<br>or 3 outside critical region. FT *their* CR in **(b)**. |
| | [Not reject $H_0$]<br>No evidence that [mean] **number of faults** has **decreased** | **A1 FT** | No contradictions. In context, not definite. |
| | | **2** | |
| 4(d) | $1 - e^{-1.5}(1 + 1.5 + 1.5^2 / 2)$ or $1 - e^{-1.5}(1 + 1.5 + 1.125)$<br>or $1 - (0.2231 + 0.3347 + 0.2510)$ | **M1** | Must see expression. FT *their* CR in **(b)**. |
| | = 0.191 (3 sf) | **A1** | |
| | | **2** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5(a) | For $X$, $\mu = 2\ \sigma^2 = 1.6$ | | |
| | Mean = 2 | **B1** | |
| | Variance $= \frac{1.6}{160}$ or $\frac{1}{100}$ or 0.01 | **B1** | Accept Var = $0.1^2$ (accept sd=0.1 if clearly identified). |
| | Normal | **B1** | |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5(b) | $\pm\frac{1.8-\frac{1}{320}-'2'}{\sqrt{'0.01'}}$ or $\pm\frac{1.8-'2'}{\sqrt{'0.01'}}$ [= –2.03 or –2]<br>or ± (287.5 – '320') / $\sqrt{'256'}$ or ± (288 – '320') / $\sqrt{'256'}$ [= –2.03 or –2] | **M1** | Allow with wrong continuity correction.<br>M1 can be implied by correct final answer or for –2.03 / –2.0 or 0.9788 / 0.9772 seen. |
| | ϕ('–2.03') = 1 – ϕ('2.03') | **M1** | Correct area consistent with their values.<br>M1 can be implied by correct final answer. |
| | = 0.0212 or 0.0228 (3 sf) | **A1** | |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(a) | E($T$) = 4 × 510 + 2 × 1015 [= 4070] | **B1** | |
| | Var($T$) = 4 × 100 + 2 × 324 [= 1048] | **B1** | or $\sqrt{(4\times100+2\times324)\left[=32.4(3\text{sf})\right]}$. |
| | $\frac{4130-4070}{\sqrt{'1048'}}$ [= 1.853] | **M1** | Standardising with their values.<br>Variance must be from a combination attempt.<br>M1 can be implied by correct final answer. |
| | Φ('1.853') | **M1** | For area consistent with their values.<br>M1 can be implied by correct final answer. |
| | = 0.968 (3 sf) | **A1** | As final answer. |
| | | **5** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(b) | E($D$) = 4 × 510 – 2 × 1015 [= 10] | **B1** | |
| | Var($D$) = 4 × 100 + 2 × 324 [= 1048] | **B1** | Or $\sqrt{(4\times100+2\times324)\left[=32.4(3\text{sf})\right]}$ . |
| | $\dfrac{0-'10'}{\sqrt{'1048'}}$ [= –0.309] | **M1** | Standardising with their values.<br>Variance must be from a combination attempt.<br>M1 can be implied by correct final answer. |
| | 1 – Φ(‘–0.309’) = Φ(‘0.309’) | **M1** | For area consistent with their values.<br>M1 can be implied by correct final answer. |
| | = 0.621 | **A1** | As final answer. |
| | | **5** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 7(a) | $1-p$ or $p-0.5$ | **M1** | SOI, e.g. on diagram. |
| | [P(–1 < $X$ < 0) =] $2p-1$ | **A1** | Clearly as final answer. |
| | | **2** | |
| 7(b)(i) | $\int_{-3}^{2}(a-b(x^2+x))\mathrm{d}x = 1$ or $\int_{-3}^{2}(ax-b(x^3+x^2))\mathrm{d}x = -0.5$ | **M1** | OE Attempt integral, with correct limits and RHS. |
| | $\left[ax-b\left(\dfrac{x^3}{3}+\dfrac{x^2}{2}\right)\right]_{-3}^{2}$ (= 1) or $\left[a\dfrac{x^2}{2}-b\left(\dfrac{x^4}{4}+\dfrac{x^3}{3}\right)\right]_{-3}^{2}$ (= –0.5) | **A1** | OE Correct integration. |
| | 2a – 8b/3 – 2b +3a – 9b +9b / 2 = 1 or 2a – 4b – 8b / 3 – 9a / 2 + 81b / 4 – 9b = –0.5 leading to 30$a$ – 55$b$ = 6 **AG** | **A1** | Correctly obtained. No errors seen. |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 7(b)(ii) | $a - b(9 - 3) = 0$ or $a - b(4 + 2) = 0$ [hence $a - 6b = 0$] | ***M1** | Use f(–3) = 0 or f(2) = 0.<br>Further attempts at integration M0. |
| | Attempt to solve $30a - 55b = 6$ and their a – 6b = 0 | **DM1** | |
| | $a = \frac{36}{125}$ or 0.288 $b = \frac{6}{125}$ or 0.048 | **A1** | |
| | | **3** | |

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Cambridge Assessment International Education

# Cambridge International A Level

**MATHEMATICS** **9709/63**

Paper 6 Probability & Statistics 2 **October/November 2022**

MARK SCHEME

Maximum Mark: 50

**Published**

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2022 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

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## Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

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| **Mathematics Specific Marking Principles** | |
|---|---|
| 1 | Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing. |
| 2 | Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected. |
| 3 | Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points. |
| 4 | Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw). |
| 5 | Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread. |
| 6 | Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear. |

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## Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

### Types of mark

**M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

**A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

**B** Mark for a correct result or statement independent of method marks.

**DM** or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

**FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

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**Abbreviations**

| | |
|---|---|
| AEF/OE | Any Equivalent Form (of answer is equally acceptable) / Or Equivalent |
| AG | Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid) |
| CAO | Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed) |
| CWO | Correct Working Only |
| ISW | Ignore Subsequent Working |
| SOI | Seen Or Implied |
| SC | Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance) |
| WWW | Without Wrong Working |
| AWRT | Answer Which Rounds To |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 1 | $\frac{62.1}{10} = 6.21$ | **B1** | OE |
| | $[\Sigma x^2 = 387.05]$<br>$\frac{10}{9}\left(\frac{\textit{their } '387.05'}{10} - \left(\textit{their } '6.21'\right)^2\right)$<br>**or** $\frac{1}{9}\left(\frac{\textit{their } '387.05'}{10} - \frac{\left(\textit{their } '6.21'\right)^2}{10}\right)$ | **M1** | Can be implied.<br>Accept alternative methods (e.g. working mean of 6).<br>Biased 0.1409 M0. |
| | = 0.157 (3 sf) or $\frac{1409}{9000}$ | **A1** | |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 2(a) | $H_0$: P(red) = 0.2 $H_1$: P(red) < 0.2 | **B1** | Allow $H_0$: $p = 0.2$ $H_1$: $p < 0.2$ . |
| | $P(X \leq 4) = 0.8^{40} + 40\times0.8^{39}\times0.2 + {}^{40}C_2\times0.8^{38}\times0.2^2$<br>$+ {}^{40}C_3\times0.8^{37}\times0.2^3 + {}^{40}C_4\times0.8^{36}\times0.2^4$ | **M1** | For full expression seen.<br>Allow one term omitted, incorrect or extra. |
| | 0.0759 | **A1** | **SC** 0.0759 without working B1. |
| | *their* '0.0759' > 0.05 | **M1** | Valid comparison (from binomial probs) of their $P(X \leqslant 4)$ with 0.05. |
| | [Do not reject $H_0$ ]. Not enough evidence that it lands on red fewer times than if it were fair or not enough evidence to suggest that the spinner is biased | **A1 FT** | FT their 0.0759.<br>In context, not definite, no contradictions. |
| | | **5** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 2(b) | P($X \leqslant 3$) = ` 0.0759 ` – $^{40}C_4 \times 0.8^{36} \times 0.2^4$ | **M1** | OE Attempted. Must be using $B(40, 0.2)$.<br>Method could be implied by correct answer here. |
| | = 0.0285 or 0.0284 | ***A1** | |
| | Largest value of $r$ is 3 | **DA1** | |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 3(a) | $\lambda = 5.2 \div 2$ [= 2.6] | **B1** | |
| | $1 - e^{-2.6}(1 + 2.6 + \frac{2.6^2}{2})$ or $1 - e^{-2.6}( 1 + 2.6 + 3.38 )$<br>**or** 1- ( 0.07427 + 0.1931 + 0.2510 ) | **M1** | Allow any $\lambda$.<br>Allow one end error.<br>Must see expression. |
| | = 0.482 (3 sf) | **B1** | |
| | | **3** | |
| 3(b) | N(120×5.2, 120×5.2) | **B1** | Stated or implied. Give at early stage. |
| | $\frac{649.5 - \textit{their } '624'}{\sqrt{\textit{their } '624'}}$ [= 1.021] | **M1** | Allow with no or wrong continuity correction. |
| | 1– Φ(*their* '1.021') | **M1** | For area consistent with *their* working. |
| | = 0.154 (3 sf) | **A1** | |
| | | **4** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4(a) | Mean = [3 (2500 + 3700)] = 18600 (kg) | **B1** | |
| | Var(Total profit) = $3(120^2 + 130^2)$ or 93900 | **M1** | or √ of this stated. |
| | sd = 306 (kg) (3 sf) | **A1** | |
| | | **3** | |
| 4(b) | E($1.5X – 0.2Y$) = 1.5x2500 – 0.20x3700 = [3010] | **B1** | Give at early stage. |
| | Var($1.5X – 0.2Y$) = $1.5^2 \times 120^2 + 0.2^2 \times 130^2$ [= 33076] | **B1** | Correct expression or result or sd = 182 (3 sf) seen. |
| | $\frac{3000 - 3010}{\sqrt{their\ '33076'}}$ [= –0.055] | **M1** | Ignore continuity correction attempts.<br>E(X) and Var must come from a combination attempt.<br>Can be implied. |
| | Φ(*their* '–0.055') = 1 – Φ(*their* '0.055') | **M1** | For area consistent with their values.<br>Can be implied. |
| | = 0.478 (3 sf) | **A1** | |
| | | **5** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5(a) | $45 \pm z \times \frac{6}{\sqrt{200}}$ | **M1** | For expression of correct form, any *z*.<br>Accept one side of interval for M1. |
| | $z = 1.96$ | **B1** | Must be seen. |
| | 44.2 to 45.8 (3 sf) | **A1** | Must be an interval. |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5(b) | $z \times \frac{11}{\sqrt{200}} = 2$ | **M1** | Or … = 4 for M1 |
| | $z = 2.571$ | **A1** | Accept 3sf if nothing better seen. |
| | $\phi$(*their* '2.571') = 0.9949<br>**and** *their* '0.9949' – (1 – *their* '0.9949') [= 0.9898] | **M1** | OE For area consistent with their values.<br>Must be seen. |
| | $\alpha$ = 99.0 (3 sf) | **A1** | Allow 99.<br>cwo<br>Final answer of 0.99 scores A0. |
| | | **4** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(a) | Curve of similar shape, $x = 0$ to $x = 4$, with highest point (2, 0.375) | **B1** | Not straight lines, not bell shaped. Must be correct at $x = 0$ and $x = 4$, highest point must be at $x = 2$, y value ± ¼ square.<br>Must not go below the x-axis. |
| | | **1** | |
| 6(b) | Curve of similar shape, from $x = 0$ to $x = 2$, highest point at $x = 1$ | **B1** | Not straight lines, not bell shaped. Must be correct at $x = 0$ and $x = 2$. Highest point must be at $x = 1$. |
| | Highest point (1, 0.75) | **B1** | |
| | | **2** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(c) | $\frac{3}{32}\int_{1+a}^{3}(3+2x-x^2)\mathrm{d}x=\frac{1}{4}$ **or** $\frac{3}{32}\int_{1-a}^{1+a}(3+2x-x^2)\mathrm{d}x=\frac{1}{2}$ | **M1** | OE<br>Attempt to integrate f(x) and correct limits with correct RHS. |
| | $\frac{3}{32}\left[3x+x^2-\frac{x^3}{3}\right]_{1+a}^{3}=\frac{1}{4}$ or $\frac{3}{32}\left[3x+x^2-\frac{x^3}{3}\right]_{1-a}^{1+a}=\frac{1}{2}$ | **A1** | Correct integration. |
| | $a^3 - 12a + 8 = 0$ | **A1** | AG Substitute limits and correctly obtain equation.<br>May see $3/32(6a+4a-6a/3-2a^3/3) = 0.5$<br>No errors seen.. |
| | | **3** | |
| 6(d) | $0.69^3 - 12\times0.69 + 8 = 0.049$ (2 sf) > 0<br>$0.70^3 - 12\times0.70 + 8 = -0.057$ (2 sf) < 0<br>Hence $0.69 < a < 0.70$ | **B1** | AG<br>Must state either the correct expression and > 0 and < 0 or both answers to 2 sf.<br>Both answers correct and conclusion.<br>Accept equivalent expressions.<br><br>OR: $a = 0.695$ (3 sf) which is between 0.69 & 0.70. |
| | | **1** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 7(a) | $H_0$: Population mean time (or $\mu$) = 32.5<br>$H_1$: Population mean time (or $\mu$) < 32.5 | **B1** | Not just "mean". |
| | $\pm\frac{31.8-32.5}{3.1\div\sqrt{50}}$ | **M1** | Must have $\sqrt{50}$.<br>Could be implied. |
| | = ± –1.597 | **A1** | |
| | '–1.597' < –1.406 [or '1.597' > 1.406] | **M1** | Valid comparison of their $z_{calc}$ with ±1.406.<br>or 0.0551 < 0.08 (or 0.0552 < 0.08). |
| | [ reject $H_0$ ]<br>There is evidence that [population] [mean ] **time** has **decreased** | **A1 FT** | In context, not definite, no contradictions.<br><br>Note: Accept critical value method 31.88 (31.9) **M1 A1** and 31.8 < 31.88 **M1** conclusion **A1**. |
| | | **5** | |
| 7(b) | $\frac{a-32.5}{3.1\div\sqrt{50}}=-1.406$ | **M1** | Standardise with 32.5 and $\sqrt{50}$ and $z$ value on RHS. |
| | $a$ = 31.88 or 31.9 | **A1** | May be seen in part **(a)**. Can score M1A1 here as well using a similar approach to **(a)**. |
| | $\frac{their\ '31.88-31.5}{3.1\div\sqrt{50}}$ [= 0.8668 to 0.8760] | **M1** | Standardise with *their* cv and mean = 31.5.<br>Must have $\sqrt{50}$. |
| | 1 – Φ('0.8668') | **M1** | For area consistent with their working. |
| | = 0.190 to 0.193 (3 sf) | **A1** | |
| | | **5** | |

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# Cambridge International AS & A Level

**MATHEMATICS** **9709/61**

Paper 6 Probability & Statistics 2 **May/June 2022**

MARK SCHEME

Maximum Mark: 50

**Published**

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|---|---|
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| 3 | Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points. |
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| 6 | Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear. |

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## Mark Scheme Notes

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### Types of mark

**M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

**A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

**B** Mark for a correct result or statement independent of method marks.

**DM** or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

**FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

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- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

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**Abbreviations**

| | |
|---|---|
| AEF/OE | Any Equivalent Form (of answer is equally acceptable) / Or Equivalent |
| AG | Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid) |
| CAO | Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed) |
| CWO | Correct Working Only |
| ISW | Ignore Subsequent Working |
| SOI | Seen Or Implied |
| SC | Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance) |
| WWW | Without Wrong Working |
| AWRT | Answer Which Rounds To |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 1(a) | Est($\mu$) = $\frac{2520}{200}$ [= 12.6] | **B1** | OE |
| | Est($\sigma^2$) = $\frac{200}{199}\left(\frac{31582}{200} - \text{'12.6'}^2\right)$ or $\frac{1}{199}\left(31852 - \frac{2520^2}{200}\right)$ | **M1** | Allow M1 if $\frac{200}{199}$ omitted |
| | = 0.5025 or 0.503 or $\frac{100}{199}$ | **A1** | CWO or $\sigma$ = 0.7088 or 0.709 |
| | $z$ = 1.96 | **B1** | |
| | '12.6' $\pm z \times \sqrt{\text{'0.5025'} \div 200}$ | **M1** | For expression of correct form Any $z$ but must be $z$ |
| | CI = 12.5 to 12.7 (3 sf) | **A1** | CWO Must be an interval<br>Note:<br>Use of biased can score maximum B1 M1 A0 B1 M1 A0 |
| | | **6** | |
| 1(b) | 0.95 × 40 [= 38] | **B1** | Give at early stage |
| | | **1** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 2 | $H_0$: P(correct) = $\frac{1}{6}$<br>$H_1$: P(correct) > $\frac{1}{6}$ | **B1** | Allow $p = \frac{1}{6}$<br>Allow $p > \frac{1}{6}$ |
| | $1 - ({}^{15}C_4 \times (\frac{5}{6})^{11} \times (\frac{1}{6})^4 + {}^{15}C_3 \times (\frac{5}{6})^{12} \times (\frac{1}{6})^3 + {}^{15}C_2 \times (\frac{5}{6})^{13} \times (\frac{1}{6})^2 + 15 \times (\frac{5}{6})^{14} \times \frac{1}{6} + (\frac{5}{6})^{15})$ | **M1** | Expression must be seen<br>Allow one end error |
| | 0.0898 or 0.0897 (3 sf) | **A1** | SC if M0 scored allow SCB1 for 0.0898 or 0.0897 |
| | 0.0898 < 0.1 | **M1** | Valid comparison<br>For valid comparison with 0.9 (0.9102 > 0.9 seen the previous M1and A1 can be recovered |
| | [Reject $H_0$] There is evidence (at the 10% level) that Arvind can predict scores | **FTA1** | Not definite, e.g. not 'He can predict' or 'Claim true'<br>In context and no contradictions |
| | | **5** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 3 | $D = X – (Y_1 + Y_2 + Y_3)$ OE<br>$E(D) = 6.2 - 2.4 \times 3$ [= –1] OE | **B1** | Give at early stage |
| | $Var(D) = 0.36 + 3 \times 0.25$ [= 1.11] | **B1** | Give at early stage |
| | $\frac{0-(-1)}{\sqrt{\text{'1.11'}}}$ [= 0.949] | **M1** | No standard deviation/variance mixes<br>Var must come from a combination attempt |
| | $1 – \Phi(\text{'0.949'})$ | **M1** | Area consistent with *their* values |
| | = 0.171 (3 s.f.) | **A1** | |
| | | **5** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4(a) | $E(Y) = \frac{20}{4}[=5]$, $Var(Y) = 20 \times \frac{1}{4} \times \frac{3}{4}\left[=\frac{15}{4}\right]$ | **B1** | Both<br>OE, SOI |
| | $Var(X) = 2$ | **B1** | SOI or standard deviation = $\sqrt{2}$ |
| | $E(X – 3Y) = –13$ | **B1** | |
| | $Var(X – 3Y) = 2 + 9 \times \text{'}\frac{15}{4}\text{'}$ [= 35.75] | **M1** | Correct formula using *their* values |
| | Standard deviation of $(X – 3Y) = 5.98$ (3 s.f.) or $\frac{1}{2}\sqrt{143}$ | **A1** | CWO |
| | | **5** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4(b) | (0, 0) and (1, 15) | **M1** | |
| | $e^{-2} \times (\frac{3}{4})^{20} + e^{-2} \times 2 \times {}^{20}C_{15} (\frac{3}{4})^5 (\frac{1}{4})^{15}$ | **M1** | |
| | 0.000430 (3 sf) | **A1** | CWO (must have evidence of addition)<br>Allow 0.00043 or $4.3(0) \times 10^{-4}$ |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5(a) | $\lambda = 4.5$ | **B1** | |
| | $1 - e^{-4.5} (1 + 4.5 + \frac{4.5^2}{2!} + \frac{4.5^3}{3!} + \frac{4.5^4}{4!})$ | **M1** | Allow one end error Allow any λ.<br>Poisson expressions must be seen |
| | = 0.468 (3 sf) | **A1** | If M0 awarded allow SC B1 for 0.468 |
| | | **3** | |
| 5(b) | $\lambda = 162$<br>($X \sim$ Po(162) $\Rightarrow X \sim$ N(162, 162)) | **B1** | |
| | $\frac{149.5 - \text{'162'}}{\sqrt{\text{'162'}}}$ and $\frac{160.5 - \text{'162'}}{\sqrt{\text{'162'}}}$<br>(= –0.982 and –0.118) | **M1** | One of these; allow with incorrect or no continuity correction |
| | Φ('0.982') – ϕ('0.118') oe | **M1** | Area consistent with *their* values (both standardisations must be seen) |
| | = 0.290 (3 sf) | **A1** | Allow 0.29 |
| | | **4** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5(c) | $\lambda = \frac{13.5}{6} + 3.6 \times \frac{2}{3}$ OE or 4.65 | **M1** | Attempt to find λ |
| | $e^{-4.65}(\frac{4.65^4}{4!} + \frac{4.65^5}{5!} + \frac{4.65^6}{6!})$ | **M1** | Allow any λ Allow one end error<br>Poisson terms not be seen |
| | 0.494 (3 sf) | **A1** | If M0 allow SC B1 for 0.494 |
| | | **3** | |
| 6(a) | $\frac{a}{2}$ | **B1** | |
| | | **1** | |
| 6(b) | $\frac{1}{4}$ | **B1** | |
| | | **1** | |
| 6(c) | $f(x) = \frac{1}{a}$ | **B1** | SOI (may be seen in part **(a)** or part **(b)**) |
| | $E(X) = \frac{a}{2}$ | **B1** | SOI |
| | $\int_0^a \frac{1}{a} x^2 dx$ | **M1** | Attempt integrate *their* $f(x) \times x^2$ with correct limits |
| | $= \left[\frac{x^3}{3a}\right]_0^a = \frac{a^2}{3}$ | **A1** | |
| | $\frac{a^2}{3} - (\frac{a}{2})^2$ or $\frac{a^2}{3} - \frac{a^2}{4}$ [$= \frac{a^2}{12}$ **AG** ] | **A1** | Must see previous line and answer<br>No errors seen |
| | | **5** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(d) | $P(X < \frac{b}{3}) = \frac{p}{3}$ | **M1** | SOI (could be on a diagram)<br>OR by integration: prob = 1-(2/3)(b/a) |
| | $P(\frac{b}{3} < X < a - .\frac{b}{3}) = 1 - \frac{2p}{3}$ | **A1** | |
| | | **2** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 7(a) | $H_0$: pop mean run time = 28.2 mins<br>$H_1$: pop mean run time < 28.2 mins | **B1** | Allow '$\mu$'. Not 'mean journey time' |
| | | **1** | |
| 7(b) | $\frac{27-28.2}{4/\sqrt{40}}$ [= –1.897] | **M1** | For standardising<br>Must have √40 |
| | Φ(< '–1.897') = 1 – Φ('1.897') | **M1** | For correct area consistent with these values |
| | 0.0289 (3 sf) | **A1** | |
| | | **3** | |
| 7(c) | $H_0$ is not rejected so… | **M1** | |
| | Type II error can be made and Type I error cannot be made | **A1** | Both needed (accept 'only a Type II error could be made') |
| | | **2** | |

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Cambridge Assessment International Education

# Cambridge International AS & A Level

**MATHEMATICS** **9709/62**

Paper 6 Probability & Statistics 2 **May/June 2022**

MARK SCHEME

Maximum Mark: 50

**Published**

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2022 series for most Cambridge IGCSE, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

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## Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

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| Mathematics Specific Marking Principles | |
|---|---|
| 1 | Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing. |
| 2 | Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected. |
| 3 | Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points. |
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| WWW | Without Wrong Working |
| AWRT | Answer Which Rounds To |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 1(a) | $72.3 \pm z\sqrt{\frac{64.3}{50}}$ | **M1** | Expression of correct form (allow only one side for M1). Must be a $z$ value |
| | $z = 1.751$ | **B1** | Accept 1.75 if nothing better seen |
| | CI is 70.3 to 74.3 metres (3 s.f.) | **A1** | Allow without units Must be an interval |
| | | **3** | |
| 1(b) | Not random sample | **B1** | Need 'random' or 'not representative/biased because…' OE |
| | | **1** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 2 | $H_0$: Pop mean height = 2.3<br>$H_1$: Pop mean height > 2.3 | **B1** | Not just 'mean'<br>Allow $\mu$ |
| | $\frac{2.4-2.3}{\frac{0.4}{\sqrt{60}}}$ | **M1** | For standardising, must have $\sqrt{60}$ |
| | 1.936 or 1.937 or 1.94 | **A1** | |
| | '1.936' < 1.96 | **M1** | Valid comparison with 1.96<br>Or 2.64% > 2.5% OE<br>Accept 1.936 < 2.24 or 2.64% > 1.25% OE if $H_1 \mu \neq 2.3$ |
| | [Do not reject $H_0$]<br>No evidence that (mean) height (with fertiliser) is more than without | **A1 FT** | FT *their z*<br>In context, not definite. E.g. not 'Mean height is not greater' with no contradictions<br>No FT for 2 tail test (max B0 M1 A1 M1 A0 3/5)<br>Accept critical values method 2.401 (M1 A1) 2.4 < 2.401 (M1) Condone 2.299 (M1 A1) < 2.3 (M1)<br>A1 conclusion |
| | | **5** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 3(a) | Poisson | **B1** | SOI |
| | Mean = 3.6 | **B1** | Can be awarded for N(3.6, …) |
| | $e^{-3.6}(1 + 3.6 + \frac{3.6^2}{2})$ | **M1** | Allow any $\lambda$<br>Allow one end error<br>Expression must be seen |
| | 0.303 (3 s.f.) | **A1** | If M0 awarded allow SC B1 for 0.303<br>SC Use of binomial: B1 for answer 0.300 (3 sf) |
| | | **4** | |
| 3(b) | [Binomial with] 200 > 50 | **B1** | |
| | [200 × 0.018 =] 3.6 < 5 or [p =] 0.018 < 0.1 | **B1** | If B0 B0 then SC *n* large, *p* small: B1 or *n* large *np* < 5: B1 or *n* > 50 and either *np* < 5 or *p* < 0.1: B1 |
| | | **2** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4(a) | $H_0$: Pop mean = 4.6 [or 9.2]<br>$H_1$: Pop mean < 4.6 [or 9.2] | **B1** | or $\lambda = 4.6$ or $\mu$ (Not just 'mean')<br>or $\lambda < 4.6$ |
| | | **1** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4(b) | Use of Poisson with $\lambda = 9.2$ | **B1** | SOI |
| | $P(X \leqslant 3) = e^{-9.2}(1 + 9.2 + \frac{9.2^2}{2} + \frac{9.2^3}{3!}) = 0.0184$ or 0.018 [< 0.02]<br>$P(X \leqslant 4) = 0.0184 + e^{-9.2} \times \frac{9.2^4}{4!} = 0.0486$ or 0.049 [> 0.02] | **M1** | At least one of these attempted correct $\lambda$<br>(with Poisson expression seen not implied) |
| | | ***A1** | Both correct<br>SC Use of $\lambda = 4.6$ scores B1 for P(X = 0) = 0.01[0][1] and $P(X \leqslant 1) = 0.056$[3]only |
| | CR is $X \leqslant 3$ | **DA1** | From CWO and at least one comparison seen<br>SC If M0 awarded allow *B1 for both 0.018 and 0.049 or better and DB1 for correct critical region from CWO and at least one comparison seen. |
| | | **4** | |
| 4(c) | 5 is not in critical region OR $P(X \leqslant 5) = 0.104 > 0.02$<br>so [not reject $H_0$] no evidence that number of cars arriving is now fewer | **M1**<br>**A1 FT** | For a comparison (i.e. 5 > 3) OE<br>In context, not definite No contradictions<br>e.g. not 'No. of cars arriving is not fewer'<br>ft *their* critical region if used (but must be from Poisson and integers) |
| | | **2** | |
| 4(d) | No, because $H_0$ was not rejected | **B1 FT** | OE, FT *their* **(c)** |
| | | **1** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4(e) | N(276, 276) | **B1** | SOI |
| | $\frac{300.5-276}{\sqrt{276}}$ [= 1.475] | **M1** | Standardising with *their* values<br>Allow with wrong or no continuity correction |
| | 1 – ϕ('1.475') = 0.0701 (3 s.f.) | **A1** | SC Use of Poisson: B1 for answer 0.0727 (3 sf) |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5(a) | $\frac{3}{16}\int_2^4(4x^2-x^3)\mathrm{d}x$ | **M1** | Attempt to integrate $xf(x)$ Ignore limits<br>(must see a power increase for attempted integration) |
| | $=\frac{3}{16}\left[\frac{4x^3}{3}-\frac{x^4}{4}\right]_2^4$ | **M1** | Attempt integrate $xf(x)$ with correct limits<br>(must see a power increase for attempted integration) |
| | $=\frac{3}{16}\left(\frac{256}{3}-64-(\frac{32}{3}-4)\right)=\frac{11}{4}$ **(AG)** | **A1** | Correct substitution of correct limits (at least 2 terms seen) and answer seen. No errors seen i.e. NO recovery of errors and no non-exact decimals (e.g. 21.33) seen |
| | | **3** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5(b) | $\frac{3}{16}\int_2^4(4x^3-x^4)\mathrm{d}x$ | ***M1** | Attempt to integrate $x^2$ f($x$) with correct limits (integration must be seen not implied. Must see a power increase for attempted integration) |
| | $=\frac{3}{16}\left[x^4-\frac{x^5}{5}\right]_2^4$ [$=\frac{39}{5}$ or 7.8]<br>Var($X$) = '$\frac{39}{5}$' $-(\frac{11}{4})^2$ | **DM1** | *their* $\int x^2\mathrm{f}(x)\mathrm{d}x-(\frac{11}{4})^2$, with $\int x^2\mathrm{f}(x)\mathrm{d}x$ evaluated, not necessarily simplified |
| | $=\frac{19}{80}$ or 0.2375 (or 0.238 (3 sf)) | **A1** | SC If M0 then score B1 for $\frac{39}{5}$ and B1 for $\frac{19}{80}$ |
| | | **3** | |
| 5(c) | $\frac{3}{16}\int_2^3(4x-x^2)\mathrm{d}x$ | **M1** | Attempt to integrate correct integral and limits<br>must see a power increase for attempted integration<br>Oe (Integrate 3 to 4)<br>OR ALTERNATIVE METHOD integrate from m to 4 or 2 to m and equate to 0.5 to obtain cubic ($m^3$ -$6m^2$ + 24 = 0 oe)<br>(NB Integrating from m to 3 and equating to 0.5M0) |
| | $=\frac{3}{16}\left[2x^2-\frac{x^3}{3}\right]_2^3$ $\left[=\frac{3}{16}\left(18-9-\left(8-\frac{8}{3}\right)\right)\right]\left[=\frac{11}{16}\right]$<br>'$\frac{11}{16}$' $-\frac{1}{2}$ | **M1** | *Their* $\int\mathrm{f}(x)\mathrm{d}x-\frac{1}{2}$ oe (1/2 – 5/16)<br>OR<br>ALTERNATIVE METHOD $m$ obtained from cubic ($m$ = 2.69459) and attempt to integrate f(x) from '*their m*' ($2<m<4$) to 3 must see a power increase for attempted integration and limits substituted |
| | $\frac{3}{16}$ or 0.1875 | **A1** | Condone 0.187 or 0.188 |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(a) | $E(D) = 53 - (4 \times 14) = -3$ | **B1** | OE Give at early stage |
| | $Var(D) = 11 + 4^2 \times 3$ [= 59] | **B1** | or $\sqrt{(11+4^2\times3)}$ (= 7.68 (3 s.f.))<br>Give at early stage |
| | $\frac{0-(-3)}{\sqrt{59}}$ [= 0.391] | **M1** | For standardising with *their* values (var must be from a combination attempt)<br>Ignore continuity correction attempts |
| | 1 – Φ('0.391') | **M1** | For area consistent with *their* values |
| | 0.348 (3 s.f.) | **A1** | As final answer |
| | | **5** | |
| 6(b) | $E(T) = 12 \times 53 + 25 \times 14$ [= 986] | **B1** | Give at early stage (N.B. accept $E(T - 1000) = -14$) |
| | $Var(T) = 12 \times 11 + 25 \times 3$ [= 207] | **B1** | Or $\sqrt{(12\times11+25\times3)}$ (= 14.4 (3sf))<br>Give at early stage |
| | $\frac{1000-986}{\sqrt{207}}$ [= 0.973] | **M1** | For standardising with *their* values (var must be from a combination attempt)<br>Ignore continuity correction attempts |
| | Φ('0.973') | **M1** | For area consistent with *their* values |
| | 0.835 (3 sf) | **A1** | As final answer |
| | | **5** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 7 | $\bar{X} \sim N(2.9, \frac{2.9}{100})$ OR Totals method N(290, 290) | **B1** | B1 for N(2.9, …) OR N(290, …) |
| | | **B1** | B1 for Var = $\frac{2.9}{100}$ OR for var = 290 SOI |
| | $\frac{2.88-2.90}{\sqrt{\frac{2.9}{100}}}$ [= –0.1174] OR $\frac{288-290}{\sqrt{290}}$ | **M1** | Standardising with *their* values Allow without –ve sign AND/OR with incorrect continuity correction<br>No mixed methods |
| | 1– Φ('0.1174') | **M1** | For area consistent with *their* values |
| | 0.453 (3 sf) | **A1** | As final answer |
| | **Alternative method for question 7** | | |
| | $\bar{X} \sim N(2.9, \frac{2.9}{100})$ OR Totals method N(290, 290) | **B1** | B1 for N(2.9, …) OR N(290, …) |
| | | **B1** | B1 for Var = $\frac{2.9}{100}$ OR Var = 290 stated or implied |
| | $\frac{(2.88-\frac{1}{200})-2.90}{\sqrt{\frac{2.9}{100}}}$ [= –0.1468] OR (287.5 – 290)/√290 | **M1** | Standardising with *their* values Allow without –ve sign AND/OR with incorrect continuity correction<br>No mixed methods |
| | 1 – ϕ('0.1468') | **M1** | For area consistent with *their* values |
| | 0.442 (3 sf) | **A1** | As final answer |
| | | **5** | |

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Cambridge Assessment International Education

# Cambridge International A Level

**MATHEMATICS** **9709/63**

Paper 6 Probability & Statistics 2 **May/June 2022**

MARK SCHEME

Maximum Mark: 50

**Published**

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2022 series for most Cambridge IGCSE, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

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**PUBLISHED**

## Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

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| Mathematics Specific Marking Principles | |
|---|---|
| 1 | Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing. |
| 2 | Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected. |
| 3 | Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points. |
| 4 | Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw). |
| 5 | Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread. |
| 6 | Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear. |

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## Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

### Types of mark

**M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

**A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

**B** Mark for a correct result or statement independent of method marks.

**DM** or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

**FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

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**Abbreviations**

| | |
|---|---|
| AEF/OE | Any Equivalent Form (of answer is equally acceptable) / Or Equivalent |
| AG | Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid) |
| CAO | Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed) |
| CWO | Correct Working Only |
| ISW | Ignore Subsequent Working |
| SOI | Seen Or Implied |
| SC | Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance) |
| WWW | Without Wrong Working |
| AWRT | Answer Which Rounds To |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 1 | $\frac{1300+\frac{1}{200}-1250}{\frac{480}{10}}$ or $\frac{1300-1250}{\frac{480}{10}}$ [= 1.042] | **M1** | Allow with incorrect or omitted continuity correction<br>Must have 10<br>Accept totals method |
| | 1 – Φ('1.042') | **M1** | For area consistent with *their* values |
| | 0.149 ( 3 s.f.) | **A1** | |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 2(a) | Conclude more than 10% of the students are left handed when this is not true | **B1** | OE. Must be in context (accept use of $p$).<br>Need the context of one tail test. |
| | | **1** | |
| 2(b) | $1-(0.9^{20}+20\times0.9^{19}\times0.1+{}^{20}C_2\times0.9^{18}\times0.1^2+{}^{20}C_3\times0.9^{17}\times0.1^3+{}^{20}C_4\times0.9^{16}\times0.1^4)$ | **M2** | M2: fully correct<br>M1: attempt 1 – P($X$ = 0, 1, 2, 3, 4);<br>allow 1 – P($X$ = 0,1,2,3,4,5) or 1 – P(X = 0,1,2,3)<br>need 1 – …<br>the method mark cannot be implied |
| | 0.0432 (3 s.f.) | **A1** | If M0 awarded allow SC B2 for 0.0432 |
| | | **3** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 2(c) | $0.7^{20} + 20 \times 0.7^{19} \times 0.3 + {}^{20}C_2 \times 0.7^{18} \times 0.3^2 + {}^{20}C_3 \times 0.7^{17} \times 0.3^3 + {}^{20}C_4 \times 0.7^{16} \times 0.3^4$ | **M1** | Attempt to find $P(\leqslant 4)$ using B(20,0.3)<br>Allow one end error<br>The method mark cannot be implied |
| | 0.238 or 0.237 (3 s.f.) | **A1** | If M0 awarded allow SC B1for 0.238 or 0.237 |
| | | **2** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 3(a) | Batteries unusable after testing or Population too big<br>or too costly or too time consuming to use the whole population oe | **B1** | |
| | | **1** | |
| 3(b) | $H_0$: $\mu = 150$<br>$H_1$: $\mu < 150$ | **B1** | Or population mean = 150; not just 'mean' = 150 |
| | $\frac{147 - 150}{\sqrt{225} \div \sqrt{120}}$ | **M1** | Allow with continuity correction<br>Need $\sqrt{120}$ |
| | –2.191 | **A1** | Condone – 2.19 |
| | –2.191 < –2.054 [or –2.055] | **M1** | OE. For valid comparison with 2.054 or 2.055<br>Or 0.0143 (or 0.0142) < 0.02<br>For two tail test allow comp –2.326 OE if $H_1$: $\mu \neq 150$<br>(can score B0M1A1M1A0 max 3/5 ) |
| | [Reject Ho] There is evidence that the (mean) life of type *B* is less than type *A* (or less than 150) | **A1 FT** | In context, not definite with no contradictions<br>Accept critical value method 147.19 M1A1<br>147 < 147.19 M1 conclusion A1<br>Or 150 > 149.81 |
| | | **5** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 3(c) | $147 \pm z \times \frac{15}{\sqrt{120}}$ | **M1** | Expression of correct form must be a $z$ value |
| | $z = 1.881$ [or 1.882] | **B1** | |
| | 144 to 150 (3 s.f.) | **A1** | Must be an interval<br>Incorrect $z$ value can only score M1B0A0 |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4(a) | $T$~N(515, 74) | **B1** | B1 for N(515, ..) give at early stage |
| | | **B1** | B1 for Var = 45 + 25 + 4 = 74 give at early stage |
| | $\frac{500 - '515'}{\sqrt{'74'}}$ [= –1.744] | **M1** | Standardise with *their* values. No standard deviation/variance mix<br>Need combination for variance. Allow continuity correction. |
| | Φ(’1.744’) | **M1** | Area consistent with *their* working |
| | 0.959 or 0.96[0] (3 s.f.) | **A1** | |
| | | **5** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4(b) | $E(S - 1.4R) = 300\text{-}1.4\text{x}200 = 20$ | **B1** | Give at early stage |
| | $\text{Var}(S - 1.4R) = 45 + 1.4^2 \times 25 = 94$ | **B1** | Give at early stage<br>SC: if B0B0 awarded allow SC B1 for 14 and 105.84 |
| | $\frac{0-(20)}{\sqrt{'94'}}$ [= –2.063] | **M1** | Standardise with *their* values. No standard deviation/variance mix. Need combination for variance. |
| | 1 – Φ('2.063') | **M1** | Area consistent with *their* working |
| | 0.0196 (3 s.f.) | **A1** | |
| | | **5** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5(a) | $\lambda = 6.6$ | **B1** | |
| | $e^{-6.6} \times \frac{6.6^6}{6!}$ | **M1** | Any $\lambda$ |
| | 0.156 (3 s.f.) | **A1** | If M0 awarded SC B1 for 0.156 |
| | | **3** | |
| 5(b) | $1 - e^{-2.2}(1 + 2.2 + \frac{2.2^2}{2} + \frac{2.2^3}{3!} + \frac{2.2^4}{4!})$ | **M1** | Allow one end error. Need 1 – …<br>Any $\lambda$ |
| | 0.0725 (3 s.f.) | **A1** | If M0 awarded SC B1 for 0.0725 |
| | | **2** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5(c) | N(26.4, 26.4) | **B1** | Give at early stage $2.2 \times 12$ |
| | $\frac{19.5 - '26.4'}{\sqrt{'26.4'}}$ [= –1.343] | **M1** | Standardising with *their* values. Allow wrong or no continuity correction |
| | $\phi('-1.343') = 1 - \phi('1.343')$ | **M1** | Area consistent with *their* working |
| | 0.0897 or 0.0896 (3 s.f.) | **A1** | |
| | | **4** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(a) | $\frac{13+a}{5}$ | **B1** | Accept $\frac{2+3+3+5+a}{5}$.<br>Do not ignore subsequent working |
| | | **1** | |
| 6(b) | $\frac{5}{4}\left(\frac{47+a^2}{5} - \left(\frac{13+a}{5}\right)^2\right) = 4$ or $\frac{1}{4}\left(47 + a^2 - \frac{(13+a)^2}{5}\right) = 4$ | **M1** | Use of correct formula using *their* value from **(a)**, in terms of $a$, and equate to 4 |
| | $2a^2 - 13a - 7 = 0$ | **A1** | Any correct three-term quadratic equation rearranged to a form ready to solve |
| | $a = 7$ | **A1** | Condone the other value of $a$ $(-\frac{1}{2})$ |
| | | **3** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 7(a)(i) | 1 | **B1** | no ambiguity |
| | | **1** | |
| 7(a)(ii) | $\frac{1}{2}$ | **B1** | No ambiguity |
| | | **1** | |
| 7(a)(iii) | $[q =]\ \frac{1}{2}p$ | **B1** | Accept $2q = p$ |
| | | **1** | |
| 7(b) | $p\int_0^a (a^2 - x^2)\mathrm{d}x = 1$ | **M1** | Attempt to integrate f($x$) and equated to 1 |
| | $\frac{2}{3}a^3 p = 1$ | **A1** | OE, simplified |
| | $"\frac{3}{2a^3}"\int_0^a (a^2x - x^3)\mathrm{d}x = 3$ or $"\frac{3}{2a^3}"\int_0^a (a^2x - x^3)\mathrm{d}x = 3$ | **M1** | Attempt to integrate $x$f($x$), with multiplier $p$ or $\frac{3}{2a^3}$ or *their* $p$, and equate to 3 |
| | $p \times \frac{a^4}{4} = 3$ | **A1** | May be implied by next line |
| | $"\frac{3}{2a^3}" \times \frac{a^4}{4} = 3$ | **M1** | OE. Substitute from one equation into the other. FT *their* equations |
| | $a = 8$ | **A1** | |
| | | **6** | |

Cambridge Assessment International Education

# Cambridge International A Level

**MATHEMATICS** **9709/62**

Paper 6 Probability and Statistics 2 **February/March 2022**

MARK SCHEME

Maximum Mark: 50

**Published**

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| Mathematics Specific Marking Principles | |
|---|---|
| 1 | Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing. |
| 2 | Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected. |
| 3 | Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points. |
| 4 | Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw). |
| 5 | Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread. |
| 6 | Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear. |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 1(a) | Est $(\mu) = \frac{1199}{6}$ or 199.833 or 200 or $\frac{2398}{12}$ [mm] | **B1** | Accept in any form |
| | Est $(\sigma^2) = \frac{12}{11}\left(\frac{479226}{12} - \frac{'1199'^2}{6}\right)$ or $\frac{1}{11}\left('479226' - \frac{'2398'^2}{6}\right)$ | **M1** | Use of their values in correct formula (may be implied) |
| | = 2.33 (3 sf) [$mm^2$] | **A1** | Accept $\frac{7}{3}$ |
| | | **3** | |
| 1(b) | Small sample | **B1** | Accept not 'not representative' unless qualified. |
| | | **1** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 2 | B(300, $\frac{1}{5}$) → N(60,48) | **B1** | SOI |
| | $\frac{45.5-60}{\sqrt{48}}$ | **M1** | Condone with wrong or no continuity correction |
| | = –2.093 | **A1** | |
| | '2.093' > 1.96 | **M1** | Valid comparison<br>Note: ϕ('–2.093') (= 0.0182),<br>0.0182< 0.025 |
| | [Evidence to reject $H_0$] There is evidence that P(landing on blue) ≠ $\frac{1}{5}$ | **A1 FT** | Allow 'There is evidence that the spinner is biased.'<br>In context, not definite, no contradictions<br>Condone critical values method (critical value 46.42 M1 A1 and 45.5 < '46.42' M1 for valid comparison A1 for correct conclusion) |
| | | | **SC:** 0.0182 unsupported:<br>0.0182 < 0.025<br>And there is evidence that the spinner is biased.<br>In context, not definite B1 only |
| | | **5** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 3 | est($p$) = 0.4 | **B1** | |
| | $'0.4' + z\sqrt{\frac{'0.4' \times (1 - '0.4')}{500}} [= 0.445]$ | **M1** | OE<br>Use of *their* 0.4 in a correct expression |
| | $z \left[ = 0.045 \div \sqrt{\frac{'0.4' \times (1 - '0.4')}{500}} \right] = 2.054$ | **A1** | Condone 2.053 and 2.05 |
| | 0.98 – (1 – 0.98) | **M1** | |
| | 96% confidence | **A1** | CWO, must be integer |
| | | **5** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4(a) | $H_0$: $\mu = 25.5$<br>$H_1$: $\mu < 25.5$ | **B1** | |
| | $\frac{23.7 - 25.5}{5.2 \div \sqrt{40}}$ | **M1** | Must have $\sqrt{40}$ |
| | $= -2.189$ | **A1** | |
| | '2.189' < 2.326 | **M1** | For valid comparison<br><br>For two-tailed test: allow compare 2.576 if $H_1$: $\mu \neq 25.5$ |
| | [Accept $H_0$] No evidence that mean time has decreased | **A1 FT** | In context, not definite, no contradictions<br>FT *their* 2.189 but no FT for two-tailed test<br>**N.B.** Use of two-tailed test can score max B0 M1 A1 M1 A0<br>Condone use of critical value method<br>(23.59 M1 A1 and 23.7 > 23.59 M1 A1 correct conclusion or 25.612 M1 A1 and 25.5 < 25.612 M1 A1 with correct conclusion) |
| | | **5** | |
| 4(b) | No, because $H_0$ was not rejected | **B1 FT** | FT *their* conclusion in **(a)** |
| | | **1** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5(a) | Mean = $5\times18.3$ and Variance = $5\times2.5^2$ [= N(91.5, 31.25)] | **B1** | SOI |
| | $\frac{95-'91.5}{\sqrt{'31.25'}}$ [= 0.626] | **M1** | FT *their* mean and variance |
| | $1-\Phi(\text{'0.626'})$ | **M1** | For finding area consistent with *their* values |
| | 0.266 (3 sf) | **A1** | |
| | | **4** | |
| 5(b) | $E(D) = 0$ | **B1** | Or $E(D-1) = -1$ |
| | $\text{Var}(D) = 2.5^2 \times 2$ [= 12.5] | **B1** | |
| | $\frac{1-0}{\sqrt{'12.5'}}$ [= 0.283] or $\frac{-1-0}{\sqrt{'12.5'}}$ [= –0.283] | **M1** | FT *their* E and Var |
| | $\Phi(\text{'0.283'}) - (1-\phi(0.283))$ [= 0.6115 – 0.3885] | **M1** | For finding area consistent with *their* values |
| | 0.223 (3 sf) | **A1** | |
| | | **5** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(a) | Quadratic curve, hence symmetrical | **B1** | OE. Allow sketch and 'symmetrical' or just 'curve symmetrical' |
| | | **1** | |
| 6(b) | $-k\int_1^3 (x^2-4x+3)\mathrm{d}x = 1$ | **M1** | Attempt to integrate f($x$) and '= 1'. Ignore limits at this stage |
| | $-k\left[\frac{x^3}{3}-2x^2+3x\right]_1^3$ | **A1** | Fully correct expression (correct integration and limits) |
| | $-k\times\left[0-\frac{4}{3}\right]=1 \quad \text{or} \quad k\times\frac{4}{3}=1$<br>$\left[k=\frac{3}{4}\right]$ | **A1** | AG, OE. Correctly substitute limits and '= 1' and correctly obtain result with no errors seen. |
| | | **3** | |
| 6(c) | $-\frac{3}{4}\int_1^3 \left(x^4-4x^3+3x^2\right)\mathrm{d}x$ | **M1** | Attempt to integrate $x^2$ f($x$) from 1 to 3 |
| | $-\frac{3}{4}\times\left[\frac{x^5}{5}-x^4+x^3\right]_1^3$<br>$\left[=\frac{3}{4}\times\frac{28}{5}=\frac{21}{5}\right]$ | **A1** | Correct integration and limits |
| | $\left[\frac{21}{5}-2^2\right]=0.2$ | **A1** | |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(d) | $-\frac{3}{4}\int_{2.5}^{3}\left(x^2-4x+3\right)\mathrm{d}x$ | **M1** | OE. Attempt to integrate f($x$), from 2.5 to 3 (or 1 to 2.5) |
| | $=-\frac{3}{4}\times\left[\frac{x^3}{3}-2x^2+3x\right]_{2.5}^{3}=\frac{5}{32}$ or 0.15625 | **A1** | |
| | $1-\left(1-'\frac{5}{32}'\right)^3$ | **M1** | OE. FT *their* $\frac{5}{32}$ . |
| | = 0.399 (3 sf) | **A1** | |
| | | **4** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 7(a)(i) | $0.024 \times 50$ [= 1.2] and $0.018 \times 60$ [= 1.08] | **B1** | |
| | $(1 - e^{-1.2}(1 + 1.2)) \times (1 - e^{-1.08}(1 + 1.08))$ | **M1** | For $(1 - e^{-\lambda}(1+\lambda)) \times (1 - e^{-\mu}(1+\mu))$ any $\lambda$, $\mu$ ($\lambda \neq \mu$)<br>Allow one end error on either or both terms |
| | = 0.0991 (3 sf) | **A1** | Unsupported answer scores maximum **SC** B1 B1<br>**SC** Use of binomial 0.0994 scores B1 only |
| | | **3** | |
| 7(a)(ii) | $\lambda = 0.024 \times 50 + 0.018 \times 60$ | **M1** | or *their* 1.2 + 1.08 (NB 0.024+0.018 is M0) |
| | $1 - e^{-2.28} \times \left(1 + 2.28 + \frac{2.28^2}{2!} + \frac{2.28^3}{3!}\right)$ | **M1** | any $\lambda$ and allow one end error |
| | = 0.197 (3 sf) | **A1** | Unsupported answer scores maximum **SC** B2 |
| | | **3** | |
| 7(b) | $e^{-\lambda} = \left[e^{-\mu}\right]^2 = e^{-2\mu}$ | **M1** | |
| | $e^{-\lambda} \times \frac{\lambda^2}{2} = k\left[e^{-\mu} \times \mu\right]^2$ | **M1** | |
| | $e^{-2\mu} \times 2\mu^2 = k \times e^{-2\mu} \times \mu^2$ | **M1** | OE. Use of $\lambda = 2\mu$ to find equation in $\mu$ and $k$ only (or $\lambda$ and $k$ only) |
| | $k = 2$ | **A1** | |
| | | **4** | |

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# Cambridge International AS & A Level

**MATHEMATICS** **9709/61**

Paper 6 Probability & Statistics 2 **October/November 2021**

MARK SCHEME

Maximum Mark: 50

**Published**

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

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**PUBLISHED**

## Generic Marking Principles

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- the specific content of the mark scheme or the generic level descriptors for the question
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- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
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GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

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| Mathematics Specific Marking Principles | |
|---|---|
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## Mark Scheme Notes

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### Types of mark

**M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

**A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

**B** Mark for a correct result or statement independent of method marks.

**DM** or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

**FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

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**Abbreviations**

| | |
|---|---|
| AEF/OE | Any Equivalent Form (of answer is equally acceptable) / Or Equivalent |
| AG | Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid) |
| CAO | Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed) |
| CWO | Correct Working Only |
| ISW | Ignore Subsequent Working |
| SOI | Seen Or Implied |
| SC | Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance) |
| WWW | Without Wrong Working |
| AWRT | Answer Which Rounds To |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 1(a) | N(12.5, ….) | **B1** | |
| | Variance = 0.4096 | **B1** | Accept 0.410 (3sf), condone $\frac{10.24}{25}$ |
| | | **2** | |
| 1(b) | $\frac{13-'12.5'}{\sqrt{'0.4096'}}$ [= 0.781] | **M1** | For standardising with *their* values. Accept standardising with 12. |
| | ϕ(‘0.781’) – (1 – Φ(‘0.781’)) | **M1** | For attempting to find *their* central area. |
| | 0.565 (3sf) | **A1** | |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 2 | N(45.2, 45.2) | **B1** | SOI |
| | $\frac{60.5-45.2}{\sqrt{45.2}}$ [= 2.276] | **M1** | Allow with wrong or no continuity correction. |
| | 1 –ϕ(‘2.276’) | **M1** | |
| | 0.0114 | **A1** | |
| | | **4** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 3 | est(p) = 0.2 accept $\frac{15}{75}$ | **B1** | SOI |
| | $2 \times z \times \sqrt{\frac{0.2\times0.8}{75}} = 0.162$ | **M1** | Expression of the correct form.<br>Condone missing $2x$. |
| | $z \left[= 0.081\times\sqrt{\frac{75}{0.2\times0.8}}\right] = 1.754$ | **A1** | Correct $z$. Condone 3sf accuracy. |
| | $\Phi$('1.754') = 0.96[03]<br>'0.96' – (1 – '0.96') | **M1** | OE. Using *their z* to find alpha. |
| | $\alpha = 92$ | **A1** | Following correct working. |
| | | **5** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4(a) | $\frac{1}{18}\int_0^{1.2}(9-x^2)\mathrm{d}x$ | **M1** | Attempt to integrate f($x$), ignore limits.<br>Must see an increase of power. |
| | $\frac{1}{18}\left[9x-\frac{x^3}{3}\right]_0^{1.2}$ | **A1** | Correct integration and correct limits. |
| | $\frac{71}{125}$ or 0.568 | **A1** | SC unsupported answer scores **B2** only. |
| | | **3** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4(b) | $\frac{1}{18}\int_0^3 (9x - x^3)\mathrm{d}x$ | **M1** | Attempt to integrate $x$f($x$), ignore limits.<br>Must see an increase of power. |
| | $\frac{1}{18}\left[\frac{9x^2}{2} - \frac{x^4}{4}\right]_0^3$ | **A1** | Correct integration and correct limits. |
| | $\frac{9}{8}$ or 1.125 | **A1** | SC unsupported answer scores **B2** only. |
| | | **3** | |
| 4(c) | $\frac{1}{18}\left[9x - \frac{x^3}{3}\right]_0^m = 0.5$ | **M1** | Attempt to integrate f(x) with correct limits and = 0.5.<br>OE. Accept limits $m$ to 3.<br>Allow $x$ instead of $m$. |
| | $\frac{1}{18}\left[9m - \frac{m^3}{3}\right] - 0.5 = 0$ | **A1** | Any correct cubic equation in $m$ or $x$. |
| | $m^3 - 27m + 27 = 0$ | **A1** | AG. Correctly obtain this equation. No errors seen. |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5(a)(i) | Po(0.025) | **B1** | For Poisson and correct parameter. |
| | $n = 2500 > 50$, $np = 0.025 < 5$ | **B1** | Must show 2500 and 0.025.<br>Accept p = $\frac{1}{100000} < 0.1$ in place of $np = 0.025 < 5$. |
| | | **2** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5(a)(ii) | $1 - e^{-0.025}$ | **M1** | Allow any $\lambda$.<br>FT *their* **(a)(i)** if normal; must have continuity correction. |
| | 0.0247 (3sf) | **A1** | Must be from Poisson.<br>Unsupported correct answer scores **B1** instead of **M1 A1**. |
| | | **2** | |
| 5(b) | $H_0$: $p = 0.3$<br>$H_1$: $p < 0.3$ | **B1** | |
| | $0.7^{28} + 28 \times 0.7^{27} \times 0.3 + {}^{28}C_2 \times 0.7^{26} \times 0.3^2 + {}^{28}C_3 \times 0.7^{25} \times 0.3^3 + {}^{28}C_4 \times 0.7^{24} \times 0.3^4$ | **M1** | Use of B(28, 0.3 ). Addition of terms must be intended.<br>Allow one term wrong or omitted or extra. |
| | 0.0474 | **A1** | Unsupported correct answer scores **B1** instead of **M1 A1**. |
| | 0.0474 > 0.02 [Not reject $H_0$ ] | **M1** | Valid comparison. |
| | No evidence that suspicion is true. | **A1 ft** | Not definite e.g. not 'Suspicion is not true', in context, no contradictions.<br><br>SC use of N(8.4, 5.88) leading to 0.054 > 0.2 OE can score **B1** only for comparison and correct conclusion.<br>Correct hypotheses with $p$ will also score B1. |
| | | **5** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(a) | est $\mu = 14$ accept $\frac{560}{40}$ | **B1** | |
| | est $\sigma^2 = \frac{40}{39}\left(\frac{7850}{40} - 14^2\right)$ or $\frac{1}{39}\left(7850 - \frac{560^2}{40}\right)$ | **M1** | |
| | 0.25641 or 0.256 (3sf) | **A1** | Accept $\frac{10}{39}$<br>Without $\frac{40}{39}$ i.e. biased: est $\sigma^2 = 0.25$ **M0 A0**. |
| | | **3** | |
| 6(b) | E($S - T$) = 14.2 – '14' [= 0.2 ] | **B1 FT** | FT *their* 14. |
| | Var($S - T$) = 0.3 + ` 0.256` [= 0.55641] | **B1 FT** | Accept $\frac{217}{390}$<br>FT *their* 0.256 including FT biased.<br>Var($S - T$) = 0.55. |
| | $\frac{0.1 - '0.2'}{\sqrt{'0.55641'}}$ [= –0.134] | **M1** | Standardising with *their* values (note biased gives –0.135). FT *their* E & Var. |
| | P($S$ – T > 0.1) = 1 – Φ('–0.134') = Φ('0.134') | **M1** | Finding correct area consistent with *their* values. |
| | 0.553 (3sf) | **A1** | Use of biased gives 0.554 (3sf) can score the **A1**.<br>Similar scheme for P($T - S$) < –0.1.<br>Similar scheme for S – T – 0.1 > 0.<br>And T – S + 0.1 < 0. |
| | | **5** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 7(a) | $H_0$: $\mu = 64.6$<br>$H_1$: $\mu < 64.6$ | **B1** | Allow population mean, not just 'mean'. |
| | $[\pm]\frac{63.5 - 64.6}{5.2 \div \sqrt{100}}$ | **M1** | Standardising. Must have $\sqrt{100}$ . |
| | $[\pm]$–2.115 | **A1** | Accept -2.12 (3sf) |
| | '2.115' > 1.96 or '–2.115' < –1.96 [do not accept $H_0$] | **M1** | Valid comparison (0.0172 < 0.025 for area comparison). |
| | There is evidence that $\mu < 64.6$ | **A1 FT** | Not definite, e.g. not '$\mu < 64.6$'. in context. No contradictions.<br>Accept critical value method leading to 63.5 < 63.58 or 64.6 > 64.52. |
| | | **5** | |
| 7(b) | $\frac{m - 64.6}{5.2 \div \sqrt{100}} = -1.96$ | **M1** | Finding the critical value using $N\left(64.6, \frac{5.2}{\sqrt{100}}\right)$ and a $z$ value. |
| | $m = 63.5808$ | **A1** | |
| | $\frac{63.5808 - 62.7}{5.2 \div \sqrt{100}}$ [= 1.694] | **M1** | Standardising using $N\left(62.7, \frac{5.2}{\sqrt{100}}\right)$ and a critical value. |
| | 1 – Φ('1.694') | **M1** | For area consistent with *their* values. |
| | 0.0451 | **A1** | Accept answers that round to 0.045. |
| | | **5** | |

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# Cambridge International AS & A Level

**MATHEMATICS** **9709/62**

Paper 6 Probability & Statistics 2 **October/November 2021**

MARK SCHEME

Maximum Mark: 50

**Published**

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| WWW | Without Wrong Working |
| AWRT | Answer Which Rounds To |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 1(a) | $\frac{20.5}{40} = 0.5125$ | **B1** | Accept 0.513 or $\frac{41}{80}$. Condone $\frac{20.5}{40}$. |
| | $\frac{40}{39}\left(\frac{10.728}{40} - \left('0.5125'^2\right)\right)$ or $\frac{1}{39}\left(10.728 - \frac{20.50^2}{40}\right)$ | **M1** | Biased variance (0.005544 or $\frac{887}{160\,000}$) scores **M0 A0.** |
| | 0.0056859 or 0.00569 (3 sf) or $\frac{887}{156\,000}$ | **A1** | CAO |
| | | **3** | |
| 1(b) | [11 × '0.5125' + 0.5]) = 6.1375 or $\frac{491}{80}$ or 6.14 (3sf) | **B1 FT** | FT *their* 0.5125 |
| | $11^2$ × '0.0056859' | **M1** | With nothing added.<br>Using *their* variance in **(a)** (no sd/var confusion) |
| | 0.688 (3sf) | **A1** | CAO |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 2(a) | E.g. Bias towards students who play instruments or only music students<br><br>or e.g. the six will possibly be friends/have similar music preferences | **B1** | OE<br>Or any reason that some are excluded e.g. because it is lunchtime or because the music building is chosen<br>or any suggestion that opinions may not be independent.<br>Note: 'not representative of all students' needs qualifying |
| | | **1** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 2(b) | 28, 119, 207 | **B1** | **B1** for 28, 119 (condone 028). |
| | | **B1** | **B1** for 207 and only 3 values stated. |
| | | **2** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 3(a) | $0.25 \pm z\sqrt{\frac{0.25\times0.75}{140}}$ | **M1** | Expression of correct form (allow **M1** for just one side stated). Must be a $z$-value. |
| | $z = 2.054$ or 2.055 | **B1** | |
| | 0.175 to 0.325 (3sf) | **A1** | Must be an interval. |
| | | **3** | |
| 3(b) | $0.90 \times 0.95 \times 0.01$<br>$+ 0.90 \times 0.05 \times 0.99$<br>$+ 0.10 \times 0.95 \times 0.99$ | **M1 M1** | **M1** for one correct triple product.<br>**M1** for all correct and added. |
| | 0.147 | **A1** | SC If zero scored award **B1** for a 2 or 3 term expression of the form $0.90 \times 0.95$ [$\times c$] OE. ($0 < c \leqslant 1$) |
| | | **3** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4(a) | Fireworks are destroyed when tested. | **B1** | |
| | | **1** | |
| 4(b) | $H_0$: Pop mean time lasted (or $\mu$) = 30<br>$H_1$: Pop mean time lasted (or $\mu$) < 30 | **B1** | Not just 'mean'. |
| | $\pm \frac{29-30}{\frac{5}{\sqrt{100}}}$ | **M1** | For standardising. Must have $\sqrt{100}$.<br>Use of totals N(3000,2500) giving $\frac{(2900-3000)}{\sqrt{2500}}$<br>scores **M1**. No mixed methods. |
| | $\pm -2$ | **A1** | |
| | $-2 > -2.326$ [Do not reject $H_0$] | **M1** | Accept –2.326 to –2.329.<br>Valid comparison or area comparison 0.0228>0.01 or 0.9772<0.99.<br>Accept CR method 28.837<29 or 30.163>30. |
| | There is not enough evidence that mean time lasted is less than 30 seconds<br>OR Not enough evidence to support the inspector's suspicion | **A1 FT** | In context (if used need mean or time / condone average instead of mean), not definite, e.g. not 'mean time lasted is not less than 30 seconds'.<br>No contradictions.<br>**Note** 2 tailed test can score **B0 M1 A1 M1** (comparison with 2.574–2.579) **A0** (no FT). |
| | | **5** | |
| 4(c) | Yes. Because population distribution is unknown [condone not Normal]. | **B1** | Both needed.<br>Condone $X$ for parent population. |
| | | **1** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5(a) | $e^{-2}(1 + 2 + \frac{2^2}{2!})$ | **M1** | P($X < 3$) any $\lambda$. Allow one end error. |
| | 0.677 (3sf) | **A1** | Unsupported correct answer scores SC **B1** only. |
| | | **2** | |
| 5(b) | N(40, 40) | **M1** | SOI |
| | $\frac{50.5 - 40}{\sqrt{40}}$ [= 1.660] | **M1** | For standardising with *their* values.<br>Allow with wrong or no cc must have square root. |
| | P($z >$ '1.660') = 1 – Φ('1.660') | **M1** | Correct area consistent with *their* working. |
| | 0.0485 or 0.0484 (3sf) | **A1** | |
| | | **4** | |
| 5(c) | $\lambda = 10$ | **B1** | Condone mean = 10. |
| | $e^{-10}\left(\frac{10^8}{8!} + \frac{10^9}{9!} + \frac{10^{10}}{10!} + \frac{10^{11}}{11!}\right)$ | **M1** | Allow any $\lambda$ (allow one end error). |
| | 0.477 (3sf) | **A1** | Unsupported correct answer scores SC **B2** only. |
| | | **3** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(a) | $H_0$: $P(0) = \frac{1}{10}$<br>$H_1$: $P(0) < \frac{1}{10}$ | **B1** | Accept p. |
| | | **1** | |
| 6(b) | For B(30,0.1) | **M1** | Used not just stated. |
| | $P(X = 0) = 0.9^{30}$ [= 0.0424] [<0.1] | **M1** | |
| | $P(X = 0 \text{ or } 1) = 0.9^{30} + 30\times0.9^{29}\times0.1=0.184$ [>0.1] | **B1** | Accept 0.184 or 0.183. |
| | Rejection region is 0 zeros | **A1** | Dependent on **M1 M1** and at least one comparison, no errors seen.<br>**SC** One unsupported correct answer 0.0424/0.184(or 0.183) and correct rejection region scores **B1**; with comparison with 0.1 scores **B2**.<br>Two unsupported correct answers 0.0424 and 0.184(or 0.183) and correct rejection region scores **B2** or if with one comparison with 0.1 scores **B3**. |
| | | **4** | |
| 6(c) | 0.0424 | **B1** | FT *their* **(b)** must have a critical region (only follow though Binomial), dependent on answer < 0.1. |
| | | **1** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(d) | Bin(30, $\frac{1}{40}$) | **B1** | SOI |
| | $1 - 0.975^{30}$ | **M1** | FT *their* rr and with Bin(30, 1/40)). |
| | 0.532 (3dp) | **A1** | SC Unsupported correct answer scores **B2** only. |
| | | **3** | |
| 6(e) | Not concluding that the probability is less than $\frac{1}{10}$, when in fact it is. | **B1** | In context. |
| | | **1** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 7(a)(i) | $k\int_0^2 (4x - x^2)\mathrm{d}x = 1$ | **M1** | Attempt integral f($x$) and = 1. Ignore limits<br>(must see a power increase for attempted integration). |
| | $k\left[\frac{4x^2}{2} - \frac{x^3}{3}\right]_0^2 = 1$ | **A1** | Correct integration and correct limits. |
| | $k \times \frac{16}{3} = 1 \left[k = \frac{3}{16}\right]$ | **A1** | OE AG<br>Convincingly obtained. At least one interim step. No errors seen. |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 7(a)(ii) | $\frac{3}{16}\int_0^2 (4x^2 - x^3)\mathrm{d}x$ | **M1** | Attempt integral $xf(x)$. Ignore limits.<br>(must see a power increase for attempted integration).<br>Condone missing $k$. |
| | $\frac{3}{16}\left[\frac{4x^2}{2} - \frac{x^3}{3}\right]_0^2$ | **A1** | Correct integration and correct limits. Condone missing $k$. |
| | $\frac{5}{4}$ | **A1** | Unsupported correct answer scores **SC B2** only. |
| | | **3** | |
| 7(b) | Symmetrical frequency density graph, 0 to 5, showing area 0.2 to left of $a$ | **B1** | With $a$ to the left of centre. |
| | Either 0.2 between $5–a$ and 5-or 0.8 between 0 and $5–a$ | **B1** | Shown on graph or stated ($5–a$ seen or implied).<br>$a$ must be non-numerical. |
| | $[P(2.5<Y<5–a)] = 0.3$ | **B1** | Must be clearly final answer.<br>$a$ must be non-numerical. |
| | | **3** | |

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# Cambridge International AS & A Level

**MATHEMATICS** **9709/63**

Paper 6 Probability & Statistics 2 **October/November 2021**

MARK SCHEME

Maximum Mark: 50

**Published**

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

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Cambridge International is publishing the mark schemes for the October/November 2021 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

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**PUBLISHED**

## Generic Marking Principles

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GENERIC MARKING PRINCIPLE 1:
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- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

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| Mathematics Specific Marking Principles | |
|---|---|
| 1 | Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing. |
| 2 | Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected. |
| 3 | Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points. |
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| 5 | Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread. |
| 6 | Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear. |

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## Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

### Types of mark

**M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

**A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

**B** Mark for a correct result or statement independent of method marks.

**DM** or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

**FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

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**Abbreviations**

| | |
|---|---|
| AEF/OE | Any Equivalent Form (of answer is equally acceptable) / Or Equivalent |
| AG | Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid) |
| CAO | Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed) |
| CWO | Correct Working Only |
| ISW | Ignore Subsequent Working |
| SOI | Seen Or Implied |
| SC | Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance) |
| WWW | Without Wrong Working |
| AWRT | Answer Which Rounds To |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 1(a) | N(12.5, ….) | **B1** | |
| | Variance = 0.4096 | **B1** | Accept 0.410 (3sf), condone $\frac{10.24}{25}$ |
| | | **2** | |
| 1(b) | $\frac{13-'12.5'}{\sqrt{'0.4096'}}$ [= 0.781] | **M1** | For standardising with *their* values. Accept standardising with 12. |
| | ϕ(‘0.781’) – (1 – Φ(‘0.781’)) | **M1** | For attempting to find *their* central area. |
| | 0.565 (3sf) | **A1** | |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 2 | N(45.2, 45.2) | **B1** | SOI |
| | $\frac{60.5-45.2}{\sqrt{45.2}}$ [= 2.276] | **M1** | Allow with wrong or no continuity correction. |
| | 1 –ϕ(‘2.276’) | **M1** | |
| | 0.0114 | **A1** | |
| | | **4** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 3 | est(p) = 0.2 accept $\frac{15}{75}$ | **B1** | SOI |
| | $2 \times z \times \sqrt{\frac{0.2\times0.8}{75}} = 0.162$ | **M1** | Expression of the correct form.<br>Condone missing $2x$. |
| | $z \left[= 0.081\times\sqrt{\frac{75}{0.2\times0.8}}\right] = 1.754$ | **A1** | Correct $z$. Condone 3sf accuracy. |
| | Φ('1.754') = 0.96[03]<br>'0.96' – (1 – '0.96') | **M1** | OE. Using *their z* to find alpha. |
| | $\alpha = 92$ | **A1** | Following correct working. |
| | | **5** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4(a) | $\frac{1}{18}\int_0^{1.2}(9-x^2)\mathrm{d}x$ | **M1** | Attempt to integrate f($x$), ignore limits.<br>Must see an increase of power. |
| | $\frac{1}{18}\left[9x-\frac{x^3}{3}\right]_0^{1.2}$ | **A1** | Correct integration and correct limits. |
| | $\frac{71}{125}$ or 0.568 | **A1** | SC unsupported answer scores **B2** only. |
| | | **3** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4(b) | $\frac{1}{18}\int_0^3 (9x - x^3)\mathrm{d}x$ | **M1** | Attempt to integrate $x\mathrm{f}(x)$, ignore limits.<br>Must see an increase of power. |
| | $\frac{1}{18}\left[\frac{9x^2}{2} - \frac{x^4}{4}\right]_0^3$ | **A1** | Correct integration and correct limits. |
| | $\frac{9}{8}$ or 1.125 | **A1** | SC unsupported answer scores **B2** only. |
| | | **3** | |
| 4(c) | $\frac{1}{18}\left[9x - \frac{x^3}{3}\right]_0^m = 0.5$ | **M1** | Attempt to integrate f(x) with correct limits and = 0.5.<br>OE. Accept limits $m$ to 3.<br>Allow $x$ instead of $m$. |
| | $\frac{1}{18}\left[9m - \frac{m^3}{3}\right] - 0.5 = 0$ | **A1** | Any correct cubic equation in $m$ or $x$. |
| | $m^3 - 27m + 27 = 0$ | **A1** | AG. Correctly obtain this equation. No errors seen. |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5(a)(i) | Po(0.025) | **B1** | For Poisson and correct parameter. |
| | $n = 2500 > 50$, $np = 0.025 < 5$ | **B1** | Must show 2500 and 0.025.<br>Accept p = $\frac{1}{100000} < 0.1$ in place of $np = 0.025 < 5$. |
| | | **2** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5(a)(ii) | $1 - e^{-0.025}$ | **M1** | Allow any $\lambda$.<br>FT *their* **(a)(i)** if normal; must have continuity correction. |
| | 0.0247 (3sf) | **A1** | Must be from Poisson.<br>Unsupported correct answer scores **B1** instead of **M1 A1**. |
| | | **2** | |
| 5(b) | $H_0$: $p = 0.3$<br>$H_1$: $p < 0.3$ | **B1** | |
| | $0.7^{28} + 28 \times 0.7^{27} \times 0.3 + {}^{28}C_2 \times 0.7^{26} \times 0.3^2 + {}^{28}C_3 \times 0.7^{25} \times 0.3^3 + {}^{28}C_4 \times 0.7^{24} \times 0.3^4$ | **M1** | Use of B(28, 0.3 ). Addition of terms must be intended.<br>Allow one term wrong or omitted or extra. |
| | 0.0474 | **A1** | Unsupported correct answer scores **B1** instead of **M1 A1**. |
| | $0.0474 > 0.02$ [Not reject $H_0$ ] | **M1** | Valid comparison. |
| | No evidence that suspicion is true. | **A1 ft** | Not definite e.g. not 'Suspicion is not true', in context, no contradictions.<br><br>SC use of N(8.4, 5.88) leading to $0.054 > 0.2$ OE can score **B1** only for comparison and correct conclusion.<br>Correct hypotheses with $p$ will also score B1. |
| | | **5** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(a) | est $\mu = 14$ accept $\frac{560}{40}$ | **B1** | |
| | est $\sigma^2 = \frac{40}{39}\left(\frac{7850}{40} - 14^2\right)$ or $\frac{1}{39}\left(7850 - \frac{560^2}{40}\right)$ | **M1** | |
| | 0.25641 or 0.256 (3sf) | **A1** | Accept $\frac{10}{39}$<br>Without $\frac{40}{39}$ i.e. biased: est $\sigma^2 = 0.25$ **M0 A0**. |
| | | **3** | |
| 6(b) | E($S - T$) = 14.2 – '14' [= 0.2 ] | **B1 FT** | FT *their* 14. |
| | Var($S - T$) = 0.3 + ` 0.256` [= 0.55641] | **B1 FT** | Accept $\frac{217}{390}$<br>FT *their* 0.256 including FT biased.<br>Var($S - T$) = 0.55. |
| | $\frac{0.1 - '0.2'}{\sqrt{'0.55641'}}$ [= –0.134] | **M1** | Standardising with *their* values (note biased gives –0.135). FT *their* E & Var. |
| | P($S$ – T > 0.1) = 1 – Φ('–0.134') = Φ('0.134') | **M1** | Finding correct area consistent with *their* values. |
| | 0.553 (3sf) | **A1** | Use of biased gives 0.554 (3sf) can score the **A1**.<br>Similar scheme for P($T - S$) < –0.1.<br>Similar scheme for S – T – 0.1 > 0.<br>And T – S + 0.1 < 0. |
| | | **5** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 7(a) | $H_0$: $\mu = 64.6$<br>$H_1$: $\mu < 64.6$ | **B1** | Allow population mean, not just 'mean'. |
| | $[\pm]\frac{63.5 - 64.6}{5.2 \div \sqrt{100}}$ | **M1** | Standardising. Must have $\sqrt{100}$ . |
| | $[\pm]$–2.115 | **A1** | Accept -2.12 (3sf) |
| | '2.115' > 1.96 or '–2.115' < –1.96 [do not accept $H_0$] | **M1** | Valid comparison (0.0172 < 0.025 for area comparison). |
| | There is evidence that $\mu < 64.6$ | **A1 FT** | Not definite, e.g. not '$\mu < 64.6$'. in context. No contradictions.<br>Accept critical value method leading to 63.5 < 63.58 or 64.6 > 64.52. |
| | | **5** | |
| 7(b) | $\frac{m - 64.6}{5.2 \div \sqrt{100}} = -1.96$ | **M1** | Finding the critical value using $N\left(64.6, \frac{5.2}{\sqrt{100}}\right)$ and a $z$ value. |
| | $m = 63.5808$ | **A1** | |
| | $\frac{63.5808 - 62.7}{5.2 \div \sqrt{100}}$ [= 1.694] | **M1** | Standardising using $N\left(62.7, \frac{5.2}{\sqrt{100}}\right)$ and a critical value. |
| | 1 – Φ('1.694') | **M1** | For area consistent with *their* values. |
| | 0.0451 | **A1** | Accept answers that round to 0.045. |
| | | **5** | |

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Cambridge Assessment International Education

# Cambridge International A Level

**MATHEMATICS** **9709/61**

Paper 6 Probability & Statistics 2 **May/June 2021**

MARK SCHEME

Maximum Mark: 50

**Published**

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

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**PUBLISHED**

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| SOI | Seen Or Implied |
| SC | Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance) |
| WWW | Without Wrong Working |
| AWRT | Answer Which Rounds To |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 1 | $\lambda = (3.1 + 1.7) \times 2$ | M1 | Attempt combined mean. Allow 3.1 + 1.7 for M1 |
| | $= 9.6$ | A1 | Correct mean |
| | $1 - e^{-9.6}\left(1 + 9.6 + \frac{9.6^2}{2} + \frac{9.6^3}{3!}\right)$ | M1 | Allow incorrect mean. Allow one end error. |
| | $= 0.986$ (3 sf) | A1 | **SC** If 9.6 seen and unsupported 0.986 M1A1B1.<br>**SC** Unsupported correct answer of 0.986 only if 9.6 also not seen scores B2 only. |
| | | **4** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 2(a) | $\pm\frac{123-125}{\frac{6}{\sqrt{40}}}$ [= –2.108...] | M1 | Must have $\sqrt{40}$<br>No standard deviation/variance mix. Ignore any continuity correction attempts for this mark. |
| | P($z <$ '–2.108') = 1 – Φ('2.108') | M1 | For correct probability area consistent with *their* working. |
| | = 0.0175 or 0.0176 (3 sf) | A1 | |
| | | **3** | |
| 2(b) | No, population is normal | B1 | Need both. |
| | | **1** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 3 | $1-\frac{20}{27}$ or $\frac{20}{27}-\frac{1}{2}$<br>$\frac{20}{27}-\left(1-\frac{20}{27}\right)$ or $\left(\frac{20}{27}-\frac{1}{2}\right)$ | M1 | For either expression seen. |
| | $\frac{13}{27}$ | A1 | OE. Accept 0.481 or 0.482. |
| | | **2** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4 | $\frac{3820}{100}$ [= 38.2] | B1 | |
| | $\frac{100}{99}\left(\frac{182200}{100}-'38.2'^2\right)$ or $\frac{1}{99}\left(182200-\frac{3820^2}{100}\right)$ | M1 | Use of biased (362.76) scores M0 |
| | $=\frac{12092}{33}$ or 366.424 or 366 (3 sf) | A1 | Accept SD=19.1422 or 19.1(3sf) |
| | '38.2' $\pm z\times\sqrt{\frac{'366.424'}{100}}$ | M1 | Expression of the correct form must be a $z$-value. |
| | $z$ = 1.881 or 1.882 | B1 | Seen. |
| | 34.6 to 41.8 (3 sf) | A1 | Allow use of biased giving (34.6,41.8)<br>Must be an interval. |
| | | **6** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5(a) | $\text{Po}\left(\frac{2}{15}\right)$ | M1 | SOI. Allow Po(0.133). |
| | $\text{P}(X \geqslant 1) = 1 - \text{e}^{-\frac{2}{15}}$ | M1 | Allow incorrect $\lambda$ allow one end error |
| | = 0.125 (3 sf) | A1 | **SC** Partially unsupported final answer:<br>$\text{Po}\left(\frac{2}{15}\right)$ stated B1 then unsupported 0.125 B1<br>**SC** Use of Binomial (0.1248) B1 only<br>Use of Normal scores M0 |
| | | **3** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5(b) | $\lambda = \dfrac{n}{75000}$ | B1 | |
| | $e^{-\frac{n}{75000}} > 0.9$ | M1 | Allow '='<br>Allow incorrect $\lambda$ |
| | $-\dfrac{n}{75000} > \ln 0.9 \quad [n < 7902.04]$ | M1 | Attempt ln both sides |
| | Largest value of $n$ is 7902 | A1 | CWO. Must be an integer. |
| | **Alternative method for Question 5(b)** | | |
| | $e^{-\mu} > 0.9$ | M1 | Allow '=' |
| | $-\mu > \ln 0.9 \quad [\mu < 0.10536]$ | M1 | Attempt ln both sides |
| | $n = \mu \times 75000$ | B1 | |
| | Largest value of $n$ is 7902 | A1 | CWO. Must be an integer. |
| | **Alternative method for Question 5(b)** | | |
| | $\dfrac{74999}{75000}$ | B1 | |
| | $\left(\dfrac{74999}{75000}\right)^n > 0.9$ | M1 | |
| | $n \ln \dfrac{74999}{75000} > \ln 0.9$ | M1 | Attempt ln or log both sides |
| | Largest value of $n$ is 7901 | A1 | CWO Must be an integer |
| | | **4** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6 | E($X$) = 3 | B1 | N.B. E($X$)=108$k$ is B0 until correct $k$ substituted in. |
| | $k\int_0^6 (6x - x^2)\mathrm{d}x = 1$<br>$k\left[3x^2 - \frac{x^3}{3}\right]_0^6$ [= 1] | M1 | Attempt integration of f($x$) and =1.<br>Ignore limits at this stage. |
| | $k\left(108 - \frac{216}{3}\right) = 1$<br>$k = \frac{3}{108}$ or $\frac{1}{36}$ | A1 | |
| | $'\frac{3}{108}'\int_0^6 (6x^3 - x^4)\mathrm{d}x$<br>$= \frac{3}{108}\left[\frac{3x^4}{2} - \frac{x^5}{5}\right]_0^6 = 10.8$ | *M1 | Attempt integration of *their* $k \times x^2$f($x$).<br>Ignore limits at this stage. Accept in terms of $k$. |
| | '10.8' – '3'$^2$ | DM1 | *Their* 10.8 (from use of limits 0 and 6) minus *their* (E($X$))$^2$.<br>Accept in terms of $k$: 388.8$k$–(108$k$)$^2$ |
| | $\frac{9}{5}$ or 1.8 | A1 | CWO. Must be convincingly obtained as AG. |
| | | **6** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 7(a) | $E(T)=3\times55+6\times27\,[=327]$ | B1 | OE. Accept unsimplified. |
| | $\text{Var}(T) = 3\times3^2 + 6\times2.5^2$ [= 64.5] | B1 | Accept unsimplified. |
| | $\frac{340-'327'}{\sqrt{'64.5'}}$ [= 1.619] | M1 | Must have √ |
| | P($z <$ '1.619') = Φ('1.619') | M1 | Correct probability area consistent with *their* working. |
| | 0.947 (3 sf) | A1 | |
| | | **5** | |
| 7(b) | $E(L–S_1–S_2) = 55 – 2 \times 27$ [=1] | B1 | OE e.g. $E(S_1+S_2 – L)= –1$. Accept unsimplified. |
| | $\text{Var}(L– S_1–S_2) = 3^2 + 2 \times 2.5^2$ [= 21.5] | B1 | Accept unsimplified. |
| | $\frac{0-'1'}{\sqrt{'21.5'}}$ [= –0.216] | M1 | Standardising with *their* values. Must come from a combination attempt. |
| | P($L–S_1–S_2 > 0$) = Φ('0.216') | M1 | Correct probability area consistent with *their* working. |
| | 0.586 or 0.585 (3 sf) | A1 | |
| | | **5** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 8(a) | Not representative (of all students in the school) | **B1** | OE idea of 'not being representative' e.g. different grades in the school have different characteristics/proportions....<br>Don't accept 'not random' or 'biased' without further explanation. |
| | | **1** | |
| 8(b) | $H_0$: P(not correct uniform) = 0.15<br>$H_1$: P(not correct uniform) < 0.15 | **B1** | Allow "$p$" |
| | | **1** | |
| 8(c) | Any two probs attempted using B(50,0.15) | **M1** | |
| | $P(X \leqslant 3) = 0.85^{50} + 50 \times 0.85^{49} \times 0.15 + {}^{50}C_2 \times 0.85^{48} \times 0.15^2 + {}^{50}C_3 \times 0.85^{47} \times 0.15^3$ | **M1** | Attempt the tail probability P(0,1,2,3) with B(50,0.15) must be added. |
| | $P(X \leqslant 4) = 0.04605 + {}^{50}C_4 \times 0.85^{46} \times 0.15^4$ | **M1** | OE. *Their* $P(X \leqslant 3) + P(X = 4)$ or P(0,1,2,3,4) with B(50,0.15) must be added. |
| | $P(X \leqslant 3) = 0.0460$ or 0.0461 [<0.05]<br>$P(X \leqslant 4) = 0.112$ or [>0.05] | **A1** | Both correct.<br>OR if $P(X \leqslant 4)$ not seen; P(4)=0.06606 **and** 0.06606>0.05 **and** $P(X \leqslant 3)$=0.0460 scores M1 A1 |
| | P(Type I) = 0.0460 or 0.0461 (3 sf) | **A1** | Dependent on second M1.<br>**SC** If M1M1M1A0 scored allow A1FT for incorrect $P(X \leqslant 3)$ as long as <0.05 |
| | | **5** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 8(d) | 4 is outside critical region ($\leqslant 3$) OE or P($X \leqslant 4$) = 0.112 which is > 0.05 | **M1** | FT working from **(c)**. |
| | No evidence that proportion not wearing the correct uniform has decreased (Accept Ho) | **A1** | In context not definite, e.g. not 'Proportion has not decreased'. No contradiction. |
| | | **2** | |
| 8(e) | Not rejected $H_0$ | ***B1 FT** | FT If Reject $H_0$ in **(d)** |
| | Type II | **DB1 FT** | FT Type I |
| | | **2** | |

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Cambridge Assessment International Education

# Cambridge International AS & A Level

**MATHEMATICS** **9709/62**

Paper 6 Probability & Statistics 2 **May/June 2021**

MARK SCHEME

Maximum Mark: 50

**Published**

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## Generic Marking Principles

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GENERIC MARKING PRINCIPLE 1:
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- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
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GENERIC MARKING PRINCIPLE 3:
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- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

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| Mathematics Specific Marking Principles | |
|---|---|
| 1 | Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing. |
| 2 | Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected. |
| 3 | Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points. |
| 4 | Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw). |
| 5 | Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread. |
| 6 | Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear. |

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## Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

### Types of mark

**M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

**A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

**B** Mark for a correct result or statement independent of method marks.

**DM** or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

**FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

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**Abbreviations**

| | |
|---|---|
| AEF/OE | Any Equivalent Form (of answer is equally acceptable) / Or Equivalent |
| AG | Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid) |
| CAO | Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed) |
| CWO | Correct Working Only |
| ISW | Ignore Subsequent Working |
| SOI | Seen Or Implied |
| SC | Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance) |
| WWW | Without Wrong Working |
| AWRT | Answer Which Rounds To |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 1(a) | $H_0: p = \frac{1}{4}$<br>$H_1: p \neq \frac{1}{4}$ | B1 | or $H_0: \mu = 25$<br>or $H_1: \mu \neq 25$ |
| | | **1** | |
| 1(b) | $N\left(25, \frac{75}{4}\right)$ | B1 | SOI.<br>Allow B1 for $N\left(25, \frac{75}{4}\right)$ or N(0.25,0.001875) SOI. |
| | $\pm \frac{15.5-25}{\sqrt{\frac{75}{4}}}$ or $\frac{\frac{15.5}{100}-0.25}{\sqrt{\frac{0.25\times0.75}{100}}}$ | M1 | Standardise with *their* N(25,…)<br>Allow with no or wrong continuity correction. |
| | $\pm$ –2.194 (2.19) | A1 | |
| | –2.326 < –2.194 or 0.0141 > 0.01 or 0.9859 < 0.99 | M1 | For valid comparison (accept 2.326 to 2.329) |
| | No evidence to reject that the probability is $\frac{1}{4}$ | A1 FT | OE must be in context and not definite, e.g. not 'Claim untrue'.<br>No contradictions.<br>FT *their z* ; dependent on two–tailed test (one-tailed test can score B1 M1 A1 M1 A0)<br>**SC** for use of Binomial B(100,0.25) P = 0.0111 for B1 and then comparison with 0.01 and correct conclusion for B1, maximum 2 out of 5 marks. |
| | | **5** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 2(a) | Var($X$) = 400 × 0.01 × 0.99 (= 3.96) | M1 | |
| | Var($4X + 2$) = 16 × Var($X$) | M1 | For 16 × *their* Var($X$) |
| | 63.36 | A1 | Accept 63.4 |
| | | **3** | |
| 2(b)(i) | Po(4) | B1 | |
| | $n = 400 > 50$ and either $np = 4 < 5$ or $p=0.01 < 0.1$ | B1 | Must quote values 400 and 4 or clearly see $n$=400 and $np$=4 (or $p$=0.01) in working |
| | | **2** | |
| 2(b)(ii) | $e^{-4}\left(\frac{4^2}{2!}+\frac{4^3}{3!}+\frac{4^4}{4!}+\frac{4^5}{5!}\right)$ | M1 | FT *their* '4'<br>Allow one end error<br>FT from **(b)(i)** Use of Normal allow M1 for attempt at standardising (with correct continuity correction) using *their* N(4,3.96) and attempt at probability.<br>FT from **(b)(i)** Use of Binomial allow M1 for attempt at P(2,3,4,5) Binomial terms clearly seen and added |
| | 0.694 (3 sf) | A1 | CWO<br>**SCB1** only for unsupported answer of 0.694 |
| | | **2** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 3(a) | $\frac{1}{2}p(p-1)=1$ | M1 | For area =1<br>For verification methods accept $\frac{1}{2}\times 2\times 1=1$ or<br>$\frac{1}{2}\times 2\times(p-1)=1$ or $\frac{1}{2}\times 1\times p=1$ as indication that area=1 |
| | $p=2$ | A1 | AG - Convincing method and answer.<br>Must see quadratic rearranged to =0 and no errors seen.<br>N.B. Accept convincing verification methods (e.g. statement such as 'assume $p=2$' or 'if $p=2$' or 'using $p=2$' or showing by clear substitution that $p=2$ fits $\frac{1}{2}p(p-1)=1$ with clear conclusion) |
| | | **2** | |
| 3(b) | Gradient = 2<br>equation of line is $y=2x+c$<br>line passes through (1, 0), hence $c=-2$ | M1 | Award for attempting equation of line $y=mx+c$ with $m$ = 2, -2, $\frac{1}{2}$ or $-\frac{1}{2}$ and numerical $c$ ($c\neq 0$) |
| | $y=2x-2$ | A1 | May be seen in **(a)**<br>M1 can be implied by correct answer |
| | $2\int_1^2 (x^2-x)\mathrm{d}x$ | M1 | For attempting $\int xf(x)\mathrm{d}x$.<br>Ignore limits, FT *their* equation. |
| | $2\left[\frac{x^3}{3}-\frac{x^2}{2}\right]_1^2$ | A1 FT | Correct integration FT *their* f($x$)and correct limits |
| | $\frac{5}{3}$ or 1.67 (3 sf) | A1 | |
| | | **5** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4(a) | Mean = 15.0+32.0+8.6 [= 55.6] | B1 | Allow unsimplified |
| | Var = $1.1^2+3.5^2+1.2^2$ [= 14.9] | B1 | Allow unsimplified |
| | | **2** | |
| 4(b) | $\frac{60-"55.6"}{\sqrt{"14.9"}}$ [= 1.140] | M1 | FT *their* 55.6 and 14.9<br>Ignore continuity correction |
| | 1 – ϕ("1.140") | M1 | For correct probability area consistent with *their* working |
| | 0.127 (3 sf) | A1 | CWO |
| | | **3** | |
| 4(c) | $\frac{54.5-"55.6"}{\sqrt{\frac{"14.9"}{15}}}$ or $\frac{817.5-834}{\sqrt{223.5}}$ [= –1.104] | **M1** | FT *their* 55.6 and 14.9<br>No mixed methods |
| | 1 – ϕ("1.104") | **M1** | For correct probability area consistent with their working |
| | 0.135 (3 sf) | **A1** | As final answer |
| | | **3** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5(a) | Conclude that (population) mean time has changed (or is not 42.4) although $\mu$ has not changed (or is still 42.4) | B1 | OE. In context. |
| | | **1** | |
| 5(b) | $H_0$: population mean (or $\mu$) = 42.4<br>$H_1$: population mean (or $\mu$) $\neq$ 42.4 | B1 | Not just 'mean'. (could be seen in **(a)**) |
| | $\pm \dfrac{45.6 - 42.4}{\sqrt{38.2 \div 20}}$ | M1 | For standardising (must have $\sqrt{20}$ ) |
| | $\pm$ 2.315 | A1 | |
| | 2.240 < '2.315' | M1 | For valid comparison (accept 2.241)<br>or P($z$ > 2.315) = 0.0103 < 0.0125 oe |
| | There is evidence that $\mu$ or mean time has changed | A1 FT | FT *their z*<br>In context, not definite. No contradictions.<br>**Note:** Accept correct alternative methods<br>**SC**: One tail test no FT. Can score B0 M1 A1 M1 (comparison with 1.96) A0 (maximum 3 out of 5) |
| | | **5** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(a) | est $(\mu) = \frac{7570}{100}$ $(= 75.7)$ | B1 | |
| | est$(\sigma^2) = \frac{100}{99}\left(\frac{\sum h^2}{100} - '75.7'^2\right)$ or $\frac{1}{99}\left(588050 - \frac{7570^2}{100}\right)$<br>$= \frac{100}{99}\left(\frac{588050}{100} - '75.7'^2\right)$ [= 151.525] | M1 | Attempted<br><br>(**Note:** Biased variance (150.01) scores M0 ) |
| | = 152 (3 sf) | A1 | Or $\frac{15001}{99}$ |
| | | **3** | |
| 6(b) | '75.7' $\pm z\sqrt{\frac{'151.525'}{100}}$ | M1 | For expression of correct form. Must be a $z$ value. Condone just + or just -. |
| | $z = 2.576$ | B1 | Accept 2.574 to 2.579 |
| | 72.5 to 78.9 | A1 FT | FT biased variance only<br>Must be an interval |
| | | **3** | |
| 6(c) | $0.99^4$ | B1 | |
| | 0.961 (3 sf) | B1 | |
| | | **2** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 7(a) | $e^{-4.2} \times \frac{4.2^4}{4!}$ | M1 | P(4), allow any $\lambda$ |
| | 0.194 (3 sf) | A1 | As final answer.<br>**SC** Unsupported correct answer scores B1 only. |
| | | **2** | |
| 7(b) | $1 - e^{-8.4}\left(1 + 8.4 + \frac{8.4^2}{2} + \frac{8.4^3}{3!}\right)$ | M1 | Allow M1 with incorrect $\lambda$. Accept one end error. |
| | 0.968 (3 sf) | A1 | As final answer.<br>**SC** Unsupported correct answer scores B1 only. |
| | | **2** | |
| 7(c) | N(50.4, 50.4) | M1 | SOI |
| | $\frac{39.5 - 50.4}{\sqrt{50.4}}$ [= –1.535] | M1 | Allow wrong or no continuity correction.<br>Must have √ |
| | Φ('–1.535') = 1 – Φ('1.535') | M1 | For correct probability area consistent with *their* working. |
| | 0.0624 (3 sf) or 0.0623 | A1 | |
| | | **4** | |

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Cambridge Assessment International Education

# Cambridge International A Level

**MATHEMATICS** **9709/63**

Paper 6 Probability & Statistics 2 **May/June 2021**

MARK SCHEME

Maximum Mark: 50

**Published**

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GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

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| Mathematics Specific Marking Principles | |
|---|---|
| 1 | Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing. |
| 2 | Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected. |
| 3 | Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points. |
| 4 | Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw). |
| 5 | Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread. |
| 6 | Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear. |

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## Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

### Types of mark

**M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

**A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

**B** Mark for a correct result or statement independent of method marks.

**DM** or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

**FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

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**Abbreviations**

| | |
|---|---|
| AEF/OE | Any Equivalent Form (of answer is equally acceptable) / Or Equivalent |
| AG | Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid) |
| CAO | Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed) |
| CWO | Correct Working Only |
| ISW | Ignore Subsequent Working |
| SOI | Seen Or Implied |
| SC | Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance) |
| WWW | Without Wrong Working |
| AWRT | Answer Which Rounds To |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 1 | $\lambda = 10 \times 1.36$ [= 13.6] | **M1** | |
| | E(amount) = $5 \times 13.6$ = [$]68 | **A1** | |
| | Var(amount) = $5^2 \times 13.6$ [= 340] | **M1** | $5^2 \times \ldots$ |
| | | **M1** | $\ldots \times$ *their* $\lambda$ |
| | Standard Deviation = [$]18.4(4) (3 s.f.) | **A1** | CAO condone $2\sqrt{85}$ |
| | | **5** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 2(a) | Conclude (mean) (journey) time has not decreased when in fact it has. | **B1** | OE in context |
| | | **1** | |
| 2(b) | $H_0$: Pop mean (or $\mu$) = 1.4<br>$H_1$: Pop mean (or $\mu$) < 1.4 | **B1** | May be seen in **(a)** |
| | $\frac{1.36-1.4}{\frac{0.12}{\sqrt{50}}}$ | **M1** | Accept totals method $\frac{68-70}{\sqrt{50}\times 0.12}$<br>No mixed methods or no standard deviation/variance mixes |
| | –2.357 or – 2.36 | **A1** | Correct $z$ or correct area if used |
| | –2.357 < –1.96 or 0.0092 < 0.025 or 0.9908 > 0.975<br>Or CV method 1.36 < 1.367 | **M1** | valid comparison |
| | There is evidence that (mean) (journey) times have decreased | **A1 FT** | in context not definite no contradictions<br>NB use of two tail test scores max B0M1A1M1A0 no ft for two tail test |
| | | **5** | |
| 2(c) | $H_0$ was rejected OE | ***B1 FT** | FT $H_0$ was accepted OE |
| | Type I | **DB1 FT** | FT Type II |
| | | **2** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 3(a)(i) | $H_0$: $\lambda = 2.4$<br>$H_1$: $\lambda > 2.4$ | **B1** | Accept $\lambda$ or $\mu$<br>Accept 2.4 or 0.8 (per year) |
| | | **1** | |
| 3(a)(ii) | $1 - e^{-2.4}(1 + 2.4 + \frac{2.4^2}{2} + \frac{2.4^3}{3!} + \frac{2.4^4}{4!})$ | **M1** | Any $\lambda$; allow one end error |
| | 0.0959 (3 sf) | **A1** | **SC** unsupported answer 0.0959 scores B1 only not M1A1 |
| | 0.0959 > 0.05 | **M1** | Valid comparison<br>Use of 0.9041 < 0.95 can recover either M1A1 or B1 |
| | There is evidence that Jane's claim not justified<br>or There is insufficient evidence to support Jane's claim | **A1 FT** | OE. In context, not definite, e.g. not 'Jane is wrong', no contradictions.<br>Condone omission of Jane. |
| | | **4** | |
| 3(b) | Mean not constant so Poisson model not valid | **B1** | |
| | | **1** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4(a) | $\frac{4509}{90}$ [= 50.1] | **B1** | |
| | $\frac{90}{89}(\frac{225950}{90} - '50.1'^2)$ or $\frac{1}{89}\left(225950 - \frac{4509^2}{90}\right)$ | **M1** | Attempted.<br>Use of biased = 0.5455 scores M0A0 |
| | $\frac{491}{890}$ or 0.552 (3 sf) | **A1** | |
| | | **3** | |
| 4(b) | '50.1' $\pm z\sqrt{\frac{'\frac{491}{890}'}{90}}$ | **M1** | Expression of the correct form, allow any $z$-value but must be a $z$-value |
| | $z = 2.326$ | **B1** | Accept 2.326 to 2.329 |
| | 49.9 to 50.3 (3 sf) | **A1** | FT from biased variance. Must be an interval. |
| | | **3** | |
| 4(c) | Population of masses is unknown | **B1** | Accept population of masses is not normal |
| | | **1** | |
| 4(d) | 1 – 0.98 | **M1** | 0.02 seen |
| | 0.02 ÷ 2 = 0.01 | **A1** | As final answer |
| | | **2** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5(a) | Po(2.5) | **B1** | Accept Poisson with mean = 2.5 not just np = 2.5 |
| | $n = 25\,000 > 50$ and $np$ (or $\lambda$) = 2.5 which is < 5<br>or $n = 25\,000 > 50$ and p = 0.0001 < 0.1 | **B1** | Must see 2.5 (or 0.0001) and 25000 OE, not just $np < 5$ (or $p < 0.1$) and $n > 50$ |
| | | **2** | |
| 5(b) | $e^{-2.5}(1 + 2.5 + \frac{2.5^2}{2} + \frac{2.5^3}{3!})$ | **M1** | Any $\lambda$, accept one end error.<br>FT binomial from part **(a)** scores M1 only for equivalent binomial expressions<br>FT normal from part **(a)** must use correct continuity correction and tables scores M1 only for complete method |
| | 0.758 (3 sf) | **A1** | Unsupported answer of 0.758 scores B1 instead of M1A1 |
| | | **2** | |
| 5(c) | $e^{-2.5} \times \frac{2.5^k}{(k)!} = 2e^{-2.5} \times \frac{2.5^{k+1}}{(k+1)!}$ | **M1** | Any $\lambda$<br>FT binomial from **(a)** scores M1 only for equivalent binomial expression<br>FT from **(a)** normal for equivalent expressions continuity correction must be included |
| | $k = 4$ | **A1** | No errors seen<br>**SC** $k = 4$ unsupported scores B1 only, but see full Poisson expressions for P(4) and P(5) and 0.134 scores M1A1 |
| | | **2** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5(d) | $1 - e^{-\lambda} = 0.963$ | **M1** | Accept *their* attempt at $\lambda$ |
| | $\lambda = -\ln 0.037$ (= 3.2968 or 3.30 or 3.3) | **M1** | Correct use of lns |
| | $n = 33\,000$ (3 sf) | **A1** | Allow $n = 32\,950$ to $33\,050$ (must be an integer)<br>**SC** use of binomial leading to 32 967 scores<br>B1 for $(0.9999)^n = 0.037$<br>B1 for 33 000 to 3sf (32 967) |
| | | **3** | |
| 6(a) | $P(X > 10) = \int_{10}^{20} \frac{3}{8000}(x-20)^2 \, dx$ | **M1** | Attempt integration of f($x$), ignore limits. |
| | $= \left[\frac{3}{8000} \times \frac{(x-20)^3}{3}\right]_{10}^{20}$ or $\frac{3}{8000}\left[\frac{x^3}{3} - \frac{40x^2}{2} + 400x\right]_{10}^{20}$<br>$= \frac{1}{8000}\left[0 - (-10)^3\right]$ | **M1** | Substitute correct limits 10 to 20<br>or 1 – … limits 0 to 10 in *their* integral |
| | $\frac{1}{8}$ or 0.125 | **A1** | **SC** Unsupported answer of $\frac{1}{8}$ scores B1 only |
| | $(\frac{1}{8})^2 = \frac{1}{64}$ or 0.0156 (3 sf) | **B1 FT** | FT *their* $P(X > 10)$ dependent on first M1 gained |
| | | **4** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(b) | $\int_0^{20} \frac{3}{8000}(x^3 - 40x^2 + 400x)\mathrm{d}x$ | **M1** | Attempt integration of $xf(x)$. Ignore limits. |
| | $\frac{3}{8000}\left[\frac{x^4}{4} - \frac{40x^3}{3} + \frac{400x^2}{2}\right]_0^{20}$<br>or $\left(\frac{3x}{8000} \times \frac{(x-20)^3}{3}\right) - \frac{1}{8000}\left(\frac{(x-20)^4}{4}\right)$ | **A1** | Correct integral<br>(by expanding or by parts) |
| | $\frac{3}{8000}\left[\frac{160000}{4} - \frac{40 \times 8000}{3} + 200 \times 400\right]$ | **M1** | Subst correct limits in their (4th degree) integral |
| | 5 | **A1** | |
| | | **4** | |
| 6(c) | $\int_0^m \frac{3}{8000}(x-20)^2 \mathrm{d}x = 0.5$ | **M1** | Attempt to integrate f($x$) and equate to 0.5. Ignore limits. |
| | $\left[\frac{3}{8000} \times \frac{(x-20)^3}{3}\right]_0^m = 0.5$ or $\frac{3}{8000}\left[\frac{x^3}{3} - \frac{40x^2}{2} + 400x\right]_0^m = 0.5$<br>$\frac{1}{8000}\left[(m-20)^3 - (-20)^3\right] = 0.5$ | **M1** | Attempt integral and substitute limits 0 and $m$<br>or $m$ and 20 and = 0.5 |
| | $(m - 20)^3 = -4000$ | **A1** | AG. Found convincingly. |
| | $(m = 20 + \sqrt[3]{-4000})$<br>$m = 4.13$ (3 sf) | **B1** | |
| | | **4** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(d) | Doesn't allow for trains > 20 mins late or Doesn't allow for trains being early | **B1** | or any relevant comment e.g. trains on Sun may be different to trains on Mon |
| | | **1** | |

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Cambridge Assessment International Education

# Cambridge International AS & A Level

**MATHEMATICS** **9709/62**

Paper 6 Probability and Statistics 2 **March 2021**

MARK SCHEME

Maximum Mark: 50

**Published**

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the March 2021 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

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**[Turn over**

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## Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

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| WWW | Without Wrong Working |
| AWRT | Answer Which Rounds To |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 1(a) | Est($\mu$) = $\frac{4820}{60}$ or $\frac{241}{3}$ or 80.3 (3 sf) | **B1** | |
| | Est($\sigma^2$) = $\frac{60}{59}(\frac{392\,050}{60} - (\frac{4820}{60})^2)$ | **M1** | Use of biased (80.72) score M0 A0. |
| | 82.0904 $\left(\frac{14530}{177}\right)$ to 82.635 or SD = 9.0604 to 9.0904 (3sf) | **A1** | |
| | $z$ = 2.326 | **B1** | |
| | $\frac{4820}{60} \pm z \times \sqrt{\frac{'82.0904'}{60}}$ | **M1** | Expression of the correct form – must be $z$ value. |
| | 77.6 to 83.1 (3 sf) | **A1** | CWO<br>Use of biased 77.6 to 83.0(3) can score B1M1A1 (max 4/6). |
| | | **6** | |
| 1(b) | Population distribution of times unknown | **B1** | Accept 'not normal'. |
| | | **1** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 2(a) | $\frac{1}{2}\times\frac{1}{2}k\times k = 1$ | **M1** | Or use of $\int_0^k\left(-\frac{1}{2}x+\frac{1}{2}k\right)\mathrm{d}x=1$ and attempt at integral. |
| | $k=2$ | **A1** | Unsupported answers M0 A0. Do not accept $\pm 2$. |
| | | **2** | |
| 2(b) | $\mathrm{f}(x)=-\frac{1}{2}x+1$ | **B1 FT** | FT *their k* from $y=-\frac{1}{2}x+\frac{1}{2}k$ . |
| | $\int_0^2(-\frac{1}{2}x^2+x)\mathrm{d}x = \left[-\frac{x^3}{6}+\frac{x^2}{2}\right]_0^2$ | **M1** | Attempt integration of $x\mathrm{f}(x)$ limits 0 to $k$.<br>FT *their* $\mathrm{f}(x)$.<br>Could be in terms of $k$. |
| | $\frac{2}{3}$ or 0.667 (3 sf) | **A1** | |
| | | **3** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 2(c) | $\int_{p}^{1}(-\frac{1}{2}x+1)\mathrm{d}x$ [= 0.25] | **M1** | FT their equation of **line**; correct integral and limits (could be reversed) stated or $\frac{1}{2}(1-p)(1-\frac{1}{2}p+\frac{1}{2})$ [= 0.25]. |
| | $\left[-\frac{x^2}{4}+x\right]_{p}^{1} = 0.25$<br>$-\frac{1}{4}+1+\frac{p^2}{4}-p = 0.25$ | **M1** | Attempt substitution of correct limits (not reversed) into their integral or attempt expand must equal 0.25.<br>OE |
| | $p^2 - 4p + 2 = 0$ | **M1** | Obtain 3-term quadratic set equal to 0, obtain at least 1 solution. |
| | $p = 2 - \sqrt{2}$ or 0.586 | **A1** | CAO |
| | | **4** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 3(a) | One-tail because investigating whether "higher" | **B1** | OE. Must have both parts. |
| | | **1** | |
| 3(b) | $H_0$: Population mean (or $\mu$) in city same as for others<br>$H_1$: Population mean (or $\mu$) in city greater than for others | **B1 FT** | If **(a)** two-tail:<br>$H_0$: Pop mean (or $\mu$) in city same as for others.<br>$H_1$: Pop mean (or $\mu$) in region different from others. |
| | 2.41 > 2.326 or 0.008 < 0.01 or 0.992 > 0.99 | **M1** | If **(a)** two-tail: 2.41 < 2.576 or 0.992 < 0.995. |
| | There is evidence that buildings are higher [on average]. | **A1 FT** | In context, not definite. No contradictions.<br>If **(a)** two-tail: There is no evidence that the [average] height of buildings is different. |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4(a) | B(1000, $\frac{1}{400}$) | **B1** | Accept Bin and $n = 1000$, $p = \frac{1}{400}$ . |
| | | **1** | |
| 4(b) | Po(2.5) | **B2** | B1 for Po. B1 for $\lambda = 2.5$ . |
| | | **2** | |
| 4(c)(i) | $e^{-2.5} \times \frac{2.5^4}{4!}$ | **M1** | FT *their* **(b)** for Normal must have a continuity correction.<br>Allow any $\lambda$ |
| | 0.134 (3 sf) | **A1** | CWO |
| | | **2** | |
| 4(c)(ii) | $e^{-2.5}(\frac{2.5^2}{2!} + \frac{2.5^3}{3!} + \frac{2.5^4}{4!})$ | **M1** | FT *their* **(b)** for Normal must have a continuity correction.<br>Allow with one term extra or omitted or wrong. Allow any $\lambda$. |
| | 0.604 (3 sf) | **A1** | CWO |
| | | **2** | |
| 4(d) | $\lambda = 2.5 \times 0.7$ or $\lambda = 700 \times \frac{1}{400}$ [= 1.75] | **M1** | Must see $\lambda$ or use of Poisson. |
| | $1 - e^{-1.75}$ | **M1** | Allow any $\lambda$. Allow 1 – P(0,1). |
| | 0.826 | **A1** | **SC B1** Use of B(700,0.0025) leading to 0.826. |
| | | **3** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5(a) | $E(L_1+L_2+L_3+S_1+S_2+S_3+S_4) = 3\times5.10+4\times2.51$ [= 25.34] | **B1** | OE (E(3L + 4S – 25.5) = -0.16) |
| | $\text{Var}(L_1+L_2+L_3+S_1+S_2+S_3+S_4) = 3\times0.0102+4\times0.0036$ [= 0.045] | **B1** | or $\text{SD} = \frac{3\sqrt{2}}{20} = 0.2121$. |
| | $\frac{25.5-'25.34'}{\sqrt{'0.045'}}$ [= 0.754] | **M1** | No SD/variance mix.<br>Standardising with *their* values (must be from a combination attempt). |
| | Φ('0.754') | **M1** | For the correct area consistent with *their* working. |
| | 0.775 (3 sf) | **A1** | |
| | | **5** | |
| 5(b) | $E(L-2S) = 5.10-2\times2.51$ [= 0.08] | **B1** | OE |
| | $\text{Var}(L-2S) = 0.0102+2^2\times0.0036$ [= 0.0246] | **B1** | Or $\text{SD} = 0.1568$. |
| | $\frac{0-'0.08'}{\sqrt{'0.0246'}}$ [= –0.510] | **M1** | No SD/variance mix.<br>Standardising with *their* values (must be from a combination attempt). |
| | $P(Z > '-0.510') = \phi('0.510')$ | **M1** | For the correct area consistent with *their* working. |
| | 0.695 (3 sf) | **A1** | |
| | | **5** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(a) | $H_0$: population proportion = 0.08 OE<br>$H_1$: population proportion > 0.08 OE | **B1** | Allow '$p = 0.08$' etc. |
| | $P(X \geq 4) = 1 - P(X \leq 3) =$<br>$1 - (0.92^{25} + 25 \times 0.92^{24} \times 0.08 + {}^{25}C_2 \times 0.92^{23} \times 0.08^2 + {}^{25}C_3 \times 0.92^{22} \times 0.08^3)$ | **M1** | Allow 1 – (one term omitted or extra or wrong). |
| | 0.135 (3 sf) | **A1** | |
| | 0.135 > 0.05 | **M1** | Valid comparison.<br>Note: '0.865'<0.95 can score M1 A1 and can recover previous M1 A1 for 0.865. |
| | There is no evidence that proportion owning Chantor has increased | **A1 FT** | In context. Not definite, e.g. not 'Proportion not increased'. No contradictions. |
| | | **5** | |
| 6(b) | $H_0$ was not rejected. | ***B1 FT** | $H_0$ was rejected (consistent with **(a)**). |
| | Hence Type II might have been made. | **DB1 FT** | Type I error. |
| | | **2** | |
| 6(c) | $P(X \geq 5) = 1 - P(X \leq 4)$<br>$= 1 - \left((1 - 0.1351) + {}^{25}C_4 \times 0.92^{21} \times 0.08^4\right)$ [= 0.0451] | ***M1** | Attempted.<br>Note: If critical region method used in **(a)** marks can be awarded here. |
| | 0.0451 < 0.05 | **A1** | Comparison of 0.045[1] with 0.05.<br>Note: If critical region method used in **(a)** marks can be awarded here. |
| | P(Type I error) = 0.0451 or 0.0452 | **A1** | Dependent on M1* only.<br>**SC** Unsupported answers score: **B1** for 0.0451<0.05 and **B1** for final answer 0.0451 only. |
| | | **3** | |

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# Cambridge International AS & A Level

**MATHEMATICS** **9709/61**

Paper 6 Probability & Statistics 2 **October/November 2020**

MARK SCHEME

Maximum Mark: 50

**Published**

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

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**[Turn over**

## Generic Marking Principles

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GENERIC MARKING PRINCIPLE 1:
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- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

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| Mathematics Specific Marking Principles | |
|---|---|
| 1 | Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing. |
| 2 | Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected. |
| 3 | Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points. |
| 4 | Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw). |
| 5 | Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread. |
| 6 | Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear. |

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## Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

**Types of mark**

**M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

**A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

**B** Mark for a correct result or statement independent of method marks.

**DM** or **DB** When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

**FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

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**Abbreviations**

| | |
|---|---|
| AEF/OE | Any Equivalent Form (of answer is equally acceptable) / Or Equivalent |
| AG | Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid) |
| CAO | Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed) |
| CWO | Correct Working Only |
| ISW | Ignore Subsequent Working |
| SOI | Seen Or Implied |
| SC | Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance) |
| WWW | Without Wrong Working |
| AWRT | Answer Which Rounds To |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 1(a) | $\text{Po}\left(\frac{2}{3}\right)$ | **B1** | Poisson with correct mean stated (to at least 3 sf) or implied in working. |
| | $1-\text{e}^{-\frac{2}{3}}\left(1+\frac{2}{3}\right)$ | **M1** | 1 – P($X$ = 0 or 1); allow incorrect $\lambda$; allow one end error |
| | = 0.144 (3 sf) | **A1** | **SC** B1 for use of binomial or no working shown leading to correct final answer. |
| | | **3** | |
| 1(b) | $n > 50$ and $np = \frac{2}{3} < 5$ **or** $n > 50$ and $p = \frac{1}{300} < 0.1$ | **B1** | Accept $p$ or $np$ clearly stated in part **(a)**.<br>Do not accept $n$ is large and $p$ is small. |
| | | **3** | |
| 1(c) | $\text{Po}\left(\frac{11}{3}\right)$ | **B1** | Poisson with correct mean stated (to at least 3sf) or implied in working. |
| | $\text{e}^{-\frac{11}{3}}\left(1+\frac{11}{3}+\frac{\left(\frac{11}{3}\right)^2}{2!}+\frac{\left(\frac{11}{3}\right)^3}{3!}\right)$ | **M1** | P($X$ = 0, 1, 2, 3); allow incorrect $\lambda$; allow one end error. Must not be multiplied by any additional values. |
| | = 0.501 (3 sf) | **A1** | As final answer. |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 2(a) | $\frac{\frac{102}{250}\times\frac{250-102}{250}}{250}$ (= 0.000966144)<br>$\frac{102}{250}\pm z\sqrt{'0.00096614'}$ | **M1** | Any $z$ but must be a $z$ value. One side of the interval scores M1. |
| | $z = 1.645$ | **B1** | |
| | Confident Interval is 0.357 to 0.459 (3 sf) | **A1** | Must be an interval. |
| | | **3** | |
| 2(b) | Estimate of mean $\left(\frac{50460}{250}\right)$ = \$201.84 | **B1** | Allow without units. Allow 3s.f. \$202. |
| | $\frac{250}{249}\left(\frac{19854200}{250}-\left(\frac{50460}{250}\right)^2\right)$ or $\frac{1}{249}\left(19854200-\frac{50460^2}{250}\right)$ | **M1** | |
| | Estimate of variance = 38 832.75 dollars$^2$ or 38 800 (3 sf) | **A1** | Allow with missing units. (Calculation of biased gives 38 700 scores M0A0) |
| | | **3** | |
| 2(c) | e.g. Every house doesn't have an equal chance of being selected or most houses have no chance of being selected. | **B1** | Or other similar e.g. Houses in streets with few houses are more likely to be selected. Not just 'biased', OE, without explanation |
| | | **1** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 3 | F – 0.5M | **M1** | SOI |
| | $\sim N(17, 27^2 + 0.25 \times 55^2)$ | **B1** | for 102 – 0.5(170) (= 17) or 34 |
| | | **B1** | for $27^2 + 0.25 \times 55^2$ (= 1485.25) or $2^2 \times 27^2 + 55^2$ (= 5941) |
| | $\frac{0 - '17'}{\sqrt{'1485.25'}}$ (= –0.4411) | **M1** | Must have an attempt at combining F and M. No standard deviation/variance errors. |
| | P(F – 0.5M < 0) = ϕ('-0.4411') = 1 – ϕ('0.4411') | **M1** | Correct area consistent with *their* figures. |
| | = 0.330 (3 sf) | **A1** | Allow 0.33 if no greater accuracy given |
| | **Alternative method for question 3** | | |
| | 2F – M | | |
| | $\sim N(34, \ 2^2 \times 27^2 + 55^2)$ | **B1** | for 102 – 0.5(170) (= 17) or 34 |
| | | **B1** | for $27^2 + 0.25 \times 55^2$ (= 1485.25) or $2^2 \times 27^2 + 55^2$ (= 5941) |
| | $\frac{0 - '34'}{\sqrt{'5941'}}$ (= –0.4411) | **M1** | Must have an attempt at combining F and M. No standard deviation/variance errors. |
| | P(2F – M < 0) = ϕ('-0.4411') = 1 – ϕ('0.4411') | **M1** | Correct area consistent with *their* figures. |
| | = 0.330 (3 sf) | **A1** | Allow 0.33 if no greater accuracy given |
| | | **6** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4(a) | $(k =)\ \frac{1}{a}$ | **B1** | |
| | | **1** | |
| 4(b) | (Mean =) *their* $k \times \frac{a^2}{2}\ \left(= \frac{a}{2}\right)$ | **B1 FT** | OE seen. FT *their k* |
| | $\frac{1}{a}\int_0^a x^2 \mathrm{d}x \left(= \frac{'a^2'}{3}\right)$ | **M1** | Attempt at correct integral and use of limits. Accept in terms of $k$ or incorrect $k$. |
| | $-\left(\frac{'a'}{2}\right)^2 \left(= \frac{a^2}{12}\right)$ | **M1** | For subtracting mean$^2$, allow if integration not complete. FT incorrect values of $k$. |
| | $\left(\frac{a^2}{12} = 3\right)\ a = 6$ | **A1** | Can be in terms of $k$. |
| | | **4** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5(a) | $\sqrt{2.1}$ or 1.45 (3 sf) | **B1** | |
| | | **1** | |
| 5(b) | $\lambda = 4.2$ | **B1** | |
| | $1 - e^{-4.2}(1 + 4.2)$ | **M1** | $1 - P(X \leqslant 1)$ any $\lambda$, allow one end error. |
| | = 0.922 (3 sf) | **A1** | |
| | | **3** | |
| 5(c) | $\lambda = 6.3$<br>$e^{-6.3}\left(\frac{6.3^5}{5!}+\frac{6.3^6}{6!}+\frac{6.3^7}{7!}\right)$ | **M1** | $P(X = 5, 6, 7)$ any $\lambda$, allow one end error. |
| | = 0.455 (3 sf) | **A1** | |
| | | **2** | |
| 5(d) | $H_0$: $\lambda = 6.3$<br>$H_1$: $\lambda < 6.3$ | **B1** | Accept $\mu$, accept 2.1 (per week) |
| | $P(X \leqslant 2) = e^{-6.3}\left(1+6.3+\frac{6.3^2}{2!}\right)$ | **M1** | |
| | = 0.0498 or 0.0499 | **A1** | Accept 0.0499 |
| | '0.0498' < 0.1 | **M1** | For valid comparison.<br>For CV method the comparison can be '2 lies in CR of $X \leqslant 2$' |
| | There is evidence that mean number of absences has decreased. | **A1 FT** | In context, not definite, e.g. not 'Mean number of absences has decreased.' No contradictions. |
| | | **5** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5(e) | $H_0$ rejected | ***B1 FT** | OE |
| | Hence Type I error possible | **DB1 FT** | |
| | | **2** | |
| 6(a) | $\frac{40-38.4}{\frac{6.9}{\sqrt{30}}} = 1.270$ $\frac{38-38.4}{\frac{6.9}{\sqrt{30}}} = -0.3175$ | **M1** | M1 for either correct expression must have √30 (condone continuity correction) |
| | | **A1** | A1 for ±1.270 or for 1.27 or AWRT |
| | | **A1** | A1 for ± (–0.3175) must be opposite sign or for 0.317 or 0.318 or AWRT |
| | Φ('1.270') – (1 – ϕ('0.3175') | **M1** | For correct method consistent with *their* values |
| | = 0.523 (3 sf) or 0.522 | **A1** | |
| | | **5** | |
| 6(b)(i) | 2-tail because looking for 'change', not decrease or increase | **B1** | OE |
| | | **1** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(b)(ii) | $H_0$: Population mean journey time (or $\mu$) = 38.4<br>$H_1$: Population mean journey time (or $\mu$) ≠ 38.4 | **B1** | Not just 'mean journey time' |
| | $\frac{40.2-38.4}{\frac{6.9}{\sqrt{30}}}$ | **M1** | For standardising (must have √30 ) |
| | = 1.429 | **A1** | |
| | '1.429' < 1.645 | **M1** | For valid comparison (area comparison 0.0765 > 0.05 ) |
| | There is no evidence that mean journey time has changed. | **A1 FT** | In context. Not definite (e.g. not 'mean journey time has not changed'). No contradictions. FT *their* '1.429'<br>(Note use of 1-tail test scores B0 M1A1M1(comparison with 1.282) A0 max) |
| | **Alternative method for question 6(b)(ii) – critical values method** | | |
| | $H_0$: Population mean journey time (or $\mu$) = 38.4<br>$H_1$: Population mean journey time (or $\mu$) ≠ 38.4 | **B1** | Not just 'mean journey time' |
| | $38+1.645\left(\frac{6.9}{\sqrt{30}}\right)$ | **M1** | |
| | = 40.47 | **A1** | |
| | 40.2 < 40.47 | **M1** | For valid comparison |
| | There is no evidence that mean journey time has changed. | **A1 FT** | In context. Not definite (e.g. not 'mean journey time has not changed'). No contradictions. |
| | | **5** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(b)(iii) | Yes, because population distribution unknown. | **B1** | Allow: Yes, because population distribution not normal. |
| | | **1** | |

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# Cambridge International AS & A Level

**MATHEMATICS** **9709/62**

Paper 6 Probability & Statistics 2 **October/November 2020**

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**Abbreviations**

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CAO Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)

CWO Correct Working Only

ISW Ignore Subsequent Working

SOI Seen Or Implied

SC Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

WWW Without Wrong Working

AWRT Answer Which Rounds To

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 1 | Poisson, any $\lambda$ | **M1** | Used |
| | $1-e^{-3}\left(1+3+\frac{3^2}{2}\right)$ | **M1** | Allow one end error |
| | = 0.577 (3sf) | **A1** | **SC** Use of Binomial (or unsupported correct answer) scores B1 only |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 2(a) | $\frac{56}{300} \pm z \times \sqrt{\frac{\frac{56}{300} \times \frac{244}{300}}{300}}$ | **M1** | For expression of the correct form. Must be a $z$ value |
| | $z$ = 2.054 or 2.055 | **B1** | |
| | 0.14(0) to 0.233 (3sf) or 0.141 to 0.233 (3sf) | **A1** | Must be an interval |
| | | **3** | |
| 2(b) | $\frac{1}{6}$ (= 0.167)<br>This is within confidence interval, so no reason to believe die is biased. | **B1 FT** | Note if confidence interval set up with $\frac{1}{6}, \frac{56}{300}$ it should be the value used here.<br><br>FT *their* confidence interval. Not definite, e.g. not 'Die not biased'. |
| | | **1** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 3(a) | $\frac{1}{2} \times 3 \times c = 1$<br>($c = \frac{2}{3}$ **AG**) | **B1** | Must see this line, oe, and result<br>(Alternative method involving equation of line $\left(y = \left(\frac{-c}{3}\right)x + c\right)$<br>must have all relevant working shown) |
| | | **1** | |
| 3(b) | $\left(\frac{1}{3}\right)^2$ | **M1** | Allow M1 for $\frac{1}{3}$ seen as a linear scale factor<br>or Attempt to find equation of line (of form y = mx + c where c ≠ 0) and interval from 2 to 3 OE<br>or Attempt to find the point (2, $\frac{2}{9}$) using the equation of the line (of form y = mx +c where c ≠ 0) and then use area of triangle |
| | $= \frac{1}{9}$ or 0.111(3sf) | **A1** | |
| | | **2** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 3(c) | Equation of line is $y = \frac{2}{3} - \left(\frac{2}{3} \div 3\right)x$ | ***M1** | OE Must be of form y = mx + c (c ≠ 0). |
| | $\mathrm{E}(X) = \int_0^3 \left(\frac{2}{3}x - \frac{2}{9}x^2\right)\mathrm{d}x$ | **DM1** | Attempt integration $x \times$ *their* f($x$), ignore limits |
| | $= \left[\frac{x^2}{3} - \frac{2x^3}{27}\right]_0^3$ | **A1 FT** | Correct integration and limits. FT *their* equation of line |
| | = 1 | **A1** | |
| | | **4** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4 | est($\mu$) = $\frac{1850}{200}$ or 9.25 | **B1** | |
| | est($\sigma^2$) = $\frac{200}{199}\left(\frac{17850}{200}-\left(\frac{1850}{200}\right)^2\right)$ or $\frac{1}{199}\left(17850-\frac{1850^2}{200}\right)$ | **M1** | |
| | = 3.71 or 3.7060 or $\frac{1475}{398}$ | **A1** | |
| | $H_0$: $\mu = 8.9$<br>$H_1$: $\mu \neq 8.9$ | **B1** | Accept Population mean (not just mean) |
| | $\frac{\frac{1850}{200}-8.9}{\sqrt{\frac{"3.706"}{200}}}$ | **M1** | Use of biased variance (3.6875) still scores M1 |
| | = 2.57(3sf) (or using areas 0.00507 – 0.0051) | **A1** | Accept 2.58 (3sf) or using areas 0.0049–0.005 where biased variance used. |
| | 2.24 < 2.57 or 0.00507 < 0.0125 | **M1** | For valid comparison with 2.240 or 2.241 or valid comparison with 0.0125<br>Accept 2.24 < 2.58 or 0.00496 < 0.0125 where biased variance used |
| | (Reject $H_0$) There is evidence that $\mu$ is not 8.9 | **A1 FT** | Not definite, e.g. NOT '$\mu \neq 8.9$' Must be in context. No contradictions.<br>(Accept cv method)<br>(**Note:** Use of 1 tail test scores Max B1M1A1B0M1A1M1A0, max 6 out of 8) |
| | | **8** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5(a) | Customers arrive independently or singly or at random | **B1** | Any one of these in context |
| | | **1** | |
| 5(b) | $e^{-2.3} \times \frac{2.3^3}{3!}$ | **M1** | Attempt correct expression seen |
| | = 0.203 (3sf) | **A1** | |
| | | **2** | |
| 5(c) | Po(4.6) | **B1** | SOI |
| | $1 - e^{-4.6}\left(1 + 4.6 + \frac{4.6^2}{2!} + \frac{4.6^3}{3!}\right)$ | **M1** | Correct expression, with any $\lambda$, allow one end error |
| | = 0.674 (3sf) | **A1** | As final answer. |
| | | **3** | |
| 5(d) | P(none arrive) = $e^{-2.3}$ (= 0.10026) | **M1** | Must be clearly *their* P(none arrive) |
| | ${}^5C_2(e^{-2.3})^2(1 - e^{-2.3})^3$ | **M1** | FT *their* $e^{-2.3}$ |
| | = 0.0732 or 0.0733 (3sf) | **A1** | |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(a) | $H_0$: P(contains offer) = $\frac{1}{3}$<br>$H_1$: P(contains offer) < $\frac{1}{3}$ | **B1** | Allow *p* for P(contains offer) but not just proportion |
| | P(0,1 or 2 offers in 20 \| $H_0$)<br>$= \left(\frac{2}{3}\right)^{20} + 20\left(\frac{2}{3}\right)^{19}\left(\frac{1}{3}\right) + {}^{20}C_2\left(\frac{2}{3}\right)^{18}\left(\frac{1}{3}\right)^2$ | **M1** | |
| | = 0.0176 (3sf) | **A1** | |
| | '0.0176' < 0.1 | **M1** | For valid comparison.<br>**SC** comparison of 0.982(4) > 0.9 scores M1 and recovers the previous M1 A1 |
| | (Reject $H_0$ ) No evidence (at 10% level) to support manufacturers claim | **A1 FT** | In context. Not definite. No contradictions.<br>(Note 2 tail test scores max B0M1A1M1A0, max 3 out of 5)<br>Accept critical region method: M1 A1 for correctly finding critical region of < 4 ; 2 in critical region M1; A1 conclusion<br>**SC** Use of Normal approximation $N\left(\frac{20}{3}, \frac{40}{9}\right)$ scores B1 M1 A0<br>M1 A1 max; the first M1 for $\frac{\left(2.5 - \frac{20}{3}\right)}{\sqrt{\left(\frac{40}{9}\right)}}$ requires use of correct continuity correction and the comparison 0.024 < 0.1 OE must be a valid comparison |
| | | **5** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(b) | $1 - P(X \leqslant 3)$ | **M1** | M1 for 1 – (one term omitted or extra or incorrect) or omit '1 – ' |
| | $=1-\left[\left(\frac{6}{7}\right)^{20}+20\left(\frac{6}{7}\right)^{19}\left(\frac{1}{7}\right)+{}^{20}C_2\left(\frac{6}{7}\right)^{18}\left(\frac{1}{7}\right)^2+{}^{20}C_3\left(\frac{6}{7}\right)^{17}\left(\frac{1}{7}\right)^3\right]$ | **A1** | for all correct expression |
| | = 0.318 (3sf) | **A1** | As final answer. |
| | | **3** | |
| 6(c) | Concluding that prop is 1 in 3 when it is actually less(1 in 7) | **B1** | OE, in context. |
| | | **1** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 7(a) | P(S > L + 200) = P(S – L > 200)<br>E(S – L) = 380–210 (=170) or E(S–L–200)= 380–210–200 (=–30) | **B1** | These may be implied by next line |
| | Var(S – L) = 140+80 (=220) or Var(S–L–200)= 140+80 (=220) | **B1** | |
| | $\frac{200-"170"}{\sqrt{"220"}}$ or $\frac{0-"-30"}{\sqrt{"220"}}$ (= 2.023) | **M1** | Standardising with *their* values (must be from a combination attempt) Allow with attempted continuity correction. |
| | $1 - \phi("2.023")$ | **M1** | Area consistent with their values |
| | = 0.0216 (3sf) | **A1** | (0.0234 with continuity correction) |
| | | **5** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 7(b) | E(total cost) = 380 × 20 + 210 × 50 (= 18 100) | **B1** | or \$181 |
| | Var(total cost) = 140 × $20^2$ + 80 × $50^2$ (= 256 000) | **B1** | or 25.6 (dollar$^2$) These may be implied by next line |
| | $\frac{19000-"18100"}{\sqrt{"256000"}}$ or $\frac{190-"181"}{\sqrt{"25.6"}}$ (= 1.778) | **M1** | Standardising with *their* values (must be from a combination attempt). No mixed methods.<br>Allow with attempted continuity correction. |
| | ϕ("1.778") | **M1** | Area consistent with *their* values |
| | = 0.962 or 0.963 (3 sf) | **A1** | (0.953 or 0.954 with continuity correction) |
| | | **5** | |

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Cambridge Assessment International Education

# Cambridge International AS & A Level

**MATHEMATICS** **9709/63**

Paper 6 Probability & Statistics 2 **October/November 2020**

MARK SCHEME

Maximum Mark: 50

**Published**

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2020 series for most Cambridge IGCSE™, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.



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**[Turn over**

## Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

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| Mathematics Specific Marking Principles | |
|---|---|
| 1 | Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing. |
| 2 | Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected. |
| 3 | Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points. |
| 4 | Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw). |
| 5 | Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread. |
| 6 | Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear. |

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## Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

### Types of mark

**M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

**A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

**B** Mark for a correct result or statement independent of method marks.

**DM** or **DB** When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

**FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

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**Abbreviations**

| | |
|---|---|
| AEF/OE | Any Equivalent Form (of answer is equally acceptable) / Or Equivalent |
| AG | Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid) |
| CAO | Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed) |
| CWO | Correct Working Only |
| ISW | Ignore Subsequent Working |
| SOI | Seen Or Implied |
| SC | Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance) |
| WWW | Without Wrong Working |
| AWRT | Answer Which Rounds To |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 1(a) | $\text{Po}\left(\frac{2}{3}\right)$ | **B1** | Poisson with correct mean stated (to at least 3 sf) or implied in working. |
| | $1-\text{e}^{-\frac{2}{3}}\left(1+\frac{2}{3}\right)$ | **M1** | $1 - \text{P}(X = 0 \text{ or } 1)$; allow incorrect $\lambda$; allow one end error |
| | $= 0.144$ (3 sf) | **A1** | **SC** B1 for use of binomial or no working shown leading to correct final answer. |
| | | **3** | |
| 1(b) | $n > 50$ and $np = \frac{2}{3} < 5$ **or** $n > 50$ and $p = \frac{1}{300} < 0.1$ | **B1** | Accept $p$ or $np$ clearly stated in part **(a)**.<br>Do not accept $n$ is large and $p$ is small. |
| | | **3** | |
| 1(c) | $\text{Po}\left(\frac{11}{3}\right)$ | **B1** | Poisson with correct mean stated (to at least 3sf) or implied in working. |
| | $\text{e}^{-\frac{11}{3}}\left(1+\frac{11}{3}+\frac{\left(\frac{11}{3}\right)^2}{2!}+\frac{\left(\frac{11}{3}\right)^3}{3!}\right)$ | **M1** | $\text{P}(X = 0, 1, 2, 3)$; allow incorrect $\lambda$; allow one end error. Must not be multiplied by any additional values. |
| | $= 0.501$ (3 sf) | **A1** | As final answer. |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 2(a) | $\frac{\frac{102}{250}\times\frac{250-102}{250}}{250}$ (= 0.000966144)<br>$\frac{102}{250}\pm z\sqrt{'0.00096614'}$ | **M1** | Any $z$ but must be a $z$ value. One side of the interval scores M1. |
| | $z = 1.645$ | **B1** | |
| | Confident Interval is 0.357 to 0.459 (3 sf) | **A1** | Must be an interval. |
| | | **3** | |
| 2(b) | Estimate of mean $\left(\frac{50460}{250}\right)$ = \$201.84 | **B1** | Allow without units. Allow 3s.f. \$202. |
| | $\frac{250}{249}\left(\frac{19854200}{250}-\left(\frac{50460}{250}\right)^2\right)$ or $\frac{1}{249}\left(19854200-\frac{50460^2}{250}\right)$ | **M1** | |
| | Estimate of variance = 38 832.75 dollars$^2$ or 38 800 (3 sf) | **A1** | Allow with missing units. (Calculation of biased gives 38 700 scores M0A0) |
| | | **3** | |
| 2(c) | e.g. Every house doesn't have an equal chance of being selected or most houses have no chance of being selected. | **B1** | Or other similar e.g. Houses in streets with few houses are more likely to be selected. Not just 'biased', OE, without explanation |
| | | **1** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 3 | F – 0.5M | **M1** | SOI |
| | $\sim N(17, 27^2 + 0.25 \times 55^2)$ | **B1** | for 102 – 0.5(170) (= 17) or 34 |
| | | **B1** | for $27^2 + 0.25 \times 55^2$ (= 1485.25) or $2^2 \times 27^2 + 55^2$ (= 5941) |
| | $\frac{0 - '17'}{\sqrt{'1485.25'}}$ (= –0.4411) | **M1** | Must have an attempt at combining F and M. No standard deviation/variance errors. |
| | P(F – 0.5M < 0) = ϕ('-0.4411') = 1 – ϕ('0.4411') | **M1** | Correct area consistent with *their* figures. |
| | = 0.330 (3 sf) | **A1** | Allow 0.33 if no greater accuracy given |
| | **Alternative method for question 3** | | |
| | 2F – M | | |
| | $\sim N(34, \ 2^2 \times 27^2 + 55^2)$ | **B1** | for 102 – 0.5(170) (= 17) or 34 |
| | | **B1** | for $27^2 + 0.25 \times 55^2$ (= 1485.25) or $2^2 \times 27^2 + 55^2$ (= 5941) |
| | $\frac{0 - '34'}{\sqrt{'5941'}}$ (= –0.4411) | **M1** | Must have an attempt at combining F and M. No standard deviation/variance errors. |
| | P(2F – M < 0) = ϕ('-0.4411') = 1 – ϕ('0.4411') | **M1** | Correct area consistent with *their* figures. |
| | = 0.330 (3 sf) | **A1** | Allow 0.33 if no greater accuracy given |
| | | **6** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4(a) | $(k =)\ \frac{1}{a}$ | **B1** | |
| | | **1** | |
| 4(b) | (Mean =) *their* $k \times \frac{a^2}{2}\ \left(= \frac{a}{2}\right)$ | **B1 FT** | OE seen. FT *their k* |
| | $\frac{1}{a}\int_0^a x^2 \mathrm{d}x \left(= \frac{'a^2'}{3}\right)$ | **M1** | Attempt at correct integral and use of limits. Accept in terms of $k$ or incorrect $k$. |
| | $-\left(\frac{'a}{2}'\right)^2 \left(= \frac{a^2}{12}\right)$ | **M1** | For subtracting mean$^2$, allow if integration not complete. FT incorrect values of $k$. |
| | $\left(\frac{a^2}{12} = 3\right)\ a = 6$ | **A1** | Can be in terms of $k$. |
| | | **4** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5(a) | $\sqrt{2.1}$ or 1.45 (3 sf) | **B1** | |
| | | **1** | |
| 5(b) | $\lambda = 4.2$ | **B1** | |
| | $1 - e^{-4.2}(1 + 4.2)$ | **M1** | $1 - P(X \leqslant 1)$ any $\lambda$, allow one end error. |
| | = 0.922 (3 sf) | **A1** | |
| | | **3** | |
| 5(c) | $\lambda = 6.3$<br>$e^{-6.3}\left(\frac{6.3^5}{5!} + \frac{6.3^6}{6!} + \frac{6.3^7}{7!}\right)$ | **M1** | $P(X = 5, 6, 7)$ any $\lambda$, allow one end error. |
| | = 0.455 (3 sf) | **A1** | |
| | | **2** | |
| 5(d) | $H_0$: $\lambda = 6.3$<br>$H_1$: $\lambda < 6.3$ | **B1** | Accept $\mu$, accept 2.1 (per week) |
| | $P(X \leqslant 2) = e^{-6.3}\left(1 + 6.3 + \frac{6.3^2}{2!}\right)$ | **M1** | |
| | = 0.0498 or 0.0499 | **A1** | Accept 0.0499 |
| | '0.0498' < 0.1 | **M1** | For valid comparison.<br>For CV method the comparison can be '2 lies in CR of $X \leqslant 2$' |
| | There is evidence that mean number of absences has decreased. | **A1 FT** | In context, not definite, e.g. not 'Mean number of absences has decreased.' No contradictions. |
| | | **5** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5(e) | $H_0$ rejected | ***B1 FT** | OE |
| | Hence Type I error possible | **DB1 FT** | |
| | | **2** | |
| 6(a) | $\frac{40-38.4}{\frac{6.9}{\sqrt{30}}} = 1.270$ $\frac{38-38.4}{\frac{6.9}{\sqrt{30}}} = -0.3175$ | **M1** | M1 for either correct expression must have √30 (condone continuity correction) |
| | | **A1** | A1 for ±1.270 or for 1.27 or AWRT |
| | | **A1** | A1 for ± (–0.3175) must be opposite sign or for 0.317 or 0.318 or AWRT |
| | Φ('1.270') – (1 – ϕ('0.3175') | **M1** | For correct method consistent with *their* values |
| | = 0.523 (3 sf) or 0.522 | **A1** | |
| | | **5** | |
| 6(b)(i) | 2-tail because looking for 'change', not decrease or increase | **B1** | OE |
| | | **1** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(b)(ii) | $H_0$: Population mean journey time (or $\mu$) = 38.4<br>$H_1$: Population mean journey time (or $\mu$) ≠ 38.4 | **B1** | Not just 'mean journey time' |
| | $\frac{40.2-38.4}{\frac{6.9}{\sqrt{30}}}$ | **M1** | For standardising (must have √30 ) |
| | = 1.429 | **A1** | |
| | '1.429' < 1.645 | **M1** | For valid comparison (area comparison 0.0765 > 0.05 ) |
| | There is no evidence that mean journey time has changed. | **A1 FT** | In context. Not definite (e.g. not 'mean journey time has not changed'). No contradictions. FT *their* '1.429'<br>(Note use of 1-tail test scores B0 M1A1M1(comparison with 1.282) A0 max) |
| | **Alternative method for question 6(b)(ii) – critical values method** | | |
| | $H_0$: Population mean journey time (or $\mu$) = 38.4<br>$H_1$: Population mean journey time (or $\mu$) ≠ 38.4 | **B1** | Not just 'mean journey time' |
| | $38+1.645\left(\frac{6.9}{\sqrt{30}}\right)$ | **M1** | |
| | = 40.47 | **A1** | |
| | 40.2 < 40.47 | **M1** | For valid comparison |
| | There is no evidence that mean journey time has changed. | **A1 FT** | In context. Not definite (e.g. not 'mean journey time has not changed'). No contradictions. |
| | | **5** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(b)(iii) | Yes, because population distribution unknown. | **B1** | Allow: Yes, because population distribution not normal. |
| | | **1** | |

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# Cambridge International AS & A Level

**MATHEMATICS** **9709/51**

Paper 5 Probability & Statistics 1 **May/June 2020**

MARK SCHEME

Maximum Mark: 50

**Published**

Students did not sit exam papers in the June 2020 series due to the Covid-19 global pandemic.

This mark scheme is published to support teachers and students and should be read together with the question paper. It shows the requirements of the exam. The answer column of the mark scheme shows the proposed basis on which Examiners would award marks for this exam. Where appropriate, this column also provides the most likely acceptable alternative responses expected from students. Examiners usually review the mark scheme after they have seen student responses and update the mark scheme if appropriate. In the June series, Examiners were unable to consider the acceptability of alternative responses, as there were no student responses to consider.

Mark schemes should usually be read together with the Principal Examiner Report for Teachers. However, because students did not sit exam papers, there is no Principal Examiner Report for Teachers for the June 2020 series.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the June 2020 series for most Cambridge IGCSE™ and Cambridge International A & AS Level components, and some Cambridge O Level components.



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**[Turn over**

**Generic Marking Principles**

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- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

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Marks awarded are always **whole marks** (not half marks, or other fractions).

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- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

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GENERIC MARKING PRINCIPLE 6:
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| Mathematics Specific Marking Principles | |
|---|---|
| 1 | Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing. |
| 2 | Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected. |
| 3 | Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points. |
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| 5 | Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread. |
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## Mark Scheme Notes

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**A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

**B** Mark for a correct result or statement independent of method marks.

**DM** or **DB** When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

**FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

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**Abbreviations**

| | |
|---|---|
| AEF/OE | Any Equivalent Form (of answer is equally acceptable) / Or Equivalent |
| AG | Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid) |
| CAO | Correct Answer Only (emphasising that no "follow through" from a previous error is allowed) |
| CWO | Correct Working Only |
| ISW | Ignore Subsequent Working |
| SOI | Seen Or Implied |
| SC | Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance) |
| WWW | Without Wrong Working |
| AWRT | Answer Which Rounds To |

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| Question | Answer | Marks |
|---|---|---|
| 1(a) | Prob of 4 (from 1,3, 3,1 or 2,2) = $\frac{3}{36}=\frac{1}{12}$ **AG** | **B1** |
| | | **1** |
| 1(b) | Mean = $\frac{1}{\frac{1}{12}}=12$ | **B1** |
| | | **1** |
| 1(c) | $\left(\frac{11}{12}\right)^5\times\frac{1}{12}=0.0539$ or $\frac{161051}{2985984}$ | **B1** |
| | | **1** |
| 1(d) | $1-\left(\frac{11}{12}\right)^7$ | **M1** |
| | 0.456 or $\frac{16344637}{35831808}$ | **A1** |
| | | **2** |

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| Question | Answer | Marks |
|---|---|---|
| 2(a) | 6! | **M1** |
| | 720 | **A1** |
| | | **2** |
| 2(b) | Total number: $\frac{9!}{3!2!}(30240)$ | **M1** |
| | Number with Ls together $= \frac{8!}{3!}(6720)$ | **M1** |
| | Number with Ls not together $= \frac{9!}{3!2!} - \frac{8!}{3!}$<br>$= 30\,240 - 6720$ | **M1** |
| | 23 520 | **A1** |
| | **Alternative method for question 2(b)** | |
| | $\frac{7!}{3!} \times \frac{8 \times 7}{2}$ | |
| | $7! \times k$ in numerator, $k$ integer $\geq 1$ | **M1** |
| | $8 \times 7 \times m$ in numerator or $8C2 \times m$, $m$ integer $\geq 1$ | **M1** |
| | 3! in denominator | **M1** |
| | 23 520 | **A1** |
| | | **4** |

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| Question | Answer | Marks |
|---|---|---|
| 3(a) | $x$: 0, 1, 2, 3<br>Probability: $\frac{1}{56}$, $\frac{15}{56}$, $\frac{30}{56}$, $\frac{10}{56}$<br>(**B1** for probability distribution table with correct outcome values) | **B1** |
| | $P(0) = \frac{3}{8}\times\frac{2}{7}\times\frac{1}{6} = \frac{1}{56}$<br>$P(1) = \frac{5}{8}\times\frac{3}{7}\times\frac{2}{6}\times 3 = \frac{15}{56}$<br>$P(2) = \frac{5}{8}\times\frac{4}{7}\times\frac{3}{6}\times 3 = \frac{30}{56}$<br>$P(3) = \frac{5}{8}\times\frac{4}{7}\times\frac{3}{6} = \frac{10}{56}$<br>(**M1** for denominator $8\times7\times6$) | **M1** |
| | Any one probability correct (with correct outcome) | **A1** |
| | All probabilities correct | **A1** |
| | | **4** |
| 3(b) | $1 - P(8, 9, 10) = 1 - \left[{}^{10}C_8\, 0.64^8 0.36^2 + {}^{10}C_9\, 0.64^9 0.36^1 + 0.64^{10}\right]$ | **M1** |
| | $1 - (0.164156 + 0.064852 + 0.11529)$ | **M1** |
| | 0.759 | **A1** |
| | | **3** |

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| Question | Answer | Marks |
|---|---|---|
| 4 | Scenarios:<br>2P 3V 2G $^8C_2 \times {}^4C_2 \times {}^6C_3 = 28 \times 6 \times 20 = 3360$<br>2P 4V 1G $^8C_2 \times {}^4C_1 \times {}^6C_4 = 28 \times 4 \times 15 = 1680$<br>3P 3V 1G $^8C_3 \times {}^4C_1 \times {}^6C_3 = 56 \times 4 \times 20 = 4480$<br>4P 2V 1G $^8C_4 \times {}^4C_1 \times {}^6C_2 = 70 \times 4 \times 15 = 4200$<br>(**M1** for $^8C_r \times {}^4C_r \times {}^6C_r$ with $\sum r = 7$) | **M1** |
| | Two unsimplified products correct | **B1** |
| | Summing the number of ways for 3 or 4 correct scenarios | **M1** |
| | Total: 13 720 | **A1** |
| | | **4** |

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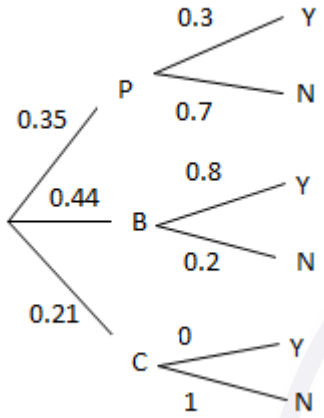
| Question | Answer | Marks |
|---|---|---|
| 5(a) |  | |
| | Fully correct labelled tree for method of transport with correct probabilities. | **B1** |
| | Fully correct labelled branches with correct probabilities for lateness with either 1 branch after W or 2 branches with the prob 0 | **B1** |
| | | **2** |
| 5(b) | $0.35 \times 0.3 + 0.44 \times 0.8 \, (+ 0)$ | **M1** |
| | 0.457 | **A1** |
| | | **2** |

| Question | Answer | Marks |
|---|---|---|
| 5(c) | $\text{P(not B}\mid\text{not fruit)} = \dfrac{\text{P}(\text{B}'\cap\text{F}')}{\text{P}(\text{F}')}$ | **M1** |
| | $\dfrac{0.35\times0.7+0.21\times1}{1-\textit{their}\,(\mathbf{b})}$ | **M1** |
| | $\dfrac{0.455}{0.543}$<br>(**M1** for 1 – *their* **(b)** or summing three appropriate 2-factor probabilities, correct or consistent with *their* tree diagram as denominator) | **M1** |
| | 0.838 or $\dfrac{455}{543}$ | **A1** |
| | | **4** |

| Question | Answer | Marks |
|---|---|---|
| 6(a) | $\text{P}\left(\dfrac{50-54}{6.1}<z<\dfrac{60-54}{6.1}\right)=\text{P}\left(-0.6557<Z<0.9836\right)$ | **M1** |
| | Both values correct | **A1** |
| | $\Phi\,(0.9836)-\Phi\,(-0.6557)=\Phi\,(0.9836)+\Phi\,(0.6557)-1$<br>$=0.8375+0.7441-1$<br>(Correct area) | **M1** |
| | 0.582 | **A1** |
| | | **4** |

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| Question | Answer | Marks |
|---|---|---|
| 6(b) | $\frac{45-\mu}{\sigma} = -0.994$ | **B1** |
| | $\frac{56-\mu}{\sigma} = 1.372$ | **B1** |
| | One appropriate standardisation equation with $\mu, \sigma$, z-value (not probability) and 45 or 56. | **M1** |
| | $11 = 2.366\,\sigma$<br>(**M1** for correct algebraic elimination of $\mu$ or $\sigma$ from *their* two simultaneous equations to form an equation in one variable) | **M1** |
| | $\sigma = 4.65$, $\mu = 49.6$ | **A1** |
| | | **5** |

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| Question | Answer | Marks |
|---|---|---|
| 7(a) | Class widths: 10, 5, 15, 20, 10 | **M1** |
| | Frequency density = frequency/*their* class width: 1.8, 4.8, 2, 1, 0.8 | **M1** |
| | All heights correct on diagram (using a linear scale) | **A1** |
| | Correct bar ends | **B1** |
| | Bar ends: 10.5, 15.5, 30.5, 50.5, 60.5 | **B1** |
| | | **5** |
| 7(b) | 11 – 15 and 31 – 50 | **B1** |
| | Greatest IQR = 50 – 11 = 39 | **B1** |
| | | **2** |
| 7(c) | $\text{Mean} = \frac{18\times5.5+24\times13+30\times23+20\times40.5+8\times55.5}{100} = \frac{2355}{100} = 23.6$ | **B1** |
| | $\text{Var} = \frac{18\times5.5^2+24\times13^2+30\times23^2+20\times40.5^2+8\times55.5^2}{100} - \text{mean}^2$ | **M1** |
| | $\frac{77917.5}{100} - \text{mean}^2 = 224.57$ | **A1** |
| | Standard deviation = 15.0<br>(**FT** *their* variance) | **A1 FT** |
| | | **4** |

Cambridge Assessment International Education

# Cambridge International AS & A Level

**MATHEMATICS** **9709/52**

Paper 5 Probability & Statistics 1 **May/June 2020**

MARK SCHEME

Maximum Mark: 50

**Published**

Students did not sit exam papers in the June 2020 series due to the Covid-19 global pandemic.

This mark scheme is published to support teachers and students and should be read together with the question paper. It shows the requirements of the exam. The answer column of the mark scheme shows the proposed basis on which Examiners would award marks for this exam. Where appropriate, this column also provides the most likely acceptable alternative responses expected from students. Examiners usually review the mark scheme after they have seen student responses and update the mark scheme if appropriate. In the June series, Examiners were unable to consider the acceptability of alternative responses, as there were no student responses to consider.

Mark schemes should usually be read together with the Principal Examiner Report for Teachers. However, because students did not sit exam papers, there is no Principal Examiner Report for Teachers for the June 2020 series.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the June 2020 series for most Cambridge IGCSE™ and Cambridge International A & AS Level components, and some Cambridge O Level components.



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**[Turn over**

**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
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- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
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- marks are not deducted for errors
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GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

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| Mathematics Specific Marking Principles | |
|---|---|
| 1 | Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing. |
| 2 | Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected. |
| 3 | Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points. |
| 4 | Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw). |
| 5 | Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread. |
| 6 | Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear. |

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## Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

### Types of mark

**M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

**A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

**B** Mark for a correct result or statement independent of method marks.

**DM** or **DB** When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

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| WWW | Without Wrong Working |
| AWRT | Answer Which Rounds To |

SAT PREP SP www.satprep.co.in

| Question | Answer | Marks |
|---|---|---|
| 1 | $\sum x - 50n = 144$ | **B1** |
| | $50n + 144 = 944$ | **M1** |
| | $n = 16$ | **A1** |
| | | **3** |

| Question | Answer | Marks |
|---|---|---|
| 2(a) | $\frac{56}{500}$ or $\frac{14}{125}$ or 0.112 | **B1** |
| | | **1** |
| 2(b) | $\mathrm{P(D \mid S)} = \frac{\mathrm{P(D \cap S)}}{\mathrm{P(S)}} = \frac{120}{280}$ | **M1** |
| | $\frac{120}{280}$ or $\frac{3}{7}$ | **A1** |
| | | **2** |

SAT PREP SP www.satprep.co.in

| Question | Answer | Marks |
|---|---|---|
| 2(c) | $P(\text{hockey}) = \frac{220}{500} = 0.44$<br>$P(\text{Amos or Benn}) = \frac{242}{500} = 0.484$<br>$P(\text{hockey} \cap \text{A or B}) = \frac{104}{500} = 0.208$<br>$P(H) \times P(A \cup B) = P(H \cap (A \cup B))$ if independent | **M1** |
| | $\frac{220}{500} \times \frac{242}{500} = \frac{1331}{6250}$ so not independent | **A1** |
| | | **2** |
| | | |

| Question | *Answer* | Marks |
|---|---|---|
| 3(a) | Median = 0.238 | **B1** |
| | UQ = 0.245, LQ = 0.231,<br>So IQR = 0.245 – 0.231 | **M1** |
| | 0.014 | **A1** |
| | | **3** |

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th></tr>
<tr><td rowspan="5">3(b)</td><td>
<table>
<tr><td></td><td></td><td>LQ</td><td>M</td><td>UQ</td><td></td></tr>
<tr><td>A</td><td>0.220</td><td>0.231<br>FT</td><td>0.238<br>FT</td><td>0.245<br>FT</td><td>0.254</td></tr>
<tr><td>B</td><td>0.211</td><td>0.224</td><td>0.232</td><td>0.243</td><td>0.256</td></tr>
</table>
</td><td></td></tr>
<tr><td>Medians and quartiles correctly plotted for A or B</td><td>B1</td></tr>
<tr><td>End points correct for A or B</td><td>B1</td></tr>
<tr><td>Completely correct, including scale</td><td>B1</td></tr>
<tr><td></td><td>3</td></tr>
<tr><td rowspan="3">3(c)</td><td>Lengths of rods produced by machine A are longer.<br>(B1 for comparison of central tendency)</td><td>B1</td></tr>
<tr><td>Lengths of rods produced by machine A are less spread out<br>(B1 for comparison of spread)</td><td>B1</td></tr>
<tr><td></td><td>2</td></tr>
</table>

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| Question | Answer | Marks |
|---|---|---|
| 4(a) | $\mathrm{P}(X < 25) = \mathrm{P}\left(z < \frac{25-40}{12}\right) = \mathrm{P}(z < -1.25)\mathrm{P}(X < 25) = \mathrm{P}(z <)$ | **M1** |
| | $1 - 0.8944$ | **M1** |
| | 0.106 | **A1** |
| | | **3** |
| 4(b) | 0.8944 divided by 3<br>(**M1** for 1 - *their* **(a)** divided by 3) | **M1** |
| | 0.298 **AG** | **A1** |
| | | **2** |
| 4(c) | 0.2981 gives $z = 0.53$ | **B1** |
| | $\frac{h-40}{12} = 0.53$ | **M1** |
| | $h = 46.4$ | **A1** |
| | | **3** |

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| Question | Answer | Marks |
|---|---|---|
| 5(a) | <table><tr><td></td><td>1</td><td>1</td><td>2</td><td>2</td><td>3</td></tr><tr><td>1</td><td>1</td><td>1</td><td>2</td><td>2</td><td>3</td></tr><tr><td>2</td><td>2</td><td>2</td><td>2</td><td>2</td><td>3</td></tr><tr><td>3</td><td>3</td><td>3</td><td>3</td><td>3</td><td>3</td></tr></table> | **M1** |
| | $\frac{7}{15}$ **AG** | **A1** |
| | | **2** |
| 5(b) | <table><tr><td>$x$</td><td>1</td><td>2</td><td>3</td></tr><tr><td>Probability</td><td>$\frac{2}{15}$</td><td>$\frac{6}{15}$</td><td>$\frac{7}{15}$</td></tr></table> | **B1** |
| | P(1) or P(2) correct | **B1** |
| | 3rd probability correct, **FT** sum to 1 | **B1** |
| | | **3** |

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| Question | Answer | Marks |
|---|---|---|
| 5(c) | $\mathrm{E}(X) = \dfrac{2+12+21}{15} = \dfrac{35}{15} = \dfrac{7}{3}$ | **B1** |
| | $\mathrm{Var}(X) = \dfrac{1^2\times 2 + 2^2\times 6 + 3^2\times 7}{15} - \left(\dfrac{7}{3}\right)^2$ | **M1** |
| | $\dfrac{22}{45}(0.489)$ | **A1** |
| | | **3** |
| | | |

| Question | *Answer* | Marks |
|---|---|---|
| 6(a) | $\dfrac{8!}{3!}$ | **M1** |
| | 6720 | **A1** |
| | | **2** |

SAT PREP SP www.satprep.co.in

| Question | Answer | Marks |
|---|---|---|
| 6(b) | Total number = $\frac{10!}{2!3!}$ $(302400)$ (A) | **B1** |
| | With Es together = $\frac{9!}{3!}$ $(60480)$ (B) | **B1** |
| | Es not together = *their* (A) – *their* (B) | **M1** |
| | 241920 | **A1** |
| | **Alternative method for question 6(b)** | |
| | $\frac{8!}{3!}\times\frac{9\times8}{2}$ | |
| | $8! \times k$ in numerator, $k$ integer $\geq 1$, denominator $\geq 1$ | **B1** |
| | $3! \times m$ in denominator, $m$ integer $\geq 1$ | **B1** |
| | *Their* $\frac{8!}{3!}$ Multiplied by ${}^{9}C_{2}$ (OE) only (no additional terms) | **M1** |
| | 241920 | **A1** |
| | | **4** |

| Question | Answer | Marks |
|---|---|---|
| 6(c) | Scenarios:<br>E M M M $^5C_0 = 1$<br>E M M _ $^5C_1 = 5$<br>E M _ _ $^5C_2 = 10$ | **M1** |
| | Summing the number of ways for 2 or 3 correct scenarios | **M1** |
| | Total = 16 | **A1** |
| | | **3** |

| Question | Answer | Marks |
|---|---|---|
| 7(a) | $1 - P(10, 11, 12)$<br>$= 1 - [^{12}C_{10}\,0.72^{10}0.28^2 + {}^{12}C_{11}\,0.72^{11}0.28^1 + 0.72^{12}]$ | **M1** |
| | $1 - (0.19372 + 0.09057 + 0.01941)$ | **A1** |
| | 0.696 | **A1** |
| | | **3** |
| 7(b) | $0.28^3 \times 0.72 = 0.0158$ | **B1** |
| | | **1** |

| Question | Answer | Marks |
|---|---|---|
| 7(c) | Mean = $100 \times 0.72 = 72$<br>Var = $100 \times 0.72 \times 0.28 = 20.16$ | **M1** |
| | P(less than 64) = $\mathrm{P}\left(z < \frac{63.5 - 72}{\sqrt{20.16}}\right)$<br>(**M1** for substituting *their* $\mu$ *and* $\sigma$ into ±standardisation formula with a numerical value for '*63.5*') | **M1** |
| | Using either 63.5 or 64.5 within a ±standardisation formula | **M1** |
| | Appropriate area Φ, from standardisation formula P($z$<…) in final solution<br>= P($z < -1.893$) | **M1** |
| | 0.0292 | **A1** |
| | | **5** |

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Cambridge Assessment International Education

# Cambridge International AS & A Level

**MATHEMATICS** **9709/53**

Paper 5 Probability & Statistics 1 **May/June 2020**

MARK SCHEME

Maximum Mark: 50

**Published**

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| 5 | Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread. |
| 6 | Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear. |

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## Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

### Types of mark

**M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

**A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

**B** Mark for a correct result or statement independent of method marks.

**DM** or **DB** When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

**FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

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**Abbreviations**

| | |
|---|---|
| AEF/OE | Any Equivalent Form (of answer is equally acceptable) / Or Equivalent |
| AG | Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid) |
| CAO | Correct Answer Only (emphasising that no "follow through" from a previous error is allowed) |
| CWO | Correct Working Only |
| ISW | Ignore Subsequent Working |
| SOI | Seen Or Implied |
| SC | Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance) |
| WWW | Without Wrong Working |
| AWRT | Answer Which Rounds To |

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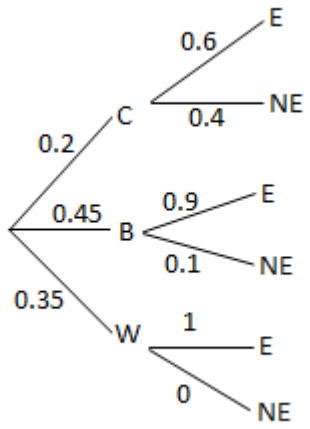
| Question | Answer | Marks |
|---|---|---|
| 1(a) |  | |
| | Fully correct labelled tree for method of transport with correct probabilities. | **B1** |
| | Fully correct labelled branches with correct probabilities for lateness with either 1 branch after W or 2 branches with the probability 0. | **B1** |
| | | **2** |
| 1(b) | $P(C\|E) = \frac{P(C \cap E)}{P(E)} = \frac{0.2\times0.6}{0.2\times0.6+0.45\times0.1+0.35\times1}$ | **M1** |
| | Summing three appropriate 2-factor probabilities | **M1** |
| | $\frac{0.12}{0.515}$ | **A1** |
| | 0.233 or $\frac{12}{515}$ | **A1** |
| | | **4** |

| Question | Answer | Marks |
|---|---|---|
| 2(a) | $0.22^3 = 0.0106$ | **B1** |
| | | **1** |
| 2(b) | $P(2, 3, 4) = {}^{16}C_2\ 0.22^2 0.78^{14} + {}^{16}C_3 0.22^3 0.78^{13} + {}^{16}C_4 0.22^4 0.78^{12}$ | **M1** |
| | $0.179205 + 0.235877 + 0.216221$ | **A1** |
| | 0.631 | **A1** |
| | | **3** |

| Question | Answer | Marks |
|---|---|---|
| 3(a) | $P(X < 21) = P\left(z < \frac{21-15.8}{4.2}\right) = \Phi(1.238)$ | **M1** |
| | 0.892 | **A1** |
| | | **2** |
| 3(b) | $z = \pm 0.674$ | **B1** |
| | $\frac{k-15.8}{4.2} = 0.674$ | **M1** |
| | 18.6 | **A1** |
| | | **3** |

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| Question | Answer | Marks |
|---|---|---|
| 4(a) | <table><tr><td>–1</td><td>0</td><td>0</td><td>1</td></tr><tr><td>0</td><td>1</td><td>1</td><td>2</td></tr><tr><td>2</td><td>3</td><td>3</td><td>4</td></tr></table><br><table><tr><td>$x$</td><td>–1</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>Probability</td><td>$\frac{1}{12}$</td><td>$\frac{3}{12}$</td><td>$\frac{3}{12}$</td><td>$\frac{2}{12}$</td><td>$\frac{2}{12}$</td><td>$\frac{1}{12}$</td></tr></table><br>Probability distribution table with correct scores with at least one probability | **B1** |
| | At least 4 probabilities correct | **B1** |
| | All probabilities correct | **B1** |
| | | **3** |
| 4(b) | $\text{E(X)} = \frac{-1+0+3+4+6+4}{12} = \frac{16}{12} = \frac{4}{3}$ | **B1** |
| | $\text{Var(X)} = \frac{1+0+3+8+18+16}{12} - \left(\frac{4}{3}\right)^2$ | **M1** |
| | $\frac{37}{18}$ $(= 2.06)$ | **A1** |
| | | **3** |

| Question | Answer | Marks |
|---|---|---|
| 5(a) | $\frac{1}{\frac{1}{4}} = 4$ | **B1** |
| | | **1** |
| 5(b) | $\frac{9}{64}$ $(= 0.141)$ | **B1** |
| | | **1** |
| 5(c) | $P(X < 6) = 1 - \left(\frac{3}{4}\right)^5$<br>(**FT** *their* probability/mean from part **(a)**) | **M1** |
| | 0.763 | **A1** |
| | | **2** |
| 5(d) | Mean $= 80 \times 0.25 = 20$<br>Var $= 80 \times 0.25 \times 0.75 = 15$ | **M1** |
| | $P(\text{more than } 25) = P\left(z > \frac{25.5 - 20}{\sqrt{15}}\right)$ | **M1** |
| | $P(z > 1.42)$ | **M1** |
| | $1 - 0.9222$ | **M1** |
| | 0.0778 | **A1** |
| | | **5** |

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| Question | Answer | Marks |
|---|---|---|
| 6(a) | A \| \| B<br>2 \| 6<br>5 2 0 \| 3 \| 0 1 5 8<br>9 7 2 1 1 \| 4 \| 1 2 2 7 9<br>3 2 \| 5 \| 2<br>4 \| 6<br>KEY 1 \| 4 \| 2 means \$41 000 for A and \$42 000 for B | |
| | Correct stem | **B1** |
| | Correct A on LHS | **B1** |
| | Correct B on same diagram | **B1** |
| | Correct key for *their* diagram, both companies identified and correct units | **B1** |
| | | **4** |
| 6(b) | Median = [\$]42 000 | **B1** |
| | LQ = [\$]35 000<br>UQ = [\$]52 000 | **B1** |
| | IQR = [\$]17 000<br>(**FT** if $49000 \leqslant UQ \leqslant 53000 - 32000 \leqslant LQ \leqslant 41000$) | **B1 FT** |
| | | **3** |

| Question | Answer | Marks |
|---|---|---|
| 6(c) | Sum of given 11 numbers is 433 000 | **M1** |
| | Sum of 12 numbers, including new = $38\,500 \times 12 = 462\,000$ | **M1** |
| | Difference = new salary = [$]29 000 | **A1** |
| | | **3** |

| Question | Answer | Marks |
|---|---|---|
| 7(a) | $\frac{9!}{2!2!} = 90\,720$ | **B1** |
| | | **1** |
| 7(b) | $\frac{6!}{2!}$ | **M1** |
| | 360 | **A1** |
| | | **2** |

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| Question | Answer | Marks |
|---|---|---|
| 7(c) | 2 Es together $= \frac{8!}{2!}(= 20160)$ | **M1** |
| | Es not together $= 90720 - 20160 = 70560$ | **M1** |
| | Probability $= \frac{70560}{90720}$ | **M1** |
| | $\frac{7}{9}$ or 0.778 | **A1** |
| | **Alternative method for question 7(c)** | |
| | _ ^ _ ^ _ ^ _ ^ _ ^ _ ^ _ ^ _<br>$\frac{7!}{2!} \times \frac{8 \times 7}{2} = 70560$ | |
| | $7! \times k$ in numerator, $k$ integer $\geqslant 1$, denominator $\geqslant 1$ | **M1** |
| | Multiplying by ${}^{8}C_{2}$ OE | **M1** |
| | Probability $= \frac{70560}{90720}$ | **M1** |
| | $\frac{7}{9}$ or 0.778 | **A1** |
| | | **4** |

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| Question | Answer | Marks |
|---|---|---|
| 7(d) | Scenarios are:<br>E L _ _ _ $^5C_3$ 10<br>E E L _ _ $^5C_2$ 10<br>E _ _ _ _ $^5C_4$ 5<br>E E _ _ _ $^5C_3$ 10 | **M1** |
| | Summing the number of ways for 3 or 4 correct scenarios | **M1** |
| | Total = 35 | **A1** |
| | | **3** |

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Cambridge Assessment International Education

# Cambridge International A Level

**MATHEMATICS** **9709/62**

Paper 6 Probability and Statistics **March 2020**

MARK SCHEME

Maximum Mark: 50

**Published**

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

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**[Turn over**

**PUBLISHED**

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| **Mathematics-Specific Marking Principles** | |
|---|---|
| 1 | Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing. |
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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 1 | $(\lambda =) \frac{5}{12} = 0.417$ or better | **B1** | |
| | $1 - e^{-\frac{5}{12}}(1 + \frac{5}{12})$ | **M1** | $1 - P(X = 0 \text{ or } 1)$, by Poisson, using any $\lambda$, allow $1 - P(X = 0 \text{ or } 1 \text{ or } 2)$ for M1 |
| | = 0.0661 or 0.0662 (3 sf) | **A1** | Final answer<br>**SC** use of Binomial (from 0.06607…) B1 only |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 2 | $2 \times z \times \frac{3.2}{10} = 1.25$ | **M1** | OE Allow without '2 ×' |
| | $z = 1.953$ | **A1** | SOI |
| | $\phi$('*their* 1.953') (= 0.9746) | **M1** | |
| | = 1 – 2(1 – '0.9746')<br>= 0.9492 | **M1** | OE |
| | $\alpha = 94.9$ or 95 | **A1** | CWO |
| | | **5** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 3(a) | est ($\mu$) = 37.6 or $\frac{1504}{40}$ or $\frac{188}{5}$ | **B1** | |
| | est ($\sigma^2$) = $\frac{40}{39}\left[\frac{57760}{40} - 37.6^2\right] = 31.0154 = \frac{2016}{65}$ | **M1** | Correct substitution in any correct formula<br>$\frac{1}{39}\left[57760 - \frac{1504^2}{40}\right]$ |
| | = 31.(0) (3 sf) | **A1** | Accept $\frac{2016}{65}$ or $31\frac{1}{65}$ |
| | | **3** | |
| 3(b) | $H_0$: Pop mean (or $\mu$) = 39.2<br>$H_1$: Pop mean (or $\mu$) < 39.2 | **B1** | Both. Not just 'mean' |
| | $\frac{'37.6' - 39.2}{\frac{\sqrt{'31.0154'}}{\sqrt{40}}}$ | **M1** | Allow use of biased variance (30.2), must have √40 |
| | = –1.817 | **A1** | **SC** FT use of biased = –1.840 for A1 |
| | '1.817' > 1.645 OE | **M1** | Valid comparison '*their* 1.817' with 1.645 or valid area comparison 0.0346 < 0.05 OE |
| | There is evidence that mean time has decreased | **A1FT** | FT *their* 1.817; in context, not definite, no contradictions<br>**SC** For 2 tail test: $H_1$: $\mu \neq 39.2$ and comp 1.96, max B0M1A1M1A0 (no FT for final mark) |
| | | **5** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4(a) | $\lambda$ (= 0.4 × 365 ÷ 50) = 2.92 | **B1** | |
| | $e^{-2.92}(1 + 2.92 + \frac{2.92^2}{2})$ | **M1** | Any $\lambda$. Allow one end error |
| | = 0.441 (3 sf) | **A1** | |
| | | **3** | |
| 4(b) | $e^{-\lambda} > 0.95$ | **M1** | Allow '=' throughout |
| | $-\lambda >$ ln 0.95 or $\lambda < 0.051293$ OE | **M1** | Attempt ln both sides |
| | '0.051293' × 50 ÷ 0.4 (= 6.411) | **M1** | |
| | Largest $n$ is 6 (3 sf)<br>Allow $n = 6$ or $n \leqslant 6$ (NOT $n < 6$ or $n \geqslant 6$ as final answer) | **A1** | **SC** Trial and Improvement<br>M1 for $e^{-\lambda} > 0.95$ SOI; M1 for $\lambda = n \times \frac{0.4}{50}$; M1 for use of both $n = 6$ giving 0.9531 and $n = 7$ giving 0.9455; A1 $n = 6$ |
| | | **4** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5(a) | $\frac{3}{4000}\int_5^{10}(100-x^2)\mathrm{d}x$<br>$=\frac{3}{4000}[100x - \frac{x^3}{3}]_5^{10}$ | **M1** | Attempt integration of f($x$), ignore limits. Condone omission of $\frac{3}{4000}$ |
| | $=\frac{3}{4000}(1000 - \frac{1000}{3} - 500 + \frac{125}{3})$ | **M1** | Correct limits 5 and 10. OE SOI |
| | = 0.156 (3 sf) or $\frac{5}{32}$ | **A1** | For fully correct working seen including substitution of limits |
| | | **3** | |
| 5(b) | $\frac{3}{4000}\int_p^{10}(100-x^2)\mathrm{d}x = \frac{1}{4}$ | **M1** | Attempt integration of f($x$) with any limits and $=\frac{1}{4}$ or $=\frac{3}{4}$ seen. Condone omission of $\frac{3}{4000}$ |
| | $\frac{3}{4000}[100x - \frac{x^3}{3}]_p^{10} = \frac{1}{4}$ | **A1** | Correct integration with correct limits seen (or implied for limits p and 10) and $= \frac{1}{4}$ OE<br>Condone omission of $\frac{3}{4000}$ |
| | $\frac{3}{4000}(1000 - \frac{1000}{3} - 100p + \frac{p^3}{3}) = \frac{1}{4}$ | **M1** | Attempt substitution correct limits in their integration of f($x$). Accept limits 0 to $p$ if clearly seen, accept limits –10 and $p$. Substitution must be seen. |
| | e.g. $\frac{2000}{3} - 100p + \frac{p^3}{3} = \frac{1000}{3}$<br>$p^3 - 300p + 1000 = 0$ | **A1** | AG<br>No errors seen |
| | | **4** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5(c) | Curve is symmetrical about $x = 0$ | **B1** | May be implied by sketch. No contradictions<br>or integrate f($x$) between $-q$ and $+q$ and equate to 0.5 leading to $q^3 - 300q + 1000 = 0$ oe |
| | $q = 3.47$ | **B1** | |
| | | **2** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(a) | N(310, 50) | **B1** | SOI |
| | $\frac{300-'310'}{\sqrt{'50'}}$ (= –1.414) | **M1** | Standardise using their values |
| | Φ(‘–1.414’) = 1 – ϕ(‘1.414’) | **M1** | Area consistent with their values |
| | = 0.0786 or 0.0787 (3 sf) | **A1** | As final answer |
| | | **4** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(b) | P($L - 2S > 0$ ) | **M1** | OE SOI |
| | E($X$) = 200-2x110 or = – 20 | **B1** | OE seen |
| | Var = $30 + 2^2 \times 20$ or = 110 | **B1** | Seen |
| | N(–20, 110)<br>$\frac{0-('-20')}{\sqrt{'110'}}$ (= 1.907) | **M1** | Standardising with their values. Mean and variance must come from a combination attempt. |
| | 1 – Φ(‘1.907’) | **M1** | Correct area consistent with their working |
| | = 0.0283 (3 sf) | **A1** | Final answer |
| | | **6** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 7(a) | P($X \leqslant n$) ($n \leqslant 20$) attempted, using B(20, 0.95) | **M1** | OE |
| | P($X \leqslant 17$) <u>or</u> P($X \leqslant 16$) attempted, using B(20, 0.95) | **M1** | OE |
| | (P($X \leqslant 17$)) = 0.0755 <u>and</u> (P($X \leqslant 16$)) = 0.0159 | **A1** | OE (0.925 and 0.984) both correct |
| | Rej region is $X \leqslant 16$ or X < 17 | **A1** | Dependent on M1M1 and previous answers correct to at least 0.075/0.076 and 0.016 **or** 0.92/0.93 and 0.98<br>Correct unsupported answers of 0.0755 and 0.0159 OE scores M1 M1 A0 |
| | | **4** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 7(b) | 0.0159 | **B1** | FT *their* rejection region, from Binomial in $a$, if P($X$ in rejection region) < 0.025 |
| | | **1** | |
| 7(c) | Use of B(20, 0.7) | **M1** | |
| | P($X > 16 \mid p = 0.7$) | **M1** | Correct method using B(20, 0.7) |
| | = 0.107 | **A1** | |
| | | **3** | |

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Cambridge International A Level

**Cambridge Assessment International Education**
Cambridge International Advanced Level

**MATHEMATICS** **9709/72**

Paper 7 **October/November 2019**

MARK SCHEME

Maximum Mark: 50

**Published**

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**M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

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**DM** or **DB** When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

**FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

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**Abbreviations**

| | |
|---|---|
| AEF/OE | Any Equivalent Form (of answer is equally acceptable) / Or Equivalent |
| AG | Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid) |
| CAO | Correct Answer Only (emphasising that no “follow through” from a previous error is allowed) |
| CWO | Correct Working Only |
| ISW | Ignore Subsequent Working |
| SOI | Seen Or Implied |
| SC | Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance) |
| WWW | Without Wrong Working |
| AWRT | Answer Which Rounds To |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 1(i) | Binomial | **B1** | |
| | $n = 500$ **and** $p = \frac{1}{150}$ or 0.00667 | **B1** | Or $B\left(500, \frac{1}{150}\right)$ for B1B1 |
| | | **2** | |
| 1(ii) | Poisson | **B1** | |
| | $n$ large and mean = $\frac{10}{3}$ or 3.3 or better, which is < 5 | **B1** | Accept $n > 50$ |
| | | **2** | |
| 1(iii) | $1 - e^{-\frac{10}{3}} \times \left(1 + \frac{10}{3} + \frac{\left(\frac{10}{3}\right)^2}{2}\right)$ | **M1** | 1-P($X$ = 0, 1, 2) |
| | = 1 – 0.353 | **A1** | Correct expression with λ=3.3 or better |
| | = 0.647 (3 sf) | **A1** | **SC** Use of Binomial scores B1 for 0.648. Use of Normal scores B1 for 0.67(0) to 0.677 |
| | | **3** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 2(i)(a) | Assume standard deviation for the region is 7.1 | **B1** | Or standard deviation is same as for whole population OE |
| | $\frac{63.2-65.2}{\frac{7.1}{\sqrt{n}}} = -2.182$ | **M1** | Attempt to find correct equation (accept +2.182) |
| | $n = \{-2.182 \times 7.1 \div (-2)\}^2$ | **A1** | Any correct expression for $n$ or $\sqrt{n}$ . SOI |
| | $n = 60$ | **A1** | CWO. Must be an integer |
| | | **4** | |
| 2(i)(b) | $H_0$: population mean (or $\mu$) = 65.2<br>$H_1$: population mean (or $\mu$) < 65.2 | **B1** | Not just 'mean' |
| | 2.182 > 1.751 | **M1** | Or valid area comparison. |
| | There is evidence that animals are shorter in this region | **A1** | CWO. No contradictions |
| | | **3** | |
| 2(ii) | Population unknown or population not given as normal | **B1** | Allow population not normal. Accept distribution of X unknown. |
| | | **1** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 3(i) | $\text{est}(\mu) = \frac{25110}{50}$ (= 502.2) | **B1** | |
| | $\text{est}\left(\sigma^2\right) = \frac{50}{49}\left(\frac{12610300}{50} - \frac{25110}{50}\right)^2 \left(= \frac{50}{49} \times \frac{58}{50} = 1.1836\right)$ | **M1** | OE |
| | 1.18 (3 sf) or $\frac{58}{49}$ | **A1** | Accept SD = 1.0879 |
| | $z$ = 2.054 or 2.055 | **B1** | |
| | $502.2 \pm z \times \frac{\sqrt{'1.1836'}}{\sqrt{50}}$ | **M1** | Must be of correct form. |
| | 501.9 to 502.5 (1dp) | **A1** | CWO. Must be in interval.<br>**SC** accept use of biased variance (1.16) for M1 A1 |
| | | **6** | |
| 3(ii) | More confident **or** $z$ would be greater,<br>Hence wider. | **B1** | OE<br>Reason needed |
| | | **1** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4(i) | $\frac{1}{2}\times a\times\frac{a}{2}=1$ **or** $\frac{1}{2}\int_0^a x\,\mathrm{d}x=1$<br>$\frac{a^2}{4}=1$ OE | **M1** | Attempt at triangle area or integral f($x$) **and** $=1$, |
| | $a=2$ | **A1** | |
| | | **2** | |
| 4(ii) | $\frac{1}{2}\int_0^2 x^2\,\mathrm{d}x$ | **M1** | Attempt integral $x$f($x$) |
| | $=\left[\frac{x^3}{6}\right]_0^2$ | **M1** | Correct integral and limits 0 to their 'a' |
| | $\left(=\frac{8}{6}\right)=\frac{4}{3}$ | **A1** | AG<br>CWO |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4(iii) | $P\left(X<\frac{4}{3}\right)=\frac{1}{2}\int_0^{\frac{4}{3}} x\,dx$ | **M1** | Attempt integral f($x$) between correct limits |
| | $=\frac{4}{9}$ | **A1** | or $\frac{5}{9}$ |
| | $P\left(E(X)<X<m\right)=\frac{1}{2}-\text{'}\frac{4}{9}\text{'}$ | **M1** | or $\frac{5}{9}-\frac{1}{2}$ |
| | $\frac{1}{18}$ | **A1** | |
| | **Alternative method for question 4(iii)** | | |
| | Attempt to find $m$ | **M1** | |
| | $m=\sqrt{2}$ | **A1** | |
| | Integrate f($x$) between $\frac{4}{3}$ and $\text{'}\sqrt{2}\text{'}$ | **M1** | |
| | $\frac{1}{18}$ | **A1** | |
| | | **4** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5(i) | mean = 3250 var. = 61 | **B1** | Or mean = 325 var. $= \frac{6.1}{10}$ |
| | $\frac{3240-3250}{\sqrt{61}}\left(=-1.280\right)$ | **M1** | Standardise with their values (no mixed methods) |
| | $\phi$('–1.280') = 1 – $\phi$('1.280) | **M1** | Area consistent with their figures |
| | 0.100 | **A1** | Allow 0.1 |
| | | **4** | |
| 5(ii) | E(D) = 325 – 2 × 167 = –9 | **B1** | Accept $\pm 9$ |
| | Var(D) = 6.1 + $2^2$ × 5.6 (= 28.5) | **B1** | |
| | $\frac{0-\left(-9\right)}{\sqrt{28.5}}\left(=1.686\right)$ | **M1** | Standardising with *their* values. Must have a combination attempt on denominator and $\sqrt{}$ |
| | 1 – $\phi$('1.686') | **M1** | Area consistent with their figures |
| | 0.0459 | **A1** | |
| | | **5** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(i) | $H_0$: Pop mean (or $\lambda$ or $\mu$) is 1.1<br>$H_1$: Pop mean (or $\lambda$ or $\mu$) is more than 1.1 | **B1** | |
| | $P(X \geqslant 4) = 1 - e^{-1.1}\left(1 + 1.1 + \frac{1.1^2}{2} + \frac{1.1^3}{3!}\right)$ | **M1** | Correct expression for either $P(X \geqslant 4)$ or $P(X \geqslant 5)$ |
| | 0.0257 | **A1** | Correct value of either $P(X \geqslant 4)$ or $P(X \geqslant 5)$ |
| | $P(X \geqslant 5) = 0.0257 - e^{-1.1} \times \frac{1.1^4}{4!} = 0.00544$ | **B1** | B1 for the other value<br>(Note use of $P(X < 4) = 0.9743$ and $P(X < 5) = 0.99456$ can score only if comparison with 0.99 seen) |
| | $0.00544 < 0.01 < 0.0257$ | **M1** | OE stated (valid comparison) |
| | There is evidence mean has increased | **B1** | **SC** $P(X \geqslant 6) = 0.000968$ M1A1<br>Conclusion B1 |
| | | **6** | |
| 6(ii) | Concluding mean has increased when it has not | **B1** | In context |
| | '0.00544' | **B1FT** | FT *their* $P(X \geqslant 5)$, dep < 0.01 |
| | | **2** | |
| 6(iii) | $e^{-7.0}\left(1 + 7 + \frac{7^2}{2} + \frac{7^3}{3!} + \frac{7^4}{4!}\right)$ | **M1** | Correct expression for $P(X \leqslant 4 \mid \lambda = 7.0)$ |
| | 0.173 (3 sf) | **A1** | |
| | | **2** | |

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Cambridge International A Level

**Cambridge Assessment International Education**
Cambridge International Advanced Level

**MATHEMATICS** **9709/73**

Paper 7 **October/November 2019**

MARK SCHEME

Maximum Mark: 50

**Published**

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

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**PUBLISHED**

## Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
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- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
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Marks must be awarded **positively**:

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- marks are awarded when candidates clearly demonstrate what they know and can do
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- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

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## Mark Scheme Notes

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### Types of mark

**M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

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| WWW | Without Wrong Working |
| AWRT | Answer Which Rounds To |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 1(i) | 9.6, 12.4 | **B1 B1** | |
| | | **2** | |
| 1(ii) | 6.6, 49.6 | **B1 B1** | |
| | | **2** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 2(i) | $\left(\lambda\left(=2\times2.4\right)=4.8\right)$<br>$e^{-4.8}\left(1+4+\frac{4.8^2}{2}+\frac{4.8^3}{3!}\right)$ | **M1** | Any $\lambda$ |
| | 0.294 (3 sf) | **A1** | |
| | | **2** | |
| 2(ii) | $\left(\lambda\left(=60\times2.4\right)=144\right)$<br>N('144', '144') | **M1** | N and $\sigma^2=\mu$ SOI |
| | $\frac{139.5-'144'}{\sqrt{'144'}}\left(=-0.375\right)$ | **M1** | Allow with no continuity correction |
| | $\phi$('0.375') | **M1** | Correct area consistent with their working |
| | 0.646 (3 sf) | **A1** | |
| | | **4** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 3(i) | Assume population is normally distributed | **B1** | |
| | $\bar{x} = 25.9$ | **B1** | Allow $\frac{259}{10}$ |
| | $z = 2.17$ | **B1** | |
| | $'25.9' \pm z \times \frac{3}{\sqrt{10}}$ | **M1** | Must have correct form and $z$. |
| | 23.8 to 28.0 (3 sf) | **A1** | CWO |
| | | **5** | |
| 3(ii) | $0.03^2$ (=0.0009) | **B1** | |
| | | **1** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4 | Assume trains are independent OR probability of being on time is constant | **B1** | Must be in context |
| | $H_0$: P(on time)=0.92<br>$H_1$: P(on time)<0.92 | **B1** | Both.<br>Allow '$p$' or $\pi$ |
| | $1 - \left( {}^{20}C_{17} \times 0.92^{17} \times 0.08^3 + {}^{20}C_{18} \times 0.92^{18} \times 0.08^2 + 20 \times 0.92^{19} \times 0.08 + 0.92^{20} \right)$ | **M1** | Allow one end error<br>Must have 1 – … |
| | =0.0706 (3 sf) | **A1** | |
| | Compare with 0.05 | **M1** | Valid comparison needed |
| | No evidence that percentage less than 92% | **A1FT** | OE<br>No contradictions.<br><br><u>Method using normal approximation:</u><br>If the first B1B1 is earned then:<br>$\text{CV} - 1.566 \left( \text{from} \dfrac{16.5 - 20 \times 0.92}{\sqrt{20 \times 0.92 \times 0.08}}, \text{with continuity correction} \right)$<br>**or**<br>CV=1.978 (without continuity correction)<br><br>comp $z$=1.645<br>No evidence that % decreased (1.566) or evidence that % decreased (1.978) is awarded **SC2** after B marks |
| | | **6** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5(i) | Po(3) | **B1** | SOI |
| | $e^{-3}\left(\frac{3^3}{3!}+\frac{3^4}{4!}+\frac{3^5}{5!}\right)$ | **M1** | Allow one or two extra terms (2 or 6 or both) |
| | 0.493 (3 sf) | **A1** | |
| | | **3** | |
| 5(ii) | A correct equation from $P(0)=P(2)$ $\left(\text{leading to } 1=\frac{\lambda^2}{2}\right)$ | **M1** | |
| | $\lambda=\sqrt{2}$ or 1.41 (3 sf) | **A1** | CWO |
| | | **2** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5(iii)(a) | Correct inequality $\left(\text{leading to } \frac{5.2^n}{n!} < \frac{5.2^{n+1}}{(n+1)!}\right)$ | **B1** | |
| | | **1** | |
| 5(iii)(b) | $n+1 < 5.2$ or $1 < \frac{5.2}{n+1}$ | **M1** | Simplify to a stage without exponentials, powers or factorials. |
| | Largest $n$ is 4 | **A1** | |
| | | **2** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(i) | $k\int_0^3 \left(3x - x^2\right) \mathrm{d}x = 1$ | **M1** | Attempt to integrate f($x$) and = 1 |
| | $k\left[\frac{3}{2}x^2 - \frac{x^3}{3}\right]_0^3$<br>$k\left(\frac{27}{2} - \frac{27}{3}\right) = 1$ | **A1** | Correct integral and limits |
| | $k = \frac{2}{9}$ | **A1** | AG<br>No errors seen |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(ii) | $\frac{2}{9}\int_1^2 \left(3x - x^2\right)\mathrm{d}x = \frac{2}{9}\left[\frac{3}{2}x^2 - \frac{x^3}{3}\right]_1^2 = \frac{2}{9}\times\left(6 - \frac{8}{3} - \frac{3}{2} + \frac{1}{3}\right)$ | **M1** | Attempt to integrate f($x$) d$x$ with limits 1 and 2<br>OE |
| | $\frac{13}{27}$ or 0.481 (3 sf) | **A1** | |
| | | **2** | |
| 6(iii) | $y = 3x - x^2$ symmetrical about $x = \frac{3}{2}$ | **M1** | Attempt $\frac{2}{9}\int_0^3 (3x^2 - x^3)\,\mathrm{d}x$ |
| | E($X$)= $\frac{3}{2}$ | **A1** | |
| | $\frac{2}{9}\int_0^3 (3x^3 - x^4)\,\mathrm{d}x$ | **M1** | Attempt to integrate $x^2$f($x$) |
| | $= \frac{2}{9}\left[\frac{3x^4}{4} - \frac{x^5}{5}\right]_0^3 \left(= \frac{2}{9}\times\frac{243}{20} = \frac{27}{10}\right)$<br>$\frac{'27'}{10} - \left(\frac{'3'}{2}\right)^2$ | **M1** | Subtract their (E($X$))$^2$ from their integral $x^2$f($x$) with correct limits substituted |
| | $\frac{9}{20}$ or 0.45 | **A1** | |
| | | **5** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 7(i) | $H_0$: Pop mean=546<br>$H_1$: Pop mean>546 | **B1** | Both. Allow just $\mu$, but not just 'mean' |
| | $\frac{581-546}{\frac{120}{\sqrt{40}}}$ | **M1** | Standardising. Need $\frac{120}{\sqrt{40}}$ |
| | =1.845 allow 1.844 | **A1** | Allow 1.84 or 1.85 AWRT |
| | 1.845<1.96 | **M1** | OE. Or area comparison 0.0325>0.025 or large probabilities |
| | No evidence that mean weekly income has increased | **A1FT** | No contradictions.<br>If $H_1$: ≠, and 2.241 used, max B0M1A1M1A0 |
| | | **5** | |
| 7(ii) | $\frac{a-546}{\frac{120}{\sqrt{40}}}=1.96$ | **M1** | Standardise to find $a$. Need $\frac{120}{\sqrt{40}}$ **and** 546 **and** a value of $z$ |
| | $a$ = 583.19 | **A1** | Allow 583 to 3sf |
| | $\frac{'583.19'-595}{\frac{120}{\sqrt{40}}}(=-0.622)$ | **M1** | Standardise. Need $\frac{120}{\sqrt{40}}$ and 595 |
| | ϕ('–0.622')=1 – ϕ('0.622') | **M1** | Consistent area |
| | 0.267 | **A1** | |
| | | **5** | |

Cambridge International A Level

**Cambridge Assessment International Education**
Cambridge International Advanced Level

**MATHEMATICS** **9709/71**

Paper 7 **May/June 2019**

MARK SCHEME

Maximum Mark: 50

**Published**

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- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

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| |
|---|
| GENERIC MARKING PRINCIPLE 5:<br><br>Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen). |
| GENERIC MARKING PRINCIPLE 6:<br><br>Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind. |

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**Mark Scheme Notes**

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

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- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

  - Note: B2 or A2 means that the candidate can earn 2 or 0.
    B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking $g$ equal to 9.8 or 9.81 instead of 10.

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The following abbreviations may be used in a mark scheme or used on the scripts:

AEF/OE Any Equivalent Form (of answer is equally acceptable) / Or Equivalent

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

SOI Seen or implied

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

**Penalties**

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 1(i) | Mean = 115 | **B1** | |
| | SD = 40 | **B1** | |
| | | **2** | |
| 1(ii) | Mean = 15 × '115' = 1725 | **B1ft** | |
| | $15 \times$ '40'$^2$ (= 24000) | **M1** | or SD = √15 × '40'. ft their **(i)** |
| | SD = √24000<br>SD = 155 (cents) (3 sf) | **A1** | Accept √24000 SC: Allow correct answers in dollars |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 2(i) | Assume sd still 4.8 or is unchanged | **B1** | or Assume the 150 times can be treated as a random sample / are independent |
| | $H_0$: Pop mean = 26.5<br>$H_1$: Pop mean > 26.5 | **B1** | Allow '$\mu$' but not just 'mean' |
| | $\frac{27.5-26.5}{\frac{4.8}{\sqrt{150}}}$ | **M1** | Standardise, with √<br>Accept CV method |
| | = 2.552 | **A1** | |
| | Comp with $z$-value<br>'2.552' > 2.326 | **M1** | or comp 1 – Φ('2.552') with 0.01<br>1 – 0.9946 = 0.0054 < 0.01 |
| | There is evidence time has increased | **A1ft** | oe No contradictions<br>(2 tail test scores max. B1 B0 M1 A1 M1 (for comparison with 2.576) A0 no ft) |
| | | **6** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 2(ii) | No because pop is normal so distr of $\bar{X}$ is normal | **B1** | Condone just 'No because pop is normal' |
| | | **1** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 3(i) | $H_0$: $P(6) = \frac{1}{6}$<br>$H_1$: $P(6) < \frac{1}{6}$ | **B1** | |
| | $(\frac{5}{6})^{30} + 30(\frac{1}{6}) \times (\frac{5}{6})^{29} + {}^{30}C_2(\frac{1}{6})^2 \times (\frac{5}{6})^{28}$ | **M1** | Allow one term incorrect, omitted or extra |
| | = 0.103 | **A1** | |
| | '0.103' > 0.05 | **M1** | |
| | No evidence (at 5% level) that die biased | **A1ft** | oe No contradictions |
| | | **5** | |
| 3(ii) | $(\frac{5}{6})^{30} + 30(\frac{1}{6}) \times (\frac{5}{6})^{29}$ | **M1** | |
| | P(Type I) = 0.0295 | **A1** | |
| | | **2** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4(a)(i) | $0.5 \times 1/a = (\frac{0.5}{a})$ | **M1** | Or attempt to integrate f($x$) (=1/a) between 0 and 0.5 |
| | $= \frac{1}{2a}$ oe | **A1** | Accept 0.5/a for A1 |
| | | **2** | |
| 4(a)(ii) | $\frac{a}{2}$ | **B1** | |
| | | **1** | |
| 4(a)(iii) | $\int_0^a \frac{x^2}{a} dx - (\text{`}\frac{a}{2}\text{'})^2$ | **M1** | Integ their $x^2$f($x$) from 0 to $a$ and sub their mean$^2$ |
| | $\text{Var}(X) = \frac{a^2}{3} - \frac{a^2}{4}$<br>$(\text{Var}(X) = \frac{a^2}{12}$ **AG**) | **A1** | Must see this line oe |
| | | **2** | |
| 4(b) | $\int_2^b \frac{3}{2(t-1)^2} dt$ | **M1** | Attempt integ g($t$) ignore limits |
| | $\left[-\frac{3}{2(t-1)}\right]_2^b$ | **A1** | Correct integral |
| | $-\frac{3}{2}(\frac{1}{(b-1)} - 1) = \frac{3}{4}$<br>$(1 - \frac{1}{(b-1)} = \frac{1}{2})$ | **M1** | Attempt subst correct limits in their integ and = $\frac{3}{4}$ |
| | $b = 3$ | **A1** | |
| | | **4** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5(a)(i) | $e^{-2.3}(\frac{2.3^2}{2} + \frac{2.3^3}{3!} + \frac{2.3^4}{4!}$ | **M1** | Allow one end error |
| | = 0.585 | **A1** | |
| | | **2** | |
| 5(a)(ii) | $(\lambda) = 4.6$ | **B1** | |
| | $1 - e^{-4.6}(1 + 4.6 + \frac{4.6^2}{2})$ | **M1** | any $\lambda$, Allow one end error |
| | = 0.837 (3 sf) | **A1** | |
| | | **3** | |
| 5(a)(iii) | $S \sim N(115, 115)$ | **B1** | May be implied |
| | $\frac{110.5-115}{\sqrt{115}}$ (= –0.420) | **M1** | Allow with wrong or no cc OR no √ |
| | 1 – Φ('0.420') (= 1 – 0.663) | **M1** | |
| | = 0.337 | **A1** | Accept alternative method using N(2.3, 2.3) no mixed methods. |
| | | **4** | |
| 5(b) | $e^{-\lambda} \times \frac{\lambda^3}{3!} = e^{-\lambda} \times \frac{\lambda^5}{5!}$ | **M1** | |
| | $\lambda^3 = \frac{\lambda^5}{4\times5}$ or $\lambda^2 = 20$ oe | **A1** | any correct simplification without $e^{-\lambda}$ or ! |
| | $\lambda = \sqrt{20}$ or $2\sqrt{5}$ or 4.47 (3 sf) | **A1** | |
| | | **3** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(i) | Biased towards people who like tennis<br>Excludes people who don't like tennis | **B1** | or other sensible |
| | | **1** | |
| 6(ii) | Obtain a list of all people in the town | **B1** | |
| | Use random numbers | **B1** | or, e.g. pick numbers from a hat or other sensible |
| | | **2** | |
| 6(iii) | Var($p$) = $\frac{\frac{47}{350}(1-\frac{47}{350})}{350}$ (= 0.000332152) | **M1** | |
| | $z$ = 1.645 | **B1** | |
| | $\frac{47}{350} \pm z\sqrt{\frac{\frac{47}{350}(1-\frac{47}{350})}{350}}$ | **M1** | Must be a $z$ value |
| | 0.104 to 0.164 (3 sf) | **A1** | Must be an interval |
| | | **4** | |
| 6(iv) | 1.25 × 1.645 (= 2.056) | **M1** | or 1.25 × their width ÷ 2 ÷ their $\sqrt{\frac{\frac{47}{350}(1-\frac{47}{350})}{350}}$ (Complete method) |
| | Φ('2.056') (= 0.980) | **M1** | Attempt Φ(their $z$) |
| | $x$ = 96 (2 sf) | **A1** | Allow 0.96 (2 sf) CWO |
| | | **3** | |

Cambridge International A Level

**Cambridge Assessment International Education**
Cambridge International Advanced Level

**MATHEMATICS** **9709/72**

Paper 7 **May/June 2019**

MARK SCHEME

Maximum Mark: 50

**Published**

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**PUBLISHED**

## Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

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- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

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- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

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| |
|---|
| GENERIC MARKING PRINCIPLE 5:<br><br>Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen). |
| GENERIC MARKING PRINCIPLE 6:<br><br>Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind. |

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**Mark Scheme Notes**

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

  - Note: B2 or A2 means that the candidate can earn 2 or 0.
    B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 1(i) | 0.0842 (3 sf) | **B1** | |
| | | **1** | |
| 1(ii) | $e^{-5} \times \frac{5^n}{n!} = e^{-5} \times \frac{5^{n+1}}{(n+1)!}$ | **B1** | or $\frac{5^n}{n!} = \frac{5^{n+1}}{(n+1)!}$ or better<br>ISW |
| | | **1** | |
| 1(iii) | $1 = \frac{5}{n+1}$<br>$n = 4$ | **B1** | |
| | | **1** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 2(i) | Normal with mean 372 | **B1** | |
| | sd = $\frac{54}{\sqrt{36}}$ | **M1** | or variance = $\frac{54^2}{36}$ M1 |
| | (= 9) | **A1** | (= 81) A1 |
| | | **3** | |
| 2(ii) | Pop normal | **B1** | Allow $X$ is normal |
| | | **1** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 3(i) | Est($\mu$) = 1.85 | **B1** | |
| | Est($\sigma^2$) = $\frac{50}{49}\left(\frac{175.25}{50} - '1.85'^2\right)$ | **M1** | Allow $\sqrt{\frac{50}{49}\left(\frac{175.25}{150} - '1.85'^2\right)}$ or 0.0290 for M1 |
| | = 0.0842 (3 sf) or $\frac{33}{392}$ | **A1** | Cao<br>If $\frac{50}{49}$ omitted (giving var = 0.0825 or sd = 0.287) M0A0 |
| | | **3** | |
| 3(ii) | $H_0$: Pop mean time = 1.9 (h)<br>$H_1$: Pop mean time < 1.9 (h) | **B1** | Allow '$\mu$' but not just 'mean' |
| | $\pm\frac{1.85-1.9}{\sqrt{\frac{'0.0842'}{50}}}$ | **M1** | $\pm\frac{1.85-1.9}{\frac{'0.290'}{\sqrt{50}}}$ Accept totals method (92.5–95) / $\sqrt{4.21}$ |
| | = –1.22 | **A1** | = –1.22 |
| | comp $z$ = –1.645 | **M1** | Or other valid comparison 0.888 or 0.889 < 0.95 OR 0.111 or 0.112>0.05 |
| | No evidence that mean time < 1.9 h | **A1** | **FT** their z. Correct conclusion. No contradictions<br>If $\frac{50}{49}$ not used in (1): var = 0.8225, sd = 0.907, cr = 1.17 can score all marks in **(ii)**<br>Note- 2 tail test can score B0 M1 A1 M1 (comparison with 1.96) A0 (no ft) max3/5 |
| | | **5** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4 | Use of $1.5X_1 - X_2$ or similar | **B1** | |
| | $E(1.5X_1 - X_2) = 1.5(110) - 110$ (= 55) | **B1** | or $E(X_1 - 1.5X_2) = 110 - 1.5(110)$ (= –55) |
| | $Var(1.5X_1 - X_2) = 1.5^2 \times 1050 + 1050$ (or 3412.5) | **M1** | Correct expression or result |
| | $\frac{0-55}{\sqrt{'3412.5'}}$ or $\frac{0-(-55)}{\sqrt{'3412.5'}}$ (= ± 0.942) | **M1** | Their '55'. Allow incorrect var (dep > 0 and ≠ 1050) |
| | $1 - \Phi$('0.942') | **M1** | Area consistent with their working |
| | = 0.173 | **A1** | |
| | Ans 0.346 (3 sf) | **B1** | **FT** double their prob (must be <1) |
| | | **7** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5(i) | $H_0$: $p = 0.1$<br>$H_1$: $p < 0.1$ | **B1** | |
| | | **1** | |
| 5(ii) | B(40, 0.1) stated or implied by use of | **B1** | e.g. by ${}^{40}C_x$ or $0.9^p \times 0.1^q$ ($p + q = 40$) |
| | $0.9^{40} + 40 \times 0.9^{39} \times 0.1$ | **M1** | Correct working (if seen). If working not seen, M1 may be implied by 0.0805 |
| | = 0.0805 | **A1** | |
| | | **3** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5(iii) | $z = 1.645$ | **B1** | seen |
| | $\frac{6}{80} \pm z\sqrt{\frac{\frac{6}{80}\times\frac{(80-6)}{80}}{80}}$ | **M1** | Formula of correct form. Must be a 'z' |
| | = 0.0266 to 0.123 (3 sfs) | **A1** | Allow 0.03 to 0.12 or better<br>Must be an interval |
| | | **3** | |
| 5(iv) | 10% (or manufacturer's claim) is within CI<br>Hence no reason to question claim | **B1** | **FT** Allow '10% is within CI, accept claim' oe<br>Must include both parts. No contradictions.<br>**FT** their CI<br>Note if CI is centred on 0.1 allow ft 0.075 is within CI, accept claim |
| | | **1** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(i) | $a\int_1^b \frac{1}{x^2}dx = 1$ | **M1** | Attempt int f($x$) and = 1, ignore limits |
| | $a[-\frac{1}{x}]_1^b = 1$ | **A1** | correct integ and limits = 1 |
| | $a[1-\frac{1}{b}] = 1$ or $a\times\frac{b-1}{b} = 1$<br>$b = \frac{a}{a-1}$ **AG** | **A1** | No errors seen |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(ii) | $a\int_1^{\frac{3}{2}} \frac{1}{x^2}dx = \frac{1}{2}$<br>$a\,[-\frac{1}{x}]_1^{\frac{3}{2}} = \frac{1}{2}$ | **M1** | Attempt int f($x$) with limits 1 to $\frac{3}{2}$ and = $\frac{1}{2}$ |
| | $a\,[1 - \frac{2}{3}] = \frac{1}{2}$ | **A1** | oe correct equn in $a$ |
| | $a = \frac{3}{2}$, $b = 3$ | **A1** | Both |
| | | **3** | |
| 6(iii) | $\frac{3}{2}\int_1^3 \frac{1}{x}dx$ | **M1** | Attempt int $x$f($x$), ignore limits – condone missing a |
| | $= \frac{3}{2}\,[\ln x]_1^3$ | **A1** | **FT** Correct integ and *their* limits 1 to b – condone missing a |
| | $= \frac{3}{2}\ln 3$ or 1.65 (3 sf) | **A1** | **FT** *their a* and *b* (valid *b* i.e. >1) |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 7(i) | Max no. of passengers plane can take oe | **B1** | oe e.g. No of passengers who bought tickets |
| | | **1** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 7(ii) | $\lambda = 3.2$ | **B1** | |
| | $e^{-3.2}(\frac{3.2^3}{3!} + \frac{3.2^4}{4!} + \frac{3.2^5}{5!})$ | **M1** | Any $\lambda$. Allow one end error |
| | = 0.5146 = 0.515 (3 sfs) | **A1** | SR Use of Bin(640,0.005) scores B1 (only) for 0.516 |
| | | **3** | |
| 7(iii) | $n > 50$ | **B1** | Accept n is large |
| | $np$ = 1.6, which is < 5 or p=0.005 which is <0.1 | **B1** | Allow $np$ = 3.2 |
| | | **2** | |
| 7(iv) | $H_0$: Pop mean (for 5 days) = 8<br>$H_1$: Pop mean (for 5 days) < 8 | **B1** | or Pop mean (for 1 day) = 1.6<br>Pop mean (for 1 day) < 1.6<br>Allow $\lambda$ or $\mu$ but not just 'mean' |
| | $e^{-8}(1 + 8 + \frac{8^2}{2!})$ | **M1** | Any $\lambda$ ($\neq$ 1.6) No end errors. Accept use of Bin(1600,0.005) P(0,1,2)=0.0136 |
| | = 0.0138 | **A1** | |
| | Comp 0.025 | **M1** | Valid comparison |
| | Evidence that mean no. failing to arrive has decreased | **A1** | **FT** their '0.0138' or '0.0136'. No contradictions |
| | | **5** | |

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Cambridge International A Level

**Cambridge Assessment International Education**
Cambridge International Advanced Level

**MATHEMATICS** **9709/73**

Paper 7 **May/June 2019**

MARK SCHEME

Maximum Mark: 50

**Published**

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| |
|---|
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  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

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The following abbreviations may be used in a mark scheme or used on the scripts:

AEF/OE Any Equivalent Form (of answer is equally acceptable) / Or Equivalent

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

SOI Seen or implied

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

**Penalties**

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 1 | $0.6 \pm z\sqrt{\frac{0.4\times0.6}{100}}$ | **M1** | Recognisable value of z |
| | $z = 2.326$ | **B1** | 2.326 to 2.329 |
| | 0.486 to 0.714 (3 sf) | **A1** | Must be an interval |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 2 | $\frac{50}{49}\left(\frac{4361}{50} - \bar{x}^2\right) = 9.62$ | **M1** | or $\left(\frac{4361}{49} - \frac{(\Sigma x)^2}{50\times49}\right) = 9.62$ BOD regarding symbols used |
| | $\bar{x}^2 = \frac{4361}{50} - 9.62\times\frac{49}{50} = 77.7924$ | **A1** | $(\Sigma x)^2 = 4361 \times 50 - 9.62 \times 50 \times 49 = 194481$or $\Sigma x = 441$ $\left(\Sigma x\right)$ or $\left(\bar{x}\right)$ must be correctly identified |
| | $\bar{x} = 8.82$ (3 sf) | **A1** | SC use of 'biased' leading to 8.81 B1 |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 3(i) | *D* more likely to be chosen | **B1** | oe, e.g. P(*D*) > P(*A*) e.g. P(A)=P(B)=P(C)=1/6 P(D)=1/2 no contradictions |
| | | **1** | |
| 3(ii) | Reject scores of 5 or 6 | **B1** | or other correct: choose D when the score is 4 |
| | | **1** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 3(iii) | AB AC AD BC BD CD | **B1** | |
| | Allocate as follows:<br>1: AB; 2: AC; 3: AD; 4: BC; 5: BD 6: CD | **B1** | or similar |
| | | **2** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4 | Total ~ N(1208, .....) | **B1** | |
| | Var(total) (= 10×1.2 + 20×0.7 (+ 0)) = 26 | **B1** | May be implied by next line |
| | $\pm \frac{1200-"1208"}{\sqrt{"26"}}$ (= –1.569) | **M1** | **FT** their mean and var of total mass, e.g. allow 1200 and 11.24 (from $10 \times 1.2^2 + 20 \times 0.7^2$) |
| | 1 – Φ(“1.569”) | **M1** | Correct area consistent with their working |
| | = 0.0583 (3 sf) | **A1** | |
| | | **5** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5 | $H_0$: Pop mean = 20<br>$H_1$: Pop mean $\neq$ 20 | **B1** | Accept $\mu$ |
| | $\frac{\Sigma x}{6}$ $(= \frac{126.9}{6} = 21.15)$ | **M1** | Attempted<br>or 126.9 and 11.64 attempted |
| | $\frac{'21.15'-20}{\sqrt{\frac{1.94}{6}}}$ | **M1** | Must have $\sqrt{6}$<br>or $\frac{120-126.9}{\sqrt{11.64}}$ no mixed method |
| | = 2.022 | **A1** | |
| | 2(1 - ϕ(‘2.022’) ) 2 (1 – '0.9784)' = 0.0432) | **M1** | **FT** $2\times(1-'.9784')$ |
| | $\alpha$ = 4.32 (3 sf) | **A1** | **FT** Allow 4.3 or 4, if correct working seen, or clearly implied, as far as 0.0216<br>**FT** their z, no error seen<br>One-tail test scores maximum 3/6 |
| | | **6** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(i) | $\frac{3}{a^3}\int_0^a x^2 dx$<br>$(= \frac{3}{a^3}\left[\frac{x^3}{3}\right]_0^a)$ | **M1** | Attempt to integrate f(x) with limits 0 and a (condone missing $\frac{3}{a^3}$) |
| | $= \frac{3a^3}{3a^3}$ | **A1** | $\frac{3a^3}{3a^3} - 0$ or better seen |
| | = 1 Hence f is pdf for all $a$ | **A1** | Answer = 1 and comment |
| | | **3** | |
| 6(ii) | $\frac{3}{a^3}\int_0^2 x^2 dx = 0.5$<br>$\frac{3}{a^3}\left[\frac{x^3}{3}\right]_0^2 = 0.5$ | **M1** | Attempt to integrate f(x)=0.5, limits 0 and 2 oe, condone missing $\frac{3}{a^3}$ |
| | $\frac{3}{a^3}\times\frac{8}{3} = 0.5$ oe | **A1** | $\frac{2^3}{3} - 0$ or better, condone missing $\frac{3}{a^3}$ |
| | $a^3 = 16$ or $a = \sqrt[3]{16}$<br>( = 2.52 **AG**) | **A1** | Convincingly obtained<br>Note: Attempt to verify 2.52, M1 as stated except not equated to 0.5.A1 as stated, A1 for evaluation to 0.499..apprx 0.5 |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(iii) | $\frac{3}{16}\int_0^{2.52} x^3 dx$ or $\frac{3}{16}\int_0^{a} x^3 dx$<br>$= \frac{3}{16}\left[\frac{x^4}{4}\right]_0^{2.52}$ or $\frac{3}{16}\left[\frac{x^4}{4}\right]_0^{a}$ | **M1** | Attempt integ $xf(x)$, correct limits, condone missing $\frac{3}{a^3}$ |
| | $= \frac{3}{16}\times\frac{40.317}{4}$ | **A1** | $\frac{2.52^4}{4} - 0$ or better, condone missing $\frac{3}{a^3}$ |
| | = 1.89 (3 sf) | **A1** | |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 7(i) | Use of Po(2.8) | **M1** | May be implied |
| | $1 - e^{-2.8}(1 + 2.8 + \frac{2.8^2}{2}))$ | **M1** | Any λ allowing one end error |
| | = 0.531or 0.53(0) (3 sf) | **A1** | SC Binomial 0.534 B1 |
| | | **3** | |
| 7(ii) | Use of Po(5.8) | **M1** | May be implied |
| | $e^{-5.8} \times \frac{5.8^6}{6!}$ | **M1** | Any λ |
| | = 0.16(0) (3 sf) | **A1** | |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 7(iii) | Use of N(58, 58) | **M1** | May be implied<br>or N(58, 55.38) |
| | $\frac{50.5-'58'}{\sqrt{'58'}}$ (= -0.985) | **M1** | Standardised with their values, allow wrong or incorrect cc |
| | Φ('0.985') | **M1** | Correct area consistent with their working<br>or $\Phi("1.008")$ |
| | = 0.838 (3 sf) | **A1** | or 0.843 |
| | | **4** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 8(i) | $H_0: p = \frac{1}{4}$<br>$H_1: p > \frac{1}{4}$ | **B1** | |
| | ${}^{10}C_6 (\frac{1}{4})^6 (\frac{3}{4})^4 + {}^{10}C_7 (\frac{1}{4})^7 (\frac{3}{4})^3 + {}^{10}C_8 (\frac{1}{4})^8 (\frac{3}{4})^2 + 10 (\frac{1}{4})^9 (\frac{3}{4}) + (\frac{1}{4})^{10}$ | **M1** | Correct terms, allow one term incorrect or omitted or extra<br>or summing all correct terms from 0 to 5 allow one term incorrect or omitted or extra |
| | = 0.0197 | **A1** | or 0.9803 |
| | comp '0.0197' with 0.01 | **M1** | Valid comparison with 0.01<br>or valid comparison with 0.99 |
| | No evidence to conclude $p > \frac{1}{4}$ | **A1** | **FT** No contradictions<br>Use of two-tail test can score BOM1A1M1(comparison with 0.005) A0 |
| | | **5** | |
| 8(ii) | ${}^{10}C_7 (\frac{1}{4})^7 (\frac{3}{4})^3 + {}^{10}C_8 (\frac{1}{4})^8 (\frac{3}{4})^2 + 10 (\frac{1}{4})^9 (\frac{3}{4}) + (\frac{1}{4})^{10}$ | **M1** | Their $P(X \geqslant 6) - {}^{10}C_6 (0.25)^6 (0.75)^4$ |
| | P(Type I) = 0.00351 (3 sf) | **A1** | Accept 0.00348 to 0.00351 |
| | | **2** | |
| 8(iii) | C.R is $X \geqslant 7$<br>P(Type II) = 1 – P($X \geqslant 7 \mid p = \frac{3}{5}$) = | **M1** | May be implied |
| | $1 - ({}^{10}C_7 (\frac{3}{5})^7 (\frac{2}{5})^3 + {}^{10}C_8 (\frac{3}{5})^8 (\frac{2}{5})^2 + 10 (\frac{3}{5})^9 (\frac{2}{5}) + (\frac{3}{5})^{10})$ | **M1** | Accept $1 - P(X \geqslant 8 \mid p = \frac{3}{5})$ or $1 - P(X \geqslant 6 \mid p = \frac{3}{5})$ |
| | = 0.618 | **A1** | |
| | | **3** | |

Cambridge International A Level

**Cambridge Assessment International Education**
Cambridge International Advanced Level

**MATHEMATICS** **9709/72**

Paper 7 Probability and Statistics **March 2019**

MARK SCHEME

Maximum Mark: 50

**Published**

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

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**PUBLISHED**

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- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

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| |
|---|
| GENERIC MARKING PRINCIPLE 5:<br><br>Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen). |
| GENERIC MARKING PRINCIPLE 6:<br><br>Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind. |

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**Mark Scheme Notes**

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

  - Note: B2 or A2 means that the candidate can earn 2 or 0.
    B2/1/0 means that the candidate can earn anything from 0 to 2.

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 1(i) | $z = 2.326$ | **B1** | |
| | $62.3 \pm z\frac{13.2}{\sqrt{200}}$ | **M1** | Any $z$. Expression of correct form. Must be a 'z' |
| | 60.1 to 64.5 (3 sfs) | **A1** | Must be an interval |
| | | **3** | |
| 1(ii) | Yes, because pop not (given to be) normal, or pop distribution unknown | **B1** | No contradictions |
| | | **1** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 2 | $E(X - 3Y) = 0.2$ | **B1** | oe |
| | $Var(X - 3Y) = 12.1 + 9 \times 8.6$ (= 89.5) | **B1** | |
| | $\frac{0 - 0.2}{\sqrt{"89.5"}}$ (= –0.021) | **M1** | For area consistent with their working |
| | Φ('0.021') | **M1** | |
| | = 0.508 (3 sfs) | **A1** | |
| | | **5** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 3 | $H_0$: $\lambda = 32$<br>$H_1$: $\lambda < 32$ | **B1** | Accept 'population mean' ($\mu$) |
| | $X \sim N(32, 32)$ | **B1** | seen or implied |
| | $\frac{21.5 - 32}{\sqrt{32}}$ | **M1** | Standardise with their values. Allow with no or wrong cc |
| | $= -1.856$<br>cv of $z = -2.054$ (or $-2.055$ or $-2.053$) | **A1** | |
| | '1.856' < 2.054 | **M1** | Valid comparison<br>or comp $\phi$("1.856") with 0.98 i.e. 0.9682 < 0.98 oe |
| | No evidence that fewer accidents at B than at A | **A1f** | No contradictions<br>Note Use of CV method<br>x = 20.38 M1 A1 comparison 21.5 > 20.38 M1 conc A1 |
| | | **6** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4(i) | $\bar{x} = \frac{420}{50} = 8.4$ | **B1** | |
| | $s^2 = \frac{50}{49}\left(\frac{27530}{50} - \left(\frac{420}{50}\right)^2\right)$ | **M1** | Or $1/49(27530 - (420)^2/50)$ |
| | $= 489.8(36....)$ | **A1** | Must see $\geqslant$ 4 sf |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4(ii) | $\Phi^{-1}(0.9377) = 1.536$ | **B1** | |
| | $\frac{5-'8.4'}{\sqrt{\frac{490}{n}}} = -1.536$ | **M1** | Attempting to standardise – must have correct form |
| | $n = (\frac{1.536}{3.4})^2 \times 490$ (= 100.0048) | **M1** | Attempting numerical expression for n or √n (must have used a 'z' value) may be implied by answer |
| | $n = 100$ | **A1** | No errors seen. Must be whole number |
| | | **4** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5(i) | $1 - e^{-1.8}(1 + 1.8)$ | **M1** | Accept any λ. Accept 1–P(0,1,2) |
| | = 0.537 (3 sf) | **A1** | |
| | | **2** | |
| 5(ii) | $\lambda = 2.2$ | **B1** | |
| | $e^{-2.2}(1 + 2.2 + \frac{2.2^2}{2!} + \frac{2.2^3}{3!} + \frac{2.2^4}{4!})$ | **M1** | Attempt expr'n for<br>$P(X \leqslant 4)$,allow one end error, allow any $\lambda$ |
| | = 0.928 (3 sf) or 0.927 | **A1** | |
| | | **3** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5(iii) | $1 - e^{-1.8t} \geqslant 0.99$ or $1 - e^{-\lambda} \geqslant 0.99$ | **M1** | Condone = signs/incorrect inequality signs |
| | $e^{-1.8t} \leqslant 0.01$ or $e^{-\lambda} \leqslant 0.01$<br>$-1.8t \leqslant \ln 0.01$ | **M1** | Valid attempt take logs (must have single term on each side) |
| | $t \geqslant 2.56$<br>She must watch for at least 2.56 (hours) | **A1** | or 2 hours, 34 mins or better. No errors seen |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(i) | Test is for "difference" oe | **B1** | Test is not for 'increase' or 'decrease' oe No contradictions |
| | | **1** | |
| 6(ii) | 0.05 | **B1** | |
| | Conclude mean time is different when it is not | **B1** | oe, in context |
| | | **2** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(iii) | Assume $\sigma = 6.4$ | **B1** | |
| | $H_0$: pop mean = 91.4<br>$H_1$: pop mean $\neq$ 91.4 | **B1** | Allow $\mu$, but not 'mean' |
| | $\bar{x} = \frac{568.5}{6}$ (= 94.75) | **B1** | |
| | $\frac{'94.75'-91.4}{\frac{6.4}{\sqrt{6}}}$ | **M1** | Must have $\sqrt{6}$ |
| | = 1.282<br>cv of $z$ = 1.96 | **A1** | |
| | '1.282' < 1.96 | **M1** | Valid comparison<br>or comp $\phi$("1.282") with 0.975 0.9(001) < 0.975 or 0.0999 (or 0.1) > 0.025<br>consistent use of one tail test can score M1 for comparison with 1.645oe but not A1ft<br>oe. No contradictions. ft their z. |
| | No evidence mean time different | **A1 ft** | CV method x = 96.52 M1 A1 94.75 < 96.52 M1 Conc A1 |
| | | **7** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 7(i) | $\sqrt{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \cos x \mathrm{d}x$<br>$= \sqrt{2}\,[\sin x]_{\frac{\pi}{6}}^{\frac{\pi}{4}}$ | **M1** | Attempt integ f($x$) with correct limits |
| | $= \frac{2-\sqrt{2}}{2}$ oe or 0.293 (3 sf) | **A1** | SC Final answer of 0.707 scores B1sc |
| | | **2** | |
| 7(ii) | $\sqrt{2}\int_0^m \cos x \mathrm{d}x = 0.5$ | **M1** | Attempt to integ f(x) & = 0.5. Ignore limits. Condone missing $\sqrt{2}$ |
| | $\sqrt{2}\,[\sin x]_0^m = 0.5$<br>$\sqrt{2}\,\sin m = 0.5$ | **A1** | Correct integral and limits 0 to unknown & = 0.5 Condone missing $\sqrt{2}$ |
| | $\sin m = \frac{1}{2\sqrt{2}}$ oe | **M1** | For rearranging their expression to the form $\sin m$=… ($\sin m$ = 0.35355.... or 0.354) seen or implied |
| | $m$ = 0.361 (3 sfs) | **A1** | No errors seen<br>(Note 20.705 can score M1 A1 M1 A0) |
| | | **4** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 7(iii) | $\sqrt{2}\int_0^{\frac{\pi}{4}} x\cos x\,dx$ | **M1** | Attempt to integ $xf(x)$. Ignore limits. Condone missing $\sqrt{2}$ |
| | $= \sqrt{2}\ \{[x(\sin x)]_0^{\frac{\pi}{4}} - \int_0^{\frac{\pi}{4}} \sin x\,dx\ \}$ | **M1** | Attempt to integ by parts leading to expression of form $\pm x\sin x \pm \cos x$ with correct limits |
| | $= \sqrt{2}\ \{\frac{\pi}{4\sqrt{2}} - 0 - [-\cos x]_0^{\frac{\pi}{4}}\}$ | **A1** | For $\sqrt{2}(x\sin x - (-\cos x))$ with correct limits |
| | $= \sqrt{2}\ \{\frac{\pi}{4\sqrt{2}} + \cos\frac{\pi}{4} - 1\}$<br><br>$= \frac{\pi}{4} + 1 - \sqrt{2}$ oe or 0.371 (3 sf) | **A1** | |
| | | **4** | |

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Cambridge International A Level

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Cambridge International Advanced Level

**MATHEMATICS** **9709/71**

Paper 7 **October/November 2018**

MARK SCHEME

Maximum Mark: 50

**Published**

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- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

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**Mark Scheme Notes**

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

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AEF/OE Any Equivalent Form (of answer is equally acceptable) / Or Equivalent

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

CAO Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)

CWO Correct Working Only – often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

SOI Seen or implied

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

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PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 1(i) | $176 \pm z \times \frac{7.2}{\sqrt{200}}$ | **M1** | need correct form must be z |
| | $z = 2.24$ | **B1** | allow 2.241 and 2.242 |
| | 175 to 177 | **A1** | cwo |
| | | **3** | |
| 1(ii) | Sample random | **B1** | oe. both words essential |
| | | **1** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 2(i) | $H_0: p = \frac{1}{3}$ $H_1: p < \frac{1}{3}$ | **B1** | |
| | | **1** | |
| 2(ii) | $0.0084 < 0.01$ | **B1** | Allow P( N ⩽ 36 ) < 0.01 or 1% |
| | There is evidence that $p$ has decreased | **B1 dep** | Allow '$p$ has decreased' or $p < \frac{1}{3}$ |
| | | **2** | |
| 2(iii) | 150 | **B1** | |
| | | **1** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 3 | $\frac{12.2 - 12}{2.5 / \sqrt{n}}$ | **M1** | Standardisation.<br>Allow cc. need correct form incl sqrt |
| | (=) 1.96 | **B1** | Correct $z$ |
| | $\sqrt{n} = 1.96 \times 2.5 \div 0.2$ | **M1** | Rearrange equation in n or sqrt n with numerical z to the stage n= or sqrt n = allow arithmetical slips only |
| | $n = 600$ | **A1** | accept 601<br>SR whole number ans from 595 to 605 can score full marks if fully justified |
| | | **4** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4(i) | $\lambda = 10\times0.25 + 10\times0.36 \quad (= 6.1)$ | **B1** | |
| | $1 - e^{-6.1}\,(1 + 6.1 + \frac{6.1^2}{2} + \frac{6.1^3}{3!})$ | **M1** | $1 - P(X \leqslant 3)$, any $\lambda$ Allow one end error |
| | $= 0.857$ | **A1** | Allow 0.858 |
| | | **3** | |
| 4(ii) | $\lambda = 61$ | **B1 ft** | Ft from (i) |
| | N('61', '61') | **M1** | N with $\mu = \lambda$, any λ. May be implied |
| | $\frac{59.5-61}{\sqrt{'61'}} \quad (= -0.192)$ | **M1** | Standardise with their mean and variance Allow no or wrong cc. not 61/100 |
| | $\Phi('-0.192') = 1 - \Phi('0.192')$ | **M1** | Correct area consistent with their working |
| | $= 0.424$ | **A1** | |
| | | **5** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5(i) | $T_1 + T_2 \sim N(5, 0.4^2 + 0.5^2)$ | **B1** | or N( 5, 0.41 ) |
| | $\frac{6-5}{\sqrt{'0.41'}} \quad (= 1.562)$ | **M1** | Allow cc |
| | $\Phi('1.562')$ | **M1** | Correct area consistent with their working |
| | $= 0.941$ | **A1** | |
| | | **4** | |
| 5(ii) | $\text{Var}(T_2 - 1.2T_1) = 0.5^2 + 1.2^2 \times 0.4^2$ $(= 0.4804)$ | **B1** | Or similar using $1.2T_1 - T_2$ |
| | $T_2 - 1.2T_1 - N(0.16, 0.4804)$ | **B1 ft** | Only ft attempt at combination. no ft for neg var. |
| | $\frac{0 - '0.16'}{\sqrt{'0.4804'}} \quad (= -0.231)$ | **M1** | Standardise with their mean and variance. Allow cc |
| | $P(T_2 - 1.2T_1) > 0$ | | |
| | $= \Phi('0.231')$ | **M1** | Correct area consistent with their working |
| | $= 0.591$ (3 sfs) | **A1** | |
| | | **5** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(i) | $k\int_2^6 x^{-1}\mathrm{d}x = 1$ | **M1** | Attempt integrate f($x$) & = 1. Ignore limits |
| | $k\left[\ln x\right]_2^6 = 1$<br>$k(\ln 6 - \ln 2) = 1$ or $k\ln 3 = 1$<br>$k = \frac{1}{\ln 3}$ **AG** | **A1** | correct sub of correct limits in correct integral leading to correct ans. No errors seen. |
| | | **2** | |
| 6(ii) | $\frac{1}{\ln 3}\int_2^6 1\,\mathrm{d}x$ | **M1** | Attempt integ $x$f($x$). Ignore limits |
| | $= \frac{1}{\ln 3}\left[x\right]_2^6$ $(= \frac{1}{\ln 3}(6-2))$ | **A1** | Correct integral and limits |
| | $= \frac{4}{\ln 3} = 3.64$ **AG** | **A1** | No errors seen |
| | | **3** | |
| 6(iii) | $\mathrm{P}(X < \mathrm{E}(X)) = \frac{1}{\ln 3}\int_2^{3.64} x^{-1}\mathrm{d}x$ | **M1** | Attempt integ f($x$) from 2 to $\frac{4}{\ln 3}$ or 3.64 oe |
| | $= \frac{1}{\ln 3}\left[\ln x\right]_2^{3.64}$<br>$= \frac{1}{\ln 3}(\ln 3.64 - \ln 2)$ $(= 0.545)$ | **A1** | Correct sub correct limits into correct integral |
| | P($m$ < X < E($X$)) = "0.545" – 0.5 | **M1** | Subt 0.5 from their P($X$ < E($X$))<br>art 0.045 . ft their P($X$ < E($X$) (> 0.5) |
| | = 0.045 (2 sfs) | **A1** | equivalent method<br>M1 method for median-need 0.5<br>and limits 2 to m or m to 6<br>A1 sqrt 12 or 3.464<br>M1 calc area from "3.464" to 3.64<br>A1 0.045 or better, not 0.046 |
| | | **4** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 7(i) | $H_0: \mu = 51$ $H_1: \mu < 51$ | **B1** | Or popn mean … |
| | $\bar{x} = \frac{7480}{150} = 49.8667 = 49.9$ | **B1** | |
| | $s^2 = \frac{150}{149}(\frac{380000}{150} - (\frac{748}{15})^2)$<br>$= 46.9620 = 47.0$ or $s = 6.85$ | **M1** | Correct subst in $s^2$ or $\sqrt{s^2}$ formula<br>Biased var scores M0 |
| | $\frac{49.8667-51}{\sqrt{\frac{'46.962'}{150}}}$ allow $\frac{49.9-51}{\sqrt{\frac{'47'}{150}}}$ | **M1** | Allow 49.8667 to 49.9 in numerator<br>Need sqrt 150 |
| | $= (-)\ 2.025 = (-)\ 1.965$ | **A1** | Accept 2.02 or 2.03<br>Accept –2.0264 –1.9651 provided correct working |
| | comp $z = 1.96$ | **M1** | or comp $1 - \phi(2.025)$ with 0.025 |
| | There is evidence that $\mu < 51$ | **A1 ft** | no contradictions<br>biased var B1B1M0M1A0M1A1ft (max 5/7)<br><br>accept cv method<br>$x_{crit} = 49.9028$ M1A1<br>49 867 < 49.9… M1A1 |
| | | **7** | |
| 7(ii) | $\frac{\bar{x}-51}{\frac{6.856}{\sqrt{150}}} = -1.96$ | **M1** | Need 51 and sqrt 150 and correct form |
| | $\bar{x} = 51 - 1.097 = 49.9$<br>Rejection region is $\bar{x} < 49.9$ | **A1** | This may have been found in part (i) |
| | $\frac{49.9-49}{\frac{6.856}{\sqrt{150}}}$ (= 1.608 to 1.614) | **M1** | Need 49 and sqrt 150 and correct form |
| | $P(\bar{x} > 49.9 \mid \mu = 49) = 1 - \Phi('1.608')$ | **M1** | |
| | P(Type II error) = 0.0539 | **A1** | Allow 0.0533 to 0.0539 |
| | | **5** | |

Cambridge International A Level

**Cambridge Assessment International Education**
Cambridge International Advanced Level

**MATHEMATICS** **9709/72**

Paper 7 **October/November 2018**

MARK SCHEME

Maximum Mark: 50

**Published**

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2018 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.



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**PUBLISHED**

## Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

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| |
|---|
| GENERIC MARKING PRINCIPLE 5:<br><br>Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen). |
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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 1 | $e^{-2.3}(\frac{2.3^2}{2}+\frac{2.3^3}{3!}+\frac{2.3^4}{4!})$ | **M2** | M1 for one term wrong or one end error or 1 – P(2, 3, 4) |
| | = 0.585 (3 sf) | **A1** | |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 2(i) | $z = 1.96$ | **B1** | seen |
| | $330.1 \pm z \times \frac{4.8}{\sqrt{180}}$ | **M1** | Must be of correct form. Any $z$ |
| | = 329.4 to 330.8 (1 dp) | **A1** | Must be to 1 dp. Must be an interval. |
| | | **3** | |
| 2(ii) | Yes, because vol of all cans not stated to be normal | **B1** | Or Yes, population not stated to be normal |
| | | **1** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 3 | $E(T) = 2 \times 250 + 5 \times 160$ (= 1300) | **B1** | |
| | $Var(T) = 2 \times 10 + 5 \times 9$ (= 65) | **B1** | |
| | $\frac{1310-'1300'}{\sqrt{'65'}}$ (= 1.240) | **M1** | Standardise using their values (must come from a combination attempt). Ignore cc |
| | $1 - \phi(\text{'1.240'})$ | **M1** | Correct area consistent with their working |
| | = 0.1075 | **A1** | Allow 0.107 to 0.108 (no errors seen) |
| | | **5** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4(i) | $\int_0^a \frac{k}{(x+1)^2} dx = 1$ | **M1** | Any attempt integ f($x$) and = 1. Ignore limits |
| | $-\left[\frac{k}{(x+1)}\right]_0^a = 1$<br>$-k(\frac{1}{a+1} - 1) = 1$ | **M1** | Attempt subst correct limits into correct integral |
| | $k \times \frac{a}{a+1} = 1$ and $k = \frac{a+1}{a}$ **AG** | **A1** | No errors seen |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4(ii) | Max time allowed by model (for runners to finish) | **B1** | Allow: All runners finish in time $a$ or less or Longest time (taken by any runner) oe |
| | | **1** | |
| 4(iii) | $\frac{a+1}{a}\int_0^{0.5}\frac{1}{(x+1)^2}\,dx = \frac{3}{4}$ | **M1** | Attempt integ f($x$) and $= \frac{3}{4}$; ignore limits oe. Condone missing / incorrect k |
| | $-\frac{a+1}{a}\left[\frac{1}{(x+1)}\right]_0^{0.5} = \frac{3}{4}$<br>$-\frac{a+1}{a}\left(\frac{2}{3}-1\right) = \frac{3}{4}$ | **M1** | Attempt subst correct limits into correct integral. Condone missing / incorrect k |
| | $a = 0.8$ oe | **A1** | |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5(i) | $\hat{\mu} = \frac{126}{70}$ or $\frac{9}{5}$ or 1.8 oe | **B1** | |
| | $\Sigma x^2 f = 286$ | **B1** | Seen or implied |
| | $\text{Est}(\sigma^2) = \frac{70}{69}\left(\frac{\Sigma x^2 f}{70} - \text{'1.8'}^2\right)$ | **M1** | oe attempted |
| | = 0.858 or 296 / 345 | **A1** | Note: Final answer for var 0.846 (biased) and no working implies B1 for 286 |
| | | **4** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5(ii) | $H_0$: $\mu = 1.9$<br>$H_1$: $\mu < 1.9$ | **B1** | Or 'pop mean'; not just 'mean' |
| | $\frac{1.8-1.9}{\sqrt{\frac{'0.858'}{70}}}$ | **M1** | Standardise with their values from **(i)**. Must have sqr 70. No SD / Var mix |
| | = –0.903 | **A1** | Accept ± |
| | 0.903 < 1.645 | **M1** | comp 1.645 allow comp 1.96 if $H_1$: $\mu \neq 1.9$<br>or comp 1 – ϕ('0.903')=0.182 or 0.183 with 0.05 (or 0.025 if $H_1$: $\mu \neq 1.9$) |
| | No evidence that mean no courts in S is less than in N | **A1ft** | No contradictions. ft their 0.903, but not comp 1.96 i.e. no ft for a 2 tail test<br>Accept cv method: cv = 1.718 M1A1 1.718 < 1.8 M1 conclusion A1 (cv centred on 1.8 gives 1.982 M1A1 and M1 for 1.982 > 1.9 A1 conclusion) |
| | | **5** | |
| 5(iii) | Type II because $H_0$ was not rejected | **B1ft** | ft their conclusion, i.e. if $H_0$ rejected, 'Type I because $H_0$ rejected' B1<br>Answer must be consistent with their conclusion. No conclusion in **(ii)** will score B0 |
| | | **1** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(i) | $H_o$: $p = 0.15$<br>$H_1$: $p < 0.15$<br><br>$(N(60 \times 0.15, 60 \times 0.15 \times 0.85))$<br><br>$= N(9, 7.65)$ | **B1** | Accept $H_0$: $\mu = 9$<br>$H_1$: $\mu < 9$<br>Use of Normal approximation:<br>$(N(0.15, \frac{0.15 \times 0.85}{60}))$<br>$= N(0.15, 0.002125)$ |
| | $\frac{6.5 - '9'}{\sqrt{'7.65'}}$ | **M1** | For standardising (or $\frac{\frac{6}{60} + \frac{0.5}{60} - '0.15'}{\sqrt{'0.002125'}} = -0.904$) Allow wrong or no cc |
| | $= -0.904$ | **A1** | Accept ± |
| | '0.904' < 1.282 | **M1** | Valid comparison of z values or $\phi('-0.904') = 0.183 > 0.1$<br>ft their 0.904 |
| | No evidence train late less often | **A1ft** | Use of Bin (60,0.15) to give Pr (< = 6) = 0.1848 M1A1<br>Valid comparison with 0.1 M1 Conclusion A1ft |
| | | **5** | |
| 6(ii) | $0.1 + z \times \sqrt{\frac{0.1 \times 0.9}{60}} = 0.150$ | **M1** | For $\sqrt{(0.1 \times 0.9 / 60)}$ seen |
| | | **M1** | for $0.1 + z \times ... = 0.150$ or $2z\ldots = 0.1$ |
| | $z = 1.291$ | **A1** | |
| | $\phi$('1.291') (= 0.90(16)) | **M1** | for correct method to find $\alpha$ |
| | $\alpha = 80$ | **A1ft** | ft their $z$. Must be a +ve non-zero integer < 100 |
| | | **5** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 7(i) | $e^{-5.6} \times \frac{5.6^3}{3!}$ | **M1** | Allow any $\lambda$ |
| | = 0.108 (3 sf) | **A1** | |
| | | **2** | |
| 7(ii) | $P(X=2 \ \& \ Y=1) = e^{-2.1} \times \frac{2.1^2}{2} \times e^{-3.5} \times 3.5$<br>$(0.2700 \times 0.10569 = 0.028538)$ | **M1** | |
| | $\frac{P(X=2 \ \& \ Y=1)}{P(X+Y=3)}$ attempted<br>$= \frac{'0.028538'}{'0.108234}$ | **M1** | For attempt at fraction with their **(i)** as denominator or $\frac{2.1^2}{2} \times 3.5 \div \frac{5.6^3}{3}$ M2 |
| | = 0.264 (3 sf) | **A1** | |
| | | **3** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 7(iii) | Var($X$) = 2.1 | **B1** | soi |
| | $\bar{X} \sim \text{N}(2.1, \frac{2.1}{100})$ or $\text{N}(210,210)$ | **B1** | soi B1 for N(2.1, ...) |
| | | **B1** | B1 for $\frac{2.1}{100}$ oe<br>Standardise with their values. Allow with or without cc or with incorrect cc |
| | $\frac{2.2-2.1}{\sqrt{\frac{2.1}{100}}}$ oe $(220 - 210) / \sqrt{210}$ (= 0.690) | **M1** | or $\frac{2.2+0.5\div100-2.1}{\sqrt{\frac{2.1}{100}}}$ or $(220.5 - 210) / \sqrt{210})$ (= 0.725)<br>no mixed methods |
| | 1 – ϕ(‘0.690’) | **M1** | Correct area consistent with their working or 1 – ϕ(‘0.725’) |
| | = 0.245 (3 sf) | **A1** | = 0.234 (3 sf) |
| | | **6** | |

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Cambridge International Advanced Level

**MATHEMATICS** **9709/73**

Paper 7 **October/November 2018**

MARK SCHEME

Maximum Mark: 50

**Published**

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Cambridge International is publishing the mark schemes for the October/November 2018 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.



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## Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

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**Mark Scheme Notes**

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

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- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

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The following abbreviations may be used in a mark scheme or used on the scripts:

AEF/OE Any Equivalent Form (of answer is equally acceptable) / Or Equivalent

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

CAO Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)

CWO Correct Working Only – often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

SOI Seen or implied

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

**Penalties**

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become 'follow through' marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 1(i) | $176 \pm z \times \frac{7.2}{\sqrt{200}}$ | **M1** | need correct form must be z |
| | $z = 2.24$ | **B1** | allow 2.241 and 2.242 |
| | 175 to 177 | **A1** | cwo |
| | | **3** | |
| 1(ii) | Sample random | **B1** | oe. both words essential |
| | | **1** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 2(i) | $H_0: p = \frac{1}{3}$ $H_1: p < \frac{1}{3}$ | **B1** | |
| | | **1** | |
| 2(ii) | $0.0084 < 0.01$ | **B1** | Allow P( N $\leqslant$ 36 ) < 0.01 or 1% |
| | There is evidence that $p$ has decreased | **B1 dep** | Allow '$p$ has decreased' or $p < \frac{1}{3}$ |
| | | **2** | |
| 2(iii) | 150 | **B1** | |
| | | **1** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 3 | $\frac{12.2 - 12}{2.5 / \sqrt{n}}$ | **M1** | Standardisation.<br>Allow cc. need correct form incl sqrt |
| | (=) 1.96 | **B1** | Correct $z$ |
| | $\sqrt{n} = 1.96 \times 2.5 \div 0.2$ | **M1** | Rearrange equation in n or sqrt n with numerical z to the stage n= or sqrt n = allow arithmetical slips only |
| | $n = 600$ | **A1** | accept 601<br>SR whole number ans from 595 to 605 can score full marks if fully justified |
| | | **4** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4(i) | $\lambda = 10\times0.25 + 10\times0.36 \quad (= 6.1)$ | **B1** | |
| | $1 - e^{-6.1}(1 + 6.1 + \frac{6.1^2}{2} + \frac{6.1^3}{3!})$ | **M1** | $1 - P(X \leqslant 3)$, any $\lambda$ Allow one end error |
| | $= 0.857$ | **A1** | Allow 0.858 |
| | | **3** | |
| 4(ii) | $\lambda = 61$ | **B1 ft** | Ft from (i) |
| | N('61', '61') | **M1** | N with $\mu = \lambda$, any $\lambda$. May be implied |
| | $\frac{59.5-61}{\sqrt{'61'}} \quad (= -0.192)$ | **M1** | Standardise with their mean and variance Allow no or wrong cc. not 61/100 |
| | $\Phi('-0.192') = 1 - \Phi('0.192')$ | **M1** | Correct area consistent with their working |
| | $= 0.424$ | **A1** | |
| | | **5** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5(i) | $T_1 + T_2 \sim N(5, 0.4^2 + 0.5^2)$ | **B1** | or N( 5, 0.41 ) |
| | $\frac{6-5}{\sqrt{'0.41'}} \quad (= 1.562)$ | **M1** | Allow cc |
| | $\Phi('1.562')$ | **M1** | Correct area consistent with their working |
| | $= 0.941$ | **A1** | |
| | | **4** | |
| 5(ii) | $\text{Var}(T_2 - 1.2T_1) = 0.5^2 + 1.2^2 \times 0.4^2$ $(= 0.4804)$ | **B1** | Or similar using $1.2T_1 - T_2$ |
| | $T_2 - 1.2T_1 - N(0.16, 0.4804)$ | **B1 ft** | Only ft attempt at combination. no ft for neg var. |
| | $\frac{0 - '0.16'}{\sqrt{'0.4804'}} \quad (= -0.231)$ | **M1** | Standardise with their mean and variance. Allow cc |
| | $P(T_2 - 1.2T_1) > 0$ | | |
| | $= \Phi('0.231')$ | **M1** | Correct area consistent with their working |
| | $= 0.591$ (3 sfs) | **A1** | |
| | | **5** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(i) | $k\int_2^6 x^{-1}\mathrm{d}x = 1$ | **M1** | Attempt integrate f($x$) & = 1. Ignore limits |
| | $k\left[\ln x\right]_2^6 = 1$<br>$k(\ln 6 - \ln 2) = 1$ or $k\ln 3 = 1$<br>$k = \frac{1}{\ln 3}$ **AG** | **A1** | correct sub of correct limits in correct integral leading to correct ans. No errors seen. |
| | | **2** | |
| 6(ii) | $\frac{1}{\ln 3}\int_2^6 1\,\mathrm{d}x$ | **M1** | Attempt integ $x$f($x$). Ignore limits |
| | $= \frac{1}{\ln 3}\left[x\right]_2^6$ $(= \frac{1}{\ln 3}(6-2))$ | **A1** | Correct integral and limits |
| | $= \frac{4}{\ln 3} = 3.64$ **AG** | **A1** | No errors seen |
| | | **3** | |
| 6(iii) | $\mathrm{P}(X < \mathrm{E}(X)) = \frac{1}{\ln 3}\int_2^{3.64} x^{-1}\mathrm{d}x$ | **M1** | Attempt integ f($x$) from 2 to $\frac{4}{\ln 3}$ or 3.64 oe |
| | $= \frac{1}{\ln 3}\left[\ln x\right]_2^{3.64}$<br>$= \frac{1}{\ln 3}(\ln 3.64 - \ln 2)$ $(= 0.545)$ | **A1** | Correct sub correct limits into correct integral |
| | P($m$ < X < E($X$)) = "0.545" – 0.5 | **M1** | Subt 0.5 from their P($X$ < E($X$))<br>art 0.045 . ft their P($X$ < E($X$) (> 0.5) |
| | = 0.045 (2 sfs) | **A1** | equivalent method<br>M1 method for median-need 0.5<br>and limits 2 to m or m to 6<br>A1 sqrt 12 or 3.464<br>M1 calc area from "3.464" to 3.64<br>A1 0.045 or better, not 0.046 |
| | | **4** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 7(i) | $H_0: \mu = 51$ $H_1: \mu < 51$ | **B1** | Or popn mean … |
| | $\bar{x} = \frac{7480}{150} = 49.8667 = 49.9$ | **B1** | |
| | $s^2 = \frac{150}{149}(\frac{380000}{150} - (\frac{748}{15})^2)$<br>$= 46.9620 = 47.0$ or $s = 6.85$ | **M1** | Correct subst in $s^2$ or $\sqrt{s^2}$ formula<br>Biased var scores M0 |
| | $\frac{49.8667-51}{\sqrt{\frac{'46.962'}{150}}}$ allow $\frac{49.9-51}{\sqrt{\frac{'47'}{150}}}$ | **M1** | Allow 49.8667 to 49.9 in numerator<br>Need sqrt 150 |
| | $= (-)\ 2.025 = (-)\ 1.965$ | **A1** | Accept 2.02 or 2.03<br>Accept –2.0264 –1.9651 provided correct working |
| | comp $z = 1.96$ | **M1** | or comp $1 - \phi(2.025)$ with 0.025 |
| | There is evidence that $\mu < 51$ | **A1 ft** | no contradictions<br>biased var B1B1M0M1A0M1A1ft (max 5/7)<br><br>accept cv method<br>$x_{crit} = 49.9028$ M1A1<br>49 867 < 49.9… M1A1 |
| | | **7** | |
| 7(ii) | $\frac{\bar{x}-51}{\frac{6.856}{\sqrt{150}}} = -1.96$ | **M1** | Need 51 and sqrt 150 and correct form |
| | $\bar{x} = 51 - 1.097 = 49.9$<br>Rejection region is $\bar{x} < 49.9$ | **A1** | This may have been found in part (i) |
| | $\frac{49.9-49}{\frac{6.856}{\sqrt{150}}}$ (= 1.608 to 1.614) | **M1** | Need 49 and sqrt 150 and correct form |
| | $P(\bar{x} > 49.9 \mid \mu = 49) = 1 - \Phi('1.608')$ | **M1** | |
| | P(Type II error) = 0.0539 | **A1** | Allow 0.0533 to 0.0539 |
| | | **5** | |

Cambridge International AS & A Level

**Cambridge Assessment International Education**
Cambridge International Advanced Subsidiary and Advanced Level

**MATHEMATICS** **9709/71**

Paper 7 **May/June 2018**

MARK SCHEME

Maximum Mark: 50

**Published**

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 1 | est($\mu$) (= 153.2 ÷ 75) = 2.04 (3 sf) | **B1** | |
| | est($\sigma^2$) = $\frac{75}{74}(\frac{340.24}{75} - "2.04267"^2)$ oe | **M1** | |
| | = 0.369 (3 sf) | **A1** | Accept 0.368 |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 2(i) | $\frac{20}{100} \pm z \times \sqrt{\frac{0.2\times(1-0.2)}{100}}$ | **M1** | Any $z$ |
| | $z$ = 1.881 or 1.882 | **B1** | |
| | = 0.125 to 0.275 | **A1** | |
| | | **3** | |
| 2(ii) | $\frac{1}{6}$ is within this range<br>No evidence of bias concerning 2 | **B1ft** | Both statements needed |
| | | **1** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 3 | N(153, 153) | **B1** | Seen or implied |
| | $\frac{139.5-153}{\sqrt{"153"}}$ (= –1.091) | **M1** | Allow with wrong or no cc |
| | $\phi("-1.091") = 1 - \phi("1.091")$ | **M1** | For area consistent with their working |
| | = 0.138 (3 sf) | **A1** | |
| | | **4** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4(i) | mean= 155.1 | **B1** | |
| | var = $1.5^2 \times 10.2$ ( = 22.95)<br>sd = $\sqrt{"22.95"}$ | **M1** | or $1.5 \times \sqrt{10.2}$ |
| | = 4.79 | **A1** | |
| | | **3** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4(ii) | mean = 103.4 + "155.1" (= 258.5)<br>var = 10.2 + "22.95" (=33.15) | **B1ft** | Both. ft their 155.1 and 22.95. Accept sd. |
| | $\frac{250-"258.5"}{\sqrt{"33.15"}}$ (= –1.476) | **M1** | Standardising – no sd/var mix. Their mean/sd must be from an attempt at combination |
| | $1-\phi(-1.476) = \phi(1.476)$ | **M1** | For area consistent with their working |
| | = 0.930 (3 sf) | **A1** | Allow 0.93 |
| | | **4** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5(i) | $\frac{14-14.2}{\frac{3.1}{\sqrt{50}}}$ (= – 0.456) | **M1** | For stand'n; must have √50 |
| | $1 - \Phi("0.456")$ | **M1** | for area consistent with their working |
| | = 0.324 (3 sfs) | **A1** | |
| | | **3** | |
| 5(ii) | No because *n* large | **B1** | Accept n > 30 |
| | | **1** | |
| 5(iii) | $H_0: \mu = 14.2$<br>$H_1: \mu < 14.2$ | **B1** | or 'pop mean', but not just 'mean' |
| | $\frac{13.5-14.2}{\frac{3.1}{\sqrt{100}}}$ | **M1** | For stand'n; must have √100 |
| | = –2.258 | **A1** | |
| | comp –2.054 (or –2.055) | **M1** | Valid comparison of z values or areas (0.0119 < 0.02) |
| | There is evidence (at 2% level) that mean mass in this area < 14.2 | **A1ft** | Ft their z. Correct conclusion no contradictions |
| | | **5** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(i) | $\int_5^{10} \frac{k}{x^2} \mathrm{d}x = 1$ | **M1** | Attempt integration f($x$) and '= 1'; ignore limits |
| | $\left[-\frac{k}{x}\right]_5^{10} = 1$ oe<br>$(\frac{k}{5} - \frac{k}{10} = 1)$ | **A1** | Correct integration and limits and '= 1' |
| | $k = 10$ **AG** | **A1** | No errors seen |
| | | **3** | |
| 6(ii) | $10\int_5^{10} \frac{1}{x} \mathrm{d}x$<br>$10\left[\ln x\right]_5^{10}$ | **M1** | Attempt integ $x$f($x$); ignore limits.<br>or 10(ln 10 – ln 5) |
| | = 10ln 2 **AG** | **A1** | No errors seen |
| | | **2** | |
| 6(iii) | $10\int_9^{10} \frac{1}{x^2} \mathrm{d}x$<br>$(10\left[-\frac{1}{x}\right]_9^{10})$ | **M1** | Attempt integ f(x) with correct limits |
| | $10\left[-\frac{1}{10} + \frac{1}{9}\right]$ | **A1** | Substitute correct limits in correct integration |
| | $= \frac{1}{9}$ or 0.111 (3 sf) | **A1** | |
| | | **3** | |
| 6(iv) | $\int_5^a \frac{k}{x^2} \mathrm{d}x = 0.6$<br>$10\left[-\frac{1}{x}\right]_5^a = 0.6$ | **M1** | Attempt integration of f($x$) with correct limits and = 0.6 |
| | $10[\frac{1}{5} - \frac{1}{a}] = 0.6$ | **A1** | Substitute correct limits in correct integration |
| | $a = \frac{50}{7}$ or 7.14 (3 sf) | **A1** | |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 7(i) | Po(1.0) | **B1** | Seen or implied |
| | $e^{-1}(1 + 1 + \frac{1^2}{2})$ | **M1** | Allow any $\lambda$. Allow one end error. |
| | = 0.920 (3 sfs) | **A1** | |
| | | **3** | |
| 7(ii) | $P(X > 3) = 1 - e^{-1.5}(1+1.5+ \frac{1.5^2}{2} + \frac{1.5^3}{3!})$ | **M1** | Allow any $\lambda$. Allow one end error |
| | = 0.0656 | **A1** | |
| | | **2** | |
| 7(iii)(a) | Incorrectly concluding that more absences than usual when there are not oe | **B1** | In context |
| | | **1** | |
| 7(iii)(b) | $H_0$: $\lambda$ = 1.5 (or 0.3)<br>$H_1$: $\lambda$ > 1.5 (or 0.3) | **B1** | Or $\mu$<br>Both |
| | $P(X > 4)$ = "0.0656" $- e^{-1.5} \times \frac{1.5^4}{4!}$<br>= 0.0186 (3 sf) | **M1** | or $1 - e^{-1.5}(1+1.5+ \frac{1.5^2}{2} + \frac{1.5^3}{3!} + \frac{1.5^4}{4!})$ |
| | P(Type I) = 0.0186 or 0.0185 | **A1ft** | Ft their $P(X > 4)$ if less than 0.05 |
| | | **3** | |
| 7(iii)(c) | $P(X > 3)$ = "0.0656" | **B1ft** | Ft their **(ii)** |
| | 0.0656 > 0.05 | **M1** | |
| | No evidence of more than usual male absences | **A1ft** | Ft their P(X>3). Correct conclusion. No contradictions. |
| | | **3** | |

Cambridge International A Level

**Cambridge Assessment International Education**
Cambridge International Advanced Level

**MATHEMATICS** **9709/72**

Paper 7 **May/June 2018**

MARK SCHEME

Maximum Mark: 50

**Published**

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|---|
| GENERIC MARKING PRINCIPLE 5:<br><br>Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen). |
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- When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

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    B2/1/0 means that the candidate can earn anything from 0 to 2.

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ISW Ignore Subsequent Working

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PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 1 | $\lambda = 4.4$ | **B1** | |
| | $P(X<4) = e^{-4.4}(1 + 4.4 + \frac{4.4^2}{2} + \frac{4.4^3}{3!})$ | **M1** | Allow any $\lambda$ allow one end error |
| | $= 0.359$ | **A1** | |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 2 | A: N(6, 4.8) | **B1 B1** | B1 for N(6, ..) for either A or B. B1 for 4.8 (or $2.19^2$) (or SD=2.19) |
| | B: N(6, 2.4) | **B1** | B1 For 2.4 (or $1.55^2$) (or SD=1.55) (SR 3/3 but error seen withhold B1 so 2/3 scored) |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 3(i) | $52 \pm z \times \frac{6.5}{\sqrt{15}}$ | **M1** | Expression of the correct form. Any $z$ |
| | $z = 1.96$ | **B1** | Seen or used |
| | 48.7 to 55.3 (3 sf) | **A1** | Must be an interval |
| | | **3** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 3(ii) | Narrower<br>because more information or<br>because $\frac{\sigma}{\sqrt{n}}$ smaller | **B1** | oe<br>Accept 'sample size is larger' 'more employees' 'width inversely proportional to sq root of n' 'if n increases width decreases' '95% CI is 49.7 to 54.3'<br>or similar. No contradictions |
| | | **1** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4(i) | Est($\mu$) = 495.9 | **B1** | Accept 496 |
| | Est($\sigma^2$) = $\frac{10}{9}(\frac{2459283}{10} - "495.9"^2)$ | **M1** | Attempt $\Sigma x^2$ and subst in correct formula (1/9("2459283" – "4959"$^2$/10)).<br>May be implied by correct answer |
| | = 12.8 (3 sf) or 383/30 | **A1** | (Note: Biased var "11.49" scores M0 A0) |
| | | **3** | |
| 4(ii) | $H_0$: $\mu = 505$<br>$H_1$: $\mu < 505$<br>$\frac{\frac{75660}{150} - 505}{3.6 \div \sqrt{150}}$ | **B1** | Allow 'Pop mean' but not just 'mean' |
| | = –2.04 | **M1** | Correct stand'n; must have √150. No sd/var mixes. Condone sample SD (3.58/3.39)<br>Accept standardisation of totals ((75660-75750)/44.091) Accept CV method |
| | | **A1** | Accept +2.04 (Note: if valid area comparison done 0.0207/0.0206 or 0.979 needed for A1) |
| | comp $z$ = –2.054 | **M1** | Valid comparison of z's or area (0.0207/6>0.02; 0.979(3)<0.98) |
| | No evidence (at 2%) that machine pkts mean mass < 505 | **A1ft** | oe No contradictions.<br>SC Two tail test can score B0 M1 A1 M1 for comparison with 2.326 A0 (max 3/5) |
| | | **5** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4(iii) | Large sample, so sample mean approx normally distr'd | **B1** | Allow just 'Sample is large' or '$n$ is large' n>30 |
| | | **1** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5(i) | $\frac{1}{2} \times a \times b = 1$ | **M1** | Attempt $\Delta$ area = 1 or $\int(b – bx/a)\, dx = 1$ with correct limits |
| | $b = \frac{2}{a}$ | **A1** | |
| | | **2** | |
| 5(ii) | grad = $-\frac{2}{a^2}$ or $-\frac{b}{a}$ | **B1** | allow without '–' sign (could be implied or seen in **(i)**) |
| | $y - (\frac{2}{a}) =$ grad $\times x$ or $y =$ grad $\times (x – a)$ | **M1** | correct use of $y = mx + c$ or $y - y_1 = m(x – x_1)$ with (0,b) or (a,0) including attempt at substitution of their b |
| | $y - (\frac{2}{a}) = -\frac{2}{a^2}x$ or $y = -\frac{2}{a^2}(x – a)$ and $y = \frac{2}{a} - \frac{2}{a^2}x$ **AG** | **A1** | No errors seen |
| | | **3** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5(iii) | $\int_0^a (\frac{2}{a}x - \frac{2}{a^2}x^2)\mathrm{d}x$ | **M1** | Attempt int $xf(x)$ ignore limits |
| | $= [\frac{1}{a}x^2 - \frac{2}{3a^2}x^3]_0^a$ | **A1** | Correct integration ignore limits |
| | $a - \frac{2}{3}a = 0.5$ | **M1** | Sub correct limits into their integral and = 0.5 |
| | $a = 1.5$ | **A1** | |
| | | **4** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(i) | Accidents occur independently or randomly | **B1** | In context. Allow 'singly'. |
| | | **1** | |
| 6(ii) | $e^{-2.5} \times \frac{2.5^4}{4!}$ | **M1** | Poisson P(4), allow any $\lambda$ |
| | = 0.134 (3 sfs) | **A1** | |
| | | **2** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(iii) | $\lambda = \frac{25}{12}$ or 2.08(333) | **B1** | |
| | $1 - e^{-\frac{25}{12}}(1 + \frac{25}{12} + \frac{\frac{25}{12}^2}{2!} + \frac{\frac{25}{12}^3}{3!})$ | **M1** | 1 – Poisson P(0, 1, 2, 3), allow any $\lambda$ allow one end error |
| | = 0.158 (3 sfs) | **A1** | As final answer |
| | | **3** | |
| 6(iv) | $N(\frac{1825}{84}, \frac{1825}{84})$ or N(21.7(26), 21.7(26)) | **B1** | Stated or implied |
| | $\frac{29.5-\frac{1825}{84}}{\sqrt{\frac{1825}{84}}}$ | **M1** | Allow with wrong or no cc with their mean/sd |
| | Φ("1.668") | **M1** | Correct area consistent with their working |
| | = 0.952 ( 3 sfs) | **A1** | |
| | | **4** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 7(i) | $H_0$: P(10) = 0.1<br>$H_1$: P(10) > 0.1 | **B1** | Both. Allow '*p*' for P(10) |
| | B(9,0.1)<br>P($X \geqslant 3$) =<br>$1 - (0.9^9 + 9\times0.9^8 \times 0.1 + {}^9C_2 \times 0.9^7 \times 0.1^2)$ | **M1** | Allow one extra term in bracket |
| | = 0.05297... or 0.053(0) | **A1** | |
| | comp 0.01 | **M1** | Valid comparison. (comparison with 0.99 can recover previous M1 A1 for 0.9470) |
| | No evidence (at 1% level) to reject $H_0$<br>Claim not justified | **A1ft** | No contradictions |
| | | **5** | |
| 7(ii) | $H_0$ not rejected oe | **B1** | |
| | | **1** | |
| 7(iii) | P($X \geqslant 4$)<br>= "0.05297" – ${}^9C_3\times0.9^6\times0.1^3$ | **M1** | or $1-(0.9^9 + 9 \times 0.9^8 \times 0.1 + {}^9C_2 \times 0.9^7 \times 0.1^2 + {}^9C_3 \times 0.9^6 \times 0.1^3)$ |
| | = 0.00833 | **A1** | Note: 0.05297 and 0.00833 both needed in **(i)** or **(iii)** to justify CV |
| | Hence crit value is 4 | **B1** | Allow without working. Or in **(i)** May be implied by attempt at P($X < 4$) below |
| | B(9,0.5)<br>P($X < 4$) | **M1** | stated or implied |
| | $= 0.5^9 + 9 \times 0.5^8 \times 0.5 + {}^9C_2 \times 0.5^7 \times 0.5^2 + {}^9C_3\times0.5^6\times0.5^3$ | **M1** | Attempt P($X < 4$) with $p = 0.5$ |
| | P(Type II) = 0.254 (3 sf) | **A1** | |
| | | **6** | |

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Cambridge International Advanced Level

**MATHEMATICS** **9709/73**

Paper 7 **May/June 2018**

MARK SCHEME

Maximum Mark: 50

**Published**

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 1(i) | Po(2.25) | **B1** | Stated or implied |
| | $e^{-2.25}(1 + 2.25 + \frac{2.25^2}{2})$ | **M1** | Allow any $\lambda$, one end error |
| | = 0.609 (3 sf) | **A1** | SC B1 Use of B(75,0.03) leading to 0.608 |
| | | **3** | |
| 1(ii) | $\mu$ = 2.25, which is less than 5; n large | **B1** | Allow np<5 and n large or p<0.1 and n>50, no contradictions |
| | | **1** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 2(i) | 213, 165, 73, 196 Allow 073 | **B1** | For 3-digit no, < 265, consisting of three consecutive integers from given digits, backwards or forward. (73 or 073 counts as a 3-digit no.) |
| | | **B1** | For another three such. Other answers may be valid. If other method used, method must be clear |
| | | **2** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 2(ii) | $\frac{510}{25} = \frac{102}{5}$ or 20.4 | **B1** | |
| | $\frac{25}{24}\left[\frac{13225}{25} - \left(\frac{102}{5}\right)^2\right]$ | **M1** | $\frac{1}{24}\left(13225 - \frac{510^2}{25}\right)$ |
| | 118 (3 sf) or $\frac{2821}{24}$ | **A1** | |
| | | **3** | |
| 2(iii) | (Average) weekly earnings of all students in Amy's year | **B1** | Not 'All students in Amy's year' |
| | | **1** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 3 | $\frac{\frac{8}{64}\times(1-\frac{8}{64})}{64}$ (= $\frac{7}{4096}$ or 0.00171) | **M1** | OE, e.g. $\frac{\frac{1}{8}\times\frac{7}{8}}{64}$ |
| | $2\times z\sqrt{"\frac{7}{4096}"} = 0.130$ | **M1** | Correct equation using their variance |
| | $z = 1.572$ | **A1** | |
| | $\phi$("1.572") (= 0.942)<br>(0.942 – (1 – 0.942) = 0.884) | **M1** | 2$\phi$(their $z$) -1 |
| | $\alpha = 88$ | **A1** | CAO |
| | | **5** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4(i) | No of males leaving (to do eng) each yr has const mean or<br>Males leave (to do eng) indep of other males leaving (to do eng) or<br>Males leave (to do eng) at random | **B1** | One of these or any equiv statement in context. |
| | | **1** | |
| 4(ii) | $\lambda = 3.9$ | **B1** | |
| | $1 - e^{-3.9}(1 + 3.9 + \frac{3.9^2}{2!} + \frac{3.9^3}{3!})$ | **M1** | Any $\lambda$. Allow one end error or extra term. |
| | 0.546753 or 0.547 (3 sf) | **A1** | |
| | | **3** | |
| 4(iii) | P($F = 0$ and $M > 3$) =<br>$e^{-0.8} \times \left[1 - e^{-3.1}\left(1 + 3.1 + \frac{3.1^2}{2!} + \frac{3.1^3}{3!}\right)\right]$<br>(= 0.16857) | **M1** | Attempt P($F = 0$) × P($M > 3$) allow one end error for P($M > 3$) provided $\lambda = 3.1$ |
| | $\frac{\text{P(F=0 and M>3)}}{\text{P(M+F>3)}}$<br>$\frac{\text{"0.16857"}}{\text{"0.54675"}}$ | **M1** | Attempted, allow any probability/their **(ii)** provided the answer is <1 |
| | = 0.308 (3 sf) | **A1** | |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5(i) | Assume (pop) sd same (0.3)<br>$H_0$: Pop mean = 2.4 | **B1** | |
| | $H_1$: Pop mean $\neq$ 2.4 | **B1** | Allow '$\mu$' but not just 'mean' |
| | $\pm\frac{2.3-2.4}{\frac{0.3}{\sqrt{30}}}$ | **M1** | Must have $\sqrt{30}$ ,<br>Critical region approach (2.293, 2.507) or (2.193, 2.407) |
| | $= \pm 1.826$ | **A1** | |
| | comp $z = \pm 1.96$ | **M1** | Valid comparison (e.g. compare 0.034 with 0.025) |
| | No evidence that mean time changed | **A1f** | In context, allow accept $H_0$ if correctly defined, no contradictions.<br><br>One-tail test can score B1, B0, M1, A1, M1, A0 Max 4/6 |
| | | **6** | |
| 5(ii)(a) | 0.05 | **B1** | |
| | | **1** | |
| 5(ii)(b) | Concluding mean time has not changed when it has. | **B1** | OE, must have e.g. conclude/accept<br>SR Allow mean has decreased if a one tailed test in Part **(i)** |
| | | **1** | |

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| Question | Answer | | Marks | Guidance |
|---|---|---|---|---|
| 6(i) | $E(T) = 4.5 + 2.3$<br>$Var(T) = 1.1^2 + 0.7^2$ | (= 6.8)<br>(= 1.7) | **M1** | Both methods seen or implied |
| | $\frac{8.5 - "6.8"}{\sqrt{"1.7"}}$ | (= 1.304) | **M1** | Correct stand'n using their $\mu$ and $\sigma^2$ must be a combination of the two variables |
| | $\phi("1.304")$ | | **M1** | Area consistent with their working |
| | = 0.904 (3 sf) | | **A1** | |
| | | | **4** | |
| 6(ii) | $E(D) = 4.5 - 2 \times 2.3$ | or –0.1 | **M1** | |
| | $Var(D) = 1.1^2 + 2^2 \times 0.7^2$ | or 3.17 | **M1** | Both can seen or implied |
| | $\frac{0 - ('-0.1')}{\sqrt{'3.17'}}$ | (= 0.056) | **M1** | Correct stand'n using their $\mu$ and $\sigma^2$ must be a Combination of the two variables |
| | $1 - \phi("0.056")$ | | **M1** | Area consistent with their working |
| | = 0.478 (3 sf) | | **A1** | |
| | | | **5** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 7(i) | $k\int_1^2 (\frac{1}{x^2}+\frac{1}{x^3})\,dx = 1$ | **M1** | Attempt integ f($x$) & '= 1'; ignore limits |
| | $k\left[-\frac{1}{x}-\frac{1}{2x^2}\right]_1^2 = 1$ | **A1** | Correct integral & limits & '= 1' |
| | $k\left[-\frac{1}{2}-\frac{1}{8}+1+\frac{1}{2}\right] = 1$<br>$k = \frac{8}{7}$ **AG** | **A1** | Sufficient working must be shown, no errors seen |
| | | **3** | |
| 7(ii) | $\frac{8}{7}\int_1^2 (\frac{1}{x}+\frac{1}{x^2})\,dx$ | **M1** | Attempt integ $x$f($x$), ignore limits |
| | $= \frac{8}{7}\left[\ln x - \frac{1}{x}\right]_1^2$ | **A1** | Correct integral & limits, condone missing k |
| | $= \frac{8}{7}(\ln 2 + \frac{1}{2})$ or 1.36 (3 sf) | **A1** | |
| | | **3** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 7(iii) | $\frac{8}{7}\int_1^{1.5}(\frac{1}{x^2}+\frac{1}{x^3})\,dx$<br>$=\frac{8}{7}\left[-\frac{1}{x}-\frac{1}{2x^2}\right]_1^{1.5}$ | **M1** | Attempt integration f($x$) between 1 and 1.5 or between 1.5 and 2 |
| | $=\frac{44}{63}$ or 0.698..... | **A1** | Or $\frac{19}{63}$ or 0.302 |
| | $'\frac{44}{63}'(1-'\frac{44}{63}')^2$ | **M1** | FT their $\frac{44}{63}$ |
| | $\times 3$ | **M1** | Independent provided answer is <1 |
| | $= 0.191$ | **A1** | |
| | | **5** | |

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Cambridge International A Level

**Cambridge Assessment International Education**
Cambridge International Advanced Level

**MATHEMATICS** **9709/72**

Paper 7 Probability and Statistics **March 2018**

MARK SCHEME

Maximum Mark: 50

**Published**

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## Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

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## Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

  - Note: B2 or A2 means that the candidate can earn 2 or 0.
    B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

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The following abbreviations may be used in a mark scheme or used on the scripts:

AEF/OE Any Equivalent Form (of answer is equally acceptable) / Or Equivalent

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

SOI Seen or implied

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

**Penalties**

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 1 | $\dfrac{5-4.9}{\dfrac{2.21}{\sqrt{75}}}$ (= 0.392) | **M1** | Correct stand'n. Must have √75 |
| | 1 – Φ("0.392") | **M1** | Correct area consistent with working |
| | = 0.348 (3 sfs) | **A1** | |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 2 | $\lambda = 98.4$ | **B1** | |
| | N(98.4, 98.4) seen or implied | **B1** | |
| | $\frac{90.5-"98.4"}{\sqrt{"98.4"}}$ (= –0.796) | **M1** | allow with wrong or no cc. No sd/var mix. |
| | ϕ("0.796") | **M1** | Correct area consistent with working |
| | = 0.787 (3 sf) | **A1** | |
| | | **5** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 3(i) | $E(H_A) = 6$ | **B1** | |
| | $Var(H_A) = 5 \times 0.03^2$ | **M1** | |
| | = 0.0045 or 9/2000 | **A1** | |
| | | **3** | |
| 3(ii) | $E(H_A - 2H_B) = 0$ | **B1** | From 6–6 |
| | $Var(H_A - 2H_B) = $ '0.0045' + $4\times5\times0.02^2$ | **M2** | Allow **M1** for '0.0045' – $4\times5\times0.02^2$ *or* '0.0045' + $2\times5\times0.02^2$ *or* '0.0045' + $4\times0.02^2$ *or* '0.0045' + $4\times5^2\times0.02^2$ |
| | = 0.0125 (3 sf) or 1/80 | **A1** | |
| | | **4** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4(i) | (Po)(2.4) | **B1** | seen or implied |
| | $e^{-2.4}(1 + 2.4 + \frac{2.4^2}{2} + \frac{2.4^3}{3!})$ | **M1** | allow + P(4)/one end error. Allow wrong $\lambda$ |
| | = 0.779 (3 sfs) | **A1** | Final answer (Note: accept combination method) |
| | | **3** | |
| 4(ii) | $H_0$: $\lambda$ (or mean) = 3.6 (or 0.9)<br>$H_1$: $\lambda$ (or mean) < 3.6 (or 0.9) | **B1** | Accept $\mu$<br>for both |
| | $e^{-3.6}(1 + 3.6)$ | **M1** | Allow any $\lambda$ |
| | = 0.126 | **A1** | |
| | 0.126 > 0.1 | **M1** | Valid comparison. (Comparison with 0.9 could recover previous **M1A1**) |
| | No evidence that fewer than usual sold | **A1FT** | Correct conclusion. No contradictions |
| | | **5** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5(i) | $H_0$: P(Orange) = 0.17 $H_1$: P(Orange) < 0.17 | **B1** | or $H_0$: $p = 0.17$ $H_1$:$p < 0.17$ |
| | | | |
| 5(ii) | Wrongly concluding that % age is less than 17% | **B1** | OE in context<br>allow "fewer than 3 orange in packet even though average 17% is correct" |
| | | **1** | |
| 5(iii) | B(30, 0.17) stated or implied | **M1** | eg by $0.17^p \times 0.83^q$ ($p + q = 30$) or ${}^{30}C_r$ ($r < 30$) |
| | $(1 - 0.17)^{30} + 30(1 - 0.17)^{29} \times 0.17 + {}^{30}C_2(1 - 0.17)^{28} \times 0.17^2$ | **M1** | correct, but allow + ${}^{30}C_3(1–0.17)^{27} \times 0.17^3$ |
| | = 0.0949 (3 sf) | **A1** | (SR: use of N(5.1,4.233) **M1** standardising (with or without cc) **M1** max 2/3) |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5(iv) | P(⩾ 3 orange \| $p = 0.05$) | **M1** | stated or attempted; can be implied |
| | $= 1 - [(0.95)^{30} + 30(0.95)^{29} \times 0.05 + {}^{30}C_2(0.95)^{28} \times 0.05^2]$ | **M1** | allow $+ {}^{30}C_3(0.95)^{27} \times 0.05^3$ in bracket, or ans 0.0608 |
| | = 0.188 (3 sfs) | **A1** | |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(i) | $1 - 6\int_{0.3}^{0.7}\left(x - x^2\right)\mathrm{d}x$ | **M1** | or $2 \times 6\int_{0}^{0.3}\left(x - x^2\right)\mathrm{d}x$ or similar<br>correct expression before integration |
| | $1 - \left[6\left(\frac{x^2}{2} - \frac{x^3}{3}\right)\right]_{0.3}^{0.7}$ | **A1** | or similar correct expression after integration |
| | $1 - 6\left[\frac{0.7^2}{2} - \frac{0.7^3}{3} - \frac{0.3^2}{2} + \frac{0.3^3}{3}\right]$ | **M1** | Attempt subst correct limits in this or other correct expression |
| | = 0.432 (or 54/125) | **A1** | (SR1 Omission of '1–' scores **B2** for 0.568 or 71/125)<br>(SR2 Omission of '2x' scores **B2** for 0.216 or 27/125) |
| | | **4** | |
| 6(ii) | Correct shape between $x = 0$ and 1 | **B1** | No curve outside this range. |
| | E($X$) = 0.5 | **B1** | |
| | | **2** | |
| 6(iii) | $6\int_0^1\left(x^3 - x^4\right)\mathrm{d}x$<br>$= \left[6\left(\frac{x^4}{4} - \frac{x^5}{5}\right)\right]_0^1$ | **M1** | attempt int $x^2$f($x$), ignore limits |
| | $6[\frac{1^4}{4} - \frac{1^5}{5}]$ (= 0.3) | **M1** | attempt subst correct limits in correct integ |
| | Var($X$) = '0.3' – '0.5'$^2$<br>= 0.05 | **A1FT** | FT their mean, dep their Var($X$) > 0 |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 7(i) | $\bar{x} = 11.83$ | **B1** | |
| | $11.83 \pm z \frac{0.1}{\sqrt{10}}$ | **M1** | any $z$ |
| | $z = 2.576$ | **B1** | accept 2.574 to 2.579 |
| | [11.75 to 11.91] | **A1** | or equiv. Accept 11.7 to 11.9 |
| | | **4** | |
| 7(ii) | No because pop normal (so $\bar{X}$ normally distr) | **B1** | |
| | | **1** | |
| 7(iii) | 11.7 not within CI | **B1FT** | |
| | | **1** | |
| 7(iv) | No because 95% CI is narrower than 99% CI | **B1** | OE |
| | | **1** | |
| 7(v) | $\Sigma x^2$ (= 1399.67) | **M1** | attempted |
| | $\text{Est}(\sigma^2) = \frac{10}{9}\left(\frac{"1399.67"}{10} - \left(\frac{"118.3"}{10}\right)^2\right)$ OE | **M1** | correct sub of their Σs into correct formula |
| | = 0.0201 (3 sf) or 181/9000 | **A1** | |
| | | **3** | |

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Cambridge International A Level

**Cambridge Assessment International Education**
Cambridge International Advanced Level

**MATHEMATICS** **9709/71**

Paper 7 **October/November 2017**

MARK SCHEME

Maximum Mark: 50

**Published**

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 1 | $\frac{40.5-31}{\sqrt{31}}$ (= 1.706) | **M1** | standn correct but allow with no or incorrect cc |
| | $1 - \phi(\text{“}1.706\text{”})$ | **M1** | indep correct area consistent with working |
| | = 0.0441 (3 sf) or 0.0440 | **A1** | not 0.044 |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 2 | Poisson | **B1** | seen or implied |
| | $\lambda = 4.03$ | **B1** | seen or implied |
| | $e^{-4.03}(1 + 4.03 + \frac{4.03^2}{2!})$ | **M1** | any $\lambda$; e.g. allow $\lambda = 4$<br>no extra or missing terms |
| | = 0.234 (3 sf) | **A1** | |
| | | **4** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 3 | $\frac{153}{200} + z \times \sqrt{\frac{\frac{153}{200} \times \frac{200-153}{200}}{200}} = 0.835$<br>($\text{Var}(P_s) = 0.000898875$)<br>(s.d. 0.02998) | **M1** | |
| | $z = 2.335$ | **A1** | allow 2.33 or 2.34 |
| | $2\Phi(z) - 1$ | **M1** | or equivalent method<br>indep |
| | $\alpha = 98$ | **A1** | allow 98.0 but not e.g. 98.04 |
| | | **4** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4(i) | $300.1 \pm z \times \frac{0.9}{\sqrt{75}}$ | **M1** | allow any value of $z$ |
| | $z = 2.576$ | **B1** | allow 2.574 to 2.579 |
| | 299.83 to 300.37 (2 dps) | **A1** | answer must be seen to 2 dps<br>need an interval |
| | | **3** | |
| 4(ii) | CI includes 300 so claim supported or justified or probably true | **B1 FT** | or equivalent<br>FT from CI in **(i)** |
| | | **1** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5(i) | $\frac{1}{4}\int_0^2 (x^2 + x)\,dx \qquad (= \frac{1}{4}\left[\frac{x^3}{3} + \frac{x^2}{2}\right]_0^2)$ | **M1** | Attempt integ $xf(x)$, ignore limits |
| | $= \frac{1}{4}(\frac{8}{3} + 2) \quad (- 0)$ | **A1** | Subst correct limits in correct integration |
| | $= \frac{7}{6}$ OE or 1.17 (3 sf) | **A1** | |
| | | **3** | |
| 5(ii) | $\frac{1}{4}\int_0^m (x+1)\,dx = 0.5 \quad (= \frac{1}{4}\left[\frac{x^2}{2} + x\right]_0^m = 0.5)$ | **M1** | attempt integ $f(x)$, limits 0 to unknown (or unknown to 2) and = 0.5 |
| | $\frac{1}{4}(\frac{m^2}{2} + m) = 0.5$<br>$m^2 + 2m - 4 = 0$<br>$m = \frac{-2 \pm \sqrt{4+16}}{2}$ OE | **A1** | a correct equation in $m$ (any form)<br><br>or $\sqrt{5} - 1$ |
| | $m = 1.24$ | **A1** | must reject the negative value if there |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(i) | Mean = $3.2 \times 90 = 288$ | **B1** | |
| | Variance = $0.4^2 \times 90^2$ | **M1** | |
| | $= 1296$ | **A1** | |
| | | **3** | |
| 6(ii) | Mean = '288' + $4.3 \times 95 = 696.5$ | **B1 FT** | |
| | Variance = '1296' + $0.6^2 \times 95^2 = 4545$ | **B1 FT** | FT their **(i)** |
| | $\frac{670-696.5}{\sqrt{4545}}$ (= -0.393) | **M1** | FT Var provided both given Vars used standardising (ignore cc)<br>no sd / Var mix |
| | $1 - \phi(\text{'}-0.393\text{'}) = \phi(\text{'}0.393)$ | **M1** | correct area consistent with their working ( i.e. their mean ) |
| | = 0.653 (3 sf) | **A1** | |
| | | **5** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 7(i) | $H_0$: mean no. sales = 3.5 | **B1** | or " ... = 0.7 (per day)" |
| | $H_1$: mean no. sales > 3.5 | **M1** | allow '$\lambda$' or '$\mu$' but not just 'mean' |
| | $P(X \geqslant 5) = 1 - e^{-3.5}(1 + 3.5 + \frac{3.5^2}{2!} + \frac{3.5^3}{3!} + \frac{3.5^4}{4!})$ | **M1** | |
| | = 0.275 | **A1** | allow 0.274 |
| | Comp with 0.10 | **M1** | valid comparison using Poisson |
| | No evidence (at 10%) to believe that sales per day have increased | **A1 FT** | correct conclusion FT<br>no contradictions |
| | | **6** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 7(ii) | $\lambda = 3.9$ | **B1** | |
| | $e^{-3.9} \times \frac{3.9^2}{2!}$ | **M1** | any $\lambda$ ($\neq 0.7$ or 0.6), single term |
| | = 0.154 (3 sf) | **A1** | |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 8(i) | $\bar{x} = 27/150$ (= 0.18) | **B1** | |
| | $s = \sqrt{\frac{150}{149}} \times \sqrt{\frac{5.01}{150} - 0.18^2}$ or variance<br>(= 0.031729)<br>(var = 3/2980 = 0.0010067) | **M1** | or var = $1/149(5.01 - 27.0^2/150)$ |
| | $H_0$: Pop mean = 0.185<br>$H_1$: Pop mean < 0.185 | **B1** | allow just '$\mu$' |
| | $\frac{0.18 - 0.185}{\frac{'0.031729'}{\sqrt{150}}}$ | **M1** | standardising, need $\sqrt{150}$ |
| | = ( – ) 1.930 (3 sfs) or 1.93 | **A1** | |
| | Comp with $z$ = ( – ) 2.326 | **M1** | consistent signs<br>or using probs 0.0268 > 0.01 or 0.9732 < 0.99<br>or using $x_{crit}$ 0.18 > 0.17897 |
| | There is no evidence (at 1% level) that concentration with drug is less than without drug | **A1 FT** | conclusion FT<br>no contradictions |
| | | **7** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 8(ii) | $\frac{cv - 0.185}{\frac{'0.031729'}{\sqrt{150}}}$ ( = – 2.326 ) | **M1** | must use 0.185 and $\sqrt{150}$ |
| | = 0.17897 or 0.179 | **A1** | acceptance region ( for $H_0$ ) is > 0.179 |
| | $\frac{"0.17897" - 0.175}{\frac{'0.031729'}{\sqrt{150}}}$ (=1.534) | **M1** | must use 0.175 and $\sqrt{150}$ |
| | 1 – ϕ(“1.534”) | **M1** | indep mark |
| | = 0.0625 (3 sf) | **A1** | Accept 0.0610 to 0.0628 |
| | | **5** | |

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**MATHEMATICS** **9709/72**

Paper 7 Paper 7 **October/November 2017**

MARK SCHEME

Maximum Mark: 50

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 1(a)(i) | Po(2.54) | **M1** | seen or implied Po(2540 × 0.001) |
| | $1 - e^{-2.54}(1 + 2.54)$ | **M1** | any $\lambda$ Allow 1 end error |
| | = 0.721 (3 sf) | **A1** | |
| | | **3** | |
| 1(a)(ii) | $n$ large and $p$ small (or $np$ (= 2.54) < 5) | **B1** | $n > 50$, $p < 0.1$ |
| | | **1** | |
| 1(b) | $\mu = 5.6$ | **B1** | |
| | $\sigma = 2.37$ (3 sf) | **B1** | Accept $\sqrt{5.6}$ |
| | | **2** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 2(i) | $4820 \pm z \times \frac{1420}{\sqrt{125}}$ | **M1** | Must be a $z$ value |
| | $z = 2.326$ | **B1** | Accept 2.326 - 2.329 |
| | 4524/4525 to 5115/5116 or 4520 to 5120 (3 sf) | **A1** | Must be an interval |
| | | **3** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 2(ii) | $\bar{x} = 4840$ | **B1** | or width = 280 or half width = 140 |
| | $4840 + 1.96 \times \frac{1420}{\sqrt{n}} = 4980$ OE | **M1** | or $140 = 1.96 \times \frac{1420}{\sqrt{n}}$ OE |
| | $n = 395$ | **A1** | CAO must be an integer |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 3(i) | $\overline{m} = \frac{98.2}{100} = 0.982$ | **B1** | Accept either |
| | $s = \sqrt{\frac{100}{99}} \times \sqrt{\frac{104.52}{100} - 0.982^2}$ (= 0.28582)<br>or var = 0.08169 | **M1** | |
| | $H_0$: Pop mean mass = 1.01<br>$H_1$: Pop mean mass < 1.01 | **B1** | not just 'mean', but allow just '$\mu$' |
| | $\pm\frac{0.982 - 1.01}{\frac{0.28582}{\sqrt{100}}}$ | **M1** | $\pm\frac{0.982 - 1.01}{\frac{0.284387}{\sqrt{100}}}$ **M1** |
| | = −0.980 (3 sf) accept ± | **A1** | = –0.985 (3 sfs) accept ± **A1** |
| | Comp with $z$ = − 1.645 (or areas 0.1635 > 0.05) | **M1** | Valid comparison of $z$'s or area's |
| | No evidence that (mean) mass is less than 1.01 | **A1 FT** | Correct conclusion FT their $z$ |
| | | **7** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 3(ii) | Distr of $X$ normal (so distr of $\overline{X}$ normal)<br>Must state or imply No | **B1** | X/parent population |
| | | **1** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4(i) | $k\int_0^a \frac{1}{\sqrt{x}}\mathrm{d}x = 1$ | **M1** | Attempt int f($x$) and = 1 ignore limits |
| | $(2k[x^{0.5}]_0^a = 1)$<br>$2ka^{0.5} = 1$ or $a = \frac{1}{4k^2}$ | **A1** | OE; a correct eqn in $k$ & $a$ after sub limits |
| | $k\int_0^a \frac{x}{\sqrt{x}}\mathrm{d}x = 3$ | **M1** | Attempt int $x$f($x$) and = 3 |
| | e.g. $\frac{2}{3}ka^{1.5} = 3$ or $a^3 = \frac{81}{4k^2}$ | **A1** | OE; a correct eqn in $k$ and $a$ after sub limits |
| | e.g. $a^2 = 81$ or e.g. $k^2 = \frac{81}{4\times 9^3}$ | **M1** | Attempt eliminate one letter |
| | $a = 9$ | **A1** | Convincingly obtained |
| | e.g. $k = \frac{9}{54}$<br>$k = \frac{1}{6}$ AG | **A1** | |
| | | **7** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4(ii) | $\frac{1}{6}\int_0^m \frac{1}{\sqrt{x}}dx = 0.5$ OE | **M1** | Attempt int f($x$), unknown limit and = 0.5 |
| | $\frac{1}{3}m^{0.5} = 0.5$ | **A1** | a correct equn in $m$ after sub limits |
| | $m = 2.25$ | **A1** | |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5(i) | E($X - Y$) = 56-43 (= 13) | **B1** | |
| | Var($X - Y$) = $6^2 + 5^2$ (= 61) | **M1** | |
| | $\frac{0-13}{\sqrt{'61'}}$ (= −1.664) | **M1** | Ignore any attempted cc/no SD/var mixes.<br>var must be attempt at a combination |
| | 1 – ϕ('−1.664') = ϕ('1.664') | **M1** | For area consistent with their working |
| | = 0.952 (3 sf) | **A1** | Similar scheme for use of $Y - X$ |
| | | **5** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5(ii) | E($M$) = 56 +1.5(43) (= 120.5) | **B1** | |
| | Var($M$) = $6^2 + 1.5^2 \times 5^2$ (= 92.25) | **M1** | |
| | $\frac{135-120.5}{\sqrt{'92.25}}$ (= 1.510) | **M1** | Ignore any attempted cc/no SD/var mixes.<br>var must be attempt at a combination |
| | $1 - \phi('1.510')$ | **M1** | For area consistent with their working |
| | = 0.0655 or 0.0656 or 6.55% or 6.56% (3 sf)<br>As final answer | **A1** | Allow 6.6% or 6.5% or 7% if correct working seen |
| | | **5** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(i) | $H_0$: Pop mean no. defectives = 5.15<br>$H_1$: Pop mean no. defectives < 5.15 | **B1** | or '= 1.03 (per day)'<br>not just 'mean', but allow just '$\lambda$' or '$\mu$' |
| | P($X \leqslant 2$) | **M1** | Attempted. Any one term error/end error/incorrect λ/expression 1–… |
| | $= e^{-5.15}\left(1 + 5.15 + \frac{5.15^2}{2}\right)$ | **M1** | Correct expression attempted |
| | = 0.113 | **A1** | |
| | Comp with 0.1 | **M1** | Valid comparison |
| | No evidence to believe mean no. of defectives has decreased | **A1 FT** | Correct conclusion (FT their value)<br>No contradictions |
| | | **6** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(ii) | BOTH P($X \leqslant 1$) = $e^{-5.15}$ (1 + 5.15) (= 0.0357) AND P($X \leqslant 2$) = = $e^{-5.15}$ (1 + 5.15 + $\frac{5.15^2}{2}$ )= (0.113) | **B1*** | (Could be seen in **(i)**) |
| | Comp either with 0.1 | **DB1** | One comparison with 0.01 (could be seen in **(i)**) |
| | P(Type I error) = 0.0357 (3 sf) | **B1** | |
| | | **3** | |
| 6(iii) | Actually mean = 1.03 but conclude that mean < 1.03 | **B1** | Mean no. of defectives not reduced, but conclude that it is reduced. |
| | | **1** | |

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Cambridge International A Level

**Cambridge Assessment International Education**
Cambridge International Advanced Level

**MATHEMATICS** **9709/73**

Paper 7 **October/November 2017**

MARK SCHEME

Maximum Mark: 50

**Published**

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

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Cambridge International is publishing the mark schemes for the October/November 2017 series for most Cambridge IGCSE®, Cambridge International A and AS Level components and some Cambridge O Level components.

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 1 | $\frac{40.5-31}{\sqrt{31}}$ (= 1.706) | **M1** | standn correct but allow with no or incorrect cc |
| | 1 – ϕ("1.706") | **M1** | indep correct area consistent with working |
| | = 0.0441 (3 sf) or 0.0440 | **A1** | not 0.044 |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 2 | Poisson | **B1** | seen or implied |
| | $\lambda = 4.03$ | **B1** | seen or implied |
| | $e^{-4.03}(1 + 4.03 + \frac{4.03^2}{2!})$ | **M1** | any $\lambda$; e.g. allow $\lambda = 4$<br>no extra or missing terms |
| | = 0.234 (3 sf) | **A1** | |
| | | **4** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 3 | $\frac{153}{200} + z \times \sqrt{\frac{\frac{153}{200} \times \frac{200-153}{200}}{200}} = 0.835$<br>(Var($P_s$) = 0.000898875)<br>(s.d. 0.02998) | **M1** | |
| | $z = 2.335$ | **A1** | allow 2.33 or 2.34 |
| | $2\Phi(z) - 1$ | **M1** | or equivalent method<br>indep |
| | $\alpha = 98$ | **A1** | allow 98.0 but not e.g. 98.04 |
| | | **4** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4(i) | $300.1 \pm z \times \frac{0.9}{\sqrt{75}}$ | **M1** | allow any value of $z$ |
| | $z = 2.576$ | **B1** | allow 2.574 to 2.579 |
| | 299.83 to 300.37 (2 dps) | **A1** | answer must be seen to 2 dps<br>need an interval |
| | | **3** | |
| 4(ii) | CI includes 300 so claim supported or justified or probably true | **B1 FT** | or equivalent<br>FT from CI in **(i)** |
| | | **1** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5(i) | $\frac{1}{4}\int_0^2 (x^2 + x)\,dx \qquad (= \frac{1}{4}\left[\frac{x^3}{3} + \frac{x^2}{2}\right]_0^2)$ | **M1** | Attempt integ $xf(x)$, ignore limits |
| | $= \frac{1}{4}(\frac{8}{3} + 2) \quad (- 0)$ | **A1** | Subst correct limits in correct integration |
| | $= \frac{7}{6}$ OE or 1.17 (3 sf) | **A1** | |
| | | **3** | |
| 5(ii) | $\frac{1}{4}\int_0^m (x+1)\,dx = 0.5 \quad (= \frac{1}{4}\left[\frac{x^2}{2} + x\right]_0^m = 0.5)$ | **M1** | attempt integ f($x$), limits 0 to unknown (or unknown to 2) and = 0.5 |
| | $\frac{1}{4}(\frac{m^2}{2} + m) = 0.5$<br>$m^2 + 2m - 4 = 0$<br>$m = \frac{-2 \pm \sqrt{4+16}}{2}$ OE | **A1** | a correct equation in $m$ (any form)<br><br>or $\sqrt{5} - 1$ |
| | $m = 1.24$ | **A1** | must reject the negative value if there |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(i) | Mean = $3.2 \times 90 = 288$ | **B1** | |
| | Variance = $0.4^2 \times 90^2$ | **M1** | |
| | $= 1296$ | **A1** | |
| | | **3** | |
| 6(ii) | Mean = '288' + $4.3 \times 95 = 696.5$ | **B1 FT** | |
| | Variance = '1296' + $0.6^2 \times 95^2 = 4545$ | **B1 FT** | FT their **(i)** |
| | $\frac{670-696.5}{\sqrt{4545}}$ (= -0.393) | **M1** | FT Var provided both given Vars used standardising (ignore cc)<br>no sd / Var mix |
| | $1 - \phi(\text{'}-0.393\text{'}) = \phi(\text{'}0.393)$ | **M1** | correct area consistent with their working ( i.e. their mean ) |
| | = 0.653 (3 sf) | **A1** | |
| | | **5** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 7(i) | $H_0$: mean no. sales = 3.5 | **B1** | or " ... = 0.7 (per day)" |
| | $H_1$: mean no. sales > 3.5 | **M1** | allow '$\lambda$' or '$\mu$' but not just 'mean' |
| | $P(X \geqslant 5) = 1 - e^{-3.5}(1 + 3.5 + \frac{3.5^2}{2!} + \frac{3.5^3}{3!} + \frac{3.5^4}{4!})$ | **M1** | |
| | = 0.275 | **A1** | allow 0.274 |
| | Comp with 0.10 | **M1** | valid comparison using Poisson |
| | No evidence (at 10%) to believe that sales per day have increased | **A1 FT** | correct conclusion FT<br>no contradictions |
| | | **6** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 7(ii) | $\lambda = 3.9$ | **B1** | |
| | $e^{-3.9} \times \frac{3.9^2}{2!}$ | **M1** | any $\lambda$ ($\neq 0.7$ or 0.6), single term |
| | = 0.154 (3 sf) | **A1** | |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 8(i) | $\bar{x} = 27/150$ (= 0.18) | **B1** | |
| | $s = \sqrt{\frac{150}{149}} \times \sqrt{\frac{5.01}{150} - 0.18^2}$ or variance<br>(= 0.031729)<br>(var = 3/2980 = 0.0010067) | **M1** | or var = $1/149(5.01 - 27.0^2/150)$ |
| | $H_0$: Pop mean = 0.185<br>$H_1$: Pop mean < 0.185 | **B1** | allow just '$\mu$' |
| | $\frac{0.18 - 0.185}{\frac{'0.031729'}{\sqrt{150}}}$ | **M1** | standardising, need $\sqrt{150}$ |
| | = ( – ) 1.930 (3 sfs) or 1.93 | **A1** | |
| | Comp with $z$ = ( – ) 2.326 | **M1** | consistent signs<br>or using probs 0.0268 > 0.01 or 0.9732 < 0.99<br>or using $x_{crit}$ 0.18 > 0.17897 |
| | There is no evidence (at 1% level) that concentration with drug is less than without drug | **A1 FT** | conclusion FT<br>no contradictions |
| | | **7** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 8(ii) | $\frac{cv - 0.185}{\frac{'0.031729'}{\sqrt{150}}}$ ( = – 2.326 ) | **M1** | must use 0.185 and $\sqrt{150}$ |
| | = 0.17897 or 0.179 | **A1** | acceptance region ( for $H_0$ ) is > 0.179 |
| | $\frac{"0.17897" - 0.175}{\frac{'0.031729'}{\sqrt{150}}}$ (=1.534) | **M1** | must use 0.175 and $\sqrt{150}$ |
| | 1 – ϕ(“1.534”) | **M1** | indep mark |
| | = 0.0625 (3 sf) | **A1** | Accept 0.0610 to 0.0628 |
| | | **5** | |

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Cambridge International A Level

**Cambridge International Examinations**
Cambridge International Advanced Level

**MATHEMATICS** **9709/71**

Paper 7 **May/June 2017**

MARK SCHEME

Maximum Mark: 50

**Published**

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 1(i) | Poisson with $\lambda = 0.2$ | **B1** | |
| | $1-e^{-0.2}\,(1 + 0.2 + \frac{0.2^2}{2})$ | **M1** | 1 – Poisson P(0, 1, 2, 3) attempted, any λ, allow one end error |
| | = 0.00115 (3 sf) | **A1** | SR: using Bin, ans 0.00115: **B1** |
| | **Total:** | **3** | |
| 1(ii) | *n* large (n > 50) | **B1** | |
| | $np = 0.2 < 5$ or *p* small | **B1** | |
| | **Total:** | **2** | |
| 2 | Assume sd still = 3.8 | **B1** | or sd unchanged |
| | $H_0$: $\mu = 64.0$ $H_1$: $\mu < 64.0$ | **B1** | |
| | $\frac{63.3-64.0}{\frac{3.8}{\sqrt{100}}}$ | **M1** | Standardising with their values (no sd / var mixes) Must have $\sqrt{100}$ |
| | = –1.842 | **A1** | |
| | comp "1.842" with *z*-value<br>"1.842" < 1.96 | **M1** | comp +ve with +ve or –ve with –ve<br>or comp Φ ("1.842") with 0.975<br>0.9672 < 0.975 OE |
| | No evidence that heights are shorter | **A1FT** | OE FT their $z_{calc}$ |
| | **Total:** | **6** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 3(a) | $7.1 \pm z \times \sqrt{\frac{2.6}{75}}$ | **M1** | Expression of correct form must be z (note MR var = 2.6² can score **M1**)<br>seen |
| | $z = 1.751$ | **B1** | |
| | 6.77 to 7.43 (3 sfs) | **A1** | Must be an interval |
| | **Total:** | **3** | |
| 3(b) | $0.04^3$ | **M1** | Allow $0.08^3$ for **M1** |
| | $= 0.000064$ | **A1** | |
| | **Total:** | **2** | |
| 3(c) | e.g. Particular day or time of day | **B1** | Allow "Not random" |
| | **Total:** | **1** | |
| 4(i) | Greater area where $x < 7.5$ than $x > 7.5$ | **B1** | Allow Graph higher for $x < 7.5$ than for $x > 7.5$<br>or Graph decreasing or equiv expl'n |
| | **Total:** | **1** | |
| 4(ii) | $\int_5^{10} \frac{k}{x^2}\,dx = 1$ | **M1** | Attempt Integ f($x$) = 1 ignore limits |
| | $k\left[-\frac{1}{x}\right]_5^{10} = 1$<br>$k \times \frac{1}{10} = 1$ | **A1** | Correct integration and limits |
| | $k = 10$ AG | **A1** | No errors seen |
| | **Total:** | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4(iii) | $10\int_5^{10} \frac{1}{x}\ \mathrm{d}x$ | **M1** | Attempt Integ $xf(x)$ ignore limits |
| | $= 10\left[\ln x\right]_5^{10}$<br>$= 10(\ln 10 - \ln 5)$ | **M1** | Correct integration and limits |
| | $= 10\ln 2$ or 6.93 (3 sf) | **A1** | OE |
| | **Total:** | **3** | |
| 4(iv) | $10\int_5^{10} 1\ \mathrm{d}x - "6.93"^2$ | **M1** | Attempt (Integ $x^2f(x)$) – $(E(x))^2$. No limits **M0** |
| | = 1.95 (accept 1.96) | **A1** | *Use of 6.93 gives 1.97* **A0** |
| | **Total:** | **2** | |
| 5(i) | $W \sim$ N(6210, 171.88) | **B2** | seen or implied. **B1** each parameter |
| | $\frac{6200 - "6210"}{\sqrt{"171.88"}}$ (= – 0.763) | **M1** | Standardising with their values. No sd / var mix |
| | 1 – Φ(“0.763”) | **M1** | For area consistent with their mean |
| | = 0.223 (3 sfs) | **A1** | |
| | **Total:** | **5** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5(ii) | E(C – 2B) = −50 | **M1** | "6210"–2(3130) (or E(2B–C)=50 |
| | Var(C – 2B) = "171.88" + $2^2 \times 12.1^2$ (= 757.52) | **M1** | |
| | $\frac{0-(-50)}{\sqrt{"757.52"}}$ (= 1.817) | **M1** | Standardising with their values |
| | Φ("1.817") | **M1** | For area consistent with their mean |
| | = 0.965 (3 sfs) | **A1** | |
| | **Total:** | **5** | |
| 6(i) | mean = 6.6 | **B1** | **B1** for 6.6 (could be scored in iii) |
| | $P(X \leqslant 1) = e^{-6.6}(1 + 6.6) = 0.0103$ | **M1** | Allow incorrect $\lambda$ in both probs |
| | $P(X \leqslant 2) = e^{-6.6}(1 + 6.6 + \frac{6.6^2}{2}) = 0.0400$ | **M1A1** | **A1** for both values |
| | CR is $X \leqslant 1$ | **DA1** | Dep on at least one **M** |
| | P(Type I error) = $P(X \leqslant 1) = 0.0103$ | **B1FT** | FT their $P(X \leqslant 1)$ |
| | **Total:** | **6** | |
| 6(ii) | Wrongly concluding that (mean) no of (sports) injuries has decreased | **B1** | Must be in context |
| | **Total:** | **1** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(iii) | $H_0$: $\lambda = 6.6$ $H_1$: $\lambda < 6.6$ | **B1** | Can be scored in (i). Allow $\mu$ or $\lambda$ / 1.1 or 6.6 or P($X \leqslant 2$) = 0.0400 > 0.02 |
| | 2 not in CR | **M1** | |
| | No evidence mean no. of injuries has decreased | **A1FT** | |
| | **Total:** | **3** | |
| 6(iv) | N(39.6, 39.6) | **B1** | May be implied |
| | $\frac{29.5 - 39.6}{\sqrt{39.6}}$ (= −1.605) | **M1** | Allow with wrong or no cc |
| | Φ("–1.605") = 1 – Φ("1.605") | **M1** | For area consistent with their mean |
| | = 0.0543 (3 sfs) | **A1** | |
| | **Total:** | **4** | |

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Cambridge International A Level

**Cambridge International Examinations**
Cambridge International Advanced Level

**MATHEMATICS** **9709/72**

Paper 7 **May/June 2017**

MARK SCHEME

Maximum Mark: 50

**Published**

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

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## Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

  - Note: B2 or A2 means that the candidate can earn 2 or 0.
    B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

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The following abbreviations may be used in a mark scheme or used on the scripts:

AEF/OE Any Equivalent Form (of answer is equally acceptable) / Or Equivalent

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

SOI Seen or implied

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

**Penalties**

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 1 | $\frac{0.801\times(1-0.801)}{2000}$ (= 0.0000797) | **M1** | |
| | $0.801 \pm z \times \sqrt{"0.0000797"}$ | **M1** | Allow any $z$-value |
| | $z = 1.96$ | **B1** | |
| | 0.784 to 0.818 (3 sf) | **A1** | As final answer. Must be an interval<br>Allow 0.783 to 0.819 |
| | **Total:** | **4** | |
| 2(i) | E(X) = 4.197 | **B1** | |
| | Var (X) = 4.196 | **B1** | Both to 3dp or better |
| | **Total:** | **4** | |
| 2(ii) | $E(X) \approx Var(X)$ | **B1** | Condone = |
| 2(iii) | $e^{-4.1968}\left(1+4.1968+\frac{4.1968^2}{2}+\frac{4.1968^3}{3!}+\frac{4.1968^4}{4!}\right)$ | **M1** | Any λ. Allow with one end error |
| | = 0.59(0) (3 sfs) | **A1** | Allow 0.591 |
| | **Total:** | **2** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 3(i) | Est ($\mu$) = 923/400 or 2.3075 or 2.31 (3 sf) | **B1** | |
| | Est($\sigma^2$) = $\frac{400}{399}\left(\frac{3170}{400} - "2.3075"^2\right)$ OE | **M1** | |
| | = 2.60696 or 2.61 (3 sf) | **A1** | (Note**:** Biased Var= 2.600 scores **M0**) |
| | **Total:** | **3** | |
| 3(ii) | $H_0$: Pop mean (or $\mu$) = "2.31" or "2310"<br>$H_1$: Pop mean (or $\mu$) > "2.31" or "2310" | **B1 FT** | |
| | $\pm \frac{2.6-"2.310"}{\sqrt{2.60696 \div 50}} = 1.27$ | **M1 A1** | Standardising using their values, Accept 1.28 |
| | Comp 1.645 (OE) | **M1** | Valid comparison *z* values or areas |
| | No evidence that incomes in the region greater | **A1 FT** | OE FT their *z*. No contradictions<br>(No FT for 2 tail test – max score **B0 M1 A1 M1** for comp 1.96 **A0**)<br>**Note:** Accept alternative CV method |
| | **Total:** | **5** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4(i) | $0.75^{20} + 20 \times 0.75^{19} \times 0.25 + {}^{20}C_2 \times 0.75^{18} \times 0.25^2$ | **M1** | No end errors |
| | = 0.0913 | **A1** | As final answer |
| | **Total:** | **2** | |
| 4(ii) | $H_0$: Pop proportion=0.25<br>$H_1$: Pop proportion<0.25 | **B1** | Allow $p$ or $\pi$, not "proportion"<br>(Accept anywhere in the question) |
| | $0.75^{25} + 25 \times 0.75^{24} \times 0.25$ | **M1** | Must be B(25,0,25) No end errors |
| | = 0.00702 | **A1** | |
| | comp 0.01 | **M1** | Valid comparison |
| | There is evidence that the claim is not justified | **A1 FT** | OE. No contradictions |
| | **Total:** | **5** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5(i) | $0.5\times1\times h=0.25$<br>$h = 0.5$<br>grad = 0.5 | **M1** | $P(X < 2) = 4 \times P(X < 1)$ **M1** |
| | $f(x) = 0.5x$ | **A1** | $P(X < 2) = 1$ **A1**<br>$a = 2$ **A1** |
| | $0.5 \times a \times 0.5a = 1$ | **M1** | $0.5 \times 2 \times h' = 1$ **M1**<br>$h' = 1$ |
| | $a = 2$ | **A1** | grad = 0.5 |
| | $P(X < 2) = 1$ | **A1** | $f(x) = 0.5x$ **A1** |
| | **Total:** | **5** | |
| 5(ii) | $\int_0^m 0.5x dx = 0.5$ | **M1** | Attempt $\int f(x)dx = 0.5$ Ignore limits |
| | $=\left[\frac{x^2}{4}\right]_0^m = 0.5$ | **A1FT** | Correct integration (ft $f(x)$ ) & limits = 0.5 |
| | $m = \sqrt{2}$ or 1.41 (3 sf) | **A1** | or by similarity $m = \frac{1}{\sqrt{2}} \times 2$ **M2**<br>$= \sqrt{2}$ **A1** |
| | **Total:** | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(i) | $e^{-2.4} \times \frac{2.4^2}{2!}$ | **M1** | Allow incorrect λ |
| | = 0.261 (3 sfs) | **A1** | |
| | **Total:** | **2** | |
| 6(ii) | N(60, 60) | **B1** | seen or implied |
| | $\frac{54.5-60}{\sqrt{60}}$ (= −0.710) | **M1** | allow with wrong or missing cc |
| | 1− ϕ("−0.710") = ϕ("0.710") | **M1** | For area consistent with their working |
| | = 0.761 (3 sf) | **A1** | |
| | **Total:** | **4** | |
| 6(iii) | $\lambda = 3.6 + 12 \div 7$ (= 186/35) (= 5.314) | **M1** | |
| | $e^{-5.314}\left(1+5.314+\frac{5.314^2}{2}+\frac{5.314^3}{3!}\right)$ | **M1** | Allow incorrect λ. Allow one end error. |
| | = 0.224 (3 sfs) | **A1** | |
| | **Total:** | **3** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 7(a) | $E(X_1+X_2) = 2 \times 4.2 = 8.4$<br>$Var(X_1+X_2) = 2 \times 1.1^2 = 2.42$ | **B1** | Both. Seen or implied (or sd = 1.56) |
| | $\frac{10-8.4}{\sqrt{2.42}}$ (= 1.029) | **M1** | Standardising with their mean and var (no sd / var mix) |
| | $1 - \phi("1.029")$ | **M1** | For area consistent with their working |
| | = 0.152 (3 sf) | **A1** | |
| | **Total:** | **4** | |
| 7(b) | $E(X) = 20.5$ | **B1** | |
| | $Var(X) = 105 + 0.5^2 \times 15$ (= 108.75) | **M1** | correct expression oe |
| | $\frac{0-"20.5"}{\sqrt{"108.75"}}$ (= −1.966) | **M1** | correct standardisation using their E & V (no sd/var mix) ignore any attempted cc |
| | $\phi("-1.966") = 1- \phi("1.966")$<br>(= (1 − 0.9754)) | **M1** | For area consistent with their working |
| | = 0.0246 or 2.46% (3 sf) | **A1** | Accept 0.0247 |
| | **Total:** | **5** | |

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Cambridge International A Level

**Cambridge International Examinations**
Cambridge International Advanced Level

**MATHEMATICS** **9709/73**

Paper 7 **May/June 2017**

MARK SCHEME

Maximum Mark: 50

**Published**

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

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**Mark Scheme Notes**

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A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

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    B2/1/0 means that the candidate can earn anything from 0 to 2.

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- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

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AEF/OE Any Equivalent Form (of answer is equally acceptable) / Or Equivalent

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CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

SOI Seen or implied

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

**Penalties**

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PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 1 | 573, 43 (or 043), 289 | **B1B1B1** | Ignore incorrect numbers. But allow other correct use of table (i.e. 573, 650, 431) |
| | **Total:** | **3** | |
| 2(i) | $z = 1.751$ | **B1** | |
| | $\frac{103}{200} \pm z\sqrt{\frac{\frac{103}{200}\times(1-\frac{103}{200})}{200}}$ oe | **M1** | all correct except for recognisable value of $z$, allow for one side only |
| | = 0.453 to 0.577 (3 sf) as final answer | **A1** | must be an interval |
| | **Total:** | **3** | |
| 2(ii) | 0.08 oe 8%, 8/100 | **B1** | |
| 3 | $10 \times 0.46^2$ (= 2.116) or $\frac{0.46}{\sqrt{10}}$ | **B1** | SOI |
| | Total mass of ore ~ N(70, 2.116) or $\sim N\left(7, \left(\frac{0.46}{\sqrt{10}}\right)^2\right)$ | **B1** | |
| | $\pm\frac{71-"70"}{\sqrt{"2.116"}}$ or $\pm\frac{7.1-"7.0"}{0.46/\sqrt{10}}$ (= 0.687) | **M1** | correct, using their sd or √(their var)<br>e.g. allow $\frac{71-"70"}{4.6}$ for **M1** |
| | $1 - \phi("0.687")$ | **M1** | for correct area consistent with their working |
| | = 0.246 (3 sf) | **A1** | |
| | **Total:** | **5** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4(i) | $\bar{x} = 6.7/200$ (= 67/2000 = 0.0335) | **B1** | |
| | $s^2 = \frac{200}{199} \times \left( \frac{0.2312}{200} - "0.0335"^2 \right)$ | **M1** | $s^2 = \frac{0.2312}{200} - 0.0335^2$ **M0** |
| | = 0.0000339(2) = 27/796000 | **A1** | = 0.00003375 **A0** |
| | **Total:** | **3** | |
| 4(ii) | $H_0$: Pop mean level = 0.034<br>$H_1$: Pop mean level ≠ 0.034 | **B1** | not just "mean", but allow just "$\mu$" |
| | $\frac{"030335" - 0.034}{\frac{\sqrt{"0.00003392"}}{\sqrt{200}}}$ | **M1** | must have $\sqrt{200}$<br>$\frac{0.0335 - 0.034}{\frac{\sqrt{"0.00003375"}}{\sqrt{200}}}$ **M1** |
| | = –1.21(4) (3 sfs) (–1.22 ↔ –1.21) | **A1** | = –1.217 (3 sfs) **A1** |
| | Comp with $z = -1.645$ (or 0.1124>0.05) | **M1** | 0.112 > 0.05<br>valid comparison $z$ or areas |
| | No evidence that (mean) pollutant level has changed, accept $H_0$ (if correctly defined) | **A1FT** | correct conclusion no contradictions<br><br>SR: One tail test: **B0**, **M1A1** as normal, **M1** (comparison with 1.282 consistent signs) **A0** |
| | **Total:** | **5** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5(i)(a) | $X \sim$ N(42, 42) | **B1** | stated or implied |
| | $\frac{39.5 - "42"}{\sqrt{"42"}}$ (= −0.386) | **M1** | allow with wrong or no cc |
| | 1 − ϕ ("−0.386") = ϕ ("0.386") | **M1** | correct area consistent with their working |
| | = 0.65(0) (3 sf) | **A1** | |
| | **Total:** | **4** | |
| 5(i)(b) | 42 > (e.g. 15) or mean is large | **B1** | $\lambda > 15$ or higher, $\lambda$ = large<br>ignore subsequent work if not undermining what already written |
| | **Total:** | **1** | |
| 5(ii)(a) | $Y \sim$ Po(1.2) | **B1** | stated or implied |
| | $1 - e^{-1.2}(1 + 1.2 + \frac{1.2^2}{2})$ | **M1** | allow any $\lambda$ allow one end error |
| | = 0.121 (3 sf) | **A1** | Using binomial: 0.119 SR **B1** |
| | **Total:** | **3** | |
| 5(ii)(b) | 60 × 0.02 = 1.2 < 5 or mean is small | **B1FT** | or large $n$ small $p$<br>FT Poisson only |
| | **Total:** | **1** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(i) | $k\int_0^1 (x - x^2)\,\mathrm{d}x = 1$ | **M1** | Attempt integ f($x$) and "= 1", ignore limits |
| | $= k\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1 = 1$ | **A1** | correct integration, limits 0 and 1 |
| | $= k\left[\frac{1}{2} - \frac{1}{3}\right] = 1$ or $\frac{k}{6} = 1$ | **A1** | correctly obtained, no errors seen |
| | **Total:** | **3** | |
| 6(ii) | E($X$) = 0.5 | **B1** | |
| | $6\int_0^1 (x^3 - x^4)\,\mathrm{d}x$ | **M1** | Attempt integ $x^2$f($x$), limits 0 to 1 |
| | $(= 6\left[\frac{1}{4} - \frac{1}{5}\right] = 0.3)$<br>"0.3" − "0.5"$^2$ | **M1** | their int $x^2$f($x$) – their (E($X$))$^2$ dep +ve result |
| | = 0.05( = 1/20) | **A1** | |
| | **Total:** | **4** | |
| 6(iii) | $6\int_{0.4}^1 (x - x^2)\,\mathrm{d}x$ | **M1** | ignore limits, eg **M1**for $6\int_{0.4}^2 (x - x^2)\,\mathrm{d}x$ |
| | $= 6\{\frac{1}{2} - \frac{1}{3} - (\frac{0.4^2}{2} - \frac{0.4^3}{3})\}$ | **A1FT** | subst correct limits into correct integration |
| | = 0.648(= 81/125) | **A1** | condone incorrect "k" for **A1** |
| | **Total:** | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 7(i) | $H_0$: Pop mean no. accidents = 5.64<br>$H_1$: Pop mean no. accidents < 5.64 | **B1** | or "= 0.47 (per month)"<br>not just "mean", but allow just "$\lambda$" or "$\mu$" |
| | Use of $\lambda = 5.64$ | **B1** | used in a Poisson calculation |
| | $= e^{-5.64}\left(1 + 5.64 + \frac{5.64^2}{2}\right)$ | **M1** | Allow incorrect $\lambda$ in otherwise correct |
| | = 0.08(0) | **A1** | |
| | Comp with 0.05 | **M1** | Valid comparison (Poisson only), no contradictions. |
| | No evidence to believe mean no. of accidents has decreased; accept $H_0$ (if correctly defined) | **A1FT** | Normal distribution: **M0M0** |
| | **Total:** | **6** | |
| 7(ii) | Mean < 0.47 but conclude that this is not so | **B1** | (Mean) no. of accidents **reduced**, but conclude not reduced. Must be in context. |
| | **Total:** | **1** | |
| 7(iii) | (Need greatest $x$ such that $P(X \leqslant x) < 0.05$ )<br>$P(X \leqslant 1) = e^{-5.64}(1 + 5.64) = 0.024$<br>$P(X \leqslant 2) = 0.08$ | **B1** | Both, could be seen in (i) |
| | Hence rejection region is $X \leqslant 1$ | **B1** | Can be implied |
| | With $\lambda = 12 \times 0.05 = 0.6$,<br>$1 - P(X \leqslant 1) = 1 - e^{-0.6}(1 + 0.6)$ | **M1** | $\lambda$=0.6 and $1 - P(X \leqslant 1)$ |
| | = 0.122 (3 sf) | **A1** | Normal scores 0 |
| | **Total:** | **4** | |

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Cambridge International Advanced Level

**MATHEMATICS** **9709/72**

Paper 7 Probability and Statistics **March 2017**

MARK SCHEME

Maximum Mark: 50

**Published**

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## Mark Scheme Notes

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AEF/OE Any Equivalent Form (of answer is equally acceptable) / Or Equivalent

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

SOI Seen or implied

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

**Penalties**

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 1 | Var($Ps$) = $\frac{0.3(1-0.3)}{120}$ (= 0.00175) | **M1** | Attempt correct values in correct formula |
| | $0.3 \pm z\sqrt{"0.00175"}$ | **M1** | must be a $z$-value, not a prob |
| | $z = 1.645$ | **B1** | |
| | CI = 0.231 to 0.369 (3 sf) | **A1** | |
| | **Total:** | **4** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 2(i) | ($H_1$): $\mu \neq 6.4$ | **B1** | |
| | **Total:** | **1** | |
| 2(ii) | comp 2.43 with a $z$-value<br>$z = 2.576$ AND | **M1** | oe valid comparison |
| | No evidence that $\mu$ is not 6.4<br>or do not reject $\mu = 6.4$ | **A1** | Allow "Accept $\mu = 6.4$"<br>Must mention $\mu$, not just "$H_0$" or "$H_1$" |
| | **Total:** | **2** | |
| 2(iii) | Testing for an increase in $\mu$, or for a decrease in $\mu$, rather than a change | **B1** | Any equiv statement |
| | **Total:** | **1** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 3(i) | $\frac{53-52}{6.1\div\sqrt{75}}$ (= 1.420) | **M1** | |
| | $\frac{51-52}{6.1\div\sqrt{75}}$ (= –1.420) | **M1** | or –"1.420" seen |
| | Φ("1.420") – Φ("–1.420") | **M1** | |
| | = 0.844 (3 sfs) | **A1** | |
| | **Total:** | **4** | |
| 3(ii) | Need to assume $\bar{X}$ (approx.) normally distributed | **B1** | or $X$ not stated to be normally distributed |
| | **Total:** | **1** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4(i) | ($\lambda =$) 4.5 | **B1** | |
| | $e^{-4.5}(1 + 4.5 + \frac{4.5^2}{2!})$ | **M1** | Allow any $\lambda$. Allow one end error |
| | = 0.174 | **A1** | |
| | **Total:** | **3** | |
| 4(ii) | Accept reduction in mean no. of missed appts although untrue | **B1** | or Mean is 0.9 (or 4.5) but < 3 missed appts. In context |
| | **Total:** | **1** | |
| 4(iii) | $P(X \geqslant 3)$ | **M1** | Attempted |
| | $= 1 - e^{-1}(1 + 1 + \frac{1^2}{2!})$ | **M1** | Allow any $\lambda$ except 4.5 or 0.9, Allow one end error |
| | = 0.0803 (3 sfs) | **A1** | |
| | **Total:** | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5(a)(i) | $k = 1$ | **B1** | |
| | **Total:** | **1** | |
| 5(a)(ii) | $f_2$: area > 1 (area ≠ 1) | **B1** | oe |
| | $f_3$: includes negative values of $f_3$ | **B1** | oe |
| | **Total:** | **2** | |
| 5(b)(i) | $6\int_{-a}^{a}(a^2 - x^2)\mathrm{d}x = 1$ | **M1** | Integ f($x$) = 1, ignore limits |
| | $6[a^2x - \frac{x^3}{3}]_{-a}^{a} = 1$ | **A1** | Correct integral and limits |
| | $6(2a^3 - \frac{2a^3}{3}) = 1$<br>$\frac{24a^3}{3} = 1$ or $8a^3 = 1$<br>$a = 1/2$ **AG** | **A1** | Correctly obtained. No errors seen.<br>(SR Verification scores M1A1 only max 2/3) |
| | **Total:** | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5(b)(ii) | 0 | **B1** | |
| | **Total:** | **1** | |
| 5(b)(iii) | $6\int_{-0.5}^{0.5}(\frac{x^2}{4}-x^4)\mathrm{d}x$<br>$(=6\left[\frac{x^3}{12}-\frac{x^5}{5}\right]_{-0.5}^{0.5}=0.05)$<br>Var = $0.05 - 0^2$ | **M1** | attempt int $x^2$f($x$) & correct limits |
| | = 0.05 oe | **A1** | cao; allow omission of $-0^2$ |
| | **Total:** | **2** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(i) | Assume cartons are random sample(s) | **B1** | or masses of cartons are independent of each other oe |
| | E($T$) = 816.4<br>Var($T$) = 1570.08 | **B1** | Both |
| | $z = \frac{900-"816.4"}{\sqrt{"1570.08"}}$ (= 2.110) | **M1** | |
| | 1 – Φ(“2.110”) | **M1** | |
| | = 0.0174 = 1.74% (3 sfs) | **A1** | % only (accept 1.7% if 0.0174 seen) |
| | **Total:** | **5** | |
| 6(ii) | P($F - S > 0$) stated or implied | **M1** | P($S - F < 0$) |
| | 62.0 – 78.8 (= –16.8)<br>& $10.0^2 + 12.6^2$ (= 258.76) | **B1** | 78.8 – 62.0 (= 16.8)<br>& $12.6^2 + 10.0^2$ (= 258.76) |
| | $z = \frac{0-("-16.8")}{\sqrt{"258.76"}}$ (= 1.044) | **M1** | $z = \frac{0-"16.8"}{\sqrt{"258.76"}}$ (= –1.044) |
| | 1 – Φ(“1.044”) | **M1** | Φ(“–1.044”) = 1 – Φ(“1.044”) |
| | (= 1 – 0.8517)<br>= 0.148 (3 sfs) | **A1** | |
| | **Total:** | **5** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 7(i) | Planes arrive at constant mean rate | **B1** | |
| | Planes arrive at random | **B1** | or Planes arrive independently<br>Must be in context |
| | **Total:** | **2** | |
| 7(ii)(a) | ($\lambda =$) $5.2 \div 4$ | **M1** | |
| | $e^{-1.3}(\frac{1.3^2}{2} + \frac{1.3^3}{3!})$ | **M1** | Allow any $\lambda$ , allow one end error |
| | = 0.330 (3 sfs) | **A1** | Accept 0.33 |
| | **Total:** | **3** | |
| 7(ii)(b) | $1 - e^{-3.467} \times (1 + 3.467 + \frac{3.467^2}{2!} + \frac{3.467^3}{3!})$ | **M1** | Allow any $\lambda$ except 5.2 or 1.3, allow one end error |
| | = 0.456 (3 sfs) | **A1** | |
| | **Total:** | **2** | |
| 7(iii) | N(52, 52) stated or implied | **B1** | |
| | $\frac{60.5-52}{\sqrt{52}}$ (= 1.179) | **M1** | ft their mean and var.<br>Allow wrong or no cc or no √ |
| | Φ(“1.179”) | **M1** | |
| | = 0.881 (3 sf) | **A1** | |
| | **Total:** | **4** | |

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Cambridge International A Level

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**MATHEMATICS** **9709/71**

Paper 7 **October/November 2016**

MARK SCHEME

Maximum Mark: 50

**Published**

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PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

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| | | | | |
|---|---|---|---|---|
| **1** | $\left(\frac{508}{8}\right) = 63.5$<br>$(\Sigma x^2 = 32360.12)$<br>$\frac{8}{7}\left(\frac{'32360.12'}{8} - '63.5'^2\right)$<br>$= 14.6$ (3 sf) or 2553/175 | **B1**<br><br>**M1**<br>**A1** | [3] | <br><br>oe<br>From correct working |
| **2 (i)** | $H_0$: P(6) = $^1/_6$ $H_1$: P(6) < $^1/_6$ | **B1** | [1] | Allow $H_0$: $p = {}^1/_6$ $H_1$: $p < {}^1/_6$ |
| **(ii)** | $\left(\frac{5}{6}\right)^{15}$<br>$= 0.065 > 0.05$ | **M1**<br>**A1** | [2] | <br>Correct result and comparison needed for A1<br>SR if 2 tail test followed allow A1 for $0.065 > 0.025$ |
| **(iii)** | $\left(\frac{5}{6}\right)^{16} = 0.054$ and $\left(\frac{5}{6}\right)^{17} = 0.045$<br><br>Smallest $n$ is 17<br><br>**OR**<br>$\left(\frac{5}{6}\right)^n < 0.05$ and attempt to solve<br>$n\ln\left(\frac{5}{6}\right) < \ln 0.05$<br>smallest $n$ is 17 | **M1**<br><br>**A1**<br><br>**M1**<br><br>**A1** | [2] | both<br><br>No errors seen |
| **3 (i)** | $(\lambda) = 3.6 \div 3 = 1.2$<br>$1 - e^{-1.2}\left(1 + 1.2 + \frac{1.2^2}{2} + \frac{1.2^3}{3!}\right)$<br>$= 0.0338$ (3 sf) | **B1**<br>**M1**<br>**A1** | [3] | 1.2 seen<br>Allow any $\lambda$<br>As final answer |
| **(ii)** | N(60 × 3.6, 60 × 3.6)<br><br>$\frac{240.5 - '216'}{\sqrt{'216'}}$ (= 1.667)<br>1– Φ('1.667')<br>= 0.0478 (3 sf) | **M1**<br><br>**M1**<br>**M1**<br>**A1** | [4] | Stated or implied<br><br>Allow with no or wrong cc (no sd/var mixes)<br>Area consistent with their working<br><br>SR use of Poisson 0.0497 scores 4/4 |
| **4 (i)** | 6080 (litres)<br>106 (litres) | **B1**<br>**B1** | [2] | |
| **(ii)** | $E(21Y - 2X) = 635$<br>$Var(21Y - 2X) =$<br>$21^2 \times 12^2 + 2^2 \times 53^2$<br>(= 74740)<br>$\frac{0 - 635}{\sqrt{'74740'}}$ (= –2.323)<br><br>1–Φ('–2.323') = Φ('2.323')<br>= 0.99(0) (3 sf) | **B1**<br>**B1**<br><br><br>**M1**<br><br>**M1**<br>**A1** | [5] | <br>correct expression or result or sd = 273 seen<br><br><br>no sd/var mixes<br><br>Area consistent with their working<br>No errors seen |
| **5 (a)** | $63 \pm z \times \frac{9}{\sqrt{100}}$<br>$z = 1.645$<br>61.5 to 64.5 (3 sf) | **M1**<br>**B1**<br>**A1** | B1<br><br>[3] | Expression of correct form, any $z$<br>Seen<br>Must be an interval |

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| | | | | |
|---|---|---|---|---|
| **(b) (i)** | $z = \frac{1.96}{2}$ (= 0.98)<br>Φ("0.98") (= 0.8365)<br>"0.8365" – (1 – "0.8365")<br>(= 0.673)<br>$\alpha = 67.3$ (3 sf) | **M1**<br>**M1**<br>**A1** | [3] | Allow $\frac{\text{any } z}{2}$<br>Allow 67 from correct working |
| **(ii)** | $4=(2x'z'x'\sigma')/\sqrt{n}$<br>$n = 200$ | **M1**<br>**A1** | [2] | Attempt to solve equ of correct form<br>SR B1 for $n = 100$ |
| **6 (i)** | $m_X, m_Y, m_Z, m_W$ or *X, Y, Z, W* | **B2** | [2] | B1 if two adjacent means interchanged, i.e. $m_Y, m_X, m_Z, m_W$ or $m_X, m_Z, m_Y, m_W$ or $m_X, m_Y, m_W, m_Z$<br>B1 for correct order reversed. |
| **(ii) (a)** | $\int_0^3 \frac{4}{81}x^4 \mathrm{d}x$<br>$= \left[\frac{4}{81}\frac{x^5}{5}\right]_0^3$<br>$= \frac{4}{81}\times\frac{3^5}{5}$ or $\frac{4}{81}\times\frac{243}{5}$ or $\frac{972}{405}$ oe<br>$= \frac{12}{5}$ or 2.4 AG | **M1**<br>**A1**<br><br>**A1** | [3] | Attempt int $xf(x)$. Ignore limits<br>Correct integration and limits (condone missing 4/81)<br>Must see correct expression as well as $\frac{12}{5}$ or 2.4<br>No errors seen |
| **(b)** | $\int_{2.4}^3 \frac{4}{81}x^3 \mathrm{d}x$ or $1 - \int_0^{2.4} \frac{4}{81}x^3 \mathrm{d}x$<br>$= \left[\frac{4}{81}\frac{x^4}{4}\right]_{2.4}^3$ or $1 - \left[\frac{4}{81}\frac{x^4}{4}\right]_0^{2.4}$<br>$= 1 - \frac{4}{81}\times\frac{2.4^4}{4}$ oe<br>$= \frac{369}{625}$ or 0.59(0) (3 sf) | **M1**<br>**A1**<br><br>**A1** | [3] | Attempt int $f(x)$ ignore limits<br>Correct integration and limits (condone missing 4/81)<br>As final answer |
| **(c)** | 1 | **B1** | [1] | |

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| | | | | |
|---|---|---|---|---|
| **7 (i)** | $H_0$: Pop mean time (or $\mu$) = 20.5<br>$H_1$: Pop mean time (or $\mu$) < 20.5 | **B1** | | Not just "mean" |
| | $\frac{20.3-20.5}{1.2\div\sqrt{100}}$ | **M1** | | Allow without √ sign |
| | = –1.667<br>or 0.0478/0.952 if areas compared | **A1** | | (accept ±1.667/1.67) |
| | '1.667' < 1.751<br>(or '–1.667' > –1.751)<br>No evidence that (pop) mean time has decreased | **M1**<br><br><br>**A1ft** | [5] | Correct comparison of their $z_{calc}$ with 1.751/1.75 oe valid comparison of areas (0.0478 > 0.04)<br>No contradictions (ft their $z$) |
| **(ii)** | $\frac{cv-20.5}{1.2\div\sqrt{100}}$ = –1.751 | **M1*** | | |
| | cv = 20.29 or 20.3 | **A1** | | |
| | $\frac{'20.29'-20.1}{1.2\div\sqrt{100}}$ (= 1.583 or 1.582) | **DM1** | | Allow $\frac{20.3-20.1}{1.2\div\sqrt{100}}$ (= 1.667) M1 |
| | 1 – Φ('1.583')<br>= 0.0567 – 0.0569 (3 sf) | **M1**<br>**A1** | [5] | 1 – Φ('1.667') M1<br>= 0.0478 (3 sf) A1 |
| **(iii)** | Concluding (mean) time not decreased when in fact it has. | **B1** | [1] | Must be in context<br>oe |

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**MATHEMATICS** **9709/72**

Paper 7 **October/November 2016**

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| | | | | |
|---|---|---|---|---|
| **1** | $\left(\frac{508}{8}\right) = 63.5$<br>$(\Sigma x^2 = 32360.12)$<br>$\frac{8}{7}\left(\frac{'32360.12'}{8} - '63.5'^2\right)$<br>$= 14.6$ (3 sf) or 2553/175 | **B1**<br><br>**M1**<br>**A1** | [3] | <br><br>oe<br>From correct working |
| **2 (i)** | $H_0$: P(6) = $^1/_6$ $H_1$: P(6) < $^1/_6$ | **B1** | [1] | Allow $H_0$: $p = {}^1/_6$ $H_1$: $p < {}^1/_6$ |
| **(ii)** | $\left(\frac{5}{6}\right)^{15}$<br>$= 0.065 > 0.05$ | **M1**<br>**A1** | [2] | <br>Correct result and comparison needed for A1<br>SR if 2 tail test followed allow A1 for $0.065 > 0.025$ |
| **(iii)** | $\left(\frac{5}{6}\right)^{16} = 0.054$ and $\left(\frac{5}{6}\right)^{17} = 0.045$<br><br>Smallest $n$ is 17<br><br>**OR**<br>$\left(\frac{5}{6}\right)^n < 0.05$ and attempt to solve<br>$n\ln\left(\frac{5}{6}\right) < \ln 0.05$<br>smallest $n$ is 17 | **M1**<br><br>**A1**<br><br>**M1**<br><br>**A1** | [2] | both<br><br>No errors seen |
| **3 (i)** | $(\lambda) = 3.6 \div 3 = 1.2$<br>$1 - e^{-1.2}\left(1 + 1.2 + \frac{1.2^2}{2} + \frac{1.2^3}{3!}\right)$<br>$= 0.0338$ (3 sf) | **B1**<br>**M1**<br>**A1** | [3] | 1.2 seen<br>Allow any $\lambda$<br>As final answer |
| **(ii)** | N(60 × 3.6, 60 × 3.6)<br><br>$\frac{240.5 - '216'}{\sqrt{'216'}}$ (= 1.667)<br>1– Φ('1.667')<br>= 0.0478 (3 sf) | **M1**<br><br>**M1**<br>**M1**<br>**A1** | [4] | Stated or implied<br><br>Allow with no or wrong cc (no sd/var mixes)<br>Area consistent with their working<br><br>SR use of Poisson 0.0497 scores 4/4 |
| **4 (i)** | 6080 (litres)<br>106 (litres) | **B1**<br>**B1** | [2] | |
| **(ii)** | $E(21Y - 2X) = 635$<br>$Var(21Y - 2X) =$<br>$21^2 \times 12^2 + 2^2 \times 53^2$<br>(= 74740)<br>$\frac{0 - 635}{\sqrt{'74740'}}$ (= –2.323)<br><br>1–Φ('–2.323') = Φ('2.323')<br>= 0.99(0) (3 sf) | **B1**<br>**B1**<br><br><br>**M1**<br><br>**M1**<br>**A1** | [5] | <br>correct expression or result or sd = 273 seen<br><br><br>no sd/var mixes<br><br>Area consistent with their working<br>No errors seen |
| **5 (a)** | $63 \pm z \times \frac{9}{\sqrt{100}}$<br>$z = 1.645$<br>61.5 to 64.5 (3 sf) | **M1**<br>**B1**<br>**A1** | B1<br><br>[3] | Expression of correct form, any $z$<br>Seen<br>Must be an interval |

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|---|---|---|---|---|
| **(b) (i)** | $z = \frac{1.96}{2}$ (= 0.98)<br>Φ("0.98") (= 0.8365)<br>"0.8365" – (1 – "0.8365")<br>(= 0.673)<br>$\alpha = 67.3$ (3 sf) | **M1**<br>**M1**<br>**A1** | [3] | Allow $\frac{\text{any } z}{2}$<br>Allow 67 from correct working |
| **(ii)** | $4=(2x'z'x'\sigma')/\sqrt{n}$<br>$n = 200$ | **M1**<br>**A1** | [2] | Attempt to solve equ of correct form<br>SR B1 for $n = 100$ |
| **6 (i)** | $m_X, m_Y, m_Z, m_W$ or $X, Y, Z, W$ | **B2** | [2] | B1 if two adjacent means interchanged, i.e. $m_Y, m_X, m_Z, m_W$ or $m_X, m_Z, m_Y, m_W$ or $m_X, m_Y, m_W, m_Z$<br>B1 for correct order reversed. |
| **(ii) (a)** | $\int_0^3 \frac{4}{81}x^4 \mathrm{d}x$<br>$= \left[\frac{4}{81}\frac{x^5}{5}\right]_0^3$<br>$= \frac{4}{81}\times\frac{3^5}{5}$ or $\frac{4}{81}\times\frac{243}{5}$ or $\frac{972}{405}$ oe<br>$= \frac{12}{5}$ or 2.4 AG | **M1**<br>**A1**<br><br>**A1** | [3] | Attempt int $xf(x)$. Ignore limits<br>Correct integration and limits (condone missing 4/81)<br>Must see correct expression as well as $\frac{12}{5}$ or 2.4<br>No errors seen |
| **(b)** | $\int_{2.4}^3 \frac{4}{81}x^3 \mathrm{d}x$ or $1 - \int_0^{2.4} \frac{4}{81}x^3 \mathrm{d}x$<br>$= \left[\frac{4}{81}\frac{x^4}{4}\right]_{2.4}^3$ or $1 - \left[\frac{4}{81}\frac{x^4}{4}\right]_0^{2.4}$<br>$= 1 - \frac{4}{81}\times\frac{2.4^4}{4}$ oe<br>$= \frac{369}{625}$ or 0.59(0) (3 sf) | **M1**<br>**A1**<br><br>**A1** | [3] | Attempt int $f(x)$ ignore limits<br>Correct integration and limits (condone missing 4/81)<br>As final answer |
| **(c)** | 1 | **B1** | [1] | |

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|---|---|---|---|---|
| **7 (i)** | $H_0$: Pop mean time (or $\mu$) = 20.5<br>$H_1$: Pop mean time (or $\mu$) < 20.5 | **B1** | | Not just "mean" |
| | $\frac{20.3-20.5}{1.2\div\sqrt{100}}$ | **M1** | | Allow without √ sign |
| | = –1.667<br>or 0.0478/0.952 if areas compared | **A1** | | (accept ±1.667/1.67) |
| | '1.667' < 1.751<br>(or '–1.667' > –1.751)<br>No evidence that (pop) mean time has decreased | **M1**<br><br><br>**A1ft** | [5] | Correct comparison of their $z_{calc}$ with 1.751/1.75 oe valid comparison of areas (0.0478 > 0.04)<br>No contradictions (ft their $z$) |
| **(ii)** | $\frac{cv-20.5}{1.2\div\sqrt{100}}$ = –1.751 | **M1*** | | |
| | cv = 20.29 or 20.3 | **A1** | | |
| | $\frac{'20.29'-20.1}{1.2\div\sqrt{100}}$ (= 1.583 or 1.582) | **DM1** | | Allow $\frac{20.3-20.1}{1.2\div\sqrt{100}}$ (= 1.667) M1 |
| | 1 – Φ('1.583')<br>= 0.0567 – 0.0569 (3 sf) | **M1**<br>**A1** | [5] | 1 – Φ('1.667') M1<br>= 0.0478 (3 sf) A1 |
| **(iii)** | Concluding (mean) time not decreased when in fact it has. | **B1** | [1] | Must be in context<br>oe |

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Cambridge International A Level

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Cambridge International Advanced Level

**MATHEMATICS** **9709/73**

Paper 7 **October/November 2016**

MARK SCHEME

Maximum Mark: 50

**Published**

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

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| | | | | |
|---|---|---|---|---|
| **1** | $e^{-3.5}(1 + 3.5 + \frac{3.5^2}{2!})$<br>= 0.321 (3 sf) | **M2**<br><br>**A1** | [3] | Allow M1 if extra term $e^{-3.5} \times \frac{3.5^3}{3!}$ or " 1 - . " or omit P(0) |
| **2 (i)** | 59 | **B1** | [1] | |
| **(ii)** | Any $x$ such that $0.687 \leqslant x \leqslant 0.693$ (3 sf) | **B1** | [1] | or 0.69 or " ... 0.686.. < 0.693 rec " |
| **(iii)** | Possible repeats | **B1** | [1] | |
| **3** | N(178, …)<br>Var = $3.2^2 + 4.1^2 + 3.8^2$ or 41.49<br><br>$\frac{175-'178'}{\sqrt{'41.49'} \div \sqrt{15}}$ (= '–1.804')<br><br>Φ('–1.804') = 1 – Φ ('1.804')<br>= 0.0356 (3 sf) | **B1**<br>**B1**<br><br>**M1**<br><br>**M1**<br>**A1** | [5] | stated or implied<br>or sd = 6.44 stated or implied<br><br>need √15 but allow var / sd mix for "41.49"<br>allow cc for method marks<br><br>independent M1 for area / prob consistent with working |
| **4** | $\frac{11.8-11}{1.6 \div \sqrt{n}} = 1.645$<br>$\frac{11.8-11}{1.6 \div \sqrt{n}} = 1.96$<br><br>$n = 10.8$ (allow 11)<br>$n = 15.4$ (allow 15)<br><br>Possible values are 11, 12, 13, 14, 15 | **M1**<br><br><br>**B1**<br>**B1**<br><br>**A1**<br><br>**A1** | [5] | M1 for $\frac{11.8-11}{1.6 \div \sqrt{n}}$ = any $z$<br>allow var / sd mix for 1.6 but need $\sqrt{n}$<br><br>B1 for each correct $z$<br><br>for both<br><br>not for just $11 \leqslant n \leqslant 15$ oe |

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|---|---|---|---|---|
| **5 (i)** | $H_0$: P(free gift) = 0.3 or $p = 0.3$<br>$H_1$: P(free gift) < 0.3 or $p < 0.3$ | **B1** | [1] | |
| **(ii)** | P($X \leqslant 2$) =<br>$0.7^{20} + 20 \times 0.7^{19} \times 0.3 + {}^{20}C_2 \times 0.7^{18} \times 0.3^2$<br>= 0.03548 or 0.0355 | **M1***<br>**A1** | | P($X \leqslant 2$) attempted |
| | P($X \leqslant 3$) =<br>'0.03548' + ${}^{20}C_3 \times 0.7^{17} \times 0.3^3$ (= 0.107 ) | **M1*** | | P($X \leqslant 3$) attempted |
| | One comparison with 0.05 seen | **M1*** | | or implied by fully correct methods for P($X \leqslant 2$) and P($X \leqslant 3$) |
| | P(Type I error) = 0.0355 (3 sf) | **DA1** √ | [5] | dep on all 3 Ms |
| **(iii)** | P($X \leqslant 3$) = '0.107'<br>'0.107' > 0.05<br>**or** cv = 2 and compare 3 >2 | **M1** | | Compare their P($X \leqslant 3$) with 0.05 |
| | No evidence to reject claim oe | **A1** √ | [2] | No evidence that 30% is not correct oe<br>ft their 0.107 |
| **6 (i)** | est($\mu$) = 3.4<br>est($\sigma^2$)= $\frac{100}{99}(\frac{1356}{100} - '3.4'^2)$<br>= 2.02(0202) | **B1**<br>**M1**<br>**A1** | | 1 / 99 (1356 – $340^2$/100 )<br>or 200/99 |
| | $z$ = 1.96 | **B1** | | |
| | $3.4 \pm z \times \sqrt{\frac{'2.020202'}{100}}$<br>= 3.12 to 3.68 ( 3 sf) | **M1**<br>**A1** | [6] | correct working only<br>allow from unbiased or biased variance |
| **(ii)** | Mean should be 3 | **B1*** | | stated or implied |
| | CI does not include 3<br>Machine probably not working properly | **DB1**√ | [2] | √ their CI<br>or evidence that…. |
| **7 (i)** | $1 - e^{-1}(1 + 1)$ (= 0.26424)<br>$1 - e^{-1.5}(1 + 1.5 + \frac{1.5^2}{2!})$ (= 0.19115) | **B1**<br>**B1** | | B1 for either λ correct.<br>B1 for either correct expression with correct λ |
| | '0.26424' × '0.19115' | **M1** | | product of their values for $\leqslant 2$ and $\leqslant 3$ from Poisson,<br>need correct form " 1 - .. " , but allow incorrect λ values and end errors |
| | = 0.0505 (3 sf) | **A1** | [4] | accept 0.0504 |

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| | | | | |
|---|---|---|---|---|
| **(ii)** | $\lambda = 30$<br>N(30, 30) | **B1**<br>**B1**√ | | seen or implied, need N(λ,λ) |
| | $\frac{35.5-30}{\sqrt{30}}$ (= 1.004) | **M1** | | allow with wrong or no cc or no √ |
| | Φ ('1.004') | **M1** | | consistent with their working |
| | = 0.842 (3 sf) | **A1** | [5] | |
| **8 (i)** | $\sigma_X$, $\sigma_Z$, $\sigma_Y$, $\sigma_W$ **or** $X$, $Z$, $Y$, $W$ | **B2** | [2] | B1 if two adjacent sds interchanged, ie $\sigma_Z$, $\sigma_X$, $\sigma_Y$, $\sigma_W$ **or** $\sigma_X$, $\sigma_Y$, $\sigma_Z$, $\sigma_W$ **or** $\sigma_X$, $\sigma_Z$, $\sigma_W$, $\sigma_Y$<br>B1 for correct order reversed |
| **(ii) (a)** | Mean = 0 stated or found<br>or "– 0" seen | **B1** | | |
| | $\frac{1}{18}\int_{-3}^{3} x^4 dx - 0$<br>$= \frac{1}{18}\left[\frac{x^5}{5}\right]_{-3}^{3}$<br>$= \frac{1}{18}\left[\frac{3^5}{5}+\frac{3^5}{5}\right]$ oe<br>= 5.4 | **M1** | | Attempt integral $^2$ f($x$). Ignore limits<br>Allow without "– 0" |
| | sd = √5.4 or $\sqrt{\frac{1}{18}\left[\frac{3^5}{5}+\frac{3^5}{5}\right]}$ or 2.324<br>sd = 2.32 (3 sf) AG | **A1** | [3] | Must see $\sqrt{\text{correct expression or 5.4}}$<br>or 2.324 or better |
| **(b)** | $\frac{1}{18}\int_{'2.324'}^{3} x^2 dx$ | **M1** | | Attempt to integrate f($x$), ignore limits |
| | $\frac{1}{18}\left[\frac{x^3}{3}\right]_{'2.324'}^{3} = \frac{1}{18}\left[\frac{3^3}{3}-\frac{'2.324'^3}{3}\right]$ | **A1** | | Sub correct limits into correct integral |
| | = 0.268 (3 sf) | **A1** | [3] | Allow 0.269 |
| **(c)** | 0 | **B1** | [1] | |

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Cambridge International A Level

**Cambridge International Examinations**
Cambridge International Advanced Level

**MATHEMATICS** **9709/71**

Paper 7 **May/June 2016**

MARK SCHEME

Maximum Mark: 50

**Published**

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CAO Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)

CWO Correct Working Only – often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

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Note: '(3 sfs)' means 'answer which rounds to … to 3 sfs'. If correct ans seen to > 3sfs, ISW for later rounding. Penalise < 3 sfs only once in paper.

| | | | | | |
|---|---|---|---|---|---|
| **1** | | B(200, $\frac{1}{6}$) → N($\frac{100}{3}$ , $\frac{250}{9}$)<br>$\frac{25.5-\frac{100}{3}}{\sqrt{\frac{250}{9}}}$<br>= –1.486<br><br>comp '1.486' with 1.282<br><br>Evidence to reject $H_0$<br>There is some evidence that $p < \frac{1}{6}$<br>or, e.g. It is likely that $p < \frac{1}{6}$ oe | **B1**<br>**M1**<br>**A1**<br><br>**M1**<br><br><br><br>**A1 ft** | [5] | seen or implied<br>allow with wrong or no cc<br>(Accept alternative correct methods)<br><br>or comp ('1.486') with 0.1<br><br><br><br>No contradictions |
| **2** | **(i)** | Each employee has an equal chance of being chosen | **B1** | [1] | oe |
| | **(ii)** | Est ($\mu$) = 4<br>Est ($\sigma^2$) = $\frac{10}{9}(\frac{199.22}{10} - '4'^2)$<br><br>= 4.36 (3 sf) | **B1**<br>**M1**<br><br>**A1** | [3] | <br>sub in correct formula attempted<br><br>working may not be seen |
| | **(iii)** | Distances travelled by all employees at the firm | **B1** | [1] | oe |
| **3** | **(i)** | ((0.5672 + 0.6528) ÷ 2)<br>= 0.61 | **B1** | [1] | |
| | **(ii)** | '0.61' + $z\sqrt{\frac{'0.61'\times(1-'0.61')}{350}}$ = 0.6528<br>$z$ = 0.0428 × $\sqrt{\frac{700}{'0.61'\times(1-'0.61')}}$ oe<br><br>= 2.321<br>98% confidence | **M1**<br>**M1**<br><br>**A1**<br>**A1 ft** | [4] | oe<br>correct rearrangement of correct equn, ft '0.61'<br><br><br>ft their $z$<br>(dep on both Ms) |

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| | | | | | |
|---|---|---|---|---|---|
| **4** | **(i)** | $H_0: \mu = 12.5$<br>$H_1: \mu \neq 12.5$ | **B1** | | |
| | | $\frac{13.5-12.5}{4.2\div\sqrt{50}}$ | **M1** | | allow $4.2 \div 50$ |
| | | $= 1.68(4)$ | **A1** | | |
| | | '1.684' < 1.96 | **M1** | | comp 1.96 allow comp 1.645 if H1: $\mu >$ 12.5<br>or comp 1 – ('1.684') with 0.025 |
| | | No evidence that mean time has changed | **A1 ft** | [5] | No contradictions<br>ft their 1.684, but not comp 1.645 |
| | **(ii)** | 0.05 | **B1** | [1] | |
| **5** | **(i)** | $T \sim N(6 \times 2.4, 6 \times 0.3^2)$<br>$(= N(14.4, 0.54)$ | **M1** | | seen or implied |
| | | $\frac{16-'14.4'}{\sqrt{'0.54'}}$ $(= 2.177)$ | **M1** | | ft their E and Var; allow without √<br>(Accept alternative method $N(2.4,(0.3^2)/6)$) |
| | | 1 – ('2.177') | **M1** | | correct area consistent with their working |
| | | $= 0.0147$ (3 sf) | **A1** | [4] | |
| | **(ii)** | $D = X_1 - 1.1X_2$<br>$E(D) = -0.24$ | **B1** | | |
| | | $Var(D) = 0.3^2 + 1.1^2 \times 0.3^2$ $(= 0.1989)$ | **M1** | | |
| | | $\frac{0-(-0.24)}{\sqrt{'0.1989'}}$ $(= 0.538)$ | **M1** | | ft their E and Var; allow without √ |
| | | ('0.538') | **M1** | | correct area consistent with their working |
| | | $= 0.705$ (3 sf) | **A1** | [5] | |
| **6** | **(i)** | 2 m | **B1** | [1] | allow without units |
| | **(ii)** | $k\int_0^2 x^2(2-x)\mathrm{d}x = 1$ | **M1** | | attempt integ f($x$) and '= 1'. Ignore limits |
| | | $k\left[\frac{2x^3}{3} - \frac{x^4}{4}\right]_0^2$ | **A1** | | correct integration and limits |
| | | $k \times \left[\frac{16}{3} - 4\right] = 1$ or $k \times \frac{4}{3} = 1$ oe | | | |
| | | $k = \frac{3}{4}$ **AG** | **A1** | [3] | No errors seen |
| | **(iii)** | $\frac{3}{4}\int_0^2 x^3(2-x)\mathrm{d}x$ | **M1** | | attempt integ $x$f($x$), condone missing $k$ |
| | | $= \frac{3}{4} \times \left[\frac{2x^4}{4} - \frac{x^5}{5}\right]_0^2$ | **A1** | | correct integration and limits, condone missing $k$ |
| | | 1.2 m oe | **A1** | [3] | allow without units |

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| | | | | |
|---|---|---|---|---|
| | **(iv)** | $\frac{3}{4}\int_0^1 x^2(2-x)\mathrm{d}x$<br>$(=\frac{3}{4}\times(\frac{2}{3}-\frac{1}{4}))$<br>$=\frac{5}{16}$ or 0.3125 oe<br><br>$400\times\frac{5}{16}=125$ | **M1**<br><br><br>**A1**<br><br>**A1 ft** [3] | attempt integ f($x$), 0 to 1, condone missing $k$<br><br><br><br>ft their $\frac{5}{16}$ |
| **7** | **(a) (i)** | $0.01\times 80$ and $0.015\times 60$<br>$(1-\mathrm{e}^{-0.8})\times(1-\mathrm{e}^{-0.9})$<br>$=0.327$ (3 sf) | **M1**<br>**M1**<br>**A1** [3] | $(1-\mathrm{e}^{-\lambda})\times(1-\mathrm{e}^{-\mu})$ any $\lambda$, $\mu$ $(\lambda\neq\mu)$<br>allow one end error |
| | **(ii)** | $\lambda=0.02\times 40+0.015\times 60$<br><br>$\mathrm{e}^{-1.7}\times(1+1.7+\frac{1.7^2}{2})$<br>$=0.757$ (3 sf) | **M1**<br><br>**M1**<br>**A1** [3] | or their 0.8 + 0.9 |
| | **(b)** | $\mathrm{e}^{-\lambda}\times\lambda=p$ and $\mathrm{e}^{-\lambda}\times\frac{\lambda^2}{2}=1.5p$<br><br>$\lambda=3$<br><br>$p=\mathrm{e}^{-3}\times 3$<br>$=0.149$ (3 sf) | **M1**<br><br>**A1**<br><br>**M1**<br>**A1** [4] | or $\mathrm{e}^{-\lambda}\times\frac{\lambda^2}{2}=1.5\times\mathrm{e}^{-\lambda}\times\lambda$ seen or implied<br><br><br>their $\lambda$ |

**[Total for paper 50]**

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Cambridge International A Level

**Cambridge International Examinations**
Cambridge International Advanced Level

**MATHEMATICS** **9709/72**

Paper 7 **May/June 2016**

MARK SCHEME

Maximum Mark: 50

**Published**

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

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Cambridge is publishing the mark schemes for the May/June 2016 series for most Cambridge IGCSE®, Cambridge International A and AS Level components and some Cambridge O Level components.

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**Mark Scheme Notes**

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously 'correct' answers or results obtained from incorrect working.

  Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking $g$ equal to 9.8 or 9.81 instead of 10.

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The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)

CWO Correct Working Only – often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

**Penalties**

MR – 1 A penalty of MR – 1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become 'follow through ' marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR – 2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA – 1 This is deducted from A or B marks in the case of premature approximation. The PA – 1 penalty is usually discussed at the meeting.

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| | | | | |
|---|---|---|---|---|
| **1** | | $\frac{6.2}{\sqrt{50}}$ or $\frac{6.2^2}{50}$ | **B1** | seen or implied |
| | | $\frac{51-53}{6.2\div\sqrt{50}}$ (= –2.281) | **M1** | allow without $\div\sqrt{50}$ |
| | | P($z >$ '–2.281') = $\phi$('2.281')<br>= 0.989 (3 sf) | **M1**<br>**A1** [4] | for finding correct area consistent with working<br>as final answer |
| **2** | **(i)** | Conclude less than 90% satisfied when this is not true oe | **B11** | In context |
| | **(ii)** | $1 - (0.9^{15} + 15 \times 0.9^{14} \times 0.1$<br>$+ {}^{15}C_2 \times 0.9^{13} \times 0.1^2 + {}^{15}C_3 \times 0.9^{12} \times 0.1^3)$<br>= 0.0556 (3 sf) or 0.0555 | **M1**<br>**M1**<br>**A1** [3] | Attempt (1–)P($X$ = 15,14,13,12) allow 1 end error<br>Attempt fully correct expression |
| **3** | **(i)** | Pop too big or takes too long oe<br>or testing destroys articles oe | **B1** [1] | or too expensive oe<br>or pop inaccessible oe |
| | **(ii) (a)** | $z$ = 1.96 | **B1** | seen |
| | | $65.7 \pm z \times \frac{\sqrt{15}}{10}$ | **M1** | Expression of correct form (must be '$z$' must be 65.7) |
| | | = 64.9 to 66.5 (3 sf) | **A1** [3] | Must be an interval |
| | **(b)** | CI does not include 64.7<br>Probably has affected (or increased) mean bounce ht. | **B1**√ [1] | allow 64.7 not within CI<br>both needed. ft their CI ft 65.7 / 64.7 mix |
| **4** | | $H_0$: $\lambda$ (or $\mu$) = 42<br>$H_1$: $\lambda$ (or $\mu$) $\neq$ 42 | **B1** | Or pop weekly mean = 2.1 etc.<br>allow 'population mean' not just 'mean' |
| | | Po(42) ~ N(42, 42) stated or implied | **B1**√ | ft their '42' (Accept alt method N(2.1,2.1 / 20) |
| | | $\frac{53.5-42}{\sqrt{42}}$ | **M1** | |
| | | = 1.77(4) (or 0.038 for area comparison) | **A1** | allow with wrong or no cc.<br>Accept alt method using N(2.1,2.1 / 20) with or without cc |
| | | comp 1.96 | **M1** | Valid comp $z$ or 1 – ('1.774') with 0.025 seen |
| | | No evidence that mean has changed | **A1**√ [6] | allow comp 1.645 if $H_1$: $\lambda$ (or $\mu$) > 42<br>No contradictions. No ft for $H_1$: $\lambda$ (or $\mu$) > 42<br>Note – accept other valid methods(e.g. cv method) |

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| | | | | |
|---|---|---|---|---|
| **5** | **(i)** | $T \sim N(520, 70)$<br>$\frac{530-520}{\sqrt{'70'}}$ (= 1.195)<br>('1.195')<br>= 0.884 (3 sf) | **B1**<br>**B1**<br>**M1**<br>**M1**<br>**A1** [5] | for N(520,..) or N(500,..) if standardising with 510<br>for Var = 70 seen or implied<br>ft their E and Var; allow without √<br>finding correct area consistent with working<br>CWO |
| | **(ii)** | E($T$) = –10<br>Var($T$) = 50 + $4.1^2$ × 20 (= 386.2)<br>$\frac{0-(-10)}{\sqrt{'386.2'}}$ (= 0.509)<br>1 – ('0.509')<br>= 0.305 (3 sf) | **B1**<br>**B1**<br>**M1**<br>**M1**<br>**A1** [5] | or +10 for T < 0<br>Seen or implied<br>ft their E and Var; allow without √<br>finding correct area consistent with working<br>CWO |
| **6** | **(i)** | $\lambda = 6.8$<br>$e^{-6.8} \times \frac{6.8^5}{5!}$<br>= 0.135 (3 sf) | **B1**<br>**M1**<br>**A1** [3] | <br>any $\lambda$ |
| | **(ii) (a)** | $e^{-3.4}(1 + 3.4 + \frac{3.4^2}{2} + \frac{3.4^3}{3!} + \frac{3.4^4}{4!})$<br>= 0.744 (3 sf) | **M1**<br>**A1** [2] | any $\lambda$, allow one end-error<br> |
| | **(b)** | '0.744' + $e^{-3.4} \times \frac{3.4^5}{5!}$<br>= 0.87(0) (3 sf) or 0.871 | **M1**<br>**A1** [2] | or complete method, any $\lambda$, allow one end-error<br> |
| | **(iii)** | P($X \leqslant 6$) = '0.870' + $e^{-3.4} \times \frac{3.4^6}{6!}$<br>= 0.94<br>Need 6 hair driers | **M1**<br>**A1**<br>**A1** [3] | or complete method, any $\lambda$<br>fully correct un-simplified expression or better<br>dep M1A1 with numerical justification (0.94 or better) |
| **7** | **(a)** | 0.3 or 1 – 0.6 or 0.4 or 0.2 seen<br>0.8 | **M1**<br>**A1** [2] | <br> |
| | **(b) (i)** | $k\int_0^{1.5}(2.25 - x^2)dx = 1$<br>$k\left[2.25x - \frac{x^3}{3}\right]_0^{1.5} = 1$<br>$k \times [3.375 - 1.125] = 1$ or $k \times \frac{9}{4} = 1$ oe<br>$k = \frac{4}{9}$ **AG** | **M1**<br>**A1**<br><br>**A1** [3] | attempt integ f($x$) and '= 1'. Ignore limits<br>correct integration and limits<br><br>No errors seen |

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| | | | |
|---|---|---|---|
| **(ii)** | $\frac{4}{9}\int_0^{1.5}(2.25x - x^3)\mathrm{d}x$ | **M1** | attempt integ $xf(x)$, ignore limits, condone missing $k$ |
| | $= \frac{4}{9}\left[2.25\frac{x^2}{2} - \frac{x^4}{4}\right]_0^{1.5}$ | **A1** | correct integration and limits, condone missing $k$ |
| | = 0.5625 or 0.563 | **A1** | |
| | Mean no. of hours = 56.25 or 56.3 56 hrs 15 mins | **A1**√ [4] | ft their 0.5625 |
| **(iii)** | Max $x$ is 1.5, less than 2.9 or $150 < 290$ | **B1** [1] | Needs numerical justification |
| **(iv)** | any $a$ such that $2.9 \leqslant a \leqslant 5$ | **B1** [1] | |
| | **Total for paper** | **50** | |

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Cambridge International A Level

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Cambridge International Advanced Level

**MATHEMATICS** **9709/73**

Paper 7 **May/June 2016**

MARK SCHEME

Maximum Mark: 50

**Published**

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  B2/1/0 means that the candidate can earn anything from 0 to 2.

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CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a 'fortuitous' answer

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**Penalties**

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PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

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| Qu | Answer | Marks | Notes |
|---|---|---|---|
| 1 | $192.4 \pm z\sqrt{\frac{43.6}{150}}$ | M1 | Allow $\frac{43.6}{\sqrt{150}}$ Allow one side for M1 |
| | $z = 2.326$ to $2.329$ | B1 | Condone √(43.6/149 ) oe |
| | 191 to 194 (3 sf) | A1 [3] | CWO |
| 2 | $H_o$: Pop mean yield = 8.2 | | or $\mu = 8.2$(not just "mean") |
| | $H_1$: Pop mean yield > 8.2 | B1 | $\mu > 8.2$ |
| | $(\pm)\frac{8.7-8.2}{1.2/\sqrt{16}}$ | M1 | Allow without √ sign (Allow cc) |
| | $= (\pm)1.667$ | A1 | |
| | Comp $z = 1.645$ Or Area comparison | M1 | Or comp 1 - Φ('1.667') with 0.05 |
| | 0.0475-0.0478) | | Valid Comparison z-values (same sign) or areas |
| | Reject $H_0$ | | No Contradictions |
| | Evidence that mean yield has increased | A1√ [5] | No follow through for 2 tail test |
| 3 (i) | Use of Poisson | B1 | |
| | Mean = 2.4 | B1 | |
| | $1 - e^{-2.4}(1 + 2.4 + \frac{2.4^2}{2})$ | M1 | Allow any $\lambda$ *(Allow one end error)* |
| | $= 0.43(0)$ (3 sf) | A1 [4] | Final answer |
| | | | SR Use of binomial: B1 for ans 0.431 (3 sf) |
| (ii) | $240 > 50$ or $n>50$ | B1 | |
| | $240 \times 0.01 = 2.4 < 5$ or $np<5$ or $p<0.1$ | B1 [2] | SR *n* large, *p* small: B1 |
| 4 (i) | $H_0$: Pop mean = 2.5 (or 7.5) | | or $\lambda = 2.5$(Not just "mean") Allow μ |
| | $H_0$: Pop mean < 2.5 (or 7.5) | B1 | or $\lambda < 2.5$ |
| | $\lambda = 7.5$ | | |
| | $P(X \leqslant 2) = e^{-7.5}(1+7.5+\frac{7.5^2}{2}) = 0.0203$ | M1 | Either $P(X\leqslant 2)$ or $P(X\leqslant 3)$ , allow any $\lambda$ |
| | $P(X\leqslant 3)=0.0203 + e^{-7.5}\times\frac{7.5^3}{3!} = 0.0591$ | A1 | *Both Correct* |
| | CR is $X \leqslant 2$ | A1 | *Clear statement* |
| | Reject $H_0$ | A1√ [5] | *Follow through their CR/their P(X⩽2)* |
| | Evidence that no of sightings fewer | | |
| (ii) | P(Type I) = 0.0203 (3 sf) | B1√ [1] | ft their $P(X \leqslant 2)$ |
| (iii) | $H_0$ was rejected oe | B1 [1] | or Type II is P(not reject $H_0$)oe |
| 5 (i) | $k\int_5^{10}(10t - t^2)\mathrm{d}t = 1$ | M1 | Attempt to integrate, ignore limits |
| | $k\left[5t^2 - \frac{t^3}{3}\right]_5^{10} = 1$ | A1 | Correct integral and limits |
| | $k(500 - \frac{1000}{3} - (125 - \frac{125}{3})) = 1$ | | |
| | $k \times \frac{250}{3} = 1$ | | |
| | $(k = \frac{3}{250}$ **AG**) | A1 [3] | No errors seen; No inexact decimals seen |

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| Qu | Answer | Marks | Notes |
|---|---|---|---|
| (ii) | $\frac{3}{250}\int_5^{10}(10t^2-t^3)\mathrm{d}t$ | M1 | Attempt to integrate, ignore limits |
| | $=\frac{3}{250}\left[\frac{10t^3}{3}-\frac{t^4}{4}\right]_5^{10}$ | A1 | Correct integral and limit. Condone missing k |
| | $=\frac{3}{250}(\frac{10000}{3}-\frac{10000}{4}-(\frac{1250}{3}-\frac{625}{4})$ | | |
| | = 6.875 or 55/8 | A1 [3] | Allow 6.88 |
| (iii) | $P(T<E(T)=\frac{3}{250}\left[5t^2-\frac{t^3}{3}\right]_5^{"6.875"}$ | M1* | ft their E($T$) |
| | = 0.5361 | | |
| | "0.5361" – 0.5 | DM1* | allow 0.036 |
| | P($T$ between E($T$) & median = 0.0361 | A1 [3] | |
| | | | **Alternative Method**<br>Integrate f(t)limits 5 and m equated to 0.5 M1*<br>Integrate f(t)limits their 6.736 (provided between 5 and 10) and their 6.875DM1<br>Allow without "minutes" |
| (iv) | 10 (minutes) | B1 [1] | |
| 6 (i) | $\lambda$= 3.9 | B1 | |
| | $e^{-3.9}\times\frac{3.9^4}{4!}$ | M1 | M1 allow any $\lambda$ |
| | = 0.195 | A1 [3] | SR Combination method<br>B1 for $\lambda$ =1.6 AND $\lambda$ =2.3 used in combination method (at least 3 combinations)<br>M1 All correctly combined and added |
| (ii) | $\bar{X}\sim N(1.6,\frac{1.6}{75})$ | B1<br>B1 [2] | B1 for N(1.6, ….)stated<br>B1 for Var = $\frac{1.6}{75}$ stated<br>SR, not stated but all implied in (iii): B1 |
| (iii) | $\frac{1.7-1.6}{\sqrt{\frac{1.6}{75}}}$ (= 0.685) | M1 | For standardising (using their values or correct values .Ignore cc |
| | 1– Φ("0.685") | M1 | Correct area consistent with their working |
| | = 0.247 (3 sf) | A1 [3] | Accept use of 1/2n correction leading to 0.233.<br>NB Use of Poisson sum Po(120) and N(120,120) with μ=127.5 leads to 0.247, or 0.233 with cc |
| (iv) | $X$ not normally distr. So CLT needed | B1 [1] | Not "it" |

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| Qu | Answer | Marks | Notes |
|---|---|---|---|
| 7 (i) | E($T$) =20.8 | **B1** | |
| | Var($T$)= $20 \times 0.03^2 + 0.01^2$(= 0.0181) | **B1** | or $\sqrt{(20 \times 0.03^2 + 0.01^2)} = 0.135$ (3sf) |
| | $\frac{20.6-20.8}{\sqrt{"0.0181"}}$ (= –1.487) | **M1** | For standardising (σ must come from combination) |
| | 1 – Φ(“1.487”) | **M1** | Area consistent with their working |
| | = 0.0684 to 0.686 | **A1** [5] | Any answer within range |
| (ii) | E($D$) = 0 | | |
| | Var($D$) = 2 × 0.0181(= 0.0362) | **B1**√ | Both (Seen or implied) |
| | $\frac{0.02-0}{\sqrt{"0.0362)}}$ ( = 0.105) | **M1** | Allow without √ |
| | Φ(“0.105”) = 0.5418 or 1-Φ(0.015) =0.4582 | **A1** | Allow to 3sf |
| | Φ(“0.105”) – (1 – Φ(“0.105”) (= 0.5418 – 0.4582) = 0.0836/0.0837 | **M1** | or 1 – 2(1 – Φ(“0.105”)) (= 1 – 2 × 0.4582) |
| | | **A1** [5] | |

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# MARK SCHEME for the March 2016 series

## 9709 MATHEMATICS

**9709/72** Paper 7 (Probability and Statistics), maximum raw mark 50

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

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**Mark Scheme Notes**

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

Note: B2 or A2 means that the candidate can earn 2 or 0.
B2/1/0 means that the candidate can earn anything from 0 to 2.

The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking $g$ equal to 9.8 or 9.81 instead of 10.

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The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

**Penalties**

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

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| | | | | |
|---|---|---|---|---|
| **1** | $E(X) = \frac{10}{3}$ oe $\quad Var(X) = \frac{25}{9}$ oe<br>$E(Y) = 10 \quad Var(Y) = 5$ | **B1**<br>**B1** | | For $E(X)$ and $Var(X)$<br>For $E(Y)$ and $Var(Y)$<br>OR For $E(X)$ and $E(Y)$<br>For $Var(X)$ and $Var(Y)$ |
| | $E(X + Y) = \frac{40}{3}$ oe or 13.3 (3 sf)<br>Var $(X + Y)$ = " $\frac{25}{9}$ " + "5"<br>sd = $\frac{\sqrt{70}}{3}$ oe or 2.79 (3 sf) | **B1**<br>**M1**<br>**A1** | [5] | <br>For adding 2 (appropriate) variances<br>or sd = or $\sqrt{2} \times \frac{5}{3}$ |
| **2** | $H_0$: P(hit target) = 0.65<br>$H_1$: P(hit target) > 0.65 | **B1** | | Allow $p = 0.65$<br>Allow $p > 0.65$ |
| | $^{20}C_2 \times 0.35^2 \times 0.65^{18} + 19 \times 0.35 \times 0.65^{19} + 0.65^{20}$<br>= 0.0121 (3 sf) | **M1**<br>**A1** | | Allow one end error. Allow p/q mix. Allow (1– ) for **M** mark<br>**A** mark recovered following valid comparison |
| | Comp 0.01<br>There is no evidence (at the 1% level) that she has improved | **M1**<br>**A1**√ | [5] | For valid comparison<br>She has probably not improved. No contradictions.<br>(SR Use of Normal **M0**, but **M1A1** for valid comparison could be awarded) |
| **3 (i)** | $H_0$: pop mean journey time = 35.2 mins<br>$H_1$: pop mean journey time < 35.2 mins | **B1** | | Allow "$\mu$". Not "mean journey time" |
| | $\frac{34.7-35.2}{5.6/\sqrt{25}}$ (= –0.446) | **M1** | | For standardising ($\sqrt{25}$ needed) |
| | $\Phi(<$ "–0.446") = 1 – $\Phi$("0.446")<br>= 0.328 (3 sf) | **M1**<br>**A1** | [4] | For correct area consistent with their working<br>As final answer |
| **(ii)** | $H_0$ *is* rejected but Type II error can only be made if $H_0$ is *not* rejected | **B1** | [1] | Allow just "$H_0$ is rejected." oe |
| **4** | $X - 2Y \sim N(0.1, 0.2^2 + 4 \times 0.1^2)$ soi (= N(0.1, 0.08)) | **B1 B1** | | **B1** for ± 0.1 **B1** for $0.2^2 + 4 \times 0.1^2$ |
| | $\frac{0-0.1}{\sqrt{"0.08"}}$ (= –0.354)<br>$\Phi$("–0.354 ") = 1 – $\Phi$("0.354")<br>= 0. 362 (3 sf) | **M1**<br>**M1**<br>**A1** | [5] | For standardising. Allow without √ sign<br>For correct area consistent with their working |
| **5 (i)** | Est($\mu$) = $\frac{14\,910}{150}$ (= 99.4)<br>Est($\sigma^2$) = $\frac{150}{149}(\frac{1525000}{150} - "99.4"^2)$<br>= 288.228 | **B1**<br>**M1**<br>**A1** | | <br>Allow **M1** if $\frac{150}{149}$ omitted<br> |
| | $z = 2.576$<br>"99.4" ± $z \times \sqrt{288.228 \div 150}$<br>CI = 95.8 to 103 (3 sf) | **B1**<br>**M1**<br>**A1** | [6] | Accept 2.574–2.579<br>Any $z$<br>(NB Use of biased Var can score 5/6 max) |
| **(ii)** | 100 lies within this CI<br>Hence yes | **B1**√ | [1] | Both needed, ft their CI |

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| | | | | |
|---|---|---|---|---|
| **(iii)** | To avoid bias or<br>Necessary to enable statistical inference | **B1** | [1] | Or any equivalent |
| **6 (i)** | $\lambda = 3.3 \times \frac{25}{30} = 2.75$ | **B1** | | |
| | $e^{-2.75}(1 + 2.75 + \frac{2.75^2}{2})$ | **M1** | | Allow any λ Allow one end error |
| | = 0.481 (3 sf) | **A1** | [3] | As final answer. Accept 0.482 |
| **(ii) (a)** | $\lambda\ (= 3.3 \times \frac{365}{30}) = 40.15$ | **B1** | | Accept 40.1 or 40.2 |
| | $(X \sim \text{Po}(40.15) \Rightarrow X \sim \text{N}(40.15, 40.15))$<br>$\frac{50.5-"40.15"}{\sqrt{"40.15"}}$ (= 1.633) | **M1** | | Allow with incorrect or no cc OR no √ sign |
| | 1 – Φ("1.633")<br>= 0.0513 (3 sf) | **M1**<br>**A1** | [4] | For correct area consistent with their working<br>Accept 0.0512 |
| **(b)** | $\lambda > 15$ | **B1** | [1] | or similar |
| **(iii)** | $\lambda = \frac{73}{30}$ oe or 1.1+1.33= 2.43 (3 sf) | **B1** | | |
| | $1 - e^{-2.43}(1 + 2.43 + \frac{2.43^2}{2} + \frac{2.43^3}{3!})$ | **M1** | | Allow any λ. Allow one end error |
| | = 0.228 (3 sf) | **A1** | [3] | |
| **7 (a) (i)** | E($X$) = 1.5 | **B1** | | |
| | $\frac{2}{9}\int_0^3 (3x^3 - x^4)\text{d}x$ | **M1** | | Attempt integ $x^2$f($x$) ignore limits |
| | $= \frac{2}{9}\left[\frac{3x^4}{4} - \frac{x^5}{5}\right]_0^3$ | | | |
| | $= \frac{2}{9}\left[\frac{243}{4} - \frac{243}{5}\right]$ (= 2.7) | **M1** | | Sub correct limits into correct integral |
| | Var($X$) (= $2.7 - 1.5^2$) = 0.45 oe | **A1**√ | [4] | Ft their E($X$), but no ft for –ve Var. |
| **(ii)** | 0.5 | **B1** | [1] | |
| **(iii)** | $(1 - \frac{13}{27}) \div 2$ | **M1** | | or $\frac{2}{9}\int_2^3 (3x - x^2)\ \text{d}x$ oe |
| | $= \frac{7}{27}$ or 0.259 | **A1** | [2] | As final answer |
| **(b)** | $\frac{1}{2} \times 2 \times 2a = \frac{1}{2}$ or $\int_0^2 ax\text{d}x = \frac{1}{2}$ | **M1** | | Attempt correct equation in 'a' |
| | $a = \frac{1}{4}$ | **A1** | | |
| | $\frac{1}{2} \times b \times \frac{1}{4}b = 1$ or $\int_0^b \frac{1}{4}x\text{d}x = 1$<br>or $b = 2 \times \sqrt{2}$ | **M1** | | or $\frac{1}{2} \times b \times ab = 1$ or $\int_0^b ax\text{d}x = 1$ attempt correct equation in (a and) b |
| | $b = 2\sqrt{2}$ | **A1**√ | [4] | Allow $b = \sqrt{8}$ or 2.83 (3 sf)<br>Ft incorrect $a$, both **M**s needed |
| | | **Total for paper 50** | | |

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# MARK SCHEME for the October/November 2015 series

## 9709 MATHEMATICS

**9709/71** Paper 7, maximum raw mark 50

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  B2/1/0 means that the candidate can earn anything from 0 to 2.

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| | | | |
|---|---|---|---|
| **1** | $\lambda = (1.2 + 2.3) \div 2$<br>$= 1.75$<br>$e^{-1.75}\left(\frac{1.75^2}{2} + \frac{1.75^3}{3!}\right)$<br>$= 0.421$ (3 sf) | **M1**<br>**A1**<br><br>**M1**<br><br>**A1** [4] | Attempt combined mean, allow 1.2 + 2.3<br>Correct mean<br><br>Allow incorrect mean.<br>Allow end errors (1 and/or 4) |
| | | **Total: 4** | |
| **2 (i)** | $\frac{6}{\sqrt{120}}$ oe seen<br><br>$\frac{30-29}{\left(\frac{6}{\sqrt{120}}\right)}$ (= 1.826)<br><br>P($z$ > '1.826') = 1 – Φ('1.826')<br>= 0.034 (2 sf) | **B1**<br><br>**M1**<br><br>**M1**<br>**A1** [4] | Or $6^2/120$ oe seen<br><br>±<br>Allow without √120. No sd/var mix<br><br>Correct tail consistent with their working<br>0.0339 |
| **(ii)** | No<br>$n$ is large (⩾30)<br><br>Sample mean is (appr) normally distrib<br>or The CLT applies oe | B1<br><br><br>B1 [2] | 1st B1 for either comment<br><br>2nd B1 for'No'with 2nd comment<br>(No mark for 'No' alone) |
| | | **Total: 6** | |
| **3 (i)** | $\frac{3420}{60}(= 57)$<br><br>$\frac{60}{59}\left(\frac{195200}{60} - '57'^2\right)$ (= 4.40678)<br>= 4.41 (3 sf) | **B1**<br><br>**M1**<br><br>**A1** [3] | <br><br>Oe<br><br>As final answer |
| **(ii)** | $'57' \pm z\sqrt{\frac{'4.40678'}{60}}$<br><br>$z$ = 2.326<br><br>[56.4 to 57.6] (3 sf) | **M1**<br><br>**B1**<br><br>**A1** [3] | <br><br>2.326 – 2.329 (accept 2.33 if no better seen)<br>NB: use of biased variance in (ii) can score in full |
| | | **Total: 6** | |

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| | | | |
|---|---|---|---|
| **4 (i)** | $k\int_1^2(3-x)dx=1$ | **M1** | Attempt $\int f(x) = 1$, ignore limits or $\frac{k}{2}(h_1 + h_2) = 1$ |
| | $k\left[3x-\frac{x^2}{2}\right]_1^2=1$ | **A1** | Correct integration & limits or $\frac{k}{2}(2 + 1) = 1$ |
| | $(k(6 – 2 – (3 – 0.5)) = 1)$ | | |
| | $k \times 1.5 = 1$ or $k \times \frac{3}{2} = 1$ or $k = \frac{1}{1.5}$ oe | | |
| | $k = \frac{2}{3}$ **AG** | **A1** [3] | No errors seen |
| **(ii)** | $\frac{2}{3}\int_1^m(3-x)dx = 0.5$ oe $\int$ from m to 2 | **M1*** | Attempt Int $f(x) = 0.5$, ignore limits oe<br>Or use of area of trapezium |
| | $\left(\frac{2}{3}\left[3x-\frac{x^2}{2}\right]_1^m = 0.5\right)$ | | |
| | $\frac{2}{3}\left[3m-\frac{m^2}{2}-2.5\right]=0.5$ | **dep M1*** | Sub of correct limits into their integral.<br>Or trapezium using 1 and m/m and 2<br>Any correct 3-term QE = 0 or $(m–3)^2 = 2.5$ |
| | $m^2 - 6m + 6.5 = 0$ oe | **A1** | |
| | $\left(m=\frac{6\pm\sqrt{36-4\times6.5}}{2}=1.42 \text{ or } 4.58\right)$ | | |
| | $m = 1.42$ (3 sf) | **A1** [4] | or $\frac{6-\sqrt{10}}{2}$ oe; single correct ans |
| | | **Total: 7** | |

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| | | | | |
|---|---|---|---|---|
| **5** | **(i)** | Po(1.6) stated or implied | **M1** | |
| | | $P(X > 3) = 1 - e^{-1.6}\left(1 + 1.6 + \frac{1.6^2}{2} + \frac{1.6^3}{3!}\right)$ | **M1** | Allow M1 for 1 – P($X \leqslant 3$), incorrect $\lambda$ and allow one end error |
| | | = 0.0788 (3 sf) | **A1** [3] | SR Use of Bin scores B1 only for 0.0788 |
| | **(ii)** | $\lambda = \frac{n}{2500}$ | **B1** | or $e^{-\mu} < 0.05$ M1 |
| | | $e^{-\frac{n}{2500}} < 0.05$ Allow = Allow incorrect $\lambda$ | **M1** | |
| | | $-\frac{n}{2500} < \ln 0.05$ Attempt ln bs | **M1** | $-\mu < \ln 0.05$ M1 ($\mu > 2.9957$) |
| | | $n > 7489.3$ (1 dp) Smallest $n = 7490$ | **A1** [4] | $n = \mu \times 2500$ B1 Smallest $n = 7490$ A1 |
| | | | | or $\frac{2499}{2500}$ B1 $\left(\frac{2499}{2500}\right)^n < 0.05$ M1 $n \ln \frac{2499}{2500} < \ln 0.05$ M1 Smallest $n = 7488$ A1 |
| | | | **Total: 7** | |
| **6** | **(i)** | $E(T) = 9 \times 78 + 7 \times 66$ (= 1164) | **B1** | Or $9 \times 78 + 7 \times 66 - 1200$ |
| | | $\text{Var}(T) = 9 \times 7^2 + 7 \times 5^2$ (= 616) | **B1** | |
| | | $\frac{1200 - '1164'}{\sqrt{'616'}}$ (= 1.450) | **M1** | ± Allow without √ |
| | | $P(z < 1.450) = \Phi(1.450)$ = 0.927 (3 sf) | **M1** **A1** [5] | Correct tail consistent with their mean |
| | **(ii)** | $E(D) = 66 - 78$ (= –12) | **B1** | Both needed |
| | | $\text{Var}(D) = 7^2 + 5^2$ (= 74) | | |
| | | $\frac{0 - ('-12')}{\sqrt{74}}$ (= 1.395) | **M1** | ± Allow without √ |
| | | $P(D > 0) = 1 - \Phi$ ('1.395') 0.0815 (3 sf) | **M1** **A1** [4] | Correct tail consistent with their mean Similar scheme for P(M – W) < 0 |
| | | | **Total: 9** | |

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| | | | |
|---|---|---|---|
| **7 (i)** | Prob could be different later in day<br>or on a different day oe | **B1** [1] | or any explanation why not random<br>or "Not random" or "Not representative" |
| **(ii)** | Looking for decrease (or improvement)<br>$H_0$: P(not arrive) = 0.2<br>$H_1$: P(not arrive) < 0.2 | **B1**<br><br>**B1** [2] | oe<br>Allow "$p = 0.2$" |
| **(iii)** | Concluding that prob has <u>decreased</u> (or publicity has worked) when it hasn't oe | **B1** [1] | In context |
| **(iv)** | P($X = 0$) and P($X = 1$) attempted<br><br>P($X \leqslant 2$) = $0.8^{30} + 30 \times 0.8^{29} \times 0.2 +$ $^{30}C_2 \times 0.8^{28} \times 0.2^2$ (= 0.0442)<br><br>P($X \leqslant 3$) = $0.8^{30} + 30 \times 0.8^{29} \times 0.2 +$ $^{30}C_2 \times 0.8^{28} \times 0.2^2 + {}^{30}C_3 \times 0.8^{27} \times 0.2^3$ = 0.123<br><br>cr is $X \leqslant 2$<br>P(Type I) = 0.0442 (3 sf) | **M1**<br><br>**M1**<br><br>**B1**<br><br>**A1**<br>**A1** [5] | B(30, 0.2) Not nec'y added<br>May be implied by calc P($X \leqslant 2$) or P($X \leqslant 3$)<br><br>Attempt P($X \leqslant 2$)<br><br>Or '0.0442' + $^{30}C_3 \times 0.8^{27} \times 0.2^3 = 0.123$ |
| **(v)** | 3 is outside cr<br><br>No evidence that $p$ has decreased (or that publicity has worked) | **M1**<br><br>**A1** √ [2] | Comparison of 3 with their cr<br>or P($X \leqslant 3$) = 0.123 which is > 0.05<br>Correct conclusion. No contradictions |
| | | **Total: 11** | |
| | | **Total for paper: 50** | |

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# MARK SCHEME for the October/November 2015 series

## 9709 MATHEMATICS

**9709/72** Paper 7, maximum raw mark 50

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**Mark Scheme Notes**

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M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

  Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking $g$ equal to 9.8 or 9.81 instead of 10.

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AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

**Penalties**

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

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| | | | |
|---|---|---|---|
| **1** | $\lambda = (1.2 + 2.3) \div 2$<br>$= 1.75$<br>$e^{-1.75}\left(\frac{1.75^2}{2} + \frac{1.75^3}{3!}\right)$<br>$= 0.421$ (3 sf) | **M1**<br>**A1**<br><br>**M1**<br><br>**A1** [4] | Attempt combined mean, allow 1.2 + 2.3<br>Correct mean<br><br>Allow incorrect mean.<br>Allow end errors (1 and/or 4) |
| | | **Total: 4** | |
| **2 (i)** | $\frac{6}{\sqrt{120}}$ oe seen | **B1** | Or $6^2/120$ oe seen |
| | $\frac{30-29}{\left(\frac{6}{\sqrt{120}}\right)}$ (= 1.826) | **M1** | ±<br>Allow without √120. No sd/var mix |
| | P($z$ > '1.826') = 1 – Φ('1.826')<br>= 0.034 (2 sf) | **M1**<br>**A1** [4] | Correct tail consistent with their working<br>0.0339 |
| **(ii)** | No<br>$n$ is large ($\geqslant$30)<br><br>Sample mean is (appr) normally distrib<br>or The CLT applies oe | B1<br><br><br>B1 [2] | 1st B1 for either comment<br><br>2nd B1 for'No'with 2nd comment<br>(No mark for 'No' alone) |
| | | **Total: 6** | |
| **3 (i)** | $\frac{3420}{60}(= 57)$ | **B1** | |
| | $\frac{60}{59}\left(\frac{195200}{60} - '57'^2\right)$ (= 4.40678)<br>= 4.41 (3 sf) | **M1**<br><br>**A1** [3] | Oe<br><br>As final answer |
| **(ii)** | $'57' \pm z\sqrt{\frac{'4.40678'}{60}}$ | **M1** | |
| | $z$ = 2.326 | **B1** | 2.326 – 2.329 (accept 2.33 if no better seen) |
| | [56.4 to 57.6] (3 sf) | **A1** [3] | NB: use of biased variance in (ii) can score in full |
| | | **Total: 6** | |

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| | | | |
|---|---|---|---|
| **4 (i)** | $k\int_1^2(3-x)dx=1$ | **M1** | Attempt $\int f(x) = 1$, ignore limits or $\frac{k}{2}(h_1 + h_2) = 1$ |
| | $k\left[3x-\frac{x^2}{2}\right]_1^2=1$ | **A1** | Correct integration & limits or $\frac{k}{2}(2 + 1) = 1$ |
| | $(k(6 – 2 – (3 – 0.5)) = 1)$ | | |
| | $k \times 1.5 = 1$ or $k \times \frac{3}{2} = 1$ or $k = \frac{1}{1.5}$ oe | | |
| | $k = \frac{2}{3}$ **AG** | **A1** [3] | No errors seen |
| **(ii)** | $\frac{2}{3}\int_1^m(3-x)dx = 0.5$ oe $\int$ from m to 2 | **M1*** | Attempt Int $f(x) = 0.5$, ignore limits oe<br>Or use of area of trapezium |
| | $\left(\frac{2}{3}\left[3x-\frac{x^2}{2}\right]_1^m = 0.5\right)$ | | |
| | $\frac{2}{3}\left[3m-\frac{m^2}{2}-2.5\right]=0.5$ | **dep M1*** | Sub of correct limits into their integral.<br>Or trapezium using 1 and m/m and 2<br>Any correct 3-term QE = 0 or $(m–3)^2 = 2.5$ |
| | $m^2 - 6m + 6.5 = 0$ oe | **A1** | |
| | $\left(m=\frac{6\pm\sqrt{36-4\times6.5}}{2}=1.42 \text{ or } 4.58\right)$ | | |
| | $m = 1.42$ (3 sf) | **A1** [4] | or $\frac{6-\sqrt{10}}{2}$ oe; single correct ans |
| | | **Total: 7** | |

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| | | | | |
|---|---|---|---|---|
| **5** **(i)** | Po(1.6) stated or implied | **M1** | | |
| | $P(X > 3) = 1 - e^{-1.6}\left(1 + 1.6 + \frac{1.6^2}{2} + \frac{1.6^3}{3!}\right)$ | **M1** | Allow M1 for 1 – P($X \leqslant 3$), incorrect $\lambda$ and allow one end error | |
| | = 0.0788 (3 sf) | **A1** [3] | SR Use of Bin scores B1 only for 0.0788 | |
| **(ii)** | $\lambda = \frac{n}{2500}$ | **B1** | or $e^{-\mu} < 0.05$ M1 | or $\frac{2499}{2500}$ B1 |
| | $e^{-\frac{n}{2500}} < 0.05$ Allow = Allow incorrect $\lambda$ | **M1** | | $\left(\frac{2499}{2500}\right)^n < 0.05$ M1 |
| | $-\frac{n}{2500} < \ln 0.05$ Attempt ln bs | **M1** | $-\mu < \ln 0.05$ M1 ($\mu > 2.9957$) | $n \ln \frac{2499}{2500} < \ln 0.05$ M1 |
| | $n > 7489.3$ (1 dp) Smallest $n = 7490$ | **A1** [4] | $n = \mu \times 2500$ B1 Smallest $n = 7490$ A1 | Smallest $n = 7488$ A1 |
| | | **Total: 7** | | |
| **6** **(i)** | E($T$) = 9 × 78 + 7 × 66 (= 1164) | **B1** | Or 9 × 78 + 7 × 66 – 1200 | |
| | Var($T$) = 9 × 7² + 7 × 5² (= 616) | **B1** | | |
| | $\frac{1200 - '1164'}{\sqrt{'616'}}$ (= 1.450) | **M1** | ± Allow without √ | |
| | P($z$ < 1.450) = Φ (1.450) = 0.927 (3 sf) | **M1** **A1** [5] | Correct tail consistent with their mean | |
| **(ii)** | E($D$) = 66 – 78 (= –12) | **B1** | Both needed | |
| | Var($D$) = 7² + 5² (= 74) | | | |
| | $\frac{0 - ('-12')}{\sqrt{74}}$ (= 1.395) | **M1** | ± Allow without √ | |
| | P($D$ > 0) = 1 – Φ ('1.395') 0.0815 (3 sf) | **M1** **A1** [4] | Correct tail consistent with their mean Similar scheme for P(M – W) < 0 | |
| | | **Total: 9** | | |

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| | | | |
|---|---|---|---|
| **7 (i)** | Prob could be different later in day<br>or on a different day oe | **B1** [1] | or any explanation why not random<br>or "Not random" or "Not representative" |
| **(ii)** | Looking for decrease (or improvement)<br>$H_0$: P(not arrive) = 0.2<br>$H_1$: P(not arrive) < 0.2 | **B1**<br><br>**B1** [2] | oe<br>Allow "$p = 0.2$" |
| **(iii)** | Concluding that prob has <u>decreased</u> (or publicity has worked) when it hasn't oe | **B1** [1] | In context |
| **(iv)** | P($X = 0$) and P($X = 1$) attempted<br><br>P($X \leqslant 2$) = $0.8^{30} + 30 \times 0.8^{29} \times 0.2 +$ $^{30}C_2 \times 0.8^{28} \times 0.2^2$ (= 0.0442)<br><br>P($X \leqslant 3$) = $0.8^{30} + 30 \times 0.8^{29} \times 0.2 +$ $^{30}C_2 \times 0.8^{28} \times 0.2^2 + {}^{30}C_3 \times 0.8^{27} \times 0.2^3$ = 0.123<br><br>cr is $X \leqslant 2$<br>P(Type I) = 0.0442 (3 sf) | **M1**<br><br>**M1**<br><br>**B1**<br><br>**A1**<br>**A1** [5] | B(30, 0.2) Not nec'y added<br>May be implied by calc P($X \leqslant 2$) or P($X \leqslant 3$)<br><br>Attempt P($X \leqslant 2$)<br><br>Or '0.0442' + $^{30}C_3 \times 0.8^{27} \times 0.2^3 = 0.123$ |
| **(v)** | 3 is outside cr<br><br>No evidence that $p$ has decreased (or that publicity has worked) | **M1**<br><br>**A1** √ [2] | Comparison of 3 with their cr<br>or P($X \leqslant 3$) = 0.123 which is > 0.05<br>Correct conclusion. No contradictions |
| | | **Total: 11** | |
| | | **Total for paper: 50** | |

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## 9709 MATHEMATICS

**9709/73** Paper 7, maximum raw mark 50

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| | | | | |
|---|---|---|---|---|
| **1 (i)** | N(352, …)<br>Variance = 2.9 | **B1**<br>**B1** | [2] | no recovery in (ii) for each B mark<br>accept sd = $\sqrt{2.9}$ = 1.70(29) stated |
| **(ii)** | $\frac{354-352}{\sqrt{2.9}}$ (= 1.174)<br><br>1 – Φ('1.174')<br><br>= 0.120 (3 sf) | **M1**<br><br>**M1**<br><br>**A1** | [3] | with their mean and var<br>Or $\frac{354.05-352}{\sqrt{2.9}}$<br>or correct restart (= 1.204)<br>(accept sd/var mix)1<br>– Φ('1.204')<br>= 0.114 (3 sf)<br>Incorrect cc can score M1M1A0 |
| **Total** | | | **[5]** | |
| **2** | ( $\Phi^{-1}(0.99)$ = ) 2.326 seen<br>N($\lambda$, $\lambda$) seen or implied<br>$\frac{55.5-\lambda}{\sqrt{\lambda}}$ = + "2.326"<br><br>$\lambda$ + "2.326" $\sqrt{\lambda}$ – 55.5 = 0<br>$\sqrt{\lambda} = \frac{-\text{"2.326"} \pm \sqrt{\text{"2.326"}^2 + 4\times 55.5}}{2}$<br>(= 6.377.. or – 8.703..))<br>$\lambda$ = 40.7 (3 sf) | **B1**<br>**M1**<br>**M1**<br><br><br><br>**M1**<br><br><br>**A1** | [5] | must be $\Phi^{-1}$, not Φ<br><br>allow with wrong or no cc & Φ(0.99) (= 0.8389)<br>must = " z " or attempt at z<br>( 0.99 / 0.01 M0 )<br><br>for correct method of solving their quad in $\sqrt{\lambda}$ and squaring to find $\lambda$<br><br>cao, one ans only<br>Without cc, $\lambda$ = 40.2: lose final A1 |
| **Total** | | | **[5]** | |
| **3 (i)** | 0.4 or 2/5 or 26/65 | **B1** | [1] | no recovery in (ii) for the B mark |
| **(ii)** | " 0.4 " + $z \times \sqrt{\frac{0.4\times 0.6}{65}}$ = 0.516 oe<br><br>$z = \left(0.116 \times \sqrt{\frac{65}{0.4\times 0.6}}\right)$ = 1.909<br><br>( Φ('1.909') = 0.97(18) )<br>2 ( '0.97' – 1)<br><br>$\alpha$ = 94 | **M1**<br><br>**A1**<br><br>**M1**<br><br>**A1** | [4] | or " 0.4 " – $z \times \sqrt{\frac{0.4\times 0.6}{65}}$ = 0.284 or<br>$z \times \sqrt{\frac{0.4\times 0.6}{65}}$ = 0.116 oe<br><br>for fully correct method to find α from their z<br>allow 94.36 or 94.4 or 94.374 |
| **Total** | | | **[5]** | |

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| | | | | |
|---|---|---|---|---|
| **4 (i)** | $k\int_{-2}^{2}(4-x^2)\mathrm{d}x = 1$ | **M1** | | attempt Integral f($x$) = 1, ignore limits |
| | $k[4x - \frac{x^3}{3}]_{-2}^{2} = 1$ | **A1** | | correct integration & limits |
| | $\left(k\left(8-\frac{8}{3}-\left(-8-\left(-\frac{8}{3}\right)\right)=1\right)\right)$ | | | |
| | $k \times \frac{32}{3} = 1$ oe Not e.g. $k \times 10.7 = k$ | | | |
| | $k = \frac{3}{32}$ **AG** | **A1** | [3] | exact answer correctly found |
| **(ii)** | Inverted parabola, vertex on $y$ axis | **B1** | | parabola must finish on x axis at ± 2, labelled (ignore markings on y axis ) |
| | E($X$) = 0 | **B1** | [2] | |
| **(iii)** | $\frac{3}{32}\int_{-2}^{1}(4-x^2)\mathrm{d}x$ | **M1** | | or $1 - \frac{3}{32}\int_{1}^{2}(4-x^2)\mathrm{d}x$ ignore limits |
| | $\frac{3}{32}[4x - \frac{x^3}{3}]_{-2}^{1}$ | **A1** | | or $1 - \frac{3}{32}[4x - \frac{x^3}{3}]_{1}^{2}$ |
| | $\frac{3}{32}(4 - \frac{1}{3} - (-8 - (-\frac{8}{3})))$ | | | correct integration and correct limits |
| | | | | $= 1 - \frac{3}{32}(8 - \frac{8}{3} - (4 - \frac{1}{3})$ |
| | $= \frac{27}{32}$ or 0.844 (3 sf) | **A1** | [3] | |
| **Total** | | | **[8]** | |

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| | | | | |
|---|---|---|---|---|
| **5 (a)** | $\lambda = 4.5$<br>$e^{-4.5}$ (= 0.011109)<br>$\left(\frac{99}{100}\right)^{450}$ (= 0.010860)<br>$\left(\frac{'0.011109' - '0.010860'}{'0.010860} \times 100\right)$<br>= 2.29% (3 sf) | **B1**<br>**M1**<br>**M1**<br><br>**A1** | [4] | alone allow any $\lambda$ |
| **(b)** | $H_0$: P(6) = $\frac{1}{6}$ or $p = \frac{1}{6}$<br>$H_1$: P(6) < $\frac{1}{6}$ or $p < \frac{1}{6}$ | **B1** | | Both needed |
| | $\left(\frac{5}{6}\right)^{25} + 25\left(\frac{5}{6}\right)^{24} \times \frac{1}{6} + {}^{25}C_2\left(\frac{5}{6}\right)^{23} \times \left(\frac{1}{6}\right)^2$ | **M1** | | allow one error ( extra term / missing term / incorrect term )<br>CR method: attempt at least P(0) and P(0 and 1) (0.010... and 0.06... < 0.1) |
| | = 0.189 (3 sf) | **A1** | | CR is 0,1 and must see 0.189 for A1 |
| | comp 0.1 | **M1** | | valid comp '0.189' with 0.1 oe<br>valid comparison of 2 with CR |
| | No reason to believe die biased | **A1** | [5] | correct conclusion, √ their 0.189<br>no contradictions |
| **Total** | | | **[9]** | |
| **6 (i)** | Ho: $\mu = 2.60$<br>$H_1$: $\mu > 2.60$ | **B1** | | allow pop mean, not just 'mean' |
| | $\pm \frac{2.64 - 2.6}{0.2 \div \sqrt{75}}$<br>= ± 1.732 | **M1**<br>**A1** | | accept ± 1.73 ( 3 sf ) |
| | '1.732' > 1.645<br>Reject Ho. There is evidence that $\mu$ has increased | **B1** √ | [4] | valid comparison with 1.645<br>( or 0.0416 < 0.05 )<br>and correct conclusion √ their 1.732<br>no contradictions<br>(or CV method $x_{crit}$ = 2.638 M1A1<br>comp 2.64 > 2.638 and concln B1 √)<br><br>SR two tail test, using 1.96 ( or using 0.025 ) can score B0M1A1B1ft max 3/4 |

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| | | | | |
|---|---|---|---|---|
| **(ii)** | $\frac{x-2.6}{0.2\div\sqrt{75}} = 1.645$ $(x = 2.638)$ | **M1** | | |
| | $\pm\frac{'2.638'-2.68}{0.2\div\sqrt{75}}$ | **M1** | | for standardising with their " 2.638 " using 2.68 |
| | $= \pm 1.819$ | **A1** | | accept 1.82 ( 3 sf ) |
| | $\Phi('-1.819') = 1 - \Phi('1.819')$ | **M1** | | indep M mark, calculate correct area / prob consistent with their working |
| | $= 0.0345$ or $0.0344$ | **A1** | [5] | |
| **Total** | | | **[9]** | |
| **7 (i)** | est $\mu = 2.087$ | **B1** | | allow 2.09 |
| | est $\sigma^2 = \frac{100}{99}\left(\frac{435.57}{100} - 2.087^2\right)$ | **M1** | | 1 / 99 (435.57 – 208.7$^2$ / 100 ) |
| | $= 0.000132(3232)$ or $131/990000$ | **A1** | [3] | without $\frac{100}{99}$ : 0.000131 M0A0 |
| **(ii)** | $E(Y-X) = 2.12 - 2.087$ ( = 0.033 ) | **B1** | | or 2.12 – 2.087 – 0.01 for Y – X – 0.01 < 0 allow 2.09 for 2.087 |
| | $Var(Y-X) = 0.000144 + '0.00013232'$ | **M1** | | or $\sqrt{(0.012^2 + '0.00013232')}$ M1 |
| | $= 0.000276(32)$ | **A1** | | = 0.016623 A1 |
| | $\frac{0.01-'0.033'}{\sqrt{'0.00027632'}}$ $(= -1.384)$ | **M1** | | their $E(Y-X)$ & $Var(Y-X)$ var must be a combination of the two vars |
| | $\Phi('-1.384') = 1 - \Phi('1.384')$ | **M1** | | correct area / prob consistent with their working |
| | $= 0.0832$ | **A1** | [6] | SR use of biased var ( 0.000131 ) in (i) and (ii) scores in (ii) B1M1 A1 for 0.000275 and M1M1 A1 for 0.0827 ( 6 / 6 available) |
| **Total** | | | **[9]** | |
| | | | | |
| | **Total for paper** | | **[50]** | |

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# MARK SCHEME for the May/June 2015 series

## 9709 MATHEMATICS

**9709/71** Paper 7, maximum raw mark 50

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A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

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AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

**Penalties**

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

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Note: "(3 sfs)" means "answer which rounds to ... to 3 sfs". If correct ans seen to $\geqslant$ 3sfs, ISW for later rounding. Penalise $<$ 3 sfs only once in paper.

| | | | |
|---|---|---|---|
| **1** | $\frac{1}{2}a^2 = 1$ | M1 | or $\int_0^a x\mathrm{d}x = 1$ |
| | $a = \sqrt{2}$ | A1 | Allow 1.41 or better |
| | $\int_0^{\sqrt{2}} x^2\mathrm{d}x$ | M1 | ignore limits |
| | $= \left[\frac{x^3}{3}\right]_0^{\sqrt{2}}$ | A1f | correct integral and limits, but ft their $a$ |
| | $= \frac{\left(\sqrt{2}\right)^3}{3} =$ or $\frac{2^{1.5}}{3}$ or $\frac{2.83}{3}$ or 0.9428<br>(= 0.943 **AG**) | A1 [5] | must see this numerical expression, or equiv<br>SR Equating $\int x\ \mathrm{f}(x)$ to 0.943 scores M1<br>Solving to find $a$ = 1.41 scores A1 |
| | | **[Total 5]** | |
| **2 (i)** | $H_0$: $p = 0.2$ or $\mu = 10$<br>$H_1$: $p > 0.2$ or $\mu > 10$ | B1 [1] | |
| **(ii)** | N(10, 8) seen or implied | B1 | or $\mathrm{N}\left(0.2, \frac{0.2\times0.8}{50}\right)$ |
| | $\frac{125-10}{\sqrt{8}}$ or $\frac{\frac{125}{50}-02}{\sqrt{\frac{0.2\times0.8}{50}}}$ | M1 | For standardising allow with no or wrong cc |
| | = 0.884 | A1 | |
| | comp 1.282 | M1f | Allow area comparison with 0.188<br>or comp 1.645 if $H_1$ $p \neq 0.2$ |
| | Claim not justified<br>or No evidence to support claim | A1f [5] | Allow accept $H_0$ provided correctly defined. Follow through their test statistic ;dep 1–tail test<br>No Contradictions<br><br>SR; Use of B(50,0.2) scores B1 provided at least two probabilities calculated. M1 For finding P($X \geqslant 13$) allow one end error. A1 for 0.186 |
| | | **[Total: 6]** | |

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| | | | | |
|---|---|---|---|---|
| **3** | **(i)** | 34<br>$2.2^2 + 1.3^2 + 2.6^2$ (=13.29) | B1<br>B1 [2] | Accept 13.3 or $3.65^2$ Allow at early stage |
| | **(ii)** | $\frac{33-'34'}{\sqrt{\frac{'13.29'}{70}}}$ (= –2.295)<br>$\frac{35-'34'}{\sqrt{\frac{'13.29'}{70}}}$ (= 2.295)<br>Φ('2.295') – Φ('–2.295')<br>= Φ('2.295') – (1 – Φ('2.295')) oe<br>= 0.978 (3 sf) | M1<br>M1<br>M1<br>A1 [4] | correct standardisation method for either<br>For attempt to use tables to find the probability between two $z$ values ,may be implied by next line<br>For a correct method to find the area between their two $z$ values |
| | | | **[Total: 6]** | |
| **4** | **(i)** | $H_0$: pop mean (or $\mu$) = 12.4<br>$H_1$: pop mean (or $\mu$) > 12.4<br>$\frac{12.9-12.4}{2.1+\sqrt{50}}$<br>1.684<br>comp cv $z$ = 1.96<br>No evidence that pop mean time has increased | B1<br>M1<br>A1<br>B1f [4] | not just "mean"<br>Allow with 50 instead of √50<br>or P($z > 1.684$) = 0.0461 $>$ 0.025<br>Allow accept $H_0$ if correctly defined.<br>Ft their test statistic. No contradictions |
| | **(ii)** | Not reject (or accept) that mean time is unchanged (or is 12.4) oe<br>although mean time has increased (or is more than 12.4) oe | B1<br>B1 [2] | |
| | **(iii)** | True (or new) mean | B1 [1] | |
| | | | **[Total: 7]** | |

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| | | | | |
|---|---|---|---|---|
| **5** | **(i)** | 4200/80 (=52.5) | B1 | |
| | | $= \frac{80}{79}\left(\frac{229\,000}{80} - '52.5'^2\right)$ (= 107.595) | M1 | |
| | | | A1 [3] | |
| | | = 108 (3 sf) | | |
| | **(ii)** | $'52.5' \pm z\sqrt{\frac{'107.595'}{80}}$ | M1 | Correct form – must be $z$-value – allow one side only |
| | | $z = 2.326$ | B1 | Seen |
| | | 49.8 to 55.2 | A1f [3] | ft their 52.5 and 107.595. Must be an interval |
| | **(iii)** | 49 | B1 [1] | |
| | | | **[Total: 7]** | |
| **6** | **(i)** | $e^{-\frac{10}{3}} \times \frac{\left(\frac{10}{3}\right)^2}{2}$ | M1 | P(2), allow any $\lambda$ |
| | | = 0.198 (3 sf) | A1 [2] | |
| | **(ii)** | $1 - e^{-2}\left(1 + 2 + \frac{2^2}{2}\right)$ | M1 | M1 allow any $\lambda$ and/or 1end error |
| | | | M1 | Correct expression , correct λ |
| | | = 0.323 (3 sf) | A1 [3] | |
| | **(iii)** | $N\left(\frac{200}{3}, \frac{200}{3}\right)$ | M1 | seen or implied |
| | | $\frac{49.5 - \frac{200}{3}}{\sqrt{\frac{200}{3}}}$ (= –2.102) | M1 | For standardising allow either wrong or no cc No sd/var mix |
| | | $\Phi('-2.102') = 1 - \Phi('2.102')$ | M1 | For finding area consistent with their working |
| | | = 0.0178 (3 sf) | A1 [4] | |
| | | | **[Total: 9]** | |

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| | | | | |
|---|---|---|---|---|
| 7 | **(i)** | $7E(X) + 5E(Y) - 2$<br>$(= 7 \times 8 + 5 \times 3) - 2$<br>$= 69$ | M1<br><br>A1 [2] | allow incorrect means |
| | **(ii)** | $Var(X) = 1.6$, $Var(Y) = 3$<br>$16Var(X) + 9Var(Y)$<br><br>$(= 16 \times 1.6 + 9 \times 3)$<br>$= 52.6$ | B1<br>M1<br>M1<br><br>A1 [4] | both<br>M1 for mult by 16 and 9; allow with '+ 3'<br>M1 for add without '+ 3'; allow incorrect multipliers |
| | **(iii)** | $X = 10$, $Y = 2$ and $X = 9$, $Y = 0$<br><br>$0.8^{10} \times e^{-3} \times \frac{3^2}{2}$ or $10 \times 0.8^9 \times 0.2 \times e^{-3}$<br><br>$0.8^{10} \times e^{-3} \times \frac{3^2}{2} + 10 \times 0.8^9 \times 0.2 \times e^{-3}$<br>$= 0.0374/5$ | B1<br><br>M1<br><br>M1<br>A1 [4] | both pairs seen or implied<br><br>or 0.0241 or 0.0134 (3sf) one correct product<br><br>all correct |
| | | | **[Total: 10]** | |

**[Total for paper 50]**

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**9709/72** Paper 7, maximum raw mark 50

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| | | | | |
|---|---|---|---|---|
| **1** | | Var = 16×9 + 25×36 (= 1044)<br><br>sd = 32.3 or 6√29 or √1044 | B1<br>M1<br>A1 | M1 for 16 (or 4²) & 25 (or 5²) used<br>M1 for add any multiples of 9 and 36 only |
| | | **Total** | **3** | |
| **2** | **(i)** | $H_0$: $\lambda = 0.5$<br><br>$H_1$: $\lambda > 0.5$ | <br><br>B1 1 | or Pop mean = 0.5, not just Mean = 0.5<br>or Pop mean (per $m^2$ )= 0.1<br>Accept $\mu$ instead of $\lambda$ |
| | **(ii)** | $1 - e^{-0.5}(1 + 0.5)$<br>= 0.0902 (3 sf)<br>comp 0.1<br>Claim justified or there is evidence to support claim | M1<br>A1<br>M1<br><br>A1√ 4 | $1 - P(X = 0,1)$ attempted, any $\lambda$. Allow 1 end error<br>Allow 0.09<br>Valid comparison NB 0.9098>0.9 recovers M1A1 M1<br>oe Accept 'Reject $H_0$' if correctly defined<br>No contradictions. |
| | | **Total** | **5** | |
| **3** | | $\lambda = 5 \times 0.15$ (= 0.75)<br>E(amount) = 200×0.75 = 150<br><br>Var(weekly no of hole–in–ones) = 0.75<br>Var(amount) = $200^2 \times 0.75$<br>= 30,000 | M1<br>A1<br><br>B1√<br>M1<br>A1 5 | <br><br><br><br>Allow $200^2$ × their variance (with nothing added/subtracted at any stage)<br>(SR probability table can score M1A0 srB1 if var rounds to 30,000 (2sf) ) |
| | | **Total** | **5** | |
| **4** | **(i)** | Conclude flight times affected<br><br>when in fact they have not been. | B1<br><br>B1 2 | Or accept pop mean changed from 6.2<br><br>although pop mean has not changed from 6.2 |
| | **(ii)** | $H_0$: Pop mean (or $\mu$) = 6.2<br>$H_0$: Pop mean (or $\mu$) ≠ 6.2<br>$\frac{5.98 - 6.2}{\frac{0.8}{\sqrt{40}}}$<br>= –1.739 (±) Accept (±)1.74<br>comp $z$ = 1.96<br><br>No evidence that flight times affected | <br>B1<br>M1<br>A1<br><br>B1√<br>4 | <br><br>Allow with 40 instead of √40 Allow SD/Var mix<br>(CV method 5.952 or 6.2279 M1 A1)<br><br>For valid comparison<br>or $P(z < -1.739) = 0.041 > 0.025$ or 5.98 > 5.952 or 6.2 < 6.228<br>and correct conclusion |
| | **(iii)** | $H_0$ was not rejected oe<br>Type II | B1*<br>B1*dep<br>2 | If in (ii) $H_0$ was rejected, then:<br>$H_0$ rejected B1; Type I B1dep |
| | | **Total** | **8** | |

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| | | | | |
|---|---|---|---|---|
| **5** | **(i)** | 14800/50 or 296<br>$\frac{50}{49}\left(\frac{4390000}{50} - '296'^2\right)$ (= 187.755)<br>= 188 (3 sf) | B1<br>M1<br><br>A1 3 | <br>Oe<br> |
| | **(ii)** | $2 \times z \times \sqrt{\frac{'187.755'}{50}} = 5.45$ oe<br>$z$ = 1.406 or 1.405<br>Φ('1.406') (= 0.92 or 0.9199)<br>$\alpha$ = 84 (2 sf) allow 83.98 | M1<br>A1<br><br><br>M1<br>A1 4 | If '2 ×' omitted: $z \times \sqrt{\frac{'187.755'}{50}} = 5.45$ M1<br>$z$ = 2.812 or 2.810 A0<br>Φ('2.812') (= 0.9975)<br>$\alpha$ = 99.5 or 99 or 100 M1 A0<br>For complete method to find $\alpha$<br>SR use of biased var(184) scores M1A1(1.4205)<br>A=84.5 M1A1 |
| | **(iii)** | $0.96^4$<br>= 0.849 (3 sf) | M1<br>A1 2 | <br> |
| | | **Total** | **9** | |
| **6** | **(i)** | $k\int_0^{15}(225 - t^2)\mathrm{d}t = 1$<br>$k\left[225t - \frac{t^3}{3}\right]_0^{15} = 1$<br>$k$×[3375 – 1125} = 1 or $k$×2250 = 1<br>($k = \frac{1}{2250}$ **AG**) | M1<br><br>A1<br>A1 3 | Attempt integ f($x$) and = 1. Ignore limits<br><br>Correct integration and limits<br>No errors seen |
| | **(ii)** | $\frac{1}{2250}\int_{10}^{15}(225 - t^2)\mathrm{d}t$<br>$(= \frac{1}{2250}\left[225t - \frac{t^3}{3}\right]_{10}^{15})$<br>$= \frac{1}{2250}$ [2250 – (2250 – $\frac{1000}{3}$)]<br>$= \frac{4}{27}$ or 0.148 (3 sf) | M1<br><br>A1<br><br>A1 3 | Attempt integ, ignore limits<br><br>Or $1 - \int_0^{10}$<br>Correct integration and limits.<br>Condone missing $k$ |
| | **(iii)** | $\frac{1}{2250}\int_0^{15}(225t - t^3)\mathrm{d}t$<br>$= \frac{1}{2250}\left[\frac{225t^2}{2} - \frac{t^4}{4}\right]_0^{15}$<br>$= \frac{1}{2250}[\frac{50625}{2} - \frac{50625}{4}]$<br>$= \frac{45}{8}$ or 5.625 or 5.63 (3 sf) | M1*<br><br>A1<br><br>M1*dep<br>A1 4 | Attempt integ $x$f($x$) , ignore limits<br><br>Correct integration and limits. Condone missing $k$<br><br>Sub correct limits into their integral<br>Accept 5 mins 37 or 38 secs |
| | | **Total** | **10** | |

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| | | | | |
|---|---|---|---|---|
| 7 | (i) | Poisson<br><br>(Actually binomial with)<br>$n > 50$<br>and $np$ (or $\lambda$) (= 2.1) which is < 5 | B1<br><br><br>B1<br>B1 | 3 | <br><br>Allow without "binomial"<br>Accept n large<br>Accept p small (p < 0.1) |
| | (ii) | $\lambda = 2.1$<br>$e^{-2.1}\left(1+2.1+\frac{2.1^2}{2}+\frac{2.1^3}{3!}\right)$<br><br>= 0.839 (3 sf) | B1<br>M1<br><br>A1 | 3 | <br>Attempt P(0,1,2,3) any $\lambda$ allow 1 end error<br>$SR_1$ Ft Normal N(2.1,2.1) B1 standardising M1 0.833 A1<br>$SR_2$ Ft Binomial B(10500,0.0002) B1 calculating binomial prob P(0,1,2,3) M1 = 0.8386 A1 |
| | (iii) | $P(X \geqslant 1) = 1 - e^{-2.1}$ (= 0. 87754)<br>$P(X = 1,2,3) = e^{-2.1}\left(2.1+\frac{2.1^2}{2}+\frac{2.1^3}{3!}\right)$ (= 0.71619)<br><br>$\frac{P(X=1,2,3)}{P(X>1)}$ $\left(=\frac{0.71619}{0.87754}\right)$<br>= 0.816 (3 sf) | M1<br><br>M1<br><br>M1<br><br>A1 | 4 | Any $\lambda$<br><br>Or '0.839' – $e^{-2.1}$<br>Any $\lambda$<br>Allow any attempted $\frac{P(X=1,2,3)}{P(X>1)}$ Any $\lambda$<br><br>$SR_1$ Ft Normal P(>0.5)=0.86523 M1 P(1,2,3)=0.698 M1 0.698/0.86523 = 0.807 M1A1<br>$SR_2$ FT Binomial M1 M1 M1 A1 |
| | | **Total** | **10** | | |
| | | **Total for paper** | **50** | | |

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**9709/73** Paper 7, maximum raw mark 50

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SOS See Other Solution (the candidate makes a better attempt at the same question)

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| | | | | |
|---|---|---|---|---|
| **1** | **(i)** | Eg: Only students who use canteen<br>The five will probably be friends | B1<br>B1 [2] | or any reason that some are excluded<br>B1 each sensible reason must be in context |
| | **(ii)** | 2–digits<br>ignore > 82 (anything too big)<br>Ignore repeats | B1<br>B1<br>B1 [3] | |
| | | | **[Total 5]** | |
| **2** | **(i)** | $H_0$: P(correct) = $\frac{1}{8}$<br>$H_1$: P(correct) > $\frac{1}{8}$ | <br><br>B1 [1] | Or $H_0$ p = 1/8<br>$H_1$p > 1/8 |
| | **(ii)** | $1-\left(\left(\frac{1}{8}\right)^{10}+10\left(\frac{1}{8}\right)^{9}\left(\frac{7}{8}\right)+{}^{10}C_2\left(\frac{1}{8}\right)^{8}\left(\frac{7}{8}\right)^{2}\right)$<br><br>= 0.120 (3 sf) or 0.119 | M1<br><br>A1<br><br>A1 [3] | M1 for attempt at correct expression accept 1 error only, e.g. 1 term extra, omitted or wrong, or omit "1–" or incorrect p/q<br>Correct expression<br><br>Note Use of Poisson in (ii) could score M1 only for expression 1 –P(0,1,2) λ =1.25 |
| | **(iii)** | 12% | B1f [1] | ft their (ii) Must be a probability |
| | | | **Total 5** | |
| **3** | **(i)** | $\text{Var}(p_s) = \frac{0.22\times(1-0.22)}{100}$<br>$\left(=\frac{429}{250\,000}\text{ or }0.001716\right)$<br><br>$0.22 \pm z\sqrt{'\frac{429}{250\,000}'}$<br><br>$z$ = 2.17 or 2.168/9 or 2.171<br><br>0.13(0) to 0.31(0) (2 sf) | M1<br><br><br><br>M1<br><br>B1<br><br>A1 [4] | pq/100<br><br><br><br>Expression of correct form with their variance<br>Any $z$ (must be a z value) accept one side only<br><br>Seen<br><br>Must be an interval |
| | **(ii)** | '2' × (1 – 0.97) × 0.97<br>= 0.0582 | M1<br>A1 [2] | |
| | | | **Total 6** | |

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| | | | |
|---|---|---|---|
| **4 (i)** | $\left(\frac{1508}{50}\right) = 30.16\ (30.2)$ | B1 | Allow any form |
| | $\frac{50}{49}\left(\frac{51\,825}{50} - (\text{'}30.16\text{'}^2)\right)$ | M1 | (129.46367) |
| | = 129 (3 sf) Or 130 | A1 [3] | |
| **(ii)** | (1.5 × '30.16' + 10)<br>= 55.24 | B1ft | ft their 30.16 |
| | ($1.5^2$ × '129….') | M1 | $1.5^2$ × their(129) with nothing added at any stage |
| | = 291 ( 3 sf) | A1ft [3] | Allow 290 |
| | | **Total 6** | |
| **5 (i)** | Cables broken<br>or not all cables can be accessed oe<br>or Too many cables oe<br>or too time consuming oe | B1 [1] | e.g. previous days' stocks may have gone |
| **(ii)** | $H_0$: Pop mean brk str (or $\mu$) = 5<br>$H_1$: Pop mean brk str (or $\mu$) < 5 | B1 | Not just "mean" |
| | $(\pm)\frac{4.95-5}{\frac{0.15}{\sqrt{60}}}$ | M1 | Allow 60 instead of √60 |
| | (= ±2.582) | A1 | |
| | comp ±2.326<br>There is evidence that mean breaking strength is less than it should be<br>Or reject $H_0$ ($H_0$ correctly defined) | B1 ft [4] | Ft their –2.582<br>(No ft 2 tailed test)<br>Correct comparison shown, no errors seen.<br>Accept area comparison 0.0049 with 0.01<br><br>[CR method ($x$ – 5)/(0.15/√60)<br>= –2.326 M1 A1<br>leading to $x$ = 4.955 compared to 4.95and correct conclusion B1ft<br>OR (($x$ – 4.95)/0.15/√60) leading to 4.995 M1 A1 compared to 5and correct conclusion B1ft] |
| **(iii)** | Population not necessarily normal<br>so yes | B1<br>B1dep [2] | SR B1 For "it" is not necc normal (no mention of population) AND Yes |
| | | **Total 7** | |

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| | | | | |
|---|---|---|---|---|
| **6** | **(i)** | $e^{-3.5} \times \frac{3.5^3}{3!}$ | M1 | P($X = 3$) any $\lambda$ |
| | | = 0.216 (3 sf) | A1 [2] | |
| | **(ii)** | N(42, 42) stated or implied | B1 | |
| | | $\frac{29.5 - 42}{\sqrt{42}}$ (= –1.929) | M1 | Allow with wrong or no cc <u>OR</u> without √ |
| | | P($z >$ '–1.929') = Φ('1.929') | M1 | For correct area consistent with their working |
| | | = 0.973 (3 sf) | A1 [4] | |
| | **(iii)** | ($\lambda$) = 2.4 | B1 | |
| | | $1 - e^{-2.4}\left(1 + 2.4 + \frac{2.4^2}{2} + \frac{2.4^3}{3!}\right)$ | M1 | for 1 – P($X \leqslant 3$), any $\lambda$ allow one end error |
| | | | M1 | Correct expression any λ |
| | | = 0.221 (3 sf) | A1 4 | NB For combination method B1 attempting 10 combinations with λ=1, λ=1.4 M1 6 expressions M1 10 expressions 0.221 A1 |
| | | | **Total 10** | |
| **7** | **(i)** | $\frac{3}{4}\int_0^c (cx - x^2)dx = 1$ | M1 | Attempt integ f($x$) and = 1. Ignore limits |
| | | $\frac{3}{4}\left[\frac{cx^2}{2} - \frac{x^3}{3}\right]_0^c = 1$ | A1 | Correct integration and limits (condone c = 2 |
| | | $\frac{3}{4}\left(\frac{c^3}{2} - \frac{c^3}{3}\right) = 1$ or $\frac{3}{4} \times \frac{c^3}{6} = 1$ or $\frac{c^3}{8} = 1$ | A1 [3] | No errors seen |
| | | ($c = 2$ **AG**) | | |
| | **(ii)** | Inverted parabola | B1 | |
| | | Through (0, 0) and (2, 0) and zero elsewhere | B1 | Must not extend beyond [0,2] |
| | | Median = 1 | B1 [3] | |
| | **(iii)** | $\frac{3}{4}\int_0^{1.5} (2x - x^2)dx$ | M1 | Attempt integ f($x$) ignore limits |
| | | $= \frac{3}{4}\left[x^2 - \frac{x^3}{3}\right]_0^{1.5}$ | A1 | Correct integration ignore limits |
| | | $\frac{3}{4}\left(1.5^2 - \frac{1.5^3}{3}\right)$ | B1 | Use of correct limits [0,1.5] or 1–[1.5,2] |
| | | $= \frac{27}{32}$ or 0.844 (3 sf) | A1 [4] | |

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| | | | |
|---|---|---|---|
| **(iv)** | $\left(\frac{27}{32} - \frac{1}{2} \text{ or } 0.844 - 0.5\right)$<br><br>$= \frac{11}{32}$ or 0.344 (3 sf) | B1f [1] | ft their (iii) For use of symmetry Note If do not use "hence" and start again B1 for cwo |
| | | **Total 11** | |

**Total for paper 50**

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# MARK SCHEME for the October/November 2014 series

## 9709 MATHEMATICS

**9709/71** Paper 7, maximum raw mark 50

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M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

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| | | | | |
|---|---|---|---|---|
| **1** | $N(-35, 60^2 + 4 \times 28^2)$ | $N(35, 60^2 + 4 \times 28^2)$ | B1<br>B1 | for $\pm(175 - 2 \times 105)$ or $\pm 35$<br>for $60^2 + 4 \times 28^2$ or 6736 |
| | $\frac{0-(-35)}{\sqrt{'6736'}}$ (= 0.426) | $\frac{0-35}{\sqrt{'6736'}}$ (= –0.426) | M1 | For standardising with their mean and variance. Allow without √<br>For use of tables and finding area consistent with working |
| | $1 - \Phi(\text{“}0.426\text{”})$<br>= 0.335 (3 sf) | | M1<br>A1 5 | |
| | | | **Total: 5** | |
| **2 (i)** | (Bin) with $n > 50$ and mean (or $np$) $< 5$<br>Po(1.5) | | B1<br>B1 | Accept n ‘large’, p ‘small’<br>Poisson with correct mean stated or implied |
| | $1 - e^{-1.5}$ | | M1 | Poisson $1 - P(X = 0)$; allow incorrect $\lambda$; allow 1 end error |
| | = 0.777 (3 sf) | | A1 4 | SR If zero scored use of Bin leading to 0.778 / 0.779 scores B1 |
| **(ii)** | 3.5<br>$e^{-3.5}\left(\frac{3.5^4}{4!} + \frac{3.5^5}{5!} + \frac{3.5^6}{6!}\right)$<br>= 0.398 (3 sf) | | B1<br>M1<br>A1 3 | Correct mean stated or implied<br>Poisson $P(X = 4, 5, 6)$; allow incorrect $\lambda$; allow 1 end error |
| | | | **Total: 7** | |
| **3 (a)** | $\int_0^{0.5}\left(1.5t - 0.75t^2\right)dt$ o.e.<br>$= \left[0.75t^2 - 0.25t^3\right]_0^{0.5}$ o.e.<br>$= \frac{5}{32}$ or 0.156 (3 sf) | | M1<br>A1<br>A1 3 | Attempt int f($t$)<br>Correct integration and limits |
| **(b) (i)** | $\frac{1}{2}\pi a^2 = 1$ or $\pi a^2 = 2$ oe<br>$a = \sqrt{\frac{2}{\pi}}$ or 0.798 (3 sf) | | M1<br>A1 2 | Attempt to find the area and equate to 1 |
| **(ii)** | 0 | | B1 1 | |
| **(iii)** | Symmetry stated, seen or implied<br>0.8 | | M1<br>A1 2 | Could be a diagram<br>As final answer |
| | | | **Total: 8** | |
| **4 (i)** | $\text{Var}(P_s) = \frac{\frac{33}{150} \times \frac{150-33}{150}}{150}$ (= 0.001144) | | M1 | |
| | $z = 2.576$ | | B1 | Seen. Accept 2.574 to 2.579 |
| | $\frac{33}{150} \pm z\sqrt{\text{‘}0.001144\text{’}}$ | | M1 | Expression of correct form. Any $z$ |
| | = 0.133 to 0.307 (3 sf) | | A1 4 | Must be an interval |

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| | | | |
|---|---|---|---|
| **(ii)** | $\frac{19035}{150}$ (= 126.9 =127(3sf))<br>$\frac{150}{149}\left(\frac{4054716}{150}-\left(\frac{19035}{150}\right)^2\right)$ o.e.<br>= 11001.17 or 11000(3 sf) | B1<br>M1<br>A1 3 | <br>For use of a correct formula<br> |
| **(iii)** | 4-digit nos. each digit 0-9<br>Ignore nos > 9526<br>Ignore repeats | B1<br>B1<br>B1 3 | Some valid way of generating 4 digit random nos<br>from valid method<br>from valid method<br>SR If zero score, full explanation of method for drawing numbers out of a hat can score B1.<br>NB Systematic sampling follows the scheme with first B1 for some way of generating a random starting point. |
| | | **Total: 10** | |
| **5 (i)** | $\frac{4.8}{\sqrt{40}}$<br>$\frac{50.3-49.5}{\frac{4.8}{\sqrt{40}}}$ (= 1.054)<br>1 – Φ('1.054')<br>= 0.146 (3 sf) | B1<br>M1<br>M1<br>A1 4 | or $\frac{4.8^2}{40}$. Accept $4.8\sqrt{40}$ or $4.8^2 \times 40$ for totals method<br>For standardising with their SD Accept ±<br>Accept totals method. No mixed methods<br>For use of tables and finding area consistent with their working |
| **(ii) (a)** | Looking for decrease | B1 1 | |
| **(b)** | $H_0$: Pop mean time spent (or $\mu$) = 49.5<br>$H_1$: Pop mean time spent (or $\mu$) < 49.5<br>$\frac{\frac{1920}{40}-49.5}{\frac{4.8}{\sqrt{40}}}$ (= –1.976)<br>'1.976' > 1.555 (or '–1.976' < –1.555)<br>There is evidence that mean time has decreased. | B1<br>M1<br>M1<br>A1 4 | Not just "mean time spent"<br>For standardising. Allow $\div\frac{4.8}{40}$<br>Accept totals method; CV method. No mixed methods<br>For valid comparison (area comparison 0.024 < 0.06)<br>CWO. No contradictions in conclusions |
| **(c)** | Population normally distr so No | B1 1 | Both needed |
| | | **Total: 10** | |

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| | | | |
|---|---|---|---|
| **6 (i)** | $\lambda = 4.65$<br>$e^{-4.65} \times \frac{4.65^4}{4!}$<br>$= 0.186$ (3 sf) | B1<br>M1<br>A1 3 | <br>Poisson P($X = 4$) with any $\lambda$ |
| **(ii)** | $\lambda = 3.875$<br>$= e^{-3.875}\left(1 + 3.875 + \frac{3.875^2}{2!}\right) = 0.257$ (3 sf) | B1<br>M1<br>A1 3 | <br>P($X = 0, 1, 2$)<br>Attempted, any $\lambda$<br>As final answer |
| **(iii)** | $\lambda = 1.5$<br>$1 - e^{-1.5}\left(1 + 1.5 + \frac{1.5^2}{2!}\right)$<br>$= 0.191$ (3 sf) | B1<br>M1<br>A1 3 | <br>1 – P($X = 0, 1, 2$)<br>Attempted, any $\lambda$<br>As final answer |
| **(iv)** | He will reject $H_0$. | B1 1 | |
| | | **Total: 10** | |

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# MARK SCHEME for the October/November 2014 series

## 9709 MATHEMATICS

**9709/72** Paper 7, maximum raw mark 50

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| | $\frac{0-(-35)}{\sqrt{'6736'}}$ (= 0.426) | $\frac{0-35}{\sqrt{'6736'}}$ (= –0.426) | M1 | For standardising with their mean and variance. Allow without √ |
| | | | | For use of tables and finding area consistent with working |
| | $1 - \Phi(\text{“}0.426\text{”})$<br>= 0.335 (3 sf) | | M1<br>A1 5 | |
| | | | **Total: 5** | |
| **2 (i)** | (Bin) with $n > 50$ and mean (or $np$) $< 5$<br>Po(1.5) | | B1<br>B1 | Accept n ‘large’, p ‘small’<br>Poisson with correct mean stated or implied |
| | $1 - e^{-1.5}$ | | M1 | Poisson $1 - P(X = 0)$; allow incorrect $\lambda$; allow 1 end error |
| | = 0.777 (3 sf) | | A1 4 | SR If zero scored use of Bin leading to 0.778 / 0.779 scores B1 |
| **(ii)** | 3.5<br>$e^{-3.5}\left(\frac{3.5^4}{4!} + \frac{3.5^5}{5!} + \frac{3.5^6}{6!}\right)$<br>= 0.398 (3 sf) | | B1<br>M1<br>A1 3 | Correct mean stated or implied<br>Poisson $P(X = 4, 5, 6)$; allow incorrect $\lambda$; allow 1 end error |
| | | | **Total: 7** | |
| **3 (a)** | $\int_0^{0.5}\left(1.5t - 0.75t^2\right)dt$ o.e.<br>$= \left[0.75t^2 - 0.25t^3\right]_0^{0.5}$ o.e.<br>$= \frac{5}{32}$ or 0.156 (3 sf) | | M1<br>A1<br>A1 3 | Attempt int f($t$)<br>Correct integration and limits |
| **(b) (i)** | $\frac{1}{2}\pi a^2 = 1$ or $\pi a^2 = 2$ oe<br>$a = \sqrt{\frac{2}{\pi}}$ or 0.798 (3 sf) | | M1<br>A1 2 | Attempt to find the area and equate to 1 |
| **(ii)** | 0 | | B1 1 | |
| **(iii)** | Symmetry stated, seen or implied<br>0.8 | | M1<br>A1 2 | Could be a diagram<br>As final answer |
| | | | **Total: 8** | |
| **4 (i)** | $\text{Var}(P_s) = \frac{\frac{33}{150} \times \frac{150-33}{150}}{150}$ (= 0.001144) | | M1 | |
| | $z = 2.576$ | | B1 | Seen. Accept 2.574 to 2.579 |
| | $\frac{33}{150} \pm z\sqrt{\text{‘}0.001144\text{’}}$ | | M1 | Expression of correct form. Any $z$ |
| | = 0.133 to 0.307 (3 sf) | | A1 4 | Must be an interval |

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| | | | |
|---|---|---|---|
| **(ii)** | $\frac{19035}{150}$ (= 126.9 =127(3sf))<br>$\frac{150}{149}\left(\frac{4054716}{150}-\left(\frac{19035}{150}\right)^2\right)$ o.e.<br>= 11001.17 or 11000(3 sf) | B1<br>M1<br>A1 3 | <br>For use of a correct formula<br> |
| **(iii)** | 4-digit nos. each digit 0-9<br>Ignore nos > 9526<br>Ignore repeats | B1<br>B1<br>B1 3 | Some valid way of generating 4 digit random nos<br>from valid method<br>from valid method<br>SR If zero score, full explanation of method for drawing numbers out of a hat can score B1.<br>NB Systematic sampling follows the scheme with first B1 for some way of generating a random starting point. |
| | | **Total: 10** | |
| **5 (i)** | $\frac{4.8}{\sqrt{40}}$<br>$\frac{50.3-49.5}{\frac{4.8}{\sqrt{40}}}$ (= 1.054)<br>1 – Φ('1.054')<br>= 0.146 (3 sf) | B1<br>M1<br>M1<br>A1 4 | or $\frac{4.8^2}{40}$. Accept $4.8\sqrt{40}$ or $4.8^2 \times 40$ for totals method<br>For standardising with their SD Accept ±<br>Accept totals method. No mixed methods<br>For use of tables and finding area consistent with their working |
| **(ii) (a)** | Looking for decrease | B1 1 | |
| **(b)** | $H_0$: Pop mean time spent (or $\mu$) = 49.5<br>$H_1$: Pop mean time spent (or $\mu$) < 49.5<br>$\frac{\frac{1920}{40}-49.5}{\frac{4.8}{\sqrt{40}}}$ (= –1.976)<br>'1.976' > 1.555 (or '–1.976' < –1.555)<br>There is evidence that mean time has decreased. | B1<br>M1<br>M1<br>A1 4 | Not just "mean time spent"<br>For standardising. Allow $\div\frac{4.8}{40}$<br>Accept totals method; CV method. No mixed methods<br>For valid comparison (area comparison 0.024 < 0.06)<br>CWO. No contradictions in conclusions |
| **(c)** | Population normally distr so No | B1 1 | Both needed |
| | | **Total: 10** | |

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| | | | |
|---|---|---|---|
| **6 (i)** | $\lambda = 4.65$<br>$e^{-4.65} \times \frac{4.65^4}{4!}$<br>$= 0.186$ (3 sf) | B1<br>M1<br>A1 3 | <br>Poisson P($X = 4$) with any $\lambda$<br> |
| **(ii)** | $\lambda = 3.875$<br>$= e^{-3.875}\left(1 + 3.875 + \frac{3.875^2}{2!}\right) = 0.257$ (3 sf) | B1<br>M1<br>A1 3 | <br>P($X = 0, 1, 2$)<br>Attempted, any $\lambda$<br>As final answer |
| **(iii)** | $\lambda = 1.5$<br>$1 - e^{-1.5}\left(1 + 1.5 + \frac{1.5^2}{2!}\right)$<br>$= 0.191$ (3 sf) | B1<br>M1<br>A1 3 | <br>1 – P($X = 0, 1, 2$)<br>Attempted, any $\lambda$<br>As final answer |
| **(iv)** | He will reject $H_0$. | B1 1 | |
| | | **Total: 10** | |

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**Cambridge International Advanced Level**

# MARK SCHEME for the October/November 2014 series

## 9709 MATHEMATICS

**9709/73** Paper 7, maximum raw mark 50

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the October/November 2014 series for most Cambridge IGCSE®, Cambridge International A and AS Level components and some Cambridge O Level components.

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**Mark Scheme Notes**

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

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The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

**Penalties**

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

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| | | | | |
|---|---|---|---|---|
| **1** **(i)** | "Different" being investigated | **B1** | **[1]** | Oe ("changed", "not equal to") |
| **(ii)** | $H_0$: Pop mean (or $\mu$) in region same as elsewhere | | | Must be "pop mean", not just "mean"<br>Can be awarded in (i) |
| | $H_1$: Pop mean (or $\mu$) in region diff from elsewhere | **B1** | | oe |
| | $1.91 < 2.054$ (or 2.055) or $-1.91 > -2.054$ | **M1** | | or $P(z > 1.91) = 0.0281 > 0.02$ or $0.0562 > 0.04$ or $0.972 < 0.98$ Accept 2.05 if nothing better seen. |
| | No evidence that mean is different | **A1** | **[3]** | inequality sign incorrect M1A0<br>no contradictions<br>"accept $H_0$" provided $H_0$ reasonably well defined |
| **Total** | | | **[4]** | |
| **2** **(i)** | $\frac{1}{2}c^2 = 1$ | **M1** | | Area of triangle = 1 or integral of $kx$ with limits 0 and $c$ and equated to 1 |
| | $c = \sqrt{2}$ or 1.41 (3 sf) | **A1** | **[2]** | |
| **(ii)** | $f(x) = x$ or $y = x$ | **B1** | | Seen or implied, e.g. by next line. Can be awarded anywhere in the question. Implied by $(a + 1)$ in area of trapezium. |
| | $\int_a^1 x\,dx = 0.1$ | **M1** | | Ignore limits. Must be integral of $kx$ and equated to 0.1. Or trapezium area. |
| | $\left[\frac{x^2}{2}\right]_a^1 = 0.1$ | **A1**√ | | Correct limits, ft incorrect $kx$. |
| | $1 - a^2 = 0.2$<br>$a = 0.894$ (3 sf) | **A1** | **[4]** | $\sqrt{\left(\frac{4}{5}\right)}$ oe |
| **(iii)** | $\int_0^{\sqrt{2}} x^2\,dx$ | **M1** | | Ignore limits; ft their f($x$) but not $\int x\,dx$ |
| | $\left[\frac{x^3}{3}\right]_0^{\sqrt{2}}$ | | | |
| | $= \frac{2}{3}\sqrt{2}$ **or 0.943 or** $\sqrt{\left(\frac{8}{3}\right)}$ | **A1**√ | **[2]** | ft their $c$, dep $0 <$ ans $<$ their $c$. Not ft their f($x$) |
| **Total** | | | **[8]** | |

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| | | | | | |
|---|---|---|---|---|---|
| **3** | **(i)** | $\text{Est}(\mu) = \frac{7220}{80}$ or 90.25 | **B1** | | Accept 90.3 |
| | | $\text{Est}(\sigma^2) = \frac{80}{79}\left(\frac{656060}{80} - \left(\frac{7220}{80}\right)^2\right)$ | **M1** | | $\frac{1}{79}\left(656060 - \frac{7220^2}{80}\right)$ |
| | | $= 56.3924$ or $\frac{4455}{79}$ | **A1** | | |
| | | $z = 2.17$ | **B1** | | Accept 56.4 |
| | | $\frac{7220}{80} \pm z \times \sqrt{\frac{'56.3924'}{80}}$ | **M1** | | Expression of correct form |
| | | = 88.4 to 92.1 (3 sf) | **A1** | **[6]** | Must be an interval<br>(N.B. biased var gives 88.4 to 92.1 scores possible B1M0A0B1M1A1) |
| | **(ii)** | Pop normal | **B1** | | *X* normal or full definition of pop normal |
| | | No | **B1dep** | **[2]** | SR B1 for "no" <u>and</u> relevant reference to normal |
| **Total** | | | | **[8]** | |
| **4** | **(i)** | $4 \times 125 + 6 \times 130$ (= 1280) | **B1** | | Give at early stage. Could be implied by 220. |
| | | $4 \times 30^2 + 6 \times 32^2$ (= 9744) | **B1** | | (If B0B0 then 1.28 and 0.009744 can score B1B1). |
| | | $(\pm)\frac{1500 - 1280}{\sqrt{9744}} (= 2.229)$ | **M1** | | Standardising. Accept sd/var mix. Must be from combination attempt. |
| | | $\Phi("2.229")$ | **M1** | | Use of tables and correct area consistent with their working |
| | | = 0.987 (3 sf) | **A1** | **[5]** | cwo |
| | **(ii)** | $125 - 0.9(130)$ (= 8) (or –8) | **B1** | | Give at early stage. (If B0B0 scored then accept |
| | | $30^2 + 0.9^2(32^2)$ (= 1729.44) | **B1** | | 0.008 and 0.0017944 for B1B1) |
| | | $(\pm)\frac{0 - '8'}{\sqrt{'1729.44'}}$ (= –0.192) | **M1** | | Accept sd/var mix. Must come from a linear combination. |
| | | $\Phi('0.192')$ | **M1** | | Use of tables and correct area consistent with their working (unclear M0) |
| | | = 0.576 (3 sf) | **A1** | **[5]** | |
| **Total** | | | | **[10]** | |

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| | | | | | |
|---|---|---|---|---|---|
| **5** | **(i)** | $H_0$: population proportion = 0.1 oe<br>$H_1$: population proportion > 0.1 oe | **B1** | | Allow "$p = 0.1$" and "$p > 0.1$" |
| | | $P(X \geqslant 4) = 1 - P(X \leqslant 3) =$<br>$1 - \left( \begin{matrix} 0.9^{18} + 18 \times 0.9^{17} \times 0.1 + \\ {}^{18}C_2 \times 0.9^{16} \times 0.1^2 + {}^{18}C_3 \times 0.9^{15} \times 0.1^3 \end{matrix} \right)$ | **M1** | | Allow 1 – (one term omitted or extra or wrong) |
| | | = 0.0982 (3 sf) | **A1** | | (note CR method 0.0982 and CR ⩾ 5 for A1) |
| | | Comp 0.08 | **M1** | | Valid comparison (0.9018 < 0.92 also recovered previous A1). Or 4 is not in CR |
| | | No evidence that more reach 1m | **A1**√ | **[5]** | Dep M1M1 no contraditions<br>"Accept $H_0$" provided $H_0$ defined |
| | **(ii)** | Not rejected $H_0$ | **B1**√ | | Ft their (i) |
| | | Type II | **B1dep**√ | **[2]** | If (i) "reject $H_0$" then ft gives Type I error |
| | **(iii)** | $P(X \geqslant 5)$ $(= 0.0282)$ | **M1** | | Attempt $P(X \geqslant 5)$ e.g. '0.0982' – ${}^{18}C_4 \times 0.9^{14} \times 0.1^4$ oe. Valid comp of their ⩾ 5 (if CR method used, could be awarded in (i)) |
| | | 0.0282 < 0.08 | **B1**√ | | |
| | | P(Type I error) = 0.0282 (3 sf) | **A1** | **[3]** | |
| **Total** | | | | **[10]** | |
| **6** | **(i)** | $e^{-3.84} \times \dfrac{3.84^4}{4!}$ | **M1** | | Poisson P($X = 4$), any $\lambda$ |
| | | = 0.195 (3 sf) | **A1** | **[2]** | |
| | **(ii)** | 1.44 | **B1** | | Seen |
| | | $1 - e^{-1.44}\left(1 + 1.44 + \dfrac{1.44^2}{2}\right)$ | **M1** | | Any $\lambda$, allow one end error, need "1 –…" |
| | | = 0.176 | **A1** | **[3]** | |
| | **(iii)** | $X$~N(41, 41) | **B1** | | Seen or implied |
| | | $\dfrac{40.5 - 41}{\sqrt{41}}$ $(= -0.078)$ $\dfrac{59.5 - 41}{\sqrt{41}}$ $(= 2.889)$ | **M1M1** | | M1M0 if no cc or incorrect cc OR no √ in both |
| | | Φ('2.889') – Φ('–0.078')<br>= Φ('2.889') – (1 – Φ('0.078'))<br>= 0.9981 – (1 – 0.5311) | **M1** | | Use of tables and correct area consistent with their working. |
| | | = 0.529 (3sf) | **A1** | **[5]** | cwo |
| **Total** | | | | **[10]** | |

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**GCE Advanced Level**

# MARK SCHEME for the May/June 2014 series

## 9709 MATHEMATICS

**9709/71** Paper 7, maximum raw mark 50

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  B2/1/0 means that the candidate can earn anything from 0 to 2.

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Note: "(3 sfs)" means "answer which rounds to ... to 3 sfs". If correct ans seen to ≥ 3sfs, ISW for later rounding. Penalise < 3 sfs only once in paper.

| | | | |
|---|---|---|---|
| **1** | N(483.2, 537.92) or N(483.2, $23.2^2$)<br><br>$\frac{436-483.2}{\sqrt{537.92}}$ or $\frac{436-483.2}{23.2}$ (= –2.035)<br><br>Φ("–2.035") = 1 – Φ("2.035")<br>= 0.021 or 2.1% | B1<br><br>M1<br><br>M1<br>A1 [4] | or $\frac{8.2}{\sqrt{8}}$ or $\frac{8.2^2}{8}$ seen or implied<br><br>or $\frac{\frac{436}{8}-60.4}{8.2/\sqrt{8}}$ standardising (no mixed methods)<br><br>Correct area consistent with their working |
| | | **[Total: 4]** | |
| **2** | $\frac{70}{69} \times 2.70$ = 2.73913<br><br>$3.61 \pm z\sqrt{\frac{"2.73913"}{70}}$<br><br>$z$ = 1.96<br>3.22 to 4.00 (3 sf) | M1A1<br><br>M1<br><br>B1<br>A1 [5] | <br><br>or $3.61 \pm z\sqrt{\frac{2.70}{69}}$ M2A1(implied)<br><br>without $\frac{70}{69}$ : $3.61 \pm z\sqrt{\frac{2.70}{70}}$ M0A0M1<br><br>$z$ = 1.96 B1<br>3.23 to 3.99(4.00) (3 sf) A1<br>Answer must be an interval |
| | | **[Total: 5]** | |
| **3** | $H_0$: $\mu = 250$<br>$H_1$: $\mu > 250$<br>$\frac{250.06-250}{0.2\div\sqrt{40}}$<br><br>= 1.90<br>comp with $z$ = 1.645<br>Claim is justified<br>or There is evidence that claim is true | B1<br><br>M1<br>A1<br><br>M1<br><br>A1√ [5] | Both hypotheses<br><br>M1 for standardising, must have √40.<br>Accept cv method<br><br>For valid comparison "1.90" with 1.645 or area comparison or CVs<br>Correct conclusion. No contradictions<br>NB 2-tail test scores B0 M1 A1 M1 (use 1.96) A0 |
| | | **[Total: 5]** | |
| **4 (i)** | B(3500, 0.001)<br>Poisson with mean = 3.5<br>$n > 50$ and $np < 5$ | B1<br>B1<br>B1 [3] | <br>or Po(3.5)<br>Both. Or $n > 50$ and $\lambda < 5$ or $3.5 < 5$ |
| **(ii)** | $e^{-3.5}(1 + 3.5 + \frac{3.5^2}{2} + \frac{3.5^3}{3!})$<br>= 0.537 (3 dp) | M1<br><br>A1 [2] | Allow any $\lambda$ |
| | | **[Total: 5)** | |

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| | | | | |
|---|---|---|---|---|
| **5** | **(i)** | $0.25(1 + 4 + 9) – 1.5^2$<br>(=1.25 **AG**) | B1 [1] | |
| | **(ii)** | $\frac{1.4–1.5}{\sqrt{\frac{5}{4}÷300}}$ (= –1.549)<br><br>Φ("–1.549") = 1 – Φ("1.549")<br>= 0.0607 (3 sf) | M1<br><br>M1<br>A1 [3] | $\frac{1.4–\frac{1}{600}–1.5}{\sqrt{\frac{5}{4}÷300}}$ (= –1.523)<br><br>Φ("–1.523") = 1 – Φ("1.523")<br>= 0.0639 (3 sf) |
| | **(iii)** | Large sample or large *n*<br>( $\bar{X}$ (approx) normally distr )<br>or<br>Central Limit Theorem | B1 [1] | |
| | | | **[Total: 5]** | |
| **6** | **(i)** | $H_0$: Rate = 0.9<br>$H_1$: Rate < 0.9<br>1 – P(17, 18, 19, 20)<br><br>$1 – ({}^{20}C_{17} \times 0.1^3 \times 0.9^{17} + {}^{20}C_{18} \times 0.1^2 \times 0.9^{18} + 20 \times 0.1 \times 0.9^{19} + 0.9^{20})$<br>= 0.133 (3 sf) | <br>B1<br>M1<br><br>M1<br><br>A1 [4] | p = 0.9<br>p < 0.9<br>Use of B(20,0.1)<br><br>Allow 1–P(18,19,20) or 1–P(16,17,18,19,20) |
| | **(ii)** | Type II<br>$H_0$ will not be rejected | B1<br>B1 [2] | <br>or Stephan will conclude standard not fallen<br>No contradictions |
| | | | **[Total: 6]** | |

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| | | | | |
|---|---|---|---|---|
| **7** | **(i)** | $\int_1^a \frac{k}{x}\,dx = 1$ | M1 | Int f($x$) & equate to 1. Ignore limits |
| | | $k[\ln x]_1^a = 1$ | A1 | Correct integration and limits and = 1 |
| | | $k\ln a = 1 \quad k = 1/\ln a$ | A1 [3] | AG |
| | **(ii)** | $\frac{1}{\ln a}\int_1^a 1\,dx$ or $k\int_1^a 1\,dx$ | M1 | Int $x$f($x$). Ignore limits |
| | | $= \frac{1}{\ln a}[x]_1^a$ or $k[x]_1^a$ | A1 | Correct integration and limits (condone missing $k$) |
| | | $= \frac{1}{\ln a}(a-1)$ | A1 [3] | |
| | **(iii)** | $\frac{1}{\ln a}\int_1^m \frac{1}{x}\,dx = 0.5$ | M1 | Int f($x$) and equate to 0.5. Ignore limits |
| | | $\frac{1}{\ln a}[\ln x]_1^m = 0.5$ | A1 | Correct integration and limits (1 to $m$ or $m$ to $a$) (condone missing $k$) |
| | | $\frac{1}{\ln a}\ln m = 0.5$ | | |
| | | $\ln m = 0.5\ln a$ | A1 | or $\ln m = \ln a^{0.5}$ |
| | | $m = \sqrt{a}$ | A1 [4] | |
| | | | **[Total: 10]** | |
| **8** | **(i)** | $V$: cannot have neg value | B1 | |
| | | $W$: cannot have non-integer value | B1 [2] | |
| | **(ii)** | (a) $e^{-\lambda} = p$ and $\lambda e^{-\lambda} = 2.5p$ (Hence $\lambda = 2.5$ **AG**) | B1 [1] | or equiv explanation |
| | **(ii)** | (b) $1 - e^{-2.5}(1 + 2.5 + \frac{2.5^2}{2})$ | M1 | Allow one end error |
| | | $= 0.456$ (3 sf) | A1 [2] | |
| | **(iii)** | $\Phi^{-1}(0.5793) = -0.2$ | B1 | |
| | | N($\mu$, $\mu$) seen or implied | M1 | |
| | | $\frac{40.5-\mu}{\sqrt{\mu}} =$ "–0.2" | M1 | Allow no cc or incorrect cc |
| | | $\mu +$ "–0.2" $\sqrt{\mu} - 40.5 = 0$ | | |
| | | $\sqrt{\mu} = \frac{"0.2" \pm \sqrt{"0.2"^2 + 4\times 40.5}}{2}$ (= 6.4647..) | M1 | For solving quadratic in $\sqrt{\mu}$ (or $\mu$) |
| | | | | Ignore other answer for $\sqrt{\mu}$, but not for $\mu$ |
| | | $\mu = 41.8$ (3 sf) | A1 [5] | |
| | | | **[Total: 10]** | |

**[Total for paper 50]**

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**CAMBRIDGE INTERNATIONAL EXAMINATIONS**

**GCE Advanced Level**

# MARK SCHEME for the May/June 2014 series

## 9709 MATHEMATICS

**9709/72** Paper 7, maximum raw mark 50

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A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

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AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

**Penalties**

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

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| | | | |
|---|---|---|---|
| **1** | $\frac{\Sigma x}{8} = \frac{2006}{8} = 250.75$ or 251 (3 s.f.)<br><br>$(\Sigma x^2 = 503274)$<br><br>$\frac{8}{7}\left(\frac{"503274"}{8} - "250.75"^2\right)$<br><br>= 38.5 o.e. (accept 6.204²) | B1<br><br><br>M1<br><br>A1 [3] | Any equivalent form<br><br><br>For use of formula of correct form<br><br>cao (as final answer) |
| **2** | $(X + Y - Z) \sim N(8, \ldots)$<br><br>$\mu$=8 (or –8)<br><br>Var $(X + Y - Z) = 2^2 + 1.5^2 + 1.8^2$ (= 9.49)<br><br>$\frac{0-8}{\sqrt{'9.49'}}$ (= –2.597)<br><br>$\Phi('-2.597') = 1 - \Phi('2.597')$<br><br>= 0.0047 | <br><br>B1<br><br>B1<br><br>M1<br><br>M1<br><br>A1 [5] | <br><br>seen or implied<br><br>– award at early stage<br><br>For standardising (accept sd/var mixes, but variance must be a combination of at least 2 of $X$, $Y$, $Z$)<br><br>For area consistent with their working |
| **3** | $H_0$: Pop mean (or $\mu$ or $\lambda$) = 50 (or 5)<br><br>$H_1$: Pop mean (or $\mu$ or $\lambda$) ≠ 50 (or 5)<br><br>$\frac{60.5-50}{\sqrt{50}}$ (±)<br><br>= (±)1.485 OR 0.0687 OR C.V<br><br><br>1.485 < 1.645 or 0.0687 > 0.05<br>No evidence that mean changed | <br><br>B1<br><br>M1<br><br>A1<br><br><br>M1<br>A1√ [5] | <br><br>Not just "mean"<br><br>For standardising with N(50,50) or N(5,5/√10)<br><br>Allow M1 with wrong or no continuity correction OR no √<br>(accept c.v method M1, A1 for 61.63 or 48.868)<br><br>For valid comparison ($z$s or areas or cv)<br>(S.R For cv comparison 61.63 only award final A1 if cc used)<br><br>or if $H_1$: $\lambda > 50$, 1.485 < 1.96 M1<br>No evid mean changed A0<br>(i.e. if one-tail test, max B0 M1 A1 M1 A0) |

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| | | | | |
|---|---|---|---|---|
| **4** | **(i)** | $\lambda = 4.5$ | B1 | seen |
| | | $1 - e^{-4.5}\left(1+4.5+\frac{4.5^2}{2}\right)$ | M1 | any $\lambda$. Allow one end error |
| | | $= 0.826$ (3 s.f.) | A1 [3] | |
| | **(ii)** | $e^{-\lambda} = 0.523$ | B1 | |
| | | $(-\lambda = \ln 0.523)$ | | |
| | | $\lambda = 0.648$ (3 s.f.) | B1 [2] | |
| | **(iii)** | $e^{-\mu} \times \frac{\mu^3}{3!} = 24 \times e^{-\mu} \times \mu$ | B1 | |
| | | $\frac{\mu^2}{6} = 24$ | M1 | For a simplified expression in $\mu^2$ with $e^{-\mu}$ and $\mu$ cancelled and no factorials. |
| | | $\mu = 12$ | A1 [3] | |
| **5** | **(i)** | $p = \frac{184}{400}$ or 0.46 | B1 | Used |
| | | $z = 1.96$ | B1 | Seen |
| | | $"0.46" \pm z \times \sqrt{\frac{"0.46"(1-"0.46")}{400}}$ | M1 | Using expression of correct form |
| | | $= 0.411$ to 0.509 | A1 [4] | Must be an interval |
| | **(ii)** | 0.5 within CI | | |
| | | Claim not supported or not justified | B1√ [1] | Both needed. No contradictions. ft their **(i)** |
| | **(iii)** | $z \times \sqrt{\frac{"0.46"(1-"0.46")}{400}} = 0.05$ | M1 | Allow M1 for $z \times \sqrt{\frac{"0.46"(1-"0.46")}{400}} = 0.1$ |
| | | $z = 2.006$ | A1 | |
| | | $\Phi(\text{'2.006'}) = 0.9775$ | | |
| | | $\alpha = \text{'0.9775'} - (1 - \text{'0.9775'})$ | M1 | or $1 - 2(1 - \text{'0.9775'})$ |
| | | $= 95.5\%$ | A1 [4] | |

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| | | | |
|---|---|---|---|
| **6 (i)** | $k\int_0^4(16t-t^3)\mathrm{d}t = 1$ | M1 | Int f($t$) = 1 ignore limits |
| | $k\left[8t^2-\frac{t^4}{4}\right]_0^4 = 1$ | A1 | correct integration with correct limits |
| | $k(128-64) = 1$ o.e. | | |
| | $k \times 64 = 1$ | | |
| | $\left(k=\frac{1}{64}\right)$ **AG** | A1 [3] | must be convinced (AG) |
| **(ii)** | $\frac{1}{64}\int_0^1(16t-t^3)\mathrm{d}t$ | M1 | Int f($t$) between 0 and 1 (accept 0 and a value < 1 , 1 and 4) |
| | $= \frac{1}{64}\left[8t^2-\frac{t^4}{4}\right]_0^1$ | A1 | correct integration and correct limits (ignore "k") |
| | $= \frac{1}{64}\left[8-\frac{1}{4}\right]$ | | |
| | $= \frac{31}{256}$ or 0.121094 | A1 | |
| | $\left(\frac{31}{256}\right)^2 = 0.0147$ (3 s.f.) o.e. | B1√ [4] | ft their "$\frac{31}{256}$" |
| **iii** | $\frac{1}{64}\int_0^4(16t^2-t^4)\mathrm{d}t$ | M1 | Int $t$f($t$) ignore limits |
| | $= \frac{1}{64}\left[\frac{16t^3}{3}-\frac{t^5}{5}\right]_0^4$ | A1 | correct integration and correct limits (ignore "k") |
| | $= \frac{1}{64}\left(\frac{1024}{3}-\frac{1024}{5}\right)$ | | |
| | $= \frac{32}{15}$ or 2.13 (3 s.f.) o.e. | A1 [3] | |

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| | | | | |
|---|---|---|---|---|
| **7 (i)** | $2^{nd}$<br><br>More representative of all appointments or Lengths may vary during the day<br>or $1^{st}$ does not include later appts so not representative | B1<br><br><br><br>B1 [2] | Any implication that times or conditions vary throughout day, e.g. doctors get tired | |
| **(ii)** | 0.01 o.e.<br><br>Concluding that times spent are too long when they are not. | B1<br><br>B1 [2] | Concluding that the mean time spent is more than 10 mins when it is not. Must be in context. | |
| **(iii)** | $H_0$: Pop mean appt time (or $\mu$) = 10<br>$H_1$: Pop mean appt time (or $\mu$) > 10<br><br>$\frac{\frac{147}{12}-10}{\frac{3.4}{\sqrt{12}}}$ (±)<br><br>= (±)2.292 or (0.0109 if area comparison done)<br><br>"2.292" < 2.326 o.e.<br><br>(No evidence to reject $H_0$.)<br>No reason to believe appts are too long | B1<br><br>M1<br><br>A1<br><br>M1<br><br>A1√ [5] | Both correct. Allow $\mu$, but not just "mean"<br><br>Allow incorrect $\frac{147}{12}$<br>Must have √12 (accept totals method)<br><br>For valid comparison<br>Comp "2.292" with 2.326<br>Or 0.0109 with 0.01<br>Or 147/12 with 12.28<br><br>Dep 2.326, ft their "2.292"<br>No contradictions | $10 + 2.326 \times \frac{3.4}{\sqrt{12}}$ M1<br><br>= 12.28 A1<br><br>$\frac{147}{12} < 12.28$ M1 |
| **(iv)** | Normal population | B1 [1] | Must have "population" or equiv | |

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**GCE Advanced Level**

# MARK SCHEME for the May/June 2014 series

## 9709 MATHEMATICS

**9709/73** Paper 7, maximum raw mark 50

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| | | | |
|---|---|---|---|
| **1** | $e^{-4}(1 + 4)$ | M1<br>M1 | M1 for P(0 or 1) using Poisson, any $\lambda$<br>Expression of correct form correct $\lambda$<br>(allow 1 end error) |
| | = 0.0916 (3 s.f.) | A1 [3] | SR Use of Bin(100000, 1/25000) scores M1 for P(0,1) allow one end error. A1 0.0916 |
| **2** | ht = $\frac{1}{2}$ seen | B1 | or $y = \frac{1}{8}x$ |
| | $\frac{1}{2} \times m \times \left(\frac{m}{4} \times "\frac{1}{2}"\right) = \frac{1}{2}$ | M1 | $\frac{1}{2} \times m \times ("\frac{1}{8}" m) = \frac{1}{2}$ or $\frac{m^2}{16} = \frac{1}{2}$ o.e. |
| | **N.B. B1 M1 must be consistent** | | Or Integrating linear function of form $y = kx$ with limits 0 and $m$ or $m$ and 4 and equated to 0.5 |
| | $m = \sqrt{8}$ or $2\sqrt{2}$ or 2.83 (3 s.f.) | A1 [3] | |
| **3** | $p = 0.56$ | B1 | Used |
| | '0.56' $\pm z \times \sqrt{\frac{0.56 \times 0.44}{100}}$ | M1 | Equation of correct form condone just +ve or –ve<br>Must be $z$ |
| | $z = 2.17$, or 2.169 or 2.171 | B1 | |
| | 0.452 to 0.668 (3 s.f.) | A1 [4] | Seen<br>Must be an interval |
| **4** | $\bar{x} = 1.65$ | B1 | |
| | $\text{est}(\sigma^2) = \frac{100}{99}\left(\frac{276.25}{100} - 1.65^2\right)$ | B1 | |
| | = 0.040404… = 4/99 | | |
| | $(\pm)\frac{1.65 - 1.6}{\sqrt{\frac{"0.040404"}{100}}}$ | M1 | Without $\frac{100}{99}$: $\frac{1.65 - 1.6}{\sqrt{\frac{"0.04"}{100}}}$ B1 B0 M1 |
| | = (±) 2.487/2.488 accept 2.49 Or 0.0065/0.0064 if area comparison done | A1 | = 2.50 A1 |
| | | | CV Method M1 must use 1.96 A1 for 1.639 or 1.6106 |
| | comp with 1.96 | M1 | For valid comparison ($z$/$z$ Signs consistent or area/area cv) |
| | There is evidence that $\mu$ is not 1.6 | A1√ [6] | Accept Reject $H_0$ No contradictions |

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| | | | |
|---|---|---|---|
| **5 (i)** | Longest lifetime | B1 [1] | Must be in context |
| **(ii)** | $\int_1^a \frac{k}{x^2}\mathrm{d}x = 1$ | M1 | Int f($x$) and equate to 1. Ignore limits |
| | $k\left[-\frac{1}{x}\right]_1^a = 1$ | A1 | Correct integral and limits |
| | $\left(k\left[-\frac{1}{a}+1\right]=1\right)$ | | |
| | $k\left[\frac{-1+a}{a}\right] = 1$ or $k(-1+a) = a$ | | |
| | $k = \frac{a}{a-1}$ **AG** | A1 [3] | Must be convinced (**AG**) |
| **(iii)** | $\frac{5}{3}\int_1^{2.5}\frac{1}{x}\mathrm{d}x$ or $k\int_1^{2.5}\frac{1}{x}\mathrm{d}x$ | M1 | Int $x$f($x$). Ignore limits |
| | $= \frac{5}{3}\,[\ln x]_1^{2.5}$ or $k[\ln x]_1^{2.5}$ | A1 | Correct integral and limits<br>(Accept "$k$" or "their $k$") |
| | $= \frac{5}{3}\ln 2.5$ or 1.53 (3 s.f.) | A1 [3] | |
| **6 (i)** | $H_0$: $p = 0.2$<br>$H_1$: $p < 0.2$ | B1 | (Allow $\pi$) |
| | P(0 or 1 5s in 25 \| $H_0$) | M1 | $0.8^{25} + 25 \times 0.8^{24} \times 0.2$ Use of B(25,1/5) and P(0) or P(1) or both – may be implied by "0.0274" |
| | = 0.0274 (3 s.f.) | A1 | |
| | Comp with 0.025 | M1 | Valid comparison |
| | No evidence (at 2.5% level) to support claim | A1√ [5] | No contradictions<br>**SR** Use of Normal N(5,4) leading to $z$ = 1.75 or 0.0401 **B1*** $H_0$ $\mu = 5$ $H_1$ $\mu < 5$ B1. Comparison 1.75 < 1.96 or 0.0401 > 0.025 **B1* dep** |
| **(ii)** | Normal | B1 | |
| | $\mu = 200$, $\sigma^2 = 160$ or $\sigma = \sqrt{160}$ | B1 [2] | |
| **(iii)** | Concluding that the machine produces the right proportion of 5s, although it doesn't. | B1 [1] | Not concluding that the machine produces too few 5s although it does. Must be in context o.e. No contradictions |

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| | | | | |
|---|---|---|---|---|
| **7** | **(i)** | Constant mean (or average) rate | B1 [1] | Constant mean per day (or week, etc.) o.e. |
| | **(ii)** | $e^{-\frac{4}{7}} \times \frac{\frac{4}{7}^2}{2!}$ or $e^{-0.571} \times \frac{0.571^2}{2!}$ | M1 | Expression for P(2) allow any $\lambda$ |
| | | = 0.0922 or 0.0921 (3 s.f.) | A1 [2] | |
| | **(iii)** | $\lambda = \frac{40}{7}$ or 5.71… | B1 | |
| | | $1 - e^{-\frac{40}{7}}\left(1+\frac{40}{7}+\frac{\frac{40}{7}^2}{2!}+\frac{\frac{40}{7}^3}{3!}\right)$ | M1 | Allow any $\lambda$ allow one end error |
| | | = 0.821 (3 s.f.) | A1 [3] | |
| | **(iv)** | $\frac{24}{7}$ o.e. 3 s.f. or better seen | B1 | |
| | | $e^{-\frac{4}{7}} \times e^{-\frac{24}{7}} \times \frac{\frac{24}{7}^5}{5!}$ | M1 | M1 for P(0) × P(5) any consistent $\lambda$ |
| | | = 0.0723 (3 s.f.) | A1 [3] | |
| **8** | **(i)** | $X + 2.5Y \sim N(127, 44.25)$ | B1<br>B1 | B1 for 127 Allow at early stage (57 + 2.5 × 28)<br>B1 for 44.25 or 6.65<br>Allow at early stage (13 + $2.5^2$ × 5)<br>May be implied by next line |
| | | $(\pm)\frac{140-"127"}{\sqrt{"44.25"}}$ | M1 | For standardising |
| | | = ±(1.954) | M1 | For area consistent with their working |
| | | 1 – Φ(“1.954”) | | |
| | | = 0.0254/0.0253 (3 s.f.) | A1 [5] | |
| | **(ii)** | $X - Y \sim N(29, 18)$ | B1<br>B1 | B1 for 29 Give at early stage (57 – 28)<br>B1 for 18 Give at early stage (13 + 5)<br>May be implied by next line |
| | | $\frac{20-"29"}{\sqrt{"18"}}$ (= –2.121) | M1 | For Standardising |
| | | 1 – Φ(“–2.121”) = Φ(“2.121”) | M1 | For area consistent with their working |
| | | = 0.983 (3 s.f.) | A1 [5] | |

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**GCE Advanced Level**

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## 9709 MATHEMATICS

**9709/71** Paper 7, maximum raw mark 50

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M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

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The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

**Penalties**

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

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| | | | |
|---|---|---|---|
| **1** | $\lambda = \frac{1}{30}$<br>$1 - e^{-\frac{1}{30}}$<br>$= 0.0328$ (3 s.f.) | B1<br><br>M1<br>M1<br>A1<br><br>[4] | o.e<br><br>$1 - P(X = 0)$ by Poisson, any $\lambda$ allow 1 end error<br>$1 - P(X = 0)$ by Poisson, correct $\lambda$ no end errors<br><br>S.R. Binomial with final answer 0.0328 B2<br>Correct answer, no working scores B2 |
| **2** | $z = 2.576$<br>$2 \times z \times \frac{0.17}{\sqrt{n}} = 0.2$ oe<br>$n = (\frac{2 \times 0.17 \times 2.576}{0.2})^2$ oe (= 19.2)<br>Smallest $n$ is 20 | B1<br><br>M1<br><br>M1<br><br>A1 [4] | Seen (accept 2.574 to 2.579)<br><br>Allow without '2 ×' OR with incorrect $z$<br><br>Attempt to arrange equ of correct form (with correct z and '2×' into the form n= or $\sqrt{n}$ = |
| **3 (i)** | est ($\mu$) = 2866 or 2870 (3 s.f.)<br>est ($\sigma^2$) = $\frac{1}{49}(410900000 - \frac{143300^2}{50})$<br>(= 4126.53)<br>= 4130 (3 sf) | B1<br><br>M1<br><br>A1 [3] | Accept 143300/50 o.e.<br><br>Correct subst in correct formula |
| **(ii)** | $H_0$: Pop mean (or $\mu$) = 2850<br>$H_1$: Pop mean (or $\mu$) ≠ 2850<br>$\frac{\frac{143300}{50} - 2850}{\frac{\sqrt{'4126.53'}}{\sqrt{50}}}$<br><br>= 1.761<br>'1.761' < 1.96<br>No evidence mean distance changed | B1<br><br>M1<br><br>A1<br>M1<br>A1f<br><br>[5] | Both. Not just 'mean'<br><br>Allow '4126.53' without $\sqrt{}$, but must have all $\sqrt{50}$<br><br>Or correct c.v. (2867.81) for alt method<br>For valid comparison of z values, areas or c.v.<br>Dep 1.96; ft their 1.761<br>If $H_1$: $\mu > 2850$ and c.f. 1.645,<br>max B0M1A1M1A0<br>(c.v. for 1 tail test 2864.94) |
| **4 (i)** | $\lambda = 2.8$<br>$e^{-2.8}(1 + 2.8 + \frac{2.8^2}{2})$<br>= 0.469 (3 s.f.) or 0.47(0) | B1<br><br>M1<br><br>A1 [3] | seen<br><br>any $\lambda$ allow one end error<br><br>As final answer |
| **(ii)** | $e^{-0.7n} \geqslant 0.99$ or $e^{-\lambda} \geqslant 0.99$<br>$-0.7n \geqslant \ln 0.99$ or $-\lambda \geqslant \ln 0.99$<br>$n \leqslant 0.01436$ or $\lambda \leqslant 0.01005$<br>'0.01436' × 150<br>or '0.01005' × 150 ÷ 0.7<br>Max period is 2.15 mins (3 sf) | M1<br>M1<br>A1<br><br>M1<br>A1 [5] | Allow '=' throughout<br>Attempt ln both sides<br>Can be implied. Accept 3 s.f.<br><br>Note $e^{-(0.7/150)n} \geqslant 0.99$ scores 1st and 3rd M1<br>T & I leading to ans 2.2 mins, SC: B2 |

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|---|---|---|---|
| **5 (i)** | $\int_0^2 k(x-2)^2\,dx = 1$<br>$\left(\left[\frac{k(x-2)^3}{3}\right]_0^2 = 1\right)$<br>$k\,[0-(-\frac{8}{3})] = 1$<br>$k = \frac{3}{8}$ **AG** | M1<br><br><br>A1<br><br>[2] | Attempt to integrate f($x$) with correct limits and = 1<br><br><br>Must see this line or better, e.g. $k \times \frac{8}{3} = 1$ |
| **(ii)** | $\frac{3}{8}\int_d^2 (x-2)^2\,dx = 0.2$<br>$\left(\frac{3}{8}\left[\frac{(x-2)^3}{3}\right]_d^2 = 0.2\right)$<br>$\frac{3}{8}\left[0-\frac{(d-2)^3}{3}\right] = 0.2$ oe<br>$((d-2)^3 = -1.6)$<br>$d = 0.83(0)$ (3 s.f.) | M1<br><br><br>M1<br><br><br>A1 [3] | $\int f(x)dx$ with limits d and 2 or 0 and d, and = 0.2 or =0.8<br>Condone missing 'k'<br><br><br>Reasonable attempt to integrate from a correct expression, with limits substituted to give expression in $d^3$.<br>Condone missing 'k' |
| **(iii)** | $\frac{3}{8}\int_0^2 x(x-2)^2\,dx$<br>$(= \frac{3}{8}\int_0^2 x^3 - 4x^2 + 4x\,dx)$<br>$= \frac{3}{8}\left[\frac{x^4}{4} - \frac{4x^3}{3} + 2x^2\right]_0^2$<br>$= \frac{1}{2}$ | M1<br><br><br><br>A1<br><br>A1 [3] | Attempt integ $xf(x)$; ignore limits, condone missing k<br>$(\frac{3}{8}\left[x \times \frac{(x-2)^3}{3} - \int \frac{(x-2)^3}{3}\,dx\right]_0^2)$<br>$= \frac{3}{8}\left[x \times \frac{(x-2)^3}{3} - \frac{(x-2)^4}{12}\right]_0^2$<br>Correct integration & limits, condone missing k |
| **6 (i)** | P(Type I) = 1 - P($\geq$ 4 assuming $p$ = 0.7)<br>$1-({}^6C_4 \times 0.7^4 \times 0.3^2 + {}^6C_5 \times 0.7^5 \times 0.3 + 0.7^6)$<br>$(= 1 - 0.744)$<br>= 0.256 (3 s.f.) | M1<br><br>M1<br><br>A1 [3] | or P($\leq$ 3 assuming $p$ = 0.7) May be implied<br${}^6C_3 \times 0.7^3 \times 0.3^3 + {}^6C_2 \times 0.7^2 \times 0.3^4 + {}^6C_1 \times 0.7 \times 0.3^5 + 0.3^6$<br>Allow one end error<br>= 0.256 (3 s.f.)<br>SR if zero scored allow B1 for use of B(6, 0.7) in any two or more terms |
| **(ii)** | P(Type II) = P($\geq$ 4 assuming $p$ = 0.35)<br>$= {}^6C_4 \times 0.35^4 \times 0.65^2 + {}^6C_5 \times 0.35^5 \times 0.65 + 0.35^6$<br>= 0.117 | M1<br><br>M1<br>A1<br>[3] | May be implied<br>Allow one end error<br><br>SR if zero scored allow B1 for use of B(6, 0.35) in any two or more terms |
| **(iii)** | Type 1<br>They will reject Luigi's belief, although it might be true. | B1<br><br>B1 [2] | <br><br>In context |

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| | | | |
|---|---|---|---|
| **7 (i)** | N(10.61, 0.1017)<br>$\frac{11-'10.61'}{\sqrt{'0.1017'}}$ (= 1.223)<br>Φ('1.223')<br>= 0.889 (3 s.f.) | B1<br><br>M1<br>M1<br><br>A1 [4] | o.e. Stated or implied (accept in un-simplified form)<br><br>Allow without √<br>For attempt to find correct area consistent with their working |
| **(ii)** | P($K - 1.2A > 0$ )<br>Var = $0.0576 + 1.2^2 \times 0.0441$<br>(= 0.121104)<br>N(–0.324, 0.121104)<br>$\frac{0-(-0.324)}{\sqrt{'0.121104'}}$ (= 0.931)<br>1 – Φ('0.931')<br>= 0.176 (3 s.f.) | M1<br><br><br>B1 B1<br><br>M1<br>M1<br><br>A1 [6] | Or similar stated or implied<br><br><br>o.e. May be implied (accept in un-simplified form)<br><br>Allow without √<br>For attempt to find correct area consistent with their working |

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**CAMBRIDGE INTERNATIONAL EXAMINATIONS**

**GCE Advanced Level**

# MARK SCHEME for the October/November 2013 series

## 9709 MATHEMATICS

**9709/72** Paper 7, maximum raw mark 50

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www.satprep.co

**Mark Scheme Notes**

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| **1** | $\lambda = \frac{1}{30}$<br>$1 - e^{-\frac{1}{30}}$<br>$= 0.0328$ (3 s.f.) | B1<br><br>M1<br>M1<br>A1<br><br>[4] | o.e<br><br>$1 - P(X = 0)$ by Poisson, any $\lambda$ allow 1 end error<br>$1 - P(X = 0)$ by Poisson, correct $\lambda$ no end errors<br><br>S.R. Binomial with final answer 0.0328 B2<br>Correct answer, no working scores B2 |
| **2** | $z = 2.576$<br>$2 \times z \times \frac{0.17}{\sqrt{n}} = 0.2$ oe<br>$n = (\frac{2 \times 0.17 \times 2.576}{0.2})^2$ oe (= 19.2)<br>Smallest $n$ is 20 | B1<br><br>M1<br><br>M1<br><br>A1 [4] | Seen (accept 2.574 to 2.579)<br><br>Allow without '2 ×' OR with incorrect $z$<br><br>Attempt to arrange equ of correct form (with correct z and '2×' into the form n= or $\sqrt{n}$ = |
| **3 (i)** | est ($\mu$) = 2866 or 2870 (3 s.f.)<br>est ($\sigma^2$) = $\frac{1}{49}(410900000 - \frac{143300^2}{50})$<br>(= 4126.53)<br>= 4130 (3 sf) | B1<br><br>M1<br><br>A1 [3] | Accept 143300/50 o.e.<br><br>Correct subst in correct formula |
| **(ii)** | $H_0$: Pop mean (or $\mu$) = 2850<br>$H_1$: Pop mean (or $\mu$) ≠ 2850<br>$\frac{\frac{143300}{50} - 2850}{\frac{\sqrt{'4126.53'}}{\sqrt{50}}}$<br><br>= 1.761<br>'1.761' < 1.96<br>No evidence mean distance changed | B1<br><br>M1<br><br><br>A1<br>M1<br>A1f<br><br>[5] | Both. Not just 'mean'<br><br>Allow '4126.53' without $\sqrt{}$, but must have all $\sqrt{50}$<br><br>Or correct c.v. (2867.81) for alt method<br>For valid comparison of z values, areas or c.v.<br>Dep 1.96; ft their 1.761<br>If $H_1$: $\mu > 2850$ and c.f. 1.645,<br>max B0M1A1M1A0<br>(c.v. for 1 tail test 2864.94) |
| **4 (i)** | $\lambda = 2.8$<br>$e^{-2.8}(1 + 2.8 + \frac{2.8^2}{2})$<br>= 0.469 (3 s.f.) or 0.47(0) | B1<br><br>M1<br><br>A1 [3] | seen<br><br>any $\lambda$ allow one end error<br><br>As final answer |
| **(ii)** | $e^{-0.7n} \geqslant 0.99$ or $e^{-\lambda} \geqslant 0.99$<br>$-0.7n \geqslant \ln 0.99$ or $-\lambda \geqslant \ln 0.99$<br>$n \leqslant 0.01436$ or $\lambda \leqslant 0.01005$<br>'0.01436' × 150<br>or '0.01005' × 150 ÷ 0.7<br>Max period is 2.15 mins (3 sf) | M1<br>M1<br>A1<br><br>M1<br>A1 [5] | Allow '=' throughout<br>Attempt ln both sides<br>Can be implied. Accept 3 s.f.<br><br>Note $e^{-(0.7/150)n} \geqslant 0.99$ scores 1st and 3rd M1<br>T & I leading to ans 2.2 mins, SC: B2 |

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| | | | |
|---|---|---|---|
| **5 (i)** | $\int_0^2 k(x-2)^2\,dx = 1$<br>$\left(\left[\frac{k(x-2)^3}{3}\right]_0^2 = 1\right)$<br>$k\,[0-(-\frac{8}{3})] = 1$<br>$k = \frac{3}{8}$ **AG** | M1<br><br><br>A1<br><br>[2] | Attempt to integrate f($x$) with correct limits and = 1<br><br><br>Must see this line or better, e.g. $k \times \frac{8}{3} = 1$ |
| **(ii)** | $\frac{3}{8}\int_d^2 (x-2)^2\,dx = 0.2$<br>$\left(\frac{3}{8}\left[\frac{(x-2)^3}{3}\right]_d^2 = 0.2\right)$<br>$\frac{3}{8}\left[0-\frac{(d-2)^3}{3}\right] = 0.2$ oe<br>$((d-2)^3 = -1.6)$<br>$d = 0.83(0)$ (3 s.f.) | M1<br><br><br>M1<br><br><br>A1 [3] | $\int f(x)dx$ with limits d and 2 or 0 and d, and = 0.2 or =0.8<br>Condone missing 'k'<br><br><br>Reasonable attempt to integrate from a correct expression, with limits substituted to give expression in $d^3$.<br>Condone missing 'k' |
| **(iii)** | $\frac{3}{8}\int_0^2 x(x-2)^2\,dx$<br>$(= \frac{3}{8}\int_0^2 x^3 - 4x^2 + 4x\,dx)$<br>$= \frac{3}{8}\left[\frac{x^4}{4} - \frac{4x^3}{3} + 2x^2\right]_0^2$<br>$= \frac{1}{2}$ | M1<br><br><br><br>A1<br><br>A1 [3] | Attempt integ $xf(x)$; ignore limits, condone missing k<br>$(\frac{3}{8}\left[x \times \frac{(x-2)^3}{3} - \int \frac{(x-2)^3}{3}\,dx\right]_0^2)$<br>$= \frac{3}{8}\left[x \times \frac{(x-2)^3}{3} - \frac{(x-2)^4}{12}\right]_0^2$<br>Correct integration & limits, condone missing k |
| **6 (i)** | P(Type I) = 1 - P($\geq$ 4 assuming $p$ = 0.7)<br>$1-({}^6C_4 \times 0.7^4 \times 0.3^2 + {}^6C_5 \times 0.7^5 \times 0.3 + 0.7^6)$<br>$(= 1 - 0.744)$<br>= 0.256 (3 s.f.) | M1<br><br>M1<br><br>A1 [3] | or P($\leq$ 3 assuming $p$ = 0.7) May be implied<br${}^6C_3 \times 0.7^3 \times 0.3^3 + {}^6C_2 \times 0.7^2 \times 0.3^4 + {}^6C_1 \times 0.7 \times 0.3^5 + 0.3^6$<br>Allow one end error<br>= 0.256 (3 s.f.)<br>SR if zero scored allow B1 for use of B(6, 0.7) in any two or more terms |
| **(ii)** | P(Type II) = P($\geq$ 4 assuming $p$ = 0.35)<br>$= {}^6C_4 \times 0.35^4 \times 0.65^2 + {}^6C_5 \times 0.35^5 \times 0.65 + 0.35^6$<br>= 0.117 | M1<br><br>M1<br>A1<br>[3] | May be implied<br>Allow one end error<br><br>SR if zero scored allow B1 for use of B(6, 0.35) in any two or more terms |
| **(iii)** | Type 1<br>They will reject Luigi's belief, although it might be true. | B1<br><br>B1 [2] | <br><br>In context |

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|---|---|---|---|
| **7 (i)** | N(10.61, 0.1017)<br>$\frac{11-'10.61'}{\sqrt{'0.1017'}}$ (= 1.223)<br>Φ('1.223')<br>= 0.889 (3 s.f.) | B1<br><br>M1<br>M1<br><br>A1 [4] | o.e. Stated or implied (accept in un-simplified form)<br><br>Allow without √<br>For attempt to find correct area consistent with their working |
| **(ii)** | P($K - 1.2A > 0$ )<br>Var = $0.0576 + 1.2^2 \times 0.0441$<br>(= 0.121104)<br>N(–0.324, 0.121104)<br>$\frac{0-(-0.324)}{\sqrt{'0.121104'}}$ (= 0.931)<br>1 – Φ('0.931')<br>= 0.176 (3 s.f.) | M1<br><br><br>B1 B1<br><br>M1<br>M1<br><br>A1 [6] | Or similar stated or implied<br><br><br>o.e. May be implied (accept in un-simplified form)<br><br>Allow without √<br>For attempt to find correct area consistent with their working |

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**CAMBRIDGE INTERNATIONAL EXAMINATIONS**

**GCE Advanced Level**

# MARK SCHEME for the October/November 2013 series

## 9709 MATHEMATICS

**9709/73** Paper 7, maximum raw mark 50

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**Penalties**

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| | | | | |
|---|---|---|---|---|
| **1** | Est($\mu$) = 1.8775 or 1.88 (3 sf) | **B1** | | Accept 751/400 ( not 150.2/80 ) |
| | Est($\sigma^2$) = $\frac{80}{79}\left(\frac{820.24}{80} - "1.8775"^2\right)$ | **M1** | | Correct subt'n in correct formula<br>1/79 ( 820.24 – 150.2$^2$/80 ) |
| | = 6.81316 or 6.81 (3 sf) | **A1** | | |
| | $z$ = 1.96 | **B1** | | Seen |
| | "1.8775" $\pm z \times \sqrt{\frac{"6.81316"}{80}}$ | **M1** | | |
| | = 1.31 to 2.45 (3 sf) | **A1** | **6** | Must be an interval.<br>NB use of biased var can still score A1. |
| **Total** | | | **[6]** | |
| **2** **(i)** | Assume sd unchanged or<br>sd = 10.4 | **B1** | | Oe e.g. var unchanged |
| | $H_0$: Pop mean speed (or $\mu$) = 62.3<br>$H_1$: Pop mean speed (or $\mu$) < 62.3 | **B1** | | Both. Not just "Mean . . " |
| | $\frac{59.9-62.3}{\frac{10.4}{\sqrt{75}}}$ | **M1** | | Accept sd/var mixes, but must have √ 75 |
| | = – 1.999 or – 2.00 ( allow + or – ) | **A1** | | Correct $z$ value ( or correct critical value ) |
| | Compare – 2.054 or –2.055 | **M1** | | Valid comparison of $z$`s/areas/critical values |
| | No evidence that mean speed decreased | **A1 ft** | **6** | No contradictions. Do not ft 2-tail test. |
| **(ii)** | Pop distribution unknown | **B1** | | |
| | Yes | **B1** | **2** | |
| **Total** | | | **[8]** | |
| **3** **(i)** | $\int_0^{10} \frac{1}{2500}\left(100t^3 - t^5\right)\mathrm{d}t$ | **M1** | | Attempt integ $t^2$f($t$) |
| | $(= \frac{1}{2500}\left[25t^4 - \frac{t^6}{6}\right]_0^{10} = \frac{100}{3})$ | | | |
| | "$\frac{100}{3}$" – $(\frac{16}{3})^2$ | **M1** | | For E ( T$^2$ ) – ( E ( T ) )$^2$ |
| | = $\frac{44}{9}$ or 4.89 (3 sf) | **A1** | **3** | |
| **(ii)** | $\int_n^{10} \frac{1}{2500}\left(100t - t^3\right)\mathrm{d}t$ | **M1** | | Attempt integ f($t$), ignore limits |
| | $\frac{1}{2500}\left[50t^2 - \frac{t_4}{4}\right] = 0.1$ | **M1** | | Attempt integ f($t$), limits $n$ to 10 or 0 to $n$<br>Equated to 0.1 or 0.9. Not need to be matched |
| | $\frac{1}{2500}\left[2500 - \left(50n^2 - \frac{n^4}{4}\right)\right] = 0.1$ | **M1** | | 0.1/0.9 matched to correct limits and used |
| | ($n^4 - 200n^2 + 9000 = 0$) | **M1** | | Correct method of solution of a QE in n$^2$ |
| | ($n^2 = 68.3772$, $n = 8.27$)<br>$n$ = 8 | **A1** | **5** | Must be single ans only |
| **Total** | | | **[8]** | |

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| | | | | | |
|---|---|---|---|---|---|
| 4 | (i) (a) | $e^{-2.1} \times \frac{2.1^3}{3!}$ alone | M1 | | Allow any $\lambda$. Allow sum of 3 or 4 rel products, e.g. P ( 3, 0) |
| | | = 0.189 | A1 | 2 | |
| | (b) | $e^{-1.2} \times \frac{1.2^3}{3!} \times e^{-0.9}$ | M1 | | P (Fem = 3) × P (Opp = 0) or P (Fem = 2) × P (Opp = 1) |
| | | $+ e^{-1.2} \times \frac{1.2^2}{2!} \times e^{-0.9} \times 0.9$ | M1 | | P ( 3, 0 ) + P ( 2, 1 ) |
| | | = 0.115 | A1 | 3 | As final answer |
| | (ii) | N (30, 30) | B1 | | seen or implied |
| | | $\frac{34.5-30}{\sqrt{30}}$ (= 0.8216) | M1 | | standardising with their N ( $\lambda$, $\lambda$ ) |
| | | $1 - \Phi$("0.822") | M1 | | Allow with no or incorrect cc or no √ |
| | | = 0.206 ( 3sf) | A1 | 4 | Area consistent with their working |
| | Total | | | | |
| 5 | (i) | E ($X$)= 3.5 | B1 | | 21/6 |
| | | $(1^2+ 2^2 + 3^2 + 4^2 + 5^2 + 6^2) \div 6 - $ "3.5"$^2$ | B1 | 2 | oe, must see correct expression and no incorrect working |
| | | $(=\frac{35}{12}$ **AG**) | | | |
| | (ii) | Attempt P ($X < 3$) or 1 – P($X \geq 3$) | M1 | | seen or implied |
| | | N (3.5, $\frac{35}{12}/50$) | M1 | | seen or implied |
| | | $\frac{3-"3.5"}{\sqrt{\frac{35}{12}/50}}$ (= –2.070) | M1 | | or $\frac{2.99-"3.5"}{\sqrt{\frac{35}{12}/50}}$ (= –2.111) |
| | | $\Phi$ ("–2.070") = 1 – $\Phi$("2.070") | | | $\Phi$ ('–2.111') = 1 – $\Phi$ ('2.111') |
| | | = 0.0192 | M1 | | = 0.0174 or 0.0173 |
| | | as final answer | A1 | | Consistent area<br>As final answer<br>or valid total method |
| | | | | 5 | Allow with incorrect cc (e.g. 2.5) OR no √.Must have ÷50 |
| | (iii) | Die is biased (towards lower numbers) | B1 indep | | Comment implying die is biased |
| | | Mean of 50 throws $\geq$ 3 (Allow > 3)<br>or Equal nos of high and low scores<br>or More high scores | B1 indep | 2 | Comment implying results of exp't do not indicate bias<br>(or indicate bias towards higher numbers)<br><br>Both must be in context |
| | Total | | | [9] | |

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| | | | | | |
|---|---|---|---|---|---|
| **6** | **(i)** | N(5100, $5\times45^2$) or N(5100, 10125) | **B1** | | seen or implied |
| | | $\frac{5200-"5100"}{\sqrt{"10125"}}$ (= 0.994) | **M1** | | standardising with their new mean and new var<br>area consistent with their working with normal |
| | | $\Phi$ ("0.994") | **M1** | | |
| | | = 0.840 (3 sf) | **A1** | **4** | |
| | **(ii)** | Use of $E - 3L$ or similar | **M1** | | |
| | | E ($E - 3L$) = –260 | **B1** | | 2800 – 3 x 1020 |
| | | Var ($E - 3L$) = $52^2 + 9 \times 45^2$ or 20929 | **B1** | | |
| | | $\frac{0-("-260")}{\sqrt{"20929"}}$ (= 1.797) | **M1** | | with a pos var with $45^2$ and $52^2$ combined |
| | | 1 – Φ ("1.797") | **M1** | | consistent area, must clearly be φ |
| | | = 0.0361 (3 sf) or 0.0362 | **A1** | **6** | |
| | | | | | P ($3L - E < 0$): similar scheme<br>SR: use of 3E – L, M1, 7380 B1, 26361 B1<br>stand 0 with these values M1, M0A0 max 4/6 |
| **Total** | | | | **[10]** | |

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**CAMBRIDGE INTERNATIONAL EXAMINATIONS**

**GCE Advanced Subsidiary Level and GCE Advanced Level**

# MARK SCHEME for the May/June 2013 series

## 9709 MATHEMATICS

**9709/71** Paper 7, maximum raw mark 50

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A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

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AEF Any Equivalent Form (of answer is equally acceptable)

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Note: "(3 sfs)" means "answer which rounds to ... to 3 sfs". If correct ans seen to ≥ 3sfs, ISW for later rounding. Penalise < 3 sfs only once in paper.

| | | | | |
|---|---|---|---|---|
| **1** | **(i)** | One of each is more likely<br>P(one of each = 0.5), P(HH) = 0.25 | B1<br>B1<br>[2] | or P(TT) = 0.25 |
| | **(ii)** | Choose Charlie only if H then T<br>Throw again if T then H | B1<br>B1<br>[2] | or similar e.g. HH for A, HT for B, TT for C<br>or vice versa |
| **2** | | $H_0$: Pop mean = 17<br>$H_1$: Pop mean ≠ 17<br><br>$\frac{18.2-17}{\frac{2.4}{\sqrt{5}}}$<br>= 1.12 (3 sf)<br><br>'1.12' < 1.96 oe<br><br>Claim can be accepted | <br>B1<br><br>M1<br><br>A1<br><br>M1<br><br>A1ft<br><br>[5] | Both correct. Allow $\mu$, but not just "mean"<br><br>Allow incorrect 18.2. Must have √5<br><br><br><br>Comp '1.12' with 1.96 or area '0.132' with 0.025<br><br>ft their '1.12'<br><br>If $H_1$: $\mu > 17$ and cf 1.645: can score max B0M1A1M1A1ft |
| | | | | $17 \pm 1.96 \frac{2.4}{\sqrt{5}}$ M1<br><br>= (14.9, 19.1) A1<br><br>'14.9'<18.2<'19.1' M1 |
| **3** | | Var(total) = $6(3.2^2 + 2.6^2)$ (+ 0)) (= 102)<br>Total ~ N(1528, 102))<br><br>$\frac{1550-"1528"}{\sqrt{"102"}}$ (= 2.178)<br>1 – Φ("2.178")<br>= 0.0147 (3 sf) | <br>B1 B1<br><br>M1<br><br>M1<br>A1<br>[5] | For mean (1528)oe and for variance (102)<br>May be implied by use of N(1528, $10.1^2$)<br><br>For standardising. No SD/Var mix<br><br>For correct area consistent with working |
| **4** | **(i)** | est($\mu$) = 2005/200 = (10.025)<br>est($\sigma^2$) = $\frac{1}{99}(20175 - \frac{2005^2}{200})$<br>= 0.376 (3 sf) | B1<br><br>M1<br><br>A1<br>[3] | <br><br>Correct subst in correct formula |
| | **(ii)** | $\frac{10-"10.025"}{\sqrt{\frac{"0.376256"}{50}}}$ (= –0.288)<br>1 – Φ('0.288')<br>= 0.387 (3 sf) | <br>M1<br><br>M1<br>A1<br>[3] | <br>Allow without √, but ÷√50 essential<br><br>(Use of 'biased' variance can still score fully in (ii) ) |

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| | | | | |
|---|---|---|---|---|
| | **(iii)** | Yes; (assumed distr of $\overline{X}$ normal)<br>although distr of $X$ unknown | B1<br>B1<br>[2] | |
| **5** | **(i)** | B(520, 0.008)<br>Po(4.16)<br>$n = 500$ which is large,<br>$np = 4.16$ which is $< 5$ or p small $<0.1$ | B1<br>B1B1<br><br>B1<br>[4] | Po: B1, $\lambda = 4.16$: B1<br><br>Both needed |
| | **(ii)** | **(a)** $1 - e^{-4.16}(1 + 4.16 + \frac{4.16^2}{2} + \frac{4.16^3}{3!})$<br>$= 0.597$ (3 sf) | <br><br>M1<br><br>A1<br>[2] | <br><br>1–P(0,1,2,3) any $\lambda$ allow one end error |
| | | **(b)** $e^{-4.16} \times \frac{4.16^n}{n!} > e^{-4.16} \times \frac{4.16^{n+1}}{(n+1)!}$<br>$1 > \frac{4.16}{n+1}$<br>$n > 3.16$<br>Smallest $n$ is 4 | M1<br><br>A1<br><br>A1<br>[3] | any $\lambda$<br><br>or equiv equn without e and without factorials<br><br>(Calculation of P(0), P(1),....P(5) scores M1for at least 3 attempted, A1 all correct, A1 for n = 4) |
| **6** | **(i)** | $\frac{1}{2}\int_4^t \frac{1}{\sqrt{t}}dt = 0.9$ or $\frac{1}{2}\int_t^9 \frac{1}{\sqrt{t}}dt = 0.1$<br>$\left[\sqrt{t}\right]_4^t = 0.9$ or $\left[\sqrt{t}\right]_t^9 = 0.1$<br>$((\sqrt{t} - 2) = 0.9$ or $(3 - \sqrt{t}) = 0.1)$<br>$t = 8.41$ (mins) (3 sf) | M1<br><br>A1<br><br>A1<br>[3] | Attempt integ f($t$) with unknown limit and 0.9/0.1.<br><br>Correct integration & limits = 0.9 or 0.1. |
| | **(ii)** | $\frac{1}{2}\int_4^9 \frac{t}{\sqrt{t}}dt$ oe<br>$\frac{1}{2}\left[\frac{t1.5}{1.5}\right]_4^9$ oe<br>$= \frac{19}{3}$<br>$\frac{1}{2}\int_4^9 \frac{t2}{\sqrt{t}}dt$ oe<br>$(= \frac{1}{2}\left[\frac{t2.5}{2.5}\right]_4^9 = \frac{211}{5})$<br>$= \frac{'211'}{5} - (\frac{'19'}{3})^2$<br>$= \frac{94}{45}$ or 2.09 (3 sf) | <br>M1<br>A1<br><br>A1<br><br>M1<br><br><br>M1<br><br>A1<br>[6] | <br>Attempt integ $tf(t)$. Ignore limits<br>Correct integration & limits<br><br><br>Attempt integ $t^2f(t)$. Ignore limits<br><br><br><br>integ $t^2f(t) - (\text{integ } tf(t))^2$ attempted |

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| | | | | |
|---|---|---|---|---|
| 7 | (i) | Conclude die is biased when it isn't oe<br>$^5C_3\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^2 + 5\left(\frac{1}{6}\right)^4\left(\frac{5}{6}\right) + \left(\frac{1}{6}\right)^5 + 5$<br>$= \frac{23}{648}$ or 0.0355 (3 sf) | B1<br>M1<br>A1 [3] | In context<br>or $1 - \left( {}^5C_2\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)3 + 5\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)^4 + \left(\frac{5}{6}\right)^5 \right)$<br>allow 1 end error |
| | (ii) | State or attempt P(0, 1, 2) with $p = \frac{2}{3}$<br>$^5C_2\left(\frac{2}{3}\right)^2\left(\frac{1}{3}\right)^3 + 5\left(\frac{2}{3}\right)\left(\frac{1}{3}\right)^4 + \left(\frac{1}{3}\right)^5$<br>$= \frac{17}{81}$ or 0.210 (3 sf) | M1<br>M1<br>A1 [3] | Or 1– P(3,4,5)<br>Attempt at correct expression<br>Allow 0.21 |
| | (iii) | Est Var($P_s$) = $\frac{0.625 \times (1 - 0.625)}{80}$<br>$(= \frac{3}{1024})$<br>$z = 2.054$ (or 2.055)<br>$0.625 \pm z \times \sqrt{\frac{'3'}{1024}}$<br>= 0.514 to 0.736 (3 sf) | M1<br>B1<br>M1<br>A1 [4] | Any $z$ |

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| | | | | |
|---|---|---|---|---|
| **1** | **(i)** | Binomial | B1 | |
| | | $n = 400, \quad p = 0.012$ | B1 [2] | Both. Not $p = 1.2\%$<br>Or B(400, 0.012): B1B1 |
| | **(ii)** | Poisson | B1 | |
| | | $n$ large and mean = 4.8, which is < 5 | B1 [2] | n large, p small |
| | **(iii)** | $1 - e^{-4.8}(1 + 4.8 + \frac{4.8^2}{2})$ | M1 | P($X$ = 0, 1, 2); allow any λ; allow one end error |
| | | = 0.857/0.858 | A1 [2] | (Normal/Binomial in (ii) can score M1 only) |
| | | | **[Total: 6]** | |
| **2** | **(i)** | $\frac{2}{3}\int_1^2 x^2 \mathrm{d}x$ | M1 | Attempt integ. $xf(x)$; ignore limits |
| | | $= \frac{2}{3}\left[\frac{x^3}{3}\right]_1^2$ | A1 | Correct integration and limits |
| | | $= \frac{14}{9}$ or 1.56 o.e. | A1 [3] | |
| | **(ii)** | $\frac{2}{3}\int_1^{\frac{14}{9}} x\,dx$ | M1 | Attempt integ. f($x$); with limits |
| | | $(= \frac{2}{3}\left[\frac{x^3}{3}\right]_1^2)$ | | |
| | | $= \frac{115}{243}$ or 0.473 (3 s.f.) | A1 [2] | |
| | **(iii)** | $\frac{115}{243} < \frac{1}{2}$ o.e. | M1 | Comparison of prob. or values |
| | | Hence mean < median | A1ft[2] | ft (i) or (ii) |
| | | | **[Total: 7]** | |

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| | | | | |
|---|---|---|---|---|
| **3** | **(i)** | $\frac{73.1-75.2}{\frac{5.7}{\sqrt{n}}} = -1.563$ | M1 | For standardising (with $\sqrt{n}$ ) |
| | | $n = \{-1.563\times5.7\div(-2.1)\}^2$ | A1 | Any correct expression for $n$ or $\sqrt{n}$. May be implied by ans. |
| | | $n = 18$ | A1 | |
| | | Assume s.d. for the region is 5.7 | B1 [4] | |
| | **(ii)** | $H_0$: pop mean (or $\mu$) = 75.2 | | |
| | | $H_0$: pop mean (or $\mu$) < 75.2 | B1 | Both (could be stated in (i)) |
| | | 1.563 comp 1.555 | M1 | For comparison of z values / areas / x values |
| | | Evidence that plants shorter | A1 [3] | CWO. No contradictions |
| | | | **[Total: 7]** | |
| **4** | **(i)** | est($\mu$) = 9750/150 = (65) | B1 | |
| | | est($\sigma^2$) = $\frac{1}{149}(647500 - \frac{9750^2}{150})$ | M1 | Correct subst. in correct formula |
| | | = 92.3 (3 s.f.) | A1 [3] | |
| | **(ii)** | $z = 2.326$ | B1 | |
| | | '65' $\pm z\times\frac{\sqrt{'92.28188'}}{\sqrt{150}}$ | M1 | Any $z$ |
| | | = 63.2 to 66.8 (3 s.f.) | A1 [3] | (Use of 'biased' can still score here) |
| | **(iii)** | $0.02^2$ | M1 | Allow M1 for 0.02 seen |
| | | = 0.0004 o.e. | A1 [2] | |
| | | | **[Total: 8]** | |

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|---|---|---|---|---|
| **5** | **(i)** | $6 \times 510 + 70 = (3130)$<br>$6 \times 12^2 + 4^2 = (880)$<br>N(3130, 880) | B1<br>B1 | |
| | | $\frac{3050-3130}{\sqrt{880}}$ $(= -2.697)$<br>$\frac{3150-3130}{\sqrt{880}}$ $(= 0.674)$ | M1 | Both. With their mean and variance($\geq$0) Allow without √ |
| | | $\Phi(\text{'0.674'}) - (1 - \Phi(\text{'2.697'}))$<br>$(= 0.7499 - 0.0035)$ | M1 | Use of tables and attempt to find area consistent with their working |
| | | $= 0.746$ (3 sf) | A1 [5] | |
| | **(ii)** | $510 - 8 \times 70 = (-50)$<br>$12^2 + 8^2 \times 4^2 = (1168)$<br>$P - 8C \sim N(-50, 1168)$ | B1<br>B1 | o.e. +50; 510/8 -70; – (510/8 – 70)<br>o.e. $(12/8)^2 + 4^2$ |
| | | $\frac{0-(-50)}{\sqrt{1168}}$ $(= 1.463)$ | M1 | For standardising with attempt "P-8C" oe with their mean and variance($\geq$0).Allow without √ |
| | | $1 - \Phi(\text{'1.463'})$ | M1 | Use of tables and attempt to find area consistent with their working |
| | | $= 0.0717$ (3 s.f.) | A1 [5] | |
| | | | **[Total: 10]** | |

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|---|---|---|---|---|
| 6 | (i) | $H_0$: Pop mean (or $\lambda$ or $\mu$) is 5.3<br>$H_1$: Pop mean (or $\lambda$ or $\mu$) is less than 5.3 | B1 | Both |
| | | $P(X \leq 1) = e^{-5.3}(1 + 5.3)$<br>$P(X \leq 2) = e^{-5.3}(1 + 5.3 + \frac{5.3^2}{2})$ /P(X=2) | M1 | Both attempted |
| | | $P(X \leq 1) = 0.0314$ or 0.0315<br>& $P(X \leq 2) = 0.102$/ P(X=2)=0.7071 | A1 | Both correct |
| | | CR is 0 or 1 cases | A1 | Dep. M1 and $P(X \leq 1) < 0.05 < P(X \leq 2)$ |
| | | No evidence mean has decreased | B1f [5] | ft their CR |
| | (ii) | Concluding mean has decreased when it hasn't | B1 | In context |
| | | '0.0314 or 0.0315' | B1ft[2] | ft their $P(X \leq 1)$, dep. < 0.05 |
| | (iii) | (Po(18.4) )<br>N(18.4, 18.4) | B1<br>B1ft | Stated or implied<br>B1 for N(18.4, ..); B1f for var. = 18.4 |
| | | $\frac{20.5-18.4}{\sqrt{18.4}}$ (= 0.490) | M1 | For standardising with or without cc.Allow without √ |
| | | 1 – Φ('0.490') | M1 | Use of tables and attempt to find area consistent with their working |
| | | = 0.312 (3 s.f.) | A1 [5] | |
| | | | **[Total: 12]** | |

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**CAMBRIDGE INTERNATIONAL EXAMINATIONS**

**GCE Advanced Subsidiary Level and GCE Advanced Level**

# MARK SCHEME for the May/June 2013 series

## 9709 MATHEMATICS

**9709/73** Paper 7, maximum raw mark 50

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the May/June 2013 series for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level components and some Ordinary Level components.

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**Mark Scheme Notes**

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

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The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

**Penalties**

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

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| | | | | |
|---|---|---|---|---|
| **1** | **(i)** | 9.3 | B1 1 | |
| | **(ii)** | 27.9 | B1 1 | |
| | **(iii)** | E ($S$) = 17.4, E($T$) = 19.4<br>E ($S - T$) = – 2.0,<br>Var ($S - T$) = 37.2 | M1<br>A1<br>B1ft 3 | For subtracting their E[S] – E[T] can be non-numerical<br>ft (i) & (ii) Adding (i) and (ii) ft non-negative answers only |
| | | | **[Total: 5]** | |
| **2** | | Assume shots independent OR prob of scoring constant<br><br>$H_0$: P(score) = 0.82<br>$H_1$: P(score) > 0.82<br><br>$20 \times 0.82^{19} \times 0.18 + 0.82^{20}$<br>= 0.102 (3 sf)<br>No evidence that improved | B1<br><br>B1<br><br>M1<br>A1<br>B1f 5 | In context<br><br>Both. Allow '$p$'<br><br>For use of Bin(20,0.82)and either P(19) and/or P(20) attempted<br>Valid comparison seen (with 0.05 if $H_1$ $p \neq 0.82$) and correct conclusion ft numerical errors in 0.102 only<br><br>Normal approx'n: B1 B1 ($\mu$= 16.4 acceptable here) if earned, then:<br>CR = 1.222 (from $\frac{18.5 - 20 \times 0.82}{\sqrt{20 \times 0.82 \times (1 - 0.82)}}$, need cc)<br>comp $z$ = 1.282<br>No evidence that improved SC 1<br>Same scheme for proportions |
| | | | **[Total: 5]** | |
| **3** | **(i)** | $\bar{x}$ = 930/15 =(62)<br>$z$ = 1.751<br>'62' $\pm z \times \frac{12}{\sqrt{15}}$<br>= 56.6 to 67.4 (3 sf) | B1<br>B1<br><br>M1<br><br>A1 4 | <br><br>Any $z$<br><br>Must be an interval |
| | **(ii)** | 92 % of such intervals will contain $\mu$ | B1 1 | Accept P(This interval contains $\mu$) = 0.92 |
| | **(iii)** | Each possible sample of this size is equally likely | B1 1 | Each member of pop equally likely to be chosen |
| | | | **[Total: 6]** | |

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|---|---|---|---|---|
| **4** | **(i)** | $e^{-2} \times 2\ (\times)\ e^{-3} \times \frac{3^4}{4!}$ | M1 | Correct exp'n for P(1) with $\lambda=2$ OR P(4) with $\lambda=3$ |
| | | $e^{-5} \times \frac{5^4}{5!}$ | B1 | Correct exp'n |
| | | $\div$ | M1 | dep M1B1 |
| | | $\frac{162}{625}$ or 0.259 (3 sf) | A1 4 | |
| | **(ii)** | $(e^{-2} \times \frac{2^r}{r!} = \frac{2}{3}e^{-2} \Rightarrow)$ | | |
| | | $3 \times 2^r = 2 \times r!$ OR $2^{r-1} = \frac{1}{3} \times r!$ | B1 | Legitimately shown |
| | | $(\Rightarrow 3 \times 2^{r-1} = r!)$ | | |
| | | $3 \times 2^3 = 24$ OR $3! = 24$ seen | B1 2 | Legitimately shown on either equation |
| | | | **[Total: 6]** | |
| **5** | **(i)** | $\int_1^\infty \frac{k}{x^3} dx = 1$ | M1 | All correct, including limits and an attempt to integrate |
| | | $\left[-\frac{k}{2x^2}\right]_1^\infty = 1$ | | |
| | | $0 - (-\frac{k}{2}) = 1$ | A1 2 | or $0 + \frac{k}{2} = 1$ or $\frac{k}{2} = 1$ AG must be convincing |
| | **(ii)** | $\int_1^2 \frac{2}{x^3} dx$ | M1 | Attempt integ f($x$); ignore limits |
| | | $= \left[-\frac{1}{x^2}\right]_1^2$ | | |
| | | $= \frac{3}{4}$ | A1 2 | |
| | **(iii)** | $\int_1^\infty \frac{2}{x^2} dx$ | M1 | Attempt integ $xf(x)$; ignore limits |
| | | $= \left[-\frac{2}{x}\right]_1^\infty$ | A1 | Correct & correct limits |
| | | $= 2$ | A1 3 | |
| | | | **[Total: 7]** | |

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|---|---|---|---|---|
| **6** | **(i)** | $\lambda$ (= 1.4 × 2.5) = 3.5 | B1 | |
| | | $1 - e^{-3.5}(1 + 3.5 + \frac{3.5^2}{2} + \frac{3.5^3}{3!})$ | M1 | Any λ allow one end error |
| | | = 0.463 (3 sf) | A1 3 | |
| | **(ii)** | ($\lambda$ = 672 × 1.4 = 940.8) | | |
| | | N(940.8, 940.8) | B1 | Seen or implied |
| | | $\frac{999.5 - 940.8}{\sqrt{940.8}}$ (= 1.914) | M1 | Allow with wrong or no cc . no sd/var mixes |
| | | Φ('1.914') | M1 | |
| | | = 0.972 (3 sf) | A1 4 | |
| | | | **[Total: 7]** | |
| **7** | **(i)** | Assume sd unchanged or 4500 | B1 | |
| | | $H_0$: Pop mean = 34600 | | |
| | | $H_1$: Pop mean > 34600 | B1 | Both. Allow just $\mu$, but not just "mean" |
| | | $\frac{35400 - 34600}{\frac{4500}{\sqrt{90}}}$ | M1 | Allow without √90 |
| | | = 1.687/1.686 (1.69) | A1 | |
| | | cf 1.645 < 1.686 | M1 | Valid comparison ( or 0.0458/0.0459 <0.05 or 35380 < 35400 or 34600 < 34620) |
| | | Evidence that mean wkly profit has increased | A1 f 6 | If $H_1$: ≠, and 1.96 used, max B1B0M1A1M1A1f No contradictions |
| | **(ii)** | Distr'n of *X* unknown. | B1* | Allow not Normal |
| | | Yes | B1* dep 2 | |
| | **(iii)** | 0.05 or 5% | B1 1 | |
| | **(iv)** | $\frac{a - 34600}{\frac{4500}{\sqrt{90}}}$ = 1.645 | M1 | Attempt to find cv must see (+) 1.645 allow without √90. If found in (i) award when used |
| | | $a$ = 35380 | A1 | |
| | | $\frac{35380 - 36500}{\frac{4500}{\sqrt{90}}}$ (= – 2.361) | M1 | |
| | | 1 – Φ('2.361') | M1 | Standardising with their " CV " must use √90 |
| | | = 0.0091 | A1 6 | Correct tail |
| | | | **[Total: 14]** | |

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