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Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

* 5 4 8 9 3 1 4 7 5 9 *

MATHEMATICS **9709/61**

Paper 6 Probability & Statistics 2 **October/November 2024**

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
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- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **12** pages. Any blank pages are indicated.

DC (EV) 347847

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DO NOT WRITE IN THIS MARGIN

1 The heights of a certain species of deer are known to have standard deviation 0.35 m. A zoologist takes a random sample of 150 of these deer and finds that the mean height of the deer in the sample is 1.42 m.

(a) Calculate a 96% confidence interval for the population mean height. [3]

(b) Bubay says that 96% of deer of this species are likely to have heights that are within this confidence interval.

Explain briefly whether Bubay is correct. [1]

2 The masses, in kilograms, of small and large bags of wheat have the independent distributions $N(16.0, 0.4)$ and $N(51.0, 0.9)$ respectively.

Find the probability that the total mass of 3 randomly chosen small bags is greater than the mass of one randomly chosen large bag. [5]

3 The times, T minutes, taken by a random sample of 75 students to complete a test were noted. The results were summarised by $\Sigma t = 230$ and $\Sigma t^2 = 930$.

(a) Calculate unbiased estimates of the population mean and variance of T. [3]

You should now assume that your estimates from part **(a)** are the true values of the population mean and variance of T.

(b) The times taken by another random sample of 75 students were noted, and the sample mean, $\overline{T}$, was found.

Find the value of a such that $P(\overline{T} > a) = 0.234$. [3]

4 A random variable X has probability density function f defined by

$$f(x) = \begin{cases} \dfrac{a}{x^2} - \dfrac{18}{x^3} & 2 \leqslant x \leqslant 3, \\ 0 & \text{otherwise,} \end{cases}$$

where a is a constant.

(a) Show that $a = \frac{27}{2}$. [3]

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(b) Show that $E(X) = \frac{27}{2}\ln\frac{3}{2} - 3$. [3]

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5 The lengths, in centimetres, of worms of a certain kind are normally distributed with mean μ and standard deviation 2.3 . An article in a magazine states that the value of μ is 12.7 . A scientist wishes to test whether this value is correct. He measures the lengths, x cm, of a random sample of 50 worms of this kind and finds that $\sum x = 597.1$. He plans to carry out a test, at the 1% significance level, of whether the true value of μ is different from 12.7 .

(a) State, with a reason, whether he should use a one-tailed or a two-tailed test. [1]

(b) Carry out the test. [5]

6 The numbers of customers arriving at service desks A and B during a 10-minute period have the independent distributions Po(1.8) and Po(2.1) respectively.

(a) Find the probability that during a randomly chosen 15-minute period more than 2 customers will arrive at desk A. [2]

(b) Find the probability that during a randomly chosen 5-minute period the total number of customers arriving at both desks is less than 4. [3]

(c) An inspector waits at desk B. She wants to wait long enough to be 90% certain of seeing at least one customer arrive at the desk.

Find the minimum time for which she should wait, giving your answer correct to the nearest minute. [4]

7 The number of accidents per year on a certain road has the distribution $Po(\lambda)$. In the past the value of λ was 3.3 . Recently, a new speed limit was imposed and the council wishes to test whether the value of λ has decreased. The council notes the total number, X, of accidents during **two** randomly chosen years after the speed limit was introduced and it carries out a test at the 5% significance level.

(a) Calculate the probability of a Type I error. [4]

(b) Given that $X = 2$, carry out the test. [3]

(c) The council decides to carry out another similar test at the 5% significance level using the same hypotheses and two different randomly chosen years.

Given that the true value of λ is 0.6, calculate the probability of a Type II error. [3]

(d) Using $\lambda = 0.6$ and a suitable approximating distribution, find the probability that there will be more than 10 accidents in 30 years. [4]

Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.

Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

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* 2 2 2 8 3 7 6 8 6 6 *

MATHEMATICS **9709/62**

Paper 6 Probability & Statistics 2 **October/November 2024**

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

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INFORMATION

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This document has **12** pages.

DC (LK) 337399/3

1 A random variable X has the distribution $B\left(4\,500\,000, \frac{1}{1\,000\,000}\right)$.

Use a Poisson distribution to calculate an estimate of $P(X \geqslant 4)$. [3]

2 The lengths of a random sample of 50 roads in a certain region were measured. Using the results, a 95% confidence interval for the mean length, in metres, of all roads in this region was found to be $[245, 263]$.

(a) Find the mean length of the 50 roads in the sample. [1]

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(b) Calculate an estimate of the standard deviation of the lengths of roads in this region. [2]

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(c) It is now given that the lengths of roads in this region are normally distributed.

State, with a reason, whether this fact would make any difference to your calculation in part **(b)**. [1]

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3 A factory owner models the number of employees who use the factory canteen on any day by the distribution $B(25, p)$. In the past the value of p was 0.8 . A new menu is introduced in the canteen and the owner wants to test whether the value of p has increased.

On a randomly chosen day he notes that the number of employees who use the canteen is 23.

(a) Use the binomial distribution to carry out the test at the 10% significance level. [5]

(b) Given that there are 30 employees at the factory comment on the suitability of the owner's model. [1]

4 A population is normally distributed with mean 35 and standard deviation 8.1 . A random sample of size 140 is chosen from this population and the sample mean is denoted by $\overline{X}$.

(a) Find $P(\overline{X} > 36)$. [3]

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(b) It is given that $P(\overline{X} < a) = 0.986$. Find the value of a. [3]

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5 A machine puts sweets into bags at random. The numbers of lemon and orange sweets in a bag have the independent distributions Po(3.7) and Po(2.6) respectively.

A bag of sweets is chosen at random.

(a) Find the probability that the number of lemon sweets in the bag is more than 2 but not more than 5. [2]

(b) Find the probability that the total number of lemon and orange sweets in the bag is less than 4. [3]

10 bags of sweets are chosen at random.

(c) Use approximating distributions to find the probability that the total number of lemon sweets in the 10 bags is less than the total number of orange sweets in the 10 bags. [6]

6 The time, X hours, taken by a large number of people to complete a challenge is modelled by the probability density function given by

$$f(x) = \begin{cases} \dfrac{1}{x^2} & a \leqslant x \leqslant b, \\ 0 & \text{otherwise,} \end{cases}$$

where a and b are constants.

(a) State what the constants a and b represent in this context. [1]

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(b) Show that $a = \dfrac{b}{b+1}$. [3]

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It is given that $E(X) = \ln 3$.

(c) Show that $b = 2$ and find the value of a. [4]

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(d) Find the median of X. [3]

7 The heights of one-year-old trees of a certain variety are known to have mean 2.3 m. A scientist believes that, on average, trees of this age and variety in her region are slightly taller than in other places. She plans to carry out a hypothesis test, at the 2% significance level, in order to test her belief.

(a) State the probability that she will make a Type I error. [1]

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She takes a random sample of 100 such trees in her region and measures their heights, h m. Her results are summarised below.

$$n = 100 \qquad \sum h = 238 \qquad \sum h^2 = 580$$

(b) Carry out the test at the 2% significance level. [7]

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(c) The scientist carries out the test correctly, but another scientist claims that she has made a Type II error.

Comment on this claim. [1]

Additional page

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Cambridge Assessment International Education

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CANDIDATE NAME

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* 8 2 7 0 6 7 3 3 1 4 *

MATHEMATICS **9709/63**

Paper 6 Probability & Statistics 2 **October/November 2024**

1 hour 15 minutes

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DC (CE) 336463/5

[Turn over

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(b) Bubay says that 96% of deer of this species are likely to have heights that are within this confidence interval.

Explain briefly whether Bubay is correct. [1]

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You should now assume that your estimates from part **(a)** are the true values of the population mean and variance of T.

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Find the value of a such that $P(\overline{T} > a) = 0.234$. [3]

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$$f(x) = \begin{cases} \frac{a}{x^2} - \frac{18}{x^3} & 2 \leqslant x \leqslant 3, \\ 0 & \text{otherwise,} \end{cases}$$

where a is a constant.

(a) Show that $a = \frac{27}{2}$. [3]

(b) Show that $E(X) = \frac{27}{2}\ln\frac{3}{2} - 3$. [3]

5 The lengths, in centimetres, of worms of a certain kind are normally distributed with mean μ and standard deviation 2.3 . An article in a magazine states that the value of μ is 12.7 . A scientist wishes to test whether this value is correct. He measures the lengths, x cm, of a random sample of 50 worms of this kind and finds that $\sum x = 597.1$. He plans to carry out a test, at the 1% significance level, of whether the true value of μ is different from 12.7 .

(a) State, with a reason, whether he should use a one-tailed or a two-tailed test. [1]

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(a) Find the probability that during a randomly chosen 15-minute period more than 2 customers will arrive at desk A. [2]

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(c) An inspector waits at desk B. She wants to wait long enough to be 90% certain of seeing at least one customer arrive at the desk.

Find the minimum time for which she should wait, giving your answer correct to the nearest minute. [4]

7 The number of accidents per year on a certain road has the distribution Po(λ). In the past the value of λ was 3.3 . Recently, a new speed limit was imposed and the council wishes to test whether the value of λ has decreased. The council notes the total number, X, of accidents during **two** randomly chosen years after the speed limit was introduced and it carries out a test at the 5% significance level.

(a) Calculate the probability of a Type I error. [4]

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(b) Given that $X = 2$, carry out the test. [3]

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(d) Using $\lambda = 0.6$ and a suitable approximating distribution, find the probability that there will be more than 10 accidents in 30 years. [4]

Additional page

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Cambridge International AS & A Level

CANDIDATE NAME

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7805754427

MATHEMATICS **9709/61**

Paper 6 Probability & Statistics 2 **May/June 2024**

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
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INFORMATION

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This document has **16** pages. Any blank pages are indicated.

DC (DE/JG) 330130/3

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1 A bus station has exactly four entrances. In the morning the numbers of passengers arriving at these entrances during a 10-second period have the independent distributions Po(0.4), Po(0.1), Po(0.2) and Po(0.5).

Find the probability that the total number of passengers arriving at the four entrances to the bus station during a randomly chosen 1-minute period in the morning is more than 3. [3]

2 The random variable X has the distribution $N(31.2, 10.4^2)$. Two independent random values of X, denoted by X_1 and X_2, are chosen.

Find $P(X_1 > 3X_2)$. [5]

3 The time taken in minutes for a certain daily train journey has a normal distribution with standard deviation 5.8 . For a random sample of 20 days the journey times were noted and the mean journey time was found to be 81.5 minutes.

(a) Calculate a 98% confidence interval for the population mean journey time. [3]

A student was asked for the meaning of this confidence interval. The student replied as follows.

'The times for 98% of these journeys are likely to be within the confidence interval.'

(b) Explain briefly whether this statement is true or not. [1]

Two independent 98% confidence intervals are found.

(c) Given that at least one of these intervals contains the population mean, find the probability that both intervals contain the population mean. [2]

4 **(a)** A random sample of 8 boxes of cereal from a certain supplier was taken. Each box was weighed and the masses in grams were as follows.

261 249 259 252 255 256 258 254

Find unbiased estimates of the population mean and variance. [3]

(b) The supplier claims that the mean mass of boxes of cereal is 253 g. A quality control officer suspects that the mean mass is actually more than 253 g. In order to test this claim, he weighs a random sample of 100 boxes of cereal and finds that the total mass is 25 360 g.

(i) Given that the population standard deviation of the masses is 3.5 g, test at the 5% significance level whether the population mean mass is more than 253 g. [5]

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An employee says, 'This test is invalid because it uses the normal distribution, but we do not know whether the masses of the boxes are normally distributed.'

(ii) Explain briefly whether this statement is true or not. [1]

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5 Sales of cell phones at a certain shop occur singly, randomly and independently.

(a) State one further condition that must be satisfied for the number of sales in a certain time period to be well modelled by a Poisson distribution. [1]

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The average number of sales per hour is 1.2 .

Assume now that a Poisson distribution is a suitable model.

(b) Find the probability that the number of sales during a randomly chosen 12-hour period will be more than 12 and less than 16. [3]

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(c) Use a suitable approximating distribution to find the probability that the number of sales during a randomly chosen 1-month period (140 hours) will be less than 150. [4]

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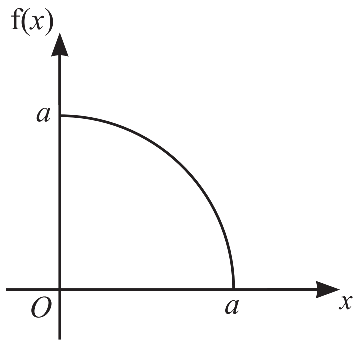

The diagram shows the graph of the probability density function, f, of a random variable X. The graph is a quarter circle entirely in the first quadrant with centre $(0, 0)$ and radius a, where a is a positive constant. Elsewhere $f(x) = 0$.

(a) Show that $a = \frac{2}{\sqrt{\pi}}$. [2]

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(b) Show that $f(x) = \sqrt{\frac{4}{\pi} - x^2}$. [2]

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(c) Show that $E(X) = \frac{8}{3\sqrt{\pi^3}}$. [4]

7 Every July, as part of a research project, Rita collects data about sightings of a particular kind of bird. Each day in July she notes whether she sees this kind of bird or not, and she records the number X of days on which she sees it. She models the distribution of X by $B(31, p)$, where p is the probability of seeing this kind of bird on a randomly chosen day in July.

Data from previous years suggests that $p = 0.3$, but in 2022 Rita suspected that the value of p had been reduced. She decided to carry out a hypothesis test.

In July 2022, she saw this kind of bird on 4 days.

(a) Use the binomial distribution to test at the 5% significance level whether Rita's suspicion is justified. [5]

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In July 2023, she noted the value of X and carried out another test at the 5% significance level using the same hypotheses.

(b) Calculate the probability of a Type I error. [2]

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Rita models the number of sightings, Y, per year of a different, very rare, kind of bird by the distribution $B(365, 0.01)$.

(c) **(i)** Use a suitable approximating distribution to find $P(Y = 4)$. [3]

(ii) Justify your approximating distribution in this context. [1]

Additional page

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CANDIDATE NAME											
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2670890949

MATHEMATICS **9709/62**

Paper 6 Probability & Statistics 2 **May/June 2024**

1 hour 15 minutes

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DC (PQ) 330131/3

1 A random variable X has the distribution $\text{Po}(145)$.

(a) Use a suitable approximating distribution to calculate $P(X \leqslant 150)$. [4]

(b) Justify the use of your approximating distribution in this case. [1]

2 Henri wants to choose a random sample from the 804 students at his college. He numbers the students from 1 to 804 and then uses random numbers generated by his calculator. The first 20 random digits produced by his calculator are as follows.

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Henri's first two student numbers are 567 and 109.

(a) Use Henri's digits to find the numbers of the next two students in the sample. [2]

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There were 30 students in Henri's sample. He asked each of them how much time, X hours, they spent on social media each week, on average. He summarised the results as follows.

$$n = 30 \qquad \Sigma x = 610 \qquad \Sigma x^2 = 12\,405$$

(b) Use this information to calculate an unbiased estimate of the mean of X and show that an unbiased estimate of the variance of X is less than 0.1 . [3]

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(c) Henri's friend claims that Henri has probably made a mistake in his calculation of Σx or Σx^2.

Use your answer to part **(b)** to comment on this claim. [1]

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3 A student wishes to estimate the proportion, p, of students at her college who have exactly one brother. She surveys a random sample of 50 students at her college and finds that 18 of them have exactly one brother. She calculates an approximate $\alpha\%$ confidence interval for p and finds that the lower limit of the confidence interval is 0.244 correct to 3 significant figures.

Find α correct to the nearest integer. [4]

4 A random variable X has the distribution N(10, 12). Two independent values of X, denoted by X_1 and X_2, are chosen at random.

(a) Write down the value of $P(X_1 > X_2)$. [1]

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(b) Find $P(X_1 > 2X_2 - 3)$. [5]

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5 The number of goals scored by a sports team in the first half of any match has the distribution $X \sim \text{Po}(3.1)$. The number of goals scored by the same team in the second half of any match has the distribution $Y \sim \text{Po}(2.4)$. You may assume that the distributions of X and Y are independent.

(a) Find $\text{P}(X < 4)$. [2]

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(b) Find the probability that, in a randomly chosen match, the team scores at least 5 goals. [3]

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(c) Given that the team scores a total of 5 goals in a randomly chosen match, find the probability that they score exactly 3 goals in the first half. [4]

6 The masses of cereal boxes filled by a certain machine have mean 510 grams. An adjustment is made to the machine and an inspector wishes to test whether the mean mass of cereal boxes filled by the machine has decreased.

After the adjustment is made, he chooses a random sample of 120 cereal boxes. The mean mass of these boxes is found to be 508 grams.

Assume that the standard deviation of the masses is 10 grams.

(a) Test at the 2.5% significance level whether the mean mass of cereal boxes filled by the machine has decreased. [5]

Later the inspector carries out a similar test at the 2.5% significance level, using the same hypotheses and another 120 randomly chosen cereal boxes.

(b) Given that the mean mass is now actually 506 grams, find the probability of a Type II error. [5]

7 The probability density function, f, of a random variable X is given by

$$f(x) = \begin{cases} k(1+\cos x) & 0 \leqslant x \leqslant \pi, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

(a) Show that $k = \frac{1}{\pi}$. [3]

(b) Verify that the median of X lies between 0.83 and 0.84 . [3]

(c) Find the exact value of $E(X)$. [4]

Additional page

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MATHEMATICS **9709/63**

Paper 6 Probability & Statistics 2 **May/June 2024**

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
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- Do **not** write on any bar codes.
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- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **12** pages.

DC (PQ) 330132/2

1 The random variable X has the distribution $B(4000, 0.001)$.

(a) Use a suitable approximating distribution to find $P(2 \leqslant X < 5)$. [3]

(b) Justify your approximating distribution in this case. [1]

2 The widths, w cm, of a random sample of 150 leaves of a certain kind were measured. The sample mean of w was found to be 3.12 cm.

Using this sample, an approximate 95% confidence interval for the population mean of the widths in centimetres was found to be [3.01, 3.23].

(a) Calculate an estimate of the population standard deviation. [3]

(b) Explain whether it was necessary to use the Central Limit theorem in your answer to part **(a)**. [1]

3 The masses in kilograms of large and small bags of cement have the independent distributions N(50, 2.4) and N(26, 1.8) respectively.

Find the probability that the total mass of 5 randomly chosen large bags of cement is greater than the total mass of 10 randomly chosen small bags of cement. [5]

4 In this question you should **not** use an approximating distribution.

At an election in Menham last year, 24% of voters supported the Today Party. A student wishes to test whether support for the Today Party has decreased since last year. He chooses a random sample of 25 voters in Menham and finds that exactly 2 of them say that they support the Today Party.

Test at the 5% significance level whether support for the Today Party has decreased. [5]

5 A random variable X has probability density function f given by

$$f(x) = \begin{cases} ax - x^3 & 0 \leqslant x \leqslant \sqrt{2}, \\ 0 & \text{otherwise,} \end{cases}$$

where a is a constant.

(a) Show that $a = 2$. [3]

(b) Find the median of X. [4]

(c) Find the exact value of $E(X)$. [3]

6 The numbers of green sweets in 200 randomly chosen packets of Frutos are summarised in the table.

Number of green sweets	0	1	2	3	> 3
Number of packets	32	50	97	21	0

(a) Calculate an unbiased estimate for the population mean of the number of green sweets in a packet of Frutos, and show that an unbiased estimate of the population variance is 0.783 correct to 3 significant figures. [3]

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The manufacturers of Frutos claim that the mean number of green sweets in a packet is 1.65 .

Anji believes that the true value of the mean, μ, is less than 1.65 . She uses the results from the 200 randomly chosen packets to test the manufacturers' claim.

(b) State suitable null and alternative hypotheses for the test. [1]

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(c) Show that the result of Anji's test is significant at the 5% level but not at the 1% level. [4]

(d) It is given that Anji made a Type I error.

Explain how this shows that the significance level that Anji used in her test was not 1%. [1]

7 The independent random variables X and Y have the distributions Po(1.9) and Po(2.2) respectively.

(a) Find $P(X+Y<4)$. [3]

(b) Find the probability that $X=2$ given that $X+Y<4$. [4]

(c) A sample of 60 randomly chosen pairs of values of X and Y is taken, and the value of $X+Y$ is calculated for each pair. The sample mean of these 60 values is found.

Find the probability that the sample mean of $X+Y$ is less than 4.0 . [6]

Additional page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

8526522996

MATHEMATICS **9709/62**

Paper 6 Probability & Statistics 2 **February/March 2024**

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS
- Answer **all** questions.
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INFORMATION
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DC (CE) 329849/3

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1 The lengths, X cm, of a sample of 100 insects of a certain type were summarised as follows.

$$n = 100 \qquad \sum x = 36.8 \qquad \sum x^2 = 17.34$$

(a) Calculate unbiased estimates for the population mean and variance of X. [3]

(b) State a necessary condition for the estimates found in part **(a)** to be reliable. [1]

2 A random sample of 250 people living in Barapet was chosen. It was found that 78 of these people owned a BETEC phone.

(a) Calculate an approximate 98% confidence interval for the proportion of people living in Barapet who own a BETEC phone. [3]

(b) Manjit claims that more than 40% of the people living in Barapet own a BETEC phone.

Use your answer to part **(a)** to comment on this claim. [1]

3 In a certain lottery, on average 1 in every 10 000 tickets is a prize-winning ticket. An agent sells 6000 tickets.

(a) Use a suitable approximating distribution to find the probability that at least 3 of the tickets sold by the agent are prize-winning tickets. [3]

(b) Justify the use of your approximating distribution in this context. [1]

4 Each year a transport firm uses X litres of gasoline and Y litres of diesel fuel, where X and Y have the independent distributions $X \sim N(10\,700, 950^2)$ and $Y \sim N(13\,400, 1210^2)$.

(a) Find the probability that in a randomly chosen year the firm uses more gasoline than diesel fuel. [5]

The costs per litre of gasoline and diesel fuel are \$0.80 and \$0.85 respectively.

(b) Find the probability that the total cost of gasoline and diesel fuel in a randomly chosen year is between \$20 000 and \$22 000. [5]

5 A teacher models the numbers of girls and boys who arrive late for her class on any day by the independent random variables $G \sim \text{Po}(0.10)$ and $B \sim \text{Po}(0.15)$ respectively.

(a) Find the probability that during a randomly chosen 2-day period no girls arrive late. [1]

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(b) Find the probability that during a randomly chosen 5-day period the total number of students who arrive late is less than 3. [3]

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(c) It is given that the values of $\text{P}(G = r)$ and $\text{P}(B = r)$ for $r \geqslant 3$ are very small and can be ignored.

Find the probability that on a randomly chosen day more girls arrive late than boys. [3]

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Following a timetable change the teacher claims that on average more students arrive late than before the change. During a randomly chosen 5-day period a total of 4 students are late.

(d) Test the teacher's claim at the 5% significance level. [5]

6 The graph of the probability density function f of a random variable X is symmetrical about the line $x = 2$. It is given that $P(2 < X < 5) = \frac{117}{256}$.

(a) Using only this information show that $P(X > -1) = \frac{245}{256}$. [2]

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It is now given that, for x in a suitable domain,

$$f(x) = k(12 + 4x - x^2), \text{ where } k \text{ is a constant.}$$

(b) Find the value of k. [3]

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(c) A different random variable X has probability density function $g(x) = \frac{2}{9}\left(2 + x - x^2\right)$. The domain of X is all values of x for which $g(x) \geqslant 0$.

Find Var(X). [5]

7 The heights, in centimetres, of adult females in Litania have mean μ and standard deviation σ. It is known that in 2004 the values of μ and σ were 163.21 and 6.95 respectively. The government claims that the value of μ this year is greater than it was in 2004. In order to test this claim a researcher plans to carry out a hypothesis test at the 1% significance level. He records the heights of a random sample of 300 adult females in Litania this year and finds the value of the sample mean.

(a) State the probability of a Type I error. [1]

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You should assume that the value of σ after 2004 remains at 6.95 .

(b) Given that the value of μ this year is actually 164.91, find the probability of a Type II error. [5]

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Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.

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Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME	

CENTRE NUMBER		CANDIDATE NUMBER	

5370356203

MATHEMATICS **9709/61**

Paper 6 Probability & Statistics 2 **October/November 2023**

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
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INFORMATION

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DC (KN) 332579

1 A random variable X has the distribution N(410, 400).

Find the probability that the mean of a random sample of 36 values of X is less than 405. [3]

2 In a survey of 300 randomly chosen adults in Rickton, 134 said that they exercised regularly. This information was used to calculate an $\alpha\%$ confidence interval for the proportion of adults in Rickton who exercise regularly. The upper bound of the confidence interval was found to be 0.487, correct to 3 significant figures.

Find the value of α correct to the nearest integer. [4]

3 A website owner finds that, on average, his website receives 0.3 hits per minute. He believes that the number of hits per minute follows a Poisson distribution.

(a) Assume that the owner is correct.

(i) Find the probability that there will be at least 4 hits during a 10-minute period. [3]

(ii) Use a suitable approximating distribution to find the probability that there will be fewer than 40 hits during a 3-hour period. [4]

A friend agrees that the website receives, on average, 0.3 hits per minute. However, she notices that the number of hits during the day-time (9.00 am to 9.00 pm) is usually about twice the number of hits during the night-time (9.00 pm to 9.00 am).

(b) **(i)** Explain why this fact contradicts the owner's belief that the number of hits per minute follows a Poisson distribution. [1]

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(ii) Specify separate Poisson distributions that might be suitable models for the number of hits during the day-time and during the night-time. [2]

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4 The masses, in kilograms, of chemicals A and B produced per day by a factory are modelled by the independent random variables X and Y respectively, where $X \sim N(10.3, 5.76)$ and $Y \sim N(11.4, 9.61)$. The income generated by the chemicals is \$2.50 per kilogram for A and \$3.25 per kilogram for B.

(a) Find the mean and variance of the daily income generated by chemical A. [2]

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(b) Find the probability that, on a randomly chosen day, the income generated by chemical A is greater than the income generated by chemical B. [6]

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5 In the past the number of enquiries per minute at a customer service desk has been modelled by a random variable with distribution Po(0.31). Following a change in the position of the desk, it is expected that the mean number of enquiries per minute will increase. In order to test whether this is the case, the total number of enquiries during a randomly chosen 5-minute period is noted. You should assume that a Poisson model is still appropriate.

Given that the total number of enquiries is 5, carry out the test at the 2.5% significance level. [5]

6 A continuous random variable X takes values from 0 to 6 only and has a probability distribution that is symmetrical.

Two values, a and b, of X are such that $P(a < X < b) = p$ and $P(b < X < 3) = \frac{13}{10}p$, where p is a positive constant.

(a) Show that $p \leqslant \frac{5}{23}$. [1]

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(b) Find $P(b < X < 6 - a)$ in terms of p. [2]

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It is now given that the probability density function of X is f, where

$$f(x) = \begin{cases} \frac{1}{36}(6x - x^2) & 0 \leqslant x \leqslant 6, \\ 0 & \text{otherwise.} \end{cases}$$

(c) Given that $b = 2$ and $p = \frac{5}{27}$, find the value of a. [5]

7 A biologist wishes to test whether the mean concentration μ, in suitable units, of a certain pollutant in a river is below the permitted level of 0.5. She measures the concentration, x, of the pollutant at 50 randomly chosen locations in the river. The results are summarised below.

$$n = 50 \qquad \Sigma x = 23.0 \qquad \Sigma x^2 = 13.02$$

(a) Carry out a test at the 5% significance level of the null hypothesis $\mu = 0.5$ against the alternative hypothesis $\mu < 0.5$. [7]

Later, a similar test is carried out at the 5% significance level using another sample of size 50 and the same hypotheses as before. You should assume that the standard deviation is unchanged.

(b) Given that, in fact, the value of μ is 0.4, find the probability of a Type II error. [5]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

7077958469

MATHEMATICS **9709/62**

Paper 6 Probability & Statistics 2 **October/November 2023**

1 hour 15 minutes

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You will need: List of formulae (MF19)

INSTRUCTIONS

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1 **(a)** A random variable X has the distribution Po(25).

Use the normal approximation to the Poisson distribution to find $P(X > 30)$. [4]

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(b) A random variable Y has the distribution $B(100, p)$ where $p < 0.05$.

Use the Poisson approximation to the binomial distribution to write down an expression, in terms of p, for $P(Y < 3)$. [2]

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2 The length, in minutes, of mathematics lectures at a certain college has mean μ and standard deviation 8.3.

(a) The total length of a random sample of 85 lectures was 4590 minutes.

Calculate a 95% confidence interval for μ. [3]

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The length, in minutes, of history lectures at the college has mean m and standard deviation s.

(b) Using a random sample of 100 history lectures, a 95% confidence interval for m was found to have width 2.8 minutes.

Find the value of s. [2]

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3 A researcher read a magazine article which stated that boys aged 1 to 3 prefer green to orange. It claimed that, when offered a green cube and an orange cube to play with, a boy is more likely to choose the green one.

The researcher disagrees with this claim. She believes that boys of this age are equally likely to choose either colour. In order to test her belief, the researcher carried out a hypothesis test at the 5% significance level. She offered a green cube and an orange cube to each of 10 randomly chosen boys aged 1 to 3, and recorded the number, X, of boys who chose the green cube.

Out of the 10 boys, 8 boys chose the green cube.

(a) **(i)** Assuming that the researcher's belief that either colour cube is equally likely to be chosen is valid, a student correctly calculates that $P(X = 8) = 0.0439$, correct to 3 significant figures. He says that, because this value is less than 0.05, the null hypothesis should be rejected.

Explain why this statement is incorrect. [1]

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(ii) Carry out the test on the researcher's claim that either colour cube is equally likely to be chosen. [5]

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(b) Another researcher claims that a Type I error was made in carrying out the test.

Explain why this cannot be true. [1]

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A similar test, at the 5% significance level, was carried out later using 10 other randomly chosen boys aged 1 to 3.

(c) Find the probability of a Type I error. [2]

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4 The height H, in metres, of mature trees of a certain variety is normally distributed with standard deviation 0.67. In order to test whether the population mean of H is greater than 4.23, the heights of a random sample of 200 trees are measured.

(a) Write down suitable null and alternative hypotheses for the test. [1]

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The sample mean height, $\bar{h}$ metres, of the 200 trees is found and the test is carried out. The result of the test is to reject the null hypothesis at the 5% significance level.

(b) Find the set of possible values of $\bar{h}$. [3]

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(c) Ajit said, 'In **(b)** we had to assume that $\overline{H}$ is normally distributed, so it was necessary to use the Central Limit Theorem.'

Explain whether you agree with Ajit. [1]

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5 The random variable X has probability density function, f, given by

$$f(x) = \begin{cases} \dfrac{1}{x^2} & a < x < b, \\ 0 & \text{otherwise,} \end{cases}$$

where a and b are positive constants.

(a) It is given that $E(X) = \ln 2$.

Show that $b = 2a$. [3]

(b) Show that $a = \frac{1}{2}$. [3]

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(c) Find the median of X. [3]

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6 A factory makes loaves of bread in batches. One batch of loaves contains X kilograms of dried yeast and Y kilograms of flour, where X and Y have the independent distributions $N(0.7, 0.02^2)$ and $N(100.0, 3.0^2)$ respectively.

Dried yeast costs \$13.50 per kilogram and flour costs \$0.90 per kilogram. For making one batch of bread the total of all other costs is \$55. The factory sells each batch of bread for \$200.

Find the probability that the profit made on one randomly chosen batch of bread is greater than \$40. [7]

7 A random variable X has the distribution Po(2.4).

(a) Find $P(2 \leqslant X < 4)$. [2]

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(b) Two independent values of X are chosen.

Find the probability that both of these values are greater than 1. [3]

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(c) It is given that $P(X = r) < P(X = r + 1)$.

(i) Find the set of possible values of r. [3]

(ii) Hence find the value of r for which $P(X = r)$ is greatest. [1]

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Cambridge International AS & A Level

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CENTRE NUMBER

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* 1 5 9 9 0 8 1 4 6 7 *

MATHEMATICS **9709/63**

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1 hour 15 minutes

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Find the value of α correct to the nearest integer. [4]

3 A website owner finds that, on average, his website receives 0.3 hits per minute. He believes that the number of hits per minute follows a Poisson distribution.

(a) Assume that the owner is correct.

(i) Find the probability that there will be at least 4 hits during a 10-minute period. [3]

(ii) Use a suitable approximating distribution to find the probability that there will be fewer than 40 hits during a 3-hour period. [4]

A friend agrees that the website receives, on average, 0.3 hits per minute. However, she notices that the number of hits during the day-time (9.00 am to 9.00 pm) is usually about twice the number of hits during the night-time (9.00 pm to 9.00 am).

(b) **(i)** Explain why this fact contradicts the owner's belief that the number of hits per minute follows a Poisson distribution. [1]

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(ii) Specify separate Poisson distributions that might be suitable models for the number of hits during the day-time and during the night-time. [2]

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4 The masses, in kilograms, of chemicals A and B produced per day by a factory are modelled by the independent random variables X and Y respectively, where $X \sim N(10.3, 5.76)$ and $Y \sim N(11.4, 9.61)$. The income generated by the chemicals is \$2.50 per kilogram for A and \$3.25 per kilogram for B.

(a) Find the mean and variance of the daily income generated by chemical A. [2]

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(b) Find the probability that, on a randomly chosen day, the income generated by chemical A is greater than the income generated by chemical B. [6]

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Given that the total number of enquiries is 5, carry out the test at the 2.5% significance level. [5]

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Two values, a and b, of X are such that $P(a < X < b) = p$ and $P(b < X < 3) = \frac{13}{10}p$, where p is a positive constant.

(a) Show that $p \leqslant \frac{5}{23}$. [1]

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(b) Find $P(b < X < 6 - a)$ in terms of p. [2]

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It is now given that the probability density function of X is f, where

$$f(x) = \begin{cases} \frac{1}{36}(6x - x^2) & 0 \leqslant x \leqslant 6, \\ 0 & \text{otherwise.} \end{cases}$$

(c) Given that $b = 2$ and $p = \frac{5}{27}$, find the value of a. [5]

7 A biologist wishes to test whether the mean concentration μ, in suitable units, of a certain pollutant in a river is below the permitted level of 0.5. She measures the concentration, x, of the pollutant at 50 randomly chosen locations in the river. The results are summarised below.

$$n = 50 \qquad \Sigma x = 23.0 \qquad \Sigma x^2 = 13.02$$

(a) Carry out a test at the 5% significance level of the null hypothesis $\mu = 0.5$ against the alternative hypothesis $\mu < 0.5$. [7]

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Later, a similar test is carried out at the 5% significance level using another sample of size 50 and the same hypotheses as before. You should assume that the standard deviation is unchanged.

(b) Given that, in fact, the value of μ is 0.4, find the probability of a Type II error. [5]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

* 0 2 5 9 9 1 0 9 4 4 *

MATHEMATICS **9709/61**

Paper 6 Probability & Statistics 2 **May/June 2023**

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS
- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION
- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **12** pages. Any blank pages are indicated.

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1 In a certain country, 20 540 adults out of a population of 6 012 300 have a degree in medicine.

(a) Use an approximating distribution to calculate the probability that, in a random sample of 1000 adults in this country, there will be fewer than 4 adults who have a degree in medicine. [4]

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(b) Justify the approximating distribution used in part **(a)**. [2]

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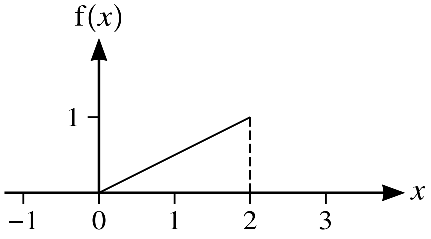
2 **(a)**



The graph of the function f is a straight line segment from (0, 0) to (2, 1).

Show that f could be a probability density function. [2]

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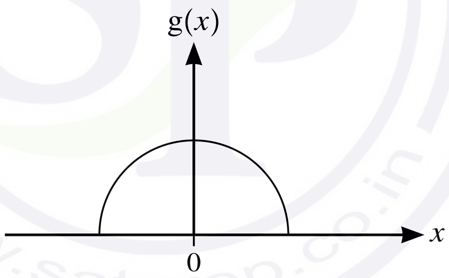
(b)



The graph of the function g is a semicircle, centre (0, 0), entirely above the *x*-axis.

Given that g is a probability density function, find the radius of the semicircle. [2]

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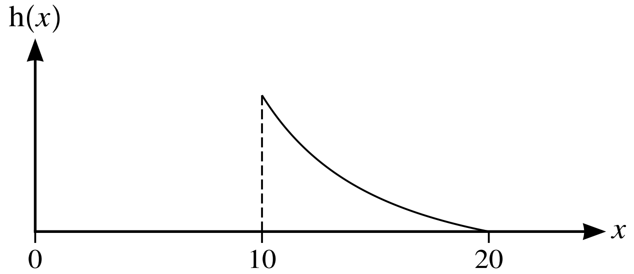
(c)



The time, X minutes, taken by a large number of students to complete a test has probability density function h, as shown in the diagram.

(i) Without calculation, use the diagram to explain how you can tell that the median time is less than 15 minutes. [1]

It is now given that

$$\mathrm{h}(x) = \begin{cases} \dfrac{40}{x^2} - \dfrac{1}{10} & 10 \leqslant x \leqslant 20, \\ 0 & \text{otherwise.} \end{cases}$$

(ii) Find the mean time. [3]

3 In the past, the annual amount of wheat produced per farm by a large number of similar sized farms in a certain region had mean 24.0 tonnes and standard deviation 5.2 tonnes. Last summer a new fertiliser was used by all the farms, and it was expected that the mean amount of wheat produced per farm would be greater than 24.0 tonnes. In order to test whether this was true, a scientist recorded the amounts of wheat produced by a random sample of 50 farms last summer. He found that the value of the sample mean was 25.8 tonnes.

Stating a necessary assumption, carry out the test at the 1% significance level. [6]

4 A certain train journey takes place every day throughout the year. The time taken, in minutes, for the journey is normally distributed with variance 11.2.

(a) The mean time for a random sample of n of these journeys was found. A 94% confidence interval for the population mean time was calculated and was found to have a width of 1.4076 minutes, correct to 4 decimal places.

Find the value of n. [3]

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(b) A passenger noted the times for 50 randomly chosen journeys in January, February and March.

Give a reason why this sample is unsuitable for use in finding a confidence interval for the population mean time. [1]

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(c) A researcher took 4 random samples and a 94% confidence interval for the population mean was found from each sample.

Find the probability that exactly 3 of these confidence intervals contain the true value of the population mean. [2]

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5 Large packets of rice are packed in cartons, each containing 20 randomly chosen packets. The masses of these packets are normally distributed with mean 1010 g and standard deviation 3.4 g. The masses of the cartons, when empty, are independently normally distributed with mean 50 g and standard deviation 2.0 g.

(a) Find the variance of the masses of full cartons. [2]

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Small packets of rice are packed in boxes. The total masses of full boxes are normally distributed with mean 6730 g and standard deviation 15.0 g. The masses of the boxes and cartons are distributed independently of each other.

(b) Find the probability that the mass of a randomly chosen full carton is more than three times the mass of a randomly chosen full box. [5]

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6 A sample of 5 randomly selected values of a variable X is as follows:

$$1 \qquad 2 \qquad 6 \qquad 1 \qquad a$$

where $a > 0$.

Given that an unbiased estimate of the variance of X calculated from this sample is $\frac{11}{2}$, find the value of a. [3]

7 The number of accidents per week at a certain factory has a Poisson distribution. In the past the mean has been 1.9 accidents per week. Last year, the manager gave all his employees a new booklet on safety. He decides to test, at the 5% significance level, whether the mean number of accidents has been reduced. He notes the number of accidents during 4 randomly chosen weeks this year.

(a) State suitable null and alternative hypotheses for the test. [1]

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(b) Find the critical region for the test and state the probability of a Type I error. [6]

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(c) State what is meant by a Type I error in this context. [1]

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(d) During the 4 randomly chosen weeks there are a total of 3 accidents.

State the conclusion that the manager should reach. Give a reason for your answer. [2]

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(e) Assuming that the mean remains 1.9 accidents per week, use a suitable approximation to calculate the probability that there will be more than 100 accidents during a 52-week period. [4]

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Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME					
CENTRE NUMBER					
CANDIDATE NUMBER					

0584750042

MATHEMATICS **9709/62**

Paper 6 Probability & Statistics 2 **May/June 2023**

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
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- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

1 In a survey of 200 randomly chosen students from a certain college, 23% of the students said that they owned a car.

Calculate an approximate 93% confidence interval for the proportion of students from the college who own a car. [3]

2 **(a)** The random variable W has a Poisson distribution.

State the relationship between $E(W)$ and $Var(W)$. [1]

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(b) The random variable X has the distribution $B(n, p)$. Jyothi wishes to use a Poisson distribution as an approximate distribution for X.

Use the formulae for $E(X)$ and $Var(X)$ to explain why it is necessary for p to be close to 0 for this to be a reasonable approximation. [1]

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(c) Given that Y has the distribution $B(20\,000, 0.000\,07)$, use a Poisson distribution to calculate an estimate of $P(Y > 2)$. [3]

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3 The masses, in kilograms, of newborn babies in country A are represented by the random variable X, with mean μ and variance σ^2. The masses of a random sample of 500 newborn babies in this country were found and the results are summarised below.

$$n = 500 \qquad \Sigma x = 1625 \qquad \Sigma x^2 = 5663.5$$

(a) Calculate unbiased estimates of μ and σ^2. [3]

A researcher wishes to test whether the mean mass of newborn babies in a neighbouring country, B, is different from that in country A. He chooses a random sample of 60 newborn babies in country B and finds that their sample mean mass is 2.95 kg.

Assume that your unbiased estimates in part **(a)** are the correct values for μ and σ^2. Assume also that the variance of the masses of newborn babies in country B is the same as in country A.

(b) Carry out the test at the 1% significance level. [5]

4 The number, X, of books received at a charity shop has a constant mean of 5.1 per day.

(a) State, in context, one condition for X to be modelled by a Poisson distribution. [1]

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Assume now that X can be modelled by a Poisson distribution.

(b) Find the probability that exactly 10 books are received in a 3-day period. [2]

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(c) Use a suitable approximating distribution to find the probability that more than 180 books are received in a 30-day period. [4]

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The number of DVDs received at the same shop is modelled by an independent Poisson distribution with mean 2.5 per day.

(d) Find the probability that the total number of books and DVDs that are received at the shop in 1 day is more than 3. [3]

5 **(a)** Two random variables X and Y have the independent distributions N(7, 3) and N(6, 2) respectively. A random value of each variable is taken.

Find the probability that the two values differ by more than 2. [5]

(b) Each candidate's overall score in a science test is calculated as follows. The mark for theory is denoted by T, the mark for practical is denoted by P, and the overall score is given by $T + 1.5P$. The variables T and P are assumed to be independent with distributions $N(62, 158)$ and $N(42, 108)$ respectively. You should assume that no continuity corrections are needed when using these distributions.

(i) A pass is awarded to candidates whose overall score is at least 90.

Find the proportion of candidates who pass. [5]

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(ii) Comment on the assumption that the variables T and P are independent. [1]

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6 When a child completes an online exercise called a Mathlit, they might be awarded a medal. The publishers claim that the probability that a randomly chosen child who completes a Mathlit will be awarded a medal is $\frac{1}{3}$. Asha wishes to test this claim. She decides that if she is awarded no medals while completing 10 Mathlits, she will conclude that the true probability is less than $\frac{1}{3}$.

(a) Use a binomial distribution to find the probability of a Type I error. [2]

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The true probability of being awarded a medal is denoted by p.

(b) Given that the probability of a Type II error is 0.8926, find the value of p. [3]

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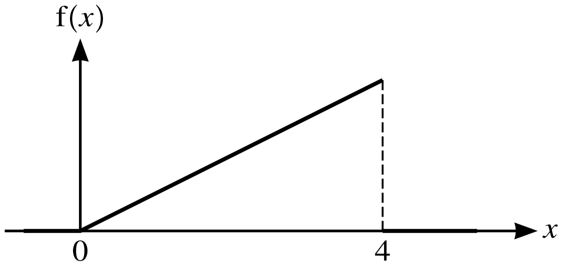
7 **(a)**



The diagram shows the graph of the probability density function, f, of a random variable X which takes values between 0 and 4 only. Between these two values the graph is a straight line.

(i) Show that $f(x) = kx$ for $0 \leqslant x \leqslant 4$, where k is a constant to be determined. [2]

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(ii) Hence, or otherwise, find $E(X)$. [3]

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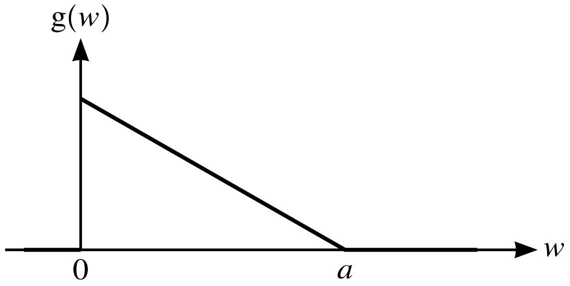
(b)



The diagram shows the graph of the probability density function, g, of a random variable W which takes values between 0 and a only, where $a > 0$. Between these two values the graph is a straight line.

Given that the median of W is 1, find the value of a. [3]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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Cambridge International AS & A Level

CANDIDATE NAME	

CENTRE NUMBER						CANDIDATE NUMBER				

1395628472

MATHEMATICS **9709/63**

Paper 6 Probability & Statistics 2 **May/June 2023**

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS
- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
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- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION
- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

1 A random variable X has probability density function f, where

$$f(x) = \begin{cases} \frac{3}{2}(1 - x^2) & 0 \leqslant x \leqslant 1, \\ 0 & \text{otherwise.} \end{cases}$$

Find $E(X)$. [3]

2 A club has 264 members, numbered from 1 to 264. Donash wants to choose a random sample of members for a survey. In order to choose the members for the sample he uses his calculator to generate random digits. His first 20 random digits are as follows.

10612 11801 21473 22759

(a) The numbers of the first two members in the sample are 106 and 121.

Write down the numbers of the next two members in the sample. [2]

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(b) To obtain the numbers for members after the 4th member, Donash starts with the second random digit, 0, and obtains the numbers 061 and 211.

Explain why this method will not produce a random sample. [1]

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3 In a random sample of 100 students at Luciana's college, x students said that they liked exams. Luciana used this result to find an approximate 90% confidence interval for the proportion, p, of all students at her college who liked exams. Her confidence interval had width 0.157 92.

(a) Find the two possible values of x. [4]

Suzma independently took another random sample and found another approximate 90% confidence interval for p.

(b) Find the probability that neither of the two confidence intervals contains the true value of p. [1]

4 The mass, in tonnes, of steel produced per day at a factory is normally distributed with mean 65.2 and standard deviation 3.6. It can be assumed that the mass of steel produced each day is independent of other days. The factory makes \$50 profit on each tonne of steel produced.

Find the probability that the total profit made in a randomly chosen 7-day week is less than \$22 000. [6]

5 Last year the mean time for pizza deliveries from Pete's Pizza Pit was 32.4 minutes. This year the time, t minutes, for pizza deliveries from Pete's Pizza Pit was recorded for a random sample of 50 deliveries. The results were as follows.

$$n = 50 \qquad \Sigma t = 1700 \qquad \Sigma t^2 = 59\,050$$

(a) Find unbiased estimates of the population mean and variance. [3]

(b) Test, at the 2% significance level, whether the mean delivery time has changed since last year. [5]

(c) Under what circumstances would it **not** be necessary to use the Central Limit Theorem in answering **(b)**? [1]

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6 It is known that 1 in 5000 people in Atalia have a certain condition. A random sample of 12 500 people from Atalia is chosen for a medical trial. The number having the condition is denoted by X.

(a) Use an appropriate approximating distribution to find $P(X \leqslant 3)$. [3]

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(b) Find the values of $E(X)$ and $Var(X)$, and explain how your answers suggest that the approximating distribution used in **(a)** is likely to be appropriate. [2]

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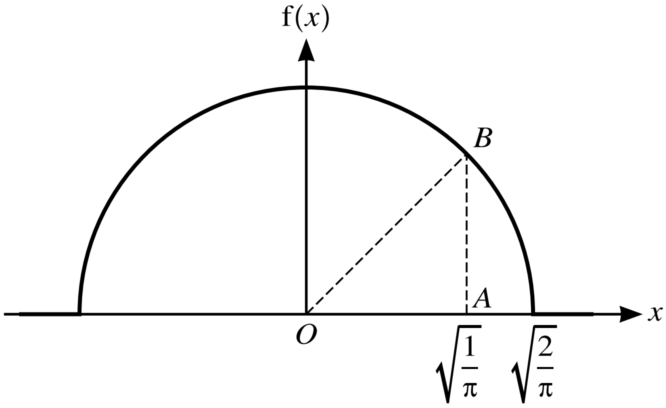

A random variable X has probability density function f, where the graph of $y = \mathrm{f}(x)$ is a semicircle with centre (0, 0) and radius $\sqrt{\frac{2}{\pi}}$, entirely above the x-axis. Elsewhere $\mathrm{f}(x) = 0$ (see diagram).

(a) Verify that f can be a probability density function. [2]

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A and B are the points where the line $x = \sqrt{\frac{1}{\pi}}$ meets the x-axis and the semicircle respectively.

(b) Show that angle AOB is $\frac{1}{4}\pi$ radians and hence find $\mathrm{P}\left(X > \sqrt{\frac{1}{\pi}}\right)$. [6]

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8 A new light was installed on a certain footpath. A town councillor decided to use a hypothesis test to investigate whether the number of people using the path in the evening had increased.

Before the light was installed, the mean number of people using the path during any 20-minute period during the evening was 1.01.

After the light was installed, the total number, n, of people using the path during 3 randomly chosen 20-minute periods during the evening was noted.

(a) Given that the value of n was 6, use a Poisson distribution to carry out the test at the 5% significance level. [6]

(b) Later a similar test, at the 5% significance level, was carried out using another 3 randomly chosen 20-minute periods during the evening.

Find the probability of a Type I error. [2]

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(c) State what is meant by a Type I error in this context. [1]

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(d) State, in context, what further information would be needed in order to find the probability of a Type II error. Do not carry out any further calculation. [2]

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Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

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7288751637

MATHEMATICS **9709/62**

Paper 6 Probability & Statistics 2 **February/March 2023**

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
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INFORMATION

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This document has **16** pages. Any blank pages are indicated.

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1 Anita carried out a survey of 140 randomly selected students at her college. She found that 49 of these students watched a TV programme called *Bunch*.

(a) Calculate an approximate 98% confidence interval for the proportion, p, of students at Anita's college who watch *Bunch*. [3]

Carlos says that the confidence interval found in **(a)** is not useful because it is too wide.

(b) Without calculation, explain briefly how Carlos can use the results of Anita's survey to find a narrower confidence interval for p. [1]

2 The number of orders arriving at a shop during an 8-hour working day is modelled by the random variable X with distribution $\mathrm{Po}(25.2)$.

(a) State **two** assumptions that are required for the Poisson model to be valid in this context. [2]

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(b) **(i)** Find the probability that the number of orders that arrive in a randomly chosen 3-hour period is between 3 and 5 inclusive. [3]

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(ii) Find the probability that, in two randomly chosen 1-hour periods, exactly 1 order will arrive in one of the 1-hour periods, and at least 2 orders will arrive in the other 1-hour period. [4]

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(c) The shop can only deal with a maximum of 120 orders during any 36-hour period.

Use a suitable approximating distribution to find the probability that, in a randomly chosen 36-hour period, there will be too many orders for the shop to deal with. [4]

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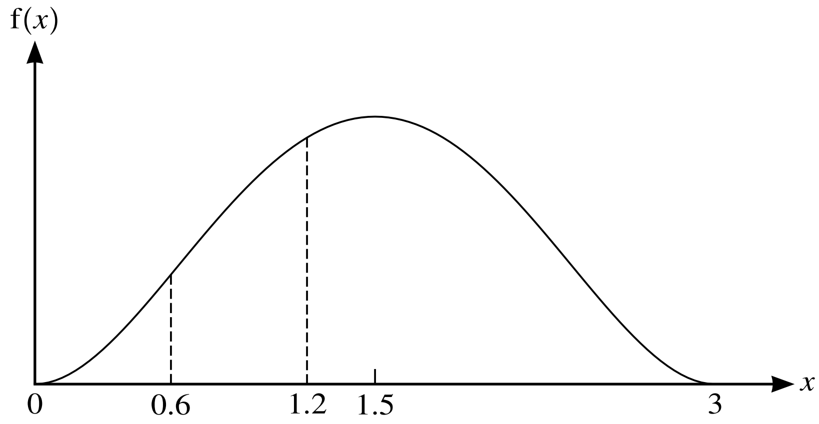

The diagram shows the graph of the probability density function, f, of a random variable X that takes values between $x = 0$ and $x = 3$ only. The graph is symmetrical about the line $x = 1.5$.

(a) It is given that $P(X < 0.6) = a$ and $P(0.6 < X < 1.2) = b$.

Find $P(0.6 < X < 1.8)$ in terms of a and b. [2]

(b) It is now given that the equation of the probability density function of X is

$$f(x) = \begin{cases} kx^2(3-x)^2 & 0 \leqslant x \leqslant 3, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

(i) Show that $k = \frac{10}{81}$. [3]

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(ii) Find Var(X). [3]

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4 The number of accidents per 3-month period on a certain road has the distribution $\text{Po}(\lambda)$. In the past the value of λ has been 5.7. Following some changes to the road, the council carries out a hypothesis test to determine whether the value of λ has decreased. If there are fewer than 3 accidents in a randomly chosen 3-month period, the council will conclude that the value of λ has decreased.

(a) Find the probability of a Type I error. [2]

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(b) Find the probability of a Type II error if the mean number of accidents per 3-month period is now actually 0.9. [3]

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5 The masses, in grams, of large and small packets of Maxwheat cereal have the independent distributions $N(410.0,\ 3.6^2)$ and $N(206.0,\ 3.7^2)$ respectively.

(a) Find the probability that a randomly chosen large packet has a mass that is more than double the mass of a randomly chosen small packet. [5]

The packets are placed in boxes. The boxes are identical in appearance. 60% of the boxes contain exactly 10 randomly chosen large packets. 40% of the boxes contain exactly 20 randomly chosen small packets.

(b) Find the probability that a randomly chosen box contains packets with a total mass of more than 4080 grams. [6]

6 Last year, the mean time taken by students at a school to complete a certain test was 25 minutes. Akash believes that the mean time taken by this year's students was less than 25 minutes. In order to test this belief, he takes a large random sample of this year's students and he notes the time taken by each student. He carries out a test, at the 2.5% significance level, for the population mean time, μ minutes. Akash uses the null hypothesis H_0: $\mu = 25$.

(a) Give a reason why Akash should use a one-tailed test. [1]

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Akash finds that the value of the test statistic is $z = -2.02$.

(b) Explain what conclusion he should draw. [2]

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In a different one-tailed hypothesis test the z-value was found to be 2.14.

(c) Given that this value would lead to a rejection of the null hypothesis at the α% significance level, find the set of possible values of α. [3]

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The population mean time taken by students at another school to complete a test last year was m minutes. Sorin carries out a one-tailed test to determine whether the population mean this year is less than m, using a random sample of 100 students. He assumes that the population standard deviation of the times is 3.9 minutes. The sample mean is 24.8 minutes, and this result just leads to the rejection of the null hypothesis at the 5% significance level.

(d) Find the value of m. [3]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

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MATHEMATICS **9709/61**

Paper 6 Probability & Statistics 2 **October/November 2022**

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
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- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

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1 The heights, in metres, of a random sample of 10 mature trees of a certain variety are given below.

5.9	6.5	6.7	5.9	6.9	6.0	6.4	6.2	5.8	5.8

Find unbiased estimates of the population mean and variance of the heights of all mature trees of this variety. [3]

2 A spinner has five sectors, each printed with a different colour. Susma and Sanjay both wish to test whether the spinner is biased so that it lands on red on fewer spins than it would if it were fair. Susma spins the spinner 40 times. She finds that it lands on red exactly 4 times.

(a) Use a binomial distribution to carry out the test at the 5% significance level. [5]

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Sanjay also spins the spinner 40 times. He finds that it lands on red r times.

(b) Use a binomial distribution to find the largest value of r that lies in the rejection region for the test at the 5% significance level. [3]

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3 Drops of water fall randomly from a leaking tap at a constant average rate of 5.2 per minute.

(a) Find the probability that at least 3 drops fall during a randomly chosen 30-second period. [3]

(b) Use a suitable approximating distribution to find the probability that at least 650 drops fall during a randomly chosen 2-hour period. [4]

4 Each month a company sells X kg of brown sugar and Y kg of white sugar, where X and Y have the independent distributions $N(2500, 120^2)$ and $N(3700, 130^2)$ respectively.

(a) Find the mean and standard deviation of the total amount of sugar that the company sells in 3 randomly chosen months. [3]

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The company makes a profit of \$1.50 per kilogram of brown sugar sold and makes a loss of \$0.20 per kilogram of white sugar sold.

(b) Find the probability that, in a randomly chosen month, the total profit is less than \$3000. [5]

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5 A builders' merchant sells stones of different sizes.

(a) The masses of size A stones have standard deviation 6 grams. The mean mass of a random sample of 200 size A stones is 45 grams.

Find a 95% confidence interval for the population mean mass of size A stones. [3]

(b) The masses of size B stones have standard deviation 11 grams. Using a random sample of size 200, an $\alpha\%$ confidence interval for the population mean mass is found to have width 4 grams.

Find α. [4]

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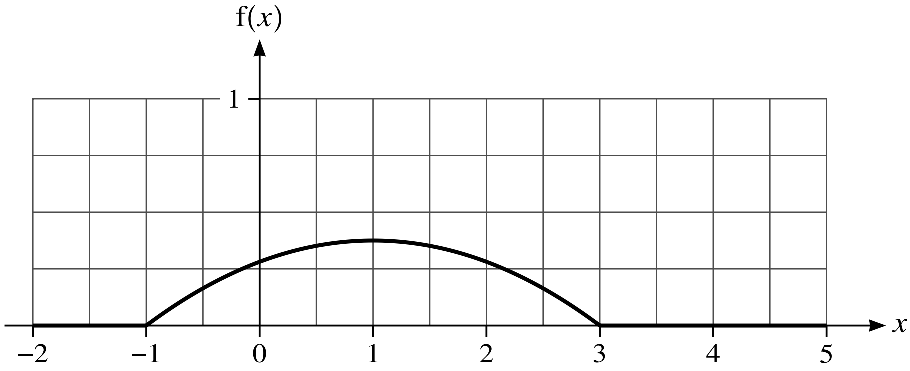
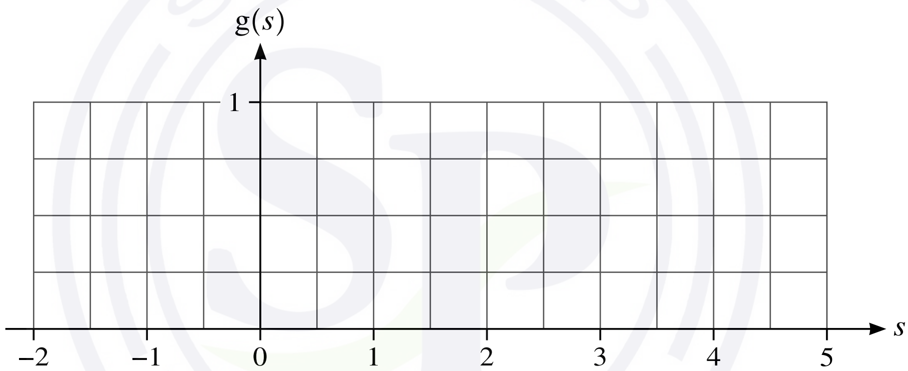


The diagram shows the graph of the probability density function of a random variable X that takes values between -1 and 3 only. It is given that the graph is symmetrical about the line $x = 1$. Between $x = -1$ and $x = 3$ the graph is a quadratic curve.

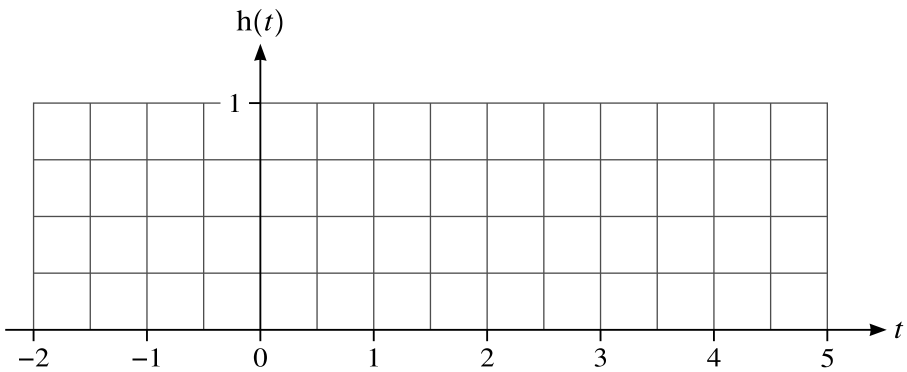
The random variable S is such that $\mathrm{E}(S) = 2 \times \mathrm{E}(X)$ and $\mathrm{Var}(S) = \mathrm{Var}(X)$.

(a) On the grid below, sketch a quadratic graph for the probability density function of S. [1]



The random variable T is such that $\mathrm{E}(T) = \mathrm{E}(X)$ and $\mathrm{Var}(T) = \frac{1}{4}\mathrm{Var}(X)$.

(b) On the grid below, sketch a quadratic graph for the probability density function of T. [2]



It is now given that

$$f(x) = \begin{cases} \frac{3}{32}(3 + 2x - x^2) & -1 \leqslant x \leqslant 3, \\ 0 & \text{otherwise.} \end{cases}$$

(c) Given that $P(1 - a < X < 1 + a) = 0.5$, show that $a^3 - 12a + 8 = 0$. [3]

(d) Hence verify that $0.69 < a < 0.70$. [1]

7 In the past Laxmi's time, in minutes, for her journey to college had mean 32.5 and standard deviation 3.1. After a change in her route, Laxmi wishes to test whether the mean time has decreased. She notes her journey times for a random sample of 50 journeys and she finds that the sample mean is 31.8 minutes. You should assume that the standard deviation is unchanged.

(a) Carry out a hypothesis test, at the 8% significance level, of whether Laxmi's mean journey time has decreased. [5]

Later Laxmi carries out a similar test with the same hypotheses, at the 8% significance level, using another random sample of size 50.

(b) Given that the population mean is now 31.5, find the probability of a Type II error. [5]

Additional Page

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Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME											
CENTRE NUMBER							CANDIDATE NUMBER				

* 7 9 9 5 2 0 8 9 3 7 *

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1 Each of a random sample of 80 adults gave an estimate, h metres, of the height of a particular building. The results were summarised as follows.

$$n = 80 \qquad \Sigma h = 2048 \qquad \Sigma h^2 = 52\,760$$

(a) Calculate unbiased estimates of the population mean and variance. [3]

(b) Using this sample, the upper boundary of an $\alpha\%$ confidence interval for the population mean is 26.0.

Find the value of α. [4]

2 In the past, the mean length of a particular variety of worm has been 10.3 cm, with standard deviation 2.6 cm. Following a change in the climate, it is thought that the mean length of this variety of worm has decreased. The lengths of a random sample of 100 worms of this variety are found and the mean of this sample is found to be 9.8 cm.

Assuming that the standard deviation remains at 2.6 cm, carry out a test at the 2% significance level of whether the mean length has decreased. [5]

3 1.6% of adults in a certain town ride a bicycle. A random sample of 200 adults from this town is selected.

(a) Use a suitable approximating distribution to find the probability that more than 3 of these adults ride a bicycle. [4]

(b) Justify your approximating distribution. [2]

4 The number of faults in cloth made on a certain machine has a Poisson distribution with mean 2.4 per $10\,\text{m}^2$. An adjustment is made to the machine. It is required to test at the 5% significance level whether the mean number of faults has decreased. A randomly selected $30\,\text{m}^2$ of cloth is checked and the number of faults is found.

(a) State suitable null and alternative hypotheses for the test. [1]

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(b) Find the probability of a Type I error. [3]

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Exactly 3 faults are found in the randomly selected $30\,m^2$ of cloth.

(c) Carry out the test at the 5% significance level. [2]

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Later a similar test was carried out at the 5% significance level, using another randomly selected $30\,m^2$ of cloth.

(d) Given that the number of faults actually has a Poisson distribution with mean 0.5 per $10\,m^2$, find the probability of a Type II error. [2]

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5 X is a random variable with distribution B(10, 0.2). A random sample of 160 values of X is taken.

(a) Find the approximate distribution of the sample mean, including the values of the parameters. [3]

(b) Hence find the probability that the sample mean is less than 1.8. [3]

6 The masses, in grams, of small and large bags of flour have the distributions N(510, 100) and N(1015, 324) respectively. André selects 4 small bags of flour and 2 large bags of flour at random.

(a) Find the probability that the total mass of these 6 bags of flour is less than 4130 g. [5]

(b) Find the probability that the total mass of the 4 small bags is more than the total mass of the 2 large bags. [5]

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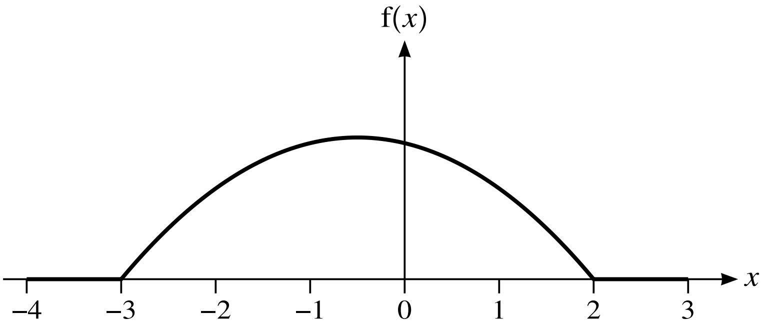

The diagram shows the graph of the probability density function, f, of a random variable X which takes values between -3 and 2 only.

(a) Given that the graph is symmetrical about the line $x = -0.5$ and that $P(X < 0) = p$, find $P(-1 < X < 0)$ in terms of p. [2]

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(b) It is now given that the probability density function shown in the diagram is given by

$$f(x) = \begin{cases} a - b(x^2 + x) & -3 \leqslant x \leqslant 2, \\ 0 & \text{otherwise,} \end{cases}$$

where a and b are positive constants.

(i) Show that $30a - 55b = 6$. [3]

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(ii) By substituting a suitable value of x into $f(x)$, find another equation relating a and b and hence determine the values of a and b. [3]

Additional Page

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Cambridge Assessment International Education

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CANDIDATE NAME

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5105685851

MATHEMATICS **9709/63**

Paper 6 Probability & Statistics 2 **October/November 2022**

1 hour 15 minutes

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Find unbiased estimates of the population mean and variance of the heights of all mature trees of this variety. [3]

2 A spinner has five sectors, each printed with a different colour. Susma and Sanjay both wish to test whether the spinner is biased so that it lands on red on fewer spins than it would if it were fair. Susma spins the spinner 40 times. She finds that it lands on red exactly 4 times.

(a) Use a binomial distribution to carry out the test at the 5% significance level. [5]

Sanjay also spins the spinner 40 times. He finds that it lands on red r times.

(b) Use a binomial distribution to find the largest value of r that lies in the rejection region for the test at the 5% significance level. [3]

3 Drops of water fall randomly from a leaking tap at a constant average rate of 5.2 per minute.

(a) Find the probability that at least 3 drops fall during a randomly chosen 30-second period. [3]

(b) Use a suitable approximating distribution to find the probability that at least 650 drops fall during a randomly chosen 2-hour period. [4]

4 Each month a company sells X kg of brown sugar and Y kg of white sugar, where X and Y have the independent distributions $N(2500, 120^2)$ and $N(3700, 130^2)$ respectively.

(a) Find the mean and standard deviation of the total amount of sugar that the company sells in 3 randomly chosen months. [3]

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The company makes a profit of \$1.50 per kilogram of brown sugar sold and makes a loss of \$0.20 per kilogram of white sugar sold.

(b) Find the probability that, in a randomly chosen month, the total profit is less than \$3000. [5]

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5 A builders' merchant sells stones of different sizes.

(a) The masses of size A stones have standard deviation 6 grams. The mean mass of a random sample of 200 size A stones is 45 grams.

Find a 95% confidence interval for the population mean mass of size A stones. [3]

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(b) The masses of size B stones have standard deviation 11 grams. Using a random sample of size 200, an α% confidence interval for the population mean mass is found to have width 4 grams.

Find α. [4]

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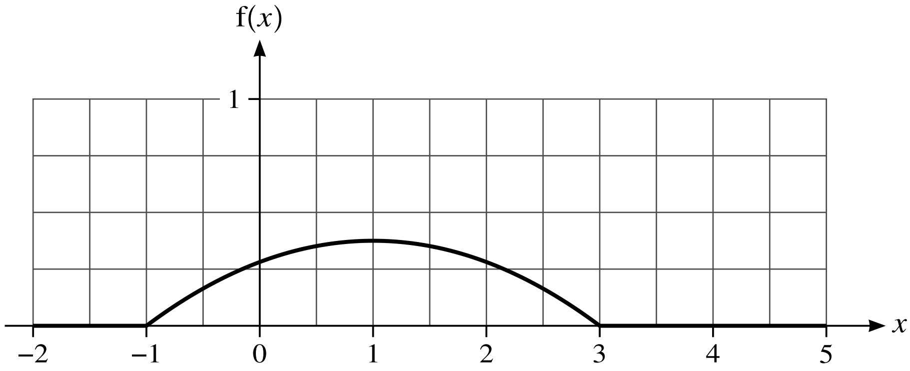

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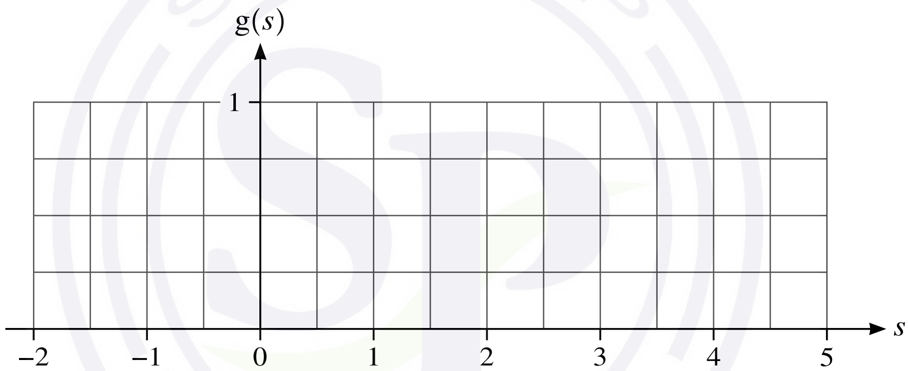
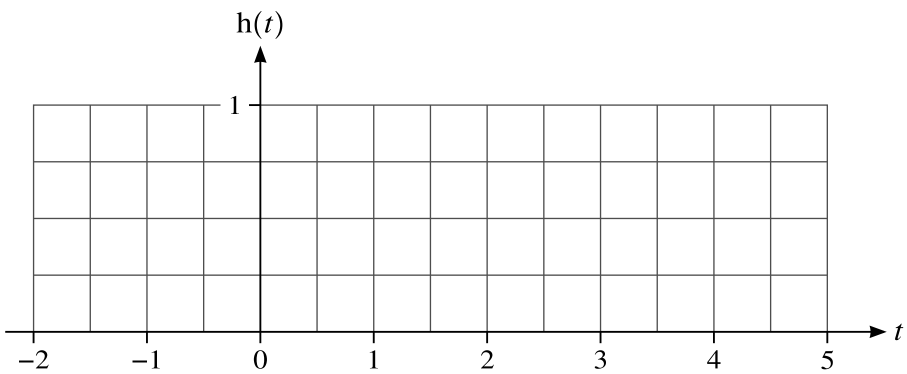
The random variable S is such that $\mathrm{E}(S) = 2 \times \mathrm{E}(X)$ and $\mathrm{Var}(S) = \mathrm{Var}(X)$.

(a) On the grid below, sketch a quadratic graph for the probability density function of S. [1]



The random variable T is such that $\mathrm{E}(T) = \mathrm{E}(X)$ and $\mathrm{Var}(T) = \frac{1}{4}\,\mathrm{Var}(X)$.

(b) On the grid below, sketch a quadratic graph for the probability density function of T. [2]



It is now given that

$$f(x) = \begin{cases} \frac{3}{32}(3 + 2x - x^2) & -1 \leqslant x \leqslant 3, \\ 0 & \text{otherwise.} \end{cases}$$

(c) Given that $P(1 - a < X < 1 + a) = 0.5$, show that $a^3 - 12a + 8 = 0$. [3]

(d) Hence verify that $0.69 < a < 0.70$. [1]

7 In the past Laxmi's time, in minutes, for her journey to college had mean 32.5 and standard deviation 3.1. After a change in her route, Laxmi wishes to test whether the mean time has decreased. She notes her journey times for a random sample of 50 journeys and she finds that the sample mean is 31.8 minutes. You should assume that the standard deviation is unchanged.

(a) Carry out a hypothesis test, at the 8% significance level, of whether Laxmi's mean journey time has decreased. [5]

Later Laxmi carries out a similar test with the same hypotheses, at the 8% significance level, using another random sample of size 50.

(b) Given that the population mean is now 31.5, find the probability of a Type II error. [5]

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5176306371

MATHEMATICS **9709/61**

Paper 6 Probability & Statistics 2 **May/June 2022**

1 hour 15 minutes

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You will need: List of formulae (MF19)

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1 The diameters, x millimetres, of a random sample of 200 discs made by a certain machine were recorded. The results are summarised below.

$$n = 200 \qquad \Sigma x = 2520 \qquad \Sigma x^2 = 31\,852$$

(a) Calculate a 95% confidence interval for the population mean diameter. [6]

(b) Jean chose 40 random samples and used each sample to calculate a 95% confidence interval for the population mean diameter.

How many of these 40 confidence intervals would be expected to include the true value of the population mean diameter? [1]

2 Arvind uses an ordinary fair 6-sided die to play a game. He believes he has a system to predict the score when the die is thrown. Before each throw of the die, he writes down what he thinks the score will be. He claims that he can write the correct score more often than he would if he were just guessing. His friend Laxmi tests his claim by asking him to write down the score before each of 15 throws of the die. Arvind writes the correct score on exactly 5 out of 15 throws.

Test Arvind's claim at the 10% significance level. [5]

3 The lengths, in centimetres, of two types of insect, A and B, are modelled by the random variables $X \sim \mathrm{N}(6.2, 0.36)$ and $Y \sim \mathrm{N}(2.4, 0.25)$ respectively.

Find the probability that the length of a randomly chosen type A insect is greater than the sum of the lengths of 3 randomly chosen type B insects. [5]

4 The independent random variables X and Y have distributions $\mathrm{Po}(2)$ and $\mathrm{B}\left(20, \frac{1}{4}\right)$ respectively.

(a) Find the mean and standard deviation of $X - 3Y$. [5]

(b) Find $P(Y = 15X)$. [3]

5 Cars arrive at a fuel station at random and at a constant average rate of 13.5 per hour.

(a) Find the probability that more than 4 cars arrive during a 20-minute period. [3]

(b) Use an approximating distribution to find the probability that the number of cars that arrive during a 12-hour period is between 150 and 160 inclusive. [4]

Independently of cars, trucks arrive at the fuel station at random and at a constant average rate of 3.6 per 15-minute period.

(c) Find the probability that the total number of cars and trucks arriving at the fuel station during a 10-minute period is more than 3 and less than 7. [3]

6 A random variable X has probability density function f. The graph of $f(x)$ is a straight line segment parallel to the x-axis from $x = 0$ to $x = a$, where a is a positive constant.

(a) State, in terms of a, the median of X. [1]

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(b) Find $P(X > \frac{3}{4}a)$. [1]

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(c) Show that $\text{Var}(X) = \frac{1}{12}a^2$. [5]

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(d) Given that $P(X < b) = p$, where $0 < b < \frac{1}{2}a$, find $P\left(\frac{1}{3}b < X < a - \frac{1}{3}b\right)$ in terms of p. [2]

7 In the past, the mean time for Jenny's morning run was 28.2 minutes. She does some extra training and she wishes to test whether her mean time has been reduced. After the training Jenny takes a random sample of 40 morning runs. She decides that if the sample mean run time is less than 27 minutes she will conclude that the training has been effective. You may assume that, after the training, Jenny's run time has a standard deviation of 4.0 minutes.

(a) State suitable null and alternative hypotheses for Jenny's test. [1]

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(b) Find the probability that Jenny will make a Type I error. [3]

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(c) Jenny found that the sample mean run time was 27.2 minutes.

Explain briefly whether it is possible for her to make a Type I error or a Type II error or both. [2]

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JC22 06_9709_62/RP

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1 **(a)** A javelin thrower noted the lengths of a random sample of 50 of her throws. The sample mean was 72.3 m and an unbiased estimate of the population variance was 64.3 m^2.

Find a 92% confidence interval for the population mean length of throws by this athlete. [3]

(b) A discus thrower wishes to calculate a 92% confidence interval for the population mean length of his throws. He bases his calculation on his first 50 throws in a week.

Comment on this method. [1]

2 In the past, the mean height of plants of a particular species has been 2.3 m. A random sample of 60 plants of this species was treated with fertiliser and the mean height of these 60 plants was found to be 2.4 m. Assume that the standard deviation of the heights of plants treated with fertiliser is 0.4 m.

Carry out a test at the 2.5% significance level of whether the mean height of plants treated with fertiliser is greater than 2.3 m. [5]

3 It is known that 1.8% of children in a certain country have not been vaccinated against measles. A random sample of 200 children in this country is chosen.

(a) Use a suitable approximating distribution to find the probability that there are fewer than 3 children in the sample who have not been vaccinated against measles. [4]

(b) Justify your approximating distribution. [2]

4 The number of cars arriving at a certain road junction on a weekday morning has a Poisson distribution with mean 4.6 per minute. Traffic lights are installed at the junction and a council officer wishes to test at the 2% significance level whether there are now fewer cars arriving. He notes the number of cars arriving during a randomly chosen 2-minute period.

(a) State suitable null and alternative hypotheses for the test. [1]

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(b) Find the critical region for the test. [4]

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The officer notes that, during a randomly chosen 2-minute period on a weekday morning, exactly 5 cars arrive at the junction.

(c) Carry out the test. [2]

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(d) State, with a reason, whether it is possible that a Type I error has been made in carrying out the test in part **(c)**. [1]

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The number of cars arriving at another junction on a weekday morning also has a Poisson distribution with mean 4.6 per minute.

(e) Use a suitable approximating distribution to find the probability that more than 300 cars will arrive at this junction in an hour. [3]

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5 A random variable X has probability density function given by

$$f(x) = \begin{cases} \frac{3}{16}(4x - x^2) & 2 \leqslant x \leqslant 4, \\ 0 & \text{otherwise.} \end{cases}$$

(a) Show that $E(X) = \frac{11}{4}$. [3]

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(b) Find Var(X). [3]

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(c) Given that the median of X is m, find $P(m < X < 3)$. [3]

6 The masses, in kilograms, of large and small sacks of grain have the distributions N(53, 11) and N(14, 3) respectively.

(a) Find the probability that the mass of a randomly chosen large sack is greater than four times the mass of a randomly chosen small sack. [5]

(b) A lift can safely carry a maximum mass of 1000 kg.

Find the probability that the lift can safely carry 12 randomly chosen large sacks and 25 randomly chosen small sacks. [5]

7 X is a random variable with distribution Po(2.90). A random sample of 100 values of X is taken.

Find the probability that the sample mean is less than 2.88. [5]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

3849072491

MATHEMATICS **9709/63**

Paper 6 Probability & Statistics 2 **May/June 2022**

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
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- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

1 The number of characters in emails sent by a particular company is modelled by the distribution $N(1250, 480^2)$.

Find the probability that the mean number of characters in a random sample of 100 emails sent by the company is more than 1300. [3]

2 Anton believes that 10% of students at his college are left-handed. Aliya believes that this is an under-estimate. She plans to carry out a hypothesis test of the null hypothesis $p = 0.1$ against the alternative hypothesis $p > 0.1$, where p is the actual proportion of students at the college that are left-handed. She chooses a random sample of 20 students from the college. She will reject the null hypothesis if at least 5 of these students are left-handed.

(a) Explain what is meant by a Type I error in this context. [1]

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(b) Find the probability of a Type I error in the test. [3]

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(c) Given that the true value of p is 0.3, find the probability of a Type II error in the test. [2]

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3 Batteries of type A are known to have a mean life of 150 hours. It is required to test whether a new type of battery, type B, has a shorter mean life than type A batteries.

(a) Give a reason for using a sample rather than the whole population in carrying out this test. [1]

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A random sample of 120 type B batteries are tested and it is found that their mean life is 147 hours, and an unbiased estimate of the population variance is 225 hours^2.

(b) Test, at the 2% significance level, whether type B batteries have a shorter mean life than type A batteries. [5]

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(c) Calculate a 94% confidence interval for the population mean life of type B batteries. [3]

4 Each box of Seeds & Raisins contains S grams of seeds and R grams of raisins. The weight of a box, when empty, is B grams. S, R and B are independent random variables, where $S \sim N(300, 45)$, $R \sim N(200, 25)$ and $B \sim N(15, 4)$. A full box of Seeds & Raisins is chosen at random.

(a) Find the probability that the total weight of the box and its contents is more than 500 grams. [5]

(b) Find the probability that the weight of seeds in the box is less than 1.4 times the weight of raisins in the box. [5]

5 The number of clients who arrive at an information desk has a Poisson distribution with mean 2.2 per 5-minute period.

(a) Find the probability that, in a randomly chosen 15-minute period, exactly 6 clients arrive at the desk. [3]

(b) If more than 4 clients arrive during a 5-minute period, they cannot all be served.

Find the probability that, during a randomly chosen 5-minute period, not all the clients who arrive at the desk can be served. [2]

(c) Use a suitable approximating distribution to find the probability that, during a randomly chosen 1-hour period, fewer than 20 clients arrive at the desk. [4]

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6 A random sample of 5 values of a variable X is given below.

$$2 \qquad 3 \qquad 3 \qquad 5 \qquad a$$

(a) Find an expression, in terms of a, for the mean of these values. [1]

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It is given that an unbiased estimate of the population variance of X, using these values, is 4. It is also given that a is positive.

(b) Find and simplify a quadratic equation in terms of a and hence find the value of a. [3]

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7 The random variables X and W have probability density functions f and g defined as follows:

$$f(x) = \begin{cases} p(a^2 - x^2) & 0 \leqslant x \leqslant a, \\ 0 & \text{otherwise,} \end{cases}$$

$$g(w) = \begin{cases} q(a^2 - w^2) & -a \leqslant w \leqslant a, \\ 0 & \text{otherwise,} \end{cases}$$

where a, p and q are constants.

(a) **(i)** Write down the value of $P(X \geqslant 0)$. [1]

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(ii) Write down the value of $P(W \geqslant 0)$. [1]

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(iii) Write down an expression for q in terms of p only. [1]

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(b) Given that $E(X) = 3$, find the value of a. [6]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

6270513510

MATHEMATICS **9709/62**

Paper 6 Probability & Statistics 2 **February/March 2022**

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
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- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **12** pages.

1 The lengths, in millimetres, of a random sample of 12 rods made by a certain machine are as follows.

200	201	198	202	200	199	199	201	197	202	200	199

(a) Find unbiased estimates of the population mean and variance. [3]

(b) Give a statistical reason why these estimates may not be reliable. [1]

2 Harry has a five-sided spinner with sectors coloured blue, green, red, yellow and black. Harry thinks the spinner may be biased. He plans to carry out a hypothesis test with the following hypotheses.

$$H_0\text{: P(the spinner lands on blue)} = \tfrac{1}{5}$$
$$H_1\text{: P(the spinner lands on blue)} \neq \tfrac{1}{5}$$

Harry spins the spinner 300 times. It lands on blue on 45 spins.

Use a suitable approximation to carry out Harry's test at the 5% significance level. [5]

3 A random sample of 500 households in a certain town was chosen. Using this sample, a confidence interval for the proportion, p, of all households in that town that owned two or more cars was found to be $0.355 < p < 0.445$.

Find the confidence level of this confidence interval. Give your answer correct to the nearest integer. [5]

4 In the past the time, in minutes, taken by students to complete a certain challenge had mean 25.5 and standard deviation 5.2. A new challenge is devised and it is expected that students will take, on average, less than 25.5 minutes to complete this challenge. A random sample of 40 students is chosen and their mean time for the new challenge is found to be 23.7 minutes.

(a) Assuming that the standard deviation of the time for the new challenge is 5.2 minutes, test at the 1% significance level whether the population mean time for the new challenge is less than 25.5 minutes. [5]

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(b) State, with a reason, whether it is possible that a Type I error was made in the test in part **(a)**. [1]

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5 The heights of buildings in a large city are normally distributed with mean 18.3 m and standard deviation 2.5 m.

(a) Find the probability that the total height of 5 randomly chosen buildings in the city is more than 95 m. [4]

(b) Find the probability that the difference between the heights of two randomly chosen buildings in the city is less than 1 m. [5]

6 In a game a ball is rolled down a slope and along a track until it stops. The distance, in metres, travelled by the ball is modelled by the random variable X with probability density function

$$f(x) = \begin{cases} -k(x-1)(x-3) & 1 \leqslant x \leqslant 3, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

(a) Without calculation, explain why $E(X) = 2$. [1]

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(b) Show that $k = \frac{3}{4}$. [3]

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(c) Find $\text{Var}(X)$. [3]

One turn consists of rolling the ball 3 times and noting the largest value of X obtained. If this largest value is greater than 2.5, the player scores a point.

(d) Find the probability that on a particular turn the player scores a point. [4]

7 **(a)** Two ponds, A and B, each contain a large number of fish. It is known that 2.4% of fish in pond A are carp and 1.8% of fish in pond B are carp. Random samples of 50 fish from pond A and 60 fish from pond B are selected.

Use appropriate Poisson approximations to find the following probabilities.

(i) The samples contain at least 2 carp from pond A and at least 2 carp from pond B. [3]

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(ii) The samples contain at least 4 carp altogether. [3]

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(b) The random variables X and Y have the distributions $\mathrm{Po}(\lambda)$ and $\mathrm{Po}(\mu)$ respectively. It is given that

- $\mathrm{P}(X = 0) = [\mathrm{P}(Y = 0)]^2$,
- $\mathrm{P}(X = 2) = k[\mathrm{P}(Y = 1)]^2$, where k is a non-zero constant.

Find the value of k. [4]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

2708090108

MATHEMATICS **9709/61**

Paper 6 Probability & Statistics 2 **October/November 2021**

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS
- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
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- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION
- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

JC21 11_9709_61/FP

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1 It is known that the height H, in metres, of trees of a certain kind has the distribution N(12.5, 10.24). A scientist takes a random sample of 25 trees of this kind and finds the sample mean, $\overline{H}$, of the heights.

(a) State the distribution of $\overline{H}$, giving the values of any parameters. [2]

(b) Find $P(12 < \overline{H} < 13)$. [3]

2 The number of enquiries received per day at a customer service desk has a Poisson distribution with mean 45.2. If more than 60 enquiries are received in a day, the customer service desk cannot deal with them all.

Use a suitable approximating distribution to find the probability that, on a randomly chosen day, the customer service desk cannot deal with all the enquiries that are received. [4]

3 A random sample of 75 students at a large college was selected for a survey. 15 of these students said that they owned a car. From this result an approximate $\alpha\%$ confidence interval for the proportion of all students at the college who own a car was calculated. The width of this interval was found to be 0.162.

Calculate the value of α correct to 2 significant figures. [5]

4 A random variable X has probability density function given by

$$f(x) = \begin{cases} \frac{1}{18}(9 - x^2) & 0 \leqslant x \leqslant 3, \\ 0 & \text{otherwise.} \end{cases}$$

(a) Find $P(X < 1.2)$. [3]

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(b) Find $E(X)$. [3]

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The median of X is m.

(c) Show that $m^3 - 27m + 27 = 0$. [3]

5 **(a)** The proportion of people having a particular medical condition is 1 in 100 000. A random sample of 2500 people is obtained. The number of people in the sample having the condition is denoted by X.

(i) State, with a justification, a suitable approximating distribution for X, giving the values of any parameters. [2]

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(ii) Use the approximating distribution to calculate $P(X > 0)$. [2]

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(b) The percentage of people having a different medical condition is thought to be 30%. A researcher suspects that the true percentage is less than 30%. In a medical trial a random sample of 28 people was selected and 4 people were found to have this condition.

Use a binomial distribution to test the researcher's suspicion at the 2% significance level. [5]

6 The random variable T denotes the time, in seconds, for 100 m races run by Tania. T is normally distributed with mean μ and variance σ^2. A random sample of 40 races run by Tania gave the following results.

$$n = 40 \qquad \Sigma t = 560 \qquad \Sigma t^2 = 7850$$

(a) Calculate unbiased estimates of μ and σ^2. [3]

The random variable S denotes the time, in seconds, for 100 m races run by Suki. S has the independent distribution $N(14.2, 0.3)$.

(b) Using your answers to part **(a)**, find the probability that, in a randomly chosen 100 m race, Suki's time will be at least 0.1 s more than Tania's time. [5]

7 The masses, in grams, of apples from a certain farm have mean μ and standard deviation 5.2. The farmer says that the value of μ is 64.6. A quality control inspector claims that the value of μ is actually less than 64.6. In order to test his claim he chooses a random sample of 100 apples from the farm.

(a) The mean mass of the 100 apples is found to be 63.5 g.

Carry out the test at the 2.5% significance level. [5]

(b) Later another test of the same hypotheses at the 2.5% significance level, with another random sample of 100 apples from the same farm, is carried out.

Given that the value of μ is in fact 62.7, calculate the probability of a Type II error. [5]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME			
CENTRE NUMBER		CANDIDATE NUMBER	

8874233119

MATHEMATICS **9709/62**

Paper 6 Probability & Statistics 2 **October/November 2021**

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
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INFORMATION

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- The number of marks for each question or part question is shown in brackets [ ].

This document has **12** pages.

JC21 11_9709_62/RP

1 The mass, in kilograms, of a block of cheese sold in a supermarket is denoted by the random variable M. The masses of a random sample of 40 blocks are summarised as follows.

$$n = 40 \qquad \Sigma m = 20.50 \qquad \Sigma m^2 = 10.7280$$

(a) Calculate unbiased estimates of the population mean and variance of M. [3]

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(b) The price, \$$P$, of a block of cheese of mass M kg is found using the formula $P = 11M + 0.50$.

Find estimates of the population mean and variance of P. [3]

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2 Andy and Jessica are doing a survey about musical preferences. They plan to choose a representative sample of six students from the 256 students at their college.

(a) Andy suggests that they go to the music building during the lunch hour and choose six students at random from the students who are there.

Give a reason why this method is unsatisfactory. [1]

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(b) Jessica decides to use another method. She numbers all the students in the college from 1 to 256. Then she uses her calculator and generates the following random numbers.

204393 162007 204028 587119 207395

From these numbers, she obtains six student numbers. The first three of her student numbers are 204, 162 and 7.

Continue Jessica's method to obtain the next three student numbers. [2]

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3 The probability that a certain spinner lands on red on any spin is p. The spinner is spun 140 times and it lands on red 35 times.

(a) Find an approximate 96% confidence interval for p. [3]

From three further experiments, Jack finds a 90% confidence interval, a 95% confidence interval and a 99% confidence interval for p.

(b) Find the probability that exactly two of these confidence intervals contain the true value of p. [3]

4 A certain kind of firework is supposed to last for 30 seconds, on average, after it is lit. An inspector suspects that the fireworks actually last a shorter time than this, on average. He takes a random sample of 100 fireworks of this kind. Each firework in the sample is lit and the time it lasts is noted.

(a) Give a reason why it is necessary to take a sample rather than testing all the fireworks of this kind. [1]

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It is given that the population standard deviation of the times that fireworks of this kind last is 5 seconds.

(b) The mean time lasted by the 100 fireworks in the sample is found to be 29 seconds.

Test the inspector's suspicion at the 1% significance level. [5]

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(c) State with a reason whether the Central Limit theorem was needed in the solution to part **(b)**. [1]

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5 In a certain large document, typing errors occur at random and at a constant mean rate of 0.2 per page.

(a) Find the probability that there are fewer than 3 typing errors in 10 randomly chosen pages. [2]

(b) Use an approximating distribution to find the probability that there are more than 50 typing errors in 200 randomly chosen pages. [4]

In the same document, formatting errors occur at random and at a constant mean rate of 0.3 per page.

(c) Find the probability that the total number of typing and formatting errors in 20 randomly chosen pages is between 8 and 11 inclusive. [3]

6 A machine is supposed to produce random digits. Bob thinks that the machine is not fair and that the probability of it producing the digit 0 is less than $\frac{1}{10}$. In order to test his suspicion he notes the number of times the digit 0 occurs in 30 digits produced by the machine. He carries out a test at the 10% significance level.

(a) State suitable null and alternative hypotheses. [1]

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(b) Find the rejection region for the test. [4]

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(c) State the probability of a Type I error. [1]

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It is now given that the machine actually produces a 0 once in every 40 digits, on average.

(d) Find the probability of a Type II error. [3]

(e) Explain the meaning of a Type II error in this context. [1]

7 **(a)** The probability density function of the random variable X is given by

$$f(x) = \begin{cases} kx(4-x) & 0 \leqslant x \leqslant 2, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

(i) Show that $k = \frac{3}{16}$. [3]

(ii) Find $E(X)$. [3]

(b) The random variable Y has the following properties.

- Y takes values between 0 and 5 only.
- The probability density function of Y is symmetrical.

Given that $P(Y < a) = 0.2$, find $P(2.5 < Y < 5 - a)$ illustrating your method with a sketch on the axes provided. [3]

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Additional Page

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Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

* 4 4 9 9 2 1 9 5 6 9 *

MATHEMATICS **9709/63**

Paper 6 Probability & Statistics 2 **October/November 2021**

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
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JC21 11_9709_63/RP

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1 It is known that the height H, in metres, of trees of a certain kind has the distribution N(12.5, 10.24). A scientist takes a random sample of 25 trees of this kind and finds the sample mean, $\overline{H}$, of the heights.

(a) State the distribution of $\overline{H}$, giving the values of any parameters. [2]

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(b) Find $P(12 < \overline{H} < 13)$. [3]

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2 The number of enquiries received per day at a customer service desk has a Poisson distribution with mean 45.2. If more than 60 enquiries are received in a day, the customer service desk cannot deal with them all.

Use a suitable approximating distribution to find the probability that, on a randomly chosen day, the customer service desk cannot deal with all the enquiries that are received. [4]

3 A random sample of 75 students at a large college was selected for a survey. 15 of these students said that they owned a car. From this result an approximate $\alpha\%$ confidence interval for the proportion of all students at the college who own a car was calculated. The width of this interval was found to be 0.162.

Calculate the value of α correct to 2 significant figures. [5]

4 A random variable X has probability density function given by

$$f(x) = \begin{cases} \frac{1}{18}(9 - x^2) & 0 \leqslant x \leqslant 3, \\ 0 & \text{otherwise.} \end{cases}$$

(a) Find $P(X < 1.2)$. [3]

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(b) Find $E(X)$. [3]

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The median of X is m.

(c) Show that $m^3 - 27m + 27 = 0$. [3]

5 **(a)** The proportion of people having a particular medical condition is 1 in 100 000. A random sample of 2500 people is obtained. The number of people in the sample having the condition is denoted by X.

(i) State, with a justification, a suitable approximating distribution for X, giving the values of any parameters. [2]

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(ii) Use the approximating distribution to calculate $P(X > 0)$. [2]

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(b) The percentage of people having a different medical condition is thought to be 30%. A researcher suspects that the true percentage is less than 30%. In a medical trial a random sample of 28 people was selected and 4 people were found to have this condition.

Use a binomial distribution to test the researcher's suspicion at the 2% significance level. [5]

6 The random variable T denotes the time, in seconds, for 100 m races run by Tania. T is normally distributed with mean μ and variance σ^2. A random sample of 40 races run by Tania gave the following results.

$$n = 40 \qquad \Sigma t = 560 \qquad \Sigma t^2 = 7850$$

(a) Calculate unbiased estimates of μ and σ^2. [3]

The random variable S denotes the time, in seconds, for 100 m races run by Suki. S has the independent distribution $N(14.2,\ 0.3)$.

(b) Using your answers to part **(a)**, find the probability that, in a randomly chosen 100 m race, Suki's time will be at least 0.1 s more than Tania's time. [5]

7 The masses, in grams, of apples from a certain farm have mean μ and standard deviation 5.2. The farmer says that the value of μ is 64.6. A quality control inspector claims that the value of μ is actually less than 64.6. In order to test his claim he chooses a random sample of 100 apples from the farm.

(a) The mean mass of the 100 apples is found to be 63.5 g.

Carry out the test at the 2.5% significance level. [5]

(b) Later another test of the same hypotheses at the 2.5% significance level, with another random sample of 100 apples from the same farm, is carried out.

Given that the value of μ is in fact 62.7, calculate the probability of a Type II error. [5]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

1417278707

MATHEMATICS **9709/61**

Paper 6 Probability & Statistics 2 **May/June 2021**

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
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- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

1 Accidents at two factories occur randomly and independently. On average, the numbers of accidents per month are 3.1 at factory A and 1.7 at factory B.

Find the probability that the total number of accidents in the two factories during a 2-month period is more than 3. [4]

2 The time, in minutes, taken by students to complete a test has the distribution N(125, 36).

(a) Find the probability that the mean time taken to complete the test by a random sample of 40 students is less than 123 minutes. [3]

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(b) Explain whether it was necessary to use the Central Limit theorem in the solution to part **(a)**. [1]

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3 The graph of the probability density function of a random variable X is symmetrical about the line $x = 4$.

Given that $P(X < 5) = \frac{20}{27}$, find $P(3 < X < 5)$. [2]

4 100 randomly chosen adults each throw a ball once. The length, l metres, of each throw is recorded. The results are summarised below.

$$n = 100 \qquad \Sigma l = 3820 \qquad \Sigma l^2 = 182\,200$$

Calculate a 94% confidence interval for the population mean length of throws by adults. [6]

5 On average, 1 in 75 000 adults has a certain genetic disorder.

(a) Use a suitable approximating distribution to find the probability that, in a random sample of 10 000 people, at least 1 has the genetic disorder. [3]

(b) In a random sample of n people, where n is large, the probability that no-one has the genetic disorder is more than 0.9.

Find the largest possible value of n. [4]

6 The probability density function, f, of a random variable X is given by

$$f(x) = \begin{cases} k(6x - x^2) & 0 \leqslant x \leqslant 6, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

State the value of $E(X)$ and show that $Var(X) = \frac{9}{5}$. [6]

7 The masses, in kilograms, of large and small sacks of flour have the distributions $N(55, 3^2)$ and $N(27, 2.5^2)$ respectively.

(a) Some sacks are loaded onto a boat. The maximum load of flour that the boat can carry safely is 340 kg.

Find the probability that the boat can carry safely 3 randomly chosen large sacks of flour and 6 randomly chosen small sacks of flour. [5]

(b) Find the probability that the mass of a randomly chosen large sack of flour is greater than the total mass of two randomly chosen small sacks of flour. [5]

  [Turn over

8 At a certain large school it was found that the proportion of students not wearing correct uniform was 0.15. The school sent a letter to parents asking them to ensure that their children wear the correct uniform. The school now wishes to test whether the proportion not wearing correct uniform has been reduced.

(a) It is suggested that a random sample of the students in Grade 12 should be used for the test.

Give a reason why this would not be an appropriate sample. [1]

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A suitable sample of 50 students is selected and the number not wearing correct uniform is noted. This figure is used to carry out a test at the 5% significance level.

(b) State suitable null and alternative hypotheses. [1]

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(c) Use a binomial distribution to find the probability of a Type I error. You must justify your answer fully. [5]

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(d) In fact 4 students out of the 50 are not wearing correct uniform.

State the conclusion of the test, explaining your answer. [2]

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(e) State, with a reason, which of the errors, Type I or Type II, may have been made. [2]

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Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

7977464529

MATHEMATICS **9709/62**

Paper 6 Probability & Statistics 2 **May/June 2021**

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
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- Do **not** write on any bar codes.
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- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **12** pages. Any blank pages are indicated.

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1 In a game, a ball is thrown and lands in one of 4 slots, labelled A, B, C and D. Raju wishes to test whether the probability that the ball will land in slot A is $\frac{1}{4}$.

(a) State suitable null and alternative hypotheses for Raju's test. [1]

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The ball is thrown 100 times and it lands in slot A 15 times.

(b) Use a suitable approximating distribution to carry out the test at the 2% significance level. [5]

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2 The random variable X has the distribution $B(400, 0.01)$.

(a) Find $\text{Var}(4X + 2)$. [3]

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(b) **(i)** State an appropriate approximating distribution for X, giving the values of any parameters. Justify your choice of approximating distribution. [2]

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(ii) Use your approximating distribution to find $P(2 \leqslant X \leqslant 5)$. [2]

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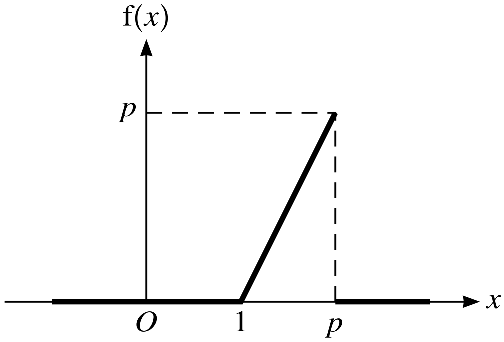

The random variable X takes values in the range $1 \leqslant x \leqslant p$, where p is a constant. The graph of the probability density function of X is shown in the diagram.

(a) Show that $p = 2$. [2]

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(b) Find $E(X)$. [5]

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  [Turn over

4 Wendy's journey to work consists of three parts: walking to the train station, riding on the train and then walking to the office. The times, in minutes, for the three parts of her journey are independent and have the distributions $N(15.0,\ 1.1^2)$, $N(32.0,\ 3.5^2)$ and $N(8.6,\ 1.2^2)$ respectively.

(a) Find the mean and variance of the total time for Wendy's journey. [2]

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If Wendy's journey takes more than 60 minutes, she is late for work.

(b) Find the probability that, on a randomly chosen day, Wendy will be late for work. [3]

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(c) Find the probability that the mean of Wendy's journey times over 15 randomly chosen days will be less than 54.5 minutes. [3]

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5 The time, in minutes, spent by customers at a particular gym has the distribution $N(\mu, 38.2)$. In the past the value of μ has been 42.4. Following the installation of some new equipment the management wishes to test whether the value of μ has changed.

(a) State what is meant by a Type I error in this context. [1]

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(b) The mean time for a sample of 20 customers is found to be 45.6 minutes.

Test at the 2.5% significance level whether the value of μ has changed. [5]

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6 The heights, h centimetres, of a random sample of 100 fully grown animals of a certain species were measured. The results are summarised below.

$$n = 100 \qquad \Sigma h = 7570 \qquad \Sigma h^2 = 588\,050$$

(a) Find unbiased estimates of the population mean and variance. [3]

(b) Calculate a 99% confidence interval for the mean height of animals of this species. [3]

Four random samples were taken and a 99% confidence interval for the population mean, μ, was found from each sample.

(c) Find the probability that all four of these confidence intervals contain the true value of μ. [2]

7 Customers arrive at a particular shop at random times. It has been found that the mean number of customers who arrive during a 5-minute interval is 2.1.

(a) Find the probability that exactly 4 customers arrive during a 10-minute interval. [2]

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(b) Find the probability that at least 4 customers arrive during a 20-minute interval. [2]

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(c) Use a suitable approximating distribution to find the probability that fewer than 40 customers arrive during a 2-hour interval. [4]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

5443685767

MATHEMATICS **9709/63**

Paper 6 Probability & Statistics 2 **May/June 2021**

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
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- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

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1 The number of goals scored by a team in a match is independent of other matches, and is denoted by the random variable X, which has a Poisson distribution with mean 1.36. A supporter offers to make a donation of \$5 to the team for each goal that they score in the next 10 matches.

Find the expectation and standard deviation of the amount that the supporter will pay. [5]

2 In the past, the time, in hours, for a particular train journey has had mean 1.40 and standard deviation 0.12. Following the introduction of some new signals, it is required to test whether the mean journey time has decreased.

(a) State what is meant by a Type II error in this context. [1]

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(b) The mean time for a random sample of 50 journeys is found to be 1.36 hours.

Assuming that the standard deviation of journey times is still 0.12 hours, test at the 2.5% significance level whether the population mean journey time has decreased. [5]

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(c) State, with a reason, which of the errors, Type I or Type II, might have been made in the test in part **(b)**. [2]

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3 The local council claims that the average number of accidents per year on a particular road is 0.8. Jane claims that the true average is greater than 0.8. She looks at the records for a random sample of 3 recent years and finds that the total number of accidents during those 3 years was 5.

(a) Assume that the number of accidents per year follows a Poisson distribution.

(i) State null and alternative hypotheses for a test of Jane's claim. [1]

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(ii) Test at the 5% significance level whether Jane's claim is justified. [4]

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(b) Jane finds that the number of accidents per year has been gradually increasing over recent years.

State how this might affect the validity of the test carried out in part **(a)(ii)**. [1]

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4 The masses, m kilograms, of flour in a random sample of 90 sacks of flour are summarised as follows.

$$n = 90 \qquad \Sigma m = 4509 \qquad \Sigma m^2 = 225\,950$$

(a) Find unbiased estimates of the population mean and variance. [3]

(b) Calculate a 98% confidence interval for the population mean. [3]

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(c) Explain why it was necessary to use the Central Limit theorem in answering part **(b)**. [1]

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(d) Find the probability that the confidence interval found in part **(b)** is wholly above the true value of the population mean. [2]

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5 Most plants of a certain type have three leaves. However, it is known that, on average, 1 in 10 000 of these plants have four leaves, and plants with four leaves are called 'lucky'. The number of lucky plants in a random sample of 25 000 plants is denoted by X.

(a) State, with a justification, an approximating distribution for X, giving the values of any parameters. [2]

Use your approximating distribution to answer parts **(b)** and **(c)**.

(b) Find $P(X \leqslant 3)$. [2]

(c) Given that $P(X = k) = 2P(X = k + 1)$, find k. [2]

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The number of lucky plants in a random sample of n plants, where n is large, is denoted by Y.

(d) Given that $P(Y \geqslant 1) = 0.963$, correct to 3 significant figures, use a suitable approximating distribution to find the value of n. [3]

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6 Alethia models the length of time, in minutes, by which her train is late on any day by the random variable X with probability density function given by

$$f(x) = \begin{cases} \frac{3}{8000}(x-20)^2 & 0 \leqslant x \leqslant 20, \\ 0 & \text{otherwise.} \end{cases}$$

(a) Find the probability that the train is more than 10 minutes late on each of two randomly chosen days. [4]

(b) Find E(X). [4]

(c) The median of X is denoted by m.

Show that m satisfies the equation $(m - 20)^3 = -4000$, and hence find m correct to 3 significant figures. [4]

(d) State one way in which Alethia's model may be unrealistic. [1]

Additional Page

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Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME	
CENTRE NUMBER	
CANDIDATE NUMBER	

9228178848

MATHEMATICS **9709/62**

Paper 6 Probability & Statistics 2 **February/March 2021**

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS
- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
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- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION
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JC21 03_9709_62/2R

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1 A construction company notes the time, t days, that it takes to build each house of a certain design. The results for a random sample of 60 such houses are summarised as follows.

$$\Sigma t = 4820 \qquad \Sigma t^2 = 392\,050$$

(a) Calculate a 98% confidence interval for the population mean time. [6]

(b) Explain why it was necessary to use the Central Limit theorem in part **(a)**. [1]

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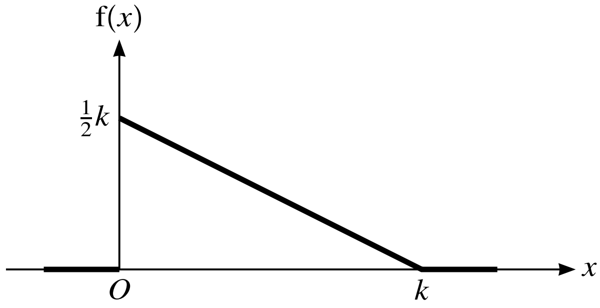

The diagram shows the graph of the probability density function, f, of a random variable X.

(a) Find the value of the constant k. [2]

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(b) Using this value of k, find $\mathrm{f}(x)$ for $0 \leqslant x \leqslant k$ and hence find $\mathrm{E}(X)$. [3]

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(c) Find the value of p such that $P(p < X < 1) = 0.25$. [4]

3 An architect wishes to investigate whether the buildings in a certain city are higher, on average, than buildings in other cities. He takes a large random sample of buildings from the city and finds the mean height of the buildings in the sample. He calculates the value of the test statistic, z, and finds that $z = 2.41$.

(a) Explain briefly whether he should use a one-tail test or a two-tail test. [1]

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(b) Carry out the test at the 1% significance level. [3]

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4 On average, 1 in 400 microchips made at a certain factory are faulty. The number of faulty microchips in a random sample of 1000 is denoted by X.

(a) State the distribution of X, giving the values of any parameters. [1]

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(b) State an approximating distribution for X, giving the values of any parameters. [2]

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(c) Use this approximating distribution to find each of the following.

(i) $P(X = 4)$. [2]

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(ii) $P(2 \leqslant X \leqslant 4)$. [2]

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(d) Use a suitable approximating distribution to find the probability that, in a random sample of 700 microchips, there will be at least 1 faulty one. [3]

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5 The volumes, in litres, of juice in large and small bottles have the distributions N(5.10, 0.0102) and N(2.51, 0.0036) respectively.

(a) Find the probability that the total volume of juice in 3 randomly chosen large bottles and 4 randomly chosen small bottles is less than 25.5 litres. [5]

(b) Find the probability that the volume of juice in a randomly chosen large bottle is at least twice the volume of juice in a randomly chosen small bottle. [5]

6 It is known that 8% of adults in a certain town own a Chantor car. After an advertising campaign, a car dealer wishes to investigate whether this proportion has increased. He chooses a random sample of 25 adults from the town and notes how many of them own a Chantor car.

(a) He finds that 4 of the 25 adults own a Chantor car.

Carry out a hypothesis test at the 5% significance level. [5]

(b) Explain which of the errors, Type I or Type II, might have been made in carrying out the test in part **(a)**. [2]

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Later, the car dealer takes another random sample of 25 adults from the town and carries out a similar hypothesis test at the 5% significance level.

(c) Find the probability of a Type I error. [3]

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Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

4890560004

MATHEMATICS **9709/61**

Paper 6 Probability & Statistics 2 **October/November 2020**

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
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- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

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1 It is known that, on average, 1 in 300 flowers of a certain kind are white. A random sample of 200 flowers of this kind is selected.

(a) Use an appropriate approximating distribution to find the probability that more than 1 flower in the sample is white. [3]

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(b) Justify the approximating distribution used in part **(a)**. [1]

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The probability that a randomly chosen flower of another kind is white is 0.02. A random sample of 150 of these flowers is selected.

(c) Use an appropriate approximating distribution to find the probability that the total number of white flowers in the two samples is less than 4. [3]

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2 In a survey, a random sample of 250 adults in Fromleigh were asked to fill in a questionnaire about their travel.

(a) It was found that 102 adults in the sample travel by bus. Find an approximate 90% confidence interval for the proportion of all the adults in Fromleigh who travel by bus. [3]

(b) The survey included a question about the amount, x dollars, spent on travel per year. The results are summarised as follows.

$$n = 250 \qquad \Sigma x = 50\,460 \qquad \Sigma x^2 = 19\,854\,200$$

Find unbiased estimates of the population mean and variance of the amount spent per year on travel. [3]

A councillor wanted to select a random sample of houses in Fromleigh. He planned to select the first house on each of the 143 streets in Fromleigh.

(c) Explain why this would not provide a random sample. [1]

3 The masses, in kilograms, of female and male animals of a certain species have the distributions $N(102, 27^2)$ and $N(170, 55^2)$ respectively.

Find the probability that a randomly chosen female has a mass that is less than half the mass of a randomly chosen male. [6]

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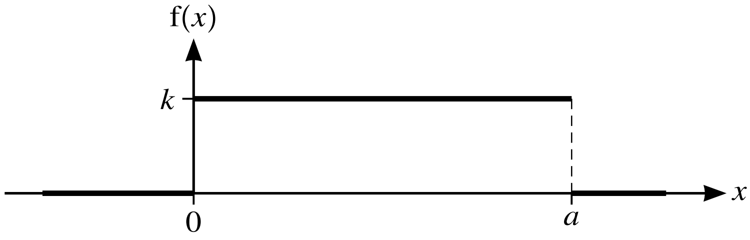

The diagram shows the probability density function, $f(x)$, of a random variable X. For $0 \leqslant x \leqslant a$, $f(x) = k$; elsewhere $f(x) = 0$.

(a) Express k in terms of a. [1]

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(b) Given that $\mathrm{Var}(X) = 3$, find a. [4]

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5 The number of absences per week by workers at a factory has the distribution Po(2.1).

(a) Find the standard deviation of the number of absences per week. [1]

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(b) Find the probability that the number of absences in a 2-week period is at least 2. [3]

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(c) Find the probability that the number of absences in a 3-week period is more than 4 and less than 8. [2]

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Following a change in working conditions, the management wished to test whether the mean number of absences has decreased. They found that, in a randomly chosen 3-week period, there were exactly 2 absences.

(d) Carry out the test at the 10% significance level. [5]

(e) State, with a reason, which of the errors, Type I or Type II, might have been made in carrying out the test in part **(d)**. [2]

6 The time, in minutes, for Anjan's journey to work on Mondays has mean 38.4 and standard deviation 6.9.

(a) Find the probability that Anjan's mean journey time for a random sample of 30 Mondays is between 38 and 40 minutes. [5]

Anjan wishes to test whether his mean journey time is different on Tuesdays. He chooses a random sample of 30 Tuesdays and finds that his mean journey time for these 30 Tuesdays is 40.2 minutes. Assume that the standard deviation for his journey time on Tuesdays is 6.9 minutes.

(b) **(i)** State, with a reason, whether Anjan should use a one-tail or a two-tail test. [1]

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(ii) Carry out the test at the 10% significance level. [5]

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(iii) Explain whether it was necessary to use the Central Limit theorem in part **(b)(ii)**. [1]

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Additional Page

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Cambridge Assessment International Education

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CANDIDATE NAME

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8854990677

MATHEMATICS **9709/62**

Paper 6 Probability & Statistics 2 **October/November 2020**

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
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INFORMATION

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- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Blank pages are indicated.

1 On average, 1 in 50 000 people have a certain gene.

Use a suitable approximating distribution to find the probability that more than 2 people in a random sample of 150 000 have the gene. [3]

2 A six-sided die has faces marked 1, 2, 3, 4, 5, 6. When the die is thrown 300 times it shows a six on 56 throws.

(a) Calculate an approximate 96% confidence interval for the probability that the die shows a six on one throw. [3]

(b) Maroulla claims that the die is biased.

Use your answer to part **(a)** to comment on this claim. [1]

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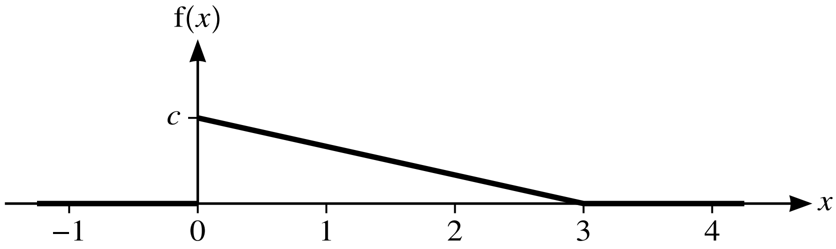

A random variable X takes values between 0 and 3 only and has probability density function as shown in the diagram, where c is a constant.

(a) Show that $c = \frac{2}{3}$. [1]

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(b) Find $P(X > 2)$. [2]

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(c) Calculate E(X). [4]

4 The areas, $X\,\text{cm}^2$, of petals of a certain kind of flower have mean $\mu\,\text{cm}^2$. In the past it has been found that $\mu = 8.9$. Following a change in the climate, a botanist claims that the mean is no longer 8.9. The areas of a random sample of 200 petals from this kind of flower are measured, and the results are summarized by

$$\Sigma x = 1850, \qquad \Sigma x^2 = 17\,850.$$

Test the botanist's claim at the 2.5% significance level. [8]

5 Customers arrive at a shop at a constant average rate of 2.3 per minute.

(a) State another condition for the number of customers arriving per minute to have a Poisson distribution. [1]

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It is now given that the number of customers arriving per minute has the distribution Po(2.3).

(b) Find the probability that exactly 3 customers arrive during a 1-minute period. [2]

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(c) Find the probability that more than 3 customers arrive during a 2-minute period. [3]

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(d) Five 1-minute periods are chosen at random. Find the probability that no customers arrive during exactly 2 of these 5 periods. [3]

6 A biscuit manufacturer claims that, on average, 1 in 3 packets of biscuits contain a prize offer. Gerry suspects that the proportion of packets containing the prize offer is less than 1 in 3. In order to test the manufacturer's claim, he buys 20 randomly selected packets. He finds that exactly 2 of these packets contain the prize offer.

(a) Carry out the test at the 10% significance level. [5]

(b) Maria also suspects that the proportion of packets containing the prize offer is less than 1 in 3. She also carries out a significance test at the 10% level using 20 randomly selected packets. She will reject the manufacturer's claim if she finds that there are 3 or fewer packets containing the prize offer.

Find the probability of a Type II error in Maria's test if the proportion of packets containing the prize offer is actually 1 in 7. [3]

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(c) Explain what is meant by a Type II error in this context. [1]

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7 Before a certain type of book is published it is checked for errors, which are then corrected. For costing purposes each error is classified as either minor or major. The numbers of minor and major errors in a book are modelled by the independent distributions N(380, 140) and N(210, 80) respectively. You should assume that no continuity corrections are needed when using these models.

A book of this type is chosen at random.

(a) Find the probability that the number of minor errors is at least 200 more than the number of major errors. [5]

The costs of correcting a minor error and a major error are 20 cents and 50 cents respectively.

(b) Find the probability that the total cost of correcting the errors in the book is less than \$190. [5]

Additional Page

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* 3 6 7 4 1 4 6 9 2 2 *

MATHEMATICS **9709/63**

Paper 6 Probability & Statistics 2 **October/November 2020**

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

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JC20 11_9709_63/RP

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1 It is known that, on average, 1 in 300 flowers of a certain kind are white. A random sample of 200 flowers of this kind is selected.

(a) Use an appropriate approximating distribution to find the probability that more than 1 flower in the sample is white. [3]

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(b) Justify the approximating distribution used in part **(a)**. [1]

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The probability that a randomly chosen flower of another kind is white is 0.02. A random sample of 150 of these flowers is selected.

(c) Use an appropriate approximating distribution to find the probability that the total number of white flowers in the two samples is less than 4. [3]

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2 In a survey, a random sample of 250 adults in Fromleigh were asked to fill in a questionnaire about their travel.

(a) It was found that 102 adults in the sample travel by bus. Find an approximate 90% confidence interval for the proportion of all the adults in Fromleigh who travel by bus. [3]

(b) The survey included a question about the amount, x dollars, spent on travel per year. The results are summarised as follows.

$$n = 250 \qquad \Sigma x = 50\,460 \qquad \Sigma x^2 = 19\,854\,200$$

Find unbiased estimates of the population mean and variance of the amount spent per year on travel. [3]

A councillor wanted to select a random sample of houses in Fromleigh. He planned to select the first house on each of the 143 streets in Fromleigh.

(c) Explain why this would not provide a random sample. [1]

3 The masses, in kilograms, of female and male animals of a certain species have the distributions $N(102, 27^2)$ and $N(170, 55^2)$ respectively.

Find the probability that a randomly chosen female has a mass that is less than half the mass of a randomly chosen male. [6]

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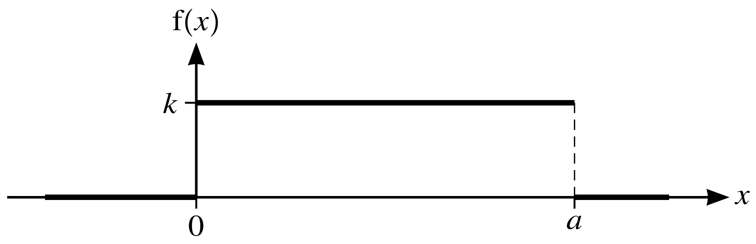

The diagram shows the probability density function, $f(x)$, of a random variable X. For $0 \leqslant x \leqslant a$, $f(x) = k$; elsewhere $f(x) = 0$.

(a) Express k in terms of a. [1]

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(b) Given that $\text{Var}(X) = 3$, find a. [4]

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5 The number of absences per week by workers at a factory has the distribution Po(2.1).

(a) Find the standard deviation of the number of absences per week. [1]

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(b) Find the probability that the number of absences in a 2-week period is at least 2. [3]

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(c) Find the probability that the number of absences in a 3-week period is more than 4 and less than 8. [2]

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Following a change in working conditions, the management wished to test whether the mean number of absences has decreased. They found that, in a randomly chosen 3-week period, there were exactly 2 absences.

(d) Carry out the test at the 10% significance level. [5]

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(e) State, with a reason, which of the errors, Type I or Type II, might have been made in carrying out the test in part **(d)**. [2]

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6 The time, in minutes, for Anjan's journey to work on Mondays has mean 38.4 and standard deviation 6.9.

(a) Find the probability that Anjan's mean journey time for a random sample of 30 Mondays is between 38 and 40 minutes. [5]

Anjan wishes to test whether his mean journey time is different on Tuesdays. He chooses a random sample of 30 Tuesdays and finds that his mean journey time for these 30 Tuesdays is 40.2 minutes. Assume that the standard deviation for his journey time on Tuesdays is 6.9 minutes.

(b) **(i)** State, with a reason, whether Anjan should use a one-tail or a two-tail test. [1]

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(ii) Carry out the test at the 10% significance level. [5]

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(iii) Explain whether it was necessary to use the Central Limit theorem in part **(b)(ii)**. [1]

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Additional Page

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Cambridge Assessment International Education

Cambridge International AS & A Level

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* 6 1 8 8 7 6 9 4 2 2 *

MATHEMATICS **9709/61**

Paper 6 Probability & Statistics 2 **May/June 2020**

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
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INFORMATION

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1 The lengths, X centimetres, of a random sample of 7 leaves from a certain variety of tree are as follows.

5.2 4.8 5.5 6.1 4.8 3.9 4.4

(a) Calculate unbiased estimates of the population mean and variance of X. [3]

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It is now given that the true value of the population variance of X is 0.55, and that X has a normal distribution.

(b) Find a 95% confidence interval for the population mean of X. [3]

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2 In the past the yield of a certain crop, in tonnes per hectare, had mean 0.56 and standard deviation 0.08. Following the introduction of a new fertilizer, the farmer intends to test at the 2.5% significance level whether the mean yield has increased. He finds that the mean yield over 10 years is 0.61 tonnes per hectare.

(a) State two assumptions that are necessary for the test. [2]

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(b) Carry out the test. [5]

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3 The masses, in kilograms, of large sacks of flour and small sacks of flour have the independent distributions $N(40, 1.5^2)$ and $N(12, 0.7^2)$ respectively.

(a) Find the probability that the total mass of 6 randomly chosen large sacks of flour is more than 245 kg. [4]

(b) Find the probability that the mass of a randomly chosen large sack of flour is less than 4 times the mass of a randomly chosen small sack of flour. [6]

4 A fair spinner has five sides numbered 1, 2, 3, 4, 5. The score on one spin is denoted by X.

(a) Show that $\mathrm{Var}(X) = 2$. [1]

Fiona has another spinner, also with five sides numbered 1, 2, 3, 4, 5. She suspects that it is biased so that the expected score is less than 3. In order to test her suspicion, she plans to spin her spinner 40 times. If the mean score is less than 2.6 she will conclude that her spinner is biased in this way.

(b) Find the probability of a Type I error. [4]

(c) State what is meant by a Type II error in this context. [1]

5 Each week a sports team plays one home match and one away match. In their home matches they score goals at a constant average rate of 2.1 goals per match. In their away matches they score goals at a constant average rate of 0.8 goals per match. You may assume that goals are scored at random times and independently of one another.

(a) A week is chosen at random.

(i) Find the probability that the team scores a total of 4 goals in their two matches. [2]

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(ii) Find the probability that the team scores a total of 4 goals, with more goals scored in the home match than in the away match. [3]

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(b) Use a suitable approximating distribution to find the probability that the team scores fewer than 25 goals in 10 randomly chosen weeks. [4]

(c) Justify the use of the approximating distribution used in part **(b)**. [1]

6 The length of time, T minutes, that a passenger has to wait for a bus at a certain bus stop is modelled by the probability density function given by

$$f(t) = \begin{cases} \frac{3}{4000}(20t - t^2) & 0 \leqslant t \leqslant 20, \\ 0 & \text{otherwise.} \end{cases}$$

(a) Sketch the graph of $y = f(t)$. [1]

(b) Hence explain, without calculation, why $E(T) = 10$. [1]

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(c) Find $Var(T)$. [3]

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(d) It is given that $P(T < 10 + a) = p$, where $0 < a < 10$.

Find $P(10 - a < T < 10 + a)$ in terms of p. [2]

(e) Find $P(8 < T < 12)$. [3]

(f) Give one reason why this model may be unrealistic. [1]

Additional Page

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3831026276

MATHEMATICS **9709/62**

Paper 6 Probability & Statistics 2 **May/June 2020**

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
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1 The masses, in grams, of plums of a certain type have the distribution $N(40.4, 5.2^2)$. The plums are packed in bags, with each bag containing 6 randomly chosen plums. If the total weight of the plums in a bag is less than 220 g the bag is rejected.

Find the percentage of bags that are rejected. [4]

2 A shop obtains apples from a certain farm. It has been found that 5% of apples from this farm are Grade A. Following a change in growing conditions at the farm, the shop management plan to carry out a hypothesis test to find out whether the proportion of Grade A apples has increased. They select 25 apples at random. If the number of Grade A apples is more than 3 they will conclude that the proportion has increased.

(a) State suitable null and alternative hypotheses for the test. [1]

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(b) Find the probability of a Type I error. [3]

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In fact 2 of the 25 apples were Grade A.

(c) Which of the errors, Type I or Type II, is possible? Justify your answer. [2]

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3 In the data-entry department of a certain firm, it is known that 0.12% of data items are entered incorrectly, and that these errors occur randomly and independently.

(a) A random sample of 3600 data items is chosen. The number of these data items that are incorrectly entered is denoted by X.

(i) State the distribution of X, including the values of any parameters. [1]

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(ii) State an appropriate approximating distribution for X, including the values of any parameters.

Justify your choice of approximating distribution. [3]

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(iii) Use your approximating distribution to find $P(X > 2)$. [2]

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(b) Another large random sample of n data items is chosen. The probability that the sample contains no data items that are entered incorrectly is more than 0.1.

Use an approximating distribution to find the largest possible value of n. [3]

4 The score on one spin of a 5-sided spinner is denoted by the random variable X with probability distribution as shown in the table.

x	0	1	2	3	4
$P(X = x)$	0.1	0.2	0.4	0.2	0.1

(a) Show that $\text{Var}(X) = 1.2$. [2]

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The spinner is spun 200 times. The score on each spin is noted and the mean, $\overline{X}$, of the 200 scores is found.

(b) Given that $P(\overline{X} > a) = 0.1$, find the value of a. [4]

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(c) Explain whether it was necessary to use the Central Limit theorem in your answer to part **(b)**. [1]

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(d) Johann has another, similar, spinner. He suspects that it is biased so that the mean score is less than 2. He spins his spinner 200 times and finds that the mean of the 200 scores is 1.86.

Given that the variance of the score on one spin of this spinner is also 1.2, test Johann's suspicion at the 5% significance level. [5]

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5 **(a)** The random variable X has the distribution $\text{Po}(\lambda)$.

(i) State the values that X can take. [1]

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It is given that $P(X = 1) = 3 \times P(X = 0)$.

(ii) Find λ. [1]

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(iii) Find $P(4 \leqslant X \leqslant 6)$. [2]

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(b) The random variable Y has the distribution $\mathrm{Po}(\mu)$ where μ is large. Using a suitable approximating distribution, it is found that $\mathrm{P}(Y < 46) = 0.0668$, correct to 4 decimal places.

Find μ. [5]

6 A random variable X has probability density function given by

$$f(x) = \begin{cases} \frac{k}{x^2} & 1 \leqslant x \leqslant a, \\ 0 & \text{otherwise,} \end{cases}$$

where k and a are positive constants.

(a) Show that $k = \frac{a}{a-1}$. [3]

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(b) Find $E(X)$ in terms of a. [3]

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(c) Find the 60th percentile of X in terms of a. [4]

Additional Page

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MATHEMATICS **9709/63**

Paper 6 Probability & Statistics 2 **May/June 2020**

1 hour 15 minutes

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JC20 06_9709_63/RP

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1 A random sample of 100 values of a variable X is taken. These values are summarised below.

$$n = 100 \qquad \Sigma x = 1556 \qquad \Sigma x^2 = 29\,004$$

Calculate unbiased estimates of the population mean and variance of X. [3]

2 Each day at the gym, Sarah completes three runs. The distances, in metres, that she completes in the three runs have the independent distributions $W \sim N(1520, 450)$, $X \sim N(2250, 720)$ and $Y \sim N(3860, 1050)$.

Find the probability that, on a particular day, Y is less than the total of W and X. [5]

3 The number of customers who visit a particular shop between 9.00 am and 10.00 am has the distribution Po(λ). In the past the value of λ was 5.2. Following some new advertising, the manager wishes to test whether the value of λ has increased. He chooses a random sample of 20 days and finds that the total number of customers who visited the shop between 9.00 am and 10.00 am on those days is 125.

Use an approximating distribution to test at the 2.5% significance level whether the value of λ has increased. [6]

4 The random variable A has the distribution Po(1.5). A_1 and A_2 are independent values of A.

(a) Find $P(A_1 + A_2 < 2)$. [3]

(b) Given that $A_1 + A_2 < 2$, find $P(A_1 = 1)$. [4]

(c) Give a reason why $A_1 - A_2$ cannot have a Poisson distribution. [1]

5 Sunita has a six-sided die with faces marked 1, 2, 3, 4, 5, 6. The probability that the die shows a six on any throw is p. Sunita throws the die 500 times and finds that it shows a six 70 times.

(a) Calculate an approximate 99% confidence interval for p. [4]

(b) Sunita believes that the die is fair. Use your answer to part **(a)** to comment on her belief. [1]

(c) Sunita uses the result of her 500 throws to calculate an $\alpha\%$ confidence interval for p. This interval has width 0.04.

Find the value of α. [5]

6 The length, X centimetres, of worms of a certain type is modelled by the probability density function

$$f(x) = \begin{cases} \frac{6}{125}(10 - x)(x - 5) & 5 \leqslant x \leqslant 10, \\ 0 & \text{otherwise.} \end{cases}$$

(a) State the value of $E(X)$. [1]

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(b) Find $\text{Var}(X)$. [3]

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(c) Two worms of this type are chosen at random.

Find the probability that exactly one of them has length less than 6 cm. [5]

7 A market researcher is investigating the length of time that customers spend at an information desk. He plans to choose a sample of 50 customers on a particular day.

(a) He considers choosing the first 50 customers who visit the information desk.

Explain why this method is unsuitable. [1]

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The actual lengths of time, in minutes, that customers spend at the information desk may be assumed to have mean μ and variance 4.8. The researcher knows that in the past the value of μ was 6.0. He wishes to test, at the 2% significance level, whether this is still true. He chooses a random sample of 50 customers and notes how long they each spend at the information desk.

(b) State the probability of making a Type I error and explain what is meant by a Type I error in this context. [2]

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(c) Given that the mean time spent at the information desk by the 50 customers is 6.8 minutes, carry out the test. [5]

(d) Give a reason why it was necessary to use the Central Limit theorem in your answer to part **(c)**. [1]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

2965364150

MATHEMATICS **9709/62**

Paper 6 Probability & Statistics 2 **February/March 2020**

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **12** pages. Blank pages are indicated.

JC20 03_9709_62/RP

1 The booklets produced by a certain publisher contain, on average, 1 incorrect letter per 30 000 letters, and these errors occur randomly. A randomly chosen booklet from this publisher contains 12 500 letters.

Use a suitable approximating distribution to find the probability that this booklet contains at least 2 errors. [3]

2 Lengths of a certain species of lizard are known to be normally distributed with standard deviation 3.2 cm. A naturalist measures the lengths of a random sample of 100 lizards of this species and obtains an $\alpha\%$ confidence interval for the population mean. He finds that the total width of this interval is 1.25 cm.

Find α. [5]

3 In the past, the mean time taken by Freda for a particular daily journey was 39.2 minutes. Following the introduction of a one-way system, Freda wishes to test whether the mean time for the journey has decreased. She notes the times, t minutes, for 40 randomly chosen journeys and summarises the results as follows.

$$n = 40 \qquad \Sigma t = 1504 \qquad \Sigma t^2 = 57\,760$$

(a) Calculate unbiased estimates of the population mean and variance of the new journey time. [3]

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(b) Test, at the 5% significance level, whether the population mean time has decreased. [5]

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4 The number of accidents on a certain road has a Poisson distribution with mean 0.4 per 50-day period.

(a) Find the probability that there will be fewer than 3 accidents during a year (365 days). [3]

(b) The probability that there will be no accidents during a period of n days is greater than 0.95.

Find the largest possible value of n. [4]

5 Bottles of Lanta contain approximately 300 ml of juice. The volume of juice, in millilitres, in a bottle is $300 + X$, where X is a random variable with probability density function given by

$$f(x) = \begin{cases} \frac{3}{4000}(100 - x^2) & -10 \leqslant x \leqslant 10, \\ 0 & \text{otherwise.} \end{cases}$$

(a) Find the probability that a randomly chosen bottle of Lanta contains more than 305 ml of juice. [3]

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(b) Given that 25% of bottles of Lanta contain more than $(300 + p)$ ml of juice, show that

$$p^3 - 300p + 1000 = 0.$$ [4]

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(c) Given that $p = 3.47$, and that 50% of bottles of Lanta contain between $(300 - q)$ and $(300 + q)$ ml of juice, find q. Justify your answer. [2]

6 The volumes, in millilitres, of large and small cups of tea are modelled by the distributions N(200, 30) and N(110, 20) respectively.

(a) Find the probability that the total volume of a randomly chosen large cup of tea and a randomly chosen small cup of tea is less than 300 ml. [4]

(b) Find the probability that the volume of a randomly chosen large cup of tea is more than twice the volume of a randomly chosen small cup of tea. [6]

7 A national survey shows that 95% of year 12 students use social media. Arvin suspects that the percentage of year 12 students at his college who use social media is less than the national percentage. He chooses a random sample of 20 students at his college and notes the number who use social media. He then carries out a test at the 2% significance level.

(a) Find the rejection region for the test. [4]

(b) Find the probability of a Type I error. [1]

(c) Jimmy believes that the true percentage at Arvin's college is 70%. Assuming that Jimmy is correct, find the probability of a Type II error. [3]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

Cambridge International A Level

Cambridge Assessment International Education
Cambridge International Advanced Level

CANDIDATE NAME		
CENTRE NUMBER		CANDIDATE NUMBER

6794697576

MATHEMATICS **9709/72**

Paper 7 Probability & Statistics 2 **(S2)** **October/November 2019**

1 hour 15 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

Write your centre number, candidate number and name in the spaces at the top of this page.
Write in dark blue or black pen.
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You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50.

This document consists of **13** printed pages and **3** blank pages.

JC19 11_9709_72/2R

Cambridge Assessment International Education

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1 On average, 1 in 150 components made by a certain machine are faulty. The random variable X denotes the number of faulty components in a random sample of 500 components.

(i) Describe fully the distribution of X. [2]

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(ii) State a suitable approximating distribution for X, giving a justification for your choice. [2]

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(iii) Use your approximating distribution to find the probability that the sample will include at least 3 faulty components. [3]

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2 The heights of a certain species of animal have been found to have mean 65.2 cm and standard deviation 7.1 cm. A researcher suspects that animals of this species in a certain region are shorter on average than elsewhere. She takes a large random sample of n animals of this species from this region and finds that their mean height is 63.2 cm. She then carries out an appropriate hypothesis test.

(i) She finds that the value of the test statistic z is -2.182, correct to 3 decimal places.

(a) Stating a necessary assumption, calculate the value of n. [4]

(b) Carry out the hypothesis test at the 4% significance level. [3]

(ii) Explain why it was necessary to use the Central Limit theorem in carrying out the test. [1]

3 The masses, in grams, of bags of flour are normally distributed with mean μ. The masses, m grams, of a random sample of 50 bags are summarised by $\Sigma m = 25\,110$ and $\Sigma m^2 = 12\,610\,300$.

(i) Calculate a 96% confidence interval for μ, giving the end-points correct to 1 decimal place. [6]

Another random sample of 50 bags of flour is taken and a 99% confidence interval for μ is calculated.

(ii) Without calculation, state whether this confidence interval will be wider or narrower than the confidence interval found in part **(i)**. Give a reason for your answer. [1]

4 A random variable X has probability density function given by

$$f(x) = \begin{cases} \frac{1}{2}x & 0 \leqslant x \leqslant a, \\ 0 & \text{otherwise,} \end{cases}$$

where a is a constant.

(i) Find a. [2]

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(ii) Show that $E(X) = \frac{4}{3}$. [3]

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The median of X is denoted by m.

(iii) Find $\mathrm{P}(\mathrm{E}(X) < X < m)$. [4]

5 The masses, in grams, of large boxes of chocolates and small boxes of chocolates have the distributions N(325, 6.1) and N(167, 5.6) respectively.

(i) Find the probability that the total mass of 10 randomly chosen large boxes of chocolates is less than 3240 g. [4]

(ii) Find the probability that the mass of a randomly chosen large box of chocolates is more than twice the mass of a randomly chosen small box of chocolates. [5]

6 The number of accidents per month, X, at a factory has a Poisson distribution. In the past the mean has been 1.1 accidents per month. Some new machinery is introduced and the management wish to test whether the mean has increased. They note the number of accidents in a randomly chosen month and carry out a hypothesis test at the 1% significance level.

(i) Show that the critical region for the test is $X \geqslant 5$. Given that the number of accidents is 6, carry out the test. [6]

Later they carry out a similar test, also at the 1% significance level.

(ii) Explain the meaning of a Type I error in this context and state the probability of a Type I error. [2]

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(iii) Given that the mean is now 7.0, find the probability of a Type II error. [2]

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Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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Cambridge International A Level

Cambridge Assessment International Education
Cambridge International Advanced Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

9931467168

MATHEMATICS **9709/73**

Paper 7 Probability & Statistics 2 **(S2)** **October/November 2019**

1 hour 15 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

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At the end of the examination, fasten all your work securely together.
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The total number of marks for this paper is 50.

This document consists of **14** printed pages and **2** blank pages.

Cambridge Assessment International Education

1 The random variable X has mean 2.4 and variance 3.1.

(i) The random variable Y is the sum of four independent values of X. Find the mean and variance of Y. [2]

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(ii) The random variable Z is defined by $Z = 4X - 3$. Find the mean and variance of Z. [2]

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2 Cars arrive at a filling station randomly and at a constant average rate of 2.4 cars per minute.

(i) Calculate the probability that fewer than 4 cars arrive in a 2-minute period. [2]

(ii) Use a suitable approximating distribution to calculate the probability that at least 140 cars arrive in a 1-hour period. [4]

3 The times, in minutes, taken by competitors to complete a puzzle have mean μ and standard deviation 3. The times taken by a random sample of 10 competitors are noted and the results are given below.

25.2 26.8 18.5 25.5 30.1 28.9 27.0 26.1 26.0 24.9

(i) Stating a necessary assumption, calculate a 97% confidence interval for μ. [5]

(ii) Two more random samples, each of 10 competitors, are taken. Their times are used to calculate two more 97% confidence intervals for μ. Find the probability that neither of these intervals contains the true value of μ. [1]

4 A train company claims that 92% of trains on a particular line arrive on time. Sanjeep suspects that the true percentage is less than 92%. He chooses a random sample of 20 trains on this line and finds that exactly 16 of them arrive on time. Making an assumption that should be stated, test at the 5% significance level whether Sanjeep's suspicion is justified. [6]

5 **(i)** The random variable X has the distribution $B(300, 0.01)$. Use a Poisson approximation to find $P(2 < X < 6)$. [3]

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(ii) The random variable Y has the distribution $Po(\lambda)$, and $P(Y = 0) = P(Y = 2)$. Find λ. [2]

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(iii) The random variable Z has the distribution Po(5.2) and it is given that $P(Z = n) < P(Z = n + 1)$.

(a) Write down an inequality in n. [1]

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(b) Hence or otherwise find the largest possible value of n. [2]

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6 A random variable X has probability density function given by

$$f(x) = \begin{cases} k(3x - x^2) & 0 \leqslant x \leqslant 3, \\ 0 & \text{otherwise.} \end{cases}$$

(i) Show that $k = \frac{2}{9}$. [3]

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(ii) Find $P(1 \leqslant X \leqslant 2)$. [2]

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(iii) Find Var(X). [5]

7 Bob is a self-employed builder. In the past his weekly income had mean \$546 and standard deviation \$120. Following a change in Bob's working pattern, his mean weekly income for 40 randomly chosen weeks was \$581. You should assume that the standard deviation remains unchanged at \$120.

(i) Test at the 2.5% significance level whether Bob's mean weekly income has increased. [5]

Bob finds his mean weekly income for another random sample of 40 weeks and carries out a similar test at the 2.5% significance level.

(ii) Given that Bob's mean weekly income is now in fact \$595, find the probability of a Type II error. [5]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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Cambridge International A Level

Cambridge Assessment International Education
Cambridge International Advanced Level

CANDIDATE NAME			
CENTRE NUMBER		CANDIDATE NUMBER	

5733406539

MATHEMATICS **9709/71**

Paper 7 Probability & Statistics 2 **(S2)** **May/June 2019**

1 hour 15 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

Write your centre number, candidate number and name in the spaces at the top of this page.
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At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50.

This document consists of **12** printed pages.

JC19 06_9709_71/2R

Cambridge Assessment International Education

[Turn over

1 At an internet café, the charge for using a computer is 5 cents per minute. The number of minutes for which people use a computer has mean 23 and standard deviation 8.

(i) Find, in cents, the mean and standard deviation of the amount people pay when using a computer. [2]

(ii) Each day, 15 people use computers independently. Find, in cents, the mean and standard deviation of the total amount paid by 15 people. [3]

2 The time, in minutes, that John takes to travel to work has a normal distribution. Last year the mean and standard deviation were 26.5 and 4.8 respectively. This year John uses a different route and he finds that the mean time for his first 150 journeys is 27.5 minutes.

(i) Stating a necessary assumption, test at the 1% significance level whether the mean time for his journey to work has increased. [6]

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(ii) State, with a reason, whether it was necessary to use the Central Limit theorem in your answer to part **(i)**. [1]

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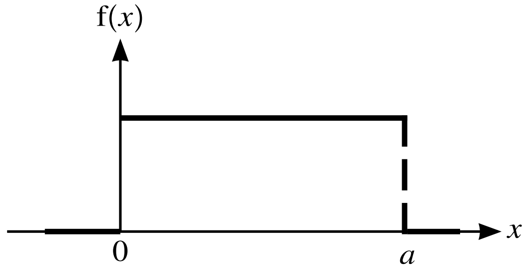
3 Sumitra has a six-sided die. She suspects that it is biased so that it shows a six less often than it would if it were fair. She decides to test the die by throwing it 30 times and noting the number of throws on which it shows a six.

(i) It shows a six on exactly 2 throws. Use a binomial distribution to carry out the test at the 5% significance level. [5]

(ii) Later, Sumitra repeats the test at the 5% significance level by throwing the die 30 times again. Find the probability of a Type I error in this second test. [2]

4 **(a)**



The diagram shows the graph of the probability density function, f, of a random variable X, where a is a constant greater than 0.5. The graph between $x = 0$ and $x = a$ is a straight line parallel to the x-axis.

(i) Find $P(X < 0.5)$ in terms of a. [2]

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(ii) Find $E(X)$ in terms of a. [1]

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(iii) Show that $\text{Var}(X) = \frac{1}{12}a^2$. [2]

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(b) A random variable T has probability density function given by

$$g(t) = \begin{cases} \dfrac{3}{2(t-1)^2} & 2 \leqslant t \leqslant 4, \\ 0 & \text{otherwise.} \end{cases}$$

Find the value of b such that $P(T \leqslant b) = \frac{3}{4}$. [4]

5 **(a)** The random variable X has the distribution Po(2.3).

(i) Find $P(2 \leqslant X \leqslant 4)$. [2]

(ii) Find the probability that the sum of two independent values of X is greater than 2. [3]

(iii) The random variable S is the sum of 50 independent values of X. Use a suitable approximating distribution to find $P(S \leqslant 110)$. [4]

(b) The random variable Y has the distribution $\text{Po}(\lambda)$. Given that $\text{P}(Y = 3) = \text{P}(Y = 5)$, find λ. [3]

6 Ramesh plans to carry out a survey in order to find out what adults in his town think about local sports facilities. He chooses a random sample from the adult members of a tennis club and gives each of them a questionnaire.

(i) Give a reason why this will not result in Ramesh having a random sample of adults who live in the town. [1]

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(ii) Describe briefly a valid method that Ramesh could use to choose a random sample of adults in the town. [2]

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Ramesh now uses a valid method to choose a random sample of 350 adults from the town. He finds that 47 adults think that the local sports facilities are good.

(iii) Calculate an approximate 90% confidence interval for the proportion of all adults in the town who think that the local sports facilities are good. [4]

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(iv) Ramesh calculates a confidence interval whose width is 1.25 times the width of this 90% confidence interval. Ramesh's new interval is an $x\%$ confidence interval. Find the value of x. [3]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

Cambridge Assessment International Education
Cambridge International Advanced Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

MATHEMATICS **9709/72**

Paper 7 Probability & Statistics 2 **(S2)** **May/June 2019**

1 hour 15 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

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This document consists of **14** printed pages and **2** blank pages.

JC19 06_9709_72/RP

1 The random variable X has the distribution Po(5).

(i) Find $P(X = 2)$. [1]

It is given that $P(X = n) = P(X = n + 1)$.

(ii) Write down an equation in n. [1]

(iii) Hence or otherwise find the value of n. [1]

2 The random variable X has mean 372 and standard deviation 54.

(i) Describe fully the distribution of the mean of a random sample of 36 values of X. [3]

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(ii) The distribution in part **(i)** might be either exact or approximate. State a condition under which the distribution is exact. [1]

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[Turn over

3 It is claimed that, on average, a particular train journey takes less than 1.9 hours. The times, t hours, taken for this journey on a random sample of 50 days were recorded. The results are summarised below.

$$n = 50 \qquad \Sigma t = 92.5 \qquad \Sigma t^2 = 175.25$$

(i) Calculate unbiased estimates of the population mean and variance. [3]

(ii) Test the claim at the 5% significance level. [5]

4 The heights of a certain variety of plant are normally distributed with mean 110 cm and variance 1050 cm^2. Two plants of this variety are chosen at random. Find the probability that the height of one of these plants is at least 1.5 times the height of the other. [7]

5 The manufacturer of a certain type of biscuit claims that 10% of packets include a free offer printed on the packet. Jyothi suspects that the true proportion is less than 10%. He plans to test the claim by looking at 40 randomly selected packets and, if the number which include the offer is less than 2, he will reject the manufacturer's claim.

(i) State suitable hypotheses for the test. [1]

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(ii) Find the probability of a Type I error. [3]

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On another occasion Jyothi looks at 80 randomly selected packets and finds that exactly 6 include the free offer.

(iii) Calculate an approximate 90% confidence interval for the proportion of packets that include the offer. [3]

(iv) Use your confidence interval to comment on the manufacturer's claim. [1]

6 X is a random variable with probability density function given by

$$f(x) = \begin{cases} \dfrac{a}{x^2} & 1 \leqslant x \leqslant b, \\ 0 & \text{otherwise,} \end{cases}$$

where a and b are constants.

(i) Show that $b = \dfrac{a}{a-1}$. [3]

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(ii) Given that the median of X is $\frac{3}{2}$, find the values of a and b. [3]

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(iii) Use your values of a and b from part **(ii)** to find $E(X)$. [3]

7 All the seats on a certain daily flight are always sold. The number of passengers who have bought seats but fail to arrive for this flight on a particular day is modelled by the distribution $B(320, 0.005)$.

(i) Explain what the number 320 represents in this context. [1]

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(ii) The total number of passengers who have bought seats but fail to arrive for this flight on 2 randomly chosen days is denoted by X. Use a suitable approximating distribution to find $P(2 < X < 6)$. [3]

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(iii) Justify the use of your approximating distribution. [2]

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After some changes, the airline wishes to test whether the mean number of passengers per day who fail to arrive for this flight has decreased.

(iv) During 5 randomly chosen days, a total of 2 passengers failed to arrive. Carry out the test at the 2.5% significance level. [5]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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Cambridge International A Level

Cambridge Assessment International Education
Cambridge International Advanced Level

CANDIDATE NAME			
CENTRE NUMBER		CANDIDATE NUMBER	

4223097642

MATHEMATICS **9709/73**

Paper 7 Probability & Statistics 2 **(S2)** **May/June 2019**

1 hour 15 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

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JC19 06_9709_73/2R

Cambridge Assessment International Education

[Turn over

1 A coin is thrown 100 times and it shows heads 60 times. Calculate an approximate 98% confidence interval for the probability, p, that the coin shows heads on any throw. [3]

2 The length of worms is denoted by X cm. The lengths of a random sample of 50 worms were measured. Some of the results were lost, but the following results are available.

- $\Sigma x^2 = 4361$
- An unbiased estimate of the population variance of X is 9.62.

Calculate the mean length of the 50 worms. [3]

3 Luis has to choose one person at random from four people, A, B, C and D. He throws a fair six-sided die. If the score is 1, he will choose A. If the score is 2 he will choose B. If the score is 3, he will choose C. If the score is 4 or more he will choose D.

(i) Explain why the choice made by this method is not random. [1]

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(ii) Describe how Luis could use a single throw of the die to make a random choice. [1]

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On another day, Luis has to choose two people at random from the same four people, A, B, C and D.

(iii) List the possible choices of two people and hence describe how Luis could use a single throw of the die to make this random choice. [2]

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4 A factory supplies boxes of children's bricks. Each box contains 10 randomly chosen large bricks and 20 randomly chosen small bricks. The masses, in grams, of large and small bricks have the distributions N(60, 1.2) and N(30, 0.7) respectively. The mass of an empty box is 8 g. Find the probability that the total weight of a box and its contents is less than 1200 g. [5]

5 The amount of money, in dollars, spent by a customer on one visit to a certain shop is modelled by the distribution $N(\mu, 1.94)$. In the past, the value of μ has been found to be 20.00, but following a rearrangement in the shop, the manager suspects that the value of μ has changed. He takes a random sample of 6 customers and notes how much they each spend, in dollars. The results are as follows.

17.60 23.50 17.30 22.00 31.00 15.50

The manager carries out a hypothesis test using a significance level of $\alpha\%$. The test does not support his suspicion. Find the largest possible value of α. [6]

6 A function f is defined by

$$f(x) = \begin{cases} \dfrac{3x^2}{a^3} & 0 \leqslant x \leqslant a, \\ 0 & \text{otherwise,} \end{cases}$$

where a is a constant.

(i) Show that f is a probability density function for all positive values of a. [3]

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The random variable X has probability density function f and the median of X is 2.

(ii) Show that $a = 2.52$, correct to 3 significant figures. [3]

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(iii) Find $E(X)$. [3]

7 Each day at a certain doctor's surgery there are 70 appointments available in the morning and 60 in the afternoon. All the appointments are filled every day. The probability that any patient misses a particular morning appointment is 0.04, and the probability that any patient misses a particular afternoon appointment is 0.05. All missed appointments are independent of each other.

Use suitable approximating distributions to answer the following.

(i) Find the probability that on a randomly chosen morning there are at least 3 missed appointments. [3]

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(ii) Find the probability that on a randomly chosen day there are a total of exactly 6 missed appointments. [3]

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(iii) Find the probability that in a randomly chosen 10-day period there are more than 50 missed appointments. [4]

8 The four sides of a spinner are A, B, C, D. The spinner is supposed to be fair, but Sonam suspects that the spinner is biased so that the probability, p, that it will land on side A is greater than $\frac{1}{4}$. He spins the spinner 10 times and finds that it lands on side A 6 times.

(i) Test Sonam's suspicion using a 1% significance level. [5]

Later Sonam carries out a similar test at the 1% significance level, using another 10 spins of the spinner.

(ii) Calculate the probability of a Type I error. [2]

(iii) Assuming that the value of p is actually $\frac{3}{5}$, calculate the probability of a Type II error. [3]

Additional Page

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Cambridge International A Level

Cambridge Assessment International Education
Cambridge International Advanced Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

7957641337

MATHEMATICS **9709/72**

Paper 7 Probability & Statistics 2 **(S2)** **February/March 2019**

1 hour 15 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

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The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50.

This document consists of **13** printed pages and **3** blank pages.

JC19 03_9709_72/RP

Cambridge Assessment International Education

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1 The masses of a certain variety of plums are known to have standard deviation 13.2 g. A random sample of 200 of these plums is taken and the mean mass of the plums in the sample is found to be 62.3 g.

(i) Calculate a 98% confidence interval for the population mean mass. [3]

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(ii) State with a reason whether it was necessary to use the Central Limit theorem in the calculation in part **(i)**. [1]

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2 The independent random variables X and Y have the distributions $N(9.2, 12.1)$ and $N(3.0, 8.6)$ respectively. Find $P(X > 3Y)$. [5]

3 At factory A the mean number of accidents per year is 32. At factory B the records of numbers of accidents before 2018 have been lost, but the number of accidents during 2018 was 21. It is known that the number of accidents per year can be well modelled by a Poisson distribution. Use an approximating distribution to test at the 2% significance level whether the mean number of accidents at factory B is less than at factory A. [6]

4 The lifetimes, X hours, of a random sample of 50 batteries of a certain kind were found. The results are summarised by $\Sigma x = 420$ and $\Sigma x^2 = 27\,530$.

(i) Calculate an unbiased estimate of the population mean of X and show that an unbiased estimate of the population variance is 490, correct to 3 significant figures. [3]

(ii) The lifetimes of a further large sample of n batteries of this kind were noted, and the sample mean, $\overline{X}$, was found. Use your estimates from part **(i)** to find the value of n such that $P(\overline{X} > 5) = 0.9377$. [4]

5 The number of eagles seen per hour in a certain location has the distribution Po(1.8). The number of vultures seen per hour in the same location has the independent distribution Po(2.6).

(i) Find the probability that, in a randomly chosen hour, at least 2 eagles are seen. [2]

(ii) Find the probability that, in a randomly chosen half-hour period, the total number of eagles and vultures seen is less than 5. [3]

Alex wants to be at least 99% certain of seeing at least 1 eagle.

(iii) Find the minimum time for which she should watch for eagles. [3]

6 The time taken by volunteers to complete a certain task is normally distributed. In the past the time, in minutes, has had mean 91.4 and standard deviation 6.4. A new, similar task is introduced and the times, t minutes, taken by a random sample of 6 volunteers to complete the new task are summarised by $\Sigma t = 568.5$. Andrea plans to carry out a test, at the 5% significance level, of whether the mean time for the new task is different from the mean time for the old task.

(i) Give a reason why Andrea should use a two-tail test. [1]

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(ii) State the probability that a Type I error is made, and explain the meaning of a Type I error in this context. [2]

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You may assume that the times taken for the new task are normally distributed.

(iii) Stating another necessary assumption, carry out the test. [7]

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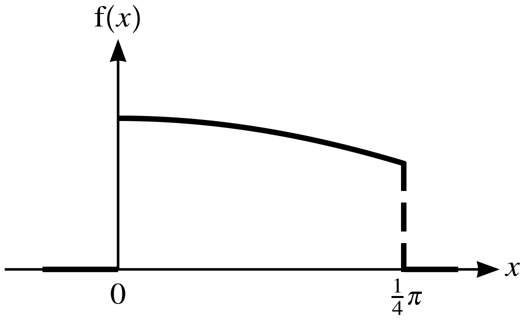

A random variable X has probability density function given by

$$f(x) = \begin{cases} (\sqrt{2})\cos x & 0 \leqslant x \leqslant \frac{1}{4}\pi, \\ 0 & \text{otherwise,} \end{cases}$$

as shown in the diagram.

(i) Find $P(X > \frac{1}{6}\pi)$. [2]

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(ii) Find the median of X. [4]

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(iii) Find E(X). [4]

Additional Page

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Cambridge International A Level

Cambridge Assessment International Education
Cambridge International Advanced Level

CANDIDATE NAME		
CENTRE NUMBER		CANDIDATE NUMBER

7957641337

MATHEMATICS **9709/72**

Paper 7 Probability & Statistics 2 **(S2)** **February/March 2019**

1 hour 15 minutes

Candidates answer on the Question Paper.

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JC19 03_9709_72/RP

Cambridge Assessment International Education

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(iii) Stating another necessary assumption, carry out the test. [7]

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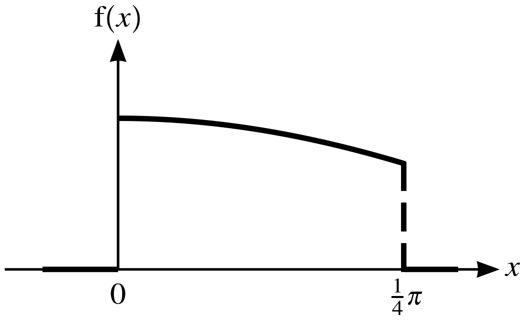

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as shown in the diagram.

(i) Find $P(X > \frac{1}{6}\pi)$. [2]

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(ii) Find the median of X. [4]

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(iii) Find E(X). [4]

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Cambridge International A Level

Cambridge International Examinations
Cambridge International Advanced Level

CANDIDATE NAME

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CANDIDATE NUMBER

0523393823

MATHEMATICS **9709/71**

Paper 7 Probability & Statistics 2 **(S2)** **October/November 2018**

1 hour 15 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

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This document consists of **12** printed pages.

CAMBRIDGE International Examinations

1 The standard deviation of the heights of adult males is 7.2 cm. The mean height of a sample of 200 adult males is found to be 176 cm.

(i) Calculate a 97.5% confidence interval for the mean height of adult males. [3]

(ii) State a necessary condition for the calculation in part **(i)** to be valid. [1]

2 A headteacher models the number of children who bring a 'healthy' packed lunch to school on any day by the distribution B(150, p). In the past, she has found that $p = \frac{1}{3}$. Following the opening of a fast food outlet near the school, she wishes to test, at the 1% significance level, whether the value of p has decreased.

(i) State the null and alternative hypotheses for this test. [1]

On a randomly chosen day she notes the number, N, of children who bring a 'healthy' packed lunch to school. She finds that $N = 36$ and then, assuming that the null hypothesis is true, she calculates that $P(N \leqslant 36) = 0.0084$.

(ii) State, with a reason, the conclusion that the headteacher should draw from the test. [2]

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(iii) According to the model, what is the largest number of children who might bring a packed lunch to school? [1]

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3 A population has mean 12 and standard deviation 2.5. A large random sample of size n is chosen from this population and the sample mean is denoted by $\overline{X}$. Given that $P(\overline{X} < 12.2) = 0.975$, correct to 3 significant figures, find the value of n. [4]

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4 Small drops of two liquids, A and B, are randomly and independently distributed in the air. The average numbers of drops of A and B per cubic centimetre of air are 0.25 and 0.36 respectively.

(i) A sample of $10\,\text{cm}^3$ of air is taken at random. Find the probability that the total number of drops of A and B in this sample is at least 4. [3]

(ii) A sample of $100\,\text{cm}^3$ of air is taken at random. Use an approximating distribution to find the probability that the total number of drops of A and B in this sample is less than 60. [5]

5 The times, in months, taken by a builder to build two types of house, P and Q, are represented by the independent variables $T_1 \sim N(2.2, 0.4^2)$ and $T_2 \sim N(2.8, 0.5^2)$ respectively.

(i) Find the probability that the total time taken to build one house of each type is less than 6 months. [4]

(ii) Find the probability that the time taken to build a type Q house is more than 1.2 times the time taken to build a type P house. [5]

6 The random variable X has probability density function given by

$$f(x) = \begin{cases} kx^{-1} & 2 \leqslant x \leqslant 6, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

(i) Show that $k = \dfrac{1}{\ln 3}$. [2]

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(ii) Show that $E(X) = 3.64$, correct to 3 significant figures. [3]

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(iii) Given that the median of X is m, find $P(m < X < E(X))$. [4]

7 A mill owner claims that the mean mass of sacks of flour produced at his mill is 51 kg. A quality control officer suspects that the mean mass is actually less than 51 kg. In order to test the owner's claim she finds the mass, x kg, of each of a random sample of 150 sacks and her results are summarised as follows.

$$n = 150 \qquad \Sigma x = 7480 \qquad \Sigma x^2 = 380\,000$$

(i) Carry out the test at the 2.5% significance level. [7]

You may now assume that the population standard deviation of the masses of sacks of flour is 6.856 kg. The quality control officer weighs another random sample of 150 sacks and carries out another test at the 2.5% significance level.

(ii) Given that the population mean mass is 49 kg, find the probability of a Type II error. [5]

Additional Page

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Cambridge International Examinations
Cambridge International Advanced Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

0635550837

MATHEMATICS **9709/72**

Paper 7 Probability & Statistics 2 **(S2)** **October/November 2018**

1 hour 15 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

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JC18 11_9709_72/2R

1 The random variable X has the distribution $\mathrm{Po}(2.3)$. Find $\mathrm{P}(2 \leqslant X < 5)$. [3]

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2 The standard deviation of the volume of drink in cans of Koola is 4.8 centilitres. A random sample of 180 cans is taken and the mean volume of drink in these 180 cans is found to be 330.1 centilitres.

(i) Calculate a 95% confidence interval for the mean volume of drink in all cans of Koola. Give the end-points of your interval correct to 1 decimal place. [3]

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(ii) Explain whether it was necessary to use the Central Limit theorem in your answer to part **(i)**. [1]

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3 Sugar and flour for making cakes are measured in cups. The mass, in grams, of one cup of sugar has the distribution $N(250, 10)$. The mass, in grams, of one cup of flour has the independent distribution $N(160, 9)$. Each cake contains 2 cups of sugar and 5 cups of flour. Find the probability that the total mass of sugar and flour in one cake exceeds 1310 grams. [5]

4 The time, X hours, taken by a large number of runners to complete a race is modelled by the probability density function given by

$$f(x) = \begin{cases} \dfrac{k}{(x+1)^2} & 0 \leqslant x \leqslant a, \\ 0 & \text{otherwise,} \end{cases}$$

where k and a are constants.

(i) Show that $k = \dfrac{a+1}{a}$. [3]

(ii) State what the constant a represents in this context. [1]

Three quarters of the runners take half an hour or less to complete the race.

(iii) Find the value of a. [3]

5 The numbers of basketball courts in a random sample of 70 schools in South Mowland are summarised in the table.

Number of basketball courts	0	1	2	3	4	>4
Number of schools	2	28	26	10	4	0

(i) Calculate unbiased estimates for the population mean and variance of the number of basketball courts per school in South Mowland. [4]

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The mean number of basketball courts per school in North Mowland is 1.9.

(ii) Test at the 5% significance level whether the mean number of basketball courts per school in South Mowland is less than the mean for North Mowland. [5]

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(iii) State, with a reason, which of the errors, Type I or Type II, might have been made in the test in part **(ii)**. [1]

6 In the past, Angus found that his train was late on 15% of his daily journeys to work. Following a timetable change, Angus found that out of 60 randomly chosen days, his train was late on 6 days.

(i) Test at the 10% significance level whether Angus' train is late less often than it was before the timetable change. [5]

Angus used his random sample to find an $\alpha\%$ confidence interval for the proportion of days on which his train is late. The upper limit of his interval was 0.150, correct to 3 significant figures.

(ii) Calculate the value of α correct to the nearest integer. [5]

7 The independent random variables X and Y have the distributions Po(2.1) and Po(3.5) respectively.

(i) Find $P(X + Y = 3)$. [2]

(ii) Given that $X + Y = 3$, find $P(X = 2)$. [3]

(iii) A random sample of 100 values of X is taken. Find the probability that the sample mean is more than 2.2. [6]

Additional Page

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Cambridge International A Level

Cambridge International Examinations
Cambridge International Advanced Level

CANDIDATE NAME		
CENTRE NUMBER		CANDIDATE NUMBER

2323984608

MATHEMATICS **9709/73**

Paper 7 Probability & Statistics 2 **(S2)** **October/November 2018**

1 hour 15 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

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JC18 11_9709_73/3R

CAMBRIDGE
International Examinations

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3 A population has mean 12 and standard deviation 2.5. A large random sample of size n is chosen from this population and the sample mean is denoted by $\overline{X}$. Given that $P(\overline{X} < 12.2) = 0.975$, correct to 3 significant figures, find the value of n. [4]

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4 Small drops of two liquids, A and B, are randomly and independently distributed in the air. The average numbers of drops of A and B per cubic centimetre of air are 0.25 and 0.36 respectively.

(i) A sample of $10\,\text{cm}^3$ of air is taken at random. Find the probability that the total number of drops of A and B in this sample is at least 4. [3]

(ii) A sample of $100\,\text{cm}^3$ of air is taken at random. Use an approximating distribution to find the probability that the total number of drops of A and B in this sample is less than 60. [5]

5 The times, in months, taken by a builder to build two types of house, P and Q, are represented by the independent variables $T_1 \sim \mathrm{N}(2.2,\ 0.4^2)$ and $T_2 \sim \mathrm{N}(2.8,\ 0.5^2)$ respectively.

(i) Find the probability that the total time taken to build one house of each type is less than 6 months. [4]

(ii) Find the probability that the time taken to build a type Q house is more than 1.2 times the time taken to build a type P house. [5]

6 The random variable X has probability density function given by

$$f(x) = \begin{cases} kx^{-1} & 2 \leqslant x \leqslant 6, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

(i) Show that $k = \dfrac{1}{\ln 3}$. [2]

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(ii) Show that $E(X) = 3.64$, correct to 3 significant figures. [3]

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(iii) Given that the median of X is m, find $\mathrm{P}(m < X < \mathrm{E}(X))$. [4]

7 A mill owner claims that the mean mass of sacks of flour produced at his mill is 51 kg. A quality control officer suspects that the mean mass is actually less than 51 kg. In order to test the owner's claim she finds the mass, x kg, of each of a random sample of 150 sacks and her results are summarised as follows.

$$n = 150 \qquad \Sigma x = 7480 \qquad \Sigma x^2 = 380\,000$$

(i) Carry out the test at the 2.5% significance level. [7]

You may now assume that the population standard deviation of the masses of sacks of flour is 6.856 kg. The quality control officer weighs another random sample of 150 sacks and carries out another test at the 2.5% significance level.

(ii) Given that the population mean mass is 49 kg, find the probability of a Type II error. [5]

Additional Page

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Cambridge International Examinations
Cambridge International Advanced Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

MATHEMATICS **9709/71**

Paper 7 Probability & Statistics 2 **(S2)** **May/June 2018**

1 hour 15 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

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At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50.

This document consists of **12** printed pages.

JC18 06_9709_71/2R

CAMBRIDGE International Examinations

1 A random sample of 75 values of a variable X gave the following results.

$$n = 75 \qquad \Sigma x = 153.2 \qquad \Sigma x^2 = 340.24$$

Find unbiased estimates for the population mean and variance of X. [3]

2 A six-sided die is suspected of bias. The die is thrown 100 times and it is found that the score is 2 on 20 throws. It is given that the probability of obtaining a score of 2 on any throw is p.

(i) Find an approximate 94% confidence interval for p. [3]

(ii) Use your answer to part **(i)** to comment on whether the die may be biased. [1]

3 The number of e-readers sold in a 10-day period in a shop is modelled by the distribution Po(5.1). Use an approximating distribution to find the probability that fewer than 140 e-readers are sold in a 300-day period. [4]

4 The volume, in millilitres, of a small cup of coffee has the distribution N(103.4, 10.2). The volume of a large cup of coffee is 1.5 times the volume of a small cup of coffee.

(i) Find the mean and standard deviation of the volume of a large cup of coffee. [3]

(ii) Find the probability that the total volume of a randomly chosen small cup of coffee and a randomly chosen large cup of coffee is greater than 250 ml. [4]

5 The mass, in kilograms, of rocks in a certain area has mean 14.2 and standard deviation 3.1.

(i) Find the probability that the mean mass of a random sample of 50 of these rocks is less than 14.0 kg. [3]

(ii) Explain whether it was necessary to assume that the population of the masses of these rocks is normally distributed. [1]

(iii) A geologist suspects that rocks in another area have a mean mass which is less than 14.2 kg. A random sample of 100 rocks in this area has sample mean 13.5 kg. Assuming that the standard deviation for rocks in this area is also 3.1 kg, test at the 2% significance level whether the geologist is correct. [5]

6 The time, in minutes, taken by people to complete a test is modelled by the continuous random variable X with probability density function given by

$$f(x) = \begin{cases} \dfrac{k}{x^2} & 5 \leqslant x \leqslant 10, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

(i) Show that $k = 10$. [3]

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(ii) Show that $E(X) = 10 \ln 2$. [2]

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(iii) Find $P(X > 9)$. [3]

(iv) Given that $P(X < a) = 0.6$, find a. [3]

7 The number of absences by girls from a certain class on any day is modelled by a random variable with distribution Po(0.2). The number of absences by boys from the same class on any day is modelled by an independent random variable with distribution Po(0.3).

(i) Find the probability that, during a randomly chosen 2-day period, the total number of absences is less than 3. [3]

(ii) Find the probability that, during a randomly chosen 5-day period, the number of absences by boys is more than 3. [2]

(iii) The teacher claims that, during the football season, there are more absences by boys than usual. In order to test this claim at the 5% significance level, he notes the number of absences by boys during a randomly chosen 5-day period during the football season.

(a) State what is meant by a Type I error in this context. [1]

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(b) State appropriate null and alternative hypotheses and find the probability of a Type I error. [3]

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(c) In fact there were 4 absences by boys during this period. Test the teacher's claim at the 5% significance level. [3]

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Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

Cambridge International A Level

Cambridge International Examinations
Cambridge International Advanced Level

CANDIDATE NAME	
CENTRE NUMBER	
CANDIDATE NUMBER	

3951791503

MATHEMATICS **9709/72**

Paper 7 Probability & Statistics 2 **(S2)** **May/June 2018**

1 hour 15 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

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This document consists of **12** printed pages.

CAMBRIDGE International Examinations

1 The numbers of alpha, beta and gamma particles emitted per minute by a certain piece of rock have independent distributions Po(0.2), Po(0.3) and Po(0.6) respectively. Find the probability that the total number of particles emitted during a 4-minute period is less than 4. [3]

2 The random variable X has the distribution N(3, 1.2). The random variable A is defined by $A = 2X$. The random variable B is defined by $B = X_1 + X_2$, where X_1 and X_2 are independent random values of X. Describe fully the distribution of A and the distribution of B. [3]

Distribution of A: ....

Distribution of B: ....

3 The management of a factory wished to find a range within which the time taken to complete a particular task generally lies. It is given that the times, in minutes, have a normal distribution with mean μ and standard deviation 6.5. A random sample of 15 employees was chosen and the mean time taken by these employees was found to be 52 minutes.

(i) Calculate a 95% confidence interval for μ. [3]

Later another 95% confidence interval for μ was found, based on a random sample of 30 employees.

(ii) State, with a reason, whether the width of this confidence interval was less than, equal to or greater than the width of the previous interval. [1]

4 The mean mass of packets of sugar is supposed to be 505 g. A random sample of 10 packets filled by a certain machine was taken and the masses, in grams, were found to be as follows.

500 499 496 495 498 490 492 501 494 494

(i) Find unbiased estimates of the population mean and variance. [3]

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The mean mass of packets produced by this machine was found to be less than 505 g, so the machine was adjusted. Following the adjustment, the masses of a random sample of 150 packets from the machine were measured and the total mass was found to be 75 660 g.

(ii) Given that the population standard deviation is 3.6 g, test at the 2% significance level whether the machine is still producing packets with mean mass less than 505 g. [5]

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(iii) Explain why the use of the normal distribution is justified in carrying out the test in part **(ii)**. [1]

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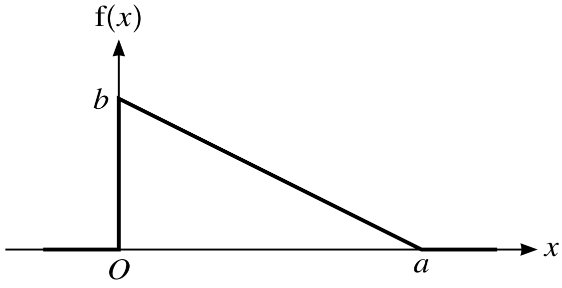

The diagram shows the probability density function, f, of a random variable X, in terms of the constants a and b.

(i) Find b in terms of a. [2]

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(ii) Show that $f(x) = \frac{2}{a} - \frac{2}{a^2}x$. [3]

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(iii) Given that $E(X) = 0.5$, find a. [4]

6 Accidents on a particular road occur at a constant average rate of 1 every 4.8 weeks.

(i) State, in context, one condition for the number of accidents in a given period to be modelled by a Poisson distribution. [1]

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Assume now that a Poisson distribution is a suitable model.

(ii) Find the probability that exactly 4 accidents will occur during a randomly chosen 12-week period. [2]

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(iii) Find the probability that more than 3 accidents will occur during a randomly chosen 10-week period. [3]

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(iv) Use a suitable approximating distribution to find the probability that fewer than 30 accidents will occur during a randomly chosen 2-year period ($104\frac{2}{7}$ weeks). [4]

7 A ten-sided spinner has edges numbered 1, 2, 3, 4, 5, 6, 7, 8, 9, 10. Sanjeev claims that the spinner is biased so that it lands on the 10 more often than it would if it were unbiased. In an experiment, the spinner landed on the 10 in 3 out of 9 spins.

(i) Test at the 1% significance level whether Sanjeev's claim is justified. [5]

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(ii) Explain why a Type I error cannot have been made. [1]

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In fact the spinner is biased so that the probability that it will land on the 10 on any spin is 0.5.

(iii) Another test at the 1% significance level, also based on 9 spins, is carried out. Calculate the probability of a Type II error. [6]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

Cambridge International A Level

Cambridge International Examinations
Cambridge International Advanced Level

CANDIDATE NAME		
CENTRE NUMBER		
CANDIDATE NUMBER		

0419852400

MATHEMATICS **9709/73**

Paper 7 Probability & Statistics 2 **(S2)** **May/June 2018**

1 hour 15 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

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The total number of marks for this paper is 50.

This document consists of **12** printed pages.

JC18 06_9709_73/3R

CAMBRIDGE International Examinations

[Turn over

1 A random variable X has the distribution B(75, 0.03).

(i) Use the Poisson approximation to the binomial distribution to calculate $P(X < 3)$. [3]

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(ii) Justify the use of the Poisson approximation. [1]

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2 Amy has to choose a random sample from the 265 students in her year at college. She numbers the students from 1 to 265 and then uses random numbers generated by her calculator. The first two random numbers produced by her calculator are 0.213 165 448 and 0.073 165 196.

(i) Use these figures to find the numbers of the first four students in her sample. [2]

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There were 25 students in Amy's sample. She asked each of them how much money, $\$x$, they earned in a week, on average. Her results are summarised below.

$$n = 25 \qquad \Sigma x = 510 \qquad \Sigma x^2 = 13\,225$$

(ii) Find unbiased estimates of the population mean and variance. [3]

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(iii) Explain briefly what is meant by 'population' in this question. [1]

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3 A researcher wishes to estimate the proportion, p, of houses in London Road that have only one occupant. He takes a random sample of 64 houses in London Road and finds that 8 houses in the sample have only one occupant. Using this sample, he calculates that an approximate $\alpha\%$ confidence interval for p has width 0.130. Find α correct to the nearest integer. [5]

4 The numbers, M and F, of male and female students who leave a particular school each year to study engineering have means 3.1 and 0.8 respectively.

(i) State, in context, one condition required for M to have a Poisson distribution. [1]

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Assume that M and F can be modelled by independent Poisson distributions.

(ii) Find the probability that the total number of students who leave to study engineering in a particular year is more than 3. [3]

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(iii) Given that the total number of students who leave to study engineering in a particular year is more than 3, find the probability that no female students leave to study engineering in that year. [3]

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5 The time taken for a particular train journey is normally distributed. In the past, the time had mean 2.4 hours and standard deviation 0.3 hours. A new timetable is introduced and on 30 randomly chosen occasions the time for this journey is measured. The mean time for these 30 occasions is found to be 2.3 hours.

(i) Stating any assumption(s), test, at the 5% significance level, whether the mean time for this journey has changed. [6]

(ii) A similar test at the 5% significance level was carried out using the times from another randomly chosen 30 occasions.

(a) State the probability of a Type I error. [1]

(b) State what is meant by a Type II error in this context. [1]

6 The times, in minutes, taken to complete the two parts of a task are normally distributed with means 4.5 and 2.3 respectively and standard deviations 1.1 and 0.7 respectively.

(i) Find the probability that the total time taken for the task is less than 8.5 minutes. [4]

(ii) Find the probability that the time taken for the first part of the task is more than twice the time taken for the second part. [5]

7 A random variable X has probability density function defined by

$$f(x) = \begin{cases} k\left(\frac{1}{x^2} + \frac{1}{x^3}\right) & 1 \leqslant x \leqslant 2, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

(i) Show that $k = \frac{8}{7}$. [3]

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(ii) Find $E(X)$. [3]

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(iii) Three values of X are chosen at random. Find the probability that one of these values is less than 1.5 and the other two are greater than 1.5. [5]

Additional Page

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Cambridge International Examinations
Cambridge International Advanced Level

CANDIDATE NAME

CENTRE NUMBER | CANDIDATE NUMBER

6786904181

MATHEMATICS **9709/72**

Paper 7 Probability & Statistics 2 **(S2)** **February/March 2018**

1 hour 15 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

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The total number of marks for this paper is 50.

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JC18 03_9709_72/4R

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1 X is a random variable with mean 4.9 and standard deviation 2.21. A random sample of 75 values of X is taken. Find the probability that the sample mean is greater than 5.0. [3]

2 The number of phone calls arriving in a 10-minute period at a switchboard is modelled by the random variable X which has the distribution $\text{Po}(4.1)$. Use an approximating distribution to find the probability that more than 90 calls arrive in a 4-hour period. [5]

3 The heights of plants of type A have mean 1.2 m and standard deviation 0.03 m. A random sample of 5 plants of type A is selected. The sum of the heights of these 5 plants is denoted by H_A m.

(i) Find the mean and variance of H_A. [3]

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The heights of plants of type B have mean 0.6 m and standard deviation 0.02 m. A random sample of 5 plants of type B is selected. The sum of the heights of these 5 plants is denoted by H_B m.

(ii) Find the mean and variance of $H_A - 2H_B$. [4]

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4 A store sells two types of computer, laptops and tablets. The number of laptops sold per hour is modelled by a random variable with distribution Po(0.9). The number of tablets sold per hour is modelled by an independent random variable with distribution Po(1.5).

(i) Find the probability that, during a randomly chosen hour, the total number of laptops and tablets sold in the store is less than 4. [3]

(ii) The manager claims that on sunny Saturdays fewer laptops than usual are sold. In order to test this claim, an employee notes the number of laptops sold during a 4-hour period on a randomly chosen sunny Saturday. In fact only 1 laptop is sold during this period. Test the manager's claim at the 10% significance level. [5]

5 Packets of Frugums contain 30 sweets. The manufacturer claims that, on average, 17% of the sweets are orange flavoured. Angela suspects that the average is actually less than 17%. In order to test the manufacturer's claim, she buys a packet of Frugums. If there are fewer than 3 orange flavoured sweets in the packet, she will conclude that the claim is false.

(i) State appropriate null and alternative hypotheses. [1]

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(ii) Explain what is meant by a Type I error in this situation. [1]

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(iii) Calculate the probability of a Type I error. [3]

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(iv) Given that the true percentage of orange flavoured sweets is 5%, calculate the probability of a Type II error. [3]

  [Turn over

6 A random variable X has probability density function given by

$$f(x) = \begin{cases} 6x(1-x) & 0 \leqslant x \leqslant 1, \\ 0 & \text{otherwise.} \end{cases}$$

(i) Find the probability that X does not lie between 0.3 and 0.7. [4]

(ii) Sketch the graph of the probability density function and hence state the value of $E(X)$. [2]

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(iii) Find $\text{Var}(X)$. [3]

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7 A nutritionist wishes to investigate the mean sugar content in some cereal bars. He takes a random sample of 10 of the bars and measures the mass, in grams, of sugar in each bar. His results are shown below.

11.9	11.7	11.8	11.9	11.6	12.1	11.7	11.9	11.8	11.9

Assume that the mass, in grams, of sugar in bars of this type has the distribution $N(\mu, 0.01)$.

(i) Calculate a 99% confidence interval for μ. [4]

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(ii) Explain whether it was necessary to use the Central Limit theorem in the calculation in part **(i)**. [1]

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(iii) The manufacturer claims that the mean mass of sugar in bars of this type is 11.7 g. Explain why your answer to part **(i)** does not support this claim. [1]

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(iv) The manufacturer suggests that a 95% confidence interval would be more likely to support his claim than a 99% confidence interval. **Without doing a calculation**, explain whether this suggestion is correct. [1]

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(v) It is thought that the value of 0.01 for the population variance may not be correct. Use the values in the sample to calculate an unbiased estimate of the population variance. [3]

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Additional Page

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Cambridge International Advanced Level

CANDIDATE NAME		
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1096435216

MATHEMATICS **9709/71**

Paper 7 Probability & Statistics 2 **(S2)** **October/November 2017**

1 hour 15 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

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JC17 11_9709_71/FP

CAMBRIDGE International Examinations

1 A random variable, X, has the distribution Po(31). Use the normal approximation to the Poisson distribution to find $P(X > 40)$. [3]

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2 An airline has found that, on average, 1 in 100 passengers do not arrive for each flight, and that this occurs randomly. For one particular flight the airline always sells 403 seats. The plane only has room for 400 passengers, so the flight is overbooked if the number of passengers who do not arrive is less than 3. Use a suitable approximation to find the probability that the flight is overbooked. [4]

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3 After an election 153 adults, from a random sample of 200 adults, said that they had voted. Using this information, an $\alpha\%$ confidence interval for the proportion of all adults who voted in the election was found to be 0.695 to 0.835, both correct to 3 significant figures. Find the value of α, correct to the nearest integer. [4]

4 The lengths, in millimetres, of rods produced by a machine are normally distributed with mean μ and standard deviation 0.9. A random sample of 75 rods produced by the machine has mean length 300.1 mm.

(i) Find a 99% confidence interval for μ, giving your answer correct to 2 decimal places. [3]

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The manufacturer claims that the machine produces rods with mean length 300 mm.

(ii) Use the confidence interval found in part **(i)** to comment on this claim. [1]

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5 A continuous random variable, X, has probability density function given by

$$f(x) = \begin{cases} \frac{1}{4}(x+1) & 0 \leqslant x \leqslant 2, \\ 0 & \text{otherwise.} \end{cases}$$

(i) Find $E(X)$. [3]

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(ii) Find the median of X. [3]

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6 The numbers of barrels of oil, in millions, extracted per day in two oil fields A and B are modelled by the independent random variables X and Y respectively, where $X \sim N(3.2, 0.4^2)$ and $Y \sim N(4.3, 0.6^2)$. The income generated by the oil from the two fields is \$90 per barrel for A and \$95 per barrel for B.

(i) Find the mean and variance of the daily income, in millions of dollars, generated by field A. [3]

(ii) Find the probability that the total income produced by the two fields in a day is at least \$670 million. [5]

7 In the past the number of cars sold per day at a showroom has been modelled by a random variable with distribution Po(0.7). Following an advertising campaign, it is hoped that the mean number of sales per day will increase. In order to test at the 10% significance level whether this is the case, the total number of sales during the first 5 days after the campaign is noted. You should assume that a Poisson model is still appropriate.

(i) Given that the total number of cars sold during the 5 days is 5, carry out the test. [6]

The number of cars sold per day at another showroom has the independent distribution Po(0.6). Assume that the distribution for the first showroom is still Po(0.7).

(ii) Find the probability that the total number of cars sold in the two showrooms during 3 days is exactly 2. [3]

8 In order to test the effect of a drug, a researcher monitors the concentration, X, of a certain protein in the blood stream of patients. For patients who are not taking the drug the mean value of X is 0.185. A random sample of 150 patients taking the drug was selected and the values of X were found. The results are summarised below.

$$n = 150 \qquad \Sigma x = 27.0 \qquad \Sigma x^2 = 5.01$$

The researcher wishes to test at the 1% significance level whether the mean concentration of the protein in the blood stream of patients taking the drug is less than 0.185.

(i) Carry out the test. [7]

(ii) Given that, in fact, the mean concentration for patients taking the drug is 0.175, find the probability of a Type II error occurring in the test. [5]

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Cambridge International Examinations
Cambridge International Advanced Level

CANDIDATE NAME

CENTRE NUMBER | CANDIDATE NUMBER

2064040879

MATHEMATICS **9709/72**

Paper 7 Probability & Statistics 2 **(S2)** **October/November 2017**

1 hour 15 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

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You are reminded of the need for clear presentation in your answers.

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The total number of marks for this paper is 50.

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JC17 11_9709_72/RP

CAMBRIDGE International Examinations

1 **(a)** **(i)** A random variable X has the distribution B(2540, 0.001). Use the Poisson approximation to the binomial distribution to find $P(X > 1)$. [3]

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(ii) Explain why the Poisson approximation is appropriate in this case. [1]

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(b) Two independent random variables, S and T, have distributions Po(2.1) and Po(3.5) respectively. Find the mean and standard deviation of $S + T$. [2]

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2 The number of words in History essays by students at a certain college has mean μ and standard deviation 1420.

(i) The mean number of words in a random sample of 125 History essays was found to be 4820. Calculate a 98% confidence interval for μ. [3]

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(ii) Another random sample of n History essays was taken. Using this sample, a 95% confidence interval for μ was found to be 4700 to 4980, both correct to the nearest integer. Find the value of n. [3]

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3 The masses, m kg, of packets of flour are normally distributed. The mean mass is supposed to be 1.01 kg. A quality control officer measures the masses of a random sample of 100 packets. The results are summarised below.

$$n = 100 \qquad \Sigma m = 98.2 \qquad \Sigma m^2 = 104.52$$

(i) Test at the 5% significance level whether the population mean mass is less than 1.01 kg. [7]

(ii) Explain whether it was necessary to use the Central Limit theorem in your answer to part **(i)**. [1]

4 The random variable X has probability density function given by

$$f(x) = \begin{cases} \dfrac{k}{\sqrt{x}} & 0 < x \leqslant a, \\ 0 & \text{otherwise,} \end{cases}$$

where k and a are constants. It is given that $E(X) = 3$.

(i) Find the value of a and show that $k = \frac{1}{6}$. [7]

(ii) Find the median of X. [3]

5 The marks in paper 1 and paper 2 of an examination are denoted by X and Y respectively, where X and Y have the independent continuous distributions $N(56, 6^2)$ and $N(43, 5^2)$ respectively.

(i) Find the probability that a randomly chosen paper 1 mark is more than a randomly chosen paper 2 mark. [5]

(ii) Each candidate's overall mark is M where $M = X + 1.5Y$. The minimum overall mark for grade A is 135. Find the proportion of students who gain a grade A. [5]

6 In a certain factory the number of items per day found to be defective has had the distribution Po(1.03). After the introduction of new quality controls, the management wished to test at the 10% significance level whether the mean number of defective items had decreased. They noted the total number of defective items produced in 5 randomly chosen days. It is assumed that defective items occur randomly and that a Poisson model is still appropriate.

(i) Given that the total number of defective items produced during the 5 days was 2, carry out the test. [6]

(ii) Using another random sample of 5 days the same test is carried out again, with the same significance level. Find the probability of a Type I error. [3]

(iii) Explain what is meant by a Type I error in this context. [1]

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Cambridge International A Level

Cambridge International Examinations
Cambridge International Advanced Level

CANDIDATE NAME

CENTRE NUMBER | CANDIDATE NUMBER

0280662099

MATHEMATICS **9709/73**

Paper 7 Probability & Statistics 2 **(S2)** **October/November 2017**

1 hour 15 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

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CAMBRIDGE International Examinations

1 A random variable, X, has the distribution Po(31). Use the normal approximation to the Poisson distribution to find $P(X > 40)$. [3]

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2 An airline has found that, on average, 1 in 100 passengers do not arrive for each flight, and that this occurs randomly. For one particular flight the airline always sells 403 seats. The plane only has room for 400 passengers, so the flight is overbooked if the number of passengers who do not arrive is less than 3. Use a suitable approximation to find the probability that the flight is overbooked. [4]

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3 After an election 153 adults, from a random sample of 200 adults, said that they had voted. Using this information, an $\alpha\%$ confidence interval for the proportion of all adults who voted in the election was found to be 0.695 to 0.835, both correct to 3 significant figures. Find the value of α, correct to the nearest integer. [4]

4 The lengths, in millimetres, of rods produced by a machine are normally distributed with mean μ and standard deviation 0.9. A random sample of 75 rods produced by the machine has mean length 300.1 mm.

(i) Find a 99% confidence interval for μ, giving your answer correct to 2 decimal places. [3]

The manufacturer claims that the machine produces rods with mean length 300 mm.

(ii) Use the confidence interval found in part **(i)** to comment on this claim. [1]

5 A continuous random variable, X, has probability density function given by

$$f(x) = \begin{cases} \frac{1}{4}(x+1) & 0 \leqslant x \leqslant 2, \\ 0 & \text{otherwise.} \end{cases}$$

(i) Find $E(X)$. [3]

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(ii) Find the median of X. [3]

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6 The numbers of barrels of oil, in millions, extracted per day in two oil fields A and B are modelled by the independent random variables X and Y respectively, where $X \sim N(3.2, 0.4^2)$ and $Y \sim N(4.3, 0.6^2)$. The income generated by the oil from the two fields is \$90 per barrel for A and \$95 per barrel for B.

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The number of cars sold per day at another showroom has the independent distribution Po(0.6). Assume that the distribution for the first showroom is still Po(0.7).

(ii) Find the probability that the total number of cars sold in the two showrooms during 3 days is exactly 2. [3]

8 In order to test the effect of a drug, a researcher monitors the concentration, X, of a certain protein in the blood stream of patients. For patients who are not taking the drug the mean value of X is 0.185. A random sample of 150 patients taking the drug was selected and the values of X were found. The results are summarised below.

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The researcher wishes to test at the 1% significance level whether the mean concentration of the protein in the blood stream of patients taking the drug is less than 0.185.

(i) Carry out the test. [7]

(ii) Given that, in fact, the mean concentration for patients taking the drug is 0.175, find the probability of a Type II error occurring in the test. [5]

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Cambridge International A Level

Cambridge International Examinations
Cambridge International Advanced Level

CANDIDATE NAME

CENTRE NUMBER | CANDIDATE NUMBER

7264703467

MATHEMATICS **9709/71**

Paper 7 Probability & Statistics 2 **(S2)** **May/June 2017**

1 hour 15 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

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JC17 06_9709_71/RP

CAMBRIDGE International Examinations

[Turn over

1 On average, 1 clover plant in 10 000 has four leaves instead of three.

(i) Use an approximating distribution to calculate the probability that, in a random sample of 2000 clover plants, more than 2 will have four leaves. [3]

(ii) Justify your approximating distribution. [2]

2 Past experience has shown that the heights of a certain variety of plant have mean 64.0 cm and standard deviation 3.8 cm. During a particularly hot summer, it was expected that the heights of plants of this variety would be less than usual. In order to test whether this was the case, a botanist recorded the heights of a random sample of 100 plants and found that the value of the sample mean was 63.3 cm. Stating a necessary assumption, carry out the test at the 2.5% significance level. [6]

3 **(a)** The waiting time at a certain bus stop has variance 2.6 minutes^2. For a random sample of 75 people, the mean waiting time was 7.1 minutes. Calculate a 92% confidence interval for the population mean waiting time. [3]

(b) A researcher used 3 random samples to calculate 3 independent 92% confidence intervals. Find the probability that all 3 of these confidence intervals contain only values that are greater than the actual population mean. [2]

(c) Another researcher surveyed the first 75 people who waited at a bus stop on a Monday morning. Give a reason why this sample is unsuitable for use in finding a confidence interval for the mean waiting time. [1]

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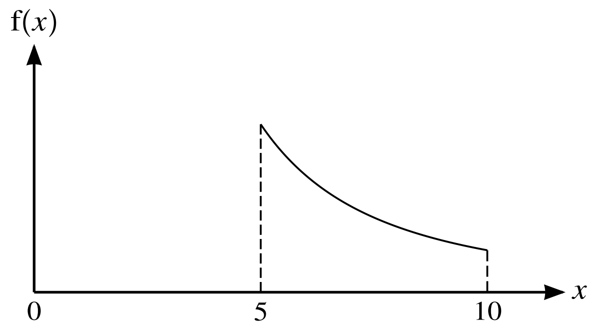

The time, X minutes, taken by a large number of runners to complete a certain race has probability density function f given by

$$f(x) = \begin{cases} \dfrac{k}{x^2} & 5 \leqslant x \leqslant 10, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant, as shown in the diagram.

(i) Without calculation, explain how you can tell that there were more runners whose times were below 7.5 minutes than above 7.5 minutes. [1]

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(ii) Show that $k = 10$. [3]

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(iii) Find $E(X)$. [3]

(iv) Find $Var(X)$. [2]

5 Large packets of sugar are packed in cartons, each containing 12 packets. The weights of these packets are normally distributed with mean 505 g and standard deviation 3.2 g. The weights of the cartons, when empty, are independently normally distributed with mean 150 g and standard deviation 7 g.

(i) Find the probability that the total weight of a full carton is less than 6200 g. [5]

Small packets of sugar are packed in boxes. The total weight of a full box has a normal distribution with mean 3130 g and standard deviation 12.1 g.

(ii) Find the probability that the weight of a randomly chosen full carton is less than double the weight of a randomly chosen full box. [5]

6 The number of sports injuries per month at a certain college has a Poisson distribution. In the past the mean has been 1.1 injuries per month. The principal recently introduced new safety guidelines and she decides to test, at the 2% significance level, whether the mean number of sports injuries has been reduced. She notes the number of sports injuries during a 6-month period.

(i) Find the critical region for the test and state the probability of a Type I error. [6]

(ii) State what is meant by a Type I error in this context. [1]

(iii) During the 6-month period there are a total of 2 sports injuries. Carry out the test. [3]

(iv) Assuming that the mean remains 1.1, calculate the probability that there will be fewer than 30 sports injuries during a 36-month period. [4]

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Cambridge International A Level

Cambridge International Examinations
Cambridge International Advanced Level

CANDIDATE NAME

CENTRE NUMBER | CANDIDATE NUMBER

4433263648

MATHEMATICS **9709/72**

Paper 7 Probability & Statistics 2 **(S2)** **May/June 2017**

1 hour 15 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

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CAMBRIDGE International Examinations

[Turn over

1 In a survey of 2000 randomly chosen adults, 1602 said that they owned a smartphone. Calculate an approximate 95% confidence interval for the proportion of adults in the whole population who own a smartphone. [4]

2 Javier writes an article containing 52 460 words. He plans to upload the article to his website, but he knows that this process sometimes introduces errors. He assumes that for each word in the uploaded version of his article, the probability that it contains an error is 0.000 08. The number of words containing an error is denoted by X.

(i) Find $E(X)$ and $\mathrm{Var}(X)$, giving your answers correct to three decimal places. [2]

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Javier wants to use the Poisson distribution as an approximating distribution to calculate the probability that there will be fewer than 5 words containing an error in his uploaded article.

(ii) Explain how your answers to part **(i)** are consistent with the use of the Poisson distribution as an approximating distribution. [1]

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(iii) Use the Poisson distribution to calculate $P(X < 5)$. [2]

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3 Household incomes, in thousands of dollars, in a certain country are represented by the random variable X with mean μ and standard deviation σ. The incomes of a random sample of 400 households are found and the results are summarised below.

$$n = 400 \qquad \Sigma x = 923 \qquad \Sigma x^2 = 3170$$

(i) Calculate unbiased estimates of μ and σ^2. [3]

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(ii) A random sample of 50 households in one particular region of the country is taken and the sample mean income, in thousands of dollars, is found to be 2.6. Using your values from part **(i)**, test at the 5% significance level whether household incomes in this region are greater, on average, than in the country as a whole. [5]

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4 It is claimed that 1 in every 4 packets of certain biscuits contains a free gift. Marisa and André both suspect that the true proportion is less than 1 in 4.

(i) Marisa chooses 20 packets at random. She decides that if fewer than 3 contain free gifts, she will conclude that the claim is not justified. Use a binomial distribution to find the probability of a Type I error. [2]

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(ii) André chooses 25 packets at random. He decides to carry out a significance test at the 1% level, using a binomial distribution. Given that only 1 of the 25 packets contains a free gift, carry out the test. [5]

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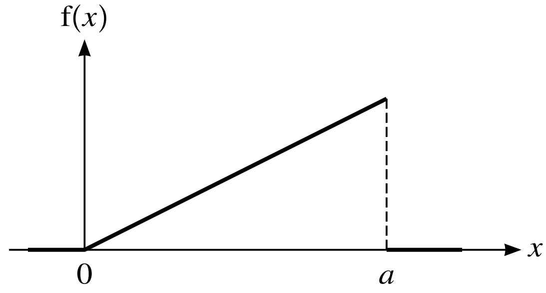

The diagram shows the graph of the probability density function, f, of a random variable X which takes values between 0 and a only. It is given that $P(X < 1) = 0.25$.

(i) Find, in any order,

(a) $P(X < 2)$,

(b) the value of a,

(c) $f(x)$.

[5]

(ii) Find the median of X. [3]

6 Old televisions arrive randomly and independently at a recycling centre at an average rate of 1.2 per day.

(i) Find the probability that exactly 2 televisions arrive in a 2-day period. [2]

(ii) Use an appropriate approximating distribution to find the probability that at least 55 televisions arrive in a 50-day period. [4]

Independently of televisions, old computers arrive randomly and independently at the same recycling centre at an average rate of 4 per 7-day week.

(iii) Find the probability that the total number of televisions and computers that arrive at the recycling centre in a 3-day period is less than 4. [3]

  [Turn over

7 **(a)** A random variable X is normally distributed with mean 4.2 and standard deviation 1.1. Find the probability that the sum of two randomly chosen values of X is greater than 10. [4]

(b) Each candidate's overall score for an essay is calculated as follows. The mark for creativity is denoted by C, the penalty mark for spelling errors is denoted by S and the overall score is defined by $C - \frac{1}{2}S$. The variables C and S are independent and have distributions $N(29, 105)$ and $N(17, 15)$ respectively. Find the proportion of candidates receiving a negative overall score. [5]

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Cambridge International Examinations
Cambridge International Advanced Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

MATHEMATICS **9709/73**

Paper 7 Probability & Statistics 2 **(S2)** **May/June 2017**

1 hour 15 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

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JC17 06_9709_73/2R

1 A residents' association has 654 members, numbered from 1 to 654. The secretary wishes to send a questionnaire to a random sample of members. In order to choose the members for the sample she uses a table of random numbers. The first line in the table is as follows.

1096 4357 3765 0431 0928 9264

The numbers of the first two members in the sample are 109 and 643. Find the numbers of the next three members in the sample. [3]

2 In a random sample of 200 shareholders of a company, 103 said that they wanted a change in the management.

(i) Find an approximate 92% confidence interval for the proportion, p, of all shareholders who want a change in the management. [3]

(ii) State the probability that a 92% confidence interval does not contain p. [1]

3 The mass, in tonnes, of iron ore produced per day at a mine is normally distributed with mean 7.0 and standard deviation 0.46. Find the probability that the total amount of iron ore produced in 10 randomly chosen days is more than 71 tonnes. [5]

4 Last year the mean level of a certain pollutant in a river was found to be 0.034 grams per millilitre. This year the levels of pollutant, X grams per millilitre, were measured at a random sample of 200 locations in the river. The results are summarised below.

$$n = 200 \qquad \Sigma x = 6.7 \qquad \Sigma x^2 = 0.2312$$

(i) Calculate unbiased estimates of the population mean and variance. [3]

(ii) Test, at the 10% significance level, whether the mean level of pollutant has changed. [5]

5 **(i)** A random variable X has the distribution Po(42).

(a) Use an appropriate approximating distribution to find $P(X \geqslant 40)$. [4]

(b) Justify your use of the approximating distribution. [1]

(ii) A random variable Y has the distribution B(60, 0.02).

(a) Use an appropriate approximating distribution to find $P(Y > 2)$. [3]

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(b) Justify your use of the approximating distribution. [1]

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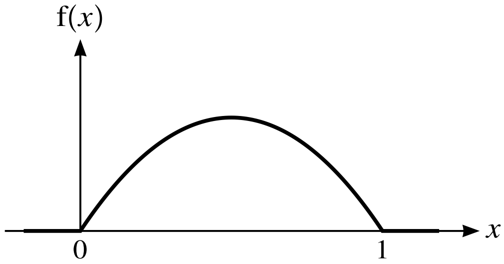

The diagram shows the graph of the probability density function, f, of a continuous random variable X, where f is defined by

$$f(x) = \begin{cases} k(x - x^2) & 0 \leqslant x \leqslant 1, \\ 0 & \text{otherwise.} \end{cases}$$

(i) Show that the value of the constant k is 6. [3]

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(ii) State the value of $E(X)$ and find $Var(X)$. [4]

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(iii) Find $P(0.4 < X < 2)$. [3]

7 In the past the number of accidents per month on a certain road was modelled by a random variable with distribution Po(0.47). After the introduction of speed restrictions, the government wished to test, at the 5% significance level, whether the mean number of accidents had decreased. They noted the number of accidents during the next 12 months. It is assumed that accidents occur randomly and that a Poisson model is still appropriate.

(i) Given that the total number of accidents during the 12 months was 2, carry out the test. [6]

(ii) Explain what is meant by a Type II error in this context. [1]

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It is given that the mean number of accidents per month is now in fact 0.05.

(iii) Using another random sample of 12 months the same test is carried out again, with the same significance level. Find the probability of a Type II error. [4]

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Cambridge International A Level

Cambridge International Examinations
Cambridge International Advanced Level

CANDIDATE NAME			
CENTRE NUMBER		CANDIDATE NUMBER	

9660799626

MATHEMATICS **9709/72**

Paper 7 Probability & Statistics 2 **(S2)** **February/March 2017**

1 hour 15 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

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JC17 03_9709_72/2R

CAMBRIDGE International Examinations

[Turn over

1 In a survey, 36 out of 120 randomly selected voters in Hungton said that if there were an election next week they would vote for the Alpha party. Calculate an approximate 90% confidence interval for the proportion of voters in Hungton who would vote for the Alpha party. [4]

2 Karim has noted the lifespans, in weeks, of a large random sample of certain insects. He carries out a test, at the 1% significance level, for the population mean, μ. Karim's null hypothesis is $\mu = 6.4$.

(i) Given that Karim's test is two-tail, state the alternative hypothesis. [1]

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Karim finds that the value of the test statistic is $z = 2.43$.

(ii) Explain what conclusion he should draw. [2]

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(iii) Explain briefly when a one-tail test is appropriate, rather than a two-tail test. [1]

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3 The length, in centimetres, of a certain type of snake is modelled by the random variable X with mean 52 and standard deviation 6.1. A random sample of 75 snakes is selected, and the sample mean, $\overline{X}$, is found.

(i) Find $P(51 < \overline{X} < 53)$. [4]

(ii) Explain why it was necessary to use the Central Limit theorem in the solution to part **(i)**. [1]

4 At a doctors' surgery, the number of missed appointments per day has a Poisson distribution. In the past the mean number of missed appointments per day has been 0.9. Following some publicity, the manager carries out a hypothesis test to determine whether this mean has decreased. If there are fewer than 3 missed appointments in a randomly chosen 5-day period, she will conclude that the mean has decreased.

(i) Find the probability of a Type I error. [3]

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(ii) State what is meant by a Type I error in this context. [1]

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(iii) Find the probability of a Type II error if the mean number of missed appointments per day is 0.2. [3]

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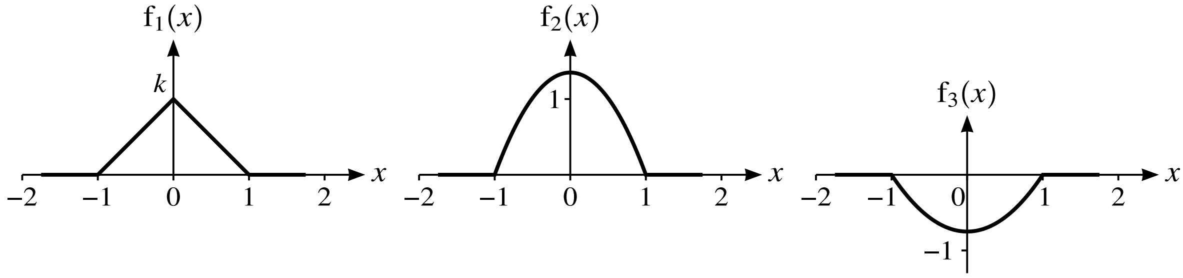
5 **(a)**



The diagram shows the graphs of three functions, f_1, f_2 and f_3. The function f_1 is a probability density function.

(i) State the value of k. [1]

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(ii) For each of the functions f_2 and f_3, state why it cannot be a probability density function. [2]

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(b) The probability density function g is defined by

$$g(x) = \begin{cases} 6(a^2 - x^2) & -a \leqslant x \leqslant a, \\ 0 & \text{otherwise,} \end{cases}$$

where a is a constant.

(i) Show that $a = \frac{1}{2}$. [3]

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(ii) State the value of E(X). [1]

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(iii) Find Var(X). [2]

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6 The masses, in kilograms, of cartons of sugar and cartons of flour have the distributions $N(78.8,\ 12.6^2)$ and $N(62.0,\ 10.0^2)$ respectively.

(i) The standard load for a certain crane is 8 cartons of sugar and 3 cartons of flour. The maximum load that can be carried safely by the crane is 900 kg. Stating a necessary assumption, find the percentage of standard loads that will exceed the maximum safe load. [5]

(ii) Find the probability that a randomly chosen carton of sugar has a smaller mass than a randomly chosen carton of flour. [5]

7 The number of planes arriving at an airport every hour during daytime is modelled by the random variable X with distribution $Po(5.2)$.

(i) State two assumptions required for the Poisson model to be valid in this context. [2]

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(ii) (a) Find the probability that the number of planes arriving in a 15-minute period is greater than 1 and less than 4, [3]

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(b) Find the probability that more than 3 planes will arrive in a 40-minute period. [2]

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(iii) The airport has enough staff to deal with a maximum of 60 planes landing during a 10-hour day. Use a suitable approximation to find the probability that, on a randomly chosen 10-hour day, staff will be able to deal with all the planes that land. [4]

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Cambridge International Examinations
Cambridge International Advanced Level

MATHEMATICS **9709/71**

Paper 7 Probability & Statistics 2 **(S2)** **October/November 2016**

1 hour 15 minutes

Additional Materials: List of Formulae (MF9)

* 1 9 6 9 7 3 4 1 0 5 *

READ THESE INSTRUCTIONS FIRST

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This document consists of **3** printed pages, **1** blank page and **1** insert.

1 The weights, in kilograms, of a random sample of eight 16-year old males are given below.

58.9 63.5 62.7 59.4 66.9 68.0 60.4 68.2

Find unbiased estimates of the population mean and variance of the weights of all 16-year old males. [3]

2 A die has six faces numbered 1, 2, 3, 4, 5, 6. Manjit suspects that the die is biased so that it shows a six on fewer throws than it would if it were fair. In order to test her suspicion, she throws the die a certain number of times and counts the number of sixes.

(i) State suitable null and alternative hypotheses for Manjit's test. [1]

(ii) There are no sixes in the first 15 throws. Show that this result is not significant at the 5% level. [2]

(iii) Find the smallest value of n such that, if there are no sixes in the first n throws, this result is significant at the 5% level. [2]

3 Particles are emitted randomly from a radioactive substance at a constant average rate of 3.6 per minute. Find the probability that

(i) more than 3 particles are emitted during a 20-second period, [3]

(ii) more than 240 particles are emitted during a 1-hour period. [4]

4 Each week a farmer sells X litres of milk and Y kg of cheese, where X and Y have the independent distributions $N(1520, 53^2)$ and $N(175, 12^2)$ respectively.

(i) Find the mean and standard deviation of the total amount of milk that the farmer sells in 4 randomly chosen weeks. [2]

During a year when milk prices are low, the farmer makes a loss of 2 cents per litre on milk and makes a profit of 21 cents per kg on cheese, so the farmer's overall weekly profit is $(21Y - 2X)$ cents.

(ii) Find the probability that, in a randomly chosen week, the farmer's overall profit is positive. [5]

5 **(a)** The masses, in grams, of certain tomatoes are normally distributed with standard deviation 9 grams. A random sample of 100 tomatoes has a sample mean of 63 grams. Find a 90% confidence interval for the population mean mass of these tomatoes. [3]

(b) The masses, in grams, of certain potatoes are normally distributed with known population standard deviation but unknown population mean. A random sample of potatoes is taken in order to find a confidence interval for the population mean. Using a sample of size 50, a 95% confidence interval is found to have width 8 grams.

(i) Using another sample of size 50, an α% confidence interval has width 4 grams. Find α. [3]

(ii) Find the sample size n, such that a 95% confidence interval has width 4 grams. [2]

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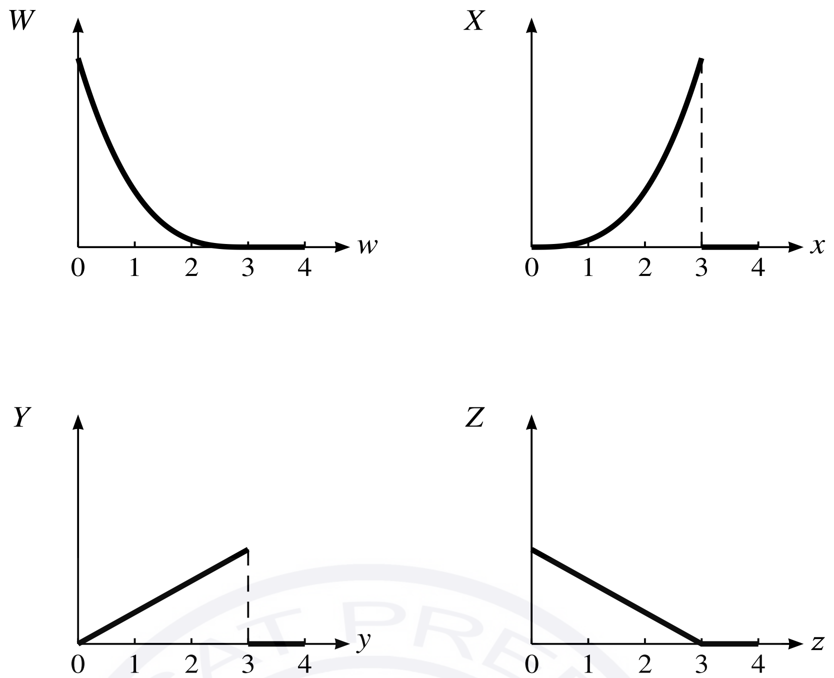

The diagrams show the probability density functions of four random variables W, X, Y and Z. Each of the four variables takes values between 0 and 3 only, and their medians are m_W, m_X, m_Y and m_Z respectively.

(i) List m_W, m_X, m_Y and m_Z in order of size, starting with the largest. [2]

(ii) The probability density function of X is given by

$$f(x) = \begin{cases} \frac{4}{81}x^3 & 0 \leqslant x \leqslant 3, \\ 0 & \text{otherwise.} \end{cases}$$

(a) Show that $E(X) = \frac{12}{5}$. [3]

(b) Calculate $P(X > E(X))$. [3]

(c) Write down the value of $P(X < 2E(X))$. [1]

7 In the past the time, in minutes, taken for a particular rail journey has been found to have mean 20.5 and standard deviation 1.2. Some new railway signals are installed. In order to test whether the mean time has decreased, a random sample of 100 times for this journey are noted. The sample mean is found to be 20.3 minutes. You should assume that the standard deviation is unchanged.

(i) Carry out a significance test, at the 4% level, of whether the population mean time has decreased. [5]

Later another significance test of the same hypotheses, using another random sample of size 100, is carried out at the 4% level.

(ii) Given that the population mean is now 20.1, find the probability of a Type II error. [5]

(iii) State what is meant by a Type II error in this context. [1]

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Cambridge International A Level

Cambridge International Examinations
Cambridge International Advanced Level

MATHEMATICS **9709/72**

Paper 7 Probability & Statistics 2 **(S2)** **October/November 2016**

1 hour 15 minutes

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Additional Materials: List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

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CAMBRIDGE International Examinations
[Turn over

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Find unbiased estimates of the population mean and variance of the weights of all 16-year old males. [3]

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During a year when milk prices are low, the farmer makes a loss of 2 cents per litre on milk and makes a profit of 21 cents per kg on cheese, so the farmer's overall weekly profit is $(21Y - 2X)$ cents.

(ii) Find the probability that, in a randomly chosen week, the farmer's overall profit is positive. [5]

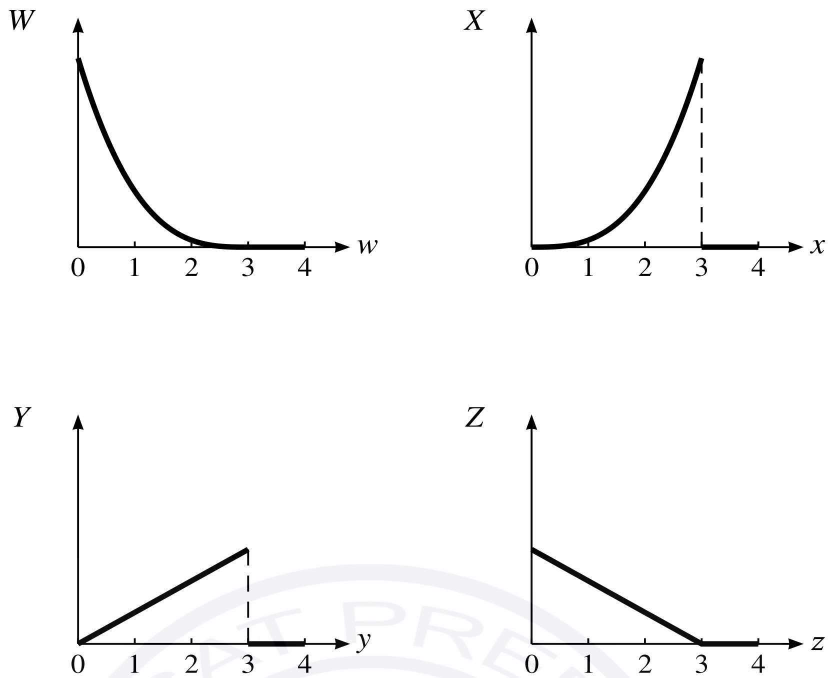
5 **(a)** The masses, in grams, of certain tomatoes are normally distributed with standard deviation 9 grams. A random sample of 100 tomatoes has a sample mean of 63 grams. Find a 90% confidence interval for the population mean mass of these tomatoes. [3]

(b) The masses, in grams, of certain potatoes are normally distributed with known population standard deviation but unknown population mean. A random sample of potatoes is taken in order to find a confidence interval for the population mean. Using a sample of size 50, a 95% confidence interval is found to have width 8 grams.

(i) Using another sample of size 50, an $\alpha\%$ confidence interval has width 4 grams. Find α. [3]

(ii) Find the sample size n, such that a 95% confidence interval has width 4 grams. [2]

6



The diagrams show the probability density functions of four random variables W, X, Y and Z. Each of the four variables takes values between 0 and 3 only, and their medians are m_W, m_X, m_Y and m_Z respectively.

(i) List m_W, m_X, m_Y and m_Z in order of size, starting with the largest. [2]

(ii) The probability density function of X is given by

$$f(x) = \begin{cases} \frac{4}{81}x^3 & 0 \leqslant x \leqslant 3, \\ 0 & \text{otherwise.} \end{cases}$$

(a) Show that $E(X) = \frac{12}{5}$. [3]

(b) Calculate $P(X > E(X))$. [3]

(c) Write down the value of $P(X < 2E(X))$. [1]

7 In the past the time, in minutes, taken for a particular rail journey has been found to have mean 20.5 and standard deviation 1.2. Some new railway signals are installed. In order to test whether the mean time has decreased, a random sample of 100 times for this journey are noted. The sample mean is found to be 20.3 minutes. You should assume that the standard deviation is unchanged.

(i) Carry out a significance test, at the 4% level, of whether the population mean time has decreased. [5]

Later another significance test of the same hypotheses, using another random sample of size 100, is carried out at the 4% level.

(ii) Given that the population mean is now 20.1, find the probability of a Type II error. [5]

(iii) State what is meant by a Type II error in this context. [1]

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Cambridge International A Level

Cambridge International Examinations
Cambridge International Advanced Level

MATHEMATICS **9709/73**

Paper 7 Probability & Statistics 2 **(S2)** **October/November 2016**

1 hour 15 minutes

Additional Materials: List of Formulae (MF9)

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READ THESE INSTRUCTIONS FIRST

An answer booklet is provided inside this question paper. You should follow the instructions on the front cover of the answer booklet. If you need additional answer paper ask the invigilator for a continuation booklet.

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The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50.

This document consists of **4** printed pages and **1** insert.

JC16 11_9709_73/2R

CAMBRIDGE International Examinations

[Turn over

1 The random variable X has the distribution $\mathrm{Po}(3.5)$. Find $\mathrm{P}(X < 3)$. [3]

2 Dominic wishes to choose a random sample of five students from the 150 students in his year. He numbers the students from 1 to 150. Then he uses his calculator to generate five random numbers between 0 and 1. He multiplies each random number by 150 and rounds up to the next whole number to give a student number.

(i) Dominic's first random number is 0.392. Find the student number that is produced by this random number. [1]

(ii) Dominic's second student number is 104. Find a possible random number that would produce this student number. [1]

(iii) Explain briefly why five random numbers may not be enough to produce a sample of five student numbers. [1]

3 A men's triathlon consists of three parts: swimming, cycling and running. Competitors' times, in minutes, for the three parts can be modelled by three independent normal variables with means 34.0, 87.1 and 56.9, and standard deviations 3.2, 4.1 and 3.8, respectively. For each competitor, the total of his three times is called the race time. Find the probability that the mean race time of a random sample of 15 competitors is less than 175 minutes. [5]

4 The manufacturer of a tablet computer claims that the mean battery life is 11 hours. A consumer organisation wished to test whether the mean is actually greater than 11 hours. They invited a random sample of members to report the battery life of their tablets. They then calculated the sample mean. Unfortunately a fire destroyed the records of this test except for the following partial document.

Test of the mean batte the table	
Sample size, n	
Sample mean (hours)	11.8
Is the result significant at the 5% level?	Yes
Is the result significant at the 2.5% level?	No

Given that the population of battery lives is normally distributed with standard deviation 1.6 hours, find the set of possible values of the sample size, n. [5]

5 It is claimed that 30% of packets of Froogum contain a free gift. Andre thinks that the actual proportion is less than 30% and he decides to carry out a hypothesis test at the 5% significance level. He buys 20 packets of Froogum and notes the number of free gifts he obtains.

(i) State null and alternative hypotheses for the test. [1]

(ii) Use a binomial distribution to find the probability of a Type I error. [5]

Andre finds that 3 of the 20 packets contain free gifts.

(iii) Carry out the test. [2]

6 A variable X takes values 1, 2, 3, 4, 5, and these values are generated at random by a machine. Each value is supposed to be equally likely, but it is suspected that the machine is not working properly. A random sample of 100 values of X, generated by the machine, gives the following results.

$$n = 100 \qquad \Sigma x = 340 \qquad \Sigma x^2 = 1356$$

(i) Find a 95% confidence interval for the population mean of the values generated by the machine. [6]

(ii) Use your answer to part **(i)** to comment on whether the machine may be working properly. [2]

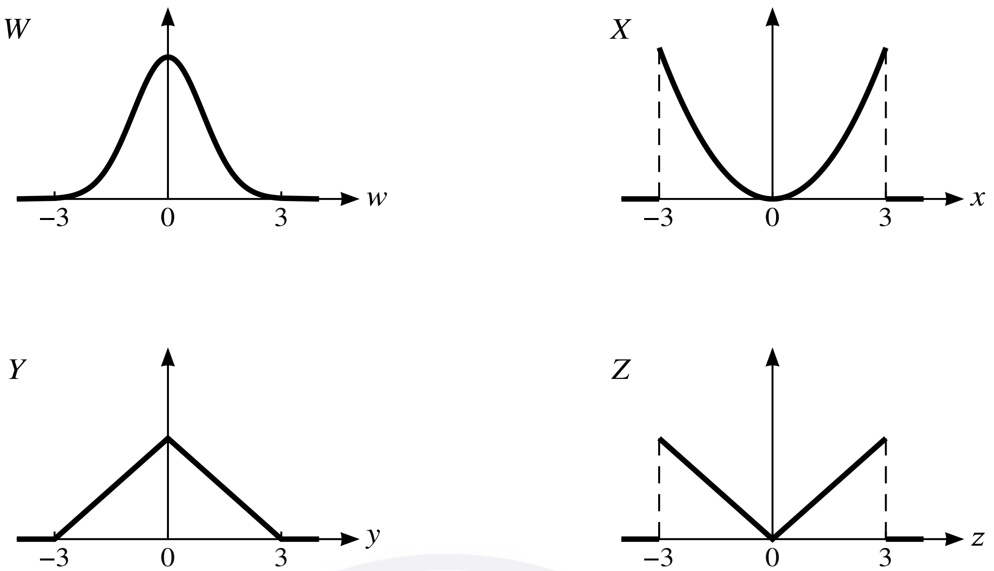
7 Men arrive at a clinic independently and at random, at a constant mean rate of 0.2 per minute. Women arrive at the same clinic independently and at random, at a constant mean rate of 0.3 per minute.

(i) Find the probability that at least 2 men and at least 3 women arrive at the clinic during a 5-minute period. [4]

(ii) Find the probability that fewer than 36 people arrive at the clinic during a 1-hour period. [5]

[Question 8 is printed on the next page.]

8



The diagrams show the probability density functions of four random variables W, X, Y and Z. Each of the four variables takes values between −3 and 3 only, and their standard deviations are σ_W, σ_X, σ_Y and σ_Z respectively.

(i) List σ_W, σ_X, σ_Y and σ_Z in order of size, starting with the largest. [2]

(ii) The probability density function of X is given by

$$\mathrm{f}(x) = \begin{cases} \frac{1}{18}x^2 & -3 \leqslant x \leqslant 3, \\ 0 & \text{otherwise.} \end{cases}$$

(a) Show that $\sigma_X = 2.32$ correct to 3 significant figures. [3]

(b) Calculate $\mathrm{P}(X > \sigma_X)$. [3]

(c) Write down the value of $\mathrm{P}(X > 2\sigma_X)$. [1]

Cambridge International A Level

Cambridge International Examinations
Cambridge International Advanced Level

MATHEMATICS **9709/71**

Paper 7 Probability & Statistics 2 **(S2)** **May/June 2016**

1 hour 15 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

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READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

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This document consists of **3** printed pages and **1** blank page.

1 A six-sided die shows a six on 25 throws out of 200 throws. Test at the 10% significance level the null hypothesis: P(throwing a six) = $\frac{1}{6}$, against the alternative hypothesis: P(throwing a six) < $\frac{1}{6}$. [5]

2 A researcher is investigating the lengths, in kilometres, of the journeys to work of the employees at a certain firm. She takes a random sample of 10 employees.

(i) State what is meant by 'random' in this context. [1]

The results of her sample are as follows.

1.5 2.0 3.6 5.9 4.8 8.7 3.5 2.9 4.1 3.0

(ii) Find unbiased estimates of the population mean and variance. [3]

(iii) State what is meant by 'population' in this context. [1]

3 Based on a random sample of 700 people living in a certain area, a confidence interval for the proportion, p, of all people living in that area who had travelled abroad was found to be $0.5672 < p < 0.6528$.

(i) Find the proportion of people in the sample who had travelled abroad. [1]

(ii) Find the confidence level of this confidence interval. Give your answer correct to the nearest integer. [4]

4 In the past, the time spent by customers in a certain shop had mean 12.5 minutes and standard deviation 4.2 minutes. Following a change of layout in the shop, the mean time spent in the shop by a random sample of 50 customers is found to be 13.5 minutes.

(i) Assuming that the standard deviation remains at 4.2 minutes, test at the 5% significance level whether the mean time spent by customers in the shop has changed. [5]

(ii) Another random sample of 50 customers is chosen and a similar test at the 5% significance level is carried out. State the probability of a Type I error. [1]

5 The thickness of books in a large library is normally distributed with mean 2.4 cm and standard deviation 0.3 cm.

(i) Find the probability that the total thickness of 6 randomly chosen books is more than 16 cm. [4]

(ii) Find the probability that the thickness of a book chosen at random is less than 1.1 times the thickness of a second book chosen at random. [5]

6 In each turn of a game, a coin is pushed and slides across a table. The distance, X metres, travelled by the coin has probability density function given by

$$f(x) = \begin{cases} kx^2(2-x) & 0 \leqslant x \leqslant 2, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

(i) State the greatest possible distance travelled by the coin in one turn. [1]

(ii) Show that $k = \frac{3}{4}$. [3]

(iii) Find the mean distance travelled by the coin in one turn. [3]

(iv) Out of 400 turns, find the expected number of turns in which the distance travelled by the coin is less than 1 metre. [3]

7 **(a)** A large number of spoons and forks made in a factory are inspected. It is found that 1% of the spoons and 1.5% of the forks are defective. A random sample of 140 items, consisting of 80 spoons and 60 forks, is chosen. Use the Poisson approximation to the binomial distribution to find the probability that the sample contains

(i) at least 1 defective spoon and at least 1 defective fork, [3]

(ii) fewer than 3 defective items. [3]

(b) The random variable X has the distribution $\text{Po}(\lambda)$. It is given that

$$P(X = 1) = p \quad \text{and} \quad P(X = 2) = 1.5p,$$

where p is a non-zero constant. Find the value of λ and hence find the value of p. [4]

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Cambridge International A Level

Cambridge International Examinations
Cambridge International Advanced Level

MATHEMATICS **9709/72**

Paper 7 Probability & Statistics 2 **(S2)** **May/June 2016**

1 hour 15 minutes

* 1 1 5 7 9 6 7 7 8 3 *

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

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JC16 06_9709_72/3R

CAMBRIDGE International Examinations

1 The length of time, in minutes, taken by people to complete a task has mean 53.0 and standard deviation 6.2. Find the probability that the mean time taken to complete the task by a random sample of 50 people is more than 51 minutes. [4]

2 Jacques is a chef. He claims that 90% of his customers are satisfied with his cooking. Marie suspects that the true percentage is lower than 90%. She asks a random sample of 15 of Jacques' customers whether they are satisfied. She then performs a hypothesis test of the null hypothesis $p = 0.9$ against the alternative hypothesis $p < 0.9$, where p is the population proportion of customers who are satisfied. She decides to reject the null hypothesis if fewer than 12 customers are satisfied.

(i) In the context of the question, explain what is meant by a Type I error. [1]

(ii) Find the probability of a Type I error in Marie's test. [3]

3 **(i)** Give a reason for using a sample rather than the whole population in carrying out a statistical investigation. [1]

(ii) Tennis balls of a certain brand are known to have a mean height of bounce of 64.7 cm, when dropped from a height of 100 cm. A change is made in the manufacturing process and it is required to test whether this change has affected the mean height of bounce. 100 new tennis balls are tested and it is found that their mean height of bounce when dropped from a height of 100 cm is 65.7 cm and the unbiased estimate of the population variance is 15 cm^2.

(a) Calculate a 95% confidence interval for the population mean. [3]

(b) Use your answer to part **(ii)(a)** to explain what conclusion can be drawn about whether the change has affected the mean height of bounce. [1]

4 At a certain company, computer faults occur randomly and at a constant mean rate. In the past this mean rate has been 2.1 per week. Following an update, the management wish to determine whether the mean rate has changed. During 20 randomly chosen weeks it is found that 54 computer faults occur. Use a suitable approximation to test at the 5% significance level whether the mean rate has changed. [6]

5 Each box of Fruity Flakes contains X grams of flakes and Y grams of fruit, where X and Y are independent random variables, having distributions N(400, 50) and N(100, 20) respectively. The weight of each box, when empty, is exactly 20 grams. A full box of Fruity Flakes is chosen at random.

(i) Find the probability that the total weight of the box and its contents is less than 530 grams. [5]

(ii) Find the probability that the weight of flakes in the box is more than 4.1 times the weight of fruit in the box. [5]

6 At a certain shop the demand for hair dryers has a Poisson distribution with mean 3.4 per week.

(i) Find the probability that, in a randomly chosen two-week period, the demand is for exactly 5 hair dryers. [3]

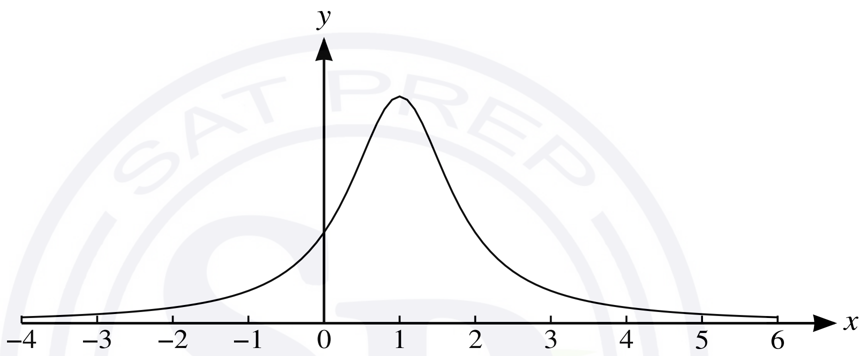
(ii) At the beginning of a week the shop has a certain number of hair dryers for sale. Find the probability that the shop has enough hair dryers to satisfy the demand for the week if

(a) they have 4 hair dryers in the shop, [2]

(b) they have 5 hair dryers in the shop. [2]

(iii) Find the smallest number of hair dryers that the shop needs to have at the beginning of a week so that the probability of being able to satisfy the demand that week is at least 0.9. [3]

7 **(a)**



The diagram shows the graph of the probability density function of a variable X. Given that the graph is symmetrical about the line $x = 1$ and that $P(0 < X < 2) = 0.6$, find $P(X > 0)$. [2]

(b) A flower seller wishes to model the length of time that tulips last when placed in a jug of water. She proposes a model using the random variable X (in hundreds of hours) with probability density function given by

$$f(x) = \begin{cases} k(2.25 - x^2) & 0 \leqslant x \leqslant 1.5, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

(i) Show that $k = \frac{4}{9}$. [3]

(ii) Use this model to find the mean number of hours that a tulip lasts in a jug of water. [4]

The flower seller wishes to create a similar model for daffodils. She places a large number of daffodils in jugs of water and the longest time that any daffodil lasts is found to be 290 hours.

(iii) Give a reason why $f(x)$ would not be a suitable model for daffodils. [1]

(iv) The flower seller considers a model for daffodils of the form

$$g(x) = \begin{cases} c(a^2 - x^2) & 0 \leqslant x \leqslant a, \\ 0 & \text{otherwise,} \end{cases}$$

where a and c are constants. State a suitable value for a. (There is no need to evaluate c.) [1]

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Cambridge International A Level

Cambridge International Examinations
Cambridge International Advanced Level

MATHEMATICS **9709/73**

Paper 7 Probability & Statistics 2 **(S2)** **May/June 2016**

1 hour 15 minutes

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Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

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This document consists of **3** printed pages and **1** blank page.

CAMBRIDGE
International Examinations

[Turn over

1 The time taken for a particular type of paint to dry was measured for a sample of 150 randomly chosen points on a wall. The sample mean was 192.4 minutes and an unbiased estimate of the population variance was 43.6 minutes^2. Find a 98% confidence interval for the mean drying time. [3]

2 In the past, the mean annual crop yield from a particular field has been 8.2 tonnes. During the last 16 years, a new fertiliser has been used on the field. The mean yield for these 16 years is 8.7 tonnes. Assume that yields are normally distributed with standard deviation 1.2 tonnes. Carry out a test at the 5% significance level of whether the mean yield has increased. [5]

3 1% of adults in a certain country own a yellow car.

(i) Use a suitable approximating distribution to find the probability that a random sample of 240 adults includes more than 2 who own a yellow car. [4]

(ii) Justify your approximation. [2]

4 The number of sightings of a golden eagle at a certain location has a Poisson distribution with mean 2.5 per week. Drilling for oil is started nearby. A naturalist wishes to test at the 5% significance level whether there are fewer sightings since the drilling began. He notes that during the following 3 weeks there are 2 sightings.

(i) Find the critical region for the test and carry out the test. [5]

(ii) State the probability of a Type I error. [1]

(iii) State why the naturalist could not have made a Type II error. [1]

5 The time, T minutes, taken by people to complete a test has probability density function given by

$$f(t) = \begin{cases} k(10t - t^2) & 5 \leqslant t \leqslant 10, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

(i) Show that $k = \frac{3}{250}$. [3]

(ii) Find $E(T)$. [3]

(iii) Find the probability that a randomly chosen value of T lies between $E(T)$ and the median of T. [3]

(iv) State the greatest possible length of time taken to complete the test. [1]

6 X and Y are independent random variables with distributions Po(1.6) and Po(2.3) respectively.

(i) Find $P(X + Y = 4)$. [3]

A random sample of 75 values of X is taken.

(ii) State the approximate distribution of the sample mean, $\overline{X}$, including the values of the parameters. [2]

(iii) Hence find the probability that the sample mean is more than 1.7. [3]

(iv) Explain whether the Central Limit theorem was needed to answer part **(ii)**. [1]

7 Bags of sugar are packed in boxes, each box containing 20 bags. The masses of the boxes, when empty, are normally distributed with mean 0.4 kg and standard deviation 0.01 kg. The masses of the bags are normally distributed with mean 1.02 kg and standard deviation 0.03 kg.

(i) Find the probability that the total mass of a full box of 20 bags is less than 20.6 kg. [5]

(ii) Two full boxes are chosen at random. Find the probability that they differ in mass by less than 0.02 kg. [5]

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Cambridge International A Level

Cambridge International Examinations
Cambridge International Advanced Level

MATHEMATICS **9709/72**

Paper 7 Probability & Statistics 2 **(S2)** **February/March 2016**

1 hour 15 minutes

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Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

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JC16 03_9709_72/2R

CAMBRIDGE
International Examinations

[Turn over

1 A fair six-sided die is thrown 20 times and the number of sixes, X, is recorded. Another fair six-sided die is thrown 20 times and the number of odd-numbered scores, Y, is recorded. Find the mean and standard deviation of $X + Y$. [5]

2 Jill shoots arrows at a target. Last week, 65% of her shots hit the target. This week Jill claims that she has improved. Out of her first 20 shots this week, she hits the target with 18 shots. Assuming shots are independent, test Jill's claim at the 1% significance level. [5]

3 In the past, Arvinder has found that the mean time for his journey to work is 35.2 minutes. He tries a different route to work, hoping that this will reduce his journey time. Arvinder decides to take a random sample of 25 journeys using the new route. If the sample mean is less than 34.7 minutes he will conclude that the new route is quicker. Assume that, for the new route, the journey time has a normal distribution with standard deviation 5.6 minutes.

(i) Find the probability that a Type I error occurs. [4]

(ii) Arvinder finds that the sample mean is 34.5 minutes. Explain briefly why it is impossible for him to make a Type II error. [1]

4 The masses, in grams, of large bags of sugar and small bags of sugar are denoted by X and Y respectively, where $X \sim N(5.1, 0.2^2)$ and $Y \sim N(2.5, 0.1^2)$. Find the probability that the mass of a randomly chosen large bag is less than twice the mass of a randomly chosen small bag. [5]

5 The 150 oranges in a random sample from a certain supplier were weighed and the masses, X grams, were recorded. The results are summarised below.

$$n = 150 \qquad \Sigma x = 14\,910 \qquad \Sigma x^2 = 1\,525\,000$$

(i) Calculate a 99% confidence interval for the population mean of X. [6]

(ii) The supplier claims that the mean mass of his oranges is 100 grams. Use your answer to part **(i)** to explain whether this claim should be accepted. [1]

(iii) State briefly why the sample should be random. [1]

6 The battery in Sue's phone runs out at random moments. Over a long period, she has found that the battery runs out, on average, 3.3 times in a 30-day period.

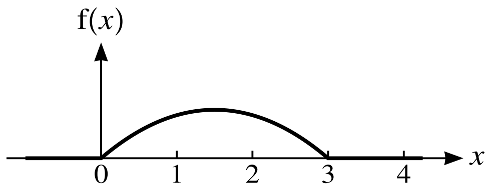
(i) Find the probability that the battery runs out fewer than 3 times in a 25-day period. [3]

(ii) **(a)** Use an approximating distribution to find the probability that the battery runs out more than 50 times in a year (365 days). [4]

(b) Justify the approximating distribution used in part **(ii)(a)**. [1]

(iii) Independently of her phone battery, Sue's computer battery also runs out at random moments. On average, it runs out twice in a 15-day period. Find the probability that the total number of times that her phone battery and her computer battery run out in a 10-day period is at least 4. [3]

7 **(a)**



The diagram shows the graph of the probability density function, f, of a random variable X, where

$$f(x) = \begin{cases} \frac{2}{9}(3x - x^2) & 0 \leqslant x \leqslant 3, \\ 0 & \text{otherwise.} \end{cases}$$

(i) State the value of $E(X)$ and find $Var(X)$. [4]

(ii) State the value of $P(1.5 \leqslant X \leqslant 4)$. [1]

(iii) Given that $P(1 \leqslant X \leqslant 2) = \frac{13}{27}$, find $P(X > 2)$. [2]

(b) A random variable, W, has probability density function given by

$$g(w) = \begin{cases} aw & 0 \leqslant w \leqslant b, \\ 0 & \text{otherwise,} \end{cases}$$

where a and b are constants. Given that the median of W is 2, find a and b. [4]

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Cambridge International Examinations
Cambridge International Advanced Level

MATHEMATICS **9709/71**

Paper 7 Probability & Statistics 2 **(S2)** **October/November 2015**

1 hour 15 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

* 6 7 8 6 8 8 4 2 6 0 *

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JC15 11_9709_71/2R

1 Failures of two computers occur at random and independently. On average the first computer fails 1.2 times per year and the second computer fails 2.3 times per year. Find the probability that the total number of failures by the two computers in a 6-month period is more than 1 and less than 4. [4]

2 The mean and standard deviation of the time spent by people in a certain library are 29 minutes and 6 minutes respectively.

(i) Find the probability that the mean time spent in the library by a random sample of 120 people is more than 30 minutes. [4]

(ii) Explain whether it was necessary to assume that the time spent by people in the library is normally distributed in the solution to part **(i)**. [2]

3 Jagdeesh measured the lengths, x minutes, of 60 randomly chosen lectures. His results are summarised below.

$$n = 60 \qquad \Sigma x = 3420 \qquad \Sigma x^2 = 195\,200$$

(i) Calculate unbiased estimates of the population mean and variance. [3]

(ii) Calculate a 98% confidence interval for the population mean. [3]

4 A random variable X has probability density function given by

$$\mathrm{f}(x) = \begin{cases} k(3-x) & 1 \leqslant x \leqslant 2, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

(i) Show that $k = \frac{2}{3}$. [3]

(ii) Find the median of X. [4]

5 On average, 1 in 2500 adults has a certain medical condition.

(i) Use a suitable approximation to find the probability that, in a random sample of 4000 people, more than 3 have this condition. [3]

(ii) In a random sample of n people, where n is large, the probability that none has the condition is less than 0.05. Find the smallest possible value of n. [4]

6 The weights, in kilograms, of men and women have the distributions $N(78, 7^2)$ and $N(66, 5^2)$ respectively.

(i) The maximum load that a certain cable car can carry safely is 1200 kg. If 9 randomly chosen men and 7 randomly chosen women enter the cable car, find the probability that the cable car can operate safely. [5]

(ii) Find the probability that a randomly chosen woman weighs more than a randomly chosen man. [4]

7 At a certain hospital it was found that the probability that a patient did not arrive for an appointment was 0.2. The hospital carries out some publicity in the hope that this probability will be reduced. They wish to test whether the publicity has worked.

(i) It is suggested that the first 30 appointments on a Monday should be used for the test. Give a reason why this is not an appropriate sample. [1]

A suitable sample of 30 appointments is selected and the number of patients that do not arrive is noted. This figure is used to carry out a test at the 5% significance level.

(ii) Explain why the test is one-tail and state suitable null and alternative hypotheses. [2]

(iii) State what is meant by a Type I error in this context. [1]

(iv) Use the binomial distribution to find the critical region, and find the probability of a Type I error. [5]

(v) In fact 3 patients out of the 30 do not arrive. State the conclusion of the test, explaining your answer. [2]

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Cambridge International Examinations
Cambridge International Advanced Level

MATHEMATICS **9709/72**

Paper 7 Probability & Statistics 2 **(S2)** **October/November 2015**

1 hour 15 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

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Cambridge International A Level

Cambridge International Examinations
Cambridge International Advanced Level

MATHEMATICS **9709/73**

Paper 7 Probability & Statistics 2 **(S2)** **October/November 2015**

1 hour 15 minutes

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Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

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1 It is known that the number, N, of words contained in the leading article each day in a certain newspaper can be modelled by a normal distribution with mean 352 and variance 29. A researcher takes a random sample of 10 leading articles and finds the sample mean, $\overline{N}$, of N.

(i) State the distribution of $\overline{N}$, giving the values of any parameters. [2]

(ii) Find $P(\overline{N} > 354)$. [3]

2 The number of calls received per 5-minute period at a large call centre has a Poisson distribution with mean λ, where $\lambda > 30$. If more than 55 calls are received in a 5-minute period, the call centre is overloaded. It has been found that the probability of being overloaded during a randomly chosen 5-minute period is 0.01. Use the normal approximation to the Poisson distribution to obtain a quadratic equation in $\sqrt{\lambda}$ and hence find the value of λ. [5]

3 From a random sample of 65 people in a certain town, the proportion who own a bicycle was noted. From this result an $\alpha\%$ confidence interval for the proportion, p, of all people in the town who own a bicycle was calculated to be $0.284 < p < 0.516$.

(i) Find the proportion of people in the sample who own a bicycle. [1]

(ii) Calculate the value of α correct to 2 significant figures. [4]

4 A random variable X has probability density function given by

$$f(x) = \begin{cases} k(4 - x^2) & -2 \leqslant x \leqslant 2, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

(i) Show that $k = \frac{3}{32}$. [3]

(ii) Sketch the graph of $y = f(x)$ and hence write down the value of $E(X)$. [2]

(iii) Find $P(X < 1)$. [3]

5 **(a)** Narika has a die which is known to be biased so that the probability of throwing a 6 on any throw is $\frac{1}{100}$. She uses an approximating distribution to calculate the probability of obtaining no 6s in 450 throws. Find the percentage error in using the approximating distribution for this calculation. [4]

(b) Johan claims that a certain six-sided die is biased so that it shows a 6 less often than it would if the die were fair. In order to test this claim, the die is thrown 25 times and it shows a 6 on only 2 throws. Test at the 10% significance level whether Johan's claim is justified. [5]

6 Parcels arriving at a certain office have weights W kg, where the random variable W has mean μ and standard deviation 0.2. The value of μ used to be 2.60, but there is a suspicion that this may no longer be true. In order to test at the 5% significance level whether the value of μ has increased, a random sample of 75 parcels is chosen. You may assume that the standard deviation of W is unchanged.

(i) The mean weight of the 75 parcels is found to be 2.64 kg. Carry out the test. [4]

(ii) Later another test of the same hypotheses at the 5% significance level, with another random sample of 75 parcels, is carried out. Given that the value of μ is now 2.68, calculate the probability of a Type II error. [5]

7 The diameter, in cm, of pistons made in a certain factory is denoted by X, where X is normally distributed with mean μ and variance σ^2. The diameters of a random sample of 100 pistons were measured, with the following results.

$$n = 100 \qquad \Sigma x = 208.7 \qquad \Sigma x^2 = 435.57$$

(i) Calculate unbiased estimates of μ and σ^2. [3]

The pistons are designed to fit into cylinders. The internal diameter, in cm, of the cylinders is denoted by Y, where Y has an independent normal distribution with mean 2.12 and variance 0.000 144. A piston will not fit into a cylinder if $Y - X < 0.01$.

(ii) Using your answers to part **(i)**, find the probability that a randomly chosen piston will not fit into a randomly chosen cylinder. [6]

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Cambridge International Examinations
Cambridge International Advanced Level

MATHEMATICS **9709/71**

Paper 7 Probability & Statistics 2 **(S2)** **May/June 2015**

1 hour 15 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

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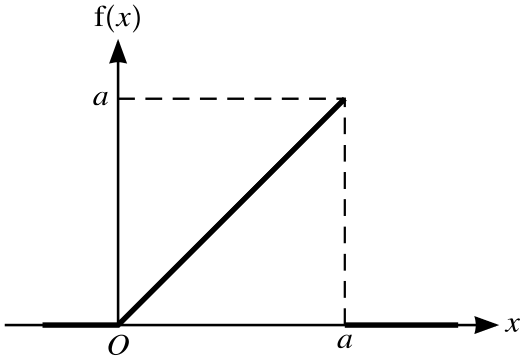
1



The random variable X has probability density function, f, as shown in the diagram, where a is a constant. Find the value of a and hence show that $E(X) = 0.943$ correct to 3 significant figures. [5]

2 Sami claims that he can read minds. He asks each of 50 people to choose one of the 5 letters A, B, C, D or E. He then tells each person which letter he believes they have chosen. He gets 13 correct. Sami says "This shows that I can read minds, because 13 is more than I would have got right if I were just guessing."

(i) State null and alternative hypotheses for a test of Sami's claim. [1]

(ii) Test at the 10% significance level whether Sami's claim is justified. [5]

3 The daily times, in minutes, that Yu Ming takes showering, getting dressed and having breakfast are independent and have the distributions $N(9, 2.2^2)$, $N(8, 1.3^2)$ and $N(17, 2.6^2)$ respectively. The total daily time that Yu Ming takes for all three activities is denoted by T minutes.

(i) Find the mean and variance of T. [2]

(ii) Yu Ming notes the value of T on each day in a random sample of 70 days and calculates the sample mean. Find the probability that the sample mean is between 33 and 35. [4]

4 In the past, the time taken by vehicles to drive along a particular stretch of road has had mean 12.4 minutes and standard deviation 2.1 minutes. Some new signs are installed and it is expected that the mean time will increase. In order to test whether this is the case, the mean time for a random sample of 50 vehicles is found. You may assume that the standard deviation is unchanged.

(i) The mean time for the sample of 50 vehicles is found to be 12.9 minutes. Test at the 2.5% significance level whether the population mean time has increased. [4]

(ii) State what is meant by a Type II error in this context. [2]

(iii) State what extra piece of information would be needed in order to find the probability of a Type II error. [1]

5 The masses, m grams, of a random sample of 80 strawberries of a certain type were measured and summarised as follows.

$$n = 80 \qquad \Sigma m = 4200 \qquad \Sigma m^2 = 229\,000$$

(i) Find unbiased estimates of the population mean and variance. [3]

(ii) Calculate a 98% confidence interval for the population mean. [3]

50 random samples of size 80 were taken and a 98% confidence interval for the population mean, μ, was found from each sample.

(iii) Find the number of these 50 confidence intervals that would be expected to include the true value of μ. [1]

6 A publishing firm has found that errors in the first draft of a new book occur at random and that, on average, there is 1 error in every 3 pages of a first draft. Find the probability that in a particular first draft there are

(i) exactly 2 errors in 10 pages, [2]

(ii) at least 3 errors in 6 pages, [3]

(iii) fewer than 50 errors in 200 pages. [4]

7 The independent variables X and Y are such that $X \sim \mathrm{B}(10, 0.8)$ and $Y \sim \mathrm{Po}(3)$. Find

(i) $\mathrm{E}(7X + 5Y - 2)$, [2]

(ii) $\mathrm{Var}(4X - 3Y + 3)$, [4]

(iii) $\mathrm{P}(2X - Y = 18)$. [4]

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Cambridge International A Level

Cambridge International Examinations
Cambridge International Advanced Level

MATHEMATICS **9709/72**

Paper 7 Probability & Statistics 2 **(S2)** **May/June 2015**

1 hour 15 minutes

* 1 7 7 6 8 4 5 9 9 8 *

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

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JC15 06_9709_72/3R

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International Examinations

[Turn over

1 The independent random variables X and Y have standard deviations 3 and 6 respectively. Calculate the standard deviation of $4X - 5Y$. [3]

2 Cloth made at a certain factory has been found to have an average of 0.1 faults per square metre. Suki claims that the cloth made by her machine contains, on average, more than 0.1 faults per square metre. In a random sample of $5\ \mathrm{m}^2$ of cloth from Suki's machine, it was found that there were 2 faults. Assuming that the number of faults per square metre has a Poisson distribution,

(i) state null and alternative hypotheses for a test of Suki's claim, [1]

(ii) test at the 10% significance level whether Suki's claim is justified. [4]

3 In a golf tournament, the number of times in a day that a 'hole-in-one' is scored is denoted by the variable X, which has a Poisson distribution with mean 0.15. Mr Crump offers to pay \$200 each time that a hole-in-one is scored during 5 days of play. Find the expectation and variance of the amount that Mr Crump pays. [5]

4 In the past, the flight time, in hours, for a particular flight has had mean 6.20 and standard deviation 0.80. Some new regulations are introduced. In order to test whether these new regulations have had any effect upon flight times, the mean flight time for a random sample of 40 of these flights is found.

(i) State what is meant by a Type I error in this context. [2]

(ii) The mean time for the sample of 40 flights is found to be 5.98 hours. Assuming that the standard deviation of flight times is still 0.80 hours, test at the 5% significance level whether the population mean flight time has changed. [4]

(iii) State, with a reason, which of the errors, Type I or Type II, might have been made in your answer to part **(ii)**. [2]

5 The volumes, v millilitres, of juice in a random sample of 50 bottles of Cooljoos are measured and summarised as follows.

$$n = 50 \qquad \Sigma v = 14\,800 \qquad \Sigma v^2 = 4\,390\,000$$

(i) Find unbiased estimates of the population mean and variance. [3]

(ii) An $\alpha\%$ confidence interval for the population mean, based on this sample, is found to have a width of 5.45 millilitres. Find α. [4]

Four random samples of size 10 are taken and a 96% confidence interval for the population mean is found from each sample.

(iii) Find the probability that these 4 confidence intervals all include the true value of the population mean. [2]

6 The waiting time, T minutes, for patients at a doctor's surgery has probability density function given by

$$f(t) = \begin{cases} k(225 - t^2) & 0 \leqslant t \leqslant 15, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

(i) Show that $k = \frac{1}{2250}$. [3]

(ii) Find the probability that a patient has to wait for more than 10 minutes. [3]

(iii) Find the mean waiting time. [4]

7 In a certain lottery, 10 500 tickets have been sold altogether and each ticket has a probability of 0.0002 of winning a prize. The random variable X denotes the number of prize-winning tickets that have been sold.

(i) State, with a justification, an approximating distribution for X. [3]

(ii) Use your approximating distribution to find $P(X < 4)$. [3]

(iii) Use your approximating distribution to find the conditional probability that $X < 4$, given that $X \geqslant 1$. [4]

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Cambridge International A Level

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Cambridge International Advanced Level

MATHEMATICS **9709/73**

Paper 7 Probability & Statistics 2 **(S2)** **May/June 2015**

1 hour 15 minutes

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Additional Materials: Answer Booklet/Paper
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List of Formulae (MF9)

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[Turn over

1 Jyothi wishes to choose a representative sample of 5 students from the 82 members of her school year.

(i) She considers going into the canteen and choosing a table with five students from her year sitting at it, and using these five people as her sample. Give two reasons why this method is unsatisfactory. [2]

(ii) Jyothi decides to use another method. She numbers all the students in her year from 1 to 82. Then she uses her calculator and generates the following random numbers.

231492 762305 346280

From these numbers, she obtains the student numbers 23, 14, 76, 5, 34 and 62. Explain how Jyothi obtained these student numbers from the list of random numbers. [3]

2 Marie claims that she can predict the winning horse at the local races. There are 8 horses in each race. Nadine thinks that Marie is just guessing, so she proposes a test. She asks Marie to predict the winners of the next 10 races and, if she is correct in 3 or more races, Nadine will accept Marie's claim.

(i) State suitable null and alternative hypotheses. [1]

(ii) Calculate the probability of a Type I error. [3]

(iii) State the significance level of the test. [1]

3 A die is biased so that the probability that it shows a six on any throw is p.

(i) In an experiment, the die shows a six on 22 out of 100 throws. Find an approximate 97% confidence interval for p. [4]

(ii) The experiment is repeated and another 97% confidence interval is found. Find the probability that exactly one of the two confidence intervals includes the true value of p. [2]

4 The marks, x, of a random sample of 50 students in a test were summarised as follows.

$$n = 50 \qquad \Sigma x = 1508 \qquad \Sigma x^2 = 51\,825$$

(i) Calculate unbiased estimates of the population mean and variance. [3]

(ii) Each student's mark is scaled using the formula $y = 1.5x + 10$. Find estimates of the population mean and variance of the scaled marks, y. [3]

5 The mean breaking strength of cables made at a certain factory is supposed to be 5 tonnes. The quality control department wishes to test whether the mean breaking strength of cables made by a particular machine is actually less than it should be. They take a random sample of 60 cables. For each cable they find the breaking strength by gradually increasing the tension in the cable and noting the tension when the cable breaks.

(i) Give a reason why it is necessary to take a sample rather then testing all the cables produced by the machine. [1]

(ii) The mean breaking strength of the 60 cables in the sample is found to be 4.95 tonnes. Given that the population standard deviation of breaking strengths is 0.15 tonnes, test at the 1% significance level whether the population mean breaking strength is less than it should be. [4]

(iii) Explain whether it was necessary to use the Central Limit theorem in the solution to part **(ii)**. [2]

6 People arrive at a checkout in a store at random, and at a constant mean rate of 0.7 per minute. Find the probability that

(i) exactly 3 people arrive at the checkout during a 5-minute period, [2]

(ii) at least 30 people arrive at the checkout during a 1-hour period. [4]

People arrive independently at another checkout in the store at random, and at a constant mean rate of 0.5 per minute.

(iii) Find the probability that a total of more than 3 people arrive at this pair of checkouts during a 2-minute period. [4]

7 The probability density function of the random variable X is given by

$$f(x) = \begin{cases} \frac{3}{4}x(c - x) & 0 \leqslant x \leqslant c, \\ 0 & \text{otherwise,} \end{cases}$$

where c is a constant.

(i) Show that $c = 2$. [3]

(ii) Sketch the graph of $y = f(x)$ and state the median of X. [3]

(iii) Find $P(X < 1.5)$. [4]

(iv) Hence write down the value of $P(0.5 < X < 1)$. [1]

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Cambridge International A Level

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Cambridge International Advanced Level

MATHEMATICS **9709/71**

Paper 7 Probability & Statistics 2 **(S2)** **October/November 2014**

1 hour 15 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

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JC14 11_9709_71/RP

CAMBRIDGE International Examinations

1 The masses, in grams, of potatoes of types A and B have the distributions $N(175, 60^2)$ and $N(105, 28^2)$ respectively. Find the probability that a randomly chosen potato of type A has a mass that is at least twice the mass of a randomly chosen potato of type B. [5]

2 The probability that a randomly chosen plant of a certain kind has a particular defect is 0.01. A random sample of 150 plants is taken.

(i) Use an appropriate approximating distribution to find the probability that at least 1 plant has the defect. Justify your approximating distribution. [4]

The probability that a randomly chosen plant of another kind has the defect is 0.02. A random sample of 100 of these plants is taken.

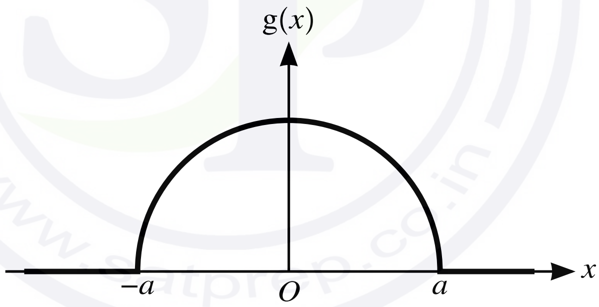
(ii) Use an appropriate approximating distribution to find the probability that the total number of plants with the defect in the two samples together is more than 3 and less than 7. [3]

3 **(a)** The time for which Lucy has to wait at a certain traffic light each day is T minutes, where T has probability density function given by

$$f(t) = \begin{cases} \frac{3}{2}t - \frac{3}{4}t^2 & 0 \leqslant t \leqslant 2, \\ 0 & \text{otherwise.} \end{cases}$$

Find the probability that, on a randomly chosen day, Lucy has to wait for less than half a minute at the traffic light. [3]

(b)



The diagram shows the graph of the probability density function, g, of a random variable X. The graph of g is a semicircle with centre (0, 0) and radius a. Elsewhere $g(x) = 0$.

(i) Find the value of a. [2]

(ii) State the value of $E(X)$. [1]

(iii) Given that $P(X < -c) = 0.2$, find $P(X < c)$. [2]

4 In a survey a random sample of 150 households in Nantville were asked to fill in a questionnaire about household budgeting.

(i) The results showed that 33 households owned more than one car. Find an approximate 99% confidence interval for the proportion of all households in Nantville with more than one car. [4]

(ii) The results also included the weekly expenditure on food, x dollars, of the households. These were summarised as follows.

$$n = 150 \qquad \Sigma x = 19\,035 \qquad \Sigma x^2 = 4\,054\,716$$

Find unbiased estimates of the mean and variance of the weekly expenditure on food of all households in Nantville. [3]

(iii) The government has a list of all the households in Nantville numbered from 1 to 9526. Describe briefly how to use random numbers to select a sample of 150 households from this list. [3]

5 The number of hours that Mrs Hughes spends on her business in a week is normally distributed with mean μ and standard deviation 4.8. In the past the value of μ has been 49.5.

(i) Assuming that μ is still equal to 49.5, find the probability that in a random sample of 40 weeks the mean time spent on her business in a week is more than 50.3 hours. [4]

Following a change in her arrangements, Mrs Hughes wishes to test whether μ has decreased. She chooses a random sample of 40 weeks and notes that the total number of hours she spent on her business during these weeks is 1920.

(ii) **(a)** Explain why a one-tail test is appropriate. [1]

(b) Carry out the test at the 6% significance level. [4]

(c) Explain whether it was necessary to use the Central Limit theorem in part **(ii)(b)**. [1]

6 The number of accidents on a certain road has a Poisson distribution with mean 3.1 per 12-week period.

(i) Find the probability that there will be exactly 4 accidents during an 18-week period. [3]

Following the building of a new junction on this road, an officer wishes to determine whether the number of accidents per week has decreased. He chooses 15 weeks at random and notes the number of accidents. If there are fewer than 3 accidents altogether he will conclude that the number of accidents per week has decreased. He assumes that a Poisson distribution still applies.

(ii) Find the probability of a Type I error. [3]

(iii) Given that the mean number of accidents per week is now 0.1, find the probability of a Type II error. [3]

(iv) Given that there were 2 accidents during the 15 weeks, explain why it is impossible for the officer to make a Type II error. [1]

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Cambridge International A Level

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Cambridge International Advanced Level

MATHEMATICS **9709/72**

Paper 7 Probability & Statistics 2 **(S2)** **October/November 2014**

1 hour 15 minutes

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JC14 11_9709_72/FP

CAMBRIDGE International Examinations

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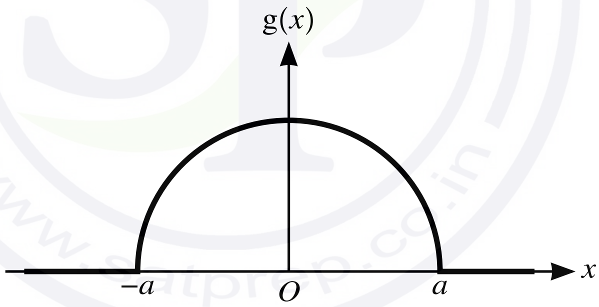
(b)



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(b) Carry out the test at the 6% significance level. [4]

(c) Explain whether it was necessary to use the Central Limit theorem in part **(ii)** **(b)**. [1]

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(ii) Find the probability of a Type I error. [3]

(iii) Given that the mean number of accidents per week is now 0.1, find the probability of a Type II error. [3]

(iv) Given that there were 2 accidents during the 15 weeks, explain why it is impossible for the officer to make a Type II error. [1]

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Cambridge International Advanced Level

MATHEMATICS **9709/73**

Paper 7 Probability & Statistics 2 **(S2)** **October/November 2014**

1 hour 15 minutes

Additional Materials: Answer Booklet/Paper
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List of Formulae (MF9)

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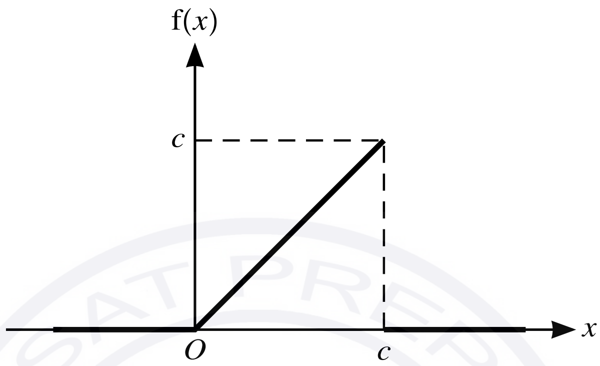
JC14 11_9709_73/FP

1 A researcher wishes to investigate whether the mean height of a certain type of plant in one region is different from the mean height of this type of plant everywhere else. He takes a large random sample of plants from the region and finds the sample mean. He calculates the value of the test statistic, z, and finds that $z = 1.91$.

(i) Explain briefly why the researcher should use a two-tail test. [1]

(ii) Carry out the test at the 4% significance level. [3]

2



The diagram shows the graph of the probability density function, f, of a random variable X.

(i) Find the value of the constant c. [2]

(ii) Find the value of a such that $P(a < X < 1) = 0.1$. [4]

(iii) Find $E(X)$. [2]

3 The times, in minutes, taken by people to complete a walk are normally distributed with mean μ. The times, t minutes, for a random sample of 80 people were summarised as follows.

$$\Sigma t = 7220 \qquad \Sigma t^2 = 656\,060$$

(i) Calculate a 97% confidence interval for μ. [6]

(ii) Explain whether it was necessary to use the Central Limit theorem in part **(i)**. [2]

4 The masses, in grams, of tomatoes of type A and type B have the distributions $N(125, 30^2)$ and $N(130, 32^2)$ respectively.

(i) Find the probability that the total mass of 4 randomly chosen tomatoes of type A and 6 randomly chosen tomatoes of type B is less than 1.5 kg. [5]

(ii) Find the probability that a randomly chosen tomato of type A has a mass that is at least 90% of the mass of a randomly chosen tomato of type B. [5]

5 It is known that when seeds of a certain type are planted, on average 10% of the resulting plants reach a height of 1 metre. A gardener wishes to investigate whether a new fertiliser will increase this proportion. He plants a random sample of 18 seeds of this type, using the fertiliser, and notes how many of the resulting plants reach a height of 1 metre.

(i) In fact 4 of the 18 plants reach a height of 1 metre. Carry out a hypothesis test at the 8% significance level. [5]

(ii) Explain which of the errors, Type I or Type II, might have been made in part **(i)**. [2]

Later, the gardener plants another random sample of 18 seeds of this type, using the fertiliser, and again carries out a hypothesis test at the 8% significance level.

(iii) Find the probability of a Type I error. [3]

6 The number of calls received at a small call centre has a Poisson distribution with mean 2.4 calls per 5-minute period. Find the probability of

(i) exactly 4 calls in an 8-minute period, [2]

(ii) at least 3 calls in a 3-minute period. [3]

The number of calls received at a large call centre has a Poisson distribution with mean 41 calls per 5-minute period.

(iii) Use an approximating distribution to find the probability that the number of calls received in a 5-minute period is between 41 and 59 inclusive. [5]

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Cambridge International A Level

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Cambridge International Advanced Level

MATHEMATICS **9709/71**

Paper 7 Probability & Statistics 2 **(S2)** **May/June 2014**

1 hour 15 minutes

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JC14 06_9709_71/2R

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International Examinations

[Turn over

1 The masses, in grams, of apples of a certain type are normally distributed with mean 60.4 and standard deviation 8.2. The apples are packed in bags, with each bag containing 8 randomly chosen apples. The bags are checked by Quality Control and any bag containing apples with a total mass of less than 436 g is rejected. Find the proportion of bags that are rejected. [4]

2 A die is biased. The mean and variance of a random sample of 70 scores on this die are found to be 3.61 and 2.70 respectively. Calculate a 95% confidence interval for the population mean score. [5]

3 The lengths, in centimetres, of rods produced in a factory have mean μ and standard deviation 0.2. The value of μ is supposed to be 250, but a manager claims that one machine is producing rods that are too long on average. A random sample of 40 rods from this machine is taken and the sample mean length is found to be 250.06 cm. Test at the 5% significance level whether the manager's claim is justified. [5]

4 The proportion of people who have a particular gene, on average, is 1 in 1000. A random sample of 3500 people in a certain country is chosen and the number of people, X, having the gene is found.

(i) State the distribution of X and state also an appropriate approximating distribution. Give the values of any parameters in each case. Justify your choice of the approximating distribution. [3]

(ii) Use the approximating distribution to find $P(X \leqslant 3)$. [2]

5 The score on one throw of a 4-sided die is denoted by the random variable X with probability distribution as shown in the table.

x	0	1	2	3
$P(X = x)$	0.25	0.25	0.25	0.25

(i) Show that $\text{Var}(X) = 1.25$. [1]

The die is thrown 300 times. The score on each throw is noted and the mean, $\overline{X}$, of the 300 scores is found.

(ii) Use a normal distribution to find $P(\overline{X} < 1.4)$. [3]

(iii) Justify the use of the normal distribution in part **(ii)**. [1]

6 Stephan is an athlete who competes in the high jump. In the past, Stephan has succeeded in 90% of jumps at a certain height. He suspects that his standard has recently fallen and he decides to carry out a hypothesis test to find out whether he is right. If he succeeds in fewer than 17 of his next 20 jumps at this height, he will conclude that his standard has fallen.

(i) Find the probability of a Type I error. [4]

(ii) In fact Stephan succeeds in 18 of his next 20 jumps. Which of the errors, Type I or Type II, is possible? Explain your answer. [2]

7 A random variable X has probability density function given by

$$f(x) = \begin{cases} \dfrac{k}{x} & 1 \leqslant x \leqslant a, \\ 0 & \text{otherwise,} \end{cases}$$

where k and a are positive constants.

(i) Show that $k = \dfrac{1}{\ln a}$. [3]

(ii) Find $E(X)$ in terms of a. [3]

(iii) Find the median of X in terms of a. [4]

8 **(i)** The following tables show the probability distributions for the random variables V and W.

v	-1	0	1	>1
$P(V = v)$	0.368	0.368	0.184	0.080

w	0	0.5	1	>1
$P(W = w)$	0.368	0.368	0.184	0.080

For each of the variables V and W state how you can tell from its probability distribution that it does NOT have a Poisson distribution. [2]

(ii) The random variable X has the distribution $\text{Po}(\lambda)$. It is given that

$$P(X = 0) = p \quad \text{and} \quad P(X = 1) = 2.5p,$$

where p is a constant.

(a) Show that $\lambda = 2.5$. [1]

(b) Find $P(X \geqslant 3)$. [2]

(iii) The random variable Y has the distribution $\text{Po}(\mu)$, where $\mu > 30$. Using a suitable approximating distribution, it is found that $P(Y > 40) = 0.5793$ correct to 4 decimal places. Find μ. [5]

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Cambridge International A Level

Cambridge International Examinations
Cambridge International Advanced Level

MATHEMATICS **9709/72**

Paper 7 Probability & Statistics 2 **(S2)** **May/June 2014**

1 hour 15 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

4606052898

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JC14 06_9709_72/RP

CAMBRIDGE International Examinations

1 The weights, in grams, of a random sample of 8 packets of cereal are as follows.

250 248 255 244 259 250 242 258

Calculate unbiased estimates of the population mean and variance. [3]

2 Each day Samuel travels from A to B and from B to C. He then returns directly from C to A. The times, in minutes, for these three journeys have the independent distributions $N(20,\ 2^2)$, $N(18,\ 1.5^2)$ and $N(30,\ 1.8^2)$, respectively. Find the probability that, on a randomly chosen day, the total time for his two journeys from A to B and B to C is less than the time for his return journey from C to A. [5]

3 The number of calls per day to an enquiry desk has a Poisson distribution. In the past the mean has been 5. In order to test whether the mean has changed, the number of calls on a random sample of 10 days was recorded. The total number of calls was found to be 61. Use an approximate distribution to test at the 10% significance level whether the mean has changed. [5]

4 **(i)** The random variable W has the distribution $\text{Po}(1.5)$. Find the probability that the sum of 3 independent values of W is greater than 2. [3]

(ii) The random variable X has the distribution $\text{Po}(\lambda)$. Given that $P(X = 0) = 0.523$, find the value of λ correct to 3 significant figures. [2]

(iii) The random variable Y has the distribution $\text{Po}(\mu)$, where $\mu \neq 0$. Given that

$$P(Y = 3) = 24 \times P(Y = 1),$$

find μ. [3]

5 Mahmoud throws a coin 400 times and finds that it shows heads 184 times. The probability that the coin shows heads on any throw is denoted by p.

(i) Calculate an approximate 95% confidence interval for p. [4]

(ii) Mahmoud claims that the coin is not fair. Use your answer to part **(i)** to comment on this claim. [1]

(iii) Mahmoud's result of 184 heads in 400 throws gives an $\alpha\%$ confidence interval for p with width 0.1. Calculate the value of α. [4]

6 The time, T hours, spent by people on a visit to a museum has probability density function

$$f(t) = \begin{cases} kt(16 - t^2) & 0 \leqslant t \leqslant 4, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

(i) Show that $k = \frac{1}{64}$. [3]

(ii) Calculate the probability that two randomly chosen people each spend less than 1 hour on a visit to the museum. [4]

(iii) Find the mean time spent on a visit to the museum. [3]

7 A researcher is investigating the actual lengths of time that patients spend with the doctor at their appointments. He plans to choose a sample of 12 appointments on a particular day.

(i) Which of the following methods is preferable, and why?

- Choose the first 12 appointments of the day.
- Choose 12 appointments evenly spaced throughout the day. [2]

Appointments are scheduled to last 10 minutes. The actual lengths of time, in minutes, that patients spend with the doctor may be assumed to have a normal distribution with mean μ and standard deviation 3.4. The researcher suspects that the actual time spent is more than 10 minutes on average. To test this suspicion, he recorded the actual times spent for a random sample of 12 appointments and carried out a hypothesis test at the 1% significance level.

(ii) State the probability of making a Type I error and explain what is meant by a Type I error in this context. [2]

(iii) Given that the total length of time spent for the 12 appointments was 147 minutes, carry out the test. [5]

(iv) Give a reason why the Central Limit theorem was not needed in part **(iii)**. [1]

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Cambridge International A Level

Cambridge International Examinations
Cambridge International Advanced Level

MATHEMATICS **9709/73**

Paper 7 Probability & Statistics 2 **(S2)** **May/June 2014**

1 hour 15 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

2538496228

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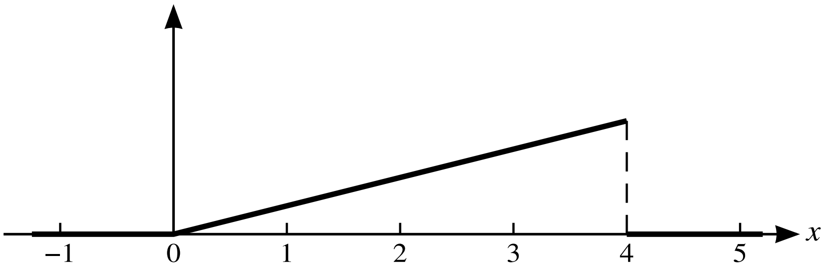
JC14 06_9709_73/FP

CAMBRIDGE International Examinations

[Turn over

1 On average 1 in 25 000 people have a rare blood condition. Use a suitable approximating distribution to find the probability that fewer than 2 people in a random sample of 100 000 have the condition. [3]

2



A random variable X takes values between 0 and 4 only and has probability density function as shown in the diagram. Calculate the median of X. [3]

3 A die is thrown 100 times and shows an odd number on 56 throws. Calculate an approximate 97% confidence interval for the probability that the die shows an odd number on one throw. [4]

4 The weights, X kilograms, of rabbits in a certain area have population mean μ kg. A random sample of 100 rabbits from this area was taken and the weights are summarised by

$$\Sigma x = 165, \qquad \Sigma x^2 = 276.25.$$

Test at the 5% significance level the null hypothesis $H_0 : \mu = 1.6$ against the alternative hypothesis $H_1 : \mu \neq 1.6$. [6]

5 The lifetime, X years, of a certain type of battery has probability density function given by

$$f(x) = \begin{cases} \dfrac{k}{x^2} & 1 \leqslant x \leqslant a, \\ 0 & \text{otherwise,} \end{cases}$$

where k and a are positive constants.

(i) State what the value of a represents in this context. [1]

(ii) Show that $k = \dfrac{a}{a-1}$. [3]

(iii) Experience has shown that the longest that any battery of this type lasts is 2.5 years. Find the mean lifetime of batteries of this type. [3]

6 A machine is designed to generate random digits between 1 and 5 inclusive. Each digit is supposed to appear with the same probability as the others, but Max claims that the digit 5 is appearing less often than it should. In order to test this claim the manufacturer uses the machine to generate 25 digits and finds that exactly 1 of these digits is a 5.

(i) Carry out a test of Max's claim at the 2.5% significance level. [5]

(ii) Max carried out a similar hypothesis test by generating 1000 digits between 1 and 5 inclusive. The digit 5 appeared 180 times. Without carrying out the test, state the distribution that Max should use, including the values of any parameters. [2]

(iii) State what is meant by a Type II error in this context. [1]

7 A Lost Property office is open 7 days a week. It may be assumed that items are handed in to the office randomly, singly and independently.

(i) State another condition for the number of items handed in to have a Poisson distribution. [1]

It is now given that the number of items handed in per week has the distribution Po(4.0).

(ii) Find the probability that exactly 2 items are handed in on a particular day. [2]

(iii) Find the probability that at least 4 items are handed in during a 10-day period. [3]

(iv) Find the probability that, during a certain week, 5 items are handed in altogether, but no items are handed in on the first day of the week. [3]

8 In an examination, the marks in the theory paper and the marks in the practical paper are denoted by the random variables X and Y respectively, where $X \sim \mathrm{N}(57, 13)$ and $Y \sim \mathrm{N}(28, 5)$. You may assume that each candidate's marks in the two papers are independent. The final score of each candidate is found by calculating $X + 2.5Y$. A candidate is chosen at random. Without using a continuity correction, find the probability that this candidate

(i) has a final score that is greater than 140, [5]

(ii) obtains at least 20 more marks in the theory paper than in the practical paper. [5]

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General Certificate of Education Advanced Level

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MATHEMATICS **9709/71**

Paper 7 Probability & Statistics 2 **(S2)** **October/November 2013**

1 hour 15 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

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JC13 11_9709_71/RP

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International Examinations

1 Each computer made in a factory contains 1000 components. On average, 1 in 30 000 of these components is defective. Use a suitable approximate distribution to find the probability that a randomly chosen computer contains at least 1 faulty component. [4]

2 Heights of a certain species of animal are known to be normally distributed with standard deviation 0.17 m. A conservationist wishes to obtain a 99% confidence interval for the population mean, with total width less than 0.2 m. Find the smallest sample size required. [4]

3 Following a change in flight schedules, an airline pilot wished to test whether the mean distance that he flies in a week has changed. He noted the distances, x km, that he flew in 50 randomly chosen weeks and summarised the results as follows.

$$n = 50 \qquad \Sigma x = 143\,300 \qquad \Sigma x^2 = 410\,900\,000$$

(i) Calculate unbiased estimates of the population mean and variance. [3]

(ii) In the past, the mean distance that he flew in a week was 2850 km. Test, at the 5% significance level, whether the mean distance has changed. [5]

4 The number of radioactive particles emitted per 150-minute period by some material has a Poisson distribution with mean 0.7.

(i) Find the probability that at most 2 particles will be emitted during a randomly chosen 10-hour period. [3]

(ii) Find, in minutes, the longest time period for which the probability that no particles are emitted is at least 0.99. [5]

5 The volume, in cm^3, of liquid left in a glass by people when they have finished drinking all they want is modelled by the random variable X with probability density function given by

$$\mathrm{f}(x) = \begin{cases} k(x-2)^2 & 0 \leqslant x \leqslant 2, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

(i) Show that $k = \frac{3}{8}$. [2]

(ii) 20% of people leave at least $d\,\text{cm}^3$ of liquid in a glass. Find d. [3]

(iii) Find $\mathrm{E}(X)$. [3]

6 At the last election, 70% of people in Apoli supported the president. Luigi believes that the same proportion support the president now. Maria believes that the proportion who support the president now is 35%. In order to test who is right, they agree on a hypothesis test, taking Luigi's belief as the null hypothesis. They will ask 6 people from Apoli, chosen at random, and if more than 3 support the president they will accept Luigi's belief.

(i) Calculate the probability of a Type I error. [3]

(ii) If Maria's belief is true, calculate the probability of a Type II error. [3]

(iii) In fact 2 of the 6 people say that they support the president. State which error, Type I or Type II, might be made. Explain your answer. [2]

7 Kieran and Andreas are long-jumpers. They model the lengths, in metres, that they jump by the independent random variables $K \sim N(5.64, 0.0576)$ and $A \sim N(4.97, 0.0441)$ respectively. They each make a jump and measure the length. Find the probability that

(i) the sum of the lengths of their jumps is less than 11 m, [4]

(ii) Kieran jumps more than 1.2 times as far as Andreas. [6]

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General Certificate of Education Advanced Level

MATHEMATICS **9709/72**

Paper 7 Probability & Statistics 2 **(S2)** **October/November 2013**

1 hour 15 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

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JC13 11_9709_72/FP

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General Certificate of Education Advanced Level

MATHEMATICS **9709/73**

Paper 7 Probability & Statistics 2 **(S2)** **October/November 2013**

1 hour 15 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

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1 A random sample of 80 values of a variable X is taken and these values are summarised below.

$$n = 80 \qquad \Sigma x = 150.2 \qquad \Sigma x^2 = 820.24$$

Calculate unbiased estimates of the population mean and variance of X and hence find a 95% confidence interval for the population mean of X. [6]

2 A traffic officer notes the speeds of vehicles as they pass a certain point. In the past the mean of these speeds has been $62.3\,\text{km h}^{-1}$ and the standard deviation has been $10.4\,\text{km h}^{-1}$. A speed limit is introduced, and following this, the mean of the speeds of 75 randomly chosen vehicles passing the point is found to be $59.9\,\text{km h}^{-1}$.

(i) Making an assumption that should be stated, test at the 2% significance level whether the mean speed has decreased since the introduction of the speed limit. [6]

(ii) Explain whether it was necessary to use the Central Limit theorem in part **(i)**. [2]

3 The waiting time, T weeks, for a particular operation at a hospital has probability density function given by

$$f(t) = \begin{cases} \frac{1}{2500}(100t - t^3) & 0 \leqslant t \leqslant 10, \\ 0 & \text{otherwise.} \end{cases}$$

(i) Given that $E(T) = \frac{16}{3}$, find $\text{Var}(T)$. [3]

(ii) 10% of patients have to wait more than n weeks for their operation. Find the value of n, giving your answer correct to the nearest integer. [5]

4 Goals scored by Femchester United occur at random with a constant average of 1.2 goals per match. Goals scored against Femchester United occur independently and at random with a constant average of 0.9 goals per match.

(i) Find the probability that in a randomly chosen match involving Femchester,

(a) a total of 3 goals are scored, [2]

(b) a total of 3 goals are scored and Femchester wins. [3]

The manager promises the Femchester players a bonus if they score at least 35 goals in the next 25 matches.

(ii) Find the probability that the players receive the bonus. [4]

5 A fair six-sided die has faces numbered 1, 2, 3, 4, 5, 6. The score on one throw is denoted by X.

(i) Write down the value of $\mathrm{E}(X)$ and show that $\mathrm{Var}(X) = \frac{35}{12}$. [2]

Fayez has a six-sided die with faces numbered 1, 2, 3, 4, 5, 6. He suspects that it is biased so that when it is thrown it is more likely to show a low number than a high number. In order to test his suspicion, he plans to throw the die 50 times. If the mean score is less than 3 he will conclude that the die is biased.

(ii) Find the probability of a Type I error. [5]

(iii) With reference to this context, describe circumstances in which Fayez would make a Type II error. [2]

6 The lifetimes, in hours, of Longlive light bulbs and Enerlow light bulbs have the independent distributions $\mathrm{N}(1020, 45^2)$ and $\mathrm{N}(2800, 52^2)$ respectively.

(i) Find the probability that the total of the lifetimes of 5 randomly chosen Longlive bulbs is less than 5200 hours. [4]

(ii) Find the probability that the lifetime of a randomly chosen Enerlow bulb is at least 3 times that of a randomly chosen Longlive bulb. [6]

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UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

MATHEMATICS **9709/71**

Paper 7 Probability & Statistics 2 **(S2)** **May/June 2013**

1 hour 15 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

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JC13 06_9709_71/FP

UNIVERSITY of CAMBRIDGE International Examinations

[Turn over

1 Marie wants to choose one student at random from Anthea, Bill and Charlie. She throws two fair coins. If both coins show tails she will choose Anthea. If both coins show heads she will choose Bill. If the coins show one of each she will choose Charlie.

(i) Explain why this is not a fair method for choosing the student. [2]

(ii) Describe how Marie could use the two coins to give a fair method for choosing the student. [2]

2 The times taken by students to complete a task are normally distributed with standard deviation 2.4 minutes. A lecturer claims that the mean time is 17.0 minutes. The times taken by a random sample of 5 students were 17.8, 22.4, 16.3, 23.1 and 11.4 minutes. Carry out a hypothesis test at the 5% significance level to determine whether the lecturer's claim should be accepted. [5]

3 Weights of cups have a normal distribution with mean 91 g and standard deviation 3.2 g. Weights of saucers have an independent normal distribution with mean 72 g and standard deviation 2.6 g. Cups and saucers are chosen at random to be packed in boxes, with 6 cups and 6 saucers in each box. Given that each empty box weighs 550 g, find the probability that the total weight of a box containing 6 cups and 6 saucers exceeds 1550 g. [5]

4 The lengths, x m, of a random sample of 200 balls of string are found and the results are summarised by $\Sigma x = 2005$ and $\Sigma x^2 = 20\,175$.

(i) Calculate unbiased estimates of the population mean and variance of the lengths. [3]

(ii) Use the values from part **(i)** to estimate the probability that the mean length of a random sample of 50 balls of string is less than 10 m. [3]

(iii) Explain whether or not it was necessary to use the Central Limit theorem in your calculation in part **(ii)**. [2]

5 The probability that a new car of a certain type has faulty brakes is 0.008. A random sample of 520 new cars of this type is chosen, and the number, X, having faulty brakes is noted.

(i) Describe fully the distribution of X and describe also a suitable approximating distribution. Justify this approximating distribution. [4]

(ii) Use your approximating distribution to find

(a) $P(X > 3)$, [2]

(b) the smallest value of n such that $P(X = n) > P(X = n + 1)$. [3]

6 The time in minutes taken by people to read a certain booklet is modelled by the random variable T with probability density function given by

$$f(t) = \begin{cases} \dfrac{1}{2\sqrt{t}} & 4 \leqslant t \leqslant 9, \\ 0 & \text{otherwise.} \end{cases}$$

(i) Find the time within which 90% of people finish reading the booklet. [3]

(ii) Find $E(T)$ and $\text{Var}(T)$. [6]

7 Leila suspects that a particular six-sided die is biased so that the probability, p, that it will show a six is greater than $\frac{1}{6}$. She tests the die by throwing it 5 times. If it shows a six on 3 or more throws she will conclude that it is biased.

(i) State what is meant by a Type I error in this situation and calculate the probability of a Type I error. [3]

(ii) Assuming that the value of p is actually $\frac{2}{3}$, calculate the probability of a Type II error. [3]

Leila now throws the die 80 times and it shows a six on 50 throws.

(iii) Calculate an approximate 96% confidence interval for p. [4]

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UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

MATHEMATICS **9709/72**

Paper 7 Probability & Statistics 2 **(S2)** **May/June 2013**

1 hour 15 minutes

Additional Materials: Answer Booklet/Paper
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0225375134

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The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

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1 It is known that 1.2% of rods made by a certain machine are bent. The random variable X denotes the number of bent rods in a random sample of 400 rods.

(i) State the distribution of X. [2]

(ii) State, with a reason, a suitable approximate distribution for X. [2]

(iii) Use your approximate distribution to find the probability that the sample will include more than 2 bent rods. [2]

2 A random variable X has probability density function given by

$$f(x) = \begin{cases} \frac{2}{3}x & 1 \leqslant x \leqslant 2, \\ 0 & \text{otherwise.} \end{cases}$$

(i) Find $E(X)$. [3]

(ii) Find $P(X < E(X))$. [2]

(iii) Hence explain whether the mean of X is less than, equal to or greater than the median of X. [2]

3 The heights of a certain variety of plant have been found to be normally distributed with mean 75.2 cm and standard deviation 5.7 cm. A biologist suspects that pollution in a certain region is causing the plants to be shorter than usual. He takes a random sample of n plants of this variety from this region and finds that their mean height is 73.1 cm. He then carries out an appropriate hypothesis test.

(i) He finds that the value of the test statistic z is -1.563, correct to 3 decimal places. Calculate the value of n. State an assumption necessary for your calculation. [4]

(ii) Use this value of the test statistic to carry out the hypothesis test at the 6% significance level. [3]

4 The masses, in grams, of a certain type of plum are normally distributed with mean μ and variance σ^2. The masses, m grams, of a random sample of 150 plums of this type were found and the results are summarised by $\Sigma m = 9750$ and $\Sigma m^2 = 647\,500$.

(i) Calculate unbiased estimates of μ and σ^2. [3]

(ii) Calculate a 98% confidence interval for μ. [3]

Two more random samples of plums of this type are taken and a 98% confidence interval for μ is calculated from each sample.

(iii) Find the probability that neither of these two intervals contains μ. [2]

5 Packets of cereal are packed in boxes, each containing 6 packets. The masses of the packets are normally distributed with mean 510 g and standard deviation 12 g. The masses of the empty boxes are normally distributed with mean 70 g and standard deviation 4 g.

(i) Find the probability that the total mass of a full box containing 6 packets is between 3050 g and 3150 g. [5]

(ii) A packet and an empty box are chosen at random. Find the probability that the mass of the packet is at least 8 times the mass of the empty box. [5]

6 The number of cases of asthma per month at a clinic has a Poisson distribution. In the past the mean has been 5.3 cases per month. A new treatment is introduced. In order to test at the 5% significance level whether the mean has decreased, the number of cases in a randomly chosen month is noted.

(i) Find the critical region for the test and, given that the number of cases is 2, carry out the test. [5]

(ii) Explain the meaning of a Type I error in this context and state the probability of a Type I error. [2]

(iii) At another clinic the mean number of cases of asthma per month has the independent distribution Po(13.1). Assuming that the mean for the first clinic is still 5.3, use a suitable approximating distribution to estimate the probability that the total number of cases in the two clinics in a particular month is more than 20. [5]

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UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

MATHEMATICS **9709/73**

Paper 7 Probability & Statistics 2 **(S2)** **May/June 2013**

1 hour 15 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

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READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
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This document consists of **3** printed pages and **1** blank page.

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[Turn over

1 The mean and variance of the random variable X are 5.8 and 3.1 respectively. The random variable S is the sum of three independent values of X. The independent random variable T is defined by $T = 3X + 2$.

(i) Find the variance of S. [1]

(ii) Find the variance of T. [1]

(iii) Find the mean and variance of $S - T$. [3]

2 A hockey player found that she scored a goal on 82% of her penalty shots. After attending a coaching course, she scored a goal on 19 out of 20 penalty shots. Making an assumption that should be stated, test at the 10% significance level whether she has improved. [5]

3 Each of a random sample of 15 students was asked how long they spent revising for an exam. The results, in minutes, were as follows.

50 70 80 60 65 110 10 70 75 60 65 45 50 70 50

Assume that the times for all students are normally distributed with mean μ minutes and standard deviation 12 minutes.

(i) Calculate a 92% confidence interval for μ. [4]

(ii) Explain what is meant by a 92% confidence interval for μ. [1]

(iii) Explain what is meant by saying that a sample is 'random'. [1]

4 The independent random variables X and Y have the distributions Po(2) and Po(3) respectively.

(i) Given that $X + Y = 5$, find the probability that $X = 1$ and $Y = 4$. [4]

(ii) Given that $P(X = r) = \frac{2}{3}P(X = 0)$, show that $3 \times 2^{r-1} = r!$ and verify that $r = 4$ satisfies this equation. [2]

5 A random variable X has probability density function given by

$$f(x) = \begin{cases} \dfrac{k}{x^3} & x \geqslant 1, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

(i) Show that $k = 2$. [2]

(ii) Find $P(1 \leqslant X \leqslant 2)$. [2]

(iii) Find $E(X)$. [3]

6 Calls arrive at a helpdesk randomly and at a constant average rate of 1.4 calls per hour. Calculate the probability that there will be

(i) more than 3 calls in $2\frac{1}{2}$ hours, [3]

(ii) fewer than 1000 calls in four weeks (672 hours). [4]

7 In the past the weekly profit at a store had mean \$34 600 and standard deviation \$4500. Following a change of ownership, the mean weekly profit for 90 randomly chosen weeks was \$35 400.

(i) Stating a necessary assumption, test at the 5% significance level whether the mean weekly profit has increased. [6]

(ii) State, with a reason, whether it was necessary to use the Central Limit theorem in part **(i)**. [2]

The mean weekly profit for another random sample of 90 weeks is found and the same test is carried out at the 5% significance level.

(iii) State the probability of a Type I error. [1]

(iv) Given that the population mean weekly profit is now \$36 500, calculate the probability of a Type II error. [5]

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