CAMBRIDGE
International Education

Cambridge International A Level

MATHEMATICS **9709/31**

Paper 3 Pure Mathematics 3 **October/November 2024**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

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Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
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- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
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- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
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4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).

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M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

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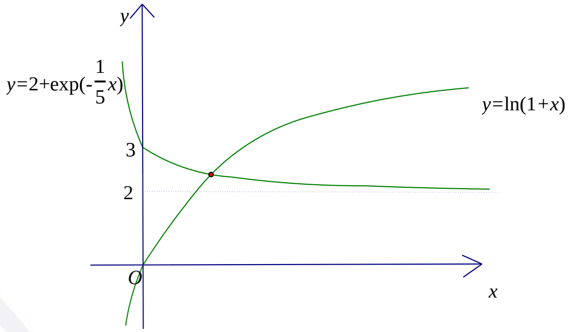
Question	Answer	Marks	Guidance
1	Substitute $x = -\frac{1}{2}$ and equate the result to zero	**M1**	
	Obtain a correct equation, e.g. $-\frac{4}{8} + \frac{a}{4} - \frac{5}{2} + b = 0$	**A1**	$\left(\frac{a}{4} + b = 3\right)$ Any equivalent form.
	Substitute $x = 2$ and $x = 4$ and use $\mathrm{p}(4) = 3\mathrm{p}(2)$	**M1**	If using long division, M1 is for correct use of two constant remainders. Condone if 3 is on the wrong side.
	Obtain a correct equation, e.g. $3(32 + 4a + 10 + b) = 256 + 16a + 20 + b$	**A1**	$(-2a + b = 75)$ Any equivalent form.
	Obtain $a = -32$ and $b = 11$	**A1**	
		5	

Question	Answer	Marks	Guidance
2	Integrate to obtain $px^3\ln 3x + q\int x^2 \mathrm{d}x$	**M1***	
	Obtain $\frac{1}{3}x^3\ln 3x - \frac{1}{3}\int x^2 \mathrm{d}x$	**A1**	Or unsimplified equivalent.
	Complete integration to obtain $\frac{1}{3}x^3\ln 3x - \frac{1}{9}x^3$	**A1**	
	Use correct limits correctly in an expression of the form $rx^3\ln 3x + sx^3$	**DM1**	$9\ln 9 - 3 - \frac{1}{3}\ln 3 + \frac{1}{9}$ An exact expression for *their* integral.
	Obtain $\frac{53}{3}\ln 3 - \frac{26}{9}$	**A1**	Or 2-term equivalent.
		5	

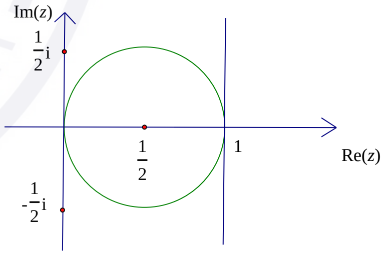
Question	Answer	Marks	Guidance
3	State or imply $\frac{1+\frac{\mathrm{d}y}{\mathrm{d}x}}{x+y}$ as the derivative of $\ln(x+y)$	**B1**	
	State or imply $6xy+3x^2\frac{\mathrm{d}y}{\mathrm{d}x}$ as the derivative of $3x^2y$	**B1**	
	Substitute $(1, 0)$ and solve for $\frac{\mathrm{d}y}{\mathrm{d}x}$	**M1**	Having the correct form for at least one of the above.
	Obtain $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{2}$ or 0.5	**A1**	
	Alternative Method for Question 3:		
	Rewrite as $x+y=\mathrm{e}^{3x^2y}$ and state or imply $1+\frac{\mathrm{d}y}{\mathrm{d}x}$ as the derivative of the LHS	**B1**	
	State or imply $\left(6xy+3x^2\frac{\mathrm{d}y}{\mathrm{d}x}\right)\mathrm{e}^{3x^2y}$ as the derivative of e^{3x^2y}	**B1**	
	Substitute $(1, 0)$ and solve for $\frac{\mathrm{d}y}{\mathrm{d}x}$	**M1**	Having the correct form for at least one of the above.
	Obtain $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{2}$ or 0.5	**A1**	
		4	

Question	Answer	Marks	Guidance
4(a)	Factorise Or obtain an expression in $\cos^2\theta$ or $\sec^2\theta$	**M1**	$(\sec^2\theta-\tan^2\theta)(\sec^2\theta+\tan^2\theta)$ Or $-1+\dfrac{2}{\cos^2\theta}$, OE.
	Use of $1+\tan^2\theta=\sec^2\theta$ (anywhere)	**M1**	The 2 method marks can appear in either order, in which case the 2nd M1 is for expanding $(1+\tan^2\theta)^2-\tan^4\theta$.
	Obtain $1\times(1+\tan^2\theta+\tan^2\theta)=1+2\tan^2\theta$	**A1**	Obtain given answer from full and correct working.
		3	
4(b)	Form an equation in $\tan 2\alpha$. Or multiply through by $\cos^4 2\alpha$ to form an equation in $\sin 2\alpha$ or $\cos 2\alpha$	**M1**	$1+2\tan^2 2\alpha=2\tan^2 2\alpha(1+\tan^2 2\alpha)$ $\cos^4 2\alpha+2\sin^2 2\alpha\cos^2 2\alpha=2\sin^2 2\alpha$ $\Rightarrow\ \sin^4 2\alpha+2\sin^2 2\alpha-1=0$ or $\cos^4 2\alpha-4\cos^2 2\alpha+2=0$
	Solve for $\tan 2\alpha$ or equivalent	**M1**	$\left(\tan 2\alpha=\pm\sqrt{\dfrac{1}{\sqrt{2}}}\right)$
	Obtain one correct solution for α, e.g. $20.0(30..)°$	**A1**	
	Obtain a second correct value for α, e.g. $70.0°\ (69.9698..°)$	**A1**	
	Obtain solutions $110°$ (110.0) and $160°$ (160.0) for α, and no others in range	**A1**	
		5	

Question	Answer	Marks	Guidance
5(a)	Sketch a relevant graph, e.g. $y = 2 + e^{-0.2x}$ For the sketches: Correct curvature Intersections with the *y*-axis approximately correct Horizontal asymptote approximately correct – need not draw in Allow scale not marked and implied by their sketch	**B1**	
	Sketch a second relevant graph, e.g. $y = \ln(1+x)$ and justify the given statement	**B1**	
		2	
5(b)	Calculate the value of a relevant expression or values of a relevant pair of expressions at $x = 7$ and $x = 9$	**M1**	
	Complete the argument correctly with correct calculated values	**A1**	E.g. $2.079 < 2.246$ and $2.302 > 2.165$, or $0.167 > 0$ and $-0.137 < 0$.
		2	
5(c)	Use the iterative process correctly at least once	**M1**	I.e., obtain one value and substitute that value back into the formula.
	Obtain final answer 8.03	**A1**	
	Show sufficient iterations to at least 4 decimal places to justify 8.03 to 2 decimal places, or show that there is a sign change in the interval $(8.025, 8.035)$	**A1**	E.g. 7, 8.4555, 7.8846, 8.0849, 8.0115, 8.0380, 8.0283, 8.0318 8, 8.0421, 8.0268, 8.0324, 9, 7.7172, 8.1490, 7.9887, 8.0463, 8.0253, 8.0329
		3	

Question	Answer	Marks	Guidance
6(a)	Correct use of product rule to differentiate	***M1**	$2\cos 2x(1+\sin 2x)+\sin 2x\times 2\cos 2x$, or $2\cos^2 x-2\sin^2 x+8\sin x\cos^3 x-8\cos x\sin^3 x$. All terms needed but could have errors in the coefficients.
	Obtain $2\cos 2x+4\cos 2x\sin 2x$	**A1**	OE
	Equate derivative to zero and solve for $2x$ or x	**DM1**	$2\cos 2x(1+2\sin 2x)=0 \Rightarrow x=\frac{1}{2}\sin^{-1}\left(-\frac{1}{2}\right)$ Condone if they only consider cos $2x = 0$.
	Obtain $x=\dfrac{7}{12}\pi,\ y=-\dfrac{1}{4}$	**A1**	Mark degrees as a misread. The Q asks for an exact answer.
		4	
6(b)	Use correct double angle formula to integrate Or use integration by parts and correct double angle formula	**M1**	$\int \sin 2x+\dfrac{1-\cos 4x}{2}\,\mathrm{d}x$ Or $-\dfrac{1}{2}\cos 2x(1+\sin 2x)+\int\dfrac{1+\cos 4x}{2}\,\mathrm{d}x$.
	Obtain $-\dfrac{1}{2}\cos 2x+\dfrac{x}{2}-\dfrac{1}{8}\sin 4x\ (+C)$	**A1**	Or $-\dfrac{1}{2}\cos 2x-\dfrac{1}{2}\cos 2x\sin 2x+\dfrac{x}{2}+\dfrac{1}{8}\sin 4x\ (+C)$
	Use limits 0 and $\dfrac{1}{2}\pi$ correctly in a solution containing $p\cos 2x$ and $q\sin 4x$	**M1**	$\dfrac{1}{2}+\dfrac{1}{4}\pi-0+\dfrac{1}{2}-0+0$
	Obtain $\dfrac{1}{4}\pi+1$	**A1**	
		4	

Question	**Answer**	**Marks**	**Guidance**
7(a)	State or imply the form $\frac{A}{1+2x}+\frac{Bx+C}{2+x^2}$	**B1**	
	Use a correct method to find a constant	**M1**	
	Obtain one of $A=1, B=2$ and $C=3$	**A1**	
	Obtain a second value	**A1**	
	Obtain the third value	**A1**	
		5	
7(b)	State $\frac{-1.-2.-3}{3!}(2x)^3$ or –8	**B1 FT**	Correct term in x^3 or coefficient of x^3 in the expansion of $A(1+2x)^{-1}$. Any equivalent form.
	Use a correct method to obtain the coefficient of x^2 in the expansion of $\left(2+x^2\right)^{-1}$ or the coefficient of x^2 in the expansion of $\left(1+\frac{x^2}{2}\right)^{-1}$.	**M1**	Do not need to deal with 2^{-1} at this stage.
	Obtain $(Bx+C)\times\frac{1}{2}\times-\frac{1}{2}x^2$ or $-\frac{B}{4}x^3$ or $-\frac{B}{4}$	**A1 FT**	Follow *their B* (and *C*).
	Obtain final answer $-8\frac{1}{2}$ or $-8\frac{1}{2}x^3$	**A1**	Or simplified equivalent. Ignore additional terms for other powers of *x*.
		4	

Question	Answer	Marks	Guidance
8(a)	Multiply numerator and denominator by $1-y\mathrm{i}$	**M1**	OE
	Obtain $\dfrac{1}{1+y^2}+\dfrac{-y}{1+y^2}\mathrm{i}$	**A1**	OE
		2	
8(b)	Express $\left(a-\dfrac{1}{2}\right)^2+b^2$ in terms of *y* and expand the bracket	**M1**	$\left(\dfrac{1}{1+y^2}-\dfrac{1}{2}\right)^2+\left(\dfrac{(-)y}{1+y^2}\right)^2$
	Obtain $\left(\dfrac{1}{\left(1+y^2\right)^2}-\dfrac{2}{2\left(1+y^2\right)}+\dfrac{1}{4}\right)+\dfrac{y^2}{\left(1+y^2\right)^2}$	**A1FT**	Follow *their* answer from **(a)** provided it gives an expression in *y*.
	Obtain $\frac{1}{4}$ from full and correct working	**A1**	AG
		3	
8(c)	Show a vertical straight line through $1+0\mathrm{i}$	**B1**	
	Show a circle centre $\frac{1}{2}+0\mathrm{i}$	**B1**	
	Show a circle with radius $\frac{1}{2}$ and centre not at the origin	**B1**	
		3	
8(d)	circle centre $\frac{1}{2}+0\mathrm{i}$ with radius $\frac{1}{2}$	**B1**	OE Condone inclusion of the origin.
		1	

Question	Answer	Marks	Guidance
9(a)	Use a correct method to form a vector equation	**M1**	Allow in column vectors.
	Obtain $\mathbf{r} = 8\mathbf{i} - 5\mathbf{j} + 6\mathbf{k} + \lambda(2\mathbf{i} + \mathbf{j} + 4\mathbf{k})$	**A1**	Need **r** = …
		2	
9(b)	State the position vector of a point on *l* in component form Or at least 2 correct components seen	**B1 FT**	Follow *their* equation $(8+2\lambda)\mathbf{i} + (-5+\lambda)\mathbf{j} + (6+4\lambda)\mathbf{k}$. Might see the correct equation for the first time in **(b).**
	Equate to $-t\mathbf{i} + 4t\mathbf{j} + 3t\mathbf{k}$ and solve for t	**M1**	
	Obtain $t = -2$	**A1**	
		3	
9(c)	Evaluate the scalar product of a pair of relevant vectors	**M1**	$(2\mathbf{i} + \mathbf{j} + 4\mathbf{k})\cdot(a\mathbf{i} - \mathbf{j} + 3\mathbf{k}) = 2a + 11$ OE, SOI
	Complete the process for finding the cosine of θ	***M1**	Divide the scalar product by the product of the moduli and equate to $\cos\theta$.
	Obtain $\dfrac{2a+11}{\sqrt{21}\sqrt{10+a^2}} = \pm\dfrac{1}{\sqrt{6}}$	**A1**	OE
	Form a 3-term quadratic equation in a and solve for a	**DM1**	$a^2 + 88a + 172 = 0$ OE
	Obtain $a = -2, a = -86$	**A1**	Correct only (both values).
		5	

Question	Answer	Marks	Guidance
10(a)	$\dfrac{dV}{dt} = \pm k\sqrt{V}$ or $\dfrac{dV}{dt} = 16\pi\dfrac{dh}{dt}$	B1	SOI
	Correct use of chain rule and $V = 16\pi h$	M1	OE, e.g. $\dfrac{dV}{dt} = 16\pi\dfrac{dh}{dt}$.
	Obtain $\dfrac{dh}{dt} = \dfrac{dV}{dt} \div \dfrac{dV}{dh} = \dfrac{-k\sqrt{16\pi h}}{16\pi}$	A1	Any equivalent form in terms of *h*.
	$= -\left(\dfrac{k}{4\sqrt{\pi}}\right)\sqrt{h} = -\lambda\sqrt{h}$ since $\dfrac{k}{4\sqrt{\pi}}$ is constant	A1	Obtain given answer from full and correct working.
		4	
10(b)	Separate variables correctly and commence integration	*M1	$\int \dfrac{1}{\sqrt{h}} dh = \int -\lambda dt$ OE
	Obtain $-\lambda t = 2\sqrt{h}(+C)$	A1	
	Use the boundary conditions in an equation containing pt and $q\sqrt{h}$ to form an equation in λ and/or C	DM1	OE, e.g. $0 = 4 + C$ or $-20\lambda = 2\times1.5 + C$.
	Use the boundary conditions in an equation containing pt and $q\sqrt{h}$ to form a second equation in λ and/or C and solve	DM1	$C = -4,\ \lambda = \dfrac{1}{20}$ OE, e.g. $-\dfrac{t}{20} = 2\sqrt{h} - 4$.
	Hence $t = 80 - 40\sqrt{h}$	A1	Must be seen.
	Time to empty the tank is 80 minutes	A1	
		6	

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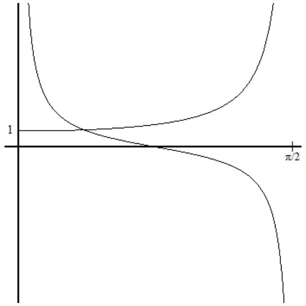
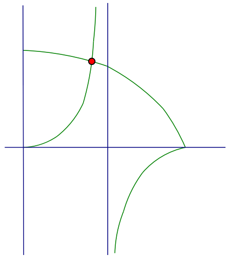
1	Obtain a correct unsimplified version of the x or x^2 term of the expansion of $(9-3x)^{\frac{1}{2}}$ or $\left(1-\frac{1}{3}x\right)^{\frac{1}{2}}$	**M1**	E.g. $-\frac{1}{2}\times\frac{1}{3}x$ or $-\frac{\frac{1}{2}\times\frac{1}{2}}{2}\times\frac{1}{9}x^2$ or $\frac{1}{2}9^{\frac{-1}{2}}(-3x)^1$ or $\frac{\frac{1}{2}\times\frac{-1}{2}}{2}9^{\frac{-3}{2}}(-3x)^2$. Not for symbolic coefficients in the form ${}^{n}\mathrm{C}_{r}$.
	State correct first term 3	**B1**	
	Obtain the next two terms $-\frac{1}{2}x-\frac{1}{24}x^2$	**A1 A1**	A1 for each term correct. Do not ISW.
			SC M1A1 for $1-\frac{1}{6}x-\frac{1}{72}x^2$ seen on its own or as a factor.
		4	

Question	Answer	Marks	Guidance
2(a)	Sketch a relevant graph, e.g. $y = \cot 2x$ 	**B1**	Alt: use $\tan 2x$ and $\cos x$. And only one root in range. (also cross at $\frac{\pi}{2}$) 
	Sketch a second relevant graph on the same axes, e.g. $y = \sec x$ and justify the given statement	**B1**	Need to mark intersection with a dot, a cross, or say roots at points of intersection, OE.
		2	
2(b)	State $x = \frac{1}{2}\tan^{-1}(\cos x)$ and rearrange to the **given equation** cot $2x = \sec x$ Note: If using the alternative approach in **(a)**, can stop at $\tan 2x = \cos x$	**B1**	Should see $\tan 2x = \cos x$ before the given conclusion. Or rearrange cot $2x = \sec x$ to $x = \frac{1}{2}\tan^{-1}(\cos x)$ and state iterative formula $x_{n+1} = \frac{1}{2}\tan^{-1}(\cos x_n)$.
		1	

Question	Answer	Marks	Guidance
3	Square $x + \mathrm{i}y$ and equate real and imaginary parts to 6 and −8 respectively	***M1**	Condone +8 in place of -8 and/or $\mathrm{i}^2 = 1$.
	Obtain equations $x^2 - y^2 = 6$ and $2xy = -8$	**A1**	OE
	Eliminate one variable and find an equation in the other (from 2 equations each in 2 unknowns)	**DM1**	Condone a slip but not seriously incorrect algebra, e.g. use of $x = -4y$ is M0.
	Obtain $x^4 - 6x^2 - 16 = 0$ or $y^4 + 6y^2 - 16 = 0$	**A1**	Accept 3-term equivalents e.g. $x^4 = 6x^2 + 16$. Condone missing '= 0' if implied by subsequent work.
	Obtain answers $\pm\left(2\sqrt{2} - \sqrt{2}\mathrm{i}\right)$ or exact equivalents	**A1**	Allow if values of x and y stated separately but the pairing is clear. Ignore additional correct solutions for x and y not real, but A0 if any additional incorrect answers.
		5	

Question	Answer	Marks	Guidance
4	Use laws of indices correctly and solve for 5^x	***M1**	E.g. obtain $5^x = \frac{5}{12}$ OE. Allow for $y = \ldots$ if they have previously stated $y = 5^x$. Could be implied if they have a correct simplified equation in 5^x, e.g. $12 \times 5^x = 5$.
	Use a correct method for solving an equation of the form $5^x = a$, where $a > 0$	**DM1**	Allow $x\ln 5 = \ln\left(\frac{10}{24}\right)$.
	Obtain answer –0.544	**A1**	CWO. If no working shown, 0/3. Note: 3 dp required.
		3	
5(a)	$\frac{4\pi}{7}$ and/or $-\frac{\pi}{7}$	**M1**	SOI Allow $\frac{4}{7}\pi - \left(-\frac{1}{7}\pi\right)$ or $\frac{4}{7}\pi - \frac{1}{7}\pi$ Note: Many multiply top and bottom by the conjugate, which is fine, but to score the M1 they need to state or imply the argument of a complex number.
	Obtain arg $u = \frac{5}{7}\pi$	**A1**	Do not accept degrees.
		2	

Question	Answer	Marks	Guidance
5(b)	Reflection (in the) real axis	**B1**	Correct non-contradictory statement. Condone *x*–axis or horizontal axis. Need ‘reflection’. Not ‘mirror’, ‘flip’.
	arg $u^* = -\frac{5}{7}\pi$	**B1FT**	FT *their* exact **(a).** Accept $2\pi -$ *their* exact **(a)**. Accept an ‘exact’ expression in place of an exact value. Need to see a value or an expression. Do not accept $\arg u^* = -\arg u$ without a value seen.
		2	
6	Form a pair of equations in *a* and *b*	***M1**	Condone sign slips but must be using the given coordinates correctly. e.g. $\begin{cases}\ln a + 8.27 = 3.4\ln b \\ \ln a + 2.24 = 0.5\ln b\end{cases}$ or $\begin{cases}ae^{2.24} = b^{0.5} \\ ae^{8.27} = b^{3.4}\end{cases}$
	Carry out a correct method for finding ln*a* or ln*b* or *a* or *b*	**DM1**	Condone sign slip.
	Obtain value *a* = 0.3	**A1**	(0.30109…)
	Obtain value *b* = 8	**A1**	(7.99895…) Allow A0A1 if both values ‘correct’ but not rounded to 1 sf. Allow 4/4 for $0.3y = 8^x$ with correct working shown.

Question	Answer	Marks	Guidance
6	**Alternative Method for Question 6:**		
	Carry out a correct method for finding ln$b$ or b (Need to link the gradient to ln$b$ at some point)	***M1**	Condone sign slips but must be using the given coordinates correctly. $\ln b = \frac{8.27-2.24}{3.4-0.5} (= 2.079....)$
	Obtain value $b = 8$	**A1**	
	Correct method to find $\pm\ln a$ or a	**DM1**	Condone sign slip $(\ln a = -1.200...)$.
	Obtain value $a = 0.3$	**A1**	Allow A0A1 if both values 'correct' but not rounded to 1 sf. Allow 4/4 for $0.3y = 8^x$ with correct working shown.
		4	

Question	Answer	Marks	Guidance
7(a)	Use correct double angle formula to obtain an equation in tan *x*	**M1**	e.g. $\tan^3 x + \frac{2\times 2\tan x}{1-\tan^2 x} - \tan x (=0)$. Allow if the correct formula is quoted but then they lose the 2 from the numerator when they use the formula.
	Obtain a correct equation in tan *x* in any form without fractions	**A1**	E.g. $\tan^3 x - \tan^5 x + 4\tan x - \tan x + \tan^3 x (=0)$. Condone if '= 0' is missing here.
	Reduce to the **given answer** of $\tan^4 x - 2\tan^2 x - 3 = 0$ correctly	**A1**	Obtain given answer from correct working but condone if never mention $\tan x \neq 0$. Condone the right terms in a different order 'Show that' so each line must be correct.
		3	
7(b)	A complete correct method to solve the equation to obtain a value for θ	**M1**	$\left(\tan 2\theta = \pm\sqrt{3}\right)$ Allow if they make a slip in copying the equation but do have a complete method to obtain a value of θ. M0 if they get a value for 2θ but never halve it.
	Obtain two of (θ =) $\frac{1}{6}\pi, \frac{1}{3}\pi, \frac{2}{3}\pi$ and $\frac{5}{6}\pi$	**A1**	
	Obtain the other two of (θ =) $\frac{1}{6}\pi, \frac{1}{3}\pi, \frac{2}{3}\pi$ and $\frac{5}{6}\pi$ and no others in the interval	**A1**	Exact, ignore any answers outside interval Accept $\frac{2}{6}\pi$ for $\frac{1}{3}\pi$ and $\frac{4}{6}\pi$ for $\frac{2}{3}\pi$. Do not need to see $\theta = \frac{\pi}{2}$ (from $\tan 2\theta = 0$).
		3	

Question	Answer	Marks	Guidance
8(a)	Use correct product rule or chain rule to find derivative of x with respect to t	**M1**	Obtain $k\tan 2t\sec^2 2t$.
	Obtain $\frac{\mathrm{d}x}{\mathrm{d}t}=4\tan 2t\sec^2 2t$ oe	**A1**	
	$\frac{\mathrm{d}y}{\mathrm{d}t}=-2\sin 2t$	**B1**	
	Use $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\mathrm{d}y}{\mathrm{d}t}\times\frac{\mathrm{d}t}{\mathrm{d}x}$ to obtain **given answer** $\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{1}{2}\cos^3 2t$	**B1**	Condone if $\frac{\mathrm{d}y}{\mathrm{d}x}$ missing.
		4	
8(b)	Obtain x = 1 and $y=\frac{\sqrt{2}}{2}$	**B1**	Accept y = 0.707…
	State or imply gradient of tangent is $\frac{-\sqrt{2}}{8}$ or gradient of normal is $4\sqrt{2}$	**B1**	Any equivalent form, e.g. $2^{\frac{5}{2}}$. Accept – 0.177 or 5.66.
	Use correct method to find equation of **normal** using *their* values	**M1**	Need a fully substituted equation for the normal (in any form) or to get at least as far as finding value for m and expression for c.
	Obtain equation of normal is $y=4\sqrt{2}x-\frac{7\sqrt{2}}{2}$ or equivalent 3 term equation	**A1**	E.g., $y = 5.66x – 4.95$. Must be $y = …$
		4	

Question	Answer	Marks	Guidance
9(a)	Use a correct method to find $\overrightarrow{OD}$	**M1**	E.g. $\overrightarrow{OC}+3\left(\overrightarrow{OA}-\overrightarrow{OB}\right)=$ $(-3\mathbf{i}-2\mathbf{j}+2\mathbf{k})+3((2\mathbf{i}+\mathbf{j}-3\mathbf{k})-(4\mathbf{j}+\mathbf{k}))$ $\left(\overrightarrow{AB}=-2\mathbf{i}+3\mathbf{j}+4\mathbf{k}\right)$ Accept column vectors throughout.
	Obtain position vector of *D* is $3\mathbf{i}-11\mathbf{j}-10\mathbf{k}$	**A1**	Accept coordinates.
		2	
9(b)	Carry out correct method for finding a vector equation for $\overrightarrow{AC}$ or $\overrightarrow{BD}$	***M1**	E.g. $2\mathbf{i}+\mathbf{j}-3\mathbf{k}+\lambda(5\mathbf{i}+3\mathbf{j}-5\mathbf{k})$ or $4\mathbf{j}+\mathbf{k}+\mu(3\mathbf{i}-15\mathbf{j}-11\mathbf{k})$. Condone missing **r** = …
	Both diagonal equations correct.	**A1ft**	Seen or implied. Follow their *D*. Condone missing **r** = …
	Equate at least two pairs of corresponding components and solve for λ or for μ	**DM1**	Dependent on using relevant lines and two different parameters.
	Obtain $\lambda=-\frac{1}{4}$ or $\mu=\frac{1}{4}$	**A1**	The values will depend on the directions of their lines
	Obtain position vector of *P* is $\frac{3}{4}\mathbf{i}+\frac{1}{4}\mathbf{j}-\frac{7}{4}\mathbf{k}$	**A1**	OE Accept coordinates. Do not ISW.

Question	Answer	Marks	Guidance
9(b)	**Alternative Method for Question 9(b):**		
	State or imply $\overrightarrow{AC} = 5\mathbf{i} - 3\mathbf{j} + 5\mathbf{k}$	**B1 FT**	Or $\overrightarrow{BD} = 3\mathbf{i} - 15\mathbf{j} - 11\mathbf{k}$ Follow *their D* if used.
	Identify similar triangles with ratio 1 : 3	**M1**	
	Use similar triangles to obtain $\overrightarrow{OP}$, e.g. $\overrightarrow{OP} = \overrightarrow{OA} + \frac{1}{4}\overrightarrow{AC}$	**M1**	Must be correct fraction.
	Obtain position vector of *P* is $\frac{3}{4}\mathbf{i} + \frac{1}{4}\mathbf{j} - \frac{7}{4}\mathbf{k}$	**A2**	OE Allow A1A0 if any two values are correct.
		5	
9(c)	Find direction vector $\overrightarrow{BA} = 2\mathbf{i} - 3\mathbf{j} - 4\mathbf{k}$ and $\overrightarrow{BC} = -3\mathbf{i} - 6\mathbf{j} + \mathbf{k}$ or equivalent	**B1FT**	Or $\overrightarrow{AB}$ and $\overrightarrow{CB}$. FT if using an incorrect $\overrightarrow{AB}$ from earlier work.
	Carry out correct process for evaluating the scalar product of two relevant vectors	**M1**	Allow if one is going in the negative direction, e.g. $\overrightarrow{AB}$ and $\overrightarrow{BC}$.
	Using the correct process for the moduli, divide *their* scalar product by the product of their moduli and evaluate the inverse cosine of the result to obtain an angle	**M1**	Independent of the first M1. For their two vectors $\theta = \cos^{-1}\frac{8}{\sqrt{29}\sqrt{46}} = \ldots$
	Obtain answer 77.3° (or 1.35 radians)	**A1**	77.347… Correctly rounded to more than 3 sf or AWRT 77.3.
		4	

Question	Answer	Marks	Guidance
10(a)	Obtain $\frac{\mathrm{d}V}{\mathrm{d}t} = 40\pi - 0.8\pi r$ or equivalent	**B1**	Need a complete correct statement seen or implied.
	Obtain $\frac{\mathrm{d}V}{\mathrm{d}r} = 4\pi r^2$ or equivalent e.g. $\frac{\mathrm{d}V}{\mathrm{d}t} = 4\pi r^2 \frac{\mathrm{d}r}{\mathrm{d}t}$	**B1**	Need a complete correct statement seen or implied.
	Use the chain rule to obtain **given answer (including the derivative)**	**B1**	Allow if $\frac{\mathrm{d}r}{\mathrm{d}t} = \frac{50-r}{5r^2}$ follows $\frac{\mathrm{d}r}{\mathrm{d}t} = \frac{40-0.8r}{4r^2}$ without further explanation (π already cancelled) and no incorrect statements seen.
		3	
10(b)	Commence division and reach quotient of the form $-5r \pm 250$ or $5r^2 = (50 - r)(Ar + B) + C$ and reach $A = -5$ and $B = \pm 250$	**M1**	Allow M1 if divide by $r - 50$ to obtain $5r \pm 250$.
	Obtain quotient $-5r - 250$	**A1**	Do not need to state which is quotient and which is remainder. However, if clearly muddled, then M1A1A0 for both expressions correct.
	Obtain remainder 12 500	**A1**	Note: 12 500 following division by $r - 50$ is correct and scores this A1 ISW.
			SC B1 only for correct use of remainder theorem to obtain correct remainder.
		3	

Question	Answer	Marks	Guidance
10(c)	Prepare to integrate e.g. separate variables correctly Or express in the form $\frac{\mathrm{d}t}{\mathrm{d}r}=\frac{5r^2}{50-r}\left(=-\left(5r+250\right)+\frac{12500}{50-r}\right)$	**B1FT**	$\int\frac{5r^2}{50-r}\mathrm{d}r=\int 1\mathrm{d}t$ Condone missing d*r*, d*t* or missing integral signs, but not both. Follow their division in **(b)** if substitute before separating.
	Obtain term *t*	**DB1**	
	Obtain terms $\frac{A}{2}r^2+Br-C\ln\left(50-r\right)$	**M1**	From *their* $Ar+B+\frac{C}{50-r}$ in **(b)** where $ABC\neq 0$. Allow a single slip in the coefficients.
	Obtain terms $-\frac{5}{2}r^2-250r-12500\ln(50-r)$	**A1FT**	FT *their* **(b)**, provided of the correct form.
	Use $t=0$, $r=0$ to evaluate a constant or as limits in a solution containing terms of the form r^2, r, $\ln(50-r)$ and t	**M1**	
	Obtain final answer $t=-\frac{5}{2}r^2-250r-12500\ln(50-r)+12500\ln 50$	**A1**	OE Must be $t=$ ….. Allow with $12500\ln 50=48900$ or better.
		6	
10(d)	Obtain $t=70.5$	**B1**	May be more accurate (70.4605…).
		1	

Question	Answer	Marks	Guidance
11(a)	Use correct quotient rule NB the question asks for $\mathrm{f}'(x)$ so need complete form	**M1**	Or correct product rule.
	Obtain correct derivative in any form, e.g. $\dfrac{4\mathrm{e}^{2x}(\mathrm{e}^{2x}-3\mathrm{e}^{x}+2)-2\mathrm{e}^{2x}(2\mathrm{e}^{2x}-3\mathrm{e}^{x})}{\left(\mathrm{e}^{2x}-3\mathrm{e}^{x}+2\right)^{2}}$	**A1**	
	Equate *their* derivative to zero	***M1**	Can be implied by numerator equated to zero for quotient rule. $\left(8=6\mathrm{e}^{x}\right)$
	Solve for *x* to obtain $x=\ln a$	**DM1**	*a* positive.
	Obtain $x=\ln\dfrac{4}{3}$ and $y=-16$	**A1**	No errors seen. Accept equivalent exact forms, e.g. $x=\ln\dfrac{8}{6}$.

Question	Answer	Marks	Guidance
11(a)	**Alternative Method for Question 11(a)**		
	Complete method to express $\mathrm{f}(x)$ in partial fractions	**M1**	As far as $p+\frac{q}{\mathrm{e}^x-2}+\frac{r}{\mathrm{e}^x-1}$ with values for p, q and r $\left(2+\frac{8}{\mathrm{e}^x-2}-\frac{2}{\mathrm{e}^x-1}\right)$. Allow in $u\left(u=\mathrm{e}^x\right)$.
	Differentiate to obtain $\mathrm{f}'(x)=\frac{s\mathrm{e}^x}{\left(\mathrm{e}^x-2\right)^2}+\frac{t\mathrm{e}^x}{\left(\mathrm{e}^x-1\right)^2}$	***M1**	Note: the question requires f'(x) so if they have substituted for e^x, they will also need chain rule.
	Obtain $\mathrm{f}'(x)=\frac{-8\mathrm{e}^x}{\left(\mathrm{e}^x-2\right)^2}+\frac{2\mathrm{e}^x}{\left(\mathrm{e}^x-1\right)^2}$	**A1**	From correct work.
	Equate derivative to zero and solve for x to obtain $x = \ln a$	**DM1**	Must follow correctly to give a positive value of a.
	Obtain $x=\ln\frac{4}{3}$ and $y = -16$	**A1**	No errors seen. Accept $x=\ln\frac{8}{6}$, or equivalent.
		5	

Question	Answer	Marks	Guidance
11(b)	State or imply $\frac{\mathrm{d}u}{\mathrm{d}x}=\mathrm{e}^x$	**B1**	
	Obtain $\int\frac{2u}{u^2-3u+2}\,\mathrm{d}u$ or equivalent Or₁ $\int\left(\frac{2}{u}+\right)\frac{8}{u(u-2)}-\frac{2}{u(u-1)}\mathrm{d}u$	**B1**	Correct expression in u. Condone missing du or missing integral but not both. Allow FT if using their partial fractions from **(a).**
	State or imply partial fractions of the form $\frac{A}{u-1}+\frac{B}{u-2}$ Or₁ $\frac{C}{u}+\frac{D}{u-2}+\frac{E}{u-1}$ Or₂ $\frac{2u-3}{u^2-3u+2}+\frac{3}{u^2-3u+2}=\frac{2u-3}{u^2-3u+2}+\frac{F}{u-2}+\frac{G}{u-1}$	**B1 FT**	Complete reduction to partial fractions. Correct form for *their* integrand.
	Use a correct method for finding a constant	**M1**	Available if they have incorrect form.
	Obtain correct $\frac{-2}{u-1}+\frac{4}{u-2}$ Or₂ $\frac{2u-3}{u^2-3u+2}+\frac{3}{u-2}-\frac{3}{u-1}$	**A1**	
	Integrate to obtain $a\ln(u-1)+b\ln(u-2)$ or equivalent	***M1**	M0 if they have additional terms that do not cancel out.
	Obtain correct $-2\ln(u-1)+4\ln(u-2)$ or equivalent	**A1FT**	FT values of *their* partial fraction coefficients.
	Correctly use limits $u = 5$ and 3 in an expression of the form $a\ln(u-1)+b\ln(u-2)$ or $x=\ln 5$ and $\ln 3$ in an expression of the form $a\ln(\mathrm{e}^x-1)+b\ln(\mathrm{e}^x-2)$	**DM1**	
	Obtain $\ln\frac{81}{4}$	**A1**	Accept $\ln 20.25$.
		9	

CAMBRIDGE
International Education

Cambridge International A Level

MATHEMATICS **9709/33**

Paper 3 Pure Mathematics 3 **October/November 2024**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners’ meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2024 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

This document consists of **26** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics-Specific Marking Principles

1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.

2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.

3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.

4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).

5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.

6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate’s intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

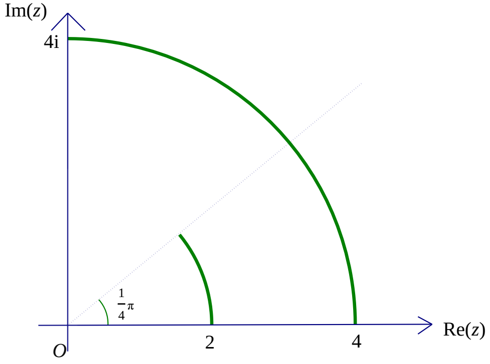
Types of mark

M	Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
A	Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
B	Mark for a correct result or statement independent of method marks.
DM or **DB**	When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
FT	Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1(a)	For all 4 marks, scales must be approximately equal, dashes can replace numbers. Arcs don’t have to be perfectly circular, mark intention.		
	Show an arc of a circle, centre the origin and radius 2. Only need 2 on Re(z) or 2i on Im(z) or r = 2 to show correct radius	**B1**	
	Show an arc centre the origin for $0 \leqslant \arg z \leqslant \frac{1}{4}\pi$ with any radius	**B1**	
	Max B1 if sector shaded		
		2	
1(b)	Show an arc of a circle, centre the origin and radius 4. Only need 4 on Re(z) or 4i on Im(z) or r = 4 to show correct radius	**B1**	
	Show an arc centre the origin for $0 \leqslant \arg z \leqslant \frac{1}{2}\pi$ with any radius	**B1**	
	Max B1 if sector shaded		
		2	

Question	Answer	Marks	Guidance
2(a)	State or imply the equation $x = \sqrt{\dfrac{4}{5-2x}}$ and square the equation	*B1	Could work with x_{n+1} throughout or with x_n throughout instead of x.
	Rearrange this with at least one intermediate step in the form $2x^3 - 5x^2 + 4 = 0$	DB1	
	Alternative Method 1 for Question 2(a)		
	Rearrange $2x^3 - 5x^2 + 4 = 0$ to $x^2(5 - 2x) = 4$ (or a different intermediate step) and to either $x^2 = \dfrac{4}{5-2x}$ or $x = \sqrt{\dfrac{4}{5-2x}}$ and then obtain the iterative formula $x_{n+1} = \sqrt{\dfrac{4}{5-2x_n}}$	B2	
	Alternative Method 2 for Question 2(a)		
	Rearrange $2x^3 - 5x^2 + 4 = 0$ to $x^2(5 - 2x) = 4$ (or a different intermediate step) and to $x^2 = \dfrac{4}{5-2x}$ and to $x_{n+1}{}^2 = \dfrac{4}{5-2x_n}$	*B1	Must have introduced x_{n+1} and x_n.
	Obtain the iterative formula $x_{n+1} = \sqrt{\dfrac{4}{5-2x_n}}$	DB1	
		2	

Question	Answer	Marks	Guidance
2(b)	Use the iterative process correctly at least once	**M1**	The question specifies initial value 1.2, so must use the formula to obtain a value and then use this value in the formula.
	Obtain final answer 1.28	**A1**	Can gain this mark even if less than 4 dp shown in iteration.
	Show sufficient iterations to at least 4 dp to justify 1.28 to 2 dp or show that there is a sign change in the interval $(1.275, 1.285)$	**A1**	$1.2,\ 1.2403,\ 1.2601,\ 1.2700, 1.2752, 1.2778, \ldots$ Allow small errors, truncation and recovery.
		3	

Question	Answer	Marks	Guidance
3(a)	State or imply that $\ln P = \ln a + kt$ or ln P = ln a + k(ln e)t $\ln P = \frac{1}{20}t + 3$ B0 until associated with a and /or k	**B1**	Can be implied by both a and k correct. $P = e^3 e^{\frac{1}{20}t}$ gets B1B1.
	State $k = \frac{1}{20}$, not from $\frac{dP}{dt} = k$	**B1**	OE. Can be embedded in $P = ae^{kt}$.
	$\ln a = 3 \Rightarrow a = 20$ to 2 sf	**B1**	Must be 2 sf, can be embedded in $P = ae^{kt}$.
		3	
3(b)	Form a correct equation in t using a and k, or *their* a and k where a will cancel (or are both numerical)	**M1**	E.g. $2a = ae^{kt}$, $2 = e^{kt}$, kt = ln 2.
	Obtain $t = 14[\text{hours}]$	**A1**	Allow 13.75 [hrs] (13 hrs 45 min) to 14 [hrs]. ISW
		2	

Question	Answer	Marks	Guidance
4	Substitute $z = x + iy$ and obtain a horizontal equation Do not allow if this would lead to an equation containing *xy* terms which do not cancel	***M1**	E.g. $5\left(x+\left(y-3\right)\mathrm{i}\right)=\left(2-9\mathrm{i}\right)\left(x+\left(y+3\right)\mathrm{i}\right)$
	Use $\mathrm{i}^2 = -1$ anywhere	**M1**	
	Obtain e.g. $5x+5\left(y-3\right)\mathrm{i}=\left(2x+9y+27\right)+\mathrm{i}\left(2y+6-9x\right)$ or e.g. $3x^2 + 3y^2 - 12y - 63 + (9x^2 + 9y^2 - 30x + 54y + 81)\mathrm{i} = 0$	**A1**	Or equivalent expression free of products of complex numbers. Terms can be in any order.
	Obtain simultaneous equations by equating real and imaginary parts	**DM1**	E.g. $3x-9y=27$ and $3y+9x=21$ $3x^2 + 3y^2 - 12y - 63 = 0$ and $9x^2 + 9y^2 - 30x + 54y + 81 = 0$
	Obtain $[z =]\ 3-2\mathrm{i}$ only	**A1**	

Question	Answer	Marks	Guidance
4	**Alternative Method for Question 4:**		
	Obtain a horizontal equation in *z* Do not allow if it would lead to an equation containing z^2 where the *xy* terms do not cancel	***M1**	E.g. $5z - 15i = 2z + 6i - 27i^2 - 9iz$. Allow errors, but no brackets.
	Use $i^2 = -1$ anywhere	**M1**	
	Obtain $z = \frac{9+7i}{1+3i}$	**A1**	OE (might have an uncancelled factor of 3)
	Multiply top and bottom by $1-3i$ or equivalent for *their z*	**DM1**	Must see working for numerator or denominator, e.g. $9-27i+21+7i$ or 1 + 9 or 10. If $\frac{9+7i}{1+3i} = 3-2i$ M0A0. If $\frac{9+7i}{1+3i} \times \frac{1-3i}{1-3i} = 3-2i$ M0A0 **SC B1**. If $\frac{9+7i}{1+3i} \times \frac{1-3i}{1-3i}$ and working in numerator or denominator and $3-2i$ M1A1.
	Obtain $[z =]\ 3-2i$ only	**A1**	
		5	

Question	Answer	Marks	Guidance
5(a)	Rewrite $\cos^4\theta$ as $\left(\frac{1+\cos 2\theta}{2}\right)^2$ or $\sin^4\theta$ as $\left(\frac{1-\cos 2\theta}{2}\right)^2$ or $4\sin^2\theta\cos^2\theta$ as $\sin^2 2\theta$	**B1**	Starting on left. Double angle for one term.
	Obtain $\left(\frac{1+\cos 2\theta}{2}\right)^2-\left(\frac{1-\cos 2\theta}{2}\right)^2-\sin^2 2\theta$	**B1**	OE, e.g. $1\times\cos 2\theta-\sin^2 2\theta$.
	Expand to $\frac{1}{4}+\frac{1}{2}\cos 2\theta+\frac{1}{4}\cos^2 2\theta-\left(\frac{1}{4}-\frac{1}{2}\cos 2\theta+\frac{1}{4}\cos^2 2\theta\right)-(1-\cos^2 2\theta)$ and simplify to obtain $\cos^2 2\theta+\cos 2\theta-1$	**B1**	AG
	Alternative Method 1 for Question 5(a):		
	Express $\cos^4\theta-\sin^4\theta$ as $\left(\cos^2\theta+\sin^2\theta\right)\left(\cos^2\theta-\sin^2\theta\right)$ or rewrite $4\sin^2\theta\cos^2\theta$ as $\sin^2 2\theta$	**B1**	Starting on left.
	Simplify to $\cos 2\theta-\sin^2 2\theta$	**B1**	If $\cos^4\theta-\sin^4\theta=\cos 2\theta$ instead of $\left(\cos^2\theta+\sin^2\theta\right)\left(\cos^2\theta-\sin^2\theta\right)=\cos 2\theta$, B0.
	Use $\sin^2 2\theta=1-\cos^2 2\theta$ to obtain $\cos^2 2\theta+\cos 2\theta-1$	**B1**	AG

Question	Answer	Marks	Guidance
5(a)	**Alternative Method 2 for Question 5(a):**		
	Use correct double angle formulae once e.g. replace $\cos 2\theta$ with $\cos^2\theta-\sin^2\theta$ $\left(\cos^2\theta-\sin^2\theta\right)^2+\left(\cos^2\theta-\sin^2\theta\right)-1$	**B1**	Starting on right. Double angle for one term.
	Expand to obtain $\cos^4\theta-2\sin^2\theta\cos^2\theta\boldsymbol{-\sin^4\theta+2\sin^4\theta}+\cos^2\theta-\sin^2\theta-1$ * or $\cos^4\theta-2\sin^2\theta\cos^2\theta+\sin^4\theta+\cos^2\theta-\sin^2\theta-1$ leading to $\cos^4\theta-2\sin^2\theta\cos^2\theta+\sin^4\theta-2\sin^2\theta$ leading to $\cos^4\theta-2\sin^2\theta\cos^2\theta+\sin^4\theta\ \boldsymbol{-2\sin^2\theta\left(\cos^2\theta+\sin^2\theta\right)}$ **	**B1**	Write $\sin^4\theta$ as $-\sin^4\theta+2\sin^4\theta$. Write $2\sin^2\theta$ as $2\sin^2\theta(\cos^2\theta+\sin^2\theta)$.
	Rewrite as * $\cos^4\theta-2\sin^2\theta\cos^2\theta-\sin^4\theta+2\sin^4\theta-2\sin^2\theta$ leading to $\cos^4\theta-2\sin^2\theta\cos^2\theta-\sin^4\theta+2\sin^2\theta\left(\sin^2\theta-1\right)$ leading to $\cos^4\theta-4\sin^2\theta\cos^2\theta-\sin^4\theta$ ** $\cos^4\theta-2\sin^2\theta\cos^2\theta+\sin^4\theta-2\sin^2\theta\cos^2\theta-2\sin^4\theta$ leading to $\cos^4\theta-4\sin^2\theta\cos^2\theta-\sin^4\theta$	**B1**	
		3	

Question	Answer	Marks	Guidance
5(b)	State a quadratic equation in $\cos 2\alpha$ and solve for α $\left(\cos^2 2\alpha + \cos 2\alpha - 1 = 0\right)$	**M1**	Alternative: form a quadratic in $\tan^2\alpha$ and solve for α $\left(\tan^4\alpha + 4\tan^2\alpha - 1 = 0\right)$.
	Obtain $\alpha = 25.9°$ or $\alpha = 154.1°$	**A1**	May be more accurate. Allow 154 for 154.1.
	Obtain $\alpha = 25.9°$ and $\alpha = 154.1°$ and no others in range	**A1**	May be more accurate. Allow 154 for 154.1. Mark answers in radians as a misread (0.452, 2.69).
		3	

Question	Answer	Marks	Guidance
6(a)	$P(2,\ 1, -3)$	**B1**	Accept *x* = 2, *y* = 1, *z* = –3. Do not accept 2**i** + **j** – 3**k** or $\begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix}$.
		1	

Question	Answer	Marks	Guidance
6(b)	Use the correct method to find the scalar product of the direction vectors	M1	$\pm\left(-1\times 2+2\times 5\right)=\pm 8$ Allow error of 0 × –1 = –1.
	Divide the scalar product by the product of the moduli to obtain $\pm\cos\theta$ using consistent vectors throughout	M1	$\dfrac{\textit{their } 8}{\sqrt{\textit{their } 5}\sqrt{\textit{their } 30}}$
	Obtain $\cos\theta=\dfrac{8}{5\sqrt{6}}$	A1	OE, e.g. $\dfrac{8}{\sqrt{150}}$ or $\dfrac{4\sqrt{6}}{15}$. If no $\dfrac{8}{5\sqrt{6}}$ seen, just 49.2, then A0. Decimal only seen, A0. ISW
6(b)	**Alternative Method for Question 6(b):**		
	Use of cosine rule: e.g. sides of $\sqrt{5},\sqrt{30}$ and $\sqrt{19}$ found	B1	Could use other points.
	e.g. $\cos\theta=\dfrac{5+30-19}{2\sqrt{5}\sqrt{30}}$	M1	
	Obtain $\cos\theta=\dfrac{8}{5\sqrt{6}}$	A1	OE, e.g. $\dfrac{8}{\sqrt{150}}$ or $\dfrac{4\sqrt{6}}{15}$ or $\dfrac{8}{\sqrt{5}\sqrt{30}}$. If no $\dfrac{8}{5\sqrt{6}}$ seen, just 49.2, then A0. Decimal only seen, A0. ISW
		3	

Question	Answer	Marks	Guidance
6(c)	Any two of $\lvert PA\rvert = 2\sqrt{5}$ $\lvert PB\rvert = \sqrt{30}$ or $\lvert AB\rvert = \sqrt{82}$ seen	**B1**	May be seen by stating or implying that $\lambda = 2$ and $\mu = -1$.
	Area $= \frac{1}{2}\times 2\sqrt{5}\times\sqrt{30}\times\sqrt{1-\frac{64}{150}}$	**M1**	Correct method for the exact area of the triangle. Note that: sin $APB = \frac{\sqrt{129}}{15}$ sin $ABP = \sqrt{\frac{86}{615}}$ cos $ABP = \frac{46}{\sqrt{2460}}$ Perp A to BP $= \frac{\sqrt{2580}}{15}$ Perp B to AP $= \frac{\sqrt{430}}{5}$
	$=\sqrt{86}$	**A1**	Or simplified exact equivalent. ISW
	Alternative Method for Question 6(c)		
	$\overrightarrow{PA}\times\overrightarrow{PB} = -4\mathrm{i} - 18\mathrm{j} - 2\mathrm{k}$	**B1**	$\overrightarrow{PA} = -2\mathrm{i} + 4\mathrm{k}$, $\overrightarrow{PB} = -2\mathrm{i} + \mathrm{j} - 5\mathrm{k}$.
	Area $= \frac{1}{2}\left\lvert\overrightarrow{PA}\times\overrightarrow{PB}\right\rvert = \frac{1}{2}\sqrt{16+324+4}$	**M1**	Correct method for the exact area of the triangle.
	$=\sqrt{86}$	**A1**	Or simplified exact equivalent. ISW
		3	

Question	Answer	Marks	Guidance
7(a)	Obtain $\frac{dx}{dt} = 6\cos 2t$	**B1**	Allow $3.2\cos 2t$. $dx = 6\cos 2t$ is B0.
	Obtain $\frac{dy}{dt} = \sec^2 t - \text{cosec}^2 t$	**B1**	Any equivalent form. $dy = \sec^2 t - \text{cosec}^2 t$ is B0, but $\frac{dy}{dx} = \frac{\sec^2 t - \text{cosec}^2 t}{6\cos 2t}$ can go on to gain M1M1A1, so 3/5 possible.
	Use $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$	***M1**	$\frac{dy}{dx} = \frac{\sec^2 t - \text{cosec}^2 t}{6\cos 2t}$
	Express as a single fraction With $\frac{dy}{dt}$ correctly simplified as a single fraction in terms of sin t and cos t Allow with $6\cos 2\text{t}$ expressed as $\frac{1}{6\cos 2t}$ outside bracket	**DM1**	Allow $\frac{\sin^2 t - \cos^2 t}{\cos^2 t \sin^2 t} \times \frac{1}{6\cos 2t}$ or $\frac{\sin^2 t - \cos^2 t}{\cos^2 t \sin^2 t} \div 6\cos 2t$ or $\frac{\frac{\sin^2 t - \cos^2 t}{\cos^2 t \sin^2 t}}{6\cos 2t}$
	Obtain $\left(\frac{-4\cos 2t}{6\cos 2t \times \sin^2 2t} = \right)\frac{-2}{3\sin^2 2t}$ from full and correct working Numerator and denominator must have identical terms before cancelling, both cos2t or both + and – ($\sin^2 t - \cos^2 t$)	**A1**	AG Allow slips in θ and x for t to recover earlier marks, provided these are corrected before the final line. Do not allow serious errors in working for the final mark.

Question	Answer	Marks	Guidance
7(a)	**Alternative Method for Question 7(a):**		
	$y = \frac{6}{x}$	**B2**	Using $y = \frac{\tan^2 t + 1}{\tan t} = \frac{\sec^2 t}{\frac{\sin t}{\cos t}} = \frac{1}{\sin t \cos t}$
	$\frac{\mathrm{d}y}{\mathrm{d}x} = -6x^{-2}$	**B1**	
	$\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{6}{9\sin^2 2t}$	**M1**	
	Obtain $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{-2}{3\sin^2 2t}$ from full and correct working	**A1**	
		5	
7(b)	Gradient of normal = $\frac{3}{2}$	**B1**	
	Use correct method to find the equation of the normal	**M1**	E.g. $(y-2) = \frac{3}{2}(x-3)$ or find c in $y = \frac{3}{2}x + c$. Allow a wrong value for x or y but not both, with *their* normal gradient.
	Obtain $2y - 3x + 5 = 0$	**A1**	Or $k(2y - 3x + 5) = 0$, where k is an integer.
		3	

Question	Answer	Marks	Guidance
8(a)	State or imply the form $\dfrac{A}{a-2x}+\dfrac{B}{3a+x}$ and use a correct method to find a constant	**M1**	
	Obtain $A=2a$ or $B=a$	**A1**	
	Obtain $A=2a$ and $B=a$	**A1**	
		3	
8(b)	Use a correct method to obtain the first two terms of the expansion of $(a-2x)^{-1}$ or $\left(1-\dfrac{2x}{a}\right)^{-1}$ or $(3a+x)^{-1}$ or $\left(1+\dfrac{x}{3a}\right)^{-1}$	**M1**	
	Obtain $+2\left(1+\dfrac{2x}{a}+\dfrac{4x^2}{a^2}+...\right)$	**A1ft**	OE. May be unsimplified. Follow *their A*, *B* for an expansion involving *a*.
	Obtain $+\dfrac{1}{3}\left(1-\dfrac{x}{3a}+\dfrac{x^2}{9a^2}+...\right)$	**A1ft**	OE. May be unsimplified. Follow *their A*, *B* for an expansion involving *a*.
	Obtain $+\dfrac{7}{3}+\dfrac{35x}{9a}+\dfrac{217x^2}{27a^2}$	**A1**	Or simplified equivalent. Final answer. Ignore any terms in higher powers of *x*. Do not ISW, e.g. multiplying by $27a^2$. Condone different order of terms.

Question	Answer	Marks	Guidance
8(b)	**Alternative Method for Question 8(b)**		
	Expanding $7a^2(a-2x)^{-1}(3a+x)^{-1}$ from the original question. Use a correct method to obtain the first two terms of the expansion of $(a-2x)^{-1}$ or $\left(1-\frac{2x}{a}\right)^{-1}$ or $(3a+x)^{-1}$ or $\left(1+\frac{x}{3a}\right)^{-1}$	**M1**	
	Obtain $+7a\left(1+\frac{2x}{a}+\frac{4x^2}{a^2}+...\right)$ or $+\frac{7a}{3}\left(1-\frac{x}{3a}+\frac{x^2}{9a^2}+...\right)$	**A1**	OE. May be unsimplified. May be implied by the expression shown for the next A1.
	Obtain $+\frac{7}{3}\left(1+\frac{2x}{a}+\frac{4x^2}{a^2}+...\right)\left(1-\frac{x}{3a}+\frac{x^2}{9a^2}+...\right)$	**A1**	OE. May be unsimplified.
	Obtain $+\frac{7}{3}+\frac{35x}{9a}+\frac{217x^2}{27a^2}$	**A1**	Or simplified equivalent. Final answer. Ignore any terms in higher powers of x. Do not ISW, e.g. multiplying by $27a^2$. Condone different order of terms.
		4	
8(c)	$\lvert x\rvert<\frac{a}{2}$	**B1**	Or $-\frac{a}{2}<x<\frac{a}{2}$. Mark final answer. Must make a clear statement.
		1	

Question	Answer	Marks	Guidance
9(a)	Divide to obtain quotient $x^2 + k$	**M1**	*k* is a constant.
	Obtain quotient $x^2 - 4$	**A1**	If quotient stated separately, mark at this stage.
	Obtain remainder 32	**A1**	If remainder stated separately, mark at this stage. Need not state which is quotient and remainder, but if stated wrongly, max 2/3. After a correct division, still allow the marks if then written as $x^2 - 4 + \dfrac{32}{x^2+4}$.
	Alternative Method for Question 9(a)		
	Expands brackets to get *B* = 0	**M1**	$(x^2+4)(x^2+Bx+C)+D=$ $x^4+Bx^3+(C+4)x^2+4Bx+4C+D$
	C = –4	**A1**	
	D = 32	**A1**	Need not state which is quotient and remainder, but if stated wrongly, max 2/3.
		3	

Question	Answer	Marks	Guidance
9(b)	$\frac{1}{3}x^3 - 4x$	**B1 FT**	Follow *their* quotient of form $Ax^2 + B$.
	Obtain $p\tan^{-1}qx$ where $q = 2$ or $q = \frac{1}{2}$	**M1**	
	Obtain $16\tan^{-1}\frac{1}{2}x$	**A1 FT**	Follow *their* constant remainder, i.e. $\left(\frac{\textit{their}\text{ constant remainder}}{2}\right)\tan^{-1}\frac{1}{2}x$.
	Use limits correctly in an expression containing $p\tan^{-1}qx$ where $q = 2$ or $q = \frac{1}{2}$ and $rx^3 + sx$	**M1**	Terms need not be evaluated, e.g. $\left[8\sqrt{3} - 8\sqrt{3}\right] + 16\tan^{-1}\sqrt{3} - \left(\frac{8}{3} - 8 + 16\tan^{-1}1\right)$ or $\frac{8}{3} - 8$ can be $-\frac{16}{3}$, $16\tan^{-1}\sqrt{3}$ can be $\frac{16\pi}{3}$, $16\tan^{-1}1$ can be 4π.
	Obtain $\frac{4}{3}(\pi + 4)$ from full and correct working	**A1**	AG
		5	

Question	Answer	Marks	Guidance
10(a)	State that $\frac{\mathrm{d}V}{\mathrm{d}t}=50000-600h$	**B1**	May be seen as $\frac{\mathrm{d}V}{\mathrm{d}t}=50000$ and $\frac{\mathrm{d}V}{\mathrm{d}t}=[-]600h$. When put together (may be in the chain rule) B1 can be awarded.
	[Use $V=40000h$ to] obtain $\frac{\mathrm{d}V}{\mathrm{d}h}=40000$ and use this and *their* $\frac{\mathrm{d}V}{\mathrm{d}t}$ in the correct chain rule to obtain $\frac{\mathrm{d}h}{\mathrm{d}t}$ or [Use $V=40000h$ to] obtain $\frac{\mathrm{d}V}{\mathrm{d}t}=40000\frac{\mathrm{d}h}{\mathrm{d}t}$ and equate to *their* $\frac{\mathrm{d}V}{\mathrm{d}t}$	**M1**	$\frac{\mathrm{d}h}{\mathrm{d}t}=\frac{\mathrm{d}V}{\mathrm{d}t}\div\frac{\mathrm{d}V}{\mathrm{d}h}$ E.g. $\frac{50000-600h}{40000}=\frac{\mathrm{d}h}{\mathrm{d}t}$ E.g. $40000\frac{\mathrm{d}h}{\mathrm{d}t}=50000-600h$

Question	Answer	Marks	Guidance
10(a)	Obtain $200\frac{dh}{dt}=250-3h$ from full and correct working	**A1**	AG $\frac{\mathrm{d}h}{\mathrm{d}t}=\frac{50000-600h}{40000}$ OE, leading to given answer with no other working, or no incorrect working seen **SC B1** (1/3). $\frac{\mathrm{d}V}{\mathrm{d}t}=50000-600h$ B1 followed by $\frac{\mathrm{d}h}{\mathrm{d}t}=\frac{50000-600h}{40000}$ OE, leading to given answer with no other working, or no incorrect working seen, **SC B1** 2/3. $\frac{\mathrm{d}V}{\mathrm{d}t}=\frac{\mathrm{d}V}{\mathrm{d}h}\times\frac{\mathrm{d}h}{\mathrm{d}t}$ followed by $50000-600h=40000\frac{\mathrm{d}h}{\mathrm{d}t}$ OE, leading to given answer with no other working, or no incorrect working seen, B1 for implied $\frac{\mathrm{d}V}{\mathrm{d}t}$ and **SC B1** 2/3.
		3	

Question	Answer	Marks	Guidance
10(b)	Separate variables correctly and integrate one side correctly	**M1**	E.g. $\int \frac{1}{250-3h}\,\mathrm{d}h = \int \frac{1}{200}\,\mathrm{d}t$. Integral signs may be omitted, 200 may be on opposite side.
	Obtain $-\frac{1}{3}\ln\lvert 250-3h\rvert = \frac{t}{200}\,(+C)$	**A1**	OE Condone missing "+C" and lack of modulus signs.
	Use $t=0,\ h=50$ in an expression containing $\ln(250-3h)$ or $\ln\lvert 250-3h\rvert$ to find the constant of integration.	**M1**	Or equivalent use of limits 50 and 80.
	Obtain $C = -\frac{1}{3}\ln 100$	**A1**	OE, e.g. $\frac{1}{3}\ln\frac{100}{250-3h} = \frac{t}{200}$, or $-\frac{200}{3}\ln\left(\frac{10}{100}\right)$. With or without modulus signs on the log terms.
	$t = 150$	**A1**	
		5	

Question	Answer	Marks	Guidance
11(a)	Use of correct product rule and correct chain rule	**M1**	$\frac{\mathrm{d}y}{\mathrm{d}x} = A\cos x\sqrt{2+\cos x} + \frac{B\sin x\sin x}{\sqrt{2+\cos x}}$
	Obtain $\frac{\mathrm{d}y}{\mathrm{d}x} = 2\cos x\sqrt{2+\cos x} - \frac{2\sin^2 x}{2\sqrt{2+\cos x}}$	**A1**	OE
	Equate the derivative to zero and obtain a horizontal 3 term quadratic equation or 4 term quartic equation in $\cos a$ If M0 earlier then needs that expression to be such that arrive at 3 term quadratic or 4 term quartic equation in cos x without further trig errors. The only error in the form of the differential allowed is for $(2+\cos x)^{-\frac{1}{2}}$ to be $(2+\cos x)^{+\frac{1}{2}}$ or $(2+\cos x)^{-\frac{3}{2}}$	***M1**	Accept in $\cos x$. E.g. $3\cos^2 x + 4\cos x - 1 = 0$. E.g. $3\cos^4 x + 16\cos^3 x + 18\cos^2 x - 1 = 0$.
	Solve for cos a	**DM1**	$\left(\cos a = \frac{-2+\sqrt{7}}{3} \text{ or } 0.215\right)$ Allow presence of other solution(s).
	Obtain $a = 4.93$	**A1**	Allow more accurate, e.g. 4.929… even though question states 2 dp. If x = 1.35 leads to x = 4.93 award A1 BOD. If x = 1.35 and x = 4.93 award A0.
		5	

Question	Answer	Marks	Guidance
11(b)	State or imply $\mathrm{d}u = -\sin x\,\mathrm{d}x$	**B1**	OE If B0, max M1M1M1.
	Substitute throughout for u and $\mathrm{d}u$	**M1**	
	Obtain $-\int 2\sqrt{u}du$	**A1**	OE. Ignore limits if $-\int 2\sqrt{u}du$, but if $+\int 2\sqrt{u}du$, then must have correct limits $\int_1^3 2\sqrt{u}\mathrm{d}u$. (See final M1)
	Integrate to obtain $ku^{\frac{3}{2}}(+C)$	**M1**	Constant of integration not required
	Use correct limits correctly in an expression of the form $ku^{\frac{3}{2}}$ or $k(2+\cos x)^{\frac{3}{2}}$	**M1**	1 and 3 for u, or 0 and π for x.
	Obtain $\frac{4}{3}\left(3\sqrt{3}-1\right)$ or $4\sqrt{3}-\frac{4}{3}$ or $\frac{4}{3}\sqrt{27}-\frac{4}{3}$	**A1**	OE. Allow, e.g., $\sqrt{3}^3$ for $\sqrt{27}$. ISW but don't ignore e.g. multiplying throughout by 3. If the answer is changed from negative to positive value at end, then A0. Last M1A1 can use modulus, providing no errors seen.
		6	

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MATHEMATICS **9709/31**

Paper 3 Pure Mathematics 3 **May/June 2024**

MARK SCHEME

Maximum Mark: 75

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Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics-Specific Marking Principles

1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.

2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.

3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.

4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).

5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.

6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M	Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
A	Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
B	Mark for a correct result or statement independent of method marks.
DM or **DB**	When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
FT	Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

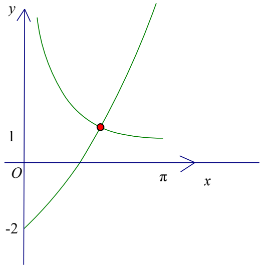
Question	Answer	Marks	Guidance
1	State correct unsimplified first two terms of the expansion of $(1-2x)^{\frac{1}{2}}$, e.g. $1+\frac{1}{2}(-2x)$	**B1**	Symbolic coefficients are not sufficient. $1-x$
	State correct unsimplified term in x^2, e.g. $\frac{\frac{1}{2}\left(\frac{1}{2}-1\right)(-2x)^2}{2!}$	**B1**	Symbolic coefficients are not sufficient. $-\frac{1}{2}x^2$
	Obtain sufficient terms of the product of $(3+x)$ and the expansion up to the term in x^2	**M1**	
	Obtain final answer $3-2x-\frac{5}{2}x^2$	**A1**	
		4	

Question	Answer	Marks	Guidance
2	Use law of logarithm of a product (or quotient) on correct terms	***M1**	
	Use correct method to eliminate logarithm	**DM1**	
	Obtain a correct quadratic in x, e.g. $x^2-5x-e^7=0$ (allow decimals)	**A1**	
	Obtain answer $x = 35.71$ only	**A1**	
		4	

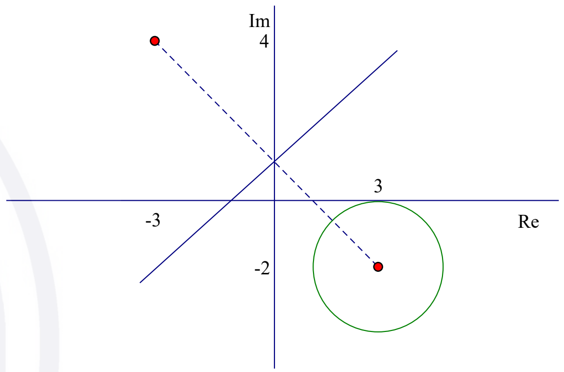
Question	Answer	Marks	Guidance
3	State or imply that $y\ln a = \ln b + \ln x$	**B1**	
	Carry out a completely correct method for finding $\ln a$ or $\ln b$	**M1**	E.g., from $\ln a = \ln b + 0.336$ $1.5\ln a = \ln b + 1.31$.
	Obtain value $a = 7$	**A1**	
	Obtain value $b = 5$	**A1**	
		4	

Question	Answer	Marks	Guidance
4(a)	State or imply $r = 2$	**B1**	
	State or imply $\theta = -\frac{2}{3}\pi$	**B1**	
		2	
4(b)	State or imply $r = \frac{5}{2}$	**B1FT**	FT $\dfrac{5}{\textit{their}\ 2}$.
	State or imply $\theta = \frac{5}{6}\pi$	**B1FT**	FT $\frac{1}{6}\pi - \textit{their} - \frac{2}{3}\pi$.
		2	

Question	Answer	Marks	Guidance
5	Use correct quotient (or product) rule	***M1**	
	Obtain correct derivative $\frac{e^{\sin x}\cos^3 x-(-2e^{\sin x}\sin x\cos x)}{\left(\cos^2 x\right)^2}$ or equivalent	**A1**	
	Equate numerator to zero	**DM1**	
	Obtain equation in one unknown	**DM1**	E.g. $\sin^2 x - 2\sin x - 1 = 0$.
	Solve a 3 term quadratic in sin x to find a value for x	**M1**	
	Obtain a correct solution to the quadratic equation, e.g. 3.57°	**A1**	At least 3sf.
	Obtain a further correct solution, e.g. $x = 5.86°$ and no others in the interval	**A1FT**	At least 3sf. FT 3π – *their* 3.57.
	Alternative Method for the first 3 marks:		
	Take logarithms of both sides and simplify	**(*M1)**	$\ln y = \sin x - 2\ln\cos x$ or equivalent.
	Obtain $\frac{1}{y}\frac{dy}{dx} = \cos x + 2\frac{\sin x}{\cos x}$	**(A1)**	Or equivalent.
	Equate $\frac{dy}{dx}$ to zero	**(DM1)**	
	Continue as for the original		
		7	

Question	Answer	Marks	Guidance
6(a)	Sketch a relevant graph, e.g. $y = e^x - 3$ Correct shape, correct vertical intercept	B1	
	Sketch a second relevant graph, e.g. $y = \operatorname{cosec}\dfrac{x}{2}$ (correct shape, minimum above the axis) and justify the given statement. Need to mark intersection with a dot, a cross, or say root at points of intersection, or equivalent	B1	
		2	
6(b)	Calculate the values of a relevant expression or pair of expressions at $x = 1$ and $x = 2$	M1	Use of degrees is M0.
	Complete the argument correctly with correct calculated values	A1	E.g. $-0.282 < 2.086,\ 4.389 > 1.188$ $1 < 1.626,\ 2 > 1.432$ $2.36 > 0,\ -3.2 < 0$ At least 2sf. Condone truncation.
		2	
6(c)	State $x = \ln\left(\operatorname{cosec}\dfrac{1}{2}x + 3\right)$ and rearrange to the given equation $\operatorname{cosec}\dfrac{x}{2} = e^x - 3$	B1	AG. Or vice versa and obtain the iterative formula.
		1	
6(d)	Use the iterative formula correctly at least twice	M1	Use of degrees in M0 (might see 1.38….).
	Obtain final answer 1.50	A1	
	Show sufficient iterations to 4 dp to justify 1.50 to 4 dp or show there is a sign change in the interval (1.495, 1.505)	A1	1.5156, 1.4940, 1.4978, 1.4971.
		3	

Question	Answer	Marks	Guidance
6(e)	4	**B1**	
		1	

Question	Answer	Marks	Guidance
7(a)	Show a circle centre (3, –2)	**B1**	
	Show a circle with radius 2 FT centre not at the origin	**B1FT**	
	Show the point representing (–3, 4) or the midpoint (0, 1)	**B1**	
	Show the perpendicular bisector of the line joining (–3, 4) and centre of the circle FT is on the position of (–3, 4) and centre of the circle	**B1FT**	
		4	
7(b)	Carry out a correct method for finding the least value of $\|z-w\|$	**M1**	(Distance (3, –2) to (0, 1)) – 2.
	Obtain answer $\sqrt{18}-2$ or $3\sqrt{2}-2$	**A1**	
		2	

Question	Answer	Marks	Guidance
8	State or imply du = – cos$x$ dx	**B1**	
	Use sin 2x = 2sin$x$cos$x$ and write the integral in terms of u	***M1**	
	Obtain $\pm 2\int \frac{(1-u)}{\sqrt{u}}\,\mathrm{d}u$ or equivalent	**A1**	
	Integrate correctly to obtain $au^{\frac{1}{2}} + bu^{\frac{3}{2}}$	**DM1**	
	Obtain correct $-4u^{\frac{1}{2}} + \frac{4}{3}u^{\frac{3}{2}}$	**A1**	
	Correctly use limits $u = 2$ and 0 in an expression of the form $au^{\frac{1}{2}} + bu^{\frac{3}{2}}$ OR limits $x = \frac{3}{2}\pi$ and $\frac{1}{2}\pi$ in an expression of the form $a(1-\sin x)^{\frac{1}{2}} + b(1-\sin x)^{\frac{3}{2}}$	**DM1**	
	Obtain $\frac{8}{3} - \frac{4}{3}\sqrt{2}$	**A1**	
		7	

Question	Answer	Marks	Guidance
9(a)	Carry out correct process for evaluating the scalar product of direction vectors, equate the result to zero and obtain given value of $a = 4$	**B1**	E.g. $2(3) + (–1)(–2) + a(–2) = 0$.
		1	
9(b)	Express general point of at least one line correctly in component form, i.e. $(1 + 2\lambda, –2 – \lambda, 3 + 4\lambda)$ or $(–1 + 3\mu, –1 – 2\mu, –1 – 2\mu)$	**B1**	The third component could be implied by a correct final answer.
	Equate at least two pairs of corresponding components and solve for λ or for μ	**M1**	
	Obtain $\lambda = –1$ or $\mu = 0$	**A1**	
	Obtain position vector of point of intersection is $–\mathbf{i} – \mathbf{j} – \mathbf{k}$	**A1**	
		4	
9(c)	Equate one component of l_1 to matching component of A and solve to find λ	**M1**	
	Use $\lambda = –3$ in equation of l_1 and show this gives position vector of A	**A1**	AG Or show $\lambda = –3$ for all three components equated.
		2	
9(d)	Method to find position vector of B	**M1**	E.g. $\pm 2\times$*their* $(–\mathbf{i} – \mathbf{j} – \mathbf{k}) \pm (–5\mathbf{i} + \mathbf{j} – 9\mathbf{k})$
	Obtain position vector of B is $3\mathbf{i} – 3\mathbf{j} + 7\mathbf{k}$	**A1**	
		2	

Question	Answer	Marks	Guidance
10(a)	Obtain $2 = \sec^2 y \dfrac{dy}{dx}$ or equivalent	**B1**	E.g. $2\dfrac{dx}{dy} = \sec^2 y$ by differentiation with respect to y.
	Use $\sec^2 y = 1 + \tan^2 y$	**M1**	
	Replace $\tan y$ with $2x$ and rearrange to obtain given answer $\dfrac{dy}{dx} = \dfrac{2}{1+4x^2}$	**A1**	
		3	
10(b)	Integrate by parts and reach $ax^2 \tan^{-1} 2x + b\int \dfrac{x^2}{1+4x^2}\,dx$	***M1**	
	Obtain $\frac{1}{2}x^2 \tan^{-1} 2x - \int \dfrac{x^2}{1+4x^2}\,dx$	**A1**	OE
	Reduce integral to expression of the form $\int m + \dfrac{n}{1+4x^2}\,dx$	**M1**	
	Complete integration and reach $px^2 \tan^{-1} 2x + qx + r\tan^{-1} 2x$	**M1**	
	Obtain $\frac{1}{2}x^2 \tan^{-1} 2x - \frac{1}{4}x + \frac{1}{8}\tan^{-1} 2x$	**A1**	OE
	Use limits of $x = \frac{1}{2}$ and $x = \frac{1}{2}\sqrt{3}$ in the correct order, having integrated twice	**DM1**	
	Obtain answer $\frac{5}{48}\pi - \frac{1}{8}\sqrt{3} + \frac{1}{8}$ or exact equivalent	**A1**	
		7	

Question	Answer	Marks	Guidance
11(a)	State or imply equation of the form $\frac{dx}{dt} = kx(300-x)$ and use $\frac{dx}{dt} = 0.2$ and $x = 1$	**M1**	M0 for verification.
	Obtain $k = \frac{1}{1495}$ and rearrange to the given answer	**A1**	$1495\frac{dx}{dt} = x(300-x)$.
		2	

Question	Answer	Marks	Guidance
11(b)	Separate variables correctly	**B1**	$\int \frac{1}{x(300-x)}\mathrm{d}x = \int \frac{1}{1495}\mathrm{d}t$
	Correct integration of *t* term	**B1**	E.g. obtain *t* or $\frac{t}{1495}$.
	State or imply partial fractions of the form $\frac{A}{x}+\frac{B}{300-x}$	**B1**	
	Correct method to find *A* or *B*	**M1**	$A=\frac{1}{300}$ and $B=\frac{1}{300}$. May see $A=B=\frac{1495}{300}=\frac{299}{60}$.
	Obtain terms $\frac{1495}{300}\ln x-\frac{1495}{300}\ln(300-x)$	**A1**	OE. May see $\frac{1}{300}\ln x-\frac{1}{300}\ln(300-x)$.
	Use $t=0$, $x=1$ to evaluate a constant or as limits in a solution containing terms of the form $\ln x$, $\ln(300-x)$ and *t*.	**M1**	
	Obtain correct answer in any form	**A1**	E.g. $\frac{1495}{300}[\ln x-\ln(300-x)]=t-\frac{1495}{300}\ln 299$.
	Use law of logarithms twice to obtain an expression for *t*	**M1**	
	Obtain final answer $t=\frac{299}{60}\ln\frac{299x}{300-x}$ or equivalent single logarithm	**A1**	
		9	

Cambridge International A Level

MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **May/June 2024**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2024 series for most Cambridge IGCSE, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

This document consists of **20** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
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DM or **DB**	When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
FT	Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

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- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
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Abbreviations

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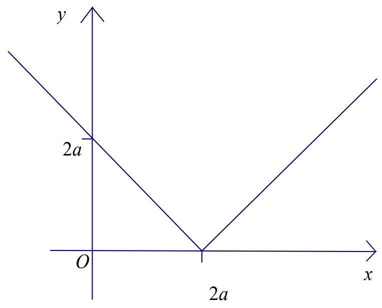
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1(a)		**B1**	Correct shape, roughly symmetrical. Both sections should be solid straight lines. If not drawn with a ruler the intention must be clear. Allow construction lines if dashed or clearly fainter. 2*a* marked on each axis (must be 2*a*, not just 2). Needs to extend into negative *x*. If *a* is given a value, then B0. Ignore $y = 2x - 3a$ if seen.
		1	
1(b)	Solve linear equation or inequality to obtain critical value $x = \frac{5}{3}a$ or exact equivalent.	**B1**	Ignore $x = a$ if seen.
	Obtain $x < \frac{5}{3}a$ or exact equivalent	**B1**	Accept $x < \frac{10}{6}a$ or $\left(-\infty, \frac{5}{3}a\right)$. Must be strict inequality. Need a clear final solution: $x > a$ or $x < a$ must be rejected if seen as part of the working. Rejection can be implied, e.g. if only the correct inequality is underlined. B0 B0 if *a* is given a value.
	Alternative Method for Question 1(b)		
	Solve quadratic equation $(2x-3a)^2 = (x-2a)^2$ to obtain critical value $x = \frac{5}{3}a$ or exact equivalent	**(B1)**	$\left(3x^2 - 8ax + 5a^2 = 0\right)$ Ignore $x = a$ if seen.
	Obtain $x < \frac{5}{3}a$ or exact equivalent	**(B1)**	Accept $x < \frac{10}{6}a$ or $\left(-\infty, \frac{5}{3}a\right)$. Must be strict inequality. Need a clear final solution: $x > a$ or $x < a$ must be rejected if seen as part of the working. Rejection can be implied, e.g. if only the correct inequality is underlined. B0 B0 if *a* is given a value.
		2	

Question	Answer	Marks	Guidance
2	State or imply the form $A+\frac{B}{2x+3}+\frac{C}{x-4}$	**B1**	$\frac{Dx+E}{2x+3}+\frac{F}{x-4}$ and $\frac{P}{2x+3}+\frac{Qx+R}{x-4}$ are also valid.
	Use a correct method for finding a constant	**M1**	**SC**: If score B0, they can score **M1 A1** for one correct constant. B0 M1 A0 available if they substitute two values to form simultaneous equations but get an incorrect answer, or they substitute one value and make an arithmetic error.
	Obtain one of $A=3$, $B=-2$ and $C=4$	**A1**	**SC**: If the horizontal equation is correct apart from an incorrect value for *A*, the other A marks may be available.
	Obtain a second value	**A1**	**SC**: If denominator factorised as $\left(x+\frac{3}{2}\right)\left(x-4\right)$ can score a maximum of **B0 M1 A1 A1 A0** for a split involving 3 terms.
	Obtain a third value	**A1**	ISW Statement of the final split is not required.

Question	Answer	Marks	Guidance
2	**Alternative method for Question 2**		
	Divide numerator by denominator	**(M1)**	
	Obtain $3\left(+\dfrac{Px+Q}{2x^2-5x-12}\right)$	**(A1)**	$\left(3+\dfrac{6x+20}{(2x+3)(x-4)}\right)$
	State or imply the form $\dfrac{Px+Q}{2x^2-5x-12}=\dfrac{D}{2x+3}+\dfrac{E}{x-4}$	**(B1)**	Must deal with the 3 separately or include it correctly on both sides in their split.
	Obtain one of $D=-2$ and $E=4$	**(A1)**	**SC**: If denominator factorised as $\left(x+\frac{3}{2}\right)\left(x-4\right)$, then can score a maximum of **B0 M1 A1 A1 A0** for a split involving three terms.
	Obtain a second value	**(A1)**	ISW Statement of the final split is not required.
		5	

Question	Answer	Marks	Guidance
3(a)	Use logarithms to obtain a correct expression without powers e.g. $(2y-1)\ln a = (x-y)\ln b$	**B1**	Could use logs to any base e.g. $2y-1=(x-y)\log_a b$. Do not condone missing brackets unless recovered later.
	Separate terms and factorise to obtain $y(2\ln a + \ln b) = x\ln b + \ln a$	**B1**	Or equivalent, e.g. $y = x\dfrac{\ln b}{\ln a^2 b} + \dfrac{\ln a}{\ln a^2 b}$ or $y(2+\log_a b) = x\log_a b + 1$.
	Clear explanation of linear form. From correct work only.	**B1**	E.g. equation matches the linear form $y = mx + c$ or $py = qx + r$. Condone if they compare with $y = mx + c$, but do not actually state that it must therefore be a straight line. Stating "this is a linear equation" without comparing to a relevant standard form scores B0. B0 if they have $m = \ldots$ and $c = \ldots$ correct but never actually mention $y = mx + c$.
		3	
3(b)	Use $a = b^3$ and log laws to simplify their equation	**M1**	$\left[y = x\dfrac{\ln b}{\ln b^7} + \dfrac{\ln b^3}{\ln b^7}\right]$ Denominator reduced to a single log term.
	Obtain $y = \dfrac{1}{7}x + \dfrac{3}{7}$	**A1**	Accept $y = \dfrac{x}{7} + \dfrac{3}{7}$ but not $y = \dfrac{x+3}{7}$.
	Alternative method for Question 3(b)		
	Use $a = b^3$ to obtain $b^{3(2y-1)} = b^{x-y}$ or equivalent	**(M1)**	Or $\log_a b = \frac{1}{3}$.
	Obtain $y = \dfrac{1}{7}x + \dfrac{3}{7}$	**(A1)**	Accept $y = \dfrac{x}{7} + \dfrac{3}{7}$ but not $y = \dfrac{x+3}{7}$.
		2	

Question	Answer	Marks	Guidance
4	Substitute $y=1$ and obtain $e^x = a$, where $a>0$	**M1**	Must come from a quadratic in e^x. $\left(e^{2x}+e^x-6=0\right)$ Ignore any negative solution.
	Obtain $e^x = 2$ only	**A1**	Or equivalent e.g. $x=\ln 2$. Condone x = 0.693…..
	State or imply $\frac{d}{dx}\left(ye^{2x}\right)=2ye^{2x}+e^{2x}\frac{dy}{dx}$	**B1**	Accept y' for $\frac{dy}{dx}$.
	State or imply $\frac{d}{dx}\left(y^2e^x\right)=y^2e^x+2ye^x\frac{dy}{dx}$	**B1**	Accept y' for $\frac{dy}{dx}$.
	Differentiate RHS of given equation to obtain zero (could be implied by subsequent work), substitute for x and y and obtain $\frac{dy}{dx}=\ldots$	**M1**	Independent. $\left[2\times4+4\frac{dy}{dx}+2+4\frac{dy}{dx}=0\right]$ NB: Could also rearrange to obtain $\frac{dy}{dx}$ then substitute. Both steps are needed for M1.
	Obtain $-\frac{5}{4}$ or -1.25	**A1**	Correct answer from correct working only Accept $\frac{-10}{8}$. -1.2499 is A0.

Question	Answer	Marks	Guidance
4	**Alternative method for Question 4: Dividing through by e^x**		
	Substitute $y=1$ and obtain $\mathrm{e}^x = a$, where $a>0$	**(M1)**	Must come from a quadratic in e^x. $\left(\mathrm{e}^{2x}+\mathrm{e}^x-6=0\right)$ Ignore any negative solution.
	Obtain $\mathrm{e}^x = 2$ only	**(A1)**	Or equivalent e.g. $x=\ln 2$. Condone $x = 0.693$…..
	State or imply $\frac{\mathrm{d}}{\mathrm{d}x}\left(y^2\right)=2y\frac{\mathrm{d}y}{\mathrm{d}x}$	**(B1)**	Accept y' for $\frac{\mathrm{d}y}{\mathrm{d}x}$.
	State or imply $\frac{\mathrm{d}}{\mathrm{d}x}\left(ye^x\right)=y\mathrm{e}^x+\mathrm{e}^x\frac{\mathrm{d}y}{\mathrm{d}x}$	**(B1)**	Accept y' for $\frac{\mathrm{d}y}{\mathrm{d}x}$.
	Differentiate RHS of given equation to obtain $-6\mathrm{e}^{-x}$, substitute for x and y and obtain $\frac{\mathrm{d}y}{\mathrm{d}x}=$.....	**(M1)**	Independent. $\left[1\times 2+2\frac{\mathrm{d}y}{\mathrm{d}x}+2\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{6}{2}\right]$ NB: Could also rearrange to obtain $\frac{\mathrm{d}y}{\mathrm{d}x}$ then substitute. Both steps are needed for M1.
	Obtain $-\frac{5}{4}$ or -1.25	**(A1)**	Correct answer from correct working only Accept $\frac{-10}{8}$. -1.2499 is A0.
		6	

Question	Answer	Marks	Guidance
5(a)	Calculate the value of a relevant expression or values of a pair of expressions at $x = 0.7$ and $x = 0.8$	**M1**	Allow if working with a smaller interval, e.g. (0.72, 0.78). Need all relevant values but condone one error. Pairings must be clear for solutions involving four values (do not accept embedded values). M0 if working in degrees e.g -1.94…, 1.04…
	Complete the argument correctly with correct calculated values. (can be using the equation in the rubric or the equation in **(b)** or equivalent)	**A1**	E.g. $4.95... > 4.26...$ and $4.06... < 4.49...$ -0.439…< 0, 0.690…> 0 1.4 < 1.5029…, 1.6 > 1.449… 1.1 > 1, 0.86 < 1 0.0515 > 0, -0.075 < 0. Allow values rounded or truncated to 2sf.
		2	
5(b)	State $2x = \ln(5 + \cos 3x)$ and take exponential of both sides to obtain $e^{2x} = 5 + \cos 3x$	**B1**	Given answer requires fully correct working or work vice versa. If working in reverse, must get to the iterative formula, including subscripts.
		1	
5(c)	Use the iterative process correctly at least once	**M1**	M0 if working in degrees (e.g. values heading for 0.89….).
	Obtain final answer 0.740	**A1**	
	Show sufficient iterations to at least 5dp to justify 0.740 to 3dp, or show that there is a sign change in the interval $(0.7395, 0.7405)$	**A1**	E.g. $0.7, 0.75150, 0.73719, 0.74105, 0.74000, 0.74028$ $0.75, 0.73759, 0.74094, 0.74003, 0.74028$ $0.8, 0.72494, 0.74443, 0.73909, 0.74053, 0.74014, 0.74025$ Allow recovery. Allow truncation or rounding and condone small differences in the final decimal place.
		3	

Question	Answer	Marks	Guidance
6(a)	Use correct product rule	***M1**	Or equivalent. Condone incorrect chain rule. M0 if a value is used for a (not equivalent work).
	Obtain correct derivative	**A1**	E.g. $\frac{\mathrm{d}y}{\mathrm{d}x} = -axe^{-ax} + e^{-ax}$
	Equate derivative to zero and solve for x	**DM1**	
	Obtain $x = \frac{1}{a}$, $y = \frac{1}{ae}$	**A1**	ISW Or exact equivalent.
		4	
6(b)	Use integration by parts to obtain $pxe^{-ax} + q\int e^{-ax}\mathrm{d}x$	***M1**	Condone sign error in parts formula and omission of dx. M0 if a value is used for a (not equivalent work).
	Obtain $-\frac{1}{a}xe^{-ax} + \frac{1}{a}\int e^{-ax}\mathrm{d}x$	**A1**	OE
	Complete integration to obtain $-\frac{1}{a}xe^{-ax} - \frac{1}{a^2}e^{-ax}$	**A1**	OE
	Correct use of limits 0 and $\frac{2}{a}$ in an expression of the form $rxe^{-ax} + se^{-ax}$	**DM1**	$\left(\frac{-2}{a^2}e^{-2} - \frac{1}{a^2}e^{-2} + 0 + \frac{1}{a^2}\right)$
	Obtain $\frac{1}{a^2}\left(1 - 3e^{-2}\right)$	**A1**	ISW Or simplified 2-term equivalent, e.g. $\frac{e^2 - 3}{a^2e^2}$.
		5	

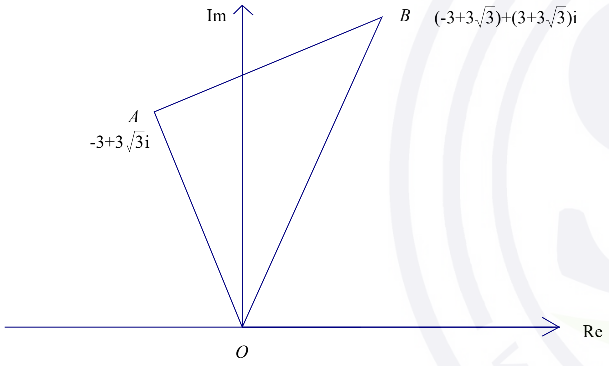
Question	Answer	Marks	Guidance
7(a)	Factorise LHS using difference of 2 squares	***M1**	$\left(\left(\cos^2\theta-\sin^2\theta\right)\left(\cos^2\theta+\sin^2\theta\right)\right)$
	Simplify	**DM1**	$\cos^2\theta+\sin^2\theta=1$ must be seen or implied, e.g. $\left(\cos^2\theta-\sin^2\theta\right)\left(\cos^2\theta+\sin^2\theta\right)=\left(\cos^2\theta-\sin^2\theta\right)$.
	Obtain $\cos^4\theta-\sin^4\theta\equiv\cos 2\theta$ from correct working	**A1**	AG
	Alternative Method for Question 7(a)		
	Use of correct rearrangements of double angle formulae	**(*M1)**	E.g. $\left(\frac{1+\cos 2\theta}{2}\right)^2-\left(\frac{1-\cos 2\theta}{2}\right)^2$ Only condone $\left(\frac{1+\cos 2\theta}{2}\right)^2-\left(\frac{\cos 2\theta-1}{2}\right)^2$ if correct expression for $\sin^2\theta$ seen.
	Expand and simplify	**(DM1)**	Collect like terms. Condone recovery from missing brackets.
	Obtain $\cos^4\theta-\sin^4\theta\equiv\cos 2\theta$ from correct working	**(A1)**	AG
	Alternative Method 2 for Question 7(a)		
	Correct use of Pythagoras	**(*M1)**	E.g. $\left(1-\sin^2\theta\right)^2-\sin^4\theta$ or $\cos^2\theta\left(1-\sin^2\theta\right)-\sin^2\theta\left(1-\cos^2\theta\right)$
	Expand and simplify	**(DM1)**	Condone recovery from missing brackets.
	Obtain $\cos^4\theta-\sin^4\theta\equiv\cos 2\theta$ from correct working	**(A1)**	AG
		3	

Question	Answer	Marks	Guidance
7(b)	Use part (a) and correct double angle formula to obtain expression involving $\int \sin^2 2\theta\,\mathrm{d}\theta$ or $\int \cos^2 2\theta\,\mathrm{d}\theta$	**M1**	$\int \cos^4\theta - \sin^4\theta + 4\sin^2\theta\cos^2\theta\,\mathrm{d}\theta = \int \cos 2\theta + \sin^2 2\theta\,\mathrm{d}\theta$ Allow BOD for $2\sin^2 2\theta$ if $\sin 2\theta = 2\sin\theta\cos\theta$ seen.
	$\int \cos 2\theta d\theta = \frac{1}{2}\sin 2\theta$	**B1**	Seen or implied.
	Use of correct double angle formula on second part of the integral to obtain a form that can be integrated directly	**M1**	e.g. $\int \sin^2 2\theta\,\mathrm{d}\theta = \int \frac{1-\cos 4\theta}{2}\,\mathrm{d}\theta$
	Obtain $\frac{1}{2}\theta - \frac{1}{8}\sin 4\theta$	**A1**	Condone a mixture of x and θ.
	Correct use of limits $\pm\frac{\pi}{8}$ in an expression of the form $p\theta + q\sin 2\theta + r\sin 4\theta$ and evaluate the trig	**M1**	$\left(2\left(\frac{1}{2}\times\frac{1}{\sqrt{2}}+\frac{\pi}{16}-\frac{1}{8}\right)\right)$
	Obtain $\frac{1}{2}\sqrt{2}+\frac{1}{8}\pi-\frac{1}{4}$	**A1**	ISW Or exact equivalent from exact working.
		6	

Question	Answer	Marks	Guidance
8(a)	Correct direction vector seen or implied ($\overrightarrow{BC} = 3\mathbf{i} + 3\mathbf{j} - 3\mathbf{k}$)	**B1**	Condone $\overrightarrow{BC} = -3\mathbf{i} - 3\mathbf{j} + 3\mathbf{k}$.
	Use a correct method to form a vector equation	**M1**	Allow for the RHS with no LHS.
	Obtain $\mathbf{r} = 5\mathbf{i} + 2\mathbf{j} + \lambda(\mathbf{i} + \mathbf{j} - \mathbf{k})$	**A1**	ISW Must have **r** = … or $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \ldots$, **not** l_1 = … Or, equivalent vector form, e.g. $\mathbf{r} = 8\mathbf{i} + 5\mathbf{j} - 3\mathbf{k} + \alpha(\mathbf{i} + \mathbf{j} - \mathbf{k})$ or $\mathbf{r} = 5\mathbf{i} + 2\mathbf{j} + \lambda(3\mathbf{i} + 3\mathbf{j} - 3\mathbf{k})$. Condone a column vector with **i**, **j**, **k**.
		3	
8(b)	Use components to form two relevant equations in 2 unknowns For their l_1 B0 if they use the same unknown for both lines.	**B1FT**	Two components of $\begin{pmatrix} 5+\lambda \\ 2+\lambda \\ -\lambda \end{pmatrix} = \begin{pmatrix} -2+3\mu \\ 1+\mu \\ 4-2\mu \end{pmatrix}$ seen or implied.
	Solve 2 relevant equations in 2 unknowns for λ or μ	**M1**	For *their* l_1.
	Obtain $\lambda = 2$ or $\mu = 3$	**A1**	Or equivalent e.g. using $\overline{BC}$ as direction vector gives $\lambda = \frac{2}{3}$.
	Obtain $(7, 4, -2)$ No need to check the third equation – the question implies that the lines intersect.	**A1**	Accept position vector. Condone a column vector with **i**, **j**, **k**. **SC**: **B1 M1 A1 A1** if one component of their line is incorrect but they do not use that component.
		4	

Question	Answer	Marks	Guidance
8(c)	State $AB = \sqrt{7^2 + 1^2 + 4^2} \left(= \sqrt{66}\right)$	**B1**	Or $(AB)^2 = 66$ Condone a sign error in $\overrightarrow{AB}$.
	State $\overrightarrow{BD}$ in component form	**B1**	$\begin{pmatrix} -7+3r \\ -1+r \\ 4-2r \end{pmatrix}$ or equivalent.
	$AB = BD \Rightarrow (3r-7)^2 + (r-1)^2 + (-2r+4)^2 = 66$ $\left(14r^2 - 60r = 0\right)$	**M1**	Or equivalent equation in one unknown for their *AB* and *their* $\overrightarrow{BD} \neq \overrightarrow{OD}$. If you never see a correct form and they go direct to $9r^2 + 49 + r^2 + 1$..... then M0.
	$\Rightarrow r = \frac{30}{7}$	**A1**	Correct only. Ignore $r = 0$ if seen.
	$\overrightarrow{OD} = \frac{76}{7}\mathbf{i} + \frac{37}{7}\mathbf{j} - \frac{32}{7}\mathbf{k}$	**A1**	Must be a vector. Condone if also have $\overrightarrow{OD} = \overrightarrow{OA}$.
		5	

Question	Answer	Marks	Guidance
9(a)	State $z\omega = \left(-3 + 3\sqrt{3}\right) + \left(3 + 3\sqrt{3}\right)\mathrm{i}$	**B1**	Or exact equivalent with real and imaginary parts collected. Need brackets around the coefficient of i. Allow for *a* =, *b* = stated correctly.
		1	

Question	Answer	Marks	Guidance
9(b)	Obtain $\lvert z\rvert = \sqrt{2}$	**B1**	
	Obtain $\arg z = -\frac{\pi}{4}$ final answer	**B1**	
	Obtain $\lvert \omega\rvert = 6$	**B1**	
	Obtain $\arg \omega = \frac{2\pi}{3}$ final answer	**B1**	
		4	
9(c)			Note: The question does not require the diagram. If they use $\frac{5\pi}{12}$ they need to demonstrate where it comes from. Complex number equivalent to *AB* is $3\sqrt{3}+3\text{i}$.
	Show $\lvert OA\rvert = \lvert AB\rvert = 6,$ hence isosceles	**B1**	One mark for 'isosceles' and one mark for 'right angle'. There will be alternatives e.g. use of Pythagoras (ratio of lengths is $1:1:\sqrt{2}$), expressing each number in "vector" form and using scalar product or explaining the effect of multiplying by 1 – i.
	$\angle AOB = \arg \omega - \arg z\omega = -\arg z = \frac{\pi}{4}$ hence third angle is a right angle	**B1**	
		2	

Question	Answer	Marks	Guidance
9(d)	$\arg z\omega = \arg z + \arg \omega \left(= \frac{2\pi}{3} - \frac{\pi}{4} = \frac{5\pi}{12}\right)$	**M1**	For showing correct use of their angles from part **(b)**. Must demonstrate where $\frac{5\pi}{12}$ comes from.
	$\arg z\omega = \tan^{-1}\dfrac{3+3\sqrt{3}}{-3+3\sqrt{3}}$	**M1**	Correct method for *their* $z\omega$ from part **(a)**. Must link to point B on diagram or to $\arg z\omega$. Need to see $\tan^{-1}\dfrac{3+3\sqrt{3}}{-3+3\sqrt{3}}$ or $\tan\theta = \dfrac{3+3\sqrt{3}}{-3+3\sqrt{3}}$ and not just $\tan^{-1}\dfrac{1+\sqrt{3}}{-1+\sqrt{3}}$.
	$\Rightarrow \tan\left(\frac{5}{12}\pi\right) = \dfrac{\sqrt{3}+1}{\sqrt{3}-1}$	**A1**	Obtain given answer from full and correct working.
		3	

Question	Answer	Marks	Guidance
10(a)	Use of correct chain rule (and correct quotient rule) and $\cos^{-3}\theta$	**M1**	Obtain $k \times (\cos\theta)^{-4} \times \sin\theta$ or equivalent.
	$\dfrac{dy}{d\theta} = -3 \times -\sin\theta(\cos\theta)^{-4} = 3\sin\theta\sec^4\theta$ Must be expressed in the given form	**A1**	Obtain given answer from full and correct working (signs must be shown), but condone $\frac{d}{d\theta}\left(\sec^3\theta\right) = ...$ and $y'(\theta)$.
		2	

Question	Answer	Marks	Guidance
10(b)	Separate variables: $\int \frac{\sin\theta}{\cos^4\theta}\,\mathrm{d}\theta = \int \frac{(x+3)}{(x^2+9)}\,\mathrm{d}x$	**B1**	Or $\int \frac{3\sin\theta}{\cos^4\theta}\,\mathrm{d}\theta = \int \frac{3x+9}{x^2+9}\,\mathrm{d}x$. Condone missing integral signs or missing dx or dθ, but not both.
	Obtain $p\sec^3\theta\,(+A)$	**B1**	Correct form, p any constant but not 0.
	Use $\int \frac{3x+9}{x^2+9}\,\mathrm{d}x = \int\left(\frac{3x}{x^2+9}+\frac{9}{x^2+9}\right)\mathrm{d}x$ and obtain $q\ln\left(x^2+9\right)$ or $r\tan^{-1}\frac{x}{3}\,(+C)$.	***M1**	Might have one third of both sides. Alt: substitute $x = 3\tan\phi$ to obtain $q\int 1+\tan\phi\,\mathrm{d}\phi$; condone if have θ in place of ϕ in this method.
	Obtain $q\ln\left(x^2+9\right)$ and $r\tan^{-1}\frac{x}{3}\,(+C)$	**DM1**	Obtain $q\left(\phi \mp \ln(\cos\phi)\right)$ OE.
	Obtain $\sec^3\theta = \frac{3}{2}\ln\left(x^2+9\right)+3\tan^{-1}\frac{x}{3}\,(+C)$ or equivalent	**A1**	Or might see a third of both sides. Must have 2 different variables.
	Use $\theta = \frac{1}{3}\pi, x = 3$ in an equation including $p\sec^3\theta$, $q\ln\left(x^2+9\right)$ and $r\tan^{-1}\frac{x}{3}$ to evaluate the constant of integration	**M1**	Or as limits in a definite integral. Limits for ϕ are 0 and $\frac{1}{4}\pi$.
	Obtain constant = $8-\frac{3}{2}\ln 18-\frac{3}{4}\pi$	**A1**	OE, e.g. 1.308… to at least 3sf.
	Obtain $\cos\theta = 0.601$	**A1**	Accept AWRT 0.601.
		8	

CAMBRIDGE
International Education

Cambridge International A Level

MATHEMATICS **9709/33**

Paper 3 Pure Mathematics 3 **May/June 2024**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2024 series for most Cambridge IGCSE, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

This document consists of **21** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics-Specific Marking Principles

1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.

2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.

3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.

4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).

5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.

6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	Use law of the logarithm of product or quotient on each side	***B1**	Allow logs to any base, as well as decimals, throughout. $\ln 8^3 + \ln 8^{-6x}$ and $\ln 4 + \ln 5^{-2x}$. Allow for $\ln \frac{8^3}{4}$ and $\ln 8^{6x} - \ln 5^{2x}$. $(3 - 6x)\ln 8$ and $\ln 4 + \ln 5^{-2x}$ gains next DB1 as well.
	Use law of logarithm of a power involving x on ONE side, e.g. $\ln 8^3 + (-)6x\ln 8$ or $(3 - 6x)\ln 8$ or $(9-18x)\ln 2$ or $\ln 4 - 2x\ln 5$	**DB1**	**SC** If *B0 DB0, then allow **B1** (1/4) for a correct logarithm law seen anywhere.
	Obtain a correct linear equation in x, e.g. $(3-6x)\ln 8 = (9-18x)\ln 2 = \ln 4 - 2x\ln 5$	**B1**	If in decimals, allow small errors in 2nd and 3rd dp.
	Obtain answer $x = 0.524$	**B1**	3dp required. No working scores 0/4 marks. After *B1 DB1 to correct answer with no more log working seen, then **SC B1** for $x = 0.524$. Maximum 3/4 possible.
	Alternative Method for Question 1		
	Use laws of indices to get to $a = b^{\pm 2x}$ or $c^{\pm x}$ in a correct form so now only ONE log power law required	**(B2)**	$(8^3/4)$ and $(5/8^3)^{-2x}$ or $(5^2/8^6)^{-x}$ opposite sides or $(4/8^3)$ and $(8^3/5)^{-2x}$ or $(8^6/5^2)^{-x}$ opposite sides
	Obtain a correct linear equation in x, e.g. $\ln\frac{8^3}{4} = 2x\ln\frac{8^3}{5}$	**(B1)**	$-2x\ln(5/8^3)$ or $2x\ln(8^3/5)$ or $x\ln(8^6/5^2)$ or $-x\ln(5^2/8^6)$. **SC**: If B0 then allow **B1** (1/4) for a correct term seen anywhere. If in decimals, allow small errors in 2nd and 3rd dp.
	Obtain answer $x = 0.524$	**(B1)**	3dp required. No working scores 0/4 marks. From the first line to correct answer with no log working seen, then B2 and **SC B1** for $x = 0.524$. Maximum 3/4 possible.

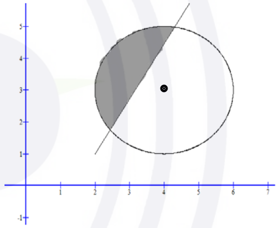
Question	Answer	Marks	Guidance
1	**Alternative Method 2 for Question 1**		
	Use laws of indices to get to any correct form with indices combined so now TWO log power laws are required	**(*B1)**	Allow 2^{7-18x} and 5^{-2x} on opposite sides or 2^{9-18x} and $2^{2-4.64x}$ on opposite sides.
	Use law of logarithm of a power involving x on ONE side, e.g. $(7-18x)\ln 2 = \ln 5^{-2x}$ or $\ln 2^{7-18x} = -2x\ln 5$ or … Allow $7-18x\ln 2$ or $9-18x\ln 2$	**(DB1)**	e.g. $(7-18x)\ln 2$ or $\left(9-18x\right)\ln 2$ or $-2x\ln 5$ or $(2-4.64x)\ln 2$ **SC:** If *B0 DB0 then allow **B1** (1/4) for a correct term seen anywhere. E.g. any term in *B1 shown above.
	Obtain a correct linear equation in x, e.g. $(7-18x)\ln 2 = -2x\ln 5$ or $(9-18x)\ln 2 = (2-4.64x)\ln 2$	**(B1)**	If in decimals, allow small errors in 2nd and 3rd dp.
	Obtain answer $x = 0.524$	**(B1)**	3dp required. No working scores 0/4 marks. From the first line to correct answer with no log working seen, then *B1 and **SC B1** for $x = 0.524$. Maximum 2/4 possible.
		4	

Question	Answer	Marks	Guidance
2	Use correct product rule $\cos 2x$ may be $1 - 2\sin^2 x$ or …	**M1**	$ae^{2x}\sin 2x + e^{2x}b\cos 2x$. Need a or $b = 2$. Allow M1 if only error is e^{x} instead of e^{2x} in one of terms, then maximum 1/5.
	Obtain correct derivative $2e^{2x}\sin 2x + 2e^{2x}\cos 2x$	**A1**	OE, e.g. $4e^{2x}\sin x\cos x + 2e^{2x}\left(\cos^2 x - \sin^2 x\right)$.
	Equate derivative of the form $ae^{2x}\sin 2x + e^{2x}b\cos 2x$ to 0 and solve for $2x$ or x using a correct method Note may have substituted for $\sin 2x$ and/or $\cos 2x$	**M1**	Obtain $2x = \tan^{-1}(-\textit{their } b/\textit{their } a)$ OE. Allow one slip in rearranging. Allow degrees. Variety of other methods available, such as solving quadratic equation in $\sin x$ or $\tan x$ e.g. $\tan^2 x - 2\tan x - 1 = 0$ leading to $x = \tan^{-1}(1+\sqrt{2})$.
	Obtain $x = \frac{3}{8}\pi$ only or exact equivalent	**A1**	CWO 67.5° gets A0. Ignore any answers outside interval $0 \leqslant x \leqslant \frac{\pi}{2}$.
	Obtain $y = \frac{1}{2}\sqrt{2}e^{\frac{3}{4}\pi}$ only or exact simplified equivalent	**A1**	CWO, ISW. Not $\sin\left(\frac{3}{4}\pi e^{\frac{3}{4}\pi}\right)$. Ignore any answers using x outside interval $0 \leqslant x \leqslant \frac{\pi}{2}$.
		5	

Question	Answer	Marks	Guidance
3	Square $x + \mathrm{i}y$ obtaining three terms when simplified and equate real and imaginary parts to 24 and −7 respectively	**M1**	Having used $\mathrm{i}^2 = -1$.
	Obtain equations $x^2 - y^2 = 24$ and $2xy = -7$	**A1**	Allow $2xy\mathrm{i} = -7\mathrm{i}$.
	Eliminate one variable by correct method and find a horizontal equation in the other	**M1**	All powers of x or y are positive and are in the numerator.
	Obtain $4x^4 - 96x^2 - 49 = 0$ or $4y^4 + 96y^2 - 49 = 0$ or 3-term equivalents	**A1**	
	Obtain answers $\frac{7\sqrt{2}}{2}-\frac{\sqrt{2}}{2}\mathrm{i}$ and $-\frac{7\sqrt{2}}{2}+\frac{\sqrt{2}}{2}\mathrm{i}$ or exact equivalents and no others	**A1**	E.g. $\pm\left(\frac{7\sqrt{2}}{2}-\frac{\sqrt{2}}{2}\mathrm{i}\right)$, but not $\pm\left(\frac{7\sqrt{2}}{2}\mp\frac{\sqrt{2}}{2}\mathrm{i}\right)$ or $\left(\pm\frac{7\sqrt{2}}{2}\mp\frac{\sqrt{2}}{2}\mathrm{i}\right)$. Allow coordinates or $x =$..., $y =$… paired correctly. ISW converting to different form. Must simplify $\sqrt{49}$.
		5	

Question	Answer	Marks	Guidance
4	State or imply that $\ln k + \ln y = cx$ or $\ln y = cx + \ln \frac{1}{k}$ etc.	**B1**	Allow $\ln k + \ln y = cx \ln e$
	Carry out a completely correct method for finding $\ln k$ or c	**M1**	Equations must have been formulated correctly.
	Obtain value $c = 0.80$	**A1**	AWRT Allow 0.8 for 0.80. Not a fraction. Accept in the equation $ky = e^{cx}$.
	Obtain value $k = 6.5$	**A1**	AWRT Not a fraction. Accept in the equation $ky = e^{cx}$.

Question	Answer	Marks	Guidance
5	State or imply the form $A+\frac{B}{x-1}+\frac{C}{2x+1}$	**B1**	
	Use a correct method for finding a constant	**M1**	Correct appropriate method.
	Obtain one of $A = 3$, $B = 2$ and $C = –3$	**A1**	
	Obtain a second value	**A1**	
	Obtain a third value	**A1**	
	Alternative Method for Question 5		
	Divide numerator by denominator to reach $A = 3$	**(M1)**	May be implied by $3\ [+]\ \frac{ax+b}{(x-1)(2x+1)}$ with a and b not both 0.
	Obtain $3+\frac{x+5}{(x-1)(2x+1)}$	**(A1)**	
	State or imply the form $\frac{D}{x-1}+\frac{E}{2x+1}$	**(B1)**	
	Obtain one of $D = 2$ and $E = -3$	**(A1)**	
	Obtain a second value	**(A1)**	
		5	

Question	Answer	Marks	Guidance
6(a)	Show a circle centre (4, 3) Allow dashes for coordinates on axes	**B1**	Note full circle is not required but must show centre and include relevant arc.
	Show a circle with radius 2. Can be implied by at least two of the points (2, 3), (6, 3), (4, 1) and (4, 5) being correct	**B1FT**	FT centre not at the origin.
	Point representing (2, 1)	**B1**	Half-line or 'correct' full line extending into the third quadrant implies point (2, 1).
	Show a half-line at their (2, 1) at an angle of $\frac{1}{3}\pi$, cutting top of circle between $x = 3$ and $x = 5$	**B1FT**	FT the point (±2, ±1) or (±1, ±2).
	Shade the correct region Needs correct half-line or "correct" full line extending into the third quadrant **AND** correct circle	**B1**	
		5	
6(b)	Carry out a correct method for finding the greatest value of arg z in the correct region in (a)	**M1**	E.g. $\sin^{-1}(2/\sqrt{(25)}) + \tan^{-1}(3/4)$ or $\sin^{-1}(2/\sqrt{(25)}) + \sin^{-1}(3/5)$. Or, e.g., substitute $y = kx$ in circle equation, solve when discriminant = 0, to get $\tan^{-1}\left(\frac{6+\sqrt{21}}{6}\right)$.
	Obtain answer 1.06, or 1.05 or 1.055 or 1.056 or 60.4° or 60.5°	**A1**	The marks in **(b)** are available even if errors in **(a)**. No working seen scores 0/2 marks.
		2	

Question	Answer	Marks	Guidance
7(a)	Show $8 \times (-7)^3 + 54 \times (-7)^2 - 17 \times (-7) - 21 = 0$ This is sufficient if no errors seen. $[-2744 + 2646 + 119 - 21 = 0]$ Or complete division of $8x^3 + 54x^2 - 17x - 21$ by $x + 7$ to get quotient $8x^2 - 2x - 3$ and remainder of 0 Or state $(x + 7)(8x^2 - 2x - 3)$ is sufficient Factors must be stated again in (b) to collect marks there	**B1**	No errors allowed. Correct division: $\begin{array}{r\|rrrr} & 8x^2 & -2x & -3 & \\ \hline x+7 & 8x^3 & +54x^2 & -17x & -21 \\ & 8x^3 & +56x^2 & & \\ \hline & & -2x^2 & -17x & \\ & & -2x^2 & -14x & \\ \hline & & & -3x & -21 \\ & & & -3x & -21 \end{array}$
		1	
7(b)	Commence division and reach partial quotient of the form $8x^2 \pm 2x$ or $8x^3 + 54x^2 - 17x - 21 = (x + 7)(Ax^2 + Bx + C)$ and reach $A = 8$ and $B = \pm 2$ or $C = -3$	**M1**	Condone no visible working.
	Obtain quotient $8x^2 - 2x - 3$ with no errors seen Stating $(x + 7)(8x^2 - 2x - 3)$ is sufficient	**A1**	Division can terminate with 0 or $-3x - 21$ stated once or twice. The working of division and finding quotient may be seen in **(a)** but results required here to collect marks.
		2	

Question	Answer	Marks	Guidance
7(c)	Solve quadratic from **(b)** to obtain a value for $\theta = \cos^{-1}\left(\frac{-1}{2}\right)$ or $\cos^{-1}\left(\frac{3}{4}\right)$	**M1**	$(x + 7)(8x^2 - 2x - 3) = (x + 7)(4x - 3)(2x + 1) = 0$ $x = \cos\theta = \frac{2 \pm \sqrt{4+96}}{16} = -\frac{1}{2}$ and $\frac{3}{4}$.
	Obtain one answer, e.g. $\theta = 120°$	**A1**	
	Obtain three further answers, e.g. $\theta = 240°$, 41.4° and 318.6° (condone 319°) and no others in the interval	**A1**	Accept more accurate answers. Answers in radians, maximum 2/3.
		3	

Question	Answer	Marks	Guidance
8(a)	State $R = \sqrt{12}$ or exact equivalent	**B1**	ISW
	Use trig formula to find α	**M1**	Allow $\alpha = 30°$ or $\tan^{-1}\left(\frac{\pm\sqrt{3}}{3}\right)$ or $\cos^{-1}\left(\pm\frac{\sqrt{3}}{2}\right)$ or $\sin^{-1}\left(\pm\frac{1}{2}\right)$ Allow M1 if $-\tan^{-1}\left(\frac{\sqrt{3}}{3}\right)$ etc. NB: If $\cos\alpha = 3$ and $\sin\alpha = \sqrt{3}$ seen then M0 A0.
	Obtain $\alpha = \frac{1}{6}\pi$	**A1**	CWO, so A0 if from $\tan^{-1}\left(-\frac{\sqrt{3}}{3}\right)$.
		3	

Question	Answer	Marks	Guidance
8(b)	Express integral in the form $A\int \sec^2(2x+...)\,dx$ or $A\int \sec^2(2x-...)\,dx$	**B1FT**	FT α from **(a)**.
	Integrate and reach $B\tan(2x+...)$ or $B\tan(2x-...)$	**B1FT**	FT α from **(a)**. Where $B = A$ or $2A$ or $0.5A$.
	Obtain $\frac{1}{8}\tan(2x+...)$	**B1FT**	OE FT α from **(a)**. Allow $\frac{1}{8}$ as $\frac{1}{4}\times\frac{1}{2}$. Coefficient must be correct.
	Use limits of $x=0$ and $x=\frac{1}{12}\pi$ in the correct order in expression of form $B\tan(2x\pm...)$ so $B\tan\left(\frac{\pi}{6}+...\right)-B\tan(...)$ or $B\tan\left(\frac{\pi}{6}-...\right)-B\tan(-...)$	**M1**	Allow with tan still present. FT α from **(a)**. **SC:** B1 $\frac{\sqrt{3}}{12}$ OE after $\frac{1}{8}\tan\left(2x+\frac{1}{6}\pi\right)$ with no working.
	Obtain answer $\frac{1}{12}\sqrt{3}$ or $\frac{1}{4\sqrt{3}}$ or $\frac{1}{\sqrt{48}}$ or single term exact equivalent	**A1**	$\frac{1}{8}\left(\sqrt{3}-\frac{1}{\sqrt{3}}\right)=\frac{1}{8}\left(\frac{3-1}{\sqrt{3}}\right)$ needs simplifying.
		5	Note: allow all marks in **(b)** even if $\alpha=\frac{1}{6}\pi$ found by an incorrect method in **(a)**.

Question	Answer	Marks	Guidance
9(a)	Obtain $\frac{\mathrm{d}V}{\mathrm{d}t}=[\pm]\frac{k}{t}$ or $\frac{\mathrm{d}V}{\mathrm{d}t}=[\pm]\frac{1}{kt}$	**B1**	
	Obtain $\frac{\mathrm{d}V}{\mathrm{d}x}=20x-3x^2$	**B1**	
	Correct use of chain rule involving k	**M1**	Use $\frac{\mathrm{d}V}{\mathrm{d}t}=\frac{\mathrm{d}V}{\mathrm{d}x}\times\frac{\mathrm{d}x}{\mathrm{d}t}$. Expressions for $\frac{\mathrm{d}V}{\mathrm{d}t}$ and $\frac{\mathrm{d}V}{\mathrm{d}x}$ must be seen to get M1.
	Obtain $\frac{\mathrm{d}x}{\mathrm{d}t}=[\pm]\frac{k}{t\left(20x-3x^2\right)}$ or equivalent,	**A1**	If this expression is first seen with numerical values, allow A1 when their value of k is substituted back into the general expression.
	Use $t=\frac{1}{10}$, $x=\frac{1}{2}$ and $\frac{\mathrm{d}x}{\mathrm{d}t}=-\frac{20}{37}$ to obtain given answer which must be stated $\frac{\mathrm{d}x}{\mathrm{d}t}=-\frac{20}{37}$ needed to score final A1	**A1**	$\frac{\mathrm{d}x}{\mathrm{d}t}=\frac{-1}{2t\left(20x-3x^2\right)}$ AG Need to at least see $-\frac{20}{37}=\frac{k}{\frac{1}{10}\left(10-\frac{3}{4}\right)}$ if $\frac{k}{t}$ or $-\frac{20}{37}=\frac{-k}{\frac{1}{10}\left(10-\frac{3}{4}\right)}$ if $-\frac{k}{t}$ in working for correct k. $\frac{\mathrm{d}x}{\mathrm{d}t}=\frac{20}{37}$ seen anywhere, then A0.
		5	

Question	Answer	Marks	Guidance
9(b)	Separate variables correctly & integrate at least one side correctly	**B1**	
	Obtain terms $10x^2 - x^3$	**B1**	May see $-10x^2 + x^3$ if negative sign moved across or e.g. $20x^2 - 2x^3$ if 2 moved across. Allow $\frac{20x^2}{2} - \frac{3x^3}{3}$.
	Obtain term $\ln t$ with 'correct' coefficient from their separation of variables, for example $a \ln t$ for $\frac{a}{t}$.	**B1FT**	FT sign and position of 2 from their separation but B0 if error from later manipulation.
	Use $t = \frac{1}{10}$, $x = \frac{1}{2}$ to evaluate a constant or as limits in a solution containing terms of the form x^2, x^3 and $\ln t$ (or $\ln 2t$)	**M1**	Allow numerical and sign errors and decimals. Allow if exponentiate before substitution, even if exponentiation done incorrectly, allow for c or e^c.
	Obtain correct answer in any form, for example $10x^2 - x^3 = -\frac{\ln t}{2} + \frac{19}{8} + \frac{\ln 0.1}{2}$	**A1**	$10x^2 - x^3 = -\frac{\ln 2t}{2} + \frac{19}{8} + \frac{\ln 0.2}{2}$ or $10x^2 - x^3 = -\frac{\ln t}{2} + 2.5 - 0.125 - 1.15\ldots$ Allow 1.14 to 1.16 for 1.15 and allow 2.44 to 2.46 for 2.45
	Obtain answer $t = \frac{1}{10}e^{2x^3 - 20x^2 + \frac{19}{4}}$ or equivalent	**A1**	ISW Need $t = \ldots\ldots\ldots$ E.g. $\frac{0.1}{e^{20x^2 - 2x^3 - \frac{19}{4}}}$, $\frac{e^{2x^3 + \frac{19}{4}}}{10e^{20x^2}}$, $\frac{1}{10}e^{\frac{19}{4}}e^{2x^3 - 20x^2}$. Allow decimals, allow 2.44 to 2.46 for 2.45, e.g. $e^{2x^3 - 20x^2 + 2.45}$. A0 if $e^{\ln\frac{1}{10}}$ present in final answer.
		6	

Question	Answer	Marks	Guidance
10(a)	Carry out correct process for evaluating the scalar product of direction vectors	***M1**	$\left[\begin{pmatrix}3\\4\\a\end{pmatrix}.\begin{pmatrix}-1\\2\\2\end{pmatrix} \text{ or } \begin{pmatrix}3\\4\\a\end{pmatrix}.\begin{pmatrix}1\\-2\\-2\end{pmatrix}\right]$ $3(-1) + 4(2) + 2a$ or $-3 + 8 + 2a$ or $5 + 2a$. Allow one slip in unsimplified form.
	Using the correct process for the moduli, divide the scalar product by the product of the moduli and equate to $\pm\frac{\sqrt{2}}{2}$, or equate the scalar product to the product of the moduli and $\pm\frac{\sqrt{2}}{2}$	***M1**	*M1 marks independent of each other, so *M0 *M1 for failure to use both direction vectors, but must be using scalar product and same 2 vectors throughout. Allow $\frac{\sqrt{2}}{2}$ or $-\frac{\sqrt{2}}{2}$ throughout question.
	State a correct equation in any form, e.g. $\frac{5+2a}{3\sqrt{25+a^2}} = [\pm]\frac{\sqrt{2}}{2}$ Allow unsimplified as in guidance	**A1**	$\frac{5+2a}{\sqrt{9+16+a^2}\sqrt{1+4+4}} = [\pm]\frac{\sqrt{2}}{2}$ OE E.g. $5 + 2a = [\pm]\frac{\sqrt{2}}{2}\sqrt{9+16+a^2}\sqrt{1+4+4}$ If moduli initially correct but later has errors, award A1 when using $\frac{\sqrt{2}}{2}$ or $\pm\frac{\sqrt{2}}{2}$ or $-\frac{\sqrt{2}}{2}$.
	Form a quadratic equation in *a* with 3 or more terms all on one side and solve for *a*. DM1 depends on BOTH *M1	**DM1**	Must square $(5 + 2a)$ to get 3 terms and must remove square roots from both terms on other side. $25 + 20a + 4a^2 = \frac{9}{2}(25 + a^2)$ $a^2 - 40a + 175 = 0$ hence $(a - 5)(a - 35) = 0$.
	Obtain $a = 5$ and $a = 35$	**A2**	A1 for each, working not needed if quadratic correct.
		6	

Question	Answer	Marks	Guidance
10(b)	Express general point of at least one line correctly in component form, i.e. $\begin{pmatrix} 1+3\lambda \\ 1+4\lambda \\ 2a+a\lambda \end{pmatrix}$ or $\begin{pmatrix} -3-\mu \\ -1+2\mu \\ 4+2\mu \end{pmatrix}$	**B1**	Often the third point on the line occurs after M1 A1 is gained.
	Equate at least two pairs of corresponding components and solve for λ or μ or a	**M1**	If solve for a first, they must have a complete method to eliminate both λ and μ. If using a to solve for λ or for μ, a must have been found from a valid method.
	Obtain $\lambda = -1$ or $\mu = -1$	**A1**	
	Obtain $a = 2$	**A1**	
	Obtain position vector of the point of intersection is $-2\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$ Two different answers for point of intersection scores A0 even if one is correct	**A1**	Accept coordinates, row or column, but not **(**$-2\mathbf{i}, -3\mathbf{j}, +2\mathbf{k}$**)** or $\begin{pmatrix} -2\mathbf{i} \\ -3\mathbf{j} \\ 2\mathbf{k} \end{pmatrix}$ but ISW after correct form seen.
		5	

Question	Answer	Marks	Guidance
11	State or imply $2\mathrm{d}x = \sec^2\theta\mathrm{d}\theta$ or $(1 + 4x^2)\mathrm{d}\theta$ or equivalent e.g. $\frac{\mathrm{d}x}{\mathrm{d}\theta} = 0.5\sec^2\theta$ or $\frac{\mathrm{d}\theta}{\mathrm{d}x} = 2\cos^2\theta$	**B1**	$2\mathrm{d}x = \sec^2\boldsymbol{x}\ \mathrm{d}\theta$ seen **B0**. Allow BOD cancellation of $\sec^2 x$ and $\sec^2\theta$ and all remaining marks, so 8/9 possible.
	Use $1 + \tan^2\theta = \sec^2\theta$ in $\mathrm{d}x$ or in $\frac{1}{\left(1+4x^2\right)^2}$	**B1**	Or $1+\frac{\sin^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$. Must substitute for $\mathrm{d}x$ or no more marks possible.
	Use $\cos^2\theta = \frac{1}{2}(1+\cos 2\theta)$	**B1**	$\mathrm{d}\theta$ seen once is sufficient. If never seen, this B1 mark is not available; max 8/9 possible.
	Obtain integral of the form $\int p(1+\cos 2\theta)\ \mathrm{d}\theta$	**M1**	Any non-zero p.
	Obtain $\int 3(1+\cos 2\theta)\ \mathrm{d}\theta$	**A1**	Allow $6\times\frac{1}{2}$.
	Integrate to obtain $p\theta + q\sin 2\theta$	***M1**	Any non-zero q but same p as before.
	Obtain correct $3\theta + \frac{3}{2}\sin 2\theta$	**A1**	
	Use limits $\theta = \frac{\pi}{4}$ and 0 correctly in an expression of the form $p\theta + q\sin 2\theta$ or $x = \frac{1}{2}$ and 0 in an appropriate expression in terms of x	**DM1**	$p\frac{\pi}{4} + q\sin\frac{\pi}{2} - [p.0 + q.0]$ May be implied.

Question	Answer	Marks	Guidance
11	Obtain $\frac{3}{2}+\frac{3}{4}\pi$ or exact equivalent e.g. $\frac{3}{4}\pi+\frac{3}{2}$ but not $3\left(\frac{1}{2}+\frac{1}{4}\pi\right)$	**A1**	Must be in the form *a* + *b*π where are *a* and *b* are rational numbers, but ISW after correct form seen (e.g. 1.5 + 0.75π scores A1).
		9	

CAMBRIDGE
International Education

Cambridge International A Level

MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **February/March 2024**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the February/March 2024 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

This document consists of **18** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles

1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.

2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.

3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.

4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).

5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.

6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

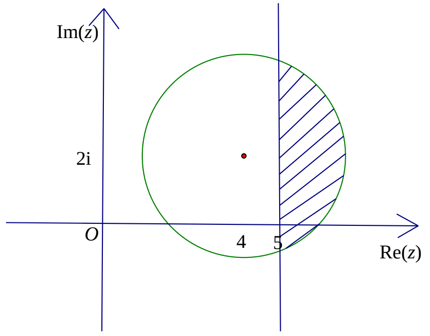
AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	Commence division and reach partial quotient of the form $x^2 \pm 3x$ or $x^4 - 3x^3 + 9x^2 - 12x + 27 = \left(x^2+5\right)\left(Ax^2 + Bx + C\right) + Dx + E$ or $Ax^4 + Bx^3 + (5A + C)x^2 + 5Bx + 5C$ and reach $A = 1$ and $B = \pm 3$	**M1**	
	Obtain quotient $x^2 - 3x + 4$	**A1**	$A = 1$, $B = -3$ [$5A + C = 9$ so $C = 4$; $5B + D = -12$ so $D = 3$; $5C + E = 27$ so $E = 7$]. A pair of incorrect statements ‘remainder $x^2 - 3x + 4$’ and ‘quotient $3x + 7$’ score M1A1A0.
	Obtain remainder $3x + 7$	**A1**	
	$\begin{array}{lllllll} & x^2 & - & 3x & + & 4 & \\ x^2+5 & x^4 & - & 3x^3 & + & 9x^2 - 12x + 27 & \\ & x^4 & & & + & 5x^2 & \\ & & - & 3x^3 & + & 4x^2 & \\ & & - & 3x^3 & & & -15x \\ & & & & + & 4x^2 + 3x & \\ & & & & + & 4x^2 & +20 \\ & & & & + & 3x & +7 \end{array}$	**3**	

Question	Answer	Marks	Guidance
2(a)	State unsimplified term in x, or its coefficient, in the expansion of $(4-x)^{\frac{1}{2}}$	**B1**	$4^{\frac{1}{2}}\times\frac{1}{2}\times\left(\frac{-x}{4}\right)=\frac{-x}{4}$.
	State unsimplifed term in x^2, or its coefficient, in the expansion of $(4-x)^{\frac{1}{2}}$	**B1**	$4^{\frac{1}{2}}\times\frac{\frac{1}{2}\times\frac{-1}{2}}{2}\times\left(\frac{-x}{4}\right)^2=\frac{-x^2}{64}$. Allow $\left(\frac{x}{4}\right)^2$.
	Multiply by $(2x-5)$ and obtain 2 terms in x^2, allow even if errors in $4^{\frac{1}{2}}$, signs, etc.	**M1**	Allow unsimplified 2x. $4^{\frac{1}{2}}\times\frac{1}{2}\times\left(\frac{-x}{4}\right)-5$. $4^{\frac{1}{2}}\times\frac{\frac{1}{2}\times\frac{-1}{2}}{2}\times\left(\frac{-x}{4}\right)^2$. Allow $\left(\frac{x}{4}\right)^2$. $2x\times\left(\frac{-x}{4}\right)(-5)\times\frac{-x^2}{64}$ or $2\times\left(\frac{-1}{4}\right)(-5)\times\left(\frac{-1}{64}\right)$.
	Obtain $-\frac{27}{64}$ or –0.421875 or $-\frac{54}{128}$	**A1**	Allow in a full expansion up to x^2, ignore extra terms even if they contain errors.
		4	
2(b)	$\lvert x\rvert<4$	**B1**	or $-4<x<4$.
		1	

Question	Answer	Marks	Guidance
3(a)	Obtain $r = 4$	**B1**	$\lvert z\rvert = \sqrt{\left(\left(-\sqrt{3}\right)^2 + 1^2\right)}$ so $r = \lvert z\rvert^2 = \left(-\sqrt{3}\right)^2 + 1^2$.
	Correct method for the argument	**M1**	$\theta = 2\tan^{-1}\left(\frac{-\sqrt{3}}{1}\right)$ or $2 \times \frac{5\pi}{6}$.
	Obtain $\theta = -\frac{\pi}{3}$	**A1**	Arg with no working **B1** instead of **M1 A1**. **A0** if decimals. Allow separate mod and arg to gain full marks
	Alternative solution for Question 3(a)		
	$z^2 = 2 - 2\sqrt{3}\,\mathrm{i}$ so $r = \sqrt{2^2 + \left(-2\sqrt{3}\right)^2} = 4$	**B1**	
	Correct method for the argument	**M1**	$\arg z^2 = \tan^{-1}\frac{-2\sqrt{3}}{2}$
	Obtain $\theta = -\frac{\pi}{3}$	**A1**	Arg with no working **B1** instead of **M1 A1**. **A0** if decimals. Allow separate mod and arg to gain full marks
		3	
3(b)	Use of $\alpha + their\,\theta = 0$ or $\alpha + their\,\theta = -\pi$ or $\alpha + their\,\theta = \pi$	**M1**	Seen or implied. Using their θ or new value calculated in **(b)**.
	Use of $R = \frac{their\ \mathrm{r}}{12}$	**M1**	Seen or implied.
	Obtain $\frac{1}{3}\mathrm{e}^{-\mathrm{i}\frac{2\pi}{3}}$ and $\frac{1}{3}\mathrm{e}^{\mathrm{i}\frac{\pi}{3}}$	**A1**	
		3	

Question	Answer	Marks	Guidance
4	Obtain $\ln p - \ln q = a$	**B1**	$\frac{p}{q} = e^a$.
	Obtain $\ln p + 2\ln q = b$	**B1**	$pq^2 = e^b$.
	Completed method to obtain $\ln\left(p^7 q\right)$	**M1**	E.g. $\ln q = \frac{b-a}{3}$, $\ln p = \frac{2a+b}{3}$ and attempt $7\ln p + \ln q$. All exponentials must be removed to obtain **M1**.
	Obtain $\frac{13a+8b}{3}$	**A1**	
	Alternative solution for Question 4		
	State $p^7 q = \left(\frac{p}{q}\right)^x \left(q^2 p\right)^y$	**B1**	Or $\ln p^7 q = x\ln\frac{p}{q} + y\ln q^2 p$.
	Equate indices to form simultaneous equations in x and y, can have errors	**M1**	$x + y = 7$ and $- x + 2y = 1$.
	Obtain $7 = x + y$ and $1 = 2y - x$	**A1**	Leading to $x = \frac{13}{3}$, $y = \frac{8}{3}$.
	Evaluate $x \times a + y \times b$ to obtain $\frac{13a+8b}{3}$	**A1**	
		4	

Question	Answer	Marks	Guidance
5(a)	Show a circle with centre $4+2\mathrm{i}$	**B1**	
	Show a circle with radius 3 and centre not at the origin	**B1**	
	Show the straight line $\mathrm{Re}(z)=5$	**B1**	
	Shade the correct region Allow even if radius 3 mark not gained or shown incorrectly	**B1**	
		4	If 4 and 6 seen on diagram and line is at mid point, but 5 not marked, allow final two B1 marks.
5(b)	Carry out a complete method for finding the greatest value of arg z	**M1**	e.g. $\tan^{-1}\dfrac{2+2\sqrt{2}}{5}$. Allow $2\sqrt{2}$ as $\sqrt{(3^2-1^2)}$.
	Obtain answer 0.768 radians or $44.0°$	**A1**	
		2	**SC B1** $\tan^{-1}(2/4)+\sin^{-1}(3/\sqrt{(4^2+2^2)})=26.565°+42.130°=68.695°$ 68.7° or [1.19896] 1.20 radians.

Question	Answer	Marks	Guidance
6(a)	State or imply $4y\frac{\mathrm{d}y}{\mathrm{d}x}$ as the derivative of $2y^2$	**B1**	**SC** If $\frac{\mathrm{d}y}{\mathrm{d}x}$ introduced instead of $\frac{\mathrm{d}}{\mathrm{d}x}$ then allow **B1** for both, followed by correct method **M1** Max 2.
	State or imply $3y+3x\frac{\mathrm{d}y}{\mathrm{d}x}$ as the derivative of $3xy$	**B1**	Allow extra $\frac{\mathrm{d}y}{\mathrm{d}x}$ = correct expression to collect all marks if correct.
	Complete the differentiation, all 4 terms, isolate 2 $\frac{\mathrm{d}y}{\mathrm{d}x}$ terms on LHS or bracket $\frac{\mathrm{d}y}{\mathrm{d}x}$ terms and solve for $\frac{\mathrm{d}y}{\mathrm{d}x}$	**M1**	
	Obtain $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{2x-3y-1}{4y+3x}$	**A1**	Answer Given – need to have seen $4y\frac{\mathrm{d}y}{\mathrm{d}x}+3x\frac{\mathrm{d}y}{\mathrm{d}x}=2x-3y-1$ or $(4y+3x)\frac{\mathrm{d}y}{\mathrm{d}x}-2x+3y=-1$. Need to see = $2x$ or = 0 consistently throughout otherwise **M1 A0**. No recovery allowed. When all terms are included then must be an equation.
		4	Allow all marks if using dx and dy.
6(b)	Equate numerator to zero, obtaining $2x = 3y + 1$ or $3y = 2x - 1$ and form equation in x only or y only from $2y^2 + 3xy + x = x^2$	**M1***	e.g. $\frac{2}{9}(2x-1)^2+x(2x-1)+x=x^2$ or $2y^2+\frac{3}{2}(1+3y)y+\frac{1}{2}(1+3y)=\frac{1}{4}(1+3y)^2$. Allow errors.
	Obtain $\frac{2}{9}(2x-1)^2=-x^2$ or a 3 term quadratic in one unknown and try to solve. If errors in quadratic formulation allow solution, applying usual rules for solution of quadratic equation, and allow **M1**	**DM1**	e.g. $17x^2-8x+2=0$ $\left(b^2-4ac=-72\right)$ or $17y^2+6y+1=0$ $\left(b^2-4ac=-32\right)$. $x = 4/17 \pm (3\sqrt{2}/17)\mathrm{i}$, $y = -3//17 \pm (2\sqrt{2}/17)\mathrm{i}$.
	Conclude that the equation has no [real] roots	**A1**	Given Answer. CWO
		3	

Question	Answer	Marks	Guidance
7(a)	Use correct product rule	**M1**	
	Obtain correct derivative in any form	**A1**	e.g. $\frac{dy}{dx} = e^{2x} + 2xe^{2x} - 5$
	Equate derivative to zero and obtain $\alpha = \frac{1}{2}\ln\left(\frac{5}{1+2\alpha}\right)$	**A1**	Given answer – need to see $e^{2x} = 5/(1 + 2x)$ or $\ln e^{2x} = \ln (5/(1 + 2x))$ in working. Must be in terms of α not x. Allow α to be used before equating to 0.
		3	
7(b)	Calculate the value of a relevant expression or values of a pair of expressions at $x = 0.4$ and $x = 0.5$	**M1**	Need to attempt BOTH values and have one correct.
	Complete the argument correctly with correct calculated values	**A1**	e.g. $0.4 < 0.51[08]$ and $0.5 > 0.458$ or 0.46 or 0.45 or $-0.11[08] < 0$ and $0.042 > 0$ If use original derivative -0.994 (0.4) and 0.437 (0.5).
		2	
7(c)	Use the iterative process $\alpha_{n+1} = \frac{1}{2}\ln\left(\frac{5}{1+2\alpha_n}\right)$ correctly at least twice anywhere in iteration process	**M1**	Obtain one value and then substitute it into the formula to obtain a second value.
	Obtain final answer 0.47	**A1**	
	Show sufficient iterations to 4 d.p. to justify 0.47 to 2 d.p. or show there is a sign change in the interval $(0.465, 0.475)$	**A1**	$0.4, 0.5108, 0.4528, 0.4823, 0.4670, 0.4749$ $0.45, 0.4838, 0.4663, 0.4753, 0.4707, 0.4730$ $0.5, 0.4581, 0.4795, 0.4685, 0.4742$ Allow self correction.
		3	**SC B1** No working 0.47

Question	Answer	Marks	Guidance
8(a)	Use the correct expansion of $\cos\left(x+\frac{1}{4}\pi\right)$ to obtain $\sin x+2\cos x$	**B1**	$3\sin x+2\sqrt{2}\left(\frac{1}{\sqrt{2}}\cos x-\frac{1}{\sqrt{2}}\sin x\right)$.
	State $R=\sqrt{5}$	**B1 FT**	ISW FT *their* $a\sin x+b\cos x$ provided this expression obtained by correct method.
	Use correct trig formulae to find α	**M1**	$\alpha = \tan^{-1}(b/a)$ from *their* $a\sin x+b\cos x$ or $\sin^{-1}$ or $\cos^{-1}$ provided this expression obtained by correct method. NB If $\cos\alpha = 1$ and $\sin\alpha = 2$ then **M0 A0**.
	Obtain $\alpha=1.107$	**A1**	3 d.p. CAO Treat answer in degrees as a misread $(63.435°)$.
		4	
8(b)	$\sin^{-1}\left(\frac{1.5}{R}\right)$	**B1 FT**	Follow *their R*.
	Use a correct method to obtain an un-simplified value of θ with *their* α	**M1**	$2\left(\sin^{-1}\left(\frac{1.5}{R}\right)-\alpha\right)$ or $2\left(\pi-\sin^{-1}\left(\frac{1.5}{R}\right)-\alpha\right)$.
	Obtain one correct answer e.g. -0.74 in the interval	**A1**	
	Obtain second correct answer e.g. 2.60 (2.5986) or 4π – 0.74 = 11.8 or 2.60 − 4π = −9.97 in the interval	**A1**	If uses 1.11° withhold first accuracy mark gained, but allow rest of accuracy marks. Allow 2.6(0).
	Obtain two more correct answers e.g. -9.97 and 11.8 and no others in the interval	**A1**	Ignore answers outside the interval. Treat answers in degrees as a misread. $(-571.1°, -42.6°, 148.9°, 677.2°)$.
		5	

Question	Answer	Marks	Guidance
9(a)	Find the scalar product of a pair of adjacent sides	**M1**	$\overrightarrow{OA}$ = (5, − 2, 1), $\overrightarrow{OB}$ = (8, 2, − 6), $\overrightarrow{OC}$ = (3, 4, −7), $\overrightarrow{CB}$ = (5, −2, 1), $\overrightarrow{AB}$ = (3, 4, −7).
	Show that the sides are perpendicular	**A1**	e.g. $\overrightarrow{OA}.\overrightarrow{OC}=15-8-7=0$. Need to see working of numerator, ignore denominator.
	Compare a pair of opposite sides	**M1**	$\overrightarrow{OA}$ and $\overrightarrow{CB}$ or $\overrightarrow{OC}$ and $\overrightarrow{AB}$.
	Show that they are parallel and equal in length and hence *OABC* is a rectangle	**A1**	e.g. $\overrightarrow{AB}=\overrightarrow{AO}+\overrightarrow{OB}=3\mathbf{i}+4\mathbf{j}-7\mathbf{k}=\overrightarrow{OC}$. If show $\overrightarrow{AB}=3\boldsymbol{i}+4\boldsymbol{j}-7\boldsymbol{k}=\overrightarrow{OC}$, then **M1 A1** since this implies parallel and of equal length. If only show lengths equal **M1**. If repeat for other pair of opposite sides then **A1.**
	Alternative solution for Question 9(a)		
	Show the diagonals $\overrightarrow{OB}$ and $\overrightarrow{AC}$ are equal in length $\left(\sqrt{104}\right)$		$\overrightarrow{\mathbf{AC}}$ = (−2, 6, −8).
	Show the diagonals bisect each other at $(4,1,-3)$		$\dfrac{\overrightarrow{\mathbf{OB}}}{2}=\overrightarrow{\mathbf{OC}}+\dfrac{1}{2}(\overrightarrow{\mathbf{OA}}-\overrightarrow{\mathbf{OC}})$ = (4, 1, −3).
	Show the quadrilateral is a parallelogram		e.g. $\overrightarrow{OB}=\overrightarrow{OA}+\overrightarrow{OC}$.
	Show both pairs of opposite sides are equal in length and a pair of adjacent sides are perpendicular		
		4	Without calculation of scalar product max is **M1 A1**.

Question	Answer	Marks	Guidance
9(b)	$\overrightarrow{AC} = \pm(-2\mathbf{i} + 6\mathbf{j} - 8\mathbf{k})$ or $\dfrac{\overrightarrow{AC}}{2} = \pm(-1\mathbf{i} + 3\mathbf{j} - 4\mathbf{k})$	**B1**	Seen or implied using diagonals.
	Scalar product of a pair of relevant vectors	**M1**	e.g. $\overrightarrow{AC}.\overrightarrow{OB} = -16 + 12 + 48$.
	Using the correct process for the moduli, divide the scalar product by the product of the moduli and obtain the inverse cosine of the result.	**M1**	$\pm\cos^{-1}\left(\dfrac{44}{104}\right)$. For any two vectors.
	Obtain answer $65.(0)°$	**A1**	Accept 1.13 radians.
	Alternative solution for Question 9(b)		
	Scalar product of a pair of relevant vectors	**M1**	e.g. $\overrightarrow{OA}.\overrightarrow{OB} = 40 - 4 - 6$ using one side and a diagonal. or $\overrightarrow{OC}.\overrightarrow{OB}$ = 24 + 8 + 42. Must use scalar product.
	Using the correct process for the moduli, divide the scalar product by the product of the moduli and obtain the inverse cosine of the result. Any two vectors.	**M1**	$\pm\cos^{-1}\left(\dfrac{\sqrt{30}}{\sqrt{104}}\right)$ or $\cos^{-1}\left(\dfrac{\sqrt{74}}{\sqrt{104}}\right)$.
	Required angle = 180° – 2 × 57.5° or 180° – 2 × 32.5° = 115° and 180° − 115° or 2× 32.5°	**B1**	OE SOI Complete method to find the acute angle.
	Obtain answer 65.0°	**A1**	Accept 1.13 radians.
		4	

Question	Answer	Marks	Guidance
10(a)	State or imply the form $\frac{A}{2a+x}+\frac{B}{2a-x}+\frac{C}{5a-2x}$	**B1**	Allow if seen prior to assigning a value for *a*.
	Use a correct method for finding a coefficient	**M1**	
	Obtain one of $A=1, B=9, C=-16$	**A1**	
	Obtain a second value	**A1**	
	Obtain the third value	**A1**	
		5	**SC** $\frac{Dx+E}{4a\wedge 2-x\wedge 2}+\frac{C}{5a-2x}$ **B0 M1** and $C = -16$ **A1** Max 2/5. **SC** Allow **M1** only for other incorrect partial fraction.

Question	Answer	Marks	Guidance
10(b)	Integrate and obtain one of the terms $\ln\|2a+x\|-9\ln\|2a-x\|+8\ln\|5a-2x\|$	**B1 FT**	Condone missing modulus signs. Use *their A*, *B* and *C*.
	Obtain a second correct term	**B1 FT**	
	Obtain the third correct term	**B1 FT**	Max 3/5 if value is assigned for *a* (award M0 A0).
	Substitute limits correctly in an integral of the form $p\ln\|2a+x\|+q\ln\|2a-x\|++r\ln\|5a-2x\|$ and remove all a's	**M1**	Either (i) collect terms with same coeeficient and remove all *a*'s e.g. *p*ln 3*a* −*p*ln *a* + *q*ln *a* − qln 3*a* + *r*ln 3*a* – *r*ln7*a* hence *p*ln 3 − *q*ln 3 + *r*ln 3 − *r*ln7 or (ii) collect same ln terms and remove all *a*'s e.g. (*p* – *q* + *r*) ln 3*a* – (*p* – *q*) ln *a* – *r*ln7*a* and − (*p* − *q*) ln *a* = (−*p* + *q* − *r*) ln *a* + *r* ln *a* hence *p ln* 3 – *q ln*3 + *r*ln 3 – *r ln* 7.
	Obtain $18\ln3-8\ln7$ from correct working	**A1**	A0 if the solution involves logarithms of negative numbers.
		5	

Question	Answer	Marks	Guidance
11	Separate variables correctly	**B1**	$\int(1+y)\mathrm{e}^{-3y}\mathrm{d}y=\int\frac{1}{1+\cos 2\theta}\mathrm{d}\theta$. Allow $1/\mathrm{e}^{3y}$ and missing integral signs.
	Integrate to obtain $p(1+y)\mathrm{e}^{-3y}+\int q\mathrm{e}^{-3y}\mathrm{d}y$	**M1**	Allow unless clear evidence that formula used has a + sign.
	Obtain $\frac{-1}{3}(1+y)\mathrm{e}^{-3y}+\int\frac{1}{3}\mathrm{e}^{-3y}\mathrm{d}y$	**A1**	Allow unsimplified.
	Obtain $\frac{-1}{3}(1+y)\mathrm{e}^{-3y}-\frac{1}{9}\mathrm{e}^{-3y}(+A)$	**A1**	Condone no constant of integration.
	Use correct double angle formula to obtain $\int\frac{1}{2\cos^2\theta}\mathrm{d}\theta$	**B1**	
	Obtain $k\tan[+B]$	**B1**	Condone no constant of integration.
	Use $y=0,\ \theta=\frac{\pi}{4}$ to evaluate a constant of integration in an expression of the form $\alpha y\mathrm{e}^{-3y}, \beta\mathrm{e}^{-3y}$ and $\gamma\tan\theta$ only.	**M1***	$\frac{1}{2}=-\frac{1}{3}-\frac{1}{9}+C\quad\left(C=\frac{17}{18}\right)$ Allow $\alpha y\mathrm{e}^{3y}$ and $\beta\mathrm{e}^{3y}$. Must have integrated LHS twice.
	Use $y=1$	**DM1**	$\frac{-(1+1)}{3\mathrm{e}^3}-1\left(9\mathrm{e}^3\right)=\frac{1}{2}\tan\theta-\frac{17}{18}$. Must have integrated LHS.
	Obtain $\tan\theta=\frac{17}{9}-\frac{14}{9}\mathrm{e}^{-3}$	**A1**	Or exact equivalent . Exact ISW. Allow $\theta=\tan^{-1}\left(\frac{17}{9}-\frac{14}{9}\mathrm{e}^{-3}\right)$. If x instead of θ then withhold final **A1**.
		9	

Cambridge Assessment International Education

Cambridge International A Level

MATHEMATICS **9709/31**

Paper 3 Pure Mathematics 3 **October/November 2023**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2023 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

This document consists of **21** printed pages.

[Turn over

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles

1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.

2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.

3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.

4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).

5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.

6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

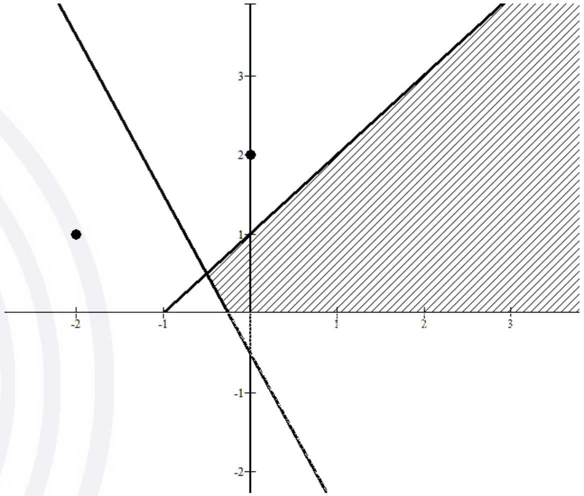
M	Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
A	Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
B	Mark for a correct result or statement independent of method marks.
DM or **DB**	When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
FT	Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	Use correct quotient or product rule	*M1	
	Obtain correct derivative in any form	A1	e.g. $\frac{(1-3x)2x-x^2(-3)}{(1-3x)^2}\left(=\frac{2x-3x^2}{(1-3x)^2}\right)$ or $3x^2(1-3x)^{-2}+2x(1-3x)^{-1}$.
	Equate derivative to 8 and solve for x	DM1	$75x^2-50x+8=(15x-4)(5x-2)$.
	Obtain answers $x=\frac{2}{5}$ and $\frac{4}{15}$	A1	Exact values required.
	Obtain answers $y=-\frac{4}{5}$ and $\frac{16}{45}$	A1	Allow A1 for one correct point.
		5	

Question	Answer	Marks	Guidance
2	Show points representing 2i and –2 + i	**B1**	Can be implied if the correct perpendicular is drawn.
	Show perpendicular bisector of *their* (2i and –2 + i)	**B1FT**	
	Show correct half–line of gradient 1 from point (–1, 0)	**B1**	Should pass through (0, 1).
	Correct loci and shade correct region	**B1**	
		4	

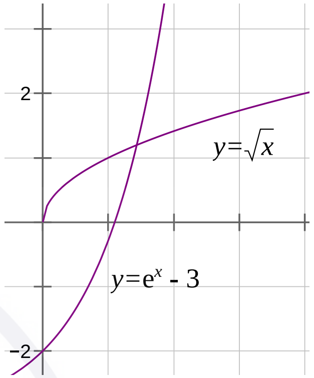
Question	Answer	Marks	Guidance
3	State or imply that $\ln y = \ln a + x \ln b$	**B1**	
	Carry out a completely correct method for finding $\ln a$ or $\ln b$	**M1**	$3.7 = \ln a + \ln b$ and $6.46 = \ln a + 2.2 \ln b$ leading to $\ln a = 1.4$, $\ln b = 2.3$.
	Obtain value $a = 4.06$	**A1**	
	Obtain value $b = 9.97$	**A1**	SC B1 for $a = e^{1.4}$ and $b = e^{2.3}$.
	Alternative Method for Question 3		
	$e^{3.7} = ab^1$ and $e^{6.46} = ab^{2.2}$	**B1**	
	Divide to obtain $e^{2.76} = b^{1.2}$ and state or imply $2.76 = 1.2 \ln b$	**M1**	
	Obtain value $a = 4.06$	**A1**	
	Obtain value $b = 9.97$	**A1**	
		4	

Question	Answer	Marks	Guidance
4(a)	Multiply numerator and denominator by $a + 5\text{i}$	**M1**	OE
	Use $\text{i}^2 = -1$	**M1**	At least once.
	Obtain answer $\frac{3a-10}{a^2+25}+\frac{2a+15}{a^2+25}\text{i}$	**A1**	
	Alternative Method for Question 4(a)		
	Multiply $x + \text{i}y$ by $a - 5\text{i}$ and use $\text{i}^2 = -1$	**M1**	
	Compare real and imaginary parts	**M1**	$3 = ax + 5y$, $2 = ay - 5x$.
	Obtain answer $\frac{3a-10}{a^2+25}+\frac{2a+15}{a^2+25}\text{i}$	**A1**	
		3	
4(b)	State or imply Im**(a)** ÷ Re**(a)** = 1	**M1**	Or Im**(a)** = Re**(a)** or equivalent for *their u*.
	Obtain answer $a = 25$	**A1**	
		2	

Question	Answer	Marks	Guidance
5(a)	Use correct trig formulae and obtain an equation in sin x and cos x	***M1**	Allow one sign error.
	Obtain a correct equation in any form	**A1**	e.g $2\cos x \sin\frac{\pi}{6} = -2\sin x \sin\frac{\pi}{3}$.
	Substitute exact trig ratios and obtain an expression for tan x	**DM1**	Allow one sign error.
	Obtain answer $\tan x = -\frac{1}{\sqrt{3}}$	**A1**	Or exact equivalent.
		4	
5(b)	Obtain answer, e.g. $x = \frac{5\pi}{6}$	**B1**	
	Obtain second answer, e.g. $x = \frac{11\pi}{6}$ and no others in the interval	**B1FT**	FT first answer + π (provided $0 \leqslant$ first answer $\leqslant \pi$). Or FT first answer – π (provided $\pi \leqslant$ first answer $\leqslant 2\pi$). Ignore any answers outside interval.
		2	

Question	Answer	Marks	Guidance
6(a)	State correct derivative of x **or** y with respect to t	**B1**	$\frac{\mathrm{d}x}{\mathrm{d}t}=\frac{1}{2}t^{-\frac{1}{2}}$, $\frac{\mathrm{d}y}{\mathrm{d}t}=\frac{1}{t}$.
	Use $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\mathrm{d}y}{\mathrm{d}t}\times\frac{\mathrm{d}t}{\mathrm{d}x}$	**M1**	Use correct chain rule.
	Obtain answer $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{2}{\sqrt{t}}$	**A1**	Or simplified equivalent e.g $2t^{-\frac{1}{2}}$ or $\frac{2\sqrt{t}}{t}$.
		3	
6(b)	State or imply *their* $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{2}$	**M1**	
	Obtain $\sqrt{t}=4$	**A1**	Or equivalent.
	Obtain answer (7, ln 16)	**A1**	Or exact equivalent. Can state the two components separately.
		3	

Question	Answer	Marks	Guidance
7	Separate variables correctly	**B1**	$\int \tan\theta \, \mathrm{d}\theta = \int \frac{x}{x^2+3} \, \mathrm{d}x$. Condone missing integral signs **or** missing dx, dθ. Can be implied by later work.
	Obtain term –ln(cos θ)	**B1**	Or equivalent e.g. ln(sec θ).
	Obtain term of the form $a\ln\left(x^2+3\right)$	**M1**	
	Obtain term $\frac{1}{2}\ln\left(x^2+3\right)$	**A1**	
	Use $x = 1$, $\theta = 0$ to evaluate a constant or as limits in a solution containing terms of the form $a\ln\left(x^2+3\right)$ and $b\ln(\cos\theta)$	**M1**	If they have rearranged then the constant must be of the correct form.
	Obtain correct answer in any form	**A1**	$\frac{1}{2}\ln\left(x^2+3\right) = -\ln\cos\theta + \ln 2$.
	Obtain final answer $x^2 = \frac{4}{\cos^2\theta} - 3$	**A1**	Or equivalent e.g. $x^2 = 4\sec^2 x - 3$. lns removed.
		7	

Question	Answer	Marks	Guidance
8(a)	Sketch a relevant graph, e.g. $y = \mathrm{e}^x - 3$ $y = \mathrm{e}^x - 3$ Should cut vertical axis at (0, –2) and have increasing gradient.	**B1**	 Ignore anything outside 1st and 4th quadrants. For second B1 need to mark intersection with a dot, a cross, or say root at point of intersection, or equivalent.
	Sketch a second relevant graph, e.g. $y = \sqrt{x}$ and justify the given statement $y = \sqrt{x}$ should start at (0, 0) and have reducing grading	**B1**	
		2	
8(b)	Calculate the values of a relevant expression or pair of expressions at $x = 1$ and $x = 2$	**M1**	
	Complete the argument correctly with correct calculated values	**A1**	e.g. $1 > -0.28..,\ 1.41 < 4.39..$ $1.28 > 0$, $-2.98 < 0$.
		2	
8(c)	State $x = \ln\left(3 + \sqrt{x}\right)$ and rearrange to the given equation $\sqrt{x} = \mathrm{e}^x - 3$	**B1**	Or rearrange $\sqrt{x} = \mathrm{e}^x - 3$ to $x = \ln(3 + \sqrt{x})$ and state iterative formula of $x_{n+1} = \ln(3 + \sqrt{x_n})$. AG
		1	

Question	Answer	Marks	Guidance
8(d)	Use the iterative process correctly at least once	**M1**	
	Obtain final answer 1.43	**A1**	
	Show sufficient iterations to at least 4 d.p. to justify 1.43 to 2 d.p. or show there is a sign change in the interval (1.425, 1.435) Condone recovery and small differences in the final figure in the iteration	**A1**	e.g. 1, 1.3864, 1.4297, 1.4341, … 1.5, 1.4210, 1.4332, 14.344, 1.4345, … 2, 1.4848, 1.4395, 1.4350, 1.4346, 1.4345, …
		3	

Question	Answer	Marks	Guidance
9(a)	Use the correct product rule	***M1**	Condone error in chain rule.
	Obtain correct derivative in any form	**A1**	e.g. $\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{x^2}{2}\mathrm{e}^{-\frac{x^2}{4}} + \mathrm{e}^{-\frac{x^2}{4}}$.
	Equate derivative to zero and solve for *x*	**DM1**	
	Obtain answer $\left(\sqrt{2}, \sqrt{2}\mathrm{e}^{-\frac{1}{2}}\right)$	**A1**	Or exact equivalent. Can state the components separately.
		4	

Question	Answer	Marks	Guidance
9(b)	State or imply $\mathrm{d}x = \frac{1}{2}u^{-\frac{1}{2}}\,\mathrm{d}u$	**B1**	Or equivalent e.g. $\mathrm{d}u = 2x\mathrm{d}x$. Alternative substitution: $u = -\frac{1}{4}x^2$.
	Substitute for x and dx	**M1**	
	Obtain correct integral $\frac{1}{2}\int \mathrm{e}^{-\frac{1}{4}u}\,\mathrm{d}u$	**A1**	OE
	Use correct limits in an integral of the form $a\mathrm{e}^{-\frac{1}{4}u}$ or $a\mathrm{e}^{-\frac{1}{4}x^2}$	**M1**	u = 9 and u = 0 or x = 3 and x = 0.
	Obtain answer $2 - 2\mathrm{e}^{-\frac{9}{4}}$	**A1**	Or exact equivalent.
	Alternative Method for Question 9(b)		
	$\int x\mathrm{e}^{-\frac{1}{4}x^2}\,\mathrm{d}x = a\mathrm{e}^{-\frac{1}{4}x^2}$	**M1**	Recognition used.
	a negative	**A1**	
	$a = -2$	**A1**	
	Use correct limits in an integral of the form $a\mathrm{e}^{-\frac{1}{4}x^2}$	**M1**	x = 3 and x = 0.
	Obtain answer $2 - 2\mathrm{e}^{-\frac{9}{4}}$	**A1**	Or exact equivalent.
		5	

Question	Answer	Marks	Guidance
10(a)	State or imply the form $\dfrac{A}{1-2x}+\dfrac{B}{2+x}+\dfrac{C}{(2+x)^2}$	**B1**	
	Use a correct method for finding a coefficient	**M1**	$A(2+x)^2+B(1-2x)(2+x)+C(1-2x)$ $=24x+13$.
	Obtain one of $A=4$, $B=2$ and $C=-7$	**A1**	If errors in equating still allow A marks for A and C.
	Obtain a second value	**A1**	
	Obtain the third value	**A1**	Mark the form $\dfrac{A}{1-2x}+\dfrac{Dx+E}{(2+x)^2}$, where $A=4$, $D=2$ and $E=-3$, B1 M1 A1 A1 A1 as above. If there are extra term in partial fractions, that is 4 unknowns A, B, D and E then B0 unless recover at end, e.g. by setting $B=0$. If B set to any value other than 0 and all coefficients correctly found to their new values then allow all A marks, but still B0 for partial fraction expression. Hence A1 for each coefficient, but nothing for coefficient set to specific value. Another case of extra term in partial fraction expression, namely $+F$, mark as above but need $F=0$ to recover B1.
		5	

Question	Answer	Marks	Guidance
10(b)	Use a correct method to find the first two terms of the expansion of $(1-2x)^{-1}$, $(2+x)^{-1}$, $(2+x)^{-2}$, $\left(1+\frac{x}{2}\right)^{-1}$ or $\left(1+\frac{x}{2}\right)^{-2}$	**M1**	Symbolic coefficients are not sufficient for the M1.
	Obtain correct un-simplified expansions up to the term in x^2 of each partial fraction	**A1 FT**	$A\left(1+(-1)(-2x)+\frac{(-1)(-2)}{2}(-2x)^2+..\right)$ $A = 4$.
		A1 FT	$\frac{B}{2}\left(1+(-1)\left(\frac{x}{2}\right)+\frac{(-1)(-2)}{2}\left(\frac{x}{2}\right)^2+...\right)$ $B = 2$.
		A1 FT	$\frac{C}{4}\left(1+(-2)\left(\frac{x}{2}\right)+\frac{(-2)(-3)}{2}\left(\frac{x}{2}\right)^2+...\right)$ $C = -7$ $= 4(1 + 2x + 4x^2) + 2/2(1 - x/2 + x^2/4) - 7/4(1 - x + 3x^2/4)$ $= (4 + 1 - 7/4) + (8 - 1/2 + 7/4)\, x + (16 + 1/4 - 21/16)\, x^2$ The FT is on *A*, *B*, *C*.

Question	Answer	Marks	Guidance
	Obtain final answer $\frac{13}{4}+\frac{37}{4}x+\frac{239}{16}x^2$	**A1**	OE $(Dx + E)/4\ [1 + (–2)(x/2) +(–2)(–3)(x/2)^2/2\ …]$ $D = 2 \quad E = –3$ The FT is on A, D, E. Maclaurin's Series f(0) = 13/4 B1 f'(0) = 37/4 B1 f''(0) = 239/8 B1. $\frac{13}{4}+\frac{37}{4}x+\frac{239}{8}x^2/2$ or equivalent M1 A1. If $1+\frac{37}{4}x+\frac{239}{8}x^2/2$ then M0 A0 unless *their* f(0) actually is 1. For the A, D, E form of fractions, give M1 A1FT A1FT for the expanded partial fractions, then, if $D \neq 0$, M1 for multiplying out fully, and A1 for the final answer. If final answer has been multiplied throughout (e.g. by 16) then A0 at the end
		5	
10(c)	$\lvert x\rvert<\frac{1}{2}$	**B1**	OE
		1	

Question	Answer	Marks	Guidance
11(a)	Obtain 3**i** + 2**j** + **k**	**B1**	Accept coordinates in place of position vector.
		1	

Question	Answer	Marks	Guidance
11(b)	$\overrightarrow{AM}$ or $\overrightarrow{AP}$ correct soi	**B1**	$\overrightarrow{AM} = 2\mathbf{j} + \mathbf{k}$, or $\overrightarrow{AP} = -2\mathbf{i} + \mathbf{j} + 2\mathbf{k}$.
	Carry out correct process for evaluating the scalar product of $\overrightarrow{AM}$ and $\overrightarrow{AP}$	**M1**	or $\overrightarrow{MA}$ and $\overrightarrow{PA}$: $0+2+2$.
	Using the correct process for the moduli, divide the scalar product by the product of the moduli and obtain the inverse cosine of the result	**M1**	For their vectors. $\theta = \cos^{-1}\left(\frac{4}{3\sqrt{5}}\right)$.
	Obtain answer 53.4° or 0.932^c	**A1**	
		4	
11(c)	Find $\overrightarrow{PQ}$ (or $\overrightarrow{QP}$) for a general point Q on the line passing through O and M,	**B1 FT**	e.g. $\boldsymbol{PQ} = -(\mathbf{i} + \mathbf{j} + 2\mathbf{k}) + \mu(3\mathbf{i} + 2\mathbf{j} + \mathbf{k})$. Follow *their M*.
	Calculate the scalar product of $\overrightarrow{PQ}$ and a direction vector for the line passing through O and M and equate to zero	***M1**	
	Solve and obtain correct solution e.g. $\mu = -\frac{1}{2}$	**A1**	
	Carry out method to calculate PQ	**DM1**	$\sqrt{.5^2 + 0 + 1.5^2}$.
	Obtain answer $\frac{\sqrt{10}}{2}$	**A1**	Or exact equivalent.
	Alternative Method 1 for Question 11(c)		
	Find $\overrightarrow{PQ}$ (or $\overrightarrow{QP}$) for a general point Q on the line passing through O and M,	**B1 FT**	e.g. $\boldsymbol{PQ} = -(\mathbf{i} + \mathbf{j} + 2\mathbf{k}) + \mu(3\mathbf{i} + 2\mathbf{j} + \mathbf{k})$. Follow *their M*.
	Use a correct method to express PQ^2 (or PQ) in terms of μ	***M1**	
	Obtain a correct equation in any form	**A1**	e.g. $PQ^2 = (1 + 3\mu)^2 + (1 + 2\mu)^2 + (2 + \mu)^2$

Question	Answer	Marks	Guidance
11(c)	Carry out a complete method for finding its minimum	**DM1**	e.g. $6(1+3\mu)+4(1+2\mu)+2(2+\mu)=0,\ \mu=-\frac{1}{2}$.
	Obtain answer $\frac{\sqrt{10}}{2}$	**A1**	Or exact equivalent.
	Alternative Method 2 for Question 11(c)		
	Calling (0, 0, 0) A, state $\overrightarrow{PA}$ (or $\overrightarrow{AP}$) in component form, e.g. $\mathbf{i}+\mathbf{j}+2\mathbf{k}$	**B1**	
	Use a scalar product to find the projection of $\overrightarrow{PA}$ (or $\overrightarrow{AP}$) on the line passing through O and M	**M1**	
	Obtain correct answer $\frac{7}{\sqrt{14}}$	**A1**	OE
	Use Pythagoras to find the perpendicular	**M1**	$d=\sqrt{AP^2-AQ^2}=\sqrt{1+1+2^2-\left(\frac{7}{\sqrt{14}}\right)^2}$.
	Obtain answer $\frac{\sqrt{10}}{2}$	**A1**	Or exact equivalent.
	Alternative Method 3 for Question 11(c)		
	Calling (0, 0, 0) A, state $\overrightarrow{PA}$ (or $\overrightarrow{AP}$) in component form, e.g. $\mathbf{i}+\mathbf{j}+2\mathbf{k}$	**B1**	
	Calculate the vector product of $\overrightarrow{PA}$ and a direction vector for the line passing through O and M	**M1**	
	Obtain correct answer, e.g. $3\mathbf{i}-5\mathbf{j}+\mathbf{k}$	**A1**	

Question	Answer	Marks	Guidance
11(c)	Divide modulus of the product by that of the direction vector	**M1**	e.g. $\dfrac{\sqrt{3^2+5^2+1^2}}{\sqrt{3^2+2^2+1^2}}$.
	Obtain answer $\dfrac{\sqrt{10}}{2}$	**A1**	Or exact equivalent.
		5	

Cambridge Assessment
International Education

Cambridge International A Level

MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **October/November 2023**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2023 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

This document consists of **24** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles

1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.

2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.

3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.

4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).

5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.

6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

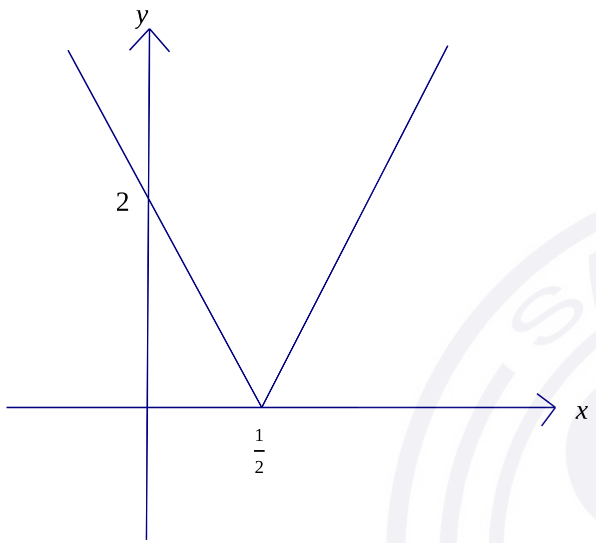
DM or **DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

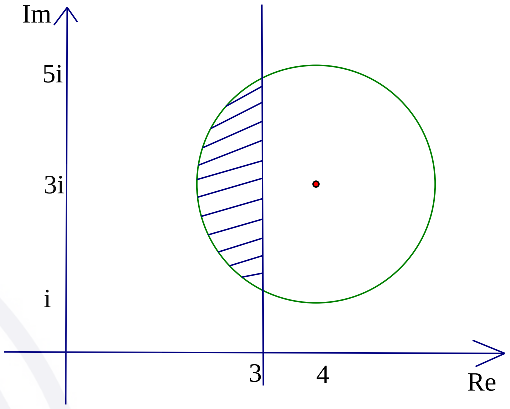
AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

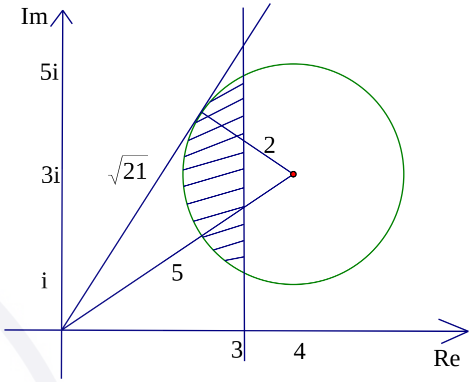
Question	Answer	Marks	Guidance
1(a)		**B1**	Show a recognizable sketch graph of $y = \|4x - 2\|$. Roughly symmetrical. Should extend into the second quadrant. Ignore $y = 4x - 2$ below the axis if intention is clear e.g. dashed or the required lines are clearly bolder. Some indication of scale on **both** axes – accept dashes. Must go beyond (0, 2) and (1, 2). Ignore any attempt to sketch $y = 1 + 3x$.
		1	

Question	Answer	Marks	Guidance
1(b)	Obtain critical value $x=3$	**B1**	Allow incorrect inequality. Allow if later rejected. Allow $\frac{21}{7}$.
	Solve the linear equation $1+3x=2-4x$	**M1**	Or corresponding linear inequality.
	Obtain critical value $\frac{1}{7}$	**A1**	Allow 0.143 or better. Allow incorrect inequality. Allow if later rejected.
	Obtain final answer $x<\frac{1}{7}$ [or] $x>3$	**A1**	Or equivalent. Allow with a comma, or nothing between. Strict inequalities only. Exact values. A0 for $\frac{1}{7}>x>3$ A0 for $x<\frac{1}{7}$ **and** $x>3$.
	Alternative method for question 1(b)		
	Solve the quadratic inequality $(4x-2)^2>(1+3x)^2$, or corresponding quadratic equation	**M1**	e.g. $7x^2-22x+3=0$. Available if they start with the correct equation / inequality, have a correct method for squaring (i.e. not $(a+b)^2=a^2+b^2$) and a correct method for solving. Need to obtain at least one critical value.
	Obtain critical value $x=3$	**A1**	Allow incorrect inequality. Allow if later rejected. Allow $\frac{21}{7}$.
	Obtain critical value $\frac{1}{7}$	**A1**	Allow 0.143 or better. Allow incorrect inequality. Allow if later rejected.
	Obtain final answer $x<\frac{1}{7}$ [or] $x>3$	**A1**	Or equivalent. Strict inequalities only. Allow with a comma, or nothing between. Exact values. A0 for $\frac{1}{7}>x>3$ A0 for $x<\frac{1}{7}$ and $x>3$.
		4	

Question	Answer	Marks	Guidance
2	Obtain $\frac{\mathrm{d}x}{\mathrm{d}t}=\frac{2}{t}\ln t$	**B1**	Any equivalent form.
	Obtain $\frac{\mathrm{d}y}{\mathrm{d}t}=-2t\mathrm{e}^{2-t^2}$	**B1**	Any equivalent form.
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\mathrm{d}y}{\mathrm{d}t}\div\frac{\mathrm{d}x}{\mathrm{d}t}$ and substitute $t=\mathrm{e}$	**M1**	Correct use of chain rule for $\frac{\mathrm{d}y}{\mathrm{d}x}\left(\frac{-2\mathrm{e}^2\mathrm{e}^{2-\mathrm{e}^2}}{2\ln\mathrm{e}}\right)$. Condone an error between correct combination of the derivatives and attempt to substitute e.
	Obtain $-\mathrm{e}^{4-\mathrm{e}^2}$	**A1**	ISW Accept $-0.0337(405..)$. Accept $-\mathrm{e}^4\mathrm{e}^{-\mathrm{e}^2}$, $\frac{-\mathrm{e}^4}{\mathrm{e}^{\mathrm{e}^2}}$ and $-\mathrm{e}^2\mathrm{e}^{2-\mathrm{e}^2}$. Allow M1A1 for a correct decimal answer following B1B1 seen.
		4	

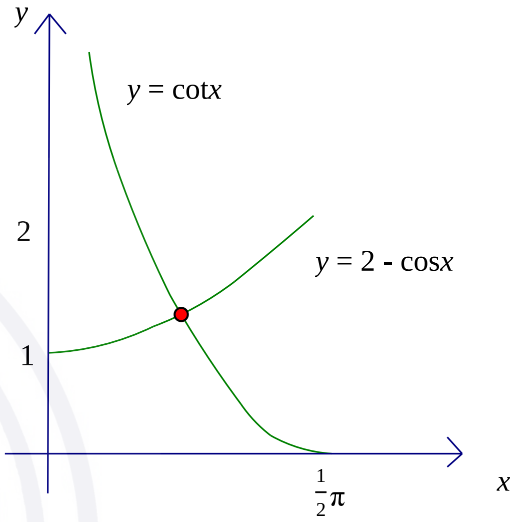
Question	Answer	Marks	Guidance
3	Substitute $x=\frac{1}{2}$ and equate the result to zero	**M1**	Or divide as far as $(2x-1)\left(x^2+px+q\right)$ and equate constant remainder to zero.
	Obtain a correct equation with powers evaluated	**A1**	e.g. $\frac{1}{4}+\frac{a}{4}-\frac{11}{2}+b=0$ or $a+4b=21$.
	Substitute $x=-1$ and equate result to 12	**M1**	Or divide as far as $(x+1)\left(2x^2+rx+s\right)$ and equate constant remainder to 12.
	Obtain a correct equation with powers evaluated	**A1**	e.g. $-2+a+11+b=12$ or $a+b=3$.
	Obtain $a=-3,\quad b=6$	**A1**	
		5	
	$\begin{array}{r\|llll} & & x^2 & \left(\frac{a+1}{2}\right)x & \frac{1}{2}\left(-\frac{21}{2}+\frac{a}{2}\right) \\ \hline 2x-1 & 2x^3 & ax^2 & -11x & b \\ & 2x^3 & -x^2 & & \\ \hline & & (a+1)x^2 & -11x & \\ & & (a+1)x^2 & -\left(\frac{a+1}{2}\right)x & \\ \hline & & & \left(-\frac{21}{2}+\frac{a}{2}\right)x & b \\ & & & \left(-\frac{21}{2}+\frac{a}{2}\right)x & -\frac{1}{2}\left(-\frac{21}{2}+\frac{a}{2}\right) \\ \hline & & & & b-\frac{21}{4}+\frac{a}{4} \\ \hline \end{array}$		$\begin{array}{r\|llll} & & 2x^2 & (a-2)x & (-9-a) \\ \hline x+1 & 2x^3 & ax^2 & -11x & b \\ & 2x^3 & 2x^2 & & \\ \hline & & (a-2)x^2 & -11x & \\ & & (a-2)x^2 & (a-2)x & \\ \hline & & & (-9-a)x & b \\ & & & (-9-a)x & (-9-a) \\ \hline & & & & b+9+a \\ \hline \end{array}$

Question	Answer	Marks	Guidance
4(a)	Show a circle with centre $4+3\mathrm{i}$. Accept a curved shape with correct point roughly in the middle.	**B1**	 Need some indication of scale e.g. label the centre, mark key points on the axes or dashes on the axes. Condone dotted lines in place of solid lines Condone correct shaded shape but not an entire circle
	Show a circle with radius 2 and centre not at the origin. The shape should be consistent with their scales	**B1**	
	Show correct vertical line. Enough to meet correct circle twice or complete line for any other circle.	**B1**	
	Shade the correct region on a correct diagram Any other shading must be accompanied by words to explain which region is required	**B1**	
		4	

Question	Answer	Marks	Guidance
4(b)	Carry out a complete method for finding the greatest value of $\arg(z)$ e.g $\tan^{-1}\frac{3}{4}+\sin^{-1}\frac{2}{5}$ $\quad(0.6435+0.4115)$	**M1**	
	Obtain answer 1.06 (accept 1.055 or 1.056) radians or $60.45°$ (accept $60.4°$ or $60.5°$)	**A1**	
	Alternative method for question 4(b)		
	Tangent to circle passing through origin has equation $y=mx$. The equation $(x-4)^2+(y-3)^2=4$ will have one root. Hence $\left(1+m^2\right)x^2-(8+6m)x+21=0$, discriminant = 0 = $48m^2-96m+20$ and $m=\dfrac{6\pm\sqrt{21}}{6}$ with the larger value needed to give greatest arg(*z*). Required angle is $\tan^{-1}m$.	**M1**	Complete method for finding the greatest value of arg(*z*).
	Obtain answer 1.06 radians or $60.45°$	**A1**	Accept 1.055 or 1.056 radians. Accept $60.4°$ or $60.5°$.
		2	

Question	Answer	Marks	Guidance
5	Split fraction to obtain $1+\frac{x-4}{x^2+4}$	**B1**	
	Attempt integration and obtain $p\ln\left(x^2+4\right)$ or $q\tan^{-1}\left(\frac{x}{2}\right)$ from correct working	**M1**	Allow for $p\ln\left(x^2+4\right)$ from $\int\frac{x}{x^2+4}\mathrm{d}x$ but only if a correct method for splitting has been used.
	Obtain $\frac{1}{2}\ln\left(x^2+4\right)$	**A1 FT**	Follow through is on their coefficients in the partial fraction. Allow from $\frac{x^2}{x^2+4}+\frac{x}{x^2+4}$ even if the split of the fraction is not complete. If $1-\frac{4}{x^2+4}+\frac{x}{x^2+4}$ later seen or implied, award the B1. Only available from a correct split, not from an approach using parts that is incomplete.
	Obtain $-2\tan^{-1}\left(\frac{x}{2}\right)$	**A1 FT**	Only available from a correct split, not from an approach using parts that is incomplete.
	Correct use of correct limits 0 and 6 in an expression involving $p\ln\left(x^2+4\right)$, $q\tan^{-1}\left(\frac{x}{2}\right)$ and no incorrect terms.	**M1**	p and q should be constants. The x term is not required at this stage.
	Obtain $6+\frac{1}{2}\ln 10-2\tan^{-1}3$	**A1**	ISW Or three term equivalent. (Must combine the ln terms.) Accept with $\frac{1}{2}\ln\lvert 10\rvert$.

Question	Answer	Marks	Guidance
5	**Alternative method for question 5**		
	Use the substitution $x = 2\tan\theta$ to obtain $\int 2\tan^2\theta + \tan\theta \, \mathrm{d}\theta$	**B1**	
	Attempt integration and obtain $p\tan\theta$ or $r\ln(\sec\theta)$ from correct working	**M1**	
	Obtain $2\tan\theta\,(-2\theta)$ and	**A1 FT**	Follow through on *their* coefficients after the substitution.
	Obtain $\ln\sec\theta$	**A1 FT**	Follow through on *their* coefficients after the substitution.
	Use correct limits 0 and $\tan^{-1}3$ in an expression involving $u\tan\theta$, $v\ln\sec\theta$ and no incorrect terms	**M1**	u and v should be constants. The θ term is not required at this stage.
	Obtain $6+\ln\left\|\sec\left(\tan^{-1}3\right)\right\| - 2\tan^{-1}3$	**A1**	ISW Or three term equivalent. Not required to simplify $\ln\left\|\sec\left(\tan^{-1}3\right)\right\|$.
		6	

Question	Answer	Marks	Guidance
6(a)	Sketch a relevant graph. e.g. $y = \cot x$: x intercept should be correct. Not touching the y-axis. No incorrect curvature. Ignore anything outside $0 < x \leqslant \frac{1}{2}\pi$.	**B1**	 2nd B1 requires a mark at the point of intersection or a suitable comment for the justification.
	Sketch a second relevant graph **and justify the given statement** e.g. $y = 2 - \cos x$: Condone if looks almost straight, but not if drawn with a ruler and not incorrect curvature. Correct y intercept. Needs to be drawn for $0 < x \leqslant \frac{1}{2}\pi$. Ignore outside this.	**B1**	
		2	
6(b)	Calculate the value of a relevant expression or values of a pair of expressions at $x = 0.6$ and $x = 0.8$ Must be working in radians. Values correct to at least 2 significant figures. Need all relevant values but only one (pair) needs to be correct to award M1. Complete set of values for their expression. If not comparing with 0 or 1 then the pairing must be clear, not just embedded values.	**M1**	e.g. $1.17 < 1.46$, $1.30 > 0.971$, $-0.29 < 0$, $0.33 > 0$. $-0.20 < 0$, $0.342 > 0$ from $\tan x(2 - \cos x) - 1 = 0$. $0.80 < 1$, $1.34 > 1$ from $\tan x(2 - \cos x) = 1$. $0.146 > 0, -0.105 < 0$ from $x - \tan^{-1}\left(\frac{1}{2-\cos x}\right)$.
	Complete the argument correctly with correct calculated values (awrt 2 s.f.). Clear comparison for their expression. Allow work on a smaller interval.	**A1**	Accept truncated values. If comparing with 0 can either indicate different signs or a negative product.
		2	

Question	Answer	Marks	Guidance
6(c)	Use the iterative process correctly at least once. Must be working in radians	**M1**	
	Obtain final answer 0.68	**A1**	Must be a clear conclusion.
	Show sufficient iterations to at least 4 d.p. to justify 0.68 to 2 d.p. or show there is a sign change in the interval (0.675, 0.685). Allow recovery. Allow truncation. Allow small differences in the 4th s.f.	**A1**	e.g. 0.7, 0.6806, 0.6855, 0.6843, 0.6846 0.6, 0.7053, 0.6792, 0.6858, 0.6842, 0.6846 0.8, 0.6545, 0.6920, 0.6826, 0.6850, 0.6844, 0.6845 .
		3	

Question	Answer	Marks	Guidance
7(a)	Use correct expansion for $\cos(2\theta+\theta)$	***M1**	
	Use correct double angle formulae to express $\cos 3\theta$ in terms of $\cos\theta$ and $\sin\theta$	**DM1**	
	Show sufficient working to confirm $\cos 3\theta \equiv 4\cos^3\theta - 3\cos\theta$	**A1**	AG
		3	

Question	Answer	Marks	Guidance
7(b)	Use the identity and correct double angle formula to obtain an equation in $\cos\theta$ only. Must come from using all three terms in the given equation.	*M1	e.g. $4\cos^3\theta - 3\cos\theta + \cos\theta(2\cos^2\theta - 1) = \cos^2\theta$ $6\cos^3\theta - \cos^2\theta - 4\cos\theta = 0$ or $6\cos^2\theta - \cos\theta - 4 = 0$.
	Obtain $\theta = 90°$	B1	Allow if $\cos\theta$ obtained correctly as a factor of *their* expression (even if there is an error in the quadratic factor). Can follow M0.
	Solve a 3-term quadratic in $\cos\theta$ to obtain a value of θ	DM1	
	Obtain one value e.g. $25.3°$	A1	Accept awrt $25.3°$.
	Obtain a second value e.g. $137.5°$ and no extras in range	A1	Accept awrt $137.5°$. Ignore values outside the range. Mark solutions in radians as a misread $(0.442, 1.57, 2.40)$.
		5	

Question	Answer	Marks	Guidance
8(a)	Multiply both sides by $a+2\mathrm{i}$ and attempt expansion of right-hand side	*M1	
	Use of $\mathrm{i}^2 = -1$ seen at least once (or implied)	DM1	e.g. $2+3a\mathrm{i} = \lambda\left(2a+2\right)+\lambda\mathrm{i}\left(-a+4\right)$
	Compare real and imaginary parts to obtain an equation in *a* only $\left[2=\lambda(2a+2),\quad 3a=\lambda\left(-a+4\right)\right]$	M1	e.g. $\frac{3a}{2}=\frac{-a+4}{2a+2}$. Any equivalent form.
	Obtain $3a^2+4a-4=0$ from correct working	A1	AG
	Alternative method for question 8(a)		
	Multiply top and bottom of the left-hand side by $a-2\mathrm{i}$ and attempt both expansions	*M1	Do not need the right-hand side at this stage.
	Use of $\mathrm{i}^2 = -1$ seen at least once or implied	DM1	e.g. $\left[\lambda\left(2-\mathrm{i}\right)=\right]\frac{8a+\mathrm{i}\left(3a^2-4\right)}{a^2+4}$.
	Compare real and imaginary parts to obtain an equation in *a* only	M1	e.g. $8a=-2\left(3a^2-4\right)$. Any equivalent form.
	Obtain $3a^2+4a-4=0$ from correct working	A1	AG
		4	
8(b)	Solve given quadratic to obtain a value of *a* and use this to form an equation in λ only (based on an equation seen in *their* working in **(a)** or **(b)**)	M1	Can be implied by relevant working seen or a correct value for λ seen.
	Obtain $a=-2,\ \lambda=-1$ or $a=\frac{2}{3},\ \lambda=\frac{3}{5}$	A1	Allow $\frac{6}{10}$ and 0.6.
	Obtain second correct pair of values	A1	
		3	

Question	Answer	Marks	Guidance
9(a)	Use correct product rule	***M1**	As far as $p\cos x\cos 2x + q\sin x\sin 2x$ or full working (*u*, *v*, d*u*/d*x*, d*v*/d*x*) shown.
	Obtain $\frac{dy}{dx} = \cos x\cos 2x - 2\sin x\sin 2x$	**A1**	OE
	Equate derivative to zero and use correct double angle formulae	**DM1**	Allow if only have one double angle in their derivative.
	Obtain $\cos x\left(1-6\sin^2 x\right)=0$ or equivalent	**A1**	e.g. $\cos x\left(6\cos^2 x-5\right)=0$, $5\tan^2 x=1$. Simplified but not necessarily factorised - like terms must be collected.
	Obtain $a = 0.42$	**A1**	Only. Accept $x = 0.42$.
	Alternative method for question 9(a)		
	Use correct double angle formula	***M1**	
	Obtain $\sin x - 2\sin^3 x$ or equivalent	**A1**	
	Use correct chain rule or product rule to differentiate and equate the derivative to zero	**DM1**	
	Obtain $\cos x\left(1-6\sin^2 x\right)=0$	**A1**	OE
	Obtain $a = 0.42$	**A1**	Only. Accept $x = 0.42$.
		5	

Question	Answer	Marks	Guidance
9(b)	Use double angle formula and obtain $p\cos^3 x + q\cos x$ correctly	***M1**	e.g. from $\int 2\cos^2 x\sin x - \sin x \, \mathrm{d}x$.
	Obtain $\pm\left(-\frac{2}{3}\cos^3 x + \cos x\right)$	**A1**	Correct for *their* integral.
	Correct use of limits $\frac{1}{4}\pi$ and $\frac{3}{4}\pi$ (or use double the integral from $\frac{1}{4}\pi$ to $\frac{1}{2}\pi$)	**DM1**	OE $\pm\left(-\frac{2}{3}\left[\left(\frac{-1}{\sqrt{2}}\right)^3 - \left(\frac{1}{\sqrt{2}}\right)^3\right] - \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right)$.
	Obtain $\frac{2\sqrt{2}}{3}$	**A1**	Or simplified exact equivalent. Final answer must be positive.
	Alternative method 1 for question 9(b)		
	Use integration by parts **twice** and obtain $r\cos x\cos 2x + s\sin x\sin 2x$	***M1**	Seen, not just implied.
	Obtain $\frac{1}{3}\cos x\cos 2x + \frac{2}{3}\sin x\sin 2x$	**A1**	Accept ± (correct for *their* integral).
	Correct use of limits $\frac{1}{4}\pi$ and $\frac{3}{4}\pi$ (or use double the integral from $\frac{1}{4}\pi$ to $\frac{1}{2}\pi$)	**DM1**	OE $\pm\frac{1}{3}\left(0 + 2\times\frac{1}{\sqrt{2}}\times -1 - 0 - 2\times\frac{1}{\sqrt{2}}\times 1\right)$.
	Obtain $\frac{2\sqrt{2}}{3}$	**A1**	Or simplified exact equivalent. Final answer must be positive.

Question	Answer	Marks	Guidance
	Alternative method 2 for question 9(b)		
	Use factor formula and integrate to obtain $g\cos 3x + h\cos x$	***M1**	$\int \frac{1}{2}(\sin 3x - \sin x)\mathrm{d}x$.
	Obtain $\pm\left(-\frac{1}{6}\cos 3x + \frac{1}{2}\cos x\right)$	**A1**	Correct for *their* integral.
	Correct use of limits $\frac{1}{4}\pi$ and $\frac{3}{4}\pi$ (or use double the integral from $\frac{1}{4}\pi$ to $\frac{1}{2}\pi$)	**DM1**	OE $\mp\frac{1}{\sqrt{2}}\left(\frac{1}{6}+\frac{1}{2}+\frac{1}{6}+\frac{1}{2}\right)$.
	Obtain $\dfrac{2\sqrt{2}}{3}$	**A1**	Or exact equivalent. Final answer must be positive.
		4	

Question	Answer	Marks	Guidance
10(a)	Use the correct process to calculate the scalar product of the direction vectors	**M1**	$(-2+4+2c)$.
	Divide the scalar product by the product of the moduli and equate the result to $\cos 60°$	**M1**	Or equivalent e.g. $2+2c = \sqrt{6}\sqrt{20+c^2}\cos 60°$. Allow for the correct process using $60°$ but the wrong vectors.
	Obtain correct equation in c	**A1**	e.g. $\dfrac{2+2c}{\sqrt{6}\sqrt{20+c^2}} = \dfrac{1}{2}$ or $10c^2 + 32c - 104 = 0$.
	Obtain $c = 2$	**A1**	Only.
		4	

Question	Answer	Marks	Guidance
10(b)	Calling $(6, -3, 6)$ *A*, find $\overrightarrow{AP}$ for a general point *P* on *l*	**B1**	e.g. $\begin{pmatrix}-3+\lambda\\1+\lambda\\-5+2\lambda\end{pmatrix}$.
	Equate the scalar product of *their* $\overrightarrow{AP}$ and a direction vector for *l* to zero and obtain an equation in λ	***M1**	e.g. $(-3+\lambda)+(1+\lambda)+(-10+4\lambda)=0$.
	Solve and obtain $\lambda=2$	**A1**	
	Carry out a method to calculate $\lvert\overrightarrow{AP}\rvert$	**DM1**	e.g. $(-1)^2+3^2+(-1)^2$ or $1^2+3^2+1^2$.
	Obtain $\sqrt{11}$ from correct working	**A1**	AG
	Alternative method 1 for question 10(b)		
	Calling $(6, -3, 6)$ *A*, find $\overrightarrow{AP}$ for a general point *P* on *l*	**B1**	e.g. $\begin{pmatrix}-3+\lambda\\1+\lambda\\-5+2\lambda\end{pmatrix}$
	Differentiate the modulus of $\overrightarrow{AP}$ or the square of the modulus and equate the derivative to zero	***M1**	e.g. $2(-3+\lambda)+2(1+\lambda)+4(-5+2\lambda)=0$
	Solve and obtain $\lambda=2$	**A1**	
	Carry out a method to calculate $\lvert\overrightarrow{AP}\rvert$	**DM1**	e.g. $(-1)^2+3^2+(-1)^2$ or $1^2+3^2+1^2$
	Obtain $\sqrt{11}$ from correct working	**A1**	AG

Question	Answer	Marks	Guidance
	Alternative method 2 for question 10(b)		
	Vector from $(6,-3,6)$ to $(3,-2,1)$ is $-3\mathbf{i}+\mathbf{j}-5\mathbf{k}$	**B1**	The method works for vector from $(6,-3,6)$ to any point on *l*.
	Use scalar product to find the angle between *their* vector and the direction of *l*	**M1**	
	Obtain $\cos\theta=\frac{3-1+10}{\sqrt{35}\sqrt{6}}\left(=\sqrt{\frac{24}{35}}\right)$ or $\sin\theta=\sqrt{\frac{11}{35}}$	**A1**	
	Correct use of trig to find the projection of their vector on the normal to *l*	**M1**	$\sqrt{35}\sin\theta=\sqrt{35}\times\sqrt{\frac{11}{35}}$.
	Obtain $\sqrt{11}$ from correct working	**A1**	AG
	Alternative method 3 for question 10(c)		
	Vector from $(6,-3,6)$ to $(3,-2,1)$ is $-3\mathbf{i}+\mathbf{j}-5\mathbf{k}$	**B1**	
	Find the vector product of *their* vector and the direction of *l*	**M1**	
	Obtain $\mathbf{i}(2+5)-\mathbf{j}(-6+5)+\mathbf{k}(-3-1)\,(=7\mathbf{i}+\mathbf{j}-4\mathbf{k})$	**A1**	
	Correct use of trig to find the perpendicular distance	**M1**	$\frac{\lvert \textit{their}\text{ vector product}\rvert}{\lvert\text{direction vector}\rvert}$.
	Distance $=\frac{\sqrt{66}}{\sqrt{6}}=\sqrt{11}$	**A1**	
		5	

Question	Answer	Marks	Guidance
11(a)	Correct separation of variables.	B1	$\int \frac{1}{y^2+y}\,\mathrm{d}y = \int -\frac{1}{x^2}\,\mathrm{d}x$. Condone missing integral signs or missing dx, dy, but not both.
	Obtain $\frac{1}{x}$	B1	
	Express $\frac{1}{y^2+y}$ in partial fractions or express the denominator of the fraction as a difference of two squares	*M1	Allow for the correct split of $\frac{\pm 1}{\left(y^2 \pm y\right)}$.
	Obtain $\frac{1}{y}-\frac{1}{y+1}$ or $\frac{1}{\left(y+\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2}$	A1	Allow if coefficients for the partial fractions are correct but followed by an error.
	Obtain $\ln y - \ln(y+1)$	A1	Or equivalent, dependent on where they left the minus sign.
	Use $x=1, y=1$ to find constant of integration or as limits in a definite integral in an expression containing terms of the form $\frac{p}{x}, q\ln y$ and $r\ln(1+y)$	DM1	$\ln\frac{1}{2} = 1 + C$ If they rearrange the equation before finding the constant of integration then the constant must be of the correct form.
	Correct equation in x and y	A1	$\ln\frac{y}{1+y} = \frac{1}{x} - 1 + \ln\frac{1}{2}$.
	Obtain $y = \frac{\mathrm{e}^{\frac{1}{x}-1}}{2-\mathrm{e}^{\frac{1}{x}-1}}$	A1	Or equivalent e.g. $y = \frac{1}{2\mathrm{e}^{1-\frac{1}{x}}-1}$, $y = \frac{1}{\mathrm{e}^{1-\frac{1}{x}+\ln 2}-1}$. Accept with decimal value for e^{-1}.
		8	

Question	Answer	Marks	Guidance
11(b)	State that y approaches $\frac{1}{2\mathrm{e}-1}$	**B1 FT**	Or exact equivalent. Condone $y=\frac{1}{2\mathrm{e}-1}$. FT on an expression in $\mathrm{e}^{\frac{1}{x}}$.
		1	

Cambridge Assessment
International Education

Cambridge International A Level

MATHEMATICS **9709/33**

Paper 3 Pure Mathematics 3 **October/November 2023**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2023 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

This document consists of **21** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles

1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.

2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.

3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.

4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).

5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.

6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M	Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
A	Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
B	Mark for a correct result or statement independent of method marks.
DM or **DB**	When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
FT	Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

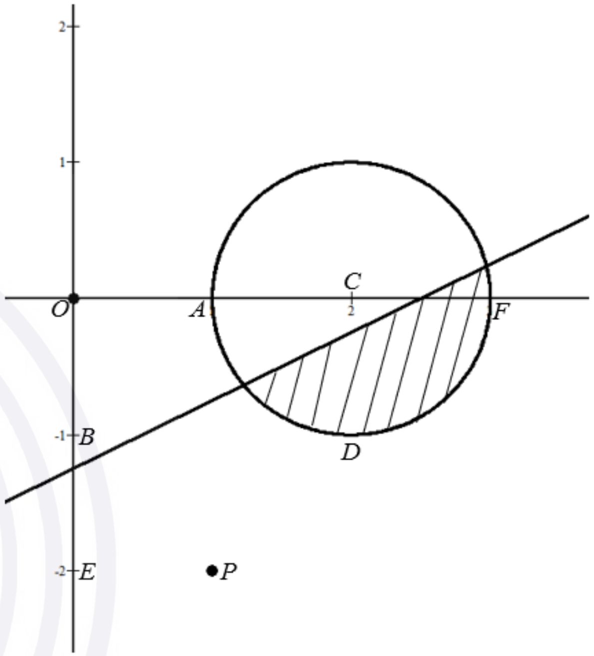
Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	State or imply non-modular inequality $-0.5 < 2^{x+1} - 2 < 0.5$, can be in two separate statements, or $\left(2^{x+1} - 2\right)^2 < 0.5^2$ or corresponding pair of linear equations $0.5 = 2^{x+1} - 2$ and $-0.5 = 2^{x+1} - 2$ or quadratic equation $\left(2^{x+1} - 2\right)^2 = 0.5^2$	**B1**	$-0.25 < 2^x - 1 < 0.25$, can be in two separate statements, or $\left(2^x - 1\right)^2 < 0.25^2$ or corresponding pair of linear equations $0.25 = 2^x - 1$ and $-0.25 = 2^x - 1$ or quadratic equation $\left(2^x - 1\right)^2 = 0.25^2$. Incorrect inequality mark recoverable by correct final answer or $x < 0.32$ and $x > -0.42$.
	Use correct method for solving an equation or inequality of the form $2^{x+1} = a$ or $2^x = b$ where $a, b > 0$	**M1**	Reach $(x + 1)\ln 2 = \ln a$ or equivalent, do not need to reach $x = \ldots$
	Obtain critical values $x = 0.322$ and -0.415 or awrt $x = 0.32$ and -0.42 or exact equivalents	**A1**	e.g. $\frac{\ln 2.5}{\ln 2} - 1$ and $\frac{\ln 1.5}{\ln 2} - 1$.
	State final answer $-0.415 < x < 0.322$ or $(-0.415, 0.322)$	**A1**	Need 3 significant figures. Need combined result, not $x < 0.32$ and $x > -0.42$. Must be strict inequalities. No working, 0/4.
	Alternative method for Question 1		
	Use correct method for solving an equation or inequality of the form $2^{x+1} = a$ or $2^x = b$ where $a, b > 0$	**M1**	May see $2^{x+1} = 1.5$ and $2^{x+1} = 2.5$. Reach $(x + 1)\ln 2 = \ln a$ or equivalent, don't need to reach $x = \ldots$
	Obtain one critical value, e.g. 0.322 or awrt $x = 0.32$ or exact equivalent	**A1**	e.g. $\frac{\ln 2.5}{\ln 2} - 1$.
	Obtain the other critical value e.g. -0.415 or awrt $x = -0.42$ or exact equivalent	**A1**	e.g. $\frac{\ln 1.5}{\ln 2} - 1$.

Question	Answer	Marks	Guidance
1	State final answer $–0.415 < x < 0.322$ or (–0.415, 0.322)	**A1**	Need 3 significant figures. Need combined result, not $x < 0.32$ and $x > – 0.42$. Must be strict inequalities. No working, 0/4.
		4	

Question	Answer	Marks	Guidance
2	Show a circle centre (2, 0)	**B1**	
	Show the relevant part of a circle with radius 1	**B1 FT**	FT centre not at the origin even if centre at 1 – 2i. Must clearly go through (1, 0) or (3, 0) (oe for FT mark).
	Show the point representing 1 – 2i	**B1**	Can be implied by correct perpendicular bisector
	Show the perpendicular bisector of the line joining 1 – 2i and the origin. Perpendicular to *OP* by eye and at midpoint of *OP* by eye sufficient. Must reach midpoint of *OP* and if extended will cut *BE.*	**B1 FT**	FT on the position of 1 – 2i.

Question	Answer	Marks	Guidance
2	Shade the correct region. Dependent on all previous marks, except in case 3 below, and the perpendicular must cut axes between *CF* and *BE*, but not actually through *C* or *F* and not through *B* or *E* Scale can be implied by dashes 1 Scale only on *y*-axis and 2*OA* = *OC* B1, B1FT, B1, B1FT, B1 2 Scale only on *x*-axis and 2*OB* = *OE* B1, B1FT, B1, B1FT, B1 3 No scale on either axis, but 2*OA* = *OC* B0, B1FT, B0, B1FT, B1 then 2*OB* = *OE*	**B1**	
		5	

Question	Answer	Marks	Guidance
3	$2(-2)^3 + a(-2)^2 + b(-2) + 6 = -38$ Allow errors $x+2 \overline{)2x^3 + ax^2 \quad + bx \quad + 6}$ with quotient $2x^2 + (a-4)x + b - 2a + 8$ $\underline{2x^3 + 4x^2}$ $(a-4)x^2 + bx$ $\underline{(a-4)x^2 + (2a-8)x}$ $(b-2a+8)x + 6$ $\underline{(b-2a+8)x + 2b - 4a + 16}$ $4a - 2b - 10$	**M1**	Substitute *x* = –2 and equate the result to –38 or divide by *x* + 2 to obtain quadratic quotient, and equate constant remainder to –38.

Question	Answer	Marks	Guidance
3	Obtain a correct evaluated equation, e.g. $-16+4a-2b+6=-38$ or $4a-2b=-28$	**A1**	
	$2\left(\frac{1}{2}\right)^3+a\left(\frac{1}{2}\right)^2+b\left(\frac{1}{2}\right)+6=\frac{19}{2}$ Allow errors $x^2+\frac{a+1}{2}x+\frac{b}{2}+\frac{a}{4}+\frac{1}{4}$ $2x-1\overline{)2x^3+ax^2\quad+bx\quad+6}$ $\underline{2x^3-x^2}$ $(a+1)x^2+bx$ $\underline{(a+1)x2-\left(\frac{a}{2}+\frac{1}{2}\right)x}$ $\left(b+\frac{a}{2}+\frac{1}{2}\right)x+6$ $\underline{\left(b+\frac{a}{2}+\frac{1}{2}\right)x-\left(\frac{b}{2}+\frac{a}{4}+\frac{1}{4}\right)}$ $6+\frac{b}{2}+\frac{a}{4}+\frac{1}{4}$	**M1**	Substitute $x=\frac{1}{2}$ and equate the result to $\frac{19}{2}$ or divide by $2x-1$ to obtain quadratic quotient, and equate constant remainder to $\frac{19}{2}$.
	Obtain a correct evaluated equation, e.g. $\frac{1}{4}+\frac{a}{4}+\frac{b}{2}+6=\frac{19}{2}$ or $\frac{a}{4}+\frac{b}{2}=\frac{13}{4}$	**A1**	
	Obtain $a=-3$ and $b=8$	**A1**	ISW
		5	

Question	Answer	Marks	Guidance
4	$\dfrac{2 \pm \sqrt{(-2)^2 - 4(3+\mathrm{i})(3-\mathrm{i})}}{2(3+\mathrm{i})}$	**M1**	Use quadratic formula to solve for *w*
	Use $\mathrm{i}^2 = -1$ in $(3+\mathrm{i})(3-\mathrm{i})$	**M1**	
	Obtain one of the answers $w = \dfrac{2+6\mathrm{i}}{6+2\mathrm{i}}$ or $w = \dfrac{2-6\mathrm{i}}{6+2\mathrm{i}}$	**A1**	Must be simplified to this form.
	Show **intention** to multiply numerator and denominator by conjugate of their denominator.	**M1**	Independent of previous M marks but must be of the same form, e.g. $\dfrac{a}{b+c\mathrm{i}}$.
	Obtain final answers $\dfrac{3}{5}+\dfrac{4}{5}\mathrm{i}$ and $-\mathrm{i}$ Accept $0.6 + 0.8\mathrm{i}$ and $0 - \mathrm{i}$	**A1**	**SC** Both correct final answers from $w = \dfrac{2+6\mathrm{i}}{6+2\mathrm{i}}$ and $w = \dfrac{2-6\mathrm{i}}{6+2\mathrm{i}}$ seen, no evidence of conjugate, then **SC B1** for both. Allow $x = \dfrac{3}{5}$, $y = \dfrac{4}{5}$ or $x = 0$, $y = -1$. A0 for $\dfrac{3+4\mathrm{i}}{5}$.
	Alternative method for Question 4		
	Multiply the equation by $3 - \mathrm{i}$	**M1**	
	Use $\mathrm{i}^2 = -1$ in $(3+\mathrm{i})(3-\mathrm{i})$	**M1**	
	Obtain $10w^2 - 2(3-\mathrm{i})w + (3-\mathrm{i})^2 = 0$ or equivalent	**A1**	
	Use quadratic formula or factorise to solve for *w*	**M1**	

Question	Answer	Marks	Guidance
4	Obtain final answers $\frac{3}{5}+\frac{4}{5}\mathrm{i}$ and $-\mathrm{i}$ Accept $0.6 + 0.8\mathrm{i}$ and $0 - \mathrm{i}$	**A1**	**SC** Both correct final answers from $10w^2 - 2(3 - \mathrm{i})w + (3 - \mathrm{i})^2 = 0$ with no working then **SC B1** for both. Allow $x = \frac{3}{5}$, $y = \frac{4}{5}$ or $x = 0$, $y = -1$. A0 for $\frac{3+4\mathrm{i}}{5}$.
	Alternative method for Question 4		
	Substitute $w = x + \mathrm{i}y$ and form equations for real and imaginary parts	**M1**	
	Use $\mathrm{i}^2 = -1$ in $(x + \mathrm{i}y)^2$	**M1**	
	Obtain $3(x^2 - y^2) - 2xy - 2x + 3 = 0$ and $x^2 - y^2 + 6xy - 2y - 1 = 0$	**A1**	OE
	Form quartic equation in x only or y only using the correct substitution and solve for x or y	**M1**	Use correct $y = \frac{(3-x)}{10x-3}$ to attempt to form and solve $50x^4 - 60x^3 + 63x^2 - 27x = 0$ $x(5x - 3)(10x^2 - 6x + 9) = 0$. Use correct $x = \frac{3(1+y)}{10y+1}$ to attempt to form and solve $100y^4 + 40y^3 - 66y^2 - 14y - 8 = 0$ $(y + 1)(5y - 4)(20y^2 + 4y + 2) = 0$.
	Obtain final answers $\frac{3}{5}+\frac{4}{5}\mathrm{i}$ and $-\mathrm{i}$ Accept $0.6 + 0.8\mathrm{i}$ and $0 - \mathrm{i}$	**A1**	**SC** Both correct final answers from $3(x^2 - y^2) - 2xy - 2x + 3 = 0$ and $x^2 - y^2 + 6xy - 2y - 1 = 0$ with no working then **SC B1** for both. Allow $x = \frac{3}{5}$, $y = \frac{4}{5}$ or $x = 0$, $y = -1$. A0 for $\frac{3+4\mathrm{i}}{5}$.
		5	

Question	Answer	Marks	Guidance
5	Use correct product or quotient rule	**M1**	Need attempt at both derivatives condone errors in chain rule. In quotient rule allow BOD in formula if ± 2x seen unless clear that incorrect formula has been used. If omit denominator or forget to square or complete reversal of signs then M0 A0 M1 A1 A1 A1.
	Obtain correct derivative in any form, e.g. $\dfrac{6x(1-x^2)e^{3x^2-1}+2xe^{3x^2-1}}{\left(1-x^2\right)^2}$	**A1**	If $6x\left(1-x^2\right)e^{3x^2-1}+2xe^{3x^2-1}=0$ from the start, with no wrong formula seen, award M1A1.
	Equate derivative (or its numerator) to zero and solve for x	**M1**	$6x - 6x^3 + 2x = 0$ and solve. Allow for just one x value. Allow if from solution of 3 term quadratic equation, but if they get $x = 0$ the x must factorise out
	Obtain the point $(0, e^{-1})$ or exact equivalent	**A1**	Or for all three x coordinates found 0, $\pm\dfrac{2\sqrt{3}}{3}$ oe and no extras but if this is the case then one pair of correct coordinates A1 and both other pairs of correct coordinates A1. Accept, e.g. $x = 0$, $y = e^{-1}$ ISW for last 3 marks.
	Obtain the point $\left(\dfrac{2\sqrt{3}}{3}, -3e^{3}\right)$ or exact equivalent	**A1**	Allow √(4/3).
	Obtain the point $\left(-\dfrac{2\sqrt{3}}{3}, -3e^{3}\right)$ or exact equivalent	**A1**	
		6	

Question	Answer	Marks	Guidance
6(a)	Use correct Pythagoras $\cot^2\theta = \mathrm{cosec}^2\theta - 1$ or $\cot^2\theta = 1/\sin^2\theta - 1$ or $\cot^2\theta = \cos^2\theta/\sin^2\theta$ and then $\cos^2\theta = 1 - \sin^2\theta$, together with double angle formula $\cos 2\theta = 1 - 2\sin^2\theta$, to obtain an equation in $\sin\theta$ or $\sin\theta$ and $\mathrm{cosec}^2\theta$	**M1**	If consistent omission of brackets, e.g. $(\sin\theta)^2$ written as $\sin\theta^2$ then **SC B1** in place of M1A1.
	Obtain a correct equation in $\sin\theta$ in any form	**A1**	e.g. $1/\sin^2\theta - 1 + 2(1 - 2\sin^2\theta) = 4$ or $\frac{1-\sin^2}{\sin^2} + 2\left(1-2\sin^2\right) = 4$. If $\frac{\cos^2}{\sin^2} + 2\left(1-2\sin^2\right) = 4$ then e.g. $1 - \sin^2 + 2\left(1-2\sin^2\right)\sin^2 = 4$. (missing $\sin^2$ on right) allow M1A1A0.
	Reduce to the given answer of $4\sin^4\theta + 3\sin^2\theta - 1 = 0$ correctly	**A1**	AG Must follow from a horizontal equation (no denominators). If $s = \sin\theta$ used and defined, allow all marks. If not defined, award M1A1A0.
		3	

Question	Answer	Marks	Guidance
6(b)	Solve the given quadratic to obtain a value for θ	M1	$(4\sin^2\theta - 1)(\sin^2\theta + 1) = 0$ and solve for θ. Incorrect sign in solution of quadratic seen, e.g. $(4\sin^2\theta - 1)(\sin^2\theta - 1) = 0$ then M0 A0 A0 but if only see $(4\sin^2\theta - 1) = 0$ and nothing incorrect seen allow 3/3.
	Obtain answer, e.g. $\theta = 30°$	A1	$\pi/6$ award A0
	Obtain three further answers, e.g. $\theta = 150°$, $210°$ and $330°$ and no others in the interval	A1	Ignore any answers outside interval. $5\pi/6$ $7\pi/6$ $11\pi/6$ award A1.
		3	

Question	Answer	Marks	Guidance
7(a)	State or imply $2y\frac{dy}{dx}$ as the derivative of y^2	B1	Allow for $3x^2dx + 2ydy$ or $F_x = 3x^2 + 6x$ and $F_y = 2y + 3$.
	Equate derivative of LHS to zero and solve for $\frac{dy}{dx}$	M1	$3x^2 + 2y\frac{dy}{dx} + 6x + 3\frac{dy}{dx} = 0$ or $3x^2dx + 2ydy + 6xdx + 3dy = 0$ or $\frac{dy}{dx} = -\frac{F_x}{F_y}$ need evidence from B1 mark or formula must be seen. Allow errors.
	Obtain the given answer	A1	AG $\frac{dy}{dx} = -\frac{3x^2 + 6x}{2y+3}$ not $\frac{-3x^2 - 6x}{2y+3}$. Must factorise with $\frac{dy}{dx}$ e.g. $3x^2 + 6x + \frac{dy}{dx}(2y+3) = 0$ or $3x^2dx + 6xdx + dy(2y+3) = 0$.
		3	

Question	Answer	Marks	Guidance
7(b)	Equate numerator to zero and solve for x	***M1**	Allow for just one x value.
	Obtain $x = 0$ and $x = –2$ only	**A1**	
	Substitute their x, [$x = 0$ or $x = –2$] in curve equation to obtain quadratic equation in y equal to 0	**DM1**	$y^2 + 3y – 4 = 0$ or $y^2 + 3y = 0$.
	Obtain $y = 1$ and $y = –4$ [when $x = 0$]	**A1**	
	Obtain $y = 0$ and $y = –3$ [when $x = –2$]	**A1**	ISW If forget $x = 0$ then max 3/5.
		5	

Question	Answer	Marks	Guidance
8	Separate variables correctly and reach $a\sec^2 3y$ or be^{-4x}	**B1**	Condone missing integral signs or dy and dx, but allow if recognisable integrals follow. Not for $1/\cos^2 3y$ and $1/e^{4x}$.
	Obtain term $-\frac{1}{4}e^{-4x}$	**B1**	Can recover the previous B1 if de^{-4x} seen here.
	Obtain only a term of the form $a\tan 3y$	**M1**	Can recover the first B1 if $a\tan 3y$ seen here.
	Obtain term $\frac{1}{3}\tan 3y$	**A1**	
	Use $x = 2$, $y = 0$ to evaluate a constant or as limits in a solution containing terms of the form $a\tan by$ and $ce^{\pm 4x}$	**M1**	May see $\tan by$ and $e^{\pm 4x}$ here.
	Obtain correct answer in any form	**A1**	e.g. $\frac{1}{3}\tan 3y = -\frac{1}{4}e^{-4x} + \frac{1}{4}e^{-8}$ or $\frac{1}{3}\tan 3y = -\frac{1}{4}e^{-4x} + 8.39 \times 10^{-5}$
	Obtain final answer $y = \frac{1}{3}\tan^{-1}\left(\frac{3}{4}e^{-8} - \frac{3}{4}e^{-4x}\right)$	**A1**	ISW OE e.g. $y = \frac{1}{3}\tan^{-1}\left(2.52\times10^{-4} - \frac{3}{4}e^{-4x}\right)$
		7	

Question	Answer	Marks	Guidance
9(a)	State or imply the form $\dfrac{Ax+B}{2+3x^2}+\dfrac{C}{2-x}$	**B1**	If incorrect partial fractions e.g. A = 0 or $Ax^2 + B$ then M1, A1 A0 for correct C. Only allow single A1 even if other coefficients correct. B1 recoverable by a correct form end statement.
	Use a correct method for finding a coefficient	**M1**	e.g. $(Ax + B)(2 - x) + C(2 + 3x^2)$ $= (3C - A)x^2 + (2A - B)x + (2B + 2C)$ $= 17x^2 - 7x + 16$.
	Obtain one of $A = -2$, $B = 3$ and $C = 5$	**A1**	If error present in above still allow A1 for C.
	Obtain a second value	**A1**	
	Obtain the third value	**A1**	Extra term in partial fractions, $D/(2 + 3x^2)$, that is 4 unknowns A, B, C and D then B0 unless recover at end, e.g. by setting B or D = 0. If B or D set to any value other than 0 and all coefficients correctly found to their new values then allow all A marks, but still B0 for partial fraction expression unless $B + D$ combined. Hence A1 for each coefficient, but nothing for coefficient set to specific value. Another case of extra term in partial fraction expression, namely + F, mark as above but need F = 0 to recover B1.
		5	

Question	Answer	Marks	Guidance
9(b)	Use a correct method to find the first two terms of the expansion $(2-x)^{-1} = 2^{-1} + (-1)\,2^{-2}(-x) + [(-1)(-2)2^{-3}(-x)^2/2!]$, $\left(1+\frac{3x^2}{2}\right)^{-1} = 1 - \frac{3x^2}{2}$ or $\left(1-\frac{x}{2}\right)^{-1} = 1 - \left(\frac{-x}{2}\right)$	**M1**	Symbolic coefficients are not sufficient for the M1.
	$\frac{Ax+B}{2}\left[1+(-1)\frac{3x^2}{2}\ldots\right]$ $A = -2$ $B = 3$ $\frac{C}{2}\left[1+(-1)\left(\frac{-x}{2}\right)+\frac{(-1)(-2)}{2}\left(\frac{-x}{2}\right)^2 + \frac{(-1)(-2)(-3)}{6}\left(\frac{-x}{2}\right)^3\ldots\right]$ $C = 5$	**A1 FT**	Obtain correct un-simplified expansions up to the term in x^3 of each partial fraction.
	$= \frac{3-2x}{2}\left(1-\frac{3x^2}{2}\right) + \frac{5}{2}\left(1+\frac{x}{2}+\frac{x^2}{4}+\frac{x^3}{8}\right)$ $=\left(\frac{3}{2}+\frac{5}{2}\right)+\left(-1+\frac{5}{4}\right)x+\left(-\frac{9}{4}+\frac{5}{8}\right)x^2+\left(\frac{3}{2}+\frac{5}{16}\right)x^3$	**A1 FT**	Unsimplified $(2-x)^{-1}$ expanded correctly, error in simplifying before their C is involved in the expression, allow A1FT when their C is introduced. The FT is on A, B, C.
	Multiply expansion of $\left(1+\frac{3x^2}{2}\right)^{-1}$ $\left(\text{must reach } 1\pm\frac{3x^2}{2}\right)$ by $Ax + B$, where $AB \neq 0$, up to the term in x^3. Allow if used $Cx + D$ ($Ax + B$ miscopied).	**M1**	Allow either ±2 or $\pm 2^{-1}$ outside bracket or missing. Allow one error in actual multiplication to acquire the 4 terms [all terms needed]. Ignore errors in higher powers.

Question	Answer	Marks	Guidance
9(b)	Obtain final answer $4+\frac{1}{4}x-\frac{13}{8}x^2+\frac{29}{16}x^3$, or equivalent [If final answer has been multiplied throughout, e.g. by 16 then A0 at the end]	**A1**	Maclaurin’s Series: f(0) = 4 B1 f’(0) = 1/4 B1. f’(0) = –13/4 and f’”(0) = 87/8 B1. $4+\frac{1}{4}x-\frac{\frac{13}{4}x^2}{2}+\frac{\frac{87}{8}x^3}{6}$ or equivalent M1 A1. If $1+\frac{1}{4}x-\frac{\frac{13}{4}x^2}{2}+\frac{\frac{87}{8}x^3}{6}$ then M0 A0 unless their f(0) actually is 1.
		5	
9(c)	State answer $\lvert x\rvert < \sqrt{\frac{2}{3}}$ or $-\sqrt{\frac{2}{3}} < x < \sqrt{\frac{2}{3}}$ clear conclusion required	**B1**	Or exact equivalent. Strict inequality.
		1	

Question	Answer	Marks	Guidance
10(a)	Use the product rule correctly on $y = x\cos 2x$	**M1**	dx/dx cos 2x + x d/dx(cos 2x) attempted.
	Obtain the correct derivative in any form	**A1**	e.g. cos 2x – 2x sin 2x. If cos 2x + x–2sin 2x, not recovered, max M1A0A1FTA0 but can recover for full marks by seeing correct substitution.
	Obtain $y=-\frac{\pi}{2}$ and $\frac{\mathrm{d}y}{\mathrm{d}x}=-1$ when $x=\frac{\pi}{2}$	**A1FT**	FT *their* $\frac{\mathrm{d}y}{\mathrm{d}x}$ with $x=\frac{\pi}{2}$ substituted.
	Obtain answer $x + y = 0$	**A1**	OE CWO Need to see y and dy/dx at $x=\frac{\pi}{2}$.
		4	

Question	Answer	Marks	Guidance
10(b)	Integrate by parts and reach $ax\sin 2x + b\int \sin 2x\,dx$	***M1**	
	Obtain $\frac{1}{2}x\sin 2x - \frac{1}{2}\int \sin 2x\,dx$	**A1**	OE
	Complete integration and obtain $\frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x$	**A1**	OE
	Use limits of $x = 0$ and $x = \frac{\pi}{4}$ in the correct order, having integrated twice to obtain *ax* sin 2*x* + *c*cos 2*x*	**DM1**	If correct, $\frac{1}{2}\left(\frac{\pi}{4}\right)\sin\frac{2\pi}{4} + \frac{1}{4}\cos\frac{2\pi}{4} - \frac{1}{4}\cos 0$ or $\frac{1}{2}\left(\frac{\pi}{4}\right)\sin\frac{2\pi}{4} - \frac{1}{4}\cos 0$. Max one substitution error.
	Obtain answer $\frac{\pi}{8} - \frac{1}{4}$ or exact simplified two term equivalent	**A1**	ISW Accept $\frac{\pi - 2}{8}$. Accept $\frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x$ then final answer.
		5	

Question	Answer	Marks	Guidance
11(a)	Use correct process for modulus on direction vector of *l*, e.g. $\sqrt{(-1)^2 + 1^2 + 2^2}$	**M1**	SOI Allow -1^2. Allow $\sqrt{(-\lambda)^2 + \lambda^2 + (2\lambda)^2}$.
	$[\pm]\frac{1}{\sqrt{6}}(-\mathrm{i} + \mathrm{j} + 2\mathrm{k})$	**A1**	OE Allow coordinates as row or column, but not row or column with **i**, **j** and **k** included.
		2	

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td rowspan="3">11(b)</td><td>Use a correct method to form an equation for line m</td><td>M1</td><td>Allow even if all signs of point incorrect, namely use $+2\mathbf{i} - 2\mathbf{j} + \mathbf{k}$ or $-3\mathbf{i} + \mathbf{j} - \mathbf{k}$.</td></tr>
<tr><td>Obtain $\mathbf{r} = -2\mathbf{i} + 2\mathbf{j} - \mathbf{k} + \mu_1(-5\mathbf{i} + 3\mathbf{j} - 2\mathbf{k})$</td><td>A1</td><td>OE, e.g. $\mathbf{r} = 3\mathbf{i} - \mathbf{j} + \mathbf{k} + \mu_2(-5\mathbf{i} + 3\mathbf{j} - 2\mathbf{k})$
Must have $\mathbf{r} = \dots$</td></tr>
<tr><td></td><td>2</td><td></td></tr>
<tr><td rowspan="6">11(c)</td><td>Justify lines are not parallel</td><td>B1</td><td>$(-5, 3, -2) \neq d(-1, 1, 2)$ or $(-5, 3, -2)\mathbf{x}(-1, 1, 2) \neq 0$.
Can find angle ($105°$, $74.6°$, 1.84^c or $1.3(0)^c$) instead but if incorrect B0 and A0 at end.
Accept direction vectors don’t have common factor but not direction vectors are not equal or direction vectors are different or $\mu \neq \lambda$ or scalar product $\neq 0$.
Not the line equations are not multiples of each other.</td></tr>
<tr><td>Express l or m in component form
e.g. $(-2 - 5\mu_1, 2 + 3\mu_1, -1 - 2\mu_1)$ or $(3 - 5\mu_2, -1 + 3\mu_2, 1 - 2\mu_2)$ or $(1 - \lambda, -2 + \lambda, -3 + 2\lambda)$</td><td>B1</td><td></td></tr>
<tr><td>Equate two pairs of components of general points on l and <i>their</i> m and solve simultaneously for λ or for μ</td><td>M1</td><td></td></tr>
<tr><td>Obtain correct answer for λ or μ, e.g. $\lambda = \frac{11}{2}$, $\mu_1 = \frac{1}{2}$</td><td>A1</td><td></td></tr>
<tr><td>Determine that all three equations are not satisfied and the lines fail to intersect and conclude the lines are skew.
Conclusion needs to follow correct working</td><td>A1</td><td>
<table>
<tr><th>1</th><th>λ</th><th>μ_1</th><th></th><th>2</th><th>λ</th><th>μ_2</th><th></th></tr>
<tr><td>ij</td><td>11/2</td><td>1/2</td><td>$8 \neq -2$</td><td>ij</td><td>11/2</td><td>3/2</td><td>$8 \neq -2$</td></tr>
<tr><td>ik</td><td>4/3</td><td>–1/3</td><td>$-2/3 \neq 1$</td><td>ik</td><td>4/3</td><td>2/3</td><td>$-2/3 \neq 1$</td></tr>
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</table>
Dependent on 4 previous marks gained.</td></tr>
<tr><td></td><td>5</td><td></td></tr>
</table>

Cambridge International A Level

MATHEMATICS **9709/31**

Paper 3 Pure Mathematics 3 **May/June 2023**

MARK SCHEME

Maximum Mark: 75

Published

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M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

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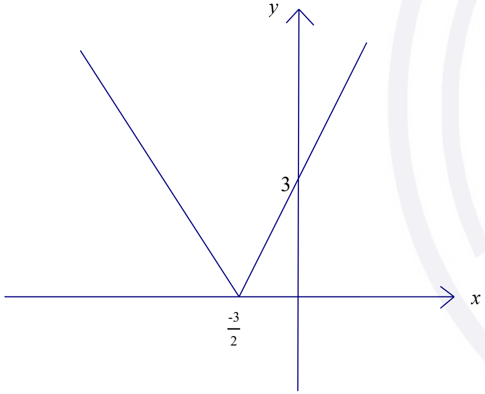
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	$3\left(e^{2x}\right)^2-5\left(e^{2x}\right)-4=0$	**B1**	OE Form 3 term quadratic in e^{2x}.
	$e^{2x}=\frac{5\pm\sqrt{73}}{6}$, $x=\frac{1}{2}\ln\left(\frac{5+\sqrt{73}}{6}\right)$	**M1**	Use correct method to solve for x.
	$x=0.407$	**A1**	Only
		3	

Question	Answer	Marks	Guidance
2(a)		**B1**	Show a recognizable sketch graph of $y=\|2x+3\|$. (Ignore any attempt to sketch $y=3x+8$). Straight lines. Vertex in approximately correct position on x axis. Symmetry.
		1	

Question	Answer	Marks	Guidance
2(b)	Find x-coordinate of intersection with $y = 3x + 8$	**M1**	
	Obtain $x = -\frac{11}{5}$	**A1**	
	State final answer $x > -\frac{11}{5}$ only	**A1**	$(x > -2.2)$ Do not condone $\geqslant$ for $>$.
	Alternative Method 1		
	Solve the linear inequality $3x + 8 > -(2x + 3)$, or corresponding linear equation	**M1**	
	Obtain critical value $x = -\frac{11}{5}$	**A1**	
	State final answer $x > -\frac{11}{5}$ only	**A1**	$(x > -2.2)$ Do not condone $\geqslant$ for $>$.
	Alternative Method 2		
	Solve the quadratic inequality $(3x + 8)^2 > (2x + 3)^2$, or corresponding quadratic equation	**(M1)**	$5x^2 + 36x + 55$.
	Obtain critical value $x = -\frac{11}{5}$	**(A1)**	Ignore -5 if seen.
	State final answer $x > -\frac{11}{5}$ only	**(A1)**	$(x > -2.2)$ Do not condone $\geqslant$ for $>$.
		3	

Question	Answer	Marks	Guidance
3	State unsimplified term in x^3, or its coefficient, in the expansion of $(1+4x)^{\frac{1}{2}}$	B1	$\dfrac{\frac{1}{2}\times\frac{-1}{2}\times\frac{-3}{2}}{6}(4x)^3\ (=4)$ Must expand binomial coefficient.
	State unsimplified term in x^2, or its coefficient, in the expansion of $(1+4x)^{\frac{1}{2}}$	B1	$\dfrac{\frac{1}{2}\times\frac{-1}{2}}{2}(4x)^2\ (=-2)$ Must expand binomial coefficient.
	Multiply by $(3+x)$ and combine terms in x^3, or their coefficients	M1	$(3\times4-1\times2)$ Allow if they expanded with x rather than $4x$.
	Obtain answer 10	A1	Accept $10x^3$
		4	

Question	Answer	Marks	Guidance
4(a)	Use correct double angle formulae	M1	e.g. $2\sin\theta\cos\theta+\cos^2\theta-\sin^2\theta=2\sin^2\theta$
	Obtain $\cos^2\theta+2\sin\theta\cos\theta-3\sin^2\theta=0$ from **full and correct** working	A1	AG Check conclusion is complete and matches the working.
		2	

Question	Answer	Marks	Guidance
4(b)	Factorise to obtain $(\cos\theta - \sin\theta)(\cos\theta + 3\sin\theta) = 0$	**B1**	OE
	Solve a quadratic in $\sin\theta$ and $\cos\theta$ to obtain a value for θ.	**M1**	$\tan\theta = 1$ *or* $\tan\theta = -\frac{1}{3}$.
	Obtain one correct value e.g. 45°	**A1**	
	Obtain a second correct value e.g. 161.6° and no others in the interval	**A1**	Mark answers in radians (0.785 and 2.82) as a misread. Accept awrt 161.6 .
	Alternative Method 1		
	Obtain $3\tan^2\theta - 2\tan\theta - 1 = 0$	**B1**	
	Solve a 3 term quadratic in $\tan\theta$ to obtain a value for θ.	**M1**	$\tan\theta = 1$ *or* $\tan\theta = -\frac{1}{3}$.
	Obtain one correct value e.g. 45°	**A1**	
	Obtain a second correct value e.g. 161.6° and no others in the interval	**A1**	Mark answers in radians (0.785 and 2.82) as a misread.
	Alternative Method 2		
	Obtain $(\cos\theta + \sin\theta)^2 = (2\sin\theta)^2$	**B1**	
	Solve to obtain a value for θ.	**M1**	$\tan\theta = 1$ *or* $\tan\theta = -\frac{1}{3}$.
	Obtain one correct value e.g. 45°	**A1**	
	Obtain a second correct value e.g. 161.6° and no others in the interval	**A1**	Mark answers in radians (0.785 and 2.82) as a misread.
		4	

Question	Answer	Marks	Guidance
5(a)	State or imply $2xy + x^2\frac{\mathrm{d}y}{\mathrm{d}x}$ as derivative of x^2y	B1	Accept partial: $\frac{\partial}{\partial x} \to 2xy$.
	State or imply $2ay\frac{\mathrm{d}y}{\mathrm{d}x}$ as derivative of ay^2	B1	Accept partial: $\frac{\partial}{\partial y} \to x^2 - 2ay$.
	Equate attempted derivative to zero and solve for $\frac{\mathrm{d}y}{\mathrm{d}x}$	M1	
	Obtain answer $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2xy}{2ay - x^2}$ from correct working	A1	AG
		4	
5(b)	State or imply $2ay - x^2 = 0$	*M1	
	Substitute into equation of curve to obtain equation in x and a or in y and a	DM1	e.g. $2ay^2 - ay^2 = 4a^3$ or $\frac{x^4}{2a} - \frac{x^4}{4a} = 4a^3$.
	Obtain one correct point	A1	e.g. $(2a, 2a)$.
	Obtain second correct point and no others	A1	e.g. $(-2a, 2a)$.
		4	**SC**: Allow A1 A0 for $x = \pm 2a$ or for $y = 2a$.

Question	Answer	Marks	Guidance
6(a)	Obtain a vector for one side of the parallelogram	**B1**	e.g. $\overrightarrow{AB}=\begin{pmatrix}2\\2\\-1\end{pmatrix}$ or $\overrightarrow{BC}=\begin{pmatrix}-1\\-5\\-6\end{pmatrix}$.
	Correct method to obtain $\pm\overrightarrow{OD}$	**M1**	e.g. $\overrightarrow{OD}=\overrightarrow{OA}+\overrightarrow{BC}$. MO if use $\overrightarrow{AB}=\overrightarrow{CD}$ or $\overrightarrow{BC}=\overrightarrow{DA}$.
	Obtain $\overrightarrow{OD}=\mathbf{i}-4\mathbf{j}-3\mathbf{k}$	**A1**	Any equivalent form. Accept coordinates.
		3	
6(b)	Using the correct process, evaluate the scalar product $\overrightarrow{BA}.\overrightarrow{BC}$	**M1**	$(2+10-6)$ Scalar product of two relevant vectors. OE
	Using the correct process for the moduli, divide the scalar product by the product of the moduli.	**M1**	$\frac{2+10-6}{\sqrt{9}\times\sqrt{62}}$.
	Obtain answer $\frac{2}{\sqrt{62}}$	**A1**	ISW Or simplified equivalent i.e. $\frac{\sqrt{62}}{31}$.
		3	

Question	Answer	Marks	Guidance
6(c)	State or imply $\sin\theta = \sqrt{\frac{58}{62}}$	**B1 FT**	Follow *their* $\cos\theta$.
	Use correct method to find the area of *ABCD*	**M1**	**e.g.** $2 \times \frac{1}{2} BA \times BC \sin\theta$. Condone decimals.
	Correct unsimplified expression for the area	**A1 FT**	**e.g.** $2 \times \frac{1}{2} \times 3 \times \sqrt{62} \times \sin\theta$. Condone decimals. Follow *their* sides and angle.
	Obtain answer $3\sqrt{58}$	**A1**	Correct only.
		4	

Question	Answer	Marks	Guidance
7	Correct separation of variables	**B1**	$\int \sin^2 3y \, dy = \int 4\sec 2x \tan 2x \, dx$ or equivalent. Condone missing integral signs or d*x* and d*y*.
	Integrate to obtain $k\sec 2x$	**M1**	
	Obtain $2\sec 2x$	**A1**	
	Use double angle formula and integrate to obtain $py + q\sin 6y$	**M1**	Or two cycles of integration by parts.
	Obtain $\frac{1}{2}y - \frac{1}{12}\sin 6y$	**A1**	
	Use $y = 0$, $x = \frac{\pi}{6}$ in a solution containing terms $\lambda\sec 2x$ and $\mu\sin 6y$ to find the constant of integration	**M1**	
	Obtain $\frac{1}{2}y - \frac{1}{12}\sin 6y = 2\sec 2x - 4$	**A1**	Or equivalent seen or implied by $\frac{\pi}{2}\left(-\frac{1}{12}\sin \pi\right) = 2\sec 2x - 4$.
	Obtain $x = 0.541$	**A1**	From correct working (not by using the calculator to integrate).
		8	

Question	Answer	Marks	Guidance
8(a)	State or imply the form $\frac{A}{2x+1}+\frac{B}{x+2}+\frac{C}{(x+2)^2}$	**B1**	Accept $\frac{A}{2x+1}+\frac{Dx+E}{(x+2)^2}$.
	Use a correct method for finding a constant	**M1**	
	Obtain one of $A=1, B=-2, C=3$	**A1**	For alternative form: $A=1, D=-2, E=-1$.
	Obtain a second value	**A1**	
	Obtain the third value	**A1**	
		5	
8(b)	Integrate and obtain one of $\frac{1}{2}\ln(2x+1), -2\ln(x+2), \frac{-3}{x+2}$	**B1 FT**	The follow through is on *their* A, B, C.
	Obtain a second term	**B1 FT**	If the alternative form is used, then either need to use integration by parts or split the fraction further.
	Obtain the third term	**B1 FT**	
	Substitute limits correctly in an integral with at least two terms of the form $\frac{1}{2}\ln(2x+1), -2\ln(x+2)$ and $\frac{-3}{x+2}$ and subtract in correct order	**M1**	The terms used need to have been obtained correctly. Must be exact values, not decimals.
	Obtain $1-\ln 3$	**A1**	
		5	

Question	Answer	Marks	Guidance
9(a)	Commence integration and reach $pxe^{-2x}+q\int e^{-2x}dx$	*M1	OE
	Obtain $-\frac{1}{2}xe^{-2x}+\frac{1}{2}\int e^{-2x}dx$	A1	OE
	Complete integration and obtain $-\frac{1}{2}xe^{-2x}-\frac{1}{4}e^{-2x}$	A1	
	Use limits correctly and equate to $\frac{1}{8}$, having integrated twice	DM1	$-\frac{1}{2}a\,e^{-2a}-\frac{1}{4}e^{-2a}+\frac{1}{4}=\frac{1}{8}$.
	Obtain $a=\frac{1}{2}\ln(4a+2)$ correctly	A1	AG
		5	
9(b)	Calculate the values of a relevant expression or pair of expressions at a = 0.5 and a = 1	M1	
	Justify the given statement with correct calculated values	A1	e.g. 0.5 < 0.69…, 1 > 0.89… 0.193 > 0, –1.105 < 0 0.066 < 0.125, 0.148 > 0.125 if put limits in the integral. Condone if they use calculator for the definite integral.
		2	
9(c)	Use the iterative process $a_{n+1}=\frac{1}{2}\ln(4a_n+2)$ correctly at least once.	M1	
	Obtain final answer 0.84	A1	
	Show sufficient iterations to at least 4 d.p. to justify 0.84 to 2 d.p. or show that there is a sign change in (0.835, 0.845)	A1	e.g. 0.75, 0.8047, 0.8261, 0.8343, 0.8373, 0.8385 1, 0.8959, 0.8599, 0.8469, 0.8420, 0.8402 .
		3	

Question	Answer	Marks	Guidance
10(a)	Substitute $x = -3$ to obtain value of $\mathrm{p}(-3)$	**M1**	
	Obtain $\mathrm{p}(-3) = 0$ and hence given result	**A1**	
	Alternative method for Question 10(a)		
	Divide $\mathrm{p}(x)$ by $(x+3)$ to obtain quotient $x^2 \pm 2x + \ldots$	**M1**	
	Obtain quotient $x^2 + 2x + 25$, with zero remainder and hence given result	**A1**	
		2	

Question	Answer	Marks	Guidance
10(b)	Substitute $z=-1+2\sqrt{6}\mathrm{i}$ and attempt expansions of z^2 and z^3	M1	$z^2=-23-4\sqrt{6}\mathrm{i},\quad z^3=-1+6\sqrt{6}\mathrm{i}+72-48\sqrt{6}\mathrm{i}$.
	Use $\mathrm{i}^2=-1$	M1	Seen at least once.
	Obtain $\mathrm{p}(z)=0$ and hence given result	A1	**SC B1** if there is no evidence of working for the square or the cube. Total 1/3.
	Alternative Method 1		
	Use roots $z=-1+2\sqrt{6}\mathrm{i}$ to form quadratic factor	M1	$z^2+2z+25$.
	Divide $\mathrm{p}(z)$ by *their* quadratic factor	M1	
	Obtain zero remainder and hence given result.	A1	
	Alternative Method 2		
	Set *their* quadratic factor from **(a)** equal to zero	M1	
	Solve for z	M1	Need to see method here as answer is given.
	Obtain $z=-1+2\sqrt{6}\mathrm{i}$ (and $z=-1-2\sqrt{6}\mathrm{i}$)	A1	
	Alternative Method 3		
	Substitute $z=-1+2\sqrt{6}\mathrm{i}$ into *their* quadratic factor and attempt expansion of z^2	M1	
	Use $\mathrm{i}^2=-1$	M1	
	Obtain 0 and hence given result	A1	
		3	

Question	Answer	Marks	Guidance
10(c)	State $z_1 = \sqrt{3}\text{i}$ and $z_2 = -\sqrt{3}\text{i}$	**B1**	
	Expand $(x+\text{i}y)^2 = -1+2\sqrt{6}\text{i}$ and compare real and imaginary parts	**M1**	Allow for use of $z^2 = -1-2\sqrt{6}\text{i}$.
	Obtain $x^2 - y^2 = -1$ and $xy = \sqrt{6}$	**A1**	
	Solve to obtain x and y	**M1**	
	Obtain $z_3 = \sqrt{2}+\sqrt{3}\text{i}$ and $z_4 = -\sqrt{2}-\sqrt{3}\text{i}$	**A1**	
	Use $z^2 = -1-2\sqrt{6}\text{i}$ to obtain z_5 and z_6	**M1**	Allow for use of $z^2 = -1+2\sqrt{6}\text{i}$.
	Obtain $z_5 = \sqrt{2}-\sqrt{3}\text{i}$ and $z_6 = -\sqrt{2}+\sqrt{3}\text{i}$	**A1**	
		7	

Cambridge Assessment
International Education

Cambridge International A Level

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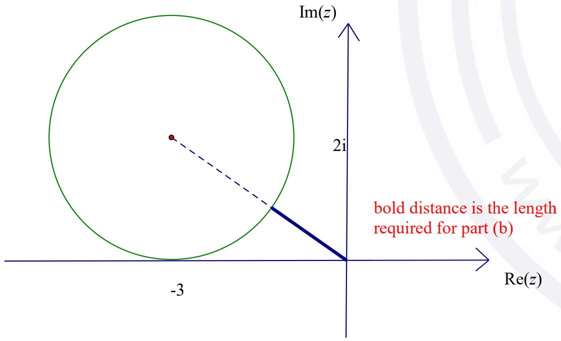
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	State or imply non-modular inequality $(5x-3)^2 < 2^2(3x-7)^2$, or corresponding quadratic equation, or pair of linear equations $(5x - 3) = \pm 2(3x - 7)$	**B1**	$11x^2 - 138x + 187 > 0$.
	Solve a 3-term quadratic, or solve **two** linear equations for x	**M1**	If no working is shown, the M1 is implied by the correct roots for an incorrect quadratic.
	Obtain critical values $x = \frac{17}{11}$ and $x = 11$	**A1**	Accept 1.55 or better.
	State **final** answer $x < \frac{17}{11},\ x > 11$	**A1**	Strict inequality required. In set notation, allow notation for open sets but not for closed sets e.g. accept $\left(-\infty, \frac{17}{11}\right) \cup (11, \infty)$ or $(-\infty, \frac{17}{11}[\ \cup\ ]11, \infty)$ but not $(-\infty, \frac{17}{11}] \cup [11, \infty)$. Allow 'or' but not 'and'. Accept $\cup$. Final A0 for $\frac{17}{11} > x > 11$. Exact values expected but ISW if exact inequalities seen followed by decimal approx.
	Alternative Method for Question 1		
	Obtain critical value $x = 11$ from a graphical method, or by inspection, or by solving a linear equation or an inequality	**B1**	
	Obtain critical value $x = \frac{17}{11}$ similarly	**B2**	Accept decimal value.
	State final answer $x < \frac{17}{11},\ x > 11$	**B1**	Strict inequality required. See notes above.
		4	

Question	Answer	Marks	Guidance
2	Use law of the logarithm of a power, quotient or product	**M1**	Must be used correctly on a **correct** term. e.g. M1 for $2\ln x = \ln x^2$ but M0 for $2\ln x - \ln 2 = 2\ln \frac{x}{2}$. M0 for $\ln(2x^2 - 3) = \ln 2x^2 - \ln 3$ $= \ln 2 + 2\ln x - \ln 3$.
	Remove logarithms and obtain a correct equation in x	**A1**	e.g. $2x^2 - 3 = \frac{x^2}{2}$.
	Obtain final answer $x = \sqrt{2}$ only	**A1**	If $x = -\sqrt{2}$ is mentioned, it must be rejected.
		3	

Question	Answer	Marks	Guidance
3(a)		**B1**	Show a circle with centre –3 + 2i. Allow for a curved figure with ‘centre’ in roughly the correct position. Accept marks or numbers on axes, coordinates of centre shown. B0B1 available for axes the wrong way round (and M1 A1 in part **(b)**).
	Show a circle with radius 2	**B1 FT**	FT centre not at the origin. Allow ‘near miss’ on x axis. Different scales on axes require an ellipse for B1 B1. Scales on the axes and any label of the radius must be consistent for B1 B1. Correct circle shaded scores B1 B0.
		2	

Question	Answer	Marks	Guidance
3(b)	Carry out a correct method for finding the least value of $\|z\|$	**M1**	e.g. distance of centre from origin – radius or find point of intersection of circle and $3y = -2x$ and use Pythagoras. If they subtract the wrong way round M0. If their diagram is a reflection or a rotation of the correct diagram, M1 A1 is available (requires equivalent work). Any other circle M0.
	Obtain answer $\sqrt{13}-2$ or $\sqrt{17-4\sqrt{13}}$	**A1**	Or exact equivalent e.g. $\sqrt{17-\frac{26}{3}\sqrt{\frac{36}{13}}}$. Correct solution only. Allow A1 if exact answer seen and then decimal given.
		2	

Question	Answer	Marks	Guidance
4	Use correct double angle formula to obtain an equation in $\cos\left(\frac{x}{2}\right)$ only	***M1**	e.g. $2\left(2\cos^2\left(\frac{x}{2}\right)-1\right)-\cos\left(\frac{x}{2}\right)=1$.
	Obtain a 3 term quadratic in $\cos\left(\frac{x}{2}\right)$,	**A1**	e.g. $4\cos^2\left(\frac{x}{2}\right)-\cos\left(\frac{x}{2}\right)-3=0$. Allow $4\cos^2 u-\cos u-3=0$. Condone $\frac{x}{2}=x$.
	Obtain $\cos\left(\frac{x}{2}\right)=-\frac{3}{4}$ and $\cos\left(\frac{x}{2}\right)=1$	**A1**	Allow answer in u e.g. $(4\cos u+3)(\cos u-1)$ and condone $\frac{x}{2}=x$.
	Solve for the **original** x	**DM1**	Must see evidence of doubling, not halving.
	Obtain $x = 0$ and 4.84 and no others in the interval	**A1**	Ignore any answers outside interval. Accept AWRT 4.84. Accept 1.54π. Must be in radians. 277.2 indicates M1 but is A0.
	Alternative Method for Question 4		
	Use correct double angle formula to obtain an equation in $\cos x$ only	***M1**	e.g. $2\cos x-1=\sqrt{\frac{\cos x+1}{2}}$.
	Obtain a 3 term quadratic in $\cos x$,	**A1**	e.g. $8\cos^2 x-9\cos x+1=0$.
	Obtain $\cos x=\frac{1}{8}$ and $\cos x = 1$	**A1**	
	Solve for x	**DM1**	
	Obtain answers $x = 0$ and 4.84 and no others in the interval	**A1**	Ignore any answers outside interval. Accept AWRT 4.84. Must be in radians. 277.2 is A0.
		5	

Question	Answer	Marks	Guidance
5(a)	Substitute $2 + y\mathrm{i}$ in $a^3 - a^2 - 2a$ and attempt expansions of a^2 and a^3	**M1**	$a^2 = 4 + 4y\mathrm{i} - y^2$ $a^3 = 8 + 12y\mathrm{i} - 6y^2 - y^3\mathrm{i}$. If using $a\left(a^2 - a - 2\right)$ must then expand fully. Must see working.
	Use $\mathrm{i}^2 = -1$	**M1**	Seen at least once (e.g. in squaring).
	Obtain **final** answer $-5y^2 + (6y - y^3)\mathrm{i}$	**A1**	Or simplified equivalent e.g. $6y\mathrm{i} - 5y^2 - y^3\mathrm{i}$. Do not ISW.
		3	No evidence of working for the square or the cube can score SC B1 for the correct answer.
5(b)	Equate *their* $-5y^2$ to -20 and solve for y	**M1**	Need to obtain a value for y. Available even if *their* y is not real.
	Obtain $y = -2$	**A1**	From correct work. Allow after incorrect f(a) if the real part was correct. Condone ± 2 with positive not rejected.
	Obtain **final** answer $\arg a = -\dfrac{\pi}{4}$	**A1**	Correct only (must have rejected y positive). OE e.g. $-\frac{\pi}{4} \pm 2n\pi$. Accept -0.785, 5.50. Allow after incorrect f(a) if the real part was correct. Accept degrees. Do not ISW.
		3	

Question	Answer	Marks	Guidance
6(a)	Calculate the values of a relevant expression or pair of expressions at $x = 0.5$ and $x = 1$	**M1**	Need to evaluate at both points, but M1 still available if one value incorrect. Use of degrees is M0. Correct use of a smaller interval is M1. If using g(x) – f(x), there needs to be a clear indication of the comparison being made e.g. by listing values in a table. Embedded values 0.5 and 1 are not sufficient. 3.92 and 1.83 alone are not sufficient.
	Complete the argument correctly with conclusion about change of sign or change of inequalities and with correct calculated values. Can all be in symbols – an explanation in words is not required.	**A1**	e.g. 3.92 > 1.5, 1.83 < 3 or 2.42 > 0, –1.17 < 0.
		2	
6(b)	State $x = \frac{1}{3}\left(x + 4\tan^{-1}\frac{1}{3x}\right)$	**M1**	Or rearrange $\cot\left(\frac{x}{2}\right) = 3x$ as far as $2x = 4\tan^{-1}\left(\frac{1}{3x}\right)$
	Rearrange to the given equation $\cot\left(\frac{x}{2}\right) = 3x$ Need intermediate step between $\frac{x}{2} = \tan^{-1}\frac{1}{3x}$ and $\cot\left(\frac{x}{2}\right) = 3x$	**A1**	Or continue rearrangement to $x = \frac{1}{3}\left(x + 4\tan^{-1}\frac{1}{3x}\right)$ and state iterative formula of $x_{n+1} = \frac{1}{3}\left(x_n + 4\tan^{-1}\frac{1}{3x_n}\right)$ **AG**
		2	

Question	Answer	Marks	Guidance
6(c)	Use the iterative process correctly at least once	**M1**	Obtain one value and substitute that back in to obtain a second value. Working in degrees is M0.
	Obtain final answer 0.79	**A1**	Must be to 2 d.p.
	Show sufficient iterations to at least 4 d.p. to justify 0.79 to 2 d.p. or show there is a sign change in the interval (0.785, 0.795)	**A1**	e.g. 1, 0.7623, 0.8037, 0.7921, 0.7951, 0.7943, 0.7945 or 0.5, 0.9506, 0.7665, 0.8024, 0.7924, 0.7950, 0.7944, 0.7945 or 0.75, 0.8076, 0.7911, 0.7954, 0.7943, 0.7946, 0.7945 . Condone truncation. Allow recovery. Condone minor differences in the final d.p.
		3	If they do the iteration in **(b)** but restate the conclusion here, no marks in **(b)** but could score 3/3 for **(c)**.

Question	Answer	Marks	Guidance
7(a)	State or imply $6y\dfrac{dy}{dx}$ as the derivative of $3y^2$	**B1**	Allow y' for $\dfrac{dy}{dx}$ throughout. Accept $\dfrac{\partial f}{\partial x} = 6x + 4y$.
	State or imply $4x\dfrac{dy}{dx} + 4y$ as the derivative of $4xy$	**B1**	Accept $\dfrac{\partial f}{\partial y} = 4x + 6y$.
	Equate derivative of LHS to zero and solve for $\dfrac{dy}{dx}$	**M1**	Allow an extra $\dfrac{dy}{dx}$ in front of their differentiated equation. Allow if '= 0' is implied but not seen. Allow $\dfrac{dy}{dx} = -\dfrac{\partial f}{\partial x}\Big/\dfrac{\partial f}{\partial y}$
	Obtain $\dfrac{dy}{dx} = -\dfrac{3x+2y}{2x+3y}$	**A1**	AG – must come from correct working. The position of the negative must be clear.
		4	

Question	Answer	Marks	Guidance
7(b)	Equate $\frac{dy}{dx}$ to –2 and solve for x in terms of y or for y in terms of x	*M1	Must be using the given derivative.
	Obtain $x = -4y$ or $y = -\frac{x}{4}$	A1	Seen or implied by correct later work.
	Substitute *their* $x = -4y$ or *their* $y = -\frac{x}{4}$ in curve equation	DM1	Allow unsimplified.
	Obtain $y = \pm\frac{1}{\sqrt{7}}$ or $x = \pm\frac{4}{\sqrt{7}}$	A1	Or exact equivalent. Or $x = \frac{4}{\sqrt{7}}$ and $y = -\frac{1}{\sqrt{7}}$ or exact equivalent.
	Obtain both pairs of values	A1	Or $x = -\frac{4}{\sqrt{7}}$ and $y = \frac{1}{\sqrt{7}}$ or exact equivalent. A1 A0 for incorrect final pairing.
		5	

Question	Answer	Marks	Guidance
8(a)	Separate variables correctly	**B1**	$\int \frac{1}{4+9y^2}\,\mathrm{d}y = \int \mathrm{e}^{-(2x+1)}\,\mathrm{d}x$. Condone missing integral signs **or** dx and dy missing.
	Obtain term $-\frac{1}{2}\mathrm{e}^{-2x-1}$	**B1**	OE e.g. $-\frac{1}{2\mathrm{e}}\mathrm{e}^{-2x}$.
	Obtain term of the form $a\tan^{-1}\left(\frac{3y}{2}\right)$	**M1**	
	Obtain term $\frac{1}{6}\tan^{-1}\left(\frac{3y}{2}\right)$	**A1**	OE e.g. $\frac{1}{9}\times\frac{3}{2}\tan^{-1}\frac{3y}{2}$.
	Use $x = 1$, $y = 0$ to evaluate a constant or as limits in a solution containing or derived from terms of the form $a\tan^{-1}(by)$ and $c\mathrm{e}^{\pm(2x+1)}$	**M1**	If they rearrange before evaluating the constant, the constant must be of the correct form.
	Obtain correct answer in any form	**A1**	e.g. $\frac{1}{6}\tan^{-1}\frac{3y}{2} = \frac{1}{2}\mathrm{e}^{-3} - \frac{1}{2}\mathrm{e}^{-(2x+1)}$.
	Obtain final answer $y = \frac{2}{3}\tan\left(3\mathrm{e}^{-3} - 3\mathrm{e}^{-2x-1}\right)$	**A1**	OE Allow with $3\mathrm{e}^{-3} = 0.149....$
		7	
8(b)	State that y approaches $\frac{2}{3}\tan\left(3\mathrm{e}^{-3}\right)$	**B1 FT**	Or exact equivalent. The FT is on correct work on a solution containing e^{-2x-1}. Condone $y = \ldots$ Accept correct answer stated with minimal wording. 0.10032… is not exact so B0.
		1	

Question	Answer	Marks	Guidance
9(a)	State or imply the form $\frac{A}{1+2x}+\frac{B}{2-x}+\frac{C}{(2-x)^2}$	B1	Alternative form: $\frac{A}{1+2x}$ + $\frac{Dx+E}{(2-x)^2}$
	Use a correct method for finding a coefficient	M1	e.g. $A(2-x)^2 + B(1+2x)(2-x) + C(1+2x)$ $= 2x^2 + 17x - 17$ and compare coefficients or substitute for x. $A(2-x)^3 + B(1+2x)(2-x)^2 + C(1+2x)(2-x)$ $= 2x^2 + 17x - 17$ scores M0.
	Obtain one of $A = -4$, $B = -3$ and $C = 5$	A1	
	Obtain a second value	A1	
	Obtain the third value	A1	Extra term in partial fractions, then B0 unless recover at end. Allow the marks for any constants found correctly. Missing terms in partial fractions, B0 but M1A1 is available for a correct method that obtains at least one correct constant (e.g. cover-up rule) Max 2/5. Ignore any substitution back into their original expression. If alternative form used: $A = -4$, $D = 3$ and $E = -1$.
		5	

Question	Answer	Marks	Guidance
9(b)	Integrate and obtain terms $-2\ln(1+2x)+3\ln(2-x)+\frac{5}{2-x}$	**B1FT** **B1FT** **B1FT**	OE The FT is on correct use of *their* A, B and C; or on A, D and E. If using the A, D, E form then B1 for the A term, but no further marks until partial fractions are used to split the second term or they use integration by parts to obtain $\frac{Dx+E}{2-x}-\int\frac{D}{2-x}\,dx$ for the 2nd B1 and 3rd B1 for correct completion. B0FT, B0FT, B0FT if they place *their* A, B, C with incorrect denominators.
	Substitute limits correctly in an integral with two terms (obtained correctly) of the form $a\ln(1+2x)+b\ln(2-x)+\frac{c}{2-x}$, where $abc\neq 0$	**M1**	Condone minor slips in substitution. Exact substitution required.
	Obtain answer $\frac{5}{2}-\ln 72$ after full and correct working	**A1**	AG – evidence of some correct work to combine or simplify logs is required e.g. allow from $-\ln 9+\ln\frac{1}{8}$ or $-\ln 2^3-\ln 3^2$.
		5	

Question	Answer	Marks	Guidance
10(a)	Use the product rule correctly to obtain $p(x+5)(3-2x)^n + q(3-2x)^{\frac{1}{2}}$	*M1	Allow with incorrect chain rule. BOD over sign errors unless an incorrect rule is quoted.
	Obtain correct derivative in any form	A1	e.g. $-(x+5)(3-2x)^{-\frac{1}{2}} + (3-2x)^{\frac{1}{2}}$.
	Equate derivative to zero and obtain a linear equation	DM1	Allow with surd factor e.g. $(3-2x)^{-\frac{1}{2}}\left(-(x+5)+(3-2x)\right)=0$.
	Obtain a correct linear equation.	A1	e.g. $-(x + 5) + 3 - 2x = 0$.
	Obtain answer $\left(-\frac{2}{3}, \frac{13\sqrt{39}}{9}\right)$.	A1	Or exact equivalent e.g. $\left(-\frac{2}{3}, \frac{13\sqrt{13}}{3\sqrt{3}}\right)$ or $\left(-\frac{2}{3}, \frac{\sqrt{2197}}{\sqrt{27}}\right)$. Accept with x, y stated separately. ISW
	Alternative Method for Question 10(a)		
	Obtain y^2 and differentiate	*M1	Ignore *their* left hand side i.e. *their* $\frac{\mathrm{d}}{\mathrm{d}x}y^2$.
	Obtain correct derivative in any form	A1	e.g. $-6x^2 - 34x - 20$.
	Equate derivative to zero and solve for x	DM1	
	Obtain $-\frac{2}{3}$	A1	Ignore –5 if seen.
	Obtain answer $\left(-\frac{2}{3}, \frac{13\sqrt{39}}{9}\right)$ only	A1	Or exact equivalent e.g. $\left(-\frac{2}{3}, \frac{13\sqrt{13}}{3\sqrt{3}}\right)$ or $\left(-\frac{2}{3}, \frac{\sqrt{2197}}{\sqrt{27}}\right)$. ISW
		5	

Question	Answer	Marks	Guidance
10(b)	Use the given substitution and reach $a\int\left(\frac{13}{2}-\frac{u}{2}\right)u^{\frac{1}{2}}\,\mathrm{d}u$	*M1	OE Need to see -2 or -½ used. Condone if du missing or the integral sign is missing. Allow M1A0 for complete substitution into $\int x\sqrt{3-2x}\,\mathrm{d}x$ to obtain first term of the line below.
	Obtain correct integral $-\frac{1}{2}\int\left(\frac{13}{2}-\frac{u}{2}\right)u^{\frac{1}{2}}\,\mathrm{d}u$	A1	OE e.g. $-\frac{1}{2}\left[\int\frac{3-u}{2}\sqrt{u}\,\mathrm{d}u+5\int\sqrt{u}\,\mathrm{d}u\right]$. Ignore limits at this stage. Condone if du missing.
	$x=-5$ and $\frac{3}{2}$	B1	SOI e.g. by u = 13 and 0. In any order and at any stage.
	Use correct limits the right way round in an integral of the form $a\left(\frac{26}{3}u^{\frac{3}{2}}-\frac{2}{5}u^{\frac{5}{2}}\right)$	DM1	
	Obtain answer $\frac{169}{15}\sqrt{13}$ or $a=\frac{169}{15}$	A1	or exact equivalents.
		5	

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td>11(a)</td><td>Carry out correct method for finding a vector equation for <i>AB</i></td><td>M1</td><td></td></tr>
<tr><td></td><td>Obtain $[\mathbf{r} =]\ \mathbf{i} + 2\mathbf{j} - 2\mathbf{k} + \lambda(\mathbf{i} - 3\mathbf{j} + 3\mathbf{k})$</td><td>A1</td><td>OE e.g. $\mathbf{r} = 2\mathbf{i} - \mathbf{j} + \mathbf{k} + \lambda(-\mathbf{i} + 3\mathbf{j} - 3\mathbf{k})$.</td></tr>
<tr><td></td><td>Equate two pairs of components of general points on <i>their AB</i> and l and evaluate λ or μ</td><td>M1</td><td>$\begin{pmatrix} 1+\lambda \\ 2-3\lambda \\ -2+3\lambda \end{pmatrix} = \begin{pmatrix} 1+2\mu \\ -1-3\mu \\ 3+4\mu \end{pmatrix}$.</td></tr>
<tr><td></td><td>Obtain correct answer for λ or μ, e.g. $\lambda = -1$, $\mu = -2$</td><td>A1</td><td>Correct value from two correct component equations.</td></tr>
<tr><td></td><td>Verify that all three equations are not satisfied and the lines fail to intersect
($\neq$ is sufficient justification e.g. $0 \neq -3$).</td><td>A1</td><td>Conclusion needs to follow correct values.
Hybrid versions are possible e.g. using j and k to get one parameter and then i to obtain the other.
or e.g. solving two pairs of simultaneous equations and showing that the results are not the same.
Alternatives:
<table>
<tr><td><i>A</i></td><td>λ</td><td>μ</td><td></td><td><i>B</i></td><td>λ</td><td>μ</td><td></td></tr>
<tr><td>ij</td><td>2</td><td>1</td><td>$4 \neq 7$</td><td>ij</td><td>1</td><td>1</td><td>$4 \neq 7$</td></tr>
<tr><td>ik</td><td>5</td><td>5/2</td><td>$-13 \neq -17/2$</td><td>ik</td><td>4</td><td>5/2</td><td>$-13 \neq -17/2$</td></tr>
<tr><td>jk</td><td>–1</td><td>–2</td><td>$0 \neq -3$</td><td>jk</td><td>–2</td><td>–2</td><td>$0 \neq -3$</td></tr>
</table>
</td></tr>
<tr><td></td><td></td><td>5</td><td></td></tr>
</table>

Question	Answer	Marks	Guidance
11(b)	Find $\overrightarrow{AP}$ for a general point P on l, e.g. $-3\mathbf{j} + 5\mathbf{k} + \mu(2\mathbf{i} - 3\mathbf{j} + 4\mathbf{k})$	**B1**	Or equivalent e.g. $\overrightarrow{PA} = -2\mu\mathbf{i} + (3\mu+3)\mathbf{j} - (4\mu+5)\mathbf{k}$.
	Calculate scalar product of *their* $\overrightarrow{AP}$ and a direction vector for l and equate the result to zero	**M1**	e.g. $4\mu + (9+9\mu) + (20+16\mu) = 0$. M0 if using $\overrightarrow{OP}$. M0 if using parallel line through A.
	Obtain $\mu = -1$	**A1**	
	Obtain answer $-\mathbf{i} + 2\mathbf{j} - \mathbf{k}$	**A1**	Accept coordinates in place of position vector.
	Alternative Method for Question 11(b)		
	Find $\overrightarrow{AP}$ for a general point P on l, e.g. $-3\mathbf{j} + 5\mathbf{k} + \mu(2\mathbf{i} - 3\mathbf{j} + 4\mathbf{k})$	**B1**	Or equivalent e.g. $\overrightarrow{PA} = -2\mu\mathbf{i} + (3\mu+3)\mathbf{j} - (4\mu+5)\mathbf{k}$.
	Use Pythagoras and differentiate with respect to μ to obtain value of μ corresponding to minimum distance. (No need to prove it is a minimum)	**M1**	$\frac{d}{d\mu}\left(4\mu^2 + 9(\mu+1)^2 + (4\mu+5)^2\right) = 0$.
	Obtain $\mu = -1$	**A1**	
	Obtain answer $-\mathbf{i} + 2\mathbf{j} - \mathbf{k}$	**A1**	Accept coordinates in place of position vector.
		4	

Cambridge Assessment International Education

Cambridge International A Level

MATHEMATICS **9709/33**

Paper 3 Pure Mathematics 3 **May/June 2023**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2023 series for most Cambridge IGCSE, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

This document consists of **26** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

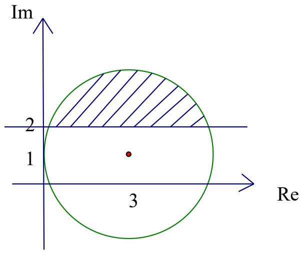
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	Use exponentials or law for the logarithm of a product, quotient or power	**M1***	$e^{\ln(5+x)} = e^{5+\ln x}$ insufficient. Need e.g. $\ln\left(\frac{x+5}{x}\right) = 5$ or $\ln(x+5) = \ln(e^5) + \ln x$ or $\ln(x + 5) = \ln(e^5x)$ or $x + 5 = e^{5 + \ln x}$ or $x + 5 = e^5 e^{\ln x}$ and others.
	Correctly remove logarithms	**DM1**	
	Obtain a correct equation in x	**A1**	e.g. $\frac{x+5}{x} = e^5$ (or 148.4…) or $x + 5 = xe^5$.
	Obtain 0.034	**A1**	CAO Final answer must be 3d.p.
		4	

Question	Answer	Marks	Guidance
2	Divide to obtain quotient $2x^2 \pm 2x + k \ (k \neq 0)$	**M1**	Obtain result in answer column, together with a linear polynomial or a constant as remainder. If correct: $\begin{array}{rl} & \underline{2x^2 - 2x - 4} \\ x^2+x+3 & 2x^4 \qquad\qquad\qquad -27 \\ & \underline{2x^4 + 2x^3 + 6x^2} \\ & -2x^3 - 6x^2 \\ & \underline{-2x^3 - 2x^2 - 6x} \\ & -4x^2 + 6x - 27 \\ & \underline{-4x^2 - 4x - 12} \\ & 10x - 15 \end{array}$
	Obtain [quotient] $2x^2 - 2x - 4$	**A1**	Allow unless quotient and remainder interchanged, then A0 A1.
	Obtain [remainder] $10x - 15$	**A1**	Allow $(x^2 + x + 3)(2x^2 - 2x - 4) + 10x - 15$.
	Alternative Method for Question 2		
	Expand $\left(x^2+x+3\right)\left(Ax^2+Bx+C\right)+\left(Dx+E\right)$ and reach $A = 2$, $B = \pm 2$, $C = k$	**M1**	Solve all 3 equations for A, B and C, allow sign errors in establishing equations and in solving. If correct, $A = 2$, $A + B = 0$, $3A + B + C = 0$, $3B + C + D = 0$, $3C + E = -27$. Obtain result in answer column, together with a linear polynomial or a constant as remainder.
	Obtain [quotient] $2x^2 - 2x - 4$	**A1**	Allow unless quotient and remainder interchanged, then A0 A1.
	Obtain [remainder] $10x - 15$	**A1**	Allow $(x^2 + x + 3)\ (2x^2 - 2x - 4) + 10x - 15$.
		3	

Question	Answer	Marks	Guidance
3	Show a circle with centre $3+\mathrm{i}$	**B1**	Must be some evidence of scale on both axes or centre stated as 3 + i or (3, 1).
	Show a circle with radius 3 and centre not at the origin	**B1**	Must be some evidence that radius = 3 or stated $r = 3$
	Show the line $y = 2$	**B1**	Line $y = 2$ can be represented by 2 or correct dashes.
	Shade the correct region	**B1**	Line and circle must be correct.
		**4**	Scales may be replaced by dashes on axes for all marks. Correct figure, with no scale on either axis then allow 1/3 and the B1 for correct shaded region Max 2/4. If B0 above for line but relatively correct position then B1 for correct shaded region Max 3/4. Re and Im axes interchanged but clearly labelled, allow **SCB1** for centre and radius of circle correct and **SCB1** for line and shading correct Max 2/4.

Question	Answer	Marks	Guidance
4	State $\dfrac{dy}{d\theta} = 1 - 2\sin\theta$	**B1**	Ignore left side throughout dx/dt, dy/dt, dx, dy but must see $\dfrac{dy}{dx}$ for final A1.
	Use correct quotient rule, or product rule if rewrite x as $\cos\theta(2 - \sin\theta)^{-1}$	**M1**	Incorrect formula seen M0 A0 otherwise BOD.
	Obtain $\dfrac{dx}{d\theta} = \dfrac{-(2-\sin\theta)\sin\theta + \cos^2\theta}{(2-\sin\theta)^2}$ o.e.	**A1**	$-\sin\theta(2-\sin\theta)^{-1} - \cos\theta(2-\sin\theta)^{-2}(-\cos\theta)$ or equivalent.
	Use $\dfrac{dy}{dx} = \dfrac{dy}{d\theta} \div \dfrac{dx}{d\theta}$	**M1**	$\left(\dfrac{dy}{dx} = (1-2\sin\theta) \div \dfrac{1-2\sin\theta}{(2-\sin\theta)^2}\right)$. Allow M1 even if errors in both derivatives.
	Obtain $\dfrac{dy}{dx} = (2-\sin\theta)^2$.	**A1**	AG – must see working in above cell to gain final A1. Allow $\cos^2\theta + \sin^2\theta = 1$ to be implied. x instead of θ or missing θ more than twice on right side then A0 final mark.
		5	

Question	Answer	Marks	Guidance
5(a)	Use correct product rule	**M1**	$\frac{\mathrm{d}}{\mathrm{d}x}(x^2)\cos(3x)+x^2\frac{\mathrm{d}}{\mathrm{d}x}(\cos 3x)$.
	Obtain correct derivative in any form	**A1**	e.g. $2x\cos 3x-3x^2\sin 3x$.
	Equate derivative to zero and obtain $a=\frac{1}{3}\tan^{-1}\left(\frac{2}{3a}\right)$.	**A1**	AG Condone $a=\frac{1}{3}\tan^{-1}\frac{2}{3a}$. Must at least reach expression $2x = 3x^2\tan(3x)$ or better before final answer to gain A1. Final answer must be in terms of a. Can work with x and switch to a at very end. Look for $\frac{2}{3}a$ or $\frac{2}{3}x$ in working not immediately corrected or as penultimate line A0.
		3	
5(b)	Use the iterative process $a_{n+1}=\frac{1}{3}\tan^{-1}\left(\frac{2}{3a_n}\right)$ correctly at least twice during successive iterations in the numerous iterations	**M1**	Degrees 0/3.
	Obtain final answer 0.36	**A1**	Must be 2d.p.
	Show sufficient iterations to 4 or more d.p. to justify 0.36 to 2 d.p. or show there is a sign change in the interval $(0.355, 0.365)$	**A1**	Allow small errors in 4th d.p. Allow errors at start if self corrects later.
	0.5 0.4 0.3 0.2 0.1 π/6 π/12 0.3091 0.3435 0.3826 0.4264 0.4740 0.3017 0.3989 0.3789 0.3650 0.3499 0.3339 0.3176 0.3820 0.3439 0.3513 0.3566 0.3625 0.3688 0.3754 0.3502 0.3649 0.3619 0.3599 0.3576 0.3552 0.3526 0.3624 0.3567 0.3578 0.3604 0.3614 0.3576 0.3580	**3**	

Question	Answer	Marks	Guidance
6(a)	Expand $\cos(x-60°)$ correctly and evaluate $3\cos x + 2\cos(x-60°)$ to obtain $4\cos x+\sqrt{3}\sin x$ or unsimplified coefficients	**B1**	Need to see $A\cos x + B\sin x$ with A and B correct A may be 4 or $3 + 2\cos 60$ and B may be $\sqrt{3}$ or $2\sin 60$.
	State $R=\sqrt{19}$ [$R\cos\alpha = 4$ $R\sin\alpha = \sqrt{3}$]	**B1 FT**	Follow through *their* 4 and $\sqrt{3}$. If coefficients are 3 and 2 then B0. $R = \sqrt{19}$ from $R = 4.36$ B0 but 4.36 seen after $\sqrt{19}$ ISW.
	Use correct trig formulae for their expansion to find α e.g. $\alpha=\tan^{-1}\frac{\sqrt{3}}{4}$ or $\cos^{-1}\frac{4}{\sqrt{19}}$ or $\sin^{-1}\frac{\sqrt{3}}{\sqrt{19}}$	**M1**	If $\sin\alpha = \sqrt{3}$ $\cos\alpha = 4$ seen then M0 A0. If $\underline{\tan\alpha} = 23.41°$ M0 A0 but can recover if $\alpha = 23.41°$ seen later. $\alpha=\tan^{-1}\frac{2}{3}$ M1 $(\alpha=33.69°)$ but $\alpha=\tan^{-1}\frac{3}{2}$ M0
	Obtain $\alpha = 23.41°$	**A1**	Allow if x instead of α.
		4	

Question	Answer	Marks	Guidance
6(b)	$\cos^{-1}\left(\frac{2.5}{R}\right)$	**B1 FT**	SOI [55.0°]. Follow through *their* $\sqrt{19}$.
	Use a correct method to find a value of 2θ (not x) in the interval. Allow sign error in moving α to right side	**M1**	$2\theta = \cos^{-1}\left(\frac{2.5}{R}\right) + 23.41°$ or $2\theta = 360° - \cos^{-1}\left(\frac{2.5}{R}\right) + 23.41°$ with R substituted.
	Obtain one correct answer e.g. $39.2°$	**A1**	If working for **M1** not seen then **M1** implied by $39.2°$ or $164.2°$ Must be at least 1d.p.
	Obtain second correct answer e.g. $164.2°$ and no others in the interval	**A1**	Must be at least 1d.p. Ignore answers outside the given interval.
		4	

Question	Answer	Marks	Guidance
7(a)	$\frac{du}{dx} = -\sin x$	**B1**	SOI
	Use double angle formula and substitute for x and dx throughout the integral	**M1**	All x's must be removed, can be coefficient errors provided 2 seen in working.
	Obtain $\pm\int 2ue^{2u}du$	**A1**	Limits may be omitted, or left as 0 and π, during the change of variable stage.
	Justify new limits and obtain $\int_{-1}^{1} 2ue^{2u}du$ from correct working	**A1**	AG Must see $x = 0$, $u = 1$ and $x = \pi$, $u = -1$. Inequalities alone e.g. $0 \leqslant x \leqslant \pi$ and $1 \leqslant u \leqslant -1$ or $-1 \leqslant u \leqslant 1$ for limits are insufficient A0 If sign in expression and order of limits incorrect then A0. If negative sign is present in the integrand then this can be removed and limits introduced in correct order in a single step.
		4	

Question	Answer	Marks	Guidance
7(b)	Commence integration and reach $aue^{2u} + b\int e^{2u}\,du$, where $ab \neq 0$, $b < 0$	**M1***	Condone d*x*.
	Complete integration and obtain $ue^{2u} - \frac{1}{2}e^{2u}$	**A1**	OE Allow $(2u\frac{1}{2}e^{2u}) - \frac{1}{2}e^{2u}$.
	Use correct limits correctly in $cue^{2u} + d\,e^{2u}$ having integrated twice or in $c\cos x\,e^{2\cos x} + d\,e^{2\cos x}$	**DM1**	1 and −1 for *u*, 0 and π for *x* e.g. $ce^2 + de^2 - (-ce^{-2} + de^{-2})$. Not decimals. Allow one sign error at most in going from $cue^{2u} + d\,e^{2u}$ or $c\cos x\,e^{2\cos x} + d\,e^{2\cos x}$ to $ce^2 + de^2 - (-ce^{-2} + de^{-2})$. $[e^2 - ½e^2 - (-e^{-2} - ½e^{-2})]$ Complete reversal of sign by converting back to cos *x* and not making *x* = 0 upper limit is DM0 A0.
	Obtain $\frac{1}{2}e^2 + \frac{3}{2}e^{-2}$	**A1**	ISW Or equivalent 2-term expression e.g. $\frac{e^4+3}{2e^2}$ or $\frac{1}{2}\left(e^2 + \frac{3}{e^2}\right)$.
		4	

Question	Answer	Marks	Guidance
8	Separate the variables correctly	**B1**	$\frac{y+4}{y^2+4}\mathrm{d}y = \frac{1}{x}\mathrm{d}x$.
	Obtain $\ln x$	**B1**	
	Split the fraction and integrate to obtain $p\ln\left(y^2+4\right)$ or $q\tan^{-1}\frac{y}{2}$ correctly	***M1**	Only following subdivision into $\frac{y}{y^2+4}+\frac{4}{y^2+4}$. If no subdivision seen then both terms $p\ln\left(y^2+4\right)$ and $q\tan^{-1}\frac{y}{2}$ must be present.
	Obtain $\frac{1}{2}\ln\left(y^2+4\right)$	**A1**	
	Obtain $2\tan^{-1}\frac{y}{2}$	**A1**	
	Use $\left(4, 2\sqrt{3}\right)$ in an expression containing at least 2 of $a\ln x, b\ln\left(y^2+4\right)$ and $c\tan^{-1}\frac{y}{2}$ to obtain constant of integration	**DM1**	Allow one sign or arithmetic error e.g. $\frac{2\pi}{3}$. May use $\left(4, 2\sqrt{3}\right)$ and $(x, 2)$ as limits to find x for the final 3 marks.
	Correct solution (any form) e.g. $\frac{1}{2}\ln\left(y^2+4\right)+2\tan^{-1}\frac{y}{2}=\ln x+\frac{2\pi}{3}$ or $\frac{1}{2}\ln\left(y^2+4\right)+2\tan^{-1}\frac{y}{2}=\ln x+2\tan^{-1}\sqrt{3}+\frac{1}{2}\ln 16-\ln 4$	**A1**	However solution not asked for so allow $\frac{1}{2}\ln 8+2\tan^{-1}1=\ln x+2\tan^{-1}\sqrt{3}+\frac{1}{2}\ln 16-\ln 4$.
	Obtain $\sqrt{8}\mathrm{e}^{-\frac{1}{6}\pi}$ or 1.68 or more accurate or $2\sqrt{2}\mathrm{e}^{-\frac{1}{6}\pi}$ or $\frac{\sqrt{8}}{\mathrm{e}^{\frac{1}{6}\pi}}$ or $\mathrm{e}^{0.516}$	**A1**	ISW Must remove ln so $x = \mathrm{e}^{(\ln 2\sqrt{2} - \pi/6)}$ A0**.**

Question	Answer	Marks	Guidance
8	**Alternative method for first *M1 A1 A1**		
	$p\left((y+4)\tan^{-1}\dfrac{y}{2}-\int\tan^{-1}\dfrac{y}{2}\mathrm{d}y\right)$	***M1**	Allow sign error.
	$(y+4)\dfrac{1}{2}\tan^{-1}\dfrac{y}{2}-\dfrac{y}{2}\tan^{-1}\dfrac{y}{2}+\int\dfrac{y}{y^2+4}\mathrm{d}y$	**A1**	
	Obtain $2\tan^{-1}\frac{y}{2}+\frac{1}{2}\ln\left(y^2+4\right)$	**A1**	
		8	

Question	Answer	Marks	Guidance
9(a)	Perform scalar product of direction vectors and set result equal to zero	**M1**	$2c+6+4=0$.
	Use P to find the value of λ	**M1**	$3-2\lambda=7 \Rightarrow \lambda=-2$ [$a+\lambda c=4$, $b+4\lambda=-2$]. Equation for line l may contain $-\lambda$ instead of $+\lambda$ leading to $\lambda=2$ all marks available.
	Obtain $c=-5$ or $b=6$	**A1**	
	$a=-6$, $b=6$ and $c=-5$ all correct	**A1**	
		4	**SC1**: Use P to find the value of λ M1 Substitute $\lambda=-2$ into point P, so $a-2c=4$, and put $\mu=-1$ and $\lambda=-1$ into l so $a-c=-1$, then solve to obtain $a=-6$, $b=6$ and $c=-5$. All 3 values correct A1. Max 2/4.

Question	Answer	Marks	Guidance
9(b)	Find $\overrightarrow{PQ}$ (or $\overrightarrow{QP}$) for a general point *Q* on *m* $= \pm((1+2\mu, 2-3\mu, 3+\mu)-(a+\lambda c, 3-2\lambda, b+4\lambda))$	**B1**	$\left[\overrightarrow{PQ} \text{ or } \overrightarrow{QP} = \pm\begin{pmatrix}-3+2\mu\\-5-3\mu\\5+\mu\end{pmatrix}\right]$ Could be *their a, b, c* and λ values provided M1 M1 gained in **(a)**. Allow expression in answer column.
	Equate the scalar product of $\overrightarrow{PQ}$ (or $\overrightarrow{QP}$) and a direction vector for *m* to zero and obtain an equation in μ	**M1***	$(2(-3+2\mu)-3(-5-3\mu)+(5+\mu))=0$. Allow $\overrightarrow{PQ} = \overrightarrow{OQ} + \overrightarrow{OP}$ sign problem.
	Solve and obtain $\mu=-1$	**A1**	$PQ^2 = (-3+2\mu)^2+(-5-3\mu)^2+(5+\mu)^2$. [$= 14(\mu+1)^2+45$]. Min when $\mu = -1$ or by differentiation.
	Obtain $\overrightarrow{OQ} = -\mathbf{i}+5\mathbf{j}+2\mathbf{k}$ or $\overrightarrow{PQ} = -5\mathbf{i}-2\mathbf{j}+4\mathbf{k}$ Must be labelled correctly	**A1**	The working may be in **(a)** provided at least this result is used in **(b)**.
	Carry out a method to find the position vector of *R* **Alternative method for DM1** $\overrightarrow{OR} = (4, 7, -2)+t(-5, -2, 4)$ $\overrightarrow{QR} = \overrightarrow{OR} - \overrightarrow{OQ}$ Solve $\lvert\mathbf{QR}\rvert^2 = \frac{9}{4}\lvert\mathbf{PQ}\rvert^2$ or $\lvert\mathbf{QR}\rvert = \frac{3}{2}\lvert\mathbf{PQ}\rvert$ $t = 2.5$	**DM1**	e.g. Use $\overrightarrow{OR} = \overrightarrow{OP} + \frac{5}{2}\overrightarrow{PQ}$ or $\overrightarrow{OR} = \overrightarrow{OQ} + \frac{3}{2}\overrightarrow{PQ}$ or $\overrightarrow{OR} = \frac{5}{2}\overrightarrow{OQ} - \frac{3}{2}\overrightarrow{OP}$ or $2\overrightarrow{QR} = 2(\overrightarrow{OR} - \overrightarrow{OQ}) = 3\overrightarrow{PQ}$ where $\overrightarrow{OR} = (x, y, z)$. $\overrightarrow{PQ}$ used in all these approaches, may be incorrect, must be in the correct direction, i.e. not using $\overrightarrow{QP}$ for $\overrightarrow{PQ}$.

Question	Answer	Marks	Guidance
9(b)	Obtain $-\frac{17}{2}\boldsymbol{i}+2\boldsymbol{j}+8\boldsymbol{k}$ from correct working	**A1**	Accept coordinates. Don't accept $-\frac{17}{2}\mathbf{i}+\frac{4}{2}\mathbf{j}+\frac{16}{2}\mathbf{k}$.
		6	**SC2** Equate lines, attempt to find $\mu=-1$ or $\lambda=-1$ M1* $\overrightarrow{OQ}=-\mathbf{i}+5\mathbf{j}+2\mathbf{k}$ A1. Attempt to find $\overrightarrow{OQ}$ using other parameter value DM1. $\overrightarrow{OQ}=-\mathbf{i}+5\mathbf{j}+2\mathbf{k}$ therefore intersect A1. Then use main scheme for the final DM1 A1.
			First **DM1 A1** are available if they show the 3 coordinates are consistent for the 2 parameter values instead of attempting to find $\overrightarrow{OQ}$ using the other parameter value and then showing intersection

Question	Answer	Marks	Guidance
10(a)	State or imply the form $\dfrac{A}{1+2x}+\dfrac{B}{3-x}+\dfrac{C}{(3-x)^2}$	**B1**	Alternative form: $\dfrac{A}{1+2x}+\dfrac{Dx+E}{(3-x)^2}$.
	Use a correct method to find a constant	**M1**	Incorrect format for partial fractions: Allow M1 and a possible A1 if obtain one of these correct values. Max 2/5 Allow M1 even if multiply up by $(1 + 2x)(3 - x)^3$.
	Obtain one of $A = 2$, $B = 2$ and $C = -3$	**A1**	Alternative form: obtain one of $A = 2$, $D = -2$ and $E = 3$.
	Obtain a second value	**A1**	
	Obtain the third value	**A1**	Do not need to substitute values back into original form.
		5	If $\dfrac{A}{1+2x}+\dfrac{B}{3-x}+\dfrac{Cx+D}{(3-x)^2}$ B0 but M1 A1 for *A*, A1 for *B* and A1 for *C* and *D*. If $C = 0$ then recovers B1 from above.

Question	Answer	Marks	Guidance
10(b)	Use a correct method to obtain the first two terms of one of the unsimplified expansions $(1+2x)^{-1}, \left(1-\frac{1}{3}x\right)^{-1}, \left(1-\frac{1}{3}x\right)^{-2}$ $(3-x)^{-1}$, $(3-x)^{-2}$	**M1**	$(1+2x)^{-1}=1+(-1)(2x)+\dots$ $\left(1-\frac{1}{3}x\right)^{-1}=1+(-1)(-x/3)+\dots$ $\left(1-\frac{1}{3}x\right)^{-2}=1+(-2)(-x/3)+\dots$ $(3-x)^{-1}=3^{-1}+(-1)3^{-2}(-x)\dots$ $(3-x)^{-2}=3^{-2}+(-2)3^{-3}(-x)+\dots$
	Obtain the correct unsimplified expansions up to the term in x^2 for each partial fraction If correct, should be working with $\frac{2}{1+2x}+\frac{2}{3-x}-\frac{3}{(3-x)^2}$ or $\frac{2}{1+2x}+\frac{-2x+3}{(3-x)^2}$	**A1 FT** **A1 FT** **A1 FT**	Follow through on *their* A, B, C $A(1+(-1)(2x)+((-1)(-2)/2)(2x)^2+\dots)$ $\frac{B}{3}(1+(-1)(-x/3)+((-1)(-2)/2)(-x/3)^2+\dots)$ $\frac{C}{3^2}(1+(-2)(-x/3)+((-2)(-3)/2)(-x/3)^2+\dots)$. Must be *their* coefficients from **(a)** but may be unsimplified expansions for FT marks. If correct, expect to see $2(1-2x+(2x)^2)$ or $2-4x+8x^2$ $\frac{2}{3}\left(1+\frac{x}{3}+\left(\frac{x}{3}\right)^2\right)$ or $\frac{2}{3}+\frac{2}{9}x+\frac{2}{27}x^2$ $-\frac{1}{3}\left(1+\frac{2x}{3}+(3)\left(\frac{x}{3}\right)^2\right)$ or $-\frac{1}{3}-\frac{2}{9}x-\frac{x^2}{9}$.
	Obtain final answer $\frac{7}{3}-4x+\frac{215}{27}x^2$	**A1**	Accept $2\frac{1}{3}-4x+7\frac{26}{27}x^2$. No ISW.

Question	Answer	Marks	Guidance
10(b)	**Alternative Method for Question 10(b)**		
	For the form $\frac{A}{1+2x}+\frac{Dx+E}{(3-x)^2}$	**M1***	For the first two terms of an expanded partial fraction, following their A, D, E.
	Obtain the correct unsimplified expansions up to the term in x^2 for each partial fraction	**A1FT** **A1FT**	$A(1 + (-1)(2x) + ((-1)(-2)/2)(2x)^2 + \ldots) +$ $(Dx + E)\frac{1}{3^2}(1 + (-2)(-x/3) + ((-2)(-3)/2)(-x/3)^2 +)$ $2(1 - 2x + (2x)^2 + \ldots)$ $+ \frac{-2x+3}{3^2}(1 + \frac{2x}{3} + (3)\left(\frac{x}{3}\right)^2 + \ldots)$.
	Multiply out fully	**DM1**	Provided $DE \neq 0$. Ignore cubic terms and above. Allow error in one term but all terms must be present. If correct, expect to see $2-4x+8x^2-\frac{2}{9}x-\frac{4}{27}x^2+\frac{1}{3}+\frac{2}{9}x+\frac{1}{9}x^2$
	Obtain final answer $\frac{7}{3}-4x+\frac{215}{27}x^2$	**A1**	Accept $2\frac{1}{3}-4x+7\frac{26}{27}x^2$. No ISW

Question	Answer	Marks	Guidance
10(b)	**Alternative Method for Question 10(b): Maclaurin's Series**		
	Correct derivatives for $A(1+2x)^{-1}$, $B(3-x)^{-1}$ and $C(3-x)^{-2}$ $(-1)(2)A(1+2x)^{-2}$, $(-1)(-1)B(3-x)^{-2}$ and $(-2)(-1)C(3-x)^{-3}$	**B1 FT**	
	One of following $(-2)(2)(-1)(2)A(1+2x)^{-3}$, $(-2)(-1)(-1)(-1)B(3-x)^{-3}$ and $(-3)(-1)(-2)(-1)C(3-x)^{-4}$	**B1 FT**	
	All correct	**B1 FT**	
	Substitute in $f(0)+xf'(0)+\frac{x^2}{2}f''(0)$	**M1**	
	Obtain final answer $\frac{7}{3}-4x+\frac{215}{27}x^2$	**A1**	Accept $2\frac{1}{3}-4x+7\frac{26}{27}x^2$. No ISW
		5	

Question	Answer	Marks	Guidance
11(a)	Multiply numerator and denominator by $(3-ai)$	**M1**	Must perform complete multiplications but need not simplify i^2. Can have errors but no term duplicated or missing. $\frac{(5a-2i)(3-ai)}{9-a^2} = \frac{13a-i(5a^2+6)}{9-a^2}$ M0 M1 A0 No working so unsure if denominator multiplied by $3-ai$ M1 M1 A0
	Use $i^2=-1$ at least once and separate real and imaginary parts	**M1**	
	Obtain $\frac{13a-i(5a^2+6)}{9+a^2}$ or $\frac{13a-5a^2i-6i}{9+a^2}$	**A1**	OE If $15a-2a=13a$ seen later award this A1.
	Use $\arg z$ to form equation in a $-\frac{5a^2+6}{13a}=\pm\tan\left(\pm\frac{\pi}{4}\right)$ or $-\frac{13a}{5a^2+6}=\pm\tan\left(\pm\frac{\pi}{4}\right)$ or $\tan^{-1}\left(-\frac{5a^2+6}{13a}\right)=\pm\frac{\pi}{4}$ or $\tan^{-1}\left(-\frac{13a}{5a^2+6}\right)=\pm\frac{\pi}{4}$	**M1**	Allow expression given in answer column or $5a^2+6=\pm 13a$ or use $-(x \pm xi) = (13a-i(5a^2+6))/(9+a^2)$ and eliminate x so $5a^2+6=\pm 13a$ M1.
	Obtain $a=2$	**A1**	Need to reject $a=\frac{3}{5}$ or ignore it in future work. May not see second root, but if present, must be $\frac{3}{5}$.
	Obtain $z=2-2i$ only	**A1**	Allow $z=-2i+2$.

Question	Answer	Marks	Guidance
11(a)	**Alternative Method 1 for the first four marks**		
	arg z = arg $(5a - 2i)$ – arg $(3 + ai)$	**M1**	
	$= \tan^{-1}\left(\frac{-2}{5a}\right) - \tan^{-1}\left(\frac{a}{3}\right)$ $= \tan^{-1}\left(\frac{-2}{5a} - \frac{a}{3}\right) / \left(1 + \left(\frac{-2}{5a}\right)\left(\frac{a}{3}\right)\right)$	**M1**	Allow one sign error in second M1.
	$= \tan^{-1}\left(-\frac{5a^2+6}{13a}\right)$ or $\tan^{-1}\left(-\frac{13a}{5a^2+6}\right)$	**A1**	
	$\pm\frac{\pi}{4} = \tan^{-1}\left(-\frac{5a^2+6}{13a}\right)$ or $\tan^{-1}\left(-\frac{13a}{5a^2+6}\right)$	**M1**	Equate *their* $\tan^{-1}\left(-\frac{5a^2+6}{13a}\right)$ to $\pm\frac{\pi}{4}$. Then as original scheme for final 2 marks.
	Alternative Method 2 for the first four marks		
	$(x + iy)(3 + ai) = 5a - 2i$ $3x - ay = 5a$ and $ax + 3y = -2$	**M1 A1**	
	$x = \pm y$ Find x or y in terms of a, e.g. $x = \frac{2}{3-a}$ or $x = \frac{5a}{3+a}$	**M1**	
	Substitute in other equation, for example $3\left(\frac{2}{3-a}\right) + a\left(\frac{2}{3-a}\right) = 5a$	**M1**	Then as original scheme for final 2 marks.
		6	

Question	Answer	Marks	Guidance
11(b)	State $\arg\left(z^3\right) = -\frac{3}{4}\pi$ or evaluate from $z = b - b\text{i}$ or from $-2b^3(1+\text{i})$	**B1**	If 2 different values given award B0. Do not ISW.
	Complete method to obtain *r* from *their z*	**M1**	$\left\lvert z^3\right\rvert = \left(\sqrt{x^2+y^2}\right)^3$. If z correct, may see $\left\lvert z^3\right\rvert = \left(\sqrt{2^2+(-2)^2}\right)^3$ or $\left\lvert z^3\right\rvert = \sqrt{(-16)^2+(-16)^2}$.
	$r = 16\sqrt{2}$	**A1**	CAO A1 if $z = 2 - 2\text{i}$ obtained correctly. or $z =$ used with $a = 2$ found correctly, otherwise A0XP. May see arg and r given in a final answer i.e. $16\sqrt{2}\text{e}^{-\frac{3}{4}\pi\text{i}}$. Allow this form for arg and r to collect full marks, even if i missing. Ignore answers outside the given interval. If 2 different values given award A0.
		3	

Cambridge Assessment
International Education

Cambridge International A Level

MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **February/March 2023**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the February/March 2023 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

This document consists of **17** printed pages.

[Turn over

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate’s intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

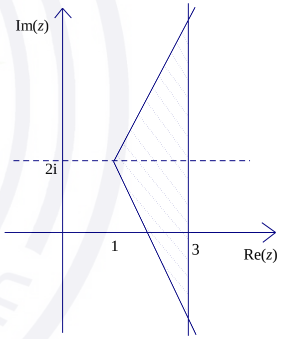
M	Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
A	Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
B	Mark for a correct result or statement independent of method marks.
DM or **DB**	When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
FT	Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	Use law of the logarithm of a quotient or express x as $\ln e^x$	**M1**	$x = \ln[(2y – 3)/(y + 4)]$ or $\ln e^x = \ln(2y – 3) − \ln (y + 4)]$.
	Remove logarithms and obtain a correct equation e.g. $e^x = \dfrac{2y-3}{y+4}$	**A1**	
	Obtain answer $y = \dfrac{3+4e^x}{2-e^x}$	**A1**	OE ISW
		3	

Question	Answer	Marks	Guidance
2(a)	Show correct half-lines from 1 + 2i, symmetrical about $y = 2i$ (drawn between $\dfrac{\pi}{4}$ and $\dfrac{5\pi}{12}$).	**B1**	
	Show the line $x = 3$ extending in both quadrants.	**B1**	
	Shade the correct region. Allow dashes on axes as scale. FT If only error is one of following: FULL lines or $x \neq 3$ or one sign error in 1 + 2i or angle outside tolerance or scale missing on one axis.	**B1 FT**	
			SC No scale on either axis allow **B1 FT** for otherwise correct figure in correct position.
		3	

Question	Answer	Marks	Guidance
2(b)	Carry out a complete method for finding the least value of arg z	**M1**	e.g. $-\tan^{-1}\frac{\left(2\sqrt{3}-2\right)}{3}$ or $\tan^{-1}\frac{\left(-2\sqrt{3}+2\right)}{3}$.
	Obtain answer –0.454 3dp	**A1**	**SC B1** 0.454 .
		2	

Question	Answer	Marks	Guidance
3	Commence division and reach partial quotient $2x^2 + (a \pm 2)x$	**M1**	$2x^2 + (a + 2)x + a$ need $2x^2 + (a \pm 2)x$ $(x^2 - x + 1)\ 2x^4 + ax^3 + 0x^2 + bx - 1$ $2x^4 - 2x^3 + 2x^2$ $(a + 2)x^3 - 2x^2 + bx$ $(a + 2)x^3 - (a + 2)x^2 + (a + 2)x$ $ax^2 + (b - (a + 2))x - 1$ $ax^2 - ax + a$ $(b - 2)x - (1 + a)$ $3x + 2$ Working backwards from remainder: $2x^2 + (\ldots.)x \pm 3$ **M1** $2x^2 - x - 3$ **A1**
	Obtain correct quotient $2x^2 + (a + 2)x + a$	**A1**	Allow sign error e.g. in $b - 2$.
	Set *their* linear remainder equal to part of "$3x + 2$" and solve for a or for b	**M1**	Remainder = $3x + 2 = (b - 2)x - 1 - a$. Allow for just equating x term or constant term.
	Obtain answer $a = -3$	**A1**	
	Obtain answer $b = 5$	**A1**	

Question	Answer	Marks	Guidance
3	**Alternative method for Question 3**		
	State $2x^4+ax^3+0x^2+bx-1=\left(x^2-x+1\right)\left(2x^2+Ax+B\right)+3x+2$ and form and solve equation(s) to obtain A or B	**M1**	e.g. $0 = B - A + 2$ and $-1 = B + 2$.
	Obtain $A = -1$, $B = -3$	**A1**	
	Form and solve equations for a or for b	**M1**	e.g. $a = A - 2$ or $b = -B + A + 3$.
	Obtain answer $a = -3$	**A1**	
	Obtain answer $b = 5$	**A1**	
	Alternative method for Question 3		
	Use remainder theorem with $x=\dfrac{1\pm\sqrt{-3}}{2}$ or $x=\dfrac{1\pm i\sqrt{3}}{2}$	**M1**	Allow for correct use of a reasonable attempt at either root in exact or decimal form in the remainder theorem $x^2=\dfrac{-1+\sqrt{-3}}{2}$ $x^3=-1$ $x^4=\dfrac{-1-\sqrt{-3}}{2}$.
	Obtain $-a+\dfrac{b}{2}\pm\dfrac{b\sqrt{-3}}{2}\mp\sqrt{-3}-2=\dfrac{7}{2}\pm\dfrac{3\sqrt{-3}}{2}$ or $-a+\dfrac{b}{2}\pm\dfrac{bi\sqrt{3}}{2}\mp i\sqrt{3}-2=\dfrac{7}{2}\pm\dfrac{3i\sqrt{3}}{2}$	**A1**	Expand brackets and obtain exact equation for either root. Accept exact equivalent.
	Solve simultaneous equations, or single equation, for a or for b	**M1**	
	Obtain answer $a = -3$ from exact working	**A1**	
	Obtain answer $b = 5$ from exact working	**A1**	
		5	

Question	Answer	Marks	Guidance
4	Substitute $z = x + iy$ and $z^* = x - iy$ to obtain a correct equation, horizontal or with (1– 2i)/(1 – 2i) seen, in x and y	**B1**	$5(x + iy) - (x + iy)(x - iy)(1 + 2i) + (30 + 10i)(1 + 2i) = 0$ $5(x + iy)(1 - 2i)/[(1 + 2i)(1 - 2i)] - (x + iy)(x - iy) + (30 + 10i) = 0$ $x - 2ix + iy + 2y - x^2 - y^2 + 30 + 10i = 0$.
	Use $i^2 = -1$ at least once and equate real and imaginary parts to zero	***M1**	OE For their horizontal equation.
	Obtain two correct equations e.g. $x + 2y - x^2 - y^2 + 30 = 0$ and $-2x + y + 10 = 0$	**A1**	$5x - (x^2 + y^2) + 10 = 0$ $5y - 2(x^2 + y^2) + 70 = 0$ $5y - 10x + 50 = 0$ $x + 2y - (x^2 + y^2) + 30 = 0$ Allow $-2ix + iy + 10i = 0$.
	Solve quadratic equation for x or for y	**DM1**	$x^2 - 9x + 18 = (x - 3)(x - 6) = 0$ $y^2 + 2y - 8 = (y + 4)(y - 2) = 0$ **DM0** If x or y imaginary.
	Obtain answers 3 – 4i and 6 + 2i	**A1**	
		5	

Question	Answer	Marks	Guidance
5(a)	Obtain $\frac{dx}{dt}=e^{2t}+2te^{2t}$	**B1**	OE
	Use $\frac{dy}{dx}=\frac{dy}{dt}\times\frac{dt}{dx}$	**M1**	$\frac{dy}{dx}=\frac{2t+1}{e^{2t}\left(1+2t\right)}$.
	Obtain the given answer $\frac{dy}{dx}$ = e $^{-2t}$	**A1**	AG Need to see e^{2t} (1 + 2t) in denominator.
		3	
5(b)	Obtain x = $-e^{-2}$ or $-\frac{1}{e^2}$ and y = 3 at t = − 1	**B1**	
	Obtain gradient of normal = $-e^{-2}$ or $-\frac{1}{e^2}$	**B1**	
	x = 0 substituted into equation of normal or use of gradients to give $y=3-\frac{1}{e^4}$ with no errors	**B1**	Equation of normal $y-3=-e^{-2}\left(x--e^{-2}\right)$. AG **SC** Decimals **B0 B1 B0** − 0.135 .
		3	

Question	Answer	Marks	Guidance
6(a)	State $R = 13$	**B1**	Allow if $\sqrt{(12^2 + (-5)^2)}$ seen.
	Use correct trig formulae to find $\alpha = \tan^{-1}(\pm 5/12) = \cos^{-1}(\pm 12/13) = \sin^{-1}(\pm 5/13)$	**M1**	$\cos(\alpha) = 12$ and $\sin(\alpha) = 5$ M0 However, $\sin(\alpha)/\cos(\alpha) = 5/12$ or $-5/12$ with no error seen, or $\tan(\alpha) = 5/12$ or $-5/12$ quoted then allow.
	Obtain $\alpha = 0.395$	**A1**	CWO If negative sign seen when finding R then A0 here. If degrees 22.6 A0 MR. Only penalise degrees once in **(a)** and **(b)**. Note $\alpha = 0.39479\ldots$
		3	
6(b)	$\cos^{-1}\left(\frac{6}{R}\right)$	**B1FT**	SOI 1.0910… FT *their* incorrect R.
	Use correct method to find a value of $2x$ in the interval	**M1**	$2x = \cos^{-1}\left(\frac{6}{R}\right) + \alpha$ or $2\pi - \cos^{-1}\left(\frac{6}{R}\right) + \alpha$. Allow if $\cos(2x + 0.395)$ seen
	Obtain answer, e.g. $x = 0.743$ or 0.742	**A1**	42.5 or 42.6 degrees.
	Obtain second answer, e.g. $x = 2.79$ and no others in the interval	**A1**	159.8, 159.9 or 160.0 degrees all possible depending whether using 3 dp or 4 dp.
		4	

Question	Answer	Marks	Guidance
7(a)	State or imply area of major sector = $\frac{1}{2}r^2(2\pi - x)$	**B1**	OE
	State or imply area of shaded segment = $\frac{1}{2}r^2 x - \frac{1}{2}r^2 \sin x$	**B1**	OE $r^2 \sin(x/2)\cos(x/2)$ B0 until changed to $(1/2)r^2 \sin x$.
	State $\frac{1}{2}r^2(2\pi - x) = 3\left(\frac{1}{2}r^2 x - \frac{1}{2}r^2 \sin x\right)$	**M1**	OE Area of major sector = 3 times (area of minor sector – area of triangle). Allow $r^2 \sin(x/2)\cos(x/2)$.
	Obtain the given answer $x = \frac{3}{4}\sin x + \frac{1}{2}\pi$ after full and correct working	**A1**	AG Allow rectified slip if before penultimate line.
		4	
7(b)	Calculate the values of a relevant expression or pair of expressions at $x = 2$ and $x = 2.5$	**M1**	$x = 2$ $\quad$ $x = 2.5$ $(3/4)\sin x + (1/2)\pi$ 2.2(5277) $\quad$ 2.0(197) $2 < 2.2$ or 2.3 $\quad$ $2.5 > 2.0$ $x - (3/4)\sin x - (1/2)\pi$ $-0.2(5277) < 0$ $\quad$ $+0.4(803) > 0$ or change of sign Attempt both values and one correct for M1.
	Complete the argument correctly with correct calculated values	**A1**	Degrees award 0/2
		2	

Question	Answer	Marks	Guidance
7(c)	Use the iterative formula correctly at least twice	**M1**	
	Obtain final answer 2.18	**A1**	
	Show sufficient iterations to 4 d.p. to justify 2.18 to 2 d.p. or show there is a sign change in the interval (2.175, 2.185)	**A1**	2 2.25 2.5 2.2528 2.1543(5) 2.0196(5) 2.1530 2.1967 2.2465 2.1972 2.1786 2.1560 2.1784 2.1865 2.1960 2.1866 2.1831 2.1789 2.1830 2.1845 2.1863 2.1846 2.1831 2.1845 Degrees award 0/3
		3	

Question	Answer	Marks	Guidance
8(a)	Use the product rule correctly	***M1**	x^3 d/dx(ln$x$) + d/dx(x^3) ln$x$.
	Obtain the correct derivative in any form	**A1**	e.g. $\frac{x^3}{x}+3x^2\ln x$.
	Equate derivative to zero and solve exactly for x	**DM1**	Reaching $x = \mathrm{e}^a$.
	Obtain answer $\left(\frac{1}{\sqrt[3]{\mathrm{e}}},-\frac{1}{3\mathrm{e}}\right)$ or exact equivalent	**A1**	ISW
		4	

Question	Answer	Marks	Guidance
8(b)	Integrate by parts and reach $ax^4 \ln x + b\int(x^4 / x)\mathrm{d}x$	***M1**	
	Obtain $\frac{x^4}{4}\ln x - \frac{1}{4}\int(x^4 / x)\mathrm{d}x$	**A1**	OE
	Complete integration and obtain $\frac{x^4}{4}\ln x - \frac{x^4}{16}$	**A1**	OE
	Use limits of $x = \frac{1}{2}$ and $x = 1$ in the correct order, having integrated twice	**DM1**	Correct substitution [(1/4)ln1 or 0 − 1/16] – [(1/64)ln(1/2) – (1/16)^2] or minus this value CWO. Allow omission of (1/4)ln1 or 0.
	Obtain answer $\frac{15}{256} - \frac{1}{64}\ln 2$ or exact equivalent final answer	**A1**	
		5	

Question	Answer	Marks	Guidance
9	Separate variables correctly and obtain e^{-3y} and $\sin^2 2x$ on the opposite sides	**B1**	
	Obtain term $-\frac{1}{3}\mathrm{e}^{-3y}$	**B1**	
	Use correct double angle formula for $\sin^2 2x$ = (1/2)[1 – cos 4x]	**M1**	
	Obtain terms $\frac{1}{2}\left[x-\frac{1}{4}\sin 4x\right]$ oe	**A1**	
	Use $x = 0$, $y = 0$ to evaluate a constant or as limits in a solution containing terms of the form ax and $b\sin 4x$ and $c\mathrm{e}^{\pm 3y}$	**M1**	
	Obtain correct answer in any form e.g. $-\frac{1}{3}\mathrm{e}^{-3y}=\frac{1}{2}\left[x-\frac{1}{4}\sin 4x\right]-\frac{1}{3}$	**A1**	
	Substitute $x = \frac{1}{2}$ and obtain $y = 0.175$ or $-\frac{1}{3}\ln\left(\frac{1}{4}+\frac{3}{8}\sin 2\right)$	**A1**	OE ISW
		7	

Question	Answer	Marks	Guidance
10(a)	Carry out correct process for evaluating the scalar product of $\overrightarrow{OA}$ and $\overrightarrow{OB}$	M1	$\pm(3, -1, 2).(1, 2, -3) = \pm(3 - 2 - 6) = [-5]$.
	Using the correct process for the moduli, divide the scalar product by the product of the moduli and obtain $\cos^{-1}\{\pm(3-2-6)/[\sqrt{(3^2+(-1)^2+2^2)}\ \sqrt{(1^2+2^2+(-3)^2)}]\}$	A1	
	Obtain answer 110.9° or 1.94^c	A1	
		3	
10(b)	Use a correct method to form an equation for line through *AB*	M1	
	Obtain $\mathbf{r} = 3\mathbf{i} - \mathbf{j} + 2\mathbf{k} + \mu_1(2\mathbf{i} - 3\mathbf{j} + 5\mathbf{k})$	A1	OE e.g. $\mathbf{r} = \mathbf{i} + 2\mathbf{j} - 3\mathbf{k} + \mu_2(-2\mathbf{i} + 3\mathbf{j} - 5\mathbf{k})$. Need **r** or (x, y, z).
		2	
10(c)	Obtain a correct equation for line through *CD* e.g. $[\mathbf{r} =]\ \mathbf{i} - 2\mathbf{j} + 5\mathbf{k} + \lambda_1(-4\mathbf{i} + 4\mathbf{j} - 6\mathbf{k})$	B1	OE e.g. $[\mathbf{r} =]\ 5\mathbf{i} - 6\mathbf{j} + 11\mathbf{k} + \lambda_2(-4\mathbf{i} + 4\mathbf{j} - 6\mathbf{k})$. **r** can be omitted or another symbol used.
	Equate two pairs of components of general points on *their l* and *their CD* and solve for λ or for μ	M1	
	Obtain e.g. $\lambda_1 = -2$ or $\mu_1 = 3$ or $\lambda_2 = -1$ or $\mu_2 = -4$	A1	
	Obtain position vector $9\mathbf{i} - 10\mathbf{j} + 17\mathbf{k}$	A1	Condone $(9, -10, 17)$ but not $(9\mathbf{i}, -10\mathbf{j}, 17\mathbf{k})$.
		4	

Question	Answer	Marks	Guidance
11(a)	State or imply the form $\dfrac{Ax+B}{4+x^2}+\dfrac{C}{1+x}$	**B1**	
	Use a correct method for finding a coefficient	**M1**	$(Ax + B)(1 + x) + C(4 + x^2) = 5x^2 + x + 11$.
	Obtain one of $A = 2$, $B = –1$ and $C = 3$	**A1**	If error present in above still allow A1 for C.
	Obtain a second value	**A1**	
	Obtain the third value	**A1**	If $A = 0$ then max M1 A1 (for C).
		5	
11(b)	Integrate and obtain terms $\left(\dfrac{A}{2}\right)\ln\left(4+x^2\right)+\dfrac{B}{2}\tan^{-1}\left(\dfrac{x}{2}\right)+C\ln\left(1+x\right)$	**B1FT +** **B1FT +** **B1FT**	The FT is on A, B and C. Integral of $\dfrac{Ax+B}{4+x^2}$ = $\dfrac{B}{2}\tan^{-1}\left(\dfrac{x}{2}\right)$ or $(A/2)\ln(4 + x^2)$ only. B0FT unless clearly from single term.
	Substitute limits 0 and 2 correctly in an integral of the form $a\ln\left(4+x^2\right)+b\tan^{-1}\left(\dfrac{x}{2}\right)+c\ln\left(1+x\right)$, where $abc\neq 0$	**M1**	$a\ln\left(4+4\right)+b\tan^{-1}\left(\dfrac{2}{2}\right)+c\ln\left(1+2\right)$ − [$a\ln 4+b\tan^{-1}0+c\ln\left(1\right)$]. Allow one sign or substitution error. Allow omission of $b\tan^{-1}0+c\ln\left(1\right)$.
	Obtain answer $\ln 54-\dfrac{\pi}{8}$ after full and correct working	**A1**	AG – work to combine or simplify at least 2 of ln terms is required CWO. A0 if any non exact value(s) seen.
		5	

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MATHEMATICS **9709/31**

Paper 3 Pure Mathematics 3 **October/November 2022**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

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This document consists of **17** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
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- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
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2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
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6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

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M	Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
A	Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
B	Mark for a correct result or statement independent of method marks.
DM or **DB**	When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
FT	Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

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AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
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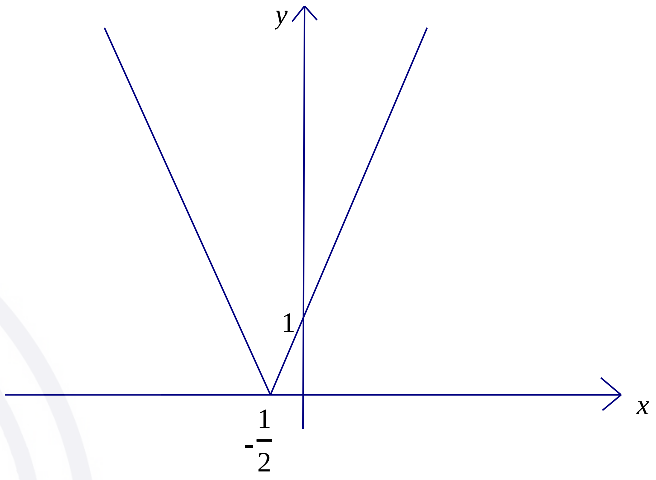
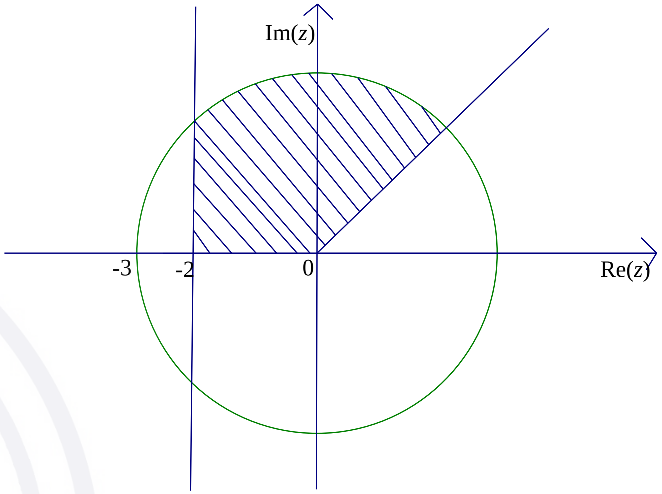
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1(a)	Show a recognisable sketch graph of $y=\lvert 2x+1\rvert$	**B1**	 Ignore $y=3x+5$ if also drawn on the sketch.
		1	

Question	Answer	Marks	Guidance
1(b)	Find x-coordinate of intersection with $y = 3x + 5$	**M1**	
	Obtain $x = -\frac{6}{5}$	**A1**	
	State final answer $x < -\frac{6}{5}$ only	**A1**	Do not condone $\le$ for $<$ in the final answer.
	Alternative method 1 for question 1(b)		
	Solve the linear inequality $3x + 5 < -(2x + 1)$, or corresponding equation	**M1**	Must solve the relevant equation.
	Obtain critical value $x = -\frac{6}{5}$	**A1**	Ignore -4 if seen.
	State final answer $x < -\frac{6}{5}$ only	**A1**	
	Alternative method 2 for question 1(b)		
	Solve the quadratic inequality $(3x+5)^2 < (2x+1)^2$, or corresponding equation	**M1**	$5x^2 + 26x + 24 < 0$
	Obtain critical value $x = -\frac{6}{5}$	**A1**	Ignore -4 if seen.
	State final answer $x < -\frac{6}{5}$ only	**A1**	
		3	

Question	Answer	Marks	Guidance
2	Show a circle with radius 3 and centre the origin	**B1**	 For the vertical line and the circle, allow the B1 marks if all you see is the relevant part.
	Show the line $x = -2$	**B1**	
	Show the correct half line for $\frac{\pi}{4}$	**B1**	
	Shade the correct region	**B1**	
		4	

Question	Answer	Marks	Guidance
3	Use law of logarithm of a product or power	**M1**	One correct application of a log law.
	Obtain a correct linear equation in any form, e.g. $(3x-1)\ln 2 = \ln 5 - x\ln 3$	**A1**	
	Solve for x	**M1**	As far as $x = \ldots$ with only minor slips in processing.
	Obtain answer $x = \dfrac{\ln 10}{\ln 24}$	**A1**	
	Alternative method for question 3		
	Use laws of indices to split at least one exponential term	**M1**	e.g. $\dfrac{2^{3x}}{2}$ or an arrangement with 8^x
	Obtain $24^x = 10$	**A1**	OE
	Solve for x	**M1**	
	Obtain answer $x = \dfrac{\ln 10}{\ln 24}$	**A1**	
		4	

Question	Answer	Marks	Guidance
4	Use correct $\tan(A+B)$ formula and obtain an equation in tan *x* or an equation in $\cos x$ and $\sin x$	**M1**	e.g. $\frac{\tan x+\tan 45°}{1-\tan x\tan 45°}=\frac{2}{\tan x}$ Allow if 2 in denominator or $\frac{\sin x\cos 45°+\cos x\sin 45°}{\cos x\cos 45°-\sin x\sin 45°}=\frac{2\cos x}{\sin x}$.
	Obtain correct 3 term equation $\tan^2 x+3\tan x-2=0$, or equivalent	**A1**	or $3\sin x\cos x=2\cos^2 x-\sin^2 x$
	Solve a 3-term quadratic in tan *x* and obtain a value for *x*	**M1**	
	Obtain answer, e.g. 29.3°	**A1**	29.316…
	Obtain second answer, e.g. 105.7° and no other	**A1**	105.583…. Ignore answers outside the given interval. Treat answers in radians as a misread.
		5	

Question	Answer	Marks	Guidance
5(a)	State or imply $u^2=4e^{\frac{1}{2}\pi i}$	**B1**	
	Obtain answer $v=\frac{4}{3}e^{\frac{1}{6}\pi i}$	**B1 + B1**	For the modulus and the argument.
		3	
5(b)	State *n* = 6	**B1**	
		1	

Question	Answer	Marks	Guidance
6(a)	Express $\cos 4\theta$ in terms of $\cos 2\theta$ and/or $\sin 2\theta$	**B1**	
	Express $\cos 2\theta$ in terms of $\cos\theta$ and/or $\sin\theta$	**B1**	Anywhere
	Expand to obtain a correct expression in terms of $\cos\theta$	**B1**	e.g. $2\left(2\cos^2\theta-1\right)^2-1+4\left(2\cos^2\theta-1\right)+3$
	Reduce correctly to $\cos 4\theta+4\cos 2\theta+3\equiv 8\cos^4\theta$	**B1**	AG
		4	
6(b)	Use the identity and carry out method to calculate a root	**M1**	$8\cos^4\theta-3=4$
	Obtain answer, e.g. 14.7°	**A1**	
	Obtain second answer, e.g.165.3°, and no other in the given interval	**A1 FT**	Ignore answers outside the given interval. Treat answers in radians as a misread.
		3	

Question	Answer	Marks	Guidance
7(a)	Use correct product or quotient rule	**M1**	
	Obtain correct derivative in any form	**A1**	e.g. $\frac{dy}{dx} = \frac{\cos^2 x + 2x\sin x\cos x}{\cos^4 x}$ or $\frac{dy}{dx} = \sec^2 x + 2x\sec^2 x\tan x$
	Equate derivative at $x = a$ to 12 and obtain $a = \cos^{-1}\left(\sqrt[3]{\frac{\cos a + 2a\sin a}{12}}\right)$	**A1**	AG
		3	
7(b)	Evaluate a relevant expression or pair of expressions at $a = 0.9$ and $a = 1$	**M1**	Must be calculated in radians.
	Complete the argument correctly with correct calculated values	**A1**	e.g. $\begin{matrix}\cos 0.9 = 0.622 > 0.553\\ \cos 1 = 0.540 < 0.570\end{matrix}$ or $\begin{matrix}0.9 < 0.985\\ 1 > 0.964\end{matrix}$ or $\begin{matrix}0.0846 > 0\\ -0.0358 < 0\end{matrix}$ or could be looking at values of the gradient 8.46 & 14.1
		2	
7(c)	Use the process $a_{n+1} = \cos^{-1}\left(\sqrt[3]{\frac{\cos a_n + 2a_n\sin a_n}{12}}\right)$ correctly at least once	**M1**	Must be working in radians.
	Obtain final answer 0.97	**A1**	
	Show sufficient iterations to 4 d.p. to justify 0.97 to 2 d.p., or show there is a sign change in the interval (0.965, 0.975)	**A1**	e.g. 0.95, 0.9743, 0.9694, 0.9704
		3	

Question	Answer	Marks	Guidance
8(a)	Separate variables correctly	**B1**	$\int \frac{1}{x}\,\mathrm{d}x = \int k\mathrm{e}^{-0.1t}\,\mathrm{d}t$
	Obtain term ln x	**B1**	
	Obtain term $-10k\mathrm{e}^{-0.1t}$	**B1**	Not from $\int x\mathrm{e}^{-0.1t}\,\mathrm{d}t$
	Use $x = 20$, $t = 0$ to evaluate a constant or as limits in a solution containing terms $a\ln x, be^{-0.1t}$ where $ab \neq 0$	**M1**	
	Obtain $\ln x = 10k\left(1-\mathrm{e}^{-0.1t}\right) + \ln 20$	**A1**	or equivalent ISW
		5	
8(b)	Use $x = 40$, $t = 10$ to find k or $10k$	**M1**	Available for their function of the correct structure even if they found no constant in **(a)**.
	Obtain $10k = 1.09654$	**A1**	or equivalent e.g. $10k = \dfrac{\ln 2}{1-e^{-1}}$
	State that x tends to 59.9	**A1**	Need a number, not an expression for that value 3 sf or better 59.87595…..
		3	

Question	Answer	Marks	Guidance
9(a)	Use correct product or quotient rule	***M1**	
	Obtain correct derivative in any form	**A1**	e.g. $\frac{dy}{dx} = -e^{-\frac{x}{3}} - \frac{1}{3}(3-x)e^{-\frac{x}{3}}$
	Equate their derivative to zero and solve for x	**DM1**	
	Obtain $x = 6$	**A1**	
	Obtain $y = -3e^{-2}$	**A1**	Or exact equivalent.
		5	
9(b)	Commence integration and reach $a(3-x)e^{-\frac{1}{3}x} + b\int e^{-\frac{1}{3}x}\, dx$, where $ab \neq 0$	***M1**	
	Obtain $-3(3-x)e^{-\frac{1}{3}x} - 3\int e^{-\frac{1}{3}x}\, dx$, or equivalent	**A1**	
	Complete integration and obtain $3xe^{-\frac{1}{3}x}$, or equivalent	**A1**	$-3e^{-\frac{x}{3}}(3-x) + 9e^{-\frac{x}{3}}$
	Substitute limits $x = 0$ and $x = 3$, having integrated twice	**DM1**	
	Obtain answer $\frac{9}{e}$, or exact equivalent	**A1**	
		5	

Question	Answer	Marks	Guidance
10(a)	State or imply the form $\dfrac{A}{1+x}+\dfrac{B}{2+x}+\dfrac{C}{(2+x)^2}$	**B1**	
	Use a correct method to find a constant	**M1**	
	Obtain one of A= 3, B = – 1 and C = – 2	**A1**	SR after B0 can score M1A1 for one correct value
	Obtain a second value	**A1**	
	Obtain the third value	**A1**	$\dfrac{A}{1+x}$ + $\dfrac{Dx+E}{(2+x)^2}$, where A = 3, D = – 1 and E = – 4, is awarded B1 M1 A1 A1 A1 as above.
		5	
10(b)	Use a correct method to find the first two terms of the expansion of $(1+x)^{-1}$, $(2+x)^{-1}$, $\left(1+\frac{1}{2}x\right)^{-1}$, $(2+x)^{-2}$ or $\left(1+\frac{1}{2}x\right)^{-2}$	**M1**	For the A, D, E form of fractions, award M1 A1FT A1FT for the expanded partial fractions, then if $D \neq 0$, M1 for multiplying out fully, and A1 for the final answer.
	Obtain correct unsimplified expansions up to the term in x^2 of each partial fraction	**A3 FT**	$3\left(1-x+x^2.....\right)$ $-\frac{1}{2}\left(1-\frac{x}{2}+\frac{x^2}{4}.....\right)$ $-\frac{2}{4}\left(1-x+\frac{3}{4}x^2\right)$
	Obtain final answer $2-\dfrac{9}{4}x+\dfrac{5}{2}x^2$	**A1**	
		5	

Question	Answer	Marks	Guidance
11(a)	State $\overrightarrow{OM} = 2\mathbf{i} + 2\mathbf{j}$ or equivalent	**B1**	Can be implied by $\overrightarrow{MB} = -2\mathbf{i} + 2\mathbf{j}$ or $\overrightarrow{MA} = 2\mathbf{i} - 2\mathbf{j}$.
	Obtain $\overrightarrow{MD} = -2\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$	**B1**	
	Use a correct method to find $\overrightarrow{ON}$	**M1**	e.g. $\overrightarrow{OC} + \frac{2}{3}\overrightarrow{CB}$
	Obtain answer $3\mathbf{j} + \mathbf{k}$	**A1**	
		4	
11(b)	Use the correct process for evaluating the scalar product of $\overrightarrow{MD}$ and $\overrightarrow{ON}$	**M1**	
	Using the correct process for the moduli, divide the scalar product by the product of the moduli and reach the inverse cosine of the result	**M1**	$\cos^{-1}\left(\dfrac{-6+3}{\sqrt{10}\sqrt{17}}\right)$
	Obtain final answer 103.3°	**A1**	
		3	

Question	Answer	Marks	Guidance
11(c)	Taking a general point *P* of *ON* to have position vector $\lambda(3\mathbf{j}+\mathbf{k})$, form an equation in λ by *either* equating the scalar product of $\overrightarrow{ON}$ and $\overrightarrow{MP}$ to zero, *or* applying Pythagoras to triangle *OMP*, *or* equating the derivative of $\lvert\overrightarrow{MP}\rvert$ to zero	**M1**	e.g. $\begin{pmatrix}-2\\-2+3\lambda\\\lambda\end{pmatrix}.\begin{pmatrix}0\\3\\1\end{pmatrix}=0$
	Solve and obtain $\lambda=\frac{3}{5}$	**A1**	
	Substitute for λ and calculate *MP*	**M1**	$\overrightarrow{MP}=-2\mathbf{i}-\frac{1}{5}\mathbf{j}+\frac{3}{5}\mathbf{k}$
	Obtain $\sqrt{\frac{22}{5}}$	**A1**	AG
	Alternative method for question 11(c)		
	Use a scalar product to find the projection *OQ* of *OM* on *OM*	**M1**	
	Obtain $OQ=\frac{6}{\sqrt{10}}$	**A1**	
	Use Pythagoras in triangle *OMQ* to find *MQ*	**M1**	
	Obtain $\sqrt{\frac{22}{5}}$	**A1**	AG
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Cambridge Assessment
International Education

Cambridge International A Level

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AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	Use law of the logarithm of a power or product	**M1**	Ignoring the 3 or the 5 is not a misread.
	Obtain a correct linear equation in any form, e.g. $(3x-1)\ln 2=\ln 5+(1-x)\ln 3$	**A1**	Condone invisible brackets if they are used correctly later.
	Solve for *x*	**M1**	Get as far as *x* = … Condone minor slips in the processing e.g. sign errors and losing a term that had been there, but award M0 for a fundamental error e.g. $3x\ln 2+x\ln 3=3x\ln 6$ or ignoring the 3 or the 5 completely. Condone working in decimals.
	Obtain **final** answer $x=\dfrac{\ln 30}{\ln 24}$	**A1**	Do not ISW
	Alternative method for question 1		
	Use laws of indices to split at least one exponential term	**M1**	e.g. $\dfrac{2^{3x}}{2}$ or an arrangement with 8^x and/or 3^x.
	Obtain $24^x=30$	**A1**	Or equivalent e.g. $3^x8^x=30$ not for $3^x2^{3x}=30$ (need two factors with the same index).
	Solve for *x*	**M1**	Get as far as *x* = …
	Obtain **final** answer $x=\dfrac{\ln 30}{\ln 24}$	**A1**	Do not ISW
		4	

Question	Answer	Marks	Guidance
2(a)	Substitute $x = -\frac{3}{2}$ and equate result to zero	**M1**	Or divide by $2x+3$ and set constant remainder equal to zero. Or state $(2x^3 - x^2 + a) = (2x+3)(x^2 + px + q)$, compare coefficients and solve for p or q.
	Obtain $a = 9$	**A1**	
		2	

Question	Answer	Marks	Guidance
2(b)	Commence division by $(2x+3)$ reaching a partial quotient x^2+kx	***M1**	The M1 is earned if inspection reaches an unknown factor: x^2+Bx+C and an equation in B and/or C, or an unknown factor Ax^2+Bx+3 and an equation in A and/or B.
	Obtain factorisation $(2x+3)(x^2-2x+3)$	**A1**	Allow if the correct quotient seen. Correct factors seen in **(a)** and quoted or used here scores M1A1.
	Show that x^2-2x+3 is always positive, or $2x^3-x^2+9$ only intersects the *x*-axis once	**DM1**	Must use their quadratic factor. SC If M0, allow B1 if state $x<-\frac{3}{2}$ and no error seen
	State **final** answer $x<-\frac{3}{2}$ from correct work	**A1**	
		4	

Question	Answer	Marks	Guidance
3	Use correct product rule on given expression	***M1**	
	Obtain correct derivative in any form	**A1**	e.g. $\cos x \sin 2x + 2\sin x \cos 2x$
	Use correct double angle formulae to express derivative in terms of $\sin x$ and $\cos x$	***M1**	
	Equate derivative to zero and obtain an equation in one trig variable	**DM1**	dependent on the 2 previous M Marks.
	Obtain $3\sin^2 x = 2$, $3\cos^2 x = 1$ or $\tan^2 x = 2$	**A1**	OE
	Solve and obtain $x = 0.955$	**A1**	3 sf only. Final answer in degrees is A0. Ignore any attempt to find the corresponding value of y.
	Alternative method for the first three marks		
	Use correct double angle formula to obtain $y = 2\cos x - 2\cos^3 x$	***M1**	or $y = 2\sin^2 x \cos x$
	Use chain rule and / or product rule	***M1**	
	Obtain derivative $y' = -2\sin x + 6\sin x \cos^2 x$	**A1**	$y' = -2\sin^3 x + 4\sin x \cos^2 x$
	Alternative method for the second and third M marks		
	Equate derivative to zero and obtain an equation in $\tan x$ and $\tan 2x$	***M1**	
	Use correct double angle formula to obtain an equation in $\tan x$	**DM1**	
		6	

Question	Answer	Marks	Guidance
4(a)	State $R = \sqrt{17}$	**B1**	Allow if working from an incorrect expansion but not from decimals.
	Use correct trig formulae to find α (Correct expansion and correct expression for trig ratio for α)	**M1**	NB: $\cos\alpha = 4$ and $\sin\alpha = 1$ scores M0A0. M0 for incorrect expansion of $\cos(x-\alpha)$ M1 for correct expression for trig ratio for α and no errors seen.
	Obtain $\alpha = 14.04°$	**A1**	2 d.p. required Allow M1A1 for correct answer with no working shown. Correct answer from incorrect working (e.g. $\tan^{-1}\left(-\frac{1}{4}\right)$) is awarded M0A0. $180° - \tan^{-1}\left(-\frac{1}{4}\right)$ is awarded M1
		3	
4(b)	Evaluate $\cos^{-1}\left(\frac{3}{\sqrt{17}}\right)$ to at least 1 d.p. (43.3138…°)	**B1 FT**	FT *their R*. Accept awrt $43.3°$ or awrt $316.7°$ Can be implied by subsequent working.
	Use correct method to find a value of *x* in the interval	**M1**	Must be working with 2*x* and *their* α .
	Obtain answer, e.g. 14.6°	**A1**	Accept overspecified answers but they need to be correct. (14.6388…. and 151.3249…).
	Use a correct method to find a second answer in the interval	**M1**	Must be working with 2*x*, *their* α and $360° -$ *their* 43.3 .
	Obtain second answer in the interval, e.g. 151.3° , and no other in the interval	**A1**	Ignore answers outside the given interval. Treat answers in radians (0.255… and 2.64…) as a misread.
		5	

Question	Answer	Marks	Guidance
5(a)	Use quadratic formula, or completing the square $\left((z-3\mathrm{i})^2-3=0\right)$ and use $\mathrm{i}^2=-1$ to find a root	**M1**	
	Obtain a root, e.g. $\sqrt{3}+3\mathrm{i}$	**A1**	Or exact 2 term equivalent e.g. $\frac{6\mathrm{i}}{2}+\frac{\sqrt{12}}{2}$ ISW.
	Obtain the other root, e.g. $-\sqrt{3}+3\mathrm{i}$	**A1**	Or exact 2 term equivalent ISW.
		3	
5(b)	Show points representing the roots correctly	**B1 FT**	2 roots consistent with *their* **(a)** and with no errors seen on the diagram. B0 if they only have one root or more than 2 roots Must match their scale and $1<\sqrt{3}<2$ Linear scales seen or implied. Need some indication of scale (numbers or dashes). Scales along an axis must be approximately consistent but scales may be different on the 2 axes.
		1	

Question	Answer	Marks	Guidance
5(c)	State modulus of either root is $2\sqrt{3}$, or simplified exact equivalent	**B1 FT**	ISW if converted to decimal . Ignore modulus of second root if seen. Follow their root(s) not on either axis (from **(a)** or **(b)**).
	Find the argument of one of their roots – get as far as $\tan^{-1}(...)$	**M1**	SOI but must be correct for their root.
	Obtain correct arguments $\frac{1}{3}\pi$ and $\frac{2}{3}\pi$, or simplified exact equivalents	**A1**	Must obtain values. Allow degrees.
		3	
5(d)	Give a **complete justification** that the **correct triangle** is equilateral	**B1**	Check *their* diagram in **(b)**. Possible justifications: 3 equal sides, or all angles equal to $\frac{\pi}{3}$, or isosceles and an angle of $\frac{\pi}{3}$.
		1	

Question	Answer	Marks	Guidance
6(a)	State or imply $\overrightarrow{AB}$ or $\overrightarrow{AC}$ correctly in component form	**B1**	$\left(\overrightarrow{AB} = 2\mathbf{i} - 2\mathbf{j} + \mathbf{k}, \quad \overrightarrow{AC} = 4\mathbf{i} - 3\mathbf{k}\right)$.
	Using the correct process with relevant vectors to evaluate the scalar product $\overrightarrow{AB}.\overrightarrow{AC}$,	**M1**	or $\overrightarrow{BA}.\overrightarrow{CA}$ $(8-3=5)$. M0 for $\overrightarrow{AB}.\overrightarrow{CA}$.
	Using the correct process for the moduli, divide *their* scalar product by the product of *their* moduli to obtain $\cos\theta$ or θ	**M1**	$\left(\frac{5}{\sqrt{9}\sqrt{25}}\right)$ Independent of the first M1.
	Obtain answer $\frac{1}{3}$	**A1**	ISW. Need to see a value for $\cos\theta$. Accept $\frac{5}{15}$ or 0.333 ($\cos^{-1}\frac{1}{3}$ alone is not sufficient)
		4	

Question	Answer	Marks	Guidance
6(b)	Use correct method to find an **exact** value for the sine of angle *BAC* from *their* **(a)**	**M1**	$\left(\sqrt{1-\frac{1}{9}}\right)$
	Obtain answer $\frac{2}{3}\sqrt{2}$, or equivalent	**A1**	
	Use correct area formula to find the area of triangle *ABC* with *their* versions of relevant vectors	**M1**	$\left(\frac{1}{2}\sqrt{9}\sqrt{25}\times \textit{their}\sin\theta\right)$ or $\frac{1}{2}\sqrt{9}\sqrt{25}\times\sin\left(\cos^{-1}\frac{1}{3}\right)$
	Obtain answer $5\sqrt{2}$ or $\sqrt{50}$	**A1**	Only ISW
	Alternative method 1 for question 6(b)		
	Use correct method to find the perpendicular distance from *A* to *BC* (or *B* to *AC* or *C* to *AB*)	**M1**	$\begin{pmatrix}2+2\lambda\\-2+2\lambda\\1-4\lambda\end{pmatrix}.\begin{pmatrix}2\\2\\-4\end{pmatrix}=0\Rightarrow\lambda=\frac{1}{6}$
	Obtain $\frac{1}{3}\sqrt{75}$	**A1**	$\left(\left\|\frac{7}{3}\mathbf{i}-\frac{5}{3}\mathbf{j}+\frac{1}{3}\mathbf{k}\right\|\right)$
	Use correct area formula to find the area of triangle *ABC*	**M1**	$\left(\frac{1}{2}\times \textit{their}\sqrt{24}\times \textit{their}\,\frac{1}{3}\sqrt{75}\right)$ The length they use for *their* base must be found correctly.
	Obtain answer $5\sqrt{2}$ or $\sqrt{50}$	**A1**	

Question	Answer	Marks	Guidance
6(b)	**Alternative method 2 for question 6(b)**		
	Correct method to find the semi-perimeter	**M1**	
	Obtain $4+\sqrt{6}$	**A1**	
	Correct application of Hero's (Heron's) formula	**M1**	$\sqrt{\left(4+\sqrt{6}\right)\left(1+\sqrt{6}\right)\left(-1+\sqrt{6}\right)\left(4-\sqrt{6}\right)}$
	Obtain answer $5\sqrt{2}$ or $\sqrt{50}$	**A1**	
		4	

Question	Answer	Marks	Guidance
7(a)	Show sufficient working to justify the given statement	**B1**	e.g. see $2\cot\theta \times -\operatorname{cosec}^2\theta$ in the working or express in terms of $\sin\theta$ and $\cos\theta$ and use quotient rule to obtain the given result. Solution must have θ present throughout and must reach the given answer.
		1	

Question	Answer	Marks	Guidance
7(b)	Separate variables correctly Check for relevant working in **(a)**	**B1**	$\int x\mathrm{d}x = \int \frac{\tan^2\theta}{\sin^2\theta} - \frac{2\cot\theta}{\sin^2\theta}\mathrm{d}\theta$ Condone incorrect notation e.g. missing dx. Need either the integral sign or the dx, dθ.
	Obtain term $\frac{1}{2}x^2$	**B1**	
	Obtain terms $\tan\theta + \cot^2\theta$	**B1 + B1**	Alternative: $\int \frac{2\cot\theta}{\sin^2\theta}\mathrm{d}\theta = \int \frac{2\cos\theta}{\sin^3\theta}\mathrm{d}\theta = -\frac{1}{\sin^2\theta}(+C)$
	Form an equation for the constant of integration, or use limits $x = 2$, $\theta = \frac{1}{4}\pi$, in a solution with **at least two correctly obtained terms** of the form ax^2, $b\tan\theta$ and $c\cot^2\theta$, where $abc \neq 0$	**M1**	Need to have 3 terms. Constant of correct form.
	State correct solution in any form, e.g. $\frac{1}{2}x^2 = \tan\theta + \cot^2\theta$	**A1**	or $\frac{1}{2}x^2 = \tan\theta + \operatorname{cosec}^2\theta - 1$ If everything else is correct, allow a correct final answer to imply this A1.
	Substitute $\theta = \frac{1}{6}\pi$ and obtain answer $x = 2.67$	**A1**	2.6748… $\sqrt{\frac{18+2\sqrt{3}}{3}}$ If see a correctly rounded value ISW.
		7	

Question	Answer	Marks	Guidance
8(a)	State (a =) π^2	**B1**	Allow 32400, 180^2 . Accept $\left(x=\right)\pi^2$.
		1	
8(b)	State or imply dx =2u du or equivalent	**B1**	e.g. $\frac{\mathrm{d}u}{\mathrm{d}x}=\frac{1}{2\sqrt{x}}$ Incorrect statements e.g. $\mathrm{d}u=\frac{1}{2\sqrt{x}}$ is B0.
	Substitute for x and dx throughout the integral	**M1**	
	Obtain $\int 2u\sin u\,\mathrm{d}u$	**A1**	Allow with missing du.
	Commence integration of $\int ku\sin u\,\mathrm{d}u$ by parts and reach $\mp ku\cos u\pm\int k\cos u\,\mathrm{d}u$	***M1**	
	Obtain integral $-ku\cos u+k\sin u$	**A1**	
	Substitute limits u = 0 and u = $\sqrt{their\ a}$, $a\neq 0$, a in radians or $x=0$ and *their a* in the complete integral	**DM1**	$-2\pi\cos\pi+2\sin\pi\left(+0-2\sin 0\right)$ Need limits stated but condone if zeros not shown in substitution.
	Obtain answer 2π	**A1**	
		7	

Question	Answer	Marks	Guidance
9(a)	State or imply angle $AOC = \pi - 2\theta$	**B1**	Might be seen on the printed diagram.
	Use correct formulae for the area of a sector and triangle, or of a segment, and find the area of the shaded region	**M1**	$\frac{1}{2}r^2(\pi-2\theta)-\frac{1}{2}r^2\sin(\pi-2\theta)$ or $\frac{1}{2}\pi r^2-\left[\frac{1}{2}r^2(2\theta)+\frac{1}{2}r^2\sin(\pi-2\theta)\right]$ M0 if subtraction the wrong way round.
	Equate to $\dfrac{1}{6}\pi r^2$ and obtain a correct equation in any form	**A1**	e.g. $\frac{1}{6}\pi r^2=\frac{1}{2}r^2(\pi-2\theta)-\frac{1}{2}r^2\sin(\pi-2\theta)$.
	Obtain $\theta=\frac{1}{3}(\pi-1.5\sin 2\theta)$ correctly	**A1**	AG Condone if state / imply $\sin(\pi-2\theta)=\sin 2\theta$.
		4	
9(b)	Evaluate a relevant expression or pair of expressions at $\theta=0.5$ and $\theta=0.7$	**M1**	Allow work on a smaller interval. Need to evaluate for both limits, with at least one correct. When using $x=\mathrm{f}(x)$ embedded values are not sufficient e.g. f(0.5) …. is accepted but $\frac{1}{3}(\pi-1.5\sin 2\times 0.5)=\ldots$ is not.
	Complete the argument correctly with correct calculated values	**A1**	e.g. $0.5<0.626, 0.7>0.554$ or $0.126>0, -0.146<0$ If using pairs then the pairing must be clear. Need to see the inequalities or an appropriate comment. Need to see values calculated to at least 2 sf.
		2	

Question	Answer	Marks	Guidance
9(c)	Use the iterative process $\theta_{n+1} = \frac{1}{3}(\pi - 1.5\sin 2\theta_n)$ correctly at least once	**M1**	i.e obtain one value and use that value to obtain a second value. Must be working in radians.
	Obtain final answer 0.586	**A1**	
	Show sufficient iterations to 5 d.p. to justify 0.586 to 3 d.p., or show there is a sign change in the interval (0.5855, 0.5865).	**A1**	e.g. $0.5, 0.62646, 0.57225, 0.59195, 0.58416, 0.58715, 0.58599, 0.58644$ $0.6, 0.58118, 0.58833, 0.58553, 0.58661, 0.58619, 0.58636$ $0.7, 0.55447, 0.59958, 0.58133, 0.58827, 0.58556, 0.58661, 0.58620, 0.58636$ Allow working to more than 5 dp, but not less.
		3	

Question	Answer	Marks	Guidance
10(a)	State or imply the form $\frac{A}{1+x}+\frac{Bx+C}{2+x^2}$	**B1**	
	Use a correct method for finding a constant	**M1**	
	Obtain one of A = 2, B = – 1 and C = 0	**A1**	**SC**: A maximum of M1A1 is available for obtaining $A=2$ after scoring B0.
	Obtain a second value	**A1**	
	Obtain the third value	**A1**	
		5	
10(b)	Integrate and obtain term $2\ln(1+x)$	**B1 FT**	$A\ln(1+x)$
	Integrate and obtain term of the form $k\ln(2+x^2)$ from an integral of the correct form	***M1**	Ignore any separate working relating to $C\neq 0$.
	Obtain term $-\frac{1}{2}\ln(2+x^2)$	**A1 FT**	$\frac{B}{2}\ln(2+x^2)$
	Substitute limits in an integral containing terms of the form $a\ln(1+x)+b\ln(2+x^2)$, where $ab\neq 0$	**DM1**	Ignore working relating to $C\neq 0$. $\left(2\ln 5-2\ln 1-\frac{1}{2}\ln 18+\frac{1}{2}\ln 2\right)$ Dependent on the first M1. Must be subtracting the correct way round. Must have an exact substitution.
	Obtain answer $\ln\left(\frac{25}{3}\right)$	**A1**	ISW Any exact equivalent e.g. $\ln\frac{25\sqrt{2}}{\sqrt{18}}$ with no number in front of the logarithm.
		5	

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International Education

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MATHEMATICS **9709/33**

Paper 3 Pure Mathematics 3 **October/November 2022**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2022 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

This document consists of **15** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:
- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:
- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M	Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
A	Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
B	Mark for a correct result or statement independent of method marks.
DM or **DB**	When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
FT	Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	Use law for the logarithm of a product, quotient or power	**M1**	
	Remove logarithms and state a correct equation, e.g. $x(2x-1)=(x+1)^2$	**A1**	
	Solve a 3-term quadratic obtaining at least one root	**M1**	
	Obtain answer 3.303 only	**A1**	
		4	

Question	Answer	Marks	Guidance
2	State a correct unsimplified term in x or x^2 of the expansion of either $(1+2x)^{\frac{1}{2}}$ or $(1-2x)^{-\frac{1}{2}}$	**B1**	
	State correct unsimplified expansion of $(1+2x)^{\frac{1}{2}}$ up to the term in x^2	**B1**	
	State correct unsimplified expansion of $(1-2x)^{-\frac{1}{2}}$ up to the term in x^2	**B1**	
	Obtain sufficient terms of the product of the expansions	**M1**	
	Obtain final answer $1+2x+2x^2$	**A1**	
	Alternative method for question 2		
	State that the expression equals $(1+2x)(1-4x^2)^{-\frac{1}{2}}$ and state a term of the expansion	**B1**	
	State correct unsimplified expansion of $(1-4x^2)^{-\frac{1}{2}}$ up to the term in x^2	**B1 + B1**	
	Obtain sufficient terms of the product of (1 + 2x) and the expansion	**M1**	
	Obtain final answer $1+2x+2x^2$	**A1**	
		5	

Question	Answer	Marks	Guidance
3	Commence integration by parts and reach $x\tan x \pm \int \tan x.1\mathrm{d}x$	***M1**	
	Use a correct method to integrate tan x	**M1**	
	Obtain integral $x\tan x - \ln\sec x$, or equivalent	**A1**	
	Use limits correctly, having integrated twice	**DM1**	
	Obtain answer $\frac{1}{4}\pi - \frac{1}{2}\ln 2$, or exact equivalent	**A1**	
		5	

Question	Answer	Marks	Guidance
4	State or imply $\frac{\mathrm{d}x}{\mathrm{d}t} = 2 - \sec^2 t$ or $\frac{\mathrm{d}y}{\mathrm{d}t} = 2\frac{\cos 2t}{\sin 2t}$	**B1**	
	Use $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}t} \div \frac{\mathrm{d}x}{\mathrm{d}t}$	**M1**	
	Obtain correct answer in any form	**A1**	
	Use double angle formula to express derivative in terms of cos x and sin x	**M1**	
	Obtain the given answer correctly	**A1**	AG
		5	

Question	Answer	Marks	Guidance
5(a)	Show a circle with centre -2	**B1**	
	Show a circle with radius 2 and centre not the origin	**B1**	
	Show the line $y = 1$	**B1**	
	Shade the correct region	**B1**	
		4	
5(b)	Identify the correct point and carry out a correct method to find the argument	**M1**	
	Obtain answer $\frac{11}{12}\pi$	**A1**	2.88 radians or 165°.
		2	

Question	Answer	Marks	Guidance
6	Use quadratic formula to solve for *z*	**M1**	**SC M1**: For substitution of $x + iy$ and multiplying out.
	Use $i^2 = -1$ throughout	**M1**	**SC M1**: Use $i^2 = -1$ throughout.
	Obtain correct answer in any form	**A1**	**SC A1**: For two correct equations $x^2 - y^2 + 6xy - 2x + y = 0$ and $-3\left(x^2 - y^2\right) + 2xy - x - 2y + 1 = 0$.
	Multiply numerator and denominator by (1 + 3i), or equivalent	**M1**	
	Obtain final answer, e.g. $-\frac{1}{2} + \frac{1}{2}i$	**A1**	
	Obtain second final answer, e.g. $\frac{2}{5} + \frac{1}{5}i$	**A1**	
		6	

Question	Answer	Marks	Guidance
7(a)	Rearrange and obtain $4\cos x - \sin x = \sqrt{5}$	**B1**	
	State $R = \sqrt{17}$	**B1**	
	Use trig formulae to find α	**M1**	
	Obtain $\alpha = 14.04°$	**A1**	
		4	

Question	Answer	Marks	Guidance
7(b)	Evaluate $\cos^{-1}\left(\frac{\sqrt{5}}{\sqrt{17}}\right)$	**B1 FT**	FT *their R.*
	Carry out a correct method to find a value of *x* in the given interval	**M1**	
	Obtain answer, e.g. 21.6°	**A1**	
	Obtain a second answer, e.g. 144.4° and no other in the interval	**A1**	Treat answers in radians as a misread. Ignore answers outside the given interval.
		4	

Question	Answer	Marks	Guidance
8(a)	Use quotient or product rule	**M1**	
	Obtain correct derivative in any form	**A1**	
	Equate derivative at $x = p$ to zero and obtain the given equation	**A1**	
		3	
8(b)	Evaluate a relevant expression or pair of relevant pair of expressions at $p = 2.5$ and $p = 3$	**M1**	
	Complete the argument with correct calculated values	**A1**	
		2	

Question	Answer	Marks	Guidance
8(c)	Use the iterative formula $p_{n+1} = 3\left(1 - \mathrm{e}^{-p_n}\right)$ correctly at least once	**M1**	
	Obtain final answer $p = 2.82$	**A1**	
	Show sufficient iterations to 4 d.p.to justify 2.82 to 2 d.p., or show there is a sign change in the interval (2.815, 2.825)	**A1**	
		3	

Question	Answer	Marks	Guidance
9(a)	State $\overrightarrow{OM} = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$	**B1**	
	Use a correct method to find $\overrightarrow{ON}$	**M1**	
	Obtain answer $\begin{pmatrix} 3 \\ -2 \\ -1 \end{pmatrix}$	**A1**	
		3	
9(b)	Carry out a correct method to form a vector equation for *MN*	**M1**	
	Obtain a correct equation in any form, e.g. $\mathbf{r} = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -3 \\ -1 \end{pmatrix}$	**A1**	OE
		2	

Question	Answer	Marks	Guidance
9(c)	State a correct vector equation for *AB* in any form, e.g. $\mathbf{r} = \begin{pmatrix}1\\0\\1\end{pmatrix} + \mu\begin{pmatrix}-1\\5\\1\end{pmatrix}$	**B1**	
	Equate components of *AB* and *MN* and solve for λ or for μ	**M1**	
	Obtain $\lambda = -3$ or $\mu = 2$	**A1**	
	Obtain position vector $\begin{pmatrix}-1\\10\\3\end{pmatrix}$, or equivalent, for *Q*	**A1**	
		4	

Question	Answer	Marks	Guidance
10(a)	$a = 30$ and $b = 0.01$	**B1**	
		1	

Question	Answer	Marks	Guidance
10(b)	Separate variables and integrate one side	**M1**	
	Obtain terms $-100\ln(30-0.01V)$ and t, or equivalent	**A1 FT** **+ A1 FT**	FT *their a* and *b*.
	Evaluate a constant, or use $t = 0$, $V = 0$ as limits, in a solution containing terms $c \ln(30-0.01V)$ and dt where $cd \neq 0$	**M1**	
	Obtain solution $100\ln 30-100\ln(30-0.01V)=t$, or equivalent	**A1**	
	Substitute $V = 1000$ and obtain answer $t = 40.5$	**A1**	
		6	
10(c)	Obtain $V=3000\left(1-\mathrm{e}^{-0.01t}\right)$	**B1**	OE
	State that V approaches 3000	**B1**	
		2	

Question	Answer	Marks	Guidance
11(a)	State or imply the form $\dfrac{A}{3-x}+\dfrac{Bx+C}{1+3x^2}$	**B1**	
	Use a correct method to find a constant	**M1**	
	Obtain one of $A = 2$, $B = 0$ and $C = 1$	**A1**	
	Obtain a second value	**A1**	
	Obtain the third value	**A1**	
		5	
11(b)	Integrate and obtain term $-2\ln(3-x)$	**B1 FT**	
	Obtain term of the form $b\tan^{-1}\left(\sqrt{3}x\right)$	**M1**	
	Obtain term $\dfrac{1}{\sqrt{3}}\tan^{-1}\left(\sqrt{3}x\right)$	**A1 FT**	
	Substitute limits correctly in an integral with terms $a\ln(3-x)$ and $b\tan^{-1}\left(\sqrt{3}x\right)$, where $ab \neq 0$	**M1**	
	Obtain answer $2\ln\dfrac{3}{2}+\dfrac{1}{3\sqrt{3}}\pi$, or equivalent	**A1**	
		5	

Cambridge Assessment International Education

Cambridge International A Level

MATHEMATICS **9709/31**

Paper 3 Pure Mathematics 3 **May/June 2022**

MARK SCHEME

Maximum Mark: 75

Published

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Abbreviations

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WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	Use law of the logarithm of a product or a quotient or a power	*M1	
	Obtain a correct linear equation in any form	A1	e.g. $\ln 2+(2x-1)\ln 3=(x+1)\ln 4$ or $\log_2 2+(2x-1)\log_2 3=(2x+2)\log_2 2$
	Solve for *x*	DM1	Allow for unsimplified expression *x* = … Allow M1 M1 for $x=1.45$ from $6^{2x-1}=4^{x+1}$.
	Obtain answer *x* = 2.21	A1	The question asks for 2 dp.
	Alternative method for question 1		
	Correct use of indices to obtain $2.25^x=6$ or $1.5^{2x}=6$	M1 A1	
	Correct use of logarithms to solve for *x*	M1	Allow solution of $2.25^x=6$ by trial and improvement as far as 2.2…
	Obtain answer *x* = 2.21	A1	Need to see an intermediate step / sequence of iterations.
		4	

Question	Answer	Marks	Guidance
2(a)	State a correct unsimplified version of the x^2 or the x^4 term of the expansion of $\left(2-x^2\right)^{-2}$ or $\left(1-\frac{1}{2}x^2\right)^{-2}$	**M1**	$\frac{1}{4}\left(1+2\frac{x^2}{2}+\frac{-2.-3}{2}\left(\frac{x^2}{2}\right)^2 \ldots\right)$ Symbolic binomial coefficients are not sufficient for the M1.
	State correct first term $\frac{1}{4}$	**B1**	Accept 2^{-2}.
	Obtain the next two terms $\frac{1}{4}x^2+\frac{3}{16}x^4$	**A1 A1**	A1 for each one correct ISW. Full marks for $\frac{1}{4}\left(1+x^2+\frac{3}{4}x^4\right)$ ISW.
			SC allow **M1 A1 A1** for $\frac{1}{4}$ and $1+x^2+\frac{3}{4}x^4$ SOI. **SC** allow **M1 A1** for $1+x^2+\frac{3}{4}x^4$
		4	
2(b)	State answer $\lvert x\rvert<\sqrt{2}$	**B1**	Or $-\sqrt{2}<x<\sqrt{2}$.
		1	

Question	Answer	Marks	Guidance
3	Use correct trigonometric formulae to form an equation in $\tan x$	***M1**	e.g. $\frac{1-\tan^2 x}{\tan x}+\frac{3}{\tan x}=5$
	Obtain a correct linear equation in any form	**A1**	$1-\tan^2 x+3=5\tan x$
	Reduce equation to a 3-term quadratic	**A1**	$\tan^2 x+5\tan x-4=0$, or 3-term equivalent
	Solve a 3-term quadratic in tan x and obtain a value of x	**DM1**	
	Obtain answer, e.g. $x = 35.1°$	**A1**	
	Obtain second answer, e.g. $x = 99.9°$, and no other in $(0°, 180°)$	**A1**	Ignore answers outside $(0°, 180°)$. Treat answers in radians $(0.612, 1.74)$ as a misread.
	Alternative method for question 3		
	Use correct formulae for $\sin 2x$ and $\cos 2x$ to form an equation in $\sin x$ and $\cos x$	***M1**	
	Obtain $4\frac{\cos x}{\sin x}-\frac{\sin x}{\cos x}=5$	**A1**	
	Reduce equation to a 3-term quadratic	**A1**	$\tan^2 x+5\tan x-4=0$, or 3-term equivalent
	Solve a 3-term quadratic in tan x and obtain a value of x	**DM1**	
	Obtain answer, e.g. $x = 35.1°$	**A1**	
	Obtain second answer, e.g. $x = 99.9°$, and no other in $(0°, 180°)$	**A1**	Ignore answers outside $(0°, 180°)$. Treat answers in radians $(0.612, 1.74)$ as a misread.
		6	

Question	Answer	Marks	Guidance
4	Separate variables correctly	B1	$\int \frac{x}{1+x^2}\,dx = \int \frac{1}{y}\,dy$ Accept without integral signs.
	Obtain term ln y	B1	
	State term of the form $k\ln\left(1+x^2\right)$	M1	
	State correct term $\frac{1}{2}\ln\left(1+x^2\right)$	A1	OE
	Evaluate a constant, or use limits $x = 0$, $y = 2$ in a solution containing terms $a\ \ln y$ and $b\ln\left(1+x^2\right)$ where $ab \neq 0$	M1	If they remove logs first the constant must be of the correct form.
	Obtain correct solution in any form	A1	e.g $\ln y + \ln\frac{1}{2} = \frac{1}{2}\ln\left(1+x^2\right)$
	Simplify and obtain $y = 2\sqrt{1+x^2}$	A1	OE The question asks for simplification, so need to deal with $\exp(\ln(\ldots))$.
		7	

Question	Answer	Marks	Guidance
5(a)	Substitute $x = 2$, equate to zero	**M1**	Or divide by $x - 2$ and equate constant remainder to zero.
	Obtain a correct equation, e.g. $8a - 40 + 2b + 8 = 0$	**A1**	Seen or implied in subsequent work.
	Differentiate p(x), substitute $x = 2$ and equate result to zero	**M1**	Or divide by $x - 2$ and equate constant remainder to zero.
	Obtain $12a - 40 + b = 0$, or equivalent	**A1**	SOI in subsequent work.
	Obtain $a = 3$ and $b = 4$	**A1**	
	Alternative method for question 5(a)		
	State or imply $(x-2)^2$ is a factor	**M1**	
	$\mathrm{p}(x)=(x-2)^2(ax+2)$	**A1**	
	Obtain an equation in b	**M1**	
	e.g. by comparing coefficients of x: $b=4a-8$	**A1**	
	Obtain $a = 3$ and $b = 4$	**A1**	
			SC If uses $x = -2$ in both equations allow **M1** and allow **A1** for $a = -3$, $b = -4$.
		5	

Question	Answer	Marks	Guidance
5(b)	Attempt division by $(x-2)$	**M1**	The M1 is earned if division reaches a partial quotient of ax^2+kx, or if inspection has an unknown factor ax^2+ex+f and an equation in *e* and/or *f*. Where *a* has the value found in part **5(a)**.
	Obtain quadratic factor $3x^2-4x-4$	**A1**	
	Obtain factorisation $(3x+2)(x-2)(x-2)$	**A1**	
	Alternative method for question 5(b)		
	State or imply $(x-2)^2$ is a factor	**B1**	
	Attempt division by $(x-2)^2$, reaching a quotient $ax+k$ or use inspection with unknown factor $cx+d$ reaching a value for *c* or for *d*	**M1**	
	Obtain factorisation $(3x+2)(x-2)^2$	**A1**	Accept $3\left(x+\frac{2}{3}\right)(x-2)^2$.
		3	

Question	Answer	Marks	Guidance
6(a)	State or imply $\mathrm{d}x = 3\sec^2\theta\,\mathrm{d}\theta$	**B1**	
	Substitute throughout for x and $\mathrm{d}x$	**M1**	
	Obtain any correct form in terms of θ	**A1**	e.g. $\int \frac{81\sec^2\theta}{\left(9+9\tan^2\theta\right)^2}\mathrm{d}\theta$
	Justify change of limits and obtain $\int_0^{\frac{\pi}{4}}\cos^2\theta\,\mathrm{d}\theta$ correctly	**A1**	AG
		4	
6(b)	Obtain indefinite integral of the form $\int a + b\cos 2\theta\,\mathrm{d}\theta$, where $ab \neq 0$	***M1**	
	Obtain $\frac{1}{2}\theta + \frac{1}{4}\sin 2\theta$	**A1**	
	Use correct limits correctly in an expression containing $p\theta$ and $q\sin 2\theta$ where $pq \neq 0$	**DM1**	$\frac{\pi}{8}+\frac{1}{4}(-0)$
	Obtain answer $\frac{1}{8}(\pi + 2)$	**A1**	Or exact equivalent e.g. $\frac{1}{8}\pi + \frac{1}{4}$.
		4	

Question	Answer	Marks	Guidance
7(a)	Multiply numerator **and** denominator by 1 – 2i, or equivalent	**M1**	At least one multiplication completed.
	Obtain correct numerator $(1-2a)\sqrt{2}-(2+a)\sqrt{2}\mathrm{i}$	**A1**	OE
	Obtain final answer $\frac{1-2a}{5}\sqrt{2}-\frac{2+a}{5}\sqrt{2}\mathrm{i}$	**A1**	OE
	Alternative method for question 7(a)		
	Multiply $x+\mathrm{i}y$ by $1+2\mathrm{i}$ and compare real and imaginary parts	**M1**	
	Obtain $x-2y=\sqrt{2}$ and $2x+y=a\sqrt{2}$	**A1**	
	Obtain final answer $\frac{1-2a}{5}\sqrt{2}-\frac{2+a}{5}\sqrt{2}\mathrm{i}$	**A1**	OE
		3	
7(b)	Obtain $r = 2$	**B1 FT**	
	Obtain $\theta=-\frac{3}{4}\pi$	**B1**	
		2	
7(c)	Use correct method to find r or θ	**M1**	
	State answer $\sqrt{2}\mathrm{e}^{-\frac{3}{8}\pi\mathrm{i}}$	**A1 FT**	
	State answer $\sqrt{2}\mathrm{e}^{\frac{5}{8}\pi\mathrm{i}}$	**A1 FT**	
		3	

Question	Answer	Marks	Guidance
8(a)	State or imply $3y^2 \dfrac{dy}{dx}$ as derivative of y^3	**B1**	
	State or imply $2y + 2x \dfrac{dy}{dx}$ as derivative of $2xy$	**B1**	
	Complete differentiation and equate attempted derivative to zero and solve for $\dfrac{dy}{dx}$	**M1**	
	Obtain answer $-\dfrac{3x^2 + 2y}{3y^2 + 2x}$	**A1**	
		4	
8(b)	Find gradient at either (0, – 2) or (– 2, 0)	**M1**	
	Obtain answers $\dfrac{1}{3}$ and 3	**A1 A1**	
	Use $\tan(A \pm B)$ formula to find $\tan\alpha$	**M1**	
	Obtain answer $\tan\alpha = \dfrac{4}{3}$	**A1**	
		5	

Question	Answer	Marks	Guidance
9(a)	Obtain $\overrightarrow{OM} = \mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$	**B1**	
	Use a correct method to find $\overrightarrow{MN}$	**M1**	e.g. $\overrightarrow{MO} + \overrightarrow{OA} + \overrightarrow{AN}$ or $\overrightarrow{MO} + \overrightarrow{ON}$
	Obtain $\overrightarrow{MN} = \mathbf{i} + \mathbf{j} - 2\mathbf{k}$	**A1**	Accept any notation.
		3	
9(b)	Use a correct method to form an equation for *MN*	**M1**	Allow without $\mathbf{r} = \ldots$
	Obtain $\mathbf{r} = 2\mathbf{i} + 3\mathbf{j} + \lambda(\mathbf{i} + \mathbf{j} - 2\mathbf{k})$	**A1 FT**	OE e.g. $\mathbf{r} = \mathbf{i} + 2\mathbf{j} + 2\mathbf{k} + \mu(\mathbf{i} + \mathbf{j} - 2\mathbf{k})$ Must have $\mathbf{r} = \ldots$ Follow *their* answers to part **9(a)**.
		2	
9(c)	State $\overrightarrow{OP}$ for a general point *P* on *MN* in component form, e.g. $(2 + \lambda, 3 + \lambda, -2\lambda)$	**B1**	
	Equate scalar product of $\overrightarrow{OP}$ and a direction vector for *MN* to zero and solve for λ	**M1**	
	Obtain $\lambda = -\dfrac{5}{6}$	**A1**	OE e.g. $\mu = \frac{1}{6}$
	Obtain $\sqrt{\frac{53}{6}}$ correctly	**A1**	AG e.g. from $\sqrt{\left(\frac{7}{6}\right)^2 + \left(\frac{13}{6}\right)^2 + \left(\frac{5}{3}\right)^2}$
		4	

Question	Answer	Marks	Guidance
10(a)	Use correct product rule	**M1**	Condone incorrect / missing chain rule
	Obtain correct derivative in any form	**A1**	e.g. $\frac{dy}{dx} = \sqrt{\sin x} + \frac{x\cos x}{2\sqrt{\sin x}}$ or $2y\frac{dy}{dx} = 2x\sin x + x^2\cos x$
	Equate derivative to zero and obtain an equation in tan x or tan a	**M1**	
	Obtain $\tan a = -\frac{1}{2}a$ correctly	**A1**	AG
		4	
10(b)	Calculate the value of a relevant expression or pair of expressions at $a = 2$ and $a = 2.5$	**M1**	Must be working in radians At least one correct
	Complete the argument correctly with correct calculated values	**A1**	e.g. $-1 > -2.18$ and $-1.25 < -0.747$
		2	
10(c)	State a suitable equation, e.g. $x = \pi - \tan^{-1}\left(\frac{1}{2}x\right)$	**B1**	A correct equation without subscripts or quote $\tan\theta = -\tan(\pi - \theta)$
	Using $\tan(A \pm B)$ formula, or otherwise, rearrange this as $\tan x = -\frac{1}{2}x$	**B1**	Complete argument correctly
		2	

Question	Answer	Marks	Guidance
10(d)	Use the iterative process correctly at least once	**M1**	Must be working in radians
	Obtain answer *a* = 2.29	**A1**	
	Show sufficient iterations to 4 dp to justify 2.29 to 2 dp or show there is a sign change in the interval (2.285, 2.295)	**A1**	e.g. 2.25, 2.2974, 2.2871, 2.2893, 2.2888, …
		3	

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Cambridge International A Level

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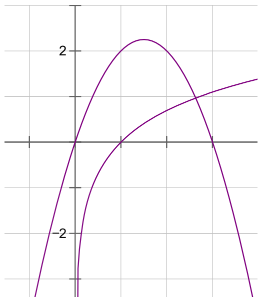
	State a correct equation without logs (in any form)	**A1**	e.g. $3+e^{2x}=3e^{2x}$
	Carry out correct method to solve an equation of the form $e^{2x}=a$, where $a>0$, or for solving $e^{x}=b$ $(b>0)$ if they have already taken the square root	**M1**	Allow for $x=\frac{1}{2}\ln\frac{3}{2}$. M1 can be implied by correct answer.
	Obtain answer $x = 0.203$	**A1**	CAO. The question requires 3 d.p. Answer only with no working shown is 0/4.
		4	

Question	Answer	Marks	Guidance
2	Use correct double-angle formula to obtain an equation in $\cos\theta$	**M1**	e.g. $3\left(2\cos^2\theta-1\right)=3\cos\theta+2$
	Obtain $6\cos^2\theta-3\cos\theta-5=0$, or 3-term equivalent	**A1**	M1 A0 is scored if they use any correct formula for $\cos 2\theta$ and make a subsequent error.
	Solve a 3-term quadratic in $\cos\theta$ for θ	**M1**	As far as $\theta=\cos^{-1}\left(\frac{3-\sqrt{129}}{12}\right)$ if quadratic correct.
	Obtain a correct answer, e.g. 134.1°	**A1**	Accept greater accuracy e.g. 134.1456, 225.8544.
	Obtain a second answer, e.g. 225.9° and no other in $\left[0°, 360°\right]$	**A1 FT**	Treat answers in radians (2.34 and 3.94) as a misread. Ignore answers outside $\left[0°, 360°\right]$. The FT is for 360° minus the first answer.
			Special Ruling: If they have an **incorrect** quadratic that leads legitimately to 4 solutions for θ, allow FT for 360° minus an answer in $(0°, 180°)$. More than 4 solutions is maximum M1 A0 M1 A0 A0. If *their* equation should have 4 solutions and the candidate only gives 3 solutions then M1 A0 M1 A0 A0. Mis-read leading to a quadratic with 4 solutions could score maximum M1 A0 M1 A1 A1 or M1 A0 M1 A1 A0 if extra/missing solution.
		5	

Question	Answer	Marks	Guidance
3	Substitute $x = \frac{1}{2}$, equate result to zero	**M1**	Or divide by $2x-1$ and equate constant remainder to zero.
	Obtain a correct simplified equation	**A1**	e.g. $\frac{1}{8}a + \frac{1}{4} + \frac{1}{2}b + 3 = 0$ or $a + 4b = -26$
	Substitute $x = -2$, equate result to 5	**M1**	Or divide by $x+2$ and equate constant remainder to 5.
	Obtain a correct simplified equation	**A1**	e.g. $-8a + 4 - 2b + 3 = 5$ or $8a + 2b = 2$
	Obtain $a = 2$ and $b = -7$	**A1**	WWW
		5	

Question	Answer	Marks	Guidance
4	Use the correct product rule and then the chain rule to differentiate either $\cos^3 x$ or $\sqrt{\sin x}$	M1	e.g. two terms with one part of $\frac{\mathrm{d}y}{\mathrm{d}x} = p\cos^2 x\sin x\sqrt{\sin x} + q\frac{\cos^3 x\cos x}{\sqrt{\sin x}}$.
	Obtain correct derivative in any form e.g. $\frac{\mathrm{d}y}{\mathrm{d}x} = -3\cos^2 x\sin x\sqrt{\sin x} + \frac{\cos^3 x\cos x}{2\sqrt{\sin x}}$	A1 A1	A1 for each correct term substituted in the complete derivative.
	Equate their derivative to zero and obtain a horizontal equation with positive integer powers of sin x and/or cos x from an equation including $\sqrt{\sin x}$ or $\frac{1}{\sqrt{\sin x}}$ using sensible algebra.	M1	e.g. $-3\cos^2 x\sin^2 x + \frac{1}{2}\cos^4 x = 0$
	Use correct formula(s) to express *their* equation/derivative in terms of one trigonometric function	M1	Can be awarded before the previous M1. May involve more than one trigonometric term.
	Obtain $7\cos^2 x = 6$, $7\sin^2 x = 1$, or $6\tan^2 x = 1$, or equivalent, and obtain answer $x = 0.388$	A1	CAO. The question asks for 3 sf. Ignore additional answers outside $\left(0, \frac{\pi}{2}\right)$. 22.2° is A0.
		6	

Question	Answer	Marks	Guidance
5(a)	Sketch a relevant graph, e.g. $y = \ln x$	B1	 $\ln(x)$: sketch should imply y-axis is an asymptote. Through (1, 0) if marked. Correct shape. $3x - x^2$: Symmetrical. Through (0, 0) and (3, 0) if marked. If ln(x) correct accept parabola for +ve y only. If ln(x) incorrect then need parabola in 3 quadrants.
	Sketch a second relevant graph, e.g. $y = 3x - x^2$, **and** justify the given statement by marking the root on the sketch or by use of a suitable comment	B1	
		2	
5(b)	Calculate the values of a relevant expression or pair of expressions at $x = 2$ and $x = 2.8$	M1	Allow for a smaller interval. At least one value correct if comparing with 0. If using pairs then the pairing must be clear.
	Complete the argument correctly with correct calculated values	A1	e.g. $0.693 < 2$ and $1.03 > 0.56$ or $1.307 > 0, -0.47 < 0$ using $\sqrt{3x - \ln x}$ $0.304 > 0, -0.085 < 0$. Need to have calculated values to at least 2 sf.
		2	

Question	Answer	Marks	Guidance
5(c)	Use the iterative process correctly at least once	**M1**	
	Obtain final answer 2.63	**A1**	
	Show sufficient iterations to at least 4 dp to justify 2.63 to 2 dp or show there is a sign change in the interval (2.625, 2.635)	**A1**	**SC** Allow **M1 A1 A0** to a candidate who starts at a point in the interval and reaches a premature conclusion
		3	

Question	Answer	Marks	Guidance
6(a)	Correct separation of variables	**B1**	$\int e^{-y}\mathrm{d}y = \int xe^{-x}\mathrm{d}x$ Condone missing integral signs.
	Obtain term $-e^{-y}$	**B1**	
	Commence integration by parts and reach $\pm xe^{-x} \pm \int e^{-x}\,\mathrm{d}x$	***M1**	M0 if clearly using differentiation of a product.
	Complete integration and obtain $-xe^{-x} - e^{-x}$	**A1**	
	Use $x = 0$ and $y = 0$ to evaluate a constant or as limits in a solution containing or derived from terms ae^{-y}, bxe^{-x} and ce^{-x}, where $abc \neq 0$	**DM1**	Must see working for this. In a correct solution they should have $-e^{-y} + C = -xe^{-x} - e^{-x}$ or equivalent. If they take logarithms before finding the constant, the constant must be of the right form.
	Correct solution in any form Must follow from correct working	**A1**	e.g. $-e^{-y} = -xe^{-x} - e^{-x}$ A0 if constant of integration ignored or assumed to be zero.
	Obtain final answer $y = -\ln\left((x+1)e^{-x}\right)$ from correct working	**A1**	OE e.g. $y = x - \ln(x+1)$, $y = \ln\left(\frac{e^{x}}{x+1}\right)$. A0 if constant of integration ignored or assumed to be zero.
		7	
6(b)	Obtain answer $(y =) 1 - \ln 2$	**B1**	Must follow from at least 6 or7 obtained in part **6(a).**
		1	

Question	Answer	Marks	Guidance
7(a)	State or imply $6xy+3x^2\dfrac{dy}{dx}$ as derivative of $3x^2y$	**B1**	
	State or imply $3y^2\dfrac{dy}{dx}$ as derivative of y^3	**B1**	Allow B1 B1 for $\left(3x^2dx+\right)6xydx+3x^2dy-3y^2dy\left[=0\right]$
	Equate attempted derivative of left-hand side to zero and solve to obtain an equation with $\dfrac{dy}{dx}$ as subject	**M1**	Allow if zero implied by subsequent working. Allow if recover from an extra $\dfrac{dy}{dx}=\ldots$ at the beginning of the left-hand side.
	Obtain $\dfrac{dy}{dx}=\dfrac{x^2+2xy}{y^2-x^2}$ correctly	**A1**	AG Accept y' for $\frac{dy}{dx}$.
		4	
7(b)	Equate numerator to zero	***M1**	Must be using the given derivative.
	Obtain $x=-2y$, or equivalent	**A1**	An equation with x or y as the subject SOI.
	Use $x^3+3x^2y-y^3=3$ to obtain an equation in x or y	**DM1**	$-8y^3+12y^3-y^3=3$ or $x^3-\frac{3}{2}x^3+\frac{1}{8}x^3=3$ or any equivalent form (do not need to evaluate powers).
	Obtain the point (– 2, 1) and no others from solving their cubic equation	**A1**	Allow if each component stated separately. ISW.
	State the point $(0,\ -\sqrt[3]{3})$, or equivalent from correct work	**B1**	Accept $(0,\ \sqrt[3]{-3})$, or (0, – 1.44) (-1.44225). Allow if each component stated separately. ISW.
		5	

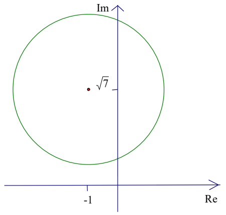
Question	Answer	Marks	Guidance
8(a)	State or imply the form $\dfrac{A}{3x-1}+\dfrac{Bx+C}{x^2+3}$	**B1**	
	Use a correct method for finding a constant	**M1**	
	Obtain one of $A=1$, $B=0$ and $C=3$ from correct working	**A1**	A maximum of M1 A1 is available after B0.
	Obtain a second value from correct working	**A1**	
	Obtain the third value from correct working	**A1**	
		5	
8(b)	Integrate and obtain term $\dfrac{1}{3}\ln(3x-1)$	**B1 FT**	OE e.g. $\dfrac{1}{3}\ln(x-\dfrac{1}{3})$. The FT is on the value of *A*.
	Obtain term of the form $k\tan^{-1}\left(\dfrac{x}{\sqrt{3}}\right)$	**M1**	
	Obtain term $\sqrt{3}\tan^{-1}\left(\dfrac{x}{\sqrt{3}}\right)$	**A1 FT**	OE. The FT is on the value of *C*.
	Substitute correct limits in an integral of the form $a\ln(3x-1)+k\tan^{-1}\left(\dfrac{x}{\sqrt{3}}\right)$, where $ak\neq 0$, and evaluate trigonometry	**M1**	Must be subtracted the right way round. $\left(\frac{1}{3}\ln 8-\frac{1}{3}\ln 2+\sqrt{3}\times\frac{\pi}{3}-\sqrt{3}\times\frac{\pi}{6}\right)$ Angles should be in radians. Condone angles as decimals.
	Obtain answer $\dfrac{2}{3}\ln 2+\dfrac{\sqrt{3}\pi}{6}$ from correct working in part **8(b)**	**A1**	Or exact 2-term equivalent e.g. $\dfrac{1}{3}\ln 4+\dfrac{\pi}{2\sqrt{3}}$ ISW.
		5	

Question	Answer	Marks	Guidance
9(a)	Express general point of *l* or *m* in component form, i.e. $(-1+2\lambda, 3-\lambda, 4-\lambda)$ or $(5+a\mu, 4+b\mu, 3+\mu)$	**B1**	
	Equate components and eliminate either λ or μ	**M1**	e.g. $\mu=\frac{2}{1-b}$, $\lambda=\frac{-1-b}{1-b}$, $\mu=\frac{-4}{2+a}$, $\lambda=\frac{a+6}{a+2}$
	Eliminate the other parameter or obtain a second expression in the first	**M1**	λ and μ are not required to be the subject of the equations.
	Show intermediate steps to obtain $2b-a=4$	**A1**	AG
	Alternative method for question 9(a)		
	Express general point of *l* or *m* in component form, i.e. $(-1+2\lambda, 3-\lambda, 4-\lambda)$ or $(5+a\mu, 4+b\mu, 3+\mu)$	**B1**	
	Express *a* or *b* in terms of λ and μ	**M1**	$a=\dfrac{2\lambda-6}{\mu}$, $b=\dfrac{-1-\lambda}{\mu}$
	Use $\lambda=1-\mu$	**M1**	
	Obtain $2b-a=4$	**A1**	AG
		4	
9(b)	Using the correct process equate the scalar product of the direction vectors to zero	***M1**	$(2\mathbf{i}-\mathbf{j}-\mathbf{k}).(a\mathbf{i}+b\mathbf{j}+\mathbf{k})=0$ SOI.
	Obtain $2a-b-1=0$	**A1**	OE e.g. $2(2b-4)-b-1=0$
	Solve simultaneous equations for *a* or for *b*	**DM1**	
	Obtain *a* = 2, *b* = 3	**A1**	
		4	

Question	Answer	Marks	Guidance
9(c)	Substitute found values in component equations and solve for λ or for μ	**M1**	
	Obtain answer 3**i** + **j** + 2**k** from either $\lambda = 2$ or $\mu = -1$	**A1**	Accept as coordinates or equivalent.
		2	

Question	Answer	Marks	Guidance
10(a)	Substitute $x = -1+\sqrt{7}\mathrm{i}$ in the equation and attempt expansions of x^2 and x^3	***M1**	
	Use $\mathrm{i}^2 = -1$ correctly at least once and solve for k	**DM1**	$2\left(20-4\sqrt{7}\mathrm{i}\right)+3\left(-6-2\sqrt{7}\mathrm{i}\right)+14\left(-1+\sqrt{7}\mathrm{i}\right)+k=0$
	Obtain answer $k = -8$	**A1**	
			SC B1 only for those who show no working for the cube and square and obtain answer $k = -8$.
	Alternative method for question 10(a)		
	Attempt division by $\left(x+1-\sqrt{7}\mathrm{i}\right)$ as far as $2x^2 + z_1x + \ldots$	***M1**	See division on next page.
	Use $\mathrm{i}^2 = -1$ correctly at least once and obtain $2x^2 + z_1x + z_2 +$ remainder	**DM1**	
	Obtain answer $k = -8$	**A1**	
		3	

Question	Answer	Marks	Guidance
10(b)	State answer $-1-\sqrt{7}\mathrm{i}$	**B1**	Can be seen simply stated on its own, or in a list of roots. Allow if stated clearly in part **10(a)**.
	Carry out a method for finding a quadratic factor with zeros $-1+\sqrt{7}\mathrm{i}$ and $-1-\sqrt{7}\mathrm{i}$	**M1**	Or state $\left(x-\left(-1+\sqrt{7}\mathrm{i}\right)\right)\left(x-\left(-1-\sqrt{7}\mathrm{i}\right)\right)(2x-p)$
	Obtain x^2+2x+8	**A1**	Or obtain $\left(-1+\sqrt{7}\mathrm{i}\right)\left(-1-\sqrt{7}\mathrm{i}\right)p=-8$ Or obtain $\left(-1+\sqrt{7}\mathrm{i}\right)+\left(-1-\sqrt{7}\mathrm{i}\right)+\frac{p}{2}=-\frac{3}{2}$
	Obtain root $x=\dfrac{1}{2}$, or equivalent, via division or inspection	**A1**	Needs to follow from the working.
		4	

Question	Answer	Marks	Guidance
10(c)	Show a circle with centre $-1+\sqrt{7}\text{i}$	**B1**	
	Show circle with radius 2 and centre not at the origin There needs to be some evidence of scale e.g. radius marked or a scale on the axes	**B1**	If the scales are very different from each other then B1 for centre in the correct position and B1 for an ellipse. If there is more than one circle the max score is B1.
		2	
10(d)	Carry out a complete method for calculating the maximum value of arg *z* for correct circle	**M1**	e.g. $\frac{\pi}{2}+\tan^{-1}\frac{1}{\sqrt{7}}+\frac{\pi}{4}$ Can be implied by 155.7°.
	Obtain answer 2.72 radians	**A1**	CAO. The question requires radians.
		2	

Cambridge Assessment International Education

Cambridge International A Level

MATHEMATICS **9709/33**

Paper 3 Pure Mathematics 3 **May/June 2022**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2022 series for most Cambridge IGCSE, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

This document consists of **23** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (ISW).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M	Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
A	Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
B	Mark for a correct result or statement independent of method marks.
DM or **DB**	When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
FT	Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	State or imply non-modular inequality $2^2(3x+a)^2 < (2x+3a)^2$, or corresponding quadratic equation, or pair of linear equations	**B1**	e.g. $(6x+2a)^2 = (2x+3a)^2$ or $32x^2 + 12xa - 5a^2 = 0$ $2(3x + a) = (2x + 3a)$ and $-2(3x + a) = (2x + 3a)$
	Solve 3-term quadratic, or solve two linear equations for x	**M1**	Apply general rules for solving quadratic equation by formula or by factors. Instead of $x = \{\text{formula}\}$, have $\{\text{formula}\} = 0$ and try to solve for a then M0
	Obtain critical values $x = \frac{1}{4}a$ and $x = -\frac{5}{8}a$	**A1**	
	State final answer $-\frac{5}{8}a < x < \frac{1}{4}a$ or $-0.625a < x < 0.25a$ or $x > -\frac{5}{8}a$ **and** $x < \frac{1}{4}a$ or $x > -\frac{5}{8}a \cap x < \frac{1}{4}a$	**A1**	Do not condone $\leqslant$ for $<$ in the final answer. Do not ISW. **SC** Set a to value, (say $a = 1$), after initial B1 gained, then $-\frac{5}{8} < x < \frac{1}{4}$ **B1** maximum 2 out of 4.
	Alternative method for question 1		
	Obtain critical value $x = \frac{1}{4}a$ from a graphical method, or by solving a linear equation or linear inequality	**B1**	
	Obtain critical value $x = -\frac{5}{8}a$ similarly	**B2**	
	State final answer $-\frac{5}{8}a < x < \frac{1}{4}a$ or $-0.625a < x < 0.25a$ or $x > -\frac{5}{8}a$ **and** $x < \frac{1}{4}a$ or $x > -\frac{5}{8}a \cap x < \frac{1}{4}a$	**B1**	Do not condone $\leqslant$ for $<$ in the final answer. Do not ISW**.** **SC** Set a to value, (say $a = 1$), after initial B1 gained, then $-\frac{5}{8} < x < \frac{1}{4}$ **B1** maximum 2 out of 4.
		4	

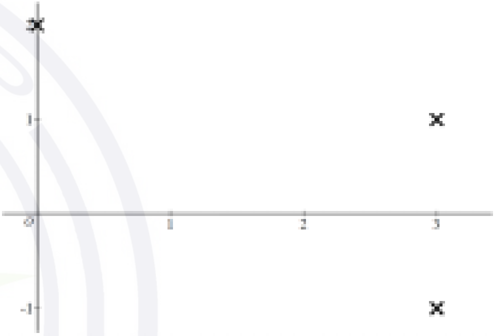
Question	Answer	Marks	Guidance
2	Use correct $\cos(A-B)$ formula to obtain an equation in $\cos\theta$ and $\sin\theta$	**B1**	$\cos\theta\cos 60 + \sin\theta\sin 60 = 3\sin\theta$
	Use trigonometric formula and substitute values for cos 60 and sin 60 to obtain an equation in $\tan\theta$ (or $\cos\theta$ or $\sin\theta$)	**M1**	Allow $\frac{1}{2}$ and $\frac{\sqrt{3}}{2}$ interchanged. $\frac{1}{2}+\frac{\sqrt{3}}{2}\tan\theta = 3\tan\theta$ $\frac{1}{4}\cos^2\theta = \left(3-\frac{\sqrt{3}}{2}\right)\left(3-\frac{\sqrt{3}}{2}\right)(1-\cos^2\theta)$ $\frac{1}{4}(1-\sin^2\theta) = \left(3-\frac{\sqrt{3}}{2}\right)\left(3-\frac{\sqrt{3}}{2}\right)\sin^2\theta$
	Obtain $\tan\theta = \frac{1}{6-\sqrt{3}}$ or $\tan\theta = \frac{6+\sqrt{3}}{33}$ or 0.2343, $\cos\theta = \frac{3\ \frac{\sqrt{3}}{2}}{\sqrt{10-3\sqrt{3}}}$ or 0.9736 or $\sin\theta = \frac{\frac{1}{2}}{\sqrt{10-3\sqrt{3}}}$ or 0.2281	**A1**	OE
	Obtain answer, e.g. $\theta = 13.2°$	**A1**	May be more accurate, allow value rounding to 13.2°. $\theta = 13.1867°$.
	Obtain second answer, e.g. $\theta = 193.2°$ and no others in the given interval	**A1 FT**	May be more accurate. Allow value rounding to 193.2°. FT is on previous value of θ, must have scored M1. Note if θ is negative (e.g. -13.2): $-13.2 + 180 = 166.8$ A0 but $-13.2 + 360 = 346.8$ A1 FT. Ignore answers outside the given interval. Treat answers in radians as a misread. 0.23015, 3.3717.

Question	Answer	Marks	Guidance
2	**Alternative method for question 2** – using $R\cos(\theta \pm \alpha)$ or equivalent		
	Use correct cos($A - B$) formula to obtain an equation in $\cos\theta$ and $\sin\theta$	**B1**	$\cos\theta\cos 60 + \sin\theta\sin 60 = 3\sin\theta$
	Correct method for finding $\tan\alpha$ from $p\cos\theta + q\sin\theta = 0$	**M1**	$\tan\alpha = \pm\dfrac{q}{p}$
	Correct value of α	**A1**	76.8° or 1.34 radians (may be more accurate).
	Obtain answer, e.g. $\theta = 13.2°$	**A1**	May be more accurate, allow value rounding to 13.2°. $\theta = 13.1867°$.
	Obtain second answer, e.g. $\theta = 193.2°$ and no others in the given interval	**A1 FT**	May be more accurate. Allow value rounding to 193.2°. FT is on previous value of θ, must have scored M1. Note if θ is negative (e.g. -13.2): $-13.2 + 180 = 166.8$ A0 but $-13.2 + 360 = 346.8$ A1 FT. Ignore answers outside the given interval. Treat answers in radians as a misread. 0.23015, 3.3717.
		5	

Question	Answer	Marks	Guidance
3(a)	Use law of logarithm of a power	**M1**	$\log_3(2x+1) = 1 + \log_3(x-1)^2$
	Use $\log_3 3 = 1$	**B1**	$\log_3(2x+1) = \log_3 3 + 2\log_3(x-1)$ $\left[\log 3\left(\frac{2x+1}{(x-1)^2}\right) = \log_3 3 \quad \text{or} \quad \left(\frac{2x+1}{(x-1)^2}\right) = 3\right]$ **SC** For candidates scoring M0 B0 due to combining logs before dealing with coefficient 2, and confusing coefficients, allow $\log_3(\ldots) = c$ leading to $(\ldots) = 3^c$ **B1**.
	Obtain $3x^2 - 8x + 2 = 0$ or $1.5x^2 - 4x + 1 = 0$	**A1**	OE 3 terms only and = 0 required.
		3	
3(b)	Solve 3-term quadratic equation from part **3(a)** or restart to find y	**M1**	$y = \frac{4 \pm \sqrt{10}}{6}$ or $y = 1.1937\ldots$ or $y = 0.1396\ldots$ ($x = 2.3874$ or $x = 0.2792$) May solve for x but must find $y = \frac{x}{2}$ to gain M1.
	Obtain answer 1.19	**A1**	CAO. 2 dp required.
		2	

Question	Answer	Marks	Guidance
4(a)	Use correct product rule or quotient rule, and attempt at chain rule	**M1**	$ke^{-4x}\tan x + e^{-4x}\sec^2 x$ or $\dfrac{e^{4x}\sec^2 x - \tan x(ke^{4x})}{(e^{4x})^2}$ Need to see d(tan x)/dx = $\sec^2 x$ (formula sheet) and attempt at ke^{-4x}, where $k \neq 1$.
	Obtain correct derivative in any form	**A1**	$-4e^{-4x}\tan x + e^{-4x}\sec^2 x$ or $\dfrac{e^{4x}\sec^2 x - \tan x(4e^{4x})}{(e^{4x})^2}$
	Use trigonometric formulae to express derivative in the form $ke^{-4x}\sin x\cos x\sec^2 x + ae^{-4x}\sec^2 x$ or $ke^{-4x}\dfrac{\sin x\cos x}{\cos x\cos x} + ae^{-4x}\sec^2 x$ or $\sec^2 x(ke^{-4x}\sin x\cos x + ae^{-4x})$ Allow $\dfrac{1}{\cos^2 x}$ instead of $\sec^2 x$	**M1**	Need to use $\dfrac{\tan x}{\sec^2 x} = \sin x\cos x$ or $\tan x = \dfrac{\sin x}{\cos x}.\dfrac{\cos x}{\cos x}$ OE. M1 is independent of previous M1, but expression must be of appropriate form.
	Obtain correct answer with $a = 1$ and $b = -2$	**A1**	At least one line of trigonometric working is required from $-4e^{-4x}\tan x + e^{-4x}\sec^2 x$ to given answer $\sec^2 x(1 - 2\sin 2x)\,e^{-4x}$ with elements in any order. If only error: $4\sin x\cos x = 4\sin 2x$ M1 A1 M1 A0.
		4	

Question	Answer	Marks	Guidance
4(b)	Equate derivative to zero and use correct method to solve for x	**M1**	$\sin 2x = \frac{1}{2}$, hence $x = \frac{1}{2}\sin^{-1}\frac{1}{2}$ or $x = \tan^{-1}(2 \pm \sqrt{3})$ Allow M1 for correct method for non-exact value.
	Obtain answer, e.g. $x = \frac{1}{12}\pi$	**A1**	[0.262 M1 A0]
	Obtain second answer, e.g. $\frac{5}{12}\pi$ and no other in the given interval	**A1 FT**	FT $\frac{\pi - \mathit{their}\ 2x}{2}$ if exact values; x must be $< \frac{\pi}{2}$. Ignore answers outside the given interval. Treat answers in degrees as a misread. 15°, 75°. **SC** No values found for a and b in **4(a)** but chooses values in **4(b)**: max **M1** for x.
		3	

Question	Answer	Marks	Guidance
5(a)	Show u and u^* in relatively correct positions. Must have sense of scale on axes	**B1**	$u = 3 - \mathrm{i}$, $u^* = 3 + \mathrm{i}$ Ignore labels.
	Show $u^* - u$ in a relatively correct position. Must have sense of scale on axes	**B1**	2i. Scale only on Imaginary axis is sufficient for this mark.
	State that *OABC* is a parallelogram [independent of previous marks]	**B1**	Ignore 'quadrilateral'. Allow 'trapezium' from correct work.
		3	

Question	Answer	Marks	Guidance
5(b)	Multiply *their* numerator and the given denominator by 3 + i and attempt to evaluate either	**M1**	Can have missing term and arithmetic errors but need $i^2 = -1$ once, seen or implied.
	Obtain numerator 8 + 6i or denominator 10	**A1**	
	State final answer $\frac{4}{5}+\frac{3}{5}i$ or $\frac{8}{10}+\frac{6}{10}i$ or 0.8 + 0.6i	**A1**	Correct answer with no working scores 0/3.
	Alternative method for question 5(b)		
	Obtain two equations in x and y, and attempt to solve for x or for y	**M1**	$3 = 3x + y$ and $1 = -x + 3y$
	Obtain $x = \frac{4}{5}$ or $\frac{8}{10}$ or 0.8 $y = \frac{3}{5}$ or $\frac{6}{10}$ or 0.6	**A1**	
	State final answer $\frac{4}{5}+\frac{3}{5}i$ or $\frac{8}{10}+\frac{6}{10}i$ or 0.8 + 0.6i	**A1**	Correct answer with no working scores 0/3.
		3	

Question	Answer	Marks	Guidance
5(c)	State or imply $\arg\frac{u^*}{u} = \arg u^* - \arg u$ or $2\arg u^*$	**M1**	
	Justify the given statement correctly	**A1**	AG $\arg\frac{u^*}{u} = \tan^{-1}\frac{3}{4}$, $\arg u^* = \tan^{-1}\frac{1}{3}$ and $\arg u = \tan^{-1} -\frac{1}{3}$ (or $\arg u = -\tan^{-1}\frac{1}{3}$), needed if use first expression in M1; or $\arg\frac{u^*}{u} = \tan^{-1}\frac{3}{4}$ and $\arg u^* = \tan^{-1}\frac{1}{3}$, needed if use second expression in M1.
	Alternative method for question 5(c)		
	Use $\tan 2A$ formula with $\tan A = \frac{1}{3}$	**M1**	$\tan 2A = \frac{2\tan A}{1-\tan^2 A}$, $\tan A = \frac{1}{3}$, hence $\tan 2A = \frac{3}{4}$.
	Justify the given statement correctly	**A1**	AG So $2A = \tan^{-1}\frac{3}{4} = \arg\frac{u^*}{u}$ and $A = \tan^{-1}\frac{1}{3} = \arg u^*$ hence $\arg\frac{u^*}{u} = 2\arg u^*$.
		2	

Question	Answer	Marks	Guidance
6(a)	Use chain rule at least once	**M1**	Needs $\frac{\mathrm{d}y}{\mathrm{d}t} = \frac{1}{\tan t}\frac{\mathrm{d}}{\mathrm{d}t}(\tan t)$ or $\frac{\mathrm{d}x}{\mathrm{d}t} = (-1)(\cos^{-2} t)\frac{\mathrm{d}}{\mathrm{d}t}(\cos t)$. BOD if + and (– 1) (– 1) not seen. $\frac{\mathrm{d}x}{\mathrm{d}t}$ = sec t tan t (from List of Formulae MF19) M1 A1. If $\frac{\mathrm{d}x}{\mathrm{d}t} = -\sec t \tan t$ M1 A0.
	Obtain $\frac{\mathrm{d}x}{\mathrm{d}t} = \sec t \tan t$	**A1**	OE e.g. $\sin t\,(\cos t)^{-2}$. If e.g. $\frac{\mathrm{d}x}{\mathrm{d}t} = \sec x \tan x$ or $\sec\theta\tan\theta$ or $\sec t\tan x$, condone recovery on next line.
	Obtain $\frac{\mathrm{d}y}{\mathrm{d}t} = \frac{\sec^2 t}{\tan t}$	**A1**	OE e.g. $\frac{1}{\sin t\,\cos t}$. If e.g. $\frac{\mathrm{d}y}{\mathrm{d}t} = \frac{\sec^2 x}{\tan x}$ or $\frac{\sec^2\theta}{\tan\theta}$, condone recovery on next line. Only penalise notation errors **once** in $\frac{\mathrm{d}x}{\mathrm{d}t}$ and $\frac{\mathrm{d}y}{\mathrm{d}t}$ if no recovery.
	Use $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}t} \div \frac{\mathrm{d}x}{\mathrm{d}t}$	**M1**	Allow even if previous M0 scored, but must be using derivatives.
	Obtain given answer $\frac{\cos t}{\sin^2 t}$	**A1**	AG After $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}t} \div \frac{\mathrm{d}x}{\mathrm{d}t}$ used, any notation error A0. Must cancel cos t correctly**.**
		5	

Question	Answer	Marks	Guidance
6(b)	State or imply $t = \frac{1}{4}\pi$ when $y = 0$	**B1**	
	Form the equation of the tangent at $y = 0$ or find c	**M1**	$x = \sqrt{2}$, $\frac{dy}{dx} = \sqrt{2}$ and $y = 0$, *their* coordinates and gradient used in $y = mx + c$.
	Obtain answer $y = \sqrt{2}x - 2$	**A1**	OE e.g. $y = \sqrt{2}\,(x - \sqrt{2})$ ISW. Allow $y = 1.41x - 2[.00]$ or $1.41(x - 1.41)$.
		3	

Question	Answer	Marks	Guidance
7(a)	State or imply the form $\frac{A}{x-2} + \frac{Bx+C}{2x^2+3}$	**B1**	If $1 - \frac{A}{x-2} + \frac{Bx+C}{2x^2+3}$ or $\frac{A}{x-2} + \frac{C}{2x^2+3}$ B0 then M1 A1 (for $A = 3$) still possible.
	Use a correct method for finding a constant	**M1**	
	Obtain one of $A = 3$, $B = -1$ and $C = 6$	**A1**	Allow all A marks obtained even if method would give errors if equations solved in a different order.
	Obtain a second value	**A1**	
	Obtain the third value	**A1**	
		5	

Question	Answer	Marks	Guidance
7(b)	Use correct method to find the first two terms of the expansion of $(x-2)^{-1}$, $\left(1-\frac{1}{2}x\right)^{-1}$, $(2x^2+3)^{-1}$ or $\left(1+\frac{2}{3}x^2\right)^{-1}$	**M1**	Symbolic binomial coefficients not sufficient for the M1.
	Obtain correct unsimplified expansions, up to the term in x^2, of each partial fraction	**A1 FT** **A1 FT**	The FT is on *A*, *B* and *C*. $-\frac{A}{2}\left[1-\left(-\frac{x}{2}\right)+\frac{(-1)(-2)}{2}\left(-\frac{x}{2}\right)^2+\ldots\right]$ $\frac{Bx+C}{3}\left[1-\frac{2x^2}{3}+\ldots\right]$
	Extract the coefficient 3 correctly from $(2x^2+3)^{-1}$ with expansion to $1\pm\frac{2}{3}x^2$ then multiply by $Bx+C$ up to the terms in x^2, where $BC\neq 0$	**M1**	$\frac{C}{3}+\frac{Bx}{3}\pm\frac{C}{3}\left(\frac{2}{3}\right)x^2$ or $\frac{1}{3}\left(C+Bx\pm C\left(\frac{2}{3}\right)x^2\right)$ Allow a slip in multiplication for M1. Allow miscopies in *B* and *C* from **7(a)**.
	Obtain final answer $\frac{1}{2}-\frac{13}{12}x-\frac{41}{24}x^2$	**A1**	Do not ISW.
		5	

Question	Answer	Marks	Guidance
8(a)	Separate variables correctly	**B1**	$\frac{dN}{N^{\frac{3}{2}}} = (k\cos 0.02t)\,dt$ Allow without integral signs.
	Obtain term $-\frac{2}{\sqrt{N}}$	**B1**	OE Ignore position of k.
	Obtain term $50\sin 0.02t$	**B1**	OE Ignore position of k.
	Use $t = 0$, $N = 100$ to evaluate a constant, or as limits, in a solution containing terms $\frac{a}{\sqrt{N}}$ and $b\sin 0.02t$, where $ab \neq 0$	**M1**	$\left[\text{e.g. } c = -0.2 \quad \text{or} \quad c = \frac{-0.2}{k}\right]$
	Obtain correct solution in any form, e.g. $-\frac{2}{\sqrt{N}} = 50k\sin 0.02t - 0.2$	**A1**	OE ISW e.g. $N = \frac{1}{(25k\sin 0.02t - 0.1)^2}$ $\quad -2N^{-\frac{1}{2}} = \frac{k}{0.02}\sin 0.02t - \frac{1}{5}$ $50k\sin 0.02t = -\frac{2}{\sqrt{N}} + \frac{1}{5}$ $\quad \frac{1}{\sqrt{N}} = -\frac{1}{2}k\left(50\sin 0.02t\right) + \frac{1}{10}$ $50\sin\left(\frac{1}{50}t\right) = -\frac{2\sqrt{N}}{kN} + \frac{20}{100k}$
		5	
8(b)	Use the substitution $N = 625$ and $t = 50$ in expression of appropriate form to evaluate k	**M1**	Expression must contain $a + bk\sin 0.02t$, $(\sqrt{N})^{\pm n}$, where $n = -1, 1, 3$ or 5 and a and b are constants $ab \neq 0$ or $(a + bk\sin 0.02t)^{\pm 2}$ and $(N)^{\pm n}$. Allow with k replaced by $\frac{1}{k}$, error due to $k(N^{-3/2})$ when separating variables in **8(a)**. If invert term by term when 3 terms shown then M0.
	Obtain $k = 0.00285[2148]$	**A1**	Must evaluate sin1. Degrees $k = 0.138$ M1 A0.
		2	

Question	Answer	Marks	Guidance
8(c)	Rearrange and obtain $N = 4(0.2-0.142(607)\sin 0.02t)^{-2}$ Substitution for *k* required	**M1**	Anything of the form $N = c(d-ek\sin 0.02t)^{-2}$, where *c*, *d* and *e* are constants $cde \neq 0$ and value of *k* substituted. Allow with *k* replaced by 1/*k*, error due to $k(N^{-3/2})$ when separating variables in **8(a)**. OE ISW e.g. $N=\left(-\frac{10}{0.7125\sin 0.02t-1}\right)^{2}$ $N=\frac{1}{(-0.0713\sin 0.02t+0.1)^{2}}$ $N=\frac{100}{\left(\left(\frac{0.6}{\sin 1}\right)\sin 0.02t-1\right)^{2}}$ $N=\frac{1}{\left(\frac{3}{-50\sin 1}\times\sin 0.02t+\frac{1}{10}\right)^{2}}$ $N=\left(-\frac{0.06}{\sin 1}\sin 0.02t+0.1\right)^{-2}$ $N=\left(\frac{800}{80-57\sin 0.02t}\right)^{2}$ Do not need to substitute for sin(0.02*t*) = 1, but must substitute for *k*.
	Accept answers between 1209 and 1215	**A1**	ISW Substitute $\sin 0.02t = 1$ or $t = 50\sin^{-1}1$ or 78.5 or 25π. Answer with no working (rubric) 0/2. **SC** *N* = … not seen but correct numerical answer **B1** 1/2.
		2	

Question	Answer	Marks	Guidance
9(a)	Using the correct process find the scalar product of direction vectors of l and **OA**	**M1**	$(1, 5, 6)\mathbf{.}(-1, 2, 3) = -1.1 + 5.2 + 6.3 = -1 + 10 + 18$
	Using the correct process for the moduli, divide the scalar product by the product of the moduli and find the inverse cosine of the result	**M1**	*Their* scalar product ÷ $[\sqrt{(1^2 + 5^2 + 6^2)}\sqrt{((-1)^2 + 2^2 + 3^2)}]$. Angle = $\cos^{-1}\dfrac{27}{\sqrt{62}\sqrt{14}}$.
	Obtain answer 23.6°.	**A1**	AWRT 23.6°. 23.5889°. Radians 0.412 scores A0 (0.4117…).
		3	
9(b)	Taking a general point P on l, state **AP** (or **PA**) in component form, e.g. $(3 - \lambda, -5 + 2\lambda, -5 + 3\lambda)$	**B1**	Note: (4, 1, 0) or (4, 1, 1), for 4**i** + **k** is not MR, but M1 possible.
	Either equate scalar product of **AP** and direction vector of l to zero and solve for λ *or* use Pythagoras in a relevant triangle and solve for λ	**M1**	$(3 - \lambda, -5 + 2\lambda, -5 + 3\lambda)\mathbf{.}(-1, 2, 3) = 0$ $-3 \ -10 \ -15 + \lambda + 4\lambda + 9\lambda = 0$ or let **OQ** = (4, 0, 1) so **AQ** = (3, – 5, – 5), **QP** = $(-\lambda, 2\lambda, 3\lambda)$, **AP** = $(3 - \lambda, -5 + 2\lambda, -5 + 3\lambda)$ hence $3^2 + (-5)^2 + (-5)^2 =$ $(3 - \lambda)^2 + (-5 + 2\lambda)^2 + (-5 + 3\lambda)^2 + (-\lambda)^2 + (2\lambda)^2 + (3\lambda)^2$ Other alternative approaches are possible, e.g. minimise AP or AP^2, either by completing the square or by differentiating.
	Obtain $\lambda = 2$	**A1**	$\lambda = 2$
	State that the position vector **OP*** of the foot is 2**i** + 4**j** + 7**k**	**A1**	OE Condone coordinates.
		4	

Question	Answer	Marks	Guidance
9(c)	Set up a correct method for finding the position vector of the reflection of *A* in *l*	**M1**	For all methods, allow a sign error in one component only: **OA′** = **OP*** + (**OP*** – **OA**) $their(2,4,7)+(their\,2,4,7-1,5,6)$ or **OA′** = **OP*** – (**OA** – **OP***) $their(2,4,7)-(1,5,6-their\,2,4,7)$ or **OA′** = **OA** + 2(**OP*** – **OA**) $\begin{pmatrix}1+2(their\,2-1)\\5+2(their\,4-5)\\6+2(their\,7-6)\end{pmatrix}$ or midpoint **OP*** = (**OA** + **OA′**)/2 with *their* λ value substituted. $\frac{1+x}{2}=their\,2\quad\frac{5+y}{2}=their\,4\quad\frac{6+z}{2}=their\,7$
	Obtain answer 3**i** + 3**j** + 8**k** or $3\left(\mathbf{i}+\mathbf{j}+\frac{8}{3}\right)$	**A1**	OE Condone coordinates $x=3, y=3, z=8$ A1. No method shown and correct answer 2/2.
		2	

Question	Answer	Marks	Guidance
10(a)	Commence integration and reach $ax^3 \ln x + b\int x^3 . \frac{1}{x}\ \mathrm{d}x$	*M1	OE Allow omission of d*x*.
	Obtain $\frac{1}{3}x^3 \ln x - \frac{1}{3}\int x^3 .\ \frac{1}{x}\ \mathrm{d}x$	A1	OE Allow omission of d*x*.
	Complete integration and obtain $\frac{1}{3}x^3 \ln x - \frac{1}{9}x^3$	A1	Allow $-\frac{1}{3}\left(\frac{1}{3}x^3\right)$.
	Use limits correctly and equate to 4, having integrated twice	DM1	$\frac{1}{3}a^3 \ln a - \frac{1}{9}a^3 - (0 - \frac{1}{9}) = 4$ allow one sign error OR one numerical error, but 0 may be absent or expressed as $\frac{a^3}{3}\ln 1$. Allow $-\frac{1}{3}\left(\frac{1}{3}ax^3\right)$ and $-\frac{1}{3}\left(\frac{1}{3}\right)$.
	Obtain given result correctly	A1	$a = \left(\frac{35}{3\ln a - 1}\right)^{\frac{1}{3}}$ AG After substitution, any errors even if corrected A0. Need to see at least one line of working between substitution and the given answer.
		5	
10(b)	Calculate the values of a relevant expression or pair of expressions at $a = 2.4$ and $a = 2.8$ All values must be correct for M1 (numerical question)	M1	
	Justify the given statement with correct calculated values	A1	$2.4 < 2.7(8)$ and $2.8 > 2.5(6)$ sign change here insufficient OR –0.3(8) and 0.2(4) $< 0, > 0$ or change of sign.
		2	

Question	Answer	Marks	Guidance
10(c)	Use the iterative process $a_{n+1} = \left(\frac{35}{3\ln a_n - 1}\right)^{\frac{1}{3}}$ correctly at least twice	**M1**	
	Obtain final answer $a = 2.64$	**A1**	Must be 2 dp.
	Show sufficient iterations to 4 dp to justify 2.64 to 2 dp, or show there is a sign change in (2.635, 2.645)	**A1**	2.635 $(35/(3\ln a - 1))^{1/3} - a = 0.0029(4) > 0$ 2.645 $(35/(3\ln a - 1))^{1/3} - a = -0.012 < 0$
		3	

Cambridge Assessment
International Education

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MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **February/March 2022**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the February/March 2022 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

This document consists of **19** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

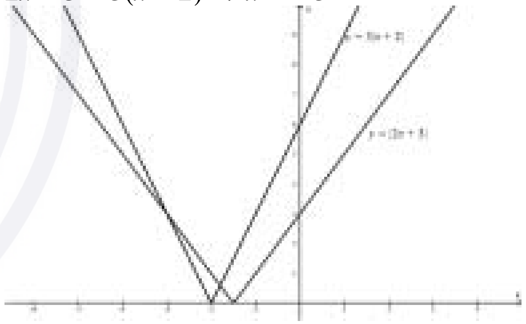
Types of mark

M	Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
A	Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
B	Mark for a correct result or statement independent of method marks.
DM or **DB**	When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
FT	Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	State or imply non-modular inequality $(2x+3)^2 > 3^2(x+2)^2$, or corresponding quadratic equation, or pair of linear equations	**B1**	
	Make a reasonable attempt at solving a 3-term quadratic, or solve two linear equations for x	**M1**	Quadratic formula or $(5x+9)(x+3)$
	Obtain critical values $x = -3$ and $x = -\frac{9}{5}$	**A1**	OE
	State final answer $-3 < x < -\frac{9}{5}$ or $x > -3$ and $x < -\frac{9}{5}$	**A1**	[Do not condone $\leqslant$ for $<$ in the final answer.] No ISW
	Alternative method for question 1		
	Obtain critical value $x = -3$ from a graphical method, or by solving a linear equation or linear inequality	**B1**	$2x + 3 = 3(x + 2) \Rightarrow x = -3$ 
	Obtain critical value $x = -\frac{9}{5}$ similarly	**B2**	
	State final answer $-3 < x < -\frac{9}{5}$ or $x > -3$ and $x < -\frac{9}{5}$	**B1**	[Do not condone $\leqslant$ for $<$ in the final answer.] No ISW
		4	

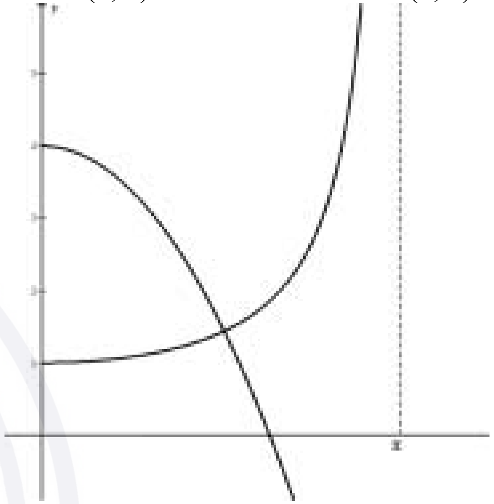
Question	Answer	Marks	Guidance
2	Show a circle with centre $-2+3\mathrm{i}$	**B1**	Must see $(-2, 3)$ or appropriate marks on axes
	Show a circle of radius 2 and centre not at the origin.	**B1**	
	Show correct **half line** from the origin	**B1**	$\frac{3\pi}{4}$ or $\frac{\pi}{4}$ seen, or half line that approximately bisects angle $\frac{\pi}{2}$.
	Shade the correct region.	**B1**	
		4	N.B. Maximum 3 out of 4 if any errors seen.

Question	**Answer**	**Marks**	**Guidance**
3	State or imply $n\ln x + 2\ln y = \ln C$	**B1**	
	Substitute values of ln y and ln x, or equate gradient of line to $\pm\frac{1}{2}n$, but not ± n, and solve for n	**M1**	Using ln$x$ and ln$y$ values
	Obtain n = 0.8[0] or 0.8[00] or $\frac{4}{5}$	**A1**	
	Solve for C	**M1**	Using ln$x$ and ln$y$ values in equation of correct form, that is ln$C$ not C. Allow C = $e^{2.668}$.
	Obtain C = 14.41	**A1**	Must be 2 d.p.
	Alternative method for question 3		
	Obtain two correct equations in n and C by substituting x and y values in the given equation	**B1**	$(2.886)^n \times (2.484)^2 = C$ and $(1.363)^n \times (3.353)^2 = C$
	Solve for n	**M1**	Using x and y values
	Obtain n = 0.8[0] or 0.8[00] or 4/5	**A1**	$\left(\frac{2.886}{1.363}\right)^n \times \left(\frac{2.484}{3.353}\right)^2 = 1$ leading to $n = 0.7995$
	Solve for C	**M1**	Using x and y values
	Obtain C = 14.41	**A1**	Must be 2 d.p.
		5	

Question	Answer	Marks	Guidance
4	State $\dfrac{\mathrm{d}x}{\mathrm{d}\theta} = \sin\theta$ or $\dfrac{\mathrm{d}y}{\mathrm{d}\theta} = -\sin\theta + \dfrac{1}{2}\sin 2\theta$	**B1**	
	Use $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{\mathrm{d}y}{\mathrm{d}\theta} \div \dfrac{\mathrm{d}x}{\mathrm{d}\theta}$	**M1**	
	Obtain correct answer in any form	**A1**	e.g. $\dfrac{-\sin\theta + \frac{1}{2}\sin 2\theta}{\sin\theta}$
	Use double angle correctly to obtain $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of θ	**M1**	$\sin 2\theta = 2\sin\theta\cos\theta$
	Obtain the given answer with no errors seen $-2\sin^2\left(\dfrac{1}{2}\theta\right)$	**A1**	AG. Requires correct cancellation of ALL $\sin\theta$ terms and $\cos\theta = 1 - 2\sin^2\left(\dfrac{1}{2}\theta\right)$ seen **SC** For incorrect signs, consistent throughout max. B0, M1, A0, M1, A1
		5	

Question	Answer	Marks	Guidance
5	Use correct $\tan(A+B)$ formula and obtain an equation in $\tan\alpha$ and $\tan\beta$	**M1**	$\frac{\tan + \tan}{1 \ \tan\tan} = 2$
	Substitute throughout for $\tan\alpha$ or for $\tan\beta$	**M1**	$\frac{3\tan + \tan}{1 \ 3\tan^2} = 2$
	Obtain $3\tan^2\beta + 2\tan\beta - 1 = 0$ or $\tan^2\alpha + 2\tan\alpha - 3 = 0$	**A1**	OE e.g. $6\tan^2\beta + 4\tan\beta - 2 = 0$ or $\frac{2}{3}\tan^2\alpha + \frac{4}{3}\tan\alpha - 2 = 0$
	Solve a 3-term quadratic and find an angle	**M1**	
	Obtain answer $\alpha = 45°$, $\beta = 18.4°$	**A1**	$\frac{\pi}{4}$ or 0.785, 0.322
	Obtain answer $\alpha = 108.4°$, $\beta = 135°$	**A1**	1.89 , $\frac{3\pi}{4}$ or 2.36. Answer in radians, max. A1A0 or vice versa. Ignore answers outside [0°, 180°]
		6	SC: If A0A0 allow **SC B1** for both α's or both β's

Question	Answer	Marks	Guidance
6	Substitute and obtain a correct equation in x and y	**B1**	$(x + \mathrm{i}y)^2 + 2\mathrm{i}(x - \mathrm{i}y) = 1$
	Use $\mathrm{i}^2 = -1$ at least once and equate real and imaginary parts	**M1**	
	Obtain two correct equations, e.g. $x^2 - y^2 + 2y = 1$ and $2xy + 2x = 0$	**A1**	
	Solve for x or for y	**M1**	
	Using $y = -1$, obtain answer $w = -2 - \mathrm{i}$ only	**A1**	A0 if $w = 2 - \mathrm{i}$ as well
	Using $x = 0$, obtain answer $w = \mathrm{i}$	**A1**	
		6	

Question	Answer	Marks	Guidance
7(a)	Sketch a relevant graph, e.g. $y = 4 - x^2$	**B1**	Needs (0, 4) or marks on axis and (2, 0) or (π, 0) 
	Sketch a second relevant graph, e.g. $y = \sec\frac{1}{2}x$, and justify the given statement	**B1**	Needs (0, 1) or mark on axis and (π, 0) Asymptote NOT required, but must NOT reach $x = \pi$. Sec graph must exist over at least interval $\left[0, \frac{3\pi}{4}\right]$ and quadratic graph over [0, 2.5].
		2	
7(b)	Calculate the value of a relevant expression or values of a pair of relevant expressions at $x = 1$ and $x = 2$.	**M1**	Need all 4 values or the 2 values correct for M1. Angles in degrees score M0.
	Complete the argument with correct calculated values	**A1**	
		2	

Question	Answer	Marks	Guidance
7(c)	Use the iterative process correctly at least twice	**M1**	
	Obtain final answer 1.60	**A1**	Must be 2 d.p.
	Show sufficient iterations to 4 d.p.to justify 1.60 to 2 d.p. or show there is a sign change in the interval (1.595, 1.605)	**A1**	
		3	

Question	Answer	Marks	Guidance
8(a)	Commence division and reach quotient of the form $2x \pm 1$	**M1**	Or by inspection $8x^3 + 4x^2 + 2x + 7 = (4x^2 + 1)(2x \pm 1) + \text{r}$
	Obtain (quotient) $2x + 1$	**A1**	
	Obtain (remainder) 6	**A1**	
		3	

Question	Answer	Marks	Guidance
8(b)	Obtain terms $x^2 + x$	**B1**	OE
	Obtain term of the form $a\tan^{-1}2x$	**M1**	
	Obtain term $3\tan^{-1}2x$	**A1**	OE
	Use $x = 0$ and $x = \frac{1}{2}$ as limits in a solution containing a term of the form $a\tan^{-1}2x$	**M1**	$\left(\frac{1}{2}\right)^2 + \frac{1}{2} + a\frac{\pi}{4}$, need $\frac{\pi}{4}$ seen or implied
	Obtain final answer $\frac{3}{4}(1+\pi)$, or exact equivalent	**A1**	ISW, Answers in degrees score A0.
		5	

Question	Answer	Marks	Guidance
9	Correctly separate variables and integrate at least one side	**M1**	To obtain $a\ln y$ or $b\ln(x+1)+c\ln(3x+1)$
	Obtain term $\ln y$ from integral of $1/y$	**B1**	
	State or imply the form $\dfrac{A}{x+1}+\dfrac{B}{3x+1}$ and use a correct method to find a constant	**M1**	
	Obtain $A=-\dfrac{1}{2}$ and $B=\dfrac{3}{2}$	**A1**	
	Obtain terms $-\dfrac{1}{2}\ln(x+1)+\dfrac{1}{2}\ln(3x+1)$ or $-\dfrac{1}{2}\ln(2x+2)+\dfrac{1}{2}\ln(6x+2)$ or combination of these terms	**A1 FT + A1 FT**	The FT is on the values of A and B.
	Use $x=1$ and $y=1$ to evaluate a constant, or expression for a constant, (decimal equivalent of ln terms allowed) or as limits, in a solution containing terms $a\ln y$, $b\ln(x+1)$ and $c\ln(3x+1)$, where $abc\neq 0$	***M1**	e.g. $\ln y=-\dfrac{1}{2}\ln(x+1)+\dfrac{1}{2}\ln(3x+1)-\dfrac{1}{2}\ln 2$
	Obtain an expression for y or y^2 and substitute $x=3$	**DM1**	Do not accept decimal equivalent of ln terms
	Obtain answer $y=\dfrac{1}{2}\sqrt{5}$ or $\sqrt{\dfrac{5}{4}}$ or $\sqrt{\dfrac{10}{8}}$	**A1**	ISW. Must be simplified and exact, do not allow 1.118 or $e^{\frac{1}{2}\ln\frac{5}{4}}$.
		9	

Question	Answer	Marks	Guidance
10(a)	Obtain direction vector $-\mathbf{i}-3\mathbf{j}+\mathbf{k}$	**B1**	OE
	Use a correct method to form a vector equation	**M1**	
	Obtain answer $\mathbf{r}=2\mathbf{i}+\mathbf{j}+\mathbf{k}+\lambda(-\mathbf{i}-3\mathbf{j}+\mathbf{k})$ or $\mathbf{r}=\ \mathbf{i}\ \ 2\mathbf{j}+2\mathbf{k}+\lambda(-\mathbf{i}-3\mathbf{j}+\mathbf{k})$	**A1**	Need **r** or r on LHS
		3	
10(b)	Carry out the correct process for evaluating the scalar product of the direction vectors.	**M1**	(–1, – 3, 1) **.** (1, –3, –2) = – 1 + 9 – 2
	Using the correct process for the moduli, divide the scalar product by the product of the moduli and find the inverse cosine of the result for any 2 vectors	**M1**	$\cos^{-1}\left(\frac{1+9\ 2}{(1+9+1)(1+9+4))}\right)$
	Obtain answer 61.1 °	**A1**	61.086°
		3	

Question	Answer	Marks	Guidance
10(c)	Express general point of *AB* or *l* in component form, e.g. $(2-\lambda, 1-3\lambda, 1+\lambda)$ or $(1+\mu, 2-3\mu, -3-2\mu)$	**B1**	
	Equate at least two pairs of components and solve for λ or for μ	**M1**	
	Obtain a correct answer for λ or μ, e.g. $\lambda = 6, \frac{1}{3}$, or $-\frac{14}{9}$; $\mu = -5, \frac{2}{3}$ or $-\frac{11}{9}$	**A1**	
	Verify that all three equations are not satisfied, and the lines do not intersect	**A1**	
	Express general point of *AB* or *l* in component form, e.g. $(1-\lambda^*, -2-3\lambda^*, 2+\lambda^*)$ or $(1+\mu^*, 2-3\mu^*, -3-2\mu^*)$	**4**	

Question	Answer	Marks	Guidance
11(a)	Use correct product rule or chain rule	**M1**	
	Obtain correct derivative in any form	**A1**	$\cos x.\cos 2x - \sin x.2\sin 2x$
	Equate derivative to zero and use a correct double angle formula	***M1**	If chain rule used then derivative set to 0 gains M1 since correct double angle formula has already been used.
	Obtain an equation in one trigonometric variable	**DM1**	Allow following from coefficient errors in differentiation only
	Obtain $6\sin^2 x = 1$, $6\cos^2 x = 5$ or $5\tan^2 x = 1$	**A1**	One of these 3 expressions
	Obtain final answer $x = 0.421$	**A1**	Must be 3s.f.
		6	

Question	Answer	Marks	Guidance
11(b)	State or imply d$u = -\sin x\ \mathrm{d}x$	**B1**	
	Using double angle formula, express integral in terms of u and du	**M1**	Use $\cos 2x = 2\cos^2 x - 1$
	Integrate and obtain $\pm\left(u - \frac{2}{3}u^3\right)$	**A1**	
	Use limits $u = 1$, $u = \frac{1}{\sqrt{2}}$ in an integral of the form $au + bu^3$, where $ab \neq 0$	**M1**	Require both limits substituted twice in $au + bu^3$ for M1. Do not condone decimals.
	Obtain $\frac{1}{3}\left(\sqrt{2} - 1\right)$ or $\frac{1}{3}\sqrt{2}\ \frac{1}{3}$ or $\frac{2}{3}\left(\frac{1}{\sqrt{2}}\right)\frac{1}{3}$ or simplified equivalent	**A1**	ISW
		5	

Cambridge Assessment
International Education

Cambridge International A Level

MATHEMATICS **9709/31**

Paper 3 Pure Mathematics 3 **October/November 2021**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2021 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

This document consists of **16** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M	Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
A	Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
B	Mark for a correct result or statement independent of method marks.
DM or **DB**	When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
FT	Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	State or imply non-modular equation $4^2\left(5^x-1\right)^2=\left(5^x\right)^2$ or pair of equations $4\left(5^x-1\right)=\pm 5^x$	M1	
	Obtain $5^x=\frac{4}{3}$ and $5^x=\frac{4}{5}$ (or $5^{x+1}=4$)	A1	
	Use correct method for solving an equation of the form $5^x=a$, or $5^{x+1}=b$ where $a > 0$, or $b > 0$	M1	
	Obtain answers $x = 0.179$ and $x = -0.139$	A1	
	Alternative method for question 1		
	Obtain $5^x=\frac{4}{3}$ by solving an equation	B1	
	Obtain $5^x=\frac{4}{5}$ (or $5^{x+1}=4$) by solving an equation	B1	
	Use correct method for solving an equation of the form $5^x=a$, or $5^{x+1}=b$ where $a > 0$, or $b > 0$	M1	
	Obtain answers $x = 0.179$ and $x = -0.139$	A1	
		4	

Question	Answer	Marks	Guidance
2(a)	State $R = \sqrt{34}$	**B1**	
	Use trig formulae to find α	**M1**	$\tan\alpha = \frac{3}{5}$ or $\sin\alpha = \frac{3}{\sqrt{34}}$ or $\cos\alpha = \frac{5}{\sqrt{34}}$.
	Obtain $\alpha = 0.54$	**A1**	30.96° scores **M1A0**.
		3	
2(b)	State greatest value 34	**B1 FT**	*Their* R^2.
	State least value 0	**B1**	
		2	

Question	Answer	Marks	Guidance
3(a)	Use correct product rule	**M1**	
	Obtain correct derivative in any form	**A1**	$\frac{dy}{dx} = e^{1-2x} - 2xe^{1-2x}$
	Equate derivative to zero and solve for x	**M1**	
	Obtain $x = \frac{1}{2}$ and $y = \frac{1}{2}$	**A1**	
		4	
3(b)	Use a correct method for determining the nature of a stationary point	**M1**	e.g. $\frac{d^2y}{dx^2} = -2e^{1-2x} - 2(1-2x)e^{1-2x}$
	Show that it is a maximum point	**A1**	
		2	

Question	Answer	Marks	Guidance
4	State that $\frac{\mathrm{d}u}{\mathrm{d}x} = \frac{1}{2\sqrt{x}}$ or $\mathrm{d}u = \frac{1}{2\sqrt{x}}\,\mathrm{d}x$	**B1**	
	Substitute throughout for x and $\mathrm{d}x$	**M1**	
	Obtain a correct integral with integrand $\frac{2}{u^2+1}$	**A1**	
	Integrate and obtain term of the form $k\tan^{-1}u$	**M1**	$\left(2\tan^{-1}u\right)$
	Use limits $\sqrt{3}$ and ∞ for u or equivalent and evaluate trig.	**A1**	e.g. $2\left(\frac{\pi}{2}-\frac{\pi}{3}\right)$ Must be working in radians.
	Obtain answer $\frac{1}{3}\pi$	**A1**	Or equivalent single term.
		6	

Question	Answer	Marks	Guidance
5(a)	Use correct trig formulae and express equation in terms of $\tan\theta$	**M1**	
	Obtain a correct equation in $\tan\theta$ in any form	**A1**	e.g. $\frac{1-\tan^2\theta}{2\tan\theta}+\frac{1}{\tan\theta}=2$
	Reduce to $\tan^2\theta+4\tan\theta-3=0$, or 3-term equivalent	**A1**	
		3	
5(b)	Solve a 3-term quadratic for tan θ and calculate θ	**M1**	$\left(\tan\theta=-2\pm\sqrt{7}\right)$
	Obtain answer, e.g. 0.573	**A1**	Must be 3 d.p.
	Obtain second answer, e.g. 1.783 and no other	**A1**	Ignore answers outside the given interval. Treat answers in degrees as a misread. $(32.9°, 102.1°)$
		3	

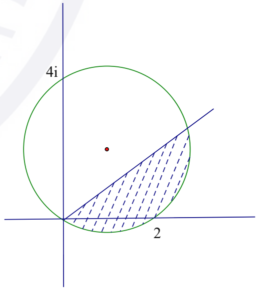
Question	Answer	Marks	Guidance
6	State or imply 1 + 2x as first terms of the expansion of $\sqrt{1+4x}$	**B1**	Allow for correct unsimplified expression.
	State or imply $-2x^2$ as third term of the expansion of $\sqrt{1+4x}$	**B1**	Allow for correct unsimplified expression.
	Form an expression for the coefficient of x or coefficient of x^2 in the expansion of $(a+bx)\sqrt{1+4x}$ and equate to given coefficient	**M1**	All relevant terms considered.
	Obtain $2a + b = 3$, or equivalent	**A1**	One correct equation.
	Obtain– $2a + 2b = -6$ or equivalent	**A1**	Second correct equation.
	Obtain answer $a = 2$ and $b = -1$	**A1**	
		6	

Question	Answer	Marks	Guidance
7(a)	Show sufficient working to justify the given answer	**B1**	
		1	
7(b)	Correct separation of variables	**B1**	e.g. $-\int \frac{1}{t}\mathrm{d}t = \int \frac{1}{x\ln x}\mathrm{d}x$
	Obtain term $\ln(\ln x)$	**B1**	
	Obtain term $-\ln t$	**B1**	
	Evaluate a constant or use $x = \mathrm{e}$ and $t = 2$ as limits in an expression involving $\ln(\ln x)$	**M1**	
	Obtain correct solution in any form, e.g. $\ln(\ln x) = -\ln t + \ln 2$	**A1**	
	Use log laws to enable removal of logarithms	**M1**	
	Obtain answer $x = \mathrm{e}^{\frac{2}{t}}$, or simplified equivalent	**A1**	
		7	
7(c)	State that x tends to 1 coming from $x = \mathrm{e}^{\frac{k}{t}}$	**B1**	
		1	

Question	Answer	Marks	Guidance
8(a)	Commence integration and reach $a\sqrt{x}\ln x + b\int\sqrt{x}\,.\,\frac{1}{x}\,\mathrm{d}x$, or equivalent	*M1	
	Obtain $2\sqrt{x}\ln x - \int 2\sqrt{x}\,.\,\frac{1}{x}\,\mathrm{d}x$, or equivalent	A1	
	Obtain integral $2\sqrt{x}\ln x - 4\sqrt{x}$, or equivalent	A1	
	Substitute limits and equate result to 6	DM1	
	Rearrange and obtain $a = \exp\left(\frac{1}{\sqrt{a}} + 2\right)$	A1	Obtain **given answer** from full and correct working.
		5	
8(b)	Calculate the values of a relevant expression or pair of expressions at $a = 9$ and $a = 11$	M1	e.g. $\begin{cases} 9 < 10.31 \\ 11 > 9.99 \end{cases}$ or $1.31 > 0, -1.01 < 0$
	Complete the argument correctly with correct values	A1	
		2	
8(c)	Use the iterative process $a_{n+1} = \exp(\frac{1}{\sqrt{a_n}} + 2)$ correctly at least once	M1	
	Obtain answer 10.12	A1	
	Show sufficient iterations to 4dp to justify 10.12 to 2dp, or show there is a sign change in the interval (10.115, 10.125)	A1	e.g. $10, 10.1374, 10.1156, 10.1190, \ldots.$ $9, 10.3123, 10.0886, 10.1233, 10.1178, \ldots$ $11, 9.9893, 10.1391, 10.1153, 10.1191, \ldots.$
		3	

Question	Answer	Marks	Guidance
9(a)	Use correct method to evaluate the scalar product of relevant vectors	**M1**	$(-4-2+6)$
	Obtain answer zero and deduce the given statement	**A1**	Need a conclusion or a statement in advance that the scalar product will be zero.
		2	
9(b)	Express general point of l or m in component form, e.g. $(3 + 4s, 2 - s, 5 + 3s)$ or $(1 - t, -1 + 2t, -2 + 2t)$	**B1**	
	Equate at least two pairs of components and solve for s or for t	**M1**	
	Obtain correct answer $s = -1$ and $t = 2$	**A1**	
	Verify that all three equations are satisfied	**A1**	
	State position vector of the intersection $-\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$, or equivalent	**A1**	Can come from 1 correct value and no contradictory statement.
		5	

Question	Answer	Marks	Guidance
9(c)	Taking a general point *P* on *m*, form an equation in *t* by ***either*** equating a relevant scalar product to zero, ***or*** equating the derivative of $\lvert\overrightarrow{OP}\rvert$ to zero, ***or*** taking a specific point *Q* on *m*, e.g. (1, – 1, – 2), using Pythagoras in triangle *OPQ*	***M1**	e.g. $\begin{pmatrix}1-t\\-1+2t\\-2+2t\end{pmatrix}.\begin{pmatrix}-1\\2\\2\end{pmatrix}=0$
	Obtain $t=\frac{7}{9}$	**A1**	
	Carry out correct method to find *OP*	**DM1**	
	Obtain $\frac{\sqrt{5}}{3}$	**A1**	Obtain the **given answer** from full and correct working.
	Alternative method for question 9(c)		
	Take a specific point *Q* on *m*, e.g. (– 1, 3, 2) and use a scalar product to find *QN*, the projection of *OQ* on *m*	***M1**	
	Obtain $QN=\frac{11}{3}$, or equivalent	**A1**	
	Use Pythagoras to obtain *ON*	**DM1**	
	Obtain the given answer correctly	**A1**	
		4	

Question	Answer	Marks	Guidance
10(a)	Substitute 1 + 2i in the polynomial and attempt expansions of x^2 and x^3	M1	$u^2 = -3+4\mathrm{i}$, $u^3 = -11-2\mathrm{i}$ Full substitution but need not simplify.
	Equate real and/or imaginary parts to zero	M1	$-18-3a+b=0$, $4+4a=0$
	Obtain $a = -1$	A1	
	Obtain $b = 15$	A1	
		4	
10(b)	State second root 1 – 2i	B1	
		1	
10(c)	State the quadratic factor x^2-2x+5	B1	
	State the linear factor $2x+3$	B1	
		2	
10(d)(i)	Show a circle with centre 1 + 2i	B1	
	Show circle passing through the origin	B1	
	Show the half line $y = x$ in the first quadrant (accept chord of circle)	B1	
	Shade the correct region on a correct diagram	B1	
		4	
10(d)(ii)	State answer $2-\sqrt{5}$	B1	
		1	

Cambridge Assessment International Education

Cambridge International A Level

MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **October/November 2021**

MARK SCHEME

Maximum Mark: 75

Published

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[Turn over

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- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

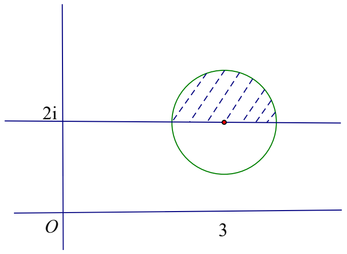
AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	Use law of the logarithm of a product, a quotient or power	***M1**	e.g. $\ln\left(7^x\right)=x\ln 7$
	Obtain a correct linear equation in any form	**A1**	e.g. $\ln 3+(1-x)\ln 2=x\ln 7$
	Solve a linear equation for x	**DM1**	
	Obtain answer $x=\dfrac{\ln 6}{\ln 14}$	**A1**	Maximum 3 out of 4 available if final answer not in required form e.g. 0.67… ISW once correct answer seen.
	Alternative method for Question 1		
	$2^{1-x}=2\times 2^{-x}$	***M1**	OE
	$6=2^x7^x\left[=14^x\right]$	**A1**	
	Use law of the logarithm of a power to solve for x	**DM1**	Must be a linear power. Allow $x=\ln_{14}(6)$.
	Obtain answer $x=\dfrac{\ln 6}{\ln 14}$	**A1**	ISW once correct answer seen.
		4	

Question	Answer	Marks	Guidance
2	State or imply non-modular inequality $(3x-a)^2 > 2^2(x+2a)^2$, or corresponding quadratic equation, or pair of linear equations or linear inequalities	**B1**	Need 2^2 seen or implied.
	Make reasonable attempt to solve a 3-term quadratic, or solve two linear equations for x in terms of a	**M1**	$(5x^2 - 22ax - 15a^2 = 0)$
	Obtain critical values $x = 5a$ and $x = -\frac{3}{5}a$ and no others	**A1**	OE Accept incorrect inequalities with correct critical values. Must state 2 values i.e. $\frac{a \pm b}{c}$ is not sufficient.
	State final answer $x > 5a$, $x < -\frac{3}{5}a$	**A1**	Do not condone $\geqslant$ for > or $\leqslant$ for < in the final answer. $5a < x < -\frac{3}{5}a$ is **A0**, 'and' is **A0**.
	Alternative method for Question 2		
	Obtain critical value $x = 5a$ from a graphical method, or by solving a linear equation or linear inequality	**B1**	
	Obtain critical value $x = -\frac{3}{5}a$ similarly	**B2**	Maximum 2 marks if more than 2 critical values.
	State final answer $x > 5a$, $x < -\frac{3}{5}a$	**B1**	Do not condone ≥ for > or ≤ for < in the final answer. $5a < x < -\frac{3}{5}a$ is **B0**, 'and' is **B0**.
		4	

Question	Answer	Marks	Guidance
3(a)	Substitute for u and w and state correct conjugate of one side	B1	
	Express the other side without conjugates and confirm $(u+w)^* = u^* + w^*$	B1	**Given answer.** Needs explicit reference to conjugate of both sides.
		2	
3(b)	Substitute and remove conjugates to obtain a correct equation in x and y	B1	e.g. $x+2-(y+1)\mathrm{i}+(2+\mathrm{i})(x+\mathrm{i}y)=0$
	Use $\mathrm{i}^2 = -1$ and equate real and imaginary parts to zero	M1	
	Obtain two correct equations in x and y	A1	e.g. $3x - y + 2 = 0$ and $x + y - 1 = 0$. Allow $x\mathrm{i} + y\mathrm{i} - \mathrm{i} = 0$.
	Solve and obtain answer $z = -\frac{1}{4} + \frac{5}{4}\mathrm{i}$	A1	Allow for real and imaginary parts stated separately.
		4	

Question	Answer	Marks	Guidance
4	State or imply the form $A + \frac{B}{2x-1} + \frac{C}{x-3}$	**B1**	$\frac{Dx+E}{2x-1} + \frac{F}{x-3}$ and $\frac{P}{2x-1} + \frac{Qx+R}{x-3}$ are also valid.
	Use a correct method for finding a constant	**M1**	
	Obtain one of $A = 2$, $B = -3$ and $C = 2$	**A1**	Allow maximum **M1A1** for one or more 'correct' values after **B0**.
	Obtain a second value	**A1**	
	Obtain the third value	**A1**	
	Alternative method for Question 4		
	Divide numerator by denominator	**M1**	
	Obtain $2\left[+\frac{Px+Q}{(2x-1)(x-3)}\right]$	**A1**	$\left[2+\frac{x+7}{(2x-1)(x-3)}\right]$
	State or imply the form $\frac{D}{2x-1} + \frac{E}{x-3}$	**B1**	
	Obtain one of $D = -3$ and $E = 2$	**A1**	
	Obtain a second value	**A1**	
		5	

Question	Answer	Marks	Guidance
5(a)	Show circle with centre 3 + 2i	**B1**	
	Show circle with radius 1. Must match *their* scales: if scales not identical should have an ellipse.	**B1**	
	Show line $y = 2$ in at least the diameter of a circle in the first quadrant	**B1**	
	Shade the correct region in a correct diagram	**B1**	
		4	
5(b)	Identify the correct point	**B1**	
	Carry out a correct method for finding the argument	**M1**	e.g. $\arg x = \tan^{-1}\frac{2}{3} + \sin^{-1}\frac{1}{\sqrt{13}}$ Exact working required.
	Obtain answer 49.8°	**A1**	Or better. 0.869 radians scores **B1M1A0**.
		3	**Special Case 1: B1M0** for 45° if they have shaded the wrong half of the circle. **Special Case 2:** 3 out of 3 available if they identify the correct point on the correct circle and it is consistent with *their* shading.

Question	Answer	Marks	Guidance
6(a)	State correct expansion of $\sin(3x+2x)$ or $\sin(3x-2x)$	**B1**	
	Substitute expansions in $\frac{1}{2}(\sin 5x+\sin x)$, or equivalent	**M1**	
	Simplify and obtain $\frac{1}{2}(\sin 5x+\sin x)=\sin 3x\cos 2x$	**A1**	Obtain the **given identity** correctly.
		3	
6(b)	Obtain integral $-\frac{1}{10}\cos 5x-\frac{1}{2}\cos x$, or equivalent	**B1**	
	Substitute limits correctly in an expression of the form $p\cos 5x+q\cos x$	**M1**	Correct limits and subtracted the right way around. Do not need values of trig functions for M1. Maximum one slip.
	Obtain $\frac{1}{5}\left(3-\sqrt{2}\right)$	**A1**	Substitute values and obtain the **given answer** following full, correct and exact working.
		3	

Question	Answer	Marks	Guidance
7	Separate variables correctly	**B1**	$\int \frac{1}{y^2}\,\mathrm{d}y = \int 4x\mathrm{e}^{-2x}\,\mathrm{d}x$
	$\int \frac{1}{y^2}\,\mathrm{d}y = -\frac{1}{y}$	**B1**	OE
	Commence the other integration and reach $ax\mathrm{e}^{-2x} + b\int \mathrm{e}^{-2x}\,\mathrm{d}x$	**M1**	
	Obtain $-2x\mathrm{e}^{-2x} + 2\int \mathrm{e}^{-2x}\,\mathrm{d}x$ or $-\frac{1}{2}x\mathrm{e}^{-2x} + \frac{1}{2}\int \mathrm{e}^{-2x}\,\mathrm{d}x$	**A1**	SOI (might have taken out factor of 4)
	Complete integration and obtain $-2x\mathrm{e}^{-2x} - \mathrm{e}^{-2x}$	**A1**	
	Evaluate a constant or use $x = 0$ and $y = 1$ as limits in a solution containing terms of the form $\frac{p}{y}$, $qx\mathrm{e}^{-2x}$, $r\mathrm{e}^{-2x}$, or equivalent.	**M1**	
	Obtain $y = \frac{\mathrm{e}^{2x}}{2x+1}$, or equivalent expression for y	**A1**	ISW
		7	

Question	Answer	Marks	Guidance
8(a)	Expand the square and equate to 1	**B1**	
	Use correct double angle formula	**M1**	Need to see $\frac{4}{2}$ or $\sin 2\theta = 2\sin\theta\cos\theta$ stated.
	Obtain $\cos^4\theta + \sin^4\theta = 1 - \frac{1}{2}\sin^2 2\theta$	**A1**	Obtain the **given result** correctly.
		3	
8(b)	Use the identity and carry out a method for finding a root	**M1**	$\left(1 - \frac{1}{2}\sin^2 2\theta = \frac{5}{9}\right)$
	Obtain answer 35.3°	**A1**	Must be correct if overspecified: 35.264...
	Obtain a second answer, e.g. 54.7°	**A1 FT**	$\left[\text{e.g } 90° - \textit{their } 35.3°\right]$ Do not FT if mixing degrees and radians.
	Obtain the remaining answers, e.g. 144.7° and 125.3° and no others in the given interval	**A1 FT**	$\left[\text{e.g.} 180° - .. \text{ and } 180° - ..\right]$ Ignore answers outside the given interval. Treat answers in radians as a misread. $(0.615, 0.955, 2.19, 2.53)$ Do not FT if mixing degrees and radians.
		4	

Question	Answer	Marks	Guidance
9(a)	State correct derivative of ye^{2x} with respect to x	**B1**	$2ye^{2x}+e^{2x}\frac{dy}{dx}$
	State correct derivative of y^2e^x with respect to x	**B1**	$2ye^x\frac{dy}{dx}+y^2e^x$
	Equate attempted derivative of the LHS to zero and solve for $\frac{dy}{dx}$	**M1**	
	Obtain $\frac{dy}{dx}=\frac{2ye^x-y^2}{2y-e^x}$	**A1**	Obtain the **given answer** correctly. Condone multiplication by $\frac{-1}{-1}$ and cancelling of e^x without comment.
	Alternative method for Question 9(a)		
	Rearrange as $y=\frac{2}{e^{2x}-ye^x} \Rightarrow \frac{d}{dx}\left(e^{2x}-ye^x\right)=2e^{2x}-ye^x-e^x\frac{dy}{dx}$	**B1**	Other rearrangements are possible e.g. $y=2e^{-2x}+y^2e^{-x} \quad \frac{d}{dx}\left(y^2e^{-x}\right)=2ye^{-x}\frac{dy}{dx}-y^2e^{-x}$
	$\frac{dy}{dx}=-\frac{2}{\left(e^{2x}-ye^x\right)^2}\times\left(2e^{2x}-ye^x-e^x\frac{dy}{dx}\right)$	**B1**	$\Rightarrow\frac{dy}{dx}=-4e^{-x}+2ye^{-x}\frac{dy}{dx}-y^2e^{-x}$
	Solve for $\frac{dy}{dx}$	**M1**	
	Obtain $\frac{dy}{dx}=\frac{2ye^x-y^2}{2y-e^x}$	**A1**	Obtain the **given answer** correctly.
		4	

Question	Answer	Marks	Guidance
9(b)	Equate denominator to zero and substitute for y or for e^x in the equation of the curve	*M1	
	Obtain equation of the form $a\mathrm{e}^{3x} = b$ or $cy^3 = d$	DM1	$\left(\mathrm{e}^{3x} = 8,\quad y^3 = 1\right)$ SOI
	Obtain $x = \ln 2$	A1	Accept $\frac{1}{3}\ln 8$ ISW
	Obtain $y = 1$	A1	
		4	

Question	Answer	Marks	Guidance
10(a)	Obtain direction vector $-\mathbf{i} + \mathbf{j} + 2\mathbf{k}$, or equivalent	B1	Accept answers as column vectors throughout.
	Use a correct method to form a vector equation	M1	
	State answer $\mathbf{r} = \mathbf{i} + 2\mathbf{j} - \mathbf{k} + \lambda(-\mathbf{i} + \mathbf{j} + 2\mathbf{k})$, or equivalent correct form	A1	e.g. $\mathbf{r} = \begin{pmatrix} 0 \\ 3 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix}$ Allow $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ for **r**.
		3	
10(b)	Use a correct method to find the position vector of C	M1	e.g. $\mathbf{OC} = \mathbf{OA} + \mathbf{AC} = \begin{pmatrix} 1-3 \\ 2+3 \\ -1+6 \end{pmatrix}$
	Obtain answer $-2\mathbf{i} + 5\mathbf{j} + 5\mathbf{k}$, or equivalent	A1	Accept as coordinates.
		2	

Question	Answer	Marks	Guidance
10(c)	State $\overrightarrow{OP}$ in component form	**B1 FT**	
	Form an equation in λ by equating the modulus of *OP* to $\sqrt{14}$, or equivalent	**M1**	
	Simplify and obtain $3\lambda^2 - \lambda - 4 = 0$, or equivalent	**A1**	$3\lambda^2 + \lambda - 4 = 0$ if using $\mathbf{i} - \mathbf{j} - 2\mathbf{k}$ in **(a)**. $3\mu^2 + 5\mu - 2 = 0$ if using $-\mathbf{i} + \mathbf{j} + 2\mathbf{k}$ in **(a)** and *OB*.
	Solve a 3-term quadratic and find a position vector	**M1**	$\left(\lambda = -1, \frac{4}{3} \text{ or } \lambda = 1, -\frac{4}{3} \text{ or } \mu = \frac{1}{3}, -2 \text{ or } \mu = -\frac{1}{3}, 2\right)$
	Obtain answers $2\mathbf{i} + \mathbf{j} - 3\mathbf{k}$ and $-\frac{1}{3}\mathbf{i} + \frac{10}{3}\mathbf{j} + \frac{5}{3}\mathbf{k}$, or equivalent	**A1**	Accept as coordinates.
		5	

Question	Answer	Marks	Guidance
11(a)	Use chain rule	**M1**	Allow if not starting with the correct index.
	Obtain correct derivative in any form	**A1**	e.g. $\frac{dy}{dx} = \frac{\sec^2 x}{2\sqrt{\tan x}}$
	Use correct Pythagoras to obtain correct derivative in terms of tan *x*	**A1**	e.g. $\frac{dy}{dx} = \frac{1 + \tan^2 x}{2\sqrt{\tan x}}$
	Use a correct derivative to obtain $\frac{dy}{dx} = 1$ when $x = \frac{1}{4}\pi$	**B1**	Confirm the **given statement** from correct work. Should see at least $\frac{2}{2} = 1$.
		4	

Question	Answer	Marks	Guidance
11(b)	Equate answer to part (**a**) to 1 and obtain a quartic equation in t or $\tan x$	***M1**	At least as far as $\left(1+\tan^2 x\right)^2 = 4\tan x$.
	Obtain correct answer, i.e. $t^4+2t^2-4t+1=0$	**A1**	Or equivalent horizontal form.
	Commence division by $t-1$	**DM1**	As far as $t^3+t^2+\ldots$by long division or inspection. Allow verification by multiplying given answer by $t-1$.
	Obtain the given answer	**A1**	
		4	
11(c)	Use the iterative process correctly with the given formula at least once	**M1**	Obtain one value and use that to obtain the next. Must be working in radians.
	Obtain final answer $a = 0.29$	**A1**	
	Show sufficient iterations to 4 d.p. to justify 0.29 to 2 d.p., or show there is a sign change in (0.285, 0.295)	**A1**	e.g. 0.3, 0.2854, 0.2894, 0.2883, ... 0.4, 0.2436, 0.2984, 0.2841, 0.2883, 0.2871, ... 0.5, 0.1776, 0.3103, 0.2805, 0.2893, 0.2868, ...
		3	

Cambridge Assessment
International Education

Cambridge International AS & A Level

MATHEMATICS **9709/33**

Paper 3 Pure Mathematics 3 **October/November 2021**

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- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
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Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
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WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	Commence division and reach partial quotient of the form $2x^2 + kx$	M1	
	Obtain quotient $2x^2 + 2x - 2$	A1	
	Obtain remainder $-6x + 5$	A1	
		3	

Question	Answer	Marks	Guidance
2(a)	Show a recognizable sketch graph of $y = \|2x - 3\|$	B1	
		1	

Question	Answer	Marks	Guidance
2(b)	Find x-coordinate of intersection with $y = 3x + 2$	**M1**	
	Obtain $x = \frac{1}{5}$	**A1**	
	State final answer $x > \frac{1}{5}$ only	**A1**	
	Alternative method for Question 2(b)		
	Solve the linear inequality $3 - 2x < 3x + 2$, or corresponding equation	**M1**	
	Obtain critical value $x = \frac{1}{5}$	**A1**	
	State final answer $x > \frac{1}{5}$ only	**A1**	
	Alternative method for Question 2(b)		
	Solve the quadratic inequality $(2x-3)^2 < (3x+2)^2$, or corresponding equation	**M1**	
	Obtain critical value $x = \frac{1}{5}$	**A1**	
	State final answer $x > \frac{1}{5}$ only	**A1**	
		3	

Question	Answer	Marks	Guidance
3	Use laws of indices correctly and solve for 4^x	M1	
	Obtain correct solution in any form, e.g. $4^x = \frac{256}{15}$	A1	
	Use a correct method for solving an equation of the form $4^x = a$, where $a > 0$	M1	
	Obtain answer 2.047	A1	
		4	

Question	Answer	Marks	Guidance
4	Commence integration and reach $ax\cos\frac{1}{2}x + b\int\cos\frac{1}{2}x\,dx$	*M1	
	Obtain $-2x\cos\frac{1}{2}x + 2\int\cos\frac{1}{2}x\,dx$	A1	OE
	Complete integration obtaining $-2x\cos\frac{1}{2}x + 4\sin\frac{1}{2}x$	A1	OE
	Use limits correctly, having integrated twice	DM1	
	Obtain answer $2 + \frac{\sqrt{3}}{3}\pi$, or exact equivalent	A1	
		5	

Question	Answer	Marks	Guidance
5	Use double angle formula and obtain an equation in $\sin\theta$	**M1**	
	Reduce to $6\sin^2\theta + \sin\theta - 5 = 0$, or 3-term equivalent	**A1**	
	Solve a 3-term quadratic in $\sin\theta$ and calculate θ	**M1**	
	Obtain answer, e.g. 56.4°	**A1**	
	Obtain second and third answers, e.g.123.6° and 270° and no others in the given interval	**A1**	Ignore answers outside the interval. Treat answers in radians as a misread.
		5	

Question	Answer	Marks	Guidance
6(a)	Use $\cos(A-B)$ formula and obtain an expression in terms of $\sin x$ and $\cos x$	**M1**	
	Collect terms and reach $2\cos x + \sqrt{3}\sin x$	**A1**	
	State $R = \sqrt{7}$	**A1**	
	Use trig formula to find α	**M1**	
	Obtain $\alpha =$ 40.89°	**A1**	
		5	
6(b)	Use correct method to find x	**M1**	
	Obtain answer $x = 220.9°$	**A1**	
		2	

Question	Answer	Marks	Guidance
7(a)	Use chain rule to differentiate LHS	***M1**	
	Obtain $\frac{1}{x+y}\left(1+\frac{\mathrm{d}y}{\mathrm{d}x}\right)$	**A1**	
	Equate derivative of LHS to $1-2\frac{\mathrm{d}y}{\mathrm{d}x}$ and solve for $\frac{\mathrm{d}y}{\mathrm{d}x}$	**DM1**	
	Obtain the given answer correctly	**A1**	
		4	
7(b)	State $x + y = 1$	**B1**	
	Obtain or imply $x - 2y = 0$	**B1**	
	Obtain coordinates $x = \frac{2}{3}$ and $y = \frac{1}{3}$	**B1**	
		3	

Question	Answer	Marks	Guidance
8(a)	State $\overrightarrow{OM} = 4\mathbf{i} + 2\mathbf{j}$	**B1**	
	Use a correct method to find $\overrightarrow{ON}$	**M1**	
	Obtain answer $3\mathbf{j} + \mathbf{k}$	**A1**	
	Use a correct method to find a line equation for MN	**M1**	
	Obtain answer $\mathbf{r} = 3\mathbf{j} + \mathbf{k} + \lambda(4\mathbf{i} - \mathbf{j} - \mathbf{k})$, or equivalent	**A1**	
		5	

Question	Answer	Marks	Guidance
8(b)	Taking a general point *P* on *MN*, form an equation in λ by *either* equating a relevant scalar product to zero *or* equating the derivative of $\overrightarrow{OP}$ to zero *or* using Pythagoras in triangle *OPM* or *OPN*	**M1**	
	Obtain $\lambda = \dfrac{2}{9}$	**A1**	OE
	Use correct method to find *OP*	**M1**	
	Obtain the given answer correctly	**A1**	
	Alternative method to Question 8(b)		
	Use a scalar product to find the projection of *OM* (or *ON*) on *MN*	**M1**	
	Obtain answer $\dfrac{14}{\sqrt{18}}\left(\text{or } \dfrac{4}{\sqrt{18}}\right)$	**A1**	
	Use Pythagoras to obtain the perpendicular	**M1**	
	Obtain the given answer correctly	**A1**	
		4	

Question	Answer	Marks	Guidance
9(a)	Use quotient or product rule	**M1**	
	Obtain correct derivative in any form	**A1**	
	Equate derivative to zero and solve for x	**M1**	
	Obtain answer $x = 3$	**A1**	
		4	
9(b)	State $\frac{\mathrm{d}u}{\mathrm{d}x} = \frac{1}{2\sqrt{x}}$, or $\mathrm{d}x = 2\sqrt{x}\mathrm{d}u$, or $2u\ \mathrm{d}u = \mathrm{d}x$	**B1**	
	Substitute and obtain integrand $\frac{2}{9-u^2}$	**B1**	
	Use given formula for the integral or integrate relevant partial fractions	**M1**	
	Obtain integral $\frac{1}{3}\ln\left(\frac{3+u}{3-u}\right)$	**A1**	OE
	Use limits $u = 0$ and $u = 2$ correctly	**M1**	
	Obtain the given answer correctly	**A1**	
		6	

Question	Answer	Marks	Guidance
10(a)	State or imply equation of the form $\frac{dx}{dt} = k\frac{x}{20-x}$	M1	
	Obtain $k = 19$	A1	AG
		2	
10(b)	Separate variables and integrate at least one side	M1	
	Obtain terms $20 \ln x - x$ and $19t$, or equivalent	A1 A1	
	Evaluate a constant or use $t = 0$ and $x = 1$ as limits in a solution containing terms $a \ln x$ and bt	M1	
	Substitute $t = 1$ and rearrange the equation in the given form	A1	AG
		5	
10(c)	Use $x_{n+1} = e^{0.9+0.05x_n}$ correctly at least once	M1	
	Obtain final answer $x = 2.83$	A1	
	Show sufficient iterations to 4 decimal places to justify 2.83 to 2 d.p. or show there is a sign change in the interval (2.825, 2.835)	A1	
		3	
10(d)	Set $x = 20$ and obtain answer $t = 2.15$	B1	
		1	

Question	Answer	Marks	Guidance
11(a)	State or imply $r = 2$	B1	
	State or imply $\theta = \frac{5}{6}\pi$	B1	
		2	
11(b)	Use a correct method for finding the modulus or argument of u^6	M1	
	Show correctly that u^6 is real and has value -64	A1	
		2	
11(c)(i)	Show half lines from the point representing $-\sqrt{3} + \mathrm{i}$	B1	
	Show correct half lines	B1	
	Show the line $x = 2$ in the first quadrant	B1	
	Shade the correct region	B1	
		4	
11(c)(ii)	Carry out a correct method to find the greatest value of $\lvert z \rvert$	M1	
	Obtain answer 5.14	A1	
		2	

Cambridge Assessment International Education

Cambridge International A Level

MATHEMATICS **9709/31**

Paper 3 Pure Mathematics 3 **May/June 2021**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2021 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

This document consists of **16** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M	Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
A	Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
B	Mark for a correct result or statement independent of method marks.
DM or **DB**	When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
FT	Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

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WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	State or imply non-modular inequality $2^2(3x-1)^2 < (x+1)^2$, or corresponding quadratic equation, or pair of linear equations	**B1**	
	Form and solve a 3-term quadratic, or solve two linear equations for x	**M1**	e.g. $35x^2 - 26x + 3 = 0$
	Obtain critical values $x = \frac{3}{5}$ and $x = \frac{1}{7}$	**A1**	Allow 0.143 or better
	State final answer $\frac{1}{7} < x < \frac{3}{5}$	**A1**	Exact values required. Accept $x > \frac{1}{7}$ **and** $x < \frac{3}{5}$ Do not condone $\leqslant$ for $<$ in the final answer. Fractions need not be in lowest terms.
	Alternative method for Question 1		
	Obtain critical value $x = \frac{3}{5}$ from a graphical method, or by solving a linear equation or linear inequality	**B1**	
	Obtain critical value $x = \frac{1}{7}$ similarly	**B2**	Allow 0.143 or better
	State final answer $\frac{1}{7} < x < \frac{3}{5}$	**B1**	OE. Exact values required. Accept $x > \frac{1}{7}$ **and** $x < \frac{3}{5}$ Do not condone $\leqslant$ for $<$ in the final answer. Fractions need not be in lowest terms.
		4	

Question	Answer	Marks	Guidance
2	Reduce to a 3-term quadratic $u^2+6u-1=0$ OE	**B1**	Allow '= 0' implied
	Solve a 3-term quadratic for *u*	**M1**	
	Obtain root $\sqrt{10}-3$	**A1**	
	Obtain answer $x = -1.818$ only	**A1**	The question asks for 3 d.p.
	Reject $-\sqrt{10}-3$ correctly	**B1**	e.g. by stating that $e^x>0$ or $\ln\left(-10-\sqrt{3}\right)$ is impossible Not "math error".
	Alternative method for Question 2		
	Rearrange to obtain a correct iterative formula	**B1**	e.g. $x_{n+1}=-\ln\left(6+e^{x_n}\right)$
	Use the iterative process at least twice	**M1**	
	Obtain answer $x = -1.818$	**A1**	
	Show sufficient iterations to at least 4 d.p. to justify $x = -1.818$	**A1**	1,−2.165…,−1.811…,−1.819…,−1.818…,−1.818…
	Clear explanation of why there is only one real root	**B1**	
		5	

Question	Answer	Marks	Guidance
3(a)	Use correct trig expansions and obtain an equation in sin x and cos x	***M1**	
	Use correct exact trig ratios for $30°$ in *their* expansion	**B1 FT**	e.g. $\cos x\left(\frac{\sqrt{3}}{2}-1\right)=\sin x\left(\sqrt{3}-\frac{1}{2}\right)$
	Obtain an equation in tan x	**DM1**	Allow if their error in line 1 was a sign error
	Obtain $\tan x=\dfrac{2-\sqrt{3}}{1-2\sqrt{3}}$ from correct working	**A1**	AG
		4	
3(b)	Obtain answer in the given interval, e.g.$173.8°$	**B1**	Accept $174°,\ 354°$ or better
	Obtain a second answer and no other in the given interval, e.g. $353.8°$	**B1**	Ignore answers outside the given interval. Treat answers in radians (3.03 and 6.17) as a misread.
		2	

Question	Answer	Marks	Guidance
4(a)	Use correct double angle formula or *t*-substitution twice	**M1**	
	Obtain $\dfrac{1-\cos 2\theta}{1+\cos 2\theta}=\tan^2\theta$ from correct working	**A1**	AG
		2	

Question	Answer	Marks	Guidance
4(b)	Express $\tan^2\theta$ in terms of $\sec^2\theta$	**M1**	$\left(\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\left(\sec^2\theta \pm 1\right)\mathrm{d}\theta\right)$
	Integrate and obtain terms $\tan\theta - \theta$	**A1**	Accept with a mixture of x and θ
	Substitute limits correctly in an integral of the form $a\tan\theta + b\theta$, where $ab \neq 0$	**M1**	$\left(\sqrt{3} - \frac{\pi}{3} - \frac{1}{\sqrt{3}} + \frac{\pi}{6}\right)$ Allow if trig. not substituted
	Obtain answer $\frac{2}{3}\sqrt{3} - \frac{1}{6}\pi$	**A1**	or equivalent exact 2-term expression
		4	

Question	Answer	Marks	Guidance
5(a)	Use quadratic formula and $\mathrm{i}^2 = -1$	**M1**	
	Obtain answers $p\mathrm{i} + \sqrt{q - p^2}$ and $p\mathrm{i} - \sqrt{q - p^2}$	**A1**	Accept $\dfrac{2p\mathrm{i} \pm \sqrt{-4p^2 + 4q}}{2}$ and ISW
		2	
5(b)	State or imply that the discriminant must be negative	**M1**	
	State condition $q < p^2$	**A1**	
		2	

Question	Answer	Marks	Guidance
5(c)	Carry out a correct method for finding a relation, e.g. use the fact that the argument of one of the roots is $(\pm)60°$	**M1**	
	State a correct relation in any form, e.g. $\frac{p}{\sqrt{q-p^2}}=(\pm)\sqrt{3}$	**A1**	
	Simplify to $q=\frac{4}{3}p^2$	**A1**	
	Alternative method for Question 5(c)		
	Carry out a correct method for finding a relation, e.g. use the fact that the sides have equal length	**M1**	
	State a correct relation in any form, e.g. $4\left(q-p^2\right)=p^2+q-p^2$	**A1**	
	Simplify to $q=\frac{4}{3}p^2$	**A1**	
		3	

Question	Answer	Marks	Guidance
6(a)	Use correct chain rule **or** correct quotient rule to differentiate x or y	**M1**	
	Obtain $\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{3}{2+3t}$ **or** $\frac{\mathrm{d}y}{\mathrm{d}t} = \frac{2}{(2+3t)^2}$	**A1**	OE
	Use $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}t} \div \frac{\mathrm{d}x}{\mathrm{d}t}$	**M1**	
	Obtain answer $\frac{2}{3(2+3t)}$	**A1**	OE. Express as a simple fraction but not necessarily fully cancelled.
	Explain why this is always positive	**A1**	For correct gradient. e.g. x is only defined for $2+3t>0$ hence gradient > 0
	Alternative method for Question 6(a)		
	Form equation in x and y only	**M1**	
	Obtain $y = \frac{e^x - 2}{3e^x}\left(= \frac{1}{3} - \frac{2}{3}e^{-x}\right)$	**A1**	OE
	Differentiate	**M1**	
	Obtain $y' = \frac{2}{3}e^{-x}$	**A1**	OE
	Explain why this is always positive	**A1**	
		5	

Question	Answer	Marks	Guidance
6(b)	Obtain $y=-\frac{1}{3}$ when $x=0$	**B1**	
	Use a correct method to form the given tangent	**M1**	$\left(\dfrac{y+\frac{1}{3}}{x}=\dfrac{2}{3}\right)$
	Obtain answer $3y=2x-1$	**A1**	OE
		3	

Question	Answer	Marks	Guidance
7(a)	Use correct quotient rule or correct product rule	**M1**	e.g. $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{\sqrt{x}.\dfrac{1}{1+x^2}-\tan^{-1}x.\dfrac{1}{2\sqrt{x}}}{x}$
	Obtain correct derivative in any form	**A1**	
	Equate derivative to zero and remove inverse tangent	**M1**	
	Obtain $a=\tan\left(\dfrac{2a}{1+a^2}\right)$ from correct working	**A1**	AG. Accept with *x* in place of *a*.
		4	

Question	Answer	Marks	Guidance
7(b)	Calculate the value of a relevant expression or pair of expressions at $a = 1.3$ and $a = 1.5$	M1	Must be using radians
	Complete the argument correctly with correct calculated values	A1	e.g. $1.3 < 1.448, \quad 1.5 > 1.322 \ (0.148, -0.178)$
		2	

Question	Answer	Marks	Guidance
7(c)	Use the iterative process $a_{n+1} = \tan\left(\frac{2a_n}{1+a_n^{\ 2}}\right)$ correctly at least twice	M1	
	Obtain final answer 1.39	A1	
	Show sufficient iterations to at least 4 d.p. to justify 1.39 to 2 d.p. or show there is a sign change in the interval (1.385, 1.395)	A1	Allow recovery
		3	

Question	Answer	Marks	Guidance
8(a)	State or imply $\overrightarrow{AB} = \begin{pmatrix} 2 \\ -1 \\ -3 \end{pmatrix}$	B1	OE. Allow ±
	Use the correct process to calculate the scalar product of a pair of relevant vectors, e.g. their $\overrightarrow{AB}$ and a direction vector for l	M1	$(2+2-3=1)$
	Using the correct process for the moduli, divide the scalar product by the product of the moduli of the two vectors and evaluate the inverse cosine of the result	M1	$\cos^{-1}\left(\frac{1}{\sqrt{6}\sqrt{14}}\right)$
	Obtain answer $83.7°$ or 1.46 radians	A1	Or answers rounding to $83.7°$ or 1.46 radians
		4	

Question	Answer	Marks	Guidance
8(b)	State or imply $\pm\overrightarrow{AP}$ and $\pm\overrightarrow{BP}$ in component form, i.e. $(1+\lambda, 1-2\lambda, \lambda)$ and $(-1+\lambda, 2-2\lambda, 3+\lambda)$, or equivalent	**B1**	
	Form an equation in λ by equating moduli or by using $\cos BAP = \cos ABP$	***M1**	
	Obtain a correct equation in any form $(1+\lambda)^2+(1-2\lambda)^2+\lambda^2=(\lambda-1)^2+(2-2\lambda)^2+(\lambda+3)^2$	**A1**	Or $(1+\lambda)\sqrt{14-4\lambda+6\lambda^2}=(13-\lambda)\sqrt{2-2\lambda+6\lambda^2}$ $\left(83\lambda^3-528\lambda^2+207\lambda-162=0\right)$
	Solve for λ and obtain position vector	**DM1**	$[\lambda = 6]$
	Obtain correct position vector for P in any form, e.g. $(8, -9, 7)$ or $8\mathbf{i}-9\mathbf{j}+7\mathbf{k}$	**A1**	Accept coordinates
		5	

Question	Answer	Marks	Guidance
9(a)	Use correct product rule or correct quotient rule	**M1**	
	Obtain correct derivative in any form	**A1**	$y'=\dfrac{x^{-\frac{2}{3}}}{x}-\dfrac{2}{3}x^{-\frac{5}{3}}\ln x$
	Equate 2 term derivative to zero and solve for x	**M1**	
	Obtain answer $x=\mathrm{e}^{\frac{3}{2}}$	**A1**	Or exact equivalent
	Obtain answer $y=\dfrac{3}{2\mathrm{e}}$	**A1**	Or exact equivalent
		5	

Question	Answer	Marks	Guidance
9(b)	Commence integration and reach $ax^{\frac{1}{3}}\ln x + b\int x^{\frac{1}{3}}.\frac{1}{x}\mathrm{d}x$	***M1**	
	Obtain $3x^{\frac{1}{3}}\ln x - 3\int x^{\frac{1}{3}}.\frac{1}{x}\mathrm{d}x$	**A1**	
	Complete the integration and obtain $3x^{\frac{1}{3}}\ln x - 9x^{\frac{1}{3}}$	**A1**	OE
	Use limits correctly in an expression of the form $px^{\frac{1}{3}}\ln x + qx^{\frac{1}{3}}$ $(pq \neq 0)$	**DM1**	$6\ln 8 - 9\times 2 - 0 + 9$
	Obtain $18\ln 2 - 9$ from full and correct working	**A1**	AG need to see $\ln 8 = 3\ln 2$
		5	

Question	Answer	Marks	Guidance
10	State a suitable form of partial fractions for $\frac{1}{x^2(1+2x)}$	**B1**	e.g. $\frac{A}{x}+\frac{B}{x^2}+\frac{C}{1+2x}$ or $\frac{Ax+B}{x^2}+\frac{C}{1+2x}$
	Use a relevant method to determine a constant	**M1**	
	Obtain one of $A=-2$, $B=1$ and $C=4$	**A1**	
	Obtain a second value	**A1**	
	Obtain the third value	**A1**	
	Separate variables correctly and integrate at least one term	**M1**	
	Obtain terms $-2\ln x-\frac{1}{x}+2\ln(1+2x)$ and t	**B3 FT**	The FT is on *A*, *B* and *C*. Withhold B1 for each error or omission.
	Evaluate a constant, or use limits $x=1$, $t=0$ in a solution containing terms t, $a\ln x$ and $b\ln(1+2x)$, where $ab\neq 0$	**M1**	
	Obtain a correct expression for t in any form, e.g. $t=-\frac{1}{x}+2\ln\left(\frac{1+2x}{3x}\right)+1$	**A1**	
		11	

Cambridge Assessment International Education

Cambridge International A Level

MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **May/June 2021**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2021 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

This document consists of **19** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

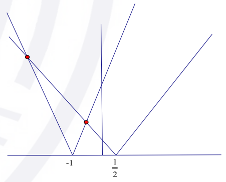
Types of mark

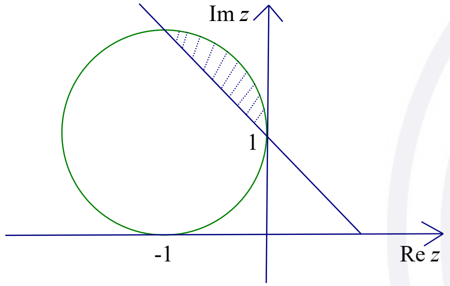
M	Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
A	Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
B	Mark for a correct result or statement independent of method marks.
DM or **DB**	When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
FT	Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	State or imply non-modular inequality $(2x-1)^2 < 3^2(x+1)^2$, or corresponding quadratic equation	**B1**	e.g. $5x^2+22x+8=0$ Allow recovery from 'invisible brackets' on RHS
	Form and solve a 3-term quadratic in x	**M1**	
	Obtain critical values $x = -4$ and $x = -\frac{2}{5}$	**A1**	
	State final answer $x < -4,\ x > -\frac{2}{5}$	**A1**	Do not condone ⩽ for <, or ⩾ for > in the final answer. Allow 'or' but not 'and'. $-\frac{2}{5} < x < -4$ scores A0. Accept equivalent forms using brackets e.g. $x \in (-\infty,-4) \cup (-0.4,\infty)$
	Alternative method for Question 1		
	Obtain critical value $x = -4$ from a graphical method, or by solving a linear equation or linear inequality	**B1**	
	Obtain critical value $x = -\frac{2}{5}$ similarly	**B2**	
	State final answer $x < -4,\ x > -\frac{2}{5}$	**B1**	Do not condone ⩽ for <, or ⩾ for > in the final answer. Allow 'or' but not 'and'. $-\frac{2}{5} < x < -4$ scores A0. Accept equivalent forms using brackets e.g. $x \in (-\infty,-4) \cup (-0.4,\infty)$
		4	

Question	Answer	Marks	Guidance
2	Show a circle with centre – 1 + i.	**B1**	Need some indication of scale or a correct label. Could just be mark(s) on the axes
	Show a circle with radius 1 and centre not at the origin (or relevant part thereof).	**B1**	
	Show correct half line from 1(or relevant part thereof) .	**B1**	
	Shade the correct region on a correct diagram.	**B1**	
		**4**	N.B. If they have very different scales on *their* 2 axes the diagram must match *their* scale - the 'circle' should be an ellipse. Allow freehand diagrams with clear correct intention.

Question	Answer	Marks	Guidance
3(a)	State or imply $\ln x = \ln A - y\ln 3$	**B1**	$\left(y = -\dfrac{1}{\ln 3}\ln x + \dfrac{\ln A}{\ln 3}\right)$
	State that the graph of y against $\ln x$ has an equation that is *linear* in y and $\ln x$, or has an equation of the standard form '$y = mx + c$' and is thus a straight line	**B1**	Must be a correct statement. Accept if the 2 equations are written side by side with no comment. An equation with $y\ln 3$ should be compared with the form $py + q\ln x = c$.
	State that the gradient is $-\dfrac{1}{\ln 3}$	**B1**	OE. Exact answer required. ISW after a correct statement.
		3	

Question	Answer	Marks	Guidance
3(b)	Substitute ln $x = 0$, $y = 1.3$ and use correct method to solve for A	**M1**	$(\ln A = 1.3\ln 3)$ Follow *their* equation in y and $\ln x$. Must be substituting $\ln x = 0$, not $x = 0$. ln 0 'used' in the solution scores M0A0.
	Obtain answer $A = 4.17$ only	**A1**	Must be 2 d.p. as specified in question
		2	

Question	Answer	Marks	Guidance
4	Commence integration and reach $ax\tan^{-1}\frac{1}{2}x + b\int x\frac{1}{c+x^2}\,\mathrm{d}x$	***M1**	OE. Denominator might be $1+\frac{x^2}{4}$ or $2+\frac{x^2}{2}$.
	Obtain $x\tan^{-1}\left(\frac{1}{2}x\right) - \int x.\ \frac{2}{4+x^2}\,\mathrm{d}x$	**A1**	OE
	Complete integration and obtain $x\tan^{-1}\left(\frac{1}{2}x\right) - \ln\left(4+x^2\right)$	**A1**	OE e.g. with $\ln\left(1+\frac{x^2}{4}\right)$
	Substitute limits correctly in an expression of the form $px\tan^{-1}x + q\ln\left(c+x^2\right)$	**DM1**	$2\tan^{-1}1 - \ln 8 + \ln 4$ OE
	Obtain final answer $\frac{1}{2}\pi - \ln 2$	**A1**	OE exact answer. Needs a value for $\tan^{-1}1$ and a single log term
	Alternative method for Question 4		
	Use the substitution $\theta = \tan^{-1}\frac{x}{2}$ to obtain $\lambda\int 2\theta\sec^2\theta\,\mathrm{d}\theta$ and reach $p\theta\tan\theta + q\int\tan\theta\,\mathrm{d}\theta$	***M1**	

Question	Answer	Marks	Guidance
4	Obtain $2\theta\tan\theta - 2\int\tan\theta\,\mathrm{d}\theta$	**A1**	OE
	Complete integration and obtain $2\theta\tan\theta + 2\ln(\cos\theta)$	**A1**	OE
	Substitute correct limits correctly in an expression of the form $r\theta\tan\theta + s\ln(\cos\theta)$	**DM1**	Limits should be $\frac{\pi}{4}$ and 0. Limits must be in radians.
	Obtain final answer $\frac{1}{2}\pi - \ln 2$	**A1**	OE exact answer. Need values for trig. functions and a single log term.
		5	

Question	Answer	Marks	Guidance
5	Square $a + \mathrm{i}b$, use $\mathrm{i}^2 = -1$ and equate real and imaginary parts to 10 and $-4\sqrt{6}$ respectively	M1	
	Obtain $a^2 - b^2 = 10$ and $2ab = -4\sqrt{6}$	A1	Allow $2ab\mathrm{i} = -4\sqrt{6}\mathrm{i}$
	Eliminate one unknown and find an equation in the other	M1	Must be sensible algebra e.g. use of $\sqrt{a^2 - b^2} = a - b$ socres M0
	Obtain $a^4 - 10a^2 - 24[=0]$, or $b^4 + 10b^2 - 24[=0]$, or 3-term equivalent	A1	Or equivalent horizontal equation from correct work
	Obtain final answers $\pm\left(2\sqrt{3} - \sqrt{2}\mathrm{i}\right)$, or exact equivalents	A1	e.g. $\pm\left(\sqrt{12} - \sqrt{2}\mathrm{i}\right)$ from correct work
	Alternative method for Question 5		
	Use the correct method to find the modulus and argument of $\sqrt{u}$	M1	
	Obtain modulus $\sqrt{14}$	A1	
	Obtain argument $\tan^{-1}\dfrac{-1}{\sqrt{6}}$ using an exact method	A1	e.g. by using half angle formula which gives $2\sqrt{6}t^2 - 10t - 2\sqrt{6} = 0$
	Convert to the required form	M1	$\pm\sqrt{14}\left(\dfrac{\sqrt{6}}{\sqrt{7}} - \dfrac{1}{\sqrt{7}}\mathrm{i}\right)$ This mark is available if working in decimals
	Obtain answers $\pm\left(2\sqrt{3} - \sqrt{2}\mathrm{i}\right)$, or exact equivalents	A1	e.g. $\pm\left(\sqrt{12} - \sqrt{2}\mathrm{i}\right)$
		5	

Question	Answer	Marks	Guidance
6(a)	Express the LHS in terms of $\cos 2\theta$ and $\sin 2\theta$	**B1**	e.g. $\frac{1}{\sin 2\theta} - \frac{\cos 2\theta}{\sin 2\theta}$
	Use correct double angle formulae to express the LHS in terms of $\cos\theta$ and $\sin\theta$	**M1**	e.g. $\frac{1-\left(1-2\sin^2\theta\right)}{2\sin\theta\cos\theta}$
	Obtain $\tan\theta$ from correct working	**A1**	AG
	Alternative method for Question 6(a)		
	Express the LHS in terms of $\sin 2\theta$ and $\tan 2\theta$	**B1**	
	Use correct double angle formulae to express the LHS in terms of $\cos\theta$ and $\sin\theta$	**M1**	e.g. $\frac{1}{2\sin\theta\cos\theta} - \frac{1-\frac{\sin^2\theta}{\cos^2\theta}}{2\frac{\sin\theta}{\cos\theta}}\left(=\frac{4\sin^2\theta}{4\sin\theta\cos\theta}\right)$
	Obtain $\tan\theta$ from correct working	**A1**	AG
	Alternative method for Question 6(a)		
	Express the LHS in terms of $\sin 2\theta$ and $\tan 2\theta$	**B1**	
	Use correct t substitution or rearrangement of $\sin 2\theta$ in terms of $\sec^2 2\theta$ and $\tan\theta$ to express the LHS in terms of $\tan\theta$.	**M1**	$\left(\frac{\sec^2\theta}{2\tan\theta} - \frac{1-\tan^2\theta}{2\tan\theta} = \right)\frac{1+\tan^2}{2\tan} - \frac{1-\tan^2}{2\tan}$
	Obtain $\tan\theta$ from correct working	**A1**	AG
		3	

Question	Answer	Marks	Guidance
6(b)	State integral of the form $\mp\ln\cos\theta$ or $\pm\ln\sec\theta$	***M1**	$[-\ln\cos\theta]_{\frac{\pi}{4}}^{\frac{\pi}{3}}$ OE
	Use correct limits correctly and insert exact values for the trigonometric ratios	**DM1**	Need to see evidence of the substitution
	Obtain a correct expression, e.g. $-\ln\frac{1}{2}+\ln\frac{1}{\sqrt{2}}$	**A1**	
	Obtain $\frac{1}{2}\ln 2$ from correct working	**A1**	AG (must see an intermediate step)
		4	

Question	Answer	Marks	Guidance
7	State equation $\frac{\mathrm{d}y}{\mathrm{d}x} = k\frac{y}{\sqrt{x+1}}$	**B1**	OE. Must be a differential equation.
	Separate variables correctly for *their* differential equation and integrate at least one side	***M1**	$\int \frac{1}{y}\mathrm{d}y = \int \frac{k}{\sqrt{x+1}}\mathrm{d}x$
	Obtain ln y	**A1**	Allow M1A1A1 if they have assumed $k = 1$ or are working with an incorrect value for k
	Obtain $2[k]\sqrt{x+1}$	**A1**	
	Use (0, 1) **and** (3, e) in an expression containing $\ln y$ and $\sqrt{x+1}$ and a constant of integration to determine k and/or a constant of integration c (or use (0, 1), (3, e) and (x, y) as limits on definite integrals)	**DM1**	If remove logs before finding the constant of integration then the constant must be of the correct form.
	Obtain $k = \frac{1}{2}$ and $c = -1$	**A1**	OE. $\left(\ln y = \sqrt{x+1} - 1\right)$ Their value of c will depend on where c is in their equation and whether they are working with $\frac{1}{k}\ln y$. The value of k must be consistent with what they integrated.
	Obtain $y = \exp\left(\sqrt{x+1} - 1\right)$	**A1**	NFWW, OE, ISW.
		7	

Question	Answer	Marks	Guidance
8	Use correct product (or quotient) rule	**M1**	At least 3 of 4 terms correct
	Obtain $\frac{dy}{dx} = -5e^{-5x}\tan^2 x + 2e^{-5x}\tan x \sec^2 x$	**A1**	OE.
	Equate *their* derivative to zero, use $\sec^2 x = 1 + \tan^2 x$ and obtain an equation in $\tan x$	**M1**	
	Obtain $2\tan^2 x - 5\tan x + 2 = 0$	**A1**	Allow $2\tan^3 x - 5\tan^2 x + 2\tan x = 0$
	State answer $x = 0$	**B1**	From correct derivative.
	Solve a 3 term quadratic in $\tan x$ and obtain a value of x	**M1**	Must be in radians
	Obtain answer, e.g. 0.464	**A1**	Must be 3 d.p. as specified in the question. Allow A1A0 if both values given to 2 d.p. or > 3 d.p.
	Obtain second non-zero answer, e.g. 1.107 and no other in the given interval	**A1**	
	Alternative method for Question 8		
	Use correct product (or quotient) rule	**M1**	At least 3 of 4 terms correct
	Obtain $\frac{dy}{dx} = -5e^{-5x}\tan^2 x + 2e^{-5x}\tan x \sec^2 x$	**A1**	OE
	Equate *their* derivative to zero and obtain an equation in $\sin x$ and $\cos x$	**M1**	
	Obtain $5\cos x \sin x = 2$	**A1**	Or simplified equivalent (i.e. cancelled)
	State answer $x = 0$	**B1**	From correct derivative.
	Use double angle formula or square both sides and solve for x	**M1**	Or equivalent method. Must be in radians.
	Obtain answer, e.g. 0.464	**A1**	Must be 3 d.p. as specified in the question. Allow A1A0 if both values given to 2 d.p. or > 3 d.p.
	Obtain second non-zero answer, e.g. 1.107 and no other in the given interval	**A1**	
		8	

Question	Answer	Marks	Guidance
9(a)	State or imply the form $\dfrac{A}{2+x}+\dfrac{B+Cx}{3+x^2}$	**B1**	
	Use a correct method for finding a constant	**M1**	SOI
	Obtain one of $A = 4$, $B = 1$ and $C = -2$	**A1**	
	Obtain a second value	**A1**	
	Obtain the third value	**A1**	ISW
		5	
9(b)	Use correct method to find the first two terms of the expansion of $(2+x)^{-1}$, $\left(1+\frac{1}{2}x\right)^{-1}$, $\left(3+x^2\right)^{-1}$ or $\left(1+\frac{1}{3}x^2\right)^{-1}$	**M1**	Allow unsimplified but not if still including ${}^nC_r$
	Obtain correct unsimplified expansions up to the term in x^2 of each partial fraction	**A1 FT** **A1 FT**	$2\left(1-\frac{1}{2}x+\left(\frac{1}{2}x\right)^2....\right)$ $+\frac{1}{3}(1-2x)\left(1-\frac{1}{3}x^2....\right)$ The FT is on *their* A, B and C
	Multiply out, up to the terms in x^2, by $B + Cx$, where $BC \neq 0$	**M1**	Allow with B and C as implied in part **(b)**
	Obtain final answer $\frac{7}{3}-\frac{5}{3}x+\frac{7}{18}x^2$	**A1**	Or equivalent in form $p+qx+rx^2$. A0 if they multiply through by 18.
		5	

Question	Answer	Marks	Guidance
10(a)	State or imply $CD = 2r - 2r\cos x$	**B1**	
	Using correct formulae for area of sector and trapezium, or equivalent, form an equation in r and x	**M1**	e.g. $2\times\frac{1}{2}r^2x = \frac{0.9}{2}(2r + 2r - 2r\cos x)r\sin x$
	Obtain $x = 0.9(2 - \cos x)\sin x$	**A1**	AG, NFWW
		3	
10(b)	Calculate the values of a relevant expression or pair of expressions at $x = 0.5$ and $x = 0.7$	**M1**	Calculated for both values and correct for one value is sufficient for M1. Must be working in radians.
	Complete the argument correctly with correct values	**A1**	Must have sufficient accuracy to support the answer e.g. $\begin{matrix}0.5 > 0.484\\0.7 < 0.716\end{matrix}$ or $\begin{matrix}0.016 > 0\\-0.016 < 0\end{matrix}$ or $\begin{matrix}0.96... < 1\\1.02... > 1\end{matrix}$
		2	
10(c)	State a suitable equation, e.g. $\cos x = \left(2 - \frac{x}{0.9\sin x}\right)$	**B1**	If working in reverse, the first B1 is for $\frac{x}{0.9\sin x} = 2 - \cos x$
	Rearrange this as $x = 0.9\sin x(2 - \cos x)$	**B1**	Need to see the complete sequence of changes.
		2	

Question	Answer	Marks	Guidance
10(d)	Use the iterative process correctly at least once	**M1**	Must be working in radians
	Obtain answer 0.62	**A1**	
	Show sufficient iterations to at least 4 d.p.to justify 0.62 to 2 d.p., or show there is a sign change in the interval (0.615, 0.625)	**A1**	Allow recovery. N.B. A candidate who starts with 0.5 and stops at 0.61 or starts at 0.7 and stops at 0.63 can score M1A0A1 if they have worked to the required accuracy.
		3	
11(a)	Show that $OA = OB = \sqrt{5}$	**B1**	CWO
	Evaluate the scalar product of the correct position vectors	**M1**	e.g. $(0-1+0)$ Condone of using AO and/or BO
	Divide *their* scalar product by the product of the moduli of *their* vectors and evaluate the inverse cosine of the result	**M1**	Much reach an angle. The question asks for the use of scalar product, so alternative methods (e.g. cosine rule) are not accepted.
	Obtain answer 101.5°	**A1**	The question asks for an answer in degrees. Accept 102° or better. Mark radians (1.77) as a misread. Do not ISW: 78.5° as final answer scores A0.
		4	

Question	Answer	Marks	Guidance
11(b)	State or imply M has position vector **i** – **k**	**B1**	OE
	Taking a general point of OM to have position vector λ**i** – λ**k**, express $AP = \sqrt{7}\ OA$ as an equation in λ	***M1**	$\lambda\left(\textit{their}\ \overline{OM}\right)$
	State a correct equation in any form	**A1**	e.g. $\sqrt{(-2+\lambda)^2+1+(-\lambda)^2}=\sqrt{7}\sqrt{5}$
	Reduce to $\lambda^2-2\lambda-15=0$	**A1**	OE
	Solve a quadratic and state a position vector	**DM1**	
	Obtain answers 5**i** – 5**k** and – 3**i** + 3**k**	**A1**	Accept coordinates
	Alternative method for Question 11(b)		
	State or imply that $OP=\gamma\sqrt{2}$	**B1**	
	State or imply that $\cos\frac{1}{2}AOB=\sqrt{\frac{2}{5}}$ and use cosine rule to form an equation in γ	***M1**	Allow $\cos\frac{1}{2}AOB=0.632...$
	State a correct equation in any form	**A1**	e.g. $35=5+2\gamma^2-2\sqrt{5}.\gamma\sqrt{2}.\frac{\sqrt{2}}{\sqrt{5}}$
	Reduce to $\gamma^2-2\gamma-15=0$	**A1**	OE
	Solve a quadratic and state a position vector	**DM1**	
	Obtain answers 5**i** – 5**k** and – 3**i** + 3**k**	**A1**	Accept coordinates

Question	Answer	Marks	Guidance
11(b)	**Alternative method for Question 11(b)**		
	State or imply M has position vector $\mathbf{i} - \mathbf{k}$	**B1**	OE
	State or imply that $AM = \sqrt{3}$	**B1**	
	Use Pythagoras to find MP	***M1**	$MP = \sqrt{35 - (AM)^2}$
	Obtain $MP = 4\sqrt{2}$	**A1**	
	Correct method to find a position vector	**DM1**	$(\mathbf{i} - \mathbf{k}) \pm 4(\mathbf{i} - \mathbf{k})$
	Obtain answers $5\mathbf{i} - 5\mathbf{k}$ and $-3\mathbf{i} + 3\mathbf{k}$	**A1**	Accept coordinates
		6	

Cambridge Assessment International Education

Cambridge International A Level

MATHEMATICS **9709/33**

Paper 3 Pure Mathematics 3 **May/June 2021**

MARK SCHEME

Maximum Mark: 75

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[Turn over

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GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:
- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M	Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
A	Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
B	Mark for a correct result or statement independent of method marks.
DM or **DB**	When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
FT	Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	State correct first two terms $1 + 2x$	**B1**	
	State a correct unsimplified version of the x^2 or x^3 term	**M1**	Symbolic binomial coefficients are not sufficient for the M mark.
	Obtain the next term $-x^2$	**A1**	
	Obtain the final term $\frac{4}{3}x^3$	**A1**	
		4	

Question	Answer	Marks	Guidance
2	State or imply $u^2 - 3u - 1 = 0$, or equivalent in 4^x	**B1**	
	Solve for u or 4^x	**M1**	
	Obtain root $\frac{1}{2}\left(3+\sqrt{13}\right)$, or decimal in [3.30, 3.31]	**A1**	
	Use correct method for finding x from a positive root	**M1**	
	Obtain answer $x = 0.862$ and no other	**A1**	
		5	

Question	Answer	Marks	Guidance
3(a)	State $\frac{\mathrm{d}x}{\mathrm{d}t} = 1 + \frac{1}{t+2}$	**B1**	
	Use product rule	**M1**	
	Obtain $\frac{\mathrm{d}y}{\mathrm{d}t} = \mathrm{e}^{-2t} - 2(t-1)\mathrm{e}^{-2t}$	**A1**	OE
	Use $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}t} \div \frac{\mathrm{d}x}{\mathrm{d}t}$	**M1**	
	Obtain correct answer in any simplified form, e.g. $\frac{(3-2t)(t+2)}{t+3}\ \mathrm{e}^{-2t}$	**A1**	
		5	
3(b)	Equate derivative to zero and solve for t	**M1**	
	Obtain $t = \frac{3}{2}$ and obtain answer $y = \frac{1}{2}\mathrm{e}^{-3}$, or exact equivalent	**A1**	
		2	

Question	Answer	Marks	Guidance
4(a)	State or imply the form $\frac{A}{1+2x}+\frac{B}{4-x}$ and use a correct method to find a constant	**M1**	
	Obtain one of $A = 4$ and $B = -1$	**A1**	
	Obtain the second value	**A1**	
		3	
4(b)	Integrate and obtain terms $2\ln(1+2x)+\ln(4-x)$	**B1FT** **+B1FT**	The FT is on *A* and *B*.
	Substitute limits correctly in an integral of the form $a\ln(1+2x)+b\ln(4-x)$, where $ab \neq 0$	**M1**	
	Obtain final answer $\ln\left(\frac{50}{27}\right)$	**A1**	
		4	

Question	Answer	Marks	Guidance
5(a)	Use double angle formula to express tan 4θ in terms of tan 2θ	**M1**	
	Use double angle formula to express result in terms of tan θ	**M1**	
	Obtain a correct equation in tan θ in any form	**A1**	
	Obtain the given answer	**A1**	
		4	

Question	Answer	Marks	Guidance
5(b)	Solve for tan θ and obtain a value of θ	**M1**	
	Obtain answer, e.g. $53.5°$	**A1**	
	Obtain second answer, e.g. $126.5°$ and no other in the interval	**A1**	Ignore answers outside the given interval. Treat answers in radians as a misread.
		3	

Question	Answer	Marks	Guidance
6(a)	Sketch a relevant graph, e.g. $y = \cot\frac{1}{2}x$	**B1**	
	Sketch a second relevant graph, e.g. $y = 1 + e^{-x}$, and justify the given statement	**B1**	
		2	
6(b)	Calculate values of a relevant expression or pair of expressions at $x = 1$ and $x = 1.5$	**M1**	
	Complete the argument correctly with correct calculated values	**A1**	
		2	
6(c)	Use the iterative formula correctly at least once	**M1**	
	Obtain final answer 1.34	**A1**	
	Show sufficient iterations to 4 d.p. to justify 1.34 to 2 d.p. or show there is a sign change in the interval (1.335, 1.345)	**A1**	
		3	

Question	Answer	Marks	Guidance
7(a)(i)	Justify the given statement $\frac{MN}{y} = \frac{\mathrm{d}y}{\mathrm{d}x}$	**B1**	
		1	
7(a)(ii)	Express the area of PMN in terms of y and $\frac{\mathrm{d}y}{\mathrm{d}x}$ and equate to $\tan x$	**M1**	
	Obtain the given equation correctly	**A1**	
		2	
7(b)	Separate variables and integrate at least one side	**M1**	
	Obtain term $\frac{1}{6}y^3$	**A1**	
	Obtain term of the form $\pm\ln\cos x$	**M1**	
	Evaluate a constant or use $x = 0$ and $y = 1$ in a solution containing terms ay^3 and $\pm\ln\cos x$, or equivalent	**M1**	
	Obtain correct answer in any form, e.g. $\frac{1}{6}y^3 = -\ln\cos x + \frac{1}{6}$	**A1**	
	Obtain final answer $y = \sqrt[3]{(1 - 6\ln\cos x)}$	**A1**	OE
		6	

Question	Answer	Marks	Guidance
8(a)	Use quotient or product rule	**M1**	
	Obtain correct derivative in any form	**A1**	
	Equate derivative to zero and solve for x	**M1**	
	Obtain $x = \sqrt[4]{e}$ and $y = \dfrac{1}{4e}$, or exact equivalents	**A1**	
		4	
8(b)	Commence integration and reach $ax^{-3}\ln x + b\int x^{-3}.\ \dfrac{1}{x}\ dx$	***M1**	
	Obtain $-\dfrac{1}{3}x^{-3}\ln x + \dfrac{1}{3}\int x^{-3}.\ \dfrac{1}{x}\ dx$	**A1**	OE
	Complete integration and obtain $-\dfrac{1}{3}x^{-3}\ln x - \dfrac{1}{9}x^{-3}$	**A1**	
	Substitute limits correctly, having integrated twice	**DM1**	
	Obtain answer $\dfrac{1}{9} - \dfrac{1}{3}a^{-3}\ln a - \dfrac{1}{9}a^{-3}$	**A1**	OE
	Justify the given statement	**A1**	
		6	

Question	Answer	Marks	Guidance
9(a)	State or imply $\overrightarrow{AB} = 2\mathbf{i} + \mathbf{j} - 2\mathbf{k}$	**B1**	OE
	Carry out a correct method to find $\overrightarrow{OD}$	**M1**	
	Obtain answer $-4\mathbf{i} - \mathbf{j} + 3\mathbf{k}$	**A1**	OE
		3	
9(b)	State $\mathbf{r} = -\mathbf{i} + 2\mathbf{j} + 3\mathbf{k} + \lambda(2\mathbf{i} + \mathbf{j} - 2\mathbf{k})$	**B1FT**	OE. The FT is on $\overrightarrow{AB}$.
		1	
9(c)	For a general point *P* on *AB*, state $\overrightarrow{CP}$ or $\overrightarrow{DP}$ in component form, e.g. $\overrightarrow{CP} = (3 - 2\lambda, -\lambda, -6 + 2\lambda)$	***M1**	
	Equate a relevant scalar product to zero *or* equate derivative of $\left\|\overrightarrow{CP}\right\|$ to zero *or* use Pythagoras in a relevant triangle and solve for λ	**DM1**	
	Obtain $\lambda = 2$	**A1**	
	Show the perpendicular is of length 3	**A1**	
	Carry out a correct method to find the area of *ABCD* and obtain the answer 18	**A1**	
	Alternative method for Question 9(c)		
	Use a scalar product to find the projection *CN* (or *DN*) of *BC* (or *AD*) on *CD*	***M1**	
	Obtain *CN* = 3 (or *DN* = 3)	**A1**	
	Use Pythagoras to obtain *BN* (or *AN*)	**DM1**	

Question	Answer	Marks	Guidance
9(c) cont'd	Obtain answer 3	**A1**	
	Carry out a correct method to find the area of *ABCD* and obtain the answer 18	**A1**	
		5	

Question	Answer	Marks	Guidance
10(a)	Substitute $-1+\sqrt{2}\mathrm{i}$ and attempt expansions of the z^2 and z^4 terms	**M1**	
	Use $\mathrm{i}^2=-1$ at least once	**M1**	
	Complete the verification correctly	**A1**	
		3	
10(b)	State second root $-1-\sqrt{2}\mathrm{i}$	**A1**	
	Carry out a method to find a quadratic factor with zeros $-1\pm\sqrt{2}\mathrm{i}$	**M1**	
	Obtain z^2+2z+3	**A1**	
	Commence division and reach partial quotient z^2+kz	**M1**	
	Obtain second quadratic factor z^2-2z+4	**A1**	
	Solve a 3-term quadratic and use $\mathrm{i}^2=-1$	**M1**	
	Obtain roots $1+\sqrt{3}\mathrm{i}$ and $1-\sqrt{3}\mathrm{i}$	**A1**	
		7	

Cambridge Assessment
International Education

Cambridge International A Level

MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **March 2021**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the March 2021 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

This document consists of **15** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

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Marks awarded are always **whole marks** (not half marks, or other fractions).

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WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	Use law of the logarithm of a product or power	**M1**	
	Obtain a correct equation free of logarithms, e.g. $3\left(x^3-3\right)=x^3$	**A1**	
	Obtain x = 1.65	**A1**	
		3	

Question	Answer	Marks	Guidance
2	Substitute $x = -2$, equate result to zero and obtain a correct equation, e.g. $-8a+20+8+b=0$	**B1**	
	Substitute $x = -1$ and equate result to 2	**M1**	
	Obtain a correct equation, e.g. $-a + 5 + 4 + b = 2$	**A1**	
	Solve for a or for b	**M1**	
	Obtain $a = 3$ and $b = -4$	**A1**	
		5	

Question	**Answer**	**Marks**	**Guidance**
3	Use correct trig formulae to obtain an equation in tan x	***M1**	
	Using $\tan 45° = 1$, obtain a horizontal equation in $\tan x$ in any form	**DM1**	
	Reduce the equation to $\tan^2 x + \tan x - 1 = 0$, or 3-term equivalent	**A1**	
	Solve a 3-term quadratic in tan x, for x	**M1**	
	Obtain answer, e.g. $x = 31.7°$	**A1**	
	Obtain second answer, e.g. $x = 121.7°$, and no other in the interval	**A1**	Ignore answers outside the given interval.
		6	

Question	**Answer**	**Marks**	**Guidance**
4(a)	Separate variables correctly and attempt integration of at least one side	**M1**	
	Obtain term $\ln y$	**A1**	
	Obtain term of the form $\pm\ln(1-\cos x)$	**M1**	
	Obtain term $\ln\left(1-\cos x\right)$	**A1**	
	Use $x=\pi$, $y = 4$ to evaluate a constant, or as limits, in a solution containing terms of the form $a\ln y$ and $b\ln(1-\cos x)$	**M1**	
	Obtain final answer $y = 2(1-\cos x)$	**A1**	OE
		6	

Question	**Answer**	**Marks**	**Guidance**
4(b)	Show a correct graph for $0 < x < 2\pi$ with the maximum at $x = \pi$	**B1 FT**	The FT is for graphs of the form $y = a(1 - \cos x)$, where a is positive.
		1	

Question	**Answer**	**Marks**	**Guidance**
5(a)	State $R = \sqrt{11}$	**B1**	
	Use trig formulae to find α	**M1**	
	Obtain $\alpha = 37.09°$	**A1**	
		3	
5(b)	Evaluate $\sin^{-1}\left(\frac{1}{\sqrt{11}}\right)$ to at least 2 dp (17.5484°)	**B1 FT**	The FT is on R.
	Use correct method to find a value of θ in the interval	**M1**	
	Obtain answer, e.g. 62.7°	**A1**	
	Use a correct method to obtain a second answer	**M1**	
	Obtain second answer, e.g. 170.2°, and no other in the interval	**A1**	Ignore answers outside the given interval.
		5	

Question	Answer	Marks	Guidance
6(a)	Carry out a relevant method to determine constants *A* and *B* such that $\frac{5a}{(2x-a)(3a-x)} = \frac{A}{2x-a} + \frac{B}{3a-x}$	**M1**	
	Obtain $A = 2$	**A1**	
	Obtain $B = 1$	**A1**	
		3	
6(b)	Integrate and obtain terms $\ln(2x-a) - \ln(3a-x)$	**B1 FT** **B1 FT**	The FT is on the values of *A* and *B*.
	Substitute limits correctly in a solution containing terms of the form $b\ln(2x-a)$ and $c\ln(3a-x)$, where $bc \neq 0$	**M1**	
	Obtain the given answer showing full and correct working	**A1**	
		4	

Question	Answer	Marks	Guidance
7(a)	Express general point of a line in component form, e.g. $(1 + 2s, 3 - s, 2 + 3s)$ or $(2 + t, 1 - t, 4 + 4t)$	**B1**	
	Equate at least two pairs of components and solve for s or for t	**M1**	
	Obtain correct answer for s or for t (possible answers are $-1, 6, \frac{2}{5}$ for s and $-3, 4, -\frac{1}{5}$ for t)	**A1**	
	Verify that all three component equations are not satisfied	**A1**	
	Show that the lines are not parallel and are thus skew	**A1**	
		5	
7(b)	Carry out correct process for evaluating the scalar product of the direction vectors	**M1**	
	Using the correct process for the moduli, divide the scalar product by the product of the moduli and evaluate the inverse cosine of the result	**M1**	
	Obtain answer $19.1°$ or 0.333 radians	**A1**	
		3	

Question	Answer	Marks	Guidance
8(a)	Multiply numerator and denominator by 3 – i	**M1**	OE
	Obtain numerator – 10 + 10i or denominator 10	**A1**	
	Obtain final answer – 1 + i	**A1**	
		3	
8(b)	State or imply $r = \sqrt{2}$	**B1 FT**	
	State or imply that $\theta = \frac{3}{4}\pi$	**B1 FT**	
		2	
8(c)	State that *OA* and *BC* are parallel	**B1**	
	State that $BC = 2OA$	**B1**	
		2	

Question	Answer	Marks	Guidance
8(d)	Use angle $AOB = \arg u - \arg v = \arg\frac{u}{v}$	**M1**	
	Obtain the given answer	**A1**	
	Alternative method for question 8(d)		
	Obtain tan AOB from gradients of OA and OB and the $\tan(A \pm B)$ formula	**M1**	
	Obtain the given answer	**A1**	
	Alternative method for question 8(d)		
	Obtain $\cos AOB$ by using the cosine rule or a scalar product	**M1**	
	Obtain the given answer	**A1**	
		2	

Question	Answer	Marks	Guidance
9(a)	Calculate the values of a relevant expression or pair of expressions at $x = 1$ and $x = 1.5$	**M1**	
	Complete the argument correctly with correct calculated values	**A1**	
		2	
9(b)	Use the iterative formula $x_{n+1} = \frac{e^{2x_n}+1}{e^{2x_n}-1}$, or equivalent, correctly at least once	**M1**	
	Obtain final answer 1.20	**A1**	
	Show sufficient iterations to 4 dp to justify 1.20 to 2 dp, or show there is a sign change in the interval (1.195,1.205)	**A1**	
		3	

Question	Answer	Marks	Guidance
9(c)	Use quotient rule	**M1**	
	Obtain correct derivative in any form	**A1**	
	Equate derivative to – 8 and obtain a quadratic in e^{2x}	**M1**	
	Obtain $2\left(e^{2x}\right)^2 - 5e^{2x} + 2 = 0$	**A1**	OE
	Solve a 3-term quadratic in e^{2x} for x	**M1**	
	Obtain answer $x = \frac{1}{2}\ln 2$, or exact equivalent, only	**A1**	
	Alternative method for question 9(c)		
	Use quotient rule	**M1**	
	Obtain correct derivative in any form	**A1**	
	Equate derivative to – 8, take square roots and obtain a quadratic in e^{x}	**M1**	
	Obtain $\sqrt{2}e^{2x} - e^{x} - \sqrt{2} = 0$	**A1**	OE
	Solve a 3-term quadratic in e^{x} for x	**M1**	
	Obtain answer $x = \frac{1}{2}\ln 2$, or exact equivalent, only	**A1**	
		6	

Question	Answer	Marks	Guidance
10(a)	State or imply d*u* = cos *x* d*x*	**B1**	
	Using double angle formula for sin2*x* and Pythagoras, express integral in terms of *u* and d*u*.	**M1**	
	Obtain integral $\int 2\left(u-u^3\right)\mathrm{d}u$	**A1**	OE
	Use limits *u* = 0 and *u* = 1 in an integral of the form au^2+bu^4, where $ab \neq 0$	**M1**	$a+b$ or $a+b-0$ $\left(a=1 \text{ and } b=-\frac{1}{2}\right)$
	Obtain answer $\frac{1}{2}$	**A1**	
		5	
10(b)	Use product rule	**M1**	
	Obtain correct derivative in any form	**A1**	
	Equate derivative to zero and use a double angle formula	***M1**	
	Obtain an equation in one trig variable	**DM1**	
	Obtain $4\sin^2 x=1$, $4\cos^2 x=3$ or $3\tan^2 x=1$	**A1**	
	Obtain answer $x=\frac{1}{6}\pi$	**A1**	
		6	

Cambridge International A Level

MATHEMATICS **9709/31**

Paper 3 Pure Mathematics 3 **October/November 2020**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

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This document consists of **21** printed pages.

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Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M	Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
A	Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
B	Mark for a correct result or statement independent of method marks.
DM or **DB**	When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
FT	Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	Make a recognisable sketch graph of $y = 2\lvert x-3\rvert$ and the line $y = 2 - 5x$	**B1**	Need to see correct V at $x = 3$, roughly symmetrical, $x = 3$ stated, domain at least $(-2, 5)$.
	Find x-coordinate of intersection with $y = 2 - 5x$	**M1**	Find point of intersection with $y = 2\lvert x - 3\rvert$ or solve $2 - 5x$ with $2(x - 3)$ or $-2(x - 3)$
	Obtain $x = -\dfrac{4}{3}$	**A1**	
	State final answer $x < -\dfrac{4}{3}$	**A1**	Do not accept $x < -1.33$ [Do not condone $\leqslant$ for $<$ in the final answer.]
	Alternative method for question 1		
	State or imply non-modular inequality/equality $(2-5x)^2 >, \geqslant, =, 2^2(x-3)^2$, or corresponding quadratic equation, or pair of linear equations $(2-5x) >, \geqslant, =, \pm 2(x-3)$	**B1**	Two correct linear equations only
	Make reasonable attempt at solving a 3-term quadratic, or solve one linear equation, or linear inequality for x	**M1**	$21x^2 + 4x - 32 = (3x+4)(7x-8) = 0$ $2 - 5x$ or $-(2 - 5x)$ with $2(x-3)$ or $-2(x-3)$
	Obtain critical value $x = -\dfrac{4}{3}$	**A1**	
	State final answer $x < -\dfrac{4}{3}$	**A1**	Do not accept $x < -1.33$ [Do not condone $\leqslant$ for $<$ in the final answer.]
		4	

Question	Answer	Marks	Guidance
2	Show a circle with centre the origin and radius 2	**B1**	
	Show the point representing 1 – i	**B1**	
	Show a circle with centre 1 – i and radius 1	**B1 FT**	The FT is on the position of 1 – i.
	Shade the appropriate region	**B1 FT**	The FT is on the position of 1 – i. Shaded region outside circle with centre the origin and radius 2 and inside circle with centre $\pm 1 \pm$ i and radius 1
		4	

Question	Answer	Marks	Guidance
3	State or imply $\frac{\mathrm{d}x}{\mathrm{d}\theta} = 2\sin 2\theta$ or $\frac{\mathrm{d}y}{\mathrm{d}\theta} = 2 + 2\cos 2\theta$	**B1**	
	Use $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}\theta} \div \frac{\mathrm{d}x}{\mathrm{d}\theta}$	**M1**	
	Obtain correct answer $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2+2\cos 2\theta}{2\sin 2\theta}$	**A1**	OE
	Use correct double angle formulae	**M1**	
	Obtain the given answer correctly $\frac{\mathrm{d}y}{\mathrm{d}x} = \cot\theta$	**A1**	AG. Must have simplified numerator in terms of cosθ.
	Alternative method for question 3		
	Start by using both correct double angle formulae e.g. $x = 3 - (2\cos^2\theta - 1)$, $y = 2\theta + 2\sin\theta\cos\theta$	**M1**	
	$\frac{\mathrm{d}x}{\mathrm{d}\theta}$ or $\frac{\mathrm{d}y}{\mathrm{d}\theta}$	**B1**	
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\left(2+2\left(\cos^2\theta - \sin^2\theta\right)\right)}{4\cos\theta\sin\theta}$	**M1 A1**	
	Simplify to given answer correctly $\frac{\mathrm{d}y}{\mathrm{d}x} = \cot\theta$	**A1**	AG

Question	Answer	Marks	Guidance
3	**Alternative method for question 3**		
	Set = *2θ*. State $\frac{\mathrm{d}x}{\mathrm{d}t} = \sin t$ or $\frac{\mathrm{d}y}{\mathrm{d}t} = 1+\cos t$	**B1**	
	Use $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}t} \div \frac{\mathrm{d}x}{\mathrm{d}t}$	**M1**	
	Obtain correct answer $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1+\cos t}{\sin t}$	**A1**	OE
	Use correct double angle formulae	**M1**	
	Obtain the given answer correctly $\frac{\mathrm{d}y}{\mathrm{d}x} = \cot\theta$	**A1**	
		5	
4	State or imply $\log_{10} 10 = 1$	**B1**	$\log_{10} 10^{-1} = -1$
	Use law of the logarithm of a power, product or quotient	**M1**	
	Obtain a correct equation in any form, free of logs	**A1**	e.g. $(2x+1)/(x+1)^2 = 10^{-1}$ or $10(2x+1)/(x+1)^2 = 10^0$ or 1 or $x^2 + 2x + 1 = 20x + 10$
	Reduce to $x^2 - 18x - 9 = 0$, or equivalent	**A1**	
	Solve a 3-term quadratic	**M1**	
	Obtain final answers $x = 18.487$ and $x = -0.487$	**A1**	Must be 3 d.p. Do not allow rejection.
		6	

Question	Answer	Marks	Guidance
5(a)	Sketch a relevant graph, e.g. $y = \text{cosec } x$	B1	cosec x, U shaped, roughly symmetrical about $x = \frac{\pi}{2}$, $y\left(\frac{\pi}{2}\right) = 1$ and domain at least $\left(\frac{\pi}{6}, \frac{5\pi}{6}\right)$.
	Sketch a second relevant graph, e.g. $y = 1 + \text{e}^{-\frac{1}{2}x}$, and justify the given statement	B1	Exponential graph needs $y(0) = 2$, negative gradient, always increasing, and $y(\pi) > 1$ Needs to mark intersections with dots, crosses, or say roots at points of intersection, or equivalent
		2	
5(b)	Use the iterative formula correctly at least twice	M1	2, 2.3217, 2.2760, 2.2824… Need to see 2 iterations and following value inserted correctly
	Obtain final answer 2.28	A1	Must be supported by iterations
	Show sufficient iterations to at least 4 d.p. to justify 2.28 to 2 d.p., or show there is a sign change in the interval (2.275, 2.285)	A1	
		3	

Question	Answer	Marks	Guidance
6(a)	State $R = \sqrt{15}$	**B1**	
	Use trig formulae to find α	**M1**	$\frac{\sin\alpha}{\cos\alpha} = \frac{3}{\sqrt{6}}$ with no error seen or $\tan\alpha = \frac{3}{\sqrt{6}}$ quoted then allow M1
	Obtain $\alpha = 50.77$	**A1**	Must be 2 d.p. If radians 0.89 A0 MR
		3	
6(b)	Evaluate $\beta = \cos^{-1}\frac{2.5}{\sqrt{15}}$ (49.797° to 4 d.p.)	**B1 FT**	The FT is on incorrect R. $\frac{x}{3} = \beta - \alpha$ [–2.9° and –301.7°]
	Use correct method to find a value of $\frac{x}{3}$ in the interval	**M1**	Needs to use $\frac{x}{3}$
	Obtain answer rounding to $x = 301.6°$ to $301.8°$	**A1**	
	Obtain second answer rounding to $x = 2.9(0)°$ to $2.9(2)°$ and no others in the interval	**A1**	
		4	

Question	Answer	Marks	Guidance
7(a)	Substitute $-1+\sqrt{5}\,\mathrm{i}$ in the equation and attempt expansions of x^2 and x^3	**M1**	All working must be seen. Allow M1 if small errors in $1-2\sqrt{5}\mathrm{i}-5$ or $1-\sqrt{5}\mathrm{i}-\sqrt{5}\mathrm{i}-5$ and $4-2\sqrt{5}\mathrm{i}+10$ or $4-4\sqrt{5}\mathrm{i}+2\sqrt{5}\mathrm{i}+10$
	Use $\mathrm{i}^2=-1$ correctly at least once	**M1**	$1-5$ or $4+10$ seen
	Complete the verification correctly	**A1**	$2\left(14-2\sqrt{5}\mathrm{i}\right)+\left(-4-2\sqrt{5}\mathrm{i}\right)+6\left(-1+\sqrt{5}\mathrm{i}\right)-18=0$
		3	
7(b)	State second root $-1-\sqrt{5}\mathrm{i}$	**B1**	
	Carry out a complete method for finding a quadratic factor with zeros $-1+\sqrt{5}\mathrm{i}$ and $-1-\sqrt{5}\mathrm{i}$	**M1**	
	Obtain x^2+2x+6	**A1**	
	Obtain root $x=\dfrac{3}{2}$	**A1**	OE
	Alternative method for question 7(b)		
	State second root $-1-\sqrt{5}\mathrm{i}$	**B1**	
	$(x+1-\sqrt{5}\mathrm{i})(x+1+\sqrt{5}\mathrm{i})(2x+a)=2x^3+x^2+6x-18$	**M1**	
	$(1-\sqrt{5}\,\mathrm{i})(1+\sqrt{5}\,\mathrm{i})\,a=-18$	**A1**	
	$6a=-18\quad a=-3\quad$ leading to $x=\dfrac{3}{2}$	**A1**	OE

Question	Answer	Marks	Guidance
7(b)	**Alternative method for question 7(b)**		
	State second root $-1-\sqrt{5}\mathrm{i}$	**B1**	
	POR = 6 SOR = – 2	**M1**	
	Obtain x^2+2x+6	**A1**	
	Obtain root $x=\frac{3}{2}$	**A1**	OE
	Alternative method for question 7(b)		
	State second root $-1-\sqrt{5}\mathrm{i}$	**B1**	
	POR $\left(-1-\sqrt{5}\mathrm{i}\right)\left(-1+\sqrt{5}\mathrm{i}\right)a=9$	**M1 A1**	
	Obtain root $x=\frac{3}{2}$	**A1**	OE
	Alternative method for question 7(b)		
	State second root $-1-\sqrt{5}\mathrm{i}$	**B1**	
	SOR $\left(-1-\sqrt{5}\mathrm{i}\right)+\left(-1+\sqrt{5}\mathrm{i}\right)+a=-\frac{1}{2}$	**M1 A1**	
	Obtain root $x=\frac{3}{2}$	**A1**	OE
		4	

Question	Answer	Marks	Guidance
8	Separate variables correctly and attempt integration of at least one side	**B1**	$\frac{1}{y}\mathrm{d}y = \frac{1-2x^2}{x}\mathrm{d}x$
	Obtain term $\ln y$	**B1**	
	Obtain terms $\ln x - x^2$	**B1**	
	Use $x = 1$, $y = 1$ to evaluate a constant, or as limits, in a solution containing at least 2 terms of the form $a\ln y$, $b\ln x$ and cx^2	**M1**	The 2 terms of required form must be from correct working e.g. $\ln y = \ln x - x^2 + 1$
	Obtain correct solution in any form	**A1**	
	Rearrange and obtain $y = xe^{1-x^2}$	**A1**	OE
		6	

Question	Answer	Marks	Guidance
9(a)	State or imply the form $\frac{A}{1-x}+\frac{B}{2+3x}+\frac{C}{(2+3x)^2}$	**B1**	
	Use a correct method for finding a coefficient	**M1**	
	Obtain one of $A = 1$, $B = -1$, $C = 6$	**A1**	
	Obtain a second value	**A1**	
	Obtain the third value	**A1**	In the form $\frac{A}{1-x}+\frac{Dx+E}{(2+3x)^2}$, where $A = 1$, $D = -3$ and $E = 4$ can score B1 M1 A1 A1 A1 as above.
		5	

Question	Answer	Marks	Guidance
9(b)	Use a correct method to find the first two terms of the expansion of $(1-x)^{-1}$, $(2+3x)^{-1}$, $\left(1+\frac{3}{2}x\right)^{-1}$, $(2+3x)^{-2}$ or $\left(1+\frac{3}{2}x\right)^{-2}$	**M1**	Symbolic coefficients are not sufficient for the M1 $A\left[\frac{1+(-1)(-x)+(-1)(-2)(-x)^2}{2\ldots}\right]$ $A=1$ $\frac{B}{2}\left[\frac{1+(-1)\left(\frac{3x}{2}\right)+(-1)(-2)\left(\frac{3x}{2}\right)^2}{2\ldots}\right]$ $B=1$ $\frac{C}{4}\left[\frac{1+(-2)\left(\frac{3x}{2}\right)+(-2)(-3)\left(\frac{3x}{2}\right)^2}{2\ldots}\right]$ $C=6$
	Obtain correct un-simplified expansions up to the term in of each partial fraction	**A1 FT** + **A1 FT** + **A1 FT**	$\left(1+x+x^2\right)+\left(-\frac{1}{2}+\left(\frac{3}{4}\right)x-\left(\frac{9}{8}\right)x^2\right)$ $+\left(\frac{6}{4}-\left(\frac{18}{4}\right)x+\left(\frac{81}{8}\right)x^2\right)$ [The FT is on *A*, *B*, *C*] $\left(1-\frac{1}{2}+\frac{6}{4}\right)+\left(1+\frac{3}{4}-\frac{18}{4}\right)x+\left(1-\frac{9}{8}+\frac{81}{8}\right)x^2$
	Obtain final answer $2-\frac{11}{4}x+10x^2$, or equivalent	**A1**	Allow unsimplified fractions $\frac{(Dx+E)}{4}\left[\frac{1+(-2)\left(\frac{3x}{2}\right)+(-2)(-3)\left(\frac{3x}{2}\right)^2}{2\ldots}\right]$ $D=-3$, $E=4$ The FT is on *A*, *D*, *E*.
		5	

Question	Answer	Marks	Guidance
10(a)	Use correct product or quotient rule	***M1**	$\frac{\mathrm{d}y}{\mathrm{d}x}=\left(-\frac{1}{2}\right)(2-x)\mathrm{e}^{-\frac{1}{2}x}-\mathrm{e}^{-\frac{1}{2}x}$ M1 requires at least one of derivatives correct
	Obtain correct derivative in any form	**A1**	
	Equate derivative to zero and solve for x	**DM1**	
	Obtain $x = 4$	**A1**	ISW
	Obtain $y = -2\mathrm{e}^{-2}$, or exact equivalent	**A1**	
		5	

Question	Answer	Marks	Guidance
10(b)	Commence integration and reach $a(2-x)e^{-\frac{1}{2}x}+b\int e^{-\frac{1}{2}x}dx$	***M1**	Condone omission of dx $-2(2-x)e^{-\frac{1}{2}x}+4e^{-\frac{1}{2}x}$ or $2xe^{-\frac{1}{2}x}$
	Obtain $-2(2-x)e^{-\frac{1}{2}x}-2\int e^{-\frac{1}{2}x}dx$	**A1**	OE
	Complete integration and obtain $2xe^{-\frac{1}{2}x}$	**A1**	OE
	Use correct limits, $x = 0$ and $x = 2$, correctly, having integrated twice	**DM1**	Ignore omission of zeros and allow max of 1 error
	Obtain answer $4e^{-1}$, or exact equivalent	**A1**	ISW
	Alternative method for question 10(b)		
	$\frac{d\left(2xe^{-\frac{1}{2}x}\right)}{dx}=2e^{-\frac{1}{2}x}-xe^{-\frac{1}{2}x}$	***M1 A1**	
	$\therefore 2xe^{-\frac{1}{2}x}$	**A1**	
	Use correct limits, $x = 0$ and $x = 2$, correctly, having integrated twice	**DM1**	Ignore omission of zeros and allow max of 1 error
	Obtain answer $4e^{-1}$, or exact equivalent	**A1**	ISW
		5	

Question	Answer	Marks	Guidance
11(a)	Express general point of at least one line correctly in component form, i.e. $(1 + a\lambda, 2 + 2\lambda, 1 - \lambda)$ or $(2 + 2\mu, 1 - \mu, -1 + \mu)$	**B1**	
	Equate at least two pairs of corresponding components and solve for λ or for μ	**M1**	May be implied $1 + a\lambda = 2 + 2\mu$ $2 + 2\lambda = 1 - \mu$ $1 - \lambda = -1 + \mu$
	Obtain $\lambda = -3$ or $\mu = 5$	**A1**	
	Obtain $a = -\frac{11}{3}$	**A1**	Allow $a = -3.667$
	State that the point of intersection has position vector $12\mathbf{i} - 4\mathbf{j} + 4\mathbf{k}$	**A1**	Allow coordinate form $(12, -4, 4)$
		5	

Question	Answer	Marks	Guidance
11(b)	Use correct process for finding the scalar product of direction vectors for the two lines	**M1**	$(a, 2, -1) \mathbf{.} (2, -1, 1) = 2a - 2 - 1$ or $2a - 3$
	Using the correct process for the moduli, divide the scalar product by the product of the moduli and equate the result to $\pm\frac{1}{6}$	***M1**	
	State a correct equation in a in any form, e.g. $\frac{2a-2-1}{\sqrt{6}\sqrt{(a^2+5)}} = \pm\frac{1}{6}$	**A1**	
	Solve for a	**DM1**	Solve 3-term quadratic for a having expanded $(2a - 3)^2$ to produce 3 terms e.g. $36(2a - 3)^2 = 6(a^2 + 5)$ $138a^2 - 432a + 294 = 0$ $23a^2 - 72a + 49 = 0$ $(23a - 49)(a - 1) = 0$
	Obtain $a = 1$	**A1**	
	Obtain $a = \frac{49}{23}$	**A1**	Allow $a = 2.13$

Question	Answer	Marks	Guidance
11(b)	**Alternative method for question 11(b)**		
	$\cos(\theta) = [\lvert a^2 + 2^2 + (-1)^2\rvert^2 + \lvert 2^2 + (-1)^2 + 1^2\rvert^2$ $- \lvert(a-2)^2 + 3^2 + (-2)^2\rvert^2] / [2\lvert a^2 + 2^2 + (-1)^2\rvert.\lvert 2^2 + (-1)^2 + 1^2\rvert]$	**M1**	Use of cosine rule. Must be correct vectors.
	Equate the result to $\pm\frac{1}{6}$	***M1** **A1**	Allow M1* here for any two vectors
	Solve for a	**DM1**	Solve 3-term quadratic for a having expanded $(2a-3)^2$ to produce 3 terms e.g. $36(2a-3)^2 = 6(a^2+5)$ $138a^2 - 432a + 294 = 0$ $23a^2 - 72a + 49 = 0$ $(23a-49)(a-1) = 0$
	Obtain $a = 1$	**A1**	
	Obtain $a = \frac{49}{23}$	**A1**	Allow $a = 2.13$
		6	

Cambridge Assessment International Education

Cambridge International A Level

MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **October/November 2020**

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[Turn over

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M	Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
A	Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
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Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
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AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	State that $1+e^{-3x}=e^2$	**B1**	With no errors seen to that point
	Use correct method to solve an equation of the form $e^{-3x}=a$, where $a > 0$, for x or equivalent	**M1**	$\left(e^{-3x}=6.389...\right)$ Evidence of method must be seen.
	Obtain answer $x = -0.618$ only	**A1**	Must be 3 decimal places
	Alternative method for question 1		
	State that $1+e^{-3x}=e^2$	**B1**	
	Rearrange to obtain an expression for e^x and solve an equation of the form $e^x=a$, where $a > 0$, or equivalent	**M1**	$e^x=\sqrt[3]{\dfrac{1}{e^2-1}}$
	Obtain answer $x = -0.618$ only	**A1**	Must be 3 decimal places
		3	

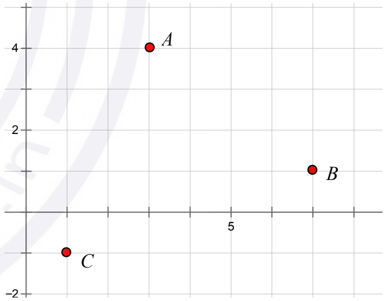
Question	Answer	Marks	Guidance
2(a)	State a correct unsimplified version of the x or x^2 or x^3 term	**M1**	For the given expression
	State correct first two terms $1 + 2x$	**A1**	
	Obtain the next two terms $-4x^2+\dfrac{40}{3}x^3$	**A1 + A1**	One mark for each correct term. ISW Accept $13\frac{1}{3}$ The question asks for simplified coefficients, so candidates should cancel fractions.
		4	
2(b)	State answer $\lvert x\rvert<\dfrac{1}{6}$	**B1**	OE. Strict inequality
		1	

Question	Answer	Marks	Guidance
3(a)	State or imply $y\log 2 = \log 3 - 2x\log 3$	**B1**	Accept $y\ln 2 = (1-2x)\ln 3$
	State that the graph of y against x has an equation which is linear in x and y, or is of the form $ay = bx + c$	**B1**	Correct equation. Need a clear statement/comparison with matching linear form.
	Clear indication that the gradient is $-\dfrac{2\ln 3}{\ln 2}$	**B1**	Must be exact. Any equivalent e.g. $-\dfrac{2\log_k 3}{\log_k 2}$, $\log_2 \dfrac{1}{9}$
		3	
3(b)	Substitute $y = 3x$ in an equation involving logarithms and solve for x	**M1**	
	Obtain answer $x = \dfrac{\ln 3}{\ln 72}$	**A1**	Allow M1A1 for the correct answer following decimals
		2	

Question	Answer	Marks	Guidance
4(a)	Use correct $\tan(A+B)$ formula and obtain an equation in $\tan\theta$	**M1**	e.g. $\dfrac{\tan\theta+\tan 60°}{1-\tan\theta\tan 60°}=\dfrac{2}{\tan\theta}$
	Use $\tan 60°=\sqrt{3}$ and obtain a correct horizontal equation in any form	**A1**	e.g. $\tan\theta\left(\tan\theta+\sqrt{3}\right)=2\left(1-\sqrt{3}\tan\theta\right)$
	Reduce to $\tan^2\theta+3\sqrt{3}\tan\theta-2=0$ correctly	**A1**	AG
		3	
4(b)	Solve the given quadratic to obtain a value for θ	**M1**	$\left(\tan\theta=\dfrac{-3\sqrt{3}\pm\sqrt{35}}{2}=0.3599,-5.556\right)$
	Obtain one correct answer e.g. $\theta=19.8°$	**A1**	Accept 1d.p. or better. If over-specified must be correct. 19.797...., 100.2029...
	Obtain second correct answer $\theta=100.2°$ and no others in the given interval	**A1**	Ignore answers outside the given interval.
		3	

Question	Answer	Marks	Guidance
5(a)	State $\frac{dx}{d\theta} = \sec^2\theta$ or $\frac{dy}{d\theta} = -2\sin\theta\cos\theta$	**B1**	CWO, AEF.
	Use $\frac{dy}{dx} = \frac{dy}{d\theta} \div \frac{dx}{d\theta}$	**M1**	
	Obtain $\frac{dy}{dx} = -2\sin\theta\cos^3\theta$ from correct working	**A1**	AG
	Alternative method for question 5(a)		
	Convert to Cartesian form and differentiate	**M1**	$y = \frac{1}{1+x^2}$
	$\frac{dy}{dx} = \frac{-2x}{\left(1+x^2\right)^2}$	**A1**	OE
	Obtain $\frac{dy}{dx} = -2\sin\theta\cos^3\theta$ from correct working	**A1**	AG
		3	

Question	Answer	Marks	Guidance
5(b)	Use correct product rule to obtain $\frac{\text{d}}{\text{d}\theta}\left(\pm 2\cos^3\theta\sin\theta\right)$	**M1**	Condone incorrect naming of the derivative For work done in correct context
	Obtain correct derivative in any form	**A1**	e.g. $\pm\left(-2\cos^4\theta + 6\sin^2\theta\cos^2\theta\right)$
	Equate derivative to zero and obtain an equation in one trig ratio	**A1**	e.g. $3\tan^2\theta = 1$, or $4\sin^2\theta = 1$ or $4\cos^2\theta = 3$
	Obtain answer $x = -\frac{1}{\sqrt{3}}$	**A1**	Or $-\frac{\sqrt{3}}{3}$
	Alternative method for question 5(b)		
	Use correct quotient rule to obtain $\frac{\text{d}^2y}{\text{d}x^2}$	**M1**	
	Obtain correct derivative in any form	**A1**	$\frac{-2\left(1+x^2\right)^2 + 2\times 2x\times 2x\left(1+x^2\right)}{\left(1+x^2\right)^4}$
	Equate derivative to zero and obtain an equation in x^2	**A1**	e.g. $6x^2 = 2$
	Obtain answer $x = -\frac{1}{\sqrt{3}}$	**A1**	
		4	

Question	Answer	Marks	Guidance
6(a)	Multiply numerator and denominator by 1 + i, or equivalent	**M1**	Must multiply out
	Obtain numerator 6 + 8i or denominator 2	**A1**	
	Obtain final answer $u = 3 + 4i$	**A1**	
	Alternative method for question 6(a)		
	Multiply out $(1-i)(x+iy)=7+i$ and compare real and imaginary parts	**M1**	
	Obtain $x+y=7$ or $y-x=1$	**A1**	
	Obtain final answer $u = 3 + 4i$	**A1**	
		3	
6(b)	Show the point A representing u in a relatively correct position	**B1 FT**	The FT is on $xy \neq 0$.
	Show the other two points B and C in relatively correct positions: approximately equal distance above / below real axis	**B1**	 Take the position of A as a guide to 'scale' if axes not marked
		2	

Question	Answer	Marks	Guidance
6(c)	State or imply arg(1 – i) = $-\frac{1}{4}\pi$	**B1**	Arg$C$
	Substitute exact arguments in arg(7 + i) – arg(1 – i) = arg u	**M1**	Must see a statement about the relationship between the Args e.g. Arg$A$ = Arg$B$ – Arg$C$ or equivalent exact method
	Obtain $\tan^{-1}\left(\frac{4}{3}\right)=\tan^{-1}\left(\frac{1}{7}\right)+\frac{1}{4}\pi$ correctly	**A1**	Obtain given answer correctly from *their* $u = k(3+4i)$
		3	

Question	Answer	Marks	Guidance
7(a)	Correct separation of variables	B1	$\int \sec^2 2x\,\mathrm{d}x = \int \mathrm{e}^{-3t}\,\mathrm{d}t$ Needs correct structure
	Obtain term $-\frac{1}{3}\mathrm{e}^{-3t}$	B1	
	Obtain term of the form $k\tan 2x$	M1	From correct working
	Obtain term $\frac{1}{2}\tan 2x$	A1	
	Use $x = 0$, $t = 0$ to evaluate a constant, or as limits in a solution containing terms of the form $a\tan 2x$ and $b\mathrm{e}^{-3t}$, where $ab \neq 0$	M1	
	Obtain correct solution in any form	A1	e.g. $\frac{1}{2}\tan 2x = -\frac{1}{3}\mathrm{e}^{-3t} + \frac{1}{3}$
	Obtain final answer $x = \frac{1}{2}\tan^{-1}\left(\frac{2}{3}\left(1-\mathrm{e}^{-3t}\right)\right)$	A1	
		7	
7(b)	State that x approaches $\frac{1}{2}\tan^{-1}\left(\frac{2}{3}\right)$	B1 FT	Correct value. Accept $x \to 0.294$ The FT is dependent on letting $\mathrm{e}^{-3t} \to 0$ in a solution containing e^{-3t}.
		1	

Question	Answer	Marks	Guidance
8(a)	Obtain $\overrightarrow{AB}=\begin{pmatrix}2\\-2\\-4\end{pmatrix}$ and $\overrightarrow{CD}=\begin{pmatrix}2\\1\\1\end{pmatrix}$	**B1**	Or equivalent seen or implied
	Use the correct process for calculating the modulus of both vectors to obtain AB and CD	**M1**	$AB=\sqrt{24}, CD=\sqrt{6}$
	Using exact values, verify that $AB=2CD$	**A1**	Obtain given statement from correct work Allow from $BA=2DC$, OE
		3	
8(b)	Use the correct process to calculate the scalar product of the relevant vectors (*their* $\overrightarrow{AB}$ and $\overrightarrow{CD}$)	**M1**	$\begin{pmatrix}2\\-2\\-4\end{pmatrix}$ and $\begin{pmatrix}2\\1\\1\end{pmatrix}$ or $\begin{pmatrix}2\\-2\\-4\end{pmatrix}$ and $\begin{pmatrix}4\\2\\2\end{pmatrix}$
	Divide the scalar product by the product of the moduli and evaluate the inverse cosine of the result	**M1**	
	Obtain answer 99.6° (or 1.74 radians) or better	**A1**	Do not ISW if go on to subtract from 180° (99.594…, 1.738...) Accept 260.4°
		3	

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td>8(c)</td><td>State correct vector equations for <i>AB</i> and <i>CD</i> in any form,
e.g. $(\mathbf{r}=)\begin{pmatrix}2\\1\\5\end{pmatrix}+\lambda\begin{pmatrix}2\\-2\\-4\end{pmatrix}$ and $(\mathbf{r}=)\begin{pmatrix}1\\1\\2\end{pmatrix}+\mu\begin{pmatrix}2\\1\\1\end{pmatrix}$</td><td>B1ft</td><td>Follow their $\overrightarrow{AB}$ and $\overrightarrow{CD}$
Alternative: $(\mathbf{r}=)\begin{pmatrix}4\\-1\\1\end{pmatrix}+\lambda\begin{pmatrix}2\\-2\\-4\end{pmatrix}$ and $(\mathbf{r}=)\begin{pmatrix}3\\2\\3\end{pmatrix}+\mu\begin{pmatrix}2\\1\\1\end{pmatrix}$</td></tr>
<tr><td></td><td>Equate at least two pairs of components of their lines and solve for λ or for μ</td><td>M1</td><td></td></tr>
<tr><td></td><td>Obtain correct pair of values from correct equations</td><td>A1</td><td>Alternatives when taking A or B as point on line
<table>
<tr><th><i>A</i></th><th>λ</th><th>μ</th><th></th><th><i>B</i></th><th>λ</th><th>μ</th><th></th></tr>
<tr><td>ij</td><td>$-\frac{1}{6}$</td><td>$\frac{1}{3}$</td><td>$\frac{17}{3}\neq\frac{7}{3}$</td><td>ij</td><td>$-\frac{7}{6}$</td><td>$-\frac{2}{3}$</td><td>$\frac{17}{3}\neq\frac{7}{3}$</td></tr>
<tr><td>ik</td><td>$\frac{1}{2}$</td><td>1</td><td>$0\neq2$</td><td>ik</td><td>$-\frac{1}{2}$</td><td>0</td><td>$0\neq2$</td></tr>
<tr><td>jk</td><td>$\frac{3}{2}$</td><td>-3</td><td>$5\neq-5$</td><td>jk</td><td>$\frac{1}{2}$</td><td>-4</td><td>$5\neq-5$</td></tr>
</table></td></tr>
<tr><td></td><td>Verify that all three equations are not satisfied and that the lines do not intersect</td><td>A1</td><td>CWO with conclusion
e.g. $\frac{17}{3}\neq\frac{7}{3}$ or $\frac{17}{3}=\frac{7}{3}$ is inconsistent or equivalent</td></tr>
<tr><td></td><td></td><td>4</td><td></td></tr>
</table>

Question	Answer	Marks	Guidance
9(a)	State or imply the form $\dfrac{A}{3x+2}+\dfrac{Bx+C}{x^2+4}$	**B1**	
	Use a correct method for finding a constant	**M1**	
	Obtain one of $A = 3$, $B = -1$, $C = 3$	**A1**	
	Obtain a second value	**A1**	
	Obtain the third value	**A1**	
		5	
9(b)	Integrate and obtain $\ln(3x+2)\ldots$	**B1 FT**	The FT is on A
	State a term of the form $k\ln\left(x^2+4\right)$.	**M1**	From $\int \dfrac{\lambda x}{x^2+4}\mathrm{d}x$
	$\ldots-\dfrac{1}{2}\ln\left(x^2+4\right)\ldots$	**A1 FT**	The FT is on B
	$\ldots+\dfrac{3}{2}\tan^{-1}\dfrac{x}{2}$	**B1 FT**	The FT is on C
	Substitute limits correctly in an integral with at least two terms of the form $a\ln(3x+2)$, $b\ln\left(x^2+4\right)$ and $c\ \tan^{-1}\left(\dfrac{x}{2}\right)$, and subtract in correct order	**M1**	Using terms that have been obtained correctly from completed integrals
	Obtain answer $\dfrac{3}{2}\ln 2+\dfrac{3}{8}\pi$, or exact 2-term equivalent	**A1**	
		6	

Question	Answer	Marks	Guidance
10(a)	Use correct product rule	**M1**	
	Obtain correct derivative in any form	**A1**	e.g. $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{2\sqrt{x}}\cos x-\sqrt{x}\sin x$. Accept in a or in x
	Equate derivative to zero and obtain $\tan a=\frac{1}{2a}$	**A1**	Obtain given answer from correct working**.** The question says 'show that ..' so there should be an intermediate step e.g. $\cos x=2x\sin x$. Allow $\tan x=\frac{1}{2x}$
		3	
10(b)	Use the iterative process correctly at least once (get one value and go on to use it in a second use of the formula)	**M1**	Must be working in radians Degrees gives 1, 12.6039, 5.4133, ... M0
	Obtain final answer 3.29	**A1**	Clear conclusion
	Show sufficient iterations to at least 4 d.p.to justify 3.29, or show there is a sign change in the interval (3.285, 3.295)	**A1**	$3, 3.3067, 3.2917, 3.2923$ Allow more than 4d.p. Condone truncation.
		3	

Question	Answer	Marks	Guidance
10(c)	State or imply the indefinite integral for the volume is $\pi\int\left(\sqrt{x}\cos x\right)^2 \mathrm{d}x$	**B1**	[If π omitted, or 2π or $\frac{1}{2}\pi$ used, give B0 and follow through. 4/6 available]
	Use correct cos 2*A* formula, commence integration by parts and reach $x(ax+b\sin 2x)\pm\int ax+b\sin 2x\mathrm{d}x$	***M1**	Alternative: $\frac{x^2}{4}+\frac{x}{4}\sin 2x-\int\frac{1}{4}\sin 2x\mathrm{d}x$
	Obtain $x(\frac{1}{2}x+\frac{1}{4}\sin 2x)-\int\frac{1}{2}x+\frac{1}{4}\sin 2x\,\mathrm{d}x$, or equivalent	**A1**	
	Complete integration and obtain $\frac{1}{4}x^2+\frac{1}{4}x\sin 2x+\frac{1}{8}\cos 2x$	**A1**	OE
	Substitute limits $x = 0$ and $x = \frac{1}{2}\pi$, having integrated twice	**DM1**	$\frac{\pi}{2}\left[\frac{\pi^2}{8}+0-\frac{1}{4}-0-0-\frac{1}{4}\right]$
	Obtain answer $\frac{1}{16}\pi\left(\pi^2-4\right)$, or exact equivalent	**A1**	CAO
		6	

Cambridge Assessment International Education

Cambridge International A Level

MATHEMATICS **9709/33**

Paper 3 Pure Mathematics 3 **October/November 2020**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2020 series for most Cambridge IGCSE™, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

This document consists of **21** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

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Question	Answer	Marks	Guidance
1	Make a recognisable sketch graph of $y = 2\|x-3\|$ and the line $y = 2 - 5x$	**B1**	Need to see correct V at $x = 3$, roughly symmetrical, $x = 3$ stated, domain at least $(-2, 5)$.
	Find x-coordinate of intersection with $y = 2 - 5x$	**M1**	Find point of intersection with $y = 2\|x - 3\|$ or solve $2 - 5x$ with $2(x-3)$ or $-2(x-3)$
	Obtain $x = -\frac{4}{3}$	**A1**	
	State final answer $x < -\frac{4}{3}$	**A1**	Do not accept $x < -1.33$ [Do not condone $\leqslant$ for $<$ in the final answer.]
	Alternative method for question 1		
	State or imply non-modular inequality/equality $(2-5x)^2 >, \geqslant, =, 2^2(x-3)^2$, or corresponding quadratic equation, or pair of linear equations $(2-5x) >, \geqslant, =, \pm 2(x-3)$	**B1**	Two correct linear equations only
	Make reasonable attempt at solving a 3-term quadratic, or solve one linear equation, or linear inequality for x	**M1**	$21x^2 + 4x - 32 = (3x+4)(7x-8) = 0$ $2 - 5x$ or $-(2-5x)$ with $2(x-3)$ or $-2(x-3)$
	Obtain critical value $x = -\frac{4}{3}$	**A1**	
	State final answer $x < -\frac{4}{3}$	**A1**	Do not accept $x < -1.33$ [Do not condone $\leqslant$ for $<$ in the final answer.]
		4	

Question	Answer	Marks	Guidance
2	Show a circle with centre the origin and radius 2	**B1**	
	Show the point representing 1 – i	**B1**	
	Show a circle with centre 1 – i and radius 1	**B1 FT**	The FT is on the position of 1 – i.
	Shade the appropriate region	**B1 FT**	The FT is on the position of 1 – i. Shaded region outside circle with centre the origin and radius 2 and inside circle with centre $\pm 1 \pm$ i and radius 1
		4	

Question	Answer	Marks	Guidance
3	State or imply $\dfrac{dx}{d\theta} = 2\sin 2\theta$ or $\dfrac{dy}{d\theta} = 2 + 2\cos 2\theta$	**B1**	
	Use $\dfrac{dy}{dx} = \dfrac{dy}{d\theta} \div \dfrac{dx}{d\theta}$	**M1**	
	Obtain correct answer $\dfrac{dy}{dx} = \dfrac{2+2\cos 2\theta}{2\sin 2\theta}$	**A1**	OE
	Use correct double angle formulae	**M1**	
	Obtain the given answer correctly $\dfrac{dy}{dx} = \cot\theta$	**A1**	AG. Must have simplified numerator in terms of $\cos\theta$.
	Alternative method for question 3		
	Start by using both correct double angle formulae e.g. $x = 3 - (2\cos^2\theta - 1)$, $y = 2\theta + 2\sin\theta\cos\theta$	**M1**	
	$\dfrac{dx}{d\theta}$ or $\dfrac{dy}{d\theta}$	**B1**	
	$\dfrac{dy}{dx} = \dfrac{\left(2+2\left(\cos^2\theta - \sin^2\theta\right)\right)}{4\cos\theta\sin\theta}$	**M1 A1**	
	Simplify to given answer correctly $\dfrac{dy}{dx} = \cot\theta$	**A1**	AG

Question	Answer	Marks	Guidance
3	**Alternative method for question 3**		
	Set $= 2\theta$. State $\frac{dx}{dt} = \sin t$ or $\frac{dy}{dt} = 1 + \cos t$	**B1**	
	Use $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$	**M1**	
	Obtain correct answer $\frac{dy}{dx} = \frac{1 + \cos t}{\sin t}$	**A1**	OE
	Use correct double angle formulae	**M1**	
	Obtain the given answer correctly $\frac{dy}{dx} = \cot\theta$	**A1**	
		5	
4	State or imply $\log_{10} 10 = 1$	**B1**	$\log_{10} 10^{-1} = -1$
	Use law of the logarithm of a power, product or quotient	**M1**	
	Obtain a correct equation in any form, free of logs	**A1**	e.g. $(2x + 1)/(x + 1)^2 = 10^{-1}$ or $10(2x + 1)/(x + 1)^2 = 10^0$ or 1 or $x^2 + 2x + 1 = 20x + 10$
	Reduce to $x^2 - 18x - 9 = 0$, or equivalent	**A1**	
	Solve a 3-term quadratic	**M1**	
	Obtain final answers $x = 18.487$ and $x = -0.487$	**A1**	Must be 3 d.p. Do not allow rejection.
		6	

Question	Answer	Marks	Guidance
5(a)	Sketch a relevant graph, e.g. $y = \operatorname{cosec} x$	**B1**	cosec x, U shaped, roughly symmetrical about $x = \frac{\pi}{2}$, $y\left(\frac{\pi}{2}\right) = 1$ and domain at least $\left(\frac{\pi}{6}, \frac{5\pi}{6}\right)$.
	Sketch a second relevant graph, e.g. $y = 1 + e^{-\frac{1}{2}x}$, and justify the given statement	**B1**	Exponential graph needs $y(0) = 2$, negative gradient, always increasing, and $y(\pi) > 1$ Needs to mark intersections with dots, crosses, or say roots at points of intersection, or equivalent
		2	
5(b)	Use the iterative formula correctly at least twice	**M1**	2, 2.3217, 2.2760, 2.2824… Need to see 2 iterations and following value inserted correctly
	Obtain final answer 2.28	**A1**	Must be supported by iterations
	Show sufficient iterations to at least 4 d.p. to justify 2.28 to 2 d.p., or show there is a sign change in the interval (2.275, 2.285)	**A1**	
		3	

Question	Answer	Marks	Guidance
6(a)	State $R = \sqrt{15}$	**B1**	
	Use trig formulae to find α	**M1**	$\frac{\sin\alpha}{\cos\alpha} = \frac{3}{\sqrt{6}}$ with no error seen or $\tan\alpha = \frac{3}{\sqrt{6}}$ quoted then allow M1
	Obtain $\alpha = 50.77$	**A1**	Must be 2 d.p. If radians 0.89 A0 MR
		3	
6(b)	Evaluate $\beta = \cos^{-1}\frac{2.5}{\sqrt{15}}$ (49.797° to 4 d.p.)	**B1 FT**	The FT is on incorrect R. $\frac{x}{3} = \beta - \alpha$ [–2.9° and –301.7°]
	Use correct method to find a value of $\frac{x}{3}$ in the interval	**M1**	Needs to use $\frac{x}{3}$
	Obtain answer rounding to $x = 301.6°$ to $301.8°$	**A1**	
	Obtain second answer rounding to $x = 2.9(0)°$ to $2.9(2)°$ and no others in the interval	**A1**	
		4	

Question	Answer	Marks	Guidance
7(a)	Substitute $-1+\sqrt{5}\,\mathrm{i}$ in the equation and attempt expansions of x^2 and x^3	**M1**	All working must be seen. Allow M1 if small errors in $1-2\sqrt{5}\mathrm{i}-5$ or $1-\sqrt{5}\mathrm{i}-\sqrt{5}\mathrm{i}-5$ and $4-2\sqrt{5}\mathrm{i}+10$ or $4-4\sqrt{5}\mathrm{i}+2\sqrt{5}\mathrm{i}+10$
	Use $\mathrm{i}^2=-1$ correctly at least once	**M1**	$1-5$ or $4+10$ seen
	Complete the verification correctly	**A1**	$2\left(14-2\sqrt{5}\mathrm{i}\right)+\left(-4-2\sqrt{5}\mathrm{i}\right)+6\left(-1+\sqrt{5}\mathrm{i}\right)-18=0$
		3	
7(b)	State second root $-1-\sqrt{5}\mathrm{i}$	**B1**	
	Carry out a complete method for finding a quadratic factor with zeros $-1+\sqrt{5}\mathrm{i}$ and $-1-\sqrt{5}\mathrm{i}$	**M1**	
	Obtain x^2+2x+6	**A1**	
	Obtain root $x=\frac{3}{2}$	**A1**	OE
	Alternative method for question 7(b)		
	State second root $-1-\sqrt{5}\mathrm{i}$	**B1**	
	$(x+1-\sqrt{5}\mathrm{i})(x+1+\sqrt{5}\mathrm{i})(2x+a)=2x^3+x^2+6x-18$	**M1**	
	$(1-\sqrt{5}\,\mathrm{i})(1+\sqrt{5}\,\mathrm{i})\,a=-18$	**A1**	
	$6a=-18$ $a=-3$ leading to $x=\frac{3}{2}$	**A1**	OE

Question	Answer	Marks	Guidance
7(b)	**Alternative method for question 7(b)**		
	State second root $-1-\sqrt{5}\mathrm{i}$	**B1**	
	POR = 6 SOR = – 2	**M1**	
	Obtain $x^2 + 2x + 6$	**A1**	
	Obtain root $x = \frac{3}{2}$	**A1**	OE
	Alternative method for question 7(b)		
	State second root $-1-\sqrt{5}\mathrm{i}$	**B1**	
	POR $\left(-1-\sqrt{5}\mathrm{i}\right)\left(-1+\sqrt{5}\mathrm{i}\right)a = 9$	**M1 A1**	
	Obtain root $x = \frac{3}{2}$	**A1**	OE
	Alternative method for question 7(b)		
	State second root $-1-\sqrt{5}\mathrm{i}$	**B1**	
	SOR $\left(-1-\sqrt{5}\mathrm{i}\right)+\left(-1+\sqrt{5}\mathrm{i}\right)+a = -\frac{1}{2}$	**M1 A1**	
	Obtain root $x = \frac{3}{2}$	**A1**	OE
		4	

Question	Answer	Marks	Guidance
8	Separate variables correctly and attempt integration of at least one side	**B1**	$\frac{1}{y}\mathrm{d}y = \frac{1-2x^2}{x}\mathrm{d}x$
	Obtain term $\ln y$	**B1**	
	Obtain terms $\ln x - x^2$	**B1**	
	Use $x = 1$, $y = 1$ to evaluate a constant, or as limits, in a solution containing at least 2 terms of the form $a\ln y$, $b\ln x$ and cx^2	**M1**	The 2 terms of required form must be from correct working e.g. $\ln y = \ln x - x^2 + 1$
	Obtain correct solution in any form	**A1**	
	Rearrange and obtain $y = xe^{1-x^2}$	**A1**	OE
		6	

Question	Answer	Marks	Guidance
9(a)	State or imply the form $\dfrac{A}{1-x}+\dfrac{B}{2+3x}+\dfrac{C}{(2+3x)^2}$	**B1**	
	Use a correct method for finding a coefficient	**M1**	
	Obtain one of $A=1$, $B=-1$, $C=6$	**A1**	
	Obtain a second value	**A1**	
	Obtain the third value	**A1**	In the form $\dfrac{A}{1-x}+\dfrac{Dx+E}{(2+3x)^2}$, where $A=1$, $D=-3$ and $E=4$ can score B1 M1 A1 A1 A1 as above.
		5	

Question	Answer	Marks	Guidance
9(b)	Use a correct method to find the first two terms of the expansion of $(1-x)^{-1}$, $(2+3x)^{-1}$, $\left(1+\frac{3}{2}x\right)^{-1}$, $(2+3x)^{-2}$ or $\left(1+\frac{3}{2}x\right)^{-2}$	M1	Symbolic coefficients are not sufficient for the M1 $A\left[\frac{1+(-1)(-x)+(-1)(-2)(-x)^2}{2\ldots}\right]$ $A=1$ $\frac{B}{2}\left[\frac{1+(-1)\left(\frac{3x}{2}\right)+(-1)(-2)\left(\frac{3x}{2}\right)^2}{2\ldots}\right]$ $B=1$ $\frac{C}{4}\left[\frac{1+(-2)\left(\frac{3x}{2}\right)+(-2)(-3)\left(\frac{3x}{2}\right)^2}{2\ldots}\right]$ $C=6$
	Obtain correct un-simplified expansions up to the term in of each partial fraction	A1 FT + A1 FT + A1 FT	$\left(1+x+x^2\right)+\left(-\frac{1}{2}+\left(\frac{3}{4}\right)x-\left(\frac{9}{8}\right)x^2\right)$ $+\left(\frac{6}{4}-\left(\frac{18}{4}\right)x+\left(\frac{81}{8}\right)x^2\right)$ [The FT is on *A*, *B*, *C*] $\left(1-\frac{1}{2}+\frac{6}{4}\right)+\left(1+\frac{3}{4}-\frac{18}{4}\right)x+\left(1-\frac{9}{8}+\frac{81}{8}\right)x^2$
	Obtain final answer $2-\frac{11}{4}x+10x^2$, or equivalent	A1	Allow unsimplified fractions $\frac{(Dx+E)}{4}\left[\frac{1+(-2)\left(\frac{3x}{2}\right)+(-2)(-3)\left(\frac{3x}{2}\right)^2}{2\ldots}\right]$ $D=-3$, $E=4$ The FT is on *A*, *D*, *E*.
		5	

Question	Answer	Marks	Guidance
10(a)	Use correct product or quotient rule	*M1	$\frac{dy}{dx}=\left(-\frac{1}{2}\right)(2-x)e^{-\frac{1}{2}x}-e^{-\frac{1}{2}x}$ M1 requires at least one of derivatives correct
	Obtain correct derivative in any form	A1	
	Equate derivative to zero and solve for x	DM1	
	Obtain $x = 4$	A1	ISW
	Obtain $y = -2e^{-2}$, or exact equivalent	A1	
		5	

Question	Answer	Marks	Guidance
10(b)	Commence integration and reach $a(2-x)\mathrm{e}^{-\frac{1}{2}x}+b\int \mathrm{e}^{-\frac{1}{2}x}\,\mathrm{d}x$	***M1**	Condone omission of dx $-2(2-x)\mathrm{e}^{-\frac{1}{2}x}+4\mathrm{e}^{-\frac{1}{2}x}$ or $2x\mathrm{e}^{-\frac{1}{2}x}$
	Obtain $-2(2-x)\mathrm{e}^{-\frac{1}{2}x}-2\int \mathrm{e}^{-\frac{1}{2}x}\,\mathrm{d}x$	**A1**	OE
	Complete integration and obtain $2x\mathrm{e}^{-\frac{1}{2}x}$	**A1**	OE
	Use correct limits, $x = 0$ and $x = 2$, correctly, having integrated twice	**DM1**	Ignore omission of zeros and allow max of 1 error
	Obtain answer $4\mathrm{e}^{-1}$, or exact equivalent	**A1**	ISW
	Alternative method for question 10(b)		
	$\dfrac{\mathrm{d}\left(2x\mathrm{e}^{-\frac{1}{2}x}\right)}{\mathrm{d}x}=2\mathrm{e}^{-\frac{1}{2}x}-x\mathrm{e}^{-\frac{1}{2}x}$	***M1 A1**	
	$\therefore 2x\mathrm{e}^{-\frac{1}{2}x}$	**A1**	
	Use correct limits, $x = 0$ and $x = 2$, correctly, having integrated twice	**DM1**	Ignore omission of zeros and allow max of 1 error
	Obtain answer $4\mathrm{e}^{-1}$, or exact equivalent	**A1**	ISW
		5	

Question	Answer	Marks	Guidance
11(a)	Express general point of at least one line correctly in component form, i.e. $(1 + a\lambda, 2 + 2\lambda, 1 - \lambda)$ or $(2 + 2\mu, 1 - \mu, -1 + \mu)$	**B1**	
	Equate at least two pairs of corresponding components and solve for λ or for μ	**M1**	May be implied $1 + a\lambda = 2 + 2\mu \;\; 2 + 2\lambda = 1 - \mu \;\; 1 - \lambda = -1 + \mu$
	Obtain $\lambda = -3$ or $\mu = 5$	**A1**	
	Obtain $a = -\dfrac{11}{3}$	**A1**	Allow $a = -3.667$
	State that the point of intersection has position vector $12\mathbf{i} - 4\mathbf{j} + 4\mathbf{k}$	**A1**	Allow coordinate form $(12, -4, 4)$
		5	

Question	Answer	Marks	Guidance
11(b)	Use correct process for finding the scalar product of direction vectors for the two lines	**M1**	$(a, 2, -1) \cdot (2, -1, 1) = 2a - 2 - 1$ or $2a - 3$
	Using the correct process for the moduli, divide the scalar product by the product of the moduli and equate the result to $\pm\frac{1}{6}$	***M1**	
	State a correct equation in a in any form, e.g. $\frac{2a-2-1}{\sqrt{6}\sqrt{(a^2+5)}} = \pm\frac{1}{6}$	**A1**	
	Solve for a	**DM1**	Solve 3-term quadratic for a having expanded $(2a - 3)^2$ to produce 3 terms e.g. $36(2a - 3)^2 = 6(a^2 + 5)$ $138a^2 - 432a + 294 = 0$ $23a^2 - 72a + 49 = 0$ $(23a - 49)(a - 1) = 0$
	Obtain $a = 1$	**A1**	
	Obtain $a = \frac{49}{23}$	**A1**	Allow $a = 2.13$

Question	Answer	Marks	Guidance
11(b)	**Alternative method for question 11(b)**		
	$\cos(\theta) = [\lvert a^2 + 2^2 + (-1)^2\rvert^2 + \lvert 2^2 + (-1)^2 + 1^2\rvert^2$ $- \lvert(a-2)^2 + 3^2 + (-2)^2\rvert^2] / [2\lvert a^2 + 2^2 + (-1)^2\rvert . \lvert 2^2 + (-1)^2 + 1^2\rvert]$	**M1**	Use of cosine rule. Must be correct vectors.
	Equate the result to $\pm\frac{1}{6}$	***M1** **A1**	Allow M1* here for any two vectors
	Solve for *a*	**DM1**	Solve 3-term quadratic for *a* having expanded $(2a-3)^2$ to produce 3 terms e.g. $36(2a-3)^2 = 6(a^2+5)$ $138a^2 - 432a + 294 = 0$ $23a^2 - 72a + 49 = 0$ $(23a-49)(a-1) = 0$
	Obtain $a = 1$	**A1**	
	Obtain $a = \frac{49}{23}$	**A1**	Allow $a = 2.13$
		6	

Cambridge Assessment International Education

Cambridge International A Level

MATHEMATICS	**9709/31**
Paper 3 Pure Mathematics 3	**May/June 2020**
MARK SCHEME
Maximum Mark: 75

Published

Students did not sit exam papers in the June 2020 series due to the Covid-19 global pandemic.

This mark scheme is published to support teachers and students and should be read together with the question paper. It shows the requirements of the exam. The answer column of the mark scheme shows the proposed basis on which Examiners would award marks for this exam. Where appropriate, this column also provides the most likely acceptable alternative responses expected from students. Examiners usually review the mark scheme after they have seen student responses and update the mark scheme if appropriate. In the June series, Examiners were unable to consider the acceptability of alternative responses, as there were no student responses to consider.

Mark schemes should usually be read together with the Principal Examiner Report for Teachers. However, because students did not sit exam papers, there is no Principal Examiner Report for Teachers for the June 2020 series.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the June 2020 series for most Cambridge IGCSE™ and Cambridge International A & AS Level components, and some Cambridge O Level components.

This document consists of **13** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics-Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate’s intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M	Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
A	Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
B	Mark for a correct result or statement independent of method marks.
DM or **DB**	When a part of a question has two or more “method” steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
FT	Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks
1	Use law of the logarithm of a product or power	**M1**
	Obtain a correct linear inequality in any form, e.g. $\ln 2 + (1 - 2x)\ln 3 < x \ln 5$	**A1**
	Solve for x	**M1**
	Obtain $x > \dfrac{\ln 6}{\ln 45}$	**A1**
		4

Question	Answer	Marks
2(a)	State a correct unsimplified version of the x or x^2 term of the expansion of $(2 - 3x)^{-2}$ or $\left(1 - \dfrac{3}{2}x\right)^{-2}$	**M1**
	State correct first term $\dfrac{1}{4}$	**B1**
	Obtain the next two terms $\dfrac{3}{4}x + \dfrac{27}{16}x^2$	**A1 + A1**
		4
2(b)	State answer $\lvert x\rvert < \dfrac{2}{3}$, or equivalent	**B1**
		1

Question	Answer	Marks
3	Use tan $(A \pm B)$ formula and obtain an equation in tan θ	**M1**
	Using $\tan 60° = \sqrt{3}$, obtain a horizontal equation in tan θ in any correct form	**A1**
	Reduce the equation to $3\tan^2\theta + 4\tan\theta - 1 = 0$, or equivalent	**A1**
	Solve a 3-term quadratic for tan θ	**M1**
	Obtain a correct answer, e.g. 12.1°	**A1**
	Obtain a second correct answer, e.g. 122.9°, and no others in the given interval	**A1**
		6

Question	Answer	Marks
4(a)	Use product rule	**M1**
	Obtain derivative in any correct form e.g. $2e^{2x}(\sin x + 3\cos x) + e^{2x}(\cos x - 3\sin x)$	**A1**
	Equate derivative to zero and obtain an equation in one trigonometric ratio	**M1**
	Obtain $x = 1.43$ only	**A1**
		4
4(b)	Use a correct method to determine the nature of the stationary point e.g. $x = 1.42,\ y' = 0.06e^{2.84} > 0$ $x = 1.44,\ y' = -0.07e^{2.88} < 0$	**M1**
	Show that it is a maximum point	**A1**
		2

Question	Answer	Marks
5(a)	Commence division and reach quotient of the form $2x+k$	**M1**
	Obtain quotient $2x-1$	**A1**
	Obtain remainder 6	**A1**
		3
5(b)	Obtain terms x^2-x (**FT** on quotient of the form $2x+k$)	**B1FT**
	Obtain term of the form $a\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)$	**M1**
	Obtain term $\frac{6}{\sqrt{3}}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)$ (**FT** on a constant remainder)	**A1FT**
	Use $x=1$ and $x=3$ as limits in a solution containing a term of the form $a\tan^{-1}(bx)$	**M1**
	Obtain final answer $\frac{1}{\sqrt{3}}\pi+6$, or exact equivalent	**A1**
		5

Question	Answer	Marks
6(a)	State or imply $AT = r\tan x$ or $BT = r\tan x$	**B1**
	Use correct area formula and form an equation in r and x	**M1**
	Rearrange in the given form	**A1**
		3
6(b)	Calculate the values of a relevant expression or pair of expressions at $x = 1$ and $x = 1.4$	**M1**
	Complete the argument correctly with correct calculated values	**A1**
		2
6(c)	Use the iterative formula correctly at least once	**M1**
	Obtain final answer 1.35	**A1**
	Show sufficient iterations to 4 d.p. to justify 1.35 to 2 d.p. or show there is a sign change in the interval (1.345, 1.355)	**A1**
		3

Question	Answer	Marks
7(a)	Use quotient or product rule	**M1**
	Obtain derivative in any correct form e.g. $\dfrac{-\sin x(1+\sin x)-\cos x(\cos x)}{(1+\sin x)^2}$	**A1**
	Use Pythagoras to simplify the derivative	**M1**
	Justify the given statement	**A1**
		4

Question	Answer	Marks
7(b)	State integral of the form $a\ln(1+\sin x)$	***M1**
	State correct integral $\ln(1+\sin x)$	**A1**
	Use limits correctly	**DM1**
	Obtain answer $\ln\frac{4}{3}$	**A1**
		4
8(a)	State $\frac{\mathrm{d}y}{\mathrm{d}x}=k\frac{y}{x\sqrt{x}}$, or equivalent	**B1**
	Separate variables correctly and attempt integration of at least one side	**M1**
	Obtain term $\ln y$, or equivalent	**A1**
	Obtain term $-2k\frac{1}{\sqrt{x}}$, or equivalent	**A1**
	Use given coordinates to find k or a constant of integration c in a solution containing terms of the form $a\ln y$ and $\frac{b}{\sqrt{x}}$, where $ab\neq 0$	**M1**
	Obtain $k=1$ and $c=2$	**A1 + A1**
	Obtain final answer $y=\exp\left(-\frac{2}{\sqrt{x}}+2\right)$, or equivalent	**A1**
		8

Question	Answer	Marks
8(b)	State that y approaches e^2 (FT *their c* in part **(a)** of the correct form)	**B1FT**
		1
9(a)	State $\overrightarrow{AB}\left(\text{or } \overrightarrow{BA}\right)$ and $\overrightarrow{BC}\left(\text{or } \overrightarrow{CB}\right)$ in vector form	**B1**
	Calculate their scalar product	**M1**
	Show product is zero and confirm angle ABC is a right angle	**A1**
		3
9(b)	Use correct method to calculate the lengths of AB and BC	**M1**
	Show that $AB = BC$ and the triangle is isosceles	**A1**
		2
9(c)	State a correct equation for the line through B and C, e.g. $\mathbf{r} = \mathbf{i} + \mathbf{j} + \mathbf{k} + \lambda(2\mathbf{i} + \mathbf{j} + 2\mathbf{k})$ or $\mathbf{r} = 3\mathbf{i} + 2\mathbf{j} + 3\mathbf{k} + \mu\left(-2\mathbf{i} - \mathbf{j} - 2\mathbf{k}\right)$	**B1**
	Taking a general point of BC to be P, form an equation in λ by **either** equating the scalar product of $\overrightarrow{OP}$ and $\overrightarrow{BC}$ to zero, **or** applying Pythagoras to triangle OBP (or OCP), *or* setting the derivative of $\left\lvert\overrightarrow{OP}\right\rvert$ to zero	**M1**
	Solve and obtain $\lambda = -\dfrac{5}{9}$	**A1**
	Obtain answer $\dfrac{1}{3}\sqrt{2}$, or equivalent	**A1**

Question	Answer	Marks
	Alternative method for question 9(c)	
	Use a scalar product to find the projection *CN* (or *BN*) of *OC* (or *OB*) on *BC*	**M1**
	Obtain answer $CN = \frac{5}{3}\left(\text{or } BN = \frac{14}{3}\right)$	**A1**
	Use Pythagoras to find *ON*	**M1**
	Obtain answer $\frac{1}{3}\sqrt{2}$, or equivalent	**A1**
		4

Question	Answer	Marks
10(a)(i)	Multiply numerator and denominator by $a - 2\text{i}$, or equivalent	**M1**
	Use $\text{i}^2 = -1$ at least once	**A1**
	Obtain answer $\frac{6}{a^2+4} + \frac{3ai}{a^2+4}$	**A1**
		3
10(a)(ii)	Either state that $\arg u = -\frac{1}{3}\pi$ or express u^* in terms of a (FT on u)	**B1**
	Use correct method to form an equation in a	**M1**
	Obtain answer $a = -2\sqrt{3}$	**A1**
		3

Question	Answer	Marks
10(b)(i)	Show the perpendicular bisector of points representing 2i and 1 + i	**B1**
	Show the point representing 2 + i	**B1**
	Show a circle with radius 2 and centre 2 + i (FT on the position of the point for 2 + i)	**B1FT**
	Shade the correct region	**B1**
		4
10(b)(ii)	State or imply the critical point 2 + 3i	**B1**
	Obtain answer 56.3° or 0.983 radians	**B1**
		2

Cambridge Assessment
International Education

Cambridge International A Level

MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **May/June 2020**

MARK SCHEME

Maximum Mark: 75

Published

Students did not sit exam papers in the June 2020 series due to the Covid-19 global pandemic.

This mark scheme is published to support teachers and students and should be read together with the question paper. It shows the requirements of the exam. The answer column of the mark scheme shows the proposed basis on which Examiners would award marks for this exam. Where appropriate, this column also provides the most likely acceptable alternative responses expected from students. Examiners usually review the mark scheme after they have seen student responses and update the mark scheme if appropriate. In the June series, Examiners were unable to consider the acceptability of alternative responses, as there were no student responses to consider.

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This document consists of **13** printed pages.

Generic Marking Principles

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GENERIC MARKING PRINCIPLE 1:

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- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
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2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
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4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M	Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
A	Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
B	Mark for a correct result or statement independent of method marks.
DM or **DB**	When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
FT	Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
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CWO	Correct Working Only
ISW	Ignore Subsequent Working
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SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks
1	Commence division and reach partial quotient $3x^2 + kx$	**M1**
	Obtain quotient $3x^2 + 2x - 1$	**A1**
	Obtain remainder $2x - 5$	**A1**
		4

Question	Answer	Marks
2	State or imply $2 \ln y = \ln A + kx$	**B1**
	Substitute values of $\ln y$ and x, or equate gradient of line to k, and solve for k	**M1**
	Obtain $k = 0.80$	**A1**
	Solve for $\ln A$	**M1**
	Obtain $A = 3.31$	**A1**
	Alternative method for question 2	
	Obtain two correct equations in y and x by substituting y- and x- values in the given equation	**B1**
	Solve for k	**M1**
	Obtain $k = 0.80$	**A1**
	Solve for A	**M1**
	Obtain $A = 3.31$	**A1**
		5

Question	Answer	Marks
3	Commence integration and reach $ax^{\frac{5}{2}}\ln x + b\int x^{\frac{5}{2}}.\frac{1}{x}\,\mathrm{d}x$	M1*
	Obtain $\frac{2}{5}x^{\frac{5}{2}}\ln x - \frac{2}{5}\int x^{\frac{5}{2}}.\frac{1}{x}\,\mathrm{d}x$	A1
	Complete the integration and obtain $\frac{2}{5}x^{\frac{5}{2}}\ln x - \frac{4}{25}x^{\frac{5}{2}}$, or equivalent	A1
	Use limits correctly, having integrated twice e.g $\frac{2}{5}\times 32\ln 4 - \frac{4}{25}\times 32 - \left(\frac{2}{5}\times 0\right) + \frac{4}{25}$	DM1
	Obtain answer $\frac{128}{5}\ln 2 - \frac{124}{25}$, or exact equivalent	A1
		5

Question	Answer	Marks
4	Use correct product rule	M1
	Obtain correct derivative in any form, e.g. $-\sin x\sin 2x + 2\cos x\cos 2x$	A1
	Use double angle formula to express derivative in terms of $\sin x$ and $\cos x$	M1
	Equate derivative to zero and obtain an equation in one trig function	M1
	Obtain $3\sin 2x = 1$, or $3\cos 2x = 2$ or $2\tan 2x = 1$	A1
	Solve and obtain $x = 0.615$	A1
		6

Question	Answer	Marks
5(a)	State $R = \sqrt{7}$	**B1**
	Use trig formulae to find α	**M1**
	Obtain $\alpha = 57.688°$	**A1**
		3
5(b)	Evaluate $\cos-1\left(\frac{1}{\sqrt{7}}\right)$ to at least 3 d.p. (67.792°) (**FT** is on *their R*)	**B1 FT**
	Use correct method to find a value of θ in the interval	**M1**
	Obtain answer, e.g. 5.1°	**A1**
	Obtain second answer, e.g.117.3°, only	**A1**
		4

6(a)	Use quotient or product rule	**M1**
	Obtain correct derivative in any form e.g. $\frac{\left(1+3x^4\right)-x\times12x^3}{\left(1+3x^4\right)^2}$	**A1**
	Equate derivative to zero and solve for x	**M1**
	Obtain answer 0.577	**A1**
		4

Question	Answer	Marks
6(b)	State or imply $\mathrm{d}u = 2\sqrt{3x}\,\mathrm{d}x$, or equivalent	**B1**
	Substitute for x and dx	**M1**
	Obtain integrand $\dfrac{1}{2\sqrt{3}\left(1+u^2\right)}$, or equivalent	**A1**
	State integral of the form $a\tan^{-1}u$ and use limits $u = 0$ and $u = \sqrt{3}$ (or $x = 0$ and $x = 1$) correctly	**M1**
	Obtain answer $\dfrac{\sqrt{3}}{18}\pi$, or exact equivalent	**A1**
		5

Question	Answer	Marks
7	Separate variables correctly and integrate at least one side	**B1**
	Obtain term $\ln(y-1)$	**B1**
	Carry out a relevant method to determine A and B such that $\frac{1}{(x+1)(x+3)} \equiv \frac{A}{x+1} + \frac{B}{x+3}$	**M1**
	Obtain $A = \frac{1}{2}$ and $B = -\frac{1}{2}$	**A1**
	Integrate and obtain terms $\frac{1}{2}\ln(x+1) - \frac{1}{2}\ln(x+3)$ $\frac{1}{2}\ln(x+1) - \frac{1}{2}\ln(x+3)$, or equivalent (**FT** is on A and B)	**A1 FT** **+ A1 FT**
	Use $x = 0$, $y = 2$ to evaluate a constant, or as limits in a solution containing terms of the form $a\ln(y-1)$, $b\ln(x+1)$ and $c\ln(x+3)$, where $abc \neq 0$	**M1**
	Obtain correct answer in any form	**A1**
	Obtain final answer $y = 1 + \sqrt{\left(\frac{3x+3}{x+3}\right)}$, or equivalent	**A1**
		9

Question	Answer	Marks
8(a)	Substitute and obtain a correct equation in x and y	**B1**
	Use $i^2 = -1$ and equate real and imaginary parts	**M1**
	Obtain two correct equations in x and y, e.g. $x - y = 3$ and $3x + y = 5$	**A1**
	Solve and obtain answer $z = 2 - i$	**A1**
		4
8(b)(i)	Show a point representing $2 + 2i$	**B1**
	Show a circle with radius 1 and centre not at the origin (**FT** is on the point representing the centre)	**B1 FT**
	Show the correct half line from 4i	**B1**
	Shade the correct region	**B1**
		4
8(b)(ii)	Carry out a complete method for finding the least value of Im z	**M1**
	Obtain answer $2 - \frac{1}{2}\sqrt{2}$, or exact equivalent	**A1**
		2

Question	Answer	Marks
9(a)	State $\cos p = \frac{k}{1+p}$	**B1**
	Differentiate both equations and equate derivatives at $x = p$	**M1**
	Obtain a correct equation in any form, e.g. $-\sin p = -\frac{k}{(1+p)^2}$	**A1**
	Eliminate k	**M1**
	Obtain the **given answer** showing sufficient working	**A1**
		5
9(b)	Use the iterative formula correctly at least once	**M1**
	Obtain final answer $p = 0.568$	**A1**
	Show sufficient iterations to justify 0.568 to 3 d.p., or show there is a sign change in the interval (0.5675, 0.5685)	**A1**
		3
9(c)	Use a correct method to find k	**M1**
	Obtain answer $k = 1.32$	**A1**
		2

Question	Answer	Marks
10(a)	State that the position vector of *M* is $3\mathbf{i} + \mathbf{j}$	**B1**
	Use a correct method to find the position vector of *N*	**M1**
	Obtain answer $\frac{10}{3}\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$	**A1**
	Use a correct method to form an equation for *MN*	**M1**
	Obtain correct answer in any form, e.g. $\mathbf{r} = 3\mathbf{i} + \mathbf{j} + \lambda\left(\frac{1}{3}\mathbf{i} + \mathbf{j} + 2\mathbf{k}\right)$	**A1**
		5
10(b)	State or imply $\mathbf{r} = \mu(2\mathbf{i} + 2\mathbf{j} + 3\mathbf{k})$ as equation for *OB*	**B1**
	Equate sufficient components of *MN* and *OB* and solve for λ or for μ	**M1**
	Obtain $\lambda = 3$ or $\mu = 2$ and position vector $4\mathbf{i} + 4\mathbf{j} + 6\mathbf{k}$ for *P*	**A1**
		3
10(c)	Carry out correct process for evaluating the scalar product of direction vectors for *OP* and *MP*, or equivalent	**M1**
	Using the correct process for the moduli, divide the scalar product by the product of the moduli and evaluate the inverse cosine of the result	**M1**
	Obtain answer 21.6°	**A1**
		3

Cambridge Assessment
International Education

Cambridge International A Level

MATHEMATICS **9709/33**

Paper 3 Pure Mathematics 3 **May/June 2020**

MARK SCHEME

Maximum Mark: 75

Published

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[Turn over

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WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks
1	State or imply non-modular inequality $(2x-1)^2 > 3^2(x+2)^2$, or corresponding quadratic equation, or pair of linear equations	**B1**
	Make reasonable attempt at solving a 3-term quadratic, or solve two linear equations for x	**M1**
	Obtain critical values $x = -7$ and $x = -1$	**A1**
	State final answer $-7 < x < -1$	**A1**
	Alternative method for question 1	
	Obtain critical value $x = -1$ from a graphical method, or by solving a linear equation or linear inequality	**B1**
	Obtain critical value $x = -7$ similarly	**B2**
	State final answer $-7 < x < -1$ [Do not condone $\leqslant$ for $<$ in the final answer.]	**B1**
		4

Question	Answer	Marks
2	Commence integration and reach $a(2-x)e^{-2x}+b\int e^{-2x}\,\mathrm{d}x$, or equivalent	**M1***
	Obtain $-\frac{1}{2}(2-x)e^{-2x}-\frac{1}{2}\int e^{-2x}\,\mathrm{d}x$, or equivalent	**A1**
	Complete integration and obtain $-\frac{1}{2}(2-x)e^{-2x}+\frac{1}{4}e^{-2x}$, or equivalent	**A1**
	Use limits correctly, having integrated twice	**DM1**
	Obtain answer $\frac{1}{4}\left(3-e^{-2}\right)$, or exact equivalent	**A1**
		5

Question	Answer	Marks
3(a)	Remove logarithms correctly and state $1 + e^{-x} = e^{-2x}$, or equivalent	**B1**
	Show equation is $u^2 + u - 1 = 0$, where $u = e^x$, or equivalent	**B1**
		2
3(b)	Solve a 3-term quadratic for u	**M1**
	Obtain root $\frac{1}{2}\left(-1+\sqrt{5}\right)$, or decimal in [0.61, 0.62]	**A1**
	Use correct method for finding x from a positive root	**M1**
	Obtain answer $x = -0.481$ only	**A1**
		4

Question	Answer	Marks
4(a)	Use the product rule	**M1**
	State or imply derivative of $\tan^{-1}(\frac{1}{2}x)$ is of the form $k/(4+x^2)$, where $k = 2$ or 4, or equivalent	**M1**
	Obtain correct derivative in any form, e.g. $\tan^{-1}\left(\frac{1}{2}x\right)+\frac{2x}{x^2+4}$, or equivalent	**A1**
		3
4(b)	State or imply y-coordinate is $\frac{1}{2}\pi$	**B1**
	Carry out a complete method for finding p, e.g. by obtaining the equation of the tangent and setting $x = 0$, or by equating the gradient at $x = 2$ to $\frac{\frac{1}{2}\pi - p}{2}$	**M1**
	Obtain answer $p = -1$	**A1**
		3

Question	Answer	Marks
5	Use tan 2*A* formula to express RHS in terms of tan θ	**M1**
	Use tan ($A \pm B$) formula to express LHS in terms of tan θ	**M1**
	Using tan 45° = 1, obtain a correct horizontal equation in any form	**A1**
	Reduce equation to $2\tan^2\theta + \tan\theta - 1 = 0$	**A1**
	Solve a 3-term quadratic and find a value of θ	**M1**
	Obtain answer $\theta = 26.6°$ and no other	**A1**
		6

Question	Answer	Marks
6(a)	Sketch a relevant graph, e.g. $y = x^5$	**B1**
	Sketch a second relevant graph, e.g. $y = x + 2$ and justify the given statement	**B1**
		2
6(b)	State a suitable equation, e.g. $x = \dfrac{4x^5 + 2}{5x^4 - 1}$	**B1**
	Rearrange this as $x^5 = 2 + x$ or commence working *vice versa*	**B1**
		2
6(c)	Use the iterative formula correctly at least once	**M1**
	Obtain final answer 1.267	**A1**
	Show sufficient iterations to 5 d.p. to justify 1.267 to 3 d.p., or show there is a sign change in the interval (1.2665, 1.2675)	**A1**
		3

Question	Answer	Marks
7(a)	State or imply the form $\frac{A}{2x-1}+\frac{B}{2x+1}$ and use a relevant method to find A or B	M1
	Obtain $A=1$, $B=-1$	A1
		2
7(b)	Square the result of part (**a**) and substitute the fractions of part (**a**)	M1
	Obtain the given answer correctly	A1
		2
7(c)	Integrate and obtain $-\frac{1}{2(2x-1)}-\frac{1}{2}\ln(2x-1)+\frac{1}{2}\ln(2x+1)-\frac{1}{2(2x+1)}$, or equivalent	B3, 2, 1, 0
	Substitute limits correctly	M1
	Obtain the given answer correctly	A1
		5

Question	Answer	Marks
8(a)	State or imply $\overrightarrow{AB}$ or $\overrightarrow{AD}$ in component form	**B1**
	Use a correct method for finding the position vector of *C*	**M1**
	Obtain answer $4\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}$, or equivalent	**A1**
	Using the correct process for the moduli, compare lengths of a pair of adjacent sides, e.g. *AB* and *AD*	**M1**
	Show that *ABCD* has a pair of unequal adjacent sides	**A1**
	Alternative method for question 8(a)	
	State or imply $\overrightarrow{AB}$ or $\overrightarrow{AD}$ in component form	**B1**
	Use a correct method for finding the position vector of *C*	**M1**
	Obtain answer $4\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}$, or equivalent	**A1**
	Use the correct process to calculate the scalar product of $\overrightarrow{AC}$ and $\overrightarrow{BD}$, or equivalent	**M1**
	Show that the diagonals of *ABCD* are not perpendicular	**A1**
		5
8(b)	Use the correct process to calculate the scalar product of a pair of relevant vectors, e.g. $\overrightarrow{AB}$ and $\overrightarrow{AD}$	**M1**
	Using the correct process for the moduli, divide the scalar product by the product of the moduli of the two vectors and evaluate the inverse cosine of the result	**M1**
	Obtain answer 100.3°	**A1**
		3

Question	Answer	Marks
8(c)	Use a correct method to calculate the area, e.g. calculate *AB.AC* sin *BAD*	**M1**
	Obtain answer 11.0 (**FT** on angle BAD)	**A1 FT**
		2

Question	Answer	Marks
9(a)	Eliminate u or w and obtain an equation w or u	**M1**
	Obtain a quadratic in u or w, e.g. $u^2 - 2iu - 6 = 0$ or $w^2 + 2iw - 6 = 0$	**A1**
	Solve a 3-term quadratic for u or for w	**M1**
	Obtain answer $u = \sqrt{5} + \mathrm{i}$, $w = \sqrt{5} - \mathrm{i}$	**A1**
	Obtain answer $u = -\sqrt{5} + \mathrm{i}$, $w = -\sqrt{5} - \mathrm{i}$	**A1**
		5
9(b)	Show the point representing $2 + 2\mathrm{i}$	**B1**
	Show a circle with centre $2 + 2\mathrm{i}$ and radius 2 (**FT** is on the position of $2 + 2\mathrm{i}$)	**B1 FT**
	Show half-line from origin at 45° to the positive x-axis	**B1**
	Show line for Re $z = 3$	**B1**
	Shade the correct region	**B1**
		5

Question	Answer	Marks
10(a)	State or imply $\frac{\mathrm{d}V}{\mathrm{d}t}=-k\sqrt{h}$	**B1**
	State or imply $\frac{\mathrm{d}V}{\mathrm{d}h}=2\pi rh-\pi h^2$, or equivalent	**B1**
	Use $\frac{\mathrm{d}V}{\mathrm{d}t}=\frac{\mathrm{d}V}{\mathrm{d}h}.\frac{\mathrm{d}h}{\mathrm{d}t}$	**M1**
	Obtain the **given answer** correctly	**A1**
		4
10(b)	Separate variables and attempt integration of at least one side	**M1**
	Obtain terms $\frac{4}{3}rh^{\frac{3}{2}}-\frac{2}{5}h^{\frac{5}{2}}$ and $-Bt$	**A3, 2, 1, 0**
	Use $t=0$, $h=r$ to find a constant of integration c	**M1**
	Use $t=14$, $h=0$ to find B	**M1**
	Obtain correct c and B, e.g. $c=\frac{14}{15}r^{\frac{5}{2}}$, $B=\frac{1}{15}r^{\frac{5}{2}}$	**A1**
	Obtain final answer $t=14-20\left(\frac{h}{r}\right)^{\frac{3}{2}}+6\left(\frac{h}{r}\right)^{\frac{5}{2}}$, or equivalent	**A1**
		8

Cambridge Assessment
International Education

Cambridge International A Level

MATHEMATICS **9709/32**

Paper 3 Pure Mathematics **March 2020**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

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This document consists of **13** printed pages.

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Marks awarded are always **whole marks** (not half marks, or other fractions).

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- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
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GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics-Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M	Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
A	Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
B	Mark for a correct result or statement independent of method marks.
DM or **DB**	When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
FT	Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

Abbreviations

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AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
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WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1(a)	Make a recognisable sketch graph of $y = \|x-2\|$	B1	
		1	
1(b)	Find x-coordinate of intersection with $y = 3x - 4$	M1	
	Obtain $x = \frac{3}{2}$	A1	
	State final answer $x > \frac{3}{2}$ only	A1	
	Alternative method for question 1(b)		
	Solve the linear inequality $3x - 4 > 2 - x$, or corresponding equation	M1	
	Obtain critical value $x = \frac{3}{2}$	A1	
	State final answer $x > \frac{3}{2}$ only	A1	
	Alternative method for question 1(b)		
	Solve the quadratic inequality $(x-2)^2 < (3x-4)^2$, or corresponding equation	M1	
	Obtain critical value $x = \frac{3}{2}$	A1	
	State final answer $x > \frac{3}{2}$ only	A1	
		3	

Question	Answer	Marks	Guidance
2	Use law of logarithm of a power and sum and remove logarithms	M1	
	Obtain a correct equation in any form, e.g. $3(2x+5)=(x+2)^2$	A1	
	Use correct method to solve a 3-term quadratic, obtaining at least one root	M1	
	Obtain final answer $x = 1+2\sqrt{3}$ or $1 + \sqrt{12}$ only	A1	
		4	

Question	Answer	Marks	Guidance
3(a)	Sketch the graph $y = \sec x$	M1	
	Sketch the graph $y=2-\frac{1}{2}x$, and justify the given statement	A1	
		2	
3(b)	Calculate the values of a relevant expression or pair of expressions at $x = 0.8$ and $x = 1$	M1	
	Complete the argument correctly with correct calculated values	A1	
		2	
3(c)	Use the iterative formula correctly at least once	M1	
	Obtain final answer 0.88	A1	
	Show sufficient iterations to 4 d.p. to justify 0.88 to 2 d.p., or show there is a sign change in the interval (0.875, 0.885)	A1	
		3	

Question	Answer	Marks	Guidance
4	Integrate by parts and reach $ax\tan x + b\int \tan x\,\mathrm{d}x$	**M1***	
	Obtain $x\tan x - \int \tan x\,\mathrm{d}x$	**A1**	
	Complete the integration, obtaining a term $\pm\ln\cos x$, or equivalent	**M1**	
	Obtain integral $x\tan x + \ln\cos x$, or equivalent	**A1**	
	Substitute limits correctly, having integrated twice	**DM1**	
	Use a law of logarithms	**M1**	
	Obtain answer $\frac{5}{18}\sqrt{3}\pi - \frac{1}{2}\ln 3$, or exact simplified equivalent	**A1**	
		7	

Question	Answer	Marks	Guidance
5(a)	Express LHS correctly as a single fraction	**B1**	
	Use $\cos(A \pm B)$ formula to simplify the numerator	**M1**	
	Use sin $2A$ formula to simplify the denominator	**M1**	
	Obtain the given result.	**A1**	
		4	

Question	Answer	Marks	Guidance
5(b)	Obtain an equation in $\tan 2x$ and use correct method to solve for x	**M1**	
	Obtain answer, e.g. 0.232	**A1**	
	Obtain second answer, e.g. 1.80	**A1**	Ignore answers outside the given interval.
		3	

Question	Answer	Marks	Guidance
6(a)	Separate variables correctly and attempt integration of at least one side	**B1**	
	Obtain term of the form $a\tan^{-1}(2y)$	**M1**	
	Obtain term $\frac{1}{2}\tan^{-1}(2y)$	**A1**	
	Obtain term $-\mathrm{e}^{-x}$	**B1**	
	Use $x = 1$, $y = 0$ to evaluate a constant or as limits in a solution containing terms of the form $a\tan^{-1}(by)$ and $c\mathrm{e}^{\pm x}$	**M1**	
	Obtain correct answer in any form	**A1**	
	Obtain final answer $y = \frac{1}{2}\tan\left(2\mathrm{e}^{-1} - 2\mathrm{e}^{-x}\right)$, or equivalent	**A1**	
		7	

Question	Answer	Marks	Guidance
6(b)	State that y approaches $\frac{1}{2}\tan\left(2\mathrm{e}^{-1}\right)$, or equivalent	**B1FT**	The FT is on correct work on a solution containing e^{-x}.
		1	

Question	Answer	Marks	Guidance
7(a)	State or imply $3y^2 + 6xy\,\frac{\mathrm{d}y}{\mathrm{d}x}$ as derivative of $3xy^2$	**B1**	
	State or imply $3y^2\,\frac{\mathrm{d}y}{\mathrm{d}x}$ as derivative of y^3	**B1**	
	Equate attempted derivative of LHS to zero and solve for $\frac{\mathrm{d}y}{\mathrm{d}x}$	**M1**	Need to see $\frac{\mathrm{d}y}{\mathrm{d}x}$ factorised out prior to AG
	Obtain the given answer correctly	**A1**	AG
		4	
7(b)	Equate denominator to zero	***M1**	
	Obtain $y = 2x$, or equivalent	**A1**	
	Obtain an equation in x or y	**DM1**	
	Obtain the point (1, 2)	**A1**	
	State the point $\left(\sqrt[3]{5},0\right)$	**B1**	Alternatively (1.71, 0).
		5	

Question	Answer	Marks	Guidance
8(a)	Obtain $\overrightarrow{OM} = 2\mathbf{i} + \mathbf{j}$	**B1**	
	Use a correct method to find $\overrightarrow{MN}$	**M1**	
	Obtain $\overrightarrow{MN} = -\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$	**A1**	
		3	
8(b)	Use a correct method to form an equation for *MN*	**M1**	
	Obtain $\mathbf{r} = 2\mathbf{i} + \mathbf{j} + \lambda(-\mathbf{i} + 2\mathbf{j} + 2\mathbf{k})$, or equivalent	**A1**	
		2	
8(c)	Find $\overrightarrow{DP}$ for a point *P* on *MN* with parameter λ, e.g. $(2-\lambda, 1+2\lambda, -2+2\lambda)$	**B1**	
	Equate scalar product of $\overrightarrow{DP}$ and a direction vector for *MN* to zero and solve for λ	**M1**	
	Obtain $\lambda = \frac{4}{9}$	**A1**	
	State that the position vector of *P* is $\frac{14}{9}\mathbf{i} + \frac{17}{9}\mathbf{j} + \frac{8}{9}\mathbf{k}$	**A1**	
		4	

Question	Answer	Marks	Guidance
9(a)	State or imply the form $\frac{A}{1+2x}+\frac{B}{1-2x}+\frac{C}{2+x}$	**B1**	
	Use a correct method for finding a constant	**M1**	
	Obtain one of $A = -2$, $B = 1$ and $C = 4$	**A1**	
	Obtain a second value	**A1**	
	Obtain the third value	**A1**	
		5	
9(b)	Use correct method to find the first two terms of the expansion of $(1+2x)^{-1}$, $(1-2x)^{-1}$, $(2+x)^{-1}$ or $\left(1+\frac{1}{2}x\right)^{-1}$	**M1**	
	Obtain correct unsimplified expansions up to the term in x^2 of each partial fraction	**A1FT** + **A1FT** + **A1FT**	The FT is on A, B and C.
	Obtain final answer $1+5x-\frac{7}{2}x^2$	**A1**	
		5	

Question	Answer	Marks	Guidance
10(a)	Solve for v or w	**M1**	
	Use $i^2 = -1$	**M1**	
	Obtain $v = -\dfrac{2i}{1+i}$ or $w = \dfrac{5+7i}{-1+i}$	**A1**	
	Multiply numerator and denominator by the conjugate of the denominator	**M1**	
	Obtain $v = -1 - i$	**A1**	
	Obtain $w = 1 - 6i$	**A1**	
		6	
10(b)(i)	Show a circle with centre 2 + 3i	**B1**	
	Show a circle with radius 1 and centre not at the origin	**B1**	
		2	
10(b)(ii)	Carry out a complete method for finding the least value of arg z	**M1**	
	Obtain answer $40.2°$ or 0.702 radians	**A1**	
		2	

Cambridge Assessment International Education
Cambridge International Advanced Level

MATHEMATICS **9709/31**

Paper 3 **October/November 2019**

MARK SCHEME

Maximum Mark: 75

Published

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[Turn over

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Question	Answer	Marks	Guidance
1	State $1+e^{2y}=e^{x}$	**B1**	
	Make *y* the subject	**M1**	Rearrange to $e^{2y}=\ldots$ and use logs
	Obtain answer $y=\frac{1}{2}\ln\left(e^{x}-1\right)$	**A1**	OE
		3	

Question	Answer	Marks	Guidance
2	State or imply non-modular inequality $(2x-3)^2 > 4^2(x+1)^2$, or corresponding quadratic equation, or pair of linear equations $(2x-3) = \pm 4(x+1)$	B1	$12x^2+44x+7<0$
	Make reasonable attempt at solving a 3-term quadratic, or solve two linear equations for x	M1	Correct method seen, or implied by correct answers
	Obtain critical values $x = -\frac{7}{2}$ and $x = -\frac{1}{6}$	A1	
	State final answer $-\frac{7}{2} < x < -\frac{1}{6}$	A1	
	Alternative method for question 2		
	Obtain critical value $x = -\frac{7}{2}$ from a graphical method, or by inspection, or by solving a linear equation or an inequality	B1	
	Obtain critical value $x = -\frac{1}{6}$ similarly	B2	
	State final answer $-\frac{7}{2} < x < -\frac{1}{6}$	B1	
		4	

Question	Answer	Marks	Guidance
3	State $\frac{dx}{dt} = 2 + 2\cos 2t$	**B1**	
	Use the chain rule to find the derivative of y	**M1**	
	Obtain $\frac{dy}{dt} = \frac{2\sin 2t}{1 - \cos 2t}$	**A1**	OE
	Use $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$	**M1**	
	Obtain $\frac{dy}{dx} = \text{cosec}2t$ correctly	**A1**	AG
		5	

Question	Answer	Marks	Guidance
4(i)	State $\frac{dN}{dt} = ke^{-0.02t}N$ and show $k = -0.01$	B1	OE $(-10 = k \times 1 \times 1000)$
		1	
4(ii)	Separate variables correctly and integrate at least one side	B1	$\int \frac{1}{N} dN = \int -0.01e^{-0.02t} dt$
	Obtain term $\ln N$	B1	OE
	Obtain term $0.5e^{-0.02t}$	B1	OE
	Use $N = 1000$, $t = 0$ to evaluate a constant, or as limits, in a solution with terms $a\ln N$ and $be^{-0.02t}$, where $ab \neq 0$	M1	
	Obtain correct solution in any form e.g. $\ln N - \ln 1000 = 0.5\left(e^{-0.02t} - 1\right)$	A1	$\ln 1000 - \frac{1}{2} = 6.41$
	Substitute $N = 800$ and obtain $t = 29.6$	A1	
		6	
4(iii)	State that N approaches $\frac{1000}{\sqrt{e}}$	B1	Accept 606 or 607 or 606.5
		1	

Question	Answer	Marks	Guidance
5(i)	Use correct product rule	**M1**	
	Obtain correct derivative in any form $\frac{\mathrm{d}y}{\mathrm{d}x} = -2\mathrm{e}^{-2x}\ln(x-1) + \frac{\mathrm{e}^{-2x}}{x-1}$	**A1**	
	Equate derivative to zero and derive $x = 1 + e^{\frac{1}{2(x-1)}}$ or $p = 1 + {}^{\frac{1}{2(p-1)}}$	**A1**	AG
		3	
5(ii)	Calculate values of a relevant expression or pair of relevant expressions at $x = 2.2$ and $x = 2.6$ $\mathrm{f}(x) = \ln(x-1) - \frac{1}{2(x-1)} \Rightarrow \mathrm{f}(2.2) = -0.234, \mathrm{f}(2.6) = 0.317$ $\mathrm{f}(x) = 2\mathrm{e}^{-2x}\ln(x-1) + \frac{\mathrm{e}^{-2x}}{x-1} \Rightarrow \mathrm{f}(2.2) = 0.005\ldots, \mathrm{f}(2.6) = -0.0017\ldots$	**M1**	
	Complete the argument correctly with correct calculated values	**A1**	
		2	

Question	Answer	Marks	Guidance
5(iii)	Use the iterative process $p_{n+1} = 1 + \exp\left(\frac{1}{2(p_n - 1)}\right)$ correctly at least once	**M1**	
	Obtain final answer 2.42	**A1**	
	Show sufficient iterations to 4 d.p. to justify 2.42 to 2 d.p., or show there is a sign change in the interval (2.415, 2.425)	**A1**	
		3	

Question	Answer	Marks	Guidance
6(i)	Use correct quotient rule	**M1**	
	Obtain $\frac{\mathrm{d}y}{\mathrm{d}x} = -\operatorname{cosec}^2 x$ correctly	**A1**	AG
		2	
6(ii)	Integrate by parts and reach $ax\cot x + b\int \cot x\,\mathrm{d}x$	***M1**	
	Obtain $-x\cot x + \int \cot x\,\mathrm{d}x$	**A1**	OE
	State $\pm\ln\sin x$ as integral of $\cot x$	**M1**	
	Obtain complete integral $-x\cot x + \ln\sin x$	**A1**	OE
	Use correct limits correctly	**DM1**	$0 + 0 + \frac{\pi}{4} - \ln\frac{1}{\sqrt{2}}$
	Obtain $\frac{1}{4}(\pi + \ln 4)$ following full and exact working	**A1**	AG
		6	

Question	Answer	Marks	Guidance
7(i)	Express general point of *l* or *m* in component form e.g. $(a+\lambda, 2-2\lambda, 3+3\lambda)$ or $(2+2\mu, 1-\mu, 2+\mu)$	**B1**	
	Equate at least two pairs of corresponding components and solve for λ or for μ	**M1**	
	Obtain **either** $\lambda = -2$ or $\mu = -5$ **or** $\lambda = \frac{1}{3}a$ or $\mu = \frac{2}{3}a - 1$ **or** $\lambda = \frac{1}{5}(a-4)$ or $\mu = \frac{1}{5}(3a-7)$	**A1**	
	Obtain $a = -6$	**A1**	
		4	
7(ii)	Use scalar product to obtain a relevant equation in a, b and c, e.g. $a - 2b + 3c = 0$	**B1**	
	Obtain a second equation, e.g. $2a - b + c = 0$ and solve for one ratio	**M1**	
	Obtain $a : b : c = 1 : 5 : 3$	**A1**	OE
	Substitute a relevant point and values of a, b, c in general equation and find d	**M1**	
	Obtain correct answer $x + 5y + 3z = 13$	**A1FT**	OE. The FT is on a from part (**i**), if used
	Alternative method for question 7(ii)		
	Attempt to calculate vector product of relevant vectors,	**M1**	e.g. $(\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}).(2\mathbf{i} - \mathbf{j} + \mathbf{k})$
	Obtain two correct components	**A1**	
	Obtain correct answer, e.g. $\mathbf{i} + 5\mathbf{j} + 3\mathbf{k}$	**A1**	
	Substitute a relevant point and find d	**M1**	
	Obtain correct answer $x + 5y + 3z = 13$	**A1FT**	OE. The FT is on a from part (**i**), if used

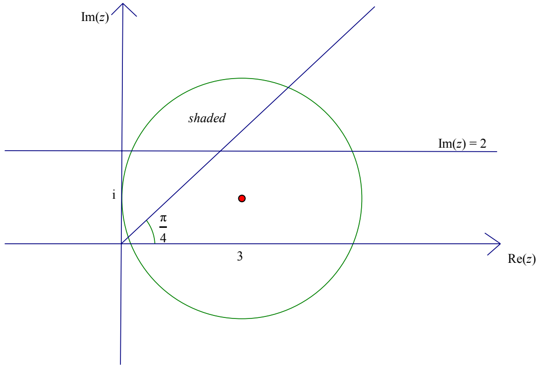
Question	Answer	Marks	Guidance
7(ii)	**Alternative method for question 7(ii)**		
	Using a relevant point and relevant vectors, form a 2-parameter equation for the plane	**M1**	
	State a correct equation, e.g. $\mathbf{r} = 2\mathbf{i} + \mathbf{j} + 2\mathbf{k} + \lambda(\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}) + \mu(2\mathbf{i} - \mathbf{j} + \mathbf{k})$	**A1FT**	
	State three correct equations in x, y, z, λ and μ	**A1FT**	
	Eliminate λ and μ	**M1**	
	Obtain correct answer $x + 5y + 3z = 13$	**A1FT**	OE. The FT is on a from part (**i**), if used
		5	

Question	Answer	Marks	Guidance
8(i)	State or imply the form $\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+2}$	**B1**	
	Use a correct method for finding a constant	**M1**	
	Obtain one of $A = -1$, $B = 3$, $C = 2$	**A1**	
	Obtain a second value	**A1**	
	Obtain the third value	**A1**	Allow in the form $\frac{Ax+B}{x^2} + \frac{C}{x+2}$
		5	

Question	Answer	Marks	Guidance
8(ii)	Integrate and obtain terms $\ln x - \frac{3}{x} + 2\ln(x+2)$	**B1FT +** **B1FT +** **B1FT**	The FT is on *A*, *B*, *C*; or on *A*, *D*, *E*.
	Substitute limits correctly in an integral with terms $a\ln x$, $\frac{b}{x}$ and $c\ln(x+2)$, where $abc \neq 0$	**M1**	$-\ln 4 - \frac{3}{4} + 2\ln 6 (+\ln 1) + 3 - 2\ln 3$
	Obtain $\frac{9}{4}$ following full and exact working	**A1**	AG – work to combine or simplify logs is required
		5	

Question	Answer	Marks	Guidance
9(i)	Use cos($A + B$) formula to express cos$3x$ in terms of trig functions of $2x$ and x	**M1**	
	Use double angle formulae and Pythagoras to obtain an expression in terms of cos x only	**M1**	
	Obtain a correct expression in terms of cos x in any form	**A1**	
	Obtain $\cos 3x \equiv 4\cos^3 x - 3\cos x$	**A1**	AG
		4	
9(ii)	Use identity and solve cubic $4\cos^3 x = -1$ for x	**M1**	$\cos x = -0.6299...$
	Obtain answer 2.25 and no other in the interval	**A1**	Accept 0.717π M1A0 for 129.0°
		2	

Question	Answer	Marks	Guidance
9(iii)	Obtain indefinite integral $\frac{1}{12}\sin 3x+\frac{3}{4}\sin x$	**B1 + B1**	
	Substitute limits in an indefinite integral of the form $a\sin 3x+b\sin x$, where $ab\neq 0$	**M1**	$\frac{1}{4}\left[\frac{1}{3}\sin\pi+3\sin\frac{\pi}{3}-\frac{1}{3}\sin\frac{\pi}{2}-3\sin\frac{\pi}{6}\right]$
	Obtain answer $\frac{1}{24}\left(9\sqrt{3}-11\right)$, or exact equivalent	**A1**	
	Alternative method for question 9(iii)		
	$\int\cos x\left(1-\sin^2 x\right)\mathrm{d}x=\sin x-\frac{1}{3}\sin^3 x\left(+C\right)$	**B1 + B1**	
	Substitute limits in an indefinite integral of the form $a\sin x+b\sin^3 x$ where $ab\neq 0$	**M1**	$\left(\frac{\sqrt{3}}{2}-\frac{1}{2}-\frac{1}{4}\frac{\sqrt{3}}{2}+\frac{1}{24}\right)$
	Obtain answer $\frac{1}{24}\left(9\sqrt{3}-11\right)$, or exact equivalent	**A1**	
		4	

Question	Answer	Marks	Guidance
10(a)	Square $a + ib$ and equate real and imaginary parts to -3 and $-2\sqrt{10}$ respectively	*M1	
	Obtain $a^2 - b^2 = -3$ and $2ab = -2\sqrt{10}$	A1	
	Eliminate one unknown and find an equation in the other	DM1	
	Obtain $a^4 + 3a^2 - 10 = 0$, or $b^4 - 3b^2 - 10 = 0$, or horizontal 3-term equivalent	A1	
	Obtain answers $\pm\left(\sqrt{2} - \sqrt{5}\mathrm{i}\right)$, or exact equivalent	A1	
		5	
10(b)	Show point representing 3 + i in relatively correct position	B1	
	Show a circle with radius 3 and centre not at the origin	B1	
	Show correct half line from the origin at $\frac{1}{4}\pi$ to the real axis	B1	
	Show horizontal line $y = 2$	B1	
	Shade the correct region	B1	
		5	

Cambridge International A Level

Cambridge Assessment International Education
Cambridge International Advanced Level

MATHEMATICS **9709/32**

Paper 3 **October/November 2019**

MARK SCHEME

Maximum Mark: 75

Published

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Cambridge Assessment International Education

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A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

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FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	Remove logarithms and state $4-3^x=e^{1.2}$, or equivalent	B1	Accept $4-3^x=3.32(01169.....)$ 3 s.f. or better
	Use correct method to solve an equation of the form $3^x=a$, where $a>0$.	M1	$\left(3^x=0.67988..\right)$ Complete method to $x=\ldots$ If using $\log_3$ the subscript can be implied
	Obtain answer $x=-0.351$ only	A1	CAO must be to 3 d.p.
		3	

Question	Answer	Marks	Guidance
2	Use correct quotient rule or correct product rule	**M1**	
	Obtain correct derivative in any form	**A1**	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{-2\mathrm{e}^{-2x}\left(1-x^2\right)+2x\mathrm{e}^{-2x}}{\left(1-x^2\right)^2}$
	Equate derivative to zero and obtain a 3 term quadratic in x	**M1**	
	Obtain a correct 3-term equation e.g. $2x^2+2x-2=0$ or $x^2+x=1$	**A1**	From correct work only
	Solve and obtain $x = 0.618$ only	**A1**	From correct work only
		5	

Question	Answer	Marks	Guidance
3	Commence division and reach partial quotient $x^2 + kx$	**M1**	
	Obtain correct quotient $x^2 + 2x - 1$	**A1**	
	Set their linear remainder equal to $2x + 3$ and solve for a or for b	**M1**	Remainder = $(a+3)x+(b-1)$
	Obtain answer $a = -1$	**A1**	
	Obtain answer $b = 4$	**A1**	
	Alternative method for question 3		
	State $x^4 + 3x^3 + ax + b = (x^2 + x - 1)(x^2 + Ax + B) + 2x + 3$ and form and solve two equations in A and B	**M1**	e.g. $3 = 1 + A$ and $0 = -1 + A + B$
	Obtain $A = 2,\ B = -1$	**A1**	
	Form and solve equations for a or b	**M1**	e.g. $a = B - A + 2, \quad b = -B + 3$
	Obtain answer $a = -1$	**A1**	
	Obtain answer $b = 4$	**A1**	
		5	

Question	Answer	Marks	Guidance
3	**Alternative method for question 3**		
	Use remainder theorem with $x = \frac{-1 \pm \sqrt{5}}{2}$	**M1**	Allow for correct use of either root in exact or decimal form.
	Obtain $-\frac{a}{2} \pm \frac{a\sqrt{5}}{2} + b = \frac{9}{2} \mp \frac{\sqrt{5}}{2}$	**A1**	Expand brackets and obtain exact equation for either root. Accept exact equivalent.
	Solve simultaneous equations for a or b	**M1**	
	Obtain answer $a = -1$ from exact working	**A1**	
	Obtain answer $b = 4$ from exact working	**A1**	
		5	

Question	Answer	Marks	Guidance
4(i)	State $R = \sqrt{7}$	**B1**	
	Use correct trig formulae to find α	**M1**	e.g. $\tan\alpha = \frac{1}{\sqrt{6}}, \sin\alpha = \frac{1}{\sqrt{7}}$, or $\cos\alpha = \frac{\sqrt{6}}{\sqrt{7}}$
	Obtain $\alpha = 22.208°$	**A1**	ISW
		3	

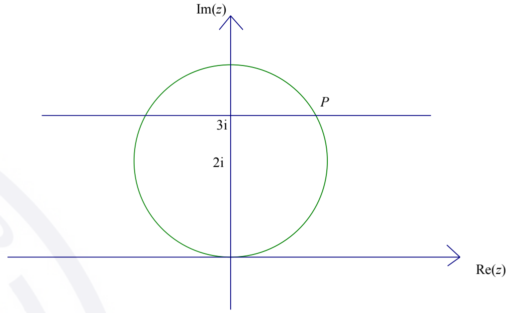
Question	Answer	Marks	Guidance
4(ii)	Evaluate $\sin^{-1}\left(\frac{2}{\sqrt{7}}\right)$ to at least 1 d.p.	**B1FT**	49.107° to 3 d.p. B1 can be implied by correct answer(s) later. The FT is on *their R*
			SC: allow B1 for a correct alternative equation e.g. $3\tan^2\theta-2\sqrt{6}\tan\theta+1=0$
	Use correct method to find a value of θ in the interval	**M1**	Must get to θ
	Obtain answer, e.g. 13.4°	**A1**	Accept correct over-specified answers. 13.449…, 54.3425…
	Obtain second answer, e.g. 54.3° and no extras in the given interval	**A1**	Ignore answers outside the given interval.
		4	

Question	Answer	Marks	Guidance
5	State $4xy+2x^2\dfrac{\mathrm{d}y}{\mathrm{d}x}$, or equivalent, as derivative of $2x^2y$	**B1**	
	State $y^2+2xy\dfrac{\mathrm{d}y}{\mathrm{d}x}$, or equivalent, as derivative of xy^2	**B1**	
	Equate attempted derivative of LHS to zero and set $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ equal to zero (or set numerator equal to zero)	***M1**	$\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{y^2-4xy}{2x^2-2xy}$
	Reject $y = 0$	**B1**	Allow from $y^2-kxy=0$
	Obtain $y = 4x$	**A1**	OE from correct numerator. ISW
	Obtain an equation in y (or in x) and solve for y (or for x) in terms of a	**DM1**	$8x^3-16x^3=a^3$ or $\dfrac{y^3}{8}-\dfrac{y^3}{4}=a^3$
	Obtain $y = -2a$	**A1**	With no errors seen
		7	

Question	Answer	Marks	Guidance
5	**Alternative method for question 5**		
	Rewrite as $y=\dfrac{a^3}{2x^2-xy}$ and differentiate	**M1**	Correct use of function of a function and implicit differentiation
	Obtain correct derivative (in any form)	**A1**	$\dfrac{dy}{dx}=\dfrac{-a^3\left(4x-y-x\dfrac{dy}{dx}\right)}{\left(2x^2-xy\right)^2}$
	set $\dfrac{dy}{dx}$ equal to zero (or set numerator equal to zero)	***M1**	
	Obtain $4x-y=0$	**A1**	
	Confirm $2x^2-xy\neq 0$	**B1**	$x=0$ and $2x=y$ both give $a=0$
	Obtain an equation in y (or in x) and solve for y (or for x)	**DM1**	$8x^3-16x^3=a^3$ or $\dfrac{y^3}{8}-\dfrac{y^3}{4}=a^3$
	Obtain $y=-2a$	**A1**	With no errors seen
		7	

Question	Answer	Marks	Guidance
6	Separate variables correctly to obtain $\int \frac{1}{x+2}\,dx = \int \cot\frac{1}{2}\theta\,d\theta$	**B1**	Or equivalent integrands. Integral signs SOI
	Obtain term $\ln(x+2)$	**B1**	Modulus signs not needed.
	Obtain term of the form $k\ln\sin\frac{1}{2}\theta$	**M1**	
	Obtain term $2\ln\sin\frac{1}{2}\theta$	**A1**	
	Use $x = 1$, $\theta = \frac{1}{3}\pi$ to evaluate a constant, or as limits, in an expression containing $p\ln(x+2)$ and $q\ln\left(\sin\frac{1}{2}\theta\right)$	**M1**	Reach C = an expression or a decimal value
	Obtain correct solution in any form e.g. $\ln(x+2) = 2\ln\sin\frac{1}{2}\theta + \ln 12$	**A1**	$\ln 12 = 2.4849$….. Accept constant to at least 3 s.f. Accept with $\ln 3 - 2\ln\frac{1}{2}$
	Remove logarithms and use correct double angle formula	**M1**	Need correct algebraic process. $\left(\frac{x+2}{12} = \frac{1-\cos\theta}{2}\right)$
	Obtain answer $x = 4 - 6\cos\theta$	**A1**	
		8	

Question	Answer	Marks	Guidance
7(a)	Substitute and obtain a correct horizontal equation in x and y in any form	**B1**	$zz^* + \mathrm{i}z - 2z^* = 0 \Rightarrow$ $x^2 + y^2 + \mathrm{i}x - y - 2x + 2\mathrm{i}y = 0$ Allow if still includes brackets and/or i^2
	Use $\mathrm{i}^2 = -1$ and equate real and imaginary parts to zero OE	***M1**	For their horizontal equation
	Obtain two correct equations e.g. $x^2 + y^2 - y - 2x = 0$ and $x + 2y = 0$	**A1**	Allow $\mathrm{i}x + 2\mathrm{i}y = 0$
	Solve for x or for y	**DM1**	
	Obtain answer $\frac{6}{5} - \frac{3}{5}\mathrm{i}$ and no other	**A1**	OE, condone $\frac{1}{5}(6 - 3\mathrm{i})$
		5	

Question	Answer	Marks	Guidance
7(b)(i)	Show a circle with centre 2i and radius 2	**B1**	
	Show horizontal line $y = 3$ – in first and second quadrant	**B1**	
			SC: For clearly labelled axes not in the conventional directions, allow B1 for a fully ‘correct’ diagram.
		2	
7(b)(ii)	Carry out a complete method for finding the argument. (Not by measuring the sketch)	**M1**	$\left(z = \sqrt{3} + 3\mathrm{i}\right)$ Must show working if using 1.7 in place of $\sqrt{3}$.
	Obtain answer $\frac{1}{3}\pi$ (or 60°)	**A1**	**SC:** Allow B2 for 60° with no working
		2	

Question	Answer	Marks	Guidance
8(i)	State or imply the form $\frac{A}{2x-1}+\frac{Bx+C}{x^2+2}$	**B1**	
	Use a correct method for finding a constant	**M1**	
	Obtain one of $A = 4$, $B = -1$, $C = 0$	**A1**	
	Obtain a second value	**A1**	
	Obtain the third value	**A1**	
		5	
8(ii)	Integrate and obtain term $2\ln(2x-1)$	**B1FT**	The FT is on A. $\frac{1}{2}A\ln(2x-1)$
	Integrate and obtain term of the form $k\ln(x^2+2)$	***M1**	From $\frac{nx}{x^2+2}$
	Obtain term $-\frac{1}{2}\ln(x^2+2)$	**A1FT**	The FT is on B
	Substitute limits correctly in an integral of the form $a\ln(2x-1)+b\ln(x^2+2)$, where $ab \neq 0$	**DM1**	$2\ln 9(-2\ln 1)-\frac{1}{2}\ln 27+\frac{1}{2}\ln 3$
	Obtain answer ln 27 after full and correct exact working	**A1**	ISW
		5	

Question	Answer	Marks	Guidance
9(i)	Commence integration by parts, reaching $ax\sin\frac{1}{3}x - b\int \sin\frac{1}{3}x\,\mathrm{d}x$	***M1**	
	Obtain $3x\sin\frac{1}{3}x - 3\int \sin\frac{1}{3}x\,\mathrm{d}x$	**A1**	
	Complete integration and obtain $3x\sin\frac{1}{3}x + 9\cos\frac{1}{3}x$	**A1**	
	Substitute limits correctly and equate result to 3 in an integral of the form $px\sin\frac{1}{3}x + q\cos\frac{1}{3}x$	**DM1**	$3 = 3a\sin\frac{a}{3} + 9\cos\frac{a}{3}\left(-0\right) - 9$
	Obtain $a = \dfrac{4 - 3\cos\frac{a}{3}}{\sin\frac{a}{3}}$ correctly	**A1**	With sufficient evidence to show how they reach the given equation
		5	
9(ii)	Calculate values at $a = 2.5$ and $a = 3$ of a relevant expression or pair of expressions.	**M1**	$2.5 < 2.679$ and $3 > 2.827$ If using 2.679 and 2.827 must be linked explicitly to 2.5 and 3. Solving f(a) = 0, f(2.5) = 0.179. and f(3) = –0.173 or if $\mathrm{f}(a) = a\sin\frac{1}{3}a + 3\cos\frac{1}{3}a - 4 \Rightarrow \mathrm{f}(2.5) = -0.13.., \mathrm{f}(3) = 0.145...$
	Complete the argument correctly with correct calculated values	**A1**	Accept values to 1 sf. or better
		2	

Question	Answer	Marks	Guidance
9(iii)	Use the iterative process $a_{n+1} = \; a_{n+1}\dfrac{4-3\cos\frac{1}{3}a_n}{\sin\frac{1}{3}a_n}$ correctly at least once	**M1**	
	Show sufficient iterations to at least 5 d.p. to justify 2.736 to 3d.p., or show a sign change in the interval (2.7355, 2.7365)	**A1**	
	Obtain final answer 2.736	**A1**	
		3	

Question	Answer	Marks	Guidance
10(i)	Express general point of *l* in component form e.g. $(1+\lambda, 3-2\lambda, -2+3\lambda)$	**B1**	
	Substitute in equation of *p* and solve for λ	**M1**	
	Obtain final answer $\frac{5}{3}\mathbf{i}+\frac{5}{3}\mathbf{j}$ from $\lambda = \frac{2}{3}$	**A1**	OE Accept $1.67\mathbf{i}+1.67\mathbf{j}$ or better
		3	

Question	Answer	Marks	Guidance
10(ii)	Use correct method to evaluate a scalar product of relevant vectors e.g. (**i** – 2**j** + 3**k**).(2**i** + **j** – 3**k**)	**M1**	
	Using the correct process for calculating the moduli, divide the scalar product by the product of the moduli and evaluate the inverse sine or cosine of the result	**M1**	$\lvert\sin\theta\rvert = \frac{9}{14}$
	Obtain answer 40.0° or 0.698 radians	**A1**	AWRT
		3	
	Alternative method for question 10(ii)		
	Use correct method to evaluate a vector product of relevant vectors e.g. (**i** – 2**j** + 3**k**)x(2**i** + **j** – 3**k**)	**M1**	
	Using the correct process for calculating the moduli, divide the modulus of the vector product by the product of the moduli of the two vectors and evaluate the inverse sine or cosine of the result	**M1**	$\cos\theta = \frac{\sqrt{115}}{14}$
	Obtain answer 40.0° or 0.698 radians	**A1**	AWRT
		3	

Question	Answer	Marks	Guidance
10(iii)	State $a - 2b + 3c = 0$ or $2a + b - 3c = 0$	**B1**	
	Obtain two relevant equations and solve for one ratio, e.g. $a : b$	**M1**	Could use $2a + b - 3c = 0$ and $\begin{cases} a + 3b - 2c = d \\ \frac{5}{3}a + \frac{5}{3}b = d \end{cases}$ i.e. use two points on the line rather than the direction of the line. The second M1 is not scored until they solve for d.
	Obtain $a : b : c = 3 : 9 : 5$	**A1**	OE
	Substitute a, b, c and a relevant point in the plane equation and evaluate d	**M1**	Using their calculated normal and a relevant point
	Obtain answer $3x + 9y + 5z = 20$	**A1**	OE
	Alternative method for question 10(iii)		
	Attempt to calculate vector product of relevant vectors, e.g. $(\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}) \times (2\mathbf{i} + \mathbf{j} - 3\mathbf{k})$	**M1**	
	Obtain two correct components	**A1**	
	Obtain correct answer, e.g. $3\mathbf{i} + 9\mathbf{j} + 5\mathbf{k}$	**A1**	
	Use the product and a relevant point to find d	**M1**	Using *their* calculated normal and a relevant point
	Obtain answer $3x + 9y + 5z = 20$, or equivalent	**A1**	OE

Question	Answer	Marks	Guidance
10(iii)	**Alternative method for question 10(iii)**		
	Attempt to form a 2-parameter equation with relevant vectors	**M1**	
	State a correct equation e.g. $\mathbf{r} = \mathbf{i} + 3\mathbf{j} - 2\mathbf{k} + \lambda(\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}) + \mu(2\mathbf{i} + \mathbf{j} - 3\mathbf{k})$	**A1**	
	State 3 equations in x, y, z, λ and μ	**A1**	
	Eliminate λ and μ	**M1**	
	Obtain answer $3x + 9y + 5z = 2$	**A1**	OE
		5	

Cambridge Assessment International Education
Cambridge International Advanced Level

MATHEMATICS **9709/33**

Paper 3 **October/November 2019**

MARK SCHEME

Maximum Mark: 75

Published

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Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2019 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

This document consists of **13** printed pages.

[Turn over

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B	Mark for a correct result or statement independent of method marks.
DM or **DB**	When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
FT	Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no “follow through” from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	State or imply non-modular inequality $(x+2)^2 > (3x-1)^2$, or corresponding quadratic equation, or pair of linear equations $2(x + 2) = \pm (3x - 1)$	**B1**	
	Make reasonable attempt at solving a 3-term quadratic, or solve two linear equations for x	**M1**	
	Obtain critical values $x = -\frac{3}{5}$ and $x = 5$	**A1**	
	State final answer $-\frac{3}{5} < x < 5$	**A1**	
	Alternative method for question 1		
	Obtain critical value $x = 5$ from a graphical method, or by inspection, or by solving a linear equation or an inequality	**B1**	
	Obtain critical value $x = -\frac{3}{5}$ similarly	**B2**	
	State final answer $-\frac{3}{5} < x < 5$	**B1**	
		4	

Question	Answer	Marks	Guidance
2	Substitute $x=-\frac{1}{2}$, equate result to zero and obtain a correct equation, e.g. $-\frac{6}{8}+\frac{1}{4}a-\frac{1}{2}b-2=0$	**B1**	
	Substitute $x=-2$ and equate result to -24	***M1**	
	Obtain a correct equation, e.g. $-48+4a-2b-2=-24$	**A1**	
	Solve for a or for b	**DM1**	
	Obtain $a=5$ and $b=-3$	**A1**	
		5	

Question	Answer	Marks	Guidance
3	Reduce the equation to a horizontal equation in 3^{3x}, 3^{3x+1} or 27^{x}	**M1**	
	Simplify and reach $3(3^{3x})=5$, $3\left(27^{x}\right)=5$, or equivalent	**A1**	
	Use correct method for finding x from a positive value of 3^{3x}, 3^{3x+1} or 27^{x}	**M1**	
	Obtain answer $x=0.155$	**A1**	
		4	

Question	Answer	Marks	Guidance
4(i)	Use tan $(A + B)$ formula to express the LHS in terms of tan $2x$ and tan x	**M1**	
	Using the tan $2A$ formula, express the entire equation in terms of tan x	**M1**	
	Obtain a correct equation in tan x in any form	**A1**	
	Obtain the given form correctly	**A1**	AG
		4	
4(ii)	Use correct method to solve the given equation for x	**M1**	
	Obtain answer, e.g. $x = 26.8°$	**A1**	
	Obtain second answer, e.g. $x = 73.7°$ and no other	**A1**	Ignore answers outside the given interval
		3	

Question	Answer	Marks	Guidance
5(i)	Sketch a relevant graph, e.g. $y = \ln(x + 2)$	**B1**	
	Sketch a second relevant graph, e.g. $y = 4e^{-x}$, and justify the given statement	**B1**	Consideration of behaviour for $x < 0$ is needed for the second B1
		2	
5(ii)	Calculate the values of a relevant expression or pair of expressions at $x = 1$ and $x = 1.5$	**M1**	
	Complete the argument correctly with correct calculated values	**A1**	
		2	

Question	Answer	Marks	Guidance
5(iii)	Use the iterative formula correctly at least **twice** using output from a previous iteration	**M1**	
	Obtain final answer 1.23	**A1**	
	Show sufficient iterations to 4 d.p. to justify 1.23 to 2 d.p., or show there is a sign change in the interval (1.225, 1.235)	**A1**	
		3	

Question	Answer	Marks	Guidance
6(i)	Obtain answer $w=\frac{1}{2}+\frac{\sqrt{3}}{2}\mathrm{i}$	**B1**	
		1	
6(ii)	Show point representing *u*	**B1**	
	Show point representing *v* in relatively correct position	**B1**	
		2	
6(iii)	Explain why the moduli are equal	**B1**	
	Explain why the arguments are equal	**B1**	
	Use $\mathrm{i}^2=-1$ and obtain 2*uw* in the given form	**M1**	
	Obtain answer $1-2\sqrt{3}+\left(2+\sqrt{3}\right)\mathrm{i}$	**A1**	
		4	

Question	Answer	Marks	Guidance
7(i)	Substitute coordinates (5, 2, – 2) in $x + 4y - 8z = d$	**M1**	
	Obtain plane equation $x + 4y - 8z = 29$, or equivalent	**A1**	
		2	
7(ii)	Attempt to use perpendicular formula to find perpendicular from (5, 2, – 2) to m	**M1**	
	Obtain a correct unsimplified expression, e.g. $\dfrac{5+8+16-2}{\sqrt{(1+16+64)}}$	**A1**	
	Obtain answer 3	**A1**	
	Alternative method 1 for question 7(ii)		
	State or imply perpendicular from O to m is $\dfrac{2}{9}$ or from O to n is $\dfrac{29}{9}$	**B1**	
	Find difference in perpendiculars	**M1**	
	Obtain answer 3	**A1**	
	Alternative method 2 for question 7(ii)		
	Obtain correct parameter value, or position vector or coordinates of the foot of the perpendicular from (5, 2, – 2) to m, e.g. $\mu = \pm\dfrac{1}{3}$; $\left(\dfrac{14}{3}, \dfrac{2}{3}, \dfrac{2}{3}\right)$	**B1**	
	Calculate the length of the perpendicular	**M1**	
	Obtain answer 3	**B1**	
		3	

Question	Answer	Marks	Guidance
7(iii)	Calling the direction vector $a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$, use a scalar product to form a relevant equation in a, b and c, e.g. $a + 4b - 8c = 0$ or $5a + 2b - 2z = 0$	**B1**	
	Solve two relevant equations for the ratio $a : b : c$	**M1**	
	Obtain $a : b : c = 4 : -19 : -9$	**A1**	OE
	State answer $\mathbf{r} = 5\mathbf{i} + 2\mathbf{j} - 2\mathbf{k} + \lambda(4\mathbf{i} - 19\mathbf{j} - 9\mathbf{k})$	**A1**	OE
	Alternative method for question 7(iii)		
	Attempt to calculate vector product of two relevant vectors, e.g. $(\mathbf{i} + 4\mathbf{j} - 8\mathbf{k})\times(5\mathbf{i} + 2\mathbf{j} - 2\mathrm{k})$	**M1**	
	Obtain two correct components	**A1**	
	Obtain $8\mathbf{i} - 38\mathbf{j} - 18\mathbf{k}$	**A1**	OE
	State answer $\mathbf{r} = 5\mathbf{i} + 2\mathbf{j} - 2\mathbf{k} + \lambda(4\mathbf{i} - 19\mathbf{j} - 9\mathbf{k})$	**A1**	OE
		4	

Question	Answer	Marks	Guidance
8(i)	State or imply ordinates 1, 1.2116…, 2.7597...	**B1**	
	Use correct formula, or equivalent, with $h = 0.6$	**M1**	
	Obtain answer 1.85	**A1**	
		3	
8(ii)	Explain why the rule gives an overestimate	**B1**	
		1	
8(iii)	Differentiate using quotient or chain rule	**M1**	
	Obtain correct derivative in terms of $\sin x$ and $\cos x$	**A1**	
	Equate derivative to 2, use Pythagoras and obtain an equation in $\sin x$	**M1**	
	Obtain $2\sin^2 x + \sin x - 2 = 0$	**A1**	OE
	Solve a 3-term quadratic for x	**M1**	
	Obtain answer $x = 0.896$ only	**A1**	
		6	

Question	Answer	Marks	Guidance
9(i)	Separate variables correctly and integrate one side	**B1**	
	Obtain term $0.2t$, or equivalent	**B1**	
	Carry out a relevant method to obtain A and B such that $\frac{1}{(20-x)(40-x)} \equiv \frac{A}{20-x}+\frac{B}{40-x}$	***M1**	OE
	Obtain $A=\frac{1}{20}$ and $B=-\frac{1}{20}$	**A1**	
	Integrate and obtain terms $-\frac{1}{20}\ln(20-x)+\frac{1}{20}\ln(40-x)$ OE	**A1FT** **+A1FT**	The FT is on A and B
	Use $x = 10$, $t = 0$ to evaluate a constant, or as limits	**DM1**	
	Obtain correct answer in any form	**A1**	
	Obtain final answer $x=\frac{60e^{4t}-40}{3e^{4t}-1}$	**A1**	OE
		9	
9(ii)	State that x approaches 20	**B1**	
		1	

Question	Answer	Marks	Guidance
10(i)	Use product rule and chain rule at least once	M1	
	Obtain correct derivative in any form	A1	
	Equate derivative to zero, use Pythagoras and obtain an equation in cos x	M1	
	Obtain $\cos^2 x + 3\cos x - 1 = 0$, or 3-term equivalent	A1	
	Obtain answer $x = 1.26$	A1	
		5	
10(ii)	Using du = ± sin x dx express integrand in terms of u and du	M1	
	Obtain integrand $e^u\left(u^2-1\right)$	A1	OE
	Commence integration by parts and reach $ae^u\left(u^2-1\right)+b\int ue^u\,du$	*M1	
	Obtain $e^u\left(u^2-1\right)-2\int ue^u\,du$	A1	OE
	Complete integration, obtaining $e^u\left(u^2-2u+1\right)$	A1	OE
	Substitute limits $u = 1$ and $u = -1$ (or $x = 0$ and $x = \pi$), having integrated completely	DM1	
	Obtain answer $\frac{4}{e}$, or exact equivalent	A1	
		7	

Cambridge Assessment International Education
Cambridge International Advanced Level

MATHEMATICS **9709/31**

Paper 3 **May/June 2019**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2019 series for most Cambridge IGCSE™, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

This document consists of **17** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

  - Note: B2 or A2 means that the candidate can earn 2 or 0.
    B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
CWO	Correct Working Only – often written by a 'fortuitous' answer
ISW	Ignore Subsequent Working
SOI	Seen or implied
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Question	Answer	Marks	Guidance
1	State or imply ordinates 3, 2, 0, 4	**B1**	These and no more Accept in unsimplified form $\left\|2^0-4\right\|$ etc.
	Use correct formula, or equivalent, with $h = 1$ and four ordinates	**M1**	
	Obtain answer 5.5	**A1**	
		3	

Question	Answer	Marks	Guidance
2	Use law for the logarithm of a product, quotient or power	**M1**	Condone $\ln\dfrac{x}{x-1}$ for M1
	Obtain a correct equation free of logarithms	**A1**	e.g. $(2x-3)(x-1)=x^2$ or $x^2-5x+3=0$
	Solve a 3-term quadratic obtaining at least one root	**M1**	Must see working if using an incorrect quadratic $\left(\dfrac{5\pm\sqrt{13}}{2}\right)$
	Obtain answer $x = 4.30$ only	**A1**	Q asks for 2 dp. Do not ISW. Overspecified answers score A0 Overspecified and no working can score M1A0
		4	

Question	Answer	Marks	Guidance
3	State or imply $3y^2+6xy\frac{dy}{dx}$ as derivative of $3xy^2$	**B1**	
	State or imply $3y^2\frac{dy}{dx}$ as derivative of y^3	**B1**	
	Equate derivative of LHS to zero, substitute (1, 3) and find the gradient	**M1**	$\left(\frac{dy}{dx}=\frac{x^2+y^2}{y^2-2xy}\right)$ For incorrect derivative need to see the substitution
	Obtain final answer $\frac{10}{3}$ or equivalent	**A1**	3.33 or better. Allow $\frac{30}{9}$ ISW after correct answer seen
		4	

Question	Answer	Marks	Guidance
4	Use correct trig formula and obtain an equation in tan θ	**M1**	Allow with 45° e.g. $\frac{1}{\tan\theta}-\frac{1}{\frac{\tan\theta+\tan 45°}{1-\tan\theta\tan 45°}}=3$
	Obtain a correct horizontal equation in any form	**A1**	e.g. $1+\tan\theta-\tan\theta(1-\tan\theta)=3\tan\theta(1+\tan\theta)$
	Reduce to $2\tan^2\theta+3\tan\theta-1=0$	**A1**	or 3-term equivalent
	Solve 3-term quadratic and find a value of θ	**M1**	Must see working if using an incorrect quadratic
	Obtain answer 15.7°	**A1**	One correct solution (degrees to at least 3 sf)
	Obtain answer 119.(3)°	**A1**	Second correct solution and no others in range (degrees to at least 3 sf) Mark 0.274, 2.082 as MR: A0A1
		6	

Question	Answer	Marks	Guidance
5(i)	Use chain rule	**M1**	$k\cos\theta\sin^{-3}\theta\left(=-k\operatorname{cosec}^2\theta\cot\theta\right)$ Allow M1 for $-2\cos\theta\sin^{-1}\theta$
	Obtain correct answer in any form	**A1**	e.g. $-2\operatorname{cosec}^2\theta\cot\theta$, $\dfrac{-2\cos\theta}{\sin^3\theta}$ Accept $\dfrac{-2\sin\theta\cos\theta}{\sin^4\theta}$
		2	
5(ii)	Separate variables correctly and integrate at least one side	**B1**	$\int x\,\mathrm{d}x=\int-\operatorname{cosec}^2\theta\cot\theta\,\mathrm{d}\theta$
	Obtain term $\dfrac{1}{2}x^2$	**B1**	
	Obtain term of the form $\dfrac{k}{\sin^2\theta}$	**M1***	or equivalent
	Obtain term $\dfrac{1}{2\sin^2\theta}$	**A1**	or equivalent
	Use $x=4$, $\theta=\dfrac{1}{6}\pi$ to evaluate a constant, or as limits, in a solution with terms ax^2 and $\dfrac{b}{\sin^2\theta}$, where $ab\neq 0$	**DM1**	Dependent on the preceding M1
	Obtain solution $x=\sqrt{\left(\operatorname{cosec}^2\theta+12\right)}$	**A1**	or equivalent
		6	

Question	Answer	Marks	Guidance
6(i)	State correct expansion of $\sin(2x+x)$	**B1**	
	Use trig formulae and Pythagoras to express $\sin 3x$ in terms of $\sin x$	**M1**	
	Obtain a correct expression in any form	**A1**	e.g. $2\sin x(1-\sin^2 x)+\sin x(1-2\sin^2 x)$
	Obtain $\sin 3x \equiv 3\sin x - 4\sin^3 x$ correctly **AG**	**A1**	Accept $=$ for $\equiv$
		4	
6(ii)	Use identity, integrate and obtain $-\frac{3}{4}\cos x + \frac{1}{12}\cos 3x$	**B1 B1**	One mark for each term correct
	Use limits correctly in an integral of the form $a\cos x + b\cos 3x$, where $ab \neq 0$	**M1**	$\left(-\frac{3}{8}-\frac{1}{12}+\frac{3}{4}-\frac{1}{12}=-\frac{11}{24}+\frac{2}{3}\right)$
	Obtain answer $\frac{5}{24}$	**A1**	Must be exact. Accept simplified equivalent e.g. $\frac{15}{72}$ Answer only with no working is 0/4
		4	

Question	Answer	Marks	Guidance
7(i)	State at least one correct derivative	B1	$-2\sin\frac{1}{2}x$, $\frac{1}{(4-x)^2}$
	Equate product of derivatives to – 1	M1	or equivalent
	Obtain a correct equation, e.g. $2\sin\frac{1}{2}x=(4-x)^2$	A1	
	Rearrange correctly to obtain $a=4-\sqrt{2\sin\frac{a}{2}}$ **AG**	A1	
		4	
7(ii)	Calculate values of a relevant expression or pair of expressions at $a = 2$ and $a = 3$	M1	e.g. $\begin{matrix} a=2 & 2<2.7027.. \\ a=3 & 3>2.587.. \end{matrix}$ $\begin{pmatrix} 0.703 \\ -0.412 \end{pmatrix}$ $\begin{pmatrix} 2.317 \\ -0.995 \end{pmatrix}$ Values correct to at least 2 dp
	Complete the argument correctly with correct calculated values	A1	
		2	
7(iii)	Use the iterative formula $a_{n+1}=4-\sqrt{(2\sin\frac{1}{2}a_n)}$ correctly at least once	M1	
	Obtain final answer 2.611	A1	
	Show sufficient iterations to 5 d.p. to justify 2.611 to 3 d.p., or show there is a sign change in the interval (2.6105, 2.6115)	A1	2, 2.70272, 2.60285, 2.61152, 2.61070, 2.61077 2.5, 2.62233, 2.60969, 2.61087, 2.61076 3, 2.58756, 2.61301, 2.61056, 2.61079 Condone truncation. Accept more than 5 dp
		3	

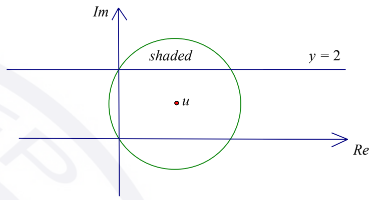
Question	Answer	Marks	Guidance
8(i)	State or imply the form $\frac{A}{2+x}+\frac{B}{3-x}+\frac{C}{(3-x)^2}$	**B1**	
	Use a correct method to obtain a constant	**M1**	
	Obtain one of $A=2$, $B=2$, $C=-7$	**A1**	
	Obtain a second value	**A1**	
	Obtain the third value	**A1**	[Mark the form $\frac{A}{2+x}+\frac{Dx+E}{(3-x)^2}$, where $A=2$, $D=-2$ and $E=-1$, B1M1A1A1A1.]
		5	
8(ii)	Use a correct method to find the first two terms of the expansion of $(2+x)^{-1}$, $(3-x)^{-1}$ or $(3-x)^{-2}$, or equivalent, e.g. $\left(1+\frac{1}{2}x\right)^{-1}$	**M1**	
	Obtain correct unsimplified expansions up to the term in x^2 of each partial fraction	**A1** **A1** **A1**	**FT** on A, B and C $1-\frac{x}{2}+\frac{x^2}{4}$ $\frac{2}{3}\left(1+\frac{x}{3}+\frac{x^2}{9}\right)$ $-\frac{7}{9}\left(1+\frac{2x}{3}+\frac{3x^2}{9}\right)$
	Obtain final answer $\frac{8}{9}-\frac{43}{54}x+\frac{7}{108}x^2$	**A1**	
			For the A, D, E form of fractions give M1A1**ft**A1**ft** for the expanded partial fractions, then, if $D \neq 0$, M1 for multiplying out fully, and A1 for the final answer.
		5	

Question	Answer	Marks	Guidance
9(i)	Obtain a vector parallel to the plane, e.g. $\overrightarrow{CB} = 2\mathbf{i} + \mathbf{j}$	**B1**	
	Use scalar product to obtain an equation in a, b, c,	**M1**	e.g. $2a + b = 0$, $a + 5c = 0$, $a + b - 5c = 0$
	Obtain two correct equations in a, b, c	**A1**	
	Solve to obtain $a : b : c$,	**M1**	or equivalent
	Obtain $a : b : c = 5 : -10 : -1$,	**A1**	or equivalent
	Obtain equation $5x - 10y - z = -25$,	**A1**	or equivalent
	Alternative method 1		
	Obtain a vector parallel to the plane, e.g. $\overrightarrow{CD} = \mathbf{i} + 5\mathbf{k}$	**B1**	$\overrightarrow{BD} = -\mathbf{i} - \mathbf{j} + 5\mathbf{k}$
	Obtain a second such vector and calculate their vector product, e.g. $(2\mathbf{i} + \mathbf{j}) \times (\mathbf{i} + 5\mathbf{k})$	**M1**	
	Obtain two correct components	**A1**	
	Obtain correct answer, e.g. $5\mathbf{i} - 10\mathbf{j} - \mathbf{k}$	**A1**	
	Substitute to find d	**M1**	
	Obtain equation $5x - 10y - z = -25$,	**A1**	or equivalent

Question	Answer	Marks	Guidance
9(i)	**Alternative method 2**		
	Obtain a vector parallel to the plane, e.g. $\overrightarrow{DB} = \mathbf{i} + \mathbf{j} - 5\mathbf{k}$	**B1**	
	Obtain a second such vector and form correctly a 2-parameter equation for the plane	**M1**	
	State a correct equation, e.g. $\mathbf{r} = 3\mathbf{i} + 4\mathbf{j} + \lambda(\mathbf{i} + 5\mathbf{k}) + \mu(\mathbf{i} + \mathbf{j} - 5\mathbf{k})$	**A1**	
	State three equations in x, y, z, λ and μ	**A1**	
	Eliminate λ and μ	**M1**	
	Obtain equation $5x - 10y - z = -25$	**A1**	or equivalent
	Alternative method 3		
	Substitute for B and C and obtain $3a + 4b = d$ and $a + 3b = d$	**B1**	
	Substitute for D to obtain a third equation and eliminate one unknown (a, b, or d) entirely	**M1**	
	Obtain two correct equations in two unknowns, e.g. a, b, c	**A1**	
	Solve to obtain their ratio, e.g. $a : b : c$	**M1**	
	Obtain $a : b : c = 5 : -10 : -1$, $a : c : d = 5 : -1 : -25$, or $b : c : d = 10 : 1 : 25$	**A1**	or equivalent
	Obtain equation $5x - 10y - z = -25$	**A1**	or equivalent

Question	Answer	Marks	Guidance
9(i)	**Alternative method 4**		
	Substitute for B and C and obtain $3a + 4b = d$ and $a + 3b = d$	**B1**	
	Solve to obtain $a : b : d$	**M2**	or equivalent
	Obtain $a : b : d = 1 : -2 : -5$	**A1**	or equivalent
	Substitute for C to obtain c	**M1**	
	Obtain equation $5x - 10y - z = -25$	**A1**	or equivalent
		6	
9(ii)	State or imply a normal vector for the plane $OABC$ is **k**	**B1**	
	Carry out correct process for evaluating a scalar product of two relevant vectors, e.g. (5**i** – 10**j** – **k**).(**k**)	**M1**	i.e. correct process using **k** and their normal
	Using the correct process for calculating the moduli, divide the scalar product by the product of the moduli and evaluate the inverse cosine of the result	**M1**	Allow M1M1 for clear use of an incorrect vector that has been stated to be the normal to $OABC$
	Obtain answer 84.9° or 1.48 radians	**A1**	
		4	

Question	Answer	Marks	Guidance
10(i)	State or imply $r = 2$	**B1**	Accept $\sqrt{4}$
	State or imply $\theta = \frac{1}{6}\pi$	**B1**	
	Use a correct method for finding the modulus or the argument of u^4	**M1**	Allow correct answers from correct u with minimal working shown
	Obtain modulus 16	**A1**	
	Obtain argument $\frac{2}{3}\pi$	**A1**	Accept $16e^{i\frac{2\pi}{3}}$
		5	
10(ii)	Substitute u and carry out a correct method for finding u^3	**M1**	$(u^3 = 8i)$ Follow *their* u^3 if found in part **(i)**
	Verify u is a root of the given equation	**A1**	
	State that the other root is $\sqrt{3} - \mathrm{i}$	**B1**	
	Alternative method		
	State that the other root is $\sqrt{3} - \mathrm{i}$	**B1**	
	Form quadratic factor and divide cubic by quadratic	**M1**	$(z-\sqrt{3}-i)(z-\sqrt{3}+i)(= z^2 - 2\sqrt{3}z + 4)$
	Verify that remainder is zero and hence that u is a root of the given equation	**A1**	
		3	

Question	Answer	Marks	Guidance
10(iii)	Show the point representing u in a relatively correct position	**B1**	
	Show a circle with centre u and radius 2	**B1**	**FT** on the point representing u. Condone near miss of origin
	Show the line $y = 2$	**B1**	
	Shade the correct region	**B1**	
	Show that the line and circle intersect on $x = 0$	**B1**	Condone near miss
		5	

Cambridge Assessment International Education
Cambridge International Advanced Level

MATHEMATICS **9709/32**

Paper 3 **May/June 2019**

MARK SCHEME

Maximum Mark: 75

Published

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Question	Answer	Marks	Guidance
1	State unsimplified term in x^2, or its coefficient in the expansion of $(1+3x)^{\frac{1}{3}}$ $\left(\dfrac{\frac{1}{3}\times\frac{-2}{3}}{2}(3x)^2\right)$	**B1**	Symbolic binomial coefficients are not sufficient for the B marks
	State unsimplified term in x^3, or its coefficient in the expansion of $(1+3x)^{\frac{1}{3}}$ $\left(\dfrac{\frac{1}{3}\times\frac{-2}{3}\times\frac{-5}{3}}{6}(3x)^3\right)$	**B1**	
	Multiply by (3 – *x)* to give 2 terms in x^3, or their coefficients	**M1**	$\left(3\times\frac{10}{6}+1\right)$ Ignore errors in terms other than x^3 $3\times x^3\text{coeff}-x^2\text{coeff}$ and no other term in x^3
	Obtain answer 6	**A1**	Not $6x^3$
		4	

Question	Answer	Marks	Guidance
2	State or imply $u^2 - u - 12 (= 0)$, or equivalent in 3^x	**B1**	Need to be convinced they know $3^{2x} = \left(3^x\right)^2$
	Solve for *u*, or for 3^x, and obtain root 4	**B1**	
	Use a correct method to solve an equation of the form $3^x = a$ where a >0	**M1**	Need to see evidence of method. Do not penalise an attempt to use the negative root as well. e.g. $x \ln 3 = \ln a, \quad x = \log_3 a$ If seen, accept solution of straight forward cases such as $3^x = 3$, $x = 1$ without working
	Obtain final answer $x = 1.26$ **only**	**A1**	The Q asks for 2 dp
		4	

Question	Answer	Marks	Guidance
3	Use correct trig formulae to obtain an equation in tan θ or equivalent (e.g all in sin θ or all in cos θ)	***M1**	$\frac{1-\tan^2\theta}{2\tan\theta}=2\tan\theta$. Allow $\frac{\cot^2\theta-1}{2\cot\theta}=\frac{2}{\cot\theta}$
	Obtain a correct simplified equation	**A1**	$5\tan^2\theta=1$ or $\sin^2\theta=\frac{1}{6}$ or $\cos^2\theta=\frac{5}{6}$
	Solve for θ	**DM1**	Dependent on the first M1
	Obtain answer 24.1° (or 155.9°)	**A1**	One correct in range to at least 3 sf
	Obtain second answer	**A1**	**FT** $180° - their\ 24.1°$ and no others in range. Correct to at least 3 sf. Accept $156°$ but not 156.0 Ignore values outside range If working in $\tan\theta$ or $\cos\theta$ need to be considering both square roots to score the second A1 Mark 0.421, 2.72 as a MR, so A0A1
		5	

Question	Answer	Marks	Guidance
4	Use correct quotient rule	**M1**	Allow use of correct product rule on $x\times\left(1+\ln x\right)^{-1}$
	Obtain correct derivative in any form	**A1**	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\left(1+\ln x\right)-x\times\frac{1}{x}}{\left(1+\ln x\right)^{2}}=\left(\frac{1}{1+\ln x}-\frac{1}{\left(1+\ln x\right)^{2}}\right)$
	Equate derivative to $\frac{1}{4}$ and obtain a quadratic in ln x or (1+ ln x)	**M1**	Horizontal form. Accept $\ln x=\frac{1}{4}\left(1+\ln x\right)^{2}$
	Reduce to $(\ln x)^2-2\ln x+1=0$	**A1**	or 3-term equivalent. Condone $\ln x^2$ if later used correctly
	Solve a 3-term quadratic in ln x for x	**M1**	Must see working if solving incorrect quadratic
	Obtain answer x = e	**A1**	Accept e^1
	Obtain answer $y=\frac{1}{2}$ e	**A1**	Exact only with no decimals seen before the exact value. Accept $\frac{\mathrm{e}^1}{2}$ but not $\frac{\mathrm{e}}{1+\ln \mathrm{e}}$
		7	

Question	Answer	Marks	Guidance
5(i)	State answer $-1-\sqrt{3}\mathrm{i}$	**B1**	If $-\frac{1}{2}$ given as well at this point, still just B1
		1	

Question	Answer	Marks	Guidance
5(ii)	Substitute $x = -1+\sqrt{3}\mathrm{i}$ in the equation and attempt expansions of x^2 and x^3	**M1**	Need to see sufficient working to be convinced that a calculator has not been used.
	Use $\mathrm{i}^2 = -1$ correctly at least once	**M1**	Allow for relevant use at any point in the solution
	Obtain $k = 2$	**A1**	
	Carry out a complete method for finding a quadratic factor with zeros $-1+\sqrt{3}\mathrm{i}$ and $-1-\sqrt{3}\mathrm{i}$	**M1**	Could use factor theorem from this point. Need to see working. M1 for correct testing of correct root or allow M1 for three unsuccessful valid attempts.
	Obtain x^2+2x+4	**A1**	Using factor theorem, obtain $\mathrm{f}\left(-\frac{1}{2}\right)=0$
	Obtain root $x = -\frac{1}{2}$, or equivalent, *via* division or inspection	**A1**	Final answer

Question	Answer	Marks	Guidance
5(ii)	**Alternative method 1**		
	Carry out a complete method for finding a quadratic factor with zeros $-1+\sqrt{3}\mathrm{i}$ and $-1-\sqrt{3}\mathrm{i}$ (multiplying two linear factors or using sum and product of roots)	**M1**	Need to see sufficient working to be convinced that a calculator has not been used.
	Use $\mathrm{i}^2=-1$ correctly at least once	**M1**	Allow for relevant use at any point in the solution
	Obtain x^2+2x+4	**A1**	Allow M1A0 for x^2+2x+3
	Obtain linear factor $kx+1$ and compare coefficients of x or x^2 and solve for k	**M1**	Can find the factor by inspection or by long division Must get to zero remainder
	Obtain $k = 2$	**A1**	
	Obtain root $x=-\frac{1}{2}$	**A1**	Final answer
			Note: Verification that $x=-\frac{1}{2}$ is a root is worth no marks without a clear demonstration of how the root was obtained

Question	Answer	Marks	Guidance
5(ii)	**Alternative method 2**		
	Use equation for sum of roots of cubic and use equation for product of roots of cubic	**M1**	
	Use $\mathrm{i}^2 = -1$ correctly at least once	**M1**	Allow for relevant use at any point in the solution
	Obtain $-\frac{5}{k} = -2 + \gamma,\quad -\frac{4}{k} = 4\gamma$	**A1**	
	Solve simultaneous equations for k and γ	**M1**	
	Obtain $k = 2$	**A1**	
	Obtain root $\gamma = -\frac{1}{2}$	**A1**	Final answer
		6	

Question	Answer	Marks	Guidance
6(i)	Correct use of trigonometry to obtain $AB = 2r\cos x$	**B1**	**AG**
		1	

Question	Answer	Marks	Guidance
6(ii)	Use correct method for finding the area of the sector and the semicircle and form an equation in x	M1	$\frac{1}{2}\times\frac{1}{2}\pi r^2=\frac{1}{2}\left(2r\cos x\right)^2 2x$
	Obtain $x=\cos^{-1}\sqrt{\frac{\pi}{16x}}$ correctly **AG**	A1	Via correct simplification e.g. from $\cos^2 x=\frac{\pi}{16x}$
		2	
6(iii)	Calculate values of a relevant expression or pair of expressions at $x = 1$ and $x = 1.5$ Must be working in radians	M1	e.g. $\begin{array}{ll} x=1 & 1\to 1.11 \\ x=1.5 & 1.5\to 1.20 \end{array}$ Accept $\begin{array}{l} f\left(1\right)=1.11 \\ f\left(1.5\right)=1.20 \end{array}$ $f\left(x\right)=x-\cos^{-1}\sqrt{\frac{\pi}{16x}}$: $f\left(1\right)=-0.111.,\ f\left(1.5\right)=0.3..$ $f(x)=\cos x-\sqrt{\frac{\pi}{16x}}$: $f(1)=0.097.,f(1.5)=-0.291.$ For $16x\cos^2 x-\pi$ $f(1)=1.529..,f(1.5)=-3.02..$ Must find values. M1 if at least one value correct
	Correct values and complete the argument correctly	A1	
		2	

Question	Answer	Marks	Guidance
6(iv)	Use $x_{n+1} = \cos^{-1}\sqrt{\left(\frac{\pi}{16x_n}\right)}$ correctly at least twice Must be working in radians	M1	$1, 1.11173, 1.13707, 1.14225, 1.14329, 1.14349, 1.14354, 1.14354$ $1.25, 1.16328, 1.14742, 1.14432, 1.14370$ $1.5, 1.20060, 1.15447, 1.14570, 1.14397, 1.14363$
	Obtain final answer 1.144	A1	
	Show sufficient iterations to at least 5 d.p. to justify 1.144 to 3 d.p. or show there is a sign change in the interval (1.1435, 1.1445)	A1	
		3	

Question	Answer	Marks	Guidance
7(i)	Separate variables correctly and attempt integration of at least one side	**B1**	$\int e^{-y}\,dy = \int xe^{x}\,dx$
	Obtain term $-e^{-y}$	**B1**	B0B1 is possible
	Commence integration by parts and reach $xe^{x} \pm \int e^{x}\,dx$	**M1**	B0B0M1A1 is possible
	Obtain $xe^{x} - e^{x}$	**A1**	or equivalent
			B1B1M1A1 is available if there is no constant of integration
	Use $x = 0$, $y = 0$ to evaluate a constant, or as limits in a definite integral, in a solution with terms ae^{-y}, bxe^{x} and ce^{x}, where $abc \neq 0$	**M1**	Must see this step
	Obtain correct solution in any form	**A1**	e.g. $e^{-y} = e^{x} - xe^{x}$
	Rearrange as $y = -\ln(1-x) - x$	**A1**	or equivalent e.g. $y = \ln\dfrac{1}{e^{x}(1-x)}$ ISW
		7	
7(ii)	Justify the given statement	**B1**	e.g. require $1 - x > 0$ for the ln term to exist, hence $x < 1$ Must be considering the range of values of x, and must be relevant to *their* y involving $\ln(1-x)$
		1	

Question	Answer	Marks	Guidance
8(i)	State or imply the form $\dfrac{A}{2x+1}+\dfrac{B}{2x+3}+\dfrac{C}{(2x+3)^2}$	**B1**	
	Use a correct method to find a constant	**M1**	
	Obtain the values $A = 1$, $B = -1$, $C = 3$	**A1 A1 A1**	
	[Mark the form $\dfrac{A}{2x+1}+\dfrac{Dx+E}{(2x+3)^2}$, where $A = 1$, $D = -2$ and $E = 0$, B1M1A1A1A1 as above.]		Full marks for the three correct constants – do not actually need to see the partial fractions
		5	
8(ii)	Integrate and obtain terms $\frac{1}{2}\ln(2x+1)-\frac{1}{2}\ln(2x+3)-\dfrac{3}{2(2x+3)}$ [Correct integration of the A, D, E form of fractions gives $\frac{1}{2}\ln(2x+1)+\dfrac{x}{2x+3}-\frac{1}{2}\ln(2x+3)$ if integration by parts is used for the second partial fraction.]	**B1 B1 B1**	**FT** on A, B and C.
	Substitute limits correctly in an integral with terms $a\ln(2x+1)$, $b\ln(2x+3)$ and $c/(2x+3)$, where $abc \neq 0$ If using alternative form: $cx/(2x+3)$	**M1**	value for upper limit – value for lower limit 1 slip in substituting can still score M1 Condone omission of $\ln(1)$
	Obtain the **given answer** following full and correct working	**A1**	Need to see at least one interim step of valid log work. **AG**
		5	

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td>9(i)</td><td>Carry out correct method for finding a vector equation for <i>AB</i></td><td>M1</td><td></td></tr>
<tr><td></td><td>Obtain $(\mathbf{r} =)\mathbf{i} + 2\mathbf{j} - \mathbf{k} + \lambda(2\mathbf{i} - \mathbf{j} + 2\mathbf{k})$, or equivalent</td><td>A1</td><td></td></tr>
<tr><td></td><td>Equate two pairs of components of general points on <i>their AB</i> and <i>l</i> and solve for λ or for μ</td><td>M1</td><td>$\begin{pmatrix} 1+2\lambda \\ 2-\lambda \\ -1+2\lambda \end{pmatrix} = \begin{pmatrix} 2+\mu \\ 1+\mu \\ 1+2\mu \end{pmatrix}$</td></tr>
<tr><td></td><td>Obtain correct answer for λ or μ, e.g. $\lambda = 0$, $\mu = -1$</td><td>A1</td><td></td></tr>
<tr><td></td><td>Verify that all three equations are not satisfied and the lines fail to intersect ($\neq$ is sufficient justification e.g. $2 \neq 0$)
Conclusion needs to follow correct values</td><td>A1</td><td>Alternatives
<table>
<tr><th><i>A</i></th><th>λ</th><th>μ</th><th></th><th><i>B</i></th><th>λ</th><th>μ</th><th></th></tr>
<tr><td>ij</td><td>$2/3$</td><td>$1/3$</td><td>$\frac{1}{3} \neq \frac{5}{3}$</td><td></td><td>$-1/3$</td><td>$1/3$</td><td>$\frac{1}{3} \neq \frac{5}{3}$</td></tr>
<tr><td>ik</td><td>0</td><td>–1</td><td>$2 \neq 0$</td><td></td><td>–1</td><td>–1</td><td>$2 \neq 0$</td></tr>
<tr><td>jk</td><td>1</td><td>0</td><td>$3 \neq 2$</td><td></td><td>0</td><td>0</td><td>$3 \neq 2$</td></tr>
</table></td></tr>
<tr><td></td><td></td><td>5</td><td></td></tr>
</table>

Question	Answer	Marks	Guidance
9(ii)	State or imply midpoint has position vector $2\mathbf{i}+\frac{3}{2}\mathbf{j}$	**B1**	
	Substitute in $2x - y + 2z = d$ and find d	**M1**	Correct use of *their* direction for *AB* and *their* midpoint
	Obtain plane equation $4x - 2y + 4z = 5$	**A1**	or equivalent e.g. $\mathbf{r}.\begin{pmatrix}2\\-1\\2\end{pmatrix}=\frac{5}{2}$
	Substitute components of l in plane equation and solve for μ	**M1**	Correct use of their plane equation.
	Obtain $\mu = -\frac{1}{2}$ and position vector $\frac{3}{2}\mathbf{i}+\frac{1}{2}\mathbf{j}$ for the point P	**A1**	Final answer Accept coordinates in place of position vector
		5	

Question	Answer	Marks	Guidance
10(i)	State correct expansion of $\sin(3x+x)$ or $\sin(3x-x)$	**B1**	B0 If their formula retains ± in the middle
	Substitute expansions in $\frac{1}{2}(\sin 4x+\sin 2x)$	**M1**	
	Obtain $\sin 3x\cos x=\frac{1}{2}(\sin 4x+\sin 2x)$ correctly	**A1**	Must see the $\sin 4x$ and $\sin 2x$ or reference to LHS and RHS for A1 **AG**
		3	
10(ii)	Integrate and obtain $-\frac{1}{8}\cos 4x-\frac{1}{4}\cos 2x$	**B1 B1**	
	Substitute limits $x = 0$ and $x=\frac{1}{3}\pi$ correctly	**M1**	In their expression
	Obtain answer $\frac{9}{16}$	**A1**	From correct working seen.
		4	

Question	Answer	Marks	Guidance
10(iii)	State correct derivative $2\cos 4x + \cos 2x$	**B1**	
	Using correct double angle formula, express derivative in terms of $\cos 2x$ and equate the result to zero	**M1**	
	Obtain $4\cos^2 2x + \cos 2x - 2 = 0$	**A1**	
	Solve for *x* or 2*x* (could be labelled *x*) $\left(\cos 2x = \dfrac{-1 \pm \sqrt{33}}{8}\right)$	**M1**	Must see working if solving an incorrect quadratic The roots of the correct quadratic are -0.843 and 0.593 Need to get as far as $x = \ldots$ The wrong value of *x* is 0.468 and can imply M1 if correct quadratic seen Could be working from a quartic in $\cos x$: $16\cos^4 x - 14\cos^2 x + 1 = 0$
	Obtain answer *x* = 1.29 only	**A1**	
		5	

Cambridge International A Level

Cambridge Assessment International Education
Cambridge International Advanced Level

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Paper 3 **May/June 2019**

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<u>Mark Scheme Notes</u>

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more “method” steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously “correct” answers or results obtained from incorrect working.

  - Note: B2 or A2 means that the candidate can earn 2 or 0.
    B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no “follow through” from a previous error is allowed)
CWO	Correct Working Only – often written by a ‘fortuitous’ answer
ISW	Ignore Subsequent Working
SOI	Seen or implied
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become “follow through” marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Question	Answer	Marks	Guidance
1	Use law of the logarithm of a product or quotient	**M1**	
	Use law of the logarithm of power **twice**	**M1**	
	Obtain a correct linear equation in x, e.g. $(3-2x)\ln 5=\ln 4+x\ln 7$	**A1**	
	Obtain answer x = 0.666	**A1**	
		4	

Question	Answer	Marks	Guidance
2	Commence integration and reach $ax^2\sin 2x + b\int x\sin 2x\,\mathrm{d}x$	**M1***	
	Obtain $\frac{1}{2}x^2\sin 2x-\int x\sin 2x\,\mathrm{d}x$, or equivalent	**A1**	
	Complete the integration and obtain $\frac{1}{2}x^2\sin 2x+\frac{1}{2}x\cos 2x-\frac{1}{4}\sin 2x$, or equivalent	**A1**	
	Use limits correctly, having integrated twice	**DM1**	
	Obtain given answer correctly	**A1**	
		5	

Question	Answer	Marks	Guidance
3(i)	Use double angle formulae and express entire fraction in terms of $\sin\theta$ and $\cos\theta$	**M1**	
	Obtain a correct expression	**A1**	
	Obtain the given answer	**A1**	
		3	
3(ii)	State integral of the form $\pm\ln\cos\theta$	**M1***	
	Use correct limits correctly and insert exact values for the trig ratios	**DM1**	
	Obtain a correct expression, e.g. $-\ln\frac{1}{\sqrt{2}} + \ln\frac{\sqrt{3}}{2}$	**A1**	
	Obtain the given answer following full and exact working	**A1**	
		4	

Question	Answer	Marks	Guidance
4(i)	Use the quotient or product rule	**M1**	
	Obtain correct derivative in any form	**A1**	
	Reduce to $-\frac{2e^{-x}}{\left(1-e^{-x}\right)^2}$, or equivalent, and explain why this is always negative	**A1**	
		3	

Question	Answer	Marks	Guidance
4(ii)	Equate derivative to – 1 and obtain the given equation	**B1**	
	State or imply $u^2 - 4u + 1 = 0$, or equivalent in e^a	**B1**	
	Solve for *a*	**M1**	
	Obtain answer $a = \ln\left(2+\sqrt{3}\right)$ and no other	**A1**	
		4	

Question	Answer	Marks	Guidance
5	Separate variables correctly and integrate at least one side	**B1**	
	Obtain term $\ln(x+1)$	**B1**	
	Obtain term of the form $a\ln\left(y^2+5\right)$	**M1**	
	Obtain term $\frac{1}{2}\ln(y^2+5)$	**A1**	
	Use $y = 2$, $x = 0$ to determine a constant, or as limits, in a solution containing terms $a\ln(y^2+5)$ and $b\ln(x+1)$, where $ab \neq 0$	**M1**	
	Obtain correct solution in any form	**A1**	
	Obtain final answer $y^2 = 9(x+1)^2 - 5$	**A1**	
		7	

Question	Answer	Marks	Guidance
6(i)	State $b = 3$	**B1**	
		1	
6(ii)	Commence division by $x - b$ and reach partial quotient $x^3 + kx^2$	**M1**	
	Obtain quotient $x^3 + x^2 + 3x + 2$	**A1**	There being no remainder
	Equate quotient to zero and rearrange to make the subject a	**M1**	
	Obtain the given equation	**A1**	
		4	
6(iii)	Use the iterative formula $a_{n+1} = -\frac{1}{3}\left(2 + a_n{}^2 + a_n{}^3\right)$ correctly at least once	**M1**	
	Obtain final answer –0.715	**A1**	
	Show sufficient iterations to 5 d.p. to justify –0.715 to 3 d.p., or show there is a sign change in the interval (–0.7145, –0.7155)	**A1**	
		3	

Question	Answer	Marks	Guidance
7(i)	Use product rule	**M1**	
	Obtain correct derivative in any form	**A1**	
		2	
7(ii)	Equate derivative to zero and use correct $\cos(A + B)$ formula	**M1**	
	Obtain the given equation	**A1**	
		2	
7(iii)	Use correct method to solve for x	**M1**	
	Obtain answer, e.g. $x = \frac{1}{12}\pi$	**A1**	
	Obtain second answer, e.g. $\frac{7}{12}\pi$, and no other	**A1**	
		3	

Question	Answer	Marks	Guidance
8(i)	Multiply numerator and denominator by $1 + \sqrt{3}\mathrm{i}$, or equivalent	**M1**	
	$4\mathrm{i} - 4\sqrt{3}$ and $3 + 1$	**A1**	
	Obtain final answer $-\sqrt{3} + \mathrm{i}$	**A1**	
		3	

Question	Answer	Marks	Guidance
8(ii)	State that the modulus of u is 2	**B1**	
	State that the argument of u is $\frac{5}{6}\pi$ (or 150°)	**B1**	
		2	
8(iii)	Show a circle with centre the origin and radius 2	**B1**	
	Show u in a relatively correct position	**B1**	**FT**
	Show the perpendicular bisector of the line joining u and the origin	**B1**	**FT**
	Shade the correct region	**B1**	
		4	

Question	Answer	Marks	Guidance
9(i)	State or imply the form $\frac{A}{3+x}+\frac{B}{1-x}+\frac{C}{(1-x)^2}$	**B1**	
	Use a correct method for finding a constant	**M1**	
	Obtain one of $A=-3$, $B=-1$, $C=2$	**A1**	
	Obtain a second value	**A1**	
	Obtain the third value	**A1**	Mark the form $\frac{A}{3+x}+\frac{Dx+E}{(1-x)^2}$, where $A=-3$, $D=1$ and $E=1$, B1M1A1A1A1 as above.
		5	
9(ii)	Use a correct method to find the first two terms of the expansion of $(3+x)^{-1}$, $(1+\frac{1}{3}x)^{-1}$, $(1-x)^{-1}$ or $(1-x)^{-2}$	**M1**	
	Obtain correct unsimplified expansions up to the term in x^3 of each partial fraction	**A1**	**FT** on A
		A1	**FT** on B
		A1	**FT** on C
	Obtain final answer $\frac{10}{3}x+\frac{44}{9}x^2+\frac{190}{27}x^3$	**A1**	For the A, D, E form of fractions give M1A1ftA1ft for the expanded partial fractions, then, if $D \neq 0$, M1 for multiplying out fully, and A1 for the final answer.
		5	

Question	Answer	Marks	Guidance
10(i)	Find $\overrightarrow{PQ}$ for a general point Q on l, e.g. $-3\mathbf{i}+6\mathbf{k}+\mu(2\mathbf{i}-\mathbf{j}-2\mathbf{k})$	**B1**	
	Calculate scalar product of $\overrightarrow{PQ}$ and a direction vector for l and equate the result to zero	**M1**	
	Solve for μ and obtain $\mu = 2$	**A1**	
	Carry out a complete method for finding the length of $\overrightarrow{PQ}$	**M1**	
	Obtain answer 3	**A1**	
	Alternative method for question 10(i)		
	Calling the point (1, 2, 3) A, state $\overrightarrow{AP}$ (or $\overrightarrow{PA}$) in component form, e.g. $3\mathbf{i} - 6\mathbf{k}$	**B1**	
	Use a scalar product with a direction vector for l to find the projection of $\overrightarrow{AP}$ (or $\overrightarrow{PA}$) on l	**M1**	
	Obtain correct answer in any form, e.g. $\frac{18}{\sqrt{9}}$	**A1**	
	Use Pythagoras to find the perpendicular	**M1**	
	Obtain answer 3	**A1**	

Question	Answer	Marks	Guidance
10(i)	**Alternative method for question 10(i)**		
	State $\overrightarrow{AP}$ (or $\overrightarrow{PA}$) in component form	**B1**	
	Calculate a vector product with a direction vector for l	**M1**	
	Obtain correct answer, e.g. $6\mathbf{i} - 6\mathbf{j} - 3\mathbf{k}$	**A1**	
	Divide modulus of the product by that of the direction vector	**M1**	
	Obtain answer 3	**A1**	
		5	

Question	Answer	Marks	Guidance
10(ii)	Substitute coordinates of a general point of *l* in the plane equation and equate constant terms	**M1**	
	Obtain a correct equation, e.g. $a + 2b + 6 = 13$	**A1**	
	Equate the coefficient of μ to zero	**M1**	
	Obtain a correct equation, e.g. $2a – b – 4 = 0$	**A1**	
	Substitute (1, 2, 3) in the plane equation	**M1**	
	Obtain a correct equation, e.g. $a + 2b + 6 = 13$	**A1**	
	Alternative method for question 10(ii)		
	Find a second point on *l* and obtain an equation in *a* and/or *b*	**M1**	
	Obtain a correct equation, e.g. $5a – 2 = 13$	**A1**	
	Equate scalar product of a direction vector for *l* and a vector normal for the plane to zero	**M1**	
	Obtain a correct equation, e.g. $2a – b – 4 = 0$	**A1**	
	Solve for *a* or for *b*	**M1**	
	Obtain $a = 3$ and $b = 2$	**A1**	
		6	

Cambridge Assessment International Education
Cambridge International Advanced Level

MATHEMATICS **9709/32**

Paper 3 Pure Mathematics **March 2019**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the March 2019 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

This document consists of **15** printed pages.

[Turn over

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

  - Note: B2 or A2 means that the candidate can earn 2 or 0.
    B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
CWO	Correct Working Only – often written by a 'fortuitous' answer
ISW	Ignore Subsequent Working
SOI	Seen or implied
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Question	Answer	Marks	Guidance
1(i)	Use law for the logarithm of a product or quotient	**M1**	
	Use $\log_{10} 100 = 2$ or $10^2 = 100$	**M1**	
	Obtain $x^2 - 4x - 100 = 0$, or equivalent	**A1**	
		3	
1(ii)	Solve a 3-term quadratic equation	**M1**	
	Obtain answer 12.2 only	**A1**	
		2	

Question	Answer	Marks	Guidance
2(i)	Use the iterative formula correctly at least once	**M1**	
	Obtain answer 1.3195	**A1**	
	Show sufficient iterations to 6 d.p. to justify 1.3195 to 4 d.p., or show there is a sign change in (1.31945, 1.31955)	**A1**	
		3	
2(ii)	State $x = \dfrac{2x^6 + 12x}{3x^5 + 8}$, or equivalent	**B1**	
	State answer $\sqrt[5]{4}$, or exact equivalent	**B1**	
		2	

Question	Answer	Marks	Guidance
3(i)	Use trig formulae and obtain an equation in sin θ and cos θ	**M1**	
	Obtain a correct equation in any form	**A1**	
	Substitute exact trig ratios and obtain an expression for tan θ	**M1**	
	Obtain answer tan $\theta = \dfrac{2\sqrt{2}-1}{1-\sqrt{6}}$, or equivalent	**A1**	
		4	
3(ii)	State answer, e.g. $\theta = 128.4°$	**B1**	
	State second answer, e.g. $\theta = 308.4°$	**B1 ft**	
		2	

Question	Answer	Marks	Guidance
4	Integrate by parts and reach $ax^{-\frac{1}{2}}\ln x + b\int x^{-\frac{1}{2}}.\frac{1}{x}\mathrm{d}x$	**M1***	
	Obtain $-2x^{-\frac{1}{2}}\ln x + 2\int x^{-\frac{1}{2}}.\frac{1}{x}\mathrm{d}x$, or equivalent	**A1**	
	Complete the integration, obtaining $-2x^{-\frac{1}{2}}\ln x - 4x^{-\frac{1}{2}}$, or equivalent	**A1**	
	Substitute limits correctly, having integrated twice	**M1(dep*)**	
	Obtain the given answer following full and correct working	**A1**	
		5	

Question	Answer	Marks	Guidance
5	State $\cos y \dfrac{dy}{dx}$ as derivative of $\sin y$	**B1**	
	State correct derivative in terms of x and y, e.g. $\sec^2 x / \cos y$	**B1**	
	State correct derivative in terms of x, e.g. $\dfrac{\sec^2 x}{\sqrt{1-\tan^2 x}}$	**B1**	
	Use double angle formula	**M1**	
	Obtain the given answer correctly	**A1**	
		5	

Question	Answer	Marks	Guidance
6	Separate variables correctly and attempt integration of at least one side	**B1**	
	Obtain term $-\dfrac{1}{2y^2}$, or equivalent	**B1**	
	Obtain term $-k\ \mathrm{e}^{-x}$	**B1**	
	Use a pair of limits, e.g. $x = 0$, $y = 1$ to obtain an equation in k and an arbitrary constant c	**M1**	
	Use a second pair of limits, e.g. $x = 1$, $y = \sqrt{\mathrm{e}}$, to obtain a second equation and solve for k or for c	**M1**	
	Obtain $k = \dfrac{1}{2}$ and $c = 0$	**A1**	
	Obtain final answer $y = \mathrm{e}^{\frac{1}{2}x}$, or equivalent	**A1**	
		7	

Question	Answer	Marks	Guidance
7(a)	Use quadratic formula to solve for z	**M1**	
	Use $\mathrm{i}^2 = -1$ throughout	**M1**	
	Obtain correct answer in any form	**A1**	
	Multiply numerator and denominator by 1 – i, or equivalent	**M1**	
	Obtain final answer, e.g. 1 – i	**A1**	
	Obtain second final answer, e.g. $\frac{5}{2}+\frac{1}{2}\mathrm{i}$	**A1**	
		6	
7(b)	Show the point representing u in relatively correct position	**B1**	
	Show the horizontal line through $z = \mathrm{i}$	**B1**	
	Show correct half-lines from u, one of gradient 1 and the other vertical	**B1ft**	
	Shade the correct region	**B1**	
		4	

Question	Answer	Marks	Guidance
8(i)	State or imply the form $A+\frac{B}{2+x}+\frac{C}{3-2x}$	**B1**	
	Use a correct method for finding a constant	**M1**	
	Obtain one of $A = 2$, $B = -4$ and $C = 6$	**A1**	
	Obtain a second value	**A1**	
	Obtain the third value	**A1**	
		5	
8(ii)	Use correct method to find the first two terms of the expansion of $(2+x)^{-1}$ or $(3-2x)^{-1}$, or equivalent	**M1**	
	Obtain correct unsimplified expansions up to the term in x^2 of each partial fraction	**A1ft +A1ft**	The ft is on *B* and *C*
	Add the value of *A* to the sum of the expansions	**M1**	
	Obtain final answer $2+\frac{7}{3}x+\frac{7}{18}x^2$	**A1**	
		5	

Question	Answer	Marks	Guidance
9(i)	State or imply a correct normal vector to either plane, e.g. 2**i** + 3**j** – **k**, or **i** – 2**j** + **k**	B1	
	Carry out correct process for evaluating the scalar product of two normal vectors	M1	
	Using the correct process for the moduli, divide the scalar product of the two normal vectors by the product of their moduli and evaluate the inverse cosine of the result	M1	
	Obtain answer 56.9° or 0.994 radians	A1	
		4	
9(ii)	*EITHER*: Carry out a complete strategy for finding a point on the line (call the line *l*)	M1	
	Obtain such a point, e.g. (1, 1, 4)	A1	
	EITHER: State a correct equation for a direction vector a**i** + b**j** + c**k** for *l*, e.g. $2a + 3b – c = 0$	B1	
	State a second equation, e.g. $a – 2b + c = 0$, and solve for one ratio, e.g. $a : b$	M1	
	Obtain $a : b : c = 1 : – 3 : – 7$, or equivalent	A1	
	State a correct answer, e.g. **r** = **i** + **j** + 4**k** + λ(**i** – 3**j** – 7**k**)	A1	
	*OR*1: Attempt to calculate the vector product of the two normal vectors	M1	
	Obtain two correct components	A1	
	Obtain **i** – 3**j** – 7**k**, or equivalent	A1	
	State a correct answer, e.g. **r** = **i** + **j** + 4**k** + λ(**i** – 3**j** – 7**k**), or equivalent	A1	

Question	Answer	Marks	Guidance
9(ii)	*OR*2: Obtain a second point on *l* e.g. (0, 4, 11)	**B1**	
	Subtract position vectors and obtain a direction vector for *l*	**M1**	
	Obtain $\mathbf{i} - 3\mathbf{j} - 7\mathbf{k}$, or equivalent	**A1**	
	State a correct answer, e.g. $\mathbf{r} = 4\mathbf{j} + 11\mathbf{k} + \mu(\mathbf{i} - 3\mathbf{j} - 7\mathbf{k})$, or equivalent	**A1**	
	*OR*3: Express one variable in terms of a second	**M1**	
	Obtain a correct simplified expression, e.g. $y = 4 - 3x$	**A1**	
	Express the third variable in terms of the second	**M1**	
	Obtain a correct simplified expression, e.g. $z = 11 - 7x$	**A1**	
	Form a vector equation for the line	**M1**	
	State a correct answer, e.g. $\mathbf{r} = 4\mathbf{j} + 11\mathbf{k} + \lambda(\mathbf{i} - 3\mathbf{j} - 7\mathbf{k})$, or equivalent	**A1**	
		6	

Question	Answer	Marks	Guidance
9(ii)	*OR*4: Express one variable in terms of a second	**M1**	
	Obtain a correct simplified expression, e.g. $x = \frac{4}{3} - \frac{y}{3}$	**A1**	
	Express the same variable in terms of the third	**M1**	
	Obtain a correct simplified expression, e.g. $x = \frac{11}{7} - \frac{z}{7}$	**A1**	
	Form a vector equation for the line	**M1**	
	Obtain a correct answer, e.g. **r** = 4**j** + 11**k** +μ(**i** – 3**j** – 7**k**), or equivalent	**A1**	
		6	

Question	Answer	Marks	Guidance
10(i)	State or imply du = – sin x dx	**B1**	
	Using Pythagoras express the integral in terms of u	**M1**	
	Obtain integrand $\pm\sqrt{u}\left(1-u^2\right)$	**A1**	
	Integrate and obtain $-\frac{2}{3}u^{\frac{3}{2}}+\frac{2}{7}u^{\frac{7}{2}}$, or equivalent	**A1**	
	Change limits correctly and substitute correctly in an integral of the form $au^{\frac{3}{2}}+bu^{\frac{7}{2}}$	**M1**	Or substitute original limits correctly in an integral of the form $a(\cos x)^{\frac{3}{2}}+b(\cos x)^{\frac{7}{2}}$
	Obtain answer $\frac{8}{21}$	**A1**	
		6	
10(ii)	Use product rule and chain rule at least once	**M1**	
	Obtain correct derivative in any form	**A1 + A1**	
	Equate derivative to zero and obtain a horizontal equation in integral powers of sin x and cos x	**M1**	
	Use correct methods to obtain an equation in one trig function	**M1**	
	Obtain $\tan^2 x=6$, $7\cos^2 x=1$ or $7\sin^2 x=6$, or equivalent, and obtain answer 1.183	**A1**	
		6	

Cambridge International A Level

Cambridge Assessment International Education
Cambridge International Advanced Level

MATHEMATICS **9709/31**

Paper 3 **October/November 2018**

MARK SCHEME

Maximum Mark: 75

Published

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[Turn over

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- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more “method” steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously “correct” answers or results obtained from incorrect working.

  - Note: B2 or A2 means that the candidate can earn 2 or 0.
    B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no “follow through” from a previous error is allowed)
CWO	Correct Working Only – often written by a ‘fortuitous’ answer
ISW	Ignore Subsequent Working
SOI	Seen or implied
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become “follow through” marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Question	Answer	Marks	Guidance
1	*EITHER*: State or imply non-modular inequality $2^2(2x-a)^2 < (x+3a)^2$, or corresponding quadratic equation, or pair of linear equations $2(2x-a) = \pm(x+3a)$	**B1**	
	Make reasonable attempt at solving a 3-term quadratic, or solve two linear equations for x	**M1**	
	Obtain critical values $x = \frac{5}{3}a$ and $x = -\frac{1}{5}a$	**A1**	
	State final answer $-\frac{1}{5}a < x < \frac{5}{3}a$	**A1**	
	OR: Obtain critical value $x = \frac{5}{3}a$ from a graphical method, or by inspection, or by solving a linear equation or an inequality	**B1**	
	Obtain critical value $x = -\frac{1}{5}a$ similarly	**B2**	
	State final answer $-\frac{1}{5}a < x < \frac{5}{3}a$ [Do not condone $\leqslant$ for $<$ in the final answer.]	**B1**	
		4	

Question	Answer	Marks	Guidance
2	Rearrange the equation in the form $ae^{2x} = b$ or $ae^{x} = be^{-x}$	**M1**	
	Obtain correct equation in either form with $a = 2$ and $b = 5$	**A1**	
	Use correct method to solve for x	**M1**	
	Obtain answer $x = 0.46$	**A1**	
		4	

Question	Answer	Marks	Guidance
3 (i)	Sketch a relevant graph, e.g. $y = x^3$	**B1**	
	Sketch a second relevant graph, e.g. $y = 3 - x$, and justify the given statement	**B1**	Consideration of behaviour for $x < 0$ is needed for the second B1
		2	
3(ii)	State or imply the equation $x = \left(2x^3 + 3\right) / \left(3x^2 + 1\right)$	**B1**	
	Rearrange this in the form $x^3 = 3 - x$, or commence work *vice versa*	**B1**	
		2	

Question	Answer	Marks	Guidance
3(iii)	Use the iterative formula correctly at least once	**M1**	
	Obtain final answer 1.213	**A1**	
	Show sufficient iterations to 5 d.p. or more to justify 1.213 to 3 d.p., or show there is a sign change in the interval (1.2125, 1.2135)	**A1**	
		3	

Question	Answer	Marks	Guidance
4(i)	Obtain $\frac{\mathrm{d}x}{\mathrm{d}\theta} = 2\cos\theta + 2\cos 2\theta$ or $\frac{\mathrm{d}y}{\mathrm{d}\theta} = -2\sin\theta - 2\sin 2\theta$	**B1**	
	Use $\mathrm{d}y/\mathrm{d}x = \mathrm{d}y/\mathrm{d}\theta \div \mathrm{d}x/\mathrm{d}\theta$	**M1**	
	Obtain correct $\frac{\mathrm{d}y}{\mathrm{d}x}$ in any form, e.g. $-\frac{2\sin\theta + 2\sin 2\theta}{2\cos\theta + 2\cos 2\theta}$	**A1**	
		3	
4(ii)	Equate denominator to zero and use any correct double angle formula	**M1***	
	Obtain correct 3-term quadratic in cos θ in any form	**A1**	
	Solve for θ	**depM1***	
	Obtain $x = 3\sqrt{3}/2$ and $y = \frac{1}{2}$, or exact equivalents	**A1**	
		4	

Question	Answer	Marks	Guidance
5	Separate variables correctly and integrate at least one side	**B1**	
	Obtain term ln y	**B1**	
	Obtain terms 2 ln $x - \frac{1}{2}x^2$	**B1+B1**	
	Use $x = 1$, $y = 1$ to evaluate a constant, or as limits	**M1**	
	Obtain correct solution in any form, e.g. $\ln y = 2\ln x - \frac{1}{2}x^2 + \frac{1}{2}$	**A1**	
	Rearrange as $y = x^2 \exp\left(\frac{1}{2} - \frac{1}{2}x^2\right)$, or equivalent	**A1**	
		7	

Question	Answer	Marks	Guidance
6(i)	Rearrange in the form $\sqrt{3}\sin x - \cos x = \sqrt{2}$	**B1**	
	State $R = 2$	**B1**	
	Use trig formulae to obtain α	**M1**	
	Obtain α = 30° with no errors seen	**A1**	
		4	

Question	Answer	Marks	Guidance
6(ii)	Evaluate $\sin^{-1}\left(\frac{\sqrt{2}}{R}\right)$	**B1ft**	
	Carry out a correct method to find a value of x in the given interval	**M1**	
	Obtain answer $x = 75°$	**A1**	
	Obtain a second answer e.g. $x = 165°$ and no others [Treat answers in radians as a misread. Ignore answers outside the given interval.]	**A1ft**	
		4	

Question	Answer	Marks	Guidance
7(i)	Use product rule	**M1***	
	Obtain correct derivative in any form	**A1**	
	Equate derivative to zero and obtain an equation in a single trig function	**depM1***	
	Obtain a correct equation, e.g. $3\tan^2 x = 2$	**A1**	
	Obtain answer $x = 0.685$	**A1**	
		5	

Question	Answer	Marks	Guidance
7(ii)	Use the given substitution and reach $a\int\left(u^2 - u^4\right)\mathrm{d}u$	**M1**	
	Obtain correct integral with $a = 5$ and limits 0 and 1	**A1**	
	Use correct limits in an integral of the form $a\left(\frac{1}{3}u^3 - \frac{1}{5}u^5\right)$	**M1**	
	Obtain answer $\frac{2}{3}$	**A1**	
		4	

Question	Answer	Marks	Guidance
8(i)	*EITHER*: Multiply numerator and denominator by 1 + 2i, or equivalent, or equate to $x + iy$, obtain two equations in x and y and solve for x or for y	**M1**	
	Obtain quotient $-\frac{4}{5}+\frac{7}{5}$i, or equivalent	**A1**	
	Use correct method to find either r or θ	**M1**	
	Obtain $r = 1.61$	**A1**	
	Obtain $\theta = 2.09$	**A1**	
	OR: Find modulus or argument of 2 + 3i or of 1 – 2i	**B1**	
	Use correct method to find r	**M1**	
	Obtain $r = 1.61$	**A1**	
	Use correct method to find θ	**M1**	
	Obtain $\theta = 2.09$	**A1**	
		5	
8(ii)	Show a circle with centre 3 – 2i	**B1**	
	Show a circle with radius 1	**B1ft**	Centre not at the origin
	Carry out a correct method for finding the least value of $\lvert z\rvert$	**M1**	
	Obtain answer $\sqrt{13}-1$	**A1**	
		4	

Question	Answer	Marks	Guidance
9(i)	State or imply the form $\frac{A}{2-x}+\frac{B}{3+2x}+\frac{C}{(3+2x)^2}$	**B1**	
	Use a correct method to find a constant	**M1**	
	Obtain one of $A = 1$, $B = -1$, $C = 3$	**A1**	
	Obtain a second value	**A1**	
	Obtain the third value [Mark the form $\frac{A}{2-x}+\frac{Dx+E}{(3+2x)^2}$, where $A = 1$, $D = -2$ and $E = 0$, B1M1A1A1A1 as above.]	**A1**	
		5	
9(ii)	Integrate and obtain terms $-\ln(2-x)-\frac{1}{2}\ln(3+2x)-\frac{3}{2(3+2x)}$	**B3ft**	The f.t is on *A*, *B*, *C*; or on *A*, *D*, *E*.
	Substitute correctly in an integral with terms $a\ln(2-x)$, $b\ln(3+2x)$ and $c/(3+2x)$ where $abc \neq 0$	**M1**	
	Obtain the given answer after full and correct working [Correct integration of the *A*, *D*, *E* form gives an extra constant term if integration by parts is used for the second partial fraction.]	**A1**	
		5	

Question	Answer	Marks	Guidance
10(i)	*EITHER*: Expand scalar product of a normal to m and a direction vector of l	**M1**	
	Verify scalar product is zero	**A1**	
	Verify that one point of l does not lie in the plane	**A1**	
	OR: Substitute coordinates of a general point of l in the equation of the plane m	**M1**	
	Obtain correct equation in λ in any form	**A1**	
	Verify that the equation is not satisfied for any value of λ	**A1**	
		3	
10(ii)	Use correct method to evaluate a scalar product of normal vectors to m and n	**M1**	
	Using the correct process for the moduli, divide the scalar product by the product of the moduli and evaluate the inverse cosine of the result	**M1**	
	Obtain answer 74.5° or 1.30 radians	**A1**	
		3	
10(iii)	*EITHER*: Using the components of a general point P of l form an equation in λ by equating the perpendicular distance from n to 2	**M1**	
	OR: Take a point Q on l, e.g. (5, 3, 3) and form an equation in λ by equating the length of the projection of QP onto a normal to plane n to 2	**M1**	
	Obtain a correct modular or non-modular equation in any form	**A1**	
	Solve for λ and obtain a position vector for P, e.g. $7\mathbf{i} + 5\mathbf{j} + 7\mathbf{j}$ from $\lambda = 3$	**A1**	
	Obtain position vector of the second point, e.g. $3\mathbf{i} + \mathbf{j} - \mathbf{k}$ from $\lambda = -1$	**A1**	
		4	

Cambridge International A Level

Cambridge Assessment International Education
Cambridge International Advanced Level

MATHEMATICS **9709/32**

Paper 3 **October/November 2018**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2018 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

This document consists of **19** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
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Question	Answer	Marks	Guidance
1	State or imply non-modular inequality $3^2(2x-1)^2 > (x+4)^2$, or corresponding quadratic equation, or pair of linear equations/inequalities $3(2x-1) = \pm(x+4)$	**B1**	$35x^2 - 44x - 7 = 0$
	Make reasonable attempt at solving a 3-term quadratic, or solve two linear equations for x	**M1**	Allow for reasonable attempt at factorising e.g. $(5x - 7)(7x + 1)$
	Obtain critical values $x = \frac{7}{5}$ and $x = -\frac{1}{7}$	**A1**	Accept 1.4 and -0.143 or better for penultimate A mark
	State final answer $x > \frac{7}{5}$, $x < -\frac{1}{7}$	**A1**	'and' is A0, $\frac{7}{5} < x < -\frac{1}{7}$ is A0. Must be exact values. Must be strict inequalities in final answer
	Alternative		
	Obtain critical value $x = \frac{7}{5}$ from a graphical method	**B1**	or by inspection, or by solving a linear equation or an inequality
	Obtain critical value $x = -\frac{1}{7}$ similarly	**B2**	
	State final answer $x > \frac{7}{5}$ or $x < -\frac{1}{7}$ or equivalent	**B1**	[Do not condone $\geqslant$ for $>$, or $\leqslant$ for $<$.]
		4	

Question	Answer	Marks	Guidance
2	Use trig formula and obtain an equation in $\sin\theta$ and $\cos\theta$	**M1***	Condone sign error in expansion and/or omission of "$+\cos\theta$" $\sin\theta\cos 30° - \cos\theta\sin 30° + \cos\theta = 2\sin\theta$
	Obtain an equation in $\tan\theta$	**M1(dep*)**	e.g. $\tan\theta = \frac{1-\sin 30°}{2-\cos 30°}$ Can be implied by correct answer following correct expansion. Otherwise need to see working
	Obtain $\tan\theta = 1/\left(4-\sqrt{3}\right)$, or equivalent	**A1**	$\frac{4+\sqrt{3}}{13}$, $0.4409....$ (2 s.f or better)
	Obtain final answer $\theta = 23.8°$ and no others in range	**A1**	At least 3 sf $(23.7939....)$ ignore extra values outside range
		4	

Question	Answer	Marks	Guidance
3(i)	Integrate by parts and reach $a\dfrac{\ln x}{x^2} + b\int \dfrac{1}{x}.\dfrac{1}{x^2}\,\mathrm{d}x$	**M1***	
	Obtain $\pm\dfrac{1}{2}\dfrac{\ln x}{x^2} \pm \int \dfrac{1}{x}.\dfrac{1}{2x^2}\,\mathrm{d}x$, or equivalent	**A1**	
	Complete integration correctly and obtain $-\dfrac{\ln x}{2x^2} - \dfrac{1}{4x^2}$, or equivalent	**A1**	Condone without '$+ C$' ISW
		3	

Question	Answer	Marks	Guidance
3(ii)	Substitute limits correctly in an expression of the form $a\frac{\ln x}{x^2}+\frac{b}{x^2}$ or equivalent	**M1(dep*)**	$-\frac{1}{8}\ln 2-\frac{1}{16}+\frac{1}{4}$
	Obtain the **given answer** following full and **exact** working	**A1**	The step $\ln 2=\frac{1}{2}\ln 4$ or 2ln2 = ln4 needs to be clear.
		2	

Question	Answer	Marks	Guidance
4	Substitute and obtain 3-term quadratic $3u^2+4u-1=0$, or equivalent	**B1**	e.g. $3\left(\mathrm{e}^x\right)^2+4\mathrm{e}^x-1=0$
	Solve a 3 term quadratic for u	**M1**	Must be an equation with real roots
	Obtain root $\left(\sqrt{7}-2\right)/3$, or decimal in [0.21, 0.22]	**A1**	Or equivalent. Ignore second root (even if incorrect)
	Use correct method for finding x **from a positive value of** e^x	**M1**	Must see some indication of method: use of $x=\ln u$
	Obtain answer $x = -$ 1.536 only	**A1**	CAO. Must be 3 dp
		5	

Question	Answer	Marks	Guidance
5(i)	Use product rule on a correct expression	**M1**	Condone with $+\frac{x}{8-x}$ unless there is clear evidence of incorrect product rule.
	Obtain correct derivative in any form	**A1**	$\frac{dy}{dx}=\ln(8-x)-\frac{x}{8-x}$
	Equate derivative to 1 and obtain $x=8-\frac{8}{\ln(8-x)}$	**A1**	**Given answer:** check carefully that it follows from correct working
			Condone the use of *a* for *x* throughout
		3	
5(ii)	Calculate values of a relevant expression or pair of relevant expressions at x = 2.9 and x = 3.1	**M1**	$8-\frac{8}{\ln 5.1}=3.09>2.9,\quad 8-\frac{8}{\ln 4.9}=2.97<3.1$ Clear linking of pairs needed for M1 by this method (0.19 and -0.13)
	Complete the argument correctly with correct calculated values	**A1**	Note: valid to consider gradient at 2.9 (1.06..) and 3.1 (0.95..) and comment on comparison with 1
		2	

Question	Answer	Marks	Guidance
5(iii)	Use the iterative process $x_{n+1} = 8 - \dfrac{8}{\ln(8 - x_n)}$ correctly to find at least two successive values. SR: Clear successive use of 0, 1, 2, 3 etc., or equivalent, scores M0.	**M1**	3, 3.0293, 3.0111, 3.0225, 3.0154, (3.0198) 2.9, 3.0897, 2.9728, 3.0460, 3.0006, 3.290, 3.0113, 3.0223, 3.0155 3.1, 2.9661, 3.0501, 2.9980, 3.0305, 3.0103, 3.0229, 3.0151 Allow M1 if values given to fewer than 4 dp
	Obtain final answer 3.02	**A1**	
	Show sufficient iterations to at least 4 d.p. to justify 3.02 to 2 d.p., or show there is a sign change in the interval (3.015, 3.025)	**A1**	Must have two consecutive values rounding correctly to 3.02
		3	

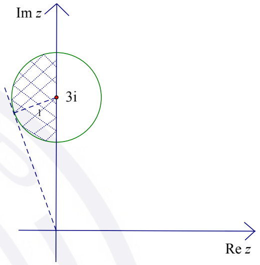
Question	Answer	Marks	Guidance
6	State equation $\frac{\mathrm{d}y}{\mathrm{d}x} = k\frac{y^2}{x}$, or equivalent	**B1**	SC: If $k = 1$ seen or implied give B0 and then allow B1B1B0M1, max 3/8.
	Separate variables correctly and integrate at least one side	**B1**	$\int \frac{k}{x}\mathrm{d}x = \int \frac{1}{y^2}\mathrm{d}y$ Allow with incorrect value substituted for k
	Obtain terms $-\frac{1}{y}$ and $k\ln x$	**B1 + B1**	Incorrect k used scores max. B1B0
	Use given coordinates correctly to find k and/or a constant of integration C in an equation containing terms $\frac{a}{y}$, $b\ln x$ and C	**M1**	SC: If an incorrect method is used to find k, M1 is allowable for a correct method to find C
	Obtain $k = \frac{1}{2}$ and $c = -1$, or equivalent	**A1 + A1**	$\frac{1}{2}\ln x = 1 - \frac{1}{y}$ A0 for fortuitous answers.
	Obtain answer $y = \frac{2}{2-\ln x}$, or equivalent, and ISW	**A1**	$y = \frac{-1}{-1+\ln\sqrt{x}}$
			SC: MR of the fraction. $\frac{\mathrm{d}y}{\mathrm{d}x} = k\frac{y^2}{x^2}$ B1 Separate variables and integrate B1 $\frac{-1}{y} = \frac{-k}{x}(+C)$ B1+B1 Substitute to find k and/or c M1 $k = \frac{\mathrm{e}}{2(\mathrm{e}-1)}$, $c = \frac{2-\mathrm{e}}{2(\mathrm{e}-1)}$ A1+A1 Answer A0
		8	

Question	Answer	Marks	Guidance
7(i)	Use correct quotient or product rule	**M1**	
	Obtain correct derivative in any form	**A1**	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{-3\sin x\left(2+\sin x\right)-3\cos x\cos x}{\left(2+\sin x\right)^2}$ Condone invisible brackets if recovery implied later.
	Equate numerator to zero	**M1**	
	Use $\cos^2 x+\sin^2 x=1$ and solve for sin x	**M1**	$-6\sin x-3=0\ \Rightarrow \sin x=\ldots$.
	Obtain coordinates $x=-\pi/6$ and $y=\sqrt{3}$ ISW	**A1 + A1**	From correct working. No others in range
			SR: A candidate who only states the numerator of the derivative, but justifies this, can have full marks. Otherwise they score M0A0M1M1A0A0
		6	
7(ii)	State indefinite integral of the form k ln (2 + sin x)	**M1***	
	Substitute limits correctly, equate result to 1 and obtain 3 ln (2 + sin a) – 3 ln 2 = 1	**A1**	or equivalent
	Use correct method to solve for a	**M1(dep*)**	Allow for a correct method to solve an incorrect equation, so long as that equation has a solution. $1+\frac{1}{2}\sin a=\mathrm{e}^{1/3}\ \Rightarrow a=\sin^{-1}\left[2\left(\mathrm{e}^{1/3}-1\right)\right]$ Can be implied by 52.3°
	Obtain answer a = 0.913 or better	**A1**	Ignore additional solutions. Must be in radians.
		4	

Question	Answer	Marks	Guidance
8(i)	State or imply the form $\dfrac{A}{1-2x}+\dfrac{B}{2-x}+\dfrac{C}{(2-x)^2}$	**B1**	
	Use a correct method for finding a constant M1 is available following a single slip in working from their form but no A marks (even if a constant is “correct”)	**M1**	$7 = A + 2B$ $-15 = -4A - 5B - 2C$ $8 = 4A + 2B + C$
	Obtain one of $A = 1$, $B = 3$, $C = -2$	**A1**	
	Obtain a second value	**A1**	
	Obtain the third value	**A1**	
	[Mark the form $\dfrac{A}{1-2x}+\dfrac{Dx+E}{(2-x)^2}$, where $A = 1$, $D = -3$ and $E = 4$, B1M1A1A1A1 as above.]		
		5	

Question	Answer	Marks	Guidance
8(ii)	Use a correct method to find the first two terms of the expansion of $(1-2x)^{-1}$, $(2-x)^{-1}$, $\left(1-\frac{1}{2}x\right)^{-1}$, $(2-x)^{-2}$ or $\left(1-\frac{1}{2}x\right)^{-2}$	**M1**	Symbolic coefficients are not sufficient for the M1
	Obtain correct unsimplified expansions up to the term in x^2 of each partial fraction	**A3ft**	The ft is on *A*, *B*, *C*. $1+2x+4x^2$ $\frac{3}{2}+\frac{3}{4}x+\frac{3}{8}x^2$ $-\frac{1}{2}-\frac{1}{2}x-\frac{3}{8}x^2$
	Obtain **final** answer $2+\frac{9}{4}x+4x^2$	**A1**	
	[For the *A*, *D*, *E* form of fractions give M1A2ft for the expanded partial fractions, then, if $D \neq 0$, M1 for multiplying out fully, and A1 for the final answer.]		[The ft is on *A*, *D*, *E*.]
		5	

Question	Answer	Marks	Guidance
9(a)(i)	Multiply numerator and denominator by 1 + 2i, or equivalent	M1	Requires at least one of $2+10\mathrm{i}+12\mathrm{i}^2$ and $1-4\mathrm{i}^2$ together with use of $\mathrm{i}^2=-1$. Can be implied by $\frac{-10+10\mathrm{i}}{5}$
	Obtain quotient – 2 + 2i	A1	
	Alternative		
	Equate to $x + \mathrm{i}y$, obtain two equations in x and y and solve for x or for y	M1	$x+2y=2,\quad y-2x=6$
	Obtain quotient – 2 + 2i	A1	
		2	
9(a)(ii)	Use correct method to find either r or θ	M1	If only finding θ, need to be looking for θ in the correct quadrant
	Obtain $r=2\sqrt{2}$, or exact equivalent	A1ft	ft their $x + \mathrm{i}y$
	Obtain $\theta=\frac{3}{4}\pi$ from exact work	A1ft	ft on $k(-1+\mathrm{i})$ for $k>0$ Do not ISW
		3	

Question	Answer	Marks	Guidance
9(b)	Show a circle with centre 3i	**B1**	
	Show a circle with radius 1	**B1ft**	Follow through their centre provided not at the origin For clearly unequal scales, should be an ellipse
	All correct with even scales and **shade** the correct region	**B1**	
	Carry out a correct method for calculating greatest value of arg z	**M1**	e.g. $\arg z = \frac{\pi}{2} + \sin^{-1}\frac{1}{3}$
	Obtain answer 1.91	**A1**	
		5	

Question	Answer	Marks	Guidance
10(i)	Substitute for **r** and expand the scalar product to obtain an equation in λ	**M1***	e.g. $3(5+\lambda)+(-3-2\lambda)+(-1+\lambda)=5\ \ (2\lambda=5-11)$ or $3(4+\lambda)+1(-5-2\lambda)+(-1+\lambda)=0$ Must attempt to deal with **i** + 2**j**
	Solve a linear equation for λ	**M1(dep*)**	
	Obtain $\lambda = -3$ and position vector $\mathbf{r}_A = 2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}$ for A	**A1**	Accept coordinates
		3	
10(ii)	State or imply a normal vector of p is 3**i** + **j** + **k**, or equivalent	**B1**	
	Use correct method to evaluate a scalar product of relevant vectors e.g. (**i** – 2**j** + **k**).(3**i** + **j** + **k**)	**M1**	
	Using the correct process for calculating the moduli, divide the scalar product by the product of the moduli and evaluate the inverse sine or cosine of the result	**M1**	$\cos\theta = \frac{2}{\sqrt{6}\sqrt{11}}$ Second M1 available if working with the wrong vectors
	Obtain answer 14.3° or 0.249 radians	**A1**	Or better

Question	Answer	Marks	Guidance
10(ii)	**Alternative 1**		
	Use of a point on l and Cartesian equation $3x+y+z=5$ to find distance of point from plane e.g. $B(5,-3,-1)$ $d=\frac{3\times5-3-1-5}{\sqrt{9+1+1}}$	**M1**	
	$=\frac{6}{\sqrt{11}}\quad(=1.809...)$	**A1**	
	Complete method to find angle e.g. $\sin\theta=\dfrac{d}{AB}$	**M1**	
	$\theta=\sin^{-1}\left(\dfrac{6}{\sqrt{11}\sqrt{54}}\right)=0.249$	**A1**	Or better
	Alternative 2		
	State or imply a normal vector of p is $3\mathbf{i}+\mathbf{j}+\mathbf{k}$, or equivalent	**B1**	
	Use correct method to evaluate a vector product of relevant vectors e.g. $(\mathbf{i}-2\mathbf{j}+\mathbf{k})\mathbf{x}(3\mathbf{i}+\mathbf{j}+\mathbf{k})$	**M1**	$3\mathbf{i}-2\mathbf{j}+7\mathbf{k}$
	Using the correct process for calculating the moduli, divide the vector product by the product of the moduli and evaluate the inverse sine or cosine of the result	**M1**	$\sin\theta=\dfrac{\sqrt{3^2+2^2+7^2}}{\sqrt{11}\sqrt{6}}$. Second M1 available if working with the wrong vectors
	Obtain answer 14.3° or 0.249 radians	**A1**	Or better
		4	

Question	Answer	Marks	Guidance
10(iii)	Taking the direction vector of the line to be $a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$, state a relevant equation in a, b, c, e.g. $3a + b + c = 0$	**B1**	
	State a second relevant equation, e.g. $a - 2b + c = 0$, and solve for one ratio, e.g. $a : b$	**M1**	
	Obtain $a : b : c = 3 : -2 : -7$, or equivalent	**A1**	
	State answer $\mathbf{r} = 2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k} + \mu(3\mathbf{i} - 2\mathbf{j} - 7\mathbf{k})$	**A1ft**	Or equivalent. The f.t. is on $\mathbf{r}_A$ Requires '$\mathbf{r} = \ldots$.'
	Alternative		
	Attempt to calculate the vector product of relevant vectors, e.g. $(3\mathbf{i} + \mathbf{j} + \mathbf{k}) \times (\mathbf{i} - 2\mathbf{j} + \mathbf{k})$	**M1**	
	Obtain two correct components of the product	**A1**	
	Obtain correct product, e.g. $3\mathbf{i} - 2\mathbf{j} - 7\mathbf{k}$	**A1**	
	State answer $\mathbf{r} = 2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k} + \mu(3\mathbf{i} - 2\mathbf{j} - 7\mathbf{k})$	**A1ft**	Or equivalent. The f.t. is on $\mathbf{r}_{A.}$ Requires "$\mathbf{r} = \ldots$."
		4	

Cambridge International A Level

Cambridge Assessment International Education
Cambridge International Advanced Level

MATHEMATICS **9709/33**

Paper 3 **October/November 2018**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2018 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

This document consists of **14** printed pages.

Cambridge Assessment International Education

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

  - Note: B2 or A2 means that the candidate can earn 2 or 0.
    B2/1/0 means that the candidate can earn anything from 0 to 2.

The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no “follow through” from a previous error is allowed)
CWO	Correct Working Only – often written by a ‘fortuitous’ answer
ISW	Ignore Subsequent Working
SOI	Seen or implied
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become “follow through” marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Question	Answer	Marks	Guidance
1	*EITHER*: State or imply non-modular inequality $2^2(2x-a)^2 < (x+3a)^2$, or corresponding quadratic equation, or pair of linear equations $2(2x - a) = \pm (x + 3a)$	**B1**	
	Make reasonable attempt at solving a 3-term quadratic, or solve two linear equations for x	**M1**	
	Obtain critical values $x=\frac{5}{3}a$ and $x=-\frac{1}{5}a$	**A1**	
	State final answer $-\frac{1}{5}a<x<\frac{5}{3}a$	**A1**	
	OR: Obtain critical value $x=\frac{5}{3}a$ from a graphical method, or by inspection, or by solving a linear equation or an inequality	**B1**	
	Obtain critical value $x=-\frac{1}{5}a$ similarly	**B2**	
	State final answer $-\frac{1}{5}a<x<\frac{5}{3}a$ [Do not condone $\leqslant$ for $<$ in the final answer.]	**B1**	
		4	

Question	Answer	Marks	Guidance
2	Rearrange the equation in the form $ae^{2x} = b$ or $ae^{x} = be^{-x}$	**M1**	
	Obtain correct equation in either form with $a = 2$ and $b = 5$	**A1**	
	Use correct method to solve for x	**M1**	
	Obtain answer $x = 0.46$	**A1**	
		4	

Question	Answer	Marks	Guidance
3 (i)	Sketch a relevant graph, e.g. $y = x^3$	**B1**	
	Sketch a second relevant graph, e.g. $y = 3 - x$, and justify the given statement	**B1**	Consideration of behaviour for $x < 0$ is needed for the second B1
		2	
3(ii)	State or imply the equation $x = \left(2x^3 + 3\right) / \left(3x^2 + 1\right)$	**B1**	
	Rearrange this in the form $x^3 = 3 - x$, or commence work *vice versa*	**B1**	
		2	

Question	Answer	Marks	Guidance
3(iii)	Use the iterative formula correctly at least once	**M1**	
	Obtain final answer 1.213	**A1**	
	Show sufficient iterations to 5 d.p. or more to justify 1.213 to 3 d.p., or show there is a sign change in the interval (1.2125, 1.2135)	**A1**	
		3	

Question	Answer	Marks	Guidance
4(i)	Obtain $\frac{dx}{d\theta} = 2\cos\theta + 2\cos 2\theta$ or $\frac{dy}{d\theta} = -2\sin\theta - 2\sin 2\theta$	**B1**	
	Use $dy/dx = dy/d\theta \div dx/d\theta$	**M1**	
	Obtain correct $\frac{dy}{dx}$ in any form, e.g. $-\frac{2\sin\theta + 2\sin 2\theta}{2\cos\theta + 2\cos 2\theta}$	**A1**	
		3	
4(ii)	Equate denominator to zero and use any correct double angle formula	**M1***	
	Obtain correct 3-term quadratic in cos θ in any form	**A1**	
	Solve for θ	**depM1***	
	Obtain $x = 3\sqrt{3}/2$ and $y = \frac{1}{2}$, or exact equivalents	**A1**	
		4	

Question	Answer	Marks	Guidance
5	Separate variables correctly and integrate at least one side	**B1**	
	Obtain term ln y	**B1**	
	Obtain terms 2 ln $x - \frac{1}{2}x^2$	**B1+B1**	
	Use $x = 1$, $y = 1$ to evaluate a constant, or as limits	**M1**	
	Obtain correct solution in any form, e.g. $\ln y = 2\ln x - \frac{1}{2}x^2 + \frac{1}{2}$	**A1**	
	Rearrange as $y = x^2 \exp\left(\frac{1}{2} - \frac{1}{2}x^2\right)$, or equivalent	**A1**	
		7	

Question	Answer	Marks	Guidance
6(i)	Rearrange in the form $\sqrt{3}\sin x - \cos x = \sqrt{2}$	**B1**	
	State $R = 2$	**B1**	
	Use trig formulae to obtain α	**M1**	
	Obtain $\alpha = 30°$ with no errors seen	**A1**	
		4	

Question	Answer	Marks	Guidance
6(ii)	Evaluate $\sin^{-1}\left(\frac{\sqrt{2}}{R}\right)$	**B1ft**	
	Carry out a correct method to find a value of x in the given interval	**M1**	
	Obtain answer $x = 75°$	**A1**	
	Obtain a second answer e.g. $x = 165°$ and no others [Treat answers in radians as a misread. Ignore answers outside the given interval.]	**A1ft**	
		4	

Question	Answer	Marks	Guidance
7(i)	Use product rule	**M1***	
	Obtain correct derivative in any form	**A1**	
	Equate derivative to zero and obtain an equation in a single trig function	**depM1***	
	Obtain a correct equation, e.g. $3\tan^2 x = 2$	**A1**	
	Obtain answer $x = 0.685$	**A1**	
		5	

Question	Answer	Marks	Guidance
7(ii)	Use the given substitution and reach $a\int\left(u^2-u^4\right)\mathrm{d}u$	**M1**	
	Obtain correct integral with $a = 5$ and limits 0 and 1	**A1**	
	Use correct limits in an integral of the form $a\left(\frac{1}{3}u^3-\frac{1}{5}u^5\right)$	**M1**	
	Obtain answer $\frac{2}{3}$	**A1**	
		4	

Question	Answer	Marks	Guidance
8(i)	*EITHER*: Multiply numerator and denominator by 1 + 2i, or equivalent, or equate to $x + iy$, obtain two equations in x and y and solve for x or for y	**M1**	
	Obtain quotient $-\frac{4}{5}+\frac{7}{5}$ i, or equivalent	**A1**	
	Use correct method to find either r or θ	**M1**	
	Obtain $r = 1.61$	**A1**	
	Obtain $\theta = 2.09$	**A1**	
	OR: Find modulus or argument of 2 + 3i or of 1 – 2i	**B1**	
	Use correct method to find r	**M1**	
	Obtain $r = 1.61$	**A1**	
	Use correct method to find θ	**M1**	
	Obtain $\theta = 2.09$	**A1**	
		5	
8(ii)	Show a circle with centre 3 – 2i	**B1**	
	Show a circle with radius 1	**B1ft**	Centre not at the origin
	Carry out a correct method for finding the least value of $\|z\|$	**M1**	
	Obtain answer $\sqrt{13} - 1$	**A1**	
		4	

Question	Answer	Marks	Guidance
9(i)	State or imply the form $\dfrac{A}{2-x}+\dfrac{B}{3+2x}+\dfrac{C}{(3+2x)^2}$	**B1**	
	Use a correct method to find a constant	**M1**	
	Obtain one of $A = 1$, $B = -1$, $C = 3$	**A1**	
	Obtain a second value	**A1**	
	Obtain the third value [Mark the form $\dfrac{A}{2-x}+\dfrac{Dx+E}{(3+2x)^2}$, where $A = 1$, $D = -2$ and $E = 0$, B1M1A1A1A1 as above.]	**A1**	
		5	
9(ii)	Integrate and obtain terms $-\ln(2-x)-\dfrac{1}{2}\ln(3+2x)-\dfrac{3}{2(3+2x)}$	**B3ft**	The f.t is on A, B, C; or on A, D, E.
	Substitute correctly in an integral with terms $a\ln(2-x)$, $b\ln(3+2x)$ and $c/(3+2x)$ where $abc \neq 0$	**M1**	
	Obtain the given answer after full and correct working [Correct integration of the A, D, E form gives an extra constant term if integration by parts is used for the second partial fraction.]	**A1**	
		5	

Question	Answer	Marks	Guidance
10(i)	*EITHER*: Expand scalar product of a normal to m and a direction vector of l	**M1**	
	Verify scalar product is zero	**A1**	
	Verify that one point of l does not lie in the plane	**A1**	
	OR: Substitute coordinates of a general point of l in the equation of the plane m	**M1**	
	Obtain correct equation in λ in any form	**A1**	
	Verify that the equation is not satisfied for any value of λ	**A1**	
		3	
10(ii)	Use correct method to evaluate a scalar product of normal vectors to m and n	**M1**	
	Using the correct process for the moduli, divide the scalar product by the product of the moduli and evaluate the inverse cosine of the result	**M1**	
	Obtain answer 74.5° or 1.30 radians	**A1**	
		3	
10(iii)	*EITHER*: Using the components of a general point P of l form an equation in λ by equating the perpendicular distance from n to 2	**M1**	
	OR: Take a point Q on l, e.g. (5, 3, 3) and form an equation in λ by equating the length of the projection of QP onto a normal to plane n to 2	**M1**	
	Obtain a correct modular or non-modular equation in any form	**A1**	
	Solve for λ and obtain a position vector for P, e.g. $7\mathbf{i} + 5\mathbf{j} + 7\mathbf{j}$ from $\lambda = 3$	**A1**	
	Obtain position vector of the second point, e.g. $3\mathbf{i} + \mathbf{j} - \mathbf{k}$ from $\lambda = -1$	**A1**	
		4	

Cambridge Assessment International Education
Cambridge International Advanced Level

MATHEMATICS **9709/31**

Paper 3 **May/June 2018**

MARK SCHEME

Maximum Mark: 75

Published

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[Turn over

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  - Note: B2 or A2 means that the candidate can earn 2 or 0.
    B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

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The following abbreviations may be used in a mark scheme or used on the scripts:

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PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Question	Answer	Marks
1	Use law for the logarithm of a product, quotient or power	**M1**
	Obtain a correct equation free of logarithms, e.g. $4(x^4-4)=x^4$	**A1**
	Solve for x	**M1**
	Obtain answer x = 1.52 only	**A1**
		4

Question	Answer	Marks
2(i)	Use trig formulae and obtain an equation in sin x and cos x	**M1***
	Obtain a correct equation in any form	**A1**
	Substitute exact trig ratios and obtain an expression for tan x	**M1(dep*)**
	Obtain answer $\tan x = \frac{-\left(6+\sqrt{6}\right)}{\left(6-\sqrt{2}\right)}$ or equivalent	**A1**
		4
2(ii)	State answer, e.g. 118.5°	**B1**
	State second answer, e.g. 298.5°	**B1ft**
		2

Question	Answer	Marks
3	Use quotient or product rule	**M1**
	Obtain correct derivative in any form	**A1**
	Equate derivative to zero and obtain a quadratic in $\tan\frac{1}{2}x$ or an equation of the form a sin x = b	**M1 ***
	Solve for x	**M1(dep*)**
	Obtain answer 0.340	**A1**
	Obtain second answer 2.802 and no other in the given interval	**A1**
		6

Question	Answer	Marks
4	*EITHER:* Commence division by $x^2 - x + 1$ and reach a partial quotient of the form $x^2 + kx$	**M1**
	Obtain quotient $x^2 + 3x + 2$	**A1**
	Either Set remainder identically equal to zero and solve for a or for b, *or* multiply given divisor and found quotient and obtain a or b	**M1**
	Obtain $a = 1$	**A1**
	Obtain $b = 2$	**A1**
	OR: Assume an unknown factor $x^2 + Bx + C$ and obtain an equation in B and/or C	**M1**
	Obtain $B = 3$ and $A = 2$	**A1**
	Either Use equations to obtain a or b *or* multiply given divisor and found factor to obtain a or b	**M1**
	Obtain $a = 1$	**A1**
	Obtain $b = 2$	**A1**
		5

Question	Answer	Marks
5(i)	State or imply $dx = -2\cos\theta\sin\theta\,d\theta$, or equivalent	**B1**
	Substitute for x and dx, and use Pythagoras	**M1**
	Obtain integrand $\pm 2\cos^2\theta$	**A1**
	Justify change of limits and obtain given answer correctly	**A1**
		4
5(ii)	Obtain indefinite integral of the form $a\theta + b\sin 2\theta$	**M1***
	Obtain $\theta + \frac{1}{2}\sin 2\theta$	**A1**
	Use correct limits correctly	**M1(dep*)**
	Obtain answer $\frac{1}{6}\pi$ with no errors seen	**A1**
		4

Question	Answer	Marks
6(i)	Separate variables correctly and integrate at least one side	**B1**
	Obtain term ln x	**B1**
	Obtain term $-\frac{2}{3}kt\sqrt{t}$, or equivalent	**B1**
	Evaluate a constant, or use limits $x = 100$ and $t = 0$, in a solution containing terms $a \ln x$ and $b\,t\sqrt{t}$	**M1**
	Obtain correct solution in any form, e.g. $\ln x = -\frac{2}{3}kt\sqrt{t} + \ln 100$	**A1**
		5
6(ii)	Substitute $x = 80$ and $t = 25$ to form equation in k	**M1**
	Substitute $x = 40$ and eliminate k	**M1**
	Obtain answer $t = 64.1$	**A1**
		3

Question	Answer	Marks
7(i)	Use quadratic formula, or completing the square, or the substitution $z = x + \mathrm{i}y$ to find a root, using $\mathrm{i}^2 = -1$	**M1**
	Obtain a root, e.g. $-\sqrt{6} - \sqrt{2}\mathrm{i}$	**A1**
	Obtain the other root, e.g. $-\sqrt{6} - \sqrt{2}\mathrm{i}$	**A1**
		3
7(ii)	Represent both roots in relatively correct positions	**B1ft**
		1
7(iii)	State or imply correct value of a relevant length or angle, e.g. OA, OB, AB, angle between OA or OB and the real axis	**B1ft**
	Carry out a complete method for finding angle OAB	**M1**
	Obtain $AOB = 60°$ correctly	**A1**
		3
7(iv)	Give a complete justification of the given statement	**B1**
		1

Question	Answer	Marks
8(i)	Integrate by parts and reach $lxe^{-\frac{1}{2}x} + m\int e^{-\frac{1}{2}x}\,dx$	**M1***
	Obtain $-2xe^{-\frac{1}{2}x} + 2\int e^{-\frac{1}{2}x}\,dx$	**A1**
	Complete the integration and obtain $-2xe^{-\frac{1}{2}x} - 4e^{-\frac{1}{2}x}$, or equivalent	**A1**
	Having integrated twice, use limits and equate result to 2	**M1(dep*)**
	Obtain the given equation correctly	**A1**
		5
8(ii)	Calculate values of a relevant expression or pair of expressions at $a = 3$ and $a = 3.5$	**M1**
	Complete the argument correctly with correct calculated values	**A1**
		2
8(iii)	Use the iterative formula $a_{n+1} = 2\ln(a_n + 2)$ correctly at least once	**M1**
	Obtain final answer 3.36	**A1**
	Show sufficient iterations to 4 d.p. to justify 3.36 to 2 d.p., or show there is a sign change in the interval (3.355, 3.365)	**A1**
		3

Question	Answer	Marks
9(i)	State or imply the form $A+\dfrac{B}{x-1}+\dfrac{C}{3x+2}$	**B1**
	State or obtain $A = 4$	**B1**
	Use a correct method to obtain a constant	**M1**
	Obtain one of $B = 3$, $C = -1$	**A1**
	Obtain the other value	**A1**
		5
9(ii)	Use correct method to find the first two terms of the expansion of $(x-1)^{-1}$ or $(3x+2)^{-1}$, or equivalent	**M1**
	Obtain correct unsimplified expansions up to the term in x^2 of each partial fraction	**A1ft + A1ft**
	Add the value of A to the sum of the expansions	**M1**
	Obtain final answer $\dfrac{1}{2}-\dfrac{9}{4}x-\dfrac{33}{8}x^2$	**A1**
		5

Question	Answer	Marks
10(a)	*EITHER*: Find $\overrightarrow{PQ}$ (or $\overrightarrow{QP}$) for a general point Q on l, e.g. $(1+\mu)\ \mathbf{i}+(4+2\mu)\mathbf{j}+(4+3\mu)\mathbf{k}$	**B1**
	Calculate the scalar product of $\overrightarrow{PQ}$ and a direction vector for l and equate to zero	**M1**
	Solve and obtain correct solution e.g. $\mu=-\frac{3}{2}$	**A1**
	Carry out method to calculate PQ	**M1**
	Obtain answer 1.22	**A1**
	*OR*1: Find $\overrightarrow{PQ}$ (or $\overrightarrow{QP}$) for a general point Q on l	**B1**
	Use a correct method to express PQ^2 (or PQ) in terms of μ	**M1**
	Obtain a correct expression in any form	**A1**
	Carry out a complete method for finding its minimum	**M1**
	Obtain answer 1.22	**A1**
	*OR*2: Calling (4, 2, 5) A, state $\overrightarrow{PA}$ (or $\overrightarrow{AP}$) in component form, e.g. $\mathbf{i}+4\mathbf{j}+4\mathbf{k}$	**B1**
	Use a scalar product to find the projection of $\overrightarrow{PA}$ (or $\overrightarrow{AP}$) on l	**M1**
	Obtain correct answer $21/\sqrt{14}$, or equivalent	**A1**
	Use Pythagoras to find the perpendicular	**M1**
	Obtain answer 1.22	**A1**
	*OR*3: State $\overrightarrow{PA}$ (or $\overrightarrow{AP}$) in component form	**B1**
	Calculate vector product of $\overrightarrow{PA}$ and a direction vector for l	**M1**
	Obtain correct answer, e.g. $4\mathbf{i}+\mathbf{j}-2\mathbf{k}$	**A1**
	Divide modulus of the product by that of the direction vector	**M1**
	Obtain answer 1.22	**A1**
		5

Question	Answer	Marks
10(ii)	*EITHER*: Use scalar product to obtain a relevant equation in a, b and c, e.g. $a + 2b + 3c = 0$	**B1**
	Obtain a second relevant equation, e.g. using $\overrightarrow{PA}$ $a + 4b + 4c = 0$, and solve for one ratio	**M1**
	Obtain $a : b : c = 4 : 1 : -2$, or equivalent	**A1**
	Substitute a relevant point and values of a, b, c in general equation and find d	**M1**
	Obtain correct answer, $4x + y - 2z = 8$, or equivalent	**A1**
	*OR*1: Attempt to calculate vector product of relevant vectors, e.g. $(\mathbf{i} + 4\mathbf{j} + 4\mathbf{k}) \times (\mathbf{i} + 2\mathbf{j} + 3\mathbf{k})$	**M1**
	Obtain two correct components	**A1**
	Obtain correct answer, e.g. $4\mathbf{i} + \mathbf{j} - 2\mathrm{k}$	**A1**
	Substitute a relevant point and find d	**M1**
	Obtain correct answer, $4x + y - 2z = 8$, or equivalent	**A1**
	*OR*2: Using a relevant point and relevant vectors form a 2-parameter equation for the plane	**M1**
	State a correct equation, e.g. $\mathbf{r} = 4\mathbf{i} + 2\mathbf{j} + 5\mathbf{k} + \lambda(\mathbf{i} + 4\mathbf{j} + 4\mathbf{k}) + \mu(\mathbf{i} + 2\mathbf{j} + 3\mathbf{k})$	**A1**
	State three correct equations in x, y, z, λ and μ	**A1**
	Eliminate λ and μ	**M1**
	Obtain correct answer $4x + y - 2z = 8$, or equivalent	**A1**
		5

Cambridge International A Level

Cambridge Assessment International Education
Cambridge International Advanced Level

MATHEMATICS **9709/32**

Paper 3 **May/June 2018**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2018 series for most Cambridge IGCSE™, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

This document consists of **18** printed pages.

Cambridge Assessment International Education

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

  - Note: B2 or A2 means that the candidate can earn 2 or 0.
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Question	Answer	Marks	Guidance
1	*EITHER*: State or imply non-modular equation $3^2\left(2^x-1\right)^2=\left(2^x\right)^2$, or pair of equations $3\left(2^x-1\right)=\pm 2^x$	M1	$8\left(2^x\right)^2-18\left(2^x\right)+9=0$
	Obtain $2^x=\frac{3}{2}$ and $2^x=\frac{3}{4}$ or equivalent	A1	
	OR: Obtain $2^x=\frac{3}{2}$ by solving an equation	B1	
	Obtain $2^x=\frac{3}{4}$ by solving an equation	B1	
	Use correct method for solving an equation of the form $2^x=a$, where $a>0$	M1	
	Obtain **final** answers $x = 0.585$ and $x = -0.415$ only	A1	The question requires 3 s.f. Do not ISW if they go on to reject one value
		4	

Question	Answer	Marks	Guidance
2	Use correct tan $(A \pm B)$ formula and obtain an equation in tan θ	**M1**	$\frac{1}{\tan\theta} + \frac{1-\tan\theta\tan 45}{\tan\theta + \tan 45} = 2$ Allow M1 with $\tan 45°$ $= \frac{1}{\tan\theta} + \frac{1-\tan\theta}{\tan\theta + 1}$
	Obtain a correct equation in any form	**A1**	With values substituted
	Reduce to $3\tan^2\theta = 1$, or equivalent	**A1**	
	Obtain answer $x = 30°$	**A1**	One correct solution
	Obtain answer $x = 150°$	**A1**	Second correct solution and no others in range
	OR: use correct $\sin(A \pm B)$ and $\cos(A \pm B)$ to form equation in $\sin\theta$ and $\cos\theta$ M1A1		
	Reduce to $\tan^2\theta = \frac{1}{3}, \sin^2\theta = \frac{1}{4}, \cos^2\theta = \frac{3}{4}$ or $\cot^2\theta = 3$ A1 etc.		
		5	

Question	Answer	Marks	Guidance
3(i)	Fully justify the given statement	**B1**	Some indication of use of gradient of curve = gradient of tangent (*PT*) and no errors seen /no incorrect statements
		1	
3(ii)	Separate variables and attempt integration of at least one side Obtain terms ln *y* and $\frac{1}{2}x$	**B1 B1**	Must be working from $\int \frac{1}{y}\mathrm{d}y = \int k\mathrm{d}x$ B marks are not available for fortuitously correct answers
	Use $x = 4$, $y = 3$ to evaluate a constant or as limits in a solution with terms $a \ln y$ and bx, where $ab \neq 0$	**M1**	
	Obtain correct solution in any form	**A1**	$\ln y = \frac{1}{2}x + \ln 3 - 2$
	Obtain answer $y = 3e^{\frac{1}{2}x-2}$, or equivalent	**A1**	Accept $y = e^{\frac{1}{2}x+\ln 3-2}$, $y = e^{\frac{x-1.80}{2}}$, $y = 3\sqrt{e^{x-4}}$ $\lvert y\rvert = \ldots$ scores A0
		5	

Question	Answer	Marks	Guidance
4(i)	Use correct double angle formulae and express LHS in terms of $\cos x$ and $\sin x$	**M1**	$\dfrac{2\sin x - 2\sin x\cos x}{1-\left(2\cos^2 x - 1\right)}$
	Obtain a correct expression	**A1**	
	Complete method to get correct denominator e.g. by factorising to remove a factor of $1-\cos x$	**M1**	
	Obtain the given RHS correctly *OR (working R to L)*:	**A1**	
	$\dfrac{\sin x}{1+\cos x}\times\dfrac{1-\cos x}{1-\cos x}=\dfrac{\sin x-\sin x\cos x}{1-\cos^2 x}$ $=\dfrac{2\sin x-2\sin x\cos x}{2-2\cos^2 x}$ M1A1		Given answer so check working carefully
	$=\dfrac{2\sin x-\sin 2x}{1-\cos 2x}$ M1A1		
		4	
4(ii)	State integral of the form $a\ \ln(1+\cos x)$	**M1***	If they use the substitution $u=1+\cos x$ allow M1A1 for $-\ln u$
	Obtain integral $-\ln(1+\cos x)$	**A1**	
	Substitute correct limits in correct order	**M1(dep)***	
	Obtain answer $\ln\left(\dfrac{3}{2}\right)$, or equivalent	**A1**	
		4	

Question	Answer	Marks	Guidance
5(i)	State or imply $3y^2\frac{\mathrm{d}y}{\mathrm{d}x}$ as derivative of y^3	**B1**	
	State or imply $6xy+3x^2\frac{\mathrm{d}y}{\mathrm{d}x}$ as derivative of $3x^2y$ *OR* State or imply $2x(x+3y)+x^2\left(1+3\frac{\mathrm{d}y}{\mathrm{d}x}\right)$ as derivative of $x^2(x+3y)$	**B1**	$3x^2+6xy+3x^2\frac{\mathrm{d}y}{\mathrm{d}x}-3y^2\frac{\mathrm{d}y}{\mathrm{d}x}=0$
	Equate derivative of the LHS to zero and solve for $\frac{\mathrm{d}y}{\mathrm{d}x}$	**M1**	Given answer so check working carefully
	Obtain the given answer	**A1**	
		4	
5(ii)	Equate derivative to – 1 and solve for y	**M1***	
	Use their $y = -2x$ or equivalent to obtain an equation in x or y	**M1(dep*)**	
	Obtain answer $(1, -2)$	**A1**	
	Obtain answer $(\sqrt[3]{3}, 0)$	**B1**	Must be exact e.g. $\mathrm{e}^{\frac{1}{3}\ln 3}$ but ISW if decimals after exact value seen
		4	

Question	Answer	Marks	Guidance
6(i)	Use correct method for finding the area of a segment and area of semicircle and form an equation in θ	**M1**	e.g. $\frac{\pi a^2}{4} = \frac{1}{2}a^2\theta - \frac{1}{2}a^2 \sin\theta$
	State a correct equation in any form	**A1**	Given answer so check working carefully
	Obtain the given answer correctly	**A1**	
		3	
6(ii)	Calculate values of a relevant expression or pair of expressions at $\theta = 2.2$ and $\theta = 2.4$	**M1**	e.g. $f(\theta) = \frac{\pi}{2} + \sin\theta \quad \begin{cases} f(2.2) = 2.37... > 2.2 \\ f(2.4) = 2.24... < 2.4 \end{cases}$ or $f(\theta) = \theta - \frac{\pi}{2} - \sin\theta \quad \begin{cases} f(2.2) = -0.17... < 0 \\ f(2.4) = +0.15... > 0 \end{cases}$
	Complete the argument correctly with correct calculated values	**A1**	
		2	

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td rowspan="4">6(iii)</td><td>Use $\theta_{n+1} = \frac{1}{2}\pi + \sin\theta_n$ correctly at least once</td><td>M1</td><td rowspan="3">e.g.
<table>
<tr><td>2.2</td><td>2.3</td><td>2.4</td></tr>
<tr><td>2.3793</td><td>2.3165</td><td>2.2463</td></tr>
<tr><td>2.2614</td><td>2.3054</td><td>2.3512</td></tr>
<tr><td>2.3417</td><td>2.3129</td><td>2.2814</td></tr>
<tr><td>2.2881</td><td>2.3079</td><td>2.3288</td></tr>
<tr><td>2.3244</td><td></td><td>2.2970</td></tr>
<tr><td>2.3000</td><td></td><td>2.3185</td></tr>
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</table>

Question	Answer	Marks	Guidance
7(i)	Substitute in *uv*, expand the product and use $i^2 = -1$	**M1**	
	Obtain answer $uv = -11 - 5\sqrt{3}i$	**A1**	
	EITHER: Substitute in *u/v* and multiply numerator and denominator by the conjugate of *v*, or equivalent	**M1**	
	Obtain numerator $-7 + 7\sqrt{3}i$ or denominator 7	**A1**	
	Obtain final answer $-1 + \sqrt{3}i$	**A1**	
	OR: Substitute in *u/v* , equate to $x + iy$ and solve for *x* or for *y*	**M1**	$\begin{cases} -3\sqrt{3} = \sqrt{3}x - 2y \\ 1 = 2x + \sqrt{3}y \end{cases}$
	Obtain $x = -1$ or $y = \sqrt{3}$	**A1**	
	Obtain final answer $-1 + \sqrt{3}\,i$	**A1**	
		5	

Question	Answer	Marks	Guidance
7(ii)	Show the points *A* and *B* representing *u* and *v* in relatively correct positions	**B1**	
	Carry out a complete method for finding angle *AOB*, e.g. calculate arg(*u*/*v*) If using $\theta = \tan^{-1}\left(-\sqrt{3}\right)$ must refer to $\arg\left(\frac{u}{v}\right)$	**M1**	*OR*: $\tan a = \frac{-1}{3\sqrt{3}}, \tan b = \frac{2}{\sqrt{3}} \Rightarrow \tan\left(a-b\right) = \frac{\frac{-1}{3\sqrt{3}} - \frac{2}{\sqrt{3}}}{1-\frac{2}{9}}$ $= -\sqrt{3}$ $\Rightarrow \theta = \frac{2\pi}{3}$ *OR*: $\cos\theta = \frac{\begin{pmatrix}-3\sqrt{3}\\1\end{pmatrix}\begin{pmatrix}\sqrt{3}\\2\end{pmatrix}}{\sqrt{7}\sqrt{28}} = \frac{-9+2}{14} = \frac{-1}{2}$ $\Rightarrow \theta = \frac{2\pi}{3}$ *OR*: $\cos\theta = \frac{28+7-49}{2\sqrt{28}\sqrt{7}} = -\frac{1}{2} \Rightarrow \theta = \frac{2\pi}{3}$
	Prove the given statement	**A1**	Given answer so check working carefully
		3	

Question	Answer	Marks	Guidance
8(i)	Use correct product or quotient rule	**M1**	$\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{1}{3}(x+1)\mathrm{e}^{-\frac{1}{3}x}+\mathrm{e}^{-\frac{1}{3}x}$ or $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\mathrm{e}^{\frac{1}{3}x}-(x+1)\frac{1}{3}\mathrm{e}^{\frac{1}{3}x}}{\mathrm{e}^{\frac{2}{3}x}}$
	Obtain complete correct derivative in any form	**A1**	
	Equate derivative to zero and solve for x	**M1**	
	Obtain answer $x = 2$ with no errors seen	**A1**	
		4	
8(ii)	Integrate by parts and reach $a(x+1)\mathrm{e}^{-\frac{1}{3}x}+b\int\mathrm{e}^{-\frac{1}{3}x}\,\mathrm{d}x$	**M1***	
	Obtain $-3(x+1)\mathrm{e}^{-\frac{1}{3}x}+3\int\mathrm{e}^{-\frac{1}{3}x}\,\mathrm{d}x$, or equivalent	**A1**	$-3xe^{-\frac{1}{3}x}+\int 3\mathrm{e}^{-\frac{1}{3}x}\mathrm{d}x-3\mathrm{e}^{-\frac{1}{3}x}$
	Complete integration and obtain $-3(x+1)\mathrm{e}^{-\frac{1}{3}x}-9\mathrm{e}^{-\frac{1}{3}x}$, or equivalent	**A1**	
	Use **correct** limits $x = -1$ and $x = 0$ **in the correct order**, having integrated twice	**M1(dep*)**	
	Obtain answer $9\mathrm{e}^{\frac{1}{3}}-12$, or equivalent	**A1**	
		5	

Question	Answer	Marks	Guidance
9(i)	Use a correct method to find a constant	**M1**	
	Obtain one of the values $A = -3$, $B = 1$, $C = 2$	**A1**	
	Obtain a second value	**A1**	
	Obtain the third value	**A1**	
		4	
9(ii)	Use a correct method to find the first two terms of the expansion of $(3-x)^{-1}$, $\left(1-\frac{1}{3}x\right)^{-1}$, $(2+x^2)^{-1}$ or $\left(1+\frac{1}{2}x^2\right)^{-1}$	**M1**	Symbolic binomial coefficients are not sufficient for the M1.
	Obtain correct unsimplified expansions up to the term in x^3 of each partial fraction	**A1Ft + A1Ft**	The ft is on A, B and C. $-1\left(1+\frac{x}{3}+\frac{x^2}{9}+\frac{x^3}{27}...\right) + \frac{x+2}{2}\left(1-\frac{x^2}{2}...\right)$ $-1-\frac{x}{3}-\frac{x^2}{9}-\frac{x^3}{27}+1-\frac{x^2}{2}+\frac{x}{2}-\frac{x^3}{4}$
	Multiply out their expansion, up to the terms in x^3, by $Bx + C$, where $BC \neq 0$	**M1**	
	Obtain **final** answer $\frac{1}{6}x-\frac{11}{18}x^2-\frac{31}{108}x^3$, or equivalent	**A1**	
		5	

Question	Answer	Marks	Guidance
10(i)	Equate at least two pairs of components and solve for *s* or for *t*	**M1**	$\begin{cases} s = \frac{-4}{3} \\ t = \frac{-5}{3} \\ -5 \neq \frac{-1}{3} \end{cases}$ or $\begin{cases} s = -6 \\ t = -11 \\ 7 \neq -7 \end{cases}$ or $\begin{cases} s = \frac{-2}{5} \\ t = \frac{-13}{5} \\ \frac{6}{5} \neq \frac{-8}{5} \end{cases}$
	Obtain correct answer for *s* or *t*, e.g. $s = -6$, $t = -11$	**A1**	
	Verify that all three equations are not satisfied and the lines fail to intersect	**A1**	
	State that the lines are not parallel	**B1**	
		4	
10(ii)	*EITHER*: Use scalar product to obtain a relevant equation in *a*, *b* and *c*, e.g. $2a + 3b - c = 0$	**B1**	
	Obtain a second equation, e.g. $a + 2b + c = 0$, and solve for one ratio, e.g. $a : b$	**M1**	
	Obtain $a : b : c$ and state correct answer, e.g. $5\mathbf{i} - 3\mathbf{j} + \mathbf{k}$, or equivalent	**A1**	
	OR: Attempt to calculate vector product of relevant vectors, e.g. $(2\mathbf{i} + 3\mathbf{j} - \mathbf{k})\times(\mathbf{i} + 2\mathbf{j} + \mathbf{k})$	**M1**	
	Obtain two correct components	**A1**	
	Obtain correct answer, e.g. $5\mathbf{i} - 3\mathbf{j} + \mathbf{k}$	**A1**	
		3	

Question	Answer		Marks	Guidance
10(iii)	*EITHER*:	State position vector or coordinates of the mid-point of a line segment joining points on *l* and *m*, e.g. $\frac{3}{2}\mathbf{i}+\mathbf{j}+\frac{5}{2}\mathbf{k}$	**B1**	*OR*: Use the result of **(ii)** to form equations of planes containing *l* and *m* B1
		Use the result of (**ii**) and the mid-point to find *d*	**M1**	Use average of distances to find equation of *p*. M1
		Obtain answer $5x - 3y + z = 7$, or equivalent	**A1**	Obtain answer $5x - 3y + z = 7$, or equivalent A1
	OR:	Using the result of part **(ii)**, form an equation in *d* by equating perpendicular distances to the plane of a point on *l* and a point on *m*	**M1**	
		State a correct equation, e.g. $\left\lvert\frac{14-d}{\sqrt{35}}\right\rvert = \left\lvert\frac{-d}{\sqrt{35}}\right\rvert$	**A1**	
		Solve for *d* and obtain answer $5x - 3y + z = 7$, or equivalent	**A1**	
			3	

Cambridge
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A Level

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Cambridge International Advanced Level

MATHEMATICS **9709/33**

Paper 3 **May/June 2018**

MARK SCHEME

Maximum Mark: 75

Published

This document consists of **9** printed pages.

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International Education

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Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

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Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

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Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously 'correct' answers or results obtained from incorrect working.

  - Note: B2 or A2 means that the candidate can earn 2 or 0.
    B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only – often written by a ‘fortuitous’ answer
ISW	Ignore Subsequent Working
SOI	Seen or implied
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become ‘follow through’ marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Question	Answer	Marks
1	Obtain a correct unsimplified version of the *x* or x^2 term of the expansion of $(4-3x)^{-\frac{1}{2}}$ or $\left(1-\frac{3}{4}x\right)^{-\frac{1}{2}}$	**M1**
	State correct first term 2	**B1**
	Obtain the next two terms $\frac{3}{4}x+\frac{27}{64}x^2$	**A1 + A1**
	Total:	**4**

Question	Answer	Marks
2	State or imply $u^2=u+5$, or equivalent in 5^x	**B1**
	Solve for *u*, or 5^x	**M1**
	Obtain root $\frac{1}{2}(1+\sqrt{21})$, or decimal in [2.79, 2.80]	**A1**
	Use correct method for finding *x* from a positive root	**M1**
	Obtain answer *x* = 0.638 and no other answer	**A1**
	Total:	**5**

Question	Answer	Marks
3	Integrate by parts and reach $ax\sin 3x+b\int\sin 3x\mathrm{d}x$	**M1***
	Obtain $\frac{1}{3}x\sin 3x-\frac{`1}{3}\int\sin 3x\mathrm{d}x$, or equivalent	**A1**
	Complete the integration and obtain $\frac{1}{3}x\sin 3x+\frac{1}{9}\cos 3x$, or equivalent	**A1**
	Substitute limits correctly having integrated twice and obtained *ax* sin 3*x* + *b* cos 3*x*	**M1(dep*)**
	Obtain answer $\frac{1}{18}(\pi-2)$ OE	**A1**
	Total:	**5**

Question	Answer	Marks
4(i)	Use the quotient or product rule	**M1**
	Obtain correct derivative in any form	**A1**
	Equate derivative to zero and obtain the given equation	**A1**
	Total:	**3**
4(ii)	Sketch a relevant graph, e.g. $y = \ln x$	**B1**
	Sketch a second relevant graph, e.g. $y = 1 + \frac{3}{x}$, and justify the given statement	**B1**
	Total:	**2**
4(iii)	Use iterative formula $x_{n+1} = \frac{3+x}{\ln x_n}$ correctly at least once	**M1**
	Obtain final answer 4.97	**A1**
	Show sufficient iterations to 4 d.p.to justify 4.97 to 2 d.p. or show there is a sign change in the interval (4.965, 4.975)	**A1**
	Total:	**3**

Question	Answer	Marks
5(i)	Attempt cubic expansion and equate to 1	**M1**
	Obtain a correct equation	**A1**
	Use Pythagoras and double angle formula in the expansion	**M1**
	Obtain the given result correctly	**A1**
	Total:	**4**
5(ii)	Use the identity and carry out a method for finding a root	**M1**
	Obtain answer 20.9°	**A1**
	Obtain a second answer, e.g. 69.1°	**A1FT**
	Obtain the remaining answers, e.g. 110.9° and 159.1°, and no others in the given interval	**A1FT**
	Total:	**4**

Question	Answer	Marks
6(i)	Carry out relevant method to find A and B such that $\frac{1}{4-y^2} \equiv \frac{A}{2+y} + \frac{B}{2-y}$	**M1**
	Obtain $A = B = \frac{1}{4}$	**A1**
	Total:	**2**
6(ii)	Separate variables correctly and integrate at least one side to obtain one of the terms $a \ln x$, $b \ln (2 + y)$ or $c \ln (2 - y)$	**M1**
	Obtain term $\ln x$	**B1**
	Integrate and obtain terms $\frac{1}{4}\ln(2+y) - \frac{1}{4}\ln(2-y)$	**A1FT**
	Use $x = 1$ and $y = 1$ to evaluate a constant, or as limits, in a solution containing at least two terms of the form $a \ln x$, $b \ln (2 + y)$ and $c \ln (2 - y)$	**M1**
	Obtain a correct solution in any form, e.g. $\ln x = \frac{1}{4}\ln(2+y) - \frac{1}{4}\ln(2-y) - \frac{1}{4}\ln 3$	**A1**
	Rearrange as $\frac{2(3x^4-1)}{(3x^4+1)}$, or equivalent	**A1**
	Total:	**6**

Question	Answer	Marks
7(i)	State answer $R = \sqrt{5}$	**B1**
	Use trig formulae to find $\tan \alpha$	**M1**
	Obtain $\tan\alpha = 2$	**A1**
	Total:	**3**
7(ii)	State that the integrand is $3\sec^2(\theta - \alpha)$	**B1FT**
	State correct indefinite integral $3\tan(\theta - \alpha)$	**B1FT**
	Substitute limits correctly	**M1**
	Use $\tan(A \pm B)$ formula	**M1**
	Obtain the given exact answer correctly	**A1**
	Total:	**5**

Question	Answer	Marks
8(i)	State or imply $3y^2 \dfrac{dy}{dx}$ as derivative of y^3	**B1**
	State or imply $3y^2 + 6xy \dfrac{dy}{dx}$ as derivative of $3xy^2$	**B1**
	Equate derivative of LHS to zero and solve for $\dfrac{dy}{dx}$	**M1**
	Obtain the given answer	**A1**
	Total:	**4**
8(ii)	Equate denominator to zero and solve for y	**M1***
	Obtain $y = 0$ and $x = a$	**A1**
	Obtain $y = \alpha x$ and substitute in curve equation to find x or to find y	**M1(dep*)**
	Obtain $x = -a$	**A1**
	Obtain $y = 2a$	**A1**
	Total:	**5**

Question	Answer	Marks
9(a)	Substitute and obtain a correct equation in x and y	**B1**
	Use $i^2 = -1$ and equate real and imaginary parts	**M1**
	Obtain two correct equations in x and y, e.g. $3x - y = 1$ and $3y - x = 5$	**A1**
	Solve and obtain answer $z = 1 +$ **2(i)**	**A1**
	Total:	**4**
9(b)	Show a circle with radius 3	**B1**
	Show the line $y = 2$ extending in both quadrants	**B1**
	Shade the correct region	**B1**
	Carry out a complete method for finding the greatest value of arg z	**M1**
	Obtain answer 2.41	**A1**
	Total:	**5**

Question	Answer	Marks
10(i)	Carry out a correct method for finding a vector equation for *AB*	**M1**
	Obtain $\mathbf{r} = 2\mathbf{i} + \mathbf{j} + 3\mathbf{k} + \lambda(2\mathbf{i} - 2\mathbf{k})$, or equivalent	**A1**
	Equate pair(s) of components *AB* and *l* and solve for λ or μ	**M1(dep*)**
	Obtain correct answer for λ or μ	**A1**
	Verify that all three component equations are not satisfied	**A1**
	Total:	**5**
10(ii)	State or imply a direction vector for *AP* has components $(2 + t, 5 + 2t, -3 - 2t)$	**B1**
	State or imply that cos 120° equals the scalar product of $\overrightarrow{AP}$ and $\overrightarrow{AB}$ divided by the product of their moduli	**M1**
	Carry out the correct processes for finding the scalar product and the product of the moduli in terms of *t*, and obtain an equation in terms of *t*	**M1**
	Obtain the given equation correctly	**A1**
	Solve the quadratic and use a root to find a position vector for *P*	**M1**
	Obtain position vector $2\mathbf{i} + 2\mathbf{j} + 4\mathbf{k}$ from $t = -2$, having rejected the root $t = -\frac{2}{3}$	**A1**
	Total:	**6**

Cambridge International A Level

Cambridge Assessment International Education
Cambridge International Advanced Level

MATHEMATICS **9709/32**

Paper 3 Pure Mathematics **March 2018**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the March 2018 series for most Cambridge IGCSE®, Cambridge International A and AS Level components and some Cambridge O Level components.

This document consists of **10** printed pages.

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1	State or imply ordinates 1, 0.8556…, 0.6501…, 0	**B1**
	Use correct formula, or equivalent, with $h = \frac{1}{12}\pi$ and four ordinates	**M1**
	Obtain answer 0.525	**A1**
		3

Question	Answer	Marks
2	State a correct unsimplified version of the x or x^2 or x^3 term	**M1**
	State correct first two terms $1 - x$	**A1**
	Obtain the next two terms $-\frac{3}{2}x^2 - \frac{7}{2}x^3$	**A1 + A1**
		4

Question	Answer	Marks
3(i)	State correct expansion of $\cos(3x + x)$ or $\cos(3x - x)$	**B1**
	Substitute in $\frac{1}{2}(\cos 4x + \cos 2x)$	**M1**
	Obtain the given identity correctly **AG**	**A1**
		3
3(ii)	Obtain integral $\frac{1}{8}\sin 4x + \frac{1}{4}\sin 2x$	**B1**
	Substitute limits correctly	**M1**
	Obtain the given answer following full, correct and exact working **AG**	**A1**
		3

Question	Answer	Marks
4(i)	State or imply $n \ln y = \ln A + 3 \ln x$	**B1**
	State that the graph of ln y against ln x has an equation which is *linear* in ln y and ln x, or has equation of the form $nY = \ln A + 3X$, where $Y = \ln y$ and $X = \ln x$, and is thus a straight line.	**B1**
		2
4(ii)	Substitute x- and y-values in $n \ln y = \ln A + 3 \ln x$ or in the given equation and solve for one of the constants	**M1**
	Obtain a correct constant, e.g. $n = 1.70$	**A1**
	Solve for a second constant	**M1**
	Obtain the other constant, e.g. $A = 2.90$	**A1**
		4

Question	Answer	Marks
5(i)	State correct derivative of x or y with respect to t	**B1**
	Use $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}t} \div \frac{\mathrm{d}x}{\mathrm{d}t}$	**M1**
	Obtain $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{4\sin 2t}{2 + 2\cos 2t}$, or equivalent	**A1**
	Use double angle formulae throughout	**M1**
	Obtain the given answer correctly **AG**	**A1**
		5
5(ii)	State or imply $t = \tan^{-1}\left(-\frac{1}{4}\right)$	**B1**
	Obtain answer $x = -0.961$	**B1**
		2

Question	Answer	Marks
6(i)	Show sufficient working to justify the given statement **AG**	**B1**
		1
6(ii)	Separate variables correctly and attempt integration of at least one side	**B1**
	Obtain term $\frac{1}{2}x^2$	**B1**
	Obtain terms $\tan^2\theta + \tan\theta$, or $\sec^2\theta + \tan\theta$	**B1 + B1**
	Evaluate a constant, or use limits $x = 1$, $\theta = \frac{1}{4}\pi$, in a solution with two terms of the form ax^2 and $b\tan\theta$, where $ab \neq 0$	**M1**
	State correct answer in any form, e.g. $\frac{1}{2}x^2 = \tan^2\theta + \tan\theta - \frac{3}{2}$	**A1**
	Substitute $\theta = \frac{1}{3}\pi$ and obtain $x = 2.54$	**A1**
		7

Question	Answer	Marks
7(i)	Sketch a relevant graph, e.g. $y = e^{2x}$	**B1**
	Sketch a second relevant graph, e.g. $y = 6 + e^{-x}$, and justify the given statement	**B1**
		2
7(ii)	Calculate the value of a relevant expression or values of a pair of relevant expressions at $x = 0.5$ and $x = 1$	**M1**
	Complete the argument correctly with correct calculated values	**A1**
		2
7(iii)	State a suitable equation, e.g. $x = \frac{1}{3}\ln\left(1 + 6e^{x}\right)$	**B1**
	Rearrange this as $e^{2x} = 6 + e^{-x}$, or commence working *vice versa*	**B1**
		2

Question	Answer	Marks
7(iv)	Use the iterative formula correctly at least once	**M1**
	Obtain final answer 0.928	**A1**
	Show sufficient iterations to 5 d.p. to justify 0.928 to 3 d.p., or show there is a sign change in the interval (0.9275, 0.9285)	**A1**
		3

Question	Answer	Marks
8(i)	State or imply the form $\frac{A}{2x+1}+\frac{Bx+C}{x^2+9}$	**B1**
	Use a correct method for finding a constant	**M1**
	Obtain one of $A = 3$, $B = 1$ and $C = 0$	**A1**
	Obtain a second value	**A1**
	Obtain the third value	**A1**
		5
8(ii)	Integrate and obtain term $\frac{3}{2}\ln(2x+1)$ (FT on A value)	**B1 FT**
	Integrate and obtain term of the form $k\ln(x^2+9)$	**M1**
	Obtain term $\frac{1}{2}\ln(x^2+9)$ (FT on B value)	**A1 FT**
	Substitute limits correctly in an integral of the form $a\ln(2x+1)+b\ln(x^2+9)$, where $ab\neq 0$	**M1**
	Obtain answer ln 45 after full and correct working	**A1**
		5

Question	Answer	Marks
9(i)(a)	Substitute x = 1 + 2i in the equation and attempt expansions of x^2 and x^3	**M1**
	Use $i^2 = -1$ correctly at least once and solve for k	**M1**
	Obtain answer k = 15	**A1**
		3
9(i)(b)	State answer 1 – 2i	**B1**
	Carry out a complete method for finding a quadratic factor with zeros 1 + 2i and 1 – 2i	**M1**
	Obtain $x^2 - 2x + 5$	**A1**
	Obtain root $-\frac{3}{2}$, or equivalent, *via* division or inspection	**A1**
		4
9(ii)	Show a circle with centre 1 + 2i	**B1**
	Show a circle with radius 1	**B1**
	Carry out a complete method for calculating the least value of arg z	**M1**
	Obtain answer 0.64	**A1**
		4

Question	Answer	Marks
10(i)	Express general point of l in component form, e.g. $\mathbf{r} = (4+\mu)\mathbf{i} + (3+2\mu)\mathbf{j} + (-1-2\mu)\mathbf{k}$, or equivalent NB: Calling the vector **a** + μ **b**, the **B1** is earned by a correct reduction of the sum to a single vector or by expressing the substitution as a distributed sum **a.n** + μ **b.n**	**B1**
	Substitute in given equation of p and solve for μ	**M1**
	Obtain final answer 2**i** – **j** + 3**k** from $\mu = -2$	**A1**
		3

Question	Answer	Marks
10(ii)	Using the correct process, evaluate the scalar product of a direction vector for *l* and a normal for *p*	**M1**
	Using the correct process for the moduli, divide the scalar product by the product of the moduli and find the inverse sine or cosine of the result	**M1**
	Obtain answer 10.3° (or 0.179 radians)	**A1**
		3
10(iii)	*EITHER*: State $a + 2b - 2c = 0$ or $2a - 3b - c = 0$	**(B1**
	Obtain two relevant equations and solve for one ratio, e.g. $a : b$	**M1**
	Obtain $a : b : c = 8 : 3 : 7$, or equivalent	**A1**
	Substitute a, b, c and given point and evaluate d	**M1**
	Obtain answer $8x + 3y + 7z = 5$	**A1)**
	*OR*1: Attempt to calculate vector product of relevant vectors, e.g. $(2\mathbf{i} - 3\mathbf{j} - \mathbf{k})\times(\mathbf{i} + 2\mathbf{j} - 2\mathbf{k})$	**(M1**
	Obtain two correct components of the product	**A1**
	Obtain correct product, e.g. $8\mathbf{i} + 3\mathbf{j} + 7\mathbf{k}$	**A1**
	Use the product and the given point to find d	**M1**
	Obtain answer $8x + 3y + 7z = 5$, or equivalent	**A1)**
	*OR*2: Attempt to form a 2-parameter equation with relevant vectors	**(M1**
	State a correct equation, e.g. $\mathbf{r} = 4\mathbf{j} - \mathbf{k} + \lambda(\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}) + \mu(2\mathbf{i} - 3\mathbf{j} - \mathbf{k})$	**A1**
	State 3 equations in x, y, z, λ and μ	**A1**
	Eliminate λ and μ	**M1**
	State answer $8x + 3y + 7z = 5$, or equivalent	**A1)**
		5

Cambridge Assessment International Education
Cambridge International Advanced Level

MATHEMATICS **9709/31**

Paper 3 **October/November 2017**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2017 series for most Cambridge IGCSE®, Cambridge International A and AS Level components and some Cambridge O Level components.

This document consists of **9** printed pages.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

  - Note: B2 or A2 means that the candidate can earn 2 or 0.
    B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
CWO	Correct Working Only – often written by a 'fortuitous' answer
ISW	Ignore Subsequent Working
SOI	Seen or implied
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Question	Answer	Marks
1	Commence division and reach a partial quotient $x^2 + kx$	M1
	Obtain quotient $x^2 - 2x + 5$	A1
	Obtain remainder $-12x + 5$	A1
		3

Question	Answer	Marks
2	Plot the four points and draw straight line	B1
	State or imply that $\ln y = \ln C + x \ln a$	B1
	Carry out a completely correct method for finding $\ln C$ or $\ln a$	M1
	Obtain answer $C = 3.7$	A1
	Obtain answer $a = 1.5$	A1
		5

Question	Answer	Marks
3(i)	Calculate value of a relevant expression or expressions at $x = 2$ and $x = 3$	M1
	Complete the argument correctly with correct calculated values	A1
		2
3(ii)	Use an iterative formula correctly at least once	M1
	Show that (B) fails to converge	A1
	Using (A), obtain final answer 2.43	A1
	Show sufficient iterations to justify 2.43 to 2 d.p., or show there is a sign change in (2.425, 2.435)	A1
		4

Question	Answer	Marks
4(i)	Use correct $\tan(A \pm B)$ formula and express the LHS in terms of $\tan x$	**M1**
	Using $\tan 45° = 1$ express LHS as a single fraction	**A1**
	Use Pythagoras or correct double angle formula	**M1**
	Obtain given answer	**A1**
		4
4(ii)	Show correct sketch for one branch	**B1**
	Both branches correct and nothing else seen in the interval	**B1**
	Show asymptote at $x = 45°$	**B1**
		3

Question	Answer	Marks
5(i)	State or imply $y^3 + 3xy^2\frac{\mathrm{d}y}{\mathrm{d}x}$ as derivative of xy^3	**B1**
	State or imply $4y^3\frac{\mathrm{d}y}{\mathrm{d}x}$ as derivative of y^4	**B1**
	Equate derivative of the LHS to zero and solve for $\frac{\mathrm{d}y}{\mathrm{d}x}$	**M1**
	Obtain the given answer	**A1**
		4
5(ii)	Equate numerator to zero	***M1**
	Obtain $y = -2x$, or equivalent	**A1**
	Obtain an equation in x or y	**DM1**
	Obtain final answer $x = -1$, $y = 2$ and $x = 1$, $y = -2$	**A1**
		4

Question	Answer	Marks
6	Separate variables correctly and attempt integration of one side	**B1**
	Obtain term $\tan y$, or equivalent	**B1**
	Obtain term of the form $k\ln\cos x$, or equivalent	**M1**
	Obtain term $-4\ln\cos x$, or equivalent	**A1**
	Use $x = 0$ and $y = \frac{1}{4}\pi$ in solution containing $a\tan y$ and $b\ln\cos x$ to evaluate a constant, or as limits	**M1**
	Obtain correct solution in any form, e.g. $\tan y = 4\ln\sec x + 1$	**A1**
	Substitute $y = \frac{1}{3}\pi$ in solution containing terms $a\tan y$ and $b\ln\cos x$, and use correct method to find x	**M1**
	Obtain answer $x = 0.587$	**A1**
		8

Question	Answer	Marks
7(a)	Square $x + \mathrm{i}y$ and equate real and imaginary parts to 8 and -15	**M1**
	Obtain $x^2 - y^2 = 8$ and $2xy = -15$	**A1**
	Eliminate one unknown and find a horizontal equation in the other	**M1**
	Obtain $4x^4 - 32x^2 - 225 = 0$ or $4y^4 + 32y^2 - 225 = 0$, or three term equivalent	**A1**
	Obtain answers $\pm\frac{1}{\sqrt{2}}(5 - 3\mathrm{i})$ or equivalent	**A1**
		5
7(b)	Show a circle with centre $2 + \mathrm{i}$ in a relatively correct position	**B1**
	Show a circle with radius 2 and centre not at the origin	**B1**
	Show line through i at an angle of $\frac{1}{4}\pi$ to the real axis	**B1**
	Shade the correct region	**B1**
		4

Question	Answer	Marks
8(i)	Use a relevant method to determine a constant	**M1**
	Obtain one of the values $A = 2$, $B = 2$, $C = -1$	**A1**
	Obtain a second value	**A1**
	Obtain the third value	**A1**
		4
8(ii)	Integrate and obtain terms $2x + 2\ln(x+2) - \frac{1}{2}\ln(2x-1)$ (deduct **B1** for each error or omission) [The FT is on A, B and C]	**B2 FT**
	Substitute limits correctly in an integral containing terms $a\ln(x+2)$ and $b\ln(2x-1)$, where $ab \neq 0$	***M1**
	Use at least one law of logarithms correctly	**DM1**
	Obtain the given answer after full and correct working	**A1**
		5

Question	Answer	Marks
9(i)	Use correct product or quotient rule	**M1**
	Obtain correct derivative in any form	**A1**
	Equate derivative to zero and obtain a 3 term quadratic equation in x	**M1**
	Obtain answers $x = 2 \pm \sqrt{3}$	**A1**
		4
9(ii)	Integrate by parts and reach $k(1+x^2)e^{-\frac{1}{2}x} + l\int xe^{-\frac{1}{2}x}\,dx$	***M1**
	Obtain $-2(1+x^2)e^{-\frac{1}{2}x} + 4\int xe^{-\frac{1}{2}x}\,dx$, or equivalent	**A1**
	Complete the integration and obtain $(-18-8x-2x^2)e^{-\frac{1}{2}x}$, or equivalent	**A1**
	Use limits $x = 0$ and $x = 2$ correctly, having fully integrated twice by parts	**DM1**
	Obtain the given answer	**A1**
		5

Question	Answer	Marks
10(i)	Equate at least two pairs of components of general points on *l* and *m* and solve for λ or for μ	**M1**
	Obtain correct answer for λ or μ, e.g. $\lambda = 3$ or $\mu = -2$; $\lambda = 0$ or $\mu = -\frac{1}{2}$; or $\lambda = \frac{3}{2}$ or $\mu = -\frac{7}{2}$	**A1**
	Verify that not all three pairs of equations are satisfied and that the lines fail to intersect	**A1**
		3
10(ii)	Carry out correct process for evaluating scalar product of direction vectors for *l* and *m*	***M1**
	Using the correct process for the moduli, divide the scalar product by the product of the moduli and evaluate the inverse cosine of the result	**DM1**
	Obtain answer 45° or $\frac{1}{4}\pi$ (0.785) radians	**A1**
		3
10(iii)	*EITHER:* Use scalar product to obtain a relevant equation in *a*, *b* and *c*, e.g. $-a + b + 4c = 0$	**B1**
	Obtain a second equation, e.g. $2a + b - 2c = 0$ and solve for one ratio, e.g. $a : b$	**M1**
	Obtain $a : b : c = 2 : -2 : 1$, or equivalent	**A1**
	Substitute (3, –2, –1) and values of *a*, *b* and *c* in general equation and find *d*	**M1**
	Obtain answer $2x - 2y + z = 9$, or equivalent	**A1**
	*OR*1: Attempt to calculate vector product of relevant vectors, e.g $(-\mathbf{i} + \mathbf{j} + 4\mathbf{k}) \times (2\mathbf{i} + \mathbf{j} - 2\mathbf{k})$	**(M1**
	Obtain two correct components	**A1**
	Obtain correct answer, e.g. $-6\mathbf{i} + 6\mathbf{j} - 3\mathbf{k}$	**A1**
	Substitute (3, –2, –1) in $-6x + 6y - 3z = d$, or equivalent, and find *d*	**M1**
	Obtain answer $-2x + 2y - z = -9$, or equivalent	**A1)**
	*OR*2: Using the relevant point and relevant vectors, form a 2-parameter equation for the plane	**(M1**
	State a correct equation, e.g. $\mathbf{r} = 3\mathbf{i} - 2\mathbf{j} - \mathbf{k} + \lambda(-\mathbf{i} + \mathbf{j} + 4\mathbf{k}) + \mu(2\mathbf{i} + \mathbf{j} - 2\mathbf{k})$	**A1**
	State three correct equations in *x*, *y*, *z*, λ and μ	**A1**
	Eliminate λ and μ	**M1**

Question	Answer	Marks
	Obtain answer $2x-2y+z=9$, or equivalent	**A1)**
	*OR*3: Using the relevant point and relevant vectors, form a determinant equation for the plane	**(M1**
	State a correct equation, e.g. $\begin{vmatrix} x-3 & y+2 & z+1 \\ -1 & 1 & 4 \\ 2 & 1 & -2 \end{vmatrix}=0$	**A1**
	Attempt to expand the determinant	**M1**
	Obtain two correct cofactors	**A1**
	Obtain answer $-2x+2y-z=-9$, or equivalent	**A1)**
		5

Cambridge Assessment International Education
Cambridge International Advanced Level

MATHEMATICS **9709/32**

Paper 3 **October/November 2017**

MARK SCHEME

Maximum Mark: 75

Published

This document consists of **9** printed pages.

Cambridge Assessment International Education

Mark Scheme Notes

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Question	Answer	Marks
1(i)	State or imply ordinates 0.915929…, 1, 1.112485…	**B1**
	Use correct formula, or equivalent, with h = 1.2 and three ordinates	**M1**
	Obtain answer 2.42 only	**A1**
		3
1(ii)	Justify the given statement	**B1**
		1

Question	Answer	Marks
2	Use law for the logarithm of a power or a quotient on the given equation	**M1**
	Use $\log_2 8 = 3$ or $2^3 = 8$	**M1**
	Obtain $x^2 - 8x - 8 = 0$, or horizontal equivalent	**A1**
	Solve a 3-term quadratic equation	**M1**
	Obtain final answer x = 8.90 only	**A1**
		5

Question	Answer	Marks
3	Use correct $\tan(A \pm B)$ formula and express LHS in terms of tan θ	**M1**
	Using tan 60° = $\sqrt{3}$ and $\cot\theta = 1/\tan\theta$, obtain a correct equation in $\tan\theta$ in any form	**A1**
	Reduce the equation to one in $\tan^2\theta$ only	**M1**
	Obtain $11\tan^2\theta = 1$, or equivalent	**A1**
	Obtain answer 16.8°	**A1**
		5

Question	Answer	Marks
4(i)	Use correct product or quotient rule or rewrite as $2\sec x - \tan x$ and differentiate	**M1**
	Obtain correct derivative in any form	**A1**
	Equate the derivative to zero and solve for x	**M1**
	Obtain $x = \frac{1}{6}\pi$	**A1**
	Obtain $y = \sqrt{3}$	**A1**
		5
4(ii)	Carry out an appropriate method for determining the nature of a stationary point	**M1**
	Show the point is a minimum point with no errors seen	**A1**
		2

Question	Answer	Marks
5	Separate variables and obtain $\int \frac{1}{y}\,dy = \int \frac{x+2}{x+1}\,dx$	**B1**
	Obtain term $\ln y$	**B1**
	Use an appropriate method to integrate $(x+2)/(x+1)$	***M1**
	Obtain integral $x + \ln(x+1)$, or equivalent, e.g. $\ln(x+1) + x + 1$	**A1**
	Use x = 1 and y = 2 to evaluate a constant, or as limits	**DM1**
	Obtain correct solution in x and y in any form e.g. $\ln y = x + \ln(x+1) - 1$	**A1**
	Obtain answer $y = (x+1)e^{x-1}$	**A1**
		7

Question	Answer	Marks
6(i)	State or imply $3x^2y + x^3\dfrac{dy}{dx}$ as derivative of x^3y	**B1**
	State or imply $9xy^2\dfrac{dy}{dx} + 3y^3$ as derivative of $3xy^3$	**B1**
	Equate derivative of the LHS to zero and solve for $\dfrac{dy}{dx}$	**M1**
	Obtain the given answer AG	**A1**
		4
6(ii)	Equate numerator to zero and use $x = -y$ to obtain an equation in x or in y	**M1**
	Obtain answer $x = a$ and $y = -a$	**A1**
	Obtain answer $x = -a$ and $y = a$	**A1**
	Consider and reject $y = 0$ and $x = y$ as possibilities	**B1**
		4

Question	Answer	Marks
7(i)	State modulus 2	**B1**
	State argument $-\frac{1}{3}\pi$ or $-60°$ ($\frac{5}{3}\pi$ or $300°$)	**B1**
		2
7(ii)	*EITHER:* Expand $(1-(\sqrt{3})i)^3$ completely and process i^2 and i^3	**(M1**
	Verify that the given relation is satisfied	**A1)**
	OR: $u^3 = 2^3\left(\cos(-\pi) + i\sin(-\pi)\right)$ or equivalent: follow their answers to **(i)**	**(M1**
	Verify that the given relation is satisfied	**A1)**
		2

Question	Answer	Marks
7(iii)	Show a circle with centre $1-(\sqrt{3})\mathrm{i}$ in a relatively correct position	**B1**
	Show a circle with radius 2 passing through the origin	**B1**
	Show the line Re $z = 2$	**B1**
	Shade the correct region	**B1**
		4

Question	Answer	Marks
8(i)	State or imply the form $\dfrac{A}{1-x}+\dfrac{B}{2x+3}+\dfrac{C}{(2x+3)^2}$	**B1**
	Use a relevant method to determine a constant	**M1**
	Obtain one of the values $A = 1$, $B = -2$, $C = 5$	**A1**
	Obtain a second value	**A1**
	Obtain the third value	**A1**
		5
	[Mark the form $\dfrac{A}{1-x}+\dfrac{Dx+E}{(2x+3)^2}$, where $A = 1$, $D = -4$, $E = -1$, **B1M1A1A1A1** as above.]	
8(ii)	Use a correct method to find the first two terms of the expansion of $(1-x)^{-1}$, $(1+\frac{2}{3}x)^{-1}$, $(2x+3)^{-1}$, $(1+\frac{2}{3}x)^{-2}$ or $(2x+3)^{-2}$	**M1**
	Obtain correct unsimplified expansions up to the term in x^2 of each partial fraction	**A3 FT**
	Obtain final answer $\frac{8}{9}+\frac{19}{27}x+\frac{13}{9}x^2$, or equivalent	**A1**
		5

Question	Answer	Marks
9(i)	Integrate by parts and reach $ax^{\frac{3}{2}}\ln x + b\int x^{\frac{3}{2}}.\frac{1}{x}\ \mathrm{d}x$	*M1
	Obtain $\frac{2}{3}x^{\frac{3}{2}}\ln x - \frac{2}{3}\int x^{\frac{1}{2}}\ \mathrm{d}x$	A1
	Obtain integral $\frac{2}{3}x^{\frac{3}{2}}\ln x - \frac{4}{9}x^{\frac{3}{2}}$, or equivalent	A1
	Substitute limits correctly and equate to 2	DM1
	Obtain the given answer correctly AG	A1
		5
9(ii)	Evaluate a relevant expression or pair of expressions at $x = 2$ and $x = 4$	M1
	Complete the argument correctly with correct calculated values	A1
		2
9(iii)	Use the iterative formula correctly at least once	M1
	Obtain final answer 3.031	A1
	Show sufficient iterations to 5 d.p. to justify 3.031 to 3 d.p., or show there is a sign change in the interval (3.0305, 3.0315)	A1
		3

Question	Answer	Marks
10(i)	State or imply a correct normal vector to either plane, e.g. $\mathbf{i}+\mathbf{j}+3\mathbf{k}$ or $2\mathbf{i}-2\mathbf{j}+\mathbf{k}$	B1
	Carry out correct process for evaluating the scalar product of two normal vectors	M1
	Using the correct process for the moduli, divide the scalar product of the two normals by the product of their moduli and evaluate the inverse cosine of the result	M1
	Obtain final answer 72.5° or 1.26 radians	A1
		4
10(ii)	*EITHER*: Substitute $y = 2$ in both plane equations and solve for x or for z	(M1
	Obtain $x = 3$ and $z = 1$	A1)
	OR: Find the equation of the line of intersection of the planes	
	Substitute $y = 2$ in line equation and solve for x or for z	(M1
	Obtain $x = 3$ and $z = 1$	A1)

Question	Answer	Marks
	EITHER: Use scalar product to obtain an equation in a, b and c, e.g. $a + b + 3c = 0$	**(B1**
	Form a second relevant equation, e.g. $2a - 2b + c = 0$, and solve for one ratio, e.g. $a : b$	***M1**
	Obtain final answer $a : b : c = 7 : 5 : -4$	**A1**
	Use coordinates of A and values of a, b and c in general equation and find d	**DM1**
	Obtain answer $7x + 5y - 4z = 27$, or equivalent	**A1 FT)**
	*OR*1: Calculate the vector product of relevant vectors, e.g. $(\mathbf{i} + \mathbf{j} + 3\mathbf{k}) \times (2\mathbf{i} - 2\mathbf{j} + \mathbf{k})$	**(*M1**
	Obtain two correct components	**A1**
	Obtain correct answer, e.g. $7\mathbf{i} + 5\mathbf{j} - 4\mathbf{k}$	**A1**
	Substitute coordinates of A in plane equation with their normal and find d	**DM1**
	Obtain answer $7x + 5y - 4z = 27$, or equivalent	**A1 FT)**
	*OR*2: Using relevant vectors, form a two-parameter equation for the plane	**(*M1**
	State a correct equation, e.g. $\mathbf{r} = 3\mathbf{i} + 2\mathbf{j} + \mathbf{k} + \lambda(\mathbf{i} + \mathbf{j} + 3\mathbf{k}) + \mu(2\mathbf{i} - 2\mathbf{j} + \mathbf{k})$	**A1 FT**
	State 3 correct equations in x, y, z, λ and μ	**A1 FT**
	Eliminate λ and μ	**DM1**
	Obtain answer $7x + 5y - 4z = 27$, or equivalent	**A1 FT)**
	*OR*3: Use the direction vector of the line of intersection of the two planes as normal vector to the plane	**(*M1**
	Two correct components	**A1**
	Three correct components	**A1**
	Substitute coordinates of A in plane equation with their normal and find d	**DM1**
	Obtain answer $7x + 5y - 4z = 27$, or equivalent	**A1 FT)**
		7

Cambridge Assessment International Education
Cambridge International Advanced Level

MATHEMATICS **9709/33**

Paper 3 **October/November 2017**

MARK SCHEME

Maximum Mark: 75

Published

This document consists of **9** printed pages.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

  - Note: B2 or A2 means that the candidate can earn 2 or 0.
    B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
CWO	Correct Working Only – often written by a 'fortuitous' answer
ISW	Ignore Subsequent Working
SOI	Seen or implied
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Question	Answer	Marks
1	Commence division and reach a partial quotient $x^2 + kx$	**M1**
	Obtain quotient $x^2 - 2x + 5$	**A1**
	Obtain remainder $-12x + 5$	**A1**
		3

Question	Answer	Marks
2	Plot the four points and draw straight line	**B1**
	State or imply that $\ln y = \ln C + x \ln a$	**B1**
	Carry out a completely correct method for finding $\ln C$ or $\ln a$	**M1**
	Obtain answer $C = 3.7$	**A1**
	Obtain answer $a = 1.5$	**A1**
		5

Question	Answer	Marks
3(i)	Calculate value of a relevant expression or expressions at $x = 2$ and $x = 3$	**M1**
	Complete the argument correctly with correct calculated values	**A1**
		2
3(ii)	Use an iterative formula correctly at least once	**M1**
	Show that (B) fails to converge	**A1**
	Using (A), obtain final answer 2.43	**A1**
	Show sufficient iterations to justify 2.43 to 2 d.p., or show there is a sign change in (2.425, 2.435)	**A1**
		4

Question	Answer	Marks
4(i)	Use correct $\tan(A \pm B)$ formula and express the LHS in terms of $\tan x$	**M1**
	Using $\tan 45° = 1$ express LHS as a single fraction	**A1**
	Use Pythagoras or correct double angle formula	**M1**
	Obtain given answer	**A1**
		4
4(ii)	Show correct sketch for one branch	**B1**
	Both branches correct and nothing else seen in the interval	**B1**
	Show asymptote at $x = 45°$	**B1**
		3

Question	Answer	Marks
5(i)	State or imply $y^3 + 3xy^2\frac{\mathrm{d}y}{\mathrm{d}x}$ as derivative of xy^3	**B1**
	State or imply $4y^3\frac{\mathrm{d}y}{\mathrm{d}x}$ as derivative of y^4	**B1**
	Equate derivative of the LHS to zero and solve for $\frac{\mathrm{d}y}{\mathrm{d}x}$	**M1**
	Obtain the given answer	**A1**
		4
5(ii)	Equate numerator to zero	***M1**
	Obtain $y = -2x$, or equivalent	**A1**
	Obtain an equation in x or y	**DM1**
	Obtain final answer $x = -1$, $y = 2$ and $x = 1$, $y = -2$	**A1**
		4

Question	Answer	Marks
6	Separate variables correctly and attempt integration of one side	**B1**
	Obtain term $\tan y$, or equivalent	**B1**
	Obtain term of the form $k \ln \cos x$, or equivalent	**M1**
	Obtain term $-4 \ln \cos x$, or equivalent	**A1**
	Use $x = 0$ and $y = \frac{1}{4}\pi$ in solution containing $a \tan y$ and $b \ln \cos x$ to evaluate a constant, or as limits	**M1**
	Obtain correct solution in any form, e.g. $\tan y = 4 \ln \sec x + 1$	**A1**
	Substitute $y = \frac{1}{3}\pi$ in solution containing terms $a \tan y$ and $b \ln \cos x$, and use correct method to find x	**M1**
	Obtain answer $x = 0.587$	**A1**
		8

Question	Answer	Marks
7(a)	Square $x + \mathrm{i}y$ and equate real and imaginary parts to 8 and -15	**M1**
	Obtain $x^2 - y^2 = 8$ and $2xy = -15$	**A1**
	Eliminate one unknown and find a horizontal equation in the other	**M1**
	Obtain $4x^4 - 32x^2 - 225 = 0$ or $4y^4 + 32y^2 - 225 = 0$, or three term equivalent	**A1**
	Obtain answers $\pm\frac{1}{\sqrt{2}}(5 - 3\mathrm{i})$ or equivalent	**A1**
		5
7(b)	Show a circle with centre $2 + \mathrm{i}$ in a relatively correct position	**B1**
	Show a circle with radius 2 and centre not at the origin	**B1**
	Show line through i at an angle of $\frac{1}{4}\pi$ to the real axis	**B1**
	Shade the correct region	**B1**
		4

Question	Answer	Marks
8(i)	Use a relevant method to determine a constant	**M1**
	Obtain one of the values $A = 2$, $B = 2$, $C = -1$	**A1**
	Obtain a second value	**A1**
	Obtain the third value	**A1**
		4
8(ii)	Integrate and obtain terms $2x + 2\ln(x+2) - \frac{1}{2}\ln(2x-1)$ (deduct **B1** for each error or omission) [The FT is on A, B and C]	**B2 FT**
	Substitute limits correctly in an integral containing terms $a\ln(x+2)$ and $b\ln(2x-1)$, where $ab \neq 0$	***M1**
	Use at least one law of logarithms correctly	**DM1**
	Obtain the given answer after full and correct working	**A1**
		5

Question	Answer	Marks
9(i)	Use correct product or quotient rule	**M1**
	Obtain correct derivative in any form	**A1**
	Equate derivative to zero and obtain a 3 term quadratic equation in x	**M1**
	Obtain answers $x = 2 \pm \sqrt{3}$	**A1**
		4
9(ii)	Integrate by parts and reach $k(1+x^2)e^{-\frac{1}{2}x} + l\int xe^{-\frac{1}{2}x}\,dx$	***M1**
	Obtain $-2(1+x^2)e^{-\frac{1}{2}x} + 4\int xe^{-\frac{1}{2}x}\,dx$, or equivalent	**A1**
	Complete the integration and obtain $(-18-8x-2x^2)e^{-\frac{1}{2}x}$, or equivalent	**A1**
	Use limits $x = 0$ and $x = 2$ correctly, having fully integrated twice by parts	**DM1**
	Obtain the given answer	**A1**
		5

Question	Answer	Marks
10(i)	Equate at least two pairs of components of general points on *l* and *m* and solve for λ or for μ	M1
	Obtain correct answer for λ or μ, e.g. $\lambda = 3$ or $\mu = -2$; $\lambda = 0$ or $\mu = -\frac{1}{2}$; or $\lambda = \frac{3}{2}$ or $\mu = -\frac{7}{2}$	A1
	Verify that not all three pairs of equations are satisfied and that the lines fail to intersect	A1
		3
10(ii)	Carry out correct process for evaluating scalar product of direction vectors for *l* and *m*	*M1
	Using the correct process for the moduli, divide the scalar product by the product of the moduli and evaluate the inverse cosine of the result	DM1
	Obtain answer 45° or $\frac{1}{4}\pi$ (0.785) radians	A1
		3
10(iii)	*EITHER:* Use scalar product to obtain a relevant equation in *a*, *b* and *c*, e.g. $-a + b + 4c = 0$	B1
	Obtain a second equation, e.g. $2a + b - 2c = 0$ and solve for one ratio, e.g. $a : b$	M1
	Obtain $a : b : c = 2 : -2 : 1$, or equivalent	A1
	Substitute (3, –2, –1) and values of *a*, *b* and *c* in general equation and find *d*	M1
	Obtain answer $2x - 2y + z = 9$, or equivalent	A1
	*OR*1: Attempt to calculate vector product of relevant vectors, e.g $(-\mathbf{i} + \mathbf{j} + 4\mathbf{k}) \times (2\mathbf{i} + \mathbf{j} - 2\mathbf{k})$	(M1
	Obtain two correct components	A1
	Obtain correct answer, e.g. $-6\mathbf{i} + 6\mathbf{j} - 3\mathbf{k}$	A1
	Substitute (3, –2, –1) in $-6x + 6y - 3z = d$, or equivalent, and find *d*	M1
	Obtain answer $-2x + 2y - z = -9$, or equivalent	A1)
	*OR*2: Using the relevant point and relevant vectors, form a 2-parameter equation for the plane	(M1
	State a correct equation, e.g. $\mathbf{r} = 3\mathbf{i} - 2\mathbf{j} - \mathbf{k} + \lambda(-\mathbf{i} + \mathbf{j} + 4\mathbf{k}) + \mu(2\mathbf{i} + \mathbf{j} - 2\mathbf{k})$	A1
	State three correct equations in *x*, *y*, *z*, λ and μ	A1
	Eliminate λ and μ	M1

Question	Answer	Marks
	Obtain answer $2x-2y+z=9$, or equivalent	**A1)**
	*OR*3: Using the relevant point and relevant vectors, form a determinant equation for the plane	**(M1**
	State a correct equation, e.g. $\begin{vmatrix} x-3 & y+2 & z+1 \\ -1 & 1 & 4 \\ 2 & 1 & -2 \end{vmatrix}=0$	**A1**
	Attempt to expand the determinant	**M1**
	Obtain two correct cofactors	**A1**
	Obtain answer $-2x+2y-z=-9$, or equivalent	**A1)**
		5

Cambridge International Examinations
Cambridge International Advanced Level

MATHEMATICS **9709/31**

Paper 3 **May/June 2017**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the May/June 2017 series for most Cambridge IGCSE®, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

This document consists of **9** printed pages.

[Turn over

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PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Question	Answer	Marks
1	**EITHER:** State or imply non-modular inequality $(2x+1)^2 < (3(x-2))^2$, or corresponding quadratic equation, or pair of linear equations $(2x+1) = \pm 3(x-2)$	**(B1**
	Make reasonable solution attempt at a 3-term quadratic e.g. $5x^2 - 40x + 35 = 0$ or solve two linear equations for x	**M1**
	Obtain critical values $x = 1$ and $x = 7$	**A1**
	State final answer $x < 1$ and $x > 7$	**A1)**
	OR: Obtain critical value $x = 7$ from a graphical method, or by inspection, or by solving a linear equation or inequality	**(B1**
	Obtain critical value $x = 1$ similarly	**B2**
	State final answer $x < 1$ and $x > 7$	**B1)**
	Total:	**4**
2	**EITHER:** State a correct unsimplified version of the x or x^2 or x^3 term in the expansion of $(1+6x)^{-\frac{1}{3}}$	**(M1**
	State correct first two terms $1-2x$	**A1**
	Obtain term $8x^2$	**A1**
	Obtain term $-\frac{112}{3}x^3$ $\left(37\frac{1}{3}x^3\right)$ in final answer	**A1)**
	OR: Differentiate expression and evaluate $\mathrm{f}(0)$ and $\mathrm{f}'(0)$, where $\mathrm{f}'(x) = k(1+6x)^{-\frac{4}{3}}$	**(M1**
	Obtain correct first two terms $1-2x$	**A1**
	Obtain term $8x^2$	**A1**
	Obtain term $-\frac{112}{3}x^3$ in final answer	**A1)**
	Total:	**4**

Question	Answer	Marks
3(i)	Remove logarithms correctly and obtain $\mathrm{e}^x = \dfrac{1-y}{y}$	**B1**
	Obtain the **given answer** $y = \dfrac{\mathrm{e}^{-x}}{1+\mathrm{e}^{-x}}$ following full working	**B1**
	Total:	**2**
3(ii)	State integral $k\ln(1+\mathrm{e}^{-x})$ where $k = \pm 1$	***M1**
	State correct integral $-\ln(1+\mathrm{e}^{-x})$	**A1**
	Use limits correctly	**DM1**
	Obtain the **given answer** $\ln\left(\dfrac{2\mathrm{e}}{\mathrm{e}+1}\right)$ following full working	**A1**
	Total:	**4**
4(i)	Use chain rule to differentiate x $\left(\dfrac{\mathrm{d}x}{\mathrm{d}\theta} = -\dfrac{\sin\theta}{\cos\theta}\right)$	**M1**
	State $\dfrac{\mathrm{d}y}{\mathrm{d}\theta} = 3 - \sec^2\theta$	**B1**
	Use $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{\mathrm{d}y}{\mathrm{d}\theta} \div \dfrac{\mathrm{d}x}{\mathrm{d}\theta}$	**M1**
	Obtain correct $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in any form e.g. $\dfrac{3-\sec^2\theta}{-\tan\theta}$	**A1**
	Obtain $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{\tan^2\theta - 2}{\tan\theta}$, or equivalent	**A1**
	Total:	**5**
4(ii)	Equate gradient to −1 and obtain an equation in $\tan\theta$	**M1**
	Solve a 3 term quadratic $\left(\tan^2\theta + \tan\theta - 2 = 0\right)$ in $\tan\theta$	**M1**
	Obtain $\theta = \dfrac{\pi}{4}$ and $y = \dfrac{3\pi}{4} - 1$ only	**A1**
	Total:	**3**

Question	Answer	Marks
5(i)	Use correct sector formula at least once and form an equation in r and x	**M1**
	Obtain a correct equation in any form	**A1**
	Rearrange in the given form	**A1**
	Total:	**3**
5(ii)	Calculate values of a relevant expression or expressions at $x = 1$ and $x = 1.5$	**M1**
	Complete the argument correctly with correct calculated values	**A1**
	Total:	**2**
5(iii)	Use the iterative formula correctly at least once	**M1**
	Obtain final answer 1.374	**A1**
	Show sufficient iterations to 5 d.p. to justify 1.374 to 3 d.p., or show there is a sign change in the interval (1.3745, 1.3755)	**A1**
	Total:	**3**
6(i)	State or obtain coordinates (1, 2, 1) for the mid-point of AB	**B1**
	Verify that the midpoint lies on m	**B1**
	State or imply a correct normal vector to the plane, e.g. $2\mathbf{i}+2\mathbf{j}-\mathbf{k}$	**B1**
	State or imply a direction vector for the segment AB, e.g. $-4\mathbf{i}-4\mathbf{j}+2\mathbf{k}$	**B1**
	Confirm that m is perpendicular to AB	**B1**
	Total:	**5**
6(ii)	State or imply that the perpendicular distance of m from the origin is $\frac{5}{3}$, or unsimplified equivalent	**B1**
	State or imply that n has an equation of the form $2x+2y-z=k$	**B1**
	Obtain answer $2x+2y-z=2$	**B1**
	Total:	**3**

Question	Answer	Marks
7(i)	State that $u - 2w = -7 - \mathrm{i}$	**B1**
	EITHER: Multiply numerator and denominator of $\frac{u}{w}$ by 3 – 4i, or equivalent	**(M1**
	Simplify the numerator to 25 + 25i or denominator to 25	**A1**
	Obtain final answer 1 + i	**A1)**
	OR: Obtain two equations in x and y and solve for x or for y	**(M1**
	Obtain $x = 1$ or $y = 1$	**A1**
	Obtain final answer 1 + i	**A1)**
	Total:	**4**
7(ii)	Find the argument of $\frac{u}{w}$	**M1**
	Obtain the given answer	**A1**
	Total:	**2**
7(iii)	State that *OB* and *CA* are parallel	**B1**
	State that *CA* = 2*OB*, or equivalent	**B1**
	Total:	**2**
8(i)	Use $\sin(A - B)$ formula and obtain an expression in terms of $\sin x$ and $\cos x$	**M1**
	Collect terms and reach $\sqrt{3}\sin x - 2\cos x$, or equivalent	**A1**
	Obtain $R = \sqrt{7}$	**A1**
	Use trig formula to find α	**M1**
	Obtain $\alpha = 49.11°$ with no errors seen	**A1**
	Total:	**5**

Question	Answer	Marks
8(ii)	Evaluate $\sin^{-1}(1/\sqrt{7})$ to at least 1 d.p. (22.21° to 2 d.p.)	**B1 FT**
	Use a correct method to find a value of x in the interval $0° < x < 180°$	**M1**
	Obtain answer 71.3°	**A1**
	[ignore answers outside given range.]	
	Total:	**3**
9(i)	Carry out a relevant method to obtain A and B such that $\frac{1}{x(2x+3)} \equiv \frac{A}{x} + \frac{B}{2x+3}$, or equivalent	**M1**
	Obtain $A = \frac{1}{3}$ and $B = -\frac{2}{3}$, or equivalent	**A1**
	Total:	**2**
9(ii)	Separate variables and integrate one side	**B1**
	Obtain term $\ln y$	**B1**
	Integrate and obtain terms $\frac{1}{3}\ln x - \frac{1}{3}\ln(2x+3)$, or equivalent	**B2 FT**
	Use $x = 1$ and $y = 1$ to evaluate a constant, or as limits, in a solution containing $a\ln y,\ b\ln x,\ c\ln(2x+3)$	**M1**
	Obtain correct solution in any form, e.g. $\ln y = \frac{1}{3}\ln x - \frac{1}{3}\ln(2x+3) + \frac{1}{3}\ln 5$	**A1**
	Obtain answer $y = 1.29$ (3s.f. only)	**A1**
	Total:	**7**
10(i)	State or imply $\mathrm{d}u = -\sin x\ \mathrm{d}x$	**B1**
	Using correct double angle formula, express the integral in terms of u and $\mathrm{d}u$	**M1**
	Obtain integrand $\pm(2u^2 - 1)^2$	**A1**
	Change limits and obtain correct integral $\int_{\frac{1}{\sqrt{2}}}^{1} (2u^2 - 1)^2\,\mathrm{d}u$ with no errors seen	**A1**
	Substitute limits in an integral of the form $au^5 + bu^3 + cu$	**M1**
	Obtain answer $\frac{1}{15}(7 - 4\sqrt{2})$, or exact simplified equivalent	**A1**
	Total:	**6**

Question	Answer	Marks
10(ii)	Use product rule and chain rule at least once	**M1**
	Obtain correct derivative in any form	**A1**
	Equate derivative to zero and use trig formulae to obtain an equation in $\cos x$ and $\sin x$	**M1**
	Use correct methods to obtain an equation in cos x or sin x only	**M1**
	Obtain $10\cos^2 x = 9$ or $10\sin^2 x = 1$, or equivalent	**A1**
	Obtain answer 0.32	**A1**
	Total:	**6**

Cambridge International A Level

Cambridge International Examinations
Cambridge International Advanced Level

MATHEMATICS **9709/32**

Paper 3 **May/June 2017**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the May/June 2017 series for most Cambridge IGCSE®, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

This document consists of **12** printed pages.

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International Examinations

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more “method” steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously “correct” answers or results obtained from incorrect working.

  - Note: B2 or A2 means that the candidate can earn 2 or 0.
    B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

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The following abbreviations may be used in a mark scheme or used on the scripts:

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no “follow through” from a previous error is allowed)
CWO	Correct Working Only – often written by a ‘fortuitous’ answer
ISW	Ignore Subsequent Working
SOI	Seen or implied
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

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PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Question	Answer	Marks
1	Use law of the logarithm of a power or a quotient	**M1**
	Remove logarithms and obtain a correct equation in x. e.g. $x^2+1=\mathrm{e}x^2$	**A1**
	Obtain answer 0.763 and no other	**A1**
	Total:	**3**
2	*EITHER*: State or imply non-modular inequality $(x-3)^2<(3x-4)^2$, or corresponding equation	**(B1**
	Make reasonable attempt at solving a three term quadratic	**M1**
	Obtain critical value $x=\frac{7}{4}$	**A1**
	State final answer $x>\frac{7}{4}$ only	**A1)**
	*OR*1: State the relevant critical inequality $3-x<3x-4$, or corresponding equation	**(B1**
	Solve for x	**M1**
	Obtain critical value $x=\frac{7}{4}$	**A1**
	State final answer $x>\frac{7}{4}$ only	**A1)**
	*OR*2: Make recognizable sketches of $y=\lvert x-3\rvert$ and $y=3x-4$ on a single diagram	**(B1**
	Find x-coordinate of the intersection	**M1**
	Obtain $x=\frac{7}{4}$	**A1**
	State final answer $x>\frac{7}{4}$ only	**A1)**
	Total:	**4**

Question	Answer	Marks
3(i)	Use correct formulae to express the equation in terms of cos θ and sin θ	**M1**
	Use Pythagoras and express the equation in terms of cos θ only	**M1**
	Obtain correct 3-term equation, e.g. $2\cos^4\theta + \cos^2\theta - 2 = 0$	**A1**
	Total:	**3**
3(ii)	Solve a 3-term quadratic in $\cos^2\theta$ for cos θ	**M1**
	Obtain answer $\theta = 152.1°$ only	**A1**
	Total:	**2**
4(i)	State $\frac{dy}{dt} = 4 + \frac{2}{2t-1}$	**B1**
	Use $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$	**M1**
	Obtain answer $\frac{dy}{dx} = \frac{8t-2}{2t(2t-1)}$, or equivalent e.g. $\frac{2}{t} + \frac{2}{4t^2 - 2t}$	**A1**
	Total:	**3**
4(ii)	Use correct method to find the gradient of the normal at $t = 1$	**M1**
	Use a correct method to form an equation for the normal at $t = 1$	**M1**
	Obtain final answer $x + 3y - 14 = 0$, or horizontal equivalent	**A1**
	Total:	**3**

Question	Answer	Marks
5(i)	State $\frac{\mathrm{d}y}{\mathrm{d}t} = -\frac{2y}{(1+t)^2}$, or equivalent	**B1**
	Separate variables correctly and attempt integration of one side	**M1**
	Obtain term $\ln y$, or equivalent	**A1**
	Obtain term $\frac{2}{(1+t)}$, or equivalent	**A1**
	Use $y = 100$ and $t = 0$ to evaluate a constant, or as limits in an expression containing terms of the form $a \ln y$ and $\frac{b}{1+t}$	**M1**
	Obtain correct solution in any form, e.g. $\ln y = \frac{2}{1+t} - 2 + \ln 100$	**A1**
	Total:	**6**
5(ii)	State that the mass of B approaches $\frac{100}{\mathrm{e}^2}$, or **exact** equivalent	**B1**
	State or imply that the mass of A tends to zero	**B1**
	Total:	**2**

Question	Answer	Marks
6(i)	*EITHER:* Substitute $x = 2 - \mathrm{i}$ (or $x = 2 + \mathrm{i}$) in the equation and attempt expansions of x^2 and x^3	(M1
	Equate real and/or imaginary parts to zero	M1
	Obtain $a = -2$	A1
	Obtain $b = 10$	A1)
	*OR*1: Substitute $x = 2 - \mathrm{i}$ in the equation and attempt expansions of x^2 and x^3	(M1
	Substitute $x = 2 + \mathrm{i}$ in the equation and add/subtract the two equations	M1
	Obtain $a = -2$	A1
	Obtain $b = 10$	A1)
	*OR*2: Factorise to obtain $(x-2+\mathrm{i})(x-2-\mathrm{i})(x-p)$ $\left(=\left(x^2-4x+5\right)(x-p)\right)$	(M1
	Compare coefficients	M1
	Obtain $a = -2$	A1
	Obtain $b = 10$	A1)
	*OR*3: Obtain the quadratic factor $(x^2 - 4x + 5)$	(M1
	Use algebraic division to obtain a real linear factor of the form $x - p$ and set the remainder equal to zero	M1
	Obtain $a = -2$	A1
	Obtain $b = 10$	A1)
	*OR*4: Use $\alpha\beta = 5$ and $\alpha + \beta = 4$ in $\alpha\beta + \beta\gamma + \gamma\alpha = -3$	(M1
	Solve for γ and use in $\alpha\beta\gamma = -b$ and/or $\alpha + \beta + \gamma = -a$	M1
	Obtain $a = -2$	A1
	Obtain $b = 10$	A1)

Question	Answer	Marks
	OR5: Factorise as $(x–- (2\text{-i}))(x^2 + ex + g)$ and compare coefficients to form an equation in a and b	**(M1**
	Equate real and/or imaginary parts to zero	**M1**
	Obtain $a = – 2$	**A1**
	Obtain $b = 10$	**A1)**
	Total:	**4**
6(ii)	Show a circle with centre $2 -$ i in a relatively correct position	**B1**
	Show a circle with radius 1 and centre not at the origin	**B1**
	Show the perpendicular bisector of the line segment joining 0 to – i	**B1**
	Shade the correct region	**B1**
	Total:	**4**
7(i)	Use quotient or chain rule	**M1**
	Obtain given answer correctly	**A1**
	Total:	**2**
7(ii)	*EITHER*: Multiply numerator and denominator of LHS by $1+\sin\theta$	**(M1**
	Use Pythagoras and express LHS in terms of sec θ and tan θ	**M1**
	Complete the proof	**A1)**
	OR1: Express RHS in terms of cos θ and sin θ	**(M1**
	Use Pythagoras and express RHS in terms of sin θ	**M1**
	Complete the proof	**A1)**
	OR2: Express LHS in terms of secθ and tanθ	**(M1**
	Multiply numerator and denominator by secθ + tanθ and use Pythagoras	**M1**
	Complete the proof	**A1)**
	Total:	**3**

Question	Answer	Marks
7(iii)	Use the identity and obtain integral $2\tan\theta + 2\sec\theta - \theta$	**B2**
	Use correct limits correctly in an integral containing terms $a\tan\theta$ and $b\sec\theta$	**M1**
	Obtain answer $2\sqrt{2} - \frac{1}{4}\pi$	**A1**
	Total:	**4**
8(i)	State or imply the form $\frac{A}{3x+2} + \frac{Bx+C}{x^2+5}$	**B1**
	Use a relevant method to determine a constant	**M1**
	Obtain one of the values $A = 2$, $B = 1$, $C = -3$	**A1**
	Obtain a second value	**A1**
	Obtain the third value	**A1**
	Total:	**5**
8(ii)	Use correct method to find the first two terms of the expansion of $(3x+2)^{-1}$, $(1+\frac{3}{2}x)^{-1}$, $(5+x^2)^{-1}$ or $(1+\frac{1}{5}x^2)^{-1}$ [Symbolic coefficients, e.g. $\begin{pmatrix}-1\\2\end{pmatrix}$ are not sufficient]	**M1**
	Obtain correct unsimplified expansions up to the term in x^2 of each partial fraction. The FT is on A, B, C. from part (i)	**A1FT + A1FT**
	Multiply out up to the term in x^2 by $Bx + C$, where $BC \neq 0$	**M1**
	Obtain **final answer** $\frac{2}{5} - \frac{13}{10}x + \frac{237}{100}x^2$, or equivalent	**A1**
	Total:	**5**
9(i)	*EITHER*: Find $\overrightarrow{AP}$ for a general point P on l with parameter λ, e.g.$(8 + 3\lambda, -3 - \lambda, 4 + 2\lambda)$	**(B1**
	Equate scalar product of $\overrightarrow{AP}$ and direction vector of l to zero and solve for λ	**M1**
	Obtain $\lambda = -\frac{5}{2}$ and foot of perpendicular $\frac{3}{2}\mathbf{i} + \frac{3}{2}\mathbf{j} + 3\mathbf{k}$	**A1**
	Carry out a complete method for finding the position vector of the reflection of A in l	**M1**
	Obtain answer $2\mathbf{i} + \mathbf{j} + 2\mathbf{k}$	**A1)**

Question	Answer	Marks
	OR: Find $\overrightarrow{AP}$ for a general point P on l with parameter λ, e.g.$(8 + 3\lambda, -3 - \lambda, 4 + 2\lambda)$	**(B1**
	Differentiate $\lvert AP \rvert^2$ and solve for λ at minimum	**M1**
	Obtain $\lambda = -\frac{5}{2}$ and foot of perpendicular $\frac{3}{2}\mathbf{i} + \frac{3}{2}\mathbf{j} + 3\mathbf{k}$	**A1**
	Carry out a complete method for finding the position vector of the reflection of A in l	**M1**
	Obtain answer $2\mathbf{i} + \mathbf{j} + 2\mathbf{k}$	**A1)**
	Total:	**5**
9(ii)	*EITHER:* Use scalar product to obtain an equation in a, b and c, e.g. $3a - b + 2c = 0$	**(B1**
	Form a second relevant equation, e.g. $9a - b + 8c = 0$ and solve for one ratio, e.g. $a : b$	**M1**
	Obtain final answer $a : b : c = 1 : 1 : -1$ and state plane equation $x + y - z = 0$	**A1)**
	*OR*1: Attempt to calculate vector product of two relevant vectors, e.g. $(3\mathbf{i} - \mathbf{j} + 2\mathbf{k}) \times (9\mathbf{i} - \mathbf{j} + 8\mathbf{k})$	**(M1**
	Obtain two correct components	**A1**
	Obtain correct answer, e.g. $-6\mathbf{i} - 6\mathbf{j} + 6\mathbf{k}$, and state plane equation $-x - y + z = 0$	**A1)**
	*OR*2: Using a relevant point and relevant vectors, attempt to form a 2-parameter equation for the plane, e.g. $\mathbf{r} = 6\mathbf{i} + 6\mathbf{k} + s(3\mathbf{i} - \mathbf{j} + 2\mathbf{k}) + t(9\mathbf{i} - \mathbf{j} + 8\mathbf{k})$	**(M1**
	State 3 correct equations in x, y, z, s and t	**A1**
	Eliminate s and t and state plane equation $x + y - z = 0$, or equivalent	**A1)**
	*OR*3: Using a relevant point and relevant vectors, attempt to form a determinant equation for the plane, e.g. $\begin{vmatrix} x-3 & y-1 & z-4 \\ 3 & -1 & 2 \\ 9 & -1 & 8 \end{vmatrix} = 0$	**(M1**
	Expand a correct determinant and obtain two correct cofactors	**A1**
	Obtain answer $-6x - 6y + 6z = 0$, or equivalent	**A1)**
	Total:	**3**

Question	Answer	Marks
9(iii)	*EITHER*: Using the correct processes, divide the scalar product of $\overrightarrow{OA}$ and a normal to the plane by the modulus of the normal or make a recognisable attempt to apply the perpendicular formula	**(M1**
	Obtain a correct expression in any form, e.g. $\dfrac{1+2-4}{\sqrt{(1^2+1^2+(-1)^2)}}$, or equivalent	**A1 FT**
	Obtain answer $1/\sqrt{3}$, or exact equivalent	**A1)**
	OR1: Obtain equation of the parallel plane through *A*, e.g. $x+y-z=-1$ [The f.t. is on the plane found in part (**ii**).]	**(B1 FT**
	Use correct method to find its distance from the origin	**M1**
	Obtain answer $1/\sqrt{3}$, or exact equivalent	**A1)**
	OR2: Form equation for the intersection of the perpendicular through *A* and the plane [FT on their **n**]	**(B1 FT**
	Solve for λ	**M1**
	$\lvert\lambda\mathbf{n}\rvert=\dfrac{1}{\sqrt{3}}$	**A1)**
	Total:	**3**
10(i)	Use correct product rule	**M1**
	Obtain correct derivative in any form $\left(y'=2x\cos 2x-2x^2\sin 2x\right)$	**A1**
	Equate to zero and derive the given equation	**A1**
	Total:	**3**
10(ii)	Use the iterative formula correctly at least once e.g. $0.5\to 0.55357\to 0.53261\to 0.54070\to 0.53755$	**M1**
	Obtain final answer 0.54	**A1**
	Show sufficient iterations to 4 d.p. to justify 0.54 to 2 d.p., or show there is a sign change in the interval (0.535, 0.545)	**A1**
	Total:	**3**

Question	Answer	Marks
10(iii)	Integrate by parts and reach $ax^2 \sin 2x + b\int x \sin 2x \, dx$	*M1
	Obtain $\frac{1}{2}x^2 \sin 2x - \int 2x.\frac{1}{2}\sin 2x \, dx$	A1
	Complete integration and obtain $\frac{1}{2}x^2 \sin 2x + \frac{1}{2}x \cos 2x - \frac{1}{4}\sin 2x$, or equivalent	A1
	Substitute limits $x = 0$, $x = \frac{1}{4}\pi$, having integrated twice	DM1
	Obtain answer $\frac{1}{32}(\pi^2 - 8)$, or exact equivalent	A1
	Total:	5

Cambridge International A Level

Cambridge International Examinations
Cambridge International Advanced Level

MATHEMATICS **9709/33**

Paper 3 **May/June 2017**

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Question	Answer	Marks
1	Express the LHS in terms of either cos x and sin x or in terms of tan x	**B1**
	Use Pythagoras	**M1**
	Obtain the given answer	**A1**
	Total:	**3**
2	*EITHER:* State a correct unsimplified version of the x or x^2 term in the expansion of $\left(1+\frac{2}{3}x\right)^{-3}$ or $\left(3+2x\right)^{-3}$ [Symbolic binomial coefficients, e.g. $\begin{pmatrix}-3\\2\end{pmatrix}$, are not sufficient for **M1**.]	**(M1**
	State correct first term $\frac{1}{27}$	**B1**
	Obtain term $-\frac{2}{27}x$	**A1**
	Obtain term $\frac{8}{81}x^2$	**A1)**
	OR: Differentiate expression and evaluate $\mathrm{f}(0)$ and $\mathrm{f}'(0)$, where $\mathrm{f}'(x)=k(3+2x)^{-4}$	**(M1**
	State correct first term $\frac{1}{27}$	**B1**
	Obtain term $-\frac{2}{27}x$	**A1**
	Obtain term $\frac{8}{81}x^2$	**A1)**
	Total:	**4**
3	Rearrange as $3u^2+4u-4=0$, or $3\mathrm{e}^{2x}+4\mathrm{e}^x-4=0$, or equivalent	**B1**
	Solve a 3-term quadratic for e^x or for u	**M1**
	Obtain $\mathrm{e}^x=\frac{2}{3}$ or $u=\frac{2}{3}$	**A1**
	Obtain answer $x=-0.405$ and no other	**A1**
	Total:	**4**

Question	Answer	Marks
4	Integrate by parts and reach $a\theta\cos\frac{1}{2}\theta + b\int\cos\frac{1}{2}\theta\ \mathrm{d}\theta$	*M1
	Complete integration and obtain indefinite integral $-2\theta\cos\frac{1}{2}\theta + 4\sin\frac{1}{2}\theta$	A1
	Substitute limits correctly, having integrated twice	DM1
	Obtain final answer $(4-\pi)/\sqrt{2}$, or exact equivalent	A1
	Total:	**4**
5(i)	Use the chain rule	M1
	Obtain correct derivative in any form	A1
	Use correct trigonometry to express derivative in terms of $\tan x$	M1
	Obtain $\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{4\tan x}{4+\tan^2 x}$, or equivalent	A1
	Total:	**4**
5(ii)	Equate derivative to –1 and solve a 3–term quadratic for tan x	M1
	Obtain answer x=1.11 and no other in the given interval	A1
	Total:	**2**
6(i)	Calculate the value of a relevant expression or expressions at $x = 2.5$ and at another relevant value, e.g. $x = 3$	M1
	Complete the argument correctly with correct calculated values	A1
	Total:	**2**
6(ii)	State a suitable equation, e.g. $x = \pi + \tan^{-1}\left(1/(1-x)\right)$ without suffices	B1
	Rearrange this as $\cot x = 1 - x$, or commence working *vice versa*	B1
	Total:	**2**
6(iii)	Use the iterative formula correctly at least once	M1
	Obtain final answer 2.576 only	A1
	Show sufficient iterations to 5 d.p. to justify 2.576 to 3 d.p., or show there is a sign change in the interval (2.5755, 2.5765)	A1
	Total:	**3**

Question	Answer	Marks
7(i)	Use correct quotient rule or product rule	**M1**
	Obtain correct derivative in any form	**A1**
	Equate derivative to zero and solve for x	**M1**
	Obtain $x = 2$	**A1**
	Total:	**4**
7(ii)	State or imply ordinates 1.6487…, 1.3591…, 1.4938…	**B1**
	Use correct formula, or equivalent, with $h = 1$ and three ordinates	**M1**
	Obtain answer 2.93 only	**A1**
	Total:	**3**
7(iii)	Explain why the estimate would be less than E	**B1**
	Total:	**1**
8(i)	Justify the given differential equation	**B1**
	Total:	**1**
8(ii)	Separate variables correctly and attempt to integrate one side	**B1**
	Obtain term kt, or equivalent	**B1**
	Obtain term $-\ln(50-x)$, or equivalent	**B1**
	Evaluate a constant, or use limits $x = 0$, $t = 0$ in a solution containing terms $a\ln(50-x)$ and bt	**M1***
	Obtain solution $-\ln(50-x) = kt - \ln 50$, or equivalent	**A1**
	Use $x = 25$, $t = 10$ to determine k	**DM1**
	Obtain correct solution in any form, e.g. $\ln 50 - \ln(50-x) = \frac{1}{10}(\ln 2)t$	**A1**
	Obtain answer $x = 50(1 - \exp(-0.0693t))$, or equivalent	**A1**
	Total:	**8**

Question	Answer	Marks
9(i)	State or imply the form $\frac{A}{x}+\frac{B}{x^2}+\frac{C}{3x+2}$	**B1**
	Use a relevant method to determine a constant	**M1**
	Obtain one of the values $A=3$, $B=-2$, $C=-6$	**A1**
	Obtain a second value	**A1**
	Obtain the third value [Mark the form $\frac{Ax+B}{x^2}+\frac{C}{3x+2}$ using same pattern of marks.]	**A1**
	Total:	**5**
9(ii)	Integrate and obtain terms $3\ln x = \frac{2}{x} - 2\ln(3x+2)$ [The FT is on A, B and C] **Note:** Candidates who integrate the partial fraction $\frac{3x-2}{x^2}$ by parts should obtain $3\ln x + \frac{2}{x} - 3$ or equivalent	**B3 FT**
	Use limits correctly, having integrated all the partial fractions, in a solution containing terms $a\ln x + \frac{b}{x} + c\ln(3x+2)$	**M1**
	Obtain the given answer following full and exact working	**A1**
	Total:	**5**
10(i)	Carry out a correct method for finding a vector equation for AB	**M1**
	Obtain $\mathbf{r}=\mathbf{i}-2\mathbf{j}+2\mathbf{k}+\lambda(2\mathbf{i}+3\mathbf{j}-\mathbf{k})$, or equivalent	**A1**
	Equate two pairs of components of general points on AB and l and solve for λ or for μ	**M1**
	Obtain correct answer for λ or μ, e.g. $\lambda=\frac{5}{7}$ or $\mu=\frac{3}{7}$	**A1**
	Obtain $m=3$	**A1**
	Total:	**5**

Question	Answer	Marks
10(ii)	*EITHER:* Use scalar product to obtain an equation in a, b and c, e.g. $a-2b-4c=0$	**(B1**
	Form a second relevant equation, e.g. $2a+3b-c=0$ and solve for one ratio, e.g. $a : b$	**M1**
	Obtain final answer $a : b : c = 14 : -7 : 7$	**A1**
	Use coordinates of a relevant point and values of a, b and c and find d	**M1**
	Obtain answer $14x-7y+7z=42$, or equivalent	**A1)**
	OR 1: Attempt to calculate the vector product of relevant vectors, e.g. $(\mathbf{i}-2\mathbf{j}-4\mathbf{k})\times(2\mathbf{i}+3\mathbf{j}-\mathbf{k})$	**(M1**
	Obtain two correct components	**A1**
	Obtain correct answer, e.g. $14\mathbf{i}-7\mathbf{j}+7\mathbf{k}$	**A1**
	Substitute coordinates of a relevant point in $14x-7y+7z=d$, or equivalent, and find d	**M1**
	Obtain answer $14x-7y+7z=42$, or equivalent	**A1)**
	OR 2: Using a relevant point and relevant vectors, form a 2–parameter equation for the plane	**(M1**
	State a correct equation, e.g. $\mathbf{r}=\mathbf{i}-2\mathbf{j}+2\mathbf{k}+s(\mathbf{i}-2\mathbf{j}-4\mathbf{k})+t(2\mathbf{i}+3\mathbf{j}-\mathbf{k})$	**A1**
	State 3 correct equations in x, y, z, s and t	**A1**
	Eliminate s and t	**M1**
	Obtain answer $2x-y+z=6$, or equivalent	**A1)**
	OR 3: Using a relevant point and relevant vectors, form a determinant equation for the plane	**(M1**
	State a correct equation, e.g. $\begin{vmatrix} x-1 & y+2 & z-1 \\ 1 & -2 & -4 \\ 2 & 3 & -1 \end{vmatrix}=0$	**A1**
	Attempt to expand the determinant	**M1**
	Obtain or imply two correct cofactors	**A1**
	Obtain answer $14x-7y+7z=42$, or equivalent	**A1)**
	Total:	**5**

Question	Answer	Marks
11(a)	Solve for z or for w	**M1**
	Use $i^2 = -1$	**M1**
	Obtain $w = \frac{i}{2-i}$ or $z = \frac{2+i}{2-i}$	**A1**
	Multiply numerator and denominator by the conjugate of the denominator	**M1**
	Obtain $w = -\frac{1}{5} + \frac{2}{5}i$	**A1**
	Obtain $z = \frac{3}{5} + \frac{4}{5}i$	**A1**
	Total:	**6**
11(b)	*EITHER:* Find $\pm\left[2 + \left(2 - 2\sqrt{3}\right)i\right]$	**(B1**
	Multiply by 2i (or –2i)	**M1***
	Add result to v	**DM1**
	Obtain answer $4\sqrt{3} - 1 + 6i$	**A1)**
	OR: State $\frac{z-v}{v-u} = ki$, or equivalent	**(M1**
	State $k = 2$	**A1**
	Substitute and solve for z even if i omitted	**M1**
	Obtain answer $4\sqrt{3} - 1 + 6i$	**A1)**
	Total:	**4**

Cambridge International A Level

Cambridge International Examinations
Cambridge International Advanced Level

MATHEMATICS **9709/32**

Paper 3 Pure Mathematics **March 2017**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the March 2017 series for most Cambridge IGCSE®, Cambridge International A and AS Level components and some Cambridge O Level components.

This document consists of **11** printed pages.

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    B2/1/0 means that the candidate can earn anything from 0 to 2.

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The following abbreviations may be used in a mark scheme or used on the scripts:

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PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Question	Answer	Marks
1	Remove logarithm and obtain $1+2^x = \mathrm{e}^2$	**B1**
	Use correct method to solve an equation of the form $2^x = a$, where $a > 0$	**M1**
	Obtain answer $x = 2.676$	**A1**
	Total:	**3**

Question	Answer	Marks
2	*EITHER*:	
	State or imply non-modular inequality $(x-4)^2 < (2(3x+1))^2$, or corresponding quadratic equation, or pair of linear equations $x-4 = \pm 2(3x+1)$	**(B1**
	Make reasonable solution attempt at a 3-term quadratic, or solve two linear equations for x	**M1**
	Obtain critical values $x = -\frac{6}{5}$ and $x = \frac{2}{7}$	**A1**
	State final answer $x < -\frac{6}{5}$, $x > \frac{2}{7}$	**A1)**
	OR:	
	Obtain critical value $x = -\frac{6}{5}$ from a graphical method, or by inspection, or by solving a linear equation or inequality	**(B1**
	Obtain critical value $x = \frac{2}{7}$ similarly	**B2**
	State final answer $x < -\frac{6}{5}$, $x > \frac{2}{7}$	**B1)**
	Total:	**4**

Question	Answer	Marks
3(i)	Sketch a relevant graph, e.g. $y = \mathrm{e}^{-\frac{1}{2}x}$	**B1**
	Sketch a second relevant graph, e.g. $y = 4 - x^2$, and justify the given statement	**B1**
	Total:	**2**
3(ii)	Calculate the value of a relevant expression or values of a pair of expressions at $x = -1$ and $x = -1.5$	**M1**
	complete the argument correctly with correct calculated values	**A1**
	Total:	**2**

Question	Answer	Marks
3(iii)	Use the iterative formula correctly at least once	**M1**
	Obtain final answer – 1.41	**A1**
	Show sufficient iterations to 4 d.p. to justify – 1.41 to 2 d.p., or show there is a sign change in the interval (– 1.415, – 1.405)	**A1**
	Total:	**3**

Question	Answer	Marks
4(i)	State $R = 17$	**B1**
	Use trig formula to find α	**M1**
	Obtain $\alpha = 61.93°$ with no errors seen	**A1**
	Total:	**3**
4(ii)	Evaluate $\cos^{-1}(4/17)$ to at least 1d.p. (76.39° to 2 d.p.)	**B1**√
	Use a correct method to find a value of x in the interval $0° < x < 180°$	**M1**
	Obtain answer, e.g. $x = 7.2°$	**A1**
	Obtain second answer, e.g. $x = 110.8°$and no others	**A1**
	[Ignore answers outside the given interval.]	
	[Treat answers in radians as a misread.]	
	Total:	**4**

Question	Answer	Marks
5	Use product rule	**M1**
	Obtain correct derivative in any form	**A1**
	Equate derivative to zero, use Pythagoras and obtain a quadratic equation in $\tan x$	**M1**
	Obtain $\tan^2 x - a\tan x + 1 = 0$, or equivalent	**A1**
	Use the condition for a quadratic to have only one root	**M1**
	Obtain answer $a = 2$	**A1**
	Obtain answer $x = \frac{1}{4}\pi$	**A1**
	Total:	**7**

Question	Answer	Marks
6(i)	Verify that the point with position vector $\mathbf{i}+2\mathbf{j}-3\mathbf{k}$ lies in the plane	**B1**
	EITHER:	
	Find a second point on *l* and substitute its coordinates in the equation of *p*	**(M1**
	Verify that the second point, e.g. (3, 1, – 2), lies in the plane	**A1)**
	OR:	
	Expand scalar product of a normal to *p* and the direction vector of *l*	**(M1**
	Verify scalar product is zero	**A1)**
	Total:	**3**

Question	**Answer**	**Marks**
6(ii)	*EITHER*:	
	Use scalar product to obtain a relevant equation in a , b and c, e.g. $2a-b+c=0$	**(B1**
	Obtain a second relevant equation, e.g. $3a+b-5c=0$, and solve for one ratio e.g. $a:b$	**M1**
	Obtain $a:b:c=4:13:5$, or equivalent	**A1**
	Substitute (3, – 1, 2) and the values of a, b and c in the general equation and find d	**M1**
	Obtain answer $4x+13y+5z=9$, or equivalent	**A1)**
	*OR*1:	
	Attempt to calculate vector product of relevant vectors, e.g. $(2\mathbf{i}-\mathbf{j}+\mathbf{k})\times(3\mathbf{i}+\mathbf{j}-5\mathbf{k})$	**(M1**
	Obtain two correct components	**A1**
	Obtain correct answer, e.g. $4\mathbf{i}+13\mathbf{j}+5\mathbf{k}$	**A1**
	Substitute (3, – 1, 2) in $4x+13y+5z=d$, or equivalent, and find d	**M1**
	Obtain answer $4x+13y+5z=9$, or equivalent	**A1)**
	*OR*2:	
	Using the relevant point and relevant vectors form a 2-parameter equation for the plane	**(M1**
	State a correct equation, e.g. $\mathbf{r}=3\mathbf{i}-\mathbf{j}+2\mathbf{k}+\lambda(2\mathbf{i}-\mathbf{j}+\mathbf{k})+\mu(3\mathbf{i}+\mathbf{j}-5\mathbf{k})$	**A1**
	State three correct equations in x, y, z, λ and μ	**A1**
	Eliminate λ and μ	**M1**
	Obtain answer $4x+13y+5z=9$, or equivalent	**A1)**
	*OR*3:	
	Using the relevant point and relevant vectors form a determinant equation for the plane	**(M1**
	State a correct equation, e.g. $\begin{vmatrix} x-3 & y+1 & z-2 \\ 2 & -1 & 1 \\ 3 & 1 & -5 \end{vmatrix}=0$	**A1**
	Attempt to expand the determinant	**M1**
	Obtain or imply two correct cofactors	**A1**

Question	Answer	Marks
	Obtain answer $4x+13y+5z=9$, or equivalent	**A1)**
	Total:	**5**

Question	Answer	Marks
7(i)	State or imply $\dfrac{\mathrm{d}V}{\mathrm{d}t}=2\dfrac{\mathrm{d}h}{\mathrm{d}t}$	**B1**
	State or imply $\dfrac{\mathrm{d}V}{\mathrm{d}t}=1-0.2\sqrt{h}$	**B1**
	Obtain the given answer correctly	**B1**
	Total:	**3**
7(ii)	State or imply $\mathrm{d}u=-\dfrac{1}{2\sqrt{h}}\ \mathrm{d}h$, or equivalent	**B1**
	Substitute for h and dh throughout	**M1**
	Obtain $T=\displaystyle\int_3^5 \frac{20(5-u)}{u}\ \mathrm{d}u$, or equivalent	**A1**
	Integrate and obtain terms $100\ln u-20u$, or equivalent	**A1**
	Substitute limits u = 3 and u = 5 correctly	**M1**
	Obtain answer 11.1, with no errors seen	**A1**
	Total:	**6**

Question	Answer	Marks
8(i)	Substitute $z = -1 + \mathrm{i}$ and attempt expansions of the z^2 and z^4 terms	**M1**
	Use $\mathrm{i}^2 = -1$ at least once	**M1**
	Complete the verification correctly	**A1**
	Total:	**3**
8(ii)	State second root $z = -1 - \mathrm{i}$	**B1**
	Carry out a complete method for finding a quadratic factor with zeros $-1+\mathrm{i}$ and $-1-\mathrm{i}$	**M1**
	Obtain $z^2 + 2z + 2$, or equivalent	**A1**
	Attempt division of p(z) by $z^2 + 2z + 2$ and reach a partial quotient $z^2 + kz$	**M1**
	Obtain quadratic factor $z^2 - 2z + 5$	**A1**
	Solve 3-term quadratic and use $\mathrm{i}^2 = -1$	**M1**
	Obtain roots 1 + 2i and 1 – 2i	**A1**
	Total:	**7**

Question	Answer	Marks
9(i)	State or imply the form $\dfrac{A}{2+x} + \dfrac{Bx+C}{4+x^2}$	**B1**
	Use a relevant method to determine a constant	**M1**
	Obtain one of the values $A = -2$, $B = 1$, $C = 4$	**A1**
	Obtain a second value	**A1**
	Obtain the third value	**A1**
	Total:	**5**

Question	Answer	Marks
9(ii)	Use correct method to obtain the first two terms of the expansion of $(1+\frac{1}{2}x)^{-1}$, $(2+x)^{-1}$, $(1+\frac{1}{4}x^2)^{-1}$ or $(4+x^2)^{-1}$	M1
	Obtain correct unsimplified expansions up to the term in x^2 of each partial fraction	A1√ + A1√
	Multiply out up to the term in x^2 by $Bx+C$, where $BC \neq 0$	M1
	Obtain final answer $\frac{3}{4}x-\frac{1}{2}x^2$	A1
	[Symbolic binomial coefficients, e.g. ${}_{-1}\mathrm{C}_2$, are not sufficient for the first M1. The f.t. is on *A*, *B*, *C*.]	
	[In the case of an attempt to expand $x(6-x)(2+x)^{-1}(4+x^2)^{-1}$, give M1A1A1 for the expansions, M1 for multiplying out fully, and A1 for the final answer.]	
	Total:	**5**

Question	Answer	Marks
10(i)	State or imply derivative is $2\frac{\ln x}{x}$	B1
	State or imply gradient of the normal at x = e is $-\frac{1}{2}\mathrm{e}$, or equivalent	B1
	Carry out a complete method for finding the x-coordinate of Q	M1
	Obtain answer $x=\mathrm{e}+\frac{2}{\mathrm{e}}$, or exact equivalent	A1
	Total:	**4**
10(ii)	Justify the given statement by integration or by differentiation	B1
	Total:	**1**
10(iii)	Integrate by parts and reach $ax(\ln x)^2+b\int x.\frac{\ln x}{x}\,\mathrm{d}x$	M1*
	Complete the integration and obtain $x(\ln x)^2-2x\ln x+2x$, or equivalent	A1
	Use limits x = 1 and x = e correctly, having integrated twice	DM1
	Obtain exact value e – 2	A1
	Use x- coordinate of Q found in part **(i)** and obtain final answer $\mathrm{e}-2+\frac{1}{\mathrm{e}}$	B1√
	Total:	**5**

Cambridge International A Level

Cambridge International Examinations
Cambridge International Advanced Level

MATHEMATICS **9709/31**

Paper 3 **October/November 2016**

MARK SCHEME

Maximum Mark: 75

Published

This document consists of **9** printed pages.

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1	Solve for 3^x and obtain $3^x = \frac{18}{7}$ Use correct method for solving an equation of the form $3^x = a$, where $a > 0$ Obtain answer $x = 0.860$ 3 d.p. only	**B1** **M1** **A1**	[3]
2	State correct unsimplified first two terms of the expansion of $(1+2x)^{-\frac{3}{2}}$, e.g. $1+(-\frac{3}{2})(2x)$ State correct unsimplified term in x^2, e.g. $(-\frac{3}{2})(-\frac{3}{2}-1)(2x)^2 / 2!$ Obtain sufficient terms of the product of $(2-x)$ and the expansion up to the term in x^2 Obtain **final** answer $2-7x+18x^2$ Do not ISW	**B1** **B1** **M1** **A1**	[4]
3	*EITHER*: Correctly restate the equation in terms of $\sin\theta$ and $\cos\theta$ Correct method to obtain a horizontal equation in $\sin\theta$ Reduce the equation to a correct quadratic in any form, e.g. $3\sin^2\theta - \sin\theta - 2 = 0$ Solve a three-term quadratic for $\sin\theta$ Obtain final answer $\theta = -41.8°$ only [Ignore answers outside the given interval.] *OR* 1: Square both sides of the equation and use $1+\tan^2\theta = \sec^2\theta$ Correct method to obtain a horizontal equation in $\sin\theta$ Reduce the equation to a correct quadratic in any form, e.g. $9\sin^2\theta - 6\sin\theta - 8 = 0$ Solve a three-term quadratic for $\sin\theta$ Obtain final answer $\theta = -41.8°$ only *OR* 2: Multiply through by $(\sec\theta + \tan\theta)$ Use $\sec^2\theta - \tan^2\theta = 1$ Obtain $1 = 3 + 3\sin\theta$ Solve for $\sin\theta$ Obtain final answer $\theta = -41.8°$ only	**B1** **M1** **A1** **M1** **A1** **B1** **M1** **A1** **M1** **A1** **M1** **B1** **A1** **M1** **A1**	[5]

4	*EITHER*: *EITHER*: State $2xy + x^2\frac{dy}{dx}$, or equivalent, as derivative of x^2y	**B1**	
	State $6y^2 + 12xy\frac{dy}{dx}$, or equivalent, as derivative of $6xy^2$	**B1**	
	OR: Differentiating LHS using correct product rule, state term $xy(1-6\frac{dy}{dx})$, or equivalent	**B1**	
	State term $(y + x\frac{dy}{dx})(x-6y)$, or equivalent	**B1**	
	Equate attempted derivative of LHS to zero and set $\frac{dy}{dx}$ equal to zero	**M1***	
	Obtain a horizontal equation, e.g. $6y^2 - 2xy = 0$ (from correct work only)	**A1**	
	Explicitly reject $y = 0$ as a possibility $py^2 - qxy = 0$	**A1**	
	Obtain an equation in x or y	**DM1**	
	Obtain answer $(-3a, -a)$	**A1**	
	OR: Rearrange to $y = \frac{9a^3}{x(x-6y)}$ and use correct quotient rule to obtain $-\frac{9a^3}{x^2(x-6y)^2} \times .....$	**B1**	
	State term $(x-6y) + x(1-6y')$, or equivalent	**B1**	
	Justify division by $x(x-6y)$	**B1**	
	Set $\frac{dy}{dx}$ equal to zero	**M1***	
	Obtain a horizontal equation, e.g. $6y^2 - 2xy = 0$ (from correct work only)	**A1**	
	Obtain an equation in x or y	**DM1**	
	Obtain answer $(-3a, -a)$	**A1**	[7]
5 (i)	*EITHER*: Use tan $2A$ formula to express LHS in terms of $\tan\theta$	**M1**	
	Express as a single fraction in any correct form	**A1**	
	Use Pythagoras or cos $2A$ formula	**M1**	
	Obtain the given result correctly	**A1**	
	OR: Express LHS in terms of sin 2θ, cos 2θ, sin θ and cos θ	**M1**	
	Express as a single fraction in any correct form	**A1**	
	Use Pythagoras or cos $2A$ formula or $\sin(A-B)$ formula	**M1**	
	Obtain the given result correctly	**A1**	[4]
(ii)	Integrate and obtain a term of the form $a\ln(\cos 2\theta)$ or $b\ln(\cos\theta)$ (or secant equivalents)	**M1***	
	Obtain integral $-\frac{1}{2}\ln(\cos 2\theta) + \ln(\cos\theta)$, or equivalent	**A1**	
	Substitute limits correctly (expect to see use of <u>both</u> limits)	**DM1**	
	Obtain the given answer following full and correct working	**A1**	[4]

6	**(i)**	Make recognizable sketch of a relevant graph	**B1**	
		Sketch the other relevant graph and justify the given statement	**B1**	[2]
	(ii)	Use calculations to consider the value of a relevant expression at $x = 1.4$ and $x = 1.6$, or the values of relevant expressions at $x = 1.4$ and $x = 1.6$	**M1**	
		Complete the argument correctly with correct calculated values	**A1**	[2]
	(iii)	State $x = 2\sin^{-1}\left(\dfrac{3}{x+3}\right)$	**B1**	
		Rearrange this in the form $\operatorname{cosec}\frac{1}{2}x = \frac{1}{3}x + 1$	**B1**	[2]
		If working in reverse, need $\sin\dfrac{x}{2} = \left(\dfrac{3}{x+3}\right)$ for first B1		
	(iv)	Use the iterative formula correctly at least once	**M1**	
		Obtain final answer 1.471	**A1**	
		Show sufficient iterations to 5 d.p. to justify 1.471 to 3 d.p., or show there is a sign change in the interval (1.4705, 1.4715)	**A1**	[3]
7	**(i)**	Use the correct product rule	**M1**	
		Obtain correct derivative in any form, e.g. $(2-2x)\mathrm{e}^{\frac{1}{2}x} + \frac{1}{2}(2x-x^2)\mathrm{e}^{\frac{1}{2}x}$	**A1**	
		Equate derivative to zero and solve for x	**M1**	
		Obtain $x = \sqrt{5} - 1$ only	**A1**	[4]
	(ii)	Integrate by parts and reach $a(2x-x^2)\mathrm{e}^{\frac{1}{2}x} + b\int(2-2x)\mathrm{e}^{\frac{1}{2}x}\,\mathrm{d}x$	**M1***	
		Obtain $2\mathrm{e}^{\frac{1}{2}x}(2x-x^2) - 2\int(2-2x)\mathrm{e}^{\frac{1}{2}x}\mathrm{d}x$, or equivalent	**A1**	
		Complete the integration correctly, obtaining $(12x-2x^2-24)\mathrm{e}^{\frac{1}{2}x}$, or equivalent	**A1**	
		Use limits $x = 0$, $x = 2$ correctly having integrated by parts twice	**DM1**	
		Obtain answer 24 – 8e, or exact simplified equivalent	**A1**	[5]

8	**(i)**	State or imply a correct normal vector to either plane, e.g. $3\mathbf{i}+\mathbf{j}-\mathbf{k}$ or $\mathbf{i}-\mathbf{j}+2\mathbf{k}$	**B1**	
		Use correct method to calculate their scalar product	**M1**	
		Show value is zero and planes are perpendicular	**A1**	[3]
	(ii)	*EITHER*: Carry out a complete strategy for finding a point on l the line of intersection	**M1**	
		Obtain such a point, e.g. (0, 7, 5) , (1, 0, 1), (5/4, –7/4, 0)	**A1**	
		EITHER: State two equations for a direction vector $a\mathbf{i}+b\mathbf{j}+c\mathbf{k}$ for l, e.g. $3a+b-c=0$ and $a-b+2c=0$	**B1**	
		Solve for one ratio, e.g. $a:b$	**M1**	
		Obtain $a:b:c = 1:-7:-4$, or equivalent	**A1**	
		State a correct answer, e.g. $\mathbf{r}=7\mathbf{j}+5\mathbf{k}+\lambda(\mathbf{i}-7\mathbf{j}-4\mathbf{k})$	**A1**√	
		*OR*1: Obtain a second point on l, e.g. (1, 0, 1)	**B1**	
		Subtract vectors and obtain a direction vector for l	**M1**	
		Obtain $-\mathbf{i}+7\mathbf{j}+4\mathbf{k}$, or equivalent	**A1**	
		State a correct answer, e.g. $\mathbf{r}=\mathbf{i}+\mathbf{k}+\lambda(-\mathbf{i}+7\mathbf{j}+4\mathbf{k})$	**A1**√	
		*OR*2: Attempt to find the vector product of the two normal vectors	**M1**	
		Obtain two correct components of the product	**A1**	
		Obtain $\mathbf{i}-7\mathbf{j}-4\mathbf{k}$, or equivalent	**A1**	
		State a correct answer, e.g. $\mathbf{r}=7\mathbf{j}+5\mathbf{k}+\lambda(\mathbf{i}-7\mathbf{j}-4\mathbf{k})$	**A1**√	
		*OR*1: Express one variable in terms of a second variable	**M1**	
		Obtain a correct simplified expression, e.g. $y = 7 - 7x$	**A1**	
		Express the third variable in terms of the second	**M1**	
		Obtain a correct simplified expression, e.g. $z = 5 - 4x$	**A1**	
		Form a vector equation for the line	**M1**	
		Obtain a correct equation, e.g. $\mathbf{r}=7\mathbf{j}+5\mathbf{k}+\lambda(\mathbf{i}-7\mathbf{j}-4\mathbf{k})$	**A1**√	
		*OR*2: Express one variable in terms of a second variable	**M1**	
		Obtain a correct simplified expression, e.g. $z=5-4x$	**A1**	
		Express the same variable in terms of the third	**M1**	
		Obtain a correct simplified expression e.g. $z=(7+4y)/7$	**A1**	
		Form a vector equation for the line	**M1**	
		Obtain a correct equation, e.g. $\mathbf{r}=\frac{5}{4}\mathbf{i}-\frac{7}{4}\mathbf{j}+\lambda(-\frac{1}{4}\mathbf{i}+\frac{7}{4}\mathbf{j}+\mathbf{k})$	**A1**√	[6]

9	(a)	EITHER:	Use quadratic formula to solve for w	M1	
			Use $i^2 = -1$	M1	
			Obtain one of the answers $w = \frac{1}{2i+1}$ and $w = -\frac{5}{2i+1}$	A1	
			Multiply numerator and denominator of an answer by $-2i + 1$, or equivalent	M1	
			Obtain final answers $\frac{1}{5} - \frac{2}{5}i$ and $-1 + 2i$	A1	
		OR1:	Multiply the equation by $1 - 2i$	M1	
			Use $i^2 = -1$	M1	
			Obtain $5w^2 + 4w(1-2i) - (1-2i)^2 = 0$, or equivalent	A1	
			Use quadratic formula or factorise to solve for w	M1	
			Obtain final answers $\frac{1}{5} - \frac{2}{5}i$ and $-1 + 2i$	A1	
		OR2:	Substitute $w = x + iy$ and form equations for real and imaginary parts	M1	
			Use $i^2 = -1$	M1	
			Obtain $(x^2 - y^2) - 4xy + 4x - 1 = 0$ and $2(x^2 - y^2) + 2xy + 4y + 2 = 0$ o.e.	A1	
			Form equation in x only or y only and solve	M1	
			Obtain final answers $\frac{1}{5} - \frac{2}{5}i$ and $-1 + 2i$	A1	[5]
	(b)	Show a circle with centre $1 + i$		B1	
		Show a circle with radius 2		B1	
		Show half-line $\arg z = \frac{1}{4}\pi$		B1	
		Show half-line $\arg z = -\frac{1}{4}\pi$		B1	
		Shade the correct region		B1	[5]

10	**(i)**	Separate variables correctly and integrate at least one side	**M1**	
		Integrate and obtain term kt, or equivalent	**A1**	
		Carry out a relevant method to obtain A and B such that $\frac{1}{x(4-x)} \equiv \frac{A}{x} + \frac{B}{4-x}$, or equivalent	**M1***	
		Obtain $A = B = \frac{1}{4}$, or equivalent	**A1**	
		Integrate and obtain terms $\frac{1}{4}\ln x - \frac{1}{4}\ln(4-x)$, or equivalent	**A1**√	
		EITHER: Use a pair of limits in an expression containing $p\ln x$, $q\ln(4-x)$ and rt and evaluate a constant	**DM1**	
		Obtain correct answer in any form, e.g. $\ln x - \ln(4-x) = 4kt - \ln 9$, or $\ln\left(\frac{x}{4-x}\right) = 4kt - 8k$	**A1**	
		Use a second pair of limits and determine k	**DM1**	
		Obtain the given exact answer correctly	**A1**	
		OR: Use both pairs of limits in a definite integral	**M1***	
		Obtain the given exact answer correctly	**A1**	
		Substitute k and either pair of limits in an expression containing $p\ln x$, $q\ln(4-x)$ and rt and evaluate a constant	**DM1**	
		Obtain $\ln\frac{x}{4-x} = t\ln 3 - \ln 9$ or equivalent	**A1**	[9]
	(ii)	Substitute $x = 3.6$ and solve for t	**M1**	
		Obtain answer $t = 4$	**A1**	[2]

Cambridge International Examinations
Cambridge International Advanced Level

MATHEMATICS **9709/32**

Paper 3 **October/November 2016**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the October/November 2016 series for most Cambridge IGCSE®, Cambridge International A and AS Level components and some Cambridge O Level components.

This document consists of **9** printed pages.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

  - Note: B2 or A2 means that the candidate can earn 2 or 0.
    B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
CWO	Correct Working Only – often written by a 'fortuitous' answer
ISW	Ignore Subsequent Working
SOI	Seen or implied
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √ " marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

1	Solve for 3^x and obtain $3^x = \frac{18}{7}$ Use correct method for solving an equation of the form $3^x = a$, where $a > 0$ Obtain answer $x = 0.860$ 3 d.p. only	**B1** **M1** **A1**	[3]
2	State correct unsimplified first two terms of the expansion of $(1+2x)^{-\frac{3}{2}}$, e.g. $1+(-\frac{3}{2})(2x)$ State correct unsimplified term in x^2, e.g. $(-\frac{3}{2})(-\frac{3}{2}-1)(2x)^2 / 2!$ Obtain sufficient terms of the product of $(2-x)$ and the expansion up to the term in x^2 Obtain **final** answer $2-7x+18x^2$ Do not ISW	**B1** **B1** **M1** **A1**	[4]
3	*EITHER*: Correctly restate the equation in terms of $\sin\theta$ and $\cos\theta$ Correct method to obtain a horizontal equation in $\sin\theta$ Reduce the equation to a correct quadratic in any form, e.g. $3\sin^2\theta - \sin\theta - 2 = 0$ Solve a three-term quadratic for $\sin\theta$ Obtain final answer $\theta = -41.8°$ only [Ignore answers outside the given interval.] *OR* 1: Square both sides of the equation and use $1+\tan^2\theta = \sec^2\theta$ Correct method to obtain a horizontal equation in $\sin\theta$ Reduce the equation to a correct quadratic in any form, e.g. $9\sin^2\theta - 6\sin\theta - 8 = 0$ Solve a three-term quadratic for $\sin\theta$ Obtain final answer $\theta = -41.8°$ only *OR* 2: Multiply through by $(\sec\theta + \tan\theta)$ Use $\sec^2\theta - \tan^2\theta = 1$ Obtain $1 = 3 + 3\sin\theta$ Solve for $\sin\theta$ Obtain final answer $\theta = -41.8°$ only	**B1** **M1** **A1** **M1** **A1** **B1** **M1** **A1** **M1** **A1** **M1** **B1** **A1** **M1** **A1**	[5]

4		*EITHER*: *EITHER*: State $2xy+x^2\frac{dy}{dx}$, or equivalent, as derivative of x^2y	B1	
		State $6y^2+12xy\frac{dy}{dx}$, or equivalent, as derivative of $6xy^2$	B1	
		OR: Differentiating LHS using correct product rule, state term $xy(1-6\frac{dy}{dx})$, or equivalent	B1	
		State term $(y+x\frac{dy}{dx})(x-6y)$, or equivalent	B1	
		Equate attempted derivative of LHS to zero and set $\frac{dy}{dx}$ equal to zero	M1*	
		Obtain a horizontal equation, e.g. $6y^2-2xy=0$ (from correct work only)	A1	
		Explicitly reject $y = 0$ as a possibility $py^2-qxy=0$	A1	
		Obtain an equation in x or y	DM1	
		Obtain answer $(-3a, -a)$	A1	
		OR: Rearrange to $y=\frac{9a^3}{x(x-6y)}$ and use correct quotient rule to obtain $-\frac{9a^3}{x^2(x-6y)^2}\times .....$	B1	
		State term $(x-6y)+x(1-6y')$, or equivalent	B1	
		Justify division by $x(x-6y)$	B1	
		Set $\frac{dy}{dx}$ equal to zero	M1*	
		Obtain a horizontal equation, e.g. $6y^2-2xy=0$ (from correct work only)	A1	
		Obtain an equation in x or y	DM1	
		Obtain answer $(-3a, -a)$	A1	[7]
5	(i)	*EITHER*: Use tan $2A$ formula to express LHS in terms of $\tan\theta$	M1	
		Express as a single fraction in any correct form	A1	
		Use Pythagoras or cos $2A$ formula	M1	
		Obtain the given result correctly	A1	
		OR: Express LHS in terms of sin 2θ, cos 2θ, sin θ and cos θ	M1	
		Express as a single fraction in any correct form	A1	
		Use Pythagoras or cos $2A$ formula or sin$(A-B)$ formula	M1	
		Obtain the given result correctly	A1	[4]
	(ii)	Integrate and obtain a term of the form $a\ln(\cos 2\theta)$ or $b\ln(\cos\theta)$ (or secant equivalents)	M1*	
		Obtain integral $-\frac{1}{2}\ln(\cos 2\theta)+\ln(\cos\theta)$, or equivalent	A1	
		Substitute limits correctly (expect to see use of <u>both</u> limits)	DM1	
		Obtain the given answer following full and correct working	A1	[4]

6	**(i)**	Make recognizable sketch of a relevant graph Sketch the other relevant graph and justify the given statement	**B1** **B1**	[2]
	(ii)	Use calculations to consider the value of a relevant expression at $x = 1.4$ and $x = 1.6$, or the values of relevant expressions at $x = 1.4$ and $x = 1.6$ Complete the argument correctly with correct calculated values	**M1** **A1**	[2]
	(iii)	State $x = 2\sin^{-1}\left(\dfrac{3}{x+3}\right)$ Rearrange this in the form $\operatorname{cosec}\frac{1}{2}x = \frac{1}{3}x + 1$ If working in reverse, need $\sin\dfrac{x}{2} = \left(\dfrac{3}{x+3}\right)$ for first B1	**B1** **B1**	[2]
	(iv)	Use the iterative formula correctly at least once Obtain final answer 1.471 Show sufficient iterations to 5 d.p. to justify 1.471 to 3 d.p., or show there is a sign change in the interval (1.4705, 1.4715)	**M1** **A1** **A1**	[3]
7	**(i)**	Use the correct product rule Obtain correct derivative in any form, e.g. $(2-2x)e^{\frac{1}{2}x} + \frac{1}{2}(2x - x^2)e^{\frac{1}{2}x}$ Equate derivative to zero and solve for x Obtain $x = \sqrt{5} - 1$ only	**M1** **A1** **M1** **A1**	[4]
	(ii)	Integrate by parts and reach $a(2x - x^2)e^{\frac{1}{2}x} + b\int(2-2x)e^{\frac{1}{2}x}\ dx$ Obtain $2e^{\frac{1}{2}x}(2x - x^2) - 2\int(2-2x)e^{\frac{1}{2}x}dx$, or equivalent Complete the integration correctly, obtaining $(12x - 2x^2 - 24)e^{\frac{1}{2}x}$, or equivalent Use limits $x = 0$, $x = 2$ correctly having integrated by parts twice Obtain answer 24 – 8e, or exact simplified equivalent	**M1*** **A1** **A1** **DM1** **A1**	[5]

8	**(i)**	State or imply a correct normal vector to either plane, e.g. $3\mathbf{i}+\mathbf{j}-\mathbf{k}$ or $\mathbf{i}-\mathbf{j}+2\mathbf{k}$	**B1**	
		Use correct method to calculate their scalar product	**M1**	
		Show value is zero and planes are perpendicular	**A1**	[3]
	(ii)	*EITHER*: Carry out a complete strategy for finding a point on *l* the line of intersection	**M1**	
		Obtain such a point, e.g. (0, 7, 5) , (1, 0, 1), (5/4, –7/4, 0)	**A1**	
		EITHER: State two equations for a direction vector $a\mathbf{i}+b\mathbf{j}+c\mathbf{k}$ for *l*,		
		e.g. $3a+b-c=0$ and $a-b+2c=0$	**B1**	
		Solve for one ratio, e.g. $a : b$	**M1**	
		Obtain $a : b : c = 1 : -7 : -4$, or equivalent	**A1**	
		State a correct answer, e.g. $\mathbf{r}=7\mathbf{j}+5\mathbf{k}+\lambda(\mathbf{i}-7\mathbf{j}-4\mathbf{k})$	**A1**√	
		*OR*1: Obtain a second point on *l*, e.g. (1, 0, 1)	**B1**	
		Subtract vectors and obtain a direction vector for *l*	**M1**	
		Obtain $-\mathbf{i}+7\mathbf{j}+4\mathbf{k}$, or equivalent	**A1**	
		State a correct answer, e.g. $\mathbf{r}=\mathbf{i}+\mathbf{k}+\lambda(-\mathbf{i}+7\mathbf{j}+4\mathbf{k})$	**A1**√	
		*OR*2: Attempt to find the vector product of the two normal vectors	**M1**	
		Obtain two correct components of the product	**A1**	
		Obtain $\mathbf{i}-7\mathbf{j}-4\mathbf{k}$, or equivalent	**A1**	
		State a correct answer, e.g. $\mathbf{r}=7\mathbf{j}+5\mathbf{k}+\lambda(\mathbf{i}-7\mathbf{j}-4\mathbf{k})$	**A1**√	
		*OR*1: Express one variable in terms of a second variable	**M1**	
		Obtain a correct simplified expression, e.g. $y = 7 - 7x$	**A1**	
		Express the third variable in terms of the second	**M1**	
		Obtain a correct simplified expression, e.g. $z = 5 - 4x$	**A1**	
		Form a vector equation for the line	**M1**	
		Obtain a correct equation, e.g. $\mathbf{r}=7\mathbf{j}+5\mathbf{k}+\lambda(\mathbf{i}-7\mathbf{j}-4\mathbf{k})$	**A1**√	
		*OR*2: Express one variable in terms of a second variable	**M1**	
		Obtain a correct simplified expression, e.g. $z=5-4x$	**A1**	
		Express the same variable in terms of the third	**M1**	
		Obtain a correct simplified expression e.g. $z=(7+4y)/7$	**A1**	
		Form a vector equation for the line	**M1**	
		Obtain a correct equation, e.g. $\mathbf{r}=\frac{5}{4}\mathbf{i}-\frac{7}{4}\mathbf{j}+\lambda(-\frac{1}{4}\mathbf{i}+\frac{7}{4}\mathbf{j}+\mathbf{k})$	**A1**√	[6]

9	(a)	EITHER: Use quadratic formula to solve for w	M1	
		Use $i^2 = -1$	M1	
		Obtain one of the answers $w = \frac{1}{2i+1}$ and $w = -\frac{5}{2i+1}$	A1	
		Multiply numerator and denominator of an answer by –2i + 1, or equivalent	M1	
		Obtain final answers $\frac{1}{5} - \frac{2}{5}i$ and –1 + 2i	A1	
		OR1: Multiply the equation by 1 – 2i	M1	
		Use $i^2 = -1$	M1	
		Obtain $5w^2 + 4w(1-2i) - (1-2i)^2 = 0$, or equivalent	A1	
		Use quadratic formula or factorise to solve for w	M1	
		Obtain final answers $\frac{1}{5} - \frac{2}{5}i$ and –1 + 2i	A1	
		OR2: Substitute $w = x + iy$ and form equations for real and imaginary parts	M1	
		Use $i^2 = -1$	M1	
		Obtain $(x^2 - y^2) - 4xy + 4x - 1 = 0$ and $2(x^2 - y^2) + 2xy + 4y + 2 = 0$ o.e.	A1	
		Form equation in x only or y only and solve	M1	
		Obtain final answers $\frac{1}{5} - \frac{2}{5}i$ and –1 + 2i	A1	[5]
	(b)	Show a circle with centre 1 + i	B1	
		Show a circle with radius 2	B1	
		Show half-line $\arg z = \frac{1}{4}\pi$	B1	
		Show half-line $\arg z = -\frac{1}{4}\pi$	B1	
		Shade the correct region	B1	[5]

10	(i)	Separate variables correctly and integrate at least one side	M1	
		Integrate and obtain term *kt*, or equivalent	A1	
		Carry out a relevant method to obtain A and B such that $\frac{1}{x(4-x)} \equiv \frac{A}{x} + \frac{B}{4-x}$, or equivalent	M1*	
		Obtain $A = B = \frac{1}{4}$, or equivalent	A1	
		Integrate and obtain terms $\frac{1}{4}\ln x - \frac{1}{4}\ln(4-x)$, or equivalent	A1√	
		EITHER: Use a pair of limits in an expression containing $p\ln x$, $q\ln(4-x)$ and rt and evaluate a constant	DM1	
		Obtain correct answer in any form, e.g. $\ln x - \ln(4-x) = 4kt - \ln 9$, or $\ln\left(\frac{x}{4-x}\right) = 4kt - 8k$	A1	
		Use a second pair of limits and determine k	DM1	
		Obtain the given exact answer correctly	A1	
		OR: Use both pairs of limits in a definite integral	M1*	
		Obtain the given exact answer correctly	A1	
		Substitute k and either pair of limits in an expression containing $p\ln x$, $q\ln(4-x)$ and rt and evaluate a constant	DM1	
		Obtain $\ln\frac{x}{4-x} = t\ln 3 - \ln 9$ or equivalent	A1	[9]
	(ii)	Substitute $x = 3.6$ and solve for t	M1	
		Obtain answer $t = 4$	A1	[2]

Cambridge International Examinations
Cambridge International Advanced Level

MATHEMATICS **9709/33**

Paper 3 **October/November 2016**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the October/November 2016 series for most Cambridge IGCSE®, Cambridge International A and AS Level components and some Cambridge O Level components.

This document consists of **6** printed pages.

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Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more “method” steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously “correct” answers or results obtained from incorrect working.

  - Note: B2 or A2 means that the candidate can earn 2 or 0.
    B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
CWO	Correct Working Only – often written by a 'fortuitous' answer
ISW	Ignore Subsequent Working
SOI	Seen or implied
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √ " marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

1		Use law of the logarithm of a quotient	**M1**	
		Remove logarithms and obtain a correct equation, e.g. $e^z = \frac{y+2}{y+1}$	**A1**	
		Obtain answer $y = \frac{2-e^z}{e^z-1}$, or equivalent	**A1**	[3]
2		Use correct quotient or product rule	**M1**	
		Obtain correct derivative in any form	**A1**	
		Use Pythagoras to simplify the derivative to $\frac{1}{1+\cos x}$, or equivalent	**A1**	
		Justify the given statement, $-1 < \cos x < 1$ statement, or equivalent	**A1**	[4]
3		Use the tan *2A* formula to obtain an equation in tan θ only	**M1**	
		Obtain a correct horizontal equation	**A1**	
		Rearrange equation as a quadratic in tan θ, e.g. $3\tan^2\theta + 2\tan\theta - 1 = 0$	**A1**	
		Solve for θ (usual requirements for solution of quadratic)	**M1**	
		Obtain answer, e.g. 18.4°	**A1**	
		Obtain second answer, e.g. 135° , and no others in the given interval	**A1**	[6]
4	**(i)**	Commence division by $x^2 - x + 2$ and reach a partial quotient $4x^2 + kx$	**M1**	
		Obtain quotient $4x^2 + 4x + a - 4$ or $4x^2 + 4x + b/2$	**A1**	
		Equate x or constant term to zero and solve for a or b	**M1**	
		Obtain $a = 1$	**A1**	
		Obtain $b = -6$	**A1**	[5]
	(ii)	Show that $x^2 - x + 2 = 0$ has no real roots	**B1**	
		Obtain roots $\frac{1}{2}$ and $-\frac{3}{2}$ from $4x^2 + 4x - 3 = 0$	**B1**	[2]
5	**(i)**	State equation $\frac{dy}{dx} = \frac{1}{2}xy$	**B1**	[1]
	(ii)	Separate variables correctly and attempts to integrate one side of equation	**M1**	
		Obtain terms of the form $a\ln y$ and bx^2	**A1**	
		Use $x = 0$ and $y = 2$ to evaluate a constant, or as limits, in expression containing a ln y or bx^2	**M1**	
		Obtain correct solution in any form, e.g. $\ln y = \frac{1}{4}x^2 + \ln 2$	**A1**	
		Obtain correct expression for y, e.g. $y = 2e^{\frac{1}{4}x^2}$	**A1**	[5]
	(iii)	Show correct sketch for $x \geqslant 0$. Needs through (0, 2) and rapidly increasing positive gradient.	**B1**	[1]

6 (i)	State or imply $\mathrm{d}u = \dfrac{1}{2\sqrt{x}}\mathrm{d}x$	B1	
	Substitute for x and dx throughout	M1	
	Justify the change in limits and obtain the given answer	A1	[3]
(ii)	Convert integrand into the form $A + \dfrac{B}{u+1}$	M1*	
	Obtain integrand $A = 1$, $B = -2$	A1	
	Integrate and obtain $u - 2\ln(u+1)$	A1√ + A1√	
	Substitute limits correctly in an integral containing terms au and $b\ln(u + 1)$, where $ab \neq 0$	DM1	
	Obtain the given answer following full and correct working	A1	[6]
	[The f.t. is on A and B.]		
7 (i)	State modulus $2\sqrt{2}$, or equivalent	B1	
	State argument $-\frac{1}{3}\pi$ (or $-60°$)	B1	[2]
(ii) (a)	State answer $3\sqrt{2} + \sqrt{6}\,\mathrm{i}$	B1	
(b)	*EITHER:* Substitute for z and multiply numerator and denominator by conjugate of iz	M1	
	Simplify the numerator to $4\sqrt{3} + 4\,\mathrm{i}$ or the denominator to 8	A1	
	Obtain final answer $\frac{1}{2}\sqrt{3} + \frac{1}{2}\mathrm{i}$	A1	
	OR: Substitute for z, obtain two equations in x and y and solve for x or for y	M1	
	Obtain $x = \frac{1}{2}\sqrt{3}$ or $y = \frac{1}{2}$	A1	
	Obtain final answer $\frac{1}{2}\sqrt{3} + \frac{1}{2}\mathrm{i}$	A1	[4]
(iii)	Show points A and B in relatively correct positions	B1	
	Carry out a complete method for finding angle AOB, e.g. calculate the argument of $\dfrac{z^*}{\mathrm{i}z}$	M1	
	Obtain the given answer	A1	[3]
8 (i)	State or imply the form $\dfrac{A}{x+2} + \dfrac{Bx+C}{x^2+4}$	B1	
	Use a correct method to determine a constant	M1	
	Obtain one of $A = 2$, $B = 1$, $C = -1$	A1	
	Obtain a second value	A1	
	Obtain a third value	A1	[5]

(ii)	Use correct method to find the first two terms of the expansion of $(x+2)^{-1}$, $(1+\frac{1}{2}x)^{-1}$, $(4+x^2)^{-1}$ or $(1+\frac{1}{4}x^2)^{-1}$ Obtain correct unsimplified expansions up to the term in x^2 of each partial fraction Multiply out fully by $Bx + C$, where $BC \neq 0$ Obtain final answer $\frac{3}{4}-\frac{1}{4}x+\frac{5}{16}x^2$, or equivalent [Symbolic binomial coefficients, e.g. $\begin{pmatrix}-1\\1\end{pmatrix}$ are not sufficient for the M1. The f.t. is on *A*, *B*, *C*.] [In the case of an attempt to expand $(3x^2+x+6)(x+2)^{-1}(x^2+4)^{-1}$, give M1A1A1 for the expansions, M1 for multiplying out fully, and A1 for the final answer.]	**M1** **A1√ + A1√** **M1** **A1**	[5]
9 (i)	Differentiate both equations and equate derivatives Obtain equation $\cos a - a\sin a = -\frac{k}{a^2}$ State $a\cos a = \frac{k}{a}$ and eliminate *k* Obtain the given answer showing sufficient working	**M1*** **A1 + A1** **DM1** **A1**	[5]
(ii)	Show clearly correct use of the iterative formula at least once Obtain answer 1.077 Show sufficient iterations to 5 d.p. to justify 1.077 to 3 d.p., or show there is a sign change in the interval (1.0765, 1.0775)	**M1** **A1** **A1**	[3]
(iii)	Use a correct method to determine *k* Obtain answer $k = 0.55$	**M1** **A1**	[2]
10 (i)	Express general point of *l* in component form e.g. $(1+2\lambda,\ 2-\lambda,\ 1+\lambda)$ Using the correct process for the modulus form an equation in λ Reduce the equation to a quadratic, e.g. $6\lambda^2+2\lambda-4=0$ Solve for λ (usual requirements for solution of a quadratic) Obtain final answers $-\mathbf{i}+3\mathbf{j}$ and $\frac{7}{3}\mathbf{i}+\frac{4}{3}\mathbf{j}+\frac{5}{3}\mathbf{k}$	**B1** **M1*** **A1** **DM1** **A1**	[5]
(ii)	Using the correct process, find the scalar product of a direction vector for *l* and a normal for *p* Using the correct process for the moduli, divide the scalar product by the product of the moduli and equate the result to $\frac{2}{3}$ State a correct equation in any form, e.g. $\frac{2a-1+1}{\sqrt{(a^2+1+1)}.\sqrt{(2^2+(-1)^2+1)}}=\pm\frac{2}{3}$ Solve for a^2 Obtain answer $a = \pm 2$	**M1** **M1** **A1** **M1** **A1**	[5]

Cambridge International A Level

Cambridge International Examinations
Cambridge International Advanced Level

MATHEMATICS **9709/31**

Paper 3 **May/June 2016**

MARK SCHEME

Maximum Mark: 75

Published

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Cambridge is publishing the mark schemes for the May/June 2016 series for most Cambridge IGCSE®, Cambridge International A and AS Level components and some Cambridge O Level components.

This document consists of **8** printed pages.

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[Turn over

Mark Scheme Notes

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A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

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The following abbreviations may be used in a mark scheme or used on the scripts:

AEF	Any Equivalent Form (of answer is equally acceptable)
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
BOD	Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)
CAO	Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
CWO	Correct Working Only – often written by a 'fortuitous' answer
ISW	Ignore Subsequent Working
MR	Misread
PA	Premature Approximation (resulting in basically correct work that is insufficiently accurate)
SOS	See Other Solution (the candidate makes a better attempt at the same question)
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

1 **(i)** *EITHER*: State or imply non-modular equation $(2(x-1))^2 = (3x)^2$, or pair of linear equations $2(x-1) = \pm 3x$ **B1**

Make reasonable solution attempt at a 3-term quadratic, or solve two linear equations **M1**

Obtain answers $x = -2$ and $x = \frac{2}{5}$ **A1**

OR: Obtain answer $x = -2$ by inspection or by solving a linear equation **(B1**

Obtain answer $x = \frac{2}{5}$ similarly **B2)**

[3]

(ii) Use correct method for solving an equation of the form $5^x = a$ or $5^{x+1} = a$, where $a > 0$ **M1**

Obtain answer $x = -0.569$ only **A1**

[2]

2 Integrate by parts and reach $axe^{-2x} + b\int e^{-2x}\,dx$ **M1**

Obtain $-\frac{1}{2}xe^{-2x} + \frac{1}{2}\int e^{-2x}\,dx$, or equivalent **A1**

Complete the integration correctly, obtaining $-\frac{1}{2}xe^{-2x} - \frac{1}{4}e^{-2x}$, or equivalent **A1**

Use limits $x = 0$ and $x = \frac{1}{2}$ correctly, having integrated twice **M1**

Obtain answer $\frac{1}{4} - \frac{1}{2}e^{-1}$, or exact equivalent **A1**

[5]

3 Correctly restate the equation in terms of sin θ and cos θ **B1**

Using Pythagoras obtain a horizontal equation in cos θ **M1**

Reduce the equation to a correct quadratic in cos θ, e.g. $3\cos^2\theta - \cos\theta - 2 = 0$ **A1**

Solve a 3-term quadratic for cos θ **M1**

Obtain answer $\theta = 131.8°$ only **A1**

[5]

[Ignore answers outside the given interval.]

4 Separate variables and attempt integration of at least one side **M1***

Obtain term $\ln y$ **A1**

Obtain terms $\ln x - x^2$ **A1**

Use $x = 1$ and $y = 2$ to evaluate a constant, or as limits **DM1***

Obtain correct solution in any form, e.g. $\ln y = \ln x - x^2 + \ln 2 + 1$ **A1**

Obtain correct expression for y, free of logarithms, i.e. $y = 2x\exp(1 - x^2)$ **A1**

[6]

5		Use product rule	**M1**
		Obtain correct derivative in any form, e.g. $\cos x\cos 2x-2\sin x\sin 2x$	**A1**
		Equate derivative to zero and use double angle formulae	**M1**
		Remove factor of cos x and reduce equation to one in a single trig function	**M1**
		Obtain $6\sin^2 x=1$, $6\cos^2 x=5$ or $5\tan^2 x=1$	**A1**
		Solve and obtain x = 0.421	**A1** [6]
		[Alternative: Use double angle formula M1.Use chain rule to differentiate M1. Obtain correct derivative e.g. $\cos\theta-6\sin^2\theta\cos\theta$ A1, then as above.]	
6	**(i)**	Make recognizable sketch of a relevant graph	**B1**
		Sketch the other relevant graph and justify the given statement	**B1** [2]
	(ii)	State $x=\frac{1}{2}\ln(25/x)$	**B1**
		Rearrange this in the form $5e^{-x}=\sqrt{x}$	**B1** [2]
	(iii)	Use the iterative formula correctly at least once	**M1**
		Obtain final answer 1.43	**A1**
		Show sufficient iterations to 4 d.p. to justify 1.43 to 2 d.p., or show there is a sign change in the interval (1.425, 1.435)	**A1** [3]
7	**(i)**	State or imply $6xy+3x^2\frac{dy}{dx}$ as derivative of $3x^2y$	**B1**
		State $3y^2\frac{dy}{dx}$ as derivative of y^3	**B1**
		Equate attempted derivative of the LHS to zero and solve for $\frac{dy}{dx}$	**M1**
		Obtain the given answer	**A1** [4]
	(ii)	Equate numerator to zero	**M1***
		Obtain $x = 2y$, or equivalent	**A1**
		Obtain an equation in x or y	**DM1***
		Obtain the point (–2, –1)	**A1**
		State the point (0, 1.44)	**B1** [5]

8	**(i)**	State or imply the form $\frac{A}{x+1}+\frac{B}{x-3}+\frac{C}{(x-3)^2}$	**B1**
		Use a correct method to determine a constant	**M1**
		Obtain one of the values $A = 1$, $B = 3$, $C = 12$	**A1**
		Obtain a second value	**A1**
		Obtain a third value	**A1**
			[5]
		[Mark the form $\frac{A}{x+1}+\frac{Dx+E}{(x-3)^2}$, where $A = 1$, $D = 3$, $E = 3$, B1M1A1A1A1 as above.]	
	(ii)	Use correct method to find the first two terms of the expansion of $(x+1)^{-1}$, $(x-3)^{-1}$, $(1-\frac{1}{3}x)^{-1}$, $(x-3)^{-2}$, or $(1-\frac{1}{3}x)^{-2}$	**M1**
		Obtain correct unsimplified expansions up to the term in x^2 of each partial fraction	**A1√ + A1√ + A1√**
		Obtain final answer $\frac{4}{3}-\frac{4}{9}x+\frac{4}{3}x^2$, or equivalent	**A1**
			[5]

9 **(i)** *EITHER*: Obtain a vector parallel to the plane, e.g. $\overrightarrow{AB} = \mathbf{i} - 2\mathbf{j} - 3\mathbf{k}$ **B1**

Use scalar product to obtain an equation in *a*, *b*, *c* e.g. $a - 2b - 3c = 0$, $a + b - c = 0$, or $3b + 2c = 0$ **M1**

State two correct equations **A1**

Solve to obtain ratio $a : b : c$ **M1**

Obtain $a : b : c = 5 : -2 : 3$ **A1**

Obtain equation $5x - 2y + 3z = 5$, or equivalent **A1**

*OR*1: Substitute for two points, e.g. *A* and *B*, and obtain $a + 3b + 2c = d$ and $2a + b - c = d$ **(B1**

Substitute for another point, e.g. *C*, to obtain a third equation and eliminate one unknown entirely from all three equations **M1**

Obtain two correct equations in three unknowns, e.g. in *a*, *b*, *c* **A1**

Solve to obtain their ratio **M1**

Obtain $a : b : c = 5 : -2 : 3$, $a : c : d = 5 : 3 : 5$, $a : b : d = 5 : -2 : 5$, or $b : c : d = -2 : 3 : 5$ **A1**

Obtain equation $5x - 2y + 3z = 5$, or equivalent **A1)**

*OR*2: Obtain a vector parallel to the plane, e.g. $\overrightarrow{AC} = \mathbf{i} + \mathbf{j} - \mathbf{k}$ **(B1**

Obtain a second such vector and calculate their vector product, e.g. $(\mathbf{i} - 2\mathbf{j} - 3\mathbf{k}) \times (\mathbf{i} + \mathbf{j} - \mathbf{k})$ **M1**

Obtain two correct components of the product **A1**

Obtain correct answer e.g. $5\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$ **A1**

Substitute in $5x - 2y + 3z = d$ to find *d* **M1**

Obtain equation $5x - 2y + 3z = 5$, or equivalent **A1)**

*OR*3: Obtain a vector parallel to the plane, e.g. $\overrightarrow{BC} = 3\mathbf{j} + 2\mathbf{k}$ **(B1**

Obtain a second such vector and form correctly a 2-parameter equation for the plane **M1**

Obtain a correct equation, e.g. $\mathbf{r} = \mathbf{i} + 3\mathbf{j} + 2\mathbf{k} + \lambda(\mathbf{i} - 2\mathbf{j} - 3\mathbf{k}) + \mu(3\mathbf{j} + 2\mathbf{k})$ **A1**

State three correct equations in x, y, z, λ, μ **A1**

Eliminate λ and μ **M1**

Obtain equation $3x - 2y + 3z = 5$, or equivalent **A1)**

[6]

(ii) Correctly form an equation for the line through *D* parallel to *OA* **M1**

Obtain a correct equation e.g. $\mathbf{r} = -3\mathbf{i} + \mathbf{j} + 2\mathbf{k} + \lambda(\mathbf{i} + 3\mathbf{j} + 2\mathbf{k})$ **A1**

Substitute components in the equation of the plane and solve for λ **M1**

Obtain $\lambda = 2$ and position vector $-\mathbf{i} + 7\mathbf{j} + 6\mathbf{k}$ for *P* **A1**

Obtain the given answer correctly **A1**

[5]

10 **(a)** Square $x + iy$ and equate real and imaginary parts to 7 and $-6\sqrt{2}$ respectively **M1**

Obtain equations $x^2 - y^2 = 7$ and $2xy = -6\sqrt{2}$ **A1**

Eliminate one variable and find an equation in the other **M1**

Obtain $x^4 - 7x^2 - 18 = 0$ or $y^4 + 7y^2 - 18 = 0$, or 3-term equivalent **A1**

Obtain answers $\pm(3 - i\sqrt{2})$ **A1**

[5]

(b) **(i)**	Show point representing 1 + 2i	**B1**
	Show circle with radius 1 and centre 1 + 2i	**B1√**
	Show a half line from the point representing 1	**B1**
	Show line making the correct angle with the real axis	**B1**
		[4]
(ii)	State or imply the relevance of the perpendicular from 1 + 2i to the line	**M1**
	Obtain answer $\sqrt{2}-1$ (or 0.414)	**A1**
		[2]

Cambridge International A Level

Cambridge International Examinations
Cambridge International Advanced Level

MATHEMATICS **9709/32**

Paper 3 **May/June 2016**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the May/June 2016 series for most Cambridge IGCSE®, Cambridge International A and AS Level components and some Cambridge O Level components.

This document consists of **7** printed pages.

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Mark Scheme Notes

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M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more “method” steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously “correct” answers or results obtained from incorrect working.

- **Note:** B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

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The following abbreviations may be used in a mark scheme or used on the scripts:

AEF	Any Equivalent Form (of answer is equally acceptable)
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<u>Penalties</u>

MR – 1	A penalty of MR – 1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become “follow through √” marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR – 2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA – 1	This is deducted from A or B marks in the case of premature approximation. The PA – 1 penalty is usually discussed at the meeting.

1		Use law of the logarithm of a product, power or quotient	M1*	
		Obtain a correct linear equation, e.g. $(3x-1)\ln 4 = \ln 3 + x\ln 5$	A1	
		Solve a linear equation for x	DM1*	
		Obtain answer $x = 0.975$	A1	[4]
2		State a correct un-simplified version of the x or x^2 or x^3 term	M1	
		State correct first two terms $1 + x$	A1	
		Obtain the next two terms $\frac{3}{2}x^2 + \frac{5}{2}x^3$	A1 A1	[4]
		[Symbolic binomial coefficients, e.g. $\begin{pmatrix} -\frac{1}{2} \\ 3 \end{pmatrix}$ are not sufficient for the M mark.]		
3		Integrate by parts and reach $ax^2\cos 2x + b\int x\cos 2x \,\mathrm{d}x$	M1*	
		Obtain $-\frac{1}{2}x^2\cos 2x + \int x\cos 2x$, or equivalent	A1	
		Complete the integration and obtain $-\frac{1}{2}x^2\cos 2x + \frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x$, or equivalent	A1	
		Use limits correctly having integrated twice	DM1*	
		Obtain answer $\frac{1}{8}(\pi^2 - 4)$, or exact equivalent, with no errors seen	A1	[5]
4		State or imply derivative of $(\ln x)^2$ is $\dfrac{2\ln x}{x}$	B1	
		Use correct quotient or product rule	M1	
		Obtain correct derivative in any form, e.g. $\dfrac{2\ln x}{x^2} - \dfrac{(\ln x)^2}{x^2}$	A1	
		Equate derivative (or its numerator) to zero and solve for $\ln x$	M1	
		Obtain the point (1, 0) with no errors seen	A1	
		Obtain the point $(\mathrm{e}^2,\ 4\mathrm{e}^{-2})$	A1	[6]
5	(i)	*EITHER*: Express cos 4θ in terms of cos 2θ and/or sin 2θ	B1	
		Use correct double angle formulae to express LHS in terms of sin θ and/or cos θ	M1	
		Obtain a correct expression in terms of sin θ alone	A1	
		Reduce correctly to the given form	A1	
		OR: Use correct double angle formula to express RHS in terms of cos 2θ	M1	
		Express $\cos^2 2\theta$ in terms of cos 4θ	B1	
		Obtain a correct expression in terms of cos 4θ and cos 2θ	A1	
		Reduce correctly to the given form	A1	[4]
	(ii)	Use the identity and carry out a method for finding a root	M1	
		Obtain answer 68.5°	A1	
		Obtain a second answer, e.g. 291.5°	A1√	
		Obtain the remaining answers, e.g. 111.5° and 248.5°, and no others in the given interval	A1√	[4]
		[Ignore answers outside the given interval. Treat answers in radians as a misread.]		

6	**(i)**	Separate variables correctly and attempt integration of at least one side	**B1**	
		Obtain term $\ln x$	**B1**	
		Obtain term of the form $k\ln(3+\cos 2\theta)$, or equivalent	**M1**	
		Obtain term $-\frac{1}{2}\ln(3+\cos 2\theta)$, or equivalent	**A1**	
		Use $x = 3$, $\theta = \frac{1}{4}\pi$ to evaluate a constant or as limits in a solution with terms $a\ln x$ and $b\ln(3+\cos 2\theta)$,where $ab \neq 0$	**M1**	
		State correct solution in any form, e.g. $\ln x = -\frac{1}{2}\ln(3+\cos 2\theta)+\frac{3}{2}\ln 3$	**A1**	
		Rearrange in a correct form, e.g. $x = \sqrt{\left(\frac{27}{3+\cos 2\theta}\right)}$	**A1**	[7]
	(ii)	State answer $x = 3\sqrt{3}/2$, or exact equivalent (accept decimal answer in [2.59, 2.60])	**B1**	[1]
7	**(i)**	State or imply the form $A + \frac{B}{2x+1} + \frac{C}{x+2}$	**B1**	
		State or obtain $A = 2$	**B1**	
		Use a correct method for finding a constant	**M1**	
		Obtain one of $B = 1$, $C = -2$	**A1**	
		Obtain the other value	**A1**	[5]
	(ii)	Integrate and obtain terms $2x + \frac{1}{2}\ln(2x+1) - 2\ln(x+2)$	**B3√**	
		Substitute correct limits correctly in an integral with terms $a\ln(2x+1)$ and $b\ln(x+2)$, where $ab \neq 0$	**M1**	
		Obtain the given answer after full and correct working	**A1**	[5]
8	**(i)**	Use correct quotient or chain rule	**M1**	
		Obtain correct derivative in any form	**A1**	
		Obtain the given answer correctly	**A1**	[3]
	(ii)	State a correct equation, e.g. $-\mathrm{e}^{-a} = -\operatorname{cosec} a \cot a$	**B1**	
		Rearrange it correctly in the given form	**B1**	[2]
	(iii)	Calculate values of a relevant expression or pair of expressions at $x = 1$ and $x = 1.5$	**M1**	
		Complete the argument correctly with correct calculated values	**A1**	[2]
	(iv)	Use the iterative formula correctly at least once	**M1**	
		Obtain final answer 1.317	**A1**	
		Show sufficient iterations to 5 d.p. to justify 1.317 to 3 d.p., or show there is a sign change in the interval (1.3165, 1,3175)	**A1**	[3]

9	**(i)**	*Either* state or imply $\overrightarrow{AB}$ or $\overrightarrow{BC}$ in component form, *or* state position vector of midpoint of $\overrightarrow{AC}$	**B1**	
		Use a correct method for finding the position vector of D	**M1**	
		Obtain answer $3\mathbf{i}+3\mathbf{j}+\mathbf{k}$, or equivalent	**A1**	
		EITHER: Using the correct process for the moduli, compare lengths of a pair of adjacent sides, e.g. AB and BC	**M1**	
		Show that $ABCD$ has a pair of adjacent sides that are equal	**A1**	
		OR: Calculate scalar product $\overrightarrow{AC}.\overrightarrow{BD}$ or equivalent	**M1**	
		Show that $ABCD$ has perpendicular diagonals	**A1**	[5]
	(ii)	*EITHER*: State $a+2b+3c=0$ or $2a+b-2c=0$	**B1**	
		Obtain two relevant equations and solve for one ratio, e.g. $a : b$	**M1**	
		Obtain $a : b : c = -7 : 8 : -3$, or equivalent	**A1**	
		Substitute coordinates of a relevant point in $-7x + 8y - 3z = d$, and evaluate	**M1**	
		Obtain answer $-7x + 8y - 3z = 29$, or equivalent	**A1**	
		*OR*1:Attempt to calculate vector product of relevant vectors, e.g. $(\mathbf{i}+2\mathbf{j}+3\mathbf{k})\times(2\mathbf{i}+\mathbf{j}-2\mathbf{k})$	**M1**	
		Obtain two correct components of the product	**A1**	
		Obtain correct product, e.g. $-7\mathbf{i}+8\mathbf{j}-3\mathbf{k}$	**A1**	
		Substitute coordinates of a relevant point in $-7x+8y-3z=d$ and evaluate d	**M1**	
		Obtain answer $-7x+8y-3z=29$ or equivalent	**A1**	
		*OR*2:Attempt to form a 2-parameter equation with relevant vectors	**M1**	
		State a correct equation, e.g. $\mathbf{r}=2\mathbf{i}+5\mathbf{j}-\mathbf{k}+\lambda(\mathbf{i}+2\mathbf{j}+3\mathbf{k})+\mu(2\mathbf{i}+\mathbf{j}-2\mathbf{k})$	**A1**	
		State 3 equations in x, y, z, λ and μ	**A1**	
		Eliminate λ and μ	**M1**	
		Obtain answer $-7x+8y-3z=29$, or equivalent	**A1**	
		*OR*3:Using a relevant point and relevant direction vectors, form a determinant equation for the plane	**M1**	
		State a correct equation, e.g. $\begin{vmatrix} x-2 & y-5 & z+1 \\ 1 & 2 & 3 \\ 2 & 1 & -2 \end{vmatrix}=0$	**A1**	
		Attempt to expand the determinant	**M1**	
		Obtain correct values of two cofactors	**A1**	
		Obtain answer $-7x+8y-3z=29$, or equivalent	**A1**	[5]

10 **(a)** *EITHER*: Use quadratic formula to solve for z **M1**

Use $\mathrm{i}^2 = -1$ **M1**

Obtain a correct answer in any form, simplified as far as $(-2 \pm \mathrm{i}\sqrt{8})/2\mathrm{i}$ **A1**

Multiply numerator and denominator by i, or equivalent **M1**

Obtain final answers $\sqrt{2} + \mathrm{i}$ and $-\sqrt{2} + \mathrm{i}$ **A1**

OR: Substitute $x + \mathrm{i}y$ and equate real and imaginary parts to zero **M1**

Use $\mathrm{i}^2 = -1$ **M1**

Obtain $-2xy + 2x = 0$ and $x^2 - y^2 + 2y - 3 = 0$, or equivalent **A1**

Solve for x and y **M1**

Obtain final answers $\sqrt{2} + \mathrm{i}$ and $-\sqrt{2} + \mathrm{i}$ **A1** [5]

(b) **(i)** *EITHER*: Show the point representing 4 + 3i in relatively correct position **B1**

Show the perpendicular bisector of the line segment joining this point to the origin **B1√** [2]

OR: Obtain correct Cartesian equation of the locus in any form, e.g. $8x + 6y = 25$ **B1**

Show this line **B1√**

[This f.t. is dependent on using a correct method to determine the equation.]

(ii) State or imply the relevant point is represented by 2 + 1.5i or is at (2, 1.5) **B1**

Obtain modulus 2.5 **B1√**

Obtain argument 0.64 (or 36.9°) (allow decimals in [0.64, 0.65] or [36.8, 36.9]) **B1√** [3]

Cambridge International Examinations
Cambridge International Advanced Level

MATHEMATICS **9709/33**

Paper 3 **May/June 2016**

MARK SCHEME

Maximum Mark: 75

Published

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[Turn over

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PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

1		*EITHER*: State or imply non-modular inequality $(2(x-2))^2 > (3x+1)^2$, or corresponding quadratic equation, or pair of linear equations $2(x-2) = \pm(3x+1)$	**B1**
		Make reasonable solution attempt at a 3-term quadratic, or solve two linear equations for x	**M1**
		Obtain critical values $x = -5$ and $x = \frac{3}{5}$	**A1**
		State final answer $-5 < x < \frac{3}{5}$	**A1**
		OR: Obtain critical value $x = -5$ from a graphical method, or by inspection, or by solving a linear equation or inequality	**(B1**
		Obtain critical value $x = \frac{3}{5}$ similarly	**B2**
		State final answer $-5 < x < \frac{3}{5}$	**B1)**
		[Do not condone ≤ for <.]	[4]
2	**(i)**	State or imply $y \ln 3 = (2 - x)\ln 4$	**B1**
		State that this is of the form $ay = bx + c$ and thus a straight line, or equivalent	**B1**
		State gradient is $-\dfrac{\ln 4}{\ln 3}$, or exact equivalent	**B1**
			[3]
	(ii)	Substitute $y = 2x$ and solve for x, using a log law correctly at least once	**M1**
		Obtain answer $x = \ln 4 / \ln 6$, or exact equivalent	**A1**
			[2]
3	**(i)**	State answer $R = 3$	**B1**
		Use trig formula to find	**M1**
		Obtain $\alpha = 41.81°$ with no errors seen	**A1**
			[3]
	(ii)	Evaluate $\cos^{-1}(0.4)$ to at least 1 d.p. (66.42° to 2 d.p.)	**B1**√
		Carry out an appropriate method to find a value of x in the given range	**M1**
		Obtain answer 216.5° only	**A1**
		[Ignore answers outside the given interval.]	[3]
4	**(i)**	State $\dfrac{dx}{dt} = 1 - \sin t$	**B1**
		Use chain rule to find the derivative of y	**M1**
		Obtain $\dfrac{dy}{dt} = \dfrac{\cos t}{1 + \sin t}$, or equivalent	**A1**
		Use $\dfrac{dy}{dx} = \dfrac{dy}{dt} \div \dfrac{dx}{dt}$	**M1**
		Obtain the given answer correctly	**A1**
			[5]
	(ii)	State or imply $t = \cos^{-1}(\frac{1}{3})$	**B1**
		Obtain answers $x = 1.56$ and $x = -0.898$	**B1 + B1**
			[3]

5		Separate variables and make reasonable attempt at integration of either integral	**M1**
		Obtain term $\frac{1}{2}e^{2y}$	**B1**
		Use Pythagoras	**M1**
		Obtain terms $\tan x - x$	**A1**
		Evaluate a constant or use $x = 0$, $y = 0$ as limits in a solution containing terms $ae^{\pm 2y}$ and $b\tan x$, ($ab \neq 0$)	**M1**
		Obtain correct solution in any form, e.g. $\frac{1}{2}e^{2y} = \tan x - x + \frac{1}{2}$	**A1**
		Set $x = \frac{1}{4}\pi$ and use correct method to solve an equation of the form $e^{\pm 2y} = a$ or $e^{\pm y} = a$, where $a > 0$	**M1**
		Obtain answer $y = 0.179$	**A1**
			[8]
6	**(i)**	Use the product rule	**M1**
		Obtain correct derivative in any form	**A1**
		Equate 2-term derivative to zero and obtain the given answer correctly	**A1**
			[3]
	(ii)	Use calculations to consider the sign of a relevant expression at $p = 2$ and $p = 2.5$, or compare values of relevant expressions at $p= 2$ and $p = 2.5$	**M1**
		Complete the argument correctly with correct calculated values	**A1**
			[2]
	(iii)	Use the iterative formula correctly at least once	**M1**
		Obtain final answer 2.15	**A1**
		Show sufficient iterations to 4 d.p. to justify 2.15 to 2 d.p., or show there is a sign change in the interval (2.145,2.155)	**A1**
			[3]
7	**(i)**	State or imply $du = 2x\,dx$, or equivalent	**B1**
		Substitute for x and dx throughout	**M1**
		Reduce to the given form and justify the change in limits	**A1**
			[3]
	(ii)	Convert integrand to a sum of integrable terms and attempt integration	**M1**
		Obtain integral $\frac{1}{2}\ln u + \frac{1}{u} - \frac{1}{4u^2}$, or equivalent (deduct A1 for each error or omission)	**A1 + A1**
		Substitute limits in an integral containing two terms of the form $a\ln u$ and bu^{-2}	**M1**
		Obtain answer $\frac{1}{2}\ln 2 - \frac{5}{16}$, exact simplified equivalent	**A1**
			[5]

8	**(i)**	State a correct equation for AB in any form, e.g. $\mathbf{r}=\mathbf{i}+\mathbf{j}+\mathbf{k}+\lambda(\mathbf{i}-\mathbf{j}+2\mathbf{k})$, or equivalent	**B1**
		Equate at least two pairs of components of AB and l and solve for λ or for μ	**M1**
		Obtain correct answer for λ or for μ, e.g. $\lambda=-1$ or $\mu=2$	**A1**
		Show that not all three equations are not satisfied and that the lines do not intersect	**A1**
			[4]
	(ii)	*EITHER*: Find $\overrightarrow{AP}$ (or $\overrightarrow{PA}$) for a general point P on l, e.g. $(1-\mu)\mathbf{i}+(-3+2\mu)\mathbf{j}+(-2+\mu)\mathbf{k}$	**B1**
		Calculate the scalar product of $\overrightarrow{AP}$ and a direction vector for l and equate to zero	**M1**
		Solve and obtain $\mu=\frac{3}{2}$	**A1**
		Carry out a method to calculate AP when $\mu=\frac{3}{2}$	**M1**
		Obtain the given answer $\dfrac{1}{\sqrt{2}}$ correctly	**A1**
		OR 1:Find $\overrightarrow{AP}$ (or $\overrightarrow{PA}$) for a general point P on l	**(B1**
		Use correct method to express AP^2 (or AP) in terms of μ	**M1**
		Obtain a correct expression in any form, e.g. $(1-\mu)^2+(-3+2\mu)^2+(-2+\mu)^2$	**A1**
		Carry out a complete method for finding its minimum	**M1**
		Obtain the given answer correctly	**A1)**
		OR 2:Calling (2, −2, −1) C, state $\overrightarrow{AC}$ (or $\overrightarrow{CA}$) in component form, e.g. $\mathbf{i}-3\mathbf{j}-2\mathbf{k}$	**(B1**
		Use a scalar product to find the projection of $\overrightarrow{AC}$ (or $\overrightarrow{CA}$) on l	**M1**
		Obtain correct answer in any form, e.g. $\dfrac{9}{\sqrt{6}}$	**A1**
		Use Pythagoras to find the perpendicular	**M1**
		Obtain the given answer correctly	**A1)**
		OR 3:State $\overrightarrow{AC}$ (or $\overrightarrow{CA}$) in component form	**(B1**
		Calculate vector product of $\overrightarrow{AC}$ and a direction vector for l, e.g. $(\mathbf{i}-3\mathbf{j}-2\mathbf{k})\times(-\mathbf{i}+2\mathbf{j}+\mathbf{k})$	**M1**
		Obtain correct answer in any form, e.g. $\mathbf{i}+\mathbf{j}-\mathbf{k}$	**A1**
		Divide modulus of the product by that of the direction vector	**M1**
		Obtain the given answer correctly	**A1)**
			[5]
9	**(i)**	*EITHER*: Multiply numerator and denominator of $\dfrac{u}{v}$ by 2 + i, or equivalent	**M1**
		Simplify the numerator to −5 +5i or denominator to 5	**A1**
		Obtain final answer −1 + I	**A1**
		OR: Obtain two equations in x and y and solve for x or for y	**(M1**
		Obtain $x=-1$ or $y=1$	**A1**
		Obtain final answer −1 + I	**A1)**
			[3]
	(ii)	Obtain $u+v=1+2\text{i}$	**B1**
		In an Argand diagram show points A, B, C representing u, v and $u+v$ respectively	**B1**√
		State that OB and AC are parallel	**B1**
		State that $OB=AC$	**B1**
			[4]

(iii) Carry out an appropriate method for finding angle AOB, e.g. find $\arg(u/v)$ **M1**

Show sufficient working to justify the given answer $\frac{3}{4}\pi$ **A1**

[2]

10 **(i)** State or imply the form $\dfrac{A}{x+3}+\dfrac{B}{x-1}+\dfrac{C}{(x-1)^2}$ **B1**

Use a correct method to determine a constant **M1**

Obtain one of the values $A = -3$, $B = 1$, $C = 2$ **A1**

Obtain a second value **A1**

Obtain the third value **A1**

[Mark the form $\dfrac{A}{x+3}+\dfrac{Dx+E}{(x-1)^2}$, where $A = -3$, $D = 1$, $E = 1$, B1M1A1A1A1 as above.] [5]

(ii) Use a correct method to find the first two terms of the expansion of $(x+3)^{-1}$, $(1+\frac{1}{3}x)^{-1}$, $(x-1)^{-1}$, $(1-x)^{-1}$, $(x-1)^{-2}$, or $(1-x)^{-2}$ **M1**

Obtain correct unsimplified expressions up to the term in x^2 of each partial fraction **A1√ + A1√ + A1√**

Obtain final answer $\frac{10}{3}x+\frac{44}{9}x^2$, or equivalent **A1**

[5]

CAMBRIDGE INTERNATIONAL EXAMINATIONS

Cambridge International Advanced Subsidiary and Advanced Level

MARK SCHEME for the March 2016 series

9709 MATHEMATICS

9709/32 Paper 3 (Pure Mathematics), maximum raw mark 75

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

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CAMBRIDGE
International Examinations

Mark Scheme Notes

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A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

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- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously “correct” answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

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AEF	Any Equivalent Form (of answer is equally acceptable)
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SOS	See Other Solution (the candidate makes a better attempt at the same question)
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become “follow through √” marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

1		Use law of the logarithm of a power, quotient or product	**M1**	
		Remove logarithms and obtain a correct equation in x, e.g. $x^2+4=4x^2$	**A1**	
		Obtain final answer $x=2/\sqrt{3}$, or exact equivalent	**A1**	[3]
2		Use tan($A \pm B$) formula and obtain an equation in tan θ	**M1**	
		Using $\tan 45°=1$, obtain a horizontal equation in tan θ in any correct form	**A1**	
		Reduce the equation to $7\tan^2\theta-2\tan\theta-1=0$, or equivalent	**A1**	
		Solve a 3-term quadratic for tan θ	**M1**	
		Obtain a correct answer, e.g. $\theta=28.7°$	**A1**	
		Obtain a second answer, e.g. $\theta=165.4°$, and no others	**A1**	[6]
		[Ignore answers outside the given interval. Treat answers in radians as a misread (0.500, 2.89).]		
3	**(i)**	Consider sign of $x^5-3x^3+x^2-4$ at $x=1$ and $x=2$, or equivalent	**M1**	
		Complete the argument correctly with correct calculated values	**A1**	[2]
	(ii)	Rearrange the given quintic equation in the given form, or work *vice versa*	**B1**	[1]
	(iii)	Use the iterative formula correctly at least once	**M1**	
		Obtain final answer 1.78	**A1**	
		Show sufficient iterations to 4 d.p. to justify 1.78 to 2 d.p., or show there is a sign change in the interval (1.775, 1.785)	**A1**	[3]
4	**(i)**	Substitute $x=-\frac{1}{2}$ and equate to zero, or divide by $(2x+1)$ and equate constant remainder to zero	**M1**	
		Obtain $a=3$	**A1**	[2]
	(ii) (a)	Commence division by $(2x+1)$ reaching a partial quotient of $2x^2+kx$	**M1**	
		Obtain factorisation $(2x+1)(2x^2-x+2)$	**A1**	[2]
		[The M1 is earned if inspection reaches an unknown factor $2x^2+Bx+C$ and an equation in B and/or C, or an unknown factor Ax^2+Bx+2 and an equation in A and/or B.]		
	(b)	State or imply critical value $x=-\frac{1}{2}$	**B1**	
		Show that $2x^2-x+2$ is always positive, or that the gradient of $4x^3+3x+2$ is always positive	**B1***	
		Justify final answer $x>-\frac{1}{2}$	**B1(dep*)**	[3]
5	**(i)**	State or imply $\mathrm{d}x=\sqrt{3}\sec^2\theta\ \mathrm{d}\theta$	**B1**	
		Substitute for x and dx throughout	**M1**	
		Obtain the given answer correctly	**A1**	[3]

	(ii)	Replace integrand by $\frac{1}{2}\cos 2\theta + \frac{1}{2}$	**B1**	
		Obtain integral $\frac{1}{4}\sin 2\theta + \frac{1}{2}\theta$	**B1√**	
		Substitute limits correctly in an integral of the form $c\sin 2\theta + b\theta$, where $cb \neq 0$	**M1**	
		Obtain answer $\frac{1}{12}\sqrt{3}\pi + \frac{3}{8}$, or exact equivalent	**A1**	[4]
		[The f.t. is on integrands of the form $a\cos 2\theta + b$, where $ab \neq 0$.]		
6	**(i)**	*EITHER*: State correct derivative of sin y with respect to x	**B1**	
		Use product rule to differentiate the LHS	**M1**	
		Obtain correct derivative of the LHS	**A1**	
		Obtain a complete and correct derived equation in any form	**A1**	
		Obtain a correct expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ in any form	**A1**	
		OR: State correct derivative of sin y with respect to x	**B1**	
		Rearrange the given equation as $\sin y = x/(\ln x + 2)$ and attempt to differentiate both sides	**B1**	
		Use quotient or product rule to differentiate the RHS	**M1**	
		Obtain correct derivative of the RHS	**A1**	
		Obtain a correct expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ in any form	**A1**	[5]
	(ii)	Equate $\frac{\mathrm{d}y}{\mathrm{d}x}$ to zero and obtain a horizontal equation in ln x or sin y	**M1**	
		Solve for ln x	**M1**	
		Obtain final answer $x = 1/\mathrm{e}$, or exact equivalent	**A1**	[3]
7	**(i)**	Separate variables and attempt integration of one side	**M1**	
		Obtain term $-\mathrm{e}^{-y}$	**A1**	
		Integrate $x\mathrm{e}^x$ by parts reaching $x\mathrm{e}^x \pm \int \mathrm{e}^x \, \mathrm{d}x$	**M1**	
		Obtain integral $x\mathrm{e}^x - \mathrm{e}^x$	**A1**	
		Evaluate a constant, or use limits $x = 0, y = 0$	**M1**	
		Obtain correct solution in any form	**A1**	
		Obtain final answer $y = -\ln(\mathrm{e}^x(1-x))$, or equivalent	**A1**	[7]
	(ii)	Justify the given statement	**B1**	[1]

8	**(i)**	*EITHER*:	Substitute for **r** in the given equation of p and expand scalar product	**M1**	
			Obtain equation in λ in any correct form	**A1**	
			Verify this is not satisfied for any value of λ	**A1**	
		*OR*1:	Substitute coordinates of a general point of l in the Cartesian equation of plane p	**M1**	
			Obtain equation in λ in any correct form	**A1**	
			Verify this is not satisfied for any value of λ	**A1**	
		*OR*2:	Expand scalar product of the normal to p and the direction vector of l	**M1**	
			Verify scalar product is zero	**A1**	
			Verify that one point of l does not lie in the plane	**A1**	
		*OR*3:	Use correct method to find the perpendicular distance of a general point of l from p	**M1**	
			Obtain a correct unsimplified expression in terms of λ	**A1**	
			Show that the perpendicular distance is $5/\sqrt{6}$, or equivalent, for all λ	**A1**	
		*OR*4:	Use correct method to find the perpendicular distance of a particular point of l from p	**M1**	
			Show that the perpendicular distance is $5/\sqrt{6}$, or equivalent	**A1**	
			Show that the perpendicular distance of a second point is also $5/\sqrt{6}$, or equivalent	**A1**	[3]
	(ii)	*EITHER*:	Calling the unknown direction vector $a\mathbf{i}+b\mathbf{j}+c\mathbf{k}$ state equation $2a+b+3c=0$	**B1**	
			State equation $2a-b-c=0$	**B1**	
			Solve for one ratio, e.g. $a:b$	**M1**	
			Obtain ratio $a:b:c=1:4:-2$, or equivalent	**A1**	
		OR:	Attempt to calculate the vector product of the direction vector of l and the normal vector of the plane p, e.g. $(2\mathbf{i}+\mathbf{j}+3\mathbf{k})\times(2\mathbf{i}-\mathbf{j}-\mathbf{k})$	**M2**	
			Obtain two correct components of the product	**A1**	
			Obtain answer $2\mathbf{i}+8\mathbf{j}-4\mathbf{k}$, or equivalent	**A1**	
			Form line equation with relevant vectors	**M1**	
			Obtain answer $\mathbf{r}=5\mathbf{i}+3\mathbf{j}+\mathbf{k}+\mu(\mathbf{i}+4\mathbf{j}-2\mathbf{k})$, or equivalent	**A1**√	[6]
9	**(i)**		State or obtain $A=3$	**B1**	
			Use a relevant method to find a constant	**M1**	
			Obtain one of $B=-4$, $C=4$ and $D=0$	**A1**	
			Obtain a second value	**A1**	
			Obtain the third value	**A1**	[5]
	(ii)		Integrate and obtain $3x-4\ln x$	**B1**√	
			Integrate and obtain term of the form $k\ln(x^2+2)$	**M1**	
			Obtain term $2\ln(x^2+2)$	**A1**√	
			Substitute limits in an integral of the form $ax+b\ln x+c\ln(x^2+2)$, where $abc\neq 0$	**M1**	
			Obtain given answer $3-\ln 4$ after full and correct working	**A1**	[5]
10	**(a)**		Substitute and obtain a correct equation in x and y	**B1**	
			Use $\mathrm{i}^2=-1$ and equate real and imaginary parts	**M1**	
			Obtain two correct equations, e.g. $x+2y+1=0$ and $y+2x=0$	**A1**	
			Solve for x or for y	**M1**	
			Obtain answer $z=\frac{1}{3}-\frac{2}{3}\mathrm{i}$	**A1**	[5]

(b)	**(i)**	Show a circle with centre $-1+3\text{ i}$	**B1**	
		Show a circle with radius 1	**B1**	
		Show the line Im $z = 3$	**B1**	
		Shade the correct region	**B1**	[4]
	(ii)	Carry out a complete method to calculate the relevant angle	**M1**	
		Obtain answer 0.588 radians (accept 33.7°)	**A1**	[2]

CAMBRIDGE INTERNATIONAL EXAMINATIONS

Cambridge International Advanced Level

MARK SCHEME for the October/November 2015 series

9709 MATHEMATICS

9709/31 Paper 3, maximum raw mark 75

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CAMBRIDGE
International Examinations

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PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

1		*EITHER*: State or imply non-modular inequality $(2x-5)^2 > (3(2x+1))^2$, or corresponding quadratic equation, or pair of linear equations $(2x-5) = \pm 3(2x+1)$	**B1**	
		Make reasonable solution attempt at a 3-term quadratic, or solve two linear equations for x	**M1**	
		Obtain critical values -2 and $\frac{1}{4}$	**A1**	
		State final answer $-2 < x < \frac{1}{4}$	**A1**	
		OR: Obtain critical value $x = -2$ from a graphical method, or by inspection, or by solving a linear equation or inequality	**B1**	
		Obtain critical value $x = \frac{1}{4}$ similarly	**B2**	
		State final answer $-2 < x < \frac{1}{4}$	**B1**	[4]
		[Do not condone $\leqslant$ for $<$]		
2		State or imply $1 + u = u^2$	**B1**	
		Solve for u	**M1**	
		Obtain root $\frac{1}{2}(1+\sqrt{5})$, or decimal in [1.61, 1.62]	**A1**	
		Use correct method for finding x from a positive root	**M1**	
		Obtain $x = 0.438$ and no other answer	**A1**	[5]
3		Use $\tan(A \pm B)$ and obtain an equation in $\tan\theta$ and $\tan\phi$	**M1***	
		Substitute throughout for $\tan\theta$ or for $\tan\phi$	**dep M1***	
		Obtain $3\tan^2\theta - \tan\theta - 4 = 0$ or $3\tan^2\phi - 5\tan\phi - 2 = 0$, or 3-term equivalent	**A1**	
		Solve a 3-term quadratic and find an angle	**M1**	
		Obtain answer $\theta = 135°$, $\phi = 63.4°$	**A1**	
		Obtain answer $\theta = 53.1°$, $\phi = 161.6°$	**A1**	[6]
		[Treat answers in radians as a misread. Ignore answers outside the given interval.]		
		[SR: Two correct values of θ (or ϕ) score A1; then A1 for both correct θ, ϕ pairs.]		
4	**(i)**	Evaluate, or consider the sign of, $x^3 - x^2 - 6$ for two integer values of x, or equivalent	**M1**	
		Obtain the pair $x = 2$ and $x = 3$, with no errors seen	**A1**	[2]
	(ii)	State a suitable equation, e.g. $x = \sqrt{(x + (6/x))}$	**B1**	
		Rearrange this as $x^3 - x^2 - 6 = 0$, or work *vice versa*	**B1**	[2]
	(iii)	Use the iterative formula correctly at least once	**M1**	
		Obtain final answer 2.219	**A1**	
		Show sufficient iterates to 5 d.p. to justify 2.219 to 3 d.p., or show there is a sign change in the interval (2.2185, 2.2195)	**A1**	[3]

5	(i)	State or imply that the derivative of e^{-2x} is $-2e^{-2x}$	B1	
		Use product or quotient rule	M1	
		Obtain correct derivative in any form	A1	
		Use Pythagoras	M1	
		Justify the given form	A1	[5]
	(ii)	Fully justify the given statement	B1	[1]
	(iii)	State answer $x = \frac{1}{4}\pi$	B1	[1]
6	(i)	Substitute $x = -1$, equate to zero and simplify at least as far as $-8 + a - b - 1 = 0$	B1	
		Substitute $x = -\frac{1}{2}$ and equate the result to 1	M1	
		Obtain a correct equation in any form, e.g. $-1 + \frac{1}{4}a - \frac{1}{2}b - 1 = 1$	A1	
		Solve for a or for b	M1	
		Obtain $a = 6$ and $b = -3$	A1	[5]
	(ii)	Commence division by $(x + 1)$ reaching a partial quotient $8x^2 + kx$	M1	
		Obtain quadratic factor $8x^2 - 2x - 1$	A1	
		Obtain factorisation $(x + 1)(4x + 1)(2x - 1)$	A1	[3]
		[The M1 is earned if inspection reaches an unknown factor $8x^2 + Bx + C$ and an equation in B and/or C, or an unknown factor $Ax^2 + Bx - 1$ and an equation in A and/or B.] [If linear factors are found by the factor theorem, give B1B1 for $(2x - 1)$ and $(4x + 1)$, and B1 for the complete factorisation.]		
7	(i)	Use correct method to form a vector equation for AB	M1	
		Obtain a correct equation, e.g. $\mathbf{r} = \mathbf{i} + 2\mathbf{j} + \lambda(2\mathbf{i} - 2\mathbf{j} + \mathbf{k})$ or $\mathbf{r} = 3\mathbf{i} + \mathbf{k} + \mu(2\mathbf{i} - 2\mathbf{j} + \mathbf{k})$	A1	[2]
	(ii)	Using a direction vector for AB and a relevant point, obtain an equation for m in any form	M1	
		Obtain answer $2x - 2y + z = 4$, or equivalent	A1	[2]
	(iii)	Express general point of AB in component form, e.g. $(1 + 2\lambda, 2 - 2\lambda, \lambda)$ or $(3 + 2\mu, -2\mu, 1 + \mu)$	B1√	
		Substitute in equation of m and solve for λ or for μ	M1	
		Obtain final answer $\frac{7}{3}\mathbf{i} + \frac{2}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}$ for the position vector of N, from $\lambda = \frac{2}{3}$ or $\mu = -\frac{1}{3}$	A1	
		Carry out a correct method for finding CN	M1	
		Obtain the given answer $\sqrt{13}$	A1	[5]
		[The f.t. is on the direction vector for AB.]		

8		Separate variables and integrate one side	B1	
		Obtain term $\ln(x+2)$	B1	
		Use cos $2A$ formula to express $\sin^2 2\theta$ in the form $a + b\cos 4\theta$	M1	
		Obtain correct form $(1-\cos 4\theta)/2$, or equivalent	A1	
		Integrate and obtain term $\frac{1}{2}\theta - \frac{1}{8}\sin 4\theta$, or equivalent	A1√	
		Evaluate a constant, or use $\theta = 0$, $x = 0$ as limits in a solution containing terms $c\ln(x+2), d\sin(4\theta), e\theta$	M1	
		Obtain correct solution in any form, e.g. $\ln(x+2) = \frac{1}{2}\theta - \frac{1}{8}\sin 4\theta + \ln 2$	A1	
		Use correct method for solving an equation of the form $\ln(x+2) = f$	M1	
		Obtain answer $x = 0.962$	A1	[9]
9	(i)	Show u in a relatively correct position	B1	
		Show u^* in a relatively correct position	B1	
		Show $u^* - u$ in a relatively correct position	B1	
		State or imply that *OABC* is a parallelogram	B1	[4]
	(ii)	*EITHER*: Substitute for u and multiply numerator and denominator by 3 + i, or equivalent	M1	
		Simplify the numerator to 8 + 6i or the denominator to 10	A1	
		Obtain final answer $\frac{4}{5} + \frac{3}{5}\mathrm{i}$, or equivalent	A1	
		OR: Substitute for u, obtain two equations in x and y and solve for x or for y	M1	
		Obtain $x = \frac{4}{5}$ or $y = \frac{3}{5}$, or equivalent	A1	
		Obtain final answer $\frac{4}{5} + \frac{3}{5}\mathrm{i}$, or equivalent	A1	[3]
	(iii)	State or imply $\arg(u^*/u) = \tan^{-1}(\frac{3}{4})$	B1	
		Substitute exact arguments in $\arg(u^*/u) = \arg u^* - \arg u$	M1	
		Fully justify the given statement using exact values	A1	[3]
10	(i)	Use the quotient rule	M1	
		Obtain correct derivative in any form	A1	
		Equate derivative to zero and solve for x	M1	
		Obtain answer $x = \sqrt[3]{2}$, or exact equivalent	A1	[4]
	(ii)	State or imply indefinite integral is of the form $k\ln(1+x^3)$	M1	
		State indefinite integral $\frac{1}{3}\ln(1+x^3)$	A1	
		Substitute limits correctly in an integral of the form $k\ln(1+x^3)$	M1	
		State or imply that the area of R is equal to $\frac{1}{3}\ln(1+p^3) - \frac{1}{3}\ln 2$, or equivalent	A1	
		Use a correct method for finding p from an equation of the form $\ln(1+p^3) = a$ or $\ln((1+p^3)/2) = b$	M1	
		Obtain answer $p = 3.40$	A1	[2]

CAMBRIDGE INTERNATIONAL EXAMINATIONS

Cambridge International Advanced Level

MARK SCHEME for the October/November 2015 series

9709 MATHEMATICS

9709/32 Paper 3, maximum raw mark 75

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- When a part of a question has two or more “method” steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

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- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

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The following abbreviations may be used in a mark scheme or used on the scripts:

AEF	Any Equivalent Form (of answer is equally acceptable)
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PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

1		*EITHER*: State or imply non-modular inequality $(2x-5)^2 > (3(2x+1))^2$, or corresponding quadratic equation, or pair of linear equations $(2x-5) = \pm 3(2x+1)$	**B1**	
		Make reasonable solution attempt at a 3-term quadratic, or solve two linear equations for x	**M1**	
		Obtain critical values -2 and $\frac{1}{4}$	**A1**	
		State final answer $-2 < x < \frac{1}{4}$	**A1**	
		OR: Obtain critical value $x = -2$ from a graphical method, or by inspection, or by solving a linear equation or inequality	**B1**	
		Obtain critical value $x = \frac{1}{4}$ similarly	**B2**	
		State final answer $-2 < x < \frac{1}{4}$	**B1**	[4]
		[Do not condone $\leqslant$ for $<$]		
2		State or imply $1 + u = u^2$	**B1**	
		Solve for u	**M1**	
		Obtain root $\frac{1}{2}(1+\sqrt{5})$, or decimal in [1.61, 1.62]	**A1**	
		Use correct method for finding x from a positive root	**M1**	
		Obtain $x = 0.438$ and no other answer	**A1**	[5]
3		Use $\tan(A \pm B)$ and obtain an equation in $\tan\theta$ and $\tan\phi$	**M1***	
		Substitute throughout for $\tan\theta$ or for $\tan\phi$	**dep M1***	
		Obtain $3\tan^2\theta - \tan\theta - 4 = 0$ or $3\tan^2\phi - 5\tan\phi - 2 = 0$, or 3-term equivalent	**A1**	
		Solve a 3-term quadratic and find an angle	**M1**	
		Obtain answer $\theta = 135°$, $\phi = 63.4°$	**A1**	
		Obtain answer $\theta = 53.1°$, $\phi = 161.6°$	**A1**	[6]
		[Treat answers in radians as a misread. Ignore answers outside the given interval.]		
		[SR: Two correct values of θ (or ϕ) score A1; then A1 for both correct θ, ϕ pairs.]		
4	**(i)**	Evaluate, or consider the sign of, $x^3 - x^2 - 6$ for two integer values of x, or equivalent	**M1**	
		Obtain the pair $x = 2$ and $x = 3$, with no errors seen	**A1**	[2]
	(ii)	State a suitable equation, e.g. $x = \sqrt{(x + (6/x))}$	**B1**	
		Rearrange this as $x^3 - x^2 - 6 = 0$, or work *vice versa*	**B1**	[2]
	(iii)	Use the iterative formula correctly at least once	**M1**	
		Obtain final answer 2.219	**A1**	
		Show sufficient iterates to 5 d.p. to justify 2.219 to 3 d.p., or show there is a sign change in the interval (2.2185, 2.2195)	**A1**	[3]

5	(i)	State or imply that the derivative of e^{-2x} is $-2e^{-2x}$	B1	
		Use product or quotient rule	M1	
		Obtain correct derivative in any form	A1	
		Use Pythagoras	M1	
		Justify the given form	A1	[5]
	(ii)	Fully justify the given statement	B1	[1]
	(iii)	State answer $x = \frac{1}{4}\pi$	B1	[1]
6	(i)	Substitute $x = -1$, equate to zero and simplify at least as far as $-8 + a - b - 1 = 0$	B1	
		Substitute $x = -\frac{1}{2}$ and equate the result to 1	M1	
		Obtain a correct equation in any form, e.g. $-1 + \frac{1}{4}a - \frac{1}{2}b - 1 = 1$	A1	
		Solve for *a* or for *b*	M1	
		Obtain $a = 6$ and $b = -3$	A1	[5]
	(ii)	Commence division by $(x + 1)$ reaching a partial quotient $8x^2 + kx$	M1	
		Obtain quadratic factor $8x^2 - 2x - 1$	A1	
		Obtain factorisation $(x+1)(4x+1)(2x-1)$	A1	[3]
		[The M1 is earned if inspection reaches an unknown factor $8x^2 + Bx + C$ and an equation in *B* and/or *C*, or an unknown factor $Ax^2 + Bx - 1$ and an equation in *A* and/or *B*.] [If linear factors are found by the factor theorem, give B1B1 for $(2x - 1)$ and $(4x + 1)$, and B1 for the complete factorisation.]		
7	(i)	Use correct method to form a vector equation for *AB*	M1	
		Obtain a correct equation, e.g. $\mathbf{r} = \mathbf{i} + 2\mathbf{j} + \lambda(2\mathbf{i} - 2\mathbf{j} + \mathbf{k})$ or $\mathbf{r} = 3\mathbf{i} + \mathbf{k} + \mu(2\mathbf{i} - 2\mathbf{j} + \mathbf{k})$	A1	[2]
	(ii)	Using a direction vector for *AB* and a relevant point, obtain an equation for *m* in any form	M1	
		Obtain answer $2x - 2y + z = 4$, or equivalent	A1	[2]
	(iii)	Express general point of *AB* in component form, e.g. $(1 + 2\lambda, 2 - 2\lambda, \lambda)$ or $(3 + 2\mu, -2\mu, 1 + \mu)$	B1√	
		Substitute in equation of *m* and solve for λ or for μ	M1	
		Obtain final answer $\frac{7}{3}\mathbf{i} + \frac{2}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}$ for the position vector of *N*, from $\lambda = \frac{2}{3}$ or $\mu = -\frac{1}{3}$	A1	
		Carry out a correct method for finding *CN*	M1	
		Obtain the given answer $\sqrt{13}$	A1	[5]
		[The f.t. is on the direction vector for *AB*.]		

8		Separate variables and integrate one side	**B1**	
		Obtain term $\ln(x+2)$	**B1**	
		Use cos 2*A* formula to express $\sin^2 2\theta$ in the form $a + b\cos 4\theta$	**M1**	
		Obtain correct form $(1-\cos 4\theta)/2$, or equivalent	**A1**	
		Integrate and obtain term $\frac{1}{2}\theta - \frac{1}{8}\sin 4\theta$, or equivalent	**A1√**	
		Evaluate a constant, or use $\theta = 0$, $x = 0$ as limits in a solution containing terms $c\ln(x+2), d\sin(4\theta), e\theta$	**M1**	
		Obtain correct solution in any form, e.g. $\ln(x+2) = \frac{1}{2}\theta - \frac{1}{8}\sin 4\theta + \ln 2$	**A1**	
		Use correct method for solving an equation of the form $\ln(x+2) = f$	**M1**	
		Obtain answer $x = 0.962$	**A1**	[9]
9	**(i)**	Show u in a relatively correct position	**B1**	
		Show u^* in a relatively correct position	**B1**	
		Show $u^* - u$ in a relatively correct position	**B1**	
		State or imply that *OABC* is a parallelogram	**B1**	[4]
	(ii)	*EITHER*: Substitute for u and multiply numerator and denominator by 3 + i, or equivalent	**M1**	
		Simplify the numerator to 8 + 6i or the denominator to 10	**A1**	
		Obtain final answer $\frac{4}{5} + \frac{3}{5}\mathrm{i}$, or equivalent	**A1**	
		OR: Substitute for u, obtain two equations in x and y and solve for x or for y	**M1**	
		Obtain $x = \frac{4}{5}$ or $y = \frac{3}{5}$, or equivalent	**A1**	
		Obtain final answer $\frac{4}{5} + \frac{3}{5}\mathrm{i}$, or equivalent	**A1**	[3]
	(iii)	State or imply $\arg(u^*/u) = \tan^{-1}(\frac{3}{4})$	**B1**	
		Substitute exact arguments in $\arg(u^*/u) = \arg u^* - \arg u$	**M1**	
		Fully justify the given statement using exact values	**A1**	[3]
10	**(i)**	Use the quotient rule	**M1**	
		Obtain correct derivative in any form	**A1**	
		Equate derivative to zero and solve for x	**M1**	
		Obtain answer $x = \sqrt[3]{2}$, or exact equivalent	**A1**	[4]
	(ii)	State or imply indefinite integral is of the form $k\ln(1+x^3)$	**M1**	
		State indefinite integral $\frac{1}{3}\ln(1+x^3)$	**A1**	
		Substitute limits correctly in an integral of the form $k\ln(1+x^3)$	**M1**	
		State or imply that the area of *R* is equal to $\frac{1}{3}\ln(1+p^3) - \frac{1}{3}\ln 2$, or equivalent	**A1**	
		Use a correct method for finding p from an equation of the form $\ln(1+p^3) = a$ or $\ln((1+p^3)/2) = b$	**M1**	
		Obtain answer $p = 3.40$	**A1**	[2]

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9709/33 Paper 3, maximum raw mark 75

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PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

1 Draw curve with increasing gradient existing for negative and positive values of x **M1**

Draw correct curve passing through the origin **A1** [2]

2 Either State correct unsimplified x^2 or x^3 term **M1**
Obtain $a = -9$ **A1**
Obtain $b = 45$ **A1**

Or Use chain rule to differentiate twice to obtain form $k(1+9x)^{-\frac{5}{3}}$ **M1**

Obtain $f''(x) = -18(1+9x)^{-\frac{5}{3}}$ and hence $a = -9$ **A1**

Obtain $f'''(x) = 270(1+9x)^{-\frac{8}{3}}$ and hence $b = 45$ **A1** [3]

3 Use correct quotient rule or equivalent to find first derivative **M1***

Obtain $\frac{-(1+\tan x)\sec^2 x - \sec^2 x(2-\tan x)}{(1+\tan x)^2}$ or equivalent **A1**

Substitute $x = \frac{1}{4}\pi$ to find gradient **dep M1***

Obtain $-\frac{3}{2}$ **A1**

Form equation of tangent at $x = \frac{1}{4}\pi$ **M1**

Obtain $y = -\frac{3}{2}x + 1.68$ or equivalent **A1** [6]

4 **(i)** Use $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\dot{y}}{\dot{x}}$ and equate $\frac{\mathrm{d}y}{\mathrm{d}x}$ to 4 **M1**

Obtain $\frac{4p^3}{2p+3} = 4$ or equivalent **A1**

Confirm given result $p = \sqrt[3]{2p+3}$ correctly **A1** [3]

(ii) Evaluate $p - \sqrt[3]{2p+3}$ or $p^3 - 2p - 3$ or equivalent at 1.8 and 2.0 **M1**
Justify result with correct calculations and argument
(–0.076 and 0.087 or –0.77 and 1 respectively) **A1** [2]

(iii) Use the iterative process correctly at least once with $1.8 \leqslant p_n \leqslant 2.0$ **M1**
Obtain final answer 1.89 **A1**
Show sufficient iterations to at least 4 d.p. to justify 1.89 or show sign change in interval (1.885, 1.895) **A1** [3]

5		State $\mathrm{d}u = 3\sin x\,\mathrm{d}x$ or equivalent	**B1**	
		Use identity $\sin 2x = 2\sin x\cos x$	**B1**	
		Carry out complete substitution, for x and $\mathrm{d}x$	**M1**	
		Obtain $\int \frac{8-2u}{\sqrt{u}}\,\mathrm{d}u$, or equivalent	**A1**	
		Integrate to obtain expression of form $au^{\frac{1}{2}} + bu^{\frac{3}{2}}$, $ab \neq 0$	**M1***	
		Obtain correct $16u^{\frac{1}{2}} - \frac{4}{3}u^{\frac{3}{2}}$	**A1**	
		Apply correct limits correctly	**dep M1***	
		Obtain $\frac{20}{3}$ or exact equivalent	**A1**	[8]
6		State or imply $\sin A \times \cos 45 + \cos A \times \sin 45 = 2\sqrt{2}\cos A$	**B1**	
		Divide by $\cos A$ to find value of $\tan A$	**M1**	
		Obtain $\tan A = 3$	**A1**	
		Use identity $\sec^2 B = 1 + \tan^2 B$	**B1**	
		Solve three-term quadratic equation and find $\tan B$	**M1**	
		Obtain $\tan B = \frac{3}{2}$ only	**A1**	
		Substitute **numerical values** in $\frac{\tan A - \tan B}{1 + \tan A \tan B}$	**M1**	
		Obtain $\frac{3}{11}$	**A1**	[8]
7	**(i)**	Either Substitute $x = -1$ and evaluate	**M1**	
		Obtain 0 and conclude $x+1$ is a factor	**A1**	
		Or Divide by $x+1$ and obtain a constant remainder	**M1**	
		Obtain remainder = 0 and conclude $x+1$ is a factor	**A1**	[2]
	(ii)	Attempt division, or equivalent, at least as far as quotient $4x^2 + kx$	**M1**	
		Obtain complete quotient $4x^2 - 5x - 6$	**A1**	
		State form $\frac{A}{x+1} + \frac{B}{x-2} + \frac{C}{4x+3}$	**A1**	
		Use relevant method for finding at least one constant	**M1**	
		Obtain one of $A = -2, B = 1, C = 8$	**A1**	
		Obtain all three values	**A1**	
		Integrate to obtain three terms each involving natural logarithm of linear form	**M1**	
		Obtain $-2\ln(x+1) + \ln(x-2) + 2\ln(4x+3)$, condoning no use of modulus signs and absence of … + c	**A1**	[8]

8	**(i)**		Express a general point on the line in single component form, e.g. $(\lambda, 2-3\lambda, -8+4\lambda)$, substitute in equation of plane and solve for λ	**M1**	
			Obtain $\lambda = 3$	**A1**	
			Obtain $(3, -7, 4)$	**A1**	[3]
	(ii)		State or imply normal vector to plane is 4**i** – **j** + 5**k**	**B1**	
			Carry out process for evaluating scalar product of two relevant vectors	**M1**	
			Using the correct process for the moduli, divide the scalar product by the product of the moduli and evaluate $\sin^{-1}$ or $\cos^{-1}$ of the result.	**M1**	
			Obtain $54.8°$ or 0.956 radians	**A1**	[4]
	(iii)	<u>Either</u>	Find at least one position of *C* by translating by appropriate multiple of direction vector **i** – 3**j** + 4**k** from *A* or *B*	**M1**	
			Obtain $(-3, 11, -20)$	**A1**	
			Obtain $(9, -25, 28)$	**A1**	
		<u>Or</u>	Form quadratic equation in λ by considering $BC^2 = 4AB^2$	**M1**	
			Obtain $26\lambda^2 - 156\lambda - 702 = 0$ or equivalent and hence $\lambda = -3, \lambda = 9$	**A1**	
			Obtain $(-3, 11, -20)$ and $(9, -25, 28)$	**A1**	[3]
9	**(a)**	<u>Either</u>	Find *w* using conjugate of $1+3\text{i}$	**M1**	
			Obtain $\frac{7-\text{i}}{5}$ or equivalent	**A1**	
			Square $x+\text{i}y$ form to find w^2	**M1**	
			Obtain $w^2 = \frac{48-14\text{i}}{25}$ and confirm modulus is 2	**A1**	
			Use correct process for finding argument of w^2	**M1**	
			Obtain –0.284 radians or $-16.3°$	**A1**	
		<u>Or 1</u>	Find *w* using conjugate of $1+3\text{i}$	**M1**	
			Obtain $\frac{7-\text{i}}{5}$ or equivalent	**A1**	
			Find modulus of *w* and hence of w^2	**M1**	
			Confirm modulus is 2	**A1**	
			Find argument of *w* and hence of w^2	**M1**	
			Obtain –0.284 radians or $-16.3°$	**A1**	
		<u>Or 2</u>	Square both sides to obtain $(-8+6\text{i})w^2 = -12+16\text{i}$	**B1**	
			Find w^2 using relevant conjugate	**M1**	
			Use correct process for finding modulus of w^2	**M1**	
			Confirm modulus is 2	**A1**	
			Use correct process for finding argument of w^2	**M1**	
			Obtain –0.284 radians or $-16.3°$	**A1**	

	Or 3	Find modulus of LHS and RHS	**M1**	
		Find argument of LHS and RHS	**M1**	
		Obtain $\sqrt{10}\,\mathrm{e}^{1.249\mathrm{i}}\ w=\sqrt{20}\,\mathrm{e}^{1.107\mathrm{i}}$ or equivalent	**A1**	
		Obtain $w=\sqrt{2}\,\mathrm{e}^{-0.1419\mathrm{i}}$ or equivalent	**A1**	
		Use correct process for finding w^2	**M1**	
		Obtain 2 and –0.284 radians or $-16.3°$	**A1**	
	Or 4	Find moduli of $2+4\mathrm{i}$ and $1+3\mathrm{i}$	**M1**	
		Obtain $\sqrt{20}$ and $\sqrt{10}$	**A1**	
		Obtain $\lvert w^2\rvert=2$ correctly	**A1**	
		Find $\arg(2+4\mathrm{i})$ and $\arg(1+3\mathrm{i})$	**M1**	
		Use correct process for $\arg(w^2)$	**A1**	
		Obtain –0.284 radians or $-16.3°$	**A1**	
	Or 5	Let $w=a+\mathrm{i}b$, form and solve simultaneous equations in a and b	**M1**	
		$a=\frac{7}{5}$ and $b=-\frac{1}{5}$	**A1**	
		Find modulus of w and hence of w^2	**M1**	
		Confirm modulus is 2	**A1**	
		Find argument of w and hence of w^2	**M1**	
		Obtain –0.284 radians or $-16.3°$	**A1**	
	Or 6	Find w using conjugate of $1+3\mathrm{i}$	**M1**	
		Obtain $\frac{7-\mathrm{i}}{5}$ or equivalent	**A1**	
		Use $\lvert w^2\rvert=w\overline{w}$	**M1**	
		Confirm modulus is 2	**A1**	
		Find argument of w and hence of w^2	**M1**	
		Obtain –0.284 radians or $-16.3°$	**A1**	[6]
(b)	Draw circle with centre the origin and radius 5		**B1**	
	Draw straight line parallel to imaginary axis in correct position		**B1**	
	Use relevant trigonometry on a correct diagram to find argument(s)		**M1**	
	Obtain $5\mathrm{e}^{\pm\frac{1}{3}\pi\mathrm{i}}$ or equivalents in required form		**A1**	[4]

10	**(i)**	State $\frac{dN}{dt} = k(N-150)$	**B1**	[1]
	(ii)	Substitute $\frac{dN}{dt} = 60$ and $N = 900$ to find value of k	**M1**	
		Obtain $k = 0.08$	**A1**	
		Separate variables and obtain general solution involving $\ln(N-150)$	**M1***	
		Obtain $\ln(N-150) = 0.08t + c$ (following their k) or $\ln(N-150) = kt + c$	**A1**√	
		Substitute $t = 0$ and $N = 650$ to find c	**dep M1***	
		Obtain $\ln(N-150) = 0.08t + \ln 500$ or equivalent	**A1**	
		Obtain $N = 500e^{0.08t} + 150$	**A1**	[7]
	(iii)	<u>Either</u> Substitute $t = 15$ to find N <u>or</u> solve for t with $N = 2000$	**M1**	
		Obtain <u>Either</u> $N = 1810$ <u>or</u> $t = 16.4$ and conclude target not met	**A1**	[2]

CAMBRIDGE INTERNATIONAL EXAMINATIONS

Cambridge International Advanced Level

MARK SCHEME for the May/June 2015 series

9709 MATHEMATICS

9709/31 Paper 3 (Paper 3), maximum raw mark 75

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CAMBRIDGE
International Examinations

Mark Scheme Notes

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A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more “method” steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously “correct” answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

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AEF	Any Equivalent Form (of answer is equally acceptable)
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ISW	Ignore Subsequent Working
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SOS	See Other Solution (the candidate makes a better attempt at the same question)
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become “follow through √” marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

1	Use law for the logarithm of a power at least once	*M1	
	Obtain correct linear equation, e.g. $5x\text{In}2 = (2x+1)\text{In}3$	A1	
	Solve a linear equation for x	M1 dep *M	
	Obtain $x = 0.866$	A1	[4]
2	Attempt calculation of at least 3 ordinates	M1	
	Obtain 9, 7, 1, 17	A1	
	Use trapezium rule with $h = 1$	M1	
	Obtain $\frac{1}{2}\left(9+14+2+17\right)$ or equivalent and hence 21	A1	[4]
3	Either Obtain correct (unsimplified) version of x^2 or x^4 term in $(1-2x^2)^{-2}$	M1	
	Obtain $1+4x^2$	A1	
	Obtain … $+12x^4$	A1	
	Obtain correct (unsimplified) version of x^2 or x^4 term in $(1+6x^2)^{\frac{2}{3}}$	M1	
	Obtain $1+4x^2-4x^4$	A1	
	Combine expansions to obtain $k = 16$ with no error seen	A1	
	Or Obtain correct (unsimplified) version of x^2 or x^4 term in $(1+6x^2)^{\frac{2}{3}}$	M1	
	Obtain $1+4x^2$	A1	
	Obtain … $-4x^4$	A1	
	Obtain correct (unsimplified) version of x^2 or x^4 term in $(1-2x^2)^{-2}$	M1	
	Obtain $1+4x^2+12x^4$	A1	
	Combine expansions to obtain $k = 16$ with no error seen	A1	[6]
4	Differentiate to obtain form $a\sin 2x + b\cos x$	M1	
	Obtain correct $-6\sin 2x + 7\cos x$	A1	
	Use identity $\sin 2x = 2\sin x\cos x$	B1	
	Solve equation of form $c\sin x\cos x + d\cos x = 0$ to find at least one value of x	M1	
	Obtain 0.623	A1	
	Obtain 2.52	A1	
	Obtain 1.57 or $\frac{1}{2}\pi$ from equation of form $c\sin x\cos x + d\cos x = 0$	A1	
	Treat answers in degrees as MR – 1 situation		[7]

5	(a)	Use identity $\tan^2 2x = \sec^2 2x - 1$	B1	
		Obtain integral of form $ax + b\tan 2x$	M1	
		Obtain correct $3x + \frac{1}{2}\tan 2x$, condoning absence of + c	A1	[3]
	(b)	State $\sin x\cos\frac{1}{2}\pi + \cos\ x\ \sin\frac{1}{6}\pi$	B1	
		Simplify integrand to $\cos\frac{1}{6}\pi + \frac{\cos x\sin\frac{1}{6}\pi}{\sin x}$ or equivalent	B1	
		Integrate to obtain at least term of form $a\,\text{In}(\sin x)$	*M1	
		Apply limits and simplify to obtain two terms	M1 dep *M	
		Obtain $\frac{1}{8}\pi\sqrt{3} - \frac{1}{2}\ln(\frac{1}{\sqrt{2}})$ or equivalent	A1	[5]
6	(i)	Obtain $\pm\begin{pmatrix}2\\-3\\-4\end{pmatrix}$ as direction vector of l_1	B1	
		State that two direction vectors are not parallel	B1	
		Express general point of l_1 or l_2 in component form, e.g. $(2\lambda, 1-3\lambda, 5-4\lambda)$ or $(7+\mu, l+2\mu, 1+5\mu)$	B1	
		Equate at least two pairs of components and solve for λ or for μ	M1	
		Obtain correct answers for λ and μ	A1	
		Verify that all three component equations are not satisfied (with no errors seen)	A1	[6]
	(ii)	Carry out correct process for evaluating scalar product of $\begin{pmatrix}1\\2\\5\end{pmatrix}$ and $\begin{pmatrix}1\\0\\0\end{pmatrix}$	M1	
		Use correct process for finding modulus and evaluating inverse cosine	M1	
		Obtain 79.5° or 1.39 radians	A1	[3]
7		Separate variables and factorise to obtain $\frac{\mathrm{d}y}{(3y+1)(y+3)} = 4x\,\mathrm{d}x$ or equivalent	B1	
		State or imply the form $\frac{A}{3y+1} + \frac{B}{y+3}$ and use a relevant method to find A or B	M1	
		Obtain $A = \frac{3}{8}$ and $B = -\frac{1}{8}$	A1	
		Integrate to obtain form $k_1\ln(3y+1) + k_2\ln(y+3)$	M1	
		Obtain correct $\frac{1}{8}\ln(3y+1) - \frac{1}{8}\ln(y=3) = 2x^2$ or equivalent	A1	
		Substitute $x = 0$ and $y = 1$ in equation of form $k_1\ln(3y+1) + k_2\ln(y+3) = k_3x^2 + c$ to find a value of c	M1	
		Obtain $c = 0$	A1	
		Use correct process to obtain equation without natural logarithm present	M1	
		Obtain $y = \frac{3e^{16x^2} - 1}{3 - e^{16x^2}}$ or equivalent	A1	[9]

8	(i)	Either Expand $(2-\mathrm{i})^2$ to obtain $3-4\mathrm{i}$ or unsimplified equivalent	B1	
		Multiply by $\frac{3+4\mathrm{i}}{3+4\mathrm{i}}$ and simplify to $x+\mathrm{i}y$ form or equivalent	M1	
		Confirm given answer $2+4\mathrm{i}$	A1	
		Or Expand $(2-\mathrm{i})^2$ to obtain $3-4\mathrm{i}$ or unsimplified equivalent	B1	
		Obtain two equations in x and y and solve for x or y	M1	
		Confirm given answer $2+4\mathrm{i}$	A1	[3]
	(ii)	Identify $4+4$ or $-4+4\mathrm{i}$ as point at either end or state $p=2$ or state $p=-6$	B1	
		Use appropriate method to find both critical values of p	M1	
		State $-6 \leqslant p \leqslant 2$	A1	[3]
	(iii)	Identify equation as of form $\lvert z-a \rvert = a$ or equivalent	M1	
		Form correct equation for a not involving modulus, e.g. $(a-2)^2+4^2=a^2$	A1	
		State $\lvert z-5 \rvert = 5$	A1	[3]
9	(i)	Use product rule to find first derivative	M1	
		Obtain $2x\mathrm{e}^{2-x} - x^2\mathrm{e}^{2-x}$	A1	
		Confirm $x=2$ at M	A1	[3]
	(ii)	Attempt integration by parts and reach $\pm x^2\mathrm{e}^{2-x} \pm \int 2x\mathrm{e}^{2-x}\,\mathrm{d}x$	*M1	
		Obtain $-x^2\mathrm{e}^{2-x} + \int 2x\mathrm{e}^{2-x}\,\mathrm{d}x$	A1	
		Attempt integration by parts and reach $\pm x^2\mathrm{e}^{2-x} \pm 2x\mathrm{e}^{2-x} \pm 2\mathrm{e}^{2-x}$	*M1	
		Obtain $-x^2\mathrm{e}^{2-x} - 2x\mathrm{e}^{2-x} - 2\mathrm{e}^{2-x}$	A1	
		Use limits 0 and 2 having integrated twice	M1 dep *M	
		Obtain $2\mathrm{e}^2 - 10$	A1	[6]
10	(i)	Obtain $\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{2}{t+2}$ and $\frac{\mathrm{d}y}{\mathrm{d}t} = 3t^2+2$	B1	
		Use $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}t} \div \frac{\mathrm{d}x}{\mathrm{d}t}$	M1	
		Obtain $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{2}(3t^2+2)(t+2)$	A1	
		Identify value of t at the origin as –1	B1	
		Substitute to obtain $\frac{5}{2}$ as gradient at the origin	A1	[5]
	(ii)	(a) Equate derivative to $\frac{1}{2}$ and confirm $p = \frac{1}{3p^2+2} - 2$	B1	[1]
		(b) Use the iterative formula correctly at least once	M1	
		Obtain value $p=-1.924$ or better (–1.92367…)	A1	
		Show sufficient iterations to justify accuracy or show a sign change in appropriate interval	A1	
		Obtain coordinates (–5.15, –7.97)	A1	[4]

CAMBRIDGE INTERNATIONAL EXAMINATIONS

Cambridge International Advanced Subsidiary and Advanced Level

MARK SCHEME for the May/June 2015 series

9709 MATHEMATICS

9709/32 Paper 3, maximum raw mark 75

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CAMBRIDGE
International Examinations

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PA – 1	This is deducted from A or B marks in the case of premature approximation. The PA – 1 penalty is usually discussed at the meeting.

1		State or imply ordinates 0, 0.405465…, 0.623810…, 0.693147…	B1	
		Use correct formula, or equivalent, with $h = \frac{1}{6}\pi$ and four ordinates	M1	
		Obtain answer 0.72	A1	**[3]**
2		Use laws of indices correctly and solve for u	M1	
		Obtain u in any correct form, e.g. $u = \frac{16}{16-1}$	A1	
		Use correct method for solving an equation of the form $4^x = a$, where $a > 0$	M1	
		Obtain answer $x = 0.0466$	A1	**[4]**
3	EITHER:	Use correct product rule	M1	
		Obtain correct derivative in any form, e.g. $-\sin x\cos 2x - 2\cos x \sin 2x$	A1	
		Use the correct double angle formulae to express derivative in cos x and sin x, or cos 2x and sin x	M1	
	*OR*1:	Use correct double angle formula to express y in terms of cos x and attempt differentiation	M1	
		Use chain rule correctly	M1	
		Obtain correct derivative in any form, e.g. $-6\cos^2 x \sin x + \sin x$	A1	
	*OR*2:	Use correct factor formula and attempt differentiation	M1	
		Obtain correct derivative in any form, e.g. $-\frac{3}{2}\sin 3x - \frac{1}{2}\sin x$	A1	
		Use correct trig formulae to express derivative in terms of cos x and sin x, or sin x	M1	
		Equate derivative to zero and obtain an equation in one trig function	M1	
		Obtain $6\cos^2 x = 1$, $6\sin^2 x = 5$, $\tan^2 x = 5$ or $3\cos 2x = -2$	A1	
		Obtain answer $x = 1.15$ (or 65.9°) and no other in the given interval	A1	**[6]**
		[Ignore answers outside the given interval.]		
		[SR: Solution attempts following the *EITHER* scheme for the first two marks can earn the second and third method marks as follows:		
		Equate derivative to zero and obtain an equation in tan 2x and tan x	M1	
		Use correct double angle formula to obtain an equation in tan x	M1]	
4	**(i)**	State $R = \sqrt{13}$	B1	
		Use trig formula to find α	M1	
		Obtain $\alpha = 33.69°$ with no errors seen	A1	**[3]**
	(ii)	Evaluate $\sin^{-1}(1/\sqrt{13})$ to at least 1 d.p. (16.10° to 2 d.p)	B1$\surd$	
		Carry out an appropriate method to find a value of θ in the interval $0° < \theta < 180°$	M1	
		Obtain answer $\theta = 130.2°$ and no other in the given interval	A1	**[3]**
		[Ignore answers outside the given interval.]		
		[Treat answers in radians as a misread and deduct A1 from the marks for the angles.]		
5	**(i)**	State or imply $AT = r\tan x$ or $BT = r\tan x$	B1	
		Use correct arc formula and form an equation in r and x	M1	
		Rearrange in the given form	A1	**[3]**
	(ii)	Calculate values of a relevant expression or expressions at $x = 1$ and $x = 1.3$	M1	
		Complete the argument correctly with correct calculated values	A1	**[2]**

	(iii)	Use the iterative formula correctly at least once	M1	
		Obtain final answer 1.11	A1	
		Show sufficient iterations to 4 d.p. to justify 1.11 to 2 d.p., or show there is a sign change in the interval (1.105, 1.115)	A1	**[3]**
6	**(i)**	State or imply $\mathrm{d}u = -\dfrac{1}{2\sqrt{x}}\mathrm{d}x$, or equivalent	B1	
		Substitute for x and dx throughout	M1	
		Obtain integrand $\dfrac{\pm 2(2-u)^2}{u}$, or equivalent	A1	
		Show correct working to justify the change in limits and obtain the given answer with no errors seen	A1	**[4]**
	(ii)	Integrate and obtain at least two terms of the form $a\ln u, bu,$ and cu^2	M1*	
		Obtain indefinite integral $8\ln u - 8u + u^2$, or equivalent	A1	
		Substitute limits correctly	M1(dep*)	
		Obtain the given answer correctly having shown sufficient working	A1	**[4]**
7	**(i)**	Square $x + \mathrm{i}y$ and equate real and imaginary parts to -1 and $4\sqrt{3}$	M1	
		Obtain $x^2 - y^2 = -1$ and $2xy = 4\sqrt{3}$	A1	
		Eliminate one unknown and find an equation in the other	M1	
		Obtain $x^4 + x^2 - 12 = 0$ or $y^4 - y^2 - 12 = 0$, or three term equivalent	A1	
		Obtain answers $\pm(\sqrt{3} + 2\mathrm{i})$	A1	**[5]**
		[If the equations are solved by inspection, give B2 for the answers and B1 for justifying them]		
	(ii)	Show a circle with centre $-1 + 4\sqrt{3}$ in a relatively correct position	B1	
		Show a circle with radius 1 and centre not at the origin	B1	
		Carry out a complete method for calculating the greatest value of arg z	M1	
		Obtain answer 1.86 or 106.4°	A1	**[4]**
8	**(i)**	State or imply the form $\dfrac{A}{3-2x} + \dfrac{Bx+C}{x^2+4}$	B1	
		Use a relevant method to determine a constant	M1	
		Obtain one of the values $A = 3$, $B = -1$, $C = -2$	A1	
		Obtain a second value	A1	
		Obtain the third value	A1	**[5]**
	(ii)	Use correct method to find the first two terms of the expansion of $(3-2x)^{-1}$, $(1-\frac{2}{3}x)^{-1}$, $(4+x^2)^{-1}$ or $(1+\frac{1}{4}x^2)^{-1}$	M1	
		Obtain correct unsimplified expansions up to the term in x^2 of each partial fraction	A1√+A1√	
		Multiply out up to the term in x^2 by $Bx + C$, where $BC \neq 0$	M1	
		Obtain final answer $\frac{1}{2} + \frac{5}{12}x + \frac{41}{72}x^2$, or equivalent	A1	**[5]**

[Symbolic coefficients, e.g. $\begin{pmatrix}-1\\2\end{pmatrix}$ are not sufficient for the first M1.The f.t. is on A, B, C.]

[In the case of an attempt to expand $(5x^2+x+6)(3-2x)^{-1}(x^2+4)^{-1}$, give M1A1A1for the expansions, M1 for multiplying out fully, and A1 for the final answer.]

9	**(i)**	Separate variables correctly and attempt integration of one side	B1	
		Obtain term ln x	B1	
		Obtain term of the form $a\ln(k+e^{-t})$	M1	
		Obtain term $-\ln(k+e^{-t})$	A1	
		Evaluate a constant or use limits $x = 10$, $t = 0$ in a solution containing terms $a\ln(k+e^{-t})$ and $b\ln x$	M1*	
		Obtain correct solution in any form, e.g. $\ln x - \ln 10 = -\ln(k+e^{-t}) + \ln(k+1)$	A1	**[6]**
	(ii)	Substitute $x = 20$, $t = 1$ and solve for k	M1(dep*)	
		Obtain the given answer	A1	**[2]**
	(iii)	Using $e^{-t} \to 0$ and the given value of k, find the limiting value of x	M1	
		Justify the given answer	A1	**[2]**

10 **(i)** Carry out a correct method for finding a vector equation for *AB* M1

Obtain $\mathbf{r} = 2\mathbf{i} - \mathbf{j} + 3\mathbf{k} + \lambda(-\mathbf{i} + 2\mathbf{j} + 2\mathbf{k})$, or equivalent A1

Equate at least two pairs of components of general points on *AB* and *l* and solve for λ or for μ M1

Obtain correct answer for λ or μ, e.g. $\lambda = 1$ or $\mu = 0$; $\lambda = -\frac{4}{5}$ or $\mu = \frac{3}{5}$;

or $\lambda = \frac{1}{4}$ or $\mu = -\frac{3}{2}$ A1

Verify that not all three pairs of equations are satisfied and that the lines fail to intersect A1 **[5]**

(ii) *EITHER*: Obtain a vector parallel to the plane and not parallel to *l*, e.g. $\mathbf{i} - 2\mathbf{j} + \mathbf{k}$ B1

Use scalar product to obtain an equation in *a*, *b* and *c*, e.g. $3a + b - c = 0$ B1

Form a second relevant equation, e.g. $a - 2b + c = 0$ and solve for one ratio, e.g. $a : b$ M1

Obtain final answer $a : b : c = 1 : 4 : 7$ A1

Use coordinates of a relevant point and values of *a*, *b* and *c* in general equation and find *d* M1

Obtain answer $x + 4y + 7z = 19$, or equivalent A1

*OR*1: Obtain a vector parallel to the plane and not parallel to *l*, e.g. $\mathbf{i} - 2\mathbf{j} + \mathbf{k}$ B1

Obtain a second relevant vector parallel to the plane and attempt to calculate their vector product, e.g. $(\mathbf{i} - 2\mathbf{j} + \mathbf{k}) \times (3\mathbf{i} + \mathbf{j} - \mathbf{k})$ M1

Obtain two correct components A1

Obtain correct answer, e.g. $\mathbf{i} + 4\mathbf{j} + 7\mathbf{k}$ A1

Substitute coordinates of a relevant point in $x + 4y + 7z = d$, or equivalent, and find *d* M1

Obtain answer $x + 4y + 7z = 19$, or equivalent A1

*OR*2: Obtain a vector parallel to the plane and not parallel to *l*, e.g. $\mathbf{i} - 2\mathbf{j} + \mathbf{k}$ B1

Using a relevant point and second relevant vector, form a 2-parameter equation for the plane M1

State a correct equation, e.g. $\mathbf{r} = 2\mathbf{i} - \mathbf{j} + 3\mathbf{k} + s(\mathbf{i} - 2\mathbf{j} + \mathbf{k}) + t(3\mathbf{i} + \mathbf{j} - \mathbf{k})$ A1

State 3 correct equations in *x*, *y*, *z*, *s* and *t* A1

Eliminate *s* and *t* M1

Obtain answer $x + 4y + 7z = 19$, or equivalent A1

*OR*3: Using the coordinates of *A* and two points on *l*, state three simultaneous equations in *a*, *b*, *c* and *d*, e.g. $a + b + 2c = d$, $2a - b + 3c = d$ and $4a + 2b + c = d$ B1

Solve and find one ratio, e.g. $a : b$ M1

State one correct ratio A1

Obtain a correct ratio of three of the unknowns, e.g. $a : b : c = 1 : 4 : 7$, or equivalent A1

Either use coordinates of a relevant point and the found ratio to find the fourth unknown, e.g. *d*, or find the ratio $a : b : c : d$ M1

Obtain answer $x + 4y + 7z = 19$, or equivalent A1

*OR*4: Obtain a vector parallel to the plane and not parallel to *l*, e.g. $\mathbf{i} - 2\mathbf{j} + \mathbf{k}$ B1

Using a relevant point and second relevant vector, form a determinant equation for the plane M1

State a correct equation, e.g. $\begin{vmatrix} x-2 & y+1 & z-3 \\ 1 & -2 & 1 \\ 3 & 1 & -1 \end{vmatrix} = 0$ A1

Attempt to expand the determinant M1

Obtain or imply two correct cofactors A1

Obtain answer $x + 4y + 7z = 19$, or equivalent A1 **[6]**

CAMBRIDGE INTERNATIONAL EXAMINATIONS

Cambridge International Advanced Level

MARK SCHEME for the May/June 2015 series

9709 MATHEMATICS

9709/33 Paper 3, maximum raw mark 75

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the May/June 2015 series for most Cambridge IGCSE®, Cambridge International A and AS Level components and some Cambridge O Level components.

CAMBRIDGE
International Examinations

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PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

1		Use law for the logarithm of a product, quotient or power	M1	
		Obtain a correct equation free of logarithms, e.g. $\dfrac{x+4}{x^2}=4$	A1	
		Solve a 3-term quadratic obtaining at least one root	M1	
		Obtain final answer $x = 1.13$ only	A1	**4**
2	*EITHER*:	State or imply non-modular inequality $(x-2)^2 > (2x-3)^2$, or corresponding equation	B1	
		Solve a 3-term quadratic, as in Q1.	M1	
		Obtain critical value $x = \frac{5}{3}$	A1	
		State final answer $x < \frac{5}{3}$ only	A1	
	*OR*1:	State the relevant critical linear inequality $(2-x) > (2x-3)$, or corresponding equation	B1	
		Solve inequality or equation for x	M1	
		Obtain critical value $x = \frac{5}{3}$	A1	
		State final answer $x < \frac{5}{3}$ only	A1	
	*OR*2:	Make recognisable sketches of $y = 2x - 3$ and $y = \lvert x-2 \rvert$ on a single diagram	B1	
		Find x-coordinate of the intersection	M1	
		Obtain $x = \frac{5}{3}$	A1	
		State final answer $x < \frac{5}{3}$ only	A1	**4**
3		Use correct tan $2A$ and cot A formulae to form an equation in tan x	M1	
		Obtain a correct equation in any form	A1	
		Reduce equation to the form $\tan^2 x + 6\tan x - 3 = 0$, or equivalent	A1	
		Solve a three term quadratic in tan x for x, **as in Q1.**	M1	
		Obtain answer, e.g. 24.9° (24.896)	A1	
		Obtain second answer, e.g. 98.8 (98.794) and no others in the given interval	A1	**6**
		[Ignore outside the given interval. Treat answers in radians as a misread.] Radian answers 0.43452, 1.7243		
4		Use correct quotient or product rule	M1	
		Obtain correct derivative in any form	A1	
		Equate derivative to zero and obtain a horizontal equation	M1	
		Carry out complete method for solving an equation of the form $a\mathrm{e}^{3x} = b$, or $a\mathrm{e}^{5x} = b\mathrm{e}^{2x}$	M1	
		Obtain $x = \ln 2$, or exact equivalent	A1	
		Obtain $y = \frac{1}{3}$, or exact equivalent	A1	**6**

5	**(i)**	State $\frac{dx}{dt} = -4a\cos^3 t\sin t$, or $\frac{dy}{dt} = 4a\sin^3 t\cos t$	B1	
		Use $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$	M1	
		Obtain correct expression for $\frac{dy}{dx}$ in a simplified form	A1	**3**
	(ii)	Form the equation of the tangent	M1	
		Obtain a correct equation in any form	A1	
		Obtain the given answer	A1	**3**
	(iii)	State the x-coordinate of P or the y-coordinate of Q in any form	B1	
		Obtain the given result correctly	B1	**2**
6	**(i)**	Integrate and reach $\pm x\sin x \mp \int \sin x\, dx$	M1*	
		Obtain integral $x\sin x + \cos x$	A1	
		Substitute limits correctly, must be seen since AG, and equate result to 0.5	M1(dep*)	
		Obtain the given form of the equation	A1	**4**
	(ii)	*EITHER*: Consider the sign of a relevant expression at $a = 1$ and at another relevant value, e.g. $a = 1.5 \leqslant \frac{\pi}{2}$	M1	
		OR: Using limits correctly, consider the sign of $[x\sin x + \cos x]_0^a - 0.5$, or compare the value of $[x\sin x + \cos x]_0^a$ with 0.5, for a =1 AND for another relevant value, e.g $a = 1.5 \leqslant \frac{\pi}{2}$.	M1	
		Complete the argument, so change of sign, or above and below stated, both with correct calculated values	A1	**2**
	(iii)	Use the iterative formula correctly at least once	M1	
		Obtain final answer 1.2461	A1	
		Show sufficient iterations to 6 d.p. to justify 1.2461 to 4 d.p., or show there is a sign change in the interval (1.24605, 1.24615)	A1	**3**
7	**(i)**	Separate variables correctly and integrate one side	B1	
		Obtain term $2\sqrt{M}$, or equivalent	B1	
		Obtain term $50k\sin(0.02t)$, or equivalent	B1	
		Evaluate a constant of integration, or use limits $M = 100$, $t = 0$ in a solution with terms of the form $a\sqrt{M}$ and $b\sin(0.02t)$	M1*	
		Obtain correct solution in any form, e.g. $2\sqrt{M} = 50k\sin(0.02t) + 20$	A1	**5**
	(ii)	Use values M=196, t =50 and calculate k	M1(dep*)	
		Obtain answer $k = 0.190$	A1	**2**
	(iii)	State an expression for M in terms of t, e.g. $M = (4.75\sin(0.02t) + 10)^2$	M1(dep*)	
		State that the least possible number of micro-organisms is 28 or 27.5 or 27.6 (27.5625)	A1	**2**

8	**(i)**	*EITHER*:	Substitute for u in $\frac{\mathrm{i}}{u}$ and multiply numerator and denominator by $1+\mathrm{i}$	M1	
			Obtain final answer $-\frac{1}{2}+\frac{1}{2}\mathrm{i}$, or equivalent	A1	
		OR:	Substitute for u, obtain two equations in x and y and solve for x or for y	M1	
			Obtain final answer $-\frac{1}{2}+\frac{1}{2}\mathrm{i}$, or equivalent	A1	**2**
	(ii)		Show a point representing u in a relatively correct position	B1	
			Show the bisector of the line segment joining u to the origin	B1	
			Show a circle with centre at the point representing i	B1	
			Show a circle with radius 2	B1	**4**
	(iii)		State argument $-\frac{1}{2}\pi$, or equivalent, e.g. 270°	B1	
			State or imply the intersection in the first quadrant represents $2+\mathrm{i}$	B1	
			State argument 0.464, (0.4636)or equivalent, e.g. 26.6° (26.5625)	B1	**3**
9	**(i)**		State or imply a correct normal vector to either plane, e.g. $\mathbf{i}+3\mathbf{j}-2\mathbf{k}$, or $2\mathbf{i}+\mathbf{j}+3\mathbf{k}$	B1	
			Carry out correct process for evaluating the scalar product of two normal vectors	M1	
			Using the correct process for the moduli, divide the scalar product of the two normals by the product of their moduli and evaluate the inverse cosine of the result	M1	
			Obtain answer 85.9° or 1.50 radians	A1	**4**

(ii) *EITHER*: Carry out a complete strategy for finding a point on l M1
Obtain such a point, e.g. (0, 2, 1) A1
EITHER: State two equations for a direction vector $a\mathbf{i}+b\mathbf{j}+c\mathbf{k}$ for l,
e.g. $a+3b-2c=0$
and $2a+b+3c=0$ B1
Solve for one ratio, e.g. $a : b$ M1
Obtain $a : b : c = 11 : -7 : -5$ A1
State a correct answer, e.g. $\mathbf{r}=2\mathbf{j}+\mathbf{k}+\lambda(11\mathbf{i}-7\mathbf{j}-5\mathbf{k})$ A1√

*OR*1: Obtain a second point on l, e.g. $\left(\frac{22}{7}, 0, -\frac{3}{7}\right)$ B1
Subtract position vectors and obtain a direction vector for l M1
Obtain $22\mathbf{i}-14\mathbf{j}-10\mathbf{k}$, or equivalent A1
State a correct answer, e.g. $\mathbf{r}=2\mathbf{j}+\mathbf{k}+\lambda(22\mathbf{i}-14\mathbf{j}-10\mathbf{k})$ A1√

*OR*2: Attempt to find the vector product of the two normal vectors M1
Obtain two correct components A1
Obtain $11\mathbf{i}-7\mathbf{j}-5\mathbf{k}$, or equivalent A1
State a correct answer, e.g. $\mathbf{r}=2\mathbf{j}+\mathbf{k}+\lambda(11\mathbf{i}-7\mathbf{j}-5\mathbf{k})$ A1√

*OR*3: Express one variable in terms of a second M1
Obtain a correct simplified expression, e.g. $x=(22-11y)/7$ A1
Express the same variable in terms of the third M1
Obtain a correct simplified expression, e.g. $x=(11-11z)/5$ A1
Form a vector equation for the line M1
State a correct answer, e.g. $\mathbf{r}=2\mathbf{j}+\mathbf{k}+\lambda\left(\mathbf{i}-\frac{7}{11}\mathbf{j}-\frac{5}{11}\mathbf{k}\right)$ A1√

*OR*4: Express one variable in terms of a second M1
Obtain a correct simplified expression, e.g. $y=(22-7x)/11$ A1
Express the third variable in terms of the second M1
Obtain a correct simplified expression, e.g. $z=(11-5x)/11$ A1
Form a vector equation for the line M1
State a correct answer, e.g. $\mathbf{r}=2\mathbf{j}+\mathbf{k}+\lambda\left(\mathbf{i}-\frac{7}{11}\mathbf{j}-\frac{5}{11}\mathbf{k}\right)$ A1√ **6**

[The √ marks are dependent on all M marks being earned.]

10 (i) State or imply $\mathrm{f}(x)\equiv\frac{A}{2x-1}+\frac{B}{x+2}+\frac{C}{(x+2)^2}$ B1
Use a relevant method to determine a constant M1
Obtain one of the values $A=2$, $B=-1$, $C=3$ A1
Obtain the remaining values A1 + A1 **5**

[Apply an analogous scheme to the form $\frac{A}{2x-1}+\frac{Dx+E}{(x+2)^2}$; the values being $A=2$, $D=-1$, $E=1$.]

(ii) Integrate and obtain terms $\frac{1}{2}\cdot 2\ln(2x-1)-\ln(x+2)-\frac{3}{x+2}$ B1√ + B1√ + B1√

Use limits correctly, namely substitution must be seen in at least two of the partial fractions to obtain M1 Integrate all 3 partial fractions and substitute in all three partial fractions for A1 since AG. M1

Obtain the given answer following full and exact working A1 **5**

[The t marks are dependent on *A*, *B*, *C* etc.]

[SR: If *B*, *C* or *E* omitted, give B1M1 in part (**i**) and B1√B1√M1 in part (**ii**).]

[NB: Candidates who follow the *A*, *D*, *E* scheme in part (**i**) and then integrate $\frac{-x+1}{(x+2)^2}$ by parts should obtain $\frac{1}{2}\cdot 2\ln(2x-1)-\ln(x+2)+\frac{x-1}{x+2}$ (the third term is equivalent to $-\frac{3}{x+2}+1$).]

CAMBRIDGE INTERNATIONAL EXAMINATIONS

Cambridge International Advanced Level

MARK SCHEME for the October/November 2014 series

9709 MATHEMATICS

9709/31 Paper 3, maximum raw mark 75

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1		Use law of the logarithm of a power	M1	
		Obtain a correct linear equation in any form, e.g. $x = (x-2)\ln 3$	A1	
		Obtain answer $x = 22.281$	A1	**[3]**
2	**(i)**	State or imply ordinates 2, 1.1547…, 1, 1.1547…	B1	
		Use correct formula, or equivalent, with $h = \frac{1}{6}\pi$ and four ordinates	M1	
		Obtain answer 1.95	A1	**[3]**
	(ii)	Make recognisable sketch of $y = \operatorname{cosec} x$ for the given interval	B1	
		Justify a statement that the estimate will be an overestimate	B1	**[2]**
3		Substitute $x = -\frac{1}{3}$, equate result to zero or divide by $3x + 1$ and equate the remainder to zero and obtain a correct equation, e.g. $-\frac{1}{27}a + \frac{1}{9}b - \frac{1}{3} + 3 = 0$	B1	
		Substitute $x = 2$ and equate result to 21 or divide by $x - 2$ and equate constant remainder to 21	M1	
		Obtain a correct equation, e.g. $8a + 4b + 5 = 21$	A1	
		Solve for a or for b	M1	
		Obtain $a = 12$ and $b = -20$	A1	**[5]**
4	**(i)**	Use chain rule correctly at least once	M1	
		Obtain either $\frac{dx}{dt} = \frac{3\sin t}{\cos^4 t}$ or $\frac{dy}{dt} = 3\tan^2 t\sec^2 t$, or equivalent	A1	
		Use $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$	M1	
		Obtain the given answer	A1	**[4]**
	(ii)	State a correct equation for the tangent in any form	B1	
		Use Pythagoras	M1	
		Obtain the given answer	A1	**[3]**
5	**(i)**	Substitute $z = 1 + i$ and obtain $w = \frac{1+2i}{1+i}$	B1	
		EITHER: Multiply numerator and denominator by the conjugate of the denominator, or equivalent	M1	
		Simplify numerator to $3 + i$ or denominator to 2	A1	
		Obtain final answer $\frac{3}{2} + \frac{1}{2}i$, or equivalent	A1	
		OR: Obtain two equations in x and y, and solve for x or for y	M1	
		Obtain $x = \frac{3}{2}$ or $y = \frac{1}{2}$, or equivalent	A1	
		Obtain final answer $\frac{3}{2} + \frac{1}{2}i$, or equivalent	A1	**[4]**

(ii) *EITHER*: Substitute $w = z$ and obtain a 3-term quadratic equation in z, e.g. $\mathrm{i}z^2 + z - \mathrm{i} = 0$ B1

Solve a 3-term quadratic for z or substitute $z = x + \mathrm{i}y$ and use a correct method to solve for x and y M1

OR: Substitute $w = x + \mathrm{i}y$ and obtain two correct equations in x and y by equating real and imaginary parts B1

Solve for x and y M1

Obtain a correct solution in any form, e.g. $z = \dfrac{-1 \pm \sqrt{3}\,\mathrm{i}}{2\mathrm{i}}$ A1

Obtain final answer $-\dfrac{\sqrt{3}}{2} + \dfrac{1}{2}\mathrm{i}$ A1 **[4]**

6 **(i)** Integrate and reach $bx\ln 2x - c\int x.\dfrac{1}{x}\,\mathrm{d}x$, or equivalent M1*

Obtain $x\ln 2x - \int x.\dfrac{1}{x}\,\mathrm{d}x$, or equivalent A1

Obtain integral $x\ln 2x - x$, or equivalent A1

Substitute limits correctly and equate to 1, having integrated twice M1(dep*)

Obtain a correct equation in any form, e.g. $a\ln 2a - a + 1 - \ln 2 = 1$ A1

Obtain the given answer A1 **[6]**

(ii) Use the iterative formula correctly at least once M1

Obtain final answer 1.94 A1

Show sufficient iterations to 4 d.p. to justify 1.94 to 2d.p. or show that there is a sign change in the interval (1.935, 1.945). A1 **[3]**

7 **(i)** Separate variables correctly and attempt to integrate at least one side B1

Obtain term $\ln R$ B1

Obtain $\ln x - 0.57x$ B1

Evaluate a constant or use limits $x = 0.5$, $R = 16.8$, in a solution containing terms of the form $a\ln R$ and $b\ln x$ M1

Obtain correct solution in any form A1

Obtain a correct expression for R, e.g. $R = xe^{(3.80 - 0.57x)}$, $R = 44.7xe^{-0.57x}$ or $R = 33.6xe^{(0.285 - 0.57x)}$ A1 **[6]**

(ii) Equate $\dfrac{\mathrm{d}R}{\mathrm{d}x}$ to zero and solve for x M1

State or imply $x = 0.57^{-1}$, or equivalent, e.g. 1.75 A1

Obtain $R = 28.8$ (allow 28.9) A1 **[3]**

8 **(i)** Use $\sin(A + B)$ formula to express $\sin 3\theta$ in terms of trig. functions of 2θ and θ M1

Use correct double angle formulae and Pythagoras to express $\sin 3\theta$ in terms of $\sin\theta$ M1

Obtain a correct expression in terms of $\sin\theta$ in any form A1

Obtain the given identity A1 **[4]**

[SR: Give M1 for using correct formulae to express RHS in terms of $\sin\theta$ and $\cos 2\theta$, then M1A1 for expressing in terms of $\sin\theta$ and $\sin 3\theta$ only, or in terms of $\cos\theta$, $\sin\theta$, $\cos 2\theta$ and $\sin 2\theta$, then A1 for obtaining the given identity.]

	(ii)	Substitute for x and obtain the given answer	B1	**[1]**
	(iii)	Carry out a correct method to find a value of x	M1	
		Obtain answers 0.322, 0.799, –1.12	A1 + A1 + A1	**[4]**
		[Solutions with more than 3 answers can only earn a maximum of A1 + A1.]		
9	**(i)**	State or imply the form $\frac{A}{1-x}+\frac{B}{2-x}+\frac{C}{(2-x)^2}$	B1	
		Use a correct method to determine a constant	M1	
		Obtain one of $A = 2$, $B = -1$, $C = 3$	A1	
		Obtain a second value	A1	
		Obtain a third value	A1	**[5]**
		[The alternative form $\frac{A}{1-x}+\frac{Dx+E}{(2-x)^2}$, where $A = 2$, $D = 1$, $E = 1$ is marked B1M1A1A1A1 as above.]		
	(ii)	Use correct method to find the first two terms of the expansion of $(1-x)^{-1}, (2-x)^{-1}, (2-x)^{-2}, (1-\frac{1}{2}x)^{-1}$ or $(1-\frac{1}{2}x)^{-2}$	M1	
		Obtain correct unsimplified expansions up to the term in x^2 of each partial fraction	A1✓ + A1✓ + A1✓	
		Obtain final answer $\frac{9}{4}+\frac{5}{2}x+\frac{39}{16}x^2$, or equivalent	A1	**[5]**
		[Symbolic binomial coefficients, e.g. $\begin{pmatrix}-1\\1\end{pmatrix}$ are not sufficient for M1. The ✓ is on A,B,C.]		
		[For the A,D,E form of partial fractions, give M1 A1✓ A1✓ for the expansions then, if $D \neq 0$, M1 for multiplying out fully and A1 for the final answer.]		
		[In the case of an attempt to expand $(x^2-8x+9)(1-x)^{-1}(2-x)^{-2}$, give M1A1A1 for the expansions, M1 for multiplying out fully, and A1 for the final answer.]		
10	**(i)**	*EITHER*: Find $\overrightarrow{AP}$ (or $\overrightarrow{PA}$) for a point P on l with parameter λ, e.g. $\mathbf{i}-17\mathbf{j}+4\mathbf{k}+\lambda(-2\mathbf{i}+\mathbf{j}-2\mathbf{k})$	B1	
		Calculate scalar product of $\overrightarrow{AP}$ and a direction vector for l and equate to zero	M1	
		Solve and obtain $\lambda = 3$	A1	
		Carry out a complete method for finding the length of AP	M1	
		Obtain the given answer 15 correctly	A1	
		*OR*1: Calling (4, –9, 9) B, state $\overrightarrow{BA}$ (or $\overrightarrow{AB}$) in component form, e.g. $-\mathbf{i}+17\mathbf{j}-4\mathbf{k}$	B1	
		Calculate vector product of $\overrightarrow{BA}$ and a direction vector for l, e.g. $(-\mathbf{i}+17\mathbf{j}-4\mathbf{k})\times(-2\mathbf{i}+\mathbf{j}-2\mathbf{k})$	M1	
		Obtain correct answer, e.g. $-30\mathbf{i}+6\mathbf{j}+33\mathbf{k}$	A1	
		Divide the modulus of the product by that of the direction vector	M1	
		Obtain the given answer correctly	A1	
		*OR*2: State $\overrightarrow{BA}$ (or $\overrightarrow{AB}$) in component form	B1	
		Use a scalar product to find the projection of BA (or AB) on l	M1	
		Obtain correct answer in any form, e.g. $\frac{27}{\sqrt{9}}$	A1	
		Use Pythagoras to find the perpendicular	M1	

	Obtain the given answer correctly	A1	
*OR*3:	State $\overrightarrow{BA}$ (or $\overrightarrow{AB}$) in component form	B1	
	Use a scalar product to find the cosine of *ABP*	M1	
	Obtain correct answer in any form, e.g. $\dfrac{27}{\sqrt{9}.\sqrt{306}}$	A1	
	Use trig. to find the perpendicular	M1	
	Obtain the given answer correctly	A1	
*OR*4:	State $\overrightarrow{BA}$ (or $\overrightarrow{AB}$) in component form	B1	
	Find a second point *C* on *l* and use the cosine rule in triangle *ABC* to find the cosine of angle *A, B*, or *C*, or use a vector product to find the area of *ABC*	M1	
	Obtain correct answer in any form	A1	
	Use trig. or area formula to find the perpendicular	M1	
	Obtain the given answer correctly	A1	
*OR*5:	State correct $\overrightarrow{AP}$ (or $\overrightarrow{PA}$) for a point *P* on *l* with parameter λ in any form	B1	
	Use correct method to express AP^2 (or *AP*) in terms of λ	M1	
	Obtain a correct expression in any form, e.g. $(1-2\lambda)^2+(-17+\lambda)^2+(4-2\lambda)^2$	A1	
	Carry out a method for finding its minimum (using calculus, algebra or Pythagoras)	M1	
	Obtain the given answer correctly	A1	**[5]**
(ii) *EITHER*:	Substitute coordinates of a general point of *l* in equation of plane and either equate constant terms or equate the coefficient of λ to zero, obtaining an equation in *a* and *b*	M1*	
	Obtain a correct equation, e.g. $4a-9b-27+1=0$	A1	
	Obtain a second correct equation, e.g. $-2a+b+6=0$	A1	
	Solve for *a* or for *b*	M1(dep*)	
	Obtain $a=2$ and $b=-2$	A1	
OR:	Substitute coordinates of a point of *l* and obtain a correct equation, e.g. $4a-9b=26$	B1	
	EITHER: Find a second point on *l* and obtain an equation in *a* and *b*	M1*	
	Obtain a correct equation	A1	
	OR: Calculate scalar product of a direction vector for *l* and a vector normal to the plane and equate to zero	M1*	
	Obtain a correct equation, e.g. $-2a+b+6=0$	A1	
	Solve for *a* or for *b*	M1(dep*)	
	Obtain $a=2$ and $b=-2$	A1	**[5]**

CAMBRIDGE INTERNATIONAL EXAMINATIONS

Cambridge International Advanced Level

MARK SCHEME for the October/November 2014 series

9709 MATHEMATICS

9709/32 Paper 3, maximum raw mark 75

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the October/November 2014 series for most Cambridge IGCSE®, Cambridge International A and AS Level components and some Cambridge O Level components.

CAMBRIDGE
International Examinations

Mark Scheme Notes

Marks are of the following three types:

M	Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
A	Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
B	Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF	Any Equivalent Form (of answer is equally acceptable)
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
BOD	Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)
CAO	Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
CWO	Correct Working Only – often written by a 'fortuitous' answer
ISW	Ignore Subsequent Working
MR	Misread
PA	Premature Approximation (resulting in basically correct work that is insufficiently accurate)
SOS	See Other Solution (the candidate makes a better attempt at the same question)
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

1		Use law of the logarithm of a power	M1	
		Obtain a correct linear equation in any form, e.g. $x = (x-2)\ln 3$	A1	
		Obtain answer $x = 22.281$	A1	**[3]**
2	**(i)**	State or imply ordinates 2, 1.1547…, 1, 1.1547…	B1	
		Use correct formula, or equivalent, with $h = \frac{1}{6}\pi$ and four ordinates	M1	
		Obtain answer 1.95	A1	**[3]**
	(ii)	Make recognisable sketch of $y = \operatorname{cosec} x$ for the given interval	B1	
		Justify a statement that the estimate will be an overestimate	B1	**[2]**
3		Substitute $x = -\frac{1}{3}$, equate result to zero or divide by $3x + 1$ and equate the remainder to zero and obtain a correct equation, e.g. $-\frac{1}{27}a + \frac{1}{9}b - \frac{1}{3} + 3 = 0$	B1	
		Substitute $x = 2$ and equate result to 21 or divide by $x - 2$ and equate constant remainder to 21	M1	
		Obtain a correct equation, e.g. $8a + 4b + 5 = 21$	A1	
		Solve for a or for b	M1	
		Obtain $a = 12$ and $b = -20$	A1	**[5]**
4	**(i)**	Use chain rule correctly at least once	M1	
		Obtain either $\frac{dx}{dt} = \frac{3\sin t}{\cos^4 t}$ or $\frac{dy}{dt} = 3\tan^2 t\sec^2 t$, or equivalent	A1	
		Use $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$	M1	
		Obtain the given answer	A1	**[4]**
	(ii)	State a correct equation for the tangent in any form	B1	
		Use Pythagoras	M1	
		Obtain the given answer	A1	**[3]**
5	**(i)**	Substitute $z = 1 + i$ and obtain $w = \frac{1+2i}{1+i}$	B1	
		EITHER: Multiply numerator and denominator by the conjugate of the denominator, or equivalent	M1	
		Simplify numerator to $3 + i$ or denominator to 2	A1	
		Obtain final answer $\frac{3}{2} + \frac{1}{2}i$, or equivalent	A1	
		OR: Obtain two equations in x and y, and solve for x or for y	M1	
		Obtain $x = \frac{3}{2}$ or $y = \frac{1}{2}$, or equivalent	A1	
		Obtain final answer $\frac{3}{2} + \frac{1}{2}i$, or equivalent	A1	**[4]**

	(ii)	*EITHER*: Substitute $w = z$ and obtain a 3-term quadratic equation in z, e.g. $\mathrm{i}z^2 + z - \mathrm{i} = 0$	B1	
		Solve a 3-term quadratic for z or substitute $z = x + \mathrm{i}y$ and use a correct method to solve for x and y	M1	
		OR: Substitute $w = x + \mathrm{i}y$ and obtain two correct equations in x and y by equating real and imaginary parts	B1	
		Solve for x and y	M1	
		Obtain a correct solution in any form, e.g. $z = \dfrac{-1 \pm \sqrt{3}\,\mathrm{i}}{2\mathrm{i}}$	A1	
		Obtain final answer $-\dfrac{\sqrt{3}}{2} + \dfrac{1}{2}\mathrm{i}$	A1	**[4]**
6	**(i)**	Integrate and reach $bx\ln 2x - c\int x.\dfrac{1}{x}\,\mathrm{d}x$, or equivalent	M1*	
		Obtain $x\ln 2x - \int x.\dfrac{1}{x}\,\mathrm{d}x$, or equivalent	A1	
		Obtain integral $x\ln 2x - x$, or equivalent	A1	
		Substitute limits correctly and equate to 1, having integrated twice	M1(dep*)	
		Obtain a correct equation in any form, e.g. $a\ln 2a - a + 1 - \ln 2 = 1$	A1	
		Obtain the given answer	A1	**[6]**
	(ii)	Use the iterative formula correctly at least once	M1	
		Obtain final answer 1.94	A1	
		Show sufficient iterations to 4 d.p. to justify 1.94 to 2d.p. or show that there is a sign change in the interval (1.935, 1.945).	A1	**[3]**
7	**(i)**	Separate variables correctly and attempt to integrate at least one side	B1	
		Obtain term $\ln R$	B1	
		Obtain $\ln x - 0.57x$	B1	
		Evaluate a constant or use limits $x = 0.5$, $R = 16.8$, in a solution containing terms of the form $a\ln R$ and $b\ln x$	M1	
		Obtain correct solution in any form	A1	
		Obtain a correct expression for R, e.g. $R = xe^{(3.80 - 0.57x)}$, $R = 44.7xe^{-0.57x}$ or $R = 33.6xe^{(0.285 - 0.57x)}$	A1	**[6]**
	(ii)	Equate $\dfrac{\mathrm{d}R}{\mathrm{d}x}$ to zero and solve for x	M1	
		State or imply $x = 0.57^{-1}$, or equivalent, e.g. 1.75	A1	
		Obtain $R = 28.8$ (allow 28.9)	A1	**[3]**
8	**(i)**	Use $\sin(A + B)$ formula to express $\sin 3\theta$ in terms of trig. functions of 2θ and θ	M1	
		Use correct double angle formulae and Pythagoras to express $\sin 3\theta$ in terms of $\sin\theta$	M1	
		Obtain a correct expression in terms of $\sin\theta$ in any form	A1	
		Obtain the given identity	A1	**[4]**
		[SR: Give M1 for using correct formulae to express RHS in terms of $\sin\theta$ and $\cos 2\theta$, then M1A1 for expressing in terms of $\sin\theta$ and $\sin 3\theta$ only, or in terms of $\cos\theta$, $\sin\theta$, $\cos 2\theta$ and $\sin 2\theta$, then A1 for obtaining the given identity.]		

	(ii)		Substitute for x and obtain the given answer	B1	**[1]**
	(iii)		Carry out a correct method to find a value of x	M1	
			Obtain answers 0.322, 0.799, –1.12	A1 + A1 + A1	**[4]**
			[Solutions with more than 3 answers can only earn a maximum of A1 + A1.]		
9	**(i)**		State or imply the form $\frac{A}{1-x}+\frac{B}{2-x}+\frac{C}{(2-x)^2}$	B1	
			Use a correct method to determine a constant	M1	
			Obtain one of $A = 2$, $B = -1$, $C = 3$	A1	
			Obtain a second value	A1	
			Obtain a third value	A1	**[5]**
			[The alternative form $\frac{A}{1-x}+\frac{Dx+E}{(2-x)^2}$, where $A = 2$, $D = 1$, $E = 1$ is marked B1M1A1A1A1 as above.]		
	(ii)		Use correct method to find the first two terms of the expansion of $(1-x)^{-1}, (2-x)^{-1}, (2-x)^{-2}, (1-\frac{1}{2}x)^{-1}$ or $(1-\frac{1}{2}x)^{-2}$	M1	
			Obtain correct unsimplified expansions up to the term in x^2 of each partial fraction	A1✓ + A1✓ + A1✓	
			Obtain final answer $\frac{9}{4}+\frac{5}{2}x+\frac{39}{16}x^2$, or equivalent	A1	**[5]**
			[Symbolic binomial coefficients, e.g. $\begin{pmatrix}-1\\1\end{pmatrix}$ are not sufficient for M1. The ✓ is on A,B,C.]		
			[For the A,D,E form of partial fractions, give M1 A1✓ A1✓ for the expansions then, if $D \neq 0$, M1 for multiplying out fully and A1 for the final answer.]		
			[In the case of an attempt to expand $(x^2-8x+9)(1-x)^{-1}(2-x)^{-2}$, give M1A1A1 for the expansions, M1 for multiplying out fully, and A1 for the final answer.]		
10	**(i)**	*EITHER*:	Find $\overrightarrow{AP}$ (or $\overrightarrow{PA}$) for a point P on l with parameter λ, e.g. $\mathbf{i}-17\mathbf{j}+4\mathbf{k}+\lambda(-2\mathbf{i}+\mathbf{j}-2\mathbf{k})$	B1	
			Calculate scalar product of $\overrightarrow{AP}$ and a direction vector for l and equate to zero	M1	
			Solve and obtain $\lambda = 3$	A1	
			Carry out a complete method for finding the length of AP	M1	
			Obtain the given answer 15 correctly	A1	
		*OR*1:	Calling (4, –9, 9) B, state $\overrightarrow{BA}$ (or $\overrightarrow{AB}$) in component form, e.g. $-\mathbf{i}+17\mathbf{j}-4\mathbf{k}$	B1	
			Calculate vector product of $\overrightarrow{BA}$ and a direction vector for l, e.g. $(-\mathbf{i}+17\mathbf{j}-4\mathbf{k})\times(-2\mathbf{i}+\mathbf{j}-2\mathbf{k})$	M1	
			Obtain correct answer, e.g. $-30\mathbf{i}+6\mathbf{j}+33\mathbf{k}$	A1	
			Divide the modulus of the product by that of the direction vector	M1	
			Obtain the given answer correctly	A1	
		*OR*2:	State $\overrightarrow{BA}$ (or $\overrightarrow{AB}$) in component form	B1	
			Use a scalar product to find the projection of BA (or AB) on l	M1	
			Obtain correct answer in any form, e.g. $\frac{27}{\sqrt{9}}$	A1	
			Use Pythagoras to find the perpendicular	M1	

	Obtain the given answer correctly	A1	
*OR*3:	State $\overrightarrow{BA}$ (or $\overrightarrow{AB}$) in component form	B1	
	Use a scalar product to find the cosine of *ABP*	M1	
	Obtain correct answer in any form, e.g. $\dfrac{27}{\sqrt{9}.\sqrt{306}}$	A1	
	Use trig. to find the perpendicular	M1	
	Obtain the given answer correctly	A1	
*OR*4:	State $\overrightarrow{BA}$ (or $\overrightarrow{AB}$) in component form	B1	
	Find a second point *C* on *l* and use the cosine rule in triangle *ABC* to find the cosine of angle *A, B*, or *C*, or use a vector product to find the area of *ABC*	M1	
	Obtain correct answer in any form	A1	
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	Obtain the given answer correctly	A1	
*OR*5:	State correct $\overrightarrow{AP}$ (or $\overrightarrow{PA}$) for a point *P* on *l* with parameter λ in any form	B1	
	Use correct method to express AP^2 (or *AP*) in terms of λ	M1	
	Obtain a correct expression in any form, e.g. $(1-2\lambda)^2+(-17+\lambda)^2+(4-2\lambda)^2$	A1	
	Carry out a method for finding its minimum (using calculus, algebra or Pythagoras)	M1	
	Obtain the given answer correctly	A1	**[5]**
(ii) *EITHER*:	Substitute coordinates of a general point of *l* in equation of plane and either equate constant terms or equate the coefficient of λ to zero, obtaining an equation in *a* and *b*	M1*	
	Obtain a correct equation, e.g. $4a-9b-27+1=0$	A1	
	Obtain a second correct equation, e.g. $-2a+b+6=0$	A1	
	Solve for *a* or for *b*	M1(dep*)	
	Obtain $a=2$ and $b=-2$	A1	
OR:	Substitute coordinates of a point of *l* and obtain a correct equation, e.g. $4a-9b=26$	B1	
	EITHER: Find a second point on *l* and obtain an equation in *a* and *b*	M1*	
	Obtain a correct equation	A1	
	OR: Calculate scalar product of a direction vector for *l* and a vector normal to the plane and equate to zero	M1*	
	Obtain a correct equation, e.g. $-2a+b+6=0$	A1	
	Solve for *a* or for *b*	M1(dep*)	
	Obtain $a=2$ and $b=-2$	A1	**[5]**

CAMBRIDGE INTERNATIONAL EXAMINATIONS

Cambridge International Advanced Level

MARK SCHEME for the October/November 2014 series

9709 MATHEMATICS

9709/33 Paper 3, maximum raw mark 75

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CAMBRIDGE
International Examinations

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PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

1		Either	State or imply non-modular inequality $(3x-1)^2<(2x+5)^2$ or corresponding quadratic equation or pair of linear equations $3x-1=\pm(2x+5)$	B1	
			Solve a three-term quadratic or two linear equations $5x^2-26x-24<0$	M1	
			Obtain $-\frac{4}{5}$ and 6	A1	
			State $-\frac{4}{5}<x<6$	A1	
		Or	Obtain value 6 from graph, inspection or solving linear equation	B1	
			Obtain value $-\frac{4}{5}$ similarly	B2	
			State $-\frac{4}{5}<x<6$	B1	[4]
2			Use correct product rule or correct chain rule to differentiate y	M1	
			Use $\frac{dy}{dx}=\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$	M*1	
			Obtain $\frac{-4\cos\theta\sin^2\theta+2\cos^3\theta}{\sec^2\theta}$ or equivalent	A1	
			Express $\frac{dy}{dx}$ in terms of $\cos\theta$	DM*1	
			Confirm given answer $6\cos^5\theta-4\cos^3\theta$ legitimately	A1	[5]
3	(i)	Either	Equate p(−1) or p(−2) to zero or divide by $(x+1)$ or $(x+2)$ and equate constant remainder to zero.	M*1	
			Obtain two equations $a-b=6$ and $4a-2b=34$ or equivalents	A1	
			Solve pair of equations for a or b	DM*1	
			Obtain $a=11$ and $b=5$	A1	
		Or	State or imply third factor is $4x-1$	B1	
			Carry out complete expansion of $(x+1)(x+2)(4x-1)$ or $(x+1)(x+2)(Cx+D)$	M1	
			Obtain $a=11$	A1	
			Obtain $b=5$	A1	[4]
	(ii)		Use division or equivalent and obtaining linear remainder	M1	
			Obtain quotient $4x+a$, following their value of a	A1√	
			Indicate remainder $x-13$	A1	[3]

4	(i)	Either	Use $\cos(A \pm B)$ correctly at least once	M1	
			State correct complete expansion	A1	
			Confirm given answer $\cos\theta$ with explicit use of $\cos 60^o = \frac{1}{2}$	A1	
			SR: "correct" answer from sign errors in both expansions is B1 only		
		Or	Use correct $\cos A + \cos B$ formula	M1	
			State correct result e.g. $2\cos\left(\frac{2\theta}{2}\right)\cos\left(\frac{-120}{2}\right)$	A1	
			Confirm given answer $\cos\theta$ with explicit use of $\cos\left(\pm 60^o\right) = \frac{1}{2}$	A1	[3]
	(ii)		State or imply $\frac{\cos 2x}{\cos x} = 3$	B1	
			Obtain equation $2\cos^2 x - 3\cos x - 1 = 0$	B1	
			Solve a three-term quadratic equation for $\cos x$	M1	
			Obtain $\frac{1}{4}\left(3 - \sqrt{17}\right)$ or exact equivalent and, finally, no other	A1	[4]
5	(i)		State or imply $\mathrm{i}w = -3 + 5\mathrm{i}$	B1	
			Carry out multiplication by $\frac{4 - \mathrm{i}}{4 - \mathrm{i}}$	M1	
			Obtain final answer $-\frac{7}{17} + \frac{23}{17}\mathrm{i}$ or equivalent	A1	[3]
	(ii)		Multiply w by z to obtain $17 + 17\mathrm{i}$	B1	
			State $\arg w = \tan^{-1}\frac{3}{5}$ or $\arg z = \tan^{-1}\frac{1}{4}$	B1	
			State $\arg wz = \arg w + \arg z$	M1	
			Confirm given result $\tan^{-1}\frac{3}{5} + \tan^{-1}\frac{1}{4} = \frac{1}{4}\pi$ legitimately	A1	[4]
6	(i)		State or imply correct ordinates 1, 0.94259…, 0.79719…, 0.62000…	B1	
			Use correct formula or equivalent with $h = 0.1$ and four y values	M1	
			Obtain 0.255 with no errors seen	A1	[3]
	(ii)		Obtain or imply $a = -6$	B1	
			Obtain x^4 term including correct attempt at coefficient	M1	
			Obtain or imply $b = 27$	A1	
		Either	Integrate to obtain $x - 2x^3 + \frac{27}{5}x^5$, following their values of a and b	B1√	
			Obtain 0.259	B1	
		Or	Use correct trapezium rule with at least 3 ordinates	M1	
			Obtain 0.259 (from 4)	A1	[5]

7	**(i)**	State at least two of the equations $1+\lambda = a+\mu$, $4 = 2+2\mu$, $-2+3\lambda = -2+3a\mu$	B1	
		Solve for λ or for μ	M1	
		Obtain $\lambda = a$ (or $\lambda = a+\mu-1$) and $\mu = 1$	A1	
		Confirm values satisfy third equation	A1	[4]
	(ii)	State or imply point of intersection is $(a+1, 4, 3a-2)$	B1	
		Use correct method for the modulus of the position vector and equate to 9, following their point of intersection	M*1	
		Solve a three-term quadratic equation in a $(a^2-a-6=0)$	DM*1	
		Obtain –2 and 3	A1	[4]
8	**(i)**	Sensibly separate variables and attempt integration of at least one side	M1	
		Obtain $2y^{\frac{1}{2}} = \ldots$ or equivalent	A1	
		Correct integration by parts of $x\sin\frac{1}{3}x$ as far as $ax\cos\frac{1}{3}x \pm \int b\cos\frac{1}{3}x\mathrm{d}x$	M1	
		Obtain $-3x\cos\frac{1}{3}x + \int 3\cos\frac{1}{3}x\mathrm{d}x$ or equivalent	A1	
		Obtain $-3x\cos\frac{1}{3}x + 9\sin\frac{1}{3}x$ or equivalent	A1	
		Obtain $y = \left(-\frac{3}{10}x\cos\frac{1}{3}x + \frac{9}{10}\sin\frac{1}{3}x + c\right)^2$ or equivalent	A1	[6]
	(ii)	Use $x = 0$ and $y = 100$ to find constant	M*1	
		Substitute 25 and calculate value of y	DM*1	
		Obtain 203	A1	[3]
9	**(i)**	Sketch increasing curve with correct curvature passing through origin, for $x \geq 0$	B1	
		Recognisable sketch of $y = 40 - x^3$, with equation stated, for $x > 0$	B1	
		Indicate in some way the one intersection, dependent on both curves being roughly correct and both existing for some $x < 0$	B1	[3]
	(ii)	Consider signs of $x^3 + \ln(x+1) - 40$ at 3 and 4 or equivalent or compare values of relevant expressions for $x = 3$ and $x = 4$	M1	
		Complete argument correctly with correct calculations (–11.6 and 25.6)	A1	[2]
	(iii)	Use the iterative formula correctly at least once	M1	
		Obtain final answer 3.377	A1	
		Show sufficient iterations to justify accuracy to 3 d.p. or show sign change in interval (3.3765, 3.3775)	A1	[3]
	(iv)	Attempt value of $\ln(x+1)$	M1	
		Obtain 1.48	A1	[2]

10	State or imply $\frac{\mathrm{d}u}{\mathrm{d}x} = \mathrm{e}^x$	B1	
	Substitute throughout for x and dx	M1	
	Obtain $\int \frac{u}{u^2 + 3u + 2}\,\mathrm{d}u$ or equivalent (ignoring limits so far)	A1	
	State or imply partial fractions of form $\frac{A}{u+2} + \frac{B}{u+1}$, following their integrand	B1	
	Carry out a correct process to find at least one constant for their integrand	M1	
	Obtain correct $\frac{2}{u+2} - \frac{1}{u+1}$	A1	
	Integrate to obtain $a\ln(u+2) + b\ln(u+1)$	M1	
	Obtain $2\ln(u+2) - \ln(u+1)$ or equivalent, follow their A and B	A1√	
	Apply appropriate limits and use at least one logarithm property correctly	M1	
	Obtain given answer $\ln\frac{8}{5}$ legitimately	A1	[10]
	SR for integrand $\frac{u^2}{u(u+1)(u+2)}$		
	State or imply partial fractions of form $\frac{A}{u} + \frac{B}{u+1} + \frac{C}{u+2}$	(B1)	
	Carry out a correct process to find at least one constant	(M1)	
	Obtain correct $\frac{2}{u+2} - \frac{1}{u+1}$	(A1)	
	…complete as above.		

CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Level

MARK SCHEME for the May/June 2014 series

9709 MATHEMATICS

9709/31 Paper 3, maximum raw mark 75

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

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CAMBRIDGE
International Examinations

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- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously “correct” answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

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PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

1	(i)	State $\sin 2\alpha = 2\sin\alpha\cos\alpha$ and $\sec\alpha = 1/\cos\alpha$	B1	
		Obtain $2\sin\alpha$	B1	[2]
	(ii)	Use $\cos 2\beta = 2\cos^2\beta - 1$ or equivalent to produce correct equation in $\cos\beta$	B1	
		Solve three-term quadratic equation for $\cos\beta$	M1	
		Obtain $\cos\beta = \frac{1}{3}$ only	A1	[3]
2		State $\frac{du}{dx} = 3\sec^2 x$ or equivalent	B1	
		Express integral in terms of u and du (accept unsimplified and without limits)	M1	
		Obtain $\int \frac{1}{3} u^{\frac{1}{2}}\, du$	A1	
		Integrate $Cu^{\frac{1}{2}}$ to obtain $\frac{2C}{3}u^{\frac{3}{2}}$	M1	
		Obtain $\frac{14}{9}$	A1	[5]
3		Obtain $\frac{2}{2t+3}$ for derivative of x	B1	
		Use quotient of product rule, or equivalent, for derivative of y	M1	
		Obtain $\frac{5}{(2t+3)^2}$ or unsimplified equivalent	A1	
		Obtain $t = -1$	B1	
		Use $\frac{dy}{dx} = \frac{dy}{dt} / \frac{dx}{dt}$ in algebraic or numerical form	M1	
		Obtain gradient $\frac{5}{2}$	A1	[6]
4		Separate variables correctly and recognisable attempt at integration of at least one side	M1	
		Obtain $\ln y$, or equivalent	B1	
		Obtain $k\ln\left(2 + e^{3x}\right)$	B1	
		Use $y(0) = 36$ to find constant in $y = A\left(2 + e^{3x}\right)^k$ or $\ln y = k\ln\left(2 + e^{3x}\right) + c$ or equivalent	M1*	
		Obtain equation correctly without logarithms from $\ln y = \ln\left(A\left(2 + e^{3x}\right)^k\right)$	*M1	
		Obtain $y = 4\left(2 + e^{3x}\right)^2$	A1	[6]

5	**(i)**	Either	Multiply numerator and denominator by $\sqrt{3}+\mathrm{i}$ and use $\mathrm{i}^2 = -1$	M1	
			Obtain correct numerator $18+18\sqrt{3}\mathrm{i}$ or correct denominator 4	B1	
			Obtain $\frac{9}{2}+\frac{9}{2}\sqrt{3}\mathrm{i}$ or $\left(18+18\sqrt{3}\mathrm{i}\right)/4$	A1	
			Obtain modulus or argument	M1	
			Obtain $9\mathrm{e}^{\frac{1}{3}\pi\mathrm{i}}$	A1	[5]
		OR	Obtain modulus and argument of numerator or denominator, or both moduli or both arguments	M1	
			Obtain moduli and argument 18 and $\frac{1}{6}\pi$ or 2 and $-\frac{1}{6}\pi$		
			or moduli 18 and 2 or arguments $\frac{1}{6}\pi$ and $-\frac{1}{6}\pi$ (allow degrees)	B1	
			Obtain $18\mathrm{e}^{\frac{1}{6}\pi\mathrm{i}} \div 2\mathrm{e}^{-\frac{1}{6}\pi\mathrm{i}}$ or equivalent	A1	
			Divide moduli and subtract arguments	M1	
			Obtain $9\mathrm{e}^{\frac{1}{3}\pi\mathrm{i}}$	A1	[5]
	(ii)	State $3\mathrm{e}^{\frac{1}{6}\pi\mathrm{i}}$, following through their answer to part (i)		B1√	
		State $3\mathrm{e}^{\frac{1}{6}\pi\mathrm{i}\pm\frac{1}{2}\pi\mathrm{i}}$, following through their answer to part (i)		B1√	
		Obtain $3\mathrm{e}^{-\frac{5}{6}\pi\mathrm{i}}$		B1	[3]
6	**(i)**	Use law for the logarithm for a product or quotient or exponentiation AND for a power		M1	
		Obtain $(4x-5)^2(x+1) = 27$		B1	
		Obtain given equation correctly $16x^3 - 24x^2 - 15x - 2 = 0$		A1	[3]
	(ii)	Obtain $x = 2$ is root or $(x-2)$ is a factor, or likewise with $x = -\frac{1}{4}$		B1	
		Divide by $(x-2)$ to reach a quotient of the form $16x^2 + kx$		M1	
		Obtain quotient $16x^2 + 8x + 1$		A1	
		Obtain $(x-2)(4x+1)^2$ or $(x-2)$, $(4x+1)$, $(4x+1)$		A1	[4]
	(iii)	State $x = 2$ only		A1	[1]
7	**(i)**	Obtain $2x - 3y + 6z$ for LHS of equation		B1	
		Obtain $2x - 3y + 6z = 23$		B1	[2]
	(ii)	Either	Use correct formula to find perpendicular distance	M1	
			Obtain unsimplified value $\frac{\pm 23}{\sqrt{2^2+(-3)^2+6^2}}$, following answer to (i)	A1√	
			Obtain $\frac{23}{7}$ or equivalent	A1	[3]

		OR 1	Use scalar product of (4, –1, 2) and a vector normal to the plane	M1	
			Use unit normal to plane to obtain $\pm\frac{(8+3+12)}{\sqrt{49}}$	A1	
			Obtain $\frac{23}{7}$ or equivalent	A1	[3]
		OR 2	Find parameter intersection of p and $\mathbf{r} = \mu(2\mathbf{i} - 3\mathbf{j} + 6\mathbf{k})$	M1	
			Obtain $\mu = \frac{23}{49}$ [and $\left(\frac{46}{49}, -\frac{69}{49}, \frac{138}{49}\right)$ as foot of perpendicular]	A1	
			Obtain distance $\frac{23}{7}$ or equivalent	A1	[3]
	(iii)	Either	Recognise that plane is $2x - 3y + 6z = k$ and attempt use of formula for perpendicular distance to plane at least once	M1	
			Obtain $\frac{\lvert 23-k \rvert}{7} = 14$ or equivalent	A1	
			Obtain $2x - 3y + 6z = 121$ and $2x - 3y + 6z = -75$	A1	[3]
		OR	Recognise that plane is $2x - 3y + 6z = k$ and attempt to find at least one point on q using l with $\lambda = \pm 2$	M1	
			Obtain $2x - 3y + 6z = 121$	A1	
			Obtain $2x - 3y + 6z = -75$	A1	[3]
8	(i)		Sketch $y = \operatorname{cosec} x$ for at least $0 , x , \pi$	B1	
			Sketch $y = x(\pi - x)$ for at least $0 , x , \pi$	B1	
			Justify statement concerning two roots, with evidence of 1 and $\frac{1}{4}\pi^2$ for y-values on graph via scales	B1	[3]
	(ii)		Use $\operatorname{cosec} x = \frac{1}{\sin x}$ and commence rearrangement	M1	
			Obtain given equation correctly, showing sufficient detail	A1	[2]
	(iii)	(a)	Use the iterative formula correctly at least once	M1	
			Obtain final answer 0.66	A1	
			Show sufficient iterations to 4 decimal places to justify answer or show a sign change in the interval (0.655, 0.665)	A1	[3]
		(b)	Obtain 2.48	B1	[1]

9	**(i)**	Either	State or imply partial fractions are of form $\frac{A}{3-x}+\frac{B}{1+2x}+\frac{C}{(1+2x)^2}$	B1	
			Use any relevant method to obtain a constant	M1	
			Obtain $A = 1$	A1	
			Obtain $B=\frac{3}{2}$	A1	
			Obtain $C=-\frac{1}{2}$	A1	[5]
		Or	State or imply partial fractions are of form $\frac{A}{3-x}+\frac{Dx+E}{(1+2x)^2}$	B1	
			Use any relevant method to obtain a constant	M1	
			Obtain $A = 1$	A1	
			Obtain $D = 3$	A1	
			Obtain $E = 1$	A1	[5]
	(ii)		Obtain the first two terms of one of the expansion of $(3-x)^{-1}$, $\left(1-\frac{1}{3}x\right)^{-1}$ $(1+2x)^{-1}$ and $(1+2x)^{-2}$	M1	
			Obtain correct unsimplified expansion up to the term in x^2 of each partial fraction, following in each case the value of A, B, C	A1√ A1√ A1√	
			Obtain answer $\frac{4}{3}-\frac{8}{9}x+\frac{1}{27}x^2$	A1	[5]
			[If A, D, E approach used in part (i), give M1A1√A1√ for the expansions, M1 for multiplying out fully and A1 for final answer]		
10	**(i)**		Use of product or quotient rule	M1	
			Obtain $-5e^{-\frac{1}{2}x}\sin 4x+40e^{-\frac{1}{2}x}\cos 4x$	A1	
			Equate $\frac{dy}{dx}$ to zero and obtain $\tan 4z = k$ or $R\cos(4x \pm \alpha)$	M1	
			Obtain $\tan 4x = 8$ or $\sqrt{65}\cos\left(4x \pm \tan^{-1}\frac{1}{8}\right)$	A1	
			Obtain 0.362 or 20.7°	A1	
			Obtain 1.147 or 65.7°	A1	[6]
	(ii)		State or imply that x-coordinates of T_n are increasing by $\frac{1}{4}\pi$ or 45°	B1	
			Attempt solution of inequality (or equation) of form $x_1 + (n-1)k\pi$. 25	M1	
			Obtain $n>\frac{4}{\pi}(25-0.362)+1$, following through on their value of x_1	A1√	
			n = 33	A1	[4]

CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Level

MARK SCHEME for the May/June 2014 series

9709 MATHEMATICS

9709/32 Paper 3, maximum raw mark 75

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		Make reasonable solution attempt at a 3-term quadratic, or solve two linear equations for x	M1	
		Obtain critical values $x = \frac{1}{4}a$ and $x = \frac{5}{2}a$	A1	
		State answer $\frac{1}{4}a < x < \frac{5}{2}a$	A1	
	OR:	Obtain critical value $x = \frac{5}{2}a$ from a graphical method, or by inspection, or by solving a linear equation or inequality	B1	
		Obtain critical value $x = \frac{1}{4}a$ similarly	B2	
		State answer $\frac{1}{4}a < x < \frac{5}{2}a$	B1	4
		[Do not condone $\leqslant$ for $<$.]		
2		Remove logarithms and obtain $5 - \mathrm{e}^{-2x} = \mathrm{e}^{\frac{1}{2}}$, or equivalent	B1	
		Obtain a correct value for e^{-2x}, e^{2x}, e^{-x} or e^{x}, e.g. $\mathrm{e}^{2x} = 1/(5 - \mathrm{e}^{\frac{1}{2}})$	B1	
		Use correct method to solve an equation of the form $\mathrm{e}^{2x} = a$, $\mathrm{e}^{-2x} = a$, $\mathrm{e}^{x} = a$ or $\mathrm{e}^{-x} = a$ where $a > 0$. [The M1 is dependent on the correct removal of logarithms.]	M1	
		Obtain answer $x = -0.605$ only.	A1	4
3		Use $\cos(A+B)$ formula to obtain an equation in $\cos x$ and $\sin x$	M1	
		Use trig formula to obtain an equation in $\tan x$ (or $\cos x$ or $\sin x$)	M1	
		Obtain $\tan x = \sqrt{3} - 4$, or equivalent (or find $\cos x$ or $\sin x$)	A1	
		Obtain answer $x = -66.2°$	A1	
		Obtain answer $x = 113.8°$ and no others in the given interval	A1	5
		[Ignore answers outside the given interval. Treat answers in radians as a misread $(-1.16, 1.99)$.]		
		[The other solution methods are *via* $\cos x = \pm 1/\sqrt{(1 + (\sqrt{3}-4)^2)}$ and $\sin x = \pm(\sqrt{3}-4)/\sqrt{(1+(\sqrt{3}-4)^2)}$.]		
4	**(i)**	State $\frac{\mathrm{d}x}{\mathrm{d}t} = 1 - \sec^2 t$, or equivalent	B1	
		Use chain rule	M1	
		Obtain $\frac{\mathrm{d}y}{\mathrm{d}t} = -\frac{\sin t}{\cos t}$, or equivalent	A1	
		Use $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}t} \div \frac{\mathrm{d}x}{\mathrm{d}t}$	M1	
		Obtain the given answer correctly.	A1	5
	(ii)	State or imply $t = \tan^{-1}(\frac{1}{2})$	B1	
		Obtain answer $x = -0.0364$	B1	2

5	**(i)**	Differentiate f(x) and obtain $\mathrm{f}'(x) = (x-2)^2\mathrm{g}'(x) + 2(x-2)\mathrm{g}(x)$	B1	
		Conclude that $(x – 2)$ is a factor of $\mathrm{f}'(x)$	B1	**2**
	(ii)	*EITHER:* Substitute $x = 2$, equate to zero and state a correct equation, e.g. $32 + 16a + 24 + 4b + a = 0$	B1	
		Differentiate polynomial, substitute $x = 2$ and equate to zero or divide by $(x-2)$ and equate constant remainder to zero	M1*	
		Obtain a correct equation, e.g. $80 + 32a + 36 + 4b = 0$	A1	
		OR1: Identify given polynomial with $(x-2)^2(x^3 + Ax^2 + Bx + C)$ and obtain an equation in a and/or b	M1*	
		Obtain a correct equation, e.g. $\frac{1}{4}a - 4(4+a) + 4 = 3$	A1	
		Obtain a second correct equation, e.g. $-\frac{3}{4}a + 4(4+a) = b$	A1	
		OR2: Divide given polynomial by $(x-2)^2$ and obtain an equation in a and b	M1*	
		Obtain a correct equation, e.g. $29 + 8a + b + 0$	A1	
		Obtain a second correct equation, e.g. $176 + 47a + 4b = 0$	A1	
		Solve for a or for b	M1(dep*)	
		Obtain $a = –4$ and $b = 3$	A1	**5**
6	**(i)**	Use correct arc formula and form an equation in r and x	M1	
		Obtain a correct equation in any form	A1	
		Rearrange in the given form	A1	**3**
	(ii)	Consider sign of a relevant expression at $x = 1$ and $x = 1.5$, or compare values of relevant expressions at $x = 1$ and $x = 1.5$	M1	
		Complete the argument correctly with correct calculated values	A1	**2**
	(iii)	Use the iterative formula correctly at least once	M1	
		Obtain final answer 1.21	A1	
		Show sufficient iterations to 4 d.p. to justify 1.21 to 2 d.p., or show there is a sign change in the interval (1.205,1.215)	A1	**3**
7	**(a)**	*EITHER*: Substitute and expand $(-1 + \sqrt{5}\,\mathrm{i})^3$ completely	M1	
		Use $\mathrm{i}^2 = -1$ correctly at least once	M1	
		Obtain $a = –12$	A1	
		State that the other complex root is $-1 - \sqrt{5}\,\mathrm{i}$	B1	
		OR1: State that the other complex root is $-1 - \sqrt{5}\,\mathrm{i}$	B1	
		State the quadratic factor $z^2 + 2z + 6$	B1	
		Divide the cubic by a 3-term quadratic, equate remainder to zero and solve for a or, using a 3-term quadratic, factorise the cubic and determine a	M1	
		Obtain $a = –12$	A1	
		OR2: State that the other complex root is $-1 - \sqrt{5}\mathrm{i}$	B1	
		State or show the third root is 2	B1	
		Use a valid method to determine a	M1	
		Obtain $a = –12$	A1	
		OR3: Substitute and use De Moivre to cube $\sqrt{6}\mathrm{cis}(114.1^\circ)$, or equivalent	M1	
		Find the real and imaginary parts of the expression	M1	
		Obtain $a = –12$	A1	
		State that the other complex root is $-1 - \sqrt{5}\mathrm{i}$	B1	**4**

	(b)	*EITHER:* Substitute $w = \cos 2\theta + \mathrm{i}\sin 2\theta$ in the given expression	B1	
		Use double angle formulae throughout	M1	
		Express numerator and denominator in terms of $\cos\theta$ and $\sin\theta$ only	A1	
		Obtain given answer correctly	A1	
		OR: Substitute $w = \mathrm{e}^{2\mathrm{i}\theta}$ in the given expression	B1	
		Divide numerator and denominator by $\mathrm{e}^{\mathrm{i}\theta}$, or equivalent	M1	
		Express numerator and denominator in terms of $\cos\theta$ and $\sin\theta$ only	A1	
		Obtain the given answer correctly	A1	**4**
8	**(i)**	Use product rule	M1	
		Obtain derivative in any correct form	A1	
		Differentiate first derivative using the product rule	M1	
		Obtain second derivative in any correct form, e.g. $-\frac{1}{2}\sin\frac{1}{2}x - \frac{1}{4}x\cos\frac{1}{2}x - \frac{1}{2}\sin\frac{1}{2}x$	A1	
		Verify the given statement	A1	**5**
	(ii)	Integrate and reach $kx\sin\frac{1}{2}x + l\int\sin\frac{1}{2}x\,\mathrm{d}x$	M1*	
		Obtain $2x\sin\frac{1}{2}x - 2\int\sin\frac{1}{2}x\,\mathrm{d}x$, or equivalent	A1	
		Obtain indefinite integral $2x\sin\frac{1}{2}x + 4\cos\frac{1}{2}x$	A1	
		Use correct limits $x = 0$, $x = \pi$ correctly	M1(dep*)	
		Obtain answer $2\pi - 4$, or exact equivalent	A1	**5**
9	**(i)**	State or imply $\frac{\mathrm{d}N}{\mathrm{d}t} = kN(1-0.01N)$ and obtain the given answer $k = 0.02$	B1	**1**
	(ii)	Separate variables and attempt integration of at least one side	M1	
		Integrate and obtain term $0.02t$, or equivalent	A1	
		Carry out a relevant method to obtain A or B such that $\frac{1}{N(1-0.01N)} \equiv \frac{A}{N} + \frac{B}{1-0.01N}$, or equivalent	M1*	
		Obtain $A = 1$ and $B = 0.01$, or equivalent	A1	
		Integrate and obtain terms $\ln N - \ln(1-0.01N)$, or equivalent	A1√	
		Evaluate a constant or use limits $t = 0$, $N = 20$ in a solution with terms $a\ln N$ and $b\ln(1-0.01N)$, $ab \neq 0$	M1(dep*)	
		Obtain correct answer in any form, e.g. $\ln N - \ln(1-0.01N) = 0.02t + \ln 25$	A1	
		Rearrange and obtain $t = 50\ln(4N/(100-N))$, or equivalent	A1	**8**
	(iii)	Substitute $N = 40$ and obtain $t = 49.0$	B1	**1**

10	**(i)**	*EITHER:*	State or imply $\overrightarrow{AB}$ and $\overrightarrow{AC}$ correctly in component form	B1	
			Using the correct processes evaluate the scalar product $\overrightarrow{AB}.\overrightarrow{AC}$, or equivalent	M1	
			Using the correct process for the moduli divide the scalar product by the product of the moduli	M1	
			Obtain answer $\frac{20}{21}$	A1	
		OR:	Use correct method to find lengths of all sides of triangle *ABC*	M1	
			Apply cosine rule correctly to find the cosine of angle *BAC*	M1	
			Obtain answer $\frac{20}{21}$	A1	**4**
	(ii)		State an exact value for the sine of angle *BAC*, e.g. $\sqrt{41}/21$	B1✓	
			Use correct area formula to find the area of triangle *ABC*	M1	
			Obtain answer $\frac{1}{2}\sqrt{41}$, or exact equivalent	A1	**3**
			[SR: Allow use of a vector product, e.g. $\overrightarrow{AB}\times\overrightarrow{AC} = -6\mathbf{i}+2\mathbf{j}-\mathbf{k}$ B1✓. Using correct process for the modulus, divide the modulus by 2 M1. Obtain answer $\frac{1}{2}\sqrt{41}$ A1.]		
	(iii)	*EITHER*:	State or obtain $b = 0$	B1	
			Equate scalar product of normal vector and $\overrightarrow{BC}$ (or $\overrightarrow{CB}$) to zero	M1	
			Obtain $a + b - 4c = 0$ (or $a - 4c = 0$)	A1	
			Substitute a relevant point in $4x + z = d$ and evaluate d	M1	
			Obtain answer $4x + z = 9$, or equivalent	A1	
		*OR*1:	Attempt to calculate vector product of relevant vectors, e.g. $(\mathbf{j})\times(\mathbf{i}+\mathbf{j}-4\mathbf{k})$	M1	
			Obtain two correct components of the product	A1	
			Obtain correct product, e.g. $-4\mathbf{i} - \mathbf{k}$	A1	
			Substitute a relevant point in $4x + z = d$ and evaluate d	M1	
			Obtain $4x + z = 9$, or equivalent	A1	
		*OR*2:	Attempt to form 2-parameter equation for the plane with relevant vectors	M1	
			State a correct equation, e.g. $\mathbf{r} = 2\mathbf{i} + 4\mathbf{j} + \mathbf{k} + \lambda(\mathbf{j}) + \mu(\mathbf{i}+\mathbf{j}-4\mathbf{k})$	A1	
			State 3 equations in x, y, z, λ and μ	A1	
			Eliminate μ	M1	
			Obtain answer $4x + z = 9$, or equivalent	A1	
		*OR*3:	State or obtain $b = 0$	B1	
			Substitute for *B* and *C* in the plane equation and obtain $2a + c = d$ and $3a - 3c = d$ (or $2a + 4b + c = d$ and $3a + 5b - 3c = d$)	B1	
			Solve for one ratio, e.g. $a : d$	M1	
			Obtain $a : c : d$, or equivalent	M1	
			Obtain answer $4x + z = 9$, or equivalent	A1	
		OR4:	Attempt to form a determinant equation for the plane with relevant vectors	M1	
			State a correct equation, e.g. $\begin{vmatrix} x-2 & y-4 & z-1 \\ 0 & 1 & 0 \\ 1 & 1 & -4 \end{vmatrix} = 0$	A1	
			Attempt to use a correct method to expand the determinant	M1	
			Obtain two correct terms of a 3-term expansion, or equivalent	A1	
			Obtain answer $4x + z = 9$, or equivalent	A1	**5**

CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Level

MARK SCHEME for the May/June 2014 series

9709 MATHEMATICS

9709/33 Paper 3, maximum raw mark 75

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the May/June 2014 series for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level components and some Ordinary Level components.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF	Any Equivalent Form (of answer is equally acceptable)
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
BOD	Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)
CAO	Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
CWO	Correct Working Only – often written by a "fortuitous" answer
ISW	Ignore Subsequent Working
MR	Misread
PA	Premature Approximation (resulting in basically correct work that is insufficiently accurate)
SOS	See Other Solution (the candidate makes a better attempt at the same question)
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

1		Use law of the logarithm of a quotient or product or $2=\log_{10}100$	M1	
		Remove logarithms and obtain $x + 9 = 100x$, or equivalent	A1	
		Obtain answer $x=\frac{1}{11}$	A1	**3**
2		State a correct unsimplified version of the x or x^2 or x^3 term	M1	
		State correct first two terms $1-x$	A1	
		Obtain the next two terms $2x^2-\frac{14}{3}x^3$	A1 + A1	**4**
		[Symbolic binomial coefficients, e.g. $\begin{pmatrix}-\frac{1}{3}\\3\end{pmatrix}$ are not sufficient for the M mark.]		
3	**(i)**	Use $\tan(A \pm B)$ formula and obtain an equation in $\tan x$	M1	
		Using $\tan 60°=\sqrt{3}$, obtain a horizontal equation in $\tan x$ in any correct form	A1	
		Reduce the equation to the given form	A1	**3**
	(ii)	Solve the given quadratic for $\tan x$	M1	
		Obtain a correct answer, e.g. $x = 21.6°$	A1	
		Obtain a second answer, e.g. $x = 128.4°$, and no others	A1	**3**
		[Ignore answers outside the given interval. Treat answers in radians as a misread (0.377, 2.24).]		
4	**(i)**	Consider sign of $x-10/(\mathrm{e}^{2x}-1)$ at $x = 1$ and $x = 2$	M1	
		Complete the argument correctly with correct calculated values	A1	**2**
	(ii)	State or imply $\alpha=\frac{1}{2}\ln(1+10/\alpha)$	B1	
		Rearrange this as $\alpha=10/(\mathrm{e}^{2\alpha}-1)$ or work *vice versa*	B1	**2**
	(iii)	Use the iterative formula correctly at least once	M1	
		Obtain final answer 1.14	A1	
		Show sufficient iterations to 4 d.p. to justify 1.14 to 2 d.p., or show there is a sign change in the interval (1.135, 1.145)	A1	**3**
5		Separate variables correctly and attempt integration of at least one side	B1	
		Obtain term in the form $a\sqrt{(2x+1)}$	M1	
		Express $1/(\cos^2\theta)$ as $\sec^2\theta$	B1	
		Obtain term of the form $k\tan\theta$	M1	
		Evaluate a constant, or use limits $x=0, \theta=\frac{1}{4}\pi$ in a solution with terms $a\sqrt{(2x+1)}$ and $k\tan\theta$, $ak\neq 0$	M1	
		Obtain correct solution in any form, e.g. $\sqrt{(2x+1)}=\frac{1}{2}\tan\theta+\frac{1}{2}$	A1	
		Rearrange and obtain $x=\frac{1}{8}(\tan\theta+1)^2-\frac{1}{2}$, or equivalent	A1	**7**

6		Obtain correct derivative of RHS in any form	B1	
		Obtain correct derivative of LHS in any form	B1	
		Set $\frac{dy}{dx}$ equal to zero and obtain a horizontal equation	M1	
		Obtain a correct equation, e.g. $x^2 + y^2 = 1$, from correct work	A1	
		By substitution in the curve equation, or otherwise, obtain an equation in x^2 or y^2	M1	
		Obtain $x = \frac{1}{2}\sqrt{3}$	A1	
		Obtain $y = \frac{1}{2}$	A1	**7**
7	**(a)**	*EITHER*: Multiply numerator and denominator by 1 – 4i, or equivalent, and use $i^2 = -1$	M1	
		Simplify numerator to $-17 - 17i$, or denominator to 17	A1	
		Obtain final answer $-1 - i$	A1	
		OR: Using $i^2 = -1$, obtain two equations in x and y, and solve for x or for y	M1	
		Obtain $x = -1$ or $y = -1$, or equivalent	A1	
		Obtain final answer $-1 - i$	A1	**3**
	(b) (i)	Show a point representing 2 + i in relatively correct position	B1	
		Show a circle with centre 2 + i and radius 1	B1√	
		Show the perpendicular bisector of the line segment joining i and 2	B1	
		Shade the correct region	B1	**4**
	(ii)	State or imply that the angle between the tangents from the origin to the circle is required	M1	
		Obtain answer 0.927 radians (or 53.1°)	A1	**2**
8	**(i)**	Use a correct method for finding a constant	M1	
		Obtain one of $A = 3$, $B = 3$, $C = 0$	A1	
		Obtain a second value	A1	
		Obtain a third value	A1	**4**
	(ii)	Integrate and obtain term $-3\ln(2 - x)$	B1√	
		Integrate and obtain term of the form $k\ln(2 + x^2)$	M1	
		Obtain term $\frac{3}{2}\ln(2 + x^2)$	A1√	
		Substitute limits correctly in an integral of the form $a\ln(2 - x) + b\ln(2 + x^2)$, where $ab \neq 0$	M1	
		Obtain given answer after full and correct working	A1	**5**
9	**(i)**	Substitute for x and dx throughout using $u = \sin x$ and $du = \cos x\, dx$, or equivalent	M1	
		Obtain integrand e^{2u}	A1	
		Obtain indefinite integral $\frac{1}{2}e^{2u}$	A1	
		Use limits $u = 0$, $u = 1$ correctly, or equivalent	M1	
		Obtain answer $\frac{1}{2}(e^2 - 1)$, or exact equivalent	A1	**5**

	(ii)	Use chain rule or product rule	M1	
		Obtain correct terms of the derivative in any form, e.g. $2\cos x\,\mathrm{e}^{2\sin x}\cos x-\mathrm{e}^{2\sin x}\sin x$	A1 + A1	
		Equate derivative to zero and obtain a quadratic equation in sin *x*	M1	
		Solve a 3-term quadratic and obtain a value of *x*	M1	
		Obtain answer 0.896	A1	**6**
10	**(i)**	Express general point of *l* in component form, e.g. $(1+3\lambda, 2-2\lambda, -1+2\lambda)$	B1	
		Substitute in given equation of *p* and solve for λ	M1	
		Obtain final answer $-\frac{1}{2}\mathbf{i}+3\mathbf{j}-2\mathbf{k}$, or equivalent, from $\lambda=-\frac{1}{2}$	A1	**3**
	(ii)	State or imply a vector normal to the plane, e.g. $2\mathbf{i}+3\mathbf{j}-5\mathbf{k}$	B1	
		Using the correct process, evaluate the scalar product of a direction vector for *l* and a normal for *p*	M1	
		Using the correct process for the moduli, divide the scalar product by the product of the moduli and find the inverse sine or cosine of the result	M1	
		Obtain answer 23.2° (or 0.404 radians)	A1	**4**
	(iii)	*EITHER*: State $2a + 3b - 5c = 0$ or $3a - 2b + 2c = 0$	B1	
		Obtain two relevant equations and solve for one ratio, e.g. $a : b$	M1	
		Obtain $a : b : c = 4 : 19 : 13$, or equivalent	A1	
		Substitute coordinates of a relevant point in $4x + 19y + 13z = d$, and evaluate d	M1	
		Obtain answer $4x + 19y + 13z = 29$, or equivalent	A1	
		*OR*1: Attempt to calculate vector product of relevant vectors, e.g. $(2\mathbf{i} + 3\mathbf{j} - 5\mathbf{k})\times(3\mathbf{i} - 2\mathbf{j} + 2\mathbf{k})$	M1	
		Obtain two correct components of the product	A1	
		Obtain correct product, e.g. $-4\mathbf{i} -19\mathbf{j} -13\mathbf{k}$	A1	
		Substitute coordinates of a relevant point in $4x + 19y + 13z = d$	M1	
		Obtain answer $4x + 19y + 13z = 29$, or equivalent	A1	
		*OR*2: Attempt to form a 2-parameter equation with relevant vectors	M1	
		State a correct equation, e.g. $\mathbf{r}=\mathbf{i}+2\mathbf{j}-\mathbf{k}+\lambda(2\mathbf{i}+3\mathbf{j}-5\mathbf{k})+\mu(3\mathbf{i}-2\mathbf{j}+2\mathbf{k})$	A1	
		State 3 equations in x, y, z, λ and μ	A1	
		Eliminate λ and μ	M1	
		Obtain answer $4x + 19y + 13z = 29$, or equivalent	A1	
		OR3: Using a relevant point and relevant direction vectors, form a determinant equation for the plane	M1	
		State a correct equation, e.g. $\begin{vmatrix} x-1 & y-2 & z+1 \\ 2 & 3 & -5 \\ 3 & -2 & 2 \end{vmatrix}=0$	A1	
		Attempt to expand the determinant	M1	
		Obtain correct values of two cofactors	A1	
		Obtain answer $4x+19y+13z=29$, or equivalent	A1	**5**

CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Level

MARK SCHEME for the October/November 2013 series

9709 MATHEMATICS

9709/31 Paper 3, maximum raw mark 75

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1		Use correct quotient or product rule	M1	
		Obtain correct derivative in any form	A1	
		Justify the given statement	A1	**[3]**
2	*EITHER:*	State or imply non-modular equation $2^2\left(3^x-1\right)^2=\left(3^x\right)^2$, or pair of equations $2\left(3^x-1\right)=\pm 3^x$	M1	
		Obtain $3^x = 2$ and $3^x=\frac{2}{3}$ (or $3^{x+1} = 2$)	A1	
	OR:	Obtain $3^x = 2$ by solving an equation or by inspection	B1	
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		Use correct method for solving an equation of the form $3^x = a$ (or $3^{x+1} = a$), where $a > 0$	M1	
		Obtain final answers 0.631 and –0.369	A1	**[4]**
3	*EITHER:*	Integrate by parts and reach $kx^{\frac{1}{2}}\ln x-m\int x^{\frac{1}{2}}.\frac{1}{x}\,\mathrm{d}x$	M1*	
		Obtain $2x^{\frac{1}{2}}\ln x-2\int\frac{1}{x^{\frac{1}{2}}}\,\mathrm{d}x$, or equivalent	A1	
		Integrate again and obtain $2x^{\frac{1}{2}}\ln x-4x^{\frac{1}{2}}$, or equivalent	A1	
		Substitute limits $x = 1$ and $x = 4$, having integrated twice	M1(dep*)	
		Obtain answer $4(\ln 4-1)$, or exact equivalent	A1	
	OR1:	Using $u = \ln x$, or equivalent, integrate by parts and reach $kue^{\frac{1}{2}u}-m\int e^{\frac{1}{2}u}\,\mathrm{d}u$	M1*	
		Obtain $2ue^{\frac{1}{2}u}-2\int e^{\frac{1}{2}u}\,\mathrm{d}u$, or equivalent	A1	
		Integrate again and obtain $2ue^{\frac{1}{2}u}-4e^{\frac{1}{2}u}$, or equivalent	A1	
		Substitute limits $u = 0$ and $u = \ln 4$, having integrated twice	M1(dep*)	
		Obtain answer $4\ln 4-4$, or exact equivalent	A1	
	OR2:	Using $u=\sqrt{x}$, or equivalent, integrate and obtain $ku\ln u-m\int u.\frac{1}{u}\mathrm{d}u$	M1*	
		Obtain $4u\ln u-4\int 1\mathrm{d}u$, or equivalent	A1	
		Integrate again and obtain $4u\ln u-4u$, or equivalent	A1	
		Substitute limits $u = 1$ and $u = 2$, having integrated twice or quoted $\int\ln u\,\mathrm{d}u$ as $u\ln u\pm u$	M1(dep*)	
		Obtain answer $8\ln 2-4$, or exact equivalent	A1	
	OR3:	Integrate by parts and reach $I=\frac{x\ln x\pm x}{\sqrt{x}}+k\int\frac{x\ln x\pm x}{x\sqrt{x}}\,\mathrm{d}x$	M1*	
		Obtain $I=\frac{x\ln x-x}{\sqrt{x}}+\frac{1}{2}I-\frac{1}{2}\int\frac{1}{\sqrt{x}}\,\mathrm{d}x$	A1	
		Integrate and obtain $I=2\sqrt{x}\ln x-4\sqrt{x}$, or equivalent	A1	
		Substitute limits $x = 1$ and $x = 4$, having integrated twice	M1(dep*)	
		Obtain answer $4\ln 4-4$, or exact equivalent	A1	**[5]**

4		Use correct product or quotient rule at least once	M1*	
		Obtain $\frac{\mathrm{d}x}{\mathrm{d}t} = \mathrm{e}^{-t}\sin t - \mathrm{e}^{-t}\cos t$ or $\frac{\mathrm{d}y}{\mathrm{d}t} = \mathrm{e}^{-t}\cos t - \mathrm{e}^{-t}\sin t$, or equivalent	A1	
		Use $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}t} \div \frac{\mathrm{d}x}{\mathrm{d}t}$	M1	
		Obtain $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\sin t - \cos t}{\sin t + \cos t}$, or equivalent	A1	
		EITHER: Express $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of tan t only	M1(dep*)	
		Show expression is identical to $\tan\left(t - \frac{1}{4}\pi\right)$	A1	
		OR: Express $\tan\left(t - \frac{1}{4}\pi\right)$ in terms of tan t	M1	
		Show expression is identical to $\frac{\mathrm{d}y}{\mathrm{d}x}$	A1	**[6]**
5	**(i)**	Use Pythagoras	M1	
		Use the sin2A formula	M1	
		Obtain the given result	A1	**[3]**
	(ii)	Integrate and obtain a $k\ln\sin\theta$ or $m\ln\cos\theta$ term, or obtain integral of the form $p\ln\tan\theta$	M1*	
		Obtain indefinite integral $\frac{1}{2}\ln\sin\theta - \frac{1}{2}\ln\cos\theta$, or equivalent, or $\frac{1}{2}\ln\tan\theta$	A1	
		Substitute limits correctly	M1(dep)*	
		Obtain the given answer correctly having shown appropriate working	A1	**[4]**
6	**(i)**	State or imply $AB = 2r\cos\theta$ or $AB^2 = 2r^2 - 2r^2\cos(\pi - 2\theta)$	B1	
		Use correct formula to express the area of sector ABC in terms of r and θ	M1	
		Use correct area formulae to express the area of a segment in terms of r and θ	M1	
		State a correct equation in r and θ in any form	A1	
		Obtain the given answer	A1	**[5]**
		[SR: If the complete equation is approached by adding two sectors to the shaded area above BO and OC give the first M1 as on the scheme, and the second M1 for using correct area formulae for a triangle AOB or AOC, and a sector AOB or AOC.]		
	(ii)	Use the iterative formula correctly at least once	M1	
		Obtain final answer 0.95	A1	
		Show sufficient iterations to 4 d.p. to justify 0.95 to 2 d.p., or show there is a sign change in the interval (0.945, 0.955)	A1	**[3]**

7	(i)	State or imply partial fractions are of the form $\dfrac{A}{x-2}+\dfrac{Bx+C}{x^2+3}$	B1	
		Use a relevant method to determine a constant	M1	
		Obtain one of the values $A = -1$, $B = 3$, $C = -1$	A1	
		Obtain a second value	A1	
		Obtain the third value	A1	**[5]**
	(ii)	Use correct method to obtain the first two terms of the expansions of $(x-2)^{-1}$, $\left(1-\dfrac{1}{2}x\right)^{-1}$, $(x^2+3)^{-1}$ or $\left(1+\dfrac{1}{3}x^2\right)^{-1}$	M1	
		Substitute correct unsimplified expansions up to the term in x^2 into each partial fraction	A1√+A1√	
		Multiply out fully by $Bx + C$, where $BC \neq 0$	M1	
		Obtain final answer $\dfrac{1}{6}+\dfrac{5}{4}x+\dfrac{17}{72}x^2$, or equivalent	A1	**[5]**
		[Symbolic binomial coefficients, e.g. $\begin{pmatrix}-1\\1\end{pmatrix}$ are not sufficient for the M1. The f.t. is on *A, B, C.*]		
		[In the case of an attempt to expand $(2x^2-7x-1)(x-2)^{-1}(x^2+3)^{-1}$, give M1A1A1 for the expansions, M1 for multiplying out fully, and A1 for the final answer.]		
		[If *B* or *C* omitted from the form of partial fractions, give B0M1A0A0A0 in **(i)**; M1A1√A1√ in **(ii)**]		
8	(a)	*EITHER:* Solve for *u* or for *v*	M1	
		Obtain $u=\dfrac{2\mathrm{i}-6}{1-2\mathrm{i}}$ or $v=\dfrac{5}{1-2\mathrm{i}}$, or equivalent	A1	
		Either: Multiply a numerator and denominator by conjugate of denominator, or equivalent *Or:* Set *u* or *v* equal to *x* + i*y*, obtain two equations by equating real and imaginary parts and solve for *x* or for *y*	M1	
		OR: Using *a* + i*b* and *c* +i*d* for *u* and *v*, equate real and imaginary parts and obtain four equations in *a, b, c* and *d*	M1	
		Obtain $b + 2d = 2$, $a + 2c = 0$, $a + d = 0$ and $-b + c = 3$, or equivalent	A1	
		Solve for one unknown	M1	
		Obtain final answer $u = -2 -2\mathrm{i}$, or equivalent	A1	
		Obtain final answer $v = 1 + 2\mathrm{i}$, or equivalent	A1	**[5]**
	(b)	Show a circle with centre $-\mathrm{i}$	B1	
		Show a circle with radius 1	B1	
		Show correct half line from 2 at an angle of $\dfrac{3}{4}\pi$ to the real axis	B1	
		Use a correct method for finding the least value of the modulus	M1	
		Obtain final answer $\dfrac{3}{\sqrt{2}}-1$, or equivalent, e.g. 1.12 (allow 1.1)	A1	**[5]**

9	(i)	*EITHER:*	Obtain a vector parallel to the plane, e.g. $\overrightarrow{AB} = -2\mathbf{i} + 4\mathbf{j} - \mathbf{k}$	B1	
			Use scalar product to obtain an equation in *a, b, c,* e.g. $-2a + 4b - c = 0$, $3a - 3b + 3c = 0$, or $a + b + 2c = 0$	M1	
			Obtain two correct equations in *a, b, c*	A1	
			Solve to obtain ratio $a : b : c$	M1	
			Obtain $a : b : c = 3 : 1 : -2$, or equivalent	A1	
			Obtain equation $3x + y - 2z = 1$, or equivalent	A1	
		OR1:	Substitute for two points, e.g. *A* and *B*, and obtain $2a - b + 2c = d$ and $3b + c = d$	B1	
			Substitute for another point, e.g. *C*, to obtain a third equation and eliminate one unknown entirely from the three equations	M1	
			Obtain two correct equations in three unknowns, e.g. in *a, b, c*	A1	
			Solve to obtain their ratio, e.g. $a : b : c$	M1	
			Obtain $a : b : c = 3 : 1 : -2$, $a : c : d = 3 : -2 : 1$, $a : b : d = 3 : 1 : 1$ or $b : c : d = -1 : -2 : 1$	A1	
			Obtain equation $3x + y - 2z = 1$, or equivalent	A1	
		OR2:	Obtain a vector parallel to the plane, e.g. $\overrightarrow{BC} = 3\mathbf{i} - 3\mathbf{j} + 3\mathbf{k}$	B1	
			Obtain a second such vector and calculate their vector product e.g. $(-2\mathbf{i} + 4\mathbf{j} - \mathbf{k}) \times (3\mathbf{i} - 3\mathbf{j} + 3\mathbf{k})$	M1	
			Obtain two correct components of the product	A1	
			Obtain correct answer, e.g. $9\mathbf{i} + 3\mathbf{j} - 6\mathbf{k}$	A1	
			Substitute in $9x + 3y - 6z = d$ to find d	M1	
			Obtain equation $9x + 3y - 6z = 3$, or equivalent	A1	
		OR3:	Obtain a vector parallel to the plane, e.g. $\overrightarrow{AC} = \mathbf{i} + \mathbf{j} + 2\mathbf{k}$	B1	
			Obtain a second such vector and form correctly a 2-parameter equation for the plane	M1	
			Obtain a correct equation, e.g. $\mathbf{r} = 3\mathbf{i} + 4\mathbf{k} + \lambda(-2\mathbf{i} + 4\mathbf{j} - \mathbf{k}) + \mu(\mathbf{i} + \mathbf{j} + 2\mathbf{k})$	A1	
			State three correct equations in x, y, z, λ, μ	A1	
			Eliminate λ and μ	M1	
			Obtain equation $3x + y - 2z = 1$, or equivalent	A1	[6]
	(ii)		Obtain answer $\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$, or equivalent	B1	[1]

	(iii)	*EITHER:*	Use $\frac{\overrightarrow{OA}.\overrightarrow{OD}}{\lvert\overrightarrow{OD}\rvert}$ to find projection *ON* of *OA* onto *OD*	M1	
			Obtain $ON = \frac{4}{3}$	A1	
			Use Pythagoras in triangle *OAN* to find *AN*	M1	
			Obtain the given answer	A1	
		OR1:	Calculate the vector product of $\overrightarrow{OA}$ and $\overrightarrow{OD}$	M1	
			Obtain answer 6**i** + 2**j** – 5**k**	A1	
			Divide the modulus of the vector product by the modulus of $\overrightarrow{OD}$	M1	
			Obtain the given answer	A1	
		OR2:	Taking general point *P* of *OD* to have position vector $\lambda(\mathbf{i}+2\mathbf{j}+2\mathbf{k})$, form an equation in λ by either equating the scalar product of $\overrightarrow{AP}$ and $\overrightarrow{OP}$ to zero, or using Pythagoras in triangle *OPA,* or setting the derivative of $\lvert\overrightarrow{AP}\rvert$ to zero	M1	
			Solve and obtain $\lambda = \frac{4}{9}$	A1	
			Carry out method to calculate *AP* when $\lambda = \frac{4}{9}$	M1	
			Obtain the given answer	A1	
		OR3:	Use a relevant scalar product to find the cosine of *AOD* or *ADO*	M1	
			Obtain $\cos AOD = \frac{4}{9}$ or $\cos ADO = \frac{5}{3\sqrt{10}}$, or equivalent	A1	
			Use trig to find the length of the perpendicular	M1	
			Obtain the given answer	A1	
		OR4:	Use cosine formula in triangle *AOD* to find cos *AOD* or cos *ADO*	M1	
			Obtain $\cos AOD = \frac{8}{18}$ or $\cos ADO = \frac{10}{6\sqrt{10}}$, or equivalent	A1	
			Use trig to find the length of the perpendicular	M1	
			Obtain the given answer	A1	**[4]**
10	**(i)**		State or imply $V = \pi h^3$	B1	
			State or imply $\frac{dV}{dt} = -k\sqrt{h}$	B1	
			Use $\frac{dV}{dt} = \frac{dV}{dh}.\frac{dh}{dt}$, or equivalent	M1	
			Obtain the given equation	A1	**[4]**
			[The M1 is only available if $\frac{dV}{dh}$ is in terms of *h* and has been obtained by a correct method.]		
			[Allow B1 for $\frac{dV}{dt} = k\sqrt{h}$ but withhold the final A1 until the polarity of the constant $\frac{k}{3\pi}$ has been justified.]		

(ii)	Separate variables and integrate at least one side	M1	
	Obtain terms $\frac{2}{5}h^{\frac{5}{2}}$ and $-At$, or equivalent	A1	
	Use $t=0, h=H$ in a solution containing terms of the form $ah^{\frac{5}{2}}$ and $bt+c$	M1	
	Use $t = 60$, $h = 0$ in a solution containing terms of the form $ah^{\frac{5}{2}}$ and $bt + c$	M1	
	Obtain a correct solution in any form, e.g. $\frac{2}{5}h^{\frac{5}{2}} = \frac{1}{150}H^{\frac{5}{2}}t + \frac{2}{5}H^{\frac{5}{2}}$	A1	
(ii)	Obtain final answer $t = 60\left(1-\left(\frac{h}{H}\right)^{\frac{5}{2}}\right)$, or equivalent	A1	**[6]**
(iii)	Substitute $h=\frac{1}{2}H$ and obtain answer $t = 49.4$	B1	**[1]**

CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Level

MARK SCHEME for the October/November 2013 series

9709 MATHEMATICS

9709/32 Paper 3, maximum raw mark 75

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the October/November 2013 series for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level components and some Ordinary Level components.

CAMBRIDGE
International Examinations

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

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	OR:	Obtain $3^x = 2$ by solving an equation or by inspection	B1	
		Obtain $3^x=\frac{2}{3}$ (or $3^{x+1} = 2$) by solving an equation or by inspection	B1	
		Use correct method for solving an equation of the form $3^x = a$ (or $3^{x+1} = a$), where $a > 0$	M1	
		Obtain final answers 0.631 and –0.369	A1	[4]
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		Obtain $2x^{\frac{1}{2}}\ln x-2\int\frac{1}{x^{\frac{1}{2}}}\,dx$, or equivalent	A1	
		Integrate again and obtain $2x^{\frac{1}{2}}\ln x-4x^{\frac{1}{2}}$, or equivalent	A1	
		Substitute limits $x = 1$ and $x = 4$, having integrated twice	M1(dep*)	
		Obtain answer $4(\ln 4-1)$, or exact equivalent	A1	
	OR1:	Using $u = \ln x$, or equivalent, integrate by parts and reach $kue^{\frac{1}{2}u}-m\int e^{\frac{1}{2}u}\,du$	M1*	
		Obtain $2ue^{\frac{1}{2}u}-2\int e^{\frac{1}{2}u}\,du$, or equivalent	A1	
		Integrate again and obtain $2ue^{\frac{1}{2}u}-4e^{\frac{1}{2}u}$, or equivalent	A1	
		Substitute limits $u = 0$ and $u = \ln 4$, having integrated twice	M1(dep*)	
		Obtain answer $4\ln 4-4$, or exact equivalent	A1	
	OR2:	Using $u=\sqrt{x}$, or equivalent, integrate and obtain $ku\ln u-m\int u.\frac{1}{u}\,du$	M1*	
		Obtain $4u\ln u-4\int 1\,du$, or equivalent	A1	
		Integrate again and obtain $4u\ln u-4u$, or equivalent	A1	
		Substitute limits $u = 1$ and $u = 2$, having integrated twice or quoted $\int\ln u\,du$ as $u\ln u\pm u$	M1(dep*)	
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	OR3:	Integrate by parts and reach $I=\frac{x\ln x\pm x}{\sqrt{x}}+k\int\frac{x\ln x\pm x}{x\sqrt{x}}\,dx$	M1*	
		Obtain $I=\frac{x\ln x-x}{\sqrt{x}}+\frac{1}{2}I-\frac{1}{2}\int\frac{1}{\sqrt{x}}\,dx$	A1	
		Integrate and obtain $I=2\sqrt{x}\ln x-4\sqrt{x}$, or equivalent	A1	
		Substitute limits $x = 1$ and $x = 4$, having integrated twice	M1(dep*)	
		Obtain answer $4\ln 4-4$, or exact equivalent	A1	[5]

4		Use correct product or quotient rule at least once	M1*	
		Obtain $\frac{\mathrm{d}x}{\mathrm{d}t} = \mathrm{e}^{-t}\sin t - \mathrm{e}^{-t}\cos t$ or $\frac{\mathrm{d}y}{\mathrm{d}t} = \mathrm{e}^{-t}\cos t - \mathrm{e}^{-t}\sin t$, or equivalent	A1	
		Use $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}t} \div \frac{\mathrm{d}x}{\mathrm{d}t}$	M1	
		Obtain $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\sin t - \cos t}{\sin t + \cos t}$, or equivalent	A1	
		EITHER: Express $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of tan *t* only	M1(dep*)	
		Show expression is identical to $\tan\left(t - \frac{1}{4}\pi\right)$	A1	
		OR: Express $\tan\left(t - \frac{1}{4}\pi\right)$ in terms of tan *t*	M1	
		Show expression is identical to $\frac{\mathrm{d}y}{\mathrm{d}x}$	A1	**[6]**
5	**(i)**	Use Pythagoras	M1	
		Use the sin2*A* formula	M1	
		Obtain the given result	A1	**[3]**
	(ii)	Integrate and obtain a $k\ln\sin\theta$ or $m\ln\cos\theta$ term, or obtain integral of the form $p\ln\tan\theta$	M1*	
		Obtain indefinite integral $\frac{1}{2}\ln\sin\theta - \frac{1}{2}\ln\cos\theta$, or equivalent, or $\frac{1}{2}\ln\tan\theta$	A1	
		Substitute limits correctly	M1(dep)*	
		Obtain the given answer correctly having shown appropriate working	A1	**[4]**
6	**(i)**	State or imply $AB = 2r\cos\theta$ or $AB^2 = 2r^2 - 2r^2\cos(\pi - 2\theta)$	B1	
		Use correct formula to express the area of sector *ABC* in terms of *r* and θ	M1	
		Use correct area formulae to express the area of a segment in terms of *r* and θ	M1	
		State a correct equation in *r* and θ in any form	A1	
		Obtain the given answer	A1	**[5]**
		[SR: If the complete equation is approached by adding two sectors to the shaded area above *BO* and *OC* give the first M1 as on the scheme, and the second M1 for using correct area formulae for a triangle *AOB* or *AOC*, and a sector *AOB* or *AOC*.]		
	(ii)	Use the iterative formula correctly at least once	M1	
		Obtain final answer 0.95	A1	
		Show sufficient iterations to 4 d.p. to justify 0.95 to 2 d.p., or show there is a sign change in the interval (0.945, 0.955)	A1	**[3]**

7	(i)	State or imply partial fractions are of the form $\dfrac{A}{x-2}+\dfrac{Bx+C}{x^2+3}$	B1	
		Use a relevant method to determine a constant	M1	
		Obtain one of the values $A=-1$, $B=3$, $C=-1$	A1	
		Obtain a second value	A1	
		Obtain the third value	A1	[5]
	(ii)	Use correct method to obtain the first two terms of the expansions of $(x-2)^{-1}$, $\left(1-\dfrac{1}{2}x\right)^{-1}$, $(x^2+3)^{-1}$ or $\left(1+\dfrac{1}{3}x^2\right)^{-1}$	M1	
		Substitute correct unsimplified expansions up to the term in x^2 into each partial fraction	A1√+A1√	
		Multiply out fully by $Bx+C$, where $BC \neq 0$	M1	
		Obtain final answer $\dfrac{1}{6}+\dfrac{5}{4}x+\dfrac{17}{72}x^2$, or equivalent	A1	[5]
		[Symbolic binomial coefficients, e.g. $\begin{pmatrix}-1\\1\end{pmatrix}$ are not sufficient for the M1. The f.t. is on *A, B, C.*]		
		[In the case of an attempt to expand $(2x^2-7x-1)(x-2)^{-1}(x^2+3)^{-1}$, give M1A1A1 for the expansions, M1 for multiplying out fully, and A1 for the final answer.]		
		[If *B* or *C* omitted from the form of partial fractions, give B0M1A0A0A0 in **(i)**; M1A1√A1√ in **(ii)**]		
8	(a)	*EITHER:* Solve for *u* or for *v*	M1	
		Obtain $u=\dfrac{2\mathrm{i}-6}{1-2\mathrm{i}}$ or $v=\dfrac{5}{1-2\mathrm{i}}$, or equivalent	A1	
		Either: Multiply a numerator and denominator by conjugate of denominator, or equivalent		
		Or: Set *u* or *v* equal to $x+\mathrm{i}y$, obtain two equations by equating real and imaginary parts and solve for *x* or for *y*	M1	
		OR: Using $a+\mathrm{i}b$ and $c+\mathrm{i}d$ for *u* and *v*, equate real and imaginary parts and obtain four equations in *a, b, c* and *d*	M1	
		Obtain $b+2d=2$, $a+2c=0$, $a+d=0$ and $-b+c=3$, or equivalent	A1	
		Solve for one unknown	M1	
		Obtain final answer $u=-2-2\mathrm{i}$, or equivalent	A1	
		Obtain final answer $v=1+2\mathrm{i}$, or equivalent	A1	[5]
	(b)	Show a circle with centre $-\mathrm{i}$	B1	
		Show a circle with radius 1	B1	
		Show correct half line from 2 at an angle of $\dfrac{3}{4}\pi$ to the real axis	B1	
		Use a correct method for finding the least value of the modulus	M1	
		Obtain final answer $\dfrac{3}{\sqrt{2}}-1$, or equivalent, e.g. 1.12 (allow 1.1)	A1	[5]

9	**(i)**	*EITHER:*	Obtain a vector parallel to the plane, e.g. $\overrightarrow{AB} = -2\mathbf{i} + 4\mathbf{j} - \mathbf{k}$	B1	
			Use scalar product to obtain an equation in *a, b, c,* e.g. $-2a + 4b - c = 0$, $3a - 3b + 3c = 0$, or $a + b + 2c = 0$	M1	
			Obtain two correct equations in *a, b, c*	A1	
			Solve to obtain ratio $a : b : c$	M1	
			Obtain $a : b : c = 3 : 1 : -2$, or equivalent	A1	
			Obtain equation $3x + y - 2z = 1$, or equivalent	A1	
		OR1:	Substitute for two points, e.g. *A* and *B*, and obtain $2a - b + 2c = d$ and $3b + c = d$	B1	
			Substitute for another point, e.g. *C*, to obtain a third equation and eliminate one unknown entirely from the three equations	M1	
			Obtain two correct equations in three unknowns, e.g. in *a, b, c*	A1	
			Solve to obtain their ratio, e.g. $a : b : c$	M1	
			Obtain $a : b : c = 3 : 1 : -2$, $a : c : d = 3 : -2 : 1$, $a : b : d = 3 : 1 : 1$ or $b : c : d = -1 : -2 : 1$	A1	
			Obtain equation $3x + y - 2z = 1$, or equivalent	A1	
		OR2:	Obtain a vector parallel to the plane, e.g. $\overrightarrow{BC} = 3\mathbf{i} - 3\mathbf{j} + 3\mathbf{k}$	B1	
			Obtain a second such vector and calculate their vector product e.g. $(-2\mathbf{i} + 4\mathbf{j} - \mathbf{k}) \times (3\mathbf{i} - 3\mathbf{j} + 3\mathbf{k})$	M1	
			Obtain two correct components of the product	A1	
			Obtain correct answer, e.g. $9\mathbf{i} + 3\mathbf{j} - 6\mathbf{k}$	A1	
			Substitute in $9x + 3y - 6z = d$ to find *d*	M1	
			Obtain equation $9x + 3y - 6z = 3$, or equivalent	A1	
		OR3:	Obtain a vector parallel to the plane, e.g. $\overrightarrow{AC} = \mathbf{i} + \mathbf{j} + 2\mathbf{k}$	B1	
			Obtain a second such vector and form correctly a 2-parameter equation for the plane	M1	
			Obtain a correct equation, e.g. $\mathbf{r} = 3\mathbf{i} + 4\mathbf{k} + \lambda(-2\mathbf{i} + 4\mathbf{j} - \mathbf{k}) + \mu(\mathbf{i} + \mathbf{j} + 2\mathbf{k})$	A1	
			State three correct equations in x, y, z, λ, μ	A1	
			Eliminate λ and μ	M1	
			Obtain equation $3x + y - 2z = 1$, or equivalent	A1	**[6]**
	(ii)		Obtain answer $\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$, or equivalent	B1	**[1]**

	(iii)	*EITHER:*	Use $\dfrac{\overrightarrow{OA}.\overrightarrow{OD}}{\left\lvert\overrightarrow{OD}\right\rvert}$ to find projection *ON* of *OA* onto *OD*	M1	
			Obtain $ON = \dfrac{4}{3}$	A1	
			Use Pythagoras in triangle *OAN* to find *AN*	M1	
			Obtain the given answer	A1	
		OR1:	Calculate the vector product of $\overrightarrow{OA}$ and $\overrightarrow{OD}$	M1	
			Obtain answer 6**i** + 2**j** – 5**k**	A1	
			Divide the modulus of the vector product by the modulus of $\overrightarrow{OD}$	M1	
			Obtain the given answer	A1	
		OR2:	Taking general point *P* of *OD* to have position vector $\lambda(\mathbf{i}+2\mathbf{j}+2\mathbf{k})$, form an equation in λ by either equating the scalar product of $\overrightarrow{AP}$ and $\overrightarrow{OP}$ to zero, or using Pythagoras in triangle *OPA,* or setting the derivative of $\left\lvert\overrightarrow{AP}\right\rvert$ to zero	M1	
			Solve and obtain $\lambda = \dfrac{4}{9}$	A1	
			Carry out method to calculate *AP* when $\lambda = \dfrac{4}{9}$	M1	
			Obtain the given answer	A1	
		OR3:	Use a relevant scalar product to find the cosine of *AOD* or *ADO*	M1	
			Obtain $\cos AOD = \dfrac{4}{9}$ or $\cos ADO = \dfrac{5}{3\sqrt{10}}$, or equivalent	A1	
			Use trig to find the length of the perpendicular	M1	
			Obtain the given answer	A1	
		OR4:	Use cosine formula in triangle *AOD* to find cos *AOD* or cos *ADO*	M1	
			Obtain $\cos AOD = \dfrac{8}{18}$ or $\cos ADO = \dfrac{10}{6\sqrt{10}}$, or equivalent	A1	
			Use trig to find the length of the perpendicular	M1	
			Obtain the given answer	A1	**[4]**
10	**(i)**		State or imply $V = \pi h^3$	B1	
			State or imply $\dfrac{\mathrm{d}V}{\mathrm{d}t} = -k\sqrt{h}$	B1	
			Use $\dfrac{\mathrm{d}V}{\mathrm{d}t} = \dfrac{\mathrm{d}V}{\mathrm{d}h}.\dfrac{\mathrm{d}h}{\mathrm{d}t}$, or equivalent	M1	
			Obtain the given equation	A1	**[4]**
			[The M1 is only available if $\dfrac{\mathrm{d}V}{\mathrm{d}h}$ is in terms of *h* and has been obtained by a correct method.]		
			[Allow B1 for $\dfrac{\mathrm{d}V}{\mathrm{d}t} = k\sqrt{h}$ but withhold the final A1 until the polarity of the constant $\dfrac{k}{3\pi}$ has been justified.]		

(ii)	Separate variables and integrate at least one side	M1	
	Obtain terms $\frac{2}{5}h^{\frac{5}{2}}$ and $-At$, or equivalent	A1	
	Use $t = 0, h = H$ in a solution containing terms of the form $ah^{\frac{5}{2}}$ and $bt + c$	M1	
	Use $t = 60$, $h = 0$ in a solution containing terms of the form $ah^{\frac{5}{2}}$ and $bt + c$	M1	
	Obtain a correct solution in any form, e.g. $\frac{2}{5}h^{\frac{5}{2}} = \frac{1}{150}H^{\frac{5}{2}}t + \frac{2}{5}H^{\frac{5}{2}}$	A1	
(ii)	Obtain final answer $t = 60\left(1 - \left(\frac{h}{H}\right)^{\frac{5}{2}}\right)$, or equivalent	A1	**[6]**
(iii)	Substitute $h = \frac{1}{2}H$ and obtain answer $t = 49.4$	B1	**[1]**

CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Level

MARK SCHEME for the October/November 2013 series

9709 MATHEMATICS

9709/33 Paper 3, maximum raw mark 75

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the October/November 2013 series for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level components and some Ordinary Level components.

CAMBRIDGE
International Examinations

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

Note: B2 or A2 means that the candidate can earn 2 or 0.
B2/1/0 means that the candidate can earn anything from 0 to 2.

The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF	Any Equivalent Form (of answer is equally acceptable)
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
BOD	Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)
CAO	Correct Answer Only (emphasising that no “follow through” from a previous error is allowed)
CWO	Correct Working Only – often written by a ‘fortuitous’ answer
ISW	Ignore Subsequent Working
MR	Misread
PA	Premature Approximation (resulting in basically correct work that is insufficiently accurate)
SOS	See Other Solution (the candidate makes a better attempt at the same question)
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become “follow through ✓” marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

1		Apply at least one logarithm property correctly	*M1	
		Obtain $\frac{(x+4)^2}{x} = x + a$ or equivalent **without logarithm** involved	A1	
		Rearrange to express x in terms of a	M1	d*M
		Obtain $\frac{16}{a-8}$ or equivalent	A1	[4]
2		Carry out complete substitution including the use of $\frac{\mathrm{d}u}{\mathrm{d}x} = 3$	M1	
		Obtain $\int\left(\frac{1}{3} - \frac{1}{3u}\right)\mathrm{d}u$	A1	
		Integrate to obtain form $k_1u + k_2\ln u$ or $k_1u + k_2\ \ln 3u$ where $k_1k_2 \neq 0$	M1	
		Obtain $\frac{1}{3}(3x+1) - \frac{1}{3}\ln(3x+1)$ or equivalent, condoning absence of modulus signs and + c	A1	[4]
3	**(i)**	Substitute –2 and equate to zero or divide by $x + 2$ and equate remainder to zero or use –2 in synthetic division	M1	
		Obtain $a = -1$	A1	[2]
	(ii)	Attempt to find quadratic factor by division reaching $x^2 + kx$, or inspection as far as $(x+2)(x^2 + Bx + c)$ and equations for one or both of B and C, or $(x+2)(Ax^2 + Bx + 7)$ and equations for one or both of A and B.	M1	
		Obtain $x^2 - 3x + 7$	A1	
		Use discriminant to obtain –19, or equivalent, and **confirm one root** cwo	A1	[3]
4		Differentiate y^3 to obtain $3y^2\frac{\mathrm{d}y}{\mathrm{d}x}$	B1	
		Use correct product rule at least once	*M1	
		Obtain $6\mathrm{e}^{2x}y + 3\mathrm{e}^{2x}\frac{\mathrm{d}y}{\mathrm{d}x} + \mathrm{e}^xy^3 + 3\mathrm{e}^xy^2\frac{\mathrm{d}y}{\mathrm{d}x}$ as derivative of LHS	A1	
		Equate derivative of LHS to zero, substitute $x = 0$ and $y = 2$ and find value of $\frac{\mathrm{d}y}{\mathrm{d}x}$	M1(d*M)	
		Obtain $-\frac{4}{3}$ or equivalent as **final answer**	A1	[5]
5	**(i)**	Use integration by parts to obtain $ax\mathrm{e}^{-\frac{1}{2}x} + \int b\mathrm{e}^{-\frac{1}{2}x}\,\mathrm{d}x$	M1*	
		Obtain $-8x\mathrm{e}^{-\frac{1}{2}x} + \int 8\mathrm{e}^{-\frac{1}{2}x}\,\mathrm{d}x$ or unsimplified equivalent	A1	
		Obtain $-8x\mathrm{e}^{-\frac{1}{2}x} - 16\mathrm{e}^{-\frac{1}{2}x}$	A1	
		Use limits correctly and equate to 9	M1(d*M)	
		Obtain given answer $p = 2\ln\left(\frac{8p+16}{7}\right)$ correctly	A1	[5]

	(ii)	Use correct iteration formula correctly at least once		M1	
		Obtain final answer 3.77		A1	
		Show sufficient iterations to 5sf or better to justify accuracy 3.77 or show sign change in interval (3.765, 3.775)		A1	[3]
		$[3.5 \rightarrow 3.6766 \rightarrow 3.7398 \rightarrow 3.7619 \rightarrow 3.7696 \rightarrow 3.7723]$			
6	**(i)**	Find scalar product of the normals to the planes		M1	
		Using the correct process for the moduli, divide the scalar product by the product of the moduli and find $\cos^{-1}$ of the result.		M1	
		Obtain 67.8° (or 1.18 radians)		A1	[3]
	(ii)	<u>EITHER</u>	Carry out complete method for finding point on line	M1	
			Obtain one such point, e.g. $(2,-3,0)$ or $\left(\frac{17}{7},0,\frac{6}{7}\right)$ or $(0,-17,-4)$ or …	A1…	
			<u>Either</u> State $3a-b+2c=0$ and $a+b-4c=0$ or equivalent	B1	
			Attempt to solve for one ratio, e.g. $a:b$	M1	
			Obtain $a:b:c=1:7:2$ or equivalent	A1	
			State a correct final answer, e.g. r = [2, –3, 0] + λ[1, 7, 2]	A1√	
			<u>Or 1</u> Obtain a second point on the line	A1	
			Subtract position vectors to obtain direction vector	M1	
			Obtain [1, 7, 2] or equivalent	A1	
			State a correct final answer, e.g. r = [2, –3, 0] + λ[1, 7, 2]	A1√	
			<u>Or 2</u> Use correct method to calculate vector product of two normals	M1	
			Obtain two correct components	A1	
			Obtain [2, 14, 4] or equivalent	A1	
			State a correct final answer, e.g. r = [2, –3, 0] + λ[1, 7, 2]	A1√	
			[√ is dependent on both M marks in all three cases]		
		<u>OR 3</u>	Express one variable in terms of a second variable	M1	
			Obtain a correct simplified expression, e.g. $x=\frac{1}{2}(4+z)$	A1	
			Express the first variable in terms of third variable	M1	
			Obtain a correct simplified expression, e.g. $x=\frac{1}{7}(17+y)$	A1	
			Form a vector equation for the line	M1	
			State a correct final answer, e.g. r = [0, –17, –4] + λ[1, 7, 2]	A1	
		<u>OR 4</u>	Express one variable in terms of a second variable	M1	
			Obtain a correct simplified expression, e.g. $z=2x-4$	A1	
			Express third variable in terms of the second variable	M1	
			Obtain a correct simplified expression, e.g. $y=7x-17$	A1	
			Form a vector equation for the line	M1	
			State a correct final answer, e.g. r = [0, –17, –4] + λ[1, 7, 2]	A1	[6]

7	**(i)**		Use $\sec\theta = \frac{1}{\cos\theta}$ and $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$	B1	
			Use $\sin 2\theta = 2\sin\theta\cos\theta$ and to form a horizontal equation in $\sin\theta$ and $\cos\theta$ or fractions with common denominators	M1	
			Obtain given equation $2\sin\theta + 4\cos\theta = 3$ correctly	A1	[3]
	(ii)		State or imply $R = \sqrt{20}$ or 4.47 or equivalent	B1	
			Use correct trigonometry to find α	M1	
			Obtain 63.43 or 63.44 with no errors seen	A1	[3]
	(iii)		Carry out a correct method to find one value in given range	M1	
			Obtain 74.4° (or 338.7°)	A1	
			Carry out a correct method to find second value in given range	M1	
			Obtain 338.7° (or 74.4°) and no others between 0° and 360°	A1	[4]
8	**(i)**	Either	State or imply form $\frac{A}{1+x} + \frac{B}{(1+x)^2} + \frac{C}{2-3x}$	B1	
			Use any relevant method to find at least one constant	M1	
			Obtain $A = -1$	A1	
			Obtain $B = 3$	A1	
			Obtain $C = 4$	A1	
		Or	State or imply form $\frac{A}{1+x} + \frac{Bx}{(1+x)^2} + \frac{C}{2-3x}$	B1	
			Use any relevant method to find at least one constant	M1	
			Obtain $A = 2$	A1	
			Obtain $B = -3$	A1	
			Obtain $C = 4$	A1	
		Or	State or imply form $\frac{Dx+E}{(1+x)^2} + \frac{F}{2-3x}$	B1	
			Use any relevant method to find at least one constant	M1	
			Obtain $D = -1$	A1	
			Obtain $E = 2$	A1	
			Obtain $F = 4$	A1	[5]

	(ii)	Either	Use correct method to find first two terms of expansion of $(1+x)^{-1}$ or $(1+x)^{-2}$ or $(2-3x)^{-1}$ or $\left(1-\frac{3}{2}x\right)^{-1}$	M1	
			Obtain correct unsimplified expansion of first partial fraction up to x^2 term	A1√	
			Obtain correct unsimplified expansion of second partial fraction up to x^2 term	A1√	
			Obtain correct unsimplified expansion of third partial fraction up to x^2 term	A1√	
			Obtain final answer $4-2x+\frac{25}{2}x^2$	A1	
		Or 1	Use correct method to find first two terms of expansion of $(1+x)^{-2}$ or $(2-3x)^{-1}$ or $\left(1-\frac{3}{2}x\right)^{-1}$	M1	
			Obtain correct unsimplified expansion of first partial fraction up to x^2 term	A1√	
			Obtain correct unsimplified expansion of second partial fraction up to x^2 term	A1√	
			Expand and obtain sufficient terms to obtain three terms	M1	
			Obtain final answer $4-2x+\frac{25}{2}x^2$	A1	
		Or 2	(expanding original expression) Use correct method to find first two terms of expansion of $(1+x)^{-2}$ or $(2-3x)^{-1}$ or $\left(1-\frac{3}{2}x\right)^{-1}$	M1	
			Obtain correct expansion $1-2x+3x^2$ or unsimplified equivalent	A1	
			Obtain correct expansion $\frac{1}{2}\left(1+\frac{3}{2}x+\frac{9}{4}x^2\right)$ or unsimplified equivalent	A1	
			Expand and obtain sufficient terms to obtain three terms	M1	
			Obtain final answer $4-2x+\frac{25}{2}x^2$	A1	
		Or 3	(McLaurin expansion) Obtain first derivative $f'(x)=(1+x)^{-2}-6(1+x)^{-3}+12(2-3x)^{-2}$	M1	
			Obtain $f'(0)=1-6+3$ or equivalent	A1	
			Obtain $f''(0)=-2+18+9$ or equivalent	A1	
			Use correct form for McLaurin expansion	M1	
			Obtain final answer $4-2x+\frac{25}{2}x^2$	A1	[5]
9	**(a)**		Solve using formula, including simplification under square root sign	M1*	
			Obtain $\frac{-2\pm 4\mathrm{i}}{2(2-\mathrm{i})}$ or similarly simplified equivalents	A1	
			Multiply by $\frac{2+\mathrm{i}}{2+\mathrm{i}}$ or equivalent in at least one case	M1(d*M)	
			Obtain final answer $-\frac{4}{5}+\frac{3}{5}\mathrm{i}$	A1	
			Obtain final answer –i	A1	[5]

(b)	Show w in first quadrant with modulus and argument relatively correct	B1	
	Show w^3 in second quadrant with modulus and argument relatively correct	B1	
	Show w^* in fourth quadrant with modulus and argument relatively correct	B1	
	Use correct method for area of triangle	M1	
	Obtain 10 by calculation	A1	[5]

10	Use $2\cos^2 x = 1 + \cos 2x$ or equivalent	B1	
	Separate variables and integrate at least one side	M1	
	Obtain $\ln\left(y^3+1\right) = \ldots$ or equivalent	A1	
	Obtain $\ldots = 2x + \sin 2x$ or equivalent	A1	
	Use $x = 0$, $y = 2$ to find constant of integration (or as limits) in an expression containing at least two terms of the form $a\ln\left(y^3+1\right)$, bx or $c \sin 2x$	M1*	
	Obtain $\ln\left(y^3+1\right) = 2x + \sin 2x + \ln 9$ or equivalent e.g. implied by correct constant	A1	
	Identify at least one of $\frac{1}{2}\pi$ and $\frac{3}{2}\pi$ as x-coordinate at stationary point	B1	
	Use correct process to find y-coordinate for at least one x-coordinate	M1(d*M)	
	Obtain 5.9	A1	
	Obtain 48.1	A1	[10]

CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level and GCE Advanced Level

MARK SCHEME for the May/June 2013 series

9709 MATHEMATICS

9709/31 Paper 3, maximum raw mark 75

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

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International Examinations

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A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF	Any Equivalent Form (of answer is equally acceptable)
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
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CAO	Correct Answer Only (emphasising that no “follow through” from a previous error is allowed)
CWO	Correct Working Only – often written by a ‘fortuitous’ answer
ISW	Ignore Subsequent Working
MR	Misread
PA	Premature Approximation (resulting in basically correct work that is insufficiently accurate)
SOS	See Other Solution (the candidate makes a better attempt at the same question)
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become “follow through √” marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

1			Carry out division or equivalent at least as far as two terms of quotient	M1	
			Obtain quotient $2x-4$	A1	
			Obtain remainder 8	A1	[3]
2			Obtain $1-x$ as first two terms of $(1+2x)^{-\frac{1}{2}}$	B1	
			Obtain $+\frac{3}{2}x^2$ or unsimplified equivalent as third term of $(1+2x)^{-\frac{1}{2}}$	B1	
			Multiply $1+3x$ by attempt at $(1+2x)^{-\frac{1}{2}}$, obtaining sufficient terms	M1	
			Obtain final answer $1+2x-\frac{3}{2}x^2$	A1	[4]
3			State or imply correct form $\dfrac{A}{x}+\dfrac{Bx+C}{x^2+1}$	B1	
			Use any relevant method to find at least one constant	M1	
			Obtain $A=2$	A1	
			Obtain $B=5$	A1	
			Obtain $C=-3$	A1	[5]
4	(i)	Either	State or imply non-modular equation $(4x-1)^2=(x-3)^2$ or pair of linear equations $4x-1=\pm(x-3)$	B1	
			Solve a three-term quadratic equation or two linear equations	M1	
			Obtain $-\dfrac{2}{3}$ and $\dfrac{4}{5}$	A1	
		Or	Obtain value $-\dfrac{2}{3}$ from inspection or solving linear equation	B1	
			Obtain value $\dfrac{4}{5}$ similarly	B2	[3]
	(ii)		State or imply at least $4^y=\dfrac{4}{5}$, following a positive answer from part (i)	B1√	
			Apply logarithms and use $\log a^b = b\log a$ property	M1	
			Obtain –0.161 and no other answer	A1	[3]
5	(i)		Use correct quotient rule or equivalent	M1	
			Obtain $\dfrac{(1+e^{2x})2x-(1+x^2)2e^{2x}}{(1+e^{2x})^2}$ or equivalent	A1	
			Substitute $x = 0$ and obtain $-\dfrac{1}{2}$ or equivalent	A1	[3]
	(ii)		Differentiate y^3 and obtain $3y^2\dfrac{dy}{dx}$	B1	
			Differentiate $5xy$ and obtain $5y+5x\dfrac{dy}{dx}$	B1	
			Obtain $6x^2+5y+5x\dfrac{dy}{dx}+3y^2\dfrac{dy}{dx}=0$	B1	

			Substitute $x = 0, y = 2$ to obtain $-\frac{5}{6}$ or equivalent following correct work	B1	[4]
6	**(i)**		State or imply A is (1, 4, –2)	B1	
			State or imply $\overrightarrow{QP} = 12\mathbf{i} + 6\mathbf{j} - 6\mathbf{k}$ or equivalent	B1	
			Use QP as normal and A as mid-point to find equation of plane	M1	
			Obtain $12x + 6y - 6z = 48$ or equivalent	A1	[4]
	(ii)	Either	State equation of PB is $\mathbf{r} = 7\mathbf{i} + 7\mathbf{j} - 5\mathbf{k} + \lambda\mathbf{i}$	B1	
			Set up and solve a relevant equation for λ.	M1	
			Obtain $\lambda = -9$ and hence B is (–2, 7, –5)	A1	
			Use correct method to find distance between A and B.	M1	
			Obtain 5.20	A1	
		Or	Obtain 12 for result of scalar product of QP and $\mathbf{i}$ or equivalent	B1	
			Use correct method involving moduli, scalar product and cosine to find angle APB	M1	
			Obtain $35.26°$ or equivalent	A1	
			Use relevant trigonometry to find AB	M1	
			Obtain 5.20	A1	[5]
7	**(a)**		State or imply $3a + 3b\mathrm{i} + 2\mathrm{i}(a - b\mathrm{i}) = 17 + 8\mathrm{i}$	B1	
			Consider real and imaginary parts to obtain two linear equations in a and b	M1*	
			Solve two simultaneous linear equations for a or b	M1 (dep*)	
			Obtain $7 - 2\mathrm{i}$	A1	[4]
	(b)	Either	Show or imply a triangle with side 2	B1	
			State at least two of the angles $\frac{1}{4}\pi, \frac{2}{3}\pi$ and $\frac{1}{12}\pi$	B1	
			State or imply argument is $\frac{1}{4}\pi$	B1	
			Use sine rule or equivalent to find r	M1	
			Obtain $6.69\mathrm{e}^{\frac{1}{4}\pi\mathrm{i}}$	A1	
		Or	State $y = x$.	B1	
			State $y = \frac{1}{\sqrt{3}}x + 2$ or $\frac{\sqrt{3}}{2} = \frac{x}{\sqrt{x^2 + (y-2)^2}}$ or $\frac{1}{2} = \frac{y-2}{\sqrt{x^2 + (y-2)^2}}$	B1	
			State or imply argument is $\frac{\pi}{4}$	B1	
			Solve for x or y.	M1	
			Obtain $6.69e^{\frac{1}{4}\pi i}$	A1	[5]
8	**(a)**		Carry out integration by parts and reach $ax^2 \ln x + b\int \frac{1}{2}x^2 \mathrm{d}x$	M1*	
			Obtain $2x^2 \ln x - \int \frac{1}{x} . 2x^2 \, \mathrm{d}x$	A1	
			Obtain $2x^2 \ln x - x^2$	A1	
			Use limits, having integrated twice	M1 (dep*)	
			Confirm given result $56 \ln 2 - 12$	A1	[5]

	(b)	State or imply $\frac{du}{dx} = 4\cos 4x$	B1	
		Carry out complete substitution except limits	M1	
		Obtain $\int(\frac{1}{4} - \frac{1}{4}u^2)\,du$ or equivalent	A1	
		Integrate to obtain form $k_1u + k_2u^3$ with non-zero constants k_1, k_2	M1	
		Use appropriate limits to obtain $\frac{11}{96}$	A1	[5]
9	**(i)**	State or imply $R = 5$	B1	
		Use relevant trigonometry to find α	M1	
		Obtain $\alpha = 0.6435$	A1	[3]
	(ii) (a)	Carry out appropriate method to find one value in given range	M1	
		Obtain 1.80	A1	
		Carry out appropriate method to find second value in given range	M1	
		Obtain 5.77 and no other value	A1	[4]
	(b)	Express integrand as $k\sec^2(\theta - \text{their } \alpha)$ for any constant k	M1	
		Integrate to obtain result $k\tan(\theta - \text{their } \alpha)$	A1	
		Obtain correct answer $2\tan(\theta - 0.6435)$	A1	[3]
10	**(i)**	State $\frac{dV}{dt} = 80 - kV$	B1	
		Correctly separate variables and attempt integration of one side	M1	
		Obtain $a\ln(80 - kV) = t$ or equivalent	M1*	
		Obtain $-\frac{1}{k}\ln(80 - kV) = t$ or equivalent	A1	
		Use $t = 0$ and $V = 0$ to find constant of integration or as limits	M1 (dep*)	
		Obtain $-\frac{1}{k}\ln(80 - kV) = t - \frac{1}{k}\ln 80$ or equivalent	A1	
		Obtain given answer $V = \frac{1}{k}(80 - 80e^{-kt})$ correctly	A1	[7]
	(ii)	Use iterative formula correctly at least once	M1	
		Obtain final answer 0.14	A1	
		Show sufficient iterations to 4 s.f. to justify answer to 2 s.f. or show a sign change in the interval (0.135, 0.145)	A1	[3]
	(iii)	State a value between 530 and 540 cm^3 inclusive	B1	
		State or imply that volume approaches 569 cm^3 (allowing any value between 567 and 571 inclusive)	B1	[2]

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9709/32 Paper 3, maximum raw mark 75

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1	*EITHER*:	State or imply non-modular equation $(x-2)^2 = \left(\frac{1}{3}x\right)^2$,		
		or pair of equations $x - 2 = \pm\frac{1}{3}x$	M1	
		Obtain answer $x = 3$	A1	
		Obtain answer $x = \frac{3}{2}$, or equivalent	A1	
	OR:	Obtain answer $x = 3$ by solving an equation or by inspection	B1	
		State or imply the equation $x - 2 = -\frac{1}{3}$, or equivalent	M1	
		Obtain answer $x = \frac{3}{2}$, or equivalent	A1	[3]
2	**(i)**	Use the iterative formula correctly at least once	M1	
		Obtain final answer 3.6840	A1	
		Show sufficient iterations to at least 6 d.p. to justify 3.6840, or show there is a sign change in the interval (3.68395, 3.68405)	A1	[3]
	(ii)	State a suitable equation, e.g. $x = \frac{x(x^3+100)}{2(x^3+25)}$	B1	
		State that the value of α is $3\sqrt{50}$, or exact equivalent	B1	[2]
3	*EITHER*:	State or imply $\ln y = \ln A - kx^2$	B1	
		Substitute values of $\ln y$ and x^2, and solve for k or $\ln A$	M1	
		Obtain $k = 0.42$ or $A = 2.80$	A1	
		Solve for $\ln A$ or k	M1	
		Obtain $A = 2.80$ or $k = 0.42$	A1	
	OR1:	State or imply $\ln y = \ln A - kx^2$	B1	
		Using values of $\ln y$ and x^2, equate gradient of line to $-k$ and solve for k	M1	
		Obtain $k = 0.42$	A1	
		Solve for $\ln A$	M1	
		Obtain $A = 2.80$	A1	
	OR2:	Obtain two correct equations in k and A and substituting y– and x^2 – values in $y = Ae^{-kx^2}$	B1	
		Solve for k	M1	
		Obtain $k = 0.42$	A1	
		Solve for A	M1	
		Obtain $A = 2.80$	A1	[5]
		[SR: If unsound substitutions are made, e.g. using $x = 0.;64$ and $y = 0.76$, give B1M0A0M1A0 in the *EITHER* and *OR1* schemes, and B0M1A0M1A0 in the *OR2* scheme.]		

4	**(i)**	Substitute $x = -\frac{1}{3}$, or divide by $3x+1$, and obtain a correct equation,		
		e.g. $-\frac{1}{27}a - \frac{20}{9} - \frac{1}{3} + 3 = 0$	B1	
		Solve for a an equation obtained by a valid method	M1	
		Obtain $a = 12$	A1	[3]
	(ii)	Commence division by $3x+1$ reaching a partial quotient $\frac{1}{3}ax^2 + kx$	M1	
		Obtain quadratic factor $4x^2 - 8x + 3$	A1	
		Obtain factorisation $(3x+1)(2x-1)(2x-3)$	A1	[3]
		[The M1 is earned if inspection reaches an unknown factor $\frac{1}{3}ax^2 + Bx + C$ and an equation in B and/or C, or an unknown factor $Ax^2 + Bx + 3$ and an equation in A and/or B, or if two coefficients with the correct moduli are stated without working.] [If linear factors are found by the factor theorem, give B1B1 for $(2x-1)$ and $(2x-3)$, and B1 for the complete factorisation.] [Synthetic division giving $12x^2 - 24x + 9$ as quadratic factor earns M1A1, but the final factorisation needs $(x+\frac{1}{3})$, or equivalent, in order to earn the second A1.] [SR: If $x = \frac{1}{3}$ is used in substitution or synthetic division, give the M1 in part **(i)** but give M0 in part **(ii)**.]		
5	*EITHER*:	State $2ay\frac{dy}{dx}$ as derivative of ay^2	B1	
		State $y^2 + 2xy\frac{dy}{dx}$ as derivative of xy^2	B1	
		Equate derivative of LHS to zero and set $\frac{dy}{dx}$ equal to zero	M1	
		Obtain $3x^2 + y^2 - 6ax = 0$, or horizontal equivalent	A1	
		Eliminate y and obtain an equation in x	M1	
		Solve for x and obtain answer $x = \sqrt{3}a$	A1	
	OR1:	Rearrange equation in the form $y^2 = \frac{3ax^2 - x^3}{x+a}$ and attempt differentiation of one side	B1	
		Use correct quotient or product rule to differentiate RHS	M1	
		Obtain correct derivative of RHS in any form	A1	
		Set $\frac{dy}{dx}$ equal to zero and obtain an equation in x	M1	
		Obtain a correct horizontal equation free of surds	A1	
		Solve for x and obtain answer $x = \sqrt{3}a$	A1	
	OR2:	Rearrange equation in the form $y = \left(\frac{3ax^2 - x^3}{x+a}\right)^{\frac{1}{2}}$ and differentiation of RHS	B1	
		Use correct quotient or product rule and chain rule	M1	
		Obtain correct derivative in any form	A1	

		Equate derivative to zero and obtain an equation in x	M1	
		Obtain a correct horizontal equation free of surds	A1	
		Solve for x and obtain answer $x = \sqrt{3}a$	A1	[6]
6	(i)	Use correct quotient or chain rule to differentiate sec x	M1	
		Obtain given derivative, sec x tan x, correctly	A1	
		Use chain rule to differentiate y	M1	
		Obtain the given answer	A1	[4]
	(ii)	Using $\mathrm{d}x\sqrt{3}\sec^2\theta\ \mathrm{d}\theta$, or equivalent, express integral in terms of θ and $\mathrm{d}\theta$	M1	
		Obtain $\int\sec\theta\ \mathrm{d}\theta$	A1	
		Use limits $\frac{1}{6}\pi$ and $\frac{1}{3}\pi$ correctly in an integral form of the form $k\ln(\sec\theta + \tan\theta)$	M1	
		Obtain a correct exact final answer in the given form, e.g. $\ln\left(\frac{2+\sqrt{3}}{\sqrt{3}}\right)$	A1	[4]
7	(i)	Use cos (A + B) formula to express the given expression in terms of cos x and sin x	M1	
		Collect terms and reach $\frac{\cos x}{\sqrt{2}} - \frac{3}{\sqrt{2}}\sin x$, or equivalent	A1	
		Obtain $R = 2.236$	A1	
		Use trig formula to find α	M1	
		Obtain $\alpha = 71.57°$ with no errors seen	A1	[5]
	(ii)	Evaluate $\cos^{-1}$ (2/2.236) to at least 1 d.p. (26.56° to 2 d.p., use of $R = \sqrt{5}$ gives 26.57°)	B1√	
		Carry out an appropriate method to find a value of x in the interval $0° < x < 360°$	M1	
		Obtain answer, e.g. $x = 315°$ (315.0°)	A1	
		Obtain second answer, e.g. 261.9° and no others in the given interval	A1	[4]
		[Ignore answers outside the given range.]		
		[Treat answers in radians as a misread and deduct A1 from the answers for the angles.]		
		[SR: Conversion of the equation to a correct quadratic in sin x, cos x, or tan x earns B1, then M1 for solving a 3-term quadratic and obtaining a value of x in the given interval, and A1 + A1 for the two correct answers (candidates must reject spurious roots to earn the final A1).]		
8	(i)	Use any relevant method to determine a constant	M1	
		Obtain one of the values $A = 1$, $B = –2$, $C = 4$	A1	
		Obtain a second value	A1	
		Obtain the third value	A1	[4]
		[If A and C are found by the cover up rule, give B1 + B1 then M1A1 for finding B. If only one is found by the rule, give B1M1A1A1.]		
	(ii)	Separate variables and obtain one term by integrating $\frac{1}{y}$ or a partial fraction	M1	
		Obtain $\ln y = -\frac{1}{2} - 2\ln(2x+1) + c$, or equivalent	A3√	

		Evaluate a constant, or use limits $x = 1$, $y = 1$, in a solution containing at least three terms of the form $k \ln y$, l/x, $m \ln x$ and $n \ln (2x + 1)$, or equivalent	M1	
		Obtain solution $\ln y = -\frac{1}{2} - 2\ln x + 2\ln(2x+1) + c$, or equivalent	A1	
		Substitute $x = 2$ and obtain $y = \frac{25}{36}e^{\frac{1}{2}}$, or exact equivalent free of logarithms	A1	[7]
		(The f.t. is on A, B, C. Give A2√ if there is only one error or omission in the integration; A1√ if two.)		
9	**(a)**	Substitute $w = x + iy$ and state a correct equation in x and y	B1	
		Use $i^2 = -1$ and equate real parts	M1	
		Obtain $y = -2$	A1	
		Equate imaginary parts and solve for x	M1	
		Obtain $x = 2\sqrt{2}$, or equivalent, only	A1	[5]
	(b)	Show a circle with centre 2i	B1	
		Show a circle with radius 2	B1	
		Show half line from –2 at $\frac{1}{4}\pi$ to real axis	B1	
		Shade the correct region	B1	
		Carry out a complete method for calculating the greatest value of $\lvert z \rvert$	M1	
		Obtain answer 3.70	A1	[6]
10	**(i)**	Carry out a correct method for finding a vector equation for AB	M1	
		Obtain $\mathbf{r} = 2\mathbf{i} - 3\mathbf{j} + 2\mathbf{k} + \lambda(3\mathbf{i} + \mathbf{j} - \mathbf{k})$ or $\mathbf{r} = \mu(2\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}) + (1 - \mu)(5\mathbf{i} - 2\mathbf{j} + \mathbf{k})$, or equivalent	A1	
		Substitute components in equation of p and solve for λ or for μ	M1	
		Obtain $\lambda = \frac{3}{2}$ or $\mu = -\frac{1}{2}$ and final answer $\frac{13}{2}\mathbf{i} - \frac{3}{2}\mathbf{j} + \frac{1}{2}\mathbf{k}$, or equivalent	A1	[4]
	(ii)	Either equate scalar product of direction vector of AB and normal to q to zero or substitute for A and B in the equation of q and subtract expressions	M1*	
		Obtain $3 + b - c = 0$, or equivalent	A1	
		Using the correct method for the moduli, divide the scalar product of the normals to p and q by the product of their moduli and equate to $\pm\frac{1}{2}$, or form horizontal equivalent	M1*	
		Obtain correct equation in any form, e.g. $\frac{1+b}{\sqrt{(1+b^2+c^2)}\sqrt{(1+1)}} = \pm\frac{1}{2}$	A1	
		Solve simultaneous equations for b or for c	M1 (dep*)	
		Obtain $b = -4$ and $c = -1$	A1	
		Use a relevant point and obtain final answer $x - 4y - z = 12$, or equivalent	A1√	[7]
		(The f.t. is on b and c.)		

CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level and GCE Advanced Level

MARK SCHEME for the May/June 2013 series

9709 MATHEMATICS

9709/33 Paper 3, maximum raw mark 75

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the May/June 2013 series for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level components and some Ordinary Level components.

CAMBRIDGE
International Examinations

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF	Any Equivalent Form (of answer is equally acceptable)
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
BOD	Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)
CAO	Correct Answer Only (emphasising that no “follow through” from a previous error is allowed)
CWO	Correct Working Only – often written by a “fortuitous” answer
ISW	Ignore Subsequent Working
MR	Misread
PA	Premature Approximation (resulting in basically correct work that is insufficiently accurate)
SOS	See Other Solution (the candidate makes a better attempt at the same question)
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become “follow through √” marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

1	*EITHER*:	State or imply non-modular inequality $(4x + 3)^2 > x^2$, or corresponding equation or pair of equations $4x + 3 = \pm x$	M1	
		Obtain a critical value, e.g. -1	A1	
		Obtain a second critical value, e.g. $-\frac{3}{5}$	A1	
		State final answer $x < -1$, $x > -\frac{3}{5}$	A1	
	OR:	Obtain critical value $x = -1$, by solving a linear equation or inequality, or from a graphical method or by inspection	B1	
		Obtain the critical value $-\frac{3}{5}$ similarly	B2	
		State final answer $x < -1$, $x > -\frac{3}{5}$	B1	[4]
		[Do not condone $\leq$ or $\geq$.]		
2		Use law for the logarithm of a product, quotient or power	M1	
		Use ln e = 1 or exp(l) = 3	M1	
		Obtain correct equation free of logarithms in any form, e.g. $\frac{y+1}{y} = \mathrm{e}x^3$	A1	
		Rearrange as $y = (\mathrm{e}x^3 - 1)^{-1}$, or equivalent	A1	[4]
3		Use correct tan $2A$ formula and $\cot x = 1/\tan x$ to form an equation in $\tan x$	M1	
		Obtain a correct horizontal equation in any form	A1	
		Solve an equation in $\tan^2 x$ for x	M1	
		Obtain answer, e.g. 40.2°	A1	
		Obtain second answer, e.g. 139.8°, and no other in the given interval	A1√	[5]
		[Ignore answers outside the given interval.]		
		[Treat answers in radians as a misread and deduct A1 from the marks for the angles.]		
		[SR: For the answer $x = 90°$ give B1 and A1 for one of the other angles.]		
4	**(i)**	State $R = 2$	B1	
		Use trig formula to find α	M1	
		Obtain $\alpha = \frac{1}{6}\pi$ with no errors seen	A1	[3]
	(ii)	Substitute denominator of integrand and state integral $k \tan(x - \alpha)$	M1*	
		State correct indefinite integral $\frac{1}{4}\tan\left(x - \frac{1}{6}\pi\right)$	A1√	
		Substitute limits	M1 (dep*)	
		Obtain the given answer correctly	A1	[4]

5	**(i)**	Substitute $x = -\frac{1}{2}$, or divide by $(2x + 1)$, and obtain a correct equation, e.g. $a - 2b + 8 = 0$	B1	
		Substitute $x = \frac{1}{2}$ and equate to 1, or divide by $(2x - 1)$ and equate constant remainder to 1	M1	
		Obtain a correct equation, e.g. $a + 2b + 12 = 0$	A1	
		Solve for a or for b	M1	
		Obtain $a = -10$ and $b = -1$	A1	[5]
	(ii)	Divide by $2x^2 - 1$ and reach a quotient of the form $4x + k$	M1	
		Obtain quotient $4x - 5$	A1	
		Obtain remainder $3x - 2$	A1	[3]
6	**(i)**	State the correct derivatives $2e^{2x-3}$ and $2/x$	B1	
		Equate derivatives and use a law of logarithms on an equation equivalent to $ke^{2x-3} = m/x$	M1	
		Obtain the given result correctly (or work *vice versa*)	A1	[3]
	(ii)	Consider the sign of $a - \frac{1}{2}(3 - \ln a)$ when $a = 1$ and $a = 2$, or equivalent	M1	
		Complete the argument with correct calculated values	A1	[2]
	(iii)	Use the iterative formula correctly at least once	M1	
		Obtain final answer 1.35	A1	
		Show sufficient iterations to 4 d.p. to justify 1.35 to 2 d.p., or show there is a sign change in the interval (1.345, 1.355)	A1	[3]
7	**(i)**	Show that $a^2 + b^2 = (a + \mathrm{i}b)(a - \mathrm{i}b)$	B1	
		Show that $(a + \mathrm{i}b - k\mathrm{i})^* = a - \mathrm{i}b + k\mathrm{i}$	B1	[2]
	(ii)	Square both sides and express the given equation in terms of z and z^*	M1	
		Obtain a correct equation in any form, e.g. $(z - 10\mathrm{i})(z^* + 10\mathrm{i}) = 4(z - 4\mathrm{i})(z^* + 4\mathrm{i})$	A1	
		Obtain the given equation	A1	
		Either express $\lvert z - 2i\rvert = 4$ in terms of z and z^* or reduce the given equation to the form $\lvert z - u\rvert = r$	M1	
		Obtain the given answer correctly	A1	[5]
	(iii)	State that the locus is a circle with centre 2i and radius 5	B1	[1]

8	**(i)**	Separate variables correctly and integrate at least one side	M1	
		Obtain term ln t, or equivalent	B1	
		Obtain term of the form $a\ln(k-x^3)$	M1	
		Obtain term $-\frac{2}{3}\ln(k-x^3)$, or equivalent	A1	
		EITHER: Evaluate a constant or use limits $t = 1$, $x = 1$ in a solution containing $a \ln t$ and $b\ln(k-x^3)$	M1*	
		Obtain correct answer in any form e.g. $\ln t = -\frac{2}{3}\ln(k-x^3)+\frac{2}{3}\ln(k-1)$	A1	
		Use limits $t = 4$, $x = 2$, and solve for k	M1(dep*)	
		Obtain $k = 9$	A1	
		OR: Using limits $t = 1$, $x = 1$ and $t = 4$, $x = 2$ in a solution containing $a \ln t$ and $b\ln(k-x^3)$ obtain an equation in k	M1*	
		Obtain a correct equation in any form, e.g. $\ln 4 = -\frac{2}{3}\ln(k-8)+\frac{2}{3}\ln(k-1)$	A1	
		Solve for k	M1(dep*)	
		Obtain $k = 9$	A1	
		Substitute $k = 9$ and obtain $x = (9-8t^{-\frac{3}{2}})^{\frac{1}{3}}$	A1	[9]
	(ii)	State that x approaches $9^{\frac{1}{3}}$, or equivalent	B1√	[1]
9	**(i)**	Use product rule	M1	
		Obtain correct derivative in any form, e.g. $4\sin 2x\cos 2x\cos x - \sin^2 2x\sin x$	A1	
		Equate derivative to zero and use a double angle formula	M1*	
		Reduce equation to one in a single trig function	M1(dep*)	
		Obtain a correct equation in any form, e.g. $10\cos^3 x = 6\cos x$, $4 = 6\tan^2 x$ or $4 = 10\sin^2 x$	A1	
		Solve and obtain $x = 0.685$	A1	[6]
	(ii)	Using $\mathrm{d}u = \pm\cos x\,\mathrm{d}x$, or equivalent, express integral in terms of u and $\mathrm{d}u$	M1	
		Obtain $\int 4u^2(1-u^2)\,\mathrm{d}u$, or equivalent	A1	
		Use limits $u = 0$ and $u = 1$ in an integral of the form $au^3 + bu^5$	M1	
		Obtain answer $\frac{8}{15}$ (or 0.533)	A1	[4]
10	**(i)**	Equate scalar product of direction vector of l and p to zero	M1	
		Solve for a and obtain $a = -6$	A1	[2]
	(ii)	Express general point of l correctly in parametric form, e.g. $3\mathbf{i} + 2\mathbf{j} + \mathbf{k} + \mu(2\mathbf{i} + \mathbf{j} + 2\mathbf{k})$ or $(1-\mu)(3\mathbf{i} + 2\mathbf{j} + \mathbf{k}) + \mu(\mathbf{i} + \mathbf{j} - \mathbf{k})$	B1	
		Equate at least two pairs of corresponding components of l and the second line and solve for λ or for μ	M1	
		Obtain either $\lambda = \frac{2}{3}$ or $\mu = \frac{1}{3}$; or $\lambda = \frac{2}{a-1}$ or $\mu = \frac{1}{a-1}$; or reach $\lambda(a-4) = 0$ or $(1+\mu)(a-4) = 0$	A1	
		Obtain $a = 4$ having ensured (if necessary) that all three component equations are satisfied	A1	[4]

(iii) Using the correct process for the moduli, divide scalar product of direction vector if *l* and normal to *p* by the product of their moduli and equate to the sine of the given angle, or form an equivalent horizontal equation M1*

Use $\frac{2}{\sqrt{5}}$ as sine of the angle A1

State equation in any form, e.g. $\frac{a+6}{\sqrt{(a^2+4+1)}\sqrt{(1+4+4)}}=\frac{2}{\sqrt{5}}$ A1

Solve for a M1 (dep*)

Obtain answers for $a = 0$ and $a = \frac{60}{31}$, or equivalent A1 [5]

[Allow use of the cosine of the angle to score M1M1.]