* 0000800000001 *

Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

* 0 7 5 9 5 7 4 0 8 2 *

MATHEMATICS **9709/31**

Paper 3 Pure Mathematics 3 **October/November 2024**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
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- Do **not** write on any bar codes.
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- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

DC (DE/SG) 336974/5

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1 The polynomial $4x^3 + ax^2 + 5x + b$, where a and b are constants, is denoted by $\mathrm{p}(x)$. It is given that $(2x+1)$ is a factor of $\mathrm{p}(x)$. When $\mathrm{p}(x)$ is divided by $(x-4)$ the remainder is equal to 3 times the remainder when $\mathrm{p}(x)$ is divided by $(x-2)$.

Find the values of a and b. [5]

2 Find the exact value of $\int_1^3 x^2 \ln 3x \, dx$. Give your answer in the form $a \ln b + c$, where a and c are rational and b is an integer. [5]

3 The equation of a curve is $\ln(x+y) = 3x^2y$.

Find the gradient of the curve at the point $(1, 0)$. [4]

4 **(a)** Show that $\sec^4\theta - \tan^4\theta \equiv 1 + 2\tan^2\theta$. [3]

(b) Hence or otherwise solve the equation $\sec^4 2\alpha - \tan^4 2\alpha = 2\tan^2 2\alpha \sec^2 2\alpha$ for $0° < \alpha < 180°$. [5]

5 **(a)** By sketching a suitable pair of graphs, show that the equation $2+e^{-0.2x}=\ln(1+x)$ has only one root. [2]

(b) Show by calculation that this root lies between 7 and 9. [2]

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(c) Use the iterative formula

$$x_{n+1} = \exp\left(2 + e^{-0.2x_n}\right) - 1$$

to determine the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places.

[exp(x) is an alternative notation for e^x.] [3]

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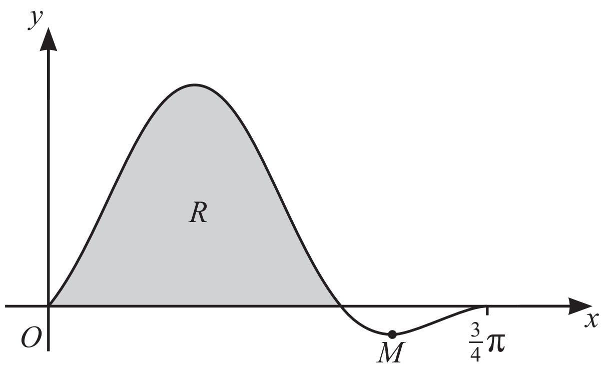

The diagram shows the curve $y = \sin 2x(1 + \sin 2x)$, for $0 \leqslant x \leqslant \frac{3}{4}\pi$, and its minimum point M. The shaded region bounded by the curve that lies above the x-axis and the x-axis itself is denoted by R.

(a) Given that the x-coordinate of M lies in the interval $\frac{1}{2}\pi < x < \frac{3}{4}\pi$, find the exact coordinates of M. [4]

(b) Find the exact area of the region R. [4]

7 Let $f(x) = \dfrac{5x^2 + 8x + 5}{(1+2x)(2+x^2)}$.

(a) Express f(x) in partial fractions. [5]

(b) Hence find the coefficient of x^3 in the expansion of $f(x)$. [4]

8 **(a)** Given that $z = 1 + yi$ and that y is a real number, express $\frac{1}{z}$ in the form $a + bi$, where a and b are functions of y. [2]

(b) Show that $\left(a - \frac{1}{2}\right)^2 + b^2 = \frac{1}{4}$, where a and b are the functions of y found in part **(a)**. [3]

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(c) On a single Argand diagram, sketch the loci given by the equations $\mathrm{Re}(z) = 1$ and $\left|z - \frac{1}{2}\right| = \frac{1}{2}$, where z is a complex number. [3]

(d) The complex number z is such that $\mathrm{Re}(z) = 1$. Use your answer to part **(b)** to give a geometrical description of the locus of $\frac{1}{z}$. [1]

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9 The position vector of point A relative to the origin O is $\overrightarrow{OA} = 8\mathbf{i} - 5\mathbf{j} + 6\mathbf{k}$.
The line l passes through A and is parallel to the vector $2\mathbf{i} + \mathbf{j} + 4\mathbf{k}$.

(a) State a vector equation for l. [2]

(b) The position vector of point B relative to the origin O is $\overrightarrow{OB} = -t\mathbf{i} + 4t\mathbf{j} + 3t\mathbf{k}$, where t is a constant.
The line l also passes through B.

Find the value of t. [3]

(c) The line m has vector equation $\mathbf{r} = 5\mathbf{i} - \mathbf{j} + 2\mathbf{k} + \mu(a\mathbf{i} - \mathbf{j} + 3\mathbf{k})$. The acute angle between the directions of l and m is θ, where $\cos\theta = \frac{1}{\sqrt{6}}$.

Find the possible values of a. [5]

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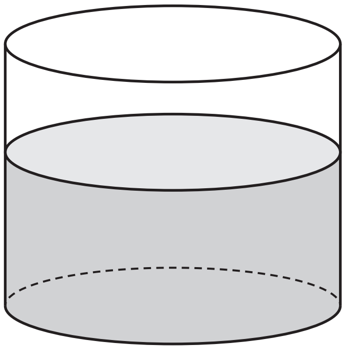

A large cylindrical tank is used to store water. The base of the tank is a circle of radius 4 metres. At time t minutes, the depth of the water in the tank is h metres. There is a tap at the bottom of the tank. When the tap is open, water flows out of the tank at a rate proportional to the square root of the volume of water in the tank.

(a) Show that $\frac{dh}{dt} = -\lambda\sqrt{h}$, where λ is a positive constant. [4]

(b) At time $t = 0$ the tap is opened. It is given that $h = 4$ when $t = 0$ and that $h = 2.25$ when $t = 20$.

Solve the differential equation to obtain an expression for t in terms of h, and hence find the time taken to empty the tank. [6]

Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.

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Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

3449856295

MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **October/November 2024**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS
- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
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- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION
- The total mark for this paper is 75.
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This document has **20** pages. Any blank pages are indicated.

DC (DE/JG) 336975/6

1 Expand $(9-3x)^{\frac{1}{2}}$ in ascending powers of x, up to and including the term in x^2, simplifying the coefficients. [4]

2 **(a)** By sketching a suitable pair of graphs, show that the equation $\cot 2x = \sec x$ has exactly one root in the interval $0 < x < \frac{1}{2}\pi$. [2]

(b) Show that if a sequence of real values given by the iterative formula

$$x_{n+1} = \frac{1}{2}\tan^{-1}(\cos x_n)$$

converges, then it converges to the root in part **(a)**. [1]

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3 The square roots of $6-8i$ can be expressed in the Cartesian form $x+iy$, where x and y are real and exact.

By first forming a quartic equation in x or y, find the square roots of $6-8i$ in exact Cartesian form. [5]

4 Solve the equation $5^x = 5^{x+2} - 10$. Give your answer correct to 3 decimal places. [3]

5 **(a)** The complex number u is given by

$$u = \frac{(\cos\frac{1}{7}\pi + i\sin\frac{1}{7}\pi)^4}{\cos\frac{1}{7}\pi - i\sin\frac{1}{7}\pi}.$$

Find the exact value of arg u. [2]

(b) The complex numbers u and u^* are plotted on an Argand diagram.

Describe the single geometrical transformation that maps u onto u^* and state the exact value of arg u^*. [2]

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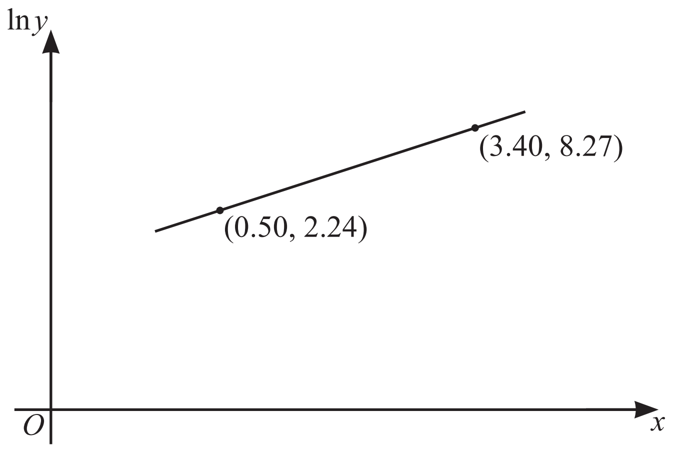

The variables x and y satisfy the equation $ay = b^x$, where a and b are constants. The graph of $\ln y$ against x is a straight line passing through the points (0.50, 2.24) and (3.40, 8.27), as shown in the diagram.

Find the values of a and b. Give each value correct to 1 significant figure. [4]

7 (a) Show that the equation $\tan^3 x + 2\tan 2x - \tan x = 0$ may be expressed as

$$\tan^4 x - 2\tan^2 x - 3 = 0$$

for $\tan x \neq 0$. [3]

(b) Hence solve the equation $\tan^3 2\theta + 2\tan 4\theta - \tan 2\theta = 0$ for $0 < \theta < \pi$. Give your answers in exact form. [3]

8 The parametric equations of a curve are

$$x = \tan^2 2t, \quad y = \cos 2t,$$

for $0 < t < \frac{1}{4}\pi$.

(a) Show that $\frac{dy}{dx} = -\frac{1}{2}\cos^3 2t$. [4]

(b) Hence find the equation of the normal to the curve at the point where $t = \frac{1}{8}\pi$. Give your answer in the form $y = mx + c$. [4]

9 With respect to the origin O, the points A, B and C have position vectors given by

$$\overrightarrow{OA} = \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 0 \\ 4 \\ 1 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} -3 \\ -2 \\ 2 \end{pmatrix}.$$

(a) The point D is such that $ABCD$ is a trapezium with $\overrightarrow{DC} = 3\overrightarrow{AB}$.

Find the position vector of D. [2]

(b) The diagonals of the trapezium intersect at the point P.

Find the position vector of P. [5]

(c) Using a scalar product, calculate angle *ABC*. [4]

10 A balloon in the shape of a sphere has volume V and radius r. Air is pumped into the balloon at a constant rate of 40π starting when time $t = 0$ and $r = 0$. At the same time, air begins to flow out of the balloon at a rate of $0.8\pi r$. The balloon remains a sphere at all times.

(a) Show that r and t satisfy the differential equation

$$\frac{dr}{dt} = \frac{50 - r}{5r^2}.$$ [3]

(b) Find the quotient and remainder when $5r^2$ is divided by $50 - r$. [3]

(c) Solve the differential equation in part **(a)**, obtaining an expression for t in terms of r. [6]

(d) Find the value of t when the radius of the balloon is 12. [1]

11 Let $f(x) = \dfrac{2e^{2x}}{e^{2x} - 3e^{x} + 2}$.

(a) Find $f'(x)$ and hence find the exact coordinates of the stationary point of the curve with equation $y = f(x)$. [5]

(b) Use the substitution $u = e^x$ and partial fractions to find the exact value of $\int_{\ln 3}^{\ln 5} f(x)\,dx$.

Give your answer in the form $\ln a$, where a is a rational number in its simplest form. [9]

Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.

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Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

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MATHEMATICS **9709/33**

Paper 3 Pure Mathematics 3 **October/November 2024**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

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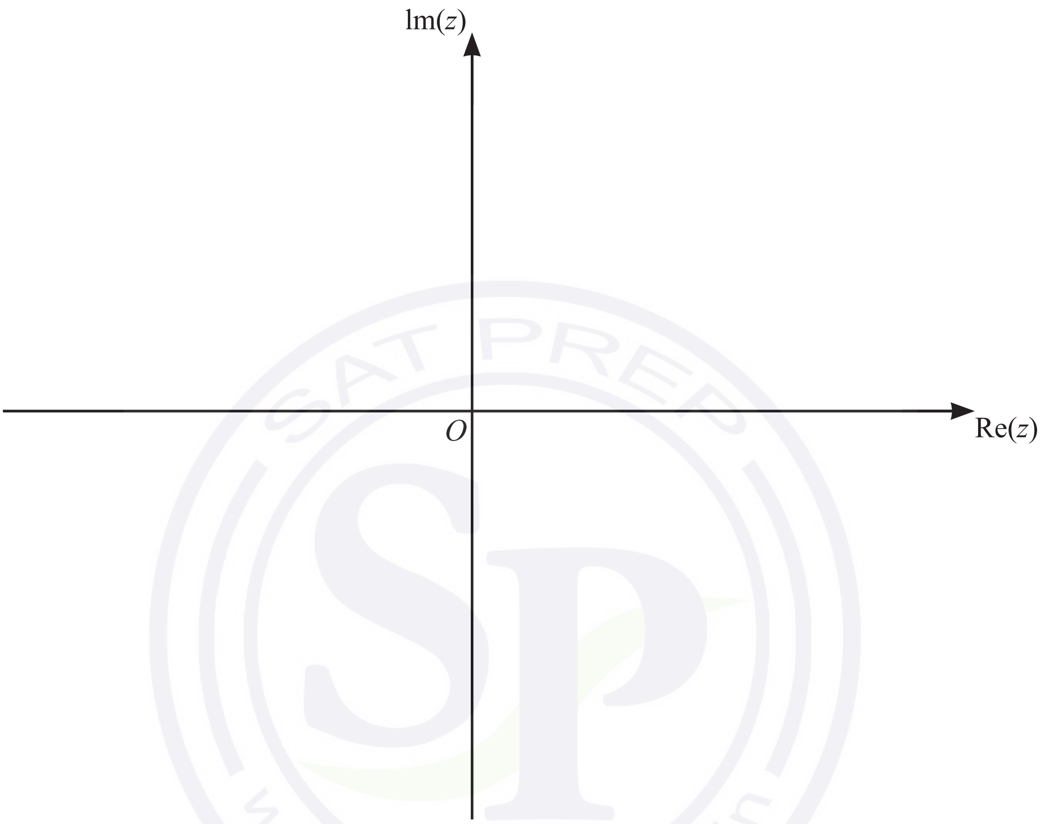
This document has **20** pages.

DC (DE/SG) 336976/3

1 The complex number z satisfies $|z| = 2$ and $0 \leqslant \arg z \leqslant \frac{1}{4}\pi$.

(a) On the Argand diagram below, sketch the locus of the points representing z. [2]

(b) On the **same diagram**, sketch the locus of the points representing z^2. [2]



2 Let $f(x) = 2x^3 - 5x^2 + 4$.

(a) Show that if a sequence of values given by the iterative formula

$$x_{n+1} = \sqrt{\frac{4}{5 - 2x_n}}$$

converges, then it converges to a root of the equation $f(x) = 0$. [2]

(b) The equation has a root close to 1.2 .

Use the iterative formula from part **(a)** and an initial value of 1.2 to determine the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

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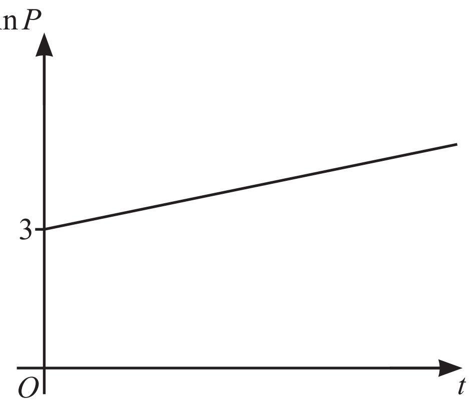

The number of bacteria in a population, P, at time t hours is modelled by the equation $P = ae^{kt}$, where a and k are constants. The graph of $\ln P$ against t, shown in the diagram, has gradient $\frac{1}{20}$ and intersects the vertical axis at $(0, 3)$.

(a) State the value of k and find the value of a correct to 2 significant figures. [3]

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(b) Find the time taken for P to double. Give your answer correct to the nearest hour. [2]

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4 Find the complex number z satisfying the equation

$$\frac{z-3\mathrm{i}}{z+3\mathrm{i}}=\frac{2-9\mathrm{i}}{5}.$$

Give your answer in the form $x+\mathrm{i}y$, where x and y are real. [5]

5 (a) Show that $\cos^4\theta - \sin^4\theta - 4\sin^2\theta\cos^2\theta \equiv \cos^2 2\theta + \cos 2\theta - 1$. [3]

(b) Solve the equation $\cos^4\alpha - \sin^4\alpha = 4\sin^2\alpha\cos^2\alpha$ for $0° \leqslant \alpha \leqslant 180°$. [3]

6 The lines l and m have vector equations

$$l: \quad \mathbf{r} = 2\mathbf{i} + \mathbf{j} - 3\mathbf{k} + \lambda(-\mathbf{i} + 2\mathbf{k}) \quad \text{and} \quad m: \quad \mathbf{r} = 2\mathbf{i} + \mathbf{j} - 3\mathbf{k} + \mu(2\mathbf{i} - \mathbf{j} + 5\mathbf{k}).$$

Lines l and m intersect at the point P.

(a) State the coordinates of P. [1]

(b) Find the exact value of the cosine of the acute angle between l and m. [3]

(c) The point A on line l has coordinates $(0, 1, 1)$. The point B on line m has coordinates $(0, 2, -8)$.

Find the exact area of triangle APB. [3]

7 The parametric equations of a curve are

$$x = 3\sin 2t, \qquad y = \tan t + \cot t,$$

for $0 < t < \frac{1}{2}\pi$.

(a) Show that $\dfrac{dy}{dx} = \dfrac{-2}{3\sin^2 2t}$. [5]

(b) Find the equation of the normal to the curve at the point where $t = \frac{1}{4}\pi$. Give your answer in the form $py + qx + r = 0$, where p, q and r are integers. [3]

8 Let $f(x) = \frac{7a^2}{(a-2x)(3a+x)}$, where a is a positive constant.

(a) Express $f(x)$ in partial fractions. [3]

(b) Hence obtain the expansion of f(x) in ascending powers of x, up to and including the term in x^2. [4]

(c) State the set of values of x for which the expansion in part **(b)** is valid. [1]

9 **(a)** Find the quotient and remainder when $x^4 + 16$ is divided by $x^2 + 4$. [3]

(b) Hence show that $\int_{2}^{2\sqrt{3}} \frac{x^4+16}{x^2+4}\,dx = \frac{4}{3}(\pi+4)$. [5]

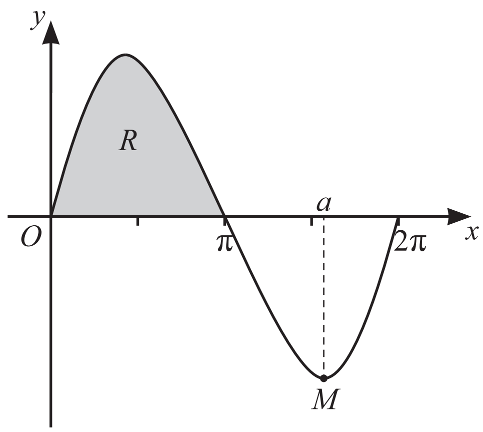
10 A water tank is in the shape of a cuboid with base area $40\,000\,\text{cm}^2$. At time t minutes the depth of water in the tank is h cm. Water is pumped into the tank at a rate of $50\,000\,\text{cm}^3$ per minute. Water is leaking out of the tank through a hole in the bottom at a rate of $600h\,\text{cm}^3$ per minute.

(a) Show that $200\dfrac{dh}{dt} = 250 - 3h$. [3]

(b) It is given that when $t = 0$, $h = 50$.

Find the time taken for the depth of water in the tank to reach 80 cm. Give your answer correct to 2 significant figures. [5]

11



The diagram shows the curve $y = 2\sin x\sqrt{2+\cos x}$, for $0 \leqslant x \leqslant 2\pi$, and its minimum point M, where $x = a$.

(a) Find the value of a correct to 2 decimal places. [5]

(b) Use the substitution $u = 2 + \cos x$ to find the exact area of the shaded region R. [6]

Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.

Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

3353608262

MATHEMATICS **9709/31**

Paper 3 Pure Mathematics 3 **May/June 2024**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS
- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
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INFORMATION
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- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

1 Expand $(3+x)(1-2x)^{\frac{1}{2}}$ in ascending powers of x, up to and including the term in x^2, simplifying the coefficients. [4]

2 Solve the equation $\ln(x-5) = 7 - \ln x$. Give your answer correct to 2 decimal places. [4]

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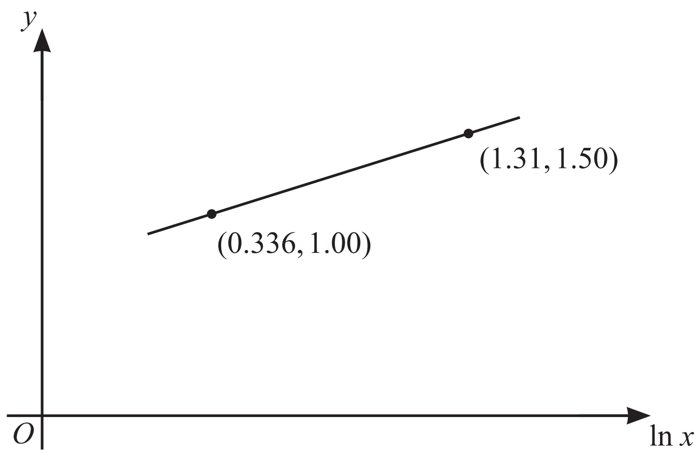

The variables x and y satisfy the equation $a^y = bx$, where a and b are constants. The graph of y against $\ln x$ is a straight line passing through the points (0.336, 1.00) and (1.31, 1.50), as shown in the diagram.

Find the values of a and b. Give each value correct to the nearest integer. [4]

4 The complex number u is given by $u = -1 - \mathrm{i}\sqrt{3}$.

(a) Express u in the form $r(\cos\theta + \mathrm{i}\sin\theta)$, where $r > 0$ and $-\pi < \theta \leqslant \pi$. Give the exact values of r and θ. [2]

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The complex number v is given by $v = 5\left(\cos\frac{1}{6}\pi + \mathrm{i}\sin\frac{1}{6}\pi\right)$.

(b) Express the complex number $\frac{v}{u}$ in the form $r\mathrm{e}^{\mathrm{i}\theta}$ where $r > 0$ and $-\pi < \theta \leqslant \pi$. [2]

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5 The equation of a curve is $y = \frac{e^{\sin x}}{\cos^2 x}$ for $0 \leqslant x \leqslant 2\pi$.

Find $\frac{dy}{dx}$ and hence find the x-coordinates of the stationary points of the curve. [7]

6 **(a)** By sketching a suitable pair of graphs, show that the equation $\operatorname{cosec}\frac{1}{2}x = e^x - 3$ has exactly one root, denoted by α, in the interval $0 < x < \pi$. [2]

(b) Verify by calculation that α lies between 1 and 2. [2]

(c) Show that if a sequence of values in the interval $0 < x < \pi$ given by the iterative formula

$$x_{n+1} = \ln\left(\operatorname{cosec}\tfrac{1}{2}x_n + 3\right)$$

converges, then it converges to α. [1]

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(d) Use this iterative formula with an initial value of 1.4 to determine α correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

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(e) State the minimum number of calculated iterations needed with this initial value to determine α correct to 2 decimal places. [1]

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7 **(a)** On a single Argand diagram sketch the loci given by the equations $|z-3+2i| = 2$ and $|w-3+2i| = |w+3-4i|$ where z and w are complex numbers. [4]

(b) Hence find the least value of $|z-w|$ for points on these loci. Give your answer in an exact form. [2]

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8 Use the substitution $u = 1 - \sin x$ to find the exact value of

$$\int_{\pi}^{\frac{3}{2}\pi} \frac{\sin 2x}{\sqrt{1-\sin x}}\,dx.$$

Give your answer in the form $a + b\sqrt{2}$ where a and b are rational numbers to be determined. [7]

9 The equations of two straight lines l_1 and l_2 are

$$l_1: \quad \mathbf{r} = \mathbf{i} - 2\mathbf{j} + 3\mathbf{k} + \lambda(2\mathbf{i} - \mathbf{j} + a\mathbf{k}) \quad \text{and} \quad l_2: \quad \mathbf{r} = -\mathbf{i} - \mathbf{j} - \mathbf{k} + \mu(3\mathbf{i} - 2\mathbf{j} - 2\mathbf{k}),$$

where a is a constant.

The lines l_1 and l_2 are perpendicular.

(a) Show that $a = 4$. [1]

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The lines l_1 and l_2 also intersect.

(b) Find the position vector of the point of intersection. [4]

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The point A has position vector $-5\mathbf{i}+\mathbf{j}-9\mathbf{k}$.

(c) Show that A lies on l_1. [2]

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The point B is the image of A after a reflection in the line l_2.

(d) Find the position vector of B. [2]

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10 **(a)** Given that $2x = \tan y$, show that $\dfrac{dy}{dx} = \dfrac{2}{1+4x^2}$. [3]

(b) Hence find the exact value of $\displaystyle\int_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}} x\tan^{-1}(2x)\,dx$. [7]

11 In a field there are 300 plants of a certain species, all of which can be infected by a particular disease. At time t after the first plant is infected there are x infected plants. The rate of change of x is proportional to the product of the number of plants infected and the number of plants that are **not** yet infected. The variables x and t are treated as continuous, and it is given that $\frac{dx}{dt} = 0.2$ and $x = 1$ when $t = 0$.

(a) Show that x and t satisfy the differential equation

$$1495 \frac{dx}{dt} = x(300 - x).$$ [2]

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(b) Using partial fractions, solve the differential equation and obtain an expression for t in terms of a single logarithm involving x. [9]

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Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.

Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

8567013635

MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **May/June 2024**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS
- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION
- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

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1 **(a)** Sketch the graph of $y = |x - 2a|$, where a is a positive constant. [1]

(b) Solve the inequality $2x - 3a < |x - 2a|$. [2]

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2 Express $\dfrac{6x^2 - 9x - 16}{2x^2 - 5x - 12}$ in partial fractions. [5]

3 The variables x and y satisfy the equation $a^{2y-1} = b^{x-y}$, where a and b are constants.

(a) Show that the graph of y against x is a straight line. [3]

(b) Given that $a = b^3$, state the equation of the straight line in the form $y = px + q$, where p and q are rational numbers in their simplest form. [2]

4 The equation of a curve is $ye^{2x} + y^2e^x = 6$.

Find the gradient of the curve at the point where $y = 1$. [6]

5 **(a)** It is given that the equation $e^{2x} = 5 + \cos 3x$ has only one root.

Show by calculation that this root lies in the interval $0.7 < x < 0.8$. [2]

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(b) Show that if a sequence of values in the interval $0.7 < x < 0.8$ given by the iterative formula

$$x_{n+1} = \frac{1}{2}\ln\left(5 + \cos 3x_n\right)$$

converges then it converges to the root of the equation in part **(a)**. [1]

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(c) Use this iterative formula to determine the root correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

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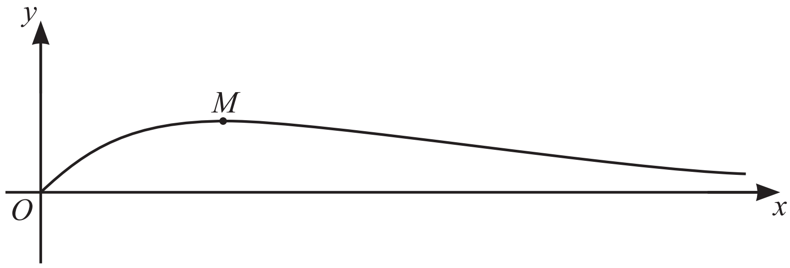

The diagram shows the curve $y = xe^{-ax}$, where a is a positive constant, and its maximum point M.

(a) Find the exact coordinates of M. [4]

(b) Find the exact value of $\int_0^{\frac{2}{a}} xe^{-ax}\,dx$. [5]

7 **(a)** Show that $\cos^4\theta - \sin^4\theta \equiv \cos 2\theta$. [3]

(b) Hence find the exact value of $\int_{-\frac{1}{8}\pi}^{\frac{1}{8}\pi}\left(\cos^4\theta - \sin^4\theta + 4\sin^2\theta\cos^2\theta\right)\mathrm{d}\theta$. [6]

8 The points A, B and C have position vectors $\overrightarrow{OA} = -2\mathbf{i} + \mathbf{j} + 4\mathbf{k}$, $\overrightarrow{OB} = 5\mathbf{i} + 2\mathbf{j}$ and $\overrightarrow{OC} = 8\mathbf{i} + 5\mathbf{j} - 3\mathbf{k}$, where O is the origin. The line l_1 passes through B and C.

(a) Find a vector equation for l_1. [3]

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The line l_2 has equation $\mathbf{r} = -2\mathbf{i} + \mathbf{j} + 4\mathbf{k} + \mu(3\mathbf{i} + \mathbf{j} - 2\mathbf{k})$.

(b) Find the coordinates of the point of intersection of l_1 and l_2. [4]

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(c) The point D on l_2 is such that $AB = BD$.

Find the position vector of D. [5]

9 The complex numbers z and ω are defined by $z = 1 - \mathrm{i}$ and $\omega = -3 + 3\sqrt{3}\mathrm{i}$.

(a) Express $z\omega$ in the form $a + b\mathrm{i}$, where a and b are real and in exact surd form. [1]

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(b) Express z and ω in the form $r\mathrm{e}^{\mathrm{i}\theta}$, where $r > 0$ and $-\pi < \theta \leqslant \pi$. Give the exact values of r and θ in each case. [4]

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(c) On an Argand diagram, the points representing ω and $z\omega$ are A and B respectively.

Prove that OAB is an isosceles right-angled triangle, where O is the origin. [2]

(d) Using your answers to part **(b)**, prove that $\tan\frac{5}{12}\pi = \frac{\sqrt{3}+1}{\sqrt{3}-1}$. [3]

10 **(a)** By writing $y = \sec^3\theta$ as $\dfrac{1}{\cos^3\theta}$, show that $\dfrac{dy}{d\theta} = 3\sin\theta\sec^4\theta$. [2]

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(b) The variables x and θ satisfy the differential equation

$$\left(x^2+9\right)\sin\theta\frac{d\theta}{dx} = (x+3)\cos^4\theta.$$

It is given that $x = 3$ when $\theta = \frac{1}{3}\pi$.

Solve the differential equation to find the value of $\cos\theta$ when $x = 0$. Give your answer correct to 3 significant figures. [8]

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CENTRE NUMBER						CANDIDATE NUMBER				

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MATHEMATICS **9709/33**

Paper 3 Pure Mathematics 3 **May/June 2024**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
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- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
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INFORMATION

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1 Solve the equation $8^{3-6x} = 4 \times 5^{-2x}$. Give your answer correct to 3 decimal places. [4]

2 Find the exact coordinates of the stationary point of the curve $y = e^{2x}\sin 2x$ for $0 \leqslant x \leqslant \frac{1}{2}\pi$. [5]

3 The square roots of $24 - 7\mathrm{i}$ can be expressed in the Cartesian form $x + \mathrm{i}y$, where x and y are real and exact.

By first forming a quartic equation in x or y, find the square roots of $24 - 7\mathrm{i}$ in exact Cartesian form. [5]

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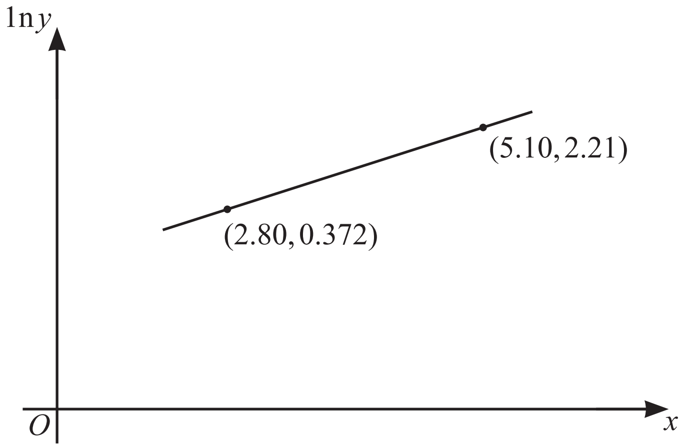

The variables x and y satisfy the equation $ky = e^{cx}$, where k and c are constants. The graph of $\ln y$ against x is a straight line passing through the points $(2.80, 0.372)$ and $(5.10, 2.21)$, as shown in the diagram.

Find the values of k and c. Give each value correct to 2 significant figures. [4]

5 Express $\dfrac{6x^2 - 2x + 2}{(x-1)(2x+1)}$ in partial fractions. [5]

6 **(a)** On an Argand diagram shade the region whose points represent complex numbers z which satisfy both the inequalities $|z-4-3i| \leqslant 2$ and $\arg(z-2-i) \geqslant \frac{1}{3}\pi$. [5]

(b) Calculate the greatest value of $\arg z$ for points in this region. [2]

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7 Let $f(x) = 8x^3 + 54x^2 - 17x - 21$.

(a) Show that $x + 7$ is a factor of $f(x)$. [1]

(b) Find the quotient when $f(x)$ is divided by $x + 7$. [2]

(c) Hence solve the equation

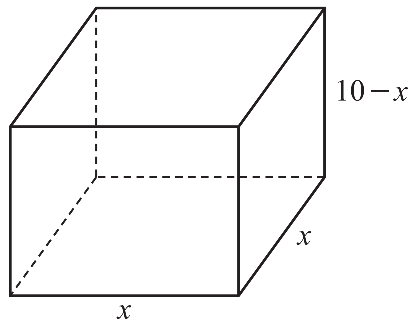
$$8\cos^3\theta + 54\cos^2\theta - 17\cos\theta - 21 = 0,$$

for $0° \leqslant \theta \leqslant 360°$. [3]

8 (a) Express $3\cos 2x - \sqrt{3}\sin 2x$ in the form $R\cos(2x + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$. Give the exact values of R and α. [3]

(b) Hence find the exact value of $\int_0^{\frac{1}{12}\pi} \frac{3}{\left(3\cos 2x - \sqrt{3}\sin 2x\right)^2}\,\mathrm{d}x$, simplifying your answer. [5]

9



A container in the shape of a cuboid has a square base of side x and a height of $(10-x)$. It is given that x varies with time, t, where $t > 0$. The container decreases in volume at a rate which is inversely proportional to t.

When $t = \frac{1}{10}$, $x = \frac{1}{2}$ and the rate of decrease of x is $\frac{20}{37}$.

(a) Show that x and t satisfy the differential equation

$$\frac{dx}{dt} = \frac{-1}{2t(20x-3x^2)}.$$ [5]

(b) Solve the differential equation, obtaining an expression for t in terms of x. [6]

10 The equations of two straight lines are

$$\mathbf{r} = \mathbf{i} + \mathbf{j} + 2a\mathbf{k} + \lambda(3\mathbf{i} + 4\mathbf{j} + a\mathbf{k}) \quad \text{and} \quad \mathbf{r} = -3\mathbf{i} - \mathbf{j} + 4\mathbf{k} + \mu(-\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}),$$

where a is a constant.

(a) Given that the acute angle between the directions of these lines is $\frac{1}{4}\pi$, find the possible values of a. [6]

(b) Given instead that the lines intersect, find the value of a and the position vector of the point of intersection. [5]

11 Use the substitution $2x = \tan\theta$ to find the exact value of

$$\int_0^{\frac{1}{2}} \frac{12}{\left(1+4x^2\right)^2}\,dx\ .$$

Give your answer in the form $a+b\pi$, where a and b are rational numbers. [9]

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5552004552

MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **February/March 2024**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
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- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

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1 Find the quotient and remainder when $x^4 - 3x^3 + 9x^2 - 12x + 27$ is divided by $x^2 + 5$. [3]

2 **(a)** Find the coefficient of x^2 in the expansion of $(2x-5)\sqrt{4-x}$. [4]

(b) State the set of values of x for which the expansion in part **(a)** is valid. [1]

3 It is given that $z = -\sqrt{3} + \mathrm{i}$.

(a) Express z^2 in the form $r\mathrm{e}^{\mathrm{i}\theta}$, where $r > 0$ and $-\pi < \theta \leqslant \pi$. [3]

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(b) The complex number ω is such that $z^2\omega$ is real and $\left|\frac{z^2}{\omega}\right| = 12$.

Find the two possible values of ω, giving your answers in the form $R\mathrm{e}^{\mathrm{i}\alpha}$, where $R > 0$ and $-\pi < \alpha \leqslant \pi$. [3]

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4 The positive numbers p and q are such that

$$\ln\left(\frac{p}{q}\right) = a \quad \text{and} \quad \ln\left(q^2 p\right) = b.$$

Express $\ln\left(p^7 q\right)$ in terms of a and b. [4]

5 **(a)** On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities $|z-4-2\mathrm{i}| \leqslant 3$ and $|z| \geqslant |10-z|$. [4]

(b) Find the greatest value of $\arg z$ for points in this region. [2]

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6 The equation of a curve is $2y^2 + 3xy + x = x^2$.

(a) Show that $\frac{dy}{dx} = \frac{2x - 3y - 1}{4y + 3x}$. [4]

(b) Hence show that the curve does **not** have a tangent that is parallel to the x-axis. [3]

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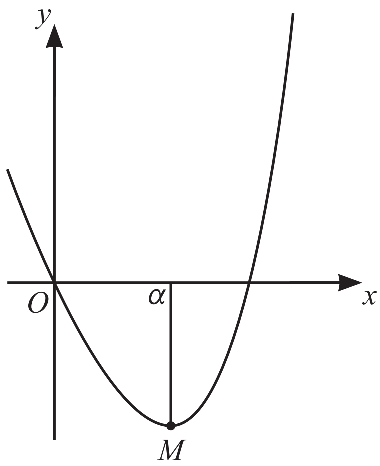

The diagram shows the curve $y = xe^{2x} - 5x$ and its minimum point M, where $x = \alpha$.

(a) Show that α satisfies the equation $\alpha = \frac{1}{2}\ln\left(\frac{5}{1+2\alpha}\right)$. [3]

(b) Verify by calculation that α lies between 0.4 and 0.5 . [2]

(c) Use an iterative formula based on the equation in part **(a)** to determine α correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

8 **(a)** Express $3\sin x + 2\sqrt{2}\cos\left(x + \frac{1}{4}\pi\right)$ in the form $R\sin(x + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$. State the exact value of R and give α correct to 3 decimal places. [4]

(b) Hence solve the equation

$$6\sin\tfrac{1}{2}\theta + 4\sqrt{2}\cos\left(\tfrac{1}{2}\theta + \tfrac{1}{4}\pi\right) = 3$$

for $-4\pi < \theta < 4\pi$. [5]

9 Relative to the origin O, the position vectors of the points A, B and C are given by

$$\overrightarrow{OA} = 5\mathbf{i} - 2\mathbf{j} + \mathbf{k}, \quad \overrightarrow{OB} = 8\mathbf{i} + 2\mathbf{j} - 6\mathbf{k} \quad \text{and} \quad \overrightarrow{OC} = 3\mathbf{i} + 4\mathbf{j} - 7\mathbf{k}.$$

(a) Show that $OABC$ is a rectangle. [4]

(b) Use a scalar product to find the acute angle between the diagonals of *OABC*. [4]

10 Let $f(x) = \dfrac{36a^2}{(2a+x)(2a-x)(5a-2x)}$, where a is a positive constant.

(a) Express $f(x)$ in partial fractions. [5]

(b) Hence find the exact value of $\int_{-a}^{a} \mathrm{f}(x)\,\mathrm{d}x$, giving your answer in the form $p\ln q + r\ln s$ where p and r are integers and q and s are prime numbers. [5]

11 The variables y and θ satisfy the differential equation

$$(1+y)(1+\cos 2\theta)\frac{\mathrm{d}y}{\mathrm{d}\theta} = \mathrm{e}^{3y}.$$

It is given that $y = 0$ when $\theta = \frac{1}{4}\pi$.

Solve the differential equation and find the exact value of $\tan\theta$ when $y = 1$. [9]

Additional page

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Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

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5051231203

MATHEMATICS **9709/31**

Paper 3 Pure Mathematics 3 **October/November 2023**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
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INFORMATION

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1 Find the exact coordinates of the points on the curve $y = \dfrac{x^2}{1 - 3x}$ at which the gradient of the tangent is equal to 8. [5]

2 On an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities $|z - 2\mathrm{i}| \leqslant |z + 2 - \mathrm{i}|$ and $0 \leqslant \arg(z + 1) \leqslant \frac{1}{4}\pi$. [4]

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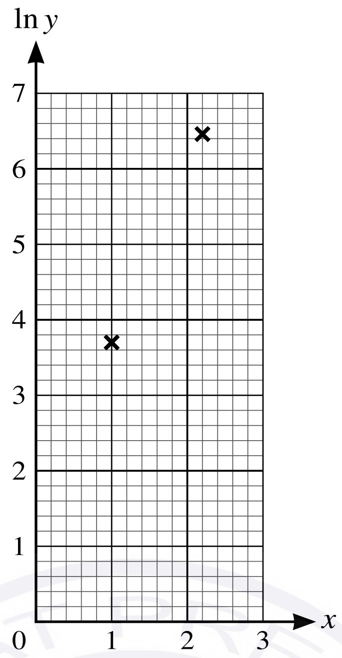

The variables x and y are related by the equation $y = ab^x$, where a and b are constants. The diagram shows the result of plotting $\ln y$ against x for two pairs of values of x and y. The coordinates of these points are (1, 3.7) and (2.2, 6.46).

Use this information to find the values of a and b. [4]

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4 The complex number u is defined by $u = \dfrac{3 + 2i}{a - 5i}$, where a is real.

(a) Express u in the Cartesian form $x + iy$, where x and y are in terms of a. [3]

(b) Given that $\arg u = \frac{1}{4}\pi$, find the value of a. [2]

5 **(a)** Given that

$$\sin\left(x + \tfrac{1}{6}\pi\right) - \sin\left(x - \tfrac{1}{6}\pi\right) = \cos\left(x + \tfrac{1}{3}\pi\right) - \cos\left(x - \tfrac{1}{3}\pi\right),$$

find the exact value of $\tan x$. [4]

(b) Hence find the exact roots of the equation

$$\sin\left(x + \tfrac{1}{6}\pi\right) - \sin\left(x - \tfrac{1}{6}\pi\right) = \cos\left(x + \tfrac{1}{3}\pi\right) - \cos\left(x - \tfrac{1}{3}\pi\right)$$

for $0 \leqslant x \leqslant 2\pi$. [2]

6 The parametric equations of a curve are

$$x = \sqrt{t} + 3, \qquad y = \ln t,$$

for $t > 0$.

(a) Obtain a simplified expression for $\dfrac{dy}{dx}$ in terms of t. [3]

(b) Hence find the exact coordinates of the point on the curve at which the gradient of the normal is -2. [3]

7 The variables x and θ satisfy the differential equation

$$\frac{x}{\tan\theta}\frac{dx}{d\theta} = x^2 + 3.$$

It is given that $x = 1$ when $\theta = 0$.

Solve the differential equation, obtaining an expression for x^2 in terms of θ. [7]

8 **(a)** By sketching a suitable pair of graphs, show that the equation

$$\sqrt{x} = e^x - 3$$

has only one root. [2]

(b) Show by calculation that this root lies between 1 and 2. [2]

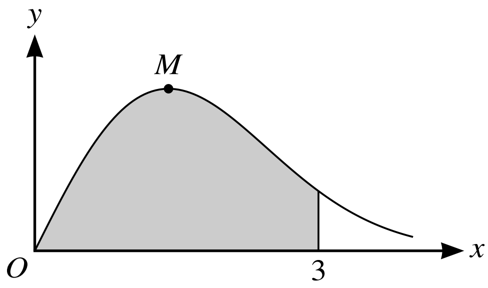
(c) Show that, if a sequence of values given by the iterative formula

$$x_{n+1} = \ln\left(3 + \sqrt{x_n}\right)$$

converges, then it converges to the root of the equation in **(a)**. [1]

(d) Use the iterative formula to calculate the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

9



The diagram shows the curve $y = xe^{-\frac{1}{4}x^2}$, for $x \geqslant 0$, and its maximum point M.

(a) Find the exact coordinates of M. [4]

(b) Using the substitution $x = \sqrt{u}$, or otherwise, find by integration the exact area of the shaded region bounded by the curve, the x-axis and the line $x = 3$. [5]

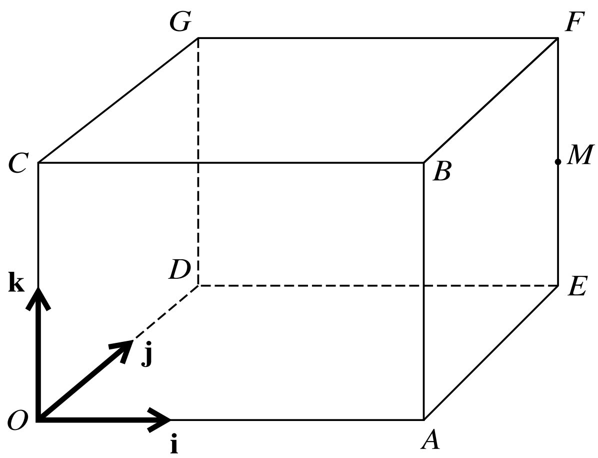
10 Let $f(x) = \dfrac{24x + 13}{(1 - 2x)(2 + x)^2}$.

(a) Express $f(x)$ in partial fractions. [5]

(b) Hence obtain the expansion of $f(x)$ in ascending powers of x, up to and including the term in x^2. [5]

(c) State the set of values of x for which the expansion in **(b)** is valid. [1]

11



In the diagram, $OABCDEFG$ is a cuboid in which $OA = 3$ units, $OC = 2$ units and $OD = 2$ units. Unit vectors $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ are parallel to OA, OD and OC respectively. M is the midpoint of EF.

(a) Find the position vector of M. [1]

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The position vector of P is $\mathbf{i} + \mathbf{j} + 2\mathbf{k}$.

(b) Calculate angle PAM. [4]

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(c) Find the exact length of the perpendicular from P to the line passing through O and M. [5]

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If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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CENTRE NUMBER

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MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **October/November 2023**

1 hour 50 minutes

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INFORMATION

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1 **(a)** Sketch the graph of $y = |4x - 2|$. [1]

(b) Solve the inequality $1 + 3x < |4x - 2|$. [4]

2 The parametric equations of a curve are

$$x = (\ln t)^2, \qquad y = e^{2-t^2},$$

for $t > 0$.

Find the gradient of the curve at the point where $t = e$, simplifying your answer. [4]

3 The polynomial $2x^3 + ax^2 - 11x + b$ is denoted by $p(x)$. It is given that $p(x)$ is divisible by $(2x - 1)$ and that when $p(x)$ is divided by $(x + 1)$ the remainder is 12.

Find the values of a and b. [5]

4 **(a)** On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities $|z - 4 - 3i| \leqslant 2$ and $\text{Re}\, z \leqslant 3$. [4]

(b) Find the greatest value of $\arg z$ for points in this region. [2]

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5 Find the exact value of $\int_0^6 \frac{x(x+1)}{x^2+4}\,dx$. [6]

6 **(a)** By sketching a suitable pair of graphs, show that the equation

$$\cot x = 2 - \cos x$$

has one root in the interval $0 < x \leqslant \frac{1}{2}\pi$. [2]

(b) Show by calculation that this root lies between 0.6 and 0.8. [2]

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(c) Use the iterative formula $x_{n+1} = \tan^{-1}\left(\frac{1}{2 - \cos x_n}\right)$ to determine the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

7 **(a)** By expressing 3θ as $2\theta + \theta$, prove the identity $\cos 3\theta \equiv 4\cos^3\theta - 3\cos\theta$. [3]

(b) Hence solve the equation

$$\cos 3\theta + \cos\theta \cos 2\theta = \cos^2\theta$$

for $0° \leqslant \theta \leqslant 180°$. [5]

8 It is given that $\frac{2+3a\mathrm{i}}{a+2\mathrm{i}} = \lambda(2-\mathrm{i})$, where a and λ are real constants.

(a) Show that $3a^2 + 4a - 4 = 0$. [4]

(b) Hence find the possible values of a and the corresponding values of λ. [3]

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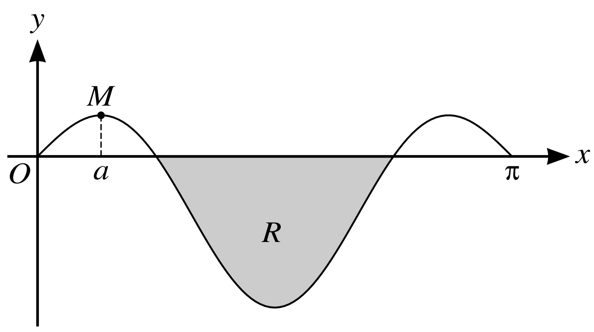

The diagram shows the curve $y = \sin x \cos 2x$, for $0 \leqslant x \leqslant \pi$, and a maximum point M, where $x = a$. The shaded region between the curve and the x-axis is denoted by R.

(a) Find the value of a correct to 2 decimal places. [5]

(b) Find the exact area of the region R, giving your answer in simplified form. [4]

10 The equations of the lines l and m are given by

$$l: \quad \mathbf{r} = \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} \qquad \text{and} \qquad m: \quad \mathbf{r} = \begin{pmatrix} 6 \\ -3 \\ 6 \end{pmatrix} + \mu \begin{pmatrix} -2 \\ 4 \\ c \end{pmatrix},$$

where c is a positive constant. It is given that the angle between l and m is $60°$.

(a) Find the value of c. [4]

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(b) Show that the length of the perpendicular from (6, −3, 6) to l is $\sqrt{11}$. [5]

11 The variables x and y satisfy the differential equation

$$x^2\frac{dy}{dx} + y^2 + y = 0.$$

It is given that $x = 1$ when $y = 1$.

(a) Solve the differential equation to obtain an expression for y in terms of x. [8]

(b) State what happens to the value of y when x tends to infinity. Give your answer in an exact form. [1]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

7354647755

MATHEMATICS **9709/33**

Paper 3 Pure Mathematics 3 **October/November 2023**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

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1 Find the set of values of x satisfying the inequality $\left|2^{x+1} - 2\right| < 0.5$, giving your answer to 3 significant figures. [4]

2 On an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities $|z - 1 + 2i| \leqslant |z|$ and $|z - 2| \leqslant 1$. [5]

3 The polynomial $2x^3 + ax^2 + bx + 6$, where a and b are constants, is denoted by $p(x)$. When $p(x)$ is divided by $(x + 2)$ the remainder is -38 and when $p(x)$ is divided by $(2x - 1)$ the remainder is $\frac{19}{2}$.

Find the values of a and b. [5]

4 Solve the quadratic equation $(3 + \mathrm{i})w^2 - 2w + 3 - \mathrm{i} = 0$, giving your answers in the form $x + \mathrm{i}y$, where x and y are real. [5]

5 Find the exact coordinates of the stationary points of the curve $y = \dfrac{e^{3x^2-1}}{1-x^2}$. [6]

6 (a) Show that the equation $\cot^2\theta + 2\cos 2\theta = 4$ can be written in the form

$$4\sin^4\theta + 3\sin^2\theta - 1 = 0.$$ [3]

(b) Hence solve the equation $\cot^2\theta + 2\cos 2\theta = 4$, for $0° < \theta < 360°$. [3]

7 The equation of a curve is $x^3 + y^2 + 3x^2 + 3y = 4$.

(a) Show that $\frac{dy}{dx} = -\frac{3x^2 + 6x}{2y + 3}$. [3]

(b) Hence find the coordinates of the points on the curve at which the tangent is parallel to the x-axis. [5]

8 The variables x and y satisfy the differential equation

$$e^{4x}\frac{dy}{dx} = \cos^2 3y.$$

It is given that $y = 0$ when $x = 2$.

Solve the differential equation, obtaining an expression for y in terms of x. [7]

9 Let $f(x) = \dfrac{17x^2 - 7x + 16}{(2 + 3x^2)(2 - x)}$.

(a) Express $f(x)$ in partial fractions. [5]

(b) Hence obtain the expansion of $f(x)$ in ascending powers of x, up to and including the term in x^3. [5]

(c) State the set of values of x for which the expansion in **(b)** is valid. Give your answer in an exact form. [1]

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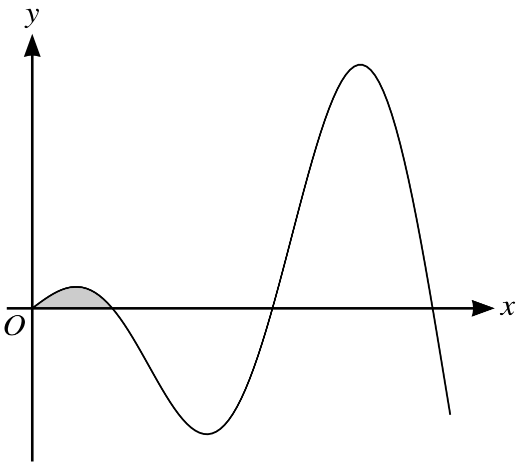

The diagram shows the curve $y = x\cos 2x$, for $x \geqslant 0$.

(a) Find the equation of the tangent to the curve at the point where $x = \frac{1}{2}\pi$. [4]

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(b) Find the exact area of the shaded region shown in the diagram, bounded by the curve and the x-axis. [5]

11 The line l has equation $\mathbf{r} = \mathbf{i} - 2\mathbf{j} - 3\mathbf{k} + \lambda(-\mathbf{i} + \mathbf{j} + 2\mathbf{k})$. The points A and B have position vectors $-2\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ and $3\mathbf{i} - \mathbf{j} + \mathbf{k}$ respectively.

(a) Find a unit vector in the direction of l. [2]

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The line m passes through the points A and B.

(b) Find a vector equation for m. [2]

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(c) Determine whether lines l and m are parallel, intersect or are skew. [5]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

* 2 9 6 8 7 5 0 1 7 5 *

MATHEMATICS **9709/31**

Paper 3 Pure Mathematics 3 **May/June 2023**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

1 Solve the equation

$$3e^{2x} - 4e^{-2x} = 5.$$

Give the answer correct to 3 decimal places. [3]

2 **(a)** Sketch the graph of $y = |2x + 3|$. [1]

(b) Solve the inequality $3x + 8 > |2x + 3|$. [3]

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3 Find the coefficient of x^3 in the binomial expansion of $(3 + x)\sqrt{1 + 4x}$. [4]

4 **(a)** Show that the equation $\sin 2\theta + \cos 2\theta = 2\sin^2\theta$ can be expressed in the form

$$\cos^2\theta + 2\sin\theta\cos\theta - 3\sin^2\theta = 0.$$ [2]

(b) Hence solve the equation $\sin 2\theta + \cos 2\theta = 2\sin^2\theta$ for $0° < \theta < 180°$. [4]

5 The equation of a curve is $x^2y - ay^2 = 4a^3$, where a is a non-zero constant.

(a) Show that $\frac{dy}{dx} = \frac{2xy}{2ay - x^2}$. [4]

(b) Hence find the coordinates of the points where the tangent to the curve is parallel to the y-axis. [4]

6 Relative to the origin O, the points A, B and C have position vectors given by

$$\overrightarrow{OA} = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}, \qquad \overrightarrow{OB} = \begin{pmatrix} 4 \\ 3 \\ 2 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} 3 \\ -2 \\ -4 \end{pmatrix}.$$

The quadrilateral $ABCD$ is a parallelogram.

(a) Find the position vector of D. [3]

(b) The angle between BA and BC is θ.

Find the exact value of $\cos\theta$. [3]

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(c) Hence find the area of $ABCD$, giving your answer in the form $p\sqrt{q}$, where p and q are integers. [4]

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7 The variables x and y satisfy the differential equation

$$\cos 2x \frac{dy}{dx} = \frac{4\tan 2x}{\sin^2 3y},$$

where $0 \leqslant x < \frac{1}{4}\pi$. It is given that $y = 0$ when $x = \frac{1}{6}\pi$.

Solve the differential equation to obtain the value of x when $y = \frac{1}{6}\pi$. Give your answer correct to 3 decimal places. [8]

8 Let $f(x) = \dfrac{3 - 3x^2}{(2x + 1)(x + 2)^2}$.

(a) Express $f(x)$ in partial fractions. [5]

(b) Hence find the exact value of $\int_0^4 f(x)\,dx$, giving your answer in the form $a + b\ln c$, where a, b and c are integers. [5]

9 The constant a is such that $\int_0^a x\mathrm{e}^{-2x}\,\mathrm{d}x = \frac{1}{8}$.

(a) Show that $a = \frac{1}{2}\ln(4a + 2)$. [5]

(b) Verify by calculation that a lies between 0.5 and 1. [2]

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(c) Use an iterative formula based on the equation in **(a)** to determine a correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

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10 The polynomial $x^3 + 5x^2 + 31x + 75$ is denoted by $p(x)$.

(a) Show that $(x + 3)$ is a factor of $p(x)$. [2]

(b) Show that $z = -1 + 2\sqrt{6}i$ is a root of $p(z) = 0$. [3]

(c) Hence find the complex numbers z which are roots of $p(z^2) = 0$. [7]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

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MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **May/June 2023**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

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1 Solve the inequality $|5x - 3| < 2|3x - 7|$. [4]

2 Solve the equation $\ln(2x^2 - 3) = 2\ln x - \ln 2$, giving your answer in an exact form. [3]

3 **(a)** On an Argand diagram, sketch the locus of points representing complex numbers z satisfying $|z + 3 - 2\mathrm{i}| = 2$. [2]

(b) Find the least value of $|z|$ for points on this locus, giving your answer in an exact form. [2]

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4 Solve the equation $2\cos x - \cos\frac{1}{2}x = 1$ for $0 \leqslant x \leqslant 2\pi$. [5]

5 The complex number $2 + y\mathrm{i}$ is denoted by a, where y is a real number and $y < 0$. It is given that $\mathrm{f}(a) = a^3 - a^2 - 2a$.

(a) Find a simplified expression for $\mathrm{f}(a)$ in terms of y. [3]

(b) Given that $\mathrm{Re}(\mathrm{f}(a)) = -20$, find $\arg a$. [3]

6 The equation $\cot \frac{1}{2}x = 3x$ has one root in the interval $0 < x < \pi$, denoted by α.

(a) Show by calculation that α lies between 0.5 and 1. [2]

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(b) Show that, if a sequence of positive values given by the iterative formula

$$x_{n+1} = \frac{1}{3}\left(x_n + 4\tan^{-1}\left(\frac{1}{3x_n}\right)\right)$$

converges, then it converges to α. [2]

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(c) Use this iterative formula to calculate α correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

7 The equation of a curve is $3x^2 + 4xy + 3y^2 = 5$.

(a) Show that $\dfrac{dy}{dx} = -\dfrac{3x + 2y}{2x + 3y}$. [4]

(b) Hence find the exact coordinates of the two points on the curve at which the tangent is parallel to $y + 2x = 0$. [5]

8 **(a)** The variables x and y satisfy the differential equation

$$\frac{dy}{dx} = \frac{4 + 9y^2}{e^{2x+1}}.$$

It is given that $y = 0$ when $x = 1$.

Solve the differential equation, obtaining an expression for y in terms of x. [7]

(b) State what happens to the value of y as x tends to infinity. Give your answer in an exact form. [1]

9 Let $f(x) = \dfrac{2x^2 + 17x - 17}{(1 + 2x)(2 - x)^2}$.

(a) Express $f(x)$ in partial fractions. [5]

(b) Hence show that $\int_0^1 f(x)\,dx = \frac{5}{2} - \ln 72$. [5]

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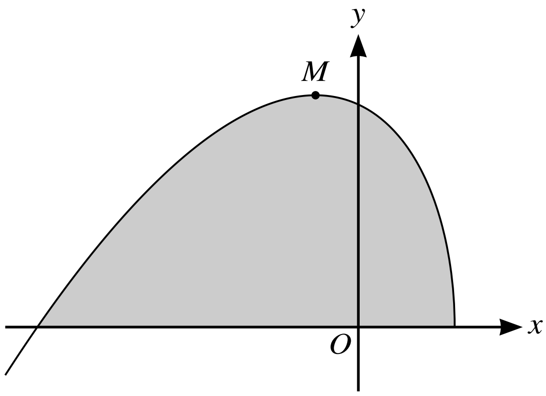

The diagram shows the curve $y = (x+5)\sqrt{3-2x}$ and its maximum point M.

(a) Find the exact coordinates of M. [5]

(b) Using the substitution $u = 3 - 2x$, find by integration the area of the shaded region bounded by the curve and the x-axis. Give your answer in the form $a\sqrt{13}$, where a is a rational number. [5]

11 The points A and B have position vectors $\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$ and $2\mathbf{i} - \mathbf{j} + \mathbf{k}$ respectively. The line l has equation $\mathbf{r} = \mathbf{i} - \mathbf{j} + 3\mathbf{k} + \mu(2\mathbf{i} - 3\mathbf{j} + 4\mathbf{k})$.

(a) Show that l does not intersect the line passing through A and B. [5]

(b) Find the position vector of the foot of the perpendicular from A to l. [4]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME			
CENTRE NUMBER		CANDIDATE NUMBER	

* 5 4 7 1 2 1 0 8 4 0 *

MATHEMATICS **9709/33**

Paper 3 Pure Mathematics 3 **May/June 2023**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages.

[Turn over

1 Solve the equation $\ln(x+5) = 5 + \ln x$. Give your answer correct to 3 decimal places. [4]

2 Find the quotient and remainder when $2x^4 - 27$ is divided by $x^2 + x + 3$. [3]

3 On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities $|z - 3 - \mathrm{i}| \leqslant 3$ and $|z| \geqslant |z - 4\mathrm{i}|$. [4]

4 The parametric equations of a curve are

$$x = \frac{\cos\theta}{2 - \sin\theta}, \qquad y = \theta + 2\cos\theta.$$

Show that $\frac{dy}{dx} = (2 - \sin\theta)^2$. [5]

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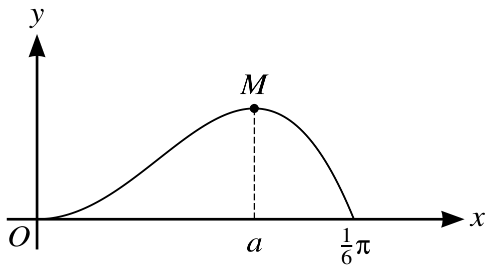

The diagram shows the part of the curve $y = x^2 \cos 3x$ for $0 \leqslant x \leqslant \frac{1}{6}\pi$, and its maximum point M, where $x = a$.

(a) Show that a satisfies the equation $a = \frac{1}{3}\tan^{-1}\left(\frac{2}{3a}\right)$. [3]

(b) Use an iterative formula based on the equation in **(a)** to determine a correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

6 **(a)** Express $3\cos x + 2\cos(x - 60°)$ in the form $R\cos(x - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. State the exact value of R and give α correct to 2 decimal places. [4]

(b) Hence solve the equation

$$3\cos 2\theta + 2\cos(2\theta - 60°) = 2.5$$

for $0° < \theta < 180°$. [4]

7 **(a)** Use the substitution $u = \cos x$ to show that

$$\int_0^{\pi} \sin 2x \, e^{2\cos x} \, dx = \int_{-1}^{1} 2u e^{2u} \, du.$$ [4]

(b) Hence find the exact value of $\int_0^{\pi} \sin 2x\, e^{2\cos x}\, dx$. [4]

8 The variables x and y satisfy the differential equation

$$\frac{dy}{dx} = \frac{y^2 + 4}{x(y + 4)}$$

for $x > 0$. It is given that $x = 4$ when $y = 2\sqrt{3}$.

Solve the differential equation to obtain the value of x when $y = 2$. [8]

9 The lines l and m have equations

$$l: \quad \mathbf{r} = a\mathbf{i} + 3\mathbf{j} + b\mathbf{k} + \lambda(c\mathbf{i} - 2\mathbf{j} + 4\mathbf{k}),$$
$$m: \quad \mathbf{r} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k} + \mu(2\mathbf{i} - 3\mathbf{j} + \mathbf{k}).$$

Relative to the origin O, the position vector of the point P is $4\mathbf{i} + 7\mathbf{j} - 2\mathbf{k}$.

(a) Given that l is perpendicular to m and that P lies on l, find the values of the constants a, b and c. [4]

(b) The perpendicular from P meets line m at Q. The point R lies on PQ extended, with $PQ : QR = 2 : 3$.

Find the position vector of R. [6]

10 Let $f(x) = \dfrac{21 - 8x - 2x^2}{(1 + 2x)(3 - x)^2}$.

(a) Express $f(x)$ in partial fractions. [5]

(b) Hence obtain the expansion of $f(x)$ in ascending powers of x, up to and including the term in x^2. [5]

11 The complex number z is defined by $z = \dfrac{5a - 2\mathrm{i}}{3 + a\mathrm{i}}$, where a is an integer. It is given that $\arg z = -\frac{1}{4}\pi$.

(a) Find the value of a and hence express z in the form $x + \mathrm{i}y$, where x and y are real. [6]

(b) Express z^3 in the form $re^{i\theta}$, where $r > 0$ and $-\pi < \theta \leqslant \pi$. Give the simplified exact values of r and θ. [3]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME			
CENTRE NUMBER		CANDIDATE NUMBER	

* 9 3 3 7 4 9 5 7 5 6 *

MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **February/March 2023**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages.

1 It is given that $x = \ln(2y - 3) - \ln(y + 4)$.

Express y in terms of x. [3]

2 **(a)** On an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities $-\frac{1}{3}\pi \leqslant \arg(z - 1 - 2\text{i}) \leqslant \frac{1}{3}\pi$ and $\text{Re}\, z \leqslant 3$. [3]

(b) Calculate the least value of $\arg z$ for points in the region from **(a)**. Give your answer in radians correct to 3 decimal places. [2]

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3 The polynomial $2x^4 + ax^3 + bx - 1$, where a and b are constants, is denoted by $p(x)$. When $p(x)$ is divided by $x^2 - x + 1$ the remainder is $3x + 2$.

Find the values of a and b. [5]

4 Solve the equation

$$\frac{5z}{1+2\mathrm{i}} - zz^* + 30 + 10\mathrm{i} = 0,$$

giving your answers in the form $x + \mathrm{i}y$, where x and y are real. [5]

5 The parametric equations of a curve are

$$x = te^{2t}, \qquad y = t^2 + t + 3.$$

(a) Show that $\dfrac{dy}{dx} = e^{-2t}$. [3]

(b) Hence show that the normal to the curve, where $t = -1$, passes through the point $\left(0,\ 3 - \dfrac{1}{e^4}\right)$. [3]

6 **(a)** Express $5\sin\theta + 12\cos\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$. [3]

(b) Hence solve the equation $5\sin 2x + 12\cos 2x = 6$ for $0 \leqslant x \leqslant \pi$. [4]

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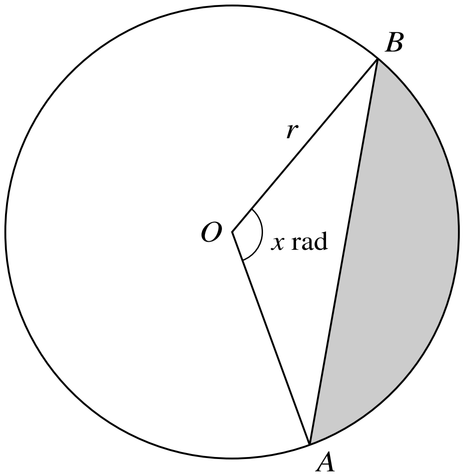

The diagram shows a circle with centre O and radius r. The angle of the **minor** sector AOB of the circle is x radians. The area of the **major** sector of the circle is 3 times the area of the shaded region.

(a) Show that $x = \frac{3}{4}\sin x + \frac{1}{2}\pi$. [4]

(b) Show by calculation that the root of the equation in **(a)** lies between 2 and 2.5. [2]

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(c) Use an iterative formula based on the equation in **(a)** to calculate this root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

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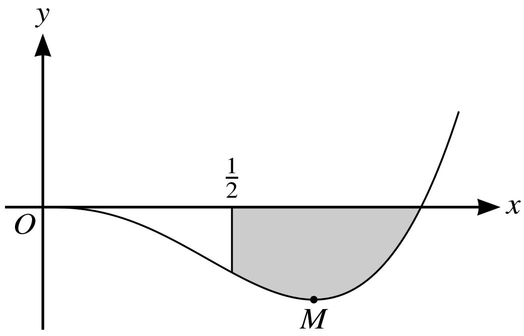

The diagram shows the curve $y = x^3 \ln x$, for $x > 0$, and its minimum point M.

(a) Find the exact coordinates of M. [4]

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(b) Find the exact area of the shaded region bounded by the curve, the x-axis and the line $x = \frac{1}{2}$. [5]

9 The variables x and y satisfy the differential equation

$$\frac{dy}{dx} = e^{3y} \sin^2 2x.$$

It is given that $y = 0$ when $x = 0$.

Solve the differential equation and find the value of y when $x = \frac{1}{2}$. [7]

10 With respect to the origin O, the points A, B, C and D have position vectors given by

$$\overrightarrow{OA} = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix}, \quad \overrightarrow{OC} = \begin{pmatrix} 1 \\ -2 \\ 5 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OD} = \begin{pmatrix} 5 \\ -6 \\ 11 \end{pmatrix}.$$

(a) Find the obtuse angle between the vectors $\overrightarrow{OA}$ and $\overrightarrow{OB}$. [3]

The line l passes through the points A and B.

(b) Find a vector equation for the line l. [2]

(c) Find the position vector of the point of intersection of the line l and the line passing through C and D. [4]

11 Let $f(x) = \dfrac{5x^2 + x + 11}{(4 + x^2)(1 + x)}$.

(a) Express $f(x)$ in partial fractions. [5]

(b) Hence show that $\int_0^2 \mathrm{f}(x)\,\mathrm{d}x = \ln 54 - \frac{1}{8}\pi$. [5]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

1087265794

MATHEMATICS **9709/31**

Paper 3 Pure Mathematics 3 **October/November 2022**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

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1 **(a)** Sketch the graph of $y = |2x + 1|$. [1]

(b) Solve the inequality $3x + 5 < |2x + 1|$. [3]

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2 On a sketch of an Argand diagram shade the region whose points represent complex numbers z satisfying the inequalities $|z| \leqslant 3$, $\text{Re}\, z \geqslant -2$ and $\frac{1}{4}\pi \leqslant \arg z \leqslant \pi$. [4]

3 Solve the equation $2^{3x-1} = 5(3^{-x})$. Give your answer in the form $\dfrac{\ln a}{\ln b}$, where a and b are integers. [4]

4 Solve the equation $\tan(x + 45°) = 2\cot x$ for $0° < x < 180°$. [5]

5 The complex numbers u and w are defined by $u = 2e^{\frac{1}{4}\pi i}$ and $w = 3e^{\frac{1}{3}\pi i}$.

(a) Find $\dfrac{u^2}{w}$, giving your answer in the form $re^{i\theta}$, where $r > 0$ and $-\pi < \theta \leqslant \pi$. Give the exact values of r and θ. [3]

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(b) State the least positive integer n such that both $\operatorname{Im} w^n = 0$ and $\operatorname{Re} w^n > 0$. [1]

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6 **(a)** Prove the identity $\cos 4\theta + 4\cos 2\theta + 3 \equiv 8\cos^4\theta$. [4]

(b) Hence solve the equation $\cos 4\theta + 4\cos 2\theta = 4$ for $0° \leqslant \theta \leqslant 180°$. [3]

7 The equation of a curve is $y = \dfrac{x}{\cos^2 x}$, for $0 \leqslant x < \frac{1}{2}\pi$. At the point where $x = a$, the tangent to the curve has gradient equal to 12.

(a) Show that $a = \cos^{-1}\left(\sqrt[3]{\dfrac{\cos a + 2a \sin a}{12}}\right)$. [3]

(b) Verify by calculation that a lies between 0.9 and 1. [2]

(c) Use an iterative formula based on the equation in part **(a)** to determine a correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

8 In a certain chemical reaction the amount, x grams, of a substance is increasing. The differential equation satisfied by x and t, the time in seconds since the reaction began, is

$$\frac{dx}{dt} = kxe^{-0.1t},$$

where k is a positive constant. It is given that $x = 20$ at the start of the reaction.

(a) Solve the differential equation, obtaining a relation between x, t and k. [5]

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(b) Given that $x = 40$ when $t = 10$, find the value of k and find the value approached by x as t becomes large. [3]

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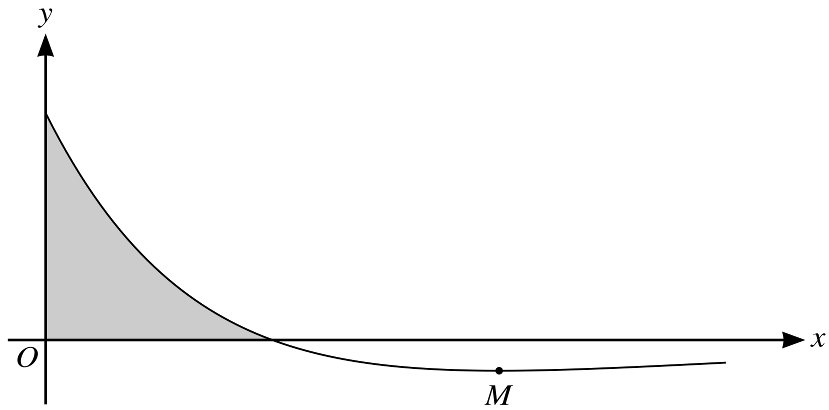

The diagram shows part of the curve $y = (3 - x)e^{-\frac{1}{3}x}$ for $x \geqslant 0$, and its minimum point M.

(a) Find the exact coordinates of M. [5]

(b) Find the area of the shaded region bounded by the curve and the axes, giving your answer in terms of e. [5]

10 Let $f(x) = \dfrac{2x^2 + 7x + 8}{(1+x)(2+x)^2}$.

(a) Express $f(x)$ in partial fractions. [5]

(b) Hence obtain the expansion of $f(x)$ in ascending powers of x, up to and including the term in x^2. [5]

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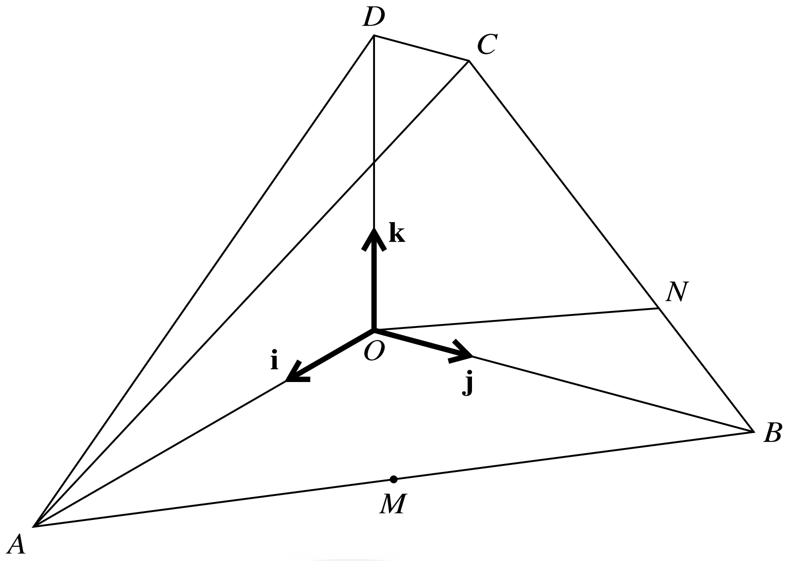

In the diagram, $OABCD$ is a solid figure in which $OA = OB = 4$ units and $OD = 3$ units. The edge OD is vertical, DC is parallel to OB and $DC = 1$ unit. The base, OAB, is horizontal and angle $AOB = 90°$. Unit vectors **i**, **j** and **k** are parallel to OA, OB and OD respectively. The midpoint of AB is M and the point N on BC is such that $CN = 2NB$.

(a) Express vectors $\overrightarrow{MD}$ and $\overrightarrow{ON}$ in terms of **i**, **j** and **k**. [4]

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(b) Calculate the angle in degrees between the directions of $\overrightarrow{MD}$ and $\overrightarrow{ON}$. [3]

(c) Show that the length of the perpendicular from M to ON is $\sqrt{\frac{22}{5}}$. [4]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME						
CENTRE NUMBER						
CANDIDATE NUMBER						

* 6 2 1 7 9 2 1 4 8 4 *

MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **October/November 2022**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS
- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION
- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

1 Solve the equation $2^{3x-1} = 5(3^{1-x})$. Give your answer in the form $\dfrac{\ln a}{\ln b}$ where a and b are integers. [4]

2 The polynomial $2x^3 - x^2 + a$, where a is a constant, is denoted by $p(x)$. It is given that $(2x + 3)$ is a factor of $p(x)$.

(a) Find the value of a. [2]

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(b) When a has this value, solve the inequality $p(x) < 0$. [4]

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3 The equation of a curve is $y = \sin x \sin 2x$. The curve has a stationary point in the interval $0 < x < \frac{1}{2}\pi$.

Find the x-coordinate of this point, giving your answer correct to 3 significant figures. [6]

4 **(a)** Express $4\cos x - \sin x$ in the form $R\cos(x + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. State the exact value of R and give α correct to 2 decimal places. [3]

(b) Hence solve the equation $4\cos 2x - \sin 2x = 3$ for $0° < x < 180°$. [5]

5 **(a)** Solve the equation $z^2 - 6\mathrm{i}z - 12 = 0$, giving the answers in the form $x + \mathrm{i}y$, where x and y are real and exact. [3]

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(b) On a sketch of an Argand diagram with origin O, show points A and B representing the roots of the equation in part **(a)**. [1]

(c) Find the exact modulus and argument of each root. [3]

(d) Hence show that the triangle OAB is equilateral. [1]

6 Relative to the origin O, the points A, B and C have position vectors given by

$$\overrightarrow{OA} = \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} 5 \\ 3 \\ -2 \end{pmatrix}.$$

(a) Using a scalar product, find the cosine of angle BAC. [4]

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(b) Hence find the area of triangle ABC. Give your answer in a simplified exact form. [4]

7 The variables x and θ satisfy the differential equation

$$x\sin^2\theta\frac{dx}{d\theta} = \tan^2\theta - 2\cot\theta,$$

for $0 < \theta < \frac{1}{2}\pi$ and $x > 0$. It is given that $x = 2$ when $\theta = \frac{1}{4}\pi$.

(a) Show that $\frac{d}{d\theta}(\cot^2\theta) = -\frac{2\cot\theta}{\sin^2\theta}$.

(You may assume without proof that the derivative of $\cot\theta$ with respect to θ is $-\operatorname{cosec}^2\theta$.) [1]

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(b) Solve the differential equation and find the value of x when $\theta = \frac{1}{6}\pi$. [7]

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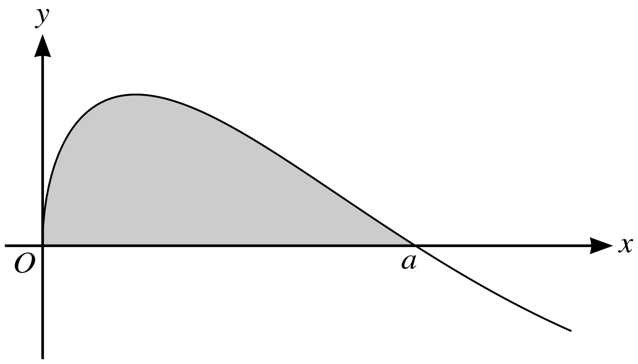

The diagram shows part of the curve $y = \sin\sqrt{x}$. This part of the curve intersects the x-axis at the point where $x = a$.

(a) State the exact value of a. [1]

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(b) Using the substitution $u = \sqrt{x}$, find the exact area of the shaded region in the first quadrant bounded by this part of the curve and the x-axis. [7]

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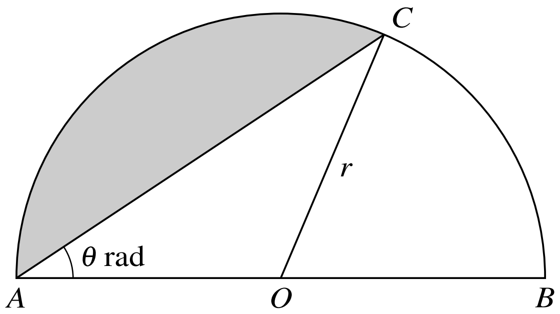

The diagram shows a semicircle with diameter AB, centre O and radius r. The shaded region is the minor segment on the chord AC and its area is one third of the area of the semicircle. The angle CAB is θ radians.

(a) Show that $\theta = \frac{1}{3}(\pi - 1.5\sin 2\theta)$. [4]

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(b) Verify by calculation that $0.5 < \theta < 0.7$. [2]

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(c) Use an iterative formula based on the equation in part **(a)** to determine θ correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

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10 Let $f(x) = \dfrac{4 - x + x^2}{(1 + x)(2 + x^2)}$.

(a) Express $f(x)$ in partial fractions. [5]

(b) Find the exact value of $\int_0^4 f(x)\,dx$. Give your answer as a single logarithm. [5]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

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MATHEMATICS **9709/33**

Paper 3 Pure Mathematics 3 **October/November 2022**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

JC22 11_9709_33/RP

1 Solve the equation $\ln(2x - 1) = 2\ln(x + 1) - \ln x$. Give your answer correct to 3 decimal places. [4]

2 Expand $\sqrt{\dfrac{1+2x}{1-2x}}$ in ascending powers of x, up to and including the term in x^2, simplifying the coefficients. [5]

3 Find the exact value of $\int_0^{\frac{1}{4}\pi} x \sec^2 x \, dx$. [5]

4 The parametric equations of a curve are

$$x = 2t - \tan t, \qquad y = \ln(\sin 2t),$$

for $0 < t < \frac{1}{2}\pi$.

Show that $\dfrac{dy}{dx} = \cot t$. [5]

5 **(a)** On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities $|z + 2| \leqslant 2$ and $\operatorname{Im} z \geqslant 1$. [4]

(b) Find the greatest value of $\arg z$ for points in the shaded region. [2]

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6 Solve the quadratic equation $(1 - 3\mathrm{i})z^2 - (2 + \mathrm{i})z + \mathrm{i} = 0$, giving your answers in the form $x + \mathrm{i}y$, where x and y are real. [6]

7 **(a)** Show that the equation $\sqrt{5}\sec x + \tan x = 4$ can be expressed as $R\cos(x + \alpha) = \sqrt{5}$, where $R > 0$ and $0° < \alpha < 90°$. Give the exact value of R and the value of α correct to 2 decimal places. [4]

(b) Hence solve the equation $\sqrt{5}\sec 2x + \tan 2x = 4$, for $0° < x < 180°$. [4]

8 The curve with equation $y = \dfrac{x^3}{e^x - 1}$ has a stationary point at $x = p$, where $p > 0$.

(a) Show that $p = 3(1 - e^{-p})$. [3]

(b) Verify by calculation that p lies between 2.5 and 3. [2]

(c) Use an iterative formula based on the equation in part **(a)** to determine p correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

9 With respect to the origin O, the position vectors of the points A, B and C are given by

$$\overrightarrow{OA} = \begin{pmatrix} 0 \\ 5 \\ 2 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} 4 \\ -3 \\ -2 \end{pmatrix}.$$

The midpoint of AC is M and the point N lies on BC, between B and C, and is such that $BN = 2NC$.

(a) Find the position vectors of M and N. [3]

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(b) Find a vector equation for the line through M and N. [2]

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(c) Find the position vector of the point Q where the line through M and N intersects the line through A and B. [4]

10 A gardener is filling an ornamental pool with water, using a hose that delivers 30 litres of water per minute. Initially the pool is empty. At time t minutes after filling begins the volume of water in the pool is V litres. The pool has a small leak and loses water at a rate of $0.01V$ litres per minute.

The differential equation satisfied by V and t is of the form $\frac{dV}{dt} = a - bV$.

(a) Write down the values of the constants a and b. [1]

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(b) Solve the differential equation and find the value of t when $V = 1000$. [6]

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(c) Obtain an expression for V in terms of t and hence state what happens to V as t becomes large. [2]

11 Let $f(x) = \dfrac{5 - x + 6x^2}{(3 - x)(1 + 3x^2)}$.

(a) Express $f(x)$ in partial fractions. [5]

(b) Find the exact value of $\int_0^1 f(x)\,dx$, simplifying your answer. [5]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

5794103083

MATHEMATICS **9709/31**

Paper 3 Pure Mathematics 3 **May/June 2022**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

JC22 06_9709_31/RP

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1 Solve the equation $2(3^{2x-1}) = 4^{x+1}$, giving your answer correct to 2 decimal places. [4]

2 **(a)** Expand $(2 - x^2)^{-2}$ in ascending powers of x, up to and including the term in x^4, simplifying the coefficients. [4]

(b) State the set of values of x for which the expansion is valid. [1]

3 Solve the equation $2\cot 2x + 3\cot x = 5$, for $0° < x < 180°$. [6]

4 The variables x and y satisfy the differential equation

$$\frac{dy}{dx} = \frac{xy}{1 + x^2},$$

and $y = 2$ when $x = 0$.

Solve the differential equation, obtaining a simplified expression for y in terms of x. [7]

5 The polynomial $ax^3 - 10x^2 + bx + 8$, where a and b are constants, is denoted by $p(x)$. It is given that $(x - 2)$ is a factor of both $p(x)$ and $p'(x)$.

(a) Find the values of a and b. [5]

(b) When a and b have these values, factorise $p(x)$ completely. [3]

6 Let $I = \int_0^3 \frac{27}{(9 + x^2)^2}\, dx$.

(a) Using the substitution $x = 3\tan\theta$, show that $I = \int_0^{\frac{1}{4}\pi} \cos^2\theta \, d\theta$. [4]

(b) Hence find the exact value of I. [4]

7 The complex number u is defined by $u = \dfrac{\sqrt{2} - a\sqrt{2}\,\mathrm{i}}{1 + 2\mathrm{i}}$, where a is a positive integer.

(a) Express u in terms of a, in the form $x + \mathrm{i}y$, where x and y are real and exact. [3]

It is now given that $a = 3$.

(b) Express u in the form $re^{i\theta}$, where $r > 0$ and $-\pi < \theta \leqslant \pi$, giving the exact values of r and θ. [2]

(c) Using your answer to part **(b)**, find the two square roots of u. Give your answers in the form $re^{i\theta}$, where $r > 0$ and $-\pi < \theta \leqslant \pi$, giving the exact values of r and θ. [3]

8 The equation of a curve is $x^3 + y^3 + 2xy + 8 = 0$.

(a) Express $\frac{dy}{dx}$ in terms of x and y. [4]

The tangent to the curve at the point where $x = 0$ and the tangent at the point where $y = 0$ intersect at the acute angle α.

(b) Find the exact value of $\tan \alpha$. [5]

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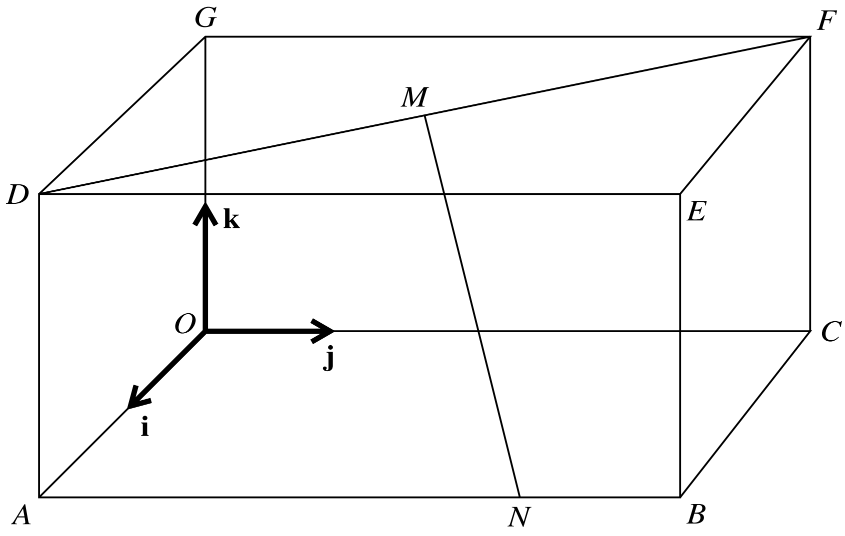

In the diagram, $OABCDEFG$ is a cuboid in which $OA = 2$ units, $OC = 4$ units and $OG = 2$ units. Unit vectors **i**, **j** and **k** are parallel to OA, OC and OG respectively. The point M is the midpoint of DF. The point N on AB is such that $AN = 3NB$.

(a) Express the vectors $\overrightarrow{OM}$ and $\overrightarrow{MN}$ in terms of **i**, **j** and **k**. [3]

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(b) Find a vector equation for the line through M and N. [2]

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(c) Show that the length of the perpendicular from O to the line through M and N is $\sqrt{\frac{53}{6}}$. [4]

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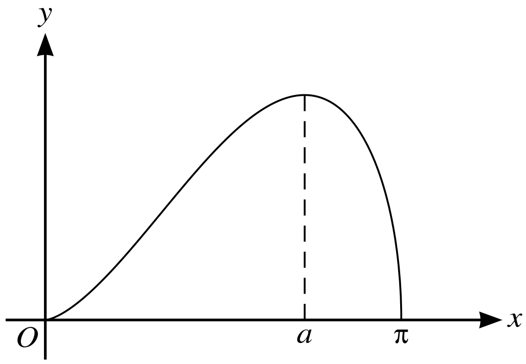

The curve $y = x\sqrt{\sin x}$ has one stationary point in the interval $0 < x < \pi$, where $x = a$ (see diagram).

(a) Show that $\tan a = -\frac{1}{2}a$. [4]

(b) Verify by calculation that a lies between 2 and 2.5. [2]

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(c) Show that if a sequence of values in the interval $0 < x < \pi$ given by the iterative formula $x_{n+1} = \pi - \tan^{-1}\left(\frac{1}{2}x_n\right)$ converges, then it converges to a, the root of the equation in part **(a)**. [2]

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(d) Use the iterative formula given in part **(c)** to determine a correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

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Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

* 0 7 8 8 3 5 4 8 5 4 *

MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **May/June 2022**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

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1 Solve the equation $\ln(e^{2x} + 3) = 2x + \ln 3$, giving your answer correct to 3 decimal places. [4]

2 Solve the equation $3\cos 2\theta = 3\cos\theta + 2$, for $0° \leqslant \theta \leqslant 360°$. [5]

3 The polynomial $ax^3 + x^2 + bx + 3$ is denoted by $p(x)$. It is given that $p(x)$ is divisible by $(2x - 1)$ and that when $p(x)$ is divided by $(x + 2)$ the remainder is 5.

Find the values of a and b. [5]

4 The equation of a curve is $y = \cos^3 x \sqrt{\sin x}$. It is given that the curve has one stationary point in the interval $0 < x < \frac{1}{2}\pi$.

Find the x-coordinate of this stationary point, giving your answer correct to 3 significant figures. [6]

5 **(a)** By sketching a suitable pair of graphs, show that the equation $\ln x = 3x - x^2$ has one real root. [2]

(b) Verify by calculation that the root lies between 2 and 2.8. [2]

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(c) Use the iterative formula $x_{n+1} = \sqrt{3x_n - \ln x_n}$ to determine the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

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6 The variables x and y satisfy the differential equation

$$\frac{dy}{dx} = xe^{y-x},$$

and $y = 0$ when $x = 0$.

(a) Solve the differential equation, obtaining an expression for y in terms of x. [7]

(b) Find the value of y when $x = 1$, giving your answer in the form $a - \ln b$, where a and b are integers. [1]

7 The equation of a curve is $x^3 + 3x^2y - y^3 = 3$.

(a) Show that $\frac{dy}{dx} = \frac{x^2 + 2xy}{y^2 - x^2}$. [4]

(b) Find the coordinates of the points on the curve where the tangent is parallel to the x-axis. [5]

8 Let $f(x) = \dfrac{x^2 + 9x}{(3x - 1)(x^2 + 3)}$.

(a) Express $f(x)$ in partial fractions. [5]

(b) Hence find $\int_1^3 f(x)\,dx$, giving your answer in a simplified exact form. [5]

9 The lines l and m have vector equations

$$\mathbf{r} = -\mathbf{i} + 3\mathbf{j} + 4\mathbf{k} + \lambda(2\mathbf{i} - \mathbf{j} - \mathbf{k}) \quad \text{and} \quad \mathbf{r} = 5\mathbf{i} + 4\mathbf{j} + 3\mathbf{k} + \mu(a\mathbf{i} + b\mathbf{j} + \mathbf{k})$$

respectively, where a and b are constants.

(a) Given that l and m intersect, show that $2b - a = 4$. [4]

(b) Given also that l and m are perpendicular, find the values of a and b. [4]

(c) When a and b have these values, find the position vector of the point of intersection of l and m. [2]

10 The complex number $-1 + \sqrt{7}\mathrm{i}$ is denoted by u. It is given that u is a root of the equation

$$2x^3 + 3x^2 + 14x + k = 0,$$

where k is a real constant.

(a) Find the value of k. [3]

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(b) Find the other two roots of the equation. [4]

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(c) On an Argand diagram, sketch the locus of points representing complex numbers z satisfying the equation $|z - u| = 2$. [2]

(d) Determine the greatest value of $\arg z$ for points on this locus, giving your answer in radians. [2]

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Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME			
CENTRE NUMBER		CANDIDATE NUMBER	

* 9 9 3 7 7 2 5 5 2 2 *

MATHEMATICS **9709/33**

Paper 3 Pure Mathematics 3 **May/June 2022**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

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1 Find, in terms of a, the set of values of x satisfying the inequality

$$2|3x + a| < |2x + 3a|,$$

where a is a positive constant. [4]

2 Solve the equation $\cos(\theta - 60°) = 3\sin\theta$, for $0° \leqslant \theta \leqslant 360°$. [5]

3 **(a)** Show that the equation $\log_3(2x + 1) = 1 + 2\log_3(x - 1)$ can be written as a quadratic equation in x. [3]

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(b) Hence solve the equation $\log_3(4y + 1) = 1 + 2\log_3(2y - 1)$, giving your answer correct to 2 decimal places. [2]

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4 The curve $y = e^{-4x} \tan x$ has two stationary points in the interval $0 \leqslant x < \frac{1}{2}\pi$.

(a) Obtain an expression for $\frac{dy}{dx}$ and show it can be written in the form $\sec^2 x(a + b \sin 2x)e^{-4x}$, where a and b are constants. [4]

(b) Hence find the exact x-coordinates of the two stationary points. [3]

5 The complex number $3 - \mathrm{i}$ is denoted by u.

(a) Show, on an Argand diagram with origin O, the points A, B and C representing the complex numbers u, u^* and $u^* - u$ respectively.

State the type of quadrilateral formed by the points O, A, B and C. [3]

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(b) Express $\dfrac{u^*}{u}$ in the form $x + \mathrm{i}y$, where x and y are real. [3]

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(c) By considering the argument of $\frac{u^*}{u}$, or otherwise, prove that $\tan^{-1}\left(\frac{3}{4}\right) = 2\tan^{-1}\left(\frac{1}{3}\right)$. [2]

6 The parametric equations of a curve are $x = \frac{1}{\cos t}$, $y = \ln \tan t$, where $0 < t < \frac{1}{2}\pi$.

(a) Show that $\frac{dy}{dx} = \frac{\cos t}{\sin^2 t}$. [5]

(b) Find the equation of the tangent to the curve at the point where $y = 0$. [3]

7 Let $f(x) = \dfrac{5x^2 + 8x - 3}{(x-2)(2x^2+3)}$.

(a) Express $f(x)$ in partial fractions. [5]

(b) Hence obtain the expansion of $f(x)$ in ascending powers of x, up to and including the term in x^2. [5]

8 At time t days after the start of observations, the number of insects in a population is N. The variation in the number of insects is modelled by a differential equation of the form $\frac{dN}{dt} = kN^{\frac{3}{2}} \cos 0.02t$, where k is a constant and N is a continuous variable. It is given that when $t = 0$, $N = 100$.

(a) Solve the differential equation, obtaining a relation between N, k and t. [5]

(b) Given also that $N = 625$ when $t = 50$, find the value of k. [2]

(c) Obtain an expression for N in terms of t, and find the greatest value of N predicted by this model. [2]

9 With respect to the origin O, the point A has position vector given by $\overrightarrow{OA} = \mathbf{i} + 5\mathbf{j} + 6\mathbf{k}$. The line l has vector equation $\mathbf{r} = 4\mathbf{i} + \mathbf{k} + \lambda(-\mathbf{i} + 2\mathbf{j} + 3\mathbf{k})$.

(a) Find in degrees the acute angle between the directions of OA and l. [3]

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(b) Find the position vector of the foot of the perpendicular from A to l. [4]

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(c) Hence find the position vector of the reflection of A in l. [2]

10 The constant a is such that $\int_1^a x^2 \ln x \, dx = 4$.

(a) Show that $a = \left(\frac{35}{3\ln a - 1}\right)^{\frac{1}{3}}$. [5]

(b) Verify by calculation that a lies between 2.4 and 2.8. [2]

(c) Use an iterative formula based on the equation in part **(a)** to determine a correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

3201784670

MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **February/March 2022**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

JC22 03_9709_32/RP

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1 Solve the inequality $|2x+3| > 3|x+2|$. [4]

2 On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities $|z + 2 - 3\mathrm{i}| \leqslant 2$ and $\arg z \leqslant \frac{3}{4}\pi$. [4]

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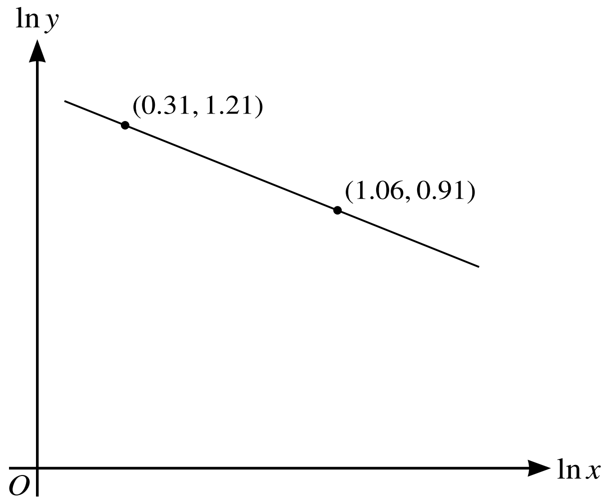

The variables x and y satisfy the equation $x^n y^2 = C$, where n and C are constants. The graph of $\ln y$ against $\ln x$ is a straight line passing through the points (0.31, 1.21) and (1.06, 0.91), as shown in the diagram.

Find the value of n and find the value of C correct to 2 decimal places. [5]

4 The parametric equations of a curve are

$$x = 1 - \cos\theta, \qquad y = \cos\theta - \tfrac{1}{4}\cos 2\theta.$$

Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = -2\sin^2\left(\tfrac{1}{2}\theta\right)$. [5]

5 The angles α and β lie between $0°$ and $180°$ and are such that

$$\tan(\alpha + \beta) = 2 \quad \text{and} \quad \tan\alpha = 3\tan\beta.$$

Find the possible values of α and β. [6]

6 Find the complex numbers w which satisfy the equation $w^2 + 2iw^* = 1$ and are such that $\text{Re}\, w \leqslant 0$. Give your answers in the form $x + iy$, where x and y are real. [6]

7 **(a)** By sketching a suitable pair of graphs, show that the equation $4 - x^2 = \sec \frac{1}{2}x$ has exactly one root in the interval $0 \leqslant x < \pi$. [2]

(b) Verify by calculation that this root lies between 1 and 2. [2]

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(c) Use the iterative formula $x_{n+1} = \sqrt{4 - \sec \frac{1}{2}x_n}$ to determine the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

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8 (a) Find the quotient and remainder when $8x^3 + 4x^2 + 2x + 7$ is divided by $4x^2 + 1$. [3]

(b) Hence find the exact value of $\int_0^{\frac{1}{2}} \frac{8x^3 + 4x^2 + 2x + 7}{4x^2 + 1}\,dx$. [5]

9 The variables x and y satisfy the differential equation

$$(x+1)(3x+1)\frac{dy}{dx} = y,$$

and it is given that $y = 1$ when $x = 1$.

Solve the differential equation and find the exact value of y when $x = 3$, giving your answer in a simplified form. [9]

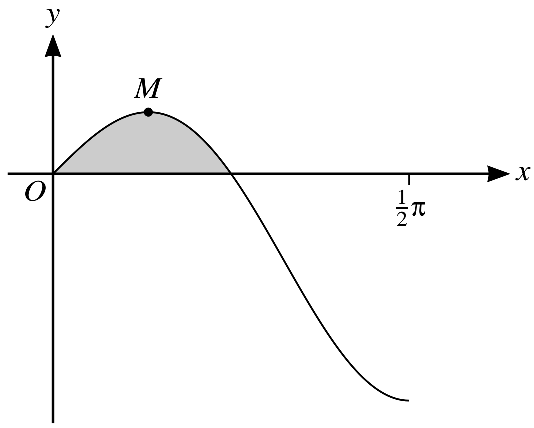
10 The points A and B have position vectors $2\mathbf{i} + \mathbf{j} + \mathbf{k}$ and $\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$ respectively. The line l has vector equation $\mathbf{r} = \mathbf{i} + 2\mathbf{j} - 3\mathbf{k} + \mu(\mathbf{i} - 3\mathbf{j} - 2\mathbf{k})$.

(a) Find a vector equation for the line through A and B. [3]

(b) Find the acute angle between the directions of AB and l, giving your answer in degrees. [3]

(c) Show that the line through A and B does not intersect the line l. [4]

11



The diagram shows the curve $y = \sin x \cos 2x$ for $0 \leqslant x \leqslant \frac{1}{2}\pi$, and its maximum point M.

(a) Find the x-coordinate of M, giving your answer correct to 3 significant figures. [6]

(b) Using the substitution $u = \cos x$, find the area of the shaded region enclosed by the curve and the x-axis in the first quadrant, giving your answer in a simplified exact form. [5]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME						
CENTRE NUMBER						
CANDIDATE NUMBER						

0746866051

MATHEMATICS **9709/31**

Paper 3 Pure Mathematics 3 **October/November 2021**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS
- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION
- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages.

JC21 11_9709_31/RP

1 Solve the equation $4|5^x - 1| = 5^x$, giving your answers correct to 3 decimal places. [4]

2 **(a)** Express $5\sin x - 3\cos x$ in the form $R\sin(x - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$. Give the exact value of R and give α correct to 2 decimal places. [3]

(b) Hence state the greatest and least possible values of $(5\sin x - 3\cos x)^2$. [2]

3 The curve with equation $y = xe^{1-2x}$ has one stationary point.

(a) Find the coordinates of this point. [4]

(b) Determine whether the stationary point is a maximum or a minimum. [2]

4 Using the substitution $u = \sqrt{x}$, find the exact value of

$$\int_3^{\infty} \frac{1}{(x+1)\sqrt{x}}\,dx.$$ [6]

[Turn over

5 **(a)** Show that the equation

$$\cot 2\theta + \cot \theta = 2$$

can be expressed as a quadratic equation in $\tan \theta$. [3]

(b) Hence solve the equation $\cot 2\theta + \cot \theta = 2$, for $0 < \theta < \pi$, giving your answers correct to 3 decimal places. [3]

6 When $(a + bx)\sqrt{1 + 4x}$, where a and b are constants, is expanded in ascending powers of x, the coefficients of x and x^2 are 3 and -6 respectively.

Find the values of a and b. [6]

7 **(a)** Given that $y = \ln(\ln x)$, show that

$$\frac{dy}{dx} = \frac{1}{x \ln x}.$$ [1]

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The variables x and t satisfy the differential equation

$$x \ln x + t\frac{dx}{dt} = 0.$$

It is given that $x = e$ when $t = 2$.

(b) Solve the differential equation obtaining an expression for x in terms of t, simplifying your answer. [7]

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(c) Hence state what happens to the value of x as t tends to infinity. [1]

8 The constant a is such that $\int_1^a \frac{\ln x}{\sqrt{x}}\,dx = 6$.

(a) Show that $a = \exp\left(\frac{1}{\sqrt{a}} + 2\right)$. [5]

[$\exp(x)$ is an alternative notation for e^x.]

(b) Verify by calculation that a lies between 9 and 11. [2]

(c) Use an iterative formula based on the equation in part **(a)** to determine a correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

9 Two lines l and m have equations $\mathbf{r} = 3\mathbf{i} + 2\mathbf{j} + 5\mathbf{k} + s(4\mathbf{i} - \mathbf{j} + 3\mathbf{k})$ and $\mathbf{r} = \mathbf{i} - \mathbf{j} - 2\mathbf{k} + t(-\mathbf{i} + 2\mathbf{j} + 2\mathbf{k})$ respectively.

(a) Show that l and m are perpendicular. [2]

(b) Show that l and m intersect and state the position vector of the point of intersection. [5]

(c) Show that the length of the perpendicular from the origin to the line m is $\frac{1}{3}\sqrt{5}$. [4]

10 The complex number $1 + 2\mathrm{i}$ is denoted by u. The polynomial $2x^3 + ax^2 + 4x + b$, where a and b are real constants, is denoted by $\mathrm{p}(x)$. It is given that u is a root of the equation $\mathrm{p}(x) = 0$.

(a) Find the values of a and b. [4]

(b) State a second complex root of this equation. [1]

(c) Find the real factors of $\mathrm{p}(x)$. [2]

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(d) **(i)** On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities $|z - u| \leqslant \sqrt{5}$ and $\arg z \leqslant \frac{1}{4}\pi$. [4]

(ii) Find the least value of $\operatorname{Im} z$ for points in the shaded region. Give your answer in an exact form. [1]

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Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

4325784148

MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **October/November 2021**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

JC21 11_9709_32/RP

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1 Find the value of x for which $3(2^{1-x}) = 7^x$. Give your answer in the form $\frac{\ln a}{\ln b}$, where a and b are integers. [4]

2 Solve the inequality $|3x - a| > 2|x + 2a|$, where a is a positive constant. [4]

3 **(a)** Given the complex numbers $u = a + \mathrm{i}b$ and $w = c + \mathrm{i}d$, where a, b, c and d are real, prove that $(u + w)^* = u^* + w^*$. [2]

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(b) Solve the equation $(z + 2 + \mathrm{i})^* + (2 + \mathrm{i})z = 0$, giving your answer in the form $x + \mathrm{i}y$ where x and y are real. [4]

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4 Express $\dfrac{4x^2 - 13x + 13}{(2x-1)(x-3)}$ in partial fractions. [5]

5 **(a)** On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities $|z - 3 - 2i| \leqslant 1$ and $\text{Im}\, z \geqslant 2$. [4]

(b) Find the greatest value of $\arg z$ for points in the shaded region, giving your answer in degrees. [3]

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6 **(a)** Using the expansions of $\sin(3x + 2x)$ and $\sin(3x - 2x)$, show that

$$\tfrac{1}{2}(\sin 5x + \sin x) \equiv \sin 3x \cos 2x.$$ [3]

(b) Hence show that $\int_0^{\frac{1}{4}\pi} \sin 3x \cos 2x \, dx = \frac{1}{5}(3 - \sqrt{2})$. [3]

7 The variables x and y satisfy the differential equation

$$e^{2x}\frac{dy}{dx} = 4xy^2,$$

and it is given that $y = 1$ when $x = 0$.

Solve the differential equation, obtaining an expression for y in terms of x. [7]

8 **(a)** By first expanding $(\cos^2\theta + \sin^2\theta)^2$, show that

$$\cos^4\theta + \sin^4\theta \equiv 1 - \tfrac{1}{2}\sin^2 2\theta. \qquad [3]$$

(b) Hence solve the equation

$$\cos^4\theta + \sin^4\theta = \tfrac{5}{9},$$

for $0° < \theta < 180°$. [4]

9 The equation of a curve is $ye^{2x} - y^2e^x = 2$.

(a) Show that $\frac{dy}{dx} = \frac{2ye^x - y^2}{2y - e^x}$. [4]

(b) Find the exact coordinates of the point on the curve where the tangent is parallel to the y-axis. [4]

10 With respect to the origin O, the position vectors of the points A and B are given by $\overrightarrow{OA} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$ and $\overrightarrow{OB} = \begin{pmatrix} 0 \\ 3 \\ 1 \end{pmatrix}$.

(a) Find a vector equation for the line l through A and B. [3]

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(b) The point C lies on l and is such that $\overrightarrow{AC} = 3\overrightarrow{AB}$.

Find the position vector of C. [2]

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(c) Find the possible position vectors of the point P on l such that $OP = \sqrt{14}$. [5]

11 The equation of a curve is $y = \sqrt{\tan x}$, for $0 \leqslant x < \frac{1}{2}\pi$.

(a) Express $\frac{dy}{dx}$ in terms of $\tan x$, and verify that $\frac{dy}{dx} = 1$ when $x = \frac{1}{4}\pi$. [4]

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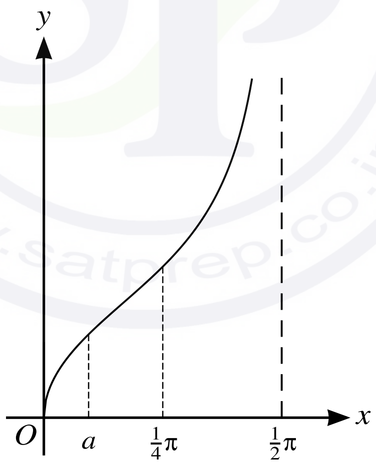
The value of $\frac{dy}{dx}$ is also 1 at another point on the curve where $x = a$, as shown in the diagram.



(b) Show that $t^3 + t^2 + 3t - 1 = 0$, where $t = \tan a$. [4]

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(c) Use the iterative formula

$$a_{n+1} = \tan^{-1}\left(\tfrac{1}{3}(1 - \tan^2 a_n - \tan^3 a_n)\right)$$

to determine a correct to 2 decimal places, giving the result of each iteration to 4 decimal places. [3]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

Cambridge International AS & A Level

CANDIDATE NAME	
CENTRE NUMBER	
CANDIDATE NUMBER	

1472641323

MATHEMATICS **9709/33**

Paper 3 Pure Mathematics 3 **October/November 2021**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

JC21 11_9709_33/RP

1 Find the quotient and remainder when $2x^4 + 1$ is divided by $x^2 - x + 2$. [3]

2 **(a)** Sketch the graph of $y = |2x - 3|$. [1]

(b) Solve the inequality $|2x - 3| < 3x + 2$. [3]

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3 Solve the equation $4^{x-2} = 4^x - 4^2$, giving your answer correct to 3 decimal places. [4]

4 Find the exact value of $\int_{\frac{1}{3}\pi}^{\pi} x \sin \tfrac{1}{2}x \, dx$. [5]

5 Solve the equation $\sin\theta = 3\cos 2\theta + 2$, for $0^\circ \leqslant \theta \leqslant 360^\circ$. [5]

6 **(a)** By first expanding $\cos(x - 60°)$, show that the expression

$$2\cos(x - 60°) + \cos x$$

can be written in the form $R\cos(x - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. Give the exact value of R and the value of α correct to 2 decimal places. [5]

(b) Hence find the value of x in the interval $0° < x < 360°$ for which $2\cos(x - 60°) + \cos x$ takes its least possible value. [2]

7 The equation of a curve is $\ln(x + y) = x - 2y$.

(a) Show that $\frac{dy}{dx} = \frac{x + y - 1}{2(x + y) + 1}$. [4]

(b) Find the coordinates of the point on the curve where the tangent is parallel to the x-axis. [3]

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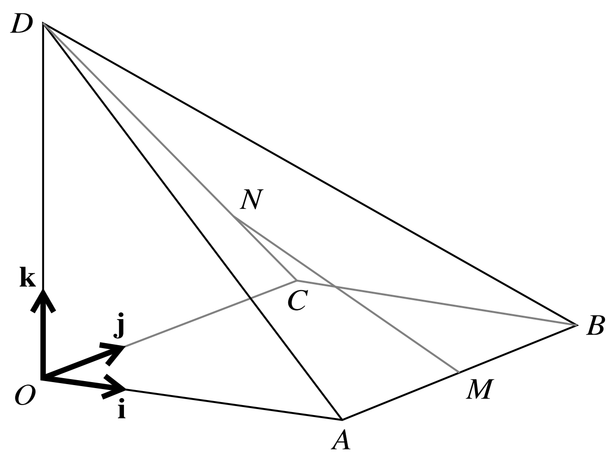

In the diagram, *OABCD* is a pyramid with vertex *D*. The horizontal base *OABC* is a square of side 4 units. The edge *OD* is vertical and $OD = 4$ units. The unit vectors **i**, **j** and **k** are parallel to *OA*, *OC* and *OD* respectively.

The midpoint of *AB* is *M* and the point *N* on *CD* is such that $DN = 3NC$.

(a) Find a vector equation for the line through *M* and *N*. [5]

(b) Show that the length of the perpendicular from O to MN is $\frac{1}{3}\sqrt{82}$. [4]

9 Let $f(x) = \frac{1}{(9-x)\sqrt{x}}$.

(a) Find the x-coordinate of the stationary point of the curve with equation $y = f(x)$. [4]

(b) Using the substitution $u = \sqrt{x}$, show that $\int_0^4 f(x)\,dx = \frac{1}{3}\ln 5$. [6]

10 A large plantation of area $20\,\text{km}^2$ is becoming infected with a plant disease. At time t years the area infected is $x\,\text{km}^2$ and the rate of increase of x is proportional to the ratio of the area infected to the area not yet infected.

When $t = 0$, $x = 1$ and $\frac{dx}{dt} = 1$.

(a) Show that x and t satisfy the differential equation

$$\frac{dx}{dt} = \frac{19x}{20 - x}.$$ [2]

(b) Solve the differential equation and show that when $t = 1$ the value of x satisfies the equation $x = e^{0.9+0.05x}$. [5]

(c) Use an iterative formula based on the equation in part **(b)**, with an initial value of 2, to determine x correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

(d) Calculate the value of t at which the entire plantation becomes infected. [1]

11 The complex number $-\sqrt{3} + \text{i}$ is denoted by u.

(a) Express u in the form $r\text{e}^{\text{i}\theta}$, where $r > 0$ and $-\pi < \theta \leqslant \pi$, giving the exact values of r and θ. [2]

(b) Hence show that u^6 is real and state its value. [2]

(c) **(i)** On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities $0 \leqslant \arg(z - u) \leqslant \frac{1}{4}\pi$ and $\mathrm{Re}\, z \leqslant 2$. [4]

(ii) Find the greatest value of $|z|$ for points in the shaded region. Give your answer correct to 3 significant figures. [2]

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Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

* 3 6 0 2 9 4 2 8 2 8 *

MATHEMATICS **9709/31**

Paper 3 Pure Mathematics 3 **May/June 2021**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

JC21 06_9709_31/RP

1 Solve the inequality $2|3x - 1| < |x + 1|$. [4]

2 Find the real root of the equation $\frac{2e^x + e^{-x}}{2 + e^x} = 3$, giving your answer correct to 3 decimal places. Your working should show clearly that the equation has only one real root. [5]

3 **(a)** Given that $\cos(x - 30°) = 2\sin(x + 30°)$, show that $\tan x = \dfrac{2 - \sqrt{3}}{1 - 2\sqrt{3}}$. [4]

(b) Hence solve the equation

$$\cos(x - 30°) = 2\sin(x + 30°),$$

for $0° < x < 360°$. [2]

4 **(a)** Prove that $\dfrac{1 - \cos 2\theta}{1 + \cos 2\theta} \equiv \tan^2 \theta$. [2]

(b) Hence find the exact value of $\displaystyle\int_{\frac{1}{6}\pi}^{\frac{1}{3}\pi} \frac{1 - \cos 2\theta}{1 + \cos 2\theta} \, d\theta$. [4]

5 **(a)** Solve the equation $z^2 - 2piz - q = 0$, where p and q are real constants. [2]

In an Argand diagram with origin O, the roots of this equation are represented by the distinct points A and B.

(b) Given that A and B lie on the imaginary axis, find a relation between p and q. [2]

(c) Given instead that triangle *OAB* is equilateral, express q in terms of p. [3]

6 The parametric equations of a curve are

$$x = \ln(2 + 3t), \qquad y = \frac{t}{2 + 3t}.$$

(a) Show that the gradient of the curve is always positive. [5]

(b) Find the equation of the tangent to the curve at the point where it intersects the y-axis. [3]

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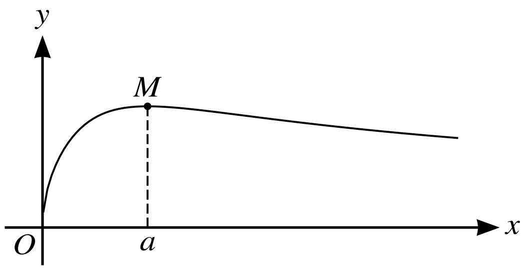

The diagram shows the curve $y = \frac{\tan^{-1} x}{\sqrt{x}}$ and its maximum point M where $x = a$.

(a) Show that a satisfies the equation

$$a = \tan\left(\frac{2a}{1 + a^2}\right).$$ [4]

(b) Verify by calculation that a lies between 1.3 and 1.5. [2]

(c) Use an iterative formula based on the equation in part **(a)** to determine a correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

8 With respect to the origin O, the points A and B have position vectors given by $\overrightarrow{OA} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$ and $\overrightarrow{OB} = \begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix}$. The line l has equation $\mathbf{r} = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$.

(a) Find the acute angle between the directions of AB and l. [4]

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(b) Find the position vector of the point P on l such that $AP = BP$. [5]

9 The equation of a curve is $y = x^{-\frac{2}{3}} \ln x$ for $x > 0$. The curve has one stationary point.

(a) Find the exact coordinates of the stationary point. [5]

(b) Show that $\int_1^8 y\,dx = 18\ln 2 - 9$. [5]

10 The variables x and t satisfy the differential equation $\frac{dx}{dt} = x^2(1 + 2x)$, and $x = 1$ when $t = 0$.

Using partial fractions, solve the differential equation, obtaining an expression for t in terms of x. [11]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

4535261400

MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **May/June 2021**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

JC21 06_9709_32/RP

1 Solve the inequality $|2x - 1| < 3|x + 1|$. [4]

2 On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities $|z + 1 - \mathrm{i}| \leqslant 1$ and $\arg(z - 1) \leqslant \frac{3}{4}\pi$. [4]

3 The variables x and y satisfy the equation $x = A(3^{-y})$, where A is a constant.

(a) Explain why the graph of y against $\ln x$ is a straight line and state the exact value of the gradient of the line. [3]

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It is given that the line intersects the y-axis at the point where $y = 1.3$.

(b) Calculate the value of A, giving your answer correct to 2 decimal places. [2]

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4 Using integration by parts, find the exact value of $\int_0^2 \tan^{-1}\left(\frac{1}{2}x\right) \mathrm{d}x$. [5]

5 The complex number u is given by $u = 10 - 4\sqrt{6}\mathrm{i}$.

Find the two square roots of u, giving your answers in the form $a + \mathrm{i}b$, where a and b are real and exact. [5]

6 **(a)** Prove that $\operatorname{cosec} 2\theta - \cot 2\theta \equiv \tan \theta$. [3]

(b) Hence show that $\int_{\frac{1}{4}\pi}^{\frac{1}{3}\pi} (\operatorname{cosec} 2\theta - \cot 2\theta)\, d\theta = \frac{1}{2} \ln 2$. [4]

7 A curve is such that the gradient at a general point with coordinates (x, y) is proportional to $\frac{y}{\sqrt{x+1}}$. The curve passes through the points with coordinates (0, 1) and (3, e).

By setting up and solving a differential equation, find the equation of the curve, expressing y in terms of x. [7]

8 The equation of a curve is $y = e^{-5x}\tan^2 x$ for $-\frac{1}{2}\pi < x < \frac{1}{2}\pi$.

Find the x-coordinates of the stationary points of the curve. Give your answers correct to 3 decimal places where appropriate. [8]

9 Let $f(x) = \dfrac{14 - 3x + 2x^2}{(2 + x)(3 + x^2)}$.

(a) Express $f(x)$ in partial fractions. [5]

(b) Hence obtain the expansion of $f(x)$ in ascending powers of x, up to and including the term in x^2. [5]

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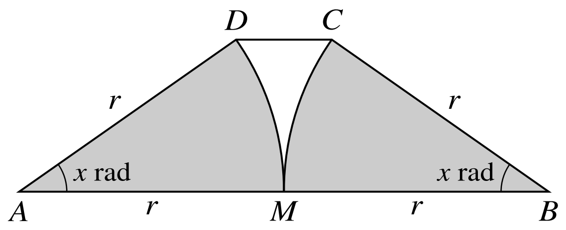

The diagram shows a trapezium $ABCD$ in which $AD = BC = r$ and $AB = 2r$. The acute angles BAD and ABC are both equal to x radians. Circular arcs of radius r with centres A and B meet at M, the midpoint of AB.

(a) Given that the sum of the areas of the shaded sectors is 90% of the area of the trapezium, show that x satisfies the equation $x = 0.9(2 - \cos x)\sin x$. [3]

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(b) Verify by calculation that x lies between 0.5 and 0.7. [2]

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(c) Show that if a sequence of values in the interval $0 < x < \frac{1}{2}\pi$ given by the iterative formula

$$x_{n+1} = \cos^{-1}\left(2 - \frac{x_n}{0.9\sin x_n}\right)$$

converges, then it converges to the root of the equation in part **(a)**. [2]

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(d) Use this iterative formula to determine x correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

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11 With respect to the origin O, the points A and B have position vectors given by $\overrightarrow{OA} = 2\mathbf{i} - \mathbf{j}$ and $\overrightarrow{OB} = \mathbf{j} - 2\mathbf{k}$.

(a) Show that $OA = OB$ and use a scalar product to calculate angle AOB in degrees. [4]

The midpoint of AB is M. The point P on the line through O and M is such that $PA : OA = \sqrt{7} : 1$.

(b) Find the possible position vectors of P. [6]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

2912457036

MATHEMATICS **9709/33**

Paper 3 Pure Mathematics 3 **May/June 2021**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
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- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

1 Expand $(1 + 3x)^{\frac{2}{3}}$ in ascending powers of x, up to and including the term in x^3, simplifying the coefficients. [4]

2 Solve the equation $4^x = 3 + 4^{-x}$. Give your answer correct to 3 decimal places. [5]

3 The parametric equations of a curve are

$$x = t + \ln(t + 2), \qquad y = (t - 1)e^{-2t},$$

where $t > -2$.

(a) Express $\frac{dy}{dx}$ in terms of t, simplifying your answer. [5]

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(b) Find the exact y-coordinate of the stationary point of the curve. [2]

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4 Let $f(x) = \dfrac{15 - 6x}{(1 + 2x)(4 - x)}$.

(a) Express $f(x)$ in partial fractions. [3]

(b) Hence find $\int_1^2 f(x)\,dx$, giving your answer in the form $\ln\left(\dfrac{a}{b}\right)$, where a and b are integers. [4]

5 **(a)** By first expanding $\tan(2\theta + 2\theta)$, show that the equation $\tan 4\theta = \frac{1}{2}\tan\theta$ may be expressed as $\tan^4\theta + 2\tan^2\theta - 7 = 0$. [4]

(b) Hence solve the equation $\tan 4\theta = \frac{1}{2}\tan\theta$, for $0° < \theta < 180°$. [3]

6 **(a)** By sketching a suitable pair of graphs, show that the equation $\cot \frac{1}{2}x = 1 + e^{-x}$ has exactly one root in the interval $0 < x \leqslant \pi$. [2]

(b) Verify by calculation that this root lies between 1 and 1.5. [2]

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(c) Use the iterative formula $x_{n+1} = 2\tan^{-1}\left(\dfrac{1}{1+e^{-x_n}}\right)$ to determine the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

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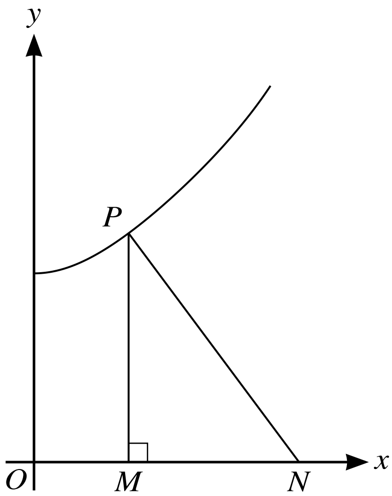

For the curve shown in the diagram, the normal to the curve at the point P with coordinates (x, y) meets the x-axis at N. The point M is the foot of the perpendicular from P to the x-axis.

The curve is such that for all values of x in the interval $0 \leqslant x < \frac{1}{2}\pi$, the area of triangle PMN is equal to $\tan x$.

(a) **(i)** Show that $\dfrac{MN}{y} = \dfrac{dy}{dx}$. [1]

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(ii) Hence show that x and y satisfy the differential equation $\frac{1}{2}y^2\dfrac{dy}{dx} = \tan x$. [2]

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(b) Given that $y = 1$ when $x = 0$, solve this differential equation to find the equation of the curve, expressing y in terms of x. [6]

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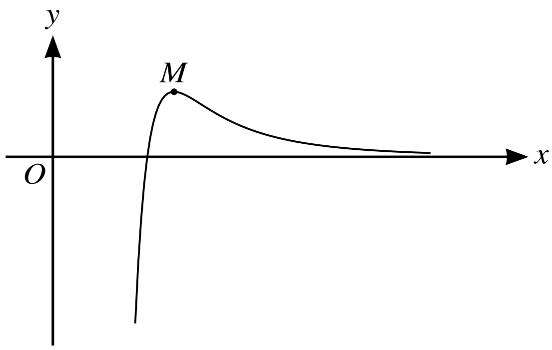

The diagram shows the curve $y = \dfrac{\ln x}{x^4}$ and its maximum point M.

(a) Find the exact coordinates of M. [4]

(b) By using integration by parts, show that for all $a > 1$, $\int_1^a \frac{\ln x}{x^4}\,dx < \frac{1}{9}$. [6]

9 The quadrilateral $ABCD$ is a trapezium in which AB and DC are parallel. With respect to the origin O, the position vectors of A, B and C are given by $\overrightarrow{OA} = -\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$, $\overrightarrow{OB} = \mathbf{i} + 3\mathbf{j} + \mathbf{k}$ and $\overrightarrow{OC} = 2\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$.

(a) Given that $\overrightarrow{DC} = 3\overrightarrow{AB}$, find the position vector of D. [3]

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(b) State a vector equation for the line through A and B. [1]

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(c) Find the distance between the parallel sides and hence find the area of the trapezium. [5]

10 **(a)** Verify that $-1 + \sqrt{2}\mathrm{i}$ is a root of the equation $z^4 + 3z^2 + 2z + 12 = 0$. [3]

(b) Find the other roots of this equation. [7]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

8483881097

MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **February/March 2021**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
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- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

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1 Solve the equation $\ln(x^3 - 3) = 3\ln x - \ln 3$. Give your answer correct to 3 significant figures. [3]

2 The polynomial $ax^3 + 5x^2 - 4x + b$, where a and b are constants, is denoted by $p(x)$. It is given that $(x + 2)$ is a factor of $p(x)$ and that when $p(x)$ is divided by $(x + 1)$ the remainder is 2.

Find the values of a and b. [5]

3 By first expressing the equation $\tan(x + 45°) = 2\cot x + 1$ as a quadratic equation in $\tan x$, solve the equation for $0° < x < 180°$. [6]

4 The variables x and y satisfy the differential equation

$$(1 - \cos x)\frac{dy}{dx} = y \sin x.$$

It is given that $y = 4$ when $x = \pi$.

(a) Solve the differential equation, obtaining an expression for y in terms of x. [6]

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(b) Sketch the graph of y against x for $0 < x < 2\pi$. [1]

5 **(a)** Express $\sqrt{7}\sin x + 2\cos x$ in the form $R\sin(x + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. State the exact value of R and give α correct to 2 decimal places. [3]

(b) Hence solve the equation $\sqrt{7}\sin 2\theta + 2\cos 2\theta = 1$, for $0° < \theta < 180°$. [5]

6 Let $f(x) = \dfrac{5a}{(2x - a)(3a - x)}$, where a is a positive constant.

(a) Express $f(x)$ in partial fractions. [3]

(b) Hence show that $\int_{a}^{2a} f(x)\,dx = \ln 6$. [4]

7 Two lines have equations $\mathbf{r} = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} + s\begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix} + t\begin{pmatrix} 1 \\ -1 \\ 4 \end{pmatrix}$.

(a) Show that the lines are skew. [5]

(b) Find the acute angle between the directions of the two lines. [3]

8 The complex numbers u and v are defined by $u = -4 + 2\text{i}$ and $v = 3 + \text{i}$.

(a) Find $\frac{u}{v}$ in the form $x + \text{i}y$, where x and y are real. [3]

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(b) Hence express $\frac{u}{v}$ in the form $r\text{e}^{\text{i}\theta}$, where r and θ are exact. [2]

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In an Argand diagram, with origin O, the points A, B and C represent the complex numbers u, v and $2u + v$ respectively.

(c) State fully the geometrical relationship between OA and BC. [2]

(d) Prove that angle $AOB = \frac{3}{4}\pi$. [2]

9 Let $f(x) = \dfrac{e^{2x} + 1}{e^{2x} - 1}$, for $x > 0$.

(a) The equation $x = f(x)$ has one root, denoted by a.

Verify by calculation that a lies between 1 and 1.5. [2]

(b) Use an iterative formula based on the equation in part **(a)** to determine a correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

(c) Find $f'(x)$. Hence find the exact value of x for which $f'(x) = -8$. [6]

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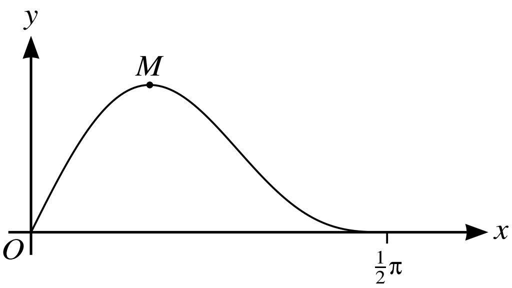

The diagram shows the curve $y = \sin 2x \cos^2 x$ for $0 \leqslant x \leqslant \frac{1}{2}\pi$, and its maximum point M.

(a) Using the substitution $u = \sin x$, find the exact area of the region bounded by the curve and the x-axis. [5]

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(b) Find the exact x-coordinate of M. [6]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

5907089706

MATHEMATICS **9709/31**

Paper 3 Pure Mathematics 3 **October/November 2020**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Blank pages are indicated.

JC20 11_9709_31/FP

[Turn over

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1 Solve the inequality $2 - 5x > 2|x - 3|$. [4]

2 On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities $|z| \geqslant 2$ and $|z - 1 + \mathrm{i}| \leqslant 1$. [4]

3 The parametric equations of a curve are

$$x = 3 - \cos 2\theta, \quad y = 2\theta + \sin 2\theta,$$

for $0 < \theta < \frac{1}{2}\pi$.

Show that $\frac{dy}{dx} = \cot \theta$. [5]

4 Solve the equation

$$\log_{10}(2x + 1) = 2\log_{10}(x + 1) - 1.$$

Give your answers correct to 3 decimal places. [6]

5 **(a)** By sketching a suitable pair of graphs, show that the equation $\operatorname{cosec} x = 1 + e^{-\frac{1}{2}x}$ has exactly two roots in the interval $0 < x < \pi$. [2]

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(b) The sequence of values given by the iterative formula

$$x_{n+1} = \pi - \sin^{-1}\left(\frac{1}{e^{-\frac{1}{2}x_n} + 1}\right),$$

with initial value $x_1 = 2$, converges to one of these roots.

Use the formula to determine this root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

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6 **(a)** Express $\sqrt{6}\cos\theta + 3\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. State the exact value of R and give α correct to 2 decimal places. [3]

(b) Hence solve the equation $\sqrt{6}\cos\frac{1}{3}x + 3\sin\frac{1}{3}x = 2.5$, for $0° < x < 360°$. [4]

7 **(a)** Verify that $-1 + \sqrt{5}\,\mathrm{i}$ is a root of the equation $2x^3 + x^2 + 6x - 18 = 0$. [3]

(b) Find the other roots of this equation. [4]

8 The coordinates (x, y) of a general point of a curve satisfy the differential equation

$$x\frac{dy}{dx} = (1 - 2x^2)y,$$

for $x > 0$. It is given that $y = 1$ when $x = 1$.

Solve the differential equation, obtaining an expression for y in terms of x. [6]

9 Let $f(x) = \dfrac{8 + 5x + 12x^2}{(1 - x)(2 + 3x)^2}$.

(a) Express $f(x)$ in partial fractions. [5]

(b) Hence obtain the expansion of $f(x)$ in ascending powers of x, up to and including the term in x^2. [5]

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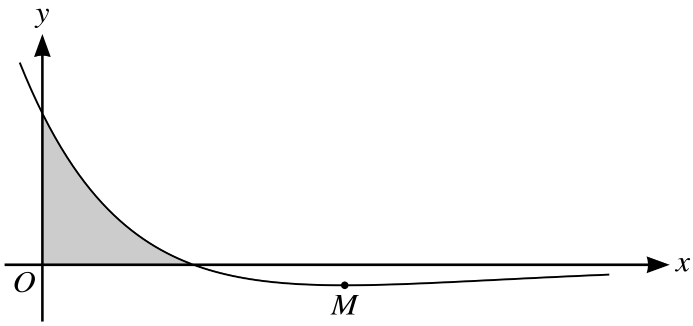

The diagram shows the curve $y = (2 - x)e^{-\frac{1}{2}x}$, and its minimum point M.

(a) Find the exact coordinates of M. [5]

(b) Find the area of the shaded region bounded by the curve and the axes. Give your answer in terms of e. [5]

11 Two lines have equations $\mathbf{r} = \mathbf{i} + 2\mathbf{j} + \mathbf{k} + \lambda(a\mathbf{i} + 2\mathbf{j} - \mathbf{k})$ and $\mathbf{r} = 2\mathbf{i} + \mathbf{j} - \mathbf{k} + \mu(2\mathbf{i} - \mathbf{j} + \mathbf{k})$, where a is a constant.

(a) Given that the two lines intersect, find the value of a and the position vector of the point of intersection. [5]

(b) Given instead that the acute angle between the directions of the two lines is $\cos^{-1}\left(\frac{1}{6}\right)$, find the two possible values of a. [6]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

* 7 6 8 8 7 0 2 0 6 7 *

MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **October/November 2020**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Blank pages are indicated.

JC20 11_9709_32/RP

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1 Solve the equation

$$\ln(1 + e^{-3x}) = 2.$$

Give the answer correct to 3 decimal places. [3]

2 **(a)** Expand $\sqrt[3]{1 + 6x}$ in ascending powers of x, up to and including the term in x^3, simplifying the coefficients. [4]

(b) State the set of values of x for which the expansion is valid. [1]

3 The variables x and y satisfy the relation $2^y = 3^{1-2x}$.

(a) By taking logarithms, show that the graph of y against x is a straight line. State the exact value of the gradient of this line. [3]

(b) Find the exact x-coordinate of the point of intersection of this line with the line $y = 3x$. Give your answer in the form $\frac{\ln a}{\ln b}$, where a and b are integers. [2]

4 **(a)** Show that the equation $\tan(\theta + 60°) = 2\cot\theta$ can be written in the form

$$\tan^2\theta + 3\sqrt{3}\tan\theta - 2 = 0.$$ [3]

(b) Hence solve the equation $\tan(\theta + 60°) = 2\cot\theta$, for $0° < \theta < 180°$. [3]

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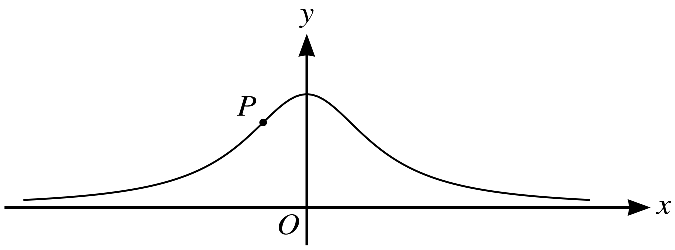

The diagram shows the curve with parametric equations

$$x = \tan\theta, \quad y = \cos^2\theta,$$

for $-\frac{1}{2}\pi < \theta < \frac{1}{2}\pi$.

(a) Show that the gradient of the curve at the point with parameter θ is $-2\sin\theta\cos^3\theta$. [3]

The gradient of the curve has its maximum value at the point P.

(b) Find the exact value of the x-coordinate of P. [4]

6 The complex number u is defined by

$$u = \frac{7 + \mathrm{i}}{1 - \mathrm{i}}.$$

(a) Express u in the form $x + \mathrm{i}y$, where x and y are real. [3]

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(b) Show on a sketch of an Argand diagram the points A, B and C representing u, $7 + \mathrm{i}$ and $1 - \mathrm{i}$ respectively. [2]

(c) By considering the arguments of $7 + \mathrm{i}$ and $1 - \mathrm{i}$, show that

$$\tan^{-1}\left(\tfrac{4}{3}\right) = \tan^{-1}\left(\tfrac{1}{7}\right) + \tfrac{1}{4}\pi.$$ [3]

7 The variables x and t satisfy the differential equation

$$e^{3t}\frac{dx}{dt} = \cos^2 2x,$$

for $t \geqslant 0$. It is given that $x = 0$ when $t = 0$.

(a) Solve the differential equation and obtain an expression for x in terms of t. [7]

(b) State what happens to the value of x when t tends to infinity. [1]

8 With respect to the origin O, the position vectors of the points A, B, C and D are given by

$$\overrightarrow{OA} = \begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 4 \\ -1 \\ 1 \end{pmatrix}, \quad \overrightarrow{OC} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OD} = \begin{pmatrix} 3 \\ 2 \\ 3 \end{pmatrix}.$$

(a) Show that $AB = 2CD$. [3]

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(b) Find the angle between the directions of $\overrightarrow{AB}$ and $\overrightarrow{CD}$. [3]

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(c) Show that the line through A and B does not intersect the line through C and D. [4]

9 Let $f(x) = \dfrac{7x + 18}{(3x + 2)(x^2 + 4)}$.

(a) Express $f(x)$ in partial fractions. [5]

(b) Hence find the exact value of $\int_0^2 f(x)\,dx$. [6]

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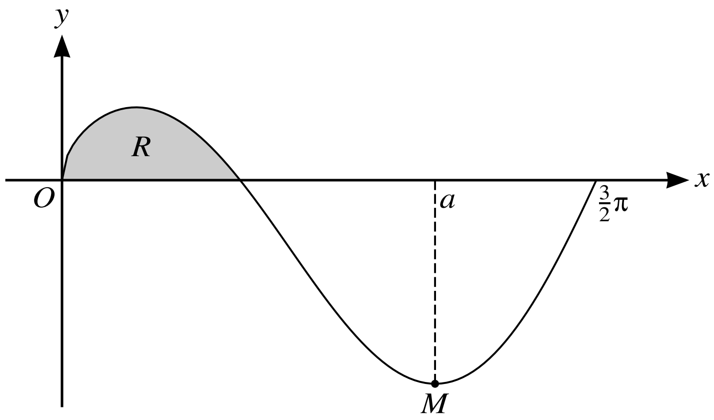

The diagram shows the curve $y = \sqrt{x}\cos x$, for $0 \leqslant x \leqslant \frac{3}{2}\pi$, and its minimum point M, where $x = a$. The shaded region between the curve and the x-axis is denoted by R.

(a) Show that a satisfies the equation $\tan a = \dfrac{1}{2a}$. [3]

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(b) The sequence of values given by the iterative formula $a_{n+1} = \pi + \tan^{-1}\left(\dfrac{1}{2a_n}\right)$, with initial value $x_1 = 3$, converges to a.

Use this formula to determine a correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

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(c) Find the volume of the solid obtained when the region R is rotated completely about the x-axis. Give your answer in terms of π. [6]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

2085422030

MATHEMATICS **9709/33**

Paper 3 Pure Mathematics 3 **October/November 2020**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Blank pages are indicated.

JC20 11_9709_33/FP

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1 Solve the inequality $2 - 5x > 2|x - 3|$. [4]

2 On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities $|z| \geqslant 2$ and $|z - 1 + \mathrm{i}| \leqslant 1$. [4]

3 The parametric equations of a curve are

$$x = 3 - \cos 2\theta, \quad y = 2\theta + \sin 2\theta,$$

for $0 < \theta < \frac{1}{2}\pi$.

Show that $\dfrac{dy}{dx} = \cot\theta$. [5]

4 Solve the equation

$$\log_{10}(2x + 1) = 2\log_{10}(x + 1) - 1.$$

Give your answers correct to 3 decimal places. [6]

5 **(a)** By sketching a suitable pair of graphs, show that the equation $\operatorname{cosec} x = 1 + e^{-\frac{1}{2}x}$ has exactly two roots in the interval $0 < x < \pi$. [2]

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(b) The sequence of values given by the iterative formula

$$x_{n+1} = \pi - \sin^{-1}\left(\frac{1}{e^{-\frac{1}{2}x_n} + 1}\right),$$

with initial value $x_1 = 2$, converges to one of these roots.

Use the formula to determine this root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

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6 **(a)** Express $\sqrt{6}\cos\theta + 3\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. State the exact value of R and give α correct to 2 decimal places. [3]

(b) Hence solve the equation $\sqrt{6}\cos\frac{1}{3}x + 3\sin\frac{1}{3}x = 2.5$, for $0° < x < 360°$. [4]

7 **(a)** Verify that $-1 + \sqrt{5}\mathrm{i}$ is a root of the equation $2x^3 + x^2 + 6x - 18 = 0$. [3]

(b) Find the other roots of this equation. [4]

8 The coordinates (x, y) of a general point of a curve satisfy the differential equation

$$x\frac{dy}{dx} = (1 - 2x^2)y,$$

for $x > 0$. It is given that $y = 1$ when $x = 1$.

Solve the differential equation, obtaining an expression for y in terms of x. [6]

9 Let $f(x) = \dfrac{8 + 5x + 12x^2}{(1 - x)(2 + 3x)^2}$.

(a) Express $f(x)$ in partial fractions. [5]

(b) Hence obtain the expansion of $f(x)$ in ascending powers of x, up to and including the term in x^2. [5]

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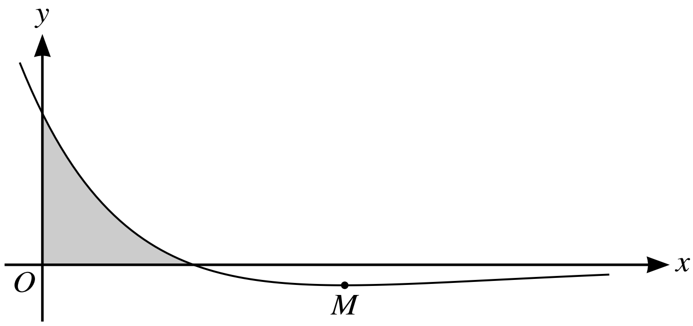

The diagram shows the curve $y = (2 - x)e^{-\frac{1}{2}x}$, and its minimum point M.

(a) Find the exact coordinates of M. [5]

(b) Find the area of the shaded region bounded by the curve and the axes. Give your answer in terms of e. [5]

11 Two lines have equations $\mathbf{r} = \mathbf{i} + 2\mathbf{j} + \mathbf{k} + \lambda(a\mathbf{i} + 2\mathbf{j} - \mathbf{k})$ and $\mathbf{r} = 2\mathbf{i} + \mathbf{j} - \mathbf{k} + \mu(2\mathbf{i} - \mathbf{j} + \mathbf{k})$, where a is a constant.

(a) Given that the two lines intersect, find the value of a and the position vector of the point of intersection. [5]

(b) Given instead that the acute angle between the directions of the two lines is $\cos^{-1}\left(\frac{1}{6}\right)$, find the two possible values of a. [6]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME			
CENTRE NUMBER		CANDIDATE NUMBER	

* 1 2 1 4 6 4 7 7 5 3 *

MATHEMATICS **9709/31**

Paper 3 Pure Mathematics 3 **May/June 2020**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Blank pages are indicated.

JC20 06_9709_31/FP

[Turn over

1 Find the set of values of x for which $2(3^{1-2x}) < 5^x$. Give your answer in a simplified exact form. [4]

2 **(a)** Expand $(2 - 3x)^{-2}$ in ascending powers of x, up to and including the term in x^2, simplifying the coefficients. [4]

(b) State the set of values of x for which the expansion is valid. [1]

3 Express the equation $\tan(\theta + 60°) = 2 + \tan(60° - \theta)$ as a quadratic equation in $\tan\theta$, and hence solve the equation for $0° \leqslant \theta \leqslant 180°$. [6]

4 The curve with equation $y = e^{2x}(\sin x + 3\cos x)$ has a stationary point in the interval $0 \leqslant x \leqslant \pi$.

(a) Find the x-coordinate of this point, giving your answer correct to 2 decimal places. [4]

(b) Determine whether the stationary point is a maximum or a minimum. [2]

5 **(a)** Find the quotient and remainder when $2x^3 - x^2 + 6x + 3$ is divided by $x^2 + 3$. [3]

(b) Using your answer to part **(a)**, find the exact value of $\int_1^3 \frac{2x^3 - x^2 + 6x + 3}{x^2 + 3}\,dx$. [5]

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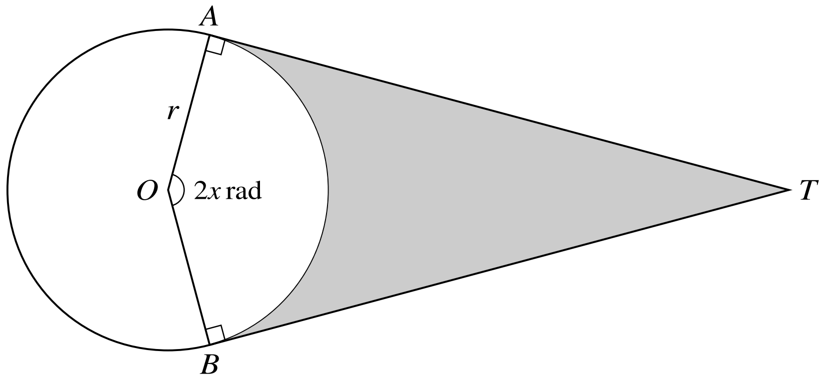

The diagram shows a circle with centre O and radius r. The tangents to the circle at the points A and B meet at T, and angle AOB is $2x$ radians. The shaded region is bounded by the tangents AT and BT, and by the minor arc AB. The area of the shaded region is equal to the area of the circle.

(a) Show that x satisfies the equation $\tan x = \pi + x$. [3]

(b) This equation has one root in the interval $0 < x < \frac{1}{2}\pi$. Verify by calculation that this root lies between 1 and 1.4. [2]

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(c) Use the iterative formula

$$x_{n+1} = \tan^{-1}(\pi + x_n)$$

to determine the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

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7 Let $f(x) = \dfrac{\cos x}{1 + \sin x}$.

(a) Show that $f'(x) < 0$ for all x in the interval $-\frac{1}{2}\pi < x < \frac{3}{2}\pi$. [4]

(b) Find $\int_{\frac{1}{6}\pi}^{\frac{1}{2}\pi} \mathrm{f}(x)\,\mathrm{d}x$. Give your answer in a simplified exact form. [4]

8 A certain curve is such that its gradient at a point (x, y) is proportional to $\frac{y}{x\sqrt{x}}$. The curve passes through the points with coordinates (1, 1) and (4, e).

(a) By setting up and solving a differential equation, find the equation of the curve, expressing y in terms of x. [8]

(b) Describe what happens to y as x tends to infinity. [1]

9 With respect to the origin O, the vertices of a triangle ABC have position vectors

$$\overrightarrow{OA} = 2\mathbf{i} + 5\mathbf{k}, \quad \overrightarrow{OB} = 3\mathbf{i} + 2\mathbf{j} + 3\mathbf{k} \quad \text{and} \quad \overrightarrow{OC} = \mathbf{i} + \mathbf{j} + \mathbf{k}.$$

(a) Using a scalar product, show that angle ABC is a right angle. [3]

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(b) Show that triangle ABC is isosceles. [2]

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(c) Find the exact length of the perpendicular from O to the line through B and C. [4]

10 **(a)** The complex number u is defined by $u = \dfrac{3\mathrm{i}}{a + 2\mathrm{i}}$, where a is real.

(i) Express u in the Cartesian form $x + \mathrm{i}y$, where x and y are in terms of a. [3]

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(ii) Find the exact value of a for which $\arg u^* = \frac{1}{3}\pi$. [3]

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(b) **(i)** On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities $|z - 2i| \leqslant |z - 1 - i|$ and $|z - 2 - i| \leqslant 2$. [4]

(ii) Calculate the least value of $\arg z$ for points in this region. [2]

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Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

4124991549

MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **May/June 2020**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Blank pages are indicated.

1 Find the quotient and remainder when $6x^4 + x^3 - x^2 + 5x - 6$ is divided by $2x^2 - x + 1$. [3]

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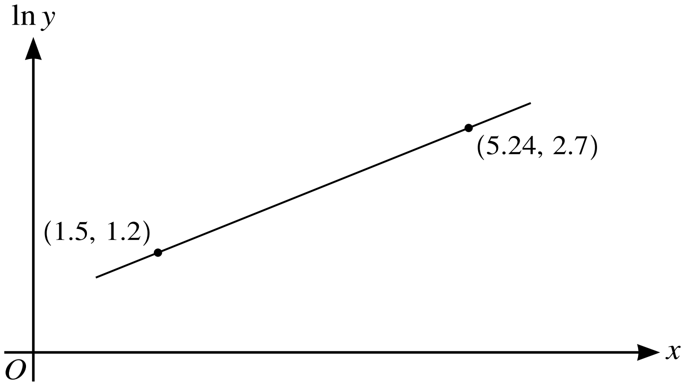


The variables x and y satisfy the equation $y^2 = Ae^{kx}$, where A and k are constants. The graph of $\ln y$ against x is a straight line passing through the points (1.5, 1.2) and (5.24, 2.7) as shown in the diagram.

Find the values of A and k correct to 2 decimal places. [5]

3 Find the exact value of

$$\int_1^4 x^{\frac{3}{2}} \ln x \, dx.$$ [5]

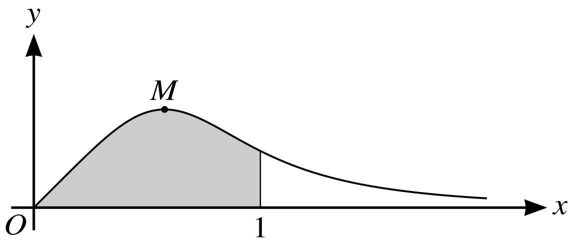
4 A curve has equation $y = \cos x \sin 2x$.

Find the x-coordinate of the stationary point in the interval $0 < x < \frac{1}{2}\pi$, giving your answer correct to 3 significant figures. [6]

5 **(a)** Express $\sqrt{2}\cos x - \sqrt{5}\sin x$ in the form $R\cos(x + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. Give the exact value of R and the value of α correct to 3 decimal places. [3]

(b) Hence solve the equation $\sqrt{2}\cos 2\theta - \sqrt{5}\sin 2\theta = 1$, for $0° < \theta < 180°$. [4]

6



The diagram shows the curve $y = \dfrac{x}{1 + 3x^4}$, for $x \geqslant 0$, and its maximum point M.

(a) Find the x-coordinate of M, giving your answer correct to 3 decimal places. [4]

(b) Using the substitution $u = \sqrt{3}x^2$, find by integration the exact area of the shaded region bounded by the curve, the x-axis and the line $x = 1$. [5]

7 The variables x and y satisfy the differential equation

$$\frac{dy}{dx} = \frac{y-1}{(x+1)(x+3)}.$$

It is given that $y = 2$ when $x = 0$.

Solve the differential equation, obtaining an expression for y in terms of x. [9]

8 **(a)** Solve the equation $(1 + 2\mathrm{i})w + \mathrm{i}w^* = 3 + 5\mathrm{i}$. Give your answer in the form $x + \mathrm{i}y$, where x and y are real. [4]

(b) **(i)** On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities $|z - 2 - 2i| \leqslant 1$ and $\arg(z - 4i) \geqslant -\frac{1}{4}\pi$. [4]

(ii) Find the least value of $\operatorname{Im} z$ for points in this region, giving your answer in an exact form. [2]

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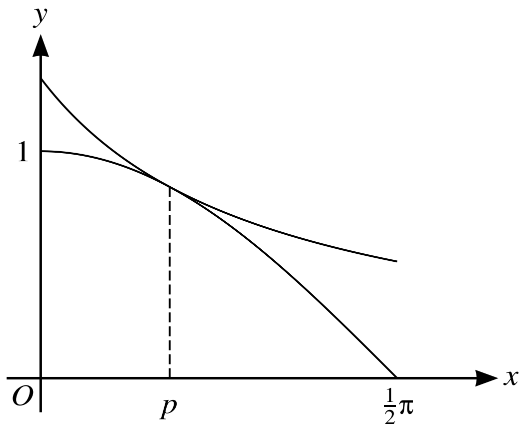

The diagram shows the curves $y = \cos x$ and $y = \dfrac{k}{1+x}$, where k is a constant, for $0 \leqslant x \leqslant \frac{1}{2}\pi$. The curves touch at the point where $x = p$.

(a) Show that p satisfies the equation $\tan p = \dfrac{1}{1+p}$. [5]

(b) Use the iterative formula $p_{n+1} = \tan^{-1}\left(\frac{1}{1 + p_n}\right)$ to determine the value of p correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

(c) Hence find the value of k correct to 2 decimal places. [2]

10 With respect to the origin O, the points A and B have position vectors given by $\overrightarrow{OA} = 6\mathbf{i} + 2\mathbf{j}$ and $\overrightarrow{OB} = 2\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$. The midpoint of OA is M. The point N lying on AB, between A and B, is such that $AN = 2NB$.

(a) Find a vector equation for the line through M and N. [5]

The line through M and N intersects the line through O and B at the point P.

(b) Find the position vector of P. [3]

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(c) Calculate angle OPM, giving your answer in degrees. [3]

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Additional Page

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Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

1862605169

MATHEMATICS **9709/33**

Paper 3 Pure Mathematics 3 **May/June 2020**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
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- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Blank pages are indicated.

1 Solve the inequality $|2x - 1| > 3|x + 2|$. [4]

2 Find the exact value of $\int_0^1 (2 - x)e^{-2x}\,dx$. [5]

  [Turn over

3 **(a)** Show that the equation

$$\ln(1 + e^{-x}) + 2x = 0$$

can be expressed as a quadratic equation in e^x. [2]

(b) Hence solve the equation $\ln(1 + e^{-x}) + 2x = 0$, giving your answer correct to 3 decimal places. [4]

4 The equation of a curve is $y = x\tan^{-1}\left(\frac{1}{2}x\right)$.

(a) Find $\frac{dy}{dx}$. [3]

(b) The tangent to the curve at the point where $x = 2$ meets the y-axis at the point with coordinates $(0, p)$.

Find p. [3]

5 By first expressing the equation

$$\tan\theta\tan(\theta + 45°) = 2\cot 2\theta$$

as a quadratic equation in $\tan\theta$, solve the equation for $0° < \theta < 90°$. [6]

6 **(a)** By sketching a suitable pair of graphs, show that the equation $x^5 = 2 + x$ has exactly one real root. [2]

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(b) Show that if a sequence of values given by the iterative formula

$$x_{n+1} = \frac{4x_n^5 + 2}{5x_n^4 - 1}$$

converges, then it converges to the root of the equation in part **(a)**. [2]

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(c) Use the iterative formula with initial value $x_1 = 1.5$ to calculate the root correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

7 Let $f(x) = \dfrac{2}{(2x-1)(2x+1)}$.

(a) Express $f(x)$ in partial fractions. [2]

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(b) Using your answer to part **(a)**, show that

$$\big(f(x)\big)^2 = \frac{1}{(2x-1)^2} - \frac{1}{2x-1} + \frac{1}{2x+1} + \frac{1}{(2x+1)^2}.$$ [2]

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(c) Hence show that $\int_1^2 (\mathrm{f}(x))^2 \, \mathrm{d}x = \frac{2}{5} + \frac{1}{2}\ln\left(\frac{5}{9}\right)$. [5]

8 Relative to the origin O, the points A, B and D have position vectors given by

$$\overrightarrow{OA} = \mathbf{i} + 2\mathbf{j} + \mathbf{k}, \quad \overrightarrow{OB} = 2\mathbf{i} + 5\mathbf{j} + 3\mathbf{k} \quad \text{and} \quad \overrightarrow{OD} = 3\mathbf{i} + 2\mathbf{k}.$$

A fourth point C is such that $ABCD$ is a parallelogram.

(a) Find the position vector of C and verify that the parallelogram is not a rhombus. [5]

(b) Find angle *BAD*, giving your answer in degrees. [3]

(c) Find the area of the parallelogram correct to 3 significant figures. [2]

9 **(a)** The complex numbers u and w are such that

$$u - w = 2\text{i} \quad \text{and} \quad uw = 6.$$

Find u and w, giving your answers in the form $x + \text{i}y$, where x and y are real and exact. [5]

(b) On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities

$$|z - 2 - 2\text{i}| \leqslant 2, \quad 0 \leqslant \arg z \leqslant \tfrac{1}{4}\pi \quad \text{and} \quad \text{Re}\, z \leqslant 3.$$ [5]

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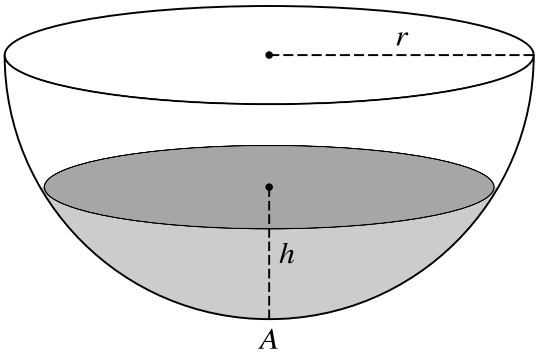

A tank containing water is in the form of a hemisphere. The axis is vertical, the lowest point is A and the radius is r, as shown in the diagram. The depth of water at time t is h. At time $t = 0$ the tank is full and the depth of the water is r. At this instant a tap at A is opened and water begins to flow out at a rate proportional to $\sqrt{h}$. The tank becomes empty at time $t = 14$.

The volume of water in the tank is V when the depth is h. It is given that $V = \frac{1}{3}\pi(3rh^2 - h^3)$.

(a) Show that h and t satisfy a differential equation of the form

$$\frac{dh}{dt} = -\frac{B}{2rh^{\frac{1}{2}} - h^{\frac{3}{2}}},$$

where B is a positive constant. [4]

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(b) Solve the differential equation and obtain an expression for t in terms of h and r. [8]

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If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

2601216071

MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **February/March 2020**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
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- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Blank pages are indicated.

1 **(a)** Sketch the graph of $y = |x - 2|$. [1]

(b) Solve the inequality $|x - 2| < 3x - 4$. [3]

2 Solve the equation $\ln 3 + \ln(2x + 5) = 2\ln(x + 2)$. Give your answer in a simplified exact form. [4]

3 **(a)** By sketching a suitable pair of graphs, show that the equation $\sec x = 2 - \frac{1}{2}x$ has exactly one root in the interval $0 \leqslant x < \frac{1}{2}\pi$. [2]

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(b) Verify by calculation that this root lies between 0.8 and 1. [2]

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(c) Use the iterative formula $x_{n+1} = \cos^{-1}\left(\dfrac{2}{4 - x_n}\right)$ to determine the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

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4 Find $\int_{\frac{1}{6}\pi}^{\frac{1}{3}\pi} x \sec^2 x \, dx$. Give your answer in a simplified exact form. [7]

5 **(a)** Show that $\dfrac{\cos 3x}{\sin x} + \dfrac{\sin 3x}{\cos x} = 2\cot 2x$. [4]

(b) Hence solve the equation $\dfrac{\cos 3x}{\sin x} + \dfrac{\sin 3x}{\cos x} = 4$, for $0 < x < \pi$. [3]

6 The variables x and y satisfy the differential equation

$$\frac{dy}{dx} = \frac{1 + 4y^2}{e^x}.$$

It is given that $y = 0$ when $x = 1$.

(a) Solve the differential equation, obtaining an expression for y in terms of x. [7]

(b) State what happens to the value of y as x tends to infinity. [1]

7 The equation of a curve is $x^3 + 3xy^2 - y^3 = 5$.

(a) Show that $\dfrac{dy}{dx} = \dfrac{x^2 + y^2}{y^2 - 2xy}$. [4]

(b) Find the coordinates of the points on the curve where the tangent is parallel to the y-axis. [5]

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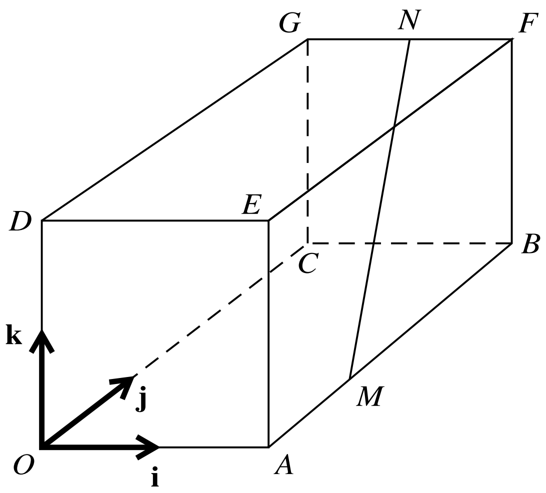

In the diagram, $OABCDEFG$ is a cuboid in which $OA = 2$ units, $OC = 3$ units and $OD = 2$ units. Unit vectors **i**, **j** and **k** are parallel to OA, OC and OD respectively. The point M on AB is such that $MB = 2AM$. The midpoint of FG is N.

(a) Express the vectors $\overrightarrow{OM}$ and $\overrightarrow{MN}$ in terms of **i**, **j** and **k**. [3]

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(b) Find a vector equation for the line through M and N. [2]

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(c) Find the position vector of P, the foot of the perpendicular from D to the line through M and N. [4]

9 Let $f(x) = \dfrac{2 + 11x - 10x^2}{(1 + 2x)(1 - 2x)(2 + x)}$.

(a) Express $f(x)$ in partial fractions. [5]

(b) Hence obtain the expansion of $f(x)$ in ascending powers of x, up to and including the term in x^2. [5]

10 **(a)** The complex numbers v and w satisfy the equations

$$v + \mathrm{i}w = 5 \qquad \text{and} \qquad (1 + 2\mathrm{i})v - w = 3\mathrm{i}.$$

Solve the equations for v and w, giving your answers in the form $x + \mathrm{i}y$, where x and y are real. [6]

(b) **(i)** On an Argand diagram, sketch the locus of points representing complex numbers z satisfying $|z - 2 - 3i| = 1$. [2]

(ii) Calculate the least value of $\arg z$ for points on this locus. [2]

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Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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Cambridge International A Level

Cambridge Assessment International Education
Cambridge International Advanced Level

CANDIDATE NAME

CENTRE NUMBER | CANDIDATE NUMBER

* 6 1 8 2 4 8 9 9 8 0 *

MATHEMATICS **9709/31**

Paper 3 Pure Mathematics 3 **(P3)** **October/November 2019**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

Write your centre number, candidate number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions in the space provided. If additional space is required, you should use the lined page at the end of this booklet. The question number(s) must be clearly shown.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.

This document consists of **19** printed pages and **1** blank page.

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1 Given that $\ln(1 + e^{2y}) = x$, express y in terms of x. [3]

2 Solve the inequality $|2x - 3| > 4|x + 1|$. [4]

3 The parametric equations of a curve are

$$x = 2t + \sin 2t, \quad y = \ln(1 - \cos 2t).$$

Show that $\dfrac{dy}{dx} = \operatorname{cosec} 2t$. [5]

4 The number of insects in a population t weeks after the start of observations is denoted by N. The population is decreasing at a rate proportional to $Ne^{-0.02t}$. The variables N and t are treated as continuous, and it is given that when $t = 0$, $N = 1000$ and $\frac{dN}{dt} = -10$.

(i) Show that N and t satisfy the differential equation

$$\frac{dN}{dt} = -0.01e^{-0.02t}N.$$ [1]

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(ii) Solve the differential equation and find the value of t when $N = 800$. [6]

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(iii) State what happens to the value of N as t becomes large. [1]

5 The curve with equation $y = e^{-2x}\ln(x-1)$ has a stationary point when $x = p$.

(i) Show that p satisfies the equation $x = 1 + \exp\left(\dfrac{1}{2(x-1)}\right)$, where $\exp(x)$ denotes e^x. [3]

(ii) Verify by calculation that p lies between 2.2 and 2.6. [2]

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(iii) Use an iterative formula based on the equation in part **(i)** to determine p correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

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6 **(i)** By differentiating $\frac{\cos x}{\sin x}$, show that if $y = \cot x$ then $\frac{dy}{dx} = -\operatorname{cosec}^2 x$. [2]

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(ii) Show that $\int_{\frac{1}{4}\pi}^{\frac{1}{2}\pi} x \operatorname{cosec}^2 x \, dx = \frac{1}{4}(\pi + \ln 4)$. [6]

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7 Two lines l and m have equations $\mathbf{r} = a\mathbf{i} + 2\mathbf{j} + 3\mathbf{k} + \lambda(\mathbf{i} - 2\mathbf{j} + 3\mathbf{k})$ and $\mathbf{r} = 2\mathbf{i} + \mathbf{j} + 2\mathbf{k} + \mu(2\mathbf{i} - \mathbf{j} + \mathbf{k})$ respectively, where a is a constant. It is given that the lines intersect.

(i) Find the value of a. [4]

(ii) When a has this value, find the equation of the plane containing l and m. [5]

8 Let $f(x) = \dfrac{x^2 + x + 6}{x^2(x + 2)}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence, showing full working, show that the exact value of $\int_1^4 f(x)\,dx$ is $\frac{9}{4}$. [5]

9 **(i)** By first expanding $\cos(2x + x)$, show that $\cos 3x \equiv 4\cos^3 x - 3\cos x$. [4]

(ii) Hence solve the equation $\cos 3x + 3\cos x + 1 = 0$, for $0 \leqslant x \leqslant \pi$. [2]

(iii) Find the exact value of $\int_{\frac{1}{6}\pi}^{\frac{1}{3}\pi} \cos^3 x \, dx$. [4]

10 **(a)** The complex number u is given by $u = -3 - (2\sqrt{10})\text{i}$. Showing all necessary working and without using a calculator, find the square roots of u. Give your answers in the form $a + \text{i}b$, where the numbers a and b are real and exact. [5]

(b) On a sketch of an Argand diagram shade the region whose points represent complex numbers z satisfying the inequalities $|z - 3 - \mathrm{i}| \leqslant 3$, $\arg z \geqslant \frac{1}{4}\pi$ and $\operatorname{Im} z \geqslant 2$, where $\operatorname{Im} z$ denotes the imaginary part of the complex number z. [5]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

Cambridge International A Level

Cambridge Assessment International Education
Cambridge International Advanced Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

4678410875

MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **(P3)** **October/November 2019**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

Write your centre number, candidate number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions in the space provided. If additional space is required, you should use the lined page at the end of this booklet. The question number(s) must be clearly shown.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.

This document consists of **19** printed pages and **1** blank page.

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Cambridge Assessment International Education

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1 Solve the equation $5\ln(4 - 3^x) = 6$. Show all necessary working and give the answer correct to 3 decimal places. [3]

2 The curve with equation $y = \frac{e^{-2x}}{1 - x^2}$ has a stationary point in the interval $-1 < x < 1$. Find $\frac{dy}{dx}$ and hence find the x-coordinate of this stationary point, giving the answer correct to 3 decimal places. [5]

3 The polynomial $x^4 + 3x^3 + ax + b$, where a and b are constants, is denoted by $p(x)$. When $p(x)$ is divided by $x^2 + x - 1$ the remainder is $2x + 3$. Find the values of a and b. [5]

4 **(i)** Express $(\sqrt{6})\sin x + \cos x$ in the form $R\sin(x + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. State the exact value of R and give α correct to 3 decimal places. [3]

(ii) Hence solve the equation $(\sqrt{6})\sin 2\theta + \cos 2\theta = 2$, for $0° < \theta < 180°$. [4]

5 The equation of a curve is $2x^2y - xy^2 = a^3$, where a is a positive constant. Show that there is only one point on the curve at which the tangent is parallel to the x-axis and find the y-coordinate of this point. [7]

6 The variables x and θ satisfy the differential equation

$$\sin\tfrac{1}{2}\theta\,\frac{dx}{d\theta} = (x+2)\cos\tfrac{1}{2}\theta$$

for $0 < \theta < \pi$. It is given that $x = 1$ when $\theta = \frac{1}{3}\pi$. Solve the differential equation and obtain an expression for x in terms of $\cos\theta$. [8]

7 **(a)** Find the complex number z satisfying the equation

$$z + \frac{iz}{z^*} - 2 = 0,$$

where z^* denotes the complex conjugate of z. Give your answer in the form $x + iy$, where x and y are real. [5]

(b) **(i)** On a single Argand diagram sketch the loci given by the equations $|z - 2i| = 2$ and $\text{Im}\, z = 3$, where $\text{Im}\, z$ denotes the imaginary part of z. [2]

(ii) In the first quadrant the two loci intersect at the point P. Find the exact argument of the complex number represented by P. [2]

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8 Let $f(x) = \dfrac{2x^2 + x + 8}{(2x - 1)(x^2 + 2)}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence, showing full working, find $\int_1^5 f(x)\,dx$, giving the answer in the form $\ln c$, where c is an integer. [5]

9 It is given that $\int_0^a x \cos \frac{1}{3}x \, dx = 3$, where the constant a is such that $0 < a < \frac{3}{2}\pi$.

(i) Show that a satisfies the equation

$$a = \frac{4 - 3\cos \frac{1}{3}a}{\sin \frac{1}{3}a}.$$ [5]

(ii) Verify by calculation that a lies between 2.5 and 3. [2]

(iii) Use an iterative formula based on the equation in part **(i)** to calculate a correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

10 The line l has equation $\mathbf{r} = \mathbf{i} + 3\mathbf{j} - 2\mathbf{k} + \lambda(\mathbf{i} - 2\mathbf{j} + 3\mathbf{k})$. The plane p has equation $2x + y - 3z = 5$.

(i) Find the position vector of the point of intersection of l and p. [3]

(ii) Calculate the acute angle between l and p. [3]

(iii) A second plane q is perpendicular to the plane p and contains the line l. Find the equation of q, giving your answer in the form $ax + by + cz = d$. [5]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

Cambridge International A Level

Cambridge Assessment International Education
Cambridge International Advanced Level

CANDIDATE NAME			
CENTRE NUMBER		CANDIDATE NUMBER	

4339107679

MATHEMATICS **9709/33**

Paper 3 Pure Mathematics 3 **(P3)** **October/November 2019**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

Write your centre number, candidate number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
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Answer **all** the questions in the space provided. If additional space is required, you should use the lined page at the end of this booklet. The question number(s) must be clearly shown.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.

This document consists of **19** printed pages and **1** blank page.

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1 Solve the inequality $2|x+2| > |3x-1|$. [4]

2 The polynomial $6x^3 + ax^2 + bx - 2$, where a and b are constants, is denoted by $p(x)$. It is given that $(2x + 1)$ is a factor of $p(x)$ and that when $p(x)$ is divided by $(x + 2)$ the remainder is -24. Find the values of a and b. [5]

3 Showing all necessary working, solve the equation $\frac{3^{2x} + 3^{-x}}{3^{2x} - 3^{-x}} = 4$. Give your answer correct to 3 decimal places. [4]

4 **(i)** By first expanding $\tan(2x + x)$, show that the equation $\tan 3x = 3 \cot x$ can be written in the form $\tan^4 x - 12 \tan^2 x + 3 = 0$. [4]

(ii) Hence solve the equation $\tan 3x = 3\cot x$ for $0° < x < 90°$. [3]

5 **(i)** By sketching a suitable pair of graphs, show that the equation $\ln(x + 2) = 4e^{-x}$ has exactly one real root. [2]

(ii) Show by calculation that this root lies between $x = 1$ and $x = 1.5$. [2]

(iii) Use the iterative formula $x_{n+1} = \ln\left(\frac{4}{\ln(x_n + 2)}\right)$ to determine the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

6 Throughout this question the use of a calculator is not permitted.

The complex number with modulus 1 and argument $\frac{1}{3}\pi$ is denoted by w.

(i) Express w in the form $x + \mathrm{i}y$, where x and y are real and exact. [1]

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The complex number $1 + 2\mathrm{i}$ is denoted by u. The complex number v is such that $|v| = 2|u|$ and $\arg v = \arg u + \frac{1}{3}\pi$.

(ii) Sketch an Argand diagram showing the points representing u and v. [2]

(iii) Explain why v can be expressed as $2uw$. Hence find v, giving your answer in the form $a + \mathrm{i}b$, where a and b are real and exact. [4]

7 The plane m has equation $x + 4y - 8z = 2$. The plane n is parallel to m and passes through the point P with coordinates $(5, 2, -2)$.

(i) Find the equation of n, giving your answer in the form $ax + by + cz = d$. [2]

(ii) Calculate the perpendicular distance between m and n. [3]

(iii) The line l lies in the plane n, passes through the point P and is perpendicular to OP, where O is the origin. Find a vector equation for l. [4]

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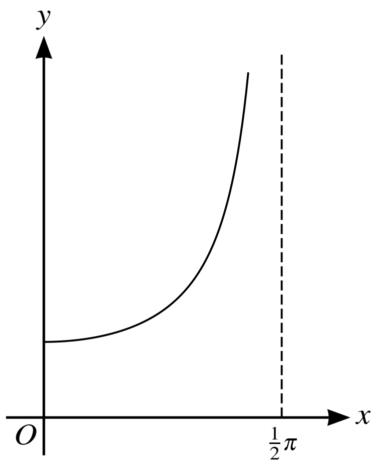

The diagram shows the graph of $y = \sec x$ for $0 \leqslant x < \frac{1}{2}\pi$.

(i) Use the trapezium rule with 2 intervals to estimate the value of $\int_0^{1.2} \sec x \, dx$, giving your answer correct to 2 decimal places. [3]

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(ii) Explain, with reference to the diagram, whether the trapezium rule gives an overestimate or an underestimate of the true value of the integral in part **(i)**. [1]

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(iii) P is the point on the part of the curve $y = \sec x$ for $0 \leqslant x < \frac{1}{2}\pi$ at which the gradient is 2. By first differentiating $\dfrac{1}{\cos x}$, find the x-coordinate of P, giving your answer correct to 3 decimal places. [6]

9 The variables x and t satisfy the differential equation $5\frac{dx}{dt} = (20 - x)(40 - x)$. It is given that $x = 10$ when $t = 0$.

(i) Using partial fractions, solve the differential equation, obtaining an expression for x in terms of t. [9]

(ii) State what happens to the value of x when t becomes large. [1]

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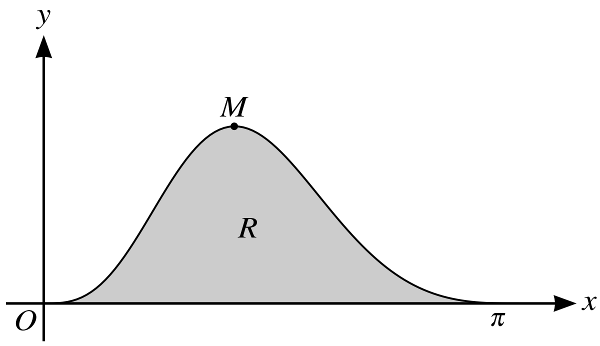

The diagram shows the graph of $y = e^{\cos x} \sin^3 x$ for $0 \leqslant x \leqslant \pi$, and its maximum point M. The shaded region R is bounded by the curve and the x-axis.

(i) Find the x-coordinate of M. Show all necessary working and give your answer correct to 2 decimal places. [5]

(ii) By first using the substitution $u = \cos x$, find the exact value of the area of R. [7]

Additional Page

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Cambridge International A Level

Cambridge Assessment International Education
Cambridge International Advanced Level

CANDIDATE NAME

CENTRE NUMBER | CANDIDATE NUMBER

2969009541

MATHEMATICS **9709/31**

Paper 3 Pure Mathematics 3 **(P3)** **May/June 2019**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

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At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.

This document consists of **18** printed pages and **2** blank pages.

1 Use the trapezium rule with 3 intervals to estimate the value of

$$\int_0^3 |2^x - 4| \, dx.$$ [3]

2 Showing all necessary working, solve the equation $\ln(2x - 3) = 2\ln x - \ln(x - 1)$. Give your answer correct to 2 decimal places. [4]

3 Find the gradient of the curve $x^3 + 3xy^2 - y^3 = 1$ at the point with coordinates $(1, 3)$. [4]

4 By first expressing the equation $\cot\theta - \cot(\theta + 45°) = 3$ as a quadratic equation in $\tan\theta$, solve the equation for $0° < \theta < 180°$. [6]

5 **(i)** Differentiate $\dfrac{1}{\sin^2\theta}$ with respect to θ. [2]

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(ii) The variables x and θ satisfy the differential equation

$$x\tan\theta\frac{\mathrm{d}x}{\mathrm{d}\theta} + \operatorname{cosec}^2\theta = 0,$$

for $0 < \theta < \frac{1}{2}\pi$ and $x > 0$. It is given that $x = 4$ when $\theta = \frac{1}{6}\pi$. Solve the differential equation, obtaining an expression for x in terms of θ. [6]

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6 **(i)** By first expanding $\sin(2x + x)$, show that $\sin 3x \equiv 3\sin x - 4\sin^3 x$. [4]

(ii) Hence, showing all necessary working, find the exact value of $\int_0^{\frac{1}{3}\pi} \sin^3 x \, dx$. [4]

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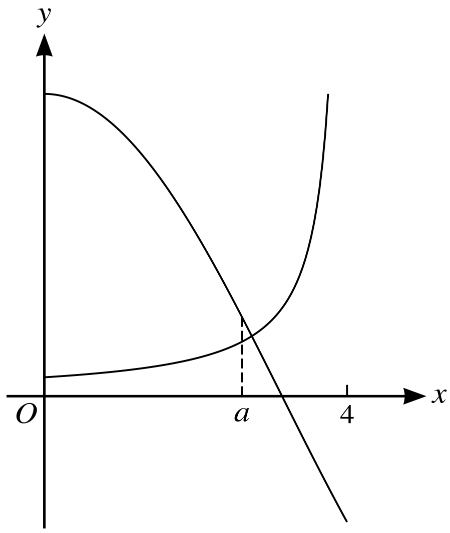

The diagram shows the curves $y = 4\cos\frac{1}{2}x$ and $y = \frac{1}{4-x}$, for $0 \leqslant x < 4$. When $x = a$, the tangents to the curves are perpendicular.

(i) Show that $a = 4 - \sqrt{\left(2\sin\frac{1}{2}a\right)}$. [4]

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(ii) Verify by calculation that a lies between 2 and 3. [2]

(iii) Use an iterative formula based on the equation in part **(i)** to determine a correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

8 Let $f(x) = \dfrac{16 - 17x}{(2 + x)(3 - x)^2}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence obtain the expansion of $f(x)$ in ascending powers of x, up to and including the term in x^2. [5]

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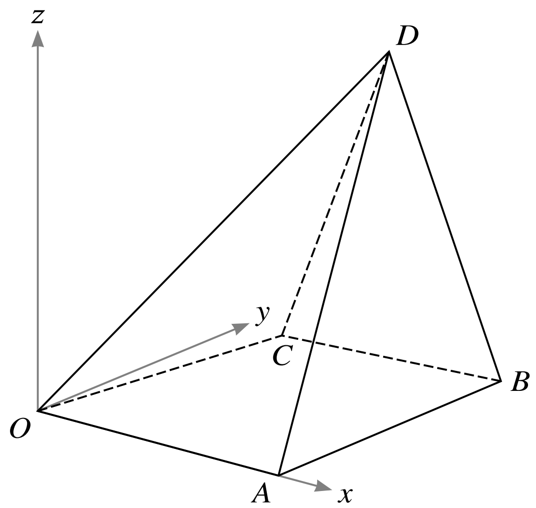

The diagram shows a set of rectangular axes Ox, Oy and Oz, and four points A, B, C and D with position vectors $\overrightarrow{OA} = 3\mathbf{i}$, $\overrightarrow{OB} = 3\mathbf{i} + 4\mathbf{j}$, $\overrightarrow{OC} = \mathbf{i} + 3\mathbf{j}$ and $\overrightarrow{OD} = 2\mathbf{i} + 3\mathbf{j} + 5\mathbf{k}$.

(i) Find the equation of the plane BCD, giving your answer in the form $ax + by + cz = d$. [6]

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(ii) Calculate the acute angle between the planes BCD and $OABC$. [4]

10 Throughout this question the use of a calculator is not permitted.

The complex number $(\sqrt{3}) + \mathrm{i}$ is denoted by u.

(i) Express u in the form $re^{\mathrm{i}\theta}$, where $r > 0$ and $-\pi < \theta \leqslant \pi$, giving the exact values of r and θ. Hence or otherwise state the exact values of the modulus and argument of u^4. [5]

(ii) Verify that u is a root of the equation $z^3 - 8z + 8\sqrt{3} = 0$ and state the other complex root of this equation. [3]

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(iii) On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities $|z - u| \leqslant 2$ and $\operatorname{Im} z \geqslant 2$, where $\operatorname{Im} z$ denotes the imaginary part of z. [5]

Additional Page

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Cambridge International A Level

Cambridge Assessment International Education
Cambridge International Advanced Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

6271108665

MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **(P3)** **May/June 2019**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

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The total number of marks for this paper is 75.

Cambridge Assessment International Education

1 Find the coefficient of x^3 in the expansion of $(3 - x)(1 + 3x)^{\frac{1}{3}}$ in ascending powers of x. [4]

2 Showing all necessary working, solve the equation $9^x = 3^x + 12$. Give your answer correct to 2 decimal places. [4]

3 Showing all necessary working, solve the equation $\cot 2\theta = 2\tan\theta$ for $0° < \theta < 180°$. [5]

4 Find the exact coordinates of the point on the curve $y = \dfrac{x}{1 + \ln x}$ at which the gradient of the tangent is equal to $\frac{1}{4}$. [7]

5 **Throughout this question the use of a calculator is not permitted.**

It is given that the complex number $-1 + (\sqrt{3})\mathrm{i}$ is a root of the equation

$$kx^3 + 5x^2 + 10x + 4 = 0,$$

where k is a real constant.

(i) Write down another root of the equation. [1]

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(ii) Find the value of k and the third root of the equation. [6]

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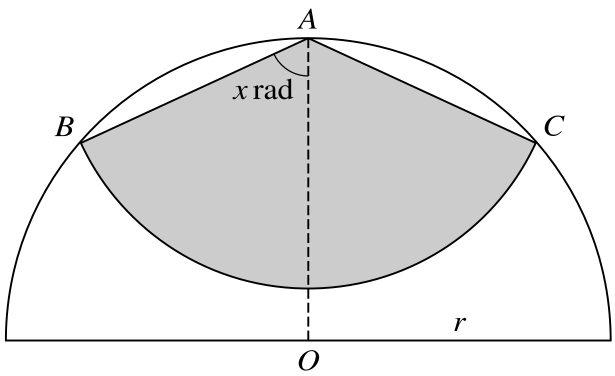

In the diagram, A is the mid-point of the semicircle with centre O and radius r. A circular arc with centre A meets the semicircle at B and C. The angle OAB is equal to x radians. The area of the shaded region bounded by AB, AC and the arc with centre A is equal to half the area of the semicircle.

(i) Use triangle OAB to show that $AB = 2r\cos x$. [1]

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(ii) Hence show that $x = \cos^{-1}\sqrt{\left(\frac{\pi}{16x}\right)}$. [2]

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(iii) Verify by calculation that x lies between 1 and 1.5. [2]

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(iv) Use an iterative formula based on the equation in part **(ii)** to determine x correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

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7 The variables x and y satisfy the differential equation $\frac{dy}{dx} = xe^{x+y}$. It is given that $y = 0$ when $x = 0$.

(i) Solve the differential equation, obtaining y in terms of x. [7]

(ii) Explain why x can only take values that are less than 1. [1]

8 Let $f(x) = \dfrac{10x + 9}{(2x + 1)(2x + 3)^2}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence show that $\int_0^1 f(x)\,dx = \frac{1}{2}\ln\frac{9}{5} + \frac{1}{5}$. [5]

9 The points A and B have position vectors $\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ and $3\mathbf{i} + \mathbf{j} + \mathbf{k}$ respectively. The line l has equation $\mathbf{r} = 2\mathbf{i} + \mathbf{j} + \mathbf{k} + \mu(\mathbf{i} + \mathbf{j} + 2\mathbf{k})$.

(i) Show that l does not intersect the line passing through A and B. [5]

(ii) The plane m is perpendicular to AB and passes through the mid-point of AB. The plane m intersects the line l at the point P. Find the equation of m and the position vector of P. [5]

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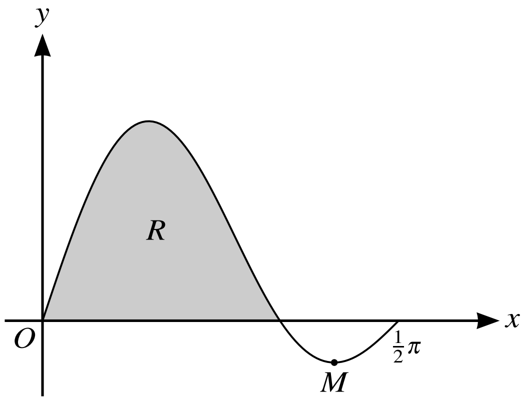

The diagram shows the curve $y = \sin 3x \cos x$ for $0 \leqslant x \leqslant \frac{1}{2}\pi$ and its minimum point M. The shaded region R is bounded by the curve and the x-axis.

(i) By expanding $\sin(3x + x)$ and $\sin(3x - x)$ show that

$$\sin 3x \cos x = \tfrac{1}{2}(\sin 4x + \sin 2x).$$ [3]

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(ii) Using the result of part **(i)** and showing all necessary working, find the exact area of the region R. [4]

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(iii) Using the result of part **(i)**, express $\frac{dy}{dx}$ in terms of $\cos 2x$ and hence find the x-coordinate of M, giving your answer correct to 2 decimal places. [5]

Additional Page

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Cambridge Assessment International Education
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CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

3275076176

MATHEMATICS **9709/33**

Paper 3 Pure Mathematics 3 **(P3)** **May/June 2019**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

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The total number of marks for this paper is 75.

This document consists of **20** printed pages.

1 Use logarithms to solve the equation $5^{3-2x} = 4(7^x)$, giving your answer correct to 3 decimal places. [4]

2 Show that $\int_0^{\frac{1}{4}\pi} x^2 \cos 2x \, dx = \frac{1}{32}(\pi^2 - 8)$. [5]

3 Let $f(\theta) = \dfrac{1 - \cos 2\theta + \sin 2\theta}{1 + \cos 2\theta + \sin 2\theta}$.

(i) Show that $f(\theta) = \tan\theta$. [3]

(ii) Hence show that $\int_{\frac{1}{6}\pi}^{\frac{1}{4}\pi} \mathrm{f}(\theta)\,\mathrm{d}\theta = \frac{1}{2}\ln\frac{3}{2}$. [4]

4 The equation of a curve is $y = \dfrac{1 + e^{-x}}{1 - e^{-x}}$, for $x > 0$.

(i) Show that $\dfrac{dy}{dx}$ is always negative. [3]

(ii) The gradient of the curve is equal to -1 when $x = a$. Show that a satisfies the equation $e^{2a} - 4e^{a} + 1 = 0$. Hence find the exact value of a. [4]

5 The variables x and y satisfy the differential equation

$$(x+1)y\frac{dy}{dx} = y^2 + 5.$$

It is given that $y = 2$ when $x = 0$. Solve the differential equation obtaining an expression for y^2 in terms of x. [7]

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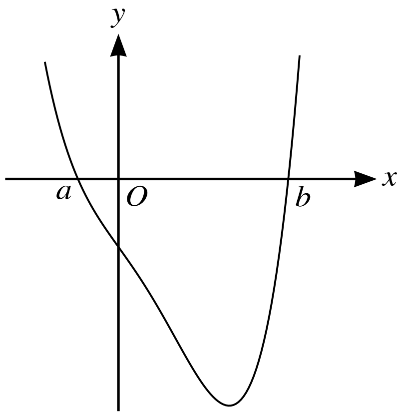

The diagram shows the curve $y = x^4 - 2x^3 - 7x - 6$. The curve intersects the x-axis at the points $(a, 0)$ and $(b, 0)$, where $a < b$. It is given that b is an integer.

(i) Find the value of b. [1]

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(ii) Hence show that a satisfies the equation $a = -\frac{1}{3}(2 + a^2 + a^3)$. [4]

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(iii) Use an iterative formula based on the equation in part **(ii)** to determine a correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

7 The curve $y = \sin\left(x + \frac{1}{3}\pi\right)\cos x$ has two stationary points in the interval $0 \leqslant x \leqslant \pi$.

(i) Find $\frac{dy}{dx}$. [2]

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(ii) By considering the formula for $\cos(A + B)$, show that, at the stationary points on the curve, $\cos\left(2x + \frac{1}{3}\pi\right) = 0$. [2]

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(iii) Hence find the exact x-coordinates of the stationary points. [3]

8 Throughout this question the use of a calculator is not permitted.

The complex number u is defined by

$$u = \frac{4\mathrm{i}}{1 - (\sqrt{3})\mathrm{i}}.$$

(i) Express u in the form $x + \mathrm{i}y$, where x and y are real and exact. [3]

(ii) Find the exact modulus and argument of u. [2]

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(iii) On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities $|z| < 2$ and $|z - u| < |z|$. [4]

9 Let $f(x) = \dfrac{2x(5-x)}{(3+x)(1-x)^2}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence obtain the expansion of $f(x)$ in ascending powers of x up to and including the term in x^3. [5]

10 The line l has equation $\mathbf{r} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k} + \mu(2\mathbf{i} - \mathbf{j} - 2\mathbf{k})$.

(i) The point P has position vector $4\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$. Find the length of the perpendicular from P to l. [5]

(ii) It is given that l lies in the plane with equation $ax + by + 2z = 13$, where a and b are constants. Find the values of a and b. [6]

Additional Page

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Cambridge International A Level

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Cambridge International Advanced Level

CANDIDATE NAME	

CENTRE NUMBER		CANDIDATE NUMBER	

4755885761

MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **(P3)** **February/March 2019**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

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The total number of marks for this paper is 75.

This document consists of **17** printed pages and **3** blank pages.

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Cambridge Assessment International Education

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1 **(i)** Show that the equation $\log_{10}(x-4) = 2 - \log_{10} x$ can be written as a quadratic equation in x. [3]

(ii) Hence solve the equation $\log_{10}(x-4) = 2 - \log_{10} x$, giving your answer correct to 3 significant figures. [2]

2 The sequence of values given by the iterative formula

$$x_{n+1} = \frac{2x_n^6 + 12x_n}{3x_n^5 + 8},$$

with initial value $x_1 = 2$, converges to α.

(i) Use the formula to calculate α correct to 4 decimal places. Give the result of each iteration to 6 decimal places. [3]

(ii) State an equation satisfied by α and hence find the exact value of α. [2]

3 **(i)** Given that $\sin(\theta + 45°) + 2\cos(\theta + 60°) = 3\cos\theta$, find the exact value of $\tan\theta$ in a form involving surds. You need not simplify your answer. [4]

(ii) Hence solve the equation $\sin(\theta + 45°) + 2\cos(\theta + 60°) = 3\cos\theta$ for $0° < \theta < 360°$. [2]

4 Show that $\int_1^4 x^{-\frac{3}{2}} \ln x \, dx = 2 - \ln 4.$ [5]

5 The variables x and y satisfy the relation $\sin y = \tan x$, where $-\frac{1}{2}\pi < y < \frac{1}{2}\pi$. Show that

$$\frac{dy}{dx} = \frac{1}{\cos x\sqrt{(\cos 2x)}}.$$ [5]

6 The variables x and y satisfy the differential equation

$$\frac{dy}{dx} = ky^3 e^{-x},$$

where k is a constant. It is given that $y = 1$ when $x = 0$, and that $y = \sqrt{e}$ when $x = 1$. Solve the differential equation, obtaining an expression for y in terms of x. [7]

7 **(a)** Showing all working and without using a calculator, solve the equation

$$(1 + i)z^2 - (4 + 3i)z + 5 + i = 0.$$

Give your answers in the form $x + iy$, where x and y are real. [6]

(b) The complex number u is given by

$$u = -1 - \mathrm{i}.$$

On a sketch of an Argand diagram show the point representing u. Shade the region whose points represent complex numbers satisfying the inequalities $|z| < |z - 2\mathrm{i}|$ and $\frac{1}{4}\pi < \arg(z - u) < \frac{1}{2}\pi$. [4]

8 Let $f(x) = \dfrac{12 + 12x - 4x^2}{(2 + x)(3 - 2x)}$.

(i) Express $f(x)$ in partial fractions. [5]

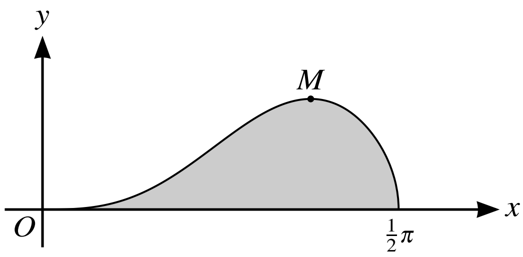
(ii) Hence obtain the expansion of $f(x)$ in ascending powers of x, up to and including the term in x^2. [5]

9 Two planes have equations $2x + 3y - z = 1$ and $x - 2y + z = 3$.

(i) Find the acute angle between the planes. [4]

(ii) Find a vector equation for the line of intersection of the planes. [6]

10



The diagram shows the curve $y = \sin^3 x \sqrt{(\cos x)}$ for $0 \leqslant x \leqslant \frac{1}{2}\pi$, and its maximum point M.

(i) Using the substitution $u = \cos x$, find by integration the exact area of the shaded region bounded by the curve and the x-axis. [6]

(ii) Showing all your working, find the x-coordinate of M, giving your answer correct to 3 decimal places. [6]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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Cambridge International A Level

Cambridge International Examinations
Cambridge International Advanced Level

CANDIDATE NAME	

CENTRE NUMBER						CANDIDATE NUMBER					

3283715323

MATHEMATICS **9709/31**

Paper 3 Pure Mathematics 3 **(P3)** **October/November 2018**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

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Answer **all** the questions in the space provided. If additional space is required, you should use the lined page at the end of this booklet. The question number(s) must be clearly shown.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.

CAMBRIDGE International Examinations

1 Find the set of values of x satisfying the inequality $2|2x - a| < |x + 3a|$, where a is a positive constant. [4]

2 Showing all necessary working, solve the equation $\frac{2e^x + e^{-x}}{e^x - e^{-x}} = 4$, giving your answer correct to 2 decimal places. [4]

3 **(i)** By sketching a suitable pair of graphs, show that the equation $x^3 = 3 - x$ has exactly one real root. [2]

(ii) Show that if a sequence of real values given by the iterative formula

$$x_{n+1} = \frac{2x_n^3 + 3}{3x_n^2 + 1}$$

converges, then it converges to the root of the equation in part **(i)**. [2]

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(iii) Use this iterative formula to determine the root correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

4 The parametric equations of a curve are

$$x = 2\sin\theta + \sin 2\theta, \quad y = 2\cos\theta + \cos 2\theta,$$

where $0 < \theta < \pi$.

(i) Obtain an expression for $\dfrac{dy}{dx}$ in terms of θ. [3]

(ii) Hence find the exact coordinates of the point on the curve at which the tangent is parallel to the y-axis. [4]

5 The coordinates (x, y) of a general point on a curve satisfy the differential equation

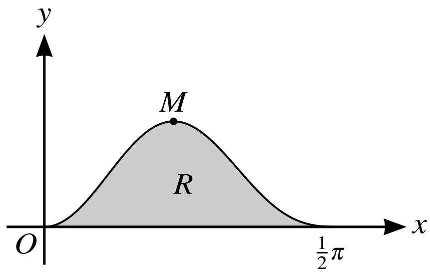
$$x\frac{dy}{dx} = (2 - x^2)y.$$

The curve passes through the point $(1, 1)$. Find the equation of the curve, obtaining an expression for y in terms of x. [7]

6 **(i)** Show that the equation $(\sqrt{2})\operatorname{cosec} x + \cot x = \sqrt{3}$ can be expressed in the form $R\sin(x - \alpha) = \sqrt{2}$, where $R > 0$ and $0° < \alpha < 90°$. [4]

(ii) Hence solve the equation $(\sqrt{2})\operatorname{cosec} x + \cot x = \sqrt{3}$, for $0° < x < 180°$. [4]

7



The diagram shows the curve $y = 5\sin^2 x \cos^3 x$ for $0 \leqslant x \leqslant \frac{1}{2}\pi$, and its maximum point M. The shaded region R is bounded by the curve and the x-axis.

(i) Find the x-coordinate of M, giving your answer correct to 3 decimal places. [5]

(ii) Using the substitution $u = \sin x$ and showing all necessary working, find the exact area of R. [4]

8 **(a)** Showing all necessary working, express the complex number $\frac{2+3\text{i}}{1-2\text{i}}$ in the form $r\text{e}^{\text{i}\theta}$, where $r > 0$ and $-\pi < \theta \leqslant \pi$. Give the values of r and θ correct to 3 significant figures. [5]

(b) On an Argand diagram sketch the locus of points representing complex numbers z satisfying the equation $|z - 3 + 2\mathrm{i}| = 1$. Find the least value of $|z|$ for points on this locus, giving your answer in an exact form. [4]

9 Let $f(x) = \dfrac{6x^2 + 8x + 9}{(2 - x)(3 + 2x)^2}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence, showing all necessary working, show that $\int_{-1}^{0} f(x)\,dx = 1 + \frac{1}{2}\ln\left(\frac{3}{4}\right)$. [5]

10 The planes m and n have equations $3x + y - 2z = 10$ and $x - 2y + 2z = 5$ respectively. The line l has equation $\mathbf{r} = 4\mathbf{i} + 2\mathbf{j} + \mathbf{k} + \lambda(\mathbf{i} + \mathbf{j} + 2\mathbf{k})$.

(i) Show that l is parallel to m. [3]

(ii) Calculate the acute angle between the planes m and n. [3]

(iii) A point P lies on the line l. The perpendicular distance of P from the plane n is equal to 2. Find the position vectors of the two possible positions of P. [4]

Additional Page

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Cambridge International A Level

Cambridge International Examinations
Cambridge International Advanced Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

9782733059

MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **(P3)** **October/November 2018**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

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The use of an electronic calculator is expected, where appropriate.
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At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.

JC18 11_9709_32/RP

CAMBRIDGE International Examinations

1 Solve the inequality $3|2x - 1| > |x + 4|$. [4]

2 Showing all necessary working, solve the equation $\sin(\theta - 30^\circ) + \cos\theta = 2\sin\theta$, for $0^\circ < \theta < 180^\circ$. [4]

3 **(i)** Find $\int \frac{\ln x}{x^3}\,dx$. [3]

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(ii) Hence show that $\int_1^2 \frac{\ln x}{x^3}\,dx = \frac{1}{16}(3 - \ln 4)$. [2]

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4 Showing all necessary working, solve the equation

$$\frac{e^x + e^{-x}}{e^x + 1} = 4,$$

giving your answer correct to 3 decimal places. [5]

5 The equation of a curve is $y = x\ln(8 - x)$. The gradient of the curve is equal to 1 at only one point, when $x = a$.

(i) Show that a satisfies the equation $x = 8 - \dfrac{8}{\ln(8 - x)}$. [3]

(ii) Verify by calculation that a lies between 2.9 and 3.1. [2]

(iii) Use an iterative formula based on the equation in part **(i)** to determine a correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

6 A certain curve is such that its gradient at a general point with coordinates (x, y) is proportional to $\frac{y^2}{x}$. The curve passes through the points with coordinates $(1, 1)$ and $(e, 2)$. By setting up and solving a differential equation, find the equation of the curve, expressing y in terms of x. [8]

7 A curve has equation $y = \dfrac{3\cos x}{2 + \sin x}$, for $-\frac{1}{2}\pi \leqslant x \leqslant \frac{1}{2}\pi$.

(i) Find the exact coordinates of the stationary point of the curve. [6]

(ii) The constant a is such that $\int_0^a \frac{3\cos x}{2+\sin x}\,dx = 1$. Find the value of a, giving your answer correct to 3 significant figures. [4]

8 Let $f(x) = \dfrac{7x^2 - 15x + 8}{(1-2x)(2-x)^2}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence obtain the expansion of $f(x)$ in ascending powers of x, up to and including the term in x^2. [5]

9 **(a)** **(i)** Without using a calculator, express the complex number $\frac{2+6i}{1-2i}$ in the form $x + iy$, where x and y are real. [2]

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(ii) Hence, without using a calculator, express $\frac{2+6i}{1-2i}$ in the form $r(\cos\theta + i\sin\theta)$, where $r > 0$ and $-\pi < \theta \leqslant \pi$, giving the exact values of r and θ. [3]

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(b) On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying both the inequalities $|z - 3\mathrm{i}| \leqslant 1$ and $\mathrm{Re}\, z \leqslant 0$, where $\mathrm{Re}\, z$ denotes the real part of z. Find the greatest value of $\arg z$ for points in this region, giving your answer in radians correct to 2 decimal places. [5]

10 The line l has equation $\mathbf{r} = 5\mathbf{i} - 3\mathbf{j} - \mathbf{k} + \lambda(\mathbf{i} - 2\mathbf{j} + \mathbf{k})$. The plane p has equation

$$(\mathbf{r} - \mathbf{i} - 2\mathbf{j}) \cdot (3\mathbf{i} + \mathbf{j} + \mathbf{k}) = 0.$$

The line l intersects the plane p at the point A.

(i) Find the position vector of A. [3]

(ii) Calculate the acute angle between l and p. [4]

[Question 10 (iii) is printed on the next page.]

(iii) Find the equation of the line which lies in p and intersects l at right angles. [4]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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Cambridge International A Level

Cambridge International Examinations
Cambridge International Advanced Level

CANDIDATE NAME

CENTRE NUMBER | CANDIDATE NUMBER

3442671267

MATHEMATICS **9709/33**

Paper 3 Pure Mathematics 3 **(P3)** **October/November 2018**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

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You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.

This document consists of **20** printed pages.

CAMBRIDGE International Examinations

1 Find the set of values of x satisfying the inequality $2|2x - a| < |x + 3a|$, where a is a positive constant. [4]

2 Showing all necessary working, solve the equation $\dfrac{2e^x + e^{-x}}{e^x - e^{-x}} = 4$, giving your answer correct to 2 decimal places. [4]

3 **(i)** By sketching a suitable pair of graphs, show that the equation $x^3 = 3 - x$ has exactly one real root. [2]

(ii) Show that if a sequence of real values given by the iterative formula

$$x_{n+1} = \frac{2x_n^3 + 3}{3x_n^2 + 1}$$

converges, then it converges to the root of the equation in part **(i)**. [2]

(iii) Use this iterative formula to determine the root correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

4 The parametric equations of a curve are

$$x = 2\sin\theta + \sin 2\theta, \quad y = 2\cos\theta + \cos 2\theta,$$

where $0 < \theta < \pi$.

(i) Obtain an expression for $\frac{dy}{dx}$ in terms of θ. [3]

(ii) Hence find the exact coordinates of the point on the curve at which the tangent is parallel to the y-axis. [4]

5 The coordinates (x, y) of a general point on a curve satisfy the differential equation

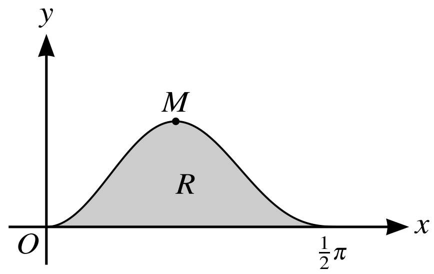
$$x\frac{dy}{dx} = (2 - x^2)y.$$

The curve passes through the point $(1, 1)$. Find the equation of the curve, obtaining an expression for y in terms of x. [7]

6 **(i)** Show that the equation $(\sqrt{2})\operatorname{cosec} x + \cot x = \sqrt{3}$ can be expressed in the form $R\sin(x - \alpha) = \sqrt{2}$, where $R > 0$ and $0° < \alpha < 90°$. [4]

(ii) Hence solve the equation $(\sqrt{2})\operatorname{cosec} x + \cot x = \sqrt{3}$, for $0° < x < 180°$. [4]

7



The diagram shows the curve $y = 5\sin^2 x \cos^3 x$ for $0 \leqslant x \leqslant \frac{1}{2}\pi$, and its maximum point M. The shaded region R is bounded by the curve and the x-axis.

(i) Find the x-coordinate of M, giving your answer correct to 3 decimal places. [5]

(ii) Using the substitution $u = \sin x$ and showing all necessary working, find the exact area of R. [4]

8 (a) Showing all necessary working, express the complex number $\frac{2+3i}{1-2i}$ in the form $re^{i\theta}$, where $r > 0$ and $-\pi < \theta \leqslant \pi$. Give the values of r and θ correct to 3 significant figures. [5]

(b) On an Argand diagram sketch the locus of points representing complex numbers z satisfying the equation $|z - 3 + 2\mathrm{i}| = 1$. Find the least value of $|z|$ for points on this locus, giving your answer in an exact form. [4]

9 Let $f(x) = \dfrac{6x^2 + 8x + 9}{(2 - x)(3 + 2x)^2}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence, showing all necessary working, show that $\int_{-1}^{0} f(x)\,dx = 1 + \frac{1}{2}\ln\left(\frac{3}{4}\right)$. [5]

10 The planes m and n have equations $3x + y - 2z = 10$ and $x - 2y + 2z = 5$ respectively. The line l has equation $\mathbf{r} = 4\mathbf{i} + 2\mathbf{j} + \mathbf{k} + \lambda(\mathbf{i} + \mathbf{j} + 2\mathbf{k})$.

(i) Show that l is parallel to m. [3]

(ii) Calculate the acute angle between the planes m and n. [3]

(iii) A point P lies on the line l. The perpendicular distance of P from the plane n is equal to 2. Find the position vectors of the two possible positions of P. [4]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

Cambridge International A Level

Cambridge International Examinations
Cambridge International Advanced Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

7084725728

MATHEMATICS **9709/31**

Paper 3 Pure Mathematics 3 **(P3)** **May/June 2018**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

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At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.

This document consists of **18** printed pages and **2** blank pages.

JC18 06_9709_31/2R

CAMBRIDGE International Examinations

1 Showing all necessary working, solve the equation $\ln(x^4 - 4) = 4\ln x - \ln 4$, giving your answer correct to 2 decimal places. [4]

2 **(i)** Given that $\sin(x - 60°) = 3\cos(x - 45°)$, find the exact value of $\tan x$. [4]

(ii) Hence solve the equation $\sin(x - 60°) = 3\cos(x - 45°)$, for $0° < x < 360°$. [2]

3 A curve has equation $y = \dfrac{e^{3x}}{\tan\frac{1}{2}x}$. Find the x-coordinates of the stationary points of the curve in the interval $0 < x < \pi$. Give your answers correct to 3 decimal places. [6]

4 The polynomial $x^4 + 2x^3 + ax + b$, where a and b are constants, is divisible by $x^2 - x + 1$. Find the values of a and b. [5]

5 Let $I = \int_{\frac{1}{4}}^{\frac{3}{4}} \sqrt{\left(\frac{x}{1-x}\right)}\,dx$.

(i) Using the substitution $x = \cos^2\theta$, show that $I = \int_{\frac{1}{6}\pi}^{\frac{1}{3}\pi} 2\cos^2\theta\,d\theta$. [4]

(ii) Hence find the exact value of I. [4]

6 In a certain chemical reaction the amount, x grams, of a substance is decreasing. The differential equation relating x and t, the time in seconds since the reaction started, is

$$\frac{dx}{dt} = -kx\sqrt{t},$$

where k is a positive constant. It is given that $x = 100$ at the start of the reaction.

(i) Solve the differential equation, obtaining a relation between x, t and k. [5]

(ii) Given that $t = 25$ when $x = 80$, find the value of t when $x = 40$. [3]

7 **(i)** Showing all working and without using a calculator, solve the equation $z^2 + (2\sqrt{6})z + 8 = 0$, giving your answers in the form $x + iy$, where x and y are real and exact. [3]

(ii) Sketch an Argand diagram showing the points representing the roots. [1]

(iii) The points representing the roots are A and B, and O is the origin. Find angle AOB. [3]

(iv) Prove that triangle AOB is equilateral. [1]

8 The positive constant a is such that $\int_0^a x\mathrm{e}^{-\frac{1}{2}x}\,\mathrm{d}x = 2$.

(i) Show that a satisfies the equation $a = 2\ln(a+2)$. [5]

(ii) Verify by calculation that a lies between 3 and 3.5. [2]

(iii) Use an iteration based on the equation in part **(i)** to determine a correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

9 Let $f(x) = \dfrac{12x^2 + 4x - 1}{(x-1)(3x+2)}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence obtain the expansion of $f(x)$ in ascending powers of x, up to and including the term in x^2. [5]

10 The point P has position vector $3\mathbf{i} - 2\mathbf{j} + \mathbf{k}$. The line l has equation $\mathbf{r} = 4\mathbf{i} + 2\mathbf{j} + 5\mathbf{k} + \mu(\mathbf{i} + 2\mathbf{j} + 3\mathbf{k})$.

(i) Find the length of the perpendicular from P to l, giving your answer correct to 3 significant figures. [5]

(ii) Find the equation of the plane containing l and P, giving your answer in the form $ax + by + cz = d$. [5]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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Cambridge International A Level

Cambridge International Examinations
Cambridge International Advanced Level

CANDIDATE NAME

CENTRE NUMBER | CANDIDATE NUMBER

9448820043

MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **(P3)** **May/June 2018**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

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The total number of marks for this paper is 75.

This document consists of **20** printed pages.

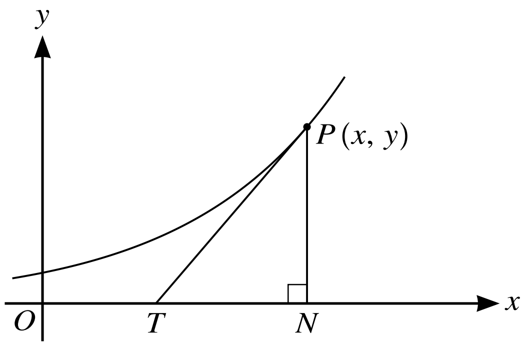
JC18 06_9709_32/RP

CAMBRIDGE International Examinations

1 Showing all necessary working, solve the equation $3|2^x - 1| = 2^x$, giving your answers correct to 3 significant figures. [4]

2 Showing all necessary working, solve the equation $\cot\theta + \cot(\theta + 45°) = 2$, for $0° < \theta < 180°$. [5]

3



In the diagram, the tangent to a curve at the point P with coordinates $(x,\ y)$ meets the x-axis at T. The point N is the foot of the perpendicular from P to the x-axis. The curve is such that, for all values of x, the gradient of the curve is positive and $TN = 2$.

(i) Show that the differential equation satisfied by x and y is $\frac{dy}{dx} = \frac{1}{2}y$. [1]

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The point with coordinates $(4,\ 3)$ lies on the curve.

(ii) Solve the differential equation to obtain the equation of the curve, expressing y in terms of x. [5]

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4 **(i)** Show that $\dfrac{2\sin x - \sin 2x}{1 - \cos 2x} \equiv \dfrac{\sin x}{1 + \cos x}$. [4]

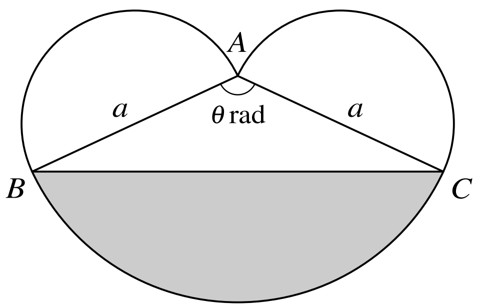
(ii) Hence, showing all necessary working, find $\int_{\frac{1}{3}\pi}^{\frac{1}{2}\pi} \frac{2\sin x - \sin 2x}{1 - \cos 2x}\, dx$, giving your answer in the form $\ln k$. [4]

5 The equation of a curve is $x^2(x + 3y) - y^3 = 3$.

(i) Show that $\frac{dy}{dx} = \frac{x^2 + 2xy}{y^2 - x^2}$. [4]

(ii) Hence find the exact coordinates of the two points on the curve at which the gradient of the normal is 1. [4]

6



The diagram shows a triangle ABC in which $AB = AC = a$ and angle $BAC = \theta$ radians. Semicircles are drawn outside the triangle with AB and AC as diameters. A circular arc with centre A joins B and C. The area of the shaded segment is equal to the sum of the areas of the semicircles.

(i) Show that $\theta = \frac{1}{2}\pi + \sin\theta$. [3]

(ii) Verify by calculation that θ lies between 2.2 and 2.4. [2]

(iii) Use an iterative formula based on the equation in part **(i)** to determine θ correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

7 **Throughout this question the use of a calculator is not permitted.**

The complex numbers $-3\sqrt{3} + \mathrm{i}$ and $\sqrt{3} + 2\mathrm{i}$ are denoted by u and v respectively.

(i) Find, in the form $x + \mathrm{i}y$, where x and y are real and exact, the complex numbers uv and $\dfrac{u}{v}$. [5]

(ii) On a sketch of an Argand diagram with origin O, show the points A and B representing the complex numbers u and v respectively. Prove that angle $AOB = \frac{2}{3}\pi$. [3]

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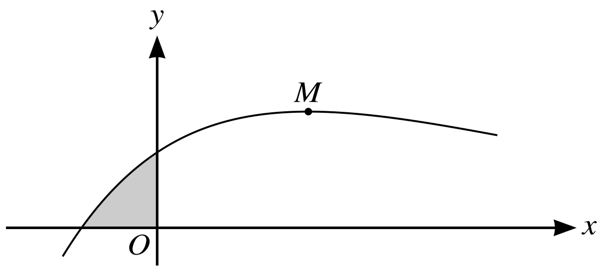

The diagram shows the curve $y = (x+1)e^{-\frac{1}{3}x}$ and its maximum point M.

(i) Find the x-coordinate of M. [4]

(ii) Find the area of the shaded region enclosed by the curve and the axes, giving your answer in terms of e. [5]

9 Let $f(x) = \dfrac{x - 4x^2}{(3 - x)(2 + x^2)}$.

(i) Express $f(x)$ in the form $\dfrac{A}{3 - x} + \dfrac{Bx + C}{2 + x^2}$. [4]

(ii) Hence obtain the expansion of $f(x)$ in ascending powers of x, up to and including the term in x^3. [5]

10 Two lines l and m have equations $\mathbf{r} = 2\mathbf{i} - \mathbf{j} + \mathbf{k} + s(2\mathbf{i} + 3\mathbf{j} - \mathbf{k})$ and $\mathbf{r} = \mathbf{i} + 3\mathbf{j} + 4\mathbf{k} + t(\mathbf{i} + 2\mathbf{j} + \mathbf{k})$ respectively.

(i) Show that the lines are skew. [4]

A plane p is parallel to the lines l and m.

(ii) Find a vector that is normal to p. [3]

(iii) Given that p is equidistant from the lines l and m, find the equation of p. Give your answer in the form $ax + by + cz = d$. [3]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

Cambridge International A Level

Cambridge International Examinations
Cambridge International Advanced Level

CANDIDATE NAME

CENTRE NUMBER | CANDIDATE NUMBER

6101943949

MATHEMATICS **9709/33**

Paper 3 Pure Mathematics 3 **(P3)** **May/June 2018**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

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Answer **all** the questions in the space provided. If additional space is required, you should use the lined page at the end of this booklet. The question number(s) must be clearly shown.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.

This document consists of **19** printed pages and **1** blank page.

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1 Expand $\dfrac{4}{\sqrt{(4-3x)}}$ in ascending powers of x, up to and including the term in x^2, simplifying the coefficients. [4]

2 Showing all necessary working, solve the equation $5^{2x} = 5^x + 5$. Give your answer correct to 3 decimal places. [5]

3 Showing all necessary working, find the value of $\int_0^{\frac{1}{6}\pi} x\cos 3x\,dx$, giving your answer in terms of π. [5]

4 The curve with equation $y = \frac{\ln x}{3 + x}$ has a stationary point at $x = p$.

(i) Show that p satisfies the equation $\ln x = 1 + \frac{3}{x}$. [3]

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(ii) By sketching suitable graphs, show that the equation in part **(i)** has only one root. [2]

(iii) It is given that the equation in part **(i)** can be written in the form $x = \dfrac{3 + x}{\ln x}$. Use an iterative formula based on this rearrangement to determine the value of p correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

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5 **(i)** By first expanding $(\cos^2 x + \sin^2 x)^3$, or otherwise, show that

$$\cos^6 x + \sin^6 x = 1 - \tfrac{3}{4}\sin^2 2x.$$ [4]

(ii) Hence solve the equation

$$\cos^6 x + \sin^6 x = \tfrac{2}{3},$$

for $0° < x < 180°$. [4]

6 **(i)** Express $\dfrac{1}{4 - y^2}$ in partial fractions. [2]

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(ii) The variables x and y satisfy the differential equation

$$x\frac{dy}{dx} = 4 - y^2,$$

and $y = 1$ when $x = 1$. Solve the differential equation, obtaining an expression for y in terms of x. [6]

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7 Throughout this question the use of a calculator is not permitted.

(i) Express $\cos\theta + 2\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$. Give the exact values of R and $\tan\alpha$. [3]

(ii) Hence, showing all necessary working, show that $\int_0^{\frac{1}{4}\pi} \frac{15}{(\cos\theta + 2\sin\theta)^2}\, d\theta = 5$. [5]

8 The equation of a curve is $2x^3 - y^3 - 3xy^2 = 2a^3$, where a is a non-zero constant.

(i) Show that $\dfrac{dy}{dx} = \dfrac{2x^2 - y^2}{y^2 + 2xy}$. [4]

(ii) Find the coordinates of the two points on the curve at which the tangent is parallel to the y-axis. [5]

9 **(a)** Find the complex number z satisfying the equation

$$3z - \mathrm{i}z^* = 1 + 5\mathrm{i},$$

where z^* denotes the complex conjugate of z. [4]

(b) On a sketch of an Argand diagram, shade the region whose points represent complex numbers z which satisfy both the inequalities $|z| \leqslant 3$ and $\text{Im}\, z \geqslant 2$, where $\text{Im}\, z$ denotes the imaginary part of z. Calculate the greatest value of $\arg z$ for points in this region. Give your answer in radians correct to 2 decimal places. [5]

10 The points A and B have position vectors $2\mathbf{i} + \mathbf{j} + 3\mathbf{k}$ and $4\mathbf{i} + \mathbf{j} + \mathbf{k}$ respectively. The line l has equation $\mathbf{r} = 4\mathbf{i} + 6\mathbf{j} + \mu(\mathbf{i} + 2\mathbf{j} - 2\mathbf{k})$.

(i) Show that l does not intersect the line passing through A and B. [5]

The point P, with parameter t, lies on l and is such that angle PAB is equal to $120°$.

(ii) Show that $3t^2 + 8t + 4 = 0$. Hence find the position vector of P. [6]

Additional Page

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Cambridge International Examinations
Cambridge International Advanced Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **(P3)** **February/March 2018**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

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You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.

1 Use the trapezium rule with three intervals to estimate the value of

$$\int_0^{\frac{1}{4}\pi} \sqrt{(1 - \tan x)}\, dx,$$

giving your answer correct to 3 decimal places. [3]

2 Expand $\sqrt[4]{(1-4x)}$ in ascending powers of x, up to and including the term in x^3, simplifying the coefficients. [4]

3 **(i)** Using the expansions of $\cos(3x + x)$ and $\cos(3x - x)$, show that

$$\tfrac{1}{2}(\cos 4x + \cos 2x) \equiv \cos 3x \cos x.$$ [3]

(ii) Hence show that $\int_{-\frac{1}{6}\pi}^{\frac{1}{6}\pi} \cos 3x \cos x \, dx = \frac{3}{8}\sqrt{3}$. [3]

4 The variables x and y satisfy the equation $y^n = Ax^3$, where n and A are constants. It is given that $y = 2.58$ when $x = 1.20$, and $y = 9.49$ when $x = 2.51$.

(i) Explain why the graph of $\ln y$ against $\ln x$ is a straight line. [2]

(ii) Find the values of n and A, giving your answers correct to 2 decimal places. [4]

5 The parametric equations of a curve are

$$x = 2t + \sin 2t, \quad y = 1 - 2\cos 2t,$$

for $-\frac{1}{2}\pi < t < \frac{1}{2}\pi$.

(i) Show that $\frac{dy}{dx} = 2\tan t$. [5]

(ii) Hence find the x-coordinate of the point on the curve at which the gradient of the normal is 2. Give your answer correct to 3 significant figures. [2]

6 The variables x and θ satisfy the differential equation

$$x\cos^2\theta\frac{\mathrm{d}x}{\mathrm{d}\theta} = 2\tan\theta + 1,$$

for $0 \leqslant \theta < \frac{1}{2}\pi$ and $x > 0$. It is given that $x = 1$ when $\theta = \frac{1}{4}\pi$.

(i) Show that $\dfrac{\mathrm{d}}{\mathrm{d}\theta}(\tan^2\theta) = \dfrac{2\tan\theta}{\cos^2\theta}$. [1]

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(ii) Solve the differential equation and calculate the value of x when $\theta = \frac{1}{3}\pi$, giving your answer correct to 3 significant figures. [7]

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7 **(i)** By sketching suitable graphs, show that the equation $e^{2x} = 6 + e^{-x}$ has exactly one real root. [2]

(ii) Verify by calculation that this root lies between 0.5 and 1. [2]

(iii) Show that if a sequence of values given by the iterative formula

$$x_{n+1} = \tfrac{1}{3}\ln\left(1 + 6e^{x_n}\right)$$

converges, then it converges to the root of the equation in part **(i)**. [2]

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(iv) Use this iterative formula to calculate the root correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

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8 Let $f(x) = \dfrac{5x^2 + x + 27}{(2x + 1)(x^2 + 9)}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence find $\int_0^4 \mathrm{f}(x)\,\mathrm{d}x$, giving your answer in the form $\ln c$, where c is an integer. [5]

9 The complex number $1 + 2\mathrm{i}$ is denoted by u.

(i) It is given that u is a root of the equation $2x^3 - x^2 + 4x + k = 0$, where k is a constant.

(a) Showing all working and without using a calculator, find the value of k. [3]

(b) Showing all working and without using a calculator, find the other two roots of this equation. [4]

(ii) On an Argand diagram sketch the locus of points representing complex numbers z satisfying the equation $|z - u| = 1$. Determine the least value of $\arg z$ for points on this locus. Give your answer in radians correct to 2 decimal places. [4]

10 The line l has equation $\mathbf{r} = 4\mathbf{i} + 3\mathbf{j} - \mathbf{k} + \mu(\mathbf{i} + 2\mathbf{j} - 2\mathbf{k})$. The plane p has equation $2x - 3y - z = 4$.

(i) Find the position vector of the point of intersection of l and p. [3]

(ii) Find the acute angle between l and p. [3]

(iii) A second plane q is parallel to l, perpendicular to p and contains the point with position vector $4\mathbf{j} - \mathbf{k}$. Find the equation of q, giving your answer in the form $ax + by + cz = d$. [5]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

Cambridge International A Level

Cambridge International Examinations
Cambridge International Advanced Level

CANDIDATE NAME			
CENTRE NUMBER		CANDIDATE NUMBER	

8927566197

MATHEMATICS **9709/31**

Paper 3 Pure Mathematics 3 **(P3)** **October/November 2017**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

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The total number of marks for this paper is 75.

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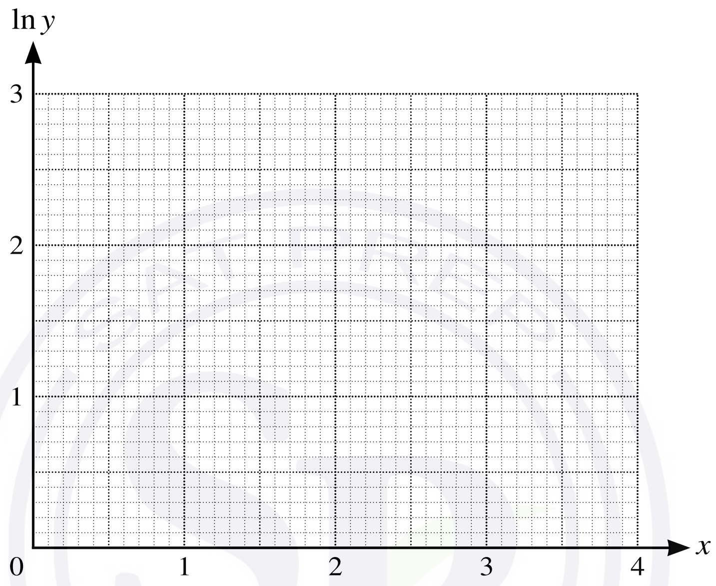
1 Find the quotient and remainder when x^4 is divided by $x^2 + 2x - 1$. [3]

2 Two variable quantities x and y are believed to satisfy an equation of the form $y = C(a^x)$, where C and a are constants. An experiment produced four pairs of values of x and y. The table below gives the corresponding values of x and $\ln y$.

x	0.9	1.6	2.4	3.2
$\ln y$	1.7	1.9	2.3	2.6

By plotting $\ln y$ against x for these four pairs of values and drawing a suitable straight line, estimate the values of C and a. Give your answers correct to 2 significant figures. [5]



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3 The equation $x^3 = 3x + 7$ has one real root, denoted by α.

(i) Show by calculation that α lies between 2 and 3. [2]

Two iterative formulae, A and B, derived from this equation are as follows:

$$x_{n+1} = (3x_n + 7)^{\frac{1}{3}}, \qquad (A)$$

$$x_{n+1} = \frac{x_n^3 - 7}{3}. \qquad (B)$$

Each formula is used with initial value $x_1 = 2.5$.

(ii) Show that one of these formulae produces a sequence which fails to converge, and use the other formula to calculate α correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [4]

4 **(i)** Prove the identity $\tan(45° + x) + \tan(45° - x) \equiv 2\sec 2x$. [4]

(ii) Hence sketch the graph of $y = \tan(45° + x) + \tan(45° - x)$ for $0° \leqslant x \leqslant 90°$. [3]

5 The equation of a curve is $2x^4 + xy^3 + y^4 = 10$.

(i) Show that $\frac{dy}{dx} = -\frac{8x^3 + y^3}{3xy^2 + 4y^3}$. [4]

(ii) Hence show that there are two points on the curve at which the tangent is parallel to the x-axis and find the coordinates of these points. [4]

6 The variables x and y satisfy the differential equation

$$\frac{dy}{dx} = 4\cos^2 y \tan x,$$

for $0 \leqslant x < \frac{1}{2}\pi$, and $x = 0$ when $y = \frac{1}{4}\pi$. Solve this differential equation and find the value of x when $y = \frac{1}{3}\pi$. [8]

7 **(a)** The complex number u is given by $u = 8 - 15\mathrm{i}$. Showing all necessary working, find the two square roots of u. Give answers in the form $a + \mathrm{i}b$, where the numbers a and b are real and exact. [5]

(b) On an Argand diagram, shade the region whose points represent complex numbers satisfying both the inequalities $|z - 2 - \mathrm{i}| \leqslant 2$ and $0 \leqslant \arg(z - \mathrm{i}) \leqslant \frac{1}{4}\pi$. [4]

8 Let $f(x) = \dfrac{4x^2 + 9x - 8}{(x+2)(2x-1)}$.

(i) Express $f(x)$ in the form $A + \dfrac{B}{x+2} + \dfrac{C}{2x-1}$. [4]

(ii) Hence show that $\int_1^4 \mathrm{f}(x)\,\mathrm{d}x = 6 + \frac{1}{2}\ln\left(\frac{16}{7}\right)$. [5]

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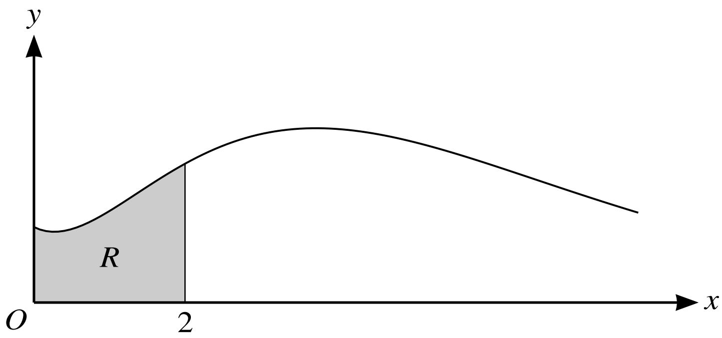

The diagram shows the curve $y = (1 + x^2)e^{-\frac{1}{2}x}$ for $x \geqslant 0$. The shaded region R is enclosed by the curve, the x-axis and the lines $x = 0$ and $x = 2$.

(i) Find the exact values of the x-coordinates of the stationary points of the curve. [4]

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(ii) Show that the exact value of the area of R is $18 - \dfrac{42}{e}$. [5]

10 The equations of two lines l and m are $\mathbf{r} = 3\mathbf{i} - \mathbf{j} - 2\mathbf{k} + \lambda(-\mathbf{i} + \mathbf{j} + 4\mathbf{k})$ and $\mathbf{r} = 4\mathbf{i} + 4\mathbf{j} - 3\mathbf{k} + \mu(2\mathbf{i} + \mathbf{j} - 2\mathbf{k})$ respectively.

(i) Show that the lines do not intersect. [3]

(ii) Calculate the acute angle between the directions of the lines. [3]

(iii) Find the equation of the plane which passes through the point $(3, -2, -1)$ and which is parallel to both l and m. Give your answer in the form $ax + by + cz = d$. [5]

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Cambridge International A Level

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Cambridge International Advanced Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

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MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **(P3)** **October/November 2017**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

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JC17 11_9709_32/RP

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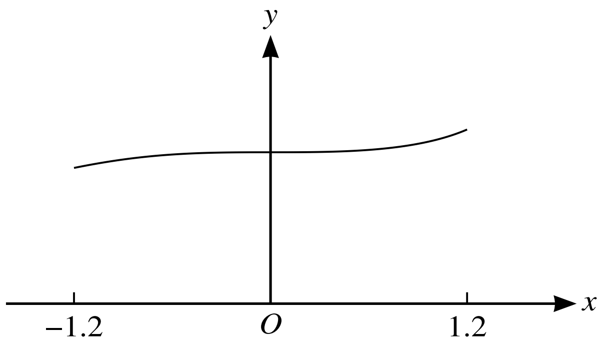

The diagram shows a sketch of the curve $y = \frac{3}{\sqrt{(9 - x^3)}}$ for values of x from −1.2 to 1.2.

(i) Use the trapezium rule, with two intervals, to estimate the value of

$$\int_{-1.2}^{1.2} \frac{3}{\sqrt{(9 - x^3)}}\, dx,$$

giving your answer correct to 2 decimal places. [3]

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(ii) Explain, with reference to the diagram, why the trapezium rule may be expected to give a good approximation to the true value of the integral in this case. [1]

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2 Showing all necessary working, solve the equation $2\log_2 x = 3 + \log_2(x + 1)$, giving your answer correct to 3 significant figures. [5]

3 By expressing the equation $\tan(\theta + 60°) + \tan(\theta - 60°) = \cot \theta$ in terms of $\tan \theta$ only, solve the equation for $0° < \theta < 90°$. [5]

4 The curve with equation $y = \dfrac{2 - \sin x}{\cos x}$ has one stationary point in the interval $-\frac{1}{2}\pi < x < \frac{1}{2}\pi$.

(i) Find the exact coordinates of this point. [5]

(ii) Determine whether this point is a maximum or a minimum point. [2]

5 The variables x and y satisfy the differential equation

$$(x+1)\frac{dy}{dx} = y(x+2),$$

and it is given that $y = 2$ when $x = 1$. Solve the differential equation and obtain an expression for y in terms of x. [7]

6 The equation of a curve is $x^3y - 3xy^3 = 2a^4$, where a is a non-zero constant.

(i) Show that $\dfrac{dy}{dx} = \dfrac{3x^2y - 3y^3}{9xy^2 - x^3}$. [4]

(ii) Hence show that there are only two points on the curve at which the tangent is parallel to the x-axis and find the coordinates of these points. [4]

7 **Throughout this question the use of a calculator is not permitted.**

The complex number $1 - (\sqrt{3})\mathrm{i}$ is denoted by u.

(i) Find the modulus and argument of u. [2]

(ii) Show that $u^3 + 8 = 0$. [2]

(iii) On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying both the inequalities $|z - u| \leqslant 2$ and $\operatorname{Re} z \geqslant 2$, where $\operatorname{Re} z$ denotes the real part of z. [4]

8 Let $f(x) = \dfrac{8x^2 + 9x + 8}{(1 - x)(2x + 3)^2}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence obtain the expansion of $f(x)$ in ascending powers of x, up to and including the term in x^2. [5]

9 It is given that $\int_1^a x^{\frac{1}{2}} \ln x \, dx = 2$, where $a > 1$.

(i) Show that $a^{\frac{3}{2}} = \dfrac{7 + 2a^{\frac{3}{2}}}{3 \ln a}$. [5]

(ii) Show by calculation that a lies between 2 and 4. [2]

(iii) Use the iterative formula

$$a_{n+1} = \left(\frac{7 + 2a_n^{\frac{3}{2}}}{3 \ln a_n} \right)^{\frac{2}{3}}$$

to determine a correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

10 Two planes p and q have equations $x + y + 3z = 8$ and $2x - 2y + z = 3$ respectively.

(i) Calculate the acute angle between the planes p and q. [4]

(ii) The point A on the line of intersection of p and q has y-coordinate equal to 2. Find the equation of the plane which contains the point A and is perpendicular to both the planes p and q. Give your answer in the form $ax + by + cz = d$. [7]

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Cambridge International A Level

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Cambridge International Advanced Level

CANDIDATE NAME			
CENTRE NUMBER		CANDIDATE NUMBER	

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MATHEMATICS **9709/33**

Paper 3 Pure Mathematics 3 **(P3)** **October/November 2017**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

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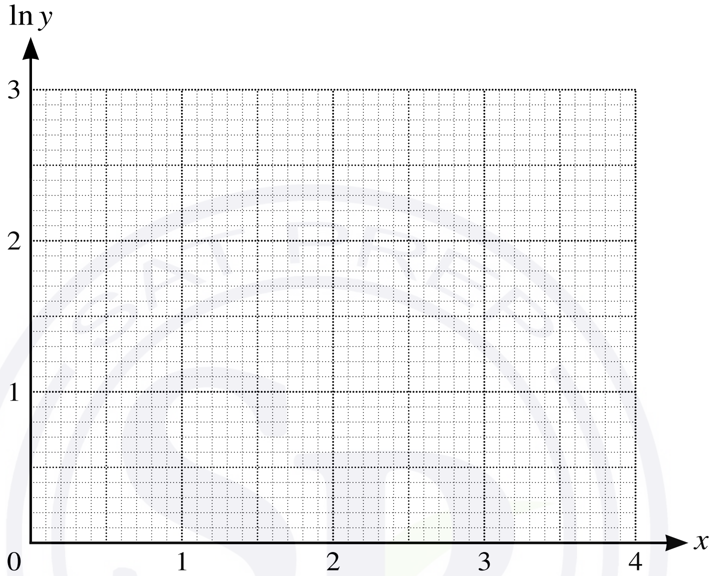
1 Find the quotient and remainder when x^4 is divided by $x^2 + 2x - 1$. [3]

2 Two variable quantities x and y are believed to satisfy an equation of the form $y = C(a^x)$, where C and a are constants. An experiment produced four pairs of values of x and y. The table below gives the corresponding values of x and $\ln y$.

x	0.9	1.6	2.4	3.2
$\ln y$	1.7	1.9	2.3	2.6

By plotting $\ln y$ against x for these four pairs of values and drawing a suitable straight line, estimate the values of C and a. Give your answers correct to 2 significant figures. [5]



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3 The equation $x^3 = 3x + 7$ has one real root, denoted by α.

(i) Show by calculation that α lies between 2 and 3. [2]

Two iterative formulae, A and B, derived from this equation are as follows:

$$x_{n+1} = (3x_n + 7)^{\frac{1}{3}}, \qquad (A)$$

$$x_{n+1} = \frac{x_n^3 - 7}{3}. \qquad (B)$$

Each formula is used with initial value $x_1 = 2.5$.

(ii) Show that one of these formulae produces a sequence which fails to converge, and use the other formula to calculate α correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [4]

4 **(i)** Prove the identity $\tan(45° + x) + \tan(45° - x) \equiv 2 \sec 2x$. [4]

(ii) Hence sketch the graph of $y = \tan(45° + x) + \tan(45° - x)$ for $0° \leqslant x \leqslant 90°$. [3]

5 The equation of a curve is $2x^4 + xy^3 + y^4 = 10$.

(i) Show that $\frac{dy}{dx} = -\frac{8x^3 + y^3}{3xy^2 + 4y^3}$. [4]

(ii) Hence show that there are two points on the curve at which the tangent is parallel to the x-axis and find the coordinates of these points. [4]

6 The variables x and y satisfy the differential equation

$$\frac{dy}{dx} = 4\cos^2 y \tan x,$$

for $0 \leqslant x < \frac{1}{2}\pi$, and $x = 0$ when $y = \frac{1}{4}\pi$. Solve this differential equation and find the value of x when $y = \frac{1}{3}\pi$. [8]

7 **(a)** The complex number u is given by $u = 8 - 15\text{i}$. Showing all necessary working, find the two square roots of u. Give answers in the form $a + \text{i}b$, where the numbers a and b are real and exact. [5]

(b) On an Argand diagram, shade the region whose points represent complex numbers satisfying both the inequalities $|z - 2 - \mathrm{i}| \leqslant 2$ and $0 \leqslant \arg(z - \mathrm{i}) \leqslant \frac{1}{4}\pi$. [4]

8 Let $f(x) = \dfrac{4x^2 + 9x - 8}{(x + 2)(2x - 1)}$.

(i) Express $f(x)$ in the form $A + \dfrac{B}{x + 2} + \dfrac{C}{2x - 1}$. [4]

(ii) Hence show that $\int_1^4 f(x)\,dx = 6 + \frac{1}{2}\ln\left(\frac{16}{7}\right)$. [5]

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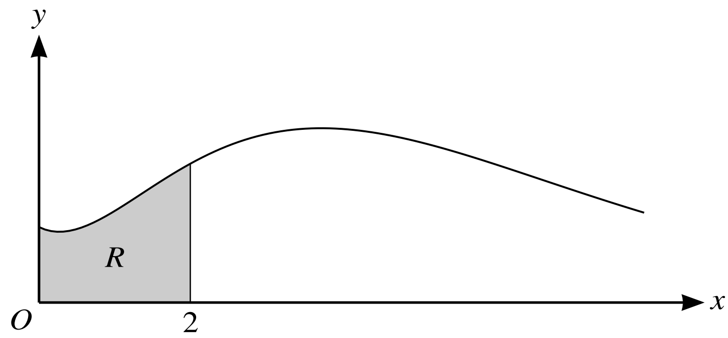

The diagram shows the curve $y = (1 + x^2)e^{-\frac{1}{2}x}$ for $x \geqslant 0$. The shaded region R is enclosed by the curve, the x-axis and the lines $x = 0$ and $x = 2$.

(i) Find the exact values of the x-coordinates of the stationary points of the curve. [4]

(ii) Show that the exact value of the area of R is $18 - \dfrac{42}{e}$. [5]

10 The equations of two lines l and m are $\mathbf{r} = 3\mathbf{i} - \mathbf{j} - 2\mathbf{k} + \lambda(-\mathbf{i} + \mathbf{j} + 4\mathbf{k})$ and $\mathbf{r} = 4\mathbf{i} + 4\mathbf{j} - 3\mathbf{k} + \mu(2\mathbf{i} + \mathbf{j} - 2\mathbf{k})$ respectively.

(i) Show that the lines do not intersect. [3]

(ii) Calculate the acute angle between the directions of the lines. [3]

(iii) Find the equation of the plane which passes through the point $(3, -2, -1)$ and which is parallel to both l and m. Give your answer in the form $ax + by + cz = d$. [5]

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Cambridge International A Level

Cambridge International Examinations
Cambridge International Advanced Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

* 4 3 8 2 1 5 3 9 8 1 *

MATHEMATICS **9709/31**

Paper 3 Pure Mathematics 3 **(P3)** **May/June 2017**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

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Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
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DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.

This document consists of **19** printed pages and **1** blank page.

JC17 06_9709_31/RP

CAMBRIDGE International Examinations

[Turn over

1 Solve the inequality $|2x + 1| < 3|x - 2|$. [4]

2 Expand $\frac{1}{\sqrt[3]{(1+6x)}}$ in ascending powers of x, up to and including the term in x^3, simplifying the coefficients. [4]

3 It is given that $x = \ln(1 - y) - \ln y$, where $0 < y < 1$.

(i) Show that $y = \dfrac{e^{-x}}{1 + e^{-x}}$. [2]

(ii) Hence show that $\int_0^1 y\,dx = \ln\left(\frac{2e}{e+1}\right)$. [4]

4 The parametric equations of a curve are

$$x = \ln \cos \theta, \quad y = 3\theta - \tan \theta,$$

where $0 \leqslant \theta < \frac{1}{2}\pi$.

(i) Express $\frac{dy}{dx}$ in terms of $\tan \theta$. [5]

(ii) Find the exact y-coordinate of the point on the curve at which the gradient of the normal is equal to 1. [3]

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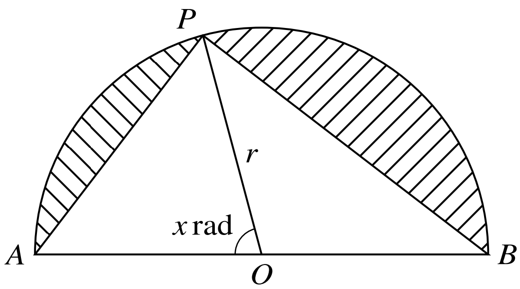

The diagram shows a semicircle with centre O, radius r and diameter AB. The point P on its circumference is such that the area of the minor segment on AP is equal to half the area of the minor segment on BP. The angle AOP is x radians.

(i) Show that x satisfies the equation $x = \frac{1}{3}(\pi + \sin x)$. [3]

(ii) Verify by calculation that x lies between 1 and 1.5. [2]

(iii) Use an iterative formula based on the equation in part **(i)** to determine x correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

6 The plane with equation $2x + 2y - z = 5$ is denoted by m. Relative to the origin O, the points A and B have coordinates $(3, 4, 0)$ and $(-1, 0, 2)$ respectively.

(i) Show that the plane m bisects AB at right angles. [5]

A second plane p is parallel to m and nearer to O. The perpendicular distance between the planes is 1.

(ii) Find the equation of p, giving your answer in the form $ax + by + cz = d$. [3]

7 Throughout this question the use of a calculator is not permitted.

The complex numbers u and w are defined by $u = -1 + 7\mathrm{i}$ and $w = 3 + 4\mathrm{i}$.

(i) Showing all your working, find in the form $x + \mathrm{i}y$, where x and y are real, the complex numbers $u - 2w$ and $\dfrac{u}{w}$. [4]

In an Argand diagram with origin O, the points A, B and C represent the complex numbers u, w and $u - 2w$ respectively.

(ii) Prove that angle $AOB = \frac{1}{4}\pi$. [2]

(iii) State fully the geometrical relation between the line segments OB and CA. [2]

8 **(i)** By first expanding $2\sin(x - 30°)$, express $2\sin(x - 30°) - \cos x$ in the form $R\sin(x - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. Give the exact value of R and the value of α correct to 2 decimal places. [5]

(ii) Hence solve the equation

$$2\sin(x - 30°) - \cos x = 1,$$

for $0° < x < 180°$. [3]

9 **(i)** Express $\dfrac{1}{x(2x+3)}$ in partial fractions. [2]

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(ii) The variables x and y satisfy the differential equation

$$x(2x+3)\frac{\mathrm{d}y}{\mathrm{d}x} = y,$$

and it is given that $y = 1$ when $x = 1$. Solve the differential equation and calculate the value of y when $x = 9$, giving your answer correct to 3 significant figures. [7]

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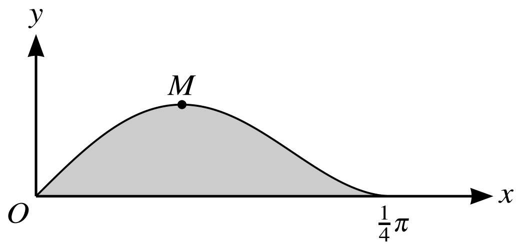

The diagram shows the curve $y = \sin x \cos^2 2x$ for $0 \leqslant x \leqslant \frac{1}{4}\pi$ and its maximum point M.

(i) Using the substitution $u = \cos x$, find by integration the exact area of the shaded region bounded by the curve and the x-axis. [6]

(ii) Find the x-coordinate of M. Give your answer correct to 2 decimal places. [6]

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Cambridge International A Level

Cambridge International Examinations
Cambridge International Advanced Level

CANDIDATE NAME

CENTRE NUMBER | CANDIDATE NUMBER

9035132693

MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **(P3)** **May/June 2017**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

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The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.

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JC17 06_9709_32/2R

CAMBRIDGE International Examinations

[Turn over

1 Solve the equation $\ln(x^2 + 1) = 1 + 2\ln x$, giving your answer correct to 3 significant figures. [3]

2 Solve the inequality $|x - 3| < 3x - 4$. [4]

3 **(i)** Express the equation $\cot\theta - 2\tan\theta = \sin 2\theta$ in the form $a\cos^4\theta + b\cos^2\theta + c = 0$, where a, b and c are constants to be determined. [3]

(ii) Hence solve the equation $\cot\theta - 2\tan\theta = \sin 2\theta$ for $90° < \theta < 180°$. [2]

4 The parametric equations of a curve are

$$x = t^2 + 1, \quad y = 4t + \ln(2t - 1).$$

(i) Express $\frac{dy}{dx}$ in terms of t. [3]

(ii) Find the equation of the normal to the curve at the point where $t = 1$. Give your answer in the form $ax + by + c = 0$. [3]

5 In a certain chemical process a substance A reacts with and reduces a substance B. The masses of A and B at time t after the start of the process are x and y respectively. It is given that $\frac{dy}{dt} = -0.2xy$ and $x = \frac{10}{(1+t)^2}$. At the beginning of the process $y = 100$.

(i) Form a differential equation in y and t, and solve this differential equation. [6]

(ii) Find the exact value approached by the mass of B as t becomes large. State what happens to the mass of A as t becomes large. [2]

6 **Throughout this question the use of a calculator is not permitted.**

The complex number $2 - \mathrm{i}$ is denoted by u.

(i) It is given that u is a root of the equation $x^3 + ax^2 - 3x + b = 0$, where the constants a and b are real. Find the values of a and b. [4]

(ii) On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying both the inequalities $|z - u| < 1$ and $|z| < |z + \mathrm{i}|$. [4]

7 **(i)** Prove that if $y = \dfrac{1}{\cos\theta}$ then $\dfrac{dy}{d\theta} = \sec\theta\tan\theta$. [2]

(ii) Prove the identity $\dfrac{1+\sin\theta}{1-\sin\theta} \equiv 2\sec^2\theta + 2\sec\theta\tan\theta - 1$. [3]

(iii) Hence find the exact value of $\displaystyle\int_0^{\frac{1}{4}\pi} \frac{1+\sin\theta}{1-\sin\theta}\,d\theta$. [4]

8 Let $f(x) = \dfrac{5x^2 - 7x + 4}{(3x + 2)(x^2 + 5)}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence obtain the expansion of $f(x)$ in ascending powers of x, up to and including the term in x^2. [5]

9 Relative to the origin O, the point A has position vector given by $\overrightarrow{OA} = \mathbf{i} + 2\mathbf{j} + 4\mathbf{k}$. The line l has equation $\mathbf{r} = 9\mathbf{i} - \mathbf{j} + 8\mathbf{k} + \mu(3\mathbf{i} - \mathbf{j} + 2\mathbf{k})$.

(i) Find the position vector of the foot of the perpendicular from A to l. Hence find the position vector of the reflection of A in l. [5]

(ii) Find the equation of the plane through the origin which contains l. Give your answer in the form $ax + by + cz = d$. [3]

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(iii) Find the exact value of the perpendicular distance of A from this plane. [3]

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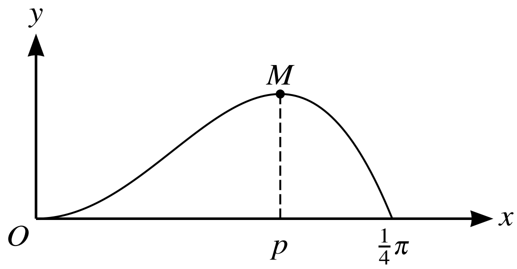

The diagram shows the curve $y = x^2 \cos 2x$ for $0 \leqslant x \leqslant \frac{1}{4}\pi$. The curve has a maximum point at M where $x = p$.

(i) Show that p satisfies the equation $p = \frac{1}{2}\tan^{-1}\left(\frac{1}{p}\right)$. [3]

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(ii) Use the iterative formula $p_{n+1} = \frac{1}{2}\tan^{-1}\left(\frac{1}{p_n}\right)$ to determine the value of p correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

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(iii) Find, showing all necessary working, the exact area of the region bounded by the curve and the x-axis. [5]

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Cambridge International A Level

Cambridge International Examinations
Cambridge International Advanced Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

5851032809

MATHEMATICS **9709/33**

Paper 3 Pure Mathematics 3 **(P3)** **May/June 2017**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

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JC17 06_9709_33/RP

CAMBRIDGE International Examinations

[Turn over

1 Prove the identity $\dfrac{\cot x - \tan x}{\cot x + \tan x} \equiv \cos 2x$. [3]

2 Expand $(3 + 2x)^{-3}$ in ascending powers of x up to and including the term in x^2, simplifying the coefficients. [4]

3 Using the substitution $u = e^x$, solve the equation $4e^{-x} = 3e^x + 4$. Give your answer correct to 3 significant figures. [4]

4 Find the exact value of $\int_0^{\frac{1}{2}\pi} \theta \sin \frac{1}{2}\theta \, d\theta$. [4]

5 A curve has equation $y = \frac{2}{3}\ln(1 + 3\cos^2 x)$ for $0 \leqslant x \leqslant \frac{1}{2}\pi$.

(i) Express $\frac{dy}{dx}$ in terms of $\tan x$. [4]

(ii) Hence find the x-coordinate of the point on the curve where the gradient is -1. Give your answer correct to 3 significant figures. [2]

6 The equation $\cot x = 1 - x$ has one root in the interval $0 < x < \pi$, denoted by α.

(i) Show by calculation that α is greater than 2.5. [2]

(ii) Show that, if a sequence of values in the interval $0 < x < \pi$ given by the iterative formula $x_{n+1} = \pi + \tan^{-1}\left(\dfrac{1}{1 - x_n}\right)$ converges, then it converges to α. [2]

(iii) Use this iterative formula to determine α correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

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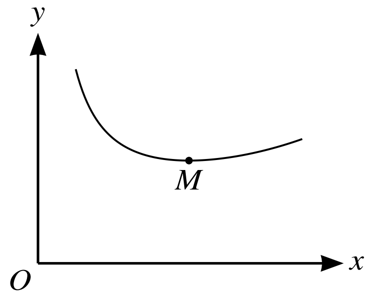

The diagram shows a sketch of the curve $y = \dfrac{e^{\frac{1}{2}x}}{x}$ for $x > 0$, and its minimum point M.

(i) Find the x-coordinate of M. [4]

(ii) Use the trapezium rule with two intervals to estimate the value of

$$\int_1^3 \frac{e^{\frac{1}{2}x}}{x}\,dx,$$

giving your answer correct to 2 decimal places. [3]

(iii) The estimate found in part **(ii)** is denoted by E. Explain, without further calculation, whether another estimate found using the trapezium rule with four intervals would be greater than E or less than E. [1]

8 In a certain chemical reaction, a compound A is formed from a compound B. The masses of A and B at time t after the start of the reaction are x and y respectively and the sum of the masses is equal to 50 throughout the reaction. At any time the rate of increase of the mass of A is proportional to the mass of B at that time.

(i) Explain why $\frac{dx}{dt} = k(50 - x)$, where k is a constant. [1]

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It is given that $x = 0$ when $t = 0$, and $x = 25$ when $t = 10$.

(ii) Solve the differential equation in part **(i)** and express x in terms of t. [8]

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9 Let $f(x) = \dfrac{3x^2 - 4}{x^2(3x + 2)}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence show that $\int_1^2 \mathrm{f}(x)\,\mathrm{d}x = \ln\left(\frac{25}{8}\right) - 1$. [5]

10 The points A and B have position vectors given by $\overrightarrow{OA} = \mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$ and $\overrightarrow{OB} = 3\mathbf{i} + \mathbf{j} + \mathbf{k}$. The line l has equation $\mathbf{r} = 2\mathbf{i} + \mathbf{j} + m\mathbf{k} + \mu(\mathbf{i} - 2\mathbf{j} - 4\mathbf{k})$, where m is a constant.

(i) Given that the line l intersects the line passing through A and B, find the value of m. [5]

(ii) Find the equation of the plane which is parallel to $\mathbf{i} - 2\mathbf{j} - 4\mathbf{k}$ and contains the points A and B. Give your answer in the form $ax + by + cz = d$. [5]

11 Throughout this question the use of a calculator is not permitted.

(a) The complex numbers z and w satisfy the equations

$$z + (1 + \mathrm{i})w = \mathrm{i} \qquad \text{and} \qquad (1 - \mathrm{i})z + \mathrm{i}w = 1.$$

Solve the equations for z and w, giving your answers in the form $x + \mathrm{i}y$, where x and y are real. [6]

(b) The complex numbers u and v are given by $u = 1 + (2\sqrt{3})i$ and $v = 3 + 2i$. In an Argand diagram, u and v are represented by the points A and B. A third point C lies in the first quadrant and is such that $BC = 2AB$ and angle $ABC = 90°$. Find the complex number z represented by C, giving your answer in the form $x + iy$, where x and y are real and exact. [4]

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Cambridge International Examinations
Cambridge International Advanced Level

CANDIDATE NAME

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MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **(P3)** **February/March 2017**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

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1 Solve the equation $\ln(1 + 2^x) = 2$, giving your answer correct to 3 decimal places. [3]

2 Solve the inequality $|x - 4| < 2|3x + 1|$. [4]

3 **(i)** By sketching suitable graphs, show that the equation $e^{-\frac{1}{2}x} = 4 - x^2$ has one positive root and one negative root. [2]

(ii) Verify by calculation that the negative root lies between −1 and −1.5. [2]

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(iii) Use the iterative formula $x_{n+1} = -\sqrt{\left(4 - e^{-\frac{1}{2}x_n}\right)}$ to determine this root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

4 **(i)** Express $8\cos\theta - 15\sin\theta$ in the form $R\cos(\theta + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$, stating the exact value of R and giving the value of α correct to 2 decimal places. [3]

(ii) Hence solve the equation

$$8\cos 2x - 15\sin 2x = 4,$$

for $0° < x < 180°$. [4]

5 The curve with equation $y = e^{-ax} \tan x$, where a is a positive constant, has only one point in the interval $0 < x < \frac{1}{2}\pi$ at which the tangent is parallel to the x-axis. Find the value of a and state the exact value of the x-coordinate of this point. [7]

6 The line l has equation $\mathbf{r} = \mathbf{i} + 2\mathbf{j} - 3\mathbf{k} + \lambda(2\mathbf{i} - \mathbf{j} + \mathbf{k})$. The plane p has equation $3x + y - 5z = 20$.

(i) Show that the line l lies in the plane p. [3]

(ii) A second plane is parallel to l, perpendicular to p and contains the point with position vector $3\mathbf{i} - \mathbf{j} + 2\mathbf{k}$. Find the equation of this plane, giving your answer in the form $ax + by + cz = d$. [5]

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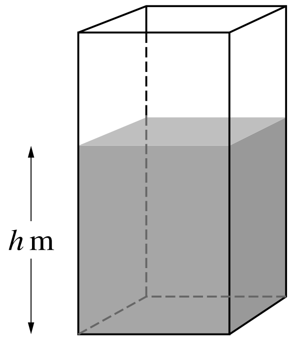

A water tank has vertical sides and a horizontal rectangular base, as shown in the diagram. The area of the base is $2\,\text{m}^2$. At time $t = 0$ the tank is empty and water begins to flow into it at a rate of $1\,\text{m}^3$ per hour. At the same time water begins to flow out from the base at a rate of $0.2\sqrt{h}\,\text{m}^3$ per hour, where h m is the depth of water in the tank at time t hours.

(i) Form a differential equation satisfied by h and t, and show that the time T hours taken for the depth of water to reach 4 m is given by

$$T = \int_0^4 \frac{10}{5 - \sqrt{h}}\,\mathrm{d}h.$$ [3]

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(ii) Using the substitution $u = 5 - \sqrt{h}$, find the value of T. [6]

8 **Throughout this question the use of a calculator is not permitted.**

The polynomial $z^4 + 3z^2 + 6z + 10$ is denoted by $\mathrm{p}(z)$. The complex number $-1 + \mathrm{i}$ is denoted by u.

(i) Showing all your working, verify that u is a root of the equation $\mathrm{p}(z) = 0$. [3]

(ii) Find the other three roots of the equation $\mathrm{p}(z) = 0$. [7]

9 Let $f(x) = \dfrac{x(6-x)}{(2+x)(4+x^2)}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence obtain the expansion of $f(x)$ in ascending powers of x, up to and including the term in x^2. [5]

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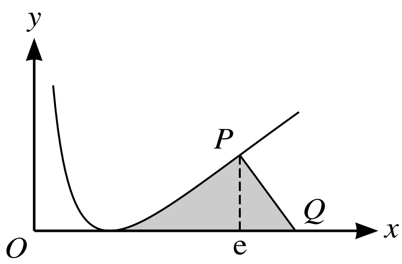

The diagram shows the curve $y = (\ln x)^2$. The x-coordinate of the point P is equal to e, and the normal to the curve at P meets the x-axis at Q.

(i) Find the x-coordinate of Q. [4]

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(ii) Show that $\int \ln x \, dx = x \ln x - x + c$, where c is a constant. [1]

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(iii) Using integration by parts, or otherwise, find the exact value of the area of the shaded region between the curve, the x-axis and the normal PQ. [5]

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Cambridge International A Level

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Cambridge International Advanced Level

MATHEMATICS **9709/31**

Paper 3 Pure Mathematics 3 **(P3)** **October/November 2016**

1 hour 45 minutes

Additional Materials: List of Formulae (MF9)

5711204026

READ THESE INSTRUCTIONS FIRST

An answer booklet is provided inside this question paper. You should follow the instructions on the front cover of the answer booklet. If you need additional answer paper ask the invigilator for a continuation booklet.

Answer **all** the questions.
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At the end of the examination, fasten all your work securely together.
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The total number of marks for this paper is 75.

This document consists of **3** printed pages, **1** blank page and **1** insert.

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1 Solve the equation $\dfrac{3^x + 2}{3^x - 2} = 8$, giving your answer correct to 3 decimal places. [3]

2 Expand $(2 - x)(1 + 2x)^{-\frac{3}{2}}$ in ascending powers of x, up to and including the term in x^2, simplifying the coefficients. [4]

3 Express the equation $\sec\theta = 3\cos\theta + \tan\theta$ as a quadratic equation in $\sin\theta$. Hence solve this equation for $-90° < \theta < 90°$. [5]

4 The equation of a curve is $xy(x - 6y) = 9a^3$, where a is a non-zero constant. Show that there is only one point on the curve at which the tangent is parallel to the x-axis, and find the coordinates of this point. [7]

5 **(i)** Prove the identity $\tan 2\theta - \tan\theta \equiv \tan\theta \sec 2\theta$. [4]

(ii) Hence show that $\displaystyle\int_0^{\frac{1}{6}\pi} \tan\theta \sec 2\theta \, d\theta = \tfrac{1}{2}\ln\tfrac{3}{2}$. [4]

6 **(i)** By sketching a suitable pair of graphs, show that the equation

$$\operatorname{cosec}\tfrac{1}{2}x = \tfrac{1}{3}x + 1$$

has one root in the interval $0 < x \leqslant \pi$. [2]

(ii) Show by calculation that this root lies between 1.4 and 1.6. [2]

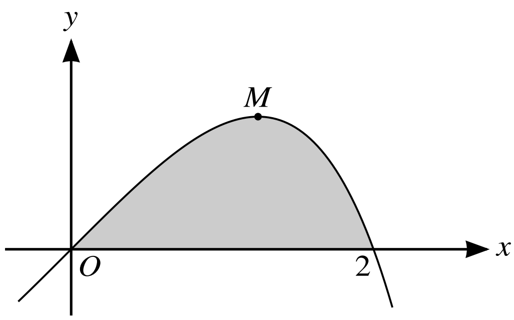
(iii) Show that, if a sequence of values in the interval $0 < x \leqslant \pi$ given by the iterative formula

$$x_{n+1} = 2\sin^{-1}\left(\frac{3}{x_n + 3}\right)$$

converges, then it converges to the root of the equation in part **(i)**. [2]

(iv) Use this iterative formula to calculate the root correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

7



The diagram shows part of the curve $y = (2x - x^2)e^{\frac{1}{2}x}$ and its maximum point M.

(i) Find the exact x-coordinate of M. [4]

(ii) Find the exact value of the area of the shaded region bounded by the curve and the positive x-axis. [5]

8 Two planes have equations $3x + y - z = 2$ and $x - y + 2z = 3$.

(i) Show that the planes are perpendicular. [3]

(ii) Find a vector equation for the line of intersection of the two planes. [6]

9 **Throughout this question the use of a calculator is not permitted.**

(a) Solve the equation $(1 + 2i)w^2 + 4w - (1 - 2i) = 0$, giving your answers in the form $x + iy$, where x and y are real. [5]

(b) On a sketch of an Argand diagram, shade the region whose points represent complex numbers satisfying the inequalities $|z - 1 - i| \leqslant 2$ and $-\frac{1}{4}\pi \leqslant \arg z \leqslant \frac{1}{4}\pi$. [5]

10 A large field of area $4\,\text{km}^2$ is becoming infected with a soil disease. At time t years the area infected is $x\,\text{km}^2$ and the rate of growth of the infected area is given by the differential equation $\frac{dx}{dt} = kx(4 - x)$, where k is a positive constant. It is given that when $t = 0$, $x = 0.4$ and that when $t = 2$, $x = 2$.

(i) Solve the differential equation and show that $k = \frac{1}{4}\ln 3$. [9]

(ii) Find the value of t when 90% of the area of the field is infected. [2]

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Cambridge International Examinations
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MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **(P3)** **October/November 2016**

1 hour 45 minutes

Additional Materials: List of Formulae (MF9)

6635872224

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[Turn over

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(ii) Show by calculation that this root lies between 1.4 and 1.6. [2]

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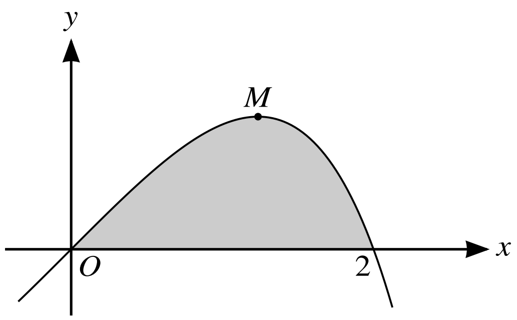
7



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(i) Solve the differential equation and show that $k = \frac{1}{4}\ln 3$. [9]

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Cambridge International Examinations
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MATHEMATICS **9709/33**

Paper 3 Pure Mathematics 3 **(P3)** **October/November 2016**

1 hour 45 minutes

* 7 2 4 0 3 6 6 8 8 2 *

Additional Materials: List of Formulae (MF9)

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1 It is given that $z = \ln(y+2) - \ln(y+1)$. Express y in terms of z. [3]

2 The equation of a curve is $y = \dfrac{\sin x}{1 + \cos x}$, for $-\pi < x < \pi$. Show that the gradient of the curve is positive for all x in the given interval. [4]

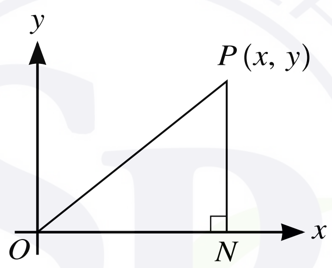
3 Express the equation $\cot 2\theta = 1 + \tan\theta$ as a quadratic equation in $\tan\theta$. Hence solve this equation for $0° < \theta < 180°$. [6]

4 The polynomial $4x^4 + ax^2 + 11x + b$, where a and b are constants, is denoted by $\mathrm{p}(x)$. It is given that $\mathrm{p}(x)$ is divisible by $x^2 - x + 2$.

(i) Find the values of a and b. [5]

(ii) When a and b have these values, find the real roots of the equation $\mathrm{p}(x) = 0$. [2]

5



The diagram shows a variable point P with coordinates (x, y) and the point N which is the foot of the perpendicular from P to the x-axis. P moves on a curve such that, for all $x \geqslant 0$, the gradient of the curve is equal in value to the area of the triangle OPN, where O is the origin.

(i) State a differential equation satisfied by x and y. [1]

The point with coordinates $(0, 2)$ lies on the curve.

(ii) Solve the differential equation to obtain the equation of the curve, expressing y in terms of x. [5]

(iii) Sketch the curve. [1]

6 Let $I = \displaystyle\int_1^4 \frac{(\sqrt{x}) - 1}{2(x + \sqrt{x})}\,\mathrm{d}x$.

(i) Using the substitution $u = \sqrt{x}$, show that $I = \displaystyle\int_1^2 \frac{u-1}{u+1}\,\mathrm{d}u$. [3]

(ii) Hence show that $I = 1 + \ln\frac{4}{9}$. [6]

7 **Throughout this question the use of a calculator is not permitted.**

The complex number z is defined by $z = (\sqrt{2}) - (\sqrt{6})\mathrm{i}$. The complex conjugate of z is denoted by z^*.

(i) Find the modulus and argument of z. [2]

(ii) Express each of the following in the form $x + \mathrm{i}y$, where x and y are real and exact:

(a) $z + 2z^*$;

(b) $\dfrac{z^*}{\mathrm{i}z}$.

[4]

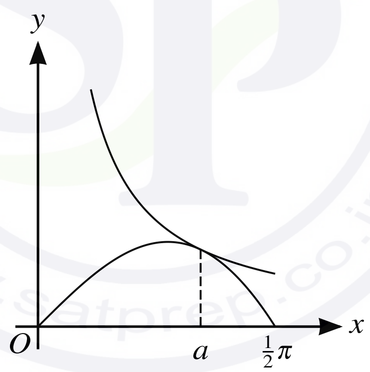
(iii) On a sketch of an Argand diagram with origin O, show the points A and B representing the complex numbers z^* and $\mathrm{i}z$ respectively. Prove that angle AOB is equal to $\frac{1}{6}\pi$. [3]

8 Let $\mathrm{f}(x) = \dfrac{3x^2 + x + 6}{(x + 2)(x^2 + 4)}$.

(i) Express $\mathrm{f}(x)$ in partial fractions. [5]

(ii) Hence obtain the expansion of $\mathrm{f}(x)$ in ascending powers of x, up to and including the term in x^2. [5]

9



The diagram shows the curves $y = x\cos x$ and $y = \dfrac{k}{x}$, where k is a constant, for $0 < x \leqslant \frac{1}{2}\pi$. The curves touch at the point where $x = a$.

(i) Show that a satisfies the equation $\tan a = \dfrac{2}{a}$. [5]

(ii) Use the iterative formula $a_{n+1} = \tan^{-1}\left(\dfrac{2}{a_n}\right)$ to determine a correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

(iii) Hence find the value of k correct to 2 decimal places. [2]

[Question 10 is printed on the next page.]

[Turn over

10 The line l has vector equation $\mathbf{r} = \mathbf{i} + 2\mathbf{j} + \mathbf{k} + \lambda(2\mathbf{i} - \mathbf{j} + \mathbf{k})$.

(i) Find the position vectors of the two points on the line whose distance from the origin is $\sqrt{(10)}$. [5]

(ii) The plane p has equation $ax + y + z = 5$, where a is a constant. The acute angle between the line l and the plane p is equal to $\sin^{-1}\left(\frac{2}{3}\right)$. Find the possible values of a. [5]

Cambridge International Examinations
Cambridge International Advanced Level

MATHEMATICS **9709/31**

Paper 3 Pure Mathematics 3 **(P3)** **May/June 2016**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

4146704902

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
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DO **NOT** WRITE IN ANY BARCODES.

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JC16 06_9709_31/RP

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[Turn over

1 **(i)** Solve the equation $2|x-1| = 3|x|$. [3]

(ii) Hence solve the equation $2|5^x - 1| = 3|5^x|$, giving your answer correct to 3 significant figures. [2]

2 Find the exact value of $\int_0^{\frac{1}{2}} x\mathrm{e}^{-2x}\,\mathrm{d}x$. [5]

3 By expressing the equation $\operatorname{cosec}\theta = 3\sin\theta + \cot\theta$ in terms of $\cos\theta$ only, solve the equation for $0° < \theta < 180°$. [5]

4 The variables x and y satisfy the differential equation

$$x\frac{\mathrm{d}y}{\mathrm{d}x} = y(1-2x^2),$$

and it is given that $y = 2$ when $x = 1$. Solve the differential equation and obtain an expression for y in terms of x in a form not involving logarithms. [6]

5 The curve with equation $y = \sin x \cos 2x$ has one stationary point in the interval $0 < x < \frac{1}{2}\pi$. Find the x-coordinate of this point, giving your answer correct to 3 significant figures. [6]

6 **(i)** By sketching a suitable pair of graphs, show that the equation

$$5\mathrm{e}^{-x} = \sqrt{x}$$

has one root. [2]

(ii) Show that, if a sequence of values given by the iterative formula

$$x_{n+1} = \frac{1}{2}\ln\left(\frac{25}{x_n}\right)$$

converges, then it converges to the root of the equation in part **(i)**. [2]

(iii) Use this iterative formula, with initial value $x_1 = 1$, to calculate the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

7 The equation of a curve is $x^3 - 3x^2y + y^3 = 3$.

(i) Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{x^2 - 2xy}{x^2 - y^2}$. [4]

(ii) Find the coordinates of the points on the curve where the tangent is parallel to the x-axis. [5]

8 Let $f(x) = \dfrac{4x^2 + 12}{(x+1)(x-3)^2}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence obtain the expansion of $f(x)$ in ascending powers of x, up to and including the term in x^2. [5]

9 With respect to the origin O, the points A, B, C, D have position vectors given by

$$\overrightarrow{OA} = \mathbf{i} + 3\mathbf{j} + 2\mathbf{k}, \quad \overrightarrow{OB} = 2\mathbf{i} + \mathbf{j} - \mathbf{k}, \quad \overrightarrow{OC} = 2\mathbf{i} + 4\mathbf{j} + \mathbf{k}, \quad \overrightarrow{OD} = -3\mathbf{i} + \mathbf{j} + 2\mathbf{k}.$$

(i) Find the equation of the plane containing A, B and C, giving your answer in the form $ax + by + cz = d$. [6]

(ii) The line through D parallel to OA meets the plane with equation $x + 2y - z = 7$ at the point P. Find the position vector of P and show that the length of DP is $2\sqrt{(14)}$. [5]

10 **(a)** Showing all your working and without the use of a calculator, find the square roots of the complex number $7 - (6\sqrt{2})\mathrm{i}$. Give your answers in the form $x + \mathrm{i}y$, where x and y are real and exact. [5]

(b) **(i)** On an Argand diagram, sketch the loci of points representing complex numbers w and z such that $|w - 1 - 2\mathrm{i}| = 1$ and $\arg(z - 1) = \frac{3}{4}\pi$. [4]

(ii) Calculate the least value of $|w - z|$ for points on these loci. [2]

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Cambridge International A Level

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Cambridge International Advanced Level

MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **(P3)** **May/June 2016**

1 hour 45 minutes

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Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

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1 Use logarithms to solve the equation $4^{3x-1} = 3(5^x)$, giving your answer correct to 3 decimal places. [4]

2 Expand $\dfrac{1}{\sqrt{(1-2x)}}$ in ascending powers of x, up to and including the term in x^3, simplifying the coefficients. [4]

3 Find the exact value of $\displaystyle\int_0^{\frac{1}{2}\pi} x^2 \sin 2x \, dx$. [5]

4 The curve with equation $y = \dfrac{(\ln x)^2}{x}$ has two stationary points. Find the exact values of the coordinates of these points. [6]

5 **(i)** Prove the identity $\cos 4\theta - 4\cos 2\theta \equiv 8\sin^4\theta - 3$. [4]

(ii) Hence solve the equation

$$\cos 4\theta = 4\cos 2\theta + 3,$$

for $0° \leqslant \theta \leqslant 360°$. [4]

6 The variables x and θ satisfy the differential equation

$$(3 + \cos 2\theta)\frac{dx}{d\theta} = x \sin 2\theta,$$

and it is given that $x = 3$ when $\theta = \frac{1}{4}\pi$.

(i) Solve the differential equation and obtain an expression for x in terms of θ. [7]

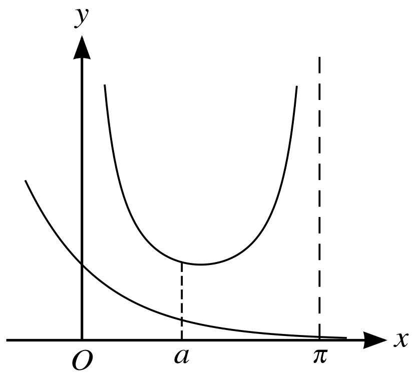
(ii) State the least value taken by x. [1]

7 Let $f(x) = \dfrac{4x^2 + 7x + 4}{(2x+1)(x+2)}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Show that $\displaystyle\int_0^4 f(x)\, dx = 8 - \ln 3$. [5]

8



The diagram shows the curve $y = \operatorname{cosec} x$ for $0 < x < \pi$ and part of the curve $y = e^{-x}$. When $x = a$, the tangents to the curves are parallel.

(i) By differentiating $\frac{1}{\sin x}$, show that if $y = \operatorname{cosec} x$ then $\frac{dy}{dx} = -\operatorname{cosec} x \cot x$. [3]

(ii) By equating the gradients of the curves at $x = a$, show that

$$a = \tan^{-1}\left(\frac{e^a}{\sin a}\right).$$ [2]

(iii) Verify by calculation that a lies between 1 and 1.5. [2]

(iv) Use an iterative formula based on the equation in part **(ii)** to determine a correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

9 The points A, B and C have position vectors, relative to the origin O, given by $\overrightarrow{OA} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$, $\overrightarrow{OB} = 4\mathbf{j} + \mathbf{k}$ and $\overrightarrow{OC} = 2\mathbf{i} + 5\mathbf{j} - \mathbf{k}$. A fourth point D is such that the quadrilateral $ABCD$ is a parallelogram.

(i) Find the position vector of D and verify that the parallelogram is a rhombus. [5]

(ii) The plane p is parallel to OA and the line BC lies in p. Find the equation of p, giving your answer in the form $ax + by + cz = d$. [5]

10 **(a)** Showing all necessary working, solve the equation $iz^2 + 2z - 3i = 0$, giving your answers in the form $x + iy$, where x and y are real and exact. [5]

(b) **(i)** On a sketch of an Argand diagram, show the locus representing complex numbers satisfying the equation $|z| = |z - 4 - 3i|$. [2]

(ii) Find the complex number represented by the point on the locus where $|z|$ is least. Find the modulus and argument of this complex number, giving the argument correct to 2 decimal places. [3]

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Cambridge International A Level

Cambridge International Examinations
Cambridge International Advanced Level

MATHEMATICS **9709/33**

Paper 3 Pure Mathematics 3 **(P3)** **May/June 2016**

1 hour 45 minutes

* 6 7 6 8 9 3 9 7 6 1 *

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

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JC16 06_9709_33/RP

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1 Solve the inequality $2|x - 2| > |3x + 1|$. [4]

2 The variables x and y satisfy the relation $3^y = 4^{2-x}$.

(i) By taking logarithms, show that the graph of y against x is a straight line. State the exact value of the gradient of this line. [3]

(ii) Calculate the exact x-coordinate of the point of intersection of this line with the line with equation $y = 2x$, simplifying your answer. [2]

3 **(i)** Express $(\sqrt{5})\cos x + 2\sin x$ in the form $R\cos(x - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$, giving the value of α correct to 2 decimal places. [3]

(ii) Hence solve the equation

$$(\sqrt{5})\cos\tfrac{1}{2}x + 2\sin\tfrac{1}{2}x = 1.2,$$

for $0° < x < 360°$. [3]

4 The parametric equations of a curve are

$$x = t + \cos t, \qquad y = \ln(1 + \sin t),$$

where $-\frac{1}{2}\pi < t < \frac{1}{2}\pi$.

(i) Show that $\dfrac{dy}{dx} = \sec t$. [5]

(ii) Hence find the x-coordinates of the points on the curve at which the gradient is equal to 3. Give your answers correct to 3 significant figures. [3]

5 The variables x and y satisfy the differential equation

$$\frac{dy}{dx} = e^{-2y}\tan^2 x,$$

for $0 \leqslant x < \frac{1}{2}\pi$, and it is given that $y = 0$ when $x = 0$. Solve the differential equation and calculate the value of y when $x = \frac{1}{4}\pi$. [8]

6 The curve with equation $y = x^2\cos\frac{1}{2}x$ has a stationary point at $x = p$ in the interval $0 < x < \pi$.

(i) Show that p satisfies the equation $\tan\frac{1}{2}p = \dfrac{4}{p}$. [3]

(ii) Verify by calculation that p lies between 2 and 2.5. [2]

(iii) Use the iterative formula $p_{n+1} = 2\tan^{-1}\left(\dfrac{4}{p_n}\right)$ to determine the value of p correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

7 Let $I = \int_0^1 \frac{x^5}{(1+x^2)^3}\,dx$.

(i) Using the substitution $u = 1 + x^2$, show that $I = \int_1^2 \frac{(u-1)^2}{2u^3}\,du$. [3]

(ii) Hence find the exact value of I. [5]

8 The points A and B have position vectors, relative to the origin O, given by $\overrightarrow{OA} = \mathbf{i} + \mathbf{j} + \mathbf{k}$ and $\overrightarrow{OB} = 2\mathbf{i} + 3\mathbf{k}$. The line l has vector equation $\mathbf{r} = 2\mathbf{i} - 2\mathbf{j} - \mathbf{k} + \mu(-\mathbf{i} + 2\mathbf{j} + \mathbf{k})$.

(i) Show that the line passing through A and B does not intersect l. [4]

(ii) Show that the length of the perpendicular from A to l is $\frac{1}{\sqrt{2}}$. [5]

9 **Throughout this question the use of a calculator is not permitted.**

The complex numbers $-1 + 3\mathrm{i}$ and $2 - \mathrm{i}$ are denoted by u and v respectively. In an Argand diagram with origin O, the points A, B and C represent the numbers u, v and $u + v$ respectively.

(i) Sketch this diagram and state fully the geometrical relationship between OB and AC. [4]

(ii) Find, in the form $x + \mathrm{i}y$, where x and y are real, the complex number $\frac{u}{v}$. [3]

(iii) Prove that angle $AOB = \frac{3}{4}\pi$. [2]

10 Let $\mathrm{f}(x) = \frac{10x - 2x^2}{(x+3)(x-1)^2}$.

(i) Express $\mathrm{f}(x)$ in partial fractions. [5]

(ii) Hence obtain the expansion of $\mathrm{f}(x)$ in ascending powers of x, up to and including the term in x^2. [5]

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Cambridge International A Level

Cambridge International Examinations
Cambridge International Advanced Level

MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **(P3)** **February/March 2016**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

0572757842

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
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Do not use staples, paper clips, glue or correction fluid.
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Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
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You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

This document consists of **3** printed pages and **1** blank page.

JC16 03_9709_32/RP

CAMBRIDGE International Examinations

1 Solve the equation $\ln(x^2 + 4) = 2\ln x + \ln 4$, giving your answer in an exact form. [3]

2 Express the equation $\tan(\theta + 45^\circ) - 2\tan(\theta - 45^\circ) = 4$ as a quadratic equation in $\tan\theta$. Hence solve this equation for $0^\circ \leqslant \theta \leqslant 180^\circ$. [6]

3 The equation $x^5 - 3x^3 + x^2 - 4 = 0$ has one positive root.

(i) Verify by calculation that this root lies between 1 and 2. [2]

(ii) Show that the equation can be rearranged in the form

$$x = \sqrt[3]{\left(3x + \frac{4}{x^2} - 1\right)}.$$ [1]

(iii) Use an iterative formula based on this rearrangement to determine the positive root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

4 The polynomial $4x^3 + ax + 2$, where a is a constant, is denoted by $\mathrm{p}(x)$. It is given that $(2x + 1)$ is a factor of $\mathrm{p}(x)$.

(i) Find the value of a. [2]

(ii) When a has this value,

(a) factorise $\mathrm{p}(x)$, [2]

(b) solve the inequality $\mathrm{p}(x) > 0$, justifying your answer. [3]

5 Let $I = \displaystyle\int_0^1 \frac{9}{(3 + x^2)^2}\,\mathrm{d}x$.

(i) Using the substitution $x = (\sqrt{3})\tan\theta$, show that $I = \sqrt{3}\displaystyle\int_0^{\frac{1}{6}\pi} \cos^2\theta\,\mathrm{d}\theta$. [3]

(ii) Hence find the exact value of I. [4]

6 A curve has equation

$$\sin y \ln x = x - 2\sin y,$$

for $-\frac{1}{2}\pi \leqslant y \leqslant \frac{1}{2}\pi$.

(i) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x and y. [5]

(ii) Hence find the exact x-coordinate of the point on the curve at which the tangent is parallel to the x-axis. [3]

7 The variables x and y satisfy the differential equation

$$\frac{dy}{dx} = xe^{x+y},$$

and it is given that $y = 0$ when $x = 0$.

(i) Solve the differential equation and obtain an expression for y in terms of x. [7]

(ii) Explain briefly why x can only take values less than 1. [1]

8 The line l has equation $\mathbf{r} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$. The plane p has equation $\mathbf{r}.\begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix} = 6$.

(i) Show that l is parallel to p. [3]

(ii) A line m lies in the plane p and is perpendicular to l. The line m passes through the point with coordinates $(5, 3, 1)$. Find a vector equation for m. [6]

9 Let $f(x) = \dfrac{3x^3 + 6x - 8}{x(x^2 + 2)}$.

(i) Express $f(x)$ in the form $A + \dfrac{B}{x} + \dfrac{Cx + D}{x^2 + 2}$. [5]

(ii) Show that $\int_1^2 f(x)\,dx = 3 - \ln 4$. [5]

10 **(a)** Find the complex number z satisfying the equation $z^* + 1 = 2iz$, where z^* denotes the complex conjugate of z. Give your answer in the form $x + iy$, where x and y are real. [5]

(b) **(i)** On a sketch of an Argand diagram, shade the region whose points represent complex numbers satisfying the inequalities $|z + 1 - 3i| \leqslant 1$ and $\text{Im}\, z \geqslant 3$, where $\text{Im}\, z$ denotes the imaginary part of z. [4]

(ii) Determine the difference between the greatest and least values of $\arg z$ for points lying in this region. [2]

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Cambridge International Examinations
Cambridge International Advanced Level

MATHEMATICS **9709/31**

Paper 3 Pure Mathematics 3 **(P3)** **October/November 2015**

1 hour 45 minutes

2390741568

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

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The total number of marks for this paper is 75.
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This document consists of **3** printed pages and **1** blank page.

JC15 11_9709_31/RP

[Turn over

1 Solve the inequality $|2x - 5| > 3|2x + 1|$. [4]

2 Using the substitution $u = 3^x$, solve the equation $3^x + 3^{2x} = 3^{3x}$ giving your answer correct to 3 significant figures. [5]

3 The angles θ and ϕ lie between 0° and 180°, and are such that

$$\tan(\theta - \phi) = 3 \quad \text{and} \quad \tan\theta + \tan\phi = 1.$$

Find the possible values of θ and ϕ. [6]

4 The equation $x^3 - x^2 - 6 = 0$ has one real root, denoted by α.

(i) Find by calculation the pair of consecutive integers between which α lies. [2]

(ii) Show that, if a sequence of values given by the iterative formula

$$x_{n+1} = \sqrt{\left(x_n + \frac{6}{x_n}\right)}$$

converges, then it converges to α. [2]

(iii) Use this iterative formula to determine α correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

5 The equation of a curve is $y = e^{-2x}\tan x$, for $0 \leqslant x < \frac{1}{2}\pi$.

(i) Obtain an expression for $\frac{dy}{dx}$ and show that it can be written in the form $e^{-2x}(a + b\tan x)^2$, where a and b are constants. [5]

(ii) Explain why the gradient of the curve is never negative. [1]

(iii) Find the value of x for which the gradient is least. [1]

6 The polynomial $8x^3 + ax^2 + bx - 1$, where a and b are constants, is denoted by $p(x)$. It is given that $(x + 1)$ is a factor of $p(x)$ and that when $p(x)$ is divided by $(2x + 1)$ the remainder is 1.

(i) Find the values of a and b. [5]

(ii) When a and b have these values, factorise $p(x)$ completely. [3]

7 The points A, B and C have position vectors, relative to the origin O, given by

$$\overrightarrow{OA} = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} 1 \\ 1 \\ 4 \end{pmatrix}.$$

The plane m is perpendicular to AB and contains the point C.

(i) Find a vector equation for the line passing through A and B. [2]

(ii) Obtain the equation of the plane m, giving your answer in the form $ax + by + cz = d$. [2]

(iii) The line through A and B intersects the plane m at the point N. Find the position vector of N and show that $CN = \sqrt{(13)}$. [5]

8 The variables x and θ satisfy the differential equation

$$\frac{dx}{d\theta} = (x+2)\sin^2 2\theta,$$

and it is given that $x = 0$ when $\theta = 0$. Solve the differential equation and calculate the value of x when $\theta = \frac{1}{4}\pi$, giving your answer correct to 3 significant figures. [9]

9 The complex number $3 - \mathrm{i}$ is denoted by u. Its complex conjugate is denoted by u^*.

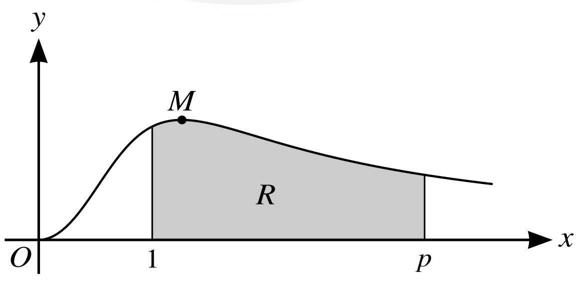
(i) On an Argand diagram with origin O, show the points A, B and C representing the complex numbers u, u^* and $u^* - u$ respectively. What type of quadrilateral is $OABC$? [4]

(ii) Showing your working and without using a calculator, express $\frac{u^*}{u}$ in the form $x + \mathrm{i}y$, where x and y are real. [3]

(iii) By considering the argument of $\frac{u^*}{u}$, prove that

$$\tan^{-1}\left(\tfrac{3}{4}\right) = 2\tan^{-1}\left(\tfrac{1}{3}\right).$$ [3]

10



The diagram shows the curve $y = \frac{x^2}{1+x^3}$ for $x \geqslant 0$, and its maximum point M. The shaded region R is enclosed by the curve, the x-axis and the lines $x = 1$ and $x = p$.

(i) Find the exact value of the x-coordinate of M. [4]

(ii) Calculate the value of p for which the area of R is equal to 1. Give your answer correct to 3 significant figures. [6]

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Cambridge International Examinations
Cambridge International Advanced Level

MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **(P3)** **October/November 2015**

1 hour 45 minutes

0107107745

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

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This document consists of **3** printed pages and **1** blank page.

JC15 11_9709_32/FP

[Turn over

1 Solve the inequality $|2x - 5| > 3|2x + 1|$. [4]

2 Using the substitution $u = 3^x$, solve the equation $3^x + 3^{2x} = 3^{3x}$ giving your answer correct to 3 significant figures. [5]

3 The angles θ and ϕ lie between 0° and 180°, and are such that

$$\tan(\theta - \phi) = 3 \quad \text{and} \quad \tan\theta + \tan\phi = 1.$$

Find the possible values of θ and ϕ. [6]

4 The equation $x^3 - x^2 - 6 = 0$ has one real root, denoted by α.

(i) Find by calculation the pair of consecutive integers between which α lies. [2]

(ii) Show that, if a sequence of values given by the iterative formula

$$x_{n+1} = \sqrt{\left(x_n + \frac{6}{x_n}\right)}$$

converges, then it converges to α. [2]

(iii) Use this iterative formula to determine α correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

5 The equation of a curve is $y = e^{-2x}\tan x$, for $0 \leqslant x < \frac{1}{2}\pi$.

(i) Obtain an expression for $\frac{dy}{dx}$ and show that it can be written in the form $e^{-2x}(a + b\tan x)^2$, where a and b are constants. [5]

(ii) Explain why the gradient of the curve is never negative. [1]

(iii) Find the value of x for which the gradient is least. [1]

6 The polynomial $8x^3 + ax^2 + bx - 1$, where a and b are constants, is denoted by $p(x)$. It is given that $(x + 1)$ is a factor of $p(x)$ and that when $p(x)$ is divided by $(2x + 1)$ the remainder is 1.

(i) Find the values of a and b. [5]

(ii) When a and b have these values, factorise $p(x)$ completely. [3]

7 The points A, B and C have position vectors, relative to the origin O, given by

$$\overrightarrow{OA} = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} 1 \\ 1 \\ 4 \end{pmatrix}.$$

The plane m is perpendicular to AB and contains the point C.

(i) Find a vector equation for the line passing through A and B. [2]

(ii) Obtain the equation of the plane m, giving your answer in the form $ax + by + cz = d$. [2]

(iii) The line through A and B intersects the plane m at the point N. Find the position vector of N and show that $CN = \sqrt{(13)}$. [5]

8 The variables x and θ satisfy the differential equation

$$\frac{dx}{d\theta} = (x + 2)\sin^2 2\theta,$$

and it is given that $x = 0$ when $\theta = 0$. Solve the differential equation and calculate the value of x when $\theta = \frac{1}{4}\pi$, giving your answer correct to 3 significant figures. [9]

9 The complex number $3 - i$ is denoted by u. Its complex conjugate is denoted by u^*.

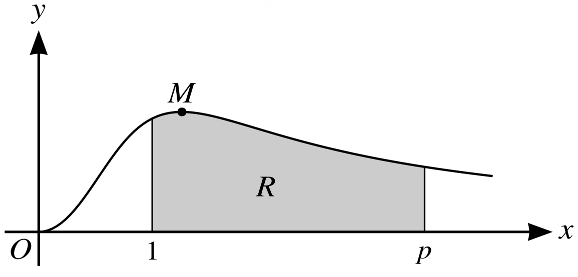
(i) On an Argand diagram with origin O, show the points A, B and C representing the complex numbers u, u^* and $u^* - u$ respectively. What type of quadrilateral is $OABC$? [4]

(ii) Showing your working and without using a calculator, express $\frac{u^*}{u}$ in the form $x + iy$, where x and y are real. [3]

(iii) By considering the argument of $\frac{u^*}{u}$, prove that

$$\tan^{-1}\left(\tfrac{3}{4}\right) = 2\tan^{-1}\left(\tfrac{1}{3}\right).$$ [3]

10



The diagram shows the curve $y = \frac{x^2}{1 + x^3}$ for $x \geqslant 0$, and its maximum point M. The shaded region R is enclosed by the curve, the x-axis and the lines $x = 1$ and $x = p$.

(i) Find the exact value of the x-coordinate of M. [4]

(ii) Calculate the value of p for which the area of R is equal to 1. Give your answer correct to 3 significant figures. [6]

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Cambridge International A Level

Cambridge International Examinations
Cambridge International Advanced Level

MATHEMATICS **9709/33**

Paper 3 Pure Mathematics 3 **(P3)** **October/November 2015**

1 hour 45 minutes

8940457897

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

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JC15 11_9709_33/RP

CAMBRIDGE
International Examinations

1 Sketch the graph of $y = e^{ax} - 1$ where a is a positive constant. [2]

2 Given that $\sqrt[3]{(1 + 9x)} \approx 1 + 3x + ax^2 + bx^3$ for small values of x, find the values of the coefficients a and b. [3]

3 A curve has equation

$$y = \frac{2 - \tan x}{1 + \tan x}.$$

Find the equation of the tangent to the curve at the point for which $x = \frac{1}{4}\pi$, giving the answer in the form $y = mx + c$ where c is correct to 3 significant figures. [6]

4 A curve has parametric equations

$$x = t^2 + 3t + 1, \qquad y = t^4 + 1.$$

The point P on the curve has parameter p. It is given that the gradient of the curve at P is 4.

(i) Show that $p = \sqrt[3]{(2p + 3)}$. [3]

(ii) Verify by calculation that the value of p lies between 1.8 and 2.0. [2]

(iii) Use an iterative formula based on the equation in part **(i)** to find the value of p correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

5 Use the substitution $u = 4 - 3\cos x$ to find the exact value of $\displaystyle\int_0^{\frac{1}{2}\pi} \frac{9\sin 2x}{\sqrt{(4 - 3\cos x)}}\,dx$. [8]

6 The angles A and B are such that

$$\sin(A + 45°) = (2\sqrt{2})\cos A \quad \text{and} \quad 4\sec^2 B + 5 = 12\tan B.$$

Without using a calculator, find the exact value of $\tan(A - B)$. [8]

7 **(i)** Show that $(x + 1)$ is a factor of $4x^3 - x^2 - 11x - 6$. [2]

(ii) Find $\displaystyle\int \frac{4x^2 + 9x - 1}{4x^3 - x^2 - 11x - 6}\,dx$. [8]

8 A plane has equation $4x - y + 5z = 39$. A straight line is parallel to the vector $\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}$ and passes through the point $A\,(0, 2, -8)$. The line meets the plane at the point B.

(i) Find the coordinates of B. [3]

(ii) Find the acute angle between the line and the plane. [4]

(iii) The point C lies on the line and is such that the distance between C and B is twice the distance between A and B. Find the coordinates of each of the possible positions of the point C. [3]

9 **(a)** It is given that $(1 + 3\text{i})w = 2 + 4\text{i}$. Showing all necessary working, prove that the exact value of $|w^2|$ is 2 and find $\arg(w^2)$ correct to 3 significant figures. [6]

(b) On a single Argand diagram sketch the loci $|z| = 5$ and $|z - 5| = |z|$. Hence determine the complex numbers represented by points common to both loci, giving each answer in the form $re^{\text{i}\theta}$. [4]

10 Naturalists are managing a wildlife reserve to increase the number of plants of a rare species. The number of plants at time t years is denoted by N, where N is treated as a continuous variable.

(i) It is given that the rate of increase of N with respect to t is proportional to $(N - 150)$. Write down a differential equation relating N, t and a constant of proportionality. [1]

(ii) Initially, when $t = 0$, the number of plants was 650. It was noted that, at a time when there were 900 plants, the number of plants was increasing at a rate of 60 per year. Express N in terms of t. [7]

(iii) The naturalists had a target of increasing the number of plants from 650 to 2000 within 15 years. Will this target be met? [2]

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Cambridge International A Level

Cambridge International Examinations
Cambridge International Advanced Level

MATHEMATICS **9709/31**

Paper 3 Pure Mathematics 3 **(P3)** **May/June 2015**

1 hour 45 minutes

8254149865

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

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JC15 06_9709_31/2R

CAMBRIDGE International Examinations

1 Use logarithms to solve the equation $2^{5x} = 3^{2x+1}$, giving the answer correct to 3 significant figures. [4]

2 Use the trapezium rule with three intervals to find an approximation to

$$\int_0^3 |3^x - 10| \, dx.$$ [4]

3 Show that, for small values of x^2,

$$\left(1 - 2x^2\right)^{-2} - \left(1 + 6x^2\right)^{\frac{2}{3}} \approx kx^4,$$

where the value of the constant k is to be determined. [6]

4 The equation of a curve is

$$y = 3\cos 2x + 7\sin x + 2.$$

Find the x-coordinates of the stationary points in the interval $0 \leqslant x \leqslant \pi$. Give each answer correct to 3 significant figures. [7]

5 **(a)** Find $\int (4 + \tan^2 2x) \, dx$. [3]

(b) Find the exact value of $\displaystyle\int_{\frac{1}{4}\pi}^{\frac{1}{2}\pi} \frac{\sin\left(x + \frac{1}{6}\pi\right)}{\sin x} \, dx$. [5]

6 The straight line l_1 passes through the points $(0, 1, 5)$ and $(2, -2, 1)$. The straight line l_2 has equation $\mathbf{r} = 7\mathbf{i} + \mathbf{j} + \mathbf{k} + \mu(\mathbf{i} + 2\mathbf{j} + 5\mathbf{k})$.

(i) Show that the lines l_1 and l_2 are skew. [6]

(ii) Find the acute angle between the direction of the line l_2 and the direction of the x-axis. [3]

7 Given that $y = 1$ when $x = 0$, solve the differential equation

$$\frac{dy}{dx} = 4x(3y^2 + 10y + 3),$$

obtaining an expression for y in terms of x. [9]

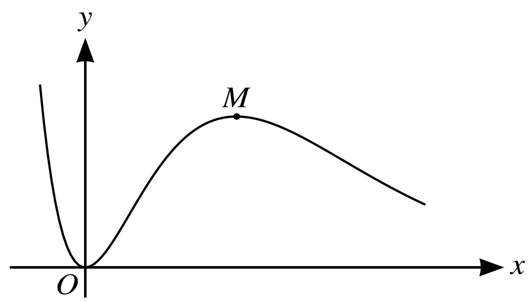
8 The complex number w is defined by $w = \dfrac{22 + 4i}{(2 - i)^2}$.

(i) Without using a calculator, show that $w = 2 + 4i$. [3]

(ii) It is given that p is a real number such that $\frac{1}{4}\pi \leqslant \arg(w + p) \leqslant \frac{3}{4}\pi$. Find the set of possible values of p. [3]

(iii) The complex conjugate of w is denoted by w^*. The complex numbers w and w^* are represented in an Argand diagram by the points S and T respectively. Find, in the form $|z - a| = k$, the equation of the circle passing through S, T and the origin. [3]

9

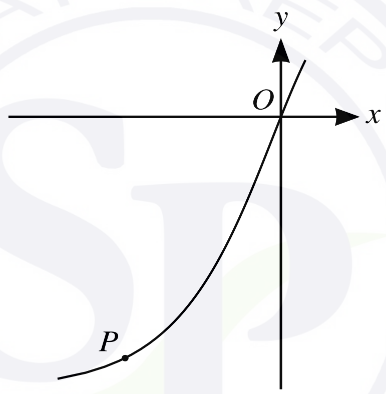


The diagram shows the curve $y = x^2 e^{2-x}$ and its maximum point M.

(i) Show that the x-coordinate of M is 2. [3]

(ii) Find the exact value of $\int_0^2 x^2 e^{2-x}\,dx$. [6]

10



The diagram shows part of the curve with parametric equations

$$x = 2\ln(t + 2), \qquad y = t^3 + 2t + 3.$$

(i) Find the gradient of the curve at the origin. [5]

(ii) At the point P on the curve, the value of the parameter is p. It is given that the gradient of the curve at P is $\frac{1}{2}$.

(a) Show that $p = \dfrac{1}{3p^2 + 2} - 2$. [1]

(b) By first using an iterative formula based on the equation in part **(a)**, determine the coordinates of the point P. Give the result of each iteration to 5 decimal places and each coordinate of P correct to 2 decimal places. [4]

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Cambridge International A Level

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MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **(P3)** **May/June 2015**

1 hour 45 minutes

2399507033

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

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JC15 06_9709_32/RP

CAMBRIDGE International Examinations

1 Use the trapezium rule with three intervals to estimate the value of

$$\int_0^{\frac{1}{2}\pi} \ln(1 + \sin x)\, dx,$$

giving your answer correct to 2 decimal places. [3]

2 Using the substitution $u = 4^x$, solve the equation $4^x + 4^2 = 4^{x+2}$, giving your answer correct to 3 significant figures. [4]

3 A curve has equation $y = \cos x \cos 2x$. Find the x-coordinate of the stationary point on the curve in the interval $0 < x < \frac{1}{2}\pi$, giving your answer correct to 3 significant figures. [6]

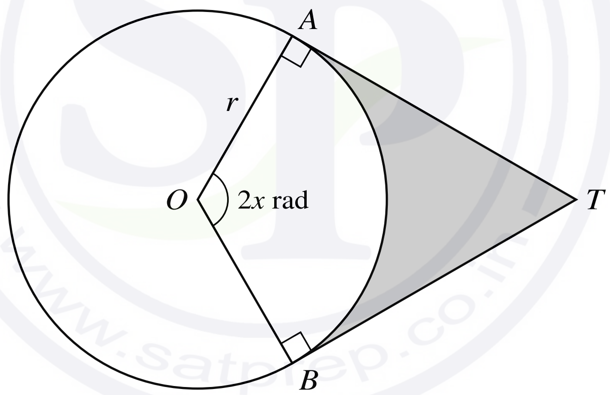
4 **(i)** Express $3\sin\theta + 2\cos\theta$ in the form $R\sin(\theta + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$, stating the exact value of R and giving the value of α correct to 2 decimal places. [3]

(ii) Hence solve the equation

$$3\sin\theta + 2\cos\theta = 1,$$

for $0° < \theta < 180°$. [3]

5



The diagram shows a circle with centre O and radius r. The tangents to the circle at the points A and B meet at T, and the angle AOB is $2x$ radians. The shaded region is bounded by the tangents AT and BT, and by the minor arc AB. The perimeter of the shaded region is equal to the circumference of the circle.

(i) Show that x satisfies the equation

$$\tan x = \pi - x.$$ [3]

(ii) This equation has one root in the interval $0 < x < \frac{1}{2}\pi$. Verify by calculation that this root lies between 1 and 1.3. [2]

(iii) Use the iterative formula

$$x_{n+1} = \tan^{-1}(\pi - x_n)$$

to determine the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

6 Let $I = \int_0^1 \frac{\sqrt{x}}{2 - \sqrt{x}} \, dx$.

(i) Using the substitution $u = 2 - \sqrt{x}$, show that $I = \int_1^2 \frac{2(2-u)^2}{u} \, du$. [4]

(ii) Hence show that $I = 8 \ln 2 - 5$. [4]

7 The complex number u is given by $u = -1 + (4\sqrt{3})i$.

(i) Without using a calculator and showing all your working, find the two square roots of u. Give your answers in the form $a + ib$, where the real numbers a and b are exact. [5]

(ii) On an Argand diagram, sketch the locus of points representing complex numbers z satisfying the relation $|z - u| = 1$. Determine the greatest value of $\arg z$ for points on this locus. [4]

8 Let $f(x) = \frac{5x^2 + x + 6}{(3 - 2x)(x^2 + 4)}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence obtain the expansion of $f(x)$ in ascending powers of x, up to and including the term in x^2. [5]

9 The number of organisms in a population at time t is denoted by x. Treating x as a continuous variable, the differential equation satisfied by x and t is

$$\frac{dx}{dt} = \frac{xe^{-t}}{k + e^{-t}},$$

where k is a positive constant.

(i) Given that $x = 10$ when $t = 0$, solve the differential equation, obtaining a relation between x, k and t. [6]

(ii) Given also that $x = 20$ when $t = 1$, show that $k = 1 - \frac{2}{e}$. [2]

(iii) Show that the number of organisms never reaches 48, however large t becomes. [2]

10 The points A and B have position vectors given by $\overrightarrow{OA} = 2\mathbf{i} - \mathbf{j} + 3\mathbf{k}$ and $\overrightarrow{OB} = \mathbf{i} + \mathbf{j} + 5\mathbf{k}$. The line l has equation $\mathbf{r} = \mathbf{i} + \mathbf{j} + 2\mathbf{k} + \mu(3\mathbf{i} + \mathbf{j} - \mathbf{k})$.

(i) Show that l does not intersect the line passing through A and B. [5]

(ii) Find the equation of the plane containing the line l and the point A. Give your answer in the form $ax + by + cz = d$. [6]

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Cambridge International Examinations
Cambridge International Advanced Level

MATHEMATICS **9709/33**

Paper 3 Pure Mathematics 3 **(P3)** **May/June 2015**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

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JC15 06_9709_33/RP

1 Solve the equation $\ln(x+4) = 2\ln x + \ln 4$, giving your answer correct to 3 significant figures. [4]

2 Solve the inequality $|x-2| > 2x-3$. [4]

3 Solve the equation $\cot 2x + \cot x = 3$ for $0° < x < 180°$. [6]

4 The curve with equation $y = \dfrac{e^{2x}}{4+e^{3x}}$ has one stationary point. Find the exact values of the coordinates of this point. [6]

5 The parametric equations of a curve are

$$x = a\cos^4 t, \quad y = a\sin^4 t,$$

where a is a positive constant.

(i) Express $\dfrac{dy}{dx}$ in terms of t. [3]

(ii) Show that the equation of the tangent to the curve at the point with parameter t is

$$x\sin^2 t + y\cos^2 t = a\sin^2 t\cos^2 t.$$ [3]

(iii) Hence show that if the tangent meets the x-axis at P and the y-axis at Q, then

$$OP + OQ = a,$$

where O is the origin. [2]

6 It is given that $\int_0^a x\cos x\,dx = 0.5$, where $0 < a < \frac{1}{2}\pi$.

(i) Show that a satisfies the equation $\sin a = \dfrac{1.5-\cos a}{a}$. [4]

(ii) Verify by calculation that a is greater than 1. [2]

(iii) Use the iterative formula

$$a_{n+1} = \sin^{-1}\left(\frac{1.5-\cos a_n}{a_n}\right)$$

to determine the value of a correct to 4 decimal places, giving the result of each iteration to 6 decimal places. [3]

7 The number of micro-organisms in a population at time t is denoted by M. At any time the variation in M is assumed to satisfy the differential equation

$$\frac{dM}{dt} = k(\sqrt{M})\cos(0.02t),$$

where k is a constant and M is taken to be a continuous variable. It is given that when $t = 0$, $M = 100$.

(i) Solve the differential equation, obtaining a relation between M, k and t. [5]

(ii) Given also that $M = 196$ when $t = 50$, find the value of k. [2]

(iii) Obtain an expression for M in terms of t and find the least possible number of micro-organisms. [2]

8 The complex number $1 - i$ is denoted by u.

(i) Showing your working and without using a calculator, express

$$\frac{i}{u}$$

in the form $x + iy$, where x and y are real. [2]

(ii) On an Argand diagram, sketch the loci representing complex numbers z satisfying the equations $|z - u| = |z|$ and $|z - i| = 2$. [4]

(iii) Find the argument of each of the complex numbers represented by the points of intersection of the two loci in part **(ii)**. [3]

9 Two planes have equations $x + 3y - 2z = 4$ and $2x + y + 3z = 5$. The planes intersect in the straight line l.

(i) Calculate the acute angle between the two planes. [4]

(ii) Find a vector equation for the line l. [6]

10 Let $f(x) = \dfrac{11x + 7}{(2x - 1)(x + 2)^2}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Show that $\int_1^2 f(x)\,dx = \frac{1}{4} + \ln\left(\frac{9}{4}\right)$. [5]

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Cambridge International Examinations
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MATHEMATICS **9709/31**

Paper 3 Pure Mathematics 3 **(P3)** **October/November 2014**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

0962340356

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JC14 11_9709_31/FP

CAMBRIDGE International Examinations

1 Use logarithms to solve the equation $e^x = 3^{x-2}$, giving your answer correct to 3 decimal places. [3]

2 **(i)** Use the trapezium rule with 3 intervals to estimate the value of

$$\int_{\frac{1}{6}\pi}^{\frac{2}{3}\pi} \operatorname{cosec} x \, dx,$$

giving your answer correct to 2 decimal places. [3]

(ii) Using a sketch of the graph of $y = \operatorname{cosec} x$, explain whether the trapezium rule gives an overestimate or an underestimate of the true value of the integral in part **(i)**. [2]

3 The polynomial $ax^3 + bx^2 + x + 3$, where a and b are constants, is denoted by $p(x)$. It is given that $(3x + 1)$ is a factor of $p(x)$, and that when $p(x)$ is divided by $(x - 2)$ the remainder is 21. Find the values of a and b. [5]

4 The parametric equations of a curve are

$$x = \frac{1}{\cos^3 t}, \quad y = \tan^3 t,$$

where $0 \leqslant t < \frac{1}{2}\pi$.

(i) Show that $\frac{dy}{dx} = \sin t$. [4]

(ii) Hence show that the equation of the tangent to the curve at the point with parameter t is $y = x \sin t - \tan t$. [3]

5 **Throughout this question the use of a calculator is not permitted.**

The complex numbers w and z satisfy the relation

$$w = \frac{z + i}{iz + 2}.$$

(i) Given that $z = 1 + i$, find w, giving your answer in the form $x + iy$, where x and y are real. [4]

(ii) Given instead that $w = z$ and the real part of z is negative, find z, giving your answer in the form $x + iy$, where x and y are real. [4]

6 It is given that $\int_1^a \ln(2x)\,dx = 1$, where $a > 1$.

(i) Show that $a = \frac{1}{2}\exp\left(1 + \frac{\ln 2}{a}\right)$, where $\exp(x)$ denotes e^x. [6]

(ii) Use the iterative formula

$$a_{n+1} = \frac{1}{2}\exp\left(1 + \frac{\ln 2}{a_n}\right)$$

to determine the value of a correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

7 In a certain country the government charges tax on each litre of petrol sold to motorists. The revenue per year is R million dollars when the rate of tax is x dollars per litre. The variation of R with x is modelled by the differential equation

$$\frac{dR}{dx} = R\left(\frac{1}{x} - 0.57\right),$$

where R and x are taken to be continuous variables. When $x = 0.5$, $R = 16.8$.

(i) Solve the differential equation and obtain an expression for R in terms of x. [6]

(ii) This model predicts that R cannot exceed a certain amount. Find this maximum value of R. [3]

8 **(i)** By first expanding $\sin(2\theta + \theta)$, show that

$$\sin 3\theta = 3\sin\theta - 4\sin^3\theta. \quad [4]$$

(ii) Show that, after making the substitution $x = \frac{2\sin\theta}{\sqrt{3}}$, the equation $x^3 - x + \frac{1}{6}\sqrt{3} = 0$ can be written in the form $\sin 3\theta = \frac{3}{4}$. [1]

(iii) Hence solve the equation

$$x^3 - x + \frac{1}{6}\sqrt{3} = 0,$$

giving your answers correct to 3 significant figures. [4]

9 Let $f(x) = \frac{x^2 - 8x + 9}{(1-x)(2-x)^2}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence obtain the expansion of $f(x)$ in ascending powers of x, up to and including the term in x^2. [5]

10 The line l has equation $\mathbf{r} = 4\mathbf{i} - 9\mathbf{j} + 9\mathbf{k} + \lambda(-2\mathbf{i} + \mathbf{j} - 2\mathbf{k})$. The point A has position vector $3\mathbf{i} + 8\mathbf{j} + 5\mathbf{k}$.

(i) Show that the length of the perpendicular from A to l is 15. [5]

(ii) The line l lies in the plane with equation $ax + by - 3z + 1 = 0$, where a and b are constants. Find the values of a and b. [5]

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Cambridge International A Level

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Cambridge International Advanced Level

MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **(P3)** **October/November 2014**

1 hour 45 minutes

9231113471

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

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CAMBRIDGE International Examinations

[Turn over

1 Use logarithms to solve the equation $e^x = 3^{x-2}$, giving your answer correct to 3 decimal places. [3]

2 **(i)** Use the trapezium rule with 3 intervals to estimate the value of

$$\int_{\frac{1}{6}\pi}^{\frac{2}{3}\pi} \operatorname{cosec} x \, dx,$$

giving your answer correct to 2 decimal places. [3]

(ii) Using a sketch of the graph of $y = \operatorname{cosec} x$, explain whether the trapezium rule gives an overestimate or an underestimate of the true value of the integral in part **(i)**. [2]

3 The polynomial $ax^3 + bx^2 + x + 3$, where a and b are constants, is denoted by $p(x)$. It is given that $(3x + 1)$ is a factor of $p(x)$, and that when $p(x)$ is divided by $(x - 2)$ the remainder is 21. Find the values of a and b. [5]

4 The parametric equations of a curve are

$$x = \frac{1}{\cos^3 t}, \quad y = \tan^3 t,$$

where $0 \leqslant t < \frac{1}{2}\pi$.

(i) Show that $\frac{dy}{dx} = \sin t$. [4]

(ii) Hence show that the equation of the tangent to the curve at the point with parameter t is $y = x \sin t - \tan t$. [3]

5 **Throughout this question the use of a calculator is not permitted.**

The complex numbers w and z satisfy the relation

$$w = \frac{z + i}{iz + 2}.$$

(i) Given that $z = 1 + i$, find w, giving your answer in the form $x + iy$, where x and y are real. [4]

(ii) Given instead that $w = z$ and the real part of z is negative, find z, giving your answer in the form $x + iy$, where x and y are real. [4]

6 It is given that $\int_1^a \ln(2x)\,\mathrm{d}x = 1$, where $a > 1$.

(i) Show that $a = \frac{1}{2}\exp\left(1 + \frac{\ln 2}{a}\right)$, where $\exp(x)$ denotes e^x. [6]

(ii) Use the iterative formula

$$a_{n+1} = \frac{1}{2}\exp\left(1 + \frac{\ln 2}{a_n}\right)$$

to determine the value of a correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

7 In a certain country the government charges tax on each litre of petrol sold to motorists. The revenue per year is R million dollars when the rate of tax is x dollars per litre. The variation of R with x is modelled by the differential equation

$$\frac{\mathrm{d}R}{\mathrm{d}x} = R\left(\frac{1}{x} - 0.57\right),$$

where R and x are taken to be continuous variables. When $x = 0.5$, $R = 16.8$.

(i) Solve the differential equation and obtain an expression for R in terms of x. [6]

(ii) This model predicts that R cannot exceed a certain amount. Find this maximum value of R. [3]

8 **(i)** By first expanding $\sin(2\theta + \theta)$, show that

$$\sin 3\theta = 3\sin\theta - 4\sin^3\theta.$$ [4]

(ii) Show that, after making the substitution $x = \frac{2\sin\theta}{\sqrt{3}}$, the equation $x^3 - x + \frac{1}{6}\sqrt{3} = 0$ can be written in the form $\sin 3\theta = \frac{3}{4}$. [1]

(iii) Hence solve the equation

$$x^3 - x + \frac{1}{6}\sqrt{3} = 0,$$

giving your answers correct to 3 significant figures. [4]

9 Let $\mathrm{f}(x) = \frac{x^2 - 8x + 9}{(1-x)(2-x)^2}$.

(i) Express $\mathrm{f}(x)$ in partial fractions. [5]

(ii) Hence obtain the expansion of $\mathrm{f}(x)$ in ascending powers of x, up to and including the term in x^2. [5]

10 The line l has equation $\mathbf{r} = 4\mathbf{i} - 9\mathbf{j} + 9\mathbf{k} + \lambda(-2\mathbf{i} + \mathbf{j} - 2\mathbf{k})$. The point A has position vector $3\mathbf{i} + 8\mathbf{j} + 5\mathbf{k}$.

(i) Show that the length of the perpendicular from A to l is 15. [5]

(ii) The line l lies in the plane with equation $ax + by - 3z + 1 = 0$, where a and b are constants. Find the values of a and b. [5]

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Cambridge International A Level

Cambridge International Examinations
Cambridge International Advanced Level

MATHEMATICS **9709/33**

Paper 3 Pure Mathematics 3 **(P3)** **October/November 2014**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

8698806885

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JC14 11_9709_33/2R

CAMBRIDGE International Examinations

[Turn over

1 Solve the inequality $|3x - 1| < |2x + 5|$. [4]

2 A curve is defined for $0 < \theta < \frac{1}{2}\pi$ by the parametric equations

$$x = \tan\theta, \quad y = 2\cos^2\theta\sin\theta.$$

Show that $\dfrac{dy}{dx} = 6\cos^5\theta - 4\cos^3\theta$. [5]

3 The polynomial $4x^3 + ax^2 + bx - 2$, where a and b are constants, is denoted by $p(x)$. It is given that $(x + 1)$ and $(x + 2)$ are factors of $p(x)$.

(i) Find the values of a and b. [4]

(ii) When a and b have these values, find the remainder when $p(x)$ is divided by $(x^2 + 1)$. [3]

4 **(i)** Show that $\cos(\theta - 60°) + \cos(\theta + 60°) \equiv \cos\theta$. [3]

(ii) Given that $\dfrac{\cos(2x - 60°) + \cos(2x + 60°)}{\cos(x - 60°) + \cos(x + 60°)} = 3$, find the exact value of $\cos x$. [4]

5 The complex numbers w and z are defined by $w = 5 + 3i$ and $z = 4 + i$.

(i) Express $\dfrac{iw}{z}$ in the form $x + iy$, showing all your working and giving the exact values of x and y. [3]

(ii) Find wz and hence, by considering arguments, show that

$$\tan^{-1}\left(\tfrac{3}{5}\right) + \tan^{-1}\left(\tfrac{1}{4}\right) = \tfrac{1}{4}\pi.$$ [4]

6 It is given that $I = \displaystyle\int_0^{0.3} \left(1 + 3x^2\right)^{-2} dx$.

(i) Use the trapezium rule with 3 intervals to find an approximation to I, giving the answer correct to 3 decimal places. [3]

(ii) For small values of x, $\left(1 + 3x^2\right)^{-2} \approx 1 + ax^2 + bx^4$. Find the values of the constants a and b. Hence, by evaluating $\displaystyle\int_0^{0.3} (1 + ax^2 + bx^4)\, dx$, find a second approximation to I, giving the answer correct to 3 decimal places. [5]

7 The equations of two straight lines are

$$\mathbf{r} = \mathbf{i} + 4\mathbf{j} - 2\mathbf{k} + \lambda(\mathbf{i} + 3\mathbf{k}) \quad \text{and} \quad \mathbf{r} = a\mathbf{i} + 2\mathbf{j} - 2\mathbf{k} + \mu(\mathbf{i} + 2\mathbf{j} + 3a\mathbf{k}),$$

where a is a constant.

(i) Show that the lines intersect for all values of a. [4]

(ii) Given that the point of intersection is at a distance of 9 units from the origin, find the possible values of a. [4]

8 The variables x and y are related by the differential equation

$$\frac{dy}{dx} = \tfrac{1}{5}xy^{\frac{1}{2}}\sin\left(\tfrac{1}{3}x\right).$$

(i) Find the general solution, giving y in terms of x. [6]

(ii) Given that $y = 100$ when $x = 0$, find the value of y when $x = 25$. [3]

9 **(i)** Sketch the curve $y = \ln(x + 1)$ and hence, by sketching a second curve, show that the equation

$$x^3 + \ln(x + 1) = 40$$

has exactly one real root. State the equation of the second curve. [3]

(ii) Verify by calculation that the root lies between 3 and 4. [2]

(iii) Use the iterative formula

$$x_{n+1} = \sqrt[3]{(40 - \ln(x_n + 1))},$$

with a suitable starting value, to find the root correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

(iv) Deduce the root of the equation

$$(e^y - 1)^3 + y = 40,$$

giving the answer correct to 2 decimal places. [2]

10 By first using the substitution $u = e^x$, show that

$$\int_0^{\ln 4} \frac{e^{2x}}{e^{2x} + 3e^x + 2}\, dx = \ln\left(\tfrac{8}{5}\right).$$ [10]

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Cambridge International A Level

Cambridge International Examinations
Cambridge International Advanced Level

MATHEMATICS **9709/31**

Paper 3 Pure Mathematics 3 **(P3)** **May/June 2014**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

5374638582

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JC14 06_9709_31/RP

CAMBRIDGE International Examinations

[Turn over

1 **(i)** Simplify $\sin 2\alpha \sec \alpha$. [2]

(ii) Given that $3\cos 2\beta + 7\cos\beta = 0$, find the exact value of $\cos\beta$. [3]

2 Use the substitution $u = 1 + 3\tan x$ to find the exact value of

$$\int_0^{\frac{1}{4}\pi} \frac{\sqrt{(1 + 3\tan x)}}{\cos^2 x}\,\mathrm{d}x.$$ [5]

3 The parametric equations of a curve are

$$x = \ln(2t + 3), \quad y = \frac{3t + 2}{2t + 3}.$$

Find the gradient of the curve at the point where it crosses the y-axis. [6]

4 The variables x and y are related by the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{6y\mathrm{e}^{3x}}{2 + \mathrm{e}^{3x}}.$$

Given that $y = 36$ when $x = 0$, find an expression for y in terms of x. [6]

5 The complex number z is defined by $z = \dfrac{9\sqrt{3} + 9\mathrm{i}}{\sqrt{3} - \mathrm{i}}$. Find, showing all your working,

(i) an expression for z in the form $r\mathrm{e}^{\mathrm{i}\theta}$, where $r > 0$ and $-\pi < \theta \leqslant \pi$, [5]

(ii) the two square roots of z, giving your answers in the form $r\mathrm{e}^{\mathrm{i}\theta}$, where $r > 0$ and $-\pi < \theta \leqslant \pi$. [3]

6 It is given that $2\ln(4x - 5) + \ln(x + 1) = 3\ln 3$.

(i) Show that $16x^3 - 24x^2 - 15x - 2 = 0$. [3]

(ii) By first using the factor theorem, factorise $16x^3 - 24x^2 - 15x - 2$ completely. [4]

(iii) Hence solve the equation $2\ln(4x - 5) + \ln(x + 1) = 3\ln 3$. [1]

7 The straight line l has equation $\mathbf{r} = 4\mathbf{i} - \mathbf{j} + 2\mathbf{k} + \lambda(2\mathbf{i} - 3\mathbf{j} + 6\mathbf{k})$. The plane p passes through the point $(4, -1, 2)$ and is perpendicular to l.

(i) Find the equation of p, giving your answer in the form $ax + by + cz = d$. [2]

(ii) Find the perpendicular distance from the origin to p. [3]

(iii) A second plane q is parallel to p and the perpendicular distance between p and q is 14 units. Find the possible equations of q. [3]

8 **(i)** By sketching each of the graphs $y = \operatorname{cosec} x$ and $y = x(\pi - x)$ for $0 < x < \pi$, show that the equation

$$\operatorname{cosec} x = x(\pi - x)$$

has exactly two real roots in the interval $0 < x < \pi$. [3]

(ii) Show that the equation $\operatorname{cosec} x = x(\pi - x)$ can be written in the form $x = \dfrac{1 + x^2 \sin x}{\pi \sin x}$. [2]

(iii) The two real roots of the equation $\operatorname{cosec} x = x(\pi - x)$ in the interval $0 < x < \pi$ are denoted by α and β, where $\alpha < \beta$.

(a) Use the iterative formula

$$x_{n+1} = \frac{1 + x_n^2 \sin x_n}{\pi \sin x_n}$$

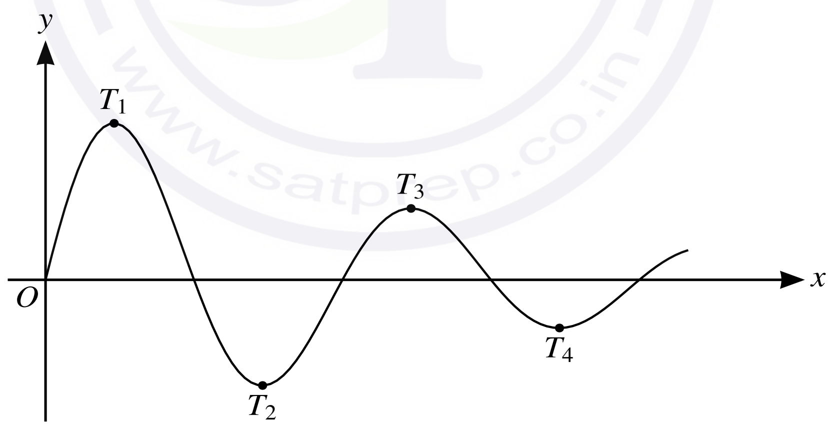
to find α correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

(b) Deduce the value of β correct to 2 decimal places. [1]

9 **(i)** Express $\dfrac{4 + 12x + x^2}{(3 - x)(1 + 2x)^2}$ in partial fractions. [5]

(ii) Hence obtain the expansion of $\dfrac{4 + 12x + x^2}{(3 - x)(1 + 2x)^2}$ in ascending powers of x, up to and including the term in x^2. [5]

10



The diagram shows the curve $y = 10e^{-\frac{1}{2}x} \sin 4x$ for $x \geqslant 0$. The stationary points are labelled T_1, T_2, T_3, … as shown.

(i) Find the x-coordinates of T_1 and T_2, giving each x-coordinate correct to 3 decimal places. [6]

(ii) It is given that the x-coordinate of T_n is greater than 25. Find the least possible value of n. [4]

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Cambridge International Examinations
Cambridge International Advanced Level

MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **(P3)** **May/June 2014**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

* 1 9 2 9 8 8 8 6 6 7 *

READ THESE INSTRUCTIONS FIRST

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Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

This document consists of **4** printed pages.

1 Find the set of values of x satisfying the inequality

$$|x + 2a| > 3|x - a|,$$

where a is a positive constant. [4]

2 Solve the equation

$$2\ln(5 - e^{-2x}) = 1,$$

giving your answer correct to 3 significant figures. [4]

3 Solve the equation

$$\cos(x + 30°) = 2\cos x,$$

giving all solutions in the interval $-180° < x < 180°$. [5]

4 The parametric equations of a curve are

$$x = t - \tan t, \quad y = \ln(\cos t),$$

for $-\frac{1}{2}\pi < t < \frac{1}{2}\pi$.

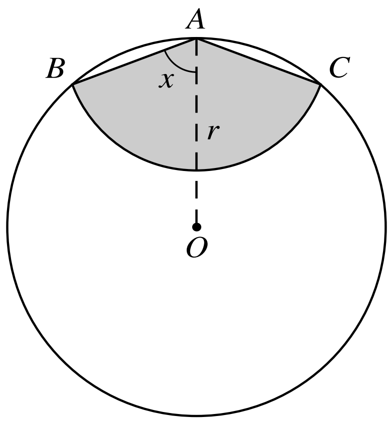
(i) Show that $\frac{dy}{dx} = \cot t$. [5]

(ii) Hence find the x-coordinate of the point on the curve at which the gradient is equal to 2. Give your answer correct to 3 significant figures. [2]

5 **(i)** The polynomial $f(x)$ is of the form $(x - 2)^2 g(x)$, where $g(x)$ is another polynomial. Show that $(x - 2)$ is a factor of $f'(x)$. [2]

(ii) The polynomial $x^5 + ax^4 + 3x^3 + bx^2 + a$, where a and b are constants, has a factor $(x - 2)^2$. Using the factor theorem and the result of part **(i)**, or otherwise, find the values of a and b. [5]

6



In the diagram, A is a point on the circumference of a circle with centre O and radius r. A circular arc with centre A meets the circumference at B and C. The angle OAB is equal to x radians. The shaded region is bounded by AB, AC and the circular arc with centre A joining B and C. The perimeter of the shaded region is equal to half the circumference of the circle.

(i) Show that $x = \cos^{-1}\left(\dfrac{\pi}{4 + 4x}\right)$. [3]

(ii) Verify by calculation that x lies between 1 and 1.5. [2]

(iii) Use the iterative formula

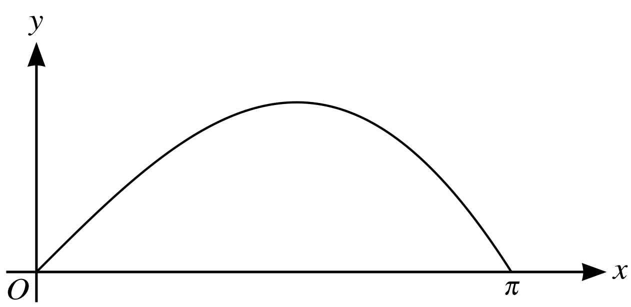
$$x_{n+1} = \cos^{-1}\left(\frac{\pi}{4 + 4x_n}\right)$$

to determine the value of x correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

7 **(a)** It is given that $-1 + (\sqrt{5})\mathrm{i}$ is a root of the equation $z^3 + 2z + a = 0$, where a is real. Showing your working, find the value of a, and write down the other complex root of this equation. [4]

(b) The complex number w has modulus 1 and argument 2θ radians. Show that $\dfrac{w - 1}{w + 1} = \mathrm{i}\tan\theta$. [4]

8



The diagram shows the curve $y = x\cos\frac{1}{2}x$ for $0 \leqslant x \leqslant \pi$.

(i) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ and show that $4\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} + y + 4\sin\frac{1}{2}x = 0$. [5]

(ii) Find the exact value of the area of the region enclosed by this part of the curve and the x-axis. [5]

9 The population of a country at time t years is N millions. At any time, N is assumed to increase at a rate proportional to the product of N and $(1 - 0.01N)$. When $t = 0$, $N = 20$ and $\frac{dN}{dt} = 0.32$.

(i) Treating N and t as continuous variables, show that they satisfy the differential equation

$$\frac{dN}{dt} = 0.02N(1 - 0.01N).$$ [1]

(ii) Solve the differential equation, obtaining an expression for t in terms of N. [8]

(iii) Find the time at which the population will be double its value at $t = 0$. [1]

10 Referred to the origin O, the points A, B and C have position vectors given by

$$\overrightarrow{OA} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}, \quad \overrightarrow{OB} = 2\mathbf{i} + 4\mathbf{j} + \mathbf{k} \quad \text{and} \quad \overrightarrow{OC} = 3\mathbf{i} + 5\mathbf{j} - 3\mathbf{k}.$$

(i) Find the exact value of the cosine of angle BAC. [4]

(ii) Hence find the exact value of the area of triangle ABC. [3]

(iii) Find the equation of the plane which is parallel to the y-axis and contains the line through B and C. Give your answer in the form $ax + by + cz = d$. [5]

Cambridge International A Level

Cambridge International Examinations
Cambridge International Advanced Level

MATHEMATICS **9709/33**

Paper 3 Pure Mathematics 3 **(P3)** **May/June 2014**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

7019745292

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The total number of marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

JC14 06_9709_33/FP

CAMBRIDGE
International Examinations

[Turn over

1 Solve the equation $\log_{10}(x+9) = 2 + \log_{10} x$. [3]

2 Expand $(1 + 3x)^{-\frac{1}{3}}$ in ascending powers of x, up to and including the term in x^3, simplifying the coefficients. [4]

3 (i) Show that the equation

$$\tan(x - 60°) + \cot x = \sqrt{3}$$

can be written in the form

$$2\tan^2 x + (\sqrt{3})\tan x - 1 = 0.$$ [3]

(ii) Hence solve the equation

$$\tan(x - 60°) + \cot x = \sqrt{3},$$

for $0° < x < 180°$. [3]

4 The equation $x = \dfrac{10}{e^{2x} - 1}$ has one positive real root, denoted by α.

(i) Show that α lies between $x = 1$ and $x = 2$. [2]

(ii) Show that if a sequence of positive values given by the iterative formula

$$x_{n+1} = \tfrac{1}{2}\ln\left(1 + \frac{10}{x_n}\right)$$

converges, then it converges to α. [2]

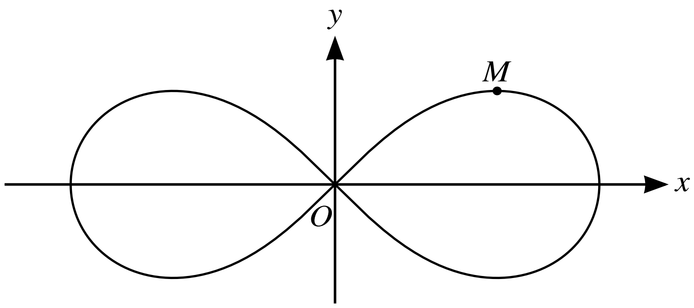
(iii) Use this iterative formula to determine α correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

5 The variables x and θ satisfy the differential equation

$$2\cos^2\theta \frac{dx}{d\theta} = \sqrt{(2x+1)},$$

and $x = 0$ when $\theta = \frac{1}{4}\pi$. Solve the differential equation and obtain an expression for x in terms of θ. [7]

6



The diagram shows the curve $(x^2 + y^2)^2 = 2(x^2 - y^2)$ and one of its maximum points M. Find the coordinates of M. [7]

7 **(a)** The complex number $\dfrac{3 - 5\mathrm{i}}{1 + 4\mathrm{i}}$ is denoted by u. Showing your working, express u in the form $x + \mathrm{i}y$, where x and y are real. [3]

(b) **(i)** On a sketch of an Argand diagram, shade the region whose points represent complex numbers satisfying the inequalities $|z - 2 - \mathrm{i}| \leqslant 1$ and $|z - \mathrm{i}| \leqslant |z - 2|$. [4]

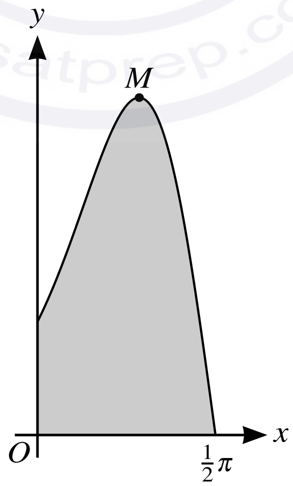
(ii) Calculate the maximum value of $\arg z$ for points lying in the shaded region. [2]

8 Let $\mathrm{f}(x) = \dfrac{6 + 6x}{(2 - x)(2 + x^2)}$.

(i) Express $\mathrm{f}(x)$ in the form $\dfrac{A}{2 - x} + \dfrac{Bx + C}{2 + x^2}$. [4]

(ii) Show that $\displaystyle\int_{-1}^{1} \mathrm{f}(x)\,\mathrm{d}x = 3\ln 3$. [5]

9



The diagram shows the curve $y = \mathrm{e}^{2\sin x}\cos x$ for $0 \leqslant x \leqslant \frac{1}{2}\pi$, and its maximum point M.

(i) Using the substitution $u = \sin x$, find the exact value of the area of the shaded region bounded by the curve and the axes. [5]

(ii) Find the x-coordinate of M, giving your answer correct to 3 decimal places. [6]

10 The line l has equation $\mathbf{r} = \mathbf{i} + 2\mathbf{j} - \mathbf{k} + \lambda(3\mathbf{i} - 2\mathbf{j} + 2\mathbf{k})$ and the plane p has equation $2x + 3y - 5z = 18$.

(i) Find the position vector of the point of intersection of l and p. [3]

(ii) Find the acute angle between l and p. [4]

(iii) A second plane q is perpendicular to the plane p and contains the line l. Find the equation of q, giving your answer in the form $ax + by + cz = d$. [5]

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

MATHEMATICS **9709/31**

Paper 3 Pure Mathematics 3 **(P3)** **October/November 2013**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

5238158802

READ THESE INSTRUCTIONS FIRST

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The total number of marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

This document consists of **4** printed pages.

JC13 11_9709_31/RP

UNIVERSITY *of* CAMBRIDGE
International Examinations

[Turn over

1 The equation of a curve is $y = \dfrac{1+x}{1+2x}$ for $x > -\frac{1}{2}$. Show that the gradient of the curve is always negative. [3]

2 Solve the equation $2|3^x - 1| = 3^x$, giving your answers correct to 3 significant figures. [4]

3 Find the exact value of $\displaystyle\int_1^4 \frac{\ln x}{\sqrt{x}}\,\mathrm{d}x$. [5]

4 The parametric equations of a curve are

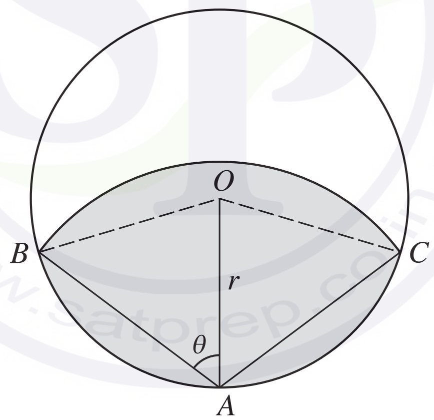
$$x = \mathrm{e}^{-t}\cos t, \quad y = \mathrm{e}^{-t}\sin t.$$

Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \tan\left(t - \frac{1}{4}\pi\right)$. [6]

5 **(i)** Prove that $\cot\theta + \tan\theta \equiv 2\operatorname{cosec} 2\theta$. [3]

(ii) Hence show that $\displaystyle\int_{\frac{1}{6}\pi}^{\frac{1}{3}\pi} \operatorname{cosec} 2\theta\,\mathrm{d}\theta = \frac{1}{2}\ln 3$. [4]

6



In the diagram, A is a point on the circumference of a circle with centre O and radius r. A circular arc with centre A meets the circumference at B and C. The angle OAB is θ radians. The shaded region is bounded by the circumference of the circle and the arc with centre A joining B and C. The area of the shaded region is equal to half the area of the circle.

(i) Show that $\cos 2\theta = \dfrac{2\sin 2\theta - \pi}{4\theta}$. [5]

(ii) Use the iterative formula

$$\theta_{n+1} = \frac{1}{2}\cos^{-1}\left(\frac{2\sin 2\theta_n - \pi}{4\theta_n}\right),$$

with initial value $\theta_1 = 1$, to determine θ correct to 2 decimal places, showing the result of each iteration to 4 decimal places. [3]

7 Let $\mathrm{f}(x) = \dfrac{2x^2 - 7x - 1}{(x-2)(x^2+3)}$.

(i) Express $\mathrm{f}(x)$ in partial fractions. [5]

(ii) Hence obtain the expansion of $\mathrm{f}(x)$ in ascending powers of x, up to and including the term in x^2. [5]

8 **Throughout this question the use of a calculator is not permitted.**

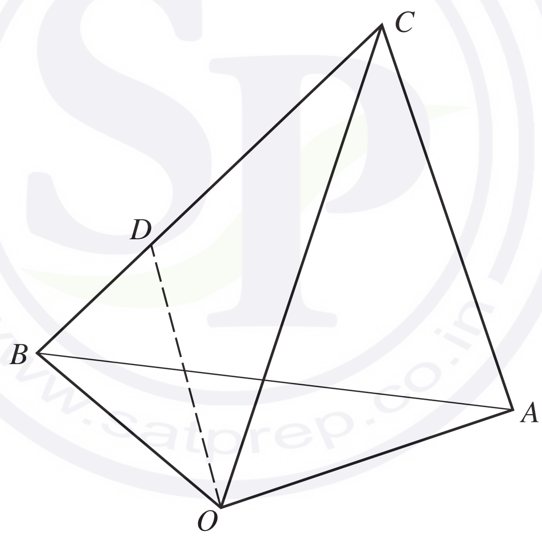
(a) The complex numbers u and v satisfy the equations

$$u + 2v = 2\mathrm{i} \quad \text{and} \quad \mathrm{i}u + v = 3.$$

Solve the equations for u and v, giving both answers in the form $x + \mathrm{i}y$, where x and y are real. [5]

(b) On an Argand diagram, sketch the locus representing complex numbers z satisfying $|z + \mathrm{i}| = 1$ and the locus representing complex numbers w satisfying $\arg(w - 2) = \frac{3}{4}\pi$. Find the least value of $|z - w|$ for points on these loci. [5]

9



The diagram shows three points A, B and C whose position vectors with respect to the origin O are given by $\overrightarrow{OA} = \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix}$, $\overrightarrow{OB} = \begin{pmatrix} 0 \\ 3 \\ 1 \end{pmatrix}$ and $\overrightarrow{OC} = \begin{pmatrix} 3 \\ 0 \\ 4 \end{pmatrix}$. The point D lies on BC, between B and C, and is such that $CD = 2DB$.

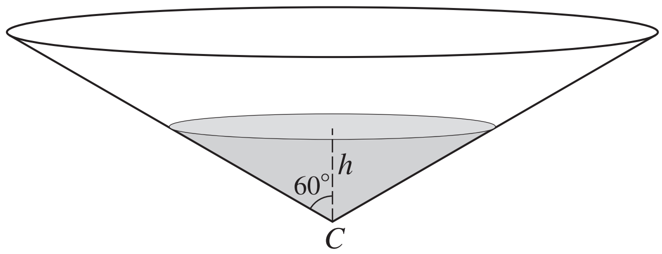
(i) Find the equation of the plane ABC, giving your answer in the form $ax + by + cz = d$. [6]

(ii) Find the position vector of D. [1]

(iii) Show that the length of the perpendicular from A to OD is $\frac{1}{3}\sqrt{(65)}$. [4]

[Question 10 is printed on the next page.]

10



A tank containing water is in the form of a cone with vertex C. The axis is vertical and the semi-vertical angle is $60°$, as shown in the diagram. At time $t = 0$, the tank is full and the depth of water is H. At this instant, a tap at C is opened and water begins to flow out. The volume of water in the tank decreases at a rate proportional to $\sqrt{h}$, where h is the depth of water at time t. The tank becomes empty when $t = 60$.

(i) Show that h and t satisfy a differential equation of the form

$$\frac{dh}{dt} = -Ah^{-\frac{3}{2}},$$

where A is a positive constant. [4]

(ii) Solve the differential equation given in part **(i)** and obtain an expression for t in terms of h and H. [6]

(iii) Find the time at which the depth reaches $\frac{1}{2}H$. [1]

[The volume V of a cone of vertical height h and base radius r is given by $V = \frac{1}{3}\pi r^2 h$.]

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **(P3)** **October/November 2013**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

4229833005

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JC13 11_9709_32/FP

UNIVERSITY of CAMBRIDGE International Examinations

1 The equation of a curve is $y = \dfrac{1+x}{1+2x}$ for $x > -\frac{1}{2}$. Show that the gradient of the curve is always negative. [3]

2 Solve the equation $2|3^x - 1| = 3^x$, giving your answers correct to 3 significant figures. [4]

3 Find the exact value of $\displaystyle\int_1^4 \frac{\ln x}{\sqrt{x}}\,dx$. [5]

4 The parametric equations of a curve are

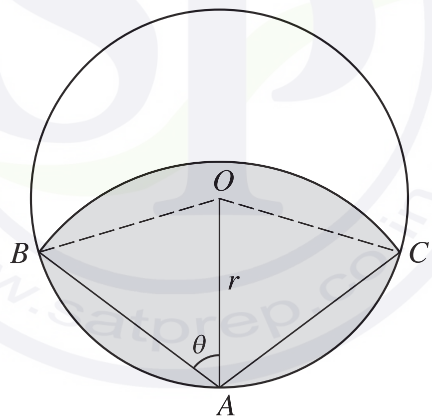
$$x = e^{-t}\cos t, \quad y = e^{-t}\sin t.$$

Show that $\dfrac{dy}{dx} = \tan\left(t - \frac{1}{4}\pi\right)$. [6]

5 **(i)** Prove that $\cot\theta + \tan\theta \equiv 2\operatorname{cosec} 2\theta$. [3]

(ii) Hence show that $\displaystyle\int_{\frac{1}{6}\pi}^{\frac{1}{3}\pi} \operatorname{cosec} 2\theta\,d\theta = \frac{1}{2}\ln 3$. [4]

6



In the diagram, A is a point on the circumference of a circle with centre O and radius r. A circular arc with centre A meets the circumference at B and C. The angle OAB is θ radians. The shaded region is bounded by the circumference of the circle and the arc with centre A joining B and C. The area of the shaded region is equal to half the area of the circle.

(i) Show that $\cos 2\theta = \dfrac{2\sin 2\theta - \pi}{4\theta}$. [5]

(ii) Use the iterative formula

$$\theta_{n+1} = \frac{1}{2}\cos^{-1}\left(\frac{2\sin 2\theta_n - \pi}{4\theta_n}\right),$$

with initial value $\theta_1 = 1$, to determine θ correct to 2 decimal places, showing the result of each iteration to 4 decimal places. [3]

7 Let $f(x) = \dfrac{2x^2 - 7x - 1}{(x-2)(x^2+3)}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence obtain the expansion of $f(x)$ in ascending powers of x, up to and including the term in x^2. [5]

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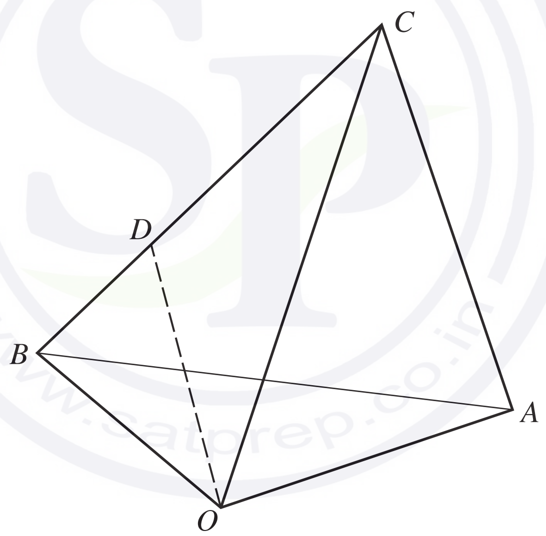
$$u + 2v = 2\mathrm{i} \quad \text{and} \quad \mathrm{i}u + v = 3.$$

Solve the equations for u and v, giving both answers in the form $x + \mathrm{i}y$, where x and y are real. [5]

(b) On an Argand diagram, sketch the locus representing complex numbers z satisfying $|z + \mathrm{i}| = 1$ and the locus representing complex numbers w satisfying $\arg(w - 2) = \frac{3}{4}\pi$. Find the least value of $|z - w|$ for points on these loci. [5]

9



The diagram shows three points A, B and C whose position vectors with respect to the origin O are given by $\overrightarrow{OA} = \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix}$, $\overrightarrow{OB} = \begin{pmatrix} 0 \\ 3 \\ 1 \end{pmatrix}$ and $\overrightarrow{OC} = \begin{pmatrix} 3 \\ 0 \\ 4 \end{pmatrix}$. The point D lies on BC, between B and C, and is such that $CD = 2DB$.

(i) Find the equation of the plane ABC, giving your answer in the form $ax + by + cz = d$. [6]

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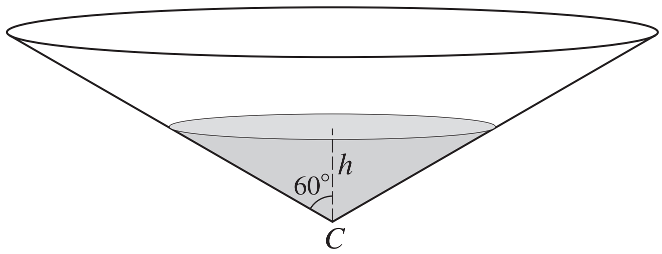
[Question 10 is printed on the next page.]

10



A tank containing water is in the form of a cone with vertex C. The axis is vertical and the semi-vertical angle is 60°, as shown in the diagram. At time $t = 0$, the tank is full and the depth of water is H. At this instant, a tap at C is opened and water begins to flow out. The volume of water in the tank decreases at a rate proportional to $\sqrt{h}$, where h is the depth of water at time t. The tank becomes empty when $t = 60$.

(i) Show that h and t satisfy a differential equation of the form

$$\frac{dh}{dt} = -Ah^{-\frac{3}{2}},$$

where A is a positive constant. [4]

(ii) Solve the differential equation given in part **(i)** and obtain an expression for t in terms of h and H. [6]

(iii) Find the time at which the depth reaches $\frac{1}{2}H$. [1]

[The volume V of a cone of vertical height h and base radius r is given by $V = \frac{1}{3}\pi r^2 h$.]

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

MATHEMATICS **9709/33**

Paper 3 Pure Mathematics 3 **(P3)** **October/November 2013**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

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READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
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The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

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1 Given that $2\ln(x+4) - \ln x = \ln(x+a)$, express x in terms of a. [4]

2 Use the substitution $u = 3x+1$ to find $\displaystyle\int \frac{3x}{3x+1}\,dx$. [4]

3 The polynomial $f(x)$ is defined by

$$f(x) = x^3 + ax^2 - ax + 14,$$

where a is a constant. It is given that $(x+2)$ is a factor of $f(x)$.

(i) Find the value of a. [2]

(ii) Show that, when a has this value, the equation $f(x) = 0$ has only one real root. [3]

4 A curve has equation $3e^{2x}y + e^x y^3 = 14$. Find the gradient of the curve at the point $(0, 2)$. [5]

5 It is given that $\displaystyle\int_0^p 4xe^{-\frac{1}{2}x}\,dx = 9$, where p is a positive constant.

(i) Show that $p = 2\ln\left(\dfrac{8p+16}{7}\right)$. [5]

(ii) Use an iterative process based on the equation in part **(i)** to find the value of p correct to 3 significant figures. Use a starting value of 3.5 and give the result of each iteration to 5 significant figures. [3]

6 Two planes have equations $3x - y + 2z = 9$ and $x + y - 4z = -1$.

(i) Find the acute angle between the planes. [3]

(ii) Find a vector equation of the line of intersection of the planes. [6]

7 **(i)** Given that $\sec\theta + 2\operatorname{cosec}\theta = 3\operatorname{cosec}2\theta$, show that $2\sin\theta + 4\cos\theta = 3$. [3]

(ii) Express $2\sin\theta + 4\cos\theta$ in the form $R\sin(\theta + \alpha)$ where $R > 0$ and $0° < \alpha < 90°$, giving the value of α correct to 2 decimal places. [3]

(iii) Hence solve the equation $\sec\theta + 2\operatorname{cosec}\theta = 3\operatorname{cosec}2\theta$ for $0° < \theta < 360°$. [4]

8 **(i)** Express $\dfrac{7x^2+8}{(1+x)^2(2-3x)}$ in partial fractions. [5]

(ii) Hence expand $\dfrac{7x^2+8}{(1+x)^2(2-3x)}$ in ascending powers of x up to and including the term in x^2, simplifying the coefficients. [5]

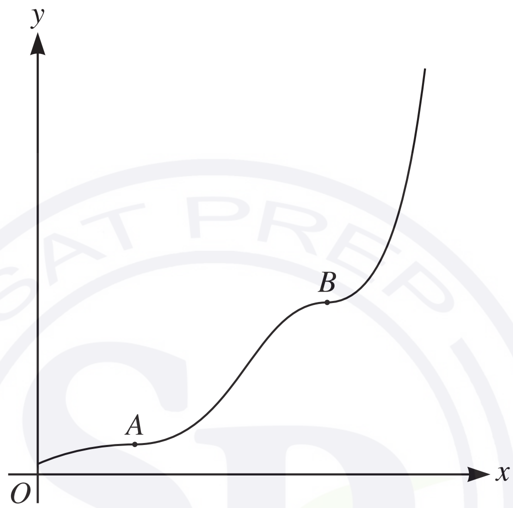
9 **(a)** Without using a calculator, use the formula for the solution of a quadratic equation to solve

$$(2-\mathrm{i})z^2 + 2z + 2 + \mathrm{i} = 0.$$

Give your answers in the form $a + b\mathrm{i}$. [5]

(b) The complex number w is defined by $w = 2\mathrm{e}^{\frac{1}{4}\pi\mathrm{i}}$. In an Argand diagram, the points A, B and C represent the complex numbers w, w^3 and w^* respectively (where w^* denotes the complex conjugate of w). Draw the Argand diagram showing the points A, B and C, and calculate the area of triangle ABC. [5]

10



A particular solution of the differential equation

$$3y^2\frac{\mathrm{d}y}{\mathrm{d}x} = 4(y^3+1)\cos^2 x$$

is such that $y = 2$ when $x = 0$. The diagram shows a sketch of the graph of this solution for $0 \leqslant x \leqslant 2\pi$; the graph has stationary points at A and B. Find the y-coordinates of A and B, giving each coordinate correct to 1 decimal place. [10]

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General Certificate of Education Advanced Level

MATHEMATICS **9709/31**

Paper 3 Pure Mathematics 3 **(P3)** **May/June 2013**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

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The total number of marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

This document consists of **3** printed pages and **1** blank page.

JC13 06_9709_31/FP

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International Examinations

1 Find the quotient and remainder when $2x^2$ is divided by $x + 2$. [3]

2 Expand $\dfrac{1 + 3x}{\sqrt{(1 + 2x)}}$ in ascending powers of x up to and including the term in x^2, simplifying the coefficients. [4]

3 Express $\dfrac{7x^2 - 3x + 2}{x(x^2 + 1)}$ in partial fractions. [5]

4 **(i)** Solve the equation $|4x - 1| = |x - 3|$. [3]

(ii) Hence solve the equation $\left|4^{y+1} - 1\right| = |4^y - 3|$ correct to 3 significant figures. [3]

5 For each of the following curves, find the gradient at the point where the curve crosses the y-axis:

(i) $y = \dfrac{1 + x^2}{1 + e^{2x}}$; [3]

(ii) $2x^3 + 5xy + y^3 = 8$. [4]

6 The points P and Q have position vectors, relative to the origin O, given by

$$\overrightarrow{OP} = 7\mathbf{i} + 7\mathbf{j} - 5\mathbf{k} \quad \text{and} \quad \overrightarrow{OQ} = -5\mathbf{i} + \mathbf{j} + \mathbf{k}.$$

The mid-point of PQ is the point A. The plane Π is perpendicular to the line PQ and passes through A.

(i) Find the equation of Π, giving your answer in the form $ax + by + cz = d$. [4]

(ii) The straight line through P parallel to the x-axis meets Π at the point B. Find the distance AB, correct to 3 significant figures. [5]

7 **(a)** Without using a calculator, solve the equation

$$3w + 2\mathrm{i}w^* = 17 + 8\mathrm{i},$$

where w^* denotes the complex conjugate of w. Give your answer in the form $a + b\mathrm{i}$. [4]

(b) In an Argand diagram, the loci

$$\arg(z - 2\mathrm{i}) = \tfrac{1}{6}\pi \quad \text{and} \quad |z - 3| = |z - 3\mathrm{i}|$$

intersect at the point P. Express the complex number represented by P in the form $r\mathrm{e}^{\mathrm{i}\theta}$, giving the exact value of θ and the value of r correct to 3 significant figures. [5]

8 **(a)** Show that $\displaystyle\int_2^4 4x \ln x \, \mathrm{d}x = 56 \ln 2 - 12$. [5]

(b) Use the substitution $u = \sin 4x$ to find the exact value of $\displaystyle\int_0^{\frac{1}{24}\pi} \cos^3 4x \, \mathrm{d}x$. [5]

9 **(i)** Express $4\cos\theta + 3\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$. Give the value of α correct to 4 decimal places. [3]

(ii) Hence

(a) solve the equation $4\cos\theta + 3\sin\theta = 2$ for $0 < \theta < 2\pi$, [4]

(b) find $\displaystyle\int \frac{50}{(4\cos\theta + 3\sin\theta)^2}\,d\theta$. [3]

10 Liquid is flowing into a small tank which has a leak. Initially the tank is empty and, t minutes later, the volume of liquid in the tank is $V\,\text{cm}^3$. The liquid is flowing into the tank at a constant rate of $80\,\text{cm}^3$ per minute. Because of the leak, liquid is being lost from the tank at a rate which, at any instant, is equal to $kV\,\text{cm}^3$ per minute where k is a positive constant.

(i) Write down a differential equation describing this situation and solve it to show that

$$V = \frac{1}{k}(80 - 80e^{-kt}).$$ [7]

(ii) It is observed that $V = 500$ when $t = 15$, so that k satisfies the equation

$$k = \frac{4 - 4e^{-15k}}{25}.$$

Use an iterative formula, based on this equation, to find the value of k correct to 2 significant figures. Use an initial value of $k = 0.1$ and show the result of each iteration to 4 significant figures. [3]

(iii) Determine how much liquid there is in the tank 20 minutes after the liquid started flowing, and state what happens to the volume of liquid in the tank after a long time. [2]

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General Certificate of Education Advanced Level

MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **(P3)** **May/June 2013**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

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JC13 06_9709_32/FP

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International Examinations

1 Solve the equation $|x - 2| = |\frac{1}{3}x|$. [3]

2 The sequence of values given by the iterative formula

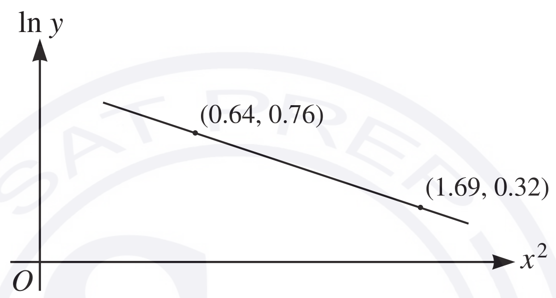
$$x_{n+1} = \frac{x_n(x_n^3 + 100)}{2(x_n^3 + 25)},$$

with initial value $x_1 = 3.5$, converges to α.

(i) Use this formula to calculate α correct to 4 decimal places, showing the result of each iteration to 6 decimal places. [3]

(ii) State an equation satisfied by α and hence find the exact value of α. [2]

3



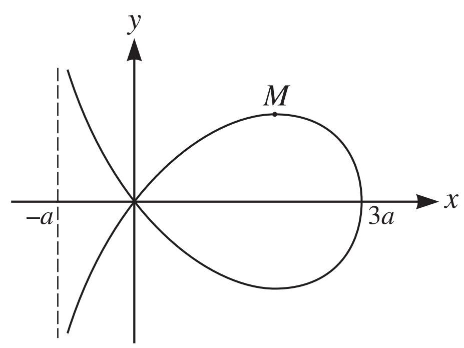
The variables x and y satisfy the equation $y = Ae^{-kx^2}$, where A and k are constants. The graph of $\ln y$ against x^2 is a straight line passing through the points (0.64, 0.76) and (1.69, 0.32), as shown in the diagram. Find the values of A and k correct to 2 decimal places. [5]

4 The polynomial $ax^3 - 20x^2 + x + 3$, where a is a constant, is denoted by $p(x)$. It is given that $(3x + 1)$ is a factor of $p(x)$.

(i) Find the value of a. [3]

(ii) When a has this value, factorise $p(x)$ completely. [3]

5



The diagram shows the curve with equation

$$x^3 + xy^2 + ay^2 - 3ax^2 = 0,$$

where a is a positive constant. The maximum point on the curve is M. Find the x-coordinate of M in terms of a. [6]

6 **(i)** By differentiating $\frac{1}{\cos x}$, show that the derivative of $\sec x$ is $\sec x \tan x$. Hence show that if $y = \ln(\sec x + \tan x)$ then $\frac{dy}{dx} = \sec x$. [4]

(ii) Using the substitution $x = (\sqrt{3})\tan\theta$, find the exact value of

$$\int_1^3 \frac{1}{\sqrt{(3 + x^2)}}\,dx,$$

expressing your answer as a single logarithm. [4]

7 **(i)** By first expanding $\cos(x + 45°)$, express $\cos(x + 45°) - (\sqrt{2})\sin x$ in the form $R\cos(x + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. Give the value of R correct to 4 significant figures and the value of α correct to 2 decimal places. [5]

(ii) Hence solve the equation

$$\cos(x + 45°) - (\sqrt{2})\sin x = 2,$$

for $0° < x < 360°$. [4]

8 **(i)** Express $\frac{1}{x^2(2x + 1)}$ in the form $\frac{A}{x^2} + \frac{B}{x} + \frac{C}{2x + 1}$. [4]

(ii) The variables x and y satisfy the differential equation

$$y = x^2(2x + 1)\frac{dy}{dx},$$

and $y = 1$ when $x = 1$. Solve the differential equation and find the exact value of y when $x = 2$. Give your value of y in a form not involving logarithms. [7]

9 **(a)** The complex number w is such that $\text{Re}\,w > 0$ and $w + 3w^* = \text{i}w^2$, where w^* denotes the complex conjugate of w. Find w, giving your answer in the form $x + \text{i}y$, where x and y are real. [5]

(b) On a sketch of an Argand diagram, shade the region whose points represent complex numbers z which satisfy both the inequalities $|z - 2\text{i}| \leqslant 2$ and $0 \leqslant \arg(z + 2) \leqslant \frac{1}{4}\pi$. Calculate the greatest value of $|z|$ for points in this region, giving your answer correct to 2 decimal places. [6]

10 The points A and B have position vectors $2\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$ and $5\mathbf{i} - 2\mathbf{j} + \mathbf{k}$ respectively. The plane p has equation $x + y = 5$.

(i) Find the position vector of the point of intersection of the line through A and B and the plane p. [4]

(ii) A second plane q has an equation of the form $x + by + cz = d$, where b, c and d are constants. The plane q contains the line AB, and the acute angle between the planes p and q is $60°$. Find the equation of q. [7]

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UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

MATHEMATICS **9709/33**

Paper 3 Pure Mathematics 3 **(P3)** **May/June 2013**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

9836805872

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1 Solve the inequality $|4x + 3| > |x|$. [4]

2 It is given that $\ln(y + 1) - \ln y = 1 + 3\ln x$. Express y in terms of x, in a form not involving logarithms. [4]

3 Solve the equation $\tan 2x = 5\cot x$, for $0° < x < 180°$. [5]

4 (i) Express $(\sqrt{3})\cos x + \sin x$ in the form $R\cos(x - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$, giving the exact values of R and α. [3]

(ii) Hence show that

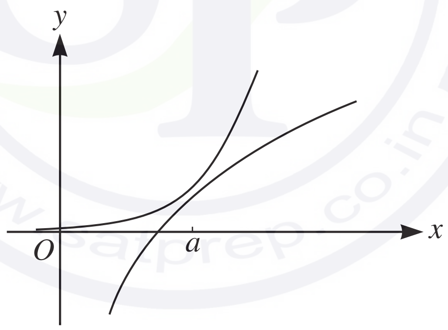
$$\int_{\frac{1}{6}\pi}^{\frac{1}{2}\pi} \frac{1}{\left((\sqrt{3})\cos x + \sin x\right)^2}\,dx = \tfrac{1}{4}\sqrt{3}.$$ [4]

5 The polynomial $8x^3 + ax^2 + bx + 3$, where a and b are constants, is denoted by $p(x)$. It is given that $(2x + 1)$ is a factor of $p(x)$ and that when $p(x)$ is divided by $(2x - 1)$ the remainder is 1.

(i) Find the values of a and b. [5]

(ii) When a and b have these values, find the remainder when $p(x)$ is divided by $2x^2 - 1$. [3]

6



The diagram shows the curves $y = e^{2x-3}$ and $y = 2\ln x$. When $x = a$ the tangents to the curves are parallel.

(i) Show that a satisfies the equation $a = \frac{1}{2}(3 - \ln a)$. [3]

(ii) Verify by calculation that this equation has a root between 1 and 2. [2]

(iii) Use the iterative formula $a_{n+1} = \frac{1}{2}(3 - \ln a_n)$ to calculate a correct to 2 decimal places, showing the result of each iteration to 4 decimal places. [3]

7 The complex number z is defined by $z = a + \mathrm{i}b$, where a and b are real. The complex conjugate of z is denoted by z^*.

(i) Show that $|z|^2 = zz^*$ and that $(z - k\mathrm{i})^* = z^* + k\mathrm{i}$, where k is real. [2]

In an Argand diagram a set of points representing complex numbers z is defined by the equation $|z - 10\mathrm{i}| = 2|z - 4\mathrm{i}|$.

(ii) Show, by squaring both sides, that

$$zz^* - 2\mathrm{i}z^* + 2\mathrm{i}z - 12 = 0.$$

Hence show that $|z - 2\mathrm{i}| = 4$. [5]

(iii) Describe the set of points geometrically. [1]

8 The variables x and t satisfy the differential equation

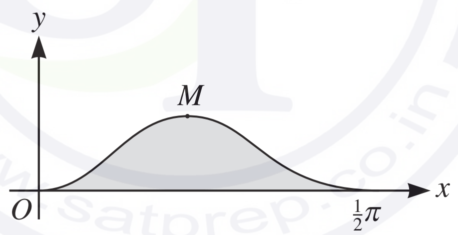
$$t\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{k - x^3}{2x^2},$$

for $t > 0$, where k is a constant. When $t = 1$, $x = 1$ and when $t = 4$, $x = 2$.

(i) Solve the differential equation, finding the value of k and obtaining an expression for x in terms of t. [9]

(ii) State what happens to the value of x as t becomes large. [1]

9



The diagram shows the curve $y = \sin^2 2x \cos x$ for $0 \leqslant x \leqslant \frac{1}{2}\pi$, and its maximum point M.

(i) Find the x-coordinate of M. [6]

(ii) Using the substitution $u = \sin x$, find by integration the area of the shaded region bounded by the curve and the x-axis. [4]

10 The line l has equation $\mathbf{r} = \mathbf{i} + \mathbf{j} + \mathbf{k} + \lambda(a\mathbf{i} + 2\mathbf{j} + \mathbf{k})$, where a is a constant. The plane p has equation $x + 2y + 2z = 6$. Find the value or values of a in each of the following cases.

(i) The line l is parallel to the plane p. [2]

(ii) The line l intersects the line passing through the points with position vectors $3\mathbf{i} + 2\mathbf{j} + \mathbf{k}$ and $\mathbf{i} + \mathbf{j} - \mathbf{k}$. [4]

(iii) The acute angle between the line l and the plane p is $\tan^{-1} 2$. [5]

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