Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Higher level
Paper 1

Tuesday 3 November 2020 (afternoon)

Candidate session number

2 hours

Instructions to candidates

- Write your session number in the boxes above.
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- You are not permitted access to any calculator for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics HL and further mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[100 marks]**.

8820 – 7201

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 4]

A discrete random variable X has the probability distribution given by the following table.

x	0	1	2	3
$\mathrm{P}(X=x)$	p	$\frac{1}{4}$	$\frac{1}{6}$	q

Given that $\mathrm{E}(X)=\frac{19}{12}$, determine the value of p and the value of q.

2. [Maximum mark: 5]

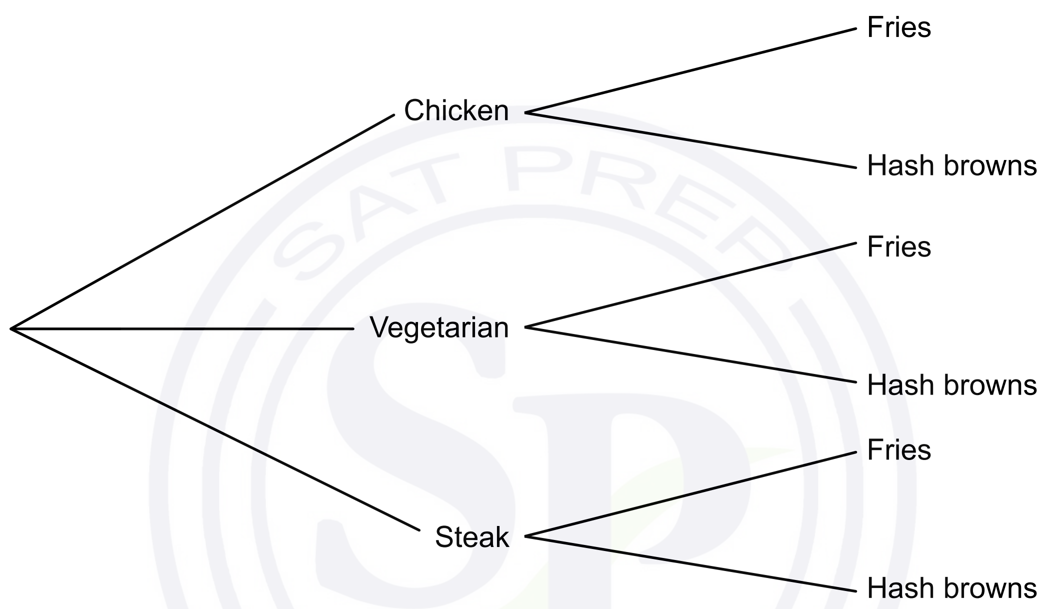
Find the equation of the tangent to the curve $y = e^{2x} - 3x$ at the point where $x = 0$.

3. [Maximum mark: 7]

At Nusaybah's Breakfast Diner, three types of omelette are available to order: chicken, vegetarian and steak. Each omelette is served with either a portion of fries or hash browns. It is known that 20% of customers choose a chicken omelette, 70% choose a vegetarian omelette and 10% choose a steak omelette.

It is also known that 65% of those ordering the chicken omelette, 70% of those ordering the vegetarian omelette and 60% of those ordering the steak omelette, order fries.

The following tree diagram represents the orders made by each customer.



(a) Complete the tree diagram by adding the respective probabilities to each branch. [2]

(b) Find the probability that a randomly selected customer orders fries. [2]

(c) Find the probability that a randomly selected customer orders fries, given that they do **not** order a chicken omelette. [3]

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4. [Maximum mark: 5]

Consider the equation $\frac{2z}{3-z^*} = \mathrm{i}$, where $z = x + \mathrm{i}y$ and x, $y \in \mathbb{R}$.

Find the value of x and the value of y.

5. [Maximum mark: 5]

The first term in an arithmetic sequence is 4 and the fifth term is $\log_2 625$.

Find the common difference of the sequence, expressing your answer in the form $\log_2 p$, where $p \in \mathbb{Q}$.

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6. [Maximum mark: 4]

Consider the equation $ax^2 + bx + c = 0$, where $a \neq 0$. Given that the roots of this equation are $x = \sin\theta$ and $x = \cos\theta$, show that $b^2 = a^2 + 2ac$.

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7. [Maximum mark: 5]

Consider the complex numbers $z_1 = \cos\frac{11\pi}{12} + i\sin\frac{11\pi}{12}$ and $z_2 = \cos\frac{\pi}{6} + i\sin\frac{\pi}{6}$.

(a) (i) Find $\frac{z_1}{z_2}$

(ii) Find $\frac{z_2}{z_1}$ [3]

(b) 0, $\frac{z_1}{z_2}$ and $\frac{z_2}{z_1}$ are represented by three points O, A and B respectively on an Argand diagram. Determine the area of the triangle OAB. [2]

8. [Maximum mark: 7]

(a) Show that $\frac{\sin x \tan x}{1-\cos x} \equiv 1+\frac{1}{\cos x}$, $x \neq 2n\pi$, $n \in \mathbb{Z}$. [3]

(b) Hence determine the range of values of k for which $\frac{\sin x \tan x}{1-\cos x} = k$ has no real solutions. [4]

9. [Maximum mark: 8]

By using the substitution $x = \tan u$, find the value of $\int_0^1 \frac{x^2}{\left(1+x^2\right)^3}\,\mathrm{d}x$.

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

10. [Maximum mark: 20]

Consider the function $f(x) = ax^3 + bx^2 + cx + d$, where $x \in \mathbb{R}$ and $a, b, c, d \in \mathbb{R}$.

(a) (i) Write down an expression for $f'(x)$.

(ii) Hence, given that f^{-1} does not exist, show that $b^2 - 3ac > 0$. [4]

(b) Consider the function $g(x) = \frac{1}{2}x^3 - 3x^2 + 6x - 8$, where $x \in \mathbb{R}$.

(i) Show that g^{-1} exists.

(ii) $g(x)$ can be written in the form $p(x-2)^3 + q$, where $p, q \in \mathbb{R}$. Find the value of p and the value of q.

(iii) Hence find $g^{-1}(x)$. [8]

The graph of $y = g(x)$ may be obtained by transforming the graph of $y = x^3$ using a sequence of three transformations.

(c) State each of the transformations in the order in which they are applied. [3]

(d) Sketch the graphs of $y = g(x)$ and $y = g^{-1}(x)$ on the same set of axes, indicating the points where each graph crosses the coordinate axes. [5]

11. [Maximum mark: 15]

Consider the curve C defined by $y^2 = \sin(xy)$, $y \neq 0$.

(a) Show that $\frac{dy}{dx} = \frac{y\cos(xy)}{2y - x\cos(xy)}$. [5]

(b) Prove that, when $\frac{dy}{dx} = 0$, $y = \pm 1$. [5]

(c) Hence find the coordinates of all points on C, for $0 < x < 4\pi$, where $\frac{dy}{dx} = 0$. [5]

Do **not** write solutions on this page.

12. [Maximum mark: 15]

Consider the function defined by $f(x)=\dfrac{kx-5}{x-k}$, where $x\in\mathbb{R}\setminus\{k\}$ and $k^2\neq 5$.

(a) State the equation of the vertical asymptote on the graph of $y=f(x)$. [1]

(b) State the equation of the horizontal asymptote on the graph of $y=f(x)$. [1]

(c) Use an algebraic method to determine whether f is a self-inverse function. [4]

Consider the case where $k=3$.

(d) Sketch the graph of $y=f(x)$, stating clearly the equations of any asymptotes and the coordinates of any points of intersections with the coordinate axes. [3]

(e) The region bounded by the x-axis, the curve $y=f(x)$, and the lines $x=5$ and $x=7$ is rotated through 2π about the x-axis. Find the volume of the solid generated, giving your answer in the form $\pi(a+b\ln 2)$, where $a,\ b\in\mathbb{Z}$. [6]

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Answers written on this page
will not be marked.

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Mathematics
Higher level
Paper 1

Monday 18 November 2019 (afternoon)

Candidate session number

2 hours

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12 pages

8819 – 7201

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Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 5]

The probability distribution of a discrete random variable, X, is given by the following table, where N and p are constants.

x	1	5	10	N
$\mathrm{P}(X = x)$	$\frac{1}{2}$	$\frac{1}{5}$	$\frac{1}{5}$	p

(a) Find the value of p. [2]

(b) Given that $\mathrm{E}(X) = 10$, find the value of N. [3]

2. [Maximum mark: 6]

Given that $\int_0^{\ln k} e^{2x}\,dx = 12$, find the value of k.

3. [Maximum mark: 5]

Three planes have equations:

$$\begin{aligned} 2x - y + z &= 5 \\ x + 3y - z &= 4 \\ 3x - 5y + az &= b \end{aligned} \quad \text{, where } a, b \in \mathbb{R}.$$

Find the set of values of a and b such that the three planes have no points of intersection.

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4. [Maximum mark: 7]

A and B are acute angles such that $\cos A = \frac{2}{3}$ and $\sin B = \frac{1}{3}$.

Show that $\cos(2A + B) = -\frac{2\sqrt{2}}{27} - \frac{4\sqrt{5}}{27}$.

5. [Maximum mark: 7]

Consider the equation $z^4 = -4$, where $z \in \mathbb{C}$.

(a) Solve the equation, giving the solutions in the form $a + \mathrm{i}b$, where $a, b \in \mathbb{R}$. [5]

(b) The solutions form the vertices of a polygon in the complex plane. Find the area of the polygon. [2]

6. [Maximum mark: 7]

Consider the function $f(x) = x\mathrm{e}^{2x}$, where $x \in \mathbb{R}$. The n^{th} derivative of $f(x)$ is denoted by $f^{(n)}(x)$.

Prove, by mathematical induction, that $f^{(n)}(x) = (2^n x + n2^{n-1})\mathrm{e}^{2x}$, $n \in \mathbb{Z}^+$.

7. [Maximum mark: 7]

(a) Write $2x - x^2$ in the form $a(x-h)^2 + k$, where $a, h, k \in \mathbb{R}$. [2]

(b) Hence, find the value of $\int_{\frac{1}{2}}^{\frac{3}{2}} \frac{1}{\sqrt{2x - x^2}}\, dx$. [5]

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8. [Maximum mark: 6]

A straight line, L_θ, has vector equation $\boldsymbol{r} = \begin{pmatrix} 5 \\ 0 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 5 \\ \sin\theta \\ \cos\theta \end{pmatrix}$, $\lambda, \theta \in \mathbb{R}$.

The plane, Π_p, has equation $x = p$, $p \in \mathbb{R}$.

Show that the angle between L_θ and Π_p is independent of both θ and p.

Do **not** write solutions on this page.

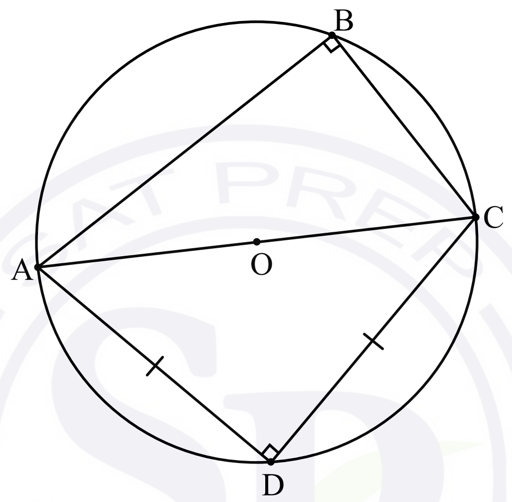
Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

9. [Maximum mark: 14]

(a) Given that $\cos 75^\circ = q$, show that $\cos 105^\circ = -q$. [1]

In the following diagram, the points A, B, C and D are on the circumference of a circle with centre O and radius r. [AC] is a diameter of the circle. $BC = r$, $AD = CD$ and $A\hat{B}C = A\hat{D}C = 90^\circ$.



(b) Show that $B\hat{A}D = 75^\circ$. [3]

(c) (i) By considering triangle ABD, show that $BD^2 = 5r^2 - 2r^2q\sqrt{6}$.

(ii) By considering triangle CBD, find another expression for BD^2 in terms of r and q. [7]

(d) Use your answers to part (c) to show that $\cos 75^\circ = \frac{1}{\sqrt{6}+\sqrt{2}}$. [3]

Do **not** write solutions on this page.

10. [Maximum mark: 19]

Consider $f\left(x\right)=\dfrac{2x-4}{x^2-1}$, $-1<x<1$.

(a) (i) Find $f'(x)$.

(ii) Show that, if $f'(x)=0$, then $x=2-\sqrt{3}$. [5]

(b) For the graph of $y=f(x)$,

(i) find the coordinates of the y-intercept;

(ii) show that there are no x-intercepts;

(iii) sketch the graph, showing clearly any asymptotic behaviour. [5]

(c) Show that $\dfrac{3}{x+1}-\dfrac{1}{x-1}=\dfrac{2x-4}{x^2-1}$. [2]

(d) The area enclosed by the graph of $y=f(x)$ and the line $y=4$ can be expressed as $\ln v$. Find the value of v. [7]

Do **not** write solutions on this page.

11. [Maximum mark: 17]

Points $\mathrm{A}(0, 0, 10)$, $\mathrm{B}(0, 10, 0)$, $\mathrm{C}(10, 0, 0)$, $V(p, p, p)$ form the vertices of a tetrahedron.

(a) (i) Show that $\overrightarrow{\mathrm{AB}} \times \overrightarrow{\mathrm{AV}} = -10\begin{pmatrix} 10-2p \\ p \\ p \end{pmatrix}$ and find a similar expression for $\overrightarrow{\mathrm{AC}} \times \overrightarrow{\mathrm{AV}}$.

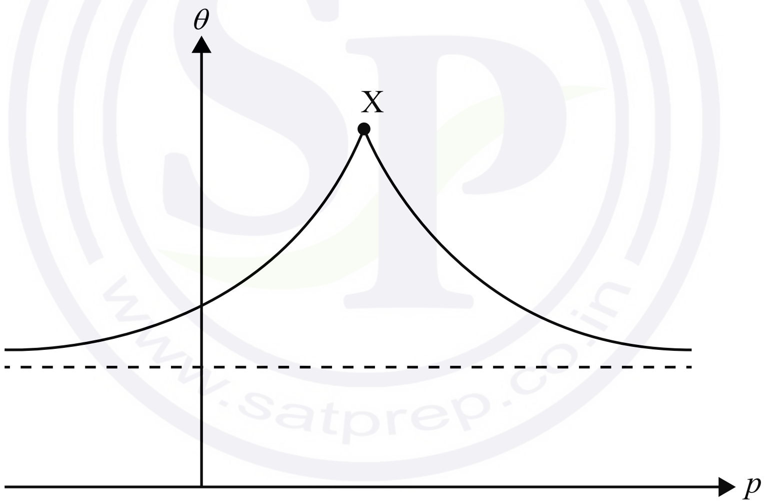
(ii) Hence, show that, if the angle between the faces ABV and ACV is θ, then $\cos\theta = \dfrac{p(3p-20)}{6p^2-40p+100}$. [8]

(b) Consider the case where the faces ABV and ACV are perpendicular.

(i) Find the two possible coordinates of V.

(ii) Comment on the positions of V in relation to the plane ABC. [4]

(c) The following diagram shows the graph of θ against p. The maximum point is shown by X.



(i) At X, find the value of p and the value of θ.

(ii) Find the equation of the horizontal asymptote of the graph. [5]

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Mathematics
Higher level
Paper 1

Monday 13 May 2019 (afternoon)

2 hours

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13 pages

2219 – 7203

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Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 4]

Let $\boldsymbol{a} = \begin{pmatrix} 2 \\ k \\ -1 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} -3 \\ k+2 \\ k \end{pmatrix}$, $k \in \mathbb{R}$.

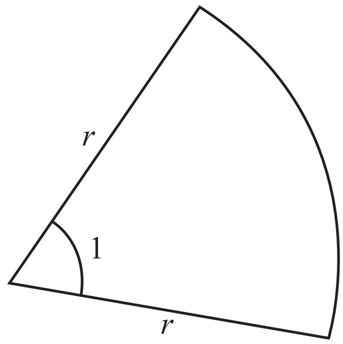
Given that $\boldsymbol{a}$ and $\boldsymbol{b}$ are perpendicular, find the possible values of k.

2. [Maximum mark: 4]

Determine the first three terms of $(1-2x)^{11}$ in ascending powers of x, giving each term in its simplest form.

3. [Maximum mark: 4]

A sector of a circle with radius $r\,\text{cm}$, where $r > 0$, is shown on the following diagram. The sector has an angle of 1 radian at the centre.



Let the area of the sector be $A\,\text{cm}^2$ and the perimeter be $P\,\text{cm}$. Given that $A = P$, find the value of r.

4. [Maximum mark: 7]

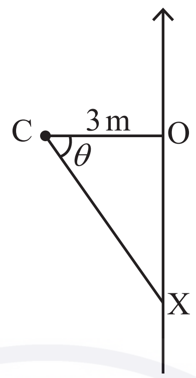
The lengths of two of the sides in a triangle are $4\,\text{cm}$ and $5\,\text{cm}$. Let θ be the angle between the two given sides. The triangle has an area of $\frac{5\sqrt{15}}{2}\,\text{cm}^2$.

(a) Show that $\sin\theta = \frac{\sqrt{15}}{4}$. [1]

(b) Find the two possible values for the length of the third side. [6]

5. [Maximum mark: 6]

A camera at point C is $3\,\mathrm{m}$ from the edge of a straight section of road as shown in the following diagram. The camera detects a car travelling along the road at $t = 0$. It then rotates, always pointing at the car, until the car passes O, the point on the edge of the road closest to the camera.



A car travels along the road at a speed of $24\,\mathrm{m\,s^{-1}}$. Let the position of the car be X and let $\mathrm{O\hat{C}X} = \theta$.

Find $\frac{\mathrm{d}\theta}{\mathrm{d}t}$, the rate of rotation of the camera, in radians per second, at the instant the car passes the point O.

6. [Maximum mark: 7]

Let X be a random variable which follows a normal distribution with mean μ. Given that $\mathrm{P}(X < \mu - 5) = 0.2$, find

(a) $\mathrm{P}(X > \mu + 5)$; [2]

(b) $\mathrm{P}(X < \mu + 5 \mid X > \mu - 5)$. [5]

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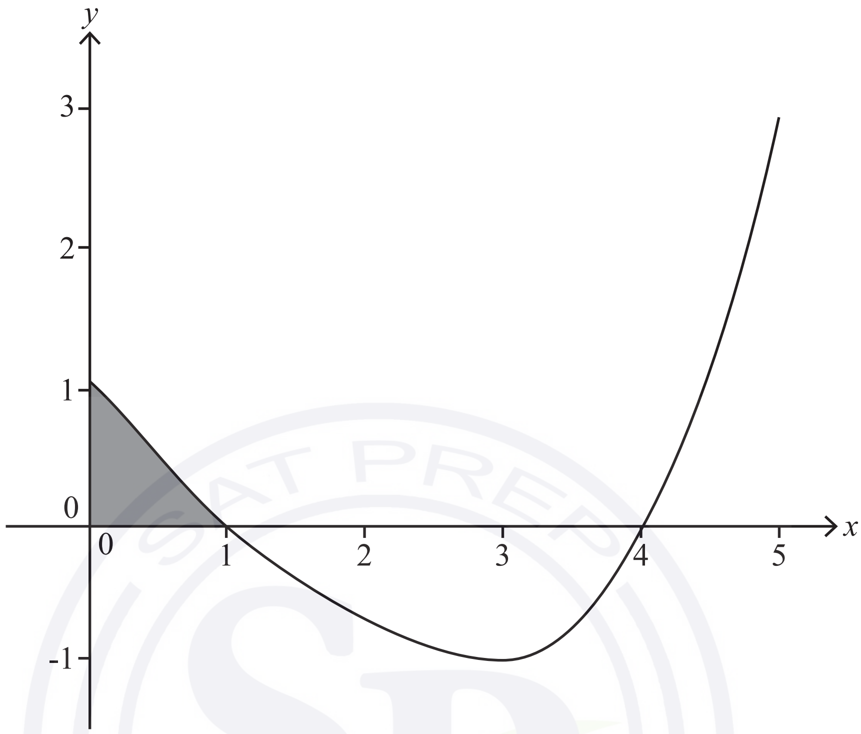
will not be marked.

7. [Maximum mark: 9]

Find the coordinates of the points on the curve $y^3 + 3xy^2 - x^3 = 27$ at which $\frac{dy}{dx} = 0$.

8. [Maximum mark: 9]

The graph of $y = f'(x)$, $0 \le x \le 5$ is shown in the following diagram. The curve intercepts the x-axis at $(1, 0)$ and $(4, 0)$ and has a local minimum at $(3, -1)$.



(a) Write down the x-coordinate of the point of inflexion on the graph of $y = f(x)$. [1]

The shaded area enclosed by the curve $y = f'(x)$, the x-axis and the y-axis is 0.5.
Given that $f(0) = 3$,

(b) find the value of $f(1)$. [3]

The area enclosed by the curve $y = f'(x)$ and the x-axis between $x = 1$ and $x = 4$ is 2.5.

(c) Find the value of $f(4)$. [2]

(d) Sketch the curve $y = f(x)$, $0 \le x \le 5$ indicating clearly the coordinates of the maximum and minimum points and any intercepts with the coordinate axes. [3]

(This question continues on the following page)

(Question 8 continued)

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

9. [Maximum mark: 15]

(a) Show that $(\sin x + \cos x)^2 = 1 + \sin 2x$. [2]

(b) Show that $\sec 2x + \tan 2x = \dfrac{\cos x + \sin x}{\cos x - \sin x}$. [4]

(c) Hence or otherwise find $\int_0^{\frac{\pi}{6}} (\sec 2x + \tan 2x)\,\mathrm{d}x$ in the form $\ln\left(a + \sqrt{b}\right)$ where $a, b \in \mathbb{Z}$. [9]

10. [Maximum mark: 15]

The function $p(x)$ is defined by $p(x) = x^3 - 3x^2 + 8x - 24$ where $x \in \mathbb{R}$.

(a) Find the remainder when $p(x)$ is divided by

(i) $(x - 2)$

(ii) $(x - 3)$. [3]

(b) Prove that $p(x)$ has only one real zero. [4]

(c) Write down the transformation that will transform the graph of $y = p(x)$ onto the graph of $y = 8x^3 - 12x^2 + 16x - 24$. [2]

The random variable X follows a Poisson distribution with a mean of λ and $6\mathrm{P}(X = 3) = 3\mathrm{P}(X = 2) - 2\mathrm{P}(X = 1) + 3\mathrm{P}(X = 0)$.

(d) Find the value of λ. [6]

Do **not** write solutions on this page.

11. [Maximum mark: 20]

Two distinct lines, l_1 and l_2, intersect at a point P. In addition to P, four distinct points are marked out on l_1 and three distinct points on l_2. A mathematician decides to join some of these eight points to form polygons.

(a) (i) Find how many sets of four points can be selected which can form the vertices of a quadrilateral.

(ii) Find how many sets of three points can be selected which can form the vertices of a triangle. [6]

The line l_1 has vector equation $\boldsymbol{r}_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}, \lambda \in \mathbb{R}$ and the line l_2 has vector equation

$$\boldsymbol{r}_2 = \begin{pmatrix} -1 \\ 0 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} 5 \\ 6 \\ 2 \end{pmatrix}, \mu \in \mathbb{R}.$$

The point P has coordinates $(4, 6, 4)$.

(b) Verify that P is the point of intersection of the two lines. [3]

The point A has coordinates $(3, 4, 3)$ and lies on l_1.

(c) Write down the value of λ corresponding to the point A. [1]

The point B has coordinates $(-1, 0, 2)$ and lies on l_2.

(d) Write down $\overrightarrow{\mathrm{PA}}$ and $\overrightarrow{\mathrm{PB}}$. [2]

Let C be the point on l_1 with coordinates $(1, 0, 1)$ and D be the point on l_2 with parameter $\mu = -2$.

(e) Find the area of the quadrilateral CDBA. [8]

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Mathematics
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13 pages

2219 – 7205

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Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 4]

In an arithmetic sequence, the sum of the 3rd and 8th terms is 1.
Given that the sum of the first seven terms is 35, determine the first term and the common difference.

2. [Maximum mark: 6]

Three points in three-dimensional space have coordinates $\mathrm{A}(0, 0, 2)$, $\mathrm{B}(0, 2, 0)$ and $\mathrm{C}(3, 1, 0)$.

(a) Find the vector

(i) $\overrightarrow{\mathrm{AB}}$;

(ii) $\overrightarrow{\mathrm{AC}}$. [2]

(b) Hence or otherwise, find the area of the triangle ABC. [4]

3. [Maximum mark: 5]

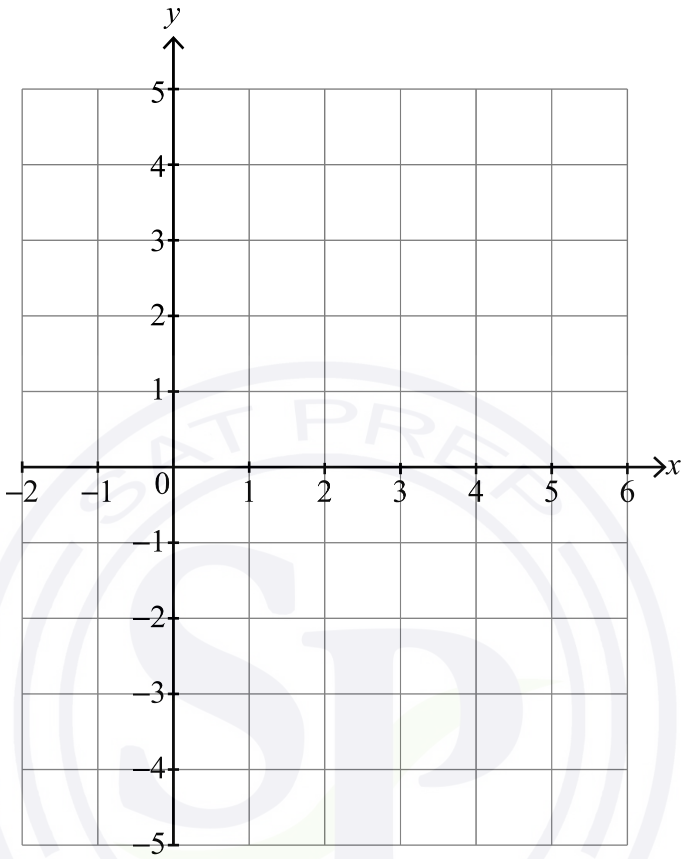
Consider the function $f(x) = x^4 - 6x^2 - 2x + 4$, $x \in \mathbb{R}$.
The graph of f is translated two units to the left to form the function $g(x)$.
Express $g(x)$ in the form $ax^4 + bx^3 + cx^2 + dx + e$ where a, b, c, d, $e \in \mathbb{Z}$.

4. [Maximum mark: 5]

Using the substitution $u = \sin x$, find $\int \frac{\cos^3 x \, dx}{\sqrt{\sin x}}$.

5. [Maximum mark: 8]

(a) Sketch the graph of $y = \dfrac{x-4}{2x-5}$, stating the equations of any asymptotes and the coordinates of any points of intersection with the axes. [5]



(b) Consider the function $f : x \to \sqrt{\dfrac{x-4}{2x-5}}$.

Write down

(i) the largest possible domain of f;

(ii) the corresponding range of f. [3]

6. [Maximum mark: 7]

The curve C is given by the equation $y = x\tan\left(\frac{\pi xy}{4}\right)$.

(a) At the point $(1, 1)$, show that $\frac{dy}{dx} = \frac{2+\pi}{2-\pi}$. [5]

(b) Hence find the equation of the normal to C at the point $(1, 1)$. [2]

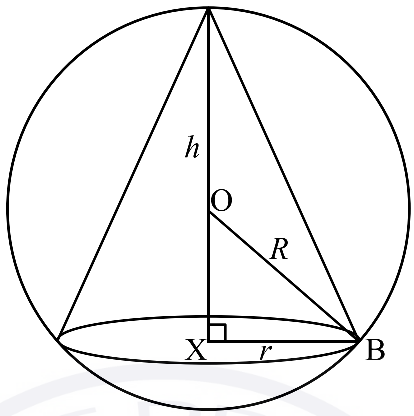
7. [Maximum mark: 7]

Solve the simultaneous equations

$$\log_2 6x = 1 + 2\log_2 y$$
$$1 + \log_6 x = \log_6(15y - 25).$$

8. [Maximum mark: 8]

A right circular cone of radius r is inscribed in a sphere with centre O and radius R as shown in the following diagram. The perpendicular height of the cone is h, X denotes the centre of its base and B a point where the cone touches the sphere.



(a) Show that the volume of the cone may be expressed by $V = \frac{\pi}{3}\left(2Rh^2 - h^3\right)$. [4]

(b) Given that there is one inscribed cone having a maximum volume, show that the volume of this cone is $\frac{32\pi R^3}{81}$. [4]

(This question continues on the following page)

(Question 8 continued)

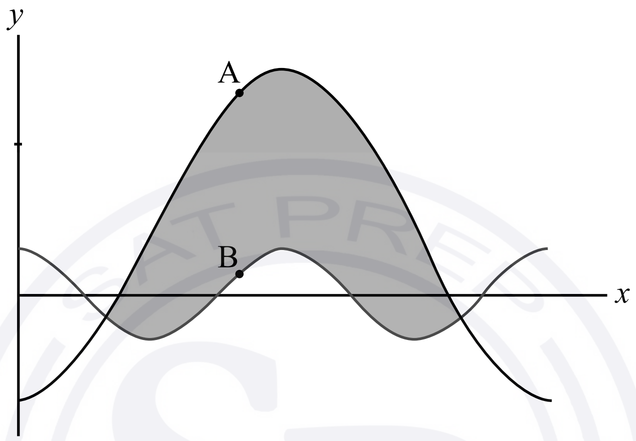
Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

9. [Maximum mark: 17]

Consider the functions f and g defined on the domain $0 < x < 2\pi$ by $f(x) = 3\cos 2x$ and $g(x) = 4 - 11\cos x$.
The following diagram shows the graphs of $y = f(x)$ and $y = g(x)$.



(a) Find the x-coordinates of the points of intersection of the two graphs. [6]

(b) Find the exact area of the shaded region, giving your answer in the form $p\pi + q\sqrt{3}$, where $p, q \in \mathbb{Q}$. [5]

At the points A and B on the diagram, the gradients of the two graphs are equal.

(c) Determine the y-coordinate of A on the graph of g. [6]

Do **not** write solutions on this page.

10. [Maximum mark: 16]

The random variable X has probability density function f given by

$$f(x)=\begin{cases} k(\pi-\arcsin x) & 0\le x\le 1 \\ 0 & \text{otherwise}\end{cases}\text{, where } k \text{ is a positive constant.}$$

(a) State the mode of X. [1]

(b) (i) Find $\int \arcsin x \,\mathrm{d}x$.

(ii) Hence show that $k=\dfrac{2}{2+\pi}$. [6]

(c) Given that $y=\left(\dfrac{x^2}{2}\right)\arcsin x-\left(\dfrac{1}{4}\right)\arcsin x+\left(\dfrac{x}{4}\right)\sqrt{1-x^2}$, show that

(i) $\dfrac{\mathrm{d}y}{\mathrm{d}x}=x\arcsin x$;

(ii) $\mathrm{E}(X)=\dfrac{3\pi}{4(\pi+2)}$. [9]

11. [Maximum mark: 17]

Consider the functions f and g defined by $f(x)=\ln|x|$, $x\in\mathbb{R}\setminus\{0\}$ and $g(x)=\ln|x+k|$, $x\in\mathbb{R}\setminus\{-k\}$, where $k\in\mathbb{R}$, $k>2$.

(a) Describe the transformation by which $f(x)$ is transformed to $g(x)$. [1]

(b) State the range of g. [1]

(c) Sketch the graphs of $y=f(x)$ and $y=g(x)$ on the same axes, clearly stating the points of intersection with any axes. [6]

The graphs of f and g intersect at the point P.

(d) Find the coordinates of P. [2]

The tangent to $y=f(x)$ at P passes through the origin $(0, 0)$.

(e) Determine the value of k. [7]

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will not be marked.

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will not be marked.

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will not be marked.

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Higher level
Paper 1

Monday 12 November 2018 (afternoon)

Candidate session number

2 hours

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics HL and further mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[100 marks]**.

8818 – 7201

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 6]

Consider two events, A and B, such that $\mathrm{P}(A) = \mathrm{P}(A' \cap B) = 0.4$ and $\mathrm{P}(A \cap B) = 0.1$.

(a) By drawing a Venn diagram, or otherwise, find $\mathrm{P}(A \cup B)$. [3]

(b) Show that the events A and B are not independent. [3]

2. [Maximum mark: 5]

A team of four is to be chosen from a group of four boys and four girls.

(a) Find the number of different possible teams that could be chosen. [3]

(b) Find the number of different possible teams that could be chosen, given that the team must include at least one girl and at least one boy. [2]

3. [Maximum mark: 7]

Consider the function $g(x) = 4\cos x + 1$, $a \le x \le \frac{\pi}{2}$ where $a < \frac{\pi}{2}$.

(a) For $a = -\frac{\pi}{2}$, sketch the graph of $y = g(x)$. Indicate clearly the maximum and minimum values of the function. [3]

(b) Write down the least value of a such that g has an inverse. [1]

(c) For the value of a found in part (b),

(i) write down the domain of g^{-1};

(ii) find an expression for $g^{-1}(x)$. [3]

4. [Maximum mark: 7]

Consider the following system of equations where $a \in \mathbb{R}$.

$$2x + 4y - z = 10$$
$$x + 2y + az = 5$$
$$5x + 12y = 2a.$$

(a) Find the value of a for which the system of equations does not have a unique solution. [2]

(b) Find the solution of the system of equations when $a = 2$. [5]

5. [Maximum mark: 6]

The vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ are defined by $\boldsymbol{a} = \begin{pmatrix} 1 \\ 1 \\ t \end{pmatrix}$, $\boldsymbol{b} = \begin{pmatrix} 0 \\ -t \\ 4t \end{pmatrix}$, where $t \in \mathbb{R}$.

(a) Find and simplify an expression for $\boldsymbol{a} \bullet \boldsymbol{b}$ in terms of t. [2]

(b) Hence or otherwise, find the values of t for which the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$ is obtuse. [4]

6. [Maximum mark: 6]

Use mathematical induction to prove that $\sum_{r=1}^{n} r(r!) = (n+1)! - 1$, for $n \in \mathbb{Z}^{+}$.

7. [Maximum mark: 6]

Consider the curves C_1 and C_2 defined as follows

$$C_1 : xy = 4 , x > 0$$
$$C_2 : y^2 - x^2 = 2 , x > 0$$

(a) Using implicit differentiation, or otherwise, find $\frac{dy}{dx}$ for each curve in terms of x and y. [4]

Let $P(a , b)$ be the unique point where the curves C_1 and C_2 intersect.

(b) Show that the tangent to C_1 at P is perpendicular to the tangent to C_2 at P. [2]

8. [Maximum mark: 7]

Consider the equation $z^4 + az^3 + bz^2 + cz + d = 0$, where $a, b, c, d \in \mathbb{R}$ and $z \in \mathbb{C}$.
Two of the roots of the equation are $\log_2 6$ and $i\sqrt{3}$ and the sum of all the roots is $3 + \log_2 3$.
Show that $6a + d + 12 = 0$.

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

9. [Maximum mark: 15]

Consider a triangle OAB such that O has coordinates $(0, 0, 0)$, A has coordinates $(0, 1, 2)$ and B has coordinates $(2b, 0, b-1)$ where $b < 0$.

(a) Find, in terms of b, a Cartesian equation of the plane Π containing this triangle. [5]

Let M be the midpoint of the line segment [OB].

(b) Find, in terms of b, the equation of the line L which passes through M and is perpendicular to the plane Π. [3]

(c) Show that L does not intersect the y-axis for any negative value of b. [7]

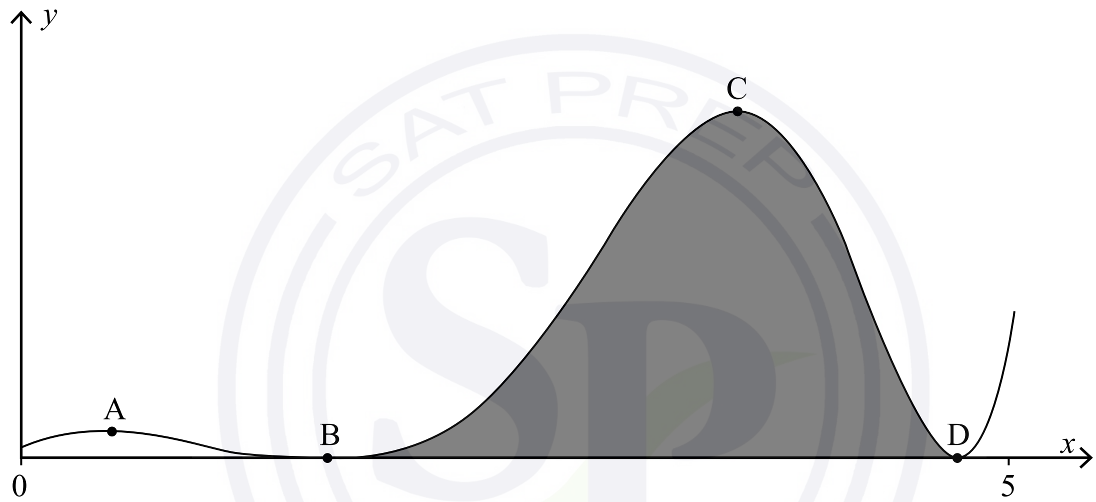
Do **not** write solutions on this page.

10. [Maximum mark: 19]

(a) Use integration by parts to show that $\int e^x \cos 2x dx = \frac{2e^x}{5}\sin 2x + \frac{e^x}{5}\cos 2x + c,\ c \in \mathbb{R}$. [5]

(b) Hence, show that $\int e^x \cos^2 x dx = \frac{e^x}{5}\sin 2x + \frac{e^x}{10}\cos 2x + \frac{e^x}{2} + c,\ c \in \mathbb{R}$. [3]

The function f is defined by $f(x) = e^x \cos^2 x$, where $0 \le x \le 5$. The curve $y = f(x)$ is shown on the following graph which has local maximum points at A and C and touches the x-axis at B and D.



(c) Find the x-coordinates of A and of C, giving your answers in the form $a + \arctan b$, where $a, b \in \mathbb{R}$. [6]

(d) Find the area enclosed by the curve and the x-axis between B and D, as shaded on the diagram. [5]

Do **not** write solutions on this page.

11. [Maximum mark: 16]

(a) Find the roots of $z^{24} = 1$ which satisfy the condition $0 < \arg(z) < \frac{\pi}{2}$, expressing your answers in the form $r\mathrm{e}^{\mathrm{i}\theta}$, where $r, \theta \in \mathbb{R}^+$. [5]

(b) Let S be the sum of the roots found in part (a).

(i) Show that $\operatorname{Re} S = \operatorname{Im} S$.

(ii) By writing $\frac{\pi}{12}$ as $\left(\frac{\pi}{4} - \frac{\pi}{6}\right)$, find the value of $\cos\frac{\pi}{12}$ in the form $\frac{\sqrt{a} + \sqrt{b}}{c}$, where a, b and c are integers to be determined.

(iii) Hence, or otherwise, show that $S = \frac{1}{2}\left(1 + \sqrt{2}\right)\left(1 + \sqrt{3}\right)(1 + \mathrm{i})$. [11]

M18/5/MATHL/HP1/ENG/TZ1/XX

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Higher level
Paper 1

Wednesday 2 May 2018 (afternoon)

2 hours

Candidate session number

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
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- A clean copy of the **mathematics HL and further mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[100 marks]**.

12 pages

2218 – 7203

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 5]

Let $f(x) = x^4 + px^3 + qx + 5$ where p, q are constants.
The remainder when $f(x)$ is divided by $(x+1)$ is 7, and the remainder when $f(x)$ is divided by $(x-2)$ is 1. Find the value of p and the value of q.

2. [Maximum mark: 7]

Let $y = \sin^2\theta,\ 0 \le \theta \le \pi$.

(a) Find $\frac{\mathrm{d}y}{\mathrm{d}\theta}$. [2]

(b) Hence find the values of θ for which $\frac{\mathrm{d}y}{\mathrm{d}\theta} = 2y$. [5]

3. [Maximum mark: 5]

Two unbiased tetrahedral (four-sided) dice with faces labelled $1, 2, 3, 4$ are thrown and the scores recorded. Let the random variable T be the maximum of these two scores. The probability distribution of T is given in the following table.

t	1	2	3	4
$\text{P}(T = t)$	$\frac{1}{16}$	a	b	$\frac{7}{16}$

(a) Find the value of a and the value of b. [3]

(b) Find the expected value of T. [2]

4. [Maximum mark: 6]

Given that $\int_{-2}^{2} f(x)\mathrm{d}x = 10$ and $\int_{0}^{2} f(x)\mathrm{d}x = 12$, find

(a) $\int_{-2}^{0} \left(f(x)+2\right)\mathrm{d}x$; [4]

(b) $\int_{-2}^{0} f(x+2)\,\mathrm{d}x$. [2]

5. [Maximum mark: 6]

Solve $(\ln x)^2 - (\ln 2)(\ln x) < 2(\ln 2)^2$.

6. [Maximum mark: 7]

Use the principle of mathematical induction to prove that

$$1+2\left(\frac{1}{2}\right)+3\left(\frac{1}{2}\right)^{2}+4\left(\frac{1}{2}\right)^{3}+\ldots+n\left(\frac{1}{2}\right)^{n-1}=4-\frac{n+2}{2^{n-1}}\text{, where } n\in\mathbb{Z}^{+}.$$

7. [Maximum mark: 9]

Let $y = \arccos\left(\frac{x}{2}\right)$.

(a) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$. [2]

(b) Find $\int_0^1 \arccos\left(\frac{x}{2}\right)\mathrm{d}x$. [7]

8. [Maximum mark: 5]

Let $a = \sin b,\ 0 < b < \frac{\pi}{2}$.

Find, in terms of b, the solutions of $\sin 2x = -a$, $0 \le x \le \pi$.

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

9. [Maximum mark: 17]

Let $f(x) = \frac{2 - 3x^5}{2x^3}, x \in \mathbb{R}, x \neq 0$.

(a) The graph of $y = f(x)$ has a local maximum at A. Find the coordinates of A. [5]

(b) (i) Show that there is exactly one point of inflexion, B, on the graph of $y = f(x)$.

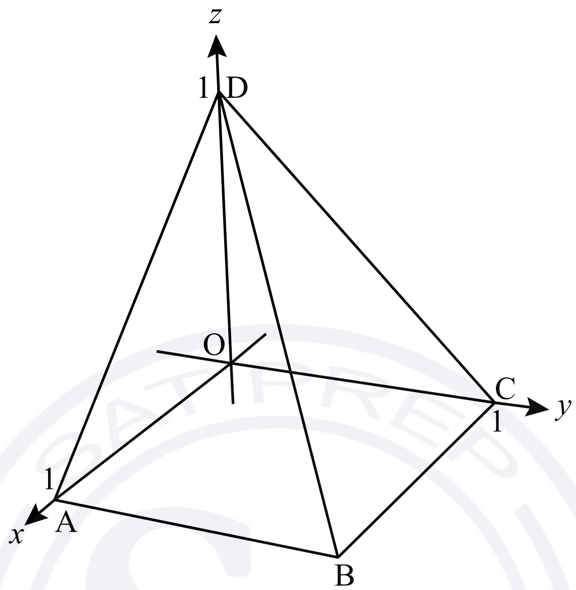
(ii) The coordinates of B can be expressed in the form $\mathrm{B}(2^a, b \times 2^{-3a})$, where $a, b \in \mathbb{Q}$. Find the value of a and the value of b. [8]

(c) Sketch the graph of $y = f(x)$ showing clearly the position of the points A and B. [4]

Do **not** write solutions on this page.

10. [Maximum mark: 19]

The following figure shows a square based pyramid with vertices at $\mathrm{O}(0, 0, 0)$, $\mathrm{A}(1, 0, 0)$, $\mathrm{B}(1, 1, 0)$, $\mathrm{C}(0, 1, 0)$ and $\mathrm{D}(0, 0, 1)$.



(a) Find the Cartesian equation of the plane Π_1, passing through the points A, B and D. [3]

The Cartesian equation of the plane Π_2, passing through the points B, C and D, is $y + z = 1$.

(b) Find the angle between the faces ABD and BCD. [4]

The plane Π_3 passes through O and is normal to the line BD.

(c) Find the Cartesian equation of Π_3. [3]

Π_3 cuts AD and BD at the points P and Q respectively.

(d) Show that P is the midpoint of AD. [4]

(e) Find the area of the triangle OPQ. [5]

Do **not** write solutions on this page.

11. [Maximum mark: 14]

Consider $w = 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$.

(a) (i) Express w^2 and w^3 in modulus-argument form.

(ii) Sketch on an Argand diagram the points represented by w^0, w^1, w^2 and w^3. [5]

These four points form the vertices of a quadrilateral, Q.

(b) Show that the area of the quadrilateral Q is $\frac{21\sqrt{3}}{2}$. [3]

Let $z = 2\left(\cos\frac{\pi}{n} + i\sin\frac{\pi}{n}\right), n \in \mathbb{Z}^+$. The points represented on an Argand diagram by $z^0, z^1, z^2, \ldots, z^n$ form the vertices of a polygon P_n.

(c) Show that the area of the polygon P_n can be expressed in the form $a\left(b^n - 1\right)\sin\frac{\pi}{n}$, where $a, b \in \mathbb{R}$. [6]

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Higher level
Paper 1

Wednesday 2 May 2018 (afternoon)

Candidate session number

2 hours

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
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- The maximum mark for this examination paper is **[100 marks]**.

11 pages

2218 – 7205

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Section A

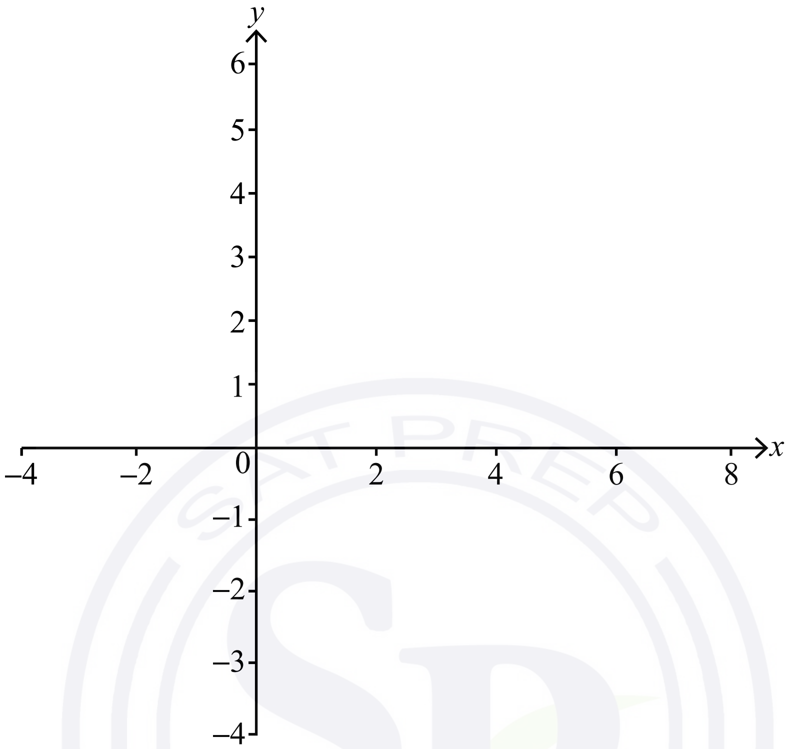
Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 4]

The acute angle between the vectors $3\boldsymbol{i} - 4\boldsymbol{j} - 5\boldsymbol{k}$ and $5\boldsymbol{i} - 4\boldsymbol{j} + 3\boldsymbol{k}$ is denoted by θ. Find $\cos\theta$.

2. [Maximum mark: 7]

(a) Sketch the graphs of $y = \frac{x}{2} + 1$ and $y = |x - 2|$ on the following axes. [3]



(b) Solve the equation $\frac{x}{2} + 1 = |x - 2|$. [4]

3. [Maximum mark: 6]

The discrete random variable X has the following probability distribution, where p is a constant.

x	0	1	2	3	4
$\mathrm{P}(X=x)$	p	$0.5-p$	0.25	0.125	p^3

(a) Find the value of p. [2]

(b) (i) Find μ, the expected value of X.

(ii) Find $\mathrm{P}(X>\mu)$. [4]

4. [Maximum mark: 6]

Consider the curve $y = \frac{1}{1-x} + \frac{4}{x-4}$.

Find the x-coordinates of the points on the curve where the gradient is zero.

5. [Maximum mark: 7]

The geometric sequence $u_1, u_2, u_3, \ldots$ has common ratio r.
Consider the sequence $A = \{a_n = \log_2 |u_n| : n \in \mathbb{Z}^+\}$.

(a) Show that A is an arithmetic sequence, stating its common difference d in terms of r. [4]

A particular geometric sequence has $u_1 = 3$ and a sum to infinity of 4.

(b) Find the value of d. [3]

6. [Maximum mark: 7]

Consider the functions f, g, defined for $x \in \mathbb{R}$, given by $f(x) = \mathrm{e}^{-x}\sin x$ and $g(x) = \mathrm{e}^{-x}\cos x$.

(a) Find

(i) $f'(x)$;

(ii) $g'(x)$. [3]

(b) Hence, or otherwise, find $\int_0^{\pi} \mathrm{e}^{-x}\sin x\,\mathrm{d}x$. [4]

7. [Maximum mark: 6]

Consider the distinct complex numbers $z = a + \mathrm{i}b$, $w = c + \mathrm{i}d$, where $a, b, c, d \in \mathbb{R}$.

(a) Find the real part of $\dfrac{z+w}{z-w}$. [4]

(b) Find the value of the real part of $\dfrac{z+w}{z-w}$ when $|z| = |w|$. [2]

8. [Maximum mark: 7]

(a) Use the substitution $u = x^{\frac{1}{2}}$ to find $\int \frac{dx}{x^{\frac{3}{2}} + x^{\frac{1}{2}}}$. [4]

(b) Hence find the value of $\frac{1}{2}\int_1^9 \frac{dx}{x^{\frac{3}{2}} + x^{\frac{1}{2}}}$, expressing your answer in the form $\arctan q$, where $q \in \mathbb{Q}$. [3]

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Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

9. [Maximum mark: 24]

The points $\mathrm{A}, \mathrm{B}, \mathrm{C}$ and D have position vectors $\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}$ and $\boldsymbol{d}$, relative to the origin O.
It is given that $\vec{\mathrm{AB}} = \vec{\mathrm{DC}}$.

(a) (i) Explain why ABCD is a parallelogram.

(ii) Using vector algebra, show that $\vec{\mathrm{AD}} = \vec{\mathrm{BC}}$. [4]

The position vectors $\vec{\mathrm{OA}}$, $\vec{\mathrm{OB}}$, $\vec{\mathrm{OC}}$ and $\vec{\mathrm{OD}}$ are given by

$$\begin{aligned} \boldsymbol{a} &= \boldsymbol{i} + 2\boldsymbol{j} - 3\boldsymbol{k} \\ \boldsymbol{b} &= 3\boldsymbol{i} - \boldsymbol{j} + p\boldsymbol{k} \\ \boldsymbol{c} &= q\boldsymbol{i} + \boldsymbol{j} + 2\boldsymbol{k} \\ \boldsymbol{d} &= -\boldsymbol{i} + r\boldsymbol{j} - 2\boldsymbol{k} \end{aligned}$$

where p, q and r are constants.

(b) Show that $p = 1$, $q = 1$ and $r = 4$. [5]

(c) Find the area of the parallelogram ABCD. [4]

The point where the diagonals of ABCD intersect is denoted by M.

(d) Find the vector equation of the straight line passing through M and normal to the plane Π containing ABCD. [4]

(e) Find the Cartesian equation of Π. [3]

The plane Π cuts the x, y and z axes at X, Y and Z respectively.

(f) (i) Find the coordinates of X, Y and Z.

(ii) Find YZ. [4]

Do **not** write solutions on this page.

10. [Maximum mark: 14]

The function f is defined by $f(x) = \dfrac{ax + b}{cx + d}$, for $x \in \mathbb{R}, x \neq -\dfrac{d}{c}$.

(a) Find the inverse function f^{-1}, stating its domain. [5]

The function g is defined by $g(x) = \dfrac{2x - 3}{x - 2}$, $x \in \mathbb{R}, x \neq 2$.

(b) (i) Express $g(x)$ in the form $A + \dfrac{B}{x - 2}$ where A, B are constants.

(ii) Sketch the graph of $y = g(x)$. State the equations of any asymptotes and the coordinates of any intercepts with the axes. [5]

The function h is defined by $h(x) = \sqrt{x}$, for $x \geq 0$.

(c) State the domain and range of $h \circ g$. [4]

11. [Maximum mark: 12]

(a) Show that $\log_{r^2} x = \dfrac{1}{2}\log_r x$ where $r, x \in \mathbb{R}^+$. [2]

It is given that $\log_2 y + \log_4 x + \log_4 2x = 0$.

(b) Express y in terms of x. Give your answer in the form $y = px^q$, where p, q are constants. [5]

The region R, is bounded by the graph of the function found in part (b), the x-axis, and the lines $x = 1$ and $x = \alpha$ where $\alpha > 1$. The area of R is $\sqrt{2}$.

(c) Find the value of α. [5]

Please **do not** write on this page.

Answers written on this page
will not be marked.

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Higher level
Paper 1

Monday 13 November 2017 (afternoon)

Candidate session number

2 hours

Instructions to candidates

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- The maximum mark for this examination paper is **[100 marks]**.

14 pages

8817 – 7201

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Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 5]

Solve the equation $\log_2(x+3)+\log_2(x-3)=4$.

2. [Maximum mark: 6]

The points A and B are given by $\mathrm{A}(0, 3, -6)$ and $\mathrm{B}(6, -5, 11)$.

The plane Π is defined by the equation $4x - 3y + 2z = 20$.

(a) Find a vector equation of the line L passing through the points A and B. [3]

(b) Find the coordinates of the point of intersection of the line L with the plane Π. [3]

3. [Maximum mark: 6]

Consider the polynomial $q(x) = 3x^3 - 11x^2 + kx + 8$.

(a) Given that $q(x)$ has a factor $(x - 4)$, find the value of k. [3]

(b) Hence or otherwise, factorize $q(x)$ as a product of linear factors. [3]

4. [Maximum mark: 4]

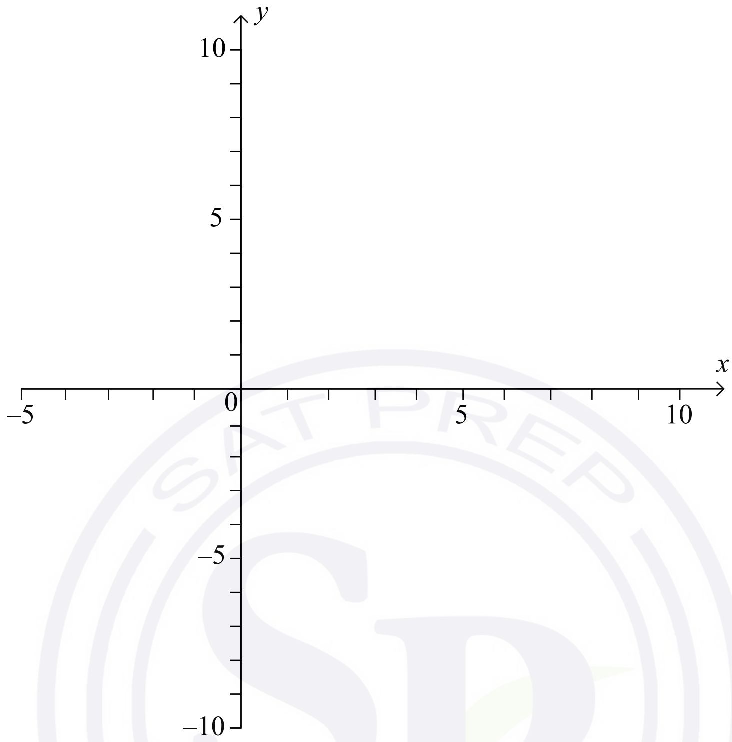
Find the coefficient of x^8 in the expansion of $\left(x^2 - \frac{2}{x}\right)^7$.

5. [Maximum mark: 5]

A particle moves in a straight line such that at time t seconds $(t \geq 0)$, its velocity v, in ms^{-1}, is given by $v = 10te^{-2t}$. Find the exact distance travelled by the particle in the first half-second.

6. [Maximum mark: 9]

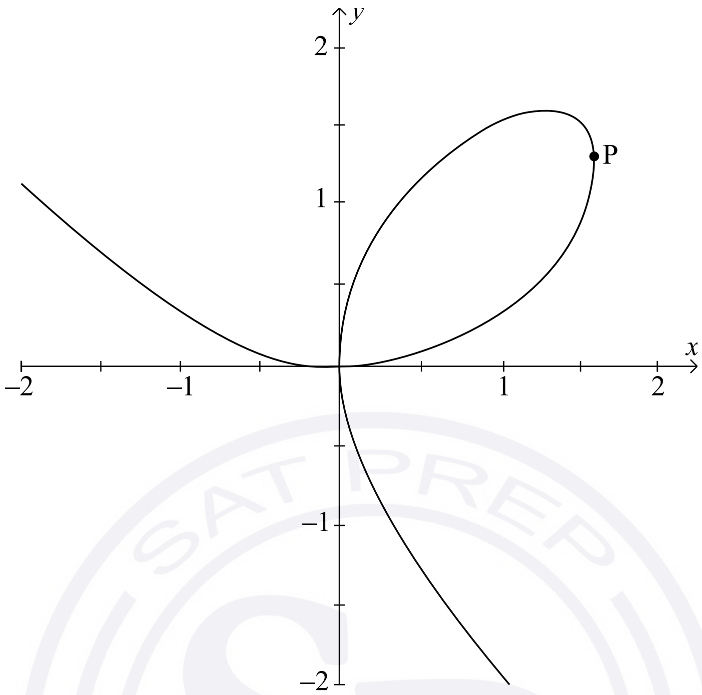
(a) Sketch the graph of $y = \dfrac{1-3x}{x-2}$, showing clearly any asymptotes and stating the coordinates of any points of intersection with the axes. [4]



(b) Hence or otherwise, solve the inequality $\left|\dfrac{1-3x}{x-2}\right| < 2$. [5]

7. [Maximum mark: 8]

The folium of Descartes is a curve defined by the equation $x^3 + y^3 - 3xy = 0$, shown in the following diagram.



Determine the exact coordinates of the point P on the curve where the tangent line is parallel to the y-axis.

8. [Maximum mark: 7]

Determine the roots of the equation $(z + 2\mathrm{i})^3 = 216\mathrm{i}$, $z \in \mathbb{C}$, giving the answers in the form $z = a\sqrt{3} + b\mathrm{i}$ where $a, b \in \mathbb{Z}$.

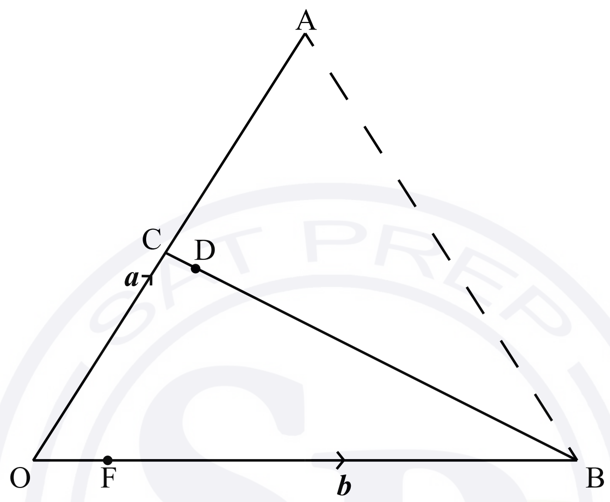
Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

9. [Maximum mark: 18]

In the following diagram, $\overrightarrow{OA} = \boldsymbol{a}$, $\overrightarrow{OB} = \boldsymbol{b}$. C is the midpoint of [OA] and $\overrightarrow{OF} = \frac{1}{6}\overrightarrow{FB}$.



(a) Find, in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$

(i) $\overrightarrow{OF}$;

(ii) $\overrightarrow{AF}$. [3]

It is given also that $\overrightarrow{AD} = \lambda\overrightarrow{AF}$ and $\overrightarrow{CD} = \mu\overrightarrow{CB}$, where $\lambda, \mu \in \mathbb{R}$.

(b) Find an expression for

(i) $\overrightarrow{OD}$ in terms of $\boldsymbol{a}$, $\boldsymbol{b}$ and λ;

(ii) $\overrightarrow{OD}$ in terms of $\boldsymbol{a}$, $\boldsymbol{b}$ and μ. [4]

(c) Show that $\mu = \frac{1}{13}$, and find the value of λ. [4]

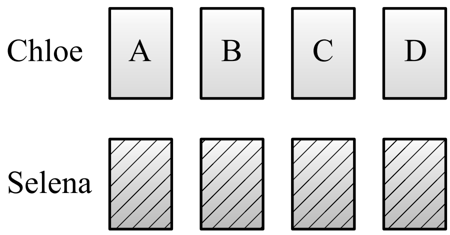
(d) Deduce an expression for $\overrightarrow{CD}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$ only. [2]

(e) Given that area $\Delta OAB = k(\text{area } \Delta CAD)$, find the value of k. [5]

Do **not** write solutions on this page.

10. [Maximum mark: 11]

Chloe and Selena play a game where each have four cards showing capital letters A, B, C and D.
Chloe lays her cards face up on the table in order A, B, C, D as shown in the following diagram.



Selena shuffles her cards and lays them face down on the table. She then turns them over one by one to see if her card matches with Chloe's card directly above.
Chloe wins if **no** matches occur; otherwise Selena wins.

(a) Show that the probability that Chloe wins the game is $\frac{3}{8}$. [6]

Chloe and Selena repeat their game so that they play a total of 50 times.
Suppose the discrete random variable X represents the number of times Chloe wins.

(b) Determine

(i) the mean of X;

(ii) the variance of X. [5]

Do **not** write solutions on this page.

11. [Maximum mark: 21]

Consider the function $f_n(x) = (\cos 2x)(\cos 4x)\ldots(\cos 2^n x)$, $n \in \mathbb{Z}^+$.

(a) Determine whether f_n is an odd or even function, justifying your answer. [2]

(b) By using mathematical induction, prove that

$$f_n(x) = \frac{\sin 2^{n+1} x}{2^n \sin 2x},\ x \neq \frac{m\pi}{2} \text{ where } m \in \mathbb{Z}.$$ [8]

(c) Hence or otherwise, find an expression for the derivative of $f_n(x)$ with respect to x. [3]

(d) Show that, for $n > 1$, the equation of the tangent to the curve $y = f_n(x)$ at $x = \frac{\pi}{4}$ is $4x - 2y - \pi = 0$. [8]

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Answers written on this page
will not be marked.

Please **do not** write on this page.

Answers written on this page
will not be marked.

M17/5/MATHL/HP1/ENG/TZ1/XX

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Higher level
Paper 1

Thursday 4 May 2017 (afternoon)

Candidate session number

2 hours

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics HL and further mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[100 marks]**.

2217 – 7203

International Baccalaureate
Baccalauréat International
Bachillerato Internacional

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 4]

Find the solution of $\log_2 x - \log_2 5 = 2 + \log_2 3$.

2. [Maximum mark: 6]

Consider the complex numbers $z_1 = 1 + \sqrt{3}\,\mathrm{i}$, $z_2 = 1 + \mathrm{i}$ and $w = \dfrac{z_1}{z_2}$.

(a) By expressing z_1 and z_2 in modulus-argument form write down

(i) the modulus of w;

(ii) the argument of w. [4]

(b) Find the smallest positive integer value of n, such that w^n is a real number. [2]

3. [Maximum mark: 5]

Solve the equation $\sec^2 x + 2\tan x = 0$, $0 \le x \le 2\pi$.

4. [Maximum mark: 5]

Three girls and four boys are seated randomly on a straight bench. Find the probability that the girls sit together and the boys sit together.

5. [Maximum mark: 7]

ABCD is a parallelogram, where $\vec{AB} = -\boldsymbol{i} + 2\boldsymbol{j} + 3\boldsymbol{k}$ and $\vec{AD} = 4\boldsymbol{i} - \boldsymbol{j} - 2\boldsymbol{k}$.

(a) Find the area of the parallelogram ABCD. [3]

(b) By using a suitable scalar product of two vectors, determine whether $\text{A}\hat{\text{B}}\text{C}$ is acute or obtuse. [4]

6. [Maximum mark: 5]

Consider the graphs of $y = |x|$ and $y = -|x| + b$, where $b \in \mathbb{Z}^{+}$.

(a) Sketch the graphs on the same set of axes. [2]

(b) Given that the graphs enclose a region of area 18 square units, find the value of b. [3]

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7. [Maximum mark: 7]

An arithmetic sequence $u_1, u_2, u_3 \ldots$ has $u_1 = 1$ and common difference $d \neq 0$. Given that u_2, u_3 and u_6 are the first three terms of a geometric sequence

(a) find the value of d. [4]

Given that $u_N = -15$

(b) determine the value of $\sum_{r=1}^{N} u_r$. [3]

8. [Maximum mark: 6]

Use the method of mathematical induction to prove that $4^n + 15n - 1$ is divisible by 9 for $n \in \mathbb{Z}^+$.

9. [Maximum mark: 5]

Find $\int \arcsin x \, dx$.

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

10. [Maximum mark: 15]

The continuous random variable X has a probability density function given by

$$f(x) = \begin{cases} k\sin\left(\frac{\pi x}{6}\right), & 0 \le x \le 6 \\ 0, & \text{otherwise.} \end{cases}$$

(a) Find the value of k. [4]

(b) By considering the graph of f write down

(i) the mean of X;

(ii) the median of X;

(iii) the mode of X. [3]

(c) (i) Show that $P(0 \le X \le 2) = \frac{1}{4}$.

(ii) Hence state the interquartile range of X. [6]

(d) Calculate $P(X \le 4 \mid X \ge 3)$. [2]

Do **not** write solutions on this page.

11. [Maximum mark: 17]

(a) (i) Express $x^2 + 3x + 2$ in the form $(x + h)^2 + k$.

(ii) Factorize $x^2 + 3x + 2$. [2]

Consider the function $f(x) = \dfrac{1}{x^2 + 3x + 2}$, $x \in \mathbb{R}$, $x \neq -2$, $x \neq -1$.

(b) Sketch the graph of $f(x)$, indicating on it the equations of the asymptotes, the coordinates of the y-intercept and the local maximum. [5]

(c) Show that $\dfrac{1}{x+1} - \dfrac{1}{x+2} = \dfrac{1}{x^2 + 3x + 2}$. [1]

(d) Hence find the value of p if $\int_0^1 f(x)\,\mathrm{d}x = \ln(p)$. [4]

(e) Sketch the graph of $y = f(|x|)$. [2]

(f) Determine the area of the region enclosed between the graph of $y = f(|x|)$, the x-axis and the lines with equations $x = -1$ and $x = 1$. [3]

Do **not** write solutions on this page.

12. [Maximum mark: 18]

Consider the polynomial $P(z) = z^5 - 10z^2 + 15z - 6$, $z \in \mathbb{C}$.

(a) Write down the sum and the product of the roots of $P(z) = 0$. [2]

(b) Show that $(z - 1)$ is a factor of $P(z)$. [2]

The polynomial can be written in the form $P(z) = (z - 1)^3\left(z^2 + bz + c\right)$.

(c) Find the value of b and the value of c. [5]

(d) Hence find the complex roots of $P(z) = 0$. [3]

Consider the function $q(x) = x^5 - 10x^2 + 15x - 6$, $x \in \mathbb{R}$.

(e) (i) Show that the graph of $y = q(x)$ is concave up for $x > 1$.

(ii) Sketch the graph of $y = q(x)$ showing clearly any intercepts with the axes. [6]

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Answers written on this page will not be marked.

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Answers written on this page will not be marked.

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Higher level
Paper 1

Thursday 4 May 2017 (afternoon)

Candidate session number

2 hours

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics HL and further mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[100 marks]**.

12 pages

2217 – 7205

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 5]

Find the term independent of x in the binomial expansion of $\left(2x^2 + \frac{1}{2x^3}\right)^{10}$.

2. [Maximum mark: 6]

The function f is defined by $f(x) = 2x^3 + 5$, $-2 \le x \le 2$.

(a) Write down the range of f. [2]

(b) Find an expression for $f^{-1}(x)$. [2]

(c) Write down the domain and range of f^{-1}. [2]

3. [Maximum mark: 5]

The 1st, 4th and 8th terms of an arithmetic sequence, with common difference d, $d \neq 0$, are the first three terms of a geometric sequence, with common ratio r. Given that the 1st term of both sequences is 9 find

(a) the value of d; [4]

(b) the value of r. [1]

4. [Maximum mark: 7]

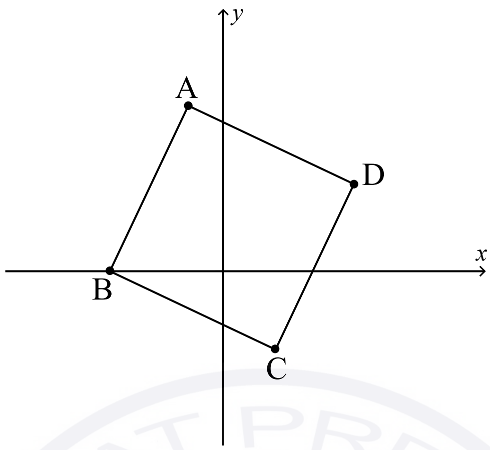
A particle moves along a straight line. Its displacement, s metres, at time t seconds is given by $s = t + \cos 2t$, $t \geq 0$. The first two times when the particle is at rest are denoted by t_1 and t_2, where $t_1 < t_2$.

(a) Find t_1 and t_2. [5]

(b) Find the displacement of the particle when $t = t_1$. [2]

5. [Maximum mark: 4]

In the following Argand diagram the point A represents the complex number $-1 + 4i$ and the point B represents the complex number $-3 + 0i$. The shape of ABCD is a square. Determine the complex numbers represented by the points C and D.



6. [Maximum mark: 7]

(a) Using the substitution $x = \tan\theta$ show that $\int_0^1 \frac{1}{\left(x^2+1\right)^2}\,dx = \int_0^{\frac{\pi}{4}} \cos^2\theta\, d\theta$. [4]

(b) Hence find the value of $\int_0^1 \frac{1}{\left(x^2+1\right)^2}\,dx$. [3]

7. [Maximum mark: 7]

(a) The random variable X has the Poisson distribution $\mathrm{Po}(m)$. Given that $\mathrm{P}(X > 0) = \frac{3}{4}$, find the value of m in the form $\ln a$ where a is an integer. [3]

(b) The random variable Y has the Poisson distribution $\mathrm{Po}(2m)$. Find $\mathrm{P}(Y > 1)$ in the form $\frac{b - \ln c}{c}$ where b and c are integers. [4]

8. [Maximum mark: 9]

Prove by mathematical induction that $\binom{2}{2}+\binom{3}{2}+\binom{4}{2}+\ldots+\binom{n-1}{2}=\binom{n}{3}$,

where $n \in \mathbb{Z}$, $n \geq 3$.

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

9. [Maximum mark: 17]

Consider the function f defined by $f(x) = x^2 - a^2, x \in \mathbb{R}$ where a is a positive constant.

(a) Showing any x and y intercepts, any maximum or minimum points and any asymptotes, sketch the following curves on separate axes.

(i) $y = f(x)$;

(ii) $y = \dfrac{1}{f(x)}$;

(iii) $y = \left|\dfrac{1}{f(x)}\right|$. [8]

(b) Find $\int f(x)\cos x\,\mathrm{d}x$. [5]

The function g is defined by $g(x) = x\sqrt{f(x)}$ for $|x| > a$.

(c) By finding $g'(x)$ explain why g is an increasing function. [4]

Do **not** write solutions on this page.

10. [Maximum mark: 11]

A window is made in the shape of a rectangle with a semicircle of radius r metres on top, as shown in the diagram. The perimeter of the window is a constant P metres.

(a) (i) Find the area of the window in terms of P and r.

(ii) Find the width of the window in terms of P when the area is a maximum, justifying that this is a maximum. [9]

(b) Show that in this case the height of the rectangle is equal to the radius of the semicircle. [2]

Do **not** write solutions on this page.

11. [Maximum mark: 22]

(a) Solve $2\sin(x+60^\circ)=\cos(x+30^\circ)$, $0^\circ \le x \le 180^\circ$. [5]

(b) Show that $\sin 105^\circ + \cos 105^\circ = \frac{1}{\sqrt{2}}$. [3]

(c) Let $z = 1 - \cos 2\theta - \mathrm{i}\sin 2\theta$, $z \in \mathbb{C}$, $0 \le \theta \le \pi$.

(i) Find the modulus and argument of z in terms of θ. Express each answer in its simplest form.

(ii) Hence find the cube roots of z in modulus-argument form. [14]

N16/5/MATHL/HP1/ENG/TZ0/XX

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Higher level
Paper 1

Thursday 10 November 2016 (afternoon)

Candidate session number

2 hours

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics HL and further mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[120 marks]**.

13 pages

8816 – 7201

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions in the boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 5]

Find the coordinates of the point of intersection of the planes defined by the equations $x+y+z=3$, $x-y+z=5$ and $x+y+2z=6$.

2. [Maximum mark: 4]

The faces of a fair six-sided die are numbered $1, 2, 2, 4, 4, 6$. Let X be the discrete random variable that models the score obtained when this die is rolled.

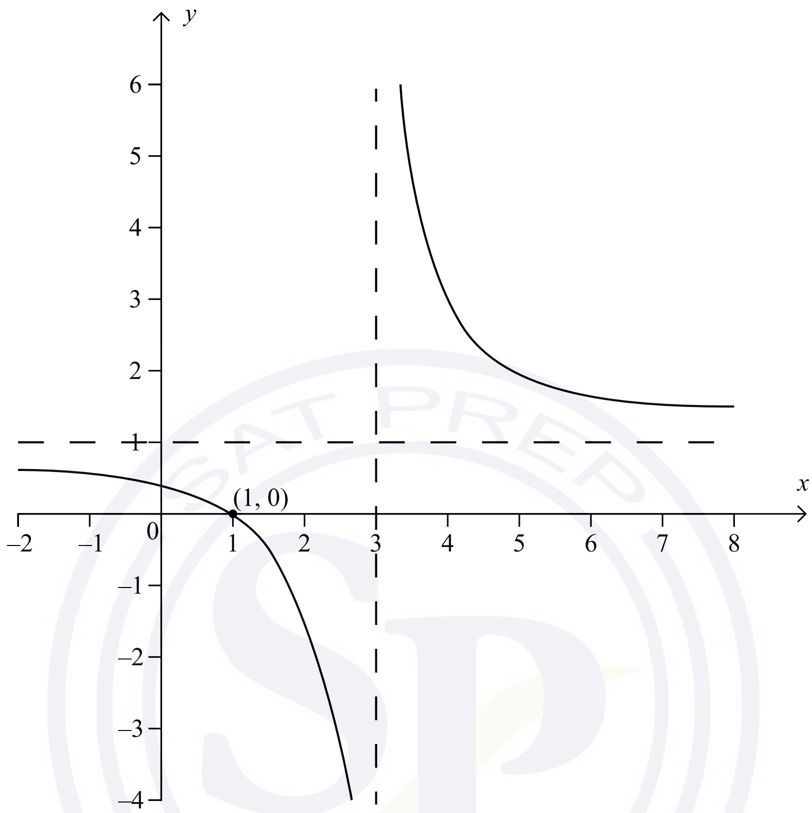
(a) Complete the probability distribution table for X. [2]

x				
$\mathrm{P}(X = x)$				

(b) Find the expected value of X. [2]

3. [Maximum mark: 4]

A rational function is defined by $f(x) = a + \dfrac{b}{x - c}$ where the parameters $a, b, c \in \mathbb{Z}$ and $x \in \mathbb{R} \setminus \{c\}$. The following diagram represents the graph of $y = f(x)$.



Using the information on the graph,

(a) state the value of a and the value of c; [2]

(b) find the value of b. [2]

. . . .

4. [Maximum mark: 5]

Consider the vectors $\boldsymbol{a} = \boldsymbol{i} - 3\boldsymbol{j} - 2\boldsymbol{k}$, $\boldsymbol{b} = -3\boldsymbol{j} + 2\boldsymbol{k}$.

(a) Find $\boldsymbol{a} \times \boldsymbol{b}$. [2]

(b) Hence find the Cartesian equation of the plane containing the vectors $\boldsymbol{a}$ and $\boldsymbol{b}$, and passing through the point $(1, 0, -1)$. [3]

5. [Maximum mark: 6]

The quadratic equation $x^2 - 2kx + (k - 1) = 0$ has roots α and β such that $\alpha^2 + \beta^2 = 4$. Without solving the equation, find the possible values of the real number k.

6. [Maximum mark: 7]

The sum of the first n terms of a sequence $\{u_n\}$ is given by $S_n = 3n^2 - 2n$, where $n \in \mathbb{Z}^+$.

(a) Write down the value of u_1. [1]

(b) Find the value of u_6. [2]

(c) Prove that $\{u_n\}$ is an arithmetic sequence, stating clearly its common difference. [4]

7. [Maximum mark: 5]

Solve the equation $4^x + 2^{x+2} = 3$.

8. [Maximum mark: 6]

Consider the lines l_1 and l_2 defined by

$$l_1 : \boldsymbol{r} = \begin{pmatrix} -3 \\ -2 \\ a \end{pmatrix} + \beta \begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix} \text{ and } l_2 : \frac{6-x}{3} = \frac{y-2}{4} = 1 - z \text{ where } a \text{ is a constant.}$$

Given that the lines l_1 and l_2 intersect at a point P,

(a) find the value of a; [4]

(b) determine the coordinates of the point of intersection P. [2]

9. [Maximum mark: 9]

A curve has equation $3x - 2y^2e^{x-1} = 2$.

(a) Find an expression for $\frac{dy}{dx}$ in terms of x and y. [5]

(b) Find the equations of the tangents to this curve at the points where the curve intersects the line $x = 1$. [4]

10. [Maximum mark: 9]

Consider two events A and B defined in the same sample space.

(a) Show that $P(A \cup B) = P(A) + P(A' \cap B)$. [3]

(b) Given that $P(A \cup B) = \frac{4}{9}$, $P(B \mid A) = \frac{1}{3}$ and $P(B \mid A') = \frac{1}{6}$,

(i) show that $P(A) = \frac{1}{3}$;

(ii) hence find $P(B)$. [6]

Turn over

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

11. [Maximum mark: 22]

Let $y = e^x \sin x$.

(a) Find an expression for $\frac{dy}{dx}$. [2]

(b) Show that $\frac{d^2y}{dx^2} = 2e^x \cos x$. [2]

Consider the function f defined by $f(x) = e^x \sin x$, $0 \le x \le \pi$.

(c) Show that the function f has a local maximum value when $x = \frac{3\pi}{4}$. [2]

(d) Find the x-coordinate of the point of inflexion of the graph of f. [2]

(e) Sketch the graph of f, clearly indicating the position of the local maximum point, the point of inflexion and the axes intercepts. [3]

(f) Find the area of the region enclosed by the graph of f and the x-axis. [6]

The curvature at any point (x, y) on a graph is defined as $\kappa = \frac{\left|\frac{d^2y}{dx^2}\right|}{\left(1 + \left(\frac{dy}{dx}\right)^2\right)^{\frac{3}{2}}}$.

(g) Find the value of the curvature of the graph of f at the local maximum point. [3]

(h) Find the value κ for $x = \frac{\pi}{2}$ and comment on its meaning with respect to the shape of the graph. [2]

Do **not** write solutions on this page.

12. [Maximum mark: 19]

Let ω be one of the non-real solutions of the equation $z^3 = 1$.

(a) Determine the value of

(i) $1 + \omega + \omega^2$;

(ii) $1 + \omega^* + (\omega^*)^2$. [4]

(b) Show that $(\omega - 3\omega^2)(\omega^2 - 3\omega) = 13$. [4]

Consider the complex numbers $p = 1 - 3\mathrm{i}$ and $q = x + (2x + 1)\mathrm{i}$, where $x \in \mathbb{R}$.

(c) Find the values of x that satisfy the equation $|p| = |q|$. [5]

(d) Solve the inequality $\mathrm{Re}(pq) + 8 < (\mathrm{Im}(pq))^2$. [6]

13. [Maximum mark: 19]

(a) Find the value of $\sin\frac{\pi}{4} + \sin\frac{3\pi}{4} + \sin\frac{5\pi}{4} + \sin\frac{7\pi}{4} + \sin\frac{9\pi}{4}$. [2]

(b) Show that $\frac{1 - \cos 2x}{2\sin x} \equiv \sin x, x \neq k\pi$ where $k \in \mathbb{Z}$. [2]

(c) Use the principle of mathematical induction to prove that
$\sin x + \sin 3x + \ldots + \sin(2n - 1)x = \frac{1 - \cos 2nx}{2\sin x}, n \in \mathbb{Z}^+, x \neq k\pi$ where $k \in \mathbb{Z}$. [9]

(d) Hence or otherwise solve the equation $\sin x + \sin 3x = \cos x$ in the interval $0 < x < \pi$. [6]

Please **do not** write on this page.

Answers written on this page will not be marked.

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Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Higher level
Paper 1

Tuesday 10 May 2016 (afternoon)

Candidate session number

2 hours

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics HL and further mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[120 marks]**.

2216 – 7203

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions in the boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 6]

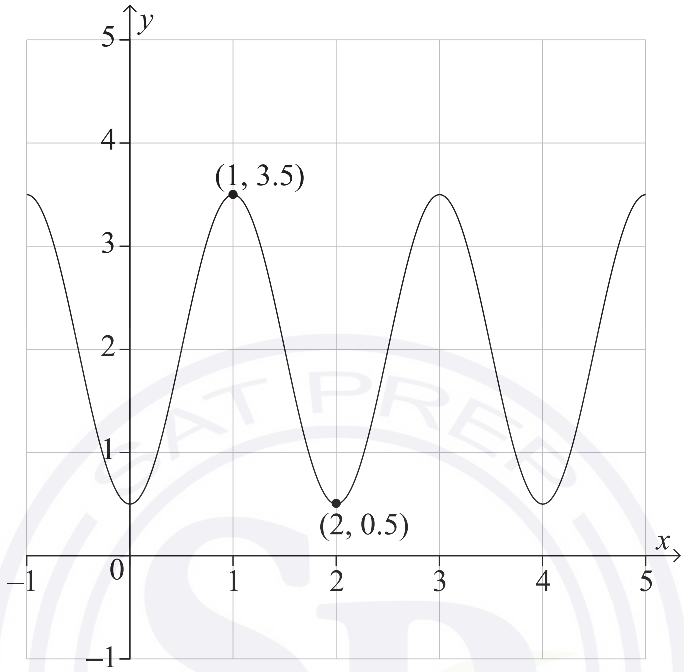
The fifth term of an arithmetic sequence is equal to 6 and the sum of the first 12 terms is 45.
Find the first term and the common difference.

2. [Maximum mark: 4]

At a skiing competition the mean time of the first three skiers is 34.1 seconds. The time for the fourth skier is then recorded and the mean time of the first four skiers is 35.0 seconds. Find the time achieved by the fourth skier.

3. [Maximum mark: 6]

The following diagram shows the curve $y = a\sin\left(b(x+c)\right) + d$, where a, b, c and d are all positive constants. The curve has a maximum point at $(1, 3.5)$ and a minimum point at $(2, 0.5)$.



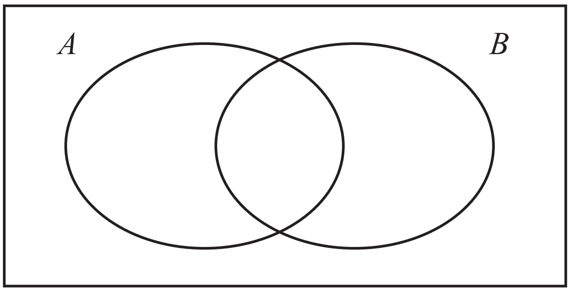
(a) Write down the value of a and the value of d. [2]

(b) Find the value of b. [2]

(c) Find the smallest possible value of c, given $c > 0$. [2]

4. [Maximum mark: 5]

(a) On the Venn diagram shade the region $A' \cap B'$. [1]



Two events A and B are such that $P(A \cap B') = 0.2$ and $P(A \cup B) = 0.9$.

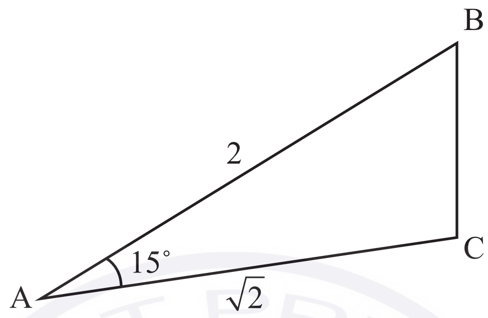
(b) Find $P(A' | B')$. [4]

5. [Maximum mark: 8]

(a) Expand and simplify $\left(1-\sqrt{3}\right)^2$. [1]

(b) By writing 15° as $60^\circ - 45^\circ$ find the value of $\cos(15^\circ)$. [3]

The following diagram shows the triangle ABC where $\text{AB} = 2$, $\text{AC} = \sqrt{2}$ and $\text{B}\hat{\text{A}}\text{C} = 15^\circ$.



(c) Find BC in the form $a + \sqrt{b}$ where $a, b \in \mathbb{Z}$. [4]

6. [Maximum mark: 4]

Find integer values of m and n for which

$$m - n\log_3 2 = 10\log_9 6$$

7. [Maximum mark: 8]

(a) Sketch on the same axes the curve $y = \left|\frac{7}{x-4}\right|$ and the line $y = x + 2$, clearly indicating any axes intercepts and any asymptotes. [3]

(b) Find the exact solutions to the equation $x + 2 = \left|\frac{7}{x-4}\right|$. [5]

8. [Maximum mark: 5]

O, A, B and C are distinct points such that $\overrightarrow{\mathrm{OA}} = \boldsymbol{a}$, $\overrightarrow{\mathrm{OB}} = \boldsymbol{b}$ and $\overrightarrow{\mathrm{OC}} = \boldsymbol{c}$.
It is given that $\boldsymbol{c}$ is perpendicular to $\overrightarrow{\mathrm{AB}}$ and $\boldsymbol{b}$ is perpendicular to $\overrightarrow{\mathrm{AC}}$.

Prove that $\boldsymbol{a}$ is perpendicular to $\overrightarrow{\mathrm{BC}}$.

9. [Maximum mark: 7]

A curve is given by the equation $y = \sin(\pi \cos x)$.

Find the coordinates of all the points on the curve for which $\frac{dy}{dx} = 0$, $0 \le x \le \pi$.

10. [Maximum mark: 7]

Find the x-coordinates of all the points on the curve $y = 2x^4 + 6x^3 + \frac{7}{2}x^2 - 5x + \frac{3}{2}$ at which the tangent to the curve is parallel to the tangent at $(-1, 6)$.

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

11. [Maximum mark: 21]

Two planes have equations

$$\Pi_1 : 4x + y + z = 8 \text{ and } \Pi_2 : 4x + 3y - z = 0$$

(a) Find the cosine of the angle between the two planes in the form $\sqrt{\frac{p}{q}}$ where $p, q \in \mathbb{Z}$. [4]

Let L be the line of intersection of the two planes.

(b) (i) Show that L has direction $\begin{pmatrix} -1 \\ 2 \\ 2 \end{pmatrix}$.

(ii) Show that the point $\mathrm{A}(1, 0, 4)$ lies on both planes.

(iii) Write down a vector equation of L. [6]

B is the point on Π_1 with coordinates $(a, b, 1)$.

(c) Given the vector $\overrightarrow{\mathrm{AB}}$ is perpendicular to L find the value of a and the value of b. [5]

(d) Show that $\mathrm{AB} = 3\sqrt{2}$. [1]

The point P lies on L and $\mathrm{A\hat{B}P} = 45°$.

(e) Find the coordinates of the two possible positions of P. [5]

Do **not** write solutions on this page.

12. [Maximum mark: 21]

(a) Use de Moivre's theorem to find the value of $\left(\cos\left(\frac{\pi}{3}\right)+\mathrm{i}\sin\left(\frac{\pi}{3}\right)\right)^3$. [2]

(b) Use mathematical induction to prove that

$$(\cos\theta - \mathrm{i}\sin\theta)^n = \cos n\theta - \mathrm{i}\sin n\theta \text{ for } n \in \mathbb{Z}^+.$$ [6]

Let $z = \cos\theta + \mathrm{i}\sin\theta$.

(c) Find an expression in terms of θ for $(z)^n + (z^*)^n$, $n \in \mathbb{Z}^+$ where z^* is the complex conjugate of z. [2]

(d) (i) Show that $zz^* = 1$.

(ii) Write down the binomial expansion of $(z + z^*)^3$ in terms of z and z^*.

(iii) Hence show that $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$. [5]

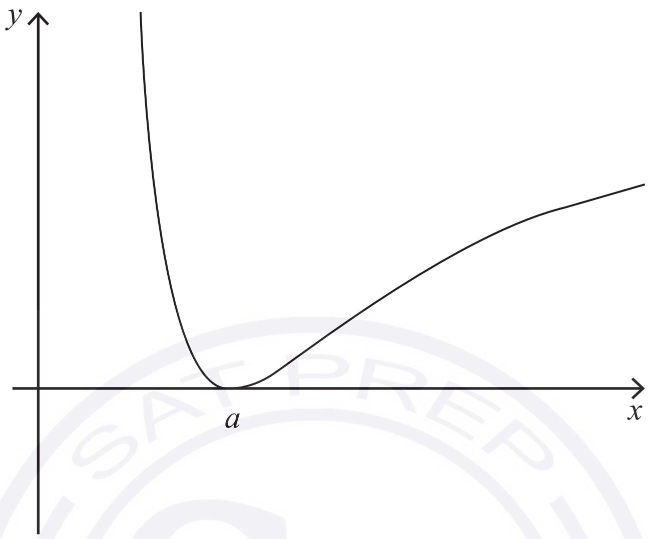
(e) Hence solve $4\cos^3\theta - 2\cos^2\theta - 3\cos\theta + 1 = 0$ for $0 \le \theta < \pi$. [6]

Turn over

Do **not** write solutions on this page.

13. [Maximum mark: 18]

The following diagram shows the graph of $y = \frac{(\ln x)^2}{x}, x > 0$.



(a) Given that the curve passes through the point $(a, 0)$, state the value of a. [1]

The region R is enclosed by the curve, the x-axis and the line $x = \mathrm{e}$.

(b) Use the substitution $u = \ln x$ to find the area of the region R. [5]

Let $I_n = \int_1^{\mathrm{e}} \frac{(\ln x)^n}{x^2} \mathrm{d}x, n \in \mathbb{N}$.

(c) (i) Find the value of I_0.

(ii) Prove that $I_n = -\frac{1}{\mathrm{e}} + nI_{n-1}, n \in \mathbb{Z}^+$.

(iii) Hence find the value of I_1. [7]

(d) Find the volume of the solid formed when the region R is rotated through 2π about the x-axis. [5]

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Answers written on this page will not be marked.

Please **do not** write on this page.

Answers written on this page will not be marked.

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Higher level
Paper 1

Tuesday 10 May 2016 (afternoon)

Candidate session number

2 hours

Instructions to candidates

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- Do not open this examination paper until instructed to do so.
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2216 – 7205

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Section A

Answer **all** questions in the boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 6]

The following system of equations represents three planes in space.

$$x + 3y + z = -1$$
$$x + 2y - 2z = 15$$
$$2x + y - z = 6$$

Find the coordinates of the point of intersection of the three planes.

2. [Maximum mark: 5]

The function f is defined as $f(x) = \frac{3x + 2}{x + 1}, x \in \mathbb{R}, x \neq -1$.

Sketch the graph of $y = f(x)$, clearly indicating and stating the equations of any asymptotes and the coordinates of any axes intercepts.

3. [Maximum mark: 5]

(a) Show that $\cot\alpha = \tan\left(\frac{\pi}{2} - \alpha\right)$ for $0 < \alpha < \frac{\pi}{2}$. [1]

(b) Hence find $\int_{\tan\alpha}^{\cot\alpha} \frac{1}{1 + x^2}\,dx$, $0 < \alpha < \frac{\pi}{2}$. [4]

4. [Maximum mark: 6]

The function f is defined as $f(x) = ax^2 + bx + c$ where $a, b, c \in \mathbb{R}$.

Hayley conjectures that $\dfrac{f(x_2) - f(x_1)}{x_2 - x_1} = \dfrac{f'(x_2) + f'(x_1)}{2}$, $x_1 \neq x_2$.

Show that Hayley's conjecture is correct.

5. [Maximum mark: 8]

A biased coin is tossed five times. The probability of obtaining a head in any one throw is p.

Let X be the number of heads obtained.

(a) Find, in terms of p, an expression for $\mathrm{P}(X=4)$. [2]

(b) (i) Determine the value of p for which $\mathrm{P}(X=4)$ is a maximum.

(ii) For this value of p, determine the expected number of heads. [6]

6. [Maximum mark: 8]

Consider the expansion of $(1 + x)^n$ in ascending powers of x, where $n \geq 3$.

(a) Write down the first four terms of the expansion. [2]

The coefficients of the second, third and fourth terms of the expansion are consecutive terms of an arithmetic sequence.

(b) (i) Show that $n^3 - 9n^2 + 14n = 0$.

(ii) Hence find the value of n. [6]

7. [Maximum mark: 6]

A and B are independent events such that $\mathrm{P}(A) = \mathrm{P}(B) = p$, $p \neq 0$.

(a) Show that $\mathrm{P}(A \cup B) = 2p - p^2$. [2]

(b) Find $\mathrm{P}(A \mid A \cup B)$ in simplest form. [4]

8. [Maximum mark: 8]

Use mathematical induction to prove that $n\left(n^2+5\right)$ is divisible by 6 for $n \in \mathbb{Z}^+$.

9. [Maximum mark: 8]

Consider the equation $\frac{\sqrt{3}-1}{\sin x}+\frac{\sqrt{3}+1}{\cos x}=4\sqrt{2},\ 0<x<\frac{\pi}{2}$. Given that $\sin\left(\frac{\pi}{12}\right)=\frac{\sqrt{6}-\sqrt{2}}{4}$ and $\cos\left(\frac{\pi}{12}\right)=\frac{\sqrt{6}+\sqrt{2}}{4}$

(a) verify that $x=\frac{\pi}{12}$ is a solution to the equation; [3]

(b) hence find the other solution to the equation for $0<x<\frac{\pi}{2}$. [5]

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

10. [Maximum mark: 18]

A line L has equation $\frac{x-2}{p} = \frac{y-q}{2} = z-1$ where $p, q \in \mathbb{R}$.

A plane Π has equation $x + y + 3z = 9$.

(a) Show that L is not perpendicular to Π. [3]

(b) Given that L lies in the plane Π, find the value of p and the value of q. [4]

Consider the different case where the acute angle between L and Π is θ where $\theta = \arcsin\left(\frac{1}{\sqrt{11}}\right)$.

(c) (i) Show that $p = -2$.

(ii) If L intersects Π at $z = -1$, find the value of q. [11]

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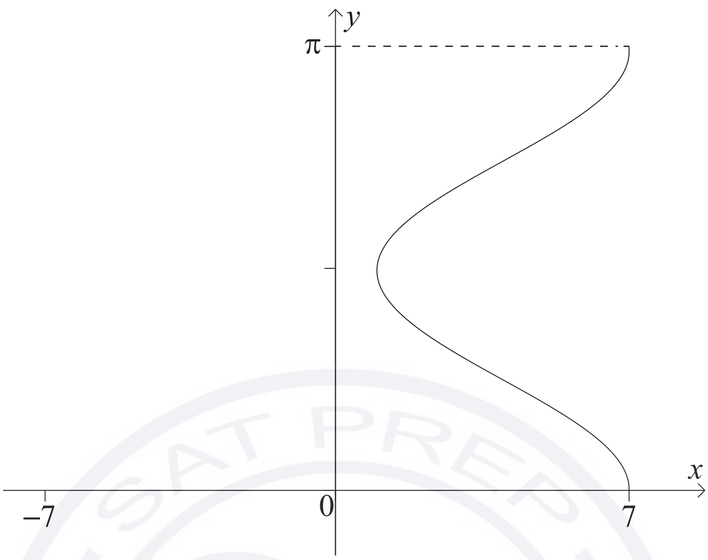
11. [Maximum mark: 19]

The following graph shows the relation $x = 3\cos 2y + 4$, $0 \le y \le \pi$.



The curve is rotated 360° about the y-axis to form a volume of revolution.

(a) Calculate the value of the volume generated. [8]

A container with this shape is made with a solid base of diameter $14\,\text{cm}$. The container is filled with water at a rate of $2\,\text{cm}^3\,\text{min}^{-1}$. At time *t* minutes, the water depth is $h\,\text{cm}$, $0 \le h \le \pi$ and the volume of water in the container is $V\,\text{cm}^3$.

(b) (i) Given that $\dfrac{\mathrm{d}V}{\mathrm{d}h} = \pi(3\cos 2h + 4)^2$, find an expression for $\dfrac{\mathrm{d}h}{\mathrm{d}t}$.

(ii) Find the value of $\dfrac{\mathrm{d}h}{\mathrm{d}t}$ when $h = \dfrac{\pi}{4}$. [4]

(c) (i) Find $\dfrac{\mathrm{d}^2h}{\mathrm{d}t^2}$.

(ii) Find the values of h for which $\dfrac{\mathrm{d}^2h}{\mathrm{d}t^2} = 0$.

(iii) By making specific reference to the shape of the container, interpret $\dfrac{\mathrm{d}h}{\mathrm{d}t}$ at the values of h found in part (c)(ii). [7]

Do **not** write solutions on this page.

12. [Maximum mark: 23]

Let $w = \cos\frac{2\pi}{7} + \mathrm{i}\sin\frac{2\pi}{7}$.

(a) Verify that w is a root of the equation $z^7 - 1 = 0$, $z \in \mathbb{C}$. [3]

(b) (i) Expand $(w-1)\left(1 + w + w^2 + w^3 + w^4 + w^5 + w^6\right)$.

(ii) Hence deduce that $1 + w + w^2 + w^3 + w^4 + w^5 + w^6 = 0$. [3]

(c) Write down the roots of the equation $z^7 - 1 = 0$, $z \in \mathbb{C}$ in terms of w and plot these roots on an Argand diagram. [3]

Consider the quadratic equation $z^2 + bz + c = 0$ where $b, c \in \mathbb{R}$, $z \in \mathbb{C}$. The roots of this equation are α and α^* where α^* is the complex conjugate of α.

(d) (i) Given that $\alpha = w + w^2 + w^4$, show that $\alpha^* = w^6 + w^5 + w^3$.

(ii) Find the value of b and the value of c. [10]

(e) Using the values for b and c obtained in part (d)(ii), find the imaginary part of α, giving your answer in surd form. [4]

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N15/5/MATHL/HP1/ENG/TZ0/XX

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Higher level
Paper 1

Wednesday 11 November 2015 (morning)

Candidate session number

2 hours

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions in the boxes provided.
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- The maximum mark for this examination paper is **[120 marks]**.

13 pages

8815 – 7201

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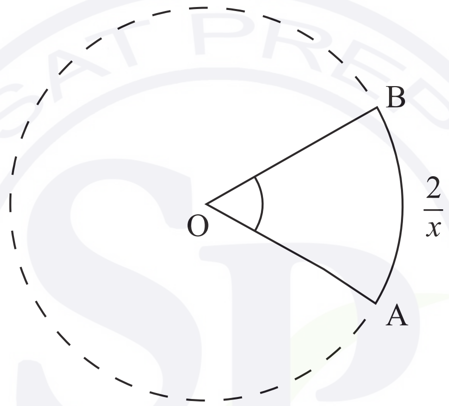
Section A

Answer **all** questions in the boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 4]

The following diagram shows a sector of a circle where $\text{A}\hat{\text{O}}\text{B} = x$ radians and the length of the arc $\text{AB} = \frac{2}{x}\text{cm}$.

Given that the area of the sector is $16\,\text{cm}^2$, find the length of the arc AB.



2. [Maximum mark: 4]

Using integration by parts find $\int x \sin x \, dx$.

3. [Maximum mark: 6]

(a) Write down and simplify the expansion of $(2+x)^4$ in ascending powers of x. [3]

(b) Hence find the exact value of $(2.1)^4$. [3]

4. [Maximum mark: 6]

Consider the curve $y = \frac{1}{1-x}$, $x \in \mathbb{R}$, $x \neq 1$.

(a) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$. [2]

(b) Determine the equation of the normal to the curve at the point $x = 3$ in the form $ax + by + c = 0$ where $a, b, c \in \mathbb{Z}$. [4]

5. [Maximum mark: 4]

Use the substitution $u = \ln x$ to find the value of $\int_{e}^{e^2} \frac{dx}{x \ln x}$.

6. [Maximum mark: 7]

A box contains four red balls and two white balls. Darren and Marty play a game by each taking it in turn to take a ball from the box, without replacement. The first player to take a white ball is the winner.

(a) Darren plays first, find the probability that he wins. [4]

The game is now changed so that the ball chosen is replaced after each turn.
Darren still plays first.

(b) Show that the probability of Darren winning has not changed. [3]

7. [Maximum mark: 8]

A curve is defined by $xy = y^2 + 4$.

(a) Show that there is no point where the tangent to the curve is horizontal. [4]

(b) Find the coordinates of the points where the tangent to the curve is vertical. [4]

. . . .

8. [Maximum mark: 8]

(a) Show that $\sin\left(\theta+\frac{\pi}{2}\right)=\cos\theta$. [1]

(b) Consider $f(x)=\sin(ax)$ where a is a constant. Prove by mathematical induction that $f^{(n)}(x)=a^n\sin\left(ax+\frac{n\pi}{2}\right)$ where $n\in\mathbb{Z}^+$ and $f^{(n)}(x)$ represents the n^{th} derivative of $f(x)$. [7]

9. [Maximum mark: 7]

Solve the equation $\sin 2x - \cos 2x = 1 + \sin x - \cos x$ for $x \in [-\pi, \pi]$.

10. [Maximum mark: 6]

A given polynomial function is defined as $f(x) = a_0 + a_1x + a_2x^2 + \ldots + a_nx^n$. The roots of the polynomial equation $f(x) = 0$ are consecutive terms of a geometric sequence with a common ratio of $\frac{1}{2}$ and first term 2.

Given that $a_{n-1} = -63$ and $a_n = 16$ find

(a) the degree of the polynomial; [4]

(b) the value of a_0. [2]

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

11. [Maximum mark: 17]

(a) Solve the equation $z^3 = 8\text{i}$, $z \in \mathbb{C}$ giving your answers in the form $z = r(\cos\theta + \text{i}\sin\theta)$ **and** in the form $z = a + b\text{i}$ where $a, b \in \mathbb{R}$. [6]

(b) Consider the complex numbers $z_1 = 1 + \text{i}$ and $z_2 = 2\left(\cos\left(\frac{\pi}{6}\right) + \text{i}\sin\left(\frac{\pi}{6}\right)\right)$.

(i) Write z_1 in the form $r(\cos\theta + \text{i}\sin\theta)$.

(ii) Calculate $z_1 z_2$ and write in the form $z = a + b\text{i}$ where $a, b \in \mathbb{R}$.

(iii) Hence find the value of $\tan\frac{5\pi}{12}$ in the form $c + d\sqrt{3}$, where $c, d \in \mathbb{Z}$.

(iv) Find the smallest value $p > 0$ such that $(z_2)^p$ is a positive real number. [11]

12. [Maximum mark: 20]

Consider the function defined by $f(x) = x\sqrt{1-x^2}$ on the domain $-1 \le x \le 1$.

(a) Show that f is an odd function. [2]

(b) Find $f'(x)$. [3]

(c) Hence find the x-coordinates of any local maximum or minimum points. [3]

(d) Find the range of f. [3]

(e) Sketch the graph of $y = f(x)$ indicating clearly the coordinates of the x-intercepts and any local maximum or minimum points. [3]

(f) Find the area of the region enclosed by the graph of $y = f(x)$ and the x-axis for $x \ge 0$. [4]

(g) Show that $\int_{-1}^{1}\left|x\sqrt{1-x^2}\right|\text{d}x > \left|\int_{-1}^{1}x\sqrt{1-x^2}\,\text{d}x\right|$. [2]

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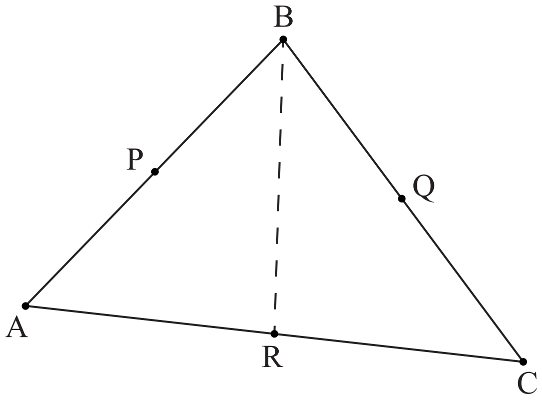
13. [Maximum mark: 23]



Consider the triangle ABC. The points P, Q and R are the midpoints of the line segments $[\mathrm{AB}]$, $[\mathrm{BC}]$ and $[\mathrm{AC}]$ respectively.

Let $\overrightarrow{\mathrm{OA}} = \boldsymbol{a}$, $\overrightarrow{\mathrm{OB}} = \boldsymbol{b}$ and $\overrightarrow{\mathrm{OC}} = \boldsymbol{c}$.

(a) Find $\overrightarrow{\mathrm{BR}}$ in terms of $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$. [2]

(b) (i) Find a vector equation of the line that passes through B and R in terms of $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$ and a parameter λ.

(ii) Find a vector equation of the line that passes through A and Q in terms of $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$ and a parameter μ.

(iii) Hence show that $\overrightarrow{\mathrm{OG}} = \frac{1}{3}(\boldsymbol{a} + \boldsymbol{b} + \boldsymbol{c})$ given that G is the point where $[\mathrm{BR}]$ and $[\mathrm{AQ}]$ intersect. [9]

(c) Show that the line segment $[\mathrm{CP}]$ also includes the point G. [3]

The coordinates of the points A, B and C are $(1, 3, 1)$, $(3, 7, -5)$ and $(2, 2, 1)$ respectively.
A point X is such that $[\mathrm{GX}]$ is perpendicular to the plane ABC.

(d) Given that the tetrahedron ABCX has volume $12\ \mathrm{units}^3$, find possible coordinates of X. [9]

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Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Higher level
Paper 1

Tuesday 12 May 2015 (morning)

Candidate session number

2 hours

Instructions to candidates

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15 pages

2215 – 7203

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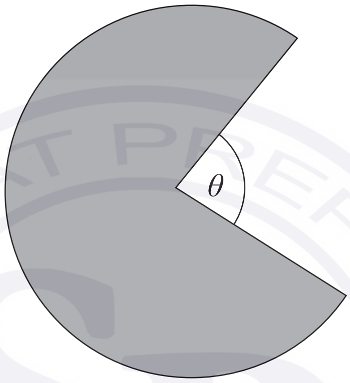
Section A

Answer **all** questions in the boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 5]

The logo, for a company that makes chocolate, is a sector of a circle of radius $2\,\text{cm}$, shown as shaded in the diagram. The area of the logo is $3\pi\,\text{cm}^2$.

diagram not to scale



(a) Find, in radians, the value of the angle θ, as indicated on the diagram. [3]

(b) Find the total length of the perimeter of the logo. [2]

2. [Maximum mark: 5]

A mathematics test is given to a class of 20 students. One student scores 0, but all the other students score 10.

(a) Find the mean score for the class. [2]

(b) Write down the median score. [1]

(c) Write down the number of students who scored

(i) above the mean score;

(ii) below the median score. [2]

3. [Maximum mark: 5]

(a) Find $\int (1 + \tan^2 x)\,dx$. [2]

(b) Find $\int \sin^2 x\,dx$. [3]

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4. [Maximum mark: 5]

(a) Expand $(x+h)^3$. [2]

(b) Hence find the derivative of $f(x)=x^3$ from first principles. [3]

5. [Maximum mark: 6]

The functions f and g are defined by $f(x) = ax^2 + bx + c$, $x \in \mathbb{R}$ and $g(x) = p\sin x + qx + r$, $x \in \mathbb{R}$ where a, b, c, p, q, r are real constants.

(a) Given that f is an even function, show that $b = 0$. [2]

(b) Given that g is an odd function, find the value of r. [2]

The function h is both odd and even, with domain $\mathbb{R}$.

(c) Find $h(x)$. [2]

6. [Maximum mark: 7]

A function f is defined by $f(x) = \dfrac{3x-2}{2x-1}$, $x \in \mathbb{R}$, $x \neq \dfrac{1}{2}$.

(a) Find an expression for $f^{-1}(x)$. [4]

(b) Given that $f(x)$ can be written in the form $f(x) = A + \dfrac{B}{2x-1}$, find the values of the constants A and B. [2]

(c) Hence, write down $\displaystyle\int \frac{3x-2}{2x-1}\,\mathrm{d}x$. [1]

7. [Maximum mark: 5]

Let $p(x) = 2x^5 + x^4 - 26x^3 - 13x^2 + 72x + 36$, $x \in \mathbb{R}$.

(a) For the polynomial equation $p(x) = 0$, state

(i) the sum of the roots;

(ii) the product of the roots. [3]

A new polynomial is defined by $q(x) = p(x + 4)$.

(b) Find the sum of the roots of the equation $q(x) = 0$. [2]

8. [Maximum mark: 7]

By using the substitution $u = e^x + 3$, find $\int \frac{e^x}{e^{2x} + 6e^x + 13} dx$.

9. [Maximum mark: 9]

The functions f and g are defined by $f(x) = 2x + \frac{\pi}{5}$, $x \in \mathbb{R}$ and $g(x) = 3\sin x + 4$, $x \in \mathbb{R}$.

(a) Show that $g \circ f(x) = 3\sin\left(2x + \frac{\pi}{5}\right) + 4$. [1]

(b) Find the range of $g \circ f$. [2]

(c) Given that $g \circ f\left(\frac{3\pi}{20}\right) = 7$, find the next value of x, greater than $\frac{3\pi}{20}$, for which $g \circ f(x) = 7$. [2]

(d) The graph of $y = g \circ f(x)$ can be obtained by applying four transformations to the graph of $y = \sin x$. State what the four transformations represent geometrically and give the order in which they are applied. [4]

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(This question continues on the following page)

(Question 9 continued)

10. [Maximum mark: 6]

A football team, Melchester Rovers are playing a tournament of five matches.
The probabilities that they win, draw or lose a match are $\frac{1}{2}$, $\frac{1}{6}$ and $\frac{1}{3}$ respectively.
These probabilities remain constant; the result of a match is independent of the results of other matches. At the end of the tournament their coach Roy loses his job if they lose three **consecutive** matches, otherwise he does not lose his job. Find the probability that Roy loses his job.

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

11. [Maximum mark: 21]

Let $y(x) = xe^{3x}, x \in \mathbb{R}$.

(a) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$. [2]

(b) Prove by induction that $\frac{\mathrm{d}^n y}{\mathrm{d}x^n} = n3^{n-1}\mathrm{e}^{3x} + x3^n\mathrm{e}^{3x}$ for $n \in \mathbb{Z}^+$. [7]

(c) Find the coordinates of any local maximum and minimum points on the graph of $y(x)$. Justify whether any such point is a maximum or a minimum. [5]

(d) Find the coordinates of any points of inflexion on the graph of $y(x)$. Justify whether any such point is a point of inflexion. [5]

(e) Hence sketch the graph of $y(x)$, indicating clearly the points found in parts (c) and (d) and any intercepts with the axes. [2]

Do **not** write solutions on this page.

12. [Maximum mark: 18]

Let $\{u_n\}$, $n \in \mathbb{Z}^+$, be an arithmetic sequence with first term equal to a and common difference of d, where $d \neq 0$. Let another sequence $\{v_n\}$, $n \in \mathbb{Z}^+$, be defined by $v_n = 2^{u_n}$.

(a) (i) Show that $\frac{v_{n+1}}{v_n}$ is a constant.

(ii) Write down the first term of the sequence $\{v_n\}$.

(iii) Write down a formula for v_n in terms of a, d and n. [4]

Let S_n be the sum of the first n terms of the sequence $\{v_n\}$.

(b) (i) Find S_n, in terms of a, d and n.

(ii) Find the values of d for which $\sum_{i=1}^{\infty} v_i$ exists.

You are now told that $\sum_{i=1}^{\infty} v_i$ does exist and is denoted by S_∞.

(iii) Write down S_∞ in terms of a and d.

(iv) Given that $S_\infty = 2^{a+1}$ find the value of d. [8]

Let $\{w_n\}$, $n \in \mathbb{Z}^+$, be a geometric sequence with first term equal to p and common ratio q, where p and q are both greater than zero. Let another sequence $\{z_n\}$ be defined by $z_n = \ln w_n$.

(c) Find $\sum_{i=1}^{n} z_i$ giving your answer in the form $\ln k$ with k in terms of n, p and q. [6]

Do **not** write solutions on this page.

13. [Maximum mark: 21]

Two lines l_1 and l_2 are given respectively by the equations $\boldsymbol{r_1} = \overrightarrow{\mathrm{OA}} + \lambda\boldsymbol{v}$ and $\boldsymbol{r_2} = \overrightarrow{\mathrm{OB}} + \mu\boldsymbol{w}$ where $\overrightarrow{\mathrm{OA}} = \boldsymbol{i} + 2\boldsymbol{j} + 3\boldsymbol{k}$, $\boldsymbol{v} = \boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k}$, $\overrightarrow{\mathrm{OB}} = 2\boldsymbol{i} + \boldsymbol{j} - \boldsymbol{k}$, $\boldsymbol{w} = \boldsymbol{i} - \boldsymbol{j} + 2\boldsymbol{k}$ and O is the origin. Let P be a point on l_1 and let Q be a point on l_2.

(a) Find $\overrightarrow{\mathrm{PQ}}$, in terms of λ and μ. [2]

(b) Find the value of λ and the value of μ for which $\overrightarrow{\mathrm{PQ}}$ is perpendicular to the direction vectors of both l_1 and l_2. [5]

(c) Hence find the shortest distance between l_1 and l_2. [3]

(d) Find the Cartesian equation of the plane Π, which contains line l_1 and is parallel to the direction vector of line l_2. [5]

Let $\overrightarrow{\mathrm{OT}} = \overrightarrow{\mathrm{OB}} + \eta(\boldsymbol{v} \times \boldsymbol{w})$.

(e) Find the value of η for which the point T lies in the plane Π. [2]

(f) For this value of η, calculate $\left|\overrightarrow{\mathrm{BT}}\right|$. [2]

(g) State what you notice about your answers to (c) and (f), and give a geometrical interpretation of this result. [2]

Please **do not** write on this page.

Answers written on this page
will not be marked.

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Higher level
Paper 1

Tuesday 12 May 2015 (morning)

Candidate session number

2 hours

Instructions to candidates

- Write your session number in the boxes above.
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- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics HL and further mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[120 marks]**.

2215 – 7205

12 pages

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions in the boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 4]

A and B are two events such that $\mathrm{P}(A) = 0.25$, $\mathrm{P}(B) = 0.6$ and $\mathrm{P}(A \cup B) = 0.7$.

(a) Find $\mathrm{P}(A \cap B)$. [2]

(b) Determine whether events A and B are independent. [2]

2. [Maximum mark: 4]

Expand $(3-x)^4$ in ascending powers of x and simplify your answer.

3. [Maximum mark: 6]

Find all solutions to the equation $\tan x + \tan 2x = 0$ where $0^\circ \le x < 360^\circ$.

4. [Maximum mark: 7]

Consider the function defined by $f(x) = x^3 - 3x^2 + 4$.

(a) Determine the values of x for which $f(x)$ is a decreasing function. [4]

There is a point of inflexion, P, on the curve $y = f(x)$.

(b) Find the coordinates of P. [3]

5. [Maximum mark: 6]

Show that $\int_1^2 x^3 \ln x \, dx = 4 \ln 2 - \frac{15}{16}$.

6. [Maximum mark: 8]

In triangle ABC, $\text{BC} = \sqrt{3}\,\text{cm}$, $\text{A}\hat{\text{B}}\text{C} = \theta$ and $\text{B}\hat{\text{C}}\text{A} = \frac{\pi}{3}$.

(a) Show that length $\text{AB} = \frac{3}{\sqrt{3}\cos\theta + \sin\theta}$. [4]

(b) Given that AB has a minimum value, determine the value of θ for which this occurs. [4]

7. [Maximum mark: 9]

(a) Find three distinct roots of the equation $8z^3 + 27 = 0$, $z \in \mathbb{C}$ giving your answers in modulus-argument form. [6]

The roots are represented by the vertices of a triangle in an Argand diagram.

(b) Show that the area of the triangle is $\frac{27\sqrt{3}}{16}$. [3]

8. [Maximum mark: 8]

By using the substitution $t = \tan x$, find $\int \frac{\mathrm{d}x}{1+\sin^2 x}$.

Express your answer in the form $m \arctan(n \tan x) + c$, where m, n are constants to be determined.

9. [Maximum mark: 8]

(a) State the set of values of a for which the function $x \mapsto \log_a x$ exists, for all $x \in \mathbb{R}^+$. [2]

(b) Given that $\log_x y = 4\log_y x$, find all the possible expressions of y as a function of x. [6]

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

10. [Maximum mark: 17]

The function f is defined by $f(x) = \frac{3x}{x-2}$, $x \in \mathbb{R}$, $x \neq 2$.

(a) Sketch the graph of $y = f(x)$, indicating clearly any asymptotes and points of intersection with the x and y axes. [4]

(b) Find an expression for $f^{-1}(x)$. [4]

(c) Find all values of x for which $f(x) = f^{-1}(x)$. [3]

(d) Solve the inequality $\left|f(x)\right| < \frac{3}{2}$. [4]

(e) Solve the inequality $f\left(\left|x\right|\right) < \frac{3}{2}$. [2]

11. [Maximum mark: 16]

Consider the functions $f(x) = \tan x$, $0 \leq x < \frac{\pi}{2}$ and $g(x) = \frac{x+1}{x-1}$, $x \in \mathbb{R}$, $x \neq 1$.

(a) Find an expression for $g \circ f(x)$, stating its domain. [2]

(b) Hence show that $g \circ f(x) = \frac{\sin x + \cos x}{\sin x - \cos x}$. [2]

(c) Let $y = g \circ f(x)$, find an exact value for $\frac{\mathrm{d}y}{\mathrm{d}x}$ at the point on the graph of $y = g \circ f(x)$ where $x = \frac{\pi}{6}$, expressing your answer in the form $a + b\sqrt{3}$, $a, b \in \mathbb{Z}$. [6]

(d) Show that the area bounded by the graph of $y = g \circ f(x)$, the x-axis and the lines $x = 0$ and $x = \frac{\pi}{6}$ is $\ln\left(1+\sqrt{3}\right)$. [6]

Do **not** write solutions on this page.

12. [Maximum mark: 14]

The cubic equation $x^3 + px^2 + qx + c = 0$, has roots α, β, γ. By expanding $(x-\alpha)(x-\beta)(x-\gamma)$ show that

(a) (i) $p = -(\alpha + \beta + \gamma)$;

(ii) $q = \alpha\beta + \beta\gamma + \gamma\alpha$;

(iii) $c = -\alpha\beta\gamma$. [3]

It is now given that $p = -6$ and $q = 18$ for parts (b) and (c) below.

(b) (i) In the case that the three roots α, β, γ form an arithmetic sequence, show that one of the roots is 2.

(ii) Hence determine the value of c. [5]

(c) In another case the three roots α, β, γ form a geometric sequence. Determine the value of c. [6]

13. [Maximum mark: 13]

(a) Show that $\dfrac{1}{\sqrt{n} + \sqrt{n+1}} = \sqrt{n+1} - \sqrt{n}$ where $n \ge 0$, $n \in \mathbb{Z}$. [2]

(b) Hence show that $\sqrt{2} - 1 < \dfrac{1}{\sqrt{2}}$. [2]

(c) Prove, by mathematical induction, that $\displaystyle\sum_{r=1}^{r=n} \frac{1}{\sqrt{r}} > \sqrt{n}$ for $n \ge 2$, $n \in \mathbb{Z}$. [9]

N14/5/MATHL/HP1/ENG/TZ0/XX

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MATHEMATICS
HIGHER LEVEL
PAPER 1

Candidate session number

Wednesday 12 November 2014 (afternoon)

2 hours

Examination code

8	8	1	4	–	7	2	0	1

INSTRUCTIONS TO CANDIDATES

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- The maximum mark for this examination paper is *[120 marks]*.

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SECTION A

*Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.*

1. *[Maximum mark: 4]*

The function f is defined by $f(x) = \frac{1}{x}$, $x \neq 0$.

The graph of the function $y = g(x)$ is obtained by applying the following transformations to the graph of $y = f(x)$:

a translation by the vector $\begin{pmatrix} -3 \\ 0 \end{pmatrix}$;

a translation by the vector $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$.

(a) Find an expression for $g(x)$. *[2]*

(b) State the equations of the asymptotes of the graph of g. *[2]*

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2. *[Maximum mark: 6]*

The quadratic equation $2x^2 - 8x + 1 = 0$ has roots α and β.

(a) Without solving the equation, find the value of

(i) $\alpha + \beta$;

(ii) $\alpha\beta$. *[2]*

Another quadratic equation $x^2 + px + q = 0$, $p, q \in \mathbb{Z}$ has roots $\frac{2}{\alpha}$ and $\frac{2}{\beta}$.

(b) Find the value of p and the value of q. *[4]*

3. *[Maximum mark: 5]*

A point P, relative to an origin O, has position vector $\overrightarrow{\mathrm{OP}} = \begin{pmatrix} 1+s \\ 3+2s \\ 1-s \end{pmatrix}$, $s \in \mathbb{R}$.

Find the minimum length of $\overrightarrow{\mathrm{OP}}$.

4. *[Maximum mark: 7]*

Events A and B are such that $P(A) = 0.2$ and $P(B) = 0.5$.

(a) Determine the value of $P(A \cup B)$ when

(i) A and B are mutually exclusive;

(ii) A and B are independent. *[4]*

(b) Determine the range of possible values of $P(A|B)$. *[3]*

5. *[Maximum mark: 6]*

A tranquilizer is injected into a muscle from which it enters the bloodstream. The concentration C in $\text{mg}\,\text{l}^{-1}$, of tranquilizer in the bloodstream can be modelled by the function $C(t) = \frac{2t}{3+t^2}$, $t \geq 0$ where t is the number of minutes after the injection.

Find the maximum concentration of tranquilizer in the bloodstream.

6. *[Maximum mark: 6]*

By using the substitution $u = 1 + \sqrt{x}$, find $\int \frac{\sqrt{x}}{1+\sqrt{x}}\,dx$.

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7. *[Maximum mark: 6]*

Consider two functions f and g and their derivatives f' and g'. The following table shows the values for the two functions and their derivatives at $x = 1$, 2 and 3.

x	1	2	3
$f(x)$	3	1	1
$f'(x)$	1	4	2
$g(x)$	2	1	4
$g'(x)$	4	2	3

Given that $p(x) = f(x)\,g(x)$ and $h(x) = g \circ f(x)$, find

(a) $p'(3)$; *[2]*

(b) $h'(2)$. *[4]*

8. *[Maximum mark: 7]*

Use mathematical induction to prove that $(2n)! \geq 2^n (n!)^2$, $n \in \mathbb{Z}^+$.

9. *[Maximum mark: 6]*

A continuous random variable T has probability density function f defined by

$$f(t) = \begin{cases} |2-t|, & 1 \le t \le 3 \\ 0, & \text{otherwise}. \end{cases}$$

(a) Sketch the graph of $y = f(t)$. *[2]*

(b) Find the interquartile range of T. *[4]*

10. *[Maximum mark: 7]*

A set of positive integers $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ is used to form a pack of nine cards. Each card displays one positive integer without repetition from this set. Grace wishes to select four cards at random from this pack of nine cards.

(a) Find the number of selections Grace could make if the largest integer drawn among the four cards is either a 5, a 6 or a 7. *[3]*

(b) Find the number of selections Grace could make if at least two of the four integers drawn are even. *[4]*

*Do **NOT** write solutions on this page.*

SECTION B

*Answer **all** questions in the answer booklet provided. Please start each question on a new page.*

11. *[Maximum mark: 23]*

The function f is defined as $f(x) = e^{3x+1}$, $x \in \mathbb{R}$.

(a) (i) Find $f^{-1}(x)$.

(ii) State the domain of f^{-1}. *[4]*

The function g is defined as $g(x) = \ln x$, $x \in \mathbb{R}^+$.
The graph of $y = g(x)$ and the graph of $y = f^{-1}(x)$ intersect at the point P.

(b) Find the coordinates of P. *[5]*

The graph of $y = g(x)$ intersects the x-axis at the point Q.

(c) Show that the equation of the tangent T to the graph of $y = g(x)$ at the point Q is $y = x - 1$. *[3]*

A region R is bounded by the graphs of $y = g(x)$, the tangent T and the line $x = e$.

(d) Find the area of the region R. *[5]*

(e) (i) Show that $g(x) \le x - 1$, $x \in \mathbb{R}^+$.

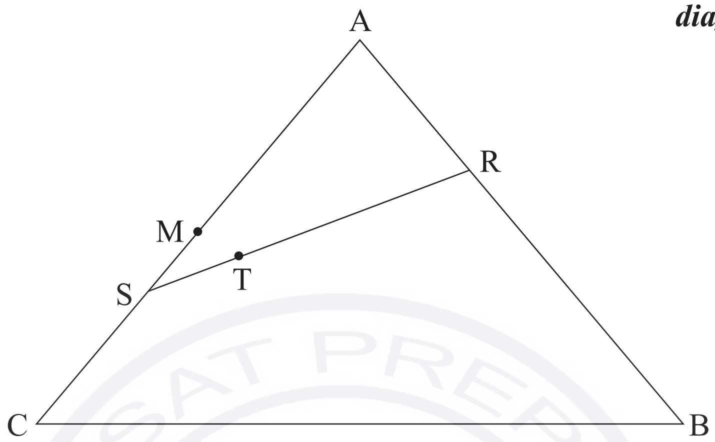
(ii) By replacing x with $\frac{1}{x}$ in part (e)(i), show that $\frac{x-1}{x} \le g(x)$, $x \in \mathbb{R}^+$. *[6]*

*Do **NOT** write solutions on this page.*

12. *[Maximum mark: 14]*

The position vectors of the points A, B and C are $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$ respectively, relative to an origin O. The following diagram shows the triangle ABC and points M, R, S and T.

diagram not to scale



M is the midpoint of [AC].

R is a point on [AB] such that $\overrightarrow{AR} = \frac{1}{3}\overrightarrow{AB}$.

S is a point on [AC] such that $\overrightarrow{AS} = \frac{2}{3}\overrightarrow{AC}$.

T is a point on [RS] such that $\overrightarrow{RT} = \frac{2}{3}\overrightarrow{RS}$.

(a) (i) Express $\overrightarrow{AM}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{c}$.

(ii) Hence show that $\overrightarrow{BM} = \frac{1}{2}\boldsymbol{a} - \boldsymbol{b} + \frac{1}{2}\boldsymbol{c}$. *[4]*

(b) (i) Express $\overrightarrow{RA}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$.

(ii) Show that $\overrightarrow{RT} = -\frac{2}{9}\boldsymbol{a} - \frac{2}{9}\boldsymbol{b} + \frac{4}{9}\boldsymbol{c}$. *[5]*

(c) Prove that T lies on [BM]. *[5]*

*Do **NOT** write solutions on this page.*

13. *[Maximum mark: 23]*

(a) (i) Show that $(1+\mathrm{i}\tan\theta)^n+(1-\mathrm{i}\tan\theta)^n=\dfrac{2\cos n\theta}{\cos^n\theta}$, $\cos\theta\neq 0$.

(ii) Hence verify that $\mathrm{i}\tan\dfrac{3\pi}{8}$ is a root of the equation $(1+z)^4+(1-z)^4=0$, $z\in\mathbb{C}$.

(iii) State another root of the equation $(1+z)^4+(1-z)^4=0$, $z\in\mathbb{C}$. *[10]*

(b) (i) Use the double angle identity $\tan 2\theta=\dfrac{2\tan\theta}{1-\tan^2\theta}$ to show that $\tan\dfrac{\pi}{8}=\sqrt{2}-1$.

(ii) Show that $\cos 4x=8\cos^4 x-8\cos^2 x+1$.

(iii) Hence find the value of $\int_0^{\frac{\pi}{8}}\dfrac{2\cos 4x}{\cos^2 x}\mathrm{d}x$. *[13]*

Please **do not** write on this page.

Answers written on this page
will not be marked.

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MATHEMATICS
HIGHER LEVEL
PAPER 1

Candidate session number

Tuesday 13 May 2014 (afternoon)

2 hours

Examination code

2	2	1	4	–	7	2	0	3

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
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- A clean copy of the ***Mathematics HL and Further Mathematics HL*** *formula booklet* is required for this paper.
- The maximum mark for this examination paper is *[120 marks]*.

14 pages

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SECTION A

*Answer **all** the questions in the boxes provided. Working may be continued below the lines, if necessary.*

1. *[Maximum mark: 5]*

When the polynomial $3x^3 + ax + b$ is divided by $(x-2)$, the remainder is 2, and when divided by $(x+1)$, it is 5. Find the value of a and the value of b.

2. *[Maximum mark: 4]*

Four numbers are such that their mean is 13, their median is 14 and their mode is 15. Find the four numbers.

3. *[Maximum mark: 5]*

Consider $a = \log_2 3 \times \log_3 4 \times \log_4 5 \times \ldots \times \log_{31} 32$. Given that $a \in \mathbb{Z}$, find the value of a.

4. *[Maximum mark: 6]*

The equation $5x^3 + 48x^2 + 100x + 2 = a$ has roots r_1, r_2 and r_3.
Given that $r_1 + r_2 + r_3 + r_1 r_2 r_3 = 0$, find the value of a.

5. *[Maximum mark: 8]*

(a) Use the identity $\cos 2\theta = 2\cos^2\theta - 1$ to prove that $\cos\frac{1}{2}x = \sqrt{\frac{1+\cos x}{2}}$, $0 \le x \le \pi$. *[2]*

(b) Find a similar expression for $\sin\frac{1}{2}x$, $0 \le x \le \pi$. *[2]*

(c) Hence find the value of $\int_0^{\frac{\pi}{2}} \left(\sqrt{1+\cos x} + \sqrt{1-\cos x}\right) \mathrm{d}x$. *[4]*

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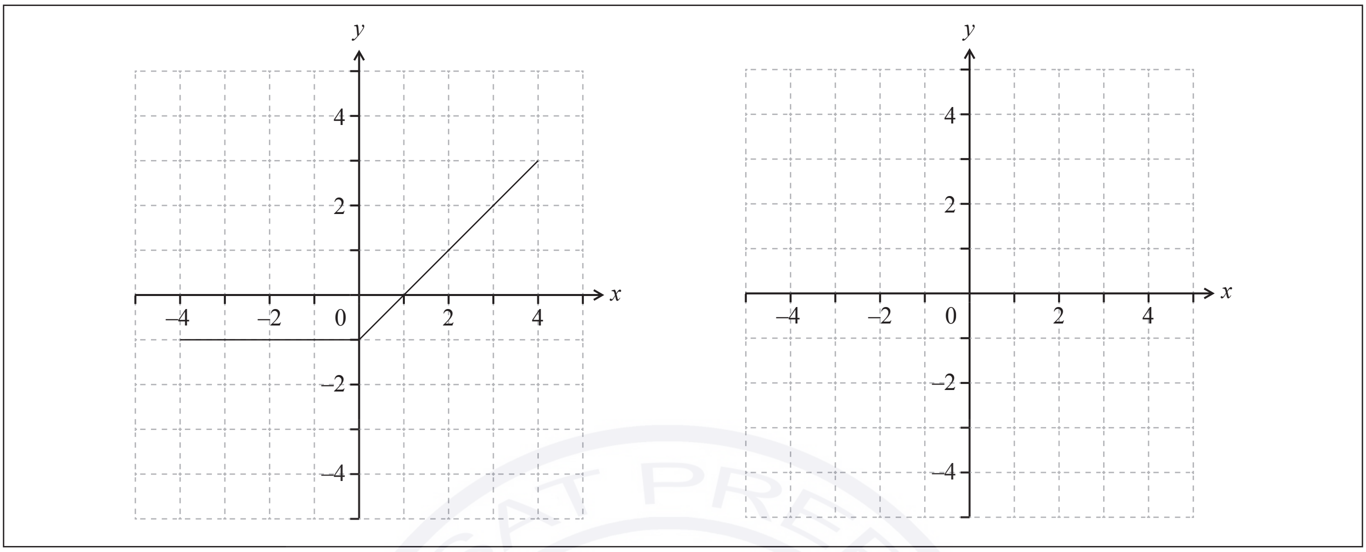
6. *[Maximum mark: 6]*

The first set of axes below shows the graph of $y = f(x)$ for $-4 \le x \le 4$.



Let $g(x) = \int_{-4}^{x} f(t)\,dt$ for $-4 \le x \le 4$.

(a) State the value of x at which $g(x)$ is a minimum. *[1]*

(b) On the second set of axes, sketch the graph of $y = g(x)$. *[5]*

7. *[Maximum mark: 5]*

The triangle ABC is equilateral of side 3 cm. The point D lies on [BC] such that $BD = 1\text{cm}$.
Find $\cos D\hat{A}C$.

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8. *[Maximum mark: 6]*

A body is moving in a straight line. When it is s metres from a fixed point O on the line its velocity, v, is given by $v = -\frac{1}{s^2}$, $s > 0$.

Find the acceleration of the body when it is 50 cm from O.

9. *[Maximum mark: 9]*

A curve has equation $\arctan x^2 + \arctan y^2 = \frac{\pi}{4}$.

(a) Find $\frac{dy}{dx}$ in terms of x and y. *[4]*

(b) Find the gradient of the curve at the point where $x = \frac{1}{\sqrt{2}}$ and $y < 0$. *[5]*

10. *[Maximum mark: 6]*

Given that $\sin x + \cos x = \frac{2}{3}$, find $\cos 4x$.

*Do **NOT** write solutions on this page.*

SECTION B

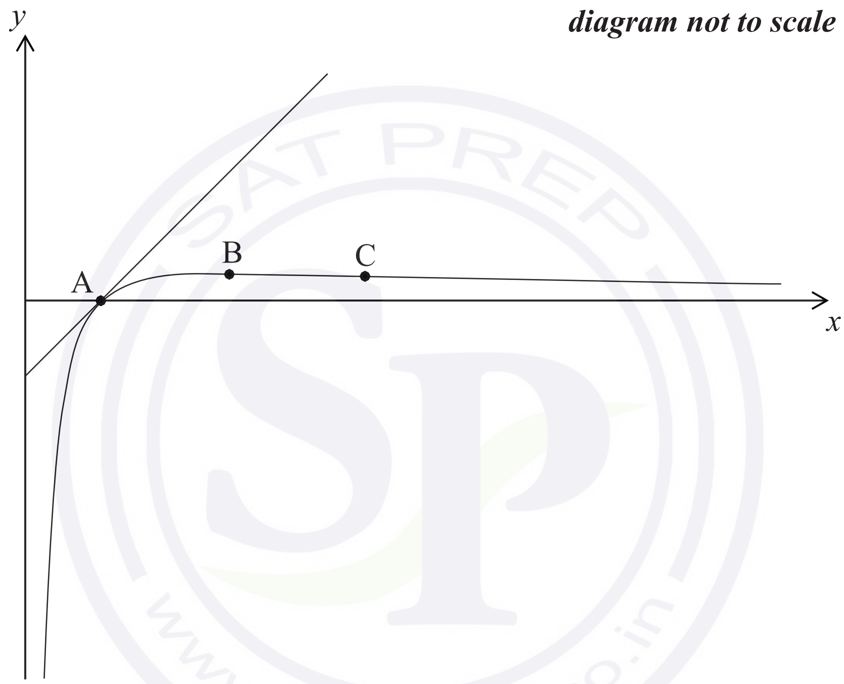
*Answer **all** questions in the answer booklet provided. Please start each question on a new page.*

11. *[Maximum mark: 21]*

Consider the function $f(x) = \frac{\ln x}{x}$, $x > 0$.

The sketch below shows the graph of $y = f(x)$ and its tangent at a point A.

diagram not to scale



(a) Show that $f'(x) = \frac{1 - \ln x}{x^2}$. *[2]*

(b) Find the coordinates of B, at which the curve reaches its maximum value. *[3]*

(c) Find the coordinates of C, the point of inflexion on the curve. *[5]*

The graph of $y = f(x)$ crosses the x-axis at the point A.

(d) Find the equation of the tangent to the graph of f at the point A. *[4]*

(e) Find the area enclosed by the curve $y = f(x)$, the tangent at A, and the line $x = \mathrm{e}$. *[7]*

12. *[Maximum mark: 22]*

(a) Show that the points $\text{O}(0, 0, 0)$, $\text{A}(6, 0, 0)$, $\text{B}\left(6, -\sqrt{24}, \sqrt{12}\right)$, $\text{C}\left(0, -\sqrt{24}, \sqrt{12}\right)$ form a square. *[3]*

(b) Find the coordinates of M, the mid-point of [OB]. *[1]*

(c) Show that an equation of the plane Π, containing the square OABC, is $y + \sqrt{2}z = 0$. *[3]*

(d) Find a vector equation of the line L, through M, perpendicular to the plane Π. *[3]*

(e) Find the coordinates of D, the point of intersection of the line L with the plane whose equation is $y = 0$. *[3]*

(f) Find the coordinates of E, the reflection of the point D in the plane Π. *[3]*

(g) (i) Find the angle $\text{O}\hat{\text{D}}\text{A}$.

(ii) State what this tells you about the solid OABCDE. *[6]*

13. *[Maximum mark: 17]*

A geometric sequence $\{u_n\}$, with complex terms, is defined by $u_{n+1} = (1+\mathrm{i})u_n$ and $u_1 = 3$.

(a) Find the fourth term of the sequence, giving your answer in the form $x + y\mathrm{i}$, $x, y \in \mathbb{R}$. *[3]*

(b) Find the sum of the first 20 terms of $\{u_n\}$, giving your answer in the form $a \times (1 + 2^m)$ where $a \in \mathbb{C}$ and $m \in \mathbb{Z}$ are to be determined. *[4]*

A second sequence $\{v_n\}$ is defined by $v_n = u_n u_{n+k}$, $k \in \mathbb{N}$.

(c) (i) Show that $\{v_n\}$ is a geometric sequence.

(ii) State the first term.

(iii) Show that the common ratio is independent of k. *[5]*

A third sequence $\{w_n\}$ is defined by $w_n = |u_n - u_{n+1}|$.

(d) (i) Show that $\{w_n\}$ is a geometric sequence.

(ii) State the geometrical significance of this result with reference to points on the complex plane. *[5]*

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Answers written on this page
will not be marked.

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MATHEMATICS
HIGHER LEVEL
PAPER 1

Candidate session number

Tuesday 13 May 2014 (afternoon)

2 hours

Examination code

2	2	1	4	–	7	2	0	5

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
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- Section A: answer all questions in the boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the ***Mathematics HL and Further Mathematics HL*** *formula booklet* is required for this paper.
- The maximum mark for this examination paper is *[120 marks]*.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

*Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.*

1. *[Maximum mark: 6]*

Events A and B are such that $P(A)=\frac{2}{5}$, $P(B)=\frac{11}{20}$ and $P(A|B)=\frac{2}{11}$.

(a) Find $P(A\cap B)$. *[2]*

(b) Find $P(A\cup B)$. *[2]*

(c) State with a reason whether or not events A and B are independent. *[2]*

2. *[Maximum mark: 5]*

Solve the equation $8^{x-1} = 6^{3x}$. Express your answer in terms of $\ln 2$ and $\ln 3$.

3. *[Maximum mark: 5]*

(a) Show that the following system of equations has an infinite number of solutions. *[2]*

$$\begin{aligned} x + y + 2z &= -2 \\ 3x - y + 14z &= 6 \\ x + 2y &= -5 \end{aligned}$$

The system of equations represents three planes in space.

(b) Find the parametric equations of the line of intersection of the three planes. *[3]*

4. *[Maximum mark: 6]*

The roots of a quadratic equation $2x^2 + 4x - 1 = 0$ are α and β.

Without solving the equation,

(a) find the value of $\alpha^2 + \beta^2$; *[4]*

(b) find a quadratic equation with roots α^2 and β^2. *[2]*

5. *[Maximum mark: 5]*

(a) Sketch the graph of $y = \left|\cos\left(\frac{x}{4}\right)\right|$ for $0 \le x \le 8\pi$. *[2]*

(b) Solve $\left|\cos\left(\frac{x}{4}\right)\right| = \frac{1}{2}$ for $0 \le x \le 8\pi$. *[3]*

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6. *[Maximum mark: 6]*

PQRS is a rhombus. Given that $\overrightarrow{PQ} = \boldsymbol{a}$ and $\overrightarrow{QR} = \boldsymbol{b}$,

(a) express the vectors $\overrightarrow{PR}$ and $\overrightarrow{QS}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$; *[2]*

(b) hence show that the diagonals in a rhombus intersect at right angles. *[4]*

7. *[Maximum mark: 7]*

Consider the complex numbers $u = 2 + 3\text{i}$ and $v = 3 + 2\text{i}$.

(a) Given that $\frac{1}{u} + \frac{1}{v} = \frac{10}{w}$, express w in the form $a + b\text{i}$, $a, b \in \mathbb{R}$. *[4]*

(b) Find w^* and express it in the form $re^{\text{i}\theta}$. *[3]*

8. *[Maximum mark: 6]*

The function f is defined by

$$f(x)=\begin{cases} 1-2x, & x\le 2 \\ \frac{3}{4}(x-2)^2-3, & x>2 \end{cases}$$

(a) Determine whether or not f is continuous. *[2]*

The graph of the function g is obtained by applying the following transformations to the graph of f :

a reflection in the y–axis followed by a translation by the vector $\begin{pmatrix}2\\0\end{pmatrix}$.

(b) Find $g(x)$. *[4]*

9. *[Maximum mark: 7]*

The first three terms of a geometric sequence are $\sin x$, $\sin 2x$ and $4\sin x\cos^2 x$, $-\frac{\pi}{2} < x < \frac{\pi}{2}$.

(a) Find the common ratio r. *[1]*

(b) Find the set of values of x for which the geometric series $\sin x + \sin 2x + 4\sin x\cos^2 x + \ldots$ converges. *[3]*

Consider $x = \arccos\left(\frac{1}{4}\right)$, $x > 0$.

(c) Show that the sum to infinity of this series is $\frac{\sqrt{15}}{2}$. *[3]*

10. *[Maximum mark: 7]*

Use the substitution $x = a\sec\theta$ to show that $\int_{a\sqrt{2}}^{2a} \frac{dx}{x^3\sqrt{x^2 - a^2}} = \frac{1}{24a^3}\left(3\sqrt{3} + \pi - 6\right)$.

*Do **NOT** write solutions on this page.*

SECTION B

*Answer **all** questions in the answer booklet provided. Please start each question on a new page.*

11. *[Maximum mark: 12]*

(a) Mobile phone batteries are produced by two machines. Machine A produces 60% of the daily output and machine B produces 40%. It is found by testing that on average 2% of batteries produced by machine A are faulty and 1% of batteries produced by machine B are faulty.

(i) Draw a tree diagram clearly showing the respective probabilities.

(ii) A battery is selected at random. Find the probability that it is faulty.

(iii) A battery is selected at random and found to be faulty. Find the probability that it was produced by machine A. *[6]*

(b) In a pack of seven transistors, three are found to be defective. Three transistors are selected from the pack at random without replacement. The discrete random variable X represents the number of defective transistors selected.

(i) Find $\mathrm{P}(X = 2)$.

(ii) **Copy** and complete the following table:

x	0	1	2	3
$\mathrm{P}(X = x)$				

(iii) Determine $\mathrm{E}(X)$. *[6]*

*Do **NOT** write solutions on this page.*

12. *[Maximum mark: 18]*

Given the points $A(1, 0, 4)$, $B(2, 3, -1)$ and $C(0, 1, -2)$,

(a) find the vector equation of the line L_1 passing through the points A and B. *[2]*

The line L_2 has Cartesian equation $\frac{x-1}{3} = \frac{y+2}{1} = \frac{z-1}{-2}$.

(b) Show that L_1 and L_2 are skew lines. *[5]*

Consider the plane Π_1, parallel to both lines L_1 and L_2. Point C lies in the plane Π_1.

(c) Find the Cartesian equation of the plane Π_1. *[4]*

The line L_3 has vector equation $\boldsymbol{r} = \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} k \\ 1 \\ -1 \end{pmatrix}$.

The plane Π_2 has Cartesian equation $x + y = 12$.

The angle between the line L_3 and the plane Π_2 is 60°.

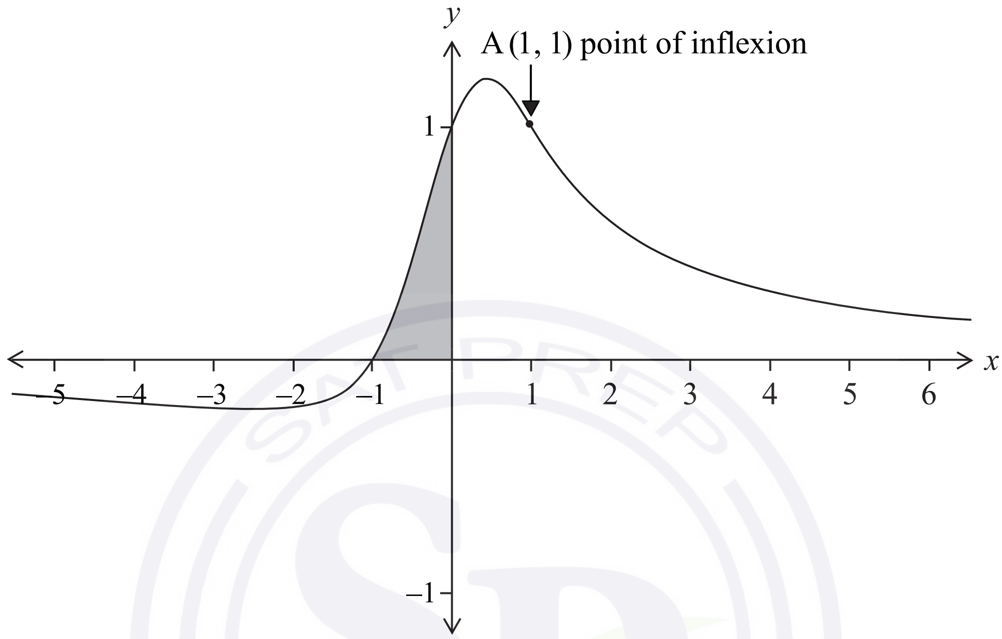
(d) (i) Find the value of k.

(ii) Find the point of intersection P of the line L_3 and the plane Π_2. *[7]*

*Do **NOT** write solutions on this page.*

13. *[Maximum mark: 16]*

The graph of the function $f(x) = \dfrac{x+1}{x^2+1}$ is shown below.



(a) Find $f'(x)$. *[2]*

(b) Hence find the x-coordinates of the points where the gradient of the graph of f is zero. *[1]*

(c) Find $f''(x)$ expressing your answer in the form $\dfrac{p(x)}{(x^2+1)^3}$, where $p(x)$ is a polynomial of degree 3. *[3]*

The point $(1, 1)$ is a point of inflexion. There are two other points of inflexion.

(d) Find the x-coordinates of the other two points of inflexion. *[4]*

(e) Find the area of the shaded region. Express your answer in the form $\dfrac{\pi}{a} - \ln\sqrt{b}$, where a and b are integers. *[6]*

*Do **NOT** write solutions on this page.*

14. *[Maximum mark: 14]*

Consider the following functions:

$$h(x) = \arctan(x),\ x \in \mathbb{R}$$
$$g(x) = \frac{1}{x},\ x \in \mathbb{R},\ x \neq 0$$

(a) Sketch the graph of $y = h(x)$. *[2]*

(b) Find an expression for the composite function $h \circ g(x)$ and state its domain. *[2]*

Given that $f(x) = h(x) + h \circ g(x)$,

(c) (i) find $f'(x)$ in simplified form;

(ii) show that $f(x) = \frac{\pi}{2}$ for $x > 0$. *[7]*

Nigel states that f is an odd function and Tom argues that f is an even function.

(d) (i) State who is correct and justify your answer.

(ii) Hence find the value of $f(x)$ for $x < 0$. *[3]*

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MATHEMATICS
HIGHER LEVEL
PAPER 1

Candidate session number

0	0							

Monday 11 November 2013 (afternoon)

Examination code

8	8	1	3	–	7	2	0	1

2 hours

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
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- Section A: answer all questions in the boxes provided.
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- A clean copy of the ***Mathematics HL and Further Mathematics SL*** *information booklet* is required for this paper.
- The maximum mark for this examination paper is *[120 marks]*.

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SECTION A

*Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.*

1. *[Maximum mark: 5]*

The cubic polynomial $3x^3 + px^2 + qx - 2$ has a factor $(x+2)$ and leaves a remainder 4 when divided by $(x+1)$. Find the value of p and the value of q.

2. *[Maximum mark: 6]*

The discrete random variable X has probability distribution:

x	0	1	2	3
$\mathrm{P}(X = x)$	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{3}{10}$	a

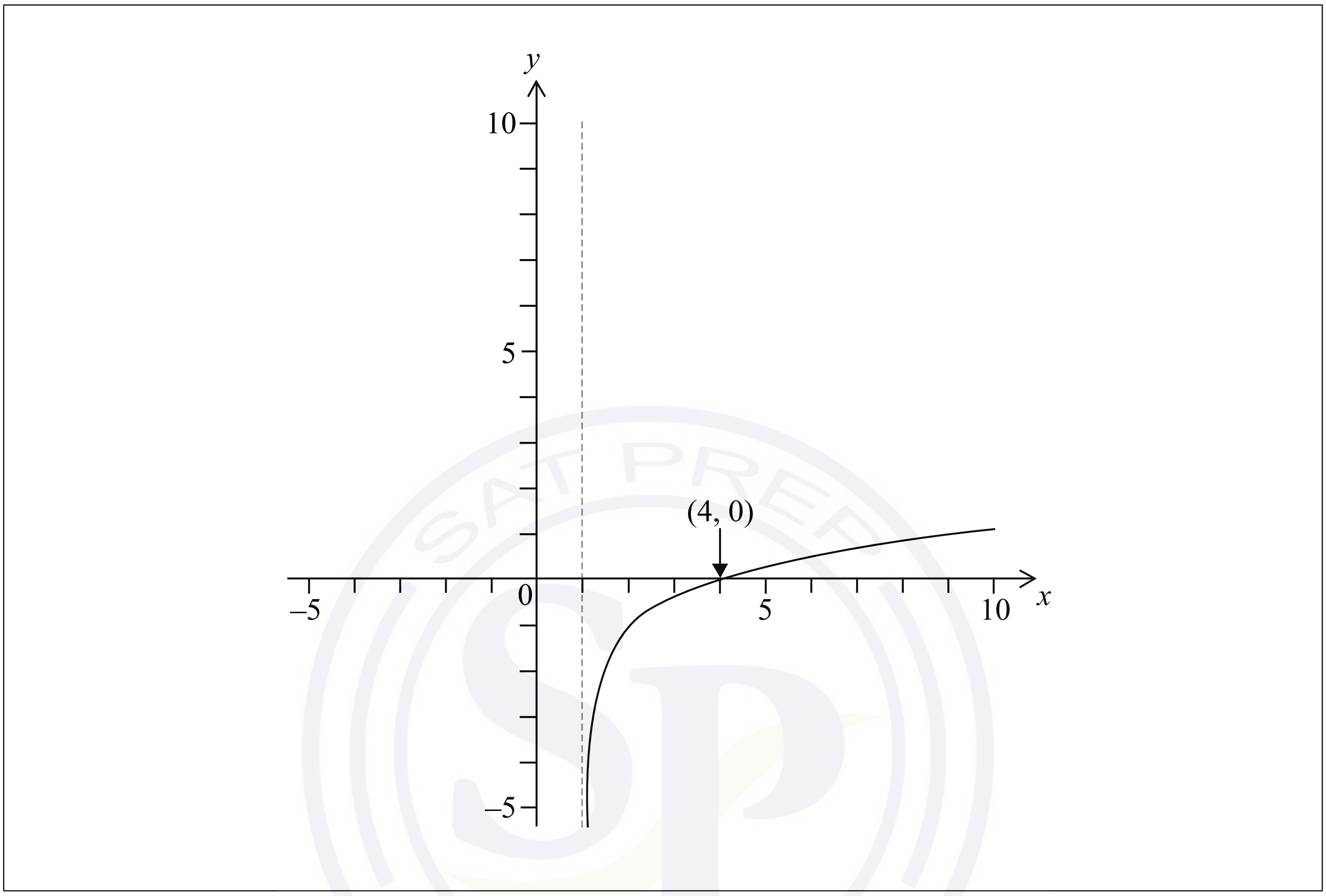
(a) Find the value of a. *[1]*

(b) Find $\mathrm{E}(X)$. *[2]*

(c) Find $\mathrm{Var}(X)$. *[3]*

3. *[Maximum mark: 7]*

The diagram below shows a sketch of the graph of $y = f(x)$.



(a) Sketch the graph of $y = f^{-1}(x)$ on the same axes. *[2]*

(b) State the range of f^{-1}. *[1]*

(c) Given that $f(x) = \ln(ax + b)$, $x > 1$, find the value of a and the value of b. *[4]*

(This question continues on the following page)

(Question 3 continued)

4. *[Maximum mark: 5]*

Consider the matrix $\boldsymbol{A} = \begin{pmatrix} 1 & a(a+1) \\ 1 & b(b+1) \end{pmatrix}$, $a \neq b$. Given that $\boldsymbol{A}$ is singular, find the value of $a+b$.

5. *[Maximum mark: 7]*

A curve has equation $x^3y^2 + x^3 - y^3 + 9y = 0$. Find the coordinates of the three points on the curve where $\frac{dy}{dx} = 0$.

6. *[Maximum mark: 7]*

Prove by mathematical induction that $n^3 + 11n$ is divisible by 3 for all $n \in \mathbb{Z}^+$.

7. *[Maximum mark: 7]*

The sum of the first two terms of a geometric series is 10 and the sum of the first four terms is 30.

(a) Show that the common ratio r satisfies $r^2 = 2$. *[4]*

(b) Given $r = \sqrt{2}$

(i) find the first term;

(ii) find the sum of the first ten terms. *[3]*

8. *[Maximum mark: 8]*

(a) Prove the trigonometric identity $\sin(x+y)\sin(x-y)=\sin^2 x-\sin^2 y$. *[4]*

(b) Given $f(x)=\sin\left(x+\frac{\pi}{6}\right)\sin\left(x-\frac{\pi}{6}\right)$, $x\in[0,\pi]$, find the range of f. *[2]*

(c) Given $g(x)=\csc\left(x+\frac{\pi}{6}\right)\csc\left(x-\frac{\pi}{6}\right)$, $x\in[0,\pi]$, $x\neq\frac{\pi}{6}$, $x\neq\frac{5\pi}{6}$, find the range of g. *[2]*

9. *[Maximum mark: 8]*

Solve the following equations:

(a) $\log_2 (x-2) = \log_4 \left(x^2 - 6x + 12\right)$; *[3]*

(b) $x^{\ln x} = \mathrm{e}^{(\ln x)^3}$. *[5]*

*Do **NOT** write solutions on this page.*

SECTION B

*Answer **all** questions in the answer booklet provided. Please start each question on a new page.*

10. *[Maximum mark: 20]*

The function f is given by $f(x) = x\mathrm{e}^{-x}$ $(x \geq 0)$.

(a) (i) Find an expression for $f'(x)$.

(ii) Hence determine the coordinates of the point A, where $f'(x) = 0$. *[3]*

(b) Find an expression for $f''(x)$ and hence show the point A is a maximum. *[3]*

(c) Find the coordinates of B, the point of inflexion. *[2]*

(d) The graph of the function g is obtained from the graph of f by stretching it in the x-direction by a scale factor 2.

(i) Write down an expression for $g(x)$.

(ii) State the coordinates of the maximum C of g.

(iii) Determine the x-coordinates of D and E, the two points where $f(x) = g(x)$. *[5]*

(e) Sketch the graphs of $y = f(x)$ and $y = g(x)$ on the same axes, showing clearly the points A, B, C, D and E. *[4]*

(f) Find an exact value for the area of the region bounded by the curve $y = g(x)$, the x-axis and the line $x = 1$. *[3]*

*Do **NOT** write solutions on this page.*

11. *[Maximum mark: 20]*

Consider the points $A(1, 0, 0)$, $B(2, 2, 2)$ and $C(0, 2, 1)$.

(a) Find the vector $\vec{CA} \times \vec{CB}$. *[4]*

(b) Find an exact value for the area of the triangle ABC. *[3]*

(c) Show that the Cartesian equation of the plane Π_1, containing the triangle ABC, is $2x + 3y - 4z = 2$. *[3]*

A second plane Π_2 is defined by the Cartesian equation $\Pi_2 : 4x - y - z = 4$. L_1 is the line of intersection of the planes Π_1 and Π_2.

(d) Find a vector equation for L_1. *[5]*

A third plane Π_3 is defined by the Cartesian equation $16x + \alpha y - 3z = \beta$.

(e) Find the value of α if all three planes contain L_1. *[3]*

(f) Find conditions on α and β if the plane Π_3 does **not** intersect with L_1. *[2]*

*Do **NOT** write solutions on this page.*

12. *[Maximum mark: 20]*

Consider the complex number $z = \cos\theta + \mathrm{i}\sin\theta$.

(a) Use De Moivre's theorem to show that $z^n + z^{-n} = 2\cos n\theta$, $n \in \mathbb{Z}^+$. *[2]*

(b) Expand $\left(z + z^{-1}\right)^4$. *[1]*

(c) Hence show that $\cos^4\theta = p\cos 4\theta + q\cos 2\theta + r$, where p, q and r are constants to be determined. *[4]*

(d) Show that $\cos^6\theta = \frac{1}{32}\cos 6\theta + \frac{3}{16}\cos 4\theta + \frac{15}{32}\cos 2\theta + \frac{5}{16}$. *[3]*

(e) Hence find the value of $\int_0^{\frac{\pi}{2}} \cos^6\theta \,\mathrm{d}\theta$. *[3]*

The region S is bounded by the curve $y = \sin x\cos^2 x$ and the x-axis between $x = 0$ and $x = \frac{\pi}{2}$.

(f) S is rotated through 2π radians about the x-axis. Find the value of the volume generated. *[4]*

(g) (i) Write down an expression for the constant term in the expansion of $\left(z + z^{-1}\right)^{2k}$, $k \in \mathbb{Z}^+$.

(ii) Hence determine an expression for $\int_0^{\frac{\pi}{2}} \cos^{2k}\theta \,\mathrm{d}\theta$ in terms of k. *[3]*

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M13/5/MATHL/HP1/ENG/TZ1/XX

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MATHEMATICS
HIGHER LEVEL
PAPER 1

Thursday 9 May 2013 (afternoon)

2 hours

Candidate session number

0	0							

Examination code

2	2	1	3	–	7	2	0	3

INSTRUCTIONS TO CANDIDATES

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SECTION A

*Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.*

1. *[Maximum mark: 6]*

(a) If $w = 2 + 2\mathrm{i}$, find the modulus and argument of w. *[2 marks]*

(b) Given $z = \cos\left(\frac{5\pi}{6}\right) + \mathrm{i}\sin\left(\frac{5\pi}{6}\right)$, find in its simplest form $w^4 z^6$. *[4 marks]*

2. *[Maximum mark: 6]*

Consider the points $A(1, 2, 3)$, $B(1, 0, 5)$ and $C(2, -1, 4)$.

(a) Find $\vec{AB} \times \vec{AC}$. *[4 marks]*

(b) Hence find the area of the triangle ABC. *[2 marks]*

3. *[Maximum mark: 5]*

Given $\boldsymbol{A} = \begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix}$ and $\boldsymbol{B} = \begin{pmatrix} 4 & -3 \\ 1 & 1 \end{pmatrix}$, find the matrix $\boldsymbol{X}$, such that $\boldsymbol{AXA}^{-1} = \boldsymbol{B}$.

4. *[Maximum mark: 5]*

The probability density function of the random variable X is defined as

$$f(x) = \begin{cases} \sin x, & 0 \le x \le \dfrac{\pi}{2} \\ 0, & \text{otherwise.} \end{cases}$$

Find $\mathrm{E}(X)$.

5. *[Maximum mark: 6]*

Paint is poured into a tray where it forms a circular pool with a uniform thickness of 0.5 cm. If the paint is poured at a constant rate of $4\,\text{cm}^3\text{s}^{-1}$, find the rate of increase of the radius of the circle when the radius is 20 cm.

6. *[Maximum mark: 6]*

The matrix $\boldsymbol{A}$ is such that $\boldsymbol{A}^2 = \boldsymbol{I}$, where $\boldsymbol{I}$ is the identity matrix. Use mathematical induction to prove that $(\boldsymbol{A}+\boldsymbol{I})^n = 2^{n-1}(\boldsymbol{A}+\boldsymbol{I})$, for all $n \in \mathbb{Z}^+$.

7. *[Maximum mark: 7]*

A curve is defined by the equation $8y\ln x - 2x^2 + 4y^2 = 7$. Find the equation of the tangent to the curve at the point where $x = 1$ and $y > 0$.

8. *[Maximum mark: 6]*

The first terms of an arithmetic sequence are $\frac{1}{\log_2 x}, \frac{1}{\log_8 x}, \frac{1}{\log_{32} x}, \frac{1}{\log_{128} x}, \ldots$

Find x if the sum of the first 20 terms of the sequence is equal to 100.

9. *[Maximum mark: 6]*

Two events A and B are such that $\mathrm{P}(A \cup B) = 0.7$ and $\mathrm{P}(A \mid B') = 0.6$.

Find $\mathrm{P}(B)$.

10. *[Maximum mark: 7]*

(a) Find all values of x for $0.1 \le x \le 1$ such that $\sin(\pi x^{-1}) = 0$. *[2 marks]*

(b) Find $\int_{\frac{1}{n+1}}^{\frac{1}{n}} \pi x^{-2} \sin(\pi x^{-1})\,dx$, showing that it takes different integer values when n is even and when n is odd. *[3 marks]*

(c) Evaluate $\int_{0.1}^{1} \left| \pi x^{-2} \sin(\pi x^{-1}) \right| dx$. *[2 marks]*

*Do **NOT** write solutions on this page.*

SECTION B

*Answer **all** questions in the answer booklet provided. Please start each question on a new page.*

11. *[Maximum mark: 19]*

(a) (i) Express $\cos\left(\frac{\pi}{6}+x\right)$ in the form $a\cos x - b\sin x$ where $a, b \in \mathbb{R}$.

(ii) Hence solve $\sqrt{3}\cos x - \sin x = 1$ for $0 \le x \le 2\pi$. *[7 marks]*

(b) Let $p(x) = 2x^3 - x^2 - 2x + 1$.

(i) Show that $x = 1$ is a zero of *p*.

(ii) Hence find all the solutions of $2x^3 - x^2 - 2x + 1 = 0$.

(iii) Express $\sin 2\theta \cos\theta + \sin^2\theta$ in terms of $\sin\theta$.

(iv) Hence solve $\sin 2\theta \cos\theta + \sin^2\theta = 1$ for $0 \le \theta \le 2\pi$. *[12 marks]*

*Do **NOT** write solutions on this page.*

12. *[Maximum mark: 19]*

(a) Express $4x^2 - 4x + 5$ in the form $a(x-h)^2 + k$ where $a, h, k \in \mathbb{Q}$. *[2 marks]*

(b) The graph of $y = x^2$ is transformed onto the graph of $y = 4x^2 - 4x + 5$. Describe a sequence of transformations that does this, making the order of transformations clear. *[3 marks]*

The function f is defined by $f(x) = \frac{1}{4x^2 - 4x + 5}$.

(c) Sketch the graph of $y = f(x)$. *[2 marks]*

(d) Find the range of f. *[2 marks]*

(e) By using a suitable substitution show that $\int f(x)\,dx = \frac{1}{4}\int \frac{1}{u^2+1}\,du$. *[3 marks]*

(f) Prove that $\int_1^{3.5} \frac{1}{4x^2 - 4x + 5}\,dx = \frac{\pi}{16}$. *[7 marks]*

*Do **NOT** write solutions on this page.*

13. *[Maximum mark: 22]*

On Saturday, Alfred and Beatrice play 6 different games against each other. In each game, one of the two wins. The probability that Alfred wins any one of these games is $\frac{2}{3}$.

(a) Show that the probability that Alfred wins exactly 4 of the games is $\frac{80}{243}$. *[3 marks]*

(b) (i) Explain why the total number of possible outcomes for the results of the 6 games is 64.

(ii) By expanding $(1+x)^6$ and choosing a suitable value for x, prove

$$64 = \binom{6}{0} + \binom{6}{1} + \binom{6}{2} + \binom{6}{3} + \binom{6}{4} + \binom{6}{5} + \binom{6}{6}.$$

(iii) State the meaning of this equality in the context of the 6 games played. *[4 marks]*

(c) The following day Alfred and Beatrice play the 6 games again. Assume that the probability that Alfred wins any one of these games is still $\frac{2}{3}$.

(i) Find an expression for the probability Alfred wins 4 games on the first day and 2 on the second day. Give your answer in the form $\binom{6}{r}^2 \left(\frac{2}{3}\right)^s \left(\frac{1}{3}\right)^t$ where the values of r, s and t are to be found.

(ii) Using your answer to (c)(i) and 6 similar expressions write down the probability that Alfred wins a total of 6 games over the two days as the sum of 7 probabilities.

(iii) Hence prove that $\binom{12}{6} = \binom{6}{0}^2 + \binom{6}{1}^2 + \binom{6}{2}^2 + \binom{6}{3}^2 + \binom{6}{4}^2 + \binom{6}{5}^2 + \binom{6}{6}^2$. *[9 marks]*

(d) Alfred and Beatrice play n games. Let A denote the number of games Alfred wins. The expected value of A can be written as $\mathrm{E}(A) = \sum_{r=0}^{n} r \binom{n}{r} \frac{a^r}{b^n}$.

(i) Find the values of a and b.

(ii) By differentiating the expansion of $(1+x)^n$, prove that the expected number of games Alfred wins is $\frac{2n}{3}$. *[6 marks]*

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will not be marked.

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MATHEMATICS
HIGHER LEVEL
PAPER 1

Thursday 9 May 2013 (afternoon)

2 hours

Candidate session number

0	0							

Examination code

2 2 1 3 – 7 2 0 5

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the ***Mathematics HL and Further Mathematics SL*** *information booklet* is required for this paper.
- The maximum mark for this examination paper is *[120 marks]*.

0116

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

*Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.*

1. *[Maximum mark: 6]*

Find the exact value of $\int_1^2 \left((x-2)^2 + \frac{1}{x} + \sin \pi x \right) dx$.

2. *[Maximum mark: 5]*

Consider the matrices $\boldsymbol{A} = \begin{pmatrix} 5 & 3 \\ 3 & 2 \end{pmatrix}$, $\boldsymbol{B} = \begin{pmatrix} 3 & -5 \\ 2 & -3 \end{pmatrix}$.

(a) Find $\det \boldsymbol{A}$ and hence write down the matrix $\boldsymbol{A}^{-1}$. *[2 marks]*

(b) Find the matrix $\boldsymbol{A}^{-1}\boldsymbol{B}$. *[3 marks]*

3. *[Maximum mark: 4]*

Expand $(2-3x)^5$ in ascending powers of x, simplifying coefficients.

4. *[Maximum mark: 5]*

Tim and Caz buy a box of 16 chocolates of which 10 are milk and 6 are dark. Caz randomly takes a chocolate and eats it. Then Tim randomly takes a chocolate and eats it.

(a) Draw a tree diagram representing the possible outcomes, clearly labelling each branch with the correct probability. *[3 marks]*

(b) Find the probability that Tim and Caz eat the same type of chocolate. *[2 marks]*

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5. *[Maximum mark: 7]*

The curve C is given by $y = \dfrac{x\cos x}{x + \cos x}$, for $x \geq 0$.

(a) Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{\cos^2 x - x^2 \sin x}{(x + \cos x)^2}$, $x \geq 0$. *[4 marks]*

(b) Find the equation of the tangent to C at the point $\left(\dfrac{\pi}{2}, 0\right)$. *[3 marks]*

6. *[Maximum mark: 7]*

A geometric sequence has first term a, common ratio r and sum to infinity 76. A second geometric sequence has first term a, common ratio r^3 and sum to infinity 36.

Find r.

7. *[Maximum mark: 7]*

Given the complex numbers $z_1 = 1 + 3\text{i}$ and $z_2 = -1 - \text{i}$.

(a) Write down the exact values of $|z_1|$ and $\arg(z_2)$. *[2 marks]*

(b) Find the minimum value of $|z_1 + \alpha z_2|$, where $\alpha \in \mathbb{R}$. *[5 marks]*

8. *[Maximum mark: 6]*

The curve C is given implicitly by the equation $\frac{x^2}{y} - 2x = \ln y$ for $y > 0$.

(a) Express $\frac{dy}{dx}$ in terms of x and y. *[4 marks]*

(b) Find the value of $\frac{dy}{dx}$ at the point on C where $y = 1$ and $x > 0$. *[2 marks]*

9. *[Maximum mark: 7]*

The function f is given by $f(x) = \dfrac{3^x + 1}{3^x - 3^{-x}}$, for $x > 0$.

(a) Show that $f(x) > 1$ for all $x > 0$. *[3 marks]*

(b) Solve the equation $f(x) = 4$. *[4 marks]*

10. *[Maximum mark: 6]*

(a) Given that $\arctan\left(\frac{1}{5}\right)+\arctan\left(\frac{1}{8}\right)=\arctan\left(\frac{1}{p}\right)$, where $p \in \mathbb{Z}^+$, find p. *[3 marks]*

(b) Hence find the value of $\arctan\left(\frac{1}{2}\right)+\arctan\left(\frac{1}{5}\right)+\arctan\left(\frac{1}{8}\right)$. *[3 marks]*

*Do **NOT** write solutions on this page.*

SECTION B

*Answer **all** questions on the answer booklet provided. Please start each question on a new page.*

11. *[Maximum mark: 21]*

The vertices of a triangle ABC have coordinates given by $A(-1, 2, 3)$, $B(4, 1, 1)$ and $C(3, -2, 2)$.

(a) (i) Find the lengths of the sides of the triangle.

(ii) Find $\cos B\hat{A}C$. *[6 marks]*

(b) (i) Show that $\overrightarrow{BC} \times \overrightarrow{CA} = -7\boldsymbol{i} - 3\boldsymbol{j} - 16\boldsymbol{k}$.

(ii) Hence, show that the area of the triangle ABC is $\frac{1}{2}\sqrt{314}$. *[5 marks]*

(c) Find the Cartesian equation of the plane containing the triangle ABC. *[3 marks]*

(d) Find a vector equation of (AB). *[2 marks]*

The point D on (AB) is such that $\overrightarrow{OD}$ is perpendicular to $\overrightarrow{BC}$ where O is the origin.

(e) (i) Find the coordinates of D.

(ii) Show that D does not lie between A and B. *[5 marks]*

*Do **NOT** write solutions on this page.*

12. *[Maximum mark: 21]*

The function f is defined by $f(x)=\frac{2x-1}{x+2}$, with domain $D=\{x:-1\le x\le 8\}$.

(a) Express $f(x)$ in the form $A+\frac{B}{x+2}$, where A and $B\in\mathbb{Z}$. *[2 marks]*

(b) Hence show that $f'(x)>0$ on D. *[2 marks]*

(c) State the range of f. *[2 marks]*

(d) (i) Find an expression for $f^{-1}(x)$.

(ii) Sketch the graph of $y=f(x)$, showing the points of intersection with both axes.

(iii) On the same diagram, sketch the graph of $y=f^{-1}(x)$. *[8 marks]*

(e) (i) On a different diagram, sketch the graph of $y=f(|x|)$ where $x\in D$.

(ii) Find all solutions of the equation $f(|x|)=-\frac{1}{4}$. *[7 marks]*

*Do **NOT** write solutions on this page.*

13. *[Maximum mark: 18]*

(a) (i) Express each of the complex numbers $z_1 = \sqrt{3} + \mathrm{i}$, $z_2 = -\sqrt{3} + \mathrm{i}$ and $z_3 = -2\mathrm{i}$ in modulus-argument form.

(ii) Hence show that the points in the complex plane representing z_1, z_2 and z_3 form the vertices of an equilateral triangle.

(iii) Show that ${z_1}^{3n} + {z_2}^{3n} = 2{z_3}^{3n}$ where $n \in \mathbb{N}$. *[9 marks]*

(b) (i) State the solutions of the equation $z^7 = 1$ for $z \in \mathbb{C}$, giving them in modulus-argument form.

(ii) If w is the solution to $z^7 = 1$ with least positive argument, determine the argument of $1 + w$. Express your answer in terms of π.

(iii) Show that $z^2 - 2z\cos\left(\frac{2\pi}{7}\right) + 1$ is a factor of the polynomial $z^7 - 1$. State the two other quadratic factors with real coefficients. *[9 marks]*

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MATHEMATICS
HIGHER LEVEL
PAPER 1

Candidate session number

0	0							

Tuesday 6 November 2012 (afternoon)

Examination code

8	8	1	2	–	7	2	0	1

2 hours

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions on the answer sheets provided. Write your session number on each answer sheet, and attach them to this examination paper and your cover sheet using the tag provided.
- At the end of the examination, indicate the number of sheets used in the appropriate box on your cover sheet.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the ***Mathematics HL and Further Mathematics SL*** *information booklet* is required for this paper.
- The maximum mark for this examination paper is *[120 marks]*.

0120

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SECTION A

*Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.*

1. *[Maximum mark: 4]*

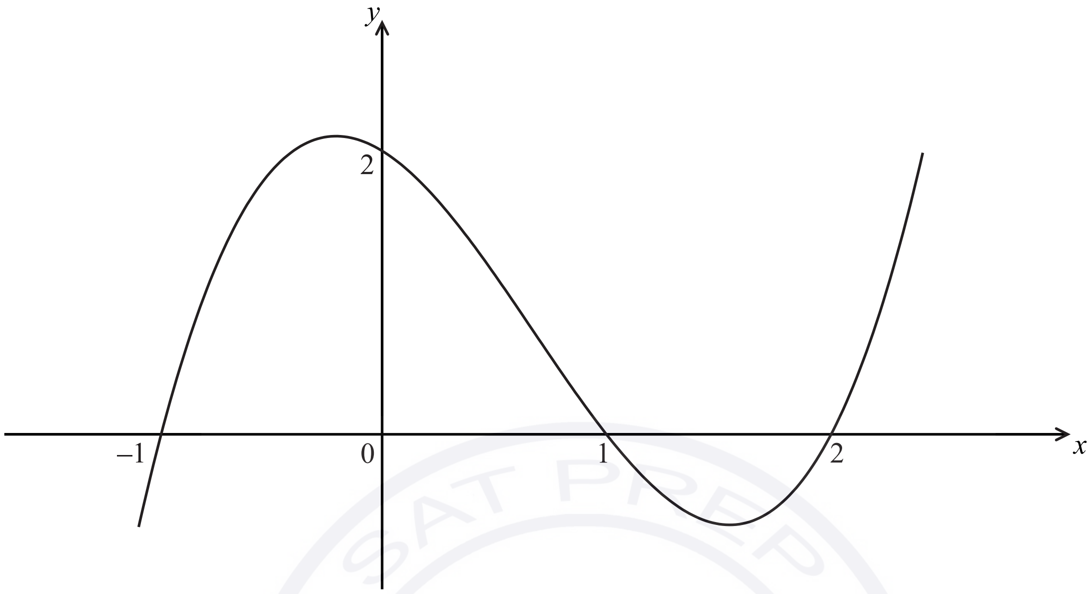
Given that $\frac{\pi}{2} < \alpha < \pi$ and $\cos\alpha = -\frac{3}{4}$, find the value of $\sin 2\alpha$.

2. *[Maximum mark: 4]*

Expand and simplify $\left(\frac{x}{y}-\frac{y}{x}\right)^4$.

3. *[Maximum mark: 7]*

Let $f(x) = x^3 + ax^2 + bx + c$, where a, b, $c \in \mathbb{Z}$. The diagram shows the graph of $y = f(x)$.



(a) Using the information shown in the diagram, find the values of a, b and c. *[4 marks]*

(This question continues on the following page)

(Question 3 continued)

(b) If $g(x) = 3f(x-2)$,

(i) state the coordinates of the points where the graph of g intercepts the x-axis.

(ii) Find the y-intercept of the graph of g. *[3 marks]*

4. *[Maximum mark: 8]*

The diagram shows the graph of the function defined by $y = x(\ln x)^2$ for $x > 0$.

The function has a local maximum at the point A and a local minimum at the point B.

(a) Find the coordinates of the points A and B. *[5 marks]*

(This question continues on the following page)

(Question 4 continued)

(b) Given that the graph of the function has exactly one point of inflexion, find its coordinates. *[3 marks]*

5. *[Maximum mark: 8]*

The continuous random variable X has probability density function given by

$$f(x) = \begin{cases} ae^{-x}, & 0 \le x \le 1 \\ 0, & \text{otherwise.} \end{cases}$$

(a) State the mode of X. *[1 mark]*

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(b) Determine the value of a. *[3 marks]*

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(Question 5 continued)

(c) Find $\mathrm{E}(X)$. *[4 marks]*

6. *[Maximum mark: 7]*

Consider the following equations, where $a, b \in \mathbb{R}$:

$$x + 3y + (a-1)z = 1$$
$$2x + 2y + (a-2)z = 1$$
$$3x + y + (a-3)z = b.$$

(a) If each of these equations defines a plane, show that, for any value of a, the planes do not intersect at a unique point. *[3 marks]*

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(Question 6 continued)

(b) Find the value of b for which the intersection of the planes is a straight line. *[4 marks]*

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7. *[Maximum mark: 7]*

In the triangle PQR, $PQ = 6$, $PR = k$ and $P\hat{Q}R = 30^\circ$.

(a) For the case $k = 4$, find the two possible values of QR. *[4 marks]*

(This question continues on the following page)

(Question 7 continued)

(b) Determine the values of k for which the conditions above define a unique triangle. *[3 marks]*

8. *[Maximum mark: 9]*

Consider the curve defined by the equation $x^2 + \sin y - xy = 0$.

(a) Find the gradient of the tangent to the curve at the point (π, π). *[6 marks]*

(This question continues on the following page)

(Question 8 continued)

(b) Hence, show that $\tan\theta = \dfrac{1}{1+2\pi}$, where θ is the acute angle between the tangent to the curve at (π, π) and the line $y = x$. *[3 marks]*

9. *[Maximum mark: 6]*

Two boats, A and B, move so that at time t hours, their position vectors, in kilometres, are $\boldsymbol{r}_A = (9t)\boldsymbol{i} + (3-6t)\boldsymbol{j}$ and $\boldsymbol{r}_B = (7-4t)\boldsymbol{i} + (7t-6)\boldsymbol{j}$.

(a) Find the coordinates of the common point of the paths of the two boats. *[4 marks]*

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(Question 9 continued)

(b) Show that the boats do not collide. *[2 marks]*

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*Do **NOT** write solutions on this page.*

SECTION B

*Answer **all** questions on the answer sheets provided. Please start each question on a new page.*

10. *[Maximum mark: 19]*

Consider the complex numbers

$$z_1 = 2\sqrt{3}\operatorname{cis}\frac{3\pi}{2} \text{ and } z_2 = -1 + \sqrt{3}\mathrm{i}.$$

(a) (i) Write down z_1 in Cartesian form.

(ii) Hence determine $(z_1 + z_2)^*$ in Cartesian form. *[3 marks]*

(b) (i) Write z_2 in modulus-argument form.

(ii) Hence solve the equation $z^3 = z_2$. *[6 marks]*

(c) Let $z = r\operatorname{cis}\theta$, where $r \in \mathbb{R}^+$ and $0 \le \theta < 2\pi$. Find all possible values of r and θ,

(i) if $z^2 = (1 + z_2)^2$;

(ii) if $z = -\frac{1}{z_2}$. *[6 marks]*

(d) Find the smallest positive value of n for which $\left(\frac{z_1}{z_2}\right)^n \in \mathbb{R}^+$. *[4 marks]*

*Do **NOT** write solutions on this page.*

11. *[Maximum mark: 18]*

Consider the matrix $\boldsymbol{A} = \begin{pmatrix} a & a-1 \\ b & b \end{pmatrix}$, $b \neq 0$.

(a) For $a = 2$ and $b = 1$, show that $(\boldsymbol{A}^2 - 3\boldsymbol{A})^2 = \boldsymbol{I}$. *[3 marks]*

(b) Find the value of a and the value of b in each of the following cases:

(i) $\boldsymbol{A}\begin{pmatrix} 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 0 \\ -2 \end{pmatrix}$;

(ii) $\boldsymbol{A}^{-1}\begin{pmatrix} 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 0 \\ -2 \end{pmatrix}$. *[6 marks]*

(c) Consider the lines l_1 and l_2 defined by the equations $ax + (a-1)y = 0$ and $bx + by = 1$, respectively, where $b \neq 0$.

(i) Find the coordinates of the intersection point of the lines l_1 and l_2 in terms of a and b.

(ii) Given that the lines are perpendicular, find the coordinates of the point of intersection in terms of b. *[9 marks]*

*Do **NOT** write solutions on this page.*

12. *[Maximum mark: 23]*

Consider a function f, defined by $f(x)=\frac{x}{2-x}$ for $0\le x\le 1$.

(a) Find an expression for $(f\circ f)(x)$. *[3 marks]*

Let $F_n(x)=\frac{x}{2^n-(2^n-1)x}$, where $0\le x\le 1$.

(b) Use mathematical induction to show that for any $n\in\mathbb{Z}^+$

$$\underbrace{(f\circ f\circ\ldots\circ f)}_{n\text{ times}}(x)=F_n(x).$$

[8 marks]

(c) Show that $F_{-n}(x)$ is an expression for the inverse of F_n. *[6 marks]*

(d) (i) State $F_n(0)$ and $F_n(1)$.

(ii) Show that $F_n(x)<x$, given $0<x<1$, $n\in\mathbb{Z}^+$.

(iii) For $n\in\mathbb{Z}^+$, let A_n be the area of the region enclosed by the graph of F_n^{-1}, the x-axis and the line $x=1$. Find the area B_n of the region enclosed by F_n and F_n^{-1} in terms of A_n. *[6 marks]*

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MATHEMATICS
HIGHER LEVEL
PAPER 1

Thursday 3 May 2012 (afternoon)

2 hours

Candidate session number

0	0							

Examination code

2	2	1	2	–	7	2	0	3

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
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- Section A: answer all questions in the boxes provided.
- Section B: answer all questions on the answer sheets provided. Write your session number on each answer sheet, and attach them to this examination paper and your cover sheet using the tag provided.
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- The maximum mark for this examination paper is *[120 marks]*.

0116

Please **do not** write on this page.

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SECTION A

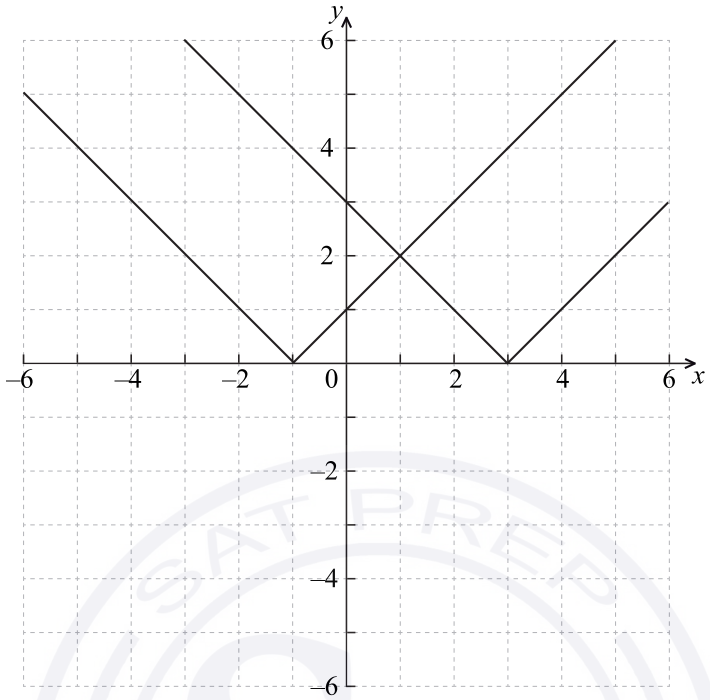
*Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.*

1. *[Maximum mark: 4]*

Find the value of k if $\sum_{r=1}^{\infty} k\left(\frac{1}{3}\right)^{r} = 7$.

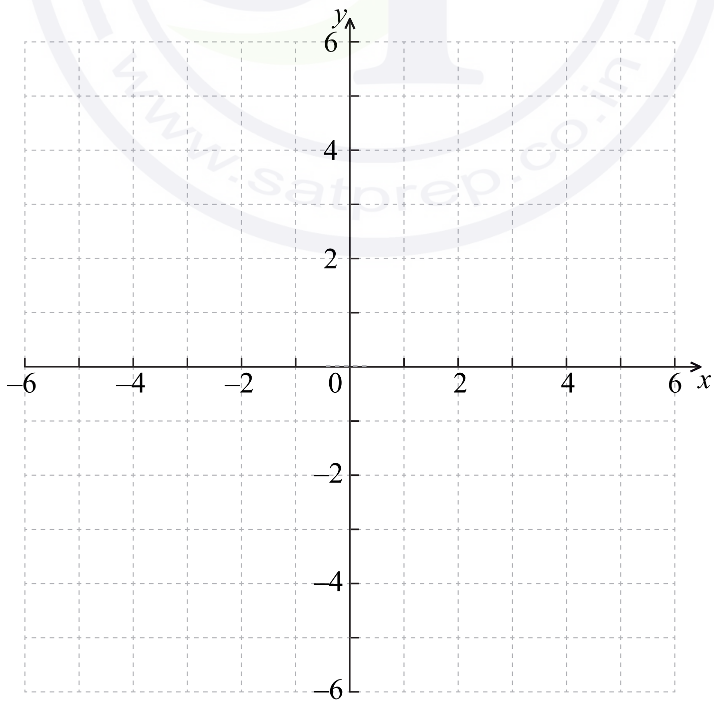
2. *[Maximum mark: 8]*

The graphs of $y = |x+1|$ and $y = |x-3|$ are shown below.



Let $f(x) = |x+1| - |x-3|$.

(a) Draw the graph of $y = f(x)$ on the blank grid below. *[4 marks]*



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(Question 2 continued)

(b) Hence state the value of

(i) $f'(-3)$;

(ii) $f'(2.7)$;

(iii) $\int_{-3}^{-2} f(x)\,\mathrm{d}x$. *[4 marks]*

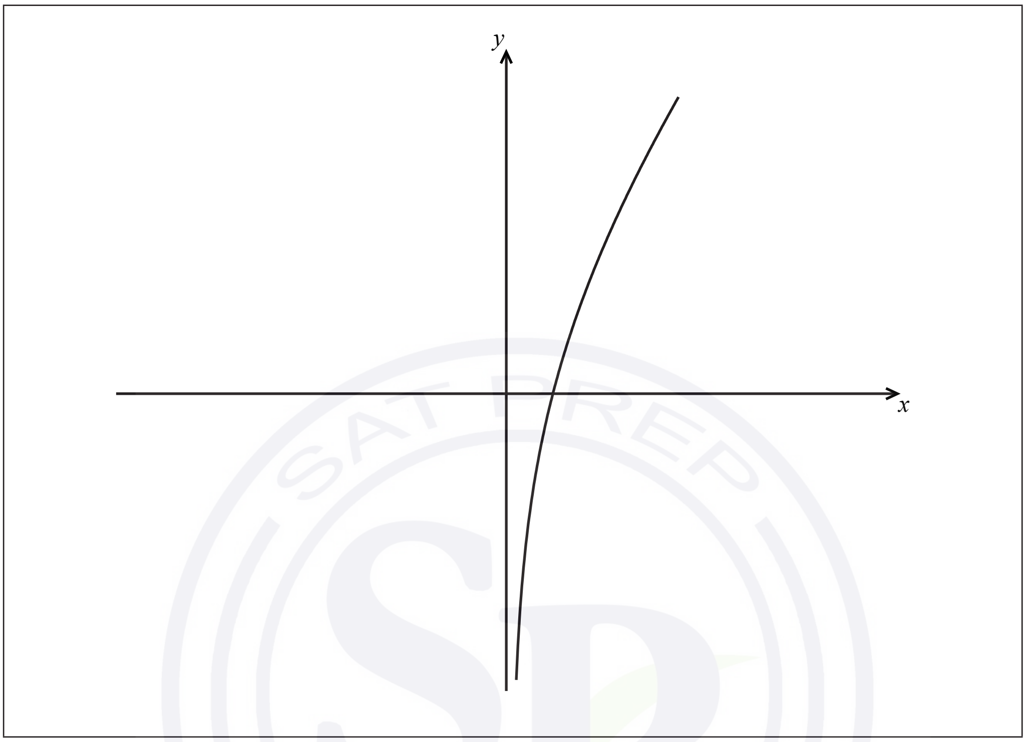
3. *[Maximum mark: 7]*

If $z_1 = a + a\sqrt{3}\,\mathrm{i}$ and $z_2 = 1 - \mathrm{i}$, where a is a real constant, express z_1 and z_2 in the form $r\,\mathrm{cis}\,\theta$, and hence find an expression for $\left(\frac{z_1}{z_2}\right)^6$ in terms of a and i.

4. *[Maximum mark: 6]*

The graph below shows $y = f(x)$, where $f(x) = x + \ln x$.

(a) On the graph below, sketch the curve $y = f^{-1}(x)$. *[2 marks]*



(b) Find the coordinates of the point of intersection of the graph of $y = f(x)$ and the graph of $y = f^{-1}(x)$. *[4 marks]*

5. *[Maximum mark: 7]*

Let $f(x) = \frac{\sin 3x}{\sin x} - \frac{\cos 3x}{\cos x}$.

(a) For what values of x does $f(x)$ not exist? *[2 marks]*

(b) Simplify the expression $\frac{\sin 3x}{\sin x} - \frac{\cos 3x}{\cos x}$. *[5 marks]*

6. *[Maximum mark: 8]*

The graph below shows the two curves $y = \frac{1}{x}$ and $y = \frac{k}{x}$, where $k > 1$.

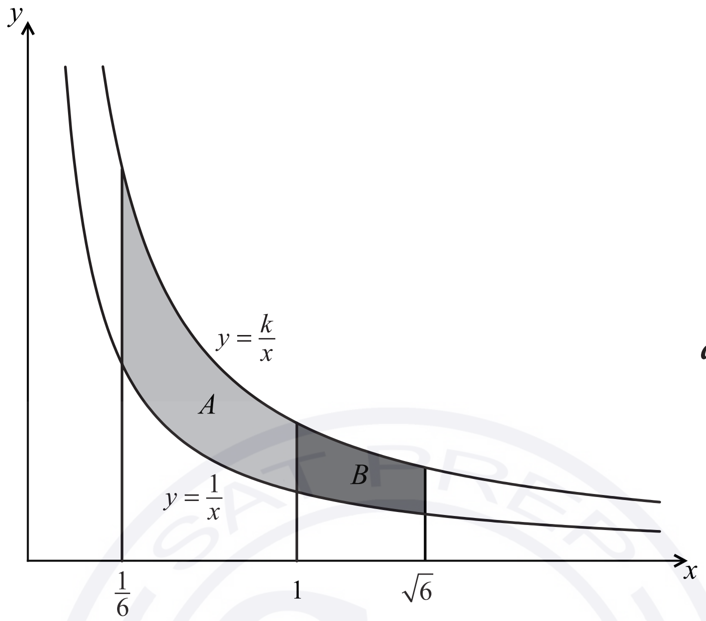


diagram not to scale

(a) Find the area of region A in terms of k. *[3 marks]*

(b) Find the area of region B in terms of k. *[2 marks]*

(c) Find the ratio of the area of region A to the area of region B. *[3 marks]*

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7. *[Maximum mark: 6]*

Given that z is the complex number $x + \mathrm{i}y$ and that $|z| + z = 6 - 2\mathrm{i}$, find the value of x and the value of y.

8. *[Maximum mark: 5]*

Solve the equation $2 - \log_3(x+7) = \log_{\frac{1}{3}} 2x$.

9. *[Maximum mark: 9]*

The curve C has equation $2x^2 + y^2 = 18$. Determine the coordinates of the four points on C at which the normal passes through the point $(1, 0)$.

*Do **NOT** write solutions on this page.*

SECTION B

*Answer **all** questions on the answer sheets provided. Please start each question on a new page.*

10. *[Maximum mark: 21]*

In the triangle ABC, $A\hat{B}C = 90^\circ$, $AC = \sqrt{2}$ and $AB = BC + 1$.

(a) Show that $\cos \hat{A} - \sin \hat{A} = \frac{1}{\sqrt{2}}$. *[3 marks]*

(b) By squaring both sides of the equation in part (a), solve the equation to find the angles in the triangle. *[8 marks]*

(c) Apply Pythagoras' theorem in the triangle ABC to find BC, and hence show that $\sin \hat{A} = \frac{\sqrt{6} - \sqrt{2}}{4}$. *[6 marks]*

(d) Hence, or otherwise, calculate the length of the perpendicular from B to [AC]. *[4 marks]*

11. *[Maximum mark: 17]*

(a) $\boldsymbol{A}$ and $\boldsymbol{U}$ are square matrices, and $\boldsymbol{X} = \boldsymbol{U}^{-1}\boldsymbol{A}\boldsymbol{U}$. Use mathematical induction to prove that $\boldsymbol{X}^n = \boldsymbol{U}^{-1}\boldsymbol{A}^n\boldsymbol{U}$, for $n \in \mathbb{Z}^+$. *[7 marks]*

(b) Let $\boldsymbol{A} = \begin{pmatrix} 2 & -3 \\ 1 & -2 \end{pmatrix}$ and $\boldsymbol{U} = \begin{pmatrix} 3 & 1 \\ 1 & 1 \end{pmatrix}$.

(i) Find the matrix $\boldsymbol{D}$ such that $\boldsymbol{A}\boldsymbol{U} = \boldsymbol{U}\boldsymbol{D}$.

(ii) Write down the matrix $\boldsymbol{D}^2$.

(iii) **Hence** prove that $\boldsymbol{A}^{2n} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, for $n \in \mathbb{Z}^+$.

(iv) Using the result from part (iii), show that $(\boldsymbol{A}^n)^{-1} = \boldsymbol{A}^n$, for $n \in \mathbb{Z}^+$. *[10 marks]*

*Do **NOT** write solutions on this page.*

12. *[Maximum mark: 22]*

Let $f(x)=\sqrt{\frac{x}{1-x}}$, $0<x<1$.

(a) Show that $f'(x)=\frac{1}{2}x^{-\frac{1}{2}}(1-x)^{-\frac{3}{2}}$ and deduce that f is an increasing function. *[5 marks]*

(b) Show that the curve $y=f(x)$ has one point of inflexion, and find its coordinates. *[6 marks]*

(c) Use the substitution $x=\sin^2\theta$ to show that $\int f(x)\,\mathrm{d}x=\arcsin\sqrt{x}-\sqrt{x-x^2}+c$. *[11 marks]*

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Answers written on this page will not be marked.

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M12/5/MATHL/HP1/ENG/TZ2/XX

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MATHEMATICS
HIGHER LEVEL
PAPER 1

Thursday 3 May 2012 (afternoon)

2 hours

Candidate session number

0	0							

Examination code

2	2	1	2	–	7	2	0	5

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions on the answer sheets provided. Write your session number on each answer sheet, and attach them to this examination paper and your cover sheet using the tag provided.
- At the end of the examination, indicate the number of sheets used in the appropriate box on your cover sheet.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the ***Mathematics HL and Further Mathematics SL*** *information booklet* is required for this paper.
- The maximum mark for this examination paper is *[120 marks]*.

0120

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

*Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.*

1. *[Maximum mark: 6]*

The same remainder is found when $2x^3+kx^2+6x+32$ and $x^4-6x^2-k^2x+9$ are divided by $x+1$. Find the possible values of k.

2. *[Maximum mark: 5]*

Find the values of x for which the vectors $\begin{pmatrix} 1 \\ 2\cos x \\ 0 \end{pmatrix}$ and $\begin{pmatrix} -1 \\ 2\sin x \\ 1 \end{pmatrix}$ are perpendicular, $0 \le x \le \frac{\pi}{2}$.

3. *[Maximum mark: 5]*

On a particular day, the probability that it rains is $\frac{2}{5}$. The probability that the "Tigers" soccer team wins on a day when it rains is $\frac{2}{7}$ and the probability that they win on a day when it does not rain is $\frac{4}{7}$.

(a) Draw a tree diagram to represent these events and their outcomes. *[1 mark]*

(b) What is the probability that the "Tigers" soccer team wins? *[2 marks]*

(c) Given that the "Tigers" soccer team won, what is the probability that it rained on that day? *[2 marks]*

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4. *[Maximum mark: 5]*

(a) Expand and simplify $\left(x-\dfrac{2}{x}\right)^4$. *[3 marks]*

(b) Hence determine the constant term in the expansion $(2x^2+1)\left(x-\dfrac{2}{x}\right)^4$. *[2 marks]*

5. *[Maximum mark: 5]*

Three non-singular 2×2 matrices $\boldsymbol{A}$, $\boldsymbol{B}$ and $\boldsymbol{X}$ satisfy $4\boldsymbol{A}-5\boldsymbol{BX}=\boldsymbol{B}$.

(a) Find $\boldsymbol{X}$ in terms of $\boldsymbol{A}$ and $\boldsymbol{B}$. *[2 marks]*

(b) Given that $\boldsymbol{A}=2\boldsymbol{B}$, find $\boldsymbol{X}$. *[3 marks]*

6. *[Maximum mark: 7]*

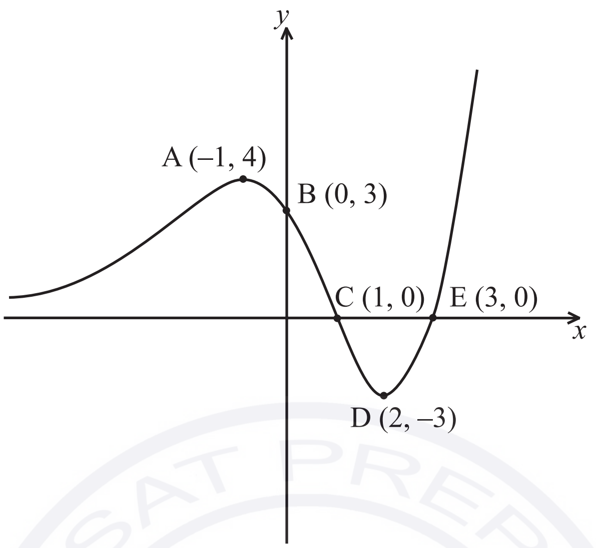
Given that $(4-5\mathrm{i})m+4n=16+15\mathrm{i}$, where $\mathrm{i}^2=-1$, find m and n if

(a) m and n are real numbers; *[3 marks]*

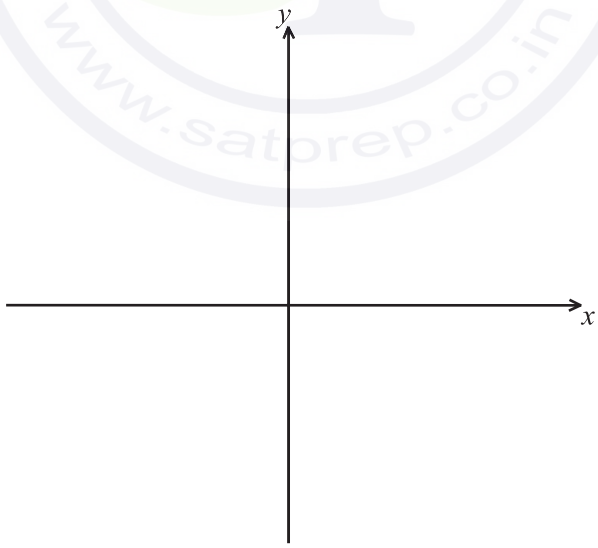
(b) m and n are conjugate complex numbers. *[4 marks]*

7. *[Maximum mark: 6]*

The graph of $y = f(x)$ is shown below, where A is a local maximum point and D is a local minimum point.



(a) On the axes below, sketch the graph of $y = \frac{1}{f(x)}$, clearly showing the coordinates of the images of the points A, B and D, labelling them A', B', and D' respectively, and the equations of any vertical asymptotes. *[3 marks]*



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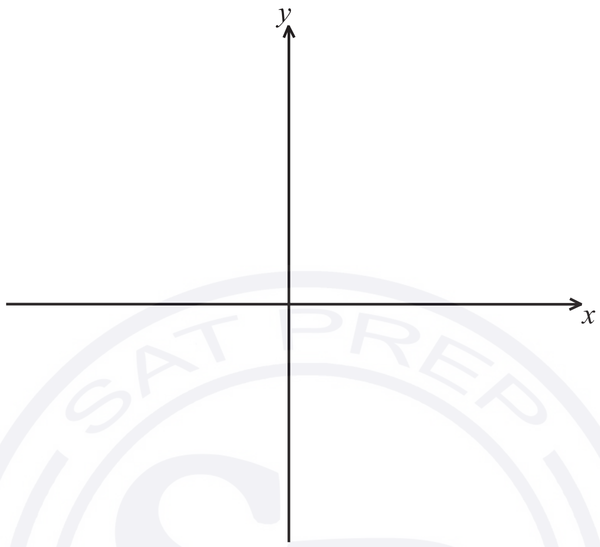
(Question 7 continued)

(b) On the axes below, sketch the graph of the derivative $y = f'(x)$, clearly showing the coordinates of the images of the points A and D, labelling them A'' and D'' respectively. *[3 marks]*



8. *[Maximum mark: 6]*

Let $x^3 y = a \sin nx$. Using implicit differentiation, show that

$$x^3 \frac{d^2 y}{dx^2} + 6x^2 \frac{dy}{dx} + (n^2 x^2 + 6)\, xy = 0 .$$

9. *[Maximum mark: 6]*

Show that $\frac{\cos A + \sin A}{\cos A - \sin A} = \sec 2A + \tan 2A$.

10. *[Maximum mark: 9]*

The function f is defined on the domain $\left[0, \frac{3\pi}{2}\right]$ by $f(x) = e^{-x}\cos x$.

(a) State the two zeros of f. *[1 mark]*

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(b) Sketch the graph of f. *[1 mark]*

(This question continues on the following page)

(Question 10 continued)

(c) The region bounded by the graph, the x-axis and the y-axis is denoted by A and the region bounded by the graph and the x-axis is denoted by B. Show that the ratio of the area of A to the area of B is

$$\frac{e^{\pi}\left(e^{\frac{\pi}{2}}+1\right)}{e^{\pi}+1}.$$

[7 marks]

Please **do not** write on this page.

Answers written on this page will not be marked.

*Do **NOT** write solutions on this page.*

SECTION B

*Answer **all** questions on the answer sheets provided. Please start each question on a new page.*

11. *[Maximum mark: 18]*

Consider the following functions:

$$f(x) = \frac{2x^2+3}{75},\ x \geq 0$$

$$g(x) = \frac{|3x-4|}{10},\ x \in \mathbb{R}.$$

(a) State the range of f and of g. *[2 marks]*

(b) Find an expression for the composite function $f \circ g(x)$ in the form $\frac{ax^2+bx+c}{3750}$, where a, b and $c \in \mathbb{Z}$. *[4 marks]*

(c) (i) Find an expression for the inverse function $f^{-1}(x)$.

(ii) State the domain and range of f^{-1}. *[4 marks]*

The domains of f and g are now restricted to $\{0, 1, 2, 3, 4\}$.

(d) By considering the values of f and g on this new domain, determine which of f and g could be used to find a probability distribution for a discrete random variable X, stating your reasons clearly. *[6 marks]*

(e) Using this probability distribution, calculate the mean of X. *[2 marks]*

*Do **NOT** write solutions on this page.*

12. *[Total mark: 29]*

Part A *[Maximum mark: 12]*

(a) Given that $(x+\mathrm{i}y)^2 = -5+12\mathrm{i}$, $x,\ y \in \mathbb{R}$. Show that

(i) $x^2 - y^2 = -5$;

(ii) $xy = 6$. *[2 marks]*

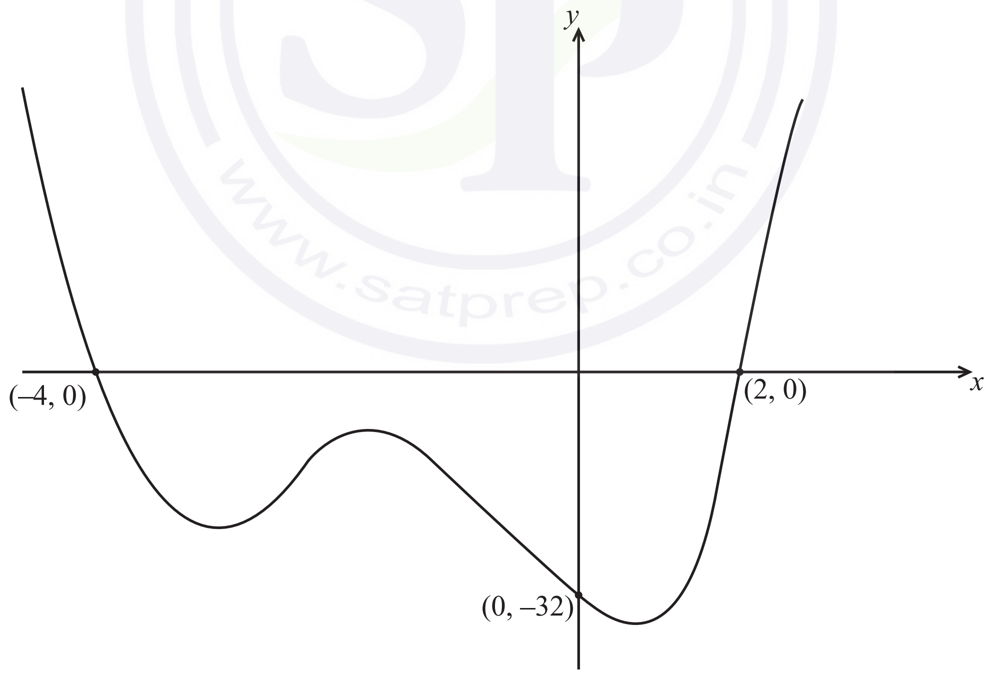
(b) Hence find the two square roots of $-5+12\mathrm{i}$. *[5 marks]*

(c) For any complex number z, show that $(z^*)^2 = (z^2)^*$. *[3 marks]*

(d) Hence write down the two square roots of $-5-12\mathrm{i}$. *[2 marks]*

Part B *[Maximum mark: 17]*

The graph of a polynomial function f of degree 4 is shown below.



(a) Explain why, of the four roots of the equation $f(x) = 0$, two are real and two are complex. *[2 marks]*

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(Question 12 continued)

(b) The curve passes through the point $(-1, -18)$. Find $f(x)$ in the form $f(x) = (x-a)(x-b)(x^2+cx+d)$, where $a, b, c, d \in \mathbb{Z}$. *[5 marks]*

(c) Find the two complex roots of the equation $f(x) = 0$ in Cartesian form. *[2 marks]*

(d) Draw the four roots on the complex plane (the Argand diagram). *[2 marks]*

(e) Express each of the four roots of the equation in the form $re^{i\theta}$. *[6 marks]*

13. *[Maximum mark: 13]*

(a) Using the definition of a derivative as $f'(x) = \lim_{h \to 0}\left(\frac{f(x+h)-f(x)}{h}\right)$, show that the derivative of $\frac{1}{2x+1}$ is $\frac{-2}{(2x+1)^2}$. *[4 marks]*

(b) Prove by induction that the n^{th} derivative of $(2x+1)^{-1}$ is $(-1)^n \frac{2^n n!}{(2x+1)^{n+1}}$. *[9 marks]*

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MATHEMATICS
HIGHER LEVEL
PAPER 1

Wednesday 2 November 2011 (afternoon)

2 hours

Candidate session number

0	0							

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all of Section A in the spaces provided.
- Section B: answer all of Section B on the answer sheets provided. Write your session number on each answer sheet, and attach them to this examination paper and your cover sheet using the tag provided.
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SECTION A

*Answer **all** the questions in the spaces provided. Working may be continued below the lines, if necessary.*

1. *[Maximum mark: 6]*

From a vertex of an equilateral triangle of side $2x$, a circular arc is drawn to divide the triangle into two regions, as shown in the diagram below.

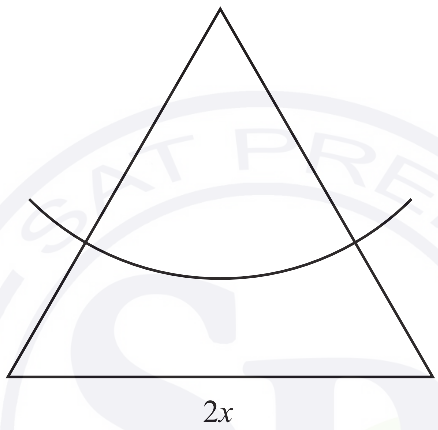

diagram not to scale

Given that the areas of the two regions are equal, find the radius of the arc in terms of x.

2. *[Maximum mark: 6]*

Find the cube roots of i in the form $a+b\mathrm{i}$, where $a,\ b\in\mathbb{R}$.

3. *[Maximum mark: 6]*

In a particular city 20 % of the inhabitants have been immunized against a certain disease. The probability of infection from the disease among those immunized is $\frac{1}{10}$, and among those not immunized the probability is $\frac{3}{4}$. If a person is chosen at random and found to be infected, find the probability that this person has been immunized.

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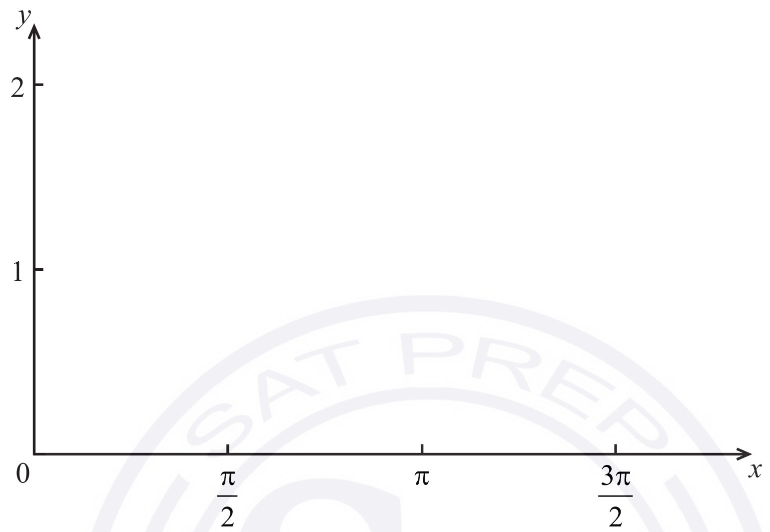
4. *[Maximum mark: 6]*

Given that $f(x) = 1 + \sin x$, $0 \le x \le \frac{3\pi}{2}$,

(a) sketch the graph of f; *[1 mark]*



(b) show that $(f(x))^2 = \frac{3}{2} + 2\sin x - \frac{1}{2}\cos 2x$; *[1 mark]*

(c) find the volume of the solid formed when the graph of f is rotated through 2π radians about the x-axis. *[4 marks]*

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5. *[Maximum mark: 6]*

A target consists of three concentric circles of radii 1 m, 3 m and 5 m respectively, as shown in the diagram.

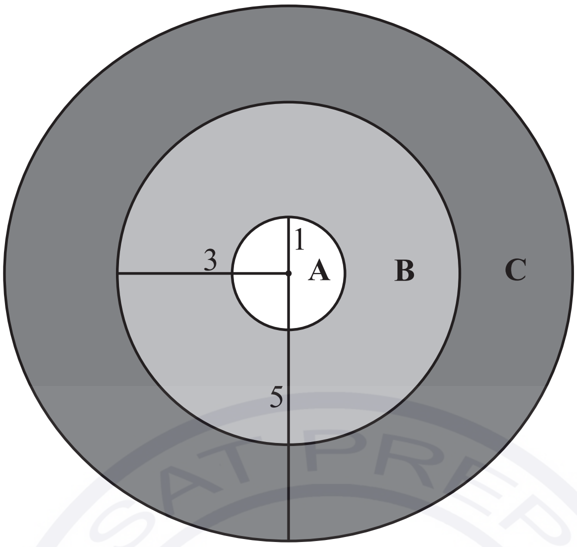

diagram not to scale

Nina shoots an arrow at the target. She has a probability of $\frac{1}{2}$ of hitting the target. If the arrow hits the target it does so at a random point on the target. Ten points are scored for hitting region A, six points for hitting region B, and three points for hitting region C. Find the expected number of points Nina scores each time she shoots an arrow at the target.

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6. *[Maximum mark: 7]*

Given that $y = \frac{1}{1-x}$, use mathematical induction to prove that $\frac{d^n y}{dx^n} = \frac{n!}{(1-x)^{n+1}}$, $n \in \mathbb{Z}^+$.

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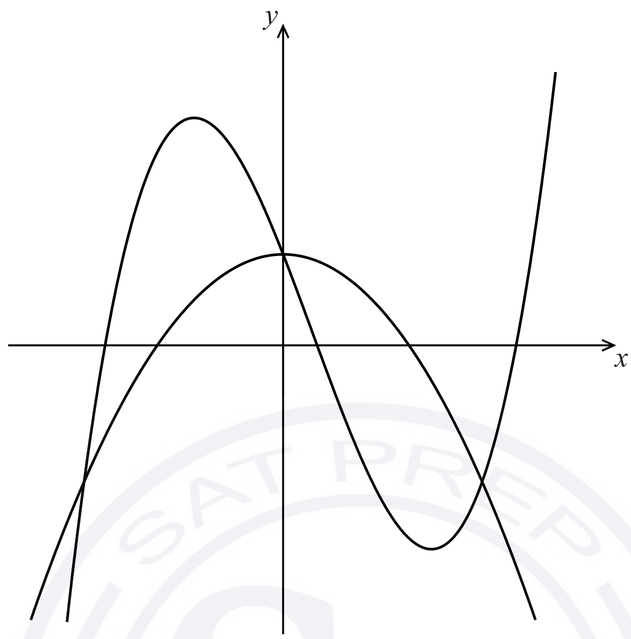
7. *[Maximum mark: 7]*

The graphs of $f(x) = -x^2 + 2$ and $g(x) = x^3 - x^2 - bx + 2$, $b > 0$, intersect and create two closed regions. Show that these two regions have equal areas.



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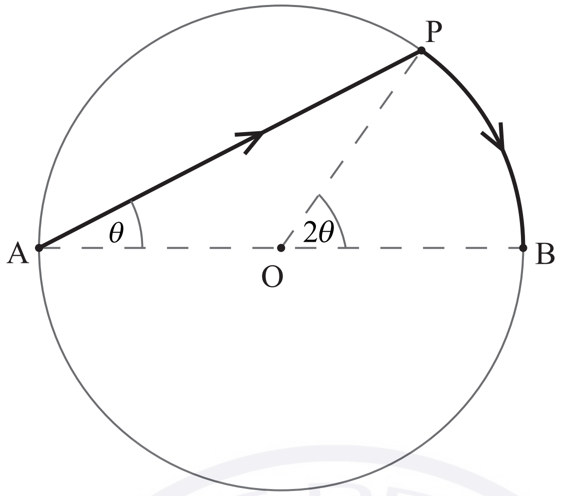
8. *[Maximum mark: 8]*

The diagram below shows a circular lake with centre O, diameter AB and radius 2 km .



Jorg needs to get from A to B as quickly as possible. He considers rowing to point P and then walking to point B. He can row at $3\ \text{km h}^{-1}$ and walk at $6\ \text{km h}^{-1}$. Let $\text{P}\hat{\text{A}}\text{B} = \theta$ radians, and t be the time in hours taken by Jorg to travel from A to B.

(a) Show that $t = \frac{2}{3}(2\cos\theta + \theta)$. *[3 marks]*

(b) Find the value of θ for which $\frac{\text{d}t}{\text{d}\theta} = 0$. *[2 marks]*

(c) What route should Jorg take to travel from A to B in the least amount of time? Give reasons for your answer. *[3 marks]*

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9. *[Maximum mark: 8]*

Consider the equation $yx^2 + (y-1)x + (y-1) = 0$.

(a) Find the set of values of y for which this equation has real roots. *[4 marks]*

(b) Hence determine the range of the function $f : x \to \frac{x+1}{x^2+x+1}$. *[3 marks]*

(c) Explain why f has no inverse. *[1 mark]*

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*Do **NOT** write solutions on this page. Any working on this page will **NOT** be marked.*

SECTION B

*Answer **all** the questions on the answer sheets provided. Please start each question on a new page.*

10. *[Maximum mark: 10]*

A continuous random variable X has the probability density function

$$f(x)=\begin{cases} k\sin x, & 0\le x\le \dfrac{\pi}{2} \\ 0, & \text{otherwise.}\end{cases}$$

(a) Find the value of k. *[2 marks]*

(b) Find $\mathrm{E}(X)$. *[5 marks]*

(c) Find the median of X. *[3 marks]*

11. *[Maximum mark: 11]*

At 12:00 a boat is 20 km due south of a freighter. The boat is travelling due east at 20 km h^{-1}, and the freighter is travelling due south at 40 km h^{-1}.

(a) Determine the time at which the two ships are closest to one another, and justify your answer. *[8 marks]*

(b) If the visibility at sea is 9 km, determine whether or not the captains of the two ships can ever see each other's ship. *[3 marks]*

*Do **NOT** write solutions on this page. Any working on this page will **NOT** be marked.*

12. *[Maximum mark: 15]*

(a) For non-zero vectors $\boldsymbol{a}$ and $\boldsymbol{b}$, show that

(i) if $|\boldsymbol{a}-\boldsymbol{b}|=|\boldsymbol{a}+\boldsymbol{b}|$, then $\boldsymbol{a}$ and $\boldsymbol{b}$ are perpendicular;

(ii) $|\boldsymbol{a}\times\boldsymbol{b}|^2=|\boldsymbol{a}|^2|\boldsymbol{b}|^2-(\boldsymbol{a}\bullet\boldsymbol{b})^2$. *[8 marks]*

(b) The points A, B and C have position vectors $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$.

(i) Show that the area of triangle ABC is $\frac{1}{2}|\boldsymbol{a} \quad \boldsymbol{b}+\boldsymbol{b} \quad \boldsymbol{c}+\boldsymbol{c} \quad \boldsymbol{a}|$.

(ii) Hence, show that the shortest distance from B to AC is

$$\frac{|\boldsymbol{a}\times\boldsymbol{b}+\boldsymbol{b}\times\boldsymbol{c}+\boldsymbol{c}\times\boldsymbol{a}|}{|\boldsymbol{c}-\boldsymbol{a}|}.$$ *[7 marks]*

13. *[Maximum mark: 24]*

The curve C with equation $y=f(x)$ satisfies the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{y}{\ln y}(x+2),\ y>1,$$

and $y=\mathrm{e}$ when $x=2$.

(a) Find the equation of the tangent to C at the point $(2, \mathrm{e})$. *[3 marks]*

(b) Find $f(x)$. *[11 marks]*

(c) Determine the largest possible domain of f. *[6 marks]*

(d) Show that the equation $f(x)=f'(x)$ has no solution. *[4 marks]*