Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Higher level
Paper 2

Wednesday 4 November 2020 (morning)

Candidate session number

2 hours

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics HL and further mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[100 marks]**.

8820 – 7202

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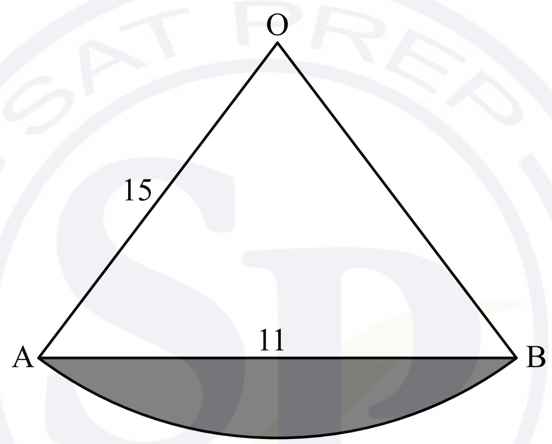
Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 5]

The following diagram shows a sector OAB of radius $15\,\mathrm{cm}$. The length of $[\mathrm{AB}]$ is $11\,\mathrm{cm}$.

diagram not to scale



Find the area of the shaded region.

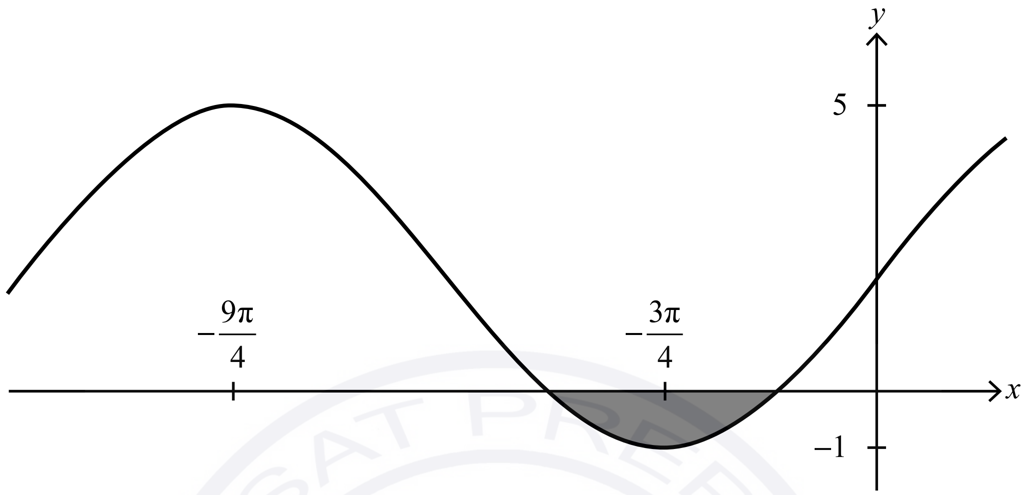
2. [Maximum mark: 4]

Jenna is a keen book reader. The number of books she reads during one week can be modelled by a Poisson distribution with mean 2.6.

Determine the expected number of weeks in one year, of 52 weeks, during which Jenna reads at least four books.

3. [Maximum mark: 8]

The following diagram shows part of the graph of $y = p + q\sin(rx)$. The graph has a local maximum point at $\left(-\frac{9\pi}{4}, 5\right)$ and a local minimum point at $\left(-\frac{3\pi}{4}, -1\right)$.



(a) Determine the values of p, q and r. [4]

(b) Hence find the area of the shaded region. [4]

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4. [Maximum mark: 6]

Find the term independent of x in the expansion of $\frac{1}{x^3}\left(\frac{1}{3x^2}-\frac{x}{2}\right)^9$.

5. [Maximum mark: 7]

A survey of British holidaymakers found that 15% of those surveyed took a holiday in the Lake District in 2019.

(a) A random sample of 16 British holidaymakers was taken. The number of people in the sample who took a holiday in the Lake District in 2019 can be modelled by a binomial distribution.

(i) State two assumptions made in order for this model to be valid.

(ii) Find the probability that at least three people from the sample took a holiday in the Lake District in 2019. [4]

(b) From a random sample of n holidaymakers, the probability that at least one of them took a holiday in the Lake District in 2019 is greater than 0.999.

Determine the least possible value of n. [3]

6. [Maximum mark: 7]

Use mathematical induction to prove that $\frac{\mathrm{d}^n}{\mathrm{d}x^n}\left(x\mathrm{e}^{px}\right) = p^{n-1}\left(px+n\right)\mathrm{e}^{px}$ for $n \in \mathbb{Z}^+$, $p \in \mathbb{Q}$.

7. [Maximum mark: 7]

At a gathering of 12 teachers, seven are male and five are female. A group of five of these teachers go out for a meal together. Determine the possible number of groups in each of the following situations:

(a) There are more males than females in the group. [4]

(b) Two of the teachers, Gary and Gerwyn, refuse to go out for a meal together. [3]

8. [Maximum mark: 6]

A small bead is free to move along a smooth wire in the shape of the curve $y = \dfrac{10}{3 - 2\mathrm{e}^{-0.5x}} \ (x \geq 0)$.

(a) Find an expression for $\dfrac{\mathrm{d}y}{\mathrm{d}x}$. [3]

At the point on the curve where $x = 4$, it is given that $\dfrac{\mathrm{d}y}{\mathrm{d}t} = -0.1 \text{ m s}^{-1}$.

(b) Find the value of $\dfrac{\mathrm{d}x}{\mathrm{d}t}$ at this exact same instant. [3]

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Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

9. [Maximum mark: 14]

The weights, in grams, of individual packets of coffee can be modelled by a normal distribution, with mean $102\,\text{g}$ and standard deviation $8\,\text{g}$.

(a) Find the probability that a randomly selected packet has a weight less than $100\,\text{g}$. [2]

(b) The probability that a randomly selected packet has a weight greater than w grams is 0.444. Find the value of w. [2]

(c) A packet is randomly selected. Given that the packet has a weight greater than $105\,\text{g}$, find the probability that it has a weight greater than $110\,\text{g}$. [3]

(d) From a random sample of 500 packets, determine the number of packets that would be expected to have a weight lying within 1.5 standard deviations of the mean. [3]

(e) Packets are delivered to supermarkets in batches of 80. Determine the probability that at least 20 packets from a randomly selected batch have a weight less than $95\,\text{g}$. [4]

Do **not** write solutions on this page.

10. [Maximum mark: 16]

The plane Π_1 has equation $3x - y + z = -13$ and the line L has vector equation

$$\boldsymbol{r} = \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} -3 \\ -1 \\ 4 \end{pmatrix}, \lambda \in \mathbb{R}.$$

(a) Given that L meets Π_1 at the point P, find the coordinates of P. [4]

(b) Find the shortest distance from the point $\text{O}(0, 0, 0)$ to Π_1. [4]

The plane Π_2 contains the point O and the line L.

(c) Find the equation of Π_2, giving your answer in the form $\boldsymbol{r}.\boldsymbol{n} = d$. [3]

(d) Determine the acute angle between Π_1 and Π_2. [5]

Do **not** write solutions on this page.

11. [Maximum mark: 20]

A particle P moves in a straight line such that after time t seconds, its velocity, v in m s^{-1}, is given by $v = \text{e}^{-3t} \sin 6t$, where $0 < t < \frac{\pi}{2}$.

(a) Find the times when P comes to instantaneous rest. [2]

At time t, P has displacement $s(t)$; at time $t = 0$, $s(0) = 0$.

(b) Find an expression for s in terms of t. [7]

(c) Find the maximum displacement of P, in metres, from its initial position. [2]

(d) Find the total distance travelled by P in the first 1.5 seconds of its motion. [2]

At successive times when the acceleration of P is $0\,\text{m s}^{-2}$, the velocities of P form a geometric sequence. The acceleration of P is zero at times t_1, t_2, t_3 where $t_1 < t_2 < t_3$ and the respective velocities are v_1, v_2, v_3.

(e) (i) Show that, at these times, $\tan 6t = 2$.

(ii) Hence show that $\frac{v_2}{v_1} = \frac{v_3}{v_2} = -\text{e}^{-\frac{\pi}{2}}$. [7]

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Diploma Programme
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Mathematics
Higher level
Paper 2

Tuesday 19 November 2019 (morning)

Candidate session number

2 hours

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8819 – 7202

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Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 5]

A geometric sequence has $u_4 = -70$ and $u_7 = 8.75$. Find the second term of the sequence.

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2. [Maximum mark: 6]

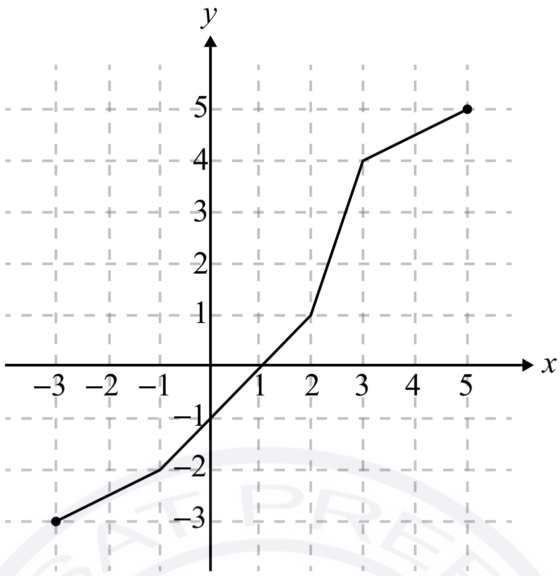
The number of marathons that Audrey runs in any given year can be modelled by a Poisson distribution with mean 1.3.

(a) Calculate the probability that Audrey will run at least two marathons in a particular year. [2]

(b) Find the probability that she will run at least two marathons in exactly four out of the following five years. [4]

3. [Maximum mark: 6]

The following diagram shows the graph of $y = f(x), -3 \le x \le 5$.



(a) Find the value of $(f \circ f)(1)$. [2]

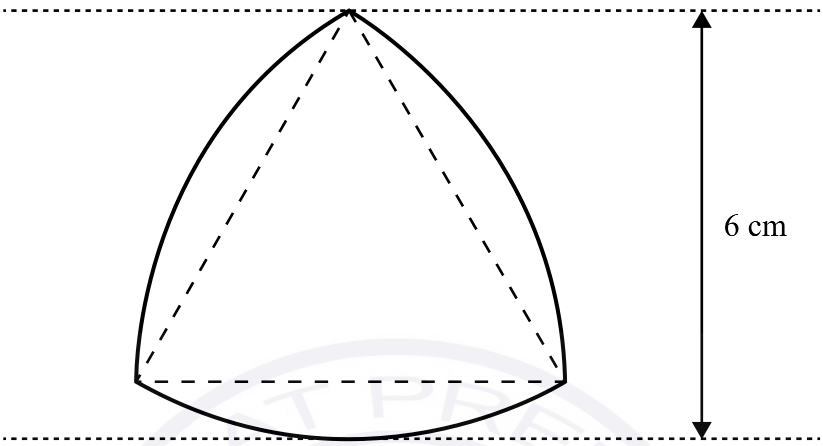
(b) Given that $f^{-1}(a) = 3$, determine the value of a. [2]

(c) Given that $g(x) = 2f(x-1)$, find the domain and range of g. [2]

4. [Maximum mark: 7]

The following shape consists of three arcs of a circle, each with centre at the opposite vertex of an equilateral triangle as shown in the diagram.

diagram not to scale



For this shape, calculate

(a) the perimeter; [2]

(b) the area. [5]

5. [Maximum mark: 6]

Consider the expansion of $(2 + x)^n$, where $n \geq 3$ and $n \in \mathbb{Z}$.

The coefficient of x^3 is four times the coefficient of x^2. Find the value of n.

6. [Maximum mark: 6]

Let $P(z) = az^3 - 37z^2 + 66z - 10$, where $z \in \mathbb{C}$ and $a \in \mathbb{Z}$.

One of the roots of $P(z) = 0$ is $3 + \mathrm{i}$. Find the value of a.

7. [Maximum mark: 6]

Runners in an athletics club have season's best times for the $100\,\text{m}$, which can be modelled by a normal distribution with mean 11.6 seconds and standard deviation 0.8 seconds. To qualify for a particular competition a runner must have a season's best time of under 11 seconds. A runner from this club who has qualified for the competition is selected at random. Find the probability that he has a season's best time of under 10.7 seconds.

8. [Maximum mark: 8]

Eight boys and two girls sit on a bench. Determine the number of possible arrangements, given that

(a) the girls do not sit together; [3]

(b) the girls do not sit on either end; [2]

(c) the girls do not sit on either end and do not sit together. [3]

Do **not** write solutions on this page.

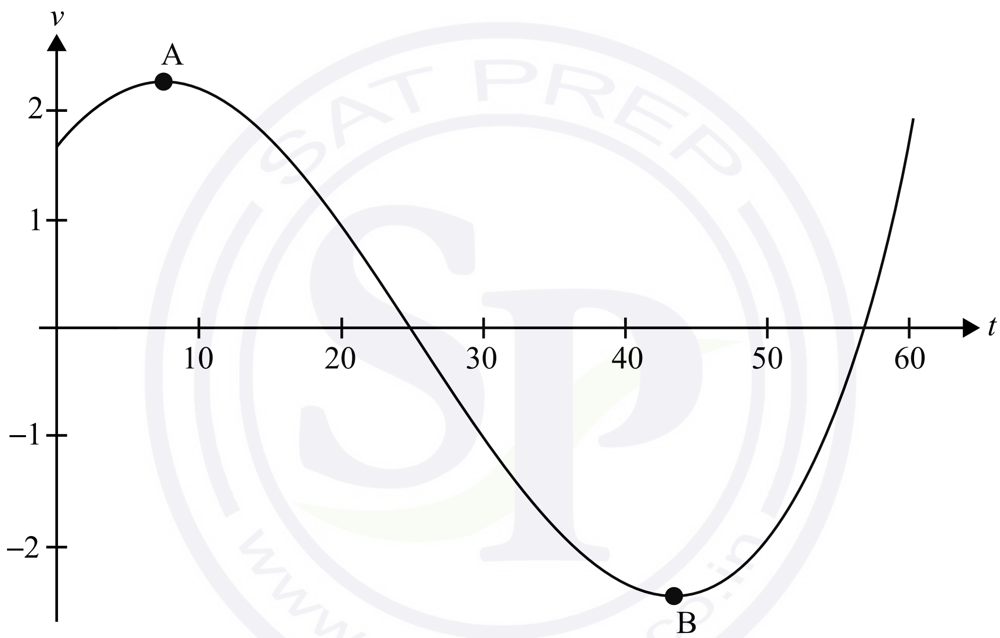
Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

9. [Maximum mark: 14]

A body moves in a straight line such that its velocity, $v\ \mathrm{m\,s^{-1}}$, after t seconds is given by $v = 2\sin\left(\frac{t}{10}+\frac{\pi}{5}\right)\csc\left(\frac{t}{30}+\frac{\pi}{4}\right)$ for $0 \le t \le 60$.

The following diagram shows the graph of v against t. Point A is a local maximum and point B is a local minimum.



(a) (i) Determine the coordinates of point A and the coordinates of point B.

(ii) Hence, write down the maximum speed of the body. [5]

(b) The body first comes to rest at time $t = t_1$. Find

(i) the value of t_1;

(ii) the distance travelled between $t = 0$ and $t = t_1$;

(iii) the acceleration when $t = t_1$. [6]

(c) Find the distance travelled in the first 30 seconds. [3]

Do **not** write solutions on this page.

10. [Maximum mark: 19]

A random variable X has probability density function

$$f(x)=\begin{cases} 3a & , \quad 0\le x<2 \\ a(x-5)(1-x) & , \quad 2\le x\le b \\ 0 & , \quad \text{otherwise} \end{cases} \quad a,\, b\in\mathbb{R}^{+},\, 3<b\le 5 .$$

(a) Find, in terms of a, the probability that X lies between 1 and 3. [4]

Consider the case where $b=5$.

(b) Sketch the graph of f. State the coordinates of the end points and any local maximum or minimum points, giving your answers in terms of a. [4]

(c) Find the value of

(i) a;

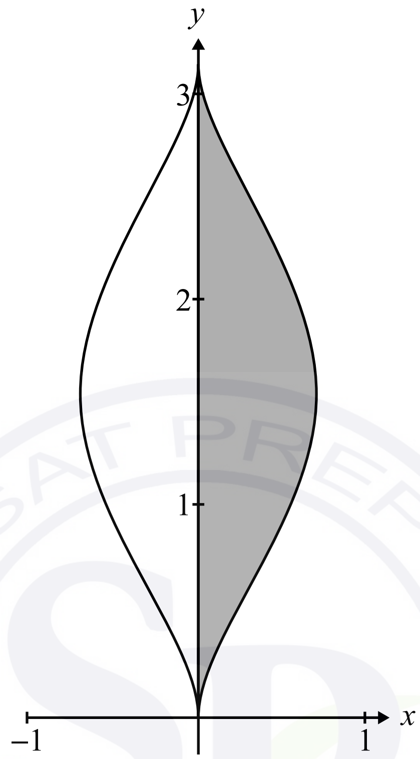
(ii) $\mathrm{E}(X)$;

(iii) the median of X. [11]

Do **not** write solutions on this page.

11. [Maximum mark: 17]

The following diagram shows part of the graph of $2x^2 = \sin^3 y$ for $0 \le y \le \pi$.



(a) (i) Using implicit differentiation, find an expression for $\frac{dy}{dx}$.

(ii) Find the equation of the tangent to the curve at the point $\left(\frac{1}{4}, \frac{5\pi}{6}\right)$. [8]

The shaded region R is the area bounded by the curve, the y-axis and the lines $y = 0$ and $y = \pi$.

(b) Find the area of R. [3]

The region R is now rotated about the y-axis, through 2π radians, to form a solid.

(c) By writing $\sin^3 y$ as $(1 - \cos^2 y)\sin y$, show that the volume of the solid formed is $\frac{2\pi}{3}$. [6]

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Mathematics
Higher level
Paper 2

Tuesday 14 May 2019 (morning)

Candidate session number

2 hours

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2219 – 7204

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Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 4]

Let l be the tangent to the curve $y = xe^{2x}$ at the point $(1, e^2)$.

Find the coordinates of the point where l meets the x-axis.

2. [Maximum mark: 5]

Solve $z^2 = 4e^{\frac{\pi}{2}i}$, giving your answers in the form

(a) $re^{i\theta}$ where $r, \theta \in \mathbb{R}, r > 0$; [3]

(b) $a + ib$ where $a, b \in \mathbb{R}$. [2]

3. [Maximum mark: 7]

The marks achieved by eight students in a class test are given in the following list.

8	4	7	6	10	9	7	3

(a) Find

(i) the mean;

(ii) the standard deviation. [2]

(b) The teacher increases all the marks by 2. Write down the new value for

(i) the mean;

(ii) the standard deviation. [2]

A ninth student also takes the test.

(c) Explain why the median is unchanged. [3]

4. [Maximum mark: 5]

The function f is defined by $f(x) = \sec x + 2,\ 0 \le x < \frac{\pi}{2}$.

(a) Write down the range of f . [1]

(b) Find $f^{-1}(x)$, stating its domain. [4]

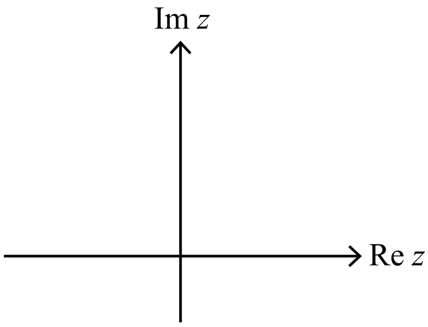
5. [Maximum mark: 6]

Use integration by parts to find $\int (\ln x)^2 \, dx$.

6. [Maximum mark: 7]

Let $z = a + b\mathrm{i}$, $a, b \in \mathbb{R}^+$ and let $\arg z = \theta$.

(a) Show the points represented by z and $z - 2a$ on the following Argand diagram. [1]



(b) Find an expression in terms of θ for

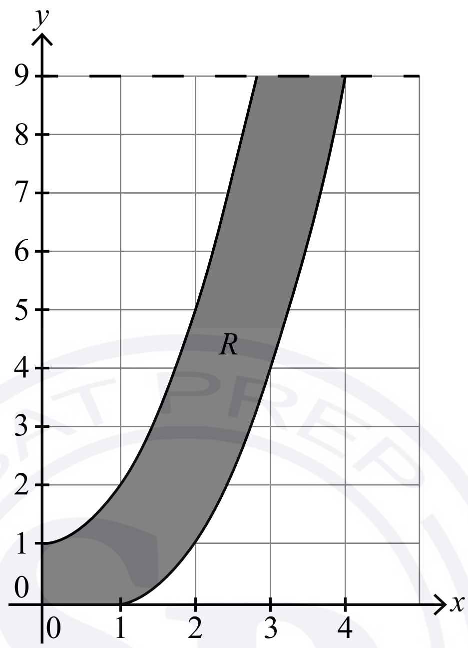
(i) $\arg(z - 2a)$;

(ii) $\arg\left(\dfrac{z}{z-2a}\right)$. [3]

(c) Hence or otherwise find the value of θ for which $\mathrm{Re}\left(\dfrac{z}{z-2a}\right) = 0$. [3]

7. [Maximum mark: 7]

The function f is defined by $f(x) = (x-1)^2, x \geq 1$ and the function g is defined by $g(x) = x^2 + 1, x \geq 0$.
The region R is bounded by the curves $y = f(x), y = g(x)$ and the lines $y = 0, x = 0$ and $y = 9$ as shown on the following diagram.



The shape of a clay vase can be modelled by rotating the region R through 360° about the y-axis.

Find the volume of clay used to make the vase.

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8. [Maximum mark: 9]

(a) Solve the inequality $x^2 > 2x + 1$. [2]

(b) Use mathematical induction to prove that $2^{n+1} > n^2$ for $n \in \mathbb{Z}, n \geq 3$. [7]

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

9. [Maximum mark: 13]

A café serves sandwiches and cakes. Each customer will choose one of the following three options; buy only a sandwich, buy only a cake or buy both a sandwich and a cake.
The probability that a customer buys a sandwich is 0.72 and the probability that a customer buys a cake is 0.45.

(a) Find the probability that a customer chosen at random will buy

(i) both a sandwich and a cake;

(ii) only a sandwich. [4]

On a typical day 200 customers come to the café.

(b) Find

(i) the expected number of cakes sold on a typical day;

(ii) the probability that more than 100 cakes will be sold on a typical day. [4]

It is known that 46 % of the customers who come to the café are male, and that 80 % of these buy a sandwich.

(c) (i) A customer is selected at random. Find the probability that the customer is male and buys a sandwich.

(ii) A female customer is selected at random. Find the probability that she buys a sandwich. [5]

Do **not** write solutions on this page.

10. [Maximum mark: 20]

The voltage v in a circuit is given by the equation

$$v(t) = 3\sin(100\pi t),\ t \geq 0 \text{ where } t \text{ is measured in seconds.}$$

(a) Write down the maximum and minimum value of v. [2]

The current i in this circuit is given by the equation

$$i(t) = 2\sin\left(100\pi(t + 0.003)\right).$$

(b) Write down two transformations that will transform the graph of $y = v(t)$ onto the graph of $y = i(t)$. [2]

The power p in this circuit is given by $p(t) = v(t) \times i(t)$.

(c) Sketch the graph of $y = p(t)$ for $0 \leq t \leq 0.02$, showing clearly the coordinates of the first maximum and the first minimum. [3]

(d) Find the total time in the interval $0 \leq t \leq 0.02$ for which $p(t) \geq 3$. [3]

The average power p_{av} in this circuit from $t = 0$ to $t = T$ is given by the equation

$$p_{av}(T) = \frac{1}{T}\int_0^T p(t)\mathrm{d}t, \text{ where } T > 0.$$

(e) Find $p_{av}(0.007)$. [2]

(f) With reference to your graph of $y = p(t)$ explain why $p_{av}(T) > 0$ for all $T > 0$. [2]

(g) Given that $p(t)$ can be written as $p(t) = a\sin(b(t - c)) + d$ where $a, b, c, d > 0$, use your graph to find the values of a, b, c and d. [6]

Do **not** write solutions on this page.

11. [Maximum mark: 17]

Consider the equation $x^5 - 3x^4 + mx^3 + nx^2 + px + q = 0$, where $m, n, p, q \in \mathbb{R}$.
The equation has three distinct real roots which can be written as $\log_2 a$, $\log_2 b$ and $\log_2 c$.
The equation also has two imaginary roots, one of which is $d\mathrm{i}$ where $d \in \mathbb{R}$.

(a) Show that $abc = 8$. [5]

The values a, b, and c are consecutive terms in a geometric sequence.

(b) Show that one of the real roots is equal to 1. [3]

(c) Given that $q = 8d^2$, find the other two real roots. [9]

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2219 – 7206

11 pages

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Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 5]

In triangle ABC, $\text{AB} = 5$, $\text{BC} = 14$ and $\text{AC} = 11$.
Find all the interior angles of the triangle. Give your answers in degrees to one decimal place.

2. [Maximum mark: 5]

Timmy owns a shop. His daily income from selling his goods can be modelled as a normal distribution, with a mean daily income of \$820, and a standard deviation of \$230. To make a profit, Timmy's daily income needs to be greater than \$1000.

(a) Calculate the probability that, on a randomly selected day, Timmy makes a profit. [2]

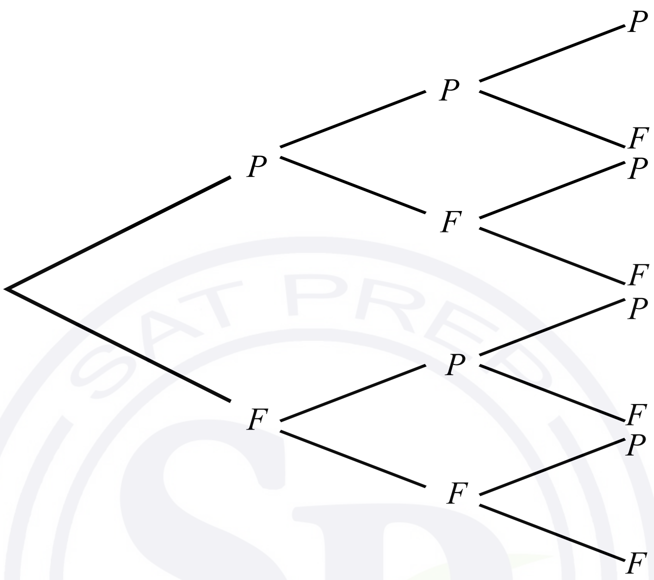
The shop is open for 24 days every month.

(b) Calculate the probability that, in a randomly selected month, Timmy makes a profit on between 5 and 10 days (inclusive). [3]

3. [Maximum mark: 8]

Iqbal attempts three practice papers in mathematics. The probability that he passes the first paper is 0.6. Whenever he gains a pass in a paper, his confidence increases so that the probability of him passing the next paper increases by 0.1. Whenever he fails a paper the probability of him passing the next paper is 0.6.

(a) Complete the given probability tree diagram for Iqbal's three attempts, labelling each branch with the correct probability. [3]

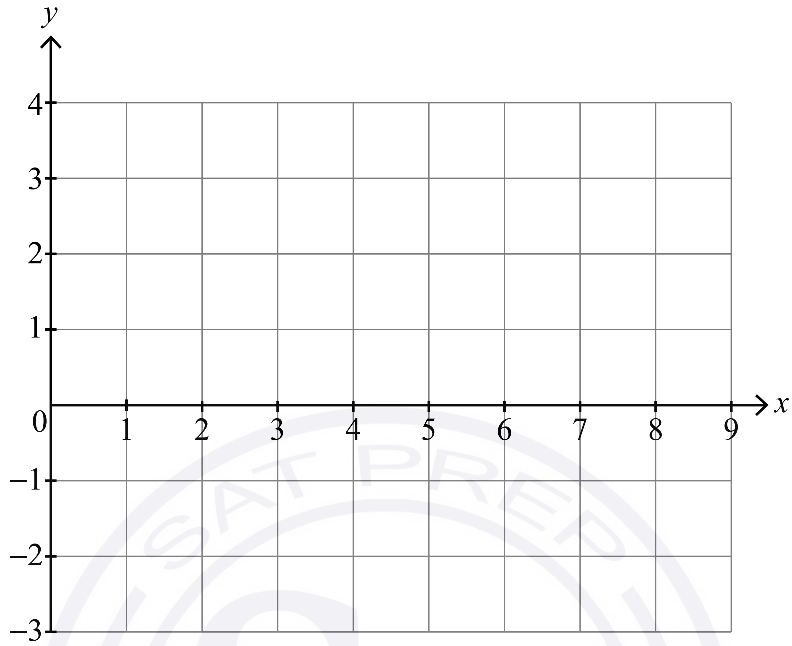


(b) Calculate the probability that Iqbal passes at least two of the papers he attempts. [2]

(c) Find the probability that Iqbal passes his third paper, given that he passed only one previous paper. [3]

4. [Maximum mark: 6]

(a) Sketch the graphs of $y = \sin^3 x + \ln x$ and $y = 1 + \cos x$ on the following axes for $0 < x \leq 9$. [2]



(b) Hence solve $\sin^3 x + \ln x - \cos x - 1 < 0$ in the range $0 < x \leq 9$. [4]

5. [Maximum mark: 6]

(a) Prove the identity $\dfrac{1+\sin 2x}{\cos 2x} \equiv \dfrac{1+\tan x}{1-\tan x}$. [4]

(b) Solve the equation $\dfrac{1+\sin 2x}{\cos 2x} = \sqrt{3}$ for $0 \le x < 2\pi$. [2]

6. [Maximum mark: 6]

A particle moves along a horizontal line such that at time t seconds, $t \geq 0$, its acceleration a is given by $a = 2t - 1$. When $t = 6$, its displacement s from a fixed origin O is 18.25 m. When $t = 15$, its displacement from O is 922.75 m. Find an expression for s in terms of t.

7. [Maximum mark: 7]

Suppose that u_1 is the first term of a geometric series with common ratio r.
Prove, by mathematical induction, that the sum of the first n terms, S_n is given by

$S_n = \dfrac{u_1\left(1-r^n\right)}{1-r}$, where $n \in \mathbb{Z}^+$.

8. [Maximum mark: 7]

(a) Find the roots of the equation $w^3 = 8\mathrm{i}$, $w \in \mathbb{C}$. Give your answers in Cartesian form. [4]

One of the roots w_1 satisfies the condition $\mathrm{Re}(w_1) = 0$.

(b) Given that $w_1 = \dfrac{z}{z - \mathrm{i}}$, express z in the form $a + b\mathrm{i}$ where $a, b \in \mathbb{Q}$. [3]

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

9. [Maximum mark: 15]

Consider the polynomial $P(z) \equiv z^4 - 6z^3 - 2z^2 + 58z - 51$, $z \in \mathbb{C}$.

(a) Express $P(z)$ in the form $(z^2 + az + b)(z^2 + cz + d)$ where $a, b, c, d \in \mathbb{R}$. [7]

(b) Sketch the graph of $y = x^4 - 6x^3 - 2x^2 + 58x - 51$, stating clearly the coordinates of any maximum and minimum points and intersections with axes. [6]

(c) Hence, or otherwise, state the condition on $k \in \mathbb{R}$ such that all roots of the equation $P(z) = k$ are real. [2]

10. [Maximum mark: 16]

Steffi the stray cat often visits Will's house in search of food. Let X be the discrete random variable "the number of times per day that Steffi visits Will's house".
The random variable X can be modelled by a Poisson distribution with mean 2.1.

(a) Find the probability that on a randomly selected day, Steffi does not visit Will's house. [2]

Let Y be the discrete random variable "the number of times per day that Steffi is fed at Will's house". Steffi is only fed on the first four occasions that she visits each day.

(b) Copy and complete the probability distribution table for Y. [4]

y	0	1	2	3	4
$P(Y = y)$					

(c) Hence find the expected number of times per day that Steffi is fed at Will's house. [3]

(d) In any given year of 365 days, the probability that Steffi does not visit Will for at most n days in total is 0.5 (to one decimal place). Find the value of n. [3]

(e) Show that the expected number of occasions per year on which Steffi visits Will's house and is not fed is at least 30. [4]

Do **not** write solutions on this page.

11. [Maximum mark: 19]

The plane Π_1 contains the points $\mathrm{P}(1, 6, -7)$, $\mathrm{Q}(0, 1, 1)$ and $\mathrm{R}(2, 0, -4)$.

(a) Find the Cartesian equation of the plane containing P, Q and R. [6]

The Cartesian equation of the plane Π_2 is given by $x - 3y - z = 3$.

(b) Given that Π_1 and Π_2 meet in a line L, verify that the vector equation of L can be given by $\boldsymbol{r} = \begin{pmatrix} \frac{5}{4} \\ 0 \\ -\frac{7}{4} \end{pmatrix} + \lambda \begin{pmatrix} \frac{1}{2} \\ 1 \\ -\frac{5}{2} \end{pmatrix}$. [3]

The Cartesian equation of the plane Π_3 is given by $ax + by + cz = 1$.

(c) Given that Π_3 is parallel to the line L, show that $a + 2b - 5c = 0$. [1]

Consider the case that Π_3 contains L.

(d) (i) Show that $5a - 7c = 4$.

(ii) Given that Π_3 is equally inclined to both Π_1 and Π_2, determine two distinct possible Cartesian equations for Π_3. [9]

Please **do not** write on this page.

Answers written on this page
will not be marked.

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Higher level
Paper 2

Tuesday 13 November 2018 (morning)

Candidate session number

2 hours

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics HL and further mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[100 marks]**.

8818 – 7202

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 5]

Consider a geometric sequence with a first term of 4 and a fourth term of -2.916.

(a) Find the common ratio of this sequence. [3]

(b) Find the sum to infinity of this sequence. [2]

2. [Maximum mark: 7]

A function f satisfies the conditions $f(0) = -4$, $f(1) = 0$ and its second derivative is $f''(x) = 15\sqrt{x} + \frac{1}{(x+1)^2}$, $x \geq 0$.

Find $f(x)$.

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3. [Maximum mark: 8]

It is known that 56 % of Infiglow batteries have a life of less than 16 hours, and 94 % have a life less than 17 hours. It can be assumed that battery life is modelled by the normal distribution $N(\mu, \sigma^2)$.

(a) Find the value of μ and the value of σ. [6]

(b) Find the probability that a randomly selected Infiglow battery will have a life of at least 15 hours. [2]

4. [Maximum mark: 5]

Find the value of the constant term in the expansion of $x^4\left(x+\dfrac{3}{x^2}\right)^5$.

5. [Maximum mark: 5]

Differentiate from first principles the function $f(x) = 3x^3 - x$.

6. [Maximum mark: 6]

Let $P(x) = 2x^4 - 15x^3 + ax^2 + bx + c$, where $a, b, c \in \mathbb{R}$.

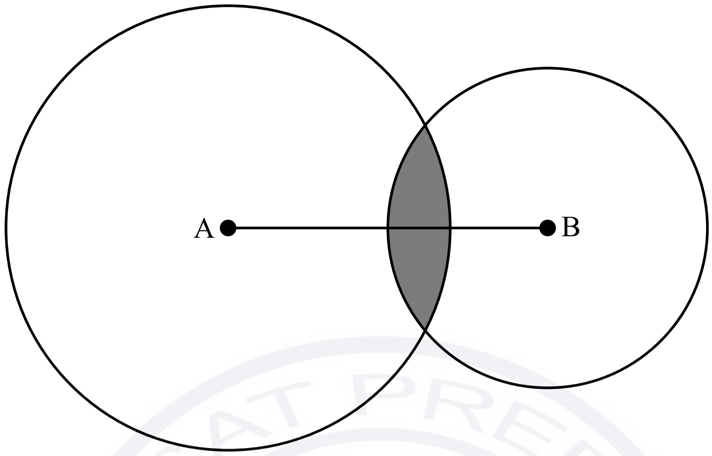
(a) Given that $(x - 5)$ is a factor of $P(x)$, find a relationship between a, b and c. [2]

(b) Given that $(x - 5)^2$ is a factor of $P(x)$, write down the value of $P'(5)$. [1]

(c) Given that $(x - 5)^2$ is a factor of $P(x)$, and that $a = 2$, find the values of b and c. [3]

7. [Maximum mark: 6]

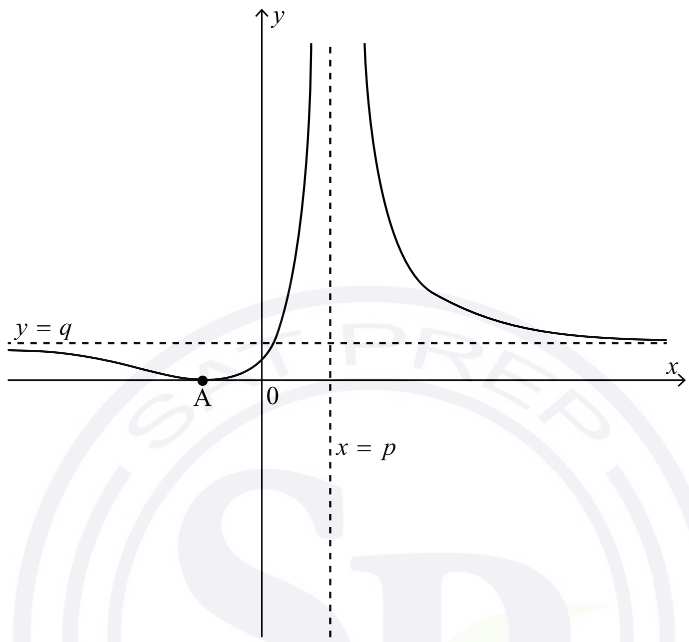
Boat A is situated 10km away from boat B, and each boat has a marine radio transmitter on board. The range of the transmitter on boat A is 7km, and the range of the transmitter on boat B is 5km. The region in which both transmitters can be detected is represented by the shaded region in the following diagram. Find the area of this region.



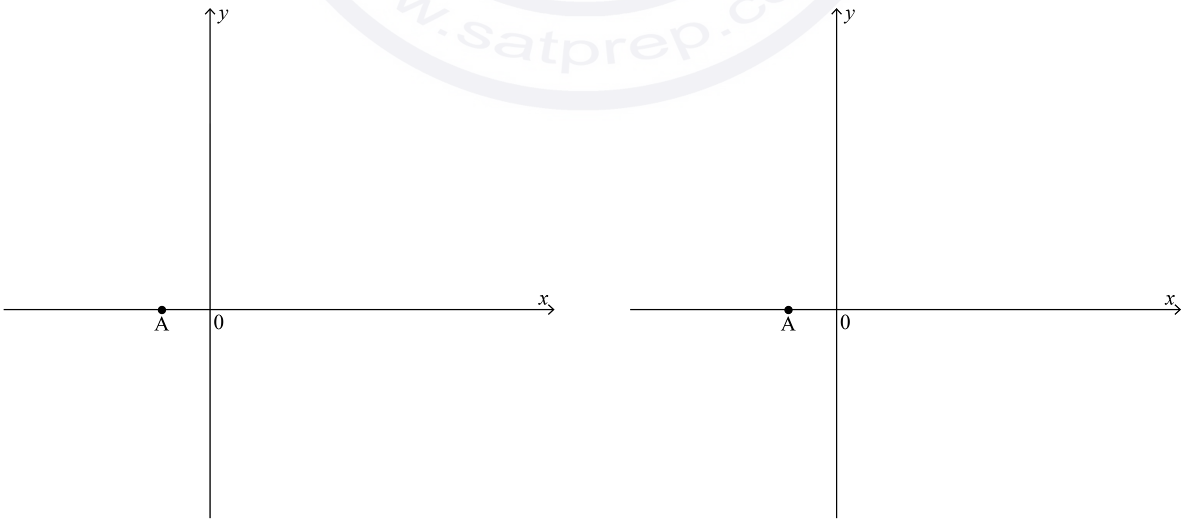
8. [Maximum mark: 8]

Consider the function $f(x) = \dfrac{ax+1}{bx+c}$, $x \neq -\dfrac{c}{b}$, where $a, b, c \in \mathbb{Z}$.

The following graph shows the curve $y = (f(x))^2$. It has asymptotes at $x = p$ and $y = q$ and meets the x-axis at A.



(a) On the following axes, sketch the two possible graphs of $y = f(x)$ giving the equations of any asymptotes in terms of p and q. [4]



(This question continues on the following page)

(Question 8 continued)

(b) Given that $p = \frac{4}{3}$, $q = \frac{4}{9}$ and A has coordinates $\left(-\frac{1}{2}, 0\right)$, determine the possible sets of values for a, b and c. [4]

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

9. [Maximum mark: 19]

The function f is defined by $f(x) = \dfrac{2\ln x + 1}{x - 3}$, $0 < x < 3$.

(a) Find $f'(x)$. [4]

(b) Hence, or otherwise, find the coordinates of the point of inflexion on the graph of $y = f(x)$. [4]

(c) Draw a set of axes showing x and y values between -3 and 3. On these axes

(i) sketch the graph of $y = f(x)$, showing clearly any axis intercepts and giving the equations of any asymptotes.

(ii) sketch the graph of $y = f^{-1}(x)$, showing clearly any axis intercepts and giving the equations of any asymptotes. [8]

(d) Hence, or otherwise, solve the inequality $f(x) > f^{-1}(x)$. [3]

Do **not** write solutions on this page.

10. [Maximum mark: 18]

Willow finds that she receives approximately 70 emails per working day.
She decides to model the number of emails received per working day using the random variable X, where X follows a Poisson distribution with mean 70.

(a) Using this distribution model, find

(i) $P(X < 60)$

(ii) the standard deviation of X. [4]

(b) In order to test her model, Willow records the number of emails she receives per working day over a period of 6 months. The results are shown in the following table.

Number of emails received (x)	**Number of days**
$40 \le x \le 49$	2
$50 \le x \le 59$	15
$60 \le x \le 69$	40
$70 \le x \le 79$	53
$80 \le x \le 89$	0
$90 \le x \le 99$	1
$100 \le x \le 109$	3
$110 \le x \le 119$	6

From the table, calculate

(i) an estimate for the mean number of emails received per working day;

(ii) an estimate for the standard deviation of the number of emails received per working day. [5]

(c) Give one piece of evidence that suggests Willow's Poisson distribution model is not a good fit. [1]

Archie works for a different company and knows that he receives emails according to a Poisson distribution, with a mean of λ emails per day.

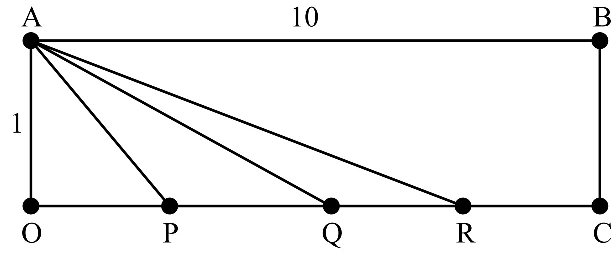
(d) Suppose that the probability of Archie receiving more than 10 emails in total on any one day is 0.99. Find the value of λ. [3]

(e) Now suppose that Archie received exactly 20 emails in total in a consecutive two day period. Show that the probability that he received exactly 10 of them on the first day is independent of λ . [5]

Do **not** write solutions on this page.

11. [Maximum mark: 13]

Consider the rectangle OABC such that $AB = OC = 10$ and $BC = OA = 1$, with the points P, Q and R placed on the line OC such that $OP = p$, $OQ = q$ and $OR = r$, such that $0 < p < q < r < 10$.



Let θ_p be the angle APO, θ_q be the angle AQO and θ_r be the angle ARO.

(a) Find an expression for θ_p in terms of p. [3]

Consider the case when $\theta_p = \theta_q + \theta_r$ and $QR = 1$.

(b) Show that $p = \dfrac{q^2 + q - 1}{2q + 1}$. [6]

(c) By sketching the graph of p as a function of q, determine the range of values of p for which there are possible values of q. [4]

Please **do not** write on this page.

Answers written on this page
will not be marked.

Please **do not** write on this page.

Answers written on this page
will not be marked.

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Higher level
Paper 2

Thursday 3 May 2018 (morning)

Candidate session number

2 hours

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
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- A clean copy of the **mathematics HL and further mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[100 marks]**.

2218 – 7204

12 pages

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 7]

The 3rd term of an arithmetic sequence is 1407 and the 10th term is 1183.

(a) Find the first term and the common difference of the sequence. [4]

(b) Calculate the number of positive terms in the sequence. [3]

2. [Maximum mark: 6]

The equation $x^2 - 5x - 7 = 0$ has roots α and β. The equation $x^2 + px + q = 0$ has roots $\alpha + 1$ and $\beta + 1$. Find the value of p and the value of q.

3. [Maximum mark: 5]

Let $f(x) = \tan(x + \pi)\cos\left(x - \frac{\pi}{2}\right)$ where $0 < x < \frac{\pi}{2}$.

Express $f(x)$ in terms of $\sin x$ and $\cos x$.

4. [Maximum mark: 5]

The age, L, in years, of a wolf can be modelled by the normal distribution $L \sim \mathrm{N}(8, 5)$.

(a) Find the probability that a wolf selected at random is at least 5 years old. [2]

Eight wolves are independently selected at random and their ages recorded.

(b) Find the probability that more than six of these wolves are at least 5 years old. [3]

5. [Maximum mark: 7]

(a) Given that $2x^3 - 3x + 1$ can be expressed in the form $Ax(x^2 + 1) + Bx + C$, find the values of the constants A, B and C. [2]

(b) Hence find $\int \frac{2x^3 - 3x + 1}{x^2 + 1} \mathrm{d}x$. [5]

6. [Maximum mark: 5]

The mean number of squirrels in a certain area is known to be 3.2 squirrels per hectare of woodland. Within this area, there is a 56 hectare woodland nature reserve. It is known that there are currently at least 168 squirrels in this reserve.

Assuming the population of squirrels follow a Poisson distribution, calculate the probability that there are more than 190 squirrels in the reserve.

7. [Maximum mark: 8]

It is known that the number of fish in a given lake will decrease by 7% each year unless some new fish are added. At the end of each year, 250 new fish are added to the lake. At the start of 2018, there are 2500 fish in the lake.

(a) Show that there will be approximately 2645 fish in the lake at the start of 2020. [3]

(b) Find the approximate number of fish in the lake at the start of 2042. [5]

8. [Maximum mark: 7]

Each of the 25 students in a class are asked how many pets they own. Two students own three pets and no students own more than three pets. The mean and standard deviation of the number of pets owned by students in the class are $\frac{18}{25}$ and $\frac{24}{25}$ respectively.

Find the number of students in the class who do not own a pet.

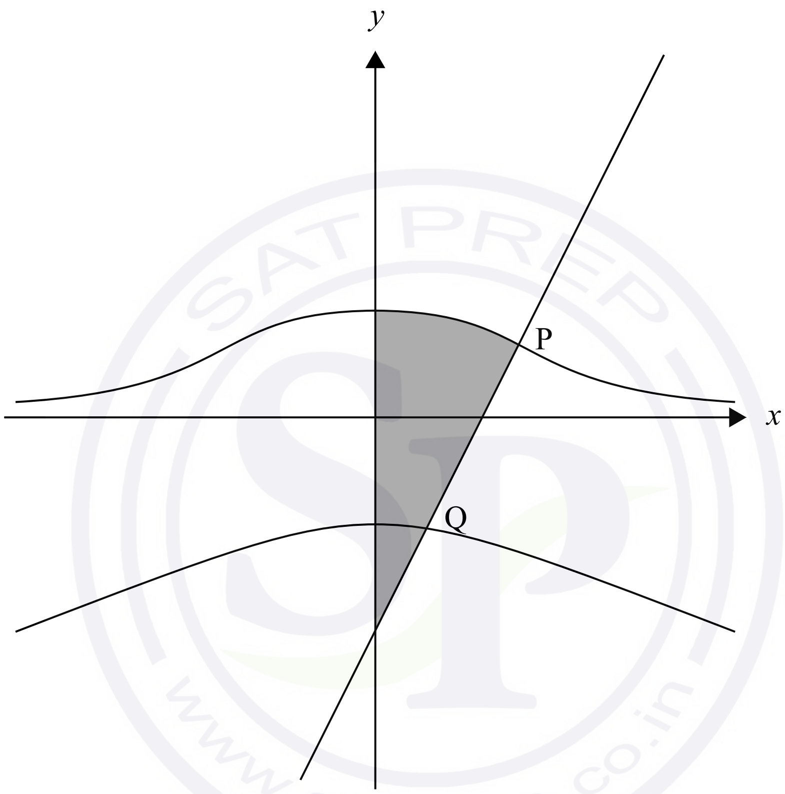
Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

9. [Maximum mark: 22]

The following graph shows the two parts of the curve defined by the equation $x^2y = 5 - y^4$, and the normal to the curve at the point $\text{P}(2, 1)$.



(a) Show that there are exactly two points on the curve where the gradient is zero. [7]

(b) Find the equation of the normal to the curve at the point P. [5]

(c) The normal at P cuts the curve again at the point Q. Find the x-coordinate of Q. [3]

(d) The shaded region is rotated by 2π about the y-axis. Find the volume of the solid formed. [7]

Do **not** write solutions on this page.

10. [Maximum mark: 13]

The continuous random variable X has probability density function f given by

$$f(x) = \begin{cases} 3ax & , \quad 0 \le x < 0.5 \\ a(2 - x) & , \quad 0.5 \le x < 2 \\ 0 & , \quad \text{otherwise} \end{cases}$$

(a) Show that $a = \frac{2}{3}$. [3]

(b) Find $\mathrm{P}(X < 1)$. [3]

(c) Given that $\mathrm{P}(s < X < 0.8) = 2 \times \mathrm{P}(2s < X < 0.8)$, and that $0.25 < s < 0.4$, find the value of s. [7]

Turn over

Do **not** write solutions on this page.

11. [Maximum mark: 15]

Two submarines A and B have their routes planned so that their positions at time t hours, $0 \le t < 20$, would be defined by the position vectors $\boldsymbol{r}_A = \begin{pmatrix} 2 \\ 4 \\ -1 \end{pmatrix} + t\begin{pmatrix} -1 \\ 1 \\ -0.15 \end{pmatrix}$ and $\boldsymbol{r}_B = \begin{pmatrix} 0 \\ 3.2 \\ -2 \end{pmatrix} + t\begin{pmatrix} -0.5 \\ 1.2 \\ 0.1 \end{pmatrix}$ relative to a fixed point on the surface of the ocean (all lengths are in kilometres).

(a) Show that the two submarines would collide at a point P and write down the coordinates of P. [4]

To avoid the collision submarine B adjusts its velocity so that its position vector is now given by

$$\boldsymbol{r}_B = \begin{pmatrix} 0 \\ 3.2 \\ -2 \end{pmatrix} + t\begin{pmatrix} -0.45 \\ 1.08 \\ 0.09 \end{pmatrix}.$$

(b) (i) Show that submarine B travels in the same direction as originally planned.

(ii) Find the value of t when submarine B passes through P. [3]

(c) (i) Find an expression for the distance between the two submarines in terms of t.

(ii) Find the value of t when the two submarines are closest together.

(iii) Find the distance between the two submarines at this time. [8]

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Higher level
Paper 2

Thursday 3 May 2018 (morning)

Candidate session number

2 hours

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics HL and further mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[100 marks]**.

2218 – 7206

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Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 6]

Consider the complex number $z = \frac{2 + 7\mathrm{i}}{6 + 2\mathrm{i}}$.

(a) Express z in the form $a + \mathrm{i}b$, where $a, b \in \mathbb{Q}$. [2]

(b) Find the exact value of the modulus of z. [2]

(c) Find the argument of z, giving your answer to 4 decimal places. [2]

2. [Maximum mark: 5]

The polynomial $x^4 + px^3 + qx^2 + rx + 6$ is exactly divisible by each of $(x - 1)$, $(x - 2)$ and $(x - 3)$.

Find the values of p, q and r.

3. [Maximum mark: 6]

The random variable X has a normal distribution with mean $\mu = 50$ and variance $\sigma^2 = 16$.

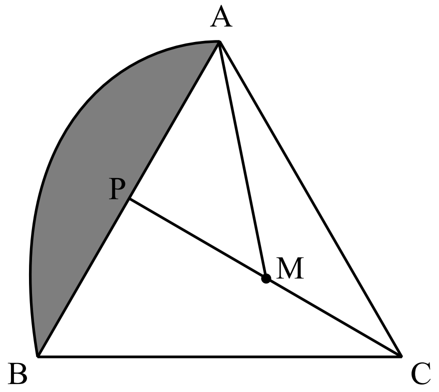
(a) Sketch the probability density function for X, and shade the region representing $\mathrm{P}(\mu - 2\sigma < X < \mu + \sigma)$. [2]

(b) Find the value of $\mathrm{P}(\mu - 2\sigma < X < \mu + \sigma)$. [2]

(c) Find the value of k for which $\mathrm{P}(\mu - k\sigma < X < \mu + k\sigma) = 0.5$. [2]

4. [Maximum mark: 8]

Consider the following diagram.



The sides of the equilateral triangle ABC have lengths 1 m. The midpoint of [AB] is denoted by P. The circular arc AB has centre, M, the midpoint of [CP].

(a) (i) Find AM.

(ii) Find $\text{A}\hat{\text{M}}\text{P}$ in radians. [5]

(b) Find the area of the shaded region. [3]

5. [Maximum mark: 6]

(a) Express the binomial coefficient $\begin{pmatrix} 3n+1 \\ 3n-2 \end{pmatrix}$ as a polynomial in n. [3]

(b) Hence find the least value of n for which $\begin{pmatrix} 3n+1 \\ 3n-2 \end{pmatrix} > 10^6$. [3]

6. [Maximum mark: 7]

Use mathematical induction to prove that $(1 - a)^n > 1 - na$ for $\{n : n \in \mathbb{Z}^+, n \geq 2\}$ where $0 < a < 1$.

7. [Maximum mark: 5]

A point P moves in a straight line with velocity v ms^{-1} given by $v(t) = e^{-t} - 8t^2e^{-2t}$ at time t seconds, where $t \geq 0$.

(a) Determine the first time t_1 at which P has zero velocity. [2]

(b) (i) Find an expression for the acceleration of P at time t.

(ii) Find the value of the acceleration of P at time t_1. [3]

8. [Maximum mark: 7]

The random variable X has a binomial distribution with parameters n and p.
It is given that $\mathrm{E}(X) = 3.5$.

(a) Find the least possible value of n. [2]

It is further given that $\mathrm{P}(X \leq 1) = 0.09478$ correct to 4 significant figures.

(b) Determine the value of n and the value of p. [5]

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Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

9. [Maximum mark: 13]

The number of taxis arriving at Cardiff Central railway station can be modelled by a Poisson distribution. During busy periods of the day, taxis arrive at a mean rate of 5.3 taxis every 10 minutes. Let T represent a random 10 minute busy period.

(a) (i) Find the probability that exactly 4 taxis arrive during T.

(ii) Find the most likely number of taxis that would arrive during T.

(iii) Given that more than 5 taxis arrive during T, find the probability that exactly 7 taxis arrive during T. [7]

During quiet periods of the day, taxis arrive at a mean rate of 1.3 taxis every 10 minutes.

(b) Find the probability that during a period of 15 minutes, of which the first 10 minutes is busy and the next 5 minutes is quiet, that exactly 2 taxis arrive. [6]

Do **not** write solutions on this page.

10. [Maximum mark: 18]

Consider the expression $f(x) = \tan\left(x + \frac{\pi}{4}\right)\cot\left(\frac{\pi}{4} - x\right)$.

(a) (i) Sketch the graph of $y = f(x)$ for $-\frac{5\pi}{8} \le x \le \frac{\pi}{8}$.

(ii) With reference to your graph, explain why f is a function on the given domain.

(iii) Explain why f has no inverse on the given domain.

(iv) Explain why f is not a function for $-\frac{3\pi}{4} \le x \le \frac{\pi}{4}$. [5]

The expression $f(x)$ can be written as $g(t)$ where $t = \tan x$.

(b) Show that $g(t) = \left(\frac{1+t}{1-t}\right)^2$. [3]

(c) Sketch the graph of $y = g(t)$ for $t \le 0$. Give the coordinates of any intercepts and the equations of any asymptotes. [3]

(d) Let α, β be the roots of $g(t) = k$, where $0 < k < 1$.

(i) Find α and β in terms of k.

(ii) Show that $\alpha + \beta < -2$. [7]

Do **not** write solutions on this page.

11. [Maximum mark: 19]

A curve C is given by the implicit equation $x + y - \cos(xy) = 0$.

(a) Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = -\left(\frac{1 + y\sin(xy)}{1 + x\sin(xy)}\right)$. [5]

(b) The curve $xy = -\frac{\pi}{2}$ intersects C at P and Q.

(i) Find the coordinates of P and Q.

(ii) Given that the gradients of the tangents to C at P and Q are m_1 and m_2 respectively, show that $m_1 \times m_2 = 1$. [7]

(c) Find the coordinates of the three points on C, nearest the origin, where the tangent is parallel to the line $y = -x$. [7]

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Higher level
Paper 2

Tuesday 14 November 2017 (morning)

Candidate session number

2 hours

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics HL and further mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[100 marks]**.

13 pages

8817 – 7202

International Baccalaureate
Baccalauréat International
Bachillerato Internacional

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 4]

Boxes of mixed fruit are on sale at a local supermarket.
Box A contains 2 bananas, 3 kiwifruit and 4 melons, and costs $\$6.58$.
Box B contains 5 bananas, 2 kiwifruit and 8 melons and costs $\$12.32$.
Box C contains 5 bananas and 4 kiwifruit and costs $\$3.00$.

Find the cost of each type of fruit.

2. [Maximum mark: 6]

Events A and B are such that $\mathrm{P}(A \cup B) = 0.95$, $\mathrm{P}(A \cap B) = 0.6$ and $\mathrm{P}(A \mid B) = 0.75$.

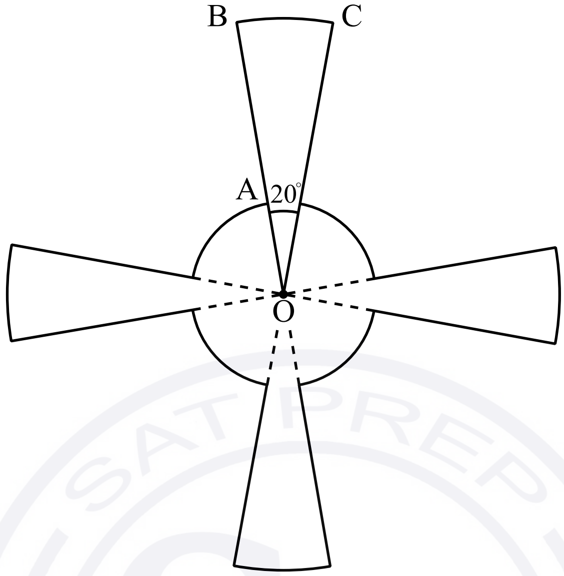
(a) Find $\mathrm{P}(B)$. [2]

(b) Find $\mathrm{P}(A)$. [2]

(c) Hence show that events A' and B are independent. [2]

3. [Maximum mark: 4]

This diagram shows a metallic pendant made out of four equal sectors of a larger circle of radius $\text{OB} = 9\,\text{cm}$ and four equal sectors of a smaller circle of radius $\text{OA} = 3\,\text{cm}$. The angle $\text{BOC} = 20^\circ$.



Find the area of the pendant.

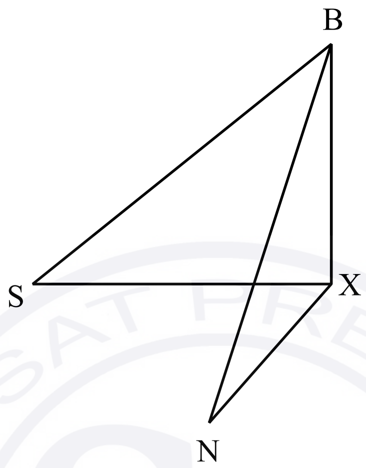
4. [Maximum mark: 6]

It is given that one in five cups of coffee contain more than $120\,\text{mg}$ of caffeine.
It is also known that three in five cups contain more than $110\,\text{mg}$ of caffeine.

Assume that the caffeine content of coffee is modelled by a normal distribution.
Find the mean and standard deviation of the caffeine content of coffee.

5. [Maximum mark: 6]

Barry is at the top of a cliff, standing $80\,\text{m}$ above sea level, and observes two yachts in the sea.
"*Seaview*" (S) is at an angle of depression of 25°.
"*Nauti Buoy*" (N) is at an angle of depression of 35°.
The following three dimensional diagram shows Barry and the two yachts at S and N.
X lies at the foot of the cliff and angle $\text{SXN} = 70^\circ$.



Find, to 3 significant figures, the distance between the two yachts.

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6. [Maximum mark: 6]

The number of bananas that Lucca eats during any particular day follows a Poisson distribution with mean 0.2.

(a) Find the probability that Lucca eats at least one banana in a particular day. [2]

(b) Find the expected number of weeks in the year in which Lucca eats no bananas. [4]

7. [Maximum mark: 5]

In the quadratic equation $7x^2 - 8x + p = 0$, $(p \in \mathbb{Q})$, one root is three times the other root. Find the value of p.

8. [Maximum mark: 7]

By using the substitution $x^2 = 2\sec\theta$, show that $\int \frac{\mathrm{d}x}{x\sqrt{x^4 - 4}} = \frac{1}{4}\arccos\left(\frac{2}{x^2}\right) + c$.

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9. [Maximum mark: 6]

Twelve students are to take an exam in advanced combinatorics.
The exam room is set out in three rows of four desks, with the invigilator at the front of the room, as shown in the following diagram.

INVIGILATOR

Desk 1	Desk 2	Desk 3	Desk 4
Desk 5	Desk 6	Desk 7	Desk 8
Desk 9	Desk 10	Desk 11	Desk 12

(a) Find the number of ways the twelve students may be arranged in the exam hall. [1]

Two of the students, Helen and Nicky, are suspected of cheating in a previous exam.

(b) Find the number of ways the students may be arranged if Helen and Nicky must sit so that one is directly behind the other (with no desk in between). For example Desk 5 and Desk 9. [2]

(c) Find the number of ways the students may be arranged if Helen and Nicky must not sit next to each other in the same row. [3]

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

10. [Maximum mark: 17]

Consider the function $f(x) = \frac{\sqrt{x}}{\sin x}$, $0 < x < \pi$.

(a) (i) Show that the x-coordinate of the minimum point on the curve $y = f(x)$ satisfies the equation $\tan x = 2x$.

(ii) Determine the values of x for which $f(x)$ is a decreasing function. [7]

(b) Sketch the graph of $y = f(x)$ showing clearly the minimum point and any asymptotic behaviour. [3]

(c) Find the coordinates of the point on the graph of f where the normal to the graph is parallel to the line $y = -x$. [4]

Consider the region bounded by the curve $y = f(x)$, the x-axis and the lines $x = \frac{\pi}{6}$, $x = \frac{\pi}{3}$.

(d) This region is now rotated through 2π radians about the x-axis. Find the volume of revolution. [3]

Do **not** write solutions on this page.

11. [Maximum mark: 18]

Consider the function $f(x) = 2\sin^2 x + 7\sin 2x + \tan x - 9$, $0 \le x < \frac{\pi}{2}$.

(a) (i) Determine an expression for $f'(x)$ in terms of x.

(ii) Sketch a graph of $y = f'(x)$ for $0 \le x < \frac{\pi}{2}$.

(iii) Find the x-coordinate(s) of the point(s) of inflexion of the graph of $y = f(x)$, labelling these clearly on the graph of $y = f'(x)$. [8]

(b) Let $u = \tan x$.

(i) Express $\sin x$ in terms of u.

(ii) Express $\sin 2x$ in terms of u.

(iii) Hence show that $f(x) = 0$ can be expressed as $u^3 - 7u^2 + 15u - 9 = 0$. [7]

(c) Solve the equation $f(x) = 0$, giving your answers in the form $\arctan k$ where $k \in \mathbb{Z}$. [3]

Do **not** write solutions on this page.

12. [Maximum mark: 15]

Phil takes out a bank loan of $\$150\,000$ to buy a house, at an annual interest rate of 3.5%. The interest is calculated at the end of each year and added to the amount outstanding.

(a) Find the amount Phil would owe the bank after 20 years. Give your answer to the nearest dollar. [3]

To pay off the loan, Phil makes annual deposits of $\$P$ at the end of every year in a savings account, paying an annual interest rate of 2%. He makes his first deposit at the end of the first year after taking out the loan.

(b) Show that the total value of Phil's savings after 20 years is $\frac{(1.02^{20}-1)P}{(1.02-1)}$. [3]

(c) Given that Phil's aim is to own the house after 20 years, find the value for P to the nearest dollar. [3]

David visits a different bank and makes a single deposit of $\$Q$, the annual interest rate being 2.8%.

(d) (i) David wishes to withdraw $\$5000$ at the end of each year for a period of n years. Show that an expression for the minimum value of Q is $\frac{5000}{1.028}+\frac{5000}{1.028^2}+\ldots+\frac{5000}{1.028^n}$.

(ii) Hence or otherwise, find the minimum value of Q that would permit David to withdraw annual amounts of $\$5000$ indefinitely. Give your answer to the nearest dollar. [6]

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will not be marked.

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will not be marked.

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Higher level
Paper 2

Friday 5 May 2017 (morning)

Candidate session number

2 hours

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
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- A clean copy of the **mathematics HL and further mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[100 marks]**.

2217 – 7204

12 pages

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Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 6]

Consider two events A and B such that $\mathrm{P}(A) = k$, $\mathrm{P}(B) = 3k$, $\mathrm{P}(A \cap B) = k^2$ and $\mathrm{P}(A \cup B) = 0.5$.

(a) Calculate k; [3]

(b) Find $\mathrm{P}(A' \cap B)$. [3]

2. [Maximum mark: 7]

The curve C is defined by equation $xy - \ln y = 1$, $y > 0$.

(a) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x and y. [4]

(b) Determine the equation of the tangent to C at the point $\left(\frac{2}{\mathrm{e}}, \mathrm{e}\right)$. [3]

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3. [Maximum mark: 6]

The coefficient of x^2 in the expansion of $\left(\frac{1}{x} + 5x\right)^8$ is equal to the coefficient of x^4 in the expansion of $(a + 5x)^7$, $a \in \mathbb{R}$. Find the value of a.

4. [Maximum mark: 6]

The region A is enclosed by the graph of $y = 2\arcsin(x-1) - \frac{\pi}{4}$, the y-axis and the line $y = \frac{\pi}{4}$.

(a) Write down a definite integral to represent the area of A. [4]

(b) Calculate the area of A. [2]

5. [Maximum mark: 6]

When carpet is manufactured, small faults occur at random. The number of faults in Premium carpets can be modelled by a Poisson distribution with mean 0.5 faults per $20\,\text{m}^2$.

Mr Jones chooses Premium carpets to replace the carpets in his office building. The office building has 10 rooms, each with the area of $80\,\text{m}^2$.

(a) Find the probability that the carpet laid in the first room has fewer than three faults. [3]

(b) Find the probability that exactly seven rooms will have fewer than three faults in the carpet. [3]

6. [Maximum mark: 6]

Consider the graph of $y = \tan\left(x + \frac{\pi}{4}\right)$, where $-\pi \le x \le 2\pi$.

(a) Write down the equations of the vertical asymptotes of the graph. [2]

The graph is reflected in the y-axis, then stretched parallel to the y-axis by a factor $\frac{1}{2}$, then translated by $\begin{pmatrix} \frac{\pi}{4} \\ -3 \end{pmatrix}$.

(b) Give the equation of the transformed graph. [4]

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7. [Maximum mark: 6]

Find the Cartesian equation of plane Π containing the points $\mathrm{A}(6, 2, 1)$ and $\mathrm{B}(3, -1, 1)$ and perpendicular to the plane $x + 2y - z - 6 = 0$.

8. [Maximum mark: 7]

A water trough which is 10 metres long has a uniform cross-section in the shape of a semicircle with radius 0.5 metres. It is partly filled with water as shown in the following diagram of the cross-section. The centre of the circle is O and the angle KOL is θ radians.

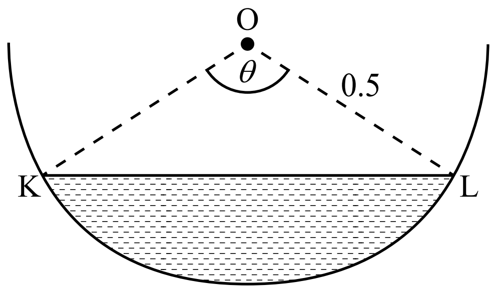

diagram not to scale

(a) Find an expression for the volume of water $V(m^3)$ in the trough in terms of θ. [3]

The volume of water is increasing at a constant rate of $0.0008\,\mathrm{m^3\,s^{-1}}$.

(b) Calculate $\frac{\mathrm{d}\theta}{\mathrm{d}t}$ when $\theta = \frac{\pi}{3}$. [4]

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

9. [Maximum mark: 8]

The times taken for male runners to complete a marathon can be modelled by a normal distribution with a mean 196 minutes and a standard deviation 24 minutes.

(a) Find the probability that a runner selected at random will complete the marathon in less than 3 hours. [2]

It is found that 5% of the male runners complete the marathon in less than T_1 minutes.

(b) Calculate T_1. [2]

The times taken for female runners to complete the marathon can be modelled by a normal distribution with a mean 210 minutes. It is found that 58% of female runners complete the marathon between 185 and 235 minutes.

(c) Find the standard deviation of the times taken by female runners. [4]

10. [Maximum mark: 15]

In triangle PQR, $\text{PR} = 12\,\text{cm}$, $\text{QR} = p\,\text{cm}$, $\text{PQ} = r\,\text{cm}$ and $\text{Q}\hat{\text{P}}\text{R} = 30°$.

(a) Use the cosine rule to show that $r^2 - 12\sqrt{3}\,r + 144 - p^2 = 0$. [2]

Consider the possible triangles with $\text{QR} = 8\,\text{cm}$.

(b) Calculate the two corresponding values of PQ. [3]

(c) Hence, find the area of the smaller triangle. [3]

Consider the case where p, the length of QR is not fixed at $8\,\text{cm}$.

(d) Determine the range of values of p for which it is possible to form two triangles. [7]

Do **not** write solutions on this page.

11. [Maximum mark: 9]

Xavier, the parachutist, jumps out of a plane at a height of h metres above the ground. After free falling for 10 seconds his parachute opens.

His velocity, $v\,\mathrm{ms}^{-1}$, t seconds after jumping from the plane, can be modelled by the function

$$v(t) = \begin{cases} 9.8t, & 0 \le t \le 10 \\ \dfrac{98}{\sqrt{1+(t-10)^2}}, & t > 10 \end{cases}.$$

(a) Find his velocity when $t = 15$. [2]

(b) Calculate the vertical distance Xavier travelled in the first 10 seconds. [2]

His velocity when he reaches the ground is $2.8\,\mathrm{ms}^{-1}$.

(c) Determine the value of h. [5]

Do **not** write solutions on this page.

12. [Maximum mark: 18]

Consider $f(x) = -1 + \ln\left(\sqrt{x^2 - 1}\right)$.

(a) Find the largest possible domain D for f to be a function. [2]

The function f is defined by $f(x) = -1 + \ln\left(\sqrt{x^2 - 1}\right),\ x \in D$.

(b) Sketch the graph of $y = f(x)$ showing clearly the equations of asymptotes and the coordinates of any intercepts with the axes. [3]

(c) Explain why f is an even function. [1]

(d) Explain why the inverse function f^{-1} does not exist. [1]

The function g is defined by $g(x) = -1 + \ln\left(\sqrt{x^2 - 1}\right),\ x \in]1, \infty[$.

(e) Find the inverse function g^{-1} and state its domain. [4]

(f) Find $g'(x)$. [3]

(g) Hence, show that there are no solutions to

(i) $g'(x) = 0$;

(ii) $\left(g^{-1}\right)'(x) = 0$. [4]

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Higher level
Paper 2

Friday 5 May 2017 (morning)

Candidate session number

2 hours

Instructions to candidates

- Write your session number in the boxes above.
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- Section A: answer all questions. Answers must be written within the answer boxes provided.
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- The maximum mark for this examination paper is **[100 marks]**.

13 pages

2217 – 7206

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Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 4]

There are 75 players in a golf club who take part in a golf tournament. The scores obtained on the 18th hole are as shown in the following table.

Score	2	3	4	5	6	7
Frequency	3	15	28	17	9	3

(a) One of the players is chosen at random. Find the probability that this player's score was 5 or more. [2]

(b) Calculate the mean score. [2]

2. [Maximum mark: 9]

Consider the curve defined by the equation $4x^2 + y^2 = 7$.

(a) Find the equation of the normal to the curve at the point $\left(1, \sqrt{3}\right)$. [6]

(b) Find the volume of the solid formed when the region bounded by the curve, the x-axis for $x \geq 0$ and the y-axis for $y \geq 0$ is rotated through 2π about the x-axis. [3]

3. [Maximum mark: 7]

Packets of biscuits are produced by a machine. The weights X, in grams, of packets of biscuits can be modelled by a normal distribution where $X \sim \mathrm{N}(\mu, \sigma^2)$. A packet of biscuits is considered to be underweight if it weighs less than 250 grams.

(a) Given that $\mu = 253$ and $\sigma = 1.5$ find the probability that a randomly chosen packet of biscuits is underweight. [2]

The manufacturer makes the decision that the probability that a packet is underweight should be 0.002. To do this μ is increased and σ remains unchanged.

(b) Calculate the new value of μ giving your answer correct to two decimal places. [3]

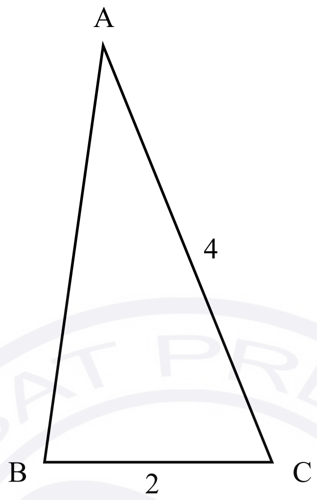
The manufacturer is happy with the decision that the probability that a packet is underweight should be 0.002, but is unhappy with the way in which this was achieved. The machine is now adjusted to reduce σ and return μ to 253.

(c) Calculate the new value of σ. [2]

4. [Maximum mark: 6]

(a) Find the set of values of k that satisfy the inequality $k^2 - k - 12 < 0$. [2]

(b) The triangle ABC is shown in the following diagram. Given that $\cos B < \frac{1}{4}$, find the range of possible values for AB. [4]



5. [Maximum mark: 9]

John likes to go sailing every day in July. To help him make a decision on whether it is safe to go sailing he classifies each day in July as windy or calm. Given that a day in July is calm, the probability that the next day is calm is 0.9. Given that a day in July is windy, the probability that the next day is calm is 0.3. The weather forecast for the 1st July predicts that the probability that it will be calm is 0.8.

(a) Draw a tree diagram to represent this information for the first three days of July. [3]

(b) Find the probability that the 3rd July is calm. [2]

(c) Find the probability that the 1st July was calm given that the 3rd July is windy. [4]

6. [Maximum mark: 5]

Given that $\log_{10}\left(\frac{1}{2\sqrt{2}}(p+2q)\right)=\frac{1}{2}\left(\log_{10}p+\log_{10}q\right)$, $p>0$, $q>0$, find p in terms of q.

7. [Maximum mark: 4]

Given that $\boldsymbol{a} \times \boldsymbol{b} = \boldsymbol{b} \times \boldsymbol{c} \neq \mathbf{0}$ prove that $\boldsymbol{a} + \boldsymbol{c} = s\boldsymbol{b}$ where s is a scalar.

8. [Maximum mark: 6]

In a trial examination session a candidate at a school has to take 18 examination papers including the physics paper, the chemistry paper and the biology paper. No two of these three papers may be taken consecutively. There is no restriction on the order in which the other examination papers may be taken.

Find the number of different orders in which these 18 examination papers may be taken.

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

9. [Maximum mark: 22]

The points A, B and C have the following position vectors with respect to an origin O.

$$\vec{\mathrm{OA}} = 2\boldsymbol{i} + \boldsymbol{j} - 2\boldsymbol{k}$$

$$\vec{\mathrm{OB}} = 2\boldsymbol{i} - \boldsymbol{j} + 2\boldsymbol{k}$$

$$\vec{\mathrm{OC}} = \boldsymbol{i} + 3\boldsymbol{j} + 3\boldsymbol{k}$$

(a) Find the vector equation of the line (BC). [3]

(b) Determine whether or not the lines (OA) and (BC) intersect. [6]

(c) Find the Cartesian equation of the plane Π_1, which passes through C and is perpendicular to $\vec{\mathrm{OA}}$. [3]

(d) Show that the line (BC) lies in the plane Π_1. [2]

The plane Π_2 contains the points O, A and B and the plane Π_3 contains the points O, A and C.

(e) Verify that $2\boldsymbol{j} + \boldsymbol{k}$ is perpendicular to the plane Π_2. [3]

(f) Find a vector perpendicular to the plane Π_3. [1]

(g) Find the acute angle between the planes Π_2 and Π_3. [4]

Do **not** write solutions on this page.

10. [Maximum mark: 15]

A continuous random variable X has probability density function f given by

$$f(x) = \begin{cases} \dfrac{x^2}{a} + b, & 0 \le x \le 4 \\ 0 & \text{otherwise} \end{cases}$$ where a and b are positive constants.

It is given that $\mathrm{P}(X \ge 2) = 0.75$.

(a) Show that $a = 32$ and $b = \dfrac{1}{12}$. [5]

(b) Find $\mathrm{E}(X)$. [2]

(c) Find $\mathrm{Var}(X)$. [2]

(d) Find the median of X. [3]

Eight independent observations of X are now taken and the random variable Y is the number of observations such that $X \ge 2$.

(e) Find $\mathrm{E}(Y)$. [2]

(f) Find $\mathrm{P}(Y \ge 3)$. [1]

Do **not** write solutions on this page.

11. [Maximum mark: 13]

It is given that $f(x) = 3x^4 + ax^3 + bx^2 - 7x - 4$ where a and b are positive integers.

(a) Given that $x^2 - 1$ is a factor of $f(x)$ find the value of a and the value of b. [4]

(b) Factorize $f(x)$ into a product of linear factors. [3]

(c) Sketch the graph of $y = f(x)$, labelling the maximum and minimum points and the x and y intercepts. [3]

(d) Using your graph state the range of values of c for which $f(x) = c$ has exactly two distinct real roots. [3]

Please **do not** write on this page.

Answers written on this page will not be marked.

Please **do not** write on this page.

Answers written on this page will not be marked.

Please **do not** write on this page.

Answers written on this page will not be marked.

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Higher level
Paper 2

Friday 11 November 2016 (morning)

Candidate session number

2 hours

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions in the boxes provided.
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- A clean copy of the **mathematics HL and further mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[120 marks]**.

8816 – 7202

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Section A

Answer **all** questions in the boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 5]

A random variable X has a probability distribution given in the following table.

x	0.5	1.5	2.5	3.5	4.5	5.5
$\mathrm{P}(X=x)$	0.12	0.18	0.20	0.28	0.14	0.08

(a) Determine the value of $\mathrm{E}(X^2)$. [2]

(b) Find the value of $\mathrm{Var}(X)$. [3]

2. [Maximum mark: 5]

Find the acute angle between the planes with equations $x + y + z = 3$ and $2x - z = 2$.

3. [Maximum mark: 6]

A discrete random variable X follows a Poisson distribution $\text{Po}(\mu)$.

(a) Show that $\text{P}(X = x + 1) = \dfrac{\mu}{x + 1} \times \text{P}(X = x), x \in \mathbb{N}$. [3]

(b) Given that $\text{P}(X = 2) = 0.241667$ and $\text{P}(X = 3) = 0.112777$, use part (a) to find the value of μ. [3]

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4. [Maximum mark: 5]

Find the constant term in the expansion of $\left(4x^2 - \frac{3}{2x}\right)^{12}$.

5. [Maximum mark: 9]

Consider the function f defined by $f(x) = 3x \arccos(x)$ where $-1 \le x \le 1$.

(a) Sketch the graph of f indicating clearly any intercepts with the axes and the coordinates of any local maximum or minimum points. [3]

(b) State the range of f. [2]

(c) Solve the inequality $|3x \arccos(x)| > 1$. [4]

6. [Maximum mark: 6]

An earth satellite moves in a path that can be described by the curve $72.5x^2 + 71.5y^2 = 1$ where $x = x(t)$ and $y = y(t)$ are in thousands of kilometres and t is time in seconds.

Given that $\frac{\mathrm{d}x}{\mathrm{d}t} = 7.75 \times 10^{-5}$ when $x = 3.2 \times 10^{-3}$, find the possible values of $\frac{\mathrm{d}y}{\mathrm{d}t}$.

Give your answers in standard form.

7. [Maximum mark: 8]

In a triangle ABC, $\text{AB} = 4\,\text{cm}$, $\text{BC} = 3\,\text{cm}$ and $\text{B}\hat{\text{A}}\text{C} = \frac{\pi}{9}$.

(a) Use the cosine rule to find the two possible values for AC. [5]

(b) Find the difference between the areas of the two possible triangles ABC. [3]

8. [Maximum mark: 8]

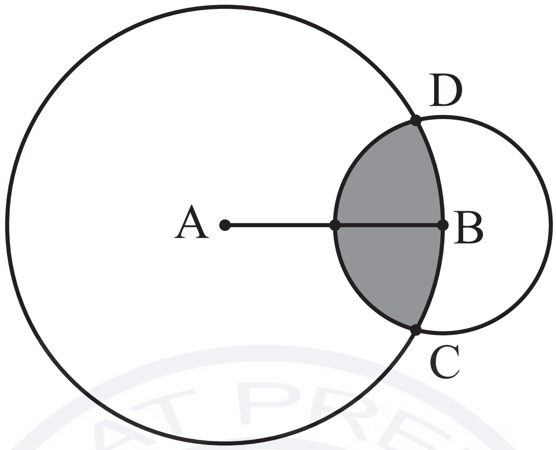
A random variable X is normally distributed with mean μ and standard deviation σ, such that $P(X < 30.31) = 0.1180$ and $P(X > 42.52) = 0.3060$.

(a) Find μ and σ. [6]

(b) Find $P(|X - \mu| < 1.2\sigma)$. [2]

9. [Maximum mark: 8]

The diagram shows two circles with centres at the points A and B and radii $2r$ and r, respectively. The point B lies on the circle with centre A. The circles intersect at the points C and D.



Let α be the measure of the angle CAD and θ be the measure of the angle CBD in radians.

(a) Find an expression for the shaded area in terms of α, θ and r. [3]

(b) Show that $\alpha = 4\arcsin\frac{1}{4}$. [2]

(c) Hence find the value of r given that the shaded area is equal to 4. [3]

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

10. [Maximum mark: 22]

Let the function f be defined by $f(x) = \dfrac{2 - e^x}{2e^x - 1}$, $x \in D$.

(a) Determine D, the largest possible domain of f. [2]

(b) Show that the graph of f has three asymptotes and state their equations. [5]

(c) Show that $f'(x) = -\dfrac{3e^x}{\left(2e^x - 1\right)^2}$. [3]

(d) Use your answers from parts (b) and (c) to justify that f has an inverse and state its domain. [4]

(e) Find an expression for $f^{-1}(x)$. [4]

(f) Consider the region R enclosed by the graph of $y = f(x)$ and the axes.
Find the volume of the solid obtained when R is rotated through 2π about the y-axis. [4]

Do **not** write solutions on this page.

11. [Maximum mark: 20]

A Chocolate Shop advertises free gifts to customers that collect three vouchers. The vouchers are placed at random into 10% of all chocolate bars sold at this shop. Kati buys some of these bars and she opens them one at a time to see if they contain a voucher. Let $\mathrm{P}(X = n)$ be the probability that Kati obtains her third voucher on the nth bar opened.

(It is assumed that the probability that a chocolate bar contains a voucher stays at 10% throughout the question.)

(a) Show that $\mathrm{P}(X = 3) = 0.001$ and $\mathrm{P}(X = 4) = 0.0027$. [3]

It is given that $\mathrm{P}(X = n) = \dfrac{n^2 + an + b}{2000} \times 0.9^{n-3}$ for $n \geq 3$, $n \in \mathbb{N}$.

(b) Find the values of the constants a and b. [5]

(c) Deduce that $\dfrac{\mathrm{P}(X = n)}{\mathrm{P}(X = n - 1)} = \dfrac{0.9(n - 1)}{n - 3}$ for $n > 3$. [4]

(d) (i) Hence show that X has two modes m_1 and m_2.

(ii) State the values of m_1 and m_2. [5]

Kati's mother goes to the shop and buys x chocolate bars. She takes the bars home for Kati to open.

(e) Determine the minimum value of x such that the probability Kati receives at least one free gift is greater than 0.5. [3]

Do **not** write solutions on this page.

12. [Maximum mark: 18]

On the day of her birth, 1st January 1998, Mary's grandparents invested $\$x$ in a savings account. They continued to deposit $\$x$ on the first day of each month thereafter.
The account paid a fixed rate of 0.4% interest per month. The interest was calculated on the last day of each month and added to the account.
Let $\$A_n$ be the amount in Mary's account on the last day of the nth month, immediately after the interest had been added.

(a) Find an expression for A_1 and show that $A_2 = 1.004^2x + 1.004x$. [2]

(b) (i) Write down a similar expression for A_3 and A_4.

(ii) Hence show that the amount in Mary's account the day before she turned 10 years old is given by $251(1.004^{120} - 1)x$. [6]

(c) Write down an expression for A_n in terms of x on the day before Mary turned 18 years old showing clearly the value of n. [1]

(d) Mary's grandparents wished for the amount in her account to be at least $\$20\,000$ the day before she was 18. Determine the minimum value of the monthly deposit $\$x$ required to achieve this. Give your answer correct to the nearest dollar. [4]

(e) As soon as Mary was 18 she decided to invest $\$15\,000$ of this money in an account of the same type earning 0.4% interest per month. She withdraws $\$1000$ every year on her birthday to buy herself a present. Determine how long it will take until there is no money in the account. [5]

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Answers written on this page will not be marked.

Please **do not** write on this page.

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M16/5/MATHL/HP2/ENG/TZ1/XX

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Higher level
Paper 2

Wednesday 11 May 2016 (morning)

2 hours

Candidate session number

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics HL and further mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[120 marks]**.

2216 – 7204

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions in the boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 4]

The points A and B have position vectors $\overrightarrow{\text{OA}} = \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$ and $\overrightarrow{\text{OB}} = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$.

(a) Find $\overrightarrow{\text{OA}} \times \overrightarrow{\text{OB}}$. [2]

(b) Hence find the area of the triangle OAB. [2]

2. [Maximum mark: 4]

(a) Express $x^2 + 4x - 2$ in the form $(x+a)^2 + b$ where $a, b \in \mathbb{Z}$. [2]

(b) If $f(x) = x + 2$ and $(g \circ f)(x) = x^2 + 4x - 2$ write down $g(x)$. [2]

3. [Maximum mark: 5]

The displacement, s, in metres, of a particle t seconds after it passes through the origin is given by the expression $s = \ln\left(2 - \mathrm{e}^{-t}\right)$, $t \geq 0$.

(a) Find an expression for the velocity, v, of the particle at time t. [2]

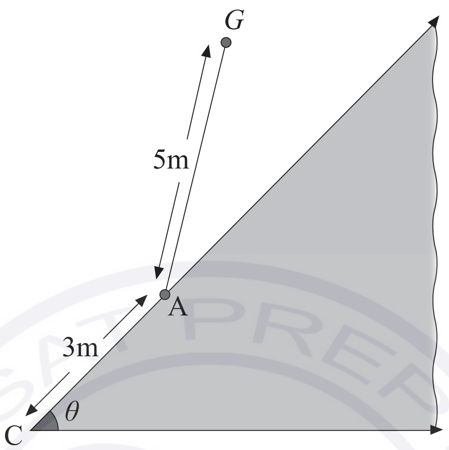
(b) Find an expression for the acceleration, a, of the particle at time t. [2]

(c) Find the acceleration of the particle at time $t = 0$. [1]

4. [Maximum mark: 6]

The diagram below shows a fenced triangular enclosure in the middle of a large grassy field. The points A and C are $3\,\mathrm{m}$ apart. A goat G is tied by a $5\,\mathrm{m}$ length of rope at point A on the outside edge of the enclosure.

Given that the corner of the enclosure at C forms an angle of θ radians and the area of field that can be reached by the goat is $44\,\mathrm{m}^2$, find the value of θ.



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5. [Maximum mark: 7]

The function f is given by $f(x) = \dfrac{3x^2 + 10}{x^2 - 4}$, $x \in \mathbb{R}$, $x \neq 2$, $x \neq -2$.

(a) Prove that f is an even function. [2]

(b) (i) Sketch the graph $y = f(x)$.

(ii) Write down the range of f. [5]

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6. [Maximum mark: 6]

The heights of students in a single year group in a large school can be modelled by a normal distribution.

It is given that 40% of the students are shorter than 1.62 m and 25% are taller than 1.79 m.

Find the mean and standard deviation of the heights of the students.

7. [Maximum mark: 8]

It has been suggested that in rowing competitions the time, T seconds taken to complete a $2000\,\text{m}$ race can be modelled by an equation of the form $T = aN^b$, where N is the number of rowers in the boat and a and b are constants for rowers of a similar standard.

To test this model the times for the finalists in all the $2000\,\text{m}$ men's races at a recent Olympic games were recorded and the mean calculated.

The results are shown in the following table for $N = 1$ and $N = 2$.

N	***T* (seconds)**
1	420.65
2	390.94

(a) Use these results to find estimates for the value of a and the value of b. Give your answers to five significant figures. [4]

(b) Use this model to estimate the mean time for the finalists in an Olympic race for boats with 8 rowers. Give your answer correct to two decimal places. [1]

It is now given that the mean time in the final for boats with 8 rowers was 342.08 seconds.

(c) Calculate the error in your estimate as a percentage of the actual value. [1]

(d) Comment on the likely validity of the model as N increases beyond 8. [2]

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8. [Maximum mark: 5]

When $x^2 + 4x - b$ is divided by $x - a$ the remainder is 2.
Given that $a, b \in \mathbb{R}$, find the smallest possible value for b.

9. [Maximum mark: 7]

Two distinct roots for the equation $z^4 - 10z^3 + az^2 + bz + 50 = 0$ are $c + \mathrm{i}$ and $2 + \mathrm{i}d$ where $a, b, c, d \in \mathbb{R}, d > 0$.

(a) Write down the other two roots in terms of c and d. [1]

(b) Find the value of c and the value of d. [6]

10. [Maximum mark: 8]

Students sign up at a desk for an activity during the course of an afternoon. The arrival of each student is independent of the arrival of any other student and the number of students arriving per hour can be modelled as a Poisson distribution with a mean of λ.

The desk is open for 4 hours. If exactly 5 people arrive to sign up for the activity during that time find the probability that exactly 3 of them arrived during the first hour.

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

11. [Maximum mark: 22]

Let $f(x) = x^4 + 0.2x^3 - 5.8x^2 - x + 4$, $x \in \mathbb{R}$.

(a) Find the solutions of $f(x) > 0$. [3]

(b) For the curve $y = f(x)$.

(i) Find the coordinates of both local minimum points.

(ii) Find the x-coordinates of the points of inflexion. [5]

The domain of f is now restricted to $[0, a]$.

(c) (i) Write down the largest value of a for which f has an inverse. Give your answer correct to 3 significant figures.

(ii) For this value of a sketch the graphs of $y = f(x)$ and $y = f^{-1}(x)$ on the same set of axes, showing clearly the coordinates of the end points of each curve.

(iii) Solve $f^{-1}(x) = 1$. [6]

Let $g(x) = 2\sin(x-1) - 3$, $-\frac{\pi}{2} + 1 \le x \le \frac{\pi}{2} + 1$.

(d) (i) Find an expression for $g^{-1}(x)$, stating the domain.

(ii) Solve $\left(f^{-1} \circ g\right)(x) < 1$. [8]

Do **not** write solutions on this page.

12. [Maximum mark: 16]

Consider the curve, C defined by the equation $y^2 - 2xy = 5 - e^x$. The point A lies on C and has coordinates $(0, a)$, $a > 0$.

(a) Find the value of a. [2]

(b) Show that $\frac{dy}{dx} = \frac{2y - e^x}{2(y - x)}$. [4]

(c) Find the equation of the normal to C at the point A. [3]

(d) Find the coordinates of the second point at which the normal found in part (c) intersects C. [4]

(e) Given that $v = y^3$, $y > 0$, find $\frac{dv}{dx}$ at $x = 0$. [3]

13. [Maximum mark: 22]

Six balls numbered $1, 2, 2, 3, 3, 3$ are placed in a bag. Balls are taken one at a time from the bag at random and the number noted. Throughout the question a ball is always replaced before the next ball is taken.

(a) A single ball is taken from the bag. Let X denote the value shown on the ball. Find $E(X)$. [2]

(b) Three balls are taken from the bag. Find the probability that

(i) the total of the three numbers is 5;

(ii) the median of the three numbers is 1. [6]

(c) Ten balls are taken from the bag. Find the probability that less than four of the balls are numbered 2. [3]

(d) Find the least number of balls that must be taken from the bag for the probability of taking out at least one ball numbered 2 to be greater than 0.95. [3]

(e) Another bag also contains balls numbered $1, 2$ or 3.
Eight balls are to be taken from this bag at random. It is calculated that the expected number of balls numbered 1 is 4.8, and the variance of the number of balls numbered 2 is 1.5.

Find the least possible number of balls numbered 3 in this bag. [8]

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Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Higher level
Paper 2

Wednesday 11 May 2016 (morning)

Candidate session number

2 hours

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
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- A clean copy of the **mathematics HL and further mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[120 marks]**.

2216 – 7206

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Section A

Answer **all** questions in the boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 5]

ABCD is a quadrilateral where $\text{AB} = 6.5$, $\text{BC} = 9.1$, $\text{CD} = 10.4$, $\text{DA} = 7.8$ and $\text{C}\hat{\text{D}}\text{A} = 90^\circ$. Find $\text{A}\hat{\text{B}}\text{C}$, giving your answer correct to the nearest degree.

2. [Maximum mark: 7]

A random variable X is normally distributed with mean 3 and variance 2^2.

(a) Find $\mathrm{P}(0 \le X \le 2)$. [2]

(b) Find $\mathrm{P}(|X| > 1)$. [3]

(c) If $\mathrm{P}(X > c) = 0.44$, find the value of c. [2]

3. [Maximum mark: 6]

Solve the simultaneous equations

$$\ln \frac{y}{x} = 2$$

$$\ln x^2 + \ln y^3 = 7\,.$$

4. [Maximum mark: 6]

The sum of the second and third terms of a geometric sequence is 96.

The sum to infinity of this sequence is 500.

Find the possible values for the common ratio, r.

5. [Maximum mark: 6]

The function f is defined as $f(x) = \sqrt{\frac{1-x}{1+x}}, -1 < x \leq 1$.

Find the inverse function, f^{-1} stating its domain and range.

6. [Maximum mark: 8]

A company produces rectangular sheets of glass of area 5 square metres. During manufacturing these glass sheets flaws occur at the rate of 0.5 per 5 square metres. It is assumed that the number of flaws per glass sheet follows a Poisson distribution.

(a) Find the probability that a randomly chosen glass sheet contains at least one flaw. [3]

Glass sheets with no flaws earn a profit of $\$5$. Glass sheets with at least one flaw incur a loss of $\$3$.

(b) Find the expected profit, P dollars, per glass sheet. [3]

This company also produces larger glass sheets of area 20 square metres. The rate of occurrence of flaws remains at 0.5 per 5 square metres.
A larger glass sheet is chosen at random.

(c) Find the probability that it contains no flaws. [2]

7. [Maximum mark: 8]

Consider the curve with equation $x^3 + y^3 = 4xy$.

(a) Use implicit differentiation to show that $\frac{dy}{dx} = \frac{4y - 3x^2}{3y^2 - 4x}$. [3]

The tangent to this curve is parallel to the x-axis at the point where $x = k$, $k > 0$.

(b) Find the value of k. [5]

8. [Maximum mark: 6]

A particle moves such that its velocity $v\,\text{ms}^{-1}$ is related to its displacement s m, by the equation $v(s) = \arctan(\sin s)$, $0 \le s \le 1$. The particle's acceleration is $a\,\text{ms}^{-2}$.

(a) Find the particle's acceleration in terms of s. [4]

(b) Using an appropriate sketch graph, find the particle's displacement when its acceleration is $0.25\,\text{ms}^{-2}$. [2]

9. [Maximum mark: 8]

OACB is a parallelogram with $\overrightarrow{\text{OA}} = \boldsymbol{a}$ and $\overrightarrow{\text{OB}} = \boldsymbol{b}$, where $\boldsymbol{a}$ and $\boldsymbol{b}$ are non-zero vectors.

(a) Show that

(i) $|\overrightarrow{\text{OC}}|^2 = |\boldsymbol{a}|^2 + 2\boldsymbol{a}\cdot\boldsymbol{b} + |\boldsymbol{b}|^2$;

(ii) $|\overrightarrow{\text{AB}}|^2 = |\boldsymbol{a}|^2 - 2\boldsymbol{a}\cdot\boldsymbol{b} + |\boldsymbol{b}|^2$. [4]

(b) Given that $|\overrightarrow{\text{OC}}| = |\overrightarrow{\text{AB}}|$, prove that OACB is a rectangle. [4]

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

10. [Maximum mark: 15]

A continuous random variable T has probability density function f defined by

$$f(t) = \begin{cases} \dfrac{t\left|\sin 2t\right|}{\pi}, & 0 \le t \le \pi \\ 0, & \text{otherwise} \end{cases}$$

(a) Sketch the graph of $y = f(t)$. [2]

(b) Use your sketch to find the mode of T. [1]

(c) Find the mean of T. [2]

(d) Find the variance of T. [3]

(e) Find the probability that T lies between the mean and the mode. [2]

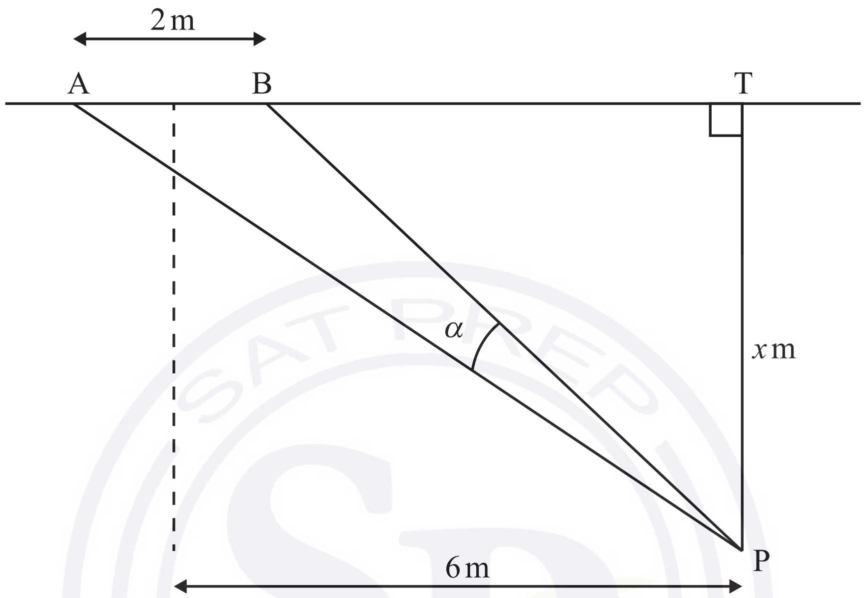
(f) (i) Find $\int_0^T f(t)\mathrm{d}t$ where $0 \le T \le \frac{\pi}{2}$.

(ii) Hence verify that the lower quartile of T is $\frac{\pi}{2}$. [5]

Do **not** write solutions on this page.

11. [Maximum mark: 22]

Points A, B and T lie on a line on an indoor soccer field. The goal, $[\text{AB}]$, is 2 metres wide. A player situated at point P kicks a ball at the goal. $[\text{PT}]$ is perpendicular to (AB) and is 6 metres from a parallel line through the centre of $[\text{AB}]$. Let PT be x metres and let $\alpha = \text{A}\hat{\text{P}}\text{B}$ measured in degrees. Assume that the ball travels along the floor.



(a) Find the value of α when $x = 10$. [4]

(b) Show that $\tan\alpha = \dfrac{2x}{x^2 + 35}$. [4]

The maximum for $\tan\alpha$ gives the maximum for α.

(c) (i) Find $\dfrac{\text{d}}{\text{d}x}(\tan\alpha)$.

(ii) Hence or otherwise find the value of α such that $\dfrac{\text{d}}{\text{d}x}(\tan\alpha) = 0$.

(iii) Find $\dfrac{\text{d}^2}{\text{d}x^2}(\tan\alpha)$ and hence show that the value of α never exceeds 10°. [11]

(d) Find the set of values of x for which $\alpha \geq 7^\circ$. [3]

Do **not** write solutions on this page.

12. [Maximum mark: 23]

The functions f and g are defined by

$$f(x) = \frac{e^x + e^{-x}}{2}, x \in \mathbb{R}$$

$$g(x) = \frac{e^x - e^{-x}}{2}, x \in \mathbb{R}$$

(a) (i) Show that $\frac{1}{4f(x) - 2g(x)} = \frac{e^x}{e^{2x} + 3}$.

(ii) Use the substitution $u = e^x$ to find $\int_0^{\ln 3} \frac{1}{4f(x) - 2g(x)} dx$. Give your answer in the form $\frac{\pi\sqrt{a}}{b}$ where $a, b \in \mathbb{Z}^+$. [9]

Let $h(x) = nf(x) + g(x)$ where $n \in \mathbb{R}, n > 1$.

(b) (i) By forming a quadratic equation in e^x, solve the equation $h(x) = k$, where $k \in \mathbb{R}^+$.

(ii) Hence or otherwise show that the equation $h(x) = k$ has two real solutions provided that $k > \sqrt{n^2 - 1}$ and $k \in \mathbb{R}^+$. [8]

Let $t(x) = \frac{g(x)}{f(x)}$.

(c) (i) Show that $t'(x) = \frac{[f(x)]^2 - [g(x)]^2}{[f(x)]^2}$ for $x \in \mathbb{R}$.

(ii) Hence show that $t'(x) > 0$ for $x \in \mathbb{R}$. [6]

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Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Higher level
Paper 2

Thursday 12 November 2015 (afternoon)

Candidate session number

2 hours

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions in the boxes provided.
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- A clean copy of the **mathematics HL and further mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[120 marks]**.

8815–7202

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions in the boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 4]

The events A and B are such that $\mathrm{P}(A) = 0.65$, $\mathrm{P}(B) = 0.48$ and $\mathrm{P}(A \cup B) = 0.818$.

(a) Find $\mathrm{P}(A \cap B)$. [2]

(b) Hence show that the events A and B are independent. [2]

2. [Maximum mark: 4]

The three planes having Cartesian equations $2x + 3y - z = 11$, $x + 2y + z = 3$ and $5x - y - z = 10$ meet at a point P. Find the coordinates of P.

3. [Maximum mark: 6]

The data of the goals scored by players in a football club during a season are given in the following table.

Goals	Frequency
0	4
1	k
2	3
3	2
4	3
8	1

(a) Given that the mean number of goals scored per player is 1.95, find the value of k. [3]

It is discovered that there is a mistake in the data and that the top scorer, who scored 22 goals, has not been included in the table.

(b) (i) Find the correct mean number of goals scored per player.

(ii) Find the correct standard deviation of the number of goals scored per player. [3]

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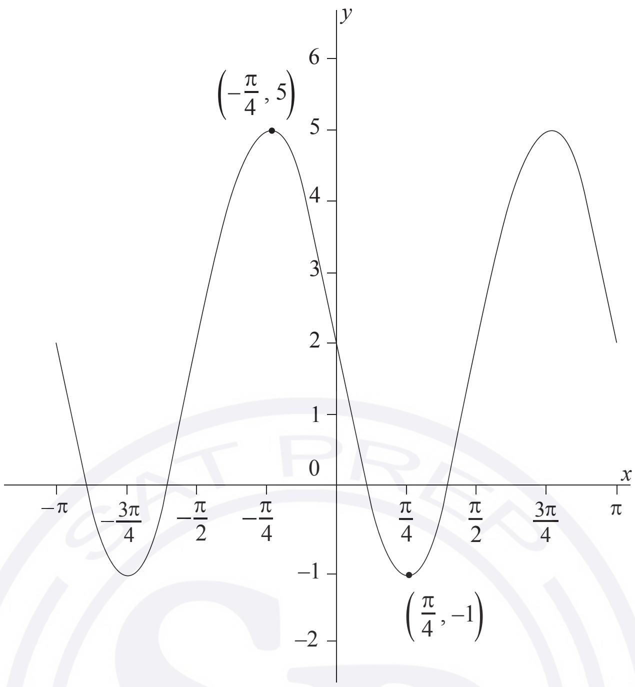
4. [Maximum mark: 6]

A function is defined by $f(x) = A\sin(Bx) + C$, $-\pi \le x \le \pi$, where $A, B, C \in \mathbb{Z}$. The following diagram represents the graph of $y = f(x)$.



(a) Find the value of

(i) A;

(ii) B;

(iii) C. [4]

(b) Solve $f(x) = 3$ for $0 \le x \le \pi$. [2]

5. [Maximum mark: 6]

A function is defined by $f(x) = x^2 + 2$, $x \geq 0$. A region R is enclosed by $y = f(x)$, the y-axis and the line $y = 4$.

(a) (i) Express the area of the region R as an integral with respect to y.

(ii) Determine the area of R, giving your answer correct to four significant figures. [3]

(b) Find the exact volume generated when the region R is rotated through 2π radians about the y-axis. [3]

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6. [Maximum mark: 6]

Josie has three ways of getting to school. 30% of the time she travels by car, 20% of the time she rides her bicycle and 50% of the time she walks.
When travelling by car, Josie is late 5% of the time. When riding her bicycle she is late 10% of the time. When walking she is late 25% of the time. Given that she was on time, find the probability that she rides her bicycle.

7. [Maximum mark: 6]

Triangle ABC has area $21\,\text{cm}^2$. The sides AB and AC have lengths $6\,\text{cm}$ and $11\,\text{cm}$ respectively. Find the two possible lengths of the side BC.

8. [Maximum mark: 6]

The continuous random variable X has the probability distribution function $f(x) = A\sin(\ln(x))$, $1 \le x \le 5$.

(a) Find the value of A to three decimal places. [2]

(b) Find the mode of X. [2]

(c) Find the value $\mathrm{P}(X \le 3 \mid X \ge 2)$. [2]

9. [Maximum mark: 8]

A particle can move along a straight line from a point O. The velocity v, in $\mathrm{m\,s^{-1}}$, is given by the function $v(t) = 1 - \mathrm{e}^{-\sin t^2}$ where time $t \geq 0$ is measured in seconds.

(a) Write down the first two times $t_1, t_2 > 0$, when the particle changes direction. [2]

(b) (i) Find the time $t < t_2$ when the particle has a maximum velocity.

(ii) Find the time $t < t_2$ when the particle has a minimum velocity. [4]

(c) Find the distance travelled by the particle between times $t = t_1$ and $t = t_2$. [2]

Please **do not** write on this page.

Answers written on this page will not be marked.

10. [Maximum mark: 8]

Ed walks in a straight line from point $P(-1, 4)$ to point $Q(4, 16)$ with constant speed. Ed starts from point P at time $t=0$ and arrives at point Q at time $t=3$, where t is measured in hours.
Given that, at time t, Ed's position vector, relative to the origin, can be given in the form, $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$,

(a) find the vectors $\boldsymbol{a}$ and $\boldsymbol{b}$. [3]

Roderick is at a point $C(11, 9)$. During Ed's walk from P to Q Roderick wishes to signal to Ed. He decides to signal when Ed is at the closest point to C.

(b) Find the time when Roderick signals to Ed. [5]

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

11. [Maximum mark: 18]

A survey is conducted in a large office building. It is found that 30% of the office workers weigh less than 62 kg and that 25% of the office workers weigh more than 98 kg. The weights of the office workers may be modelled by a normal distribution with mean μ and standard deviation σ.

(a) (i) Determine two simultaneous linear equations satisfied by μ and σ.

(ii) Find the values of μ and σ. [6]

(b) Find the probability that an office worker weighs more than 100 kg. [1]

There are elevators in the office building that take the office workers to their offices. Given that there are 10 workers in a particular elevator,

(c) find the probability that at least four of the workers weigh more than 100 kg. [2]

Given that there are 10 workers in an elevator and at least one weighs more than 100 kg,

(d) find the probability that there are fewer than four workers exceeding 100 kg. [3]

The arrival of the elevators at the ground floor between $08{:}00$ and $09{:}00$ can be modelled by a Poisson distribution. Elevators arrive on average every 36 seconds.

(e) Find the probability that in any half hour period between $08{:}00$ and $09{:}00$ more than 60 elevators arrive at the ground floor. [3]

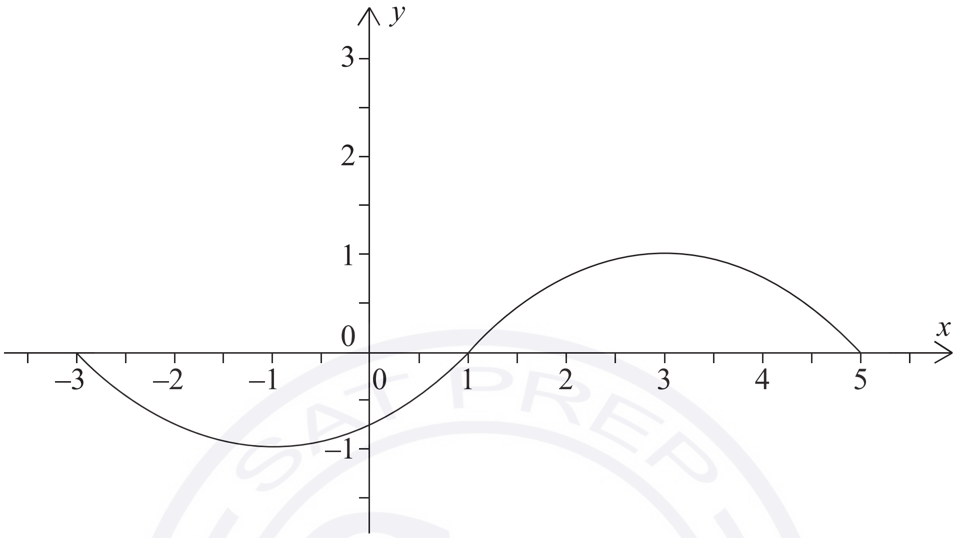
An elevator can take a maximum of 10 workers. Given that 400 workers arrive in a half hour period independently of each other,

(f) find the probability that there are sufficient elevators to take them to their offices. [3]

Do **not** write solutions on this page.

12. [Maximum mark: 21]

The following graph represents a function $y = f(x)$, where $-3 \le x \le 5$.
The function has a maximum at $(3, 1)$ and a minimum at $(-1, -1)$.



(a) The functions u and v are defined as $u(x) = x - 3$, $v(x) = 2x$ where $x \in \mathbb{R}$.

(i) State the range of the function $u \circ f$.

(ii) State the range of the function $u \circ v \circ f$.

(iii) Find the largest possible domain of the function $f \circ v \circ u$. [7]

(b) (i) Explain why f does not have an inverse.

(ii) The domain of f is restricted to define a function g so that it has an inverse g^{-1}. State the largest possible domain of g.

(iii) Sketch a graph of $y = g^{-1}(x)$, showing clearly the y-intercept and stating the coordinates of the endpoints. [6]

Consider the function defined by $h(x) = \dfrac{2x-5}{x+d}$, $x \ne -d$ and $d \in \mathbb{R}$.

(c) (i) Find an expression for the inverse function $h^{-1}(x)$.

(ii) Find the value of d such that h is a self-inverse function.

For this value of d, there is a function k such that $h \circ k(x) = \dfrac{2x}{x+1}$, $x \ne -1$.

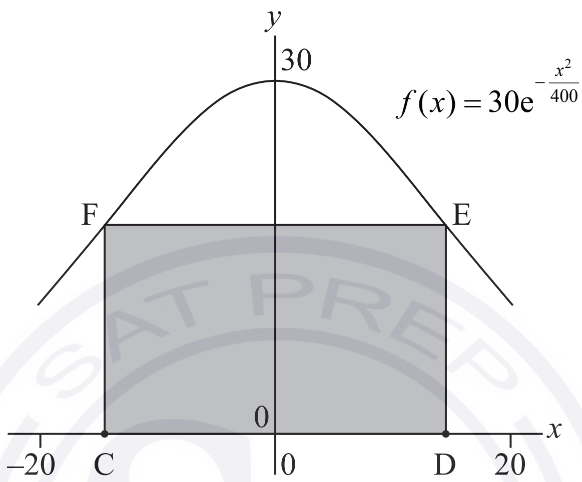
(iii) Find $k(x)$. [8]

Do **not** write solutions on this page.

13. [Maximum mark: 21]

The following diagram shows a vertical cross section of a building. The cross section of the roof of the building can be modelled by the curve $f(x) = 30e^{-\frac{x^2}{400}}$, where $-20 \le x \le 20$.

Ground level is represented by the x-axis.



(a) Find $f''(x)$. [4]

(b) Show that the gradient of the roof function is greatest when $x = -\sqrt{200}$. [3]

The cross section of the living space under the roof can be modelled by a rectangle CDEF with points $C(-a, 0)$ and $D(a, 0)$, where $0 < a \le 20$.

(c) Show that the maximum area A of the rectangle CDEF is $600\sqrt{2}e^{-\frac{1}{2}}$. [5]

(d) A function I is known as the Insulation Factor of CDEF. The function is defined as $I(a) = \dfrac{P(a)}{A(a)}$ where P = Perimeter and A = Area of the rectangle.

(i) Find an expression for P in terms of a.

(ii) Find the value of a which minimizes I.

(iii) Using the value of a found in part (ii) calculate the percentage of the cross sectional area under the whole roof that is not included in the cross section of the living space. [9]

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Higher level
Paper 2

Wednesday 13 May 2015 (afternoon)

2 hours

Candidate session number

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- A graphic display calculator is required for this paper.
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- The maximum mark for this examination paper is **[120 marks].**

2215 – 7204

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.

1. [Maximum mark: 4]

The region R is enclosed by the graph of $y = \mathrm{e}^{-x^2}$, the x-axis and the lines $x = -1$ and $x = 1$. Find the volume of the solid of revolution that is formed when R is rotated through 2π about the x-axis.

2. [Maximum mark: 4]

The finishing times in a marathon race follow a normal distribution with mean 210 minutes and standard deviation 22 minutes.

(a) Find the probability that a runner finishes the race in under three hours. [2]

The fastest 90% of the finishers receive a certificate.

(b) Find the time, below which a competitor has to complete the race, in order to gain a certificate. [2]

3. [Maximum mark: 5]

A mosaic is going to be created by randomly selecting 1000 small tiles, each of which is either black or white. The probability that a tile is white is 0.1. Let the random variable W be the number of white tiles.

(a) State the distribution of W, including the values of any parameters. [2]

(b) Write down the mean of W. [1]

(c) Find $\mathrm{P}(W > 89)$. [2]

4. [Maximum mark: 6]

A triangle ABC has $\hat{A} = 50°$, $AB = 7\,\text{cm}$ and $BC = 6\,\text{cm}$. Find the area of the triangle given that it is smaller than $10\,\text{cm}^2$.

5. [Maximum mark: 5]

A bicycle inner tube can be considered as a joined up cylinder of fixed length $200\,\text{cm}$ and radius $r\,\text{cm}$. The radius r increases as the inner tube is pumped up. Air is being pumped into the inner tube so that the volume of air in the tube increases at a constant rate of $30\,\text{cm}^3\,\text{s}^{-1}$. Find the rate at which the radius of the inner tube is increasing when $r = 2\,\text{cm}$.

6. [Maximum mark: 4]

A function f is defined by $f(x) = x^3 + e^x + 1$, $x \in \mathbb{R}$. By considering $f'(x)$ determine whether f is a one-to-one or a many-to-one function.

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7. [Maximum mark: 7]

The random variable X follows a Poisson distribution with mean $m \neq 0$.

(a) Given that $2\mathrm{P}(X = 4) = \mathrm{P}(X = 5)$, show that $m = 10$. [3]

(b) Given that $X \leq 11$, find the probability that $X = 6$. [4]

8. [Maximum mark: 8]

Let $\boldsymbol{v} = \begin{pmatrix} 2 \\ 3 \\ 5 \end{pmatrix}$ and $\boldsymbol{w} = \begin{pmatrix} 4 \\ \lambda \\ 10 \end{pmatrix}$.

(a) Find the value of λ for $\boldsymbol{v}$ and $\boldsymbol{w}$ to be parallel. [2]

(b) Find the value of λ for $\boldsymbol{v}$ and $\boldsymbol{w}$ to be perpendicular. [2]

(c) Find the two values of λ if the angle between $\boldsymbol{v}$ and $\boldsymbol{w}$ is 10°. [4]

9. [Maximum mark: 7]

Find the equation of the normal to the curve $y = \dfrac{e^x \cos x \ln(x+e)}{\left(x^{17}+1\right)^5}$ at the point where $x = 0$.

In your answer give the value of the gradient, of the normal, to three decimal places.

10. **[**Maximum mark: 10]

A function f is defined by $f(x) = (x+1)(x-1)(x-5)$, $x \in \mathbb{R}$.

(a) Find the values of x for which $f(x) < |f(x)|$. [3]

A function g is defined by $g(x) = x^2 + x - 6$, $x \in \mathbb{R}$.

(b) Find the values of x for which $g(x) < \frac{1}{g(x)}$. [7]

Turn over

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

11. [Maximum mark: 20]

The probability density function of a continuous random variable X is given by

$$f(x)=\begin{cases} 0, & x<0 \\ \dfrac{\sin x}{4}, & 0 \le x \le \pi \\ a(x-\pi), & \pi < x \le 2\pi \\ 0, & 2\pi < x \end{cases}.$$

(a) Sketch the graph of $y=f(x)$. [2]

(b) Find $\mathrm{P}(X \le \pi)$. [2]

(c) Show that $a=\dfrac{1}{\pi^2}$. [3]

(d) Write down the median of X. [1]

(e) Calculate the mean of X. [3]

(f) Calculate the variance of X. [3]

(g) Find $\mathrm{P}\left(\dfrac{\pi}{2} \le X \le \dfrac{3\pi}{2}\right)$. [2]

(h) Given that $\dfrac{\pi}{2} \le X \le \dfrac{3\pi}{2}$ find the probability that $\pi \le X \le 2\pi$. [4]

Do **not** write solutions on this page.

12. [Maximum mark: 19]

(a) (i) Use the binomial theorem to expand $(\cos\theta + \mathrm{i}\sin\theta)^5$.

(ii) Hence use De Moivre's theorem to prove

$$\sin 5\theta = 5\cos^4\theta\sin\theta - 10\cos^2\theta\sin^3\theta + \sin^5\theta.$$

(iii) State a similar expression for $\cos 5\theta$ in terms of $\cos\theta$ and $\sin\theta$. [6]

Let $z = r(\cos\alpha + \mathrm{i}\sin\alpha)$, where α is measured in degrees, be the solution of $z^5 - 1 = 0$ which has the smallest positive argument.

(b) Find the value of r and the value of α. [4]

(c) Using (a) (ii) and your answer from (b) show that $16\sin^4\alpha - 20\sin^2\alpha + 5 = 0$. [4]

(d) Hence express $\sin 72^\circ$ in the form $\dfrac{\sqrt{a + b\sqrt{c}}}{d}$ where $a, b, c, d \in \mathbb{Z}$. [5]

Do **not** write solutions on this page.

13. [Maximum mark: 21]

Richard, a marine soldier, steps out of a stationary helicopter, 1000 m above the ground, at time $t = 0$. Let his height, in metres, above the ground be given by $s(t)$. For the first 10 seconds his velocity, $v(t)\text{ ms}^{-1}$, is given by $v(t) = -10t$.

(a) (i) Find his acceleration $a(t)$ for $t < 10$.

(ii) Calculate $v(10)$.

(iii) Show that $s(10) = 500$. [6]

At $t = 10$ his parachute opens and his acceleration $a(t)$ is subsequently given by $a(t) = -10 - 5v$, $t \geq 10$.

(b) Given that $\frac{\mathrm{d}t}{\mathrm{d}v} = \frac{1}{\frac{\mathrm{d}v}{\mathrm{d}t}}$, write down $\frac{\mathrm{d}t}{\mathrm{d}v}$ in terms of v. [1]

You are told that Richard's acceleration, $a(t) = -10 - 5v$, is always positive, for $t \geq 10$.

(c) Hence show that $t = 10 + \frac{1}{5}\ln\left(\frac{98}{-2-v}\right)$. [5]

(d) Hence find an expression for the velocity, v, for $t \geq 10$. [2]

(e) Find an expression for his height, s, above the ground for $t \geq 10$. [5]

(f) Find the value of t when Richard lands on the ground. [2]

Please **do not** write on this page.

Answers written on this page
will not be marked.

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Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Higher level
Paper 2

Wednesday 13 May 2015 (afternoon)

Candidate session number

2 hours

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2215 – 7206

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Section A

Answer **all** questions in the boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 4]

In triangle ABC, $\mathrm{AB} = 5\,\mathrm{cm}$, $\mathrm{BC} = 12\,\mathrm{cm}$ and $\mathrm{A\hat{B}C} = 100^\circ$.

(a) Find the area of the triangle. [2]

(b) Find AC. [2]

2. [Maximum mark: 6]

From a group of five males and six females, four people are chosen.

(a) Determine how many possible groups can be chosen. [2]

(b) Determine how many groups can be formed consisting of two males and two females. [2]

(c) Determine how many groups can be formed consisting of at least one female. [2]

3. [Maximum mark: 5]

(a) Sketch the graph of $y=(x-5)^2-2|x-5|-9$, for $0\le x\le 10$. [3]

(b) Hence, or otherwise, solve the equation $(x-5)^2-2|x-5|-9=0$. [2]

4. [Maximum mark: 7]

Emma acquires a new cell phone for her birthday and receives texts from her friends. It is assumed that the daily number of texts Emma receives follows a Poisson distribution with mean $m = 5$.

(a) (i) Find the probability that on a certain day Emma receives more than 7 texts.

(ii) Determine the expected number of days in a week on which Emma receives more than 7 texts. [4]

(b) Find the probability that Emma receives fewer than 30 texts during a week. [3]

5. [Maximum mark: 7]

Consider the vectors given by $\boldsymbol{u} = \boldsymbol{i} + 2\boldsymbol{j} - 2\boldsymbol{k}$ and $\boldsymbol{v} = a\boldsymbol{i} + b\boldsymbol{j}$, where a and b are constants.

It is given that $\boldsymbol{u} \times \boldsymbol{v} = 4\boldsymbol{i} + b\boldsymbol{j} + c\boldsymbol{k}$, where c is a constant.

(a) Find the value of each of the constants a, b and c. [5]

(b) Hence find the Cartesian equation of the plane containing the vectors $\boldsymbol{u}$ and $\boldsymbol{v}$ and passing through the point $(0, 0, 0)$. [2]

6. [Maximum mark: 6]

The graph of $y = \ln(5x + 10)$ is obtained from the graph of $y = \ln x$ by a translation of a units in the direction of the x-axis followed by a translation of b units in the direction of the y-axis.

(a) Find the value of a and the value of b. [4]

(b) The region bounded by the graph of $y = \ln(5x + 10)$, the x-axis and the lines $x = \mathrm{e}$ and $x = 2\mathrm{e}$, is rotated through 2π radians about the x-axis. Find the volume generated. [2]

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7. [Maximum mark: 9]

Consider the following system of equations

$$2x + y + 6z = 0$$
$$4x + 3y + 14z = 4$$
$$2x - 2y + (\alpha - 2)z = \beta - 12 .$$

(a) Find conditions on α and β for which

(i) the system has no solutions;

(ii) the system has only one solution;

(iii) the system has an infinite number of solutions. [6]

(b) In the case where the number of solutions is infinite, find the general solution of the system of equations in Cartesian form. [3]

(This question continues on the following page)

(Question 7 continued)

8. [Maximum mark: 10]

Farmer Bill owns a rectangular field, 10 m by 4 m. Bill attaches a rope to a wooden post at one corner of his field, and attaches the other end to his goat Gruff.

(a) Given that the rope is 5 m long, calculate the percentage of Bill's field that Gruff is able to graze. Give your answer correct to the nearest integer. [4]

(b) Bill replaces Gruff's rope with another, this time of length a, $4 < a < 10$, so that Gruff can now graze exactly one half of Bill's field.

Show that a satisfies the equation

$$a^2 \arcsin\left(\frac{4}{a}\right) + 4\sqrt{a^2 - 16} = 40.$$ [4]

(c) Find the value of a. [2]

(This question continues on the following page)

(Question 8 continued)

9. [Maximum mark: 6]

Natasha lives in Chicago and has relatives in Nashville and St. Louis.
Each time she visits her relatives, she either flies or drives.

When travelling to Nashville, the probability that she drives is $\frac{4}{5}$, and when travelling to St. Louis, the probability that she flies is $\frac{1}{3}$.

Given that the probability that she drives when visiting her relatives is $\frac{13}{18}$, find the probability that for a particular trip,

(a) she travels to Nashville; [3]

(b) she is on her way to Nashville, given that she is flying. [3]

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Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

10. [Maximum mark: 12]

Farmer Suzie grows turnips and the weights of her turnips are normally distributed with a mean of $122\,\text{g}$ and standard deviation of $14.7\,\text{g}$.

(a) (i) Calculate the percentage of Suzie's turnips that weigh between $110\,\text{g}$ and $130\,\text{g}$.

(ii) Suzie has 100 turnips to take to market. Find the expected number weighing more than $130\,\text{g}$.

(iii) Find the probability that at least 30 of the 100 turnips weigh more than $130\,\text{g}$. [6]

Farmer Ray also grows turnips and the weights of his turnips are normally distributed with a mean of $144\,\text{g}$. Ray only takes to market turnips that weigh more than $130\,\text{g}$. Over a period of time, Ray finds he has to reject 1 in 15 turnips due to their being underweight.

(b) (i) Find the standard deviation of the weights of Ray's turnips.

(ii) Ray has 200 turnips to take to market. Find the expected number weighing more than $150\,\text{g}$. [6]

11. [Maximum mark: 15]

A curve is defined by $x^2 - 5xy + y^2 = 7$.

(a) Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{5y - 2x}{2y - 5x}$. [3]

(b) Find the equation of the normal to the curve at the point $(6\,,\,1)$. [4]

(c) Find the distance between the two points on the curve where each tangent is parallel to the line $y = x$. [8]

Do **not** write solutions on this page.

12. [Maximum mark: 15]

A particle moves in a straight line, its velocity $v\ \text{ms}^{-1}$ at time t seconds is given by $v = 9t - 3t^2,\ 0 \le t \le 5$.

At time $t = 0$, the displacement s of the particle from an origin O is $3\,\text{m}$.

(a) Find the displacement of the particle when $t = 4$. [3]

(b) Sketch a displacement/time graph for the particle, $0 \le t \le 5$, showing clearly where the curve meets the axes and the coordinates of the points where the displacement takes greatest and least values. [5]

For $t > 5$, the displacement of the particle is given by $s = a + b\cos\frac{2\pi t}{5}$ such that s is continuous for all $t \ge 0$.

(c) Given further that $s = 16.5$ when $t = 7.5$, find the values of a and b. [3]

(d) Find the times t_1 and t_2 $(0 < t_1 < t_2 < 8)$ when the particle returns to its starting point. [4]

13. [Maximum mark: 18]

The equations of the lines L_1 and L_2 are

$$L_1 : \boldsymbol{r}_1 = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix}$$

$$L_2 : \boldsymbol{r}_2 = \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 1 \\ 6 \end{pmatrix}.$$

(a) Show that the lines L_1 and L_2 are skew. [4]

(b) Find the acute angle between the lines L_1 and L_2. [4]

(c) (i) Find a vector perpendicular to both lines.

(ii) Hence determine an equation of the line L_3 that is perpendicular to both L_1 and L_2 and intersects both lines. [10]

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MATHEMATICS
HIGHER LEVEL
PAPER 2

Candidate session number

Thursday 13 November 2014 (morning)

2 hours

Examination code

8	8	1	4	–	7	2	0	2

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- The maximum mark for this examination paper is *[120 marks]*.

16 pages

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SECTION A

Answer ***all*** *questions in the boxes provided. Working may be continued below the lines if necessary.*

1. *[Maximum mark: 6]*

Consider the two planes

$$\pi_1 : 4x + 2y - z = 8$$

$$\pi_2 : x + 3y + 3z = 3 .$$

Find the angle between π_1 and π_2, giving your answer correct to the nearest degree.

2. *[Maximum mark: 5]*

The wingspans of a certain species of bird can be modelled by a normal distribution with mean 60.2 cm and standard deviation 2.4 cm.

According to this model, 99 % of wingspans are greater than x cm.

(a) Find the value of x. *[2]*

In a field experiment, a research team studies a large sample of these birds. The wingspans of each bird are measured correct to the nearest 0.1 cm.

(b) Find the probability that a randomly selected bird has a wingspan measured as 60.2 cm. *[3]*

3. *[Maximum mark: 6]*

Consider the data set $\{2, x, y, 10, 17\}$, $x, y \in \mathbb{Z}^+$ and $x < y$.

The mean of the data set is 8 and its variance is 27.6.

Find the value of x and the value of y.

4. *[Maximum mark: 5]*

Two cyclists are at the same road intersection. One cyclist travels north at $20\,\text{km}\,\text{h}^{-1}$. The other cyclist travels west at $15\,\text{km}\,\text{h}^{-1}$.

Use calculus to show that the rate at which the distance between the two cyclists changes is independent of time.

5. *[Maximum mark: 8]*

The lines l_1 and l_2 are defined as

$$l_1 : \frac{x-1}{3} = \frac{y-5}{2} = \frac{z-12}{-2}$$

$$l_2 : \frac{x-1}{8} = \frac{y-5}{11} = \frac{z-12}{6}.$$

The plane π contains both l_1 and l_2.

(a) Find the Cartesian equation of π. *[4]*

The line l_3 passing through the point $(4, 0, 8)$ is perpendicular to π.

(b) Find the coordinates of the point where l_3 meets π. *[4]*

6. *[Maximum mark: 6]*

Consider $p(x) = 3x^3 + ax + 5a$, $a \in \mathbb{R}$.

The polynomial $p(x)$ leaves a remainder of -7 when divided by $(x - a)$.

Show that only one value of a satisfies the above condition and state its value.

7. *[Maximum mark: 9]*

The seventh, third and first terms of an arithmetic sequence form the first three terms of a geometric sequence.

The arithmetic sequence has first term a and non-zero common difference d.

(a) Show that $d = \frac{a}{2}$. *[3]*

The seventh term of the arithmetic sequence is 3. The sum of the first n terms in the arithmetic sequence exceeds the sum of the first n terms in the geometric sequence by at least 200.

(b) Find the least value of n for which this occurs. *[6]*

8. *[Maximum mark: 7]*

A particle moves in a straight line such that its velocity, $v\,\text{m s}^{-1}$, at time t seconds, is given by

$$v(t)=\begin{cases}5-(t-2)^2, & 0\le t\le 4\\ 3-\dfrac{t}{2}, & t>4\end{cases}.$$

(a) Find the value of t when the particle is instantaneously at rest. *[2]*

The particle returns to its initial position at $t=T$.

(b) Find the value of T. *[5]*

9. *[Maximum mark: 8]*

Compactness is a measure of how compact an enclosed region is.

The compactness, C, of an enclosed region can be defined by $C = \frac{4A}{\pi d^2}$, where A is the area of the region and d is the maximum distance between any two points in the region.

For a circular region, $C = 1$.

Consider a regular polygon of n sides constructed such that its vertices lie on the circumference of a circle of diameter x units.

(a) If $n > 2$ and even, show that $C = \frac{n}{2\pi}\sin\frac{2\pi}{n}$. *[3]*

If $n > 1$ and odd, it can be shown that $C = \frac{n\sin\frac{2\pi}{n}}{\pi\left(1+\cos\frac{\pi}{n}\right)}$.

(b) Find the regular polygon with the least number of sides for which the compactness is more than 0.99. *[4]*

(c) Comment briefly on whether C is a good measure of compactness. *[1]*

(This question continues on the following page)

(Question 9 continued)

*Do **NOT** write solutions on this page.*

SECTION B

*Answer **all** questions in the answer booklet provided. Please start each question on a new page.*

10. *[Maximum mark: 12]*

Consider the triangle PQR where $Q\hat{P}R = 30^\circ$, $PQ = (x+2)\,\text{cm}$ and $PR = (5-x)^2\,\text{cm}$, where $-2 < x < 5$.

(a) Show that the area, A cm^2, of the triangle is given by $A = \frac{1}{4}\left(x^3 - 8x^2 + 5x + 50\right)$. *[2]*

(b) (i) State $\frac{dA}{dx}$.

(ii) Verify that $\frac{dA}{dx} = 0$ when $x = \frac{1}{3}$. *[3]*

(c) (i) Find $\frac{d^2A}{dx^2}$ and hence justify that $x = \frac{1}{3}$ gives the maximum area of triangle PQR.

(ii) State the maximum area of triangle PQR.

(iii) Find QR when the area of triangle PQR is a maximum. *[7]*

*Do **NOT** write solutions on this page.*

11. *[Maximum mark: 10]*

The number of complaints per day received by customer service at a department store follows a Poisson distribution with a mean of 0.6.

(a) On a randomly chosen day, find the probability that

(i) there are no complaints;

(ii) there are at least three complaints. *[3]*

(b) In a randomly chosen five-day week, find the probability that there are no complaints. *[2]*

(c) On a randomly chosen day, find the most likely number of complaints received. Justify your answer. *[3]*

The department store introduces a new policy to improve customer service. The number of complaints received per day now follows a Poisson distribution with mean λ.

On a randomly chosen day, the probability that there are no complaints is now 0.8.

(d) Find the value of λ. *[2]*

12. *[Maximum mark: 11]*

Ava and Barry play a game with a bag containing one green marble and two red marbles. Each player in turn randomly selects a marble from the bag, notes its colour and replaces it. Ava wins the game if she selects a green marble. Barry wins the game if he selects a red marble. Ava starts the game.

Find the probability that

(a) Ava wins on her first turn; *[1]*

(b) Barry wins on his first turn; *[2]*

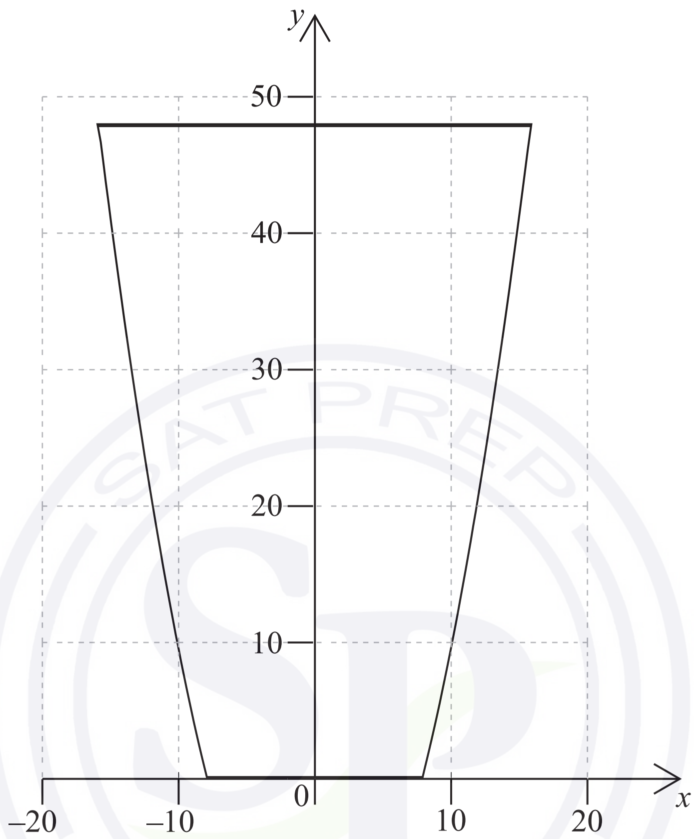
(c) Ava wins in one of her first three turns; *[4]*

(d) Ava eventually wins. *[4]*

*Do **NOT** write solutions on this page.*

13. *[Maximum mark: 16]*

The vertical cross-section of a container is shown in the following diagram.



The curved sides of the cross-section are given by the equation $y = 0.25x^2 - 16$. The horizontal cross-sections are circular. The depth of the container is $48\,\text{cm}$.

(a) If the container is filled with water to a depth of h cm, show that the volume, V cm^3, of the water is given by $V = 4\pi\left(\dfrac{h^2}{2} + 16h\right)$. *[3]*

(This question continues on the following page)

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(Question 13 continued)

The container, initially full of water, begins leaking from a small hole at a rate given by $\frac{dV}{dt} = -\frac{250\sqrt{h}}{\pi(h+16)}$ where t is measured in seconds.

(b) (i) Show that $\frac{dh}{dt} = -\frac{250\sqrt{h}}{4\pi^2(h+16)^2}$.

(ii) State $\frac{dt}{dh}$ and hence show that $t = \frac{-4\pi^2}{250}\int\left(h^{\frac{3}{2}} + 32h^{\frac{1}{2}} + 256h^{-\frac{1}{2}}\right)dh$.

(iii) Find, correct to the nearest minute, the time taken for the container to become empty. (60 seconds = 1 minute) *[10]*

Once empty, water is pumped back into the container at a rate of $8.5\,\text{cm}^3\,\text{s}^{-1}$. At the same time, water continues leaking from the container at a rate of $\frac{250\sqrt{h}}{\pi(h+16)}\text{cm}^3\,\text{s}^{-1}$.

(c) Using an appropriate sketch graph, determine the depth at which the water ultimately stabilizes in the container. *[3]*

Turn over

*Do **NOT** write solutions on this page.*

14. *[Maximum mark: 11]*

In triangle ABC,

$$3\sin B + 4\cos C = 6 \text{ and}$$
$$4\sin C + 3\cos B = 1.$$

(a) Show that $\sin(B+C) = \frac{1}{2}$. *[6]*

Robert conjectures that $\text{C}\hat{\text{A}}\text{B}$ can have two possible values.

(b) Show that Robert's conjecture is incorrect by proving that $\text{C}\hat{\text{A}}\text{B}$ has only one possible value. *[5]*

M14/5/MATHL/HP2/ENG/TZ1/XX

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MATHEMATICS
HIGHER LEVEL
PAPER 2

Candidate session number

Wednesday 14 May 2014 (morning)

Examination code

2	2	1	4	–	7	2	0	4

2 hours

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions in the boxes provided.
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15 pages

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SECTION A

*Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.*

1. *[Maximum mark: 4]*

One root of the equation $x^2 + ax + b = 0$ is $2 + 3\mathrm{i}$ where a, $b \in \mathbb{R}$. Find the value of a and the value of b.

2. *[Maximum mark: 5]*

A student sits a national test and is told that the marks follow a normal distribution with mean 100. The student receives a mark of 124 and is told that he is at the 68^{th} percentile. Calculate the variance of the distribution.

3. *[Maximum mark: 4]*

Find the number of ways in which seven different toys can be given to three children, if the youngest is to receive three toys and the others receive two toys each.

4. *[Maximum mark: 6]*

A system of equations is given below.

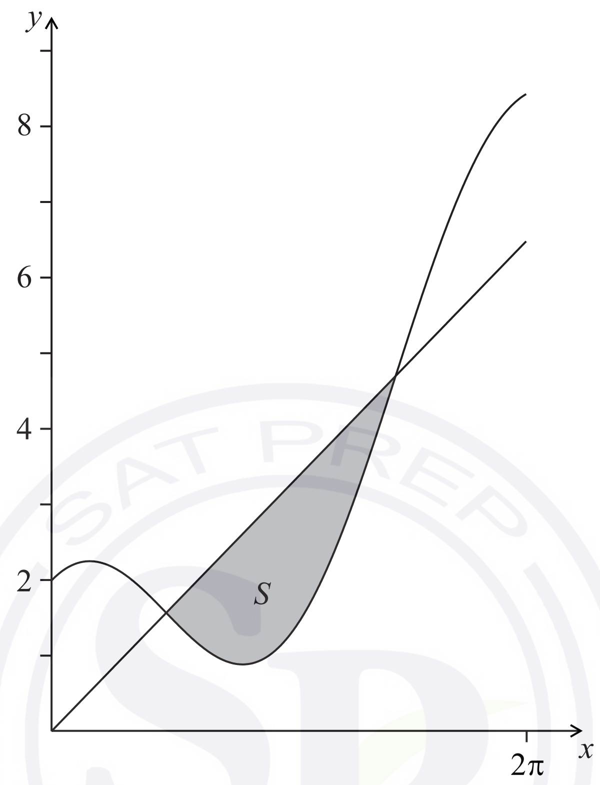
$$x+2y-z=2$$
$$2x+y+z=1$$
$$-x+4y+az=4$$

(a) Find the value of a so that the system does not have a unique solution. *[4]*

(b) Show that the system has a solution for any value of a. *[2]*

5. *[Maximum mark: 8]*

The shaded region S is enclosed between the curve $y = x + 2\cos x$, for $0 \le x \le 2\pi$, and the line $y = x$, as shown in the diagram below.



(a) Find the coordinates of the points where the line meets the curve. *[3]*

The region S is rotated by 2π about the x-axis to generate a solid.

(b) (i) Write down an integral that represents the volume V of the solid.

(ii) Find the volume V. *[5]*

(This question continues on the following page)

(Question 5 continued)

6. *[Maximum mark: 10]*

Let $f(x) = x(x+2)^6$.

(a) Solve the inequality $f(x) > x$. *[5]*

(b) Find $\int f(x)\,dx$. *[5]*

7. *[Maximum mark: 8]*

Prove, by mathematical induction, that $7^{8n+3} + 2$, $n \in \mathbb{N}$, is divisible by 5.

8. *[Maximum mark: 8]*

(a) Find the term in x^5 in the expansion of $(3x+A)(2x+B)^6$. *[4]*

Mina and Norbert each have a fair cubical die with faces labelled 1, 2, 3, 4, 5 and 6; they throw it to decide if they are going to eat a cookie.

Mina throws her die just once and she eats a cookie if she throws a four, a five or a six. Norbert throws his die six times and each time eats a cookie if he throws a five or a six.

(b) Calculate the probability that five cookies are eaten. *[4]*

9. *[Maximum mark: 7]*

The number of birds seen on a power line on any day can be modelled by a Poisson distribution with mean 5.84.

(a) Find the probability that during a certain seven-day week, more than 40 birds have been seen on the power line. *[2]*

(b) On Monday there were more than 10 birds seen on the power line. Show that the probability of there being more than 40 birds seen on the power line from that Monday to the following Sunday, inclusive, can be expressed as:

$$\frac{\mathrm{P}(X > 40) + \sum_{r=11}^{40} \mathrm{P}(X = r)\mathrm{P}(Y > 40 - r)}{\mathrm{P}(X > 10)} \text{ where } X \sim \mathrm{Po}(5.84) \text{ and } Y \sim \mathrm{Po}(35.04).$$ *[5]*

*Do **NOT** write solutions on this page.*

SECTION B

*Answer **all** questions in the answer booklet provided. Please start each question on a new page.*

10. *[Maximum mark: 21]*

Let $f(x)=\dfrac{e^{2x}+1}{e^{x}-2}$.

(a) Find the equations of the horizontal and vertical asymptotes of the curve $y=f(x)$. *[4]*

(b) (i) Find $f'(x)$.

(ii) Show that the curve has exactly one point where its tangent is horizontal.

(iii) Find the coordinates of this point. *[8]*

(c) Find the equation of L_1, the normal to the curve at the point where it crosses the y-axis. *[4]*

The line L_2 is parallel to L_1 and tangent to the curve $y=f(x)$.

(d) Find the equation of the line L_2. *[5]*

*Do **NOT** write solutions on this page.*

11. *[Maximum mark: 21]*

A random variable X has probability density function

$$f(x)=\begin{cases} ax+b, & 2\le x\le 3 \\ 0, & \text{otherwise} \end{cases},\ a,b\in\mathbb{R}.$$

(a) Show that $5a+2b=2$. *[4]*

Let $\mathrm{E}(X)=\mu$.

(b) (i) Show that $a=12\mu-30$.

(ii) Find a similar expression for b in terms of μ. *[7]*

Let the median of the distribution be 2.3.

(c) (i) Find the value of μ.

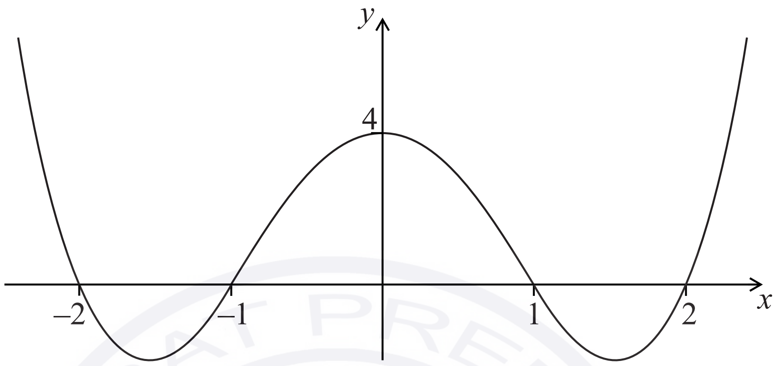
(ii) Find the value of the standard deviation of X. *[10]*

*Do **NOT** write solutions on this page.*

12. *[Maximum mark: 18]*

Let $f(x) = |x| - 1$.

(a) The graph of $y = g(x)$ is drawn below.



(i) Find the value of $(f \circ g)(1)$.

(ii) Find the value of $(f \circ g \circ g)(1)$.

(iii) Sketch the graph of $y = (f \circ g)(x)$. *[5]*

(b) (i) Sketch the graph of $y = f(x)$.

(ii) State the zeros of f. *[3]*

(c) (i) Sketch the graph of $y = (f \circ f)(x)$.

(ii) State the zeros of $f \circ f$. *[3]*

(This question continues on the following page)

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(Question 12 continued)

(d) Given that we can denote $\underbrace{f \circ f \circ f \circ \ldots \circ f}_{n \text{ times}}$ as f^n,

(i) find the zeros of f^3;

(ii) find the zeros of f^4;

(iii) deduce the zeros of f^8. *[3]*

(e) The zeros of f^{2n} are $a_1, a_2, a_3, \ldots, a_N$.

(i) State the relation between n and N;

(ii) Find, and simplify, an expression for $\sum_{r=1}^{N} |a_r|$ in terms of n. *[4]*

Please **do not** write on this page.

Answers written on this page
will not be marked.

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MATHEMATICS
HIGHER LEVEL
PAPER 2

Candidate session number

Wednesday 14 May 2014 (morning)

Examination code

2	2	1	4	–	7	2	0	6

2 hours

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SECTION A

*Answer **all** the questions in the boxes provided. Working may be continued below the lines if necessary.*

1. *[Maximum mark: 6]*

(a) (i) Find the sum of all integers, between 10 and 200, which are divisible by 7.

(ii) Express the above sum using sigma notation. *[4]*

An arithmetic sequence has first term 1000 and common difference of -6. The sum of the first n terms of this sequence is negative.

(b) Find the least value of n. *[2]*

2. *[Maximum mark: 5]*

The weights, in kg, of one-year-old bear cubs are modelled by a normal distribution with mean μ and standard deviation σ.

(a) Given that the upper quartile weight is 21.3 kg and the lower quartile weight is 17.1 kg, calculate the value of μ and the value of σ. *[4]*

A random sample of 100 of these bear cubs is selected.

(b) Find the expected number of bear cubs weighing more than 22 kg. *[1]*

3. *[Maximum mark: 5]*

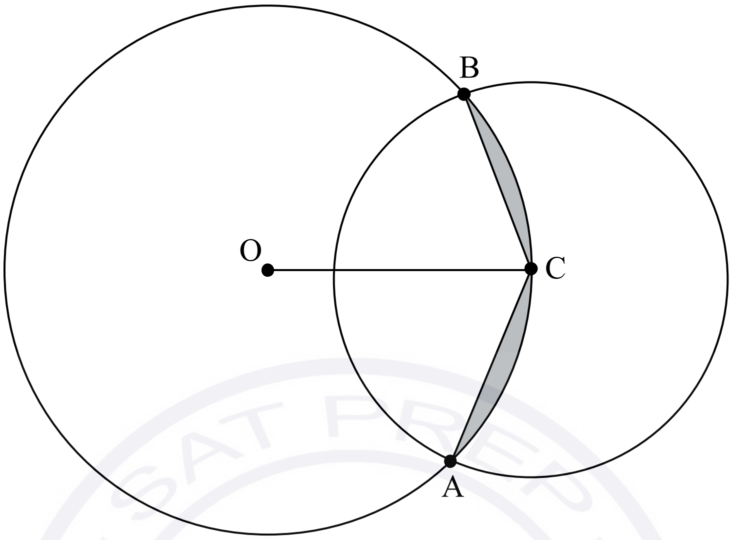
The graphs of $y = x^2 e^{-x}$ and $y = 1 - 2\sin x$ for $2 \le x \le 7$ intersect at points A and B. The x-coordinates of A and B are x_A and x_B.

(a) Find the value of x_A and the value of x_B. *[2]*

(b) Find the area enclosed between the two graphs for $x_A \le x \le x_B$. *[3]*

4. *[Maximum mark: 6]*

The following diagram shows two intersecting circles of radii 4 cm and 3 cm. The centre C of the smaller circle lies on the circumference of the bigger circle. O is the centre of the bigger circle and the two circles intersect at points A and B.



Find:

(a) $\text{B}\hat{\text{O}}\text{C}$; *[2]*

(b) the area of the shaded region. *[4]*

. . . .

5. *[Maximum mark: 6]*

Find the coefficient of x^{-2} in the expansion of $(x-1)^3\left(\frac{1}{x}+2x\right)^6$.

6. *[Maximum mark: 7]*

Six customers wait in a queue in a supermarket. A customer can choose to pay with cash or a credit card. Assume that whether or not a customer pays with a credit card is independent of any other customers' methods of payment.

It is known that 60% of customers choose to pay with a credit card.

(a) Find the probability that:

(i) the first three customers pay with a credit card and the next three pay with cash;

(ii) exactly three of the six customers pay with a credit card. *[4]*

There are n customers waiting in another queue in the same supermarket. The probability that at least one customer pays with cash is greater than 0.995.

(b) Find the minimum value of n. *[3]*

7. *[Maximum mark: 8]*

The function f is defined as $f(x) = -3 + \dfrac{1}{x-2}, x \neq 2$.

(a) (i) Sketch the graph of $y = f(x)$, clearly indicating any asymptotes and axes intercepts.

(ii) Write down the equations of any asymptotes and the coordinates of any axes intercepts. *[4]*

(b) Find the inverse function f^{-1}, stating its domain. *[4]*

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8. *[Maximum mark: 4]*

The random variable X has a Poisson distribution with mean μ.

Given that $\mathrm{P}(X=2)+\mathrm{P}(X=3)=\mathrm{P}(X=5)$,

(a) find the value of μ; *[2]*

(b) find the probability that X lies within one standard deviation of the mean. *[2]*

9. *[Maximum mark: 5]*

Sand is being poured to form a cone of height h cm and base radius r cm. The height remains equal to the base radius at all times. The height of the cone is increasing at a rate of $0.5\,\text{cm}\,\text{min}^{-1}$.

Find the rate at which sand is being poured, in $\text{cm}^3\,\text{min}^{-1}$, when the height is 4 cm.

10. *[Maximum mark: 8]*

Consider the curve with equation $\left(x^2 + y^2\right)^2 = 4xy^2$.

(a) Use implicit differentiation to find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$. *[5]*

(b) Find the equation of the normal to the curve at the point $(1, 1)$. *[3]*

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*Do **NOT** write solutions on this page.*

SECTION B

*Answer **all** questions in the answer booklet provided. Please start each question on a new page.*

11. *[Maximum mark: 13]*

The probability density function of a random variable X is defined as:

$$f(x) = \begin{cases} a\,x\cos x, & 0 \le x \le \frac{\pi}{2}, \text{ where } a \in \mathbb{R}. \\ 0, & \text{elsewhere} \end{cases}$$

(a) Show that $a = \dfrac{2}{\pi - 2}$. *[5]*

(b) Find $P\left(X < \dfrac{\pi}{4}\right)$. *[2]*

(c) Find:

(i) the mode of X;

(ii) the median of X. *[4]*

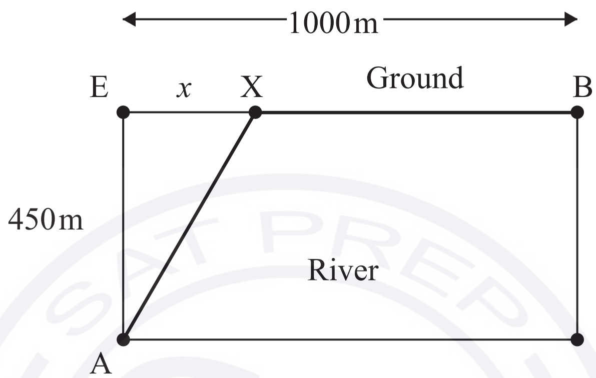
(d) Find $P\left(X < \dfrac{\pi}{8} \mid X < \dfrac{\pi}{4}\right)$. *[2]*

*Do **NOT** write solutions on this page.*

12. *[Maximum mark: 15]*

Engineers need to lay pipes to connect two cities A and B that are separated by a river of width 450 metres as shown in the following diagram. They plan to lay the pipes under the river from A to X and then under the ground from X to B. The cost of laying the pipes under the river is five times the cost of laying the pipes under the ground.

Let $\text{EX} = x$.



Let k be the cost, in dollars per metre, of laying the pipes under the ground.

(a) Show that the total cost C, in dollars, of laying the pipes from A to B is given by $C = 5k\sqrt{202500 + x^2} + (1000 - x)k$. *[2]*

(b) (i) Find $\frac{\mathrm{d}C}{\mathrm{d}x}$.

(ii) Hence find the value of x for which the total cost is a minimum, justifying that this value is a minimum. *[7]*

(c) Find the minimum total cost in terms of k. *[1]*

The angle at which the pipes are joined is $\text{A}\hat{\text{X}}\text{B} = \theta$.

(d) Find θ for the value of x calculated in (b). *[2]*

For safety reasons θ must be at least 120°.

Given this new requirement,

(e) (i) find the new value of x which minimises the total cost;

(ii) find the percentage increase in the minimum total cost. *[3]*

*Do **NOT** write solutions on this page.*

13. *[Maximum mark: 20]*

Consider $z = r(\cos\theta + \mathrm{i}\sin\theta)$, $z \in \mathbb{C}$.

(a) Use mathematical induction to prove that $z^n = r^n(\cos n\theta + \mathrm{i}\sin n\theta)$, $n \in \mathbb{Z}^+$. *[7]*

Given $u = 1 + \sqrt{3}\mathrm{i}$ and $v = 1 - \mathrm{i}$,

(b) (i) express u and v in modulus-argument form;

(ii) hence find u^3v^4. *[4]*

The complex numbers u and v are represented by point A and point B respectively on an Argand diagram.

(c) Plot point A and point B on the Argand diagram. *[1]*

Point A is rotated through $\frac{\pi}{2}$ in the anticlockwise direction about the origin O to become point A'. Point B is rotated through $\frac{\pi}{2}$ in the clockwise direction about O to become point B'.

(d) Find the area of triangle $\mathrm{OA}'\mathrm{B}'$. *[3]*

Given that u and v are roots of the equation $z^4 + bz^3 + cz^2 + dz + e = 0$, where $b, c, d, e \in \mathbb{R}$,

(e) find the values of b, c, d and e. *[5]*

*Do **NOT** write solutions on this page.*

14. *[Maximum mark: 12]*

Particle A moves such that its velocity v ms^{-1}, at time t seconds, is given by $v(t) = \frac{t}{12+t^4}$, $t \geq 0$.

(a) Sketch the graph of $y = v(t)$. Indicate clearly the local maximum and write down its coordinates. *[2]*

(b) Use the substitution $u = t^2$ to find $\int \frac{t}{12+t^4}\,\text{d}t$. *[4]*

(c) Find the exact distance travelled by particle A between $t = 0$ and $t = 6$ seconds. Give your answer in the form $k\arctan(b)$, $k, b \in \mathbb{R}$. *[3]*

Particle B moves such that its velocity v ms^{-1} is related to its displacement s m, by the equation $v(s) = \arcsin\left(\sqrt{s}\right)$.

(d) Find the acceleration of particle B when $s = 0.1\,\text{m}$. *[3]*

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Answers written on this page
will not be marked.

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MATHEMATICS
HIGHER LEVEL
PAPER 2

Candidate session number

0	0							

Tuesday 12 November 2013 (morning)

Examination code

8	8	1	3	–	7	2	0	2

2 hours

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the ***Mathematics HL and Further Mathematics SL*** *information booklet* is required for this paper.
- The maximum mark for this examination paper is *[120 marks]*.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, for example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

*Answer **all** the questions in the spaces provided. Working may be continued below the lines if necessary.*

1. *[Maximum mark: 4]*

Consider the matrices $\boldsymbol{A}=\begin{pmatrix}1 & 1 & 1\\ 2 & 0 & 1\\ 1 & -1 & 1\end{pmatrix}$ and $\boldsymbol{B}=\begin{pmatrix}2\\ -2\\ 1\end{pmatrix}$. Find the matrix $\boldsymbol{X}$ such that $\boldsymbol{AX}=\boldsymbol{B}$.

2. *[Maximum mark: 7]*

The fourth term in an arithmetic sequence is 34 and the tenth term is 76.

(a) Find the first term and the common difference. *[3]*

(b) The sum of the first n terms exceeds 5000. Find the least possible value of n. *[4]*

3. *[Maximum mark: 7]*

Consider $f(x) = \ln x - e^{\cos x}$, $0 < x \le 10$.

(a) Sketch the graph of $y = f(x)$, stating the coordinates of any maximum and minimum points and points of intersection with the x-axis. *[5]*

(b) Solve the inequality $\ln x \le e^{\cos x}$, $0 < x \le 10$. *[2]*

4. *[Maximum mark: 6]*

The duration of direct flights from London to Singapore in a particular year followed a normal distribution with mean μ and standard deviation σ.

92% of flights took under 13 hours, while only 12% of flights took under 12 hours 35 minutes.

Find μ and σ to the nearest minute.

5. *[Maximum mark: 5]*

At the start of each week, Eric and Marina pick a night at random on which they will watch a movie.

If they choose a Saturday night, the probability that they watch a French movie is $\frac{7}{9}$ and if they choose any other night the probability that they watch a French movie is $\frac{4}{9}$.

(a) Find the probability that they watch a French movie. *[3]*

(b) Given that last week they watched a French movie, find the probability that it was on a Saturday night. *[2]*

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6. *[Maximum mark: 6]*

A complex number z is given by $z = \dfrac{a+\mathrm{i}}{a-\mathrm{i}}$, $a \in \mathbb{R}$.

(a) Determine the set of values of a such that

(i) z is real;

(ii) z is purely imaginary. *[4]*

(b) Show that $|z|$ is constant for all values of a. *[2]*

7. *[Maximum mark: 6]*

The vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ are such that $\boldsymbol{a} = (3\cos\theta + 6)\boldsymbol{i} + 7\boldsymbol{j}$ and $\boldsymbol{b} = (\cos\theta - 2)\boldsymbol{i} + (1 + \sin\theta)\boldsymbol{j}$.

Given that $\boldsymbol{a}$ and $\boldsymbol{b}$ are perpendicular,

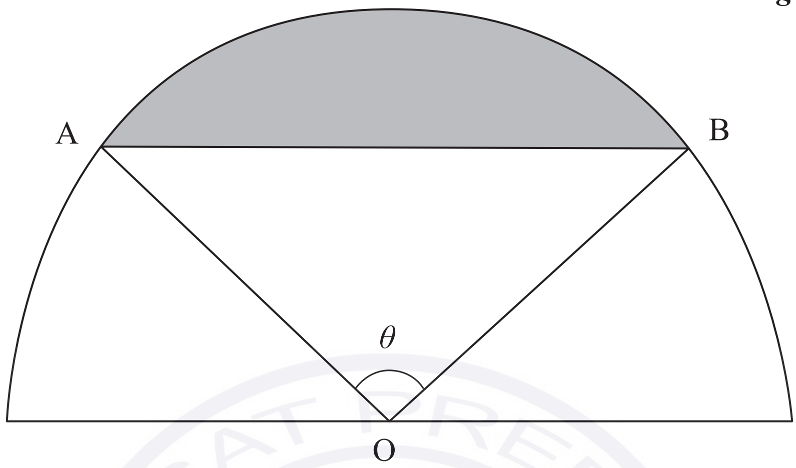
(a) show that $3\sin^2\theta - 7\sin\theta + 2 = 0$; *[3]*

(b) find the smallest possible positive value of θ. *[3]*

8. *[Maximum mark: 5]*

The diagram below shows a semi-circle of diameter 20 cm, centre O and two points A and B such that $A\hat{O}B = \theta$, where θ is in radians.

diagram not to scale



(a) Show that the shaded area can be expressed as $50\theta - 50\sin\theta$. *[2]*

(b) Find the value of θ for which the shaded area is equal to half that of the unshaded area, giving your answer correct to four significant figures. *[3]*

9. *[Maximum mark: 7]*

A line L_1 has equation $\boldsymbol{r} = \begin{pmatrix} -5 \\ -3 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 2 \\ 2 \end{pmatrix}$.

A line L_2 passing through the origin intersects L_1 and is perpendicular to L_1.

(a) Find a vector equation of L_2. *[5]*

(b) Determine the shortest distance from the origin to L_1. *[2]*

10. *[Maximum mark: 7]*

By using the substitution $x = 2\tan u$, show that $\int \frac{\mathrm{d}x}{x^2\sqrt{x^2+4}} = \frac{-\sqrt{x^2+4}}{4x} + C$.

*Do **NOT** write solutions on this page.*

SECTION B

*Answer **all** questions in the answer booklet provided. Please start each question on a new page.*

11. *[Maximum mark: 18]*

(a) The number of cats visiting Helena's garden each week follows a Poisson distribution with mean $\lambda = 0.6$.

Find the probability that

(i) in a particular week no cats will visit Helena's garden;

(ii) in a particular week at least three cats will visit Helena's garden;

(iii) over a four-week period no more than five cats in total will visit Helena's garden;

(iv) over a twelve-week period there will be exactly four weeks in which at least one cat will visit Helena's garden. *[9]*

(b) A continuous random variable X has probability distribution function f given by

$$f(x) = k \ln x \quad 1 \le x \le 3$$
$$f(x) = 0 \quad \text{otherwise}$$

(i) Find the value of k to six decimal places.

(ii) Find the value of $\mathrm{E}(X)$.

(iii) State the mode of X.

(iv) Find the median of X. *[9]*

*Do **NOT** write solutions on this page.*

12. *[Maximum mark: 20]*

(a) A particle P moves in a straight line with velocity $v\,\text{ms}^{-1}$. At time $t=0$, P is at the point O and has velocity $12\,\text{ms}^{-1}$. Its acceleration at time t seconds is given by $\frac{\text{d}v}{\text{d}t} = 3\cos\frac{t}{4}\,\text{ms}^{-2}$, $(t \ge 0)$.

(i) Find an expression for the particle's velocity v, in terms of t.

(ii) Sketch a velocity/time graph for the particle for $0 \le t \le 8\pi$, showing clearly where the curve meets the axes and any maximum or minimum points.

(iii) Find the distance travelled by the particle before first coming to rest. *[8]*

(b) Another particle Q moves in a straight line with displacement s metres and velocity $v\,\text{ms}^{-1}$. Its acceleration is given by $a = -\left(v^2+4\right)\text{ms}^{-2}$, $(0 \le t \le 1)$. At time $t=0$, Q is at the point O and has velocity $2\,\text{ms}^{-1}$.

(i) Show that the velocity v at time t is given by $v = 2\tan\left(\frac{\pi - 8t}{4}\right)$.

(ii) Show that $\frac{\text{d}v}{\text{d}s} = -\frac{\left(v^2+4\right)}{v}$.

(iii) Find the distance travelled by the particle before coming to rest. *[12]*

*Do **NOT** write solutions on this page.*

13. *[Maximum mark: 22]*

A function f is defined by $f(x) = \frac{1}{2}\left(e^{x} + e^{-x}\right)$, $x \in \mathbb{R}$.

(a) (i) Explain why the inverse function f^{-1} does not exist.

(ii) Show that the equation of the normal to the curve at the point P where $x = \ln 3$ is given by $9x + 12y - 9\ln 3 - 20 = 0$.

(iii) Find the x-coordinates of the points Q and R on the curve such that the tangents at Q and R pass through $(0, 0)$. *[14]*

(b) The domain of f is now restricted to $x \geq 0$.

(i) Find an expression for $f^{-1}(x)$.

(ii) Find the volume generated when the region bounded by the curve $y = f(x)$ and the lines $x = 0$ and $y = 5$ is rotated through an angle of 2π radians about the y-axis. *[8]*

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MATHEMATICS
HIGHER LEVEL
PAPER 2

Friday 10 May 2013 (morning)

2 hours

Candidate session number

0	0							

Examination code

2	2	1	3	–	7	2	0	4

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
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SECTION A

*Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.*

1. *[Maximum mark: 4]*

The marks obtained by a group of students in a class test are shown below.

Marks	Frequency
5	6
6	k
7	3
8	1
9	2
10	1

Given the mean of the marks is 6.5, find the value of k.

2. *[Maximum mark: 5]*

Find the value of k such that the following system of equations does not have a unique solution.

$$\begin{aligned} kx + y + 2z &= 4 \\ -y + 4z &= 5 \\ 3x + 4y + 2z &= 1 \end{aligned}$$

3. *[Maximum mark: 5]*

Emily walks to school every day. The length of time this takes can be modelled by a normal distribution with a mean of 11 minutes and a standard deviation of 3 minutes. She is late if her journey takes more than 15 minutes.

(a) Find the probability she is late next Monday. *[2 marks]*

(b) Find the probability she is late at least once during the next week (Monday to Friday). *[3 marks]*

4. *[Maximum mark: 6]*

Let $f(x) = \sin(x-1)$, $0 \le x \le \frac{\pi}{2} + 1$

Find the volume of the solid formed when the region bounded by $y = f(x)$, and the lines $x = 0$, $y = 0$ and $y = 1$ is rotated by 2π about the y-axis.

5. *[Maximum mark: 6]*

A rectangle is drawn around a sector of a circle as shown. If the angle of the sector is 1 radian and the area of the sector is $7\,\text{cm}^2$, find the dimensions of the rectangle, giving your answers to the nearest millimetre.

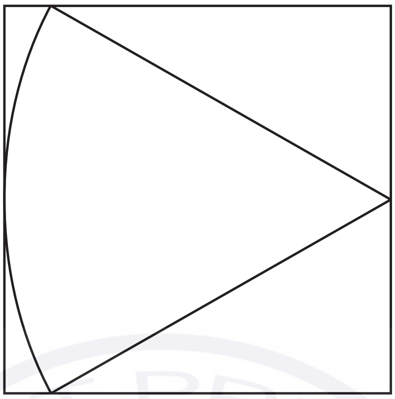


diagram not to scale

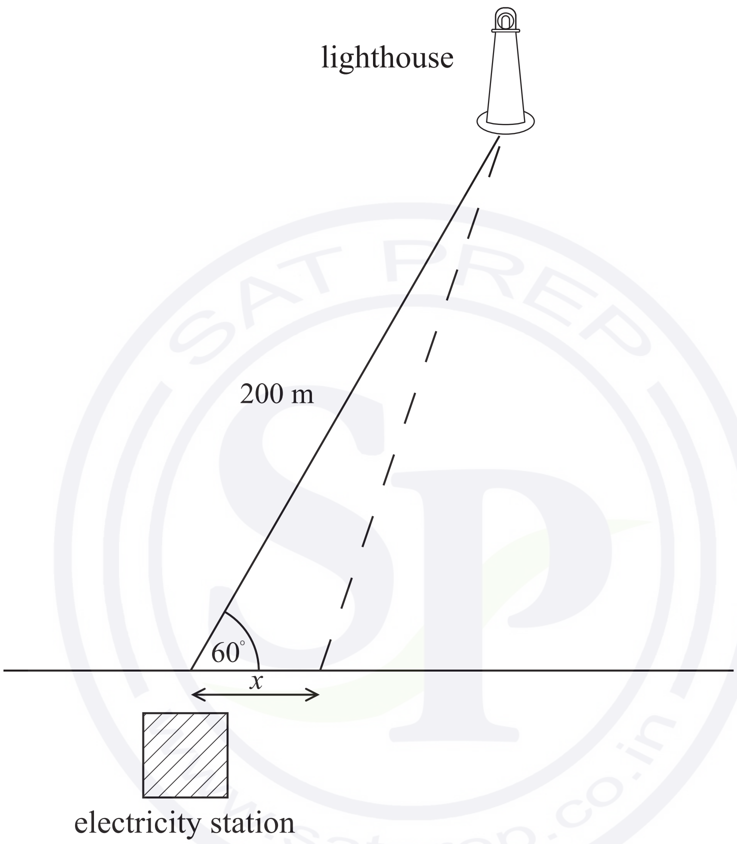
6. *[Maximum mark: 7]*

A polynomial $p(x)$ with real coefficients is of degree five. The equation $p(x) = 0$ has a complex root $2 + \mathrm{i}$. The graph of $y = p(x)$ has the x-axis as a tangent at $(2, 0)$ and intersects the coordinate axes at $(-1, 0)$ and $(0, 4)$.

Find $p(x)$ in factorised form with real coefficients.

7. *[Maximum mark: 6]*

An electricity station is on the edge of a straight coastline. A lighthouse is located in the sea 200 m from the electricity station. The angle between the coastline and the line joining the lighthouse with the electricity station is 60°. A cable needs to be laid connecting the lighthouse to the electricity station. It is decided to lay the cable in a straight line to the coast and then along the coast to the electricity station. The length of cable laid along the coastline is x metres. This information is illustrated in the diagram below.



(This question continues on the following page)

(Question 7 continued)

The cost of laying the cable along the sea bed is US$80 per metre, and the cost of laying it on land is US$20 per metre.

(a) Find, in terms of x, an expression for the cost of laying the cable. *[4 marks]*

(b) Find the value of x, to the nearest metre, such that this cost is minimized. *[2 marks]*

8. *[Maximum mark: 7]*

Three boys and three girls are to sit on a bench for a photograph.

(a) Find the number of ways this can be done if the three girls must sit together. *[3 marks]*

(b) Find the number of ways this can be done if the three girls must all sit apart. *[4 marks]*

9. *[Maximum mark: 7]*

(a) Prove that the equation $3x^2 + 2kx + k - 1 = 0$ has two distinct real roots for all values of $k \in \mathbb{R}$. *[4 marks]*

(b) Find the value of k for which the two roots of the equation are closest together. *[3 marks]*

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10. *[Maximum mark: 7]*

A ferry carries cars across a river. There is a fixed time of T minutes between crossings. The arrival of cars at the crossing can be assumed to follow a Poisson distribution with a mean of one car every four minutes. Let X denote the number of cars that arrive in T minutes.

(a) Find T, to the nearest minute, if $P(X \le 3) = 0.6$. *[3 marks]*

It is now decided that the time between crossings, T, will be 10 minutes. The ferry can carry a maximum of three cars on each trip.

(b) One day all the cars waiting at 13:00 get on the ferry. Find the probability that all the cars that arrive in the next 20 minutes will get on either the 13:10 or the 13:20 ferry. *[4 marks]*

*Do **NOT** write solutions on this page.*

SECTION B

*Answer **all** questions in the answer booklet provided. Please start each question on a new page.*

11. *[Maximum mark: 20]*

Consider the points $\text{P}(-3, -1, 2)$ and $\text{Q}(5, 5, 6)$.

(a) Find a vector equation for the line, L_1, which passes through the points P and Q. *[3 marks]*

The line L_2 has equation

$$\boldsymbol{r} = \begin{pmatrix} -4 \\ 0 \\ 4 \end{pmatrix} + s\begin{pmatrix} 5 \\ 2 \\ 0 \end{pmatrix}.$$

(b) Show that L_1 and L_2 intersect at the point $\text{R}(1, 2, 4)$. *[4 marks]*

(c) Find the acute angle between L_1 and L_2. *[3 marks]*

Let S be a point on L_2 such that $|\overrightarrow{\text{RP}}| = |\overrightarrow{\text{RS}}|$.

(d) Show that one of the possible positions for S is $\text{S}_1(-4, 0, 4)$ and find the coordinates of the other possible position, S_2. *[6 marks]*

(e) Find a vector equation of the line which passes through R and bisects $\text{P}\widehat{\text{R}}\text{S}_1$. *[4 marks]*

*Do **NOT** write solutions on this page.*

12. *[Maximum mark: 21]*

A particle, A, is moving along a straight line. The velocity, v_A ms^{-1}, of A t seconds after its motion begins is given by

$$v_A = t^3 - 5t^2 + 6t .$$

(a) Sketch the graph of $v_A = t^3 - 5t^2 + 6t$ for $t \geq 0$, with v_A on the vertical axis and t on the horizontal. Show on your sketch the local maximum and minimum points, and the intercepts with the t-axis. *[3 marks]*

(b) Write down the times for which the velocity of the particle is increasing. *[2 marks]*

(c) Write down the times for which the magnitude of the velocity of the particle is increasing. *[3 marks]*

At $t = 0$ the particle is at point O on the line.

(d) Find an expression for the particle's displacement, x_A m, from O at time t. *[3 marks]*

A second particle, B, moving along the same line, has position x_B m, velocity v_B ms^{-1} and acceleration, a_B ms^{-2}, where $a_B = -2v_B$ for $t \geq 0$. At $t = 0$, $x_B = 20$ and $v_B = -20$.

(e) Find an expression for v_B in terms of t. *[4 marks]*

(f) Find the value of t when the two particles meet. *[6 marks]*

*Do **NOT** write solutions on this page.*

13. *[Maximum mark: 19]*

The function f has inverse f^{-1} and derivative $f'(x)$ for all $x \in \mathbb{R}$. For all functions with these properties you are given the result that for $a \in \mathbb{R}$ with $b = f(a)$ and $f'(a) \neq 0$

$$(f^{-1})'(b) = \frac{1}{f'(a)}.$$

(a) Verify that this is true for $f(x) = x^3 + 1$ at $x = 2$. *[6 marks]*

(b) Given that $g(x) = xe^{x^2}$, show that $g'(x) > 0$ for all values of x. *[3 marks]*

(c) Using the result given at the start of the question, find the value of the gradient function of $y = g^{-1}(x)$ at $x = 2$. *[4 marks]*

(d) (i) With f and g as defined in parts (a) and (b), solve $g \circ f(x) = 2$.

(ii) Let $h(x) = (g \circ f)^{-1}(x)$. Find $h'(2)$. *[6 marks]*

Please **do not** write on this page.

Answers written on this page
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M13/5/MATHL/HP2/ENG/TZ2/XX

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MATHEMATICS
HIGHER LEVEL
PAPER 2

Friday 10 May 2013 (morning)

2 hours

Candidate session number

0	0							

Examination code

2	2	1	3	–	7	2	0	6

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14 pages

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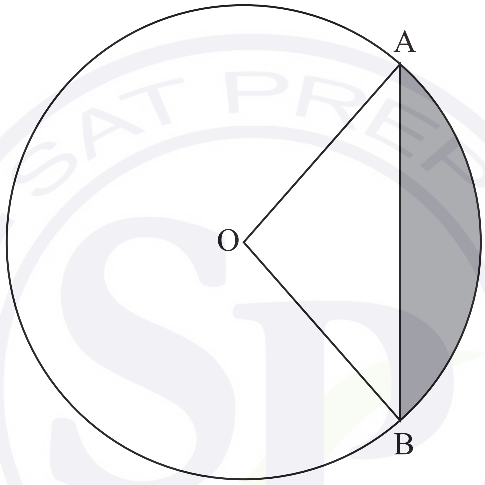
SECTION A

*Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.*

1. *[Maximum mark: 5]*

A circle of radius 4 cm, centre O, is cut by a chord [AB] of length 6 cm.

diagram not to scale



(a) Find $A\hat{O}B$, expressing your answer in radians correct to four significant figures. *[2 marks]*

(b) Determine the area of the shaded region. *[3 marks]*

2. *[Maximum mark: 5]*

Consider the system of equations

$$\begin{aligned} 0.1x - 1.7y + 0.9z &= -4.4 \\ -2.4x + 0.3y + 3.2z &= 1.2 \\ 2.5x + 0.6y - 3.7z &= 0.8\,. \end{aligned}$$

(a) Express the system of equations in matrix form. *[2 marks]*

(b) Find the solution to the system of equations. *[3 marks]*

3. *[Maximum mark: 5]*

It is believed that the lifespans of Manx cats are normally distributed with a mean of 13.5 years and a variance of 9.5 years^2.

(a) Calculate the range of lifespans of Manx cats whose lifespans are within one standard deviation of the mean. *[2 marks]*

(b) Estimate the number of Manx cats in a population of 10 000 that will have a lifespan of less than 10 years. Give your answer to the nearest whole number. *[3 marks]*

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4. *[Maximum mark: 6]*

(a) Find $\int x\sec^2 x \, dx$. *[4 marks]*

(b) Determine the value of m if $\int_0^m x\sec^2 x \, dx = 0.5$, where $m > 0$. *[2 marks]*

5. *[Maximum mark: 6]*

The arithmetic sequence $\{u_n : n \in \mathbb{Z}^+\}$ has first term $u_1 = 1.6$ and common difference $d = 1.5$. The geometric sequence $\{v_n : n \in \mathbb{Z}^+\}$ has first term $v_1 = 3$ and common ratio $r = 1.2$.

(a) Find an expression for $u_n - v_n$ in terms of n. *[2 marks]*

(b) Determine the set of values of n for which $u_n > v_n$. *[3 marks]*

(c) Determine the greatest value of $u_n - v_n$. Give your answer correct to four significant figures. *[1 mark]*

6. *[Maximum mark: 6]*

(a) Solve the equation $3\cos^2 x - 8\cos x + 4 = 0$, where $0 \le x \le 180^\circ$, expressing your answer(s) to the nearest degree. *[3 marks]*

(b) Find the exact values of $\sec x$ satisfying the equation $3\sec^4 x - 8\sec^2 x + 4 = 0$. *[3 marks]*

7. *[Maximum mark: 7]*

The length, X metres, of a species of fish has the probability density function

$$f(x) = \begin{cases} ax^2, \text{ for } 0 \le x \le 0.5 \\ 0.5a(1-x), \text{ for } 0.5 \le x \le 1 \\ 0, \text{ otherwise}. \end{cases}$$

(a) Show that $a = 9.6$. *[3 marks]*

(b) Sketch the graph of the distribution. *[2 marks]*

(c) Find $\mathrm{P}(X < 0.6)$. *[2 marks]*

8. *[Maximum mark: 7]*

Use the method of mathematical induction to prove that $5^{2n} - 24n - 1$ is divisible by 576 for $n \in \mathbb{Z}^+$.

9. *[Maximum mark: 7]*

A small car hire company has two cars. Each car can be hired for one whole day at a time. The rental charge is US$60 per car per day. The number of requests to hire a car for one whole day may be modelled by a Poisson distribution with mean 1.2.

(a) Find the probability that on a particular weekend, three requests are received on Saturday and none are received on Sunday. *[2 marks]*

Over a weekend of two days, it is given that a total of three requests are received.

(b) Find the expected total rental income for the weekend. *[5 marks]*

10. *[Maximum mark: 6]*

The acceleration of a car is $\frac{1}{40}(60-v)$ m s^{-2}, when its velocity is v m s^{-1}. Given the car starts from rest, find the velocity of the car after 30 seconds.

*Do **NOT** write solutions on this page.*

SECTION B

*Answer **all** questions on the answer booklet provided. Please start each question on a new page.*

11. *[Maximum mark: 19]*

(a) (i) Express the sum of the first n positive odd integers using sigma notation.

(ii) Show that the sum stated above is n^2.

(iii) Deduce the value of the difference between the sum of the first 47 positive odd integers and the sum of the first 14 positive odd integers. *[4 marks]*

(b) A number of distinct points are marked on the circumference of a circle, forming a polygon. Diagonals are drawn by joining all pairs of non-adjacent points.

(i) Show on a diagram all diagonals if there are 5 points.

(ii) Show that the number of diagonals is $\frac{n(n-3)}{2}$ if there are n points, where $n > 2$.

(iii) Given that there are more than one million diagonals, determine the least number of points for which this is possible. *[7 marks]*

(c) The random variable $X \sim B(n, p)$ has mean 4 and variance 3.

(i) Determine n and p.

(ii) Find the probability that in a single experiment the outcome is 1 or 3. *[8 marks]*

*Do **NOT** write solutions on this page.*

12. *[Maximum mark: 22]*

Consider the differential equation $y\frac{\mathrm{d}y}{\mathrm{d}x} = \cos 2x$.

(a) (i) Show that the function $y = \cos x + \sin x$ satisfies the differential equation.

(ii) Find the general solution of the differential equation. Express your solution in the form $y = f(x)$, involving a constant of integration.

(iii) For which value of the constant of integration does your solution coincide with the function given in part (i)? *[10 marks]*

(b) A different solution of the differential equation, satisfying $y = 2$ when $x = \frac{\pi}{4}$, defines a curve C.

(i) Determine the equation of C in the form $y = g(x)$, and state the range of the function g.

A region R in the xy plane is bounded by C, the x-axis and the vertical lines $x = 0$ and $x = \frac{\pi}{2}$.

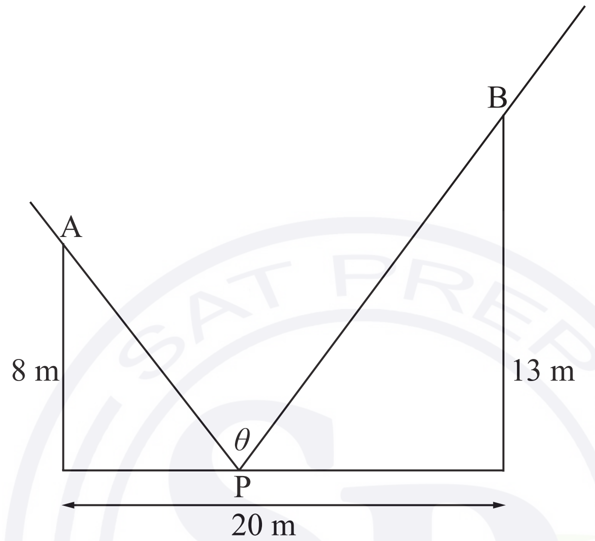
(ii) Find the area of R.

(iii) Find the volume generated when that part of R above the line $y = 1$ is rotated about the x-axis through 2π radians. *[12 marks]*

*Do **NOT** write solutions on this page.*

13. *[Maximum mark: 19]*

A straight street of width 20 metres is bounded on its parallel sides by two vertical walls, one of height 13 metres, the other of height 8 metres. The intensity of light at point P at ground level on the street is proportional to the angle θ where $\theta = \text{A}\hat{\text{P}}\text{B}$, as shown in the diagram.



(a) Find an expression for θ in terms of x, where x is the distance of P from the base of the wall of height 8m. *[2 marks]*

(b) (i) Calculate the value of θ when $x = 0$.

(ii) Calculate the value of θ when $x = 20$. *[2 marks]*

(c) Sketch the graph of θ, for $0 \le x \le 20$. *[2 marks]*

(d) Show that $\dfrac{\mathrm{d}\theta}{\mathrm{d}x} = \dfrac{5(744 - 64x - x^2)}{(x^2 + 64)(x^2 - 40x + 569)}$. *[6 marks]*

(e) Using the result in part (d), or otherwise, determine the value of x corresponding to the maximum light intensity at P. Give your answer to four significant figures. *[3 marks]*

(f) The point P moves across the street with speed $0.5\ \text{m s}^{-1}$. Determine the rate of change of θ with respect to time when P is at the midpoint of the street. *[4 marks]*

Please **do not** write on this page.

Answers written on this page
will not be marked.

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N12/5/MATHL/HP2/ENG/TZ0/XX

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MATHEMATICS
HIGHER LEVEL
PAPER 2

Candidate session number

0	0							

Wednesday 7 November 2012 (morning)

Examination code

8	8	1	2	–	7	2	0	2

2 hours

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions on the answer sheets provided. Write your session number on each answer sheet, and attach them to this examination paper and your cover sheet using the tag provided.
- At the end of the examination, indicate the number of sheets used in the appropriate box on your cover sheet.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the ***Mathematics HL and Further Mathematics SL*** *information booklet* is required for this paper.
- The maximum mark for this examination paper is *[120 marks]*.

0116

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SECTION A

*Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.*

1. *[Maximum mark: 4]*

Find the sum of all the multiples of 3 between 100 and 500.

2. *[Maximum mark: 4]*

Show that the quadratic equation $x^2 - (5-k)x - (k+2) = 0$ has two distinct real roots for all real values of k.

3. *[Maximum mark: 5]*

Consider the matrix $\boldsymbol{A} = \begin{pmatrix} \ln x & \ln(5-x) \\ 2 & 3 \end{pmatrix}$, where $0 < x < 5$. Find the value of x for which $\boldsymbol{A}$ is singular.

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4. *[Maximum mark: 6]*

A set of 15 observations has mean 11.5 and variance 9.3. One observation of 22.1 is considered unreliable and is removed. Find the mean and variance of the remaining 14 observations.

5. *[Maximum mark: 6]*

A metal rod 1 metre long is cut into 10 pieces, the lengths of which form a geometric sequence. The length of the longest piece is 8 times the length of the shortest piece. Find, to the nearest millimetre, the length of the shortest piece.

6. *[Maximum mark: 6]*

A particle moves along a straight line so that after t seconds its displacement s, in metres, satisfies the equation $s^2 + s - 2t = 0$. Find, in terms of s, expressions for its velocity and its acceleration.

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7. *[Maximum mark: 8]*

Kathy plays a computer game in which she has to find the path through a maze within a certain time. The first time she attempts the game, the probability of success is known to be 0.75. In subsequent attempts, if Kathy is successful, the difficulty increases and the probability of success is half the probability of success on the previous attempt. However, if she is unsuccessful, the probability of success remains the same. Kathy plays the game three times consecutively.

(a) Find the probability that she is successful in all three games. *[2 marks]*

(b) Assuming that she is successful in the first game, find the probability that she is successful in exactly two games. *[6 marks]*

8. *[Maximum mark: 7]*

By using the substitution $x = \sin t$, find $\int \frac{x^3}{\sqrt{1-x^2}}\,dx$.

9. *[Maximum mark: 7]*

Find the area of the region enclosed by the curves $y = x^3$ and $x = y^2 - 3$.

10. *[Maximum mark: 7]*

Let $\omega = \cos\theta + \mathrm{i}\sin\theta$. Find, in terms of θ, the modulus and argument of $(1-\omega^2)^*$.

*Do **NOT** write solutions on this page.*

SECTION B

*Answer **all** questions on the answer sheets provided. Please start each question on a new page.*

11. *[Maximum mark: 18]*

The number of visitors that arrive at a museum every minute can be modelled by a Poisson distribution with mean 2.2.

(a) If the museum is open 6 hours daily, find the expected number of visitors in 1 day. *[2 marks]*

(b) Find the probability that the number of visitors arriving during an hour exceeds 100. *[3 marks]*

(c) Find the probability that the number of visitors in each of the 6 hours the museum is open exceeds 100. *[2 marks]*

The ages of the visitors to the museum can be modelled by a normal distribution with mean μ and variance σ^2. The records show that 29 % of the visitors are under 35 years of age and 23 % are at least 55 years of age.

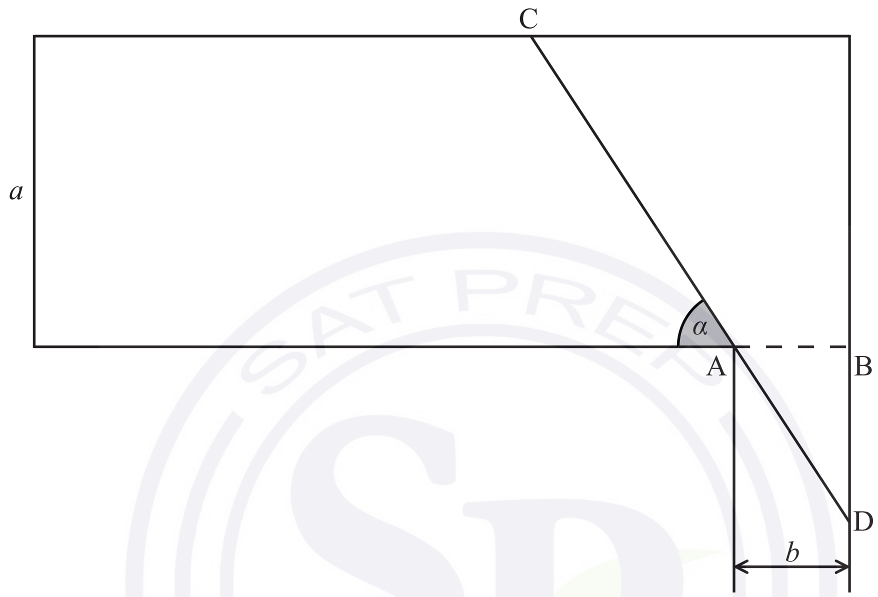
(d) Find the values of μ and σ. *[6 marks]*

(e) One day, 100 visitors under 35 years of age come to the museum. Estimate the number of visitors under 50 years of age that were at the museum on that day. *[5 marks]*

*Do **NOT** write solutions on this page.*

12. *[Maximum mark: 18]*

The diagram shows the plan of an art gallery a metres wide. [AB] represents a doorway, leading to an exit corridor b metres wide. In order to remove a painting from the art gallery, CD (denoted by L) is measured for various values of α, as represented in the diagram.



(a) If α is the angle between [CD] and the wall, show that $L = \frac{a}{\sin\alpha} + \frac{b}{\cos\alpha}$, $0 < \alpha < \frac{\pi}{2}$. *[3 marks]*

(b) If $a = 5$ and $b = 1$, find the maximum length of a painting that can be removed through this doorway. *[4 marks]*

Let $a = 3k$ and $b = k$.

(c) Find $\frac{dL}{d\alpha}$. *[3 marks]*

(d) Find, in terms of k, the maximum length of a painting that can be removed from the gallery through this doorway. *[6 marks]*

(e) Find the minimum value of k if a painting 8 metres long is to be removed through this doorway. *[2 marks]*

*Do **NOT** write solutions on this page.*

13. *[Maximum mark: 24]*

Consider the planes $\pi_1 : x - 2y - 3z = 2$ and $\pi_2 : 2x - y - z = k$.

(a) Find the angle between the planes π_1 and π_2. *[4 marks]*

(b) The planes π_1 and π_2 intersect in the line L_1. Show that the vector equation of L_1 is $\boldsymbol{r} = \begin{pmatrix} 0 \\ 2-3k \\ 2k-2 \end{pmatrix} + t\begin{pmatrix} 1 \\ 5 \\ -3 \end{pmatrix}$. *[5 marks]*

(c) The line L_2 has Cartesian equation $5 - x = y + 3 = 2 - 2z$. The lines L_1 and L_2 intersect at a point X. Find the coordinates of X. *[5 marks]*

(d) Determine a Cartesian equation of the plane π_3 containing both lines L_1 and L_2. *[5 marks]*

(e) Let Y be a point on L_1 and Z be a point on L_2 such that XY is perpendicular to YZ and the area of the triangle XYZ is 3. Find the perimeter of the triangle XYZ. *[5 marks]*

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MATHEMATICS
HIGHER LEVEL
PAPER 2

Friday 4 May 2012 (morning)

2 hours

Candidate session number

0	0							

Examination code

2	2	1	2	–	7	2	0	4

INSTRUCTIONS TO CANDIDATES

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0116

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SECTION A

*Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.*

1. *[Maximum mark: 5]*

Given that the graph of $y = x^3 - 6x^2 + kx - 4$ has exactly one point at which the gradient is zero, find the value of k.

2. *[Maximum mark: 4]*

The probability density function of the continuous random variable X is given by

$$f(x)=\begin{cases} k2^{\frac{1}{x}}, & 1\le x\le 2 \\ 0, & \text{otherwise} \end{cases}$$

where k is a constant. Find the expected value of X.

3. *[Maximum mark: 7]*

A team of 6 players is to be selected from 10 volleyball players, of whom 8 are boys and 2 are girls.

(a) In how many ways can the team be selected? *[2 marks]*

(b) In how many of these selections is exactly one girl in the team? *[3 marks]*

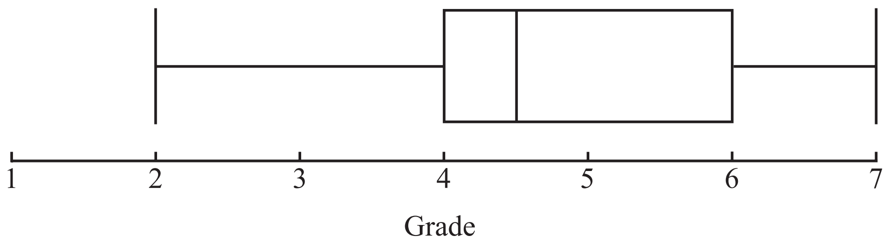
(c) If the selection of the team is made at random, find the probability that exactly one girl is in the team. *[2 marks]*

4. *[Maximum mark: 5]*

The planes $2x+3y-z=5$ and $x-y+2z=k$ intersect in the line $5x+1=9-5y=-5z$.
Find the value of k.

5. *[Maximum mark: 5]*

The box and whisker plot below illustrates the IB grades obtained by 100 students.



IB grades can only take integer values.

(a) How many students obtained a grade of more than 4? *[1 mark]*

(b) State, with reasons, the maximum possible number and minimum possible number of students who obtained a 4 in the exam. *[4 marks]*

6. *[Maximum mark: 5]*

Let $f(x) = \ln x$. The graph of f is transformed into the graph of the function g by a translation of $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$, followed by a reflection in the x-axis. Find an expression for $g(x)$, giving your answer as a single logarithm.

7. *[Maximum mark: 5]*

A fisherman notices that in any hour of fishing, he is equally likely to catch exactly two fish, as he is to catch less than two fish. Assuming the number of fish caught can be modelled by a Poisson distribution, calculate the expected value of the number of fish caught when he spends four hours fishing.

8. *[Maximum mark: 9]*

A cone has height h and base radius r. Deduce the formula for the volume of this cone by rotating the triangular region, enclosed by the line $y = h - \frac{h}{r}x$ and the coordinate axes, through 2π about the y-axis.

9. *[Maximum mark: 7]*

Find the constant term in the expansion of $\left(x-\frac{2}{x}\right)^4\left(x^2+\frac{2}{x}\right)^3$.

10. *[Maximum mark: 8]*

A triangle is formed by the three lines $y = 10 - 2x$, $y = mx$ and $y = -\frac{1}{m}x$, where $m > \frac{1}{2}$.

Find the value of m for which the area of the triangle is a minimum.

*Do **NOT** write solutions on this page.*

SECTION B

*Answer **all** questions on the answer sheets provided. Please start each question on a new page.*

11. *[Maximum mark: 14]*

The function $f(x) = 3\sin x + 4\cos x$ is defined for $0 < x < 2\pi$.

(a) Write down the coordinates of the minimum point on the graph of f. *[1 mark]*

(b) The points $\mathrm{P}(p, 3)$ and $\mathrm{Q}(q, 3)$, $q > p$, lie on the graph of $y = f(x)$. Find p and q. *[2 marks]*

(c) Find the coordinates of the point, on $y = f(x)$, where the gradient of the graph is 3. *[4 marks]*

(d) Find the coordinates of the point of intersection of the normals to the graph at the points P and Q. *[7 marks]*

12. *[Maximum mark: 22]*

A ski resort finds that the mean number of accidents on any given weekday (Monday to Friday) is 2.2. The number of accidents can be modelled by a Poisson distribution.

(a) Find the probability that in a certain week (Monday to Friday only)

(i) there are fewer than 12 accidents;

(ii) there are more than 8 accidents, given that there are fewer than 12 accidents. *[6 marks]*

Due to the increased usage, it is found that the probability of more than 3 accidents in a day at the weekend (Saturday and Sunday) is 0.24.

(b) Assuming a Poisson model,

(i) calculate the mean number of accidents per day at the weekend (Saturday and Sunday);

(ii) calculate the probability that, in the four weekends in February, there will be more than 5 accidents during at least two of the weekends. *[10 marks]*

It is found that 20 % of skiers having accidents are at least 25 years of age and 40 % are under 18 years of age.

(c) Assuming that the ages of skiers having accidents are normally distributed, find the mean age of skiers having accidents. *[6 marks]*

*Do **NOT** write solutions on this page.*

13. *[Maximum mark: 24]*

The coordinates of points A, B and C are given as $(5, -2, 5)$, $(5, 4, -1)$ and $(-1, -2, -1)$ respectively.

(a) Show that $\text{AB} = \text{AC}$ and that $\text{B}\hat{\text{A}}\text{C} = 60^\circ$. *[4 marks]*

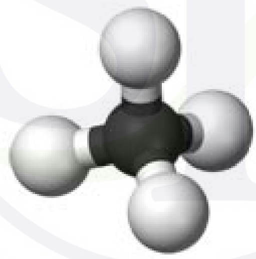
(b) Find the Cartesian equation of Π, the plane passing through A, B, and C. *[4 marks]*

(c) (i) Find the Cartesian equation of Π_1, the plane perpendicular to (AB) passing through the midpoint of [AB].

(ii) Find the Cartesian equation of Π_2, the plane perpendicular to (AC) passing through the midpoint of [AC]. *[4 marks]*

(d) Find the vector equation of L, the line of intersection of Π_1 and Π_2, and show that it is perpendicular to Π. *[3 marks]*

A methane molecule consists of a carbon atom with four hydrogen atoms symmetrically placed around it in three dimensions.



The positions of the centres of three of the hydrogen atoms are A, B and C as given. The position of the centre of the fourth hydrogen atom is D.

(e) Using the fact that $\text{AB} = \text{AD}$, show that the coordinates of one of the possible positions of the fourth hydrogen atom is $(-1, 4, 5)$. *[3 marks]*

(f) Letting D be $(-1, 4, 5)$, show that the coordinates of G, the position of the centre of the carbon atom, are $(2, 1, 2)$. Hence calculate $\text{D}\hat{\text{G}}\text{A}$, the bonding angle of carbon. *[6 marks]*

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MATHEMATICS
HIGHER LEVEL
PAPER 2

Friday 4 May 2012 (morning)

2 hours

Candidate session number

0	0							

Examination code

2	2	1	2	–	7	2	0	6

INSTRUCTIONS TO CANDIDATES

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0116

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SECTION A

*Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.*

1. *[Maximum mark: 7]*

The sum of the first 16 terms of an arithmetic sequence is 212 and the fifth term is 8.

(a) Find the first term and the common difference. *[4 marks]*

(b) Find the smallest value of n such that the sum of the first n terms is greater than 600. *[3 marks]*

2. *[Maximum mark: 5]*

The random variable X has the distribution $B(30,\ p)$. Given that $E(X) = 10$, find

(a) the value of p; *[1 mark]*

(b) $P(X = 10)$; *[2 marks]*

(c) $P(X \geq 15)$. *[2 marks]*

3. *[Maximum mark: 8]*

Consider a triangle ABC with $\text{B}\hat{\text{A}}\text{C} = 45.7^\circ$, $\text{AB} = 9.63 \text{ cm}$ and $\text{BC} = 7.5 \text{ cm}$.

(a) By drawing a diagram, show why there are two triangles consistent with this information. *[2 marks]*

(b) Find the possible values of AC. *[6 marks]*

4. *[Maximum mark: 6]*

Fifteen boys and ten girls sit in a single line.

(a) In how many ways can they be seated in a single line so that the boys and girls are in two separate groups? *[3 marks]*

(b) Two boys and three girls are selected to go the theatre. In how many ways can this selection be made? *[3 marks]*

5. *[Maximum mark: 5]*

The random variable X has the distribution $\mathrm{Po}(m)$.
Given that $\mathrm{P}(X = 5) = \mathrm{P}(X = 3) + \mathrm{P}(X = 4)$, find

(a) the value of m; *[3 marks]*

(b) $\mathrm{P}(X > 2)$. *[2 marks]*

6. *[Maximum mark: 8]*

(a) Sketch the curve $y = \dfrac{\cos x}{\sqrt{x^2+1}}$, $-4 \le x \le 4$ showing clearly the coordinates of the x-intercepts, any maximum points and any minimum points. *[4 marks]*

(b) Write down the gradient of the curve at $x = 1$. *[1 mark]*

(c) Find the equation of the normal to the curve at $x = 1$. *[3 marks]*

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7. *[Maximum mark: 5]*

The probability density function of a continuous random variable X is given by

$$f(x) = \frac{1}{1+x^4}, \; 0 \le x \le a.$$

(a) Find the value of a. *[3 marks]*

(b) Find the mean of X. *[2 marks]*

8. *[Maximum mark: 8]*

Each time a ball bounces, it reaches 95 % of the height reached on the previous bounce. Initially, it is dropped from a height of 4 metres.

(a) What height does the ball reach after its fourth bounce? *[2 marks]*

(b) How many times does the ball bounce before it no longer reaches a height of 1 metre? *[3 marks]*

(c) What is the total distance travelled by the ball? *[3 marks]*

9. *[Maximum mark: 8]*

Two discs, one of radius 8 cm and one of radius 5 cm, are placed such that they touch each other. A piece of string is wrapped around the discs. This is shown in the diagram below.

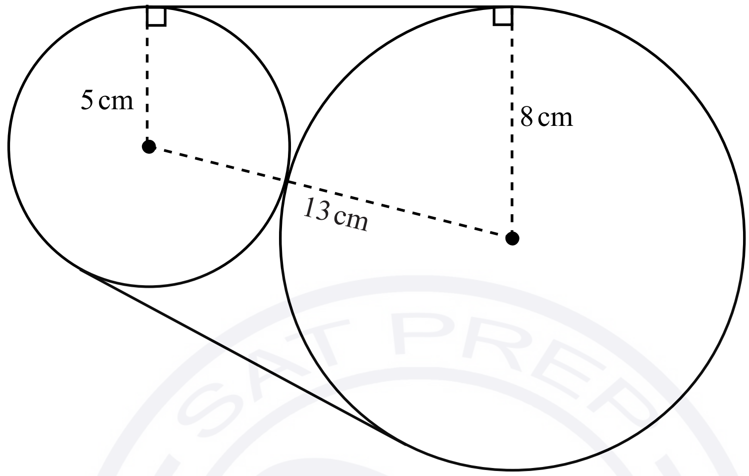

diagram not to scale

Calculate the length of string needed to go around the discs.

*Do **NOT** write solutions on this page.*

SECTION B

*Answer **all** questions on the answer sheets provided. Please start each question on a new page.*

10. *[Maximum mark: 14]*

A market stall sells apples, pears and plums.

(a) The weights of the apples are normally distributed with a mean of 200 grams and a standard deviation of 25 grams.

(i) Given that there are 450 apples on the stall, what is the expected number of apples with a weight of more than 225 grams?

(ii) Given that 70 % of the apples weigh less than m grams, find the value of m. *[5 marks]*

(b) The weights of the pears are normally distributed with a mean of $\propto$ grams and a standard deviation of σ grams. Given that 8 % of these pears have a weight of more than 270 grams and 15 % have a weight less than 250 grams, find $\propto$ and σ. *[6 marks]*

(c) The weights of the plums are normally distributed with a mean of 80 grams and a standard deviation of 4 grams. 5 plums are chosen at random. What is the probability that exactly 3 of them weigh more than 82 grams? *[3 marks]*

*Do **NOT** write solutions on this page.*

11. *[Maximum mark: 24]*

(a) Find the values of k for which the following system of equations has no solutions and the value of k for the system to have an infinite number of solutions.

$$x - 3y + z = 3$$
$$x + 5y - 2z = 1$$
$$16y - 6z = k$$

[5 marks]

(b) Given that the system of equations can be solved, find the solutions in the form of a vector equation of a line, $\boldsymbol{r} = \boldsymbol{a} + \lambda \boldsymbol{b}$, where the components of $\boldsymbol{b}$ are integers. *[7 marks]*

(c) The plane ÷ is parallel to both the line in part (b) and the line $\frac{x-4}{3} = \frac{y-6}{-2} = \frac{z-2}{0}$.

Given that ÷ contains the point $(1, 2, 0)$, show that the Cartesian equation of ÷ is $16x + 24y - 11z = 64$. *[5 marks]*

(d) The z-axis meets the plane ÷ at the point P. Find the coordinates of P. *[2 marks]*

(e) Find the angle between the line $\frac{x-2}{3} = \frac{y+5}{4} = \frac{z}{2}$ and the plane ÷ . *[5 marks]*

*Do **NOT** write solutions on this page.*

12. *[Maximum mark: 22]*

A particle moves in a straight line with velocity v metres per second. At any time t seconds, $0 \le t < \frac{3\pi}{4}$, the velocity is given by the differential equation $\frac{dv}{dt} + v^2 + 1 = 0$. It is also given that $v = 1$ when $t = 0$.

(a) Find an expression for v in terms of t. *[7 marks]*

(b) Sketch the graph of v against t, clearly showing the coordinates of any intercepts, and the equations of any asymptotes. *[3 marks]*

(c) (i) Write down the time T at which the velocity is zero.

(ii) Find the distance travelled in the interval $[0, T]$. *[3 marks]*

(d) Find an expression for s, the displacement, in terms of t, given that $s = 0$ when $t = 0$. *[5 marks]*

(e) Hence, or otherwise, show that $s = \frac{1}{2}\ln\frac{2}{1+v^2}$. *[4 marks]*

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N11/5/MATHL/HP2/ENG/TZ0/XX

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International Baccalaureate®
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MATHEMATICS
HIGHER LEVEL
PAPER 2

Thursday 3 November 2011 (morning)

2 hours

Candidate session number

0	0							

INSTRUCTIONS TO CANDIDATES

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SECTION A

*Answer **all** the questions in the spaces provided. Working may be continued below the lines, if necessary.*

1. *[Maximum mark: 5]*

Consider the graph of $y = x + \sin(x - 3)$, $-\pi \le x \le \pi$.

(a) Sketch the graph, clearly labelling the x and y intercepts with their values. *[3 marks]*

(b) Find the area of the region bounded by the graph and the x and y axes. *[2 marks]*

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2. *[Maximum mark: 7]*

Given the following system of linear equations,

$$ax + y + z = 1$$
$$x + ay + z = a$$
$$x + y + az = a^2$$

find the values of the real constant, a, for which the system has a unique solution.

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3. *[Maximum mark: 5]*

The number of vehicles passing a particular junction can be modelled using the Poisson distribution. Vehicles pass the junction at an average rate of 300 per hour.

(a) Find the probability that no vehicles pass in a given minute. *[2 marks]*

(b) Find the expected number of vehicles which pass in a given two minute period. *[1 mark]*

(c) Find the probability that more than this expected number actually pass in a given two minute period. *[2 marks]*

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4. *[Maximum mark: 5]*

(a) Given that $\arctan\frac{1}{2} - \arctan\frac{1}{3} = \arctan a$, $a \in \mathbb{Q}^+$, find the value of a. *[3 marks]*

(b) Hence, or otherwise, solve the equation $\arcsin x = \arctan a$. *[2 marks]*

5. *[Maximum mark: 5]*

The probability that the 08:00 train will be delayed on a work day (Monday to Friday) is $\frac{1}{10}$. Assuming that delays occur independently,

(a) find the probability that the 08:00 train is delayed exactly twice during any period of five work days; *[2 marks]*

(b) find the minimum number of work days for which the probability of the 08:00 train being delayed at least once exceeds 90 %. *[3 marks]*

6. *[Maximum mark: 7]*

The complex numbers z_1 and z_2 have arguments between 0 and π radians. Given that $z_1 z_2 = -\sqrt{3} + \mathrm{i}$ and $\frac{z_1}{z_2} = 2\mathrm{i}$, find the modulus and argument of z_1 and of z_2.

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7. *[Maximum mark: 6]*

(a) Find the set of values of x for which the series $\sum_{n=1}^{\infty}\left(\frac{2x}{x+1}\right)^n$ has a finite sum. *[4 marks]*

(b) Hence find the sum in terms of x. *[2 marks]*

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8. *[Maximum mark: 7]*

Given that $f(x) = \dfrac{1}{1 + e^{-x}}$,

(a) find $f^{-1}(x)$, stating its domain; *[6 marks]*

(b) find the value of x such that $f(x) = f^{-1}(x)$. *[1 mark]*

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9. *[Maximum mark: 6]*

A stalactite has the shape of a circular cone. Its height is 200 mm and is increasing at a rate of 3 mm per century. Its base radius is 40 mm and is decreasing at a rate of 0.5 mm per century. Determine if its volume is increasing or decreasing, and the rate at which the volume is changing.

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10. *[Maximum mark: 7]*

Given that $z = \frac{2-\text{i}}{1+\text{i}} - \frac{6+8\text{i}}{u+\text{i}}$, find the values of u, $u \in \mathbb{R}$, such that $\text{Re}\,z = \text{Im}\,z$.

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*Do **NOT** write solutions on this page. Any working on this page will **NOT** be marked.*

SECTION B

*Answer **all** the questions on the answer sheets provided. Please start each question on a new page.*

11. *[Maximum mark: 15]*

Jan and Sia have been selected to represent their country at an international discus throwing competition. Assume that the distance thrown by each athlete is normally distributed. The mean distance thrown by Jan in the past year was 60.33 metres with a standard deviation of 1.95 metres.

(a) In the past year, 80 % of Jan's throws have been longer than x metres. Find x correct to two decimal places. *[2 marks]*

(b) In the past year, 80 % of Sia's throws have been longer than 56.52 metres. If the mean distance of her throws was 59.39 metres, find the standard deviation of her throws. *[3 marks]*

(c) This year, Sia's throws have a mean of 59.50 metres and a standard deviation of 3.00 metres. The mean and standard deviation of Jan's throws have remained the same. In the competition, an athlete must have at least one throw of 65 metres or more in the first round to qualify for the final round. Each athlete is allowed three throws in the first round.

(i) Determine whether Jan or Sia is more likely to qualify for the final on their first throw.

(ii) Find the probability that both athletes qualify for the final. *[10 marks]*

12. *[Maximum mark: 16]*

(a) In an arithmetic sequence the first term is 8 and the common difference is $\frac{1}{4}$. If the sum of the first $2n$ terms is equal to the sum of the next n terms, find n. *[9 marks]*

(b) If $a_1, a_2, a_3, \ldots$ are terms of a geometric sequence with common ratio $r \neq 1$, show that $(a_1 - a_2)^2 + (a_2 - a_3)^2 + (a_3 - a_4)^2 + \ldots + (a_n - a_{n+1})^2 = \frac{a_1^{\,2}(1-r)(1-r^{2n})}{1+r}$. *[7 marks]*

*Do **NOT** write solutions on this page. Any working on this page will **NOT** be marked.*

13. *[Maximum mark: 16]*

Two planes Π_1 and Π_2 have equations $2x + y + z = 1$ and $3x + y - z = 2$ respectively.

(a) Find the vector equation of L, the line of intersection of Π_1 and Π_2. *[6 marks]*

(b) Show that the plane Π_3 which is perpendicular to Π_1 and contains L, has equation $x - 2z = 1$. *[4 marks]*

(c) The point P has coordinates $(-2, 4, 1)$, the point Q lies on Π_3 and PQ is perpendicular to Π_2. Find the coordinates of Q. *[6 marks]*

14. *[Maximum mark: 13]*

(a) Show that $\left| e^{i\theta} \right| = 1$. *[1 mark]*

Consider the geometric series $1 + \frac{1}{3}e^{i\theta} + \frac{1}{9}e^{2i\theta} + \ldots$.

(b) Write down the common ratio, z, of the series, and show that $|z| = \frac{1}{3}$. *[2 marks]*

(c) Find an expression for the sum to infinity of this series. *[2 marks]*

(d) Hence, show that $\sin\theta + \frac{1}{3}\sin 2\theta + \frac{1}{9}\sin 3\theta + \ldots = \frac{9\sin\theta}{10 - 6\cos\theta}$. *[8 marks]*