Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Standard level
Paper 1

Monday 18 November 2019 (afternoon)

Candidate session number

1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics SL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[90 marks]**.

8819 – 7301

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines if necessary.

1. [Maximum mark: 6]

In an arithmetic sequence, $u_2 = 5$ and $u_3 = 11$.

(a) Find the common difference. [2]

(b) Find the first term. [2]

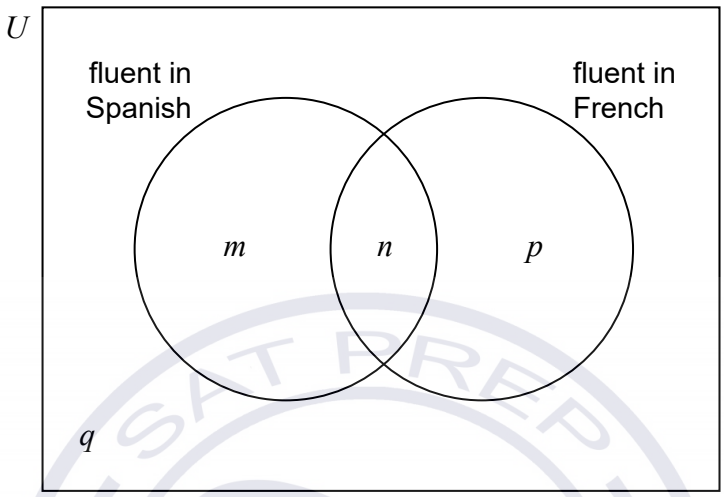
(c) Find the sum of the first 20 terms. [2]

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

2. [Maximum mark: 6]

In a class of 30 students, 18 are fluent in Spanish, 10 are fluent in French, and 5 are not fluent in either of these languages. The following Venn diagram shows the events "fluent in Spanish" and "fluent in French".

The values m, n, p and q represent numbers of students.



(a) Write down the value of q. [1]

(b) Find the value of n. [2]

(c) Write down the value of m and of p. [3]

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

3. [Maximum mark: 7]

Let $g(x) = x^2 + bx + 11$. The point $(-1, 8)$ lies on the graph of g.

(a) Find the value of b. [3]

(b) The graph of $f(x) = x^2$ is transformed to obtain the graph of g.

Describe this transformation. [4]

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

4. [Maximum mark: 6]

Consider $\binom{11}{a} = \frac{11!}{a!\,9!}$.

(a) Find the value of a. [2]

(b) Hence or otherwise find the coefficient of the term in x^9 in the expansion of $(x+3)^{11}$. [4]

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

5. [Maximum mark: 6]

Consider the function f, with derivative $f'(x) = 2x^2 + 5kx + 3k^2 + 2$ where x, $k \in \mathbb{R}$.

(a) Show that the discriminant of $f'(x)$ is $k^2 - 16$. [2]

(b) Given that f is an increasing function, find all possible values of k. [4]

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

6. [Maximum mark: 8]

Let $f(x) = 4\cos\left(\frac{x}{2}\right) + 1$, for $0 \le x \le 6\pi$. Find the values of x for which $f(x) > 2\sqrt{2} + 1$.

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

7. [Maximum mark: 6]

Let X and Y be normally distributed with $X \sim \mathrm{N}(14, a^2)$ and $Y \sim \mathrm{N}(22, a^2)$, $a > 0$.

(a) Find b so that $\mathrm{P}(X > b) = \mathrm{P}(Y < b)$. [2]

It is given that $\mathrm{P}(X > 20) = 0.112$.

(b) Find $\mathrm{P}(16 < Y < 28)$. [4]

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

Do **not** write solutions on this page.

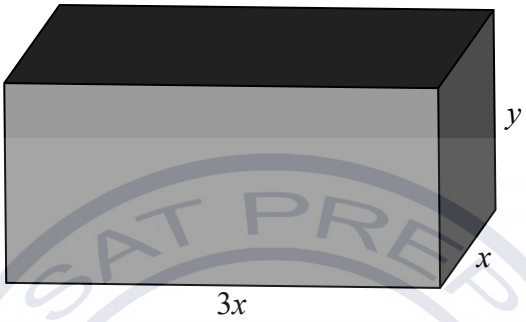
Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 14]

A small cuboid box has a rectangular base of length $3x$ cm and width x cm, where $x > 0$. The height is y cm, where $y > 0$.

diagram not to scale



The sum of the length, width and height is 12 cm.

(a) Write down an expression for y in terms of x. [1]

The volume of the box is V cm^3.

(b) Find an expression for V in terms of x. [2]

(c) Find $\frac{dV}{dx}$. [2]

(d) (i) Find the value of x for which V is a maximum.

(ii) Justify your answer. [7]

(e) Find the maximum volume. [2]

Do **not** write solutions on this page.

9. [Maximum mark: 17]

The points A and B have position vectors $\begin{pmatrix} -2 \\ 4 \\ -4 \end{pmatrix}$ and $\begin{pmatrix} 6 \\ 8 \\ 0 \end{pmatrix}$ respectively.

Point C has position vector $\begin{pmatrix} -1 \\ k \\ 0 \end{pmatrix}$. Let O be the origin.

(a) Find, in terms of k,

(i) $\vec{OA} \bullet \vec{OC}$;

(ii) $\vec{OB} \bullet \vec{OC}$. [3]

(b) Given that $A\hat{O}C = B\hat{O}C$, show that $k = 7$. [8]

(c) Calculate the area of triangle AOC. [6]

Do **not** write solutions on this page.

10. [Maximum mark: 14]

Let $g(x) = p^x + q$, for $x, p, q \in \mathbb{R}, p > 1$. The point $\mathrm{A}(0, a)$ lies on the graph of g.

Let $f(x) = g^{-1}(x)$. The point B lies on the graph of f and is the reflection of point A in the line $y = x$.

(a) Write down the coordinates of B. [2]

The line L_1 is tangent to the graph of f at B.

(b) Given that $f'(a) = \dfrac{1}{\ln p}$, find the equation of L_1 **in terms of** x, p and q. [5]

The line L_2 is tangent to the graph of g at A and has equation $y = (\ln p)x + q + 1$.

The line L_2 passes through the point $(-2, -2)$.

The gradient of the normal to g at A is $\dfrac{1}{\ln\left(\dfrac{1}{3}\right)}$.

(c) Find the equation of L_1 in terms of x. [7]

Please **do not** write on this page.

Answers written on this page
will not be marked.

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Standard level
Paper 1

Monday 13 May 2019 (afternoon)

Candidate session number

1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics SL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[90 marks]**.

2219 – 7303

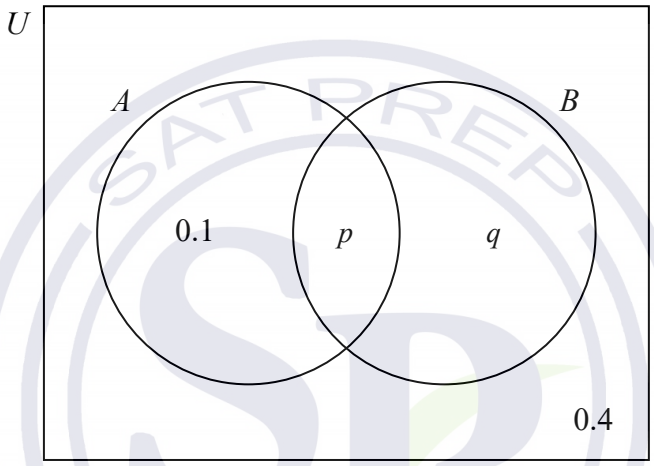
Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines if necessary.

1. [Maximum mark: 6]

The following Venn diagram shows the events A and B, where $\mathrm{P}(A) = 0.3$. The values shown are probabilities.



(a) Find the value of p. [2]

(b) Find the value of q. [2]

(c) Find $\mathrm{P}(A' \cup B)$. [2]

(This question continues on the following page)

(Question 1 continued)

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

2. [Maximum mark: 6]

A line, L_1, has equation $\boldsymbol{r} = \begin{pmatrix} -3 \\ 9 \\ 10 \end{pmatrix} + s \begin{pmatrix} 6 \\ 0 \\ 2 \end{pmatrix}$. Point $\mathrm{P}(15, 9, c)$ lies on L_1.

(a) Find c. [4]

A second line, L_2, is parallel to L_1 and passes through $(1, 2, 3)$.

(b) Write down a vector equation for L_2. [2]

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

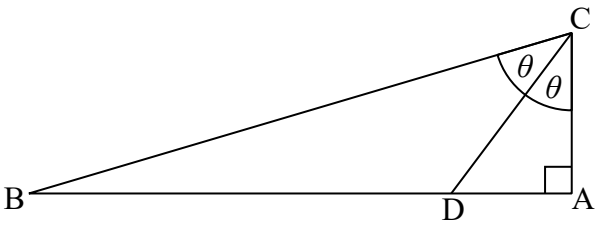
. . . . . .

. . . . . .

3. [Maximum mark: 7]

The following diagram shows a right triangle ABC. Point D lies on AB such that CD bisects $A\hat{C}B$.

diagram not to scale



$A\hat{C}D = \theta$ and $AC = 14\,\text{cm}$

(a) Given that $\sin\theta = \frac{3}{5}$, find the value of $\cos\theta$. [3]

(b) Find the value of $\cos 2\theta$. [2]

(c) Hence or otherwise, find BC. [2]

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

4. [Maximum mark: 6]

Let $f(x) = \frac{2x-1}{x+3}$, $x \neq -3$.

(a) Write down the equation of the vertical asymptote of the graph of f. [1]

(b) Find $f^{-1}(x)$. [3]

(c) Find the equation of the horizontal asymptote of the graph of f^{-1}. [2]

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

5. [Maximum mark: 5]

The derivative of a function f is given by $f'(x) = 2e^{-3x}$. The graph of f passes through $\left(\frac{1}{3}, 5\right)$.

Find $f(x)$.

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

Please **do not** write on this page.

Answers written on this page
will not be marked.

6. [Maximum mark: 7]

The magnitudes of two vectors, $\boldsymbol{u}$ and $\boldsymbol{v}$, are 4 and $\sqrt{3}$ respectively. The angle between $\boldsymbol{u}$ and $\boldsymbol{v}$ is $\frac{\pi}{6}$.

Let $\boldsymbol{w} = \boldsymbol{u} - \boldsymbol{v}$. Find the magnitude of $\boldsymbol{w}$.

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

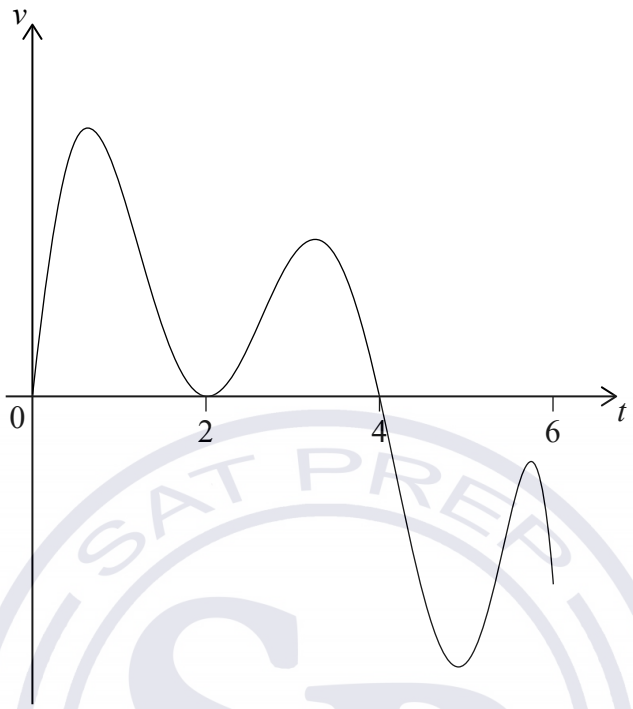
. . . . . .

. . . . . .

. . . . . .

7. [Maximum mark: 7]

A particle P starts from point O and moves along a straight line. The graph of its velocity, $v\,\text{ms}^{-1}$ after t seconds, for $0 \le t \le 6$, is shown in the following diagram.



The graph of v has t-intercepts when $t = 0, 2$ and 4.

The function $s(t)$ represents the displacement of P from O after t seconds.

It is known that P travels a distance of 15 metres in the first 2 seconds. It is also known that $s(2) = s(5)$ and $\int_2^4 v\,\mathrm{d}t = 9$.

(a) Find the value of $s(4) - s(2)$. [2]

(b) Find the total distance travelled in the first 5 seconds. [5]

(This question continues on the following page)

(Question 7 continued)

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

Do **not** write solutions on this page.

Section B

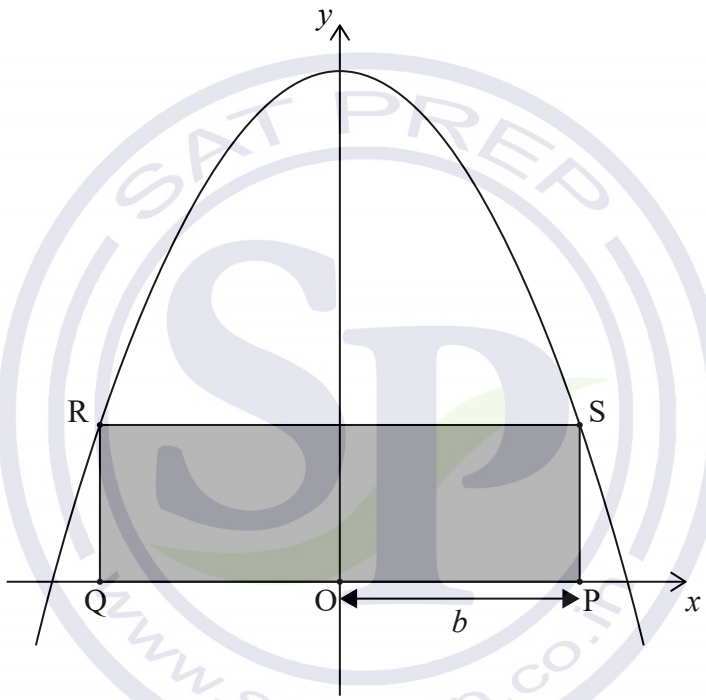
Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 16]

Let $f(x) = 9 - x^2$, $x \in \mathbb{R}$.

(a) Find the x-intercepts of the graph of f. [2]

The following diagram shows part of the graph of f.



Rectangle PQRS is drawn with P and Q on the x-axis and R and S on the graph of f.

Let $\mathrm{OP} = b$.

(b) Show that the area of PQRS is $18b - 2b^3$. [2]

(c) Hence find the value of b such that the area of PQRS is a maximum. [5]

Consider another function $g(x) = (x-3)^2 + k$, $x \in \mathbb{R}$.

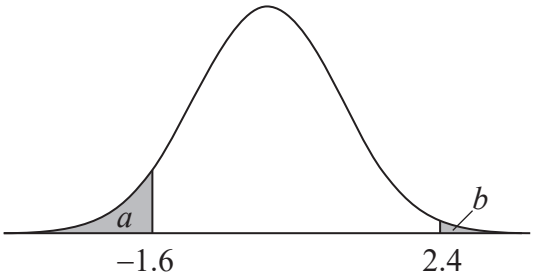
(d) Show that when the graphs of f and g intersect, $2x^2 - 6x + k = 0$. [2]

(e) Given that the graphs of f and g intersect only once, find the value of k. [5]

Do **not** write solutions on this page.

9. [Maximum mark: 13]

A random variable Z is normally distributed with mean 0 and standard deviation 1. It is known that $\mathrm{P}(z < -1.6) = a$ and $\mathrm{P}(z > 2.4) = b$. This is shown in the following diagram.



(a) Find $\mathrm{P}(-1.6 < z < 2.4)$. Write your answer in terms of a and b. [2]

(b) Given that $z > -1.6$, find the probability that $z < 2.4$. Write your answer in terms of a and b. [4]

A second random variable X is normally distributed with mean m and standard deviation s.

It is known that $\mathrm{P}(x < 1) = a$.

(c) Write down the standardized value for $x = 1$. [1]

It is also known that $\mathrm{P}(x > 2) = b$.

(d) Find s. [6]

Do **not** write solutions on this page.

10. [Maximum mark: 17]

Consider $f(x) = \sqrt{x}\sin\left(\frac{\pi}{4}x\right)$ and $g(x) = \sqrt{x}$ for $x \geq 0$. The first time the graphs of f and g intersect is at $x = 0$.

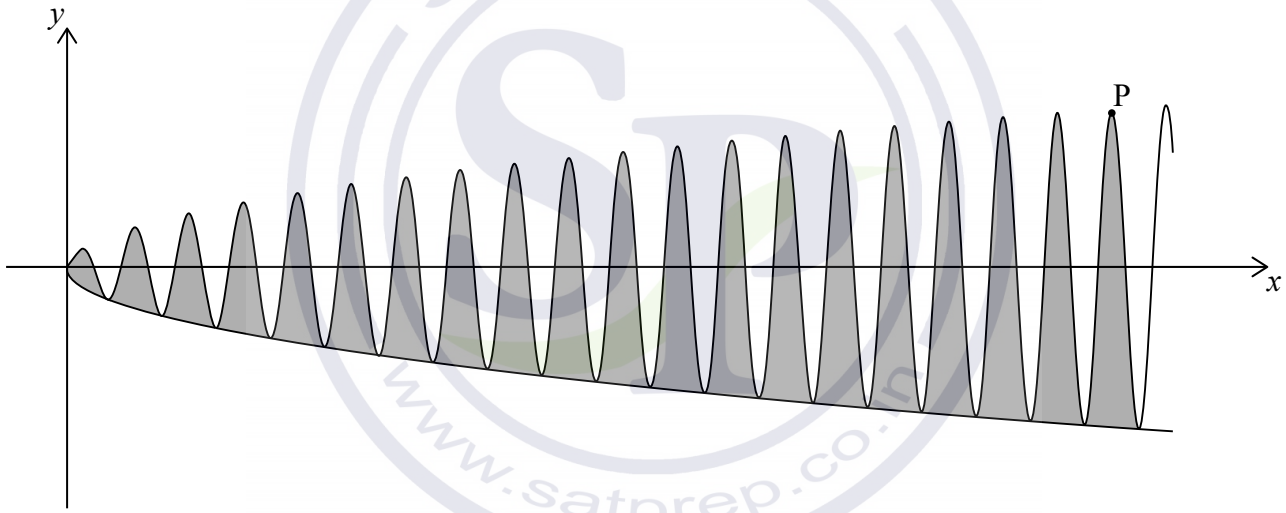
(a) Find the **two** smallest non-zero values of x for which $f(x) = g(x)$. [5]

The set of all non-zero values that satisfy $f(x) = g(x)$ can be described as an arithmetic sequence, $u_n = a + bn$ where $n \geq 1$.

(b) Find the value of a and of b. [4]

(c) At point P, the graphs of f and g intersect for the 21st time. Find the coordinates of P. [4]

The following diagram shows part of the graph of g **reflected** in the x-axis. It also shows part of the graph of f and the point P.



(d) Find an expression for the area of the shaded region. Do not calculate the value of the expression. [4]

Please **do not** write on this page.

Answers written on this page
will not be marked.

Please **do not** write on this page.

Answers written on this page
will not be marked.

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Standard level
Paper 1

Monday 13 May 2019 (afternoon)

Candidate session number

1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics SL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[90 marks]**.

2219 – 7305

11 pages

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines if necessary.

1. [Maximum mark: 6]

The following table shows the probability distribution of a discrete random variable X.

X	0	1	2	3
$\mathrm{P}(X = x)$	$\frac{3}{13}$	$\frac{1}{13}$	$\frac{4}{13}$	k

(a) Find the value of k. [3]

(b) Find $\mathrm{E}(X)$. [3]

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

2. [Maximum mark: 6]

Consider the vectors $\boldsymbol{a} = \begin{pmatrix} 0 \\ 3 \\ p \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} 0 \\ 6 \\ 18 \end{pmatrix}$.

Find the value of p for which $\boldsymbol{a}$ and $\boldsymbol{b}$ are

(a) parallel; [2]

(b) perpendicular. [4]

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

3. [Maximum mark: 6]

Consider the function $f(x) = \frac{3x+1}{x-2}$, $x \neq 2$.

(a) For the graph of f,

(i) write down the equation of the vertical asymptote;

(ii) find the equation of the horizontal asymptote. [3]

Let $g(x) = x^2 + 4$, $x \in \mathbb{R}$.

(b) Find $(f \circ g)(1)$. [3]

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

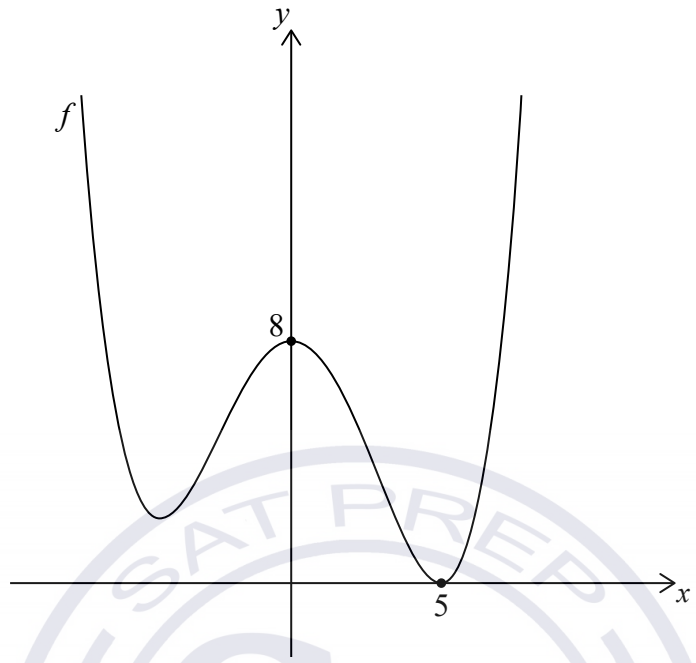
. . . . . .

. . . . . .

. . . . . .

4. [Maximum mark: 7]

The following diagram shows part of the graph of f with x-intercept $(5, 0)$ and y-intercept $(0, 8)$.



(a) Find the y-intercept of the graph of

(i) $f(x) + 3$;

(ii) $f(4x)$. [3]

(b) Find the x-intercept of the graph of $f(2x)$. [2]

(c) Describe the transformation $f(x + 1)$. [2]

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

5. [Maximum mark: 6]

Consider the function $f(x) = (1-k)x^2 + x + k$, $x \in \mathbb{R}$. Find the value of k for which $f(x)$ has two equal real roots.

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

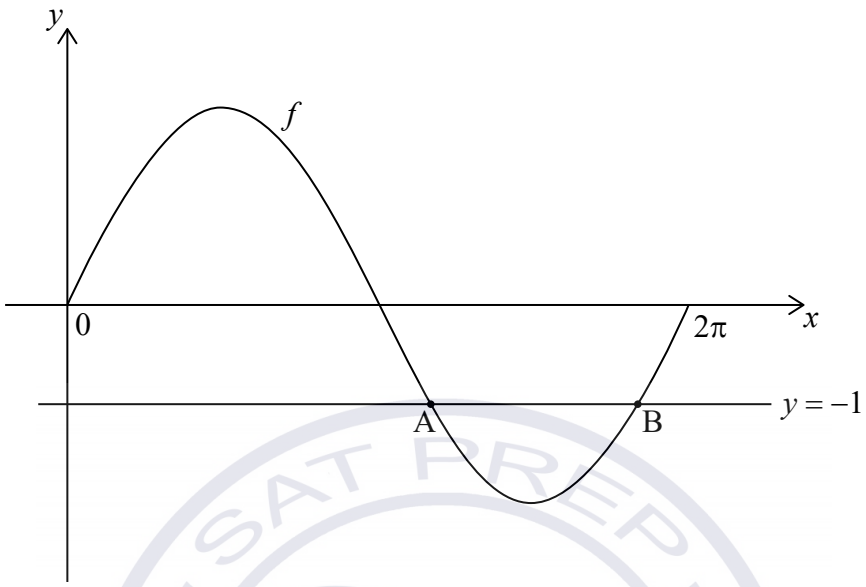
6. [Maximum mark: 7]

Solve $\log_4(2-x) = \log_{16}(13-4x)$.

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

7. [Maximum mark: 7]

Consider the graph of the function $f(x) = 2\sin x$, $0 \le x < 2\pi$. The graph of f intersects the line $y = -1$ exactly twice, at point A and point B. This is shown in the following diagram.



(a) Find the x-coordinate of A and of B. [4]

Consider the graph of $g(x) = 2\sin px$, $0 \le x < 2\pi$, where $p > 0$.

(b) Find the greatest value of p such that the graph of g does not intersect the line $y = -1$. [3]

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

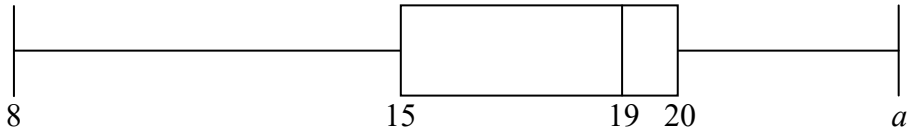
Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 16]

A group of 10 girls recorded the number of hours they spent watching television during a particular week. Their results are summarized in the box-and-whisker plot below.



(a) The range of the data is 16. Find the value of a. [2]

(b) Find the value of the interquartile range. [2]

The group of girls watched a total of 180 hours of television.

(c) Find the mean number of hours that the girls in this group spent watching television that week. [2]

A group of 20 boys also recorded the number of hours they spent watching television that same week. Their results are summarized in the table below.

$\bar{x} = 21$	$\sigma = 3$

(d) (i) Find the total number of hours the group of boys spent watching television that week.

(ii) Find the mean number of hours that **all 30** girls and boys spent watching television that week. [5]

The following week, the group of boys had exams. During this exam week, the boys spent half as much time watching television compared to the previous week.

(e) For this exam week, find

(i) the mean number of hours that the group of boys spent watching television;

(ii) the variance in the number of hours the group of boys spent watching television. [5]

Do **not** write solutions on this page.

9. [Maximum mark: 15]

Let θ be an **obtuse** angle such that $\sin\theta = \frac{3}{5}$.

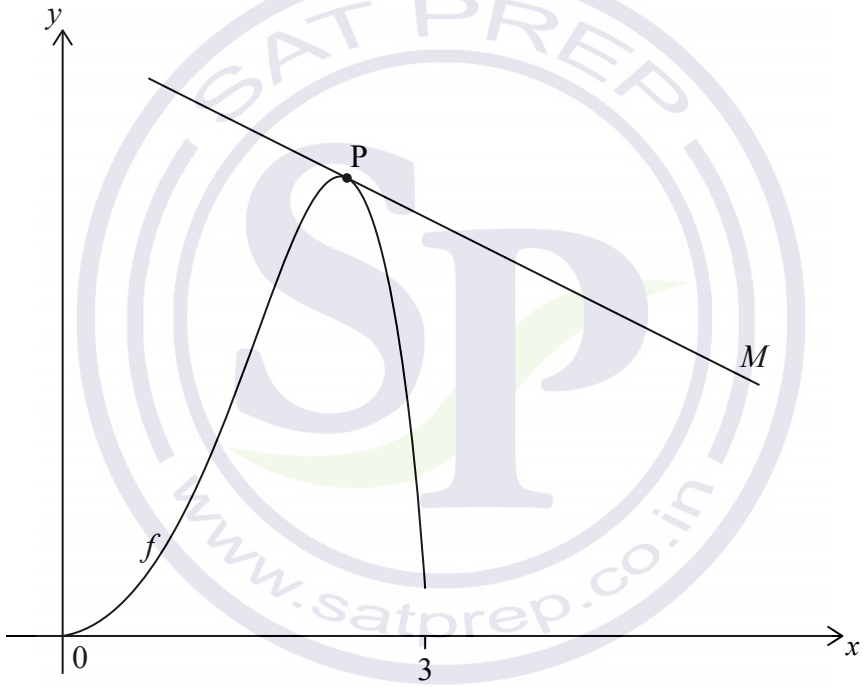
(a) Find the value of $\tan\theta$. [4]

(b) Line L passes through the origin and has a gradient of $\tan\theta$. Find the equation of L. [2]

Let $f(x) = e^x \sin x - \frac{3x}{4}$.

(c) Find the derivative of f. [5]

The following diagram shows the graph of f for $0 \leq x \leq 3$. Line M is a tangent to the graph of f at point P.



(d) Given that M is parallel to L, find the x-coordinate of P. [4]

Do **not** write solutions on this page.

10. [Maximum mark: 14]

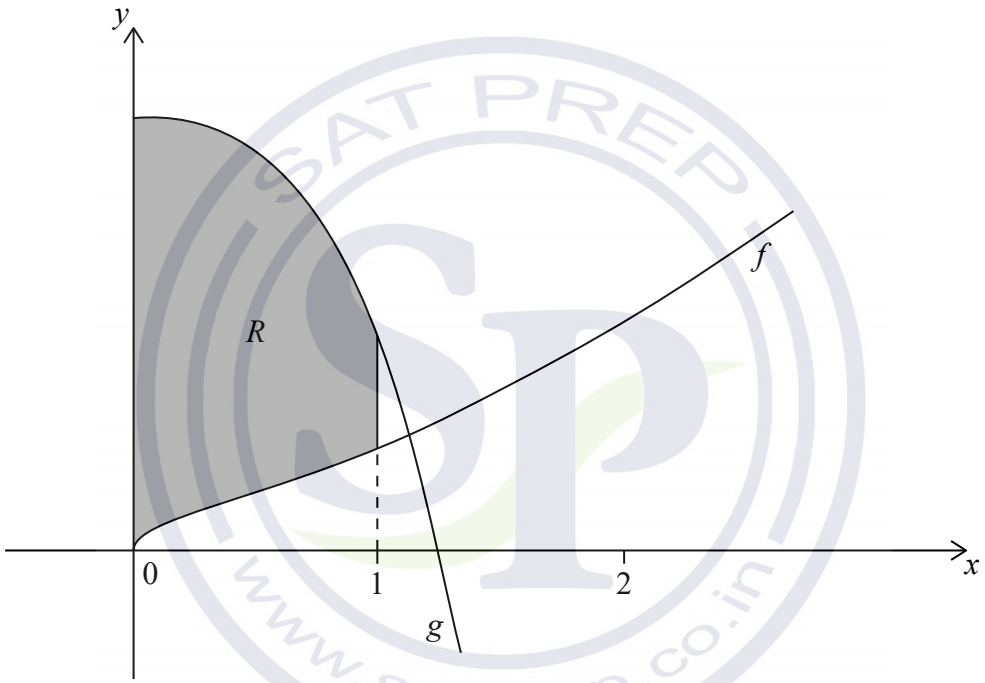
Let $y = \left(x^3 + x\right)^{\frac{3}{2}}$.

(a) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$. [3]

(b) Hence find $\int \left(3x^2 + 1\right)\sqrt{x^3 + x}\ \mathrm{d}x$. [3]

Consider the functions $f(x) = \sqrt{x^3 + x}$ and $g(x) = 6 - 3x^2\sqrt{x^3 + x}$, for $x \geq 0$.

The graphs of f and g are shown in the following diagram.



The shaded region R is enclosed by the graphs of f, g, the y-axis and $x = 1$.

(c) Write down an expression for the area of R. [2]

(d) Hence find the exact area of R. [6]

Please **do not** write on this page.

Answers written on this page
will not be marked.

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Standard level
Paper 1

Monday 12 November 2018 (afternoon)

Candidate session number

1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics SL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[90 marks]**.

8818 – 7301

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

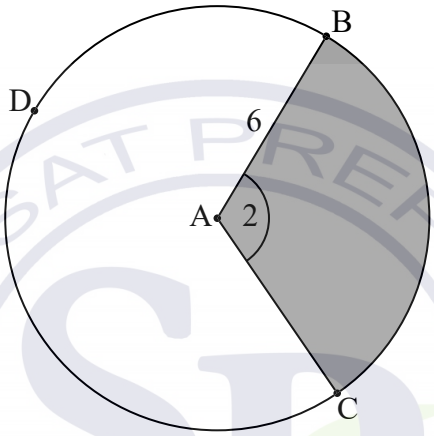
Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines if necessary.

1. [Maximum mark: 6]

The following diagram shows a circle with centre A and radius $6\,\text{cm}$.

diagram not to scale



The points B, C, and D lie on the circle, and $\text{B}\hat{\text{A}}\text{C} = 2$ radians.

(a) Find the area of the shaded sector. [2]

(b) Find the perimeter of the non-shaded sector ABDC. [4]

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

2. [Maximum mark: 5]

Two functions, f and g, are defined in the following table.

x	–2	1	3	6
$f(x)$	6	3	1	–2
$g(x)$	–7	–2	5	9

(a) Write down the value of $f(1)$. [1]

(b) Find the value of $(g \circ f)(1)$. [2]

(c) Find the value of $g^{-1}(-2)$. [2]

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

3. [Maximum mark: 6]

In an arithmetic sequence, $u_1 = -5$ and $d = 3$.

(a) Find u_8. [2]

(b) Find the value of n for which $u_n = 67$. [4]

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

4. [Maximum mark: 6]

Let $b = \log_2 a$, where $a > 0$. Write down each of the following expressions in terms of b.

(a) $\log_2 a^3$ [2]

(b) $\log_2 8a$ [2]

(c) $\log_8 a$ [2]

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

5. [Maximum mark: 6]

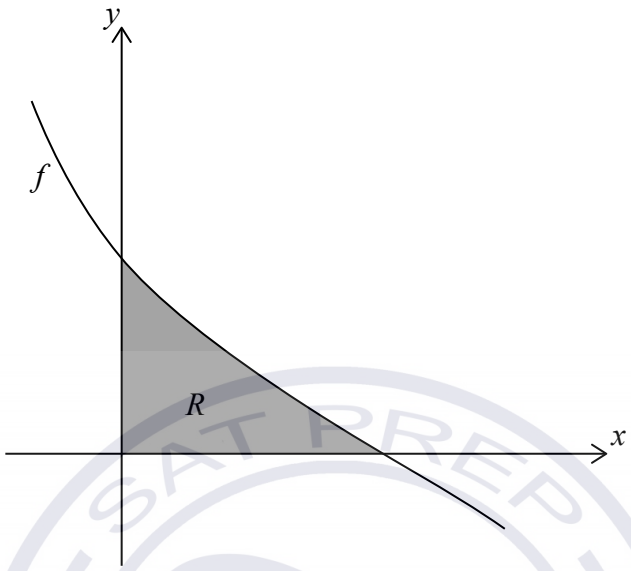
Consider the vectors $\boldsymbol{a} = \begin{pmatrix} 3 \\ 2p \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} p+1 \\ 8 \end{pmatrix}$.

Find the possible values of p for which $\boldsymbol{a}$ and $\boldsymbol{b}$ are parallel.

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

6. [Maximum mark: 8]

Let $f(x) = \frac{6-2x}{\sqrt{16+6x-x^2}}$. The following diagram shows part of the graph of f.



The region R is enclosed by the graph of f, the x-axis, and the y-axis. Find the area of R.

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

7. [Maximum mark: 6]

Given that $\sin x = \frac{1}{3}$, where $0 < x < \frac{\pi}{2}$, find the value of $\cos 4x$.

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

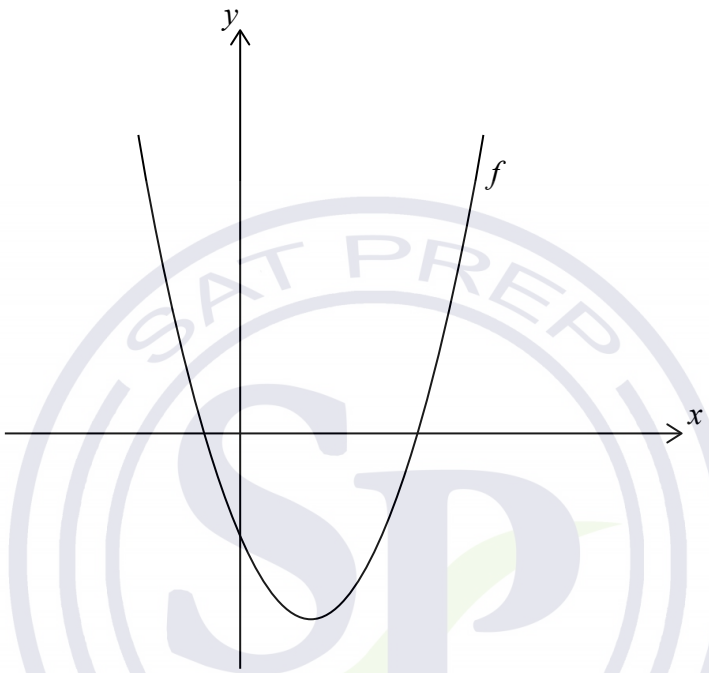
Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 16]

Let $f(x) = x^2 - 4x - 5$. The following diagram shows part of the graph of f.



(a) Find the x-intercepts of the graph of f. [5]

(b) Find the equation of the axis of symmetry of the graph of f. [2]

(c) The function can be written in the form $f(x) = (x - h)^2 + k$.

(i) Write down the value of h.

(ii) Find the value of k. [4]

The graph of a second function, g, is obtained by a reflection of the graph of f in the y-axis, followed by a translation of $\begin{pmatrix} -3 \\ 6 \end{pmatrix}$.

(d) Find the coordinates of the vertex of the graph of g. [5]

Do **not** write solutions on this page.

9. [Maximum mark: 15]

A bag contains n marbles, two of which are blue. Hayley plays a game in which she randomly draws marbles out of the bag, one after another, without replacement. The game ends when Hayley draws a blue marble.

(a) Find the probability, in terms of n, that the game will end on her

(i) first draw;

(ii) second draw. [4]

(b) Let $n = 5$. Find the probability that the game will end on her

(i) third draw;

(ii) fourth draw. [4]

Hayley plays the game when $n = 5$. She pays $\$20$ to play and can earn money back depending on the number of draws it takes to obtain a blue marble. She earns no money back if she obtains a blue marble on her first draw. Let M be the amount of money that she earns back playing the game. This information is shown in the following table.

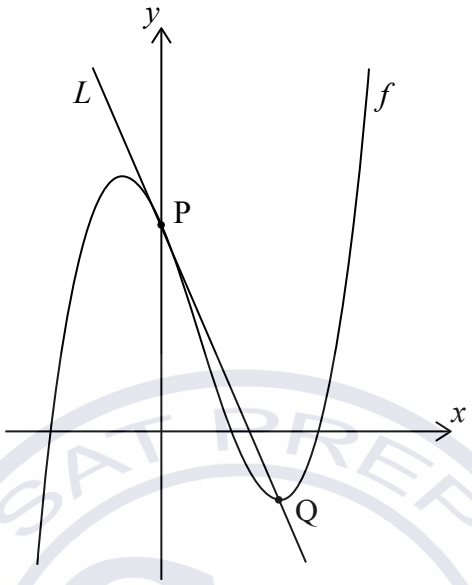
Number of draws	1	2	3	4
Money earned back ($M)	0	20	$8k$	$12k$

(c) Find the value of k so that this is a fair game. [7]

Do **not** write solutions on this page.

10. [Maximum mark: 16]

Let $f(x) = x^3 - 2x^2 + ax + 6$. Part of the graph of f is shown in the following diagram.



The graph of f crosses the y-axis at the point P. The line L is tangent to the graph of f at P.

(a) Find the coordinates of P. [2]

(b) (i) Find $f'(x)$.

(ii) Hence, find the equation of L in terms of a. [6]

The graph of f has a local minimum at the point Q. The line L passes through Q.

(c) Find the value of a. [8]

Please **do not** write on this page.

Answers written on this page
will not be marked.

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Standard level
Paper 1

Wednesday 2 May 2018 (afternoon)

Candidate session number

1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics SL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[90 marks]**.

2218 – 7303

11 pages

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines if necessary.

1. [Maximum mark: 6]

Let $f(x) = \sqrt{x+2}$ for $x \geq -2$ and $g(x) = 3x - 7$ for $x \in \mathbb{R}$.

(a) Write down $f(14)$. [1]

(b) Find $(g \circ f)(14)$. [2]

(c) Find $g^{-1}(x)$. [3]

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

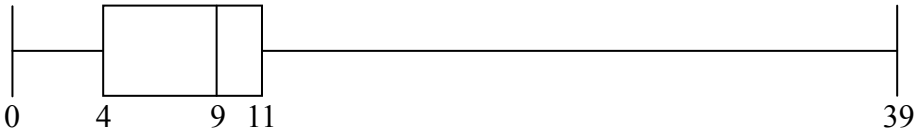
. . . . . .

. . . . . .

. . . . . .

2. [Maximum mark: 6]

The following box-and-whisker plot shows the number of text messages sent by students in a school on a particular day.



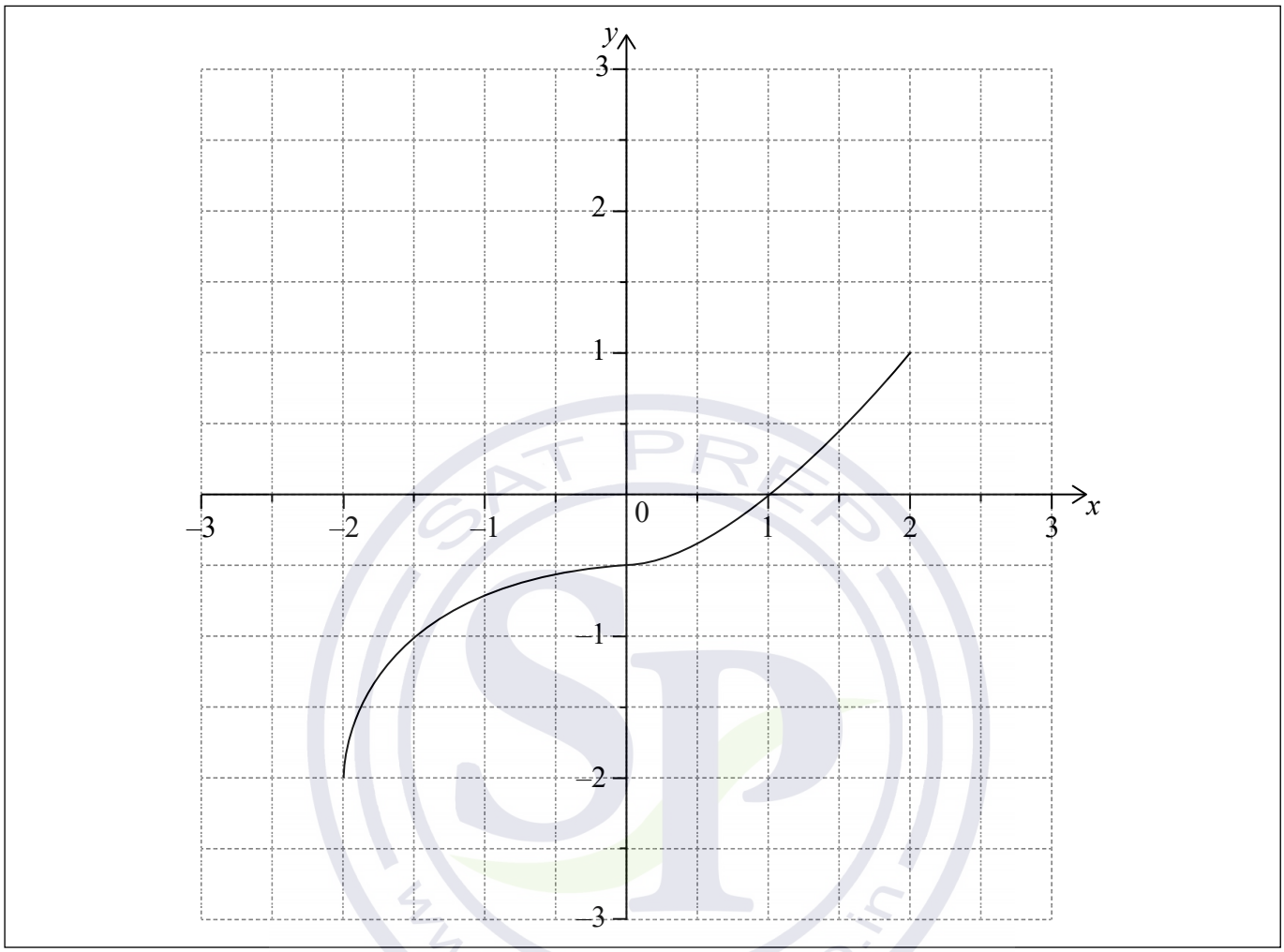
(a) Find the value of the interquartile range. [2]

(b) One student sent k text messages, where $k > 11$. Given that k is an outlier, find the least value of k. [4]

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

3. [Maximum mark: 7]

Consider a function $f(x)$, for $-2 \le x \le 2$. The following diagram shows the graph of f.



(a) Write down the value of

(i) $f(0)$;

(ii) $f^{-1}(1)$. [2]

(b) Write down the range of f^{-1}. [1]

(c) On the grid above, sketch the graph of f^{-1}. [4]

(This question continues on the following page)

(Question 3 continued)

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

4. [Maximum mark: 7]

Let $f(x) = ax^2 - 4x - c$. A horizontal line, L, intersects the graph of f at $x = -1$ and $x = 3$.

(a) (i) The equation of the axis of symmetry is $x = p$. Find p.

(ii) Hence, show that $a = 2$. [4]

(b) The equation of L is $y = 5$. Find the value of c. [3]

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

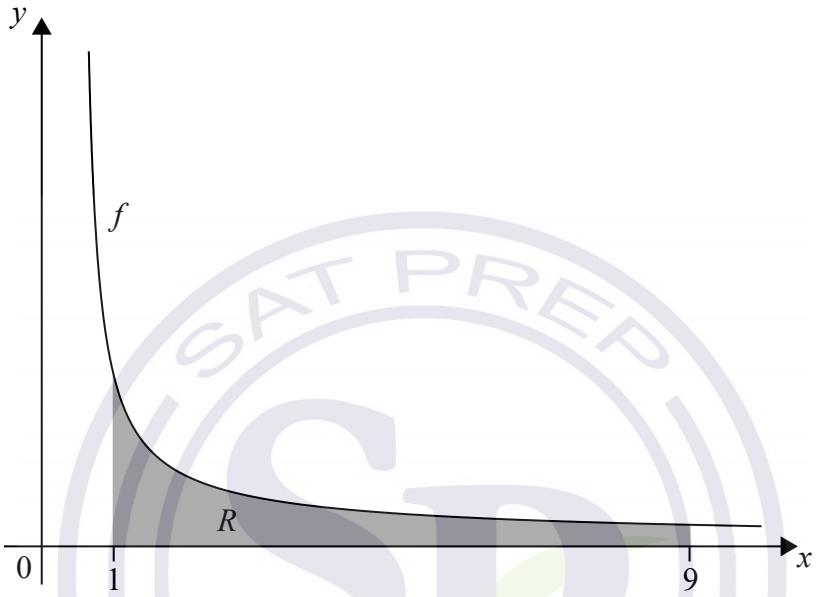
. . . . . .

5. [Maximum mark: 7]

Let $f(x) = \frac{1}{\sqrt{2x-1}}$, for $x > \frac{1}{2}$.

(a) Find $\int (f(x))^2 \, dx$. [3]

(b) Part of the graph of f is shown in the following diagram.



The shaded region R is enclosed by the graph of f, the x-axis, and the lines $x = 1$ and $x = 9$. Find the volume of the solid formed when R is revolved 360° about the x-axis. [4]

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

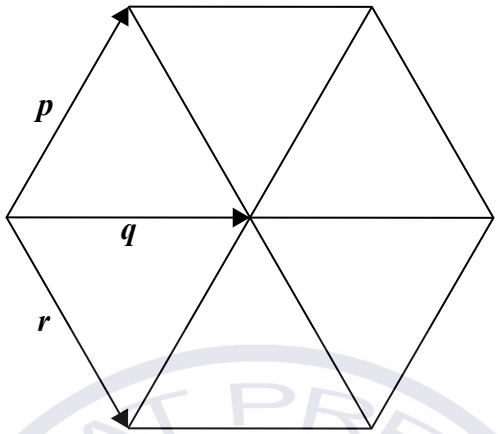
. . . . . .

. . . . . .

6. [Maximum mark: 6]

Six equilateral triangles, each with side length $3\,\text{cm}$, are arranged to form a hexagon. This is shown in the following diagram.

diagram not to scale



The vectors $\boldsymbol{p}$, $\boldsymbol{q}$ and $\boldsymbol{r}$ are shown on the diagram.

Find $\boldsymbol{p} \bullet (\boldsymbol{p} + \boldsymbol{q} + \boldsymbol{r})$.

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

7. [Maximum mark: 7]

Consider $f(x)$, $g(x)$ and $h(x)$, for $x \in \mathbb{R}$ where $h(x) = (f \circ g)(x)$.

Given that $g(3) = 7$, $g'(3) = 4$ and $f'(7) = -5$, find the gradient of the normal to the curve of h at $x = 3$.

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 13]

A function $f(x)$ has derivative $f'(x) = 3x^2 + 18x$. The graph of f has an x-intercept at $x = -1$.

(a) Find $f(x)$. [6]

(b) The graph of f has a point of inflexion at $x = p$. Find p. [4]

(c) Find the values of x for which the graph of f is concave-down. [3]

9. [Maximum mark: 16]

Point A has coordinates $(-4, -12, 1)$ and point B has coordinates $(2, -4, -4)$.

(a) Show that $\overrightarrow{AB} = \begin{pmatrix} 6 \\ 8 \\ -5 \end{pmatrix}$. [1]

(b) The line L passes through A and B.

(i) Find a vector equation for L.

(ii) Point $C(k, 12, -k)$ is on L. Show that $k = 14$. [6]

(c) (i) Find $\overrightarrow{OB} \bullet \overrightarrow{AB}$

(ii) Write down the value of angle OBA. [3]

Point D is also on L and has coordinates $(8, 4, -9)$.

(d) Find the area of triangle OCD. [6]

Do **not** write solutions on this page.

10. [Maximum mark: 15]

The first two terms of an infinite geometric sequence are $u_1 = 18$ and $u_2 = 12\sin^2\theta$, where $0 < \theta < 2\pi$, and $\theta \neq \pi$.

(a) (i) Find an expression for r in terms of θ.

(ii) Find the possible values of r. [5]

(b) Show that the sum of the infinite sequence is $\frac{54}{2+\cos(2\theta)}$. [4]

(c) Find the values of θ which give the greatest value of the sum. [6]

Please **do not** write on this page.

Answers written on this page
will not be marked.

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Standard level
Paper 1

Wednesday 2 May 2018 (afternoon)

Candidate session number

1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics SL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[90 marks]**.

12 pages

2218 – 7305

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines if necessary.

1. [Maximum mark: 5]

Let $\vec{OA} = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$ and $\vec{AB} = \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix}$, where O is the origin. L_1 is the line that passes through A and B.

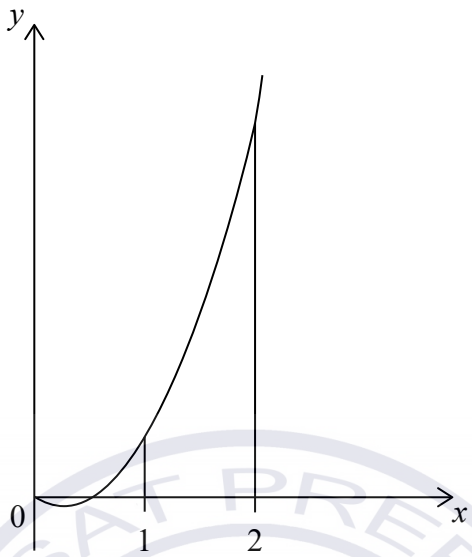
(a) Find a vector equation for L_1. [2]

(b) The vector $\begin{pmatrix} 2 \\ p \\ 0 \end{pmatrix}$ is perpendicular to $\vec{AB}$. Find the value of p. [3]

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

2. [Maximum mark: 6]

Let $f(x) = 6x^2 - 3x$. The graph of f is shown in the following diagram.



(a) Find $\int (6x^2 - 3x)\,dx$. [2]

(b) Find the area of the region enclosed by the graph of f, the x-axis and the lines $x = 1$ and $x = 2$. [4]

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

3. [Maximum mark: 6]

A data set has n items. The sum of the items is 800 and the mean is 20.

(a) Find n. [2]

The standard deviation of this data set is 3. Each value in the set is multiplied by 10.

(b) (i) Write down the value of the new mean.

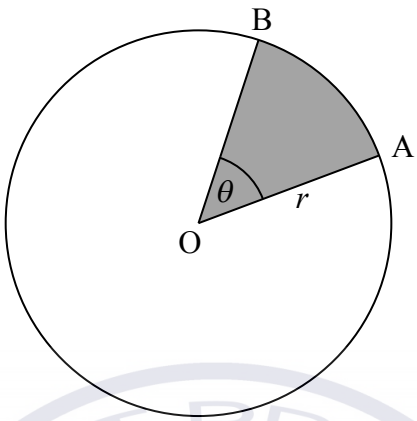
(ii) Find the value of the new variance. [4]

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

4. [Maximum mark: 7]

The following diagram shows a circle with centre O and radius r cm.

diagram not to scale



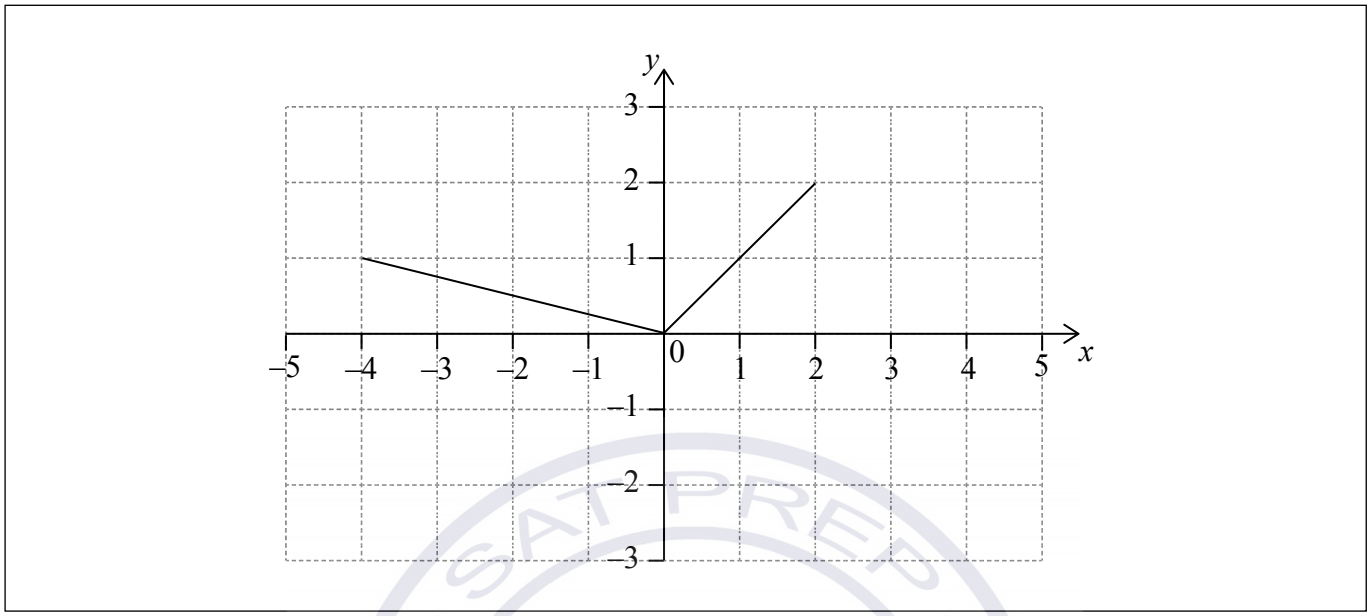
The points A and B lie on the circumference of the circle, and $\hat{AOB} = \theta$. The area of the shaded sector AOB is $12\,\text{cm}^2$ and the length of arc AB is $6\,\text{cm}$.

Find the value of r.

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

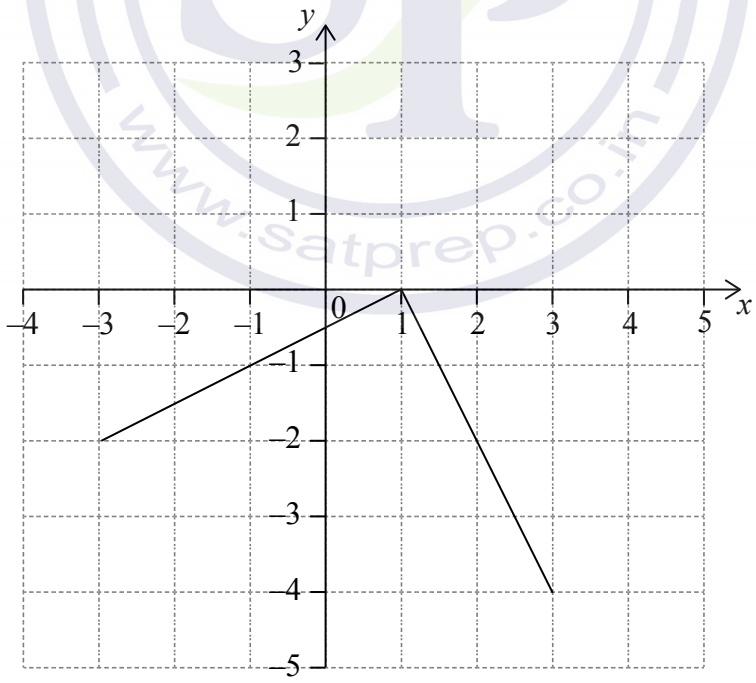
5. [Maximum mark: 6]

The following diagram shows the graph of a function f, for $-4 \le x \le 2$.



(a) On the same axes, sketch the graph of $f(-x)$. [2]

(b) Another function, g, can be written in the form $g(x) = a \times f(x + b)$. The following diagram shows the graph of g.



Write down the value of a and of b. [4]

(This question continues on the following page)

(Question 5 continued)

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

6. [Maximum mark: 7]

Let $f(x) = px^2 + qx - 4p$, where $p \neq 0$. Find the number of roots for the equation $f(x) = 0$. Justify your answer.

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

7. [Maximum mark: 8]

An arithmetic sequence has $u_1 = \log_c(p)$ and $u_2 = \log_c(pq)$, where $c > 1$ and $p, q > 0$.

(a) Show that $d = \log_c(q)$. [2]

(b) Let $p = c^2$ and $q = c^3$. Find the value of $\sum_{n=1}^{20} u_n$. [6]

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

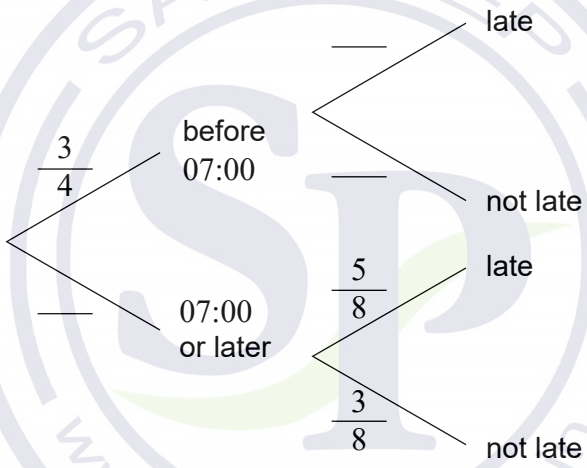
8. [Maximum mark: 14]

Pablo drives to work. The probability that he leaves home before $07{:}00$ is $\frac{3}{4}$.

If he leaves home before $07{:}00$ the probability he will be late for work is $\frac{1}{8}$.

If he leaves home at $07{:}00$ or later the probability he will be late for work is $\frac{5}{8}$.

(a) **Copy** and complete the following tree diagram. [3]



(b) Find the probability that Pablo leaves home before $07{:}00$ and is late for work. [2]

(c) Find the probability that Pablo is late for work. [3]

(d) Given that Pablo is late for work, find the probability that he left home before $07{:}00$. [3]

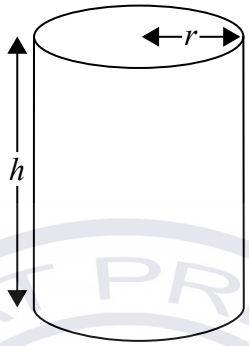
(e) Two days next week Pablo will drive to work. Find the probability that he will be late at least once. [3]

Do **not** write solutions on this page.

9. [Maximum mark: 15]

A closed cylindrical can with radius r centimetres and height h centimetres has a volume of $20\pi \text{ cm}^3$.

diagram not to scale



(a) Express h in terms of r. [2]

The material for the base and top of the can costs 10 cents per cm^2 and the material for the curved side costs 8 cents per cm^2. The total cost of the material, in cents, is C.

(b) Show that $C = 20\pi r^2 + \frac{320\pi}{r}$. [4]

(c) Given that there is a minimum value for C, find this minimum value in terms of π. [9]

Do **not** write solutions on this page.

10. [Maximum mark: 16]

Consider a function f. The line L_1 with equation $y = 3x + 1$ is a tangent to the graph of f when $x = 2$.

(a) (i) Write down $f'(2)$.

(ii) Find $f(2)$. [4]

Let $g(x) = f(x^2 + 1)$ and P be the point on the graph of g where $x = 1$.

(b) Show that the graph of g has a gradient of 6 at P. [5]

(c) Let L_2 be the tangent to the graph of g at P. L_1 intersects L_2 at the point Q. Find the y-coordinate of Q. [7]

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Standard level
Paper 1

Monday 13 November 2017 (afternoon)

Candidate session number

1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics SL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[90 marks]**.

8817 – 7301

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

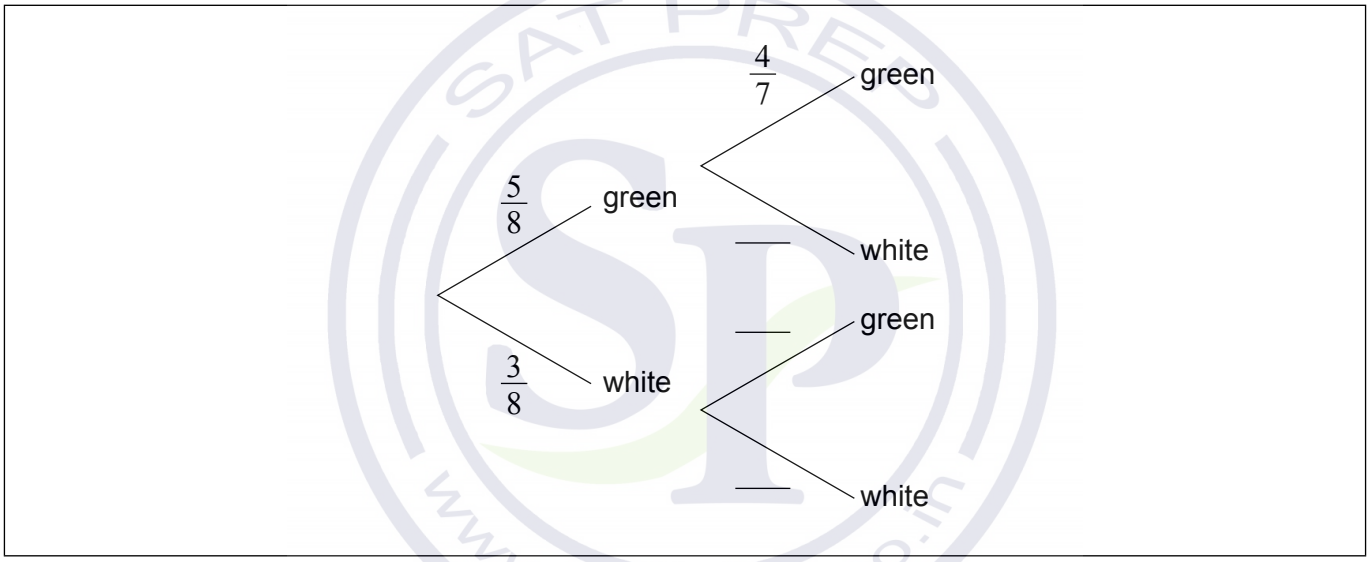
Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines if necessary.

1. [Maximum mark: 6]

A bag contains 5 green balls and 3 white balls. Two balls are selected at random without replacement.

(a) Complete the following tree diagram. [3]



(b) Find the probability that exactly one of the selected balls is green. [3]

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

2. [Maximum mark: 6]

In an arithmetic sequence, the first term is 8 and the second term is 5.

(a) Find the common difference. [2]

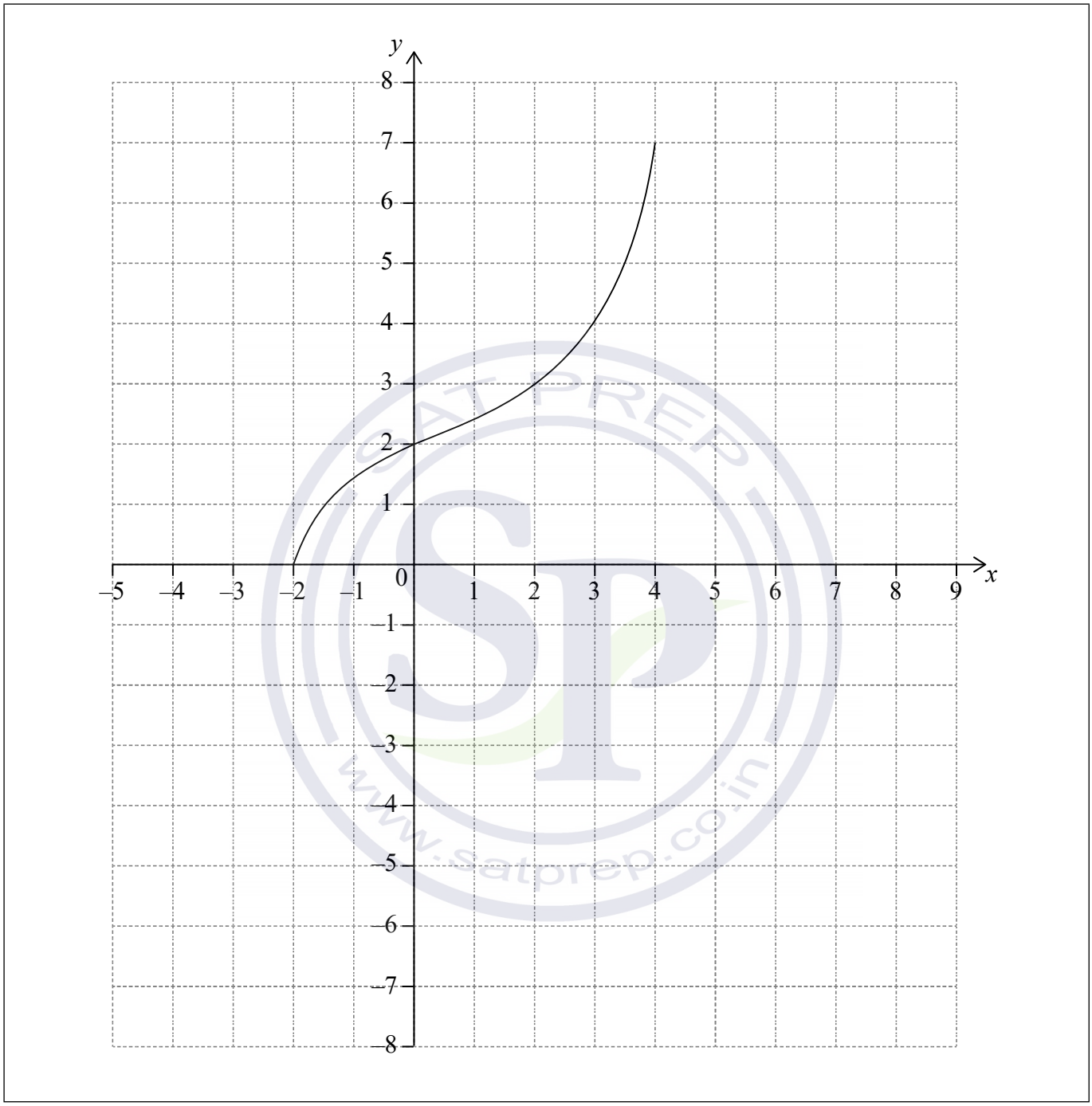
(b) Find the tenth term. [2]

(c) Find the sum of the first ten terms. [2]

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

3. [Maximum mark: 6]

The following diagram shows the graph of a function f, with domain $-2 \leq x \leq 4$.



The points $(-2, 0)$ and $(4, 7)$ lie on the graph of f.

(This question continues on the following page)

(Question 3 continued)

(a) Write down the range of f. [1]

(b) Write down

(i) $f(2)$;

(ii) $f^{-1}(2)$. [2]

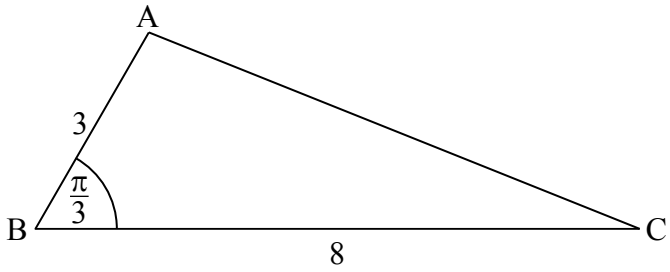
(c) On the grid opposite, sketch the graph of f^{-1}. [3]

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

4. [Maximum mark: 7]

The following diagram shows triangle ABC, with $AB = 3\,\text{cm}$, $BC = 8\,\text{cm}$, and $A\hat{B}C = \frac{\pi}{3}$.

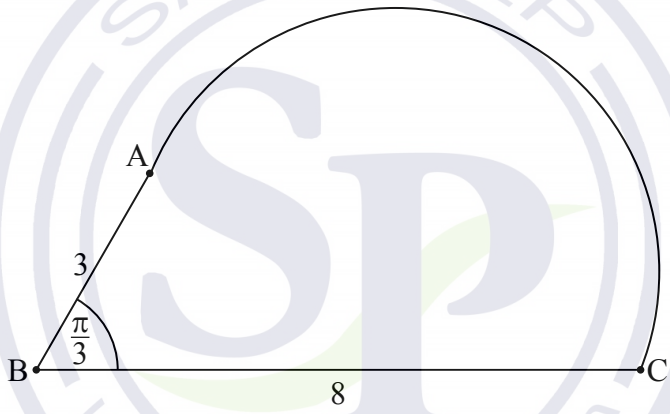
diagram not to scale



(a) Show that $AC = 7\,\text{cm}$. [4]

(b) The shape in the following diagram is formed by adding a semicircle with diameter $[AC]$ to the triangle.

diagram not to scale



Find the exact perimeter of this shape. [3]

(This question continues on the following page)

(Question 4 continued)

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

5. [Maximum mark: 6]

Let $f(x) = 1 + e^{-x}$ and $g(x) = 2x + b$, for $x \in \mathbb{R}$, where b is a constant.

(a) Find $(g \circ f)(x)$. [2]

(b) Given that $\lim_{x \to +\infty} (g \circ f)(x) = -3$, find the value of b. [4]

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

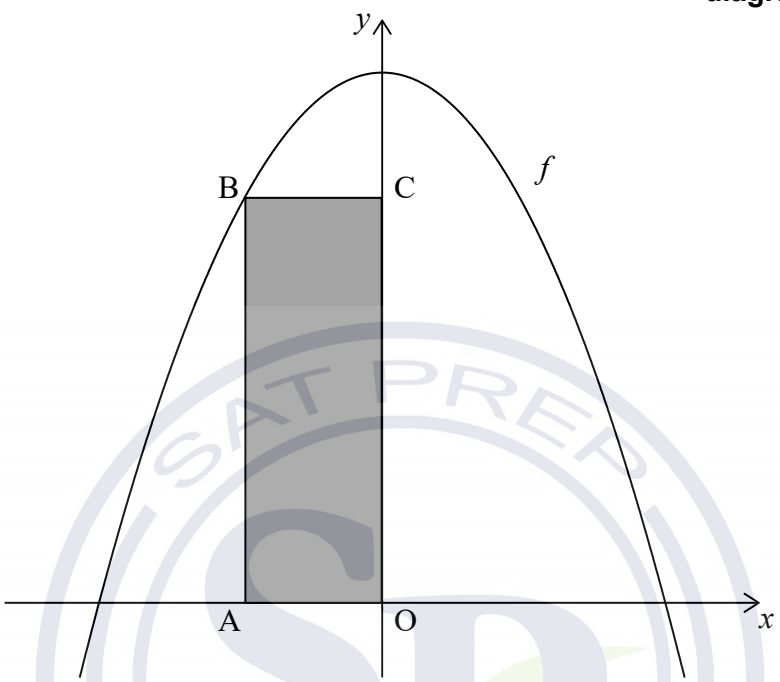
. . . . . .

. . . . . .

6. [Maximum mark: 7]

Let $f(x) = 15 - x^2$, for $x \in \mathbb{R}$. The following diagram shows part of the graph of f and the rectangle OABC, where A is on the negative x-axis, B is on the graph of f, and C is on the y-axis.

diagram not to scale



Find the x-coordinate of A that gives the maximum area of OABC.

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

7. [Maximum mark: 7]

Consider $f(x) = \log_k(6x - 3x^2)$, for $0 < x < 2$, where $k > 0$.
The equation $f(x) = 2$ has exactly one solution. Find the value of k.

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

Do **not** write solutions on this page.

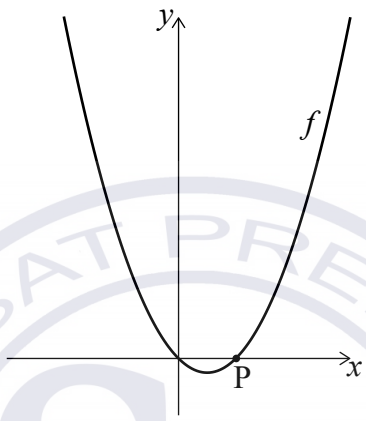
Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 16]

Let $f(x) = x^2 - x$, for $x \in \mathbb{R}$. The following diagram shows part of the graph of f.

diagram not to scale



The graph of f crosses the x-axis at the origin and at the point $\mathrm{P}(1, 0)$.

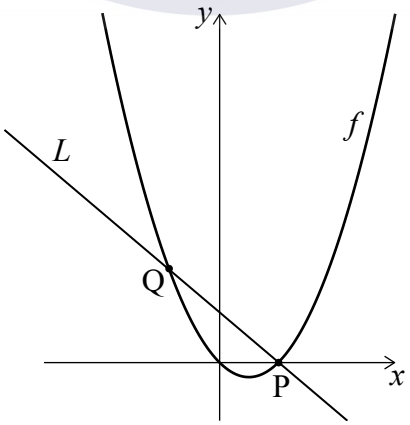
(a) Show that $f'(1) = 1$. [3]

The line L is the normal to the graph of f at P.

(b) Find the equation of L in the form $y = ax + b$. [3]

The line L intersects the graph of f at another point Q, as shown in the following diagram.

diagram not to scale



(c) Find the x-coordinate of Q. [4]

(d) Find the area of the region enclosed by the graph of f and the line L. [6]

Do **not** write solutions on this page.

9. [Maximum mark: 15]

A line L passes through points $\mathrm{A}(-3, 4, 2)$ and $\mathrm{B}(-1, 3, 3)$.

(a) (i) Show that $\vec{\mathrm{AB}} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$.

(ii) Find a vector equation for L. [3]

The line L also passes through the point $\mathrm{C}(3, 1, p)$.

(b) Find the value of p. [5]

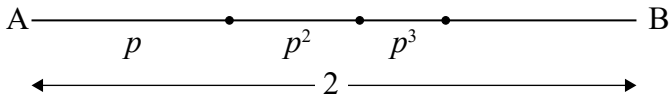
(c) The point D has coordinates $(q^2, 0, q)$. Given that $\vec{\mathrm{DC}}$ is perpendicular to L, find the possible values of q. [7]

Do **not** write solutions on this page.

10. [Maximum mark: 14]

(a) The following diagram shows [AB], with length $2\,\text{cm}$. The line is divided into an infinite number of line segments. The diagram shows the first three segments.

diagram not to scale

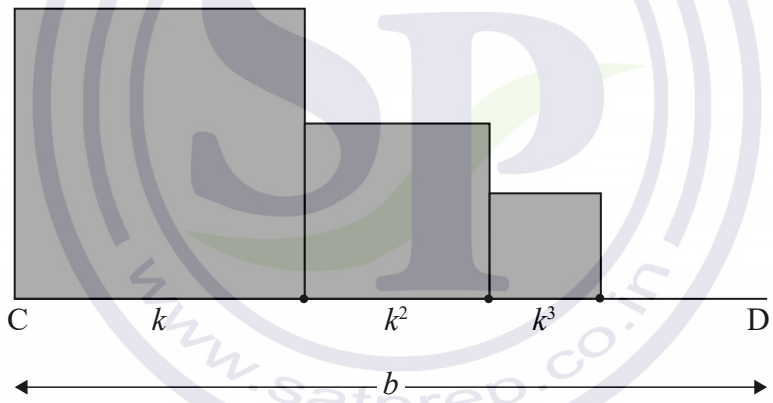


The length of the line segments are $p\,\text{cm}, p^2\,\text{cm}, p^3\,\text{cm}, \ldots$, where $0 < p < 1$.

Show that $p = \frac{2}{3}$. [5]

(b) The following diagram shows [CD], with length $b\,\text{cm}$, where $b > 1$. Squares with side lengths $k\,\text{cm}, k^2\,\text{cm}, k^3\,\text{cm}, \ldots$, where $0 < k < 1$, are drawn along [CD]. This process is carried on indefinitely. The diagram shows the first three squares.

diagram not to scale



The **total** sum of the areas of all the squares is $\frac{9}{16}$. Find the value of b. [9]

Please **do not** write on this page.

Answers written on this page
will not be marked.

Please **do not** write on this page.

Answers written on this page
will not be marked.

Please **do not** write on this page.

Answers written on this page
will not be marked.

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Standard level
Paper 1

Thursday 4 May 2017 (afternoon)

Candidate session number

1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics SL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[90 marks]**.

2217 – 7303

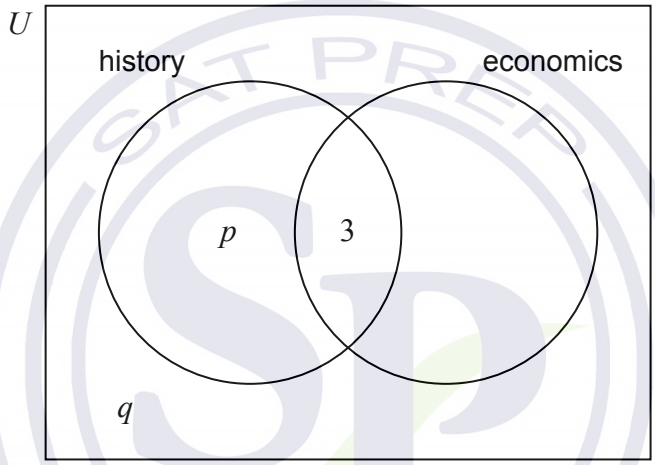
Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines if necessary.

1. [Maximum mark: 6]

In a group of 20 girls, 13 take history and 8 take economics. Three girls take both history and economics, as shown in the following Venn diagram. The values p and q represent numbers of girls.



(a) Find the value of

(i) p;

(ii) q. [4]

(b) A girl is selected at random. Find the probability that she takes economics but not history. [2]

(This question continues on the following page)

(Question 1 continued)

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

2. [Maximum mark: 5]

Let $f(x) = 5x$ and $g(x) = x^2 + 1$, for $x \in \mathbb{R}$.

(a) Find $f^{-1}(x)$. [2]

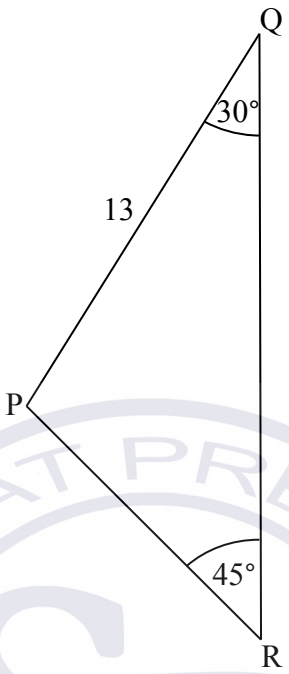
(b) Find $(f \circ g)(7)$. [3]

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

3. [Maximum mark: 6]

The following diagram shows triangle PQR.

diagram not to scale



$\text{P}\hat{\text{Q}}\text{R} = 30°$, $\text{Q}\hat{\text{R}}\text{P} = 45°$ and $\text{PQ} = 13\,\text{cm}$.

Find PR.

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

4. [Maximum mark: 6]

Jim heated a liquid until it boiled. He measured the temperature of the liquid as it cooled. The following table shows its temperature, d degrees Celsius, t minutes after it boiled.

t (min)	0	4	8	12	16	20
d (°C)	105	98.4	85.4	74.8	68.7	62.1

(a) (i) Write down the independent variable.

(ii) Write down the boiling temperature of the liquid. [2]

Jim believes that the relationship between d and t can be modelled by a linear regression equation.

(b) Jim describes the correlation as **very strong**. Circle the value below which best represents the correlation coefficient. [2]

0.992	0.251	0	–0.251	–0.992

(c) Jim's model is $d = -2.24t + 105$, for $0 \leq t \leq 20$. Use his model to predict the decrease in temperature for any 2 minute interval. [2]

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

5. [Maximum mark: 7]

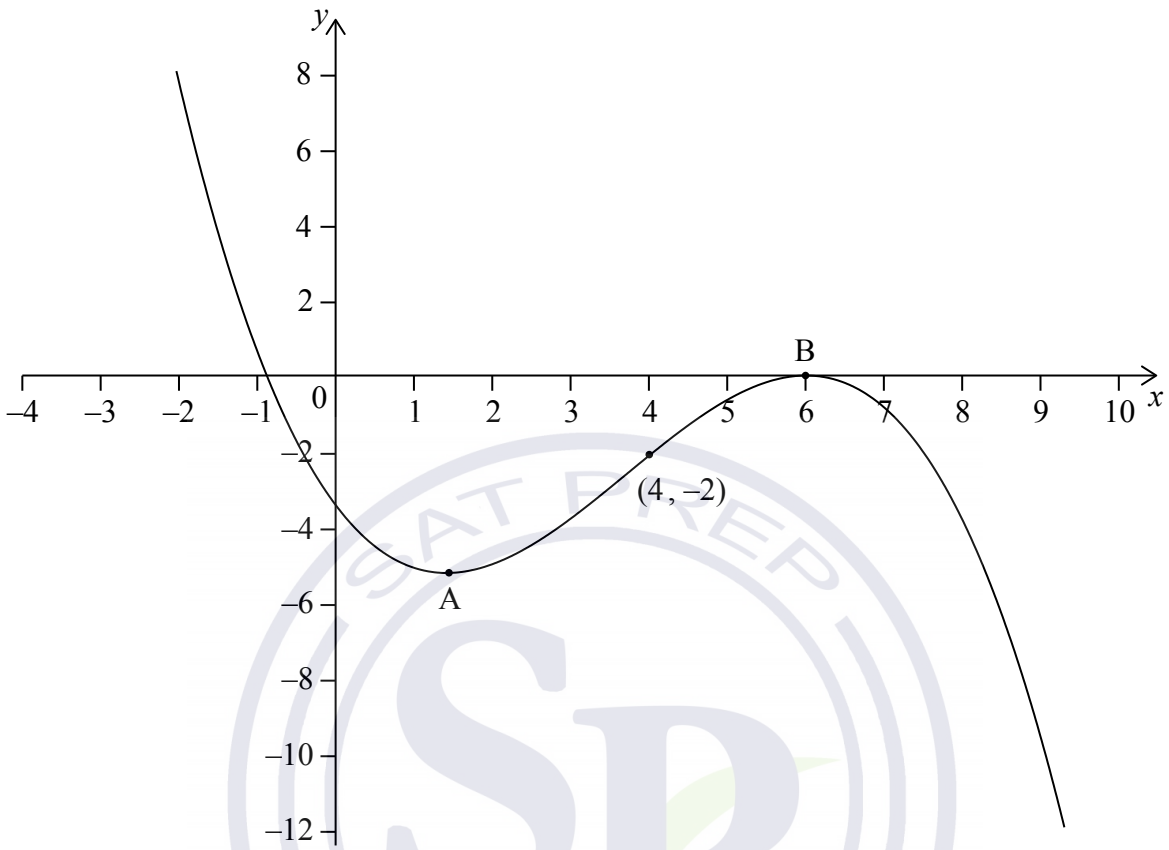
(a) Find $\int x\mathrm{e}^{x^2-1}\mathrm{d}x$. [4]

(b) Find $f(x)$, given that $f'(x) = x\mathrm{e}^{x^2-1}$ and $f(-1) = 3$. [3]

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

6. [Maximum mark: 6]

The following diagram shows the graph of f', the derivative of f.



The graph of f' has a local minimum at A, a local maximum at B and passes through $(4, -2)$.

(a) The point $P(4, 3)$ lies on the graph of the function, f.

(i) Write down the gradient of the curve of f at P.

(ii) Find the equation of the normal to the curve of f at P. [4]

(b) Determine the concavity of the graph of f when $4 < x < 5$ **and** justify your answer. [2]

(This question continues on the following page)

(Question 6 continued)

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

7. [Maximum mark: 8]

The first three terms of a geometric sequence are $\ln x^{16}$, $\ln x^{8}$, $\ln x^{4}$, for $x > 0$.

(a) Find the common ratio. [3]

(b) Solve $\sum_{k=1}^{\infty} 2^{5-k} \ln x = 64$. [5]

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 17]

A line L_1 passes through the points $\mathrm{A}(0, 1, 8)$ and $\mathrm{B}(3, 5, 2)$.

(a) (i) Find $\overrightarrow{\mathrm{AB}}$.

(ii) Hence, write down a vector equation for L_1. [4]

(b) A second line L_2, has equation $\boldsymbol{r} = \begin{pmatrix} 1 \\ 13 \\ -14 \end{pmatrix} + s\begin{pmatrix} p \\ 0 \\ 1 \end{pmatrix}$.

Given that L_1 and L_2 are perpendicular, show that $p = 2$. [3]

(c) The lines L_1 and L_2 intersect at $\mathrm{C}(9, 13, z)$. Find z. [5]

(d) (i) Find a unit vector in the direction of L_2.

(ii) Hence or otherwise, find one point on L_2 which is $\sqrt{5}$ units from C. [5]

9. [Maximum mark: 14]

A quadratic function f can be written in the form $f(x) = a(x - p)(x - 3)$. The graph of f has axis of symmetry $x = 2.5$ and y-intercept at $(0, -6)$.

(a) Find the value of p. [3]

(b) Find the value of a. [3]

(c) The line $y = kx - 5$ is a tangent to the curve of f. Find the values of k. [8]

Do **not** write solutions on this page.

10. [Maximum mark: 15]

The following table shows the probability distribution of a discrete random variable A, in terms of an angle θ.

a	1	2
$\mathrm{P}(A = a)$	$\cos\theta$	$2\cos 2\theta$

(a) Show that $\cos\theta = \frac{3}{4}$. [6]

(b) Given that $\tan\theta > 0$, find $\tan\theta$. [3]

(c) Let $y = \frac{1}{\cos x}$, for $0 < x < \frac{\pi}{2}$. The graph of y between $x = \theta$ and $x = \frac{\pi}{4}$ is rotated 360° about the x-axis. Find the volume of the solid formed. [6]

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Standard level
Paper 1

Thursday 4 May 2017 (afternoon)

Candidate session number

1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics SL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[90 marks]**.

2217 – 7305

Please **do not** write on this page.

Answers written on this page
will not be marked.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines if necessary.

1. [Maximum mark: 6]

In an arithmetic sequence, the first term is 3 and the second term is 7.

(a) Find the common difference. [2]

(b) Find the tenth term. [2]

(c) Find the sum of the first ten terms of the sequence. [2]

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

2. [Maximum mark: 7]

The vectors $\boldsymbol{a} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} k+3 \\ k \end{pmatrix}$ are perpendicular to each other.

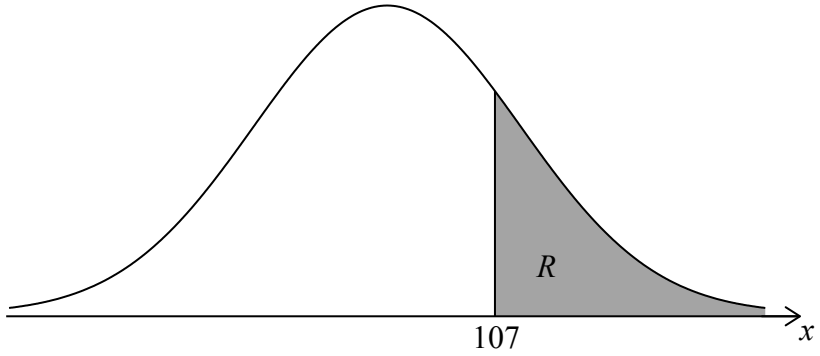
(a) Find the value of k. [4]

(b) Given that $\boldsymbol{c} = \boldsymbol{a} + 2\boldsymbol{b}$, find $\boldsymbol{c}$. [3]

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

3. [Maximum mark: 6]

The random variable X is normally distributed with a mean of 100. The following diagram shows the normal curve for X.



Let R be the shaded region under the curve, to the right of 107. The area of R is 0.24.

(a) Write down $\mathrm{P}(X > 107)$. [1]

(b) Find $\mathrm{P}(100 < X < 107)$. [3]

(c) Find $\mathrm{P}(93 < X < 107)$. [2]

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

4. [Maximum mark: 6]

The following figures consist of rows and columns of squares. The figures form a continuing pattern.

Figure 1 has two rows and one column. Figure 2 has three rows and two columns.

Figure 1 Figure 2 Figure 3 Figure 4

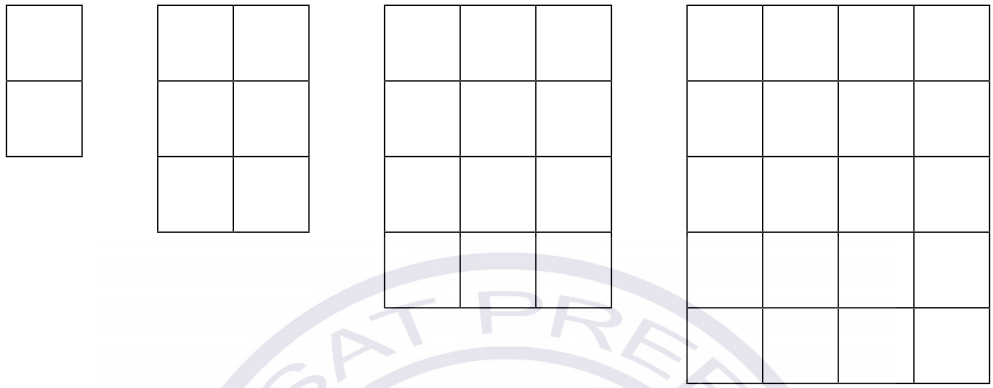


Figure 5 has p rows and q columns.

(a) Write down the value of

(i) p;

(ii) q. [2]

Each small square has an area of $1\,\text{cm}^2$. Let A_n be the total area of Figure n. The following table gives the first five values of A_n.

n	1	2	3	4	5
$A_n\,(\text{cm}^2)$	2	6	12	20	k

(b) Find the value of k. [2]

(c) Find an expression for A_n in terms of n. [2]

(This question continues on the following page)

(Question 4 continued)

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

5. [Maximum mark: 6]

Let $f'(x) = \frac{3x^2}{\left(x^3+1\right)^5}$. Given that $f(0) = 1$, find $f(x)$.

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

6. [Maximum mark: 5]

The values of the functions f and g and their derivatives for $x = 1$ and $x = 8$ are shown in the following table.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	2	4	9	–3
8	4	–3	2	5

Let $h(x) = f(x)g(x)$.

(a) Find $h(1)$. [2]

(b) Find $h'(8)$. [3]

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

Please **do not** write on this page.

Answers written on this page
will not be marked.

7. [Maximum mark: 7]

Solve $\log_2(2\sin x) + \log_2(\cos x) = -1$, for $2\pi < x < \frac{5\pi}{2}$.

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

Do **not** write solutions on this page.

Section B

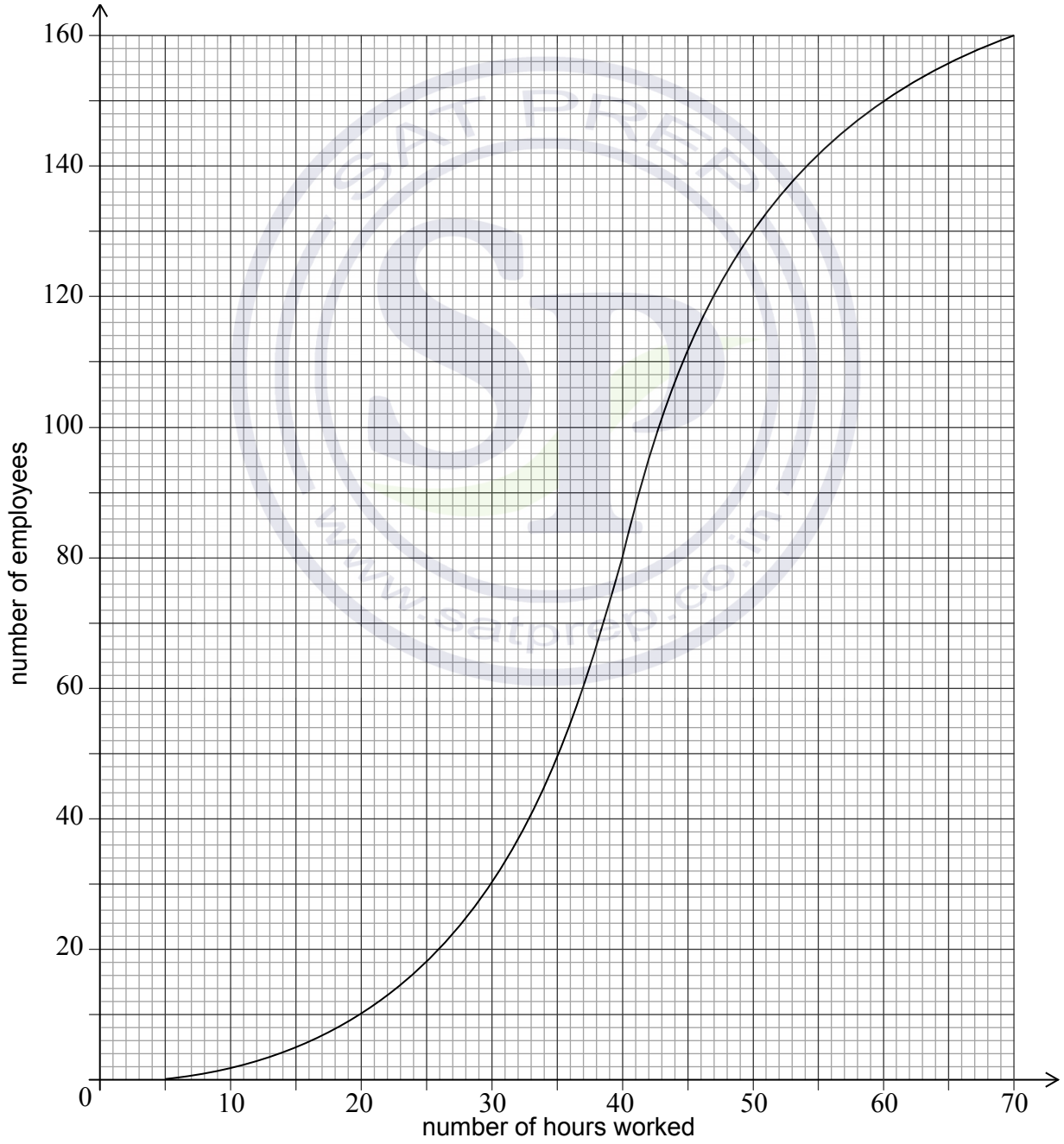
Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 14]

A city hired 160 employees to work at a festival. The following cumulative frequency curve shows the number of hours employees worked during the festival.

(a) (i) Find the median number of hours worked by the employees.

(ii) Write down the number of employees who worked 50 hours or less. [3]



(This question continues on the following page)

Do **not** write solutions on this page.

(Question 8 continued)

The city paid each of the employees £8 per hour for the first 40 hours worked, and £10 per hour for each hour they worked after the first 40 hours.

(b) Find the amount of money an employee earned for working

(i) 40 hours;

(ii) 43 hours. [4]

(c) Find the number of employees who earned £200 or less. [3]

(d) Only 10 employees earned more than £k. Find the value of k. [4]

Do **not** write solutions on this page.

9. [Maximum mark: 16]

Note: In this question, distance is in metres and time is in seconds.

Two particles P_1 and P_2 start moving from a point A at the same time, along different straight lines.

After t seconds, the position of P_1 is given by $\boldsymbol{r} = \begin{pmatrix} 4 \\ -1 \\ 3 \end{pmatrix} + t \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$.

(a) Find the coordinates of A. [2]

Two seconds after leaving A, P_1 is at point B.

(b) Find

(i) $\vec{\text{AB}}$;

(ii) $\left|\vec{\text{AB}}\right|$. [5]

Two seconds after leaving A, P_2 is at point C, where $\vec{\text{AC}} = \begin{pmatrix} 3 \\ 0 \\ 4 \end{pmatrix}$.

(c) Find $\cos \text{B}\hat{\text{A}}\text{C}$. [5]

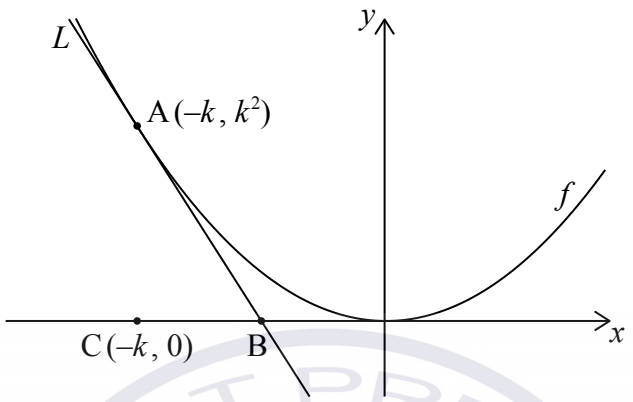
(d) Hence or otherwise, find the distance between P_1 and P_2 two seconds after they leave A. [4]

Do **not** write solutions on this page.

10. [Maximum mark: 17]

Let $f(x) = x^2$. The following diagram shows part of the graph of f.

diagram not to scale



The line L is the tangent to the graph of f at the point $A(-k, k^2)$, and intersects the x-axis at point B. The point C is $(-k, 0)$.

(a) (i) Write down $f'(x)$.

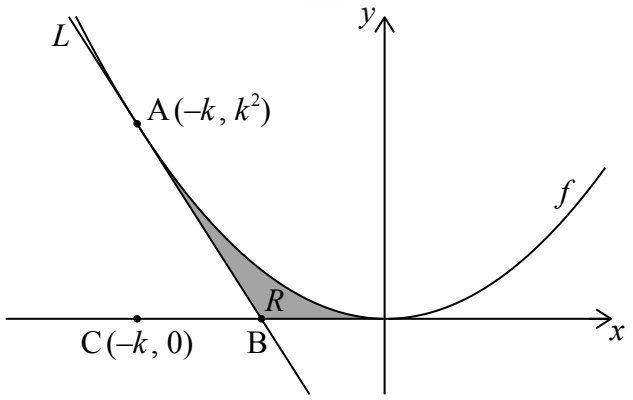
(ii) Find the gradient of L. [3]

(b) Show that the x-coordinate of B is $-\frac{k}{2}$. [5]

(c) Find the area of triangle ABC, giving your answer in terms of k. [2]

The region R is enclosed by L, the graph of f, and the x-axis. This is shown in the following diagram.

diagram not to scale



(d) Given that the area of triangle ABC is p times the area of R, find the value of p. [7]

Please **do not** write on this page.

Answers written on this page
will not be marked.

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Standard level
Paper 1

Thursday 10 November 2016 (afternoon)

Candidate session number

1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics SL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[90 marks]**.

8816 – 7301

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.

1. [Maximum mark: 6]

Let $f(x) = x^2 - 4x + 5$.

(a) Find the equation of the axis of symmetry of the graph of f. [2]

The function can also be expressed in the form $f(x) = (x - h)^2 + k$.

(b) (i) Write down the value of h.

(ii) Find the value of k. [4]

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

2. [Maximum mark: 5]

Let $\sin\theta = \frac{\sqrt{5}}{3}$, where θ is acute.

(a) Find $\cos\theta$. [3]

(b) Find $\cos 2\theta$. [2]

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

3. [Maximum mark: 7]

The values in the fourth row of Pascal's triangle are shown in the following table.

1	4	6	4	1

(a) Write down the values in the fifth row of Pascal's triangle. [2]

(b) Hence or otherwise, find the term in x^3 in the expansion of $(2x + 3)^5$. [5]

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

4. [Maximum mark: 7]

The position vectors of points P and Q are $\boldsymbol{i} + 2\boldsymbol{j} - \boldsymbol{k}$ and $7\boldsymbol{i} + 3\boldsymbol{j} - 4\boldsymbol{k}$ respectively.

(a) Find a vector equation of the line that passes through P and Q. [4]

(b) The line through P and Q is perpendicular to the vector $2\boldsymbol{i} + n\boldsymbol{k}$. Find the value of n. [3]

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

5. [Maximum mark: 6]

Events A and B are independent with $\mathrm{P}(A \cap B) = 0.2$ and $\mathrm{P}(A' \cap B) = 0.6$.

(a) Find $\mathrm{P}(B)$. [2]

(b) Find $\mathrm{P}(A \cup B)$. [4]

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

6. [Maximum mark: 7]

Let $f'(x) = \sin^3(2x)\cos(2x)$. Find $f(x)$, given that $f\left(\frac{\pi}{4}\right) = 1$.

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

7. [Maximum mark: 7]

Let $f(x) = m - \frac{1}{x}$, for $x \neq 0$. The line $y = x - m$ intersects the graph of f in two distinct points. Find the possible values of m.

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 16]

Let $\overrightarrow{OA} = \begin{pmatrix} -1 \\ 0 \\ 4 \end{pmatrix}$ and $\overrightarrow{OB} = \begin{pmatrix} 4 \\ 1 \\ 3 \end{pmatrix}$.

(a) (i) Find $\overrightarrow{AB}$.

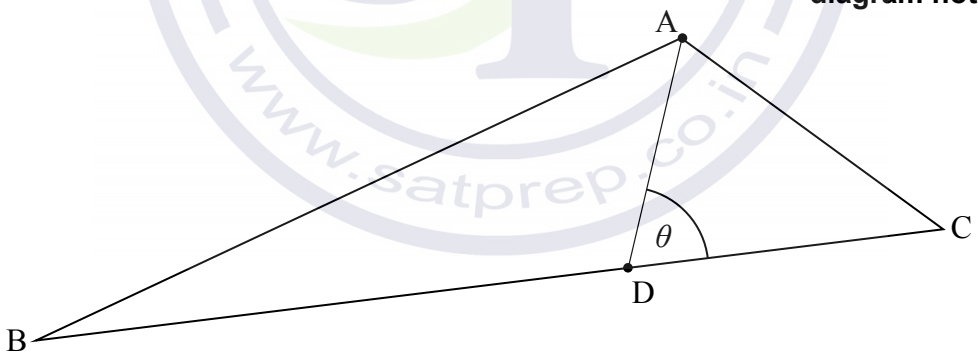
(ii) Find $\left|\overrightarrow{AB}\right|$. [4]

The point C is such that $\overrightarrow{AC} = \begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix}$.

(b) Show that the coordinates of C are $(-2, 1, 3)$. [1]

The following diagram shows triangle ABC. Let D be a point on [BC], with acute angle $ADC = \theta$.

diagram not to scale



(c) Write down an expression in terms of θ for

(i) angle ADB;

(ii) area of triangle ABD. [2]

(d) Given that $\frac{\text{area } \Delta ABD}{\text{area } \Delta ACD} = 3$, show that $\frac{BD}{BC} = \frac{3}{4}$. [5]

(e) Hence or otherwise, find the coordinates of point D. [4]

Do **not** write solutions on this page.

9. [Maximum mark: 13]

The first two terms of an infinite geometric sequence, in order, are

$$2\log_2 x,\ \log_2 x, \text{ where } x > 0.$$

(a) Find r. [2]

(b) Show that the sum of the infinite sequence is $4\log_2 x$. [2]

The first three terms of an arithmetic sequence, in order, are

$$\log_2 x,\ \log_2\left(\frac{x}{2}\right),\ \log_2\left(\frac{x}{4}\right), \text{ where } x > 0.$$

(c) Find d, giving your answer as an integer. [4]

Let S_{12} be the sum of the first 12 terms of the arithmetic sequence.

(d) Show that $S_{12} = 12\log_2 x - 66$. [2]

(e) Given that S_{12} is equal to half the sum of the infinite geometric sequence, find x, giving your answer in the form 2^p, where $p \in \mathbb{Q}$. [3]

Do **not** write solutions on this page.

10. [Maximum mark: 16]

Let $f(x) = \cos x$.

(a) (i) Find the first four derivatives of $f(x)$.

(ii) Find $f^{(19)}(x)$. [4]

Let $g(x) = x^k$, where $k \in \mathbb{Z}^+$.

(b) (i) Find the first three derivatives of $g(x)$.

(ii) Given that $g^{(19)}(x) = \frac{k!}{(k-p)!}\left(x^{k-19}\right)$, find p. [5]

Let $k = 21$ and $h(x) = \left(f^{(19)}(x) \times g^{(19)}(x)\right)$.

(c) (i) Find $h'(x)$.

(ii) Hence, show that $h'(\pi) = \frac{-21!}{2}\pi^2$. [7]

Please **do not** write on this page.

Answers written on this page will not be marked.

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Standard level
Paper 1

Tuesday 10 May 2016 (afternoon)

Candidate session number

1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics SL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[90 marks]**.

2216 – 7303

Please **do not** write on this page.

Answers written on this page will not be marked.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.

1. [Maximum mark: 5]

Let $f(x) = 8x + 3$ and $g(x) = 4x$, for $x \in \mathbb{R}$.

(a) Write down $g(2)$. [1]

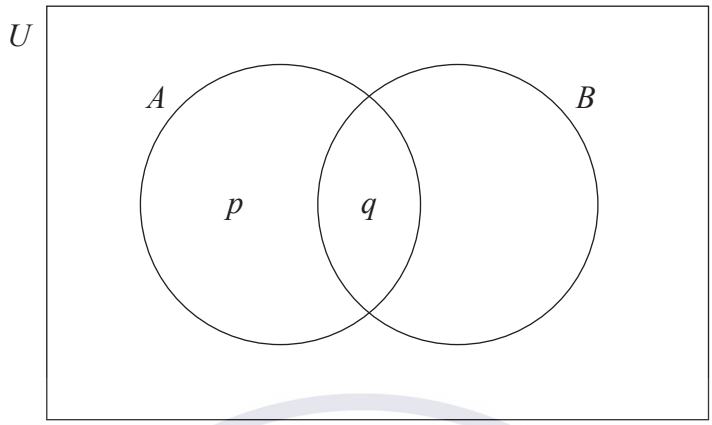
(b) Find $(f \circ g)(x)$. [2]

(c) Find $f^{-1}(x)$. [2]

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

2. [Maximum mark: 6]

The following Venn diagram shows the events A and B, where $\mathrm{P}(A) = 0.4$, $\mathrm{P}(A \cup B) = 0.8$ and $\mathrm{P}(A \cap B) = 0.1$. The values p and q are probabilities.



(a) (i) Write down the value of q.

(ii) Find the value of p. [3]

(b) Find $\mathrm{P}(B)$. [3]

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

3. [Maximum mark: 7]

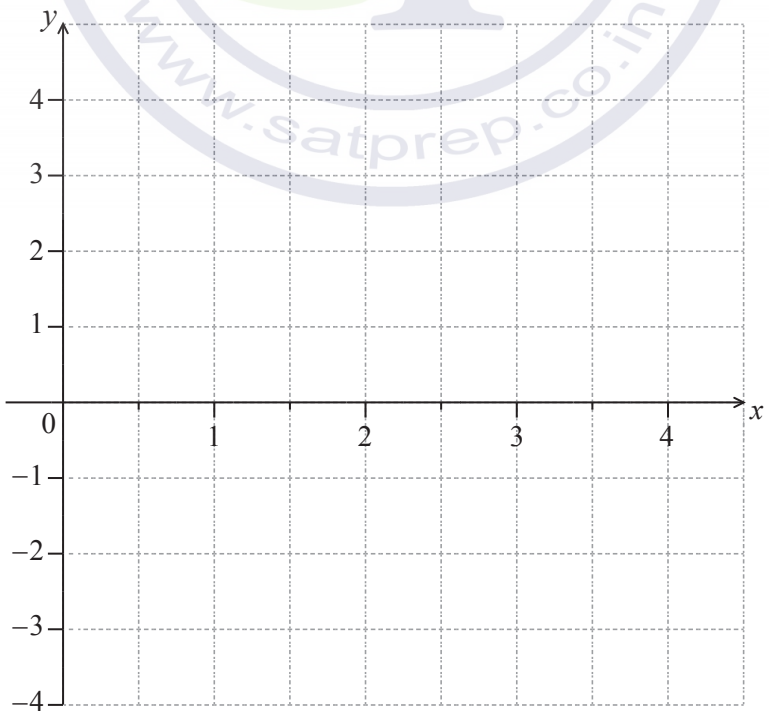
Let $f(x) = 3\sin\left(\frac{\pi}{2}x\right)$, for $0 \le x \le 4$.

(a) (i) Write down the amplitude of f.

(ii) Find the period of f. [3]

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

(b) On the following grid sketch the graph of f. [4]



4. [Maximum mark: 6]

Consider the following sequence of figures.

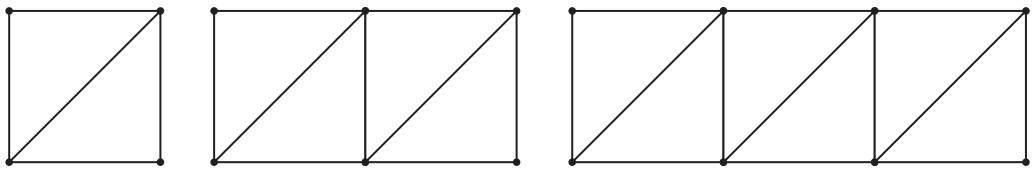


Figure 1 Figure 2 Figure 3

Figure 1 contains 5 line segments.

(a) Given that Figure n contains 801 line segments, show that $n = 200$. [3]

(b) Find the total number of line segments in the first 200 figures. [3]

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

5. [Maximum mark: 6]

Consider $f(x) = x^2 + qx + r$. The graph of f has a minimum value when $x = -1.5$.
The distance between the two zeros of f is 9.

(a) Show that the two zeros are 3 and -6. [2]

(b) Find the value of q and of r. [4]

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

6. [Maximum mark: 7]

The following diagram shows triangle ABC. The point D lies on [BC] so that [AD] bisects $\hat{BAC}$.

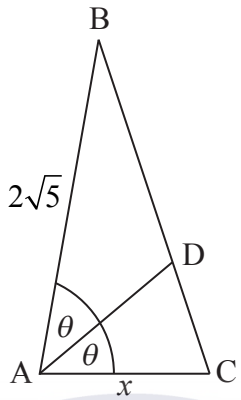


diagram not to scale

$AB = 2\sqrt{5}$ cm, $AC = x$ cm, and $D\hat{A}C = \theta$, where $\sin\theta = \frac{2}{3}$

The area of triangle ABC is $5\,\text{cm}^2$. Find the value of x.

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

7. [Maximum mark: 8]

Let $f(x) = 3\tan^4 x + 2k$ and $g(x) = -\tan^4 x + 8k\tan^2 x + k$, for $0 \le x \le 1$, where $0 < k < 1$. The graphs of f and g intersect at exactly one point. Find the value of k.

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 15]

A school collects cans for recycling to raise money. Sam's class has 20 students.
The number of cans collected by each student in Sam's class is shown in the following stem and leaf diagram.

Stem	Leaf
2	0, 1, 4, 9, 9
3	1, 7, 7, 7, 8, 8
4	1, 2, 2, 3, 5, 6, 7, 8
5	0

Key: 3|1 represents 31 cans

(a) Find the median number of cans collected. [2]

The following box-and-whisker plot also displays the number of cans collected by students in Sam's class.

(b) (i) Write down the value of a.

(ii) The interquartile range is 14. Find the value of b. [3]

(c) Sam's class collected 745 cans. They want an average of 40 cans per student.
How many more cans need to be collected to achieve this target? [3]

There are 80 students in the school.

(d) The students raise $\$0.10$ for each recycled can.

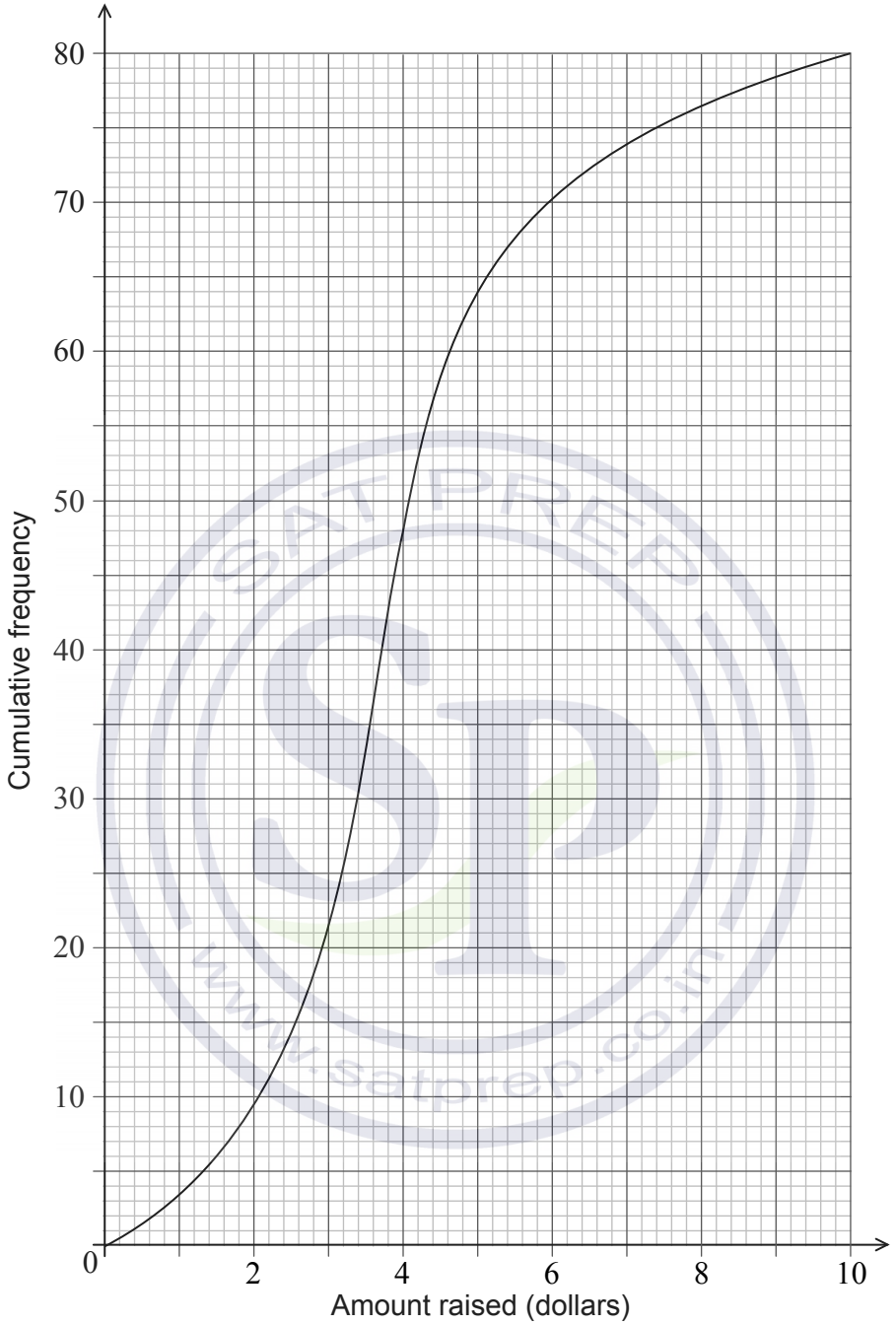
(i) Find the largest amount raised by a student in Sam's class.

(ii) The following cumulative frequency curve shows the amounts in dollars raised by all the students in the school. Find the percentage of students in the school who raised more money than anyone in Sam's class. [5]

(This question continues on the following page)

Do **not** write solutions on this page.

(Question 8 continued)



(e) The mean number of cans collected is 39.4. The standard deviation is 18.5. Each student then collects 2 more cans.

(i) Write down the new mean.

(ii) Write down the new standard deviation. [2]

Do **not** write solutions on this page.

9. [Maximum mark: 15]

Let $f'(x) = \frac{6-2x}{6x-x^2}$, for $0 < x < 6$.

The graph of f has a maximum point at P.

(a) Find the x-coordinate of P. [3]

The y-coordinate of P is $\ln 27$.

(b) Find $f(x)$, expressing your answer as a single logarithm. [8]

(c) The graph of f is transformed by a vertical stretch with scale factor $\frac{1}{\ln 3}$. The image of P under this transformation has coordinates (a, b).

Find the value of a and of b, where $a, b \in \mathbb{N}$. [4]

10. [Maximum mark: 15]

Let $f(x) = \sqrt{4x+5}$, for $x \geq -1.25$.

(a) Find $f'(1)$. [4]

Consider another function g. Let R be a point on the graph of g. The x-coordinate of R is 1. The equation of the tangent to the graph at R is $y = 3x + 6$.

(b) Write down $g'(1)$. [2]

(c) Find $g(1)$. [2]

(d) Let $h(x) = f(x) \times g(x)$. Find the equation of the tangent to the graph of h at the point where $x = 1$. [7]

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Standard level
Paper 1

Tuesday 10 May 2016 (afternoon)

Candidate session number

1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics SL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[90 marks]**.

2216 – 7305

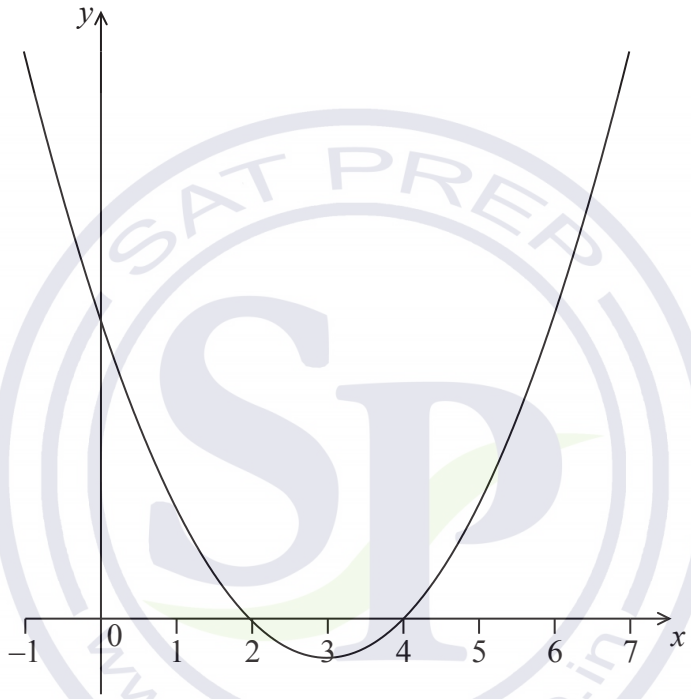
Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.

1. [Maximum mark: 6]

The following diagram shows part of the graph of a quadratic function f.



The vertex is at $(3, -1)$ and the x-intercepts at 2 and 4.

The function f can be written in the form $f(x) = (x - h)^2 + k$.

(a) Write down the value of h and of k. [2]

The function can also be written in the form $f(x) = (x - a)(x - b)$.

(b) Write down the value of a and of b. [2]

(c) Find the y-intercept. [2]

(This question continues on the following page)

(Question 1 continued)

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

2. [Maximum mark: 5]

There are 10 items in a data set. The sum of the items is 60.

(a) Find the mean. [2]

The variance of this data set is 3. Each value in the set is multiplied by 4.

(b) (i) Write down the value of the new mean.

(ii) Find the value of the new variance. [3]

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

3. [Maximum mark: 6]

Let $x = \ln 3$ and $y = \ln 5$. Write the following expressions in terms of x and y.

(a) $\ln\left(\frac{5}{3}\right)$. [2]

(b) $\ln 45$. [4]

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

4. [Maximum mark: 6]

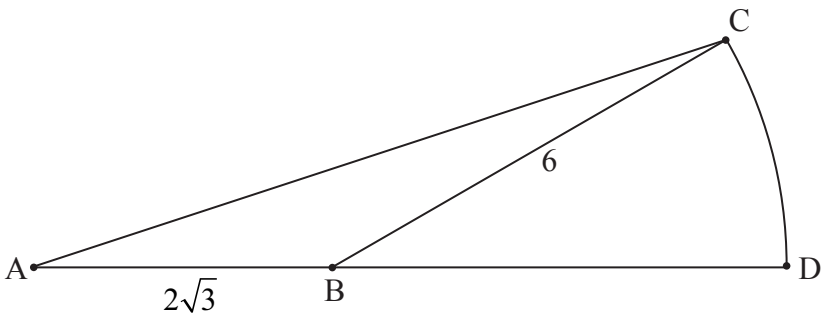
Three consecutive terms of a geometric sequence are $x - 3$, 6 and $x + 2$.
Find the possible values of x.

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

5. [Maximum mark: 8]

The following diagram shows a triangle ABC and a sector BDC of a circle with centre B and radius 6 cm. The points A, B and D are on the same line.

diagram not to scale



$AB = 2\sqrt{3}$ cm, $BC = 6$ cm, area of triangle $ABC = 3\sqrt{3}$ cm^2, $A\hat{B}C$ is obtuse.

(a) Find $A\hat{B}C$. [5]

(b) Find the exact area of the sector BDC. [3]

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

6. [Maximum mark: 7]

Let $f(x) = 6x\sqrt{1-x^2}$, for $-1 \leq x \leq 1$, and $g(x) = \cos(x)$, for $0 \leq x \leq \pi$.
Let $h(x) = (f \circ g)(x)$.

(a) Write $h(x)$ in the form $a\sin(bx)$, where $a, b \in \mathbb{Z}$. [5]

(b) Hence find the range of h. [2]

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

7. [Maximum mark: 7]

Let $\boldsymbol{u} = -3\boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k}$ and $\boldsymbol{v} = m\boldsymbol{j} + n\boldsymbol{k}$, where $m, n \in \mathbb{R}$. Given that $\boldsymbol{v}$ is a unit vector perpendicular to $\boldsymbol{u}$, find the possible values of m and of n.

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

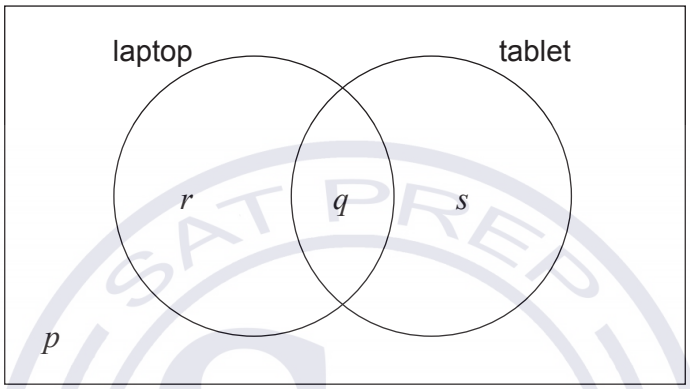
Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 13]

In a class of 21 students, 12 own a laptop, 10 own a tablet, and 3 own neither.
The following Venn diagram shows the events "own a laptop" and "own a tablet".
The values p, q, r and s represent numbers of students.



(a) (i) Write down the value of p.

(ii) Find the value of q.

(iii) Write down the value of r and of s. [5]

(b) A student is selected at random from the class.

(i) Write down the probability that this student owns a laptop.

(ii) Find the probability that this student owns a laptop or a tablet but not both. [4]

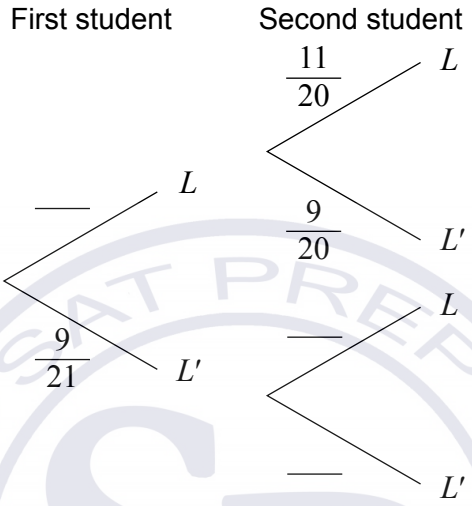
(This question continues on the following page)

Do **not** write solutions on this page.

(Question 8 continued)

(c) Two students are randomly selected from the class. Let L be the event a "student owns a laptop".

(i) **Copy** and complete the following tree diagram. (Do **not** write on this page.)

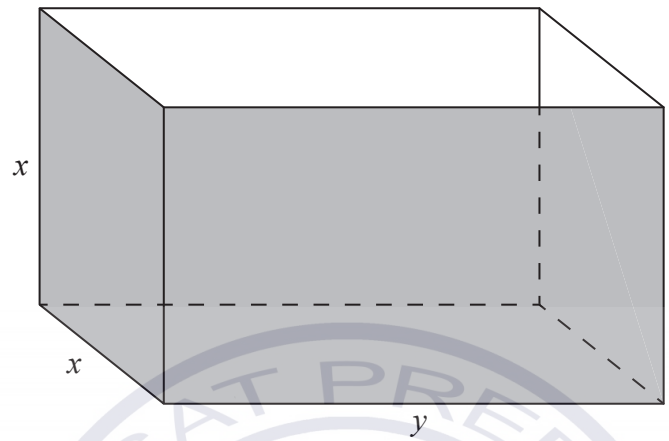


(ii) Write down the probability that the second student owns a laptop given that the first owns a laptop. [4]

Do **not** write solutions on this page.

9. [Maximum mark: 16]

Fred makes an open metal container in the shape of a cuboid, as shown in the following diagram.



The container has height x m, width x m and length y m. The volume is $36\,\text{m}^3$.

Let $A(x)$ be the outside surface area of the container.

(a) Show that $A(x) = \frac{108}{x} + 2x^2$. [4]

(b) Find $A'(x)$. [2]

(c) Given that the outside surface area is a minimum, find the height of the container. [5]

(d) Fred paints the outside of the container. A tin of paint covers a surface area of $10\,\text{m}^2$ and costs $\$20$. Find the total cost of the tins needed to paint the container. [5]

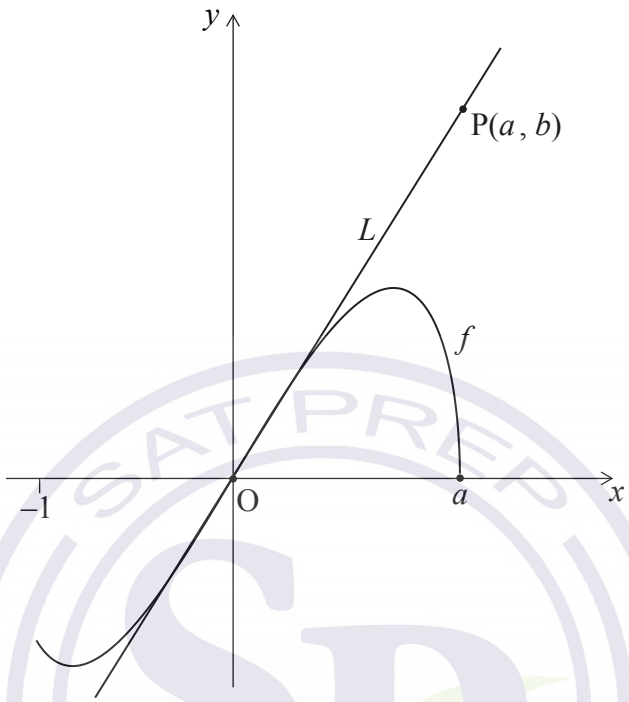
Please **do not** write on this page.

Answers written on this page will not be marked.

Do **not** write solutions on this page.

10. [Maximum mark: 16]

The following diagram shows the graph of $f(x) = 2x\sqrt{a^2 - x^2}$, for $-1 \leq x \leq a$, where $a > 1$.



The line L is the tangent to the graph of f at the origin, O. The point $\mathrm{P}(a, b)$ lies on L.

(a) (i) Given that $f'(x) = \dfrac{2a^2 - 4x^2}{\sqrt{a^2 - x^2}}$, for $-1 \leq x < a$, find the equation of L.

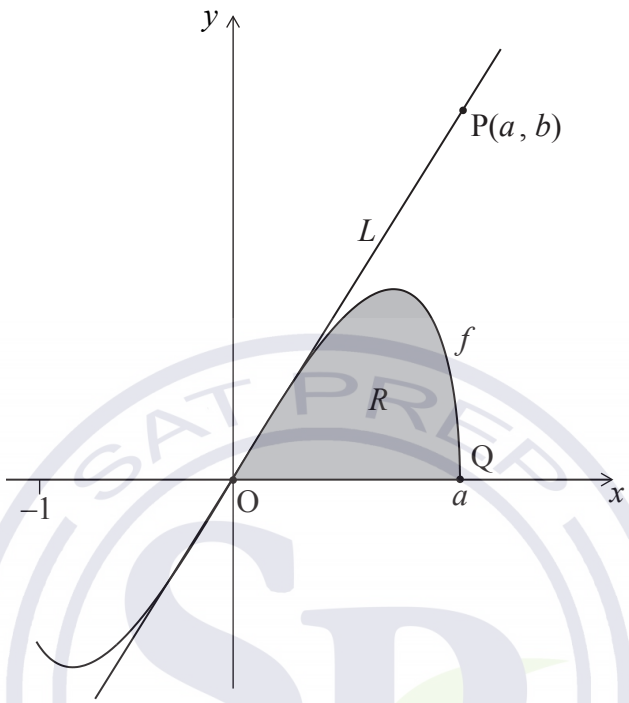
(ii) Hence or otherwise, find an expression for b in terms of a. [6]

(This question continues on the following page)

Do **not** write solutions on this page.

(Question 10 continued)

The point $\mathrm{Q}(a, 0)$ lies on the graph of f. Let R be the region enclosed by the graph of f and the x-axis. This information is shown in the following diagram.



Let A_R be the area of the region R.

(b) Show that $A_R = \frac{2}{3}a^3$. [6]

(c) Let A_T be the area of the triangle OPQ. Given that $A_T = kA_R$, find the value of k. [4]

Please **do not** write on this page.

Answers written on this page will not be marked.

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Standard level
Paper 1

Wednesday 11 November 2015 (morning)

Candidate session number

1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics SL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[90 marks]**.

8815–7301

Please **do not** write on this page.

Answers written on this page will not be marked.

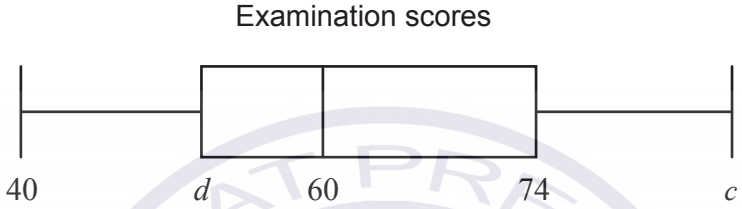
Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.

1. [Maximum mark: 5]

The following box-and-whisker plot represents the examination scores of a group of students.



(a) Write down the median score. [1]

The range of the scores is 47 marks, and the interquartile range is 22 marks.

(b) Find the value of

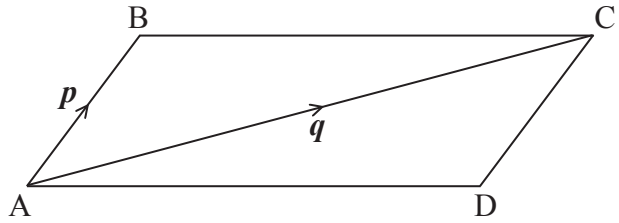
(i) c;

(ii) d. [4]

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

2. [Maximum mark: 7]

The following diagram shows the parallelogram ABCD.



Let $\overrightarrow{AB} = \boldsymbol{p}$ and $\overrightarrow{AC} = \boldsymbol{q}$. Find each of the following vectors in terms of $\boldsymbol{p}$ and/or $\boldsymbol{q}$.

(a) $\overrightarrow{CB}$ [2]

(b) $\overrightarrow{CD}$ [2]

(c) $\overrightarrow{DB}$ [3]

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

3. [Maximum mark: 6]

Let $f'(x) = 6x^2 - 5$. Given that $f(2) = -3$, find $f(x)$.

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

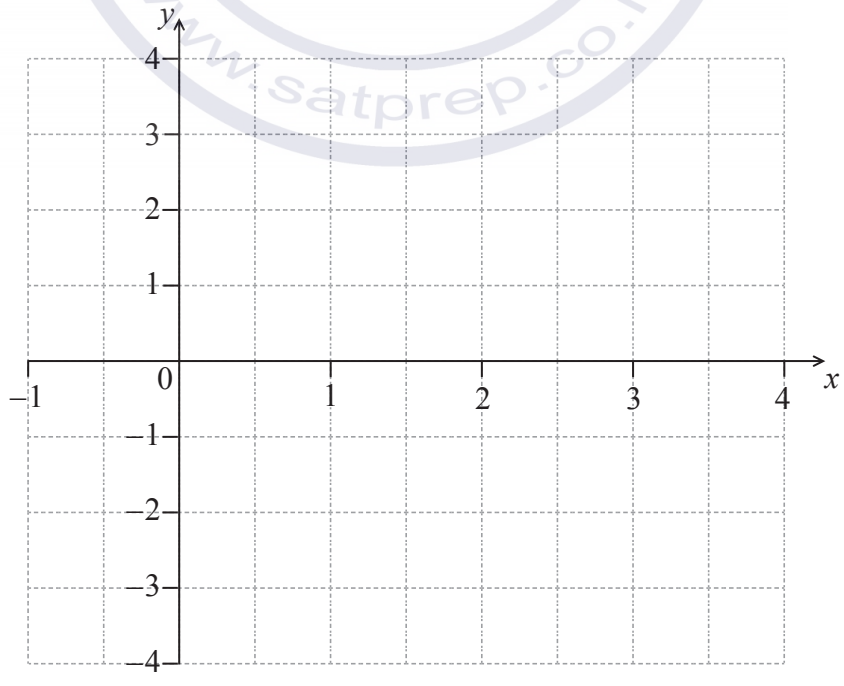
4. [Maximum mark: 7]

Let $f(x) = 3\sin(\pi x)$.

(a) Write down the amplitude of f. [1]

(b) Find the period of f. [2]

(c) On the following grid, sketch the graph of $y = f(x)$, for $0 \le x \le 3$. [4]



5. [Maximum mark: 6]

Let $f(x) = (x-5)^3$, for $x \in \mathbb{R}$.

(a) Find $f^{-1}(x)$. [3]

(b) Let g be a function so that $(f \circ g)(x) = 8x^6$. Find $g(x)$. [3]

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

6. [Maximum mark: 7]

In the expansion of $(3x+1)^n$, the coefficient of the term in x^2 is $135n$, where $n \in \mathbb{Z}^+$. Find n.

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

7. [Maximum mark: 6]

An arithmetic sequence has the first term $\ln a$ and a common difference $\ln 3$.
The 13th term in the sequence is $8\ln 9$. Find the value of a.

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

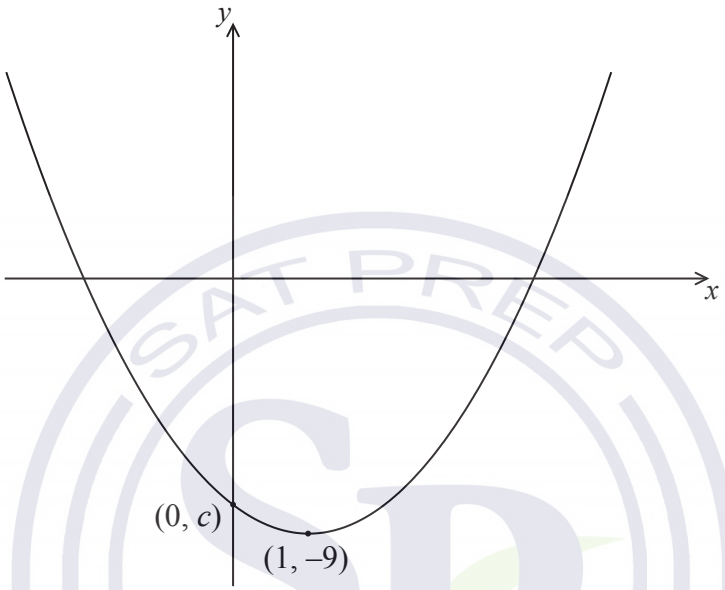
Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 16]

The following diagram shows part of the graph of a quadratic function f.



The vertex is at $(1, -9)$, and the graph crosses the y-axis at the point $(0, c)$.

The function can be written in the form $f(x) = (x - h)^2 + k$.

(a) Write down the value of h and of k. [2]

(b) Find the value of c. [2]

Let $g(x) = -(x - 3)^2 + 1$. The graph of g is obtained by a reflection of the graph of f in the x-axis, followed by a translation of $\begin{pmatrix} p \\ q \end{pmatrix}$.

(c) Find the value of p and of q. [5]

(d) Find the x-coordinates of the points of intersection of the graphs of f and g. [7]

Do **not** write solutions on this page.

9. [Maximum mark: 15]

A line L_1 passes through the points $\mathrm{A}(0, -3, 1)$ and $\mathrm{B}(-2, 5, 3)$.

(a) (i) Show that $\vec{\mathrm{AB}} = \begin{pmatrix} -2 \\ 8 \\ 2 \end{pmatrix}$.

(ii) Write down a vector equation for L_1. [3]

A line L_2 has equation $\boldsymbol{r} = \begin{pmatrix} -1 \\ 7 \\ -4 \end{pmatrix} + s\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$. The lines L_1 and L_2 intersect at a point C.

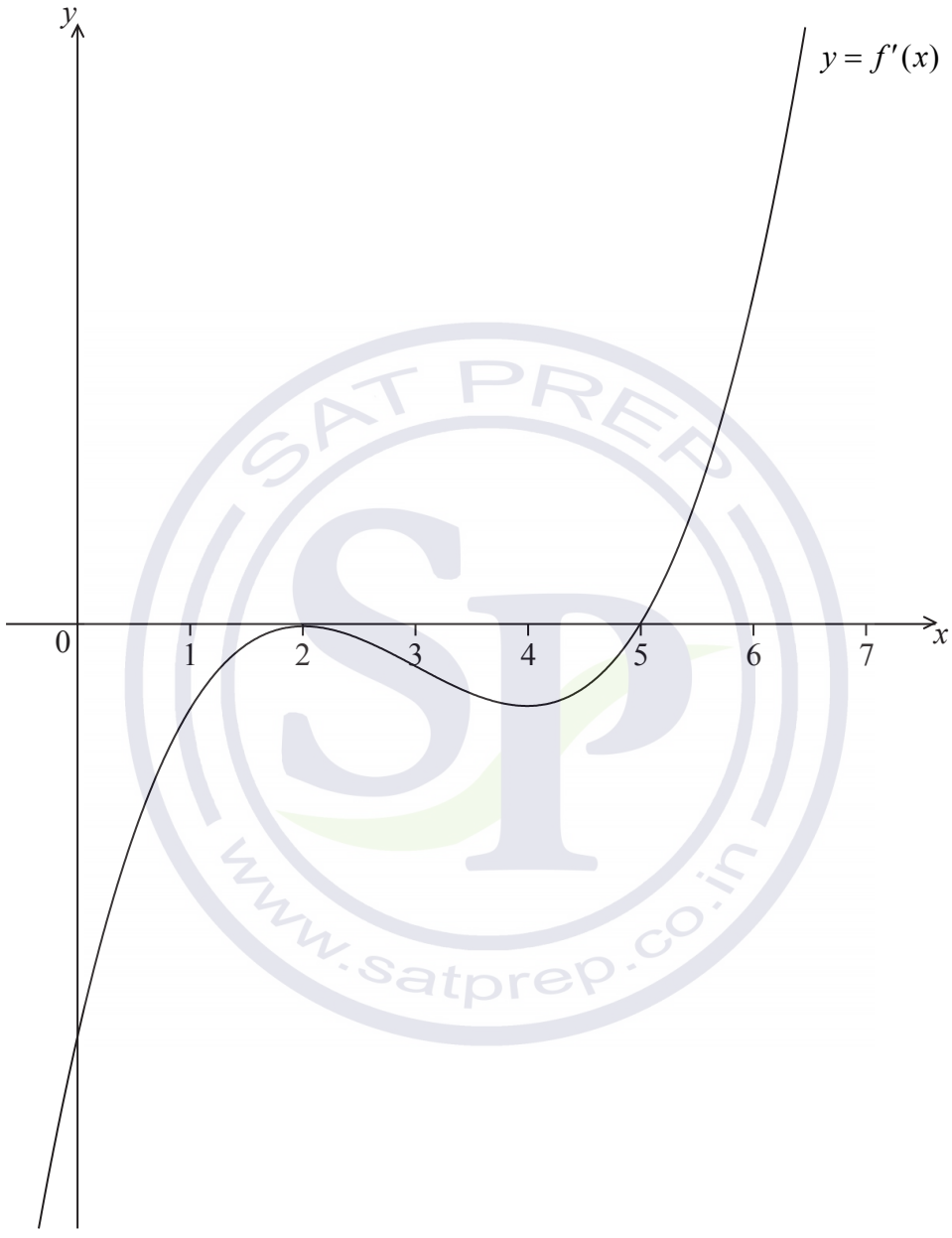
(b) Show that the coordinates of C are $(-1, 1, 2)$. [5]

(c) A point D lies on line L_2 so that $\left|\vec{\mathrm{CD}}\right| = \sqrt{18}$ and $\vec{\mathrm{CA}} \bullet \vec{\mathrm{CD}} = -9$. Find $\mathrm{A\hat{C}D}$. [7]

Do **not** write solutions on this page.

10. [Maximum mark: 15]

Let $y = f(x)$, for $-0.5 \le x \le 6.5$. The following diagram shows the graph of f', the derivative of f.



The graph of f' has a local maximum when $x = 2$, a local minimum when $x = 4$, and it crosses the x-axis at the point $(5, 0)$.

(a) Explain why the graph of f has a local minimum when $x = 5$. [2]

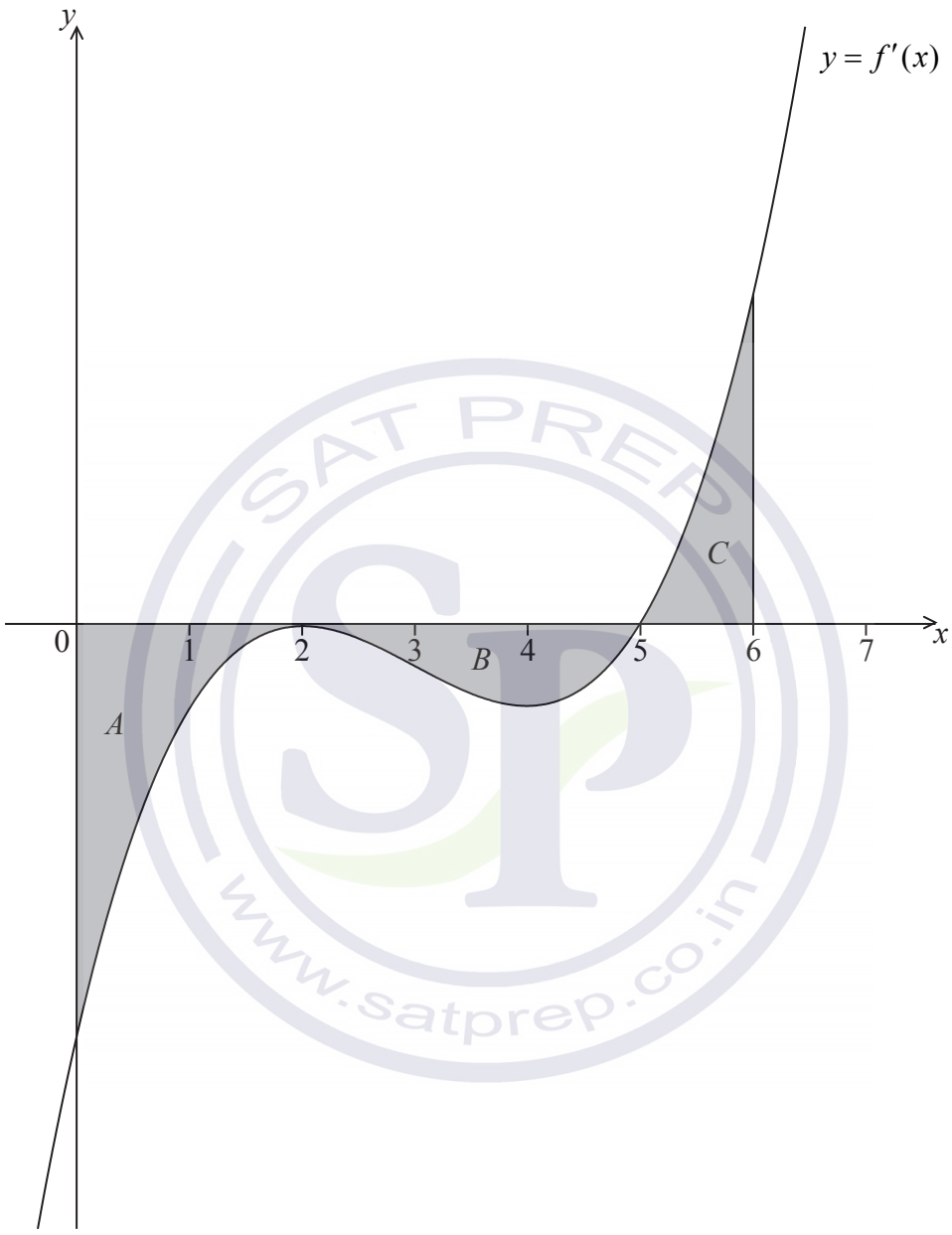
(b) Find the set of values of x for which the graph of f is concave down. [2]

(This question continues on the following page)

Do **not** write solutions on this page.

(Question 10 continued)

The following diagram shows the shaded regions A, B and C.



The regions are enclosed by the graph of f', the x-axis, the y-axis, and the line $x = 6$.
The area of region A is 12, the area of region B is 6.75 and the area of region C is 6.75.

(c) Given that $f(0) = 14$, find $f(6)$. [5]

(d) Let $g(x) = \left(f(x)\right)^2$. Given that $f'(6) = 16$, find the equation of the tangent to the graph of g at the point where $x = 6$. [6]

Please **do not** write on this page.

Answers written on this page will not be marked.

Please **do not** write on this page.

Answers written on this page will not be marked.

Please **do not** write on this page.

Answers written on this page will not be marked.

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Standard level
Paper 1

Tuesday 12 May 2015 (morning)

Candidate session number

1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **Mathematics SL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[90 marks]**.

2215–7303

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.

1. [Maximum mark: 6]

A discrete random variable X has the following probability distribution.

x	0	1	2	3
$\mathrm{P}(X=x)$	$\frac{3}{10}$	$\frac{4}{10}$	$\frac{2}{10}$	p

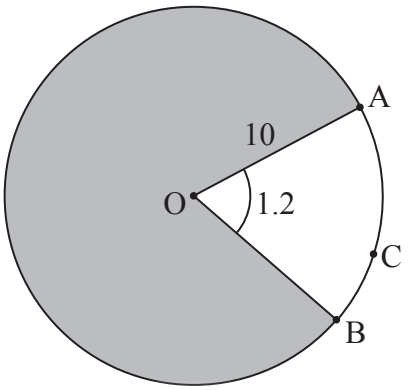
(a) Find p. [3]

(b) Find $\mathrm{E}(X)$. [3]

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

2. [Maximum mark: 5]

The following diagram shows a circle with centre O and a radius of $10\,\text{cm}$. Points A, B and C lie on the circle.



Angle AOB is 1.2 radians.

(a) Find the length of arc ACB. [2]

(b) Find the perimeter of the shaded region. [3]

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

3. [Maximum mark: 6]

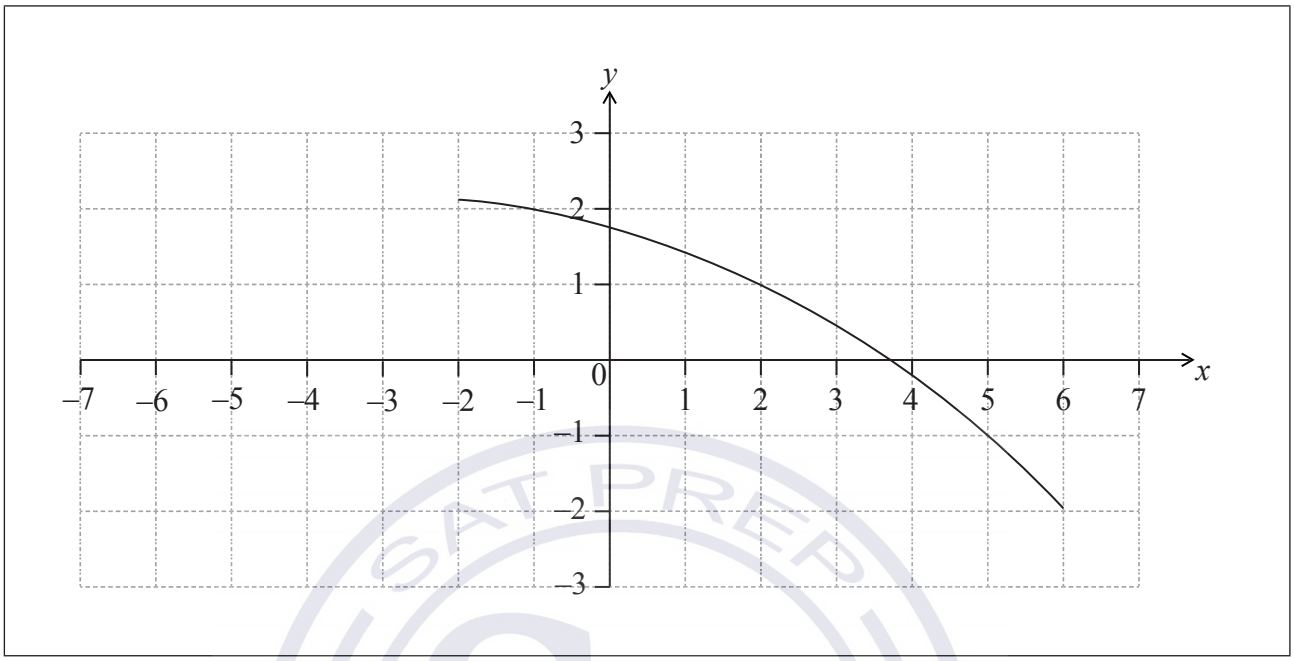
(a) Given that $2^m = 8$ and $2^n = 16$, write down the value of m and of n. [2]

(b) Hence or otherwise solve $8^{2x+1} = 16^{2x-3}$. [4]

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

4. [Maximum mark: 7]

The following diagram shows the graph of a function f.



(a) Find $f^{-1}(-1)$. [2]

(b) Find $(f \circ f)(-1)$. [3]

(c) On the same diagram, sketch the graph of $y = f(-x)$. [2]

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

5. [Maximum mark: 7]

Given that $\sin x = \frac{3}{4}$, where x is an obtuse angle, find the value of

(a) $\cos x$; [4]

(b) $\cos 2x$. [3]

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

6. [Maximum mark: 6]

Let $f(x) = px^2 + (10 - p)x + \frac{5}{4}p - 5$.

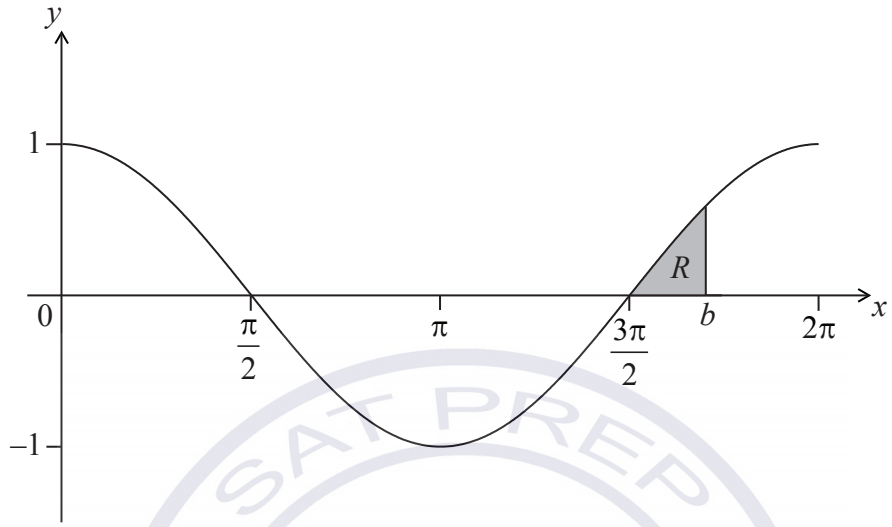
(a) Show that the discriminant of $f(x)$ is $100 - 4p^2$. [3]

(b) Find the values of p so that $f(x) = 0$ has two **equal** roots. [3]

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

7. [Maximum mark: 8]

Let $f(x) = \cos x$, for $0 \le x \le 2\pi$. The following diagram shows the graph of f.
There are x-intercepts at $x = \frac{\pi}{2}, \frac{3\pi}{2}$.



The shaded region R is enclosed by the graph of f, the line $x = b$, where $b > \frac{3\pi}{2}$, and the x-axis. The area of R is $\left(1 - \frac{\sqrt{3}}{2}\right)$. Find the value of b.

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 16]

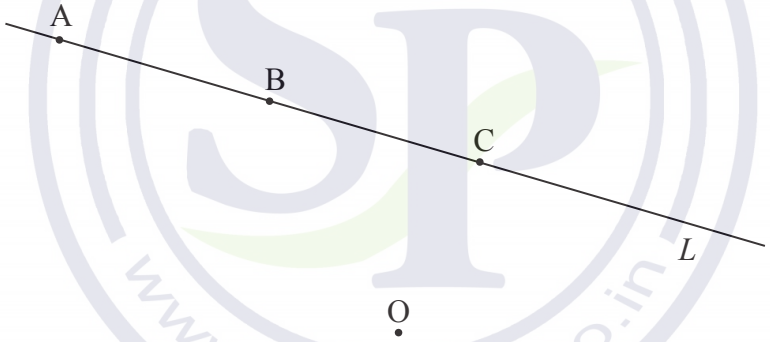
A line L passes through points $\mathrm{A}(-2, 4, 3)$ and $\mathrm{B}(-1, 3, 1)$.

(a) (i) Show that $\vec{\mathrm{AB}} = \begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix}$.

(ii) Find $\left|\vec{\mathrm{AB}}\right|$. [3]

(b) Find a vector equation for L. [2]

The following diagram shows the line L and the origin O. The point C also lies on L.



Point C has position vector $\begin{pmatrix} 0 \\ y \\ -1 \end{pmatrix}$.

(c) Show that $y = 2$. [4]

(d) (i) Find $\vec{\mathrm{OC}} \bullet \vec{\mathrm{AB}}$.

(ii) Hence, write down the size of the angle between OC and L. [3]

(e) Hence or otherwise, find the area of triangle OAB. [4]

Do **not** write solutions on this page.

9. [Maximum mark: 14]

A function f has its derivative given by $f'(x) = 3x^2 - 2kx - 9$, where k is a constant.

(a) Find $f''(x)$. [2]

The graph of f has a point of inflexion when $x = 1$.

(b) Show that $k = 3$. [3]

(c) Find $f'(-2)$. [2]

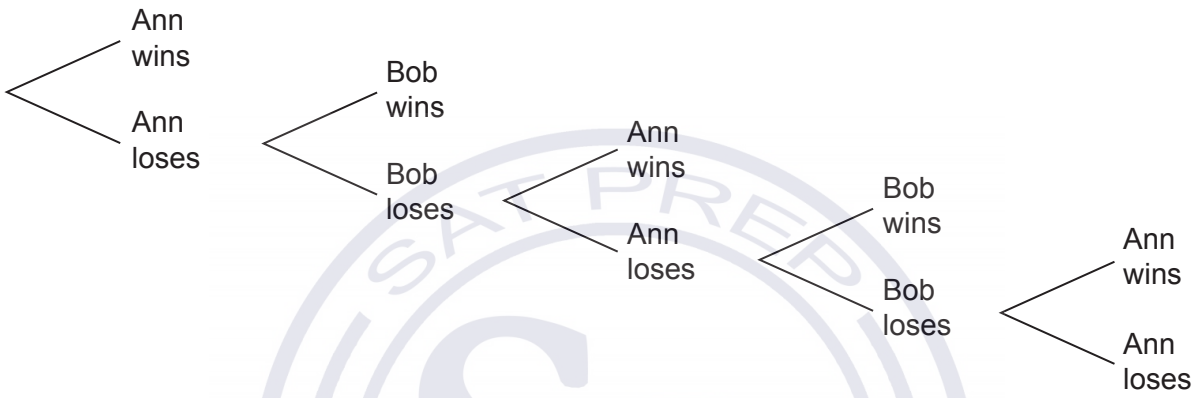
(d) Find the equation of the tangent to the curve of f at $(-2, 1)$, giving your answer in the form $y = ax + b$. [4]

(e) Given that $f'(-1) = 0$, explain why the graph of f has a local maximum when $x = -1$. [3]

Do **not** write solutions on this page.

10. [Maximum mark: 15]

Ann and Bob play a game where they each have an eight-sided die. Ann's die has three green faces and five red faces; Bob's die has four green faces and four red faces. They take turns rolling their own die and note what colour faces up. The first player to roll green wins. Ann rolls first. Part of a tree diagram of the game is shown below.



(a) Find the probability that Ann wins on her first roll. [2]

(b) (i) The probability that Ann wins on her third roll is $\frac{5}{8}\times\frac{4}{8}\times p\times q\times\frac{3}{8}$.

Write down the value of p and of q.

(ii) The probability that Ann wins on her tenth roll is $\frac{3}{8}r^k$ where $r\in\mathbb{Q}$, $k\in\mathbb{Z}$.

Find the value of r and of k. [6]

(c) Find the probability that Ann wins the game. [7]

Please **do not** write on this page.

Answers written on this page
will not be marked.

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Standard level
Paper 1

Tuesday 12 May 2015 (morning)

Candidate session number

1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **Mathematics SL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[90 marks]**.

2215–7305

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.

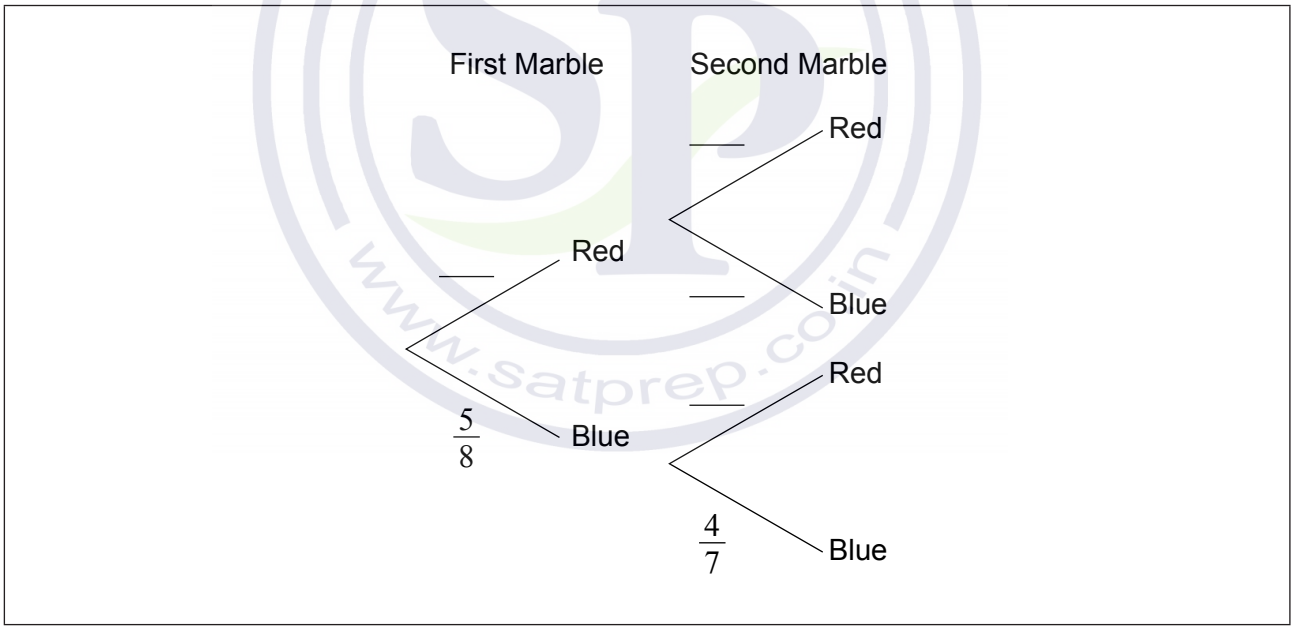
1. [Maximum mark: 6]

A bag contains eight marbles. Three marbles are red and five are blue. Two marbles are drawn from the bag without replacement.

(a) Write down the probability that the first marble drawn is red. [1]

. . . . . .
. . . . . .

(b) Complete the following tree diagram. [3]

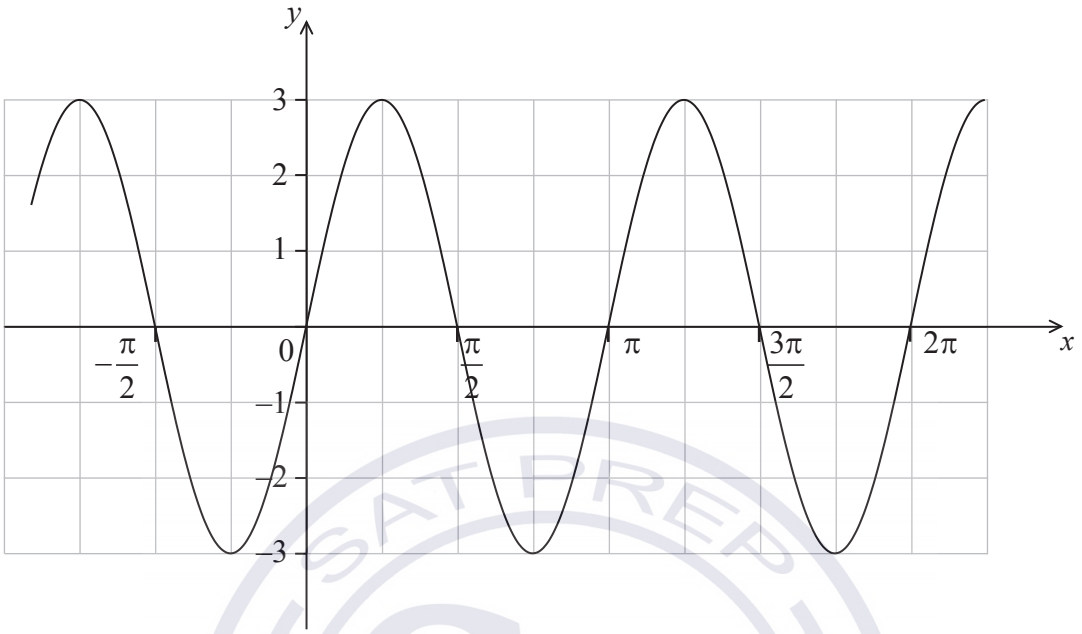


(c) Find the probability that both marbles are blue. [2]

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

2. [Maximum mark: 6]

Let $f(x) = a\sin bx$, where $b > 0$. The following diagram shows part of the graph of f.



(a) (i) Find the period of f.

(ii) Write down the amplitude of f. [3]

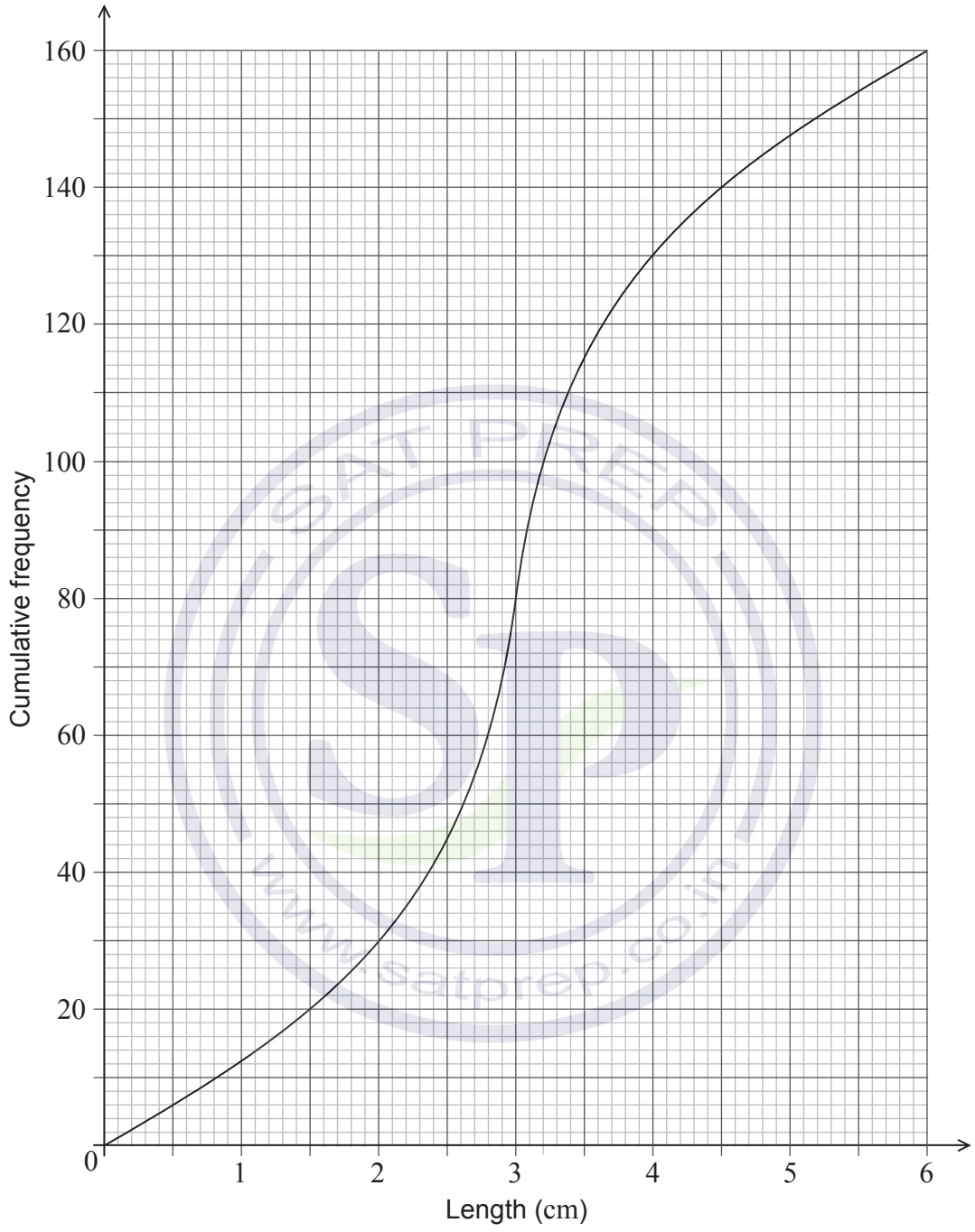
(b) (i) Write down the value of a.

(ii) Find the value of b. [3]

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

3. [Maximum mark: 6]

The following cumulative frequency diagram shows the lengths of 160 fish, in cm.



(This question continues on the following page)

(Question 3 continued)

(a) Find the median length. [2]

The following frequency table also gives the lengths of the 160 fish.

Length x cm	$0 \leq x \leq 2$	$2 < x \leq 3$	$3 < x \leq 4.5$	$4.5 < x \leq 6$
Frequency	p	50	q	20

(b) (i) Write down the value of p.

(ii) Find the value of q. [4]

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

4. [Maximum mark: 7]

Let $g(x) = \frac{\ln x}{x}$.

(a) Find $g'(x)$. [4]

(b) Find $\int g(x)\,dx$. [3]

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

5. [Maximum mark: 6]

Let $f(x) = e^{-2x}$.

(a) Write down $f'(x)$, $f''(x)$, and $f^{(3)}(x)$. [3]

(b) Find an expression for $f^{(n)}(x)$. [3]

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

6. [Maximum mark: 8]

Let $f(x) = ax^3 + bx$. At $x = 0$, the gradient of the curve of f is 3. Given that $f^{-1}(7) = 1$, find the value of a and of b.

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

7. [Maximum mark: 7]

A bag contains black and white chips. Rose pays $\$10$ to play a game where she draws a chip from the bag. The following table gives the probability of choosing each colour chip.

Outcome	black	white
Probability	0.4	0.6

Rose gets no money if she draws a white chip, and gets $\$k$ if she draws a black chip. The game is fair. Find the value of k.

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

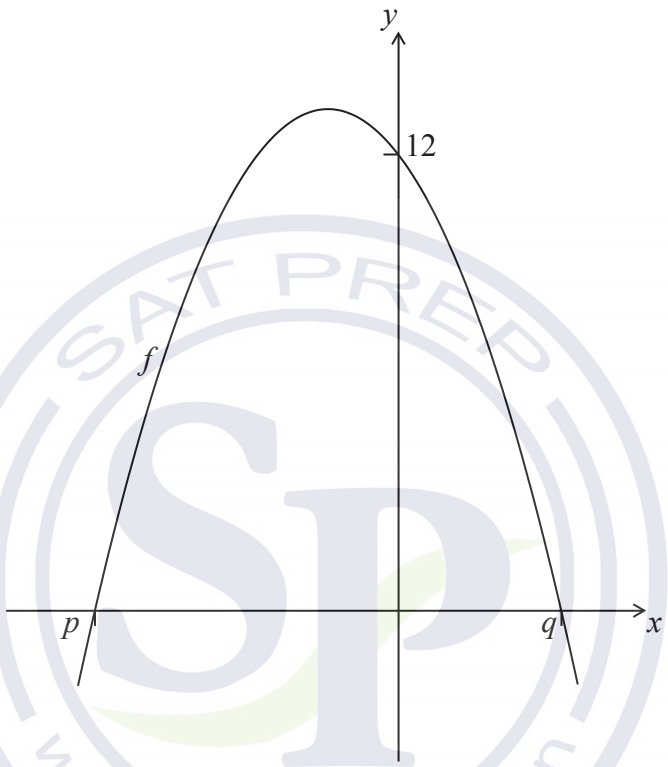
Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 15]

Let $f(x) = a(x+3)(x-1)$. The following diagram shows part of the graph of f.



The graph has x-intercepts at $(p, 0)$ and $(q, 0)$, and a y-intercept at $(0, 12)$.

(a) (i) Write down the value of p and of q.

(ii) Find the value of a. [6]

(b) Find the equation of the axis of symmetry of the graph of f. [3]

(c) Find the largest value of f. [3]

The function f can also be written as $f(x) = a(x-h)^2 + k$.

(d) Find the value of h and of k. [3]

Do **not** write solutions on this page.

9. [Maximum mark: 15]

Let P and Q have coordinates $(1, 0, 2)$ and $(-11, 8, m)$ respectively.

(a) Express $\overrightarrow{PQ}$ in terms of m. [2]

Let $\boldsymbol{a}$ and $\boldsymbol{b}$ be perpendicular vectors, where $\boldsymbol{a} = \begin{pmatrix} 1 \\ 1 \\ n \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} -3 \\ 2 \\ 1 \end{pmatrix}$.

(b) Find n. [4]

(c) Given that $\overrightarrow{PQ}$ is parallel to $\boldsymbol{b}$,

(i) express $\overrightarrow{PQ}$ in terms of $\boldsymbol{b}$;

(ii) hence find m. [5]

In part (d), distance is in metres, time is in seconds.

(d) A particle moves along a straight line through Q so that its position is given by $\boldsymbol{r} = \boldsymbol{c} + t\boldsymbol{a}$.

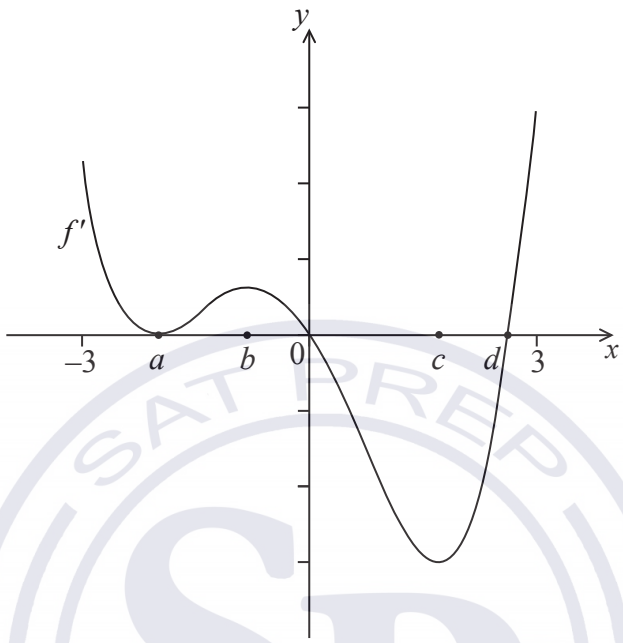
(i) Write down a possible vector $\boldsymbol{c}$.

(ii) Find the speed of the particle. [4]

Do **not** write solutions on this page.

10. [Maximum mark: 14]

Consider a function f with domain $-3 < x < 3$. The following diagram shows the graph of f', the **derivative** of f.



The graph of f' has x-intercepts at $x = a$, $x = 0$, and $x = d$. There is a local maximum at $x = b$ and local minima at $x = a$ and at $x = c$.

(a) Find all possible values of x where the graph of f is decreasing. [3]

(b) (i) Find the value of x where the graph of f has a local minimum.

(ii) Justify your answer. [3]

(c) The total area of the region enclosed by the graph of f' and the x-axis is 15. Given that $f(a) = 3$ and $f(d) = -1$, find the value of $f(0)$. [8]

88147301

International Baccalaureate®
Baccalauréat International
Bachillerato Internacional

MATHEMATICS
STANDARD LEVEL
PAPER 1

Candidate session number

Wednesday 12 November 2014 (afternoon)

1 hour 30 minutes

Examination code

8	8	1	4	–	7	3	0	1

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the ***Mathematics SL*** *formula booklet* is required for this paper.
- The maximum mark for this examination paper is *[90 marks]*.

12EP01

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

*Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.*

1. *[Maximum mark: 7]*

Let $f(x) = x^2 + x - 6$.

(a) Write down the y-intercept of the graph of f. *[1]*

(b) Solve $f(x) = 0$. *[3]*

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

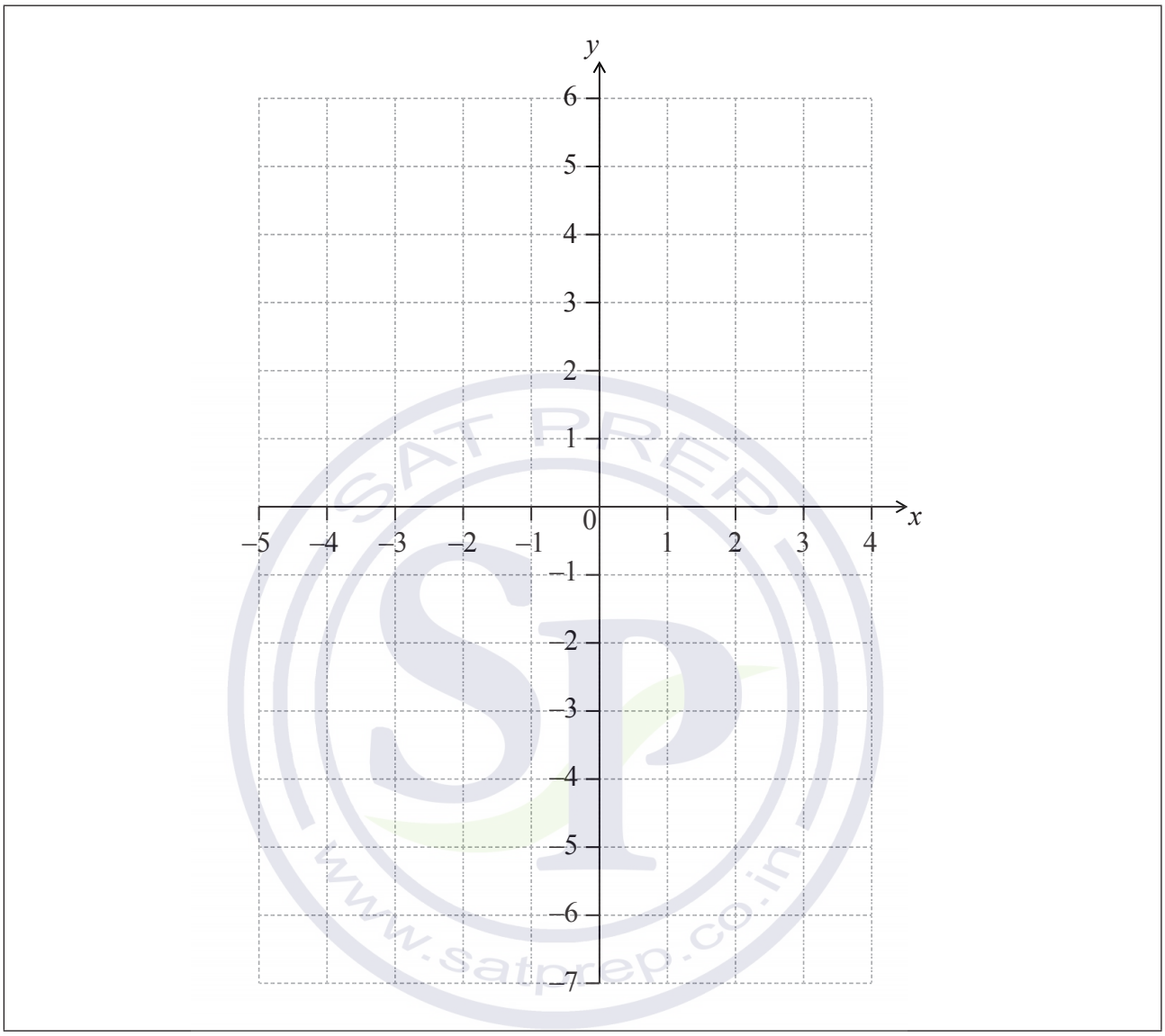
. . . . . .

. . . . . .

(This question continues on the following page)

(Question 1 continued)

(c) On the following grid, sketch the graph of f, for $-4 \le x \le 3$. *[3]*



2. *[Maximum mark: 6]*

In an arithmetic sequence, the first term is 2 and the second term is 5.

(a) Find the common difference. *[2]*

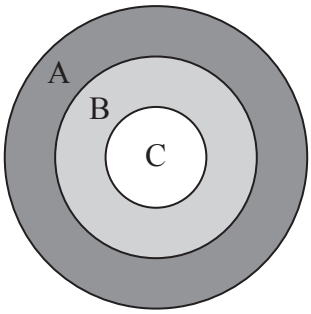
(b) Find the eighth term. *[2]*

(c) Find the sum of the first eight terms of the sequence. *[2]*

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

3. *[Maximum mark: 7]*

The following diagram shows a board which is divided into three regions A, B and C.



A game consists of a contestant throwing one dart at the board. The probability of hitting each region is given in the following table.

Region	A	B	C
Probability	$\frac{5}{20}$	$\frac{4}{20}$	$\frac{1}{20}$

(a) Find the probability that the dart does **not** hit the board. *[3]*

The contestant scores points as shown in the following table.

Region	A	B	C	Does not hit the board
Points	0	q	10	–3

(b) Given that the game is fair, find the value of q. *[4]*

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

4. *[Maximum mark: 6]*

(a) Write the expression $3\ln 2 - \ln 4$ in the form $\ln k$, where $k \in \mathbb{Z}$. *[3]*

(b) Hence or otherwise, solve $3\ln 2 - \ln 4 = -\ln x$. *[3]*

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

5. *[Maximum mark: 6]*

Let $f(x) = p + \frac{9}{x-q}$, for $x \neq q$. The line $x = 3$ is a vertical asymptote to the graph of f.

(a) Write down the value of q. *[1]*

The graph of f has a y-intercept at $(0, 4)$.

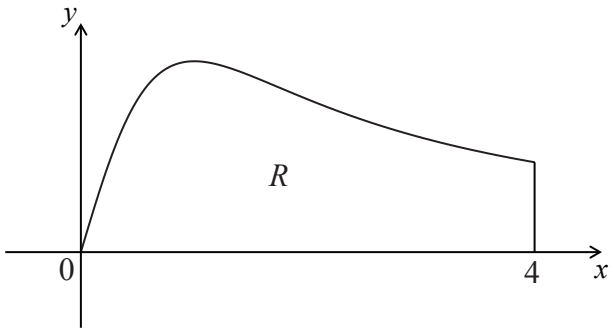
(b) Find the value of p. *[4]*

(c) Write down the equation of the horizontal asymptote of the graph of f. *[1]*

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

6. *[Maximum mark: 6]*

The following diagram shows the graph of $f(x) = \frac{x}{x^2+1}$, for $0 \le x \le 4$, and the line $x = 4$.



Let R be the region enclosed by the graph of f, the x-axis and the line $x = 4$.

Find the area of R.

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

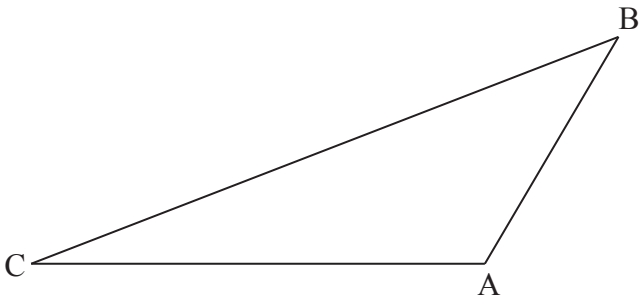
. . . . . .

. . . . . .

7. *[Maximum mark: 6]*

The following diagram shows triangle ABC.

diagram not to scale



Let $\vec{AB} \bullet \vec{AC} = -5\sqrt{3}$ and $\left|\vec{AB}\right|\left|\vec{AC}\right| = 10$. Find the area of triangle ABC.

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

*Do **NOT** write solutions on this page.*

SECTION B

*Answer **all** questions in the answer booklet provided. Please start each question on a new page.*

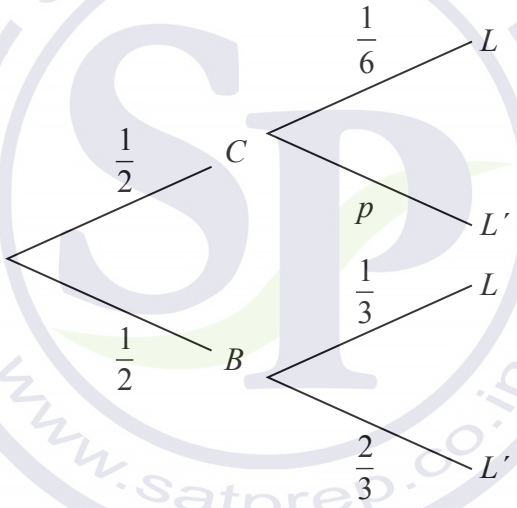
8. *[Maximum mark: 15]*

Adam travels to school by car (C) or by bicycle (B). On any particular day he is equally likely to travel by car or by bicycle.

The probability of being late (L) for school is $\frac{1}{6}$ if he travels by car.

The probability of being late for school is $\frac{1}{3}$ if he travels by bicycle.

This information is represented by the following tree diagram.



(a) Find the value of p. *[2]*

(b) Find the probability that Adam will travel by car and be late for school. *[2]*

(c) Find the probability that Adam will be late for school. *[4]*

(d) Given that Adam is late for school, find the probability that he travelled by car. *[3]*

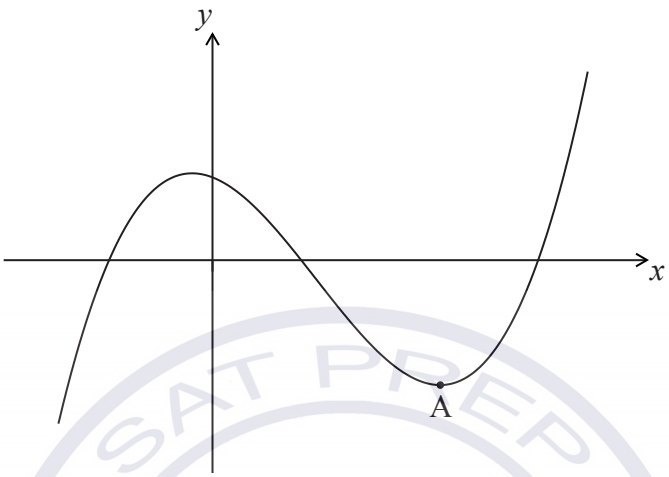
Adam will go to school three times next week.

(e) Find the probability that Adam will be late exactly once. *[4]*

*Do **NOT** write solutions on this page.*

9. *[Maximum mark: 14]*

The following diagram shows the graph of a function f. There is a local minimum point at A, where $x > 0$.



The derivative of f is given by $f'(x) = 3x^2 - 8x - 3$.

(a) Find the x-coordinate of A. *[5]*

(b) The y-intercept of the graph is at $(0, 6)$. Find an expression for $f(x)$. *[6]*

The graph of a function g is obtained by reflecting the graph of f in the y-axis, followed by a translation of $\begin{pmatrix} m \\ n \end{pmatrix}$.

(c) Find the x-coordinate of the local minimum point on the graph of g. *[3]*

*Do **NOT** write solutions on this page.*

10. *[Maximum mark: 17]*

Let L_x be a family of lines with equation given by $\boldsymbol{r} = \begin{pmatrix} x \\ \frac{2}{x} \end{pmatrix} + t\begin{pmatrix} x^2 \\ -2 \end{pmatrix}$, where $x > 0$.

(a) Write down the equation of L_1. *[2]*

A line L_a crosses the y-axis at a point P.

(b) Show that P has coordinates $\left(0, \frac{4}{a}\right)$. *[6]*

The line L_a crosses the x-axis at $\mathrm{Q}(2a, 0)$. Let $d = \mathrm{PQ}^2$.

(c) Show that $d = 4a^2 + \frac{16}{a^2}$. *[2]*

(d) There is a minimum value for d. Find the value of a that gives this minimum value. *[7]*

International Baccalaureate®
Baccalauréat International
Bachillerato Internacional

MATHEMATICS
STANDARD LEVEL
PAPER 1

Candidate session number

Tuesday 13 May 2014 (afternoon)

1 hour 30 minutes

Examination code

2	2	1	4	–	7	3	0	3

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the ***Mathematics SL*** *formula booklet* is required for this paper.
- The maximum mark for this examination paper is *[90 marks]*.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

*Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.*

1. *[Maximum mark: 5]*

Let $f(x) = a(x-h)^2 + k$. The vertex of the graph of f is at $(2, 3)$ and the graph passes through $(1, 7)$.

(a) Write down the value of h and of k. *[2]*

(b) Find the value of a. *[3]*

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

2. *[Maximum mark: 7]*

In an arithmetic sequence, the third term is 10 and the fifth term is 16.

(a) Find the common difference. *[2]*

(b) Find the first term. *[2]*

(c) Find the sum of the first 20 terms of the sequence. *[3]*

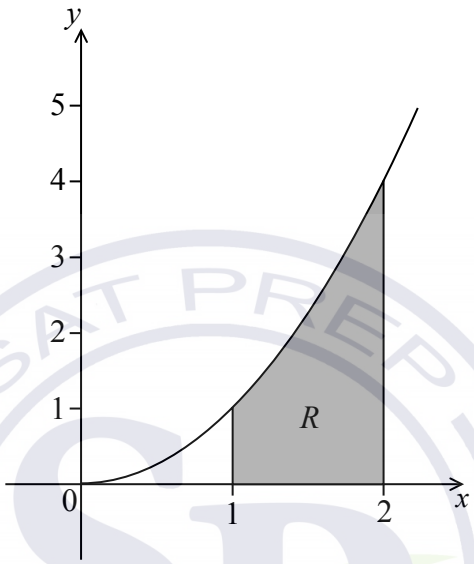
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

3. *[Maximum mark: 6]*

Let $f(x) = x^2$.

(a) Find $\int_1^2 \left(f(x)\right)^2 \mathrm{d}x$. *[4]*

(b) The following diagram shows part of the graph of f.



The shaded region R is enclosed by the graph of f, the x-axis and the lines $x = 1$ and $x = 2$.

Find the volume of the solid formed when R is revolved 360° about the x-axis. *[2]*

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

4. *[Maximum mark: 6]*

(a) Write down the value of

(i) $\log_3 27$;

(ii) $\log_8 \frac{1}{8}$;

(iii) $\log_{16} 4$. *[3]*

(b) Hence, solve $\log_3 27 + \log_8 \frac{1}{8} - \log_{16} 4 = \log_4 x$. *[3]*

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

5. *[Maximum mark: 6]*

Celeste wishes to hire a taxicab from a company which has a large number of taxicabs. The taxicabs are randomly assigned by the company.

The probability that a taxicab is yellow is 0.4.
The probability that a taxicab is a Fiat is 0.3.
The probability that a taxicab is yellow or a Fiat is 0.6.

Find the probability that the taxicab hired by Celeste is **not** a yellow Fiat.

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

6. *[Maximum mark: 7]*

Let $\int_{\pi}^{a} \cos 2x \, dx = \frac{1}{2}$, where $\pi < a < 2\pi$. Find the value of a.

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

7. *[Maximum mark: 7]*

Let $f(x) = px^3 + px^2 + qx$.

(a) Find $f'(x)$. *[2]*

(b) Given that $f'(x) \geq 0$, show that $p^2 \leq 3pq$. *[5]*

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

*Do **NOT** write solutions on this page.*

SECTION B

*Answer **all** questions in the answer booklet provided. Please start each question on a new page.*

8. *[Maximum mark: 17]*

The line L_1 passes through the points $\mathrm{A}(2, 1, 4)$ and $\mathrm{B}(1, 1, 5)$.

(a) Show that $\overrightarrow{\mathrm{AB}} = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$. *[1]*

(b) Hence, write down

(i) a direction vector for L_1;

(ii) a vector equation for L_1. *[3]*

Another line L_2 has equation $\boldsymbol{r} = \begin{pmatrix} 4 \\ 7 \\ -4 \end{pmatrix} + s\begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$. The lines L_1 and L_2 intersect at the point P.

(c) Find the coordinates of P. *[6]*

(d) (i) Write down a direction vector for L_2.

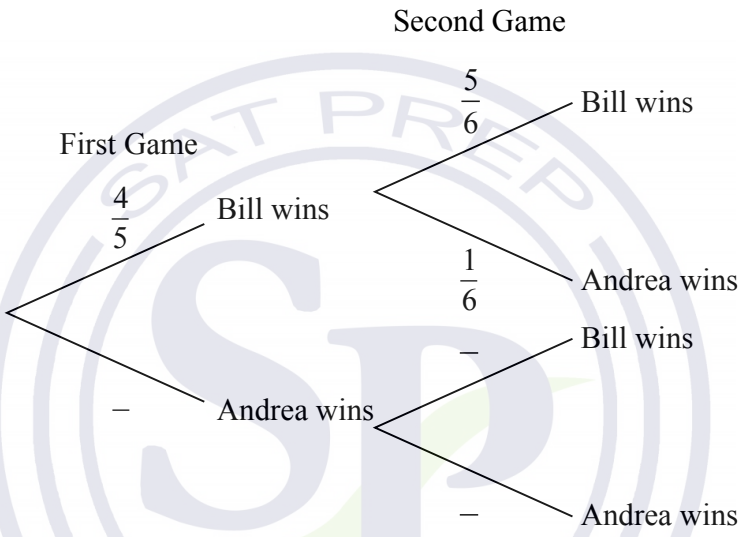
(ii) Hence, find the angle between L_1 and L_2. *[7]*

*Do **NOT** write solutions on this page.*

9. *[Maximum mark: 14]*

Bill and Andrea play two games of tennis. The probability that Bill wins the first game is $\frac{4}{5}$.
If Bill wins the first game, the probability that he wins the second game is $\frac{5}{6}$.
If Bill loses the first game, the probability that he wins the second game is $\frac{2}{3}$.

(a) **Copy** and complete the following tree diagram. *(Do **not** write on this page.)* *[3]*



(b) Find the probability that Bill wins the first game and Andrea wins the second game. *[2]*

(c) Find the probability that Bill wins at least one game. *[4]*

(d) Given that Bill wins at least one game, find the probability that he wins both games. *[5]*

*Do **NOT** write solutions on this page.*

10. *[Maximum mark: 15]*

The sides of a square are 16 cm in length. The midpoints of the sides of this square are joined to form a new square and four triangles (diagram 1). The process is repeated twice, as shown in diagrams 2 and 3.

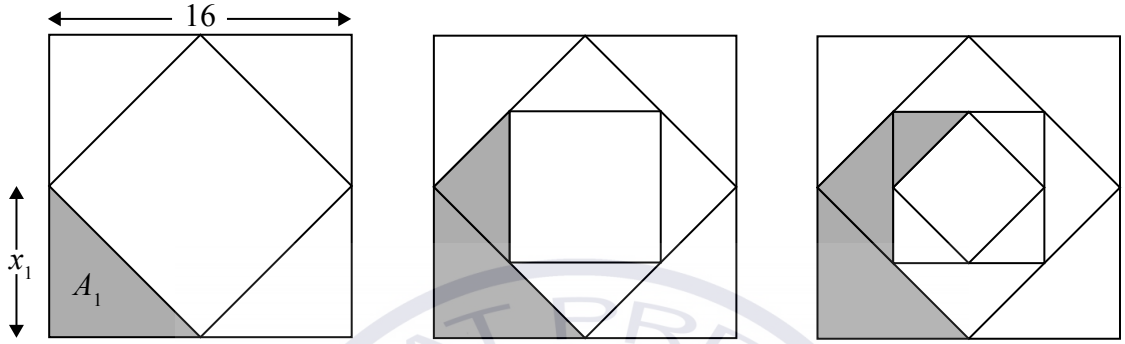


diagram 1 diagram 2 diagram 3

Let x_n denote the length of one of the equal sides of each new triangle.
Let A_n denote the area of each new triangle.

(a) The following table gives the values of x_n and A_n, for $1 \le n \le 3$. **Copy** and complete the table. *(Do **not** write on this page.)* *[4]*

n	1	2	3
x_n	8		4
A_n	32	16	

(b) The process described above is repeated. Find A_6. *[4]*

(c) Consider an initial square of side length k cm. The process described above is repeated indefinitely. The total area of the shaded regions is $k\,\text{cm}^2$. Find the value of k. *[7]*

Please **do not** write on this page.

Answers written on this page
will not be marked.

22147305

International Baccalaureate®
Baccalauréat International
Bachillerato Internacional

MATHEMATICS
STANDARD LEVEL
PAPER 1

Candidate session number

Tuesday 13 May 2014 (afternoon)

1 hour 30 minutes

Examination code

2	2	1	4	–	7	3	0	5

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the ***Mathematics SL*** *formula booklet* is required for this paper.
- The maximum mark for this examination paper is *[90 marks]*.

16EP01

Please **do not** write on this page.

Answers written on this page
will not be marked.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

*Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.*

1. *[Maximum mark: 5]*

The following diagram shows a right-angled triangle, ABC, where $\sin A = \frac{5}{13}$.

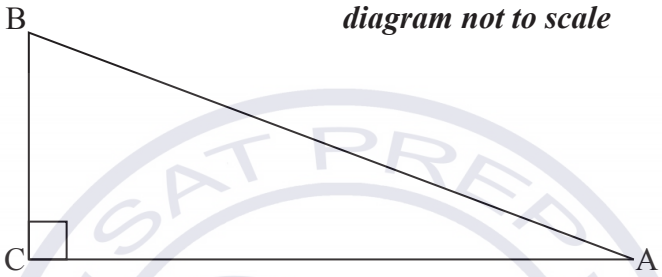


diagram not to scale

(a) Show that $\cos A = \frac{12}{13}$. *[2]*

(b) Find $\cos 2A$. *[3]*

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

2. *[Maximum mark: 7]*

Find the value of each of the following, giving your answer as an integer.

(a) $\log_6 36$ *[2]*

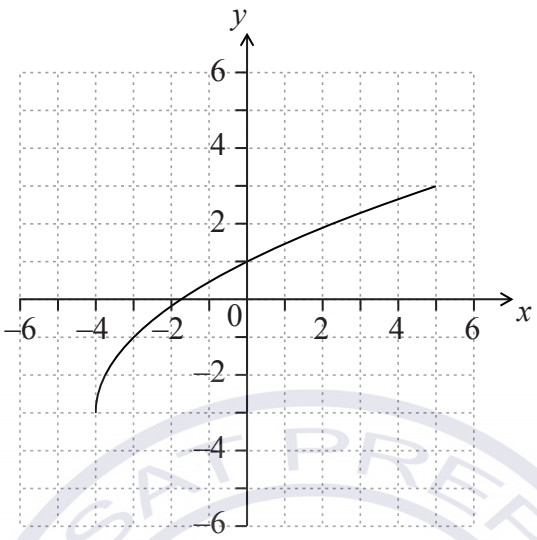
(b) $\log_6 4 + \log_6 9$ *[2]*

(c) $\log_6 2 - \log_6 12$ *[3]*

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

3. *[Maximum mark: 6]*

The following diagram shows the graph of $y = f(x)$, for $-4 \le x \le 5$.



(a) Write down the value of

(i) $f(-3)$;

(ii) $f^{-1}(1)$. *[2]*

(b) Find the domain of f^{-1}. *[2]*

(c) On the grid above, sketch the graph of f^{-1}. *[2]*

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

4. *[Maximum mark: 7]*

The line L is parallel to the vector $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$.

(a) Find the gradient of the line L. *[2]*

The line L passes through the point $(9, 4)$.

(b) Find the equation of the line L in the form $y = ax + b$. *[3]*

(c) Write down a vector equation for the line L. *[2]*

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

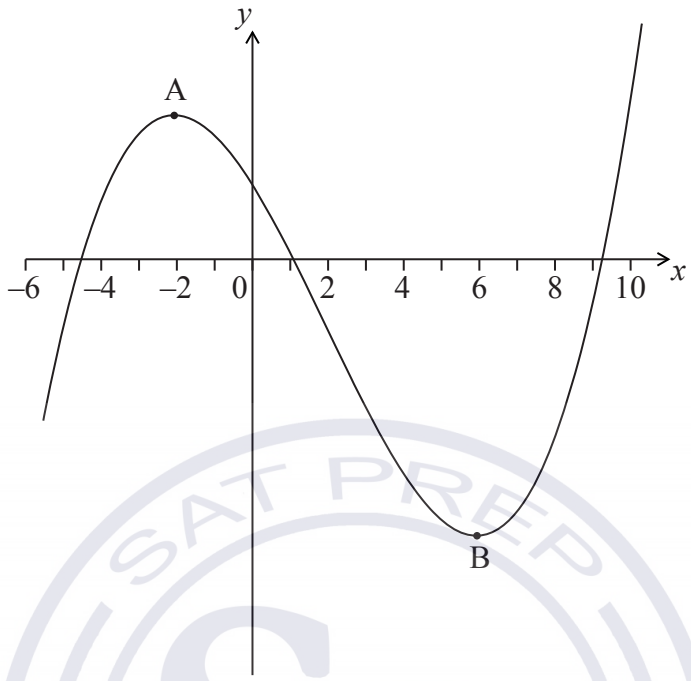
5. *[Maximum mark: 6]*

The graph of a function h passes through the point $\left(\frac{\pi}{12}, 5\right)$.

Given that $h'(x) = 4\cos 2x$, find $h(x)$.

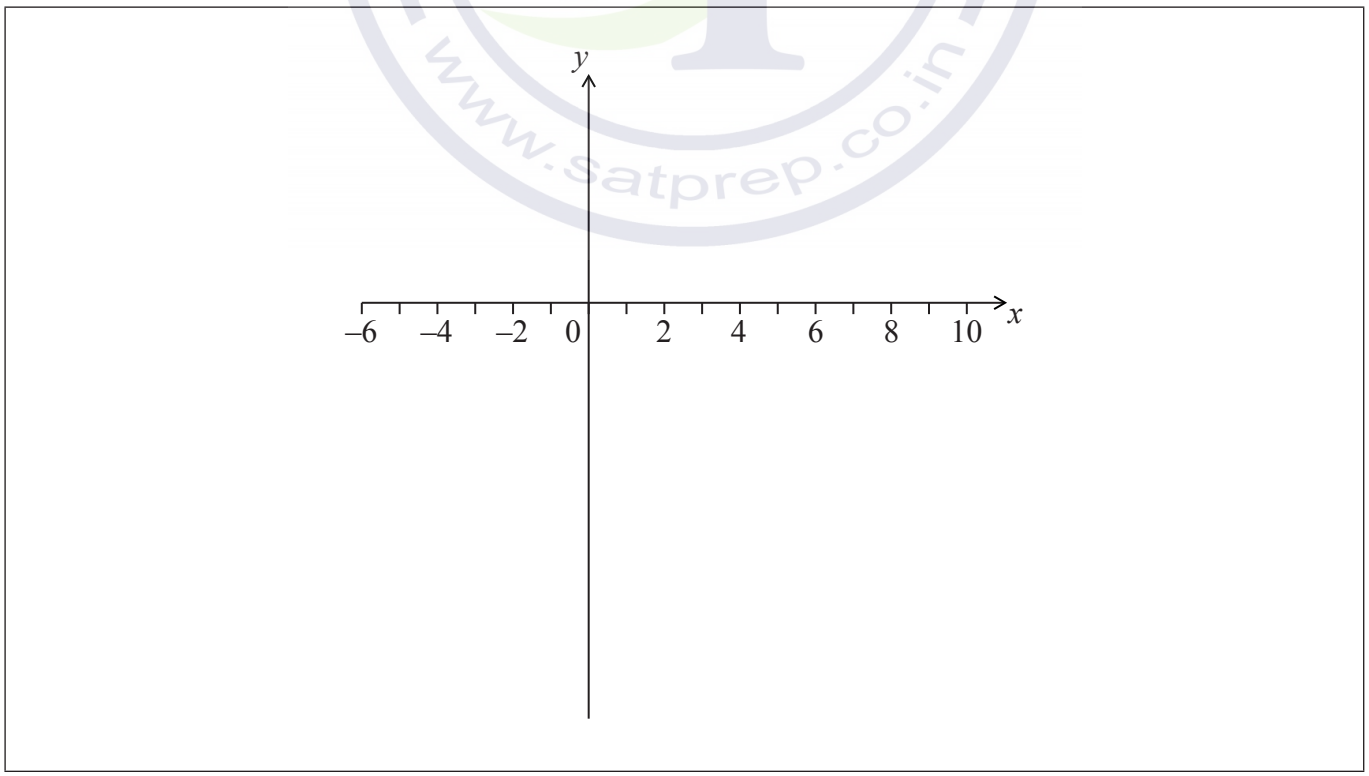
6. *[Maximum mark: 6]*

The following diagram shows part of the graph of $y = f(x)$.



The graph has a local maximum at A, where $x = -2$, and a local minimum at B, where $x = 6$.

(a) On the following axes, sketch the graph of $y = f'(x)$. *[4]*



(This question continues on the following page)

(Question 6 continued)

(b) Write down the following in order from least to greatest: $f(0)$, $f'(6)$, $f''(-2)$. *[2]*

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

7. *[Maximum mark: 8]*

The sums of the terms of a sequence follow the pattern

$S_1 = 1 + k$, $S_2 = 5 + 3k$, $S_3 = 12 + 7k$, $S_4 = 22 + 15k$, …, where $k \in \mathbb{Z}$.

(a) Given that $u_1 = 1 + k$, find u_2, u_3 and u_4. *[4]*

(b) Find a general expression for u_n. *[4]*

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

*Do **NOT** write solutions on this page.*

SECTION B

*Answer **all** questions in the answer booklet provided. Please start each question on a new page.*

8. *[Maximum mark: 15]*

Let $f(x) = 3x^2 - 6x + p$. The equation $f(x) = 0$ has two equal roots.

(a) (i) Write down the **value** of the discriminant.

(ii) Hence, show that $p = 3$. *[3]*

The graph of f has its vertex on the x-axis.

(b) Find the coordinates of the vertex of the graph of f. *[4]*

(c) Write down the solution of $f(x) = 0$. *[1]*

(d) The function can be written in the form $f(x) = a(x-h)^2 + k$. Write down the value of

(i) a;

(ii) h;

(iii) k. *[3]*

(e) The graph of a function g is obtained from the graph of f by a reflection of f in the x-axis, followed by a translation by the vector $\begin{pmatrix} 0 \\ 6 \end{pmatrix}$. Find g, giving your answer in the form $g(x) = Ax^2 + Bx + C$. *[4]*

*Do **NOT** write solutions on this page.*

9. *[Maximum mark: 15]*

Distances in this question are in metres.

Ryan and Jack have model airplanes, which take off from level ground. Jack's airplane takes off after Ryan's.

The position of Ryan's airplane t seconds after it takes off is given by $\boldsymbol{r} = \begin{pmatrix} 5 \\ 6 \\ 0 \end{pmatrix} + t \begin{pmatrix} -4 \\ 2 \\ 4 \end{pmatrix}$.

(a) Find the speed of Ryan's airplane. *[3]*

(b) Find the height of Ryan's airplane after two seconds. *[2]*

The position of Jack's airplane s seconds after **it** takes off is given by $\boldsymbol{r} = \begin{pmatrix} -39 \\ 44 \\ 0 \end{pmatrix} + s \begin{pmatrix} 4 \\ -6 \\ 7 \end{pmatrix}$.

(c) Show that the paths of the airplanes are perpendicular. *[5]*

The two airplanes collide at the point $(-23, 20, 28)$.

(d) How long after Ryan's airplane takes off does Jack's airplane take off? *[5]*

*Do **NOT** write solutions on this page.*

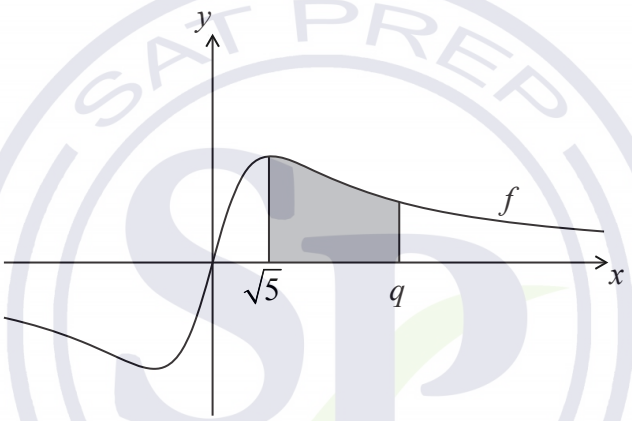
10. *[Maximum mark: 15]*

Let $f(x) = \frac{2x}{x^2+5}$.

(a) Use the quotient rule to show that $f'(x) = \frac{10-2x^2}{(x^2+5)^2}$. *[4]*

(b) Find $\int \frac{2x}{x^2+5}\,dx$. *[4]*

The following diagram shows part of the graph of f.



(c) The shaded region is enclosed by the graph of f, the x-axis, and the lines $x = \sqrt{5}$ and $x = q$. This region has an area of $\ln 7$. Find the value of q. *[7]*

Please **do not** write on this page.

Answers written on this page
will not be marked.

Please **do not** write on this page.

Answers written on this page
will not be marked.

Please **do not** write on this page.

Answers written on this page
will not be marked.

88137301

International Baccalaureate®
Baccalauréat International
Bachillerato Internacional

MATHEMATICS
STANDARD LEVEL
PAPER 1

Candidate session number

0	0							

Monday 11 November 2013 (afternoon)

Examination code

8	8	1	3	–	7	3	0	1

1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the ***Mathematics SL*** *information booklet* is required for this paper.
- The maximum mark for this examination paper is *[90 marks]*.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

*Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.*

1. *[Maximum mark: 5]*

In the following diagram, $\overrightarrow{OP} = \boldsymbol{p}$, $\overrightarrow{OQ} = \boldsymbol{q}$ and $\overrightarrow{PT} = \frac{1}{2}\overrightarrow{PQ}$.

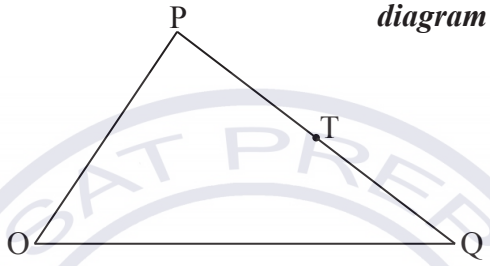


diagram not to scale

Express each of the following vectors in terms of $\boldsymbol{p}$ and $\boldsymbol{q}$,

(a) $\overrightarrow{QP}$; *[2]*

(b) $\overrightarrow{OT}$. *[3]*

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

2. *[Maximum mark: 6]*

Let $\boldsymbol{A} = \begin{pmatrix} 1 & 2 \\ 3 & 0 \end{pmatrix}$, $\boldsymbol{B} = \begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix}$ and $\boldsymbol{C} = \begin{pmatrix} 2 & -4 \\ 0 & 1 \end{pmatrix}$.

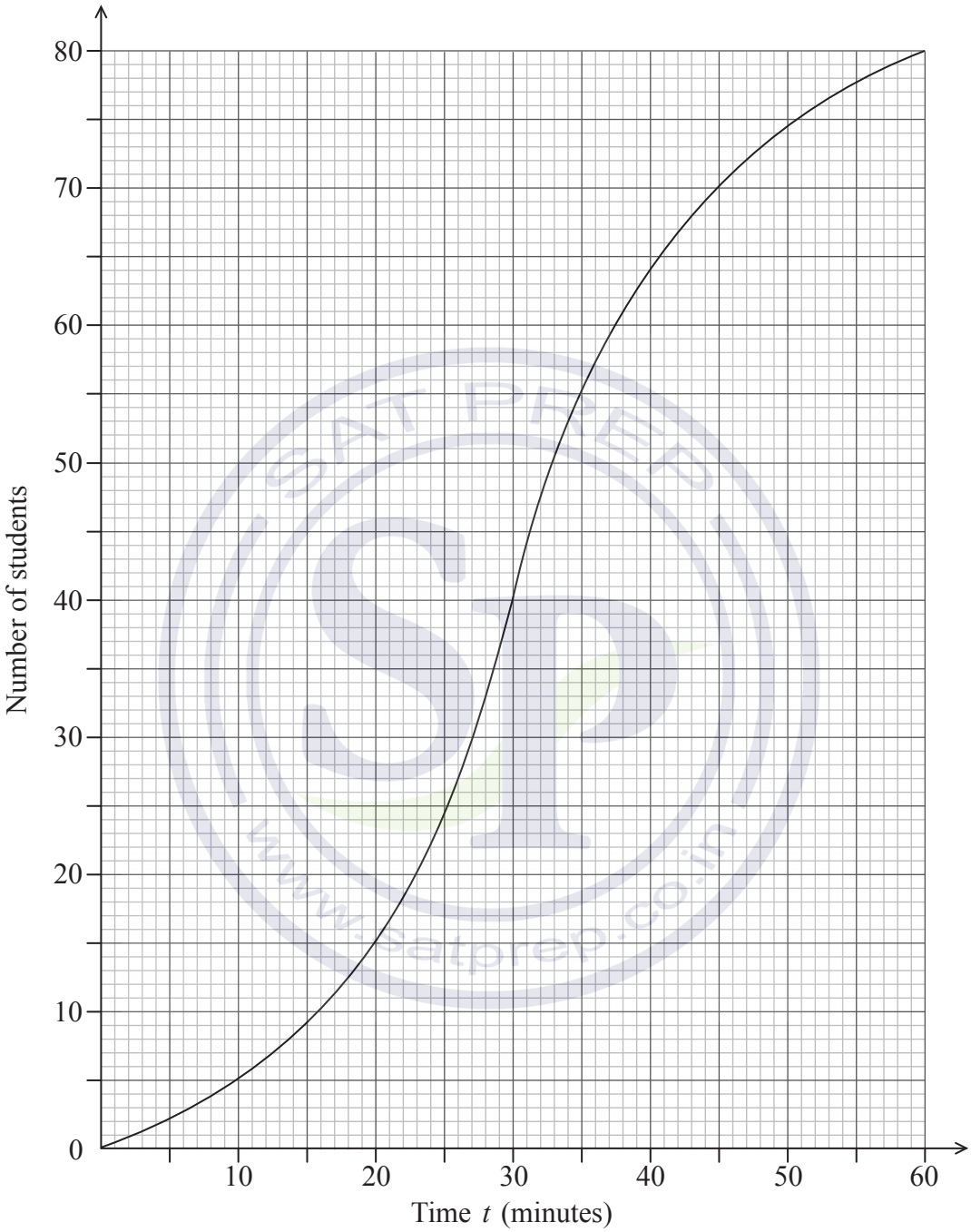
(a) Find $\boldsymbol{AB}$. *[3]*

(b) Find $\det(\boldsymbol{AB} + \boldsymbol{C})$. *[3]*

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

3. *[Maximum mark: 7]*

The following is a cumulative frequency diagram for the time t, in minutes, taken by 80 students to complete a task.



(This question continues on the following page)

(Question 3 continued)

(a) Find the number of students who completed the task in less than 45 minutes. *[2]*

(b) Find the number of students who took between 35 and 45 minutes to complete the task. *[3]*

(c) Given that 50 students take less than k minutes to complete the task, find the value of k. *[2]*

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

4. *[Maximum mark: 6]*

Consider a function $f(x)$ such that $\int_1^6 f(x)\mathrm{d}x = 8$.

(a) Find $\int_1^6 2f(x)\mathrm{d}x$. *[2]*

(b) Find $\int_1^6 \big(f(x)+2\big)\mathrm{d}x$. *[4]*

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

5. *[Maximum mark: 7]*

Let $f(x) = \sin\left(x + \frac{\pi}{4}\right) + k$. The graph of f passes through the point $\left(\frac{\pi}{4}, 6\right)$.

(a) Find the value of k. *[3]*

(b) Find the minimum value of $f(x)$. *[2]*

Let $g(x) = \sin x$. The graph of g is translated to the graph of f by the vector $\begin{pmatrix} p \\ q \end{pmatrix}$.

(c) Write down the value of p and of q. *[2]*

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

6. *[Maximum mark: 6]*

Let $f(x) = e^{2x}$. The line L is the tangent to the curve of f at $(1, e^2)$.

Find the equation of L in the form $y = ax + b$.

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

7. *[Maximum mark: 8]*

The equation $x^2 + (k+2)x + 2k = 0$ has two distinct real roots.

Find the possible values of k.

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

*Do **NOT** write solutions on this page.*

SECTION B

*Answer **all** questions in the answer booklet provided. Please start each question on a new page.*

8. *[Maximum mark: 14]*

Let $f(x) = 3x - 2$ and $g(x) = \frac{5}{3x}$, for $x \neq 0$.

(a) Find $f^{-1}(x)$. *[2]*

(b) Show that $\left(g \circ f^{-1}\right)(x) = \frac{5}{x+2}$. *[2]*

Let $h(x) = \frac{5}{x+2}$, for $x \geq 0$. The graph of h has a horizontal asymptote at $y = 0$.

(c) (i) Find the y-intercept of the graph of h.

(ii) Hence, sketch the graph of h. *[5]*

(d) For the graph of h^{-1},

(i) write down the x-intercept;

(ii) write down the equation of the vertical asymptote. *[2]*

(e) Given that $h^{-1}(a) = 3$, find the value of a. *[3]*

*Do **NOT** write solutions on this page.*

9. *[Maximum mark: 16]*

The first three terms of a infinite geometric sequence are $m-1$, 6, $m+4$, where $m \in \mathbb{Z}$.

(a) (i) Write down an expression for the common ratio, r.

(ii) Hence, show that m satisfies the equation $m^2 + 3m - 40 = 0$. *[4]*

(b) (i) Find the two possible values of m.

(ii) Find the possible values of r. *[6]*

(c) The sequence has a finite sum.

(i) State which value of r leads to this sum **and** justify your answer.

(ii) Calculate the sum of the sequence. *[6]*

*Do **NOT** write solutions on this page.*

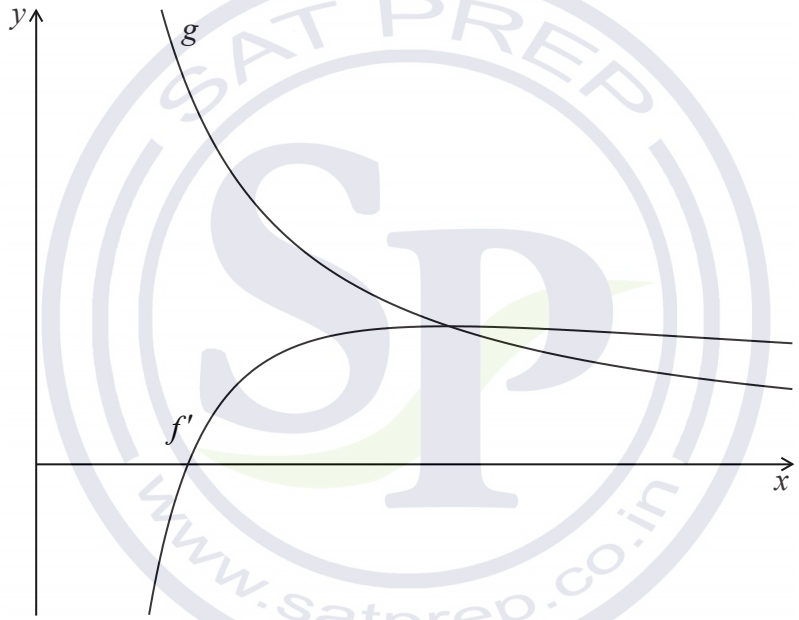
10. *[Maximum mark: 15]*

Let $f(x) = \frac{(\ln x)^2}{2}$, for $x > 0$.

(a) Show that $f'(x) = \frac{\ln x}{x}$. *[2]*

(b) There is a minimum on the graph of f. Find the x-coordinate of this minimum. *[3]*

Let $g(x) = \frac{1}{x}$. The following diagram shows parts of the graphs of f' and g.



The graph of f' has an x-intercept at $x = p$.

(c) Write down the value of p. *[2]*

The graph of g intersects the graph of f' when $x = q$.

(d) Find the value of q. *[3]*

(e) Let R be the region enclosed by the graph of f', the graph of g and the line $x = p$. Show that the area of R is $\frac{1}{2}$. *[5]*

22137303

International Baccalaureate®
Baccalauréat International
Bachillerato Internacional

MATHEMATICS
STANDARD LEVEL
PAPER 1

Candidate session number

0	0							

Thursday 9 May 2013 (afternoon)

1 hour 30 minutes

Examination code

2	2	1	3	–	7	3	0	3

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the ***Mathematics SL*** *information booklet* is required for this paper.
- The maximum mark for this examination paper is *[90 marks]*.

0112

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

*Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.*

1. *[Maximum mark: 6]*

Consider the vectors $\boldsymbol{a} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$.

(a) Find

(i) $2\boldsymbol{a} + \boldsymbol{b}$;

(ii) $\left|2\boldsymbol{a} + \boldsymbol{b}\right|$. *[4 marks]*

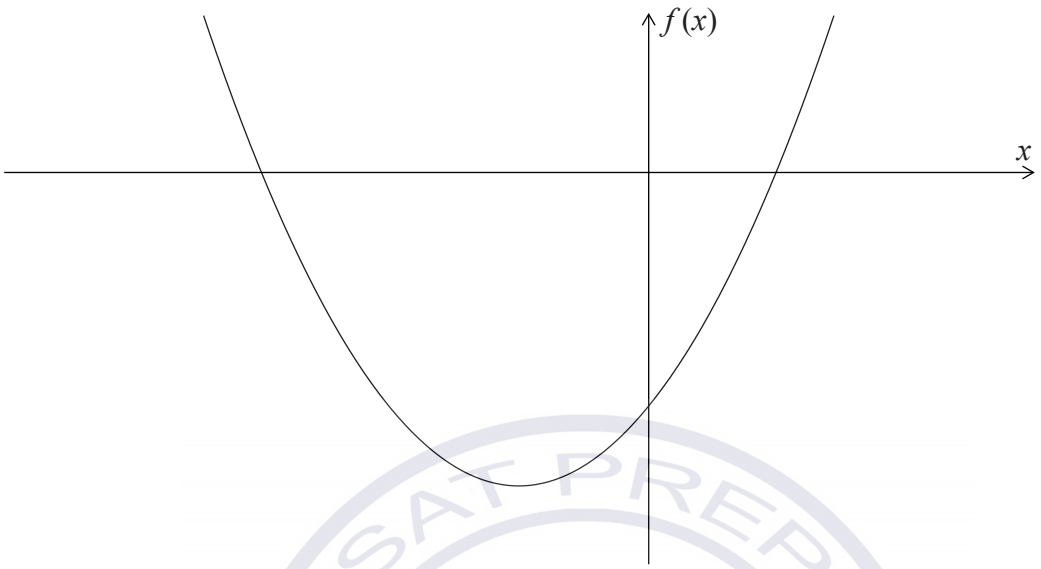
Let $2\boldsymbol{a} + \boldsymbol{b} + \boldsymbol{c} = \boldsymbol{0}$, where $\boldsymbol{0}$ is the zero vector.

(b) Find $\boldsymbol{c}$. *[2 marks]*

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

2. *[Maximum mark: 6]*

The diagram below shows part of the graph of $f(x) = (x-1)(x+3)$.



(a) Write down the x-intercepts of the graph of f. *[2 marks]*

(b) Find the coordinates of the vertex of the graph of f. *[4 marks]*

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

3. *[Maximum mark: 7]*

Consider $f(x) = x^2 \sin x$.

(a) Find $f'(x)$. *[4 marks]*

(b) Find the gradient of the curve of f at $x = \frac{\pi}{2}$. *[3 marks]*

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

4. *[Maximum mark: 7]*

Let $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$ and $\boldsymbol{X}$ be square matrices, such that $\boldsymbol{XA} + \boldsymbol{B} = \boldsymbol{C}$.

(a) Find an expression for $\boldsymbol{X}$ in terms of $\boldsymbol{A}$, $\boldsymbol{B}$ and $\boldsymbol{C}$. *[2 marks]*

(b) Given that $\boldsymbol{A} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, $\boldsymbol{B} = \begin{pmatrix} 0 & 1 \\ 1 & 2 \end{pmatrix}$ and $\boldsymbol{C} = \begin{pmatrix} 1 & 3 \\ -3 & 4 \end{pmatrix}$, find $\boldsymbol{X}$. *[5 marks]*

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

5. *[Maximum mark: 6]*

Let $f(x) = \sqrt{x-5}$, for $x \geq 5$.

(a) Find $f^{-1}(2)$. *[3 marks]*

(b) Let g be a function such that g^{-1} exists for all real numbers. Given that $g(30) = 3$, find $(f \circ g^{-1})(3)$. *[3 marks]*

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

6. *[Maximum mark: 6]*

Let $f(x) = \int \frac{12}{2x-5} \mathrm{d}x$, for $x > \frac{5}{2}$. The graph of f passes through $(4, 0)$.

Find $f(x)$.

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

7. *[Maximum mark: 7]*

Find the value of

(a) $\log_2 40 - \log_2 5$; *[3 marks]*

(b) $8^{\log_2 5}$. *[4 marks]*

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

*Do **NOT** write solutions on this page.*

SECTION B

*Answer **all** questions in the answer booklet provided. Please start each question on a new page.*

8. *[Maximum mark: 14]*

Consider points $\mathrm{A}(1, -2, -1)$, $\mathrm{B}(7, -4, 3)$ and $\mathrm{C}(1, -2, 3)$. The line L_1 passes through C and is parallel to $\overrightarrow{\mathrm{AB}}$.

(a) (i) Find $\overrightarrow{\mathrm{AB}}$.

(ii) Hence, write down a vector equation for L_1. *[4 marks]*

A second line, L_2, is given by $\boldsymbol{r} = \begin{pmatrix} -1 \\ 2 \\ 15 \end{pmatrix} + s \begin{pmatrix} 3 \\ -3 \\ p \end{pmatrix}$.

(b) Given that L_1 is perpendicular to L_2, show that $p = -6$. *[3 marks]*

(c) The line L_1 intersects the line L_2 at point Q. Find the x-coordinate of Q. *[7 marks]*

*Do **NOT** write solutions on this page.*

9. *[Maximum mark: 16]*

Jar A contains three red marbles and five green marbles. Two marbles are drawn from the jar, one after the other, without replacement.

(a) Find the probability that

(i) none of the marbles are green;

(ii) exactly one marble is green. *[5 marks]*

(b) Find the expected number of green marbles drawn from the jar. *[3 marks]*

Jar B contains six red marbles and two green marbles. A fair six-sided die is tossed. If the score is 1 or 2, a marble is drawn from jar A. Otherwise, a marble is drawn from jar B.

(c) (i) Write down the probability that the marble is drawn from jar B.

(ii) Given that the marble was drawn from jar B, write down the probability that it is red. *[2 marks]*

(d) Given that the marble is red, find the probability that it was drawn from jar A. *[6 marks]*

*Do **NOT** write solutions on this page.*

10. *[Maximum mark: 15]*

Consider $f(x) = \ln(x^4 + 1)$.

(a) Find the value of $f(0)$. *[2 marks]*

(b) Find the set of values of x for which f is increasing. *[5 marks]*

The second derivative is given by $f''(x) = \frac{4x^2(3 - x^4)}{(x^4 + 1)^2}$.

The equation $f''(x) = 0$ has only three solutions, when $x = 0$, $\pm\sqrt[4]{3}$ $(\pm 1.316\ldots)$.

(c) (i) Find $f''(1)$.

(ii) **Hence**, show that there is no point of inflexion on the graph of f at $x = 0$. *[5 marks]*

(d) There is a point of inflexion on the graph of f at $x = \sqrt[4]{3}$ $(x = 1.316\ldots)$. Sketch the graph of f, for $x \geq 0$. *[3 marks]*

Please **do not** write on this page.

Answers written on this page
will not be marked.

22137305

International Baccalaureate®
Baccalauréat International
Bachillerato Internacional

MATHEMATICS
STANDARD LEVEL
PAPER 1

Candidate session number

0	0							

Thursday 9 May 2013 (afternoon)

1 hour 30 minutes

Examination code

2	2	1	3	–	7	3	0	5

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the ***Mathematics SL*** *information booklet* is required for this paper.
- The maximum mark for this examination paper is *[90 marks]*.

0116

Please **do not** write on this page.

Answers written on this page
will not be marked.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

*Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.*

1. *[Maximum mark: 6]*

Let $f(x) = 4x - 2$ and $g(x) = -2x^2 + 8$.

(a) Find $f^{-1}(x)$. *[3 marks]*

(b) Find $(f \circ g)(1)$. *[3 marks]*

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

2. *[Maximum mark: 6]*

Let $\boldsymbol{A} = \begin{pmatrix} 1 & 2 \\ 3 & 0 \end{pmatrix}$, $\boldsymbol{B} = \begin{pmatrix} 2 & 1 \\ 1 & q \end{pmatrix}$, $\boldsymbol{C} = \begin{pmatrix} p & -1 \\ 6 & 3 \end{pmatrix}$, so that $\boldsymbol{AB} = \boldsymbol{C}$.

(a) Find the value of p. *[3 marks]*

(b) Find the value of q. *[3 marks]*

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

3. *[Maximum mark: 7]*

Let $\log_3 p = 6$ and $\log_3 q = 7$.

(a) Find $\log_3 p^2$. *[2 marks]*

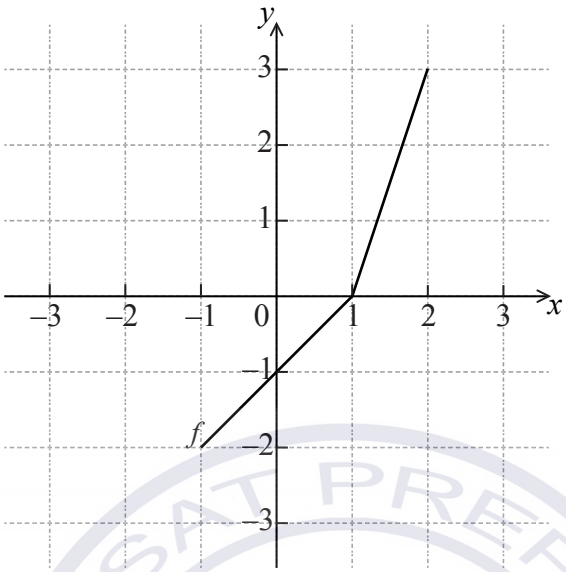
(b) Find $\log_3\left(\frac{p}{q}\right)$. *[2 marks]*

(c) Find $\log_3(9p)$. *[3 marks]*

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

4. *[Maximum mark: 6]*

The diagram below shows the graph of a function f, for $-1 \le x \le 2$.



(a) Write down the value of

(i) $f(2)$;

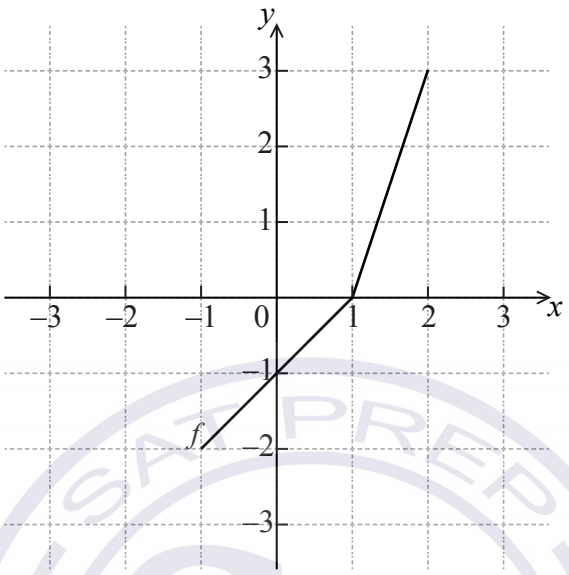
(ii) $f^{-1}(-1)$. *[3 marks]*

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

(This question continues on the following page)

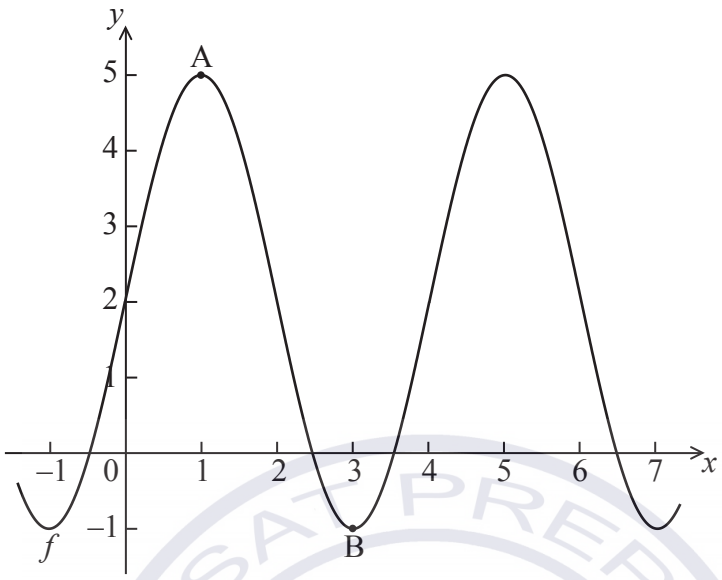
(Question 4 continued)

(b) Sketch the graph of f^{-1} on the grid below. *[3 marks]*



5. *[Maximum mark: 6]*

The diagram below shows part of the graph of a function f.



The graph has a maximum at $\mathrm{A}(1, 5)$ and a minimum at $\mathrm{B}(3, -1)$.

The function f can be written in the form $f(x) = p\sin(qx) + r$. Find the value of

(a) p; *[2 marks]*

(b) q; *[2 marks]*

(c) r. *[2 marks]*

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

6. *[Maximum mark: 7]*

A rocket moving in a straight line has velocity v km s^{-1} and displacement s km at time t seconds. The velocity v is given by $v(t) = 6e^{2t} + t$. When $t = 0$, $s = 10$. Find an expression for the displacement of the rocket in terms of t.

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

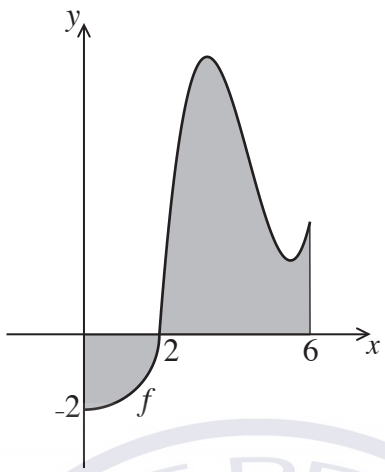
. . . . . .

. . . . . .

. . . . . .

7. *[Maximum mark: 7]*

The following is the graph of a function f, for $0 \le x \le 6$.



The first part of the graph is a quarter circle of radius 2 with centre at the origin.

(a) Find $\int_0^2 f(x)\,dx$. *[4 marks]*

(b) The shaded region is enclosed by the graph of f, the x-axis, the y-axis and the line $x = 6$. The area of this region is 3π.

Find $\int_2^6 f(x)\,dx$. *[3 marks]*

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

Please **do not** write on this page.

Answers written on this page
will not be marked.

*Do **NOT** write solutions on this page.*

SECTION B

*Answer **all** questions in the answer booklet provided. Please start each question on a new page.*

8. *[Maximum mark: 15]*

A running club organizes a race to select girls to represent the club in a competition. The times taken by the group of girls to complete the race are shown in the table below.

Time t minutes	$10 \le t < 12$	$12 \le t < 14$	$14 \le t < 20$	$20 \le t < 26$	$26 \le t < 28$	$28 \le t < 30$
Frequency	50	20	p	40	20	20
Cumulative Frequency	50	70	120	q	180	200

(a) Find the value of p and of q. *[4 marks]*

(b) A girl is chosen at random.

(i) Find the probability that the time she takes is less than 14 minutes.

(ii) Find the probability that the time she takes is at least 26 minutes. *[3 marks]*

A girl is selected for the competition if she takes less than x minutes to complete the race.

(c) Given that 40 % of the girls are not selected,

(i) find the number of girls who are not selected;

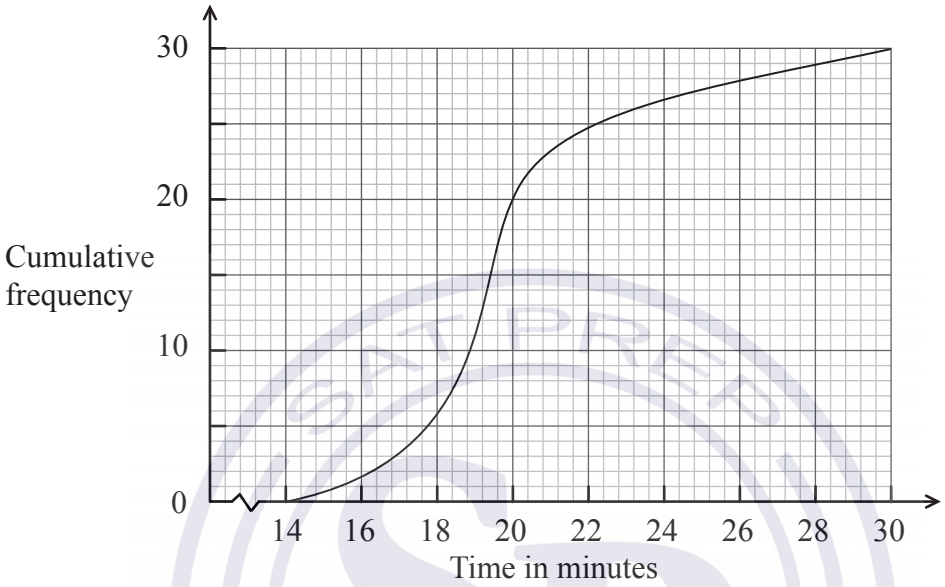
(ii) find x. *[4 marks]*

(This question continues on the following page)

*Do **NOT** write solutions on this page.*

(Question 8 continued)

Girls who are not selected, but took less than 25 minutes to complete the race, are allowed another chance to be selected. The new times taken by these girls are shown in the cumulative frequency diagram below.



(d) (i) Write down the number of girls who were allowed another chance.

(ii) Find the percentage of the **whole** group who were selected. *[4 marks]*

*Do **NOT** write solutions on this page.*

9. *[Maximum mark: 16]*

Let $f(x) = \sin x + \frac{1}{2}x^2 - 2x$, for $0 \le x \le \pi$.

(a) Find $f'(x)$. *[3 marks]*

Let g be a quadratic function such that $g(0) = 5$. The line $x = 2$ is the axis of symmetry of the graph of g.

(b) Find $g(4)$. *[3 marks]*

The function g can be expressed in the form $g(x) = a(x-h)^2 + 3$.

(c) (i) Write down the value of h.

(ii) Find the value of a. *[4 marks]*

(d) Find the value of x for which the tangent to the graph of f is parallel to the tangent to the graph of g. *[6 marks]*

*Do **NOT** write solutions on this page.*

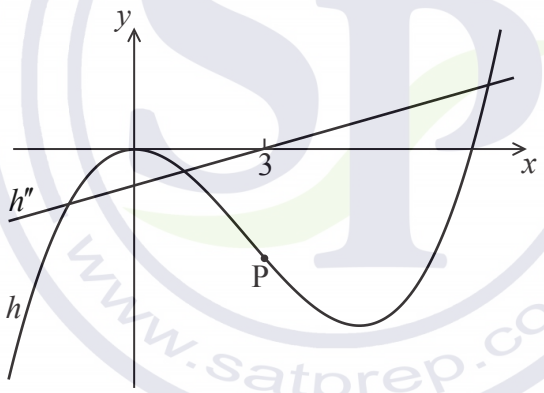
10. *[Maximum mark: 14]*

Consider the functions $f(x)$, $g(x)$ and $h(x)$. The following table gives some values associated with these functions.

x	2	3
$f(x)$	2	3
$g(x)$	–14	–18
$f'(x)$	1	1
$g'(x)$	–5	–3
$h''(x)$	–6	0

(a) Write down the value of $g(3)$, of $f'(3)$, and of $h''(2)$. *[3 marks]*

The following diagram shows parts of the graphs of h and h''.



There is a point of inflexion on the graph of h at P, when $x = 3$.

(b) Explain why P is a point of inflexion. *[2 marks]*

Given that $h(x) = f(x) \times g(x)$,

(c) find the y-coordinate of P; *[2 marks]*

(d) find the equation of the normal to the graph of h at P. *[7 marks]*

Please **do not** write on this page.

Answers written on this page
will not be marked.

88127301

International Baccalaureate®
Baccalauréat International
Bachillerato Internacional

MATHEMATICS
STANDARD LEVEL
PAPER 1

Candidate session number

0	0							

Tuesday 6 November 2012 (afternoon)

Examination code

8	8	1	2	–	7	3	0	1

1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions on the answer sheets provided. Write your session number on each answer sheet, and attach them to this examination paper and your cover sheet using the tag provided.
- At the end of the examination, indicate the number of sheets used in the appropriate box on your cover sheet.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the ***Mathematics SL*** *information booklet* is required for this paper.
- The maximum mark for this examination paper is *[90 marks]*.

0116

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

*Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.*

1. *[Maximum mark: 6]*

Let $\boldsymbol{A} = \begin{pmatrix} 0 & 3 \\ -2 & 4 \end{pmatrix}$ and $\boldsymbol{B} = \begin{pmatrix} -4 & 0 \\ 5 & 1 \end{pmatrix}$.

(a) Find $\boldsymbol{AB}$. *[3 marks]*

(b) Given that $\boldsymbol{X} - 2\boldsymbol{A} = \boldsymbol{B}$, find $\boldsymbol{X}$. *[3 marks]*

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

2. *[Maximum mark: 6]*

The following table shows the probability distribution of a discrete random variable X.

x	0	2	5	9
$\mathrm{P}(X = x)$	0.3	k	$2k$	0.1

(a) Find the value of k. *[3 marks]*

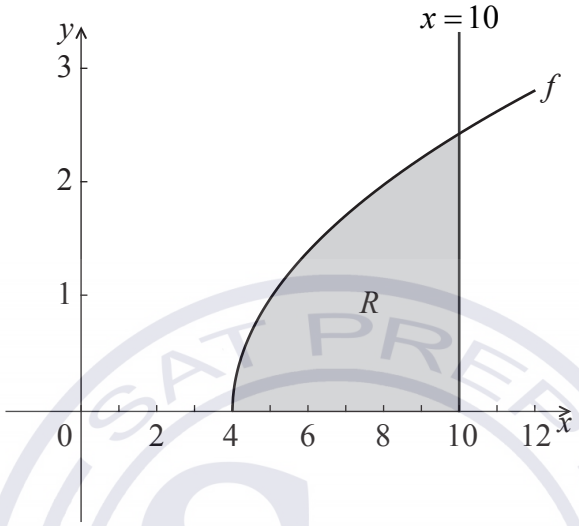
(b) Find $\mathrm{E}(X)$. *[3 marks]*

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

3. *[Maximum mark: 7]*

(a) Find $\int_4^{10} (x-4)\,\mathrm{d}x$. *[4 marks]*

(b) Part of the graph of $f(x) = \sqrt{x-4}$, for $x \geq 4$, is shown below. The shaded region R is enclosed by the graph of f, the line $x = 10$, and the x-axis.

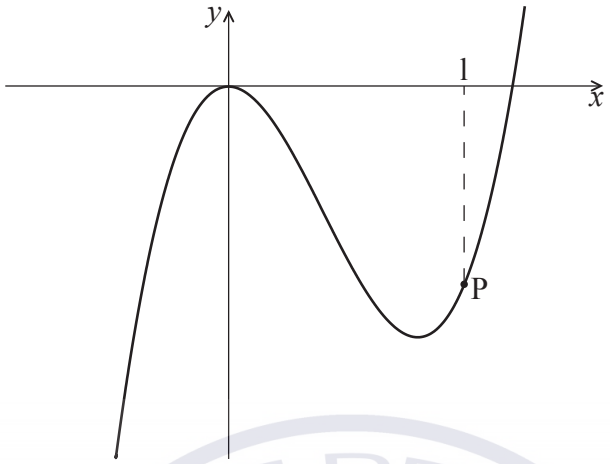


The region R is rotated 360° about the x-axis. Find the volume of the solid formed. *[3 marks]*

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

4. *[Maximum mark: 6]*

Part of the graph of $f(x) = ax^3 - 6x^2$ is shown below.



The point P lies on the graph of f. At P, $x = 1$.

(a) Find $f'(x)$. *[2 marks]*

(b) The graph of f has a gradient of 3 at the point P. Find the value of a. *[4 marks]*

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

5. *[Maximum mark: 6]*

Let $\sin 100^\circ = m$. Find expressions for each of the following in terms of m

(a) $\cos 100^\circ$; *[3 marks]*

(b) $\tan 100^\circ$; *[1 mark]*

(c) $\sin 200^\circ$. *[2 marks]*

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

6. *[Maximum mark: 8]*

The line L passes through the point $(5, -4, 10)$ and is parallel to the vector $\begin{pmatrix} 4 \\ -2 \\ 5 \end{pmatrix}$.

(a) Write down a vector equation for line L. *[2 marks]*

(b) The line L intersects the x-axis at the point P. Find the x-coordinate of P. *[6 marks]*

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

7. *[Maximum mark: 6]*

The equation $x^2 - 3x + k^2 = 4$ has two distinct real roots. Find the possible values of k.

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

Please **do not** write on this page.

Answers written on this page
will not be marked.

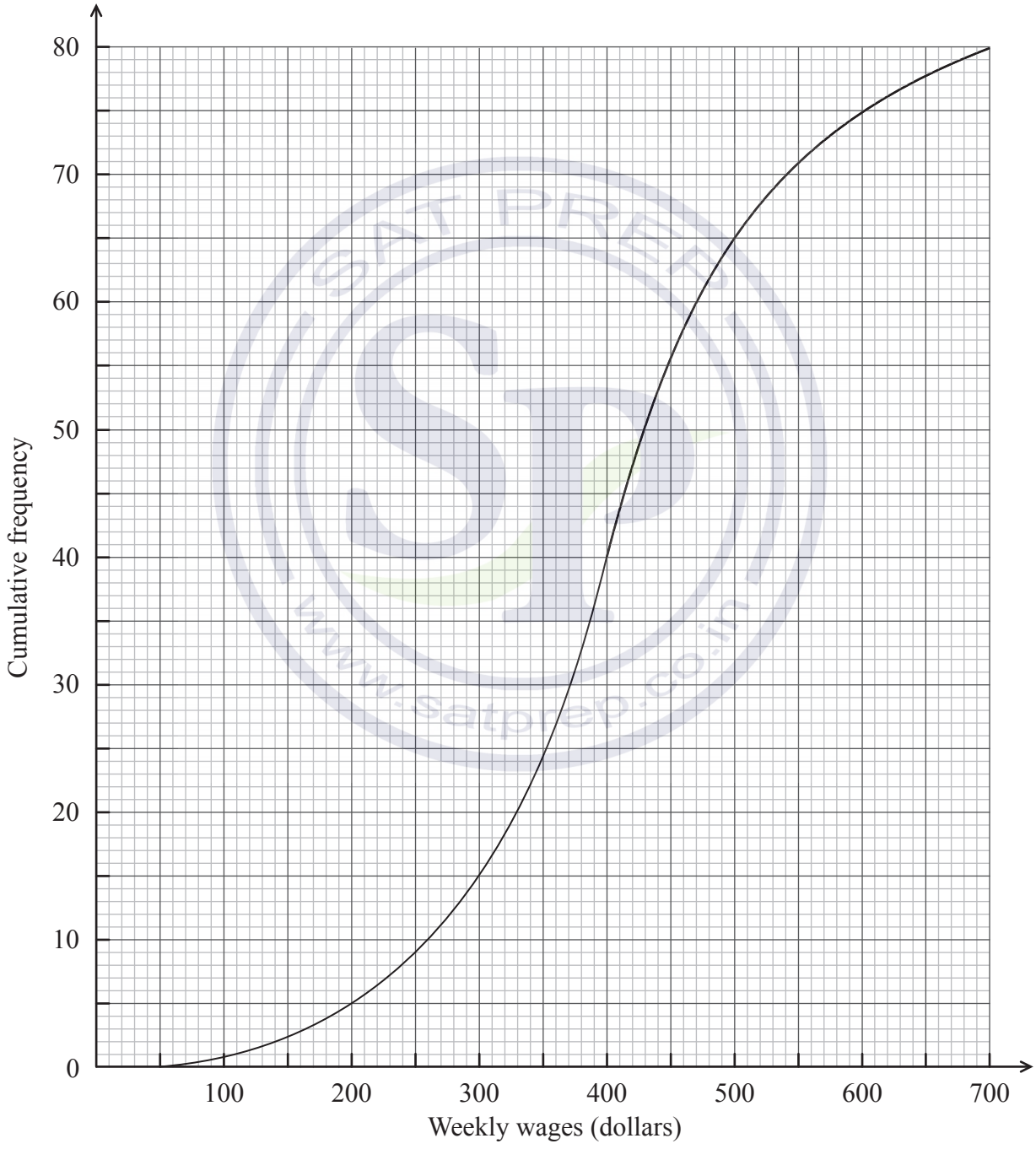
*Do **NOT** write solutions on this page.*

SECTION B

*Answer **all** questions on the answer sheets provided. Please start each question on a new page.*

8. *[Maximum mark: 15]*

The weekly wages (in dollars) of 80 employees are displayed in the cumulative frequency curve below.



(This question continues on the following page)

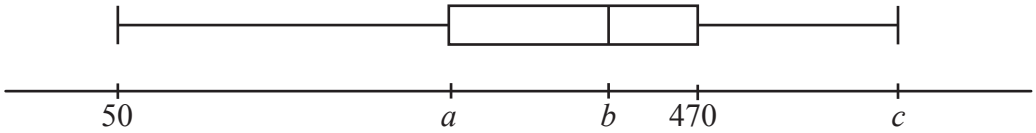
*Do **NOT** write solutions on this page.*

(Question 8 continued)

(a) (i) Write down the median weekly wage.

(ii) Find the interquartile range of the weekly wages. *[4 marks]*

The box-and-whisker plot below displays the weekly wages of the employees.



(b) Write down the value of

(i) a;

(ii) b;

(iii) c. *[3 marks]*

Employees are paid \$20 per hour.

(c) Find the median number of **hours** worked per week. *[3 marks]*

(d) Find the number of employees who work more than 25 hours per week. *[5 marks]*

*Do **NOT** write solutions on this page.*

9. *[Maximum mark: 14]*

Let A and B be points such that $\vec{OA} = \begin{pmatrix} 5 \\ 2 \\ 1 \end{pmatrix}$ and $\vec{OB} = \begin{pmatrix} 6 \\ 0 \\ 3 \end{pmatrix}$.

(a) Show that $\vec{AB} = \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$. *[1 mark]*

Let C and D be points such that ABCD is a **rectangle**.

(b) Given that $\vec{AD} = \begin{pmatrix} 4 \\ p \\ 1 \end{pmatrix}$, show that $p = 3$. *[4 marks]*

(c) Find the coordinates of point C. *[4 marks]*

(d) Find the area of rectangle ABCD. *[5 marks]*

*Do **NOT** write solutions on this page.*

10. *[Maximum mark: 16]*

Let $f(x) = \frac{6x}{x+1}$, for $x > 0$.

(a) Find $f'(x)$. *[5 marks]*

Let $g(x) = \ln\left(\frac{6x}{x+1}\right)$, for $x > 0$.

(b) Show that $g'(x) = \frac{1}{x(x+1)}$. *[4 marks]*

(c) Let $h(x) = \frac{1}{x(x+1)}$. The area enclosed by the graph of h, the x-axis and the lines $x = \frac{1}{5}$ and $x = k$ is $\ln 4$. Given that $k > \frac{1}{5}$, find the value of k. *[7 marks]*

Please **do not** write on this page.

Answers written on this page
will not be marked.

Please **do not** write on this page.

Answers written on this page
will not be marked.

Please **do not** write on this page.

Answers written on this page
will not be marked.

22127303

International Baccalaureate®
Baccalauréat International
Bachillerato Internacional

MATHEMATICS
STANDARD LEVEL
PAPER 1

Candidate session number

0	0							

Thursday 3 May 2012 (afternoon)

1 hour 30 minutes

Examination code

2	2	1	2	–	7	3	0	3

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions on the answer sheets provided. Write your session number on each answer sheet, and attach them to this examination paper and your cover sheet using the tag provided.
- At the end of the examination, indicate the number of sheets used in the appropriate box on your cover sheet.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the ***Mathematics SL*** *information booklet* is required for this paper.
- The maximum mark for this examination paper is *[90 marks]*.

0112

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

*Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.*

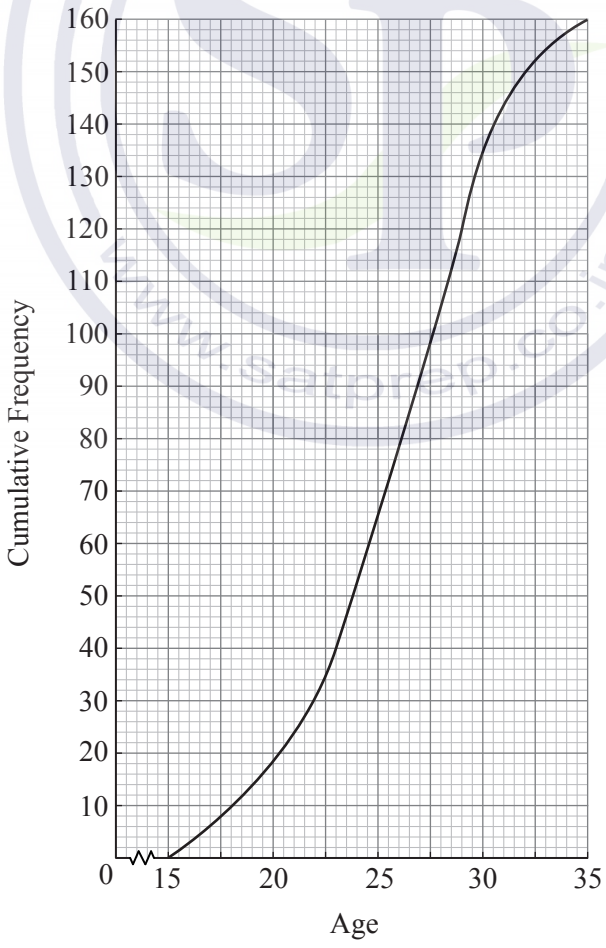
1. *[Maximum mark: 7]*

The ages of people attending a music concert are given in the table below.

Age	$15 \leq x < 19$	$19 \leq x < 23$	$23 \leq x < 27$	$27 \leq x < 31$	$31 \leq x < 35$
Frequency	14	26	52	52	16
Cumulative Frequency	14	40	92	p	160

(a) Find p. *[2 marks]*

The cumulative frequency diagram is given below.



(This question continues on the following page)

(Question 1 continued)

(b) Use the diagram to estimate

(i) the 80th percentile;

(ii) the interquartile range. *[5 marks]*

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

2. *[Maximum mark: 6]*

Let $\boldsymbol{A}$ be a 2×2 matrix and $\boldsymbol{B}$ an $m\times n$ matrix, where $\boldsymbol{A}=\begin{pmatrix}-2 & 0\\ 1 & 3\end{pmatrix}$ and $\boldsymbol{B}=\begin{pmatrix}1 & 0 & 3\\ -1 & 3 & 0\end{pmatrix}$.

(a) Write down the value of m and of n. *[2 marks]*

(b) Find $\boldsymbol{AB}$. *[3 marks]*

(c) Let $\boldsymbol{C}$ be a $p\times4$ matrix. Given that the product $\boldsymbol{BC}$ exists, write down the value of p. *[1 mark]*

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

3. *[Maximum mark: 6]*

Let $f(x) = e^{6x}$.

(a) Write down $f'(x)$. *[1 mark]*

The tangent to the graph of f at the point $P(0, b)$ has gradient m.

(b) (i) Show that $m = 6$.

(ii) Find b. *[4 marks]*

(c) Hence, write down the equation of this tangent. *[1 mark]*

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

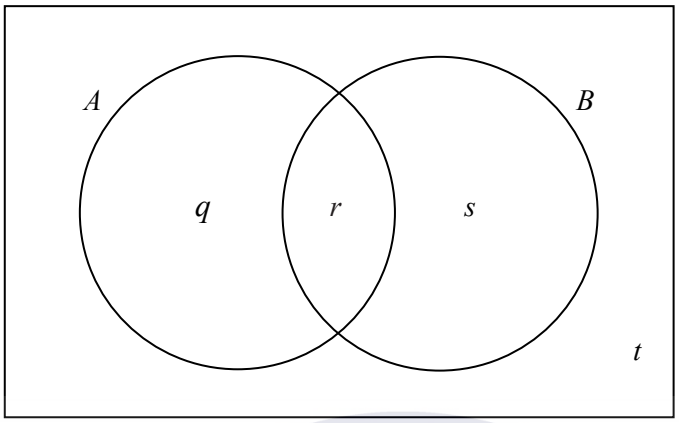
. . . . . .

. . . . . .

. . . . . .

4. *[Maximum mark: 7]*

Events A and B are such that $\mathrm{P}(A) = 0.3$, $\mathrm{P}(B) = 0.6$ and $\mathrm{P}(A \cup B) = 0.7$.



The values q, r, s and t represent probabilities.

(a) Write down the value of t. *[1 mark]*

(b) (i) Show that $r = 0.2$.

(ii) Write down the value of q and of s. *[3 marks]*

(c) (i) Write down $\mathrm{P}(B')$.

(ii) Find $\mathrm{P}(A|B')$. *[3 marks]*

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

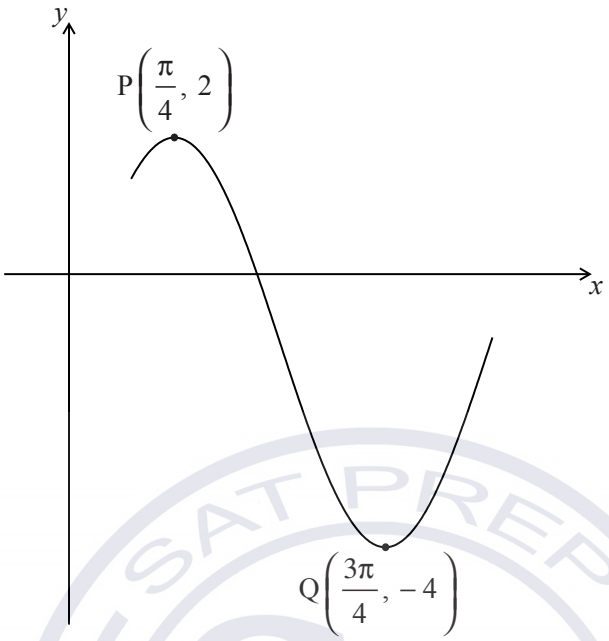
. . . . . .

. . . . . .

. . . . . .

5. *[Maximum mark: 7]*

The diagram below shows part of the graph of $f(x) = a\cos(b(x-c)) - 1$, where $a > 0$.



The point $P\left(\frac{\pi}{4}, 2\right)$ is a maximum point and the point $Q\left(\frac{3\pi}{4}, -4\right)$ is a minimum point.

(a) Find the value of a. *[2 marks]*

(b) (i) Show that the period of f is π.

(ii) Hence, find the value of b. *[4 marks]*

(c) Given that $0 < c < \pi$, write down the value of c. *[1 mark]*

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

6. *[Maximum mark: 6]*

Given that $\int_0^5 \frac{2}{2x+5}\mathrm{d}x = \ln k$, find the value of k.

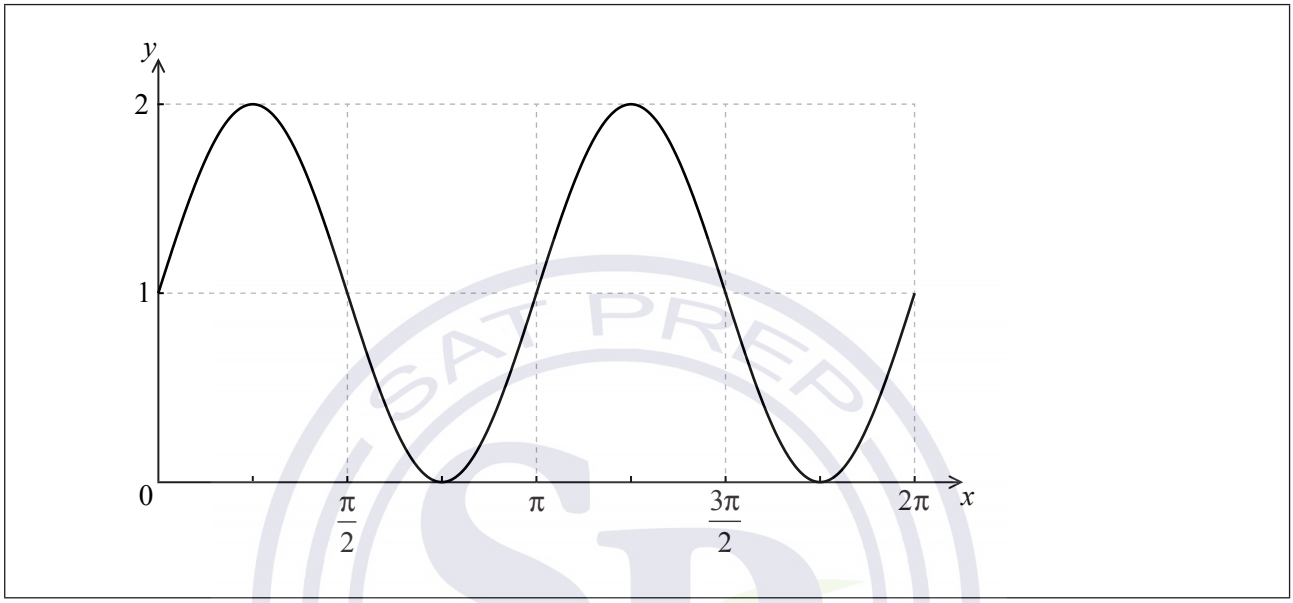
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

7. *[Maximum mark: 6]*

Let $f(x) = (\sin x + \cos x)^2$.

(a) Show that $f(x)$ can be expressed as $1 + \sin 2x$. *[2 marks]*

The graph of f is shown below for $0 \le x \le 2\pi$.



(b) Let $g(x) = 1 + \cos x$. On the same set of axes, sketch the graph of g for $0 \le x \le 2\pi$. *[2 marks]*

The graph of g can be obtained from the graph of f under a horizontal stretch of scale factor p followed by a translation by the vector $\begin{pmatrix} k \\ 0 \end{pmatrix}$.

(c) Write down the value of p and a possible value of k. *[2 marks]*

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

*Do **NOT** write solutions on this page.*

SECTION B

*Answer **all** questions on the answer sheets provided. Please start each question on a new page.*

8. *[Maximum mark: 17]*

A line L_1 passes though points $\mathrm{P}(-1, 6, -1)$ and $\mathrm{Q}(0, 4, 1)$.

(a) (i) Show that $\overrightarrow{\mathrm{PQ}} = \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$.

(ii) Hence, write down an equation for L_1 in the form $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$. *[3 marks]*

A second line L_2 has equation $\boldsymbol{r} = \begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix} + s\begin{pmatrix} 3 \\ 0 \\ -4 \end{pmatrix}$.

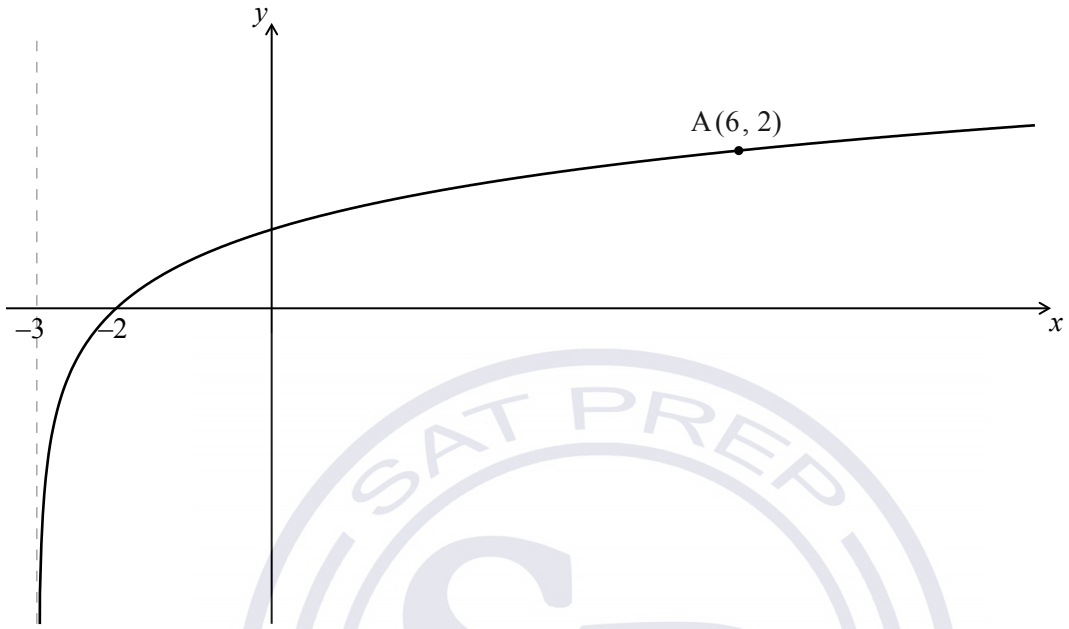
(b) Find the cosine of the angle between $\overrightarrow{\mathrm{PQ}}$ and L_2. *[7 marks]*

(c) The lines L_1 and L_2 intersect at the point R. Find the coordinates of R. *[7 marks]*

*Do **NOT** write solutions on this page.*

9. *[Maximum mark: 13]*

Let $f(x) = \log_p(x+3)$ for $x > -3$. Part of the graph of f is shown below.



The graph passes through $A(6, 2)$, has an x-intercept at $(-2, 0)$ and has an asymptote at $x = -3$.

(a) Find p. *[4 marks]*

The graph of f is reflected in the line $y = x$ to give the graph of g.

(b) (i) Write down the y-intercept of the graph of g.

(ii) Sketch the graph of g, noting clearly any asymptotes and the image of A. *[5 marks]*

(c) Find $g(x)$. *[4 marks]*

*Do **NOT** write solutions on this page.*

10. *[Maximum mark: 15]*

In this question, you are given that $\cos\frac{\pi}{3}=\frac{1}{2}$, *and* $\sin\frac{\pi}{3}=\frac{\sqrt{3}}{2}$.

The displacement of an object from a fixed point, O is given by $s(t)=t-\sin 2t$ for $0\le t\le\pi$.

(a) Find $s'(t)$. *[3 marks]*

In this interval, there are only two values of t for which the object is not moving. One value is $t=\frac{\pi}{6}$.

(b) Find the other value. *[4 marks]*

(c) Show that $s'(t)>0$ between these two values of t. *[3 marks]*

(d) Find the distance travelled between these two values of t. *[5 marks]*

22127305

International Baccalaureate®
Baccalauréat International
Bachillerato Internacional

MATHEMATICS
STANDARD LEVEL
PAPER 1

Candidate session number

0	0							

Thursday 3 May 2012 (afternoon)

1 hour 30 minutes

Examination code

2	2	1	2	–	7	3	0	5

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions on the answer sheets provided. Write your session number on each answer sheet, and attach them to this examination paper and your cover sheet using the tag provided.
- At the end of the examination, indicate the number of sheets used in the appropriate box on your cover sheet.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the ***Mathematics SL*** *information booklet* is required for this paper.
- The maximum mark for this examination paper is *[90 marks]*.

0116

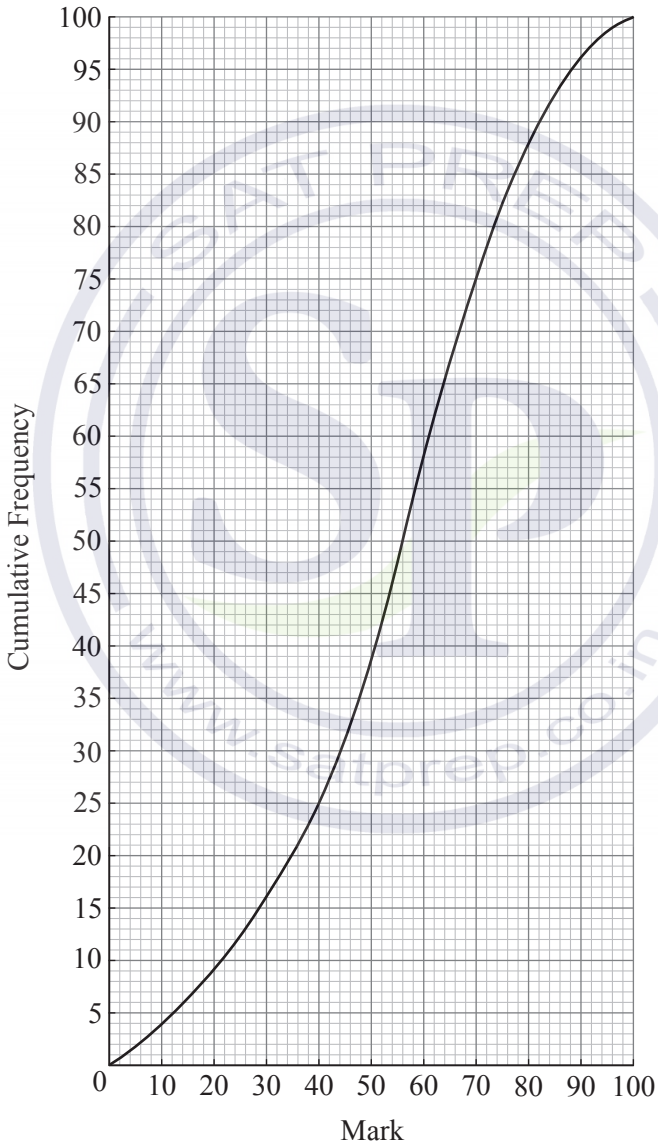
Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

*Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.*

1. *[Maximum mark: 5]*

The cumulative frequency curve below represents the marks obtained by 100 students.



(This question continues on the following page)

(Question 1 continued)

(a) Find the median mark. *[2 marks]*

(b) Find the interquartile range. *[3 marks]*

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

2. *[Maximum mark: 6]*

Let $f(x) = 2x - 1$ and $g(x) = 3x^2 + 2$.

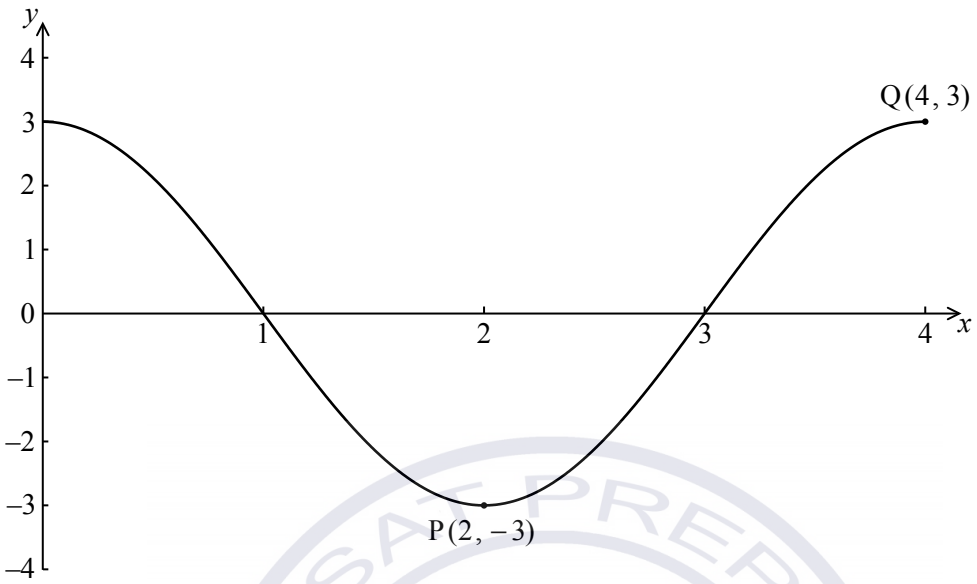
(a) Find $f^{-1}(x)$. *[3 marks]*

(b) Find $(f \circ g)(1)$. *[3 marks]*

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

3. *[Maximum mark: 6]*

The following diagram shows the graph of $f(x) = a\cos(bx)$, for $0 \le x \le 4$.



There is a minimum point at $P(2, -3)$ and a maximum point at $Q(4, 3)$.

(a) (i) Write down the value of a.

(ii) Find the value of b. *[3 marks]*

(b) Write down the gradient of the curve at P. *[1 mark]*

(c) Write down the equation of the normal to the curve at P. *[2 marks]*

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

4. *[Maximum mark: 8]*

The random variable X has the following probability distribution, with $\mathrm{P}(X > 1) = 0.5$.

x	0	1	2	3
$\mathrm{P}(X = x)$	p	q	r	0.2

(a) Find the value of r. *[2 marks]*

(b) Given that $\mathrm{E}(X) = 1.4$, find the value of p and of q. *[6 marks]*

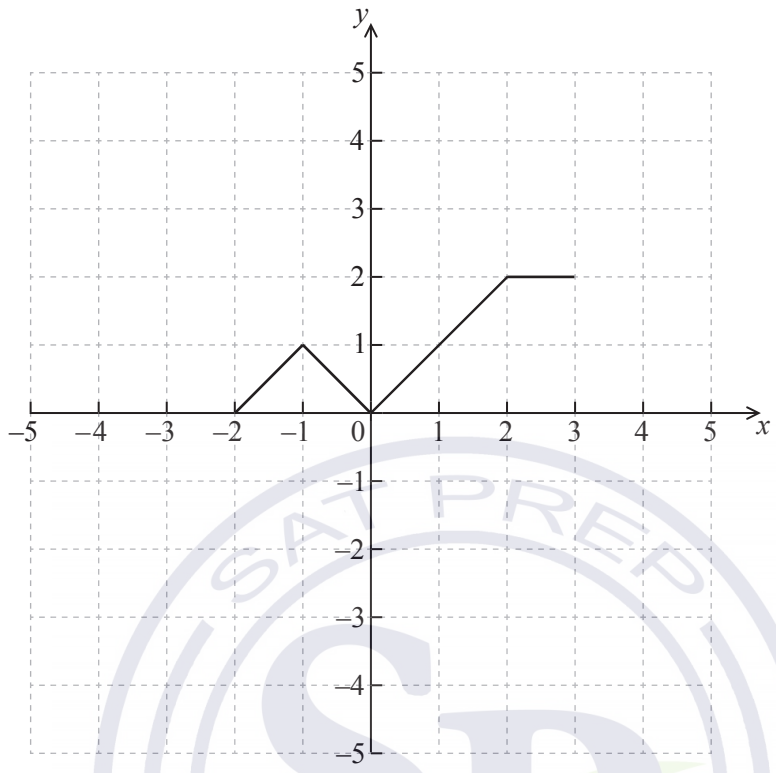
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

Please **do not** write on this page.

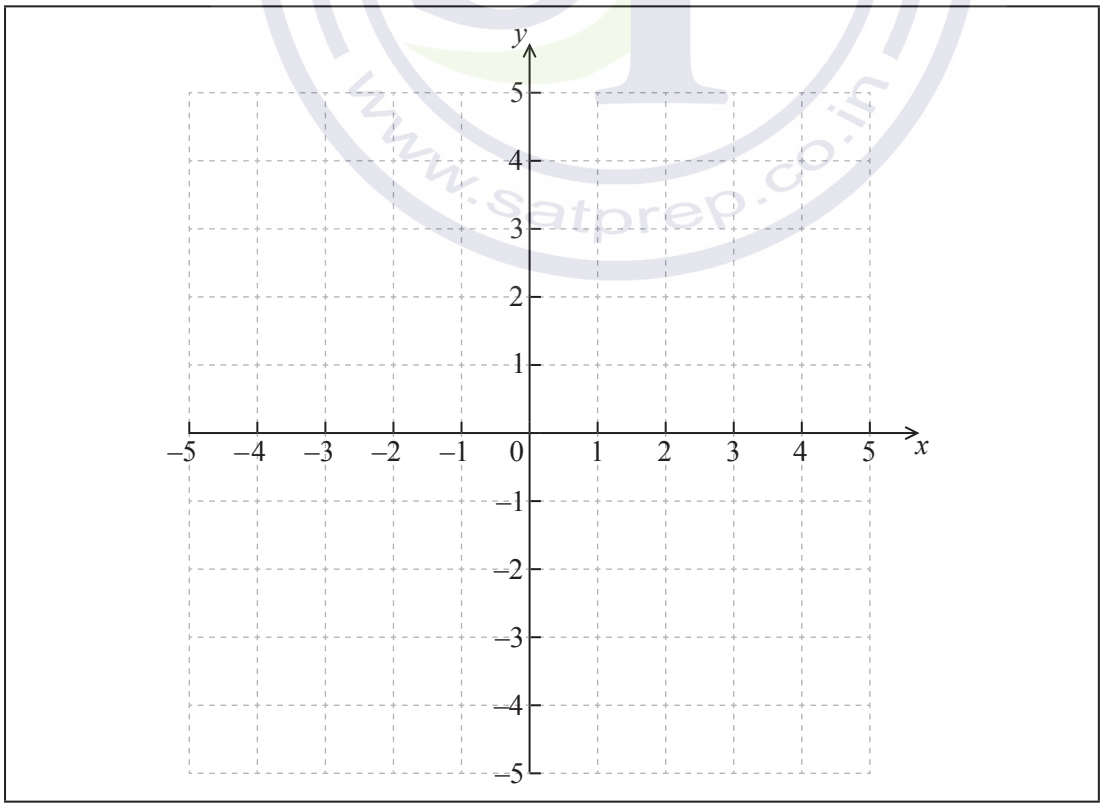
Answers written on this page
will not be marked.

5. *[Maximum mark: 6]*

The diagram below shows the graph of a function $f(x)$, for $-2 \le x \le 3$.



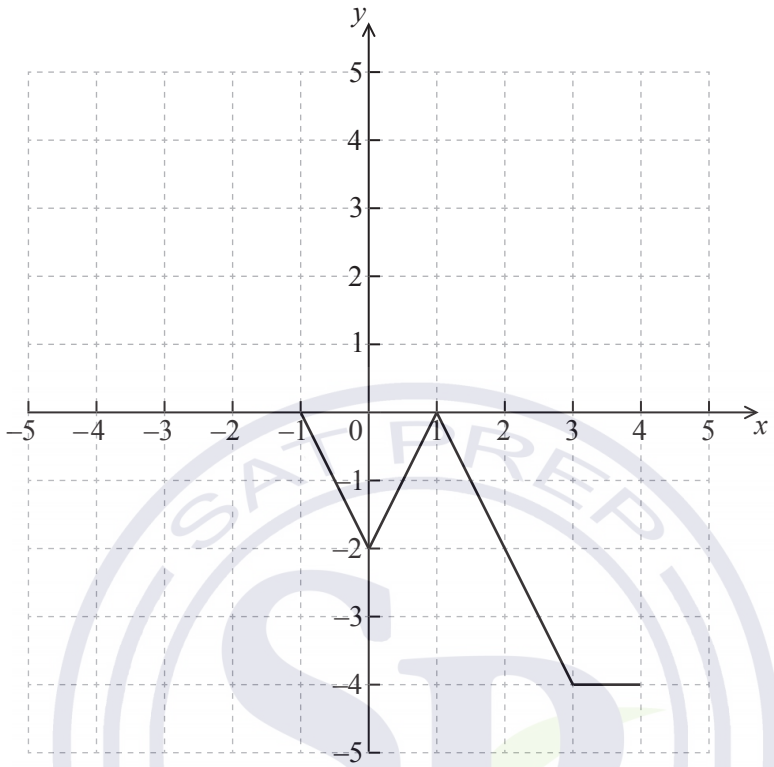
(a) Sketch the graph of $f(-x)$ on the grid below. *[2 marks]*



(This question continues on the following page)

(Question 5 continued)

(b) The graph of f is transformed to obtain the graph of g. The graph of g is shown below.



The function g can be written in the form $g(x) = af(x+b)$. Write down the value of a and of b. *[4 marks]*

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

6. *[Maximum mark: 7]*

Consider the equation $x^2 + (k-1)x + 1 = 0$, where k is a real number.

Find the values of k for which the equation has two **equal** real solutions.

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

7. *[Maximum mark: 7]*

Given that $\left(1+\frac{2}{3}x\right)^n(3+nx)^2 = 9+84x+\ldots$, find the value of n.

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

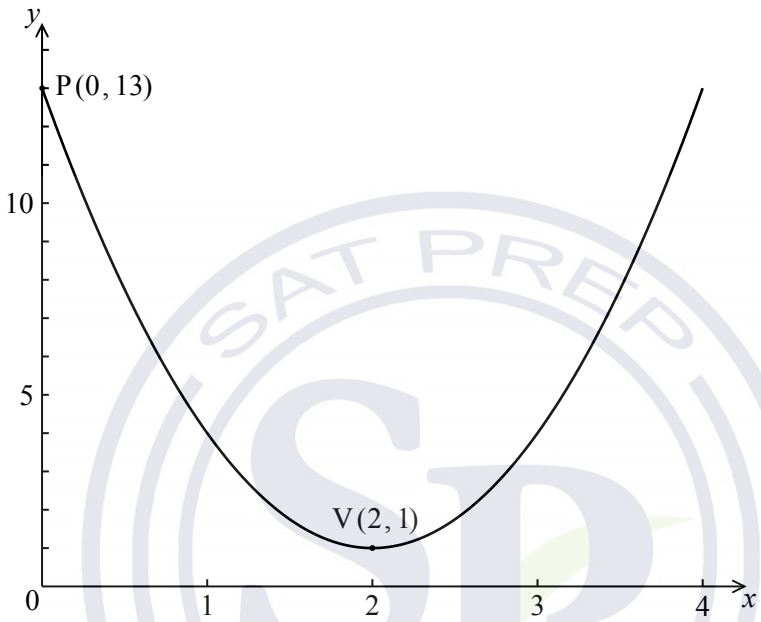
*Do **NOT** write solutions on this page.*

SECTION B

*Answer **all** questions on the answer sheets provided. Please start each question on a new page.*

8. *[Maximum mark: 15]*

The following diagram shows the graph of a quadratic function f, for $0 \le x \le 4$.



The graph passes through the point $\mathrm{P}(0, 13)$, and its vertex is the point $\mathrm{V}(2, 1)$.

(a) The function can be written in the form $f(x) = a(x-h)^2 + k$.

(i) Write down the value of h and of k.

(ii) Show that $a = 3$. *[4 marks]*

(b) Find $f(x)$, giving your answer in the form $Ax^2 + Bx + C$. *[3 marks]*

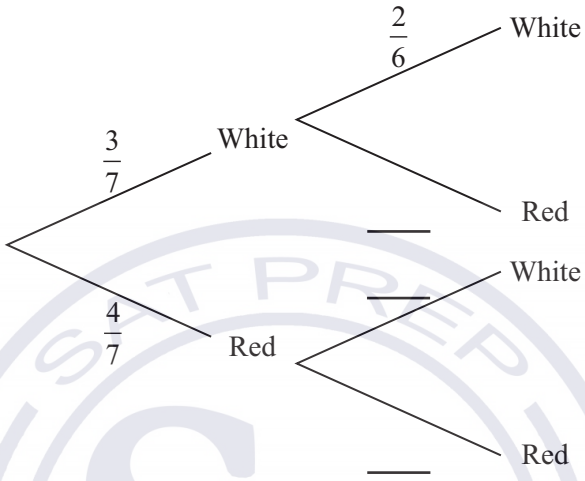
(c) Calculate the area enclosed by the graph of f, the x-axis, and the lines $x = 2$ and $x = 4$. *[8 marks]*

*Do **NOT** write solutions on this page.*

9. *[Maximum mark: 14]*

Bag A contains three white balls and four red balls. Two balls are chosen at random without replacement.

(a) (i) **Copy** and complete the following tree diagram. *(Do **not** write on this page.)*



(ii) Find the probability that two white balls are chosen. *[5 marks]*

Bag B contains four white balls and three red balls. When two balls are chosen at random without replacement from bag B, the probability that they are both white is $\frac{2}{7}$.

A standard die is rolled. If 1 or 2 is obtained, two balls are chosen without replacement from bag A, otherwise they are chosen from bag B.

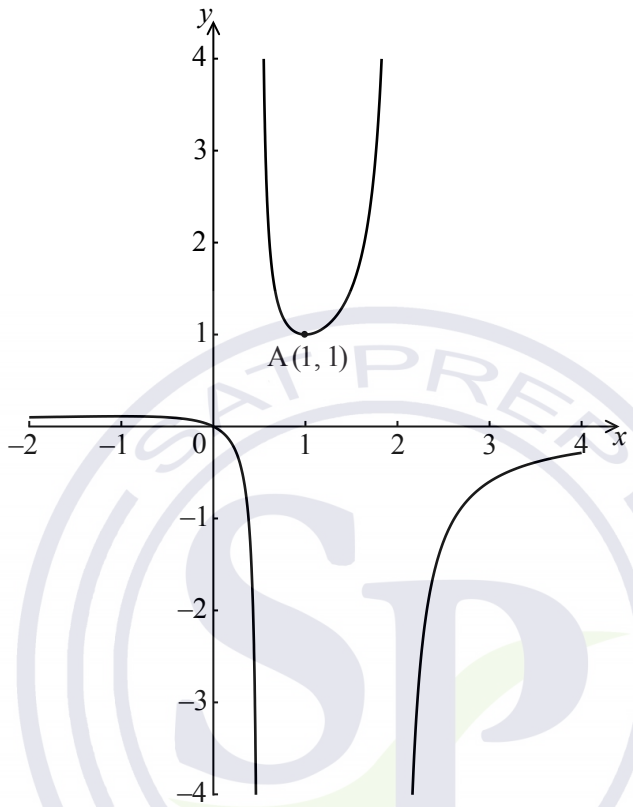
(b) Find the probability that the two balls are white. *[5 marks]*

(c) Given that both balls are white, find the probability that they were chosen from bag A. *[4 marks]*

*Do **NOT** write solutions on this page.*

10. *[Maximum mark: 16]*

Let $f(x) = \frac{x}{-2x^2 + 5x - 2}$ for $-2 \le x \le 4$, $x \ne \frac{1}{2}$, $x \ne 2$. The graph of f is given below.



The graph of f has a local minimum at $A(1, 1)$ and a local maximum at B.

(a) Use the quotient rule to show that $f'(x) = \frac{2x^2 - 2}{(-2x^2 + 5x - 2)^2}$. *[6 marks]*

(b) Hence find the coordinates of B. *[7 marks]*

(c) Given that the line $y = k$ does not meet the graph of f, find the possible values of k. *[3 marks]*

Please **do not** write on this page.

Answers written on this page
will not be marked.

Please **do not** write on this page.

Answers written on this page
will not be marked.

88117301

International Baccalaureate®
Baccalauréat International
Bachillerato Internacional

MATHEMATICS
STANDARD LEVEL
PAPER 1

Candidate session number

0	0							

Wednesday 2 November 2011 (afternoon)

Examination code

8	8	1	1	–	7	3	0	1

1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions on the answer sheets provided. Write your session number on each answer sheet, and attach them to this examination paper and your cover sheet using the tag provided.
- At the end of the examination, indicate the number of sheets used in the appropriate box on your cover sheet.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.

0112

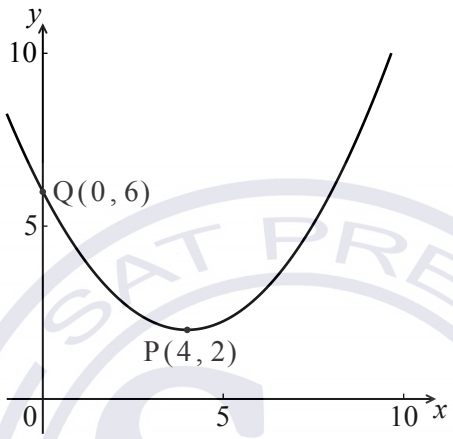
Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

*Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.*

1. *[Maximum mark: 6]*

Let f be a quadratic function. Part of the graph of f is shown below.



The vertex is at $P(4, 2)$ and the y-intercept is at $Q(0, 6)$.

(a) Write down the equation of the axis of symmetry. *[1 mark]*

The function f can be written in the form $f(x) = a(x-h)^2 + k$.

(b) Write down the value of h and of k. *[2 marks]*

(c) Find a. *[3 marks]*

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

2. *[Maximum mark: 5]*

Let $\boldsymbol{P} = \begin{pmatrix} 3 & 1 \\ 5 & 2 \end{pmatrix}$ and $\boldsymbol{Q} = \begin{pmatrix} 4 & -2 \\ -10 & 6 \end{pmatrix}$.

(a) Find $\boldsymbol{PQ}$. *[3 marks]*

(b) Hence, or otherwise, write down $\boldsymbol{P}^{-1}$. *[2 marks]*

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

3. *[Maximum mark: 6]*

A box contains six red marbles and two blue marbles. Anna selects a marble from the box. She replaces the marble and then selects a second marble.

(a) Write down the probability that the first marble Anna selects is red. *[1 mark]*

(b) Find the probability that Anna selects two red marbles. *[2 marks]*

(c) Find the probability that one marble is red and one marble is blue. *[3 marks]*

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

4. *[Maximum mark: 6]*

Let $f'(x) = 3x^2 + 2$. Given that $f(2) = 5$, find $f(x)$.

5. *[Maximum mark: 6]*

The random variable X has the following probability distribution.

x	1	2	3
$\mathrm{P}(X = x)$	s	0.3	q

Given that $\mathrm{E}(X) = 1.7$, find q.

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

6. *[Maximum mark: 8]*

Let $\sin\theta = \frac{2}{\sqrt{13}}$, where $\frac{\pi}{2} < \theta < \pi$.

(a) Find $\cos\theta$. *[3 marks]*

(b) Find $\tan 2\theta$. *[5 marks]*

. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .
. . . . . .

7. *[Maximum mark: 8]*

Let $f(x) = \frac{1}{2}x^2 + kx + 8$, where $k \in \mathbb{Z}$.

(a) Find the values of k such that $f(x) = 0$ has two equal roots. *[4 marks]*

(b) Each value of k is equally likely for $-5 \leq k \leq 5$. Find the probability that $f(x) = 0$ has no roots. *[4 marks]*

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

. . . . . .

*Do **NOT** write solutions on this page. Any working on this page will **NOT** be marked.*

SECTION B

*Answer **all** the questions on the answer sheets provided. Please start each question on a new page.*

8. *[Maximum mark: 18]*

The line L_1 passes through the points $\mathrm{P}(2, 4, 8)$ and $\mathrm{Q}(4, 5, 4)$.

(a) (i) Find $\overrightarrow{\mathrm{PQ}}$.

(ii) Hence write down a vector equation for L_1 in the form $\boldsymbol{r} = \boldsymbol{a} + s\boldsymbol{b}$. *[4 marks]*

The line L_2 is perpendicular to L_1, and parallel to $\begin{pmatrix} 3p \\ 2p \\ 4 \end{pmatrix}$, where $p \in \mathbb{Z}$.

(b) (i) Find the value of p.

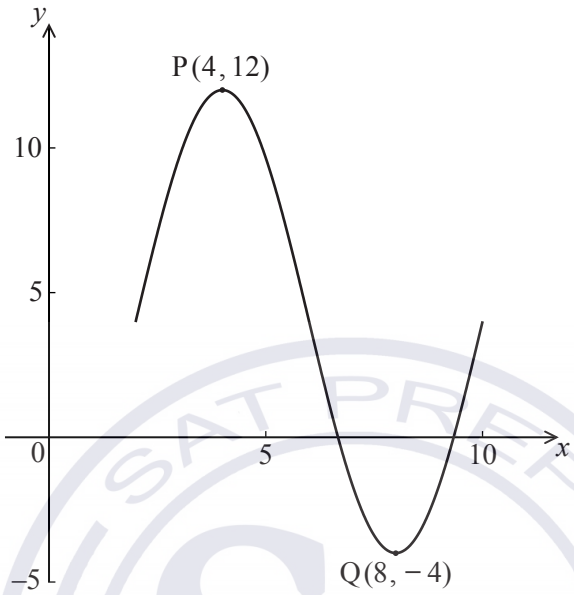
(ii) Given that L_2 passes through $\mathrm{R}(10, 6, -40)$, write down a vector equation for L_2. *[7 marks]*

(c) The lines L_1 and L_2 intersect at the point A. Find the x-coordinate of A. *[7 marks]*

*Do **NOT** write solutions on this page. Any working on this page will **NOT** be marked.*

9. *[Maximum mark: 14]*

The following diagram shows the graph of $f(x) = a\sin(b(x-c)) + d$, for $2 \le x \le 10$.



There is a maximum point at $\mathrm{P}(4, 12)$ and a minimum point at $\mathrm{Q}(8, -4)$.

(a) Use the graph to write down the value of

(i) a;

(ii) c;

(iii) d. *[3 marks]*

(b) Show that $b = \frac{\pi}{4}$. *[2 marks]*

(c) Find $f'(x)$. *[3 marks]*

(d) At a point R, the gradient is -2π. Find the x-coordinate of R. *[6 marks]*

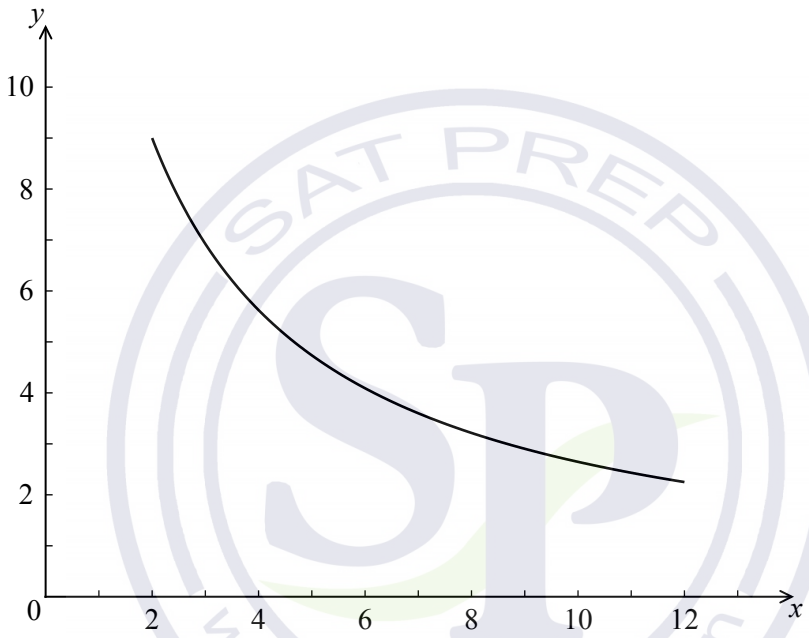
*Do **NOT** write solutions on this page. Any working on this page will **NOT** be marked.*

10. *[Maximum mark: 13]*

Let $f(x) = \frac{1}{4}x^2 + 2$. The line L is the tangent to the curve of f at $(4, 6)$.

(a) Find the equation of L. *[4 marks]*

Let $g(x) = \frac{90}{3x+4}$, for $2 \le x \le 12$. The following diagram shows the graph of g.



(b) Find the area of the region enclosed by the curve of g, the x-axis, and the lines $x = 2$ and $x = 12$. Give your answer in the form $a \ln b$, where $a, b \in \mathbb{Z}$. *[6 marks]*

(c) The graph of g is reflected in the x-axis to give the graph of h. The area of the region enclosed by the lines L, $x = 2$, $x = 12$ and the x-axis is 120 cm^2.

Find the area enclosed by the lines L, $x = 2$, $x = 12$ and the graph of h. *[3 marks]*

Please **do not** write on this page.

Answers written on this page will not be marked.