Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

November 2019

Mathematics

Standard level

Paper 1

18 pages

International Baccalaureate
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Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a valid **Method**; working must be seen.

(M) Marks awarded for a valid **Method**; may be implied by **correct** subsequent working.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

(A) Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.

R Marks awarded for clear **Reasoning**.

N Marks awarded for **correct** answers if **no** working shown.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Mark according to RM assessor instructions.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any. An exception to this rule is when work for ***M1*** is missing, as opposed to incorrect (see point 4).
- Where ***M*** and ***A*** marks are noted on the same line, *eg* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*eg* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***(M2)***, ***N3***, *etc*., do **not** split the marks, unless there is a note.
- Most ***M*** marks are for a **valid** method, ie a method which can lead to the answer: it must indicate some form of progress towards the answer.
- Once a correct answer to a question or part-question is seen, ignore further correct working. However, if further working indicates a lack of mathematical understanding do not award final ***A1***.

3 *N* marks

*If **no** working shown, award **N** marks for **correct** answers. In this case, ignore mark breakdown (**M**, **A**, **R**).*

- Do **not** award a mixture of ***N*** and other marks.
- There may be fewer ***N*** marks available than the total of ***M***, ***A*** and ***R*** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.
- There may not be a direct relationship between the ***N*** marks and the implied marks. There are times when all the marks are implied, but the ***N*** marks are not the full marks: this indicates that we want to see some of the working, without specifying what.

- For consistency within the markscheme, ***N*** marks are noted for every part, even when these match the mark breakdown.
- If a candidate has incorrect working, which somehow results in a correct answer, do **not** award the ***N*** marks for this correct answer. However, if the candidate has indicated (usually by crossing out) that the working is to be ignored, award the ***N*** marks for the correct answer.

4 Implied and must be seen marks

Implied marks appear in ***brackets*** *eg* ***(M1)****.*

- Implied marks can only be awarded if the work is seen or if implied in subsequent working (a correct final answer does not necessarily mean that the implied marks are all awarded). There are questions where some working is required, but as it is accepted that not everyone will write the same steps, all the marks are implied, but the ***N*** marks are not the full marks for the question.
- Normally the correct work is seen in the next line.
- Where there is an (***M1***) followed by ***A1*** for each correct answer, if no working shown, one correct answer is sufficient evidence to award the (***M1***).

Must be seen marks appear without ***brackets*** *eg* ***M1***.

- Must be seen marks can only be awarded if the work is seen.
- If a must be seen mark is not awarded because work is missing (as opposed to ***M0*** or ***A0*** for incorrect work) all subsequent marks may be awarded if appropriate.

5 Follow through marks (only applied after an error is made)

*Follow through (****FT****) marks are awarded where an incorrect answer (final or intermediate) from one* ***part*** *of a question is used correctly in* ***subsequent*** *part(s) or subpart(s). Usually, to award* ***FT*** *marks,* ***there must be working present*** *and not just a final answer based on an incorrect answer to a previous part. However, if the only marks awarded in a subpart are for the* ***final*** *answer, then* ***FT*** *marks should be awarded if appropriate. Examiners are expected to check student work in order to award* ***FT*** *marks where appropriate.*

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** and ***R*** marks may be awarded if appropriate. (However, as noted above, if an ***A*** mark is not awarded because work is missing, all subsequent marks may be awarded if appropriate).
- Exceptions to this rule will be explicitly noted on the markscheme.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*eg* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word "**their**" in a description, to indicate that candidates may be using an incorrect value.
- If a candidate makes an error in one part, but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the question says hence. It is often possible to use a different approach in subsequent parts that does not depend on the answer to previous parts.
- In a "show that" question, if an error in a previous subpart leads to not showing the required answer, do not award the final ***A1***. Note that if the error occurs within the same subpart, the ***FT*** rules may result in further loss of marks.

6 Mis-read

*If a candidate incorrectly copies information from the question, this is a mis-read (**MR**). A candidate should be penalized only once for a particular mis-read. Use the **MR** stamp to indicate that this is a misread. Do not award the first mark in the question, even if this is an **M** mark, but award all others (if appropriate) so that the candidate only loses one mark for the misread.*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*eg* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.

7 Discretionary marks *(d)*

*An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. In such cases the annotation DM should be used and a brief **note** written next to the mark explaining this decision.*

8 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete parts are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**. Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9 Alternative forms

*Unless the question specifies otherwise, **accept** equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

10 Calculators

No calculator is allowed. The use of any calculator on paper 1 is malpractice, and will result in no grade awarded. If you see work that suggests a candidate has used any calculator, please follow the procedures for malpractice. Examples: finding an angle, given a trig ratio of 0.4235.

11 Style

The markscheme aims to present answers using good communication, eg if the question asks to find the value of k, the markscheme will say $k = 3$*, but the marks will be for the correct value 3 – there is usually no need for the "* $k =$ *". In these cases, it is also usually acceptable to have another variable, as long as there is no ambiguity in the question, eg if the question asks to find the value of p and of q, then the student answer needs to be clear. Generally, the only situation where the full answer is required is in a question which asks for equations – in this case the markscheme will say "must be an equation".*

The markscheme often uses words to describe what the marks are for, followed by examples, using the eg notation. These examples are not exhaustive, and examiners should check what candidates have written, to see if they satisfy the description. Where these marks are ***M*** *marks, the examples may include ones using poor notation, to indicate what is acceptable.*

12 Candidate work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

Candidates are meant to write their answers to Section A on the question paper (QP), and Section B on answer booklets. Sometimes, they need more room for Section A, and use the booklet (and often comment to this effect on the QP), or write outside the box. That is fine, and this work should be marked.

The instructions tell candidates not to write on Section B of the QP. Thus they may well have done some rough work here which they assume will be ignored. If they have solutions on the answer booklets, there is no need to look at the QP. However, if there are whole questions or whole part solutions missing on answer booklets, please check to make sure that they are not on the QP, and if they are, mark those whole questions or whole part solutions that have not been written on answer booklets.

13. Diagrams

The notes on how to allocate marks for sketches usually refer to passing through particular points or having certain features. These marks can only be awarded if the sketch is approximately the correct shape. All values given will be an approximate guide to where these points/features occur. In some questions, the first ***A1*** is for the shape, in others, the marks are only for the points and/or features. In both cases, unless the shape is approximately correct, no marks can be awarded (unless otherwise stated). However, if the graph is based on previous calculations, ***FT*** marks should be awarded if appropriate.

14. Accuracy of Answers

If the level of accuracy is specified in the question, a mark will be allocated for giving the final answer to the required accuracy. When this is not specified in the question, all numerical answers should be given exactly or correct to three significant figures.

Do not accept unfinished numerical final answers such as 3/0.1 (unless otherwise stated). As a rule, numerical answers with more than one part (such as fractions) should be given using integers (eg 6/8). Calculations which lead to integers should be completed, with the exception of fractions which are not whole numbers. Intermediate values do not need to be given to the correct three significant figures. But, if candidates work with rounded values, this could lead to an incorrect answer, in which case award A0 for the final answer.

Section A

1. (a) valid approach *(M1)*

eg $11-5$, $11=5+d$

$d=6$ **A1 N2**

[2 marks]

(b) valid approach *(M1)*

eg u_2-d, $5-6$, $u_1+(3-1)(6)=11$

$u_1=-1$ **A1 N2**

[2 marks]

(c) correct substitution into sum formula

eg $\frac{20}{2}\left(2(-1)+19(6)\right)$, $\frac{20}{2}(-1+113)$ *(A1)*

$S_{20}=1120$ **A1 N2**

[2 marks]

Total [6 marks]

2. (a) $q=5$ **A1 N1**

[1 mark]

(b) valid approach *(M1)*

eg $(18+10+5)-30$, $28-25$, $18+10-n=25$

$n=3$ **A1 N2**

[2 marks]

(c) valid approach for finding m or p (may be seen in part (b)) *(M1)*

eg $18-3$, $3+p=10$

$m=15$, $p=7$ **A1A1 N3**

[3 marks]

[Total 6 marks]

3. (a) valid attempt to substitute coordinates *(M1)*

eg $g(-1) = 8$

correct substitution *(A1)*

eg $(-1)^2 + b(-1) + 11 = 8,\ 1 - b + 11 = 8$

$b = 4$ ***A1 N2***

[3 marks]

(b) valid attempt to solve *(M1)*

eg $\left(x^2 + 4x + 4\right) + 7,\ h = \frac{-4}{2},\ k = g(-2)$

correct working ***A1***

eg $(x+2)^2 + 7,\ h = -2,\ k = 7$

translation or shift (do not accept move) of vector $\begin{pmatrix} -2 \\ 7 \end{pmatrix}$ (accept left by 2 and up by 7)

A1A1 N2

[4 marks]

Total [7 marks]

4. (a) valid approach ***(M1)***

eg $11-a=9$, $\frac{11!}{9!(11-9)!}$

$a=2$ ***A1*** ***N2***

[2 marks]

(b) valid approach for expansion using $n=11$ ***(M1)***

eg $\binom{11}{r}x^{11-r}3^r$, $a^{11}b^0+\binom{11}{1}a^{10}b^1+\binom{11}{2}a^9b^2+\ldots$

evidence of choosing correct term ***A1***

eg $\binom{11}{2}3^2$, $\binom{11}{2}x^9 3^2$, $\binom{11}{9}3^2$

correct working for binomial coefficient (seen anywhere, do not accept factorials) ***A1***

eg 55, $\binom{11}{2}=55$, 55×3^2, $(55\times9)\,x^9$, $\frac{11\times10}{2}\times9$

495 ***A1*** ***N2***

Note: If there is clear evidence of adding instead of multiplying, award ***A1*** for the correct working for binomial coefficient, but no other marks.
For example, $55x^9+3^2$ would earn ***M0A0A1A0***.

Do not award final ***A1*** for a final answer of $495x^9$, even if 495 is seen previously. If no working shown, award ***N1*** for $495x^9$.

[4 marks]

Total [6 marks]

5. (a) correct substitution into $b^2 - 4ac$ ***(A1)***

eg $(5k)^2 - 4(2)(3k^2 + 2)$, $(5k)^2 - 8(3k^2 + 2)$

correct expansion of each term ***A1***

eg $25k^2 - 24k^2 - 16$, $25k^2 - (24k^2 + 16)$

$k^2 - 16$ ***AG N0***

[2 marks]

(b) valid approach ***M1***

eg $f'(x) > 0$, $f'(x) \geq 0$

recognizing discriminant <0 or ≤ 0 ***M1***

eg $D < 0$, $k^2 - 16 \leq 0$, $k^2 < 16$

two correct values for k/endpoints (even if inequalities are incorrect) ***(A1)***

eg $k = \pm 4$, $k < -4$ and $k > 4$, $|k| < 4$

correct interval ***A1 N2***

eg $-4 < k < 4$, $-4 \leq k \leq 4$

Note: Candidates may work with an equation, then write the intervals with inequalities at the end. If inequalities are not seen until the candidate's final correct answer, ***M0M0A1A1*** may be awarded.

If candidate is working with incorrect inequalitie(s) at the beginning, then gets the correct final answer, award ***M0M0A1A0*** or ***M1M0A1A0*** or ***M0M1A1A0*** in line with the markscheme.

[4 marks]

Total [6 marks]

6. **METHOD 1 – FINDING INTERVALS FOR x**

$$4\cos\left(\frac{x}{2}\right)+1>2\sqrt{2}+1$$

correct working **(A1)**

eg $4\cos\left(\frac{x}{2}\right)=2\sqrt{2},\ \cos\left(\frac{x}{2}\right)>\frac{\sqrt{2}}{2}$

recognizing $\cos^{-1}\frac{\sqrt{2}}{2}=\frac{\pi}{4}$ **(A1)**

one additional correct value for $\frac{x}{2}$ (ignoring domain and equation/inequalities) **(A1)**

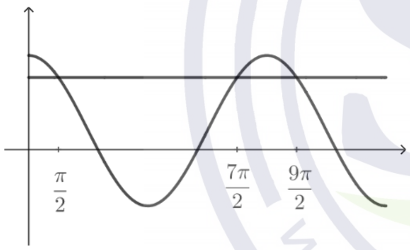
eg $-\frac{\pi}{4},\ \frac{7\pi}{4},\ 315°,\ \frac{9\pi}{4},\ -45°,\ \frac{15\pi}{4}$

three correct values for x ***A1A1***

eg $\frac{\pi}{2},\ \frac{7\pi}{2},\ \frac{9\pi}{2}$

valid approach to find intervals **(M1)**

eg



correct intervals (must be in radians) ***A1A1*** ***N2***

$0\le x<\frac{\pi}{2},\ \frac{7\pi}{2}<x<\frac{9\pi}{2}$

Note: If working shown, award ***A1A0*** if inclusion/exclusion of endpoints is incorrect. If no working shown award ***N1***.
If working shown, award ***A1A0*** if both correct intervals are given, **and** additional intervals are given. If no working shown award ***N1***.
Award ***A0A0*** if inclusion/exclusion of endpoints are incorrect **and** additional intervals are given.

continued…

Question 6 continued

METHOD 2 – FINDING INTERVALS FOR $\frac{x}{2}$

$4\cos\left(\frac{x}{2}\right)+1>2\sqrt{2}+1$

correct working ***(A1)***

eg $4\cos\left(\frac{x}{2}\right)=2\sqrt{2},\ \cos\left(\frac{x}{2}\right)>\frac{\sqrt{2}}{2}$

recognizing $\cos^{-1}\frac{\sqrt{2}}{2}=\frac{\pi}{4}$ ***(A1)***

one additional correct value for $\frac{x}{2}$ (ignoring domain and equation/inequalities) ***(A1)***

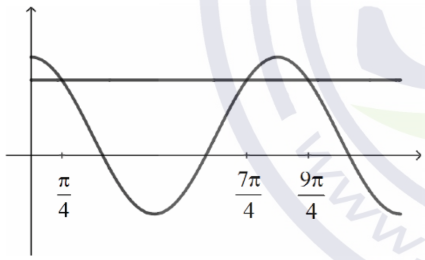
eg $-\frac{\pi}{4},\ \frac{7\pi}{4}, 315°,\ \frac{9\pi}{4}, -45°,\ \frac{15\pi}{4}$

three correct values for $\frac{x}{2}$ ***A1***

eg $\frac{\pi}{4},\ \frac{7\pi}{4},\ \frac{9\pi}{4}$

valid approach to find intervals ***(M1)***

eg



one correct interval for $\frac{x}{2}$ ***A1***

eg $0\le\frac{x}{2}<\frac{\pi}{4},\ \frac{7\pi}{4}<\frac{x}{2}<\frac{9\pi}{4}$

correct intervals (must be in radians) ***A1A1*** ***N2***

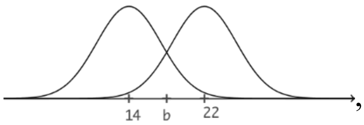
$0\le x<\frac{\pi}{2},\ \frac{7\pi}{2}<x<\frac{9\pi}{2}$

Note:	If working shown, award ***A1A0*** if inclusion/exclusion of endpoints is incorrect. If no working shown award ***N1***. If working shown, award ***A1A0*** if both correct intervals are given, **and** additional intervals are given. If no working shown award ***N1***. Award ***A0A0*** if inclusion/exclusion of endpoints are incorrect **and** additional intervals are given.

Total [8 marks]

7. (a) **METHOD 1**

recognizing that b is midway between the means of 14 and 22. ***(M1)***

eg 

$b = \frac{14+22}{2}$

$b = 18$ ***A1 N2***

METHOD 2

valid attempt to compare distributions ***(M1)***

eg $\frac{b-14}{a} = -\frac{b-22}{a}$, $b-14 = 22-b$

$b = 18$ ***A1 N2***

[2 marks]

(b) valid attempt to compare distributions (seen anywhere) ***(M1)***

eg Y is a horizontal translation of X of 8 units to the right,
$\mathrm{P}(16 < Y < 28) = \mathrm{P}(8 < X < 20)$, $\mathrm{P}(Y > 22+6) = \mathrm{P}(X > 14+6)$

valid approach using symmetry ***(M1)***

eg $1-2\mathrm{P}(X > 20)$, $1-2\mathrm{P}(Y < 16)$, $2\times\mathrm{P}(14 < x < 20)$, $\mathrm{P}(X < 8) = \mathrm{P}(X > 20)$

correct working ***(A1)***

eg $1-2(0.112)$, $2\times(0.5-0.112)$, 2×0.388, $0.888-0.112$

$\mathrm{P}(16 < Y < 28) = 0.776$ ***A1 N3***

[4 marks]

Total [6 marks]

Section B

8. (a) $y = 12 - 4x$ ***A1*** ***N1***

[1 mark]

(b) correct substitution into volume formula ***(A1)***

eg $3x \times x \times y$, $x \times 3x \times (12 - x - 3x)$, $(12-4x)(x)(3x)$

$V = 3x^2(12 - 4x)\ \left(= 36x^2 - 12x^3\right)$ ***A1*** ***N2***

Note: Award ***A0*** for unfinished answers such as $3x^2(12 - x - 3x)$.

[2 marks]

(c) $\frac{\mathrm{d}V}{\mathrm{d}x} = 72x - 36x^2$ ***A1A1*** ***N2***

Note: Award ***A1*** for $72x$ and ***A1*** for $-36x^2$.

[2 marks]

(d) (i) valid approach to find maximum ***(M1)***

eg $V' = 0$, $72x - 36x^2 = 0$

correct working ***(A1)***

eg $x(72 - 36x)$, $\frac{-72 \pm \sqrt{72^2 - 4 \cdot (-36) \cdot 0}}{2(-36)}$, $36x = 72$, $36x(2 - x) = 0$

$x = 2$ ***A2*** ***N2***

Note: Award ***A1*** for $x = 2$ and $x = 0$.

(ii) valid approach to explain that V is maximum when $x = 2$ ***(M1)***

eg attempt to find V'', sign chart (must be labelled V')

correct value/s ***A1***

eg $V''(2) = 72 - 72 \times 2$, $V'(a)$ where $a < 2$ **and** $V'(b)$ where $b > 2$

correct reasoning ***R1***

eg $V''(2) < 0$, V' is positive for $x < 2$ **and** negative for $x > 2$

Note: Do not award ***R1*** unless ***A1*** has been awarded.

V is maximum when $x = 2$ ***AG*** ***N0***

[7 marks]

(e) correct substitution into **their** expression for volume ***A1***

eg $3 \times 2^2(12 - 4 \times 2)$, $36(2^2) - 12(2^3)$

$V = 48\ (\text{cm}^3)$ ***A1*** ***N1***

[2 marks]

Total [14 marks]

9. (a) (i) correct substitution into either $\vec{OA} \bullet \vec{OC}$ or into $\vec{OB} \bullet \vec{OC}$ (in (ii)) ***(A1)***

eg $-2\times(-1)+4\times k$, $6\times(-1)+8\times k$

correct expression ***A1 N1***

eg $2+4k$, $4k+2$

(ii) correct expression ***A1 N1***

eg $8k-6$, $-6+8k$

[3 marks]

(b) finding magnitudes (seen anywhere) ***A1A1***

eg $\sqrt{(-2)^2+(4)^2+(-4)^2}$ (=6), $\sqrt{(6)^2+(8)^2+0^2}$ (=10)

correct substitution of their values into formula for angle AOC ***(A1)***

eg $\cos\theta = \dfrac{2+4k}{\sqrt{(-2)^2+(4)^2+(-4)^2}\left|\vec{OC}\right|}$

correct substitution of their values into formula for angle BOC ***(A1)***

eg $\cos\theta = \dfrac{8k-6}{\sqrt{(6)^2+(8)^2+0^2}\left|\vec{OC}\right|}$

recognizing that $\cos A\hat{O}C = \cos B\hat{O}C$ (seen anywhere) ***(M1)***

eg $\dfrac{2+4k}{\left|\vec{OC}\right|\sqrt{(-2)^2+(4)^2+(-4)^2}} = \dfrac{8k-6}{\left|\vec{OC}\right|\sqrt{6^2+(8)^2+0^2}}$, $\dfrac{2+4k}{6\sqrt{1+k^2}} = \dfrac{8k-6}{10\sqrt{1+k^2}}$

correct working (without radicals) ***(A2)***

eg $10(2+4k)=6(8k-6)$, $11k^2-79k+14=0$

correct working clearly leading to the required answer ***A1***

eg $20+36=48k-40k$, $56=8k$, $k=7$ and $k=\dfrac{2}{11}$, $(k-7)(11k-2)=0$

$k=7$ ***AG N0***

[8 marks]

continued…

Question 9 continued

(c) finding magnitude of $\overrightarrow{OC}$ (seen anywhere) ***A1***

eg $\sqrt{(-1)^2+7^2+0^2}$, $\sqrt{50}$

valid attempt to find $\cos\theta$ ***(M1)***

eg $\cos\theta = \dfrac{2+28}{6\sqrt{(-1)^2+7^2+0^2}}$, $\cos\theta = \dfrac{56-6}{10\sqrt{(-1)^2+7^2+0^2}}$,

$\left(\sqrt{26}\right)^2 = 6^2 + \left(\sqrt{50}\right)^2 - 2(6)\sqrt{50}\cos\theta$

finding $\cos\theta$ ***A1***

eg $\cos\theta = \dfrac{5}{\sqrt{50}}$ $\left(=\dfrac{1}{\sqrt{2}}\right)$

valid approach to find $\sin\theta$ (seen anywhere) ***(M1)***

eg $\theta = \dfrac{\pi}{4}$, $\sin\theta = \cos\theta$, $\sin\theta = \sqrt{1-\dfrac{25}{50}}$, $\sin\theta = \sqrt{1-\cos^2\theta}$, $\sin\theta = \dfrac{\sqrt{2}}{2}$

correct substitution of **their** values into $\dfrac{1}{2}ab\sin C$ ***(A1)***

eg $\dfrac{1}{2}\times 6\times\sqrt{50}\times\sqrt{1-\dfrac{25}{50}}$, $\dfrac{1}{2}\times 6\times\sqrt{50}\times\dfrac{5}{\sqrt{50}}$

area is 15 ***A1*** ***N3***

[6 marks]

Total [17 marks]

10. (a) $\mathrm{B}(a, 0)$ (accept $\mathrm{B}(q+1, 0)$) ***A2*** ***N2***

[2 marks]

(b)

> **Note:** There are many approaches to this part, and the steps may be done in any order. Please check working and award marks in line with the markscheme, noting that candidates may work with the equation of the line before finding a.

FINDING *a*

valid attempt to find an expression for a in terms of q ***(M1)***

$g(0) = a$, $p^0 + q = a$

$a = q + 1$ ***(A1)***

FINDING THE EQUATION OF L_1

EITHER

attempt to substitute tangent gradient and coordinates into equation of straight line ***(M1)***

eg $y - 0 = f'(a)(x - a)$, $y = f'(a)\left(x - (q+1)\right)$

correct equation in terms of a and p ***(A1)***

eg $y - 0 = \dfrac{1}{\ln(p)}(x - a)$

OR

attempt to substitute tangent gradient and coordinates to find b ***(M1)***

eg $0 = \dfrac{1}{\ln(p)}(a) + b$

$b = \dfrac{-a}{\ln(p)}$ ***(A1)***

THEN (must be in terms of **both** p and q)

$y = \dfrac{1}{\ln p}(x - q - 1)$, $y = \dfrac{1}{\ln p}x - \dfrac{q+1}{\ln p}$ ***A1*** ***N3***

> **Note:** Award ***A0*** for final answers in the form $L_1 = \dfrac{1}{\ln p}(x - q - 1)$.

[5 marks]

continued…

Question 10 continued

(c)

Note: There are many approaches to this part, and the steps may be done in any order. Please check working and award marks in line with the markscheme, noting that candidates may find q in terms of p before finding a value for p.

FINDING p

valid approach to find the gradient of the tangent ***(M1)***

eg $m_1 m_2 = -1$, $-\dfrac{1}{\dfrac{1}{\ln\left(\frac{1}{3}\right)}}$, $-\ln\left(\dfrac{1}{3}\right)$, $-\dfrac{1}{\ln p} = \dfrac{1}{\ln\left(\frac{1}{3}\right)}$

correct application of log rule (seen anywhere) ***(A1)***

eg $\ln\left(\dfrac{1}{3}\right)^{-1}$, $-(\ln(1)-\ln(3))$

correct equation (seen anywhere) ***A1***

eg $\ln p = \ln 3$, $p = 3$

FINDING q

correct substitution of $(-2, -2)$ into L_2 equation ***(A1)***

eg $-2 = (\ln p)(-2) + q + 1$

$q = 2\ln p - 3$, $q = 2\ln 3 - 3$ (seen anywhere) ***A1***

FINDING L_1

correct substitution of **their** p and q into **their** L_1 ***(A1)***

eg $y = \dfrac{1}{\ln 3}\left(x - (2\ln 3 - 3) - 1\right)$

$y = \dfrac{1}{\ln 3}(x - 2\ln 3 + 2)$, $y = \dfrac{1}{\ln 3}x - \dfrac{2\ln 3 - 2}{\ln 3}$ ***A1*** ***N2***

Note: Award ***A0*** for final answers in the form $L_1 = \dfrac{1}{\ln 3}(x - 2\ln 3 + 2)$.

[7 marks]

Total [14 marks]

Markscheme

May 2019

Mathematics

Standard level

Paper 1

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a valid **Method**; working must be seen.

(M) Marks awarded for a valid **Method**; may be implied by **correct** subsequent working.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

(A) Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.

R Marks awarded for clear **Reasoning**.

N Marks awarded for **correct** answers if **no** working shown.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Mark according to RM assessor instructions.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any. An exception to this rule is when work for ***M1*** is missing, as opposed to incorrect (see point 4).
- Where ***M*** and ***A*** marks are noted on the same line, *eg* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*eg* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***(M2)***, ***N3***, *etc*., do **not** split the marks, unless there is a note.
- Most ***M*** marks are for a **valid** method, ie a method which can lead to the answer: it must indicate some form of progress towards the answer.
- Once a correct answer to a question or part-question is seen, ignore further correct working. However, if further working indicates a lack of mathematical understanding do not award final ***A1***.

3 *N* marks

*If **no** working shown, award **N** marks for **correct** answers. In this case, ignore mark breakdown (**M**, **A**, **R**).*

- Do **not** award a mixture of ***N*** and other marks.
- There may be fewer ***N*** marks available than the total of ***M***, ***A*** and ***R*** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.
- There may not be a direct relationship between the ***N*** marks and the implied marks. There are times when all the marks are implied, but the ***N*** marks are not the full marks: this indicates that we want to see some of the working, without specifying what.

- For consistency within the markscheme, ***N*** marks are noted for every part, even when these match the mark breakdown.
- If a candidate has incorrect working, which somehow results in a correct answer, do **not** award the ***N*** marks for this correct answer. However, if the candidate has indicated (usually by crossing out) that the working is to be ignored, award the ***N*** marks for the correct answer.

4 Implied and must be seen marks

*Implied marks appear in **brackets** eg **(M1)**.*

- Implied marks can only be awarded if the work is seen or if implied in subsequent working (a correct final answer does not necessarily mean that the implied marks are all awarded). There are questions where some working is required, but as it is accepted that not everyone will write the same steps, all the marks are implied, but the ***N*** marks are not the full marks for the question.
- Normally the correct work is seen in the next line.
- Where there is an (***M1***) followed by ***A1*** for each correct answer, if no working shown, one correct answer is sufficient evidence to award the (***M1***).

*Must be seen marks appear without **brackets** eg **M1**.*

- Must be seen marks can only be awarded if the work is seen.
- If a must be seen mark is not awarded because work is missing (as opposed to ***M0*** or ***A0*** for incorrect work) all subsequent marks may be awarded if appropriate.

5 Follow through marks (only applied after an error is made)

*Follow through (**FT**) marks are awarded where an incorrect answer (final or intermediate) from one **part** of a question is used correctly in **subsequent** part(s) or subpart(s). Usually, to award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if the only marks awarded in a subpart are for the **final** answer, then **FT** marks should be awarded if appropriate. Examiners are expected to check student work in order to award **FT** marks where appropriate.*

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** and ***R*** marks may be awarded if appropriate. (However, as noted above, if an ***A*** mark is not awarded because work is missing, all subsequent marks may be awarded if appropriate).
- Exceptions to this rule will be explicitly noted on the markscheme.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*eg* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word "**their**" in a description, to indicate that candidates may be using an incorrect value.
- If a candidate makes an error in one part, but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the question says hence. It is often possible to use a different approach in subsequent parts that does not depend on the answer to previous parts.
- In a "show that" question, if an error in a previous subpart leads to not showing the required answer, do not award the final ***A1***. Note that if the error occurs within the same subpart, the ***FT*** rules may result in further loss of marks.

6 Mis-read

*If a candidate incorrectly copies information from the question, this is a mis-read (**MR**). A candidate should be penalized only once for a particular mis-read. Use the **MR** stamp to indicate that this is a misread. Do not award the first mark in the question, even if this is an **M** mark, but award all others (if appropriate) so that the candidate only loses one mark for the misread.*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*eg* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.

7 Discretionary marks *(d)*

*An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. In such cases the annotation DM should be used and a brief **note** written next to the mark explaining this decision.*

8 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete parts are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**. Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9 Alternative forms

*Unless the question specifies otherwise, **accept** equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

10 Calculators

No calculator is allowed. The use of any calculator on paper 1 is malpractice, and will result in no grade awarded. If you see work that suggests a candidate has used any calculator, please follow the procedures for malpractice. Examples: finding an angle, given a trig ratio of 0.4235.

11 Style

The markscheme aims to present answers using good communication, eg if the question asks to find the value of k, the markscheme will say $k = 3$, but the marks will be for the correct value 3 – there is usually no need for the "$k =$". In these cases, it is also usually acceptable to have another variable, as long as there is no ambiguity in the question, eg if the question asks to find the value of p and of q, then the student answer needs to be clear. Generally, the only situation

where the full answer is required is in a question which asks for equations – in this case the markscheme will say "must be an equation".

*The markscheme often uses words to describe what the marks are for, followed by examples, using the eg notation. These examples are not exhaustive, and examiners should check what candidates have written, to see if they satisfy the description. Where these marks are **M** marks, the examples may include ones using poor notation, to indicate what is acceptable.*

12 Candidate work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

Candidates are meant to write their answers to Section A on the question paper (QP), and Section B on answer booklets. Sometimes, they need more room for Section A, and use the booklet (and often comment to this effect on the QP), or write outside the box. That is fine, and this work should be marked.

The instructions tell candidates not to write on Section B of the QP. Thus they may well have done some rough work here which they assume will be ignored. If they have solutions on the answer booklets, there is no need to look at the QP. However, if there are whole questions or whole part solutions missing on answer booklets, please check to make sure that they are not on the QP, and if they are, mark those whole questions or whole part solutions that have not been written on answer booklets.

13. Diagrams

The notes on how to allocate marks for sketches usually refer to passing through particular points or having certain features. These marks can only be awarded if the sketch is approximately the correct shape. All values given will be an approximate guide to where these points/features occur. In some questions, the first ***A1*** is for the shape, in others, the marks are only for the points and/or features. In both cases, unless the shape is approximately correct, no marks can be awarded (unless otherwise stated). However, if the graph is based on previous calculations, ***FT*** marks should be awarded if appropriate.

14. Accuracy of Answers

If the level of accuracy is specified in the question, a mark will be allocated for giving the final answer to the required accuracy. When this is not specified in the question, all numerical answers should be given exactly or correct to three significant figures.

Do not accept unfinished numerical final answers such as 3/0.1 (unless otherwise stated). As a rule, numerical answers with more than one part (such as fractions) should be given using integers (eg 6/8). Calculations which lead to integers should be completed, with the exception of fractions which are not whole numbers. Intermediate values do not need to be given to the correct three significant figures. But, if candidates work with rounded values, this could lead to an incorrect answer, in which case award A0 for the final answer.

Section A

1. (a) valid approach *(M1)*

eg $0.3-0.1$, $p+0.1=0.3$

$p=0.2$ ***A1 N2***

[2 marks]

(b) valid approach *(M1)*

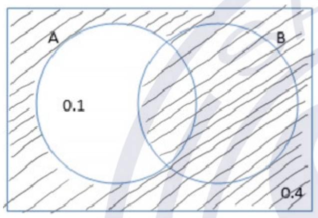
eg $1-(0.3+0.4)$, $1-0.4-0.1-p$

$q=0.3$ ***A1 N2***

[2 marks]

(c) valid approach *(M1)*

eg $0.7+0.5-0.3$, $p+q+0.4$, $1-0.1$, $\mathrm{P}(A'\cup B)=\mathrm{P}(A')+\mathrm{P}(B)-\mathrm{P}(A'\cap B)$,



$\mathrm{P}(A'\cup B)=0.9$ ***A1 N2***

[2 marks]

Total [6 marks]

2. (a) correct equation *(A1)*

eg $-3+6s=15$, $6s=18$

$s=3$ *(A1)*

substitute their s value into z component *(M1)*

eg $10+3(2)$, $10+6$

$c=16$ ***A1*** ***N3***

[4 marks]

(b) $r=\begin{pmatrix}1\\2\\3\end{pmatrix}+t\begin{pmatrix}6\\0\\2\end{pmatrix}$ $\left(=\left(\boldsymbol{i}+2\boldsymbol{j}+3\boldsymbol{k}\right)+t\left(6\boldsymbol{i}+2\boldsymbol{k}\right)\right)$ ***A2*** ***N2***

> **Note:** Accept any scalar multiple of $\begin{pmatrix}6\\0\\2\end{pmatrix}$ for the direction vector.
>
> Award ***A1*** for $\begin{pmatrix}1\\2\\3\end{pmatrix}+t\begin{pmatrix}6\\0\\2\end{pmatrix}$, ***A1*** for $L_2=\begin{pmatrix}1\\2\\3\end{pmatrix}+t\begin{pmatrix}6\\0\\2\end{pmatrix}$, ***A0*** for $r=\begin{pmatrix}6\\0\\2\end{pmatrix}+t\begin{pmatrix}1\\2\\3\end{pmatrix}$.

[2 marks]

Total [6 marks]

3. (a) valid approach ***(M1)***

eg labelled sides on separate triangle, $\sin^2 x + \cos^2 x = 1$

correct working ***(A1)***

eg missing side is 4, $\sqrt{1-\left(\frac{3}{5}\right)^2}$

$\cos\theta = \frac{4}{5}$ ***A1 N3***

[3 marks]

(b) correct substitution into $\cos 2\theta$ ***(A1)***

eg $2\left(\frac{16}{25}\right)-1$, $1-2\left(\frac{3}{5}\right)^2$, $\frac{16}{25}-\frac{9}{25}$

$\cos 2\theta = \frac{7}{25}$ ***A1 N2***

[2 marks]

(c) correct working ***(A1)***

eg $\frac{7}{25}=\frac{14}{\text{BC}}$, $\text{BC}=\frac{14\times 25}{7}$

$\text{BC} = 50 \text{ (cm)}$ ***A1 N2***

[2 marks]

Total [7 marks]

4. (a) $x = -3$ (must be an equation) ***A1*** ***N1***

[1 mark]

(b) interchanging x and y (seen anywhere) ***(M1)***

eg $x = \frac{2y-1}{y+3}$, $x(y+3) = 2y-1$

evidence of correct manipulation ***(A1)***

eg $yx + 3x = 2y - 1$, $y(x-2) = -3x - 1$, $2 - \frac{7}{y+3}$

$f^{-1}(x) = \frac{-3x-1}{x-2}$ $\left(= \frac{3x+1}{2-x}, \frac{7}{2-x} - 3\right)$ (accept $y =$) ***A1*** ***N3***

[3 marks]

(c) valid approach to find horizontal asymptote ***(M1)***

eg $\frac{-3}{1}$, vert.asymp of f becomes horiz.asymp of f^{-1}, $\frac{-3(x-2)+5}{x-2}$, $x \to \infty$

$y = -3$ (must be an equation) ***A1*** ***N2***

[2 marks]

Total [6 marks]

5. recognizing to integrate ***(M1)***

eg $\int f'$, $\int 2\mathrm{e}^{-3x}\mathrm{d}x$, $\mathrm{d}u = -3$

correct integral (do not penalize for missing $+C$) ***(A2)***

eg $-\frac{2}{3}\mathrm{e}^{-3x} + C$

substituting $\left(\frac{1}{3}, 5\right)$ (in any order) into **their** integrated expression (must have $+C$) ***M1***

eg $-\frac{2}{3}\mathrm{e}^{-3(1/3)} + C = 5$

Note: Award ***M0*** if they substitute into original or differentiated function.

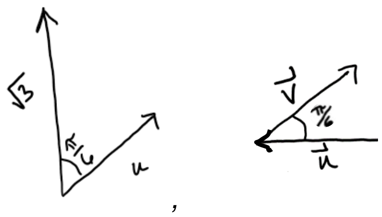
$f(x) = -\frac{2}{3}\mathrm{e}^{-3x} + 5 + \frac{2}{3}\mathrm{e}^{-1}$ (or *any* equivalent form, eg $-\frac{2}{3}\mathrm{e}^{-3x} + 5 - \frac{2}{-3\mathrm{e}}$) ***A1*** ***N4***

[5 marks]

6. **METHOD 1 (cosine rule)**

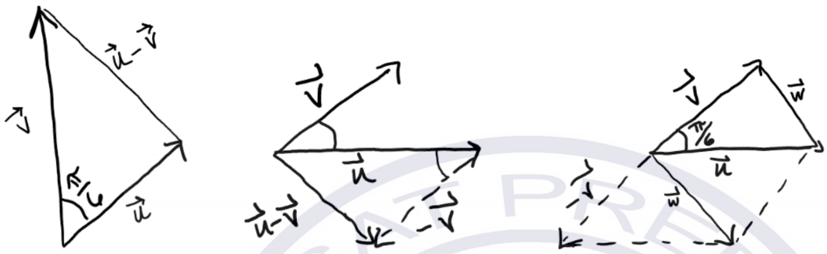
diagram including $\boldsymbol{u}$, $\boldsymbol{v}$ and included angle of $\frac{\pi}{6}$ ***(M1)***

eg



sketch of triangle with $\boldsymbol{w}$ (does not need to be to scale) ***(A1)***

eg



choosing cosine rule ***(M1)***

eg $a^2+b^2-2ab\cos C$

correct substitution ***A1***

eg $4^2+\left(\sqrt{3}\right)^2-2(4)\left(\sqrt{3}\right)\cos\frac{\pi}{6}$

$\cos\frac{\pi}{6}=\frac{\sqrt{3}}{2}$ (seen anywhere) ***(A1)***

correct working ***(A1)***

eg $16+3-12$

$|\boldsymbol{w}|=\sqrt{7}$ ***A1*** ***N2***

continued…

Question 6 continued

METHOD 2 (scalar product)

valid approach, in terms of u and v (seen anywhere) ***(M1)***

eg $|\boldsymbol{w}|^2 = (\boldsymbol{u}-\boldsymbol{v})\bullet(\boldsymbol{u}-\boldsymbol{v})$, $|\boldsymbol{w}|^2 = \boldsymbol{u}\bullet\boldsymbol{u} - 2\boldsymbol{u}\bullet\boldsymbol{v} + \boldsymbol{v}\bullet\boldsymbol{v}$, $|\boldsymbol{w}|^2 = (u_1 - v_1)^2 + (u_2 - v_2)^2$,
$|\boldsymbol{w}| = \sqrt{(u_1 - v_1)^2 + (u_2 - v_2)^2 + (u_3 - v_3)^2}$

correct value for $\boldsymbol{u}\bullet\boldsymbol{u}$ (seen anywhere) ***(A1)***

eg $|\boldsymbol{u}|^2 = 16$, $\boldsymbol{u}\bullet\boldsymbol{u} = 16$, $u_1{}^2 + u_2{}^2 = 16$

correct value for $\boldsymbol{v}\bullet\boldsymbol{v}$ (seen anywhere) ***(A1)***

eg $|\boldsymbol{v}|^2 = 3$, $\boldsymbol{v}\bullet\boldsymbol{v} = 3$, $v_1{}^2 + v_2{}^2 + v_3{}^2 = 3$

$\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$ (seen anywhere) ***(A1)***

$\boldsymbol{u}\bullet\boldsymbol{v} = 4 \times \sqrt{3} \times \frac{\sqrt{3}}{2}$ $(= 6)$ (seen anywhere) ***A1***

correct substitution into $\boldsymbol{u}\bullet\boldsymbol{u} - 2\boldsymbol{u}\bullet\boldsymbol{v} + \boldsymbol{v}\bullet\boldsymbol{v}$ or $u_1{}^2 + u_2{}^2 + v_1{}^2 + v_2{}^2 - 2(u_1v_1 + u_2v_2)$ (2 or 3 dimensions) ***(A1)***

eg $16 - 2(6) + 3$ $(= 7)$

$|\boldsymbol{w}| = \sqrt{7}$ ***A1*** ***N2***

[7 marks]

7. (a) recognizing relationship between v and s ***(M1)***

eg $\int v = s$, $s' = v$

$s(4) - s(2) = 9$ ***A1*** ***N2***

[2 marks]

(b) correctly interpreting distance travelled in first 2 seconds (seen anywhere, including part (a) or the area of 15 indicated on diagram) ***(A1)***

eg $\int_0^2 v = 15$, $s(2) = 15$

valid approach to find total distance travelled ***(M1)***

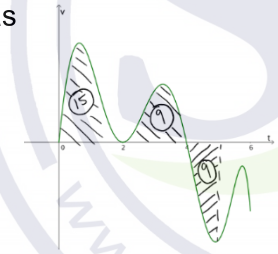
eg sum of 3 areas, $\int_0^4 v + \int_4^5 v$, shaded areas in diagram between 0 and 5

Note: Award ***M0*** if only $\int_0^5 \lvert v \rvert$ is seen.

correct working towards finding distance travelled between 2 and 5 (seen anywhere including within total area expression or on diagram) ***(A1)***

eg $\int_2^4 v - \int_4^5 v$, $\int_2^4 v = \int_4^5 \lvert v \rvert$, $\int_4^5 v\,\mathrm{d}t = -9$, $s(4) - s(2) - [s(5) - s(4)]$,

equal areas



correct working using $s(5) = s(2)$ ***(A1)***

eg $15 + 9 - (-9)$, $15 + 2[s(4) - s(2)]$, $15 + 2(9)$, $2 \times s(4) - s(2)$, $48 - 15$

total distance travelled = 33 (m) ***A1*** ***N2***

[5 marks]

Total [7 marks]

Section B

8. (a) valid approach *(M1)*

eg $f(x)=0$, $9-x^2=0$, one correct solution

$x=-3, 3$ (accept $(3, 0), (-3, 0)$) *A1* *N2*

[2 marks]

(b) valid approach *(M1)*

eg height $= f(b)$, base $= 2(\text{OP})$ or $2b$, $2b(9-x^2)$, $2b\times f(b)$

correct working that clearly leads to given answer *A1*

eg $2b(9-b^2)$

Note: Do not accept sloppy notation eg $2b \times 9-b^2$.

area $=18b-2b^3$ *AG* *N0*

[2 marks]

(c) setting derivative $=0$ (seen anywhere) *(M1)*

eg $A'=0$, $\left[18b-2b^3\right]'=0$

correct derivative (must be in terms of b only) (seen anywhere) *A2*

eg $18-6b^2$, $2b(-2b)+(9-b^2)\times 2$

correct working *(A1)*

eg $6b^2=18$, $b=\pm\sqrt{3}$

$b=\sqrt{3}$ *A1* *N3*

[5 marks]

(d) valid approach *(M1)*

eg $f=g$, $9-x^2=(x-3)^2+k$

correct working *(A1)*

eg $9-x^2=x^2-6x+9+k$, $9-x^2-x^2+6x-9-k=0$

$2x^2-6x+k=0$ *AG* *N0*

[2 marks]

continued…

Question 8 continued

(e) **METHOD 1 (discriminant)**

recognizing to use discriminant (seen anywhere) ***(M1)***

eg $\Delta,\ b^2-4ac$

discriminant $=0$ (seen anywhere) ***M1***

correct substitution into discriminant (do not accept only in quadratic formula) ***(A1)***

eg $(-6)^2-4(2)(k)$, $(6)^2-4(2)(k)$

correct working ***(A1)***

eg $36-8k=0$, $8k=36$

$k=\frac{36}{8}\ \left(=\frac{9}{2},\ 4.5\right)$ ***A1 N2***

METHOD 2 (completing the square)

valid approach to complete the square ***(M1)***

eg $2\left(x^2-3x+\frac{9}{4}\right)=-k+\frac{18}{4}$, $x^2-3x+\frac{9}{4}-\frac{9}{4}+\frac{k}{2}=0$

correct working ***(A1)***

eg $2\left(x-\frac{3}{2}\right)^2=-k+\frac{18}{4}$, $\left(x-\frac{3}{2}\right)^2-\frac{9}{4}+\frac{k}{2}=0$

recognizing condition for one solution ***M1***

eg $\left(x-\frac{3}{2}\right)^2=0$, $-\frac{9}{4}+\frac{k}{2}=0$

correct working ***(A1)***

eg $-k=-\frac{18}{4}$, $\frac{k}{2}=\frac{9}{4}$

$k=\frac{18}{4}\ \left(=\frac{9}{2},\ 4.5\right)$ ***A1 N2***

continued…

Question 8 continued

METHOD 3 (using vertex)

valid approach to find vertex (seen anywhere) ***M1***

eg $\left(2x^2-6x+k\right)'=0$, $-\dfrac{b}{2a}$

correct working ***(A1)***

eg $\left(2x^2-6x+k\right)'=4x-6$, $-\dfrac{(-6)}{2(2)}$

$x=\dfrac{6}{4}\ \left(=\dfrac{3}{2}\right)$ ***(A1)***

correct substitution ***(A1)***

eg $2\left(\dfrac{3}{2}\right)^2-6\left(\dfrac{3}{2}\right)+k=0$

$k=\dfrac{18}{4}\ \left(=\dfrac{9}{2},\ 4.5\right)$ ***A1*** ***N2***

[5 marks]

Total [16 marks]

9. (a) recognizing area under curve = 1 ***(M1)***

eg $a+x+b=1$, $100-a-b$, $1-a+b$

$\mathrm{P}(-1.6<z<2.4)=1-a-b \ \left(=1-(a+b)\right)$ ***A1*** ***N2***

[2 marks]

(b) $\mathrm{P}(z>-1.6)=1-a$ (seen anywhere) ***(A1)***

recognizing conditional probability ***(M1)***

eg $\mathrm{P}(A|B)$, $\mathrm{P}(B|A)$

correct working ***(A1)***

eg $\dfrac{\mathrm{P}(z<2.4\cap z>-1.6)}{\mathrm{P}(z>-1.6)}$, $\dfrac{\mathrm{P}(-1.6<z<2.4)}{\mathrm{P}(z>-1.6)}$

$\mathrm{P}(z<2.4|z>-1.6)=\dfrac{1-a-b}{1-a}$ ***A1*** ***N4***

Note: Do not award the final ***A1*** if correct answer is seen followed by incorrect simplification.

[4 marks]

(c) $z=-1.6$ (may be seen in part (d)) ***A1*** ***N1***

Note: Depending on the candidate's interpretation of the question, they may give $\dfrac{1-m}{s}$ as the answer to part (c). Such answers should be awarded the first ***(M1)*** in part (d), even when part (d) is left blank. If the candidate goes on to show $z=-1.6$ as part of their working in part (d), the ***A1*** in part (c) may be awarded.

[1 mark]

continued…

Question 9 continued

(d) attempt to standardize x (do not accept $\frac{x-\mu}{\sigma}$) ***(M1)***

eg $\frac{1-m}{s}$ (may be seen in part (c)), $\frac{m-2}{s}$, $\frac{x-m}{\sigma}$

correct equation with each z-value ***(A1)(A1)***

eg $-1.6=\frac{1-m}{s}$, $2.4=\frac{2-m}{s}$, $m+2.4s=2$

valid approach (to set up equation in one variable) ***M1***

eg $2.4=\frac{2-(1.6s+1)}{s}$, $\frac{1-m}{-1.6}=\frac{2-m}{2.4}$

correct working ***(A1)***

eg $1.6s+1=2-2.4s$, $4s=1$, $m=\frac{7}{5}$

$s=\frac{1}{4}$ ***A1*** ***N2***

[6 marks]

Total [13 marks]

10. (a) correct working *(A1)*

eg $\sin\left(\frac{\pi}{4}x\right)=1$, $\sqrt{x}\left(1-\sin\left(\frac{\pi}{4}x\right)\right)=0$

$\sin\left(\frac{\pi}{2}\right)=1$ (seen anywhere) *(A1)*

correct working (ignore additional values) *(A1)*

eg $\frac{\pi}{4}x=\frac{\pi}{2}$, $\frac{\pi}{4}x=\frac{\pi}{2}+2\pi$

$x=2, 10$ ***A1A1*** ***N1N1***

[5 marks]

(b) correct working *(A1)*

eg $d=10-2$, $a+b=2$, $a+2b=10$

valid approach *(M1)*

eg $2+(n-1)8$, $a+2(2-a)=10$, b = common difference

$a=-6, b=8$ (accept $-6+8n$) ***A1A1*** ***N2N2***

[4 marks]

(c) valid approach *(M1)*

eg first intersection at $x=0$, $n=20$

correct working ***A1***

eg $-6+8\times20$, $2+(20-1)\times8$, $u_{20}=154$

$\mathrm{P}\left(154, \sqrt{154}\right)$ (accept $x=154$ and $y=\sqrt{154}$) ***A1A1*** ***N3***

[4 marks]

continued…

Question 10 continued

(d) valid attempt to find upper boundary ***(M1)***

eg half way between u_{20} and u_{21}, $u_{20}+\frac{d}{2}$, $154+4$, $-2+8n$, at least two values of new sequence $\{6, 14, ...\}$

upper boundary at $x=158$ (seen anywhere) ***(A1)***

correct integral expression (accept missing $\mathrm{d}x$) ***A1A1*** ***N4***

eg $\int_0^{158}\left(\sqrt{x}\sin\left(\frac{\pi}{4}x\right)+\sqrt{x}\right)\mathrm{d}x$, $\int_0^{158}(g+f)\,\mathrm{d}x$), $\int_0^{158}\sqrt{x}\sin\left(\frac{\pi}{4}x\right)\mathrm{d}x-\int_0^{158}-\sqrt{x}\,\mathrm{d}x$

Note: Award ***A1*** for two correct limits and ***A1*** for correct integrand. The ***A1*** for correct integrand may be awarded independently of all the other marks.

[4 marks]

Total [17 marks]

Markscheme

May 2019

Mathematics

Standard level

Paper 1

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a valid **Method**; working must be seen.

(M) Marks awarded for a valid **Method**; may be implied by **correct** subsequent working.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

(A) Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.

R Marks awarded for clear **Reasoning**.

N Marks awarded for **correct** answers if **no** working shown.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Mark according to RM assessor instructions.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any. An exception to this rule is when work for ***M1*** is missing, as opposed to incorrect (see point 4).
- Where ***M*** and ***A*** marks are noted on the same line, *eg* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*eg* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***(M2)***, ***N3***, *etc*., do **not** split the marks, unless there is a note.
- Most ***M*** marks are for a **valid** method, ie a method which can lead to the answer: it must indicate some form of progress towards the answer.
- Once a correct answer to a question or part-question is seen, ignore further correct working. However, if further working indicates a lack of mathematical understanding do not award final ***A1***.

3 *N* marks

*If **no** working shown, award **N** marks for **correct** answers. In this case, ignore mark breakdown (**M**, **A**, **R**).*

- Do **not** award a mixture of ***N*** and other marks.
- There may be fewer ***N*** marks available than the total of ***M***, ***A*** and ***R*** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.
- There may not be a direct relationship between the ***N*** marks and the implied marks. There are times when all the marks are implied, but the ***N*** marks are not the full marks: this indicates that we want to see some of the working, without specifying what.

- For consistency within the markscheme, ***N*** marks are noted for every part, even when these match the mark breakdown.
- If a candidate has incorrect working, which somehow results in a correct answer, do **not** award the ***N*** marks for this correct answer. However, if the candidate has indicated (usually by crossing out) that the working is to be ignored, award the ***N*** marks for the correct answer.

4 Implied and must be seen marks

*Implied marks appear in **brackets** eg **(M1)**.*

- Implied marks can only be awarded if the work is seen or if implied in subsequent working (a correct final answer does not necessarily mean that the implied marks are all awarded). There are questions where some working is required, but as it is accepted that not everyone will write the same steps, all the marks are implied, but the ***N*** marks are not the full marks for the question.
- Normally the correct work is seen in the next line.
- Where there is an (***M1***) followed by ***A1*** for each correct answer, if no working shown, one correct answer is sufficient evidence to award the (***M1***).

*Must be seen marks appear without **brackets** eg **M1**.*

- Must be seen marks can only be awarded if the work is seen.
- If a must be seen mark is not awarded because work is missing (as opposed to ***M0*** or ***A0*** for incorrect work) all subsequent marks may be awarded if appropriate.

5 Follow through marks (only applied after an error is made)

*Follow through (**FT**) marks are awarded where an incorrect answer (final or intermediate) from one **part** of a question is used correctly in **subsequent** part(s) or subpart(s). Usually, to award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if the only marks awarded in a subpart are for the **final** answer, then **FT** marks should be awarded if appropriate. Examiners are expected to check student work in order to award **FT** marks where appropriate.*

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** and ***R*** marks may be awarded if appropriate. (However, as noted above, if an ***A*** mark is not awarded because work is missing, all subsequent marks may be awarded if appropriate).
- Exceptions to this rule will be explicitly noted on the markscheme.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*eg* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word "**their**" in a description, to indicate that candidates may be using an incorrect value.
- If a candidate makes an error in one part, but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the question says hence. It is often possible to use a different approach in subsequent parts that does not depend on the answer to previous parts.
- In a "show that" question, if an error in a previous subpart leads to not showing the required answer, do not award the final ***A1***. Note that if the error occurs within the same subpart, the ***FT*** rules may result in further loss of marks.

6 Mis-read

*If a candidate incorrectly copies information from the question, this is a mis-read (**MR**). A candidate should be penalized only once for a particular mis-read. Use the **MR** stamp to indicate that this is a misread. Do not award the first mark in the question, even if this is an **M** mark, but award all others (if appropriate) so that the candidate only loses one mark for the misread.*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*eg* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.

7 Discretionary marks *(d)*

*An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. In such cases the annotation DM should be used and a brief **note** written next to the mark explaining this decision.*

8 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete parts are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**. Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9 Alternative forms

*Unless the question specifies otherwise, **accept** equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

10 Calculators

No calculator is allowed. The use of any calculator on paper 1 is malpractice, and will result in no grade awarded. If you see work that suggests a candidate has used any calculator, please follow the procedures for malpractice. Examples: finding an angle, given a trig ratio of 0.4235.

11 Style

The markscheme aims to present answers using good communication, eg if the question asks to find the value of k, the markscheme will say $k = 3$, but the marks will be for the correct value 3 – there is usually no need for the "$k =$". In these cases, it is also usually acceptable to have another variable, as long as there is no ambiguity in the question, eg if the question asks to find the value of p and of q, then the student answer needs to be clear. Generally, the only situation

where the full answer is required is in a question which asks for equations – in this case the markscheme will say "must be an equation".

*The markscheme often uses words to describe what the marks are for, followed by examples, using the eg notation. These examples are not exhaustive, and examiners should check what candidates have written, to see if they satisfy the description. Where these marks are **M** marks, the examples may include ones using poor notation, to indicate what is acceptable.*

12 Candidate work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

Candidates are meant to write their answers to Section A on the question paper (QP), and Section B on answer booklets. Sometimes, they need more room for Section A, and use the booklet (and often comment to this effect on the QP), or write outside the box. That is fine, and this work should be marked.

The instructions tell candidates not to write on Section B of the QP. Thus they may well have done some rough work here which they assume will be ignored. If they have solutions on the answer booklets, there is no need to look at the QP. However, if there are whole questions or whole part solutions missing on answer booklets, please check to make sure that they are not on the QP, and if they are, mark those whole questions or whole part solutions that have not been written on answer booklets.

13. Diagrams

The notes on how to allocate marks for sketches usually refer to passing through particular points or having certain features. These marks can only be awarded if the sketch is approximately the correct shape. All values given will be an approximate guide to where these points/features occur. In some questions, the first ***A1*** is for the shape, in others, the marks are only for the points and/or features. In both cases, unless the shape is approximately correct, no marks can be awarded (unless otherwise stated). However, if the graph is based on previous calculations, ***FT*** marks should be awarded if appropriate.

14. Accuracy of Answers

If the level of accuracy is specified in the question, a mark will be allocated for giving the final answer to the required accuracy. When this is not specified in the question, all numerical answers should be given exactly or correct to three significant figures.

Do not accept unfinished numerical final answers such as 3/0.1 (unless otherwise stated). As a rule, numerical answers with more than one part (such as fractions) should be given using integers (eg 6/8). Calculations which lead to integers should be completed, with the exception of fractions which are not whole numbers. Intermediate values do not need to be given to the correct three significant figures. But, if candidates work with rounded values, this could lead to an incorrect answer, in which case award A0 for the final answer.

Section A

1. (a) evidence of using $\sum p = 1$ ***(M1)***

correct working ***(A1)***

eg $\frac{3}{13}+\frac{1}{13}+\frac{4}{13}+k=1,\ 1-\frac{8}{13}$

$k=\frac{5}{13}$ ***A1*** ***N2***

[3 marks]

(b) valid approach to find $\mathrm{E}(X)$ ***(M1)***

eg $1\times\frac{1}{13}+2\times\frac{4}{13}+3\times k$, $0\times\frac{3}{13}+1\times\frac{1}{13}+2\times\frac{4}{13}+3\times\frac{5}{13}$

correct working ***(A1)***

eg $\frac{1}{13}+\frac{8}{13}+\frac{15}{13}$

$\mathrm{E}(X)=\frac{24}{13}$ ***A1*** ***N2***

[3 marks]

Total [6 marks]

2. (a) valid approach ***(M1)***

eg $\boldsymbol{b}=2\boldsymbol{a}$, $\boldsymbol{a}=k\boldsymbol{b}$, $\cos\theta=1$, $\boldsymbol{a}\bullet\boldsymbol{b}=-|\boldsymbol{a}||\boldsymbol{b}|$, $2p=18$

$p=9$ ***A1*** ***N2***

[2 marks]

(b) evidence of scalar product ***(M1)***

eg $\boldsymbol{a}\bullet\boldsymbol{b}$, $(0)(0)+(3)(6)+p(18)$

recognizing $\boldsymbol{a}\bullet\boldsymbol{b}=0$ (seen anywhere) ***(M1)***

correct working ***(A1)***

eg $18+18p=0$, $18p=-18$

$p=-1$ ***A1*** ***N3***

[4 marks]

Total [6 marks]

3. (a) (i) $x = 2$ (must be an equation) ***A1*** ***N1***

(ii) valid approach ***(M1)***

eg $3+\frac{7}{x-2}$, $x \to \infty$, $\frac{3x}{x}$, $\frac{3}{1}$, $\frac{3+\frac{1}{x}}{1-\frac{2}{x}}$, $\frac{3(x-2)+7}{x-2}$

$y = 3$ (must be an equation) ***A1*** ***N2***

[3 marks]

(b) **METHOD 1**

attempt to substitute 1 into $g(x)$ or $f(x)$ ***(M1)***

eg 1^2+4, $\frac{3+1}{1-2}$

$g(1) = 5$ ***(A1)***

$(f \circ g)(1) = \frac{16}{3}$ ***A1*** ***N2***

METHOD 2

attempt to form composite function (in any order) ***(M1)***

eg $\frac{3(x^2+4)+1}{x^2+4-2}$, $\left(\frac{3x+1}{x-2}\right)^2+4$

correct substitution ***(A1)***

eg $\frac{3(5)+1}{5-2}$

$(f \circ g)(1) = \frac{16}{3}$ ***A1*** ***N2***

[3 marks]

Total [6 marks]

4. (a) (i) y-intercept is 11 (accept $(0, 11)$) ***A1*** ***N1***

(ii) valid approach ***(M1)***

eg $f(4 \times 0) = f(0)$, recognizing stretch of $\frac{1}{4}$ in x-direction

y-intercept is 8 (accept $(0, 8)$) ***A1*** ***N2***

[3 marks]

(b) x-intercept is $\frac{5}{2}$ $(= 2.5)$ (accept $\left(\frac{5}{2}, 0\right)$ or $(2.5, 0)$) ***A2*** ***N2***

[2 marks]

(c) correct name, correct magnitude **and** direction ***A1A1*** ***N2***

eg name: translation, (horizontal) shift (do not accept move)

eg magnitude and direction: 1 unit to the left, $\begin{pmatrix} -1 \\ 0 \end{pmatrix}$, horizontal by -1

[2 marks]

Total [7 marks]

5. correct substitution into discriminant (do not accept only in quadratic formula) ***(A1)***

eg $1 - 4(1-k)k$

correct expansion of discriminant (do not accept only in quadratic formula) ***A1***

eg $1 - 4k + 4k^2$, $4k^2 - 4k = -1$

recognizing discriminant equals 0 (seen anywhere) ***M1***

eg $\Delta = 0$, $b^2 - 4ac = 0$

valid attempt to solve **their** quadratic in k ***(M1)***

eg factorizing equation, use of quadratic formula,
completing the square, recognizing vertex on x-axis

correct working ***(A1)***

eg $(2k-1)^2$, $\frac{-(-4) \pm \sqrt{16 - 4(4)(1)}}{2(4)}$, $\left(k - \frac{1}{2}\right)^2 = 0$, $k = \frac{-(-4)}{2(4)}$

$k = \frac{1}{2}$ ***A1*** ***N2***

[6 marks]

6.

Note: The first three ***A*** marks are awarded for correct application of log properties, including with incorrect expressions, and in any order.

correct application of change of base (accept any base) ***(A1)***

eg $\frac{\log_4(13-4x)}{\log_4 16}$, $\frac{\log_{16}(2-x)}{\log_{16} 4}$, $\frac{\log_2(2-x)}{\log_2 4}$, $\frac{\log(13-4x)}{\log 16}$

correct numerical value ***(A1)***

eg $\log_4 16 = 2$, $\log_{16} 4 = \frac{1}{2}$

correct application of $r\log_c a = \log_c a^r$ ***(A1)***

eg $\log_4(2-x)^2$

correct equation without logs ***A1***

eg $(2-x)^2 = 13-4x$, $(2-x)^4 = (13-4x)^2$, $4-4x+x^2 = 13-4x$

correct working ***A1***

eg $x^2 = 9$

$x = -3$ ***A2*** ***N2***

[7 marks]

7. (a) correct equation ***(A1)***

eg $2\sin x = -1$, $\sin x = -\frac{1}{2}$

one correct value for $\sin^{-1}\left(\frac{1}{2}\right)$ or $\sin^{-1}\left(-\frac{1}{2}\right)$ (seen anywhere) ***(A1)***

eg $\frac{\pi}{6}$, $\frac{5\pi}{6}$, 30°, 150°, 210°, 330°

$x = \frac{7\pi}{6}, \frac{11\pi}{6}$ $\left(\text{accept } \left(\frac{7\pi}{6}, -1\right), \left(\frac{11\pi}{6}, -1\right)\right)$ ***A1A1*** ***N1N1***

Note: Award ***A1A1A1A0*** if more solutions given in addition to both correct answers.

[4 marks]

(b) recognizing period of g is larger than the period of f ***(M1)***

eg sketch of g with larger period (may be seen on diagram), A at $x = 2\pi$,

image of A when $x > 2\pi$, $\frac{7\pi}{6} \to 2\pi$, $2\sin(2\pi p) = -1$, $\frac{7\pi}{6} \times k = 2\pi$

correct working ***(A1)***

eg $\frac{7\pi}{6} \cdot \frac{1}{p} = 2\pi$, $2\pi p = \frac{7\pi}{6}$, $\frac{12}{7}$

$p = \frac{7}{12}$ $\left(\text{accept } p < \frac{7}{12} \text{ or } p \le \frac{7}{12}\right)$ ***A1*** ***N2***

[3 marks]

Total [7 marks]

Section B

8. (a) valid approach *(M1)*

eg $16+8$, $a-8$

24 (hours) *A1* *N2*

[2 marks]

(b) valid approach *(M1)*

eg $20-15$, Q_3-Q_1, $15-20$

IQR $=5$ *A1* *N2*

[2 marks]

(c) correct working *(A1)*

eg $\frac{180}{10}, \frac{180}{n}, \frac{\sum x}{10}$

mean $=18$ (hours) *A1* *N2*

[2 marks]

(d) (i) attempt to find total hours for group B *(M1)*

eg $\bar{x}\times n$

group B total hours $=420$ (seen anywhere) *A1* *N2*

(ii) attempt to find sum for combined group (may be seen in working) *(M1)*

eg $180+420$, 600

correct working *(A1)*

eg $\frac{180+420}{30}, \frac{600}{30}$

mean $=20$ (hours) *A1* *N2*

[5 marks]

continued…

Question 8 continued

(e) (i) valid approach to find the new mean **(M1)**

eg $\frac{1}{2}\mu$, $\frac{1}{2}\times 21$

mean $=\frac{21}{2}(=10.5)$ (hours) **A1** **N2**

(ii) variance = σ^2 (seen anywhere) **(A1)**

eg $\sigma^2 = 9$, $3^2 = 9$, $\left(\frac{3}{2}\right)^2$, 3^2

valid attempt to find new standard deviation or variance **(M1)**

eg $\frac{1}{4}\times 3^2$, $\frac{1}{2}\times 3$, $\frac{3}{2}$

variance = $\frac{9}{4}$ $(=2.25)$ (hours) **A1** **N2**

[5 marks]

Total [16 marks]

9. (a) evidence of valid approach *(M1)*

eg sketch of triangle with sides 3 and 5, $\cos^2\theta = 1 - \sin^2\theta$

correct working *(A1)*

eg missing side is 4 (may be seen in sketch), $\cos\theta = \frac{4}{5}$, $\cos\theta = -\frac{4}{5}$

$\tan\theta = -\frac{3}{4}$ ***A2*** ***N4***

[4 marks]

(b) correct substitution of either gradient **or** origin into equation of line *(A1)*
(do not accept $y = mx + b$)

eg $y = x\tan\theta$, $y - 0 = m(x - 0)$, $y = mx$

$y = -\frac{3}{4}x$ ***A1*** ***N2***

> **Note:** Award ***A1A0*** for $L = -\frac{3}{4}x$.

[2 marks]

(c) $\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{-3x}{4}\right) = -\frac{3}{4}$ (seen anywhere, including answer) ***A1***

choosing product rule *(M1)*
eg $uv' + vu'$

correct derivatives (must be seen in a correct product rule) ***A1A1***
eg $\cos x$, e^x

$f'(x) = \mathrm{e}^x\cos x + \mathrm{e}^x\sin x - \frac{3}{4}\left(= \mathrm{e}^x(\cos x + \sin x) - \frac{3}{4}\right)$ ***A1*** ***N5***

[5 marks]

continued…

Question 9 continued

(d) valid approach to equate **their** gradients ***(M1)***

eg $f' = \tan\theta$, $f' = -\frac{3}{4}$, $e^x \cos x + e^x \sin x - \frac{3}{4} = -\frac{3}{4}$,

$e^x(\cos x + \sin x) - \frac{3}{4} = -\frac{3}{4}$

correct equation without e^x ***(A1)***

eg $\sin x = -\cos x$, $\cos x + \sin x = 0$, $\frac{-\sin x}{\cos x} = 1$

correct working ***(A1)***

eg $\tan\theta = -1$, $x = 135^\circ$

$x = \frac{3\pi}{4}$ (do not accept 135°) ***A1*** ***N1***

Note: Do not award the final ***A1*** if additional answers are given.

[4 marks]

Total [15 marks]

10. (a) evidence of choosing chain rule ***(M1)***

eg $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$, $u = x^3 + x$, $u' = 3x^2 + 1$

$\frac{dy}{dx} = \frac{3}{2}\left(x^3 + x\right)^{\frac{1}{2}}\left(3x^2 + 1\right)$ $\left(= \frac{3}{2}\sqrt{x^3 + x}\left(3x^2 + 1\right)\right)$ ***A2*** ***N3***

[3 marks]

(b) integrating by inspection from (a) or by substitution ***(M1)***

eg $\frac{2}{3}\int \frac{3}{2}\left(3x^2 + 1\right)\sqrt{x^3 + x}\, dx$, $u = x^3 + x$, $\frac{du}{dx} = 3x^2 + 1$, $\int u^{\frac{1}{2}}$, $\frac{u^{\frac{3}{2}}}{1.5}$

correct integrated expression in terms of x ***A2*** ***N3***

eg $\frac{2}{3}(x^3 + x)^{\frac{3}{2}} + C$, $\frac{(x^3 + x)^{1.5}}{1.5} + C$

[3 marks]

(c) integrating and subtracting functions (in any order) ***(M1)***

eg $\int g - f$, $\int f - \int g$

correct integral (including limits, accept absence of dx) ***A1*** ***N2***

eg $\int_0^1 (g - f)\, dx$, $\int_0^1 6 - 3x^2\sqrt{x^3 + x} - \sqrt{x^3 + x}\, dx$, $\int_0^1 g(x) - \int_0^1 f(x)$

[2 marks]

continued…

Question 10 continued

(d) recognizing $\sqrt{x^3+x}$ is a common factor (seen anywhere, may be seen in part (c)) ***(M1)***

eg $\left(-3x^2-1\right)\sqrt{x^3+x}$, $\int 6-\left(3x^2+1\right)\sqrt{x^3+x}$, $\left(3x^2-1\right)\sqrt{x^3+x}$

correct integration ***(A1)(A1)***

eg $6x-\frac{2}{3}\left(x^3+x\right)^{\frac{3}{2}}$

> **Note:** Award ***A1*** for $6x$ and award ***A1*** for $-\frac{2}{3}\left(x^3+x\right)^{\frac{3}{2}}$.

substituting limits into **their** integrated function and subtracting (in any order) ***(M1)***

eg $6-\frac{2}{3}(1^3+1)^{\frac{3}{2}}$, $0-\left[6-\frac{2}{3}(1^3+1)^{\frac{3}{2}}\right]$

correct working ***(A1)***

eg $6-\frac{2}{3}\times 2\sqrt{2}$, $6-\frac{2}{3}\times\sqrt{4}\times\sqrt{2}$

area of $R=6-\frac{4\sqrt{2}}{3}$ $\left(=6-\frac{2}{3}\sqrt{8},\quad 6-\frac{2}{3}\times 2^{\frac{3}{2}},\quad \frac{18-4\sqrt{2}}{3}\right)$ ***A1*** ***N3***

[6 marks]

Total [14 marks]

Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

November 2018

Mathematics

Standard level

Paper 1

18 pages

Instructions to Examiners

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- Do **not** award a mixture of ***N*** and other marks.
- There may be fewer ***N*** marks available than the total of ***M***, ***A*** and ***R*** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.
- There may not be a direct relationship between the ***N*** marks and the implied marks. There are times when all the marks are implied, but the ***N*** marks are not the full marks: this indicates that we want to see some of the working, without specifying what.

- For consistency within the markscheme, ***N*** marks are noted for every part, even when these match the mark breakdown.
- If a candidate has incorrect working, which somehow results in a correct answer, do **not** award the ***N*** marks for this correct answer. However, if the candidate has indicated (usually by crossing out) that the working is to be ignored, award the ***N*** marks for the correct answer.

4 Implied and must be seen marks

Implied marks appear in ***brackets*** *eg* ***(M1)****.*

- Implied marks can only be awarded if the work is seen or if implied in subsequent working (a correct final answer does not necessarily mean that the implied marks are all awarded). There are questions where some working is required, but as it is accepted that not everyone will write the same steps, all the marks are implied, but the ***N*** marks are not the full marks for the question.
- Normally the correct work is seen in the next line.
- Where there is an (***M1***) followed by ***A1*** for each correct answer, if no working shown, one correct answer is sufficient evidence to award the (***M1***).

Must be seen marks appear without ***brackets*** *eg* ***M1****.*

- Must be seen marks can only be awarded if the work is seen.
- If a must be seen mark is not awarded because work is missing (as opposed to ***M0*** or ***A0*** for incorrect work) all subsequent marks may be awarded if appropriate.

5 Follow through marks (only applied after an error is made)

*Follow through (****FT****) marks are awarded where an incorrect answer (final or intermediate) from one* ***part*** *of a question is used correctly in* ***subsequent*** *part(s) or subpart(s). Usually, to award* ***FT*** *marks,* ***there must be working present*** *and not just a final answer based on an incorrect answer to a previous part. However, if the only marks awarded in a subpart are for the* ***final*** *answer, then* ***FT*** *marks should be awarded if appropriate. Examiners are expected to check student work in order to award* ***FT*** *marks where appropriate.*

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** and ***R*** marks may be awarded if appropriate. (However, as noted above, if an ***A*** mark is not awarded because work is missing, all subsequent marks may be awarded if appropriate).
- Exceptions to this rule will be explicitly noted on the markscheme.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*eg* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word "**their**" in a description, to indicate that candidates may be using an incorrect value.
- If a candidate makes an error in one part, but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the question says hence. It is often possible to use a different approach in subsequent parts that does not depend on the answer to previous parts.
- In a "show that" question, if an error in a previous subpart leads to not showing the required answer, do not award the final ***A1***. Note that if the error occurs within the same subpart, the ***FT*** rules may result in further loss of marks.

6 Mis-read

*If a candidate incorrectly copies information from the question, this is a mis-read (**MR**). A candidate should be penalized only once for a particular mis-read. Use the **MR** stamp to indicate that this is a misread. Do not award the first mark in the question, even if this is an **M** mark, but award all others (if appropriate) so that the candidate only loses one mark for the misread.*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*eg* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.

7 Discretionary marks *(d)*

*An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. In such cases the annotation DM should be used and a brief **note** written next to the mark explaining this decision.*

8 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete parts are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**. Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9 Alternative forms

*Unless the question specifies otherwise, **accept** equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

10 Calculators

No calculator is allowed. The use of any calculator on paper 1 is malpractice, and will result in no grade awarded. If you see work that suggests a candidate has used any calculator, please follow the procedures for malpractice. Examples: finding an angle, given a trig ratio of 0.4235.

11 Style

The markscheme aims to present answers using good communication, eg if the question asks to find the value of k, the markscheme will say $k = 3$, but the marks will be for the correct value 3 – there is usually no need for the "$k =$". In these cases, it is also usually acceptable to have another variable, as long as there is no ambiguity in the question, eg if the question asks to find the value of p and of q, then the student answer needs to be clear. Generally, the only situation

where the full answer is required is in a question which asks for equations – in this case the markscheme will say "must be an equation".

The markscheme often uses words to describe what the marks are for, followed by examples, using the eg notation. These examples are not exhaustive, and examiners should check what candidates have written, to see if they satisfy the description. Where these marks are ***M*** *marks, the examples may include ones using poor notation, to indicate what is acceptable.*

12 Candidate work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

Candidates are meant to write their answers to Section A on the question paper (QP), and Section B on answer booklets. Sometimes, they need more room for Section A, and use the booklet (and often comment to this effect on the QP), or write outside the box. That is fine, and this work should be marked.

The instructions tell candidates not to write on Section B of the QP. Thus they may well have done some rough work here which they assume will be ignored. If they have solutions on the answer booklets, there is no need to look at the QP. However, if there are whole questions or whole part solutions missing on answer booklets, please check to make sure that they are not on the QP, and if they are, mark those whole questions or whole part solutions that have not been written on answer booklets.

13. Diagrams

The notes on how to allocate marks for sketches usually refer to passing through particular points or having certain features. These marks can only be awarded if the sketch is approximately the correct shape. All values given will be an approximate guide to where these points/features occur. In some questions, the first ***A1*** is for the shape, in others, the marks are only for the points and/or features. In both cases, unless the shape is approximately correct, no marks can be awarded (unless otherwise stated). However, if the graph is based on previous calculations, ***FT*** marks should be awarded if appropriate.

14. Accuracy of Answers

If the level of accuracy is specified in the question, a mark will be allocated for giving the final answer to the required accuracy. When this is not specified in the question, all numerical answers should be given exactly or correct to three significant figures.

Do not accept unfinished numerical final answers such as 3/0.1 (unless otherwise stated). As a rule, numerical answers with more than one part (such as fractions) should be given using integers (eg 6/8). Calculations which lead to integers should be completed, with the exception of fractions which are not whole numbers. Intermediate values do not need to be given to the correct three significant figures. But, if candidates work with rounded values, this could lead to an incorrect answer, in which case award A0 for the final answer.

Section A

1. (a) correct substitution *(A1)*

eg $\frac{1}{2}(2)\left(6^2\right)$

area $= 36$ $\left(\text{cm}^2\right)$ ***A1*** ***N2***

[2 marks]

(b) valid approach to find major arc length *(M1)*

eg angle $= 2\pi - 2$, circumference – arc BC

correct working for major arc length *(A1)*

eg $6(2\pi - 2)$, $(2\times 6\times \pi) - (6\times 2)$, $12\pi - 12$

valid approach to find perimeter of a sector (seen anywhere) *(M1)*

eg arc $+ 2$(radius), $12\pi - 12 + 2(6)$

perimeter $= 12\pi$ ***A1*** ***N1***

[4 marks]

Total [6 marks]

2. (a) $f(1) = 3$ ***A1*** ***N1***

[1 mark]

(b) attempt to form the composite (including value) *(M1)*

eg $g(3)$, $g\left(f(1)\right)$

$(g\circ f)(1) = 5$ ***A1*** ***N2***

[2 marks]

(c) valid approach *(M1)*

eg $g(x) = -2$

$g^{-1}(-2) = 1$ ***A1*** ***N2***

[2 marks]

Total [5 marks]

3. (a) correct working *(A1)*
eg $-5+(8-1)(3)$

$u_8 = 16$ ***A1*** ***N2***
[2 marks]

(b) correct substitution into u_n formula *(A1)*
eg $-5+3(n-1)$, $3n-8$

correct equation *(A1)*
eg $-5+3(n-1)=67$, $3n-8=67$, $3(n-1)=72$

correct working *(A1)*
eg $3n=75$, $n-1=24$

$n=25$ ***A1*** ***N3***
[4 marks]

Total [6 marks]

4. (a) correct approach *(A1)*
eg $3\log_2 a$

$\log_2 a^3 = 3b$ ***A1*** ***N2***
[2 marks]

(b) correct working *(A1)*
eg $\log_2 8 + \log_2 a$, $\log_2 8 = 3$

$\log_2 8a = 3+b$ ***A1*** ***N2***
[2 marks]

(c) correct working *(A1)*
eg $\dfrac{\log_2 a}{\log_2 8}$, $\dfrac{1}{3}\log_2 a$, $b\log_8 2$

$\log_8 a = \dfrac{b}{3}$ ***A1*** ***N2***

[2 marks]

Total [6 marks]

5. **METHOD 1** (eliminating k)

recognizing parallel vectors are multiples of each other ***(M1)***

eg $\boldsymbol{a} = k\boldsymbol{b}$, $\begin{pmatrix} 3 \\ 2p \end{pmatrix} = k\begin{pmatrix} p+1 \\ 8 \end{pmatrix}$, $\frac{p+1}{3} = \frac{8}{2p}$, $3k = p+1$ and $2kp = 8$

correct working (must be quadratic) ***(A1)***

eg $2p^2 + 2p = 24$, $p^2 + p - 12$, $3 = \frac{p^2 + p}{4}$

valid attempt to solve **their** quadratic equation ***(M1)***
eg factorizing, formula, completing the square

evidence of correct working ***(A1)***

eg $(p+4)(p-3)$, $x = \frac{-2 \pm \sqrt{4 - 4(2)(-24)}}{4}$

$p = -4,\ p = 3$ ***A1A1*** ***N4***

METHOD 2 (solving for k)

recognizing parallel vectors are multiples of each other ***(M1)***

eg $\boldsymbol{a} = k\boldsymbol{b}$, $\begin{pmatrix} 3 \\ 2p \end{pmatrix} = k\begin{pmatrix} p+1 \\ 8 \end{pmatrix}$, $3k = p+1$ and $2kp = 8$

correct working (must be quadratic) ***(A1)***
eg $3k^2 - k = 4$, $3k^2 - k - 4$, $4k^2 = 3 - k$

one correct value for k ***(A1)***

eg $k = -1$, $k = \frac{4}{3}$, $k = \frac{3}{4}$

substituting **their** value(s) of k ***(M1)***

eg $\begin{pmatrix} 3 \\ 2p \end{pmatrix} = \frac{3}{4}\begin{pmatrix} p+1 \\ 8 \end{pmatrix}$, $3\left(\frac{4}{3}\right) = p+1$ and $2\left(\frac{4}{3}\right)p = 8$, $(-1)\begin{pmatrix} 3 \\ 2p \end{pmatrix} = \begin{pmatrix} p+1 \\ 8 \end{pmatrix}$

$p = -4,\ p = 3$ ***A1A1*** ***N4***

continued…

Question 5 continued

METHOD 3 (working with angles and cosine formula)

recognizing angle between parallel vectors is 0 and/or 180° ***M1***

eg $\cos\theta = \pm 1$, $a \bullet b = |a||b|$

correct substitution of scalar product and magnitudes into equation ***(A1)***

eg $\dfrac{3(p+1)+2p(8)}{\sqrt{3^2+(2p)^2}\sqrt{(p+1)^2+8^2}} = \pm 1$, $19p+3=\sqrt{4p^2+9}\sqrt{p^2+2p+65}$

correct working (must include both $\pm$) ***(A1)***

eg $3(p+1)+2p(8) = \pm\sqrt{3^2+(2p)^2}\sqrt{(p+1)^2+8^2}$, $19p+3=\pm\sqrt{4p^2+9}\sqrt{p^2+2p+65}$

correct quartic equation ***(A1)***

eg $361p^2+114p+9=4p^4+8p^3+269p^2+18p+585$, $4p^4+8p^3-92p^2-96p+576=0$,
$p^4+2p^3-23p^2-24p+144=0$, $(p+4)^2(p-3)^2=0$

$p=-4$, $p=3$ ***A2*** ***N4***

Total [6 marks]

6. **METHOD 1** (limits in terms of x)

valid approach to find x-intercept *(M1)*

eg $f(x)=0$, $\dfrac{6-2x}{\sqrt{16+6x-x^2}}=0$, $6-2x=0$

x-intercept is 3 *(A1)*

valid approach using substitution or inspection *(M1)*

eg $u=16+6x-x^2$, $\int_0^3 \dfrac{6-2x}{\sqrt{u}}\,dx$, $du=6-2x$, $\int \dfrac{1}{\sqrt{u}}$, $2u^{\frac{1}{2}}$,

$u=\sqrt{16+6x-x^2}$, $\dfrac{du}{dx}=(6-2x)\dfrac{1}{2}\left(16+6x-x^2\right)^{-\frac{1}{2}}$, $\int 2\,du$, $2u$

$\int f(x)\,dx = 2\sqrt{16+6x-x^2}$ *(A2)*

substituting **both** of **their** limits into **their** integrated function and subtracting *(M1)*

eg $2\sqrt{16+6(3)-3^2}-2\sqrt{16+6(0)^2-0^2}$, $2\sqrt{16+18-9}-2\sqrt{16}$

Note: Award ***M0*** if they substitute into original or differentiated function. Do not accept only "– 0" as evidence of substituting lower limit.

correct working *(A1)*

eg $2\sqrt{25}-2\sqrt{16}$, $10-8$

area $=2$ ***A1*** ***N2***

continued…

Question 6 continued

METHOD 2 (limits in terms of u)

valid approach to find x-intercept ***(M1)***

eg $f(x)=0$, $\frac{6-2x}{\sqrt{16+6x-x^2}}=0$, $6-2x=0$

x-intercept is 3 ***(A1)***

valid approach using substitution or inspection ***(M1)***

eg $u=16+6x-x^2$, $\int_0^3 \frac{6-2x}{\sqrt{u}}\,dx$, $du=6-2x$, $\int \frac{1}{\sqrt{u}}$,

$u=\sqrt{16+6x-x^2}$, $\frac{du}{dx}=(6-2x)\frac{1}{2}\left(16+6x-x^2\right)^{-\frac{1}{2}}$, $\int 2\,du$

correct integration ***(A2)***

eg $\int \frac{1}{\sqrt{u}}\,du=2u^{\frac{1}{2}}$, $\int 2\,du=2u$

both correct limits for u ***(A1)***

eg $u=16$ and $u=25$, $\int_{16}^{25}\frac{1}{\sqrt{u}}\,du$, $\left[2u^{\frac{1}{2}}\right]_{16}^{25}$, $u=4$ and $u=5$, $\int_4^5 2\,du$, $[2u]_4^5$

substituting **both** of **their** limits for u (do not accept 0 and 3) into **their** integrated function and subtracting ***(M1)***

eg $2\sqrt{25}-2\sqrt{16}$, $10-8$

Note: Award ***M0*** if they substitute into original or differentiated function, or if they have not attempted to find limits for u.

area $=2$ ***A1*** ***N2***

Total [8 marks]

7. METHOD 1

correct substitution into formula for $\cos(2x)$ or $\sin(2x)$ ***(A1)***

eg $1-2\left(\frac{1}{3}\right)^2$, $2\left(\frac{\sqrt{8}}{3}\right)^2-1$, $2\left(\frac{1}{3}\right)\left(\frac{\sqrt{8}}{3}\right)$, $\left(\frac{\sqrt{8}}{3}\right)^2-\left(\frac{1}{3}\right)^2$

$\cos(2x)=\frac{7}{9}$ or $\sin(2x)=\frac{2\sqrt{8}}{9}$ $\left(=\frac{\sqrt{32}}{9}=\frac{4\sqrt{2}}{9}\right)$ (may be seen in substitution) ***A2***

recognizing $4x$ is double angle of $2x$ (seen anywhere) ***(M1)***

eg $\cos(2(2x))$, $2\cos^2(2\theta)-1$, $1-2\sin^2(2\theta)$, $\cos^2(2\theta)-\sin^2(2\theta)$

correct substitution of **their** value of $\cos(2x)$ and/or $\sin(2x)$ into formula for $\cos(4x)$ ***(A1)***

eg $2\left(\frac{7}{9}\right)^2-1$, $\frac{98}{81}-1$, $1-2\left(\frac{2\sqrt{8}}{9}\right)^2$, $1-\frac{64}{81}$, $\left(\frac{7}{9}\right)^2-\left(\frac{2\sqrt{8}}{9}\right)^2$, $\frac{49}{81}-\frac{32}{81}$

$\cos(4x)=\frac{17}{81}$ ***A1 N2***

METHOD 2

recognizing $4x$ is double angle of $2x$ (seen anywhere) ***(M1)***

eg $\cos(2(2x))$

double angle identity for $2x$ ***(M1)***

eg $2\cos^2(2\theta)-1$, $1-2\sin^2(2x)$, $\cos^2(2\theta)-\sin^2(2\theta)$

correct expression for $\cos(4x)$ in terms of $\sin x$ and/or $\cos x$ ***(A1)***

eg $2(1-2\sin^2\theta)^2-1$, $1-2(2\sin x\cos x)^2$, $\left(1-2\sin^2\theta\right)^2-\left(2\sin\theta\cos\theta\right)^2$

correct substitution for $\sin x$ and/or $\cos x$ ***A1***

eg $2\left(1-2\left(\frac{1}{3}\right)^2\right)^2-1$, $2\left(1-4\left(\frac{1}{3}\right)^2+4\left(\frac{1}{3}\right)^4\right)-1$, $1-2\left(2\times\frac{1}{3}\times\frac{\sqrt{8}}{3}\right)^2$

correct working ***(A1)***

eg $2\left(\frac{49}{81}\right)-1$, $1-2\left(\frac{32}{81}\right)$, $\frac{49}{81}-\frac{32}{81}$

$\cos(4x)=\frac{17}{81}$ ***A1 N2***

Total [6 marks]

Section B

8. (a) valid approach *(M1)*

eg $f(x)=0$, $x^2-4x-5=0$

valid attempt to solve quadratic equation *(M1)*

eg factorizing, formula, completing the square

evidence of correct working *(A1)*

eg $(x-5)(x+1)$, $x=\dfrac{4\pm\sqrt{16-4(-5)}}{2}$

$x=-1$, $x=5$ (accept $(-1, 0)$, $(5, 0)$) *A1A1* *N3*

[5 marks]

(b) correct working *(A1)*

eg $\dfrac{-(-4)}{2(1)}$, $\dfrac{-1+5}{2}$

$x=2$ (must be an equation with $x=$) *A1* *N2*

[2 marks]

(c) (i) $h=2$ *A1* *N1*

(ii) **METHOD 1**

valid approach *(M1)*

eg $f(2)$

correct substitution *(A1)*

eg $(2)^2-4(2)-5$

$k=-9$ *A1* *N2*

METHOD 2

valid attempt to complete the square *(M1)*

eg x^2-4x+4

correct working *(A1)*

eg $(x^2-4x+4)-4-5$, $(x-2)^2-9$

$k=-9$ *A1* *N2*

[4 marks]

continued…

Question 8 continued

(d) **METHOD 1** (working with vertex)

vertex of f is at $(2, -9)$ ***(A1)***

correct horizontal reflection ***(A1)***
eg $x = -2$, $(-2, -9)$

valid approach for translation of **their** x **or** y value ***(M1)***
eg $x - 3$, $y + 6$, $\begin{pmatrix} -2 \\ -9 \end{pmatrix} + \begin{pmatrix} -3 \\ 6 \end{pmatrix}$, one correct coordinate for vertex

vertex of g is $(-5, -3)$ (accept $x = -5$, $y = -3$) ***A1A1*** ***N1N1***

METHOD 2 (working with function)

correct approach for horizontal reflection ***(A1)***

eg $f(-x)$

correct horizontal reflection ***(A1)***
eg $(-x)^2 - 4(-x) - 5$, $x^2 + 4x - 5$, $(-x - 2)^2 - 9$

valid approach for translation of **their** x **or** y value ***(M1)***
eg $(x+3)^2 + 4(x+3) - 5 + 6$, $x^2 + 10x + 22$, $(x+5)^2 - 3$, one correct coordinate for vertex

vertex of g is $(-5, -3)$ (accept $x = -5$, $y = -3$) ***A1A1*** ***N1N1***

[5 marks]

Total [16 marks]

9. (a) (i) $\frac{2}{n}$ ***A1*** ***N1***

(ii) correct probability for one of the draws ***A1***

eg $\mathrm{P}(\text{not blue first}) = \frac{n-2}{n}$, blue second $= \frac{2}{n-1}$

valid approach ***(M1)***

eg recognizing loss on first in order to win on second, $\mathrm{P}(B' \text{ then } B)$, $\mathrm{P}(B') \times \mathrm{P}(B \mid B')$, tree diagram

correct expression in terms of n ***A1*** ***N3***

eg $\frac{n-2}{n} \times \frac{2}{n-1}$, $\frac{2n-4}{n^2-n}$, $\frac{2(n-2)}{n(n-1)}$

[4 marks]

(b) (i) correct working ***(A1)***

eg $\frac{3}{5} \times \frac{2}{4} \times \frac{2}{3}$

$\frac{12}{60} \left(= \frac{1}{5}\right)$ ***A1*** ***N2***

(ii) correct working ***(A1)***

eg $\frac{3}{5} \times \frac{2}{4} \times \frac{1}{3} \times \frac{2}{2}$

$\frac{6}{60} \left(= \frac{1}{10}\right)$ ***A1*** ***N2***

[4 marks]

continued…

Question 9 continued

(c) correct probabilities (seen anywhere) ***(A1)(A1)***

eg $\text{P}(1) = \frac{2}{5}$, $\text{P}(2) = \frac{6}{20}$ (may be seen on tree diagram)

valid approach to find $\text{E}(M)$ or expected winnings using **their** probabilities ***(M1)***

eg $\text{P}(1)\times(0) + \text{P}(2)\times(20) + \text{P}(3)\times(8k) + \text{P}(4)\times(12k)$,
$\text{P}(1)\times(-20) + \text{P}(2)\times(0) + \text{P}(3)\times(8k-20) + \text{P}(4)\times(12k-20)$

correct working to find $\text{E}(M)$ or expected winnings ***(A1)***

eg $\frac{2}{5}(0) + \frac{3}{10}(20) + \frac{1}{5}(8k) + \frac{1}{10}(12k)$,
$\frac{2}{5}(-20) + \frac{3}{10}(0) + \frac{1}{5}(8k-20) + \frac{1}{10}(12k-20)$

correct equation for fair game ***A1***

eg $\frac{3}{10}(20) + \frac{1}{5}(8k) + \frac{1}{10}(12k) = 20$, $\frac{2}{5}(-20) + \frac{1}{5}(8k-20) + \frac{1}{10}(12k-20) = 0$

correct working to combine terms in k ***(A1)***

eg $-8 + \frac{14}{5}k - 4 - 2 = 0$, $6 + \frac{14}{5}k = 20$, $\frac{14}{5}k = 14$

$k = 5$ ***A1*** ***N0***

Note: Do not award the final ***A1*** if the candidate's ***FT*** probabilities do not sum to 1.

[7 marks]

Total [15 marks]

10. (a) valid approach *(M1)*

eg $f(0)$, $0^3 - 2(0)^2 + a(0) + 6$, $f(0) = 6$, $(0,\ y)$

$(0,\ 6)$ (accept $x = 0$ **and** $y = 6$) *A1* *N2*

[2 marks]

(b) (i) $f' = 3x^2 - 4x + a$ *A2* *N2*

(ii) valid approach *(M1)*

eg $f'(0)$

correct working *(A1)*

eg $3(0)^2 - 4(0) + a$, slope $= a$, $f'(0) = a$

attempt to substitute gradient and coordinates into linear equation *(M1)*

eg $y - 6 = a(x - 0)$, $y - 0 = a(x - 6)$, $6 = a(0) + c$, $L = ax + 6$

correct equation *A1* *N3*

eg $y = ax + 6$, $y - 6 = ax$, $y - 6 = a(x - 0)$

[6 marks]

(c) valid approach to find intersection *(M1)*

eg $f(x) = L$

correct equation *(A1)*

eg $x^3 - 2x^2 + ax + 6 = ax + 6$

correct working *(A1)*

eg $x^3 - 2x^2 = 0$, $x^2(x - 2) = 0$

$x = 2$ at Q *(A1)*

valid approach to find minimum *(M1)*

eg $f'(x) = 0$

correct equation *(A1)*

eg $3x^2 - 4x + a = 0$

substitution of **their** value of x at Q into **their** $f'(x) = 0$ equation *(M1)*

eg $3(2)^2 - 4(2) + a = 0$, $12 - 8 + a = 0$

$a = -4$ *A1* *N0*

[8 marks]

Total [16 marks]

M18/5/MATME/SP1/ENG/TZ1/XX/M

Markscheme

May 2018

Mathematics

Standard level

Paper 1

23 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a valid **Method**; working must be seen.

(M) Marks awarded for a valid **Method**; may be implied by **correct** subsequent working.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

(A) Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.

R Marks awarded for clear **Reasoning**.

N Marks awarded for **correct** answers if **no** working shown.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Mark according to RM assessor instructions.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any. An exception to this rule is when work for ***M1*** is missing, as opposed to incorrect (see point 4).
- Where ***M*** and ***A*** marks are noted on the same line, *eg* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*eg* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***(M2)***, ***N3***, *etc*., do **not** split the marks, unless there is a note.
- Most ***M*** marks are for a **valid** method, ie a method which can lead to the answer: it must indicate some form of progress towards the answer.
- Once a correct answer to a question or part-question is seen, ignore further correct working. However, if further working indicates a lack of mathematical understanding do not award final ***A1***.

3 *N* marks

*If **no** working shown, award **N** marks for **correct** answers – this includes acceptable answers (see accuracy booklet). In this case, ignore mark breakdown (**M, A, R**). Where a student only shows a final incorrect answer with no working, even if that answer is a correct intermediate answer, award **N0.***

- Do **not** award a mixture of ***N*** and other marks.
- There may be fewer ***N*** marks available than the total of ***M***, ***A*** and ***R*** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.
- There may not be a direct relationship between the ***N*** marks and the implied marks. There are times when all the marks are implied, but the ***N*** marks are not the full marks: this indicates that we want to see some of the working, without specifying what.

- For consistency within the markscheme, ***N*** marks are noted for every part, even when these match the mark breakdown.
- If a candidate has incorrect working, which somehow results in a correct answer, do **not** award the ***N*** marks for this correct answer. However, if the candidate has indicated (usually by crossing out) that the working is to be ignored, award the ***N*** marks for the correct answer.

4 Implied and must be seen marks

Implied marks appear in ***brackets*** *eg* ***(M1)****.*

- Implied marks can only be awarded if the work is seen or if implied in subsequent working (a correct final answer does not necessarily mean that the implied marks are all awarded). There are questions where some working is required, but as it is accepted that not everyone will write the same steps, all the marks are implied, but the ***N*** marks are not the full marks for the question.
- Normally the correct work is seen in the next line.
- Where there is an (***M1***) followed by ***A1*** for each correct answer, if no working shown, one correct answer is sufficient evidence to award the (***M1***).

Must be seen marks appear without ***brackets*** *eg* ***M1****.*

- Must be seen marks can only be awarded if the work is seen.
- If a must be seen mark is not awarded because work is missing (as opposed to ***M0*** or ***A0*** for incorrect work) all subsequent marks may be awarded if appropriate.

5 Follow through marks (only applied after an error is made)

*Follow through (****FT****) marks are awarded where an incorrect answer (final or intermediate) from one* ***part*** *of a question is used correctly in* ***subsequent*** *part(s) or subpart(s). Usually, to award* ***FT*** *marks,* ***there must be working present*** *and not just a final answer based on an incorrect answer to a previous part. However, if the only marks awarded in a subpart are for the* ***final*** *answer, then* ***FT*** *marks should be awarded if appropriate. Examiners are expected to check student work in order to award* ***FT*** *marks where appropriate.*

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** and ***R*** marks may be awarded if appropriate. (However, as noted above, if an ***A*** mark is not awarded because work is missing, all subsequent marks may be awarded if appropriate).
- Exceptions to this rule will be explicitly noted on the markscheme.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*eg* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word "**their**" in a description, to indicate that candidates may be using an incorrect value.
- If a candidate makes an error in one part, but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the question says hence. It is often possible to use a different approach in subsequent parts that does not depend on the answer to previous parts.
- In a "show that" question, if an error in a previous subpart leads to not showing the required answer, do not award the final ***A1***. Note that if the error occurs within the same subpart, the ***FT*** rules may result in further loss of marks.

6 Mis-read

*If a candidate incorrectly copies information from the question, this is a mis-read (**MR**). A candidate should be penalized only once for a particular mis-read. Use the **MR** stamp to indicate that this is a misread. Do not award the first mark in the question, even if this is an **M** mark, but award all others (if appropriate) so that the candidate only loses one mark for the misread.*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*eg* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.

7 Discretionary marks *(d)*

*An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. In such cases the annotation DM should be used and a brief **note** written next to the mark explaining this decision.*

8 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete parts are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**. Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9 Alternative forms

*Unless the question specifies otherwise, **accept** equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

10 Calculators

No calculator is allowed. The use of any calculator on paper 1 is malpractice, and will result in no grade awarded. If you see work that suggests a candidate has used any calculator, please follow the procedures for malpractice. Examples: finding an angle, given a trig ratio of 0.4235.

11 Style

The markscheme aims to present answers using good communication, eg if the question asks to find the value of k, the markscheme will say $k = 3$, but the marks will be for the correct value 3 – there is usually no need for the "$k =$". In these cases, it is also usually acceptable to have another variable, as long as there is no ambiguity in the question, eg if the question asks to find the value of p and of q, then the student answer needs to be clear. Generally, the only situation

where the full answer is required is in a question which asks for equations – in this case the markscheme will say "must be an equation".

*The markscheme often uses words to describe what the marks are for, followed by examples, using the eg notation. These examples are not exhaustive, and examiners should check what candidates have written, to see if they satisfy the description. Where these marks are **M** marks, the examples may include ones using poor notation, to indicate what is acceptable. A valid method is one which will allow candidate to proceed to the next step eg if a quadratic function is given in factorised form, and the question asks for the zeroes, then multiplying the factors does not necessarily help to find the zeros, and would not on its own count as a valid method.*

12 Candidate work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

Candidates are meant to write their answers to Section A on the question paper (QP), and Section B on answer booklets. Sometimes, they need more room for Section A, and use the booklet (and often comment to this effect on the QP), or write outside the box. That is fine, and this work should be marked.

The instructions tell candidates not to write on Section B of the QP. Thus they may well have done some rough work here which they assume will be ignored. If they have solutions on the answer booklets, there is no need to look at the QP. However, if there are whole questions or whole part solutions missing on answer booklets, please check to make sure that they are not on the QP, and if they are, mark those whole questions or whole part solutions that have not been written on answer booklets.

13. Diagrams

The notes on how to allocate marks for sketches usually refer to passing through particular points or having certain features. These marks can only be awarded if the sketch is approximately the correct shape. All values given will be an approximate guide to where these points/features occur. In some questions, the first ***A1*** is for the shape, in others, the marks are only for the points and/or features. In both cases, unless the shape is approximately correct, no marks can be awarded (unless otherwise stated). However, if the graph is based on previous calculations, ***FT*** marks should be awarded if appropriate.

14. Accuracy of Answers

If the level of accuracy is specified in the question, a mark will be allocated for giving the final answer to the required accuracy. When this is not specified in the question, all numerical answers should be given exactly or correct to three significant figures.

Do not accept unfinished numerical final answers such as 3/0.1 (unless otherwise stated). As a rule, numerical answers with more than one part (such as fractions) should be given using integers (eg 6/8). Calculations which lead to integers should be completed, with the exception of fractions which are not whole numbers. Intermediate values do not need to be given to the correct three significant figures. But, if candidates work with rounded values, this could lead to an incorrect answer, in which case award A0 for the final answer. Where numerical answers are required as the final answer to a part of a question in the markscheme, the markscheme will show

a truncated 6 sf value
the exact value if applicable, the correct 3 sf answer
Units will appear in brackets at the end.

Section A

1. (a) $f(14) = 4$ ***A1*** ***N1***
[1 mark]

(b) attempt to substitute ***(M1)***
eg $g(4)$, $3 \times 4 - 7$

5 ***A1*** ***N2***
[2 marks]

(c) interchanging x and y (seen anywhere) ***(M1)***
eg $x = 3y - 7$

evidence of correct manipulation ***(A1)***
eg $x + 7 = 3y$

$g^{-1}(x) = \frac{x+7}{3}$ ***A1*** ***N3***

[3 marks]

Total [6 marks]

2. (a) recognizing Q_1 or Q_3 (seen anywhere) ***(M1)***
eg $4, 11$, indicated on diagram

$\text{IQR} = 7$ ***A1*** ***N2***
[2 marks]

(b) recognizing the need to find $1.5\ \text{IQR}$ ***(M1)***
eg $1.5 \times \text{IQR}$, 1.5×7

valid approach to find k ***(M1)***
eg $10.5 + 11$, $1.5 \times \text{IQR} + Q_3$

21.5 ***(A1)***

$k = 22$ ***A1*** ***N3***

Note: If no working shown, award ***N2*** for an answer of 21.5.

[4 marks]

Total [6 marks]

3. (a) (i) $f(0) = -\frac{1}{2}$ ***A1*** ***N1***

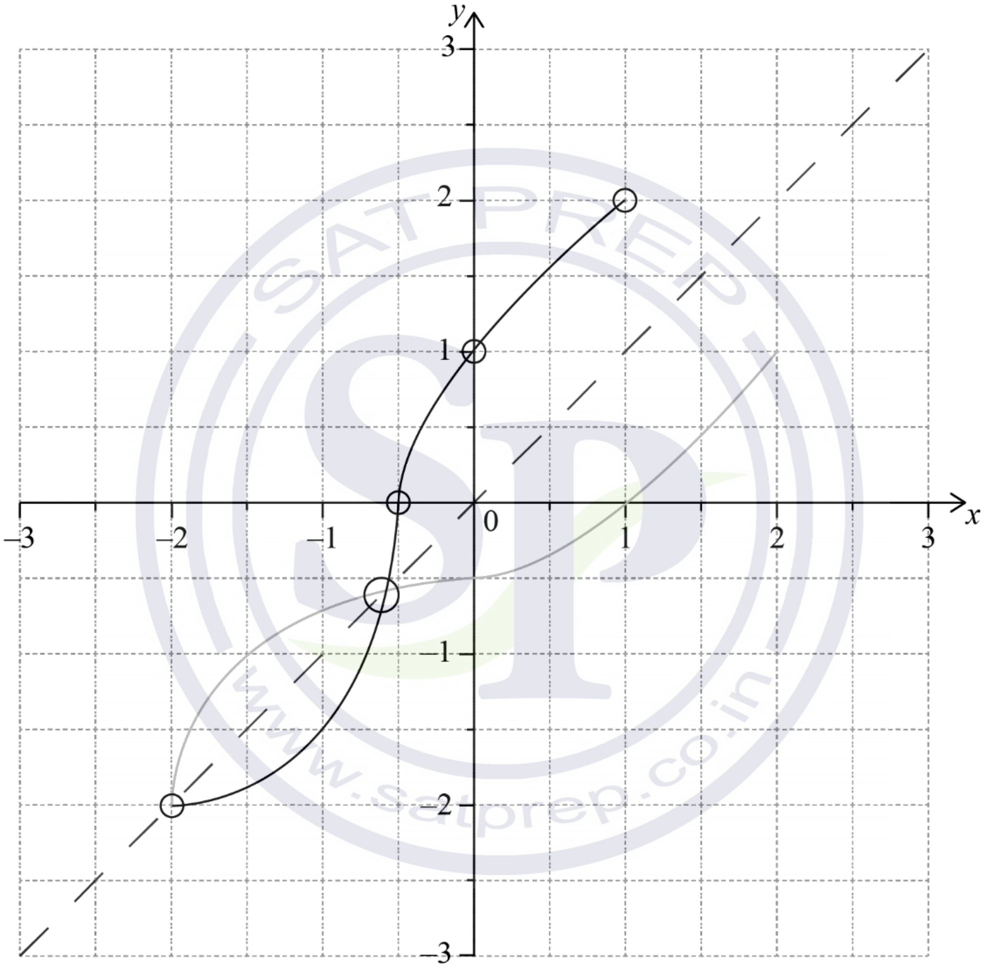
(ii) $f^{-1}(1) = 2$ ***A1*** ***N1***

[2 marks]

(b) $-2 \le y \le 2$, $y \in [-2, 2]$ (accept $-2 \le x \le 2$) ***A1*** ***N1***

[1 mark]

(c)



A1
A1A1A1 ***N4***

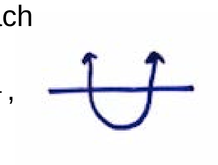
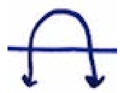
Note: Award ***A1*** for evidence of approximately correct reflection in $y = x$ with correct curvature. ($y = x$ does not need to be explicitly seen)
Only if this mark is awarded, award marks as follows:
A1 for both correct invariant points in circles,
A1 for the three other points in circles,
A1 for correct domain.

[4 marks]

Total [7 marks]

4. (a) **METHOD 1** (using symmetry to find p)

(i) valid approach ***(M1)***

eg $\frac{-1+3}{2}$,  , 

$p = 1$ ***A1*** ***N2***

> **Note:** Award no marks if they work backwards by substituting $a = 2$ into $-\frac{b}{2a}$ to find p.
>
> Do not accept $p = \frac{2}{a}$.

(ii) valid approach ***M1***

eg $-\frac{b}{2a}$, $\frac{4}{2a}$ (might be seen in (i)), $f'(1) = 0$

correct equation ***A1***

eg $\frac{4}{2a} = 1$, $2a(1) - 4 = 0$

$a = 2$ ***AG*** ***N0***

METHOD 2 (calculating a first)

(i) & (ii) valid approach to calculate a ***M1***

eg $a + 4 - c = a(3^2) - 4(3) - c$, $f(-1) = f(3)$

correct working ***A1***

eg $8a = 16$

$a = 2$ ***AG*** ***N0***

valid approach to find p ***(M1)***

eg $-\frac{b}{2a}$, $\frac{4}{2(2)}$

$p = 1$ ***A1*** ***N2***

[4 marks]

(b) valid approach ***(M1)***

eg $f(-1) = 5$, $f(3) = 5$

correct working ***(A1)***

eg $2 + 4 - c = 5$, $18 - 12 - c = 5$

$c = 1$ ***A1*** ***N2***

[3 marks]

Total [7 marks]

5. (a) correct working ***(A1)***

eg $\int \frac{1}{2x-1}\mathrm{d}x$, $\int (2x-1)^{-1}$, $\frac{1}{2x-1}$, $\int \left(\frac{1}{\sqrt{u}}\right)^2 \frac{\mathrm{d}u}{2}$

$\int (f(x))^2\,\mathrm{d}x = \frac{1}{2}\ln(2x-1)+c$ ***A2*** ***N3***

Note: Award ***A1*** for $\frac{1}{2}\ln(2x-1)$.

[3 marks]

(b) attempt to substitute either limits or the function into formula involving f^2 (accept absence of π / $\mathrm{d}x$) ***(M1)***

eg $\int_1^9 y^2\mathrm{d}x$, $\pi\int\left(\frac{1}{\sqrt{2x-1}}\right)^2\mathrm{d}x$, $\left[\frac{1}{2}\ln(2x-1)\right]_1^9$

substituting limits into **their** integral and subtracting (in any order) ***(M1)***

eg $\frac{\pi}{2}(\ln(17)-\ln(1))$, $\pi\left(0-\frac{1}{2}\ln(2\times 9-1)\right)$

correct working involving calculating a log value or using log law ***(A1)***

eg $\ln(1)=0$, $\ln\left(\frac{17}{1}\right)$

$\frac{\pi}{2}\ln 17$ (accept $\pi\ln\sqrt{17}$) ***A1*** ***N3***

Note: Full ***FT*** may be awarded as normal, from their incorrect answer in part (a), however, do not award the final two ***A*** marks unless they involve logarithms.

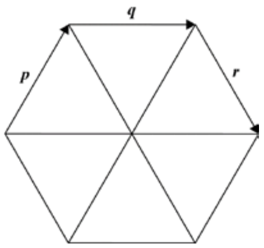
[4 marks]

Total [7 marks]

6. **METHOD 1** (using $|\boldsymbol{p}||2\boldsymbol{q}|\cos\theta$)

finding $\boldsymbol{p}+\boldsymbol{q}+\boldsymbol{r}$ ***(A1)***

eg $2\boldsymbol{q}$,



$|\boldsymbol{p}+\boldsymbol{q}+\boldsymbol{r}|=2\times3\ (=6)$ (seen anywhere) ***A1***

correct angle between $\boldsymbol{p}$ and $\boldsymbol{q}$ (seen anywhere) ***(A1)***

$\frac{\pi}{3}$ (accept 60°)

substitution of **their** values ***(M1)***

eg $3\times6\times\cos\left(\frac{\pi}{3}\right)$

correct value for $\cos\left(\frac{\pi}{3}\right)$ (seen anywhere) ***(A1)***

eg $\frac{1}{2}$, $3\times6\times\frac{1}{2}$

$\boldsymbol{p}\bullet(\boldsymbol{p}+\boldsymbol{q}+\boldsymbol{r})=9$ ***A1*** ***N3***

METHOD 2 (scalar product using distributive law)

correct expression for scalar distribution ***(A1)***

eg $\boldsymbol{p}\bullet\boldsymbol{p}+\boldsymbol{p}\bullet\boldsymbol{q}+\boldsymbol{p}\bullet\boldsymbol{r}$

three correct angles between the vector pairs (seen anywhere) ***(A2)***

eg 0° between $\boldsymbol{p}$ and $\boldsymbol{p}$, $\frac{\pi}{3}$ between $\boldsymbol{p}$ and $\boldsymbol{q}$, $\frac{2\pi}{3}$ between $\boldsymbol{p}$ and $\boldsymbol{r}$

Note: Award ***A1*** for only two correct angles.

substitution of **their** values ***(M1)***

eg $3.3.\cos0+3.3.\cos\frac{\pi}{3}+3.3.\cos120$

one correct value for $\cos0$, $\cos\left(\frac{\pi}{3}\right)$ or $\cos\left(\frac{2\pi}{3}\right)$ (seen anywhere) ***A1***

eg $\frac{1}{2}$, $3\times6\times\frac{1}{2}$

$\boldsymbol{p}\bullet(\boldsymbol{p}+\boldsymbol{q}+\boldsymbol{r})=9$ ***A1*** ***N3***

continued…

Question 6 continued

METHOD 3 (scalar product using relative position vectors)

valid attempt to find one component of $\boldsymbol{p}$ or $\boldsymbol{r}$ ***(M1)***

eg $\sin 60 = \frac{x}{3}$, $\cos 60 = \frac{x}{3}$, one correct value $\frac{3}{2}$, $\frac{3\sqrt{3}}{2}$, $\frac{-3\sqrt{3}}{2}$

one correct vector (two or three dimensions) (seen anywhere) ***A1***

eg $p = \begin{pmatrix} \frac{3}{2} \\ \frac{3\sqrt{3}}{2} \end{pmatrix}$, $q = \begin{pmatrix} 3 \\ 0 \end{pmatrix}$, $r = \begin{pmatrix} \frac{3}{2} \\ -\frac{3\sqrt{3}}{2} \\ 0 \end{pmatrix}$

three correct vectors or $\boldsymbol{p}+\boldsymbol{q}+\boldsymbol{r} = 2\boldsymbol{q}$ ***(A1)***

$\boldsymbol{p}+\boldsymbol{q}+\boldsymbol{r} = \begin{pmatrix} 6 \\ 0 \end{pmatrix}$ or $\begin{pmatrix} 6 \\ 0 \\ 0 \end{pmatrix}$ (seen anywhere, including scalar product) ***(A1)***

correct working ***(A1)***

eg $\left(\frac{3}{2}\times 6\right) + \left(\frac{3\sqrt{3}}{2}\times 0\right)$, $9+0+0$

$\boldsymbol{p}\bullet(\boldsymbol{p}+\boldsymbol{q}+\boldsymbol{r}) = 9$ ***A1*** ***N3***

Total [6 marks]

7. recognizing the need to find h' ***(M1)***

recognizing the need to find $h'(3)$ (seen anywhere) ***(M1)***

evidence of choosing chain rule ***(M1)***

eg $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$, $f'(g(3)) \times g'(3)$, $f'(g) \times g'$

correct working ***(A1)***

eg $f'(7) \times 4$, -5×4

$h'(3) = -20$ ***(A1)***

evidence of taking **their** negative reciprocal for normal ***(M1)***

eg $-\frac{1}{h'(3)}$, $m_1 m_2 = -1$

gradient of normal is $\frac{1}{20}$ ***A1*** ***N4***

Total [7 marks]

Section B

8. (a) evidence of integration *(M1)*

eg $\int f'(x)$

correct integration (accept absence of C) *(A1)(A1)*

eg $x^3+\frac{18}{2}x^2+C$, x^3+9x^2

attempt to substitute $x=-1$ into **their** $f=0$ (must have C) *M1*

eg $(-1)^3+9(-1)^2+C=0$, $-1+9+C=0$

Note: Award ***M0*** if they substitute into original or differentiated function.

correct working *(A1)*

eg $8+C=0$, $C=-8$

$f(x)=x^3+9x^2-8$ *A1* *N5*

[6 marks]

(b) **METHOD 1** (using 2^{nd} derivative)

recognizing that $f''=0$ (seen anywhere) *M1*

correct expression for f'' *(A1)*

eg $6x+18$, $6p+18$

correct working *(A1)*

$6p+18=0$

$p=-3$ *A1* *N3*

METHOD 2 (using 1^{st} derivative)

recognizing the vertex of f' is needed *(M2)*

eg $-\frac{b}{2a}$ (must be clear this is for f')

correct substitution *(A1)*

eg $\frac{-18}{2\times 3}$

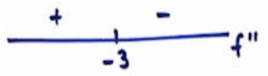
$p=-3$ *A1* *N3*

[4 marks]

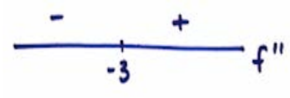
continued…

Question 8 continued

(c) valid attempt to use $f''(x)$ to determine concavity **(M1)**

eg $f''(x) < 0$, $f''(-2)$, $f''(-4)$, $6x + 18 \le 0$, 

correct working **(A1)**

eg $6x + 18 < 0$, $f''(-2) = 6$, $f''(-4) = -6$, 

f concave down for $x < -3$ (do not accept $x \le -3$) **A1** **N2**

[3 marks]

Total [13 marks]

9. (a) correct approach **A1**

eg $\vec{AO}+\vec{OB}$, $B-A$, $\begin{pmatrix}2\\-4\\-4\end{pmatrix}-\begin{pmatrix}-4\\-12\\1\end{pmatrix}$

$\vec{AB}=\begin{pmatrix}6\\8\\-5\end{pmatrix}$ **AG** **N0**

[1 mark]

(b) (i) any correct equation in the form $\boldsymbol{r}=\boldsymbol{a}+t\boldsymbol{b}$ (any parameter for t) **A2** **N2**

where $\boldsymbol{a}$ is $\begin{pmatrix}2\\-4\\-4\end{pmatrix}$ or $\begin{pmatrix}-4\\-12\\1\end{pmatrix}$ and $\boldsymbol{b}$ is a scalar multiple of $\begin{pmatrix}6\\8\\-5\end{pmatrix}$

eg $\boldsymbol{r}=\begin{pmatrix}-4\\-12\\1\end{pmatrix}+t\begin{pmatrix}6\\8\\-5\end{pmatrix}$, $(x, y, z)=(2, -4, -4)+t(6, 8, -5)$, $\boldsymbol{r}=\begin{pmatrix}-4+6t\\-12+8t\\1-5t\end{pmatrix}$

Note: Award ***A1*** for the form $\boldsymbol{a}+t\boldsymbol{b}$, ***A1*** for the form $\boldsymbol{L}=\boldsymbol{a}+t\boldsymbol{b}$, ***A0*** for the form $\boldsymbol{r}=\boldsymbol{b}+t\boldsymbol{a}$.

(ii) **METHOD 1** (solving for t)

valid approach **(M1)**

eg $\begin{pmatrix}k\\12\\-k\end{pmatrix}=\begin{pmatrix}2\\-4\\-4\end{pmatrix}+t\begin{pmatrix}6\\8\\-5\end{pmatrix}$, $\begin{pmatrix}k\\12\\-k\end{pmatrix}=\begin{pmatrix}-4\\-12\\1\end{pmatrix}+t\begin{pmatrix}6\\8\\-5\end{pmatrix}$

one correct equation **A1**

eg $-4+8t=12$, $-12+8t=12$

correct value for t **(A1)**

eg $t=2$ or 3

correct substitution **A1**

eg $2+6(2)$, $-4+6(3)$, $-[1+3(-5)]$

$k=14$ **AG** **N0**

continued…

Question 9 continued

METHOD 2 (solving simultaneously)

valid approach **(M1)**

eg $\begin{pmatrix} k \\ 12 \\ -k \end{pmatrix} = \begin{pmatrix} 2 \\ -4 \\ -4 \end{pmatrix} + t\begin{pmatrix} 6 \\ 8 \\ -5 \end{pmatrix}$, $\begin{pmatrix} k \\ 12 \\ -k \end{pmatrix} = \begin{pmatrix} -4 \\ -12 \\ 1 \end{pmatrix} + t\begin{pmatrix} 6 \\ 8 \\ -5 \end{pmatrix}$

two correct equations in **A1**

eg $k = -4 + 6t,\ -k = 1 - 5t$

EITHER (eliminating k)

correct value for t **(A1)**

eg $t = 2 \text{ or } 3$

correct substitution **A1**

eg $2 + 6(2),\ -4 + 6(3)$

OR (eliminating t)

correct equation(s) **(A1)**

eg $5k + 20 = 30t$ **and** $-6k - 6 = -30t$, $-k = 1 - 5\left(\frac{k+4}{6}\right)$

correct working clearly leading to $k = 14$ **A1**

eg $-k + 14 = 0$, $-6k = 6 - 5k - 20$, $5k = -20 + 6(1 + k)$

THEN

$k = 14$ **AG** **N0**

[6 marks]

continued…

Question 9 continued

(c) (i) correct substitution into scalar product **A1**

eg $(2)(6)-(4)(8)-(4)(-5)$, $12-32+20$

$\overrightarrow{OB}\bullet\overrightarrow{AB}=0$ **A1** **N0**

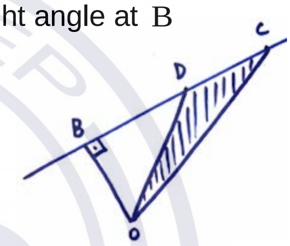
(ii) $\hat{OBA}=\frac{\pi}{2},\ 90^\circ\ \left(\text{accept } \frac{3\pi}{2},\ 270^\circ\right)$ **A1** **N1**

[3 marks]

(d) **METHOD 1** $(\frac{1}{2}\times\text{height}\times\text{CD})$

recognizing that OB is altitude of triangle with base CD (seen anywhere) **M1**

eg $\frac{1}{2}\times\left|\overrightarrow{OB}\right|\times\left|\overrightarrow{CD}\right|$, $OB\perp CD$, sketch showing right angle at B



$\overrightarrow{CD}=\begin{pmatrix}-6\\-8\\5\end{pmatrix}$ or $\overrightarrow{DC}=\begin{pmatrix}6\\8\\-5\end{pmatrix}$ (seen anywhere) **(A1)**

correct magnitudes (seen anywhere) **(A1)(A1)**

$\left|\overrightarrow{OB}\right|=\sqrt{(2)^2+(-4)^2+(-4)^2}\ \left(=\sqrt{36}\right)$

$.\left|\overrightarrow{CD}\right|=\sqrt{(-6)^2+(-8)^2+(5)^2}\ \left(=\sqrt{125}\right)$

correct substitution into $\frac{1}{2}bh$ **A1**

eg $\frac{1}{2}\times6\times\sqrt{125}$

area $=3\sqrt{125}$, $15\sqrt{5}$ **A1** **N3**

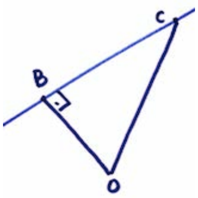
continued…

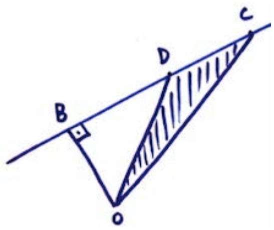
Question 9 continued

METHOD 2 (subtracting triangles)

recognizing that OB is altitude of either ΔOBD or ΔOBC (seen anywhere) ***M1***

eg $\frac{1}{2}\times\left|\vec{\text{OB}}\right|\times\left|\vec{\text{BD}}\right|$, $\text{OB}\perp\text{BC}$, sketch of triangle showing right angle at B





one correct vector $\vec{\text{BD}}$ or $\vec{\text{DB}}$ or $\vec{\text{BC}}$ or $\vec{\text{CB}}$ (seen anywhere) ***(A1)***

eg $\vec{\text{BD}}=\begin{pmatrix}6\\8\\-5\end{pmatrix}$, $\vec{\text{CB}}=\begin{pmatrix}-12\\-16\\10\end{pmatrix}$

$\left|\vec{\text{OB}}\right|=\sqrt{(2)^2+(-4)^2+(-4)^2}\left(=\sqrt{36}\right)$ (seen anywhere) ***(A1)***

one correct magnitude of a base (seen anywhere) ***(A1)***

$\left|\vec{\text{BD}}\right|=\sqrt{(6)^2+(8)^2+(5)^2}\left(=\sqrt{125}\right)$, $\left|\vec{\text{BC}}\right|=\sqrt{144+256+100}\left(=\sqrt{500}\right)$

correct working ***A1***

eg $\frac{1}{2}\times6\times\sqrt{500}-\frac{1}{2}\times6\times5\sqrt{5}$, $\frac{1}{2}\times6\times\sqrt{500}\times\sin 90-\frac{1}{2}\times6\times5\sqrt{5}\times\sin 90$

area $=3\sqrt{125}$, $15\sqrt{5}$ ***A1*** ***N3***

continued…

Question 9 continued

METHOD 3 (using $\frac{1}{2}ab\sin C$ with ΔOCD)

two correct side lengths (seen anywhere) ***(A1)(A1)***

$\left|\overrightarrow{\text{OD}}\right| = \sqrt{(8)^2+(4)^2+(-9)^2}\ \left(=\sqrt{161}\right)$, $\left|\overrightarrow{\text{CD}}\right| = \sqrt{(-6)^2+(-8)^2+(5)^2}\ \left(=\sqrt{125}\right)$,

$\left|\overrightarrow{\text{OC}}\right| = \sqrt{(14)^2+(12)^2+(-14)^2}\ \left(=\sqrt{536}\right)$

attempt to find cosine ratio (seen anywhere) ***M1***

eg $\dfrac{536-286}{-2\sqrt{161}\sqrt{125}}$, $\dfrac{\text{OD}\bullet\text{DC}}{|\text{OD}||\text{DC}|}$

correct working for sine ratio ***A1***

eg $\dfrac{(125)^2}{161\times125}+\sin^2 D=1$

correct substitution into $\frac{1}{2}ab\sin C$ ***A1***

eg $0.5\times\sqrt{161}\times\sqrt{125}\times\dfrac{6}{\sqrt{161}}$

area $=3\sqrt{125}$, $15\sqrt{5}$ ***A1*** ***N3***

[6 marks]

Total [16 marks]

10. (a) (i) valid approach *(M1)*

eg $\frac{u_2}{u_1}, \frac{u_1}{u_2}$

$r = \frac{12\sin^2\theta}{18} \left(= \frac{2\sin^2\theta}{3}\right)$ ***A1*** ***N2***

(ii) recognizing that $\sin\theta$ is bounded *(M1)*

eg $0 \le \sin^2\theta \le 1$, $-1 \le \sin\theta \le 1$, $-1 < \sin\theta < 1$

$0 < r \le \frac{2}{3}$ ***A2*** ***N3***

Note:	If working shown, award ***M1A1*** for correct values with incorrect inequality sign(s). If no working shown, award ***N1*** for correct values with incorrect inequality sign(s).

[5 marks]

(b) correct substitution into formula for infinite sum ***A1***

eg $\frac{18}{1-\frac{2\sin^2\theta}{3}}$

evidence of choosing an appropriate rule for $\cos 2\theta$ (seen anywhere) *(M1)*

eg $\cos 2\theta = 1 - 2\sin^2\theta$

correct substitution of identity/working (seen anywhere) *(A1)*

eg $\frac{18}{1-\frac{2}{3}\left(\frac{1-\cos 2\theta}{2}\right)}$, $\frac{54}{3-2\left(\frac{1-\cos 2\theta}{2}\right)}$, $\frac{18}{\frac{3-2\sin^2\theta}{3}}$

correct working that clearly leads to the given answer ***A1***

eg $\frac{18\times 3}{2+(1-2\sin^2\theta)}$, $\frac{54}{3-(1-\cos 2\theta)}$

$\frac{54}{2+\cos(2\theta)}$ ***AG*** ***N0***

[4 marks]

continued…

Question 10 continued

(c) **METHOD 1** (using differentiation)

recognizing $\frac{\mathrm{d}S_\infty}{\mathrm{d}\theta}=0$ (seen anywhere) ***(M1)***

finding any correct expression for $\frac{\mathrm{d}S_\infty}{\mathrm{d}\theta}$ ***(A1)***

eg $\frac{0-54\times(-2\sin 2\theta)}{(2+\cos 2\theta)^2}$, $-54(2+\cos 2\theta)^{-2}(-2\sin 2\theta)$

correct working ***(A1)***
eg $\sin 2\theta = 0$

any correct value for $\sin^{-1}(0)$ (seen anywhere) ***(A1)***
eg $0,\ \pi,\ \ldots$, sketch of sine curve with x-intercept(s) marked

both correct values for 2θ (ignore additional values) ***(A1)***
$2\theta=\pi,\ 3\pi$ (accept values in degrees)

both correct answers $\theta=\frac{\pi}{2},\ \frac{3\pi}{2}$ ***A1*** ***N4***

Note:	Award ***A0*** if either or both correct answers are given in degrees. Award ***A0*** if additional values are given.

continued…

Question 10 continued

METHOD 2 (using denominator)

recognizing when S_∞ is greatest *(M1)*

eg $2+\cos 2\theta$ is a minimum, $1-r$ is smallest

correct working *(A1)*

eg minimum value of $2+\cos 2\theta$ is 1, minimum $r=\frac{2}{3}$

correct working *(A1)*

eg $\cos 2\theta=-1$, $\frac{2}{3}\sin^2\theta=\frac{2}{3}$, $\sin^2\theta=1$

EITHER (using $\cos 2\theta$)

any correct value for $\cos^{-1}(-1)$ (seen anywhere) *(A1)*

eg π, 3π, ... (accept values in degrees), sketch of cosine curve with *x*-intercept(s) marked

both correct values for 2θ (ignore additional values) *(A1)*

$2\theta=\pi$, 3π (accept values in degrees)

OR (using $\sin\theta$**)**

$\sin\theta=\pm 1$ *(A1)*

$\sin^{-1}(1)=\frac{\pi}{2}$ (accept values in degrees) (seen anywhere) ***A1***

THEN

both correct answers $\theta=\frac{\pi}{2}, \frac{3\pi}{2}$ ***A1*** ***N4***

Note:	Award ***A0*** if either or both correct answers are given in degrees. Award ***A0*** if additional values are given.

[6 marks]

Total [15 marks]

Markscheme

May 2018

Mathematics

Standard level

Paper 1

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a valid **Method**; working must be seen.

(M) Marks awarded for a valid **Method**; may be implied by **correct** subsequent working.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

(A) Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.

R Marks awarded for clear **Reasoning**.

N Marks awarded for **correct** answers if **no** working shown.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Mark according to RM assessor instructions.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any. An exception to this rule is when work for ***M1*** is missing, as opposed to incorrect (see point 4).
- Where ***M*** and ***A*** marks are noted on the same line, *eg* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*eg* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***(M2)***, ***N3***, *etc*., do **not** split the marks, unless there is a note.
- Most ***M*** marks are for a **valid** method, ie a method which can lead to the answer: it must indicate some form of progress towards the answer.
- Once a correct answer to a question or part-question is seen, ignore further correct working. However, if further working indicates a lack of mathematical understanding do not award final ***A1***.

3 *N* marks

*If **no** working shown, award **N** marks for **correct** answers – this includes acceptable answers (see accuracy booklet). In this case, ignore mark breakdown (**M**, **A**, **R**). Where a student only shows a final incorrect answer with no working, even if that answer is a correct intermediate answer, award **N0.***

- Do **not** award a mixture of ***N*** and other marks.
- There may be fewer ***N*** marks available than the total of ***M***, ***A*** and ***R*** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.
- There may not be a direct relationship between the ***N*** marks and the implied marks. There are times when all the marks are implied, but the ***N*** marks are not the full marks: this indicates that we want to see some of the working, without specifying what.

- For consistency within the markscheme, ***N*** marks are noted for every part, even when these match the mark breakdown.
- If a candidate has incorrect working, which somehow results in a correct answer, do **not** award the ***N*** marks for this correct answer. However, if the candidate has indicated (usually by crossing out) that the working is to be ignored, award the ***N*** marks for the correct answer.

4 Implied and must be seen marks

*Implied marks appear in **brackets** eg **(M1)**.*

- Implied marks can only be awarded if the work is seen or if implied in subsequent working (a correct final answer does not necessarily mean that the implied marks are all awarded). There are questions where some working is required, but as it is accepted that not everyone will write the same steps, all the marks are implied, but the ***N*** marks are not the full marks for the question.
- Normally the correct work is seen in the next line.
- Where there is an (***M1***) followed by ***A1*** for each correct answer, if no working shown, one correct answer is sufficient evidence to award the (***M1***).

*Must be seen marks appear without **brackets** eg **M1**.*

- Must be seen marks can only be awarded if the work is seen.
- If a must be seen mark is not awarded because work is missing (as opposed to ***M0*** or ***A0*** for incorrect work) all subsequent marks may be awarded if appropriate.

5 Follow through marks (only applied after an error is made)

*Follow through (**FT**) marks are awarded where an incorrect answer (final or intermediate) from one **part** of a question is used correctly in **subsequent** part(s) or subpart(s). Usually, to award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if the only marks awarded in a subpart are for the **final** answer, then **FT** marks should be awarded if appropriate. Examiners are expected to check student work in order to award **FT** marks where appropriate.*

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** and ***R*** marks may be awarded if appropriate. (However, as noted above, if an ***A*** mark is not awarded because work is missing, all subsequent marks may be awarded if appropriate).
- Exceptions to this rule will be explicitly noted on the markscheme.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*eg* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word "**their**" in a description, to indicate that candidates may be using an incorrect value.
- If a candidate makes an error in one part, but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the question says hence. It is often possible to use a different approach in subsequent parts that does not depend on the answer to previous parts.
- In a "show that" question, if an error in a previous subpart leads to not showing the required answer, do not award the final ***A1***. Note that if the error occurs within the same subpart, the ***FT*** rules may result in further loss of marks.

6 Mis-read

*If a candidate incorrectly copies information from the question, this is a mis-read (**MR**). A candidate should be penalized only once for a particular mis-read. Use the **MR** stamp to indicate that this is a misread. Do not award the first mark in the question, even if this is an **M** mark, but award all others (if appropriate) so that the candidate only loses one mark for the misread.*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*eg* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.

7 Discretionary marks *(d)*

*An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. In such cases the annotation DM should be used and a brief **note** written next to the mark explaining this decision.*

8 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete parts are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**. Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9 Alternative forms

*Unless the question specifies otherwise, **accept** equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

10 Calculators

No calculator is allowed. The use of any calculator on paper 1 is malpractice, and will result in no grade awarded. If you see work that suggests a candidate has used any calculator, please follow the procedures for malpractice. Examples: finding an angle, given a trig ratio of 0.4235.

11 Style

The markscheme aims to present answers using good communication, eg if the question asks to find the value of k, the markscheme will say $k = 3$, *but the marks will be for the correct value 3 – there is usually no need for the "* $k =$ *". In these cases, it is also usually acceptable to have another variable, as long as there is no ambiguity in the question, eg if the question asks to find the value of p and of q, then the student answer needs to be clear. Generally, the only situation*

where the full answer is required is in a question which asks for equations – in this case the markscheme will say "must be an equation".

The markscheme often uses words to describe what the marks are for, followed by examples, using the eg notation. These examples are not exhaustive, and examiners should check what candidates have written, to see if they satisfy the description. Where these marks are ***M*** *marks, the examples may include ones using poor notation, to indicate what is acceptable. A valid method is one which will allow candidate to proceed to the next step eg if a quadratic function is given in factorised form, and the question asks for the zeroes, then multiplying the factors does not necessarily help to find the zeros, and would not on its own count as a valid method.*

12 Candidate work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

Candidates are meant to write their answers to Section A on the question paper (QP), and Section B on answer booklets. Sometimes, they need more room for Section A, and use the booklet (and often comment to this effect on the QP), or write outside the box. That is fine, and this work should be marked.

The instructions tell candidates not to write on Section B of the QP. Thus they may well have done some rough work here which they assume will be ignored. If they have solutions on the answer booklets, there is no need to look at the QP. However, if there are whole questions or whole part solutions missing on answer booklets, please check to make sure that they are not on the QP, and if they are, mark those whole questions or whole part solutions that have not been written on answer booklets.

13. Diagrams

The notes on how to allocate marks for sketches usually refer to passing through particular points or having certain features. These marks can only be awarded if the sketch is approximately the correct shape. All values given will be an approximate guide to where these points/features occur. In some questions, the first ***A1*** is for the shape, in others, the marks are only for the points and/or features. In both cases, unless the shape is approximately correct, no marks can be awarded (unless otherwise stated). However, if the graph is based on previous calculations, ***FT*** marks should be awarded if appropriate.

14. Accuracy of Answers

If the level of accuracy is specified in the question, a mark will be allocated for giving the final answer to the required accuracy. When this is not specified in the question, all numerical answers should be given exactly or correct to three significant figures.

Do not accept unfinished numerical final answers such as 3/0.1 (unless otherwise stated). As a rule, numerical answers with more than one part (such as fractions) should be given using integers (eg 6/8). Calculations which lead to integers should be completed, with the exception of fractions which are not whole numbers. Intermediate values do not need to be given to the correct three significant figures. But, if candidates work with rounded values, this could lead to an incorrect answer, in which case award A0 for the final answer. Where numerical answers are required as the final answer to a part of a question in the markscheme, the markscheme will show

a truncated 6 sf value
the exact value if applicable, the correct 3 sf answer
Units will appear in brackets at the end.

Section A

1. (a) any correct equation in the form $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$ (accept any parameter for t)

where $\boldsymbol{a}$ is $\begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$, and $\boldsymbol{b}$ is a scalar multiple of $\begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix}$ ***A2*** ***N2***

eg $\boldsymbol{r} = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} + t\begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix}$, $\boldsymbol{r} = 2\boldsymbol{i} + \boldsymbol{j} + 3\boldsymbol{k} + s(\boldsymbol{i} + 3\boldsymbol{j} + \boldsymbol{k})$

> **Note:** Award ***A1*** for the form $\boldsymbol{a} + t\boldsymbol{b}$, ***A1*** for the form $L = \boldsymbol{a} + t\boldsymbol{b}$, ***A0*** for the form $\boldsymbol{r} = \boldsymbol{b} + t\boldsymbol{a}$.

[2 marks]

(b) **METHOD 1**

correct scalar product ***(A1)***
eg $(1\times 2) + (3\times p) + (1\times 0)$, $2 + 3p$

evidence of equating **their** scalar product to zero ***(M1)***
eg $\boldsymbol{a}\bullet\boldsymbol{b} = 0$, $2 + 3p = 0$, $3p = -2$

$p = -\frac{2}{3}$ ***A1*** ***N3***

METHOD 2

valid attempt to find angle between vectors ***(M1)***

correct substitution into numerator and/or angle ***(A1)***
eg $\cos\theta = \frac{(1\times 2) + (3\times p) + (1\times 0)}{|a||b|}$, $\cos\theta = 0$

$p = -\frac{2}{3}$ ***A1*** ***N3***

[3 marks]

[Total: 5 marks]

2. (a) $2x^3 - \frac{3x^2}{2} + c$ $\left(\text{accept } \frac{6x^3}{3} - \frac{3x^2}{2} + c\right)$ ***A1A1*** ***N2***

Notes:	Award ***A1A0*** for both correct terms if $+c$ is omitted. Award ***A1A0*** for one correct term *eg* $2x^3 + c$. Award ***A1A0*** if both terms are correct, but candidate attempts further working to solve for c.

[2 marks]

(b) substitution of limits or function ***(A1)***

eg $\int_1^2 f(x)\,dx$, $\left[2x^3 - \frac{3x^2}{2}\right]_1^2$

substituting limits into their integrated function and subtracting ***(M1)***

eg $\frac{6\times 2^3}{3} - \frac{3\times 2^2}{2} - \left(\frac{6\times 1^3}{3} - \frac{3\times 1^2}{2}\right)$

Note: Award ***M0*** if substituted into original function.

correct working ***(A1)***

eg $\frac{6\times 8}{3} - \frac{3\times 4}{2} - \frac{6\times 1}{3} + \frac{3\times 1}{2}$, $(16-6)-\left(2-\frac{3}{2}\right)$

$\frac{19}{2}$ ***A1*** ***N3***

[4 marks]

[Total: 6 marks]

3. (a) correct approach *(A1)*

eg $\frac{800}{n} = 20$

40 ***A1*** ***N2***

[2 marks]

(b) (i) 200 ***A1*** ***N1***

(ii) **METHOD 1**

recognizing variance $= \sigma^2$ *(M1)*

eg $3^2 = 9$

correct working to find new variance *(A1)*

eg $\sigma^2 \times 10^2$, 9×100

900 ***A1*** ***N3***

METHOD 2

new standard deviation is 30 *(A1)*

recognizing variance $= \sigma^2$ *(M1)*

eg $3^2 = 9$, 30^2

900 ***A1*** ***N3***

[4 marks]

[Total: 6 marks]

4. evidence of correctly substituting into circle formula (may be seen later) ***A1A1***

eg $\frac{1}{2}\theta r^2 = 12$, $r\theta = 6$

attempt to eliminate one variable *(M1)*

eg $r = \frac{6}{\theta}$, $\theta = \frac{l}{r}$, $\frac{\frac{1}{2}\theta r^2}{r\theta} = \frac{12}{6}$

correct elimination *(A1)*

eg $\frac{1}{2} \times \frac{6}{r} \times r^2 = 12$, $\frac{1}{2}\theta \times \left(\frac{6}{\theta}\right)^2 = 12$, $A = \frac{1}{2} \times r^2 \times \frac{l}{r}$, $\frac{r^2}{2r} = 2$

correct equation *(A1)*

eg $\frac{1}{2} \times 6r = 12$, $\frac{1}{2} \times \frac{36}{\theta} = 12$, $12 = \frac{1}{2} \times r^2 \times \frac{6}{r}$

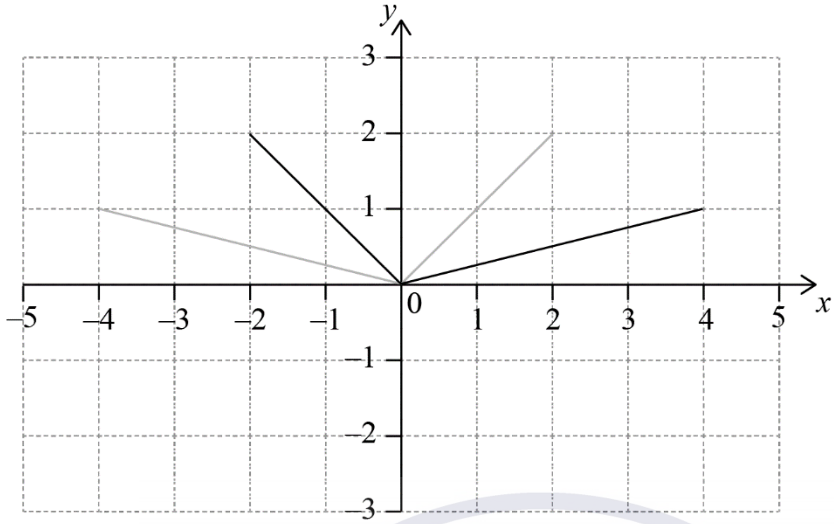
correct working *(A1)*

eg $3r = 12$, $\frac{18}{\theta} = 12$, $\frac{r}{2} = 2$, $24 = 6r$

$r = 4$ (cm) ***A1*** ***N2***

[7 marks]

5. (a)



A2 ***N2***
[2 marks]

(b) recognizing horizontal shift/translation of 1 unit ***(M1)***
eg $b = 1$, moved 1 right

recognizing vertical stretch/dilation with scale factor 2 ***(M1)***
eg $a = 2$, $y \times (-2)$

$a = -2$, $b = -1$ ***A1A1*** ***N2N2***

[4 marks]

[Total: 6 marks]

6. METHOD 1

evidence of discriminant **(M1)**
eg $b^2 - 4ac$, Δ

correct substitution into discriminant **(A1)**
eg $q^2 - 4p(-4p)$

correct discriminant **A1**
eg $q^2 + 16p^2$

$16p^2 > 0$ (accept $p^2 > 0$) **A1**
$q^2 \geq 0$ (do not accept $q^2 > 0$) **A1**

$q^2 + 16p^2 > 0$ **A1**

f has 2 roots **A1** **N0**

METHOD 2

y-intercept $= -4p$ (seen anywhere) **A1**

if p is positive, then the y-intercept will be negative **A1**

an upward-opening parabola with a negative y-intercept **R1**
eg sketch that must indicate $p > 0$.

if p is negative, then the y-intercept will be positive **A1**

a downward-opening parabola with a positive y-intercept **R1**
eg sketch that must indicate $p < 0$.

f has 2 roots **A2** **N0**

[7 marks]

7. (a) valid approach involving addition or subtraction *M1*

eg $u_2 = \log_c p + d$, $u_1 - u_2$

correct application of log law *A1*

eg $\log_c (pq) = \log_c p + \log_c q$, $\log_c \left(\dfrac{pq}{p}\right)$

$d = \log_c q$ *AG* *N0*

[2 marks]

(b) **METHOD 1** (finding u_1 and d)

recognizing $\sum = S_{20}$ (seen anywhere) *(A1)*

attempt to find u_1 or d using $\log_c c^k = k$ *(M1)*

eg $2\log_c c$, $3\log_c c$, correct value of u_1 or d

$u_1 = 2$, $d = 3$ (seen anywhere) *(A1)(A1)*

correct working *(A1)*

eg $S_{20} = \dfrac{20}{2}(2 \times 2 + 19 \times 3)$, $S_{20} = \dfrac{20}{2}(2 + 59)$, $10(61)$

$\sum_{n=1}^{20} u_n = 610$ *A1* *N2*

METHOD 2 (expressing S in terms of c)

recognizing $\sum = S_{20}$ (seen anywhere) *(A1)*

correct expression for S in terms of c *(A1)*

eg $10(2\log_c c^2 + 19\log_c c^3)$

$\log_c c^2 = 2$, $\log_c c^3 = 3$ (seen anywhere) *(A1)(A1)*

correct working *(A1)*

eg $S_{20} = \dfrac{20}{2}(2 \times 2 + 19 \times 3)$, $S_{20} = \dfrac{20}{2}(2 + 59)$, $10(61)$

$\sum_{n=1}^{20} u_n = 610$ *A1* *N2*

continued…

Question 7 continued

METHOD 3 (expressing S in terms of c)

recognizing $\sum = S_{20}$ (seen anywhere) ***(A1)***

correct expression for S in terms of c ***(A1)***

eg $10(2\log_c c^2 + 19\log_c c^3)$

correct application of log law ***(A1)***

eg $2\log_c c^2 = \log_c c^4$, $19\log_c c^3 = \log_c c^{57}$, $10\left(\log_c \left(c^2\right)^2 + \log_c \left(c^3\right)^{19}\right)$,

$10\left(\log_c c^4 + \log_c c^{57}\right)$, $10\left(\log_c c^{61}\right)$

correct application of definition of log ***(A1)***

eg $\log_c c^{61} = 61$, $\log_c c^4 = 4$, $\log_c c^{57} = 57$

correct working ***(A1)***

eg $S_{20} = \frac{20}{2}(4+57)$, $10(61)$

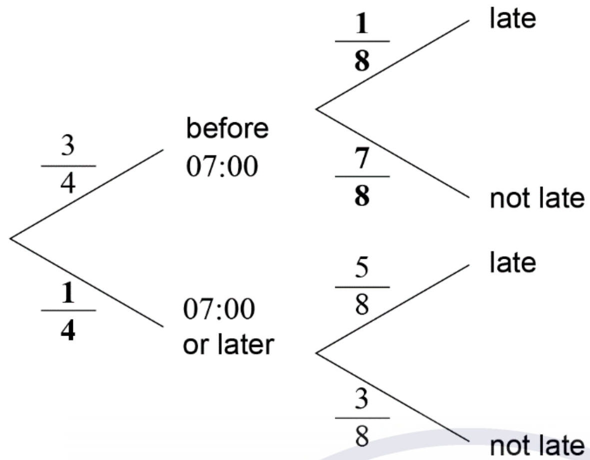
$\sum_{n=1}^{20} u_n = 610$ ***A1*** ***N2***

[6 marks]

[Total: 8 marks]

Section B

8. (a)



A1A1A1 ***N3***

Note: Award ***A1*** for each bold fraction.

[3 marks]

(b) multiplying along correct branches ***(A1)***

eg $\frac{3}{4}\times\frac{1}{8}$

$\mathrm{P}(\text{leaves before 07:00} \cap \text{late}) = \frac{3}{32}$ ***A1*** ***N2***

[2 marks]

(c) multiplying along other "late" branch ***(M1)***

eg $\frac{1}{4}\times\frac{5}{8}$

adding probabilities of two mutually exclusive late paths ***(A1)***

eg $\left(\frac{3}{4}\times\frac{1}{8}\right)+\left(\frac{1}{4}\times\frac{5}{8}\right)$, $\frac{3}{32}+\frac{5}{32}$

$\mathrm{P}(L) = \frac{8}{32}\ \left(=\frac{1}{4}\right)$ ***A1*** ***N2***

[3 marks]

continued…

Question 8 continued

(d) recognizing conditional probability (seen anywhere) ***(M1)***
eg $\mathrm{P}(A|\ B)$, $\mathrm{P}(\text{before } 7|\text{late})$

correct substitution of **their** values into formula ***(A1)***

eg $$\frac{\frac{3}{32}}{\frac{1}{4}}$$

$\mathrm{P}(\text{left before } 07{:}00|\text{late}) = \frac{3}{8}$ ***A1 N2***

[3 marks]

(e) valid approach ***(M1)***
eg $1-\mathrm{P}(\text{not late twice})$, $\mathrm{P}(\text{late once})+\mathrm{P}(\text{late twice})$

correct working ***(A1)***
eg $1-\left(\frac{3}{4}\times\frac{3}{4}\right)$, $2\times\frac{1}{4}\times\frac{3}{4}+\frac{1}{4}\times\frac{1}{4}$

$\frac{7}{16}$ ***A1 N2***

[3 marks]

[Total: 14 marks]

9. (a) correct equation for volume ***(A1)***

eg $\pi r^2 h = 20\pi$

$h = \dfrac{20}{r^2}$ ***A1*** ***N2***

[2 marks]

(b) attempt to find formula for cost of parts ***(M1)***

eg 10 × two circles, 8 × curved side

correct expression for cost of two circles in terms of r (seen anywhere) ***A1***

eg $2\pi r^2 \times 10$

correct expression for cost of curved side (seen anywhere) ***(A1)***

eg $2\pi r \times h \times 8$

correct expression for cost of curved side in terms of r ***A1***

eg $8 \times 2\pi r \times \dfrac{20}{r^2}$, $\dfrac{320\pi r}{r^2}$

$C = 20\pi r^2 + \dfrac{320\pi}{r}$ ***AG*** ***N0***

[4 marks]

(c) recognize $C' = 0$ at minimum ***(R1)***

eg $C' = 0$, $\dfrac{\mathrm{d}C}{\mathrm{d}r} = 0$

correct differentiation (may be seen in equation)

$C' = 40\pi r - \dfrac{320\pi}{r^2}$ ***A1A1***

correct equation ***A1***

eg $40\pi r - \dfrac{320\pi}{r^2} = 0$, $40\pi r = \dfrac{320\pi}{r^2}$

correct working ***(A1)***

eg $40r^3 = 320$, $r^3 = 8$

$r = 2$ (m) ***A1***

attempt to substitute **their** value of r into C

eg $20\pi \times 4 + 320 \times \dfrac{\pi}{2}$ ***(M1)***

correct working

eg $80\pi + 160\pi$ ***(A1)***

240π (cents) ***A1*** ***N3***

Note: Do not accept 753.6, 753.98 or 754, even if 240π is seen.

[9 marks]

[Total: 15 marks]

10. (a) (i) recognize that $f'(x)$ is the gradient of the tangent at x ***(M1)***

eg $f'(x) = m$

$f'(2) = 3$ (accept $m = 3$) ***A1 N2***

(ii) recognize that $f(2) = y(2)$ ***(M1)***

eg $f(2) = 3 \times 2 + 1$

$f(2) = 7$ ***A1 N2***

[4 marks]

(b) recognize that the gradient of the graph of g is $g'(x)$ ***(M1)***

choosing chain rule to find $g'(x)$ ***(M1)***

eg $\frac{dy}{du} \times \frac{du}{dx}$, $u = x^2 + 1$, $u' = 2x$

$g'(x) = f'(x^2 + 1) \times 2x$ ***A2***

$g'(1) = 3 \times 2$ ***A1***

$g'(1) = 6$ ***AG N0***

[5 marks]

(c) at Q, $L_1 = L_2$ (seen anywhere) ***(M1)***

recognize that the gradient of L_2 is $g'(1)$ (seen anywhere) ***(M1)***

eg $m = 6$

finding $g(1)$ (seen anywhere) ***(A1)***

eg $g(1) = f(2)$, $g(1) = 7$

attempt to substitute gradient and/or coordinates into equation of a straight line ***M1***

eg $y - g(1) = 6(x - 1)$, $y - 1 = g'(1)(x - 7)$, $7 = 6(1) + b$

correct equation for L_2

eg $y - 7 = 6(x - 1)$, $y = 6x + 1$ ***A1***

correct working to find Q ***(A1)***

eg same y-intercept, $3x = 0$

$y = 1$ ***A1 N2***

[7 marks]

[Total: 16 marks]

N17/5/MATME/SP1/ENG/TZ0/XX/M

Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

November 2017

Mathematics

Standard level

Paper 1

16 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a valid **Method**; working must be seen.

(M) Marks awarded for a valid **Method**; may be implied by **correct** subsequent working.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

(A) Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.

R Marks awarded for clear **Reasoning**.

N Marks awarded for **correct** answers if **no** working shown.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Mark according to RM assessor instructions.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any. An exception to this rule is when work for ***M1*** is missing, as opposed to incorrect (see point 4).
- Where ***M*** and ***A*** marks are noted on the same line, *eg* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*eg* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***(M2)***, ***N3***, *etc*., do **not** split the marks, unless there is a note.
- Most ***M*** marks are for a **valid** method, ie a method which can lead to the answer: it must indicate some form of progress towards the answer.
- Once a correct answer to a question or part-question is seen, ignore further correct working. However, if further working indicates a lack of mathematical understanding do not award final ***A1***.

3 *N* marks

*If **no** working shown, award **N** marks for **correct** answers – this includes acceptable answers (see accuracy booklet). In this case, ignore mark breakdown (**M, A, R**). Where a student only shows a final incorrect answer with no working, even if that answer is a correct intermediate answer, award **N0.***

- Do **not** award a mixture of ***N*** and other marks.
- There may be fewer ***N*** marks available than the total of ***M***, ***A*** and ***R*** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.
- There may not be a direct relationship between the ***N*** marks and the implied marks. There are times when all the marks are implied, but the ***N*** marks are not the full marks: this indicates that we want to see some of the working, without specifying what.

- For consistency within the markscheme, ***N*** marks are noted for every part, even when these match the mark breakdown.
- If a candidate has incorrect working, which somehow results in a correct answer, do **not** award the ***N*** marks for this correct answer. However, if the candidate has indicated (usually by crossing out) that the working is to be ignored, award the ***N*** marks for the correct answer.

4 Implied and must be seen marks

*Implied marks appear in **brackets** eg **(M1)**.*

- Implied marks can only be awarded if the work is seen or if implied in subsequent working (a correct final answer does not necessarily mean that the implied marks are all awarded). There are questions where some working is required, but as it is accepted that not everyone will write the same steps, all the marks are implied, but the ***N*** marks are not the full marks for the question.
- Normally the correct work is seen in the next line.
- Where there is an (***M1***) followed by ***A1*** for each correct answer, if no working shown, one correct answer is sufficient evidence to award the (***M1***).

*Must be seen marks appear without **brackets** eg **M1**.*

- Must be seen marks can only be awarded if the work is seen.
- If a must be seen mark is not awarded because work is missing (as opposed to ***M0*** or ***A0*** for incorrect work) all subsequent marks may be awarded if appropriate.

5 Follow through marks (only applied after an error is made)

*Follow through (**FT**) marks are awarded where an incorrect answer (final or intermediate) from one **part** of a question is used correctly in **subsequent** part(s) or subpart(s). Usually, to award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if the only marks awarded in a subpart are for the **final** answer, then **FT** marks should be awarded if appropriate. Examiners are expected to check student work in order to award **FT** marks where appropriate.*

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** and ***R*** marks may be awarded if appropriate. (However, as noted above, if an ***A*** mark is not awarded because work is missing, all subsequent marks may be awarded if appropriate).
- Exceptions to this rule will be explicitly noted on the markscheme.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*eg* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word "**their**" in a description, to indicate that candidates may be using an incorrect value.
- If a candidate makes an error in one part, but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the question says hence. It is often possible to use a different approach in subsequent parts that does not depend on the answer to previous parts.
- In a "show that" question, if an error in a previous subpart leads to not showing the required answer, do not award the final ***A1***. Note that if the error occurs within the same subpart, the ***FT*** rules may result in further loss of marks.

6 Mis-read

*If a candidate incorrectly copies information from the question, this is a mis-read (**MR**). A candidate should be penalized only once for a particular mis-read. Use the **MR** stamp to indicate that this is a misread. Do not award the first mark in the question, even if this is an **M** mark, but award all others (if appropriate) so that the candidate only loses one mark for the misread.*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*eg* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.

7 Discretionary marks *(d)*

*An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. In such cases the annotation DM should be used and a brief **note** written next to the mark explaining this decision.*

8 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete parts are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**. Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9 Alternative forms

*Unless the question specifies otherwise, **accept** equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

10 Calculators

No calculator is allowed. The use of any calculator on paper 1 is malpractice, and will result in no grade awarded. If you see work that suggests a candidate has used any calculator, please follow the procedures for malpractice. Examples: finding an angle, given a trig ratio of 0.4235.

11 Style

The markscheme aims to present answers using good communication, eg if the question asks to find the value of k, the markscheme will say $k = 3$, but the marks will be for the correct value 3 – there is usually no need for the "$k =$". In these cases, it is also usually acceptable to have another variable, as long as there is no ambiguity in the question, eg if the question asks to find the value of p and of q, then the student answer needs to be clear. Generally, the only situation

where the full answer is required is in a question which asks for equations – in this case the markscheme will say "must be an equation".

*The markscheme often uses words to describe what the marks are for, followed by examples, using the eg notation. These examples are not exhaustive, and examiners should check what candidates have written, to see if they satisfy the description. Where these marks are **M** marks, the examples may include ones using poor notation, to indicate what is acceptable. A valid method is one which will allow candidate to proceed to the next step eg if a quadratic function is given in factorised form, and the question asks for the zeroes, then multiplying the factors does not necessarily help to find the zeros, and would not on its own count as a valid method.*

12 Candidate work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

Candidates are meant to write their answers to Section A on the question paper (QP), and Section B on answer booklets. Sometimes, they need more room for Section A, and use the booklet (and often comment to this effect on the QP), or write outside the box. That is fine, and this work should be marked.

The instructions tell candidates not to write on Section B of the QP. Thus they may well have done some rough work here which they assume will be ignored. If they have solutions on the answer booklets, there is no need to look at the QP. However, if there are whole questions or whole part solutions missing on answer booklets, please check to make sure that they are not on the QP, and if they are, mark those whole questions or whole part solutions that have not been written on answer booklets.

13. Diagrams

The notes on how to allocate marks for sketches usually refer to passing through particular points or having certain features. These marks can only be awarded if the sketch is approximately the correct shape. All values given will be an approximate guide to where these points/features occur. In some questions, the first ***A1*** is for the shape, in others, the marks are only for the points and/or features. In both cases, unless the shape is approximately correct, no marks can be awarded (unless otherwise stated). However, if the graph is based on previous calculations, ***FT*** marks should be awarded if appropriate.

14. Accuracy of Answers

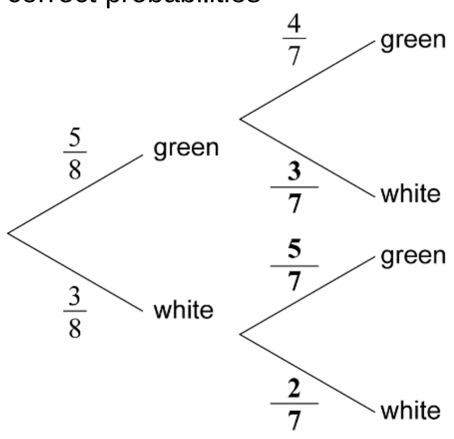
If the level of accuracy is specified in the question, a mark will be allocated for giving the final answer to the required accuracy. When this is not specified in the question, all numerical answers should be given exactly or correct to three significant figures.

Do not accept unfinished numerical final answers such as 3/0.1 (unless otherwise stated). As a rule, numerical answers with more than one part (such as fractions) should be given using integers (eg 6/8). Calculations which lead to integers should be completed, with the exception of fractions which are not whole numbers. Intermediate values do not need to be given to the correct three significant figures. But, if candidates work with rounded values, this could lead to an incorrect answer, in which case award A0 for the final answer. Where numerical answers are required as the final answer to a part of a question in the markscheme, the markscheme will show

a truncated 6 sf value
the exact value if applicable, the correct 3 sf answer
Units will appear in brackets at the end.

Section A

1. (a) correct probabilities



A1A1A1 ***N3***

Note: Award ***A1*** for each correct **bold** answer.

[3 marks]

(b) multiplying along branches ***(M1)***

eg $\frac{5}{8}\times\frac{3}{7},\ \frac{3}{8}\times\frac{5}{7},\ \frac{15}{56}$

adding probabilities of correct mutually exclusive paths ***(A1)***

eg $\frac{5}{8}\times\frac{3}{7}+\frac{3}{8}\times\frac{5}{7},\ \frac{15}{56}+\frac{15}{56}$

$\frac{30}{56}\left(=\frac{15}{28}\right)$ ***A1*** ***N2***

[3 marks]

Total [6 marks]

2. (a) subtracting terms ***(M1)***

eg $5-8,\ u_2-u_1$

$d=-3$ ***A1*** ***N2***

[2 marks]

(b) correct substitution into formula ***(A1)***

eg $u_{10}=8+(10-1)(-3),\ 8-27,\ -3(10)+11$

$u_{10}=-19$ ***A1*** ***N2***

[2 marks]

(c) correct substitution into formula for sum ***(A1)***

eg $S_{10}=\frac{10}{2}(8-19),\ 5\big(2(8)+(10-1)(-3)\big)$

$S_{10}=-55$ ***A1*** ***N2***

[2 marks]

Total [6 marks]

3. (a) correct range (do not accept $0 \le x \le 7$) ***A1*** ***N1***
eg $[0,\ 7],\ 0 \le y \le 7$

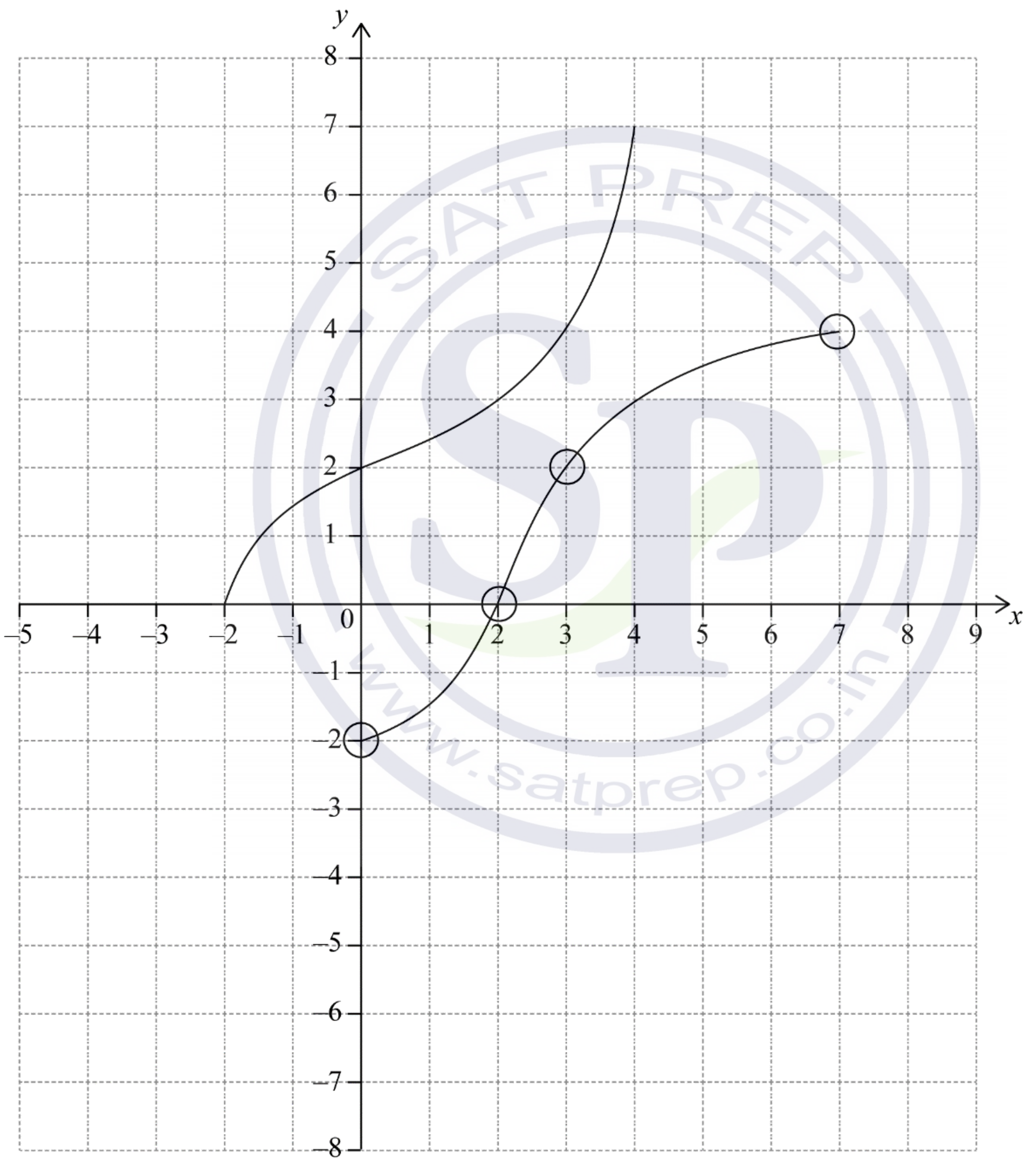
[1 mark]

(b) (i) $f(2) = 3$ ***A1*** ***N1***

(ii) $f^{-1}(2) = 0$ ***A1*** ***N1***

[2 marks]

(c)



A1A1A1 ***N3***

Notes: Award ***A1*** for both end points within circles,
A1 for images of $(2, 3)$ and $(0, 2)$ within circles,
A1 for approximately correct reflection in $y = x$, concave up then concave down shape (do not accept line segments).

[3 marks]

Total [6 marks]

4. (a) evidence of choosing the cosine rule ***(M1)***

eg $c^2 = a^2 + b^2 - 2ab\cos C$

correct substitution into RHS of cosine rule ***(A1)***

eg $3^2 + 8^2 - 2 \times 3 \times 8 \times \cos\frac{\pi}{3}$

evidence of correct value for $\cos\frac{\pi}{3}$ (may be seen anywhere, including in cosine rule) ***A1***

eg $\cos\frac{\pi}{3} = \frac{1}{2}$, $\text{AC}^2 = 9 + 64 - \left(48 \times \frac{1}{2}\right)$, $9 + 64 - 24$

correct working clearly leading to answer ***A1***

eg $\text{AC}^2 = 49$, $b = \sqrt{49}$

$\text{AC} = 7$ (cm) ***AG*** ***N0***

> **Note:** Award no marks if the only working seen is $\text{AC}^2 = 49$ or $\text{AC} = \sqrt{49}$ (or similar).

[4 marks]

(b) correct substitution for semicircle ***(A1)***

eg semicircle $= \frac{1}{2}(2\pi \times 3.5)$, $\frac{1}{2} \times \pi \times 7$, 3.5π

valid approach (seen anywhere) ***(M1)***

eg perimeter $= \text{AB} + \text{BC} +$ semicircle, $3 + 8 + \left(\frac{1}{2} \times 2 \times \pi \times \frac{7}{2}\right)$, $8 + 3 + 3.5\pi$

$11 + \frac{7}{2}\pi \quad (= 3.5\pi + 11)$ (cm) ***A1*** ***N2***

[3 marks]

Total [7 marks]

5. (a) attempt to form composite ***(M1)***

eg $g(1+e^{-x})$

correct function ***A1*** ***N2***

eg $(g \circ f)(x) = 2+b+2e^{-x}$, $2(1+e^{-x})+b$

[2 marks]

(b) evidence of $\lim_{x\to\infty}(2+b+2e^{-x}) = 2+b+\lim_{x\to\infty}(2e^{-x})$ ***(M1)***

eg $2+b+2e^{-\infty}$, graph with horizontal asymptote when $x \to \infty$

Note: Award ***M0*** if candidate clearly has incorrect limit, such as $x \to 0$, e^{∞}, $2e^{0}$.

evidence that $e^{-x} \to 0$ (seen anywhere) ***(A1)***

eg $\lim_{x\to\infty}(e^{-x}) = 0$, $1+e^{-x} \to 1$, $2(1)+b=-3$, $e^{\text{large negative number}} \to 0$, graph of $y=e^{-x}$ or $y=2e^{-x}$ with asymptote $y=0$, graph of composite function with asymptote $y=-3$

correct working ***(A1)***

eg $2+b=-3$

$b=-5$ ***A1*** ***N2***

[4 marks]

Total [6 marks]

6. attempt to find the area of OABC ***(M1)***

eg $\mathrm{OA}\times\mathrm{OC}$, $x \times f(x)$, $f(x)\times(-x)$

correct expression for area in one variable ***(A1)***

eg area $= x(15-x^2)$, $15x-x^3$, x^3-15x

valid approach to find maximum **area** (seen anywhere) ***(M1)***

eg $A'(x)=0$

correct derivative ***A1***

eg $15-3x^2$, $(15-x^2)+x(-2x)=0$, $-15+3x^2$

correct working ***(A1)***

eg $15=3x^2$, $x^2=5$, $x=\sqrt{5}$

$x=-\sqrt{5}$ $\left(\text{accept } A\left(-\sqrt{5},0\right)\right)$ ***A2*** ***N3***

[7 marks]

7. METHOD 1 – using discriminant

correct equation without logs *(A1)*

eg $6x-3x^2=k^2$

valid approach *(M1)*

eg $-3x^2+6x-k^2=0$, $3x^2-6x+k^2=0$

recognizing discriminant must be zero (seen anywhere) *M1*

eg $\Delta=0$

correct discriminant *(A1)*

eg $6^2-4(-3)(-k^2)$, $36-12k^2=0$

correct working *(A1)*

eg $12k^2=36$, $k^2=3$

$k=\sqrt{3}$ *A2 N2*

METHOD 2 – completing the square

correct equation without logs *(A1)*

eg $6x-3x^2=k^2$

valid approach to complete the square *(M1)*

eg $3\left(x^2-2x+1\right)=-k^2+3$, $x^2-2x+1-1+\frac{k^2}{3}=0$

correct working *(A1)*

eg $3(x-1)^2=-k^2+3$, $(x-1)^2-1+\frac{k^2}{3}=0$

recognizing conditions for one solution *M1*

eg $(x-1)^2=0$, $-1+\frac{k^2}{3}=0$

correct working *(A1)*

eg $\frac{k^2}{3}=1$, $k^2=3$

$k=\sqrt{3}$ *A2 N2*

[7 marks]

Section B

8. (a) $f'(x) = 2x - 1$ ***A1A1***

correct substitution ***A1***
eg $2(1) - 1,\ 2 - 1$

$f'(1) = 1$ ***AG*** ***N0***
[3 marks]

(b) correct approach to find the gradient of the normal ***(A1)***
eg $\dfrac{-1}{f'(1)},\ m_1 m_2 = -1,$ slope $= -1$

attempt to substitute correct normal gradient and coordinates into equation of a line ***(M1)***
eg $y - 0 = -1(x - 1),\ 0 = -1 + b,\ b = 1,\ L = -x + 1$

$y = -x + 1$ ***A1*** ***N2***
[3 marks]

(c) equating expressions ***(M1)***
eg $f(x) = L,\ -x + 1 = x^2 - x$

correct working (must involve combining terms) ***(A1)***
eg $x^2 - 1 = 0,\ x^2 = 1,\ x = 1$

$x = -1$ (accept $Q(-1, 2)$) ***A2*** ***N3***
[4 marks]

(d) valid approach ***(M1)***
eg $\int L - f,\ \int_{-1}^{1} (1 - x^2)\,\mathrm{d}x$, splitting area into triangles and integrals

correct integration ***(A1)(A1)***
eg $\left[x - \dfrac{x^3}{3}\right]_{-1}^{1},\ -\dfrac{x^3}{3} - \dfrac{x^2}{2} + \dfrac{x^2}{2} + x$

substituting **their** limits into **their** integrated function and subtracting (in any order) ***(M1)***
eg $1 - \dfrac{1}{3} - \left(-1 - \dfrac{-1}{3}\right)$

Note: Award ***M0*** for substituting into original or differentiated function.

area $= \dfrac{4}{3}$ ***A2*** ***N3***
[6 marks]

Total [16 marks]

9. (a) (i) correct approach **A1**

eg $\begin{pmatrix}-1\\3\\3\end{pmatrix}-\begin{pmatrix}-3\\4\\2\end{pmatrix}$, $\begin{pmatrix}3\\-4\\-2\end{pmatrix}+\begin{pmatrix}-1\\3\\3\end{pmatrix}$

$\vec{AB}=\begin{pmatrix}2\\-1\\1\end{pmatrix}$ **AG** **N0**

(ii) any correct equation in the form $r=a+tb$ (any parameter for t)

where a is $\begin{pmatrix}-3\\4\\2\end{pmatrix}$ or $\begin{pmatrix}-1\\3\\3\end{pmatrix}$ and b is a scalar multiple of $\begin{pmatrix}2\\-1\\1\end{pmatrix}$ **A2** **N2**

eg $r=\begin{pmatrix}-3\\4\\2\end{pmatrix}+t\begin{pmatrix}2\\-1\\1\end{pmatrix}$, $(x,\ y,\ z)=(-1,\ 3,\ 3)+s(-2,\ 1,\ -1)$, $r=\begin{pmatrix}-3+2t\\4-t\\2+t\end{pmatrix}$

> **Note:** Award ***A1*** for the form $a+tb$, ***A1*** for the form $L=a+tb$, ***A0*** for the form $r=b+ta$.

[3 marks]

(b) **METHOD 1 – finding value of parameter**

valid approach ***(M1)***

eg $\begin{pmatrix}-3\\4\\2\end{pmatrix}+t\begin{pmatrix}2\\-1\\1\end{pmatrix}=\begin{pmatrix}3\\1\\p\end{pmatrix}$, $(-1,\ 3,\ 3)+s(-2,\ 1,\ -1)=(3,\ 1,\ p)$

one correct equation (not involving p) ***(A1)***

eg $-3+2t=3$, $-1-2s=3$, $4-t=1$, $3+s=1$

correct parameter from their equation (may be seen in substitution) ***A1***

eg $t=3$, $s=-2$

correct substitution ***(A1)***

eg $\begin{pmatrix}-3\\4\\2\end{pmatrix}+3\begin{pmatrix}2\\-1\\1\end{pmatrix}=\begin{pmatrix}3\\1\\p\end{pmatrix}$, $3-(-2)$

$p=5$ $\left(\text{accept }\begin{pmatrix}3\\1\\5\end{pmatrix}\right)$ ***A1*** ***N2***

continued…

Question 9 continued

METHOD 2 – eliminating parameter

valid approach **(M1)**

eg $\begin{pmatrix}-3\\4\\2\end{pmatrix}+t\begin{pmatrix}2\\-1\\1\end{pmatrix}=\begin{pmatrix}3\\1\\p\end{pmatrix}$, $(-1,3,3)+s(-2,1,-1)=(3,1,p)$

one correct equation (not involving p) **(A1)**
eg $-3+2t=3$, $-1-2s=3$, $4-t=1$, $3+s=1$

correct equation (with p) **A1**
eg $2+t=p$, $3-s=p$

correct working to solve for p **(A1)**
eg $7=2p-3$, $6=1+p$

$p=5$ $\left(\text{accept }\begin{pmatrix}3\\1\\5\end{pmatrix}\right)$ **A1** **N2**

[5 marks]

(c) valid approach to find $\overrightarrow{DC}$ or $\overrightarrow{CD}$ **(M1)**

eg $\begin{pmatrix}3\\1\\5\end{pmatrix}-\begin{pmatrix}q^2\\0\\q\end{pmatrix}$, $\begin{pmatrix}q^2\\0\\q\end{pmatrix}-\begin{pmatrix}3\\1\\5\end{pmatrix}$, $\begin{pmatrix}q^2\\0\\q\end{pmatrix}-\begin{pmatrix}3\\1\\p\end{pmatrix}$

correct vector for $\overrightarrow{DC}$ or $\overrightarrow{CD}$ (may be seen in scalar product) **A1**

eg $\begin{pmatrix}3-q^2\\1\\5-q\end{pmatrix}$, $\begin{pmatrix}q^2-3\\-1\\q-5\end{pmatrix}$, $\begin{pmatrix}3-q^2\\1\\p-q\end{pmatrix}$

recognizing scalar product of $\overrightarrow{DC}$ or $\overrightarrow{CD}$ with direction vector of L is zero (seen anywhere) **(M1)**

eg $\begin{pmatrix}3-q^2\\1\\p-q\end{pmatrix}\bullet\begin{pmatrix}2\\-1\\1\end{pmatrix}=0$, $\overrightarrow{DC}\bullet\overrightarrow{AC}=0$, $\begin{pmatrix}3-q^2\\1\\5-q\end{pmatrix}\bullet\begin{pmatrix}2\\-1\\1\end{pmatrix}=0$

correct scalar product in terms of only q **A1**
eg $6-2q^2-1+5-q$, $2q^2+q-10=0$, $2\left(3-q^2\right)-1+5-q$

correct working to solve quadratic **(A1)**

eg $(2q+5)(q-2)$, $\dfrac{-1\pm\sqrt{1-4(2)(-10)}}{2(2)}$

$q=-\dfrac{5}{2}, 2$ **A1A1** **N3**

[7 marks]
Total [15 marks]

10. (a) infinite sum of segments is 2 (seen anywhere) ***(A1)***

eg $p+p^2+p^3+\ldots=2$, $\frac{u_1}{1-r}=2$

recognizing GP ***(M1)***

eg ratio is p, $\frac{u_1}{1-r}$, $u_n=u_1\times r^{n-1}$, $\frac{u_1\left(r^n-1\right)}{r-1}$

correct substitution into S_∞ formula (may be seen in equation) ***A1***

eg $\frac{p}{1-p}$

correct equation ***(A1)***

eg $\frac{p}{1-p}=2$, $p=2-2p$

correct working leading to answer ***A1***

eg $3p=2$, $2-3p=0$

$p=\frac{2}{3}$ (cm) ***AG*** ***N0***

[5 marks]

continued…

Question 10 continued

(b) recognizing infinite geometric series with squares ***(M1)***

eg $k^2 + k^4 + k^6 + \ldots,\ \dfrac{k^2}{1-k^2}$

correct substitution into $S_\infty = \dfrac{9}{16}$ (must substitute into formula) ***(A2)***

eg $\dfrac{k^2}{1-k^2} = \dfrac{9}{16}$

correct working ***(A1)***

eg $16k^2 = 9 - 9k^2,\ 25k^2 = 9,\ k^2 = \dfrac{9}{25}$

$k = \dfrac{3}{5}$ (seen anywhere) ***A1***

valid approach with segments and CD (may be seen earlier) ***(M1)***

eg $r = k,\ S_\infty = b$

correct expression for b in terms of k (may be seen earlier) ***(A1)***

eg $b = \dfrac{k}{1-k},\ b = \sum_{n=1}^{\infty} k^n,\ b = k + k^2 + k^3 + \ldots$

substituting **their** value of k into **their** formula for b ***(M1)***

eg $\dfrac{\frac{3}{5}}{1-\frac{3}{5}},\ \dfrac{\left(\frac{3}{5}\right)}{\left(\frac{2}{5}\right)}$

$b = \dfrac{3}{2}$ ***A1 N3***

[9 marks]

Total [14 marks]

Markscheme

May 2017

Mathematics

Standard level

Paper 1

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a valid **Method**; working must be seen.

(M) Marks awarded for a valid **Method**; may be implied by **correct** subsequent working.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

(A) Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.

R Marks awarded for clear **Reasoning**.

N Marks awarded for **correct** answers if **no** working shown.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Mark according to RM assessor instructions.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any. An exception to this rule is when work for ***M1*** is missing, as opposed to incorrect (see point 4).
- Where ***M*** and ***A*** marks are noted on the same line, *eg* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*eg* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***(M2)***, ***N3***, *etc*., do **not** split the marks, unless there is a note.
- Most ***M*** marks are for a **valid** method, ie a method which can lead to the answer: it must indicate some form of progress towards the answer.
- Once a correct answer to a question or part-question is seen, ignore further correct working. However, if further working indicates a lack of mathematical understanding do not award final ***A1***.

3 *N* marks

*If **no** working shown, award **N** marks for **correct** answers – this includes acceptable answers (see accuracy booklet). In this case, ignore mark breakdown (**M**, **A**, **R**). Where a student only shows a final incorrect answer with no working, even if that answer is a correct intermediate answer, award **N0.***

- Do **not** award a mixture of ***N*** and other marks.
- There may be fewer ***N*** marks available than the total of ***M***, ***A*** and ***R*** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.
- There may not be a direct relationship between the ***N*** marks and the implied marks. There are times when all the marks are implied, but the ***N*** marks are not the full marks: this indicates that we want to see some of the working, without specifying what.

- For consistency within the markscheme, ***N*** marks are noted for every part, even when these match the mark breakdown.
- If a candidate has incorrect working, which somehow results in a correct answer, do **not** award the ***N*** marks for this correct answer. However, if the candidate has indicated (usually by crossing out) that the working is to be ignored, award the ***N*** marks for the correct answer.

4 Implied and must be seen marks

*Implied marks appear in **brackets** eg **(M1)**.*

- Implied marks can only be awarded if the work is seen or if implied in subsequent working (a correct final answer does not necessarily mean that the implied marks are all awarded). There are questions where some working is required, but as it is accepted that not everyone will write the same steps, all the marks are implied, but the ***N*** marks are not the full marks for the question.
- Normally the correct work is seen in the next line.
- Where there is an (***M1***) followed by ***A1*** for each correct answer, if no working shown, one correct answer is sufficient evidence to award the (***M1***).

*Must be seen marks appear without **brackets** eg **M1**.*

- Must be seen marks can only be awarded if the work is seen.
- If a must be seen mark is not awarded because work is missing (as opposed to ***M0*** or ***A0*** for incorrect work) all subsequent marks may be awarded if appropriate.

5 Follow through marks (only applied after an error is made)

*Follow through (**FT**) marks are awarded where an incorrect answer (final or intermediate) from one **part** of a question is used correctly in **subsequent** part(s) or subpart(s). Usually, to award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if the only marks awarded in a subpart are for the **final** answer, then **FT** marks should be awarded if appropriate. Examiners are expected to check student work in order to award **FT** marks where appropriate.*

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** and ***R*** marks may be awarded if appropriate. (However, as noted above, if an ***A*** mark is not awarded because work is missing, all subsequent marks may be awarded if appropriate).
- Exceptions to this rule will be explicitly noted on the markscheme.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*eg* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word "**their**" in a description, to indicate that candidates may be using an incorrect value.
- If a candidate makes an error in one part, but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the question says hence. It is often possible to use a different approach in subsequent parts that does not depend on the answer to previous parts.
- In a "show that" question, if an error in a previous subpart leads to not showing the required answer, do not award the final ***A1***. Note that if the error occurs within the same subpart, the ***FT*** rules may result in further loss of marks.

6 Mis-read

*If a candidate incorrectly copies information from the question, this is a mis-read (**MR**). A candidate should be penalized only once for a particular mis-read. Use the **MR** stamp to indicate that this is a misread. Do not award the first mark in the question, even if this is an **M** mark, but award all others (if appropriate) so that the candidate only loses one mark for the misread.*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*eg* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.

7 Discretionary marks *(d)*

*An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. In such cases the annotation DM should be used and a brief **note** written next to the mark explaining this decision.*

8 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete parts are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**. Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9 Alternative forms

*Unless the question specifies otherwise, **accept** equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

10 Calculators

No calculator is allowed. The use of any calculator on paper 1 is malpractice, and will result in no grade awarded. If you see work that suggests a candidate has used any calculator, please follow the procedures for malpractice. Examples: finding an angle, given a trig ratio of 0.4235.

11 Style

The markscheme aims to present answers using good communication, eg if the question asks to find the value of k, the markscheme will say $k = 3$, but the marks will be for the correct value 3 – there is usually no need for the "$k =$". In these cases, it is also usually acceptable to have another variable, as long as there is no ambiguity in the question, eg if the question asks to find the value of p and of q, then the student answer needs to be clear. Generally, the only situation

where the full answer is required is in a question which asks for equations – in this case the markscheme will say "must be an equation".

*The markscheme often uses words to describe what the marks are for, followed by examples, using the eg notation. These examples are not exhaustive, and examiners should check what candidates have written, to see if they satisfy the description. Where these marks are **M** marks, the examples may include ones using poor notation, to indicate what is acceptable. A valid method is one which will allow candidate to proceed to the next step eg if a quadratic function is given in factorised form, and the question asks for the zeroes, then multiplying the factors does not necessarily help to find the zeros, and would not on its own count as a valid method.*

12 Candidate work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

Candidates are meant to write their answers to Section A on the question paper (QP), and Section B on answer booklets. Sometimes, they need more room for Section A, and use the booklet (and often comment to this effect on the QP), or write outside the box. That is fine, and this work should be marked.

The instructions tell candidates not to write on Section B of the QP. Thus they may well have done some rough work here which they assume will be ignored. If they have solutions on the answer booklets, there is no need to look at the QP. However, if there are whole questions or whole part solutions missing on answer booklets, please check to make sure that they are not on the QP, and if they are, mark those whole questions or whole part solutions that have not been written on answer booklets.

13. Diagrams

The notes on how to allocate marks for sketches usually refer to passing through particular points or having certain features. These marks can only be awarded if the sketch is approximately the correct shape. All values given will be an approximate guide to where these points/features occur. In some questions, the first ***A1*** is for the shape, in others, the marks are only for the points and/or features. In both cases, unless the shape is approximately correct, no marks can be awarded (unless otherwise stated). However, if the graph is based on previous calculations, ***FT*** marks should be awarded if appropriate.

14. Accuracy of Answers

If the level of accuracy is specified in the question, a mark will be allocated for giving the final answer to the required accuracy. When this is not specified in the question, all numerical answers should be given exactly or correct to three significant figures.

Do not accept unfinished numerical final answers such as 3/0.1 (unless otherwise stated). As a rule, numerical answers with more than one part (such as fractions) should be given using integers (eg 6/8). Calculations which lead to integers should be completed, with the exception of fractions which are not whole numbers. Intermediate values do not need to be given to the correct three significant figures. But, if candidates work with rounded values, this could lead to an incorrect answer, in which case award A0 for the final answer. Where numerical answers are required as the final answer to a part of a question in the markscheme, the markscheme will show

a truncated 6 sf value
the exact value if applicable, the correct 3 sf answer
Units will appear in brackets at the end.

Section A

1. (a) (i) valid approach *(M1)*

eg $p+3=13$, $13-3$

$p=10$ *A1* *N2*

(ii) valid approach *(M1)*

eg $p+3+5+q=20$, $20-10-8$

$q=2$ *A1* *N2*

[4 marks]

(b) valid approach *(M1)*

eg $20-p-q-3$, $1-\frac{15}{20}$, $n(E\cap H')=5$

$\frac{5}{20}$ $\left(\frac{1}{4}\right)$ *A1* *N2*

[2 marks]

Total [6 marks]

2. (a) interchanging x and y *(M1)*

eg $x=5y$

$f^{-1}(x)=\frac{x}{5}$ *A1* *N2*

[2 marks]

(b) **METHOD 1**

attempt to substitute 7 into $g(x)$ or $f(x)$ *(M1)*

eg 7^2+1, 5×7

$g(7)=50$ *(A1)*

$f(50)=250$ *A1* *N2*

METHOD 2

attempt to form composite function (in any order) *(M1)*

eg $5(x^2+1)$, $(5x)^2+1$

correct substitution *(A1)*

eg $5\times(7^2+1)$

$(f\circ g)(7)=250$ *A1* *N2*

[3 marks]

Total [5 marks]

3. METHOD 1

evidence of choosing the sine rule *(M1)*

eg $\frac{a}{\sin A} = \frac{b}{\sin B}$

correct substitution *A1*

eg $\frac{x}{\sin 30} = \frac{13}{\sin 45}$, $\frac{13 \sin 30}{\sin 45}$

$\sin 30 = \frac{1}{2}$, $\sin 45 = \frac{1}{\sqrt{2}}$ *(A1)(A1)*

correct working *A1*

eg $\frac{1}{2} \times \frac{13}{\frac{1}{\sqrt{2}}}$, $\frac{1}{2} \times 13 \times \frac{2}{\sqrt{2}}$, $13 \times \frac{1}{2} \times \sqrt{2}$

correct answer *A1* *N3*

eg $\text{PR} = \frac{13\sqrt{2}}{2}$, $\frac{13}{\sqrt{2}}$ (cm)

METHOD 2 (using height of ΔPQR)

valid approach to find height of ΔPQR *(M1)*

eg $\sin 30 = \frac{x}{13}$, $\cos 60 = \frac{x}{13}$

$\sin 30 = \frac{1}{2}$ or $\cos 60 = \frac{1}{2}$ *(A1)*

height $= 6.5$ *A1*

correct working *A1*

eg $\sin 45 = \frac{6.5}{\text{PR}}$, $\sqrt{6.5^2 + 6.5^2}$

correct working *(A1)*

eg $\sin 45 = \frac{1}{\sqrt{2}}$, $\cos 45 = \frac{1}{\sqrt{2}}$, $\sqrt{\frac{169 \times 2}{4}}$

correct answer *A1* *N3*

eg $\text{PR} = \frac{13\sqrt{2}}{2}$, $\frac{13}{\sqrt{2}}$ (cm)

[6 marks]

4. (a) (i) t ***A1*** ***N1***

(ii) 105 ***A1*** ***N1***

[2 marks]

(b) -0.992 ***A2*** ***N2***

[2 marks]

(c) valid approach ***(M1)***

eg $\frac{\mathrm{d}d}{\mathrm{d}t} = -2.24$; 2×2.24, 2×-2.24, $d(2) = -2 \times 2.24 + 105$,

finding $d(t_2) - d(t_1)$ where $t_2 = t_1 + 2$

4.48 (degrees) ***A1*** ***N2***

Notes:	Award no marks for answers that **directly** use the table to find the decrease in temperature for 2 minutes *eg* $\frac{105 - 98.4}{2} = 3.3$.

[2 marks]

Total [6 marks]

5. (a) valid approach to set up integration by substitution/inspection ***(M1)***

eg $u = x^2 - 1$, $\mathrm{d}u = 2x$, $\int 2x\mathrm{e}^{x^2-1}\mathrm{d}x$

correct expression ***(A1)***

eg $\frac{1}{2}\int 2x\mathrm{e}^{x^2-1}\mathrm{d}x$, $\frac{1}{2}\int \mathrm{e}^u \mathrm{d}u$

$\frac{1}{2}\mathrm{e}^{x^2-1} + c$ ***A2*** ***N4***

Notes:	Award ***A1*** if missing "$+c$".

[4 marks]

(b) substituting $x = -1$ into **their** answer from (a) ***(M1)***

eg $\frac{1}{2}\mathrm{e}^0$, $\frac{1}{2}\mathrm{e}^{1-1} = 3$

correct working ***(A1)***

eg $\frac{1}{2} + c = 3$, $c = 2.5$

$f(x) = \frac{1}{2}\mathrm{e}^{x^2-1} + 2.5$ ***A1*** ***N2***

[3 marks]

Total [7 marks]

6. (a) (i) -2 ***A1*** ***N1***

(ii) gradient of normal = $\frac{1}{2}$ ***(A1)***

attempt to substitute their normal gradient and coordinates of P (in any order) ***(M1)***

eg $y-4=\frac{1}{2}(x-3)$, $3=\frac{1}{2}(4)+b$, $b=1$

$y-3=\frac{1}{2}(x-4)$, $y=\frac{1}{2}x+1$, $x-2y+2=0$ ***A1*** ***N3***

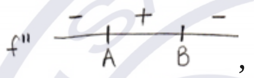
[4 marks]

(b) correct answer **and** valid reasoning ***A2*** ***N2***

answer: *eg* graph of f is concave up, concavity is positive (between $4<x<5$)

reason: *eg* slope of f' is positive, f' is increasing, $f''>0$,

sign chart (must clearly be for f'' and show A and B)



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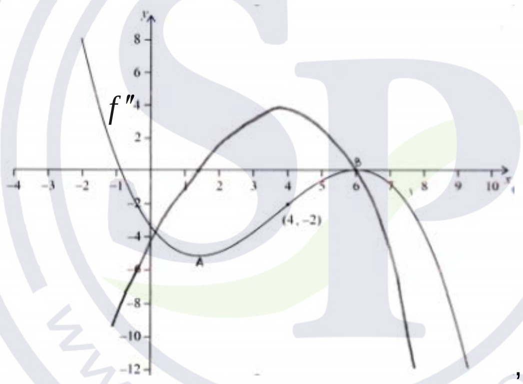

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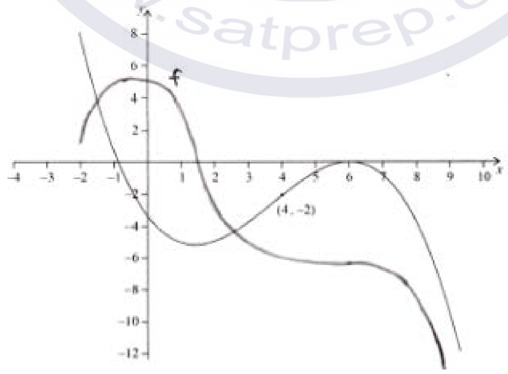


Note:	The reason given must refer to a specific function/graph. Referring to "the graph" or "it" is not sufficient.

[2 marks]
Total [6 marks]

7. (a) correct use $\log x^n = n \log x$ ***A1***

eg $16 \ln x$

valid approach to find r ***(M1)***

eg $\frac{u_{n+1}}{u_n}$, $\frac{\ln x^8}{\ln x^{16}}$, $\frac{4 \ln x}{8 \ln x}$, $\ln x^4 = \ln x^{16} \times r^2$

$r = \frac{1}{2}$ ***A1*** ***N2***

[3 marks]

(b) recognizing a sum (finite or infinite) ***(M1)***

eg $2^4 \ln x + 2^3 \ln x$, $\frac{a}{1-r}$, S_∞, $16 \ln x + \ldots$

valid approach (seen anywhere) ***(M1)***

eg recognizing GP is the same as part (a), using **their** r value from part (a), $r = \frac{1}{2}$

correct substitution into infinite sum (only if $|r|$ is a constant and less than 1) ***A1***

eg $\frac{2^4 \ln x}{1-\frac{1}{2}}$, $\frac{\ln x^{16}}{\frac{1}{2}}$, $32 \ln x$

correct working ***(A1)***

eg $\ln x = 2$

$x = e^2$ ***A1*** ***N3***

[5 marks]

Total [8 marks]

Section B

8. (a) (i) valid approach *(M1)*

eg $\text{A}-\text{B},\ -\begin{pmatrix}0\\1\\8\end{pmatrix}+\begin{pmatrix}3\\5\\2\end{pmatrix}$

$\overrightarrow{\text{AB}}=\begin{pmatrix}3\\4\\-6\end{pmatrix}$ ***A1*** ***N2***

(ii) **any** correct equation in the form $\boldsymbol{r}=\boldsymbol{a}+t\boldsymbol{b}$ (any parameter for t) ***A2*** ***N2***

where $\boldsymbol{a}$ is $\begin{pmatrix}0\\1\\8\end{pmatrix}$ or $\begin{pmatrix}3\\5\\2\end{pmatrix}$, and $\boldsymbol{b}$ is a scalar multiple of $\begin{pmatrix}3\\4\\-6\end{pmatrix}$

eg $\boldsymbol{r}=\begin{pmatrix}0\\1\\8\end{pmatrix}+t\begin{pmatrix}3\\4\\-6\end{pmatrix},\ \boldsymbol{r}=\begin{pmatrix}3+3t\\5+4t\\2-6t\end{pmatrix},\ \boldsymbol{r}=\boldsymbol{j}+8\boldsymbol{k}+t(3\boldsymbol{i}+4\boldsymbol{j}-6\boldsymbol{k})$

Note: Award ***A1*** for the form $\boldsymbol{a}+t\boldsymbol{b}$, ***A1*** for the form $\boldsymbol{L}=\boldsymbol{a}+t\boldsymbol{b}$, ***A0*** for the form $\boldsymbol{r}=\boldsymbol{b}+t\boldsymbol{a}$.

[4 marks]

(b) valid approach *(M1)*

eg $a\bullet b=0$

choosing correct direction vectors (may be seen in scalar product) ***A1***

eg $\begin{pmatrix}3\\4\\-6\end{pmatrix}$ and $\begin{pmatrix}p\\0\\1\end{pmatrix}$, $\begin{pmatrix}3\\4\\-6\end{pmatrix}\bullet\begin{pmatrix}p\\0\\1\end{pmatrix}=0$

correct working/equation ***A1***

eg $3p-6=0$

$p=2$ ***AG*** ***N0***

[3 marks]

continued…

Question 8 continued

(c) valid approach **(M1)**

eg $L_1 = \begin{pmatrix} 9 \\ 13 \\ z \end{pmatrix}$, $L_1 = L_2$

one correct equation (must be different parameters if both lines used) **(A1)**

eg $3t = 9$, $1 + 2s = 9$, $5 + 4t = 13$, $3t = 1 + 2s$

one correct value **A1**

eg $t = 3$, $s = 4$, $t = 2$

valid approach to substitute their t or s value **(M1)**

eg $8 + 3(-6)$, $-14 + 4(1)$

$z = -10$ **A1** **N3**

[5 marks]

(d) (i) $\left|\vec{d}\right| = \sqrt{2^2 + 1}$ $\left(= \sqrt{5}\right)$ **(A1)**

$\frac{1}{\sqrt{5}}\begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$ $\left(\text{accept } \begin{pmatrix} \frac{2}{\sqrt{5}} \\ \frac{0}{\sqrt{5}} \\ \frac{1}{\sqrt{5}} \end{pmatrix}\right)$ **A1** **N2**

Question 8 continued

(ii) **METHOD 1 (using unit vector)**

valid approach ***(M1)***

eg $\begin{pmatrix} 9 \\ 13 \\ -10 \end{pmatrix} \pm \sqrt{5}\,\hat{d}$

correct working ***(A1)***

eg $\begin{pmatrix} 9 \\ 13 \\ -10 \end{pmatrix} + \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 9 \\ 13 \\ -10 \end{pmatrix} - \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$

one correct point ***A1*** ***N2***

eg $(11, 13, -9), (7, 13, -11)$

METHOD 2 (distance between points)

attempt to use distance between $(1+2s, 13, -14+s)$ and $(9, 13, -10)$ ***(M1)***

eg $(2s-8)^2 + 0^2 + (s-4)^2 = 5$

solving $5s^2 - 40s + 75 = 0$ leading to $s = 5$ or $s = 3$ ***(A1)***

one correct point ***A1*** ***N2***

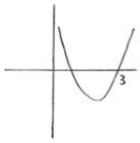
eg $(11, 13, -9), (7, 13, -11)$

[5 marks]

Total [17 marks]

9. (a) **METHOD 1 (using *x*-intercept)**

determining that 3 is an x-intercept ***(M1)***

eg $x-3=0$, 

valid approach ***(M1)***

eg $3-2.5$, $\frac{p+3}{2}=2.5$

$p=2$ ***A1*** ***N2***

METHOD 2 (expanding $f(x)$)

correct expansion (accept absence of a) ***(A1)***

eg $ax^2-a(3+p)x+3ap$, $x^2-(3+p)x+3p$

valid approach involving equation of axis of symmetry ***(M1)***

eg $\frac{-b}{2a}=2.5$, $\frac{a(3+p)}{2a}=\frac{5}{2}$, $\frac{3+p}{2}=\frac{5}{2}$

$p=2$ ***A1*** ***N2***

METHOD 3 (using derivative)

correct derivative (accept absence of a) ***(A1)***

eg $a(2x-3-p)$, $2x-3-p$

valid approach ***(M1)***

eg $f'(2.5)=0$

$p=2$ ***A1*** ***N2***

[3 marks]

(b) attempt to substitute (0, –6) ***(M1)***

eg $-6=a(0-2)(0-3)$, $0=a(-8)(-9)$, $a(0)^2-5a(0)+6a=-6$

correct working ***(A1)***

eg $-6=6a$

$a=-1$ ***A1*** ***N2***

[3 marks]

continued…

Question 9 continued

(c) **METHOD 1 (using discriminant)**

recognizing tangent intersects curve once ***(M1)***

recognizing one solution when discriminant = 0 ***M1***

attempt to set up equation ***(M1)***
eg $g = f$, $kx - 5 = -x^2 + 5x - 6$

rearranging their equation to equal zero ***(M1)***
eg $x^2 - 5x + kx + 1 = 0$

correct discriminant (if seen explicitly, not just in quadratic formula) ***A1***
eg $(k-5)^2 - 4$, $25 - 10k + k^2 - 4$

correct working ***(A1)***
eg $k - 5 = \pm 2$, $(k-3)(k-7) = 0$, $\frac{10 \pm \sqrt{100 - 4 \times 21}}{2}$

$k = 3, 7$ ***A1A1*** ***N0***

METHOD 2 (using derivatives)

attempt to set up equation ***(M1)***
eg $g = f$, $kx - 5 = -x^2 + 5x - 6$

recognizing derivative/slope are equal ***(M1)***
eg $f' = m_T$, $f' = k$

correct derivative of f ***(A1)***
eg $-2x + 5$

attempt to set up equation in terms of either x or k ***M1***
eg $(-2x+5)x - 5 = -x^2 + 5x - 6$, $k\left(\frac{5-k}{2}\right) - 5 = -\left(\frac{5-k}{2}\right)^2 + 5\left(\frac{5-k}{2}\right) - 6$

rearranging their equation to equal zero ***(M1)***
eg $x^2 - 1 = 0$, $k^2 - 10k + 21 = 0$

correct working ***(A1)***
eg $x = \pm 1$, $(k-3)(k-7) = 0$, $\frac{10 \pm \sqrt{100 - 4 \times 21}}{2}$

$k = 3, 7$ ***A1A1*** ***N0***

[8 marks]

Total [14 marks]

10. (a) evidence of summing to 1 ***(M1)***

eg $\sum p = 1$

correct equation ***A1***

eg $\cos\theta + 2\cos 2\theta = 1$

correct equation in $\cos\theta$ ***A1***

eg $\cos\theta + 2\left(2\cos^2\theta - 1\right) = 1$, $4\cos^2\theta + \cos\theta - 3 = 0$

evidence of valid approach to solve quadratic ***(M1)***

eg factorizing equation set equal to 0, $\dfrac{-1 \pm \sqrt{1 - 4\times 4\times(-3)}}{8}$

correct working, clearly leading to required answer ***A1***

eg $(4\cos\theta - 3)(\cos\theta + 1)$, $\dfrac{-1 \pm 7}{8}$

correct reason for rejecting $\cos\theta \neq -1$ ***R1***

eg $\cos\theta$ is a probability (value must lie between 0 and 1), $\cos\theta > 0$

Note: Award ***R0*** for $\cos\theta \neq -1$ without a reason.

$\cos\theta = \dfrac{3}{4}$ ***AG*** ***N0***

[6 marks]

(b) valid approach ***(M1)***

eg sketch of right triangle with sides 3 and 4, $\sin^2 x + \cos^2 x = 1$

correct working ***(A1)***

eg missing side $= \sqrt{7}$, $\dfrac{\frac{\sqrt{7}}{4}}{\frac{3}{4}}$

$\tan\theta = \dfrac{\sqrt{7}}{3}$ ***A1*** ***N2***

[3 marks]

continued…

Question 10 continued

(c) attempt to substitute either limits or the function into formula involving f^2 ***(M1)***

eg $\pi\int_{\theta}^{\frac{\pi}{4}} f^2$, $\int\left(\frac{1}{\cos x}\right)^2$

correct substitution of both limits and function ***(A1)***

eg $\pi\int_{\theta}^{\frac{\pi}{4}}\left(\frac{1}{\cos x}\right)^2 \mathrm{d}x$

correct integration ***(A1)***

eg $\tan x$

substituting **their** limits into **their** integrated function and subtracting ***(M1)***

eg $\tan\frac{\pi}{4}-\tan\theta$

Note: Award ***M0*** if they substitute into original or differentiated function.

$\tan\frac{\pi}{4}=1$ ***(A1)***

eg $1-\tan\theta$

$V=\pi-\frac{\pi\sqrt{7}}{3}$ ***A1*** ***N3***

[6 marks]

[Total: 15 marks]

Markscheme

May 2017

Mathematics

Standard level

Paper 1

17 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a valid **Method**; working must be seen.

(M) Marks awarded for a valid **Method**; may be implied by **correct** subsequent working.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

(A) Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.

R Marks awarded for clear **Reasoning**.

N Marks awarded for **correct** answers if **no** working shown.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Mark according to RM assessor instructions.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any. An exception to this rule is when work for ***M1*** is missing, as opposed to incorrect (see point 4).
- Where ***M*** and ***A*** marks are noted on the same line, *eg* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*eg* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***(M2)***, ***N3***, *etc*., do **not** split the marks, unless there is a note.
- Most ***M*** marks are for a **valid** method, ie a method which can lead to the answer: it must indicate some form of progress towards the answer.
- Once a correct answer to a question or part-question is seen, ignore further correct working. However, if further working indicates a lack of mathematical understanding do not award final ***A1***.

***N* marks**

*If **no** working shown, award **N** marks for **correct** answers – this includes acceptable answers (see accuracy booklet). In this case, ignore mark breakdown (**M, A, R**). Where a student only shows a final incorrect answer with no working, even if that answer is a correct intermediate answer, award **N0.***

- Do **not** award a mixture of ***N*** and other marks.
- There may be fewer ***N*** marks available than the total of ***M***, ***A*** and ***R*** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.
- There may not be a direct relationship between the ***N*** marks and the implied marks. There are times when all the marks are implied, but the ***N*** marks are not the full marks: this indicates that we want to see some of the working, without specifying what.

- For consistency within the markscheme, ***N*** marks are noted for every part, even when these match the mark breakdown.
- If a candidate has incorrect working, which somehow results in a correct answer, do **not** award the ***N*** marks for this correct answer. However, if the candidate has indicated (usually by crossing out) that the working is to be ignored, award the ***N*** marks for the correct answer.

4 Implied and must be seen marks

*Implied marks appear in **brackets** eg **(M1)**.*

- Implied marks can only be awarded if the work is seen or if implied in subsequent working (a correct final answer does not necessarily mean that the implied marks are all awarded). There are questions where some working is required, but as it is accepted that not everyone will write the same steps, all the marks are implied, but the ***N*** marks are not the full marks for the question.
- Normally the correct work is seen in the next line.
- Where there is an (***M1***) followed by ***A1*** for each correct answer, if no working shown, one correct answer is sufficient evidence to award the (***M1***).

*Must be seen marks appear without **brackets** eg **M1**.*

- Must be seen marks can only be awarded if the work is seen.
- If a must be seen mark is not awarded because work is missing (as opposed to ***M0*** or ***A0*** for incorrect work) all subsequent marks may be awarded if appropriate.

5 Follow through marks (only applied after an error is made)

*Follow through (**FT**) marks are awarded where an incorrect answer (final or intermediate) from one **part** of a question is used correctly in **subsequent** part(s) or subpart(s). Usually, to award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if the only marks awarded in a subpart are for the **final** answer, then **FT** marks should be awarded if appropriate. Examiners are expected to check student work in order to award **FT** marks where appropriate.*

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** and ***R*** marks may be awarded if appropriate. (However, as noted above, if an ***A*** mark is not awarded because work is missing, all subsequent marks may be awarded if appropriate).
- Exceptions to this rule will be explicitly noted on the markscheme.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*eg* probability greater than 1, use of $r>1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word "**their**" in a description, to indicate that candidates may be using an incorrect value.
- If a candidate makes an error in one part, but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the question says hence. It is often possible to use a different approach in subsequent parts that does not depend on the answer to previous parts.
- In a "show that" question, if an error in a previous subpart leads to not showing the required answer, do not award the final ***A1***. Note that if the error occurs within the same subpart, the ***FT*** rules may result in further loss of marks.

6 Mis-read

*If a candidate incorrectly copies information from the question, this is a mis-read (**MR**). A candidate should be penalized only once for a particular mis-read. Use the **MR** stamp to indicate that this is a misread. Do not award the first mark in the question, even if this is an **M** mark, but award all others (if appropriate) so that the candidate only loses one mark for the misread.*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*eg* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.

7 Discretionary marks *(d)*

*An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. In such cases the annotation DM should be used and a brief **note** written next to the mark explaining this decision.*

8 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete parts are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**. Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9 Alternative forms

*Unless the question specifies otherwise, **accept** equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

10 Calculators

No calculator is allowed. The use of any calculator on paper 1 is malpractice, and will result in no grade awarded. If you see work that suggests a candidate has used any calculator, please follow the procedures for malpractice. Examples: finding an angle, given a trig ratio of 0.4235.

11 Style

The markscheme aims to present answers using good communication, eg if the question asks to find the value of k, the markscheme will say $k = 3$, but the marks will be for the correct value 3 – there is usually no need for the "$k =$". In these cases, it is also usually acceptable to have another variable, as long as there is no ambiguity in the question, eg if the question asks to find the value of p and of q, then the student answer needs to be clear. Generally, the only situation

where the full answer is required is in a question which asks for equations – in this case the markscheme will say “must be an equation”.

*The markscheme often uses words to describe what the marks are for, followed by examples, using the eg notation. These examples are not exhaustive, and examiners should check what candidates have written, to see if they satisfy the description. Where these marks are **M** marks, the examples may include ones using poor notation, to indicate what is acceptable. A valid method is one which will allow candidate to proceed to the next step eg if a quadratic function is given in factorised form, and the question asks for the zeroes, then multiplying the factors does not necessarily help to find the zeros, and would not on its own count as a valid method.*

12 Candidate work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

Candidates are meant to write their answers to Section A on the question paper (QP), and Section B on answer booklets. Sometimes, they need more room for Section A, and use the booklet (and often comment to this effect on the QP), or write outside the box. That is fine, and this work should be marked.

The instructions tell candidates not to write on Section B of the QP. Thus they may well have done some rough work here which they assume will be ignored. If they have solutions on the answer booklets, there is no need to look at the QP. However, if there are whole questions or whole part solutions missing on answer booklets, please check to make sure that they are not on the QP, and if they are, mark those whole questions or whole part solutions that have not been written on answer booklets.

13. Diagrams

The notes on how to allocate marks for sketches usually refer to passing through particular points or having certain features. These marks can only be awarded if the sketch is approximately the correct shape. All values given will be an approximate guide to where these points/features occur. In some questions, the first ***A1*** is for the shape, in others, the marks are only for the points and/or features. In both cases, unless the shape is approximately correct, no marks can be awarded (unless otherwise stated). However, if the graph is based on previous calculations, ***FT*** marks should be awarded if appropriate.

14. Accuracy of Answers

If the level of accuracy is specified in the question, a mark will be allocated for giving the final answer to the required accuracy. When this is not specified in the question, all numerical answers should be given exactly or correct to three significant figures.

Do not accept unfinished numerical final answers such as 3/0.1 (unless otherwise stated). As a rule, numerical answers with more than one part (such as fractions) should be given using integers (eg 6/8). Calculations which lead to integers should be completed, with the exception of fractions which are not whole numbers. Intermediate values do not need to be given to the correct three significant figures. But, if candidates work with rounded values, this could lead to an incorrect answer, in which case award A0 for the final answer. Where numerical answers are required as the final answer to a part of a question in the markscheme, the markscheme will show

a truncated 6 sf value
the exact value if applicable, the correct 3 sf answer
Units will appear in brackets at the end.

Section A

1. (a) attempt to subtract terms *(M1)*

eg $d = u_2 - u_1$, $7 - 3$

$d = 4$ *A1* *N2*

[2 marks]

(b) correct approach *(A1)*

eg $u_{10} = 3 + 9(4)$

$u_{10} = 39$ *A1* *N2*

[2 marks]

(c) correct substitution into sum *(A1)*

eg $S_{10} = 5(3+39)$, $S_{10} = \frac{10}{2}(2\times 3 + 9\times 4)$

$S_{10} = 210$ *A1* *N2*

[2 marks]

[Total 6 marks]

2. (a) evidence of scalar product *M1*

eg $\boldsymbol{a} \bullet \boldsymbol{b}$, $4(k+3)+2k$

recognizing scalar product must be zero *(M1)*

eg $\boldsymbol{a} \bullet \boldsymbol{b} = 0$, $4k + 12 + 2k = 0$

correct working (must involve combining terms) *(A1)*

eg $6k + 12$, $6k = -12$

$k = -2$ *A1* *N2*

[4 marks]

(b) attempt to substitute **their** value of k (seen anywhere) *(M1)*

eg $\boldsymbol{b} = \begin{pmatrix} -2+3 \\ -2 \end{pmatrix}$, $2\boldsymbol{b} = \begin{pmatrix} 2 \\ -4 \end{pmatrix}$

correct working *(A1)*

eg $\begin{pmatrix} 4 \\ 2 \end{pmatrix} + \begin{pmatrix} 2 \\ -4 \end{pmatrix}$, $\begin{pmatrix} 4+2k+6 \\ 2+2k \end{pmatrix}$

$\boldsymbol{c} = \begin{pmatrix} 6 \\ -2 \end{pmatrix}$ *A1* *N2*

[3 marks]

[Total 7 marks]

3. (a) $\text{P}(X > 107) = 0.24 \left(= \frac{6}{25}, 24\%\right)$ ***A1*** ***N1***

[1 mark]

(b) valid approach ***(M1)***

eg $\text{P}(X > 100) = 0.5$, $\text{P}(X > 100) - \text{P}(X > 107)$

correct working ***(A1)***

eg $0.5 - 0.24$, $0.76 - 0.5$

$\text{P}(100 < X < 107) = 0.26 \left(= \frac{13}{50}, 26\%\right)$ ***A1*** ***N2***

[3 marks]

(c) valid approach ***(M1)***

eg 2×0.26, $1 - 2(0.24)$, $\text{P}(93 < X < 100) = \text{P}(100 < X < 107)$

$\text{P}(93 < X < 107) = 0.52 \left(= \frac{13}{25}, 52\%\right)$ ***A1*** ***N2***

[2 marks]

[Total 6 marks]

4. (a) (i) $p = 6$ ***A1*** ***N1***

(ii) $q = 5$ ***A1*** ***N1***

[2 marks]

(b) correct approach ***(A1)***

eg $p \times q$, 5×6

$k = 30$ ***A1*** ***N2***

[2 marks]

(c) correct approach ***(A1)***

eg rows $= n + 1$, columns $= n$

$A(n) = n(n+1) \ (= n^2 + n) \ (\text{cm}^2)$ ***A1*** ***N2***

[2 marks]

[Total 6 marks]

5. valid approach ***(M1)***

eg $\int f'\,\mathrm{d}x$, $\int \frac{3x^2}{\left(x^3+1\right)^5}\mathrm{d}x$

correct integration by substitution/inspection ***A2***

eg $f(x) = -\frac{1}{4}\left(x^3+1\right)^{-4}+c$, $\frac{-1}{4\left(x^3+1\right)^4}$

correct substitution into **their** integrated function (must include c) ***M1***

eg $1 = \frac{-1}{4\left(0^3+1\right)^4}+c$, $-\frac{1}{4}+c=1$

Note: Award ***M0*** if candidates substitute into f' or f''.

$c = \frac{5}{4}$ ***(A1)***

$f(x) = -\frac{1}{4}\left(x^3+1\right)^{-4}+\frac{5}{4} \left(= \frac{-1}{4\left(x^3+1\right)^4}+\frac{5}{4}, \frac{5\left(x^3+1\right)^4-1}{4\left(x^3+1\right)^4}\right)$ ***A1 N4***

[6 marks]

6. (a) expressing $h(1)$ as a product of $f(1)$ and $g(1)$ ***(A1)***

eg $f(1)\times g(1)$, $2(9)$

$h(1)=18$ ***A1 N2***

[2 marks]

(b) attempt to use product rule (do **not** accept $h' = f' \times g'$) ***(M1)***

eg $h' = fg' + gf'$, $h'(8) = f'(8)\,g(8) + g'(8)\,f(8)$

correct substitution of values into product rule ***(A1)***

eg $h'(8) = 4(5)+2(-3)$, $-6+20$

$h'(8)=14$ ***A1 N2***

[3 marks]

[Total 5 marks]

7. correct application of $\log a + \log b = \log ab$ ***(A1)***

eg $\log_2(2\sin x\cos x)$, $\log 2 + \log(\sin x) + \log(\cos x)$

correct equation without logs ***A1***

eg $2\sin x\cos x = 2^{-1}$, $\sin x\cos x = \frac{1}{4}$, $\sin 2x = \frac{1}{2}$

recognizing double-angle identity (seen anywhere) ***A1***

eg $\log(\sin 2x)$, $2\sin x\cos x = \sin 2x$, $\sin 2x = \frac{1}{2}$

evaluating $\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$ (30°) ***(A1)***

correct working ***A1***

eg $x = \frac{\pi}{12} + 2\pi$, $2x = \frac{25\pi}{6}$, $\frac{29\pi}{6}$, 750°, 870°, $x = \frac{\pi}{12}$ **and** $x = \frac{5\pi}{12}$, one correct final answer

$x = \frac{25\pi}{12}, \frac{29\pi}{12}$ (do not accept additional values) ***A2*** ***N0***

[7 marks]

Section B

8. (a) (i) evidence of median position *(M1)*
eg 80th employee

40 hours ***A1*** ***N2***

(ii) 130 employees ***A1*** ***N1***
[3 marks]

(b) (i) £320 ***A1*** ***N1***

(ii) splitting into 40 and 3 *(M1)*
eg 3 hours more, 3×10

correct working *(A1)*
eg $320+3\times10$

£350 ***A1*** ***N3***
[4 marks]

(c) valid approach *(M1)*
eg 200 is less than 320 so 8 pounds/hour, $200\div8$, 25, $\frac{200}{320}=\frac{x}{40}$

18 employees ***A2*** ***N3***
[3 marks]

(d) valid approach *(M1)*
eg $160-10$

60 hours worked *(A1)*

correct working *(A1)*
eg $40(8)+20(10)$, $320+200$

$k=520$ ***A1*** ***N3***
[4 marks]

[Total 14 marks]

9. (a) recognizing $t=0$ at A *(M1)*

A is $(4, -1, 3)$ *A1* *N2*

[2 marks]

(b) (i) **METHOD 1**

valid approach *(M1)*

eg $\begin{pmatrix}4\\-1\\3\end{pmatrix}+2\begin{pmatrix}1\\2\\-2\end{pmatrix}$, $(6, 3, -1)$

correct approach to find $\overrightarrow{AB}$ *(A1)*

eg $AO+OB$, $B-A$, $\begin{pmatrix}6\\3\\-1\end{pmatrix}-\begin{pmatrix}4\\-1\\3\end{pmatrix}$

$\overrightarrow{AB}=\begin{pmatrix}2\\4\\-4\end{pmatrix}$ *A1* *N2*

METHOD 2

recognizing $\overrightarrow{AB}$ is two times the direction vector *(M1)*

correct working *(A1)*

eg $\overrightarrow{AB}=2\begin{pmatrix}1\\2\\-2\end{pmatrix}$

$\overrightarrow{AB}=\begin{pmatrix}2\\4\\-4\end{pmatrix}$ *A1* *N2*

(ii) correct substitution *(A1)*

eg $\left|\overrightarrow{AB}\right|=\sqrt{2^2+4^2+4^2}$, $\sqrt{4+16+16}$, $\sqrt{36}$

$\left|\overrightarrow{AB}\right|=6$ *A1* *N2*

[5 marks]

continued…

Question 9 continued

(c) **METHOD 1 (vector approach)**

valid approach involving $\overrightarrow{AB}$ and $\overrightarrow{AC}$ ***(M1)***

eg $\overrightarrow{AB} \bullet \overrightarrow{AC}$, $\dfrac{\overrightarrow{BA} \bullet \overrightarrow{AC}}{AB \times AC}$

finding scalar product and $\left|\overrightarrow{AC}\right|$ ***(A1)(A1)***

scalar product $2(3)+4(0)-4(4) \quad (=-10)$

$\left|\overrightarrow{AC}\right| = \sqrt{3^2+0^2+4^2} \quad (=5)$

substitution of **their** scalar product and magnitudes into cosine formula ***(M1)***

eg $\cos B\hat{A}C = \dfrac{6+0-16}{6\sqrt{3^2+4^2}}$

$\cos B\hat{A}C = -\dfrac{10}{30} \quad \left(= -\dfrac{1}{3}\right)$ ***A1 N2***

METHOD 2 (triangle approach)

valid approach involving cosine rule ***(M1)***

eg $\cos B\hat{A}C = \dfrac{AB^2+AC^2-BC^2}{2 \times AB \times AC}$

finding lengths AC and BC ***(A1)(A1)***

$AC = 5$, $BC = 9$

substitution of **their** lengths into cosine formula ***(M1)***

eg $\cos B\hat{A}C = \dfrac{5^2+6^2-9^2}{2 \times 5 \times 6}$

$\cos B\hat{A}C = -\dfrac{20}{60} \quad \left(= -\dfrac{1}{3}\right)$ ***A1 N2***

[5 marks]

continued…

Question 9 continued

(d)

Note: Award relevant marks for working seen to find BC in part (c) (if cosine rule used in part (c)).

METHOD 1 (using cosine rule)

recognizing need to find BC ***(M1)***

choosing cosine rule ***(M1)***

eg $c^2 = a^2 + b^2 - 2ab\cos C$

correct substitution into RHS ***A1***

eg $\text{BC}^2 = (6)^2 + (5)^2 - 2(6)(5)\left(-\frac{1}{3}\right)$, $36 + 25 + 20$

distance is 9 ***A1*** ***N2***

METHOD 2 (finding magnitude of $\overrightarrow{\text{BC}}$)

recognizing need to find BC ***(M1)***

valid approach ***(M1)***

eg attempt to find $\overrightarrow{\text{OB}}$ or $\overrightarrow{\text{OC}}$, $\overrightarrow{\text{OB}} = \begin{pmatrix} 6 \\ 3 \\ -1 \end{pmatrix}$ or $\overrightarrow{\text{OC}} = \begin{pmatrix} 7 \\ -1 \\ 7 \end{pmatrix}$, $\overrightarrow{\text{BA}} + \overrightarrow{\text{AC}}$

correct working ***A1***

eg $\overrightarrow{\text{BC}} = \begin{pmatrix} 1 \\ -4 \\ 8 \end{pmatrix}$, $\overrightarrow{\text{CB}} = \begin{pmatrix} -1 \\ 4 \\ -8 \end{pmatrix}$, $\sqrt{1^2 + 4^2 + 8^2} = \sqrt{81}$

distance is 9 ***A1*** ***N2***

METHOD 3 (finding coordinates and using distance formula)

recognizing need to find BC ***(M1)***

valid approach ***(M1)***

eg attempt to find coordinates of B or C, $\text{B}(6, 3, -1)$ or $\text{C}(7, -1, 7)$

correct substitution into distance formula ***A1***

eg $\text{BC} = \sqrt{(6-7)^2 + \left(3-(-1)\right)^2 + (-1-7)^2}$, $\sqrt{1^2 + 4^2 + 8^2} = \sqrt{81}$

distance is 9 ***A1*** ***N2***

[4 marks]

[Total 16 marks]

10. (a) (i) $f'(x) = 2x$ ***A1*** ***N1***

(ii) attempt to substitute $x = -k$ into their derivative ***(M1)***

gradient of L is $-2k$ ***A1*** ***N2***

[3 marks]

(b) **METHOD 1**

attempt to substitute coordinates of A and their gradient into equation of a line ***(M1)***

eg $k^2 = -2k(-k) + b$

correct equation of L in any form ***(A1)***

eg $y - k^2 = -2k(x + k)$, $y = -2kx - k^2$

valid approach ***(M1)***

eg $y = 0$

correct substitution into L equation ***A1***

eg $-k^2 = -2kx - 2k^2$, $0 = -2kx - k^2$

correct working ***A1***

eg $2kx = -k^2$

$x = -\frac{k}{2}$ ***AG*** ***N0***

METHOD 2

valid approach ***(M1)***

eg $\text{gradient} = \frac{y_2 - y_1}{x_2 - x_1}$, $-2k = \frac{\text{rise}}{\text{run}}$

recognizing $y = 0$ at B ***(A1)***

attempt to substitute coordinates of A and B into slope formula ***(M1)***

eg $\frac{k^2 - 0}{-k - x}$, $\frac{-k^2}{x + k}$

correct equation ***A1***

eg $\frac{k^2 - 0}{-k - x} = -2k$, $\frac{-k^2}{x + k} = -2k$, $-k^2 = -2k(x + k)$

correct working ***A1***

eg $2kx = -k^2$

$x = -\frac{k}{2}$ ***AG*** ***N0***

[5 marks]

continued…

Question 10 continued

(c) valid approach to find area of triangle ***(M1)***

eg $\frac{1}{2}(k^2)\left(\frac{k}{2}\right)$

area of $\text{ABC} = \frac{k^3}{4}$ ***A1*** ***N2***

[2 marks]

(d) **METHOD 1** **(** $\int f - \text{triangle}$ **)**

valid approach to find area from $-k$ to 0 ***(M1)***

eg $\int_{-k}^{0} x^2 \text{d}x$, $\int_{0}^{-k} f$

correct integration (seen anywhere, even if ***M0*** awarded) ***A1***

eg $\frac{x^3}{3}$, $\left[\frac{1}{3}x^3\right]_{-k}^{0}$

substituting **their** limits into **their** integrated function and subtracting ***(M1)***

eg $0 - \frac{(-k)^3}{3}$, area from $-k$ to 0 is $\frac{k^3}{3}$

Note: Award ***M0*** for substituting into original or differentiated function.

attempt to find area of R ***(M1)***

eg $\int_{-k}^{0} f(x)\,\text{d}x - \text{triangle}$

correct working for R ***(A1)***

eg $\frac{k^3}{3} - \frac{k^3}{4}$, $R = \frac{k^3}{12}$

correct substitution into $\text{triangle} = pR$ ***(A1)***

eg $\frac{k^3}{4} = p\left(\frac{k^3}{3} - \frac{k^3}{4}\right)$, $\frac{k^3}{4} = p\left(\frac{k^3}{12}\right)$

$p = 3$ ***A1*** ***N2***

continued…

Question 10 continued

METHOD 2 $\left(\int (f - L)\right)$

valid approach to find area of R ***(M1)***

eg $\int_{-k}^{-\frac{k}{2}} x^2 - (-2kx - k^2)\,\mathrm{d}x + \int_{-\frac{k}{2}}^{0} x^2\mathrm{d}x$, $\int_{-k}^{-\frac{k}{2}} (f - L) + \int_{-\frac{k}{2}}^{0} f$

correct integration (seen anywhere, even if ***M0*** awarded) ***A2***

eg $\frac{x^3}{3} + kx^2 + k^2x$, $\left[\frac{x^3}{3} + kx^2 + k^2x\right]_{-k}^{-\frac{k}{2}} + \left[\frac{x^3}{3}\right]_{-\frac{k}{2}}^{0}$

substituting **their** limits into **their** integrated function and subtracting ***(M1)***

eg $$\left(\frac{\left(-\frac{k}{2}\right)^3}{3} + k\left(-\frac{k}{2}\right)^2 + k^2\left(-\frac{k}{2}\right)\right) - \left(\frac{(-k)^3}{3} + k(-k)^2 + k^2(-k)\right) + (0) - \left(\frac{\left(-\frac{k}{2}\right)^3}{3}\right)$$

Note: Award ***M0*** for substituting into original or differentiated function.

correct working for R ***(A1)***

eg $\frac{k^3}{24} + \frac{k^3}{24}$, $-\frac{k^3}{24} + \frac{k^3}{4} - \frac{k^3}{2} + \frac{k^3}{3} - k^3 + k^3 + \frac{k^3}{24}$, $R = \frac{k^3}{12}$

correct substitution into triangle $= pR$ ***(A1)***

eg $\frac{k^3}{4} = p\left(\frac{k^3}{24} + \frac{k^3}{24}\right)$, $\frac{k^3}{4} = p\left(\frac{k^3}{12}\right)$

$p = 3$ ***A1*** ***N2***

[7 marks]

[Total 17 marks]

Markscheme

November 2016

Mathematics

Standard level

Paper 1

16 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a valid **Method**; working must be seen.

(M) Marks awarded for a valid **Method**; may be implied by **correct** subsequent working.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

(A) Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.

R Marks awarded for clear **Reasoning**.

N Marks awarded for **correct** answers if **no** working shown.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Mark according to RM assessor instructions.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any. An exception to this rule is when work for ***M1*** is missing, as opposed to incorrect (see point 4).
- Where ***M*** and ***A*** marks are noted on the same line, *eg* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*eg* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***(M2)***, ***N3***, *etc*., do **not** split the marks, unless there is a note.
- Most ***M*** marks are for a **valid** method, ie a method which can lead to the answer: it must indicate some form of progress towards the answer.
- Once a correct answer to a question or part-question is seen, ignore further correct working. However, if further working indicates a lack of mathematical understanding do not award final ***A1***.

3 *N* marks

*If **no** working shown, award **N** marks for **correct** answers – this includes acceptable answers (see accuracy booklet). In this case, ignore mark breakdown (**M, A, R**). Where a student only shows a final incorrect answer with no working, even if that answer is a correct intermediate answer, award **N0.***

- Do **not** award a mixture of ***N*** and other marks.
- There may be fewer ***N*** marks available than the total of ***M***, ***A*** and ***R*** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.
- There may not be a direct relationship between the ***N*** marks and the implied marks. There are times when all the marks are implied, but the ***N*** marks are not the full marks: this indicates that we want to see some of the working, without specifying what.

- For consistency within the markscheme, ***N*** marks are noted for every part, even when these match the mark breakdown.
- If a candidate has incorrect working, which somehow results in a correct answer, do **not** award the ***N*** marks for this correct answer. However, if the candidate has indicated (usually by crossing out) that the working is to be ignored, award the ***N*** marks for the correct answer.

4 Implied and must be seen marks

*Implied marks appear in **brackets** eg **(M1)**.*

- Implied marks can only be awarded if the work is seen or if implied in subsequent working (a correct final answer does not necessarily mean that the implied marks are all awarded). There are questions where some working is required, but as it is accepted that not everyone will write the same steps, all the marks are implied, but the ***N*** marks are not the full marks for the question.
- Normally the correct work is seen in the next line.
- Where there is an (***M1***) followed by ***A1*** for each correct answer, if no working shown, one correct answer is sufficient evidence to award the (***M1***).

*Must be seen marks appear without **brackets** eg **M1**.*

- Must be seen marks can only be awarded if the work is seen.
- If a must be seen mark is not awarded because work is missing (as opposed to ***M0*** or ***A0*** for incorrect work) all subsequent marks may be awarded if appropriate.

5 Follow through marks (only applied after an error is made)

*Follow through (**FT**) marks are awarded where an incorrect answer (final or intermediate) from one **part** of a question is used correctly in **subsequent** part(s) or subpart(s). Usually, to award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if the only marks awarded in a subpart are for the **final** answer, then **FT** marks should be awarded if appropriate. Examiners are expected to check student work in order to award **FT** marks where appropriate.*

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** and ***R*** marks may be awarded if appropriate. (However, as noted above, if an ***A*** mark is not awarded because work is missing, all subsequent marks may be awarded if appropriate).
- Exceptions to this rule will be explicitly noted on the markscheme.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*eg* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word "**their**" in a description, to indicate that candidates may be using an incorrect value.
- If a candidate makes an error in one part, but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the question says hence. It is often possible to use a different approach in subsequent parts that does not depend on the answer to previous parts.
- In a "show that" question, if an error in a previous subpart leads to not showing the required answer, do not award the final ***A1***. Note that if the error occurs within the same subpart, the ***FT*** rules may result in further loss of marks.

6 Mis-read

*If a candidate incorrectly copies information from the question, this is a mis-read (**MR**). A candidate should be penalized only once for a particular mis-read. Use the **MR** stamp to indicate that this is a misread. Do not award the first mark in the question, even if this is an **M** mark, but award all others (if appropriate) so that the candidate only loses one mark for the misread.*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*eg* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.

7 Discretionary marks *(d)*

*An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. In such cases the annotation DM should be used and a brief **note** written next to the mark explaining this decision.*

8 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete parts are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**. Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9 Alternative forms

*Unless the question specifies otherwise, **accept** equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

10 Calculators

No calculator is allowed. The use of any calculator on paper 1 is malpractice, and will result in no grade awarded. If you see work that suggests a candidate has used any calculator, please follow the procedures for malpractice. Examples: finding an angle, given a trig ratio of 0.4235.

11 Style

The markscheme aims to present answers using good communication, eg if the question asks to find the value of k, the markscheme will say $k = 3$, but the marks will be for the correct value 3 – there is usually no need for the "$k =$". In these cases, it is also usually acceptable to have another variable, as long as there is no ambiguity in the question, eg if the question asks to find the value of p and of q, then the student answer needs to be clear. Generally, the only situation

where the full answer is required is in a question which asks for equations – in this case the markscheme will say "must be an equation".

*The markscheme often uses words to describe what the marks are for, followed by examples, using the eg notation. These examples are not exhaustive, and examiners should check what candidates have written, to see if they satisfy the description. Where these marks are **M** marks, the examples may include ones using poor notation, to indicate what is acceptable. A valid method is one which will allow candidate to proceed to the next step eg if a quadratic function is given in factorised form, and the question asks for the zeroes, then multiplying the factors does not necessarily help to find the zeros, and would not on its own count as a valid method.*

12 Candidate work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

Candidates are meant to write their answers to Section A on the question paper (QP), and Section B on answer booklets. Sometimes, they need more room for Section A, and use the booklet (and often comment to this effect on the QP), or write outside the box. That is fine, and this work should be marked.

The instructions tell candidates not to write on Section B of the QP. Thus they may well have done some rough work here which they assume will be ignored. If they have solutions on the answer booklets, there is no need to look at the QP. However, if there are whole questions or whole part solutions missing on answer booklets, please check to make sure that they are not on the QP, and if they are, mark those whole questions or whole part solutions that have not been written on answer booklets.

13. Diagrams

The notes on how to allocate marks for sketches usually refer to passing through particular points or having certain features. These marks can only be awarded if the sketch is approximately the correct shape. All values given will be an approximate guide to where these points/features occur. In some questions, the first ***A1*** is for the shape, in others, the marks are only for the points and/or features. In both cases, unless the shape is approximately correct, no marks can be awarded (unless otherwise stated). However, if the graph is based on previous calculations, ***FT*** marks should be awarded if appropriate.

14. Accuracy of Answers

If the level of accuracy is specified in the question, a mark will be allocated for giving the final answer to the required accuracy. When this is not specified in the question, all numerical answers should be given exactly or correct to three significant figures.

Do not accept unfinished numerical final answers such as 3/0.1 (unless otherwise stated). As a rule, numerical answers with more than one part (such as fractions) should be given using integers (eg 6/8). Calculations which lead to integers should be completed, with the exception of fractions which are not whole numbers. Intermediate values do not need to be given to the correct three significant figures. But, if candidates work with rounded values, this could lead to an incorrect answer, in which case award A0 for the final answer. Where numerical answers are required as the final answer to a part of a question in the markscheme, the markscheme will show

a truncated 6 sf value
the exact value if applicable, the correct 3 sf answer
Units will appear in brackets at the end.

Section A

1. (a) correct approach *(A1)*

eg $\frac{-(-4)}{2}$, $f'(x) = 2x - 4 = 0$, $\left(x^2 - 4x + 4\right) + 5 - 4$

$x = 2$ (must be an equation) *A1* *N2*

[2 marks]

(b) (i) $h = 2$ *A1* *N1*

(ii) **METHOD 1**

valid attempt to find k *(M1)*

eg $f(2)$

correct substitution into **their** function *(A1)*

eg $(2)^2 - 4(2) + 5$

$k = 1$ *A1* *N2*

METHOD 2

valid attempt to complete the square *(M1)*

eg $x^2 - 4x + 4$

correct working *(A1)*

eg $\left(x^2 - 4x + 4\right) - 4 + 5$, $(x-2)^2 + 1$

$k = 1$ *A1* *N2*

[4 marks]

[Total 6 marks]

2. (a) evidence of valid approach *(M1)*

eg right triangle, $\cos^2\theta = 1 - \sin^2\theta$

correct working *(A1)*

eg missing side is 2, $\sqrt{1 - \left(\frac{\sqrt{5}}{3}\right)^2}$

$\cos\theta = \frac{2}{3}$ *A1* *N2*

[3 marks]

(b) correct substitution into formula for $\cos 2\theta$ *(A1)*

eg $2 \times \left(\frac{2}{3}\right)^2 - 1, 1 - 2\left(\frac{\sqrt{5}}{3}\right)^2, \left(\frac{2}{3}\right)^2 - \left(\frac{\sqrt{5}}{3}\right)^2$

$\cos 2\theta = -\frac{1}{9}$ *A1* *N2*

[2 marks]

[Total 5 marks]

3. (a) $1, 5, 10, 10, 5, 1$ ***A2*** ***N2***

[2 marks]

(b) evidence of binomial expansion with binomial coefficient ***(M1)***

eg $\binom{n}{r}a^{n-r}b^{r}$, selecting correct term, $(2x)^5(3)^0+5(2x)^4(3)^1+10(2x)^3(3)^2+\ldots$

correct substitution into correct term ***(A1)(A1)(A1)***

eg $10(2)^3(3)^2, \binom{5}{3}(2x)^3(3)^2$

Note: Award ***A1*** for each factor.

$720x^3$ ***A1*** ***N2***

Notes: Do not award any marks if there is clear evidence of adding instead of multiplying. Do not award final ***A1*** for a final answer of 720, even if $720x^3$ is seen previously.

[5 marks]

[Total 7 marks]

4. (a) valid attempt to find direction vector ***(M1)***

eg $\vec{PQ}, \vec{QP}$

correct direction vector (or multiple of) ***(A1)***

eg $6\boldsymbol{i}+\boldsymbol{j}-3\boldsymbol{k}$

any correct equation in the form $\boldsymbol{r}=\boldsymbol{a}+t\boldsymbol{b}$ (any parameter for t) ***A2*** ***N3***

where $\boldsymbol{a}$ is $\boldsymbol{i}+2\boldsymbol{j}-\boldsymbol{k}$ or $7\boldsymbol{i}+3\boldsymbol{j}-4\boldsymbol{k}$, and $\boldsymbol{b}$ is a scalar multiple of $6\boldsymbol{i}+\boldsymbol{j}-3\boldsymbol{k}$

eg $\boldsymbol{r}=7\boldsymbol{i}+3\boldsymbol{j}-4\boldsymbol{k}+t(6\boldsymbol{i}+\boldsymbol{j}-3\boldsymbol{k})$, $\boldsymbol{r}=\begin{pmatrix}1+6s\\2+1s\\-1-3s\end{pmatrix}$, $r=\begin{pmatrix}1\\2\\-1\end{pmatrix}+t\begin{pmatrix}-6\\-1\\3\end{pmatrix}$

Notes: Award ***A1*** for the form $\boldsymbol{a}+t\boldsymbol{b}$, ***A1*** for the form $\boldsymbol{L}=\boldsymbol{a}+t\boldsymbol{b}$, ***A0*** for the form $\boldsymbol{r}=\boldsymbol{b}+t\boldsymbol{a}$.

[4 marks]

(b) correct expression for scalar product ***(A1)***

eg $6\times2+1\times0+(-3)\times n$, $-3n+12$

setting scalar product equal to zero (seen anywhere) ***(M1)***

eg $\boldsymbol{u}\bullet\boldsymbol{v}=0$, $-3n+12=0$

$n=4$ ***A1*** ***N2***

[3 marks]

[Total 7 marks]

5. (a) valid interpretation (may be seen on a Venn diagram) *(M1)*

eg $\mathrm{P}(A \cap B) + \mathrm{P}(A' \cap B)$, $0.2 + 0.6$

$\mathrm{P}(B) = 0.8$ ***A1*** ***N2***

[2 marks]

(b) valid attempt to find $\mathrm{P}(A)$ *(M1)*

eg $\mathrm{P}(A \cap B) = \mathrm{P}(A) \times \mathrm{P}(B)$, $0.8 \times A = 0.2$

correct working for $\mathrm{P}(A)$ *(A1)*

eg $0.25, \dfrac{0.2}{0.8}$

correct working for $\mathrm{P}(A \cup B)$ *(A1)*

eg $0.25 + 0.8 - 0.2$, $0.6 + 0.2 + 0.05$

$\mathrm{P}(A \cup B) = 0.85$ ***A1*** ***N3***

[4 marks]

[Total 6 marks]

6. evidence of integration *(M1)*

eg $\int f'(x)\,\mathrm{d}x$

correct integration (accept missing C) *(A2)*

eg $\dfrac{1}{2} \times \dfrac{\sin^4(2x)}{4}, \dfrac{1}{8}\sin^4(2x) + C$

substituting initial condition into their integrated expression (must have $+C$) ***M1***

eg $1 = \dfrac{1}{8}\sin^4\left(\dfrac{\pi}{2}\right) + C$

Note: Award ***M0*** if they substitute into the original or differentiated function.

recognizing $\sin\left(\dfrac{\pi}{2}\right) = 1$ *(A1)*

eg $1 = \dfrac{1}{8}(1)^4 + C$

$C = \dfrac{7}{8}$ *(A1)*

$f(x) = \dfrac{1}{8}\sin^4(2x) + \dfrac{7}{8}$ ***A1*** ***N5***

[7 marks]

7. valid approach *(M1)*

eg $f = y$, $m - \frac{1}{x} = x - m$

correct working to eliminate denominator *(A1)*

eg $mx - 1 = x(x - m)$, $mx - 1 = x^2 - mx$

correct quadratic equal to zero ***A1***

eg $x^2 - 2mx + 1 = 0$

correct reasoning ***R1***

eg for two solutions, $b^2 - 4ac > 0$

correct substitution into the discriminant formula *(A1)*

eg $(-2m)^2 - 4$

correct working *(A1)*

eg $4m^2 > 4$, $m^2 = 1$, sketch of positive parabola on the x-axis

correct interval ***A1*** ***N4***

eg $|m| > 1$, $m < -1$ or $m > 1$

Section B

8. (a) (i) valid approach to find $\vec{AB}$ ***(M1)***

eg $\vec{OB} - \vec{OA}$, $\begin{pmatrix} 4-(-1) \\ 1-0 \\ 3-4 \end{pmatrix}$

$\vec{AB} = \begin{pmatrix} 5 \\ 1 \\ -1 \end{pmatrix}$ ***A1 N2***

(ii) valid approach to find $\left|\vec{AB}\right|$ ***(M1)***

eg $\sqrt{(5)^2 + (1)^2 + (-1)^2}$

$\left|\vec{AB}\right| = \sqrt{27}$ ***A1 N2***

[4 marks]

(b) correct approach ***A1***

eg $\vec{OC} = \begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix} + \begin{pmatrix} -1 \\ 0 \\ 4 \end{pmatrix}$

C has coordinates $(-2, 1, 3)$ ***AG N0***

[1 mark]

(c) (i) $A\hat{D}B = \pi - \theta$, $\hat{D} = 180 - \theta$ ***A1 N1***

(ii) any correct expression for the area involving θ ***A1 N1***

eg $\text{area} = \frac{1}{2} \times AD \times BD \times \sin(180 - \theta)$, $\frac{1}{2} ab \sin\theta$, $\frac{1}{2}\left|\vec{DA}\right|\left|\vec{DB}\right| \sin(\pi - \theta)$

[2 marks]

continued…

Question 8 continued

(d) **METHOD 1** (using sine formula for area)

correct expression for the area of triangle ACD (seen anywhere) ***(A1)***

eg $\frac{1}{2}\text{AD}\times\text{DC}\times\sin\theta$

correct equation involving areas ***A1***

eg $\frac{\frac{1}{2}\text{AD}\times\text{BD}\times\sin(\pi-\theta)}{\frac{1}{2}\text{AD}\times\text{DC}\times\sin\theta}=3$

recognizing that $\sin(\pi-\theta)=\sin\theta$ (seen anywhere) ***(A1)***

$\frac{\text{BD}}{\text{DC}}=3$ (seen anywhere) ***(A1)***

correct approach using ratio ***A1***

eg $3\overrightarrow{\text{DC}}+\overrightarrow{\text{DC}}=\overrightarrow{\text{BC}}$, $\overrightarrow{\text{BC}}=4\overrightarrow{\text{DC}}$

correct ratio $\frac{\text{BD}}{\text{BC}}=\frac{3}{4}$ ***AG*** ***N0***

METHOD 2 (Geometric approach)

recognising ΔABD and ΔACD have same height ***(A1)***

eg use of h for both triangles, $\frac{\frac{1}{2}\text{BD}\times h}{\frac{1}{2}\text{CD}\times h}=3$

correct approach ***A2***

eg $\text{BD}=3x$ and $\text{DC}=x$, $\frac{\text{BD}}{\text{DC}}=3$

correct working ***A2***

eg $\text{BC}=4x$, $\text{BD}+\text{DC}=4\text{DC}$, $\frac{\text{BD}}{\text{BC}}=\frac{3x}{4x}$, $\frac{\text{BD}}{\text{BC}}=\frac{3\text{DC}}{4\text{DC}}$

$\frac{\text{BD}}{\text{BC}}=\frac{3}{4}$ ***AG*** ***N0***

[5 marks]

continued…

Question 8 continued

(e) correct working (seen anywhere) ***(A1)***

eg $\vec{BD} = \frac{3}{4}\vec{BC}$, $\vec{OD} = \vec{OB} + \frac{3}{4}\begin{pmatrix}-6\\0\\0\end{pmatrix}$, $\vec{CD} = \frac{1}{4}\vec{CB}$

valid approach (seen anywhere) ***(M1)***

eg $\vec{OD} = \vec{OB} + \vec{BD}$, $\overrightarrow{BC} = \begin{pmatrix}-6\\0\\0\end{pmatrix}$

correct working to find x-coordinate ***(A1)***

eg $\begin{pmatrix}4\\1\\3\end{pmatrix} + \frac{3}{4}\begin{pmatrix}-6\\0\\0\end{pmatrix}$, $x = 4 + \frac{3}{4}(-6)$, $-2 + \frac{1}{4}(6)$

D is $\left(-\frac{1}{2}, 1, 3\right)$ ***A1 N3***

[4 marks]

[Total 16 marks]

9. (a) evidence of dividing terms (in any order) ***(M1)***

eg $\frac{u_2}{u_1}$, $\frac{2\log_2 x}{\log_2 x}$

$r = \frac{1}{2}$ ***A1 N2***

[2 marks]

(b) correct substitution ***(A1)***

eg $\frac{2\log_2 x}{1-\frac{1}{2}}$

correct working ***A1***

eg $\frac{2\log_2 x}{\frac{1}{2}}$

$S_\infty = 4\log_2 x$ ***AG N0***

[2 marks]

continued…

Question 9 continued

(c) evidence of subtracting two terms (in any order) ***(M1)***

eg $u_3 - u_2$, $\log_2 x - \log_2 \frac{x}{2}$

correct application of the properties of logs ***(A1)***

eg $\log_2\left(\frac{\frac{x}{2}}{x}\right)$, $\log_2\left(\frac{x}{2} \times \frac{1}{x}\right)$, $(\log_2 x - \log_2 2) - \log_2 x$

correct working ***(A1)***

eg $\log_2 \frac{1}{2}$, $-\log_2 2$

$d = -1$ ***A1 N3***

[4 marks]

(d) correct substitution into the formula for the sum of an arithmetic sequence ***(A1)***

eg $\frac{12}{2}\left(2\log_2 x + (12-1)(-1)\right)$

correct working ***A1***

eg $6\left(2\log_2 x - 11\right)$, $\frac{12}{2}\left(2\log_2 x - 11\right)$

$12\log_2 x - 66$ ***AG N0***

[2 marks]

(e) correct equation ***(A1)***

eg $12\log_2 x - 66 = 2\log_2 x$

correct working ***(A1)***

eg $10\log_2 x = 66$, $\log_2 x = 6.6$, $2^{66} = x^{10}$, $\log_2\left(\frac{x^{12}}{x^2}\right) = 66$

$x = 2^{6.6}$ (accept $p = \frac{66}{10}$) ***A1 N2***

[3 marks]

[Total 13 marks]

10. (a) (i) $f'(x) = -\sin x,\ f''(x) = -\cos x,\ f^{(3)}(x) = \sin x,\ f^{(4)}(x) = \cos x$ ***A2*** ***N2***

(ii) valid approach ***(M1)***

eg recognizing that 19 is one less than a multiple of 4, $f^{(19)}(x) = f^{(3)}(x)$

$f^{(19)}(x) = \sin x$ ***A1*** ***N2***

[4 marks]

(b) (i) $g'(x) = kx^{k-1}$

$g''(x) = k(k-1)x^{k-2},\ g^{(3)}(x) = k(k-1)(k-2)x^{k-3}$ ***A1A1*** ***N2***

(ii) **METHOD 1**

correct working that leads to the correct answer, involving the correct expression for the 19th derivative ***A2***

eg $k(k-1)(k-2)\ldots(k-18)\times\dfrac{(k-19)!}{(k-19)!},\ {}_kP_{19}$

$p = 19$ (accept $\dfrac{k!}{(k-19)!}x^{k-19}$) ***A1*** ***N1***

METHOD 2

correct working involving recognizing patterns in coefficients of first three derivatives (may be seen in part (b)(i)) leading to a general rule for 19th coefficient ***A2***

eg $g'' = 2!\begin{pmatrix}k\\2\end{pmatrix},\ k(k-1)(k-2) = \dfrac{k!}{(k-3)!},\ g^{(3)}(x) = {}_kP_3\left(x^{k-3}\right),$

$g^{(19)}(x) = 19!\begin{pmatrix}k\\19\end{pmatrix}, 19!\times\dfrac{k!}{(k-19)!\times 19!},\ {}_kP_{19}$

$p = 19$ (accept $\dfrac{k!}{(k-19)!}x^{k-19}$) ***A1*** ***N1***

[5 marks]

continued…

Question 10 continued

(c) (i) valid approach using product rule ***(M1)***

eg $uv' + vu'$, $f^{(19)}g^{(20)} + f^{(20)}g^{(19)}$

correct 20th derivatives (must be seen in product rule) ***(A1)(A1)***

eg $g^{(20)}(x) = \frac{21!}{(21-20)!}x$, $f^{(20)}(x) = \cos x$

$h'(x) = \sin x(21!x) + \cos x\left(\frac{21!}{2}x^2\right)$ $\left(\text{accept } \sin x\left(\frac{21!}{1!}x\right) + \cos x\left(\frac{21!}{2!}x^2\right)\right)$ ***A1*** ***N3***

(ii) substituting $x = \pi$ (seen anywhere) ***(A1)***

eg $f^{(19)}(\pi)g^{(20)}(\pi) + f^{(20)}(\pi)g^{(19)}(\pi)$, $\sin\pi\frac{21!}{1!}\pi + \cos\pi\frac{21!}{2!}\pi^2$

evidence of one correct value for $\sin\pi$ or $\cos\pi$ (seen anywhere) ***(A1)***

eg $\sin\pi = 0$, $\cos\pi = -1$

evidence of correct values substituted into $h'(\pi)$ ***A1***

eg $21!(\pi)\left(0 - \frac{\pi}{2!}\right)$, $21!(\pi)\left(-\frac{\pi}{2}\right)$, $0 + (-1)\frac{21!}{2}\pi^2$

Note: If candidates write only the first line followed by the answer, award ***A1A0A0***.

$\frac{-21!}{2}\pi^2$ ***AG*** ***N0***

[7 marks]

[Total 16 marks]

M16/5/MATME/SP1/ENG/TZ1/XX/M

Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

May 2016

Mathematics

Standard level

Paper 1

14 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a valid **Method**; working must be seen.

(M) Marks awarded for a valid **Method**; may be implied by **correct** subsequent working.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

(A) Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.

R Marks awarded for clear **Reasoning**.

N Marks awarded for **correct** answers if **no** working shown.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Mark according to RM assessor instructions.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any. An exception to this rule is when work for ***M1*** is missing, as opposed to incorrect (see point 4).
- Where ***M*** and ***A*** marks are noted on the same line, *eg* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*eg* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***(M2)***, ***N3***, *etc*., do **not** split the marks, unless there is a note.
- Most ***M*** marks are for a **valid** method, ie a method which can lead to the answer: it must indicate some form of progress towards the answer.
- Once a correct answer to a question or part-question is seen, ignore further correct working. However, if further working indicates a lack of mathematical understanding do not award final ***A1***.

3 *N* marks

*If **no** working shown, award **N** marks for **correct** answers – this includes acceptable answers (see accuracy booklet). In this case, ignore mark breakdown (**M, A, R**). Where a student only shows a final incorrect answer with no working, even if that answer is a correct intermediate answer, award **N0.***

- Do **not** award a mixture of ***N*** and other marks.
- There may be fewer ***N*** marks available than the total of ***M***, ***A*** and ***R*** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.
- There may not be a direct relationship between the ***N*** marks and the implied marks. There are times when all the marks are implied, but the ***N*** marks are not the full marks: this indicates that we want to see some of the working, without specifying what.

- For consistency within the markscheme, ***N*** marks are noted for every part, even when these match the mark breakdown.
- If a candidate has incorrect working, which somehow results in a correct answer, do **not** award the ***N*** marks for this correct answer. However, if the candidate has indicated (usually by crossing out) that the working is to be ignored, award the ***N*** marks for the correct answer.

4 Implied and must be seen marks

*Implied marks appear in **brackets** eg **(M1)**.*

- Implied marks can only be awarded if the work is seen or if implied in subsequent working (a correct final answer does not necessarily mean that the implied marks are all awarded). There are questions where some working is required, but as it is accepted that not everyone will write the same steps, all the marks are implied, but the ***N*** marks are not the full marks for the question.
- Normally the correct work is seen in the next line.
- Where there is an (***M1***) followed by ***A1*** for each correct answer, if no working shown, one correct answer is sufficient evidence to award the (***M1***).

*Must be seen marks appear without **brackets** eg **M1**.*

- Must be seen marks can only be awarded if the work is seen.
- If a must be seen mark is not awarded because work is missing (as opposed to ***M0*** or ***A0*** for incorrect work) all subsequent marks may be awarded if appropriate.

5 Follow through marks (only applied after an error is made)

*Follow through (**FT**) marks are awarded where an incorrect answer (final or intermediate) from one **part** of a question is used correctly in **subsequent** part(s) or subpart(s). Usually, to award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if the only marks awarded in a subpart are for the **final** answer, then **FT** marks should be awarded if appropriate. Examiners are expected to check student work in order to award **FT** marks where appropriate.*

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** and ***R*** marks may be awarded if appropriate. (However, as noted above, if an ***A*** mark is not awarded because work is missing, all subsequent marks may be awarded if appropriate).
- Exceptions to this rule will be explicitly noted on the markscheme.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*eg* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word "**their**" in a description, to indicate that candidates may be using an incorrect value.
- If a candidate makes an error in one part, but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the question says hence. It is often possible to use a different approach in subsequent parts that does not depend on the answer to previous parts.
- In a "show that" question, if an error in a previous subpart leads to not showing the required answer, do not award the final ***A1***. Note that if the error occurs within the same subpart, the ***FT*** rules may result in further loss of marks.

6 Mis-read

*If a candidate incorrectly copies information from the question, this is a mis-read (**MR**). A candidate should be penalized only once for a particular mis-read. Use the **MR** stamp to indicate that this is a misread. Do not award the first mark in the question, even if this is an **M** mark, but award all others (if appropriate) so that the candidate only loses one mark for the misread.*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*eg* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.

7 Discretionary marks *(d)*

*An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. In such cases the annotation DM should be used and a brief **note** written next to the mark explaining this decision.*

8 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete parts are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**. Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9 Alternative forms

*Unless the question specifies otherwise, **accept** equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

10 Calculators

No calculator is allowed. The use of any calculator on paper 1 is malpractice, and will result in no grade awarded. If you see work that suggests a candidate has used any calculator, please follow the procedures for malpractice. Examples: finding an angle, given a trig ratio of 0.4235.

11 Style

The markscheme aims to present answers using good communication, eg if the question asks to find the value of k, the markscheme will say $k = 3$, but the marks will be for the correct value 3 – there is usually no need for the "$k =$". In these cases, it is also usually acceptable to have another variable, as long as there is no ambiguity in the question, eg if the question asks to find the value of p and of q, then the student answer needs to be clear. Generally, the only situation

where the full answer is required is in a question which asks for equations – in this case the markscheme will say "must be an equation".

*The markscheme often uses words to describe what the marks are for, followed by examples, using the eg notation. These examples are not exhaustive, and examiners should check what candidates have written, to see if they satisfy the description. Where these marks are **M** marks, the examples may include ones using poor notation, to indicate what is acceptable. A valid method is one which will allow candidate to proceed to the next step eg if a quadratic function is given in factorised form, and the question asks for the zeroes, then multiplying the factors does not necessarily help to find the zeros, and would not on its own count as a valid method.*

12 Candidate work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

Candidates are meant to write their answers to Section A on the question paper (QP), and Section B on answer booklets. Sometimes, they need more room for Section A, and use the booklet (and often comment to this effect on the QP), or write outside the box. That is fine, and this work should be marked.

The instructions tell candidates not to write on Section B of the QP. Thus they may well have done some rough work here which they assume will be ignored. If they have solutions on the answer booklets, there is no need to look at the QP. However, if there are whole questions or whole part solutions missing on answer booklets, please check to make sure that they are not on the QP, and if they are, mark those whole questions or whole part solutions that have not been written on answer booklets.

13. Diagrams

The notes on how to allocate marks for sketches usually refer to passing through particular points or having certain features. These marks can only be awarded if the sketch is approximately the correct shape. All values given will be an approximate guide to where these points/features occur. In some questions, the first ***A1*** is for the shape, in others, the marks are only for the points and/or features. In both cases, unless the shape is approximately correct, no marks can be awarded (unless otherwise stated). However, if the graph is based on previous calculations, ***FT*** marks should be awarded if appropriate.

14. Accuracy of Answers

If the level of accuracy is specified in the question, a mark will be allocated for giving the final answer to the required accuracy. When this is not specified in the question, all numerical answers should be given exactly or correct to three significant figures.

Do not accept unfinished numerical final answers such as 3/0.1 (unless otherwise stated). As a rule, numerical answers with more than one part (such as fractions) should be given using integers (eg 6/8). Calculations which lead to integers should be completed, with the exception of fractions which are not whole numbers. Intermediate values do not need to be given to the correct three significant figures. But, if candidates work with rounded values, this could lead to an incorrect answer, in which case award A0 for the final answer. Where numerical answers are required as the final answer to a part of a question in the markscheme, the markscheme will show

a truncated 6 sf value
the exact value if applicable, the correct 3 sf answer
Units will appear in brackets at the end.

Section A

1. (a) $g(2) = 8$ ***A1*** ***N1***

[1 mark]

(b) attempt to form composite (in any order) ***(M1)***

eg $f(4x)$, $4 \times (8x + 3)$

$(f \circ g)(x) = 32x + 3$ ***A1*** ***N2***

[2 marks]

(c) interchanging x and y (may be seen at any time) ***(M1)***

eg $x = 8y + 3$

$f^{-1}(x) = \frac{x-3}{8}$ (accept $\frac{x-3}{8}$, $y = \frac{x-3}{8}$) ***A1*** ***N2***

[2 marks]

Total [5 marks]

2. (a) (i) $q = 0.1$ ***A1*** ***N1***

(ii) appropriate approach ***(M1)***

eg $\mathrm{P}(A) - q$, $0.4 - 0.1$

$p = 0.3$ ***A1*** ***N2***

[3 marks]

(b) valid approach ***(M1)***

eg $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$, $\mathrm{P}(A \cap B) + \mathrm{P}(B \cap A')$

correct values ***(A1)***

eg $0.8 = 0.4 + \mathrm{P}(B) - 0.1$, $0.1 + 0.4$

$\mathrm{P}(B) = 0.5$ ***A1*** ***N2***

[3 marks]

Total [6 marks]

3. (a) (i) 3 ***A1*** ***N1***

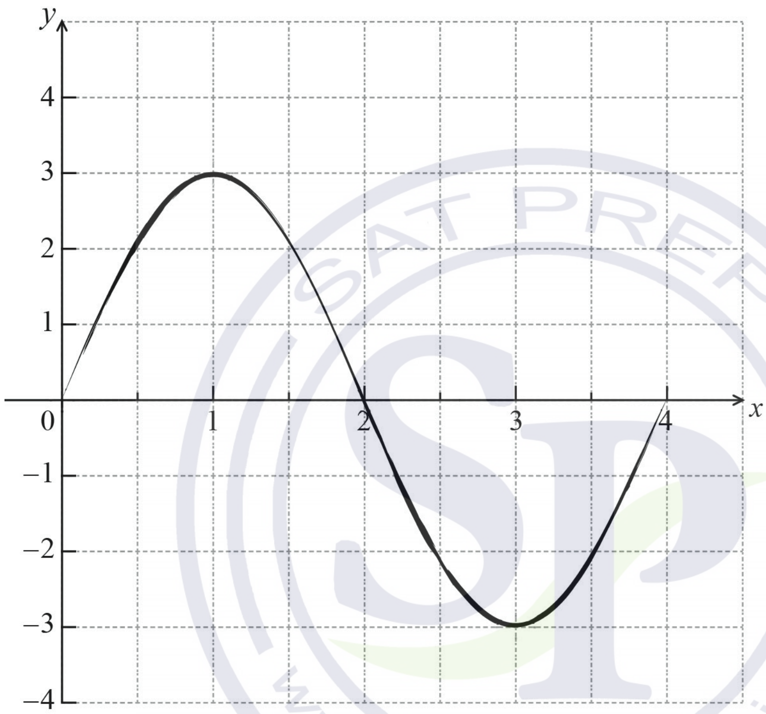
(ii) valid attempt to find the period ***(M1)***

eg $\frac{2\pi}{b},\ \frac{2\pi}{\frac{\pi}{2}}$

period $= 4$ ***A1*** ***N2***

[3 marks]

(b)



A1A1A1A1 ***N4***

[4 marks]

Total [7 marks]

4. (a) recognizing that it is an arithmetic sequence *(M1)*

eg $5, 5+4, 5+4+4, \ldots, d=4$, $u_n = u_1 + (n-1)d$, $4n+1$

correct equation ***A1***

eg $5+4(n-1)=801$

correct working (do not accept substituting $n=200$) ***A1***

eg $4n-4=796$, $n-1=\frac{796}{4}$

$n=200$ ***AG*** ***N0***

[3 marks]

(b) recognition of sum *(M1)*

eg S_{200}, $u_1+u_2+\ldots+u_{200}$, $5+9+13+\ldots+801$

correct working for AP *(A1)*

eg $\frac{200}{2}(5+801)$, $\frac{200}{2}\left(2(5)+199(4)\right)$

$80\,600$ ***A1*** ***N2***

[3 marks]

Total [6 marks]

5. (a) recognition that the x-coordinate of the vertex is -1.5 (seen anywhere) ***(M1)***
eg axis of symmetry is –1.5, sketch, $f'(-1.5)=0$

correct working to find the zeroes ***A1***
eg -1.5 ± 4.5

$x=-6$ and $x=3$ ***AG*** ***N0***
[2 marks]

(b) **METHOD 1 (using factors)**

attempt to write factors ***(M1)***
eg $(x-6)(x+3)$

correct factors ***A1***
eg $(x-3)(x+6)$

$q=3,\ r=-18$ ***A1A1*** ***N3***

METHOD 2 (using derivative or vertex)

valid approach to find q ***(M1)***

eg $f'(-1.5)=0,\ -\frac{q}{2a}=-1.5$

$q=3$ ***A1***
correct substitution ***A1***
eg $3^2+3(3)+r=0,\ (-6)^2+3(-6)+r=0$

$r=-18$ ***A1***
$q=3,\ r=-18$ ***N3***

METHOD 3 (solving simultaneously)

valid approach setting up system of two equations ***(M1)***
eg $9+3q+r=0,\ 36-6q+r=0$

one correct value
eg $q=3,\ r=-18$ ***A1***

correct substitution ***A1***
eg $3^2+3(3)+r=0,\ (-6)^2+3(-6)+r=0,\ 3^2+3q-18=0,\ 36-6q-18=0$

second correct value ***A1***
eg $q=3,\ r=-18$

$q=3,\ r=-18$ ***N3***

[4 marks]

Total [6 marks]

6. attempt to substitute side lengths **or** $\sin 2\theta$ into $\frac{1}{2}ab\sin C$ (seen anywhere) ***(M1)***

eg $\frac{1}{2}\times 2\sqrt{5}\times x\times\sin\theta$, $\frac{1}{2}ab\sin 2\theta$, $\frac{1}{2}\times 2\sqrt{5}\times x\sin 2\theta$

attempt to find $\cos\theta$ (seen anywhere) ***(M1)***

eg sketch of right triangle with sides 2 and 3, $\sqrt{1-\sin^2\theta}$

Note: Do not award the ***M1*** if ΔADC is assumed to be a right triangle.

correct working (seen anywhere) ***(A1)***

eg $\sqrt{5}$ on sketch, $\sqrt{1-\frac{4}{9}}$

$\cos\theta=\frac{\sqrt{5}}{3}$ (seen anywhere) ***A1***

correct equation ***A1***

eg $\frac{1}{2}\times 2\sqrt{5}\times x\times 2\times\frac{2}{3}\times\frac{\sqrt{5}}{3}=5$, $\frac{20x}{9}=5$

$x=\frac{9}{4}$ ***A2*** ***N2***

[7 marks]

7. discriminant $=0$ (seen anywhere) ***M1***

valid approach ***(M1)***

eg $f=g$, $3\tan^4 x+2k=-\tan^4 x+8k\tan^2 x+k$

rearranging their equation (to equal zero) ***(M1)***

eg $4\tan^4 x-8k\tan^2 x+k=0$, $4\tan^4 x-8k\tan^2 x+k$

recognizing LHS is quadratic ***(M1)***

eg $4\left(\tan^2 x\right)^2-8k\tan^2 x+k=0$, $4m^2-8km+k$

correct substitution into discriminant ***A1***

eg $(-8k)^2-4(4)(k)$

correct working to find discriminant or solve discriminant $=0$ ***(A1)***

eg $64k^2-16k$, $\frac{-(-16)\pm\sqrt{16^2}}{2\times 64}$

correct simplification ***(A1)***

eg $16k(4k-1)$, $\frac{32}{2\times 64}$

$k=\frac{1}{4}$ ***A1*** ***N2***

[8 marks]

Section B

8. (a) valid approach *(M1)*

eg between 10th and 11th, $\frac{8+8}{2}$

median $= 38$ *A1* *N2*

[2 marks]

(b) (i) $a = 20$ *A1* *N1*

(ii) valid approach *(M1)*

eg $Q_3 - Q_1$, $Q_1 + 14$, $b - 30 = 14$

$b = 44$ *A1* *N2*

[3 marks]

(c) valid approach *(M1)*

eg 40×20, $\frac{x+745}{20}$, $40 - \frac{745}{20}$

correct working *(A1)*

eg $800 - 745$, 20×2.75

55 (more cans) *A1* *N2*

[3 marks]

(d) (i) most cans in Sam's class $= 50$ *(A1)*

5 ($\$$) *A1* *N2*

(ii) correct value of 64 or 16 *A1*

valid approach *(M1)*

eg $\frac{64}{80}, 80\%, 80-64, \frac{16}{80}$

20% *A1* *N2*

[5 marks]

(e) (i) 41.4 (exact) *A1* *N1*

(ii) 18.5 *A1* *N1*

[2 marks]

Total [15 marks]

9. (a) recognizing $f'(x)=0$ ***(M1)***

correct working ***(A1)***

eg $6-2x=0$

$x=3$ ***A1 N2***

[3 marks]

(b) evidence of integration ***(M1)***

eg $\int f'$, $\int \frac{6-2x}{6x-x^2}\mathrm{d}x$

using substitution ***(A1)***

eg $\int \frac{1}{u}du$ where $u=6x-x^2$

correct integral ***A1***

eg $\ln(u)+c$, $\ln\left(6x-x^2\right)$

substituting $(3, \ln 27)$ into **their** integrated expression (must have c) ***(M1)***

eg $\ln\left(6\times3-3^2\right)+c=\ln 27$, $\ln(18-9)+\ln k=\ln 27$

correct working ***(A1)***

eg $c=\ln 27-\ln 9$

EITHER

$c=\ln 3$ ***(A1)***

attempt to substitute **their** value of c into $f(x)$ ***(M1)***

eg $f(x)=\ln\left(6x-x^2\right)+\ln 3$

$f(x)=\ln\left(3\left(6x-x^2\right)\right)$ ***A1 N4***

OR

attempt to substitute **their** value of c into $f(x)$ ***(M1)***

eg $f(x)=\ln\left(6x-x^2\right)+\ln 27-\ln 9$

correct use of a log law ***(A1)***

eg $f(x)=\ln\left(6x-x^2\right)+\ln\left(\frac{27}{9}\right)$, $f(x)=\ln\left(27\left(6x-x^2\right)\right)-\ln 9$

$f(x)=\ln\left(3\left(6x-x^2\right)\right)$ ***A1 N4***

[8 marks]

(c) $a=3$ ***A1 N1***

correct working ***A1***

eg $\frac{\ln 27}{\ln 3}$

correct use of log law ***(A1)***

eg $\frac{3\ln 3}{\ln 3}$, $\log_3 27$

$b=3$ ***A1 N2***

[4 marks]

Total [15 marks]

10. (a) choosing chain rule **(M1)**

eg $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$, $u = 4x+5$, $u' = 4$

correct derivative of f **A2**

eg $\frac{1}{2}(4x+5)^{-\frac{1}{2}} \times 4$, $f'(x) = \frac{2}{\sqrt{4x+5}}$

$f'(1) = \frac{2}{3}$ **A1** **N2**

[4 marks]

(b) recognize that $g'(x)$ is the gradient of the tangent **(M1)**

eg $g'(x) = m$

$g'(1) = 3$ **A1** **N2**

[2 marks]

(c) recognize that R is on the tangent **(M1)**

eg $g(1) = 3 \times 1 + 6$, sketch

$g(1) = 9$ **A1** **N2**

[2 marks]

(d) $f(1) = \sqrt{4+5}$ $(= 3)$ (seen anywhere) **A1**

$h(1) = 3 \times 9$ $(= 27)$ (seen anywhere) **A1**

choosing product rule to find $h'(x)$ **(M1)**

eg $uv' + u'v$

correct substitution to find $h'(1)$ **(A1)**

eg $f(1) \times g'(1) + f'(1) \times g(1)$

$h'(1) = 3 \times 3 + \frac{2}{3} \times 9$ $(= 15)$ **A1**

EITHER

attempt to substitute coordinates (in any order) into the equation of a straight line **(M1)**

eg $y - 27 = h'(1)(x-1)$, $y - 1 = 15(x - 27)$

$y - 27 = 15(x-1)$ **A1** **N2**

OR

attempt to substitute coordinates (in any order) to find the y-intercept **(M1)**

eg $27 = 15 \times 1 + b$, $1 = 15 \times 27 + b$

$y = 15x + 12$ **A1** **N2**

[7 marks]

Markscheme

May 2016

Mathematics

Standard level

Paper 1

Instructions to Examiners

Abbreviations

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(A) Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.

R Marks awarded for clear **Reasoning**.

N Marks awarded for **correct** answers if **no** working shown.

AG Answer given in the question and so no marks are awarded.

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1 General

Mark according to RM assessor instructions.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
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- Where ***M*** and ***A*** marks are noted on the same line, *eg* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*eg* substitution into a formula) and ***A1*** for using the **correct** values.
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- Where the markscheme specifies ***(M2)***, ***N3***, *etc*., do **not** split the marks, unless there is a note.
- Most ***M*** marks are for a **valid** method, ie a method which can lead to the answer: it must indicate some form of progress towards the answer.
- Once a correct answer to a question or part-question is seen, ignore further correct working. However, if further working indicates a lack of mathematical understanding do not award final ***A1***.

3 *N* marks

*If **no** working shown, award **N** marks for **correct** answers – this includes acceptable answers (see accuracy booklet). In this case, ignore mark breakdown (**M, A, R**). Where a student only shows a final incorrect answer with no working, even if that answer is a correct intermediate answer, award **N0.***

- Do **not** award a mixture of ***N*** and other marks.
- There may be fewer ***N*** marks available than the total of ***M***, ***A*** and ***R*** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.
- There may not be a direct relationship between the ***N*** marks and the implied marks. There are times when all the marks are implied, but the ***N*** marks are not the full marks: this indicates that we want to see some of the working, without specifying what.

- For consistency within the markscheme, ***N*** marks are noted for every part, even when these match the mark breakdown.
- If a candidate has incorrect working, which somehow results in a correct answer, do **not** award the ***N*** marks for this correct answer. However, if the candidate has indicated (usually by crossing out) that the working is to be ignored, award the ***N*** marks for the correct answer.

4 Implied and must be seen marks

Implied marks appear in ***brackets*** *eg* ***(M1)****.*

- Implied marks can only be awarded if the work is seen or if implied in subsequent working (a correct final answer does not necessarily mean that the implied marks are all awarded). There are questions where some working is required, but as it is accepted that not everyone will write the same steps, all the marks are implied, but the ***N*** marks are not the full marks for the question.
- Normally the correct work is seen in the next line.
- Where there is an (***M1***) followed by ***A1*** for each correct answer, if no working shown, one correct answer is sufficient evidence to award the (***M1***).

Must be seen marks appear without ***brackets*** *eg* ***M1***.

- Must be seen marks can only be awarded if the work is seen.
- If a must be seen mark is not awarded because work is missing (as opposed to ***M0*** or ***A0*** for incorrect work) all subsequent marks may be awarded if appropriate.

5 Follow through marks (only applied after an error is made)

*Follow through (**FT**) marks are awarded where an incorrect answer (final or intermediate) from one **part** of a question is used correctly in **subsequent** part(s) or subpart(s). Usually, to award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if the only marks awarded in a subpart are for the **final** answer, then **FT** marks should be awarded if appropriate. Examiners are expected to check student work in order to award **FT** marks where appropriate.*

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** and ***R*** marks may be awarded if appropriate. (However, as noted above, if an ***A*** mark is not awarded because work is missing, all subsequent marks may be awarded if appropriate).
- Exceptions to this rule will be explicitly noted on the markscheme.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*eg* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word "**their**" in a description, to indicate that candidates may be using an incorrect value.
- If a candidate makes an error in one part, but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the question says hence. It is often possible to use a different approach in subsequent parts that does not depend on the answer to previous parts.
- In a "show that" question, if an error in a previous subpart leads to not showing the required answer, do not award the final ***A1***. Note that if the error occurs within the same subpart, the ***FT*** rules may result in further loss of marks.

6 Mis-read

*If a candidate incorrectly copies information from the question, this is a mis-read (**MR**). A candidate should be penalized only once for a particular mis-read. Use the **MR** stamp to indicate that this is a misread. Do not award the first mark in the question, even if this is an **M** mark, but award all others (if appropriate) so that the candidate only loses one mark for the misread.*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*eg* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.

7 Discretionary marks *(d)*

*An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. In such cases the annotation DM should be used and a brief **note** written next to the mark explaining this decision.*

8 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete parts are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**. Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9 Alternative forms

*Unless the question specifies otherwise, **accept** equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

10 Calculators

No calculator is allowed. The use of any calculator on paper 1 is malpractice, and will result in no grade awarded. If you see work that suggests a candidate has used any calculator, please follow the procedures for malpractice. Examples: finding an angle, given a trig ratio of 0.4235.

11 Style

The markscheme aims to present answers using good communication, eg if the question asks to find the value of k, the markscheme will say $k = 3$, but the marks will be for the correct value 3 – there is usually no need for the "$k =$". In these cases, it is also usually acceptable to have another variable, as long as there is no ambiguity in the question, eg if the question asks to find the value of p and of q, then the student answer needs to be clear. Generally, the only situation

where the full answer is required is in a question which asks for equations – in this case the markscheme will say "must be an equation".

*The markscheme often uses words to describe what the marks are for, followed by examples, using the eg notation. These examples are not exhaustive, and examiners should check what candidates have written, to see if they satisfy the description. Where these marks are **M** marks, the examples may include ones using poor notation, to indicate what is acceptable. A valid method is one which will allow candidate to proceed to the next step eg if a quadratic function is given in factorised form, and the question asks for the zeroes, then multiplying the factors does not necessarily help to find the zeros, and would not on its own count as a valid method.*

12 Candidate work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

Candidates are meant to write their answers to Section A on the question paper (QP), and Section B on answer booklets. Sometimes, they need more room for Section A, and use the booklet (and often comment to this effect on the QP), or write outside the box. That is fine, and this work should be marked.

The instructions tell candidates not to write on Section B of the QP. Thus they may well have done some rough work here which they assume will be ignored. If they have solutions on the answer booklets, there is no need to look at the QP. However, if there are whole questions or whole part solutions missing on answer booklets, please check to make sure that they are not on the QP, and if they are, mark those whole questions or whole part solutions that have not been written on answer booklets.

13. Diagrams

The notes on how to allocate marks for sketches usually refer to passing through particular points or having certain features. These marks can only be awarded if the sketch is approximately the correct shape. All values given will be an approximate guide to where these points/features occur. In some questions, the first ***A1*** is for the shape, in others, the marks are only for the points and/or features. In both cases, unless the shape is approximately correct, no marks can be awarded (unless otherwise stated). However, if the graph is based on previous calculations, ***FT*** marks should be awarded if appropriate.

14. Accuracy of Answers

If the level of accuracy is specified in the question, a mark will be allocated for giving the final answer to the required accuracy. When this is not specified in the question, all numerical answers should be given exactly or correct to three significant figures.

Do not accept unfinished numerical final answers such as 3/0.1 (unless otherwise stated). As a rule, numerical answers with more than one part (such as fractions) should be given using integers (eg 6/8). Calculations which lead to integers should be completed, with the exception of fractions which are not whole numbers. Intermediate values do not need to be given to the correct three significant figures. But, if candidates work with rounded values, this could lead to an incorrect answer, in which case award A0 for the final answer. Where numerical answers are required as the final answer to a part of a question in the markscheme, the markscheme will show

a truncated 6 sf value
the exact value if applicable, the correct 3 sf answer
Units will appear in brackets at the end.

Section A

1. (a) $h=3,\ k=-1$ *A1A1* *N2*
[2 marks]

(b) $a=2,\ b=4$ (or $a=4,\ b=2$) *A1A1* *N2*
[2 marks]

(c) attempt to substitute $x=0$ into their f *(M1)*
eg $(0-3)^2-1,\ (0-2)(0-4)$

$y=8$ *A1* *N2*
[2 marks]

Total [6 marks]

2. (a) correct approach *(A1)*
eg $\frac{60}{10}$

mean $=6$ *A1* *N2*
[2 marks]

(b) (i) new mean $=24$ *A1* *N1*

(ii) valid approach *(M1)*
eg variance $\times(4)^2$, 3×16, new standard deviation $=4\sqrt{3}$

new variance $=48$ *A1* *N2*
[3 marks]

Total [5 marks]

3. (a) correct approach *(A1)*
eg $\ln5-\ln3$

$\ln\left(\frac{5}{3}\right)=y-x$ *A1* *N2*

[2 marks]

(b) recognizing factors of 45 (may be seen in log expansion) *(M1)*
eg $\ln(9\times5)$, $3\times3\times5$, $\log3^2\times\log5$

correct application of $\log(ab)=\log a+\log b$ *(A1)*
eg $\ln9+\ln5$, $\ln3+\ln3+\ln5$, $\ln3^2+\ln5$

correct working *(A1)*
eg $2\ln3+\ln5$, $x+x+y$

$\ln45=2x+y$ *A1* *N3*
[4 marks]

Total [6 marks]

4. **METHOD 1**

valid approach **(M1)**

eg $r=\dfrac{6}{x-3}$, $(x-3)\times r=6$, $(x-3)r^2=x+2$

correct equation in terms of *x* only **A1**

eg $\dfrac{6}{x-3}=\dfrac{x+2}{6}$, $(x-3)(x+2)=6^2$, $36=x^2-x-6$

correct working **(A1)**

eg x^2-x-42, $x^2-x=42$

valid attempt to solve **their** quadratic equation **(M1)**

eg factorizing, formula, completing the square

evidence of correct working **(A1)**

eg $(x-7)(x+6)$, $\dfrac{1\pm\sqrt{169}}{2}$

$x=7$, $x=-6$ **A1** **N4**

METHOD 2 (finding *r* first)

valid approach **(M1)**

eg $r=\dfrac{6}{x-3}$, $6r=x+2$, $(x-3)r^2=x+2$

correct equation in terms of *r* only **A1**

eg $\dfrac{6}{r}+3=6r-2$, $6+3r=6r^2-2r$, $6r^2-5r-6=0$

evidence of correct working **(A1)**

eg $(3r+2)(2r-3)$, $\dfrac{5\pm\sqrt{25+144}}{12}$

$r=-\dfrac{2}{3}$, $r=\dfrac{3}{2}$ **A1**

substituting their values of *r* to find *x* **(M1)**

eg $(x-3)\left(\dfrac{2}{3}\right)=6$, $x=6\left(\dfrac{3}{2}\right)-2$

$x=7$, $x=-6$ **A1** **N4**

5. (a) **METHOD 1**

correct substitution into formula for area of triangle *(A1)*

eg $\frac{1}{2}(6)\left(2\sqrt{3}\right)\sin B$, $6\sqrt{3}\sin B$, $\frac{1}{2}(6)\left(2\sqrt{3}\right)\sin B = 3\sqrt{3}$

correct working *(A1)*

eg $6\sqrt{3}\sin B = 3\sqrt{3}$, $\sin B = \frac{3\sqrt{3}}{\frac{1}{2}(6)2\sqrt{3}}$

$\sin B = \frac{1}{2}$ *(A1)*

$\frac{\pi}{6}$ (30°) *(A1)*

$\hat{ABC} = \frac{5\pi}{6}$ (150°) *A1* *N3*

METHOD 2

(using height of triangle ABC by drawing perpendicular segment from C to AD)

correct substitution into formula for area of triangle *(A1)*

eg $\frac{1}{2}\left(2\sqrt{3}\right)(h) = 3\sqrt{3}$, $h\sqrt{3}$

correct working *(A1)*

eg $h\sqrt{3} = 3\sqrt{3}$

height of triangle is 3 *A1*

$\hat{CBD} = \frac{\pi}{6}$ (30°) *(A1)*

$\hat{ABC} = \frac{5\pi}{6}$ (150°) *A1* *N3*

[5 marks]

(b) recognizing supplementary angle *(M1)*

eg $\hat{CBD} = \frac{\pi}{6}$, sector $= \frac{1}{2}\left(180 - \hat{ABC}\right)\left(6^2\right)$

correct substitution into formula for area of sector *(A1)*

eg $\frac{1}{2}\times\frac{\pi}{6}\times 6^2$, $\pi\left(6^2\right)\left(\frac{30}{360}\right)$

area $= 3\pi$ (cm^2) *A1* *N2*

[3 marks]

Total [8 marks]

6. (a) attempt to form composite in any order ***(M1)***

eg $f\left(g(x)\right)$, $\cos\left(6x\sqrt{1-x^2}\right)$

correct working ***(A1)***

eg $6\cos x\sqrt{1-\cos^2 x}$

correct application of Pythagorean identity (do not accept $\sin^2 x+\cos^2 x=1$) ***(A1)***

eg $\sin^2 x=1-\cos^2 x$, $6\cos x\sin x$, $6\cos x|\sin x|$

valid approach (do not accept $2\sin x\cos x=\sin 2x$) ***(M1)***

eg $3(2\cos x\sin x)$

$h(x)=3\sin 2x$ ***A1*** ***N3***

[5 marks]

(b) valid approach ***(M1)***

eg amplitude $=3$, sketch with max and min y-values labelled, $-3<y<3$

correct range ***A1*** ***N2***

eg $-3\le y\le 3$, $[-3, 3]$, from -3 to 3

Note: Do not award ***A1*** for $-3<y<3$ or for "between –3 and 3".

[2 marks]

Total [7 marks]

7. correct scalar product ***(A1)***

eg $m+n$

setting up their scalar product equal to 0 (seen anywhere) ***(M1)***

eg $\boldsymbol{u}\bullet\boldsymbol{v}=0,\ -3(0)+1(m)+1(n)=0,\ m=-n$

correct interpretation of unit vector ***(A1)***

eg $\sqrt{0^2+m^2+n^2}=1,\ m^2+n^2=1$

valid attempt to solve their equations (must be in one variable) ***M1***

eg $(-n)^2+n^2=1,\ \sqrt{1-n^2}+n=0,\ m^2+(-m)^2=1,\ m-\sqrt{1-m^2}=0$

correct working ***A1***

eg $2n^2=1,\ 2m^2=1,\ \sqrt{2}=\frac{1}{n},\ m=\pm\frac{1}{\sqrt{2}}$

both correct pairs ***A2*** ***N3***

eg $m=\frac{1}{\sqrt{2}}$ and $n=-\frac{1}{\sqrt{2}}$, $m=-\frac{1}{\sqrt{2}}$ and $n=\frac{1}{\sqrt{2}}$,

$m=(0.5)^{\frac{1}{2}}$. and $n=-(0.5)^{\frac{1}{2}}$., $m=-\sqrt{\frac{1}{2}}$ and $n=\sqrt{\frac{1}{2}}$

Note:	Award ***A0*** for $m=\pm\frac{1}{\sqrt{2}}$, $n=\pm\frac{1}{\sqrt{2}}$, or any other answer that does not clearly indicate the correct pairs.

[7 marks]

Section B

8. (a) (i) $p = 3$ *A1* *N1*

(ii) valid approach *(M1)*

eg $(12+10+3)-21,\ 22-18$

$q = 4$ *A1* *N2*

(iii) $r = 8,\ s = 6$ *A1A1* *N2*

[5 marks]

(b) (i) $\frac{12}{21}\ \left(=\frac{4}{7}\right)$ *A2* *N2*

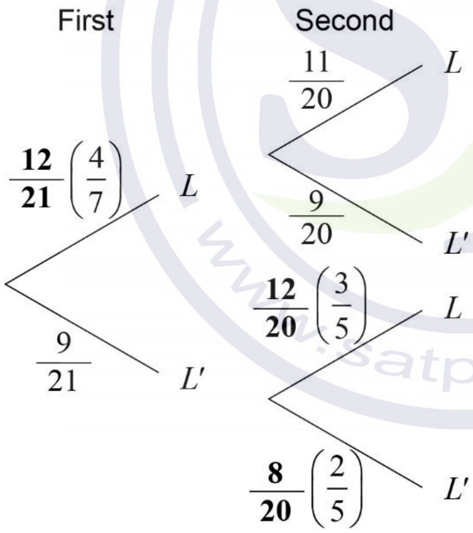
(ii) valid approach *(M1)*

eg $8+6,\ r+s$

$\frac{14}{21}\ \left(=\frac{2}{3}\right)$ *A1* *N2*

[4 marks]

(c) (i)



A1A1A1 *N3*

Note: Award ***A1*** for each correct **bold** answer.

(ii) $\frac{11}{20}$ *A1* *N1*

[4 marks]

Total [13 marks]

9. (a) correct substitution into the formula for volume **A1**

eg $36 = y \times x \times x$

valid approach to eliminate y (may be seen in formula/substitution) **M1**

eg $y = \frac{36}{x^2}$, $xy = \frac{36}{x}$

correct expression for surface area **A1**

eg $xy + xy + xy + x^2 + x^2$, area $= 3xy + 2x^2$

correct expression in terms of x only **A1**

eg $3x\left(\frac{36}{x^2}\right) + 2x^2$, $x^2 + x^2 + \frac{36}{x} + \frac{36}{x} + \frac{36}{x}$, $2x^2 + 3\left(\frac{36}{x}\right)$

$A(x) = \frac{108}{x} + 2x^2$ **AG** **N0**

[4 marks]

(b) $A'(x) = -\frac{108}{x^2} + 4x$, $4x - 108x^{-2}$ **A1A1** **N2**

Note: Award ***A1*** for each term.

[2 marks]

(c) recognizing that minimum is when $A'(x) = 0$ ***(M1)***

correct equation ***(A1)***

eg $-\frac{108}{x^2} + 4x = 0$, $4x = \frac{108}{x^2}$

correct simplification ***(A1)***

eg $-108 + 4x^3 = 0$, $4x^3 = 108$

correct working ***(A1)***

eg $x^3 = 27$

height $= 3$ (m) (accept $x = 3$) **A1** **N2**

[5 marks]

continued…

Question 9 continued

(d) attempt to find area using **their** height ***(M1)***

eg $\frac{108}{3}+2(3)^2$, $9+9+12+12+12$

minimum surface area $=54\text{m}^2$ (may be seen in part (c)) ***A1***

attempt to find the number of tins ***(M1)***

eg $\frac{54}{10}$, 5.4

6 (tins) ***(A1)***

$\$120$ ***A1*** ***N3***

[5 marks]

Total [16 marks]

10. (a) (i) recognizing the need to find the gradient when $x = 0$ (seen anywhere) ***R1***

eg $f'(0)$

correct substitution ***(A1)***

$$f'(0) = \frac{2a^2 - 4(0)}{\sqrt{a^2 - 0}}$$

$f'(0) = 2a$ ***(A1)***

correct equation with gradient $2a$ (do not accept equations of the form $L = 2ax$) ***A1*** ***N3***

eg $y = 2ax$, $y - b = 2a(x - a)$, $y = 2ax - 2a^2 + b$

(ii) **METHOD 1**

attempt to substitute $x = a$ into **their** equation of L ***(M1)***

eg $y = 2a \times a$

$b = 2a^2$ ***A1*** ***N2***

METHOD 2

equating gradients ***(M1)***

eg $\frac{b}{a} = 2a$

$b = 2a^2$ ***A1*** ***N2***

[6 marks]

continued…

Question 10 continued

(b) **METHOD 1**

recognizing that area $= \int_0^a f(x)\,\mathrm{d}x$ (seen anywhere) ***R1***

valid approach using substitution or inspection ***(M1)***

eg $\int 2x\sqrt{u}\,\mathrm{d}x$, $u = a^2 - x^2$, $\mathrm{d}u = -2x\mathrm{d}x$, $\frac{2}{3}\left(a^2 - x^2\right)^{\frac{3}{2}}$

correct working ***(A1)***

eg $\int 2x\sqrt{a^2 - x^2}\,\mathrm{d}x = \int -\sqrt{u}\,\mathrm{d}u$

$\int -\sqrt{u}\,\mathrm{d}u = -\frac{u^{\frac{3}{2}}}{\frac{3}{2}}$ ***(A1)***

$\int f(x)\mathrm{d}x = -\frac{2}{3}\left(a^2 - x^2\right)^{\frac{3}{2}} + c$ ***(A1)***

substituting limits and subtracting ***A1***

eg $A_R = -\frac{2}{3}(a^2 - a^2)^{\frac{3}{2}} + \frac{2}{3}(a^2 - 0)^{\frac{3}{2}}$, $\frac{2}{3}(a^2)^{\frac{3}{2}}$

$A_R = \frac{2}{3}a^3$ ***AG*** ***N0***

METHOD 2

recognizing that area $= \int_0^a f(x)\,\mathrm{d}x$ (seen anywhere) ***R1***

valid approach using substitution or inspection ***(M1)***

eg $\int 2x\sqrt{u}\,\mathrm{d}x$, $u = a^2 - x^2$, $\mathrm{d}u = -2x\mathrm{d}x$, $\frac{2}{3}\left(a^2 - x^2\right)^{\frac{3}{2}}$

correct working ***(A1)***

eg $\int 2x\sqrt{a^2 - x^2}\,dx = \int -\sqrt{u}\,du$

$\int -\sqrt{u}\,du = -\frac{u^{\frac{3}{2}}}{\frac{3}{2}}$ ***(A1)***

new limits for u (even if integration is incorrect) ***(A1)***

eg $u = 0$ and $u = a^2$, $\int_0^{a^2} u^{\frac{1}{2}}\mathrm{d}u$, $\left[-\frac{2}{3}u^{\frac{3}{2}}\right]_{a^2}^{0}$

substituting limits and subtracting ***A1***

eg $A_R = -\left(0 - \frac{2}{3}a^3\right)$, $\frac{2}{3}(a^2)^{\frac{3}{2}}$

$A_R = \frac{2}{3}a^3$ ***AG*** ***N0***

[6 marks]

continued…

Question 10 continued

(c) **METHOD 1**

valid approach to find area of triangle ***(M1)***

eg $\frac{1}{2}(\text{OQ})(\text{PQ})$, $\frac{1}{2}ab$

correct substitution into formula for A_T (seen anywhere) ***(A1)***

eg $A_T = \frac{1}{2} \times a \times 2a^2$, a^3

valid attempt to find k (must be in terms of a) ***(M1)***

eg $a^3 = k\frac{2}{3}a^3$, $k = \frac{a^3}{\frac{2}{3}a^3}$

$k = \frac{3}{2}$ ***A1 N2***

METHOD 2

valid approach to find area of triangle ***(M1)***

eg $\int_0^a (2ax)\,dx$

correct working ***(A1)***

eg $\left[ax^2\right]_0^a$, a^3

valid attempt to find k (must be in terms of a) ***(M1)***

eg $a^3 = k\frac{2}{3}a^3$, $k = \frac{a^3}{\frac{2}{3}a^3}$

$k = \frac{3}{2}$ ***A1 N2***

[4 marks]

Total [16 marks]

Markscheme

November 2015

Mathematics

Standard level

Paper 1

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a valid **Method**; working must be seen.

(M) Marks awarded for a valid **Method**; may be implied by **correct** subsequent working.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

(A) Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.

R Marks awarded for clear **Reasoning**.

N Marks awarded for **correct** answers if **no** working shown.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Mark according to RM assessor instructions.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any. An exception to this rule is when work for ***M1*** is missing, as opposed to incorrect (see point 4).
- Where ***M*** and ***A*** marks are noted on the same line, *eg* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*eg* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***(M2)***, ***N3***, *etc*., do **not** split the marks, unless there is a note.
- Most ***M*** marks are for a **valid** method, ie a method which can lead to the answer: it must indicate some form of progress towards the answer.
- Once a correct answer to a question or part-question is seen, ignore further correct working. However, if further working indicates a lack of mathematical understanding do not award the final ***A1***.

3 *N* marks

*If **no** working shown, award **N** marks for **correct** answers – this includes acceptable answers (see accuracy booklet). In this case, ignore mark breakdown (**M, A, R**). Where a student only shows a final incorrect answer with no working, even if that answer is a correct intermediate answer, award **N0**.*

- Do **not** award a mixture of ***N*** and other marks.
- There may be fewer ***N*** marks available than the total of ***M***, ***A*** and ***R*** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.

- There may not be a direct relationship between the ***N*** marks and the implied marks. There are times when all the marks are implied, but the ***N*** marks are not the full marks: this indicates that we want to see some of the working, without specifying what.
- For consistency within the markscheme, ***N*** marks are noted for every part, even when these match the mark breakdown.
- If a candidate has incorrect working, which somehow results in a correct answer, do **not** award the ***N*** marks for this correct answer. However, if the candidate has indicated (usually by crossing out) that the working is to be ignored, award the ***N*** marks for the correct answer.

4 Implied and must be seen marks

*Implied marks appear in **brackets** eg **(M1)**.*
- Implied marks can only be awarded if the work is seen or if implied in subsequent working (a correct final answer does not necessarily mean that the implied marks are all awarded). There are questions where some working is required, but as it is accepted that not everyone will write the same steps, all the marks are implied, but the ***N*** marks are not the full marks for the question.
- Normally the correct work is seen in the next line.
- Where there is an (***M1***) followed by ***A1*** for each correct answer, if no working shown, one correct answer is sufficient evidence to award the (***M1***).

*Must be seen marks appear without **brackets** eg **M1**.*
- Must be seen marks can only be awarded if the work is seen.
- If a must be seen mark is not awarded because work is missing (as opposed to ***M0*** or ***A0*** for incorrect work) all subsequent marks may be awarded if appropriate.

5 Follow through marks (only applied after an error is made)

*Follow through (**FT**) marks are awarded where an incorrect answer (final or intermediate) from one **part** of a question is used correctly in **subsequent** part(s) or subpart(s). Usually, to award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if the only marks awarded in a subpart are for the **final** answer, then **FT** marks should be awarded if appropriate. Examiners are expected to check student work in order to award **FT** marks where appropriate.*

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** and ***R*** marks may be awarded if appropriate. (However, as noted above, if an ***A*** mark is not awarded because work is missing, all subsequent marks may be awarded if appropriate).
- Exceptions to this rule will be explicitly noted on the markscheme.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*eg* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word "**their**" in a description, to indicate that candidates may be using an incorrect value.
- If a candidate makes an error in one part, but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the question says hence. It is often possible to use a different approach in subsequent parts that does not depend on the answer to previous parts.
- In a "show that" question, if an error in a previous subpart leads to not showing the required answer, do not award the final ***A1***. Note that if the error occurs within the same subpart, the ***FT*** rules may result in further loss of marks.

6 Mis-read

*If a candidate incorrectly copies information from the question, this is a mis-read (**MR**). A candidate should be penalized only once for a particular mis-read. Use the **MR** stamp to indicate that this is a misread. Do not award the first mark in the question, even if this is an **M** mark, but award all others (if appropriate) so that the candidate only loses one mark for the misread.*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*eg* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.

7 Discretionary marks *(d)*

*An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. In such cases the annotation DM should be used and a brief **note** written next to the mark explaining this decision.*

8 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete parts are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**. Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9 Alternative forms

*Unless the question specifies otherwise, **accept** equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

10 Calculators

No calculator is allowed. The use of any calculator on paper 1 is malpractice, and will result in no grade awarded. If you see work that suggests a candidate has used any calculator, please follow the procedures for malpractice. Examples: finding an angle, given a trig ratio of 0.4235.

11 Style

The markscheme aims to present answers using good communication, eg if the question asks to find the value of k, the markscheme will say $k = 3$, but the marks will be for the correct value 3 – there is usually no need for the "$k =$". In these cases, it is also usually acceptable to have another variable, as long as there is no ambiguity in the question, eg if the question asks to find the value of p and of q, then the student answer needs to be clear. Generally, the only situation

where the full answer is required is in a question which asks for equations – in this case the markscheme will say “must be an equation”.
*The markscheme often uses words to describe what the marks are for, followed by examples, using the eg notation. These examples are not exhaustive, and examiners should check what candidates have written, to see if they satisfy the description. Where these marks are **M** marks, the examples may include ones using poor notation, to indicate what is acceptable. A valid method is one which will allow candidate to proceed to the next step eg if a quadratic function is given in factorised form, and the question asks for the zeroes, then multiplying the factors does not necessarily help to find the zeros, and would not on its own count as a valid method.*

12 Candidate work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

Candidates are meant to write their answers to Section A on the question paper (QP), and Section B on answer booklets. Sometimes, they need more room for Section A, and use the booklet (and often comment to this effect on the QP), or write outside the box. That is fine, and this work should be marked.

The instructions tell candidates not to write on Section B of the QP. Thus they may well have done some rough work here which they assume will be ignored. If they have solutions on the answer booklets, there is no need to look at the QP. However, if there are whole questions or whole part solutions missing on answer booklets, please check to make sure that they are not on the QP, and if they are, mark those whole questions or whole part solutions that have not been written on answer booklets.

13. Diagrams

The notes on how to allocate marks for sketches usually refer to passing through particular points or having certain features. These marks can only be awarded if the sketch is approximately the correct shape. All values given will be an approximate guide to where these points/features occur. In some questions, the first ***A1*** is for the shape, in others, the marks are only for the points and/or features. In both cases, unless the shape is approximately correct, no marks can be awarded (unless otherwise stated). However, if the graph is based on previous calculations, ***FT*** marks should be awarded if appropriate.

14. Accuracy of Answers

If the level of accuracy is specified in the question, a mark will be allocated for giving the final answer to the required accuracy. When this is not specified in the question, all numerical answers should be given exactly or correct to three significant figures.

Do not accept unfinished numerical final answers such as 3/0.1 (unless otherwise stated). As a rule, numerical answers with more than one part (such as fractions) should be given using integers (*eg* 6/8). Calculations which lead to integers should be completed, with the exception of fractions which are not whole numbers.

Intermediate values do not need to be given to the correct three significant figures. But, if candidates work with rounded values, this could lead to an incorrect answer, in which case award ***A0*** for the final answer

Where numerical answers are required as the **final** answer to a part of a question in the markscheme, the markscheme will show a truncated 6 sf value, the exact value if applicable, the correct 3 sf answer. Units will appear in brackets at the end.

Section A

1. (a) 60 *A1* *N1*
[1 mark]

(b) (i) valid approach *(M1)*
eg max − min = range, $c = 40 + 47$

$c = 87$ *A1* *N2*

(ii) valid approach *(M1)*
eg $Q3 - Q1 = IQR$, $74 - 22$

$d = 52$ *A1* *N2*
[4 marks]

Total [5 marks]

2. (a) correct approach *(A1)*
eg $\overrightarrow{\mathrm{CB}} = \overrightarrow{\mathrm{CA}} + \overrightarrow{\mathrm{AB}}$, $\overrightarrow{\mathrm{AB}} - \overrightarrow{\mathrm{AC}}$, $\overrightarrow{\mathrm{AC}} + \overrightarrow{\mathrm{CB}} = \overrightarrow{\mathrm{AB}}$

$\overrightarrow{\mathrm{CB}} = -\boldsymbol{q} + \boldsymbol{p}$ *A1* *N2*
[2 marks]

(b) correct approach *(A1)*
eg $\overrightarrow{\mathrm{CD}} = \overrightarrow{\mathrm{BA}}$

$\overrightarrow{\mathrm{CD}} = -\boldsymbol{p}$ *A1* *N2*
[2 marks]

(c) correct approach *(A1)*
eg $\overrightarrow{\mathrm{DB}} = \overrightarrow{\mathrm{DC}} + \overrightarrow{\mathrm{CB}}$, $\overrightarrow{\mathrm{DA}} + \overrightarrow{\mathrm{AB}}$

correct working *(A1)*
eg $\overrightarrow{\mathrm{DB}} = \boldsymbol{p} - (\boldsymbol{q} - \boldsymbol{p})$, $\boldsymbol{p} + \boldsymbol{p} - \boldsymbol{q}$

$\overrightarrow{\mathrm{DB}} = 2\boldsymbol{p} - \boldsymbol{q}$ *A1* *N2*
[3 marks]

[Total 7 marks]

3. evidence of antidifferentiation ***(M1)***

eg $f = \int f'$

correct integration (accept absence of C) ***(A1)(A1)***

$f(x) = \frac{6x^3}{3} - 5x + C$, $2x^3 - 5x$

attempt to substitute $(2, -3)$ into **their** integrated expression (must have C) ***M1***

eg $2(2)^3 - 5(2) + C = -3$, $16 - 10 + C = -3$

Note: Award ***M0*** if substituted into original or differentiated function.

correct working to find C ***(A1)***

eg $16 - 10 + C = -3$, $6 + C = -3$, $C = -9$

$f(x) = 2x^3 - 5x - 9$ ***A1 N4***

[6 marks]

4. (a) amplitude is 3 ***A1 N1***

[1 mark]

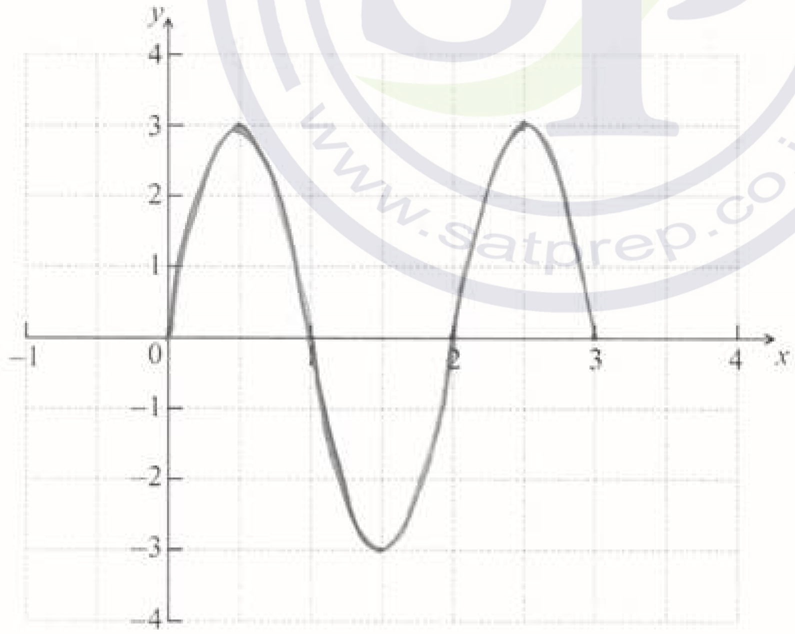
(b) valid approach ***(M1)***

eg period $= \frac{2\pi}{\pi}$, $\frac{360}{\pi}$

period is 2 ***A1 N2***

[2 marks]

(c)



A1A1 A1A1 N4

Note: Award ***A1*** for sine curve starting at (0, 0) and correct period. Only if this ***A1*** is awarded, award the following ***A1*** for correct x-intercepts; ***A1*** for correct max and min points; ***A1*** for correct domain.

[4 marks]

Total [7 marks]

5. (a) interchanging x and y (seen anywhere) ***(M1)***

eg $x = (y-5)^3$

evidence of correct manipulation ***(A1)***

eg $y - 5 = \sqrt[3]{x}$

$f^{-1}(x) = \sqrt[3]{x} + 5$ (accept $5 + x^{\frac{1}{3}}$, $y = 5 + \sqrt[3]{x}$) ***A1*** ***N2***

[3 marks]

(b) **METHOD 1**

attempt to form composite (in any order) ***(M1)***

eg $g\left((x-5)^3\right), \left(g(x) - 5\right)^3 = 8x^6$

correct working ***(A1)***

eg $g - 5 = 2x^2$, $\left(\left(2x^2+5\right)-5\right)^3$

$g(x) = 2x^2 + 5$ ***A1*** ***N2***

METHOD 2

recognising inverse relationship ***(M1)***

eg $f^{-1}\left(8x^6\right) = g\left(x\right)$, $f^{-1}\left(f \circ g\right)(x) = f^{-1}\left(8x^6\right)$

correct working

eg $g(x) = \sqrt[3]{\left(8x^6\right)} + 5$ ***(A1)***

$g(x) = 2x^2 + 5$ ***A1*** ***N2***

[3 marks]

Total [6 marks]

6. evidence of valid binomial expansion with binomial coefficients ***(M1)***

eg $\binom{n}{r}(3x)^r(1)^{n-r}$, $(3x)^n + n(3x)^{n-1} + \binom{n}{2}(3x)^{n-2} + \ldots$, $\binom{n}{r}(1)^{n-r}(3x)^r$

attempt to identify correct term ***(M1)***

eg $\binom{n}{n-2}$, $(3x)^2$, $n-r=2$

setting **correct** coefficient or term equal to $135n$ (may be seen later) ***A1***

eg $9\binom{n}{2} = 135n$, $\frac{9n(n-1)}{2}x^2 = 135nx^2$

correct working for binomial coefficient (using ${}_n\mathrm{C}_r$ formula) ***(A1)***

eg $\frac{n(n-1)(n-2)(n-3)\ldots}{2\times 1\times(n-2)(n-3)(n-4)\ldots}$, $\frac{n(n-1)}{2}$

EITHER

evidence of correct working (with linear equation in n) ***(A1)***

eg $\frac{9(n-1)}{2} = 135$, $\frac{9(n-1)}{2}x^2 = 135x^2$

correct simplification ***(A1)***

eg $n-1 = \frac{135\times 2}{9}$, $\frac{(n-1)}{2} = 15$

$n = 31$ ***A1 N2***

OR

evidence of correct working (with quadratic equation in n) ***(A1)***

eg $9n^2 - 279n = 0$, $n^2 - n = 30n$, $\left(9n^2 - 9n\right)x^2 = 270nx^2$

evidence of solving ***(A1)***

eg $9n(n-31) = 0$, $9n^2 = 279n$

$n = 31$ ***A1 N2***

[7 marks]

7.

> **Note:** There are many approaches to this question, and the steps may be done in any order. There are 3 relationships they may need to apply at some stage, for the 3rd, 4th and 5th marks. These are
>
> equating bases *eg* recognising 9 is 3^2
>
> log rules: $\ln b + \ln c = \ln(bc)$, $\ln b - \ln c = \ln\left(\frac{b}{c}\right)$,
>
> exponent rule: $\ln b^n = n \ln b$.

correct substitution into u_{13} formula **(A1)**

eg $\ln a + (13-1)\ln 3$

set up equation for u_{13} in any form (seen anywhere) **(M1)**

eg $\ln a + 12\ln 3 = 8\ln 9$

correct application of relationships (examples below) **(A1)(A1)(A1)**

$a = 81$ **A1 N3**

[6 marks]

Examples of application of relationships

Example 1

correct application of exponent rule for logs **(A1)**

eg $\ln a + \ln 3^{12} = \ln 9^8$

correct application of addition rule for logs **(A1)**

eg $\ln\left(a3^{12}\right) = \ln 9^8$

substituting for 9 or 3 in ln expression in equation **(A1)**

eg $\ln\left(a3^{12}\right) = \ln 3^{16}$, $\ln\left(a9^6\right) = \ln 9^8$

Example 2

recognising $9 = 3^2$ **(A1)**

eg $\ln a + 12\ln 3 = 8\ln 3^2$, $\ln a + 12\ln 9^{\frac{1}{2}} = 8\ln 9$

one correct application of exponent rule for logs relating $\ln 9$ to $\ln 3$ **(A1)**

eg $\ln a + 12\ln 3 = 16\ln 3$, $\ln a + 6\ln 9 = 8\ln 9$

another correct application of exponent rule for logs **(A1)**

eg $\ln a = \ln 3^4$, $\ln a = \ln 9^2$

Section B

8. (a) $h = 1$, $k = -9$ $\left(\text{accept } (x-1)^2 - 9\right)$ ***A1A1*** ***N2***

[2 marks]

(b) **METHOD 1**

attempt to substitute $x = 0$ into **their** quadratic function ***(M1)***

eg $f(0)$, $(0-1)^2 - 9$

$c = -8$ ***A1*** ***N2***

METHOD 2

attempt to expand **their** quadratic function ***(M1)***

eg $x^2 - 2x + 1 - 9$, $x^2 - 2x - 8$

$c = -8$ ***A1*** ***N2***

[2 marks]

(c) evidence of correct reflection ***A1***

eg $-\left((x-1)^2 - 9\right)$, vertex at $(1, 9)$, y-intercept at $(0, 8)$

valid attempt to find horizontal shift ***(M1)***

eg $1 + p = 3$, $1 \rightarrow 3$

$p = 2$ ***A1*** ***N2***

valid attempt to find vertical shift ***(M1)***

eg $9 + q = 1$, $9 \rightarrow 1$, $-9 + q = 1$

$q = -8$ ***A1*** ***N2***

[5 marks]

(d) valid approach ***M1***

eg $f(x) = g(x)$, $(x-1)^2 - 9 = -(x-3)^2 + 1$

correct expansion of both binomials ***(A1)***

eg $x^2 - 2x + 1$, $x^2 - 6x + 9$

correct working ***(A1)***

eg $x^2 - 2x - 8 = -x^2 + 6x - 8$

correct equation ***(A1)***

eg $2x^2 - 8x = 0$, $2x^2 = 8x$

correct working ***(A1)***

eg $2x(x-4) = 0$

$x = 0$, $x = 4$ ***A1A1*** ***N3***

[7 marks]

Total [16 marks]

9. (a) (i) correct approach **A1**

eg $\text{OB}-\text{OA}$, $\begin{pmatrix}-2\\5\\3\end{pmatrix}-\begin{pmatrix}0\\-3\\1\end{pmatrix}$

$\overrightarrow{\text{AB}}=\begin{pmatrix}-2\\8\\2\end{pmatrix}$ **AG** **N0**

(ii) **any** correct equation in the form $\boldsymbol{r}=\boldsymbol{a}+t\boldsymbol{b}$ (accept any parameter for t)

where $\boldsymbol{a}$ is $\begin{pmatrix}0\\-3\\1\end{pmatrix}$ or $\begin{pmatrix}-2\\5\\3\end{pmatrix}$, and $\boldsymbol{b}$ is a scalar multiple of $\begin{pmatrix}-2\\8\\2\end{pmatrix}$ **A2** **N2**

eg $\boldsymbol{r}=\begin{pmatrix}0\\-3\\1\end{pmatrix}+t\begin{pmatrix}-2\\8\\2\end{pmatrix}$, $\boldsymbol{r}=\begin{pmatrix}-2-2s\\5+8s\\3+2s\end{pmatrix}$, $\boldsymbol{r}=-2\boldsymbol{i}+5\boldsymbol{j}+3\boldsymbol{k}+t(-2\boldsymbol{i}+8\boldsymbol{j}+2\boldsymbol{k})$

Note: Award ***A1*** for the form $\boldsymbol{a}+t\boldsymbol{b}$, ***A1*** for the form $L=\boldsymbol{a}+t\boldsymbol{b}$, ***A0*** for the form $\boldsymbol{r}=\boldsymbol{b}+t\boldsymbol{a}$.

[3 marks]

(b) valid approach ***(M1)***
eg equating lines, $L_1=L_2$

one correct equation in one variable **A1**
eg $-2t=-1$, $-2-2t=-1$

valid attempt to solve ***(M1)***
eg $2t=1$, $-2t=1$

one correct parameter **A1**
eg $t=\frac{1}{2}$, $t=-\frac{1}{2}$, $s=-6$

correct substitution of either parameter **A1**
eg $r=\begin{pmatrix}0\\-3\\1\end{pmatrix}+\frac{1}{2}\begin{pmatrix}-2\\8\\2\end{pmatrix}$, $r=\begin{pmatrix}-2\\5\\3\end{pmatrix}-\frac{1}{2}\begin{pmatrix}-2\\8\\2\end{pmatrix}$, $r=\begin{pmatrix}-1\\7\\-4\end{pmatrix}-6\begin{pmatrix}0\\1\\-1\end{pmatrix}$

the coordinates of C are $(-1, 1, 2)$, or position vector of C is $\begin{pmatrix}-1\\1\\2\end{pmatrix}$ **AG** **N0**

Note: If candidate uses the same parameter in both vector equations and working shown, award ***M1A1M1A0A0***.

[5 marks]

continued…

Question 9 continued

(c) valid approach **(M1)**

eg attempt to find $\overrightarrow{CA}$, $\cos A\hat{C}D = \frac{\overrightarrow{CA} \bullet \overrightarrow{CD}}{\left|\overrightarrow{CA}\right|\left|\overrightarrow{CD}\right|}$, $A\hat{C}D$ formed by $\overrightarrow{CA}$ and $\overrightarrow{CD}$

$\overrightarrow{CA} = \begin{pmatrix} 1 \\ -4 \\ -1 \end{pmatrix}$ **(A1)**

finding $\left|\overrightarrow{CA}\right|$ (may be seen in cosine formula) ***A1***

eg $\sqrt{1^2 + (-4)^2 + (-1)^2}$, $\sqrt{18}$

correct substitution into cosine formula **(A1)**

eg $\frac{-9}{\sqrt{18}\sqrt{18}}$

finding $\cos A\hat{C}D = -\frac{1}{2}$ **(A1)**

$A\hat{C}D = \frac{2\pi}{3}$ (120°) ***A2*** ***N2***

Notes: Award ***A1*** if additional answers are given.

[7 marks]

Total [15 marks]

10. (a) **METHOD 1**

$f'(5) = 0$ ***(A1)***

valid reasoning including reference to the graph of f' ***R1***

eg f' changes sign from negative to positive at $x = 5$, labelled sign chart for f'

so f has a local minimum at $x = 5$ ***AG*** ***N0***

Note: It must be clear that any description is referring to the graph of f', simply giving the conditions for a minimum without relating them to f' does not gain the ***R1***.

METHOD 2

$f'(5) = 0$ ***A1***

valid reasoning referring to second derivative ***R1***

eg $f''(5) > 0$

so f has a local minimum at $x = 5$ ***AG*** ***N0***

[2 marks]

(b) attempt to find relevant interval ***(M1)***

eg f' is decreasing, gradient of f' is negative, $f'' < 0$

$2 < x < 4$ ***A1*** ***N2***

Notes: If no other working shown, award ***M1A0*** for incorrect inequalities such as $2 \le x \le 4$.

[2 marks]

(c) **METHOD 1 (one integral)**

correct application of Fundamental Theorem of Calculus ***(A1)***

eg $\int_0^6 f'(x)\mathrm{d}x = f(6) - f(0)$, $f(6) = 14 + \int_0^6 f'(x)\mathrm{d}x$

attempt to link definite integral with areas ***(M1)***

eg $\int_0^6 f'(x)\mathrm{d}x = -12 - 6.75 + 6.75$, $\int_0^6 f'(x)\mathrm{d}x =$ Area $A +$ Area $B +$ Area C

correct value for $\int_0^6 f'(x)\mathrm{d}x$ ***(A1)***

eg $\int_0^6 f'(x)\mathrm{d}x = -12$

correct working ***A1***

eg $f(6) - 14 = -12$, $f(6) = -12 + f(0)$

$f(6) = 2$ ***A1*** ***N3***

continued…

Question 10 continued

METHOD 2 (more than one integral)

correct application of Fundamental Theorem of Calculus ***(A1)***

eg $\int_0^2 f'(x)\mathrm{d}x = f(2) - f(0)$, $f(2) = 14 + \int_0^2 f'(x)$

attempt to link definite integrals with areas ***(M1)***

eg $\int_0^2 f'(x)\mathrm{d}x = 12$, $\int_2^5 f'(x)\mathrm{d}x = -6.75$, $\int_2^6 f'(x) = 0$

correct values for integrals ***(A1)***

eg $\int_0^2 f'(x)\mathrm{d}x = -12$, $\int_5^2 f'(x)\mathrm{d}x = 6.75$, $f(6) - f(2) = 0$

one correct intermediate value ***A1***

eg $f(2) = 2$, $f(5) = -4.75$

$f(6) = 2$ ***A1*** ***N3***

[5 marks]

(d) correct calculation of $g(6)$ (seen anywhere) ***A1***

eg 2^2, $g(6) = 4$

choosing chain rule or product rule ***(M1)***

eg $g'\left(f(x)\right) f'(x)$, $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}u} \times \frac{\mathrm{d}u}{\mathrm{d}x}$, $f(x)f'(x) + f'(x)f(x)$

correct derivative ***(A1)***

eg $g'(x) = 2f(x)f'(x)$, $f(x)f'(x) + f'(x)f(x)$

correct calculation of $g'(6)$ (seen anywhere) ***A1***

eg $2(2)(16)$, $g'(6) = 64$

attempt to substitute **their** values of $g'(6)$ and $g(6)$ into equation of a line ***(M1)***

eg $2^2 = (2 \times 2 \times 16)6 + b$

correct equation in any form ***A1*** ***N2***

eg $y - 4 = 64(x - 6)$, $y = 64x - 380$

[6 marks]

[Total 15 marks]

Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

May 2015

Mathematics

Standard level

Paper 1

17 pages

International Baccalaureate
Baccalauréat International
Bachillerato Internacional

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a valid **Method**; working must be seen.

(M) Marks awarded for a valid **Method**; may be implied by **correct** subsequent working.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

(A) Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.

R Marks awarded for clear **Reasoning**.

N Marks awarded for **correct** answers if **no** working shown.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Mark according to RM assessor instructions and the document "**Mathematics SL: Guidance for e-marking May 2015**". It is **essential** that you read this document before you start marking. In particular, please note the following. Marks must be recorded using the annotation stamps, using the RM assessor tool. Please check that you are entering marks for the right question. All the marks will be added and recorded by RM assessor.

If a part is **completely correct**, (and gains all the "must be seen" marks), use the ticks with numbers to stamp full marks. Do **not** use the ticks with numbers for anything else.

- If a part is completely wrong, stamp ***A0*** by the final answer.
- If a part gains anything else, all the working **must** have annotations stamped to show what marks are awarded. This includes any zero marks.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any. An exception to this rule is when work for ***M1*** is missing, as opposed to incorrect (see point 4).
- Where ***M*** and ***A*** marks are noted on the same line, *eg* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*eg* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***(M2)***, ***N3***, *etc*., do **not** split the marks, unless there is a note.
- Most ***M*** marks are for a **valid** method, ie a method which can lead to the answer: it must indicate some form of progress towards the answer.
- Once a correct answer to a question or part-question is seen, ignore further correct working. However, if further working indicates a lack of mathematical understanding do not award the final ***A1***. An exception to this may be in numerical answers, where a correct exact value is followed by an incorrect decimal (see examples on next page).

Examples

	Correct answer seen	Further working seen	Action
1.	$8\sqrt{2}$	5.65685 *(incorrect decimal value)*	Award the final ***A1*** *(ignore the further working)*
2.	$\frac{1}{4}\sin 4x$	$\sin x$	Do not award the final ***A1***
3.	$\log a - \log b$	$\log(a-b)$	Do not award the final ***A1***

3 *N* marks

*If **no** working shown, award **N** marks for **correct** answers – this includes acceptable answers (see accuracy booklet). In this case, ignore mark breakdown (**M, A, R**). Where a student only shows a final incorrect answer with no working, even if that answer is a correct intermediate answer, award **N0**.*

- Do **not** award a mixture of ***N*** and other marks.
- There may be fewer ***N*** marks available than the total of ***M***, ***A*** and ***R*** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.
- There may not be a direct relationship between the ***N*** marks and the implied marks. There are times when all the marks are implied, but the ***N*** marks are not the full marks: this indicates that we want to see some of the working, without specifying what.
- For consistency within the markscheme, ***N*** marks are noted for every part, even when these match the mark breakdown.
- If a candidate has incorrect working, which somehow results in a correct answer, do **not** award the ***N*** marks for this correct answer. However, if the candidate has indicated (usually by crossing out) that the working is to be ignored, award the ***N*** marks for the correct answer.

4 Implied and must be seen marks

*Implied marks appear in **brackets** eg **(M1)**.*

- Implied marks can only be awarded if the work is seen or if implied in subsequent working (a correct final answer does not necessarily mean that the implied marks are all awarded). There are questions where some working is required, but as it is accepted that not everyone will write the same steps, all the marks are implied, but the ***N*** marks are not the full marks for the question.
- Normally the correct work is seen in the next line.
- Where there is an (***M1***) followed by ***A1*** for each correct answer, if no working shown, one correct answer is sufficient evidence to award the (***M1***).

*Must be seen marks appear without **brackets** eg **M1**.*

- Must be seen marks can only be awarded if the work is seen.
- If a must be seen mark is not awarded because work is missing (as opposed to ***M0*** or ***A0*** for incorrect work) all subsequent marks may be awarded if appropriate.

5 Follow through marks (only applied after an error is made)

*Follow through (**FT**) marks are awarded where an incorrect answer (final or intermediate) from one **part** of a question is used correctly in **subsequent** part(s) or subpart(s). Usually, to award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if the only marks awarded in a subpart are for the **final** answer, then **FT** marks should be awarded if appropriate. Examiners are expected to check student work in order to award **FT** marks where appropriate.*

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** and ***R*** marks may be awarded if appropriate. (However, as noted above, if an ***A*** mark is not awarded because work is missing, all subsequent marks may be awarded if appropriate).
- Exceptions to this rule will be explicitly noted on the markscheme.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*eg* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word "**their**" in a description, to indicate that candidates may be using an incorrect value.
- If a candidate makes an error in one part, but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the question says hence. It is often possible to use a different approach in subsequent parts that does not depend on the answer to previous parts.
- In a "show that" question, if an error in a previous subpart leads to not showing the required answer, do not award the final ***A1***. Note that if the error occurs within the same subpart, the ***FT*** rules may result in further loss of marks.
- Where there are anticipated common errors, the ***FT*** answers are often noted on the markscheme, to help examiners. It should be stressed that these are not the only ***FT*** answers accepted, neither should ***N*** marks be awarded for these answers.

6 Mis-read

*If a candidate incorrectly copies information from the question, this is a mis-read (**MR**). A candidate should be penalized only once for a particular mis-read. Use the **MR** stamp to indicate that this is a misread. Do not award the first mark in the question, even if this is an **M** mark, but award all others (if appropriate) so that the candidate only loses one mark for the misread.*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*eg* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.

7 Discretionary marks *(d)*

*An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. In such cases the annotation DM should be used and a brief **note** written next to the mark explaining this decision.*

8 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete parts are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**. Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9 Alternative forms

*Unless the question specifies otherwise, **accept** equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

10 Calculators

No calculator is allowed. The use of any calculator on paper 1 is malpractice, and will result in no grade awarded. If you see work that suggests a candidate has used any calculator, please follow the procedures for malpractice. Examples: finding an angle, given a trig ratio of 0.4235.

11 Style

The markscheme aims to present answers using good communication, eg if the question asks to find the value of k, the markscheme will say $k = 3$, but the marks will be for the correct value 3 – there is usually no need for the "$k =$". In these cases, it is also usually acceptable to have another variable, as long as there is no ambiguity in the question, eg if the question asks to find the value of p and of q, then the student answer needs to be clear. Generally, the only situation where the full answer is required is in a question which asks for equations – in this case the markscheme will say "must be an equation". Accept sloppy notation in the working, where this is followed by correct working eg $-2^2 = 4$ where they should have written $(-2)^2 = 4$.

*The markscheme often uses words to describe what the marks are for, followed by examples, using the eg notation. These examples are not exhaustive, and examiners should check what candidates have written, to see if they satisfy the description. Where these marks are **M** marks, the examples may include ones using poor notation, to indicate what is acceptable. A valid method is one which will allow candidate to proceed to the next step eg if a quadratic function is given in factorised form, and the question asks for the zeroes, then multiplying the factors does not necessarily help to find the zeros, and would not on its own count as a valid method.*

12 Candidate work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

Candidates are meant to write their answers to Section A on the question paper (QP), and Section B on answer booklets. Sometimes, they need more room for Section A, and use the booklet (and often comment to this effect on the QP), or write outside the box. That is fine, and this work should be marked.

The instructions tell candidates not to write on Section B of the QP. Thus they may well have done some rough work here which they assume will be ignored. If they have solutions on the answer booklets, there is no need to look at the QP. However, if there are whole questions or whole part solutions missing on answer booklets, please check to make sure that they are not on the QP, and if they are, mark those whole questions or whole part solutions that have not been written on answer booklets.

13. Diagrams

The notes on how to allocate marks for sketches usually refer to passing through particular points or having certain features. These marks can only be awarded if the sketch is approximately the correct shape. All values given will be an approximate guide to where these points/features occur. In some questions, the first ***A1*** is for the shape, in others, the marks are only for the points and/or features. In both cases, unless the shape is approximately correct, no marks can be awarded (unless otherwise stated). However, if the graph is based on previous calculations, ***FT*** marks should be awarded if appropriate.

14. Accuracy of Answers

If the level of accuracy is specified in the question, a mark will be allocated for giving the final answer to the required accuracy. When this is not specified in the question, all numerical answers should be given exactly or correct to three significant figures.

Do not accept unfinished numerical final answers such as 3/0.1 (unless otherwise stated). As a rule, numerical answers with more than one part (such as fractions) should be given using integers (*eg* 6/8). Calculations which lead to integers should be completed, with the exception of fractions which are not whole numbers.

Intermediate values do not need to be given to the correct three significant figures. But, if candidates work with rounded values, this could lead to an incorrect answer, in which case award ***A0*** for the final answer.

Where numerical answers are required as the **final** answer to a part of a question in the markscheme, the markscheme will show

a truncated 6 sf value, the exact value if applicable, and the correct 3 sf answer.

Units (which are generally not required) will appear in brackets at the end.

Section A

1. (a) summing probabilities to 1 *(M1)*

eg $\sum = 1,\ 3+4+2+x=10$

correct working *(A1)*

$\frac{3}{10}+\frac{4}{10}+\frac{2}{10}+p=1,\ p=1-\frac{9}{10}$

$p=\frac{1}{10}$ ***A1 N3***

[3 marks]

(b) correct substitution into formula for $\mathrm{E}(X)$ *(A1)*

eg $0\left(\frac{3}{10}\right)+\ldots+3(p)$

correct working *(A1)*

eg $\frac{4}{10}+\frac{4}{10}+\frac{3}{10}$

$\mathrm{E}(X)=\frac{11}{10}\ (1.1)$ ***A1 N2***

[3 marks]

Total [6 marks]

2. (a) correct substitution *(A1)*

eg $10(1.2)$

ACB is 12 (cm) ***A1 N2***

[2 marks]

(b) valid approach to find major arc *(M1)*

eg circumference $-\mathrm{AB}$, major angle $\mathrm{AOB}\times$ radius

correct working for arc length *(A1)*

eg $2\pi(10)-12,\ 10(2\times 3.142-1.2),\ 2\pi(10)-12+20$

perimeter is $20\pi+8\ \ (=70.8)\ (\mathrm{cm})$ ***A1 N2***

[3 marks]

Total [5 marks]

3. (a) $m=3$, $n=4$ — ***A1A1*** ***N2***

[2 marks]

(b) attempt to apply $\left(2^a\right)^b = 2^{ab}$ — ***(M1)***

eg $6x+3$, $4(2x-3)$

equating **their** powers of 2 (seen anywhere) — ***M1***

eg $3(2x+1)=8x-12$

correct working — ***A1***

eg $8x-12=6x+3$, $2x=15$

$x=\frac{15}{2}\ (7.5)$ — ***A1*** ***N2***

[4 marks]

Total [6 marks]

4. (a) valid approach — ***(M1)***

eg horizontal line on graph at -1, $f(a)=-1$, $(-1, 5)$

$f^{-1}(-1)=5$ — ***A1*** ***N2***

[2 marks]

(b) attempt to find $f(-1)$ — ***(M1)***

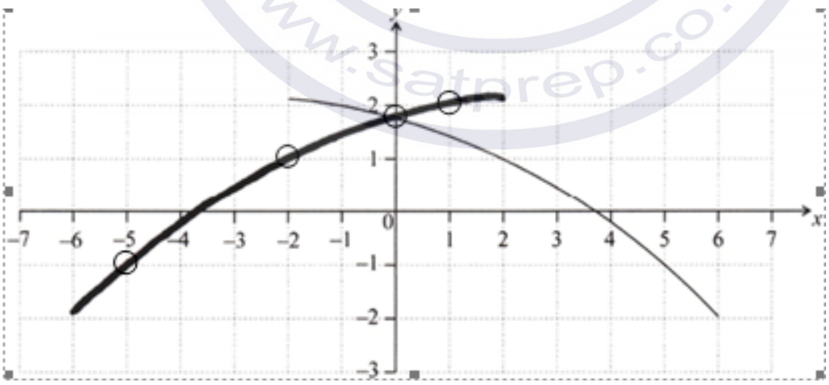
eg line on graph

$f(-1)=2$ — ***(A1)***

$(f\circ f)(-1)=1$ — ***A1*** ***N3***

[3 marks]

(c)



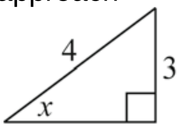
A1A1 ***N2***

> **Note:** The shape **must** be an approximately correct shape (concave down and increasing). **Only** if the shape is approximately correct, award the following for points in circles:
> ***A1*** for the y-intercept,
> ***A1*** for any **two** of these points $(-5, -1)$, $(-2, 1)$, $(1, 2)$.

[2 marks]

Total [7 marks]

5. (a) valid approach ***(M1)***

eg 

, $\sin^2 x + \cos^2 x = 1$

correct working ***(A1)***

eg $4^2 - 3^2$, $\cos^2 x = 1 - \left(\frac{3}{4}\right)^2$

correct calculation ***(A1)***

eg $\frac{\sqrt{7}}{4}$, $\cos^2 x = \frac{7}{16}$

$\cos x = -\frac{\sqrt{7}}{4}$ ***A1*** ***N3***

[4 marks]

(b) correct substitution (accept missing minus with cos) ***(A1)***

eg $1 - 2\left(\frac{3}{4}\right)^2$, $2\left(-\frac{\sqrt{7}}{4}\right)^2 - 1$, $\left(\frac{\sqrt{7}}{4}\right)^2 - \left(\frac{3}{4}\right)^2$

correct working ***A1***

eg $2\left(\frac{7}{16}\right) - 1$, $1 - \frac{18}{16}$, $\frac{7}{16} - \frac{9}{16}$

$\cos 2x = -\frac{2}{16}$ $\left(= -\frac{1}{8}\right)$ ***A1*** ***N2***

[3 marks]

Total [7 marks]

6. (a) correct substitution into $b^2 - 4ac$ ***A1***

eg $(10 - p)^2 - 4(p)\left(\frac{5}{4}p - 5\right)$

correct expansion of each term ***A1A1***

eg $100 - 20p + p^2 - 5p^2 + 20p$, $100 - 20p + p^2 - \left(5p^2 - 20p\right)$

$100 - 4p^2$ ***AG*** ***N0***

[3 marks]

(b) recognizing discriminant is zero for equal roots ***(R1)***

eg $D = 0$, $4p^2 = 100$

correct working ***(A1)***

eg $p^2 = 25$, 1 correct value of p

both correct values $p = \pm 5$ ***A1*** ***N2***

[3 marks]

Total [6 marks]

7. attempt to set up integral (accept missing or incorrect limits and missing $\mathrm{d}x$) ***M1***

eg $\int_{\frac{3\pi}{2}}^{b} \cos x \,\mathrm{d}x$, $\int_{a}^{b} \cos x \,\mathrm{d}x$, $\int_{\frac{3\pi}{2}}^{b} f \,\mathrm{d}x$, $\int \cos x$

correct integration (accept missing or incorrect limits) ***(A1)***

eg $[\sin x]_{\frac{3\pi}{2}}^{b}$, $\sin x$

substituting correct limits into **their** integrated function and subtracting (in any order) ***(M1)***

eg $\sin b - \sin\left(\frac{3\pi}{2}\right)$, $\sin\left(\frac{3\pi}{2}\right) - \sin b$

$\sin\left(\frac{3\pi}{2}\right) = -1$ (seen anywhere) ***(A1)***

setting **their** result from an integrated function equal to $\left(1 - \frac{\sqrt{3}}{2}\right)$ ***M1***

eg $\sin b = -\frac{\sqrt{3}}{2}$

evaluating $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$ or $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = -\frac{\pi}{3}$ ***(A1)***

eg $b = \frac{\pi}{3}$, $-60°$

identifying correct value ***(A1)***

eg $2\pi - \frac{\pi}{3}$, $360 - 60$

$b = \frac{5\pi}{3}$ ***A1*** ***N3***

[8 marks]

Section B

8. (a) (i) correct approach *A1*

eg $\text{B}-\text{A}$, $\text{AO}+\text{OB}$

$\vec{\text{AB}} = \begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix}$ *AG* *N0*

(ii) correct substitution *(A1)*

eg $\sqrt{(1)^2+(-1)^2+(-2)^2}$, $\sqrt{1+1+4}$

$\left|\vec{\text{AB}}\right| = \sqrt{6}$ *A1* *N2*

[3 marks]

(b) any correct equation in the form $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$ (any parameter for t)

where $\boldsymbol{a}$ is $\begin{pmatrix} -2 \\ 4 \\ 3 \end{pmatrix}$ or $\begin{pmatrix} -1 \\ 3 \\ 1 \end{pmatrix}$ and $\boldsymbol{b}$ is a scalar multiple of $\begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix}$ *A2* *N2*

eg $\boldsymbol{r} = \begin{pmatrix} -2 \\ 4 \\ 3 \end{pmatrix} + t\begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix}$, $(x, y, z) = (-1, 3, 1) + t(1, -1, -2)$, $\boldsymbol{r} = \begin{pmatrix} -1+t \\ 3-t \\ 1-2t \end{pmatrix}$

Note: Award ***A1*** for the form $\boldsymbol{a} + t\boldsymbol{b}$, ***A1*** for the form $L = \boldsymbol{a} + t\boldsymbol{b}$, ***A0*** for the form $\boldsymbol{r} = \boldsymbol{b} + t\boldsymbol{a}$.

[2 marks]

continued…

Question 8 continued

(c) **METHOD 1**

valid approach *(M1)*

eg $\begin{pmatrix}-1\\3\\1\end{pmatrix}+t\begin{pmatrix}1\\-1\\-2\end{pmatrix}=\begin{pmatrix}0\\y\\-1\end{pmatrix},\ \begin{pmatrix}0\\y\\-1\end{pmatrix}=\begin{pmatrix}-2\\4\\3\end{pmatrix}+s\begin{pmatrix}1\\-1\\-2\end{pmatrix}$

one correct equation from **their** approach ***A1***

eg $-1+t=0,\ 1-2t=-1,\ -2+s=0,\ 3-2s=-1$

one correct value for **their** parameter and equation ***A1***

eg $t=1,\ s=2$

correct substitution ***A1***

eg $3+1(-1),\ 4+2(-1)$

$y=2$ ***AG*** ***N0***

METHOD 2

valid approach *(M1)*

eg $\vec{AC}=k\,\vec{AB}$

correct working ***A1***

eg $\vec{AC}=\begin{pmatrix}2\\y-4\\-4\end{pmatrix},\ \begin{pmatrix}2\\y-4\\-4\end{pmatrix}=k\begin{pmatrix}1\\-1\\-2\end{pmatrix}$

$k=2$ ***A1***

correct substitution ***A1***

eg $y-4=-2$

$y=2$ ***AG*** ***N0***

[4 marks]

(d) (i) correct substitution ***A1***

eg $0(1)+2(-1)-1(-2),\ 0-2+2$

$\vec{OC}\bullet\vec{AB}=0$ ***A1*** ***N1***

(ii) 90° or $\frac{\pi}{2}$ ***A1*** ***N1***

[3 marks]

continued…

Question 8 continued

(e) **METHOD 1** (area = $0.5 \times$ height $\times$ base)

$\left|\vec{OC}\right| = \sqrt{0+2^2+(-1)^2}\ \left(=\sqrt{5}\right)$ (seen anywhere) ***A1***

valid approach ***(M1)***

eg $\frac{1}{2}\times\left|\vec{AB}\right|\times\left|\vec{OC}\right|$, $\left|\vec{OC}\right|$ is height of triangle

correct substitution ***A1***

eg $\frac{1}{2}\times\sqrt{6}\times\sqrt{0+(2)^2+(-1)^2}$, $\frac{1}{2}\times\sqrt{6}\times\sqrt{5}$

area is $\frac{\sqrt{30}}{2}$ ***A1*** ***N2***

METHOD 2 (difference of two areas)

one correct magnitude (seen anywhere) ***A1***

eg $\left|\vec{OC}\right| = \sqrt{2^2+(-1)^2}\ \left(=\sqrt{5}\right), \left|\vec{AC}\right| = \sqrt{4+4+16}\ \left(=\sqrt{24}\right), \left|\vec{BC}\right| = \sqrt{6}$

valid approach ***(M1)***

eg $\Delta OAC - \Delta OBC$

correct substitution ***A1***

eg $\frac{1}{2}\times\sqrt{24}\times\sqrt{5}-\frac{1}{2}\times\sqrt{5}\times\sqrt{6}$

area is $\frac{\sqrt{30}}{2}$ ***A1*** ***N2***

METHOD 3 (area = $\frac{1}{2}ab\sin C$ for ΔOAB)

one correct magnitude of $\vec{OA}$ or $\vec{OB}$ (seen anywhere) ***A1***

eg $\left|\vec{OA}\right| = \sqrt{(-2)^2+4^2+3^2}\ \left(=\sqrt{29}\right),\ \left|\vec{OB}\right| = \sqrt{1+9+1}\ \left(=\sqrt{11}\right)$

valid attempt to find $\cos\theta$ ***or*** $\sin\theta$ ***(M1)***

eg $\cos C = \frac{-1-3-2}{\sqrt{6}\times\sqrt{11}}\left(=\frac{-6}{\sqrt{66}}\right)$, $29 = 6+11-2\sqrt{6}\sqrt{11}\cos\theta$, $\frac{\sin\theta}{\sqrt{5}} = \frac{\sin 90}{\sqrt{29}}$

correct substitution into $\frac{1}{2}ab\sin C$ ***A1***

eg $\frac{1}{2}\times\sqrt{6}\times\sqrt{11}\times\sqrt{1-\frac{36}{66}}$, $0.5\times\sqrt{6}\times\sqrt{29}\times\frac{\sqrt{5}}{\sqrt{29}}$

area is $\frac{\sqrt{30}}{2}$ ***A1*** ***N2***

[4 marks]
Total [16 marks]

9. (a) $f''(x) = 6x - 2k$ ***A1A1*** ***N2***

[2 marks]

(b) substituting $x = 1$ into f'' ***(M1)***

eg $f''(1)$, $6(1) - 2k$

recognizing $f''(x) = 0$ (seen anywhere) ***M1***

correct equation ***A1***

eg $6 - 2k = 0$

$k = 3$ ***AG*** ***N0***

[3 marks]

(c) correct substitution into $f'(x)$ ***(A1)***

eg $3(-2)^2 - 6(-2) - 9$

$f'(-2) = 15$ ***A1*** ***N2***

[2 marks]

(d) recognizing gradient value (may be seen in equation) ***M1***

eg $a = 15$, $y = 15x + b$

attempt to substitute $(-2, 1)$ into equation of a straight line ***M1***

eg $1 = 15(-2) + b$, $(y - 1) = m(x + 2)$, $(y + 2) = 15(x - 1)$

correct working ***(A1)***

eg $31 = b$, $y = 15x + 30 + 1$

$y = 15x + 31$ ***A1*** ***N2***

[4 marks]

(e) **METHOD 1** (2nd derivative)

recognizing $f'' < 0$ (seen anywhere) ***R1***

substituting $x = -1$ into f'' ***(M1)***

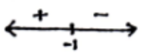
eg $f''(-1)$, $6(-1) - 6$

$f''(-1) = -12$ ***A1***

therefore the graph of f has a local maximum when $x = -1$ ***AG*** ***N0***

METHOD 2 (1st derivative)

recognizing change of sign of $f'(x)$ (seen anywhere) ***R1***

eg *sign chart* 

correct value of f' for $-1 < x < 3$ ***A1***

eg $f'(0) = -9$

correct value of f' for x value to the left of -1 ***A1***

eg $f'(-2) = 15$

therefore the graph of f has a local maximum when $x = -1$ ***AG*** ***N0***

[3 marks]

Total [14 marks]

10. (a) recognizing Ann rolls green ***(M1)***
eg P(G)

$\frac{3}{8}$ ***A1*** ***N2***

[2 marks]

(b) (i) $p=\frac{4}{8},\ q=\frac{5}{8}$ or $q=\frac{4}{8},\ p=\frac{5}{8}$ ***A1A1*** ***N2***

(ii) recognizes Ann and Bob lose 9 times ***(M1)***

eg $\overbrace{A_L\ B_L}\ \overbrace{A_L\ B_L}\ \ldots\ \overbrace{A_L\ B_L}$ 9 times, $\underbrace{\left(\frac{5}{8}\times\frac{4}{8}\right)\times\ldots\times\left(\frac{5}{8}\times\frac{4}{8}\right)}_{\text{9 times}}$

$k=9$ (seen anywhere) ***A1*** ***N2***

correct working ***(A1)***

eg $\left(\frac{5}{8}\times\frac{4}{8}\right)^9\times\frac{3}{8},\ \left(\frac{5}{8}\times\frac{4}{8}\right)\times\ldots\times\left(\frac{5}{8}\times\frac{4}{8}\right)\times\frac{3}{8}$

$r=\frac{20}{64}\ \left(=\frac{5}{16}\right)$ ***A1*** ***N2***

[6 marks]

(c) recognize the probability is an infinite sum ***(M1)***
eg Ann wins on her 1st roll or 2nd roll or 3rd roll…, S_∞

recognizing GP ***(M1)***

$u_1=\frac{3}{8}$ (seen anywhere) ***A1***

$r=\frac{20}{64}$ (seen anywhere) ***A1***

correct substitution into infinite sum of GP ***A1***

eg $\frac{\frac{3}{8}}{1-\frac{5}{16}},\ \frac{3}{8}\left(\frac{1}{1-\left(\frac{5}{8}\times\frac{4}{8}\right)}\right),\ \frac{1}{1-\frac{5}{16}}$

correct working ***(A1)***

eg $\frac{\frac{3}{8}}{\frac{11}{16}},\ \frac{3}{8}\times\frac{16}{11}$

P (Ann wins) $=\frac{48}{88}\ \left(=\frac{6}{11}\right)$ ***A1*** ***N1***

[7 marks]
Total [15 marks]

Diploma Programme
Programme du diplôme
Programa del Diploma

MARKSCHEME

May 2015

MATHEMATICS

Standard level

Paper 1

16 pages

International Baccalaureate®
Baccalauréat International
Bachillerato Internacional

Instructions to Examiners (red changed since M13, green new for M15)

Abbreviations

M Marks awarded for attempting to use a valid **Method**; working must be seen.

(M) Marks awarded for a valid **Method**; may be implied by **correct** subsequent working.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

(A) Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.

R Marks awarded for clear **Reasoning**.

N Marks awarded for **correct** answers if **no** working shown.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Mark according to RM assessor instructions and the document "**Mathematics SL: Guidance for e-marking May 2015**". It is **essential** that you read this document before you start marking. In particular, please note the following. Marks must be recorded using the annotation stamps, using the RM assessor tool. Please check that you are entering marks for the right question. All the marks will be added and recorded by RM assessor.

If a part is **completely correct**, (and gains all the "must be seen" marks), use the ticks with numbers to stamp full marks. Do **not** use the ticks with numbers for anything else.

- If a part is completely wrong, stamp ***A0*** by the final answer.
- If a part gains anything else, all the working **must** have annotations stamped to show what marks are awarded. This includes any zero marks.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any. An exception to this rule is when work for ***M1*** is missing, as opposed to incorrect (see point 4).
- Where ***M*** and ***A*** marks are noted on the same line, *eg* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*eg* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***(M2)***, ***N3***, *etc*., do **not** split the marks, unless there is a note.
- Most ***M*** marks are for a **valid** method, ie a method which can lead to the answer: it must indicate some form of progress towards the answer.
- Once a correct answer to a question or part-question is seen, ignore further correct working. However, if further working indicates a lack of mathematical understanding do not award the final ***A1***. An exception to this may be in numerical answers, where a correct exact value is followed by an incorrect decimal (see examples on next page).

Examples

	Correct answer seen	Further working seen	Action
1.	$8\sqrt{2}$	5.65685 *(incorrect decimal value)*	Award the final ***A1*** *(ignore the further working)*
2.	$\frac{1}{4}\sin 4x$	$\sin x$	Do not award the final ***A1***
3.	$\log a - \log b$	$\log(a-b)$	Do not award the final ***A1***

3 *N* marks

*If **no** working shown, award **N** marks for **correct** answers – this includes acceptable answers (see accuracy booklet). In this case, ignore mark breakdown (**M, A, R**).* Where a student only shows a final incorrect answer with no working, even if that answer is a correct intermediate answer, award ***N0***.

- Do **not** award a mixture of ***N*** and other marks.
- There may be fewer ***N*** marks available than the total of ***M***, ***A*** and ***R*** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.
- There may not be a direct relationship between the ***N*** marks and the implied marks. There are times when all the marks are implied, but the ***N*** marks are not the full marks: this indicates that we want to see some of the working, without specifying what.
- For consistency within the markscheme, ***N*** marks are noted for every part, even when these match the mark breakdown.
- If a candidate has incorrect working, which somehow results in a correct answer, do **not** award the ***N*** marks for this correct answer. However, if the candidate has indicated (usually by crossing out) that the working is to be ignored, award the ***N*** marks for the correct answer.

4 Implied and must be seen marks

*Implied marks appear in **brackets** eg **(M1)**.*

- Implied marks can only be awarded if the work is seen or if implied in subsequent working (a correct final answer does not necessarily mean that the implied marks are all awarded). There are questions where some working is required, but as it is accepted that not everyone will write the same steps, all the marks are implied, but the ***N*** marks are not the full marks for the question.
- Normally the correct work is seen in the next line.
- Where there is an (***M1***) followed by ***A1*** for each correct answer, if no working shown, one correct answer is sufficient evidence to award the (***M1***).

*Must be seen marks appear without **brackets** eg **M1**.*

- Must be seen marks can only be awarded if the work is seen.
- If a must be seen mark is not awarded because work is missing (as opposed to ***M0*** or ***A0*** for incorrect work) all subsequent marks may be awarded if appropriate.

5 Follow through marks (only applied after an error is made)

*Follow through (**FT**) marks are awarded where an incorrect answer (final or intermediate) from one **part** of a question is used correctly in **subsequent** part(s) or subpart(s). Usually, to award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if the only marks awarded in a subpart are for the **final** answer, then **FT** marks should be awarded if appropriate. Examiners are expected to check student work in order to award **FT** marks where appropriate.*

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** and ***R*** marks may be awarded if appropriate. (However, as noted above, if an ***A*** mark is not awarded because work is missing, all subsequent marks may be awarded if appropriate).
- Exceptions to this rule will be explicitly noted on the markscheme.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*eg* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word "**their**" in a description, to indicate that candidates may be using an incorrect value.
- If a candidate makes an error in one part, but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the question says hence. It is often possible to use a different approach in subsequent parts that does not depend on the answer to previous parts.
- In a "show that" question, if an error in a previous subpart leads to not showing the required answer, do not award the final ***A1***. Note that if the error occurs within the same subpart, the ***FT*** rules may result in further loss of marks.
- Where there are anticipated common errors, the ***FT*** answers are often noted on the markscheme, to help examiners. It should be stressed that these are not the only ***FT*** answers accepted, neither should ***N*** marks be awarded for these answers.

6 Mis-read

*If a candidate incorrectly copies information from the question, this is a mis-read (**MR**). A candidate should be penalized only once for a particular mis-read. Use the **MR** stamp to indicate that this is a misread. Do not award the first mark in the question, even if this is an **M** mark, but award all others (if appropriate) so that the candidate only loses one mark for the misread.*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*eg* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.

7 Discretionary marks *(d)*

*An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. In such cases the annotation DM should be used and a brief **note** written next to the mark explaining this decision.*

8 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete parts are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**. Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9 Alternative forms

Unless the question specifies otherwise, ***accept*** *equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

10 Calculators

No calculator is allowed. The use of any calculator on paper 1 is malpractice, and will result in no grade awarded. If you see work that suggests a candidate has used any calculator, please follow the procedures for malpractice. Examples: finding an angle, given a trig ratio of 0.4235.

11 Style

The markscheme aims to present answers using good communication, eg if the question asks to find the value of k, the markscheme will say $k = 3$, but the marks will be for the correct value 3 – there is usually no need for the "$k =$". In these cases, it is also usually acceptable to have another variable, as long as there is no ambiguity in the question, eg if the question asks to find the value of p and of q, then the student answer needs to be clear. Generally, the only situation where the full answer is required is in a question which asks for equations – in this case the markscheme will say "must be an equation". Accept sloppy notation in the working, where this is followed by correct working eg $-2^2 = 4$ where they should have written $(-2)^2 = 4$.

The markscheme often uses words to describe what the marks are for, followed by examples, using the eg notation. These examples are not exhaustive, and examiners should check what candidates have written, to see if they satisfy the description. Where these marks are ***M*** *marks, the examples may include ones using poor notation, to indicate what is acceptable. A valid method is one which will allow candidate to proceed to the next step eg if a quadratic function is given in factorised form, and the question asks for the zeroes, then multiplying the factors does not necessarily help to find the zeros, and would not on its own count as a valid method.*

12 Candidate work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

Candidates are meant to write their answers to Section A on the question paper (QP), and Section B on answer booklets. Sometimes, they need more room for Section A, and use the booklet (and often comment to this effect on the QP), or write outside the box. That is fine, and this work should be marked.

The instructions tell candidates not to write on Section B of the QP. Thus they may well have done some rough work here which they assume will be ignored. If they have solutions on the answer booklets, there is no need to look at the QP. However, if there are whole questions or whole part solutions missing on answer booklets, please check to make sure that they are not on the QP, and if they are, mark those whole questions or whole part solutions that have not been written on answer booklets.

13. Diagrams

The notes on how to allocate marks for sketches usually refer to passing through particular points or having certain features. These marks can only be awarded if the sketch is approximately the correct shape. All values given will be an approximate guide to where these points/features occur. In some questions, the first ***A1*** is for the shape, in others, the marks are only for the points and/or features. In both cases, unless the shape is approximately correct, no marks can be awarded (unless otherwise stated). However, if the graph is based on previous calculations, ***FT*** marks should be awarded if appropriate.

14. Accuracy of Answers

If the level of accuracy is specified in the question, a mark will be allocated for giving the final answer to the required accuracy. When this is not specified in the question, all numerical answers should be given exactly or correct to three significant figures.

Do not accept unfinished numerical final answers such as 3/0.1 (unless otherwise stated). As a rule, numerical answers with more than one part (such as fractions) should be given using integers (*eg* 6/8). Calculations which lead to integers should be completed, with the exception of fractions which are not whole numbers.

Intermediate values do not need to be given to the correct three significant figures. But, if candidates work with rounded values, this could lead to an incorrect answer, in which case award ***A0*** for the final answer.

Where numerical answers are required as the **final** answer to a part of a question in the markscheme, the markscheme will show

a truncated 6 sf value, the exact value if applicable, and the correct 3 sf answer.

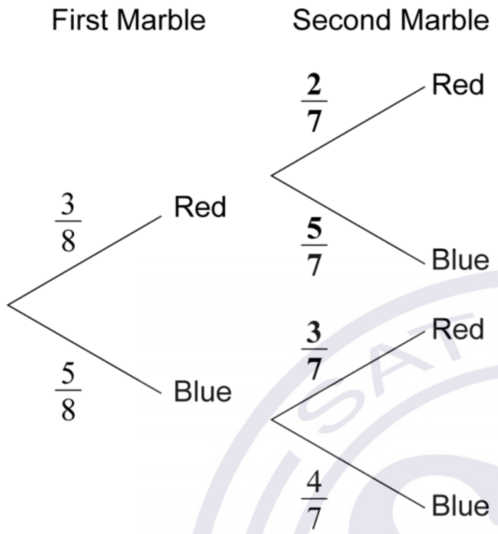
Units (which are generally not required) will appear in brackets at the end.

Section A

1. (a) $\frac{3}{8}$ ***A1*** ***N1***

[1 mark]

(b)



A1A1A1 ***N3***

Note: Award ***A1*** for each correct **bold** value.

[3 marks]

(c) multiplying along the blue branches ***(M1)***

eg $\frac{5}{8}\times\frac{4}{7}$

$\frac{20}{56}\left(=\frac{5}{14}\right)$ ***A1*** ***N2***

[2 marks]

Total [6 marks]

2. (a) (i) valid approach ***(M1)***

eg two cycles is 2π, $2\times\left(\pi-\frac{\pi}{2}\right)$

period is π ***A1*** ***N2***

(ii) amplitude is 3 ***A1*** ***N1***

[3 marks]

(b) (i) $a=3$ ***A1*** ***N1***

(ii) valid approach to find b ***(M1)***

eg correctly substituting the coordinates of a point, $b=\frac{2\pi}{\text{period}}$, $\text{period}=\frac{2\pi}{|b|}$

$b=2$ ***A1*** ***N2***

Note: If no working shown, award ***N3*** for $3\sin 2x$.

[3 marks]

[3 marks]

Total [6 marks]

3. (a) evidence of approach (may be seen on graph) ***(M1)***

eg 80, $(3,80)$

Note: Award ***M0*** for an incorrect approach such as $\frac{0+6}{2}$, which leads to the correct answer, even if $(3,80)$ is indicated on graph.

median $=3$ ***A1*** ***N2***

[2 marks]

(b) (i) $p=30$ ***A1*** ***N1***

(ii) attempt to set up an expression to find q ***(M1)***

eg cumulative frequency for 4.5 indicated on graph

correct expression to find q ***(A1)***

eg $160-20-50-30$, $140-50-p$, $140-80$

$q=60$ ***A1*** ***N2***

[4 marks]

Total [6 marks]

4. (a) **METHOD 1**

choosing quotient rule *(M1)*

eg $\dfrac{vu' - uv'}{v^2}$

$(\ln x)' = \dfrac{1}{x}$, seen in rule *(A1)*

correct substitution into the quotient rule *(A1)*

eg $\dfrac{x \times \frac{1}{x} - \ln x \times 1}{x^2}$

$g'(x) = \dfrac{1 - \ln x}{x^2}$ ***A1 N4***

METHOD 2

choosing product rule *(M1)*

eg $uv' + vu'$

one correct derivative, seen in rule *(A1)*

eg $(\ln x)' = \dfrac{1}{x}$, $-x^{-2}$

correct substitution into the product rule *(A1)*

eg $\ln x\left(-x^{-2}\right) + x^{-1}\left(\dfrac{1}{x}\right)$, $\dfrac{1}{x^2} - \dfrac{\ln x}{x^2}$

$g'(x) = \dfrac{1 - \ln x}{x^2}$ ***A1 N4***

[4 marks]

(b) attempt to use substitution or inspection *(M1)*

eg $u = \ln x$ so $\dfrac{\mathrm{d}u}{\mathrm{d}x} = \dfrac{1}{x}$, $\int u\,du$

$\int g(x)\mathrm{d}x = \dfrac{(\ln x)^2}{2} + C$ (accept absence of $+C$) ***A2 N3***

[3 marks]

Total [7 marks]

5. (a) $f'(x) = -2e^{-2x}$, $f''(x) = 4e^{-2x}$, $f^{(3)}(x) = -8e^{-2x}$ ***A1A1A1*** ***N3***

[3 marks]

(b) $f^{(n)}(x) = (-2)^n e^{-2x}$ $\left(\text{accept } (-1)^n 2^n e^{-2x}, (-2)^n f(x)\right)$ ***A2A1*** ***N3***

[3 marks]

Total [6 marks]

6. recognizing derivative ***(M1)***

eg $f'(x)$, $f'(0) = 3$

correct derivative $3ax^2 + b$ ***A1A1***

$b = 3$ ***A1*** ***N2***

recognizing inverse relationship (seen anywhere) ***(M1)***

eg $(1, 7)$, $f(1) = 7$, swapping x and y **and** substituting $(7, 1)$

correct equation

eg $a + b = 7$, $a + 3 = 7$ ***A1***

substituting **their** b ***(M1)***

eg $ax^3 + 3x$, $a + 3 = 7$

$a = 4$ ***A1*** ***N2***

Notes:	If working shown, award relevant marks for $4x^3 + 3x$.
	If no working shown, award ***N4*** for $4x^3 + 3x$.

[8 marks]

7. recognizing fair game (seen anywhere) ***(M1)***

eg $E(X) = 10$, $E(X) = 0$, money spent = money gained

correct substitution ***(A2)***

eg $0(0.6) + k(0.4)$, $0.4(k - 10) + 0.6(-10)$

correct equation ***(A2)***

eg $0(0.6) + k(0.4) = 10$, $0.4(k - 10) + 0.6(-10) = 0$, $k(0.4) = 10$

correct work towards solving equation ***(A1)***

eg $k = \frac{10}{0.4}, \frac{100}{4}$

$k = 25$ ***A1*** ***N3***

[7 marks]

Section B

8.

Note: The values of p and q found in (a)(i) are used throughout the question. Please check ***FT*** carefully on **their** values.

(a) (i) recognizing intercepts occur when $f(x)=0$ ***(M1)***

eg $p=1,\ q=-3$

$p=-3,\ q=1$ ***A1A1*** ***N3***

(ii) attempt to substitute $(0, 12)$ into **their** f to find a ***(M1)***

eg $f(0)=12$

correct working ***(A1)***

eg $12=a(3)(-1)$

$a=-4$ ***A1*** ***N2***

[6 marks]

(b) attempt to find x-value ***(M1)***

eg $\frac{p+q}{2},\ -\frac{b}{2a},\ f'(x)=0$

correct working ***(A1)***

eg $\frac{-3+1}{2},\ \frac{8}{2(-4)},\ -1,\ -8x-8=0$

$x=-1$ (must be equation) ***A1*** ***N3***

[3 marks]

continued…

Question 8 continued

(c) **METHOD 1**

substituting **their** x to find y-value *(M1)*

eg $f(-1)$, $-4(-1+3)(-1-1)$

correct calculation *(A1)*

eg $-4(2)(-2)$

largest value is 16 ***A1*** ***N2***

METHOD 2

valid attempt to complete the square *(M1)*

eg $-4\left(x^2+2x+1\right)+12+4$, $-4\left(x^2+2x+1\right)+12-1$

correct vertex form *(A1)*

eg $-4(x+1)^2+16$

largest value is 16 ***A1*** ***N2***

METHOD 3

valid approach (may be seen in (b)) *(M1)*

eg $f'(x)=0$, $-8x-8=0$

substituting $x=-1$ into $f(x)$ *(A1)*

eg $-4(-1)^2-8(-1)+12$

largest value is 16 ***A1*** ***N2***

[3 marks]

(d) **METHOD 1**

recognizing coordinates of vertex *(M1)*

eg $(-1, 16)$

$h=-1$, $k=16$ $\left(\text{accept } -4(x+1)^2+16\right)$ ***A1A1*** ***N3***

METHOD 2

valid attempt to complete the square (may be seen in (c)) *(M1)*

eg $-4\left(x^2+2x+1\right)+12+4$, $-4\left(x^2+2x+1\right)+12-1$

$h=-1$, $k=16$ $\left(\text{accept } -4(x+1)^2+16\right)$ ***A1A1*** ***N3***

[3 marks]

Total [15 marks]

9. (a) valid approach to find $\overrightarrow{PQ}$ ***(M1)***

eg $\overrightarrow{OQ}-\overrightarrow{OP}$, $P-Q$

$\overrightarrow{PQ}=\begin{pmatrix}-12\\8\\m-2\end{pmatrix}$ ***A1 N2***

[2 marks]

(b) valid approach (seen anywhere) ***(M1)***

eg $\boldsymbol{b}\bullet\boldsymbol{c}=0$, $\cos\frac{\pi}{2}=\frac{\boldsymbol{b}\bullet\boldsymbol{c}}{|\boldsymbol{b}||\boldsymbol{c}|}$

correct substitution ***(A1)***

eg $(-3)(1)+(2)(1)+(1)(n)$, $\frac{-1+n}{\sqrt{14}\sqrt{n^2+2}}$

Note:	Award ***A0*** for incorrect denominator in cosine formula, but subsequent marks may be awarded.

correct working ***(A1)***

eg $-1+n=0$

$n=1$ ***A1 N3***

[4 marks]

continued…

Question 9 continued

(c) **METHOD 1**

(i) recognizing that $\overrightarrow{PQ}$ is a scalar multiple of $\boldsymbol{b}$ ***(M1)***

eg $\overrightarrow{PQ} = k\boldsymbol{b}$

correct approach to find the scalar multiple ***(A1)***

eg $-12 = -3k$, $8 = 2x$, $\frac{1}{4}\overrightarrow{PQ} = \boldsymbol{b}$

$\overrightarrow{PQ} = 4\boldsymbol{b}$ ***A1 N3***

(ii) $m - 2 = 4(1)$ ***(A1)***

$m = 6$ ***A1 N2***

METHOD 2

(i) correct expression $\overrightarrow{PQ} = k\boldsymbol{b}$ ***A1 N1***

(ii) correct approach to find the scalar multiple ***(A1)***

eg $-12 = -3k$, $8 = 2x$, $\frac{1}{4}\overrightarrow{PQ} = \boldsymbol{b}$

correct working ***(A1)***

eg $\overrightarrow{PQ} = 4\boldsymbol{b}$, $\boldsymbol{b} = \frac{1}{4}\overrightarrow{PQ}$

$m - 2 = 4(1)$ ***(A1)***

$m = 6$ ***A1 N3***

[5 marks]

(d) (i) any correct vector (accept in equation) ***A1 N1***

eg $\boldsymbol{c} = \begin{pmatrix} -11 \\ 8 \\ 6 \end{pmatrix}$, $\begin{pmatrix} -10 \\ 9 \\ 7 \end{pmatrix}$, $\begin{pmatrix} -13 \\ 6 \\ 4 \end{pmatrix}$

(ii) recognize speed $= |\boldsymbol{a}|$ ***(M1)***

correct substitution ***(A1)***

eg $\sqrt{1^2 + 1^2 + 1^2}$, $\sqrt{1 + 1 + n^2}$

speed $= \sqrt{3}$ (m s^{-1}) ***A1 N2***

[4 marks]

Total [15 marks]

10. (a) valid reasoning *(M1)*

eg $f' \le 0$, derivative is negative

correct interval, from 0 to d, with any combination of $\le$ or $<$ *A2* *N3*

eg $0 < x < d$, $0 \le x \le d$

[3 marks]

(b) (i) recognizing that $f' = 0$ *(M1)*

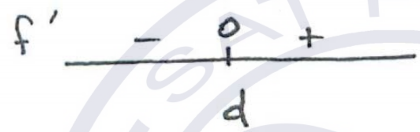
eg $x = a$, $x = 0$

$x = d$ *A1* *N2*

Note: Do not award ***A1*** if additional answers given.

(ii) complete valid reasoning for min (may be seen in (i)) *R1* *N1*

eg sign of f' changes from negative to positive, labelled sign diagram



[3 marks]

(c) recognizing two enclosed regions *(M1)*

eg area a to 0 + area 0 to d

correct expression for area (may be seen in equation, accept absence of $\mathrm{d}x$) *A1*

eg $\int_a^0 f'(x)\,\mathrm{d}x - \int_0^d f'(x)\,\mathrm{d}x$, $\int_a^d |f'(x)|\,\mathrm{d}x$, $[f(x)]_a^0 + [f(x)]_d^0$

equating integral expression to15 (must have limits, may be seen after integration) *(M1)*

eg $\int_a^0 f'(x)\,\mathrm{d}x + \left|\int_0^d f'(x)\,\mathrm{d}x\right| = 15$, $\int_a^0 f'(x)\,\mathrm{d}x + \int_0^d f'(x)\,\mathrm{d}x = 15$

recognizing integral of f' is f (seen anywhere) *(M1)*

eg $\int f'(x)\,\mathrm{d}x = f(x) + C$

considers Fundamental Theorem of Calculus *(M1)*

eg $\int_a^b f'(x)\,\mathrm{d}x = f(b) - f(a)$

correct equation in terms of f *A1*

eg $(f(0) - f(a)) - (f(d) - f(0)) = 15$, $2f(0) - f(a) - f(d) = 15$

correct simplification *(A1)*

eg $2f(0) - 3 - (-1) = 15$, $2f(0) = 17$

$f(0) = 8.5$ *A1* *N2*

[8 marks]

Total [14 marks]

N14/5/MATME/SP1/ENG/TZ0/XX/M

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MARKSCHEME

November 2014

MATHEMATICS

Standard Level

Paper 1

17 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**; working must be seen.

(M) Marks awarded for **Method**; may be implied by **correct** subsequent working.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

(A) Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.

R Marks awarded for clear **Reasoning**.

N Marks awarded for **correct** answers if **no** working shown.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Mark according to RM assessor instructions and the document "**Mathematics SL: Guidance for e-marking May 2014**". It is **essential** that you read this document before you start marking. In particular, please note the following. Marks must be recorded using the annotation stamps, using the RM assessor tool. Please check that you are entering marks for the right question.

- If a part is **completely correct**, (and gains all the "must be seen" marks), use the ticks with numbers to stamp full marks. Do **not** use the ticks with numbers for anything else.
- If a part is completely wrong, stamp ***A0*** by the final answer.
- If a part gains anything else, all the working **must** have annotations stamped to show what marks are awarded. This includes any zero marks.

All the marks will be added and recorded by RM assessor.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any. An exception to this rule is when work for ***M1*** is missing, as opposed to incorrect (see point 4).
- Where ***M*** and ***A*** marks are noted on the same line, *eg* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*eg* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***(M2)***, ***N3***, *etc*., do **not** split the marks, unless there is a note.
- Once a correct answer to a question or part-question is seen, ignore further working.
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3 *N* marks

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5 Follow through marks (only applied after an error is made)

*Follow through (**FT**) marks are awarded where an incorrect answer (final or intermediate) from one **part** of a question is used correctly in **subsequent** part(s) or subpart(s). Usually, to award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if the only marks awarded in a subpart are for the **final** answer, then **FT** marks should be awarded if appropriate. Examiners are expected to check student work in order to award **FT** marks where appropriate.*

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** and ***R*** marks may be awarded if appropriate. (However, as noted above, if an ***A*** mark is not awarded because work is missing, all subsequent marks may be awarded if appropriate).
- Exceptions to this rule will be explicitly noted on the markscheme.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.

- If the error leads to an inappropriate value (*eg* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word "**their**" in a description, to indicate that candidates may be using an incorrect value.
- If a candidate makes an error in one part, but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the question says hence. It is often possible to use a different approach in subsequent parts that does not depend on the answer to previous parts.
- In a "show that" question, if an error in a previous subpart leads to not showing the required answer, do not award the final ***A1***. Note that if the error occurs within the same subpart, the ***FT*** rules may result in further loss of marks.
- Where there are anticipated common errors, the ***FT*** answers are often noted on the markscheme, to help examiners. It should be stressed that these are not the only ***FT*** answers accepted, neither should ***N*** marks be awarded for these answers.

6 Mis-read

*If a candidate incorrectly copies information from the question, this is a mis-read (**MR**). A candidate should be penalized only once for a particular mis-read. Use the **MR** stamp to indicate that this is a misread. Do not award the first mark in the question, even if this is an **M** mark, but award all others (if appropriate) so that the candidate only loses one mark for the misread.*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*eg* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.

7 Discretionary marks *(d)*

*An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. In such cases the annotation DM should be used and a brief **note** written next to the mark explaining this decision.*

8 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete parts are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**. Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9 Alternative forms

*Unless the question specifies otherwise, **accept** equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

10 Calculators

No calculator is allowed. The use of any calculator on paper 1 is malpractice, and will result in no grade awarded. If you see work that suggests a candidate has used any calculator, please follow the procedures for malpractice. Examples: finding an angle, given a trig ratio of 0.4235.

11 Style

The markscheme aims to present answers using good communication, eg if the question asks to find the value of k, the markscheme will say $k = 3$*, but the marks will be for the correct value 3 – there is usually no need for the " k = ". In these cases, it is also usually acceptable to have another variable, as long as there is no ambiguity in the question, eg if the question asks to find the value of p and of q, then the student answer needs to be clear. Generally, the only situation where the full answer is required is in a question which asks for equations – in this case the markscheme will say "must be an equation".*

The markscheme often uses words to describe what the marks are for, followed by examples, using the eg notation. These examples are not exhaustive, and examiners should check what candidates have written, to see if they satisfy the description. Where these marks are ***M*** *marks, the examples may include ones using poor notation, to indicate what is acceptable. A valid method is one which will allow candidate to proceed to the next step eg if a quadratic function is given in factorised form, and the question asks for the zeroes, then multiplying the factors does not necessarily help to find the zeros, and would not on its own count as a valid method.*

12 Candidate work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

Candidates are meant to write their answers to Section A on the question paper (QP), and Section B on answer booklets. Sometimes, they need more room for Section A, and use the booklet (and often comment to this effect on the QP), or write outside the box. That is fine, and this work should be marked.

The instructions tell candidates not to write on Section B of the QP. Thus they may well have done some rough work here which they assume will be ignored. If they have solutions on the answer booklets, there is no need to look at the QP. However, if there are whole questions or whole part solutions missing on answer booklets, please check to make sure that they are not on the QP, and if they are, mark those whole questions or whole part solutions that have not been written on answer booklets

13. Diagrams

The notes on how to allocate marks for sketches usually refer to passing through particular points or having certain features. These marks can only be awarded if the sketch is approximately the correct shape. All values given will be an approximate guide to where these points/features occur. In some questions, the first ***A1*** is for the shape, in others, the marks are only for the points and/or features. In both cases, unless the shape is approximately correct, no marks can be awarded (unless otherwise stated). However, if the graph is based on previous calculations, ***FT*** marks should be awarded if appropriate.

14. Accuracy of Answers

If the level of accuracy is specified in the question, a mark will be allocated for giving the final answer to the required accuracy. When this is not specified in the question, all numerical answers should be given exactly or correct to three significant figures

*Candidates should NO LONGER be penalized for an accuracy error **(AP)**. Examiners should award marks according to the rules given in these instructions and the markscheme. Accuracy is not the same as correctness – an incorrect value does not achieve relevant **A** marks. It is only final answers which may lose marks for accuracy errors, not intermediate values. Please check work carefully for **FT**.*

Do not accept unfinished numerical final answers such as 3/0.1 (unless otherwise stated). As a rule, numerical answers with more than one part (such as fractions) should be given using integers (*eg* 6/8). Calculations which lead to integers should be completed, with the exception of fractions which are not whole numbers.

Clarification of intermediate values accuracy instructions

Intermediate values do not need to be given to the correct three significant figures. But, if candidates work with rounded values, this could lead to an incorrect answer, in which case award ***A0*** for the final answer. However, do not penalise inaccurate intermediate values that lead to an acceptable final answer.

All examiners must read this section carefully, as there are some changes (in red) since M13.

These instructions apply when answers need to be rounded, they do not apply to exact answers which have 3 or fewer figures. The answers will give a range of acceptable values, and any answer given to 3 or more sf that lies in this range will be accepted as well as answers given to the correct 2 sf (which will usually not be in the acceptable range). Answers which are given to 1 sf are not acceptable. There is also a change to the awarding of N marks for acceptable answers.

Where numerical answers are required as the **final** answer to a part of a question in the markscheme, the markscheme will show

a truncated 6 sf value

the exact value if applicable, the correct 3 sf answer and the range of acceptable values. This range includes both end values. Once an acceptable value is seen, ignore any subsequent values (even if rounded incorrectly).

Units (which are generally not required) will appear in brackets at the end.

Example

1.73205

$\sqrt{3}$ (exact), 1.73 [1.73, 1.74] (m)

Note that 1.73 is the correct 3 sf, 1.74 is incorrectly rounded but acceptable, 1.7 is the correct 2 sf value but 1.72 is wrong.

For subsequent parts, the markscheme will show the answers obtained from using unrounded values, and the answers from using previous **correct** 3 sf answers. Examiners will need to check the work carefully if candidates use any other acceptable answers. If other acceptable answers lead to an incorrect final answer (ie outside the range), do not award the final ***A1***. This should not be considered as ***FT***.

Intermediate values do **not** need to be given to the correct 3 sf. If candidates work with fewer than 3 sf, or with incorrectly rounded values, this could lead to an incorrect answer, in which case award ***A0*** for the final answer. However, do not penalise intermediate inaccurate values that lead to an acceptable final answer.

In questions where the final answer gains ***A2***, if other working shown, award ***A1*** for a correctly rounded 1 sf answer.

If there is **no** working shown, award the ***N*** marks for **any** acceptable answer, eg in the example above, if 1.73 achieves ***N4***, then 1.74, 1.7, 1.7320 all achieve ***N4,*** but 2 achieves ***N0*** .

The following table shows what achieves the final mark if this is the **only** numerical answer seen, as long as there is other working.

	Correctly rounded	Incorrectly rounded
1sf	No	No
2sf	Yes	No
3sf	Yes	Yes (if in the acceptable range)
4 or more sf	Yes (if in the acceptable range)	Yes (if in the acceptable range)

SECTION A

1. (a) y-intercept is -6, $(0, -6)$, $y = -6$ ***A1*** ***N1***

[1 mark]

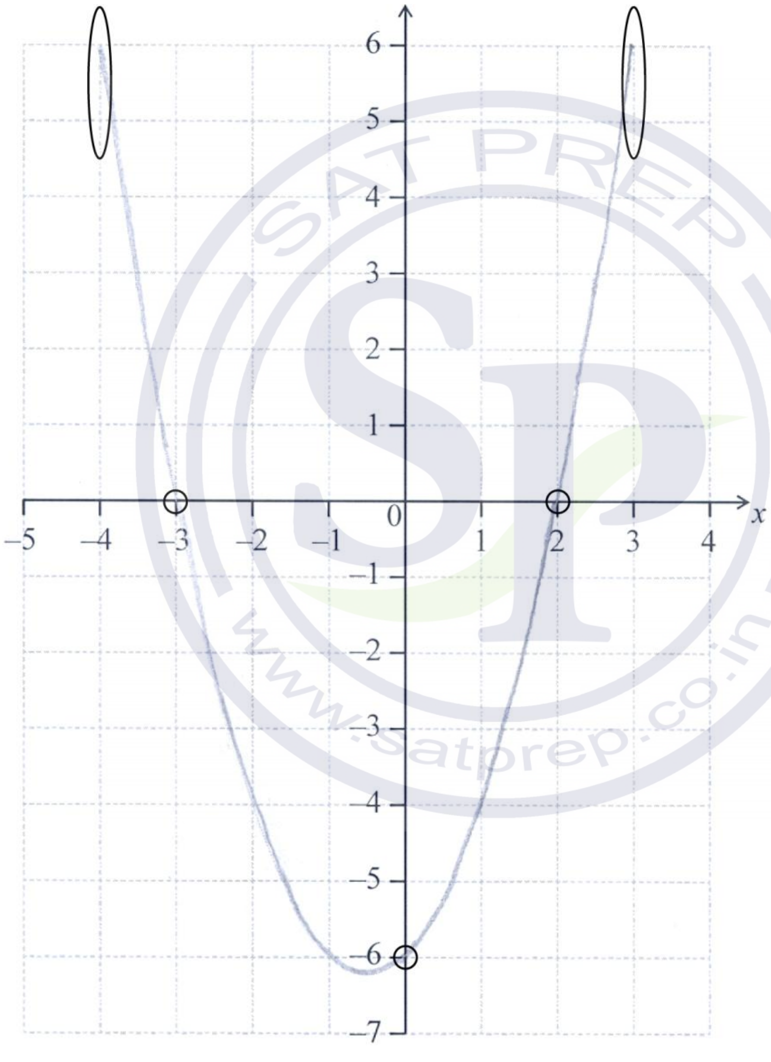
(b) valid attempt to solve ***(M1)***

eg $(x-2)(x+3)=0$, $x = \frac{-1 \pm \sqrt{1+24}}{2}$, one correct answer

$x = 2$, $x = -3$ ***A1A1*** ***N3***

[3 marks]

(c)



A1A1A1 ***N3***

Note: The shape must be an approximately correct concave up parabola. Only if the shape is correct, award the following:

A1 for the y-intercept in circle **and** the vertex approximately on $x = -\frac{1}{2}$, below $y = -6$,

A1 for **both** the x-intercepts in circles,
A1 for **both** end points in ovals.

[3 marks]

Total [7 marks]

2. (a) correct approach *(A1)*

eg $d = u_2 - u_1$, $5 - 2$

$d = 3$ ***A1*** ***N2***

[2 marks]

(b) correct approach *(A1)*

eg $u_8 = 2 + 7 \times 3$, listing terms

$u_8 = 23$ ***A1*** ***N2***

[2 marks]

(c) correct approach *(A1)*

eg $S_8 = \frac{8}{2}(2+23)$, listing terms, $\frac{8}{2}\left(2(2)+7(3)\right)$

$S_8 = 100$ ***A1*** ***N2***

[2 marks]

Total [6 marks]

3. (a) evidence of summing probabilities to 1 *(M1)*

eg $\frac{5}{20} + \frac{4}{20} + \frac{1}{20} + p = 1$, $\sum = 1$

correct working *(A1)*

eg $p = 1 - \frac{10}{20}$

$p = \frac{10}{20}\left(= \frac{1}{2}\right)$ ***A1*** ***N2***

[3 marks]

(b) correct substitution into $\mathrm{E}(X)$ *(A1)*

eg $\frac{4}{20}(q) + \frac{1}{20}(10) + \frac{10}{20}(-3)$

valid reasoning for fair game (seen anywhere, including equation) *(M1)*

eg $\mathrm{E}(X) = 0$, points lost $=$ points gained

correct working *(A1)*

eg $4q + 10 - 30 = 0$, $\frac{4}{20}q + \frac{10}{20} = \frac{30}{20}$

$q = 5$ ***A1*** ***N2***

[4 marks]

Total [7 marks]

4. (a) correct application of $\ln a^b = b\ln a$ (seen anywhere) *(A1)*

eg $\ln 4 = 2\ln 2$, $3\ln 2 = \ln 2^3$, $3\log 2 = \log 8$

correct working *(A1)*

eg $3\ln 2 - 2\ln 2$, $\ln 8 - \ln 4$

$\ln 2$ (accept $k = 2$) ***A1*** ***N2***

[3 marks]

(b) **METHOD 1**

attempt to substitute **their** answer into the equation *(M1)*

eg $\ln 2 = -\ln x$

correct application of a log rule *(A1)*

eg $\ln\frac{1}{x}$, $\ln\frac{1}{2} = \ln x$, $\ln 2 + \ln x = \ln 2x\ (= 0)$

$x = \frac{1}{2}$ ***A1*** ***N2***

METHOD 2

attempt to rearrange equation, with $3\ln 2$ written as $\ln 2^3$ or $\ln 8$ *(M1)*

eg $\ln x = \ln 4 - \ln 2^3$, $\ln 8 + \ln x = \ln 4$, $\ln 2^3 = \ln 4 - \ln x$

correct working applying $\ln a \pm \ln b$ *(A1)*

eg $\frac{4}{8}$, $8x = 4$, $\ln 2^3 = \ln\frac{4}{x}$

$x = \frac{1}{2}$ ***A1*** ***N2***

[3 marks]

Total [6 marks]

5. (a) $q = 3$ ***A1*** ***N1***

[1 mark]

(b) correct expression for $f(0)$ *(A1)*

eg $p + \frac{9}{0-3}$, $4 = p + \frac{9}{-q}$

recognizing that $f(0) = 4$ (may be seen in equation) *(M1)*

correct working *(A1)*

eg $4 = p - 3$

$p = 7$ ***A1*** ***N3***

[4 marks]

(c) $y = 7$ (must be an equation, do not accept $p = 7$) ***A1*** ***N1***

[1 mark]

Total [6 marks]

6. substitution of limits or function ***(A1)***

eg $A=\int_0^4 f(x)\,,\int \frac{x}{x^2+1}\mathrm{d}x$

correct integration by substitution/inspection ***A2***

$\frac{1}{2}\ln(x^2+1)$

substituting limits into **their** integrated function and subtracting (in any order) ***(M1)***

eg $\frac{1}{2}\left(\ln(4^2+1)-\ln(0^2+1)\right)$

correct working ***A1***

eg $\frac{1}{2}\left(\ln(4^2+1)-\ln(0^2+1)\right),\ \frac{1}{2}\left(\ln(17)-\ln(1)\right),\ \frac{1}{2}\ln 17-0$

$A=\frac{1}{2}\ln(17)$ ***A1*** ***N3***

Note: Exception to ***FT*** rule. Allow full ***FT*** on incorrect integration involving a ln function.

[6 marks]

7. attempt to find $\cos \mathrm{C\hat{A}B}$ (seen anywhere) ***(M1)***

eg $\cos\theta=\frac{\overrightarrow{\mathrm{AB}}\bullet\overrightarrow{\mathrm{AC}}}{|\overrightarrow{\mathrm{AB}}||\overrightarrow{\mathrm{AC}}|}$

$\cos \mathrm{C\hat{A}B}=\frac{-5\sqrt{3}}{10}\quad\left(=-\frac{\sqrt{3}}{2}\right)$ ***A1***

valid attempt to find $\sin \mathrm{C\hat{A}B}$ ***(M1)***

eg triangle, Pythagorean identity, $\mathrm{C\hat{A}B}=\frac{5\pi}{6}, 150^\circ$

$\sin \mathrm{C\hat{A}B}=\frac{1}{2}$ ***(A1)***

correct substitution into formula for area ***(A1)***

eg $\frac{1}{2}\times 10\times\frac{1}{2},\frac{1}{2}\times 10\times\sin\frac{\pi}{6}$

area $=\frac{10}{4}\quad\left(=\frac{5}{2}\right)$ ***A1*** ***N3***

[6 marks]

SECTION B

8. (a) correct working *(A1)*

eg $1-\frac{1}{6}$

$p=\frac{5}{6}$ ***A1*** ***N2***

[2 marks]

(b) multiplying along correct branches *(A1)*

eg $\frac{1}{2}\times\frac{1}{6}$

$\text{P}(C\cap L)=\frac{1}{12}$ ***A1*** ***N2***

[2 marks]

(c) multiplying along the other branch *(M1)*

eg $\frac{1}{2}\times\frac{1}{3}$

adding probabilities of their 2 mutually exclusive paths *(M1)*

eg $\frac{1}{2}\times\frac{1}{6}+\frac{1}{2}\times\frac{1}{3}$

correct working *(A1)*

eg $\frac{1}{12}+\frac{1}{6}$

$\text{P}(L)=\frac{3}{12}\quad\left(=\frac{1}{4}\right)$ ***A1*** ***N3***

[4 marks]

continued …

Question 8 continued

(d) recognizing conditional probability (seen anywhere) ***(M1)***

eg $\mathrm{P}(C \mid L)$

correct substitution of **their** values into formula ***(A1)***

eg $\dfrac{\frac{1}{12}}{\frac{3}{12}}$

$\mathrm{P}(C \mid L) = \dfrac{1}{3}$ ***A1 N2***

[3 marks]

(e) valid approach ***(M1)***

eg $X \sim \mathrm{B}\left(3, \frac{1}{4}\right)$, $\left(\frac{1}{4}\right)\left(\frac{3}{4}\right)^2$, $\binom{3}{1}$, three ways it could happen

correct substitution ***(A1)***

eg $\binom{3}{1}\left(\frac{1}{4}\right)^1\left(\frac{3}{4}\right)^2$, $\frac{1}{4}\times\frac{3}{4}\times\frac{3}{4}+\frac{3}{4}\times\frac{1}{4}\times\frac{3}{4}+\frac{3}{4}\times\frac{3}{4}\times\frac{1}{4}$

correct working ***(A1)***

eg $3\left(\frac{1}{4}\right)\left(\frac{9}{16}\right)$, $\frac{9}{64}+\frac{9}{64}+\frac{9}{64}$

$\dfrac{27}{64}$ ***A1 N2***

[4 marks]

Total [15 marks]

9. (a) recognizing that the local minimum occurs when $f'(x)=0$ ***(M1)***

valid attempt to solve $3x^2-8x-3=0$ ***(M1)***

eg factorization, formula

correct working ***A1***

$(3x+1)(x-3)$, $x=\frac{8\pm\sqrt{64+36}}{6}$

$x=3$ ***A2*** ***N3***

Note: Award ***A1*** if both values $x=\frac{-1}{3}$, $x=3$ are given.

[5 marks]

(b) valid approach ***(M1)***

$f(x)=\int f'(x)\,\mathrm{d}x$

$f(x)=x^3-4x^2-3x+c$ (do not penalize for missing "$+c$") ***A1A1A1***

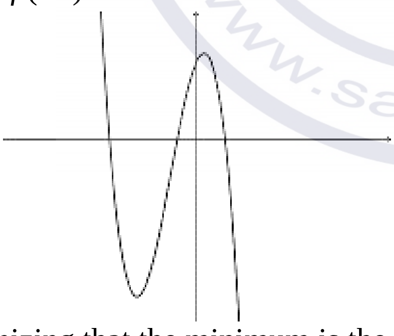
$c=6$ ***(A1)***

$f(x)=x^3-4x^2-3x+6$ ***A1*** ***N6***

[6 marks]

(c) applying reflection ***(A1)***

eg $f(-x)$



recognizing that the minimum is the image of A ***(M1)***

eg $x=-3$

correct expression for x ***A1*** ***N3***

eg $-3+m$, $\begin{pmatrix}-3+m\\-12+n\end{pmatrix}$, $(m-3, n-12)$

[3 marks]

Total [14 marks]

10. (a) attempt to substitute $x=1$ ***(M1)***

eg $\boldsymbol{r}=\begin{pmatrix}1\\ \frac{2}{1}\end{pmatrix}+t\begin{pmatrix}1^2\\ -2\end{pmatrix}$, $L_1=\begin{pmatrix}1\\ 2\end{pmatrix}+t\begin{pmatrix}1\\ -2\end{pmatrix}$

correct equation (vector or Cartesian, but do not accept "$L_1=$")

eg $\boldsymbol{r}=\begin{pmatrix}1\\ 2\end{pmatrix}+t\begin{pmatrix}1\\ -2\end{pmatrix}$, $y=-2x+4$ (must be an equation) ***A1*** ***N2***

[2 marks]

(b) appropriate approach ***(M1)***

eg $\begin{pmatrix}0\\ y\end{pmatrix}=\begin{pmatrix}a\\ \frac{2}{a}\end{pmatrix}+t\begin{pmatrix}a^2\\ -2\end{pmatrix}$

correct equation for *x*-coordinate ***A1***

eg $0=a+ta^2$

$t=\frac{-1}{a}$ ***A1***

substituting **their** parameter to find y ***(M1)***

eg $y=\frac{2}{a}-2\left(\frac{-1}{a}\right)$, $\begin{pmatrix}a\\ \frac{2}{a}\end{pmatrix}-\frac{1}{a}\begin{pmatrix}a^2\\ -2\end{pmatrix}$

correct working ***A1***

eg $y=\frac{2}{a}+\frac{2}{a}$, $\begin{pmatrix}a\\ \frac{2}{a}\end{pmatrix}-\begin{pmatrix}a\\ -\frac{2}{a}\end{pmatrix}$

finding correct expression for y ***A1***

eg $y=\frac{4}{a}$, $\begin{pmatrix}0\\ \frac{4}{a}\end{pmatrix}$

$\text{P}\left(0,\frac{4}{a}\right)$ ***AG*** ***N0***

[6 marks]

continued …

Question 10 continued

(c) valid approach **M1**

eg distance formula, Pythagorean Theorem, $\overrightarrow{PQ} = \begin{pmatrix} 2a \\ -\frac{4}{a} \end{pmatrix}$

correct simplification **A1**

eg $(2a)^2 + \left(\frac{4}{a}\right)^2$

$d = 4a^2 + \frac{16}{a^2}$ **AG** **N0**

[2 marks]

(d) recognizing need to find derivative ***(M1)***

eg d', $d'(a)$

correct derivative ***A2***

eg $8a - \frac{32}{a^3}$, $8x - \frac{32}{x^3}$

setting **their** derivative equal to 0 ***(M1)***

eg $8a - \frac{32}{a^3} = 0$

correct working ***(A1)***

eg $8a = \frac{32}{a^3}$, $8a^4 - 32 = 0$

working towards solution ***(A1)***

eg $a^4 = 4$, $a^2 = 2$, $a = \pm\sqrt{2}$

$a = \sqrt[4]{4}$ $\left(a = \sqrt{2}\right)$ $\left(\text{do not accept } \pm\sqrt{2}\right)$ ***A1*** ***N3***

[7 marks]

Total [17 marks]

M14/5/MATME/SP1/ENG/TZ1/XX/M

International Baccalaureate®
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MARKSCHEME

May 2014

MATHEMATICS

Standard Level

Paper 1

17 pages

Instructions to Examiners

All examiners must read these instructions carefully, as there are some changes since M13.

Abbreviations

M Marks awarded for attempting to use a correct **Method**; working must be seen.

(M) Marks awarded for **Method**; may be implied by **correct** subsequent working.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

(A) Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.

R Marks awarded for clear **Reasoning**.

N Marks awarded for **correct** answers if **no** working shown.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Mark according to scoris instructions and the document "**Mathematics SL: Guidance for e-marking May 2014**". It is **essential** that you read this document before you start marking. In particular, please note the following. Marks must be recorded using the annotation stamps, using the new scoris tool. Please check that you are entering marks for the right question.

- If a part is **completely correct**, (and gains all the "must be seen" marks), use the ticks with numbers to stamp full marks. Do **not** use the ticks with numbers for anything else.
- If a part is completely wrong, stamp ***A0*** by the final answer.
- If a part gains anything else, all the working **must** have annotations stamped to show what marks are awarded. This includes any zero marks.

All the marks will be added and recorded by scoris.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any. An exception to this rule is when work for ***M1*** is missing, as opposed to incorrect (see point 4).
- Where ***M*** and ***A*** marks are noted on the same line, *eg* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*eg* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***(M2)***, ***N3***, *etc*., do **not** split the marks, unless there is a note.
- Once a correct answer to a question or part-question is seen, ignore further working.

- Most ***M*** marks are for a **valid** method, ie a method which can lead to the answer: it must indicate some form of progress towards the answer.

3 *N* marks

*If **no** working shown, award **N** marks for **correct** answers – this includes acceptable answers (see accuracy booklet). In this case, ignore mark breakdown (**M**, **A**, **R**).*

- Do **not** award a mixture of ***N*** and other marks.
- There may be fewer ***N*** marks available than the total of ***M***, ***A*** and ***R*** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.
- There may not be a direct relationship between the ***N*** marks and the implied marks. There are times when all the marks are implied, but the ***N*** marks are not the full marks: this indicates that we want to see some of the working, without specifying what.
- For consistency within the markscheme, ***N*** marks are noted for every part, even when these match the mark breakdown.
- If a candidate has incorrect working, which somehow results in a correct answer, do **not** award the ***N*** marks for this correct answer. However, if the candidate has indicated (usually by crossing out) that the working is to be ignored, award the ***N*** marks for the correct answer.

4 Implied and must be seen marks

*Implied marks appear in **brackets** eg **(M1)**.*

- Implied marks can only be awarded if the work is seen or if implied in subsequent working (a correct final answer does not necessarily mean that the implied marks are all awarded). There are questions where some working is required, but as it is accepted that not everyone will write the same steps, all the marks are implied, but the ***N*** marks are not the full marks for the question.
- Normally the correct work is seen in the next line.
- Where there is an (***M1***) followed by ***A1*** for each correct answer, if no working shown, one correct answer is sufficient evidence to award the (***M1***).

*Must be seen marks appear without **brackets** eg **M1**.*

- Must be seen marks can only be awarded if the work is seen.
- If a must be seen mark is not awarded because work is missing (as opposed to ***M0*** or ***A0*** for incorrect work) all subsequent marks may be awarded if appropriate.

5 Follow through marks (only applied after an error is made)

*Follow through (**FT**) marks are awarded where an incorrect answer (final or intermediate) from one **part** of a question is used correctly in **subsequent** part(s) or subpart(s). Usually, to award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if the only marks awarded in a subpart are for the **final** answer, then **FT** marks should be awarded if appropriate. Examiners are expected to check student work in order to award **FT** marks where appropriate.*

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- Exceptions to this rule will be explicitly noted on the markscheme.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*eg* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word "**their**" in a description, to indicate that candidates may be using an incorrect value.
- If a candidate makes an error in one part, but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the question says hence. It is often possible to use a different approach in subsequent parts that does not depend on the answer to previous parts.
- In a "show that" question, if an error in a previous subpart leads to not showing the required answer, do not award the final ***A1***. Note that if the error occurs within the same subpart, the ***FT*** rules may result in further loss of marks.
- Where there are anticipated common errors, the ***FT*** answers are often noted on the markscheme, to help examiners. It should be stressed that these are not the only ***FT*** answers accepted, neither should ***N*** marks be awarded for these answers.

6 Mis-read

*If a candidate incorrectly copies information from the question, this is a mis-read (**MR**). A candidate should be penalized only once for a particular mis-read. Use the **MR** stamp to indicate that this is a misread. Do not award the first mark in the question, even if this is an **M** mark, but award all others (if appropriate) so that the candidate only loses one mark for the misread.*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*eg* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.

7 Discretionary marks *(d)*

An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. In such cases the annotation DM should be used and a brief ***note*** *written next to the mark explaining this decision.*

8 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete parts are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**. Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9 Alternative forms

Unless the question specifies otherwise, ***accept*** *equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

10 Accuracy of Answers

If the level of accuracy is specified in the question, a mark will be allocated for giving the final answer to the required accuracy. When this is not specified in the question, all numerical answers should be given exactly or correct to three significant figures

Candidates should NO LONGER be penalized for an accuracy error ***(AP)****. Examiners should award marks according to the rules given in these instructions and the markscheme. Accuracy is not the same as correctness – an incorrect value does not achieve relevant* ***A*** *marks. It is only final answers which may lose marks for accuracy errors, not intermediate values. Please check work carefully for* ***FT****. Further information on which answers are accepted is given in a separate booklet, along with examples. It is* ***essential*** *that you read this carefully.*

Do not accept unfinished numerical final answers such as 3/0.1 (unless otherwise stated). As a rule, numerical answers with more than one part (such as fractions) should be given using integers (*eg* 6/8). Calculations which lead to integers should be completed, with the exception of fractions which are not whole numbers.

Clarification of intermediate values accuracy instructions

Intermediate values do not need to be given to the correct three significant figures. But, if candidates work with rounded values, this could lead to an incorrect answer, in which case award ***A0*** for the final answer. However, do not penalise inaccurate intermediate values that lead to an acceptable final answer.

11 Calculators

No calculator is allowed. The use of any calculator on paper 1 is malpractice, and will result in no grade awarded. If you see work that suggests a candidate has used any calculator, please follow the procedures for malpractice. Examples: finding an angle, given a trig ratio of 0.4235.

12 Style

The markscheme aims to present answers using good communication, eg if the question asks to find the value of k, the markscheme will say $k=3$*, but the marks will be for the correct value 3 – there is usually no need for the " k = ". In these cases, it is also usually acceptable to have another variable, as long as there is no ambiguity in the question, eg if the question asks to find the value of p and of q, then the student answer needs to be clear. Generally, the only situation where the full answer is required is in a question which asks for equations – in this case the markscheme will say "must be an equation".*

*The markscheme often uses words to describe what the marks are for, followed by examples, using the eg notation. These examples are not exhaustive, and examiners should check what candidates have written, to see if they satisfy the description. Where these marks are **M** marks, the examples may include ones using poor notation, to indicate what is acceptable. A valid method is one which will allow candidate to proceed to the next step eg if a quadratic function is given in factorised form, and the question asks for the zeroes, then multiplying the factors does not necessarily help to find the zeros, and would not on its own count as a valid method.*

13 Candidate work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

Candidates are meant to write their answers to Section A on the question paper (QP), and Section B on answer booklets. Sometimes, they need more room for Section A, and use the booklet (and often comment to this effect on the QP), or write outside the box. That is fine, and this work should be marked.

The instructions tell candidates not to write on Section B of the QP. Thus they may well have done some rough work here which they assume will be ignored. If they have solutions on the answer booklets, there is no need to look at the QP. However, if there are whole questions or whole part solutions missing on answer booklets, please check to make sure that they are not on the QP, and if they are, mark those whole questions or whole part solutions that have not been written on answer booklets

14. Diagrams

The notes on how to allocate marks for sketches usually refer to passing through particular points or having certain features. These marks can only be awarded if the sketch is approximately the correct shape. All values given will be an approximate guide to where these points/features occur. In some questions, the first ***A1*** is for the shape, in others, the marks are only for the points and/or features. In both cases, unless the shape is approximately correct, no marks can be awarded (unless otherwise stated). However, if the graph is based on previous calculations, ***FT*** marks should be awarded if appropriate.

SECTION A

1. (a) $h = 2, k = 3$ ***A1A1*** ***N2***

[2 marks]

(b) attempt to substitute $(1, 7)$ in any order into **their** $f(x)$ ***(M1)***

eg $7 = a(1-2)^2 + 3$, $7 = a(1-3)^2 + 2$, $1 = a(7-2)^2 + 3$

correct equation ***(A1)***

eg $7 = a + 3$

$a = 4$ ***A1*** ***N2***

[3 marks]

Total [5 marks]

2. (a) attempt to find d ***(M1)***

eg $\frac{16-10}{2}$, $10-2d = 16-4d$, $2d = 6$, $d = 6$

$d = 3$ ***A1*** ***N2***

[2 marks]

(b) correct approach ***(A1)***

eg $10 = u_1 + 2 \times 3, 10-3-3$

$u_1 = 4$ ***A1*** ***N2***

[2 marks]

(c) correct substitution into sum or term formula ***(A1)***

eg $\frac{20}{2}\left(2 \times 4 + 19 \times 3\right)$, $u_{20} = 4 + 19 \times 3$

correct simplification ***(A1)***

eg $8 + 57$, $4 + 61$

$S_{20} = 650$ ***A1*** ***N2***

[3 marks]

Total [7 marks]

3. (a) substituting for $(f(x))^2$ (may be seen in integral) *A1*

eg $(x^2)^2$, x^4

correct integration, $\int x^4\,\mathrm{d}x = \frac{1}{5}x^5$ *(A1)*

substituting limits into **their integrated** function and subtracting (in any order) *(M1)*

eg $\frac{2^5}{5} - \frac{1}{5}$, $\frac{1}{5}(1-4)$

$\int_1^2 (f(x))^2\,\mathrm{d}x = \frac{31}{5}$ $(= 6.2)$ *A1* *N2*

[4 marks]

(b) attempt to substitute limits or function into formula involving f^2 *(M1)*

eg $\int_1^2 (f(x))^2\,\mathrm{d}x$, $\pi\int x^4\mathrm{d}x$

$\frac{31}{5}\pi$ $(= 6.2\pi)$ *A1* *N2*

[2 marks]

Total [6 marks]

4. (a) (i) $\log_3 27 = 3$ *A1* *N1*

(ii) $\log_8 \frac{1}{8} = -1$ *A1* *N1*

(iii) $\log_{16} 4 = \frac{1}{2}$ *A1* *N1*

[3 marks]

(b) correct equation with **their** three values *(A1)*

eg $\frac{3}{2} = \log_4 x$, $3 + (-1) - \frac{1}{2} = \log_4 x$

correct working involving powers *(A1)*

eg $x = 4^{\frac{3}{2}}$, $4^{\frac{3}{2}} = 4^{\log_4 x}$

$x = 8$ *A1* *N2*

[3 marks]

Total [6 marks]

5. recognize need for intersection of Y and F ***(R1)***
eg $\mathrm{P}(Y \cap F)$, 0.3×0.4

valid approach to find $\mathrm{P}(Y \cap F)$ ***(M1)***
eg $\mathrm{P}(Y) + \mathrm{P}(F) - \mathrm{P}(Y \cup F)$, Venn diagram

correct working (may be seen in Venn diagram) ***(A1)***
eg $0.4 + 0.3 - 0.6$

$\mathrm{P}(Y \cap F) = 0.1$ ***A1***

recognize need for complement of $Y \cap F$ ***(M1)***
eg $1 - \mathrm{P}(Y \cap F)$, $1 - 0.1$

$\mathrm{P}\left((Y \cap F)'\right) = 0.9$ ***A1*** ***N3***

[6 marks]

6. correct integration (ignore absence of limits and "+*C*") ***(A1)***

eg $\frac{\sin(2x)}{2}$, $\int_{\pi}^{a} \cos 2x = \left[\frac{1}{2}\sin(2x)\right]_{\pi}^{a}$

substituting limits into **their** integrated function and subtracting (in any order) ***(M1)***

eg $\frac{1}{2}\sin(2a) - \frac{1}{2}\sin(2\pi)$, $\sin(2\pi) - \sin(2a)$

$\sin(2\pi) = 0$ ***(A1)***

setting **their** result from an integrated function equal to $\frac{1}{2}$ ***M1***

eg $\frac{1}{2}\sin 2a = \frac{1}{2}$, $\sin(2a) = 1$

recognizing $\sin^{-1} 1 = \frac{\pi}{2}$ ***(A1)***

eg $2a = \frac{\pi}{2}$, $a = \frac{\pi}{4}$

correct value ***(A1)***

eg $\frac{\pi}{2} + 2\pi$, $2a = \frac{5\pi}{2}$, $a = \frac{\pi}{4} + \pi$

$a = \frac{5\pi}{4}$ ***A1*** ***N3***

[7 marks]

7. (a) $f'(x) = 3px^2 + 2px + q$ ***A2*** ***N2***

> **Note:** Award ***A1*** if only 1 error.

[2 marks]

(b) evidence of discriminant (must be seen explicitly, not in quadratic formula) ***(M1)***
eg $b^2 - 4ac$

correct substitution into discriminant (may be seen in inequality) ***A1***
eg $(2p)^2 - 4 \times 3p \times q$, $4p^2 - 12pq$

$f'(x) \geq 0$ then f' has two equal roots or no roots ***(R1)***

recognizing discriminant less or equal than zero ***R1***
eg $\Delta \leq 0, 4p^2 - 12pq \leq 0$

correct working that clearly leads to the required answer ***A1***
eg $p^2 - 3pq \leq 0$, $4p^2 \leq 12pq$

$p^2 \leq 3pq$ ***AG*** ***N0***

[5 marks]

Total [7 marks]

SECTION B

8. (a) correct approach *A1*

eg $\begin{pmatrix}1\\1\\5\end{pmatrix}-\begin{pmatrix}2\\1\\4\end{pmatrix}$, $\text{AO}+\text{OB}$, $\boldsymbol{b}-\boldsymbol{a}$

$\vec{\text{AB}}=\begin{pmatrix}-1\\0\\1\end{pmatrix}$ *AG* *N0*

[1 mark]

(b) (i) correct vector (or any multiple) *A1* *N1*

eg $\boldsymbol{d}=\begin{pmatrix}-1\\0\\1\end{pmatrix}$

(ii) **any** correct equation in the form $\boldsymbol{r}=\boldsymbol{a}+t\boldsymbol{b}$ (accept any parameter for t)

where $\boldsymbol{a}$ is $\begin{pmatrix}2\\1\\4\end{pmatrix}$ or $\begin{pmatrix}1\\1\\5\end{pmatrix}$, and $\boldsymbol{b}$ is a scalar multiple of $\begin{pmatrix}-1\\0\\1\end{pmatrix}$ *A2* *N2*

eg $\boldsymbol{r}=\begin{pmatrix}1\\1\\5\end{pmatrix}+t\begin{pmatrix}-1\\0\\1\end{pmatrix}$, $\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}2-s\\1\\4+s\end{pmatrix}$

Note: Award *A1* for $\boldsymbol{a}+t\boldsymbol{b}$, *A1* for $L_1=\boldsymbol{a}+t\boldsymbol{b}$, *A0* for $\boldsymbol{r}=\boldsymbol{b}+t\boldsymbol{a}$.

[3 marks]

continued …

Question 8 continued

(c) valid approach ***(M1)***

eg $r_1 = r_2$, $\begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix} + t\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 4 \\ 7 \\ -4 \end{pmatrix} + s\begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$

one correct equation in one parameter ***A1***
eg $2 - t = 4, 1 = 7 - s, 1 - t = 4$

attempt to solve ***(M1)***
eg $2 - 4 = t, s = 7 - 1, t = 1 - 4$

one correct parameter ***A1***
eg $t = -2, s = 6, t = -3$,

attempt to substitute **their** parameter into vector equation ***(M1)***

eg $\begin{pmatrix} 4 \\ 7 \\ -4 \end{pmatrix} + 6\begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$

$\mathrm{P}(4, 1, 2)$ (accept position vector) ***A1*** ***N2***

[6 marks]

(d) (i) correct direction vector for L_2 ***A1*** ***N1***

eg $\begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 2 \\ -2 \end{pmatrix}$

(ii) correct scalar product and magnitudes for **their** direction vectors ***(A1)(A1)(A1)***

scalar product $= 0 \times -1 + -1 \times 0 + 1 \times 1 \; (= 1)$

magnitudes $= \sqrt{0^2 + (-1)^2 + 1^2}, \sqrt{-1^2 + 0^2 + 1^2} \; \left(\sqrt{2}, \sqrt{2}\right)$

attempt to substitute **their** values into formula ***M1***

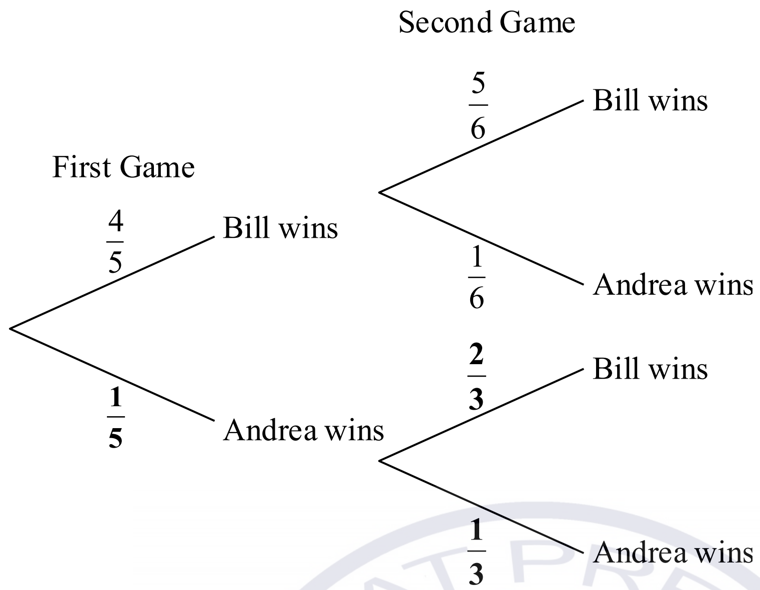
eg $\dfrac{0 + 0 + 1}{\left(\sqrt{0^2 + (-1)^2 + 1^2}\right) \times \left(\sqrt{-1^2 + 0^2 + 1^2}\right)}, \dfrac{1}{\sqrt{2} \times \sqrt{2}}$

correct value for cosine, $\dfrac{1}{2}$ ***A1***

angle is $\dfrac{\pi}{3}$ $(= 60^\circ)$ ***A1*** ***N1***

[7 marks]
Total [17 marks]

9. (a)



A1A1A1 ***N3***

Note: Award *A1* for each correct **bold** probability.

[3 marks]

(b) multiplying along the branches (may be seen on diagram) ***(M1)***

eg $\frac{4}{5}\times\frac{1}{6}$

$\frac{4}{30}\left(\frac{2}{15}\right)$ ***A1*** ***N2***

[2 marks]

(c) **METHOD 1**

multiplying along the branches (may be seen on diagram) ***(M1)***

eg $\frac{4}{5}\times\frac{5}{6}$, $\frac{4}{5}\times\frac{1}{6}$, $\frac{1}{5}\times\frac{2}{3}$

adding their probabilities of three mutually exclusive paths ***(M1)***

eg $\frac{4}{5}\times\frac{5}{6}+\frac{4}{5}\times\frac{1}{6}+\frac{1}{5}\times\frac{2}{3}$, $\frac{4}{5}+\frac{1}{5}\times\frac{2}{3}$

correct simplification ***(A1)***

eg $\frac{20}{30}+\frac{4}{30}+\frac{2}{15}$, $\frac{2}{3}+\frac{2}{15}+\frac{2}{15}$

$\frac{28}{30}\left(=\frac{14}{15}\right)$ ***A1*** ***N3***

continued …

Question 9 continued

METHOD 2

recognizing "Bill wins at least one" is complement of "Andrea wins 2" ***(R1)***
eg finding P (Andrea wins 2)

P (Andrea wins both) $= \frac{1}{5} \times \frac{1}{3}$ ***(A1)***

evidence of complement ***(M1)***

eg $1 - p,\ 1 - \frac{1}{15}$

$\frac{14}{15}$ ***A1*** ***N3***

[4 marks]

(d) P (B wins both) $= \frac{4}{5} \times \frac{5}{6} \left(= \frac{2}{3} \right)$ ***A1***

evidence of recognizing conditional probability ***(R1)***
eg $\mathrm{P}(A \mid B)$, P (Bill wins both |Bill wins at least one), tree diagram

correct substitution ***(A2)***

eg $\dfrac{\frac{4}{5} \times \frac{5}{6}}{\frac{14}{15}}$

$\frac{20}{28} \left(= \frac{5}{7} \right)$ ***A1*** ***N3***

[5 marks]

Total [14 marks]

10. (a) valid method for finding side length *(M1)*

eg $8^2+8^2=c^2$, $45-45-90$ side ratios, $8\sqrt{2}$, $\frac{1}{2}s^2=16$, $x^2+x^2=8^2$

correct working for area *(A1)*

eg $\frac{1}{2}\times 4\times 4$

n	1	2	3
x_n	8	$\sqrt{\mathbf{32}}$	4
A_n	32	16	**8**

A1A1 *N2N2*
[4 marks]

(b) **METHOD 1**

recognize geometric progression for A_n *(R1)*

eg $u_n=u_1r^{n-1}$

$r=\frac{1}{2}$ *(A1)*

correct working *(A1)*

eg $32\left(\frac{1}{2}\right)^5$; 4, 2, 1, $\frac{1}{2}$, $\frac{1}{4}$, …

$A_6=1$ *A1* *N3*

METHOD 2

attempt to find x_6 *(M1)*

eg $8\left(\frac{1}{\sqrt{2}}\right)^5$, $2\sqrt{2}$, 2, $\sqrt{2}$, 1, …

$x_6=\sqrt{2}$ *(A1)*

correct working *(A1)*

eg $\frac{1}{2}\left(\sqrt{2}\right)^2$

$A_6=1$ *A1* *N3*
[4 marks]

continued …

Question 10 continued

(c) **METHOD 1**

recognize infinite geometric series ***(R1)***

eg $S_n = \dfrac{a}{1-r}$, $|r| < 1$

area of first triangle in terms of k ***(A1)***

eg $\dfrac{1}{2}\left(\dfrac{k}{2}\right)^2$

attempt to substitute into sum of infinite geometric series (must have k) ***(M1)***

eg $\dfrac{\frac{1}{2}\left(\frac{k}{2}\right)^2}{1-\frac{1}{2}}$, $\dfrac{k}{1-\frac{1}{2}}$

correct equation ***A1***

eg $\dfrac{\frac{1}{2}\left(\frac{k}{2}\right)^2}{1-\frac{1}{2}} = k$, $k = \dfrac{\frac{k^2}{8}}{\frac{1}{2}}$

correct working ***(A1)***

eg $k^2 = 4k$

valid attempt to solve **their** quadratic ***(M1)***

eg $k(k-4)$, $k = 4$ or $k = 0$

$k = 4$ ***A1*** ***N2***

METHOD 2

recognizing that there are four sets of infinitely shaded regions with equal area ***R1***

area of original square is k^2 ***(A1)***

so total shaded area is $\dfrac{k^2}{4}$ ***(A1)***

correct equation $\dfrac{k^2}{4} = k$ ***A1***

$k^2 = 4k$ ***(A1)***

valid attempt to solve **their** quadratic ***(M1)***

eg $k(k-4)$, $k = 4$ or $k = 0$

$k = 4$ ***A1*** ***N2***

[7 marks]

Total [15 marks]

International Baccalaureate®
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MARKSCHEME

May 2014

MATHEMATICS

Standard Level

Paper 1

19 pages

Instructions to Examiners

All examiners must read these instructions carefully, as there are some changes since M13.

Abbreviations

M Marks awarded for attempting to use a correct **Method**; working must be seen.

(M) Marks awarded for **Method**; may be implied by **correct** subsequent working.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

(A) Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.

R Marks awarded for clear **Reasoning**.

N Marks awarded for **correct** answers if **no** working shown.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Mark according to scoris instructions and the document "**Mathematics SL: Guidance for e-marking May 2014**". It is **essential** that you read this document before you start marking. In particular, please note the following. Marks must be recorded using the annotation stamps, using the new scoris tool. Please check that you are entering marks for the right question.

- If a part is **completely correct**, (and gains all the "must be seen" marks), use the ticks with numbers to stamp full marks. Do **not** use the ticks with numbers for anything else.
- If a part is completely wrong, stamp ***A0*** by the final answer.
- If a part gains anything else, all the working **must** have annotations stamped to show what marks are awarded. This includes any zero marks.

All the marks will be added and recorded by scoris.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any. An exception to this rule is when work for ***M1*** is missing, as opposed to incorrect (see point 4).
- Where ***M*** and ***A*** marks are noted on the same line, *eg* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*eg* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***(M2)***, ***N3***, *etc*., do **not** split the marks, unless there is a note.
- Once a correct answer to a question or part-question is seen, ignore further working.
- Most ***M*** marks are for a **valid** method, ie a method which can lead to the answer: it must indicate some form of progress towards the answer.

3 *N* marks

*If **no** working shown, award N marks for **correct** answers – this includes acceptable answers (see accuracy booklet). In this case, ignore mark breakdown (**M**, **A**, **R**).*

- Do **not** award a mixture of ***N*** and other marks.
- There may be fewer ***N*** marks available than the total of ***M***, ***A*** and ***R*** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.
- There may not be a direct relationship between the ***N*** marks and the implied marks. There are times when all the marks are implied, but the ***N*** marks are not the full marks: this indicates that we want to see some of the working, without specifying what.
- For consistency within the markscheme, ***N*** marks are noted for every part, even when these match the mark breakdown.
- If a candidate has incorrect working, which somehow results in a correct answer, do **not** award the ***N*** marks for this correct answer. However, if the candidate has indicated (usually by crossing out) that the working is to be ignored, award the ***N*** marks for the correct answer.

4 Implied and must be seen marks

*Implied marks appear in **brackets** eg **(M1)**.*

- Implied marks can only be awarded if the work is seen or if implied in subsequent working (a correct final answer does not necessarily mean that the implied marks are all awarded). There are questions where some working is required, but as it is accepted that not everyone will write the same steps, all the marks are implied, but the ***N*** marks are not the full marks for the question.
- Normally the correct work is seen in the next line.
- Where there is an (***M1***) followed by ***A1*** for each correct answer, if no working shown, one correct answer is sufficient evidence to award the (***M1***).

*Must be seen marks appear without **brackets** eg **M1**.*

- Must be seen marks can only be awarded if the work is seen.
- If a must be seen mark is not awarded because work is missing (as opposed to ***M0*** or ***A0*** for incorrect work) all subsequent marks may be awarded if appropriate.

5 Follow through marks (only applied after an error is made)

*Follow through (**FT**) marks are awarded where an incorrect answer (final or intermediate) from one **part** of a question is used correctly in **subsequent** part(s) or subpart(s). Usually, to award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if the only marks awarded in a subpart are for the **final** answer, then **FT** marks should be awarded if appropriate. Examiners are expected to check student work in order to award **FT** marks where appropriate.*

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** and ***R*** marks may be awarded if appropriate. (However, as noted above, if an ***A*** mark is not awarded because work is missing, all subsequent marks may be awarded if appropriate).
- Exceptions to this rule will be explicitly noted on the markscheme.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*eg* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non integer value where integer required), do not award the mark(s) for the final answer(s).
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- In a "show that" question, if an error in a previous subpart leads to not showing the required answer, do not award the final ***A1***. Note that if the error occurs within the same subpart, the ***FT*** rules may result in further loss of marks.
- Where there are anticipated common errors, the ***FT*** answers are often noted on the markscheme, to help examiners. It should be stressed that these are not the only ***FT*** answers accepted, neither should ***N*** marks be awarded for these answers.

6 Mis-read

*If a candidate incorrectly copies information from the question, this is a mis-read (**MR**). A candidate should be penalized only once for a particular mis-read. Use the **MR** stamp to indicate that this is a misread. Do not award the first mark in the question, even if this is an **M** mark, but award all others (if appropriate) so that the candidate only loses one mark for the misread.*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*eg* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.

7 Discretionary marks *(d)*
*An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. In such cases the annotation DM should be used and a brief **note** written next to the mark explaining this decision.*

8 Alternative methods
Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete parts are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**. Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9 Alternative forms
*Unless the question specifies otherwise, **accept** equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

10 Accuracy of Answers

If the level of accuracy is specified in the question, a mark will be allocated for giving the final answer to the required accuracy. When this is not specified in the question, all numerical answers should be given exactly or correct to three significant figures

*Candidates should NO LONGER be penalized for an accuracy error **(AP)**. Examiners should award marks according to the rules given in these instructions and the markscheme. Accuracy is not the same as correctness – an incorrect value does not achieve relevant **A** marks. It is only final answers which may lose marks for accuracy errors, not intermediate values. Please check work carefully for **FT**. Further information on which answers are accepted is given in a separate booklet, along with examples. It is **essential** that you read this carefully.*

Do not accept unfinished numerical final answers such as 3/0.1 (unless otherwise stated). As a rule, numerical answers with more than one part (such as fractions) should be given using integers (*eg* 6/8). Calculations which lead to integers should be completed, with the exception of fractions which are not whole numbers.

Clarification of intermediate values accuracy instructions

Intermediate values do not need to be given to the correct three significant figures. But, if candidates work with rounded values, this could lead to an incorrect answer, in which case award ***A0*** for the final answer. However, do not penalise inaccurate intermediate values that lead to an acceptable final answer.

11 Calculators

No calculator is allowed. The use of any calculator on paper 1 is malpractice, and will result in no grade awarded. If you see work that suggests a candidate has used any calculator, please follow the procedures for malpractice. Examples: finding an angle, given a trig ratio of 0.4235.

12 Style

The markscheme aims to present answers using good communication, eg if the question asks to find the value of k, the markscheme will say $k = 3$, but the marks will be for the correct value 3 – there is usually no need for the " k = ". In these cases, it is also usually acceptable to have another variable, as long as there is no ambiguity in the question, eg if the question asks to find the value of p and of q, then the student answer needs to be clear. Generally, the only situation where the full answer is required is in a question which asks for equations – in this case the markscheme will say "must be an equation".

The markscheme often uses words to describe what the marks are for, followed by examples, using the eg notation. These examples are not exhaustive, and examiners should check what candidates have written, to see if they satisfy the description. Where these marks are ***M*** *marks, the examples may include ones using poor notation, to indicate what is acceptable. A valid method is one which will allow candidate to proceed to the next step eg if a quadratic function is given in factorised form, and the question asks for the zeroes, then multiplying the factors does not necessarily help to find the zeros, and would not on its own count as a valid method.*

13 Candidate work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

Candidates are meant to write their answers to Section A on the question paper (QP), and Section B on answer booklets. Sometimes, they need more room for Section A, and use the booklet (and often comment to this effect on the QP), or write outside the box. That is fine, and this work should be marked.

The instructions tell candidates not to write on Section B of the QP. Thus they may well have done some rough work here which they assume will be ignored. If they have solutions on the answer booklets, there is no need to look at the QP. However, if there are whole questions or whole part solutions missing on answer booklets, please check to make sure that they are not on the QP, and if they are, mark those whole questions or whole part solutions that have not been written on answer booklets

14. Diagrams

The notes on how to allocate marks for sketches usually refer to passing through particular points or having certain features. These marks can only be awarded if the sketch is approximately the correct shape. All values given will be an approximate guide to where these points/features occur. In some questions, the first ***A1*** is for the shape, in others, the marks are only for the points and/or features. In both cases, unless the shape is approximately correct, no marks can be awarded (unless otherwise stated). However, if the graph is based on previous calculations, ***FT*** marks should be awarded if appropriate.

SECTION A

1. (a) **METHOD 1**

approach involving Pythagoras' theorem *(M1)*

eg $5^2 + x^2 = 13^2$, labelling correct sides on triangle

finding third side is 12 (may be seen on diagram) *A1*

$\cos A = \frac{12}{13}$ *AG* *N0*

METHOD 2

approach involving $\sin^2\theta + \cos^2\theta = 1$ *(M1)*

eg $\left(\frac{5}{13}\right)^2 + \cos^2\theta = 1,\ x^2 + \frac{25}{169} = 1$

correct working *A1*

eg $\cos^2\theta = \frac{144}{169}$

$\cos A = \frac{12}{13}$ *AG* *N0*

[2 marks]

(b) correct substitution into $\cos 2\theta$ *(A1)*

eg $1 - 2\left(\frac{5}{13}\right)^2,\ 2\left(\frac{12}{13}\right)^2 - 1,\ \left(\frac{12}{13}\right)^2 - \left(\frac{5}{13}\right)^2$

correct working *(A1)*

eg $1 - \frac{50}{169},\ \frac{288}{169} - 1,\ \frac{144}{169} - \frac{25}{169}$

$\cos 2A = \frac{119}{169}$ *A1* *N2*

[3 marks]

Total [5 marks]

2. (a) correct approach *(A1)*

eg $6^x = 36$, 6^2

2 ***A1*** ***N2***

[2 marks]

(b) correct simplification *(A1)*

eg $\log_6 36$, $\log(4 \times 9)$

2 ***A1*** ***N2***

[2 marks]

(c) correct simplification *(A1)*

eg $\log_6 \frac{2}{12}$, $\log(2 \div 12)$

correct working *(A1)*

eg $\log_6 \frac{1}{6}$, $6^{-1} = \frac{1}{6}$, $6^x = \frac{1}{6}$

-1 ***A1*** ***N2***

[3 marks]

Total [7 marks]

3. (a) (i) $f(-3) = -1$ *A1* *N1*

(ii) $f^{-1}(1) = 0$ (accept $y = 0$) *A1* *N1*

[2 marks]

(b) domain of f^{-1} is range of f *(R1)*

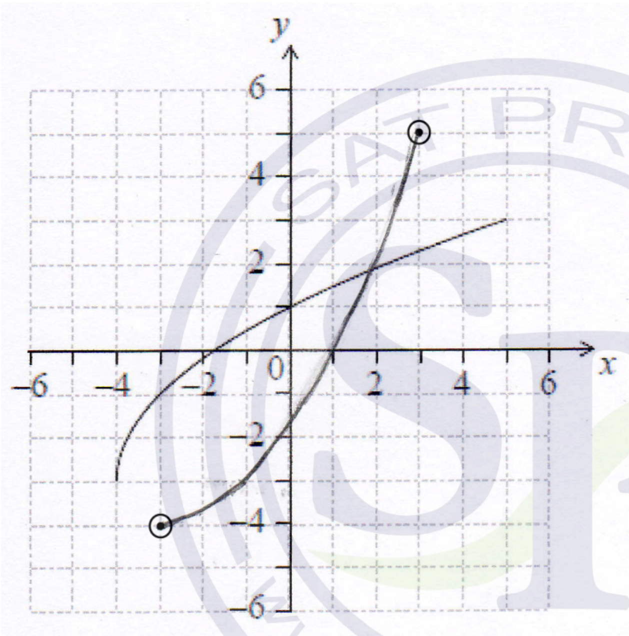
eg $\mathrm{R}f = \mathrm{D}f^{-1}$

correct answer *A1* *N2*

eg $-3 \le x \le 3$, $x \in [-3, 3]$ (accept $-3 < x < 3, -3 \le y \le 3$)

[2 marks]

(c)



A1A1 *N2*

Note:	Graph must be approximately correct reflection in $y = x$. **Only** if the shape is approximately correct, award the following: ***A1*** for x-intercept at 1, and ***A1*** for endpoints within circles.

[2 marks]

Total [6 marks]

4. (a) attempt to find gradient *(M1)*

eg reference to change in x is 3 and/or y is 2, $\frac{3}{2}$

gradient $=\frac{2}{3}$ ***A1*** ***N2***

[2 marks]

(b) attempt to substitute coordinates and/or gradient into Cartesian equation for a line *(M1)*

eg $y-4=m(x-9)$, $y=\frac{2}{3}x+b$, $9=a(4)+c$

correct substitution *(A1)*

eg $4=\frac{2}{3}(9)+c$, $y-4=\frac{2}{3}(x-9)$

$y=\frac{2}{3}x-2$ $\left(\text{accept } a=\frac{2}{3}, b=-2\right)$ ***A1*** ***N2***

[3 marks]

(c) **any** correct equation in the form $\boldsymbol{r}=\boldsymbol{a}+t\boldsymbol{b}$ (any parameter for t), where $\boldsymbol{a}$ indicates position *eg* $\begin{pmatrix}9\\4\end{pmatrix}$ or $\begin{pmatrix}0\\-2\end{pmatrix}$, and $\boldsymbol{b}$ is a scalar multiple of $\begin{pmatrix}3\\2\end{pmatrix}$

eg $\boldsymbol{r}=\begin{pmatrix}9\\4\end{pmatrix}+t\begin{pmatrix}3\\2\end{pmatrix}$, $\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}3t+9\\2t+4\end{pmatrix}$, $\boldsymbol{r}=0i-2j+s(3i+2j)$ ***A2*** ***N2***

Note: Award ***A1*** for $\boldsymbol{a}+t\boldsymbol{b}$, ***A1*** for $L=\boldsymbol{a}+t\boldsymbol{b}$, ***A0*** for $\boldsymbol{r}=\boldsymbol{b}+t\boldsymbol{a}$.

[2 marks]

Total [7 marks]

5. evidence of anti-differentiation *(M1)*

eg $\int h'(x)$, $\int 4\cos 2x\,dx$

correct integration *(A2)*

eg $h(x) = 2\sin 2x + c$, $\frac{4\sin 2x}{2}$

attempt to substitute $\left(\frac{\pi}{12}, 5\right)$ into their equation *(M1)*

eg $2\sin\left(2\times\frac{\pi}{12}\right) + c = 5$, $2\sin\left(\frac{\pi}{6}\right) = 5$

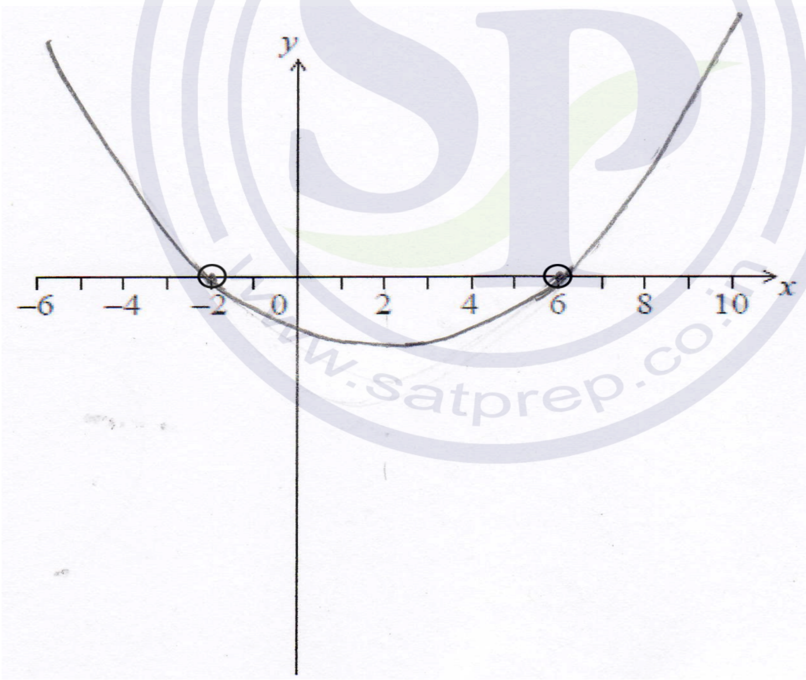
correct working *(A1)*

eg $2\left(\frac{1}{2}\right) + c = 5$, $c = 4$

$h(x) = 2\sin 2x + 4$ ***A1*** ***N5***

Total [6 marks]

6. (a)



A1A1A1A1 ***N4***

Note:	Award ***A1*** for *x*-intercept in circle at -2, ***A1*** for *x*-intercept in circle at 6. Award ***A1*** for approximately correct shape. **Only** if this ***A1*** is awarded, award ***A1*** for a negative *y*-intercept.

[4 marks]

(b) $f''(-2)$, $f'(6)$, $f(0)$ ***A2*** ***N2***

[2 marks]

Total [6 marks]

7. (a) valid method *(M1)*

eg $u_2 = S_2 - S_1$, $1 + k + u_2 = 5 + 3k$

$u_2 = 4 + 2k$, $u_3 = 7 + 4k$, $u_4 = 10 + 8k$ ***A1A1A1*** ***N4***

[4 marks]

(b) correct AP **or** GP *(A1)*

eg finding common difference is 3, common ratio is 2

valid approach using arithmetic **and** geometric formulas *(M1)*

eg $1 + 3(n-1)$ **and** $r^{n-1}k$

$u_n = 3n - 2 + 2^{n-1}k$ ***A1A1*** ***N4***

Note: Award ***A1*** for $3n-2$, ***A1*** for $2^{n-1}k$.

[4 marks]

Total [8 marks]

SECTION B

8. (a) (i) correct value 0, or $36-12p$ ***A2*** ***N2***

(ii) correct equation which clearly leads to $p=3$ ***A1***

eg $36-12p=0$, $36=12p$

$p=3$ ***AG*** ***N0***

[3 marks]

(b) **METHOD 1**

valid approach ***(M1)***

eg $x=-\frac{b}{2a}$

correct working ***A1***

eg $-\frac{(-6)}{2(3)}$, $x=\frac{6}{6}$

correct answers ***A1A1*** ***N2***

eg $x=1$, $y=0$; $(1, 0)$

METHOD 2

valid approach ***(M1)***

eg $f(x)=0$, factorisation, completing the square

correct working ***A1***

eg $x^2-2x+1=0$, $(3x-3)(x-1)$, $f(x)=3(x-1)^2$

correct answers ***A1A1*** ***N2***

eg $x=1$, $y=0$; $(1, 0)$

METHOD 3

valid approach using derivative ***(M1)***

eg $f'(x)=0$, $6x-6$

correct equation ***A1***

eg $6x-6=0$

correct answers ***A1A1*** ***N2***

eg $x=1$, $y=0$; $(1, 0)$

[4 marks]

continued …

Question 8 continued

(c) $x = 1$ ***A1*** ***N1***

[1 mark]

(d) (i) $a = 3$ ***A1*** ***N1***

(ii) $h = 1$ ***A1*** ***N1***

(iii) $k = 0$ ***A1*** ***N1***

[3 marks]

(e) attempt to apply vertical reflection ***(M1)***

eg $-f(x)$, $-3(x-1)^2$, sketch

attempt to apply vertical shift 6 units up ***(M1)***

eg $-f(x)+6$, vertex $(1, 6)$

transformations performed correctly (in correct order) ***(A1)***

eg $-3(x-1)^2+6$, $-3x^2+6x-3+6$

$g(x) = -3x^2+6x+3$ ***A1*** ***N3***

[4 marks]

Total [15 marks]

9. (a) valid approach *(M1)*

eg magnitude of direction vector

correct working *(A1)*

eg $\sqrt{(-4)^2+2^2+4^2}$, $\sqrt{-4^2+2^2+4^2}$

6 (ms^{-1}) ***A1*** ***N2***

[3 marks]

(b) substituting 2 for t *(A1)*

eg $0+2(4)$, $\boldsymbol{r}=\begin{pmatrix}5\\6\\0\end{pmatrix}+2\begin{pmatrix}-4\\2\\4\end{pmatrix}$, $\begin{pmatrix}-3\\10\\8\end{pmatrix}$, $y=10$

8 (metres) ***A1*** ***N2***

[2 marks]

(c) **METHOD 1**

choosing correct direction vectors $\begin{pmatrix}-4\\2\\4\end{pmatrix}$ and $\begin{pmatrix}4\\-6\\7\end{pmatrix}$ *(A1)(A1)*

evidence of scalar product ***M1***

eg $\boldsymbol{a}\bullet\boldsymbol{b}$

correct substitution into scalar product *(A1)*

eg $(-4\times 4)+(2\times -6)+(4\times 7)$

evidence of correct calculation of the scalar product as 0 ***A1***

eg $-16-12+28=0$

directions are perpendicular ***AG*** ***N0***

continued …

Question 9 continued

METHOD 2

choosing correct direction vectors $\begin{pmatrix} -4 \\ 2 \\ 4 \end{pmatrix}$ and $\begin{pmatrix} 4 \\ -6 \\ 7 \end{pmatrix}$ ***(A1)(A1)***

attempt to find angle between vectors ***M1***

correct substitution into numerator ***A1***

eg $\cos\theta = \dfrac{-16-12+28}{|a||b|}$, $\cos\theta = 0$

$\theta = 90^\circ$ ***A1***

directions are perpendicular ***AG*** ***N0***

[5 marks]

(d) **METHOD 1**

one correct equation for Ryan's airplane ***(A1)***

eg $5-4t=-23,\ 6+2t=20,\ 0+4t=28$

$t=7$ ***A1***

one correct equation for Jack's airplane ***(A1)***

eg $-39+4s=-23,\ 44-6s=20,\ 0+7s=28$

$s=4$ ***A1***

3 (seconds later) ***A1*** ***N2***

METHOD 2

valid approach ***(M1)***

eg $\begin{pmatrix} 5 \\ 6 \\ 0 \end{pmatrix} + t\begin{pmatrix} -4 \\ 2 \\ 4 \end{pmatrix} = \begin{pmatrix} -39 \\ 44 \\ 0 \end{pmatrix} + s\begin{pmatrix} 4 \\ -6 \\ 7 \end{pmatrix}$, one correct equation

two correct equations ***(A1)***

eg $5-4t=-39+4s$, $6+2t=44-6s$, $4t=7s$

$t=7$ ***A1***

$s=4$ ***A1***

3 (seconds later) ***A1*** ***N2***

[5 marks]

Total [15 marks]

10. (a) derivative of $2x$ is 2 (must be seen in quotient rule) ***(A1)***

derivative of x^2+5 is $2x$ (must be seen in quotient rule) ***(A1)***

correct substitution into quotient rule ***A1***

eg $\dfrac{(x^2+5)(2)-(2x)(2x)}{(x^2+5)^2}$, $\dfrac{2(x^2+5)-4x^2}{(x^2+5)^2}$

correct working which clearly leads to given answer ***A1***

eg $\dfrac{2x^2+10-4x^2}{(x^2+5)^2}$, $\dfrac{2x^2+10-4x^2}{x^4+10x^2+25}$

$f'(x)=\dfrac{10-2x^2}{(x^2+5)^2}$ ***AG*** ***N0***

[4 marks]

(b) valid approach using substitution or inspection ***(M1)***

eg $u=x^2+5$, $\mathrm{d}u=2x\mathrm{d}x$, $\dfrac{1}{2}\ln(x^2+5)$

$\displaystyle\int\frac{2x}{x^2+5}\mathrm{d}x=\int\frac{1}{u}\mathrm{d}u$ ***(A1)***

$\displaystyle\int\frac{1}{u}\mathrm{d}u=\ln u+c$ ***(A1)***

$\ln(x^2+5)+c$ ***A1*** ***N4***

[4 marks]

continued …

Question 10 continued

(c) correct expression for area **(A1)**

eg $\left[\ln\left(x^2+5\right)\right]_{\sqrt{5}}^{q}$, $\int_{\sqrt{5}}^{q}\frac{2x}{x^2+5}dx$

substituting limits into **their** integrated function and subtracting (in either order) **(M1)**

eg $\ln(q^2+5)-\ln\left(\sqrt{5}^2+5\right)$

correct working **(A1)**

eg $\ln(q^2+5)-\ln 10$, $\ln\frac{q^2+5}{10}$

equating **their** expression to $\ln 7$ (seen anywhere) **(M1)**

eg $\ln(q^2+5)-\ln 10=\ln 7$, $\ln\frac{q^2+5}{10}=\ln 7$, $\ln(q^2+5)=\ln 7+\ln 10$

correct equation without logs **(A1)**

eg $\frac{q^2+5}{10}=7$, $q^2+5=70$

$q^2=65$ **(A1)**

$q=\sqrt{65}$ **A1** **N3**

Note: Award ***A0*** for $q=\pm\sqrt{65}$.

[7 marks]

Total [15 marks]

International Baccalaureate®
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MARKSCHEME

November 2013

MATHEMATICS

Standard Level

Paper 1

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**; working must be seen.

(M) Marks awarded for **Method**; may be implied by **correct** subsequent working.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

(A) Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.

R Marks awarded for clear **Reasoning.**

N Marks awarded for **correct** answers if **no** working shown.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Mark according to scoris instructions and the document "**Mathematics SL: Guidance for e-marking November 2013**". It is **essential** that you read this document before you start marking. In particular, please note the following. Marks must be recorded using the annotation stamps, using the new scoris tool. Please check that you are entering marks for the right question.

- If a part is **completely correct**, (and gains all the "must be seen" marks), use the ticks with numbers to stamp full marks. Do **not** use the ticks with numbers for anything else.
- If a part is completely wrong, stamp ***A0*** by the final answer.
- If a part gains anything else, all the working **must** have annotations stamped to show what marks are awarded. This includes any zero marks.

All the marks will be added and recorded by scoris.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any. An exception to this rule is when work for ***M1*** is missing, as opposed to incorrect (see point 4).
- Where ***M*** and ***A*** marks are noted on the same line, *eg* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*eg* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***(M2)***, ***N3***, *etc*., do **not** split the marks, unless there is a note.
- Once a correct answer to a question or part-question is seen, ignore further working.
- Most ***M*** marks are for a **valid** method, ie a method which can lead to the answer: it must indicate some form of progress towards the answer.

3 *N* marks

*If **no** working shown, award **N** marks for **correct** answers – this includes acceptable answers (see accuracy booklet). In this case, ignore mark breakdown (**M**, **A**, **R**).*

- Do **not** award a mixture of ***N*** and other marks.
- There may be fewer ***N*** marks available than the total of ***M***, ***A*** and ***R*** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.
- There may not be a direct relationship between the ***N*** marks and the implied marks. There are times when all the marks are implied, but the ***N*** marks are not the full marks: this indicates that we want to see some of the working, without specifying what.
- For consistency within the markscheme, ***N*** marks are noted for every part, even when these match the mark breakdown.
- If a candidate has incorrect working, which somehow results in a correct answer, do **not** award the ***N*** marks for this correct answer. However, if the candidate has indicated (usually by crossing out) that the working is to be ignored, award the ***N*** marks for the correct answer.

4 Implied and must be seen marks

*Implied marks appear in **brackets** eg **(M1)**.*

- Implied marks can only be awarded if the work is seen or if implied in subsequent working (a correct final answer does not necessarily mean that the implied marks are all awarded). There are questions where some working is required, but as it is accepted that not everyone will write the same steps, all the marks are implied, but the ***N*** marks are not the full marks for the question.
- Normally the correct work is seen in the next line.
- Where there is an (***M1***) followed by ***A1*** for each correct answer, if no working shown, one correct answer is sufficient evidence to award the (***M1***).

*Must be seen marks appear without **brackets** eg **M1**.*

- Must be seen marks can only be awarded if the work is seen.
- If a must be seen mark is not awarded because work is missing (as opposed to ***M0*** or ***A0*** for incorrect work) all subsequent marks may be awarded if appropriate.

5 Follow through marks (only applied after an error is made)

*Follow through (**FT**) marks are awarded where an incorrect answer (final or intermediate) from one **part** of a question is used correctly in **subsequent** part(s) or subpart(s). Usually, to award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if the only marks awarded in a subpart are for the **final** answer, then **FT** marks should be awarded if appropriate. Examiners are expected to check student work in order to award **FT** marks where appropriate.*

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** and ***R*** marks may be awarded if appropriate. (However, as noted above, if an ***A*** mark is not awarded because work is missing, all subsequent marks may be awarded if appropriate)
- Exceptions to this rule will be explicitly noted on the markscheme.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*eg* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word "**their**" in a description, to indicate that candidates may be using an incorrect value.
- If a candidate makes an error in one part, but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the question says hence. It is often possible to use a different approach in subsequent parts that does not depend on the answer to previous parts.

- In a "show that" question, if an error in a previous subpart leads to not showing the required answer, do not award the final ***A1***. Note that if the error occurs within the same subpart, the ***FT*** rules may result in further loss of marks.
- Where there are anticipated common errors, the ***FT*** answers are often noted on the markscheme, to help examiners. It should be stressed that these are not the only ***FT*** answers accepted, neither should ***N*** marks be awarded for these answers.

6 Mis-read

*If a candidate incorrectly copies information from the question, this is a mis-read (**MR**). A candidate should be penalized only once for a particular mis-read. Use the **MR** stamp to indicate that this is a misread. Do not award the first mark in the question, even if this is an **M** mark, but award all others (if appropriate) so that the candidate only loses one mark for the misread.*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*eg* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.

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An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. In such cases the annotation DM should be used and a brief **note** *written next to the mark explaining this decision.*

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Unless the question specifies otherwise, ***accept*** *equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
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If the level of accuracy is specified in the question, a mark will be allocated for giving the final answer to the required accuracy. When this is not specified in the question, all numerical answers should be given exactly or correct to three significant figures

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The notes on how to allocate marks for sketches usually refer to passing through particular points or having certain features. These marks can only be awarded if the sketch is approximately the correct shape. All values given will be an approximate guide to where these points/features occur. In some questions, the first ***A1*** is for the shape, in others, the marks are only for the points and/or features. In both cases, unless the shape is approximately correct, no marks can be awarded (unless otherwise stated). However, if the graph is based on previous calculations, ***FT*** marks should be awarded if appropriate.

SECTION A

1. (a) appropriate approach *(M1)*

eg $\vec{QP} = \vec{QO} + \vec{OP}$, $P - Q$

$\vec{QP} = \boldsymbol{p} - \boldsymbol{q}$ *A1* *N2*

[2 marks]

(b) recognizing correct vector for $\vec{QT}$ or $\vec{PT}$ *(A1)*

eg $\vec{QT} = \frac{1}{2}(\boldsymbol{p} - \boldsymbol{q})$, $\vec{PT} = \frac{1}{2}(\boldsymbol{q} - \boldsymbol{p})$

appropriate approach *(M1)*

eg $\vec{OT} = \vec{OP} + \vec{PT}$, $\vec{OQ} + \vec{QT}$, $\vec{OP} + \frac{1}{2}\vec{PQ}$

$\vec{OT} = \frac{1}{2}(\boldsymbol{p} + \boldsymbol{q})$ $\left(\text{accept } \frac{\boldsymbol{p}+\boldsymbol{q}}{2}\right)$ *A1* *N2*

[3 marks]

[Total 5 marks]

2. (a) evidence of matrix multiplication (in any order) *(M1)*

eg $(1\times 2)+(2\times 1)$, one correct element

$\boldsymbol{AB} = \begin{pmatrix} 4 & 7 \\ 6 & 3 \end{pmatrix}$ *A2* *N3*

> **Note:** Award ***A1*** for three correct elements.

[3 marks]

(b) $\boldsymbol{AB} + \boldsymbol{C} = \begin{pmatrix} 6 & 3 \\ 6 & 4 \end{pmatrix}$ *A1*

correct substitution into formula for determinant *(A1)*

eg $(6\times 4)-(6\times 3)$

$\det(\boldsymbol{AB}+\boldsymbol{C}) = 6$ *A1* *N2*

> **Note:** Exception to ***FT***: if working shown, award ***FT*** on an incorrect 2×2 matrix $\boldsymbol{AB}+\boldsymbol{C}$.

[3 marks]

[Total 6 marks]

3. (a) attempt to find number who took less than 45 minutes ***(M1)***
eg line on graph (vertical at approx 45, or horizontal at approx 70)

70 students (accept 69) ***A1 N2***
[2 marks]

(b) 55 students completed task in less than 35 minutes ***(A1)***

subtracting **their** values ***(M1)***
eg 70 – 55

15 students ***A1 N2***
[3 marks]

(c) correct approach ***(A1)***
eg line from *y*-axis on 50

$k = 33$ ***A1 N2***
[2 marks]

[Total 7 marks]

4. (a) appropriate approach ***(M1)***
eg $2\int f(x)$, $2(8)$

$\int_1^6 2f(x)\,dx = 16$ ***A1 N2***
[2 marks]

(b) appropriate approach ***(M1)***
eg $\int f(x) + \int 2$, $8 + \int 2$

$\int 2dx = 2x$ (seen anywhere) ***(A1)***

substituting limits into **their** integrated function and subtracting
(in any order) ***(M1)***
eg $2(6) - 2(1)$, $8 + 12 - 2$

$\int_1^6 \left(f(x) + 2\right)dx = 18$ ***A1 N3***

[4 marks]

[Total 6 marks]

5. (a) **METHOD 1**

attempt to substitute both coordinates (in any order) into f ***(M1)***

eg $f\left(\frac{\pi}{4}\right)=6,\ \frac{\pi}{4}=\sin\left(6+\frac{\pi}{4}\right)+k$

correct working ***(A1)***

eg $\sin\frac{\pi}{2}=1,\ 1+k=6$

$k=5$ ***A1*** ***N2***

[3 marks]

METHOD 2

recognizing shift of $\frac{\pi}{4}$ left means maximum at 6 ***(R1)***

recognizing k is difference of maximum and amplitude ***(A1)***

eg $6-1$

$k=5$ ***A1*** ***N2***

[3 marks]

(b) evidence of appropriate approach ***(M1)***

eg minimum value of $\sin x$ is $-1,\ -1+k$, $f'(x)=0$, $\left(\frac{5\pi}{4},\ 4\right)$

minimum value is 4 ***A1*** ***N2***

[2 marks]

(c) $p=-\frac{\pi}{4},\ q=5$ $\left(\text{accept } \begin{pmatrix} -\frac{\pi}{4} \\ 5 \end{pmatrix}\right)$ ***A1A1*** ***N2***

[2 marks]

[Total 7 marks]

6. recognising need to differentiate (seen anywhere) ***R1***

eg f', $2e^{2x}$

attempt to find the gradient when $x=1$ ***(M1)***

eg $f'(1)$

$f'(1)=2e^2$ ***(A1)***

attempt to substitute coordinates (in any order) into equation of a straight line ***(M1)***

eg $y-e^2=2e^2(x-1)$, $e^2=2e^2(1)+b$

correct working ***(A1)***

eg $y-e^2=2e^2x-2e^2$, $b=-e^2$

$y=2e^2x-e^2$ ***A1*** ***N3***

[6 marks]

7. evidence of discriminant ***(M1)***

eg b^2-4ac, $\Delta=0$

correct substitution into discriminant ***(A1)***

eg $(k+2)^2-4(2k)$, $k^2+4k+4-8k$

correct discriminant ***A1***

eg k^2-4k+4, $(k-2)^2$

recognizing discriminant is positive ***R1***

eg $\Delta>0$, $(k+2)^2-4(2k)>0$

attempt to solve **their** quadratic in k ***(M1)***

eg factorizing, $k=\frac{4\pm\sqrt{16-16}}{2}$

correct working ***A1***

eg $(k-2)^2>0$, $k=2$, sketch of positive parabola on the *x*-axis

correct values ***A2*** ***N4***

eg $k\in\mathbb{R}$ and $k\neq 2$, $\mathbb{R}\setminus 2$, $]-\infty, 2[\cup]2, \infty[$

[8 marks]

SECTION B

8. (a) interchanging x and y ***(M1)***

eg $x = 3y - 2$

$f^{-1}(x) = \frac{x+2}{3}$ $\left(\text{accept } y = \frac{x+2}{3}, \frac{x+2}{3}\right)$ ***A1*** ***N2***

[2 marks]

(b) attempt to form composite (in any order) ***(M1)***

eg $g\left(\frac{x+2}{3}\right)$, $\frac{\frac{5}{3x}+2}{3}$

correct substitution ***A1***

eg $\frac{5}{3\left(\frac{x+2}{3}\right)}$

$\left(g \circ f^{-1}\right)(x) = \frac{5}{x+2}$ ***AG*** ***N0***

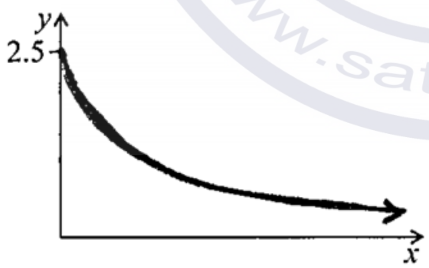
[2 marks]

(c) (i) valid approach ***(M1)***

eg $h(0)$, $\frac{5}{0+2}$

$y = \frac{5}{2}$ $\left(\text{accept } (0, 2.5)\right)$ ***A1*** ***N2***

(ii)



A1A2 ***N3***

Notes: Award ***A1*** for approximately correct shape (reciprocal, decreasing, concave up).

Only if this ***A1*** is awarded, award ***A2*** for all the following approximately correct features: *y*-intercept at (0, 2.5), asymptotic to *x*-axis, correct domain $x \geq 0$.
If only two of these features are correct, award ***A1***.

[5 marks]

Continued …

Question 8 continued

(d) (i) $x = \frac{5}{2}$ (accept $(2.5, 0)$) ***A1*** ***N1***

(ii) $x = 0$ (must be an equation) ***A1*** ***N1***
[2 marks]

(e) **METHOD 1**

attempt to substitute 3 into h (seen anywhere) ***(M1)***

eg $h(3)$, $\frac{5}{3+2}$

correct equation ***(A1)***

eg $a = \frac{5}{3+2}$, $h(3) = a$

$a = 1$ ***A1*** ***N2***
[3 marks]

METHOD 2

attempt to find inverse (may be seen in (d)) ***(M1)***

eg $x = \frac{5}{y+2}$, $h^{-1} = \frac{5}{x} - 2$, $\frac{5}{x} + 2$

correct equation, $\frac{5}{x} - 2 = 3$ ***(A1)***

$a = 1$ ***A1*** ***N2***
[3 marks]

Total [14 marks]

9. (a) (i) correct expression for r ***A1*** ***N1***

eg $r = \frac{6}{m-1}, \frac{m+4}{6}$

(ii) correct equation ***A1***

eg $\frac{6}{m-1} = \frac{m+4}{6}, \frac{6}{m+4} = \frac{m-1}{6}$

correct working ***(A1)***

eg $(m+4)(m-1) = 36$

correct working ***A1***

eg $m^2 - m + 4m - 4 = 36$, $m^2 + 3m - 4 = 36$

$m^2 + 3m - 40 = 0$ ***AG*** ***N0***

[4 marks]

(b) (i) valid attempt to solve ***(M1)***

eg $(m+8)(m-5) = 0$, $m = \frac{-3 \pm \sqrt{9+4\times 40}}{2}$

$m = -8$, $m = 5$ ***A1A1*** ***N3***

(ii) attempt to substitute **any** value of m to find r ***(M1)***

eg $\frac{6}{-8-1}, \frac{5+4}{6}$

$r = \frac{3}{2}, r = -\frac{2}{3}$ ***A1A1*** ***N3***

[6 marks]

(c) (i) $r = -\frac{2}{3}$ (may be seen in justification) ***A1***

valid reason ***R1*** ***N0***

eg $|r| < 1$, $-1 < \frac{-2}{3} < 1$

Notes: Award ***R1*** for $|r| < 1$ only if ***A1*** awarded.

finding the first term of the sequence which has $|r| < 1$ ***(A1)***

eg $-8-1$, $6 \div \frac{-2}{3}$

$u_1 = -9$ (may be seen in formula) ***(A1)***

correct substitution of u_1 and **their** r into $\frac{u_1}{1-r}$, as long as $|r| < 1$ ***A1***

eg $S_\infty = \frac{-9}{1-\left(-\frac{2}{3}\right)}, \frac{-9}{\frac{5}{3}}$

$S_\infty = -\frac{27}{5}\ (= -5.4)$ ***A1*** ***N3***

[6 marks]

Total [16 marks]

10. (a) **METHOD 1**

correct use of chain rule *A1A1*

eg $\frac{2\ln x}{2}\times\frac{1}{x}$, $\frac{2\ln x}{2x}$

Note: Award ***A1*** for $\frac{2\ln x}{2}$, ***A1*** for $\times\frac{1}{x}$.

$f'(x)=\frac{\ln x}{x}$ *AG* *N0*

[2 marks]

METHOD 2

correct substitution into quotient rule, with derivatives seen *A1*

eg $\frac{2\times 2\ln x\times\frac{1}{x}-0\times(\ln x)^2}{4}$

correct working *A1*

eg $\frac{4\ln x\times\frac{1}{x}}{4}$

$f'(x)=\frac{\ln x}{x}$ *AG* *N0*

[2 marks]

(b) setting derivative $=0$ *(M1)*

eg $f'(x)=0$, $\frac{\ln x}{x}=0$

correct working *(A1)*

eg $\ln x=0$, $x=\mathrm{e}^0$

$x=1$ *A1* *N2*

[3 marks]

(c) intercept when $f'(x)=0$ *(M1)*

$p=1$ *A1* *N2*

[2 marks]

Continued …

Question 10 continued

(d) equating functions ***(M1)***

eg $f' = g$, $\frac{\ln x}{x} = \frac{1}{x}$

correct working ***(A1)***

eg $\ln x = 1$

$q = \mathrm{e}$ (accept $x = \mathrm{e}$) ***A1*** ***N2***

[3 marks]

(e) evidence of integrating and subtracting functions (in any order, seen anywhere) ***(M1)***

eg $\int_1^e \left(\frac{1}{x} - \frac{\ln x}{x}\right) \mathrm{d}x$, $\int f' - g$

correct integration $\ln x - \frac{(\ln x)^2}{2}$ ***A2***

substituting limits into **their** integrated function and subtracting (in any order) ***(M1)***

eg $(\ln \mathrm{e} - \ln 1) - \left(\frac{(\ln \mathrm{e})^2}{2} - \frac{(\ln 1)^2}{2}\right)$

Note: Do not award ***M1*** if the integrated function has only one term.

correct working ***A1***

eg $(1-0) - \left(\frac{1}{2} - 0\right)$, $1 - \frac{1}{2}$

area $= \frac{1}{2}$ ***AG*** ***N0***

Notes: Candidates may work with two separate integrals, and only combine them at the end. Award marks in line with the markscheme.

[5 marks]

Total [15 marks]

International Baccalaureate
Baccalauréat International
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MARKSCHEME

May 2013

MATHEMATICS

Standard Level

Paper 1

18 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**; working must be seen.

(M) Marks awarded for **Method**; may be implied by **correct** subsequent working.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

(A) Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.

R Marks awarded for clear **Reasoning.**

N Marks awarded for **correct** answers if **no** working shown.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Mark according to scoris instructions and the document "**Mathematics SL: Guidance for e-marking May 2013**". It is **essential** that you read this document before you start marking. In particular, please note the following. Marks must be recorded using the annotation stamps, using the new scoris tool. Please check that you are entering marks for the right question.

- If a part is **completely correct**, (and gains all the "must be seen" marks), use the ticks with numbers to stamp full marks.
- If a part is completely wrong, stamp ***A0*** by the final answer.
- If a part gains anything else, it **must** be recorded using **all** the annotations.

All the marks will be added and recorded by scoris.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any. An exception to this rule is when work for ***M1*** is missing, as opposed to incorrect (see point 4).
- Where ***M*** and ***A*** marks are noted on the same line, *eg* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*eg* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***(M2)***, ***N3***, *etc*., do **not** split the marks, unless there is a note.
- Once a correct answer to a question or part-question is seen, ignore further working.
- Most ***M*** marks are for a **valid** method, *ie* a method which can lead to the answer: it must indicate some form of progress towards the answer.
- ***A*** marks are often dependent on the ***R*** mark being awarded for justification for the ***A*** mark, in which case it is not possible to award ***A1R0***. Hence the ***A1*** is not awarded for a correct answer if no justification or the wrong justification is given.

3 *N* marks

*If **no** working shown, award **N** marks for **correct** answers. In this case, ignore mark breakdown (**M**, **A**, **R**).*

- Do **not** award a mixture of ***N*** and other marks.
- There may be fewer ***N*** marks available than the total of ***M***, ***A*** and ***R*** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.
- There may not be a direct relationship between the ***N*** marks and the implied marks. There are times when all the marks are implied, but the ***N*** marks are not the full marks: this indicates that we want to see some of the working, without specifying what.
- For consistency within the markscheme, ***N*** marks are noted for every part, even when these match the mark breakdown.
- If a candidate has incorrect working, which somehow results in a correct answer, do **not** award the ***N*** marks for this correct answer. However, if the candidate has indicated (usually by crossing out) that the working is to be ignored, award the ***N*** marks for the correct answer.

4 Implied and must be seen marks

*Implied marks appear in **brackets eg (M1)**.*

- Implied marks can only be awarded if **correct** work is seen or if implied in subsequent working (a correct final answer does not necessarily mean that the implied marks are all awarded). There are questions where some working is required, but as it is accepted that not everyone will write the same steps, all the marks are implied, but the ***N*** marks are not the full marks for the question.
- Normally the correct work is seen or implied in the next line.
- Where there is an (***M1***) followed by ***A1*** for each correct answer, if no working shown, one correct answer is sufficient evidence to award the (***M1)***. An exception to this is where at least one numerical final answer is not given to the correct three significant figures (see the accuracy booklet).

*Must be seen marks appear without **brackets eg M1**.*

- Must be seen marks can only be awarded if **correct** work is seen.
- If a must be seen mark is not awarded because work is missing (as opposed to ***M0*** or ***A0*** for incorrect work) all subsequent marks may be awarded if appropriate.

5 Follow through marks (only applied after an error is made)

*Follow through (**FT**) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) or subpart(s). Usually, to award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if the only marks awarded in a subpart are for the **final** answer, then **FT** marks should be awarded if appropriate. Examiners are expected to check student work in order to award **FT** marks where appropriate.*

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate. (However, as noted above, if an ***A*** mark is not awarded because work is missing, all subsequent marks may be awarded if appropriate)
- Exceptions to this rule will be explicitly noted on the markscheme.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*eg* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word "**their**" in a description, to indicate that candidates may be using an incorrect value.

- If a candidate makes an error in one part, but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the question says hence. It is often possible to use a different approach in subsequent parts that does not depend on the answer to previous parts.
- In a "show that" question, if an error in a previous subpart leads to not showing the required answer, do not award the final ***A1***. Note that if the error occurs within the same subpart, the ***FT*** rules may result in further loss of marks.
- Where there are anticipated common errors, the ***FT*** answers are often noted on the markscheme, to help examiners. It should be stressed that these are not the only ***FT*** answers accepted, neither should ***N*** marks be awarded for these answers.

6 Mis-read

*If a candidate incorrectly copies information from the question, this is a mis-read (**MR**). A candidate should be penalized only once for a particular mis-read. Use the **MR** stamp to indicate that this is a misread. Do not award the first mark in the question, even if this is an **M** mark, but award all others (if appropriate) so that the candidate only loses one mark for the misread.*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*eg* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.

7 Discretionary marks *(d)*

An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. In such cases the annotation DM should be used and a brief **note** *written next to the mark explaining this decision.*

8 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for part-questions are indicated by **EITHER** . . . **OR**.
- Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9 Alternative forms

Unless the question specifies otherwise, ***accept*** *equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

10 Accuracy of Answers

If the level of accuracy is specified in the question, a mark will be allocated for giving the final answer to the required accuracy. When this is not specified in the question, all numerical answers should be given exactly or correct to three significant figures

*Candidates should NO LONGER be penalized for an accuracy error **(AP)**. Examiners should award marks according to the rules given in these instructions and the markscheme. Accuracy is not the same as correctness – an incorrect value does not achieve relevant **A** marks. It is only final answers which may lose marks for accuracy errors, not intermediate values. Please check work carefully for **FT**. Further information on which answers are accepted is given in a separate booklet, along with examples. It is **essential** that you read this carefully, as there are a number of changes.*

Do not accept unfinished numerical final answers such as 3/0.1 (unless otherwise stated). As a rule, numerical answers with more than one part (such as fractions) should be given using integers (*eg* 6/8). Calculations which lead to integers should be completed, with the exception of fractions which are not whole numbers.

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The markscheme aims to present answers using good communication, eg if the question asks to find the value of k, the markscheme will say $k = 3$, but the marks will be for the correct value 3 – there is usually no need for the "$k =$". In these cases, it is also usually acceptable to have another variable, as long as there is no ambiguity in the question, eg if the question asks to find the value of p and of q, then the student answer needs to be clear. Generally, the only situation where the full answer is required is in a question which asks for equations – in this case the markscheme will say "must be an equation".

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1. (a) (i) $2\boldsymbol{a} = \begin{pmatrix} 4 \\ -6 \end{pmatrix}$ ***(A1)***

correct expression for $2\boldsymbol{a} + \boldsymbol{b}$ ***A1*** ***N2***

eg $\begin{pmatrix} 5 \\ -2 \end{pmatrix}$, $(5, -2)$, $5\boldsymbol{i} - 2\boldsymbol{j}$

(ii) correct substitution into length formula ***(A1)***

eg $\sqrt{5^2 + 2^2}$, $\sqrt{5^2 + -2^2}$

$|2\boldsymbol{a} + \boldsymbol{b}| = \sqrt{29}$ ***A1*** ***N2***

[4 marks]

(b) valid approach ***(M1)***

eg $\boldsymbol{c} = -(2\boldsymbol{a} + \boldsymbol{b})$, $5 + x = 0, -2 + y = 0$

$\boldsymbol{c} = \begin{pmatrix} -5 \\ 2 \end{pmatrix}$ ***A1*** ***N2***

[2 marks]

Total [6 marks]

2. (a) $x = 1$, $x = -3$ $\left(\text{accept } (1, 0), (-3, 0)\right)$ ***A1A1*** ***N2***

[2 marks]

(b) **METHOD 1**

attempt to find *x*-coordinate ***(M1)***

eg $\frac{1 + -3}{2}$, $x = \frac{-b}{2a}$, $f'(x) = 0$

correct value, $x = -1$ (may be seen as a coordinate in the answer) ***A1***

attempt to find **their** *y*-coordinate ***(M1)***

eg $f(-1)$, -2×2, $y = \frac{-D}{4a}$

$y = -4$ ***A1***

vertex $(-1, -4)$ ***N3***

[4 marks]

METHOD 2

attempt to complete the square ***(M1)***

eg $x^2 + 2x + 1 - 1 - 3$

attempt to put into vertex form ***(M1)***

eg $(x + 1)^2 - 4$, $(x - 1)^2 + 4$

vertex $(-1, -4)$ ***A1A1*** ***N3***

[4 marks]

Total [6 marks]

3. (a) evidence of choosing product rule *(M1)*

eg $uv' + vu'$

correct derivatives (must be seen in the product rule) $\cos x$, $2x$ *(A1)(A1)*

$f'(x) = x^2 \cos x + 2x \sin x$ ***A1*** ***N4***

[4 marks]

(b) substituting $\frac{\pi}{2}$ into **their** $f'(x)$ *(M1)*

eg $f'\left(\frac{\pi}{2}\right)$, $\left(\frac{\pi}{2}\right)^2 \cos\left(\frac{\pi}{2}\right) + 2\left(\frac{\pi}{2}\right)\sin\left(\frac{\pi}{2}\right)$

correct values for **both** $\sin\frac{\pi}{2}$ and $\cos\frac{\pi}{2}$ seen in $f'(x)$ *(A1)*

eg $0 + 2\left(\frac{\pi}{2}\right) \times 1$

$f'\left(\frac{\pi}{2}\right) = \pi$ ***A1*** ***N2***

[3 marks]

Total [7 marks]

4. (a) attempt to solve for $\boldsymbol{X}$ ***(M1)***

eg $\boldsymbol{XA} = \boldsymbol{C} - \boldsymbol{B}$, $\boldsymbol{X} + \boldsymbol{B} = \boldsymbol{CA}^{-1}$, $\boldsymbol{A}^{-1}(\boldsymbol{C} - \boldsymbol{B})$, $\boldsymbol{A}^{-1}\boldsymbol{C} - \boldsymbol{B}$

$\boldsymbol{X} = (\boldsymbol{C} - \boldsymbol{B})\boldsymbol{A}^{-1} \quad (= \boldsymbol{CA}^{-1} - \boldsymbol{BA}^{-1})$ ***A1*** ***N2***

[2 marks]

(b) **METHOD 1**

$\boldsymbol{C} - \boldsymbol{B} = \begin{pmatrix} 1 & 2 \\ -4 & 2 \end{pmatrix}$ (seen anywhere) ***A1***

correct substitution into formula for 2×2 inverse ***A1***

eg $\boldsymbol{A}^{-1} = \frac{1}{4-6}\begin{pmatrix} 4 & -2 \\ -3 & 1 \end{pmatrix}$, $\begin{pmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{pmatrix}$

attempt to multiply $(\boldsymbol{C} - \boldsymbol{B})$ and $\boldsymbol{A}^{-1}$ (in any order) ***(M1)***

eg $\begin{pmatrix} -2+3 & 1-1 \\ 8+3 & -4-1 \end{pmatrix}$, $\begin{pmatrix} 4-6 & -2+2 \\ -16-6 & 8+2 \end{pmatrix}$, two correct elements

$\boldsymbol{X} = \begin{pmatrix} 1 & 0 \\ 11 & -5 \end{pmatrix}$ ***A2*** ***N3***

Note: Award ***A1*** for three correct elements.

[5 marks]

METHOD 2

correct substitution into formula for 2×2 inverse ***A1***

eg $\boldsymbol{A}^{-1} = \frac{1}{4-6}\begin{pmatrix} 4 & -2 \\ -3 & 1 \end{pmatrix}$, $\begin{pmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{pmatrix}$

attempt to multiply either $\boldsymbol{BA}^{-1}$ or $\boldsymbol{CA}^{-1}$ (in any order) ***(M1)***

eg $\frac{-1}{2}\begin{pmatrix} 0-3 & 0+1 \\ 4-6 & -2+2 \end{pmatrix}$, $\frac{-1}{2}\begin{pmatrix} -2-3 & -6+4 \\ \frac{3}{2}+\frac{3}{2} & \frac{9}{2}-2 \end{pmatrix}$, two correct entries

one correct multiplication ***A1***

eg $\frac{-1}{2}\begin{pmatrix} -3 & 1 \\ -2 & 0 \end{pmatrix}$, $\begin{pmatrix} \frac{5}{2} & -\frac{1}{2} \\ 12 & -5 \end{pmatrix}$

$\boldsymbol{X} = \begin{pmatrix} 1 & 0 \\ 11 & -5 \end{pmatrix}$ ***A2*** ***N3***

Note: Award ***A1*** for three correct elements. .

[5 marks]

Total [7 marks]

5. (a) **METHOD 1**

attempt to set up equation *(M1)*

eg $2 = \sqrt{y-5}$, $2 = \sqrt{x-5}$

correct working *(A1)*

eg $4 = y-5$, $x = 2^2 + 5$

$f^{-1}(2) = 9$ ***A1*** ***N2***

[3 marks]

METHOD 2

interchanging x and y (seen anywhere) *(M1)*

eg $x = \sqrt{y-5}$

correct working *(A1)*

eg $x^2 = y-5$, $y = x^2 + 5$

$f^{-1}(2) = 9$ ***A1*** ***N2***

[3 marks]

(b) recognizing $g^{-1}(3) = 30$ *(M1)*

eg $f(30)$

correct working *(A1)*

eg $(f \circ g^{-1})(3) = \sqrt{30-5}$, $\sqrt{25}$

$(f \circ g^{-1})(3) = 5$ ***A1*** ***N2***

Note: Award ***A0*** for multiple values, *eg* ± 5.

[3 marks]

Total [6 marks]

6. attempt to integrate which involves ln ***(M1)***

eg $\ln(2x-5), 12\ln 2x-5, \ln 2x$

correct expression (accept absence of C)

eg $12\ln(2x-5)\frac{1}{2}+C$, $6\ln(2x-5)$ ***A2***

attempt to substitute $(4, 0)$ into **their** integrated f ***(M1)***

eg $0=6\ln(2\times 4-5)$, $0=6\ln(8-5)+C$

$C=-6\ln 3$ ***(A1)***

$f(x)=6\ln(2x-5)-6\ln 3$ $\left(=6\ln\left(\frac{2x-5}{3}\right)\right)$ (accept $6\ln(2x-5)-\ln 3^6$) ***A1*** ***N5***

Note: Exception to the ***FT*** rule. Allow full ***FT*** on incorrect integration which must involve ln.

Total [6 marks]

7. (a) evidence of correct formula ***(M1)***

eg $\log a-\log b=\log\frac{a}{b}$, $\log\left(\frac{40}{5}\right)$, $\log 8+\log 5-\log 5$

Note: Ignore missing or incorrect base.

correct working ***(A1)***

eg $\log_2 8$, $2^3=8$

$\log_2 40-\log_2 5=3$ ***A1*** ***N2***

[3 marks]

(b) attempt to write 8 as a power of 2 (seen anywhere) ***(M1)***

eg $(2^3)^{\log_2 5}$, $2^3=8$, 2^a

multiplying powers ***(M1)***

eg $2^{3\log_2 5}$, $a\log_2 5$

correct working ***(A1)***

eg $2^{\log_2 125}$, $\log_2 5^3$, $\left(2^{\log_2 5}\right)^3$

$8^{\log_2 5}=125$ ***A1*** ***N3***

[4 marks]

Total [7 marks]

SECTION B

8. (a) (i) valid approach *(M1)*

eg $\begin{pmatrix}7\\-4\\3\end{pmatrix}-\begin{pmatrix}1\\-2\\-1\end{pmatrix}$, $\text{A}-\text{B}$, $\overrightarrow{\text{AB}}=\overrightarrow{\text{AO}}+\overrightarrow{\text{OB}}$

$\overrightarrow{\text{AB}}=\begin{pmatrix}6\\-2\\4\end{pmatrix}$ *A1* *N2*

(ii) any correct equation in the form $\boldsymbol{r}=\boldsymbol{a}+t\boldsymbol{b}$ (accept any parameter for t)

where $\boldsymbol{a}=\begin{pmatrix}1\\-2\\3\end{pmatrix}$ and $\boldsymbol{b}$ is a scalar multiple of $\overrightarrow{\text{AB}}$ *A2* *N2*

eg $\boldsymbol{r}=\begin{pmatrix}1\\-2\\3\end{pmatrix}+t\begin{pmatrix}6\\-2\\4\end{pmatrix}$, $(x, y, z)=(1, -2, 3)+t(3, -1, 2)$, $\boldsymbol{r}=\begin{pmatrix}1+6t\\-2-2t\\3+4t\end{pmatrix}$

Note: Award ***A1*** for $\boldsymbol{a}+t\boldsymbol{b}$, ***A1*** for $L_1=\boldsymbol{a}+t\boldsymbol{b}$, ***A0*** for $\boldsymbol{r}=\boldsymbol{b}+t\boldsymbol{a}$.

[4 marks]

(b) recognizing that scalar product $=0$ (seen anywhere) *R1*

correct calculation of scalar product *(A1)*
eg $6(3)-2(-3)+4p$, $18+6+4p$

correct working *A1*
eg $24+4p=0$, $4p=-24$

$p=-6$ *AG* *N0*

[3 marks]

continued …

Question 8 continued

(c) setting lines equal ***(M1)***

eg $L_1 = L_2$, $\begin{pmatrix}1\\-2\\3\end{pmatrix}+t\begin{pmatrix}6\\-2\\4\end{pmatrix}=\begin{pmatrix}-1\\2\\15\end{pmatrix}+s\begin{pmatrix}3\\-3\\-6\end{pmatrix}$

any two correct equations with **different** parameters ***A1A1***
eg $1+6t=-1+3s$, $-2-2t=2-3s$, $3+4t=15-6s$

attempt to solve **their** simultaneous equations ***(M1)***

one correct parameter ***A1***
eg $t=\frac{1}{2}$, $s=\frac{5}{3}$

attempt to substitute parameter into vector equation ***(M1)***
eg $\begin{pmatrix}1\\-2\\3\end{pmatrix}+\frac{1}{2}\begin{pmatrix}6\\-2\\4\end{pmatrix}$, $1+\frac{1}{2}\times 6$

$x=4$ (accept $(4, -3, 5)$, ignore incorrect values for y and z) ***A1*** ***N3***

[7 marks]

Total [14 marks]

9. (a) (i) attempt to find $\text{P}(\text{red})\times\text{P}(\text{red})$ ***(M1)***

eg $\frac{3}{8}\times\frac{2}{7}$, $\frac{3}{8}\times\frac{3}{8}$, $\frac{3}{8}\times\frac{2}{8}$

$\text{P}(\text{none green}) = \frac{6}{56}\ \left(=\frac{3}{28}\right)$ ***A1*** ***N2***

(ii) attempt to find $\text{P}(\text{red})\times\text{P}(\text{green})$ ***(M1)***

eg $\frac{5}{8}\times\frac{3}{7}$, $\frac{3}{8}\times\frac{5}{8}$, $\frac{15}{56}$

recognizing two ways to get one red, one green ***(M1)***

eg $2\text{P}(R)\times\text{P}(G)$, $\frac{5}{8}\times\frac{3}{7}+\frac{3}{8}\times\frac{5}{7}$, $\frac{3}{8}\times\frac{5}{8}\times 2$

$\text{P}(\text{exactly one green}) = \frac{30}{56}\ \left(=\frac{15}{28}\right)$ ***A1*** ***N2***

[5 marks]

(b) $\text{P}(\text{both green}) = \frac{20}{56}$ (seen anywhere) ***(A1)***

correct substitution into formula for $\text{E}(X)$ ***A1***

eg $0\times\frac{6}{56}+1\times\frac{30}{56}+2\times\frac{20}{56}$, $\frac{30}{64}+\frac{50}{64}$

expected number of green marbles is $\frac{70}{56}\ \left(=\frac{5}{4}\right)$ ***A1*** ***N2***

[3 marks]

continued …

Question 9 continued

(c) (i) $\mathrm{P}(\text{jar B}) = \frac{4}{6} \quad \left(= \frac{2}{3}\right)$ ***A1*** ***N1***

(ii) $\mathrm{P}(\text{red} \mid \text{jar B}) = \frac{6}{8} \quad \left(= \frac{3}{4}\right)$ ***A1*** ***N1***

[2 marks]

(d) recognizing conditional probability ***(M1)***

eg $\mathrm{P}(A|R)$, $\frac{\mathrm{P}(\text{jar A and red})}{\mathrm{P}(\text{red})}$, tree diagram

attempt to multiply along either branch (may be seen on diagram) ***(M1)***

eg $\mathrm{P}(\text{jar A and red}) = \frac{1}{3} \times \frac{3}{8} \quad \left(= \frac{1}{8}\right)$

attempt to multiply along **other** branch ***(M1)***

eg $\mathrm{P}(\text{jar B and red}) = \frac{2}{3} \times \frac{6}{8} \quad \left(= \frac{1}{2}\right)$

adding the probabilities of two mutually exclusive paths ***(A1)***

eg $\mathrm{P}(\text{red}) = \frac{1}{3} \times \frac{3}{8} + \frac{2}{3} \times \frac{6}{8} \quad \left(= \frac{5}{8}\right)$

correct substitution

eg $\mathrm{P}(\text{jar A} \mid \text{red}) = \frac{\frac{1}{3} \times \frac{3}{8}}{\frac{1}{3} \times \frac{3}{8} + \frac{2}{3} \times \frac{6}{8}}, \ \frac{\frac{1}{8}}{\frac{5}{8}}$ ***A1***

$\mathrm{P}(\text{jar A} \mid \text{red}) = \frac{1}{5}$ ***A1*** ***N3***

[6 marks]

Total [16 marks]

10. (a) substitute 0 into f *(M1)*
eg $\ln(0+1)$, $\ln 1$

$f(0)=0$ *A1* *N2*
[2 marks]

(b) $f'(x)=\frac{1}{x^4+1}\times 4x^3$ (seen anywhere) *A1A1*

Note: Award ***A1*** for $\frac{1}{x^4+1}$ and ***A1*** for $4x^3$.

recognizing f increasing where $f'(x)>0$ (seen anywhere) *R1*
eg $f'(x)>0$, diagram of signs

attempt to solve $f'(x)>0$ *(M1)*
eg $4x^3=0$, $x^3>0$

f increasing for $x>0$ (accept $x\geq 0$) *A1* *N1*
[5 marks]

(c) (i) substituting $x=1$ into f'' *(A1)*
eg $\frac{4(3-1)}{(1+1)^2}$, $\frac{4\times 2}{4}$

$f''(1)=2$ *A1* *N2*

(ii) valid interpretation of point of inflexion (seen anywhere) *R1*
eg no change of sign in $f''(x)$, no change in concavity,
f' increasing both sides of zero

attempt to find $f''(x)$ for $x<0$ *(M1)*
eg $f''(-1)$, $\frac{4(-1)^2\left(3-(-1)^4\right)}{\left((-1)^4+1\right)^2}$, diagram of signs

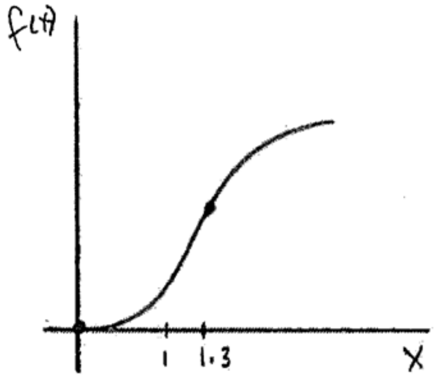
correct working leading to positive value *A1*
eg $f''(-1)=2$, discussing signs of numerator **and** denominator

there is no point of inflexion at $x=0$ *AG* *N0*
[5 marks]

continued …

Question 10 continued

(d)



A1A1A1 ***N3***

Notes: Award ***A1*** for shape concave up left of POI and concave down right of POI. Only if this ***A1*** is awarded, then award the following: ***A1*** for curve through $(0, 0)$, ***A1*** for increasing throughout. Sketch need not be drawn to scale. Only essential features need to be clear.

[3 marks]

Total [15 marks]

International Baccalaureate®
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MARKSCHEME

May 2013

MATHEMATICS

Standard Level

Paper 1

17 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**; working must be seen.

(M) Marks awarded for **Method**; may be implied by **correct** subsequent working.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

(A) Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.

R Marks awarded for clear **Reasoning.**

N Marks awarded for **correct** answers if **no** working shown.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Mark according to scoris instructions and the document "**Mathematics SL: Guidance for e-marking May 2013**". It is **essential** that you read this document before you start marking. In particular, please note the following. Marks must be recorded using the annotation stamps, using the new scoris tool. Please check that you are entering marks for the right question.

- If a part is **completely correct**, (and gains all the "must be seen" marks), use the ticks with numbers to stamp full marks.
- If a part is completely wrong, stamp ***A0*** by the final answer.
- If a part gains anything else, it **must** be recorded using **all** the annotations.

All the marks will be added and recorded by scoris.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any. An exception to this rule is when work for ***M1*** is missing, as opposed to incorrect (see point 4).
- Where ***M*** and ***A*** marks are noted on the same line, *eg* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*eg* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***(M2)***, ***N3***, *etc*., do **not** split the marks, unless there is a note.
- Once a correct answer to a question or part-question is seen, ignore further working.
- Most ***M*** marks are for a **valid** method, ie a method which can lead to the answer: it must indicate some form of progress towards the answer.
- ***A*** marks are often dependent on the ***R*** mark being awarded for justification for the ***A*** mark, in which case it is not possible to award ***A1R0***. Hence the ***A1*** is not awarded for a correct answer if no justification or the wrong justification is given.

3 *N* marks

*If **no** working shown, award **N** marks for **correct** answers. In this case, ignore mark breakdown (**M**, **A**, **R**).*

- Do **not** award a mixture of ***N*** and other marks.
- There may be fewer ***N*** marks available than the total of ***M***, ***A*** and ***R*** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.
- There may not be a direct relationship between the ***N*** marks and the implied marks. There are times when all the marks are implied, but the ***N*** marks are not the full marks: this indicates that we want to see some of the working, without specifying what.
- For consistency within the markscheme, ***N*** marks are noted for every part, even when these match the mark breakdown.
- If a candidate has incorrect working, which somehow results in a correct answer, do **not** award the ***N*** marks for this correct answer. However, if the candidate has indicated (usually by crossing out) that the working is to be ignored, award the ***N*** marks for the correct answer.

4 Implied and must be seen marks

*Implied marks appear in **brackets eg (M1)**.*

- Implied marks can only be awarded if **correct** work is seen or if implied in subsequent working (a correct final answer does not necessarily mean that the implied marks are all awarded). There are questions where some working is required, but as it is accepted that not everyone will write the same steps, all the marks are implied, but the ***N*** marks are not the full marks for the question.
- Normally the correct work is seen or implied in the next line.
- Where there is an (***M1***) followed by ***A1*** for each correct answer, if no working shown, one correct answer is sufficient evidence to award the (***M1)***. An exception to this is where at least one numerical final answer is not given to the correct three significant figures (see the accuracy booklet).

*Must be seen marks appear without **brackets eg M1**.*

- Must be seen marks can only be awarded if **correct** work is seen.
- If a must be seen mark is not awarded because work is missing (as opposed to ***M0*** or ***A0*** for incorrect work) all subsequent marks may be awarded if appropriate.

5 Follow through marks (only applied after an error is made)

*Follow through (**FT**) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) or subpart(s). Usually, to award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if the only marks awarded in a subpart are for the **final** answer, then **FT** marks should be awarded if appropriate. Examiners are expected to check student work in order to award **FT** marks where appropriate.*

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate. (However, as noted above, if an ***A*** mark is not awarded because work is missing, all subsequent marks may be awarded if appropriate)
- Exceptions to this rule will be explicitly noted on the markscheme.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*eg* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word "**their**" in a description, to indicate that candidates may be using an incorrect value.

- If a candidate makes an error in one part, but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the question says hence. It is often possible to use a different approach in subsequent parts that does not depend on the answer to previous parts.
- In a "show that" question, if an error in a previous subpart leads to not showing the required answer, do not award the final ***A1***. Note that if the error occurs within the same subpart, the ***FT*** rules may result in further loss of marks.
- Where there are anticipated common errors, the ***FT*** answers are often noted on the markscheme, to help examiners. It should be stressed that these are not the only ***FT*** answers accepted, neither should ***N*** marks be awarded for these answers.

6 Mis-read

*If a candidate incorrectly copies information from the question, this is a mis-read (**MR**). A candidate should be penalized only once for a particular mis-read. Use the **MR** stamp to indicate that this is a misread. Do not award the first mark in the question, even if this is an **M** mark, but award all others (if appropriate) so that the candidate only loses one mark for the misread.*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*eg* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.

7 Discretionary marks *(d)*

An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. In such cases the annotation DM should be used and a brief **note** *written next to the mark explaining this decision.*

8 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for part-questions are indicated by **EITHER** . . . **OR**.
- Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9 Alternative forms

*Unless the question specifies otherwise, **accept** equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

10 Accuracy of Answers

If the level of accuracy is specified in the question, a mark will be allocated for giving the final answer to the required accuracy. When this is not specified in the question, all numerical answers should be given exactly or correct to three significant figures

Candidates should NO LONGER be penalized for an accuracy error ***(AP)****. Examiners should award marks according to the rules given in these instructions and the markscheme. Accuracy is not the same as correctness – an incorrect value does not achieve relevant* ***A*** *marks. It is only final answers which may lose marks for accuracy errors, not intermediate values. Please check work carefully for* ***FT****. Further information on which answers are accepted is given in a separate booklet, along with examples. It is* ***essential*** *that you read this carefully, as there are a number of changes.*

Do not accept unfinished numerical final answers such as 3/0.1 (unless otherwise stated). As a rule, numerical answers with more than one part (such as fractions) should be given using integers (*eg* 6/8). Calculations which lead to integers should be completed, with the exception of fractions which are not whole numbers.

Clarification of intermediate values accuracy instructions

Intermediate values do not need to be given to the correct three significant figures. If candidates work with **any** rounded values, this could lead to an incorrect answer, in which case award ***A0*** for the final answer. However, do not penalise inaccurate intermediate values that lead to an acceptable final answer.

11 Calculators

No calculator is allowed. The use of any calculator on paper 1 is malpractice, and will result in no grade awarded. If you see work that suggests a candidate has used any calculator, please follow the procedures for malpractice. Examples: finding an angle, given a trig ratio of 0.4235.

12 Style

The markscheme aims to present answers using good communication, eg if the question asks to find the value of k, the markscheme will say $k = 3$*, but the marks will be for the correct value 3 – there is usually no need for the "* $k =$ *". In these cases, it is also usually acceptable to have another variable, as long as there is no ambiguity in the question, eg if the question asks to find the value of p and of q, then the student answer needs to be clear. Generally, the only situation where the full answer is required is in a question which asks for equations – in this case the markscheme will say "must be an equation".*

The markscheme often uses words to describe what the marks are for, followed by examples, using the eg notation. These examples are not exhaustive, and examiners should check what candidates have written, to see if they satisfy the description. Where these marks are ***M*** *marks, the examples may include ones using poor notation, to indicate what is acceptable. A valid method is one which will allow candidate to proceed to the next step eg if a quadratic function is given in factorised form, and the question asks for the zeroes, then multiplying the factors does not necessarily help to find the zeros, and would not on its own count as a valid method.*

13 Candidate work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

14. Diagrams

The notes on how to allocate marks for sketches usually refer to passing through particular points or having certain features. These marks can only be awarded if the sketch is approximately the correct shape. All values given will be an approximate guide to where these points/features occur. In some questions, the first ***A1*** is for the shape, in others, the marks are only for the points and/or features. In both cases, unless the shape is approximately correct, no marks can be awarded (unless otherwise stated). However, if the graph is based on previous calculations, ***FT*** marks should be awarded if appropriate.

SECTION A

1. (a) interchanging x and y (seen anywhere) ***(M1)***

eg $x = 4y - 2$

evidence of correct manipulation ***(A1)***

eg $x + 2 = 4y$

$f^{-1}(x) = \frac{x+2}{4}$ $\left(\text{accept } y = \frac{x+2}{4}, \frac{x+2}{4}, f^{-1}(x) = \frac{1}{4}x + \frac{1}{2}\right)$ ***A1*** ***N2***

[3 marks]

(b) **METHOD 1**

attempt to substitute 1 into $g(x)$ ***(M1)***

eg $g(1) = -2 \times 1^2 + 8$

$g(1) = 6$ ***(A1)***

$f(6) = 22$ ***A1*** ***N3***

METHOD 2

attempt to form composite function (in any order) ***(M1)***

eg $(f \circ g)(x) = 4(-2x^2 + 8) - 2$ $(= -8x^2 + 30)$

correct substitution

eg $(f \circ g)(1) = 4(-2 \times 1^2 + 8) - 2$, $-8 + 30$ ***(A1)***

$f(6) = 22$ ***A1*** ***N3***

[3 marks]

Total [6 marks]

2. (a) evidence of multiplying matrices ***A*** and ***B*** (in any order), (may be seen in (b)) ***(M1)***

eg $1 \times 2 + 2 \times 1$, row times column, one correct value in the first row

evidence of correct multiplication (***AB*** may be seen in (b)) ***(A1)***

eg $2 + 2 (= p)$, $\boldsymbol{AB} = \begin{pmatrix} 4 & 1+2q \\ 6 & 3 \end{pmatrix}$

$p = 4$ ***A1 N2***

[3 marks]

(b) correct equation for q ***(A1)***

eg $1 + 2q = -1$, $\begin{pmatrix} 4 & 1+2q \\ 6 & 3 \end{pmatrix} = \begin{pmatrix} 4 & -1 \\ 6 & 3 \end{pmatrix}$

working towards solving equation ***(A1)***

eg $2q = -2$

$q = -1$ ***A1 N2***

[3 marks]

Total [6 marks]

3. (a) **METHOD 1**

evidence of correct formula *(M1)*

eg $\log u^n = n\log u$, $2\log_3 p$

$\log_3(p^2) = 12$ ***A1*** ***N2***

METHOD 2

valid method using $p = 3^6$ *(M1)*

eg $\log_3(3^6)^2$, $\log 3^{12}$, $12\log_3 3$

$\log_3(p^2) = 12$ ***A1*** ***N2***

[2 marks]

(b) **METHOD 1**

evidence of correct formula *(M1)*

eg $\log\left(\frac{p}{q}\right) = \log p - \log q$, $6-7$

$\log_3\left(\frac{p}{q}\right) = -1$ ***A1*** ***N2***

METHOD 2

valid method using $p = 3^6$ and $q = 3^7$ *(M1)*

eg $\log_3\left(\frac{3^6}{3^7}\right)$, $\log 3^{-1}$, $-\log_3 3$

$\log_3\left(\frac{p}{q}\right) = -1$ ***A1*** ***N2***

[2 marks]

(c) **METHOD 1**

evidence of correct formula *(M1)*

eg $\log_3 uv = \log_3 u + \log_3 v$, $\log 9 + \log p$

$\log_3 9 = 2$ (may be seen in expression) ***A1***

eg $2 + \log p$

$\log_3(9p) = 8$ ***A1*** ***N2***

METHOD 2

valid method using $p = 3^6$ *(M1)*

eg $\log_3(9\times 3^6)$, $\log_3(3^2\times 3^6)$

correct working ***A1***

eg $\log_3 9 + \log_3 3^6$, $\log_3 3^8$

$\log_3(9p) = 8$ ***A1*** ***N2***

[3 marks]

Total [7 marks]

4. (a) (i) $f(2) = 3$ *A1* *N1*

(ii) $f^{-1}(-1) = 0$ *A2* *N2*

[3 marks]

(b) **EITHER**

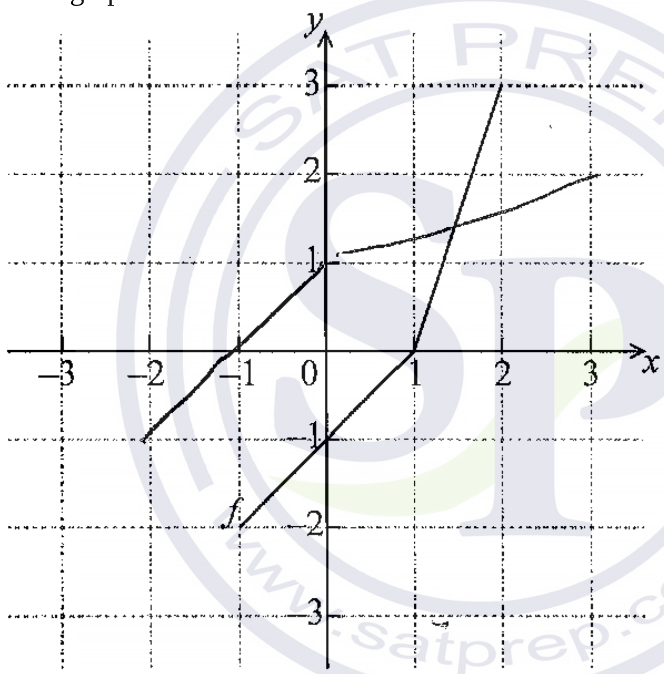
attempt to draw $y = x$ on grid *(M1)*

OR

attempt to reverse *x* and *y* coordinates *(M1)*
eg writing or plotting **at least two** of the points
$(-2, -1)$, $(-1, 0)$, $(0, 1)$, $(3, 2)$

THEN

correct graph *A2* *N3*



[3 marks]

Total [6 marks]

5. (a) valid approach to find p ***(M1)***

eg amplitude $= \frac{\max - \min}{2}$, $p = 6$

$p = 3$ ***A1*** ***N2***

[2 marks]

(b) valid approach to find q ***(M1)***

eg period $= 4$, $q = \frac{2\pi}{\text{period}}$

$q = \frac{\pi}{2}$ ***A1*** ***N2***

[2 marks]

(c) valid approach to find r ***(M1)***

eg axis $= \frac{\max + \min}{2}$, sketch of horizontal axis, $f(0)$

$r = 2$ ***A1*** ***N2***

[2 marks]

Total [6 marks]

6. evidence of antidifferentiation ***(M1)***

eg $\int (6e^{2t} + t)$

$s = 3e^{2t} + \frac{t^2}{2} + C$ ***A2A1***

> **Note:** Award ***A2*** for $3e^{2t}$, ***A1*** for $\frac{t^2}{2}$.

attempt to substitute $(0, 10)$ into **their** integrated expression (even if C is missing) ***(M1)***

correct working ***(A1)***

eg $10 = 3 + C$, $C = 7$

$s = 3e^{2t} + \frac{t^2}{2} + 7$ ***A1*** ***N6***

> **Note:** Exception to the ***FT*** rule. If working shown, allow full ***FT*** on incorrect integration which must involve a power of e.

[7 marks]

7. (a) attempt to find quarter circle area ***(M1)***

eg $\frac{1}{4}(4\pi)$, $\frac{\pi r^2}{4}$, $\int_0^2 \sqrt{4-x^2}\,dx$

area of region $= \pi$ ***(A1)***

$\int_0^2 f(x)\,dx = -\pi$ ***A2*** ***N3***

[4 marks]

(b) attempted set up with both regions ***(M1)***

eg shaded area – quarter circle , $3\pi - \pi$, $3\pi - \int_0^2 f = \int_2^6 f$

$\int_2^6 f(x)\,dx = 2\pi$ ***A2*** ***N2***

[3 marks]

Total [7 marks]

SECTION B

8. (a)	attempt to find p		*(M1)*	
	eg $120-70$, $50+20+x=120$			
	$p=50$		*A1*	*N2*
	attempt to find q		*(M1)*	
	eg $180-20$, $200-20-20$			
	$q=160$		*A1*	*N2*
				[4 marks]
(b)	(i)	$\frac{70}{200}\left(=\frac{7}{20}\right)$	*A1*	*N1*
	(ii)	valid approach	*(M1)*	
		eg $20+20$, $200-160$		
		$\frac{40}{200}\left(=\frac{1}{5}\right)$	*A1*	*N2*
				[3 marks]
(c)	(i)	attempt to find number of girls	*(M1)*	
		eg 0.4, $\frac{40}{100}\times 200$		
		80 are not selected	*A1*	*N2*
	(ii)	120 are selected	*(A1)*	
		$x=20$	*A1*	*N2*
				[4 marks]
(d)	(i)	30 given second chance	*A1*	*N1*
	(ii)	20 took less than 20 minutes	*(A1)*	
		attempt to find **their** selected total (may be seen in % calculation)	*(M1)*	
		eg $120+20\ (=140)$, $120+$ their answer from (d)(i)		
		70 (%)	*A1*	*N3*
				[4 marks]

Total [15 marks]

9. (a) $f'(x) = \cos x + x - 2$ ***A1A1A1*** ***N3***

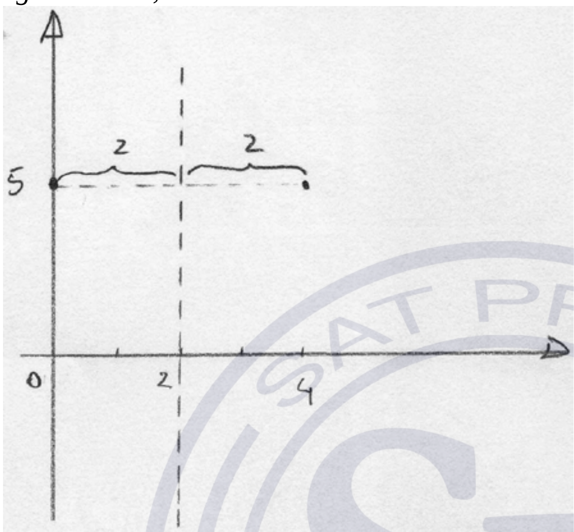
Note: Award ***A1*** for each term.

[3 marks]

(b) recognizing $g(0) = 5$ gives the point $(0, 5)$ ***(R1)***

recognize symmetry ***(M1)***

eg vertex, sketch



$g(4) = 5$ ***A1*** ***N3***

[3 marks]

(c) (i) $h = 2$ ***A1*** ***N1***

(ii) substituting into $g(x) = a(x-2)^2 + 3$ (not the vertex) ***(M1)***

eg $5 = a(0-2)^2 + 3$, $5 = a(4-2)^2 + 3$

working towards solution ***(A1)***

eg $5 = 4a + 3$, $4a = 2$

$a = \frac{1}{2}$ ***A1*** ***N2***

[4 marks]

continued …

Question 9 continued

(d) $g(x) = \frac{1}{2}(x-2)^2 + 3 = \frac{1}{2}x^2 - 2x + 5$

correct derivative of g ***A1A1***

eg $2 \times \frac{1}{2}(x-2),\ x-2$

evidence of equating both derivatives ***(M1)***

eg $f' = g'$

correct equation ***(A1)***

eg $\cos x + x - 2 = x - 2$

working towards a solution ***(A1)***

eg $\cos x = 0$, combining like terms

$x = \frac{\pi}{2}$ ***A1*** ***N0***

Note: Do not award final ***A1*** if additional values are given.

[6 marks]

Total [16 marks]

10. (a) $g(3) = -18$, $f'(3) = 1$, $h''(2) = -6$ ***A1A1A1*** ***N3***

[3 marks]

(b) $h''(3) = 0$ ***(A1)***

valid reasoning ***R1***

eg h'' changes sign at $x = 3$, change in concavity of h at $x = 3$

so P is a point of inflexion ***AG*** ***N0***

[2 marks]

(c) writing $h(3)$ as a product of $f(3)$ and $g(3)$ ***A1***

eg $f(3) \times g(3)$, $3 \times (-18)$

$h(3) = -54$ ***A1*** ***N1***

[2 marks]

(d) recognising need to find derivative of h ***(R1)***

eg h', $h'(3)$

attempt to use the product rule (do **not** accept $h' = f' \times g'$) ***(M1)***

eg $h' = fg' + gf'$, $h'(3) = f(3) \times g'(3) + g(3) \times f'(3)$

correct substitution ***(A1)***

eg $h'(3) = 3(-3) + (-18) \times 1$

$h'(3) = -27$ ***A1***

attempt to find the gradient of the normal ***(M1)***

eg $-\frac{1}{m}$, $-\frac{1}{27}x$

attempt to substitute **their** coordinates and **their** normal gradient into the equation of a line ***(M1)***

eg $-54 = \frac{1}{27}(3) + b$, $0 = \frac{1}{27}(3) + b$, $y + 54 = 27(x - 3)$, $y - 54 = \frac{1}{27}(x + 3)$

correct equation in any form ***A1*** ***N4***

eg $y + 54 = \frac{1}{27}(x - 3)$, $y = \frac{1}{27}x - 54\frac{1}{9}$

[7 marks]

Total [14 marks]

N12/5/MATME/SP1/ENG/TZ0/XX/M

International Baccalaureate
Baccalauréat International
Bachillerato Internacional

MARKSCHEME

November 2012

MATHEMATICS

Standard Level

Paper 1

16 pages

Note: Changes linked to e-marking are noted in red. Other marking changes since November 2011 are noted in green. In particular, please note the removal of the accuracy and misread penalties and the revised accuracy instructions.

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**; working must be seen.

(M) Marks awarded for **Method**; may be implied by **correct** subsequent working.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

(A) Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.

R Marks awarded for clear **Reasoning**.

N Marks awarded for **correct** answers if **no** working shown.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Mark according to scoris instructions and the document "**Mathematics SL: Guidance for e-marking May 2011**". It is **essential** that you read this document before you start marking. In particular, please note the following. Marks must be recorded using the annotation stamps, using new scoris assessor marking tool. Please check that you are entering marks for the right question.

- If a part is **completely correct**, (and gains all the "must be seen" marks), use the ticks with numbers to stamp full marks.
- If a part is completely wrong, stamp ***A0*** by the final answer.
- If a part gains anything else, it **must** be recorded using **all** the annotations.

All the marks will be added and recorded by scoris.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any. An exception to this rule is when work for ***M1*** is missing, as opposed to incorrect (see point 4).
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***(M2)***, ***N3***, *etc*., do **not** split the marks, unless there is a note.
- Once a correct answer to a question or part-question is seen, ignore further working.

3 *N* marks

*If **no** working shown, award **N** marks for **correct** answers. In this case, ignore mark breakdown (**M**, **A**, **R**).*

- Do **not** award a mixture of ***N*** and other marks.
- There may be fewer ***N*** marks available than the total of ***M***, ***A*** and ***R*** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.
- There may not be a direct relationship between the ***N*** marks and the implied marks. There are times when all the marks are implied, but the ***N*** marks are not the full marks: this indicates that we want to see some of the working, without specifying what.
- For consistency within the markscheme, ***N*** marks are noted for every part, even when these match the mark breakdown.
- If a candidate has incorrect working, which somehow results in a correct answer, do **not** award the ***N*** marks for this correct answer. However, if the candidate has indicated (usually by crossing out) that the working is to be ignored, award the ***N*** marks for the correct answer.

4 Implied and must be seen marks

*Implied marks appear in **brackets e.g. (M1)**.*

- Implied marks can only be awarded if **correct** work is seen or if implied in subsequent working (a correct answer does not necessarily mean that the implied marks are all awarded). There are questions where some working is required, but as it is accepted that not everyone will write the same steps, all the marks are implied, but the N marks are not the full marks for the question.
- Normally the correct work is seen or implied in the next line.
- Where there is an (***M1***) followed by ***A1*** for each correct answer, if no working shown, one correct answer is sufficient evidence to award the (***M1***).

*Must be seen marks appear without **brackets e.g. M1**.*

- Must be seen marks can only be awarded if **correct** work is seen.
- If a must be seen mark is not awarded because work is missing (as opposed to ***M0*** or ***A0*** for incorrect work) all subsequent marks may be awarded if appropriate.

5 Follow through marks (only applied after an error is made)

*Follow through (**FT**) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) or subpart(s). Usually, to award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if the only marks awarded in a subpart are for the answer (i.e. there is no working expected), then **FT** marks should be awarded if appropriate. Examiners are expected to check student work in order to award **FT** marks where appropriate.*

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate. (However, as noted above, if an ***A*** mark is not awarded because work is missing, all subsequent marks may be awarded if appropriate).
- Exceptions to this rule will be explicitly noted on the markscheme.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*e.g.* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word “their” in a description, to indicate that candidates may be using an incorrect value.

- If a candidate makes an error in one part, but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the question says hence. It is often possible to use a different approach in subsequent parts that does not depend on the answer to previous parts.
- In a "show that" question, if an error leads to not showing the required answer, there is a 1 mark penalty. Note that if the error occurs within the same subpart, the *FT* rules may result in further loss of marks.
- Where there are anticipated common errors, the ***FT*** answers are often noted on the markscheme, to help examiners. It should be stressed that these are not the only ***FT*** answers accepted.

6 Mis-read

*If a candidate incorrectly copies information from the question, this is a mis-read (**MR**). A candidate should be penalized only once for a particular mis-read. Use the **MR** stamp to indicate that this is a misread. Do not award the first mark in the question, even if this is an **M** mark, but award all others (if appropriate) so that the candidate only loses one mark for the misread.*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.

7 Discretionary marks *(d)*

*An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. In such cases the annotation DM should be used and a brief **note** written next to the mark explaining this decision.*

8 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for part-questions are indicated by **EITHER** . . . **OR**.
- Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9 Alternative forms

*Unless the question specifies otherwise, **accept** equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

10 Accuracy of Answers

If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy. When this is not specified in the question, all numerical answers should be given exactly or correct to three significant figures.

*Candidates should NO LONGER be penalized for an accuracy error **(AP)**. Examiners should award marks according to the rules given in these instructions and the markscheme. Accuracy is not the same as correctness – an incorrect value does not achieve relevant **A** marks. It is only final answers which may lose marks for accuracy errors, not intermediate values. Please check work carefully for **FT**. Further information on which answers are accepted is given in a separate booklet, along with examples. It is **essential** that you read this carefully, as there are a number of changes.*

Do not accept unfinished numerical answers such as 3/0.1 (unless otherwise stated). As a rule, numerical answers with more than one part (such as fractions) should be given using integers (*e.g.* 6/8). Calculations which lead to integers should be completed, with the exception of fractions which are not whole numbers.

11 Calculators

No calculator is allowed. The use of any calculator on paper 1 is malpractice, and will result in no grade awarded. If you see work that suggests a candidate has used any calculator, please follow the procedures for malpractice. Examples: finding an angle, given a trig ratio of 0.4235.

12 Style

The markscheme aims to present answers using good communication, e.g. if the question asks to find the value of k, the markscheme will say $k = 3$, but the marks will be for the correct value 3 – there is usually no need for the "$k =$". In these cases, it is also usually acceptable to have another variable, as long as there is no ambiguity in the question, e.g. if the question asks to find the value of p and of q, then the student answer needs to be clear. Generally, the only situation where the full answer is required is in a question which asks for equations – in this case the markscheme will say "must be an equation".

*The markscheme often uses words to describe what the marks are for, followed by examples, using the e.g. notation. These examples are not exhaustive, and examiners should check what candidates have written, to see if they satisfy the description. Where these marks are **M** marks, the examples may include ones using poor notation, to indicate what is acceptable.*

13 Candidate work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

Candidates are meant to write their answers to Section A on the question paper (QP), and Section B on lined paper. Sometimes, they need more room for Section A, and use lined paper (and often comment to this effect on the QP), or write outside the box. That is fine, and this work should be marked.

The instructions tell candidates not to write on Section B of the QP. Thus they may well have done some rough work here which they assume will be ignored. If they have solutions on the lined paper, there is no need to look at the QP. However, if there are whole questions or whole part solutions missing on the lined paper, please check to make sure that they are not on the QP, and if they are, mark those whole questions or whole part solutions that have not been written on the lined paper.

14 Diagrams

The notes on how to allocated marks for sketches usually refer to passing through particular points are having certain features. These marks can only be awarded if the sketch is approximately the correct shape. All values given will be an approximate guide to where these points/features occur. In some questions, the first ***A1*** is for the shape, in others, the marks are only for the points and/or features. In both cases, unless the shape is approximately correct, no marks can be awarded. However, if the graph is based on previous calculations, ***FT*** marks should be awarded if appropriate.

SECTION A

1. (a) evidence of multiplying *(M1)*

e.g. one correct element, $(0\times -4)+(3\times 5)$

$\boldsymbol{AB}=\begin{pmatrix}15 & 3\\ 28 & 4\end{pmatrix}$ ***A2*** ***N3***

Note: Award ***A1*** for three correct elements.

[3 marks]

(b) finding $2\boldsymbol{A}=\begin{pmatrix}0 & 6\\ -4 & 8\end{pmatrix}$ *(A1)*

adding $2\boldsymbol{A}$ to both sides (may be seen first) *(M1)*

e.g. $\boldsymbol{X}=\boldsymbol{B}+2\boldsymbol{A}$

$\boldsymbol{X}=\begin{pmatrix}-4 & 6\\ 1 & 9\end{pmatrix}$ ***A1*** ***N2***

[3 marks]

Total [6 marks]

2. (a) evidence of summing to 1 *(M1)*

e.g. $\sum p=1,\ 0.3+k+2k+0.1=1$

correct working *(A1)*

e.g. $0.4+3k,\ 3k=0.6$

$k=0.2$ ***A1*** ***N2***

[3 marks]

(b) correct substitution into $\mathrm{E}(X)$ formula *(A1)*

e.g. $0(0.3)+2(k)+5(2k)+9(0.1),\ 12k+0.9$

correct working *(A1)*

e.g. $0(0.3)+2(0.2)+5(0.4)+9(0.1),\ 0.4+2.0+0.9$

$\mathrm{E}(X)=3.3$ ***A1*** ***N2***

[3 marks]

Total [6 marks]

3. (a) correct integration *A1A1*

e.g. $\frac{x^2}{2}-4x$, $\left[\frac{x^2}{2}-4x\right]_4^{10}$, $\frac{(x-4)^2}{2}$

> **Notes**: In the first 2 examples, award ***A1*** for each correct term.
> In the third example, award ***A1*** for $\frac{1}{2}$ and ***A1*** for $(x-4)^2$.

substituting limits into **their** integrated function and subtracting (in any order) ***(M1)***

e.g. $\left(\frac{10^2}{2}-4(10)\right)-\left(\frac{4^2}{2}-4(4)\right)$, $10-(-8)$, $\frac{1}{2}\left(6^2-0\right)$

$\int_4^{10}(x-4)\,dx=18$ ***A1*** ***N2***

[4 marks]

(b) attempt to substitute either limits or the function into volume formula ***(M1)***

e.g. $\pi\int_4^{10} f^2 dx$, $\int_a^b\left(\sqrt{x-4}\right)^2$, $\pi\int_4^{10}\sqrt{x-4}$

> **Note:** Do not penalise for missing π or dx.

correct substitution (accept absence of dx and π) ***(A1)***

e.g. $\pi\int_4^{10}\left(\sqrt{x-4}\right)^2$, $\pi\int_4^{10}(x-4)\,dx$, $\int_4^{10}(x-4)\,dx$

volume $=18\pi$ ***A1*** ***N2***

[3 marks]

Total [7 marks]

4. (a) $f'(x)=3ax^2-12x$ ***A1A1*** ***N2***

> **Note**: Award ***A1*** for each correct term.

[2 marks]

(b) setting **their** derivative equal to 3 (seen anywhere) ***A1***

e.g. $f'(x)=3$

attempt to substitute $x=1$ into $f'(x)$ ***(M1)***

e.g. $3a(1)^2-12(1)$

correct substitution into $f'(x)$ ***(A1)***

e.g. $3a-12$, $3a=15$

$a=5$ ***A1*** ***N2***

[4 marks]

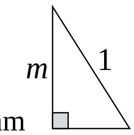
Total [6 marks]

5.

> **Note:** All answers must be given in terms of m. If a candidate makes an error that means there is no m in their answer, do not award the final ***A1FT*** mark.

METHOD 1

(a) valid approach involving Pythagoras ***(M1)***

e.g. $\sin^2 x + \cos^2 x = 1$, labelled diagram 

correct working (may be on diagram) ***(A1)***

e.g. $m^2 + (\cos 100)^2 = 1$, $\sqrt{1-m^2}$

$\cos 100 = -\sqrt{1-m^2}$ ***A1 N2***

[3 marks]

(b) $\tan 100 = -\dfrac{m}{\sqrt{1-m^2}}$ $\left(\text{accept } \dfrac{m}{-\sqrt{1-m^2}}\right)$ ***A1 N1***

[1 mark]

(c) valid approach involving double angle formula ***(M1)***

e.g. $\sin 2\theta = 2\sin\theta\cos\theta$

$\sin 200 = -2m\sqrt{1-m^2}$ $\left(\text{accept } 2m\left(-\sqrt{1-m^2}\right)\right)$ ***A1 N2***

> **Note:** If candidates find $\cos 100 = \sqrt{1-m^2}$, award full ***FT*** in parts (b) and (c), even though the values may not have appropriate signs for the angles.

[2 marks]

Total [6 marks]

METHOD 2

(a) valid approach involving tan identity ***(M1)***

e.g. $\tan = \dfrac{\sin}{\cos}$

correct working ***(A1)***

e.g. $\cos 100 = \dfrac{\sin 100}{\tan 100}$

$\cos 100 = \dfrac{m}{\tan 100}$ ***A1 N2***

[3 marks]

continued …

Question 5 continued

(b) $\tan 100 = \dfrac{m}{\cos 100}$ ***A1*** ***N1***

[1 mark]

(c) valid approach involving double angle formula ***(M1)***

e.g. $\sin 2\theta = 2\sin\theta\cos\theta$, $2m \times \dfrac{m}{\tan 100}$

$\sin 200 = \dfrac{2m^2}{\tan 100} \left(= 2m\cos 100\right)$ ***A1*** ***N2***

[2 marks]

Total [6 marks]

6. (a) **any** correct equation in the form $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$ (accept any parameter for t)

where $\boldsymbol{a}$ is $\begin{pmatrix} 5 \\ -4 \\ 10 \end{pmatrix}$, and $\boldsymbol{b}$ is a scalar multiple of $\begin{pmatrix} 4 \\ -2 \\ 5 \end{pmatrix}$ ***A2*** ***N2***

e.g. $\boldsymbol{r} = \begin{pmatrix} 5 \\ -4 \\ 10 \end{pmatrix} + t\begin{pmatrix} 4 \\ -2 \\ 5 \end{pmatrix}$, $\boldsymbol{r} = 5\boldsymbol{i} - 4\boldsymbol{j} + 10\boldsymbol{k} + t(-8\boldsymbol{i} + 4\boldsymbol{j} - 10\boldsymbol{k})$

Note: Award ***A1*** for the form $\boldsymbol{a} + t\boldsymbol{b}$, ***A1*** for $L = \boldsymbol{a} + t\boldsymbol{b}$, ***A0*** for $\boldsymbol{r} = \boldsymbol{b} + t\boldsymbol{a}$.

[2 marks]

(b) recognizing that $y = 0$ or $z = 0$ at x-intercept (seen anywhere) ***(R1)***

attempt to set up equation for x-intercept (must suggest $x \neq 0$) ***(M1)***

e.g. $L = \begin{pmatrix} x \\ 0 \\ 0 \end{pmatrix}$, $5 + 4t = x$, $r = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

one correct equation in one variable ***(A1)***

e.g. $-4 - 2t = 0$, $10 + 5t = 0$

finding $t = -2$ ***A1***

correct working ***(A1)***

e.g. $x = 5 + (-2)(4)$

$x = -3$ (accept $(-3, 0, 0)$) ***A1*** ***N3***

[6 marks]

Total [8 marks]

7. evidence of rearranged quadratic equation (may be seen in working) ***A1***

e.g. $x^2-3x+k^2-4=0$, k^2-4

evidence of discriminant (must be seen explicitly, not in quadratic formula) ***(M1)***

e.g. b^2-4ac, $\Delta=(-3)^2-4(1)(k^2-4)$

recognizing that discriminant is greater than zero (seen anywhere, including answer) ***R1***

e.g. $b^2-4ac>0$, $9+16-4k^2>0$

correct working (accept equality) ***A1***

e.g. $25-4k^2>0$, $4k^2<25$, $k^2=\frac{25}{4}$

both correct values (even if inequality never seen) ***(A1)***

e.g. $\pm\sqrt{\frac{25}{4}}$, ± 2.5

correct interval ***A1*** ***N3***

e.g. $-\frac{5}{2}<k<\frac{5}{2}$, $-2.5<k<2.5$

Note: Do not award the final mark for unfinished values, or for incorrect or reversed inequalities, including $\leq$, $k>-2.5, k<2.5$.

Special cases:

If working shown, and candidates attempt to rearrange the quadratic equation to equal zero, but find an incorrect value of *c*, award ***A1M1R1A0A0A0***.

If working shown, and candidates do not rearrange the quadratic equation to equal zero, but find $c=k^2$ or $c=\pm 4$, award ***A0M1R1A0A0A0***.

[6 marks]

SECTION B

8. (a) (i) median weekly wage $= 400$ (dollars) ***A1*** ***N1***

(ii) lower quartile $= 330$, upper quartile $= 470$ ***(A1)(A1)***

IQR $= 140$ (dollars) (accept any notation suggesting interval 330 to 470) ***A1*** ***N3***

> **Note:** Exception to the ***FT*** rule. Award ***A1(FT)*** for an incorrect IQR **only** if both quartiles are explicitly noted.

[4 marks]

(b) (i) 330 (dollars) ***A1*** ***N1***

(ii) 400 (dollars) ***A1*** ***N1***

(iii) 700 (dollars) ***A1*** ***N1***

[3 marks]

(c) valid approach ***(M1)***

e.g. $\text{hours} = \frac{\text{wages}}{\text{rate}}$

correct substitution ***(A1)***

e.g. $\frac{400}{20}$

median hours per week $= 20$ ***A1*** ***N2***

[3 marks]

(d) attempt to find wages for 25 hours per week ***(M1)***

e.g. wages $=$ hours $\times$ rate

correct substitution ***(A1)***

e.g. 25×20

finding wages $= 500$ ***(A1)***

65 people (earn ≤ 500) ***(A1)***

15 people (work more than 25 hours) ***A1*** ***N3***

[5 marks]

Total [15 marks]

9. (a) correct approach *A1*

e.g. $\vec{AO}+\vec{OB}$, $\begin{pmatrix}6\\0\\3\end{pmatrix}-\begin{pmatrix}5\\2\\1\end{pmatrix}$

$\vec{AB}=\begin{pmatrix}1\\-2\\2\end{pmatrix}$ *AG* *N0*

[1 mark]

(b) recognizing $\vec{AD}$ is perpendicular to $\vec{AB}$ (may be seen in sketch) *(R1)*

e.g. adjacent sides of rectangle are perpendicular

recognizing dot product must be zero *(R1)*

e.g. $\vec{AD}\bullet\vec{AB}=0$

correct substitution *(A1)*

e.g. $(1\times 4)+(-2\times p)+(2\times 1)$, $4-2p+2=0$

equation which clearly leads to $p=3$ *A1*

e.g. $6-2p=0$, $2p=6$

$p=3$ *AG* *N0*

[4 marks]

(c) correct approach (seen anywhere including sketch) *(A1)*

e.g. $\vec{OC}=\vec{OB}+\vec{BC}$, $\vec{OD}+\vec{DC}$

recognizing opposite sides are equal vectors (may be seen in sketch) *(R1)*

e.g. $\vec{BC}=\vec{AD}$, $\vec{DC}=\vec{AB}$, $\begin{pmatrix}6\\0\\3\end{pmatrix}+\begin{pmatrix}4\\3\\1\end{pmatrix}$, $\begin{pmatrix}9\\5\\2\end{pmatrix}+\begin{pmatrix}1\\-2\\2\end{pmatrix}$

coordinates of point C are $(10, 3, 4)$ $\left(\text{accept }\begin{pmatrix}10\\3\\4\end{pmatrix}\right)$ *A2* *N4*

Note: Award ***A1*** for two correct values.

[4 marks]

continued …

Question 9 continued

(d) attempt to find one side of the rectangle **(M1)**
e.g. substituting into magnitude formula

two correct magnitudes **A1A1**
e.g. $\sqrt{(1)^2+(-2)^2+2^2}$, 3; $\sqrt{16+9+1}$, $\sqrt{26}$

multiplying magnitudes **(M1)**
e.g. $\sqrt{26}\times\sqrt{9}$

area $=\sqrt{234}\left(=3\sqrt{26}\right)$ $\left(\text{accept } 3\times\sqrt{26}\right)$ **A1** **N3**

[5 marks]

Total [14 marks]

10. (a) **METHOD 1**

evidence of choosing quotient rule **(M1)**
e.g. $\frac{u'v-uv'}{v^2}$

evidence of correct differentiation (must be seen in quotient rule) **(A1)(A1)**
e.g. $\frac{\mathrm{d}}{\mathrm{d}x}(6x)=6$, $\frac{\mathrm{d}}{\mathrm{d}x}(x+1)=1$

correct substitution into quotient rule **A1**
e.g $\frac{(x+1)6-6x}{(x+1)^2}$, $\frac{6x+6-6x}{(x+1)^2}$

$f'(x)=\frac{6}{(x+1)^2}$ **A1** **N4**

[5 marks]

METHOD 2

evidence of choosing product rule **(M1)**
e.g. $6x(x+1)^{-1}$, $uv'+vu'$

evidence of correct differentiation (must be seen in product rule) **(A1)(A1)**
e.g. $\frac{\mathrm{d}}{\mathrm{d}x}(6x)=6$, $\frac{\mathrm{d}}{\mathrm{d}x}(x+1)^{-1}=-1(x+1)^{-2}\times 1$

correct working **A1**
e.g. $6x\times-(x+1)^{-2}+(x+1)^{-1}\times 6$, $\frac{-6x+6(x+1)}{\left(x+1\right)^2}$

$f'(x)=\frac{6}{(x+1)^2}$ **A1** **N4**

[5 marks]

continued …

Question 10 continued

(b) **METHOD 1**

evidence of choosing chain rule **(M1)**

e.g. formula, $\frac{1}{\left(\frac{6x}{x+1}\right)} \times \left(\frac{6x}{x+1}\right)'$

correct reciprocal of $\frac{1}{\left(\frac{6x}{x+1}\right)}$ is $\frac{x+1}{6x}$ (seen anywhere) **A1**

correct substitution into chain rule **A1**

e.g. $\frac{1}{\left(\frac{6x}{x+1}\right)} \times \frac{6}{(x+1)^2}$, $\left(\frac{6}{(x+1)^2}\right)\left(\frac{x+1}{6x}\right)$

working that clearly leads to the answer **A1**

e.g. $\left(\frac{6}{(x+1)}\right)\left(\frac{1}{6x}\right)$, $\left(\frac{1}{(x+1)^2}\right)\left(\frac{x+1}{x}\right)$, $\frac{6(x+1)}{6x(x+1)^2}$

$g'(x) = \frac{1}{x(x+1)}$ **AG** **N0**

[4 marks]

METHOD 2

attempt to subtract logs **(M1)**

e.g. $\ln a - \ln b$, $\ln 6x - \ln(x+1)$

correct derivatives (must be seen in correct expression) **A1A1**

e.g. $\frac{6}{6x} - \frac{1}{x+1}$, $\frac{1}{x} - \frac{1}{x+1}$

working that clearly leads to the answer **A1**

e.g. $\frac{x+1-x}{x(x+1)}$, $\frac{6x+6-6x}{6x(x+1)}$, $\frac{6(x+1-x)}{6x(x+1)}$

$g'(x) = \frac{1}{x(x+1)}$ **AG** **N0**

[4 marks]

continued …

Question 10 continued

(c) valid method using integral of $h(x)$ (accept missing/incorrect limits or missing $\mathrm{d}x$) ***(M1)***

e.g. $\text{area} = \int_{\frac{1}{5}}^{k} h(x)\,\mathrm{d}x$, $\int\left(\frac{1}{x(x+1)}\right)$

recognizing that integral of derivative will give original function ***(R1)***

e.g. $\int\left(\frac{1}{x(x+1)}\right)\mathrm{d}x = \ln\left(\frac{6x}{x+1}\right)$

correct substitution and subtraction ***A1***

e.g. $\ln\left(\frac{6k}{k+1}\right) - \ln\left(\frac{6\times\frac{1}{5}}{\frac{1}{5}+1}\right)$, $\ln\left(\frac{6k}{k+1}\right) - \ln(1)$

setting **their** expression equal to $\ln 4$ ***(M1)***

e.g. $\ln\left(\frac{6k}{k+1}\right) - \ln(1) = \ln 4$, $\ln\left(\frac{6k}{k+1}\right) = \ln 4$, $\int_{\frac{1}{5}}^{k} h(x)\,\mathrm{d}x = \ln 4$

correct equation without logs ***A1***

e.g. $\frac{6k}{k+1} = 4$, $6k = 4(k+1)$

correct working ***(A1)***

e.g. $6k = 4k + 4$, $2k = 4$

$k = 2$ ***A1*** ***N4***

[7 marks]

Total [16 marks]

M12/5/MATME/SP1/ENG/TZ1/XX/M

International Baccalaureate®
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MARKSCHEME

May 2012

MATHEMATICS

Standard Level

Paper 1

16 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**; working must be seen.

(M) Marks awarded for **Method**; may be implied by **correct** subsequent working.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

(A) Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.

R Marks awarded for clear **Reasoning**.

N Marks awarded for **correct** answers if **no** working shown.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Mark according to scoris instructions and the document "**Mathematics SL: WA Guidance for e-marking May 2012**". It is **essential** that you read this document before you start marking. In particular, please note the following.

Marks must be recorded using the annotation stamps. Please check that you are entering marks for the right question.

- If a part is **completely correct**, (and gains all the "must be seen" marks), use the ticks with numbers to stamp full marks.
- If a part is completely wrong, stamp ***A0*** by the final answer.
- If a part gains anything else, it **must** be recorded using **all** the annotations.

All the marks will be added and recorded by scoris.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any. An exception to this rule is when work for ***M1*** is missing, as opposed to incorrect (see point 4).
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***(M2)***, ***N3***, *etc*., do **not** split the marks, unless there is a note.
- Once a correct answer to a question or part-question is seen, ignore further working.

3 *N* marks

*If **no** working shown, award **N** marks for **correct** answers. In this case, ignore mark breakdown (**M**, **A**, **R**).*

- Do **not** award a mixture of ***N*** and other marks.
- There may be fewer ***N*** marks available than the total of ***M***, ***A*** and ***R*** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.
- There may not be a direct relationship between the ***N*** marks and the implied marks. There are times when all the marks are implied, but the ***N*** marks are not the full marks: this indicates that we want to see some of the working, without specifying what.
- For consistency within the markscheme, ***N*** marks are noted for every part, even when these match the mark breakdown.
- If a candidate has incorrect working, which somehow results in a correct answer, do **not** award the ***N*** marks for this correct answer. However, if the candidate has indicated (usually by crossing out) that the working is to be ignored, award the ***N*** marks for the correct answer.

4 Implied and must be seen marks

*Implied marks appear in **brackets e.g. (M1)**.*

- Implied marks can only be awarded if **correct** work is seen or if implied in subsequent working (a correct answer does not necessarily mean that the implied marks are all awarded). There are questions where some working is required, but as it is accepted that not everyone will write the same steps, all the marks are implied, but the N marks are not the full marks for the question.
- Normally the correct work is seen or implied in the next line.
- Where there is an (***M1***) followed by ***A1*** for each correct answer, if no working shown, one correct answer is sufficient evidence to award the (***M1***).

*Must be seen marks appear without **brackets e.g. M1**.*

- Must be seen marks can only be awarded if **correct** work is seen.
- If a must be seen mark is not awarded because work is missing (as opposed to ***M0*** or ***A0*** for incorrect work) all subsequent marks may be awarded if appropriate.

5 Follow through marks (only applied after an error is made)

*Follow through (**FT**) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) or subpart(s). Usually, to award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if the only marks awarded in a subpart are for the answer (i.e. there is no working expected), then **FT** marks should be awarded if appropriate. Examiners are expected to check student work in order to award **FT** marks where appropriate.*

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate. (However, as noted above, if an ***A*** mark is not awarded because work is missing, all subsequent marks may be awarded if appropriate)
- Exceptions to this rule will be explicitly noted on the markscheme.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*e.g.* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non integer value where integer required), do not award the mark(s) for the final answer(s).

- The markscheme may use the word "their" in a description, to indicate that candidates may be using an incorrect value.
- If a candidate makes an error in one part, but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the question says hence. It is often possible to use a different approach in subsequent parts that does not depend on the answer to previous parts.
- In a "show that" question, if an error leads to not showing the required answer, there is a 1 mark penalty. Note that if the error occurs within the same subpart, the *FT* rules may result in further loss of marks.
- Where there are anticipated common errors, the ***FT*** answers are often noted on the markscheme, to help examiners. It should be stressed that these are not the only ***FT*** answers accepted.

6 Mis-read

*If a candidate incorrectly copies information from the question, this is a mis-read (**MR**). Apply a **MR** penalty of 1 mark to that question. Award the marks as usual and then stamp **MR** against the answer. Scoris will automatically deduct 1 mark from the question total. A candidate should be penalized only once for a particular mis-read. Do not stamp **MR** again for that question, unless the candidate makes another mis-read.*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.

7 Discretionary marks *(d)*

*An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. In such cases the annotation DM should be used and a brief **note** written next to the mark explaining this decision.*

8 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for part-questions are indicated by **EITHER** . . . **OR**.
- Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9 Alternative forms

*Unless the question specifies otherwise, **accept** equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

10 Accuracy of Answers

If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy. When this is not specified in the question, all numerical answers should be given exactly or correct to three significant figures

*Candidates should NO LONGER be penalized for an accuracy error **(AP)**. Examiners should award marks according to the rules given in these instructions and the markscheme. Accuracy is not the same as correctness – an incorrect value does not achieve relevant **A** marks. It is only final answers which may lose marks for accuracy errors, not intermediate values. Please check work carefully for **FT**. Further information on which answers are accepted is given in a separate booklet, along with examples. It is **essential** that you read this carefully, as there are a number of changes.*

Do not accept unfinished numerical answers such as 3/0.1 (unless otherwise stated). As a rule, numerical answers with more than one part (such as fractions) should be given using integers (*e.g.* 6/8). Calculations which lead to integers should be completed, with the exception of fractions which are not whole numbers.

11 Calculators

No calculator is allowed. The use of any calculator on paper 1 is malpractice, and will result in no grade awarded. If you see work that suggests a candidate has used any calculator, please follow the procedures for malpractice. Examples: finding an angle, given a trig ratio of 0.4235.

12 Style

The markscheme aims to present answers using good communication, e.g. if the question asks to find the value of k, the markscheme will say $k = 3$, but the marks will be for the correct value 3 – there is usually no need for the "$k =$". In these cases, it is also usually acceptable to have another variable, as long as there is no ambiguity in the question, e.g. if the question asks to find the value of p and of q, then the student answer needs to be clear. Generally, the only situation where the full answer is required is in a question which asks for equations – in this case the markscheme will say "must be an equation".

*The markscheme often uses words to describe what the marks are for, followed by examples, using the e.g. notation. These examples are not exhaustive, and examiners should check what candidates have written, to see if they satisfy the description. Where these marks are **M** marks, the examples may include ones using poor notation, to indicate what is acceptable.*

13 Candidate work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

SECTION A

1. (a) evidence of valid approach *(M1)*

e.g. $92+52$, line on graph at $x=31$

$p=144$ *A1* *N2*

[2 marks]

(b) (i) evidence of valid approach *(M1)*

e.g. line on graph, 0.8×160, using complement

$=29.5$ *A1* *N2*

(ii) $Q_1=23$; $Q_3=29$ *(A1)(A1)*

IQR $=6$ (accept any notation that suggests an interval) *A1* *N3*

[5 marks]

Total [7 marks]

2. (a) $m=2$, $n=3$ *A1A1* *N2*

[2 marks]

(b) attempt to multiply elements *(M1)*

$$\boldsymbol{AB}=\begin{pmatrix}-2 & 0 & -6\\ -2 & 9 & 3\end{pmatrix}$$ *A2* *N3*

[3 marks]

(c) $p=3$ *A1* *N1*

[1 mark]

Total [6 marks]

3. (a) $f'(x) = 6e^{6x}$ ***A1*** ***N1***

[1 mark]

(b) (i) evidence of valid approach ***(M1)***

e.g. $f'(0)$, $6e^{6\times 0}$

correct manipulation ***A1***

e.g. $6e^{0}$, 6×1

$m = 6$ ***AG*** ***N0***

(ii) evidence of finding $f(0)$ ***(M1)***

e.g. $y = e^{6(0)}$

$b = 1$ ***A1*** ***N2***

[4 marks]

(c) $y = 6x + 1$ ***A1*** ***N1***

[1 mark]

Total [6 marks]

4. (a) $t = 0.3$ ***A1*** ***N1***

[1 mark]

(b) (i) correct values ***A1***

e.g. $0.3 + 0.6 - 0.7$; $0.9 - 0.7$

$r = 0.2$ ***AG*** ***N0***

(ii) $q = 0.1$, $s = 0.4$ ***A1A1*** ***N2***

[3 marks]

(c) (i) 0.4 ***A1*** ***N1***

(ii) $\mathrm{P}(A|B') = \frac{1}{4}$ ***A2*** ***N2***

[3 marks]

Total [7 marks]

5. (a) evidence of valid approach *(M1)*

e.g. $\frac{\max y \text{ value} - \min y \text{ value}}{2}$, distance from $y = -1$

$a = 3$ ***A1*** ***N2***

[2 marks]

(b) (i) evidence of valid approach *(M1)*

e.g. finding difference in *x*-coordinates, $\frac{\pi}{2}$

evidence of doubling ***A1***

e.g. $2 \times \left(\frac{\pi}{2}\right)$

period $= \pi$ ***AG*** ***N0***

(ii) evidence of valid approach *(M1)*

e.g. $b = \frac{2\pi}{\pi}$

$b = 2$ ***A1*** ***N2***

[4 marks]

(c) $c = \frac{\pi}{4}$ ***A1*** ***N1***

[1 mark]

Total [7 marks]

6. correct integration, $2 \times \frac{1}{2} \ln(2x+5)$ ***A1A1***

Note: Award ***A1*** for $2 \times \frac{1}{2} (=1)$ and ***A1*** for $\ln(2x+5)$.

evidence of substituting limits into integrated function and subtracting *(M1)*

e.g. $\ln(2 \times 5 + 5) - \ln(2 \times 0 + 5)$

correct substitution ***A1***

e.g. $\ln 15 - \ln 5$

correct working *(A1)*

e.g. $\ln \frac{15}{5}$, $\ln 3$

$k = 3$ ***A1*** ***N3***

[6 marks]

7. (a) attempt to expand ***(M1)***

e.g. $(\sin x + \cos x)(\sin x + \cos x)$; at least 3 terms

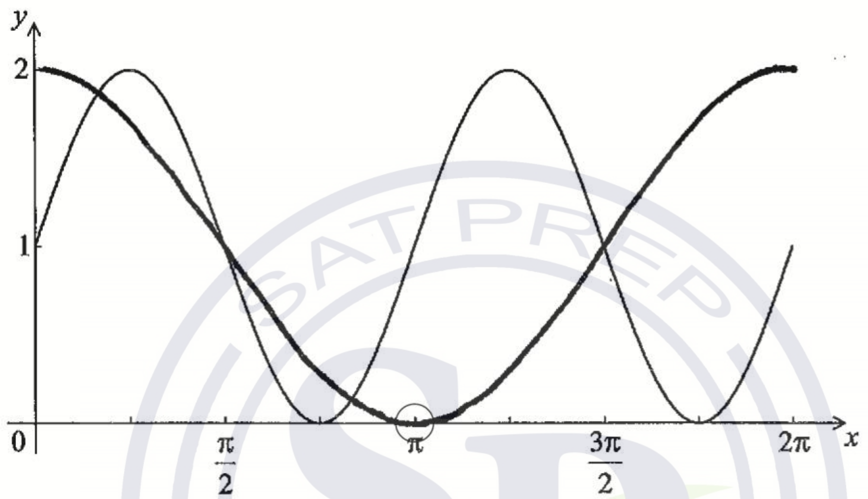
correct expansion ***A1***

e.g. $\sin^2 x + 2\sin x \cos x + \cos^2 x$

$f(x) = 1 + \sin 2x$ ***AG*** ***N0***

[2 marks]

(b)



A1A1 ***N2***

Note:	Award ***A1*** for correct sinusoidal shape with period 2π and range [0, 2], ***A1*** for minimum in circle.

[2 marks]

(c) $p = 2,\ k = -\frac{\pi}{2}$ ***A1A1*** ***N2***

[2 marks]

Total [6 marks]

SECTION B

8. (a) (i) evidence of correct approach *A1*

e.g. $\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP}$, $Q - P$

$$\overrightarrow{PQ} = \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$$ *AG* *N0*

(ii) any correct equation in the form $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$ *A2* *N2*

where $\boldsymbol{a}$ is either $\overrightarrow{OP}$ or $\overrightarrow{OQ}$ and $\boldsymbol{b}$ is a scalar multiple of $\overrightarrow{PQ}$

e.g. $\boldsymbol{r} = \begin{pmatrix} -1 \\ 6 \\ -1 \end{pmatrix} + t\begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$, $\boldsymbol{r} = \begin{pmatrix} t \\ 4-2t \\ 1+2t \end{pmatrix}$, $\boldsymbol{r} = 4\boldsymbol{j} + \boldsymbol{k} + t(\boldsymbol{i} - 2\boldsymbol{j} + 2\boldsymbol{k})$

[3 marks]

(b) choosing a correct direction vector for L_2 *(A1)*

e.g. $\begin{pmatrix} 3 \\ 0 \\ -4 \end{pmatrix}$

finding scalar products and magnitudes *(A1)(A1)(A1)*

scalar product $= 1(3) - 2(0) + 2(-4)$ $(= -5)$

magnitudes $= \sqrt{1^2 + (-2)^2 + 2^2}$ $(= 3)$, $\sqrt{3^2 + 0^2 + (-4)^2}$ $(= 5)$

substitution into formula *M1*

e.g. $\cos\theta = \dfrac{-5}{\sqrt{9} \times \sqrt{25}}$

$\cos\theta = -\dfrac{1}{3}$ *A2* *N5*

[7 marks]

continued …

Question 8 continued

(c) evidence of valid approach ***(M1)***

e.g. equating lines, $L_1 = L_2$

EITHER

one correct equation in one variable ***A2***

e.g. $6 - 2t = 2$

OR

two correct equations in two variables ***A1A1***

e.g. $2t + 4s = 0,\ t - 3s = 5$

THEN

attempt to solve ***(M1)***

one correct parameter ***A1***

e.g. $t = 2,\ s = -1$

correct substitution of either parameter ***(A1)***

e.g. $\boldsymbol{r} = \begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix} + (-1)\begin{pmatrix} 3 \\ 0 \\ -4 \end{pmatrix},\ \boldsymbol{r} = \begin{pmatrix} -1 \\ 6 \\ -1 \end{pmatrix} + (2)\begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$

coordinates $\text{R}(1, 2, 3)$ ***A1*** ***N3***

[7 marks]

Total [17 marks]

9. (a) evidence of substituting the point A *(M1)*
e.g. $2 = \log_p(6+3)$

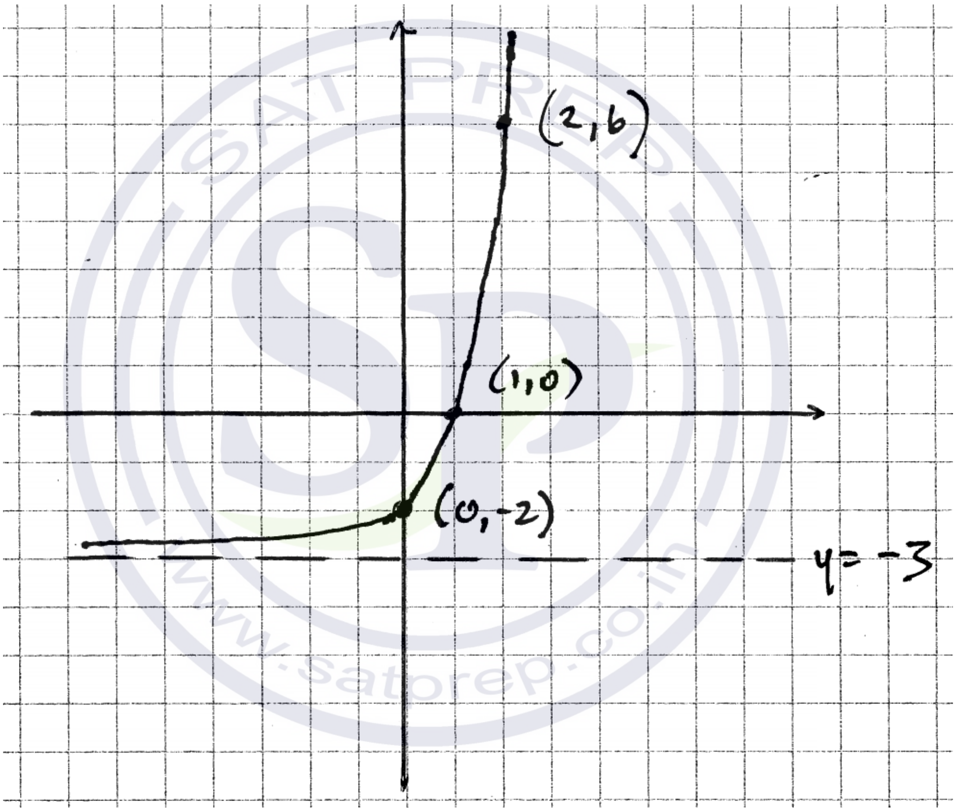
manipulating logs ***A1***
e.g. $p^2 = 9$

$p = 3$ ***A2*** ***N2***

[4 marks]

(b) (i) $y = -2$ $\left(\text{accept } (0, -2)\right)$ ***A1*** ***N1***

(ii)



A1A1A1A1 ***N4***

Note: Award ***A1*** for asymptote at $y = -3$, ***A1*** for an increasing function that is concave up, ***A1*** for a positive *x*-intercept and a negative *y*-intercept, ***A1*** for passing through the point $(2, 6)$.

[5 marks]

continued …

Question 9 continued

(c) **METHOD 1**

recognizing that $g = f^{-1}$ ***(R1)***

evidence of valid approach ***(M1)***
e.g. switching *x* and *y* (seen anywhere), solving for *x*

correct manipulation ***(A1)***
e.g. $3^x = y + 3$

$g(x) = 3^x - 3$ ***A1*** ***N3***
[4 marks]

METHOD 2

recognizing that $g(x) = a^x + b$ ***(R1)***

identifying vertical translation ***(A1)***
e.g. graph shifted down 3 units, $f(x) - 3$

evidence of valid approach ***(M1)***
e.g. substituting point to identify the base

$g(x) = 3^x - 3$ ***A1*** ***N3***
[4 marks]

Total [13 marks]

10. (a) $s'(t) = 1 - 2\cos 2t$ ***A1A2*** ***N3***

Note: Award ***A1*** for 1, ***A2*** for $-2\cos 2t$.

[3 marks]

(b) evidence of valid approach ***(M1)***

e.g. setting $s'(t) = 0$

correct working ***A1***

e.g. $2\cos 2t = 1$, $\cos 2t = \frac{1}{2}$

$2t = \frac{\pi}{3}, \frac{5\pi}{3}, \ldots$ ***(A1)***

$t = \frac{5\pi}{6}$ ***A1*** ***N3***

[4 marks]

(c) evidence of valid approach ***(M1)***

e.g. choosing a value in the interval $\frac{\pi}{6} < t < \frac{5\pi}{6}$

correct substitution ***A1***

e.g. $s'\left(\frac{\pi}{2}\right) = 1 - 2\cos\pi$

$s'\left(\frac{\pi}{2}\right) = 3$ ***A1***

$s'(t) > 0$ ***AG*** ***N0***

[3 marks]

continued …

Question 10 continued

(d) evidence of approach using s or integral of s' ***(M1)***

e.g. $\int s'(t)\,\mathrm{d}t$; $s\left(\frac{5\pi}{6}\right)$, $s\left(\frac{\pi}{6}\right)$; $\left[t - \sin 2t\right]_{\frac{\pi}{6}}^{\frac{5\pi}{6}}$

substituting values and subtracting ***(M1)***

e.g. $s\left(\frac{5\pi}{6}\right) - s\left(\frac{\pi}{6}\right)$, $\left(\frac{\pi}{6} - \frac{\sqrt{3}}{2}\right) - \left(\frac{5\pi}{6} - \left(-\frac{\sqrt{3}}{2}\right)\right)$

correct substitution ***A1***

e.g. $\frac{5\pi}{6} - \sin\frac{5\pi}{3} - \left[\frac{\pi}{6} - \sin\frac{\pi}{3}\right]$, $\left(\frac{5\pi}{6} - \left(-\frac{\sqrt{3}}{2}\right)\right) - \left(\frac{\pi}{6} - \frac{\sqrt{3}}{2}\right)$

distance is $\frac{2\pi}{3} + \sqrt{3}$ ***A1A1*** ***N3***

Note: Award ***A1*** for $\frac{2\pi}{3}$, ***A1*** for $\sqrt{3}$.

[5 marks]

Total [15 marks]

International Baccalaureate®
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MARKSCHEME

May 2012

MATHEMATICS

Standard Level

Paper 1

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**; working must be seen.

(M) Marks awarded for **Method**; may be implied by **correct** subsequent working.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

(A) Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.

R Marks awarded for clear **Reasoning**.

N Marks awarded for **correct** answers if **no** working shown.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Mark according to scoris instructions and the document "**Mathematics SL : WA Guidance for e-marking May 2012**". It is **essential** that you read this document before you start marking. In particular, please note the following.

Marks must be recorded using the annotation stamps. Please check that you are entering marks for the right question.

- If a part is **completely correct**, (and gains all the 'must be seen' marks), use the ticks with numbers to stamp full marks.
- If a part is completely wrong, stamp ***A0*** by the final answer.
- If a part gains anything else, it **must** be recorded using **all** the annotations.

All the marks will be added and recorded by scoris.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any. An exception to this rule is when work for ***M1*** is missing, as opposed to incorrect (see point 4).
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***(M2)***, ***N3***, *etc*., do **not** split the marks, unless there is a note.
- Once a correct answer to a question or part-question is seen, ignore further working.

3 *N* marks

*If **no** working shown, award **N** marks for **correct** answers. In this case, ignore mark breakdown (**M**, **A**, **R**).*

- Do **not** award a mixture of ***N*** and other marks.
- There may be fewer ***N*** marks available than the total of ***M***, ***A*** and ***R*** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.
- There may not be a direct relationship between the ***N*** marks and the implied marks. There are times when all the marks are implied, but the ***N*** marks are not the full marks: this indicates that we want to see some of the working, without specifying what.
- For consistency within the markscheme, ***N*** marks are noted for every part, even when these match the mark breakdown.
- If a candidate has incorrect working, which somehow results in a correct answer, do **not** award the ***N*** marks for this correct answer. However, if the candidate has indicated (usually by crossing out) that the working is to be ignored, award the ***N*** marks for the correct answer.

4 Implied and must be seen marks

*Implied marks appear in **brackets e.g. (M1)**.*

- Implied marks can only be awarded if **correct** work is seen or if implied in subsequent working (a correct answer does not necessarily mean that the implied marks are all awarded). There are questions where some working is required, but as it is accepted that not everyone will write the same steps, all the marks are implied, but the N marks are not the full marks for the question.
- Normally the correct work is seen or implied in the next line.
- Where there is an (***M1***) followed by ***A1*** for each correct answer, if no working shown, one correct answer is sufficient evidence to award the (***M1***).

*Must be seen marks appear without **brackets e.g. M1**.*

- Must be seen marks can only be awarded if **correct** work is seen.
- If a must be seen mark is not awarded because work is missing (as opposed to ***M0*** or ***A0*** for incorrect work) all subsequent marks may be awarded if appropriate.

5 Follow through marks (only applied after an error is made)

*Follow through (**FT**) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) or subpart(s). Usually, to award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if the only marks awarded in a subpart are for the answer (ie there is no working expected), then **FT** marks should be awarded if appropriate. Examiners are expected to check student work in order to award FT marks where appropriate.*

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate. (However, as noted above, if an ***A*** mark is not awarded because work is missing, all subsequent marks may be awarded if appropriate)
- Exceptions to this rule will be explicitly noted on the markscheme.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*e.g.* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non integer value where integer required), do not award the mark(s) for the final answer(s).

- The markscheme may use the word "their" in a description, to indicate that candidates may be using an incorrect value.
- If a candidate makes an error in one part, but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the question says hence. It is often possible to use a different approach in subsequent parts that does not depend on the answer to previous parts.
- In a "show that" question, if an error leads to not showing the required answer, there is a 1 mark penalty. Note that if the error occurs within the same subpart, the *FT* rules may result in further loss of marks.
- Where there are anticipated common errors, the FT answers are often noted on the markscheme, to help examiners. It should be stressed that these are not the only FT answers accepted.

6 Mis-read

*If a candidate incorrectly copies information from the question, this is a mis-read (**MR**). Apply a **MR** penalty of 1 mark to that question. Award the marks as usual and then stamp **MR** against the answer. Scoris will automatically deduct 1 mark from the question total. A candidate should be penalized only once for a particular mis-read. Do not stamp **MR** again for that question, unless the candidate makes another mis-read.*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.

7 Discretionary marks *(d)*

An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. In such cases the annotation DM should be used and a brief **note** *written next to the mark explaining this decision.*

8 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for part-questions are indicated by **EITHER** . . . **OR**.
- Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9 Alternative forms

*Unless the question specifies otherwise, **accept** equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

10 Accuracy of Answers

If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy. When this is not specified in the question, all numerical answers should be given exactly or correct to three significant figures

*Candidates should NO LONGER be penalized for an accuracy error **(AP)**. Examiners should award marks according to the rules given in these instructions and the markscheme. Accuracy is not the same as correctness – an incorrect value does not achieve relevant **A** marks. It is only final answers which may lose marks for accuracy errors, not intermediate values. Please check work carefully for **FT**. Further information on which answers are accepted is given in a separate booklet, along with examples. It is **essential** that you read this carefully, as there are a number of changes.*

Do not accept unfinished numerical answers such as 3/0.1 (unless otherwise stated). As a rule, numerical answers with more than one part (such as fractions) should be given using integers (*e.g.* 6/8). Calculations which lead to integers should be completed, with the exception of fractions which are not whole numbers.

11 Calculators

No calculator is allowed. The use of any calculator on paper 1 is malpractice, and will result in no grade awarded. If you see work that suggests a candidate has used any calculator, please follow the procedures for malpractice. Examples: finding an angle, given a trig ratio of 0.4235.

12 Style

The markscheme aims to present answers using good communication, e.g. if the question asks to find the value of k, the markscheme will say $k = 3$, but the marks will be for the correct value 3 – there is usually no need for the "$k =$". In these cases, it is also usually acceptable to have another variable, as long as there is no ambiguity in the question, e.g. if the question asks to find the value of p and of q, then the student answer needs to be clear. Generally, the only situation where the full answer is required is in a question which asks for equations – in this case the markscheme will say "must be an equation".

*The markscheme often uses words to describe what the marks are for, followed by examples, using the e.g. notation. These examples are not exhaustive, and examiners should check what candidates have written, to see if they satisfy the description. Where these marks are **M** marks, the examples may include ones using poor notation, to indicate what is acceptable.*

13 Candidate work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

SECTION A

1. (a) evidence of median position *(M1)*
e.g. 50, line on sketch

median is 56 ***A1*** ***N2***
[2 marks]

(b) lower quartile $= 40$, upper quartile $= 70$ *(A1)(A1)*

interquartile range $= 30$ ***A1*** ***N3***

[3 marks]

Total [5 marks]

2. (a) interchanging x and y (seen anywhere) *(M1)*
e.g. $x = 2y - 1$

correct manipulation *(A1)*
e.g. $x + 1 = 2y$

$f^{-1}(x) = \frac{x+1}{2}$ ***A1*** ***N2***

[3 marks]

(b) **METHOD 1**

attempt to find $g(1)$ or $f(1)$ *(M1)*
$g(1) = 5$ *(A1)*
$f(5) = 9$ ***A1*** ***N2***
[3 marks]

METHOD 2

attempt to form composite (in any order) *(M1)*
e.g. $2(3x^2 + 2) - 1,\ 3(2x - 1)^2 + 2$

$(f \circ g)(1) = 2(3 \times 1^2 + 2) - 1 \quad (= 6 \times 1^2 + 3)$ *(A1)*
$(f \circ g)(1) = 9$ ***A1*** ***N2***
[3 marks]

Total [6 marks]

3. (a) (i) $a = 3$ ***A1*** ***N1***

(ii) **METHOD 1**

attempt to find period ***(M1)***

e.g. $4,\ b = 4,\ \frac{2\pi}{b}$

$b = \frac{2\pi}{4}\ \left(= \frac{\pi}{2}\right)$ ***A1*** ***N2***

[3 marks]

METHOD 2

attempt to substitute coordinates ***(M1)***

e.g. $3\cos(2b) = -3,\ 3\cos(4b) = 3$

$b = \frac{2\pi}{4}\ \left(= \frac{\pi}{2}\right)$ ***A1*** ***N2***

[3 marks]

(b) 0 ***A1*** ***N1***

[1 mark]

(c) recognizing that normal is perpendicular to tangent ***(M1)***

e.g. $m_1 \times m_2 = -1,\ m = -\frac{1}{0}$, sketch of vertical line on diagram

$x = 2$ (do not accept 2 or $y = 2$) ***A1*** ***N2***

[2 marks]

Total [6 marks]

4. (a) attempt to substitute $\mathrm{P}(X > 1) = 0.5$ ***(M1)***

e.g. $r + 0.2 = 0.5$

$r = 0.3$ ***A1*** ***N2***

[2 marks]

(b) correct substitution into $\mathrm{E}(X)$ (seen anywhere) ***(A1)***

e.g. $0 \times p + 1 \times q + 2 \times r + 3 \times 0.2$

correct equation ***A1***

e.g. $q + 2 \times 0.3 + 3 \times 0.2 = 1.4$, $q + 1.2 = 1.4$

$q = 0.2$ ***A1*** ***N1***

evidence of choosing $\sum p_i = 1$ ***M1***

e.g. $p + 0.2 + 0.3 + 0.2 = 1$, $p + q = 0.5$

correct working ***(A1)***

$p + 0.7 = 1, 1 - 0.2 - 0.3 - 0.2$, $p + 0.2 = 0.5$

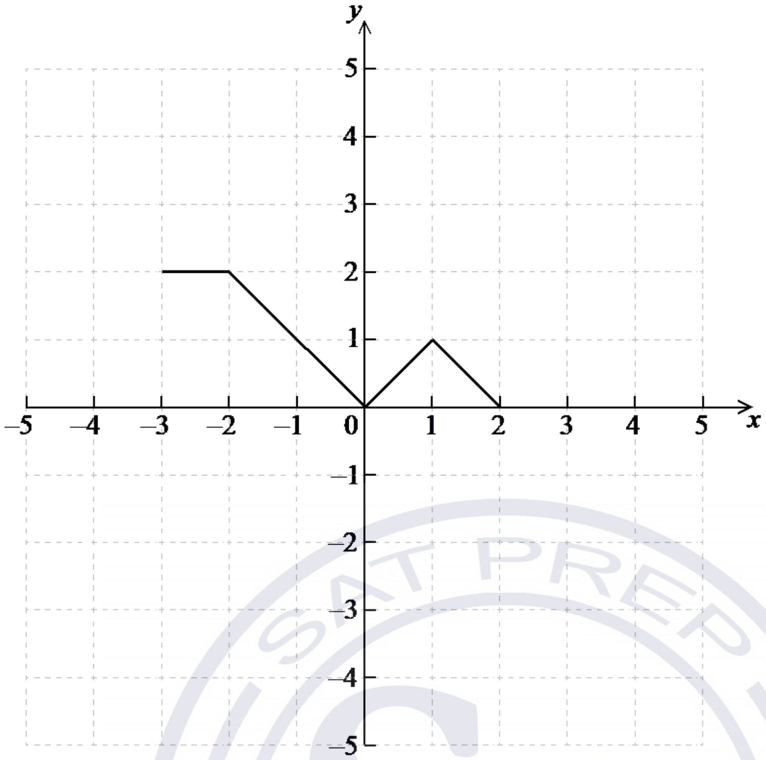
$p = 0.3$ ***A1*** ***N2***

Note: Exception to the ***FT*** rule. Award ***FT*** marks on an incorrect value of *q*, even if *q* is an inappropriate value. Do not award the final ***A*** mark for an inappropriate value of *p*.

[6 marks]

Total [8 marks]

5. (a)



A2 ***N2***

[2 marks]

(b) $a = -2, b = -1$ ***A2A2*** ***N4***

Note: Award ***A1*** for $a = 2$, ***A1*** for $b = 1$.

[4 marks]

Total [6 marks]

6. **METHOD 1**

evidence of valid approach *(M1)*
e.g. $b^2 - 4ac$, quadratic formula

correct substitution into $b^2 - 4ac$ (may be seen in formula) *(A1)*
e.g. $(k-1)^2 - 4\times1\times1$; $(k-1)^2 - 4$; $k^2 - 2k - 3$

setting **their** discriminant equal to zero *M1*
e.g. $\Delta = 0, (k-1)^2 - 4 = 0$

attempt to solve the quadratic *(M1)*
e.g. $(k-1)^2 = 4$, factorizing

correct working *A1*
e.g. $(k-1) = \pm 2$, $(k-3)(k+1)$

$k = -1$, $k = 3$ (do not accept inequalities) *A1A1* *N2*

[7 marks]

METHOD 2

recognizing perfect square *(M1)*
e.g. $(x+1)^2 = 0$, $(x-1)^2$

correct expansion *(A1)(A1)*
e.g. $x^2 + 2x + 1 = 0$, $x^2 - 2x + 1$

equating coefficients of x *A1A1*
e.g. $k - 1 = -2$, $k - 1 = 2$

$k = -1$, $k = 3$ *A1A1* *N2*

[7 marks]

7. attempt to expand $\left(1+\frac{2}{3}x\right)^n$ ***(M1)***

e.g. Pascal's triangle, $\left(1+\frac{2}{3}x\right)^n = 1+\frac{2}{3}nx+\ldots$

correct first two terms of $\left(1+\frac{2}{3}x\right)^n$ (seen anywhere) ***(A1)***

e.g. $1+\frac{2}{3}nx$

correct first two terms of quadratic (seen anywhere) ***(A1)***

e.g. $9,\ 6nx$; $(9+6nx+n^2x^2)$

correct calculation for the *x*-term ***A2***

e.g. $\frac{2}{3}nx\times 9+6nx$; $6n+6n,\ 12n$

correct equation ***A1***

e.g. $6n+6n=84,\ 12nx=84x$

$n=7$ ***A1*** ***N1***

[7 marks]

SECTION B

8. (a) (i) $h=2$, $k=1$ ***A1A1*** ***N2***

(ii) attempt to substitute coordinates of any point (except the vertex) on the graph into f ***M1***

e.g. $13=a(0-2)^2+1$

working towards solution ***A1***

e.g. $13=4a+1$

$a=3$ ***AG*** ***N0***

[4 marks]

(b) attempting to expand **their** binomial ***(M1)***

e.g. $f(x)=3(x^2-2\times 2x+4)+1$, $(x-2)^2=x^2-4x+4$

correct working ***(A1)***

e.g. $f(x)=3x^2-12x+12+1$

$f(x)=3x^2-12x+13$ (accept $A=3$, $B=-12$, $C=13$) ***A1*** ***N2***

[3 marks]

(c) **METHOD 1**

integral expression ***(A1)***

e.g. $\int_2^4(3x^2-12x+13)$, $\int f\,\mathrm{d}x$

Area $=\left[x^3-6x^2+13x\right]_2^4$ ***A1A1A1***

Note: Award ***A1*** for x^3, ***A1*** for $-6x^2$, ***A1*** for $13x$.

correct substitution of **correct** limits into **their** expression ***A1A1***

e.g. $(4^3-6\times 4^2+13\times 4)-(2^3-6\times 2^2+13\times 2)$, $64-96+52-(8-24+26)$

Note: Award ***A1*** for substituting 4, ***A1*** for substituting 2.

correct working ***(A1)***

e.g. $64-96+52-8+24-26, 20-10$

Area $=10$ ***A1*** ***N3***

[8 marks]

continued …

Question 8 continued

METHOD 2

integral expression *(A1)*

e.g. $\int_2^4 \left(3(x-2)^2+1\right)$, $\int f\,\mathrm{d}x$

Area $= \left[(x-2)^3 + x\right]_2^4$ ***A2A1***

Note: Award ***A2*** for $(x-2)^3$, ***A1*** for x.

correct substitution of **correct** limits into **their** expression ***A1A1***

e.g. $(4-2)^3+4-\left[(2-2)^3+2\right]$, $2^3+4-\left(0^3+2\right)$, 2^3+4-2

Note: Award ***A1*** for substituting 4, ***A1*** for substituting 2.

correct working *(A1)*

e.g. $8+4-2$

Area $=10$ ***A1*** ***N3***

[8 marks]

METHOD 3

recognizing area from 0 to 2 is same as area from 2 to 4 *(R1)*

e.g. sketch, $\int_2^4 f = \int_0^2 f$

integral expression *(A1)*

e.g. $\int_0^2 \left(3x^2-12x+13\right)$, $\int f\,\mathrm{d}x$

Area $= \left[x^3-6x^2+13x\right]_0^2$ ***A1A1A1***

Note: Award ***A1*** for x^3, ***A1*** for $-6x^2$, ***A1*** for $13x$.

correct substitution of **correct** limits into **their** expression ***A1(A1)***

e.g. $(2^3-6\times 2^2+13\times 2)-(0^3-6\times 0^2+13\times 0)$, $8-24+26$

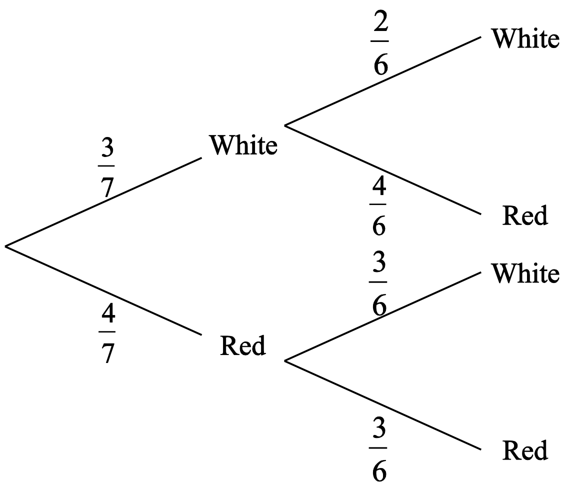
Note: Award ***A1*** for substituting 2, ***(A1)*** for substituting 0.

Area $=10$ ***A1*** ***N3***

[8 marks]

Total [15 marks]

9. (a) (i)



$\frac{4}{6}, \frac{3}{6}$ and $\frac{3}{6}$ $\left(\frac{2}{3}, \frac{1}{2} \text{ and } \frac{1}{2}\right)$ ***A1A1A1*** ***N3***

(ii) multiplying along the correct branches (may be seen on diagram) ***(A1)***

e.g. $\frac{3}{7} \times \frac{2}{6}$

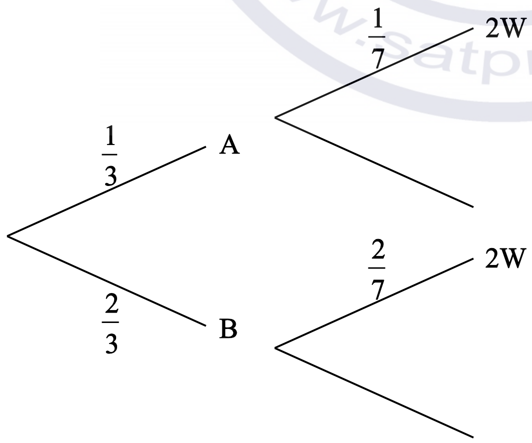
$\frac{6}{42}$ $\left(= \frac{1}{7}\right)$ ***A1*** ***N2***

[5 marks]

(b) $\text{P}(\text{bag A}) = \frac{2}{6}\left(= \frac{1}{3}\right)$, $\text{P}(\text{bag B}) = \frac{4}{6}\left(= \frac{2}{3}\right)$ (seen anywhere) ***(A1)(A1)***

appropriate approach ***(M1)***

e.g. $\text{P}(WW \cap A) + \text{P}(WW \cap B)$



correct calculation ***A1***

e.g. $\frac{1}{3} \times \frac{1}{7} + \frac{2}{3} \times \frac{2}{7}, \frac{2}{42} + \frac{8}{42}$

$\text{P}(2W) = \frac{60}{252}\left(= \frac{5}{21}\right)$ ***A1*** ***N3***

[5 marks]

continued …

Question 9 continued

(c) recognizing conditional probability **(M1)**

e.g. $\frac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$, $\mathrm{P}(A|WW) = \frac{\mathrm{P}(WW \cap A)}{\mathrm{P}(WW)}$

correct numerator **(A1)**

e.g. $\mathrm{P}(A \cap WW) = \frac{6}{42} \times \frac{2}{6}$, $\frac{1}{21}$

correct denominator **(A1)**

e.g. $\frac{60}{252}$, $\frac{5}{21}$

probability $\frac{84}{420}\left(=\frac{1}{5}\right)$ ***A1*** ***N3***

[4 marks]

10. (a) correct derivatives **applied** in quotient rule *(A1)A1A1*
$1,\ -4x+5$

> **Note:** Award *(A1)* for 1, *A1* for $-4x$ and *A1* for 5, **only** if it is clear candidates are using the quotient rule.

correct substitution into quotient rule *A1*

e.g. $\dfrac{1\times(-2x^2+5x-2)-x(-4x+5)}{(-2x^2+5x-2)^2}$, $\dfrac{-2x^2+5x-2-x.-4x+5}{(-2x^2+5x-2)^2}$

correct working *(A1)*

e.g. $\dfrac{-2x^2+5x-2-(-4x^2+5x)}{(-2x^2+5x-2)^2}$

expression clearly leading to the answer *A1*

e.g. $\dfrac{-2x^2+5x-2+4x^2-5x}{(-2x^2+5x-2)^2}$

$f'(x)=\dfrac{2x^2-2}{(-2x^2+5x-2)^2}$ *AG* *N0*

[6 marks]

(b) evidence of attempting to solve $f'(x)=0$ *(M1)*

e.g. $2x^2-2=0$

evidence of correct working *A1*

e.g. $x^2=1$, $\dfrac{\pm\sqrt{16}}{4}$, $2(x-1)(x+1)$

correct solution to quadratic *(A1)*

e.g. $x=\pm1$

correct x-coordinate $x=-1$ $\left(\text{may be seen in coordinate form } \left(-1, \dfrac{1}{9}\right)\right)$ *A1* *N2*

attempt to substitute -1 into f (do not accept any other value) *(M1)*

e.g. $f(-1)=\dfrac{-1}{-2\times(-1)^2+5\times(-1)-2}$

correct working *A1*

e.g. $\dfrac{-1}{-2-5-2}$

correct y-coordinate $y=\dfrac{1}{9}$ $\left(\text{may be seen in coordinate form } \left(-1, \dfrac{1}{9}\right)\right)$ *A1* *N2*

[7 marks]

continued …

Question 10 continued

(c) recognizing values between max and min ***(R1)***

$\frac{1}{9} < k < 1$ ***A2*** ***N3***

[3 marks]

Total [16 marks]

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MARKSCHEME

November 2011

MATHEMATICS

Standard Level

Paper 1

17 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**; working must be seen.

(M) Marks awarded for **Method**; may be implied by **correct** subsequent working.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

(A) Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.

R Marks awarded for clear **Reasoning**.

N Marks awarded for **correct** answers if **no** working shown.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Mark according to scoris instructions and parts of the document "**Mathematics SL : Guidance for e-marking November 2011".** It is **essential** that you read this document before you start marking. In particular, please note the following.

Marks must be recorded using the annotation stamps. Please check that you are entering marks for the right question.

- If a part is **completely correct**, (and gains all the 'must be seen' marks), use the ticks with numbers to stamp full marks.
- If a part is completely wrong, stamp ***A0*** by the final answer.
- If a part gains anything else, it **must** be recorded using **all** the annotations.

All the marks will be added and recorded by scoris.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any. An exception to this rule is when work for ***M1*** is missing, as opposed to incorrect (see point 4).
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***(M2)***, ***N3***, *etc*., do **not** split the marks, unless there is a note.
- Once a correct answer to a question or part-question is seen, ignore further working.

3 *N* marks

*If **no** working shown, award **N** marks for **correct** answers. In this case, ignore mark breakdown (**M**, **A**, **R**).*

- Do **not** award a mixture of ***N*** and other marks.
- There may be fewer ***N*** marks available than the total of ***M***, ***A*** and ***R*** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.
- There may not be a direct relationship between the ***N*** marks and the implied marks. There are times when all the marks are implied, but the ***N*** marks are not the full marks: this indicates that we want to see some of the working, without specifying what.
- For consistency within the markscheme, ***N*** marks are noted for every part, even when these match the mark breakdown.
- If a candidate has incorrect working, which somehow results in a correct answer, do **not** award the ***N*** marks for this correct answer. However, if the candidate has indicated (usually by crossing out) that the working is to be ignored, award the ***N*** marks for the correct answer.

4 Implied and must be seen marks

*Implied marks appear in **brackets e.g. (M1)**.*

- Implied marks can only be awarded if **correct** work is seen or if implied in subsequent working (a correct answer does not necessarily mean that the implied marks are all awarded). There are questions where some working is required, but as it is accepted that not everyone will write the same steps, all the marks are implied, but the N marks are not the full marks for the question.
- Normally the correct work is seen or implied in the next line.
- Where there is an (***M1***) followed by ***A1*** for each correct answer, if no working shown, one correct answer is sufficient evidence to award the (***M1***).

*Must be seen marks appear without **brackets e.g. M1**.*

- Must be seen marks can only be awarded if **correct** work is seen.
- If a must be seen mark is not awarded because work is missing (as opposed to ***M0*** or ***A0*** for incorrect work) all subsequent marks may be awarded if appropriate.

5 Follow through marks (only applied after an error is made)

*Follow through (**FT**) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) or subpart(s). Usually, to award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if the only marks awarded in a subpart are for the answer (ie there is no working expected), then **FT** marks should be awarded if appropriate. Examiners are expected to check student work in order to award FT marks where appropriate.*

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate. (However, as noted above, if an ***A*** mark is not awarded because work is missing, all subsequent marks may be awarded if appropriate)
- Exceptions to this rule will be explicitly noted on the markscheme.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*e.g.* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word "their" in a description, to indicate that candidates may be using an incorrect value.

- If a candidate makes an error in one part, but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the question says hence. It is often possible to use a different approach in subsequent parts that does not depend on the answer to previous parts.
- In a "show that" question, if an error leads to not showing the required answer, there is a 1 mark penalty. Note that if the error occurs within the same subpart, the *FT* rules may result in further loss of marks.
- Where there are anticipated common errors, the ***FT*** answers are often noted on the markscheme, to help examiners. It should be stressed that these are not the only ***FT*** answers accepted.

6 Mis-read

*If a candidate incorrectly copies information from the question, this is a mis-read (**MR**). Apply a **MR** penalty of 1 mark to that question. Award the marks as usual and then stamp **MR** against the answer. Scoris will automatically deduct 1 mark from the question total. A candidate should be penalized only once for a particular mis-read. Do not stamp **MR** again for that question, unless the candidate makes another mis-read.*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.

7 Discretionary marks *(d)*

*An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. In such cases the annotation D should be used and a brief **note** written next to the mark explaining this decision.*

8 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for part-questions are indicated by **EITHER** . . . **OR**.
- Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9 Alternative forms

*Unless the question specifies otherwise, **accept** equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

10 Accuracy of Answers

If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy. When this is not specified in the question, all numerical answers should be given exactly or correct to three significant figures

*Candidates should NO LONGER be penalized for an accuracy error **(AP)**. Examiners should award marks according to the rules given in these instructions and the markscheme. Accuracy is not the same as correctness – an incorrect value does not achieve relevant **A** marks. It is only final answers which may lose marks for accuracy errors, not intermediate values. Please check work carefully for **FT**.*

Do not accept unfinished numerical answers such as 3/0.1 (unless otherwise stated). As a rule, numerical answers with more than one part (such as fractions) should be given using integers (*e.g.* 6/8). Calculations which lead to integers should be completed, with the exception of fractions which are not whole numbers.

11 Calculators

No calculator is allowed. The use of any calculator on paper 1 is malpractice, and will result in no grade awarded. If you see work that suggests a candidate has used any calculator, please follow the procedures for malpractice. Examples: finding an angle, given a trig ratio of 0.4235.

12 Style

The markscheme aims to present answers using good communication, e.g. if the question asks to find the value of k, the markscheme will say $k = 3$, but the marks will be for the correct value 3 – there is usually no need for the "$k =$". In these cases, it is also usually acceptable to have another variable, as long as there is no ambiguity in the question, e.g. if the question asks to find the value of p and of q, then the student answer needs to be clear. Generally, the only situation where the full answer is required is in a question which asks for equations – in this case the markscheme will say "must be an equation".

*The markscheme often uses words to describe what the marks are for, followed by examples, using the e.g. notation. These examples are not exhaustive, and examiners should check what candidates have written, to see if they satisfy the description. Where these marks are **M** marks, the examples may include ones using poor notation, to indicate what is acceptable.*

13 Crossed out work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

SECTION A

1. (a) $x = 4$ (must be an equation) *A1* *N1*

[1 mark]

(b) $h = 4$, $k = 2$ *A1A1* *N2*

[2 marks]

(c) attempt to substitute coordinates of any point on the graph into f *(M1)*

e.g. $f(0) = 6$, $6 = a(0-4)^2 + 2$, $f(4) = 2$

correct equation (do **not** accept an equation that results from $f(4) = 2$) *(A1)*

e.g. $6 = a(-4)^2 + 2$, $6 = 16a + 2$

$a = \frac{4}{16}\left(= \frac{1}{4}\right)$ *A1* *N2*

[3 marks]

Total [6 marks]

2. (a) evidence of matrix multiplication (in any order) *(M1)*

e.g. $\boldsymbol{PQ} = \begin{pmatrix} 3(4)+1(-10) & 3(-2)+1(6) \\ 5(4)+2(-10) & 5(-2)+2(6) \end{pmatrix}$

$\boldsymbol{PQ} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$, $2\boldsymbol{I}$ *A2* *N3*

[3 marks]

(b) $\boldsymbol{P}^{-1} = \frac{1}{2}\boldsymbol{Q}$, $\begin{pmatrix} 2 & -1 \\ -5 & 3 \end{pmatrix}$ *A2* *N2*

[2 marks]

Total [5 marks]

3. **Note:** In this question, method marks may be awarded for selecting without replacement, as noted in the examples.

(a) $\mathrm{P}(R) = \frac{6}{8}\ \left(= \frac{3}{4}\right)$ ***A1*** ***N1***

[1 mark]

(b) attempt to find $\mathrm{P}(\text{Red}) \times \mathrm{P}(\text{Red})$ ***(M1)***

e.g. $\mathrm{P}(R) \times \mathrm{P}(R),\ \frac{3}{4} \times \frac{3}{4},\ \frac{6}{8} \times \frac{5}{7}$

$\mathrm{P}(2R) = \frac{36}{64}\ \left(= \frac{9}{16}\right)$ ***A1*** ***N2***

[2 marks]

(c) **METHOD 1**

attempt to find $\mathrm{P}(\text{Red}) \times \mathrm{P}(\text{Blue})$ ***(M1)***

e.g. $\mathrm{P}(R) \times \mathrm{P}(B),\ \frac{6}{8} \times \frac{2}{8},\ \frac{6}{8} \times \frac{2}{7}$

recognizing two ways to get one red, one blue ***(M1)***

e.g. $\mathrm{P}(RB) + \mathrm{P}(BR),\ 2\left(\frac{12}{64}\right),\ \frac{6}{8} \times \frac{2}{7} + \frac{2}{8} \times \frac{6}{7}$

$\mathrm{P}(1R, 1B) = \frac{24}{64}\ \left(= \frac{3}{8}\right)$ ***A1*** ***N2***

[3 marks]

METHOD 2

recognizing that $\mathrm{P}(1R, 1B)$ is $1 - \mathrm{P}(2B) - \mathrm{P}(2R)$ ***(M1)***

attempt to find $\mathrm{P}(2R)$ and $\mathrm{P}(2B)$ ***(M1)***

e.g. $\mathrm{P}(2R) = \frac{3}{4} \times \frac{3}{4},\ \frac{6}{8} \times \frac{5}{7};\ \mathrm{P}(2B) = \frac{1}{4} \times \frac{1}{4},\ \frac{2}{8} \times \frac{1}{7}$

$\mathrm{P}(1R, 1B) = \frac{24}{64}\ \left(= \frac{3}{8}\right)$ ***A1*** ***N2***

[3 marks]

Total [6 marks]

4. evidence of anti-differentiation *(M1)*
e.g. $\int f'(x)$, $\int (3x^2+2)\,\mathrm{d}x$

$f(x)=x^3+2x+c$ (seen anywhere, including the answer) *A1A1*

Attempt to substitute $(2,5)$ *(M1)*
e.g. $f(2)=(2)^3+2(2)$, $5=8+4+c$

finding the value of c *(A1)*
e.g. $5=12+c$, $c=-7$

$f(x)=x^3+2x-7$ *A1* *N5*
[6 marks]

5. correct substitution into $\mathrm{E}(X)=\sum px$ (seen anywhere) *A1*
e.g. $1s+2\times 0.3+3q=1.7$, $s+3q=1.1$

recognizing $\sum p=1$ (seen anywhere) *(M1)*
correct substitution into $\sum p=1$ *A1*
e.g. $s+0.3+q=1$

attempt to solve simultaneous equations *(M1)*

correct working *(A1)*
e.g. $0.3+2q=0.7, 2s=1$

$q=0.2$ *A1* *N4*
[6 marks]

6. (a) **METHOD 1**

evidence of choosing $\sin^2\theta + \cos^2\theta = 1$ ***(M1)***

correct working ***(A1)***

e.g. $\cos^2\theta = \frac{9}{13}$, $\cos\theta = \pm\frac{3}{\sqrt{13}}$, $\cos\theta = \sqrt{\frac{9}{13}}$

$\cos\theta = -\frac{3}{\sqrt{13}}$ ***A1*** ***N2***

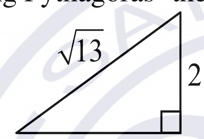
Note: If no working shown, award ***N1*** for $\frac{3}{\sqrt{13}}$.

[3 marks]

METHOD 2

approach involving Pythagoras' theorem ***(M1)***

e.g. $2^2 + x^2 = 13$,



finding third side equals 3 ***(A1)***

$\cos\theta = -\frac{3}{\sqrt{13}}$ ***A1*** ***N2***

Note: If no working shown, award ***N1*** for $\frac{3}{\sqrt{13}}$.

[3 marks]

continued …

Question 6 continued

(b) correct substitution into $\sin 2\theta$ (seen anywhere) ***(A1)***

e.g. $2\left(\frac{2}{\sqrt{13}}\right)\left(-\frac{3}{\sqrt{13}}\right)$

correct substitution into $\cos 2\theta$ (seen anywhere) ***(A1)***

e.g. $\left(-\frac{3}{\sqrt{13}}\right)^2-\left(\frac{2}{\sqrt{13}}\right)^2$, $2\left(-\frac{3}{\sqrt{13}}\right)^2-1$, $1-2\left(\frac{2}{\sqrt{13}}\right)^2$

valid attempt to find $\tan 2\theta$ ***(M1)***

e.g. $\dfrac{2\left(\frac{2}{\sqrt{13}}\right)\left(-\frac{3}{\sqrt{13}}\right)}{\left(-\frac{3}{\sqrt{13}}\right)^2-\left(\frac{2}{\sqrt{13}}\right)^2}$, $\dfrac{2\left(-\frac{2}{3}\right)}{1-\left(-\frac{2}{3}\right)^2}$

correct working ***A1***

e.g. $\dfrac{\frac{(2)(2)(-3)}{13}}{\frac{9}{13}-\frac{4}{13}}$, $\dfrac{-\frac{12}{\left(\sqrt{13}\right)^2}}{\frac{18}{13}-1}$, $\dfrac{-\frac{12}{13}}{\frac{5}{13}}$

$\tan 2\theta = -\frac{12}{5}$ ***A1*** ***N4***

Note: If students find answers for $\cos\theta$ which are not in the range [-1, 1], award full ***FT*** in (b) for correct ***FT*** working shown.

[5 marks]

Total [8 marks]

7. (a) **METHOD 1**

evidence of discriminant *(M1)*

e.g. $b^2 - 4ac$, discriminant $= 0$

correct substitution into discriminant ***A1***

e.g. $k^2 - 4 \times \frac{1}{2} \times 8$, $k^2 - 16 = 0$

$k = \pm 4$ ***A1A1*** ***N3***

METHOD 2

Recognising that equal roots means perfect square ***(R1)***

e.g. attempt to complete the square, $\frac{1}{2}\left(x^2 + 2kx + 16\right)$

correct working

e.g $\frac{1}{2}(x+k)^2$, $\frac{1}{2}k^2 = 8$ ***A1***

$k = \pm 4$ ***A1A1*** ***N3***

[4 marks]

(b) evidence of appropriate approach ***(M1)***

e.g. $b^2 - 4ac < 0$

correct working for k ***A1***

e.g. $-4 < k < 4$, $k^2 < 16$, list all correct values of k

$p = \frac{7}{11}$ ***A2*** ***N3***

[4 marks]

Total [8 marks]

8. (a) (i) evidence of approach *(M1)*

e.g. $\vec{PO}+\vec{OQ}$, $P-Q$

$\vec{PQ}=\begin{pmatrix}2\\1\\-4\end{pmatrix}$ ***A1*** ***N2***

(ii) **any** correct equation in the form $\boldsymbol{r}=\boldsymbol{a}+s\boldsymbol{b}$ (accept any parameter for s)

where $\boldsymbol{a}$ is $\begin{pmatrix}2\\4\\8\end{pmatrix}$ or $\begin{pmatrix}4\\5\\4\end{pmatrix}$, and $\boldsymbol{b}$ is a scalar multiple of $\begin{pmatrix}2\\1\\-4\end{pmatrix}$ ***A2*** ***N2***

e.g. $\boldsymbol{r}=\begin{pmatrix}2\\4\\8\end{pmatrix}+s\begin{pmatrix}2\\1\\-4\end{pmatrix}$, $\boldsymbol{r}=\begin{pmatrix}4+2s\\5+1s\\4-4s\end{pmatrix}$, $\boldsymbol{r}=2\boldsymbol{i}+4\boldsymbol{j}+8\boldsymbol{k}+s(2\boldsymbol{i}+1\boldsymbol{j}-4\boldsymbol{k})$

Note: Award ***A1*** for the form $\boldsymbol{a}+s\boldsymbol{b}$, ***A1*** for $\boldsymbol{L}=\boldsymbol{a}+s\boldsymbol{b}$, ***A0*** for $\boldsymbol{r}=\boldsymbol{b}+s\boldsymbol{a}$.

[4 marks]

(b) (i) choosing correct direction vectors for L_1 and L_2 *(A1)(A1)*

e.g. $\begin{pmatrix}2\\1\\-4\end{pmatrix}$, $\begin{pmatrix}3p\\2p\\4\end{pmatrix}$

evidence of equating scalar product to 0 *(M1)*

correct calculation of scalar product ***A1***

e.g. $2\times3p+1\times2p+(-4)\times4$, $8p-16=0$

$p=2$ ***A1*** ***N3***

(ii) **any** correct expression in the form $\boldsymbol{r}=\boldsymbol{a}+t\boldsymbol{b}$ (accept any parameter for t)

where $\boldsymbol{a}$ is $\begin{pmatrix}10\\6\\-40\end{pmatrix}$, and $\boldsymbol{b}$ is a scalar multiple of $\begin{pmatrix}6\\4\\4\end{pmatrix}$ ***A2*** ***N2***

e.g. $\boldsymbol{r}=\begin{pmatrix}10\\6\\-40\end{pmatrix}+t\begin{pmatrix}6\\4\\4\end{pmatrix}$, $\boldsymbol{r}=\begin{pmatrix}10+6s\\6+4s\\-40+4s\end{pmatrix}$, $\boldsymbol{r}=10\boldsymbol{i}+6\boldsymbol{j}-40\boldsymbol{k}+s(6\boldsymbol{i}+4\boldsymbol{j}+4\boldsymbol{k})$

Note: Award ***A1*** for the form $\boldsymbol{a}+t\boldsymbol{b}$, ***A1*** for $\boldsymbol{L}=\boldsymbol{a}+t\boldsymbol{b}$ (unless they have been penalised for $\boldsymbol{L}=\boldsymbol{a}+s\boldsymbol{b}$ in part (a)), ***A0*** for $\boldsymbol{r}=\boldsymbol{b}+t\boldsymbol{a}$.

[7 marks]

continued …

Question 8 continued

(c) appropriate approach ***(M1)***

e.g. $\begin{pmatrix}2\\4\\8\end{pmatrix}+s\begin{pmatrix}2\\1\\-4\end{pmatrix}=\begin{pmatrix}10\\6\\-40\end{pmatrix}+t\begin{pmatrix}6\\4\\4\end{pmatrix}$

any two correct equations with **different** parameters ***A1A1***

e.g. $2+2s=10+6t$, $4+s=6+4t$, $8-4s=-40+4t$

attempt to solve simultaneous equations ***(M1)***

correct working ***(A1)***

e.g. $-6=-2-2t$, $4=2t$, $-4+5s=46$, $5s=50$

one correct parameter $s=10$, $t=2$ ***A1***

$x=22$ $\left(\text{accept } (22, 14, -32)\right)$ ***A1*** ***N4***

[7 marks]

Total [18 marks]

9. (a) (i) $a = 8$ ***A1*** ***N1***

(ii) $c = 2$ ***A1*** ***N1***

(iii) $d = 4$ ***A1*** ***N1***

[3 marks]

(b) **METHOD 1**

recognizing that period $= 8$ ***(A1)***

correct working ***A1***

e.g. $8 = \frac{2\pi}{b}, b = \frac{2\pi}{8}$

$b = \frac{\pi}{4}$ ***AG*** ***N0***

[2 marks]

METHOD 2

attempt to substitute ***M1***

e.g. $12 = 8\sin\left(b(4-2)\right) + 4$

correct working ***A1***

e.g. $\sin 2b = 1$

$b = \frac{\pi}{4}$ ***AG*** ***N0***

[2 marks]

(c) evidence of attempt to differentiate or choosing chain rule ***(M1)***

e.g. $\cos\frac{\pi}{4}(x-2)$, $\frac{\pi}{4}\times 8$

$f'(x) = 2\pi\cos\left(\frac{\pi}{4}(x-2)\right)$ $\left(\text{accept } 2\pi\cos\frac{\pi}{4}(x-2)\right)$ ***A2*** ***N3***

[3 marks]

continued …

Question 9 continued

(d) recognizing that gradient is $f'(x)$ ***(M1)***
e.g. $f'(x) = m$

correct equation ***A1***
e.g. $-2\pi = 2\pi\cos\left(\frac{\pi}{4}(x-2)\right)$, $-1 = \cos\left(\frac{\pi}{4}(x-2)\right)$

correct working ***(A1)***
e.g. $\cos^{-1}(-1) = \frac{\pi}{4}(x-2)$

using $\cos^{-1}(-1) = \pi$ (seen anywhere) ***(A1)***
e.g. $\pi = \frac{\pi}{4}(x-2)$

simplifying ***(A1)***
e.g. $4 = (x-2)$

$x = 6$ ***A1*** ***N4***

[6 marks]

Total [14 marks]

10. (a) finding $f'(x) = \frac{1}{2}x$ ***A1***

attempt to find $f'(4)$ ***(M1)***

correct value $f'(4) = 2$ ***A1***

correct equation in any form ***A1*** ***N2***
e.g. $y - 6 = 2(x - 4)$, $y = 2x - 2$

[4 marks]

(b) area $= \int_2^{12} \frac{90}{3x+4} dx$

correct integral ***A1A1***
e.g. $30\ln(3x+4)$

substituting limits and subtracting ***(M1)***
e.g. $30\ln(3 \times 12 + 4) - 30\ln(3 \times 2 + 4)$, $30\ln 40 - 30\ln 10$

correct working ***(A1)***
e.g. $30(\ln 40 - \ln 10)$

correct application of $\ln b - \ln a$ ***(A1)***
e.g. $30\ln\frac{40}{10}$

area $= 30\ln 4$ ***A1*** ***N4***

[6 marks]

(c) valid approach ***(M1)***
e.g. sketch, area h = area g, 120 + **their** answer from (b)

area $= 120 + 30\ln 4$ ***A2*** ***N3***

[3 marks]

Total [13 marks]