Diploma Programme
Programme du diplôme
Programa del Diploma

Diploma Programme
Programme du diplôme
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Mathematics
Standard level
Paper 2

Tuesday 19 November 2019 (morning)

Candidate session number

1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics SL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[90 marks]**.

8819 – 7302

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, for example if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines if necessary.

1. [Maximum mark: 6]

The number of messages, M, that six randomly selected teenagers sent during the month of October is shown in the following table. The table also shows the time, T, that they spent talking on their phone during the same month.

Time spent talking on their phone (T minutes)	50	55	105	128	155	200
Number of messages (M)	358	340	740	731	800	992

The relationship between the variables can be modelled by the regression equation $M = aT + b$.

(a) Write down the value of a and of b. [3]

(b) Use your regression equation to predict the number of messages sent by a teenager that spent 154 minutes talking on their phone in October. [3]

2. [Maximum mark: 5]

Consider the lines L_1 and L_2 with respective equations

$$L_1 : y = -\frac{2}{3}x + 9 \text{ and } L_2 : y = \frac{2}{5}x - \frac{19}{5}.$$

(a) Find the point of intersection of L_1 and L_2. [2]

A third line, L_3, has gradient $-\frac{3}{4}$.

(b) Write down a direction vector for L_3. [1]

L_3 passes through the intersection of L_1 and L_2.

(c) Write down a vector equation for L_3. [2]

3. [Maximum mark: 7]

Let $f(x) = x - 8$, $g(x) = x^4 - 3$ and $h(x) = f(g(x))$.

(a) Find $h(x)$. [2]

Let C be a point on the graph of h. The tangent to the graph of h at C is parallel to the graph of f.

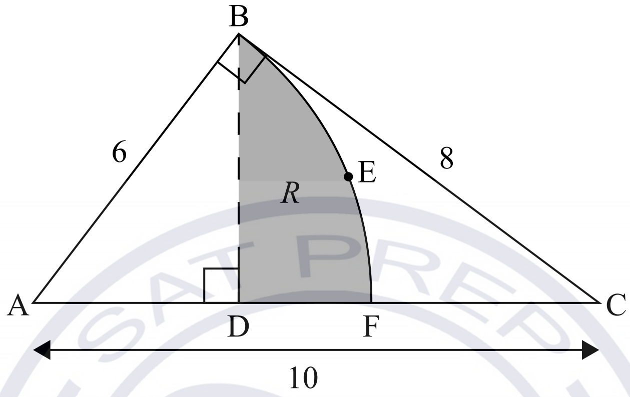
(b) Find the x-coordinate of C. [5]

4. [Maximum mark: 7]

The following diagram shows a right-angled triangle, ABC, with $\mathrm{AC} = 10\,\mathrm{cm}$, $\mathrm{AB} = 6\,\mathrm{cm}$ and $\mathrm{BC} = 8\,\mathrm{cm}$.

The points D and F lie on $[\mathrm{AC}]$.
$[\mathrm{BD}]$ is perpendicular to $[\mathrm{AC}]$.
BEF is the arc of a circle, centred at A.
The region R is bounded by $[\mathrm{BD}]$, $[\mathrm{DF}]$ and arc BEF.

diagram not to scale



(a) Find $\mathrm{B\hat{A}C}$. [2]

(b) Find the area of R. [5]

5. [Maximum mark: 7]

The first two terms of a geometric sequence are $u_1 = 2.1$ and $u_2 = 2.226$.

(a) Find the value of r. [2]

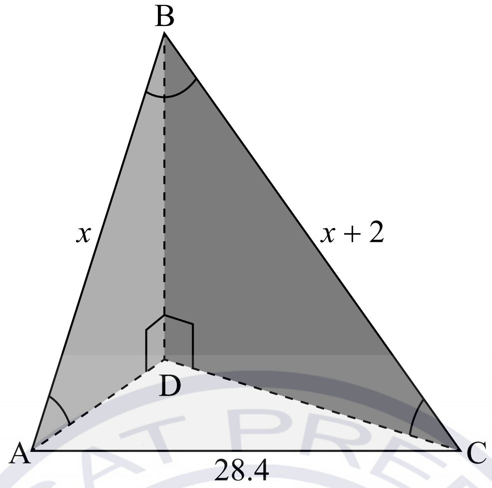
(b) Find the value of u_{10}. [2]

(c) Find the least value of n such that $S_n > 5543$. [3]

6. [Maximum mark: 6]

The diagram below shows a triangular-based pyramid with base ADC.
Edge BD is perpendicular to the edges AD and CD.

diagram not to scale



$AC = 28.4\,cm,\ AB = x\,cm,\ BC = x + 2\,cm,\ A\hat{B}C = 0.667,\ B\hat{A}D = 0.611$

Calculate AD.

7. [Maximum mark: 7]

The following table shows the probability distribution of a discrete random variable X, where $a \geq 0$ and $b \geq 0$.

x	1	4	a	$a + b - 0.5$
$\mathrm{P}(X = x)$	0.2	0.5	b	a

(a) Show that $b = 0.3 - a$. [1]

(b) Find the difference between the greatest possible expected value and the least possible expected value. [6]

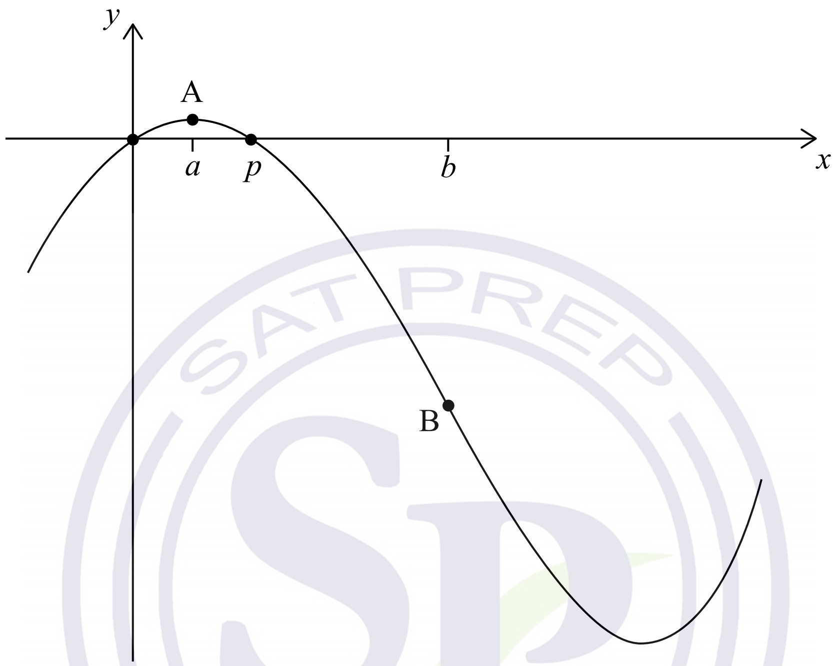
Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 16]

Let $f(x) = x^4 - 54x^2 + 60x$, for $-1 \le x \le 6$. The following diagram shows the graph of f.



There are x-intercepts at $x = 0$ and at $x = p$. There is a maximum at point A where $x = a$, and a point of inflexion at point B where $x = b$.

(a) Find the value of p. [2]

(b) (i) Write down the coordinates of A.

(ii) Find the equation of the tangent to the graph of f at A. [4]

(c) (i) Find the coordinates of B.

(ii) Find the rate of change of f at B. [7]

(d) Let R be the region enclosed by the graph of f, the x-axis and the lines $x = p$ and $x = b$. The region R is rotated 360° about the x-axis. Find the volume of the solid formed. [3]

Do **not** write solutions on this page.

9. [Maximum mark: 15]

SpeedWay airline flies from city A to city B. The flight time is normally distributed with a mean of 260 minutes and a standard deviation of 15 minutes.

A flight is considered late if it takes longer than 275 minutes.

(a) Calculate the probability a flight is **not** late. [2]

The flight is considered to be **on time** if it takes between m and 275 minutes. The probability that a flight is on time is 0.830.

(b) Find the value of m. [3]

During a week, SpeedWay has 12 flights from city A to city B. The time taken for any flight is independent of the time taken by any other flight.

(c) (i) Calculate the probability that at least 7 of these flights are **on time**.

(ii) Given that at least 7 of these flights are on time, find the probability that exactly 10 flights are on time. [7]

SpeedWay increases the number of flights from city A to city B to 20 flights each week, and improves their efficiency so that more flights are on time. The probability that at least 19 flights are on time is 0.788.

(d) A flight is chosen at random. Calculate the probability that it is on time. [3]

Do **not** write solutions on this page.

10. [Maximum mark: 14]

A rocket is travelling in a straight line, with an initial velocity of $140\,\text{m s}^{-1}$. It accelerates to a new velocity of $500\,\text{m s}^{-1}$ in two stages.

During the first stage its acceleration, $a\,\text{m s}^{-2}$, after t seconds is given by $a(t) = 240\sin(2t)$, where $0 \le t \le k$.

(a) Find an expression for the velocity, $v\,\text{m s}^{-1}$, of the rocket during the first stage. [4]

The first stage continues for k seconds until the velocity of the rocket reaches $375\,\text{m s}^{-1}$.

(b) Find the distance that the rocket travels during the first stage. [4]

During the second stage, the rocket accelerates at a constant rate. The distance which the rocket travels during the second stage is the same as the distance it travels during the first stage.

(c) Find the total time taken for the two stages. [6]

Please **do not** write on this page.

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Mathematics
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Paper 2

Tuesday 14 May 2019 (morning)

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2219 – 7304

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Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines if necessary.

1. [Maximum mark: 5]

Ten students were asked for the distance, in km, from their home to school. Their responses are recorded below.

$$0.3 \quad 0.4 \quad 3 \quad 3 \quad 3.5 \quad 5 \quad 7 \quad 8 \quad 8 \quad 10$$

(a) For these data, find the mean distance from a student's home to school. [2]

The following box-and-whisker plot represents this data.

(b) Find the value of p. [1]

(c) Find the interquartile range. [2]

2. [Maximum mark: 6]

Consider the graph of the function $f(x) = a(x+10)^2 + 15$, $x \in \mathbb{R}$.

(a) Write down the coordinates of the vertex. [2]

(b) The graph of f has a y-intercept at -20. Find a. [2]

(c) Point $\mathrm{P}(8, b)$ lies on the graph of f. Find b. [2]

3. [Maximum mark: 6]

Consider the function $f(x) = x^2 e^{3x}$, $x \in \mathbb{R}$.

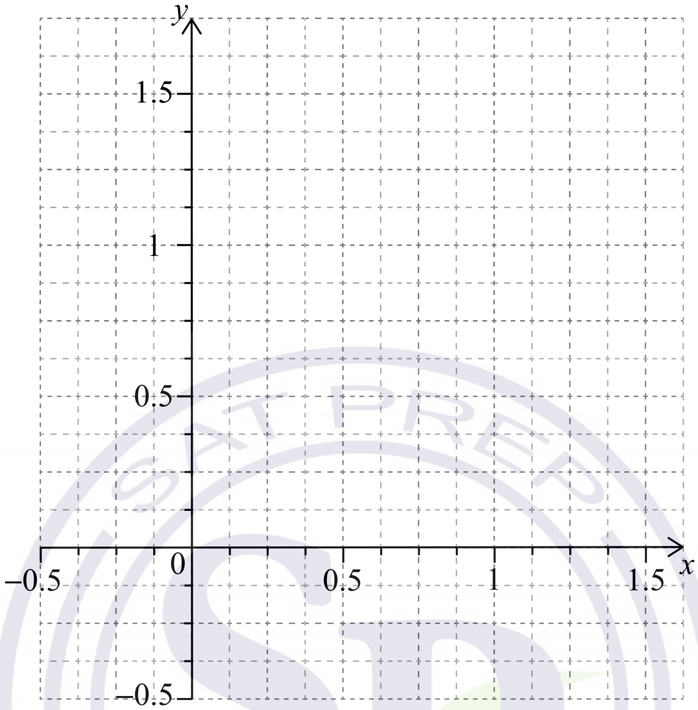
(a) Find $f'(x)$. [4]

(b) The graph of f has a horizontal tangent line at $x = 0$ and at $x = a$. Find a. [2]

4. [Maximum mark: 8]

Let $f''(x) = (\cos 2x)(\sin 6x)$, for $0 \le x \le 1$.

(a) Sketch the graph of f'' on the grid below: [3]

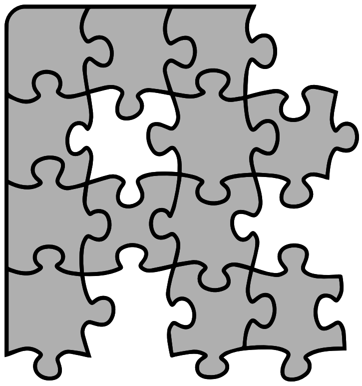


(b) Find the x-coordinates of the points of inflexion of the graph of f. [3]

(c) Hence find the values of x for which the graph of f is concave-down. [2]

5. [Maximum mark: 6]

A jigsaw puzzle consists of many differently shaped pieces that fit together to form a picture.



Jill is doing a 1000-piece jigsaw puzzle. She started by sorting the edge pieces from the interior pieces. Six times she stopped and counted how many of each type she had found. The following table indicates this information.

Edge pieces (x)	16	31	39	55	84	115
Interior pieces (y)	89	239	297	402	580	802

Jill models the relationship between these variables using the regression equation $y = ax + b$.

(a) Write down the value of a and of b. [3]

(b) Use the model to predict how many edge pieces she had found when she had sorted a **total** of 750 pieces. [3]

6. [Maximum mark: 7]

Consider the expansion of $\left(x^2 + 1.2\right)^n$ where $n \in \mathbb{Z},\ n \geq 3$. Given that the coefficient of the term containing x^6 is greater than $200\,000$, find the smallest possible value of n.

7. [Maximum mark: 7]

The first terms of an infinite geometric sequence, u_n, are $2, 6, 18, 54, \ldots$.
The first terms of a second infinite geometric sequence, v_n, are $2, -6, 18, -54, \ldots$.

The terms of a third sequence, w_n, are defined as $w_n = u_n + v_n$.

(a) Write down the first three **non-zero** terms of w_n. [3]

The finite series, $\sum_{k=1}^{225} w_k$, can also be written in the form $\sum_{k=0}^{m} 4r^k$.

(b) Find the value of

(i) r;

(ii) m. [4]

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 13]

Let $f(x) = 2\sin(3x) + 4$ for $x \in \mathbb{R}$.

(a) The range of f is $k \le f(x) \le m$. Find k and m. [3]

Let $g(x) = 5f(2x)$.

(b) Find the range of g. [2]

The function g can be written in the form $g(x) = 10\sin(bx) + c$.

(c) (i) Find the value of b and of c.

(ii) Find the period of g. [5]

(d) The equation $g(x) = 12$ has two solutions where $\pi \le x \le \frac{4\pi}{3}$. Find both solutions. [3]

Do **not** write solutions on this page.

9. [Maximum mark: 16]

Let $f(x)=\frac{16}{x}$. The line L is tangent to the graph of f at $x=8$.

(a) Find the gradient of L. [2]

L can be expressed in the form $\boldsymbol{r}=\begin{pmatrix}8\\2\end{pmatrix}+t\boldsymbol{u}$.

(b) Find $\boldsymbol{u}$. [2]

The direction vector of $y=x$ is $\begin{pmatrix}1\\1\end{pmatrix}$.

(c) Find the acute angle between $y=x$ and L. [5]

(d) (i) Find $(f\circ f)(x)$.

(ii) Hence, write down $f^{-1}(x)$.

(iii) Hence or otherwise, find the obtuse angle formed by the tangent line to f at $x=8$ and the tangent line to f at $x=2$. [7]

Do **not** write solutions on this page.

10. [Maximum mark: 16]

There are three fair six-sided dice. Each die has two green faces, two yellow faces and two red faces.

All three dice are rolled.

(a) (i) Find the probability of rolling exactly one red face.

(ii) Find the probability of rolling two or more red faces. [5]

Ted plays a game using these dice. The rules are:

- Having a turn means to roll all three dice.
- He wins $\$10$ for each green face rolled and adds this to his winnings.
- After a turn Ted can either:
  - end the game (and keep his winnings), or
  - have another turn (and try to increase his winnings).
- If two or more red faces are rolled in a turn, all winnings are lost and the game ends.

(b) Show that, after a turn, the probability that Ted adds exactly $\$10$ to his winnings is $\frac{1}{3}$. [5]

The random variable D ($\$$) represents how much is added to his winnings after a turn.

The following table shows the distribution for D, where $\$w$ represents his winnings in the game so far.

D ($\$$)	$-w$	0	10	20	30
$\mathrm{P}(D=d)$	x	y	$\frac{1}{3}$	$\frac{2}{9}$	$\frac{1}{27}$

(c) (i) Write down the value of x.

(ii) Hence, find the value of y. [3]

Ted will always have another turn if he expects an increase to his winnings.

(d) Find the least value of w for which Ted should end the game instead of having another turn. [3]

Please **do not** write on this page.

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Mathematics
Standard level
Paper 2

Tuesday 14 May 2019 (morning)

Candidate session number

1 hour 30 minutes

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2219 – 7306

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Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines if necessary.

1. [Maximum mark: 6]

A group of 7 adult men wanted to see if there was a relationship between their Body Mass Index (BMI) and their waist size. Their waist sizes, in centimetres, were recorded and their BMI calculated. The following table shows the results.

Waist (x cm)	58	63	75	82	93	98	105
BMI (y)	19	20	22	23	25	24	26

The relationship between x and y can be modelled by the regression equation $y = ax + b$.

(a) (i) Write down the value of a and of b.

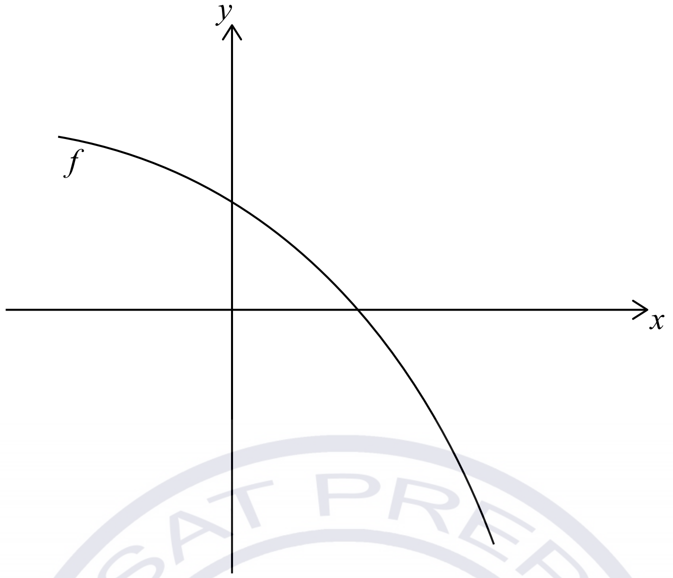
(ii) Find the correlation coefficient. [4]

(b) Use the regression equation to estimate the BMI of an adult man whose waist size is 95 cm. [2]

. . . .

2. [Maximum mark: 5]

Let $f(x) = 4 - 2e^x$. The following diagram shows part of the graph of f.

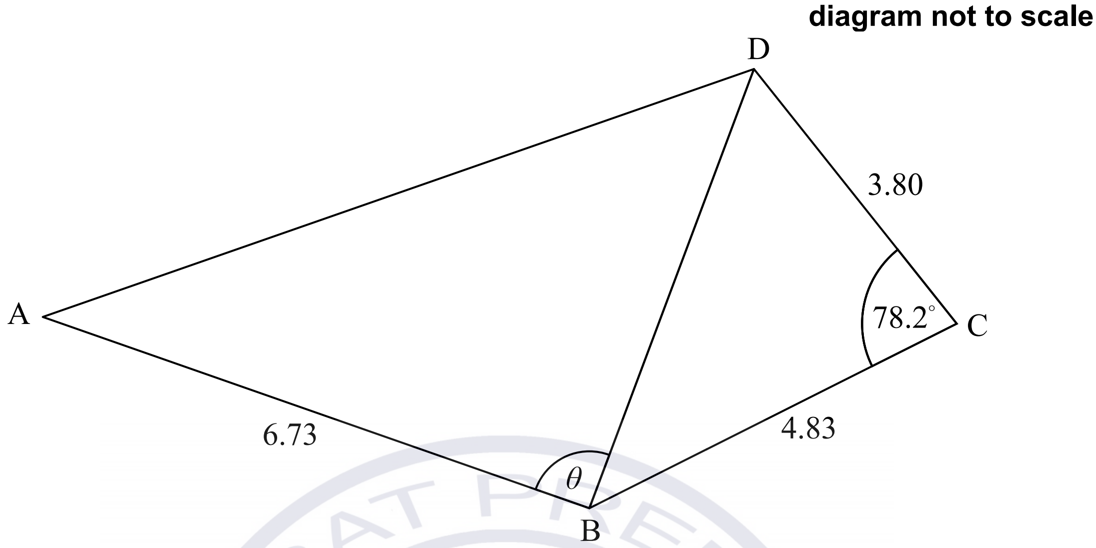


(a) Find the x-intercept of the graph of f. [2]

(b) The region enclosed by the graph of f, the x-axis and the y-axis is rotated 360° about the x-axis. Find the volume of the solid formed. [3]

3. [Maximum mark: 7]

The following diagram shows the quadrilateral ABCD.



$\mathrm{AB} = 6.73\,\mathrm{cm}$, $\mathrm{BC} = 4.83\,\mathrm{cm}$, $\mathrm{B\hat{C}D} = 78.2^\circ$ and $\mathrm{CD} = 3.80\,\mathrm{cm}$.

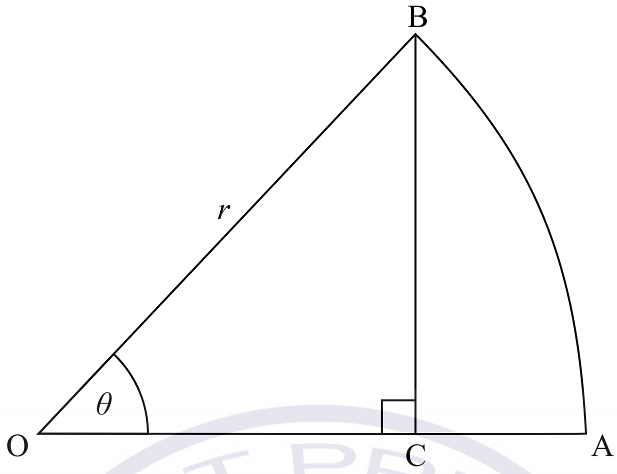
(a) Find BD. [3]

(b) The area of triangle ABD is $18.5\,\mathrm{cm}^2$. Find the possible values of θ. [4]

4. [Maximum mark: 7]

OAB is a sector of the circle with centre O and radius r, as shown in the following diagram.

diagram not to scale



The angle AOB is θ radians, where $0 < \theta < \frac{\pi}{2}$.

The point C lies on OA and OA is perpendicular to BC.

(a) Show that $OC = r\cos\theta$. [1]

(b) Find the area of triangle OBC in terms of r and θ. [2]

(c) Given that the area of triangle OBC is $\frac{3}{5}$ of the area of sector OAB, find θ. [4]

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5. [Maximum mark: 6]

The population of fish in a lake is modelled by the function

$$f(t) = \frac{1000}{1 + 24e^{-0.2t}},\ 0 \le t \le 30, \text{ where } t \text{ is measured in months.}$$

(a) Find the population of fish at $t = 10$. [2]

(b) Find the rate at which the population of fish is increasing at $t = 10$. [2]

(c) Find the value of t for which the population of fish is increasing most rapidly. [2]

6. [Maximum mark: 7]

In the expansion of the following expression, find the exact value of the constant term.

$$x^3\left(\frac{1}{2x}+x^2\right)^{15}$$

7. [Maximum mark: 6]

The vector equation of line L is given by $\boldsymbol{r} = \begin{pmatrix} -1 \\ 3 \\ 8 \end{pmatrix} + t\begin{pmatrix} 4 \\ 5 \\ -1 \end{pmatrix}$.

Point P is the point on L that is closest to the origin. Find the coordinates of P.

Do **not** write solutions on this page.

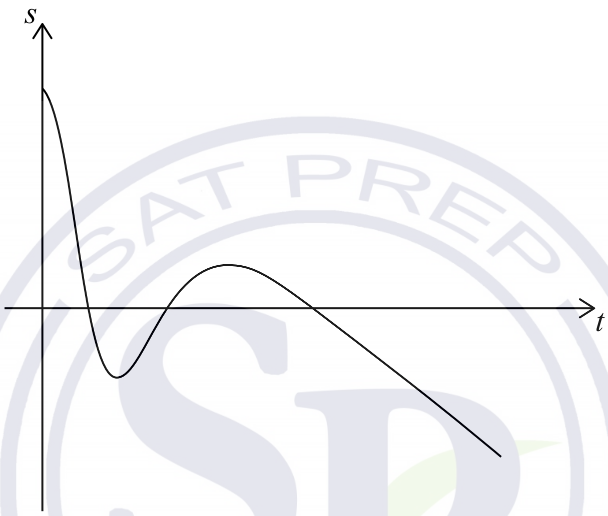
Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 16]

In this question distance is in centimetres and time is in seconds.

Particle A is moving along a straight line such that its displacement from a point P, after t seconds, is given by $s_A = 15 - t - 6t^3e^{-0.8t}$, $0 \le t \le 25$. This is shown in the following diagram.



(a) Find the initial displacement of particle A from point P. [2]

(b) Find the value of t when particle A first reaches point P. [2]

(c) Find the value of t when particle A first changes direction. [2]

(d) Find the total distance travelled by particle A in the first 3 seconds. [3]

Another particle, B, moves along the same line, starting at the same time as particle A. The velocity of particle B is given by $v_B = 8 - 2t$, $0 \le t \le 25$.

(e) (i) Given that particles A and B start at the same point, find the displacement function s_B for particle B.

(ii) Find the other value of t when particles A and B meet. [7]

Turn over

Do **not** write solutions on this page.

9. [Maximum mark: 14]

At Penna Airport the probability, $\mathrm{P}(A)$, that all passengers arrive on time for a flight is 0.70. The probability, $\mathrm{P}(D)$, that a flight departs on time is 0.85. The probability that all passengers arrive on time for a flight and it departs on time is 0.65.

(a) Show that event A and event D are **not** independent. [2]

(b) (i) Find $\mathrm{P}(A \cap D')$.

(ii) Given that all passengers for a flight arrive on time, find the probability that the flight does **not** depart on time. [5]

The number of hours that pilots fly per week is normally distributed with a mean of 25 hours and a standard deviation σ. 90% of pilots fly less than 28 hours in a week.

(c) Find the value of σ. [3]

(d) All flights have two pilots. Find the percentage of flights where **both** pilots flew more than 30 hours last week. [4]

Do **not** write solutions on this page.

10. [Maximum mark: 16]

In an arithmetic sequence, $u_1 = 1.3$, $u_2 = 1.4$ and $u_k = 31.2$.

(a) Find the value of k. [4]

(b) Find the exact value of S_k. [2]

Consider the terms, u_n, of this sequence such that $n \le k$.

Let F be the sum of the terms for which n is not a multiple of 3.

(c) Show that $F = 3240$. [5]

An infinite geometric series is given as $S_\infty = a + \frac{a}{\sqrt{2}} + \frac{a}{2} + \ldots,\ a \in \mathbb{Z}^+$.

(d) Find the largest value of a such that $S_\infty < F$. [5]

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Mathematics
Standard level
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8818 – 7302

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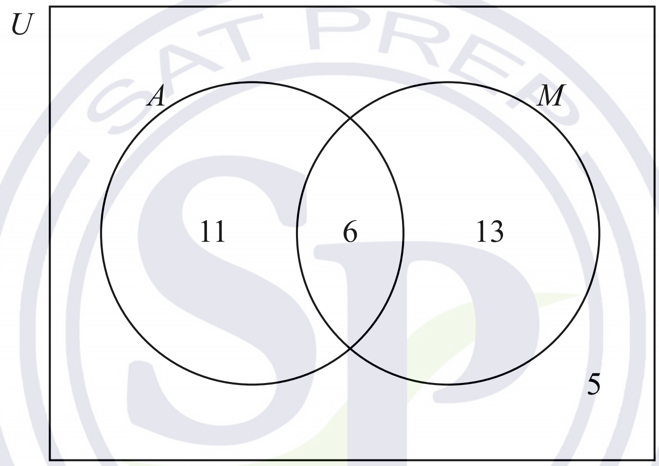
Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines if necessary.

1. [Maximum mark: 6]

In a group of 35 students, some take art class (A) and some take music class (M). 5 of these students do not take either class. This information is shown in the following Venn diagram.



(a) Write down the number of students in the group who take art class. [2]

(b) One student from the group is chosen at random. Find the probability that

(i) the student does not take art class;

(ii) the student takes either art class or music class, but not both. [4]

(This question continues on the following page)

(Question 1 continued)

2. [Maximum mark: 6]

The following table shows the hand lengths and the heights of five athletes on a sports team.

Hand length (x cm)	21.0	21.9	21.0	20.3	20.8
Height (y cm)	178.3	185.0	177.1	169.0	174.6

The relationship between x and y can be modelled by the regression line with equation $y = ax + b$.

(a) (i) Find the value of a and of b.

(ii) Write down the correlation coefficient. [4]

(b) Another athlete on this sports team has a hand length of 21.5 cm. Use the regression equation to estimate the height of this athlete. [2]

3. [Maximum mark: 7]

Let $f(x)=\frac{6x-1}{2x+3}$, for $x\neq -\frac{3}{2}$.

(a) For the graph of f,

(i) find the y-intercept;

(ii) find the equation of the vertical asymptote;

(iii) find the equation of the horizontal asymptote. [5]

(b) Hence or otherwise, write down $\lim_{x\to\infty}\left(\frac{6x-1}{2x+3}\right)$. [2]

4. [Maximum mark: 7]

A particle moves along a straight line so that its velocity, $v\ \mathrm{m\,s^{-1}}$, after t seconds is given by $v(t) = 1.4^t - 2.7$, for $0 \le t \le 5$.

(a) Find when the particle is at rest. [2]

(b) Find the acceleration of the particle when $t = 2$. [2]

(c) Find the total distance travelled by the particle. [3]

5. [Maximum mark: 6]

The sum of an infinite geometric sequence is 33.25. The second term of the sequence is 7.98. Find the possible values of r.

6. [Maximum mark: 7]

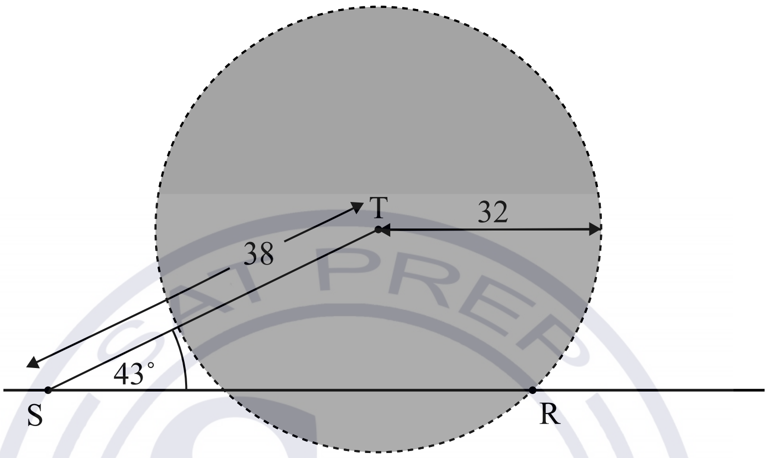
Consider the expansion of $\left(2x^4 + \frac{x^2}{k}\right)^{12}$, $k \neq 0$. The coefficient of the term in x^{40} is five times the coefficient of the term in x^{38}. Find k.

7. [Maximum mark: 6]

A communication tower, T, produces a signal that can reach cellular phones within a radius of 32 km. A straight road passes through the area covered by the tower's signal.

The following diagram shows a line representing the road and a circle representing the area covered by the tower's signal. Point R is on the circumference of the circle and points S and R are on the road. Point S is 38 km from the tower and $\text{R}\hat{\text{S}}\text{T} = 43^\circ$.

diagram not to scale



(a) Let $\text{SR} = x$. Use the cosine rule to show that $x^2 - (76\cos 43^\circ)\,x + 420 = 0$. [2]

(b) Hence or otherwise, find the total distance along the road where the signal from the tower can reach cellular phones. [4]

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Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 16]

Consider the points $\mathrm{A}(-3, 4, 2)$ and $\mathrm{B}(8, -1, 5)$.

(a) (i) Find $\overrightarrow{\mathrm{AB}}$.

(ii) Find $\left|\overrightarrow{\mathrm{AB}}\right|$. [4]

A line L has vector equation $r = \begin{pmatrix} 2 \\ 0 \\ -5 \end{pmatrix} + t\begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$. The point $\mathrm{C}(5, y, 1)$ lies on line L.

(b) (i) Find the value of y.

(ii) Show that $\overrightarrow{\mathrm{AC}} = \begin{pmatrix} 8 \\ -10 \\ -1 \end{pmatrix}$. [5]

(c) Find the angle between $\overrightarrow{\mathrm{AB}}$ and $\overrightarrow{\mathrm{AC}}$. [5]

(d) Find the area of triangle ABC. [2]

Do **not** write solutions on this page.

9. [Maximum mark: 15]

A nationwide study on reaction time is conducted on participants in two age groups. The participants in Group X are less than 40 years old. Their reaction times are normally distributed with mean 0.489 seconds and standard deviation 0.07 seconds.

(a) A person is selected at random from Group X. Find the probability that their reaction time is greater than 0.65 seconds. [2]

The participants in Group Y are 40 years or older. Their reaction times are normally distributed with mean 0.592 seconds and standard deviation σ seconds.

(b) The probability that the reaction time of a person in Group Y is greater than 0.65 seconds is 0.396. Find the value of σ. [4]

In the study, 38% of the participants are in Group X.

(c) A randomly selected participant has a reaction time greater than 0.65 seconds. Find the probability that the participant is in Group X. [6]

(d) Ten of the participants with reaction times greater than 0.65 are selected at random. Find the probability that at least two of them are in Group X. [3]

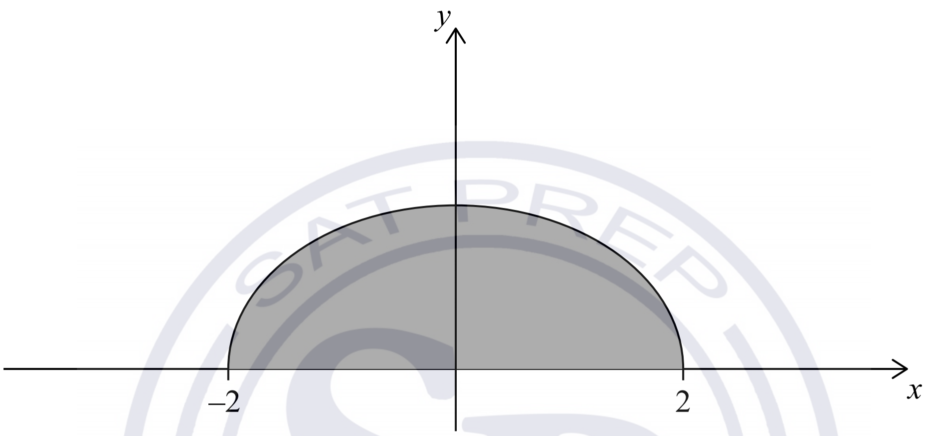
Do **not** write solutions on this page.

10. [Maximum mark: 14]

All lengths in this question are in metres.

Consider the function $f(x)=\sqrt{\frac{4-x^2}{8}}$, for $-2 \le x \le 2$. In the following diagram, the shaded region is enclosed by the graph of f and the x-axis.

diagram not to scale



A container can be modelled by rotating this region by 360° about the x-axis.

(a) Find the volume of the container. [3]

Water can flow in and out of the container.

The volume of water in the container is given by the function $g(t)$, for $0 \le t \le 4$, where t is measured in hours and $g(t)$ is measured in m^3. The rate of change of the volume of water in the container is given by $g'(t) = 0.9 - 2.5\cos(0.4t^2)$.

(b) The volume of water in the container is increasing only when $p < t < q$.

(i) Find the value of p and of q.

(ii) During the interval $p < t < q$, the volume of water in the container increases by $k\,m^3$. Find the value of k. [6]

When $t = 0$, the volume of water in the container is $2.3\,m^3$. It is known that the container is never completely full of water during the 4 hour period.

(c) Find the minimum volume of empty space in the container during the 4 hour period. [5]

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Standard level
Paper 2

Thursday 3 May 2018 (morning)

Candidate session number

1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics SL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[90 marks]**.

2218 – 7304

Please **do not** write on this page.

Answers written on this page will not be marked.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, for example if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines if necessary.

1. [Maximum mark: 5]

Let $f(x) = \ln x - 5x$, for $x > 0$.

(a) Find $f'(x)$. [2]

(b) Find $f''(x)$. [1]

(c) Solve $f'(x) = f''(x)$. [2]

2. [Maximum mark: 6]

A biased four-sided die is rolled. The following table gives the probability of each score.

Score	1	2	3	4
Probability	0.28	k	0.15	0.3

(a) Find the value of k. [2]

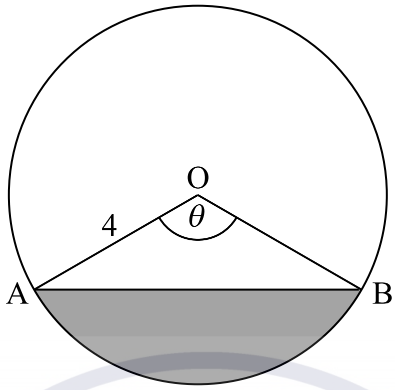
(b) Calculate the expected value of the score. [2]

(c) The die is rolled 80 times. On how many rolls would you expect to obtain a three? [2]

3. [Maximum mark: 6]

The diagram shows a circle, centre O, with radius $4\,\text{cm}$. Points A and B lie on the circumference of the circle and $\text{A}\hat{\text{O}}\text{B} = \theta$, where $0 \le \theta \le \pi$.

diagram not to scale



(a) Find the area of the shaded region, in terms of θ. [3]

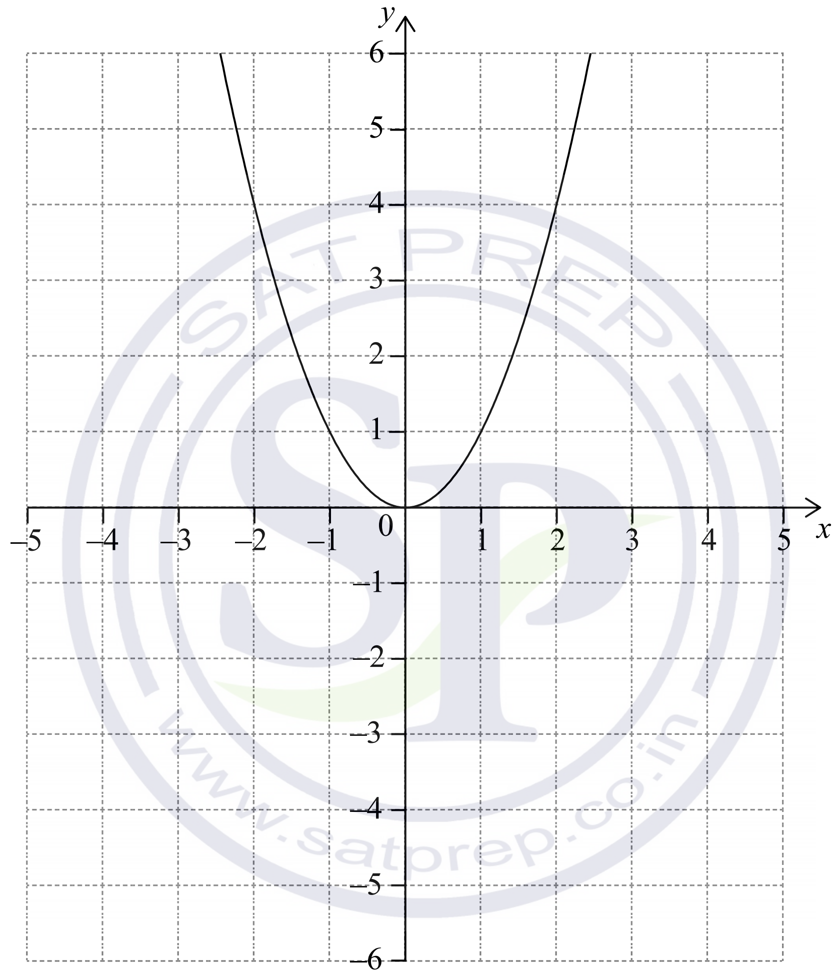
(b) The area of the shaded region is $12\,\text{cm}^2$. Find the value of θ. [3]

4. [Maximum mark: 7]

Let $g(x) = -(x-1)^2 + 5$.

(a) Write down the coordinates of the vertex of the graph of g. [1]

Let $f(x) = x^2$. The following diagram shows part of the graph of f.



The graph of g intersects the graph of f at $x = -1$ and $x = 2$.

(b) On the grid above, sketch the graph of g for $-2 \le x \le 4$. [3]

(c) Find the area of the region enclosed by the graphs of f and g. [3]

(This question continues on the following page)

(Question 4 continued)

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Answers written on this page
will not be marked.

5. [Maximum mark: 6]

Two events A and B are such that $\mathrm{P}(A) = 0.62$ and $\mathrm{P}(A \cap B) = 0.18$.

(a) Find $\mathrm{P}(A \cap B')$. [2]

(b) Given that $\mathrm{P}\left((A \cup B)'\right) = 0.19$, find $\mathrm{P}(A \mid B')$. [4]

6. [Maximum mark: 7]

Triangle ABC has $a = 8.1\,\text{cm}$, $b = 12.3\,\text{cm}$ and area $15\,\text{cm}^2$. Find the largest possible perimeter of triangle ABC.

7. [Maximum mark: 8]

Let $f(x) = e^{2\sin\left(\frac{\pi x}{2}\right)}$, for $x > 0$.

The kth maximum point on the graph of f has x-coordinate x_k where $k \in \mathbb{Z}^+$.

(a) Given that $x_{k+1} = x_k + a$, find a. [4]

(b) Hence find the value of n such that $\sum_{k=1}^{n} x_k = 861$. [4]

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Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 13]

The following table shows values of $\ln x$ and $\ln y$.

$\ln x$	1.10	2.08	4.30	6.03
$\ln y$	5.63	5.22	4.18	3.41

The relationship between $\ln x$ and $\ln y$ can be modelled by the regression equation $\ln y = a \ln x + b$.

(a) Find the value of a and of b. [3]

(b) Use the regression equation to estimate the value of y when $x = 3.57$. [3]

The relationship between x and y can be modelled using the formula $y = kx^n$, where $k \neq 0$, $n \neq 0$, $n \neq 1$.

(c) By expressing $\ln y$ in terms of $\ln x$, find the value of n and of k. [7]

Do **not** write solutions on this page.

9. [Maximum mark: 17]

The weights, in grams, of oranges grown in an orchard, are normally distributed with a mean of 297 g. It is known that 79% of the oranges weigh more than 289 g and 9.5% of the oranges weigh more than 310 g.

(a) Find the probability that an orange weighs between 289 g and 310 g. [2]

The weights of the oranges have a standard deviation of σ.

(b) (i) Find the standardized value for 289 g.

(ii) Hence, find the value of σ. [5]

The grocer at a local grocery store will buy the oranges whose weights exceed the 35th percentile.

(c) To the nearest gram, find the minimum weight of an orange that the grocer will buy. [3]

The orchard packs oranges in boxes of 36.

(d) Find the probability that the grocer buys more than half the oranges in a box selected at random. [5]

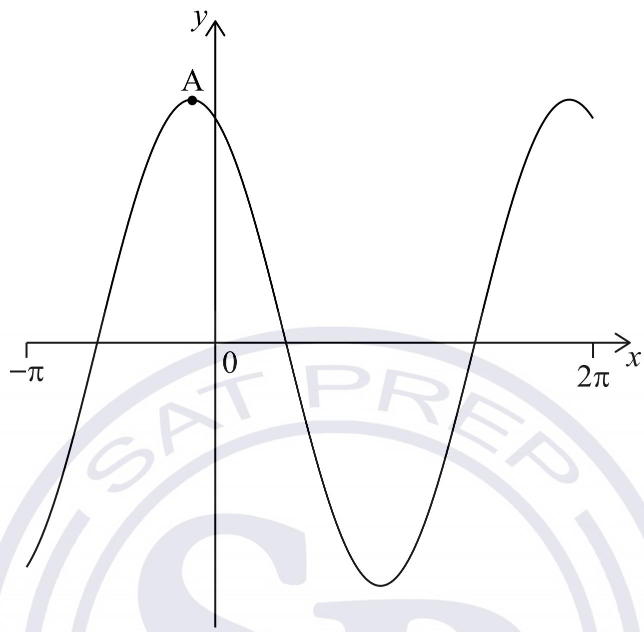
The grocer selects two boxes at random.

(e) Find the probability that the grocer buys more than half the oranges in each box. [2]

Do **not** write solutions on this page.

10. [Maximum mark: 15]

Let $f(x) = 12\cos x - 5\sin x$, $-\pi \le x \le 2\pi$, be a periodic function with $f(x) = f(x + 2\pi)$. The following diagram shows the graph of f.



There is a maximum point at A. The minimum value of f is -13.

(a) Find the coordinates of A. [2]

(b) For the graph of f, write down

(i) the amplitude;

(ii) the period. [2]

(c) Hence, write $f(x)$ in the form $p\cos(x + r)$. [3]

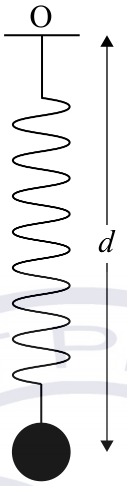
(This question continues on the following page)

Do **not** write solutions on this page.

(Question 10 continued)

A ball on a spring is attached to a fixed point O. The ball is then pulled down and released, so that it moves back and forth vertically.

diagram not to scale



The distance, d centimetres, of the centre of the ball from O at time t seconds, is given by

$$d(t) = f(t) + 17, \ 0 \le t \le 5.$$

(d) Find the maximum speed of the ball. [3]

(e) Find the first time when the ball's speed is changing at a rate of $2\,\text{cm}\,\text{s}^{-2}$. [5]

Please **do not** write on this page.

Answers written on this page
will not be marked.

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Standard level
Paper 2

Thursday 3 May 2018 (morning)

Candidate session number

1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics SL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[90 marks]**.

2218 – 7306

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, for example if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines if necessary.

1. [Maximum mark: 6]

The following table shows the mean weight, y kg, of children who are x years old.

Age (x years)	1.25	2.25	3.5	4.4	5.85
Weight (y kg)	10	13	14	17	19

The relationship between the variables is modelled by the regression line with equation $y = ax + b$.

(a) (i) Find the value of a and of b.

(ii) Write down the correlation coefficient. [4]

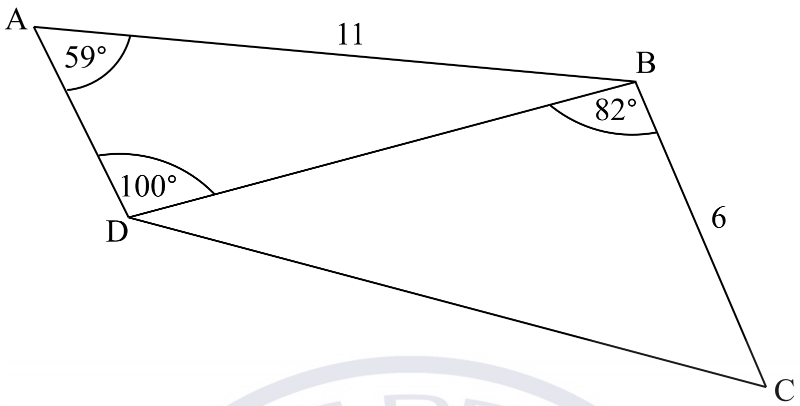
(b) Use your equation to estimate the mean weight of a child that is 1.95 years old. [2]

. . . .

2. [Maximum mark: 6]

The following diagram shows quadrilateral ABCD.

diagram not to scale



$AB = 11\text{ cm}$, $BC = 6\text{ cm}$, $B\hat{A}D = 59^\circ$, $A\hat{D}B = 100^\circ$, and $C\hat{B}D = 82^\circ$

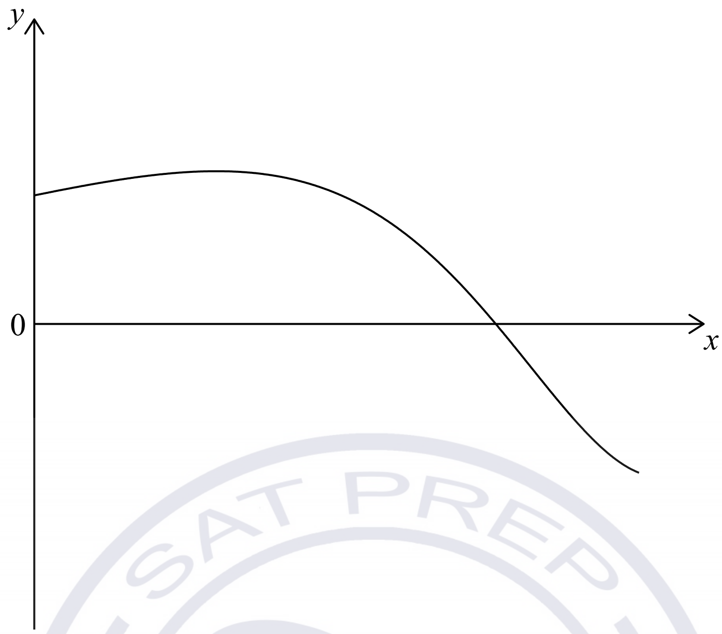
(a) Find DB. [3]

(b) Find DC. [3]

Turn over

3. [Maximum mark: 5]

Let $f(x) = \sin(e^x)$ for $0 \le x \le 1.5$. The following diagram shows the graph of f.



(a) Find the x-intercept of the graph of f. [2]

(b) The region enclosed by the graph of f, the y-axis and the x-axis is rotated 360° about the x-axis.

Find the volume of the solid formed. [3]

4. [Maximum mark: 7]

The first term of an infinite geometric sequence is 4. The sum of the infinite sequence is 200.

(a) Find the common ratio. [2]

(b) Find the sum of the first 8 terms. [2]

(c) Find the least value of n for which $S_n > 163$. [3]

Please **do not** write on this page.

Answers written on this page
will not be marked.

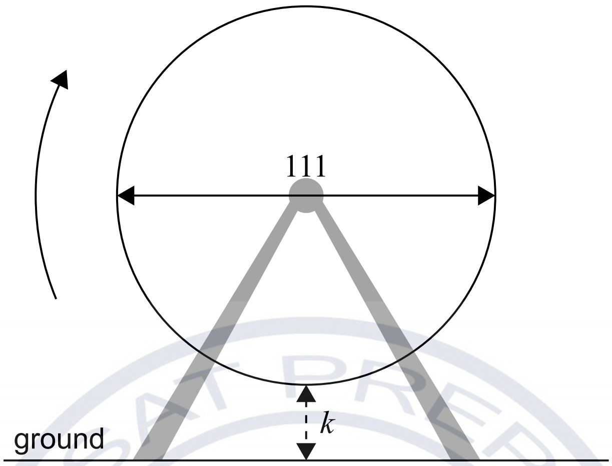
5. [Maximum mark: 6]

Consider the expansion of $\left(2x+\frac{k}{x}\right)^9$, where $k>0$. The coefficient of the term in x^3 is equal to the coefficient of the term in x^5. Find k.

6. [Maximum mark: 8]

At an amusement park, a Ferris wheel with diameter 111 metres rotates at a constant speed. The bottom of the wheel is k metres above the ground. A seat starts at the bottom of the wheel.

diagram not to scale



The wheel completes one revolution in 16 minutes.

(a) After 8 minutes, the seat is $117\,\text{m}$ above the ground. Find k. [2]

After t minutes, the height of the seat above ground is given by $h(t) = 61.5 + a\cos\left(\frac{\pi}{8}t\right)$, for $0 \le t \le 32$.

(b) Find the value of a. [3]

(c) Find when the seat is $30\,\text{m}$ above the ground for the third time. [3]

(This question continues on the following page)

(Question 6 continued)

7. [Maximum mark: 7]

Let $f(x) = \dfrac{8x-5}{cx+6}$ for $x \neq -\dfrac{6}{c}$, $c \neq 0$.

(a) The line $x = 3$ is a vertical asymptote to the graph of f. Find the value of c. [2]

(b) Write down the equation of the horizontal asymptote to the graph of f. [2]

(c) The line $y = k$, where $k \in \mathbb{R}$ intersects the graph of $|f(x)|$ at exactly one point. Find the possible values of k. [3]

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 13]

Two points P and Q have coordinates $(3, 2, 5)$ and $(7, 4, 9)$ respectively.

(a) (i) Find $\overrightarrow{PQ}$.

(ii) Find $\left|\overrightarrow{PQ}\right|$. [4]

Let $\overrightarrow{PR} = 6\boldsymbol{i} - \boldsymbol{j} + 3\boldsymbol{k}$.

(b) Find the angle between PQ and PR. [4]

(c) Find the area of triangle PQR. [2]

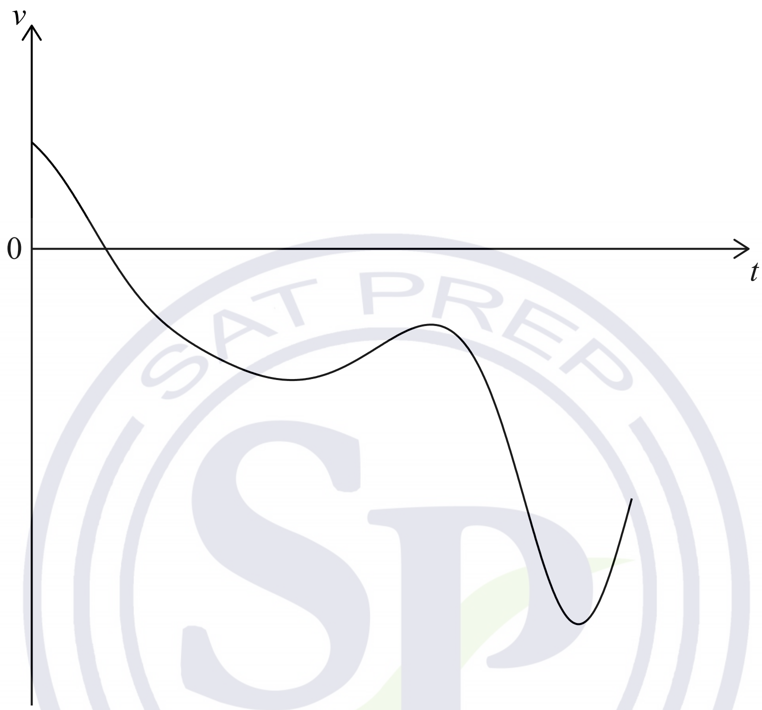
(d) Hence or otherwise find the shortest distance from R to the line through P and Q. [3]

Do **not** write solutions on this page.

9. [Maximum mark: 15]

A particle P moves along a straight line. The velocity $v\,\mathrm{m\,s^{-1}}$ of P after t seconds is given by $v(t) = 7\cos t - 5t^{\cos t}$, for $0 \le t \le 7$.

The following diagram shows the graph of v.



(a) Find the initial velocity of P. [2]

(b) Find the maximum speed of P. [3]

(c) Write down the number of times that the acceleration of P is $0\,\mathrm{m\,s^{-2}}$. [3]

(d) Find the acceleration of P when it changes direction. [4]

(e) Find the total distance travelled by P. [3]

Do **not** write solutions on this page.

10. [Maximum mark: 17]

The mass M of apples in grams is normally distributed with mean μ. The following table shows probabilities for values of M.

Values of M	$M < 93$	$93 \le M \le 119$	$M > 119$
$\mathbf{P(X)}$	k	0.98	0.01

(a) (i) Write down the value of k.

(ii) Show that $\mu = 106$. [4]

(b) Find $P(M < 95)$. [5]

The apples are packed in bags of ten.

Any apples with a mass less than 95 g are classified as small.

(c) Find the probability that a bag of apples selected at random contains at most one small apple. [3]

(d) A crate contains 50 bags of apples. A crate is selected at random.

(i) Find the expected number of bags in this crate that contain at most one small apple.

(ii) Find the probability that at least 48 bags in this crate contain at most one small apple. [5]

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will not be marked.

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Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Standard level
Paper 2

Tuesday 14 November 2017 (morning)

Candidate session number

1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics SL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[90 marks]**.

8817 – 7302

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Answers written on this page
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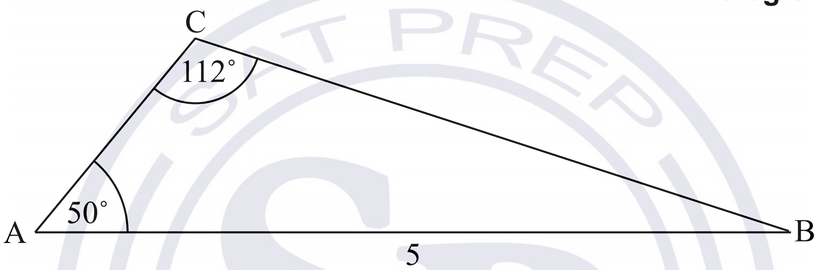
Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines if necessary.

1. [Maximum mark: 6]

The following diagram shows a triangle ABC.

diagram not to scale



$AB = 5\,cm$, $C\hat{A}B = 50°$ and $A\hat{C}B = 112°$

(a) Find BC. [3]

(b) Find the area of triangle ABC. [3]

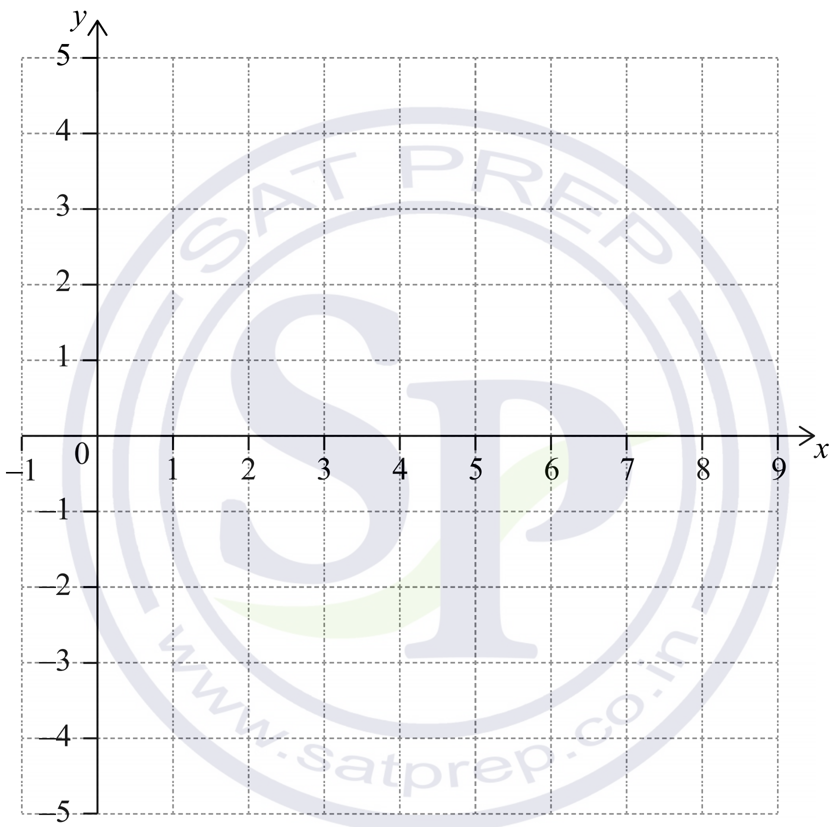
2. [Maximum mark: 7]

Let $f(x)=\dfrac{6x^2-4}{e^x}$, for $0 \le x \le 7$.

(a) Find the x-intercept of the graph of f. [2]

(b) The graph of f has a maximum at the point A. Write down the coordinates of A. [2]

(c) On the following grid, sketch the graph of f. [3]



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3. [Maximum mark: 6]

Let $\overrightarrow{AB} = \begin{pmatrix} 4 \\ 1 \\ 2 \end{pmatrix}$.

(a) Find $\left|\overrightarrow{AB}\right|$. [2]

(b) Let $\overrightarrow{AC} = \begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix}$. Find $B\hat{A}C$. [4]

4. [Maximum mark: 8]

A discrete random variable X has the following probability distribution.

X	0	1	2	3
$\mathrm{P}(X=x)$	0.475	$2k^2$	$\frac{k}{10}$	$6k^2$

(a) Find the value of k. [4]

(b) Write down $\mathrm{P}(X=2)$. [1]

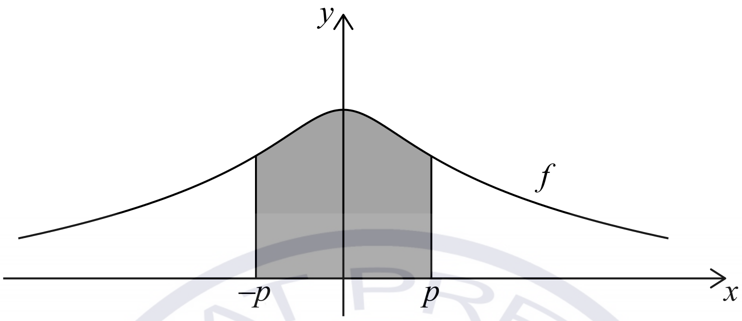
(c) Find $\mathrm{P}(X=2 \mid X>0)$. [3]

5. [Maximum mark: 5]

Let $f(x) = 6 - \ln(x^2 + 2)$, for $x \in \mathbb{R}$. The graph of f passes through the point $(p, 4)$, where $p > 0$.

(a) Find the value of p. [2]

(b) The following diagram shows part of the graph of f.



The region enclosed by the graph of f, the x-axis and the lines $x = -p$ and $x = p$ is rotated 360° about the x-axis. Find the volume of the solid formed. [3]

6. [Maximum mark: 6]

In the expansion of $ax^3(2+ax)^{11}$, the coefficient of the term in x^5 is 11880. Find the value of a.

7. [Maximum mark: 7]

The heights of adult males in a country are normally distributed with a mean of 180 cm and a standard deviation of $\sigma\text{ cm}$. 17% of these men are shorter than 168 cm. 80% of them have heights between $(192 - h)\text{ cm}$ and 192 cm.

Find the value of h.

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 14]

Adam is a beekeeper who collected data about monthly honey production in his bee hives. The data for six of his hives is shown in the following table.

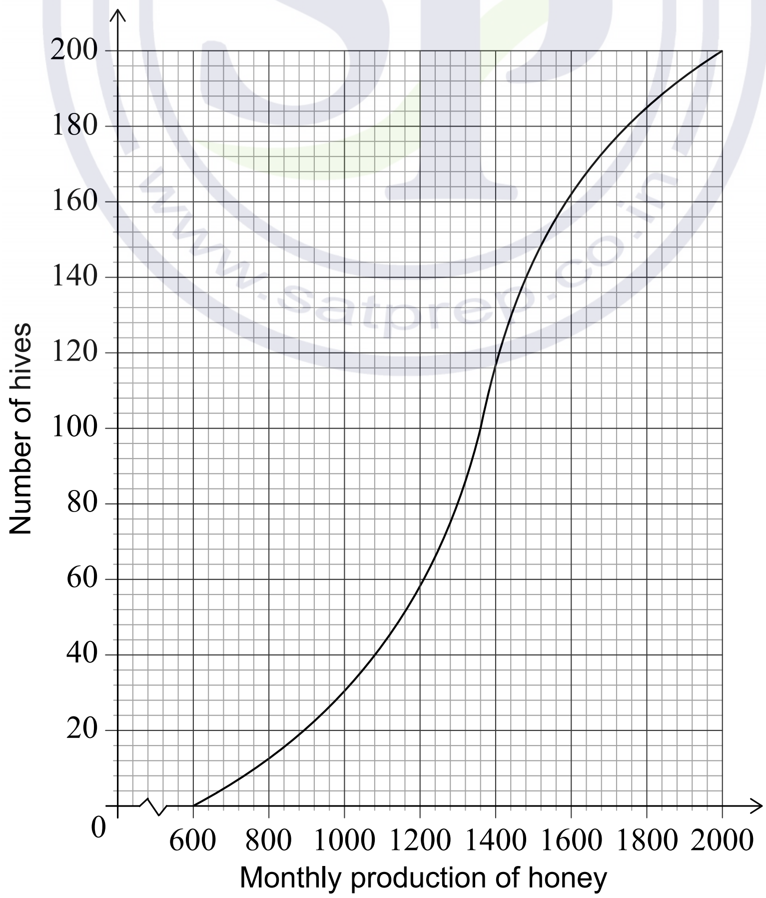
Number of bees (N)	190	220	250	285	305	320
Monthly honey production in grams (P)	900	1100	1200	1500	1700	1800

The relationship between the variables is modelled by the regression line with equation $P = aN + b$.

(a) Write down the value of a and of b. [3]

(b) Use this regression line to estimate the monthly honey production from a hive that has 270 bees. [2]

Adam has 200 hives in total. He collects data on the monthly honey production of all the hives. This data is shown in the following cumulative frequency graph.



(This question continues on the following page)

Do **not** write solutions on this page.

(Question 8 continued)

Adam's hives are labelled as low, regular or high production, as defined in the following table.

Type of hive	low	regular	high
Monthly honey production in grams (P)	$P \le 1080$	$1080 < P \le k$	$P > k$

(c) Write down the number of low production hives. [1]

Adam knows that 128 of his hives have a regular production.

(d) Find

(i) the value of k;

(ii) the number of hives that have a high production. [5]

(e) Adam decides to increase the number of bees in each low production hive. Research suggests that there is a probability of 0.75 that a low production hive becomes a regular production hive. Calculate the probability that 30 low production hives become regular production hives. [3]

Please **do not** write on this page.

Answers written on this page
will not be marked.

Do **not** write solutions on this page.

9. [Maximum mark: 14]

Note: In this question, distance is in metres and time is in seconds.

A particle P moves in a straight line for five seconds. Its acceleration at time t is given by $a = 3t^2 - 14t + 8$, for $0 \le t \le 5$.

(a) Write down the values of t when $a = 0$. [2]

(b) Hence or otherwise, find all possible values of t for which the velocity of P is decreasing. [2]

When $t = 0$, the velocity of P is $3\,\text{m s}^{-1}$.

(c) Find an expression for the velocity of P at time t. [6]

(d) Find the total distance travelled by P when its velocity is increasing. [4]

Do **not** write solutions on this page.

10. [Maximum mark: 17]

Note: In this question, distance is in millimetres.

Let $f(x)=x+a\sin\left(x-\frac{\pi}{2}\right)+a$, for $x\geq 0$.

(a) Show that $f(2\pi)=2\pi$. [3]

The graph of f passes through the origin. Let P_k be any point on the graph of f with x-coordinate $2k\pi$, where $k\in\mathbb{N}$. A straight line L passes through all the points P_k.

(b) (i) Find the coordinates of P_0 and of P_1.

(ii) Find the equation of L. [6]

(c) Show that the distance between the x-coordinates of P_k and P_{k+1} is 2π. [2]

(This question continues on the following page)

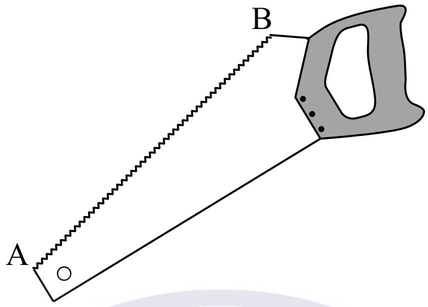
Do **not** write solutions on this page.

(Question 10 continued)

Diagram 1 shows a saw. The length of the toothed edge is the distance AB.

Diagram 1

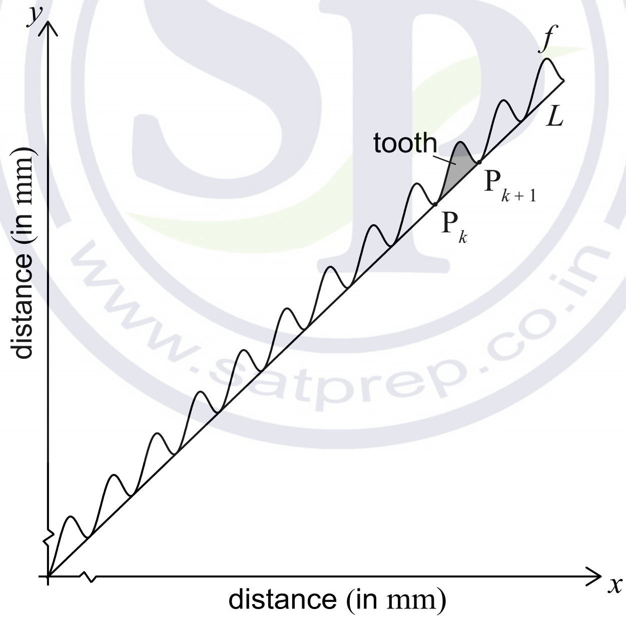
diagram not to scale



The toothed edge of the saw can be modelled using the graph of f and the line L.
Diagram 2 represents this model.

Diagram 2

diagram not to scale



The shaded part on the graph is called a tooth. A tooth is represented by the region enclosed by the graph of f and the line L, between P_k and P_{k+1}.

(d) A saw has a toothed edge which is 300 mm long. Find the number of complete teeth on this saw. [6]

Please **do not** write on this page.

Answers written on this page
will not be marked.

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Standard level
Paper 2

Friday 5 May 2017 (morning)

1 hour 30 minutes

Candidate session number

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics SL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[90 marks]**.

2217 – 7304

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, for example if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines if necessary.

1. [Maximum mark: 7]

Consider the following frequency table.

x	Frequency
2	8
4	15
7	21
10	28
11	3

(a) (i) Write down the mode.

(ii) Find the value of the range. [3]

(b) (i) Find the mean.

(ii) Find the variance. [4]

2. [Maximum mark: 6]

Let $\boldsymbol{v} = \begin{pmatrix} -10 \\ 2 \\ 1 \end{pmatrix}$ and $\boldsymbol{w} = \begin{pmatrix} 3 \\ -4 \\ 0 \end{pmatrix}$. Find the angle between $\boldsymbol{v}$ and $\boldsymbol{w}$, giving your answer correct to one decimal place.

3. [Maximum mark: 6]

Consider the graph of $f(x)=\dfrac{e^x}{5x-10}+3$, for $x \neq 2$.

(a) Find the y-intercept. [2]

(b) Find the equation of the vertical asymptote. [2]

(c) Find the minimum value of $f(x)$ for $x > 2$. [2]

4. [Maximum mark: 6]

In a large university the probability that a student is left handed is 0.08. A sample of 150 students is randomly selected from the university. Let k be the expected number of left-handed students in this sample.

(a) Find k. [2]

(b) Hence, find the probability that

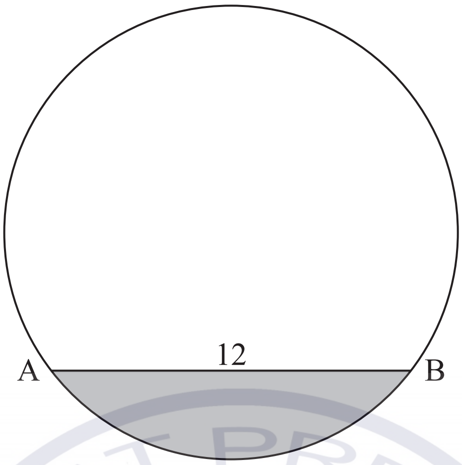
(i) exactly k students are left handed;

(ii) fewer than k students are left handed. [4]

5. [Maximum mark: 7]

The following diagram shows the chord [AB] in a circle of radius 8 cm, where $\text{AB} = 12\text{ cm}$.

diagram not to scale



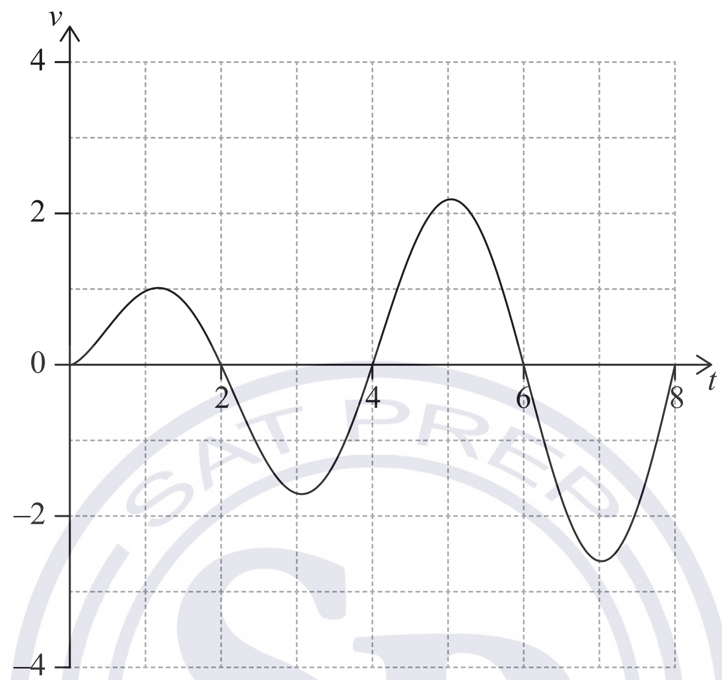
Find the area of the shaded segment.

6. [Maximum mark: 7]

Let $f(x) = (x^2 + 3)^7$. Find the term in x^5 in the expansion of the derivative, $f'(x)$.

7. [Maximum mark: 7]

A particle P moves along a straight line. Its velocity $v_{\mathrm{P}}\ \mathrm{m\,s^{-1}}$ after t seconds is given by $v_{\mathrm{P}} = \sqrt{t}\sin\left(\frac{\pi}{2}t\right)$, for $0 \le t \le 8$. The following diagram shows the graph of v_{P}.



(a) (i) Write down the first value of t at which P changes direction.

(ii) Find the **total** distance travelled by P, for $0 \le t \le 8$. [3]

(b) A second particle Q also moves along a straight line. Its velocity, $v_{\mathrm{Q}}\ \mathrm{m\,s^{-1}}$ after t seconds is given by $v_{\mathrm{Q}} = \sqrt{t}$ for $0 \le t \le 8$. After k seconds Q has travelled the same total distance as P.

Find k. [4]

(This question continues on the following page)

(Question 7 continued)

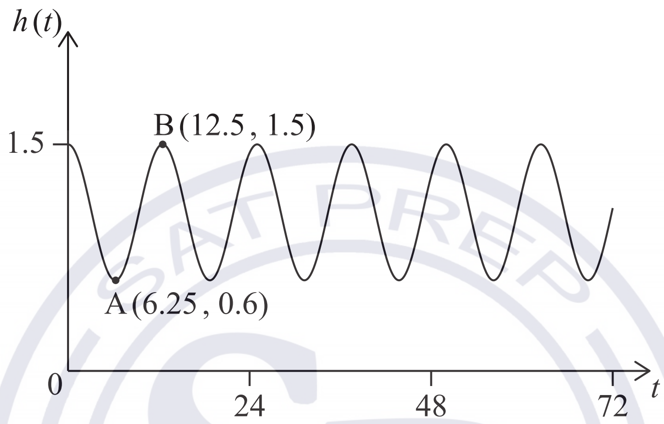
Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 14]

At Grande Anse Beach the height of the water in metres is modelled by the function $h(t) = p\cos(q \times t) + r$, where t is the number of hours after $21{:}00$ hours on 10 December 2017. The following diagram shows the graph of h, for $0 \le t \le 72$.



The point $\mathrm{A}(6.25, 0.6)$ represents the first low tide and $\mathrm{B}(12.5, 1.5)$ represents the next high tide.

(a) (i) How much time is there between the first low tide and the next high tide?

(ii) Find the difference in height between low tide and high tide. [4]

(b) Find the value of

(i) p;

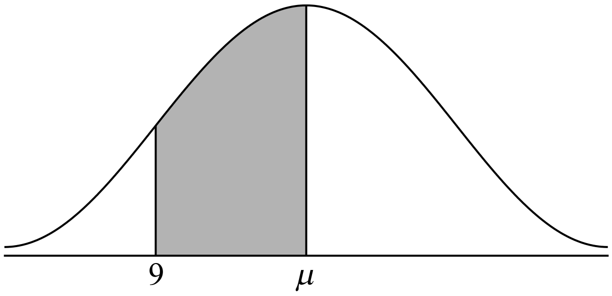
(ii) q;

(iii) r. [7]

(c) There are two high tides on 12 December 2017. At what time does the second high tide occur? [3]

9. [Maximum mark: 15]

A random variable X is normally distributed with mean, μ. In the following diagram, the shaded region between 9 and μ represents 30% of the distribution.



(a) Find $P(X < 9)$. [2]

The standard deviation of X is 2.1.

(b) Find the value of μ. [3]

The random variable Y is normally distributed with mean λ and standard deviation 3.5. The events $X > 9$ and $Y > 9$ are independent, and $P\left((X > 9) \cap (Y > 9)\right) = 0.4$.

(c) Find λ. [5]

(d) Given that $Y > 9$, find $P(Y < 13)$. [5]

10. [Maximum mark: 15]

Let $f(x) = \ln x$ and $g(x) = 3 + \ln\left(\frac{x}{2}\right)$, for $x > 0$.

The graph of g can be obtained from the graph of f by two transformations:

a horizontal stretch of scale factor q followed by

a translation of $\begin{pmatrix} h \\ k \end{pmatrix}$.

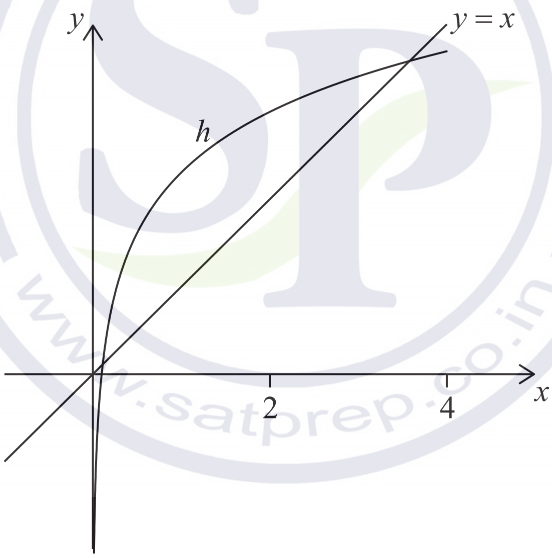
(a) Write down the value of

(i) q;

(ii) h;

(iii) k. [3]

Let $h(x) = g(x) \times \cos(0.1x)$, for $0 < x < 4$. The following diagram shows the graph of h and the line $y = x$.



The graph of h intersects the graph of h^{-1} at two points. These points have x coordinates 0.111 and 3.31, correct to three significant figures.

(b) (i) Find $\int_{0.111}^{3.31} (h(x) - x)\,dx$.

(ii) Hence, find the area of the region enclosed by the graphs of h and h^{-1}. [5]

(c) Let d be the vertical distance from a point on the graph of h to the line $y = x$. There is a point $P(a, b)$ on the graph of h where d is a maximum.
Find the coordinates of P, where $0.111 < a < 3.31$. [7]

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Standard level
Paper 2

Friday 5 May 2017 (morning)

1 hour 30 minutes

Candidate session number

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics SL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[90 marks]**.

2217 – 7306

14 pages

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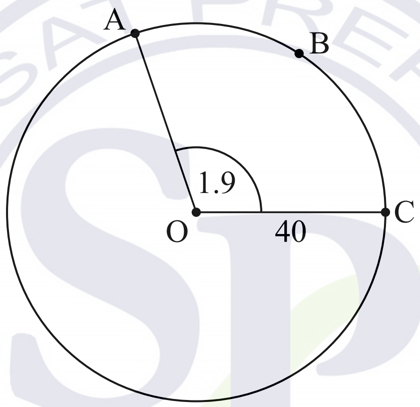
Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines if necessary.

1. [Maximum mark: 6]

The following diagram shows a circle with centre O and radius 40 cm.

diagram not to scale



The points A, B and C are on the circumference of the circle and $A\hat{O}C = 1.9$ radians .

(a) Find the length of arc ABC. [2]

(b) Find the perimeter of sector OABC. [2]

(c) Find the area of sector OABC. [2]

(This question continues on the following page)

(Question 1 continued)

2. [Maximum mark: 7]

The maximum temperature T, in degrees Celsius, in a park on six randomly selected days is shown in the following table. The table also shows the number of visitors, N, to the park on each of those six days.

Maximum temperature (T)	4	5	17	31	29	11
Number of visitors (N)	24	26	36	38	46	28

The relationship between the variables can be modelled by the regression equation $N = aT + b$.

(a) (i) Find the value of a and of b.

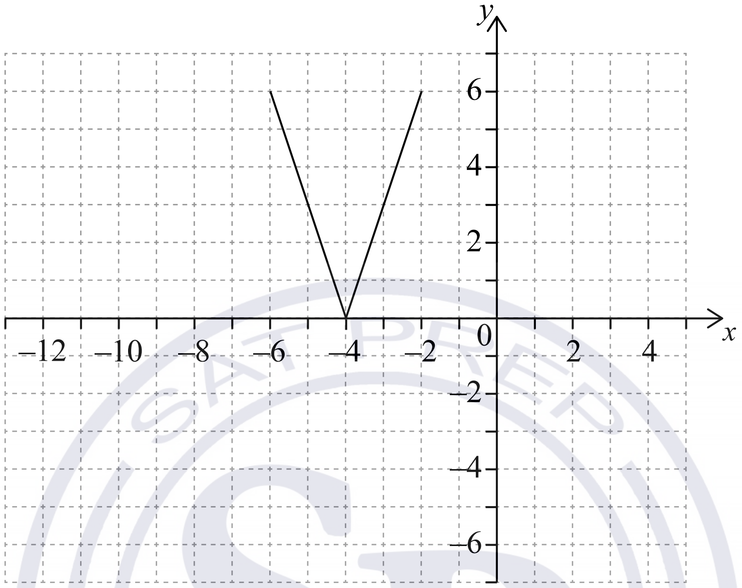
(ii) Write down the value of r. [4]

(b) Use the regression equation to estimate the number of visitors on a day when the maximum temperature is $15\,^{\circ}\mathrm{C}$. [3]

3. [Maximum mark: 6]

The following diagram shows the graph of a function $y = f(x)$, for $-6 \le x \le -2$.
The points $(-6, 6)$ and $(-2, 6)$ lie on the graph of f. There is a minimum point at $(-4, 0)$.

(a) Write down the range of f. [2]



Let $g(x) = f(x-5)$.

(b) On the grid above, sketch the graph of g. [2]

(c) Write down the domain of g. [2]

Turn over

4. [Maximum mark: 6]

The depth of water in a port is modelled by the function $d(t) = p\cos qt + 7.5$, for $0 \le t \le 12$, where t is the number of hours after high tide.

At high tide, the depth is 9.7 metres.

At low tide, which is 7 hours later, the depth is 5.3 metres.

(a) Find the value of p. [2]

(b) Find the value of q. [2]

(c) Use the model to find the depth of the water 10 hours after high tide. [2]

5. [Maximum mark: 6]

Consider a geometric sequence where the first term is 768 and the second term is 576.

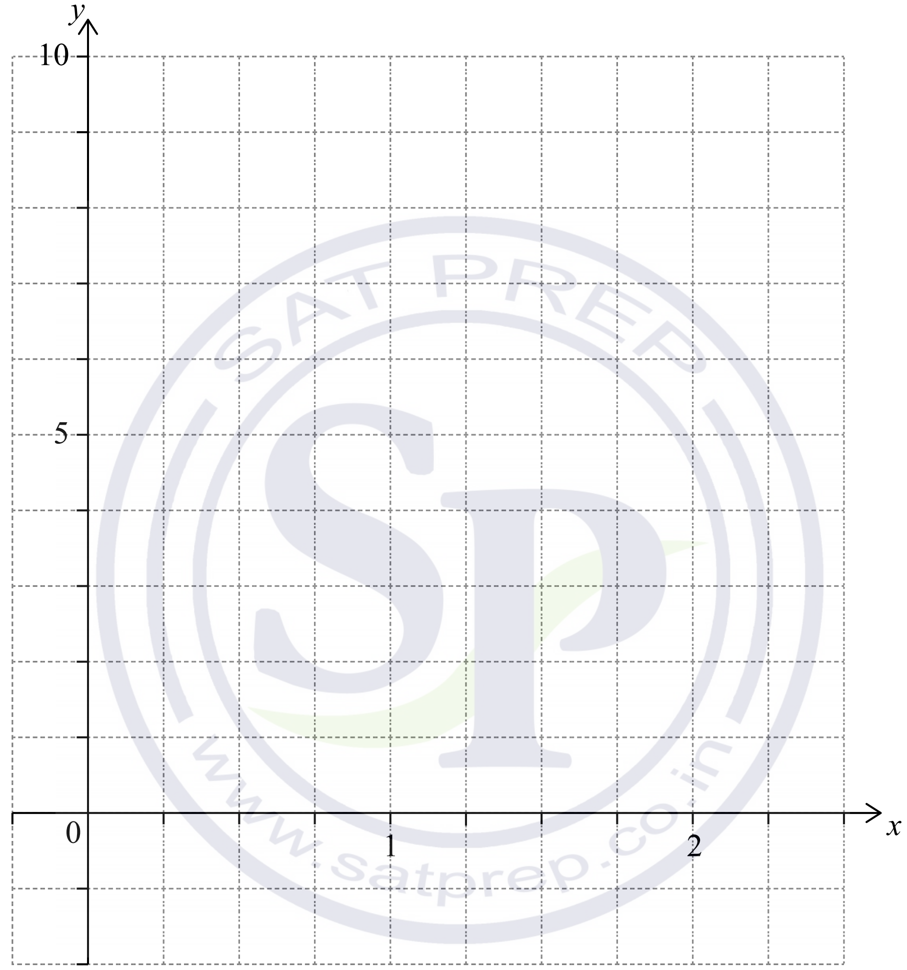
Find the least value of n such that the nth term of the sequence is less than 7.

6. [Maximum mark: 8]

Let $f(x) = x^2 - 1$ and $g(x) = x^2 - 2$, for $x \in \mathbb{R}$.

(a) Show that $(f \circ g)(x) = x^4 - 4x^2 + 3$. [2]

(b) On the following grid, sketch the graph of $(f \circ g)(x)$, for $0 \le x \le 2.25$. [3]



(c) The equation $(f \circ g)(x) = k$ has exactly two solutions, for $0 \le x \le 2.25$. Find the possible values of k. [3]

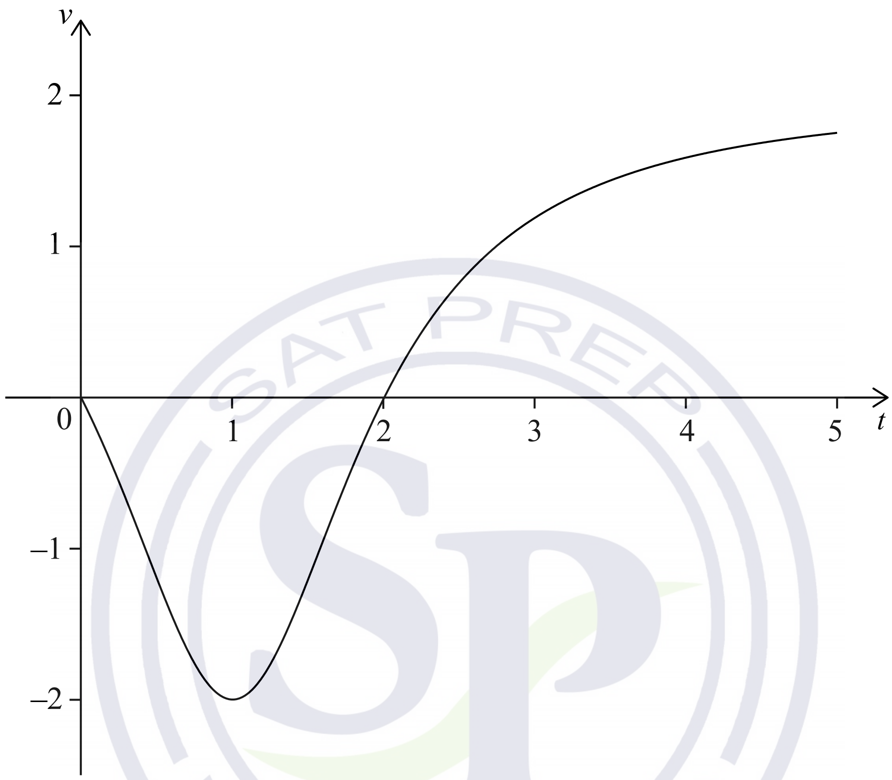
(This question continues on the following page)

(Question 6 continued)

7. [Maximum mark: 6]

Note: In this question, distance is in metres and time is in seconds.

A particle moves along a horizontal line starting at a fixed point A. The velocity v of the particle, at time t, is given by $v(t)=\dfrac{2t^2-4t}{t^2-2t+2}$, for $0 \le t \le 5$. The following diagram shows the graph of v



There are t-intercepts at $(0, 0)$ and $(2, 0)$.

Find the maximum distance of the particle from A during the time $0 \le t \le 5$ and justify your answer.

(This question continues on the following page)

(Question 7 continued)

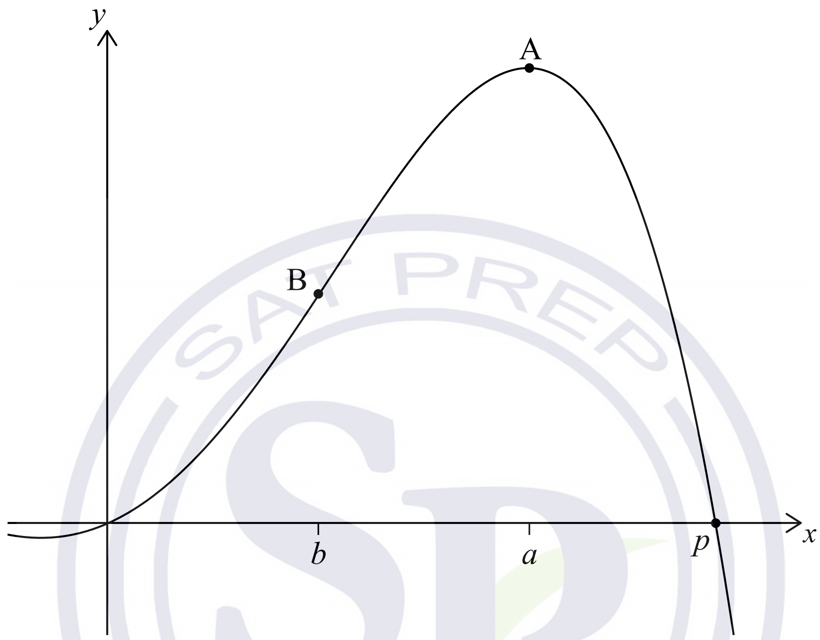
Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 15]

Let $f(x) = -0.5x^4 + 3x^2 + 2x$. The following diagram shows part of the graph of f.



There are x-intercepts at $x = 0$ and at $x = p$. There is a maximum at A where $x = a$, and a point of inflexion at B where $x = b$.

(a) Find the value of p. [2]

(b) (i) Write down the coordinates of A.

(ii) Write down the rate of change of f at A. [3]

(c) (i) Find the coordinates of B.

(ii) Find the the rate of change of f at B. [7]

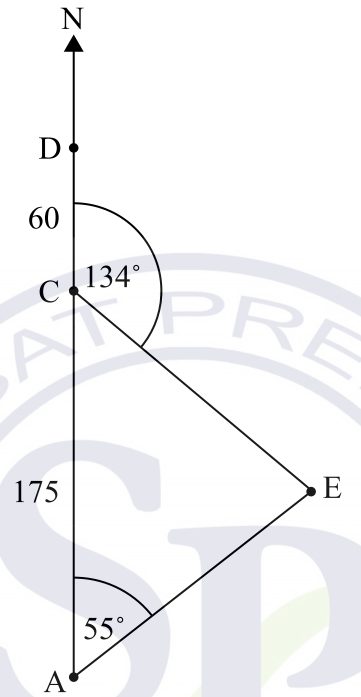
(d) Let R be the region enclosed by the graph of f, the x-axis, the line $x = b$ and the line $x = a$. The region R is rotated 360° about the x-axis. Find the volume of the solid formed. [3]

Do **not** write solutions on this page.

9. [Maximum mark: 15]

A ship is sailing north from a point A towards point D. Point C is $175\,\text{km}$ north of A. Point D is $60\,\text{km}$ north of C. There is an island at E. The bearing of E from A is 055°. The bearing of E from C is 134°. This is shown in the following diagram.

diagram not to scale



(a) Find the bearing of A from E. [2]

(b) Find CE. [5]

(c) Find DE. [3]

(d) When the ship reaches D, it changes direction and travels directly to the island at $50\,\text{km}$ per hour. At the same time as the ship changes direction, a boat starts travelling to the island from a point B. This point B lies on (AC), between A and C, and is the closest point to the island. The ship and the boat arrive at the island at the same time. Find the speed of the boat. [5]

Do **not** write solutions on this page.

10. [Maximum mark: 15]

The following table shows a probability distribution for the random variable X, where $\mathrm{E}(X) = 1.2$.

x	0	1	2	3
$\mathrm{P}(X = x)$	p	$\frac{1}{2}$	$\frac{3}{10}$	q

(a) (i) Find q.

(ii) Find p. [4]

A bag contains white and blue marbles, with at least three of each colour. Three marbles are drawn from the bag, without replacement. The number of blue marbles drawn is given by the random variable X.

(b) (i) Write down the probability of drawing three blue marbles.

(ii) Explain why the probability of drawing three white marbles is $\frac{1}{6}$.

(iii) The bag contains a total of ten marbles of which w are white. Find w. [5]

A game is played in which three marbles are drawn from the bag of ten marbles, without replacement. A player wins a prize if three white marbles are drawn.

(c) Jill plays the game nine times. Find the probability that she wins exactly two prizes. [2]

(d) Grant plays the game until he wins two prizes. Find the probability that he wins his second prize on his eighth attempt. [4]

Please **do not** write on this page.

Answers written on this page
will not be marked.

Please **do not** write on this page.

Answers written on this page
will not be marked.

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Standard level
Paper 2

Friday 11 November 2016 (morning)

1 hour 30 minutes

Candidate session number

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
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- The maximum mark for this examination paper is **[90 marks]**.

8816 – 7302

Please **do not** write on this page.

Answers written on this page will not be marked.

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Section A

Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.

1. [Maximum mark: 7]

Let $f(x) = x^2 + 2x + 1$ and $g(x) = x - 5$, for $x \in \mathbb{R}$.

(a) Find $f(8)$. [2]

(b) Find $(g \circ f)(x)$. [2]

(c) Solve $(g \circ f)(x) = 0$. [3]

2. [Maximum mark: 7]

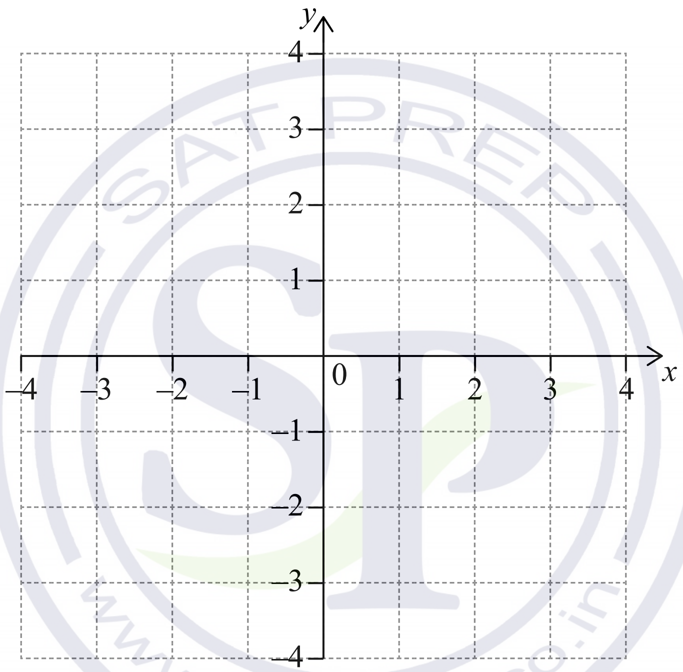
Let $f(x) = 0.225x^3 - 2.7x$, for $-3 \le x \le 3$. There is a local minimum point at A.

(a) Find the coordinates of A. [2]

(b) On the following grid,

(i) sketch the graph of f, clearly indicating the point A;

(ii) sketch the tangent to the graph of f at A. [5]



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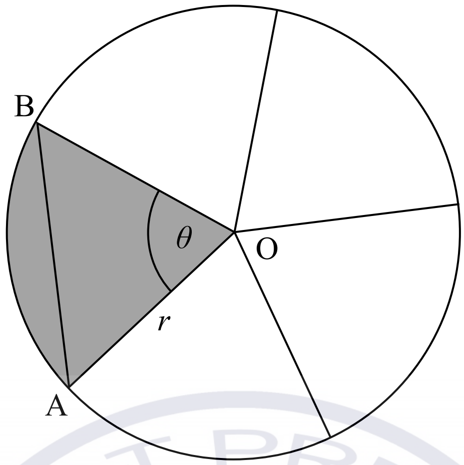
3. [Maximum mark: 7]

The following diagram shows a circle, centre O and radius r mm. The circle is divided into five equal sectors.

diagram not to scale



One sector is OAB, and $A\hat{O}B = \theta$.

(a) Write down the **exact** value of θ in radians. [1]

The area of sector AOB is $20\pi\,\text{mm}^2$.

(b) Find the value of r. [3]

(c) Find AB. [3]

4. [Maximum mark: 6]

Let $f(x) = x\mathrm{e}^{-x}$ and $g(x) = -3f(x) + 1$.

The graphs of f and g intersect at $x = p$ and $x = q$, where $p < q$.

(a) Find the value of p and of q. [3]

(b) Hence, find the area of the region enclosed by the graphs of f and g. [3]

5. [Maximum mark: 6]

The weights, W, of newborn babies in Australia are normally distributed with a mean 3.41 kg and standard deviation 0.57 kg. A newborn baby has a low birth weight if it weighs less than $w\text{ kg}$.

(a) Given that 5.3% of newborn babies have a low birth weight, find w. [3]

(b) A newborn baby has a low birth weight.
Find the probability that the baby weighs at least 2.15 kg. [3]

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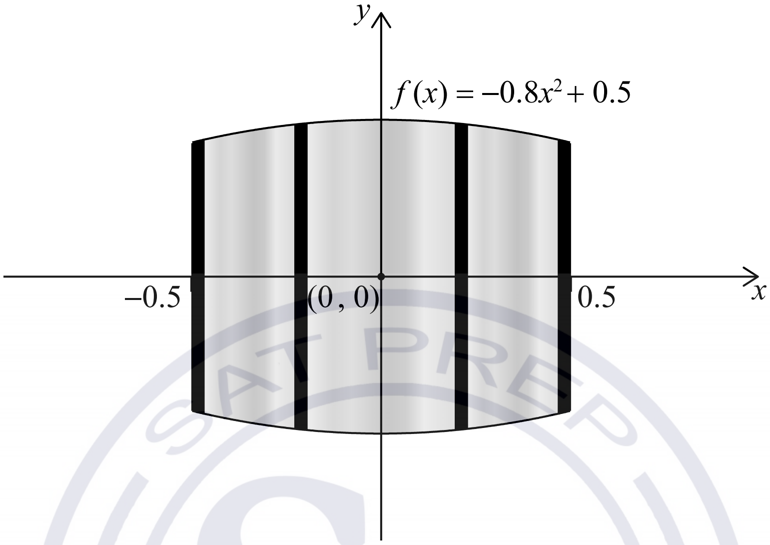
6. [Maximum mark: 6]

All lengths in this question are in metres.

Let $f(x) = -0.8x^2 + 0.5$, for $-0.5 \le x \le 0.5$. Mark uses $f(x)$ as a model to create a barrel. The region enclosed by the graph of f, the x-axis, the line $x = -0.5$ and the line $x = 0.5$ is rotated 360° about the x-axis. This is shown in the following diagram.



(a) Use the model to find the volume of the barrel. [3]

(b) The empty barrel is being filled with water. The volume $V\,\text{m}^3$ of water in the barrel after t minutes is given by $V = 0.8\left(1 - e^{-0.1t}\right)$. How long will it take for the barrel to be half-full? [3]

7. [Maximum mark: 6]

A jar contains 5 red discs, 10 blue discs and m green discs. A disc is selected at random and replaced. This process is performed four times.

(a) Write down the probability that the first disc selected is red. [1]

(b) Let X be the number of red discs selected. Find the smallest value of m for which $\text{Var}(X) < 0.6$. [5]

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 16]

Ten students were surveyed about the number of hours, x, they spent browsing the Internet during week 1 of the school year. The results of the survey are given below.

$$\sum_{i=1}^{10} x_i = 252, \ \sigma = 5 \text{ and median} = 27.$$

(a) Find the mean number of hours spent browsing the Internet. [2]

(b) During week 2, the students worked on a major project and they each spent an additional five hours browsing the Internet. For week 2, write down

(i) the mean;

(ii) the standard deviation. [2]

(c) During week 3 each student spent 5% less time browsing the Internet than during week 1. For week 3, find

(i) the median;

(ii) the variance. [6]

(d) During week 4, the survey was extended to all 200 students in the school. The results are shown in the cumulative frequency graph on the following page.

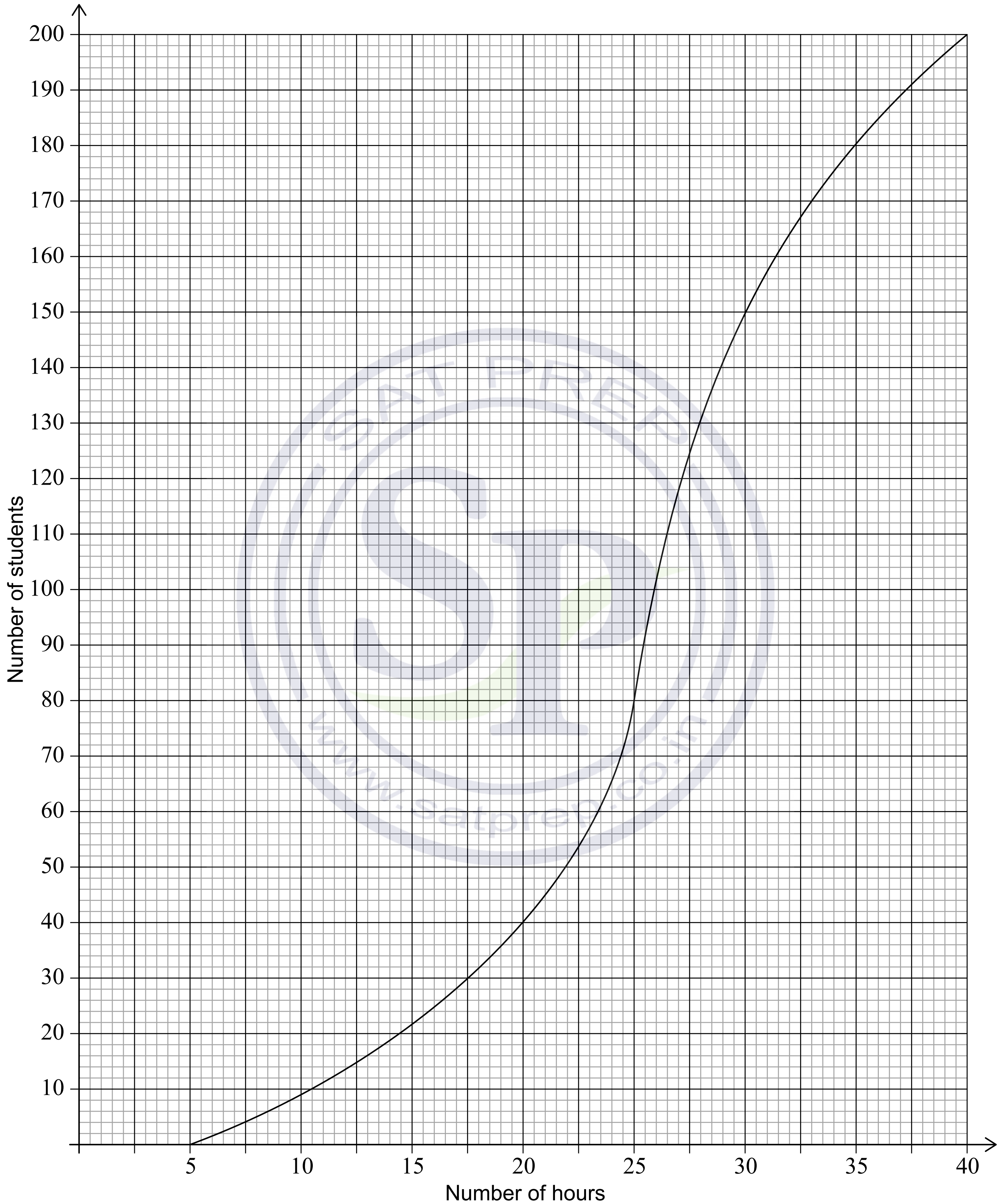
(i) Find the number of students who spent between 25 and 30 hours browsing the Internet.

(ii) Given that 10% of the students spent more than k hours browsing the Internet, find the maximum value of k. [6]

(This question continues on the following page)

Do **not** write solutions on this page.

(Question 8 continued)



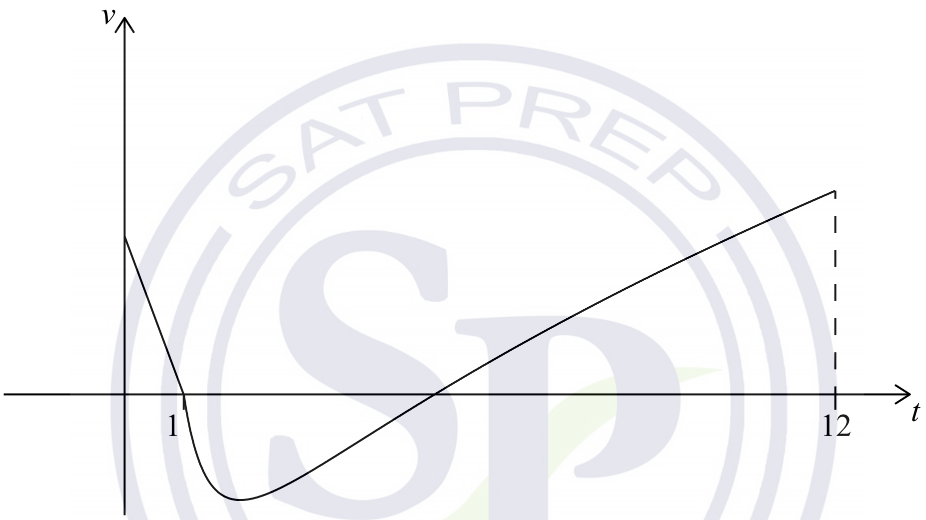
Do **not** write solutions on this page.

9. [Maximum mark: 14]

A particle P starts from a point A and moves along a horizontal straight line. Its velocity $v\ \mathrm{cm\,s^{-1}}$ after t seconds is given by

$$v(t)=\begin{cases}-2t+2, \text{ for } 0\le t\le 1\\ 3\sqrt{t}+\dfrac{4}{t^2}-7, \text{ for } 1\le t\le 12\end{cases}$$

The following diagram shows the graph of v.



(a) Find the initial velocity of P. [2]

P is at rest when $t=1$ and $t=p$.

(b) Find the value of p. [2]

When $t=q$, the acceleration of P is zero.

(c) (i) Find the value of q.

(ii) Hence, find the **speed** of P when $t=q$. [4]

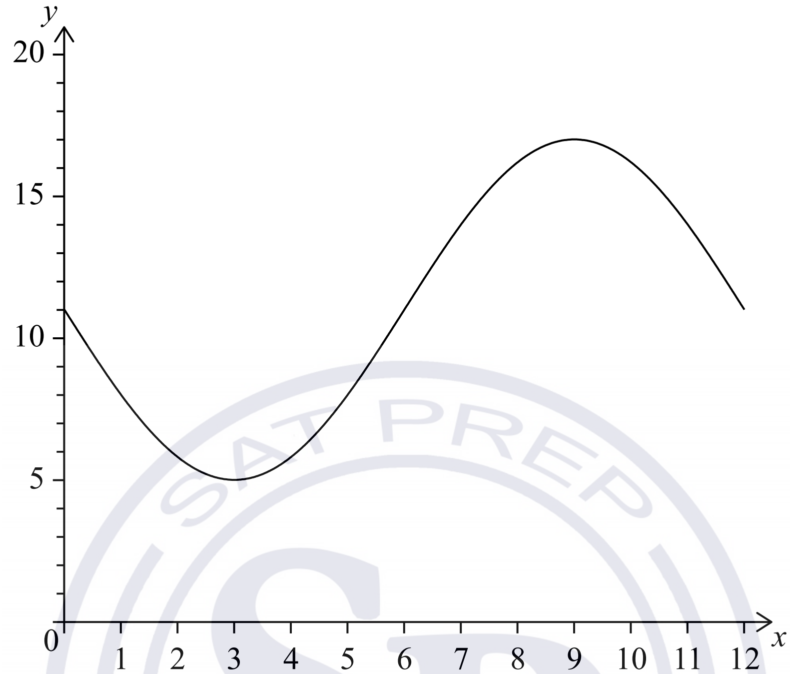
(d) (i) Find the total distance travelled by P between $t=1$ and $t=p$.

(ii) Hence or otherwise, find the displacement of P from A when $t=p$. [6]

Do **not** write solutions on this page.

10. [Maximum mark: 15]

The following diagram shows the graph of $f(x) = a\sin bx + c$, for $0 \le x \le 12$.



The graph of f has a minimum point at $(3, 5)$ and a maximum point at $(9, 17)$.

(a) (i) Find the value of c.

(ii) Show that $b = \frac{\pi}{6}$.

(iii) Find the value of a. [6]

The graph of g is obtained from the graph of f by a translation of $\begin{pmatrix} k \\ 0 \end{pmatrix}$. The maximum point on the graph of g has coordinates $(11.5, 17)$.

(b) (i) Write down the value of k.

(ii) Find $g(x)$. [3]

The graph of g changes from concave-up to concave-down when $x = w$.

(c) (i) Find w.

(ii) Hence or otherwise, find the maximum positive rate of change of g. [6]

Please **do not** write on this page.

Answers written on this page will not be marked.

Please **do not** write on this page.

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Please **do not** write on this page.

Answers written on this page will not be marked.

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Standard level
Paper 2

Wednesday 11 May 2016 (morning)

1 hour 30 minutes

Candidate session number

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics SL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[90 marks]**.

Please **do not** write on this page.

Answers written on this page will not be marked.

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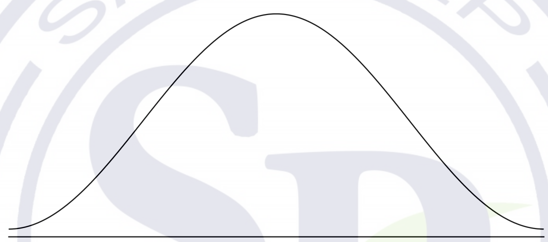
Section A

Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.

1. [Maximum mark: 6]

A random variable X is distributed normally with a mean of 20 and standard deviation of 4.

(a) On the following diagram, shade the region representing $\mathrm{P}(X \le 25)$. [2]



(b) Write down $\mathrm{P}(X \le 25)$, correct to two decimal places. [2]

(c) Let $\mathrm{P}(X \le c) = 0.7$. Write down the value of c. [2]

2. [Maximum mark: 6]

Let $f(x) = x^2$ and $g(x) = 3\ln(x+1)$, for $x > -1$.

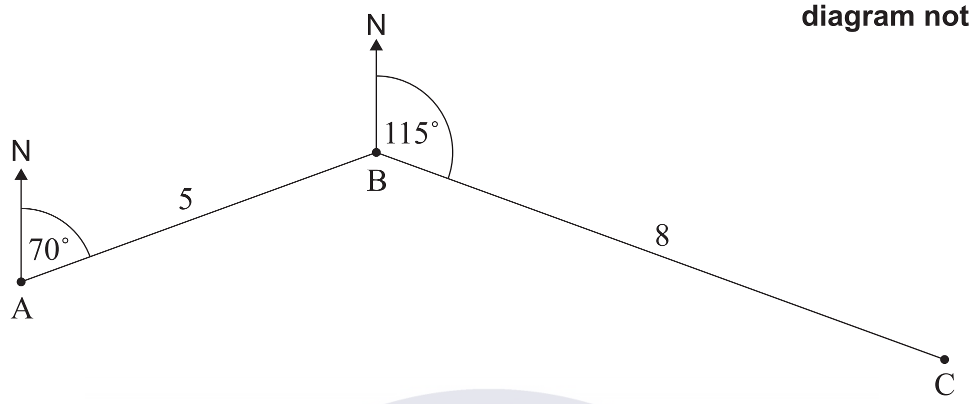
(a) Solve $f(x) = g(x)$. [3]

(b) Find the area of the region enclosed by the graphs of f and g. [3]

3. [Maximum mark: 7]

The following diagram shows three towns A, B and C. Town B is $5\,\text{km}$ from Town A, on a bearing of 070°. Town C is $8\,\text{km}$ from Town B, on a bearing of 115°.

diagram not to scale



(a) Find $\text{A}\hat{\text{B}}\text{C}$. [2]

(b) Find the distance from Town A to Town C. [3]

(c) Use the sine rule to find $\text{A}\hat{\text{C}}\text{B}$. [2]

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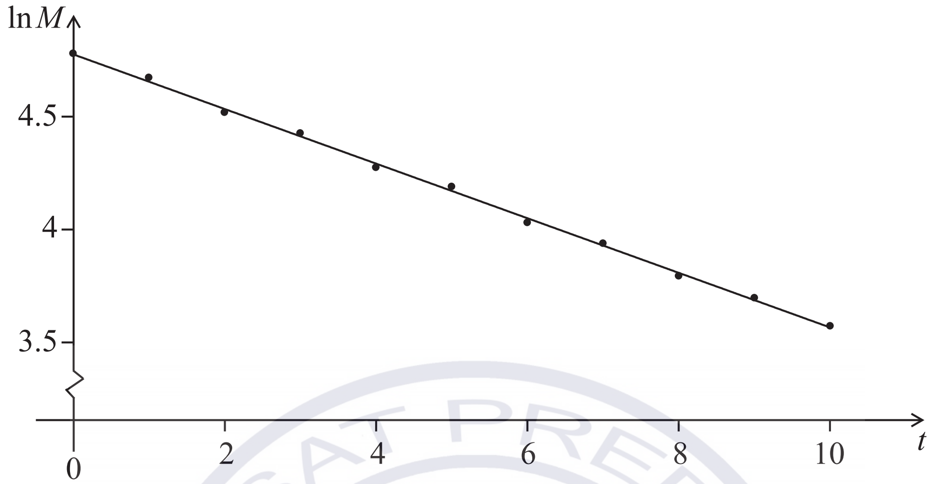
4. [Maximum mark: 6]

(a) Find the term in x^6 in the expansion of $(x+2)^9$. [4]

(b) Hence, find the term in x^7 in the expansion of $5x(x+2)^9$. [2]

5. [Maximum mark: 6]

The mass M of a decaying substance is measured at one minute intervals. The points $(t, \ln M)$ are plotted for $0 \le t \le 10$, where t is in minutes. The line of best fit is drawn. This is shown in the following diagram.



The correlation coefficient for this linear model is $r = -0.998$.

(a) State **two** words that describe the linear correlation between $\ln M$ and t. [2]

(b) The equation of the line of best fit is $\ln M = -0.12t + 4.67$. Given that $M = a \times b^t$, find the value of b. [4]

6. [Maximum mark: 6]

In a geometric sequence, the fourth term is 8 times the first term. The sum of the first 10 terms is 2557.5. Find the 10th term of this sequence.

7. [Maximum mark: 8]

Note: One decade is 10 years

A population of rare birds, P_t, can be modelled by the equation $P_t = P_0 e^{kt}$, where P_0 is the initial population, and t is measured in decades. After one decade, it is estimated that $\frac{P_1}{P_0} = 0.9$.

(a) (i) Find the value of k.

(ii) Interpret the meaning of the value of k. [3]

(b) Find the least number of **whole** years for which $\frac{P_t}{P_0} < 0.75$. [5]

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 15]

A factory has two machines, A and B. The number of breakdowns of each machine is independent from day to day.

Let A be the number of breakdowns of Machine A on any given day. The probability distribution for A can be modelled by the following table.

a	0	1	2	3
$\mathrm{P}(A=a)$	0.55	0.3	0.1	k

(a) Find k. [2]

(b) (i) A day is chosen at random. Write down the probability that Machine A has no breakdowns.

(ii) Five days are chosen at random. Find the probability that Machine A has no breakdowns on exactly four of these days. [3]

Let B be the number of breakdowns of Machine B on any given day. The probability distribution for B can be modelled by the following table.

b	0	1	2	3
$\mathrm{P}(B=b)$	0.7	0.2	0.08	0.02

(c) Find $\mathrm{E}(B)$. [2]

On Tuesday, the factory uses both Machine A and Machine B. The variables A and B are independent.

(d) (i) Find the probability that there are exactly two breakdowns on Tuesday.

(ii) Given that there are exactly two breakdowns on Tuesday, find the probability that both breakdowns are of Machine A. [8]

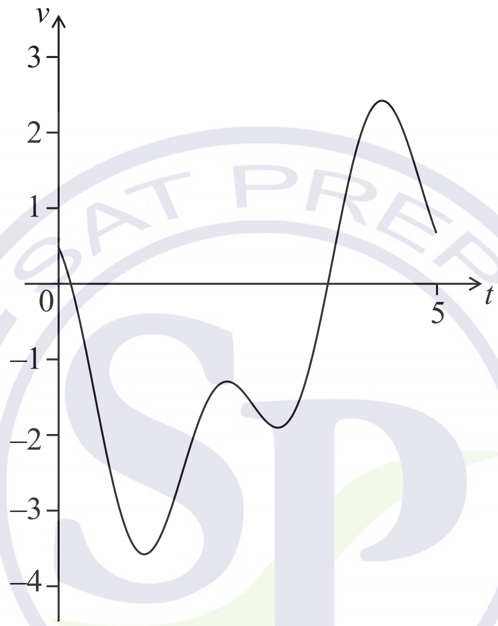
Do **not** write solutions on this page.

9. [Maximum mark: 14]

A particle P moves along a straight line so that its velocity, $v \text{ ms}^{-1}$, after t seconds, is given by $v = \cos 3t - 2\sin t - 0.5$, for $0 \le t \le 5$. The initial displacement of P from a fixed point O is 4 metres.

(a) Find the displacement of P from O after 5 seconds. [5]

The following sketch shows the graph of v.



(b) Find when P is first at rest. [2]

(c) Write down the number of times P changes direction. [2]

(d) Find the acceleration of P after 3 seconds. [2]

(e) Find the maximum speed of P. [3]

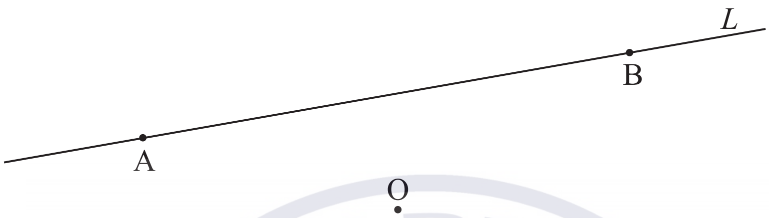
Do **not** write solutions on this page.

10. [Maximum mark: 16]

The points A and B lie on a line L, and have position vectors $\begin{pmatrix} -3 \\ -2 \\ 2 \end{pmatrix}$ and $\begin{pmatrix} 6 \\ 4 \\ -1 \end{pmatrix}$ respectively.

Let O be the origin. This is shown on the following diagram.

diagram not to scale



(a) Find $\overrightarrow{AB}$. [2]

The point C also lies on L, such that $\overrightarrow{AC} = 2\overrightarrow{CB}$.

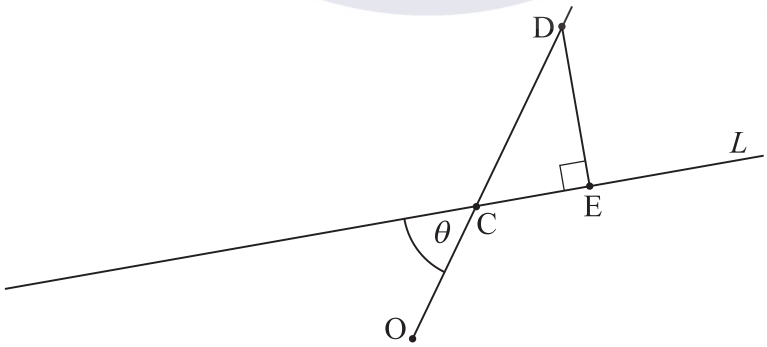
(b) Show that $\overrightarrow{OC} = \begin{pmatrix} 3 \\ 2 \\ 0 \end{pmatrix}$. [3]

Let θ be the angle between $\overrightarrow{AB}$ and $\overrightarrow{OC}$.

(c) Find θ. [5]

Let D be a point such that $\overrightarrow{OD} = k\overrightarrow{OC}$, where $k > 1$. Let E be a point on L such that $\hat{CED}$ is a right angle. This is shown on the following diagram.

diagram not to scale



(d) (i) Show that $\left|\overrightarrow{DE}\right| = (k-1)\left|\overrightarrow{OC}\right|\sin\theta$.

(ii) The distance from D to line L is less than 3 units. Find the possible values of k. [6]

M16/5/MATME/SP2/ENG/TZ2/XX

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Standard level
Paper 2

Wednesday 11 May 2016 (morning)

Candidate session number

1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics SL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[90 marks]**.

2216 – 7306

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, for example if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.

1. [Maximum mark: 6]

The first three terms of an arithmetic sequence are $u_1 = 0.3$, $u_2 = 1.5$, $u_3 = 2.7$.

(a) Find the common difference. [2]

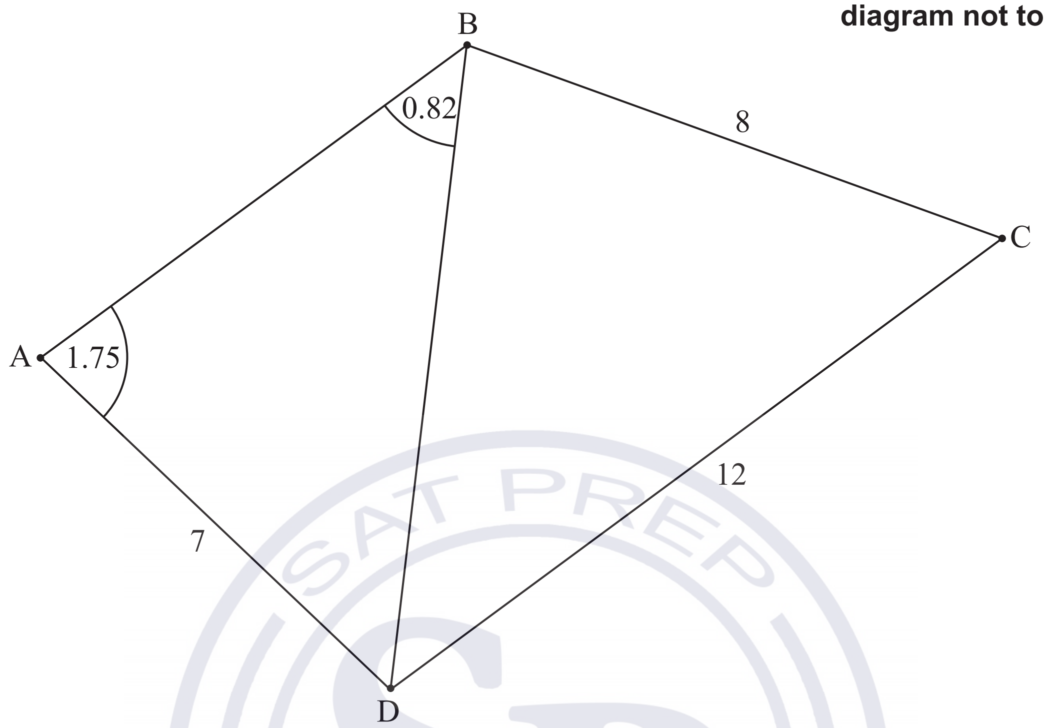
(b) Find the 30th term of the sequence. [2]

(c) Find the sum of the first 30 terms. [2]

2. [Maximum mark: 6]

The following diagram shows a quadrilateral ABCD.

diagram not to scale



$AD = 7\,\text{cm},\ BC = 8\,\text{cm},\ CD = 12\,\text{cm},\ D\hat{A}B = 1.75$ radians, $A\hat{B}D = 0.82$ radians.

(a) Find BD. [3]

(b) Find $D\hat{B}C$. [3]

3. [Maximum mark: 7]

Let $f(x) = e^{0.5x} - 2$.

(a) For the graph of f

(i) write down the y-intercept;

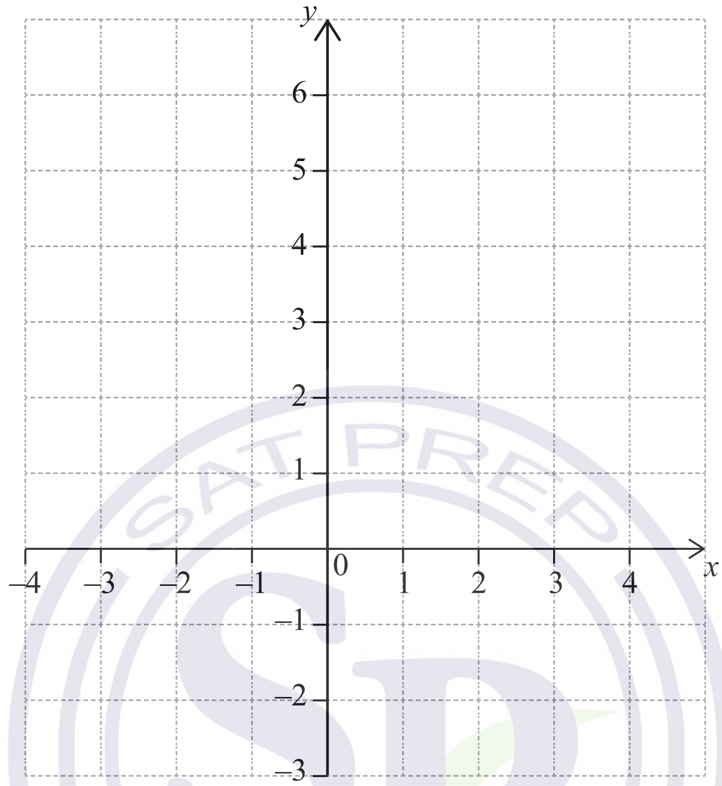
(ii) find the x-intercept;

(iii) write down the equation of the horizontal asymptote. [4]

(This question continues on the following page)

(Question 3 continued)

(b) On the following grid, sketch the graph of f, for $-4 \le x \le 4$. [3]



4. [Maximum mark: 8]

The height, h metres, of a seat on a Ferris wheel after t minutes is given by

$$h(t) = -15\cos 1.2t + 17, \text{ for } t \geq 0.$$

(a) Find the height of the seat when $t = 0$. [2]

(b) The seat first reaches a height of 20 m after k minutes. Find k. [3]

(c) Calculate the time needed for the seat to complete a full rotation, giving your answer correct to one decimal place. [3]

5. [Maximum mark: 6]

Consider the expansion of $\left(x^2+\frac{2}{x}\right)^{10}$.

(a) Write down the number of terms of this expansion. [1]

(b) Find the coefficient of x^8. [5]

6. [Maximum mark: 6]

A competition consists of two independent events, shooting at 100 targets and running for one hour.

The number of targets a contestant hits is the S score. The S scores are normally distributed with mean 65 and standard deviation 10.

(a) A contestant is chosen at random. Find the probability that their S score is less than 50. [2]

The distance in km that a contestant runs in one hour is the R score. The R scores are normally distributed with mean 12 and standard deviation 2.5. The R score is independent of the S score.

Contestants are disqualified if their S score is less than 50 **and** their R score is less than x km.

(b) Given that 1% of the contestants are disqualified, find the value of x. [4]

7. [Maximum mark: 7]

A particle moves in a straight line. Its velocity $v\,\text{m s}^{-1}$ after t seconds is given by

$$v = 6t - 6, \text{ for } 0 \le t \le 2.$$

After p seconds, the particle is $2\,\text{m}$ from its initial position. Find the possible values of p.

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 15]

The price of a used car depends partly on the distance it has travelled. The following table shows the distance and the price for seven cars on 1 January 2010.

Distance, x km	11 500	7500	13 600	10 800	9500	12 200	10 400
Price, y dollars	15 000	21 500	12 000	16 000	19 000	14 500	17 000

The relationship between x and y can be modelled by the regression equation $y = ax + b$.

(a) (i) Find the correlation coefficient.

(ii) Write down the value of a and of b. [4]

On 1 January 2010, Lina buys a car which has travelled 11 000 km.

(b) Use the regression equation to estimate the price of Lina's car, giving your answer to the nearest 100 dollars. [3]

The price of a car decreases by 5 % each year.

(c) Calculate the price of Lina's car after 6 years. [4]

Lina will sell her car when its price reaches 10 000 dollars.

(d) Find the year when Lina sells her car. [4]

Do **not** write solutions on this page.

9. [Maximum mark: 14]

Let $f(x)=\frac{1}{x-1}+2$, for $x>1$.

(a) Write down the equation of the horizontal asymptote of the graph of f. [2]

(b) Find $f'(x)$. [2]

Let $g(x)=a\mathrm{e}^{-x}+b$, for $x\geq 1$. The graphs of f and g have the same horizontal asymptote.

(c) Write down the value of b. [2]

(d) Given that $g'(1)=-\mathrm{e}$, find the value of a. [4]

(e) There is a value of x, for $1<x<4$, for which the graphs of f and g have the same gradient. Find this gradient. [4]

Do **not** write solutions on this page.

10. [Maximum mark: 15]

Consider the points $\mathrm{A}(1, 5, -7)$ and $\mathrm{B}(-9, 9, -6)$.

(a) Find $\overrightarrow{\mathrm{AB}}$. [2]

Let C be a point such that $\overrightarrow{\mathrm{AC}} = \begin{pmatrix} 6 \\ -4 \\ 0 \end{pmatrix}$.

(b) Find the coordinates of C. [2]

The line L passes through B and is parallel to (AC).

(c) Write down a vector equation for L. [2]

(d) Given that $\left|\overrightarrow{\mathrm{AB}}\right| = k\left|\overrightarrow{\mathrm{AC}}\right|$, find k. [3]

(e) The point D lies on L such that $\left|\overrightarrow{\mathrm{AB}}\right| = \left|\overrightarrow{\mathrm{BD}}\right|$. Find the possible coordinates of D. [6]

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Standard level
Paper 2

Thursday 12 November 2015 (afternoon)

1 hour 30 minutes

Candidate session number

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
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- A clean copy of the **mathematics SL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[90 marks]**.

12 pages

8815–7302

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, for example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

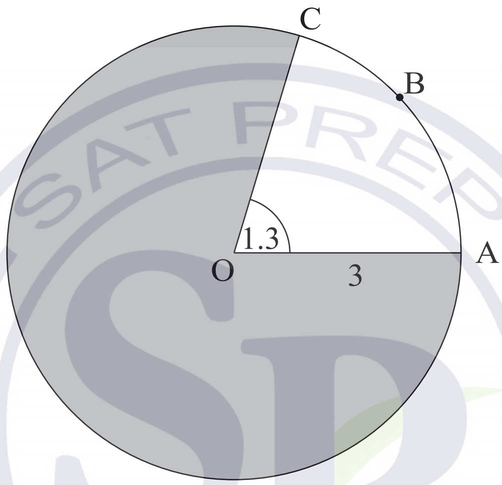
Section A

Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.

1. [Maximum mark: 6]

The following diagram shows a circle with centre O and radius 3 cm.

diagram not to scale



Points A, B, and C lie on the circle, and $\text{A}\hat{\text{O}}\text{C} = 1.3$ radians.

(a) Find the length of arc ABC. [2]

(b) Find the area of the shaded region. [4]

2. [Maximum mark: 5]

The following table shows the probability distribution of a discrete random variable X.

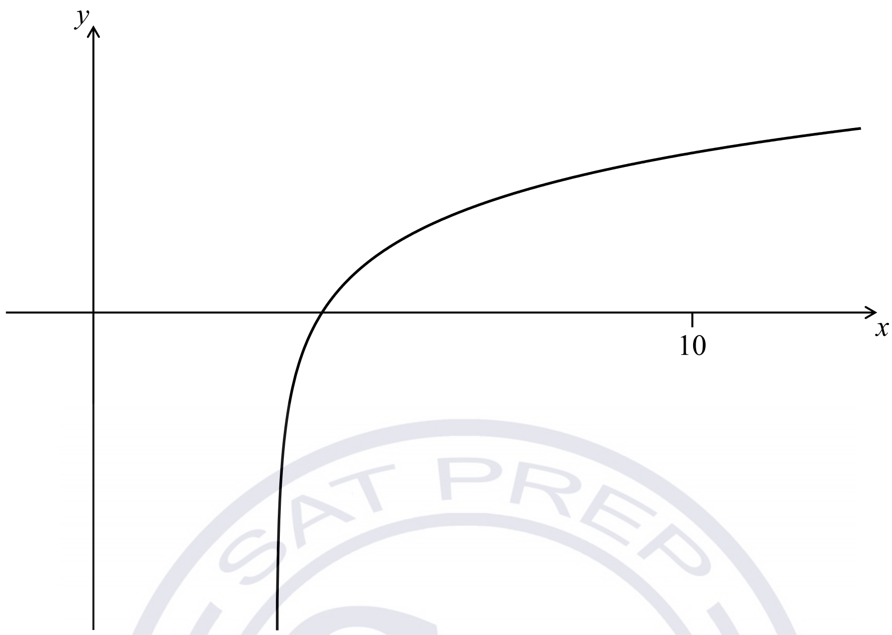
x	0	1	2	3
$\mathrm{P}(X=x)$	0.15	k	0.1	$2k$

(a) Find the value of k. [3]

(b) Find $\mathrm{E}(X)$. [2]

3. [Maximum mark: 7]

Let $f(x) = 2\ln(x-3)$, for $x > 3$. The following diagram shows part of the graph of f.



(a) Find the equation of the vertical asymptote to the graph of f. [2]

(b) Find the x-intercept of the graph of f. [2]

(c) The region enclosed by the graph of f, the x-axis and the line $x = 10$ is rotated 360° about the x-axis. Find the volume of the solid formed. [3]

(This question continues on the following page)

(Question 3 continued)

4. [Maximum mark: 7]

The first three terms of a geometric sequence are $u_1 = 0.64$, $u_2 = 1.6$, and $u_3 = 4$.

(a) Find the value of r. [2]

(b) Find the value of S_6. [2]

(c) Find the least value of n such that $S_n > 75\,000$. [3]

5. [Maximum mark: 7]

Let C and D be independent events, with $\mathrm{P}(C) = 2k$ and $\mathrm{P}(D) = 3k^2$, where $0 < k < 0.5$.

(a) Write down an expression for $\mathrm{P}(C \cap D)$ in terms of k. [2]

(b) Given that $\mathrm{P}(C \cap D) = 0.162$, find k. [2]

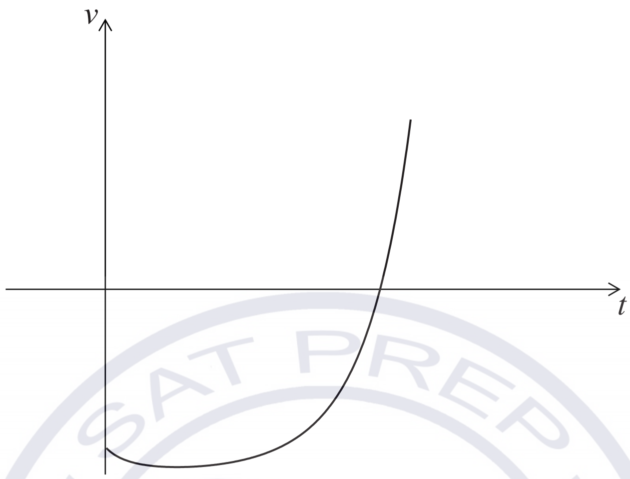
(c) Find $\mathrm{P}\left(C' \middle| D\right)$. [3]

6. [Maximum mark: 6]

The velocity $v\,\text{m s}^{-1}$ of a particle after t seconds is given by

$$v(t) = (0.3t + 0.1)^t - 4\text{, for } 0 \le t \le 5\,.$$

The following diagram shows the graph of v.



(a) Find the value of t when the particle is at rest. [3]

(b) Find the value of t when the acceleration of the particle is 0. [3]

7. [Maximum mark: 8]

Let $f(x) = \ln(x^2)$, for $x \neq 0$.

(a) Show that $f'(x) = \dfrac{2}{x}$. [2]

(b) The tangent to the graph of f at a point $\mathrm{P}\left(d,\, f(d)\right)$ passes through another point $\mathrm{Q}(1, -3)$. Find the value of d. [6]

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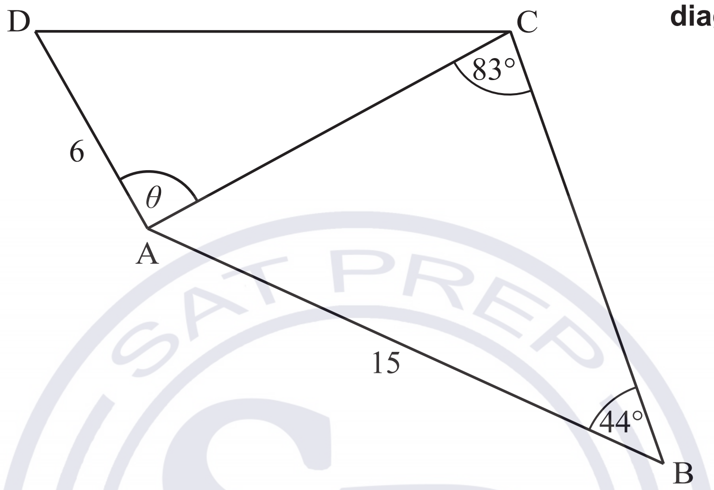
Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 14]

The following diagram shows the quadrilateral ABCD.



$AD = 6\text{ cm}$, $AB = 15\text{ cm}$, $A\hat{B}C = 44^\circ$, $A\hat{C}B = 83^\circ$ and $D\hat{A}C = \theta$

(a) Find AC. [3]

(b) Find the area of triangle ABC. [3]

The area of triangle ACD is half the area of triangle ABC.

(c) Find the possible values of θ. [5]

(d) Given that θ is obtuse, find CD. [3]

Do **not** write solutions on this page.

9. [Maximum mark: 16]

An environmental group records the numbers of coyotes and foxes in a wildlife reserve after t years, starting on 1 January 1995.

Let c be the number of coyotes in the reserve after t years. The following table shows the number of coyotes after t years.

number of years (t)	0	2	10	15	19
number of coyotes (c)	115	197	265	320	406

The relationship between the variables can be modelled by the regression equation $c = at + b$.

(a) Find the value of a and of b. [3]

(b) Use the regression equation to estimate the number of coyotes in the reserve when $t = 7$. [3]

Let f be the number of foxes in the reserve after t years. The number of foxes can be modelled by the equation $f = \dfrac{2000}{1+99\mathrm{e}^{-kt}}$, where k is a constant.

(c) Find the number of foxes in the reserve on 1 January 1995. [3]

(d) After five years, there were 64 foxes in the reserve. Find k. [3]

(e) During which year were the number of coyotes the same as the number of foxes? [4]

Do **not** write solutions on this page.

10. [Maximum mark: 14]

The masses of watermelons grown on a farm are normally distributed with a mean of 10 kg. The watermelons are classified as small, medium or large.

A watermelon is small if its mass is less than 4 kg. Five percent of the watermelons are classified as small.

(a) Find the standard deviation of the masses of the watermelons. [4]

The following table shows the percentages of small, medium and large watermelons grown on the farm.

small	medium	large
5%	57%	38%

A watermelon is large if its mass is greater than $w\text{ kg}$.

(b) Find the value of w. [2]

All the medium and large watermelons are delivered to a grocer.

(c) The grocer selects a watermelon at random from **this** delivery. Find the probability that it is medium. [3]

(d) The grocer sells all the medium watermelons for $\$1.75$ each, and all the large watermelons for $\$3.00$ each. His costs on this delivery are $\$300$, and his total profit is $\$150$. Find the number of watermelons in the delivery. [5]

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Standard level
Paper 2

Wednesday 11 May 2016 (morning)

1 hour 30 minutes

Candidate session number

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics SL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[90 marks]**.

2216 – 7304

Please **do not** write on this page.

Answers written on this page will not be marked.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, for example if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

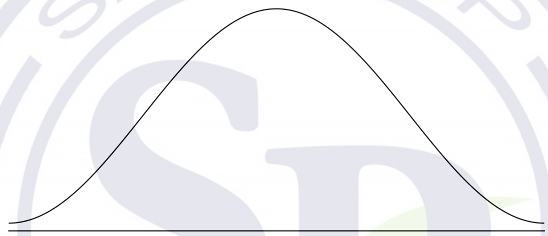
Section A

Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.

1. [Maximum mark: 6]

A random variable X is distributed normally with a mean of 20 and standard deviation of 4.

(a) On the following diagram, shade the region representing $\mathrm{P}(X \le 25)$. [2]



(b) Write down $\mathrm{P}(X \le 25)$, correct to two decimal places. [2]

(c) Let $\mathrm{P}(X \le c) = 0.7$. Write down the value of c. [2]

2. [Maximum mark: 6]

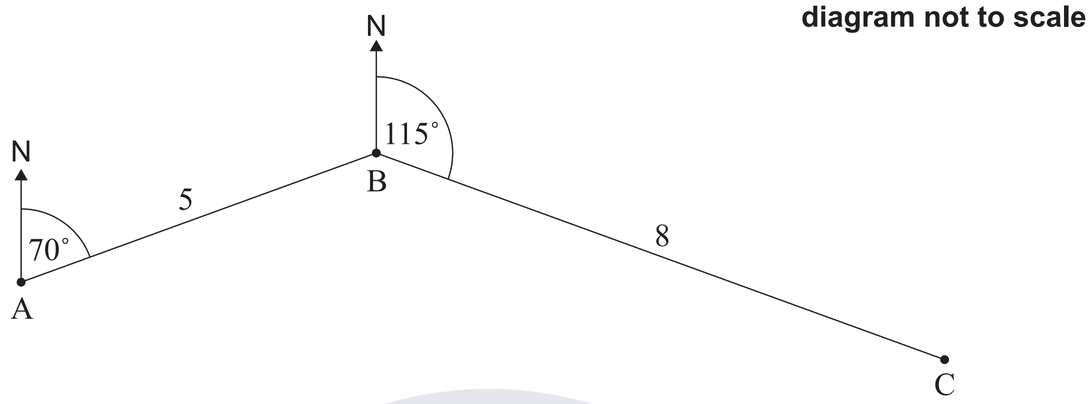
Let $f(x) = x^2$ and $g(x) = 3\ln(x + 1)$, for $x > -1$.

(a) Solve $f(x) = g(x)$. [3]

(b) Find the area of the region enclosed by the graphs of f and g. [3]

3. [Maximum mark: 7]

The following diagram shows three towns A, B and C. Town B is 5 km from Town A, on a bearing of 070˚. Town C is 8 km from Town B, on a bearing of 115˚.



(a) Find $A\hat{B}C$. [2]

(b) Find the distance from Town A to Town C. [3]

(c) Use the sine rule to find $A\hat{C}B$. [2]

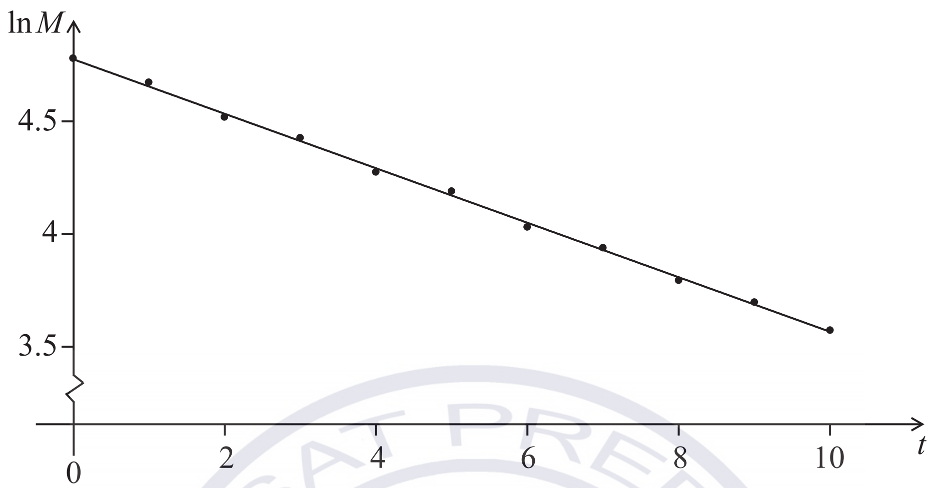
4. [Maximum mark: 6]

(a) Find the term in x^6 in the expansion of $(x+2)^9$. [4]

(b) Hence, find the term in x^7 in the expansion of $5x(x+2)^9$. [2]

5. [Maximum mark: 6]

The mass M of a decaying substance is measured at one minute intervals. The points $(t, \ln M)$ are plotted for $0 \le t \le 10$, where t is in minutes. The line of best fit is drawn. This is shown in the following diagram.



The correlation coefficient for this linear model is $r = -0.998$.

(a) State **two** words that describe the linear correlation between $\ln M$ and t. [2]

(b) The equation of the line of best fit is $\ln M = -0.12t + 4.67$. Given that $M = a \times b^t$, find the value of b. [4]

6. [Maximum mark: 6]

In a geometric sequence, the fourth term is 8 times the first term. The sum of the first 10 terms is 2557.5. Find the 10th term of this sequence.

7. [Maximum mark: 8]

Note: One decade is 10 years

A population of rare birds, P_t, can be modelled by the equation $P_t = P_0 e^{kt}$, where P_0 is the initial population, and t is measured in decades. After one decade, it is estimated that $\frac{P_1}{P_0} = 0.9$.

(a) (i) Find the value of k.

(ii) Interpret the meaning of the value of k. [3]

(b) Find the least number of **whole** years for which $\frac{P_t}{P_0} < 0.75$. [5]

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 15]

A factory has two machines, A and B. The number of breakdowns of each machine is independent from day to day.

Let A be the number of breakdowns of Machine A on any given day. The probability distribution for A can be modelled by the following table.

a	0	1	2	3
$\mathrm{P}(A=a)$	0.55	0.3	0.1	k

(a) Find k. [2]

(b) (i) A day is chosen at random. Write down the probability that Machine A has no breakdowns.

(ii) Five days are chosen at random. Find the probability that Machine A has no breakdowns on exactly four of these days. [3]

Let B be the number of breakdowns of Machine B on any given day. The probability distribution for B can be modelled by the following table.

b	0	1	2	3
$\mathrm{P}(B=b)$	0.7	0.2	0.08	0.02

(c) Find $\mathrm{E}(B)$. [2]

On Tuesday, the factory uses both Machine A and Machine B. The variables A and B are independent.

(d) (i) Find the probability that there are exactly two breakdowns on Tuesday.

(ii) Given that there are exactly two breakdowns on Tuesday, find the probability that both breakdowns are of Machine A. [8]

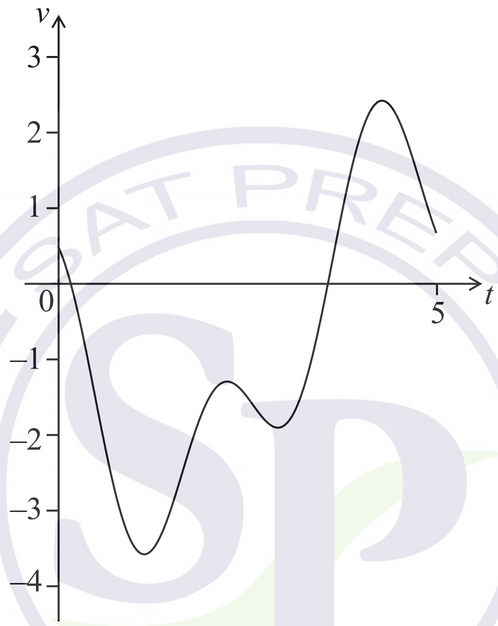
Do **not** write solutions on this page.

9. [Maximum mark: 14]

A particle P moves along a straight line so that its velocity, v ms^{-1}, after t seconds, is given by $v = \cos 3t - 2\sin t - 0.5$, for $0 \le t \le 5$. The initial displacement of P from a fixed point O is 4 metres.

(a) Find the displacement of P from O after 5 seconds. [5]

The following sketch shows the graph of v.



(b) Find when P is first at rest. [2]

(c) Write down the number of times P changes direction. [2]

(d) Find the acceleration of P after 3 seconds. [2]

(e) Find the maximum speed of P. [3]

Turn over

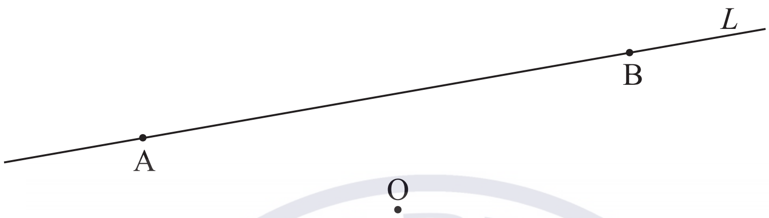
Do **not** write solutions on this page.

10. [Maximum mark: 16]

The points A and B lie on a line L, and have position vectors $\begin{pmatrix} -3 \\ -2 \\ 2 \end{pmatrix}$ and $\begin{pmatrix} 6 \\ 4 \\ -1 \end{pmatrix}$ respectively.

Let O be the origin. This is shown on the following diagram.

diagram not to scale



(a) Find $\overrightarrow{AB}$. [2]

The point C also lies on L, such that $\overrightarrow{AC} = 2\overrightarrow{CB}$.

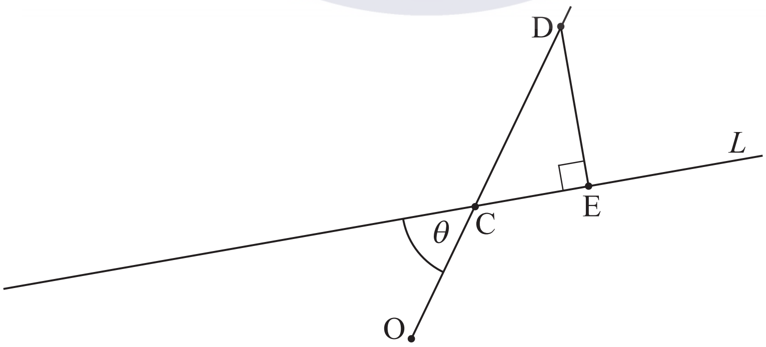
(b) Show that $\overrightarrow{OC} = \begin{pmatrix} 3 \\ 2 \\ 0 \end{pmatrix}$. [3]

Let θ be the angle between $\overrightarrow{AB}$ and $\overrightarrow{OC}$.

(c) Find θ. [5]

Let D be a point such that $\overrightarrow{OD} = k\overrightarrow{OC}$, where $k > 1$. Let E be a point on L such that $\text{C}\hat{\text{E}}\text{D}$ is a right angle. This is shown on the following diagram.

diagram not to scale



(d) (i) Show that $\left|\overrightarrow{DE}\right| = (k-1)\left|\overrightarrow{OC}\right|\sin\theta$.

(ii) The distance from D to line L is less than 3 units. Find the possible values of k. [6]

Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics
Standard level
Paper 2

Wednesday 11 May 2016 (morning)

Candidate session number

1 hour 30 minutes

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2216 – 7306

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Section A

Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.

1. [Maximum mark: 6]

The first three terms of an arithmetic sequence are $u_1 = 0.3$, $u_2 = 1.5$, $u_3 = 2.7$.

(a) Find the common difference. [2]

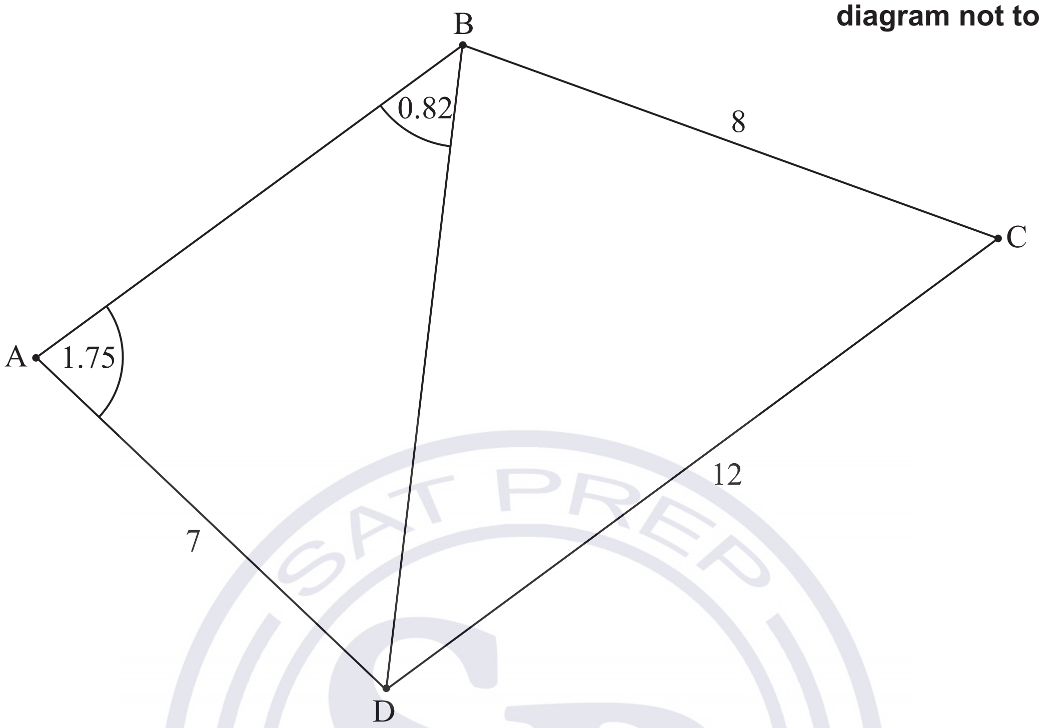
(b) Find the 30th term of the sequence. [2]

(c) Find the sum of the first 30 terms. [2]

2. [Maximum mark: 6]

The following diagram shows a quadrilateral ABCD.

diagram not to scale



$AD = 7\text{ cm}$, $BC = 8\text{ cm}$, $CD = 12\text{ cm}$, $D\hat{A}B = 1.75$ radians, $A\hat{B}D = 0.82$ radians.

(a) Find BD. [3]

(b) Find $D\hat{B}C$. [3]

Turn over

3. [Maximum mark: 7]

Let $f(x) = e^{0.5x} - 2$.

(a) For the graph of f

(i) write down the y-intercept;

(ii) find the x-intercept;

(iii) write down the equation of the horizontal asymptote. [4]

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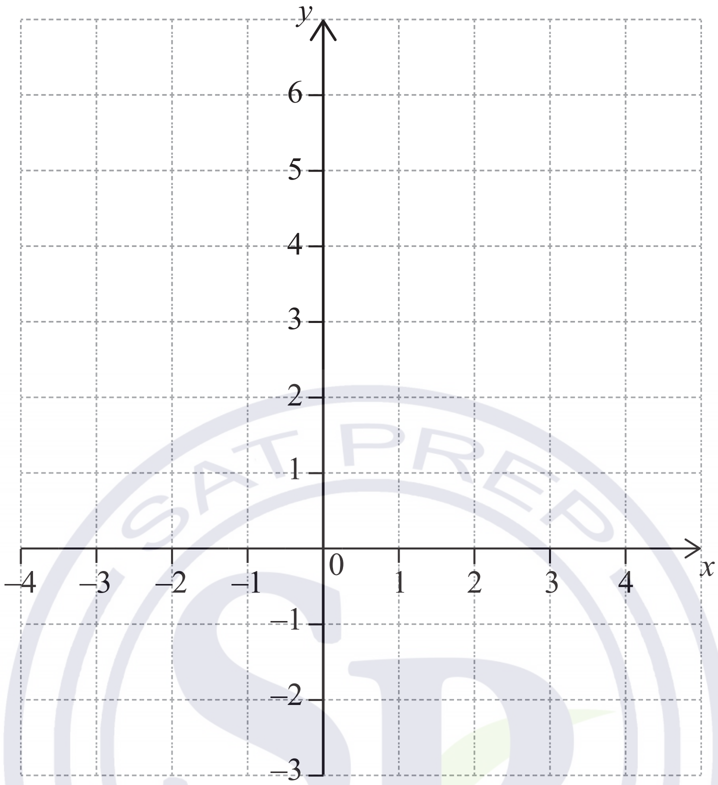
(This question continues on the following page)

(Question 3 continued)

(b) On the following grid, sketch the graph of f, for $-4 \le x \le 4$. [3]



4. [Maximum mark: 8]

The height, h metres, of a seat on a Ferris wheel after t minutes is given by

$$h(t) = -15\cos 1.2t + 17, \text{ for } t \geq 0.$$

(a) Find the height of the seat when $t = 0$. [2]

(b) The seat first reaches a height of $20\,\text{m}$ after k minutes. Find k. [3]

(c) Calculate the time needed for the seat to complete a full rotation, giving your answer correct to one decimal place. [3]

5. [Maximum mark: 6]

Consider the expansion of $\left(x^2 + \frac{2}{x}\right)^{10}$.

(a) Write down the number of terms of this expansion. [1]

(b) Find the coefficient of x^8. [5]

6. [Maximum mark: 6]

A competition consists of two independent events, shooting at 100 targets and running for one hour.

The number of targets a contestant hits is the S score. The S scores are normally distributed with mean 65 and standard deviation 10.

(a) A contestant is chosen at random. Find the probability that their S score is less than 50. [2]

The distance in km that a contestant runs in one hour is the R score. The R scores are normally distributed with mean 12 and standard deviation 2.5. The R score is independent of the S score.

Contestants are disqualified if their S score is less than 50 **and** their R score is less than x km.

(b) Given that 1% of the contestants are disqualified, find the value of x. [4]

7. [Maximum mark: 7]

A particle moves in a straight line. Its velocity $v\,\text{m s}^{-1}$ after t seconds is given by

$$v = 6t - 6\text{, for } 0 \le t \le 2.$$

After p seconds, the particle is $2\,\text{m}$ from its initial position. Find the possible values of p.

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer booklet provided. Please start each question on a new page.

8. [Maximum mark: 15]

The price of a used car depends partly on the distance it has travelled. The following table shows the distance and the price for seven cars on 1 January 2010.

Distance, x km	11 500	7500	13 600	10 800	9500	12 200	10 400
Price, y dollars	15 000	21 500	12 000	16 000	19 000	14 500	17 000

The relationship between x and y can be modelled by the regression equation $y = ax + b$.

(a) (i) Find the correlation coefficient.

(ii) Write down the value of a and of b. [4]

On 1 January 2010, Lina buys a car which has travelled 11 000 km.

(b) Use the regression equation to estimate the price of Lina's car, giving your answer to the nearest 100 dollars. [3]

The price of a car decreases by 5% each year.

(c) Calculate the price of Lina's car after 6 years. [4]

Lina will sell her car when its price reaches 10 000 dollars.

(d) Find the year when Lina sells her car. [4]

Do **not** write solutions on this page.

9. [Maximum mark: 14]

Let $f(x) = \dfrac{1}{x-1} + 2$, for $x > 1$.

(a) Write down the equation of the horizontal asymptote of the graph of f. [2]

(b) Find $f'(x)$. [2]

Let $g(x) = a\mathrm{e}^{-x} + b$, for $x \geq 1$. The graphs of f and g have the same horizontal asymptote.

(c) Write down the value of b. [2]

(d) Given that $g'(1) = -\mathrm{e}$, find the value of a. [4]

(e) There is a value of x, for $1 < x < 4$, for which the graphs of f and g have the same gradient. Find this gradient. [4]

Do **not** write solutions on this page.

10. [Maximum mark: 15]

Consider the points $\mathrm{A}(1, 5, -7)$ and $\mathrm{B}(-9, 9, -6)$.

(a) Find $\overrightarrow{\mathrm{AB}}$. [2]

Let C be a point such that $\overrightarrow{\mathrm{AC}} = \begin{pmatrix} 6 \\ -4 \\ 0 \end{pmatrix}$.

(b) Find the coordinates of C. [2]

The line L passes through B and is parallel to (AC).

(c) Write down a vector equation for L. [2]

(d) Given that $\left|\overrightarrow{\mathrm{AB}}\right| = k\left|\overrightarrow{\mathrm{AC}}\right|$, find k. [3]

(e) The point D lies on L such that $\left|\overrightarrow{\mathrm{AB}}\right| = \left|\overrightarrow{\mathrm{BD}}\right|$. Find the possible coordinates of D. [6]

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MATHEMATICS
STANDARD LEVEL
PAPER 2

Candidate session number

Thursday 13 November 2014 (morning)

1 hour 30 minutes

Examination code

8	8	1	4	–	7	3	0	2

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
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SECTION A

Answer ***all*** *questions in the boxes provided. Working may be continued below the lines if necessary.*

1. *[Maximum mark: 5]*

Let $f(x) = 2x + 3$ and $g(x) = x^3$.

(a) Find $(f \circ g)(x)$. *[2]*

(b) Solve the equation $(f \circ g)(x) = 0$. *[3]*

2. *[Maximum mark: 6]*

The following table shows the Diploma score x and university entrance mark y for seven IB Diploma students.

Diploma score (x)	28	30	27	31	32	25	27
University entrance mark (y)	73.9	78.1	70.2	82.2	85.5	62.7	69.4

(a) Find the correlation coefficient. *[2]*

The relationship can be modelled by the regression line with equation $y = ax + b$.

(b) Write down the value of a and of b. *[2]*

Rita scored a total of 26 in her IB Diploma.

(c) Use your regression line to estimate Rita's university entrance mark. *[2]*

3. *[Maximum mark: 7]*

The following diagram shows a circle with centre O and radius 8 cm.

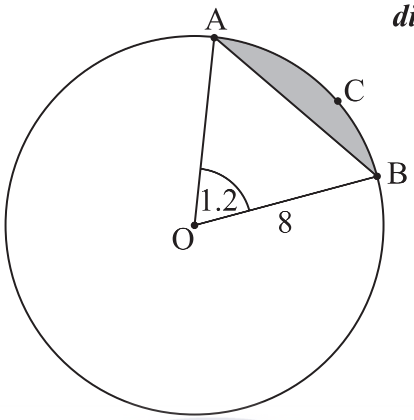


diagram not to scale

The points A, B and C are on the circumference of the circle, and $\hat{AOB} = 1.2$ radians.

(a) Find the length of arc ACB. *[2]*

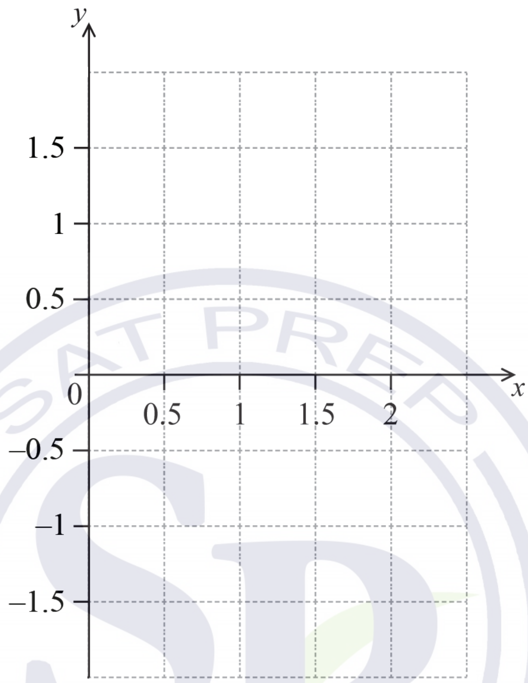
(b) Find AB. *[3]*

(c) Hence, find the perimeter of the shaded segment ABC. *[2]*

4. *[Maximum mark: 8]*

Let $f(x) = -x^4 + 2x^3 - 1$, for $0 \le x \le 2$.

(a) Sketch the graph of f on the following grid. *[3]*



(b) Solve $f(x) = 0$. *[2]*

(c) The region enclosed by the graph of f and the x-axis is rotated 360° about the x-axis. Find the volume of the solid formed. *[3]*

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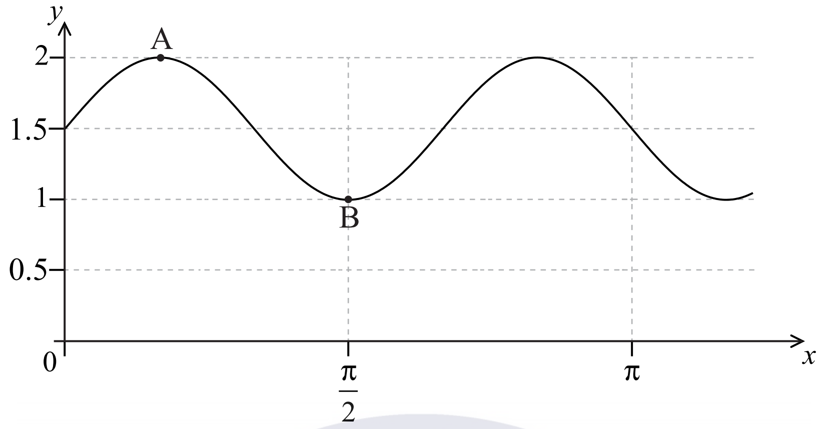
5. *[Maximum mark: 7]*

The following diagram shows part of the graph of $y = p\sin(qx) + r$.



The point $A\left(\frac{\pi}{6}, 2\right)$ is a maximum point and the point $B\left(\frac{\pi}{2}, 1\right)$ is a minimum point. Find the value of

(a) p; *[2]*

(b) r; *[2]*

(c) q. *[3]*

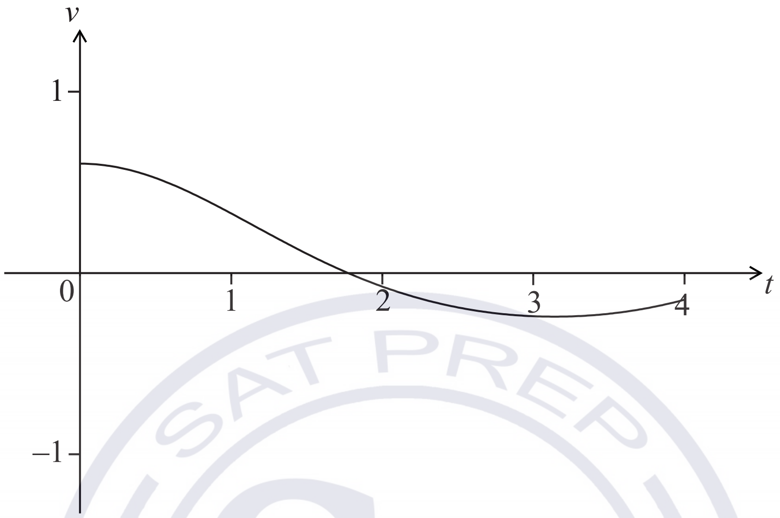
6. *[Maximum mark: 6]*

Consider the expansion of $\left(\frac{x^3}{2}+\frac{p}{x}\right)^8$. The constant term is 5103. Find the possible values of p.

7. *[Maximum mark: 6]*

A particle starts from point A and moves along a straight line. Its velocity, $v\,\text{m}\,\text{s}^{-1}$, after t seconds is given by $v(t) = e^{\frac{1}{2}\cos t} - 1$, for $0 \le t \le 4$. The particle is at rest when $t = \frac{\pi}{2}$.

The following diagram shows the graph of v.



(a) Find the distance travelled by the particle for $0 \le t \le \frac{\pi}{2}$. *[2]*

(b) Explain why the particle passes through A again. *[4]*

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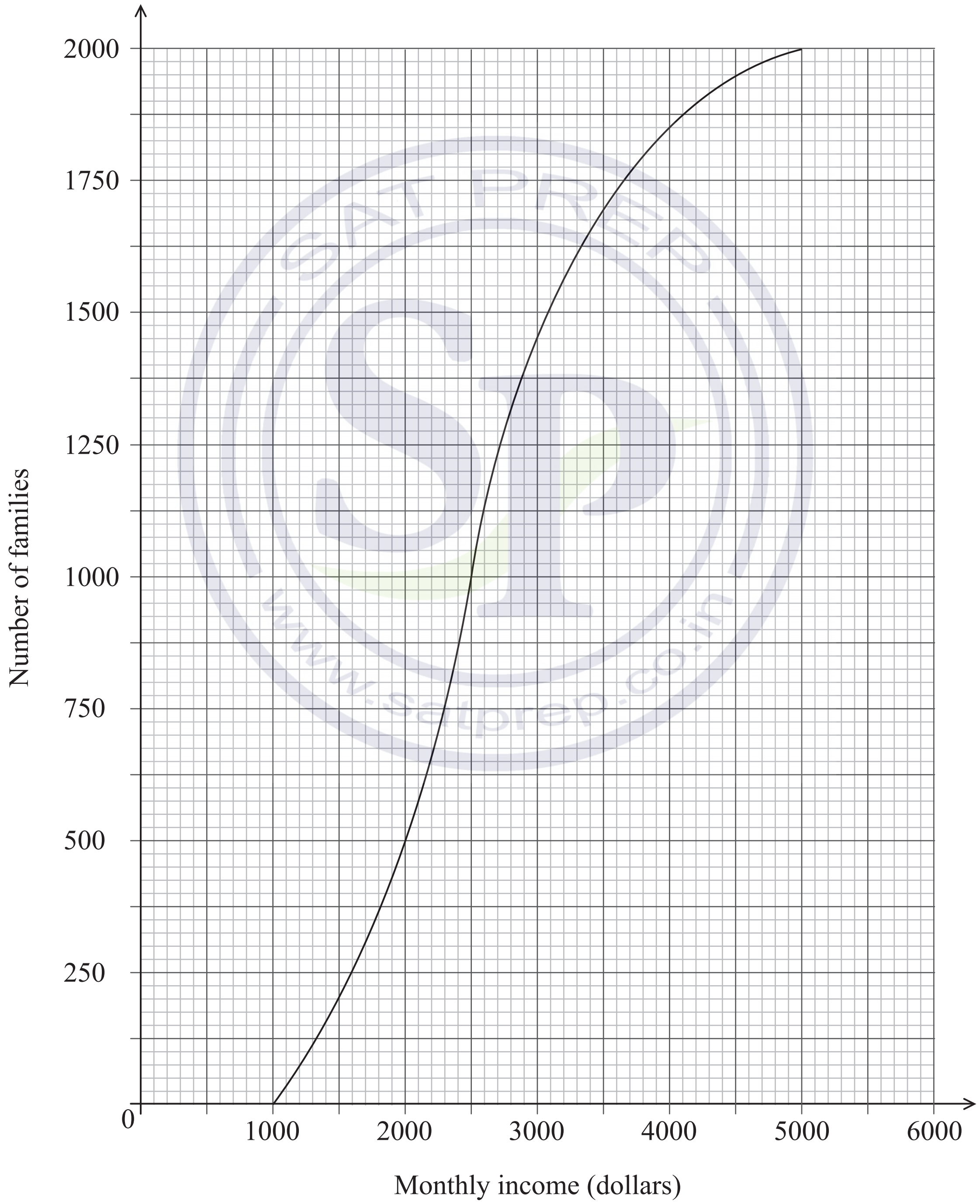
*Do **NOT** write solutions on this page.*

SECTION B

*Answer **all** questions in the answer booklet provided. Please start each question on a new page.*

8. *[Maximum mark: 15]*

The following cumulative frequency graph shows the monthly income, I dollars, of 2000 families.



(This question continues on the following page)

*Do **NOT** write solutions on this page.*

(Question 8 continued)

(a) Find the median monthly income. *[2]*

(b) (i) Write down the number of families who have a monthly income of 2000 dollars or less.

(ii) Find the number of families who have a monthly income of more than 4000 dollars. *[4]*

The 2000 families live in two different types of housing. The following table gives information about the number of families living in each type of housing and their monthly income I.

	$1000 < I \le 2000$	$2000 < I \le 4000$	$4000 < I \le 5000$
Apartment	436	765	28
Villa	64	p	122

(c) Find the value of p. *[2]*

(d) A family is chosen at random.

(i) Find the probability that this family lives in an apartment.

(ii) Find the probability that this family lives in an apartment, given that its monthly income is greater than 4000 dollars. *[4]*

(e) Estimate the mean monthly income for families living in a villa. *[3]*

*Do **NOT** write solutions on this page.*

9. *[Maximum mark: 14]*

The first two terms of a geometric sequence u_n are $u_1 = 4$ and $u_2 = 4.2$.

(a) (i) Find the common ratio.

(ii) Hence or otherwise, find u_5. *[5]*

Another sequence v_n is defined by $v_n = an^k$, where $a, k \in \mathbb{R}$, and $n \in \mathbb{Z}^+$, such that $v_1 = 0.05$ and $v_2 = 0.25$.

(b) (i) Find the value of a.

(ii) Find the value of k. *[5]*

(c) Find the smallest value of n for which $v_n > u_n$. *[4]*

*Do **NOT** write solutions on this page.*

10. *[Maximum mark: 16]*

The weights of fish in a lake are normally distributed with a mean of 760 g and standard deviation σ. It is known that 78.87% of the fish have weights between 705 g and 815 g.

(a) (i) Write down the probability that a fish weighs more than 760 g.

(ii) Find the probability that a fish weighs less than 815 g. *[4]*

(b) (i) Write down the standardized value for 815 g.

(ii) Hence or otherwise, find σ. *[4]*

A fishing contest takes place in the lake. Small fish, called tiddlers, are thrown back into the lake. The maximum weight of a tiddler is 1.5 standard deviations below the mean.

(c) Find the maximum weight of a tiddler. *[2]*

(d) A fish is caught at random. Find the probability that it is a tiddler. *[2]*

(e) 25% of the fish in the lake are salmon. 10% of the salmon are tiddlers. Given that a fish caught at random is a tiddler, find the probability that it is a salmon. *[4]*

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Answers written on this page
will not be marked.

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M14/5/MATME/SP2/ENG/TZ1/XX

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MATHEMATICS
STANDARD LEVEL
PAPER 2

Candidate session number

Wednesday 14 May 2014 (morning)

1 hour 30 minutes

Examination code

2	2	1	4	–	7	3	0	4

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the ***Mathematics SL*** *formula booklet* is required for this paper.
- The maximum mark for this examination paper is *[90 marks]*.

12 pages

Please **do not** write on this page.

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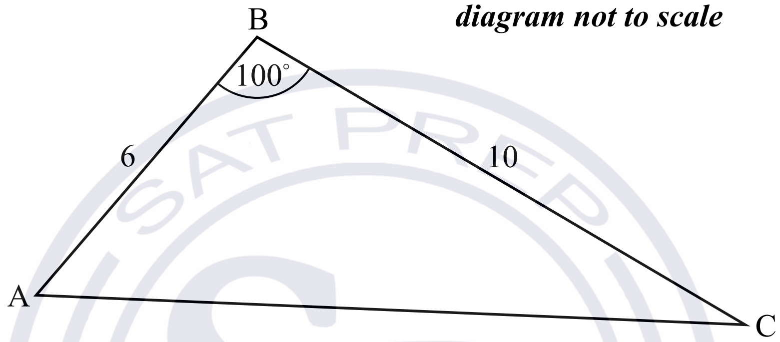
SECTION A

*Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.*

1. *[Maximum mark: 6]*

The following diagram shows triangle ABC.



$AB = 6\,cm$, $BC = 10\,cm$, and $A\hat{B}C = 100^\circ$.

(a) Find AC. *[3]*

(b) Find $B\hat{C}A$. *[3]*

2. *[Maximum mark: 5]*

Consider the expansion of $(x+3)^{10}$.

(a) Write down the number of terms in this expansion. *[1]*

(b) Find the term containing x^3. *[4]*

. . . .

3. *[Maximum mark: 7]*

The following table shows the average weights (y kg) for given heights (x cm) in a population of men.

Heights (x cm)	165	170	175	180	185
Weights (y kg)	67.8	70.0	72.7	75.5	77.2

(a) The relationship between the variables is modelled by the regression equation $y = ax + b$.

(i) Write down the value of a and of b.

(ii) Hence, estimate the weight of a man whose height is 172 cm. *[4]*

(b) (i) Write down the correlation coefficient.

(ii) State which **two** of the following describe the correlation between the variables. *[3]*

strong	zero	positive
negative	no correlation	weak

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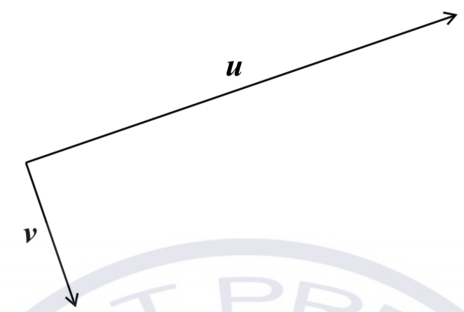
4. *[Maximum mark: 6]*

The following diagram shows two perpendicular vectors $\boldsymbol{u}$ and $\boldsymbol{v}$.



(a) Let $\boldsymbol{w} = \boldsymbol{u} - \boldsymbol{v}$. Represent $\boldsymbol{w}$ on the diagram above. *[2]*

(b) Given that $\boldsymbol{u} = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$ and $\boldsymbol{v} = \begin{pmatrix} 5 \\ n \\ 3 \end{pmatrix}$, where $n \in \mathbb{Z}$, find n. *[4]*

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5. *[Maximum mark: 6]*

The population of deer in an enclosed game reserve is modelled by the function $P(t) = 210\sin(0.5t - 2.6) + 990$, where t is in months, and $t = 1$ corresponds to 1 January 2014.

(a) Find the number of deer in the reserve on 1 May 2014. *[3]*

(b) (i) Find the rate of change of the deer population on 1 May 2014.

(ii) Interpret the answer to part (i) with reference to the deer population size on 1 May 2014. *[3]*

6. *[Maximum mark: 8]*

Ramiro and Lautaro are travelling from Buenos Aires to El Moro.

Ramiro travels in a vehicle whose velocity in m s^{-1} is given by $V_R = 40 - t^2$, where t is in seconds.

Lautaro travels in a vehicle whose displacement from Buenos Aires in metres is given by $S_L = 2t^2 + 60$.

When $t = 0$, both vehicles are at the same point.

Find Ramiro's displacement from Buenos Aires when $t = 10$.

7. *[Maximum mark: 7]*

Let $f(x) = \frac{g(x)}{h(x)}$, where $g(2) = 18$, $h(2) = 6$, $g'(2) = 5$, and $h'(2) = 2$. Find the equation of the normal to the graph of f at $x = 2$.

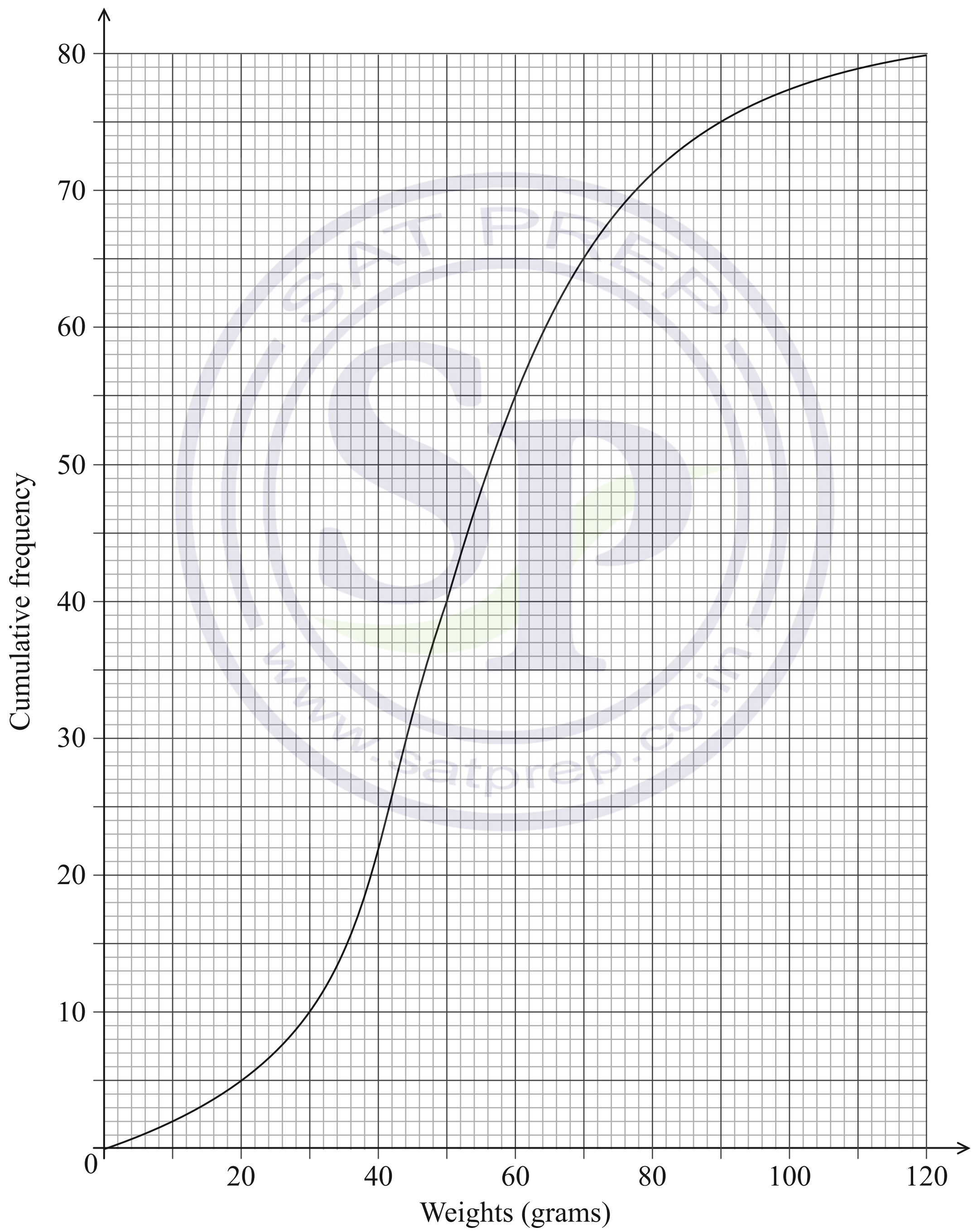
*Do **NOT** write solutions on this page.*

SECTION B

*Answer **all** questions in the answer booklet provided. Please start each question on a new page.*

8. *[Maximum mark: 16]*

The weights in grams of 80 rats are shown in the following cumulative frequency diagram.



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*Do **NOT** write solutions on this page.*

(Question 8 continued)

(a) (i) Write down the median weight of the rats.

(ii) Find the percentage of rats that weigh 70 grams or less. *[4]*

The same data is presented in the following table.

Weights *w* grams	$0 \le w \le 30$	$30 < w \le 60$	$60 < w \le 90$	$90 < w \le 120$
Frequency	p	45	q	5

(b) (i) Write down the value of p.

(ii) Find the value of q. *[4]*

(c) Use the values from the table to estimate the mean and standard deviation of the weights. *[3]*

Assume that the weights of these rats are normally distributed with the mean and standard deviation estimated in part (c).

(d) Find the percentage of rats that weigh 70 grams or less. *[2]*

(e) A sample of five rats is chosen at random. Find the probability that at most three rats weigh 70 grams or less. *[3]*

*Do **NOT** write solutions on this page.*

9. *[Maximum mark: 15]*

Let $f(x)=\cos\left(\frac{\pi}{4}x\right)+\sin\left(\frac{\pi}{4}x\right)$, for $-4\le x\le 4$.

(a) Sketch the graph of f. *[3]*

(b) Find the values of x where the function is decreasing. *[5]*

(c) The function f can also be written in the form $f(x)=a\sin\left(\frac{\pi}{4}(x+c)\right)$, where $a\in\mathbb{R}$, and $0\le c\le 2$. Find the value of

(i) a;

(ii) c. *[7]*

10. *[Maximum mark: 14]*

Let $f(x)=\frac{3x}{x-q}$, where $x\ne q$.

(a) Write down the equations of the vertical and horizontal asymptotes of the graph of f. *[2]*

The vertical and horizontal asymptotes to the graph of f intersect at the point $\mathrm{Q}(1, 3)$.

(b) Find the value of q. *[2]*

(c) The point $\mathrm{P}(x, y)$ lies on the graph of f. Show that $\mathrm{PQ}=\sqrt{(x-1)^2+\left(\frac{3}{x-1}\right)^2}$. *[4]*

(d) Hence find the coordinates of the points on the graph of f that are closest to $(1, 3)$. *[6]*

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MATHEMATICS
STANDARD LEVEL
PAPER 2

Candidate session number

Wednesday 14 May 2014 (morning)

1 hour 30 minutes

Examination code

2	2	1	4	–	7	3	0	6

INSTRUCTIONS TO CANDIDATES

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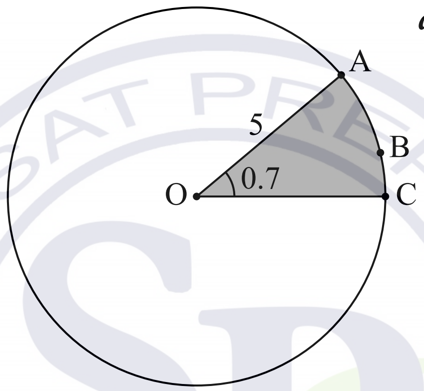
SECTION A

*Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.*

1. *[Maximum mark: 6]*

The following diagram shows a circle with centre O and radius 5 cm.

diagram not to scale



The points A, B and C lie on the circumference of the circle, and $\hat{AOC} = 0.7$ radians .

(a) (i) Find the length of the arc ABC.

(ii) Find the perimeter of the shaded sector. *[4]*

(b) Find the area of the shaded sector. *[2]*

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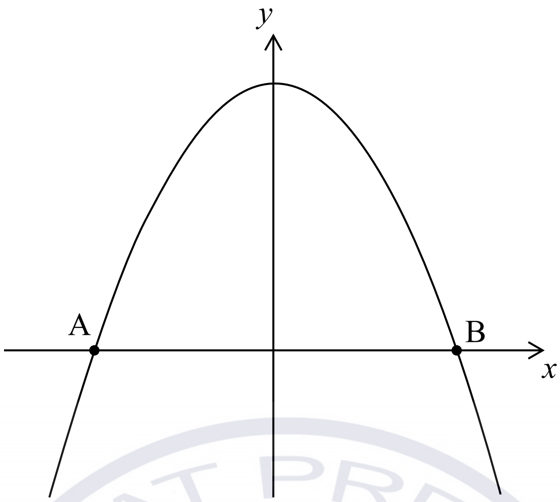
(Question 1 continued)

2. *[Maximum mark: 6]*

Let $f(x) = 5 - x^2$. Part of the graph of f is shown in the following diagram.



The graph crosses the x-axis at the points A and B.

(a) Find the x-coordinate of A and of B. *[3]*

(b) The region enclosed by the graph of f and the x-axis is revolved 360° about the x-axis. Find the volume of the solid formed. *[3]*

3. *[Maximum mark: 5]*

The following table shows the amount of fuel (y litres) used by a car to travel certain distances (x km).

Distance (x km)	40	75	120	150	195
Amount of fuel (y litres)	3.6	6.5	9.9	13.1	16.2

This data can be modelled by the regression line with equation $y = ax + b$.

(a) (i) Write down the value of a and of b.

(ii) Explain what the gradient a represents. *[3]*

(b) Use the model to estimate the amount of fuel the car would use if it is driven 110 km. *[2]*

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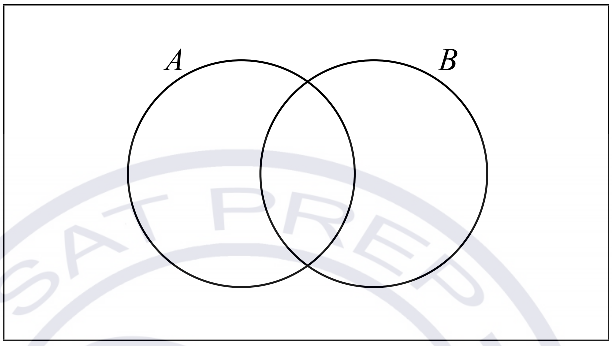
4. *[Maximum mark: 7]*

Let A and B be independent events, where $\mathrm{P}(A) = 0.3$ and $\mathrm{P}(B) = 0.6$.

(a) Find $\mathrm{P}(A \cap B)$. *[2]*

(b) Find $\mathrm{P}(A \cup B)$. *[2]*

(c) (i) On the following Venn diagram, shade the region that represents $A \cap B'$.



(ii) Find $\mathrm{P}(A \cap B')$. *[3]*

5. *[Maximum mark: 7]*

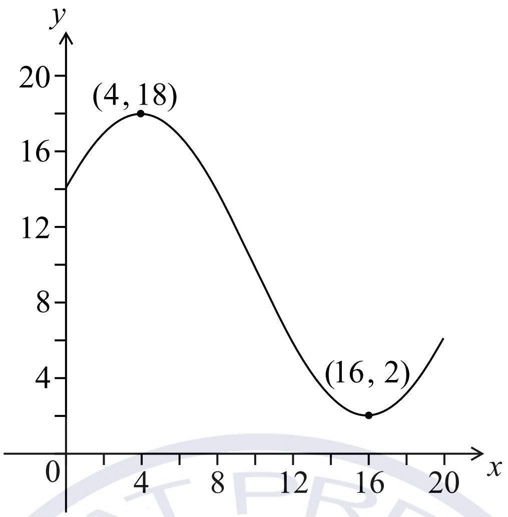
In triangle ABC, $AB = 6\text{ cm}$ and $AC = 8\text{ cm}$. The area of the triangle is 16 cm^2.

(a) Find the two possible values for $\hat{A}$. *[4]*

(b) Given that $\hat{A}$ is obtuse, find BC. *[3]*

6. *[Maximum mark: 8]*

Let $f(x) = p\cos\left(q(x+r)\right)+10$, for $0 \le x \le 20$. The following diagram shows the graph of f.



The graph has a maximum at $(4, 18)$ and a minimum at $(16, 2)$.

(a) Write down the value of r. *[2]*

(b) (i) Find p.

(ii) Find q. *[4]*

(c) Solve $f(x) = 7$. *[2]*

7. *[Maximum mark: 7]*

Consider the expansion of $x^2\left(3x^2+\frac{k}{x}\right)^8$. The constant term is 16 128.

Find k.

*Do **NOT** write solutions on this page.*

SECTION B

*Answer **all** questions in the answer booklet provided. Please start each question on a new page.*

8. *[Maximum mark: 15]*

The number of bacteria in two colonies, A and B, starts increasing at the same time.

The number of bacteria in colony A after t hours is modelled by the function $A(t) = 12e^{0.4t}$.

(a) Find the initial number of bacteria in colony A. *[2]*

(b) Find the number of bacteria in colony A after four hours. *[3]*

(c) How long does it take for the number of bacteria in colony A to reach 400? *[3]*

The number of bacteria in colony B after t hours is modelled by the function $B(t) = 24e^{kt}$.

(d) After four hours, there are 60 bacteria in colony B. Find the value of k. *[3]*

(e) The number of bacteria in colony A first exceeds the number of bacteria in colony B after n hours, where $n \in \mathbb{Z}$. Find the value of n. *[4]*

*Do **NOT** write solutions on this page.*

9. *[Maximum mark: 15]*

A particle moves in a straight line. Its velocity, $v\,\text{ms}^{-1}$, at time t seconds, is given by

$$v = (t^2 - 4)^3\text{, for } 0 \le t \le 3.$$

(a) Find the velocity of the particle when $t = 1$. *[2]*

(b) Find the value of t for which the particle is at rest. *[3]*

(c) Find the total distance the particle travels during the first three seconds. *[3]*

(d) Show that the acceleration of the particle is given by $a = 6t(t^2 - 4)^2$. *[3]*

(e) Find all possible values of t for which the velocity and acceleration are both positive or both negative. *[4]*

*Do **NOT** write solutions on this page.*

10. *[Maximum mark: 14]*

A forest has a large number of tall trees. The heights of the trees are normally distributed with a mean of 53 metres and a standard deviation of 8 metres. Trees are classified as giant trees if they are more than 60 metres tall.

(a) A tree is selected at random from the forest.

(i) Find the probability that this tree is a giant.

(ii) Given that this tree is a giant, find the probability that it is taller than 70 metres. *[6]*

(b) Two trees are selected at random. Find the probability that they are both giants. *[2]*

(c) 100 trees are selected at random.

(i) Find the expected number of these trees that are giants.

(ii) Find the probability that at least 25 of these trees are giants. *[6]*

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MATHEMATICS
STANDARD LEVEL
PAPER 2

Tuesday 12 November 2013 (morning)

1 hour 30 minutes

Candidate session number

0	0							

Examination code

8	8	1	3	–	7	3	0	2

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
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11 pages

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SECTION A

*Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.*

1. *[Maximum mark: 5]*

Let $\boldsymbol{A} = \begin{pmatrix} 0 & 5 & 4 \\ 1 & 2 & 1 \\ 2 & 2 & 0 \end{pmatrix}$, and $\boldsymbol{B} = \begin{pmatrix} 11 \\ 7 \\ 10 \end{pmatrix}$.

(a) Write down $\boldsymbol{A}^{-1}$. *[2]*

(b) Hence or otherwise, solve the equation $\boldsymbol{AX} = \boldsymbol{B}$. *[3]*

2. *[Maximum mark: 6]*

Let $f(x) = (x-1)(x-4)$.

(a) Find the x-intercepts of the graph of f. *[3]*

(b) The region enclosed by the graph of f and the x-axis is rotated 360° about the x-axis. Find the volume of the solid formed. *[3]*

3. *[Maximum mark: 6]*

Let $f(x) = \sqrt[3]{x^4} - \frac{1}{2}$.

(a) Find $f'(x)$. *[2]*

(b) Find $\int f(x)\mathrm{d}x$. *[4]*

4. *[Maximum mark: 6]*

Two events A and B are such that $\mathrm{P}(A)=0.2$ and $\mathrm{P}(A\cup B)=0.5$.

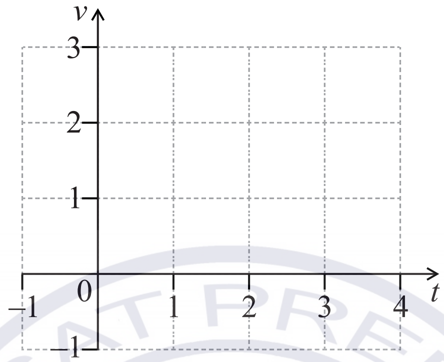
(a) Given that A and B are mutually exclusive, find $\mathrm{P}(B)$. *[2]*

(b) Given that A and B are independent, find $\mathrm{P}(B)$. *[4]*

5. *[Maximum mark: 8]*

A particle moves along a straight line such that its velocity, $v\,\mathrm{m\,s^{-1}}$, is given by $v(t) = 10t\mathrm{e}^{-1.7t}$, for $t \geq 0$.

(a) On the grid below, sketch the graph of v, for $0 \leq t \leq 4$. *[3]*



(b) Find the distance travelled by the particle in the first three seconds. *[2]*

(c) Find the velocity of the particle when its acceleration is zero. *[3]*

6. *[Maximum mark: 7]*

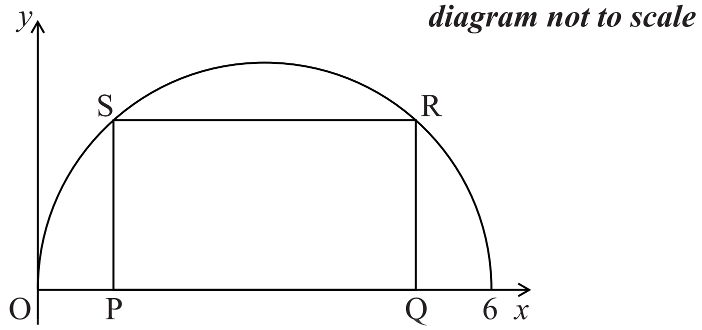
The time taken for a student to complete a task is normally distributed with a mean of 20 minutes and a standard deviation of 1.25 minutes.

(a) A student is selected at random. Find the probability that the student completes the task in less than 21.8 minutes. *[2]*

(b) The probability that a student takes between k and 21.8 minutes is 0.3. Find the value of k. *[5]*

7. *[Maximum mark: 7]*

Consider the graph of the semicircle given by $f(x) = \sqrt{6x - x^2}$, for $0 \le x \le 6$. A rectangle PQRS is drawn with upper vertices R and S on the graph of f, and PQ on the x-axis, as shown in the following diagram.



(a) Let $\text{OP} = x$.

(i) Find PQ, giving your answer in terms of x.

(ii) Hence, write down an expression for the area of the rectangle, giving your answer in terms of x. *[3]*

(b) (i) Find the rate of change of area when $x = 2$.

(ii) The area is decreasing for $a < x < b$. Find the value of a and of b. *[4]*

. . . .

*Do **NOT** write solutions on this page.*

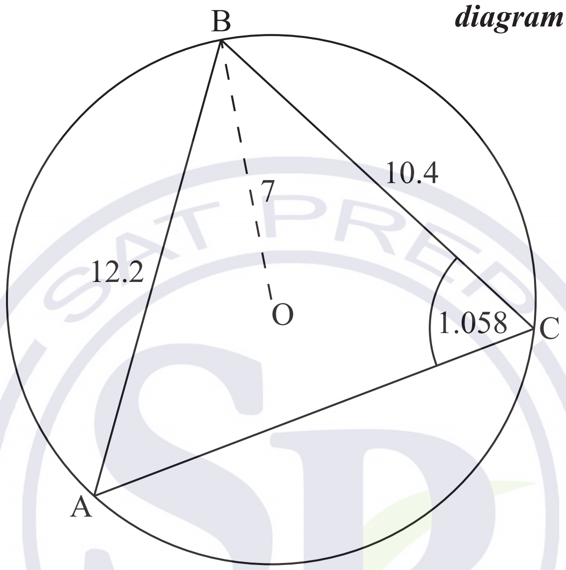
SECTION B

*Answer **all** questions in the answer booklet provided. Please start each question on a new page.*

8. *[Maximum mark: 14]*

Consider a circle with centre O and radius 7 cm. Triangle ABC is drawn such that its vertices are on the circumference of the circle.

diagram not to scale



$AB = 12.2\text{ cm}$, $BC = 10.4\text{ cm}$ and $A\hat{C}B = 1.058$ radians.

(a) Find $B\hat{A}C$. *[3]*

(b) Find AC. *[5]*

(c) Hence or otherwise, find the length of arc ABC. *[6]*

Turn over

*Do **NOT** write solutions on this page.*

9. *[Maximum mark: 17]*

Consider the lines L_1 and L_2 with equations $L_1 : \boldsymbol{r} = \begin{pmatrix} 11 \\ 8 \\ 2 \end{pmatrix} + s\begin{pmatrix} 4 \\ 3 \\ -1 \end{pmatrix}$ and $L_2 : \boldsymbol{r} = \begin{pmatrix} 1 \\ 1 \\ -7 \end{pmatrix} + t\begin{pmatrix} 2 \\ 1 \\ 11 \end{pmatrix}$.

The lines intersect at point P.

(a) Find the coordinates of P. *[6]*

(b) Show that the lines are perpendicular. *[5]*

(c) The point Q(7, 5, 3) lies on L_1. The point R is the reflection of Q in the line L_2. Find the coordinates of R. *[6]*

*Do **NOT** write solutions on this page.*

10. *[Maximum mark: 14]*

Samantha goes to school five days a week. When it rains, the probability that she goes to school by bus is 0.5. When it does not rain, the probability that she goes to school by bus is 0.3. The probability that it rains on any given day is 0.2.

(a) On a randomly selected school day, find the probability that Samantha goes to school by bus. *[4]*

(b) Given that Samantha went to school by bus on Monday, find the probability that it was raining. *[3]*

(c) In a randomly chosen school week, find the probability that Samantha goes to school by bus on exactly three days. *[2]*

(d) After n school days, the probability that Samantha goes to school by bus at least once is greater than 0.95. Find the smallest value of n. *[5]*

Please **do not** write on this page.

Answers written on this page
will not be marked.

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MATHEMATICS
STANDARD LEVEL
PAPER 2

Candidate session number

0	0							

Friday 10 May 2013 (morning)

Examination code

2	2	1	3	–	7	3	0	4

1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

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SECTION A

*Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.*

1. *[Maximum mark: 7]*

An arithmetic sequence is given by $5, 8, 11, \ldots$.

(a) Write down the value of d. *[1 mark]*

(b) Find

(i) u_{100};

(ii) S_{100}. *[4 marks]*

(c) Given that $u_n = 1502$, find the value of n. *[2 marks]*

2. *[Maximum mark: 6]*

Consider the following cumulative frequency table.

x	**Frequency**	**Cumulative frequency**
5	2	2
15	10	12
25	14	26
35	p	35
45	6	41

(a) Find the value of p. *[2 marks]*

(b) Find

(i) the mean;

(ii) the variance. *[4 marks]*

3. *[Maximum mark: 5]*

In the expansion of $(3x-2)^{12}$, the term in x^5 can be expressed as $\binom{12}{r}\times(3x)^p\times(-2)^q$.

(a) Write down the value of p, of q and of r. *[3 marks]*

(b) Find the coefficient of the term in x^5. *[2 marks]*

4. *[Maximum mark: 6]*

Consider the system of equations

$$\begin{aligned} -x - y + z &= 2.5 \\ x + y &= 1 \\ -2x - y + 2z &= -3 \end{aligned}$$

This system can be represented by the matrix equation $\boldsymbol{AX} = \boldsymbol{B}$, where $\boldsymbol{X} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$.

(a) (i) Write down the matrix $\boldsymbol{A}$.

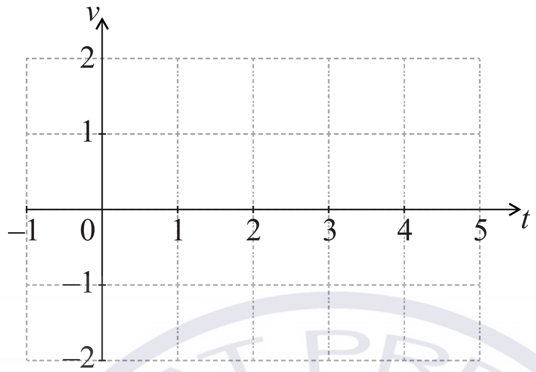
(ii) Write down the matrix $\boldsymbol{A}^{-1}$. *[3 marks]*

(b) Hence, find $\boldsymbol{X}$. *[3 marks]*

5. *[Maximum mark: 8]*

The velocity of a particle in m s^{-1} is given by $v = e^{\sin t} - 1$, for $0 \le t \le 5$.

(a) On the grid below, sketch the graph of v. *[3 marks]*



(b) (i) Write down the positive t-intercept.

(ii) Find the total distance travelled by the particle in the first five seconds. *[5 marks]*

. . . .

6. *[Maximum mark: 6]*

Let f and g be functions such that $g(x) = 2f(x+1) + 5$.

(a) The graph of f is mapped to the graph of g under the following transformations:

vertical stretch by a factor of k, followed by a translation $\begin{pmatrix} p \\ q \end{pmatrix}$.

Write down the value of

(i) k;

(ii) p;

(iii) q. *[3 marks]*

(b) Let $h(x) = -g(3x)$. The point $\mathrm{A}(6, 5)$ on the graph of g is mapped to the point A' on the graph of h. Find A'. *[3 marks]*

7. *[Maximum mark: 7]*

A random variable X is normally distributed with $\mu = 150$ and $\sigma = 10$.

Find the interquartile range of X.

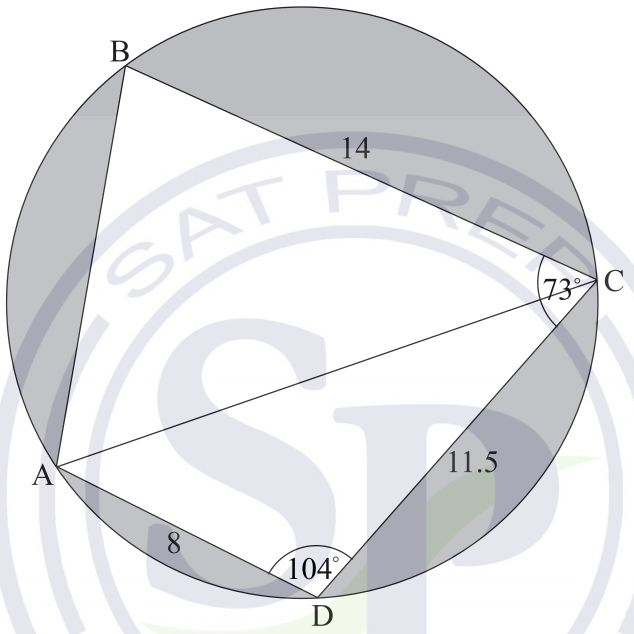
*Do **NOT** write solutions on this page.*

SECTION B

*Answer **all** questions in the answer booklet provided. Please start each question on a new page.*

8. *[Maximum mark: 14]*

The diagram shows a circle of radius 8 metres. The points ABCD lie on the circumference of the circle.



BC = 14 m, CD = 11.5 m, AD = 8 m, $A\hat{D}C = 104^\circ$, and $B\hat{C}D = 73^\circ$

(a) Find AC. *[3 marks]*

(b) (i) Find $A\hat{C}D$.

(ii) Hence, find $A\hat{C}B$. *[5 marks]*

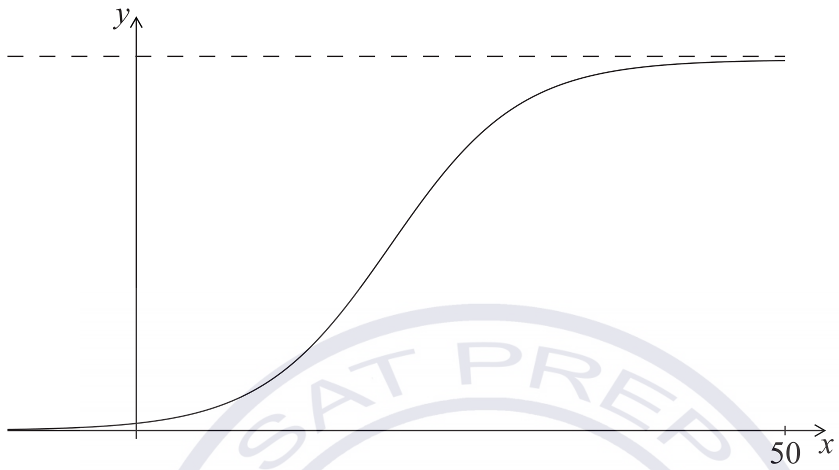
(c) Find the area of triangle ADC. *[2 marks]*

(d) Hence or otherwise, find the total area of the shaded regions. *[4 marks]*

*Do **NOT** write solutions on this page.*

9. *[Maximum mark: 15]*

Let $f(x) = \dfrac{100}{\left(1 + 50\mathrm{e}^{-0.2x}\right)}$. Part of the graph of f is shown below.



(a) Write down $f(0)$. *[1 mark]*

(b) Solve $f(x) = 95$. *[2 marks]*

(c) Find the range of f. *[3 marks]*

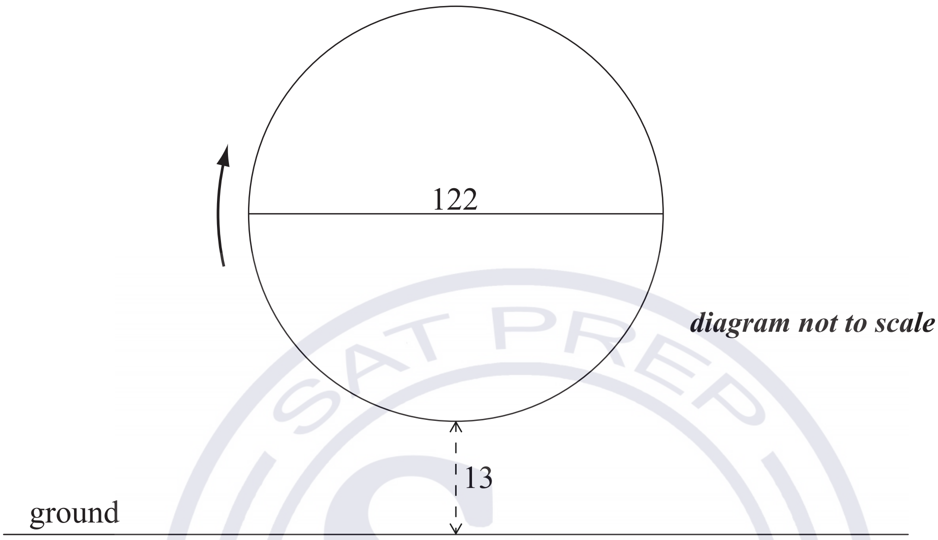
(d) Show that $f'(x) = \dfrac{1000\mathrm{e}^{-0.2x}}{\left(1 + 50\mathrm{e}^{-0.2x}\right)^2}$. *[5 marks]*

(e) Find the maximum rate of change of f. *[4 marks]*

*Do **NOT** write solutions on this page.*

10. *[Maximum mark: 16]*

A Ferris wheel with diameter 122 metres rotates clockwise at a constant speed. The wheel completes 2.4 rotations every hour. The bottom of the wheel is 13 metres above the ground.



A seat starts at the bottom of the wheel.

(a) Find the maximum height above the ground of the seat. *[2 marks]*

After t minutes, the height h metres above the ground of the seat is given by

$$h = 74 + a\cos bt .$$

(b) (i) Show that the period of h is 25 minutes.

(ii) Write down the **exact** value of b. *[2 marks]*

(c) Find the value of a. *[3 marks]*

(d) Sketch the graph of h, for $0 \le t \le 50$. *[4 marks]*

(e) In one rotation of the wheel, find the probability that a randomly selected seat is at least 105 metres above the ground. *[5 marks]*

Please **do not** write on this page.

Answers written on this page will not be marked.

M13/5/MATME/SP2/ENG/TZ2/XX

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MATHEMATICS
STANDARD LEVEL
PAPER 2

Friday 10 May 2013 (morning)

1 hour 30 minutes

Candidate session number

0	0							

Examination code

2	2	1	3	–	7	3	0	6

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the ***Mathematics SL*** *information booklet* is required for this paper.
- The maximum mark for this examination paper is *[90 marks]*.

0112

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, for example if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

*Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.*

1. *[Maximum mark: 5]*

Let $\boldsymbol{A} = \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \\ -1 & 1 & 1 \end{pmatrix}$ and $\boldsymbol{B} = \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix}$.

(a) Write down $\boldsymbol{A}^{-1}$. *[2 marks]*

(b) Solve $\boldsymbol{AX} = \boldsymbol{B}$. *[3 marks]*

2. *[Maximum mark: 6]*

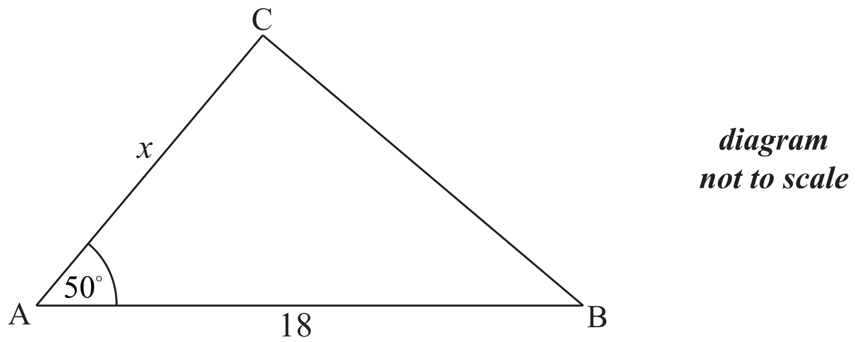
The random variable X is normally distributed with mean 20 and standard deviation 5.

(a) Find $\mathrm{P}(X \leq 22.9)$. *[3 marks]*

(b) Given that $\mathrm{P}(X < k) = 0.55$, find the value of k. *[3 marks]*

3. *[Maximum mark: 6]*

The following diagram shows a triangle ABC.



The area of triangle ABC is 80 cm^2, $\text{AB} = 18\text{ cm}$, $\text{AC} = x\text{ cm}$ and $\text{B}\hat{\text{A}}\text{C} = 50^\circ$.

(a) Find x. *[3 marks]*

(b) Find BC. *[3 marks]*

4. *[Maximum mark: 7]*

Line L_1 has equation $\boldsymbol{r}_1 = \begin{pmatrix} 10 \\ 6 \\ -1 \end{pmatrix} + s\begin{pmatrix} 2 \\ -5 \\ -2 \end{pmatrix}$ and line L_2 has equation $\boldsymbol{r}_2 = \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix} + t\begin{pmatrix} 3 \\ 5 \\ 2 \end{pmatrix}$.

Lines L_1 and L_2 intersect at point A. Find the coordinates of A.

5. *[Maximum mark: 6]*

The sum of the first three terms of a geometric sequence is 62.755, and the sum of the infinite sequence is 440. Find the common ratio.

6. *[Maximum mark: 7]*

The constant term in the expansion of $\left(\frac{x}{a}+\frac{a^2}{x}\right)^6$, where $a \in \mathbb{Z}$, is 1280. Find a.

7. *[Maximum mark: 8]*

The following diagram shows a circle with centre O and radius r cm.

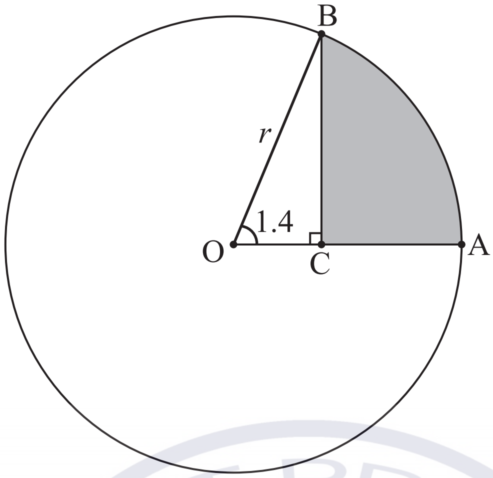


diagram not to scale

Points A and B are on the circumference of the circle and $A\hat{O}B = 1.4$ radians.
The point C is on [OA] such that $B\hat{C}O = \frac{\pi}{2}$ radians.

(a) Show that $OC = r\cos 1.4$. *[1 mark]*

(b) The area of the shaded region is 25 cm^2. Find the value of r. *[7 marks]*

*Do **NOT** write solutions on this page.*

SECTION B

*Answer **all** questions in the answer booklet provided. Please start each question on a new page.*

8. *[Maximum mark: 15]*

Consider the points $\text{A}(5, 2, 1)$, $\text{B}(6, 5, 3)$, and $\text{C}(7, 6, a+1)$, where $a \in \mathbb{R}$.

(a) Find

(i) $\overrightarrow{\text{AB}}$;

(ii) $\overrightarrow{\text{AC}}$. *[3 marks]*

Let α be the angle between $\overrightarrow{\text{AB}}$ and $\overrightarrow{\text{AC}}$.

(b) Find the value of a for which $\theta = \frac{\pi}{2}$. *[4 marks]*

(c) (i) Show that $\cos\theta = \frac{2a+14}{\sqrt{14a^2+280}}$.

(ii) Hence, find the value of a for which $\alpha = 1.2$. *[8 marks]*

*Do **NOT** write solutions on this page.*

9. *[Maximum mark: 15]*

A bag contains four gold balls and six silver balls.

(a) Two balls are drawn at random from the bag, with replacement. Let X be the number of gold balls drawn from the bag.

(i) Find $\mathrm{P}(X=0)$.

(ii) Find $\mathrm{P}(X=1)$.

(iii) Hence, find $\mathrm{E}(X)$. *[8 marks]*

Fourteen balls are drawn from the bag, with replacement.

(b) Find the probability that exactly five of the balls are gold. *[2 marks]*

(c) Find the probability that at most five of the balls are gold. *[2 marks]*

(d) Given that at most five of the balls are gold, find the probability that exactly five of the balls are gold. Give the answer correct to two decimal places. *[3 marks]*

*Do **NOT** write solutions on this page.*

10. *[Maximum mark: 15]*

Let $f(x) = e^{\frac{x}{4}}$ and $g(x) = mx$, where $m \geq 0$, and $-5 \leq x \leq 5$. Let R be the region enclosed by the y-axis, the graph of f, and the graph of g.

(a) Let $m = 1$.

(i) Sketch the graphs of f and g on the same axes.

(ii) Find the area of R. *[7 marks]*

(b) Consider all values of m such that the graphs of f and g intersect. Find the value of m that gives the greatest value for the area of R. *[8 marks]*

Please **do not** write on this page.

Answers written on this page
will not be marked.

N12/5/MATME/SP2/ENG/TZ0/XX

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MATHEMATICS
STANDARD LEVEL
PAPER 2

Wednesday 7 November 2012 (morning)

1 hour 30 minutes

Candidate session number

0	0							

Examination code

8	8	1	2	–	7	3	0	2

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions on the answer sheets provided. Write your session number on each answer sheet, and attach them to this examination paper and your cover sheet using the tag provided.
- At the end of the examination, indicate the number of sheets used in the appropriate box on your cover sheet.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the ***Mathematics SL*** *information booklet* is required for this paper.
- The maximum mark for this examination paper is *[90 marks]*.

0112

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

*Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.*

1. *[Maximum mark: 6]*

The first three terms of an arithmetic sequence are 5, 6.7, 8.4.

(a) Find the common difference. *[2 marks]*

(b) Find the 28th term of the sequence. *[2 marks]*

(c) Find the sum of the first 28 terms. *[2 marks]*

2. *[Maximum mark: 5]*

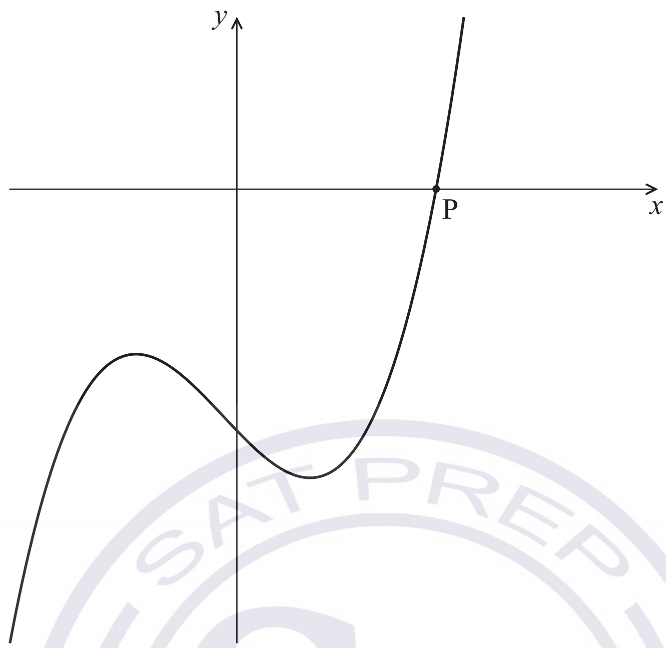
Let $\boldsymbol{A} = \begin{pmatrix} 2 & 2 & 1 \\ 3 & 1 & 0 \\ 4 & 2 & 1 \end{pmatrix}$.

(a) Write down $\boldsymbol{A}^{-1}$. *[2 marks]*

(b) Hence or otherwise, find $\boldsymbol{B}$, given that $\boldsymbol{AB} = \begin{pmatrix} -1 & 6 & -1 \\ 5 & -1 & 3 \\ 5 & 2 & 7 \end{pmatrix}$. *[3 marks]*

3. *[Maximum mark: 6]*

Let $f(x) = x^3 - 2x - 4$. The following diagram shows part of the curve of f.



The curve crosses the x-axis at the point P.

(a) Write down the x-coordinate of P. *[1 mark]*

(b) Write down the gradient of the curve at P. *[2 marks]*

(c) Find the equation of the normal to the curve at P, giving your equation in the form $y = ax + b$. *[3 marks]*

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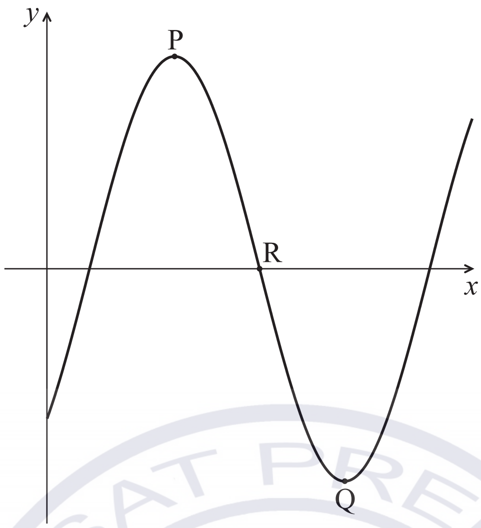
4. *[Maximum mark: 7]*

The third term in the expansion of $(2x+p)^6$ is $60x^4$. Find the possible values of p.

5. *[Maximum mark: 6]*

Let $f(x) = a\cos(b(x-c))$. The diagram below shows part of the graph of f, for $0 \le x \le 10$.



The graph has a local maximum at $P(3, 5)$, a local minimum at $Q(7, -5)$, and crosses the x-axis at R.

(a) Write down the value of

(i) a;

(ii) c. *[2 marks]*

(b) Find the value of b. *[2 marks]*

(c) Find the x-coordinate of R. *[2 marks]*

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6. *[Maximum mark: 8]*

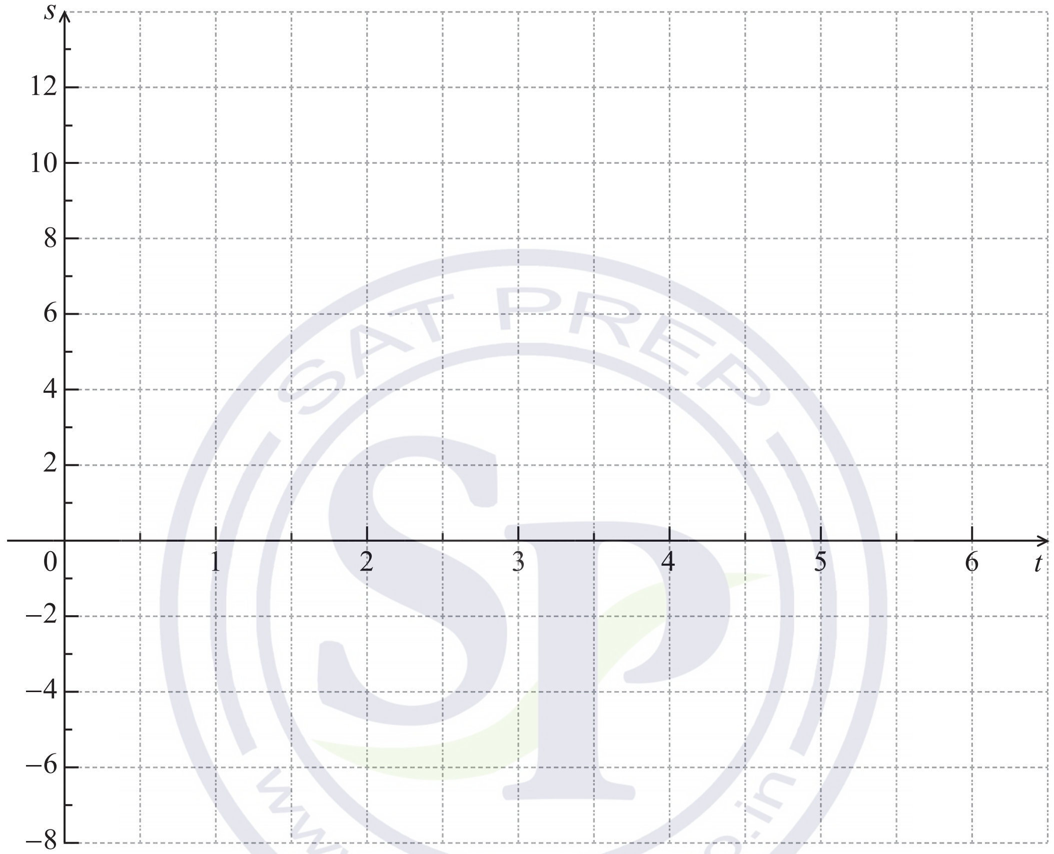
In a large city, the time taken to travel to work is normally distributed with mean μ and standard deviation σ. It is found that 4 % of the population take less than 5 minutes to get to work, and 70 % take less than 25 minutes.

Find the value of μ and of σ.

7. *[Maximum mark: 7]*

A particle's displacement, in metres, is given by $s(t) = 2t\cos t$, for $0 \le t \le 6$, where t is the time in seconds.

(a) On the grid below, sketch the graph of s. *[4 marks]*



(This question continues on the following page)

(Question 7 continued)

(b) Find the maximum velocity of the particle. *[3 marks]*

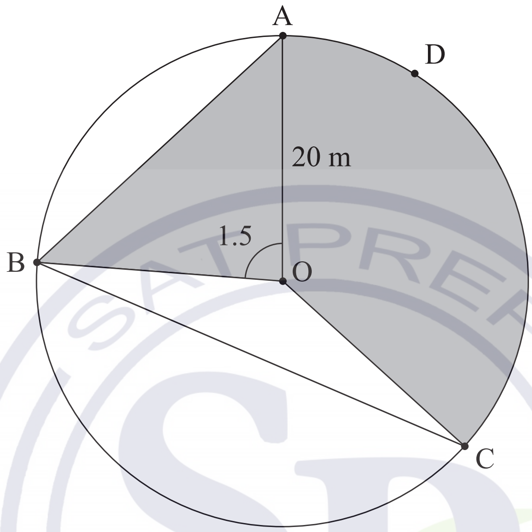
*Do **NOT** write solutions on this page.*

SECTION B

*Answer **all** questions on the answer sheets provided. Please start each question on a new page.*

8. *[Maximum mark: 15]*

The following diagram shows a circular play area for children.



The circle has centre O and a radius of 20 m, and the points A, B, C and D lie on the circle. Angle AOB is 1.5 radians.

(a) Find the length of the chord [AB]. *[3 marks]*

(b) Find the area of triangle AOB. *[2 marks]*

Angle BOC is 2.4 radians.

(c) Find the length of arc ADC. *[3 marks]*

(d) Find the area of the shaded region. *[3 marks]*

(e) The shaded region is to be painted red. Red paint is sold in cans which cost \$32 each. One can covers 140 m^2. How much does it cost to buy the paint? *[4 marks]*

*Do **NOT** write solutions on this page.*

9. *[Maximum mark: 15]*

Consider the function $f(x) = x^2 - 4x + 1$.

(a) Sketch the graph of f, for $-1 \le x \le 5$. *[4 marks]*

This function can also be written as $f(x) = (x - p)^2 - 3$.

(b) Write down the value of p. *[1 mark]*

The graph of g is obtained by reflecting the graph of f in the x-axis, followed by a translation of $\begin{pmatrix} 0 \\ 6 \end{pmatrix}$.

(c) Show that $g(x) = -x^2 + 4x + 5$. *[4 marks]*

The graphs of f and g intersect at two points.

(d) Write down the x-coordinates of these two points. *[3 marks]*

Let R be the region enclosed by the graphs of f and g.

(e) Find the area of R. *[3 marks]*

*Do **NOT** write solutions on this page.*

10. *[Maximum mark: 15]*

At a large school, students are required to learn at least one language, Spanish or French. It is known that 75 % of the students learn Spanish, and 40 % learn French.

(a) Find the percentage of students who learn **both** Spanish and French. *[2 marks]*

(b) Find the percentage of students who learn Spanish, but not French. *[2 marks]*

At this school, 52 % of the students are girls, and 85 % of the girls learn Spanish.

(c) A student is chosen at random. Let G be the event that the student is a girl, and let S be the event that the student learns Spanish.

(i) Find $\mathrm{P}(G \cap S)$.

(ii) Show that G and S are **not** independent. *[5 marks]*

(d) A boy is chosen at random. Find the probability that he learns Spanish. *[6 marks]*

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MATHEMATICS
STANDARD LEVEL
PAPER 2

Friday 4 May 2012 (morning)

1 hour 30 minutes

Candidate session number

0	0							

Examination code

2	2	1	2	–	7	3	0	4

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions on the answer sheets provided. Write your session number on each answer sheet, and attach them to this examination paper and your cover sheet using the tag provided.
- At the end of the examination, indicate the number of sheets used in the appropriate box on your cover sheet.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the ***Mathematics SL*** *information booklet* is required for this paper.
- The maximum mark for this examination paper is *[90 marks]*.

0116

Please **do not** write on this page.

Answers written on this page
will not be marked.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

*Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.*

1. *[Maximum mark: 6]*

The first three terms of an arithmetic sequence are $36, 40, 44, \ldots$.

(a) (i) Write down the value of d.

(ii) Find u_8. *[3 marks]*

(b) (i) Show that $S_n = 2n^2 + 34n$.

(ii) Hence, write down the value of S_{14}. *[3 marks]*

2. *[Maximum mark: 7]*

Let $f(x) = 2x^2 - 8x - 9$.

(a) (i) Write down the coordinates of the vertex.

(ii) Hence or otherwise, express the function in the form $f(x) = 2(x - h)^2 + k$. *[4 marks]*

(b) Solve the equation $f(x) = 0$. *[3 marks]*

3. *[Maximum mark: 6]*

Let $\boldsymbol{M} = \begin{pmatrix} x & 2x \\ x^2 & 1 \end{pmatrix}$ and $\boldsymbol{N} = \begin{pmatrix} 2 & 3 & -1 \\ 4 & 2 & 0 \\ 1 & 5 & 1 \end{pmatrix}$.

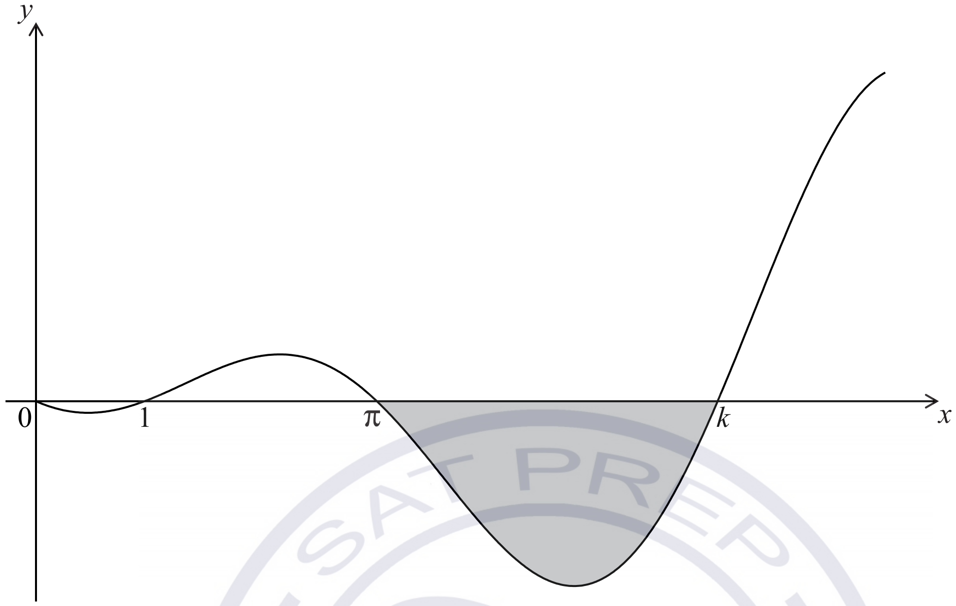
(a) Find $\det \boldsymbol{M}$. *[2 marks]*

(b) Write down $\det \boldsymbol{N}$. *[1 mark]*

(c) Find the value of x for which $\det \boldsymbol{M} = \det \boldsymbol{N}$. *[3 marks]*

4. *[Maximum mark: 7]*

The graph of $y = (x-1)\sin x$, for $0 \le x \le \frac{5\pi}{2}$, is shown below.



The graph has x-intercepts at 0, 1, π and k.

(a) Find k. *[2 marks]*

The shaded region is rotated 360° about the x-axis. Let V be the volume of the solid formed.

(b) Write down an expression for V. *[3 marks]*

(c) Find V. *[2 marks]*

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5. *[Maximum mark: 6]*

Let $\boldsymbol{M} = \begin{pmatrix} p & -1 & -2 \\ 1 & 1 & -2 \\ 1 & q & -1 \end{pmatrix}$ and $\boldsymbol{M}^{-1} = \frac{1}{2}\begin{pmatrix} 3 & -5 & 4 \\ -1 & 1 & 0 \\ 1 & -3 & 2 \end{pmatrix}$.

(a) Find the value of p and of q. *[3 marks]*

(b) Solve the system of linear equations.

$$\begin{aligned} px - y - 2z &= 7 \\ x + y - 2z &= 2 \\ x + qy - z &= -3 \end{aligned}$$

[3 marks]

6. *[Maximum mark: 6]*

Consider the expansion of $\left(2x^3+\dfrac{b}{x}\right)^8 = 256x^{24}+3072x^{20}+\ldots+kx^0+\ldots$.

(a) Find b. *[3 marks]*

(b) Find k. *[3 marks]*

7. *[Maximum mark: 7]*

The probability of obtaining “tails” when a biased coin is tossed is 0.57. The coin is tossed ten times. Find the probability of obtaining

(a) **at least** four tails; *[4 marks]*

(b) the fourth tail on the tenth toss. *[3 marks]*

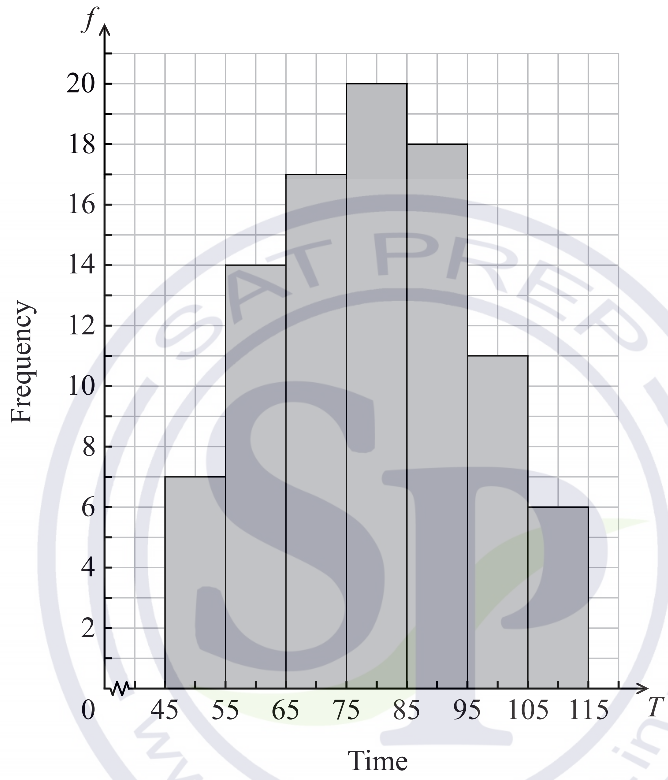
*Do **NOT** write solutions on this page.*

SECTION B

*Answer **all** questions on the answer sheets provided. Please start each question on a new page.*

8. *[Maximum mark: 13]*

The histogram below shows the time T seconds taken by 93 children to solve a puzzle.



The following is the frequency distribution for T.

Time	$45 \le T < 55$	$55 \le T < 65$	$65 \le T < 75$	$75 \le T < 85$	$85 \le T < 95$	$95 \le T < 105$	$105 \le T < 115$
Frequency	7	14	p	20	18	q	6

(a) (i) Write down the value of p and of q.

(ii) Write down the median class. *[3 marks]*

(b) A child is selected at random. Find the probability that the child takes less than 95 seconds to solve the puzzle. *[2 marks]*

(This question continues on the following page)

*Do **NOT** write solutions on this page.*

(Question 8 continued)

Consider the class interval $45 \le T < 55$.

(c) (i) Write down the interval width.

(ii) Write down the mid-interval value. *[2 marks]*

(d) Hence find an estimate for the

(i) mean;

(ii) standard deviation. *[4 marks]*

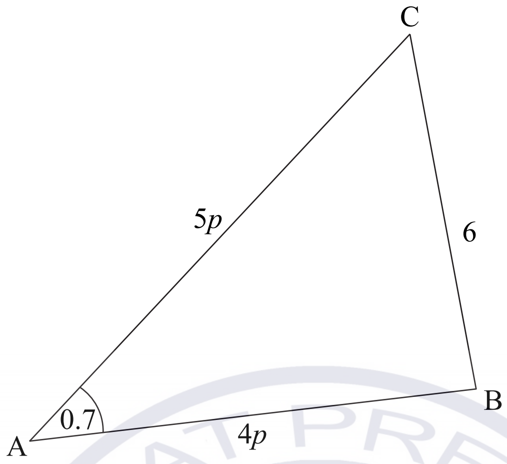
John assumes that T is normally distributed and uses this to estimate the probability that a child takes less than 95 seconds to solve the puzzle.

(e) Find John's estimate. *[2 marks]*

*Do **NOT** write solutions on this page.*

9. *[Maximum mark: 15]*

The following diagram shows a triangle ABC.

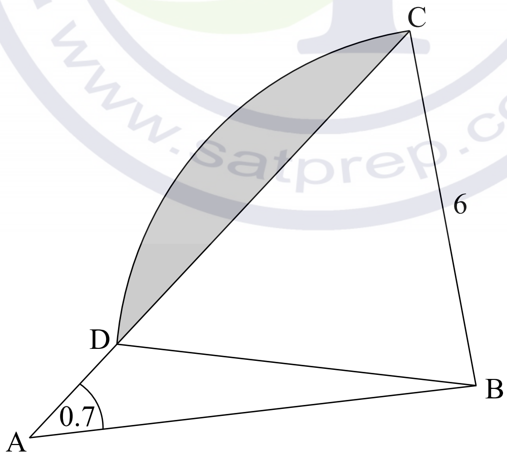


$BC = 6$, $C\hat{A}B = 0.7$ radians, $AB = 4p$, $AC = 5p$, where $p > 0$.

(a) (i) Show that $p^2(41 - 40\cos 0.7) = 36$.

(ii) Find p. *[4 marks]*

Consider the circle with centre B that passes through the point C. The circle cuts the line CA at D, and $A\hat{D}B$ is obtuse. Part of the circle is shown in the following diagram.



(b) Write down the length of BD. *[1 mark]*

(c) Find $A\hat{D}B$. *[4 marks]*

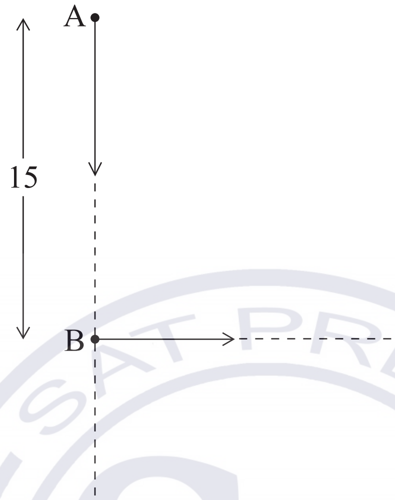
(d) (i) Show that $C\hat{B}D = 1.29$ radians, correct to 2 decimal places.

(ii) Hence, find the area of the shaded region. *[6 marks]*

*Do **NOT** write solutions on this page.*

10. *[Maximum mark: 17]*

The following diagram shows two ships A and B. At noon, ship A was 15 km due north of ship B. Ship A was moving south at 15 km h^{-1} and ship B was moving east at 11 km h^{-1}.



(a) Find the distance between the ships

(i) at 13:00;

(ii) at 14:00. *[5 marks]*

Let $s(t)$ be the distance between the ships t hours after noon, for $0 \le t \le 4$.

(b) Show that $s(t) = \sqrt{346t^2 - 450t + 225}$. *[6 marks]*

(c) Sketch the graph of $s(t)$. *[3 marks]*

(d) Due to poor weather, the captain of ship A can only see another ship if they are less than 8 km apart. Explain why the captain cannot see ship B between noon and 16:00. *[3 marks]*

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Answers written on this page
will not be marked.

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MATHEMATICS
STANDARD LEVEL
PAPER 2

Friday 4 May 2012 (morning)

1 hour 30 minutes

Candidate session number

0	0							

Examination code

2	2	1	2	–	7	3	0	6

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions on the answer sheets provided. Write your session number on each answer sheet, and attach them to this examination paper and your cover sheet using the tag provided.
- At the end of the examination, indicate the number of sheets used in the appropriate box on your cover sheet.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the ***Mathematics SL*** *information booklet* is required for this paper.
- The maximum mark for this examination paper is *[90 marks]*.

0112

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

*Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.*

1. *[Maximum mark: 6]*

The following diagram shows ΔPQR, where $RQ = 9$ cm, $P\hat{R}Q = 70^\circ$ and $P\hat{Q}R = 45^\circ$.

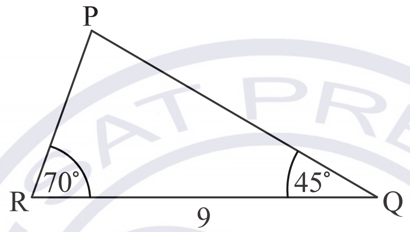

diagram not to scale

(a) Find $R\hat{P}Q$. *[1 mark]*

(b) Find PR. *[3 marks]*

(c) Find the area of ΔPQR. *[2 marks]*

2. *[Maximum mark: 6]*

Let $f(x) = \cos(e^x)$, for $-2 \le x \le 2$.

(a) Find $f'(x)$. *[2 marks]*

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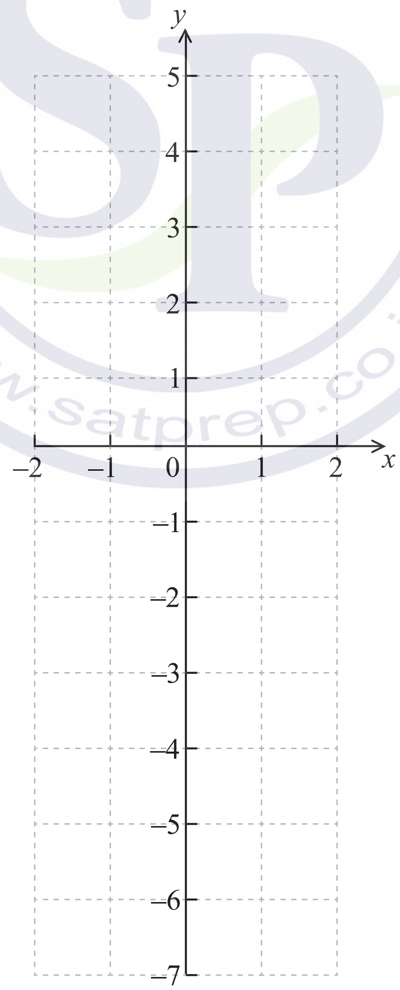
(b) On the grid below, sketch the graph of $f'(x)$. *[4 marks]*



0312

3. *[Maximum mark: 6]*

The first term of a geometric sequence is 200 and the sum of the first four terms is 324.8.

(a) Find the common ratio. *[4 marks]*

(b) Find the tenth term. *[2 marks]*

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4. *[Maximum mark: 6]*

The heights of a group of seven-year-old children are normally distributed with mean 117 cm and standard deviation 5 cm. A child is chosen at random from the group.

(a) Find the probability that this child is taller than 122.5 cm. *[3 marks]*

(b) The probability that this child is shorter than k cm is 0.65. Find the value of k. *[3 marks]*

5. *[Maximum mark: 6]*

A particle moves in a straight line with velocity $v = 12t - 2t^3 - 1$, for $t \geq 0$, where v is in centimetres per second and t is in seconds.

(a) Find the acceleration of the particle after 2.7 seconds. *[3 marks]*

(b) Find the displacement of the particle after 1.3 seconds. *[3 marks]*

6. *[Maximum mark: 7]*

Let $\boldsymbol{A} = \begin{pmatrix} -1 & -1 & 2 \\ -1 & 0 & 1 \\ 2 & 1 & -2 \end{pmatrix}$ and $\boldsymbol{B} = \begin{pmatrix} 1 & 0 & 2 \\ 0 & -1 & 0 \\ 3 & 1 & 2 \end{pmatrix}$.

(a) Write down $\boldsymbol{A}^{-1}$. *[2 marks]*

(b) Let $\boldsymbol{C}$ be a 3×3 matrix such that $\boldsymbol{ACA}^{-1} = \boldsymbol{B}$. Find $\boldsymbol{C}$. *[5 marks]*

7. *[Maximum mark: 8]*

A factory makes lamps. The probability that a lamp is defective is 0.05. A random sample of 30 lamps is tested.

(a) Find the probability that there is at least one defective lamp in the sample. *[4 marks]*

(b) Given that there is at least one defective lamp in the sample, find the probability that there are at most two defective lamps. *[4 marks]*

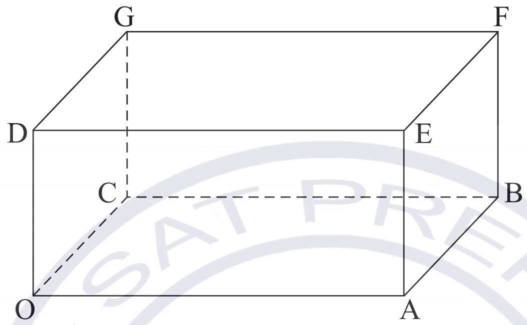
*Do **NOT** write solutions on this page.*

SECTION B

*Answer **all** questions on the answer sheets provided. Please start each question on a new page.*

8. *[Maximum mark: 16]*

The following diagram shows the cuboid (rectangular solid) OABCDEFG, where O is the origin, and $\vec{OA} = 4\boldsymbol{i}$, $\vec{OC} = 3\boldsymbol{j}$, $\vec{OD} = 2\boldsymbol{k}$.



(a) (i) Find $\vec{OB}$.

(ii) Find $\vec{OF}$.

(iii) Show that $\vec{AG} = -4\boldsymbol{i} + 3\boldsymbol{j} + 2\boldsymbol{k}$. *[5 marks]*

(b) Write down a vector equation for

(i) the line OF;

(ii) the line AG. *[4 marks]*

(c) Find the obtuse angle between the lines OF and AG. *[7 marks]*

*Do **NOT** write solutions on this page.*

9. *[Maximum mark: 13]*

Let $f(x) = ax^3 + bx^2 + c$, where a, b and c are real numbers. The graph of f passes through the point $(2, 9)$.

(a) Show that $8a + 4b + c = 9$. *[2 marks]*

The graph of f has a local minimum at $(1, 4)$.

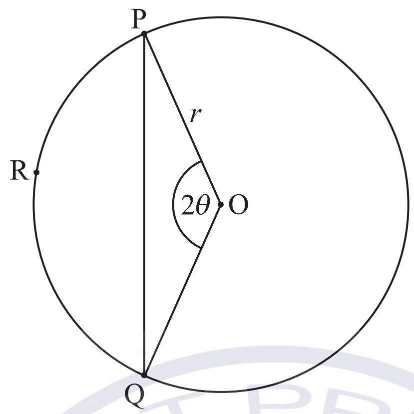
(b) Find two other equations in a, b and c, giving your answers in a similar form to part (a). *[7 marks]*

(c) Find the value of a, of b and of c. *[4 marks]*

*Do **NOT** write solutions on this page.*

10. *[Maximum mark: 16]*

Consider the following circle with centre O and radius r.



The points P, R and Q are on the circumference, $\text{P}\hat{\text{O}}\text{Q} = 2\theta$, for $0 < \theta < \frac{\pi}{2}$.

(a) Use the cosine rule to show that $\text{PQ} = 2r\sin\theta$. *[4 marks]*

Let l be the length of the arc PRQ.

(b) Given that $1.3\text{PQ} - l = 0$, find the value of θ. *[5 marks]*

Consider the function $f(\theta) = 2.6\sin\theta - 2\theta$, for $0 < \theta < \frac{\pi}{2}$.

(c) (i) Sketch the graph of f.

(ii) Write down the root of $f(\theta) = 0$. *[4 marks]*

(d) Use the graph of f to find the values of θ for which $l < 1.3\text{PQ}$. *[3 marks]*

Please **do not** write on this page.

Answers written on this page
will not be marked.

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MATHEMATICS
STANDARD LEVEL
PAPER 2

Candidate session number

0	0							

Thursday 3 November 2011 (morning)

Examination code

8	8	1	1	–	7	3	0	2

1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
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0112

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SECTION A

*Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.*

1. *[Maximum mark: 7]*

Let $f(x) = 2x + 4$ and $g(x) = 7x^2$.

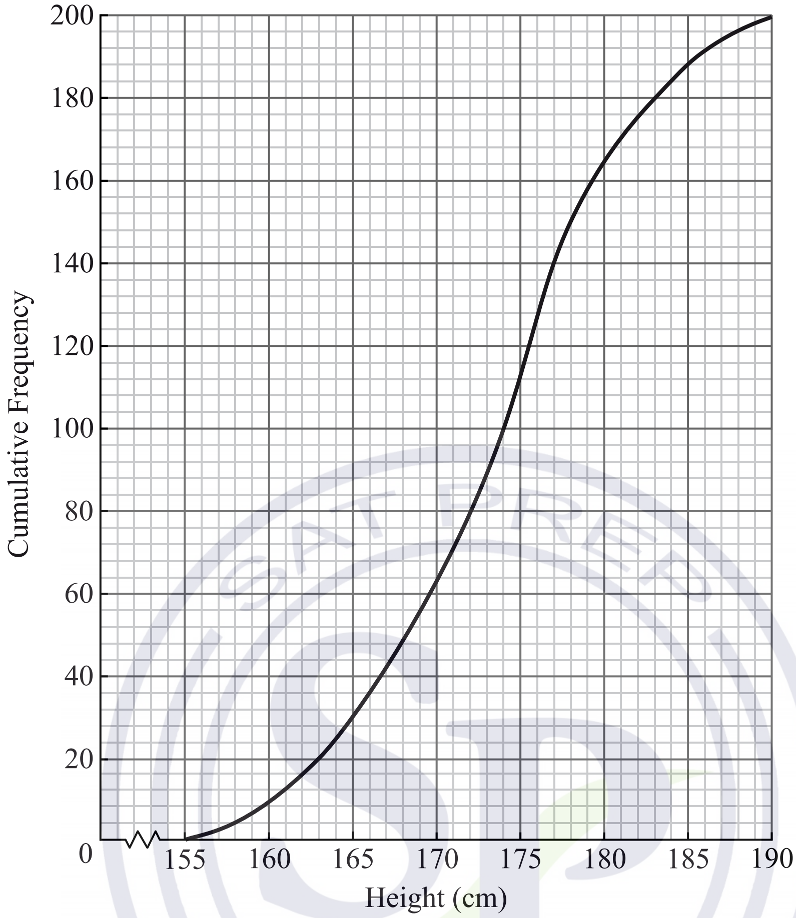
(a) Find $f^{-1}(x)$. *[3 marks]*

(b) Find $(f \circ g)(x)$. *[2 marks]*

(c) Find $(f \circ g)(3.5)$. *[2 marks]*

2. *[Maximum mark: 6]*

The cumulative frequency curve below represents the heights of 200 sixteen-year-old boys.



Use the graph to answer the following.

(a) Write down the median value. *[1 mark]*

(b) A boy is chosen at random. Find the probability that he is shorter than 161 cm. *[2 marks]*

(c) Given that 82 % of the boys are taller than h cm, find h. *[3 marks]*

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3. *[Maximum mark: 6]*

Consider the following circle with centre O and radius 6.8 cm.

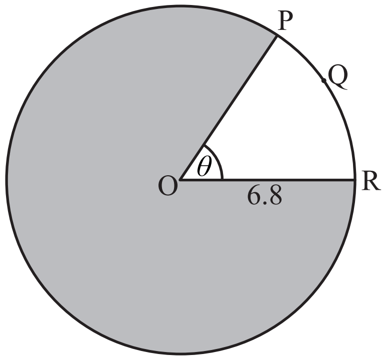


diagram not to scale

The length of the arc PQR is 8.5 cm.

(a) Find the value of θ. *[2 marks]*

(b) Find the area of the shaded region. *[4 marks]*

4. *[Maximum mark: 6]*

Consider the triangle ABC, where $\text{AB} = 10$, $\text{BC} = 7$ and $\text{C}\hat{\text{A}}\text{B} = 30^\circ$.

(a) Find the two possible values of $\text{A}\hat{\text{C}}\text{B}$. *[4 marks]*

(b) Hence, find $\text{A}\hat{\text{B}}\text{C}$, given that it is acute. *[2 marks]*

5. *[Maximum mark: 5]*

Consider the expansion of $(3x^2+2)^9$.

(a) Write down the number of terms in the expansion. *[1 mark]*

(b) Find the term in x^4. *[4 marks]*

6. *[Maximum mark: 8]*

Jose takes medication. After t minutes, the concentration of medication left in his bloodstream is given by $A(t) = 10(0.5)^{0.014t}$, where A is in milligrams per litre.

(a) Write down $A(0)$. *[1 mark]*

(b) Find the concentration of medication left in his bloodstream after 50 minutes. *[2 marks]*

(c) At 13:00, when there is no medication in Jose's bloodstream, he takes his first dose of medication. He can take his medication again when the concentration of medication reaches 0.395 milligrams per litre. What time will Jose be able to take his medication again? *[5 marks]*

7. *[Maximum mark: 7]*

Let $f(t)=2t^2+7$, where $t>0$. The function v is obtained when the graph of f is transformed by

a stretch by a scale factor of $\frac{1}{3}$ parallel to the y-axis,

followed by a translation by the vector $\begin{pmatrix} 2 \\ -4 \end{pmatrix}$.

(a) Find $v(t)$, giving your answer in the form $a(t-b)^2+c$. *[4 marks]*

(b) A particle moves along a straight line so that its velocity in m s^{-1}, at time t seconds, is given by v. Find the distance the particle travels between $t=5.0$ and $t=6.8$. *[3 marks]*

*Do **NOT** write solutions on this page. Any working on this page will **NOT** be marked.*

SECTION B

*Answer **all** the questions on the answer sheets provided. Please start each question on a new page.*

8. *[Maximum mark: 14]*

(a) Consider an infinite geometric sequence with $u_1 = 40$ and $r = \frac{1}{2}$.

(i) Find u_4.

(ii) Find the sum of the infinite sequence. *[4 marks]*

Consider an arithmetic sequence with n terms, with first term (-36) and eighth term (-8).

(b) (i) Find the common difference.

(ii) Show that $S_n = 2n^2 - 38n$. *[5 marks]*

(c) The sum of the infinite geometric sequence is equal to twice the sum of the arithmetic sequence. Find n. *[5 marks]*

*Do **NOT** write solutions on this page. Any working on this page will **NOT** be marked.*

9. *[Maximum mark: 16]*

A company produces a large number of water containers. Each container has two parts, a bottle and a cap. The bottles and caps are tested to check that they are not defective.

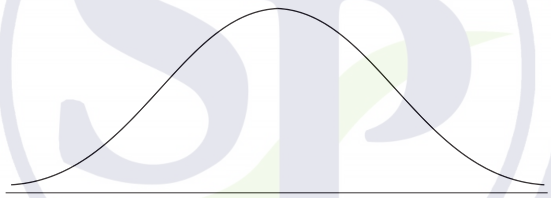
A cap has a probability of 0.012 of being defective. A random sample of 10 caps is selected for inspection.

(a) Find the probability that exactly one cap in the sample will be defective. *[2 marks]*

(b) The sample of caps passes inspection if at most one cap is defective. Find the probability that the sample passes inspection. *[2 marks]*

The heights of the bottles are normally distributed with a mean of 22 cm and a standard deviation of 0.3 cm.

(c) (i) **Copy** and complete the following diagram, shading the region representing where the heights are less than 22.63 cm.



(ii) Find the probability that the height of a bottle is less than 22.63 cm. *[5 marks]*

(d) (i) A bottle is accepted if its height lies between 21.37 cm and 22.63 cm. Find the probability that a bottle selected at random is accepted.

(ii) A sample of 10 bottles passes inspection if all of the bottles in the sample are accepted. Find the probability that the sample passes inspection. *[5 marks]*

(e) The bottles and caps are manufactured separately. A sample of 10 bottles and a sample of 10 caps are randomly selected for testing. Find the probability that both samples pass inspection. *[2 marks]*

*Do **NOT** write solutions on this page. Any working on this page will **NOT** be marked.*

10. *[Maximum mark: 15]*

Let $f(x) = \dfrac{20x}{e^{0.3x}}$, for $0 \le x \le 20$.

(a) Sketch the graph of f. *[3 marks]*

(b) (i) Write down the x-coordinate of the maximum point on the graph of f.

(ii) Write down the interval where f is increasing. *[3 marks]*

(c) Show that $f'(x) = \dfrac{20 - 6x}{e^{0.3x}}$. *[5 marks]*

(d) Find the interval where the rate of change of f is increasing. *[4 marks]*

Please **do not** write on this page.

Answers written on this page will not be marked.