Markscheme

November 2019

Mathematics

Standard level

Paper 2

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a valid **Method**; working must be seen.

(M) Marks awarded for a valid **Method**; may be implied by **correct** subsequent working.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

(A) Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.

R Marks awarded for clear **Reasoning**.

N Marks awarded for **correct** answers if **no** working shown.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Mark according to RM assessor instructions.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any. An exception to this rule is when work for ***M1*** is missing, as opposed to incorrect (see point 4).
- Where ***M*** and ***A*** marks are noted on the same line, *eg* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*eg* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***(M2)***, ***N3***, *etc*., do **not** split the marks, unless there is a note.
- Most ***M*** marks are for a **valid** method, ie a method which can lead to the answer: it must indicate some form of progress towards the answer.
- Once a correct answer to a question or part-question is seen, ignore further correct working. However, if further working indicates a lack of mathematical understanding do not award final ***A1***.

3 *N* marks

*If **no** working shown, award **N** marks for **correct** answers. In this case, ignore mark breakdown (**M, A, R**).*

- Do **not** award a mixture of ***N*** and other marks.
- There may be fewer ***N*** marks available than the total of ***M***, ***A*** and ***R*** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.
- There may not be a direct relationship between the ***N*** marks and the implied marks. There are times when all the marks are implied, but the ***N*** marks are not the full marks: this indicates that we want to see some of the working, without specifying what.
- For consistency within the markscheme, ***N*** marks are noted for every part, even when these match the mark breakdown.

- If a candidate has incorrect working, which somehow results in a correct answer, do **not** award the ***N*** marks for this correct answer. However, if the candidate has indicated (usually by crossing out) that the working is to be ignored, award the ***N*** marks for the correct answer.

4 Implied and must be seen marks

*Implied marks appear in **brackets** eg **(M1)**.*

- Implied marks can only be awarded if the work is seen or if implied in subsequent working (a correct final answer does not necessarily mean that the implied marks are all awarded). There are questions where some working is required, but as it is accepted that not everyone will write the same steps, all the marks are implied, but the ***N*** marks are not the full marks for the question.
- Normally the correct work is seen in the next line.
- Where there is an (***M1***) followed by ***A1*** for each correct answer, if no working shown, one correct answer is sufficient evidence to award the (***M1***).

*Must be seen marks appear without **brackets** eg **M1**.*

- Must be seen marks can only be awarded if the work is seen.
- If a must be seen mark is not awarded because work is missing (as opposed to ***M0*** or ***A0*** for incorrect work) all subsequent marks may be awarded if appropriate.

5 Follow through marks (only applied after an error is made)

*Follow through (**FT**) marks are awarded where an incorrect answer (final or intermediate) from one **part** of a question is used correctly in **subsequent** part(s) or subpart(s). Usually, to award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if the only marks awarded in a subpart are for the **final** answer, then **FT** marks should be awarded if appropriate. Examiners are expected to check student work in order to award **FT** marks where appropriate.*

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** and ***R*** marks may be awarded if appropriate. (However, as noted above, if an ***A*** mark is not awarded because work is missing, all subsequent marks may be awarded if appropriate).
- Exceptions to this rule will be explicitly noted on the markscheme.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*eg* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word "**their**" in a description, to indicate that candidates may be using an incorrect value.
- If a candidate makes an error in one part, but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the question says hence. It is often possible to use a different approach in subsequent parts that does not depend on the answer to previous parts.
- In a "show that" question, if an error in a previous subpart leads to not showing the required answer, do not award the final ***A1***. Note that if the error occurs within the same subpart, the ***FT*** rules may result in further loss of marks.

6 Mis-read

*If a candidate incorrectly copies information from the question, this is a mis-read (**MR**). A candidate should be penalized only once for a particular mis-read. Use the **MR** stamp to indicate that this is a misread. Do not award the first mark in the question, even if this is an **M** mark, but award all others (if appropriate) so that the candidate only loses one mark for the misread.*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*eg* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.

7 Discretionary marks *(d)*

*An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. In such cases the annotation DM should be used and a brief **note** written next to the mark explaining this decision.*

8 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete parts are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**. Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9 Alternative forms

*Unless the question specifies otherwise, **accept** equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

10 Calculators

A GDC is required for paper 2, but calculators with symbolic manipulation features (eg TI-89) are not allowed.

Calculator notation The mathematics SL guide says:
Students must always use correct mathematical notation, not calculator notation.
Do **not** accept final answers written using calculator notation. However, do not penalize the use of calculator notation in the working.

11 Style

The markscheme aims to present answers using good communication, eg if the question asks to find the value of k, the markscheme will say $k = 3$, but the marks will be for the correct value 3 – there is usually no need for the "$k =$". In these cases, it is also usually acceptable to have

another variable, as long as there is no ambiguity in the question, eg if the question asks to find the value of p and of q, then the student answer needs to be clear. Generally, the only situation where the full answer is required is in a question which asks for equations – in this case the markscheme will say "must be an equation".

*The markscheme often uses words to describe what the marks are for, followed by examples, using the eg notation. These examples are not exhaustive, and examiners should check what candidates have written, to see if they satisfy the description. Where these marks are **M** marks, the examples may include ones using poor notation, to indicate what is acceptable.*

12 Candidate work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

Candidates are meant to write their answers to Section A on the question paper (QP), and Section B on answer booklets. Sometimes, they need more room for Section A, and use the booklet (and often comment to this effect on the QP), or write outside the box. That is fine, and this work should be marked.

The instructions tell candidates not to write on Section B of the QP. Thus they may well have done some rough work here which they assume will be ignored. If they have solutions on the answer booklets, there is no need to look at the QP. However, if there are whole questions or whole part solutions missing on answer booklets, please check to make sure that they are not on the QP, and if they are, mark those whole questions or whole part solutions that have not been written on answer booklets.

13. Diagrams

The notes on how to allocate marks for sketches usually refer to passing through particular points or having certain features. These marks can only be awarded if the sketch is approximately the correct shape. All values given will be an approximate guide to where these points/features occur. In some questions, the first ***A1*** is for the shape, in others, the marks are only for the points and/or features. In both cases, unless the shape is approximately correct, no marks can be awarded (unless otherwise stated). However, if the graph is based on previous calculations, ***FT*** marks should be awarded if appropriate.

14. Accuracy of Answers

If the level of accuracy is specified in the question, a mark will be allocated for giving the final answer to the required accuracy. When this is not specified in the question, all numerical answers should be given exactly or correct to three significant figures.

Do not accept unfinished numerical final answers such as 3/0.1 (unless otherwise stated). As a rule, numerical answers with more than one part (such as fractions) should be given using integers (eg 6/8). Calculations which lead to integers should be completed, with the exception of fractions which are not whole numbers. Intermediate values do not need to be given to the correct three significant figures. But, if candidates work with rounded values, this could lead to an incorrect answer, in which case award A0 for the final answer. Where numerical answers are required as the final answer to a part of a question in the markscheme, the markscheme will show
a truncated 6 sf value
the exact value if applicable, the correct 3 sf answer

Section A

1. (a) evidence of set up **(M1)**
eg correct value for a or b (accept $r = 0.966856$)

$4.30161,\ 163.330$
$a = 4.30$, $b = 163$ (accept $y = 4.30x + 163$) **A1A1** **N3**
[3 marks]

(b) valid approach **(M1)**
eg $4.30(154) + 163$

825.778 (825.2 from 3 sf values) **(A1)**

number of messages $= 826$ (must be an integer) **A1** **N3**
[3 marks]

Total [6 marks]

2. (a) valid approach **(M1)**
eg $L_1 = L_2$, $x = 12$, $y = 1$

$(12, 1)$ (exact) **A1** **N2**
[2 marks]

(b) $\begin{pmatrix} -4 \\ 3 \end{pmatrix}$ (or any multiple of $\begin{pmatrix} -4 \\ 3 \end{pmatrix}$) **A1** **N1**

[1 mark]

(c) any correct equation in the form $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$ (accept any parameter for t) where $\boldsymbol{a}$ is a position vector for a point on L_1, and $\boldsymbol{b}$ is a scalar multiple of $\begin{pmatrix} -4 \\ 3 \end{pmatrix}$ **A2** **N2**

eg $\boldsymbol{r} = \begin{pmatrix} 12 \\ 1 \end{pmatrix} + t\begin{pmatrix} -4 \\ 3 \end{pmatrix}$

Note:	Award ***A1*** for the form $\boldsymbol{a} + t\boldsymbol{b}$, ***A1*** for the form $L = \boldsymbol{a} + t\boldsymbol{b}$, ***A0*** for the form $\boldsymbol{r} = \boldsymbol{b} + t\boldsymbol{a}$.

[2 marks]

Total [5 marks]

3. (a) attempt to form composite (in any order) ***(M1)***

eg $f(x^4-3)$, $(x-8)^4-3$

$h(x)=x^4-11$ ***A1*** ***N2***

[2 marks]

(b) recognizing that the gradient of the tangent is the derivative ***(M1)***

eg h'

correct derivative (seen anywhere) ***(A1)***

$h'(x)=4x^3$

correct value for gradient of f (seen anywhere) ***(A1)***

$f'(x)=1$, $m=1$

setting **their** derivative equal to 1 ***(M1)***

$4x^3=1$

0.629960

$x=\sqrt[3]{\frac{1}{4}}$ (exact), 0.630 ***A1*** ***N3***

[5 marks]

Total [7 marks]

4. (a) correct working *(A1)*

eg $\sin\alpha = \frac{8}{10}$, $\cos\theta = \frac{6}{10}$, $\cos \mathrm{B\hat{A}C} = \frac{6^2+10^2-8^2}{2\times 6\times 10}$

0.927295

$\mathrm{B\hat{A}C} = 0.927 \quad (=53.1^\circ)$ *(A1)* **N2**

[2 marks]

(b)

> **Note:** There may be slight differences in the final answer, depending on the approach the candidate uses in part (b). Accept a final answer that is consistent with their working.

correct area of sector ABF (seen anywhere) *(A1)*

eg $\frac{1}{2}\times 6^2 \times 0.927$, $\frac{53.1301^\circ}{360^\circ}\times\pi\times 6^2$, 16.6913

correct expression (or value) for either [AD] or [BD] (seen anywhere) *(A1)*

eg $\mathrm{AD} = 6\cos(\mathrm{B\hat{A}C})\ (=3.6)$

$\mathrm{BD} = 6\sin(53.1^\circ)\ (=4.8)$

correct area of triangle ABD (seen anywhere) *(A1)*

eg $\frac{1}{2}\times 6\cos \mathrm{B\hat{A}D}\times 6\sin \mathrm{B\hat{A}D}$, $9\sin(2\mathrm{B\hat{A}C})$, 8.64 (exact)

appropriate approach (seen anywhere) *(M1)*

eg $\mathrm{A}_{\text{triangle ABD}}$ - $\mathrm{A}_{\text{sector}}$, their sector - their triangle ABD

8.05131

area of shaded region = $8.05\ (\mathrm{cm}^2)$ **A1** **N2**

[5 marks]

Total [7 marks]

5. (a) valid approach *(M1)*

eg $\frac{u_1}{u_2}$, $\frac{2.226}{2.1}$, $2.226 = 2.1r$

$r = 1.06$ (exact) ***A1*** ***N2***

[2 marks]

(b) correct substitution *(A1)*

eg 2.1×1.06^9

3.54790 ***A1*** ***N2***

$u_{10} = 3.55$

[2 marks]

(c) correct substitution into S_n formula *(A1)*

eg $\frac{2.1(1.06^n - 1)}{1.06 - 1}$, $\frac{2.1(1.06^n - 1)}{1.06 - 1} > 5543$, $2.1(1.06^n - 1) = 332.58$, sketch of S_n and $y = 5543$

correct inequality for n **or** crossover values ***A1***

eg $n > 87.0316$, $S_{87} = 5532.73$ **and** $S_{88} = 5866.79$

$n = 88$ ***A1*** ***N2***

[3 marks]

Total [7 marks]

6. evidence of choosing cosine rule *(M1)*

eg $a^2 = b^2 + c^2 - 2bc\cos A$

correct substitution to find AB *(A1)*

eg $28.4^2 = x^2 + (x+2)^2 - 2x(x+2)\cos(0.667)$

$x = 42.2822$ ***A2***

appropriate approach to find AD *(M1)*

eg $\text{AD} = x\cos(0.611)$, $\cos(0.611) = \frac{\text{AD}}{42.2822}$

34.6322

$\text{AD} = 34.6$ ***A1*** ***N3***

Total [6 marks]

7. (a) correct approach **A1**

eg $0.2+0.5+b+a=1$, $0.7+a+b=1$

$b=0.3-a$ **AG** **N0**

[1 mark]

(b) correct substitution into $\mathrm{E}(X)$ ***(A1)***

eg $0.2+4\times0.5+a\times b+(a+b-0.5)\times a$, $0.2+2+a\times b-0.2a$

valid attempt to express $\mathrm{E}(X)$ in one variable ***M1***

eg $0.2+4\times0.5+a\times(0.3-a)+(-0.2)\times a$, $2.2+0.1a-a^2$,
$0.2+4\times0.5+(0.3-b)\times b+(-0.2)\times(0.3-b)$, $2.14+0.5b-b^2$

correct value of greatest $\mathrm{E}(X)$ ***(A1)***

2.2025 (exact)

valid attempt to find least value ***(M1)***

eg graph with minimum indicated, $\mathrm{E}(0)$ **and** $\mathrm{E}(0.3)$,
$(0, 2.2)$ **and** $(0.3, 2.14)$ if $\mathrm{E}(X)$ in terms of a
$(0, 2.14)$ **and** $(0.3, 2.2)$ if $\mathrm{E}(X)$ in terms of b

correct value of least $\mathrm{E}(X)$ ***(A1)***

eg 2.14 (exact)

difference $=0.0625$ (exact) ***A1*** ***N2***

[6 marks]

Total [7 marks]

Section B

8. (a) evidence of valid approach *(M1)*
eg $f(x)=0$, $y=0$

1.13843
$p=1.14$ ***A1*** ***N2***
[2 marks]

(b) (i) 0.562134, 16.7641
$(0.562, 16.8)$ ***A2*** ***N2***

(ii) valid approach *(M1)*
eg tangent at maximum point is horizontal, $f'=0$

$y=16.8$ (must be an equation) ***A1*** ***N2***
[4 marks]

(c) (i) **METHOD 1 (using GDC)**

valid approach ***M1***
eg $f''=0$, max/min on f', $x=-3$

sketch of either f' or f'', with max/min or root (respectively) *(A1)*

$x=3$ ***A1*** ***N1***

substituting **their** x value into f *(M1)*
eg $f(3)$

$y=-225$ (exact) (accept $(3,-225)$) ***A1*** ***N1***

METHOD 2 (analytical)

$f''=12x^2-108$ ***A1***

valid approach *(M1)*

eg $f''=0$, $x=\pm 3$

$x=3$ ***A1*** ***N1***

substituting **their** x value into f *(M1)*
eg $f(3)$

$y=-225$ (exact) (accept $(3,-225)$) ***A1*** ***N1***

(ii) recognizing rate of change is f' *(M1)*
eg y', $f'(3)$
rate of change is -156 (exact) ***A1*** ***N2***
[7 marks]

continued…

Question 8 continued

(d) attempt to substitute **either their** limits **or** the function into volume formula ***(M1)***

eg $\int_{1.14}^{3} f^2$, $\pi\int (x^4 - 54x^2 + 60x)^2\,\mathrm{d}x$, $25\,752.0$

$80\,902.3$

volume $= 80\,900$ ***A2*** ***N3***

[3 marks]

Total [16 marks]

9. (a) valid approach ***(M1)***

eg $\mathrm{P}(X < 275)$, $1 - 0.158655$

0.841344

0.841 ***A1*** ***N2***

[2 marks]

(b) valid approach ***(M1)***

eg $\mathrm{P}(X < 275) - \mathrm{P}(X < m) = 0.830$

correct working ***(A1)***

eg $\mathrm{P}(X < m) = 0.0113447$

225.820

226 (minutes) ***A1*** ***N3***

[3 marks]

(c) (i) evidence of recognizing binomial distribution (seen anywhere) ***(M1)***

eg ${}_nC_a \times p^a \times q^{n-a}$, $\mathrm{B}(n,\ p)$

evidence of summing probabilities from 7 to 12 ***(M1)***

eg $\mathrm{P}(X = 7) + \mathrm{P}(X = 8) + \ldots + \mathrm{P}(X = 12)$, $1 - \mathrm{P}(X \le 6)$

0.991248

0.991 ***A1*** ***N2***

continued…

Question 9 continued

(ii) finding $\mathrm{P}(X=10)$ (seen anywhere) ***A1***

eg $\binom{12}{10}\times 0.83^{10}\times 0.17^{2}\ (=0.295952)$

recognizing conditional probability ***(M1)***

eg $\mathrm{P}(A\,|\,B)$, $\mathrm{P}(X=10\,|\,X\geq 7)$, $\dfrac{\mathrm{P}(X=10\cap X\geq 7)}{\mathrm{P}(X\geq 7)}$

correct working ***(A1)***

eg $\dfrac{0.295952}{0.991248}$

0.298565

0.299 ***A1*** ***N1***

Note:	**Exception to the *FT* rule:** if the candidate uses an incorrect value for the probability that a flight is on time in (i) and working shown, award full ***FT*** in (ii) as appropriate.

[7 marks]

(d) correct equation ***(A1)***

eg $\binom{20}{19}p^{19}(1-p)+p^{20}=0.788$

valid attempt to solve ***(M1)***

eg graph

0.956961

0.957 ***A1*** ***N1***

[3 marks]

Total [15 marks]

10. (a) recognizing that $v = \int a$ ***(M1)***

correct integration ***A1***
eg $-120\cos(2t) + c$

attempt to find c using their $v(t)$ ***(M1)***
eg $-120\cos(0) + c = 140$

$v(t) = -120\cos(2t) + 260$ ***A1 N3***
[4 marks]

(b) evidence of valid approach to find time taken in first stage ***(M1)***
eg graph, $-120\cos(2t) + 260 = 375$

$k = 1.42595$ ***A1***

attempt to substitute **their** v and/or **their** limits into distance formula ***(M1)***
eg $\int_0^{1.42595} |v|$, $\int 260 - 120\cos(2t)$, $\int_0^k (260 - 120\cos(2t))\,\mathrm{d}t$

353.608
distance is 354 (m) ***A1 N3***
[4 marks]

(c) recognizing velocity of second stage is linear (seen anywhere) ***R1***
eg graph, $s = \frac{1}{2}h(a+b)$, $v = mt + c$

valid approach ***(M1)***
eg $\int v = 353.608$

correct equation ***(A1)***
eg $\frac{1}{2}h(375 + 500) = 353.608$

time for stage two = 0.808248 (0.809142 from 3 sf) ***A2***

2.23420 (2.23914 from 3 sf)
2.23 (seconds) (2.24 from 3 sf) ***A1 N3***
[6 marks]

Total [14 marks]

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- In a "show that" question, if an error in a previous subpart leads to not showing the required answer, do not award the final ***A1***. Note that if the error occurs within the same subpart, the ***FT*** rules may result in further loss of marks.

6 Mis-read

*If a candidate incorrectly copies information from the question, this is a mis-read (**MR**). A candidate should be penalized only once for a particular mis-read. Use the **MR** stamp to indicate that this is a misread. Do not award the first mark in the question, even if this is an **M** mark, but award all others (if appropriate) so that the candidate only loses one mark for the misread.*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*eg* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.

7 Discretionary marks *(d)*

*An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. In such cases the annotation DM should be used and a brief **note** written next to the mark explaining this decision.*

8 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete parts are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**. Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9 Alternative forms

*Unless the question specifies otherwise, **accept** equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

10 Calculators

A GDC is required for paper 2, but calculators with symbolic manipulation features (eg TI-89) are not allowed.

Calculator notation The mathematics SL guide says:
Students must always use correct mathematical notation, not calculator notation.
Do **not** accept final answers written using calculator notation. However, do not penalize the use of calculator notation in the working.

11 Style

The markscheme aims to present answers using good communication, eg if the question asks to find the value of k, the markscheme will say $k = 3$, but the marks will be for the correct value 3 – there is usually no need for the "$k =$". In these cases, it is also usually acceptable to have

another variable, as long as there is no ambiguity in the question, eg if the question asks to find the value of p and of q, then the student answer needs to be clear. Generally, the only situation where the full answer is required is in a question which asks for equations – in this case the markscheme will say "must be an equation".

*The markscheme often uses words to describe what the marks are for, followed by examples, using the eg notation. These examples are not exhaustive, and examiners should check what candidates have written, to see if they satisfy the description. Where these marks are **M** marks, the examples may include ones using poor notation, to indicate what is acceptable.*

12 Candidate work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

Candidates are meant to write their answers to Section A on the question paper (QP), and Section B on answer booklets. Sometimes, they need more room for Section A, and use the booklet (and often comment to this effect on the QP), or write outside the box. That is fine, and this work should be marked.

The instructions tell candidates not to write on Section B of the QP. Thus they may well have done some rough work here which they assume will be ignored. If they have solutions on the answer booklets, there is no need to look at the QP. However, if there are whole questions or whole part solutions missing on answer booklets, please check to make sure that they are not on the QP, and if they are, mark those whole questions or whole part solutions that have not been written on answer booklets.

13. Diagrams

The notes on how to allocate marks for sketches usually refer to passing through particular points or having certain features. These marks can only be awarded if the sketch is approximately the correct shape. All values given will be an approximate guide to where these points/features occur. In some questions, the first ***A1*** is for the shape, in others, the marks are only for the points and/or features. In both cases, unless the shape is approximately correct, no marks can be awarded (unless otherwise stated). However, if the graph is based on previous calculations, ***FT*** marks should be awarded if appropriate.

14. Accuracy of Answers

If the level of accuracy is specified in the question, a mark will be allocated for giving the final answer to the required accuracy. When this is not specified in the question, all numerical answers should be given exactly or correct to three significant figures.

Do not accept unfinished numerical final answers such as 3/0.1 (unless otherwise stated). As a rule, numerical answers with more than one part (such as fractions) should be given using integers (eg 6/8). Calculations which lead to integers should be completed, with the exception of fractions which are not whole numbers. Intermediate values do not need to be given to the correct three significant figures. But, if candidates work with rounded values, this could lead to an incorrect answer, in which case award A0 for the final answer. Where numerical answers are required as the final answer to a part of a question in the markscheme, the markscheme will show

a truncated 6 sf value

the exact value if applicable, the correct 3 sf answer

Section A

1. (a) evidence of finding $\dfrac{\sum x}{n}$ *(M1)*

eg $\dfrac{0.3+0.4+3+\ldots+10}{10}$, $\dfrac{48.2}{10}$

$\bar{x}=4.82$ (exact) ***A1 N2***

[2 marks]

(b) $p=4.25$ (exact) ***A1 N1***

[1 mark]

(c) valid approach *(M1)*

eg $\mathrm{Q}_3-\mathrm{Q}_1$ $3-8$, 3 to 8

$\mathrm{IQR}=5$ ***A1 N2***

[2 marks]

Total [5 marks]

2. (a) vertex is $(-10, 15)$ ***A1A1 N1N1***

[2 marks]

(b) valid approach *(M1)*

eg $f(0)=-20$, $-20=a(0+10)^2+15$

$a=-0.35$ (exact) ***A1 N2***

[2 marks]

(c) valid approach *(M1)*

eg $f(8)$, $-0.35(8+10)^2+15$

$b=-98.4$ (exact) (accept $f(8)=-98.4$) ***A1 N2***

[2 marks]

Total [6 marks]

3. (a) choosing product rule *(M1)*

eg $uv' + vu'$, $\left(x^2\right)'(\mathrm{e}^{3x}) + \left(\mathrm{e}^{3x}\right)' x^2$

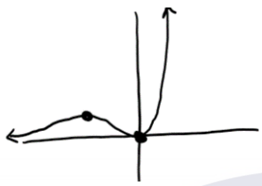
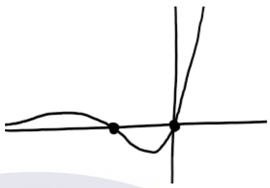
correct derivatives (must be seen in the rule) *A1A1*

eg $2x$, $3\mathrm{e}^{3x}$

$f'(x) = 2x\mathrm{e}^{3x} + 3x^2\mathrm{e}^{3x}$ ***A1*** ***N4***

[4 marks]

(b) valid method *(M1)*

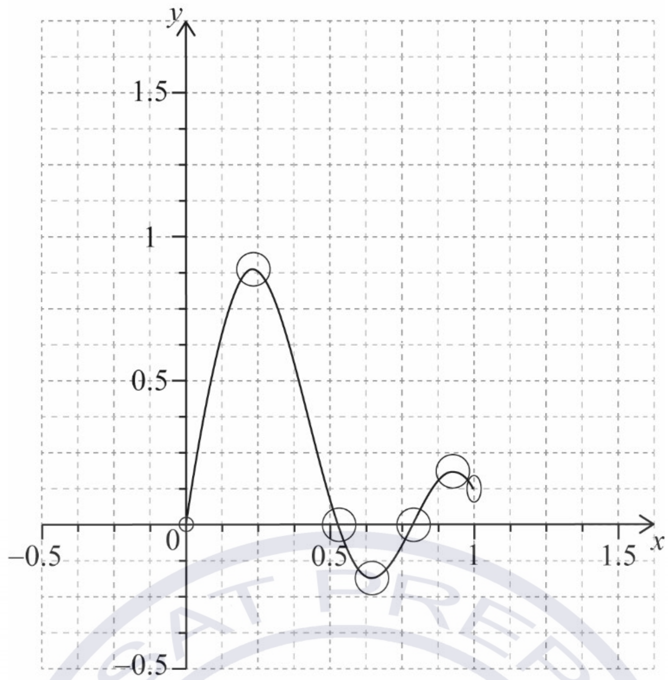
eg $f'(x) = 0$,  , 

$a = -0.667\left(= -\dfrac{2}{3}\right)$ (accept $x = -0.667$) ***A1*** ***N2***

[2 marks]

Total [6 marks]

4. (a)



A1A1A1 *N3*

> **Note:** Only if the shape is approximately correct with exactly 2 maximums and 1 minimum on the interval $0 \le x \le 1$, award the following:
> ***A1*** for correct domain with both endpoints within circle and oval.
> ***A1*** for passing through the other x-intercepts within the circles.
> ***A1*** for passing through the three turning points within circles (ignore x-intercepts and extrema outside of the domain).

[3 marks]

(b) evidence of reasoning (may be seen on graph) *(M1)*
eg $f'' = 0$, $(0.524, 0)$, $(0.785, 0)$

$0.523598, 0.785398$

$x = 0.524 \left(= \frac{\pi}{6}\right)$, $x = 0.785 \left(= \frac{\pi}{4}\right)$ *A1A1* *N3*

> **Note:** Award ***M1A1A0*** if any solution outside domain (eg $x = 0$) is also included.

[3 marks]

(c) $0.524 < x < 0.785 \left(\frac{\pi}{6} < x < \frac{\pi}{4}\right)$ *A2* *N2*

> **Note:** Award ***A1*** if any correct interval outside domain also included, unless additional solutions already penalized in (b).
> Award ***A0*** if any incorrect intervals are also included.

[2 marks]

Total [8 marks]

5. (a) valid approach *(M1)*

eg correct value for a or b (ignore incorrect labels)

$a = 6.92986$, $b = 8.80769$

$a = 6.93$, $b = 8.81$ (accept $y = 6.93x + 8.81$) ***A1A1*** ***N3***

[3 marks]

(b) valid approach *(M1)*

eg $750 = x + y$, edge + interior $= 750$

correct working *(A1)*

eg $750 - x = 6.9298x + 8.807$, 93.4684

93 (pieces) (accept 94) ***A1*** ***N3***

[3 marks]

Total [6 marks]

6. valid approach for expansion (must have correct substitution for parameters, but accept an incorrect value for r) ***(M1)***

eg $\binom{n}{r}\left(x^2\right)^{n-r}(1.2)^r$, $\binom{n}{0}\left(x^2\right)^n+\binom{n}{1}\left(x^2\right)^{n-1}(1.2)+\binom{n}{2}\left(x^2\right)^{n-2}(1.2)^2+\ldots$

attempt to identify correct term ***(M1)***

eg $2r=6$, $n-r=3$, $\binom{n}{3}$, $\binom{n}{n-3}$

correct expression ***(A1)***

eg $\binom{n}{n-3}\times 1.2^{n-3}x^6$, $\binom{n}{n-3}\times 1.2^{n-3}$

EITHER (solving inequality)

attempt to set up inequality in terms of n (accept equation) ***M1***

eg $\binom{n}{3}\times 1.2^{n-3}>200\,000$, $1.2^{n-3}\binom{n}{3}x^6=200\,000$

correct working for binomial coefficient (may be seen in equation) ***(A1)***

eg $\dfrac{n(n-1)(n-2)(n-3)!}{3!(n-3)!}$, $\dfrac{n(n-1)(n-2)}{6}\times 1.2^{n-3}=200\,000$

$n>26.4959$ (accept 26.4959 or $n=26.4959$) ***A1***

$n=27$ ***A1*** ***N2***

Note: If no working shown, award ***N1*** for 26.4959.

OR (using table)

valid approach ***(M1)***

eg $\binom{n}{3}\times 1.2^{n-3}>200\,000$, one correct coefficient of x^6 for a value of n

correct crossover values for $n=26$ and $n=27$ ***A1A1***

eg 172243, 232528

$n=27$ ***A1*** ***N2***

[7 marks]

7. (a) attempt to add corresponding terms *(M1)*

eg $2+2$, $6+(-6)$, $2(3)^{n-1}+2(-3)^{n-1}$

correct value for w_5 *(A1)*

eg 324

$4, 36, 324$ (accept $4+36+324$) ***A1*** ***N3***

[3 marks]

(b) (i) valid approach *(M1)*

eg $4\times r^1=36$, $4\times 9^{n-1}$

$r=9$ (accept $\sum_{k=0}^{m} 4\times 9^k$; m may be incorrect) ***A1*** ***N2***

(ii) recognition that 225 terms of w_n consists of 113 non-zero terms *(M1)*

eg $\sum_{1}^{113}$, $\sum_{0}^{112}$, 113

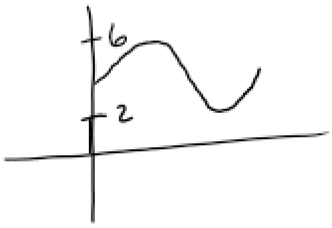
$m=112$ (accept $\sum_{k=0}^{112} 4\times r^k$; r may be incorrect) ***A1*** ***N2***

[4 marks]

Total [7 marks]

Section B

8. (a) valid attempt to find range **(M1)**

eg 

, max $= 6$ min $= 2$,

$$2\sin\left(3\times\frac{\pi}{6}\right)+4 \text{ and } 2\sin\left(3\times\frac{\pi}{2}\right)+4,\ 2(1)+4 \text{ and } 2(-1)+4,$$

$k = 2$, $m = 6$ **A1A1** **N3**

[3 marks]

(b) $10 \le y \le 30$ **A2** **N2**

[2 marks]

(c) (i) evidence of substitution (may be seen in part (b)) **(M1)**

eg $5\left(2\sin\left(3(2x)\right)+4\right)$, $3(2x)$

$b = 6$, $c = 20$ (accept $10\sin(6x)+20$) **A1A1** **N3**

Note: If no working shown, award ***N2*** for one correct value.

(ii) correct working **(A1)**

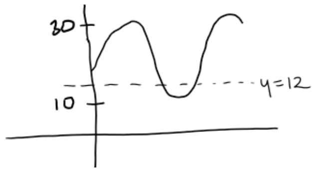
eg $\frac{2\pi}{b}$

1.04719

$\frac{2\pi}{6}\left(=\frac{\pi}{3}\right)$, 1.05 **A1** **N2**

[5 marks]

(d) valid approach **(M1)**

eg 

, $\sin^{-1}\left(-\frac{8}{10}\right)$, $6x = -0.927$, -0.154549, $x = 0.678147$

Note: Award ***M1*** for any correct value for x or $6x$ which lies outside the domain of f.

3.81974, 4.03424

$x = 3.82$, $x = 4.03$ (do not accept answers in degrees) **A1A1** **N3**

Total [13 marks]

9. (a) attempt to find $f'(8)$ *(M1)*

eg $f'(x)$, y', $-16x^{-2}$

-0.25 (exact) ***A1*** ***N2***

[2 marks]

(b) $\boldsymbol{u} = \begin{pmatrix} 4 \\ -1 \end{pmatrix}$ or any scalar multiple ***A2*** ***N2***

[2 marks]

(c) correct scalar product and magnitudes *(A1)(A1)(A1)*

scalar product $= 1 \times 4 + 1 \times -1 \ (= 3)$

magnitudes = $\sqrt{1^2 + 1^2}, \sqrt{4^2 + (-1)^2} \ \left(= \sqrt{2}, \sqrt{17}\right)$

substitution of **their** values into correct formula *(M1)*

eg $\frac{4-1}{\sqrt{1^2+1^2}\sqrt{4^2+(-1)^2}}$, $\frac{-3}{\sqrt{2}\sqrt{17}}$, 2.1112, 120.96°

1.03037, 59.0362°

angle $= 1.03$, 59.0° ***A1*** ***N4***

[5 marks]

continued…

Question 9 continued

(d) (i) attempt to form composite $(f \circ f)(x)$ ***(M1)***

eg $f(f(x))$, $f\left(\frac{16}{x}\right)$, $\frac{16}{f(x)}$

correct working ***(A1)***

eg $\frac{16}{16/x}$, $16 \times \frac{x}{16}$

$(f \circ f)(x) = x$ ***A1*** ***N2***

(ii) $f^{-1}(x) = \frac{16}{x}$ (accept $y = \frac{16}{x}$, $\frac{16}{x}$) ***A1*** ***N1***

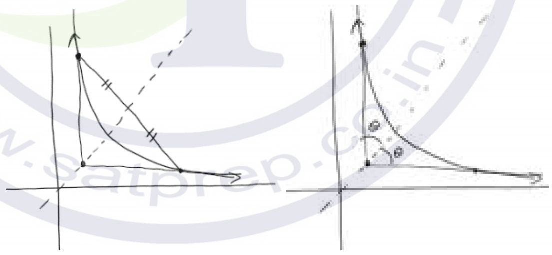
Note:	Award ***A0*** in part (ii) if part (i) is incorrect.
	Award ***A0*** in part (ii) if the candidate has found $f^{-1}(x) = \frac{16}{x}$ by interchanging x and y.

(iii) **METHOD 1**

recognition of symmetry about $y = x$ ***(M1)***

eg ,

$(2, 8) \Leftrightarrow (8, 2)$



evidence of doubling **their** angle ***(M1)***

eg 2×1.03, 2×59.0

2.06075, 118.072°

2.06 (radians) (118 degrees) ***A1*** ***N2***

continued…

Question 9 continued

METHOD 2

finding direction vector for tangent line at $x = 2$ ***(A1)***

eg $\begin{pmatrix} -1 \\ 4 \end{pmatrix}, \begin{pmatrix} 1 \\ -4 \end{pmatrix}$

substitution of **their** values into correct formula (must be from vectors) ***(M1)***

eg $\dfrac{-4-4}{\sqrt{1^2+4^2}\sqrt{4^2+(-1)^2}}, \dfrac{8}{\sqrt{17}\sqrt{17}}$

$2.06075, 118.072^\circ$

2.06 (radians) (118 degrees) ***A1*** ***N2***

METHOD 3

using trigonometry to find an angle with the horizontal ***(M1)***

eg $\tan\theta = -\dfrac{1}{4}, \tan\theta = -4$

finding both angles of rotation ***(A1)***

eg $\theta_1 = 0.244978, 14.0362^\circ, \theta_2 = 1.81577, 104.036^\circ$

$2.06075, 118.072^\circ$

2.06 (radians) (118 degrees) ***A1*** ***N2***

[7 marks]

Total [16 marks]

10. (a) (i) valid approach to find P(one red) ***(M1)***

eg $\quad {}_nC_a \times p^a \times q^{n-a}$, $\mathrm{B}(n,\ p)$, $3\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^2$, $\begin{pmatrix}3\\1\end{pmatrix}$

listing all possible cases for exactly one red (may be indicated on tree diagram)

$\mathrm{P}(1\text{ red}) = 0.444\left(=\frac{4}{9}\right)$ $[0.444,\ 0.445]$ ***A1*** ***N2***

(ii) valid approach ***(M1)***

eg $\quad \mathrm{P}(X=2)+\mathrm{P}(X=3)$, $1-\mathrm{P}(X\le 1)$, binomcdf$\left(3,\ \frac{1}{3},\ 2,\ 3\right)$

correct working ***(A1)***

eg $\quad \frac{2}{9}+\frac{1}{27}$, $0.222+0.037$, $1-\left(\frac{2}{3}\right)^3-\frac{4}{9}$

0.259259

$\mathrm{P}(\text{at least two red}) = 0.259\left(=\frac{7}{27}\right)$ ***A1*** ***N3***

[5 marks]

(b) recognition that winning $\$10$ means rolling exactly one green ***(M1)***

recognition that winning $\$10$ also means rolling at most 1 red ***(M1)***
eg "cannot have 2 or more reds"

correct approach ***A1***
eg $\quad \mathrm{P}(1\mathrm{G}\cap 0\mathrm{R})+\mathrm{P}(1\mathrm{G}\cap 1\mathrm{R})$, $\mathrm{P}(1\mathrm{G})-\mathrm{P}(1\mathrm{G}\cap 2\mathrm{R})$,
"one green and two yellows or one of each colour"

Note: Because this is a "show that" question, do not award this ***A1*** for purely numerical expressions.

one correct probability for their approach ***(A1)***

eg $\quad 3\left(\frac{1}{3}\right)\left(\frac{1}{3}\right)^2$, $\frac{6}{27}$, $3\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^2$, $\frac{1}{9}$, $\frac{2}{9}$

correct working leading to $\frac{1}{3}$ ***A1***

eg $\quad \frac{3}{27}+\frac{6}{27}$, $\frac{12}{27}-\frac{3}{27}$, $\frac{1}{9}+\frac{2}{9}$

probability $=\frac{1}{3}$ ***AG*** ***N0***

[5 marks]

continued…

Question 10 continued

(c) (i) $x = \frac{7}{27}$, 0.259 (check ***FT*** from (a)(ii)) ***A1*** ***N1***

(ii) evidence of summing probabilities to 1 ***(M1)***

eg $\sum = 1,\ x + y + \frac{1}{3} + \frac{2}{9} + \frac{1}{27} = 1,\ 1 - \frac{7}{27} - \frac{9}{27} - \frac{6}{27} - \frac{1}{27}$

0.148147 (0.148407 if working with **their** x value to 3 sf)

$y = \frac{4}{27}$ (exact), 0.148 ***A1*** ***N2***

[3 marks]

(d) correct substitution into the formula for expected value ***(A1)***

eg $-w \cdot \frac{7}{27} + 10 \cdot \frac{9}{27} + 20 \cdot \frac{6}{27} + 30 \cdot \frac{1}{27}$

correct critical value (accept inequality) ***A1***

eg $w = 34.2857 \left(= \frac{240}{7} \right),\ w > 34.2857$

$\$40$ ***A1*** ***N2***

[3 marks]

Total [16 marks]

Markscheme

May 2019

Mathematics

Standard level

Paper 2

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a valid **Method**; working must be seen.

(M) Marks awarded for a valid **Method**; may be implied by **correct** subsequent working.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

(A) Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.

R Marks awarded for clear **Reasoning**.

N Marks awarded for **correct** answers if **no** working shown.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Mark according to RM assessor instructions.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any. An exception to this rule is when work for ***M1*** is missing, as opposed to incorrect (see point 4).
- Where ***M*** and ***A*** marks are noted on the same line, *eg* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*eg* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***(M2)***, ***N3***, *etc*., do **not** split the marks, unless there is a note.
- Most ***M*** marks are for a **valid** method, ie a method which can lead to the answer: it must indicate some form of progress towards the answer.
- Once a correct answer to a question or part-question is seen, ignore further correct working. However, if further working indicates a lack of mathematical understanding do not award final ***A1***.

3 *N* marks

*If **no** working shown, award **N** marks for **correct** answers. In this case, ignore mark breakdown (**M, A, R**).*

- Do **not** award a mixture of ***N*** and other marks.
- There may be fewer ***N*** marks available than the total of ***M***, ***A*** and ***R*** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.
- There may not be a direct relationship between the ***N*** marks and the implied marks. There are times when all the marks are implied, but the ***N*** marks are not the full marks: this indicates that we want to see some of the working, without specifying what.
- For consistency within the markscheme, ***N*** marks are noted for every part, even when these match the mark breakdown.

- If a candidate has incorrect working, which somehow results in a correct answer, do **not** award the ***N*** marks for this correct answer. However, if the candidate has indicated (usually by crossing out) that the working is to be ignored, award the ***N*** marks for the correct answer.

4 Implied and must be seen marks

*Implied marks appear in **brackets** eg **(M1)**.*

- Implied marks can only be awarded if the work is seen or if implied in subsequent working (a correct final answer does not necessarily mean that the implied marks are all awarded). There are questions where some working is required, but as it is accepted that not everyone will write the same steps, all the marks are implied, but the ***N*** marks are not the full marks for the question.
- Normally the correct work is seen in the next line.
- Where there is an (***M1***) followed by ***A1*** for each correct answer, if no working shown, one correct answer is sufficient evidence to award the (***M1***).

*Must be seen marks appear without **brackets** eg **M1**.*

- Must be seen marks can only be awarded if the work is seen.
- If a must be seen mark is not awarded because work is missing (as opposed to ***M0*** or ***A0*** for incorrect work) all subsequent marks may be awarded if appropriate.

5 Follow through marks (only applied after an error is made)

*Follow through (**FT**) marks are awarded where an incorrect answer (final or intermediate) from one **part** of a question is used correctly in **subsequent** part(s) or subpart(s). Usually, to award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if the only marks awarded in a subpart are for the **final** answer, then **FT** marks should be awarded if appropriate. Examiners are expected to check student work in order to award **FT** marks where appropriate.*

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** and ***R*** marks may be awarded if appropriate. (However, as noted above, if an ***A*** mark is not awarded because work is missing, all subsequent marks may be awarded if appropriate).
- Exceptions to this rule will be explicitly noted on the markscheme.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*eg* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word "**their**" in a description, to indicate that candidates may be using an incorrect value.
- If a candidate makes an error in one part, but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the question says hence. It is often possible to use a different approach in subsequent parts that does not depend on the answer to previous parts.
- In a "show that" question, if an error in a previous subpart leads to not showing the required answer, do not award the final ***A1***. Note that if the error occurs within the same subpart, the ***FT*** rules may result in further loss of marks.

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*If a candidate incorrectly copies information from the question, this is a mis-read (**MR**). A candidate should be penalized only once for a particular mis-read. Use the **MR** stamp to indicate that this is a misread. Do not award the first mark in the question, even if this is an **M** mark, but award all others (if appropriate) so that the candidate only loses one mark for the misread.*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*eg* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.

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*An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. In such cases the annotation DM should be used and a brief **note** written next to the mark explaining this decision.*

8 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete parts are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**. Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9 Alternative forms

*Unless the question specifies otherwise, **accept** equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

10 Calculators

A GDC is required for paper 2, but calculators with symbolic manipulation features (eg TI-89) are not allowed.

Calculator notation The mathematics SL guide says:
Students must always use correct mathematical notation, not calculator notation.
Do **not** accept final answers written using calculator notation. However, do not penalize the use of calculator notation in the working.

11 Style

The markscheme aims to present answers using good communication, eg if the question asks to find the value of k, the markscheme will say $k = 3$, but the marks will be for the correct value 3 – there is usually no need for the "$k =$". In these cases, it is also usually acceptable to have

another variable, as long as there is no ambiguity in the question, eg if the question asks to find the value of p and of q, then the student answer needs to be clear. Generally, the only situation where the full answer is required is in a question which asks for equations – in this case the markscheme will say "must be an equation".

*The markscheme often uses words to describe what the marks are for, followed by examples, using the eg notation. These examples are not exhaustive, and examiners should check what candidates have written, to see if they satisfy the description. Where these marks are **M** marks, the examples may include ones using poor notation, to indicate what is acceptable.*

12 Candidate work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

Candidates are meant to write their answers to Section A on the question paper (QP), and Section B on answer booklets. Sometimes, they need more room for Section A, and use the booklet (and often comment to this effect on the QP), or write outside the box. That is fine, and this work should be marked.

The instructions tell candidates not to write on Section B of the QP. Thus they may well have done some rough work here which they assume will be ignored. If they have solutions on the answer booklets, there is no need to look at the QP. However, if there are whole questions or whole part solutions missing on answer booklets, please check to make sure that they are not on the QP, and if they are, mark those whole questions or whole part solutions that have not been written on answer booklets.

13. Diagrams

The notes on how to allocate marks for sketches usually refer to passing through particular points or having certain features. These marks can only be awarded if the sketch is approximately the correct shape. All values given will be an approximate guide to where these points/features occur. In some questions, the first ***A1*** is for the shape, in others, the marks are only for the points and/or features. In both cases, unless the shape is approximately correct, no marks can be awarded (unless otherwise stated). However, if the graph is based on previous calculations, ***FT*** marks should be awarded if appropriate.

14. Accuracy of Answers

If the level of accuracy is specified in the question, a mark will be allocated for giving the final answer to the required accuracy. When this is not specified in the question, all numerical answers should be given exactly or correct to three significant figures.

Do not accept unfinished numerical final answers such as 3/0.1 (unless otherwise stated). As a rule, numerical answers with more than one part (such as fractions) should be given using integers (eg 6/8). Calculations which lead to integers should be completed, with the exception of fractions which are not whole numbers. Intermediate values do not need to be given to the correct three significant figures. But, if candidates work with rounded values, this could lead to an incorrect answer, in which case award A0 for the final answer. Where numerical answers are required as the final answer to a part of a question in the markscheme, the markscheme will show

a truncated 6 sf value

the exact value if applicable, the correct 3 sf answer

Section A

1. (a) (i) valid approach *(M1)*

eg correct value for a or b (or for correct r or $r^2 = 0.955631$ seen in (ii))

$0.141120,\ 11.1424$

$a = 0.141,\ b = 11.1$ ***A1A1*** ***N3***

(ii) 0.977563

$r = 0.978$ ***A1*** ***N1***

[4 marks]

(b) correct substitution into **their** regression equation *(A1)*

eg $0.141(95) + 11.1$

24.5488

24.5 ***A1*** ***N2***

[2 marks]

Total [6 marks]

2. (a) valid approach *(M1)*

eg $f(x) = 0,\ 4 - 2e^x = 0$

0.693147

$x = \ln 2$ (exact), 0.693 ***A1*** ***N2***

[2 marks]

(b) attempt to substitute either their correct limits or the function into formula involving f^2 *(M1)*

eg $\int_0^{0.693} f^2,\quad \pi\int\left(4 - 2e^x\right)^2 dx,\ \int_0^{\ln 2}\left(4 - 2e^x\right)^2$

3.42545

volume $= 3.43$ ***A2*** ***N3***

[3 marks]

Total [5 marks]

3. (a) choosing cosine rule *(M1)*

eg $c^2 = a^2 + b^2 - 2ab\cos C$

correct substitution into RHS *(A1)*

eg $4.83^2 + 3.80^2 - 2 \times 4.83 \times 3.80 \times \cos 78.2$, 30.2622, $4.83^2 + 3.80^2 - 2(4.83)(3.80)\cos 1.36$

5.50111

5.50 (cm) ***A1*** ***N2***

[3 marks]

(b) correct substitution for area of triangle ABD *(A1)*

eg $\frac{1}{2} \times 6.73 \times 5.50111 \sin\theta$

correct equation ***A1***

eg $\frac{1}{2} \times 6.73 \times 5.50111 \sin\theta = 18.5$, $\sin\theta = 0.999393$

88.0023, 91.9976, 1.53593, 1.60566

$\theta = 88.0$ (degrees) or 1.54 (radians)

$\theta = 92.0$ (degrees) or 1.61 (radians)

A1A1 ***N2***

[4 marks]

Total [7 marks]

4. (a) $\cos\theta = \dfrac{\mathrm{OC}}{r}$ ***A1***

$\mathrm{OC} = r\cos\theta$ ***AG*** ***N0***

[1 mark]

(b) valid approach ***(M1)***

eg $\dfrac{1}{2}\mathrm{OC}\times\mathrm{OB}\sin\theta,\ \mathrm{BC} = r\sin\theta$, $\dfrac{1}{2}r\cos\theta\times\mathrm{BC}$, $\dfrac{1}{2}r\sin\theta\times\mathrm{OC}$

area $= \dfrac{1}{2}r^2\sin\theta\cos\theta$ $\left(= \dfrac{1}{4}r^2\sin(2\theta)\right)$ (must be in terms of r and θ) ***A1*** ***N2***

[2 marks]

(c) valid attempt to express the relationship between the areas (seen anywhere) ***(M1)***

eg $\mathrm{OCB} = \dfrac{3}{5}\mathrm{OBA}$, $\dfrac{1}{2}r^2\sin\theta\cos\theta = \dfrac{3}{5}\times\dfrac{1}{2}r^2\theta$, $\dfrac{1}{4}r^2\sin 2\theta = \dfrac{3}{10}r^2\theta$

correct equation in terms of θ only ***A1***

eg $\sin\theta\cos\theta = \dfrac{3}{5}\theta$, $\dfrac{1}{4}\sin 2\theta = \dfrac{3}{10}\theta$

valid attempt to solve **their** equation ***(M1)***

eg sketch, -0.830017 , 0

0.830017

$\theta = 0.830$ ***A1*** ***N2***

Note: Do not award final ***A1*** if additional answers given.

[4 marks]

Total [7 marks]

5. (a) valid approach *(M1)*
eg $f(10)$

235.402
235 (fish) (must be an integer) ***A1*** ***N2***
[2 marks]

(b) recognizing rate of change is derivative *(M1)*
eg rate $= f'$, $f'(10)$, sketch of f', 35 (fish per month)

35.9976
36.0 (fish per month) ***A1*** ***N2***
[2 marks]

(c) valid approach *(M1)*
eg maximum of f', $f'' = 0$

15.890
15.9 (months) ***A1*** ***N2***
[2 marks]

Total [6 marks]

6. valid approach for expanding binomial (must have correct substitution for parameters, but accept "r" or an incorrect value for r) ***(M1)***

eg $\binom{15}{r}\left(\frac{1}{2x}\right)^{(15-r)}\left(x^2\right)^r$, $\left(\frac{1}{2x}\right)^{15}\left(x^2\right)^0+15\left(\frac{1}{2x}\right)^{14}\left(x^2\right)^1+\binom{15}{2}\left(\frac{1}{2x}\right)^{13}\left(x^2\right)^2+\ldots$

recognizing need to find the term containing x^{-3} in the expansion of $\left(\frac{1}{2x}+x^2\right)^{15}$ ***(M1)***

correct equation ***(A1)***

eg $\left(x^{-1}\right)^{15-r}\left(x^2\right)^r=x^{-3}$, $\left(x^{-1}\right)^r\left(x^2\right)^{15-r}=x^{-3}$, $-15+r+2r=-3$

identifying the correct term (seen anywhere) ***A1***

eg $r=4$, $r=11$, $n-r=4$

correct working ***(A1)(A1)***

eg $\binom{15}{4}\left(\frac{1}{2x}\right)^{15-4}$, $1365\times\frac{1}{2^{11}}$

Note: Award ***A1*** for each factor.

$\frac{1365}{2048}$ ***A1*** ***N2***

[7 marks]

7. METHOD 1 (Distance between the origin and P)

correct position vector for OP *(A1)*

eg $\vec{\mathrm{OP}} = \begin{pmatrix} -1+4t \\ 3+5t \\ 8-t \end{pmatrix}$, $\mathrm{P} = (-1+4t,\ 3+5t,\ 8-t)$

correct expression for OP or OP^2 (seen anywhere) ***A1***

eg $\sqrt{(-1+4t)^2+(3+5t)^2+(8-t)^2}$, $(-1+4x)^2+(3+5x)^2+(8-x)^2$

valid attempt to find the minimum of OP *(M1)*

eg $d'=0$, root on sketch of d', min indicated on sketch of d

$t = -\frac{1}{14}, -0.0714285$ *(A1)*

substitute their value of t into L (only award if there is working to find t) *(M1)*

eg one correct coordinate, $-1+4\left(-\frac{1}{14}\right)$

$(-1.28571,\ 2.64285,\ 8.07142)$

$\left(-\frac{9}{7}, \frac{37}{14}, \frac{113}{14}\right) = (-1.29,\ 2.64,\ 8.07)$ ***A1*** ***N2***

METHOD 2 (Perpendicular vectors)

recognizing that closest implies perpendicular *(M1)*

eg $\vec{\mathrm{OP}} \perp L$ (may be seen on sketch), $a \bullet b = 0$

valid approach involving $\vec{\mathrm{OP}}$ *(M1)*

eg $\vec{\mathrm{OP}} = \begin{pmatrix} -1+4t \\ 3+5t \\ 8-t \end{pmatrix}$, $\begin{pmatrix} 4 \\ 5 \\ -1 \end{pmatrix} \bullet \vec{\mathrm{OP}}$, $\begin{pmatrix} 4 \\ 5 \\ -1 \end{pmatrix} \perp \vec{\mathrm{OP}}$

correct scalar product ***A1***

eg $4(-1+4t)+5(3+5t)-1(8-t)$, $-4+16t+15+25t-8+t=0$, $42t+3$

$t = -\frac{1}{14}, -0.0714285$ *(A1)*

substitute their value of t into L or $\vec{\mathrm{OP}}$ (only award if scalar product used to find t) *(M1)*

eg one correct coordinate, $-1+4\left(-\frac{1}{14}\right)$

$(-1.28571,\ 2.64285,\ 8.07142)$

$\left(-\frac{9}{7}, \frac{37}{14}, \frac{113}{14}\right) = (-1.29,\ 2.64,\ 8.07)$ ***A1*** ***N2***

[6 marks]

Section B

8. (a) valid approach *(M1)*

eg $s_A(0)$, $s(0)$, $t = 0$

15 (cm) *A1* *N2*

[2 marks]

(b) valid approach *(M1)*

eg $s_A = 0$, $s = 0$, 6.79321, 14.8651

2.46941

$t = 2.47$ (seconds) *A1* *N2*

[2 marks]

(c) recognizing when change in direction occurs *(M1)*

eg slope of s changes sign, $s' = 0$, minimum point, 10.0144, (4.08, -4.66)

4.07702

$t = 4.08$ (seconds) *A1* *N2*

[2 marks]

(d) **METHOD 1 (using displacement)**

correct displacement or distance from P at $t = 3$ (seen anywhere) *(A1)*

eg -2.69630, 2.69630

valid approach *(M1)*

eg $15 + 2.69630$, $s(3) - s(0)$, -17.6963

17.6963

17.7 (cm) *A1* *N2*

METHOD 2 (using velocity)

attempt to substitute either limits or the velocity function into distance formula involving $|v|$ *(M1)*

eg $\int_0^3 |v|\,\mathrm{d}t$, $\int \left|-1-18t^2\mathrm{e}^{-0.8t}+4.8t^3\mathrm{e}^{-0.8t}\right|$

17.6963

17.7 (cm) *A2* *N2*

[3 marks]

continued…

Question 8 continued

(e) (i) recognize the need to integrate velocity **(M1)**

eg $\int v(t)$

$8t - \frac{2t^2}{2} + c$ (accept x instead of t and missing c) **(A2)**

substituting initial condition into their integrated expression (must have c) **(M1)**

eg $15 = 8(0) - \frac{2(0)^2}{2} + c$, $c = 15$

$s_B(t) = 8t - t^2 + 15$ ***A1*** ***N3***

(ii) valid approach **(M1)**

eg $s_A = s_B$, sketch, $(9.30404, 2.86710)$

9.30404

$t = 9.30$ (seconds) ***A1*** ***N2***

Note: If candidates obtain $s_B(t) = 8t - t^2$ in part (e)(i), there are 2 solutions for part (e)(ii), 1.32463 and 7.79009. Award the last ***A1*** in part (e)(ii) only if both solutions are given.

[7 marks]

Total [16 marks]

9. (a) **METHOD 1**

multiplication of $\mathrm{P}(A)$ and $\mathrm{P}(D)$ ***(A1)***

eg 0.70×0.85, 0.595

correct reasoning for their probabilities ***R1***

eg $0.595 \neq 0.65$, $0.70 \times 0.85 \neq \mathrm{P}(A \cap D)$

A and D are not independent ***AG*** ***N0***

METHOD 2

calculation of $\mathrm{P}(D \mid A)$ ***(A1)***

eg $\frac{13}{14}$, 0.928

correct reasoning for their probabilities ***R1***

eg $0.928 \neq 0.85$, $\frac{0.65}{0.7} \neq \mathrm{P}(D)$

A and D are not independent ***AG*** ***N0***

[2 marks]

(b) (i) correct working ***(A1)***

eg $\mathrm{P}(A) - \mathrm{P}(A \cap D)$, $0.7 - 0.65$, correct shading and/or value on Venn diagram

$\mathrm{P}(A \cap D') = 0.05$ ***A1*** ***N2***

(ii) recognizing conditional probability (seen anywhere) ***(M1)***

eg $\frac{\mathrm{P}(D' \cap A)}{\mathrm{P}(A)}$, $\mathrm{P}(A \mid B)$

correct working ***(A1)***

eg $\frac{0.05}{0.7}$

0.071428

$\mathrm{P}(D' \mid A) = \frac{1}{14}$, 0.0714 ***A1*** ***N2***

[5 marks]

continued…

Question 9 continued

(c) finding standardized value for 28 hours (seen anywhere) ***(A1)***

eg $z = 1.28155$

correct working to find σ ***(A1)***

eg $1.28155 = \frac{28-25}{\sigma}$, $\frac{28-25}{1.28155}$

2.34091

$\sigma = 2.34$ ***A1 N2***

[3 marks]

(d) $\mathrm{P}(X > 30) = 0.0163429$ ***(A1)***

valid approach (seen anywhere) ***(M1)***

eg $\left[\mathrm{P}\left(X > 30\right)\right]^2$, $(0.01634)^2$, $\mathrm{B}(2, 0.0163429)$, $2.67\mathrm{E}\text{-}4$, $2.66\mathrm{E}\text{-}4$

0.0267090

0.0267% ***A2 N3***

[4 marks]

Total [14 marks]

10. (a) attempt to find d ***(M1)***

eg $1.4-1.3$, u_1-u_2, $1.4=1.3+(2-1)d$

$d=0.1$ (may be seen in expression for u_n) ***(A1)***

correct equation ***(A1)***

eg $1.3+(k-1)\times 0.1=31.2$, $0.1k=30$

$k=300$ ***A1*** ***N3***

[4 marks]

(b) correct substitution ***(A1)***

eg $\frac{300}{2}(1.3+31.2)$, $\frac{300}{2}\left[2(1.3)+(300-1)(0.1)\right]$, $\frac{300}{2}\left[2.6+299(0.1)\right]$

$S_k=4875$ ***A1*** ***N2***

[2 marks]

(c) recognizing need to find the sequence of multiples of 3 (seen anywhere) ***(M1)***

eg first term is u_3 $(=1.5)$ (accept notation $u_1=1.5$), $d=0.1\times 3$ $(=0.3)$, 100 terms (accept $n=100$), last term is 31.2 (accept notation $u_{100}=31.2$), $u_3+u_6+u_9+\ldots$ (accept $F=u_3+u_6+u_9+\ldots$)

correct working for sum of sequence where n is a multiple of 3 ***A2***

$\frac{100}{2}(1.5+31.2)$, $50(2\times 1.5+99\times 0.3)$, 1635

valid approach (seen anywhere) ***(M1)***

eg $S_k-(u_3+u_6+\ldots)$, $S_k-\frac{100}{2}(1.5+31.2)$, S_k-(their sum for $(u_3+u_6+\ldots)$)

correct working (seen anywhere) ***A1***

eg S_k-1635, $4875-1635$

$F=3240$ ***AG*** ***N0***

[5 marks]

continued…

Question 10 continued

(d) attempt to find r ***(M1)***

eg dividing consecutive terms

correct value of r (seen anywhere, including in formula)

eg $\frac{1}{\sqrt{2}}$, $0.707106\ldots$, $\frac{a}{0.293\ldots}$ ***A1***

correct working (accept equation) ***(A1)***

eg $\frac{a}{1-\frac{1}{\sqrt{2}}}<3240$

correct working ***A1***

METHOD 1 (analytical)

eg $3240\times\left(1-\frac{1}{\sqrt{2}}\right)$, $a<948.974$, 948.974

METHOD 2 (using table, must find both S_∞ values)

eg when $a=948$, $S_\infty=3236.67\ldots$ **AND** when $a=949$, $S_\infty=3240.08\ldots$

$a=948$ ***A1*** ***N2***

[5 marks]

Total [16 marks]

Markscheme

November 2018

Mathematics

Standard level

Paper 2

15 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a valid **Method**; working must be seen.

(M) Marks awarded for a valid **Method**; may be implied by **correct** subsequent working.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

(A) Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.

R Marks awarded for clear **Reasoning**.

N Marks awarded for **correct** answers if **no** working shown.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Mark according to RM assessor instructions.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any. An exception to this rule is when work for ***M1*** is missing, as opposed to incorrect (see point 4).
- Where ***M*** and ***A*** marks are noted on the same line, *eg* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*eg* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***(M2)***, ***N3***, *etc.*, do **not** split the marks, unless there is a note.
- Most ***M*** marks are for a **valid** method, ie a method which can lead to the answer: it must indicate some form of progress towards the answer.
- Once a correct answer to a question or part-question is seen, ignore further correct working. However, if further working indicates a lack of mathematical understanding do not award final ***A1***.

3 *N* marks

*If **no** working shown, award **N** marks for **correct** answers. In this case, ignore mark breakdown (**M, A, R**).*

- Do **not** award a mixture of ***N*** and other marks.
- There may be fewer ***N*** marks available than the total of ***M***, ***A*** and ***R*** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.
- There may not be a direct relationship between the ***N*** marks and the implied marks. There are times when all the marks are implied, but the ***N*** marks are not the full marks: this indicates that we want to see some of the working, without specifying what.
- For consistency within the markscheme, ***N*** marks are noted for every part, even when these match the mark breakdown.

- If a candidate has incorrect working, which somehow results in a correct answer, do **not** award the ***N*** marks for this correct answer. However, if the candidate has indicated (usually by crossing out) that the working is to be ignored, award the ***N*** marks for the correct answer.

4 Implied and must be seen marks

Implied marks appear in ***brackets*** *eg* ***(M1)****.*

- Implied marks can only be awarded if the work is seen or if implied in subsequent working (a correct final answer does not necessarily mean that the implied marks are all awarded). There are questions where some working is required, but as it is accepted that not everyone will write the same steps, all the marks are implied, but the ***N*** marks are not the full marks for the question.
- Normally the correct work is seen in the next line.
- Where there is an (***M1***) followed by ***A1*** for each correct answer, if no working shown, one correct answer is sufficient evidence to award the (***M1***).

Must be seen marks appear without ***brackets*** *eg* ***M1***.

- Must be seen marks can only be awarded if the work is seen.
- If a must be seen mark is not awarded because work is missing (as opposed to ***M0*** or ***A0*** for incorrect work) all subsequent marks may be awarded if appropriate.

5 Follow through marks (only applied after an error is made)

*Follow through (****FT****) marks are awarded where an incorrect answer (final or intermediate) from one* ***part*** *of a question is used correctly in* ***subsequent*** *part(s) or subpart(s). Usually, to award* ***FT*** *marks,* ***there must be working present*** *and not just a final answer based on an incorrect answer to a previous part. However, if the only marks awarded in a subpart are for the* ***final*** *answer, then* ***FT*** *marks should be awarded if appropriate. Examiners are expected to check student work in order to award* ***FT*** *marks where appropriate.*

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** and ***R*** marks may be awarded if appropriate. (However, as noted above, if an ***A*** mark is not awarded because work is missing, all subsequent marks may be awarded if appropriate).
- Exceptions to this rule will be explicitly noted on the markscheme.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
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- The markscheme may use the word "**their**" in a description, to indicate that candidates may be using an incorrect value.
- If a candidate makes an error in one part, but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the question says hence. It is often possible to use a different approach in subsequent parts that does not depend on the answer to previous parts.
- In a "show that" question, if an error in a previous subpart leads to not showing the required answer, do not award the final ***A1***. Note that if the error occurs within the same subpart, the ***FT*** rules may result in further loss of marks.

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11 Style

The markscheme aims to present answers using good communication, eg if the question asks to find the value of k, the markscheme will say $k = 3$, but the marks will be for the correct value 3 – there is usually no need for the "$k =$". In these cases, it is also usually acceptable to have

another variable, as long as there is no ambiguity in the question, eg if the question asks to find the value of p and of q, then the student answer needs to be clear. Generally, the only situation where the full answer is required is in a question which asks for equations – in this case the markscheme will say "must be an equation".

*The markscheme often uses words to describe what the marks are for, followed by examples, using the eg notation. These examples are not exhaustive, and examiners should check what candidates have written, to see if they satisfy the description. Where these marks are **M** marks, the examples may include ones using poor notation, to indicate what is acceptable.*

12 Candidate work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

Candidates are meant to write their answers to Section A on the question paper (QP), and Section B on answer booklets. Sometimes, they need more room for Section A, and use the booklet (and often comment to this effect on the QP), or write outside the box. That is fine, and this work should be marked.

The instructions tell candidates not to write on Section B of the QP. Thus they may well have done some rough work here which they assume will be ignored. If they have solutions on the answer booklets, there is no need to look at the QP. However, if there are whole questions or whole part solutions missing on answer booklets, please check to make sure that they are not on the QP, and if they are, mark those whole questions or whole part solutions that have not been written on answer booklets.

13. Diagrams

The notes on how to allocate marks for sketches usually refer to passing through particular points or having certain features. These marks can only be awarded if the sketch is approximately the correct shape. All values given will be an approximate guide to where these points/features occur. In some questions, the first ***A1*** is for the shape, in others, the marks are only for the points and/or features. In both cases, unless the shape is approximately correct, no marks can be awarded (unless otherwise stated). However, if the graph is based on previous calculations, ***FT*** marks should be awarded if appropriate.

14. Accuracy of Answers

If the level of accuracy is specified in the question, a mark will be allocated for giving the final answer to the required accuracy. When this is not specified in the question, all numerical answers should be given exactly or correct to three significant figures.

Do not accept unfinished numerical final answers such as 3/0.1 (unless otherwise stated). As a rule, numerical answers with more than one part (such as fractions) should be given using integers (eg 6/8). Calculations which lead to integers should be completed, with the exception of fractions which are not whole numbers. Intermediate values do not need to be given to the correct three significant figures. But, if candidates work with rounded values, this could lead to an incorrect answer, in which case award A0 for the final answer. Where numerical answers are required as the final answer to a part of a question in the markscheme, the markscheme will show

a truncated 6 sf value

the exact value if applicable, the correct 3 sf answer

Section A

1. (a) valid approach *(M1)*

$(A \cap M') + (A \cap M)$, $\frac{17}{35}$, $11+6$

number of students taking art class $= 17$ *A1* *N2*

[2 marks]

(b) (i) valid approach *(M1)*

$13+5$, $35-17$, 18, $1-\mathrm{P}(A)$

0.514285

$\mathrm{P}(A') = \frac{18}{35}$ (exact), 0.514 *A1* *N2*

(ii) valid approach *(M1)*

$11+13$, $35-6-5$, 24

0.685714

$\mathrm{P}(A$ or M but not both$) = \frac{24}{35}$ (exact), 0.686 *A1* *N2*

[4 marks]

Total [6 marks]

2. (a) (i) evidence of set up *(M1)*

eg correct value for a or b or r (seen in (ii)) or $r^2 (= 0.973)$

9.91044, -31.3194

$a = 9.91$, $b = -31.3$, $y = 9.91x - 31.3$ *A1A1* *N3*

(ii) 0.986417

$r = 0.986$ *A1* *N1*

[4 marks]

(b) substituting $x = 21.5$ into **their** equation *(M1)*

eg $9.91(21.5) - 31.3$

181.755

182 (cm) *A1* *N2*

[2 marks]

Total [6 marks]

3. (a) (i) valid method *(M1)*

eg $f(0)$, sketch of graph

y-intercept is $-\frac{1}{3}$ (exact), -0.333, $\left(0, -\frac{1}{3}\right)$ ***A1*** ***N2***

(ii) $x = -\frac{3}{2}$ (must be an equation) ***A1*** ***N1***

(iii) valid method *(M1)*

eg $\frac{6}{2}$, $f(x) = 3 - \frac{10}{2x+3}$, sketch of graph

$y = 3$ (must be an equation) ***A1*** ***N2***

[5 marks]

(b) valid approach *(M1)*

eg recognizing that $\lim_{x\to\infty} f(x)$ is related to the horizontal asymptote, table with large values of x, their y value from (a)(iii), L'Hopital's rule $\lim_{x\to\infty} f(x) = 3$.

$\lim_{x\to\infty}\left(\frac{6x-1}{2x+3}\right) = 3$ ***A1*** ***N2***

[2 marks]

Total [7 marks]

4. (a) valid approach *(M1)*

eg $v(t) = 0$, sketch of graph

2.95195

$t = \log_{1.4} 2.7$ (exact), $t = 2.95$ (s) ***A1*** ***N2***

[2 marks]

(b) valid approach *(M1)*

eg $a(t) = v'(t)$, $v'(2)$

0.659485

$a(2) = 1.96\ln 1.4$ (exact), $a(2) = 0.659$ $(\mathrm{m\,s^{-2}})$ ***A1*** ***N2***

[2 marks]

(c) correct approach *(A1)*

eg $\int_0^5 |v(t)|\,\mathrm{d}t$, $\int_0^{2.95}(-v(t))\,\mathrm{d}t + \int_{2.95}^5 v(t)\,\mathrm{d}t$

5.3479

distance = 5.35 (m) ***A2*** ***N3***

[3 marks]

Total [7 marks]

5. correct substitution into formula for infinite geometric series ***(A1)***

eg $33.25 = \dfrac{u_1}{1-r}$

correct substitution into formula for u_n (seen anywhere) ***(A1)***

eg $7.98 = u_1 r$

attempt to express u_1 in terms of r (or vice-versa) ***(M1)***

eg $u_1 = \dfrac{7.98}{r}$, $u_1 = 33.25(1-r)$, $r = \dfrac{7.98}{u_1}$, $r = \dfrac{33.25-u_1}{33.25}$

correct working ***(A1)***

eg $\dfrac{\left(\dfrac{7.98}{r}\right)}{1-r} = 33.25,\ 33.25(1-r) = \dfrac{7.98}{r},\ (0.4, 19.95), (0.6, 13.3), \dfrac{u_1}{1-\dfrac{7.98}{u_1}} = 33.25$

$r = 0.4 \left(= \dfrac{2}{5}\right), r = 0.6 \left(= \dfrac{3}{5}\right)$ ***A1A1*** ***N3***

Total [6 marks]

6. valid approach for expanding binomial ***(M1)***

eg $\binom{12}{r}(2x^4)^{12-r}\left(\dfrac{x^2}{k}\right)^r$, $(2x^4)^{12} + \binom{12}{1}(2x^4)^{11}\left(\dfrac{x^2}{k}\right)^1 + \binom{12}{2}(2x^4)^{10}\left(\dfrac{x^2}{k}\right)^2 + \ldots$

valid attempt to find r for x^{40} or x^{38} ***(M1)***

eg $(x^4)^{12-r}(x^2)^r = (x)^{40}, (x^4)^r (x^2)^{12-r} = (x)^{40}$,

$\binom{12}{r}(2^r)\left(\dfrac{1}{k}\right)^{12-r}(x^4)^r(x^2)^{12-r} = \binom{12}{r}(2^r)\left(\dfrac{1}{k}\right)^{12-r} x^{38}$

correct equation for finding one value of r ***(A1)***

eg $48-2r=40,\ 48-2r=38,\ 24+2r=40,\ 2r+24=38$

correct values for r (seen anywhere) ***(A1)(A1)***

eg $r=4, r=5$ **OR** $r=7, r=8$

correct equation to solve for k ***A1***

eg $\binom{12}{4}(2^8)\left(\dfrac{1}{k}\right)^4 = 5\binom{12}{5}(2^7)\left(\dfrac{1}{k}\right)^5$, $\dfrac{126\,720}{k^4} = 5 \times \dfrac{792(128)}{k^5}$, $990k = 3960$

$k = 4$ ***A1*** ***N2***

Total [7 marks]

7. (a) recognizing $\mathrm{TR} = 32$ (seen anywhere, including diagram) ***A1***

correct working ***A1***

eg $32^2 = x^2 + 38^2 - 2(x)(38)\cos 43^\circ$, $1024 = 1444 + x^2 - 76(x)\cos 43^\circ$

$x^2 - (76\cos 43^\circ)x + 420 = 0$ ***AG*** ***N0***

[2 marks]

(b)

Note: There are many approaches to this question, depending on which triangle the candidate has used, and whether they used the cosine rule and/or the sine rule. Please check working carefully and award marks in line with the markscheme.

METHOD 1

correct values for x (seen anywhere) ***A1A1***

$x = 9.02007$, 46.5628

recognizing the need to find difference in values of x ***(M1)***

eg $46.5 - 9.02$, $x_1 - x_2$

37.5427

37.5 (km) ***A1*** ***N2***

METHOD 2

correct use of sine rule in $\Delta\mathrm{SRT}$

eg $\dfrac{\sin \mathrm{S\hat{R}T}}{38} = \dfrac{\sin 43^\circ}{32}$, $\mathrm{S\hat{R}T} = 54.0835^\circ$ ***(A1)***

recognizing isosceles triangle (seen anywhere) ***(M1)***

eg $\hat{T} = 180^\circ - 2\cdot 54.0835^\circ$, two sides of 32

correct working to find distance ***A1***

eg $\sqrt{32^2 + 32^2 - 2\cdot 32\cdot 32\cos(180^\circ - 2\cdot 54.0835^\circ)}$,

$\dfrac{\sin 71.8329^\circ}{d} = \dfrac{\sin 54.0835^\circ}{32}$, $32^2 = 32^2 + x^2 - 2\cdot 32x\cos(0.944)$

37.5427

37.5 (km) ***A1*** ***N2***

[4 marks]

Total [6 marks]

Section B

8.

> **Note:** There may be slight differences in answers, depending on which combination of unrounded values and previous correct 3 sf values the candidates carry through in subsequent parts. Accept answers that are consistent with their working.

(a) (i) valid approach ***(M1)***

eg $\mathrm{B}-\mathrm{A}$, $\mathrm{AO}+\mathrm{OB}$, $\begin{pmatrix}8\\-1\\5\end{pmatrix}-\begin{pmatrix}-3\\4\\2\end{pmatrix}$

$\overrightarrow{\mathrm{AB}}=\begin{pmatrix}11\\-5\\3\end{pmatrix}$ ***A1*** ***N2***

(ii) correct substitution into formula ***(A1)***

eg $\sqrt{11^2+(-5)^2+3^2}$

12.4498

$\left|\overrightarrow{\mathrm{AB}}\right|=\sqrt{155}$ (exact), 12.4 ***A1*** ***N2***

[4 marks]

(b) (i) valid approach to find t ***(M1)***

eg $\begin{pmatrix}5\\y\\1\end{pmatrix}=\begin{pmatrix}2\\0\\-5\end{pmatrix}+t\begin{pmatrix}1\\-2\\2\end{pmatrix}$, $5=2+t$, $1=-5+2t$

$t=3$ (seen anywhere) ***(A1)***

attempt to substitute **their** parameter into the vector equation ***(M1)***

eg $\begin{pmatrix}5\\y\\1\end{pmatrix}=\begin{pmatrix}2\\0\\-5\end{pmatrix}+3\begin{pmatrix}1\\-2\\2\end{pmatrix}$, $3\cdot(-2)$

$y=-6$ ***A1*** ***N2***

continued…

Question 8 continued

(ii) correct approach **A1**

eg $\begin{pmatrix} 5 \\ -6 \\ 1 \end{pmatrix} - \begin{pmatrix} -3 \\ 4 \\ 2 \end{pmatrix}$, $\text{AO} + \text{OC}$, $c - a$

$\overrightarrow{\text{AC}} = \begin{pmatrix} 8 \\ -10 \\ -1 \end{pmatrix}$ **AG** **N0**

Note: Do not award ***A1*** in part (ii) unless answer in part (i) is correct and does not result from working backwards.

[5 marks]

(c) finding scalar product and magnitude ***(A1)(A1)***

scalar product $= 11 \times 8 + -5 \times -10 + 3 \times -1 \quad (= 135)$

$\left| \overrightarrow{\text{AC}} \right| = \sqrt{8^2 + (-10)^2 + (-1)^2} \quad \left(= \sqrt{165}, 12.8452 \right)$

evidence of substitution into formula ***(M1)***

eg $\cos\theta = \dfrac{11 \times 8 + -5 \times -10 + 3 \times -1}{\left| \overrightarrow{\text{AB}} \right| \times \sqrt{8^2 + (-10)^2 + (-1)^2}}$, $\cos\theta = \dfrac{\overrightarrow{\text{AB}} \bullet \overrightarrow{\text{AC}}}{\sqrt{155} \times \sqrt{8^2 + (-10)^2 + (-1)^2}}$

correct substitution ***(A1)***

eg $\cos\theta = \dfrac{11 \times 8 + -5 \times -10 + 3 \times -1}{\sqrt{155} \times \sqrt{8^2 + (-10)^2 + (-1)^2}}$, $\cos\theta = \dfrac{135}{159.921\ldots}$,

$\cos\theta = 0.844162\ldots$

$0.565795, \; 32.4177^\circ$

$\hat{A} = 0.566, \; 32.4^\circ$ ***A1*** ***N3***

[5 marks]

(d) correct substitution into area formula ***(A1)***

eg $\frac{1}{2} \times \sqrt{155} \times \sqrt{165} \times \sin(0.566\ldots)$, $\frac{1}{2} \times \sqrt{155 \times 165} \times \sin(32.4)$

42.8660

area $= 42.9$ ***A1*** ***N2***

[2 marks]

Total [16 marks]

9. (a) 0.010724
0.0107 ***A2*** ***N2***
[2 marks]

(b) correct z-value ***(A1)***
$0.263714\ldots$

evidence of appropriate approach ***(M1)***
eg $\dfrac{0.65-0.592}{\sigma}$, $0.264=\dfrac{x-\mu}{\sigma}$

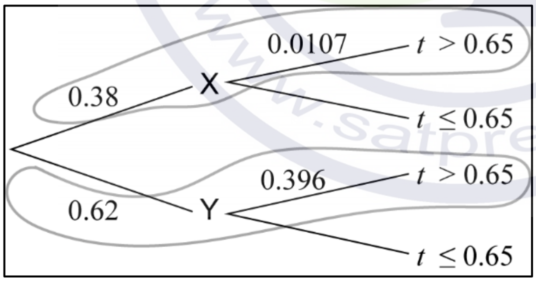
correct substitution ***(A1)***
eg $0.263714=\dfrac{0.65-0.592}{\sigma}$, $\sigma=\dfrac{0.65-0.592}{0.264}$

0.219934
$\sigma=0.220$ ***A1*** ***N3***
[4 marks]

(c) correct work for $\mathrm{P}(\text{group X and } t>0.65)$ or $\mathrm{P}(\text{group Y and } t>0.65)$ (may be seen anywhere) ***(A1)***
eg $\mathrm{P}(\text{group X})\times\mathrm{P}(t>0.65\,|\,\mathrm{X})$, $\mathrm{P}(X\cap t>0.65)=0.0107\times0.38\,(=0.004075)$, $\mathrm{P}(Y\cap t>0.65)=0.396\times0.62$

recognizing conditional probability (seen anywhere) ***(M1)***
eg $\mathrm{P}(\mathrm{X}\,|\,t>0.65)$, $\mathrm{P}(A\,|\,B)=\dfrac{\mathrm{P}(A\cap B)}{\mathrm{P}(B)}$

valid approach to find $\mathrm{P}(t>0.65)$ ***(M1)***

eg 

, $\mathrm{P}(\text{X and } t>0.65)+\mathrm{P}(\text{Y and } t>0.65)$

correct work for $\mathrm{P}(t>0.65)$ ***(A1)***
eg $0.0107\times0.38+0.396\times0.62$, 0.249595

correct substitution into conditional probability formula ***A1***
eg $\dfrac{0.0107\times0.38}{0.0107\times0.38+0.396\times0.62}$, $\dfrac{0.004075}{0.249595}$

0.016327
$\mathrm{P}(\mathrm{X}\,|\,t>0.65)=0.0163270$ ***A1*** ***N3***
[6 marks]

continued…

Question 9 continued

(d) recognizing binomial probability **(M1)**

eg $X \sim \mathrm{B}(n,\ p)$, $\binom{n}{r} p^r q^{n-r}$, $(0.016327)^2 (0.983672)^8$, $\binom{10}{2}$

valid approach **(M1)**

eg $\mathrm{P}(X \geq 2) = 1 - \mathrm{P}(X \leq 1)$, $1 - \mathrm{P}(X < a)$, summing terms from 2 to 10 (accept binomcdf(10, 0.0163, 2, 10))

0.010994

$\mathrm{P}(X \geq 2) = 0.0110$ ***A1*** ***N2***

[3 marks]

Total [15 marks]

10. (a) attempt to substitute correct limits or the function into formula involving f^2 **(M1)**

eg $\pi \int_{-2}^{2} y^2 \, \mathrm{d}y$, $\pi \int \left(\sqrt{\frac{4 - x^2}{8}} \right)^2 \mathrm{d}x$

4.18879

volume $= 4.19$, $\frac{4}{3}\pi$ (exact) (m^3) ***A2*** ***N3***

> **Note:** If candidates have their GDC incorrectly set in degrees, award ***M*** marks where appropriate, but no ***A*** marks may be awarded. Answers from degrees are $p = 13.1243$ and $q = 26.9768$ in (b)(i) and 12.3130 or 28.3505 in (b)(ii).

[3 marks]

(b) (i) recognizing the volume increases when g' is positive **(M1)**

eg $g'(t) > 0$, sketch of graph of g' indicating correct interval

1.73387, 3.56393

$p = 1.73$, $q = 3.56$ ***A1A1*** ***N3***

(ii) valid approach to find change in volume **(M1)**

eg $g(q) - g(p)$, $\int_p^q g'(t) \, \mathrm{d}t$

3.74541

total amount $= 3.75$ (m^3) ***A2*** ***N3***

[6 marks]

continued…

Question 10 continued

(c)

Note: There may be slight differences in the final answer, depending on which values candidates carry through from previous parts. Accept answers that are consistent with correct working.

recognizing when the volume of water is a maximum ***(M1)***

eg maximum when $t = q$, $\int_0^q g'(t)\,\mathrm{d}t$

valid approach to find maximum volume of water ***(M1)***

eg $2.3 + \int_0^q g'(t)\,\mathrm{d}t$, $2.3 + \int_0^p g'(t)\,\mathrm{d}t + 3.74541$, 3.85745

correct expression for the difference between volume of container and maximum value ***(A1)***

eg $4.18879 - \left(2.3 + \int_0^q g'(t)\,\mathrm{d}t\right)$, $4.19 - 3.85745$

0.331334

0.331 (m^3) ***A2 N3***

[5 marks]

Total [14 marks]

Markscheme

May 2018

Mathematics

Standard level

Paper 2

16 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a valid **Method**; working must be seen.

(M) Marks awarded for a valid **Method**; may be implied by **correct** subsequent working.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

(A) Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.

R Marks awarded for clear **Reasoning**.

N Marks awarded for **correct** answers if **no** working shown.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Mark according to RM assessor instructions.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any. An exception to this rule is when work for ***M1*** is missing, as opposed to incorrect (see point 4).
- Where ***M*** and ***A*** marks are noted on the same line, *eg* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*eg* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***(M2)***, ***N3***, *etc*., do **not** split the marks, unless there is a note.
- Most ***M*** marks are for a **valid** method, ie a method which can lead to the answer: it must indicate some form of progress towards the answer.
- Once a correct answer to a question or part-question is seen, ignore further correct working. However, if further working indicates a lack of mathematical understanding do not award final ***A1***.

3 *N* marks

*If **no** working shown, award **N** marks for **correct** answers – this includes acceptable answers (see accuracy booklet). In this case, ignore mark breakdown (**M, A, R**). Where a student only shows a final incorrect answer with no working, even if that answer is a correct intermediate answer, award **N0.***

- Do **not** award a mixture of ***N*** and other marks.
- There may be fewer ***N*** marks available than the total of ***M***, ***A*** and ***R*** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.
- There may not be a direct relationship between the ***N*** marks and the implied marks. There are times when all the marks are implied, but the ***N*** marks are not the full marks: this indicates that we want to see some of the working, without specifying what.

- For consistency within the markscheme, ***N*** marks are noted for every part, even when these match the mark breakdown.
- If a candidate has incorrect working, which somehow results in a correct answer, do **not** award the ***N*** marks for this correct answer. However, if the candidate has indicated (usually by crossing out) that the working is to be ignored, award the ***N*** marks for the correct answer.

4 Implied and must be seen marks

*Implied marks appear in **brackets** eg **(M1)**.*

- Implied marks can only be awarded if the work is seen or if implied in subsequent working (a correct final answer does not necessarily mean that the implied marks are all awarded). There are questions where some working is required, but as it is accepted that not everyone will write the same steps, all the marks are implied, but the ***N*** marks are not the full marks for the question.
- Normally the correct work is seen in the next line.
- Where there is an (***M1***) followed by ***A1*** for each correct answer, if no working shown, one correct answer is sufficient evidence to award the (***M1***).

*Must be seen marks appear without **brackets** eg **M1**.*

- Must be seen marks can only be awarded if the work is seen.
- If a must be seen mark is not awarded because work is missing (as opposed to ***M0*** or ***A0*** for incorrect work) all subsequent marks may be awarded if appropriate.

5 Follow through marks (only applied after an error is made)

*Follow through (**FT**) marks are awarded where an incorrect answer (final or intermediate) from one **part** of a question is used correctly in **subsequent** part(s) or subpart(s). Usually, to award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if the only marks awarded in a subpart are for the **final** answer, then **FT** marks should be awarded if appropriate. Examiners are expected to check student work in order to award **FT** marks where appropriate.*

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** and ***R*** marks may be awarded if appropriate. (However, as noted above, if an ***A*** mark is not awarded because work is missing, all subsequent marks may be awarded if appropriate).
- Exceptions to this rule will be explicitly noted on the markscheme.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*eg* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word "**their**" in a description, to indicate that candidates may be using an incorrect value.
- If a candidate makes an error in one part, but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the question says hence. It is often possible to use a different approach in subsequent parts that does not depend on the answer to previous parts.
- In a "show that" question, if an error in a previous subpart leads to not showing the required answer, do not award the final ***A1***. Note that if the error occurs within the same subpart, the ***FT*** rules may result in further loss of marks.

6 Mis-read

*If a candidate incorrectly copies information from the question, this is a mis-read (**MR**). A candidate should be penalized only once for a particular mis-read. Use the **MR** stamp to indicate that this is a misread. Do not award the first mark in the question, even if this is an **M** mark, but award all others (if appropriate) so that the candidate only loses one mark for the misread.*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*eg* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.

7 Discretionary marks *(d)*

*An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. In such cases the annotation DM should be used and a brief **note** written next to the mark explaining this decision.*

8 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete parts are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**. Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9 Alternative forms

*Unless the question specifies otherwise, **accept** equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

10 Calculators

A GDC is required for paper 2, but calculators with symbolic manipulation features (eg TI-89) are not allowed.

Calculator notation The mathematics SL guide says:
Students must always use correct mathematical notation, not calculator notation.
Do **not** accept final answers written using calculator notation. However, do not penalize the use of calculator notation in the working.

11 Style

The markscheme aims to present answers using good communication, eg if the question asks to find the value of k, the markscheme will say $k = 3$, but the marks will be for the correct value 3 – there is usually no need for the "$k =$". In these cases, it is also usually acceptable to have

another variable, as long as there is no ambiguity in the question, eg if the question asks to find the value of p and of q, then the student answer needs to be clear. Generally, the only situation where the full answer is required is in a question which asks for equations – in this case the markscheme will say "must be an equation".

*The markscheme often uses words to describe what the marks are for, followed by examples, using the eg notation. These examples are not exhaustive, and examiners should check what candidates have written, to see if they satisfy the description. Where these marks are **M** marks, the examples may include ones using poor notation, to indicate what is acceptable. A valid method is one which will allow candidate to proceed to the next step eg if a quadratic function is given in factorised form, and the question asks for the zeroes, then multiplying the factors does not necessarily help to find the zeros, and would not on its own count as a valid method.*

12 Candidate work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

Candidates are meant to write their answers to Section A on the question paper (QP), and Section B on answer booklets. Sometimes, they need more room for Section A, and use the booklet (and often comment to this effect on the QP), or write outside the box. That is fine, and this work should be marked.

The instructions tell candidates not to write on Section B of the QP. Thus they may well have done some rough work here which they assume will be ignored. If they have solutions on the answer booklets, there is no need to look at the QP. However, if there are whole questions or whole part solutions missing on answer booklets, please check to make sure that they are not on the QP, and if they are, mark those whole questions or whole part solutions that have not been written on answer booklets.

13. Diagrams

The notes on how to allocate marks for sketches usually refer to passing through particular points or having certain features. These marks can only be awarded if the sketch is approximately the correct shape. All values given will be an approximate guide to where these points/features occur. In some questions, the first ***A1*** is for the shape, in others, the marks are only for the points and/or features. In both cases, unless the shape is approximately correct, no marks can be awarded (unless otherwise stated). However, if the graph is based on previous calculations, ***FT*** marks should be awarded if appropriate.

14. Accuracy of Answers

If the level of accuracy is specified in the question, a mark will be allocated for giving the final answer to the required accuracy. When this is not specified in the question, all numerical answers should be given exactly or correct to three significant figures.

Do not accept unfinished numerical final answers such as 3/0.1 (unless otherwise stated). As a rule, numerical answers with more than one part (such as fractions) should be given using integers (eg 6/8). Calculations which lead to integers should be completed, with the exception of fractions which are not whole numbers. Intermediate values do not need to be given to the correct three significant figures. But, if candidates work with rounded values, this could lead to an incorrect answer, in which case award A0 for the final answer. Where numerical answers are required as the final answer to a part of a question in the markscheme, the markscheme will show

- a truncated 6 sf value
- the exact value if applicable, the correct 3 sf answer

Units will appear in brackets at the end.

Section A

1. (a) $f'(x) = \frac{1}{x} - 5$ ***A1A1*** ***N2***

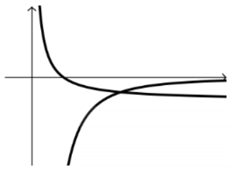
[2 marks]

(b) $f''(x) = -x^{-2}$ ***A1*** ***N1***

[1 mark]

(c) **METHOD 1 (using GDC)**

valid approach ***(M1)***

eg  , -0.358

0.558257

$x = 0.558$ ***A1*** ***N2***

Note: Do not award ***A1*** if additional answers given.

METHOD 2 (analytical)

attempt to solve their equation $f'(x) = f''(x)$ $\left(\text{do not accept } \frac{1}{x} - 5 = -\frac{1}{x^2}\right)$ ***(M1)***

eg $5x^2 - x - 1 = 0$, $\frac{1 \pm \sqrt{21}}{10}$, $\frac{1}{x} = \frac{-1 \pm \sqrt{21}}{2}$, -0.358

0.558257

$x = 0.558$ ***A1*** ***N2***

Note: Do not award ***A1*** if additional answers given.

[2 marks]

Total [5 marks]

2. (a) evidence of summing to 1 ***(M1)***

eg $0.28+k+0.15+0.3=1$, $0.73+k=1$

$k=0.27$ ***A1 N2***

[2 marks]

(b) correct substitution into formula for $\mathrm{E}(X)$ ***(A1)***

eg $1\times0.28+2\times k+3\times0.15+4\times0.3$

$\mathrm{E}(X)=2.47$ (exact) ***A1 N2***

[2 marks]

(c) valid approach ***(M1)***

eg np, 80×0.15

12 ***A1 N2***

[2 marks]

Total [6 marks]

3. (a) valid approach to find area of segment ***(M1)***

eg area of sector – area of triangle, $\frac{1}{2}r^2(\theta-\sin\theta)$

correct substitution ***(A1)***

eg $\frac{1}{2}(4)^2\theta-\frac{1}{2}(4)^2\sin\theta$, $\frac{1}{2}\times16[\theta-\sin\theta]$

area $=8\theta-8\sin\theta$, $8(\theta-\sin\theta)$ ***A1 N2***

[3 marks]

(b) setting **their** area expression equal to 12 ***(M1)***

eg $12=8(\theta-\sin\theta)$

2.26717

$\theta=2.27$ (do not accept an answer in degrees) ***A2 N3***

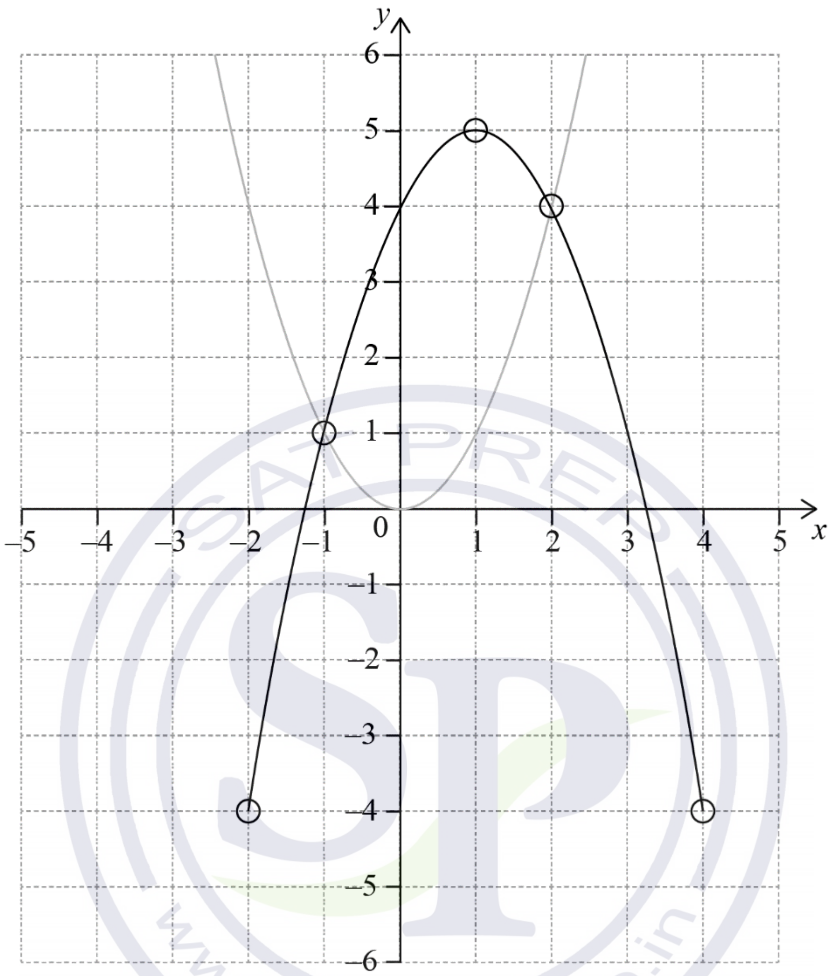
[3 marks]

Total [6 marks]

4. (a) $(1, 5)$ (exact) ***A1*** ***N1***

[1 mark]

(b)



A1A1A1 ***N3***

Note:	The shape must be a concave-down parabola. Only if the shape is correct, award the following for points in circles: ***A1*** for vertex, ***A1*** for correct intersection points, ***A1*** for correct endpoints.

[3 marks]

(c) integrating and subtracting functions (in any order) ***(M1)***

eg $\int f-g$

correct substitution of limits or functions (accept missing $\mathrm{d}x$, but do not accept any errors, including extra bits) ***(A1)***

eg $\int_{-1}^{2} g-f$, $\int -(x-1)^2+5-x^2$

area $=9$ (exact) ***A1*** ***N2***

[3 marks]

Total [7 marks]

5. (a) valid approach

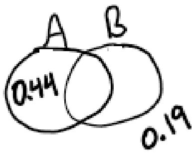
eg Venn diagram, $\mathrm{P}(A)-\mathrm{P}(A\cap B)$, $0.62-0.18$ ***(M1)***

$\mathrm{P}(A\cap B')=0.44$ ***A1*** ***N2***

[2 marks]

(b) valid approach to find either $\mathrm{P}(B')$ or $\mathrm{P}(B)$ ***(M1)***

eg



(seen anywhere), $1-\mathrm{P}(A\cap B')-\mathrm{P}\left((A\cup B)'\right)$

correct calculation for $\mathrm{P}(B')$ or $\mathrm{P}(B)$ ***(A1)***

eg $0.44+0.19$, $0.81-0.62+0.18$

correct substitution into $\dfrac{\mathrm{P}(A\cap B')}{\mathrm{P}(B')}$ ***(A1)***

eg $\dfrac{0.44}{0.19+0.44}$, $\dfrac{0.44}{1-0.37}$

0.698412

$\mathrm{P}(A\mid B')=\dfrac{44}{63}$ (exact), 0.698 ***A1*** ***N3***

[4 marks]

Total [6 marks]

6. correct substitution into the formula for area of a triangle ***(A1)***

eg $15 = \frac{1}{2} \times 8.1 \times 12.3 \times \sin C$

correct working for angle C ***(A1)***

eg $\sin C = 0.301114$, $17.5245\ldots$, 0.305860

recognizing that obtuse angle needed ***(M1)***

eg 162.475, 2.83573, $\cos C < 0$

evidence of choosing the cosine rule ***(M1)***

eg $a^2 = b^2 + c^2 - 2bc\cos(A)$

correct substitution into cosine rule to find c ***(A1)***

eg $c^2 = (8.1)^2 + (12.3)^2 - 2(8.1)(12.3)\cos C$

$c = 20.1720$ ***(A1)***

$8.1 + 12.3 + 20.1720 = 40.5720$

perimeter $= 40.6$ ***A1*** ***N4***

Total [7 marks]

7. (a) valid approach to find maxima ***(M1)***

eg one correct value of x_k, sketch of f

any two correct consecutive values of x_k ***(A1)(A1)***

eg $x_1 = 1$, $x_2 = 5$

$a = 4$ ***A1*** ***N3***

[4 marks]

(b) recognizing the sequence $x_1, x_2, x_3, \ldots, x_n$ is arithmetic ***(M1)***

eg $d = 4$

correct expression for sum ***(A1)***

eg $\frac{n}{2}\left(2(1) + 4(n-1)\right)$

valid attempt to solve for n ***(M1)***

eg graph, $2n^2 - n - 861 = 0$

$n = 21$ ***A1*** ***N2***

[4 marks]

Total [8 marks]

Section B

8. (a) valid approach **(M1)**
eg one correct value

-0.453620, 6.14210
$a = -0.454$, $b = 6.14$ **A1A1** **N3**
[3 marks]

(b) correct substitution **(A1)**
eg $-0.454 \ln 3.57 + 6.14$

correct working **(A1)**
eg $\ln y = 5.56484$

261.083 (260.409 from 3 sf)

$y = 261$, ($y = 260$ from 3sf) **A1** **N3**

Note: If no working shown, award ***N1*** for 5.56484. If no working shown, award ***N2*** for $\ln y = 5.56484$.

[3 marks]

(c) **METHOD 1**
valid approach for expressing $\ln y$ in terms of $\ln x$ **(M1)**
eg $\ln y = \ln(kx^n)$, $\ln(kx^n) = a \ln x + b$

correct application of addition rule for logs **(A1)**
eg $\ln k + \ln(x^n)$

correct application of exponent rule for logs **A1**
eg $\ln k + n \ln x$

comparing one term with regression equation (check ***FT***) **(M1)**
eg $n = a$, $b = \ln k$

correct working for k **(A1)**
eg $\ln k = 6.14210$, $k = e^{6.14210}$

465.030

$n = -0.454$, $k = 465$ (464 from 3sf) **A1A1** **N2N2**

continued…

Question 8(c) continued

METHOD 2

valid approach **(M1)**

eg $\mathrm{e}^{\ln y} = \mathrm{e}^{a \ln x + b}$

correct use of exponent laws for $\mathrm{e}^{a \ln x + b}$ **(A1)**

eg $\mathrm{e}^{a \ln x} \times \mathrm{e}^{b}$

correct application of exponent rule for $a \ln x$ **(A1)**

eg $\ln x^a$

correct equation in y ***A1***

eg $y = x^a \times \mathrm{e}^b$

comparing one term with equation of model (check ***FT***) **(M1)**

eg $k = \mathrm{e}^b$, $n = a$

465.030

$n = -0.454$, $k = 465$ (464 from 3sf) ***A1A1*** ***N2N2***

METHOD 3

valid approach for expressing $\ln y$ in terms of $\ln x$ (seen anywhere) **(M1)**

eg $\ln y = \ln(kx^n)$, $\ln(kx^n) = a \ln x + b$

correct application of exponent rule for logs (seen anywhere) **(A1)**

eg $\ln(x^a) + b$

correct working for b (seen anywhere) **(A1)**

eg $b = \ln(\mathrm{e}^b)$

correct application of addition rule for logs ***A1***

eg $\ln(\mathrm{e}^b x^a)$

comparing one term with equation of model (check ***FT***) **(M1)**

eg $k = \mathrm{e}^b$, $n = a$

465.030

$n = -0.454$, $k = 465$ (464 from 3sf) ***A1A1*** ***N2N2***

[7 marks]

Total [13 marks]

9. (a) correct approach indicating subtraction *(A1)*

eg $0.79-0.095$, appropriate shading in diagram

$\mathrm{P}(289 < w < 310) = 0.695$ (exact), 69.5% *A1* *N2*

[2 marks]

(b) **METHOD 1**

(i) valid approach *(M1)*

eg $1-p$, 21

-0.806421

$z=-0.806$ *A1* *N2*

(ii) attempt to standardize *(M1)*

eg $\sigma = \dfrac{289-297}{z}$, $\dfrac{289-297}{\sigma}$

correct substitution with their z (do not accept a probability) *A1*

eg $-0.806 = \dfrac{289-297}{\sigma}$, $\dfrac{289-297}{-0.806}$

9.92037

$\sigma = 9.92$ *A1* *N2*

METHOD 2

(i) & (ii)

correct expression for z (seen anywhere) *(A1)*

eg $\dfrac{289-\mu}{\sigma}$

valid approach *(M1)*

eg $1-p$, 21

-0.806421

$z=-0.806$ (seen anywhere) *A1* *N2*

valid attempt to set up an equation with **their** z
(do not accept a probability) *(M1)*

eg $-0.806 = \dfrac{289-297}{\sigma}$, $\dfrac{289-297}{-0.806}$

9.92037

$\sigma = 9.92$ *A1* *N2*

[5 marks]

continued…

Question 9 continued

(c) valid approach **(M1)**

eg $\mathrm{P}(W < w) = 0.35$, -0.385320 (accept 0.385320), diagram showing values in a standard normal distribution

correct score at the 35th percentile **(A1)**

eg 293.177

294 (g) **A1** **N2**

> **Note:** If working shown, award ***(M1)(A1)A0*** for 293.
> If no working shown, award ***N1*** for 293.177, ***N1*** for 293.
>
> Exception to the ***FT*** rule: If the score is incorrect, and working shown, award ***A1FT*** for correctly finding their minimum weight (by rounding up)

[3 marks]

(d) evidence of recognizing binomial (seen anywhere) **(M1)**

eg $X \sim \mathrm{B}(36,\ p)$, ${}_n\mathrm{C}_a \times p^a \times q^{n-a}$

correct probability (seen anywhere) **(A1)**

eg 0.65

EITHER

finding $\mathrm{P}(X \le 18)$ from GDC **(A1)**

eg 0.045720

evidence of using complement **(M1)**

eg $1 - \mathrm{P}(X \le 18)$

0.954279

$\mathrm{P}(X > 18) = 0.954$ **A1** **N2**

OR

recognizing $\mathrm{P}(X > 18) = \mathrm{P}(X \ge 19)$ **(M1)**

summing terms from 19 to 36 **(A1)**

eg $\mathrm{P}(X = 19) + \mathrm{P}(X = 20) + \ldots + \mathrm{P}(X = 36)$

0.954279

$\mathrm{P}(X > 18) = 0.954$ **A1** **N2**

[5 marks]

(e) correct calculation **(A1)**

0.954^2, $\binom{2}{2} 0.954^2 (1 - 0.954)^0$

0.910650

0.911 **A1** **N2**

[2 marks]

Total [17 marks]

10. (a) $-0.394791, 13$

$\mathrm{A}(-0.395, 13)$ ***A1A1*** ***N2***

[2 marks]

(b) (i) 13 ***A1*** ***N1***

(ii) 2π, 6.28 ***A1*** ***N1***

[2 marks]

(c) valid approach ***(M1)***

eg recognizing that amplitude is p or shift is r

$f(x) = 13\cos(x + 0.395)$ (accept $p = 13$, $r = 0.395$) ***A1A1*** ***N3***

Note: Accept any value of r of the form $0.395 + 2\pi k$, $k \in \mathbb{Z}$

[3 marks]

(d) recognizing need for $d'(t)$ ***(M1)***

eg $-12\sin(t) - 5\cos(t)$

correct approach (accept any variable for t) ***(A1)***

eg $-13\sin(t + 0.395)$, sketch of d', $(1.18, -13)$, $t = 4.32$

maximum speed $= 13$ ($\mathrm{cm\,s^{-1}}$) ***A1*** ***N2***

[3 marks]

(e) recognizing that acceleration is needed ***(M1)***

eg $a(t)$, $d''(t)$

correct equation (accept any variable for t) ***(A1)***

eg $a(t) = -2$, $\left|\frac{\mathrm{d}}{\mathrm{d}t}\left(d'(t)\right)\right| = 2$, $-12\cos(t) + 5\sin(t) = -2$

valid attempt to solve **their** equation ***(M1)***

eg sketch, 1.33

1.02154

1.02 ***A2*** ***N3***

[5 marks]

Total [15 marks]

Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

May 2018

Mathematics

Standard level

Paper 2

16 pages

International Baccalaureate
Baccalauréat International
Bachillerato Internacional

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a valid **Method**; working must be seen.

(M) Marks awarded for a valid **Method**; may be implied by **correct** subsequent working.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

(A) Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.

R Marks awarded for clear **Reasoning**.

N Marks awarded for **correct** answers if **no** working shown.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Mark according to RM assessor instructions.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any. An exception to this rule is when work for ***M1*** is missing, as opposed to incorrect (see point 4).
- Where ***M*** and ***A*** marks are noted on the same line, *eg* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*eg* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***(M2)***, ***N3***, *etc*., do **not** split the marks, unless there is a note.
- Most ***M*** marks are for a **valid** method, ie a method which can lead to the answer: it must indicate some form of progress towards the answer.
- Once a correct answer to a question or part-question is seen, ignore further correct working. However, if further working indicates a lack of mathematical understanding do not award final ***A1***.

3 *N* marks

*If **no** working shown, award **N** marks for **correct** answers – this includes acceptable answers (see accuracy booklet). In this case, ignore mark breakdown (**M**, **A**, **R**). Where a student only shows a final incorrect answer with no working, even if that answer is a correct intermediate answer, award **N0.***

- Do **not** award a mixture of ***N*** and other marks.
- There may be fewer ***N*** marks available than the total of ***M***, ***A*** and ***R*** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.
- There may not be a direct relationship between the ***N*** marks and the implied marks. There are times when all the marks are implied, but the ***N*** marks are not the full marks: this indicates that we want to see some of the working, without specifying what.

- For consistency within the markscheme, ***N*** marks are noted for every part, even when these match the mark breakdown.
- If a candidate has incorrect working, which somehow results in a correct answer, do **not** award the ***N*** marks for this correct answer. However, if the candidate has indicated (usually by crossing out) that the working is to be ignored, award the ***N*** marks for the correct answer.

4 Implied and must be seen marks

*Implied marks appear in **brackets** eg **(M1)**.*

- Implied marks can only be awarded if the work is seen or if implied in subsequent working (a correct final answer does not necessarily mean that the implied marks are all awarded). There are questions where some working is required, but as it is accepted that not everyone will write the same steps, all the marks are implied, but the ***N*** marks are not the full marks for the question.
- Normally the correct work is seen in the next line.
- Where there is an (***M1***) followed by ***A1*** for each correct answer, if no working shown, one correct answer is sufficient evidence to award the (***M1***).

*Must be seen marks appear without **brackets** eg **M1**.*

- Must be seen marks can only be awarded if the work is seen.
- If a must be seen mark is not awarded because work is missing (as opposed to ***M0*** or ***A0*** for incorrect work) all subsequent marks may be awarded if appropriate.

5 Follow through marks (only applied after an error is made)

*Follow through (**FT**) marks are awarded where an incorrect answer (final or intermediate) from one **part** of a question is used correctly in **subsequent** part(s) or subpart(s). Usually, to award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if the only marks awarded in a subpart are for the **final** answer, then **FT** marks should be awarded if appropriate. Examiners are expected to check student work in order to award **FT** marks where appropriate.*

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** and ***R*** marks may be awarded if appropriate. (However, as noted above, if an ***A*** mark is not awarded because work is missing, all subsequent marks may be awarded if appropriate).
- Exceptions to this rule will be explicitly noted on the markscheme.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*eg* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word "**their**" in a description, to indicate that candidates may be using an incorrect value.
- If a candidate makes an error in one part, but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the question says hence. It is often possible to use a different approach in subsequent parts that does not depend on the answer to previous parts.
- In a "show that" question, if an error in a previous subpart leads to not showing the required answer, do not award the final ***A1***. Note that if the error occurs within the same subpart, the ***FT*** rules may result in further loss of marks.

6 Mis-read

*If a candidate incorrectly copies information from the question, this is a mis-read (**MR**). A candidate should be penalized only once for a particular mis-read. Use the **MR** stamp to indicate that this is a misread. Do not award the first mark in the question, even if this is an **M** mark, but award all others (if appropriate) so that the candidate only loses one mark for the misread.*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*eg* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.

7 Discretionary marks *(d)*

*An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. In such cases the annotation DM should be used and a brief **note** written next to the mark explaining this decision.*

8 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete parts are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**. Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9 Alternative forms

*Unless the question specifies otherwise, **accept** equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

10 Calculators

A GDC is required for paper 2, but calculators with symbolic manipulation features (eg TI-89) are not allowed.

Calculator notation The mathematics SL guide says:
Students must always use correct mathematical notation, not calculator notation.
Do **not** accept final answers written using calculator notation. However, do not penalize the use of calculator notation in the working.

11 Style

The markscheme aims to present answers using good communication, eg if the question asks to find the value of k, the markscheme will say $k = 3$, but the marks will be for the correct value 3 – there is usually no need for the "$k =$". In these cases, it is also usually acceptable to have another variable, as long as there is no ambiguity in the question, eg if the question asks to find *the value of p and of q, then the student answer needs to be clear. Generally, the only situation where the full answer is required is in a question which asks for equations – in this case the markscheme will say "must be an equation".*

The markscheme often uses words to describe what the marks are for, followed by examples, using the eg notation. These examples are not exhaustive, and examiners should check what candidates have written, to see if they satisfy the description. Where these marks are ***M*** *marks, the examples may include ones using poor notation, to indicate what is acceptable. A valid method is one which will allow candidate to proceed to the next step eg if a quadratic function is given in factorised form, and the question asks for the zeroes, then multiplying the factors does not necessarily help to find the zeros, and would not on its own count as a valid method.*

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- a truncated 6 sf value
- the exact value if applicable, the correct 3 sf answer

Units will appear in brackets at the end.

Section A

1. (a) (i) valid approach *(M1)*

eg correct value for a or b (or for r seen in (ii))

$a = 1.91966 \;\; b = 7.97717$

$a = 1.92,\; b = 7.98$ ***A1A1*** ***N3***

(ii) 0.984674

$r = 0.985$ ***A1*** ***N1***

[4 marks]

(b) correct substitution into their equation *(A1)*

eg $1.92 \times 1.95 + 7.98$

11.7205

11.7 (kg) ***A1*** ***N2***

[2 marks]

[Total: 6 marks]

2. (a) evidence of choosing sine rule *(M1)*

eg $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

correct substitution *(A1)*

eg $\dfrac{\mathrm{DB}}{\sin 59^\circ} = \dfrac{11}{\sin 100^\circ}$

9.57429

$\mathrm{DB} = 9.57$ (cm) ***A1*** ***N2***

[3 marks]

(b) evidence of choosing cosine rule *(M1)*

eg $a^2 = b^2 + c^2 - 2bc\cos(A)$, $\mathrm{DC}^2 = \mathrm{DB}^2 + \mathrm{BC}^2 - 2\mathrm{DB} \times \mathrm{BC} \times \cos(\mathrm{D\hat{B}C})$

correct substitution into RHS *(A1)*

eg $9.57^2 + 6^2 - 2 \times 9.57 \times 6 \times \cos 82^\circ$, 111.677

10.5677

$\mathrm{DC} = 10.6$ (cm) ***A1*** ***N2***

[3 marks]

[Total: 6 marks]

3. (a) valid approach *(M1)*

eg $f(x)=0$, $e^x = 180$ or $0 \ldots$

1.14472

$x = \ln \pi$ (exact), 1.14 ***A1*** ***N2***

[2 marks]

(b) attempt to substitute either **their** limits or the function into formula involving f^2 *(M1)*

eg $\int_0^{1.14} f^2$, $\pi\int\left(\sin\left(e^x\right)\right)^2 dx$, 0.795135

2.49799

volume $= 2.50$ ***A2*** ***N3***

[3 marks]

[Total: 5 marks]

4. (a) correct substitution into infinite sum *(A1)*

eg $200 = \dfrac{4}{1-r}$

$r = 0.98$ (exact) ***A1*** ***N2***

[2 marks]

(b) correct substitution *(A1)*

$\dfrac{4(1-0.98^{8})}{1-0.98}$

29.8473

29.8 ***A1*** ***N2***

[2 marks]

(c) attempt to set up inequality (accept equation) *(M1)*

eg $\dfrac{4(1-0.98^{n})}{1-0.98} > 163$, $\dfrac{4(1-0.98^{n})}{1-0.98} = 163$

correct inequality for n (accept equation) or crossover values *(A1)*

eg $n > 83.5234$, $n = 83.5234$, $S_{83} = 162.606$ **and** $S_{84} = 163.354$

$n = 84$ ***A1*** ***N1***

[3 marks]

[Total: 7 marks]

5. valid approach to find one of the required terms (must have correct substitution for parameters but accept " r " or an incorrect value for r) ***(M1)***

eg $\binom{9}{r}(2x)^{9-r}\left(\frac{k}{x}\right)^r$, $\binom{9}{6}(2x)^6\left(\frac{k}{x}\right)^3$, $\binom{9}{0}(2x)^0\left(\frac{k}{x}\right)^9+\binom{9}{1}(2x)^1\left(\frac{k}{x}\right)^8+\ldots$, Pascal's triangle to 9th row

Note: Award ***M0*** if there is clear evidence of adding instead of multiplying.

identifying correct terms (must be clearly indicated if only seen in expansion) ***(A1)(A1)***

eg for x^3 term: $r=3$, $r=6$, 7th term, $\binom{9}{6}$, $\binom{9}{3}$, $(2x)^6\left(\frac{k}{x}\right)^3$, $5376k^3$

for x^5 term: $r=2$, $r=7$, 8th term, $\binom{9}{7}$, $\binom{9}{2}$, $(2x)^7\left(\frac{k}{x}\right)^2$, $4608k^2$

correct equation (may include powers of x) ***A1***

eg $\binom{9}{3}(2x)^6\left(\frac{k}{x}\right)^3=\binom{9}{2}(2x)^7\left(\frac{k}{x}\right)^2$

valid attempt to solve their equation in terms of k only ***(M1)***

eg sketch, $84\times 64k^3-36\times 128k^2=0$, $5376k-4608=0$, $\binom{9}{3}2^6k^3=\binom{9}{2}2^7k^2$

0.857142

$k=\frac{4608}{5376}$ $\left(=\frac{6}{7}\right)$ (exact), 0.857 ***A1*** ***N4***

[6 marks]

6. (a) valid approach to find k ***(M1)***

eg 8 minutes is half a turn, $k+$ diameter, $k+111=117$

$k=6$ ***A1*** ***N2***

[2 marks]

(b) **METHOD 1**

valid approach ***(M1)***

eg $\frac{\max - \min}{2}$, $a=$ radius

$|a| = \frac{117-6}{2}$, 55.5 ***(A1)***

$a=-55.5$ ***A1*** ***N2***

METHOD 2

attempt to substitute valid point into equation for f ***(M1)***

eg $h(0)=6$, $h(8)=117$

correct equation ***(A1)***

eg $6=61.5+a\cos\left(\frac{\pi}{8}\times 0\right)$, $117=61.5+a\cos\left(\frac{\pi}{8}\times 8\right)$, $6=61.5+a$

$a=-55.5$ ***A1*** ***N2***

[3 marks]

(c) valid approach ***(M1)***

eg sketch of h and $y=30$, $h=30$, $61.5-55.5\cos\left(\frac{\pi}{8}t\right)=30$, $t=2.46307$, $t=13.5369$

18.4630

$t=18.5$ (minutes) ***A2*** ***N3***

[3 marks]

[Total: 8 marks]

7. (a) valid approach *(M1)*

eg $cx+6=0, -\frac{6}{c}=3$

$c=-2$ ***A1*** ***N2***

[2 marks]

(b) valid approach *(M1)*

eg $\lim_{x\to\infty} f(x),\ y=\frac{8}{c}$

$y=-4$ (must be an equation) ***A1*** ***N2***

[2 marks]

(c) valid approach to analyze modulus function *(M1)*

eg sketch, horizontal asymptote at $y=4,\ y=0$

$k=4,\ k=0$ ***A2*** ***N3***

[3 marks]

[Total: 7 marks]

Section B

8. (a) (i) valid approach *(M1)*

eg $(7, 4, 9)-(3, 2, 5)$, $A-B$

$\overrightarrow{\mathrm{PQ}} = 4\boldsymbol{i}+2\boldsymbol{j}+4\boldsymbol{k}\ \left(=\begin{pmatrix}4\\2\\4\end{pmatrix}\right)$ ***A1*** ***N2***

(ii) correct substitution into magnitude formula *(A1)*

eg $\sqrt{4^2+2^2+4^2}$

$\left|\overrightarrow{\mathrm{PQ}}\right| = 6$ ***A1*** ***N2***

[4 marks]

(b) finding scalar product and magnitudes *(A1)(A1)*

scalar product $=(4\times6)+\left(2\times(-1)\right)+(4\times3)\ (=34)$

magnitude of $\mathrm{PR}=\sqrt{36+1+9}\ (=6.782)$

correct substitution of **their** values to find cos $\mathrm{Q\hat{P}R}$ ***M1***

eg $\cos\mathrm{Q\hat{P}R}=\dfrac{24-2+12}{(6)\times\left(\sqrt{46}\right)}$, 0.8355

0.581746

$\mathrm{Q\hat{P}R}=0.582$ radians or $\mathrm{Q\hat{P}R}=33.3^\circ$ ***A1*** ***N3***

[4 marks]

(c) correct substitution *(A1)*

eg $\dfrac{1}{2}\times\left|\overrightarrow{\mathrm{PQ}}\right|\times\left|\overrightarrow{\mathrm{PR}}\right|\times\sin P$, $\dfrac{1}{2}\times6\times\sqrt{46}\times\sin0.582$

11.1803

area is 11.2 (sq. units) ***A1*** ***N2***

[2 marks]

(d) recognizing shortest distance is perpendicular distance from R to line through P and Q *(M1)*

eg sketch, height of triangle with base $[\mathrm{PQ}]$, $\dfrac{1}{2}\times6\times h$, $\sin33.3^\circ=\dfrac{h}{\sqrt{46}}$

correct working *(A1)*

eg $\dfrac{1}{2}\times6\times d=11.2$, $\left|\overrightarrow{\mathrm{PR}}\right|\times\sin P$, $\sqrt{46}\sin33.3^\circ$

3.72677

distance $=3.73$ (units) ***A1*** ***N2***

[3 marks]

[Total 13 marks]

9. (a) initial velocity when $t=0$ ***(M1)***

eg $v(0)$

$v=7\ (\mathrm{m\,s^{-1}})$ ***A1*** ***N2***

[2 marks]

(b) recognizing maximum speed when $|v|$ is greatest ***(M1)***

eg minimum, maximum, $v'=0$

one correct coordinate for minimum ***(A1)***

eg $6.37896,\ -24.6571$

$24.7\ \ (\mathrm{m\,s^{-1}})$ ***A1*** ***N2***

[3 marks]

(c) recognizing $a=v'$ ***(M1)***

eg $a=\frac{\mathrm{d}v}{\mathrm{d}t}$, correct derivative of first term

identifying when $a=0$ ***(M1)***

eg turning points of v, t-intercepts of v'

3 ***A1*** ***N3***

[3 marks]

(d) recognizing P changes direction when $v=0$ ***(M1)***

$t=0.863851$ ***(A1)***

-9.24689

$a=-9.25\,(\mathrm{m\,s^{-2}})$ ***A2*** ***N3***

[4 marks]

(e) correct substitution of limits or function into formula ***(A1)***

eg $\int_0^7 |v|,\ \int_0^{0.8638} v\mathrm{d}t-\int_{0.8638}^{7} v\,\mathrm{d}t,\ \int\left|7\cos x-5x^{\cos x}\right|\mathrm{d}x,\ 3.32+60.6$

63.8874

63.9 (metres) ***A2*** ***N3***

[3 marks]

[Total: 15 Marks]

10. (a) (i) evidence of using $\sum p_i = 1$ ***(M1)***

eg $k + 0.98 + 0.01 = 1$

$k = 0.01$ ***A1*** ***N2***

(ii) recognizing that 93 and 119 are symmetrical about μ ***(M1)***

eg μ is midpoint of 93 and 119

correct working to find μ ***A1***

$\frac{119+93}{2}$

$\mu = 106$ ***AG*** ***N0***

[4 marks]

(b) finding standardized value for 93 or 119 ***(A1)***

eg $z = -2.32634,\ z = 2.32634$

correct substitution using **their** z value ***(A1)***

eg $\frac{93-106}{\sigma} = -2.32634,\ \frac{119-106}{2.32634} = \sigma$

$\sigma = 5.58815$ ***(A1)***

0.024508

$\mathrm{P}(X < 95) = 0.0245$ ***A2*** ***N3***

[5 marks]

(c) evidence of recognizing binomial ***(M1)***

eg ${}_nC_a \times p^a \times q^{n-a}$, $n = 10$ and $p = 0.0245$, $\mathrm{B}(n,\ p)$

valid approach ***(M1)***

eg $\mathrm{P}(X \le 1),\ \mathrm{P}(X = 0) + \mathrm{P}(X = 1)$

0.976285

0.976 ***A1*** ***N2***

[3 marks]

continued…

Question 10 continued

(d) (i) recognizing **new** binomial probability ***(M1)***

eg $\mathrm{B}(50,\ 0.976)$

correct substitution ***(A1)***

eg $\mathrm{E}(X)=50(0.976285)$

48.81425

48.8 ***A1*** ***N2***

(ii) valid approach ***(M1)***

eg $\mathrm{P}(\mathrm{X}\geq 48),\ \ 1-\mathrm{P}(\mathrm{X}\leq 47)$

0.884688

0.885 ***A1*** ***N2***

[5 marks]

[Total: 17 marks]

Diploma Programme
Programme du diplôme
Programa del Diploma

Markscheme

November 2017

Mathematics

Standard level

Paper 2

16 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a valid **Method**; working must be seen.

(M) Marks awarded for a valid **Method**; may be implied by **correct** subsequent working.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

(A) Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.

R Marks awarded for clear **Reasoning**.

N Marks awarded for **correct** answers if **no** working shown.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Mark according to RM assessor instructions.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any. An exception to this rule is when work for ***M1*** is missing, as opposed to incorrect (see point 4).
- Where ***M*** and ***A*** marks are noted on the same line, *eg* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*eg* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***(M2)***, ***N3***, *etc*., do **not** split the marks, unless there is a note.
- Most ***M*** marks are for a **valid** method, ie a method which can lead to the answer: it must indicate some form of progress towards the answer.
- Once a correct answer to a question or part-question is seen, ignore further correct working. However, if further working indicates a lack of mathematical understanding do not award final ***A1***.

3 *N* marks

*If **no** working shown, award **N** marks for **correct** answers – this includes acceptable answers (see accuracy booklet). In this case, ignore mark breakdown (**M**, **A**, **R**). Where a student only shows a final incorrect answer with no working, even if that answer is a correct intermediate answer, award **N0.***

- Do **not** award a mixture of ***N*** and other marks.
- There may be fewer ***N*** marks available than the total of ***M***, ***A*** and ***R*** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.
- There may not be a direct relationship between the ***N*** marks and the implied marks. There are times when all the marks are implied, but the ***N*** marks are not the full marks: this indicates that we want to see some of the working, without specifying what.

- For consistency within the markscheme, ***N*** marks are noted for every part, even when these match the mark breakdown.
- If a candidate has incorrect working, which somehow results in a correct answer, do **not** award the ***N*** marks for this correct answer. However, if the candidate has indicated (usually by crossing out) that the working is to be ignored, award the ***N*** marks for the correct answer.

4 Implied and must be seen marks

*Implied marks appear in **brackets** eg **(M1)**.*

- Implied marks can only be awarded if the work is seen or if implied in subsequent working (a correct final answer does not necessarily mean that the implied marks are all awarded). There are questions where some working is required, but as it is accepted that not everyone will write the same steps, all the marks are implied, but the ***N*** marks are not the full marks for the question.
- Normally the correct work is seen in the next line.
- Where there is an (***M1***) followed by ***A1*** for each correct answer, if no working shown, one correct answer is sufficient evidence to award the (***M1***).

*Must be seen marks appear without **brackets** eg **M1**.*

- Must be seen marks can only be awarded if the work is seen.
- If a must be seen mark is not awarded because work is missing (as opposed to ***M0*** or ***A0*** for incorrect work) all subsequent marks may be awarded if appropriate.

5 Follow through marks (only applied after an error is made)

*Follow through (**FT**) marks are awarded where an incorrect answer (final or intermediate) from one **part** of a question is used correctly in **subsequent** part(s) or subpart(s). Usually, to award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if the only marks awarded in a subpart are for the **final** answer, then **FT** marks should be awarded if appropriate. Examiners are expected to check student work in order to award **FT** marks where appropriate.*

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** and ***R*** marks may be awarded if appropriate. (However, as noted above, if an ***A*** mark is not awarded because work is missing, all subsequent marks may be awarded if appropriate).
- Exceptions to this rule will be explicitly noted on the markscheme.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*eg* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word "**their**" in a description, to indicate that candidates may be using an incorrect value.
- If a candidate makes an error in one part, but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the question says hence. It is often possible to use a different approach in subsequent parts that does not depend on the answer to previous parts.
- In a "show that" question, if an error in a previous subpart leads to not showing the required answer, do not award the final ***A1***. Note that if the error occurs within the same subpart, the ***FT*** rules may result in further loss of marks.

6 Mis-read

*If a candidate incorrectly copies information from the question, this is a mis-read (**MR**). A candidate should be penalized only once for a particular mis-read. Use the **MR** stamp to indicate that this is a misread. Do not award the first mark in the question, even if this is an **M** mark, but award all others (if appropriate) so that the candidate only loses one mark for the misread.*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*eg* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.

7 Discretionary marks *(d)*

*An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. In such cases the annotation DM should be used and a brief **note** written next to the mark explaining this decision.*

8 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete parts are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**. Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9 Alternative forms

*Unless the question specifies otherwise, **accept** equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

10 Calculators

A GDC is required for paper 2, but calculators with symbolic manipulation features (eg TI-89) are not allowed.

Calculator notation The mathematics SL guide says:
Students must always use correct mathematical notation, not calculator notation.
Do **not** accept final answers written using calculator notation. However, do not penalize the use of calculator notation in the working.

11 Style

The markscheme aims to present answers using good communication, eg if the question asks to find the value of k, the markscheme will say $k = 3$, but the marks will be for the correct value 3 – there is usually no need for the "$k =$". In these cases, it is also usually acceptable to have

another variable, as long as there is no ambiguity in the question, eg if the question asks to find the value of p and of q, then the student answer needs to be clear. Generally, the only situation where the full answer is required is in a question which asks for equations – in this case the markscheme will say "must be an equation".

The markscheme often uses words to describe what the marks are for, followed by examples, using the eg notation. These examples are not exhaustive, and examiners should check what candidates have written, to see if they satisfy the description. Where these marks are ***M*** *marks, the examples may include ones using poor notation, to indicate what is acceptable. A valid method is one which will allow candidate to proceed to the next step eg if a quadratic function is given in factorised form, and the question asks for the zeroes, then multiplying the factors does not necessarily help to find the zeros, and would not on its own count as a valid method.*

12 Candidate work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

Candidates are meant to write their answers to Section A on the question paper (QP), and Section B on answer booklets. Sometimes, they need more room for Section A, and use the booklet (and often comment to this effect on the QP), or write outside the box. That is fine, and this work should be marked.

The instructions tell candidates not to write on Section B of the QP. Thus they may well have done some rough work here which they assume will be ignored. If they have solutions on the answer booklets, there is no need to look at the QP. However, if there are whole questions or whole part solutions missing on answer booklets, please check to make sure that they are not on the QP, and if they are, mark those whole questions or whole part solutions that have not been written on answer booklets.

13. Diagrams

The notes on how to allocate marks for sketches usually refer to passing through particular points or having certain features. These marks can only be awarded if the sketch is approximately the correct shape. All values given will be an approximate guide to where these points/features occur. In some questions, the first ***A1*** is for the shape, in others, the marks are only for the points and/or features. In both cases, unless the shape is approximately correct, no marks can be awarded (unless otherwise stated). However, if the graph is based on previous calculations, ***FT*** marks should be awarded if appropriate.

14. Accuracy of Answers

If the level of accuracy is specified in the question, a mark will be allocated for giving the final answer to the required accuracy. When this is not specified in the question, all numerical answers should be given exactly or correct to three significant figures.

Do not accept unfinished numerical final answers such as 3/0.1 (unless otherwise stated). As a rule, numerical answers with more than one part (such as fractions) should be given using integers (eg 6/8). Calculations which lead to integers should be completed, with the exception of fractions which are not whole numbers. Intermediate values do not need to be given to the correct three significant figures. But, if candidates work with rounded values, this could lead to an incorrect answer, in which case award A0 for the final answer. Where numerical answers are required as the final answer to a part of a question in the markscheme, the markscheme will show
  a truncated 6 sf value
  the exact value if applicable, the correct 3 sf answer
Units will appear in brackets at the end.

Section A

1. (a) evidence of choosing sine rule *(M1)*

eg $\frac{\sin A}{a} = \frac{\sin B}{b}$

correct substitution *(A1)*

eg $\frac{\text{BC}}{\sin 50} = \frac{5}{\sin 112}$

4.13102

$\text{BC} = 4.13\ (\text{cm})$ ***A1*** ***N2***

[3 marks]

(b) correct working *(A1)*

eg $\hat{B} = 180 - 50 - 112$, $18°$, $\text{AC} = 1.66642$

correct substitution into area formula *(A1)*

eg $\frac{1}{2} \times 5 \times 4.13 \times \sin 18$, $0.5(5)(1.66642)\sin 50$, $\frac{1}{2}(4.13)(1.66642)\sin 112$

3.19139

area $= 3.19\ \ (\text{cm}^2)$ ***A1*** ***N2***

[3 marks]

Total [6 marks]

2. (a) valid approach ***(M1)***

eg $f(x) = 0$, ± 0.816

0.816496

$x = \sqrt{\dfrac{2}{3}}$ (exact), 0.816 ***A1*** ***N2***

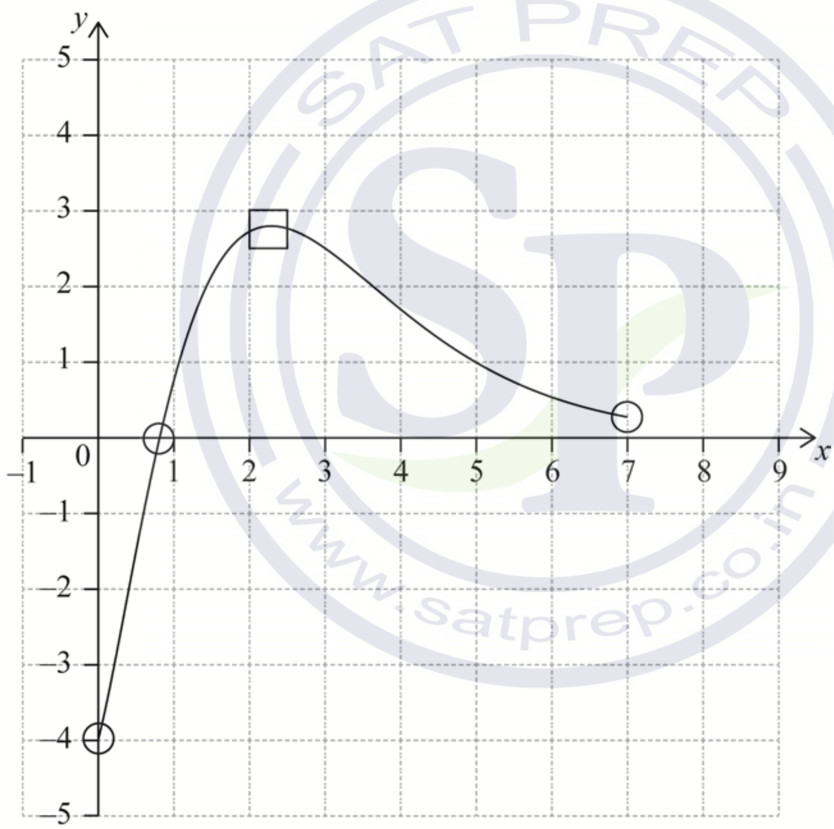
[2 marks]

(b) $(2.29099,\ 2.78124)$

$\mathrm{A}\,(2.29,\ 2.78)$ ***A1A1*** ***N2***

[2 marks]

(c)



A1A1A1 ***N3***

Notes: Award ***A1*** for correct domain and endpoints at $x = 0$ and $x = 7$ in circles,
A1 for maximum in square,
A1 for approximately correct shape that passes through **their** x-intercept in circle and has changed from concave down to concave up between 2.29 and 7.

[3 marks]

Total [7 marks]

3. (a) correct substitution ***(A1)***

eg $\sqrt{4^2+1^2+2^2}$

4.58257

$\left|\overrightarrow{AB}\right| = \sqrt{21}$ (exact), 4.58 ***A1*** ***N2***

[2 marks]

(b) finding scalar product and $\left|\overrightarrow{AC}\right|$ ***(A1)(A1)***

scalar product = $(4\times 3)+(1\times 0)+(2\times 0)\ \ (=12)$

$\left|\overrightarrow{AC}\right| = \sqrt{3^2+0+0}\ \ (=3)$

substituting **their** values into cosine formula ***(M1)***

eg $\cos \mathrm{B\hat{A}C} = \dfrac{4\times 3+0+0}{\sqrt{3^2}\times\sqrt{21}}$, $\dfrac{4}{\sqrt{21}}$, $\cos\theta = 0.873$

$0.509739\ \ (29.2059^\circ)$

$\mathrm{B\hat{A}C} = 0.510\ \ (29.2^\circ)$ ***A1*** ***N2***

[4 marks]

Total [6 marks]

4. (a) valid approach **(M1)**

eg total probability $= 1$

correct equation **(A1)**

eg $0.475 + 2k^2 + \frac{k}{10} + 6k^2 = 1,\ 8k^2 + 0.1k - 0.525 = 0$

$k = 0.25$ **A2 N3**

[4 marks]

(b) $\mathrm{P}(X = 2) = 0.025$ **A1 N1**

[1 mark]

(c) valid approach for finding $\mathrm{P}(X > 0)$ **(M1)**

eg $1 - 0.475,\ 2(0.25^2) + 0.025 + 6(0.25^2),\ 1 - \mathrm{P}(X = 0),\ 2k^2 + \frac{k}{10} + 6k^2$

correct substitution into formula for conditional probability **(A1)**

eg $\frac{0.025}{1 - 0.475},\ \frac{0.025}{0.525}$

0.0476190

$\mathrm{P}(X = 2 \mid X > 0) = \frac{1}{21}$ (exact), 0.0476 **A1 N2**

[3 marks]

Total [8 marks]

5. (a) valid approach **(M1)**

eg $f(p) = 4$, intersection with $y = 4$, ± 2.32

2.32143

$p = \sqrt{\mathrm{e}^2 - 2}$ (exact), 2.32 **A1 N2**

[2 marks]

(b) attempt to substitute **either their** limits **or** the function into volume formula (must involve f^2, accept reversed limits and absence of π and/or $\mathrm{d}x$, but do not accept any other errors) **(M1)**

eg $\int_{-2.32}^{2.32} f^2,\ \pi\int\left(6 - \ln(x^2 + 2)\right)^2 \mathrm{d}x,\ 105.675$

331.989

volume $= 332$ **A2 N3**

[3 marks]

Total [5 marks]

6. valid approach for expansion (must have correct substitution for parameters, but accept an incorrect value for *r*) ***(M1)***

eg $\binom{11}{r}(2)^{11-r}\,ax^r$, $\binom{11}{3}(2)^8(ax)^3$, $2^{11}+\binom{11}{1}(2)^{10}(ax)^1+\binom{11}{2}(2)^9(ax)^2+\ldots$

recognizing need to find term in x^2 in binomial expansion ***(A1)***

eg $r=2$, $(ax)^2$

correct term or coefficient in binomial expansion (may be seen in equation) ***(A1)***

eg $\binom{11}{2}(ax)^2(2)^9$, $55(a^2x^2)(512)$, $28160a^2$

setting up equation in x^5 with **their** coefficient/term (do not accept other powers of *x*) ***(M1)***

eg $ax^3\binom{11}{2}(ax)^2(2)^9=11880x^5$

correct equation ***(A1)***

eg $28160a^3=11880$

$a=\dfrac{3}{4}$ ***A1 N3***

[6 marks]

7. finding the z-value for 0.17 ***(A1)***
eg $z = -0.95416$

setting up equation to find σ, ***(M1)***
eg $z = \frac{168-180}{\sigma}$, $-0.954 = \frac{-12}{\sigma}$

$\sigma = 12.5765$ ***(A1)***

EITHER (Properties of the Normal curve)
correct value (seen anywhere) ***(A1)***
eg $\mathrm{P}(X < 192) = 0.83$, $\mathrm{P}(X > 192) = 0.17$

correct working ***(A1)***
eg $\mathrm{P}(X < 192-h) = 0.83 - 0.8$, $\mathrm{P}(X < 192-h) = 1 - 0.8 - 0.17$,
$\mathrm{P}(X > 192-h) = 0.8 + 0.17$

correct equation in h
eg $\frac{(192-h)-180}{12.576} = -1.88079$, $192 - h = 156.346$ ***(A1)***

35.6536
$h = 35.7$ ***A1*** ***N3***

OR (Trial and error using different values of *h*)

two correct probabilities whose 2 sf will round up **and** down, respectively, to 0.8 ***A2***
eg $\mathrm{P}(192 - 35.6 < X < 192) = 0.799706$, $\mathrm{P}(157 < X < 192) = 0.796284$,
$\mathrm{P}(192 - 36 < X < 192) = 0.801824$

$h = 35.7$ ***A2***

[7 marks]

Section B

8. (a) evidence of setup *(M1)*

eg correct value for a or b

$a = 6.96103$, $b = -454.805$

$a = 6.96$, $b = -455$ (accept $6.96x - 455$) ***A1A1*** ***N3***

[3 marks]

(b) substituting $N = 270$ into **their** equation *(M1)*

eg $6.96(270) - 455$

1424.67

$P = 1420$ (g) ***A1*** ***N2***

[2 marks]

(c) 40 (hives) ***A1*** ***N1***

[1 mark]

(d) (i) valid approach *(M1)*

eg $128 + 40$

168 hives have a production less than k *(A1)*

$k = 1640$ ***A1*** ***N3***

(ii) valid approach *(M1)*

eg $200 - 168$

32 (hives) ***A1*** ***N2***

[5 marks]

(e) recognize binomial distribution (seen anywhere) *(M1)*

eg $X \sim \mathrm{B}(n,\ p)$, $\binom{n}{r} p^r (1-p)^{n-r}$

correct values *(A1)*

eg $n = 40$ (check ***FT***) and $p = 0.75$ and $r = 30$, $\binom{40}{30} 0.75^{30} (1-0.75)^{10}$

0.144364

0.144 ***A1*** ***N2***

[3 marks]

Total [14 marks]

9. (a) $t = \frac{2}{3}$ (exact), 0.667, $t = 4$ ***A1A1*** ***N2***

[2 marks]

(b) recognizing that v is decreasing when a is negative ***(M1)***

eg $a < 0$, $3t^2 - 14t + 8 \leq 0$, sketch of a

correct interval ***A1*** ***N2***

eg $\frac{2}{3} < t < 4$

[2 marks]

(c) valid approach (do not accept a definite integral) ***(M1)***

eg $v = \int a$

correct integration (accept missing c) ***(A1)(A1)(A1)***

$t^3 - 7t^2 + 8t + c$

substituting $t = 0$, $v = 3$ (must have c) ***(M1)***

eg $3 = 0^3 - 7(0^2) + 8(0) + c$, $c = 3$

$v = t^3 - 7t^2 + 8t + 3$ ***A1*** ***N6***

[6 marks]

(d) recognizing that v increases outside the interval found in part (b) ***(M1)***

eg $0 < t < \frac{2}{3}$, $4 < t < 5$, diagram

one correct substitution into distance formula ***(A1)***

eg $\int_0^{\frac{2}{3}} |v|$, $\int_4^5 |v|$, $\int_{\frac{2}{3}}^4 |v|$, $\int_0^5 |v|$

one correct pair ***(A1)***

eg 3.13580 and 11.0833, 20.9906 and 35.2097

14.2191 ***A1*** ***N2***

$d = 14.2$ (m)

[4 marks]

Total [14 marks]

10. (a) substituting $x = 2\pi$ ***M1***

eg $2\pi + a\sin\left(2\pi - \frac{\pi}{2}\right) + a$

$2\pi + a\sin\left(\frac{3\pi}{2}\right) + a$ ***(A1)***

$2\pi - a + a$ ***A1***

$f(2\pi) = 2\pi$ ***AG*** ***N0***

[3 marks]

(b) (i) substituting the value of k ***(M1)***

$P_0(0,\ 0),\ P_1(2\pi,\ 2\pi)$ ***A1A1*** ***N3***

(ii) attempt to find the gradient ***(M1)***

eg $\frac{2\pi - 0}{2\pi - 0},\ m = 1$

correct working ***(A1)***

eg $\frac{y - 2\pi}{x - 2\pi} = 1,\ b = 0,\ y - 0 = 1(x - 0)$

$y = x$ ***A1*** ***N3***

[6 marks]

(c) subtracting x-coordinates of P_{k+1} and P_k (in any order) ***(M1)***

eg $2(k+1)\pi - 2k\pi,\ 2k\pi - 2k\pi - 2\pi$

correct working (must be in correct order) ***A1***

eg $2k\pi + 2\pi - 2k\pi,\ \left|2k\pi - 2(k+1)\pi\right|$

distance is 2π ***AG*** ***N0***

[2 marks]

continued…

Question 10 continued

(d) **METHOD 1**

recognizing the toothed-edge as the hypotenuse ***(M1)***
eg $300^2 = x^2 + y^2$, sketch

correct working (using their equation of L) ***(A1)***
eg $300^2 = x^2 + x^2$

$x = \dfrac{300}{\sqrt{2}}$ (exact), 212.132 ***(A1)***

dividing their value of x by 2π $\left(\text{do not accept } \dfrac{300}{2\pi}\right)$ ***(M1)***

eg $\dfrac{212.132}{2\pi}$

33.7618 ***(A1)***

33 (teeth) ***A1*** ***N2***

METHOD 2

vertical distance of a tooth is 2π (may be seen anywhere) ***(A1)***

attempt to find the hypotenuse for one tooth ***(M1)***
eg $x^2 = (2\pi)^2 + (2\pi)^2$

$x = \sqrt{8\pi^2}$ (exact), 8.88576 ***(A1)***

dividing 300 by their value of x ***(M1)***
eg

33.7618 ***(A1)***

33 (teeth) ***A1*** ***N2***
[6 marks]

Total [17 marks]

Markscheme

May 2017

Mathematics

Standard level

Paper 2

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a valid **Method**; working must be seen.

(M) Marks awarded for a valid **Method**; may be implied by **correct** subsequent working.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

(A) Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.

R Marks awarded for clear **Reasoning**.

N Marks awarded for **correct** answers if **no** working shown.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Mark according to RM assessor instructions.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any. An exception to this rule is when work for ***M1*** is missing, as opposed to incorrect (see point 4).
- Where ***M*** and ***A*** marks are noted on the same line, *eg* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*eg* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***(M2)***, ***N3***, *etc*., do **not** split the marks, unless there is a note.
- Most ***M*** marks are for a **valid** method, ie a method which can lead to the answer: it must indicate some form of progress towards the answer.
- Once a correct answer to a question or part-question is seen, ignore further correct working. However, if further working indicates a lack of mathematical understanding do not award final ***A1***.

3 *N* marks

*If **no** working shown, award **N** marks for **correct** answers – this includes acceptable answers (see accuracy booklet). In this case, ignore mark breakdown (**M**, **A**, **R**). Where a student only shows a final incorrect answer with no working, even if that answer is a correct intermediate answer, award **N0.***

- Do **not** award a mixture of ***N*** and other marks.
- There may be fewer ***N*** marks available than the total of ***M***, ***A*** and ***R*** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.
- There may not be a direct relationship between the ***N*** marks and the implied marks. There are times when all the marks are implied, but the ***N*** marks are not the full marks: this indicates that we want to see some of the working, without specifying what.

- For consistency within the markscheme, ***N*** marks are noted for every part, even when these match the mark breakdown.
- If a candidate has incorrect working, which somehow results in a correct answer, do **not** award the ***N*** marks for this correct answer. However, if the candidate has indicated (usually by crossing out) that the working is to be ignored, award the ***N*** marks for the correct answer.

4 Implied and must be seen marks

*Implied marks appear in **brackets** eg **(M1)**.*

- Implied marks can only be awarded if the work is seen or if implied in subsequent working (a correct final answer does not necessarily mean that the implied marks are all awarded). There are questions where some working is required, but as it is accepted that not everyone will write the same steps, all the marks are implied, but the ***N*** marks are not the full marks for the question.
- Normally the correct work is seen in the next line.
- Where there is an (***M1***) followed by ***A1*** for each correct answer, if no working shown, one correct answer is sufficient evidence to award the (***M1***).

*Must be seen marks appear without **brackets** eg **M1**.*

- Must be seen marks can only be awarded if the work is seen.
- If a must be seen mark is not awarded because work is missing (as opposed to ***M0*** or ***A0*** for incorrect work) all subsequent marks may be awarded if appropriate.

5 Follow through marks (only applied after an error is made)

*Follow through (**FT**) marks are awarded where an incorrect answer (final or intermediate) from one **part** of a question is used correctly in **subsequent** part(s) or subpart(s). Usually, to award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if the only marks awarded in a subpart are for the **final** answer, then **FT** marks should be awarded if appropriate. Examiners are expected to check student work in order to award **FT** marks where appropriate.*

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** and ***R*** marks may be awarded if appropriate. (However, as noted above, if an ***A*** mark is not awarded because work is missing, all subsequent marks may be awarded if appropriate).
- Exceptions to this rule will be explicitly noted on the markscheme.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*eg* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word "**their**" in a description, to indicate that candidates may be using an incorrect value.
- If a candidate makes an error in one part, but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the question says hence. It is often possible to use a different approach in subsequent parts that does not depend on the answer to previous parts.
- In a "show that" question, if an error in a previous subpart leads to not showing the required answer, do not award the final ***A1***. Note that if the error occurs within the same subpart, the ***FT*** rules may result in further loss of marks.

6 Mis-read

*If a candidate incorrectly copies information from the question, this is a mis-read (**MR**). A candidate should be penalized only once for a particular mis-read. Use the **MR** stamp to indicate that this is a misread. Do not award the first mark in the question, even if this is an **M** mark, but award all others (if appropriate) so that the candidate only loses one mark for the misread.*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*eg* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.

7 Discretionary marks *(d)*

*An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. In such cases the annotation DM should be used and a brief **note** written next to the mark explaining this decision.*

8 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete parts are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**. Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9 Alternative forms

*Unless the question specifies otherwise, **accept** equivalent forms.*
- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

10 Calculators

A GDC is required for paper 2, but calculators with symbolic manipulation features (eg TI-89) are not allowed.

Calculator notation The mathematics SL guide says:
Students must always use correct mathematical notation, not calculator notation.
Do **not** accept final answers written using calculator notation. However, do not penalize the use of calculator notation in the working.

11 Style

The markscheme aims to present answers using good communication, eg if the question asks to find the value of k, the markscheme will say $k = 3$, but the marks will be for the correct value 3 – there is usually no need for the "$k =$". In these cases, it is also usually acceptable to have

another variable, as long as there is no ambiguity in the question, eg if the question asks to find the value of p and of q, then the student answer needs to be clear. Generally, the only situation where the full answer is required is in a question which asks for equations – in this case the markscheme will say "must be an equation".

*The markscheme often uses words to describe what the marks are for, followed by examples, using the eg notation. These examples are not exhaustive, and examiners should check what candidates have written, to see if they satisfy the description. Where these marks are **M** marks, the examples may include ones using poor notation, to indicate what is acceptable. A valid method is one which will allow candidate to proceed to the next step eg if a quadratic function is given in factorised form, and the question asks for the zeroes, then multiplying the factors does not necessarily help to find the zeros, and would not on its own count as a valid method.*

12 Candidate work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

Candidates are meant to write their answers to Section A on the question paper (QP), and Section B on answer booklets. Sometimes, they need more room for Section A, and use the booklet (and often comment to this effect on the QP), or write outside the box. That is fine, and this work should be marked.

The instructions tell candidates not to write on Section B of the QP. Thus they may well have done some rough work here which they assume will be ignored. If they have solutions on the answer booklets, there is no need to look at the QP. However, if there are whole questions or whole part solutions missing on answer booklets, please check to make sure that they are not on the QP, and if they are, mark those whole questions or whole part solutions that have not been written on answer booklets.

13. Diagrams

The notes on how to allocate marks for sketches usually refer to passing through particular points or having certain features. These marks can only be awarded if the sketch is approximately the correct shape. All values given will be an approximate guide to where these points/features occur. In some questions, the first ***A1*** is for the shape, in others, the marks are only for the points and/or features. In both cases, unless the shape is approximately correct, no marks can be awarded (unless otherwise stated). However, if the graph is based on previous calculations, ***FT*** marks should be awarded if appropriate.

14. Accuracy of Answers

If the level of accuracy is specified in the question, a mark will be allocated for giving the final answer to the required accuracy. When this is not specified in the question, all numerical answers should be given exactly or correct to three significant figures.

Do not accept unfinished numerical final answers such as 3/0.1 (unless otherwise stated). As a rule, numerical answers with more than one part (such as fractions) should be given using integers (eg 6/8). Calculations which lead to integers should be completed, with the exception of fractions which are not whole numbers. Intermediate values do not need to be given to the correct three significant figures. But, if candidates work with rounded values, this could lead to an incorrect answer, in which case award A0 for the final answer. Where numerical answers are required as the final answer to a part of a question in the markscheme, the markscheme will show
a truncated 6 sf value
the exact value if applicable, the correct 3 sf answer.
Units will appear in brackets at the end.

Section A

1. (a) (i) mode $= 10$ *A1* *N1*

(ii) valid approach *(M1)*

eg $x_{\text{max}} - x_{\text{min,}}$ interval 2 to 11

range $= 9$ *A1* *N2*

[3 marks]

(b) (i) 7.14666

mean $= 7.15$ *A2* *N2*

(ii) recognizing that variance is $(\text{sd})^2$ *(M1)*

eg $\text{var} = \sigma^2$, 2.90605^2 , 2.92562^2

$\sigma^2 = 8.44515$

$\sigma^2 = 8.45$ *A1* *N2*

[4 marks]

Total [7 marks]

2. finding scalar product and magnitudes *(A1)(A1)(A1)*

scalar product $= (-10 \times 3) + (2 \times -4) + (1 \times 0) \quad (= -38)$

magnitudes $= \sqrt{10^2 + 2^2 + 1^2}$, $\sqrt{3^2 + (-4)^2 + (0)^2} \quad \left(\sqrt{105}, \sqrt{25}\right)$

substituting **their** values into formula *M1*

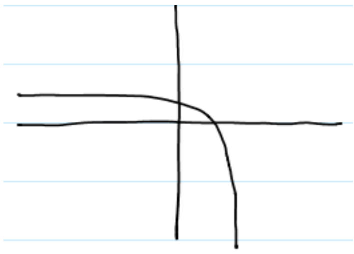
eg $\cos\theta = \dfrac{-30 - 8 + 0}{\left(\sqrt{10^2 + 2^2 + 1^2}\right) \times \left(\sqrt{3^2 + (-4)^2 + (0)^2}\right)}$

2.40637; $137.875°$

$\theta = 2.4$; $137.9°$ *A2* *N4*

[6 marks]

3. (a) valid approach *(M1)*

eg $f(0)$, 

y-intercept is 2.9 ***A1*** ***N2***

[2 marks]

(b) valid approach involving equation or inequality *(M1)*

eg $5x - 10 = 0$, 2, $x \neq 2$

$x = 2$ (must be an equation) ***A1*** ***N2***

[2 marks]

(c) 7.01710

min value $= 7.02$ ***A2*** ***N2***

Note: If candidate gives the minimum point as their final answer, award ***A1*** for $(3, 7.02)$.

Total [6 marks]

4. (a) evidence of binomial distribution (may be seen in part (b)) *(M1)*

eg np, 150×0.08

$k = 12$ ***A1*** ***N2***

[2 marks]

(b) (i) $\mathrm{P}(X = 12) = \begin{pmatrix} 150 \\ 12 \end{pmatrix} (0.08)^{12} (0.92)^{138}$ *(A1)*

0.119231

probability $= 0.119$ ***A1*** ***N2***

(ii) recognition that $X \leq 11$ *(M1)*

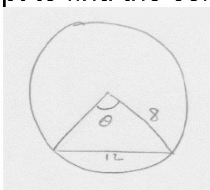
0.456800

$\mathrm{P}(X < 12) = 0.457$ ***A1*** ***N2***

[4 marks]

Total [6 marks]

5. attempt to find the central angle or half central angle ***(M1)***

eg 

, cosine rule, right triangle

correct working ***(A1)***

eg $\cos\theta = \frac{8^2+8^2-12^2}{2\cdot 8\cdot 8}$, $\sin^{-1}\left(\frac{6}{8}\right)$, 0.722734, $41.4096°$, $\frac{\pi}{2}-\sin^{-1}\left(\frac{6}{8}\right)$

correct angle $\mathrm{A\hat{O}B}$ (seen anywhere)

eg 1.69612, $97.1807°$, $2\times\sin^{-1}\left(\frac{6}{8}\right)$ ***(A1)***

correct sector area

eg $\frac{1}{2}(8)(8)(1.70)$, $\frac{97.1807}{360}(64\pi)$, 54.2759 ***(A1)***

area of triangle (seen anywhere) ***(A1)***

eg $\frac{1}{2}(8)(8)\sin 1.70$, $\frac{1}{2}(8)(12)\sin 0.722$, $\frac{1}{2}\times\sqrt{64-36}\times 12$, 31.7490

appropriate approach (seen anywhere) ***(M1)***

eg $A_{\text{triangle}} - A_{\text{sector}}$, their sector-their triangle

22.5269

area of shaded region $= 22.5\ (\text{cm}^2)$ ***A1*** ***N4***

Note:	Award ***M0A0A0A0A1*** then ***M1A0*** (if appropriate) for correct triangle area without any attempt to find an angle in triangle OAB.

[7 marks]

6. METHOD 1

derivative of $f(x)$ ***A2***

$7\left(x^2+3\right)^6(2x)$

recognizing need to find x^4 term in $\left(x^2+3\right)^6$ (seen anywhere) ***R1***

eg $14x$ $\left(\text{term in } x^4\right)$

valid approach to find the terms in $\left(x^2+3\right)^6$ ***(M1)***

eg $\binom{6}{r}\left(x^2\right)^{6-r}(3)^r$, $\left(x^2\right)^6(3)^0+\left(x^2\right)^5(3)^1+\ldots$, Pascal's triangle to 6th row

identifying correct term (may be indicated in expansion) ***(A1)***

eg 5th term, $r=2$, $\binom{6}{4}$, $\left(x^2\right)^2(3)^4$

correct working (may be seen in expansion) ***(A1)***

eg $\binom{6}{4}\left(x^2\right)^2(3)^4$, 15×3^4, $14x\times15\times81\left(x^2\right)^2$

$17010x^5$ ***A1*** ***N3***

METHOD 2

recognition of need to find x^6 in $\left(x^2+3\right)^7$ (seen anywhere) ***R1***

valid approach to find the terms in $\left(x^2+3\right)^7$ ***(M1)***

eg $\binom{7}{r}\left(x^2\right)^{7-r}(3)^r$, $\left(x^2\right)^7(3)^0+\left(x^2\right)^6(3)^1+\ldots$, Pascal's triangle to 7th row

identifying correct term (may be indicated in expansion) ***(A1)***

eg 6th term, $r=3$, $\binom{7}{3}$, $\left(x^2\right)^3(3)^4$

correct working (may be seen in expansion) ***(A1)***

eg $\binom{7}{4}\left(x^2\right)^3(3)^4$, 35×3^4

correct term ***(A1)***

$2835x^6$

differentiating their term in x^6 ***(M1)***

eg $\left(2835x^6\right)'$, $(6)\left(2835x^5\right)$

$17010x^5$ ***A1*** ***N3***

[7 marks]

7. (a) (i) $t = 2$ ***A1*** ***N1***

(ii) substitution of limits or function into formula or correct sum ***(A1)***

eg $\int_0^8 |v| \mathrm{d}t$, $\int |v_{\mathrm{Q}}| \mathrm{d}t$, $\int_0^2 v\mathrm{d}t - \int_2^4 v\mathrm{d}t + \int_4^6 v\mathrm{d}t - \int_6^8 v\mathrm{d}t$

9.64782

distance $= 9.65$ (metres) ***A1*** ***N2***

[3 marks]

(b) correct approach ***(A1)***

eg $s = \int \sqrt{t}$, $\int_0^k \sqrt{t}\,\mathrm{d}t$, $\int_0^k |v_Q| \mathrm{d}t$

correct integration ***(A1)***

eg $\int \sqrt{t} = \frac{2}{3} t^{\frac{3}{2}} + c$, $\left[\frac{2}{3} x^{\frac{3}{2}}\right]_0^k$, $\frac{2}{3} k^{\frac{3}{2}}$

equating their expression to the distance travelled by their P ***(M1)***

eg $\frac{2}{3} k^{\frac{3}{2}} = 9.65$, $\int_0^k \sqrt{t}\,\mathrm{d}t = 9.65$

5.93855

5.94 (seconds) ***A1*** ***N3***

[4 marks]

Total [7 marks]

Section B

8. (a) (i) attempt to find the difference of x-values of A and B **(M1)**

eg $6.25–12.5$

6.25 (hours), (6 hours 15 minutes) ***A1*** ***N2***

(ii) attempt to find the difference of y-values of A and B ***(M1)***

eg $1.5–0.6$

0.9 (m) ***A1*** ***N2***

[4 marks]

(b) (i) valid approach ***(M1)***

eg $\frac{\max - \min}{2}$, $0.9 \div 2$

$p = 0.45$ ***A1*** ***N2***

(ii) **METHOD 1**

period = 12.5 (seen anywhere) ***(A1)***

valid approach (seen anywhere) ***(M1)***

eg $\text{period} = \frac{2\pi}{b}$, $q = \frac{2\pi}{\text{period}}$, $\frac{2\pi}{12.5}$

0.502654

$q = \frac{4\pi}{25}$, 0.503 $\left(\text{or } -\frac{4\pi}{25}, -0.503\right)$ ***A1*** ***N2***

METHOD 2

attempt to use a coordinate to make an equation ***(M1)***

e.g. $p\cos(6.25q) + r = 0.6$, $p\cos(12.5q) + r = 1.5$

correct substitution ***(A1)***

eg $0.45\cos(6.25q) + 1.05 = 0.6$, $0.45\cos(12.5q) + 1.05 = 1.5$

0.502654

$q = \frac{4\pi}{25}$, 0.503 $\left(\text{or } -\frac{4\pi}{25}, -0.503\right)$ ***A1*** ***N2***

(iii) valid method to find r ***(M1)***

eg $\frac{\max + \min}{2}$, $0.6 + 0.45$

$r = 1.05$ ***A1*** ***N2***

[7 marks]

continued…

Question 8 continued

(c) **METHOD 1**

attempt to find start or end t-values for 12 December *(M1)*

eg $3+24$, $t=27$, $t=51$

finds t-value for second max *(A1)*

$t=50$

23:00 (or 11 pm) ***A1*** ***N3***

METHOD 2

valid approach to list either the times of high tides after 21:00 or the t-values of high tides after 21:00, showing at least two times *(M1)*

eg 21:00 + 12.5, 21:00 + 25, 12.5 + 12.5, 25 + 12.5

correct time of first high tide on 12 December *(A1)*

eg 10:30 (or 10:30 am)

time of second high tide = 23:00 ***A1*** ***N3***

METHOD 3

attempt to set **their** h equal to 1.5 *(M1)*

eg $h(t)=1.5$, $0.45\cos\left(\frac{4\pi}{25}t\right)+1.05=1.5$

correct working to find second max *(A1)*

eg $0.503t=8\pi$, $t=50$

23:00 (or 11 pm) ***A1*** ***N3***

[3 marks]

Total [14 marks]

9. (a) valid approach *(M1)*

eg $\mathrm{P}(X < \mu) = 0.5$, $0.5 - 0.3$

$\mathrm{P}(X < 9) = 0.2$ (exact) ***A1*** ***N2***

[2 marks]

(b) $z = -0.841621$ (may be seen in equation) *(A1)*

valid attempt to set up an equation with **their** z *(M1)*

eg $-0.842 = \dfrac{\mu - X}{\sigma}$, $-0.842 = \dfrac{X - \mu}{\sigma}$, $z = \dfrac{9 - \mu}{2.1}$

10.7674

$\mu = 10.8$ ***A1*** ***N3***

[3 marks]

(c) $\mathrm{P}(X > 9) = 0.8$ (seen anywhere) *(A1)*

valid approach *(M1)*

eg $\mathrm{P}(A) \times \mathrm{P}(B)$

correct equation *(A1)*

eg $0.8 \times \mathrm{P}(Y > 9) = 0.4$

$\mathrm{P}(Y > 9) = 0.5$ ***A1***

$\lambda = 9$ ***A1*** ***N3***

[5 marks]

(d) finding $\mathrm{P}(9 < Y < 13) = 0.373450$ (seen anywhere) *(A2)*

recognizing conditional probability *(M1)*

eg $\mathrm{P}(A \mid B)$, $\mathrm{P}(Y < 13 \mid Y > 9)$

correct working *(A1)*

eg $\dfrac{0.373}{0.5}$

0.746901

0.747 ***A1*** ***N3***

[5 marks]

Total [15 marks]

10. (a) (i) $q = 2$ ***A1*** ***N1***

(ii) $h = 0$ ***A1*** ***N1***

(iii) $k = 3$ ***A1*** ***N1***

Note: Accept $q = 1$, $h = 0$, and $k = 3 - \ln(2)$, 2.31 as candidate may have rewritten $g(x)$ as equal to $3 + \ln(x) - \ln(2)$.

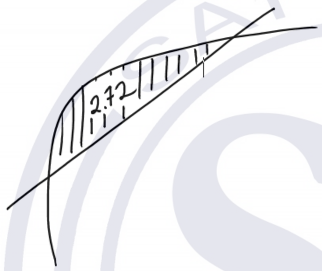
[3 marks]

(b) (i) 2.72409

2.72 ***A2*** ***N2***

(ii) recognizing area between $y = x$ and h equals 2.72 ***(M1)***

eg



recognizing graphs of h and h^{-1} are reflections of each other in $y = x$ ***(M1)***

eg area between $y = x$ and h equals between $y = x$ and h^{-1}

2×2.72, $\int_{0.111}^{3.31} \left(x - h^{-1}(x)\right) \mathrm{d}x = 2.72$

5.44819

5.45 ***A1*** ***N3***

[5 marks]

continued…

Question 10 continued

(c) valid attempt to find d ***(M1)***

eg difference in y-coordinates, $d = h(x) - x$

correct expression for d ***(A1)***

eg $\left(\ln\frac{1}{2}x + 3\right)(\cos 0.1x) - x$

valid approach to find when d is a maximum ***(M1)***

eg max on sketch of d, attempt to solve $d' = 0$

0.973679

$x = 0.974$ ***A2*** ***N4***

substituting **their** x value into $h(x)$ ***(M1)***

2.26938

$y = 2.27$ ***A1*** ***N2***

[7 marks]

[15 marks]

Markscheme

May 2017

Mathematics

Standard level

Paper 2

17 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a valid **Method**; working must be seen.

(M) Marks awarded for a valid **Method**; may be implied by **correct** subsequent working.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

(A) Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.

R Marks awarded for clear **Reasoning**.

N Marks awarded for **correct** answers if **no** working shown.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Mark according to RM assessor instructions.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any. An exception to this rule is when work for ***M1*** is missing, as opposed to incorrect (see point 4).
- Where ***M*** and ***A*** marks are noted on the same line, *eg* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*eg* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***(M2)***, ***N3***, *etc*., do **not** split the marks, unless there is a note.
- Most ***M*** marks are for a **valid** method, ie a method which can lead to the answer: it must indicate some form of progress towards the answer.
- Once a correct answer to a question or part-question is seen, ignore further correct working. However, if further working indicates a lack of mathematical understanding do not award final ***A1***.

3 *N* marks

*If **no** working shown, award **N** marks for **correct** answers – this includes acceptable answers (see accuracy booklet). In this case, ignore mark breakdown (**M**, **A**, **R**). Where a student only shows a final incorrect answer with no working, even if that answer is a correct intermediate answer, award **N0.***

- Do **not** award a mixture of ***N*** and other marks.
- There may be fewer ***N*** marks available than the total of ***M***, ***A*** and ***R*** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.
- There may not be a direct relationship between the ***N*** marks and the implied marks. There are times when all the marks are implied, but the ***N*** marks are not the full marks: this indicates that we want to see some of the working, without specifying what.

- For consistency within the markscheme, ***N*** marks are noted for every part, even when these match the mark breakdown.
- If a candidate has incorrect working, which somehow results in a correct answer, do **not** award the ***N*** marks for this correct answer. However, if the candidate has indicated (usually by crossing out) that the working is to be ignored, award the ***N*** marks for the correct answer.

4 Implied and must be seen marks

*Implied marks appear in **brackets** eg **(M1)**.*
- Implied marks can only be awarded if the work is seen or if implied in subsequent working (a correct final answer does not necessarily mean that the implied marks are all awarded). There are questions where some working is required, but as it is accepted that not everyone will write the same steps, all the marks are implied, but the ***N*** marks are not the full marks for the question.
- Normally the correct work is seen in the next line.
- Where there is an (***M1***) followed by ***A1*** for each correct answer, if no working shown, one correct answer is sufficient evidence to award the (***M1***).

*Must be seen marks appear without **brackets** eg **M1**.*
- Must be seen marks can only be awarded if the work is seen.
- If a must be seen mark is not awarded because work is missing (as opposed to ***M0*** or ***A0*** for incorrect work) all subsequent marks may be awarded if appropriate.

5 Follow through marks (only applied after an error is made)

*Follow through (**FT**) marks are awarded where an incorrect answer (final or intermediate) from one **part** of a question is used correctly in **subsequent** part(s) or subpart(s). Usually, to award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if the only marks awarded in a subpart are for the **final** answer, then **FT** marks should be awarded if appropriate. Examiners are expected to check student work in order to award **FT** marks where appropriate.*

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** and ***R*** marks may be awarded if appropriate. (However, as noted above, if an ***A*** mark is not awarded because work is missing, all subsequent marks may be awarded if appropriate).
- Exceptions to this rule will be explicitly noted on the markscheme.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*eg* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word "**their**" in a description, to indicate that candidates may be using an incorrect value.
- If a candidate makes an error in one part, but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the question says hence. It is often possible to use a different approach in subsequent parts that does not depend on the answer to previous parts.
- In a "show that" question, if an error in a previous subpart leads to not showing the required answer, do not award the final ***A1***. Note that if the error occurs within the same subpart, the ***FT*** rules may result in further loss of marks.

6 Mis-read

*If a candidate incorrectly copies information from the question, this is a mis-read (**MR**). A candidate should be penalized only once for a particular mis-read. Use the **MR** stamp to indicate that this is a misread. Do not award the first mark in the question, even if this is an **M** mark, but award all others (if appropriate) so that the candidate only loses one mark for the misread.*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*eg* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.

7 Discretionary marks *(d)*

*An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. In such cases the annotation DM should be used and a brief **note** written next to the mark explaining this decision.*

8 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete parts are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**. Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9 Alternative forms

*Unless the question specifies otherwise, **accept** equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

10 Calculators

A GDC is required for paper 2, but calculators with symbolic manipulation features (eg TI-89) are not allowed.

Calculator notation The mathematics SL guide says:
Students must always use correct mathematical notation, not calculator notation.
Do **not** accept final answers written using calculator notation. However, do not penalize the use of calculator notation in the working.

11 Style

The markscheme aims to present answers using good communication, eg if the question asks to find the value of k, the markscheme will say $k = 3$, but the marks will be for the correct value 3 – there is usually no need for the "$k =$". In these cases, it is also usually acceptable to have

another variable, as long as there is no ambiguity in the question, eg if the question asks to find the value of p and of q, then the student answer needs to be clear. Generally, the only situation where the full answer is required is in a question which asks for equations – in this case the markscheme will say "must be an equation".

*The markscheme often uses words to describe what the marks are for, followed by examples, using the eg notation. These examples are not exhaustive, and examiners should check what candidates have written, to see if they satisfy the description. Where these marks are **M** marks, the examples may include ones using poor notation, to indicate what is acceptable. A valid method is one which will allow candidate to proceed to the next step eg if a quadratic function is given in factorised form, and the question asks for the zeroes, then multiplying the factors does not necessarily help to find the zeros, and would not on its own count as a valid method.*

12 Candidate work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

Candidates are meant to write their answers to Section A on the question paper (QP), and Section B on answer booklets. Sometimes, they need more room for Section A, and use the booklet (and often comment to this effect on the QP), or write outside the box. That is fine, and this work should be marked.

The instructions tell candidates not to write on Section B of the QP. Thus they may well have done some rough work here which they assume will be ignored. If they have solutions on the answer booklets, there is no need to look at the QP. However, if there are whole questions or whole part solutions missing on answer booklets, please check to make sure that they are not on the QP, and if they are, mark those whole questions or whole part solutions that have not been written on answer booklets.

13. Diagrams

The notes on how to allocate marks for sketches usually refer to passing through particular points or having certain features. These marks can only be awarded if the sketch is approximately the correct shape. All values given will be an approximate guide to where these points/features occur. In some questions, the first ***A1*** is for the shape, in others, the marks are only for the points and/or features. In both cases, unless the shape is approximately correct, no marks can be awarded (unless otherwise stated). However, if the graph is based on previous calculations, ***FT*** marks should be awarded if appropriate.

14. Accuracy of Answers

If the level of accuracy is specified in the question, a mark will be allocated for giving the final answer to the required accuracy. When this is not specified in the question, all numerical answers should be given exactly or correct to three significant figures.

Do not accept unfinished numerical final answers such as 3/0.1 (unless otherwise stated). As a rule, numerical answers with more than one part (such as fractions) should be given using integers (eg 6/8). Calculations which lead to integers should be completed, with the exception of fractions which are not whole numbers. Intermediate values do not need to be given to the correct three significant figures. But, if candidates work with rounded values, this could lead to an incorrect answer, in which case award A0 for the final answer. Where numerical answers are required as the final answer to a part of a question in the markscheme, the markscheme will show
 a truncated 6 sf value
 the exact value if applicable, the correct 3 sf answer
Units will appear in brackets at the end.

Section A

1. (a) correct substitution into arc length formula *(A1)*
eg $(40)(1.9)$

arc length $= 76$ (cm) ***A1*** ***N2***
[2 marks]

(b) valid approach *(M1)*
eg arc $+2r$, $76+40+40$

perimeter $= 156$ (cm) ***A1*** ***N2***
[2 marks]

(c) correct substitution into area formula *(A1)*
eg $\frac{1}{2}(1.9)(40)^2$

area $= 1520$ (cm^2) ***A1*** ***N2***
[2 marks]

[Total 6 marks]

2. (a) (i) evidence of set up *(M1)*
eg correct value for a or b

0.667315, 22.2117
$a = 0.667$, $b = 22.2$ ***A1A1*** ***N3***

(ii) 0.922958
$r = 0.923$ ***A1*** ***N1***
[4 marks]

(b) valid approach *(M1)*
eg $0.667(15)+22.2$, $N(15)$

32.2214 *(A1)*
32 (visitors) (must be an integer) ***A1*** ***N2***
[3 marks]

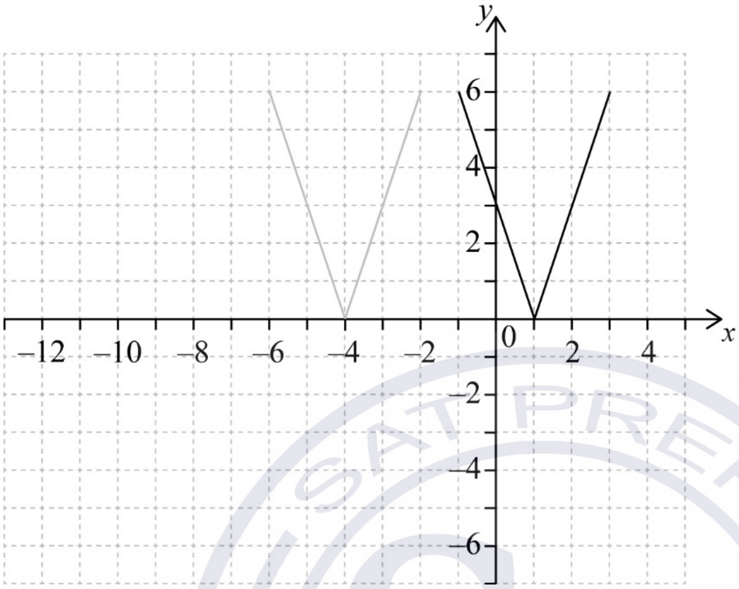
[Total 7 marks]

3. (a) correct interval ***A2*** ***N2***

eg $0 \le y \le 6$, $[0, 6]$, from 0 to 6

[2 marks]

(b)



M1A1 ***N2***

Note: Award ***M1*** for a horizontal shift of the whole shape, 5 units to the left or right and ***A1*** for the correct graph.

[2 marks]

(c) correct interval ***A2*** ***N2***

eg $-1 \le x \le 3$, $[-1, 3]$, from -1 to 3

[2 marks]

[Total 6 marks]

4. (a) valid approach ***(M1)***

eg $\frac{\text{max} - \text{min}}{2}$, sketch of graph, $9.7 = p\cos(0) + 7.5$

$p = 2.2$ ***A1*** ***N2***

[2 marks]

(b) valid approach ***(M1)***

eg $B = \frac{2\pi}{\text{period}}$, period is 14, $\frac{360}{14}$, $5.3 = 2.2\cos 7q + 7.5$

0.448798

$q = \frac{2\pi}{14}\left(\frac{\pi}{7}\right)$, 0.449 (do not accept degrees) ***A1*** ***N2***

[2 marks]

(c) valid approach ***(M1)***

eg $d(10)$, $2.2\cos\left(\frac{20\pi}{14}\right) + 7.5$

7.01045

7.01 (m) ***A1*** ***N2***

[2 marks]

[Total 6 marks]

5. attempt to find r ***(M1)***

eg $\frac{576}{768}$, $\frac{768}{576}$, 0.75

correct expression for u_n ***(A1)***

eg $768(0.75)^{n-1}$

EITHER (solving inequality)

valid approach (accept equation) ***(M1)***

eg $u_n < 7$

valid approach to find n ***M1***

eg $768(0.75)^{n-1} = 7$, $n - 1 > \log_{0.75}\left(\frac{7}{768}\right)$, sketch

correct value

eg $n = 17.3301$ ***(A1)***

$n = 18$ (must be an integer) ***A1 N2***

OR (table of values)

valid approach ***(M1)***

eg $u_n < 7$,one correct crossover value

both crossover values, $u_{17} = 7.69735$ **and** $u_{18} = 5.77301$ ***A2***

$n = 18$ (must be an integer) ***A1 N2***

OR (sketch of functions)

valid approach ***M1***

eg sketch of appropriate functions

valid approach ***(M1)***

eg finding intersections or roots (depending on function sketched)

correct value

eg $n = 17.3301$ ***(A1)***

$n = 18$ (must be an integer) ***A1 N2***

[6 marks]

6. (a) attempt to form composite in either order *(M1)*

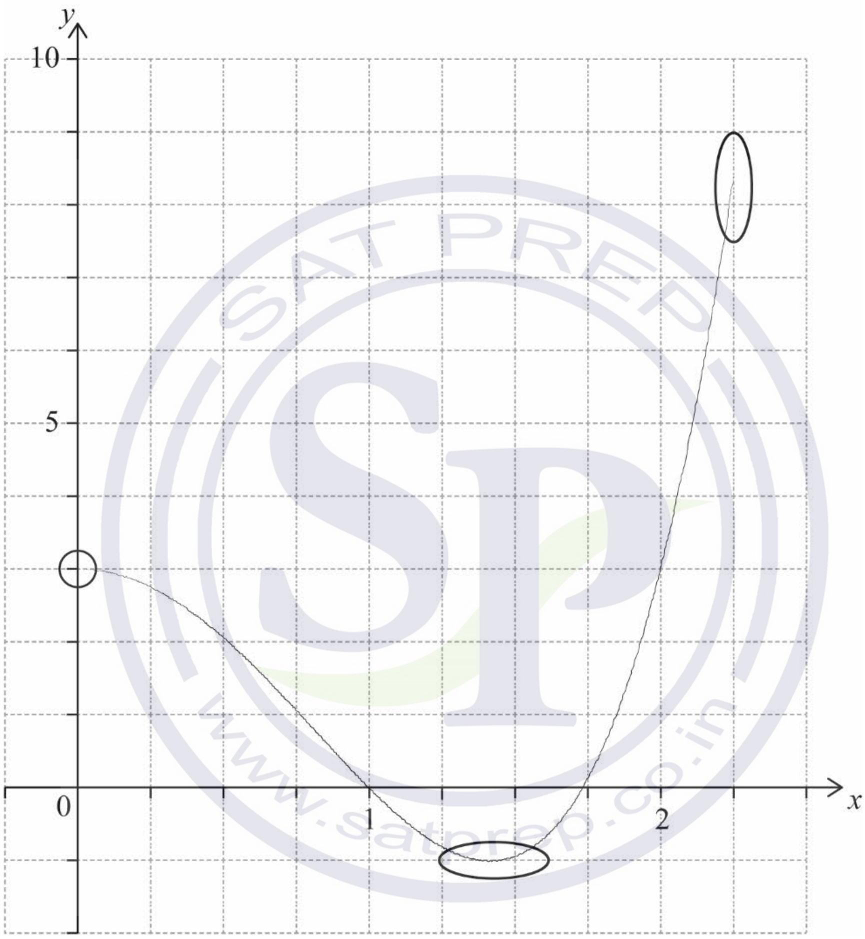
eg $f\left(x^2-2\right)$, $\left(x^2-1\right)^2-2$

$\left(x^4-4x^2+4\right)-1$ ***A1***

$(f \circ g)(x) = x^4-4x^2+3$ ***AG*** ***N0***

[2 marks]

(b)



A1
A1A1 ***N3***

Note:	Award ***A1*** for approximately correct shape which changes from concave down to concave up. Only if this ***A1*** is awarded, award the following: ***A1*** for left hand endpoint in circle **and** right hand endpoint in oval, ***A1*** for minimum in oval.

[3 marks]

(c) evidence of identifying max/min as relevant points *(M1)*

eg $x = 0, 1.41421,\ y = -1, 3$

correct interval (inclusion/exclusion of endpoints must be correct) ***A2*** ***N3***

eg $-1 < k \le 3$, $]-1, 3]$, $(-1, 3]$

[3 marks]

[Total 8 marks]

7. **METHOD 1 (displacement)**

recognizing $s = \int v \, \mathrm{d}t$ ***(M1)***

consideration of displacement at $t = 2$ **and** $t = 5$ (seen anywhere) ***M1***

eg $\int_0^2 v$ and $\int_0^5 v$

Note: Must have both for any further marks.

correct displacement at $t = 2$ and $t = 5$ (seen anywhere) ***A1A1***

-2.28318 (accept 2.28318), 1.55513

valid reasoning comparing correct displacements ***R1***

eg $\left|-2.28\right| > \left|1.56\right|$, more left than right

2.28 (m) ***A1*** ***N1***

Note: Do not award the final ***A1*** without the ***R1***.

METHOD 2 (distance travelled)

recognizing distance $= \int \left| v \right| \mathrm{d}t$ ***(M1)***

consideration of distance travelled from $t = 0$ to 2 **and** $t = 2$ to 5 (seen anywhere) ***M1***

eg $\int_0^2 v$ and $\int_2^5 v$

Note: Must have both for any further marks.

correct distances travelled (seen anywhere) ***A1A1***

2.28318, (accept -2.28318), 3.83832

valid reasoning comparing correct distance values ***R1***

eg $3.84 - 2.28 < 2.28, 3.84 < 2 \times 2.28$

2.28 (m) ***A1*** ***N1***

Note: Do not award the final ***A1*** without the ***R1***.

[6 marks]

Section B

8. (a) evidence of valid approach *(M1)*
eg $f(x)=0$, $y=0$

2.73205
$p=2.73$ ***A1*** ***N2***
[2 marks]

(b) (i) $1.87938,\ 8.11721$
$(1.88,\ 8.12)$ ***A2*** ***N2***

(ii) rate of change is 0 (do not accept decimals) ***A1*** ***N1***
[3 marks]

(c) (i) **METHOD 1 (using GDC)**

valid approach ***M1***
eg $f''=0$, max/min on f', $x=-1$

sketch of either f' or f'', with max/min or root (respectively) *(A1)*

$x=1$ ***A1*** ***N1***

Substituting **their** x value into f *(M1)*
eg $f(1)$

$y=4.5$ ***A1*** ***N1***

METHOD 2 (analytical)

$f''=-6x^2+6$ ***A1***

setting $f''=0$ *(M1)*

$x=1$ ***A1*** ***N1***

substituting **their** x value into f *(M1)*
eg $f(1)$

$y=4.5$ ***A1*** ***N1***

continued…

Question 8 continued

(ii) recognizing rate of change is f' ***(M1)***

eg y', $f'(1)$

rate of change is 6 ***A1*** ***N2***

[7 marks]

(d) attempt to substitute either limits or the function into formula involving f^2 (accept absence of π and/or $\mathrm{d}x$) ***(M1)***

eg $\pi\int\left(-0.5x^4+3x^2+2x\right)^2\mathrm{d}x$, $\int_1^{1.88} f^2$

128.890

volume $= 129$ ***A2*** ***N3***

[3 marks]

[Total 15 marks]

9. (a) valid method *(M1)*

eg $180+55$, $360-90-35$

235° (accept S55W, W35S) *A1* *N2*

[2 marks]

(b) valid approach to find $\hat{AEC}$ (may be seen in (a)) *(M1)*

eg $A\hat{E}C = 180-55-A\hat{C}E$, $134 = E+55$

correct working to find $A\hat{E}C$ (may be seen in (a)) *(A1)*

eg $180-55-46$, $134-55$, $A\hat{E}C = 79^\circ$

evidence of choosing sine rule (seen anywhere) *(M1)*

eg $\frac{a}{\sin A} = \frac{b}{\sin B}$

correct substitution into sine rule *(A1)*

eg $\frac{CE}{\sin 55^\circ} = \frac{175}{\sin A\hat{E}C}$

146.034

$CE = 146$ (km) *A1* *N2*

[5 marks]

(c) evidence of choosing cosine rule *(M1)*

eg $DE^2 = DC^2 + CE^2 - 2\times DC\times CE\times\cos\theta$

correct substitution into right-hand side *(A1)*

eg $60^2 + 146.034^2 - 2\times 60\times 146.034\cos 134$

192.612

$DE = 193$ (km) *A1* *N2*

[3 marks]

(d) valid approach for locating B *(M1)*

eg BE is perpendicular to ship's path, angle $B = 90$

correct working for BE *(A1)*

eg $\sin 46^\circ = \frac{BE}{146.034}$, $BE = 146.034\sin 46^\circ$, 105.048

valid approach for expressing time *(M1)*

eg $t = \frac{d}{s}$, $t = \frac{d}{r}$, $t = \frac{192.612}{50}$

correct working equating time *(A1)*

eg $\frac{146.034\sin 46^\circ}{r} = \frac{192.612}{50}$, $\frac{s}{105.048} = \frac{50}{192.612}$

27.2694

27.3 (km per hour) *A1* *N3*

[5 marks]

[Total 15 marks]

10. (a) (i) correct substitution into $\mathrm{E}(X)$ formula *(A1)*

eg $0(p)+1(0.5)+2(0.3)+3(q)=1.2$

$q=\frac{1}{30}$, 0.0333 **A1** **N2**

(ii) evidence of summing probabilities to 1 *(M1)*

eg $p+0.5+0.3+q=1$

$p=\frac{1}{6}$, 0.167 **A1** **N2**

[4 marks]

(b) (i) P (3 blue) $=\frac{1}{30}$, 0.0333 **A1** **N1**

(ii) valid reasoning **R1**

eg P (3 white) $=$ P(0 blue)

P(3 white) $=\frac{1}{6}$ **AG** **N0**

(iii) valid method *(M1)*

eg $\mathrm{P}(3\text{ white})=\frac{w}{10}\times\frac{w-1}{9}\times\frac{w-2}{8}$, $\frac{{}_{w}C_3}{{}_{10}C_3}$

correct equation **A1**

eg $\frac{w}{10}\times\frac{w-1}{9}\times\frac{w-2}{8}=\frac{1}{6}$, $\frac{{}_{w}C_3}{{}_{10}C_3}=0.167$

$w=6$ **A1** **N2**

[5 marks]

(c) valid approach *(M1)*

eg $\mathrm{B}(n,\ p)$, $\binom{n}{r}p^r q^{n-r}$, $(0.167)^2(0.833)^7$, $\binom{9}{2}$

0.279081

0.279 **A1** **N2**

[2 marks]

continued…

Question 10 continued

(d) recognizing one prize in first seven attempts **(M1)**

eg $\binom{7}{1}, \left(\frac{1}{6}\right)^1\left(\frac{5}{6}\right)^6$

correct working **(A1)**

eg $\binom{7}{1}\left(\frac{1}{6}\right)^1\left(\frac{5}{6}\right)^6$, 0.390714

correct approach **(A1)**

eg $\binom{7}{1}\left(\frac{1}{6}\right)^1\left(\frac{5}{6}\right)^6 \times \frac{1}{6}$

0.065119
0.0651 ***A1*** ***N2***

[4 marks]

[Total 15 marks]

Markscheme

November 2016

Mathematics

Standard level

Paper 2

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a valid **Method**; working must be seen.

(M) Marks awarded for a valid **Method**; may be implied by **correct** subsequent working.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

(A) Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.

R Marks awarded for clear **Reasoning**.

N Marks awarded for **correct** answers if **no** working shown.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Mark according to RM assessor instructions.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any. An exception to this rule is when work for ***M1*** is missing, as opposed to incorrect (see point 4).
- Where ***M*** and ***A*** marks are noted on the same line, *eg* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*eg* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***(M2)***, ***N3***, *etc*., do **not** split the marks, unless there is a note.
- Most ***M*** marks are for a **valid** method, ie a method which can lead to the answer: it must indicate some form of progress towards the answer.
- Once a correct answer to a question or part-question is seen, ignore further correct working. However, if further working indicates a lack of mathematical understanding do not award final ***A1***.

3 *N* marks

*If **no** working shown, award **N** marks for **correct** answers – this includes acceptable answers (see accuracy booklet). In this case, ignore mark breakdown (**M**, **A**, **R**). Where a student only shows a final incorrect answer with no working, even if that answer is a correct intermediate answer, award **N0**.*

- Do **not** award a mixture of ***N*** and other marks.
- There may be fewer ***N*** marks available than the total of ***M***, ***A*** and ***R*** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.
- There may not be a direct relationship between the ***N*** marks and the implied marks. There are times when all the marks are implied, but the ***N*** marks are not the full marks: this indicates that we want to see some of the working, without specifying what.

- For consistency within the markscheme, ***N*** marks are noted for every part, even when these match the mark breakdown.
- If a candidate has incorrect working, which somehow results in a correct answer, do **not** award the ***N*** marks for this correct answer. However, if the candidate has indicated (usually by crossing out) that the working is to be ignored, award the ***N*** marks for the correct answer.

4 Implied and must be seen marks

*Implied marks appear in **brackets** eg **(M1)**.*

- Implied marks can only be awarded if the work is seen or if implied in subsequent working (a correct final answer does not necessarily mean that the implied marks are all awarded). There are questions where some working is required, but as it is accepted that not everyone will write the same steps, all the marks are implied, but the ***N*** marks are not the full marks for the question.
- Normally the correct work is seen in the next line.
- Where there is an (***M1***) followed by ***A1*** for each correct answer, if no working shown, one correct answer is sufficient evidence to award the (***M1***).

*Must be seen marks appear without **brackets** eg **M1**.*

- Must be seen marks can only be awarded if the work is seen.
- If a must be seen mark is not awarded because work is missing (as opposed to ***M0*** or ***A0*** for incorrect work) all subsequent marks may be awarded if appropriate.

5 Follow through marks (only applied after an error is made)

*Follow through (**FT**) marks are awarded where an incorrect answer (final or intermediate) from one **part** of a question is used correctly in **subsequent** part(s) or subpart(s). Usually, to award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if the only marks awarded in a subpart are for the **final** answer, then **FT** marks should be awarded if appropriate. Examiners are expected to check student work in order to award **FT** marks where appropriate.*

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** and ***R*** marks may be awarded if appropriate. (However, as noted above, if an ***A*** mark is not awarded because work is missing, all subsequent marks may be awarded if appropriate).
- Exceptions to this rule will be explicitly noted on the markscheme.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*eg* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word "**their**" in a description, to indicate that candidates may be using an incorrect value.
- If a candidate makes an error in one part, but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the question says hence. It is often possible to use a different approach in subsequent parts that does not depend on the answer to previous parts.
- In a "show that" question, if an error in a previous subpart leads to not showing the required answer, do not award the final ***A1***. Note that if the error occurs within the same subpart, the ***FT*** rules may result in further loss of marks.

6 Mis-read

*If a candidate incorrectly copies information from the question, this is a mis-read (**MR**). A candidate should be penalized only once for a particular mis-read. Use the **MR** stamp to indicate that this is a misread. Do not award the first mark in the question, even if this is an **M** mark, but award all others (if appropriate) so that the candidate only loses one mark for the misread.*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*eg* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.

7 Discretionary marks *(d)*

*An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. In such cases the annotation DM should be used and a brief **note** written next to the mark explaining this decision.*

8 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete parts are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**. Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9 Alternative forms

*Unless the question specifies otherwise, **accept** equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

10 Calculators

A GDC is required for paper 2, but calculators with symbolic manipulation features (eg TI-89) are not allowed.

Calculator notation The mathematics SL guide says:
Students must always use correct mathematical notation, not calculator notation.
Do **not** accept final answers written using calculator notation. However, do not penalize the use of calculator notation in the working.

11 Style

The markscheme aims to present answers using good communication, eg if the question asks to find the value of k, the markscheme will say $k = 3$, but the marks will be for the correct value 3 – there is usually no need for the "$k =$". In these cases, it is also usually acceptable to have

another variable, as long as there is no ambiguity in the question, eg if the question asks to find the value of p and of q, then the student answer needs to be clear. Generally, the only situation where the full answer is required is in a question which asks for equations – in this case the markscheme will say "must be an equation".

*The markscheme often uses words to describe what the marks are for, followed by examples, using the eg notation. These examples are not exhaustive, and examiners should check what candidates have written, to see if they satisfy the description. Where these marks are **M** marks, the examples may include ones using poor notation, to indicate what is acceptable. A valid method is one which will allow candidate to proceed to the next step eg if a quadratic function is given in factorised form, and the question asks for the zeroes, then multiplying the factors does not necessarily help to find the zeros, and would not on its own count as a valid method.*

12 Candidate work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

Candidates are meant to write their answers to Section A on the question paper (QP), and Section B on answer booklets. Sometimes, they need more room for Section A, and use the booklet (and often comment to this effect on the QP), or write outside the box. That is fine, and this work should be marked.

The instructions tell candidates not to write on Section B of the QP. Thus they may well have done some rough work here which they assume will be ignored. If they have solutions on the answer booklets, there is no need to look at the QP. However, if there are whole questions or whole part solutions missing on answer booklets, please check to make sure that they are not on the QP, and if they are, mark those whole questions or whole part solutions that have not been written on answer booklets.

13. Diagrams

The notes on how to allocate marks for sketches usually refer to passing through particular points or having certain features. These marks can only be awarded if the sketch is approximately the correct shape. All values given will be an approximate guide to where these points/features occur. In some questions, the first ***A1*** is for the shape, in others, the marks are only for the points and/or features. In both cases, unless the shape is approximately correct, no marks can be awarded (unless otherwise stated). However, if the graph is based on previous calculations, ***FT*** marks should be awarded if appropriate.

14. Accuracy of Answers

If the level of accuracy is specified in the question, a mark will be allocated for giving the final answer to the required accuracy. When this is not specified in the question, all numerical answers should be given exactly or correct to three significant figures.

Do not accept unfinished numerical final answers such as 3/0.1 (unless otherwise stated). As a rule, numerical answers with more than one part (such as fractions) should be given using integers (eg 6/8). Calculations which lead to integers should be completed, with the exception of fractions which are not whole numbers. Intermediate values do not need to be given to the correct three significant figures. But, if candidates work with rounded values, this could lead to an incorrect answer, in which case award A0 for the final answer. Where numerical answers are required as the final answer to a part of a question in the markscheme, the markscheme will show

a truncated 6 sf value

the exact value if applicable, the correct 3 sf answer

Units will appear in brackets at the end.

Section A

1. (a) attempt to substitute $x = 8$ ***(M1)***

eg $8^2 + 2 \times 8 + 1$

$f(8) = 81$ ***A1*** ***N2***

[2 marks]

(b) attempt to form composition (in any order) ***(M1)***

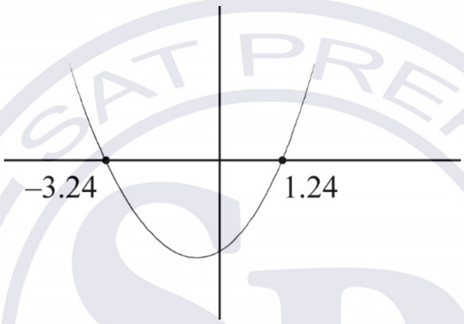
eg $f(x-5),\ g\left(f(x)\right),\ \left(x^2 + 2x + 1\right) - 5$

$(g \circ f)(x) = x^2 + 2x - 4$ ***A1*** ***N2***

[2 marks]

(c) valid approach ***(M1)***

eg $x = \frac{-2 \pm \sqrt{20}}{2}$,



$1.23606,\ -3.23606$

$x = 1.24,\ x = -3.24$ ***A1A1*** ***N3***

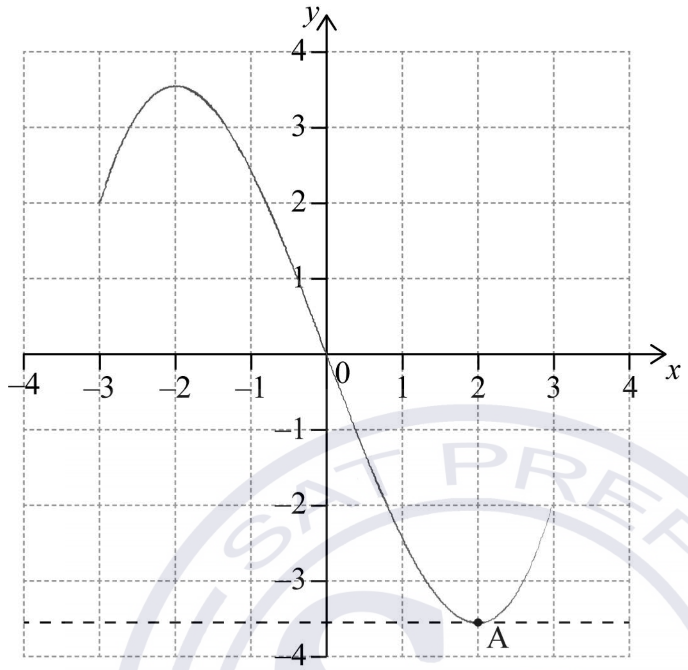
[3 marks]

[Total 7 marks]

2. (a) $A(2, -3.6)$ ***A1A1*** ***N2***

[2 marks]

(b) (i) (ii)



A1

A1A1A1 ***N4***

A1 ***N1***

Notes:	(i)	Award ***A1*** for correct cubic shape with correct curvature. Only if this ***A1*** is awarded, award the following: ***A1*** for passing through **their** point A and the origin, ***A1*** for endpoints, ***A1*** for maximum.
	(ii)	Award ***A1*** for horizontal line through **their** A.

[5 marks]

[Total 7 marks]

3. (a) $\theta = \frac{2\pi}{5}$ *A1* *N1*

[1 mark]

(b) correct expression for area *(A1)*

eg $A = \frac{1}{2}r^2\left(\frac{2\pi}{5}\right),\ \frac{\pi r^2}{5}$

evidence of equating their expression to 20π *(M1)*

eg $\frac{1}{2}r^2\left(\frac{2\pi}{5}\right) = 20\pi$, $r^2 = 100$, $r = \pm 10$

$r = 10$ *A1* *N2*

[3 marks]

(c) **METHOD 1**

evidence of choosing cosine rule *(M1)*

eg $a^2 = b^2 + c^2 - 2bc\cos A$

correct substitution of **their** r and θ into RHS *(A1)*

eg $10^2 + 10^2 - 2\times 10\times 10\cos\left(\frac{2\pi}{5}\right)$

11.7557

$\text{AB} = 11.8\ \text{(mm)}$ *A1* *N2*

METHOD 2

evidence of choosing sine rule *(M1)*

eg $\frac{\sin A}{a} = \frac{\sin B}{b}$

correct substitution of **their** r and θ *(A1)*

eg $\frac{\sin\frac{2\pi}{5}}{\text{AB}} = \frac{\sin\left(\frac{1}{2}\left(\pi - \frac{2\pi}{5}\right)\right)}{10}$

11.7557

$\text{AB} = 11.8 \quad \text{(mm)}$ *A1* *N2*

[3 marks]

[Total 7 marks]

4. (a) valid attempt to find the intersection ***(M1)***

eg $f = g$, sketch, one correct answer

$p = 0.357402$, $q = 2.15329$

$p = 0.357$, $q = 2.15$ ***A1A1*** ***N3***

[3 marks]

(b) attempt to set up an integral involving subtraction (in any order) ***(M1)***

eg $\int_p^q [f(x) - g(x)]\mathrm{d}x$, $\int_p^q f(x)\mathrm{d}x - \int_p^q g(x)\mathrm{d}x$

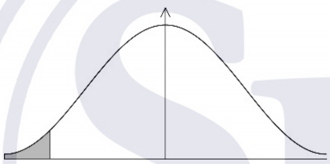
0.537667

area $= 0.538$ ***A2*** ***N3***

[3 marks]

[Total 6 marks]

5. (a) valid approach ***(M1)***

eg $z = -1.61643$, 

2.48863

$w = 2.49$ (kg) ***A2*** ***N3***

[3 marks]

(b) correct value or expression (seen anywhere)

eg $0.053 - \mathrm{P}(X \le 2.15)$, 0.039465 ***(A1)***

evidence of conditional probability ***(M1)***

eg $\dfrac{\mathrm{P}(2.15 \le X \le w)}{\mathrm{P}(X \le w)}$, $\dfrac{0.039465}{0.053}$

0.744631

0.745 ***A1*** ***N2***

[3 marks]

[Total 6 marks]

6. (a) attempt to substitute correct limits or the function into the formula involving y^2 *(M1)*

eg $\pi\int_{-0.5}^{0.5} y^2 \mathrm{d}x$, $\pi\int\left(-0.8x^2+0.5\right)^2 \mathrm{d}x$

0.601091

volume = 0.601 (m^3) *A2* *N3*

[3 marks]

(b) attempt to equate half **their** volume to V *(M1)*

eg $0.30055 = 0.8\left(1-\mathrm{e}^{-0.1t}\right)$, graph

4.71104

4.71 (minutes) *A2* *N3*

[3 marks]

[Total 6 marks]

7. (a) $\mathrm{P}(\text{red}) = \dfrac{5}{15+m}$ *A1* *N1*

[1 mark]

(b) recognizing binomial distribution *(M1)*

eg $X \sim \mathrm{B}(n,\ p)$

correct value for the complement of **their** p (seen anywhere) *A1*

eg $1-\dfrac{5}{15+m}, \dfrac{10+m}{15+m}$

correct substitution into $\mathrm{Var}(X) = np(1-p)$ *(A1)*

eg $4\left(\dfrac{5}{15+m}\right)\left(\dfrac{10+m}{15+m}\right), \dfrac{20(10+m)}{(15+m)^2} < 0.6$

$m > 12.2075$ *(A1)*

$m = 13$ *A1* *N3*

[5 marks]

[Total 6 marks]

Section B

8. (a) attempt to substitute into formula for mean *(M1)*

eg $\frac{\sum x}{10}, \frac{252}{n}, \frac{252}{10}$

mean $= 25.2$ (hours) ***A1 N2***

[2 marks]

(b) (i) mean $= 30.2$ (hours) ***A1 N1***

(ii) $\sigma = 5$ (hours) ***A1 N1***

[2 marks]

(c) (i) valid approach *(M1)*

eg 95%, 5% of 27

correct working *(A1)*

eg 0.95×27, $27 - (5\%$ of $27)$

median $= 25.65$ (exact), 25.7 (hours) ***A1 N2***

(ii) **METHOD 1**

variance = (standard deviation)^2 (seen anywhere) *(A1)*

valid attempt to find new standard deviation *(M1)*

eg $\sigma_{new} = 0.95 \times 5$, 4.75

variance $= 22.5625$ (exact), 22.6 ***A1 N2***

METHOD 2

variance = (standard deviation)^2 (seen anywhere) *(A1)*

valid attempt to find new variance *(M1)*

eg 0.95^2, $0.9025 \times \sigma^2$

new variance $= 22.5625$ (exact), 22.6 ***A1 N2***

[6 marks]

continued…

Question 8 continued

(d) (i) both correct frequencies ***(A1)***
eg $80,\ 150$

subtracting **their** frequencies in either order ***(M1)***
eg $150-80,\ 80-150$

70 (students) ***A1*** ***N2***

(ii) evidence of a valid approach ***(M1)***
eg 10% of 200, 90%

correct working ***(A1)***
eg $0.90\times200,\ 200-20$, 180 students

$k=35$ ***A1*** ***N3***

[6 marks]

[Total 16 marks]

9. (a) valid attempt to substitute $t = 0$ into the correct function ***(M1)***
eg $-2(0)+2$

2 ***A1*** ***N2***
[2 marks]

(b) recognizing $v = 0$ when P is at rest ***(M1)***

5.21834

$p = 5.22$ (seconds) ***A1*** ***N2***
[2 marks]

(c) (i) recognizing that $a = v'$ ***(M1)***
eg $v' = 0$, minimum on graph

1.95343

$q = 1.95$ ***A1*** ***N2***

(ii) valid approach to find **their** minimum ***(M1)***
eg $v(q)$, -1.75879, reference to min on graph

1.75879
speed $= 1.76$ $\left(\text{cm s}^{-1}\right)$ ***A1*** ***N2***
[4 marks]

(d) (i) substitution of **correct** $v(t)$ into distance formula, ***(A1)***
eg $\int_1^p \left| 3\sqrt{t} + \frac{4}{t^2} - 7 \right| \mathrm{d}t$, $\left| \int 3\sqrt{t} + \frac{4}{t^2} - 7\,\mathrm{d}t \right|$,

4.45368

distance $= 4.45$ (cm) ***A1*** ***N2***

(ii) displacement from $t = 1$ to $t = p$ (seen anywhere) ***(A1)***
eg -4.45368, $\int_1^p \left(3\sqrt{t} + \frac{4}{t^2} - 7 \right) \mathrm{d}t$

displacement from $t = 0$ to $t = 1$ ***(A1)***
eg $\int_0^1 (-2t+2)\,\mathrm{d}t$, $0.5 \times 1 \times 2$, 1

valid approach to find displacement for $0 \le t \le p$ ***M1***
eg $\int_0^1 (-2t+2)\,\mathrm{d}t + \int_1^p \left(3\sqrt{t} + \frac{4}{t^2} - 7 \right) \mathrm{d}t$, $\int_0^1 (-2t+2)\,\mathrm{d}t - 4.45$

-3.45368
displacement $= -3.45$ (cm) ***A1*** ***N2***
[6 marks]

[Total 14 marks]

10. (a) (i) valid approach *(M1)*

eg $\frac{5+17}{2}$

$c = 11$ ***A1*** ***N2***

(ii) valid approach *(M1)*

eg period is 12, per $= \frac{2\pi}{b}$, $9-3$

$b = \frac{2\pi}{12}$ ***A1***

$b = \frac{\pi}{6}$ ***AG*** ***N0***

(iii) **METHOD 1**

valid approach *(M1)*

eg $5 = a\sin\left(\frac{\pi}{6}\times 3\right)+11$, substitution of points

$a = -6$ ***A1*** ***N2***

METHOD 2

valid approach *(M1)*

eg $\frac{17-5}{2}$, amplitude is 6

$a = -6$ ***A1*** ***N2***

[6 marks]

(b) (i) $k = 2.5$ ***A1*** ***N1***

(ii) $g(x) = -6\sin\left(\frac{\pi}{6}(x-2.5)\right)+11$ ***A2*** ***N2***

[3 marks]

continued…

Question 10 continued

(c) (i) **METHOD 1** Using g

recognizing that a point of inflexion is required ***M1***
eg sketch, recognizing change in concavity

evidence of valid approach ***(M1)***
eg $g''(x) = 0$, sketch, coordinates of max/min on g'

$w = 8.5$ (exact) ***A1*** ***N2***

METHOD 2 Using f

recognizing that a point of inflexion is required ***M1***
eg sketch, recognizing change in concavity

evidence of valid approach involving translation ***(M1)***
eg $x = w - k$, sketch, $6 + 2.5$

$w = 8.5$ (exact) ***A1*** ***N2***

(ii) valid approach involving the derivative of g or f (seen anywhere) ***(M1)***
eg $g'(w)$, $-\pi\cos\left(\frac{\pi}{6}x\right)$, max on derivative, sketch of derivative

attempt to find max value on derivative ***M1***
eg $-\pi\cos\left(\frac{\pi}{6}(8.5 - 2.5)\right)$, $f'(6)$, dot on max of sketch

3.14159
max rate of change $= \pi$ (exact), 3.14 ***A1*** ***N2***

[6 marks]

[Total 15 marks]

Markscheme

May 2016

Mathematics

Standard level

Paper 2

Instructions to Examiners

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3 *N* marks

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4 Implied and must be seen marks

*Implied marks appear in **brackets** eg **(M1)**.*

- Implied marks can only be awarded if the work is seen or if implied in subsequent working (a correct final answer does not necessarily mean that the implied marks are all awarded). There are questions where some working is required, but as it is accepted that not everyone will write the same steps, all the marks are implied, but the ***N*** marks are not the full marks for the question.
- Normally the correct work is seen in the next line.
- Where there is an (***M1***) followed by ***A1*** for each correct answer, if no working shown, one correct answer is sufficient evidence to award the (***M1***).

*Must be seen marks appear without **brackets** eg **M1**.*

- Must be seen marks can only be awarded if the work is seen.
- If a must be seen mark is not awarded because work is missing (as opposed to ***M0*** or ***A0*** for incorrect work) all subsequent marks may be awarded if appropriate.

5 Follow through marks (only applied after an error is made)

*Follow through (**FT**) marks are awarded where an incorrect answer (final or intermediate) from one **part** of a question is used correctly in **subsequent** part(s) or subpart(s). Usually, to award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if the only marks awarded in a subpart are for the **final** answer, then **FT** marks should be awarded if appropriate. Examiners are expected to check student work in order to award **FT** marks where appropriate.*

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** and ***R*** marks may be awarded if appropriate. (However, as noted above, if an ***A*** mark is not awarded because work is missing, all subsequent marks may be awarded if appropriate).
- Exceptions to this rule will be explicitly noted on the markscheme.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*eg* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word "**their**" in a description, to indicate that candidates may be using an incorrect value.
- If a candidate makes an error in one part, but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the question says hence. It is often possible to use a different approach in subsequent parts that does not depend on the answer to previous parts.
- In a "show that" question, if an error in a previous subpart leads to not showing the required answer, do not award the final ***A1***. Note that if the error occurs within the same subpart, the ***FT*** rules may result in further loss of marks.

6 Mis-read

*If a candidate incorrectly copies information from the question, this is a mis-read (**MR**). A candidate should be penalized only once for a particular mis-read. Use the **MR** stamp to indicate that this is a misread. Do not award the first mark in the question, even if this is an **M** mark, but award all others (if appropriate) so that the candidate only loses one mark for the misread.*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*eg* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.

7 Discretionary marks *(d)*

*An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. In such cases the annotation DM should be used and a brief **note** written next to the mark explaining this decision.*

8 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete parts are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**. Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9 Alternative forms

*Unless the question specifies otherwise, **accept** equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

10 Calculators

A GDC is required for paper 2, but calculators with symbolic manipulation features (eg TI-89) are not allowed.

Calculator notation The mathematics SL guide says:
Students must always use correct mathematical notation, not calculator notation.
Do **not** accept final answers written using calculator notation. However, do not penalize the use of calculator notation in the working.

11 Style

The markscheme aims to present answers using good communication, eg if the question asks to find the value of k, the markscheme will say $k = 3$, but the marks will be for the correct value 3 –

there is usually no need for the "k =". In these cases, it is also usually acceptable to have another variable, as long as there is no ambiguity in the question, eg if the question asks to find the value of p and of q, then the student answer needs to be clear. Generally, the only situation where the full answer is required is in a question which asks for equations – in this case the markscheme will say "must be an equation".

The markscheme often uses words to describe what the marks are for, followed by examples, using the eg notation. These examples are not exhaustive, and examiners should check what candidates have written, to see if they satisfy the description. Where these marks are ***M*** *marks, the examples may include ones using poor notation, to indicate what is acceptable. A valid method is one which will allow candidate to proceed to the next step eg if a quadratic function is given in factorised form, and the question asks for the zeroes, then multiplying the factors does not necessarily help to find the zeros, and would not on its own count as a valid method.*

12 Candidate work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

Candidates are meant to write their answers to Section A on the question paper (QP), and Section B on answer booklets. Sometimes, they need more room for Section A, and use the booklet (and often comment to this effect on the QP), or write outside the box. That is fine, and this work should be marked.

The instructions tell candidates not to write on Section B of the QP. Thus they may well have done some rough work here which they assume will be ignored. If they have solutions on the answer booklets, there is no need to look at the QP. However, if there are whole questions or whole part solutions missing on answer booklets, please check to make sure that they are not on the QP, and if they are, mark those whole questions or whole part solutions that have not been written on answer booklets.

13. Diagrams

The notes on how to allocate marks for sketches usually refer to passing through particular points or having certain features. These marks can only be awarded if the sketch is approximately the correct shape. All values given will be an approximate guide to where these points/features occur. In some questions, the first ***A1*** is for the shape, in others, the marks are only for the points and/or features. In both cases, unless the shape is approximately correct, no marks can be awarded (unless otherwise stated). However, if the graph is based on previous calculations, ***FT*** marks should be awarded if appropriate.

14. Accuracy of Answers

If the level of accuracy is specified in the question, a mark will be allocated for giving the final answer to the required accuracy. When this is not specified in the question, all numerical answers should be given exactly or correct to three significant figures.

Do not accept unfinished numerical final answers such as 3/0.1 (unless otherwise stated). As a rule, numerical answers with more than one part (such as fractions) should be given using integers (*eg* 6/8). Calculations which lead to integers should be completed, with the exception of fractions which are not whole numbers.

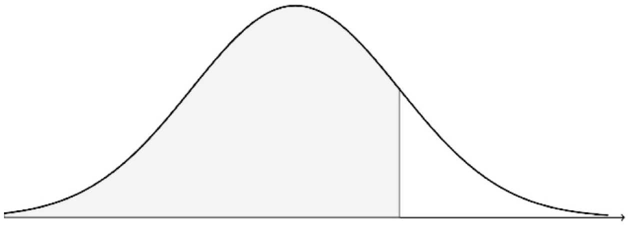
Intermediate values do not need to be given to the correct three significant figures. But, if candidates work with rounded values, this could lead to an incorrect answer, in which case award ***A0*** for the final answer.

Where numerical answers are required as the **final** answer to a part of a question in the markscheme, the markscheme will show

a truncated 6 sf value)
the exact value if applicable, the correct 3 sf answer
Units will appear in brackets at the end.

Section A

1. (a)



A1A1 ***N2***

> **Note:** Award ***A1*** for vertical line clearly to right of mean,
> ***A1*** for shading to left of their vertical line.

[2 marks]

(b) $\mathrm{P}(X \le 25) = 0.894350$ ***(A1)***

$\mathrm{P}(X \le 25) = 0.89$ (must be 2 d.p.) ***A1*** ***N2***

[2 marks]

(c) $c = 22.0976$

$c = 22.1$ ***A2*** ***N2***

[2 marks]

Total [6 marks]

2. (a) valid approach ***(M1)***

eg sketch

$0, 1.73843$

$x = 0$, $x = 1.74$ (accept $(0, 0)$ and $(1.74, 3.02)$) ***A1A1*** ***N3***

[3 marks]

(b) integrating and subtracting functions (in any order) ***(M1)***

eg $\int g - f$

correct substitution of their limits **or** function (accept missing $\mathrm{d}x$) ***(A1)***

eg $\int_0^{1.74} g - f$, $\int 3\ln(x+1) - x^2$

> **Note:** Do not award ***A1*** if there is an error in the substitution.

1.30940

1.31 ***A1*** ***N3***

[3 marks]

Total [6 marks]

3. (a) valid approach *(M1)*

eg $70+(180-115),\ 360-(110+115)$

$\mathrm{A\hat{B}C}=135^{\circ}$ ***A1*** ***N2***

[2 marks]

(b) choosing cosine rule *(M1)*

eg $c^2=a^2+b^2-2ab\cos C$

correct substitution into RHS *(A1)*

eg $5^2+8^2-2\times5\times8\cos135$

12.0651

12.1 (km) ***A1*** ***N2***

[3 marks]

(c) correct substitution (**must** be into sine rule) ***A1***

eg $\dfrac{\sin \mathrm{A\hat{C}B}}{5}=\dfrac{\sin 135}{\mathrm{AC}}$

17.0398

$\mathrm{A\hat{C}B}=17.0$ ***A1*** ***N1***

[2 marks]

Total [7 marks]

4. (a) valid approach to find the required term ***(M1)***

eg $\binom{9}{r}(x)^{9-r}(2)^r$, $x^9+9x^8(2)+\binom{9}{2}x^7(2)^2+\ldots$, Pascal's triangle to the 9th row

identifying correct term (may be indicated in expansion) ***(A1)***

eg 4th term, $r=6$, $\binom{9}{3}$, $(x)^6(2)^3$

correct calculation (may be seen in expansion) ***(A1)***

eg $\binom{9}{3}(x)^6(2)^3$, 84×2^3

$672x^6$ ***A1*** ***N3***

[4 marks]

(b) valid approach ***(M1)***

eg recognizing x^7 is found when multiplying $5x\times 672x^6$

$3360x^7$ ***A1*** ***N2***

[2 marks]

Total [6 marks]

5. (a) strong, negative (both required) ***A2*** ***N2***

[2 marks]

(b) **METHOD 1**

valid approach ***(M1)***

eg $e^{\ln M} = e^{-0.12t + 4.67}$

correct use of exponent laws for $e^{-0.12t+4.67}$ ***(A1)***

eg $e^{-0.12t} \times e^{4.67}$

comparing coefficients/terms ***(A1)***

eg $b^t = e^{-0.12t}$

$b = e^{-0.12}$ (exact), 0.887 ***A1*** ***N3***

METHOD 2

valid approach ***(M1)***

eg $\ln\left(a \times b^t\right) = -0.12t + 4.67$

correct use of log laws for $\ln\left(ab^t\right)$ ***(A1)***

eg $\ln a + t \ln b$

comparing coefficients ***(A1)***

eg $-0.12 = \ln b$

$b = e^{-0.12}$ (exact), 0.887 ***A1*** ***N3***

[4 marks]

Total [6 marks]

6. correct equation to find r ***(A1)***

eg $u_1 r^3 = 8u_1,\ r^3 = 8$

$r = 2$ (seen anywhere) ***(A1)***

correct equation to find u_1 ***A1***

eg $u_1\left(2^{10}-1\right) = 2557.5,\ u_1 = \dfrac{2557.5}{r^{10}-1}(r-1)$

$u_1 = 2.5$ ***(A1)***

$u_{10} = 2.5(2)^9$ ***(M1)***

1280 ***A1*** ***N4***

[6 marks]

7. (a) (i) valid approach ***(M1)***

eg $0.9 = \mathrm{e}^{k(1)}$

$k = -0.105360$

$k = \ln 0.9$ (exact), -0.105 ***A1*** ***N2***

(ii) correct interpretation ***R1*** ***N1***

eg population is decreasing, growth rate is negative

[3 marks]

(b) **METHOD 1**

valid approach (accept an equality, but do not accept 0.74) ***(M1)***

eg $P < 0.75P_0,\ P_0\mathrm{e}^{kt} < 0.75P_0,\ 0.75 = \mathrm{e}^{t\ln 0.9}$

valid approach to solve **their** inequality ***(M1)***

eg logs, graph

$t > 2.73045$ (accept $t = 2.73045$) (2.73982 from -0.105) ***A1***

28 years ***A2*** ***N2***

METHOD 2

valid approach which gives both crossover values accurate to at least 2 sf ***A2***

eg $\dfrac{P_{2.7}}{P_0} = 0.75241\ldots,\ \dfrac{P_{2.8}}{P_0} = 0.74452\ldots$

$t = 2.8$ ***(A1)***

28 years ***A2*** ***N2***

[5 marks]

Total [8 marks]

Section B

8. (a) evidence of summing to 1 ***(M1)***

eg $0.55+0.3+0.1+k=1$

$k=0.05$ (exact) ***A1 N2***

[2 marks]

(b) (i) 0.55 ***A1 N1***

(ii) recognizing binomial probability ***(M1)***

eg $X : B(n,\ p),\ \binom{5}{4},\ (0.55)^4(1-0.55),\ \binom{n}{r}p^r q^{n-r}$

$\mathrm{P}(X=4)=0.205889$

$\mathrm{P}(X=4)=0.206$ ***A1 N2***

[3 marks]

(c) correct substitution into formula for $\mathrm{E}(X)$ ***(A1)***

eg $0.2+(2\times0.08)+(3\times0.02)$

$\mathrm{E}(B)=0.42$ (exact) ***A1 N2***

[2 marks]

(d) (i) valid attempt to find one possible way of having 2 breakdowns ***(M1)***

eg $2A$, $2B$, $1A$ and $1B$, tree diagram

one correct calculation for 1 way (seen anywhere) ***(A1)***

eg 0.1×0.7, 0.55×0.08, 0.3×0.2

recognizing there are 3 ways of having 2 breakdowns ***(M1)***

eg A twice or B twice or one breakdown each

correct working ***(A1)***

eg $(0.1\times0.7)+(0.55\times0.08)+(0.3\times0.2)$

$\mathrm{P}(2\text{ breakdowns})=0.174$ (exact) ***A1 N3***

(ii) recognizing conditional probability ***(M1)***

eg $\mathrm{P}\left(A|B\right)$, $\mathrm{P}\left(2A|2\text{ breakdowns}\right)$

correct working ***(A1)***

eg $\dfrac{0.1\times0.7}{0.174}$

$\mathrm{P}\left(A=2\,|\,\text{two breakdowns}\right)=0.402298$

$\mathrm{P}\left(A=2\,|\,\text{two breakdowns}\right)=0.402$ ***A1 N2***

[8 marks]

Total [15 marks]

9. (a) **METHOD 1**

recognizing $s = \int v$ ***(M1)***

recognizing displacement of P in first 5 seconds (seen anywhere) ***A1***
(accept missing $\mathrm{d}t$)

eg $\int_0^5 v\mathrm{d}t$, -3.71591

valid approach to find total displacement ***(M1)***

eg $4 + (-3.7159)$, $s = 4 + \int_0^5 v$

0.284086
0.284 (m) ***A2*** ***N3***

METHOD 2

recognizing $s = \int v$ ***(M1)***

correct integration ***A1***

eg $\frac{1}{3}\sin 3t + 2\cos t - \frac{t}{2} + c$ (do not penalize missing "c")

attempt to find c ***(M1)***

eg $4 = \frac{1}{3}\sin(0) + 2\cos(0) - \frac{0}{2} + c$, $4 = \frac{1}{3}\sin 3t + 2\cos t - \frac{t}{2} + c$, $2 + c = 4$

attempt to substitute $t = 5$ into their expression with c ***(M1)***

eg $s(5)$, $\frac{1}{3}\sin(15) + 2\cos(5) - \frac{5}{2} + 2$

0.284086
0.284 (m) ***A1*** ***N3***
[5 marks]

(b) recognizing that at rest, $v = 0$ ***(M1)***

$t = 0.179900$

$t = 0.180$ (secs) ***A1*** ***N2***
[2 marks]

(c) recognizing when change of direction occurs ***(M1)***
eg v crosses t axis

2 (times) ***A1*** ***N2***
[2 marks]

continued…

Question 9 continued

(d) acceleration is v' (seen anywhere) ***(M1)***

eg $v'(3)$

0.743631

$0.744 \ (\text{ms}^{-2})$ ***A1 N2***

[2 marks]

(e) valid approach involving max or min of v ***(M1)***

eg $v'=0$, $a=0$, graph

one correct co-ordinate for min ***(A1)***

eg $1.14102, -3.27876$

$3.28 \ (\text{ms}^{-1})$ ***A1 N2***

[3 marks]

Total [14 marks]

10. (a) valid approach (addition or subtraction) ***(M1)***

eg $\text{AO}+\text{OB}$, $\text{B}-\text{A}$

$\overrightarrow{\text{AB}} = \begin{pmatrix} 9 \\ 6 \\ -3 \end{pmatrix}$ ***A1 N2***

[2 marks]

(b) **METHOD 1**

valid approach using $\overrightarrow{\text{OC}} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ ***(M1)***

eg $\overrightarrow{\text{AC}} = \begin{pmatrix} x+3 \\ y+2 \\ z-2 \end{pmatrix}$, $\overrightarrow{\text{CB}} = \begin{pmatrix} 6-x \\ 4-y \\ -1-z \end{pmatrix}$

correct working ***A1***

eg $\begin{pmatrix} x+3 \\ y+2 \\ z-2 \end{pmatrix} = \begin{pmatrix} 12-2x \\ 8-2y \\ -2-2z \end{pmatrix}$

all three equations ***A1***

eg $x+3=12-2x$, $y+2=8-2y$, $z-2=-2-2z$,

$\overrightarrow{\text{OC}} = \begin{pmatrix} 3 \\ 2 \\ 0 \end{pmatrix}$ ***AG N0***

continued…

Question 10 continued

METHOD 2

valid approach **(M1)**

eg $\vec{OC} - \vec{OA} = 2\left(\vec{OB} - \vec{OC}\right)$

correct working **A1**

eg $3\vec{OC} = 2\vec{OB} + \vec{OA}$

correct substitution of $\vec{OB}$ and $\vec{OA}$ **A1**

eg $3\vec{OC} = 2\begin{pmatrix} 6 \\ 4 \\ -1 \end{pmatrix} + \begin{pmatrix} -3 \\ -2 \\ 2 \end{pmatrix}$, $3\vec{OC} = \begin{pmatrix} 9 \\ 6 \\ 0 \end{pmatrix}$

$\vec{OC} = \begin{pmatrix} 3 \\ 2 \\ 0 \end{pmatrix}$ **AG** **N0**

METHOD 3

valid approach **(M1)**

eg $\vec{AC} = \frac{2}{3}\vec{AB}$, diagram, $\vec{CB} = \frac{1}{3}\vec{AB}$

correct working **A1**

eg $\vec{AC} = \begin{pmatrix} 6 \\ 4 \\ -2 \end{pmatrix}$, $\vec{CB} = \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix}$

correct working involving $\vec{OC}$ **A1**

eg $\vec{OC} = \begin{pmatrix} -3 \\ -2 \\ 2 \end{pmatrix} + \begin{pmatrix} 6 \\ 4 \\ -2 \end{pmatrix}$, $\begin{pmatrix} 6 \\ 4 \\ -1 \end{pmatrix} - \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix}$

$\vec{OC} = \begin{pmatrix} 3 \\ 2 \\ 0 \end{pmatrix}$ **AG** **N0**

[3 marks]

continued…

Question 10 continued

(c) finding scalar product and magnitudes **(A1)(A1)(A1)**

scalar product $= (9 \times 3) + (6 \times 2) + (-3 \times 0) \quad (= 39)$

magnitudes $\sqrt{81+36+9} \quad (= 11.22), \sqrt{9+4} \quad (= 3.605)$

substitution into formula **M1**

eg $\cos\theta = \dfrac{(9 \times 3) + 12}{\sqrt{126} \times \sqrt{13}}$

$\theta = 0.270549$ (accept 15.50135°)

$\theta = 0.271$ (accept 15.5°) **A1 N4**

[5 marks]

(d) (i) attempt to use a trig ratio **M1**

eg $\sin\theta = \dfrac{\text{DE}}{\text{CD}}, \left|\overrightarrow{\text{CE}}\right| = \left|\overrightarrow{\text{CD}}\right| \cos\theta$

attempt to express $\overrightarrow{\text{CD}}$ in terms of $\overrightarrow{\text{OC}}$ **M1**

eg $\overrightarrow{\text{OC}} + \overrightarrow{\text{CD}} = \overrightarrow{\text{OD}}$, $\text{OC} + \text{CD} = \text{OD}$

correct working **A1**

eg $\left|k\overrightarrow{\text{OC}} - \overrightarrow{\text{OC}}\right| \sin\theta$

$\left|\overrightarrow{\text{DE}}\right| = (k-1)\left|\overrightarrow{\text{OC}}\right| \sin\theta$ **AG N0**

(ii) valid approach involving the segment DE **(M1)**

eg recognizing $\left|\overrightarrow{\text{DE}}\right| < 3$, $\text{DE} = 3$

correct working (accept equation) **(A1)**

eg $(k-1)\left(\sqrt{13}\right)\sin 0.271 < 3$, $k - 1 = 3.11324$

$1 < k < 4.11$ (accept $k < 4.11$ but not $k = 4.11$) **A1 N2**

[6 marks]

Total [16 marks]

Markscheme

May 2016

Mathematics

Standard level

Paper 2

16 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a valid **Method**; working must be seen.

(M) Marks awarded for a valid **Method**; may be implied by **correct** subsequent working.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

(A) Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.

R Marks awarded for clear **Reasoning**.

N Marks awarded for **correct** answers if **no** working shown.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Mark according to RM assessor instructions

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any. An exception to this rule is when work for ***M1*** is missing, as opposed to incorrect (see point 4).
- Where ***M*** and ***A*** marks are noted on the same line, *eg* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*eg* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***(M2)***, ***N3***, *etc*., do **not** split the marks, unless there is a note.
- Most ***M*** marks are for a **valid** method, ie a method which can lead to the answer: it must indicate some form of progress towards the answer.

Once a correct answer to a question or part-question is seen, ignore further correct working. However, if further working indicates a lack of mathematical understanding do not award final ***A1***.

3 *N* marks

*If **no** working shown, award **N** marks for **correct** answers – this includes acceptable answers (see accuracy booklet). In this case, ignore mark breakdown (**M, A, R**). Where a student only shows a final incorrect answer with no working, even if that answer is a correct intermediate answer, award **N0.***

- Do **not** award a mixture of ***N*** and other marks.
- There may be fewer ***N*** marks available than the total of ***M***, ***A*** and ***R*** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.
- There may not be a direct relationship between the ***N*** marks and the implied marks. There are times when all the marks are implied, but the ***N*** marks are not the full marks: this indicates that we want to see some of the working, without specifying what.

- For consistency within the markscheme, ***N*** marks are noted for every part, even when these match the mark breakdown.
- If a candidate has incorrect working, which somehow results in a correct answer, do **not** award the ***N*** marks for this correct answer. However, if the candidate has indicated (usually by crossing out) that the working is to be ignored, award the ***N*** marks for the correct answer.

4 Implied and must be seen marks

*Implied marks appear in **brackets** eg **(M1)**.*

- Implied marks can only be awarded if the work is seen or if implied in subsequent working (a correct final answer does not necessarily mean that the implied marks are all awarded). There are questions where some working is required, but as it is accepted that not everyone will write the same steps, all the marks are implied, but the ***N*** marks are not the full marks for the question.
- Normally the correct work is seen in the next line.
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*Must be seen marks appear without **brackets** eg **M1**.*

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*Follow through (**FT**) marks are awarded where an incorrect answer (final or intermediate) from one **part** of a question is used correctly in **subsequent** part(s) or subpart(s). Usually, to award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if the only marks awarded in a subpart are for the **final** answer, then **FT** marks should be awarded if appropriate. Examiners are expected to check student work in order to award **FT** marks where appropriate.*

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- In a "show that" question, if an error in a previous subpart leads to not showing the required answer, do not award the final ***A1***. Note that if the error occurs within the same subpart, the ***FT*** rules may result in further loss of marks.

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*If a candidate incorrectly copies information from the question, this is a mis-read (**MR**). A candidate should be penalized only once for a particular mis-read. Use the **MR** stamp to indicate that this is a misread. Do not award the first mark in the question, even if this is an **M** mark, but award all others (if appropriate) so that the candidate only loses one mark for the misread.*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*eg* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.

7 Discretionary marks *(d)*

*An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. In such cases the annotation DM should be used and a brief **note** written next to the mark explaining this decision.*

8 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete parts are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**. Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9 Alternative forms

*Unless the question specifies otherwise, **accept** equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

10 Calculators

A GDC is required for paper 2, but calculators with symbolic manipulation features (eg TI-89) are not allowed.

Calculator notation The mathematics SL guide says:
Students must always use correct mathematical notation, not calculator notation.
Do **not** accept final answers written using calculator notation. However, do not penalize the use of calculator notation in the working.

11 Style

The markscheme aims to present answers using good communication, eg if the question asks to find the value of k, the markscheme will say $k = 3$, but the marks will be for the correct value 3 –

there is usually no need for the "k =". In these cases, it is also usually acceptable to have another variable, as long as there is no ambiguity in the question, eg if the question asks to find the value of p and of q, then the student answer needs to be clear. Generally, the only situation where the full answer is required is in a question which asks for equations – in this case the markscheme will say "must be an equation".

*The markscheme often uses words to describe what the marks are for, followed by examples, using the eg notation. These examples are not exhaustive, and examiners should check what candidates have written, to see if they satisfy the description. Where these marks are **M** marks, the examples may include ones using poor notation, to indicate what is acceptable. A valid method is one which will allow candidate to proceed to the next step eg if a quadratic function is given in factorised form, and the question asks for the zeroes, then multiplying the factors does not necessarily help to find the zeros, and would not on its own count as a valid method.*

12 Candidate work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

Candidates are meant to write their answers to Section A on the question paper (QP), and Section B on answer booklets. Sometimes, they need more room for Section A, and use the booklet (and often comment to this effect on the QP), or write outside the box. That is fine, and this work should be marked.

The instructions tell candidates not to write on Section B of the QP. Thus they may well have done some rough work here which they assume will be ignored. If they have solutions on the answer booklets, there is no need to look at the QP. However, if there are whole questions or whole part solutions missing on answer booklets, please check to make sure that they are not on the QP, and if they are, mark those whole questions or whole part solutions that have not been written on answer booklets.

13. Diagrams

The notes on how to allocate marks for sketches usually refer to passing through particular points or having certain features. These marks can only be awarded if the sketch is approximately the correct shape. All values given will be an approximate guide to where these points/features occur. In some questions, the first ***A1*** is for the shape, in others, the marks are only for the points and/or features. In both cases, unless the shape is approximately correct, no marks can be awarded (unless otherwise stated). However, if the graph is based on previous calculations, ***FT*** marks should be awarded if appropriate.

14. Accuracy of Answers

If the level of accuracy is specified in the question, a mark will be allocated for giving the final answer to the required accuracy. When this is not specified in the question, all numerical answers should be given exactly or correct to three significant figures.

Do not accept unfinished numerical final answers such as 3/0.1 (unless otherwise stated). As a rule, numerical answers with more than one part (such as fractions) should be given using integers (*eg* 6/8). Calculations which lead to integers should be completed, with the exception of fractions which are not whole numbers.

Intermediate values do not need to be given to the correct three significant figures. But, if candidates work with rounded values, this could lead to an incorrect answer, in which case award ***A0*** for the final answer.

Where numerical answers are required as the **final** answer to a part of a question in the markscheme, the markscheme will show

a truncated 6 sf value
the exact value if applicable, the correct 3 sf answer
Units will appear in brackets at the end.

Section A

1. (a) valid approach *(M1)*

eg $1.5-0.3,\ 1.5-2.7,\ 2.7=0.3+2d$

$d=1.2$ ***A1*** ***N2***

[2 marks]

(b) correct substitution into term formula *(A1)*

eg $0.3+1.2(30-1)$, $u_{30}=0.3+29(1.2)$

$u_{30}=35.1$ ***A1*** ***N2***

[2 marks]

(c) correct substitution into sum formula *(A1)*

eg $S_{30}=\frac{30}{2}(0.3+35.1)$, $\frac{30}{2}\left(2(0.3)+29(1.2)\right)$

$S_{30}=531$ ***A1*** ***N2***

[2 marks]

Total [6 marks]

2. (a) evidence of choosing sine rule *(M1)*

eg $\frac{a}{\sin A}=\frac{b}{\sin B}$

correct substitution *(A1)*

eg $\frac{a}{\sin 1.75}=\frac{7}{\sin 0.82}$

9.42069

$\text{BD}=9.42\ \ (\text{cm})$ ***A1*** ***N2***

[3 marks]

(b) evidence of choosing cosine rule *(M1)*

eg $\cos B=\frac{d^2+c^2-b^2}{2dc}$, $a^2=b^2+c^2-2bc\cos B$

correct substitution *(A1)*

eg $\frac{8^2+9.42069^2-12^2}{2\times 8\times 9.42069}$, $144=64+\text{BD}^2-16\,\text{BD}\cos B$

1.51271

$\text{D}\hat{\text{B}}\text{C}=1.51$ (radians) (accept 86.7°) ***A1*** ***N2***

[3 marks]

Total [6 marks]

3. (a) (i) $y = -1$ ***A1*** ***N1***

(ii) valid attempt to find x-intercept ***(M1)***

eg $f(x) = 0$

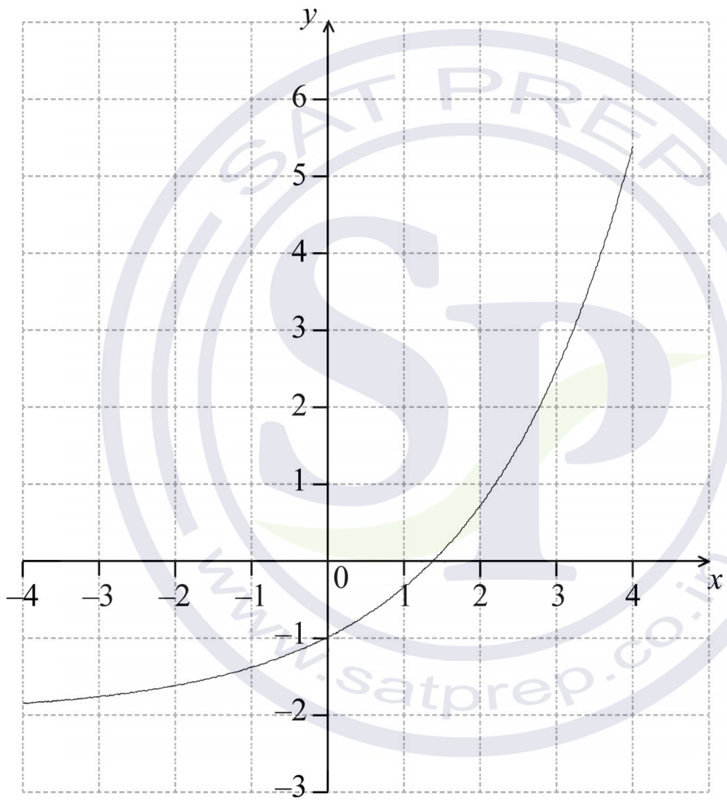
1.38629 ***A1*** ***N2***

$x = 2\ln 2\,(\text{exact}),\ 1.39$

(iii) $y = -2$ (must be equation) ***A1*** ***N1***

[4 marks]

(b)



A1A1A1 ***N3***

[3 marks]

Total [7 marks]

4. (a) valid approach *(M1)*

eg $h(0)$, $-15\cos(1.2\times 0)+17$, $-15(1)+17$

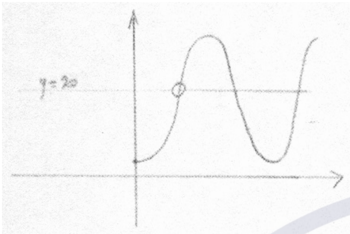
$h(0)=2$ (m) ***A1*** ***N2***

[2 marks]

(b) correct substitution into equation *(A1)*

eg $20=-15\cos 1.2t+17$, $-15\cos 1.2k=3$

valid attempt to solve for k *(M1)*

eg 

, $\cos 1.2k=-\dfrac{3}{15}$

1.47679

$k=1.48$ ***A1*** ***N2***

[3 marks]

(c) recognize the need to find the period (seen anywhere) *(M1)*

eg next t value when $h=20$

correct value for period *(A1)*

eg period $=\dfrac{2\pi}{1.2}$, 5.23598, $6.7-1.48$

5.2 (min) (must be 1 dp) ***A1*** ***N2***

[3 marks]

Total [8 marks]

5. (a) 11 terms ***A1*** ***N1***

[1 mark]

(b) valid approach *(M1)*

eg $\binom{10}{r}\left(x^2\right)^{10-r}\left(\dfrac{2}{x}\right)^r$, $a^{10}b^0+\binom{10}{1}a^9b^1+\binom{10}{2}a^8b^2+\ldots$

Pascal's triangle to 11^{th} row

valid attempt to find value of r which gives term in x^8 *(M1)*

eg $\left(x^2\right)^{10-r}\left(\dfrac{1}{x^r}\right)=x^8$, $x^{2r}\left(\dfrac{2}{x}\right)^{10-r}=x^8$

identifying required term (may be indicated in expansion) *(A1)*

eg $r=6$, 5th term, 7th term

correct working (may be seen in expansion) *(A1)*

eg $\binom{10}{6}\left(x^2\right)^6\left(\dfrac{2}{x}\right)^4$, 210×16

3360 ***A1*** ***N3***

[5 marks]

Total [6 marks]

6. (a) 0.0668072

$\mathrm{P}(S < 50) = 0.0668$ (accept $\mathrm{P}(S \le 49) = 0.0548$) ***A2*** ***N2***

[2 marks]

(b) valid approach ***(M1)***

Eg $\mathrm{P}(S < 50) \times \mathrm{P}(R < x)$

correct equation (accept any variable) ***A1***

eg $\mathrm{P}(S < 50) \times \mathrm{P}(R < x) = 1\%$, $0.0668072 \times p = 0.01$, $\mathrm{P}(R < x) = \dfrac{0.01}{0.0668}$

finding the value of $\mathrm{P}(R < x)$ ***(A1)***

eg $\dfrac{0.01}{0.0668}$, 0.149684

9.40553

$x = 9.41$ (accept $x = 9.74$ from 0.0548) ***A1*** ***N3***

Total [6 marks]

7. correct approach ***(A1)***

eg $s = \int v$, $\int_0^p 6t - 6\mathrm{d}t$

correct integration ***(A1)***

eg $\int 6t - 6\mathrm{d}t = 3t^2 - 6t + C$, $\left[3t^2 - 6t\right]_0^p$

recognizing that there are two possibilities ***(M1)***

eg 2 correct answers, $s = \pm 2$, $c \pm 2$

two correct equations in *p* ***A1A1***

eg $3p^2 - 6p = 2$, $3p^2 - 6p = -2$

0.42265, 1.57735

$p = 0.423$ or $p = 1.58$ ***A1A1*** ***N3***

[7 marks]

Section B

8. **Note:** There may be slight differences in answers, depending on which values candidates carry through in subsequent parts. Accept answers that are consistent with their working.

(a) (i) valid approach *(M1)*
eg correct value for r (or for a or b seen in (ii))

-0.994347
$r = -0.994$ ***A1*** ***N2***

(ii) $-1.58095,\ 33480.3$
$a = -1.58,\quad b = 33500$ ***A1A1*** ***N2***
[4 marks]

(b) correct substitution into **their** regression equation
eg $-1.58095(11000) + 33480.3$ ***(A1)***

$16\,089.85$ ($16\,120$ from 3sf) ***(A1)***

price $= 16100$ (dollars) (must be rounded to the nearest 100 dollars) ***A1*** ***N3***
[3 marks]

(c) **METHOD 1**

valid approach ***(M1)***
eg $P \times (\text{rate})^t$

rate $= 0.95$ (may be seen in their expression) ***(A1)***

correct expression ***(A1)***
eg 16100×0.95^6

$11\,834.97$
$11\,800$ (dollars) ***A1*** ***N2***

METHOD 2

attempt to find all six terms ***(M1)***
eg $\left(\left((16100 \times 0.95) \times 0.95\right)\ldots\right) \times 0.95$, table of values

5 correct values (accept values that round correctly to the nearest dollar)
$15\,295,\ 14\,530,\ 13\,804,\ 13\,114,\ 12\,458$ ***A2***

$11\,835$
$11\,800$ (dollars) ***A1*** ***N2***
[4 marks]

continued…

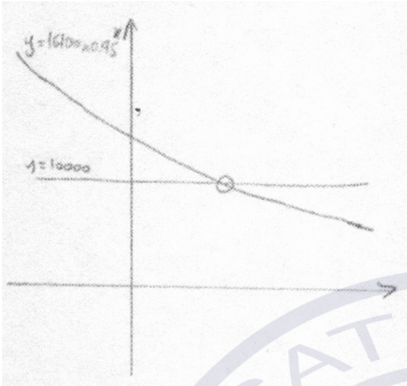
Question 8 continued

(d) **METHOD 1**

correct equation *(A1)*

eg $16100 \times 0.95^x = 10000$

valid attempt to solve *(M1)*

eg 

, using logs

9.28453 *(A1)*

year 2019 ***A1*** ***N2***

METHOD 2

valid approach using table of values *(M1)*

both crossover values (accept values that round correctly to the nearest dollar) ***A2***

eg $\mathrm{P} = 10147$ (1 Jan 2019), $\mathrm{P} = 9639.7$ (1 Jan 2020)

year 2019 ***A1*** ***N2***

[4 marks]

Total [15 marks]

9. (a) $y = 2$ (correct equation only) ***A2*** ***N2***

[2 marks]

(b) valid approach ***(M1)***

eg $(x-1)^{-1} + 2$, $f'(x) = \dfrac{0(x-1)-1}{(x-1)^2}$

$-(x-1)^{-2}$, $f'(x) = \dfrac{-1}{(x-1)^2}$ ***A1*** ***N2***

[2 marks]

(c) correct equation for the asymptote of g

eg $y = b$ ***(A1)***

$b = 2$ ***A1*** ***N2***

[2 marks]

(d) correct derivative of g (seen anywhere) ***(A2)***

eg $g'(x) = -a\mathrm{e}^{-x}$

correct equation ***(A1)***

eg $-\mathrm{e} = -a\mathrm{e}^{-1}$

7.38905

$a = \mathrm{e}^2$ (exact), 7.39 ***A1*** ***N2***

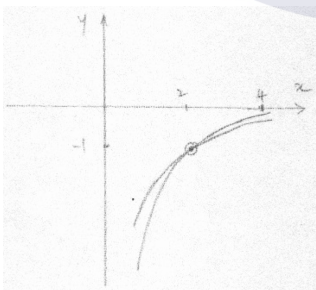
[4 marks]

(e) attempt to equate **their** derivatives ***(M1)***

eg $f'(x) = g'(x)$, $\dfrac{-1}{(x-1)^2} = -a\mathrm{e}^{-x}$

valid attempt to solve **their** equation ***(M1)***

eg correct value outside the domain of f such as 0.522 or 4.51,



correct solution (may be seen in sketch) ***(A1)***

eg $x = 2$, $(2, -1)$

gradient is -1 ***A1*** ***N3***

[4 marks]

Total [14 marks]

10. (a) valid approach *(M1)*

eg $\quad \mathrm{B}-\mathrm{A}$, $\mathrm{AO}+\mathrm{OB}$, $\begin{pmatrix} -9 \\ 9 \\ -6 \end{pmatrix} - \begin{pmatrix} 1 \\ 5 \\ -7 \end{pmatrix}$

$\overrightarrow{\mathrm{AB}} = \begin{pmatrix} -10 \\ 4 \\ 1 \end{pmatrix}$ ***A1*** ***N2***

[2 marks]

(b) valid approach *(M1)*

eg $\quad \mathrm{OC} = \mathrm{OA} + \mathrm{AC}$, $\begin{pmatrix} 1+6 \\ 5-4 \\ -7+0 \end{pmatrix}$

$\mathrm{C}(7, 1, -7)$ ***A1*** ***N2***

[2 marks]

(c) any correct equation in the form $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$ (accept any parameter for t)

where $\boldsymbol{a}$ is $\begin{pmatrix} -9 \\ 9 \\ -6 \end{pmatrix}$, and $\boldsymbol{b}$ is a scalar multiple of $\begin{pmatrix} 6 \\ -4 \\ 0 \end{pmatrix}$ ***A2*** ***N2***

eg $\quad \boldsymbol{r} = \begin{pmatrix} -9 \\ 9 \\ -6 \end{pmatrix} + t\begin{pmatrix} 6 \\ -4 \\ 0 \end{pmatrix}$, $\boldsymbol{r} = -9\boldsymbol{i} + 9\boldsymbol{j} - 6\boldsymbol{k} + s(6\boldsymbol{i} - 4\boldsymbol{j} + 0\boldsymbol{k})$

[2 marks]

(d) correct magnitudes *(A1)(A1)*

eg $\quad \sqrt{(-10)^2 + (-4)^2 + 1^2}$, $\sqrt{6^2 + (-4)^2 + (0)^2}$, $\sqrt{10^2 + 4^2 + 1}$, $\sqrt{6^2 + 4^2}$

$k = \dfrac{\sqrt{117}}{\sqrt{52}}$ $(=1.5)$ (exact) ***A1*** ***N3***

[3 marks]

continued…

Question 10 continued

(e) correct interpretation of relationship between magnitudes ***(A1)***

eg $\text{AB} = 1.5\text{AC}$, $\text{BD} = 1.5\text{AC}$, $\sqrt{117} = \sqrt{52t^2}$

recognizing D can have two positions (may be seen in working) ***R1***

eg $\overrightarrow{\text{BD}} = 1.5\overrightarrow{\text{AC}}$ and $\overrightarrow{\text{BD}} = -1.5\overrightarrow{\text{AC}}$, $t = \pm 1.5$, diagram, two answers

valid approach (seen anywhere) ***(M1)***

eg $\overrightarrow{\text{OD}} = \overrightarrow{\text{OB}} + \overrightarrow{\text{BD}}$, $\begin{pmatrix} -9 \\ 9 \\ -6 \end{pmatrix} + t\begin{pmatrix} 6 \\ -4 \\ 0 \end{pmatrix}$, $\overrightarrow{\text{BD}} = k\begin{pmatrix} 6 \\ -4 \\ 0 \end{pmatrix}$

one correct expression for $\overrightarrow{\text{OD}}$ ***(A1)***

eg $\overrightarrow{\text{OD}} = \begin{pmatrix} -9 \\ 9 \\ -6 \end{pmatrix} + 1.5\begin{pmatrix} 6 \\ -4 \\ 0 \end{pmatrix}$, $\begin{pmatrix} -9 \\ 9 \\ -6 \end{pmatrix} - 1.5\begin{pmatrix} 6 \\ -4 \\ 0 \end{pmatrix}$

$\text{D} = (0,\ 3,\ -6)$, $\text{D} = (-18,\ 15,\ -6)$ (accept position vectors) ***A1A1*** ***N3***

[6 marks]

Total [15 marks]

Markscheme

November 2015

Mathematics

Standard level

Paper 2

13 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a valid **Method**; working must be seen.

(M) Marks awarded for a valid **Method**; may be implied by **correct** subsequent working.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

(A) Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.

R Marks awarded for clear **Reasoning**.

N Marks awarded for **correct** answers if **no** working shown.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Mark according to RM assessor instructions.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any. An exception to this rule is when work for ***M1*** is missing, as opposed to incorrect (see point 4).
- Where ***M*** and ***A*** marks are noted on the same line, *eg* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*eg* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***(M2)***, ***N3***, *etc*., do **not** split the marks, unless there is a note.
- Most ***M*** marks are for a **valid** method, ie a method which can lead to the answer: it must indicate some form of progress towards the answer.
- Once a correct answer to a question or part-question is seen, ignore further correct working. However, if further working indicates a lack of mathematical understanding do not award the final ***A1***.

3 *N* marks

*If **no** working shown, award **N** marks for **correct** answers – this includes acceptable answers (see accuracy booklet). In this case, ignore mark breakdown (**M, A, R**). Where a student only shows a final incorrect answer with no working, even if that answer is a correct intermediate answer, award **N0**.*

- Do **not** award a mixture of ***N*** and other marks.
- There may be fewer ***N*** marks available than the total of ***M***, ***A*** and ***R*** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.

- There may not be a direct relationship between the ***N*** marks and the implied marks. There are times when all the marks are implied, but the ***N*** marks are not the full marks: this indicates that we want to see some of the working, without specifying what.
- For consistency within the markscheme, ***N*** marks are noted for every part, even when these match the mark breakdown.
- If a candidate has incorrect working, which somehow results in a correct answer, do **not** award the ***N*** marks for this correct answer. However, if the candidate has indicated (usually by crossing out) that the working is to be ignored, award the ***N*** marks for the correct answer.

4 Implied and must be seen marks

*Implied marks appear in **brackets** eg **(M1)**.*

- Implied marks can only be awarded if the work is seen or if implied in subsequent working (a correct final answer does not necessarily mean that the implied marks are all awarded). There are questions where some working is required, but as it is accepted that not everyone will write the same steps, all the marks are implied, but the ***N*** marks are not the full marks for the question.
- Normally the correct work is seen in the next line.
- Where there is an (***M1***) followed by ***A1*** for each correct answer, if no working shown, one correct answer is sufficient evidence to award the (***M1***).

*Must be seen marks appear without **brackets** eg **M1**.*

- Must be seen marks can only be awarded if the work is seen.
- If a must be seen mark is not awarded because work is missing (as opposed to ***M0*** or ***A0*** for incorrect work) all subsequent marks may be awarded if appropriate.

5 Follow through marks (only applied after an error is made)

*Follow through (**FT**) marks are awarded where an incorrect answer (final or intermediate) from one **part** of a question is used correctly in **subsequent** part(s) or subpart(s). Usually, to award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if the only marks awarded in a subpart are for the **final** answer, then **FT** marks should be awarded if appropriate. Examiners are expected to check student work in order to award **FT** marks where appropriate.*

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** and ***R*** marks may be awarded if appropriate. (However, as noted above, if an ***A*** mark is not awarded because work is missing, all subsequent marks may be awarded if appropriate).
- Exceptions to this rule will be explicitly noted on the markscheme.
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If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

Candidates are meant to write their answers to Section A on the question paper (QP), and Section B on answer booklets. Sometimes, they need more room for Section A, and use the booklet (and often comment to this effect on the QP), or write outside the box. That is fine, and this work should be marked.

The instructions tell candidates not to write on Section B of the QP. Thus they may well have done some rough work here which they assume will be ignored. If they have solutions on the answer booklets, there is no need to look at the QP. However, if there are whole questions or whole part solutions missing on answer booklets, please check to make sure that they are not on the QP, and if they are, mark those whole questions or whole part solutions that have not been written on answer booklets.

13. Diagrams

The notes on how to allocate marks for sketches usually refer to passing through particular points or having certain features. These marks can only be awarded if the sketch is approximately the correct shape. All values given will be an approximate guide to where these points/features occur. In some questions, the first ***A1*** is for the shape, in others, the marks are only for the points and/or features. In both cases, unless the shape is approximately correct, no marks can be awarded (unless otherwise stated). However, if the graph is based on previous calculations, ***FT*** marks should be awarded if appropriate.

14. Accuracy of Answers

If the level of accuracy is specified in the question, a mark will be allocated for giving the final answer to the required accuracy. When this is not specified in the question, all numerical answers should be given exactly or correct to three significant figures.

Do not accept unfinished numerical final answers such as 3/0.1 (unless otherwise stated). As a rule, numerical answers with more than one part (such as fractions) should be given using integers (*eg* 6/8). Calculations which lead to integers should be completed, with the exception of fractions which are not whole numbers. Intermediate values do not need to be given to the correct three significant figures. But, if candidates work with rounded values, this could lead to an incorrect answer, in which case award ***A0*** for the final answer.

Where numerical answers are required as the **final** answer to a part of a question in the markscheme, the markscheme will show a truncated 6 sf value, the exact value if applicable, the correct 3 sf answer. Units will appear in brackets at the end.

Section A

1. (a) correct substitution *(A1)*
eg $l = 1.3 \times 3$

$l = 3.9$ (cm) ***A1 N2***
[2 marks]

(b) **METHOD 1**

valid approach *(M1)*
eg finding reflex angle, $2\pi - \mathrm{C\hat{O}A}$

correct angle *(A1)*
eg $2\pi - 1.3$, 4.98318

correct substitution *(A1)*
eg $\frac{1}{2}(2\pi - 1.3)3^2$

22.4243
area $= 9\pi - 5.85$ (exact), 22.4 (cm^2) ***A1 N3***

METHOD 2

correct area of small sector *(A1)*
eg $\frac{1}{2}(1.3)3^2$, 5.85

valid approach *(M1)*
eg circle – small sector, $\pi r^2 - \frac{1}{2}\theta r^2$

correct substitution *(A1)*
eg $\pi(3^2) - \frac{1}{2}(1.3)3^2$

22.4243
area $= 9\pi - 5.85$ (exact), 22.4 (cm^2) ***A1 N3***
[4 marks]
Total [6 marks]

2. (a) evidence of using $\sum p_i = 1$ *(M1)*

correct substitution ***A1***
eg $0.15 + k + 0.1 + 2k = 1$, $3k + 0.25 = 1$

$k = 0.25$ ***A1 N2***
[3 marks]

(b) correct substitution *(A1)*
eg $0 \times 0.15 + 1 \times 0.25 + 2 \times 0.1 + 3 \times 0.5$

$\mathrm{E}(X) = 1.95$ ***A1 N2***
[2 marks]
Total [5 marks]

3. (a) valid approach *(M1)*

eg horizontal translation 3 units to the right

$x = 3$ (must be an equation) ***A1 N2***

[2 marks]

(b) valid approach *(M1)*

eg $f(x) = 0$, $e^0 = x - 3$

4, $x = 4$, $(4, 0)$ ***A1 N2***

[2 marks]

(c) attempt to substitute either **their correct** limits or the function into formula involving f^2 *(M1)*

eg $\int_4^{10} f^2$, $\pi\int\left(2\ln\left(x-3\right)\right)^2 \mathrm{d}x$

141.537

volume $= 142$ ***A2 N3***

[3 marks]

Total [7 marks]

4. (a) valid approach *(M1)*

eg $\frac{u_1}{u_2}$, $\frac{4}{1.6}$, $1.6 = r(0.64)$

$r = 2.5 \left(= \frac{5}{2}\right)$ ***A1 N2***

[2 marks]

(b) correct substitution into S_6 *(A1)*

eg $\frac{0.64(2.5^6 - 1)}{2.5 - 1}$

$S_6 = 103.74$ (exact), 104 ***A1 N2***

[2 marks]

(c) **METHOD 1 (analytic)**

valid approach *(M1)*

eg $\frac{0.64(2.5^n - 1)}{2.5 - 1} > 75000$, $\frac{0.64(2.5^n - 1)}{2.5 - 1} = 75000$

correct inequality (accept equation) *(A1)*

eg $n > 13.1803$, $n = 13.2$

$n = 14$ ***A1 N1***

METHOD 2 (table of values)

both crossover values ***A2***

eg $S_{13} = 63577.8$, $S_{14} = 158945$

$n = 14$ ***A1 N1***

[3 marks]

Total [7 marks]

5. (a) $\mathrm{P}(C \cap D) = 2k \times 3k^2$ ***(A1)***

$\mathrm{P}(C \cap D) = 6k^3$ ***A1 N2***

[2 marks]

(b) **their** correct equation ***(A1)***

eg $2k \times 3k^2 = 0.162$, $6k^3 = 0.162$

$k = 0.3$ ***A1 N2***

[2 marks]

(c) **METHOD 1**

finding **their** $\mathrm{P}(C' \cap D)$ (seen anywhere) ***(A1)***

eg 0.4×0.27, $0.27 - 0.162$, 0.108

correct substitution into conditional probability formula ***(A1)***

eg $\mathrm{P}(C' | D) = \dfrac{\mathrm{P}(C' \cap D)}{0.27}$, $\dfrac{(1-2k)(3k^2)}{3k^2}$

$\mathrm{P}(C' | D) = 0.4$ ***A1 N2***

METHOD 2

recognizing $\mathrm{P}(C' | D) = \mathrm{P}(C')$ ***A1***

finding **their** $\mathrm{P}(C') = 1 - \mathrm{P}(C)$ (only if first line seen) ***(A1)***

eg $1 - 2k$, $1 - 0.6$

$\mathrm{P}(C' | D) = 0.4$ ***A1 N2***

[3 marks]

Total [7 marks]

6. (a) recognizing particle at rest when $v = 0$ ***(M1)***

eg $(0.3t + 0.1)^t - 4 = 0$, x-intercept on graph of v

$t = 4.27631$

$t = 4.28$ (seconds) ***A2 N3***

[3 marks]

(b) valid approach to find t when a is 0 ***(M1)***

eg $v'(t) = 0$, v minimum,

$t = 1.19236$

$t = 1.19$ (seconds) ***A2 N3***

[3 marks]

Total [6 marks]

7. (a) correct substitution into chain rule **A2**

eg $f'(x) = \frac{1}{x^2} \times 2x$

$f'(x) = \frac{2}{x}$ **AG** **N0**

[2 marks]

There are many approaches to this question, especially the steps to set up the correct equation, for the two M marks. There are a few processes they may need to apply at some stage, for the ***M1M1***. These include substituting $f'(d)$ and points P and/or Q into the gradient of PQ or equation of the tangent line PQ. There may be other approaches, please check working and award marks in line with markscheme.

(b) at P, $y = \ln\left(d^2\right)$ (seen anywhere) **A1**

gradient of tangent at P is $\frac{2}{d}$ (seen anywhere) **A1**

substituting $(1, -3)$, $(d, \ln d^2)$ or gradient $\frac{2}{d}$ into equation of tangent at P ***(M1)***

eg $y - (-3) = m\left(x - 1\right)$, $y = \left(\frac{2}{d}\right)x + b$, $y - \ln d^2 = m\left(x - d\right)$

second substitution ***(M1)***

eg $y + 3 = \left(\frac{2}{d}\right)(x - 1)$, $-3 = \left(\frac{2}{d}\right)1 + b$, $m = \frac{\ln d^2 + 3}{d - 1}$

any correct equation (in *d* or *x*) **A1**

eg $-3 - \ln\left(d^2\right) = \left(\frac{2}{d}\right)\left(1 - d\right)$, $\ln\left(x^2\right) + 1 + \left(\frac{2}{x}\right) = 0$

-1.30505

$d = -1.31$ (accept $x = -1.31$) **A1** **N2**

[6 marks]

Total [8 marks]

Section B

8. (a) evidence of choosing sine rule *(M1)*

eg $\dfrac{AC}{\sin C\hat{B}A} = \dfrac{AB}{\sin A\hat{C}B}$

correct substitution *(A1)*

eg $\dfrac{AC}{\sin 44^\circ} = \dfrac{15}{\sin 83^\circ}$

10.4981

$AC = 10.5$ (cm) ***A1*** ***N2***

[3 marks]

(b) finding $C\hat{A}B$ (seen anywhere) *(A1)*

eg $180^\circ - 44^\circ - 83^\circ$, $C\hat{A}B = 53^\circ$

correct substitution for area of triangle ABC ***A1***

eg $\dfrac{1}{2} \times 15 \times 10.4981 \times \sin 53^\circ$

62.8813

area = 62.9 (cm^2) ***A1*** ***N2***

[3 marks]

(c) correct substitution for area of triangle DAC *(A1)*

eg $\dfrac{1}{2} \times 6 \times 10.4981 \times \sin\theta$

attempt to equate area of triangle ACD to half the area of triangle ABC *(M1)*

eg area $ACD = \dfrac{1}{2} \times$ area ABC; $2ACD = ABC$

correct equation ***A1***

eg $\dfrac{1}{2} \times 6 \times 10.4981 \times \sin\theta = \dfrac{1}{2}(62.9)$, $62.9887\sin\theta = 62.8813$, $\sin\theta = 0.998294$

$86.6531, 93.3468$

$\theta = 86.7^\circ$, $\theta = 93.3^\circ$ ***A1A1*** ***N2***

[5 marks]

(d)

Note: Note: If candidates use an acute angle from part (c) in the cosine rule , award ***M1A0A0*** in part (d).

evidence of choosing cosine rule *(M1)*

eg $CD^2 = AD^2 + AC^2 - 2 \times AD \times AC \times \cos\theta$

correct substitution into rhs *(A1)*

eg $CD^2 = 6^2 + 10.498^2 - 2(6)(10.498)\cos 93.336^\circ$

12.3921

12.4 (cm) ***A1*** ***N2***

[3 marks]

Total [14 marks]

9. (a) evidence of setup *(M1)*

eg correct value for a or b

$13.3823, 137.482$

$a = 13.4$, $b = 137$ *A1A1* *N3*

[3 marks]

(b) correct substitution into **their** regression equation

eg $13.3823 \times 7 + 137.482$ *(A1)*

correct calculation

231.158 *(A1)*

231 (coyotes) (must be an integer) *A1* *N2*

[3 marks]

(c) recognizing $t = 0$ *(M1)*

eg $f(0)$

correct substitution into the model

eg $\frac{2000}{1+99e^{-k(0)}}$, $\frac{2000}{100}$ *(A1)*

20 (foxes) *A1* *N2*

[3 marks]

(d) recognizing $(5, 64)$ satisfies the equation *(M1)*

eg $f(5) = 64$

correct substitution into the model

eg $64 = \frac{2000}{1+99e^{-k(5)}}$, $64\left(1+99e^{-5k}\right) = 2000$ *(A1)*

0.237124

$k = -\frac{1}{5}\ln\left(\frac{11}{36}\right)$ (exact), 0.237 *A1* *N2*

[3 marks]

(e) valid approach *(M1)*

eg $c = f$, sketch of graphs

correct working *(A1)*

eg $\frac{2000}{1+99e^{-0.237124t}} = 13.382t + 137.482$, sketch of graphs, table of values

$t = 12.0403$ *(A1)*

2007 *A1* *N2*

Note: Exception to the ***FT*** rule. Award ***A1FT*** on their value of *t*.

[4 marks]

Total [16 marks]

10. (a) finding standardized value for $4\,\text{kg}$ (seen anywhere) ***(A1)***

eg $z = -1.64485$

attempt to standardize ***(M1)***

eg $\sigma = \dfrac{x-\mu}{z}$, $\dfrac{4-10}{\sigma}$

correct substitution ***(A1)***

eg $-1.64 = \dfrac{4-10}{\sigma}$, $\dfrac{4-10}{-1.64}$

$\sigma = 3.64774$

$\sigma = 3.65$ ***A1*** ***N2***

[4 marks]

(b) valid approach ***(M1)***

eg $1-p$, 0.62, $\dfrac{w-10}{3.65} = 0.305$

$w = 11.1143$

$w = 11.1$ ***A1*** ***N2***

[2 marks]

(c) attempt to restrict melon population ***(M1)***

eg 95% are delivered, P(medium | delivered), 57 + 38

correct probability for medium watermelons ***(A1)***

eg $\dfrac{0.57}{0.95}$

$\dfrac{57}{95}$, 0.6, 60% ***A1*** ***N3***

[3 marks]

(d) proportion of large watermelons (seen anywhere) ***(A1)***

eg P(large) = 0.4, 40%

correct approach to find total sales (seen anywhere) ***(A1)***

eg 150 = sales – 300, total sales = $450

correct expression ***(A1)***

eg $1.75(0.6x)+3(0.4x)$, $1.75(0.6)+3(0.4)$

evidence of correct working ***(A1)***

eg $1.75(0.6x)+3(0.4x)=450$, $2.25x=450$

200 watermelons in the delivery ***A1*** ***N2***

[5 marks]

Total [14 marks]

Markscheme

May 2015

Mathematics

Standard level

Paper 2

17 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a valid **Method**; working must be seen.

(M) Marks awarded for a valid **Method**; may be implied by **correct** subsequent working.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

(A) Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.

R Marks awarded for clear **Reasoning**.

N Marks awarded for **correct** answers if **no** working shown.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Mark according to RM assessor instructions and the document "**Mathematics SL: Guidance for e-marking May 2015**". It is **essential** that you read this document before you start marking. In particular, please note the following. Marks must be recorded using the annotation stamps, using the RM assessor tool. Please check that you are entering marks for the right question. All the marks will be added and recorded by RM assessor.

If a part is **completely correct**, (and gains all the "must be seen" marks), use the ticks with numbers to stamp full marks. Do **not** use the ticks with numbers for anything else.
- If a part is completely wrong, stamp ***A0*** by the final answer.
- If a part gains anything else, all the working **must** have annotations stamped to show what marks are awarded. This includes any zero marks.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any. An exception to this rule is when work for ***M1*** is missing, as opposed to incorrect (see point 4).
- Where ***M*** and ***A*** marks are noted on the same line, *eg* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*eg* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***(M2)***, ***N3***, *etc*., do **not** split the marks, unless there is a note.
- Most ***M*** marks are for a **valid** method, ie a method which can lead to the answer: it must indicate some form of progress towards the answer.
- Once a correct answer to a question or part-question is seen, ignore further correct working. However, if further working indicates a lack of mathematical understanding do not award the final ***A1***. An exception to this may be in numerical answers, where a correct exact value is followed by an incorrect decimal (see examples on next page).

Examples

	Correct answer seen	Further working seen	Action
1.	$8\sqrt{2}$	5.65685 *(incorrect decimal value)*	Award the final ***A1*** *(ignore the further working)*
2.	$\frac{1}{4}\sin 4x$	$\sin x$	Do not award the final ***A1***
3.	$\log a - \log b$	$\log(a-b)$	Do not award the final ***A1***

3 *N* marks

*If **no** working shown, award **N** marks for **correct** answers – this includes acceptable answers (see accuracy booklet). In this case, ignore mark breakdown (**M**, **A**, **R**). Where a student only shows a final incorrect answer with no working, even if that answer is a correct intermediate answer, award **N0**.*

- Do **not** award a mixture of ***N*** and other marks.
- There may be fewer ***N*** marks available than the total of ***M***, ***A*** and ***R*** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.
- There may not be a direct relationship between the ***N*** marks and the implied marks. There are times when all the marks are implied, but the ***N*** marks are not the full marks: this indicates that we want to see some of the working, without specifying what.
- For consistency within the markscheme, ***N*** marks are noted for every part, even when these match the mark breakdown.
- If a candidate has incorrect working, which somehow results in a correct answer, do **not** award the ***N*** marks for this correct answer. However, if the candidate has indicated (usually by crossing out) that the working is to be ignored, award the ***N*** marks for the correct answer.

4 Implied and must be seen marks

*Implied marks appear in **brackets** eg **(M1)**.*

- Implied marks can only be awarded if the work is seen or if implied in subsequent working (a correct final answer does not necessarily mean that the implied marks are all awarded). There are questions where some working is required, but as it is accepted that not everyone will write the same steps, all the marks are implied, but the ***N*** marks are not the full marks for the question.
- Normally the correct work is seen in the next line.
- Where there is an (***M1***) followed by ***A1*** for each correct answer, if no working shown, one correct answer is sufficient evidence to award the (***M1)***.

*Must be seen marks appear without **brackets** eg **M1***.

- Must be seen marks can only be awarded if the work is seen.
- If a must be seen mark is not awarded because work is missing (as opposed to ***M0*** or ***A0*** for incorrect work) all subsequent marks may be awarded if appropriate.

5 Follow through marks (only applied after an error is made)

*Follow through (**FT**) marks are awarded where an incorrect answer (final or intermediate) from one **part** of a question is used correctly in **subsequent** part(s) or subpart(s). Usually, to award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if the only marks awarded in a subpart are for the **final** answer, then **FT** marks should be awarded if appropriate. Examiners are expected to check student work in order to award **FT** marks where appropriate.*

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** and ***R*** marks may be awarded if appropriate. (However, as noted above, if an ***A*** mark is not awarded because work is missing, all subsequent marks may be awarded if appropriate).
- Exceptions to this rule will be explicitly noted on the markscheme.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*eg* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word "**their**" in a description, to indicate that candidates may be using an incorrect value.
- If a candidate makes an error in one part, but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the question says hence. It is often possible to use a different approach in subsequent parts that does not depend on the answer to previous parts.
- In a "show that" question, if an error in a previous subpart leads to not showing the required answer, do not award the final ***A1***. Note that if the error occurs within the same subpart, the ***FT*** rules may result in further loss of marks.
- Where there are anticipated common errors, the ***FT*** answers are often noted on the markscheme, to help examiners. It should be stressed that these are not the only ***FT*** answers accepted, neither should ***N*** marks be awarded for these answers.

6 Mis-read

*If a candidate incorrectly copies information from the question, this is a mis-read (**MR**). A candidate should be penalized only once for a particular mis-read. Use the **MR** stamp to indicate that this is a misread. Do not award the first mark in the question, even if this is an **M** mark, but award all others (if appropriate) so that the candidate only loses one mark for the misread.*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*eg* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.

7 Discretionary marks *(d)*

*An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. In such cases the annotation DM should be used and a brief **note** written next to the mark explaining this decision.*

8 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete parts are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**. Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9 Alternative forms

Unless the question specifies otherwise, ***accept*** *equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

10 Calculators

A GDC is required for paper 2, but calculators with symbolic manipulation features (eg TI-89) are not allowed.

Calculator notation The mathematics SL guide says:
Students must always use correct mathematical notation, not calculator notation.
Do **not** accept final answers written using calculator notation. However, do not penalize the use of calculator notation in the working.

11 Style

The markscheme aims to present answers using good communication, eg if the question asks to find the value of k, the markscheme will say $k = 3$, but the marks will be for the correct value 3 – there is usually no need for the "$k =$". In these cases, it is also usually acceptable to have another variable, as long as there is no ambiguity in the question, eg if the question asks to find the value of p and of q, then the student answer needs to be clear. Generally, the only situation where the full answer is required is in a question which asks for equations – in this case the markscheme will say "must be an equation". Accept sloppy notation in the working, where this is followed by correct working eg $-2^2 = 4$ where they should have written $(-2)^2 = 4$.

The markscheme often uses words to describe what the marks are for, followed by examples, using the eg notation. These examples are not exhaustive, and examiners should check what candidates have written, to see if they satisfy the description. Where these marks are ***M*** *marks, the examples may include ones using poor notation, to indicate what is acceptable. A valid method is one which will allow candidate to proceed to the next step eg if a quadratic function is given in factorised form, and the question asks for the zeroes, then multiplying the factors does not necessarily help to find the zeros, and would not on its own count as a valid method.*

12 Candidate work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

Candidates are meant to write their answers to Section A on the question paper (QP), and Section B on answer booklets. Sometimes, they need more room for Section A, and use the booklet (and often comment to this effect on the QP), or write outside the box. That is fine, and this work should be marked.

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Intermediate values do not need to be given to the correct three significant figures. But, if candidates work with rounded values, this could lead to an incorrect answer, in which case award ***A0*** for the final answer.

Where numerical answers are required as the **final** answer to a part of a question in the markscheme, the markscheme will show

a truncated 6 sf value, the exact value if applicable, and the correct 3 sf answer.

Units (which are generally not required) will appear in brackets at the end.

Section A

1. (a) (i) evidence of valid approach ***(M1)***
eg 1 correct value for r, (or for a or b, seen in (ii))

0.946591
$\mathbf{r = 0.947}$ ***A1*** ***N2***

(ii) $a = 0.500957$, $b = 0.803544$
$\mathbf{a = 0.501, b = 0.804}$ ***A1A1*** ***N2***
[4 marks]

(b) substituting $x = 3.7$ into **their** equation ***(M1)***
eg $0.501(3.7) + 0.804$

2.65708 (2 hours 39.4252 minutes) ***(A1)***
$y = 2.7$ (hours)(**must** be correct 1 dp, accept 2 hours 39.4 minutes) ***A1*** ***N3***
[3 marks]

Total [7marks]

2. (a) 9 terms ***A1*** ***N1***
[1 mark]

(b) valid approach to find the required term ***(M1)***
eg $\binom{8}{r}(2x)^{8-r}(3)^r$, $(2x)^8(3)^0 + (2x)^7(3)^1 + \ldots$, Pascal's triangle to 8th row

identifying correct term (may be indicated in expansion) ***(A1)***
eg 6th term, $r = 5$, $\binom{8}{5}$, $(2x)^3(3)^5$

correct working (may be seen in expansion) ***(A1)***
eg $\binom{8}{5}(2x)^3(3)^5$, $56 \times 2^3 \times 3^5$

$108864x^3$ (accept $109000x^3$) ***A1*** ***N3***

[4 marks]

Notes: Do not award any marks if there is clear evidence of adding instead of multiplying.
Do not award final ***A1*** for a final answer of 108864, even if $108864x^3$ is seen previously.
If no working shown award ***N2*** for 108864.

Total [5 marks]

3. (a) $d = -1.5$ ***A1*** ***N1***
[1 mark]

(b) **METHOD 1**

valid approach ***(M1)***
eg $u_{10} = u_1 + 9d$, $8 = u_1 - 9(-1.5)$

correct working ***(A1)***
eg $8 = u_1 + 9d$, $6.5 = u_1 + 10d$, $u_1 = 8 - 9(-1.5)$

$u_1 = 21.5$ ***A1*** ***N2***

METHOD 2

attempt to list 3 or more terms in either direction ***(M1)***
eg $9.5, 11, 12.5, \ldots ; \ 5, 3.5, 2, \ldots \ \ldots$

correct list of 4 or more terms in **correct** direction ***(A1)***
eg 9.5 , 11 , 12.5 , 14

$u_1 = 21.5$ ***A1*** ***N2***
[3 marks]

(c) correct expression ***(A1)***
eg $\frac{50}{2}\left(2(21.5) + 49(-1.5)\right)$, $\frac{50}{2}(21.5 - 52)$, $\sum_{k=1}^{50} 21.5 + (k-1)(-1.5)$

$\text{sum} = -762.5$ **(exact)** ***A1*** ***N2***
[2 marks]

Total [6 marks]

4. (a) (i) valid approach ***(M1)***
eg sketch, $f(x) = 0$, $0 = 2x - 6$

$x = 3$ or $(3, 0)$ ***A1*** ***N2***

(ii) $x = 1$ (must be equation) ***A1*** ***N1***

(iii) valid approach ***(M1)***
eg sketch, $\frac{2x}{-1x}$, inputting large values of x , L'Hopital's rule

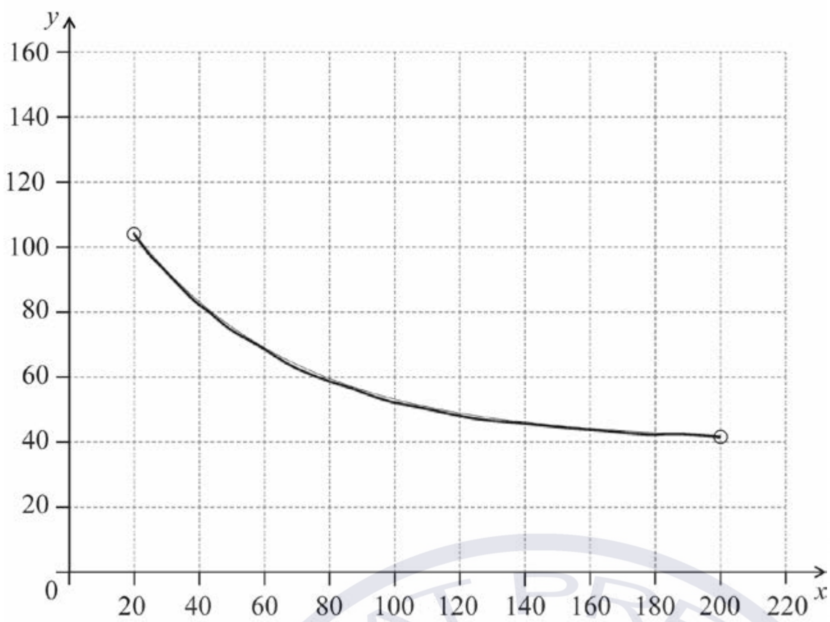
$y = -2$ (must be equation) ***A1*** ***N2***
[5 marks]

(b) valid approach ***(M1)***
eg recognizing that $\lim_{x\to\infty}$ is related to the horizontal asymptote,
table with large values of x , their y value from (a)(iii), L'Hopital's rule

$\lim_{x\to\infty} f(x) = -2$ ***A1*** ***N2***

[2 marks]
Total [7 marks]

5. (a)



A1A1A1 ***N3***

Note: Curve must be approximately correct exponential shape (concave up and decreasing). Only if the shape is approximately correct, award the following:
A1 for left endpoint in circle,
A1 for right endpoint in circle,
A1 for asymptotic to $y = 40$ (must not go below $y = 40$).

[3 marks]

(b) attempt to find $G(45)$ ***(M1)***

eg 78.6241, value read from **their** graph

multiplying cost times number of people ***(M1)***

eg 45×78.6241, $G(45) \times 45$

3538.08

3540 (dollars) ***A1*** ***N2***

[3 marks]

Total [6 marks]

6. recognizing that the gradient of tangent is the derivative *(M1)*

eg f'

finding the gradient of f at P *(A1)*

eg $f'(0.25) = 16$

evidence of taking negative reciprocal of **their** gradient at P *(M1)*

eg $\dfrac{-1}{m}$, $-\dfrac{1}{f'(0.25)}$

equating derivatives **M1**

eg $f'(x) = \dfrac{-1}{16}$, $f' = -\dfrac{1}{m}$, $\dfrac{x\left(\frac{1}{x}\right) - \ln(4x)}{x^2} = 16$

finding the x-coordinate of Q, $x = 0.700750$

$\boldsymbol{x = 0.701}$ **A1** **N3**

attempt to substitute **their** x into f to find the y-coordinate of Q *(M1)*

eg $f(0.7)$

$y = 1.47083$

$\boldsymbol{y = 1.47}$ **A1** **N2**

[7 marks]

7. (a) $(-0.3, -0.967)$

$x = -0.3$ (exact), $y = -0.967$ (exact) **A1A1** **N2**

[2 marks]

(b) y-coordinate of local maximum is $y = 11.2$ *(A1)*

negating the y-coordinate of one of the max/min *(M1)*

eg $y = 0.967$, $y = -11.2$

recognizing that the solution set has two intervals **R1**

eg two answers,

$k < -11.2$, $k > 0.967$ **A1A1** **N3N2**

[5 marks]

Notes: If working shown, do not award the final mark if strict inequalities are not used.
If no working shown, award ***N2*** for $k \le -11.2$ or ***N1*** for $k \ge 0.967$

Total [7 marks]

Section B

8. (a) valid approach *(M1)*

eg $\text{speed} = \frac{\text{distance}}{\text{time}}$, 6×1.5

$\text{SL} = 9 \ \ (\text{km})$ *A1* *N2*

[2 marks]

(b) evidence of choosing sine rule *(M1)*

eg $\frac{\sin A}{a} = \frac{\sin B}{b}$, $\sin\theta = \frac{(\text{SL})\sin 20^\circ}{5}$

correct substitution *(A1)*

eg $\frac{\sin\theta}{9} = \frac{\sin 20^\circ}{5}$

37.9981

$\text{S}\hat{\text{P}}\text{L} = 38.0^\circ$ *A1* *N2*

recognition that second angle is the supplement of first *(M1)*

eg $180 - x$

142.001

$\text{S}\hat{\text{Q}}\text{L} = 142^\circ$ *A1* *N2*

[5 marks]

continued…

Question 8 continued

(c) (i) new store is at Q **A1** **N1**

(ii) **METHOD 1**

attempt to find third angle **(M1)**

eg $\mathrm{S\hat{L}P} = 180 - 20 - 38$, $\mathrm{S\hat{L}Q} = 180 - 20 - 142$

$\mathrm{S\hat{L}Q} = 17.998^\circ$ (seen anywhere) **A1**

evidence of choosing sine rule or cosine rule **(M1)**
correct substitution into sine rule or cosine rule **(A1)**

eg $\dfrac{x}{\sin 17.998} = \dfrac{5}{\sin 20}\left(= \dfrac{9}{\sin 142}\right)$, $9^2 + 5^2 - 2(9)(5)\cos 17.998^\circ$

$4.51708\,\mathrm{km}$
4.52 (km) **A1** **N3**

METHOD 2

evidence of choosing cosine rule **(M1)**
correct substitution into cosine rule **A1**

eg $9^2 = x^2 + 5^2 - 2(x)(5)\cos 142^\circ$

attempt to solve **(M1)**
eg sketch; setting quadratic equation equal to zero;
$0 = x^2 + 7.88x - 56$

one correct value for *x* **(A1)**
eg $x = -12.3973$, $x = 4.51708$

4.51708
4.52 (km) **A1** **N3**

[6 marks]

Total [13 marks]

9. (a) 0.0477903

probability $= 0.0478$ *A2* *N2*

[2 marks]

(b) $\text{P}(\text{volume} < 250) = 0.02$ *(M1)*

$z = -2.05374$ (may be seen in equation) *A1*

attempt to set up equation with z *(M1)*

eg $\dfrac{\mu - 260}{\sigma} = z$, $260 - 2.05(\sigma) = 250$

4.86914

$\sigma = 4.87$ (ml) *A1* *N3*

[4 marks]

(c) (i) 0.968062

$\text{P}(250 < \text{Vol} < 271) = 0.968$ *A2* *N2*

(ii) recognizing conditional probability (seen anywhere, including in correct working) *R1*

eg $\text{P}\left(A \middle| B\right), \dfrac{\text{P}(A \cap B)}{\text{P}(B)}, \ \text{P}(A \cap B) = \text{P}\left(A \middle| B\right)\text{P}(B)$

correct value or expression for P(not underfilled) *(A1)*

eg $0.98, 1 - 0.02, 1 - P(X < 250)$

probability $= \dfrac{0.968}{0.98}$ *A1*

0.987818

probability $= 0.988$ *A1* *N2*

[6 marks]

continued…

Question 9 continued

(d) **METHOD 1**

evidence of recognizing binomial distribution (seen anywhere) ***(M1)***
eg $X \quad \mathrm{B}(50, 0.968)$, binomial cdf, $p = 0.968$, $r = 47$

$\mathrm{P}(X \le 47) = 0.214106$ ***(A1)***

evidence of using complement ***(M1)***
eg $1 - \mathrm{P}(X \le 47)$

0.785894
probability $= 0.786$ ***A1 N3***

METHOD 2

evidence of recognizing binomial distribution (seen anywhere) ***(M1)***
eg $X \quad \mathrm{B}(50, 0.968)$, binomial cdf, $p = 0.968$, $r = 47$

$\mathrm{P}(\text{not pass}) = 1 - \mathrm{P}(\text{pass}) = 0.0319378$ ***(A1)***
evidence of attempt to find P $(2$ or fewer fail$)$ ***(M1)***
eg 0, 1, or 2 not pass, $\mathrm{B}(50, 2)$

0.785894
probability $= 0.786$ ***A1 N3***

METHOD 3

evidence of recognizing binomial distribution (seen anywhere) ***(M1)***
eg $X \quad \mathrm{B}(50, 0.968)$, binomial cdf, $p = 0.968$, $r = 47$

evidence of summing probabilities ***(M1)***
eg $\mathrm{P}(X = 48) + \mathrm{P}(X = 49) + \mathrm{P}(X = 50)$

correct working
eg $0.263088 + 0.325488 + 0.197317$ ***(A1)***

0.785894
probability $= 0.786$ ***A1 N3***

[4 marks]

Total [16 marks]

10.	(a)	$p = 6$	***A1***	***N1***
		recognising that turning points occur when $f'(x) = 0$	***R1***	***N1***
		eg correct sign diagram		
		f' changes from positive to negative at $x = 6$	***R1***	***N1***
				[3 marks]
	(b)	$f'(2) = -2$	***A1***	***N1***
				[1 mark]
	(c)	attempt to apply chain rule	***(M1)***	
		eg $\ln(x)' \times f'(x)$		
		correct expression for $g'(x)$	***(A1)***	
		eg $g'(x) = \frac{1}{f(x)} \times f'(x)$		
		substituting $x = 2$ into **their** g'	***(M1)***	
		eg $\frac{f'(2)}{f(2)}$		
		-0.666667		
		$\mathbf{g'(2) = -\frac{2}{3}\ (exact), -0.667}$	***A1***	***N3***
				[4 marks]
	(d)	evidence of integrating $g'(x)$	***(M1)***	
		eg $g(x)\Big\vert_2^a$, $g(x)\Big\vert_a^2$		
		applying the fundamental theorem of calculus (seen anywhere)	***R1***	
		eg $\int_2^a g'(x) = g(a) - g(2)$		
		correct substitution into integral	***(A1)***	
		eg $\ln 3 + g(a) - g(2)$, $\ln 3 + g(a) - \ln\left(f(2)\right)$		
		$\ln 3 + g(a) - \ln 3$	***A1***	
		$\ln 3 + \int_2^a g'(x) = g(a)$	***AG***	***N0***
				[4 marks]

continued…

Question 10 continued

(e) **METHOD 1**

substituting $a = 5$ into the formula for $g(a)$ ***(M1)***

eg $\int_2^5 g'(x)\,\mathrm{d}x$, $g(5) = \ln 3 + \int_2^5 g'(x)\,\mathrm{d}x$ (do not accept only $g(5)$)

attempt to substitute areas ***(M1)***

eg $\ln 3 + 0.66 - 0.21$, $\ln 3 + 0.66 + 0.21$

correct working ***(A1)***

eg $g(5) = \ln 3 + (-0.66 + 0.21)$

0.648612

$g(5) = \ln 3 - 0.45$ (exact), $\mathbf{0.649}$ ***A1 N3***

METHOD 2

attempt to set up an equation for one shaded region ***(M1)***

eg $\int_4^5 g'(x)\,\mathrm{d}x = 0.21$, $\int_2^4 g'(x)\,\mathrm{d}x = -0.66$, $\int_2^5 g'(x)\,\mathrm{d}x = -0.45$

two correct equations ***(A1)***

eg $g(5) - g(4) = 0.21$, $g(2) - g(4) = 0.66$

combining equations to eliminate $g(4)$ ***(M1)***

eg $g(5) - [\ln 3 - 0.66] = 0.21$

0.648612

$g(5) = \ln 3 - 0.45$ (exact), $\mathbf{0.649}$ ***A1 N3***

METHOD 3

attempt to set up a definite integral ***(M1)***

eg $\int_2^5 g'(x)\,\mathrm{d}x = -0.66 + 0.21$, $\int_2^5 g'(x)\,\mathrm{d}x = -0.45$

correct working ***(A1)***

eg $g(5) - g(2) = -0.45$

correct substitution ***(A1)***

eg $g(5) - \ln 3 = -0.45$

0.648612

$g(5) = \ln 3 - 0.45$ (exact), $\mathbf{0.649}$ ***A1 N3***

[4 marks]

Total [16 marks]

M15/5/MATME/SP2/ENG/TZ2/XX/M

MARKSCHEME

May 2015

MATHEMATICS

Standard level

Paper 2

18 pages

Instructions to Examiners (red changed since M13)

Abbreviations

M Marks awarded for attempting to use a valid **Method**; working must be seen.

(M) Marks awarded for a valid **Method**; may be implied by **correct** subsequent working.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

(A) Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.

R Marks awarded for clear **Reasoning**.

N Marks awarded for **correct** answers if **no** working shown.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Mark according to RM assessor instructions and the document "**Mathematics SL: Guidance for e-marking May 2015**". It is **essential** that you read this document before you start marking. In particular, please note the following. Marks must be recorded using the annotation stamps, using the RM assessor tool. Please check that you are entering marks for the right question. All the marks will be added and recorded by RM assessor.

If a part is **completely correct**, (and gains all the "must be seen" marks), use the ticks with numbers to stamp full marks. Do **not** use the ticks with numbers for anything else.

- If a part is completely wrong, stamp ***A0*** by the final answer.
- If a part gains anything else, all the working **must** have annotations stamped to show what marks are awarded. This includes any zero marks.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any. An exception to this rule is when work for ***M1*** is missing, as opposed to incorrect (see point 4).
- Where ***M*** and ***A*** marks are noted on the same line, *eg* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*eg* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***(M2)***, ***N3***, *etc*., do **not** split the marks, unless there is a note.
- Most ***M*** marks are for a **valid** method, ie a method which can lead to the answer: it must indicate some form of progress towards the answer.
- Once a correct answer to a question or part-question is seen, ignore further correct working. However, if further working indicates a lack of mathematical understanding do not award the final ***A1***. An exception to this may be in numerical answers, where a correct exact value is followed by an incorrect decimal (see examples on next page).

Examples

	Correct answer seen	Further working seen	Action
1.	$8\sqrt{2}$	5.65685 *(incorrect decimal value)*	Award the final ***A1*** *(ignore the further working)*
2.	$\frac{1}{4}\sin 4x$	$\sin x$	Do not award the final ***A1***
3.	$\log a - \log b$	$\log(a-b)$	Do not award the final ***A1***

3 *N* marks

*If **no** working shown, award **N** marks for **correct** answers – this includes acceptable answers (see accuracy booklet). In this case, ignore mark breakdown (**M**, **A**, **R**). Where a student only shows a final incorrect answer with no working, even if that answer is a correct intermediate answer, award **N0**.*

- Do **not** award a mixture of ***N*** and other marks.
- There may be fewer ***N*** marks available than the total of ***M***, ***A*** and ***R*** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.
- There may not be a direct relationship between the ***N*** marks and the implied marks. There are times when all the marks are implied, but the ***N*** marks are not the full marks: this indicates that we want to see some of the working, without specifying what.
- For consistency within the markscheme, ***N*** marks are noted for every part, even when these match the mark breakdown.
- If a candidate has incorrect working, which somehow results in a correct answer, do **not** award the ***N*** marks for this correct answer. However, if the candidate has indicated (usually by crossing out) that the working is to be ignored, award the ***N*** marks for the correct answer.

4 Implied and must be seen marks

*Implied marks appear in **brackets** eg **(M1)**.*

- Implied marks can only be awarded if the work is seen or if implied in subsequent working (a correct final answer does not necessarily mean that the implied marks are all awarded). There are questions where some working is required, but as it is accepted that not everyone will write the same steps, all the marks are implied, but the ***N*** marks are not the full marks for the question.
- Normally the correct work is seen in the next line.
- Where there is an (***M1***) followed by ***A1*** for each correct answer, if no working shown, one correct answer is sufficient evidence to award the (***M1)***.

*Must be seen marks appear without **brackets** eg **M1**.*

- Must be seen marks can only be awarded if the work is seen.
- If a must be seen mark is not awarded because work is missing (as opposed to ***M0*** or ***A0*** for incorrect work) all subsequent marks may be awarded if appropriate.

5 Follow through marks (only applied after an error is made)

*Follow through (**FT**) marks are awarded where an incorrect answer (final or intermediate) from one **part** of a question is used correctly in **subsequent** part(s) or subpart(s). Usually, to award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if the only marks awarded in a subpart are for the **final** answer, then **FT** marks should be awarded if appropriate. Examiners are expected to check student work in order to award **FT** marks where appropriate.*

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** and ***R*** marks may be awarded if appropriate. (However, as noted above, if an ***A*** mark is not awarded because work is missing, all subsequent marks may be awarded if appropriate).
- Exceptions to this rule will be explicitly noted on the markscheme.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*eg* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word "**their**" in a description, to indicate that candidates may be using an incorrect value.
- If a candidate makes an error in one part, but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the question says hence. It is often possible to use a different approach in subsequent parts that does not depend on the answer to previous parts.
- In a "show that" question, if an error in a previous subpart leads to not showing the required answer, do not award the final ***A1***. Note that if the error occurs within the same subpart, the ***FT*** rules may result in further loss of marks.
- Where there are anticipated common errors, the ***FT*** answers are often noted on the markscheme, to help examiners. It should be stressed that these are not the only ***FT*** answers accepted, neither should ***N*** marks be awarded for these answers.

6 Mis-read

*If a candidate incorrectly copies information from the question, this is a mis-read (**MR**). A candidate should be penalized only once for a particular mis-read. Use the **MR** stamp to indicate that this is a misread. Do not award the first mark in the question, even if this is an **M** mark, but award all others (if appropriate) so that the candidate only loses one mark for the misread.*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*eg* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.

7 Discretionary marks *(d)*

*An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. In such cases the annotation DM should be used and a brief **note** written next to the mark explaining this decision.*

8 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete parts are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**. Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9 Alternative forms

*Unless the question specifies otherwise, **accept** equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

10 Calculators

A GDC is required for paper 2, but calculators with symbolic manipulation features (eg TI-89) are not allowed.

Calculator notation The mathematics SL guide says:
Students must always use correct mathematical notation, not calculator notation.
Do **not** accept final answers written using calculator notation. However, do not penalize the use of calculator notation in the working.

11 Style

The markscheme aims to present answers using good communication, eg if the question asks to find the value of k, the markscheme will say $k = 3$, but the marks will be for the correct value 3 – there is usually no need for the "$k =$". In these cases, it is also usually acceptable to have another variable, as long as there is no ambiguity in the question, eg if the question asks to find the value of p and of q, then the student answer needs to be clear. Generally, the only situation where the full answer is required is in a question which asks for equations – in this case the markscheme will say "must be an equation". Accept sloppy notation in the working, where this is followed by correct working eg $-2^2 = 4$ where they should have written $(-2)^2 = 4$.

*The markscheme often uses words to describe what the marks are for, followed by examples, using the eg notation. These examples are not exhaustive, and examiners should check what candidates have written, to see if they satisfy the description. Where these marks are **M** marks, the examples may include ones using poor notation, to indicate what is acceptable. A valid method is one which will allow candidate to proceed to the next step eg if a quadratic function is given in factorised form, and the question asks for the zeroes, then multiplying the factors does not necessarily help to find the zeros, and would not on its own count as a valid method.*

12 Candidate work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

Candidates are meant to write their answers to Section A on the question paper (QP), and Section B on answer booklets. Sometimes, they need more room for Section A, and use the booklet (and often comment to this effect on the QP), or write outside the box. That is fine, and this work should be marked.

The instructions tell candidates not to write on Section B of the QP. Thus they may well have done some rough work here which they assume will be ignored. If they have solutions on the answer booklets, there is no need to look at the QP. However, if there are whole questions or whole part solutions missing on answer booklets, please check to make sure that they are not on the QP, and if they are, mark those whole questions or whole part solutions that have not been written on answer booklets.

13. Diagrams

The notes on how to allocate marks for sketches usually refer to passing through particular points or having certain features. These marks can only be awarded if the sketch is approximately the correct shape. All values given will be an approximate guide to where these points/features occur. In some questions, the first ***A1*** is for the shape, in others, the marks are only for the points and/or features. In both cases, unless the shape is approximately correct, no marks can be awarded (unless otherwise stated). However, if the graph is based on previous calculations, ***FT*** marks should be awarded if appropriate.

14. Accuracy of Answers

If the level of accuracy is specified in the question, a mark will be allocated for giving the final answer to the required accuracy. When this is not specified in the question, all numerical answers should be given exactly or correct to three significant figures.

Do not accept unfinished numerical final answers such as 3/0.1 (unless otherwise stated). As a rule, numerical answers with more than one part (such as fractions) should be given using integers (*eg* 6/8). Calculations which lead to integers should be completed, with the exception of fractions which are not whole numbers.

Intermediate values do not need to be given to the correct three significant figures. But, if candidates work with rounded values, this could lead to an incorrect answer, in which case award ***A0*** for the final answer.

Where numerical answers are required as the **final** answer to a part of a question in the markscheme, the markscheme will show

a truncated 6 sf value, the exact value if applicable, and the correct 3 sf answer.

Units (which are generally not required) will appear in brackets at the end.

Section A

1. (a) evidence of choosing sine rule *(M1)*

eg $\dfrac{\text{AC}}{\sin\left(\text{A}\hat{\text{B}}\text{C}\right)} = \dfrac{\text{BC}}{\sin\left(\text{B}\hat{\text{A}}\text{C}\right)}$

correct substitution *(A1)*

eg $\dfrac{\text{AC}}{\sin 80^\circ} = \dfrac{10}{\sin 35^\circ}$

$\text{AC} = 17.1695$

$\text{AC} = 17.2 \quad (\text{cm})$ ***A1*** ***N2***

[3 marks]

(b) $\text{A}\hat{\text{C}}\text{B} = 65^\circ$ (seen anywhere) *(A1)*

correct substitution *(A1)*

eg $\dfrac{1}{2} \times 10 \times 17.1695 \times \sin 65^\circ$

$\text{area} = 77.8047$

$\text{area} = 77.8 \quad (\text{cm}^2)$ ***A1*** ***N2***

[3 marks]

Total [6 marks]

2. (a) (i) correct substitution *(A1)*

eg $6 \times 2 + 3 \times 2 + 6 \times 1$

$\boldsymbol{u} \bullet \boldsymbol{v} = 24$ ***A1*** ***N2***

(ii) correct substitution into magnitude formula for $\boldsymbol{u}$ or $\boldsymbol{v}$ *(A1)*

eg $\sqrt{6^2 + 3^2 + 6^2}$, $\sqrt{2^2 + 2^2 + 1^2}$, correct value for $|v|$

$|\boldsymbol{u}| = 9$ ***A1*** ***N2***

(iii) $|\boldsymbol{v}| = 3$ ***A1*** ***N1***

[5 marks]

(b) correct substitution into angle formula *(A1)*

eg $\dfrac{24}{9 \times 3}$, $0.\bar{8}$

$0.475882,\ 27.26604^\circ$ ***A1*** ***N2***

0.476, 27.3º

[2 marks]

Total [7 marks]

3. (a) (i) evidence of set up *(M1)*

eg correct value for a, b or r

$a = 4.8$, $b = 1.2$ ***A1A1*** ***N3***

(ii) $r = 0.988064$

$r = 0.988$ ***A1*** ***N1***

[4 marks]

(b) correct substitution into **their** regression equation *(A1)*

eg $4.8 \times 7 + 1.2$

34.8 (millions of dollars) (accept 35 and 34800000) ***A1*** ***N2***

[2 marks]

Total [6 marks]

4. valid approach to find the required term *(M1)*

eg $\binom{8}{r} x^{8-r} k^r$, Pascal's triangle to 8th row, $x^8 + 8x^7k + 28x^6k^2 + \ldots$

identifying correct term (may be indicated in expansion) *(A1)*

eg $\binom{8}{2} x^6 k^2$, $\binom{8}{6} x^6 k^2$, $r = 2$

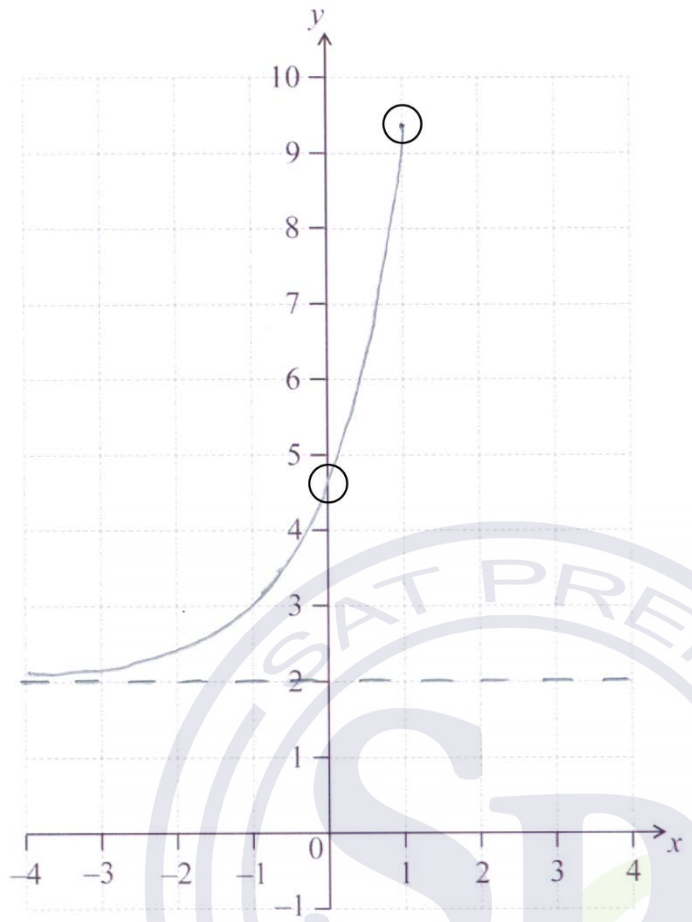
setting up equation in k with **their** coefficient/term *(M1)*

eg $28k^2x^6 = 63x^6$, $\binom{8}{6} k^2 = 63$

$k = \pm 1.5$ (exact) ***A1A1*** ***N3***

[5 marks]

5. (a)



A1A1A1 ***N3***

Note:	Curve must be approximately correct exponential shape (increasing and concave up). Only if the shape is approximately correct, award the following: ***A1*** for right end point in circle, ***A1*** for y-intercept in circle, ***A1*** for asymptotic to $y = 2$, (must be above $y = 2$).

[3 marks]

(b) valid attempt to find g ***(M1)***

eg $f(x-3)-1$, $g(x) = e^{x+1-3} + 2 - 1$, e^{x+1-3}, $2-1$, sketch

$g(x) = e^{x-2} + 1$ ***A2*** ***N3***

[3 marks]

Total [6 marks]

6. **METHOD 1**

recognize that the distance walked each minute is a geometric sequence ***(M1)***
eg $r = 0.9$, valid use of 0.9

recognize that total distance walked is the sum of a geometric sequence ***(M1)***
eg S_n, $a\left(\frac{1-r^n}{1-r}\right)$

correct substitution into the sum of a geometric sequence ***(A1)***
eg $80\left(\frac{1-0.9^n}{1-0.9}\right)$

any correct equation with sum of a geometric sequence ***(A1)***
eg $80\left(\frac{0.9^n-1}{0.9-1}\right)=660$, $1-0.9^n=\frac{66}{80}$

attempt to solve **their** equation involving the sum of a GP ***(M1)***
eg graph, algebraic approach

$n = 16.54290788$ ***A1***

since $n > 15$ ***R1***
he will be late ***AG*** ***N0***

Note: Do not award the ***R*** mark without the preceding ***A*** mark.

continued...

Question 6 continued

METHOD 2

recognize that the distance walked each minute is a geometric sequence ***(M1)***
eg $r = 0.9$, valid use of 0.9

recognize that total distance walked is the sum of a geometric sequence ***(M1)***

eg S_n, $a\left(\frac{1-r^n}{1-r}\right)$

correct substitution into the sum of a geometric sequence ***(A1)***

eg $80\left(\frac{1-0.9^n}{1-0.9}\right)$

attempt to substitute $n = 15$ into sum of a geometric sequence ***(M1)***
eg S_{15}

correct substitution ***(A1)***

eg $80\left(\frac{0.9^{15}-1}{0.9-1}\right)$

$S_{15} = 635.287$ ***A1***

since $S < 660$ ***R1***
he will not be there on time ***AG*** ***N0***

Note: Do not award the ***R*** mark without the preceding ***A*** mark.

METHOD 3
recognize that the distance walked each minute is a geometric sequence ***(M1)***
eg $r = 0.9$, valid use of 0.9

recognize that total distance walked is the sum of a geometric sequence ***(M1)***

eg S_n, $a\left(\frac{1-r^n}{1-r}\right)$

listing at least 5 correct terms of the GP ***(M1)***

15 correct terms ***A1***
80, 72, 64.8, 58.32, 52.488, 47.2392, 42.5152, 38.2637, 34.4373, 30.9936, 27.8942, 25.1048, 22.59436, 20.3349, 18.3014

attempt to find the sum of the terms ***(M1)***

eg S_{15}, $80 + 72 + 64.8 + 58.32 + 52.488 + \ldots + 18.301433$

$S_{15} = 635.287$ ***A1***

since $S < 660$ ***R1***
he will not be there on time ***AG*** ***N0***

Note: Do not award the ***R*** mark without the preceding ***A*** mark.

[7 marks]

7. attempt to set up equation *(M1)*

eg $f = g$, $kx^2 + kx = x - 0.8$

rearranging **their** equation to equal zero ***M1***

eg $kx^2 + kx - x + 0.8 = 0$, $kx^2 + x(k-1) + 0.8 = 0$

evidence of discriminant (if seen explicitly, not just in quadratic formula) *(M1)*

eg $b^2 - 4ac$, $\Delta = (k-1)^2 - 4k \times 0.8$, $D = 0$

correct discriminant *(A1)*

eg $(k-1)^2 - 4k \times 0.8$, $k^2 - 5.2k + 1$

evidence of correct discriminant greater than zero ***R1***

eg $k^2 - 5.2k + 1 > 0$, $(k-1)^2 - 4k \times 0.8 > 0$, correct answer

both correct values *(A1)*

eg 0.2, 5

correct answer ***A2*** ***N3***

eg $k < 0.2$, $k \neq 0$, $k > 5$

[8 marks]

Section B

8.

Note: The values of p and q found in (a) are used throughout the question. Please check ***FT*** carefully on **their** values.

(a) attempt to find intersection ***(M1)***
eg $f = g$

$p = 1,\ q = 3$ ***A1A1*** ***N3***
[3 marks]

(b) $f'(p) = -1$ ***A2*** ***N2***
[2 marks]

(c) (i) correct approach to find the gradient of the normal ***(A1)***
eg $m_1 m_2 = -1,\ -\dfrac{1}{f'(p)}$, correct value of 1

EITHER

attempt to substitute coordinates (in any order) and correct normal gradient to find c ***(M1)***

eg $3 = -\dfrac{1}{f'(p)} \times 1 + c,\ 1 = 1 \times 3 + c$

$c = 2$ ***(A1)***

$y = x + 2$ ***A1*** ***N2***

OR

attempt to substitute coordinates (in any order) and correct normal gradient into equation of a straight line ***(M1)***

eg $y - 3 = -\dfrac{1}{f'(p)}(x - 1),\ y - 1 = 1 \times (x - 3)$

correct working ***(A1)***
eg $y = (x - 1) + 3$

$y = x + 2$ ***A1*** ***N2***

(ii) $(0,\ 2)$ ***A1*** ***N1***
[5 marks]

(d) appropriate approach involving subtraction ***(M1)***
eg $\int_a^b (L - g)\,\mathrm{d}x,\ \int \left(3x^2 - (x + 2)\right)$

substitution of **their** limits or function ***(A1)***
eg $\int_0^p (L - g)\,\mathrm{d}x,\ \int \left((x + 2) - 3x^2\right)$

$\text{area} = 1.5$ ***A1*** ***N2***
[3 marks]
Total [13 marks]

9.

> **Note:** There may be slight differences in answers, depending on which values candidates carry through in subsequent parts. In particular there are a number of ways of doing (d). Accept answers that are consistent with their working.

(a) valid approach **(M1)**

eg $\frac{L-\mu}{\sigma}$, using a value for σ, using 68% and 95%

correct working

$\mathrm{P}(-1<Z<2)$, correct probabilities $(0.6826\ldots+0.1359\ldots)$ **(A1)**

$\mathrm{P}(50-\sigma<L<50+2\sigma)=0.818594$

$\mathrm{P}(50-\sigma<L<50+2\sigma)=0.819$ ***A1*** ***N2***

[3 marks]

(b) $z=1.95996$ **(A1)**

correct equation ***A1***

eg $\frac{53.92-50}{\sigma}=1.95996$, $\sigma=2.00004$

$\sigma=2.00$ ***AG*** ***N0***

[2 marks]

(c) valid set up ***M1***

eg $\mathrm{P}(L>t)=0.75$, right tail, 

, 0.25

$t=48.6510$

$t=48.7$ (do not accept 48.5 from using $z=-0.75$) ***A2*** ***N2***

[3 marks]

continued…

Question 9 continued

(d) (i) correct approach ***(A1)***

eg from t to 50.1, $\mathrm{P}(48.7 < X < 50.1)$, 0.269942

recognize conditional probability (seen anywhere, including in correct working) ***R1***

eg $\mathrm{P}(\mathrm{A}|\mathrm{B})$

correct substitution ***(A1)***

eg $\dfrac{\mathrm{P}(48.7 < X < 50.1)}{\mathrm{P}(X > 48.7)}$, $\dfrac{0.269942}{0.75}$

0.359923

0.360 ***A1*** ***N3***

(ii) $\mathrm{P}(X \geq 2)$ ***(A1)***

attempt to find $\mathrm{P}(X \geq 2)$ ***(M1)***

eg $1 - \mathrm{P}(X = 0) - \mathrm{P}(X = 1)$, $\mathrm{P}(X = 2) + \mathrm{P}(X = 3) + \ldots$

recognize binomial distribution ***(M1)***

eg $X \sim \mathrm{B}(n, p)$

0.923741

0.924 ***A1*** ***N2***

[8 marks]

Total [16 marks]

10. (a) area of $\mathrm{ABCD} = \mathrm{AB}^2$ (seen anywhere) ***(A1)***

choose cosine rule to find a side of the square ***(M1)***

eg $a^2 = b^2 + c^2 - 2bc\cos\theta$

correct substitution (for triangle AOB) ***A1***

eg $r^2 + r^2 - 2\times r\times r\cos\theta$, $\mathrm{OA}^2 + \mathrm{OB}^2 - 2\times\mathrm{OA}\times\mathrm{OB}\cos\theta$

correct working for AB^2 ***A1***

eg $2r^2 - 2r^2\cos\theta$

area $= 2r^2(1-\cos\theta)$ ***AG*** ***N0***

Note: Award no marks if the only working is $2r^2 - 2r^2\cos\theta$.

[4 marks]

(b) (i) $\frac{1}{2}\alpha r^2$ (accept $2r^2(1-\cos\alpha)$) ***A1*** ***N1***

(ii) correct equation in one variable ***(A1)***

eg $2(1-\cos\alpha) = \frac{1}{2}\alpha$

$\alpha = 0.511024$

$\alpha = 0.511$ (accept $\theta = 0.511$) ***A2*** ***N2***

Note: Award ***A1*** for $\alpha = 0.511$ and additional answers.

[4 marks]

continued…

Question 10 continued

(c)

Note: In this part, accept θ instead of β, and the use of equations instead of inequalities in the working.

attempt to find R ***(M1)***
eg subtraction of areas, $\text{square} - \text{segment}$

correct expression for segment area ***(A1)***
eg $\frac{1}{2}\beta r^2 - \frac{1}{2}r^2\sin\beta$

correct expression for R ***(A1)***
eg $2r^2(1-\cos\beta) - \left(\frac{1}{2}\beta r^2 - \frac{1}{2}r^2\sin\beta\right)$

correct inequality ***(A1)***
eg $2r^2(1-\cos\beta) - \left(\frac{1}{2}\beta r^2 - \frac{1}{2}r^2\sin\beta\right) > 2\left(\frac{1}{2}\beta r^2\right)$

correct inequality in terms of angle only ***A1***
eg $2(1-\cos\beta) - \left(\frac{1}{2}\beta - \frac{1}{2}\sin\beta\right) > \beta$

attempt to solve their inequality, must represent R> twice sector ***(M1)***
eg sketch, one correct value

Note: Do not award the second ***(M1)*** unless the first ***(M1)*** for attempting to find R has been awarded.

both correct values 1.30573 and 2.67369 ***(A1)***

correct inequality $1.31 < \beta < 2.67$ ***A1*** ***N3***

[8 marks]

Total [16 marks]

N14/5/MATME/SP2/ENG/TZ0/XX/M

International Baccalaureate®
Baccalauréat International
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MARKSCHEME

November 2014

MATHEMATICS

Standard Level

Paper 2

21 pages

Instructions to Examiners (red new, green check carefully)

Abbreviations

M Marks awarded for attempting to use a correct **Method**; working must be seen.

(M) Marks awarded for **Method**; may be implied by **correct** subsequent working.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

(A) Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.

R Marks awarded for clear **Reasoning**.

N Marks awarded for **correct** answers if **no** working shown.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Mark according to RM assessor instructions and the document "**Mathematics SL: Guidance for e-marking May 2014**". It is **essential** that you read this document before you start marking. In particular, please note the following. Marks must be recorded using the annotation stamps, using the RM assessor tool. Please check that you are entering marks for the right question.

- If a part is **completely correct**, (and gains all the "must be seen" marks), use the ticks with numbers to stamp full marks. Do **not** use the ticks with numbers for anything else.
- If a part is completely wrong, stamp ***A0*** by the final answer.
- If a part gains anything else, all the working **must** have annotations stamped to show what marks are awarded. This includes any zero marks.

All the marks will be added and recorded by RM assessor.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any. An exception to this rule is when work for ***M1*** is missing, as opposed to incorrect (see point 4).
- Where ***M*** and ***A*** marks are noted on the same line, *eg* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*eg* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***(M2)***, ***N3***, *etc*., do **not** split the marks, unless there is a note.
- Once a correct answer to a question or part-question is seen, ignore further working.
- Most ***M*** marks are for a **valid** method, ie a method which can lead to the answer: it must indicate some form of progress towards the answer.

3 *N* marks

*If **no** working shown, award **N** marks for **correct** answers – this includes acceptable answers (see accuracy booklet). In this case, ignore mark breakdown (**M, A, R**).*

- Do **not** award a mixture of ***N*** and other marks.
- There may be fewer ***N*** marks available than the total of ***M***, ***A*** and ***R*** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.
- There may not be a direct relationship between the ***N*** marks and the implied marks. There are times when all the marks are implied, but the ***N*** marks are not the full marks: this indicates that we want to see some of the working, without specifying what.
- For consistency within the markscheme, ***N*** marks are noted for every part, even when these match the mark breakdown.
- If a candidate has incorrect working, which somehow results in a correct answer, do **not** award the ***N*** marks for this correct answer. However, if the candidate has indicated (usually by crossing out) that the working is to be ignored, award the ***N*** marks for the correct answer.

4 Implied and must be seen marks

*Implied marks appear in **brackets** eg **(M1)**.*

- Implied marks can only be awarded if the work is seen or if implied in subsequent working (a correct final answer does not necessarily mean that the implied marks are all awarded). There are questions where some working is required, but as it is accepted that not everyone will write the same steps, all the marks are implied, but the ***N*** marks are not the full marks for the question.
- Normally the correct work is seen in the next line.
- Where there is an (***M1***) followed by ***A1*** for each correct answer, if no working shown, one correct answer is sufficient evidence to award the (***M1***).

*Must be seen marks appear without **brackets** eg **M1**.*

- Must be seen marks can only be awarded if the work is seen.
- If a must be seen mark is not awarded because work is missing (as opposed to ***M0*** or ***A0*** for incorrect work) all subsequent marks may be awarded if appropriate.

5 Follow through marks (only applied after an error is made)

*Follow through (**FT**) marks are awarded where an incorrect answer (final or intermediate) from one **part** of a question is used correctly in **subsequent** part(s) or subpart(s). Usually, to award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if the only marks awarded in a subpart are for the **final** answer, then **FT** marks should be awarded if appropriate. Examiners are expected to check student work in order to award **FT** marks where appropriate.*

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** and ***R*** marks may be awarded if appropriate. (However, as noted above, if an ***A*** mark is not awarded because work is missing, all subsequent marks may be awarded if appropriate).
- Exceptions to this rule will be explicitly noted on the markscheme.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*eg* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non integer value where integer required), do not award the mark(s) for the final answer(s).

- The markscheme may use the word "**their**" in a description, to indicate that candidates may be using an incorrect value.
- If a candidate makes an error in one part, but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the question says hence. It is often possible to use a different approach in subsequent parts that does not depend on the answer to previous parts.
- In a "show that" question, if an error in a previous subpart leads to not showing the required answer, do not award the final ***A1***. Note that if the error occurs within the same subpart, the ***FT*** rules may result in further loss of marks.
- Where there are anticipated common errors, the ***FT*** answers are often noted on the markscheme, to help examiners. It should be stressed that these are not the only ***FT*** answers accepted, neither should ***N*** marks be awarded for these answers.

6 Mis-read

*If a candidate incorrectly copies information from the question, this is a mis-read (**MR**). A candidate should be penalized only once for a particular mis-read. Use the **MR** stamp to indicate that this is a misread. Do not award the first mark in the question, even if this is an **M** mark, but award all others (if appropriate) so that the candidate only loses one mark for the misread.*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*eg* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.

7 Discretionary marks *(d)*

*An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. In such cases the annotation DM should be used and a brief **note** written next to the mark explaining this decision.*

8 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete parts are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**. Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9 Alternative forms

*Unless the question specifies otherwise, **accept** equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

10 Calculators

A GDC is required for paper 2, but calculators with symbolic manipulation features (eg TI-89) are not allowed.

Calculator notation The mathematics SL guide says:

Students must always use correct mathematical notation, not calculator notation.

Do **not** accept final answers written using calculator notation. However, do not penalize the use of calculator notation in the working.

11 Style

The markscheme aims to present answers using good communication, eg if the question asks to find the value of k, the markscheme will say $k = 3$*, but the marks will be for the correct value 3 – there is usually no need for the "* $k =$ *". In these cases, it is also usually acceptable to have another variable, as long as there is no ambiguity in the question, eg if the question asks to find the value of p and of q, then the student answer needs to be clear. Generally, the only situation where the full answer is required is in a question which asks for equations – in this case the markscheme will say "must be an equation".*

The markscheme often uses words to describe what the marks are for, followed by examples, using the eg notation. These examples are not exhaustive, and examiners should check what candidates have written, to see if they satisfy the description. Where these marks are ***M*** *marks, the examples may include ones using poor notation, to indicate what is acceptable. A valid method is one which will allow candidate to proceed to the next step eg if a quadratic function is given in factorised form, and the question asks for the zeroes, then multiplying the factors does not necessarily help to find the zeros, and would not on its own count as a valid method.*

12 Candidate work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

Candidates are meant to write their answers to Section A on the question paper (QP), and Section B on answer booklets. Sometimes, they need more room for Section A, and use the booklet (and often comment to this effect on the QP), or write outside the box. That is fine, and this work should be marked. The instructions tell candidates not to write on Section B of the QP. Thus they may well have done some rough work here which they assume will be ignored. If they have solutions on the answer booklets, there is no need to look at the QP. However, if there are whole questions or whole part solutions missing on answer booklets, please check to make sure that they are not on the QP, and if they are, mark those whole questions or whole part solutions that have not been written on answer booklets

13. Diagrams

The notes on how to allocate marks for sketches usually refer to passing through particular points or having certain features. These marks can only be awarded if the sketch is approximately the correct shape. All values given will be an approximate guide to where these points/features occur. In some questions, the first ***A1*** is for the shape, in others, the marks are only for the points and/or features. In both cases, unless the shape is approximately correct, no marks can be awarded (unless otherwise stated). However, if the graph is based on previous calculations, ***FT*** marks should be awarded if appropriate.

14. Accuracy of Answers

If the level of accuracy is specified in the question, a mark will be allocated for giving the final answer to the required accuracy. When this is not specified in the question, all numerical answers should be given exactly or correct to three significant figures

*Candidates should NO LONGER be penalized for an accuracy error **(AP)**. Examiners should award marks according to the rules given in these instructions and the markscheme. Accuracy is not the same as correctness – an incorrect value does not achieve relevant **A** marks. It is only final answers which may lose marks for accuracy errors, not intermediate values. Please check work carefully for **FT**.*

Do not accept unfinished numerical final answers such as 3/0.1 (unless otherwise stated). As a rule, numerical answers with more than one part (such as fractions) should be given using integers (*eg* 6/8). Calculations which lead to integers should be completed, with the exception of fractions which are not whole numbers.

Clarification of intermediate values accuracy instructions

Intermediate values do not need to be given to the correct three significant figures. But, if candidates work with rounded values, this could lead to an incorrect answer, in which case award ***A0*** for the final answer. However, do not penalise inaccurate intermediate values that lead to an acceptable final answer.

All examiners must read this section carefully, as there are some changes (in red) since M13.

These instructions apply when answers need to be rounded, they do not apply to exact answers which have 3 or fewer figures. The answers will give a range of acceptable values, and any answer given to 3 or more sf that lies in this range will be accepted as well as answers given to the correct 2 sf (which will usually not be in the acceptable range). Answers which are given to 1 sf are not acceptable. There is also a change to the awarding of N marks for acceptable answers.

Where numerical answers are required as the **final** answer to a part of a question in the markscheme, the markscheme will show

a truncated 6 sf value

the exact value if applicable, the correct 3 sf answer and the range of acceptable values. This range includes both end values. Once an acceptable value is seen, ignore any subsequent values (even if rounded incorrectly).

Units (which are generally not required) will appear in brackets at the end.

Example

1.73205

$\sqrt{3}$ (exact), 1.73 [1.73, 1.74] (m)

Note that 1.73 is the correct 3 sf, 1.74 is incorrectly rounded but acceptable, 1.7 is the correct 2 sf value but 1.72 is wrong.

For subsequent parts, the markscheme will show the answers obtained from using unrounded values, and the answers from using previous **correct** 3 sf answers. Examiners will need to check the work carefully if candidates use any other acceptable answers. If other acceptable answers lead to an incorrect final answer (ie outside the range), do not award the final ***A1***. This should not be considered as ***FT***.

Intermediate values do **not** need to be given to the correct 3 sf. If candidates work with fewer than 3 sf, or with incorrectly rounded values, this could lead to an incorrect answer, in which case award ***A0*** for the final answer. However, do not penalise intermediate inaccurate values that lead to an acceptable final answer.

In questions where the final answer gains ***A2***, if other working shown, award ***A1*** for a correctly rounded 1 sf answer.

If there is **no** working shown, award the ***N*** marks for **any** acceptable answer, eg in the example above, if 1.73 achieves ***N4***, then 1.74, 1.7, 1.7320 all achieve ***N4,*** but 2 achieves ***N0*** .

The following table shows what achieves the final mark if this is the **only** numerical answer seen, as long as there is other working.

	Correctly rounded	Incorrectly rounded
1sf	No	No
2sf	Yes	No
3sf	Yes	Yes (if in the acceptable range)
4 or more sf	Yes (if in the acceptable range)	Yes (if in the acceptable range)

Examples: The correct marking is given at the end of this section. Please decide what marks you would give each answer, and then check. Assume that working is shown unless otherwise indicated. If you disagree, please discuss with your team leader.

Example 1 (awards ***A1*** for final answer)

Markscheme		
7.43798		
7.44 [7.43, 7.44]	***A1***	***N3***

	Candidate's Script	Marking
(i)	7.43798 followed by anything	
(ii)	7.5	
(iii)	7.4	
(iv)	7.4 (with no working)	
(v)	7	
(vi)	7.438	
(vii)	7.43	
(viii)	7.43 (with no working)	
(ix)	7.437	
(x)	7.433	

Example 2 (awards ***A2*** for final answer)

Markscheme		
8.43482		
8.43 [8.43, 8.44]	***A2***	***N3***

	Candidate's Script	Marking
(i)	8.433016	
(ii)	8.44	
(iii)	8	
(iv)	8.42	
(v)	8.4 (with no working)	
(vi)	8 (with no working)	
(vii)	8.44 (with no working)	
(viii)	8.43 (with no working)	

Answers to the examples.

Example 1 (awards *A1* for final answer)

(i)	7.43798 followed by anything	***A1***
(ii)	7.5 (wrong)	***A0***
(iii)	7.4 (correct 2 sf)	***A1***
(iv)	7.4 (with no working)	***N3***
(v)	7 (1 sf)	***A0***
(vi)	7.438 (in acceptable range)	***A1***
(vii)	7.43 (acceptable 3 sf)	***A1***
(viii)	7.43 (with no working)	***N3***
(ix)	7.437 (in acceptable range)	***A1***
(x)	7.433 (in acceptable range)	***A1***

Example 2 (awards *A2* for final answer)

	Candidate's Script	Marking
(i)	8.433016 (in acceptable range)	***A2***
(ii)	8.44	***A2***
(iii)	8 (1 sf, penalise 1 mark)	***A1***
(iv)	8.42 (outside acceptable range)	***A0***
(v)	8.4 (with no working)	***N3***
(vi)	8 (with no working)	***N0***
(vii)	8.44 (with no working)	***N3***
(viii)	8.43 (with no working)	***N3***

SECTION A

1. (a) attempt to form composite (in any order) ***(M1)***

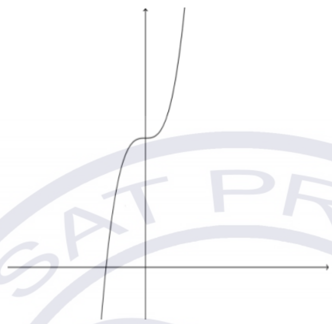
eg $f\left(x^3\right)$, $\left(2x+3\right)^3$

$(f \circ g)(x) = 2x^3 + 3$, $2\left(x\right)^3 + 3$ ***A1 N2***

[2 marks]

(b) evidence of appropriate approach ***(M1)***

eg $2x^3 = -3$, sketch



correct working ***(A1)***

eg $x^3 = \frac{-3}{2}$, sketch

-1.14471

$x = \sqrt[3]{\frac{-3}{2}}$ (exact), -1.14 $[-1.15, -1.14]$ ***A1 N3***

[3 marks]

Total [5 marks]

2. (a) evidence of set up *(M1)*
eg correct value for r (or for a or b, seen in (b))

0.996010
$r = 0.996$ [0.996, 0.997] *A1* *N2*
[2 marks]

(b) $a = 3.15037$, $b = -15.4393$
$a = 3.15$ [3.15, 3.16], $b = -15.4$ [-15.5, -15.4] *A1A1* *N2*
[2 marks]

(c) substituting 26 into **their** equation *(M1)*
eg $y = 3.15(26) - 15.4$

66.4704
66.5 $[66.4, 66.5]$ *A1* *N2*
[2 marks]

Total [6 marks]

3. (a) correct substitution into formula *(A1)*
eg $l = 1.2 \times 8$

9.6 (cm) *A1* *N2*
[2 marks]

(b) **METHOD 1**

evidence of choosing cosine rule *(M1)*
eg $2r^2 - 2 \times r^2 \times \cos\left(\text{A}\hat{\text{O}}\text{B}\right)$

correct substitution into right hand side *(A1)*
eg $8^2 + 8^2 - 2 \times 8 \times 8 \times \cos(1.2)$

9.0342795
AB = 9.03 [9.03, 9.04] (cm) *A1* *N2*

METHOD 2

evidence of choosing sine rule *(M1)*
eg $\dfrac{\text{AB}}{\sin\left(\text{A}\hat{\text{O}}\text{B}\right)} = \dfrac{\text{OB}}{\sin\left(\text{O}\hat{\text{A}}\text{B}\right)}$

finding angle OAB or OBA (may be seen in substitution) *(A1)*
eg $\dfrac{\pi - 1.2}{2}$, 0.970796

AB = 9.03 [9.03, 9.04] (cm) *A1* *N2*

[3 marks]

continued …

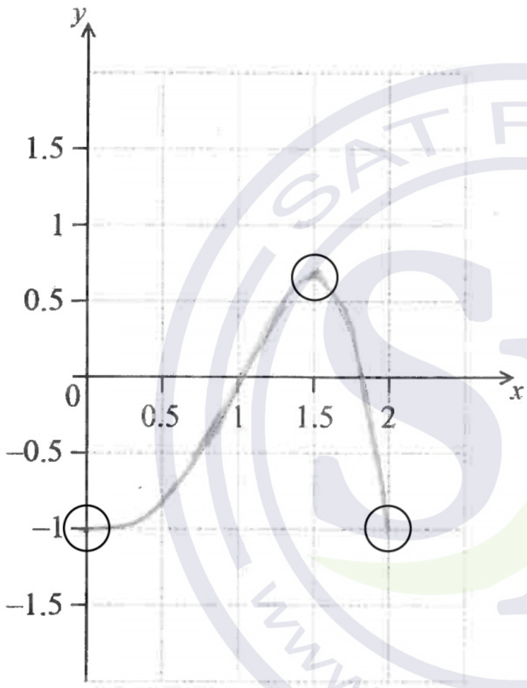
Question 3 continued

(c) correct working ***(A1)***
eg $P = 9.6 + 9.03$

18.6342
18.6 [18.6, 18.7] (cm) ***A1*** ***N2***
[2 marks]

Total [7 marks]

4. (a)



A1A1A1 ***N3***
[3 marks]

Note: Award ***A1*** for both endpoints in circles,
A1 for approximately correct shape (concave up to concave down).
Only if this ***A1*** for shape is awarded, award ***A1*** for maximum point in circle.

(b) $x = 1$ $x = 1.83928$
$x = 1$ (exact) $x = 1.84$ [1.83, 1.84] ***A1A1*** ***N2***
[2 marks]

(c) attempt to substitute either (***FT***) limits or function into formula with f^2 ***(M1)***
(accept absence of π or d*x*, but do not accept any errors, including extra bits)
eg $V = \pi\int_1^{1.84} f^2$, $\int\left(-x^4 + 2x^3 - 1\right)^2 \mathrm{d}x$

0.636581
$V = 0.637$ [0.636, 0.637] ***A2*** ***N3***
[3 marks]

Total [8 marks]

5. (a) valid approach *(M1)*

eg $\frac{2-1}{2}$, $2-1.5$

$p = 0.5$ *A1* *N2*

[2 marks]

(b) valid approach *(M1)*

eg $\frac{1+2}{2}$

$r = 1.5$ *A1* *N2*

[2 marks]

(c) **METHOD 1**

valid approach (seen anywhere) *M1*

eg $q = \frac{2\pi}{\text{period}}$, $\frac{2\pi}{\left(\frac{2\pi}{3}\right)}$

period $= \frac{2\pi}{3}$ (seen anywhere) *(A1)*

$q = 3$ *A1* *N2*

METHOD 2

attempt to substitute one point and **their** values for p and r into y *M1*

eg $2 = 0.5\sin\left(q\frac{\pi}{6}\right) + 1.5$, $\frac{\pi}{2} = 0.5\sin(q1) + 1.5$

correct equation in q *(A1)*

eg $q\frac{\pi}{6} = \frac{\pi}{2}$, $q\frac{\pi}{2} = \frac{3\pi}{2}$

$q = 3$ *A1* *N2*

METHOD 3

valid reasoning comparing the graph with that of $\sin x$ *R1*

eg position of max/min, graph goes faster

correct working *(A1)*

eg max at $\frac{\pi}{6}$ not at $\frac{\pi}{2}$, graph goes 3 times as fast

$q = 3$ *A1* *N2*

[3 marks]

Total [7 marks]

6. valid approach to find the required term *(M1)*

eg $\binom{8}{r}\left(\frac{x^3}{2}\right)^{8-r}\left(\frac{p}{x}\right)^r$, $\left(\frac{x^3}{2}\right)^8\left(\frac{p}{x}\right)^0+\binom{8}{1}\left(\frac{x^3}{2}\right)^7\left(\frac{p}{x}\right)^1+\ldots$, Pascal's triangle to required value

identifying constant term (may be indicated in expansion) *(A1)*

eg 7^{th} term, $r=6$, $\left(\frac{1}{2}\right)^2$, $\binom{8}{6}$, $\left(\frac{x^3}{2}\right)^2\left(\frac{p}{x}\right)^6$

correct calculation (may be seen in expansion) *(A1)*

eg $\binom{8}{6}\left(\frac{x^3}{2}\right)^2\left(\frac{p}{x}\right)^6$, $\frac{8\times 7}{2}\times\frac{p^6}{2^2}$

setting up equation with **their** constant term equal to 5103 *M1*

eg $\binom{8}{6}\left(\frac{x^3}{2}\right)^2\left(\frac{p}{x}\right)^6=5103$, $p^6=\frac{5103}{7}$

$p=\pm 3$ *A1A1* *N3*

[6 marks]

7. (a) correct substitution of function and/or limits into formula *(A1)*
(accept absence of d*t*, but do not accept any errors)

eg $\int_0^{\frac{\pi}{2}} v$, $\int \left| e^{\frac{1}{2}\cos t} - 1 \right| \mathrm{d}t$, $\int \left(e^{\frac{1}{2}\cos t} - 1 \right)$

0.613747
distance is 0.614 [0.613, 0.614] (m) ***A1*** ***N2***
[2 marks]

(b) **METHOD 1**

valid attempt to find the distance travelled between $t = \frac{\pi}{2}$ and $t = 4$ *(M1)*

eg $\int_{\frac{\pi}{2}}^{4} \left(e^{\frac{1}{2}\cos t} - 1 \right)$, $\int_0^4 \left| e^{\frac{1}{2}\cos t} - 1 \right| \mathrm{d}t - 0.614$

distance is 0.719565 ***A1***

valid reason, referring to change of direction (may be seen in explanation) ***R1***

valid explanation comparing **their** distances ***R1***
eg $0.719565 > 0.614$, distance moving back is more than distance moving forward

Note: Do not award the final ***R1*** unless the ***A1*** is awarded.

particle passes through A again ***AG*** ***N0***

METHOD 2

valid attempt to find displacement *(M1)*

eg $\int_{\frac{\pi}{2}}^{4} \left(e^{\frac{1}{2}\cos t} - 1 \right)$, $\int_0^4 \left(e^{\frac{1}{2}\cos t} - 1 \right)$

correct displacement ***A1***
eg –0.719565, –0.105817

recognising that displacement from 0 to $\frac{\pi}{2}$ is positive ***R1***

eg displacement = distance from 0 to $\frac{\pi}{2}$

valid explanation referring to positive and negative displacement ***R1***
eg $0.719565 > 0.614$, overall displacement is negative, since displacement after $\frac{\pi}{2}$ is negative, then particle gone backwards more than forwards

Note: Do not award the final ***R1*** unless the ***A1*** and the first ***R1*** are awarded.

particle passes through A again ***AG*** ***N0***
[4 marks]

continued …

Question 7 continued

Note: Special Case. If all working shown, and candidates seem to have misread the question, using $v = e^{\frac{1}{2}\cos t}$, award marks as follows:

(a) correct substitution of function and/or limits into formula (accept absence of d*t*, but do not accept any errors) ***A0MR***

eg $\int_0^{\frac{\pi}{2}}\left(e^{\frac{1}{2}\cos t}\right)$, $\int\left|e^{\frac{1}{2}\cos t}\right|dt$, $\int\left(e^{\frac{1}{2}\cos t}\right)$

2.184544

distance is 2.18 [2.18, 2.19] (m) ***A1*** ***N0***

(b) **METHOD 1**

valid attempt to find the distance travelled between $t = \frac{\pi}{2}$ and $t = 4$ ***M1***

eg $\int_{\frac{\pi}{2}}^4\left(e^{\frac{1}{2}\cos t}\right)$, $\int_0^4\left|e^{\frac{1}{2}\cos t}\right|dt - 2.18$

distance is 1.709638 ***A1***

reference to change of direction (may be seen in explanation) ***R1***

reasoning/stating particle passes/does not pass through A again ***R0***

METHOD 2

valid attempt to find displacement ***M1***

eg $\int_{\frac{\pi}{2}}^4\left(e^{\frac{1}{2}\cos t}\right)$, $\int_0^4\left(e^{\frac{1}{2}\cos t}\right)$

correct displacement ***A1***

eg 1.709638, 3.894182

recognising that displacement from 0 to $\frac{\pi}{2}$ is positive ***R0***

reasoning/stating particle passes/does not pass through A again ***R0***

With method 2, there is no valid reasoning about whether the particle passes through A again or not, so they cannot gain the ***R*** marks.

Total [6 marks]

SECTION B

8. (a) recognizing that the median is at half the total frequency *(M1)*

eg $\frac{2000}{2}$

$m = 2500$ (dollars) *A1* *N2*

[2 marks]

(b) (i) 500 families have a monthly income less than 2000 *A1* *N1*

(ii) correct cumulative frequency, 1850 *(A1)*

subtracting **their** cumulative frequency from 2000 *(M1)*

eg $2000 - 1850$

150 families have a monthly income of more than 4000 dollars *A1* *N2*

Note: If working shown, award ***M1A1A1*** for $128 + 22 = 150$, using the table.

[4 marks]

(c) correct calculation *(A1)*

eg $2000 - (436 + 64 + 765 + 28 + 122)$, $1850 - 500 - 765$

$p = 585$ *A1* *N2*

[2 marks]

(d) (i) correct working *(A1)*

eg $436 + 765 + 28$

0.6145 (exact)

$\frac{1229}{2000}$, 0.615 [0.614, 0.615] *A1* *N2*

(ii) correct working/probability for number of families *(A1)*

eg $122 + 28$, $\frac{150}{2000}$, 0.075

0.186666

$\frac{28}{150}\left(=\frac{14}{75}\right)$, 0.187 [0.186, 0.187] *A1* *N2*

[4 marks]

(e) evidence of using correct mid-interval values (1500, 3000, 4500) *(A1)*

attempt to substitute into $\frac{\sum fx}{\sum f}$ *(M1)*

eg $\frac{1500 \times 64 + 3000 \times p + 4500 \times 122}{64 + 585 + 122}$

3112.84

3110 [3110, 3120] (dollars) *A1* *N2*

[3 marks]

Total [15 marks]

9. (a) (i) valid approach *(M1)*

eg $r = \frac{u_2}{u_1}$, $\frac{4}{4.2}$

$r = 1.05$ (exact) ***A1*** ***N2***

(ii) attempt to substitute into formula, with **their** r *(M1)*

eg 4×1.05^n, $4 \times 1.05 \times 1.05 \ldots$

correct substitution *(A1)*

eg 4×1.05^4, $4 \times 1.05 \times 1.05 \times 1.05 \times 1.05$

$u_5 = 4.862025(\text{exact})$, 4.86 $[4.86, 4.87]$ ***A1*** ***N2***

[5 marks]

(b) (i) attempt to substitute $n = 1$ *(M1)*

eg $0.05 = a \times 1^k$

$a = 0.05$ ***A1*** ***N2***

(ii) correct substitution of $n = 2$ into v_2 ***A1***

eg $0.25 = a \times 2^k$

correct work *(A1)*

eg finding intersection point, $k = \log_2\left(\frac{0.25}{0.05}\right)$, $\frac{\log 5}{\log 2}$

2.32192

$k = \log_2 5$ (exact), 2.32 [2.32, 2.33] ***A1*** ***N2***

[5 marks]

(c) correct expression for u_n *(A1)*

eg $4 \times 1.05^{n-1}$

EITHER

correct substitution into inequality (accept equation) *(A1)*

eg $0.05 \times n^k > 4 \times 1.05^{n-1}$

valid approach to solve inequality (accept equation) *(M1)*

eg finding point of intersection, $n = 7.57994$ (7.59508 from 2.32)

$n = 8$ (must be an integer) ***A1*** ***N2***

OR

table of values

when $n = 7$, $u_7 = 5.3604$, $v_7 = 4.5836$ ***A1***

when $n = 8$, $u_8 = 5.6284$, $v_8 = 6.2496$ ***A1***

$n = 8$ (must be an integer) ***A1*** ***N2***

[4 marks]

Total [14 marks]

10. **Note:** There may be slight differences in answers, depending on which values candidates carry through in subsequent parts. Accept answers that are consistent with their working.

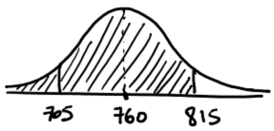
(a) (i) $P(X > 760) = 0.5$ (exact), [0.499, 0.500] ***A1*** ***N1***

(ii) evidence of valid approach ***(M1)***

recognising symmetry, $\frac{0.7887}{2}$, $1 - P(W < 815)$, $\frac{21.13}{2} + 78.87\%$

correct working ***(A1)***

eg $0.5 + 0.39435$, $1 - 0.10565$,



0.89435 (exact), 0.894 [0.894, 0.895] ***A1*** ***N2***

[4 marks]

(b) (i) 1.24999

$z = 1.25$ [1.24, 1.25] ***A1*** ***N1***

(ii) evidence of appropriate approach ***(M1)***

eg $\sigma = \frac{x - \mu}{1.25}$, $\frac{815 - 760}{\sigma}$

correct substitution ***(A1)***

eg $1.25 = \frac{815 - 760}{\sigma}$, $\frac{815 - 760}{1.24999}$

44.0003

$\sigma = 44.0$ [44.0, 44.1] (g) ***A1*** ***N2***

[4 marks]

(c) correct working ***(A1)***

eg $760 - 1.5 \times 44$

693.999

694 [693, 694] (g) ***A1*** ***N2***

[2 marks]

(d) 0.0668056

$P(X < 694) = 0.0668$ [0.0668, 0.0669] ***A2*** ***N2***

[2 marks]

continued…

Question 10 continued

(e) recognizing conditional probability (seen anywhere) ***(M1)***

eg $\mathrm{P}(\mathrm{A}|\mathrm{B})$, $\frac{0.025}{0.0668}$

appropriate approach involving conditional probability ***(M1)***

eg $\mathrm{P}(S\,|\,T) = \frac{\mathrm{P}(S \text{ and } T)}{\mathrm{P}(T)}$,

correct working

eg P (salmon and tiddler) $= 0.25 \times 0.1$, $\frac{0.25 \times 0.1}{0.0668}$ ***(A1)***

0.374220
0.374 [0.374, 0.375] ***A1 N2***

[4 marks]

Total [16 marks]

M14/5/MATME/SP2/ENG/TZ1/XX/M

International Baccalaureate®
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MARKSCHEME

May 2014

MATHEMATICS

Standard Level

Paper 2

17 pages

Instructions to Examiners

All examiners must read these instructions carefully, as there are some changes since M13.

Abbreviations

M Marks awarded for attempting to use a correct **Method**; working must be seen.

(M) Marks awarded for **Method**; may be implied by **correct** subsequent working.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

(A) Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.

R Marks awarded for clear **Reasoning**.

N Marks awarded for **correct** answers if **no** working shown.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Mark according to scoris instructions and the document "**Mathematics SL: Guidance for e-marking May 2014**". It is **essential** that you read this document before you start marking. In particular, please note the following. Marks must be recorded using the annotation stamps, using the new scoris tool. Please check that you are entering marks for the right question.

- If a part is **completely correct**, (and gains all the "must be seen" marks), use the ticks with numbers to stamp full marks. Do **not** use the ticks with numbers for anything else.
- If a part is completely wrong, stamp ***A0*** by the final answer.
- If a part gains anything else, all the working **must** have annotations stamped to show what marks are awarded. This includes any zero marks.

All the marks will be added and recorded by scoris.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any. An exception to this rule is when work for ***M1*** is missing, as opposed to incorrect (see point 4).
- Where ***M*** and ***A*** marks are noted on the same line, *eg* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*eg* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***(M2)***, ***N3***, *etc*., do **not** split the marks, unless there is a note.
- Once a correct answer to a question or part-question is seen, ignore further working.
- Most ***M*** marks are for a **valid** method, ie a method which can lead to the answer: it must indicate some form of progress towards the answer.

3 *N* marks

*If **no** working shown, award N marks for **correct** answers – this includes acceptable answers (see accuracy booklet). In this case, ignore mark breakdown (**M**, **A**, **R**).*

- Do **not** award a mixture of ***N*** and other marks.
- There may be fewer ***N*** marks available than the total of ***M***, ***A*** and ***R*** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.
- There may not be a direct relationship between the ***N*** marks and the implied marks. There are times when all the marks are implied, but the ***N*** marks are not the full marks: this indicates that we want to see some of the working, without specifying what.
- For consistency within the markscheme, ***N*** marks are noted for every part, even when these match the mark breakdown.
- If a candidate has incorrect working, which somehow results in a correct answer, do **not** award the ***N*** marks for this correct answer. However, if the candidate has indicated (usually by crossing out) that the working is to be ignored, award the ***N*** marks for the correct answer.

4 Implied and must be seen marks

*Implied marks appear in **brackets** eg **(M1)**.*

- Implied marks can only be awarded if the work is seen or if implied in subsequent working (a correct final answer does not necessarily mean that the implied marks are all awarded). There are questions where some working is required, but as it is accepted that not everyone will write the same steps, all the marks are implied, but the ***N*** marks are not the full marks for the question.
- Normally the correct work is seen in the next line.
- Where there is an (***M1***) followed by ***A1*** for each correct answer, if no working shown, one correct answer is sufficient evidence to award the (***M1***).

*Must be seen marks appear without **brackets** eg **M1**.*

- Must be seen marks can only be awarded if the work is seen.
- If a must be seen mark is not awarded because work is missing (as opposed to ***M0*** or ***A0*** for incorrect work) all subsequent marks may be awarded if appropriate.

5 Follow through marks (only applied after an error is made)

*Follow through (**FT**) marks are awarded where an incorrect answer (final or intermediate) from one **part** of a question is used correctly in **subsequent** part(s) or subpart(s). Usually, to award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if the only marks awarded in a subpart are for the **final** answer, then **FT** marks should be awarded if appropriate. Examiners are expected to check student work in order to award **FT** marks where appropriate.*

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** and ***R*** marks may be awarded if appropriate. (However, as noted above, if an ***A*** mark is not awarded because work is missing, all subsequent marks may be awarded if appropriate).
- Exceptions to this rule will be explicitly noted on the markscheme.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*eg* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word "**their**" in a description, to indicate that candidates may be using an incorrect value.
- If a candidate makes an error in one part, but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the question says hence. It is often possible to use a different approach in subsequent parts that does not depend on the answer to previous parts.
- In a "show that" question, if an error in a previous subpart leads to not showing the required answer, do not award the final ***A1***. Note that if the error occurs within the same subpart, the ***FT*** rules may result in further loss of marks.
- Where there are anticipated common errors, the ***FT*** answers are often noted on the markscheme, to help examiners. It should be stressed that these are not the only ***FT*** answers accepted, neither should ***N*** marks be awarded for these answers.

6 Mis-read

*If a candidate incorrectly copies information from the question, this is a mis-read (**MR**). A candidate should be penalized only once for a particular mis-read. Use the **MR** stamp to indicate that this is a misread. Do not award the first mark in the question, even if this is an **M** mark, but award all others (if appropriate) so that the candidate only loses one mark for the misread.*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*eg* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.

7 Discretionary marks *(d)*
An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. In such cases the annotation DM should be used and a brief ***note*** *written next to the mark explaining this decision.*

8 Alternative methods
Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete parts are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**. Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9 Alternative forms
Unless the question specifies otherwise, ***accept*** *equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

10 Accuracy of Answers

If the level of accuracy is specified in the question, a mark will be allocated for giving the final answer to the required accuracy. When this is not specified in the question, all numerical answers should be given exactly or correct to three significant figures

Candidates should NO LONGER be penalized for an accuracy error ***(AP)****. Examiners should award marks according to the rules given in these instructions and the markscheme. Accuracy is not the same as correctness – an incorrect value does not achieve relevant* ***A*** *marks. It is only final answers which may lose marks for accuracy errors, not intermediate values. Please check work carefully for* ***FT****. Further information on which answers are accepted is given in a separate booklet, along with examples. It is* ***essential*** *that you read this carefully.*

Do not accept unfinished numerical final answers such as 3/0.1 (unless otherwise stated). As a rule, numerical answers with more than one part (such as fractions) should be given using integers (*eg* 6/8). Calculations which lead to integers should be completed, with the exception of fractions which are not whole numbers.

Clarification of intermediate values accuracy instructions

Intermediate values do not need to be given to the correct three significant figures. But, if candidates work with rounded values, this could lead to an incorrect answer, in which case award ***A0*** for the final answer. However, do not penalise inaccurate intermediate values that lead to an acceptable final answer.

11 Calculators

A GDC is required for paper 2, but calculators with symbolic manipulation features (eg TI-89) are not allowed.

Calculator notation
The Mathematics SL guide says:

Students must always use correct mathematical notation, not calculator notation.

Do **not** accept final answers written using calculator notation. However, do not penalize the use of calculator notation in the working.

12 Style

The markscheme aims to present answers using good communication, eg if the question asks to find the value of k, the markscheme will say $k = 3$, but the marks will be for the correct value 3 – there is usually no need for the "$k =$". In these cases, it is also usually acceptable to have another variable, as long as there is no ambiguity in the question, eg if the question asks to find the value of p and of q, then the student answer needs to be clear. Generally, the only situation where the full answer is required is in a question which asks for equations – in this case the markscheme will say "must be an equation".

*The markscheme often uses words to describe what the marks are for, followed by examples, using the eg notation. These examples are not exhaustive, and examiners should check what candidates have written, to see if they satisfy the description. Where these marks are **M** marks, the examples may include ones using poor notation, to indicate what is acceptable. A valid method is one which will allow candidate to proceed to the next step eg if a quadratic function is given in factorised form, and the question asks for the zeroes, then multiplying the factors does not necessarily help to find the zeros, and would not on its own count as a valid method.*

13 Candidate work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

Candidates are meant to write their answers to Section A on the question paper (QP), and Section B on answer booklets. Sometimes, they need more room for Section A, and use the booklet (and often comment to this effect on the QP), or write outside the box. That is fine, and this work should be marked.

The instructions tell candidates not to write on Section B of the QP. Thus they may well have done some rough work here which they assume will be ignored. If they have solutions on the answer booklets, there is no need to look at the QP. However, if there are whole questions or whole part solutions missing on answer booklets, please check to make sure that they are not on the QP, and if they are, mark those whole questions or whole part solutions that have not been written on answer booklets

14. Diagrams

The notes on how to allocate marks for sketches usually refer to passing through particular points or having certain features. These marks can only be awarded if the sketch is approximately the correct shape. All values given will be an approximate guide to where these points/features occur. In some questions, the first ***A1*** is for the shape, in others, the marks are only for the points and/or features. In both cases, unless the shape is approximately correct, no marks can be awarded (unless otherwise stated). However, if the graph is based on previous calculations, ***FT*** marks should be awarded if appropriate.

SECTION A

1. (a) evidence of choosing cosine rule *(M1)*

eg $AC^2 = AB^2 + BC^2 - 2(AB)(BC)\cos(A\hat{B}C)$

correct substitution into the right-hand side *(A1)*

eg $6^2 + 10^2 - 2(6)(10)\cos 100^\circ$

$AC = 12.5234$

$AC = 12.5$ (cm) ***A1*** ***N2***

[3 marks]

(b) evidence of choosing a valid approach *(M1)*

eg sine rule, cosine rule

correct substitution *(A1)*

eg $\frac{\sin(B\hat{C}A)}{6} = \frac{\sin 100^\circ}{12.5}$, $\cos(B\hat{C}A) = \frac{(AC)^2 + 10^2 - 6^2}{2(AC)(10)}$

$B\hat{C}A = 28.1525$

$B\hat{C}A = 28.2^\circ$ ***A1*** ***N2***

[3 marks]

Total [6 marks]

2. (a) 11 terms ***A1*** ***N1***

[1 mark]

(b) evidence of binomial expansion *(M1)*

eg $\binom{n}{r} a^{n-r} b^r$, attempt to expand

evidence of choosing correct term *(A1)*

eg 8^{th} term, $r = 7$, $\binom{10}{7}$, $(x)^3(3)^7$

correct working *(A1)*

eg $\binom{10}{7}(x)^3(3)^7$, $\binom{10}{3}(x)^3(3)^7$,

$262440x^3$ (accept $262000x^3$) ***A1*** ***N3***

[4 marks]

Total [5 marks]

3. (a) (i) $a = 0.486$ (exact) *A1* *N1*
$b = -12.41$ (exact), –12.4 *A1* *N1*

(ii) correct substitution *(A1)*
eg $0.486(172) - 12.41$
71.182
71.2 (kg) *A1* *N2*

[4 marks]

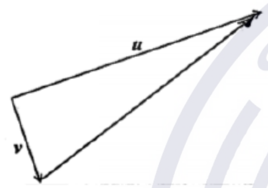
(b) (i) $r = 0.997276$
$r = 0.997$ *A1* *N1*

(ii) strong, positive (must have both correct) *A2* *N2*

[3 marks]

Total [7 marks]

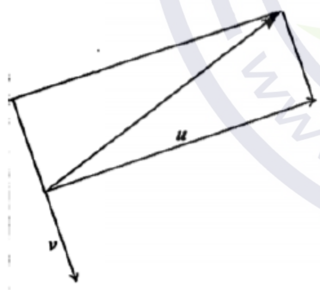
4. (a) **METHOD 1**



A1A1 *N2*

Note: Award ***A1*** for segment connecting endpoints and ***A1*** for direction (must see arrow).

METHOD 2



A1A1 *N2*

Notes: Award ***A1*** for segment connecting endpoints and ***A1*** for direction (must see arrow). Additional lines not required.

[2 marks]

(b) evidence of setting scalar product equal to zero (seen anywhere) ***R1***
eg $\boldsymbol{u} \bullet \boldsymbol{v} = 0$, $15 + 2n + 3 = 0$

correct expression for scalar product *(A1)*
eg $3 \times 5 + 2 \times n + 1 \times 3$, $2n + 18 = 0$

attempt to solve equation *(M1)*
eg $2n = -18$

$n = -9$ *A1* *N3*

[4 marks]

Total [6 marks]

5. (a) $t = 5$ ***(A1)***

correct substitution into formula ***(A1)***
eg $210\sin(0.5\times 5 - 2.6) + 990$, $P(5)$

969.034982…
969 (deer) (must be an integer) ***A1*** ***N3***

[3 marks]

(b) (i) evidence of considering derivative ***(M1)***
eg P'

104.475
104 (deer per month) ***A1*** ***N2***

(ii) (the deer population size is) **increasing** ***A1*** ***N1***

[3 marks]

Total [6 marks]

6. **METHOD 1**

$S_L(0) = 60$ (seen anywhere) ***(A1)***
recognizing need to integrate V_R ***(M1)***
eg $S_R(t) = \int V_R \, \mathrm{d}t$

correct expression
eg $40t - \frac{1}{3}t^3 + C$ ***A1A1***

Note: Award ***A1*** for $40t$, and ***A1*** for $-\frac{1}{3}t^3$.

equate displacements to find C ***(R1)***
eg $40(0) - \frac{1}{3}(0)^3 + C = 60$, $S_L(0) = S_R(0)$

$C = 60$ ***A1***

attempt to find displacement ***(M1)***
eg $S_R(10)$, $40(10) - \frac{1}{3}(10)^3 + 60$

126.666
$126\frac{2}{3}$ (exact), 127 (m) ***A1*** ***N5***

continued …

Question 6 continued

METHOD 2

recognizing need to integrate V_R ***(M1)***

eg $S_R(t) = \int V_R \mathrm{d}t$

valid approach involving a definite integral ***(M1)***

eg $\int_a^b V_R \, \mathrm{d}t$

correct expression with limits ***(A1)***

eg $\int_0^{10} \left(40 - t^2\right) \mathrm{d}t$, $\int_0^{10} V_R \, \mathrm{d}t$, $\left[40t - \frac{1}{3}t^3\right]_0^{10}$

66.6666 ***A2***

$S_L(0) = 60$ (seen anywhere) ***(A1)***

valid approach to find total displacement ***(M1)***

eg $60 + 66.666$

126.666

$126\frac{2}{3}$ (exact), 127 (m) ***A1*** ***N5***

METHOD 3

$S_L(0) = 60$ (seen anywhere) ***(A1)***

recognizing need to integrate V_R ***(M1)***

eg $S_R(t) = \int V_R \, \mathrm{d}t$

correct expression ***A1A1***

eg $40t - \frac{1}{3}t^3 + C$

Note: Award ***A1*** for $40t$, and ***A1*** for $-\frac{1}{3}t^3$.

correct expression for Ramiro displacement ***A1***

eg $S_R(10) - S_R(0)$, $\left[40t - \frac{1}{3}t^3 + C\right]_0^{10}$

66.6666 ***A1***

valid approach to find total displacement ***(M1)***

eg $60 + 66.6666$

126.666

$126\frac{2}{3}$ (exact), 127 (m) ***A1*** ***N5***

[8 marks]

7. recognizing need to find $f(2)$ or $f'(2)$ ***(R1)***

$f(2)=\frac{18}{6}$ (seen anywhere) ***(A1)***

correct substitution into the quotient rule ***(A1)***

eg $\frac{6(5)-18(2)}{6^2}$

$f'(2)=-\frac{6}{36}$ ***A1***

gradient of normal is 6 ***(A1)***

attempt to use the point and gradient to find equation of straight line ***(M1)***

eg $y-f(2)=-\frac{1}{f'(2)}(x-2)$

correct equation in any form ***A1*** ***N4***

eg $y-3=6(x-2)$, $y=6x-9$

[7 marks]

SECTION B

8. (a) (i) 50 (g) ***A1*** ***N1***

(ii) 65 rats weigh less than 70 grams ***(A1)***

attempt to find a percentage ***(M1)***

eg $\frac{65}{80}, \frac{65}{80}\times 100$

81.25 (%) (exact), 81.3 ***A1*** ***N3***

[4 marks]

(b) (i) $p = 10$ ***A2*** ***N2***

(ii) subtracting to find q ***(M1)***

eg $75 - 45 - 10$

$q = 20$ ***A1*** ***N2***

[4 marks]

(c) evidence of mid-interval values ***(M1)***

eg 15, 45, 75, 105

$\bar{x} = 52.5$ (exact), $\sigma = 22.5$ (exact) ***A1A1*** ***N3***

[3 marks]

(d) 0.781650

78.2 (%) ***A2*** ***N2***

[2 marks]

(e) recognize binomial probability ***(M1)***

eg $X \sim \mathrm{B}(n, p)$, $\binom{5}{r}\times 0.782^r \times 0.218^{5-r}$

valid approach ***(M1)***

eg $\mathrm{P}(X \le 3)$

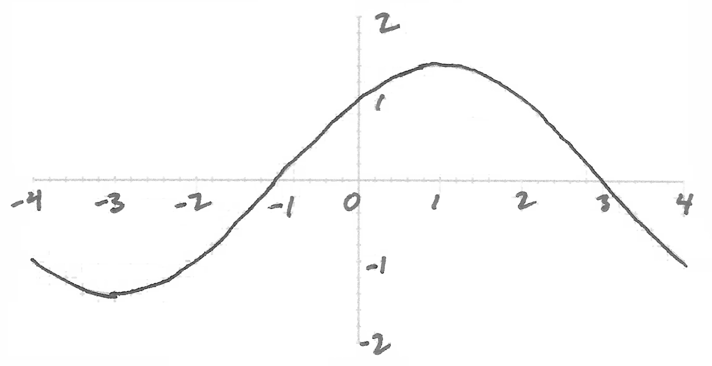
0.30067

0.301 ***A1*** ***N2***

[3 marks]

Total [16 marks]

9. (a)



A1A1A1 *N3*

> **Note:** Award *A1* for approximately correct sinusoidal shape.
> **Only** if this *A1* is awarded, award the following:
> *A1* for correct domain,
> *A1* for approximately correct range.

[3 marks]

(a) recognizes decreasing to the left of minimum or right of maximum,
eg $f'(x) < 0$ ***(R1)***

x-values of minimum and maximum (may be seen on sketch in part (a)) ***(A1)(A1)***
eg $x = -3$, $(1, 1.4)$

two correct intervals *A1A1* *N5*
eg $-4 < x < -3$, $1 \le x \le 4$; $x < -3$, $x \ge 1$

[5 marks]

(c) (i) recognizes that a is found from amplitude of wave ***(R1)***

y-value of minimum or maximum ***(A1)***
eg $(-3, -1.41)$, $(1, 1.41)$

$a = 1.41421$
$a = \sqrt{2}$, (exact), 1.41, *A1* *N3*

continued ...

Question 9 continued

(ii) **METHOD 1**

recognize that shift for sine is found at x-intercept ***(R1)***

attempt to find x-intercept ***(M1)***

eg $\cos\left(\frac{\pi}{4}x\right)+\sin\left(\frac{\pi}{4}x\right)=0$, $x=3+4k$, $k\in\mathbb{Z}$

$x=-1$ ***(A1)***

$c=1$ ***A1*** ***N4***

METHOD 2

attempt to use a coordinate to make an equation ***(R1)***

eg $\sqrt{2}\sin\left(\frac{\pi}{4}c\right)=1$, $\sqrt{2}\sin\left(\frac{\pi}{4}(3-c)\right)=0$

attempt to solve resulting equation ***(M1)***

eg sketch, $x=3+4k$, $k\in\mathbb{Z}$

$x=-1$ ***(A1)***

$c=1$ ***A1*** ***N4***

[7 marks]

Total [15 marks]

10. (a) $x = q$, $y = 3$ (must be equations) *A1A1* *N2*

[2 marks]

(b) recognizing connection between point of intersection and asymptote *(R1)*
eg $x = 1$

$q = 1$ *A1* *N2*

[2 marks]

(c) correct substitution into distance formula *A1*
eg $\sqrt{(x-1)^2 + (y-3)^2}$

attempt to substitute $y = \dfrac{3x}{x-1}$ *(M1)*

eg $\sqrt{(x-1)^2 + \left(\dfrac{3x}{x-1} - 3\right)^2}$

correct simplification of $\left(\dfrac{3x}{x-1} - 3\right)$ *(A1)*

eg $\dfrac{3x - 3(x-1)}{x-1}$

correct expression clearly leading to the required answer *A1*

eg $\dfrac{3x - 3x + 3}{x-1}$, $\sqrt{(x-1)^2 + \left(\dfrac{3x - 3x + 3}{x-1}\right)^2}$

$\text{PQ} = \sqrt{(x-1)^2 + \left(\dfrac{3}{x-1}\right)^2}$ *AG* *N0*

[4 marks]

(d) recognizing that closest is when PQ is a minimum *(R1)*
eg sketch of PQ, $(\text{PQ})'(x) = 0$

$x = -0.73205$ $x = 2.73205$ (seen anywhere) *A1A1*

attempt to find y-coordinates *(M1)*
eg $f(-0.73205)$

$(-0.73205, 1.267949)$, $(2.73205, 4.73205)$
$(-0.732, 1.27)$, $(2.73, 4.73)$ *A1A1* *N4*

[6 marks]

Total [14 marks]

International Baccalaureate®
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MARKSCHEME

May 2014

MATHEMATICS

Standard Level

Paper 2

Instructions to Examiners

All examiners must read these instructions carefully, as there are some changes since M13.

Abbreviations

M Marks awarded for attempting to use a correct **Method**; working must be seen.

(M) Marks awarded for **Method**; may be implied by **correct** subsequent working.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

(A) Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.

R Marks awarded for clear **Reasoning**.

N Marks awarded for **correct** answers if **no** working shown.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Mark according to scoris instructions and the document "**Mathematics SL: Guidance for e-marking May 2014**". It is **essential** that you read this document before you start marking. In particular, please note the following. Marks must be recorded using the annotation stamps, using the new scoris tool. Please check that you are entering marks for the right question.

- If a part is **completely correct**, (and gains all the "must be seen" marks), use the ticks with numbers to stamp full marks. Do **not** use the ticks with numbers for anything else.
- If a part is completely wrong, stamp ***A0*** by the final answer.
- If a part gains anything else, all the working **must** have annotations stamped to show what marks are awarded. This includes any zero marks.

All the marks will be added and recorded by scoris.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any. An exception to this rule is when work for ***M1*** is missing, as opposed to incorrect (see point 4).
- Where ***M*** and ***A*** marks are noted on the same line, *eg* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*eg* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***(M2)***, ***N3***, *etc*., do **not** split the marks, unless there is a note.
- Once a correct answer to a question or part-question is seen, ignore further working.
- Most ***M*** marks are for a **valid** method, ie a method which can lead to the answer: it must indicate some form of progress towards the answer.

3 *N* marks

If ***no*** *working shown, award* ***N*** *marks for* **correct** *answers – this includes acceptable answers (see accuracy booklet). In this case, ignore mark breakdown (****M, A, R****).*

- Do **not** award a mixture of *N* and other marks.
- There may be fewer ***N*** marks available than the total of ***M***, ***A*** and ***R*** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.
- There may not be a direct relationship between the ***N*** marks and the implied marks. There are times when all the marks are implied, but the ***N*** marks are not the full marks: this indicates that we want to see some of the working, without specifying what.
- For consistency within the markscheme, ***N*** marks are noted for every part, even when these match the mark breakdown.
- If a candidate has incorrect working, which somehow results in a correct answer, do **not** award the ***N*** marks for this correct answer. However, if the candidate has indicated (usually by crossing out) that the working is to be ignored, award the ***N*** marks for the correct answer.

4 Implied and must be seen marks

Implied marks appear in ***brackets*** *eg* ***(M1)****.*

- Implied marks can only be awarded if the work is seen or if implied in subsequent working (a correct final answer does not necessarily mean that the implied marks are all awarded). There are questions where some working is required, but as it is accepted that not everyone will write the same steps, all the marks are implied, but the ***N*** marks are not the full marks for the question.
- Normally the correct work is seen in the next line.
- Where there is an (***M1***) followed by ***A1*** for each correct answer, if no working shown, one correct answer is sufficient evidence to award the (***M1)***.

Must be seen marks appear without ***brackets*** *eg* ***M1****.*

- Must be seen marks can only be awarded if the work is seen.
- If a must be seen mark is not awarded because work is missing (as opposed to ***M0*** or ***A0*** for incorrect work) all subsequent marks may be awarded if appropriate.

5 Follow through marks (only applied after an error is made)

*Follow through (****FT****) marks are awarded where an incorrect answer (final or intermediate) from one* ***part*** *of a question is used correctly in* ***subsequent*** *part(s) or subpart(s). Usually, to award* ***FT*** *marks,* ***there must be working present*** *and not just a final answer based on an incorrect answer to a previous part. However, if the only marks awarded in a subpart are for the* ***final*** *answer, then* ***FT*** *marks should be awarded if appropriate. Examiners are expected to check student work in order to award* ***FT*** *marks where appropriate.*

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** and ***R*** marks may be awarded if appropriate. (However, as noted above, if an ***A*** mark is not awarded because work is missing, all subsequent marks may be awarded if appropriate).
- Exceptions to this rule will be explicitly noted on the markscheme.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*eg* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word "**their**" in a description, to indicate that candidates may be using an incorrect value.
- If a candidate makes an error in one part, but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the question says hence. It is often possible to use a different approach in subsequent parts that does not depend on the answer to previous parts.
- In a "show that" question, if an error in a previous subpart leads to not showing the required answer, do not award the final ***A1***. Note that if the error occurs within the same subpart, the ***FT*** rules may result in further loss of marks.
- Where there are anticipated common errors, the ***FT*** answers are often noted on the markscheme, to help examiners. It should be stressed that these are not the only ***FT*** answers accepted, neither should ***N*** marks be awarded for these answers.

6 Mis-read

*If a candidate incorrectly copies information from the question, this is a mis-read (**MR**). A candidate should be penalized only once for a particular mis-read. Use the **MR** stamp to indicate that this is a misread. Do not award the first mark in the question, even if this is an **M** mark, but award all others (if appropriate) so that the candidate only loses one mark for the misread.*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*eg* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.

7 Discretionary marks *(d)*

*An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. In such cases the annotation DM should be used and a brief **note** written next to the mark explaining this decision.*

8 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete parts are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**. Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9 Alternative forms

*Unless the question specifies otherwise, **accept** equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

10 Accuracy of Answers

If the level of accuracy is specified in the question, a mark will be allocated for giving the final answer to the required accuracy. When this is not specified in the question, all numerical answers should be given exactly or correct to three significant figures

Candidates should NO LONGER be penalized for an accuracy error ***(AP)****. Examiners should award marks according to the rules given in these instructions and the markscheme. Accuracy is not the same as correctness – an incorrect value does not achieve relevant* ***A*** *marks. It is only final answers which may lose marks for accuracy errors, not intermediate values. Please check work carefully for* ***FT****. Further information on which answers are accepted is given in a separate booklet, along with examples. It is* **essential** *that you read this carefully.*

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Intermediate values do not need to be given to the correct three significant figures. But, if candidates work with rounded values, this could lead to an incorrect answer, in which case award ***A0*** for the final answer. However, do not penalise inaccurate intermediate values that lead to an acceptable final answer.

11 Calculators

A GDC is required for paper 2, but calculators with symbolic manipulation features (eg TI-89) are not allowed.

Calculator notation
The Mathematics SL guide says:

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The instructions tell candidates not to write on Section B of the QP. Thus they may well have done some rough work here which they assume will be ignored. If they have solutions on the answer booklets, there is no need to look at the QP. However, if there are whole questions or whole part solutions missing on answer booklets, please check to make sure that they are not on the QP, and if they are, mark those whole questions or whole part solutions that have not been written on answer booklets

14. Diagrams

The notes on how to allocate marks for sketches usually refer to passing through particular points or having certain features. These marks can only be awarded if the sketch is approximately the correct shape. All values given will be an approximate guide to where these points/features occur. In some questions, the first ***A1*** is for the shape, in others, the marks are only for the points and/or features. In both cases, unless the shape is approximately correct, no marks can be awarded (unless otherwise stated). However, if the graph is based on previous calculations, ***FT*** marks should be awarded if appropriate.

SECTION A

1. (a) (i) correct substitution into arc length formula *(A1)*

eg 0.7×5

arc length $= 3.5$ (cm) ***A1*** ***N2***

(ii) valid approach *(M1)*

eg $3.5 + 5 + 5$, $\text{arc} + 2r$

perimeter $= 13.5$ (cm) ***A1*** ***N2***

[4 marks]

(b) correct substitution into area formula *(A1)*

eg $\frac{1}{2}(0.7)(5)^2$

area $= 8.75\,(\text{cm}^2)$ ***A1*** ***N2***

[2 marks]

Total [6 marks]

2. (a) recognizing $f(x) = 0$ *(M1)*

eg $f = 0$, $x^2 = 5$

$x = \pm 2.23606$

$x = \pm\sqrt{5}$ (exact), $x = \pm 2.24$ ***A1A1*** ***N3***

[3 marks]

(b) attempt to substitute either limits or the function into formula involving f^2 *(M1)*

eg $\pi\int\left(5 - x^2\right)^2 \mathrm{d}x$, $\pi\int_{-2.24}^{2.24}\left(x^4 - 10x^2 + 25\right)$, $2\pi\int_0^{\sqrt{5}} f^2$

187.328

volume $= 187$ ***A2*** ***N3***

[3 marks]

Total [6 marks]

3. (a) (i) $a = 0.0823604$, $b = 0.306186$

$a = 0.0824$, $b = 0.306$ *A1A1* *N2*

(ii) correct explanation with reference to number of litres required for 1 km *A1* *N1*

eg a represents the (average) amount of fuel (litres) required to drive 1 km, (average) litres per kilometre, (average) rate of change in fuel used for each km travelled

[3 marks]

(b) valid approach *(M1)*

eg $y = 0.0824(110) + 0.306$, sketch

9.36583

9.37 (litres) *A1* *N2*

[2 marks]

Total [5 marks]

4. (a) correct substitution *(A1)*

eg 0.3×0.6

$\mathrm{P}(A \cap B) = 0.18$ *A1* *N2*

[2 marks]

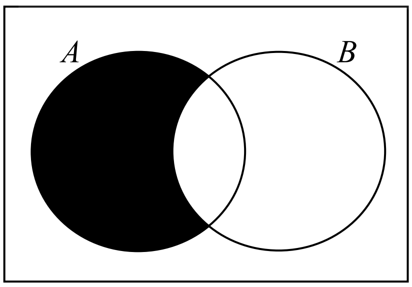
(b) correct substitution *(A1)*

eg $\mathrm{P}(A \cup B) = 0.3 + 0.6 - 0.18$

$\mathrm{P}(A \cup B) = 0.72$ *A1* *N2*

[2 marks]

(c) (i)



A1 *N1*

(ii) appropriate approach *(M1)*

eg $0.3 - 0.18$, $\mathrm{P}(A) \times \mathrm{P}(B')$

$\mathrm{P}(A \cap B') = 0.12$ (may be seen in Venn diagram) *A1* *N2*

[3 marks]

Total [7 marks]

5. (a) correct substitution into area formula *(A1)*

eg $\frac{1}{2}(6)(8)\sin A = 16$, $\sin A = \frac{16}{24}$

correct working *(A1)*

eg $A = \arcsin\left(\frac{2}{3}\right)$

$A = 0.729727656\ldots, 2.41186499\ldots$; $(41.8103149^\circ, 138.1896851^\circ)$

$A = 0.730$; 2.41 *A1A1* *N3*

(accept degrees *ie* 41.8°; 138°)

[4 marks]

(b) evidence of choosing cosine rule *(M1)*

eg $\mathrm{BC}^2 = \mathrm{AB}^2 + \mathrm{AC}^2 - 2(\mathrm{AB})(\mathrm{AC})\cos A$, $a^2 + b^2 - 2ab\cos C$

correct substitution into RHS (angle must be obtuse) *(A1)*

eg $\mathrm{BC}^2 = 6^2 + 8^2 - 2(6)(8)\cos 2.41$, $6^2 + 8^2 - 2(6)(8)\cos 138^\circ$,

$\mathrm{BC} = \sqrt{171.55}$

$\mathrm{BC} = 13.09786$

$\mathrm{BC} = 13.1\,\mathrm{cm}$ *A1* *N2*

[3 marks]

Total [7 marks]

6. (a) $r = -4$ ***A2*** ***N2***

> **Note:** Award ***A1*** for $r = 4$.

[2 marks]

(b) (i) evidence of valid approach ***(M1)***

eg $\frac{\text{max } y \text{ value} - \text{min } y \text{ value}}{2}$, distance from $y = 10$

$p = 8$ ***A1*** ***N2***

(ii) valid approach ***(M1)***

eg period is 24, $\frac{360}{24}$, substitute a point into **their** $f(x)$

$q = \frac{2\pi}{24}$ $\left(\frac{\pi}{12}, \text{exact}\right)$, 0.262 (do not accept degrees) ***A1*** ***N2***

[4 marks]

(c) valid approach ***(M1)***

eg line on graph at $y = 7$, $8\cos\left(\frac{2\pi}{24}(x-4)\right) + 10 = 7$

$x = 11.46828$

$x = 11.5$ (accept (11.5, 7)) ***A1*** ***N2***

[2 marks]

> **Note:** Do not award the final ***A1*** if additional values are given. If an incorrect value of q leads to multiple solutions, award the final ***A1*** only if **all** solutions within the domain are given.

Total [8 marks]

7. valid approach **(M1)**

eg $\binom{8}{r}(3x^2)^{8-r}\left(\frac{k}{x}\right)^r$,

$(3x^2)^8 + \binom{8}{1}(3x^2)^7\left(\frac{k}{x}\right) + \binom{8}{2}(3x^2)^6\left(\frac{k}{x}\right)^2 + \ldots$, Pascal's triangle to 9th line

attempt to find value of r which gives term in x^0 **(M1)**

eg exponent in binomial must give x^{-2}, $x^2(x^2)^{8-r}\left(\frac{k}{x}\right)^r = x^0$

correct working **(A1)**

eg $2(8-r)-r=-2$, $18-3r=0$, $2r+(-8+r)=-2$

evidence of correct term **(A1)**

eg $\binom{8}{2}$, $\binom{8}{6}(3x^2)^2\left(\frac{k}{x}\right)^6$, $r=6$, $r=2$

equating **their** term and 16128 to solve for k ***M1***

eg $x^2\binom{8}{6}(3x^2)^2\left(\frac{k}{x}\right)^6 = 16128$, $k^6 = \frac{16128}{28(9)}$

$k = \pm 2$ ***A1A1*** ***N2***

Note: If no working shown, award ***N0*** for $k=2$.

Total [7 marks]

SECTION B

8. (a) correct substitution into formula *(A1)*

eg $12e^{0.4(0)}$

12 bacteria in the dish ***A1*** ***N2***

[2 marks]

(b) correct substitution into formula *(A1)*

eg $12e^{0.4(4)}$

59.4363 *(A1)*

59 bacteria in the dish (integer answer only) ***A1*** ***N3***

[3 marks]

(c) correct equation *(A1)*

eg $A(t) = 400$, $12e^{0.4t} = 400$

valid attempt to solve *(M1)*

eg graph, use of logs

8.76639

8.77 (hours) ***A1*** ***N3***

[3 marks]

(d) valid attempt to solve *(M1)*

eg $n(4) = 60$, $60 = 24e^{4k}$, use of logs

correct working *(A1)*

eg sketch of intersection, $4k = \ln 2.5$

$k = 0.229072$

$k = \frac{\ln 2.5}{4}$ (exact), $k = 0.229$ ***A1*** ***N3***

[3 marks]

continued …

Question 8 continued

(e) **METHOD 1**

setting up an equation or inequality (accept any variable for n) *(M1)*

eg $A(t) > B(t)$, $12\mathrm{e}^{0.4n} = 24\mathrm{e}^{0.229n}$, $\mathrm{e}^{0.4n} = 2\mathrm{e}^{0.229n}$

correct working *(A1)*

eg sketch of intersection, $\mathrm{e}^{0.171n} = 2$

4.05521 (accept 4.05349) *(A1)*

$n = 5$ (integer answer only) ***A1*** ***N3***

METHOD 2

$A(4) = 59$, $B(4) = 60$ (from earlier work)

$A(5) = 88.668$, $B(5) = 75.446$ ***A1A1***

valid reasoning *(R1)*

eg $A(4) < B(4)$ **and** $A(5) > B(5)$

$n = 5$ (integer answer only) ***A1*** ***N3***

[4 marks]

Total [15 marks]

9. (a) substituting $t=1$ into v *(M1)*

eg $v(1), \left(1^2-4\right)^3$

velocity $=-27 \ \left(\text{ms}^{-1}\right)$ *A1* *N2*

[2 marks]

(b) valid reasoning *(R1)*

eg $v=0$, $\left(t^2-4\right)^3=0$

correct working *(A1)*

eg $t^2-4=0$, $t=\pm 2$, sketch

$t=2$ *A1* *N2*

[3 marks]

(c) correct integral expression for distance *(A1)*

eg $\int_0^3 |v|$, $\int \left|\left(t^2-4\right)^3\right|$, $-\int_0^2 v\,dt+\int_2^3 v\,dt$,

$\int_0^2 \left(4-t^2\right)^3 \mathrm{d}t+\int_2^3 \left(t^2-4\right)^3 \mathrm{d}t$ (do not accept $\int_0^3 v\,\mathrm{d}t$)

86.2571

distance $=86.3$ (m) *A2* *N3*

[3 marks]

(d) evidence of differentiating velocity *(M1)*

eg $v'(t)$

$a=3\left(t^2-4\right)^2(2t)$ *A2*

$a=6t\left(t^2-4\right)^2$ *AG* *N0*

[3 marks]

(e) **METHOD 1**

valid approach *M1*

eg graphs of v and a

correct working *(A1)*

eg areas of same sign indicated on graph

$2<t\le 3$ (accept $t>2$) *A2* *N2*

METHOD 2

recognizing that $a\ge 0$ (accept *a is* always positive) (seen anywhere) *R1*

recognizing that v is positive when $t>2$ (seen anywhere) *(R1)*

$2<t\le 3$ (accept $t>2$) *A2* *N2*

[4 marks]

Total [15 marks]

10. (a) (i) valid approach ***(M1)***

eg $\mathrm{P}(G)=\mathrm{P}(H>60)$, $z=0.875$, $\mathrm{P}(H>60)=1-0.809$, $\mathrm{N}(53, 8^2)$

0.190786

$\mathrm{P}(G)=0.191$ ***A1*** ***N2***

(ii) finding $\mathrm{P}(H>70)=0.01679$ (seen anywhere) ***(A1)***

recognizing conditional probability ***(R1)***

eg $\mathrm{P}(A\,|\,B)$, $\mathrm{P}(H>70\,|\,H>60)$

correct working ***(A1)***

eg $\dfrac{0.01679}{0.191}$

0.0880209

$\mathrm{P}(X>70\,|\,G)=0.0880$ ***A1*** ***N3***

[6 marks]

(b) attempt to square their $\mathrm{P}(G)$ ***(M1)***

eg 0.191^2

0.0363996

$\mathrm{P}(\text{both } G)=0.0364$ ***A1*** ***N2***

[2 marks]

(c) (i) correct substitution into formula for $\mathrm{E}(X)$ ***(A1)***

eg $100(0.191)$

$\mathrm{E}(G)=19.1$ [19.0, 19.1] ***A1*** ***N2***

(ii) recognizing binomial probability (may be seen in part (c)(i)) ***(R1)***

eg $X \sim \mathrm{B}(n, p)$

valid approach (seen anywhere) ***(M1)***

eg $\mathrm{P}(X\geq 25)=1-\mathrm{P}(X\leq 24)$, $1-\mathrm{P}(X<a)$

correct working ***(A1)***

eg $\mathrm{P}(X\leq 24)=0.913\ldots$, $1-0.913\ldots$

0.0869002

$P(X\geq 25)=0.0869$ ***A1*** ***N2***

[6 marks]

Total [14 marks]

International Baccalaureate®
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MARKSCHEME

November 2013

MATHEMATICS

Standard Level

Paper 2

16 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**; working must be seen.

(M) Marks awarded for **Method**; may be implied by **correct** subsequent working.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

(A) Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.

R Marks awarded for clear **Reasoning.**

N Marks awarded for **correct** answers if **no** working shown.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Mark according to scoris instructions and the document "**Mathematics SL: Guidance for e-marking November 2013**". It is **essential** that you read this document before you start marking. In particular, please note the following. Marks must be recorded using the annotation stamps, using the new scoris tool. Please check that you are entering marks for the right question.

- If a part is **completely correct**, (and gains all the "must be seen" marks), use the ticks with numbers to stamp full marks. Do **not** use the ticks with numbers for anything else.
- If a part is completely wrong, stamp ***A0*** by the final answer.
- If a part gains anything else, all the working **must** have annotations stamped to show what marks are awarded. This includes any zero marks.

All the marks will be added and recorded by scoris.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any. An exception to this rule is when work for ***M1*** is missing, as opposed to incorrect (see point 4).
- Where ***M*** and ***A*** marks are noted on the same line, *eg* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*eg* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***(M2)***, ***N3***, *etc*., do **not** split the marks, unless there is a note.
- Once a correct answer to a question or part-question is seen, ignore further working.
- Most ***M*** marks are for a **valid** method, ie a method which can lead to the answer: it must indicate some form of progress towards the answer.

3 *N* marks

*If **no** working shown, award **N** marks for **correct** answers – this includes acceptable answers (see accuracy booklet). In this case, ignore mark breakdown (**M**, **A**, **R**).*

- Do **not** award a mixture of ***N*** and other marks.
- There may be fewer ***N*** marks available than the total of ***M***, ***A*** and ***R*** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.
- There may not be a direct relationship between the ***N*** marks and the implied marks. There are times when all the marks are implied, but the ***N*** marks are not the full marks: this indicates that we want to see some of the working, without specifying what.
- For consistency within the markscheme, ***N*** marks are noted for every part, even when these match the mark breakdown.
- If a candidate has incorrect working, which somehow results in a correct answer, do **not** award the ***N*** marks for this correct answer. However, if the candidate has indicated (usually by crossing out) that the working is to be ignored, award the ***N*** marks for the correct answer.

4 Implied and must be seen marks

*Implied marks appear in **brackets** eg **(M1)**.*

- Implied marks can only be awarded if the work is seen or if implied in subsequent working (a correct final answer does not necessarily mean that the implied marks are all awarded). There are questions where some working is required, but as it is accepted that not everyone will write the same steps, all the marks are implied, but the ***N*** marks are not the full marks for the question.
- Normally the correct work is seen in the next line.
- Where there is an (***M1***) followed by ***A1*** for each correct answer, if no working shown, one correct answer is sufficient evidence to award the (***M1***).

*Must be seen marks appear without **brackets** eg **M1**.*

- Must be seen marks can only be awarded if the work is seen.
- If a must be seen mark is not awarded because work is missing (as opposed to ***M0*** or ***A0*** for incorrect work) all subsequent marks may be awarded if appropriate.

5 Follow through marks (only applied after an error is made)

*Follow through (**FT**) marks are awarded where an incorrect answer (final or intermediate) from one **part** of a question is used correctly in **subsequent** part(s) or subpart(s). Usually, to award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if the only marks awarded in a subpart are for the **final** answer, then **FT** marks should be awarded if appropriate. Examiners are expected to check student work in order to award **FT** marks where appropriate.*

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** and ***R*** marks may be awarded if appropriate. (However, as noted above, if an ***A*** mark is not awarded because work is missing, all subsequent marks may be awarded if appropriate)
- Exceptions to this rule will be explicitly noted on the markscheme.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*eg* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word "**their**" in a description, to indicate that candidates may be using an incorrect value.
- If a candidate makes an error in one part, but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the question says hence. It is often possible to use a different approach in subsequent parts that does not depend on the answer to previous parts.

- In a "show that" question, if an error in a previous subpart leads to not showing the required answer, do not award the final ***A1***. Note that if the error occurs within the same subpart, the ***FT*** rules may result in further loss of marks.
- Where there are anticipated common errors, the ***FT*** answers are often noted on the markscheme, to help examiners. It should be stressed that these are not the only ***FT*** answers accepted, neither should ***N*** marks be awarded for these answers.

6 Mis-read

*If a candidate incorrectly copies information from the question, this is a mis-read (**MR**). A candidate should be penalized only once for a particular mis-read. Use the **MR** stamp to indicate that this is a misread. Do not award the first mark in the question, even if this is an **M** mark, but award all others (if appropriate) so that the candidate only loses one mark for the misread.*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*eg* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.

7 Discretionary marks *(d)*

*An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. In such cases the annotation DM should be used and a brief **note** written next to the mark explaining this decision.*

8 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete parts are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**. Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9 Alternative forms

*Unless the question specifies otherwise, **accept** equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

10 Accuracy of Answers

If the level of accuracy is specified in the question, a mark will be allocated for giving the final answer to the required accuracy. When this is not specified in the question, all numerical answers should be given exactly or correct to three significant figures

*Candidates should NO LONGER be penalized for an accuracy error **(AP)**. Examiners should award marks according to the rules given in these instructions and the markscheme. Accuracy is not the same as correctness – an incorrect value does not achieve relevant **A** marks. It is only final answers which may lose marks for accuracy errors, not intermediate values. Please check work carefully for **FT**. Further information on which answers are accepted is given in a separate booklet, along with examples. It is **essential** that you read this carefully.*

Do not accept unfinished numerical final answers such as 3/0.1 (unless otherwise stated). As a rule, numerical answers with more than one part (such as fractions) should be given using integers (*eg* 6/8). Calculations which lead to integers should be completed, with the exception of fractions which are not whole numbers.

Clarification of intermediate values accuracy instructions

Intermediate values do not need to be given to the correct three significant figures. But, if candidates work with rounded values, this could lead to an incorrect answer, in which case award ***A0*** for the final answer. However, do not penalise inaccurate intermediate values that lead to an acceptable final answer.

11 Calculators

A GDC is required for paper 2, but calculators with symbolic manipulation features (eg TI-89) are not allowed.

Calculator notation

The Mathematics SL guide says:

> *Students must always use correct mathematical notation, not calculator notation.*

Do **not** accept final answers written using calculator notation. However, do not penalize the use of calculator notation in the working.

12 Style

The markscheme aims to present answers using good communication, eg if the question asks to find the value of k, the markscheme will say $k = 3$*, but the marks will be for the correct value 3 – there is usually no need for the "* $k =$ *". In these cases, it is also usually acceptable to have another variable, as long as there is no ambiguity in the question, eg if the question asks to find the value of p and of q, then the student answer needs to be clear. Generally, the only situation where the full answer is required is in a question which asks for equations – in this case the markscheme will say "must be an equation".*

*The markscheme often uses words to describe what the marks are for, followed by examples, using the eg notation. These examples are not exhaustive, and examiners should check what candidates have written, to see if they satisfy the description. Where these marks are **M** marks, the examples may include ones using poor notation, to indicate what is acceptable. A valid method is one which will allow candidate to proceed to the next step eg if a quadratic function is given in factorised form, and the question asks for the zeroes, then multiplying the factors does not necessarily help to find the zeros, and would not on its own count as a valid method.*

13 Candidate work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

14. Diagrams

The notes on how to allocate marks for sketches usually refer to passing through particular points or having certain features. These marks can only be awarded if the sketch is approximately the correct shape. All values given will be an approximate guide to where these points/features occur. In some questions, the first ***A1*** is for the shape, in others, the marks are only for the points and/or features. In both cases, unless the shape is approximately correct, no marks can be awarded (unless otherwise stated). However, if the graph is based on previous calculations, ***FT*** marks should be awarded if appropriate.

SECTION A

1. (a) $\boldsymbol{A}^{-1} = \begin{pmatrix} -1 & 4 & -1.5 \\ 1 & -4 & 2 \\ -1 & 5 & -2.5 \end{pmatrix}$ ***A2*** ***N2***

Note: Award ***A1*** for 6, 7 or 8 correct elements.

[2 marks]

(b) attempt to solve equation ***(M1)***

eg multiplying by $\boldsymbol{A}^{-1}$, setting up system of equations

$\boldsymbol{X} = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}$ (accept $x = 2,\ y = 3,\ z = -1$) ***A2*** ***N3***

[3 marks]

Total [5 marks]

2. (a) valid approach ***(M1)***

eg $f(x) = 0$, sketch of parabola showing two *x*-intercepts

$x = 1,\ x = 4$ $\left(\text{accept } (1, 0), (4, 0)\right)$ ***A1A1*** ***N3***

[3 marks]

(b) attempt to substitute either limits or the function into formula involving f^2 ***(M1)***

eg $\int_1^4 \left(f(x)\right)^2 \mathrm{d}x$, $\pi\int\left((x-1)(x-4)\right)^2$

volume $= 8.1\pi$ (exact), 25.4 ***A2*** ***N3***

[3 marks]

Total [6 marks]

3. (a) expressing f as $x^{\frac{4}{3}}$ *(M1)*

$f'(x)=\frac{4}{3}x^{\frac{1}{3}}\ \left(=\frac{4}{3}\sqrt[3]{x}\right)$ *A1* *N2*

[2 marks]

(b) attempt to integrate $\sqrt[3]{x^4}$ *(M1)*

eg $\frac{x^{\frac{4}{3}+1}}{\frac{4}{3}+1}$

$\int f(x)\mathrm{d}x=\frac{3}{7}x^{\frac{7}{3}}-\frac{x}{2}+c$ *A1A1A1* *N4*

[4 marks]

Total [6 marks]

4. (a) correct approach *(A1)*

eg $0.5=0.2+\mathrm{P}(B)$, $\mathrm{P}(A\cap B)=0$

$\mathrm{P}(B)=0.3$ *A1* *N2*

[2 marks]

(b) Correct expression for $\mathrm{P}(A\cap B)$ (seen anywhere) *A1*

eg $\mathrm{P}(A\cap B)=0.2\mathrm{P}(B)$, $0.2x$

attempt to substitute into correct formula for $\mathrm{P}(A\cup B)$ *(M1)*

eg $\mathrm{P}(A\cup B)=0.2+\mathrm{P}(B)-\mathrm{P}(A\cap B)$, $\mathrm{P}(A\cup B)=0.2+x-0.2x$

correct working *(A1)*

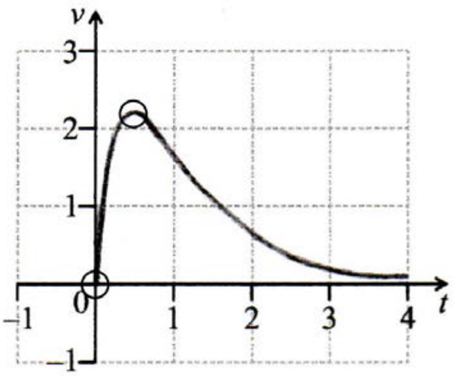
eg $0.5=0.2+\mathrm{P}(B)-0.2\mathrm{P}(B)$, $0.8x=0.3$

$\mathrm{P}(B)=\frac{3}{8}$ $(=0.375$, exact$)$ *A1* *N3*

[4 marks]

Total [6 marks]

5. (a)



A1A2 ***N3***

Notes: Award ***A1*** for approximately correct domain $0 \le t \le 4$.

The shape must be approximately correct, with maximum skewed left. **Only** if the shape is approximately correct, award ***A2*** for all the following approximately correct features, in circle of tolerance where drawn (accept seeing correct coordinates for the maximum, even if point outside circle):

Maximum point, passes through origin, asymptotic to t-axis (but must not touch the axis).

If only two of these features are correct, award ***A1.***

[3 marks]

(b) valid approach (including 0 and 3) ***(M1)***

eg $\int_0^3 10te^{-1.7t}\mathrm{d}t$, $\int_0^3 f(x)$, area from 0 to 3 (may be shaded in diagram)

distance $= 3.33$ (m) ***A1*** ***N2***

[2 marks]

(c) recognizing acceleration is derivative of velocity ***(R1)***

eg $a = \frac{\mathrm{d}v}{\mathrm{d}t}$, attempt to find $\frac{\mathrm{d}v}{\mathrm{d}t}$, reference to maximum on the graph of v

valid approach to find v when $a = 0$ (may be seen on graph) ***(M1)***

eg $\frac{\mathrm{d}v}{\mathrm{d}t} = 0$, $10e^{-1.7t} - 17te^{-1.7t} = 0$, $t = 0.588$

velocity $= 2.16$ $(\mathrm{m\,s^{-1}})$ ***A1*** ***N3***

Note: Award ***R1M1A0*** for $(0.588, 2.16)$ if velocity is not identified as final answer

[3 marks]

Total [8 marks]

6.

> **Note**: There may be slight differences in answers, depending on whether candidates use tables or GDCs, or their 3 sf answers in subsequent parts. Do not penalise answers that are consistent with **their** working and check carefully for ***FT***.

(a) attempt to standardize *(M1)*

eg $z = \dfrac{21.8 - 20}{1.25}$, 1.44

$\mathrm{P}(T < 21.8) = 0.925$ ***A1*** ***N2***

[2 marks]

(b) attempt to subtract probabilities *(M1)*

eg $\mathrm{P}(T < 21.8) - \mathrm{P}(T < k) = 0.3$, 0.925–0.3

$\mathrm{P}(T < k) = 0.625$ ***A1***

EITHER

finding the z-value for 0.625 *(A1)*

eg $z = 0.3186$ (from tables), $z = 0.3188$

attempt to set up equation using **their** z-value *(M1)*

eg $0.3186 = \dfrac{k - 20}{1.25}$, $-0.524 \times 1.25 = k - 20$

$k = 20.4$ ***A1*** ***N3***

OR

$k = 20.4$ ***A3*** ***N3***

[5 marks]

Total [7 marks]

7. (a) (i) valid approach (may be seen on diagram) *(M1)*

eg Q to 6 is x

$\mathrm{PQ} = 6 - 2x$ ***A1*** ***N2***

(ii) $A = (6 - 2x)\sqrt{6x - x^2}$ ***A1*** ***N1***

[3 marks]

(b) (i) recognising $\dfrac{\mathrm{d}A}{\mathrm{d}x}$ at $x = 2$ needed (must be the derivative of area) *(M1)*

$\dfrac{\mathrm{d}A}{\mathrm{d}x} = -\dfrac{7\sqrt{2}}{2}$, -4.95 ***A1*** ***N2***

(ii) $a = 0.879$ $b = 3$ ***A1A1*** ***N2***

[4 marks]

Total [7 marks]

SECTION B

8. **Notes:** In this question, there may be slight differences in answers, depending on which values candidates carry through in subsequent parts. Accept answers that are consistent with their working.
Candidates may have their GDCs in degree mode, leading to incorrect answers. If working shown, award marks in line with the markscheme, with ***FT*** as appropriate.
Ignore missing or incorrect units.

(a) evidence of choosing sine rule ***(M1)***

eg $\dfrac{\sin \hat{A}}{a} = \dfrac{\sin \hat{B}}{b}$

correct substitution ***(A1)***

eg $\dfrac{\sin \hat{A}}{10.4} = \dfrac{\sin 1.058}{12.2}$

$\text{B}\hat{\text{A}}\text{C} = 0.837$ ***A1*** ***N2***

[3 marks]

(b) **METHOD 1**

evidence of subtracting angles from π ***(M1)***

eg $\text{A}\hat{\text{B}}\text{C} = \pi - A - C$

correct angle (seen anywhere) ***A1***

$\text{A}\hat{\text{B}}\text{C} = \pi - 1.058 - 0.837$, $1.246, 71.4^\circ$

attempt to substitute into cosine or sine rule ***(M1)***

correct substitution ***(A1)***

eg $12.2^2 + 10.4^2 - 2 \times 12.2 \times 10.4 \cos 71.4$, $\dfrac{\text{AC}}{\sin 1.246} = \dfrac{12.2}{\sin 1.058}$

$\text{AC} = 13.3$ (cm) ***A1*** ***N3***

METHOD 2

evidence of choosing cosine rule ***M1***

eg $a^2 = b^2 + c^2 - 2bc \cos A$

correct substitution ***(A2)***

eg $12.2^2 = 10.4^2 + b^2 - 2 \times 10.4 b \cos 1.058$

$\text{AC} = 13.3$ (cm) ***A2*** ***N3***

[5 marks]

continued …

Question 8 continued

(c) **METHOD 1**

valid approach *(M1)*

eg $\cos \mathrm{A\hat{O}C} = \dfrac{\mathrm{OA}^2 + \mathrm{OC}^2 - \mathrm{AC}^2}{2 \times \mathrm{OA} \times \mathrm{OC}}$, $\mathrm{A\hat{O}C} = 2 \times \mathrm{A\hat{B}C}$

correct working *(A1)*

eg $13.3^2 = 7^2 + 7^2 - 2 \times 7 \times 7 \cos \mathrm{A\hat{O}C}$, $O = 2 \times 1.246$

$\mathrm{A\hat{O}C} = 2.492 \quad (142.8^\circ)$ *(A1)*

EITHER

correct substitution for arc length (seen anywhere) ***A1***

eg $2.492 = \dfrac{l}{7}$, $l = 17.4$, $14\pi \times \dfrac{142.8}{360}$

subtracting arc from circumference *(M1)*

eg $2\pi r - l$, $14\pi - 17.4$

OR

attempt to find $\mathrm{A\hat{O}C}$ reflex *(M1)*

eg $2\pi - 2.492$, 3.79, $360 - 142.8$

correct substitution for arc length (seen anywhere) ***A1***

eg $l = 7 \times 3.79$, $14\pi \times \dfrac{217.2}{360}$

THEN

arc $\mathrm{ABC} = 26.5$ ***A1*** ***N4***

METHOD 2

valid approach to find $\mathrm{A\hat{O}B}$ or $\mathrm{B\hat{O}C}$ *(M1)*

eg choosing cos rule, twice angle at circumference

correct working for finding **one** value, $\mathrm{A\hat{O}B}$ or $\mathrm{B\hat{O}C}$ *(A1)*

eg $\cos \mathrm{A\hat{O}B} = \dfrac{7^2 + 7^2 - 12.2^2}{2 \times 7 \times 7}$, $\mathrm{A\hat{O}B} = 2.116$, $\mathrm{B\hat{O}C} = 1.6745$

two correct calculations for arc lengths

eg $\mathrm{AB} = 7 \times 2 \times 1.058 (= 14.8135)$, $7 \times 1.6745 (= 11.7216)$ *(A1)(A1)*

adding **their** arc lengths (seen anywhere)

eg $r\mathrm{A\hat{O}B} + r\,\mathrm{B\hat{O}C}$, $14.8135 + 11.7216$, $7(2.116 + 1.6745)$ ***M1***

arc $\mathrm{ABC} = 26.5$ (cm) ***A1*** ***N4***

Note: Candidates may work with other interior triangles using a similar method. Check calculations carefully and award marks in line with markscheme.

[6 marks]

Total [14 marks]

9. (a) appropriate approach *(M1)*

eg $\begin{pmatrix}11\\8\\2\end{pmatrix}+s\begin{pmatrix}4\\3\\-1\end{pmatrix}=\begin{pmatrix}1\\1\\-7\end{pmatrix}+t\begin{pmatrix}2\\1\\11\end{pmatrix}$, $L_1=L_2$

any **two** correct equations *A1A1*

eg $11+4s=1+2t$, $8+3s=1+t$, $2-s=-7+11t$

attempt to solve system of equations *(M1)*

eg $10+4s=2(7+3s)$, $\begin{cases}4s-2t=-10\\3s-t=-7\end{cases}$

one correct parameter *A1*

eg $s=-2$, $t=1$

$\mathrm{P}(3, 2, 4)$ (accept position vector) *A1* *N3*

[6 marks]

(b) choosing correct direction vectors for L_1 and L_2 *(A1)(A1)*

eg $\begin{pmatrix}4\\3\\-1\end{pmatrix}$, $\begin{pmatrix}2\\1\\11\end{pmatrix}$ (or any scalar multiple)

evidence of scalar product (with any vectors) *(M1)*

eg $a\bullet b$, $\begin{pmatrix}4\\3\\-1\end{pmatrix}\bullet\begin{pmatrix}2\\1\\11\end{pmatrix}$

correct substitution *A1*

eg $4(2)+3(1)+(-1)(11)$, $8+3-11$

calculating $a\bullet b=0$ *A1*

Note: Do not award the final ***A1*** without evidence of calculation.

vectors are perpendicular *AG* *N0*

[5 marks]

continued …

Question 9 continued

(c)

Note: Candidates may take different approaches, which do not necessarily involve vectors. In particular, most of the working could be done on a diagram. Award marks in line with the markscheme.

METHOD 1

attempt to find $\overrightarrow{QP}$ or $\overrightarrow{PQ}$ ***(M1)***

correct working (may be seen on diagram) ***A1***

eg $\overrightarrow{QP} = \begin{pmatrix} -4 \\ -3 \\ 1 \end{pmatrix}$, $\overrightarrow{PQ} = \begin{pmatrix} 7 \\ 5 \\ 3 \end{pmatrix} - \begin{pmatrix} 3 \\ 2 \\ 4 \end{pmatrix}$

recognizing R is on L_1 (seen anywhere) ***(R1)***
eg on diagram

Q and R are equidistant from P (seen anywhere) ***(R1)***
eg $\overrightarrow{QP} = \overrightarrow{PR}$, marked on diagram

correct working ***(A1)***

eg $\begin{pmatrix} 3 \\ 2 \\ 4 \end{pmatrix} - \begin{pmatrix} 7 \\ 5 \\ 3 \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} - \begin{pmatrix} 3 \\ 2 \\ 4 \end{pmatrix}$, $\begin{pmatrix} -4 \\ -3 \\ 1 \end{pmatrix} + \begin{pmatrix} 3 \\ 2 \\ 4 \end{pmatrix}$

R $(-1, -1, 5)$ (accept position vector) ***A1 N3***

METHOD 2

recognizing R is on L_1 (seen anywhere) ***(R1)***
eg on diagram

Q and R are equidistant from P (seen anywhere) ***(R1)***
eg P midpoint of QR, marked on diagram

valid approach to find **one** coordinate of mid-point ***(M1)***

eg $x_P = \dfrac{x_Q + x_R}{2}$, $2y_P = y_Q + y_R$, $\dfrac{1}{2}(z_Q + z_R)$

one correct substitution ***A1***

eg $x_{\text{R}} = 3 + (3 - 7)$, $2 = \dfrac{5 + y_R}{2}$, $4 = \dfrac{1}{2}(z + 3)$

correct working for one coordinate ***(A1)***
eg $x_{\text{R}} = 3 - 4$, $4 - 5 = y_R$, $8 = (z + 3)$

R $(-1, -1, 5)$ (accept position vector) ***A1 N3***

[6 marks]

Total [17 marks]

10. (a) appropriate approach *(M1)*

eg $\mathrm{P}(R \cap B) + \mathrm{P}(R' \cap B)$, tree diagram,

one correct multiplication *(A1)*

eg 0.2×0.5, 0.24

correct working *(A1)*

eg $0.2 \times 0.5 + 0.8 \times 0.3$, $0.1 + 0.24$

$\mathrm{P}(\text{bus}) = 0.34$ (exact) ***A1*** ***N3***

[4 marks]

(b) recognizing conditional probability *(R1)*

eg $\mathrm{P}(A|B) = \dfrac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$

correct working ***A1***

eg $\dfrac{0.2 \times 0.5}{0.34}$

$\mathrm{P}(R|B) = \dfrac{5}{17}$, 0.294 ***A1*** ***N2***

[3 marks]

(c) recognizing binomial probability *(R1)*

eg $X \sim \mathrm{B}(n,\ p)$, $\binom{5}{3}(0.34)^3$, $(0.34)^3(1-0.34)^2$

$\mathrm{P}(X = 3) = 0.171$ ***A1*** ***N2***

[2 marks]

(d) **METHOD 1**

evidence of using complement (seen anywhere) *(M1)*

eg $1 - \mathrm{P}(\text{none})$, $1 - 0.95$

valid approach *(M1)*

eg $1 - \mathrm{P}(\text{none}) > 0.95$, $\mathrm{P}(\text{none}) < 0.05$, $1 - \mathrm{P}(\text{none}) = 0.95$

correct inequality (accept equation) ***A1***

eg $1 - (0.66)^n > 0.95$, $(0.66)^n = 0.05$

$n > 7.209$ (accept $n = 7.209$) *(A1)*

$n = 8$ ***A1*** ***N3***

METHOD 2

valid approach using guess and check/trial and error *(M1)*

eg finding $\mathrm{P}(X \geq 1)$ for various values of n

seeing the "cross over" values for the probabilities ***A1A1***

$n = 7, \mathrm{P}(X \geq 1) = 0.9454$, $n = 8, \mathrm{P}(X \geq 1) = 0.9639$

recognising $0.9639 > 0.95$ *(R1)*

$n = 8$ ***A1*** ***N3***

[5 marks]

Total [14 marks]

M13/5/MATME/SP2/ENG/TZ1/XX/M

International Baccalaureate
Baccalauréat International
Bachillerato Internacional

MARKSCHEME

May 2013

MATHEMATICS

Standard Level

Paper 2

16 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**; working must be seen.

(M) Marks awarded for **Method**; may be implied by **correct** subsequent working.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

(A) Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.

R Marks awarded for clear **Reasoning.**

N Marks awarded for **correct** answers if **no** working shown.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Mark according to scoris instructions and the document "**Mathematics SL: Guidance for e-marking May 2013**". It is **essential** that you read this document before you start marking. In particular, please note the following. Marks must be recorded using the annotation stamps, using the new scoris tool. Please check that you are entering marks for the right question.

- If a part is **completely correct**, (and gains all the "must be seen" marks), use the ticks with numbers to stamp full marks.
- If a part is completely wrong, stamp ***A0*** by the final answer.
- If a part gains anything else, it **must** be recorded using **all** the annotations.

All the marks will be added and recorded by scoris.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any. An exception to this rule is when work for ***M1*** is missing, as opposed to incorrect (see point 4).
- Where ***M*** and ***A*** marks are noted on the same line, *eg* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*eg* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***(M2)***, ***N3***, *etc*., do **not** split the marks, unless there is a note.
- Once a correct answer to a question or part-question is seen, ignore further working.
- Most ***M*** marks are for a **valid** method, ie a method which can lead to the answer: it must indicate some form of progress towards the answer.
- ***A*** marks are often dependent on the ***R*** mark being awarded for justification for the ***A*** mark, in which case it is not possible to award ***A1R0***. Hence the ***A1*** is not awarded for a correct answer if no justification or the wrong justification is given.

3 ***N* marks**

*If **no** working shown, award **N** marks for **correct** answers. In this case, ignore mark breakdown (**M**, **A**, **R**).*

- Do **not** award a mixture of ***N*** and other marks.
- There may be fewer ***N*** marks available than the total of ***M***, ***A*** and ***R*** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.
- There may not be a direct relationship between the ***N*** marks and the implied marks. There are times when all the marks are implied, but the ***N*** marks are not the full marks: this indicates that we want to see some of the working, without specifying what.
- For consistency within the markscheme, ***N*** marks are noted for every part, even when these match the mark breakdown.
- If a candidate has incorrect working, which somehow results in a correct answer, do **not** award the ***N*** marks for this correct answer. However, if the candidate has indicated (usually by crossing out) that the working is to be ignored, award the ***N*** marks for the correct answer.

4 **Implied and must be seen marks**

*Implied marks appear in **brackets eg (M1)**.*

- Implied marks can only be awarded if **correct** work is seen or if implied in subsequent working (a correct final answer does not necessarily mean that the implied marks are all awarded). There are questions where some working is required, but as it is accepted that not everyone will write the same steps, all the marks are implied, but the ***N*** marks are not the full marks for the question.
- Normally the correct work is seen or implied in the next line.
- Where there is an (***M1***) followed by ***A1*** for each correct answer, if no working shown, one correct answer is sufficient evidence to award the (***M1)***. An exception to this is where at least one numerical final answer is not given to the correct three significant figures (see the accuracy booklet).

*Must be seen marks appear without **brackets eg M1**.*

- Must be seen marks can only be awarded if **correct** work is seen.
- If a must be seen mark is not awarded because work is missing (as opposed to ***M0*** or ***A0*** for incorrect work) all subsequent marks may be awarded if appropriate.

5 **Follow through marks (only applied after an error is made)**

*Follow through (**FT**) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) or subpart(s). Usually, to award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if the only marks awarded in a subpart are for the **final** answer, then **FT** marks should be awarded if appropriate. Examiners are expected to check student work in order to award **FT** marks where appropriate.*

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate. (However, as noted above, if an ***A*** mark is not awarded because work is missing, all subsequent marks may be awarded if appropriate)
- Exceptions to this rule will be explicitly noted on the markscheme.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*eg* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non integer value where integer required), do not award the mark(s) for the final answer(s).

- The markscheme may use the word "**their**" in a description, to indicate that candidates may be using an incorrect value.
- If a candidate makes an error in one part, but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the question says hence. It is often possible to use a different approach in subsequent parts that does not depend on the answer to previous parts.
- In a "show that" question, if an error in a previous subpart leads to not showing the required answer, do not award the final ***A1***. Note that if the error occurs within the same subpart, the ***FT*** rules may result in further loss of marks.
- Where there are anticipated common errors, the ***FT*** answers are often noted on the markscheme, to help examiners. It should be stressed that these are not the only ***FT*** answers accepted, neither should ***N*** marks be awarded for these answers.

6 Mis-read

*If a candidate incorrectly copies information from the question, this is a mis-read (**MR**). A candidate should be penalized only once for a particular mis-read. Use the **MR** stamp to indicate that this is a misread. Do not award the first mark in the question, even if this is an **M** mark, but award all others (if appropriate) so that the candidate only loses one mark for the misread.*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*eg* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.

7 Discretionary marks *(d)*

An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. In such cases the annotation DM should be used and a brief **note** *written next to the mark explaining this decision.*

8 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for part-questions are indicated by **EITHER** . . . **OR**.
- Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9 Alternative forms

*Unless the question specifies otherwise, **accept** equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

10 Accuracy of Answers

If the level of accuracy is specified in the question, a mark will be allocated for giving the final answer to the required accuracy. When this is not specified in the question, all numerical answers should be given exactly or correct to three significant figures

*Candidates should NO LONGER be penalized for an accuracy error **(AP)**. Examiners should award marks according to the rules given in these instructions and the markscheme. Accuracy is not the same as correctness – an incorrect value does not achieve relevant **A** marks. It is only final answers which may lose marks for accuracy errors, not intermediate values. Please check work carefully for **FT**. Further information on which answers are accepted is given in a separate booklet, along with examples. It is **essential** that you read this carefully, as there are a number of changes.*

Do not accept unfinished numerical final answers such as 3/0.1 (unless otherwise stated). As a rule, numerical answers with more than one part (such as fractions) should be given using integers (*eg* 6/8). Calculations which lead to integers should be completed, with the exception of fractions which are not whole numbers.

Clarification of intermediate values accuracy instructions

Intermediate values do not need to be given to the correct three significant figures. If candidates work with **any** rounded values, this could lead to an incorrect answer, in which case award ***A0*** for the final answer. However, do not penalise inaccurate intermediate values that lead to an acceptable final answer.

11 Calculators

A GDC is required for paper 2, but calculators with symbolic manipulation features (eg TI-89) are not allowed.

Calculator notation
The Mathematics SL guide says:

> *Students must always use correct mathematical notation, not calculator notation.*

Do **not** accept final answers written using calculator notation. However, do not penalize the use of calculator notation in the working.

12 Style

The markscheme aims to present answers using good communication, eg if the question asks to find the value of k, the markscheme will say $k = 3$, but the marks will be for the correct value 3 – there is usually no need for the "$k =$". In these cases, it is also usually acceptable to have another variable, as long as there is no ambiguity in the question, eg if the question asks to find the value of p and of q, then the student answer needs to be clear. Generally, the only situation where the full answer is required is in a question which asks for equations – in this case the markscheme will say "must be an equation".

*The markscheme often uses words to describe what the marks are for, followed by examples, using the eg notation. These examples are not exhaustive, and examiners should check what candidates have written, to see if they satisfy the description. Where these marks are **M** marks, the examples may include ones using poor notation, to indicate what is acceptable. A valid method is one which will allow candidate to proceed to the next step eg if a quadratic function is given in factorised form, and the question asks for the zeroes, then multiplying the factors does not necessarily help to find the zeros, and would not on its own count as a valid method.*

13 Candidate work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

14. Diagrams

The notes on how to allocate marks for sketches usually refer to passing through particular points or having certain features. These marks can only be awarded if the sketch is approximately the correct shape. All values given will be an approximate guide to where these points/features occur. In some questions, the first ***A1*** is for the shape, in others, the marks are only for the points and/or features. In both cases, unless the shape is approximately correct, no marks can be awarded (unless otherwise stated). However, if the graph is based on previous calculations, ***FT*** marks should be awarded if appropriate.

SECTION A

1. (a) $d = 3$ ***A1*** ***N1***

[1 mark]

(b) (i) correct substitution into term formula ***(A1)***

eg $u_{100} = 5 + 3(99)$, $5 + 3(100 - 1)$

$u_{100} = 302$ ***A1*** ***N2***

(ii) correct substitution into sum formula ***(A1)***

eg $S_{100} = \frac{100}{2}\left(2(5) + 99(3)\right)$, $S_{100} = \frac{100}{2}(5 + 302)$

$S_{100} = 15350$ ***A1*** ***N2***

[4 marks]

(c) correct substitution into term formula ***(A1)***

eg $1502 = 5 + 3(n - 1)$, $1502 = 3n + 2$

$n = 500$ ***A1*** ***N2***

[2 marks]

Total [7 marks]

2. (a) valid approach ***(M1)***

eg $35 - 26$, $26 + p = 35$

$p = 9$ ***A1*** ***N2***

[2 marks]

(b) (i) $\text{mean} = 26.7$ ***A2*** ***N2***

(ii) recognizing that variance is $(\text{sd})^2$ ***(M1)***

eg $11.021\ldots^2$, $\sigma = \sqrt{\text{var}}$, $11.158\ldots^2$

$\sigma^2 = 121$ ***A1*** ***N2***

[4 marks]

Total [6 marks]

3. (a) $p = 5$, $q = 7$, $r = 7$ (accept $r = 5$) ***A1A1A1*** ***N3***

[3 marks]

(b) correct working ***(A1)***

eg $\binom{12}{7} \times (3x)^5 \times (-2)^7$, 792, 243, -2^7, 24634368

coefficient of term in x^5 is -24634368 ***A1*** ***N2***

Note: Do not award the final ***A1*** for an answer that contains *x*.

[2 marks]

Total [5 marks]

4. (a) (i) $\boldsymbol{A} = \begin{pmatrix} -1 & -1 & 1 \\ 1 & 1 & 0 \\ -2 & -1 & 2 \end{pmatrix}$ ***A1*** ***N1***

(ii) $\boldsymbol{A}^{-1} = \begin{pmatrix} 2 & 1 & -1 \\ -2 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$ ***A2*** ***N2***

Note: Award ***A1*** for 6, 7 or 8 correct elements.

[3 marks]

(b) evidence of multiplying by $\boldsymbol{A}^{-1}$ (in any order) ***(M1)***

eg $\boldsymbol{X} = \boldsymbol{A}^{-1}\boldsymbol{B}$, $\boldsymbol{BA}^{-1}$, one correct element

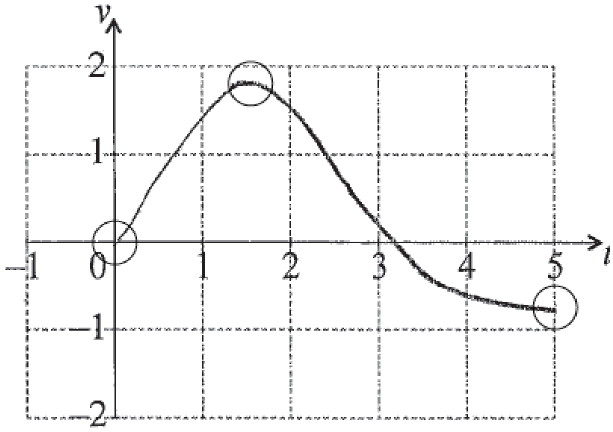
$\boldsymbol{X} = \begin{pmatrix} 9 \\ -8 \\ 3.5 \end{pmatrix}$ (accept $x = 9$, $y = -8$, $z = 3.5$) ***A2*** ***N3***

Note: Award ***A1*** for two correct elements.

[3 marks]

Total [6 marks]

5. (a)



A1A1A1 *N3*

> **Note:** Award ***A1*** for approximately correct shape crossing *x*-axis with $3 < x < 3.5$.
> **Only** if this ***A1*** is awarded, award the following:
> ***A1*** for maximum in circle, ***A1*** for endpoints in circle.

[3 marks]

(b) (i) $t = \pi\,(\text{exact}),\ 3.14$ *A1* *N1*

(ii) recognizing distance is area under velocity curve *(M1)*

eg $s = \int v$, shading on diagram, attempt to integrate v

valid approach to find the total area *(M1)*

eg area A $+$ area B, $\int v\mathrm{d}t - \int v\mathrm{d}t$, $\int_0^{3.14} v\mathrm{d}t + \int_{3.14}^{5} v\mathrm{d}t$, $\int |v|$

correct working with integration and limits (accept dx or missing dt) *(A1)*

eg $\int_0^{3.14} v\mathrm{d}t + \int_5^{3.14} v\mathrm{d}t$, $3.067\ldots + 0.878\ldots$, $\int_0^5 \left|\mathrm{e}^{\sin t} - 1\right|$

distance $= 3.95\,(\mathrm{m})$ *A1* *N3*

[5 marks]

Total [8 marks]

6. (a) (i) $k = 2$ *A1* *N1*

(ii) $p = -1$ *A1* *N1*

(iii) $q = 5$ *A1* *N1*

[3 marks]

(b) recognizing one transformation *(M1)*

eg horizontal stretch by $\frac{1}{3}$, reflection in *x*-axis,

A′ is $(2, -5)$ *A1A1* *N3*

[3 marks]

Total [6 marks]

7. recognizing one quartile probability (may be seen in a sketch) ***(M1)***

eg $\mathrm{P}(X < Q_3) = 0.75$, 0.25

finding standardized value for either quartile ***(A1)***

eg $z = 0.67448\ldots$, $z = -0.67448\ldots$

attempt to set up equation (must be with z – values) ***(M1)***

eg $0.67 = \dfrac{Q_3 - 150}{10}$, $-0.67448 = \dfrac{x - 150}{10}$

one correct quartile

eg $Q_3 = 156.74\ldots$, $Q_1 = 143.25\ldots$ ***(A1)***

correct working ***(A1)***

eg other correct quartile, $Q_3 - \mu = 6.744\ldots$

valid approach for IQR (seen anywhere) ***(A1)***

eg $Q_3 - Q_1$, $2(Q_3 - \mu)$

$\text{IQR} = 13.5$ ***A1*** ***N4***

[7 marks]

SECTION B

8. (a) evidence of choosing cosine rule *(M1)*

eg $c^2 = a^2 + b^2 - 2ab\cos C$, $\text{CD}^2 + \text{AD}^2 - 2 \times \text{CD} \times \text{AD}\cos D$

correct substitution ***A1***

eg $11.5^2 + 8^2 - 2 \times 11.5 \times 8\cos 104$, $196.25 - 184\cos 104$

$\text{AC} = 15.5\,(\text{m})$ ***A1*** ***N2***

[3 marks]

(b) (i) **METHOD 1**

evidence of choosing sine rule *(M1)*

eg $\dfrac{\sin A}{a} = \dfrac{\sin B}{b}$, $\dfrac{\sin \text{A}\hat{\text{C}}\text{D}}{\text{AD}} = \dfrac{\sin D}{\text{AC}}$

correct substitution ***A1***

eg $\dfrac{\sin \text{A}\hat{\text{C}}\text{D}}{8} = \dfrac{\sin 104}{15.516\ldots}$

$\text{A}\hat{\text{C}}\text{D} = 30.0^\circ$ ***A1*** ***N2***

METHOD 2

evidence of choosing cosine rule *(M1)*

eg $c^2 = a^2 + b^2 - 2ab\cos C$

correct substitution ***A1***

eg $8^2 = 11.5^2 + 15.516\ldots^2 - 2(11.5)(15.516\ldots)\cos C$

$\text{A}\hat{\text{C}}\text{D} = 30.0^\circ$ ***A1*** ***N2***

(ii) subtracting **their** $\text{A}\hat{\text{C}}\text{D}$ from 73 *(M1)*

eg $73 - \text{A}\hat{\text{C}}\text{D}$, $70 - 30.017...$

$\text{A}\hat{\text{C}}\text{B} = 43.0^\circ$ ***A1*** ***N2***

[5 marks]

(c) correct substitution *(A1)*

eg area $\Delta\,\text{ADC} = \dfrac{1}{2}(8)(11.5)\sin 104$

$\text{area} = 44.6\ (\text{m}^2)$ ***A1*** ***N2***

[2 marks]

(d) attempt to subtract *(M1)*

eg $\text{circle} - \text{ABCD}$, $\pi r^2 - \Delta\,\text{ADC} - \Delta\,\text{ACB}$

area $\Delta\,\text{ACB} = \dfrac{1}{2}(15.516\ldots)(14)\sin 42.98\ (= 74.0517\ldots)$ *(A1)*

correct working ***A1***

eg $\pi(8)^2 - 44.6336\ldots - \dfrac{1}{2}(15.516\ldots)(14)\sin 42.98$, $64\pi - 44.6 - 74.1$

shaded area is 82.4 (m^2) ***A1*** ***N3***

[4 marks]

Total [14 marks]

9. (a) $f(0)=\frac{100}{51}$ (exact), 1.96 ***A1*** ***N1***

[1 mark]

(b) setting up equation ***(M1)***

eg $95=\frac{100}{1+50\mathrm{e}^{-0.2x}}$, sketch of graph with horizontal line at $y=95$

$x=34.3$ ***A1*** ***N2***

[2 marks]

(c) upper bound of y is 100 ***(A1)***

lower bound of y is 0 ***(A1)***

range is $0<y<100$ ***A1*** ***N3***

[3 marks]

(d) **METHOD 1**

setting function ready to apply the chain rule ***(M1)***

eg $100\left(1+50\mathrm{e}^{-0.2x}\right)^{-1}$

evidence of correct differentiation (must be substituted into chain rule) ***(A1)(A1)***

eg $u'=-100\left(1+50\mathrm{e}^{-0.2x}\right)^{-2}$, $v'=\left(50\mathrm{e}^{-0.2x}\right)(-0.2)$

correct chain rule derivative ***A1***

eg $f'(x)=-100\left(1+50\mathrm{e}^{-0.2x}\right)^{-2}\left(50\mathrm{e}^{-0.2x}\right)(-0.2)$

correct working clearly leading to the required answer ***A1***

eg $f'(x)=1000\mathrm{e}^{-0.2x}\left(1+50\mathrm{e}^{-0.2x}\right)^{-2}$

$f'(x)=\frac{1000\mathrm{e}^{-0.2x}}{\left(1+50\mathrm{e}^{-0.2x}\right)^{2}}$ ***AG*** ***N0***

METHOD 2

attempt to apply the quotient rule (accept reversed numerator terms) ***(M1)***

eg $\frac{vu'-uv'}{v^2}$, $\frac{uv'-vu'}{v^2}$

evidence of correct differentiation inside the quotient rule ***(A1)(A1)***

eg $f'(x)=\frac{\left(1+50e^{-0.2x}\right)(0)-100\left(50e^{-0.2x}\times-0.2\right)}{\left(1+50e^{-0.2x}\right)^2}$, $\frac{100(-10)\mathrm{e}^{-0.2x}-0}{\left(1+50\mathrm{e}^{-0.2x}\right)^2}$

any correct expression for derivative (0 may not be explicitly seen) ***(A1)***

eg $\frac{-100\left(50\mathrm{e}^{-0.2x}\times-0.2\right)}{\left(1+50\mathrm{e}^{-0.2x}\right)^2}$

correct working clearly leading to the required answer ***A1***

eg $f'(x)=\frac{0-100(-10)\mathrm{e}^{-0.2x}}{\left(1+50\mathrm{e}^{-0.2x}\right)^2}$, $\frac{-100(-10)\mathrm{e}^{-0.2x}}{\left(1+50\mathrm{e}^{-0.2x}\right)^2}$

$f'(x)=\frac{1000\mathrm{e}^{-0.2x}}{\left(1+50\mathrm{e}^{-0.2x}\right)^{2}}$ ***AG*** ***N0***

[5 marks]

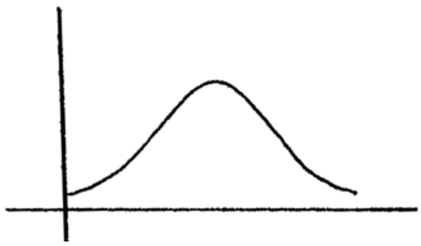
continued …

Question 9 continued

(e) **METHOD 1**

sketch of $f'(x)$ ***(A1)***

eg



recognizing maximum on $f'(x)$ ***(M1)***

eg dot on max of sketch

finding maximum on graph of $f'(x)$ ***A1***

eg $(19.6, 5)$, $x = 19.560\ldots$

maximum rate of increase is 5 ***A1*** ***N2***

[4 marks]

METHOD 2

recognizing $f''(x) = 0$ ***(M1)***

finding any correct expression for $f''(x)$ ***(A1)***

eg $$\frac{\left(1+50\mathrm{e}^{-0.2x}\right)^2\left(-200\mathrm{e}^{-0.2x}\right)-\left(1000\mathrm{e}^{-0.2x}\right)\left(2\left(1+50\mathrm{e}^{-0.2x}\right)\left(-10\mathrm{e}^{-0.2x}\right)\right)}{\left(1+50\mathrm{e}^{-0.2x}\right)^4}$$

finding $x = 19.560\ldots$ ***A1***

maximum rate of increase is 5 ***A1*** ***N2***

[4 marks]

Total [15 marks]

10. (a) valid approach *(M1)*

eg $13+\text{diameter},\ 13+122$

maximum height $=135$ (m) ***A1*** ***N2***

[2 marks]

(b) (i) $\text{period}=\dfrac{60}{2.4}$ ***A1***

$\text{period}=25$ (minutes) ***AG*** ***N0***

(ii) $b=\dfrac{2\pi}{25}\quad (=0.08\pi)$ ***A1*** ***N1***

[2 marks]

(c) **METHOD 1**

valid approach *(M1)*

eg $\max-74,\ |a|=\dfrac{135-13}{2},\ 74-13$

$|a|=61$ (accept $a=61$) *(A1)*

$a=-61$ ***A1*** ***N2***

[3 marks]

METHOD 2

attempt to substitute valid point into equation for h *(M1)*

eg $135=74+a\cos\left(\dfrac{2\pi\times 12.5}{25}\right)$

correct equation *(A1)*

eg $135=74+a\cos(\pi),\ 13=74+a$

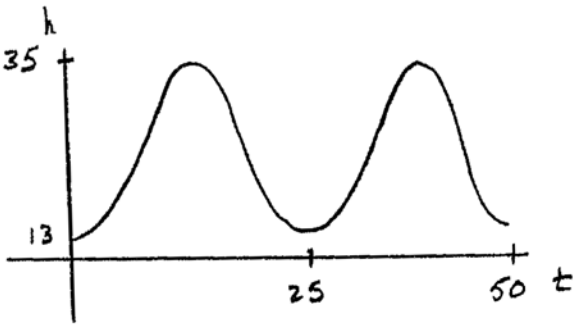
$a=-61$ ***A1*** ***N2***

[3 marks]

continued …

Question 10 continued

(d)



A1A1A1A1 ***N4***

> **Note:** Award ***A1*** for approximately correct domain, ***A1*** for approximately correct range, ***A1*** for approximately correct sinusoidal shape with 2 cycles.
> **Only** if this last ***A1*** awarded, award ***A1*** for max/min in approximately correct positions.

[4 marks]

(e) setting up inequality (accept equation) ***(M1)***

eg $h > 105$, $105 = 74 + a\cos bt$, sketch of graph with line $y = 105$

any **two** correct values for t (seen anywhere) ***A1A1***

eg $t = 8.371\ldots$, $t = 16.628\ldots$, $t = 33.371\ldots$, $t = 41.628\ldots$,

valid approach ***M1***

eg $\frac{16.628 - 8.371}{25}$, $\frac{t_1 - t_2}{25}$, $\frac{2 \times 8.257}{50}$, $\frac{2(12.5 - 8.371)}{25}$

$p = 0.330$ ***A1*** ***N2***

[5 marks]

Total [16 marks]

M13/5/MATME/SP2/ENG/TZ2/XX/M

MARKSCHEME

May 2013

MATHEMATICS

Standard Level

Paper 2

15 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**; working must be seen.

(M) Marks awarded for **Method**; may be implied by **correct** subsequent working.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

(A) Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.

R Marks awarded for clear **Reasoning.**

N Marks awarded for **correct** answers if **no** working shown.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Mark according to scoris instructions and the document "**Mathematics SL: Guidance for e-marking May 2013**". It is **essential** that you read this document before you start marking. In particular, please note the following. Marks must be recorded using the annotation stamps, using the new scoris tool. Please check that you are entering marks for the right question.

- If a part is **completely correct**, (and gains all the "must be seen" marks), use the ticks with numbers to stamp full marks.
- If a part is completely wrong, stamp ***A0*** by the final answer.
- If a part gains anything else, it **must** be recorded using **all** the annotations.

All the marks will be added and recorded by scoris.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any. An exception to this rule is when work for ***M1*** is missing, as opposed to incorrect (see point 4).
- Where ***M*** and ***A*** marks are noted on the same line, *eg* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*eg* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***(M2)***, ***N3***, *etc*., do **not** split the marks, unless there is a note.
- Once a correct answer to a question or part-question is seen, ignore further working.
- Most ***M*** marks are for a valid method, *ie* a method which can lead to the answer: it must indicate some form of progress towards the answer.
- *A* marks are often dependent on the ***R*** mark being awarded for justification for the *A* mark, in which case it is not possible to award ***A1R0***. Hence the ***A1*** is not awarded for a correct answer if no justification or the wrong justification is given.

3 ***N* marks**

*If **no** working shown, award **N** marks for **correct** answers. In this case, ignore mark breakdown (**M**, **A**, **R**).*

- Do **not** award a mixture of ***N*** and other marks.
- There may be fewer ***N*** marks available than the total of ***M***, ***A*** and ***R*** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.
- There may not be a direct relationship between the ***N*** marks and the implied marks. There are times when all the marks are implied, but the ***N*** marks are not the full marks: this indicates that we want to see some of the working, without specifying what.
- For consistency within the markscheme, ***N*** marks are noted for every part, even when these match the mark breakdown.
- If a candidate has incorrect working, which somehow results in a correct answer, do **not** award the ***N*** marks for this correct answer. However, if the candidate has indicated (usually by crossing out) that the working is to be ignored, award the ***N*** marks for the correct answer.

4 **Implied and must be seen marks**

*Implied marks appear in **brackets eg (M1)**.*

- Implied marks can only be awarded if **correct** work is seen or if implied in subsequent working (a correct final answer does not necessarily mean that the implied marks are all awarded). There are questions where some working is required, but as it is accepted that not everyone will write the same steps, all the marks are implied, but the ***N*** marks are not the full marks for the question.
- Normally the correct work is seen or implied in the next line.
- Where there is an (***M1***) followed by ***A1*** for each correct answer, if no working shown, one correct answer is sufficient evidence to award the ***(M1)***. An exception to this is where at least one numerical final answer is not given to the correct three significant figures (see the accuracy booklet).

*Must be seen marks appear without **brackets eg M1**.*

- Must be seen marks can only be awarded if **correct** work is seen.
- If a must be seen mark is not awarded because work is missing (as opposed to ***M0*** or ***A0*** for incorrect work) all subsequent marks may be awarded if appropriate.

5 **Follow through marks (only applied after an error is made)**

*Follow through **(FT)** marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) or subpart(s). Usually, to award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if the only marks awarded in a subpart are for the **final** answer, then **FT** marks should be awarded if appropriate. Examiners are expected to check student work in order to award **FT** marks where appropriate.*

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate. (However, as noted above, if an ***A*** mark is not awarded because work is missing, all subsequent marks may be awarded if appropriate)
- Exceptions to this rule will be explicitly noted on the markscheme.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*eg* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non integer value where integer required), do not award the mark(s) for the final answer(s).

- The markscheme may use the word "**their**" in a description, to indicate that candidates may be using an incorrect value.
- If a candidate makes an error in one part, but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the question says hence. It is often possible to use a different approach in subsequent parts that does not depend on the answer to previous parts.
- In a "show that" question, if an error in a previous subpart leads to not showing the required answer, do not award the final ***A1***. Note that if the error occurs within the same subpart, the ***FT*** rules may result in further loss of marks.
- Where there are anticipated common errors, the ***FT*** answers are often noted on the markscheme, to help examiners. It should be stressed that these are not the only ***FT*** answers accepted, neither should ***N*** marks be awarded for these answers.

6 Mis-read

*If a candidate incorrectly copies information from the question, this is a mis-read (**MR**). A candidate should be penalized only once for a particular mis-read. Use the **MR** stamp to indicate that this is a misread. Do not award the first mark in the question, even if this is an **M** mark, but award all others (if appropriate) so that the candidate only loses one mark for the misread.*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*eg* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.

7 Discretionary marks *(d)*

*An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. In such cases the annotation DM should be used and a brief **note** written next to the mark explaining this decision.*

8 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for part-questions are indicated by **EITHER** . . . **OR**.
- Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9 Alternative forms

*Unless the question specifies otherwise, **accept** equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

10 Accuracy of Answers

If the level of accuracy is specified in the question, a mark will be allocated for giving the final answer to the required accuracy. When this is not specified in the question, all numerical answers should be given exactly or correct to three significant figures

*Candidates should NO LONGER be penalized for an accuracy error **(AP)**. Examiners should award marks according to the rules given in these instructions and the markscheme. Accuracy is not the same as correctness – an incorrect value does not achieve relevant **A** marks. It is only final answers which may lose marks for accuracy errors, not intermediate values. Please check work carefully for **FT**. Further information on which answers are accepted is given in a separate booklet, along with examples. It is **essential** that you read this carefully, as there are a number of changes.*

Do not accept unfinished numerical final answers such as 3/0.1 (unless otherwise stated). As a rule, numerical answers with more than one part (such as fractions) should be given using integers (*eg* 6/8). Calculations which lead to integers should be completed, with the exception of fractions which are not whole numbers.

Clarification of intermediate values accuracy instructions

Intermediate values do not need to be given to the correct three significant figures. If candidates work with **any** rounded values, this could lead to an incorrect answer, in which case award ***A0*** for the final answer. However, do not penalise inaccurate intermediate values that lead to an acceptable final answer.

11 Calculators

A GDC is required for paper 2, but calculators with symbolic manipulation features (eg TI-89) are not allowed.

Calculator notation
The Mathematics SL guide says:

Students must always use correct mathematical notation, not calculator notation.

Do **not** accept final answers written using calculator notation. However, do not penalize the use of calculator notation in the working.

12 Style

The markscheme aims to present answers using good communication, eg if the question asks to find the value of k, the markscheme will say $k = 3$, but the marks will be for the correct value 3 – there is usually no need for the "$k =$". In these cases, it is also usually acceptable to have another variable, as long as there is no ambiguity in the question, eg if the question asks to find the value of p and of q, then the student answer needs to be clear. Generally, the only situation where the full answer is required is in a question which asks for equations – in this case the markscheme will say "must be an equation".

*The markscheme often uses words to describe what the marks are for, followed by examples, using the eg notation. These examples are not exhaustive, and examiners should check what candidates have written, to see if they satisfy the description. Where these marks are **M** marks, the examples may include ones using poor notation, to indicate what is acceptable. A valid method is one which will allow candidate to proceed to the next step eg if a quadratic function is given in factorised form, and the question asks for the zeroes, then multiplying the factors does not necessarily help to find the zeros, and would not on its own count as a valid method.*

13 Candidate work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

14. Diagrams

The notes on how to allocate marks for sketches usually refer to passing through particular points or having certain features. These marks can only be awarded if the sketch is approximately the correct shape. All values given will be an approximate guide to where these points/features occur. In some questions, the first ***A1*** is for the shape, in others, the marks are only for the points and/or features. In both cases, unless the shape is approximately correct, no marks can be awarded (unless otherwise stated). However, if the graph is based on previous calculations, ***FT*** marks should be awarded if appropriate.

SECTION A

1. (a) $\boldsymbol{A}^{-1} = \begin{pmatrix} 0 & 1 & -1 \\ 1 & 0 & 1 \\ -1 & 1 & -1 \end{pmatrix}$ ***A2*** ***N2***

> **Note:** Award ***A1*** for 6, 7, or 8 correct elements.

[2 marks]

(b) evidence of attempting to solve equation ***(M1)***

eg multiply by $\boldsymbol{A}^{-1}$ (on left or right), setting up system of equations, 1 or 2 correct elements

$\boldsymbol{X} = \begin{pmatrix} -4 \\ 4 \\ -5 \end{pmatrix}$ (accept $x = -4$, $y = 4$, $z = -5$) ***A2*** ***N3***

> **Note:** Award ***A1*** for two correct elements.

[3 marks]

Total [5 marks]

2. (a) evidence of appropriate approach ***(M1)***

eg $z = \dfrac{22.9 - 20}{5}$

$z = 0.58$ ***(A1)***

$\mathrm{P}(X \le 22.9) = 0.719$ ***A1*** ***N3***

[3 marks]

(b) z-score for 0.55 is 0.12566… ***(A1)***

valid approach (must be with z – values) ***(M1)***

eg using inverse normal, $0.1257 = \dfrac{k - 20}{5}$

$k = 20.6$ ***A1*** ***N3***

[3 marks]

Total [6 marks]

3. (a) correct substitution into area formula *(A1)*

eg $\frac{1}{2}(18x)\sin 50$

setting **their** area expression equal to 80 *(M1)*

eg $9x\sin 50 = 80$

$x = 11.6$ ***A1*** ***N2***

[3 marks]

(b) evidence of choosing cosine rule *(M1)*

eg $c^2 = a^2 + b^2 + 2ab\sin C$

correct substitution into right hand side (may be in terms of x) *(A1)*

eg $11.6^2 + 18^2 - 2(11.6)(18)\cos 50$

$\text{BC} = 13.8$ ***A1*** ***N2***

[3 marks]

Total [6 marks]

4. appropriate approach *(M1)*

eg $\begin{pmatrix} 10 \\ 6 \\ -1 \end{pmatrix} + s\begin{pmatrix} 2 \\ -5 \\ -2 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix} + t\begin{pmatrix} 3 \\ 5 \\ 2 \end{pmatrix}$, $L_1 = L_2$

any two correct equations ***A1A1***

eg $10 + 2s = 2 + 3t$, $6 - 5s = 1 + 5t$, $-1 - 2s = -3 + 2t$

attempt to solve *(M1)*

eg substituting one equation into another

one correct parameter ***A1***

eg $s = -1$, $t = 2$

correct substitution *(A1)*

eg $2 + 3(2)$, $1 + 5(2)$, $-3 + 2(2)$

$\text{A} = (8, 11, 1)$ (accept column vector) ***A1*** ***N4***

[7 marks]

5. correct substitution into sum of a geometric sequence *A1*

eg $62.755 = u_1\left(\dfrac{1-r^3}{1-r}\right)$, $u_1 + u_1 r + u_1 r^2 = 62.755$

correct substitution into sum to infinity *A1*

eg $\dfrac{u_1}{1-r} = 440$

attempt to eliminate one variable *(M1)*

eg substituting $u_1 = 440(1-r)$

correct equation in one variable *(A1)*

eg $62.755 = 440(1-r)\left(\dfrac{1-r^3}{1-r}\right)$, $440(1-r)(1+r+r^2) = 62.755$

evidence of attempting to solve the equation in a single variable *(M1)*

eg sketch, setting equation equal to zero, $62.755 = 440\left(1-r^3\right)$

$r = 0.95 = \dfrac{19}{20}$ *A1* *N4*

[6 marks]

6. evidence of binomial expansion *(M1)*

eg selecting correct term, $\left(\dfrac{x}{a}\right)^6\left(\dfrac{a^2}{x}\right)^0 + \dbinom{6}{1}\left(\dfrac{x}{a}\right)^5\left(\dfrac{a^2}{x}\right)^1 + \ldots$

evidence of identifying constant term in expansion for power 6 *(A1)*

eg $r = 3$, 4^{th} term

evidence of correct term (may be seen in equation) *A2*

eg $20\dfrac{a^6}{a^3}$, $\dbinom{6}{3}\left(\dfrac{x}{a}\right)^3\left(\dfrac{a^2}{x}\right)^3$

attempt to set up **their** equation *(M1)*

eg $\dbinom{6}{3}\left(\dfrac{x}{a}\right)^3\left(\dfrac{a^2}{x}\right)^3 = 1280$, $a^3 = 1280$

correct equation in one variable *a* *(A1)*

eg $20a^3 = 1280$, $a^3 = 64$

$a = 4$ *A1* *N4*

[7 marks]

7. (a) use right triangle trigonometry **_A1_**

eg $\cos 1.4 = \frac{\text{OC}}{r}$

$\text{OC} = r\cos 1.4$ **_AG_** **_N0_**

[1 mark]

(b) correct value for BC

eg $\text{BC} = r\sin 1.4$, $\sqrt{r^2 - (r\cos 1.4)^2}$ **_(A1)_**

area of $\Delta\text{OBC} = \frac{1}{2} r\sin 1.4 \times r\cos 1.4 \ \left(= \frac{1}{2} r^2 \sin 1.4 \times \cos 1.4\right)$ **_A1_**

area of sector $\text{OAB} = \frac{1}{2} r^2 \times 1.4 \ \ (= 0.7r^2)$ **_A1_**

attempt to subtract in any order **_(M1)_**

eg sector – triangle, $\frac{1}{2} r^2 \sin 1.4 \times \cos 1.4 - 0.7r^2$

correct equation **_A1_**

eg $0.7r^2 - \frac{1}{2} r\sin 1.4 \times r\cos 1.4 = 25$

attempt to solve **their** equation **_(M1)_**

eg sketch, writing as quadratic, $\frac{25}{0.616\ldots}$

$r = 6.37$ **_A1_** **_N4_**

[7 marks]

Note: Exception to ***FT*** rule. Award ***A1FT*** for a correct ***FT*** answer from a quadratic equation involving two trigonometric functions.

Total [8 marks]

SECTION B

8. (a) (i) appropriate approach *(M1)*

eg $\overrightarrow{AO}+\overrightarrow{OB}$, $B-A$

$\overrightarrow{AB}=\begin{pmatrix}1\\3\\2\end{pmatrix}$ ***A1*** ***N2***

(ii) $\overrightarrow{AC}=\begin{pmatrix}2\\4\\a\end{pmatrix}$ ***A1*** ***N1***

[3 marks]

(b) valid reasoning (seen anywhere) ***R1***

eg scalar product is zero, $\cos\frac{\pi}{2}=\frac{\boldsymbol{u}\cdot\boldsymbol{v}}{|\boldsymbol{u}||\boldsymbol{v}|}$

correct scalar product of **their** $\overrightarrow{AB}$ and $\overrightarrow{AC}$ (may be seen in part (c)) *(A1)*

eg $1(2)+3(4)+2(a)$

correct working for **their** $\overrightarrow{AB}$ and $\overrightarrow{AC}$ *(A1)*

eg $2a+14$, $2a=-14$

$a=-7$ ***A1*** ***N3***

[4 marks]

(c) (i) correct magnitudes (may be seen in (b)) *(A1)(A1)*

$\sqrt{1^2+3^2+2^2}\left(=\sqrt{14}\right)$, $\sqrt{2^2+4^2+a^2}\left(=\sqrt{20+a^2}\right)$

substitution into formula *(M1)*

eg $\cos\theta=\frac{1\times2+3\times4+2\times a}{\sqrt{1^2+3^2+2^2}\sqrt{2^2+4^2+a^2}}$, $\frac{14+2a}{\sqrt{14}\sqrt{4+16+a^2}}$

simplification leading to required answer ***A1***

eg $\cos\theta=\frac{14+2a}{\sqrt{14}\sqrt{20+a^2}}$

$\cos\theta=\frac{2a+14}{\sqrt{14a^2+280}}$ ***AG*** ***N0***

(ii) correct setup *(A1)*

eg $\cos1.2=\frac{2a+14}{\sqrt{14a^2+280}}$

valid attempt to solve *(M1)*

eg sketch, $\frac{2a+14}{\sqrt{14a^2+280}}-\cos1.2=0$, attempt to square

$a=-3.25$ ***A2*** ***N3***

[8 marks]

Total [15 marks]

9. (a) **METHOD 1**

(i) appropriate approach *(M1)*

eg $\frac{6}{10}\times\frac{6}{10}$, $\frac{6}{10}\times\frac{5}{9}$, $\frac{6}{10}\times\frac{5}{10}$

$\mathrm{P}(X=0)=\frac{9}{25}=0.36$ ***A1*** ***N2***

(ii) multiplying one pair of gold and silver probabilities *(M1)*

eg $\frac{6}{10}\times\frac{4}{10}$, $\frac{6}{10}\times\frac{4}{9}$, 0.24

adding the product of both pairs of gold and silver probabilities *(M1)*

eg $\frac{6}{10}\times\frac{4}{10}\times 2$, $\frac{6}{10}\times\frac{4}{9}+\frac{4}{10}\times\frac{6}{9}$

$\mathrm{P}(X=1)=\frac{12}{25}=0.48$ ***A1*** ***N3***

(iii) $\mathrm{P}(X=2)=0.16$ (seen anywhere) *(A1)*

correct substitution into formula for $\mathrm{E}(X)$ *(A1)*

eg $0\times 0.36+1\times 0.48+2\times 0.16$, $0.48+0.32$

$\mathrm{E}(X)=\frac{4}{5}=0.8$ ***A1*** ***N3***

[8 marks]

METHOD 2

(i) evidence of recognizing binomial (may be seen in part (ii)) *(M1)*

eg $X\sim\mathrm{B}(2,0.6)$, $\binom{2}{0}(0.4)^2(0.6)^0$

correct probability for use in binomial *(A1)*

eg $p=0.4$, $X\sim\mathrm{B}(2,0.4)$, ${}^2C_0(0.4)^0(0.6)^2$

$\mathrm{P}(X=0)=\frac{9}{25}=0.36$ ***A1*** ***N3***

(ii) correct set up *(A1)*

eg ${}_2C_1(0.4)^1(0.6)^1$

$\mathrm{P}(X=1)=\frac{12}{25}=0.48$ ***A1*** ***N2***

continued …

Question 9 continued

(iii) attempt to substitute into *np* ***(M1)***
eg 2×0.6

correct substitution into *np* ***(A1)***
eg 2×0.4

$\mathrm{E}(X) = \frac{4}{5} = 0.8$ ***A1*** ***N3***

[8 marks]

Let Y be the number of gold balls drawn from the bag in parts (b), (c), and (d).

(b) evidence of recognizing binomial (seen anywhere) ***(M1)***
eg ${}_{14}C_5 (0.4)^5 (0.6)^9$, $\mathrm{B}(14, 0.4)$

$\mathrm{P}(Y = 5) = 0.207$ ***A1*** ***N2***

[2 marks]

(c) recognize need to find $\mathrm{P}(Y \le 5)$ ***(M1)***

$\mathrm{P}(Y \le 5) = 0.486$ ***A1*** ***N2***

[2 marks]

(d) recognizing conditional probability ***(M1)***
eg $\mathrm{P}(A \mid B)$, $\mathrm{P}(Y = 5 \mid Y \le 5)$, $\frac{\mathrm{P}(Y = 5)}{\mathrm{P}(Y \le 5)}$, $\frac{0.207}{0.486}$

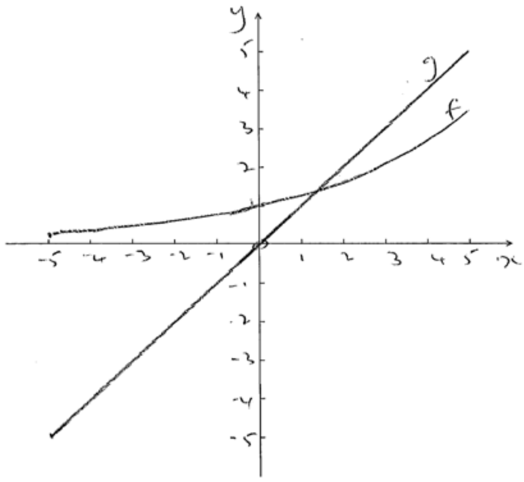
$\mathrm{P}(Y = 5 \mid Y \le 5) = 0.42522518$ ***(A1)***

$\mathrm{P}(Y = 5 \mid Y \le 5) = 0.43$ (to 2 dp) ***A1*** ***N2***

[3 marks]

Total [15 marks]

10. (a) (i)



A1A1 ***N2***

Notes:	Award ***A1*** for the graph of *f* positive, increasing and concave up. Award ***A1*** for graph of g increasing and linear with y-intercept of 0. Penalize one mark if domain is not [–5, 5] and/or if *f* and g do not intersect in the first quadrant.

(ii) attempt to find intersection of the graphs of f and g ***(M1)***

eg $\mathrm{e}^{\frac{x}{4}} = x$

$x = 1.42961\ldots$ ***A1***

valid attempt to find area of R ***(M1)***

eg $\int (x - \mathrm{e}^{\frac{x}{4}})\,\mathrm{d}x$, $\int_0^1 (g - f)$, $\int (f - g)$

Area = 0.697 ***A2*** ***N3***

[7 marks]

(b) recognize that area of R is a maximum at point of tangency ***(R1)***

eg $m = f'(x)$

equating functions ***(M1)***

eg $f(x) = g(x)$, $\mathrm{e}^{\frac{x}{4}} = mx$

$f'(x) = \frac{1}{4}\mathrm{e}^{\frac{x}{4}}$ ***(A1)***

equating gradients ***(A1)***

eg $f'(x) = g'(x)$, $\frac{1}{4}\mathrm{e}^{\frac{x}{4}} = m$

attempt to solve system of two equations for x ***(M1)***

eg $\frac{1}{4}\mathrm{e}^{\frac{x}{4}} \times x = \mathrm{e}^{\frac{x}{4}}$

$x = 4$ ***(A1)***

attempt to find m ***(M1)***

eg $f'(4)$, $\frac{1}{4}\mathrm{e}^{\frac{4}{4}}$

$m = \frac{1}{4}\mathrm{e}$ (exact), 0.680 ***A1*** ***N3***

[8 marks]

Total [15 marks]

N12/5/MATME/SP2/ENG/TZ0/XX/M

International Baccalaureate
Baccalauréat International
Bachillerato Internacional

MARKSCHEME

November 2012

MATHEMATICS

Standard Level

Paper 2

16 pages

Instructions to Examiners

Note: Changes linked to e-marking are noted in red. Other marking changes since November 2011 are noted in green. In particular, please note the removal of the accuracy and misread penalties and the revised accuracy instructions.

Abbreviations

M Marks awarded for attempting to use a correct **Method**; working must be seen.

(M) Marks awarded for **Method**; may be implied by **correct** subsequent working.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

(A) Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.

R **Marks awarded for clear** Reasoning**.**

N Marks awarded for **correct** answers if **no** working shown.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Mark according to scoris instructions and the document "**Mathematics SL: Guidance for e-marking November 2012**". It is **essential** that you read this document before you start marking. In particular, please note the following. **Marks must be recorded using the annotation stamps, using the new scoris assessor marking tool. Please check that you are entering marks for the right question.**

- If a part is **completely correct**, (and gains all the "must be seen" marks), use the ticks with numbers to stamp full marks.
- If a part is completely wrong, stamp ***A0*** by the final answer.
- If a part gains anything else, it **must** be recorded using **all** the annotations.

All the marks will be added and recorded by scoris.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any. An exception to this rule is when work for ***M1*** is missing, as opposed to incorrect (see point 4).
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***(M2)***, ***N3***, *etc.*, do **not** split the marks, unless there is a note.
- Once a correct answer to a question or part-question is seen, ignore further working.

3 *N* marks

*If **no** working shown, award **N** marks for **correct** answers. In this case, ignore mark breakdown (**M**, **A**, **R**).*

- Do **not** award a mixture of ***N*** and other marks.
- There may be fewer ***N*** marks available than the total of ***M***, ***A*** and ***R*** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.
- There may not be a direct relationship between the ***N*** marks and the implied marks. There are times when all the marks are implied, but the ***N*** marks are not the full marks: this indicates that we want to see some of the working, without specifying what.
- For consistency within the markscheme, ***N*** marks are noted for every part, even when these match the mark breakdown.
- If a candidate has incorrect working, which somehow results in a correct answer, do **not** award the ***N*** marks for this correct answer. However, if the candidate has indicated (usually by crossing out) that the working is to be ignored, award the ***N*** marks for the correct answer.

4 Implied and must be seen marks

*Implied marks appear in **brackets e.g. (M1)**.*

- Implied marks can only be awarded if **correct** work is seen or if implied in subsequent working (a correct answer does not necessarily mean that the implied marks are all awarded). There are questions where some working is required, but as it is accepted that not everyone will write the same steps, all the marks are implied, but the N marks are not the full marks for the question.
- Normally the correct work is seen or implied in the next line.
- Where there is an (***M1***) followed by ***A1*** for each correct answer, if no working shown, one correct answer is sufficient evidence to award the (***M1***).

*Must be seen marks appear without **brackets e.g. M1**.*

- Must be seen marks can only be awarded if **correct** work is seen.
- If a must be seen mark is not awarded because work is missing (as opposed to ***M0*** or ***A0*** for incorrect work) all subsequent marks may be awarded if appropriate.

5 Follow through marks (only applied after an error is made)

*Follow through (**FT**) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) or subpart(s). Usually, to award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if the only marks awarded in a subpart are for the answer (i.e. there is no working expected), then **FT** marks should be awarded if appropriate. Examiners are expected to check student work in order to award **FT** marks where appropriate.*

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate. (However, as noted above, if an ***A*** mark is not awarded because work is missing, all subsequent marks may be awarded if appropriate)
- Exceptions to this rule will be explicitly noted on the markscheme.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*e.g.* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word “their” in a description, to indicate that candidates may be using an incorrect value.

- If a candidate makes an error in one part, but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the question says hence. It is often possible to use a different approach in subsequent parts that does not depend on the answer to previous parts.
- In a "show that" question, if an error leads to not showing the required answer, there is a 1 mark penalty. Note that if the error occurs within the same subpart, the *FT* rules may result in further loss of marks.
- Where there are anticipated common errors, the ***FT*** answers are often noted on the markscheme, to help examiners. It should be stressed that these are not the only ***FT*** answers accepted.

6 Mis-read

*If a candidate incorrectly copies information from the question, this is a mis-read (**MR**). A candidate should be penalized only once for a particular mis-read. Use the **MR** stamp to indicate that this is a misread. Do not award the first mark in the question, even if this is an **M** mark, but award all others (if appropriate) so that the candidate only loses one mark for the misread.*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.

7 Discretionary marks *(d)*

*An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. In such cases the annotation DM should be used and a brief **note** written next to the mark explaining this decision.*

8 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for part-questions are indicated by **EITHER** . . . **OR**.
- Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9 Alternative forms

*Unless the question specifies otherwise, **accept** equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

10 Accuracy of Answers

If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy. When this is not specified in the question, all numerical answers should be given exactly or correct to three significant figures

*Candidates should NO LONGER be penalized for an accuracy error **(AP)**. Examiners should award marks according to the rules given in these instructions and the markscheme. Accuracy is not the same as correctness – an incorrect value does not achieve relevant **A** marks. It is only final answers which may lose marks for accuracy errors, not intermediate values. Please check work carefully for **FT**. Further information on which answers are accepted is given in a separate booklet, along with examples. It is **essential** that you read this carefully, as there are a number of changes.*

Do not accept unfinished numerical answers such as 3/0.1 (unless otherwise stated). As a rule, numerical answers with more than one part (such as fractions) should be given using integers (*e.g.* 6/8). Calculations which lead to integers should be completed, with the exception of fractions which are not whole numbers.

11 Calculators

A GDC is required for paper 2, but calculators with symbolic manipulation features (e.g. TI-89) are not allowed.

Calculator notation
The Mathematics SL guide says:

Students must always use correct mathematical notation, not calculator notation.

Do **not** accept final answers written using calculator notation. However, do not penalize the use of calculator notation in the working.

12 Style

The markscheme aims to present answers using good communication, e.g. if the question asks to find the value of k, the markscheme will say $k = 3$, but the marks will be for the correct value 3 – there is usually no need for the "$k =$". In these cases, it is also usually acceptable to have another variable, as long as there is no ambiguity in the question, e.g. if the question asks to find the value of p and of q, then the student answer needs to be clear. Generally, the only situation where the full answer is required is in a question which asks for equations – in this case the markscheme will say "must be an equation".

*The markscheme often uses words to describe what the marks are for, followed by examples, using the e.g. notation. These examples are not exhaustive, and examiners should check what candidates have written, to see if they satisfy the description. Where these marks are **M** marks, the examples may include ones using poor notation, to indicate what is acceptable.*

13 Candidate work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

Candidates are meant to write their answers to Section A on the question paper (QP), and Section B on lined paper. Sometimes, they need more room for Section A, and use lined paper (and often comment to this effect on the QP), or write outside the box. That is fine, and this work should be marked.
The instructions tell candidates not to write on Section B of the QP. Thus they may well have done some rough work here which they assume will be ignored. If they have solutions on the lined paper, there is no need to look at the QP. However, if there are whole questions or whole part solutions missing on the lined paper, please check to make sure that they are not on the QP, and if they are, mark those whole questions or whole part solutions that have not been written on the lined paper.

14. Diagrams

The notes on how to allocate marks for sketches usually refer to passing through particular points or having certain features. These marks can only be awarded if the sketch is approximately the correct shape. All values given will be an approximate guide to where these points/features occur. In some questions, the first ***A1*** is for the shape, in others, the marks are only for the points and/or features. In both cases, unless the shape is approximately correct, no marks can be awarded. However, if the graph is based on previous calculations, ***FT*** marks should be awarded if appropriate.

SECTION A

1. (a) valid method ***(M1)***

e.g. subtracting terms, using sequence formula

$d = 1.7$ ***A1*** ***N2***

[2 marks]

(b) correct substitution into term formula ***(A1)***

e.g. $5 + 27(1.7)$

28^{th} term is 50.9 (exact) ***A1*** ***N2***

[2 marks]

(c) correct substitution into sum formula ***(A1)***

e.g. $S_{28} = \frac{28}{2}\left(2(5) + 27(1.7)\right)$, $\frac{28}{2}(5 + 50.9)$

$S_{28} = 782.6$ (exact) [782, 783] ***A1*** ***N2***

[2 marks]

Total [6 marks]

2. (a) $\boldsymbol{A}^{-1} = \begin{pmatrix} -0.5 & 0 & 0.5 \\ 1.5 & 1 & -1.5 \\ -1 & -2 & 2 \end{pmatrix} \left(= \begin{pmatrix} -\frac{1}{2} & 0 & \frac{1}{2} \\ \frac{3}{2} & 1 & -\frac{3}{2} \\ -1 & -2 & 2 \end{pmatrix} \right)$ ***A2*** ***N2***

Note: Award ***A1*** for 6, 7 or 8 correct elements.

[2 marks]

(b) evidence of multiplying ***AB*** by $\boldsymbol{A}^{-1}$ (on left or right) ***(M1)***

e.g. 6, 7 or 8 correct elements

$\boldsymbol{B} = \begin{pmatrix} 3 & -2 & 4 \\ -4 & 5 & -9 \\ 1 & 0 & 9 \end{pmatrix}$ ***A2*** ***N3***

Notes: Award ***A1*** for 6, 7 or 8 correct elements. Award ***M1A1*** if correct answer follows from working where matrices are written in reversed order.

[3 marks]

Total [5 marks]

3. (a) $x = 2$ $\left(\text{accept } (2, 0)\right)$ *A1* *N1*

[1 mark]

(b) evidence of finding gradient of f at $x = 2$ *(M1)*

e.g. $f'(2)$

the gradient is 10 *A1* *N2*

[2 marks]

(c) evidence of negative reciprocal of gradient *(M1)*

e.g. $\frac{-1}{f'(x)}$, $-\frac{1}{10}$

evidence of correct substitution into equation of a line *(A1)*

e.g. $y - 0 = \frac{-1}{10}(x-2)$, $0 = -0.1(2) + b$

$y = -\frac{1}{10}x + \frac{2}{10}$ (accept $a = -0.1, b = 0.2$) *A1* *N2*

[3 marks]

Total [6 marks]

4. attempt to expand binomial *(M1)*

e.g. $(2x)^6 p^0 + \binom{6}{1}(2x)^5 (p)^1 + \ldots$, $\binom{n}{r}(2x)^r (p)^{n-r}$

one correct calculation for term in x^4 in the expansion for power 6 *(A1)*

e.g. 15, $16x^4$

correct expression for term in x^4 *(A1)*

e.g. $\binom{6}{2}(2x)^4 (p)^2$, $15.2^4\ p^2$

Notes:	Accept sloppy notation *e.g.* omission of brackets around $2x$. Accept absence of x in middle factor.

correct term *(A1)*

e.g. $240p^2x^4$ (accept absence of x^4)

setting up equation with **their** coefficient equal to 60 *M1*

e.g. $\binom{6}{2}(2)^4 (p)^2 = 60$, $240p^2x^4 = 60x^4$, $p^2 = \frac{60}{240}$

$p = \pm\frac{1}{2} (p = \pm 0.5)$ *A1A1* *N3*

[7 marks]

5. (a) (i) $a = 5$ (accept -5) *A1* *N1*

(ii) $c = 3$ (accept $c = 7$, if $a = -5$) *A1* *N1*

Note: Accept other correct values of *c*, such as 11, -5, *etc.*

[2 marks]

(b) attempt to find period *(M1)*

e.g. 8, $b = \frac{2\pi}{\text{period}}$

0.785398…

$b = \frac{2\pi}{8}$ (exact), $\frac{\pi}{4}$, 0.785 [0.785, 0.786] (do not accept 45) *A1* *N2*

[2 marks]

(c) valid approach *(M1)*

e.g. $f(x) = 0$, symmetry of curve

$x = 5$ (accept $(5, 0)$) *A1* *N2*

[2 marks]

Total [6 marks]

6. correct *z*-values *(A1)(A1)*

$-1.750686\ldots$, $0.524400\ldots$

attempt to set up **their** equations, must involve *z*-values, not % *(M1)*

e.g. one correct equation

two correct equations *A1A1*

e.g. $\mu - 1.750686\sigma = 5$, $0.5244005 = \frac{25 - \mu}{\sigma}$

attempt to solve **their** equations *(M1)*

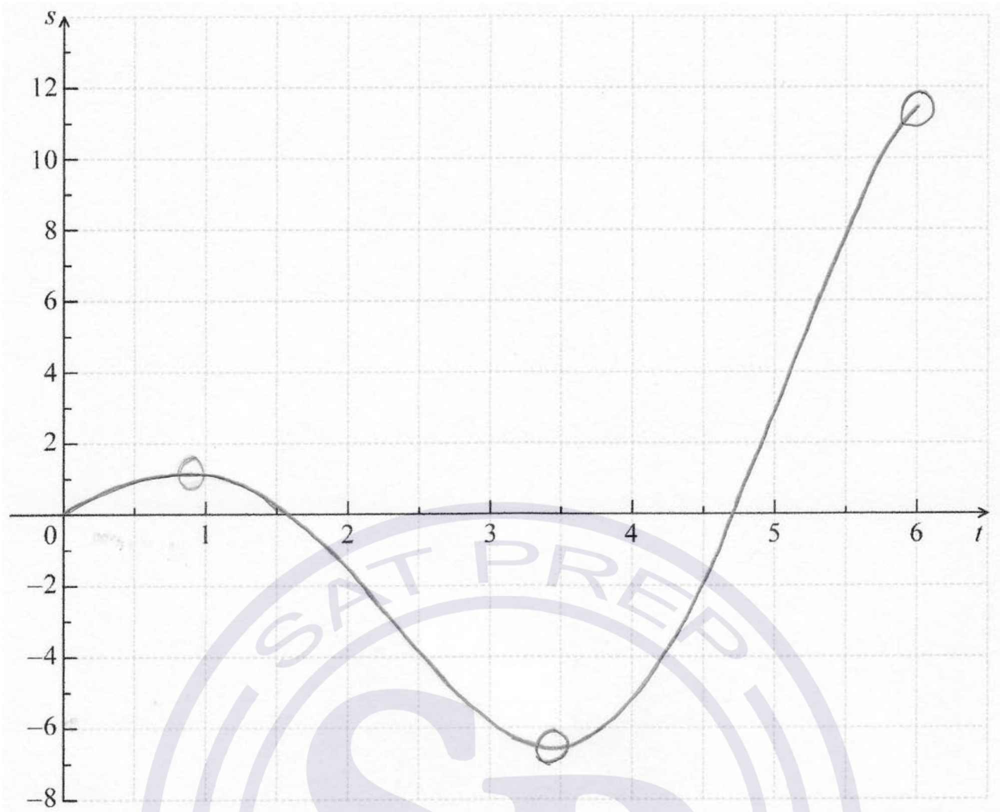
e.g. substitution, matrices, one correct value

$\mu = 20.39006\ldots$, $\sigma = 8.790874\ldots$

$\mu = 20.4\ [20.3, 20.4]$, $\sigma = 8.79\ [8.79, 8.80]$ *A1A1* *N4*

[8 marks]

7. (a)



A1A1A1A1 *N4*

> **Note:** Award ***A1*** for approximately correct shape (do not accept line segments).
> **Only** if this ***A1*** is awarded, award the following:
> ***A1*** for maximum and minimum within circles,
> ***A1*** for *x*-intercepts between 1 and 2 **and** between 4 and 5,
> ***A1*** for left endpoint at $(0, 0)$ and right endpoint within circle.

[4 marks]

(b) appropriate approach ***(M1)***

e.g. recognizing that $v = s'$, finding derivative, $a = s''$

valid method to find maximum ***(M1)***

e.g. sketch of v, $v'(t) = 0$, $t = 5.08698\ldots$

$v = 10.20025\ldots$

$v = 10.2$ [10.2,10.3] ***A1*** ***N2***

[3 marks]

[7 marks]

SECTION B

8. **Note:** In this question, do not penalise for missing or incorrect units. They are not included in the markscheme, to avoid complex answer lines.

(a) **METHOD 1**

choosing cosine rule (must have cos in it) ***(M1)***

e.g. $c^2 = a^2 + b^2 - 2ab\cos C$

correct substitution (into rhs) ***A1***

e.g. $20^2 + 20^2 - 2(20)(20)\cos 1.5$, $\text{AB} = \sqrt{800 - 800\cos 1.5}$

$\text{AB} = 27.26555\ldots$

$\text{AB} = 27.3\ [27.2, 27.3]$ ***A1*** ***N2***

[3 marks]

METHOD 2

choosing sine rule ***(M1)***

e.g. $\dfrac{\sin A}{a} = \dfrac{\sin B}{b}$, $\dfrac{\text{AB}}{\sin O} = \dfrac{\text{AO}}{\sin B}$

correct substitution ***A1***

e.g. $\dfrac{\text{AB}}{\sin 1.5} = \dfrac{20}{\sin\left(0.5(\pi - 1.5)\right)}$

$\text{AB} = 27.26555\ldots$

$\text{AB} = 27.3\ [27.2, 27.3]$ ***A1*** ***N2***

[3 marks]

(b) correct substitution into area formula ***A1***

e.g. $\dfrac{1}{2}(20)(20)\sin 1.5$, $\dfrac{1}{2}(20)(27.2655504...)\sin\left(0.5(\pi - 1.5)\right)$

area $= 199.498997\ldots$ (accept $199.75106 = 200$, from using 27.3)

area $= 199\ [199, 200]$ ***A1*** ***N1***

[2 marks]

continued …

Question 8 continued

(c) appropriate method to find angle AOC ***(M1)***

e.g. $2\pi - 1.5 - 2.4$

correct substitution into arc length formula ***(A1)***

e.g. $(2\pi - 3.9) \times 20$, $2.3831853\ldots \times 20$

arc length $= 47.6637\ldots$

arc length $= 47.7$ $(47.6, 47.7]$ (*i.e.* do **not** accept 47.6) ***A1*** ***N2***

Notes:	Candidates may misread the question and use $\hat{AOC} = 2.4$. If working shown, award ***M0*** then ***A0MRA1*** for the answer 48. Do not then penalize $\hat{AOC}$ in part (d) which, if used, leads to the answer 679.498… **However**, if they use the prematurely rounded value of 2.4 for $\hat{AOC}$, penalise 1 mark for premature rounding for the answer 48 in (c). Do not then penalize for this in (d).

[3 marks]

(d) calculating sector area using **their** angle AOC ***(A1)***

e.g. $\frac{1}{2}(2.38\ldots)(20^2)$, $200(2.38\ldots)$, $476.6370614\ldots$

shaded area $=$ **their** area of triangle AOB $+$ **their** area of sector ***(M1)***

e.g. $199.4989973\ldots + 476.6370614\ldots$, $199 + 476.637$

shaded area $= 676.136\ldots$ (accept $675.637\ldots = 676$ from using 199)

shaded area $= 676$ [676, 677], ***A1*** ***N2***

[3 marks]

(e) dividing to find number of cans ***(M1)***

e.g. $\frac{676}{140}$, $4.82857\ldots$

5 cans must be purchased ***(A1)***

multiplying to find cost of cans ***(M1)***

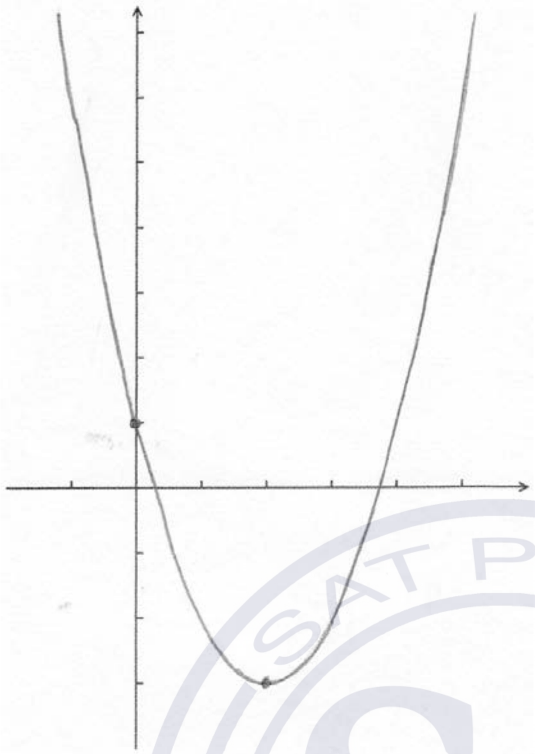
e.g. $5(32)$, $\frac{676}{140} \times 32$

cost is 160 (dollars) ***A1*** ***N3***

[4 marks]

Total [15 marks]

9. (a)



A1A1A1A1 *N4*

Note: The shape **must** be an approximately correct upwards parabola.
Only if the shape is approximately correct, award the following:
A1 for vertex $x \approx 2$, *A1* for x-intercepts between 0 and 1, and 3 and 4,
A1 for correct y-intercept $(0, 1)$, *A1* for correct domain. $[-1, 5]$.
Scale not required on the axes, but approximate positions need to be clear.

[4 marks]

(b) $p = 2$ *A1* *N1*

[1 mark]

(c) correct vertical reflection, correct vertical translation *(A1)(A1)*

e.g. $-f(x)$, $-\left((x-2)^2-3\right)$, $-y$, $-f(x)+6$, $y+6$

transformations in correct order *(A1)*

e.g. $-(x^2-4x+1)+6$, $-\left((x-2)^2-3\right)+6$

simplification which clearly leads to given answer *A1*

e.g. $-x^2+4x-1+6$, $-(x^2-4x+4-3)+6$

$g(x) = -x^2+4x+5$ *AG* *N0*

Note: If working shown, award ***A1A1A0A0*** if transformations correct, but done in reverse order, *e.g.* $-(x^2-4x+1+6)$.

[4 marks]

continued …

Question 9 continued

(d) valid approach ***(M1)***
e.g. sketch, $f = g$

$-0.449489\ldots$, $4.449489\ldots$

$\left(2 \pm \sqrt{6}\right)$ (exact), $-0.449[-0.450, -0.449]$; $4.45[4.44, 4.45]$ ***A1A1*** ***N3***

[3 marks]

(e) attempt to substitute limits or functions into area formula (accept absence of $\mathrm{d}x$) ***(M1)***
e.g. $\int_a^b \left((-x^2+4x+5)-(x^2-4x+1)\right)\mathrm{d}x$, $\int_{4.45}^{-0.449} (f-g)$,
$\int (-2x^2+8x+4)\mathrm{d}x$

approach involving subtraction of integrals/areas (accept absence of $\mathrm{d}x$) ***(M1)***
e.g. $\int_a^b (-x^2+4x+5)-\int_a^b (x^2-4x+1)$, $\int (f-g)\mathrm{d}x$

$\text{area} = 39.19183\ldots$

$\text{area} = 39.2$ [39.1, 39.2] ***A1*** ***N3***

[3 marks]

Total [15 marks]

10. (a) valid approach *(M1)*

e.g. Venn diagram with intersection, union formula, $\mathrm{P}(S \cap F) = 0.75 + 0.40 - 1$

15 (accept 15 %) *A1* *N2*

[2 marks]

(b) valid approach involving subtraction *(M1)*

e.g. Venn diagram, $75 - 15$

60 (accept 60 %) *A1* *N2*

[2 marks]

(c) (i) valid approach *(M1)*

e.g. tree diagram, multiplying probabilities, $\mathrm{P}(S \mid G) \times \mathrm{P}(G)$

correct calculation *(A1)*

e.g. 0.52×0.85

$\mathrm{P}(G \cap S) = 0.442$ (exact) *A1* *N3*

(ii) valid reasoning, with words, symbols or numbers (seen anywhere) *R1*

e.g. $\mathrm{P}(G) \times \mathrm{P}(S) \neq \mathrm{P}(G \cap S)$, $\mathrm{P}(S \mid G) \neq \mathrm{P}(S)$, not equal,

one correct value *A1*

e.g. $\mathrm{P}(G) \times \mathrm{P}(S) = 0.39$, $P\left(S \middle| G\right) = 0.85$, $0.39 \neq 0.442$

G and S are not independent *AG* *N0*

[5 marks]

(d) **METHOD 1**

48 % are boys (seen anywhere) *A1*

e.g. $\mathrm{P}(B) = 0.48$

appropriate approach *(M1)*

e.g. $\mathrm{P}(\text{girl and Spanish}) + \mathrm{P}(\text{boy and Spanish}) = \mathrm{P}(\text{Spanish})$

correct approach to find P(boy and Spanish) *(A1)*

e.g. $\mathrm{P}(B \cap S) = \mathrm{P}(S) - \mathrm{P}(G \cap S)$, $\mathrm{P}(B \cap S) = \mathrm{P}(S \mid B) \times \mathrm{P}(B)$, 0.308

correct substitution *(A1)*

e.g. $0.442 + 0.48x = 0.75$, $0.48x = 0.308$

correct manipulation *(A1)*

e.g. $\mathrm{P}(S \mid B) = \dfrac{0.308}{0.48}$

$\mathrm{P}(\text{Spanish} \mid \text{boy}) = 0.641666\ldots, 0.641\overline{6}$

$\mathrm{P}(\text{Spanish} \mid \text{boy}) = 0.642$ $[0.641, 0.642]$ *A1* *N3*

[6 marks]

continued …

Question 10 continued

METHOD 2

48 % are boys (seen anywhere) ***A1***
e.g. 0.48 used in tree diagram

appropriate approach ***(M1)***
e.g. tree diagram

correctly labelled branches on tree diagram ***(A1)***
e.g. first branches are boy/girl, second branches are Spanish/not Spanish

correct substitution ***(A1)***
e.g. $0.442 + 0.48x = 0.75$

correct manipulation ***(A1)***
e.g. $0.48x = 0.308$, $\mathrm{P}(S \mid B) = \dfrac{0.308}{0.48}$

$\mathrm{P}(\text{Spanish} \mid \text{boy}) = 0.641666\ldots, 0.641\overline{6}$

$\mathrm{P}(\text{Spanish} \mid \text{boy}) = 0.642$ $[0.641, 0.642]$ ***A1*** ***N3***
[6 marks]

Total [15 marks]

M12/5/MATME/SP2/ENG/TZ1/XX/M

International Baccalaureate
Baccalauréat International
Bachillerato Internacional

MARKSCHEME

May 2012

MATHEMATICS

Standard Level

Paper 2

15 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**; working must be seen.

(M) Marks awarded for **Method**; may be implied by **correct** subsequent working.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

(A) Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.

R Marks awarded for clear **Reasoning**.

N Marks awarded for **correct** answers if **no** working shown.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Mark according to scoris instructions and the document "**Mathematics SL: Guidance for e-marking May 2012**". It is **essential** that you read this document before you start marking. In particular, please note the following.

Marks must be recorded using the annotation stamps. Please check that you are entering marks for the right question.

- If a part is **completely correct**, (and gains all the "must be seen" marks), use the ticks with numbers to stamp full marks.
- If a part is completely wrong, stamp ***A0*** by the final answer.
- If a part gains anything else, it **must** be recorded using **all** the annotations.

All the marks will be added and recorded by scoris.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any. An exception to this rule is when work for ***M1*** is missing, as opposed to incorrect (see point 4).
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***(M2)***, ***N3***, *etc*., do **not** split the marks, unless there is a note.
- Once a correct answer to a question or part-question is seen, ignore further working.

3 *N* marks

*If **no** working shown, award **N** marks for **correct** answers. In this case, ignore mark breakdown (**M**, **A**, **R**).*

- Do **not** award a mixture of ***N*** and other marks.
- There may be fewer ***N*** marks available than the total of ***M***, ***A*** and ***R*** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.
- There may not be a direct relationship between the ***N*** marks and the implied marks. There are times when all the marks are implied, but the ***N*** marks are not the full marks: this indicates that we want to see some of the working, without specifying what.
- For consistency within the markscheme, ***N*** marks are noted for every part, even when these match the mark breakdown.
- If a candidate has incorrect working, which somehow results in a correct answer, do **not** award the ***N*** marks for this correct answer. However, if the candidate has indicated (usually by crossing out) that the working is to be ignored, award the ***N*** marks for the correct answer.

4 Implied and must be seen marks

*Implied marks appear in **brackets e.g. (M1)**.*

- Implied marks can only be awarded if **correct** work is seen or if implied in subsequent working (a correct answer does not necessarily mean that the implied marks are all awarded). There are questions where some working is required, but as it is accepted that not everyone will write the same steps, all the marks are implied, but the N marks are not the full marks for the question.
- Normally the correct work is seen or implied in the next line.
- Where there is an (***M1***) followed by ***A1*** for each correct answer, if no working shown, one correct answer is sufficient evidence to award the (***M1***).

*Must be seen marks appear without **brackets e.g. M1**.*

- Must be seen marks can only be awarded if **correct** work is seen.
- If a must be seen mark is not awarded because work is missing (as opposed to ***M0*** or ***A0*** for incorrect work) all subsequent marks may be awarded if appropriate.

5 Follow through marks (only applied after an error is made)

*Follow through (**FT**) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) or subpart(s). Usually, to award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if the only marks awarded in a subpart are for the answer (i.e. there is no working expected), then **FT** marks should be awarded if appropriate. Examiners are expected to check student work in order to award **FT** marks where appropriate.*

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate. (However, as noted above, if an ***A*** mark is not awarded because work is missing, all subsequent marks may be awarded if appropriate)
- Exceptions to this rule will be explicitly noted on the markscheme.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*e.g.* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word "their" in a description, to indicate that candidates may be using an incorrect value.

- If a candidate makes an error in one part, but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the question says hence. It is often possible to use a different approach in subsequent parts that does not depend on the answer to previous parts.
- In a "show that" question, if an error leads to not showing the required answer, there is a 1 mark penalty. Note that if the error occurs within the same subpart, the *FT* rules may result in further loss of marks.
- Where there are anticipated common errors, the ***FT*** answers are often noted on the markscheme, to help examiners. It should be stressed that these are not the only ***FT*** answers accepted.

6 Mis-read

*If a candidate incorrectly copies information from the question, this is a mis-read (**MR**). Apply a **MR** penalty of 1 mark to that question. Award the marks as usual and then stamp **MR** against the answer. Scoris will automatically deduct 1 mark from the question total. A candidate should be penalized only once for a particular mis-read. Do not stamp **MR** again for that question, unless the candidate makes another mis-read.*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.

7 Discretionary marks *(d)*

An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. In such cases the annotation DM should be used and a brief **note** *written next to the mark explaining this decision.*

8 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for part-questions are indicated by **EITHER** . . . **OR**.
- Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9 Alternative forms

Unless the question specifies otherwise, ***accept*** *equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

10 Accuracy of Answers

If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy. When this is not specified in the question, all numerical answers should be given exactly or correct to three significant figures

*Candidates should NO LONGER be penalized for an accuracy error **(AP)**. Examiners should award marks according to the rules given in these instructions and the markscheme. Accuracy is not the same as correctness – an incorrect value does not achieve relevant **A** marks. It is only final answers which may lose marks for accuracy errors, not intermediate values. Please check work carefully for **FT**. Further information on which answers are accepted is given in a separate booklet, along with examples. It is **essential** that you read this carefully, as there are a number of changes.*

Do not accept unfinished numerical answers such as 3/0.1 (unless otherwise stated). As a rule, numerical answers with more than one part (such as fractions) should be given using integers (*e.g.* 6/8). Calculations which lead to integers should be completed, with the exception of fractions which are not whole numbers.

11 Calculators

A GDC is required for paper 2, but calculators with symbolic manipulation features (e.g. TI-89) are not allowed.

Calculator notation
The Mathematics SL guide says:

Students must always use correct mathematical notation, not calculator notation.

Do **not** accept final answers written using calculator notation. However, do not penalize the use of calculator notation in the working.

12 Style

The markscheme aims to present answers using good communication, e.g. if the question asks to find the value of k, the markscheme will say $k = 3$, but the marks will be for the correct value 3 – there is usually no need for the "$k =$". In these cases, it is also usually acceptable to have another variable, as long as there is no ambiguity in the question, e.g. if the question asks to find the value of p and of q, then the student answer needs to be clear. Generally, the only situation where the full answer is required is in a question which asks for equations – in this case the markscheme will say "must be an equation".

*The markscheme often uses words to describe what the marks are for, followed by examples, using the e.g. notation. These examples are not exhaustive, and examiners should check what candidates have written, to see if they satisfy the description. Where these marks are **M** marks, the examples may include ones using poor notation, to indicate what is acceptable.*

13 Candidate work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

SECTION A

1. (a) (i) $d = 4$ *A1* *N1*

(ii) evidence of valid approach *(M1)*

e.g. $u_8 = 36 + 7(4)$, repeated addition of d from 36

$u_8 = 64$ *A1* *N2*

[3 marks]

(b) (i) correct substitution into sum formula *A1*

e.g. $S_n = \frac{n}{2}\{2(36) + (n-1)(4)\}$, $\frac{n}{2}\{72 + 4n - 4\}$

evidence of simplifying

e.g. $\frac{n}{2}\{4n + 68\}$ *A1*

$S_n = 2n^2 + 34n$ *AG* *N0*

(ii) 868 *A1* *N1*

[3 marks]

Total [6 marks]

2. (a) (i) $(2, -17)$ or $x = 2$, $y = -17$ *A1A1* *N2*

(ii) evidence of valid approach *(M1)*

e.g. graph, completing the square, equating coefficients

$f(x) = 2(x-2)^2 - 17$ *A1* *N2*

[4 marks]

(b) evidence of valid approach *(M1)*

e.g. graph, quadratic formula

–0.9154759…, 4.915475…

$x =$ -0.915, 4.92 *A1A1* *N3*

[3 marks]

Total [7 marks]

3. (a) correct substitution into formula for determinant *(A1)*

e.g. $(x)(1)-(2x)(x^2)$

det $\boldsymbol{M} = x - 2x^3$ ***A1*** ***N2***

[2 marks]

(b) det $\boldsymbol{N} = -26$ ***A1*** ***N1***

[1 mark]

evidence of valid approach *(M1)*

e.g. $x - 2x^3 = -26$, graph

2.42219559…

$x = 2.42$ ***A2*** ***N3***

[3 marks]

Total [6 marks]

4. (a) evidence of valid approach *(M1)*

e.g. $y = 0$, $\sin x = 0$

$2\pi = 6.283185\ldots$

$k = 6.28$ ***A1*** ***N2***

[2 marks]

(b) attempt to substitute either limits or the function into formula (accept absence of dx) *(M1)*

e.g. $V = \pi\int_{\pi}^{k}\left(f(x)\right)^2 \mathrm{d}x$, $\pi\int\left((x-1)\sin x\right)^2$, $\pi\int_{\pi}^{6.28\ldots} y^2\,\mathrm{d}x$

correct expression ***A2*** ***N3***

e.g. $\pi\int_{\pi}^{6.28}(x-1)^2\sin^2 x\,\mathrm{d}x$, $\pi\int_{\pi}^{2\pi}\left((x-1)\sin x\right)^2 \mathrm{d}x$

[3 marks]

(c) $V = 69.60192562\ldots$

$V = 69.6$ ***A2*** ***N2***

[2 marks]

Total [7 marks]

5. (a) evidence of valid approach *(M1)*

e.g. finding the inverse of $\boldsymbol{M}^{-1}$, $\boldsymbol{MM}^{-1} = \boldsymbol{I}$

$p = 1,\ q = 2$ ***A1A1*** ***N3***

[3 marks]

(b) evidence of attempt to solve system *(M1)*

e.g. $\boldsymbol{X} = \boldsymbol{M}^{-1}\begin{pmatrix} 7 \\ 2 \\ -3 \end{pmatrix}$, 1 or 2 correct values, substitution

$x = -0.5,\ y = -2.5,\ z = -2.5$ ***A2*** ***N3***

[3 marks]

Total [6 marks]

6. (a) Valid attempt to find term in x^{20} *(M1)*

e.g. $\binom{8}{1}(2^7)(b)$, $(2x^3)^7\left(\frac{b}{x}\right) = 3072$

correct equation ***A1***

e.g. $\binom{8}{1}(2^7)(b) = 3072$

$b = 3$ ***A1*** ***N2***

[3 marks]

(b) evidence of choosing correct term *(M1)*

e.g. 7th term, $r = 6$

correct expression ***A1***

e.g. $\binom{8}{6}(2x^3)^2\left(\frac{3}{x}\right)^6$

$k = 81648$ (accept 81600) ***A1*** ***N2***

[3 marks]

Total [6 marks]

7. (a) evidence of recognizing binomial distribution *(M1)*

e.g. $X \sim \mathrm{B}(10, 0.57)$, $p = 0.57$, $q = 0.43$

EITHER

$\mathrm{P}(X \le 3) = 2.16 \times 10^{-4} + 0.00286 + 0.01709 + 0.06041 \quad (= 0.08057\ldots)$ *(A1)*

evidence of using complement *(M1)*

e.g. $1 -$ any probability, $\mathrm{P}(X \ge 4) = 1 - \mathrm{P}(X \le 3)$

0.919423...

$\mathrm{P}(X \ge 4) = 0.919$ ***A1*** ***N3***

OR

summing the probabilities from $X = 4$ to $X = 10$ *(M1)*

correct expression or values *(A1)*

e.g. $\sum_{r=4}^{10} \binom{10}{r} (0.57)^r (0.43)^{10-r}$, $0.14013 + 0.2229 + \ldots + 0.02731 + 0.00362$

0.919424

$\mathrm{P}(X \ge 4) = 0.919$ ***A1*** ***N3***

[4 marks]

(b) evidence of valid approach *(M1)*

e.g. three tails in nine tosses, $\binom{9}{3}(0.57)^3(0.43)^6$

correct calculation

e.g. $\binom{9}{3}(0.57)^3 (0.43)^6 \times 0.57$, 0.09834×0.57 *(A1)*

0.05605178…

$\mathrm{P}(4^{\text{th}} \text{ tail on } 10^{\text{th}} \text{ toss}) = 0.0561$ ***A1*** ***N2***

[3 marks]

Total [7 marks]

SECTION B

8. (a) (i) $p = 17,\ q = 11$ *A1A1* *N2*

(ii) $75 \le T < 85$ *A1* *N1*

[3 marks]

(b) evidence of valid approach *(M1)*

e.g. adding frequencies

$\frac{76}{93} = 0.8172043\ldots$

$\mathrm{P}(T < 95) = \frac{76}{93} = 0.817$ *A1* *N2*

[2 marks]

(c) (i) 10 *A1* *N1*

(ii) 50 *A1* *N1*

[2 marks]

(d) (i) evidence of approach using mid-interval values (may be seen in part (ii)) *(M1)*

79.1397849

$\bar{x} = 79.1$ *A2* *N3*

(ii) 16.4386061

$\sigma = 16.4$ *A1* *N1*

[4 marks]

(e) evidence of valid approach *(M1)*

e.g. standardizing, $z = 0.9648...$

0.8326812

$\mathrm{P}(T < 95) = 0.833$ *A1* *N2*

[2 marks]

Total [13 marks]

9. (a) (i) evidence of valid approach *(M1)*

e.g. choosing cosine rule

correct substitution *(A1)*

e.g. $6^2 = (5p)^2 + (4p)^2 - 2 \times (4p) \times (5p)\cos 0.7$

simplification *A1*

e.g. $36 = 25p^2 + 16p^2 - 40p^2 \cos 0.7$

$p^2(41 - 40\cos 0.7) = 36$ *AG* *N0*

(ii) $1.85995\ldots$

$p = 1.86$ *A1* *N1*

Note: Award ***A0*** for $p = \pm 1.86$, *i.e.* not rejecting the negative value.

[4 marks]

(b) $\text{BD} = 6$ *A1* *N1*

[1 mark]

(c) evidence of valid approach *(M1)*

e.g. choosing sine rule

correct substitution *A1*

e.g. $\dfrac{\sin A\hat{D}B}{4p} = \dfrac{\sin 0.7}{6}$

acute $\text{A}\hat{\text{D}}\text{B} = 0.9253166\ldots$ *(A1)*

$\pi - 0.9253166\ldots = 2.216275\ldots$

$\text{A}\hat{\text{D}}\text{B} = 2.22$ *A1* *N3*

[4 marks]

continued …

Question 9 continued

(d) (i) evidence of valid approach ***(M1)***
e.g. recognize isosceles triangle, base angles equal

$\pi - 2(0.9253\ldots)$ ***A1***

$\hat{\text{CBD}} = 1.29$ ***AG*** ***N0***

(ii) area of sector BCD ***(A1)***
e.g. $0.5 \times (1.29) \times (6)^2$

area of triangle BCD ***(A1)***
e.g. $0.5 \times (6)^2 \sin 1.29$

evidence of subtraction ***M1***
$5.92496\ldots$

$5.937459\ldots$
area $= 5.94$ ***A1*** ***N3***

[6 marks]

Total [15 marks]

10. (a) (i) evidence of valid approach *(M1)*

e.g. ship A where B was, B 11 km away

distance $= 11$ ***A1*** ***N2***

(ii) evidence of valid approach *(M1)*

e.g. new diagram, Pythagoras, vectors

$s = \sqrt{15^2 + 22^2}$ *(A1)*

$\sqrt{709} = 26.62705\ldots$

$s = 26.6$ ***A1*** ***N2***

Note: Award ***M0A0A0*** for using the formula given in part (b).

[5 marks]

(b) evidence of valid approach *(M1)*

e.g. a table, diagram, formula $d = r \times t$

distance ship A travels t hours after noon is $15(t-1)$ *(A2)*

distance ship B travels in t hours after noon is $11t$ *(A1)*

evidence of valid approach ***M1***

e.g. $s(t) = \sqrt{[15(t-1)]^2 + (11t)^2}$

correct simplification ***A1***

e.g. $\sqrt{225(t^2 - 2t + 1) + 121t^2}$

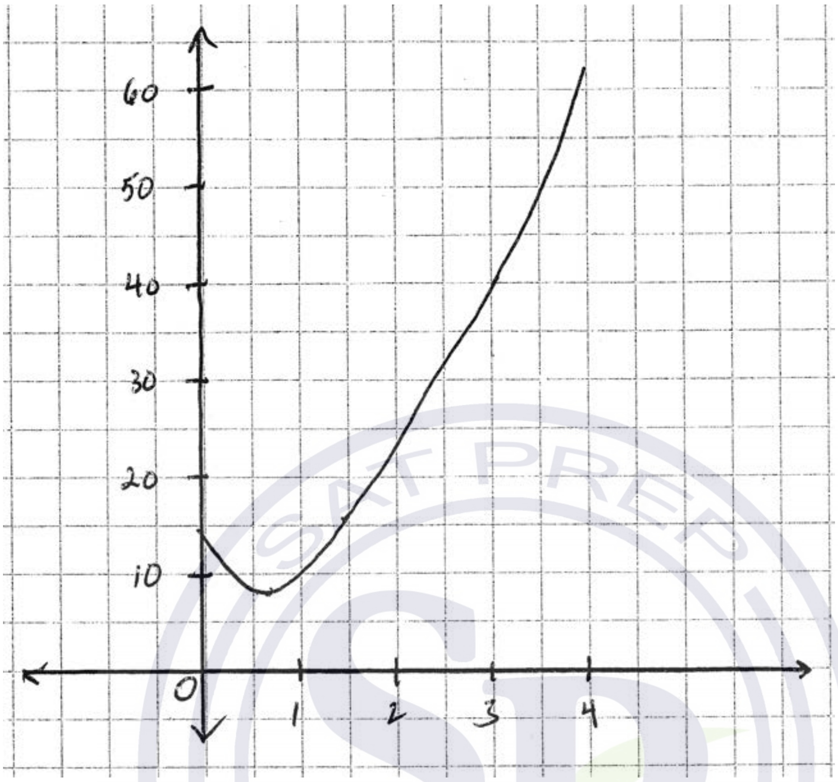
$s(t) = \sqrt{346t^2 - 450t + 225}$ ***AG*** ***N0***

[6 marks]

continued …

Question 10 continued

(c)



A1A1A1 ***N3***

Note: Award ***A1*** for shape, ***A1*** for minimum at approximately $(0.7, 9)$, ***A1*** for domain.

[3 marks]

(d) evidence of valid approach ***(M1)***
e.g. $s'(t) = 0$, find minimum of $s(t)$, graph, reference to "more than 8 km"

$\min = 8.870455...$ (accept 2 or more sf) ***A1***
since $s_{\min} > 8$, captain cannot see ship B ***R1*** ***N0***

[3 marks]

Total [17 marks]

International Baccalaureate®
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Bachillerato Internacional

MARKSCHEME

May 2012

MATHEMATICS

Standard Level

Paper 2

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**; working must be seen.

(M) Marks awarded for **Method**; may be implied by **correct** subsequent working.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

(A) Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.

R Marks awarded for clear **Reasoning**.

N Marks awarded for **correct** answers if **no** working shown.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Mark according to scoris instructions and the document "**Mathematics SL: Guidance for e-marking May 2011**". It is **essential** that you read this document before you start marking. In particular, please note the following.

Marks must be recorded using the annotation stamps. Please check that you are entering marks for the right question.

- If a part is **completely correct**, (and gains all the "must be seen" marks), use the ticks with numbers to stamp full marks.

- If a part is completely wrong, stamp ***A0*** by the final answer.

- If a part gains anything else, it **must** be recorded using **all** the annotations.

All the marks will be added and recorded by scoris.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any. An exception to this rule is when work for ***M1*** is missing, as opposed to incorrect (see point 4).
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***(M2)***, ***N3***, *etc*., do **not** split the marks, unless there is a note.
- Once a correct answer to a question or part-question is seen, ignore further working.

3 *N* marks

*If **no** working shown, award **N** marks for **correct** answers. In this case, ignore mark breakdown (**M**, **A**, **R**).*

- Do **not** award a mixture of ***N*** and other marks.
- There may be fewer ***N*** marks available than the total of ***M***, ***A*** and ***R*** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.
- There may not be a direct relationship between the ***N*** marks and the implied marks. There are times when all the marks are implied, but the ***N*** marks are not the full marks: this indicates that we want to see some of the working, without specifying what.
- For consistency within the markscheme, ***N*** marks are noted for every part, even when these match the mark breakdown.
- If a candidate has incorrect working, which somehow results in a correct answer, do **not** award the ***N*** marks for this correct answer. However, if the candidate has indicated (usually by crossing out) that the working is to be ignored, award the ***N*** marks for the correct answer.

4 Implied and must be seen marks

*Implied marks appear in **brackets e.g. (M1)**.*

- Implied marks can only be awarded if **correct** work is seen or if implied in subsequent working (a correct answer does not necessarily mean that the implied marks are all awarded). There are questions where some working is required, but as it is accepted that not everyone will write the same steps, all the marks are implied, but the N marks are not the full marks for the question.
- Normally the correct work is seen or implied in the next line.
- Where there is an (***M1***) followed by ***A1*** for each correct answer, if no working shown, one correct answer is sufficient evidence to award the (***M1***).

*Must be seen marks appear without **brackets e.g. M1**.*

- Must be seen marks can only be awarded if **correct** work is seen.
- If a must be seen mark is not awarded because work is missing (as opposed to ***M0*** or ***A0*** for incorrect work) all subsequent marks may be awarded if appropriate.

5 Follow through marks (only applied after an error is made)

*Follow through (**FT**) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) or subpart(s). Usually, to award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if the only marks awarded in a subpart are for the answer (i.e. there is no working expected), then **FT** marks should be awarded if appropriate. Examiners are expected to check student work in order to award **FT** marks where appropriate.*

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate. (However, as noted above, if an ***A*** mark is not awarded because work is missing, all subsequent marks may be awarded if appropriate)
- Exceptions to this rule will be explicitly noted on the markscheme.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*e.g.* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word “their” in a description, to indicate that candidates may be using an incorrect value.

- If a candidate makes an error in one part, but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the question says hence. It is often possible to use a different approach in subsequent parts that does not depend on the answer to previous parts.
- In a “show that” question, if an error leads to not showing the required answer, there is a 1 mark penalty. Note that if the error occurs within the same subpart, the *FT* rules may result in further loss of marks.
- Where there are anticipated common errors, the ***FT*** answers are often noted on the markscheme, to help examiners. It should be stressed that these are not the only ***FT*** answers accepted.

6 Mis-read

*If a candidate incorrectly copies information from the question, this is a mis-read (**MR**). Apply a **MR** penalty of 1 mark to that question. Award the marks as usual and then stamp **MR** against the answer. Scoris will automatically deduct 1 mark from the question total. A candidate should be penalized only once for a particular mis-read. Do not stamp **MR** again for that question, unless the candidate makes another mis-read.*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates’ own work does **not** constitute a misread, it is an error.

7 Discretionary marks *(d)*

An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. In such cases the annotation DM should be used and a brief **note** *written next to the mark explaining this decision.*

8 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for part-questions are indicated by **EITHER** . . . **OR**.
- Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9 Alternative forms

*Unless the question specifies otherwise, **accept** equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

10 Accuracy of Answers

If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy. When this is not specified in the question, all numerical answers should be given exactly or correct to three significant figures

*Candidates should NO LONGER be penalized for an accuracy error **(AP)**. Examiners should award marks according to the rules given in these instructions and the markscheme. Accuracy is not the same as correctness – an incorrect value does not achieve relevant **A** marks. It is only final answers which may lose marks for accuracy errors, not intermediate values. Please check work carefully for **FT**. Further information on which answers are accepted is given in a separate booklet, along with examples. It is **essential** that you read this carefully, as there are a number of changes.*

Do not accept unfinished numerical answers such as 3/0.1 (unless otherwise stated). As a rule, numerical answers with more than one part (such as fractions) should be given using integers (*e.g.* 6/8). Calculations which lead to integers should be completed, with the exception of fractions which are not whole numbers.

11 Calculators

A GDC is required for paper 2, but calculators with symbolic manipulation features (e.g. TI-89) are not allowed.

Calculator notation
The Mathematics SL guide says:

Students must always use correct mathematical notation, not calculator notation.

Do **not** accept final answers written using calculator notation. However, do not penalize the use of calculator notation in the working.

12 Style

The markscheme aims to present answers using good communication, e.g. if the question asks to find the value of k, the markscheme will say $k=3$, but the marks will be for the correct value 3 – there is usually no need for the "$k=$". In these cases, it is also usually acceptable to have another variable, as long as there is no ambiguity in the question, e.g. if the question asks to find the value of p and of q, then the student answer needs to be clear. Generally, the only situation where the full answer is required is in a question which asks for equations – in this case the markscheme will say "must be an equation".

*The markscheme often uses words to describe what the marks are for, followed by examples, using the e.g. notation. These examples are not exhaustive, and examiners should check what candidates have written, to see if they satisfy the description. Where these marks are **M** marks, the examples may include ones using poor notation, to indicate what is acceptable.*

13 Candidate work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

SECTION A

1. (a) $R\hat{P}Q = 65^\circ$ ***A1*** ***N1***

[1 mark]

(b) evidence of choosing sine rule ***(M1)***

correct substitution ***A1***

e.g. $\dfrac{PR}{\sin 45^\circ} = \dfrac{9}{\sin 65^\circ}$

7.021854078

PR=7.02 ***A1*** ***N2***

[3 marks]

(c) correct substitution ***(A1)***

e.g. area $= \dfrac{1}{2} \times 9 \times 7.02\ldots \times \sin 70^\circ$

29.69273008

area = 29.7 ***A1*** ***N2***

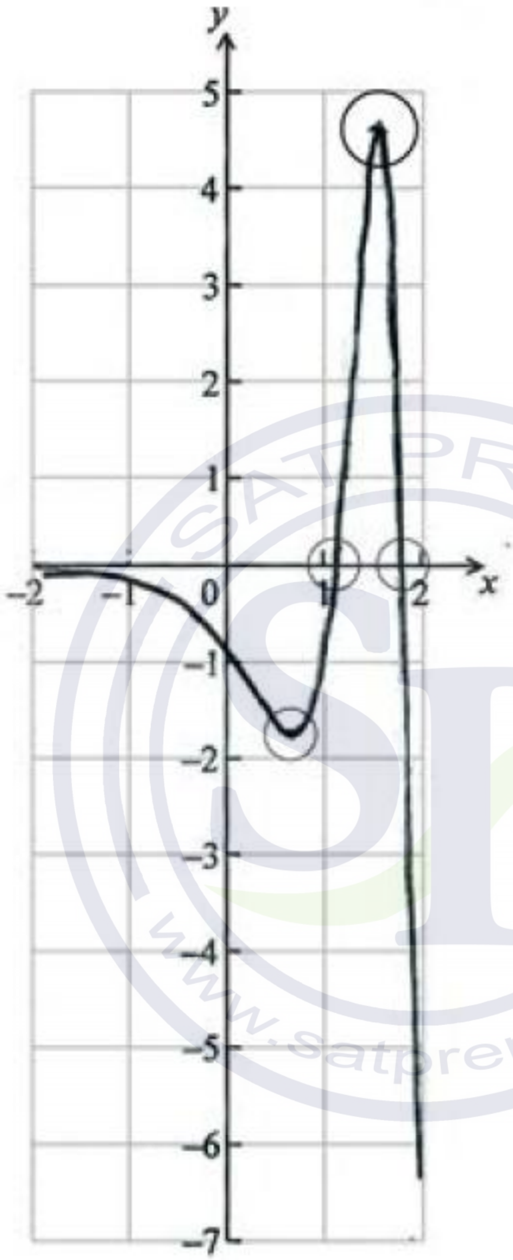
[2 marks]

Total [6 marks]

2. (a) $f'(x) = -e^x \sin(e^x)$ ***A1A1*** ***N2***

[2 marks]

(b)



A1A1A1A1 ***N4***

Note:	Award ***A1*** for shape that must have the correct domain (from -2 to $+2$) and correct range (from -6 to 4), ***A1*** for minimum in circle, ***A1*** for maximum in circle and ***A1*** for intercepts in circles.

[4 marks]
Total [6 marks]

3. (a) correct substitution into sum of a geometric sequence *(A1)*

e.g. $200\left(\frac{1-r^4}{1-r}\right)$, $200+200r+200r^2+200r^3$

attempt to set up an equation involving a sum and 324.8 ***M1***

e.g. $200\left(\frac{1-r^4}{1-r}\right)=324.8$, $200+200r+200r^2+200r^3=324.8$

$r=0.4$ (exact) ***A2*** ***N3***

[4 marks]

(b) correct substitution into formula ***A1***

e.g. $u_{10}=200\times0.4^9$

u_{10} = 0.0524288 (exact), 0.0524 ***A1*** ***N1***

[2 marks]

Total [6 marks]

4. (a) evidence of appropriate method *(M1)*

e.g. $z=\frac{122.5-117}{5}$, sketch of normal curve showing mean and 122.5, 1.1

$\mathrm{P}(Z<1.1)=0.8643$ *(A1)*

0.135666

P (H>122.5) = 0.136 ***A1*** ***N3***

[3 marks]

(b) $z=0.3853$ *(A1)*

set up equation *(M1)*

e.g. $\frac{X-117}{5}=0.3853$, sketch

$k=118.926602$

k = 199 ***A1*** ***N3***

[3 marks]

Total [6 marks]

5. (a) recognizing that acceleration is the derivative of velocity (seen anywhere) ***(R1)***

e.g. $a = \dfrac{\mathrm{d}^2 s}{\mathrm{d}t^2}$, v', $12 - 6t^2$

correctly substituting 2.7 into their expression for a (not into v) ***(A1)***

e.g. $s''(2.7)$

acceleration = −31.74 (exact), −31.7 ***A1*** ***N3***

[3 marks]

(b) recognizing that displacement is the integral of velocity ***R1***

e.g. $s = \int v$

Correctly substituting 1.3 ***(A1)***

e.g. $\int_0^{1.3} v\,\mathrm{d}t$

displacement = 7.41195 (exact), 7.41 (cm) ***A1*** ***N2***

[3 marks]

Total [6 marks]

6. (a) $\boldsymbol{A}^{-1} = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 1 \\ 1 & 1 & 1 \end{pmatrix}$ ***A2*** ***N2***

[2 marks]

(b) **Note** : The first two steps may be done in any order.

evidence of premultiplying by $\boldsymbol{A}^{-1}$ ***(M1)***

evidence of postmultiplying by $\boldsymbol{A}$ ***(M1)***

correct expression ***(A1)***

e.g. $\boldsymbol{A}^{-1}\boldsymbol{B}\boldsymbol{A}$, $\boldsymbol{A}^{-1}\boldsymbol{A}\boldsymbol{C}\boldsymbol{A}^{-1}\boldsymbol{A} = \boldsymbol{A}^{-1}\boldsymbol{B}\boldsymbol{A}$

$\boldsymbol{C} = \begin{pmatrix} 3 & 0 & 1 \\ 2 & -1 & 1 \\ 4 & 0 & 0 \end{pmatrix}$ ***A2*** ***N3***

[5 marks]

Total [7 marks]

7.

(a) evidence of recognizing binomial (seen anywhere) ***(M1)***
e.g. $\mathrm{B}(n,\ p)$, 0.95^{30}

finding $\mathrm{P}(X=0)=0.21463876$ ***(A1)***

appropriate approach ***(M1)***
e.g. complement, summing probabilities

0.785361
probability is 0.785 ***A1*** ***N3***
[4 marks]

(b) identifying correct outcomes (seen anywhere) ***(A1)***
e.g. $\mathrm{P}(X=1)+\mathrm{P}(X=2)$, 1 or 2 defective, $0.3389\ldots+0.2586\ldots$

recognizing conditional probability (seen anywhere) ***R1***
e.g. $\mathrm{P}(A|B)$, $\mathrm{P}(X\le 2|X\ge 1)$, $\mathrm{P}(\text{at most } 2|\text{at least } 1)$

appropriate approach involving conditional probability ***(M1)***
e.g. $\dfrac{\mathrm{P}(X=1)+\mathrm{P}(X=2)}{\mathrm{P}(X\ge 1)}$, $\dfrac{0.3389\ldots+0.2586\ldots}{0.785\ldots}$, $\dfrac{1 \text{ or } 2}{0.785}$

0.760847
probability is 0.761 ***A1*** ***N2***
[4 marks]

Total [8 marks]

SECTION B

8. (a) (i) valid approach *(M1)*

e.g. $\text{OA} + \text{AB}$

$\overrightarrow{\text{OB}} = 4i + 3j$ ***A1*** ***N2***

(ii) valid approach *(M1)*

e.g. $\overrightarrow{\text{OA}} + \overrightarrow{\text{AB}} + \overrightarrow{\text{BF}}$; $\overrightarrow{\text{OB}} + \overrightarrow{\text{BF}}$; $\overrightarrow{\text{OC}} + \overrightarrow{\text{CG}} + \overrightarrow{\text{GF}}$

$\overrightarrow{\text{OF}} = 4i + 3j + 2k$ ***A1*** ***N2***

(iii) correct approach ***A1***

e.g. $\overrightarrow{\text{AO}} + \overrightarrow{\text{OC}} + \overrightarrow{\text{CG}}$; $\overrightarrow{\text{AB}} + \overrightarrow{\text{BF}} + \overrightarrow{\text{FG}}$; $\overrightarrow{\text{AB}} + \overrightarrow{\text{BC}} + \overrightarrow{\text{CG}}$

$\overrightarrow{\text{AG}} = -4i + 3j + 2k$ ***AG*** ***N0***

[5 marks]

(b) (i) **any** correct equation for (OF) in the form $\boldsymbol{r} = \boldsymbol{a} + t\boldsymbol{b}$ ***A2*** ***N2***

where $\boldsymbol{a}$ is 0 or $4i + 3j + 2k$, and $\boldsymbol{b}$ is a scalar multiple of $4i + 3j + 2k$

e.g. $\boldsymbol{r} = t(4, 3, 2)$, $\boldsymbol{r} = \begin{pmatrix} 4t \\ 3t \\ 2t \end{pmatrix}$, $\boldsymbol{r} = 4i + 3j + 2k + t(4i + 3j + 2k)$

(ii) **any** correct equation for (AG) in the form $\boldsymbol{r} = \boldsymbol{a} + s\boldsymbol{b}$ ***A2*** ***N2***

where $\boldsymbol{a}$ is $4i$ or $3j + 2k$ and $\boldsymbol{b}$ is a scalar multiple of $-4i + 3j + 2k$

e.g. $\boldsymbol{r} = (4, 0, 0) + s(-4, 3, 2)$, $\boldsymbol{r} = \begin{pmatrix} 4-4s \\ 3s \\ 2s \end{pmatrix}$, $\boldsymbol{r} = 3j + 2k + s(-4i + 3j + 2k)$

[4 marks]

continued …

Question 8 continued

(c) choosing correct direction vectors, $\overrightarrow{\text{OF}}$ and $\overrightarrow{\text{AG}}$ ***(A1)(A1)***

scalar product $= -16 + 9 + 4 \ (= -3)$ ***(A1)***

magnitudes $\sqrt{4^2 + 3^2 + 2^2}$, $\sqrt{(-4)^2 + 3^2 + 2^2}$ $\left(\sqrt{29}, \sqrt{29}\right)$ ***(A1)(A1)***

substitution into formula ***M1***

e.g. $\cos\theta = \dfrac{-16 + 9 + 4}{\left(\sqrt{4^2 + 3^2 + 2^2}\right) \times \sqrt{(-4)^2 + 3^2 + 2^2}} = \left(-\dfrac{3}{29}\right)$

95.93777°, 1.67443 radians

$\theta = 95.9°$ or 1.67 ***A1*** ***N4***

[7 marks]

Total [16 marks]

9. (a) attempt to substitute coordinates in f *(M1)*
e.g. $f(2)=9$

correct substitution *A1*
e.g. $a\times 2^3+b\times 2^2+c=9$

$8a+4b+c=9$ *AG* *N0*
[2 marks]

(b) recognizing that $(1, 4)$ is on the graph of f *(M1)*
e.g. $f(1)=4$

correct equation *A1*
e.g. $a+b+c=4$

recognizing that $f'=0$ at minimum (seen anywhere) *(M1)*
e.g. $f'(1)=0$

$f'(x)=3ax^2+2bx$ (seen anywhere) *A1A1*

correct substitution into derivative *(A1)*
e.g. $3a\times 1^2+2b\times 1=0$

correct simplified equation *A1*
e.g. $3a+2b=0$

[7 marks]

(c) valid method for solving system of equations *(M1)*
e.g. inverse of a matrix, substitution

$a=2,\ b=-3,\ c=5$ *A1A1A1* *N4*
[4 marks]

Total [13 marks]

10. (a) correct substitution into cosine rule ***A1***

e.g. $\text{PQ}^2 = r^2 + r^2 - 2(r)(r)\cos(2\theta)$, $\text{PQ}^2 = 2r^2 - 2r^2\left(\cos(2\theta)\right)$

substituting $1 - 2\sin^2\theta$ for $\cos 2\theta$ (seen anywhere) ***A1***

e.g. $\text{PQ}^2 = 2r^2 - 2r^2(1 - 2\sin^2\theta)$

working towards answer ***(A1)***

e.g. $\text{PQ}^2 = 2r^2 - 2r^2 + 4r^2\sin^2\theta$

recognizing $2r^2 - 2r^2 = 0$ (including crossing out) (seen anywhere) ***A1***

e.g. $\text{PQ}^2 = 4r^2\sin^2\theta$, $\text{PQ} = \sqrt{4r^2\sin^2\theta}$

$\text{PQ} = 2r\sin\theta$ ***AG*** ***N0***

[4 marks]

(b) $\text{PRQ} = r \times 2\theta$ (seen anywhere) ***(A1)***

correct set up ***A1***

e.g. $1.3 \times 2r\sin\theta - r \times (2\theta) = 0$

attempt to eliminate r ***(M1)***

correct equation in terms of the one variable θ ***(A1)***

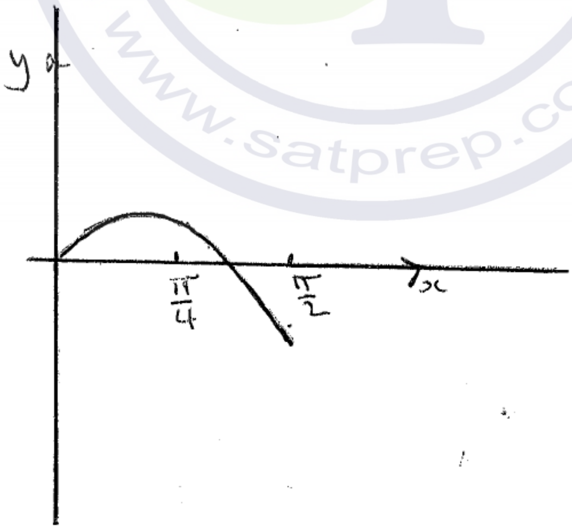
e.g. $1.3 \times 2\sin\theta - 2\theta = 0$

1.221496215

$\theta = 1.22$ (accept 70.0^0 (69.9) ***A1*** ***N3***

[5 marks]

(c) (i)



A1A1A1 ***N3***

Note:	Award ***A1*** for approximately correct shape, ***A1*** for x-intercept in approximately correct position, ***A1*** for domain. Do not penalise if sketch starts at origin.

(ii) 1.221496215

$\theta = 1.22$ ***A1*** ***N1***

[4 marks]

(d) evidence of appropriate approach (may be seen earlier) ***M2***
e.g. $2\theta < 2.6\sin\theta$, $0 < f(\theta)$, showing positive part of sketch

$0 < \theta < 1.221496215$

$0 < \theta = 1.22$ (accept $\theta < 1.22$) ***A1*** ***N1***

[3 marks]

Total [16 marks]

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MARKSCHEME

November 2011

MATHEMATICS

Standard Level

Paper 2

16 pages

Instructions to Examiners

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Using the markscheme

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All the marks will be added and recorded by scoris.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any. An exception to this rule is when work for ***M1*** is missing, as opposed to incorrect (see point 4).
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3 *N* marks

*If **no** working shown, award **N** marks for **correct** answers. In this case, ignore mark breakdown (**M**, **A**, **R**).*

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4 Implied and must be seen marks

*Implied marks appear in **brackets e.g. (M1)**.*

- Implied marks can only be awarded if **correct** work is seen or if implied in subsequent working (a correct answer does not necessarily mean that the implied marks are all awarded). There are questions where some working is required, but as it is accepted that not everyone will write the same steps, all the marks are implied, but the N marks are not the full marks for the question.
- Normally the correct work is seen or implied in the next line.
- Where there is an (***M1***) followed by ***A1*** for each correct answer, if no working shown, one correct answer is sufficient evidence to award the (***M1***).

*Must be seen marks appear without **brackets e.g. M1**.*

- Must be seen marks can only be awarded if **correct** work is seen.
- If a must be seen mark is not awarded because work is missing (as opposed to ***M0*** or ***A0*** for incorrect work) all subsequent marks may be awarded if appropriate.

5 Follow through marks (only applied after an error is made)

*Follow through (**FT**) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) or subpart(s). Usually, to award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if the only marks awarded in a subpart are for the answer (ie there is no working expected), then **FT** marks should be awarded if appropriate. Examiners are expected to check student work in order to award FT marks where appropriate.*

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate. (However, as noted above, if an ***A*** mark is not awarded because work is missing, all subsequent marks may be awarded if appropriate)
- Exceptions to this rule will be explicitly noted on the markscheme.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.

- If the error leads to an inappropriate value (*e.g.* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word "their" in a description, to indicate that candidates may be using an incorrect value.
- If a candidate makes an error in one part, but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the question says hence. It is often possible to use a different approach in subsequent parts that does not depend on the answer to previous parts.
- In a "show that" question, if an error leads to not showing the required answer, there is a 1 mark penalty. Note that if the error occurs within the same subpart, the *FT* rules may result in further loss of marks.
- Where there are anticipated common errors, the ***FT*** answers are often noted on the markscheme, to help examiners. It should be stressed that these are not the only ***FT*** answers accepted.

6 Mis-read

*If a candidate incorrectly copies information from the question, this is a mis-read (**MR**). Apply a **MR** penalty of 1 mark to that question. Award the marks as usual and then stamp **MR** against the answer. Scoris will automatically deduct 1 mark from the question total. A candidate should be penalized only once for a particular mis-read. Do not stamp **MR** again for that question, unless the candidate makes another mis-read.*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.

7 Discretionary marks *(d)*

*An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. In such cases the annotation DM should be used and a brief **note** written next to the mark explaining this decision.*

8 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for part-questions are indicated by **EITHER** . . . **OR**.
- Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9 Alternative forms

*Unless the question specifies otherwise, **accept** equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.

- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

10 Accuracy of Answers

If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy. When this is not specified in the question, all numerical answers should be given exactly or correct to three significant figures

Candidates should NO LONGER be penalized for an accuracy error ***(AP)****. Examiners should award marks according to the rules given in these instructions and the markscheme. Accuracy is not the same as correctness – an incorrect value does not achieve relevant* ***A*** *marks. It is only final answers which may lose marks for accuracy errors, not intermediate values. Please check work carefully for* ***FT****.*

Do not accept unfinished numerical answers such as 3/0.1 (unless otherwise stated). As a rule, numerical answers with more than one part (such as fractions) should be given using integers (*e.g.* 6/8). Calculations which lead to integers should be completed, with the exception of fractions which are not whole numbers.

11 Calculators

A GDC is required for paper 2, but calculators with symbolic manipulation features (e.g. TI-89) are not allowed.

Calculator notation

The Mathematics SL guide says:

Students must always use correct mathematical notation, not calculator notation.

Do **not** accept final answers written using calculator notation. However, do not penalize the use of calculator notation in the working.

12 Style

The markscheme aims to present answers using good communication, e.g. if the question asks to find the value of k, the markscheme will say $k = 3$*, but the marks will be for the correct value 3 – there is usually no need for the "* $k =$ *". In these cases, it is also usually acceptable to have another variable, as long as there is no ambiguity in the question, e.g. if the question asks to find the value of p and of q, then the student answer needs to be clear. Generally, the only situation where the full answer is required is in a question which asks for equations – in this case the markscheme will say "must be an equation".*

The markscheme often uses words to describe what the marks are for, followed by examples, using the e.g. notation. These examples are not exhaustive, and examiners should check what candidates have written, to see if they satisfy the description. Where these marks are ***M*** *marks, the examples may include ones using poor notation, to indicate what is acceptable.*

13 Crossed out work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

1. (a) interchanging x and y (may be seen at any time) *(M1)*

evidence of correct manipulation *(A1)*
e.g. $x = 2y + 4$

$f^{-1}(x) = \frac{x-4}{2}$ $\left(\text{accept } y = \frac{x-4}{2}, \frac{x-4}{2}\right)$ ***A1*** ***N2***

[3 marks]

(b) attempt to form composite (in any order) *(M1)*
e.g. $f(7x^2)$, $2(7x^2)+4$, $7(2x+4)^2$

$(f \circ g)(x) = 14x^2 + 4$ ***A1*** ***N2***
[2 marks]

(c) correct substitution *(A1)*
e.g. 7×3.5^2, $14(3.5)^2 + 4$

$(f \circ g)(3.5) = 175.5$ (accept 176) ***A1*** ***N2***
[2 marks]

Total [7 marks]

2. (a) median $= 174$ (cm) ***A1*** ***N1***
[1 mark]

(b) attempt to find number shorter than 161 *(M1)*
e.g. line on graph, 12 boys

$p = \frac{12}{200}$ $(= 0.06)$ ***A1*** ***N2***

[2 marks]

(c) **METHOD 1**

18 % have a height less than h *(A1)*

$0.18 \times 200 = 36$ (36 may be seen as a line on the graph) *(A1)*

$h = 166$ (cm) ***A1*** ***N2***
[3 marks]

METHOD 2

$0.82 \times 200 = 164$ (164 may be seen as a line on the graph) *(A1)*

$200 - 164 = 36$ *(A1)*

$h = 166$ (cm) ***A1*** ***N2***
[3 marks]

Total [6 marks]

3. (a) correct substitution *(A1)*

e.g. $8.5 = \theta(6.8)$, $\theta = \dfrac{8.5}{6.8}$

$\theta = 1.25$ (accept 71.6°) *A1* *N2*

[2 marks]

(b) **METHOD 1**

correct substitution into area formula (seen anywhere) *(A1)*

e.g. $A = \pi(6.8)^2$, $145.267\ldots$

correct substitution into area formula (seen anywhere) *(A1)*

e.g. $A = \dfrac{1}{2}(1.25)(6.8^2)$, 28.9

valid approach *M1*

e.g. $\pi(6.8)^2 - \dfrac{1}{2}(1.25)(6.8^2)$; $145.267\ldots - 28.9$; $\pi r^2 - \dfrac{1}{2}r^2 \sin\theta$

$A = 116\ (\text{cm}^2)$ *A1* *N2*

[4 marks]

METHOD 2

attempt to find reflex angle *(M1)*

e.g. $2\pi - \theta$, $360 - 1.25$

correct reflex angle *(A1)*

$\text{A}\hat{\text{O}}\text{B} = 2\pi - 1.25\ (= 5.03318\ldots)$

correct substitution into area formula *A1*

e.g. $A = \dfrac{1}{2}(5.03318...)(6.8^2)$

$A = 116\ (\text{cm}^2)$ *A1* *N2*

[4 marks]

Total [6 marks]

4. **Note**: accept answers given in degrees, and minutes.

(a) evidence of choosing sine rule *(M1)*

e.g. $\frac{\sin A}{a} = \frac{\sin B}{b}$

correct substitution *A1*

e.g. $\frac{\sin\theta}{10} = \frac{\sin 30^\circ}{7}$, $\sin\theta = \frac{5}{7}$

$\mathrm{A\hat{C}B} = 45.6^\circ$, $\mathrm{A\hat{C}B} = 134^\circ$ *A1A1* *N1N1*

Note: If candidates only find the acute angle in part (a), award no marks for (b).

[4 marks]

(b) Attempt to substitute their larger value into angle sum of triangle *(M1)*

e.g. $180^\circ - (134.415...^\circ + 30^\circ)$

$\mathrm{A\hat{B}C} = 15.6^\circ$ *A1* *N2*

[2 marks]

Total [6 marks]

5. (a) 10 terms *A1* *N1*

[1 mark]

(b) evidence of binomial expansion *(M1)*

e.g. $a^9b^0 + \binom{9}{1}a^8b + \binom{9}{2}a^7b^2 + \ldots$, $\binom{9}{r}(a)^{n-r}(b)^r$, Pascal's triangle

evidence of correct term *(A1)*

e.g. 8$^{\text{th}}$ term, $r = 7$, $\binom{9}{7}$, $(3x^2)^2 2^7$

correct expression of complete term *(A1)*

e.g. $\binom{9}{7}(3x^2)^2(2)^7$, ${}^9_2C(3x^2)^2(2)^7$, $36\times9\times128$

$41472\,x^4$ (accept $41500\,x^4$) *A1* *N2*

[4 marks]

Total [5 marks]

6. (a) $A(0) = 10$ ***A1*** ***N1***

[1 mark]

(b) substitution into formula ***(A1)***

e.g. $10(0.5)^{0.014(50)}$, $A(50)$

$A(50) = 6.16$ ***A1*** ***N2***

[2 marks]

(c) set up equation ***(M1)***

e.g. $A(t) = 0.395$

attempting to solve ***(M1)***

e.g. graph, use of logs

correct working ***(A1)***

e.g. sketch of intersection, $0.014t \log 0.5 = \log 0.0395$

$t = 333.00025..$ ***A1***

correct time 18:33 or 18:34 (accept 6:33 or 6:34 but nothing else) ***A1*** ***N3***

[5 marks]

Total [8 marks]

7. (a) applies vertical stretch parallel to the *y*-axis factor of $\frac{1}{3}$ ***(M1)***

e.g. multiply by $\frac{1}{3}$, $\frac{1}{3}f(t)$, $\frac{1}{3}\times 2$

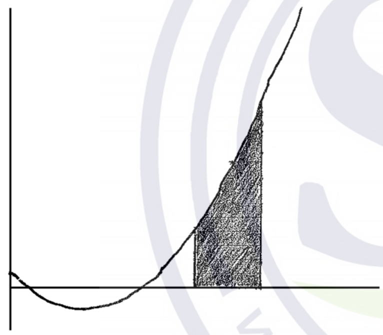
applies horizontal shift 2 units to the right ***(M1)***
e.g. $f(t-2)$, $t-2$

applies a vertical shift 4 units down ***(M1)***
e.g. subtracting 4, $f(t)-4$, $\frac{7}{3}-4$

$v(t)=\frac{2}{3}(t-2)^2-\frac{5}{3}$ ***A1*** ***N4***

[4 marks]

(b) recognizing that distance travelled is area under the curve ***M1***
e.g. $\int v$, $\frac{2}{9}(t-2)^3-\frac{5}{3}t$, sketch



distance $=15.576$ (accept 15.6) ***A2*** ***N2***
[3 marks]

Total [7 marks]

8. (a) (i) correct approach *(A1)*

e.g. $u_4 = (40)\frac{1}{2}^{(4-1)}$, listing terms

$u_4 = 5$ ***A1*** ***N2***

(ii) correct substitution into formula for infinite sum *(A1)*

e.g. $S_\infty = \frac{40}{1-0.5}$, $S_\infty = \frac{40}{0.5}$

$S_\infty = 80$ ***A1*** ***N2***

[4 marks]

(b) (i) attempt to set up expression for u_8 *(M1)*

e.g. $-36 + (8-1)d$

correct working ***A1***

e.g. $-8 = -36 + (8-1)d$, $\frac{-8-(-36)}{7}$

$d = 4$ ***A1*** ***N2***

(ii) correct substitution into formula for sum *(A1)*

e.g. $S_n = \frac{n}{2}\left(2(-36) + (n-1)4\right)$

correct working ***A1***

e.g. $S_n = \frac{n}{2}(4n-76)$, $-36n + 2n^2 - 2n$

$S_n = 2n^2 - 38n$ ***AG*** ***N0***

[5 marks]

(c) multiplying S_n (AP) by 2 or dividing S (infinite GP) by 2 *(M1)*

e.g. $2S_n$, $\frac{S_\infty}{2}$, 40

evidence of substituting into $2S_n = S_\infty$ ***A1***

e.g. $2n^2 - 38n = 40$, $4n^2 - 76n - 80\ (=0)$

attempt to solve **their** quadratic (equation) *(M1)*

e.g. intersection of graphs, formula

$n = 20$ ***A2*** ***N3***

[5 marks]

Total [14 marks]

9.

> **Note:** There may be slight differences in answers, depending on whether candidates use tables or GDCs, or their 3 sf answers in subsequent parts. Do not penalise answers that are consistent with **their** working and check carefully for ***FT***.

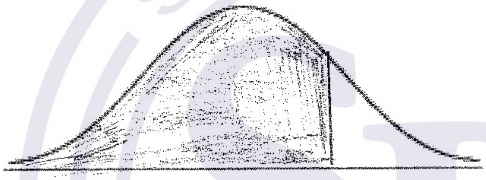
(a) evidence of recognizing binomial (seen anywhere in the question) ***(M1)***
e.g. ${}_nC_r\, p^r\, q^{n-r}$, $\mathrm{B}(n,\ p)$, ${}^{10}C_1\,(0.012)^1\,(0.988)^9$

$p = 0.108$ ***A1*** ***N2***
[2 marks]

(b) valid approach ***(M1)***
e.g. $\mathrm{P}(X \le 1)$, $0.88627\ldots + \ 0.10764\ldots$

$p = 0.994$ ***A1*** ***N2***
[2 marks]

(c) (i)



A1A1 ***N2***

> **Note:** Award ***A1*** for vertical line to right of mean,
> ***A1*** for shading to left of **their** vertical line.

(ii) valid approach ***(M1)***
e.g. $\mathrm{P}(X < 22.63)$

working to find standardized value ***(A1)***
e.g. $\dfrac{22.63-22}{0.3}$, 2.1

$p = 0.982$ ***A1*** ***N3***
[5 marks]

continued …

Question 9 continued

(d) (i) valid approach **(M1)**

e.g. $\mathrm{P}(21.37 < X < 22.63)$, $\mathrm{P}(-2.1 < z < 2.1)$

correct working **(A1)**

e.g. $0.982 - (1 - 0.982))$

$p = 0.964$ **A1** **N3**

(ii) correct working **(A1)**

e.g. $X \sim \mathrm{B}(10,\ 0.964)$, $(0.964)^{10}$

$p = 0.695$ (accept 0.694 from tables) **A1** **N2**

[5 marks]

(e) valid approach **(M1)**

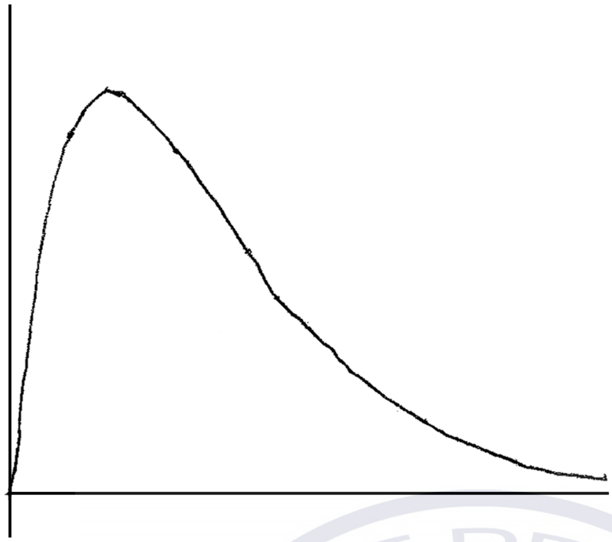
e.g. $\mathrm{P}(A \cap B) = \mathrm{P}(A)\mathrm{P}(B)$, $(0.994) \times (0.964)^{10}$

$p = 0.691$ (accept 0.690 from tables) **A1** **N2**

[2 marks]

Total [16 marks]

10. (a)



A1A1A1 *N3*

> **Note:** Award ***A1*** for approximately correct shape with inflexion/ change of curvature,
> ***A1*** for maximum skewed to the left,
> ***A1*** for asymptotic behaviour to the right.

[3 marks]

(b) (i) $x = 3.33$ *A1* *N1*

(ii) correct interval, with right end point $3\frac{1}{3}$ *A1A1* *N2*

e.g. $0 < x \le 3.33,\ 0 \le x < 3\frac{1}{3}$

> **Note**: Accept any inequalities in the right direction.

[3 marks]

(c) valid approach *(M1)*

e.g. quotient rule, product rule

2 correct derivatives (must be seen in product or quotient rule) *(A1)(A1)*

e.g. $20,\ 0.3\mathrm{e}^{0.3x}$ or $-0.3\mathrm{e}^{-0.3x}$

correct substitution into product or quotient rule *A1*

e.g. $\dfrac{20\mathrm{e}^{0.3x} - 20x(0.3)\mathrm{e}^{0.3x}}{(\mathrm{e}^{0.3x})^2},\ 20\mathrm{e}^{-0.3x} + 20x(-0.3)\mathrm{e}^{-0.3x}$

correct working *A1*

e.g. $\dfrac{20\mathrm{e}^{0.3x} - 6x\mathrm{e}^{0.3x}}{\mathrm{e}^{0.6x}},\ \dfrac{\mathrm{e}^{0.3x}\left(20 - 20x(0.3)\right)}{(\mathrm{e}^{0.3x})^2},\ \mathrm{e}^{-0.3x}\left(20 + 20x(-0.3)\right)$

$f'(x) = \dfrac{20 - 6x}{\mathrm{e}^{0.3x}}$ *AG* *N0*

[5 marks]

continued …

Question 10 continued

(d) consideration of f' or f'' ***(M1)***

valid reasoning ***R1***
e.g. sketch of f', f'' is positive, $f''=0$, reference to minimum of f'

correct value $6.6666666\ldots\left(6\frac{2}{3}\right)$ ***(A1)***

correct interval, with **both** end points ***A1*** ***N3***
e.g $6.67 < x \le 20$, $6\frac{2}{3} \le x < 20$

[4 marks]

Total [15 marks]