

## MATHEMATICS LEVEL 2

For each of the following problems, decide which is the BEST of the choices given. If the exact numerical value is not one of the choices, select the choice that best approximates this value. Then fill in the corresponding oval on the answer sheet.

**Notes:** (1) A scientific or graphing calculator will be necessary for answering some (but not all) of the questions on this test. For each question, you will have to decide whether or not you should use a calculator.

(2) For some questions in this test you may have to decide whether your calculator should be in the radian mode or the degree mode.

(3) Figures that accompany problems on this test are intended to provide information useful in solving the problems. They are drawn as accurately as possible EXCEPT when it is stated in a specific problem that its figure is not drawn to scale. All figures lie in a plane unless otherwise indicated.

(4) Unless otherwise specified, the domain of any function  $f$  is assumed to be the set of all real numbers  $x$  for which  $f(x)$  is a real number. The range of  $f$  is assumed to be the set of all real numbers  $f(x)$ , where  $x$  is in the domain of  $f$ .

(5) Reference information that may be useful in answering the questions on this test can be found below.

THE FOLLOWING INFORMATION IS FOR YOUR REFERENCE IN ANSWERING SOME OF THE QUESTIONS ON THIS TEST.

Volume of a right circular cone with radius  $r$  and height  $h$ :

$$V = \frac{1}{3}\pi r^2 h$$

Lateral area of a right circular cone with circumference of

the base  $c$  and slant height  $\ell$ :  $S = \frac{1}{2}c\ell$

Volume of a sphere with radius  $r$ :  $V = \frac{4}{3}\pi r^3$

Surface area of a sphere with radius  $r$ :  $S = 4\pi r^2$

Volume of a pyramid with base area  $B$  and height  $h$ :

$$V = \frac{1}{3}Bh$$

USE THIS SPACE FOR SCRATCHWORK.

1. If  $xy \neq 0$  and  $3x = 0.3y$ , then  $\frac{y}{x} =$
- (A) 0.1 (B) 1.0 (C) 3.0 (D) 9.0 (E) 10.0

2. If  $f(x) = (3\sqrt{x} - 4)^2$ , then how much does  $f(x)$  increase as  $x$  goes from 2 to 3?
- (A) 1.43  
 (B) 1.37  
 (C) 1.00  
 (D) 0.74  
 (E) 0.06

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## MATHEMATICS LEVEL 2—Continued

3. What is the equation of a line with a  $y$ -intercept of 3 and an  $x$ -intercept of  $-5$  ?

(A)  $y = 0.6x + 3$   
(B)  $y = 1.7x - 3$   
(C)  $y = 3x + 5$   
(D)  $y = 3x - 5$   
(E)  $y = -5x + 3$

USE THIS SPACE FOR SCRATCHWORK.

4. For what positive value of  $a$  does  $a - \sqrt{5a + 18}$  equal  $-4$  ?

(A) 0.56  
(B) 1.00  
(C) 1.12  
(D) 2.06  
(E) 4.12

5. If the second term in an arithmetic sequence is 4, and the tenth term is 15, what is the first term in the sequence?

(A) 1.18  
(B) 1.27  
(C) 1.38  
(D) 2.63  
(E) 2.75

6. If  $g(x) = |5x^2 - x^3|$ , then  $g(6) =$

(A)  $-54$  (B)  $-36$  (C)  $36$  (D)  $216$  (E)  $396$

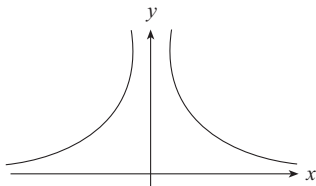
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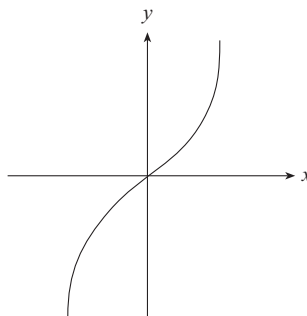
7. Which of the following graphs of functions is symmetrical with respect to the line  $y = x$  ?

USE THIS SPACE FOR SCRATCHWORK.

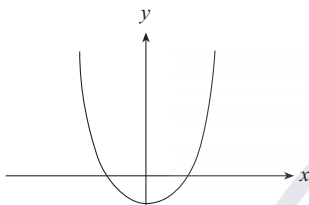
(A)



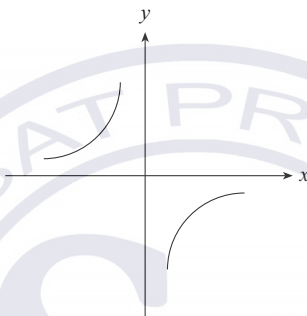
(D)



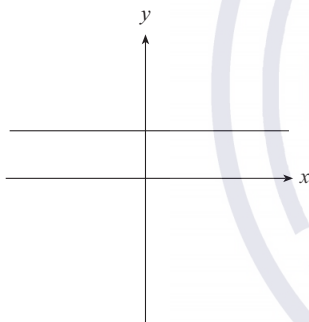
(B)



(E)



(C)



8. If  $0^\circ \leq A \leq 90^\circ$  and  $\sin A = \frac{1}{3} \sin 75^\circ$ , then  $A =$

- (A)  $12.9^\circ$  (B)  $18.8^\circ$  (C)  $25.0^\circ$  (D)  $32.2^\circ$  (E)  $45.0^\circ$

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## MATHEMATICS LEVEL 2—Continued

9. If  $f(x) = \frac{1}{2}x^2 - 6x + 11$ , then what is the minimum value of  $f(x)$  ?

- (A) -8.0  
(B) -7.0  
(C) 3.2  
(D) 6.0  
(E) 11.0

USE THIS SPACE FOR SCRATCHWORK.

10.  $|x - y| + |y - x| =$

- (A) 0  
(B)  $x - y$   
(C)  $y - x$   
(D)  $2|x - y|$   
(E)  $2|x + y|$

$0^\circ \leq A \leq 90^\circ$   
 $0^\circ \leq B \leq 90^\circ$

11. If  $\sin A = \cos B$ , then which of the following must be true?

- (A)  $A = B$   
(B)  $A = 2B$   
(C)  $A = B + 45$   
(D)  $A = 90 - B$   
(E)  $A = B + 180$

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## MATHEMATICS LEVEL 2—Continued

USE THIS SPACE FOR SCRATCHWORK.

	Total Units Production	Flawed Units
April	569	15
May	508	18
June	547	16

Figure 1

12. Each month, some of the automobiles produced at the Carco plant have flawed catalytic converters. According to the chart in Figure 1, what is the probability that a car produced in one of the three months shown will be flawed?
- (A) 0.01  
(B) 0.02  
(C) 0.03  
(D) 0.04  
(E) 0.05

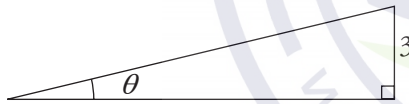


Figure 2

13. Adamsville building codes require that a wheelchair ramp must rise at an angle ( $\theta$ ) of no less than  $5^\circ$  and no more than  $7^\circ$  from the horizontal. If a wheelchair ramp rises exactly 3 feet as shown in Figure 2, which of the following could be the length of the ramp?
- (A) 19.0 feet  
(B) 24.0 feet  
(C) 28.0 feet  
(D) 35.0 feet  
(E) 42.0 feet

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MATHEMATICS LEVEL 2—Continued

USE THIS SPACE FOR SCRATCHWORK.

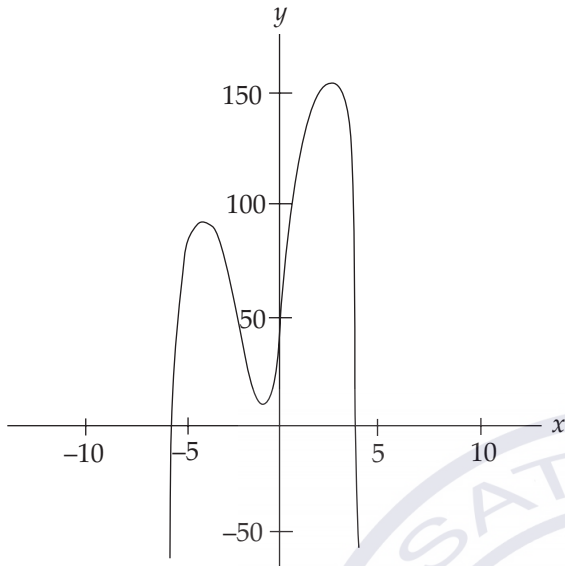


Figure 3

14. Figure 3 represents the graph of the function  $y = -x^4 - 4x^3 + 14x^2 + 45x - n$ . Which of the following could be the value of  $n$ ?
- (A) -50  
 (B) -18  
 (C) 50  
 (D) 100  
 (E) 150

15. What value does  $\frac{x^2 - x - 6}{3x + 6}$  approach as  $x$  approaches  $-2$ ?
- (A) -1.67  
 (B) -0.60  
 (C) 0  
 (D) 1.00  
 (E) 2.33

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## MATHEMATICS LEVEL 2—Continued

USE THIS SPACE FOR SCRATCHWORK.

16. In Titheland, the first 1,000 florins of any inheritance are untaxed. After the first 1,000 florins, inheritances are taxed at a rate of 65%. How large must an inheritance be, to the nearest florin, in order to amount to 2,500 florins after the inheritance tax?

- (A) 7,143  
(B) 5,286  
(C) 4,475  
(D) 3,475  
(E) 3,308

17. In an engineering test, a rocket sled is propelled into a target. If the sled's distance  $d$  in meters from the target is given by the formula  $d = -1.5t^2 + 120$ , where  $t$  is the number of seconds after rocket ignition, then how many seconds have passed since rocket ignition when the sled is 10 meters from the target?

- (A) 2.58  
(B) 8.56  
(C) 8.94  
(D) 9.31  
(E) 11.26

18.  $\sum_{k=1}^{10} 3k - 2 =$

- (A) 25 (B) 28 (C) 145 (D) 280 (E) 290

19. If  $e^x = 5$ , then  $x =$

- (A) 0.23  
(B) 1.61  
(C) 1.84  
(D) 7.76  
(E) 13.59

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20. If the greatest possible distance between two points within a certain rectangular solid is 12, then which of the following could be the dimensions of this solid?

(A)  $3 \times 3 \times 9$   
(B)  $3 \times 6 \times 7$   
(C)  $3 \times 8 \times 12$   
(D)  $4 \times 7 \times 9$   
(E)  $4 \times 8 \times 8$

USE THIS SPACE FOR SCRATCHWORK.

21. Runner A travels  $a$  feet every minute. Runner B travels  $b$  feet every second. In one hour, runner A travels how much farther than runner B, in feet?

(A)  $a - 60b$   
(B)  $a^2 - 60b^2$   
(C)  $360a - b$   
(D)  $60(a - b)$   
(E)  $60(a - 60b)$

22. A right triangle has sides in the ratio of 5:12:13. What is the measure of the smallest angle in the triangle, in degrees?

(A) 13.34  
(B) 22.62  
(C) 34.14  
(D) 42.71  
(E) 67.38

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## MATHEMATICS LEVEL 2—Continued

23. If  $f(x) = \frac{1}{x+1}$ , and  $g(x) = \frac{1}{x} + 1$ , then  $g(f(x)) =$

USE THIS SPACE FOR SCRATCHWORK.

(A) 2

(B)  $x + 2$

(C)  $2x + 2$

(D)  $\frac{x+2}{x+1}$

(E)  $\frac{2x+1}{x+1}$

24. If  $f(x) = (x - \pi)(x - 3)(x - e)$ , then what is the greatest possible distance between points at which the graph of  $y = f(x)$  intersects the  $x$ -axis?

(A) 0.14

(B) 0.28

(C) 0.36

(D) 0.42

(E) 0.72

25.  $\frac{x!}{(x-2)!} =$

(A) 0.5

(B) 2.0

(C)  $x$

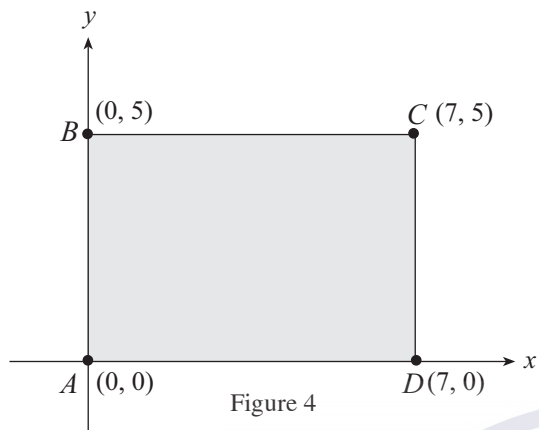
(D)  $x^2 - x$

(E)  $x^2 - 2x + 1$

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MATHEMATICS LEVEL 2—Continued

USE THIS SPACE FOR SCRATCHWORK.



26. What is the volume of the solid created by rotating rectangle  $ABCD$  in Figure 4 around the  $y$ -axis?
- (A) 219.91  
 (B) 245.00  
 (C) 549.78  
 (D) 769.69  
 (E) 816.24

27. If  $f(x, y) = \frac{x^2 - 2xy + y^2}{x^2 - y^2}$ , then  $f(-x, -y) =$

- (A) 1  
 (B)  $\frac{1}{x+y}$   
 (C)  $\frac{-x+y}{x+y}$   
 (D)  $\frac{-x+y}{x-y}$   
 (E)  $\frac{x-y}{x+y}$

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## MATHEMATICS LEVEL 2—Continued

28. In order to disprove the hypothesis, “No number divisible by 5 is less than 5,” it would be necessary to

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- (A) prove the statement false for all numbers divisible by 5
- (B) demonstrate that numbers greater than 5 are often divisible by 5
- (C) indicate that infinitely many numbers greater than 5 are divisible by 5
- (D) supply one case in which a number divisible by 5 is less than 5
- (E) show that a statement true of numbers greater than 5 is also true of numbers less than 5

29. A parallelogram has vertices at  $(0, 0)$ ,  $(5, 0)$ , and  $(2, 3)$ . What are the coordinates of the fourth vertex?

- (A)  $(3, -2)$
- (B)  $(5, 3)$
- (C)  $(7, 3)$
- (D)  $(10, 5)$
- (E) It cannot be determined from the information given.

30. The expression  $\frac{x^2 + 3x - 4}{2x^2 + 10x + 8}$  is undefined for what values of  $x$ ?

- (A)  $x = \{-1, -4\}$
- (B)  $x = \{-1\}$
- (C)  $x = \{0\}$
- (D)  $x = \{1, -4\}$
- (E)  $x = \{0, 1, 4\}$

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MATHEMATICS LEVEL 2—Continued

31. For which of the following functions is  $f(x) > 0$  for all real values of  $x$  ?

- I.  $f(x) = x^2 + 1$   
 II.  $f(x) = 1 - \sin x$   
 III.  $f(x) = \pi(\pi^{x-1})$

- (A) I only  
 (B) II only  
 (C) I and III only  
 (D) II and III only  
 (E) I, II, and III

USE THIS SPACE FOR SCRATCHWORK.

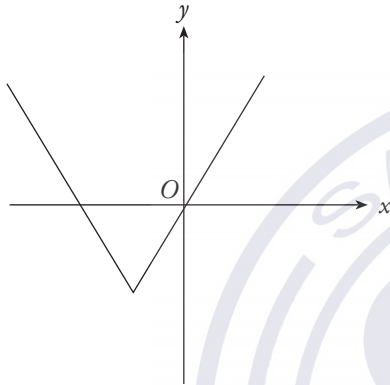
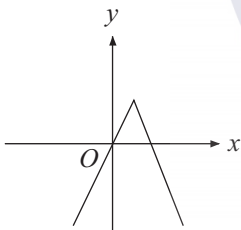


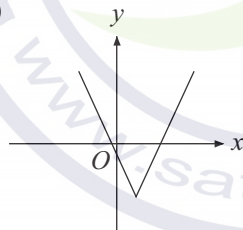
Figure 5

32. The graph of  $y = f(x)$  is shown in Figure 5. Which of the following could be the graph of  $y = -f(-x)$  ?

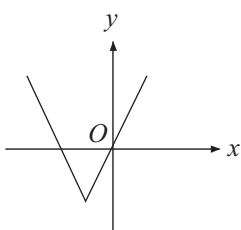
(A)



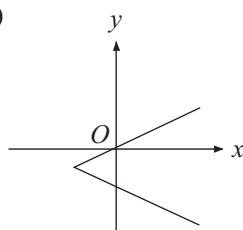
(B)



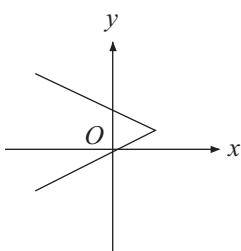
(C)



(D)



(E)



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## MATHEMATICS LEVEL 2—Continued

33. A wire is stretched from the top of a two-foot-tall anchor to the top of a 50-foot-tall antenna. If the wire is straight and has a slope of  $\frac{2}{5}$ , then what is the length of the wire in feet?

(A) 89.18  
(B) 120.00  
(C) 123.26  
(D) 129.24  
(E) 134.63

USE THIS SPACE FOR SCRATCHWORK.

34. If  $\frac{3\pi}{2} < \theta < 2\pi$  and  $\sec \theta = 4$ , then  $\tan \theta =$

(A) -3.93  
(B) -3.87  
(C) 0.26  
(D) 3.87  
(E) 3.93

35. Circle  $O$  is centered at  $(-3, 1)$  and has a radius of 4. Circle  $P$  is centered at  $(4, -4)$  and has a radius of  $n$ . If circle  $O$  is externally tangent to circle  $P$ , then what is the value of  $n$ ?

(A) 4.00  
(B) 4.37  
(C) 4.60  
(D) 5.28  
(E) 6.25

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MATHEMATICS LEVEL 2—Continued

36. In triangle  $ABC$ ,  $\frac{\sin A}{\sin B} = \frac{7}{10}$  and  $\frac{\sin B}{\sin C} = \frac{5}{2}$ . If angles  $A$ ,  $B$ , and  $C$  are opposite sides  $a$ ,  $b$ , and  $c$ , respectively, and the triangle has a perimeter of 16, then what is the length of  $a$ ?

USE THIS SPACE FOR SCRATCHWORK.

- (A) 2.7  
 (B) 4.7  
 (C) 5.3  
 (D) 8.0  
 (E) 14.0

$x$	$h(x)$
-1	0
0	3
1	0
2	3

37. The table of values above shows selected coordinate pairs on the graph of  $h(x)$ . Which of the following could be  $h(x)$ ?
- (A)  $x(x+1)(x-1)$   
 (B)  $(x+1)^2(x-1)$   
 (C)  $(x-1)(x+2)^2$   
 (D)  $(x-1)^2(x+3)$   
 (E)  $(x-1)(x+1)(2x-3)$

$$\begin{aligned} a + b + 2c &= 7 \\ a - 2b &= 8 \\ 3b + 2c &= n \end{aligned}$$

38. For what values of  $n$  does the system of equations above have no real solutions?
- (A)  $n \neq -1$   
 (B)  $n \leq 0$   
 (C)  $n \geq 1$   
 (D)  $n > 7$   
 (E)  $n = -15$

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## MATHEMATICS LEVEL 2—Continued

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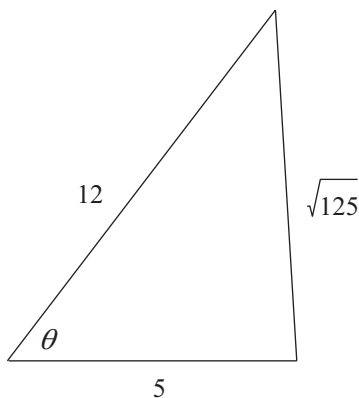


Figure 6

Note: Figure not drawn to scale.39. In Figure 6, what is the value of  $\theta$  in degrees?

- (A) 62.00  
 (B) 65.38  
 (C) 65.91  
 (D) 68.49  
 (E) 68.70

40. If  $\begin{vmatrix} l & m & n \\ p & q & r \\ s & t & u \end{vmatrix} = A$ , then  $\begin{vmatrix} 2l & 2m & 2n \\ 2p & 2q & 2r \\ 2s & 2t & 2u \end{vmatrix} =$

- (A)  $2A$   
 (B)  $4A$   
 (C)  $6A$   
 (D)  $8A$   
 (E)  $18A$

41. In the function  $g(x) = A[\sin(Bx + C)] + D$ , constants are represented by  $A$ ,  $B$ ,  $C$ , and  $D$ . If  $g(x)$  is to be altered in such a way that both its period and amplitude are increased, which of the following constants must be increased?

- (A)  $A$  only  
 (B)  $B$  only  
 (C)  $C$  only  
 (D)  $A$  and  $B$  only  
 (E)  $C$  and  $D$  only

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## MATHEMATICS LEVEL 2—Continued

42. All of the elements of list  $M$  and list  $N$  are arranged in exactly 20 pairs, such that every element from list  $M$  is paired with a distinct element from list  $N$ . If in each such pair, the element from list  $M$  is larger than the element from list  $N$ , then which of the following statements must be true?

- (A) The median of the elements in  $M$  is greater than the median of the elements in  $N$ .  
(B) Any element of  $M$  is greater than any element of  $N$ .  
(C) The mode of the elements in  $M$  is greater than the mode of the elements in  $N$ .  
(D) The range of the elements in  $M$  is greater than the range of the elements in  $N$ .  
(E) The standard deviation of the elements in  $M$  is greater than the standard deviation of the elements in  $N$ .

USE THIS SPACE FOR SCRATCHWORK.

43. If 3, 5, 8.333, and 13.889 are the first four terms of a sequence, then which of the following could define that sequence?

- (A)  $a_0 = 3; a_{n+1} = a_n + 2$   
(B)  $a_0 = 3; a_{n+1} = 2a_n - 1$   
(C)  $a_0 = 3; a_n = a_{n-1} + \frac{40}{9}$   
(D)  $a_0 = 3; a_n = \frac{5}{3}a_{n-1}$   
(E)  $a_0 = 3; a_n = \frac{7}{3}a_{n-1} - \frac{40}{9}a_{n-1}$

44. If  $0 \leq n \leq \frac{\pi}{2}$  and  $\cos(\cos n) = 0.8$ , then  $\tan n =$

- (A) 0.65  
(B) 0.75  
(C) 0.83  
(D) 1.19  
(E) 1.22

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## MATHEMATICS LEVEL 2—Continued

45. The height of a cylinder is equal to one-half of  $n$ , where  $n$  is equal to one-half of the cylinder's diameter. What is the surface area of this cylinder in terms of  $n$ ?

- (A)  $\frac{3\pi n^2}{2}$   
(B)  $2\pi n^2$   
(C)  $3\pi n^2$   
(D)  $2\pi n^2 + \frac{\pi n}{2}$   
(E)  $2\pi n^2 + \pi n$

USE THIS SPACE FOR SCRATCHWORK.

46. If  $(\tan \theta - 1)^2 = 4$ , then which of the following could be the value of  $\theta$  in radian measure?

- (A)  $-0.785$   
(B)  $1.373$   
(C)  $1.504$   
(D)  $1.512$   
(E)  $3$

47. Which of the following expresses the range of values of  $y = g(x)$ , if  $g(x) = \frac{5}{x+4}$ ?

- (A)  $\{y: y \neq 0\}$   
(B)  $\{y: y \neq 1.25\}$   
(C)  $\{y: y \neq -4.00\}$   
(D)  $\{y: y > 0\}$   
(E)  $\{y: y \leq -1 \text{ or } y \geq 1\}$

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## MATHEMATICS LEVEL 2—Continued

48. If  $\csc \theta = \frac{1}{3t}$ , then where defined,  $\cos \theta =$

USE THIS SPACE FOR SCRATCHWORK.

(A)  $3t$

(B)  $\sqrt{1 - 3t^2}$

(C)  $\sqrt{1 - 9t^2}$

(D)  $\frac{3t}{\sqrt{1 - 3t^2}}$

(E)  $\frac{3t}{\sqrt{1 - 9t^2}}$

49. If  $f(x, y) = \frac{xy + y}{x + y}$ , then which of the following

statements must be true?

I. If  $x = 0$  and  $y \neq 0$ , then  $f(x, y) = 1$ .

II. If  $x = 1$ , then  $f(x, x) = 1$ .

III.  $f(x, y) = f(y, x)$

(A) I only

(B) II only

(C) I and II only

(D) I and III only

(E) I, II, and III

50. A triangle is formed by the  $x$ -axis, the  $y$ -axis, and the line  $y = mx + b$ . If  $m = -b^3$ , then what is the volume of the cone generated by rotating this triangle around the  $x$ -axis?

(A)  $\frac{\pi}{9}$

(B)  $\frac{\pi}{3}$

(C)  $\pi$

(D)  $3\pi$

(E)  $9\pi$

**STOP**

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON THIS TEST ONLY.  
DO NOT WORK ON ANY OTHER TEST IN THIS BOOK.