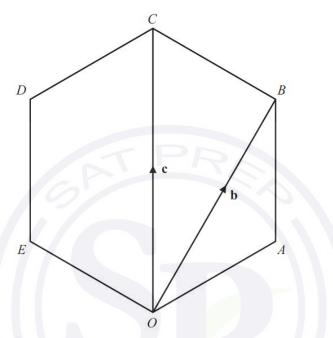
Extended Mathematics Topic : Vector-Function-Transformation Year :May 2013 -May 2024 Paper - 2 Question Booklet

Question 1



OABCDE is a regular polygon.

(a) Write down the geometrical name for this polygon.

Answer(a)[1]

(b) *O* is the origin. $\overrightarrow{OB} = \mathbf{b}$ and $\overrightarrow{OC} = \mathbf{c}$.

Find, in terms of **b** and **c**, in their simplest form,

(i) \overrightarrow{BC} ,

Answer(b)(i) $\overrightarrow{BC} =$ [1]

(ii) \overrightarrow{OA} ,

$$Answer(b)(ii) \ \overrightarrow{OA} = \dots [2]$$

(iii) the position vector of E.

$$f(x) = x + \frac{2}{x} - 3, x , 0$$
 $g(x) = \frac{x}{2} - 5$

Find

(a) fg(18),

(b) $g^{-1}(x)$.

Question 3

$$f(x) = 5x + 4$$
 $g(x) = \frac{1}{2x}, x, 0$ $h(x) = \left(\frac{1}{2}\right)^{x}$

Find

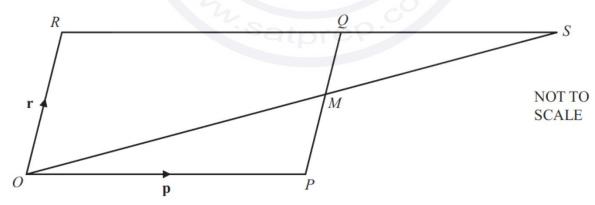
(a) fg(5),

- (b) gg(x) in its simplest form,
- Answer(b) $gg(x) = \dots$ [2]

(c) $f^{-1}(x)$,

- (d) the value of x when h(x) = 8.
- Answer(d) x = [2]

Question 4



OPQR is a parallelogram, with *O* the origin. *M* is the midpoint of *PQ*. *OM* and *RQ* are extended to meet at *S*. $\overrightarrow{OP} = \mathbf{p}$ and $\overrightarrow{OR} = \mathbf{r}$. Continue on the next page.. (a) Find, in terms of **p** and **r**, in its simplest form,

(i) \overrightarrow{OM} ,

	-		
Answer(a)(i)	OM =		[1]
211151101 (4)(1)	0111	••••••••••••••••••	L * J

(ii) the position vector of S.

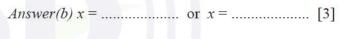
(b) When $\overrightarrow{PT} = -\frac{1}{2}\mathbf{p} + \mathbf{r}$, what can you write down about the position of T?

Answer(b)[1]

Answer(a)

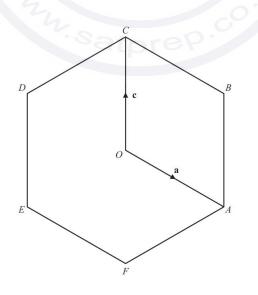
- Question 5
 - f(x) = 2x + 3 $g(x) = x^2$
 - (a) Find fg(6).
 - (b) Solve the equation gf(x) = 100.
- (c) Find $f^{-1}(x)$.
- (d) Find $ff^{-1}(5)$.

Question 6



......[2]

- - Answer(d)[1]



O is the origin. *ABCDEF* is a regular hexagon and *O* is the midpoint of *AD*.

$$\overrightarrow{OA} = \mathbf{a}$$
 and $\overrightarrow{OC} = \mathbf{c}$.

Find, in terms of \mathbf{a} and \mathbf{c} , in their simplest form

(a) \overrightarrow{BE} ,

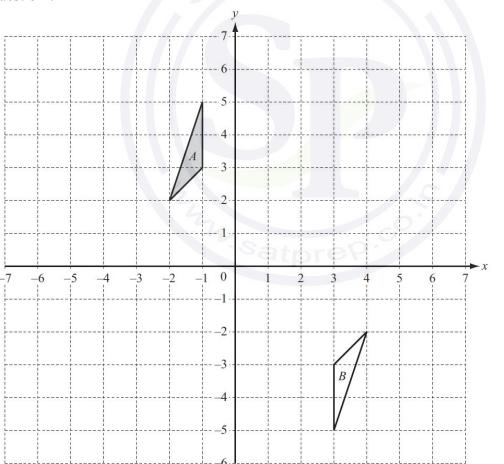
Answer(a)
$$\overrightarrow{BE} = \dots [2]$$

(b) \overrightarrow{DB} ,

Answer(c)

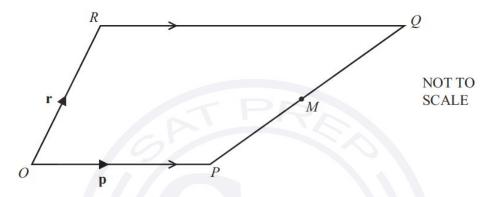
(c) the position vector of E.

Question 7



- (a) Draw the image of triangle A after a translation by the vector $\begin{pmatrix} 3 \\ -4 \end{pmatrix}$. [2]
- (b) Describe fully the single transformation which maps triangle A onto triangle B.



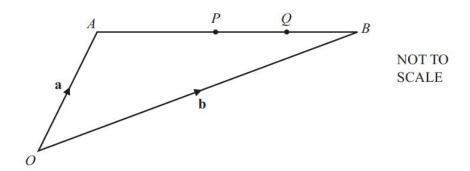


OPQR is a trapezium with *RQ* parallel to *OP* and *RQ* = 2*OP*. *O* is the origin, $\overrightarrow{OP} = \mathbf{p}$ and $\overrightarrow{OR} = \mathbf{r}$. *M* is the midpoint of *PQ*.

Find, in terms of **p** and **r**, in its simplest form

(a) \overrightarrow{PQ} ,

(b) \overrightarrow{OM} , the position vector of M.



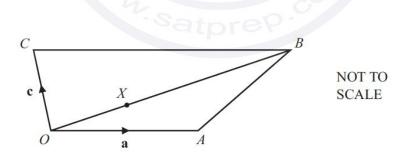
The diagram shows two points, *P* and *Q*, on a straight line *AB*. *P* is the midpoint of *AB* and *Q* is the midpoint of *PB*. *O* is the origin, $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.

Write down, in terms of a and b, in its simplest form

(a) \overrightarrow{AP} ,

Question 10

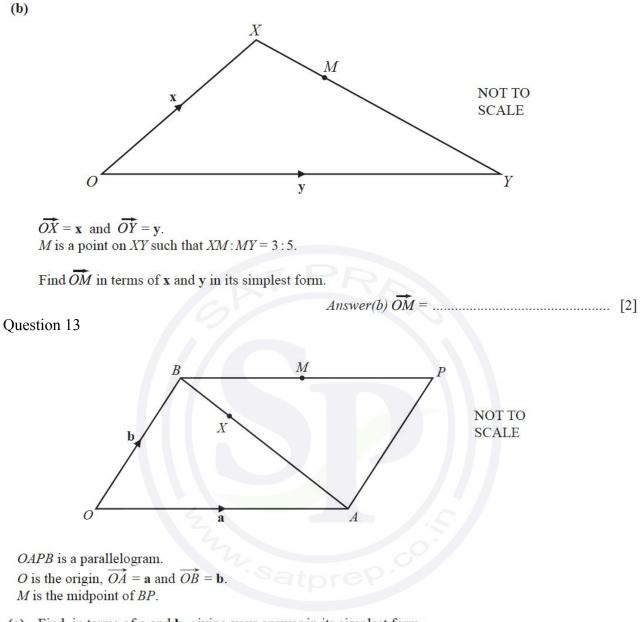
(b) the position vector of Q.



The diagram shows a quadrilateral *OABC*. $\overrightarrow{OA} = \mathbf{a}, \overrightarrow{OC} = \mathbf{c}$ and $\overrightarrow{CB} = 2\mathbf{a}$. X is a point on *OB* such that OX: XB = 1:2.

(a) Find, in terms of a and c, in its simplest form Continue on the next page.

(i) \overrightarrow{AC} , Answer(a)(i) $\overrightarrow{AC} = \dots$ [1] (ii) \overrightarrow{AX} . Answer(a)(ii) $\overrightarrow{AX} =$ [3] (b) Explain why the vectors \overrightarrow{AC} and \overrightarrow{AX} show that C, X and A lie on a straight line. Answer(b) [2] Question 11 $\mathbf{f}(x) = 5x - 3$ $g(x) = x^2$ (a) Find fg(-2). (b) Find gf(x), in terms of x, in its simplest form. (c) Find $f^{-1}(x)$. Question 12 (a) S R NOT TO SCALE 0 2**a** *PQRS* is a trapezium with PQ = 2SR. $\overrightarrow{PQ} = 2\mathbf{a}$ and $\overrightarrow{PS} = \mathbf{b}$. Find \overrightarrow{QR} in terms of **a** and **b** in its simplest form.



(a) Find, in terms of a and b, giving your answer in its simplest form,

(i) \overrightarrow{BA} ,

Answer(a)(i) $\overrightarrow{BA} = \dots$ [1]

(ii) the position vector of M.

(b)	X is on BA so that $BX:XA = 1:2$.		
	Show that X lies on OM.		
	Answer(b)		
Ques	stion 14	I	[4]
Que	$f(x) = 3x + 5$ $g(x) = x^2$		
(a)			
(a)	Find $g(3x)$.	4	[1]
		Answer(a)	[1]
(b)	Find $f^{-1}(x)$, the inverse function.		
		Answer(b) $f^{-1}(x) =$	[2]
(c)	Find $ff(x)$.		
	Give your answer in its simplest form.		5.4.7
		Answer(c)	[2]
Ques	stion 15		
	$\mathbf{f}(x) = 5 - 3x$		
(a)	Find f(6).		
		Answer(a)	[1]
(b)	Find $f(x + 2)$.		
(~)	1 ma ((r + 2)).		
		Answer(b)	[1]
(c)	Find $ff(x)$, in its simplest form.		
		Answer(c)	[2]
(d)	Find $f^{-1}(x)$, the inverse of $f(x)$.		
		Answer(d) $f^{-1}(x) =$	[2]
Ques	stion 16		
		R NOT TO SCALE	

M

S

Continue on the next page..

P

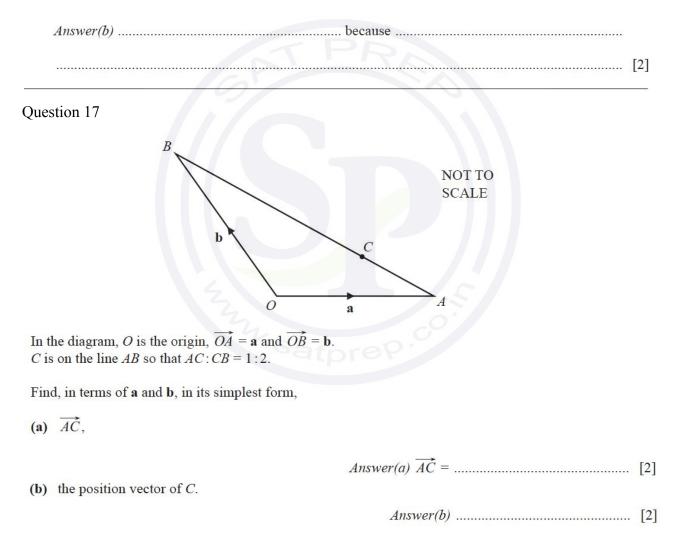
PQRS is a quadrilateral and *M* is the midpoint of *PS*. $\overrightarrow{PQ} = \mathbf{a}, \ \overrightarrow{QR} = \mathbf{b} \text{ and } \overrightarrow{SQ} = \mathbf{a} - 2\mathbf{b}.$

(a) Show that $\overrightarrow{PS} = 2\mathbf{b}$.

Answer(a)

[1]

(b) Write down the mathematical name for the quadrilateral PQRM, giving reasons for your answer.



$$f(x) = x^3$$
 $g(x) = 3x - 5$ $h(x) = 2x + 1$

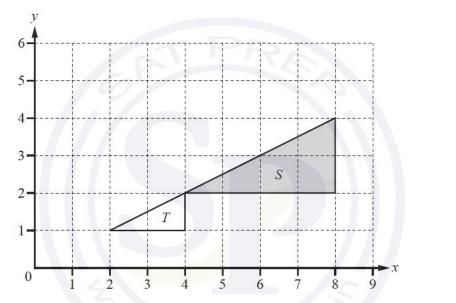
Work out

(a) ff(2),

- **(b)** gh(x) and simplify your answer,
- (c) $h^{-1}(x)$, the inverse of h(x).

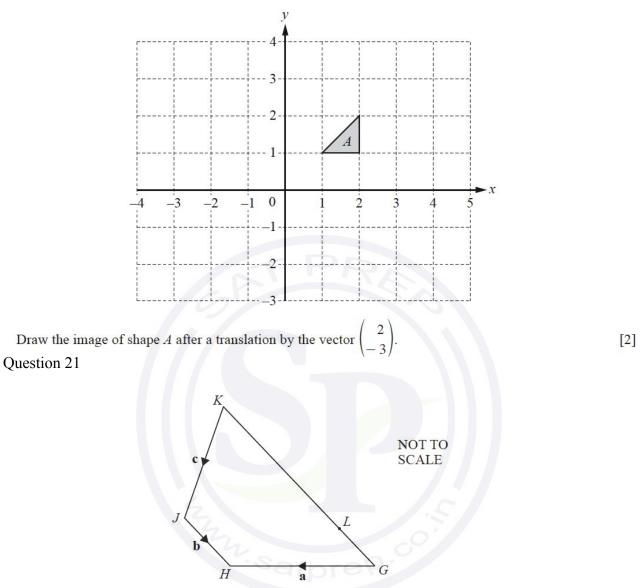


Question 19



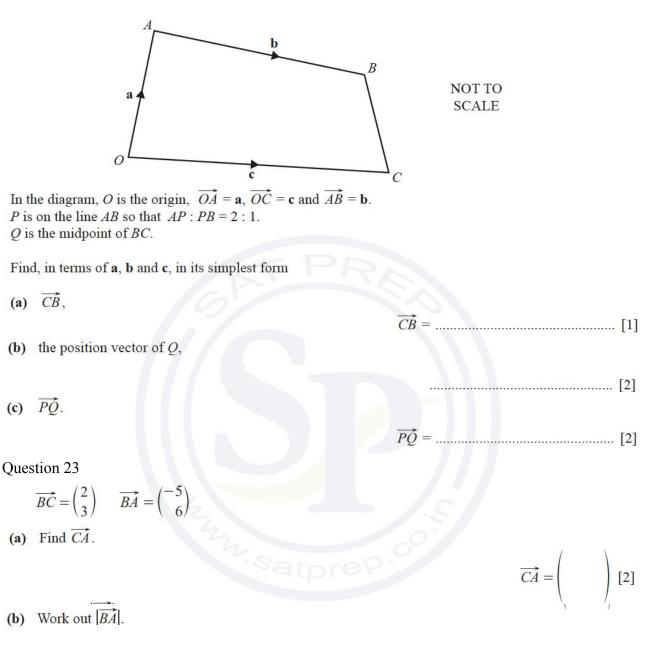
Describe fully the single transformation that maps triangle S onto triangle T.





 $\begin{array}{l} GHJK \text{ is a quadrilateral.} \\ \overrightarrow{GH} = \mathbf{a}, \overrightarrow{JH} = \mathbf{b} \text{ and } \overrightarrow{KJ} = \mathbf{c}. \\ L \text{ lies on } GK \text{ so that } LK = 3GL. \end{array}$

Find an expression, in terms of **a**, **b** and **c**, for \overrightarrow{GL} .



.....[2]

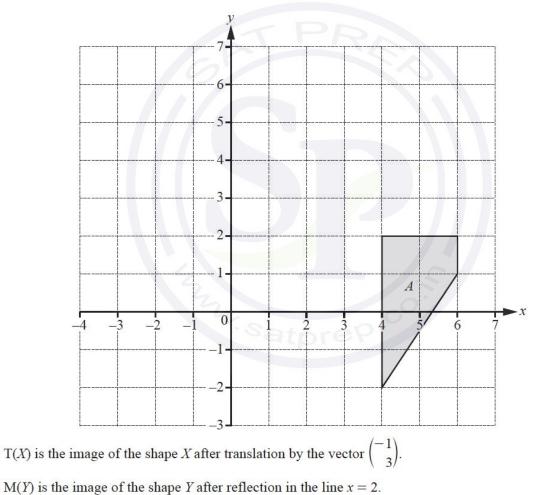
$$f(x) = \frac{x}{4} - 3$$
 $g(x) = 6x - 7$ $h(x) = 2^{3}$

- (a) Work out the value of x when f(x) = -0.5.
- **(b)** Find $g^{-1}(x)$.
- (c) Work out the value of x when h(x) = f(13).

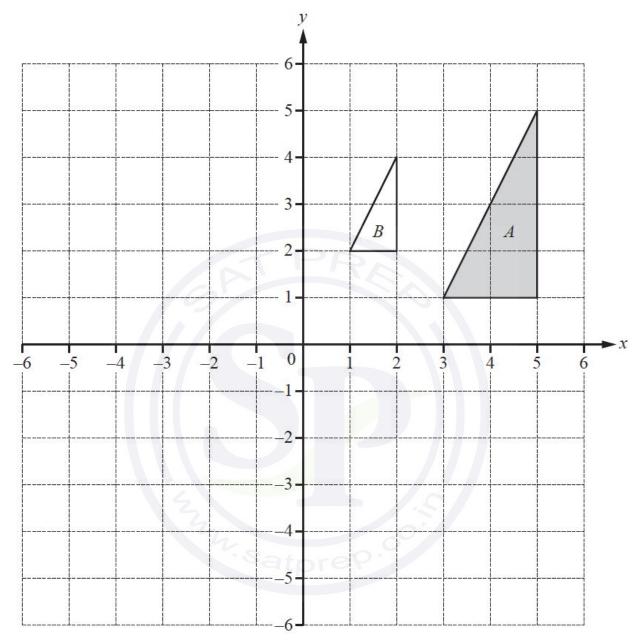
Question 25



$$g^{-1}(x) = \dots [2]$$

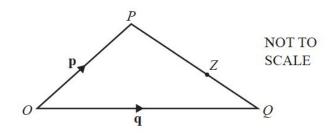


On the grid, draw MT(A), the image of shape A after the transformation MT. [3]



Describe fully the **single** transformation that maps triangle *A* onto triangle *B*.

.....[3]



O is the origin, $\overrightarrow{OP} = \mathbf{p}$ and $\overrightarrow{OQ} = \mathbf{q}$. *Z* is a point on *PQ* such that *PZ* : *ZQ* = 5 : 2.

Work out, in terms of \mathbf{p} and \mathbf{q} , the position vector of Z. Give your answer in its simplest form.

Question 28

f(x) = 3 + 4x g(x) = 6x + 7

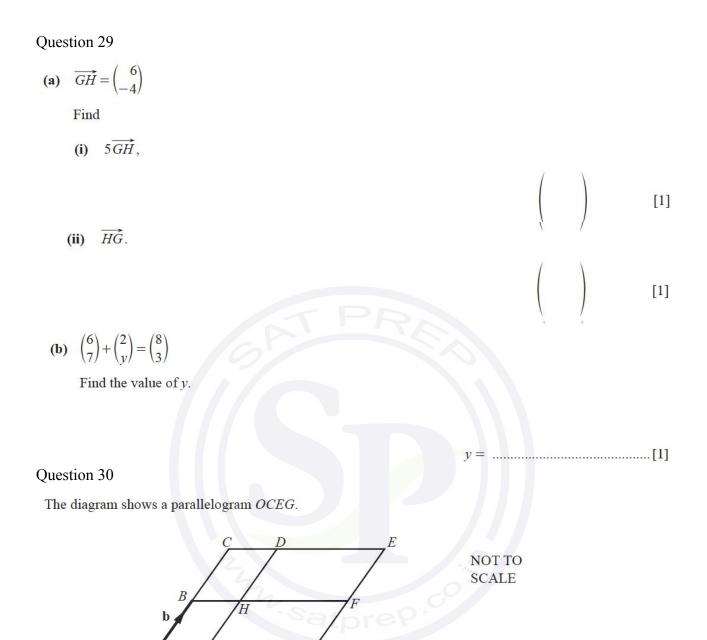
Find, in its simplest form,

(a) f(3x),

(b) fg(x).

.....[1]

.....[3]



O is the origin, $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$. *BHF* and *AHD* are straight lines parallel to the sides of the parallelogram.

G

$$\overrightarrow{OG} = 3\overrightarrow{OA}$$
 and $\overrightarrow{OC} = 2\overrightarrow{OB}$

(a) Write the vector \overrightarrow{HE} in terms of **a** and **b**.

a A

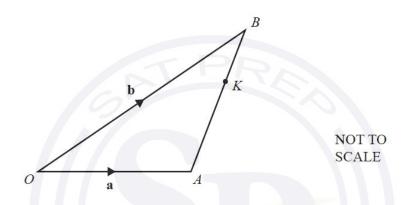
 \overrightarrow{HE} =[1]

(b) Complete this statement.

	$\mathbf{a} + 2\mathbf{b}$ is the position vector of point	[1]
(c)	Write down two vectors that can be written as $3\mathbf{a} - \mathbf{b}$.	

..... and [2]

Question 31

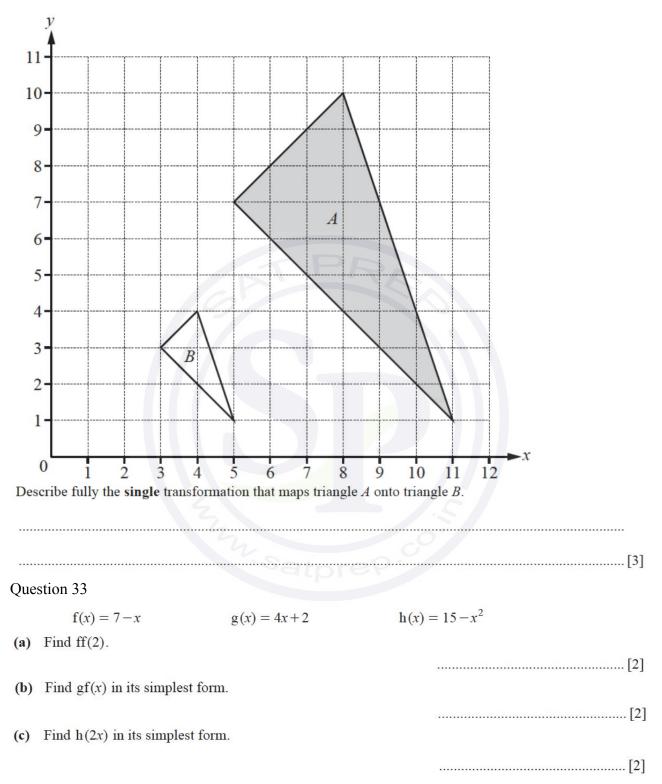


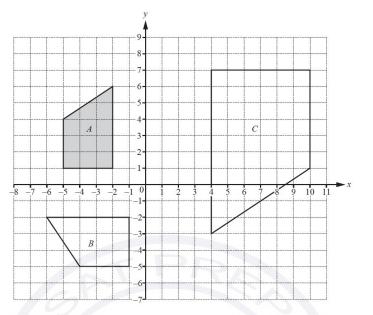
 \overrightarrow{O} is the origin and K is the point on AB so that AK : KB = 2 : 1. $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.

Find the position vector of K. Give your answer in terms of **a** and **b** in its simplest form.

.....[3]

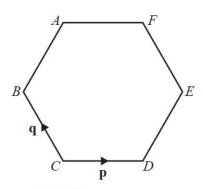






Describe fully the single transformation that maps

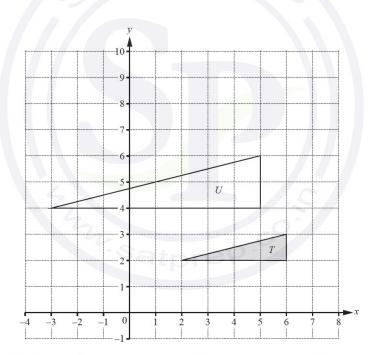
(a) shape A onto shape B,
(b) shape A onto shape C.
[3]



The diagram shows a regular hexagon *ABCDEF*. $\overrightarrow{CD} = \mathbf{p}$ and $\overrightarrow{CB} = \mathbf{q}$.

Find \overrightarrow{CA} , in terms of **p** and **q**, giving your answer in its simplest form.

Question 36

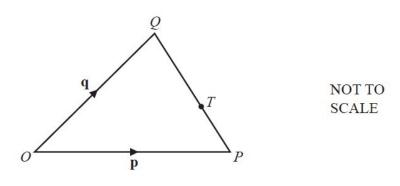


(a) Describe fully the single transformation that maps triangle T onto triangle U.

[3]

(b) On the grid, draw the image of triangle T after a rotation through 90° clockwise about the point (7, 3).

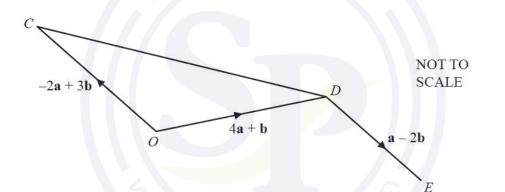
[3]



O is the origin, $\overrightarrow{OP} = \mathbf{p}$ and $\overrightarrow{OQ} = \mathbf{q}$. QT: TP = 2: 1

Find the position vector of T. Give your answer in terms of \mathbf{p} and \mathbf{q} , in its simplest form.





In the diagram, O is the origin, $\overrightarrow{OC} = -2\mathbf{a} + 3\mathbf{b}$ and $\overrightarrow{OD} = 4\mathbf{a} + \mathbf{b}$.

(a) Find \overrightarrow{CD} , in terms of a and b, in its simplest form.

$$\overrightarrow{CD}$$
 =[2]

.....[2]

(b)
$$\overrightarrow{DE} = \mathbf{a} - 2\mathbf{b}$$

Find the position vector of *E*, in terms of **a** and **b**, in its simplest form.

.....[2]

Question 39

f(x) = 5 - 2x $g(x) = x^2 + 8$

(a) Calculate
$$ff(-3)$$
.

Continue on the next page..

.....[2]

(b) Find

(i) g(2x),

(ii)
$$f^{-1}(x)$$
.
Question 40
 $f^{-1}(x) = \dots [2]$
NOT TO
SCALE
In the diagram, *OABC* is a parallelogram.
OP and *CA* intersect at *X* and *CP* : *PB* = 2 : 1.
 $\overrightarrow{OA} = a$ and $\overrightarrow{OC} = c$.
(a) Find \overrightarrow{OP} , in terms of a and c, in its simplest form.
(b) $CX: XA = 2 : 3$
(c) Find \overrightarrow{OX} , in terms of a and c, in its simplest form.
(i) Find \overrightarrow{OX} , in terms of a and c, in its simplest form.
(ii) Find \overrightarrow{OX} , in terms of a and c , in its simplest form.
(iii) Find \overrightarrow{OX} , in terms of a and c , in its simplest form.
(ii) Find \overrightarrow{OX} , xP .
Question 41
(a) $f(x) = x^3$ $g(x) = 5x + 2$
(b) Find $g^{-1}(x)$.
(c) $g^{-1}(x) = \dots [2]$

(b) $h(x) = ax^2 + 1$

Find the value of *a* when h(-2) = 21.

 $k = \dots [2]$

 $x = \dots [2]$

......[2]

Question 42

f(x) = 7 + 3x $g(x) = x^4$ $h(x) = 3^x$

(a) $h(3x) = k^x$

Find the value of k.

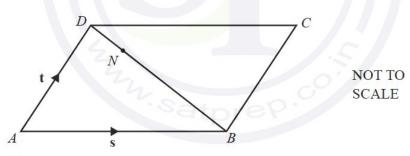
- (b) Find the value of x when f(x) = g(2).
- (c) Find $f^{-1}(x)$.

Question 43

O is the origin, $\overrightarrow{OA} = 2\mathbf{x} + 3\mathbf{y}$ and $\overrightarrow{BA} = \mathbf{x} - 4\mathbf{y}$.

Find the position vector of B, in terms of x and y, in its simplest form.

Question 44



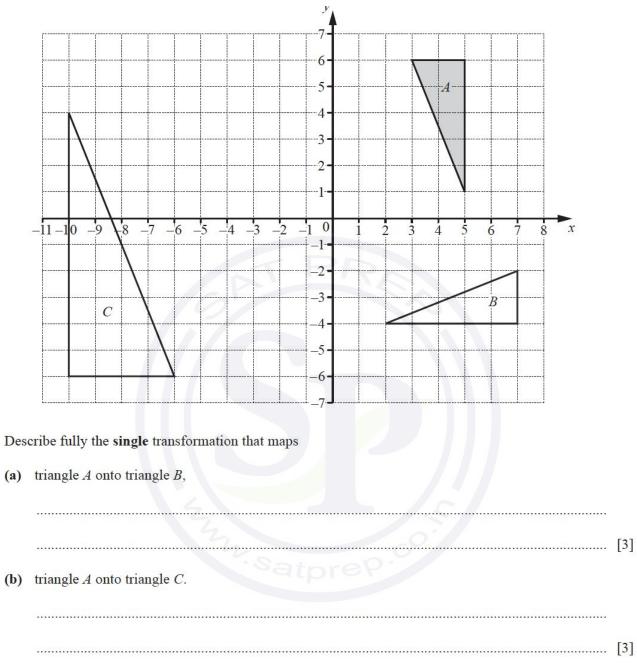
ABCD is a parallelogram. *N* is the point on *BD* such that BN : ND = 4 : 1. $\overrightarrow{AB} = \mathbf{s}$ and $\overrightarrow{AD} = \mathbf{t}$.

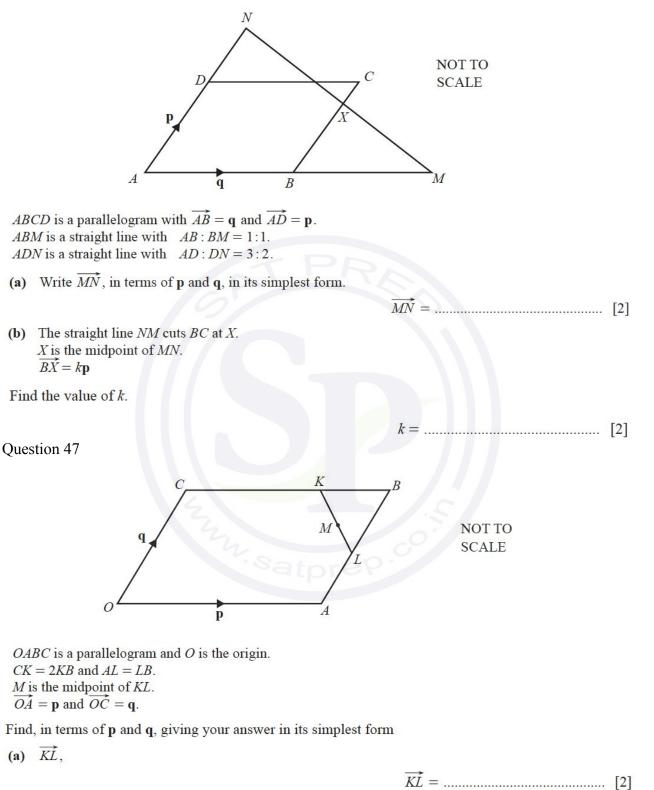
Find, in terms of s and t, an expression in its simplest form for

(a) \overrightarrow{BD} ,

 $\overrightarrow{BD} = \dots$ [1]

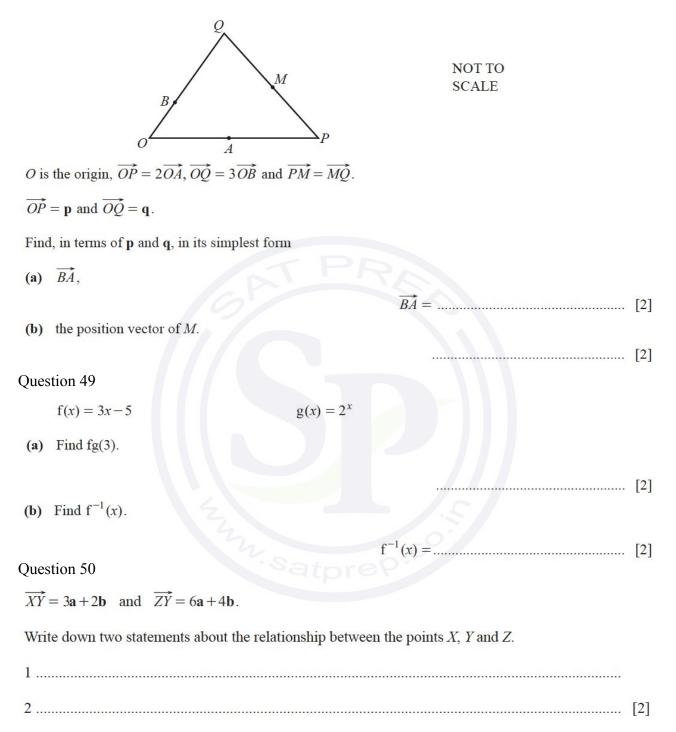
(b) \overrightarrow{CN} .

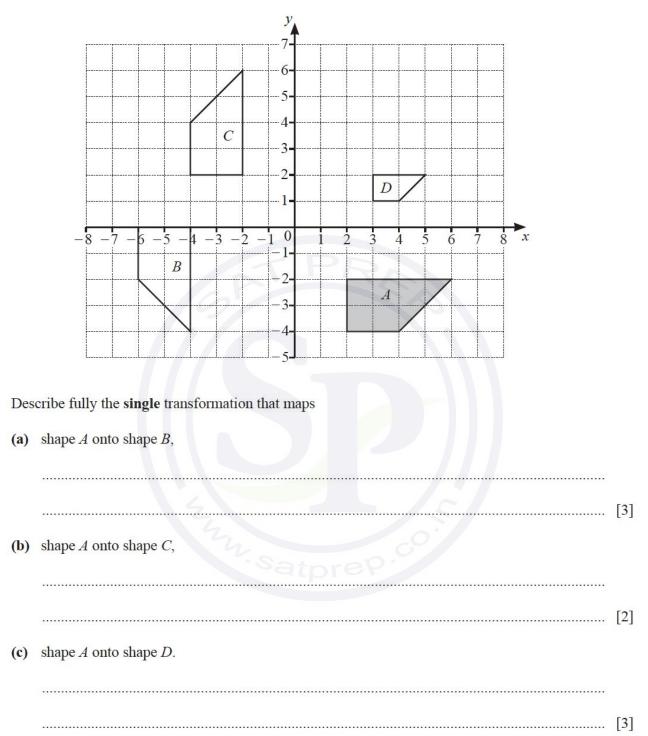


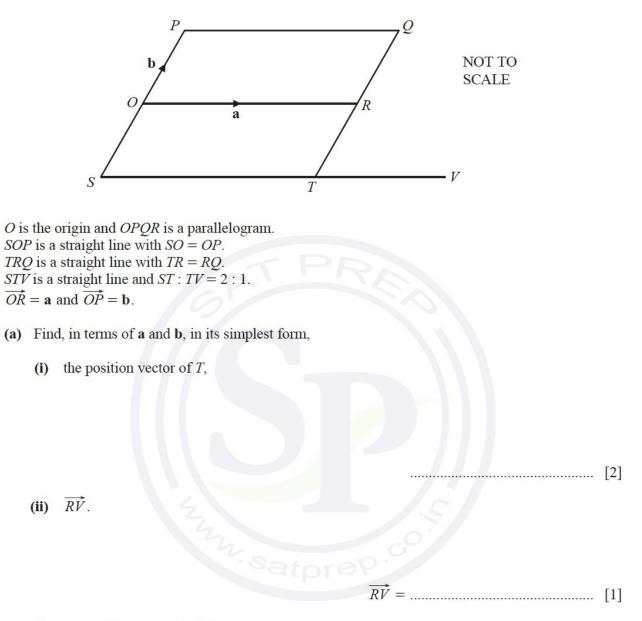


(b) the position vector of M.

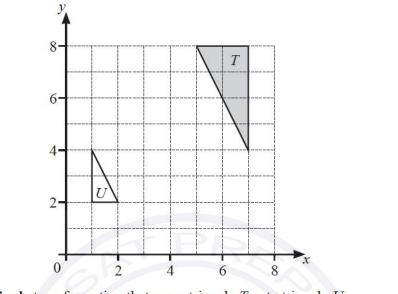
......[2]





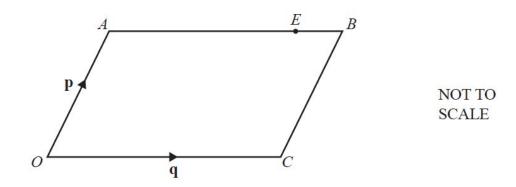


(b) Show that PT is parallel to RV.



Describe fully the single transformation that maps triangle T onto triangle U.

Question 54 (a) (i) $\mathbf{m} = \begin{pmatrix} 5 \\ 7 \end{pmatrix}$ Find 3**m**. (b) $\overrightarrow{VW} = \begin{pmatrix} 10 \\ -24 \end{pmatrix}$ Find $|\overrightarrow{VW}|$. (c) [1]



OABC is a parallelogram. $\overrightarrow{OA} = \mathbf{p}$ and $\overrightarrow{OC} = \mathbf{q}$. *E* is the point on *AB* such that *AE* : *EB* = 3 : 1.

Find \overrightarrow{OE} , in terms of **p** and **q**, in its simplest form.



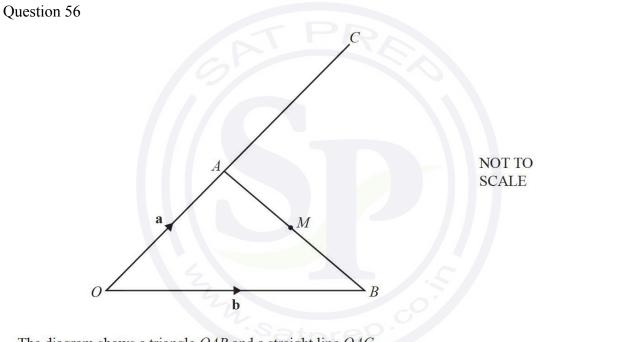
(a) f(x) = 4x + 3 g(x) = 5x - 4fg(x) = 20x + p

Find the value of p.

$$p =$$
 [2]

(b) $h(x) = \frac{5x-1}{3}$

Find $h^{-1}(x)$.



The diagram shows a triangle *OAB* and a straight line *OAC*. OA : OC = 2 : 5 and *M* is the midpoint of *AB*. $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.

Find, in terms of **a** and **b**, in its simplest form

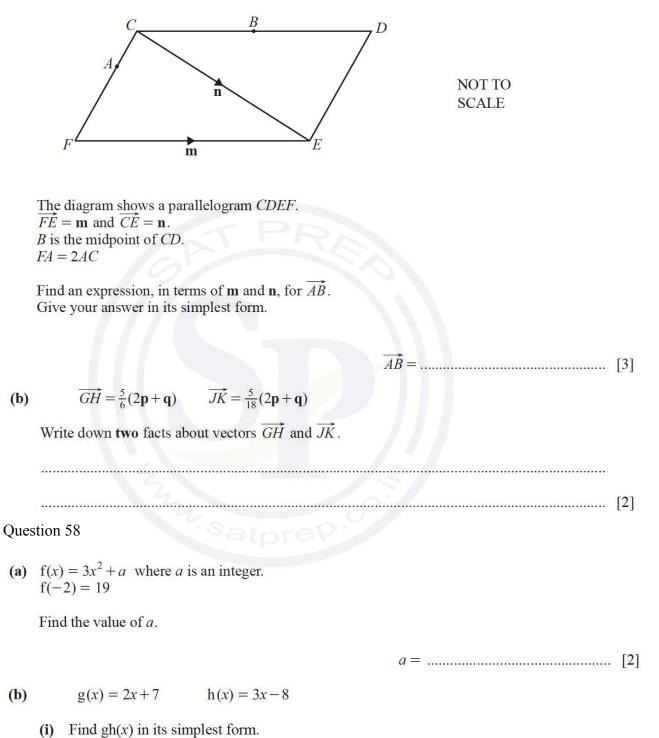
(a) \overrightarrow{AB} ,

$$\overrightarrow{AB} = \dots \qquad [1]$$

(b) \overrightarrow{MC} .

$$\overrightarrow{MC} = \dots \qquad [3]$$

(a)



(ii) Find $g^{-1}(x)$.

 $g^{-1}(x) = \dots [2]$

Question 59

Ahmed finds the magnitude of the vector $\begin{pmatrix} 2 \\ -3 \end{pmatrix}$.

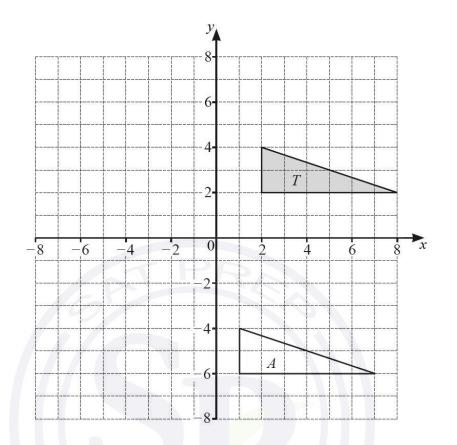
From this list, put a ring around the correct calculation.

$$\sqrt{2^2 + -3^2}$$
 $2^2 - 3^2$ $\sqrt{2^2 - 3^2}$ $2^2 + (-3)^2$ $\sqrt{2^2 + (-3)^2}$

Question 60

The magnitude of the vector $\begin{pmatrix} 20\\k \end{pmatrix}$ is 29. Find the value of k.

[1]



(a) Describe fully the single transformation that maps triangle T onto triangle A.

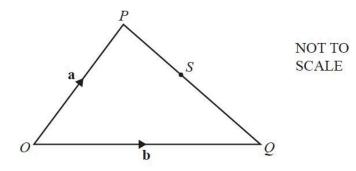
[2]

(b) Draw the image of triangle *T* after an enlargement, scale factor $-\frac{1}{2}$, centre (0, 0). [2]

Question 62

 $f(x) = x^2 - 25$ g(x) = x + 4

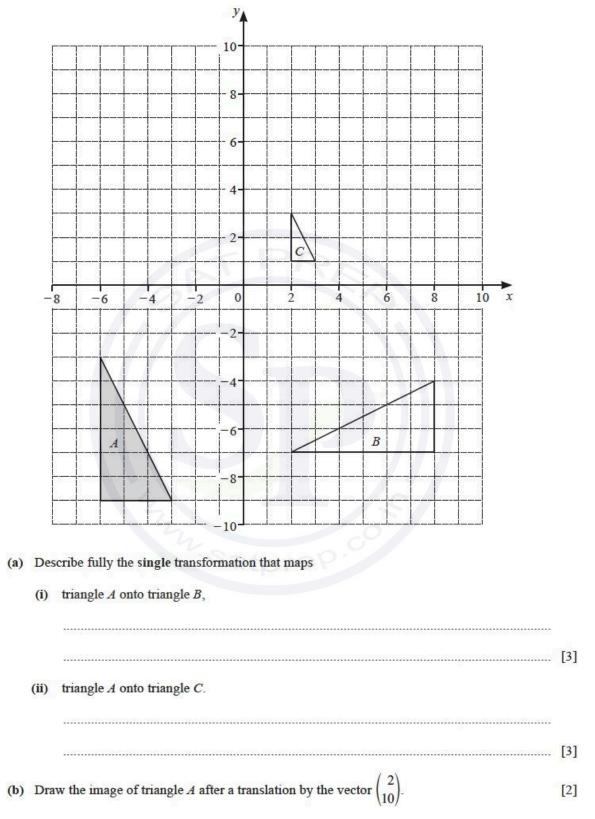
Solve fg(x+1) = gf(x).

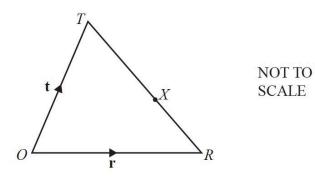


S is a point on PQ such that PS : SQ = 4 : 5.

Find \overrightarrow{OS} , in terms of **a** and **b**, in its simplest form.







ORT is a triangle. X is a point on *TR* so that TX: XR = 3:2. *O* is the origin, $\overrightarrow{OR} = \mathbf{r}$ and $\overrightarrow{OT} = \mathbf{t}$.

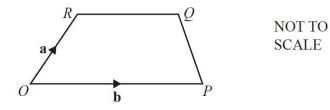
Find the position vector of X. Give your answer in terms of \mathbf{r} and \mathbf{t} in its simplest form.

Question 66

$$f(x) = 2^{x-3}$$
 $g(x) = 2x-1$ $h(x) = \frac{5}{x-3}$

- (a) Find ff(6).
- **(b)** Find $g^{-1}g(x+21)$.

(c) Find x when f(x) = h(84).



The diagram shows a trapezium *OPQR*. *O* is the origin, $\overrightarrow{OR} = \mathbf{a}$ and $\overrightarrow{OP} = \mathbf{b}$.

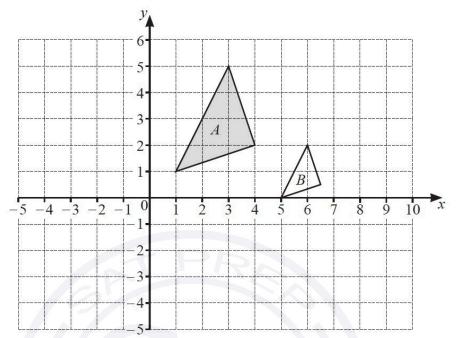
- $\left|\overrightarrow{RQ}\right| = \frac{3}{5}\left|\overrightarrow{OP}\right|$
- (a) Find \overrightarrow{PQ} in terms of **a** and **b** in its simplest form.

$$P\dot{Q} =$$
 [2]

.....[2]

(b) When PQ and OR are extended, they intersect at W.

Find the position vector of *W*.



(a) On the grid, draw the image of

[1]
[2]

$$f(x) = kx^2$$
 $g(x) = \frac{1}{x}$ $h(x) = \frac{7x-2}{5}$ $j(x) = \frac{3-10x}{14}$

(a) f(-5k) = 675

Find the value of k.

$$k = \dots [2]$$

(b) Find gh(x).

- (c) Find $h^{-1}(x) + j(x)$. Give your answer in its simplest form.

.....[4]

Question 70

$$f(x) = 7x - 8$$
 $g(x) = \frac{4}{x} + 5$ $h(x) = 2^{x} + 3$

(a) Find $f^{-1}(x)$.

- $f^{-1}(x) = \dots$ [2]
- (b) Find the value of x when $h(x) = g\left(\frac{1}{3}\right)$.

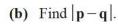


$$\mathbf{p} = \begin{pmatrix} 2\\ 8 \end{pmatrix} \qquad \qquad \mathbf{q} = \begin{pmatrix} -1\\ 4 \end{pmatrix}$$

- (a) Find
 - (i) p-q,

(ii) 6p.





B

b

0









-L

[1]

[1]

The diagram shows a triangle OAB and a parallelogram OALK. The position vector of A is **a** and the position vector of B is **b**. K is a point on AB so that AK : KB = 1 : 2.

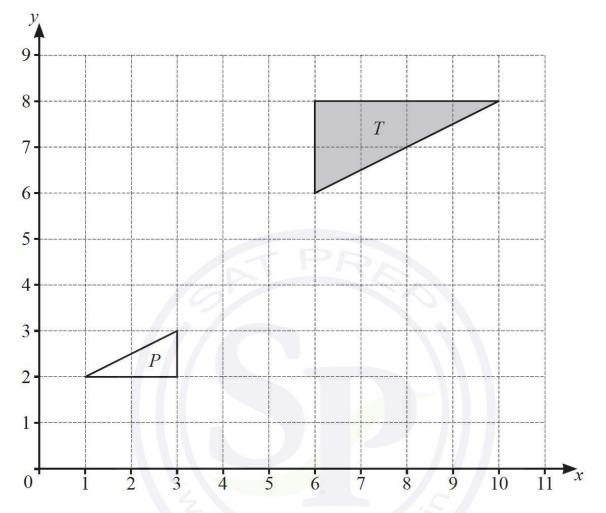
a

A

Find the position vector of L, in terms of **a** and **b**. Give your answer in its simplest form.

......[4]





Describe fully the single transformation that maps triangle T onto triangle P.

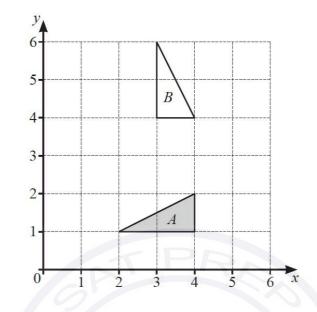
[3]

$$f(x) = 5x - 3, x > 1$$
$$g(x) = \frac{10}{x - 2}, x \neq 2$$

(a) Find gf(x).

Give your answer in its simplest form.

$$x = \dots \qquad [3]$$



Describe fully the single transformation that maps triangle A onto triangle B.

Question 77

$$f(x) = x^3 + 1$$

Find $f^{-1}(x)$.

Question 78

$$\mathbf{v} = \begin{pmatrix} -1\\ 3 \end{pmatrix} \qquad \mathbf{y} = \begin{pmatrix} 2\\ 5 \end{pmatrix}$$

Find

(a) v-y

) [1]

[1]

 $f^{-1}(x) = \dots$ [2]

(b) 2**v**.

$$f(x) = 6x - 7$$
 $g(x) = x^{-3}$

(a) Find f(x+2). Give your answer in its simplest form.

(b) Find $f^{-1}(x)$.

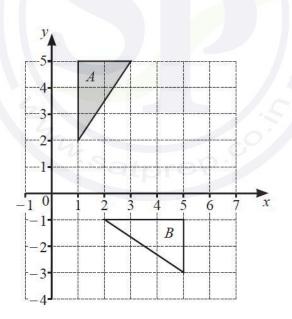
$$f^{-1}(x) = \dots$$
[2]

x =

- (c) Find x when g(x) = f(22).
- Question 80

The position vector of A is $\begin{pmatrix} 5\\ 3 \end{pmatrix}$ and $\overrightarrow{BA} = \begin{pmatrix} 4\\ 8 \end{pmatrix}$. Show that $|\overrightarrow{OB}| = 5.1$, correct to 1 decimal place.

Question 81

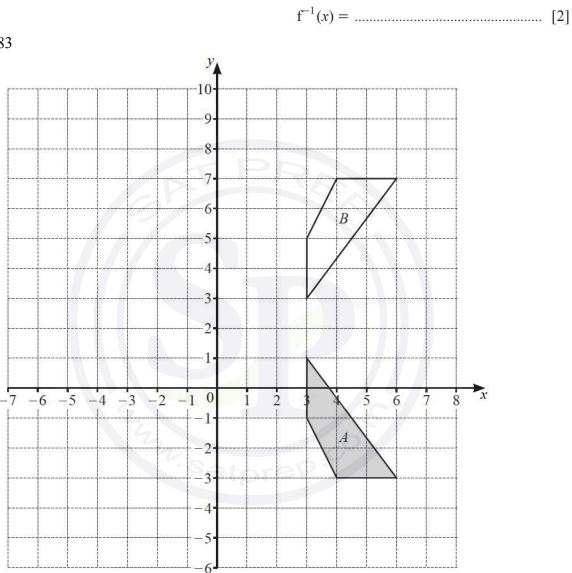


Describe fully the single transformation that maps triangle A onto triangle B.
[3]

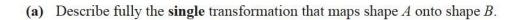
[3]

f(x) = 5x + 2

Find $f^{-1}(x)$.



Question 83

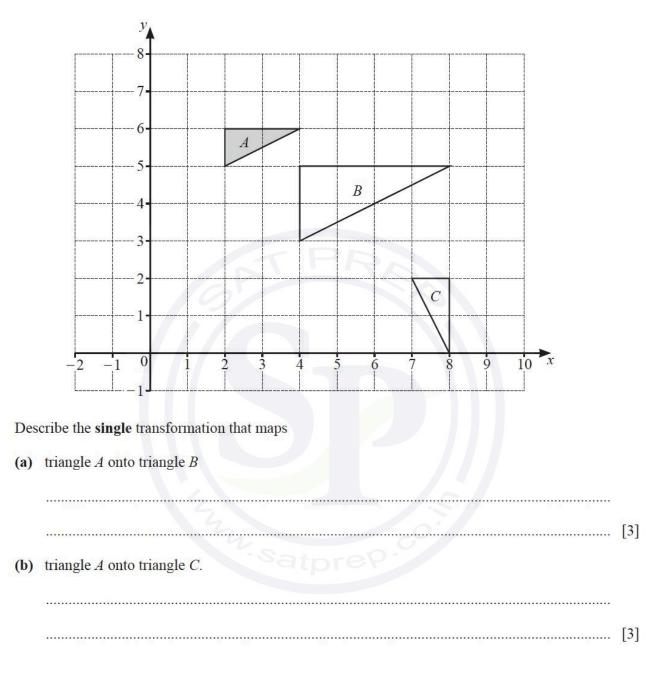


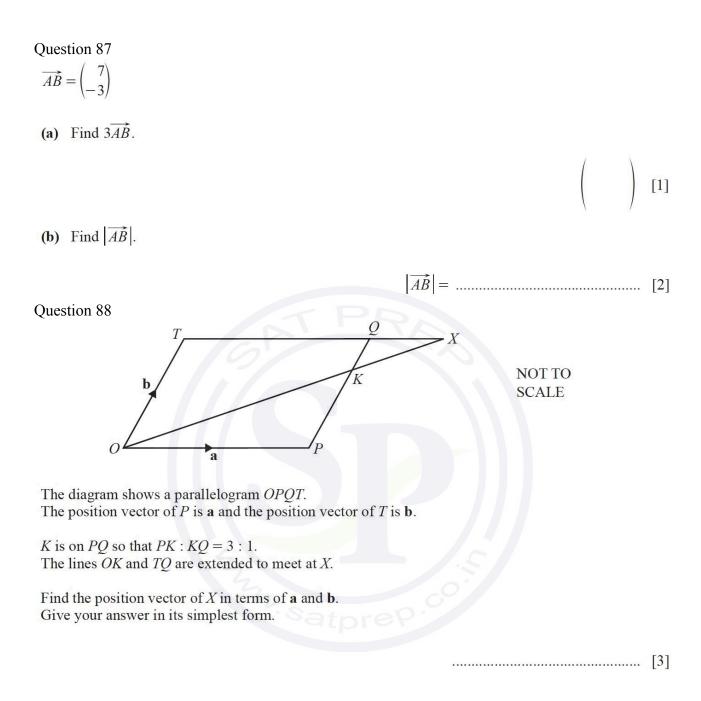
[2]

(b) Rotate shape $A 90^{\circ}$ clockwise about the point (-1, 2). [2]

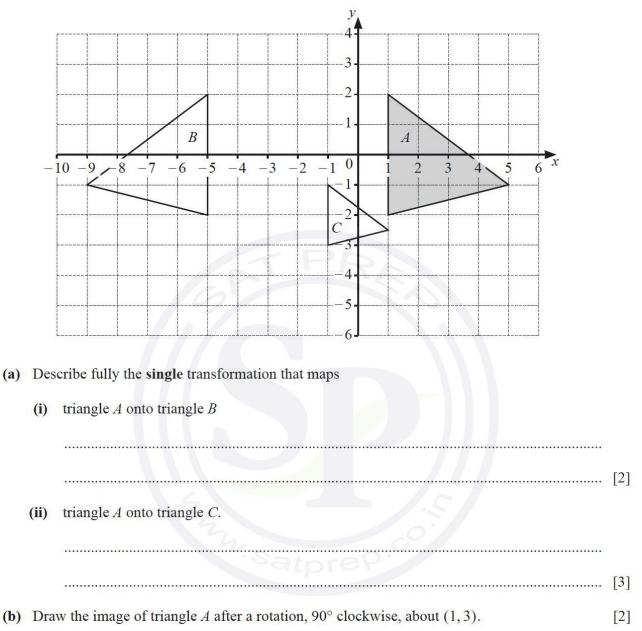
(c) Enlarge shape A by scale factor -2, centre (2, 0). [2]

Question 84 \rightarrow (8) \rightarrow (-12)			
<i>F</i> is the point (1, -4), $\overrightarrow{FG} = \begin{pmatrix} 8 \\ -3 \end{pmatrix}$ and $\overrightarrow{GH} = \begin{pmatrix} -12 \\ 35 \end{pmatrix}$.			
Find			
(a) $3\overrightarrow{FG}$			
		1	
) [1]
(b) $\overrightarrow{FG} + \overrightarrow{GH}$			
		1	
) [1]
(c) the coordinates of the point G			
	(,) [1]
(d) the magnitude of vector \overrightarrow{GH} .			
			[2]
Question 85			[2]
(a) Find the magnitude of the vector $\begin{pmatrix} -4\\ 5 \end{pmatrix}$.			
	· · · · · · · · · · · · · · · · · · ·		[2]
(b) Satore?			
\sim			
y X	NOT TO		
	SCALE		
$O \qquad \mathbf{x} \qquad A \qquad B$ The diagram change a triangle OAC			
The diagram shows a triangle <i>OAC</i> . <u>A</u> is the midpoint of the straight line <i>OB</i> . $\overrightarrow{OA} = \mathbf{x}$ and $\overrightarrow{OC} = \mathbf{y}$.			
Find \overrightarrow{CB} in terms of x and y .			
\rightarrow			

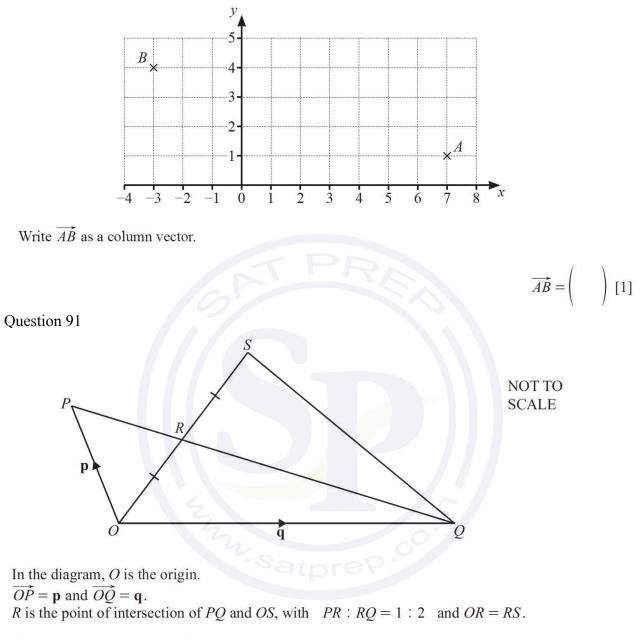




Ouestion 89







Find the position vector of S in terms of \mathbf{p} and \mathbf{q} . Give your answer in its simplest form.

.....[4]

 $f(x) = 3^x + 2$

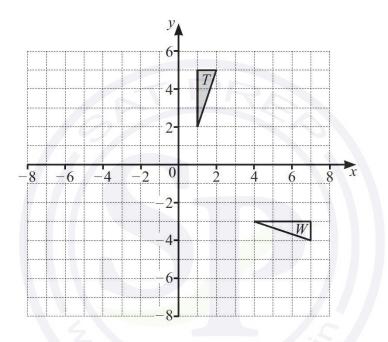
(a) Find x when f(x) = 245.

 $x = \dots [2]$

(b) Find x when $f^{-1}(x) = 7$.

 $x = \dots [2]$

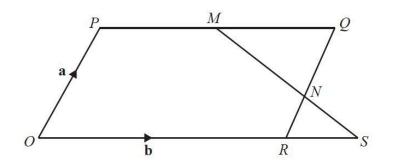
Question 93



(a) Describe fully the single transformation that maps triangle T onto triangle W.

[3]

(b) Draw the enlargement of triangle T with scale factor -2 and centre of enlargement (-1, 1). [2]



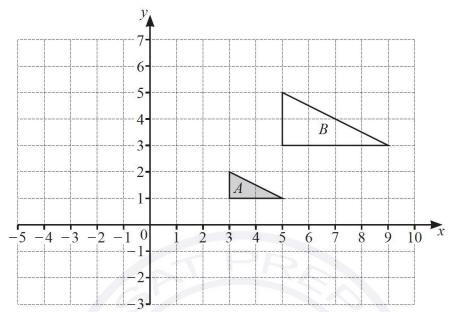
NOT TO SCALE

- *O* is the origin and *OPQR* is a parallelogram. *M* is the midpoint of *PQ* and *N* divides *QR* in the ratio 2 : 1. $\overrightarrow{OP} = \mathbf{a}$ and $\overrightarrow{OR} = \mathbf{b}$.
- (a) Find \overrightarrow{MN} . Give your answer in terms of **a** and/or **b** and in its simplest form.

(b) The lines MN and OR are extended to meet at S.

Find the position vector of *S*.

Give your answer in terms of **a** and/or **b** and in its simplest form.



- (a) Describe fully the single transformation that maps triangle A onto triangle B.[3] (b) On the grid, draw the image of triangle A after a translation by the vector $\begin{pmatrix} -4\\ 3 \end{pmatrix}$.

[2]