# Extended Mathematics <br> Topic :Graph <br> Year :May 2013 -May 2023 <br> Paper-4 <br> Answers 

Question 1

| (a) | $-5.04,1.75,0$ |
| :--- | :--- |
| (b) | Fully correct curve |
|  |  |
| (c) | -1.6 to -1.5 |
|  | -0.4 to -0.3 |
|  | 1.8 to 1.9 |
| (d) | -2.6 to -2.5 www |
|  |  |
| 1.0 .4 to -0.3 |  |
| (e) | 3.25 to 4.25 with correct tangent |


| 3 | B1 for each correct value |
| :--- | :--- |
| 5 | B3FT for 10 correct plots from their (a) <br> B2FT for 8 or 9 correct plots <br> or B1FT for 6 or 7 correct plots <br> and SC1 for two branches not joined |
| 1 |  |
| 1 |  |
| 1 | After $\mathbf{0}$ scored, M1 for $y=2 x-2$ drawn |
| 1 | B1 for correct tangent |
| 1 |  |

B2 for answer in range dep on close attempt at tangent

M1dep for $[-] \frac{\text { rise }}{\text { run }}$ used with values soi from tangent, dep on correct or close attempt at tangent

## Question 2

(a)
$0,2,0,-3$
(b)
(c)
(d) (i) line drawn from $(0,2)$ to touch curve
(ii)
ii) rise/run e.g. (their $y$ 2)/their $x$

A1

B3FT for 8 points
B2FT for 7 or 6 points
B1FT for 5 or 4 points
B1 e.g. Could be mark[s] on curve
isw other lines if not clearly used
dep on attempt at a tangent from $(0,2)$ in (d)(i) and uses scales correctly Can be implied from answer- check on tangent for their rise for a run of 1 ( $1 / 2$ small square)
B2 for $\mathbf{3}$ correct or B1 for 2 correct

No daylight at point of contact
If short, must cross at $(0,2)$ within $1 / 2$ small square when extended
ww2 dep on attempt at a tangent from $(0,2)$ in (d)(i)

## Question 3



Question 4

| (i) | $\frac{3}{2}$ or 1.5 |
| :--- | :--- |
| (ii) | $y=\frac{3}{2} x+2$ oe |

## Question 5

(a) $7,11.5,4.5$
(b) Correct curve cao
(c) (i) $0.69<x<0.81$
(ii) $-2.3<x<-2.2$
$-0.8<x<-0.6$ $0.35<x<0.5$
(d) (i) $y=10-3 x$ ruled correctly

$$
\begin{aligned}
-0.55 & <x<-0.45 \\
0.35 & <x<0.45
\end{aligned}
$$

(ii) $\begin{array}{llll}10 & 1 & -2\end{array}$

2 M1 for $\frac{14-(-4)}{8-(-4)}$ oe
B1 for $y=$ their $\frac{3}{2} x+c \quad$ o.e.
or $y=m x+2, m \neq 0$
SC1 for $\frac{3}{2} x+2$

## 1,1,1

B3FT for 10 correct plots, on correct vertical grid line and within correct 2 mm square vertically
Or B2FT for 8 or 9 correct plots Or B1FT for 6 or 7 correct plots and B1 indep for two separate branches on either side of $y$-axis

B1 for each correct
After 0 scored, allow SC1 for drawing line $y=7.5$ long enough to cross curve at least once

B2 long enough to cross curve twice.
B1 for ruled line gradient -3 or $y$ intercept at 10 but not $y=10$
Or B1 for 'correct' but freehand

B1dep Dependent on at least B1 scored for line

After 0 scored, SC2 for -0.5 and 0.4 [from solving equation]

3

B2 for $2-x-10 x^{2}[=0]$ oe
Or B1 for $\frac{2}{x^{2}}-\frac{1}{x}-10=0$ oe Correctly eliminating $-3 x$ Or B1 for $2-x-3 x^{3}=10 x^{2}-3 x^{3}$ oe Correctly clearing fractions

## Question 6

(i) 1.4 to 1.6
(ii) 1.15 to 1.25
(iii) - 1
(iv) -2.25 to -2.1
-0.9 to -0.75
2.2 to 2.35

B2 for 2 correct or B1 for one correct or B1 for $y=x$ drawn ruled to cut curve 3 times

Question 7
(a)
$0,4.5,3.11[1 \ldots]$
Complete correct curve with
minimum below $y=2$
Correct line or no line and
-0.7 to -0.6 nfww

3
B3 FT for 9 points correctly plotted B2 FT for 7 or 8 points correctly plotted or B1 FT 5 or 6 points correctly plotted
and B1 indep two separate branches not touching or cutting $y$-axis
if $0 \mathbf{S C} 1$ for $y=3$ indicated
Must check line - not if wrong line B2 for $y=1-x$ ruled correctly
or SC1 for ruled line with either gradient -1 or $y$-intercept 1 but not line $\mathrm{y}=1$ or correct freehand line

## Question 8



## Question 8

| (a) | 2.125 and 2.375 | 2 | B1 for one correct value |
| :---: | :---: | :---: | :---: |
| (b) | Correct curve | B4 | B3FT for 11 correct plots or B2FT for 9 or 10 correct plots or B1FT for 7 or 8 correct plots |
| (c) | Ruled tangent at $x=2$ | B1 | No daylight at $x=2$. Consider point of contact as midpoint between two vertices of daylight, this must be between $x=1.8$ and 2.2 |
|  | Gradient from 7.8 to 10.2 | 2 | Dep on B1 awarded <br> Allow integer/integer or a mixed number if within range or <br> M1 dep for (change in $y$ ) $\div($ change in $x$ ) Dependent on any tangent drawn or close attempt at a tangent at any point <br> Must see correct or implied calculation from a drawn tangent |
| (d) | 0 and -1.75 to -1.65 and 1.65 to 1.75 | 2 | B1 for two correct values |
| (e) | -1.2 to $-0.8<k<2.8$ to 3.2 | 2 | B1 for each correct or SC1 for reversed answers |

## Question 9

(i) $\left|\begin{array}{l|c|l}(1,2) & \mathbf{1 + 1} & \\ \text { (ii) } & y=3 x-1 \text { cao final answer } & \mathbf{3}\end{array}\right| \begin{aligned} & \text { M1 for gradient }=\frac{8--4}{3--1} \text { oe } \\ & \text { and } \mathbf{M 1} \text { for substituting }(3,8) \text { or }(-1,-4) \text { into their } \\ & y=3 x+\mathrm{c} \text { or for finding } y \text {-intercept is }-1\end{aligned}$

## Question 10

| (a) | $-3,7.375,8.875$ |
| :---: | :---: |
| (b) | Correct curve |
| (c) (i) | Any integer less than 7 or greater than 10 |
| (ii) | 7,8 or 9 |
| (d) | $y=15 x+2$ ruled and fit for purpose |
|  | -1.45 to -1.35 and 0.4 to 0.5 |
| (e) | Tangent ruled at $x=1.5$ |
|  | 7 to 12 |

$\mathbf{1 , 1 , 1}$ Accept 7.4 or 7.37 or 7.38 for 7.375 and 8.9 or 8.87 or 8.88 for 8.875

B3FT for 8 or 9 correct plots B2FT for 6 or 7 correct plots B1FT for 4 or 5 correct plots Point must touch line if exact or be in correct square if not exact (including boundaries)

B2
B1 for short line but correct or freehand full length correct line or for ruled line through $(0,2)$ (but not $y=2$ ) or for ruled line with gradient 15 (acc $\pm 1 \mathrm{~mm}$ vertically for 1 horizontal unit)

B1 for each
No daylight at point of contact. Consider point of contact as midpoint between two vertices of daylight, the midpoint must be between $x=1.4$ and 1.6

Dep on B1 or close attempt at tangent at $x=1.5$ M1 for $y-$ step $/ x-$ step for their tangent

## Question 11



Question 12


## Question 13


or for $x^{3}-5 x^{2}-x+12[=0]$ oe
or
M1 for $x^{2}-2 x+\frac{12}{x}=3 x+1$

## Question 14

$\left.\begin{array}{l|l|l|ll}\text { (a) } & \begin{array}{ll}1.5 \quad 1.25-0.75 \quad 0.5 & \mathbf{4} \\ \text { (b) } & \text { B1 for each } \\ \text { Fully correct curve } & \mathbf{5}\end{array} \begin{array}{l}\text { B5 for correct curve over full domain } \\ \text { or } \\ \text { B3 FT for } 11 \text { or } 12 \text { points } \\ \text { or B2 FT for } 9 \text { or } 10 \text { points } \\ \text { or B1 FT for } 7 \text { or } 8 \text { points }\end{array} \\ \text { and } \\ \text { B1 independent for one complete branch on } \\ \text { each side of the } y \text {-axis and not touching or } \\ \text { crossing the } y \text {-axis } \\ \text { SC4 for correct curve with branches joined }\end{array}\right]$

## Question 15

$\left.\left.\begin{array}{l|l|l|l|l}\text { (a) } & -1.5,0.5 & \mathbf{2} & \text { B1, B1 } \\ \text { (b) } & & \text { Correct curve } & & \mathbf{5}\end{array} \begin{array}{l}\text { B3 FT for } 10 \text { or } 11 \text { points } \\ \text { or B2FT for } 8 \text { or } 9 \text { points } \\ \text { or B1FT for } 6 \text { or } 7 \text { points } \\ \text { and } \\ \text { B1 independent for two branches }\end{array}\right] \begin{array}{lll}\text { SC4 for correct curve but branches joined }\end{array}\right\}$

Question 16

| (a) | 0 4 0.625 <br> (b) 0.875  <br> Fully correct smooth curve   <br> (c) line $y=x+1$ <br> and <br> 0.2 to 0.3 <br> and <br> 1.8 to 1.95 $\|$ |
| :--- | :--- | ---: | :--- |

$\mathbf{1 , 1 , 1 , 1}$

4

3

B3 FT for 8 or 9 points
or B2 FT for 6 or 7 points
or B1 FT for 4 or 5 points

Line must be fit for purpose ie at least from $x=0$
to $x=2$
B2 for correct line and 1 correct value or $\mathbf{B 1}$ for correct line or SC1 for no/wrong line and 2 correct values
(d) $\quad$ Tangent ruled at $x=-1.5$
2.2 to 5

B1 No daylight between tangent and curve at point of contact. Consider point of contact as midpoint between two vertices of daylight, the midpoint must be between $x=-1.6$ and $x=-1.4$
dep on B1
M1 for $\frac{\text { rise }}{\text { run }}$ also dep on any tangent drawn or close attempt at tangent at any point
Must see correct or implied calculation from a drawn tangent

## Question 17

| (i) | 0.25 oe and 1 | $\mathbf{2}$ | $\mathbf{B}$ |
| :--- | :--- | :---: | :---: |
| (ii) | Correct curve | $\mathbf{4}$ | B <br> or <br> o |
| (iii) | 2.3 | $\mathbf{1 F T}$ | C |
| (iv) | $y=3 x-1$ oe 3 term equation | $\mathbf{3}$ | B <br> or <br> (v) |
| (v) | -1.7 to -1.5 and 2 | $\mathbf{2}$ | B |

B1 for each

B3FT for 6 or 7 correct plots or B2FT for 4 or 5 correct plots or B1FT for 2 or 3 correct plots

Correct or FT where $y=5$ on their graph
B2 for $3 x-1$ or $y=3 x[+c]$ oe or for $m=3$ and $c=-1$
or M1 for [gradient $=$ ] $\frac{8-2}{3-1}$ oe soi by $3 x$
and M1 for substitution of $(1,2)$ or $(3,8)$ into their $y=m x+c$

B1 for either or M1 for $y=x+2$ seen or drawn

Question 18

| (a) | $3.5[0] \quad 1.94 \quad 3.11$ |
| :--- | :--- | :--- | :--- |
| (b) | Fully correct curve |
| (c) | -0.7 to -0.6 |

B1 for each
B3 FT for 10 or 11 points or B2 FT for 8 or 9 points or B1 FT for 6 or 7 points

B1 indep two separate branches not touching or cutting $y$-axis

SC4 for correct curve, but branches joined

| (d) (i) | $\begin{gathered} -1 \\ 2.5 \end{gathered}$ | 1 |
| :---: | :---: | :---: |
| (ii) | -0.6 to -0.5 with correct ruled line | 3 |
| (e) | Correct tangent and $0.5 \leqslant \operatorname{grad} \leqslant 0.85$ | 3 |

If 0,0, M1 for $y=2.5-x$ oe seen in working B2FT for drawing their ruled line from (d)(i) or M1 for ruled line through $(0,2.5) \mathrm{FT}$ or gradient -1 FT
B2 for close attempt at tangent at $x=2$ and answer in range
OR
B1 for ruled tangent at $x=2$, no daylight at $x=2$
Consider point of contact as midpoint between two vertices of daylight, the midpoint must be between $x=1.8$ and 2.2
and M1 (dep on B1 or close attempt at tangent [at any point] for $\frac{\text { rise }}{\text { run }}$

Question 19

| (a) | (i) | 10 |
| ---: | ---: | :--- |
|  | (ii) | -3.4 to -3.3 and -0.4 to -0.3 <br> and 1.6 to 1.7 |
|  | (iii) | $y=-2.3$ to -2.1 oe <br> $y=10$ to 10.1 oe |
| (b) | (i) | $2,-1,4$ |
|  | (ii) | Fully correct curve drawn |
| (c) |  | 3.2 oe |
| (ciii) | -3.4 to -3.2 and 1.8 to 1.9 |  |
| (d) | 1 |  |

SC3 for correct curves but branches joined or touching $y$-axis
or B2FT for 8 or 9 correct plots or B1FT for 6 or 7 correct plots
and B1 indep for two separate branches not touching or crossing $y$ - axis

B1 for each
FT $2 \div$ their (a)(i) +3
M1 for $\mathrm{f}(-2)=10$ or their $(\mathrm{a})(\mathrm{i})$ used

## Question 20

(a) $\quad 19[.0]$ or $18.97 . . \mathrm{nfww}$
(b)
$[y=] 3 x+1$
(c)
$y=3 x-5$ oe
(d)
$y=-\frac{1}{3} x+\frac{13}{3}$ oe isw

3
M2 for $\sqrt{(4--2)^{2}+(13--5)^{2}}$ oe or M1 for $(4--2)^{2}+(13--5)^{2}$ oe

3 B2 for answer $[y=] 3 x+c$ oe or answer $k x+1(k \neq 0)$
or M1 for $\frac{13--5}{4--2}$ oe or 3
and M1 for correct substitution of $(-2,-5)$
or $(4,13)$ into $y=($ their $m) x+c$ oe
FT their gradient from (b)
M1 for $y=m x-5$ with other $m, m \neq 0$
or $y=\{$ their gradient from (b) $\} x+c$
If 0 scored, SC1 for answer $3 x-5$
5 B2FT for $-\frac{1}{3} x+c$ ( $c$ can be numeric or algebraic)
FT - $1 /$ their gradient from (b) or M1 for $-1 /$ their gradient from (b) soi
and
B1 for [midpoint of $A B=$ ] $(1,4)$
and M1 for substitution of $(1, k)$ or $(k, 4)$ into a linear equation

## Question 21

(a) (i) $\mid-2,-0.5$ or $-\frac{1}{2}$
(ii)
Complete correct curve


SC4 for correct curves but branches joined or touching $y$-axis or B3FT 9 or 10 points or B2FT for 7 or 8 points or B1FT for 5 or 6 points
B1 for each
and B1indep two separate branches not touching or crossing $y$-axis

B1 for each

M2 for correct ruled line or M1 for correct line but freehand or for ruled line gradient - 5 or ruled line $y$-intercept -2 , but not $y=-2$ and A1 for each correct solution dependent on at least M1

If 0 scored, SC1 for both correct with no line drawn

B1 for one correct value or M1 for $x^{3}+5 x^{2}-2 x-1=0$ seen

Question 22


## Question 23

| (a) | $\begin{aligned} & 9 \\ & 10.5 \end{aligned}$ | $1$ |  |
| :---: | :---: | :---: | :---: |
| (b) | Fully correct curve | 5 | SC4 for correct curve, but branches joined <br> B3 FT for 9 or 10 points plotted or B2 FT for 7 or 8 points plotted or B1 FT for 5 or 6 points plotted <br> and B1 for two separate branches not touching or cutting $y$-axis |
| (c) | 2.1 to 2.6 | 1 |  |
|  | 8.5 to 9 | 1 |  |
| (d) | 2, 3, 5, 7 | 2 | SC1 for correct 4 values and no more than one extra positive integer or $\pm 2, \pm 3, \pm 5$, $\pm 7$ or 3 correct values and no extras |
| (e) | (-2,-12) | 1 |  |
| (f) (i) | $20+x^{2}=x^{3}$ | M1 | Multiplication by $x$ |
|  | $x^{3}-x^{2}-20=0$ | A1 | No errors or omissions |
| (ii) | Fully correct curve $y=x^{2}$ | 2 | SC1 for U - shaped parabola, vertex at origin |
| (iii) | 2.5 to 3.5 | 1 |  |
| (iv) | $3 .[0]$ to 3.1 or FT their answer to (iii) | 1FT | FT dep on (iii) $>0$ |

## Question 24

(a)
-4.5 and 10.5
(b) Correct curve

5

## B1 for each value

B4 for correct curve with branches joined
OR
B3 FT for 9 or 10 points
or B2 FT for 7 or 8 points
or B1 FT for 5 or 6 points

## and

B1 independent for one branch on each side of the $y$-axis and not touching or crossing the $y$-axis
(c)
(d) (i)
Line $y=15-3 x$ ruled
and
-0.4 to -0.31
0.35 to 0.45
2.2 to 2.3
(ii)
$[a=] 6$
$[b=]-14$
[ $c=] 0$

B2 for $6 x^{3}-14 x^{2}+2=0$ oe or
M1 for correct removal of denominator or collection of terms on one side
B3 for correct line and 2 correct values or $\mathbf{B 2}$ for correct line
or M1 for ruled line with gradient -3 or through $(0,15)$
or SC2 for no/wrong line and three correct values or SC1 for no/wrong line and two correct values or for correct freehand line

Question 25

(a)
(b)
(c) (i)

$$
\begin{aligned}
& -1<\text { ans }<-0.8 \\
& 1.25<\text { ans }<1.45
\end{aligned}
$$

$$
2.5<\text { ans }<2.6
$$

(ii)

$$
-0.7<\text { ans }<-0.5
$$

(d) (i)
$y=1$ to 1.1 oe
$y=-0.4$ to -0.33 oe
-0.4 to -0.33 oe

| (i) | $y=1$ to 1.1 oe |
| :--- | :--- |
| (ii) | $\begin{array}{l}y=-0.4 \text { to }-0.33 \mathrm{oe} \\ -0.4 \text { to }-0.33 \mathrm{oe}\end{array}$ |

## Question 26

FT only if a clear minimum point
1FT
M1 for evidence of $y=-x$ or $\frac{x^{3}}{3}-x^{2}+1=-x$
FT only if a clear maximum point

Correct or FT their graph $\mathbf{3}\left|\begin{array}{l}\text { B2 for } 4 \text { or } 5 \text { correct } \\ \text { or } \mathbf{B 1} \text { for } 2 \text { or } 3 \text { correct }\end{array}\right| \begin{aligned} & \text { B4 for correct graph but branches joined } \\ & \text { OR } \\ & \text { B3FT for } 11 \text { or } 12 \text { correct points } \\ & \text { or B2FT for } 9 \text { or } 10 \text { correct points } \\ & \text { or B1FT for } 7 \text { or } 8 \text { correct points }\end{aligned}$

| $\mathbf{1}$ |  |
| :--- | :--- |
| $\mathbf{1}$ |  |
| $\mathbf{4}$ | B3FT for 6 or 7 points plotted <br> or B2FT for 4 or 5 points plotted <br> or B1FT for 2 or 3 points plotted |
| $\mathbf{1}$ |  |
| $\mathbf{1}$ |  |
| $\mathbf{1}$ |  |
| $\mathbf{2}$ | M1 for evidence of $y=-x$ or $\frac{x^{3}}{3}-x^{2}+1=-x$ |
| $\mathbf{1 F T}$ | FT only if a clear maximum point |
| 1FT | FT only if a clear minimum point |
| 1FT | Correct or $\mathbf{F T}$ their graph |


(c) (i) $\begin{aligned} & \text { Correct ruled line through }(-2,1) \text { and } \\ & (2,-3)\end{aligned}$
(ii) 0.7 to 0.95
(iii) $[p=] 2$ and $[q=]-2$
(d) (i) (1.3 to $1.6,0)$
(ii) Ruled line from $(0,-2)$ to intersection of their graph with positive $x$-axis
(iii) Tangent [ to curve ] $A$ or (1.3 to 1.6, 0)

B1indep for a branch on each side of the $y$-axis, without touching it

B2 for $x^{3}+2 x^{2}-2=0$ oe or B1 for $x^{2}-2=-x^{3}-x^{2}$ oe or better or $1+1-\frac{2}{x^{2}}+x[=0]$ or better
B1 for straight line with gradient -1 or cutting $y$-axis at -1 or correct line but freehand or short correct ruled line

| 4 | B3 FT for 7 or 8 points or B2 FT for 5 or 6 points or B1 FT for 3 or 4 points |
| :---: | :---: |
| 2 | M1 for $y=3.5$ soi |
| M1 |  |
| A1 A1 | If 0 scored $\mathbf{S C} \mathbf{1}$ for $y=x+1$ stated or implied or for 2 correct values given |

(e) (i) $\quad$ Point plotted at $(5,5)$
(ii) Tangent ruled from $A$
(iii)
1.2 to 1.4

1
1
B2

B2 and M1 dep on reasonable attempt at tangent from $(5,5)$

M1 for change in $y$ / change in $x$ of their ruled line

## Question 28

| ;(a) | $\begin{array}{llll}0 & 2.25 & 2 & 1.25\end{array}$ | 4 | B1 for each |
| :---: | :---: | :---: | :---: |
| (b) | Fully correct smooth curve | 4 | B3 FT for 7 or 8 points or B2 FT for 5 or 6 points or B1 FT for 3 or 4 points |
| 3(c) | 1 | 1 |  |
| 3(d)(i) | $[y=] x+1$ | 1 |  |
| (d)(ii) | -2.2 to -2.1 | 1 |  |
|  | -0.45 to -0.4 | 1 |  |
|  | 0.51 to 0.6 | 1 | If zero scored, SC1 for their line in (d)(i) drawn. <br> It must be of the form $y=m x+c(m \neq 0)$ and drawn 'fit for purpose' |
| 3(e) | $-1.33<k<0$ to 0.1 | 2FT | FT Strict ft of their max point and min point dep on cubic graph or accept correct answer from calculus <br> B1 for each If zero scored, $\mathbf{S C 1}$ for two correct values reversed |

Question 29

| 4(a) | -1.75 to -1.7 | $\mathbf{1}$ |  |
| :--- | :--- | ---: | :--- |
|  | 1.7 to 1.75 | $\mathbf{1}$ |  |
| (b)(i) | Correct ruled solid tangent at <br> $(-1.5,3.5)$ | $\mathbf{1}$ |  |
| (b)(ii) | -7 to -5 | $\mathbf{2}$ dep | dep on close attempt at ruled solid tangent at $x=-1.5$ in <br> part (b)(i) <br> M1 for rise/run dep on close attempt at ruled solid tangent <br> at $x=-1.5$ |
| (c)(i) | 1 | $\mathbf{1}$ |  |
| (c)(ii) | Correct curve | $\mathbf{3}$ | B2 for 4 or 5 correct points <br> or $\mathbf{B 1}$ for 2 or 3 correct points |


| (d)(i) | -0.95 to -0.8 | $\mathbf{1}$ |  |
| :--- | :--- | ---: | :--- |
|  | 1.1 to 1.45 | $\mathbf{1}$ |  |
| (d)(ii) | their $(-0.95$ to -0.8$)<x<$ <br> their $(1.1$ to 1.45$)$ oe | $\mathbf{1 F T}$ | correct or FT their (d)(i) |
| (e)(i) | 0.125 oe and 0.03125 oe and <br> 0.000976 to 0.000977 oe | $\mathbf{1}$ |  |
| (e)(ii) | 0 | $\mathbf{1}$ | accept zero, nought, etc |

Question 30

| (a) | -1.6 to -1.4 | $\mathbf{1}$ |  |
| :--- | :--- | ---: | :--- |
| (b) | -0.5 | $\mathbf{1}$ |  |
| (c) | $k>-4$ | $\mathbf{2}$ | B1 for identifying the -4 <br> or for horizontal line drawn $y=-4$ |
| (d) | $y=x-5$ ruled <br> and <br> -2.3 to -2.1 <br> -1.2 to -1.1 <br> 1.3 to 1.4 | $\mathbf{3}$ | B2 for correct line and 2 correct values or <br> no line and 3 correct values <br> or $\mathbf{B 1}$ for no line and 2 correct values <br> or B1 for correct line |
| (e) | Tangent ruled at $x=1$ | B1 | No daylight at point of contact. Consider <br> point of contact as midpoint between two <br> vertices of daylight, the midpoint must be <br> between $x=0.8$ and 1.2 |
|  | $\mathbf{2}$ | Dep on $\mathbf{B 1}$ or close attempt at tangent at <br> $x=1$ <br> M1 for rise/run for their tangent at $x=1$ |  |

Question 31

| (a)(i) | 5 | $\mathbf{1}$ |  |
| :--- | :--- | ---: | :--- |
| (a)(ii) | $-\frac{3}{2}$ oe | $\mathbf{1}$ |  |
| 8(b) | $\left(\frac{4}{5}, 0\right)$ oe | $\mathbf{2}$ | M1 for $5 x-4=0$ soi |


| (c) | $y=-0.2 x+11$ final answer | 4 | M2 for $y=-0.2 x+c$ oe (any form) FT their <br> (a) <br> or <br> B1FT for grad $=\frac{-1}{\text { their }(\mathbf{a})(\mathbf{i})}$ soi <br> and M1 for substitution of $(10,9)$ into their equation |
| :---: | :---: | :---: | :---: |
| (d) | $(2,6)$ | 3 | M1 for elimination of one variable A1 for $x=2$ or $y=6$ |
| (e) | 13 | 3 | M2 for $(4+9) \times$ their $2 \div 2$ oe or <br> B1 for 9 oe or 4 or -4 seen |

Question 32
$\left.\begin{array}{l|l|r|l}\text { (a) } & 9,-6,9 & \mathbf{3} & \text { B1 for each } \\ \hline \text { (b) } & \text { Correct graph } & \mathbf{4} & \begin{array}{l}\text { B3FT for } 6 \text { or } 7 \text { correct points } \\ \text { or B2FT for } 4 \text { or } 5 \text { correct points } \\ \text { or B1FT for } 2 \text { or } 3 \text { correct points }\end{array} \\ \hline \text { (c) } & -3.5 \text { to }-3.35 \text { and } 0.8 \text { to } 0.9 . . & \text { 2FT } & \begin{array}{l}\text { FT } \text { their graph } \\ \text { B1FT for either }\end{array} \\ \hline \text { (d) } & \begin{array}{l}a=\frac{5}{4} \text { or } 1 \frac{1}{4} \text { or } 1.25 \\ b=-\frac{49}{8} \text { or }-6 \frac{1}{8} \text { or }-6.125\end{array} & \mathbf{3} & \left.\begin{array}{l}\text { B2 for either correct } \\ \text { or } \mathbf{M 1} \text { for }[2] ~\end{array} x+\frac{5}{4}\right)^{2} \text { seen isw } \\ \text { or for } 2 x^{2}+4 a x+2 a^{2}+b\end{array}\right]$

## Question 33

| 5(a) | 3.2 or 3.15 or 3.152 to 3.153 <br> 5.2 or 5.19 or 5.20 or $5.196 \ldots$ | $\mathbf{2}$ | B1 for each |
| :--- | :--- | ---: | :--- |
| $5(\mathrm{~b})$ | Correct graph for $0.5 \leqslant x \leqslant 3.5$ | $\mathbf{4}$ | B3FT for 6 or 7 correct points <br> or B2FT for 4 or 5 correct points <br> or B1FT for 2 or 3 correct points |
| 5(c) | 1.7 to 1.8 | $\mathbf{1 F T}$ | FT their graph if one answer |
| (d)(i) | Any integer $k \geqslant-1$ | $\mathbf{1}$ |  |
| d)(ii) | Any integer $k<-1$ | $\mathbf{1}$ |  |
| 5(e) | Tangent ruled at $x=-3$ | B1 |  |
|  | 2.5 to 4 | B2 | dep on tangent drawn at $x=-3$ or close attempt at <br> tangent at $x=-3$ <br> M1 for rise/run also dep on tangent at <br> $x=-3$ or close attempt at tangent at $x=-3$ |


| (f)(i) | $y=6-x$ ruled accurately | M2 | M1 for correct line but freehand or ruled line <br> gradient -1.1 to -0.9, or through $(0,6)$ <br> but not $y=6$ |
| :--- | :--- | ---: | :--- |
| (f)(ii) | $[a=] 8$ <br> $[b=]-48$ <br> $[c=]-16$ | $\mathbf{A 1}$ | B3 for 2 correct <br> or $x^{5}+8 x^{3}-48 x^{2}-16=0$ seen <br> or $-x^{5}-8 x^{3}+48 x^{2}+16=0$ seen <br> or M2 for correct multiplication by $8 x^{2}$ <br> or $\mathbf{B 1}$ for answers $\pm 8, \pm 48, \pm 16$ <br> or M1 for $\frac{x^{2} \times x^{3}-8 \times 2}{x^{2} \times 8}=6-x$ <br> or M1 for correct multiplication by 8 <br> or M1 for correct multiplication by $x^{2}$ |

Question 34

| 0(a) | 10.8 or 10.81 to 10.82 | $\mathbf{3}$ | M2 for $\sqrt{(6--3)^{2}+(-2-4)^{2}}$ oe <br> or $\mathbf{M 1}$ for $(6--3)^{2}+(-2-4)^{2}$ oe |
| :--- | :--- | ---: | :--- |
| (b)(i) | $(6,4)$ | $\mathbf{2}$ | $\mathbf{B 1}$ for each |
| (b)(ii) | 2 | $\mathbf{2}$ | M1 for $\frac{12-(-4)}{10-2}$ oe |
| b)(iii) | $y=-\frac{1}{2} x+4$ oe final answer | $\mathbf{3}$ | M1 for gradient $=-\frac{1}{2}$ or $-\frac{1}{\text { their }(\mathbf{b})(\text { ii) }}$ <br> $\mathbf{M 1}$ for $(2,3)$ substituted into their $y=m x+c$ <br> $y-y_{1}=m\left(x-x_{1}\right)$ oe or |

Question 35
$\left.\begin{array}{l|l|r|l}\text { (a) } & 0-0.172 .4 & \mathbf{3} & \text { B1 for each } \\ \hline \text { (b) } & \text { Fully correct smooth curve } & \mathbf{4} & \begin{array}{l}\text { B3FT for } 9 \text { or } 10 \text { correct points } \\ \text { or B2FT for } 7 \text { or } 8 \text { correct points } \\ \text { or B1FT for } 5 \text { or } 6 \text { correct points }\end{array} \\ \hline \text { (c) } & \begin{array}{l}x \leqslant 0.17 \text { to } 0.25 \\ \text { and } x \geqslant 2.25\end{array} \text { to } 2.3\end{array} \quad \mathbf{3} \begin{array}{l}\text { B2 for strict inequalities or one correct } \\ \text { or B1 for } 0.17 \text { to } 0.25 \text { and } 2.25 \text { to } 2.3 \text { seen }\end{array}\right\}$

Question 36

| !(a)(i) | $1, \ldots \ldots, \ldots . . \ldots, 16$ | 2 | B1 for each |
| :---: | :---: | :---: | :---: |
| (a)(ii) | $14, \ldots \ldots, \ldots \ldots, \ldots,-2$ | 2 | B1 for each |
| 2(b) | Fully correct smooth curves | 6 | B3 for correct curve of $y=2^{x}$ <br> or B2FT for 4 or 5 correct points or B1FT for 2 or 3 correct points <br> B3 for correct curve of $y=14-x^{2}$ or B2FT for 4 or 5 correct points or B1FT for 2 or 3 correct points |
| !(c)(i) | 3.5 to 3.7 | 1 |  |
| (c)(ii) | 2.65 to 2.8 | 1 |  |
| (d)(i) | Correct line | 1 | Ruled, through (4,2) and gradient -4 |
| (d)(ii) | Tangent $(2,10)$ | 2 | B1 for each |

Question 37
$\left.\left.\begin{array}{l}\text { (a) } \begin{array}{l|l|l|l}\text { (b) } & -2[.0],-0.2,2.5 & \text { Fully correct curve } & \mathbf{5}\end{array} \\ \begin{array}{l|l|l|l}\text { B1 for each }\end{array} \\ \text { B4 for correct curve, but branches joined } \\ \text { or B3FT for } 9 \text { or } 10 \text { correct plots }\end{array}\right] \begin{array}{l}\text { or B2FT for } 7 \text { or } 8 \text { correct plots } \\ \text { or B1FT for } 5 \text { or } 6 \text { correct plots } \\ \text { and } \\ \text { B1 indep two separate branches not } \\ \text { touching or cutting } y \text {-axis }\end{array}\right]$
$\left.\left.\begin{array}{l|l|l|l}\text { (d)(i) } & 1.05 \text { to } 1.25 & \mathbf{1} & \\ \hline \text { (d)(ii) } & \begin{array}{l}-2.3 \text { to }-2.2 \\ -0.4 \text { to }-0.3 \\ 0.3 \text { to } 0.4\end{array} & \mathbf{3} & \begin{array}{l}\text { B1 for each } \\ \text { After } 0 \text { scored } \mathbf{B 1} \text { for } y=-4 \text { ruled }\end{array} \\ \text { (e) } & \begin{array}{ll|l}{[a=] 2} \\ {[b=] 24} \\ {[n=] 5}\end{array} & \mathbf{3} & \begin{array}{l}\mathbf{B 2} \text { for } 2 \text { correct or for } \\ 2 x^{5}+24 x^{2}[-3=0]\end{array} \\ \text { or } \mathbf{B 1} \text { for } 1 \text { correct or for } \\ \frac{2 x^{5}-3+4\left(6 x^{2}\right)}{6 x^{2}}[=0] \text { oe }\end{array}\right] \begin{array}{l}\text { If } 0 \text { scored } \mathbf{S C 1} \text { for } 2 x^{5} \text { seen in final line } \\ \text { of algebra }\end{array}\right]$

Question 38
'(a) $\quad x=0$
(b) Tangent ruled at $x=0.5$
$\left|\begin{array}{ll}-9 \text { to }-6.5 \\ & \end{array}\right|$
(c)(i) $\quad \begin{array}{lll}0 & 2.4 \text { or better } & 4\end{array}$
(c)(ii) Correct smooth curve
Correct smooth curve
(d) $\quad \begin{aligned} & x^{3}+3 x+4=10-8 x^{2} \text { and correctly } \\ & \text { completed }\end{aligned}$
(e) line $y=-2 x+2$ drawn and -0.45 to -0.35 nfww

1
B1 No daylight between tangent and curve at point of contact

2 dep on ruled tangent or close attempt at tangent at $x=0.5$

M1 for rise/run also dep on tangent or close attempt at tangent at $x=0.5$

3 B1 for each
4 B3FT for 6 or 7 correct plots or B2 FT for 4 or 5 correct plots or B1 FT for 2 or 3 correct plots

FT their table

Question 39

| (a) | -3, 17 | 2 | B1 for each |
| :---: | :---: | :---: | :---: |
| (b) | Fully correct curve | 4 | B3 FT for 10 or 11 points or B2 FT for 8 or 9 points or B1 FT for 6 or 7 points |
| i(c)(i) | Correct ruled tangent for their curve through ( $0,-17$ ) | 1 |  |
| (c)(ii) | (1.7 to 2.2, -1 to 2.5) | 1 |  |
| (c)(iii) | $[y=] 9 x-17$ final answer | $3$ | M2dep for answer $[y=] 9 x[+]-c$ <br> OR <br> M1dep for gradient $=\frac{\text { rise }}{\text { run }}$ for their tangent at any point <br> B1 for answer $[y=] k x[+]-17(k \neq 0)$ |
| (d) | $\begin{aligned} & y=3 x+2 \text { ruled correctly and } \\ & -2.2 \ldots \text { to }-2.1 \\ & -0.6 \text { to }-0.4 \\ & 2.6 \text { to } 2.8 \end{aligned}$ | 4 | B2 for $y=3 x+2$ ruled or $\mathbf{B 1}$ for $[y=] 3 x+2$ soi or $y=3 x+k$ ruled or $y=k x+2$ but not $y=2$ <br> B2 for all 3 values or B1 for 2 values |

Question 40

| (a) | $(5,6)$ | 1 |  |
| :---: | :---: | :---: | :---: |
| b) | $[y=]-\frac{4}{5} x+3 \mathrm{nfww}$ | 3 | B2 for $[y=]-\frac{4}{5} x+c$ nfww or $\mathbf{M 1}$ for $\frac{\text { rise }}{\text { run }}$ using any two of $(-5,7)$ $(0,3)$ and $(5,-1)$ and B1 for $[y=] m x+3(m \neq 0)$ |
| (c) | $y=-\frac{4}{5} x-2 \text { oe }$ | 2 | FT their gradient from 8(b) <br> $\mathbf{B 1}$ for $y=($ their gradient $) x+c(c$ not 0$)$ or for $y=m x-2 \quad(m \neq 0)$ <br> or for $-\frac{4}{5} x-2$ alone |


| (d)(i) | $y=\frac{5}{4} x+4$ oe | $\mathbf{3}$ | M1 for $-\frac{1}{\text { their gradient }}$ from 8(b) <br> $\mathbf{M 1}$ for $(8,14)$ substituted into <br> their $y=m x+c$ or $\frac{y-14}{x-8}=m$ or better |
| :--- | :--- | ---: | :--- |
| (d)(ii) | 8.54 or $8.544 \ldots$ | $\mathbf{3}$ | M2 for $(14-\text { their } 6)^{2}+(8-\text { their } 5)^{2}$ or <br> better <br> or M1 for $14-$ their 6 and $8-$ their 5 seen |
| d)(iii) | $(4,6)$ | $\mathbf{2}$ | $\mathbf{B 1}$ for each |

## Question 41

| (a) | $\begin{array}{llll}0 & -2 & 0.9\end{array}$ | 3 | B1 for each |
| :---: | :---: | :---: | :---: |
| (b) |  | $4$ | B3 FT for 9 or 10 points or B2 FT for 7 or 8 points or B1 FT for 5 or 6 points |
| (c) | $\begin{aligned} & -0.45 \text { to }-0.35 \\ & 1 \\ & 2.35 \text { to } 2.45 \end{aligned}$ | 3 | FT their graph <br> B1 for each in the correct position If zero scored, SC1FT for 3 correct values |
| (d)(i) | $y=1-x$ oe | 2 | B1 for $y=1-k x$ oe, $k \neq 0$ or $y=k-x$ oe or $1-x$ |
| (d)(ii) | Correct ruled line and 2.25 to 2.4 | 3 | B2FTdep for correct ruled line <br> or B1 dep for line through $(0,1)$ when extended but not $y=1$ or with gradient -1.1 to -0.9 or correct line but freehand or SC2 for $y=x-1$ ruled after answer [ $y=] x-1$ in (d)(i) and <br> B1 for 2.25 to 2.4 |
| (e) | Correct tangent and 1.7 to 3.7 | 3 | No daylight between tangent and curve at $x=-0.25$. Point of contact is the midpoint between two vertices of daylight and this point of contact must be between -0.35 and -0.15 <br> B2 for close attempt at tangent at $x=-0.25$ and answer in range |

## Question 42

\(\left.$$
\begin{array}{l|l|r|l}\text { (a) } & -2.1,1.6,-1.7,2.1 & \mathbf{3} & \begin{array}{l}\text { B2 for 3 correct } \\
\text { or } \mathbf{B 1} \text { for } 2 \text { correct }\end{array} \\
\hline \text { (b) } & \text { Fully correct curve } & \mathbf{4} & \begin{array}{l}\text { B3FT for } 8 \text { or } 9 \text { correct plots } \\
\text { or B2FT for } 6 \text { or } 7 \text { correct plots } \\
\text { or B1FT for } 4 \text { or } 5 \text { correct plots }\end{array}
$$ <br>

\hline (c) \& line y=\frac{1}{2}(1-x) ruled \& M2 \& M1 for line with gradient-\frac{1}{2}\end{array}\right]\)| M1 for line through $\left(0, \frac{1}{2}\right)$ but not $y=\frac{1}{2}$ |
| :--- |

Question 43

| i(a)(i) | -3 | 1 |  |
| :---: | :---: | :---: | :---: |
| (a)(ii) | 6.2 to 6.4 oe | 2 | M1 for 3 seen or used |
| (b) | $y=5-3 x$ ruled | 2 | B1 for $y=5-3 x$ soi or ruled line with gradient -3 or with $y$-intercept at 5 (but not $y=5$ ) or <br> B1FT for incorrect line equation/expression shown in working and their line correctly drawn |
|  | $\begin{array}{\|l\|} \hline-0.3 \text { to }-0.2 \\ 1.65 \text { to } 1.8 \end{array}$ | 2 | B1 for each, dep on $y=5-3 x$ drawn or FT their line provided equation/expression shown in working, dep on B1FT for line |
| (c) | Tangent ruled at $x=-2$ | 1 | B1 for correct tangent |
|  | -4.5 to -2.5 | 2 | Dep on B1 for tangent or close attempt at tangent at $x=-2$ <br> M1 for rise/run also dep on tangent drawn or close attempt at correct tangent Must see correct or implied calculation from a drawn tangent |
| 5(d)(i) | 8, 4, 0.25 oe | 3 | B1 for each |
| j(d)(ii) | Correct graph | 3 | B2FT for 6 or 7 correct plots or B1FT for 4 or 5 correct plots |
| i(d)(iii) | 1.8 to 1.9 | 1 |  |

Question 44

| (a) | $2.45,0.25,-0.25$ | 3 | B1 for each |
| :---: | :---: | :---: | :---: |
| (b) | Fully correct smooth curve | 4 | B3FT for 6 or 7 points or B2 FT for 4 or 5 points or B1 FT for 2 or 3 points |
| (c) | 0.7 to 0.8 | 1 | FT their curve |
| (d)(i) | Correct ruled line | 2 | M1 for good freehand, or ruled line with gradient -1.05 to -0.95 or ruled line through $(0,2)$ but not line $y=2$ |
| (d)(ii) | Both intersections of their (b) and their (d)(i) | 2 | Strict FT intersection of their (b) and their (d)(i) <br> B1FT for one correct OR B2 for 0.27 to 0.28 and 2.38 to 2.39 |
| (e) | Substitutes $x=\sqrt{2}$ into $\frac{1}{2 x}-\frac{x}{4}$ OR <br> Identifies $y=0$ oe <br> OR <br> Correctly manipulates to a single fraction <br> e.g. $\frac{2-x^{2}}{4 x}$ oe seen | M1 |  |
|  | Concludes 'read the graph at $y=0$ ' oe <br> OR <br> Manipulates $0=\frac{1}{2 x}-\frac{x}{4}$ oe leading to $x^{2}=2$ <br> OR <br> States $\frac{2-x^{2}}{4 x}$ oe $=0$ leading to $x^{2}=2$ | A1 |  |

Question 45

| (a)(i) | 1.5 oe | 1 |  |
| :---: | :---: | :---: | :---: |
| (a)(ii) | $(0,2)$ | 1 |  |
| (b)(i) | $y=-2 x+6$ oe final answer | 3 | B2 for $y=-2 x+c$ oe or $y=m x+6$ oe $m \neq 0$ or for answer $-2 x+6$ or $\mathbf{B 1}$ for [gradient $=$ ] $-\frac{6}{3}$ oe or $c=+6$ soi |
| (b)(ii) | $y=0.5 x-1.5$ oe final answer | 3 | B1 for [gradient = ] -1 divided by their gradient from (b)(i) evaluated soi <br> M1 for substitution of $(9,3)$ into $y=($ their $m) x+c$ seen in working |
| (c)(i) | 12.6 or 12.64 to 12.65 | 3 | $\begin{aligned} & \text { M2 for } \sqrt{(8--4)^{2}+(5-1)^{2}} \text { oe } \\ & \text { or M1 for }(8--4)^{2}+(5-1)^{2} \text { oe } \end{aligned}$ |
| (c)(ii) | $(2,3)$ | 2 | B1 for each |

Question 46

| (a) | $2,2,6$ | $\mathbf{3}$ | B1 for each |
| :--- | :--- | ---: | :--- |
| (b) | Correct graph | $\mathbf{4}$ | B3FT for 10 or 11 correct plots <br> or B2FT for 8 or 9 correct plots <br> or B1FT for 6 or 7 correct plots |
| (c) | -3.3 to -3.1 | $\mathbf{1}$ | FT their graph |
| (d) | $y=-2 x$ ruled | M1 | or B1 for $y=-2 x$ stated |
|  | -2.6 to -2.45 | A1 |  |
| (e) | 3 or 4 or 5 | $\mathbf{1}$ | FT their graph <br> Allow more than one correct value |

Question 47

| (a) | $5,-3,21$ | $\mathbf{3}$ | B1 for each |
| :--- | :--- | ---: | :--- |
| (b) | Fully correct curve | $\mathbf{4}$ | B3 FT for 9 or 10 points <br> or B2 FT for 7 or 8 points <br> or B1 FT for 5 or 6 points |
| (c) | -2.9 to -2.7 <br> 0 <br> 1.7 to 1.9 | $\mathbf{2}$ | B1 for 2 correct values |

$\begin{array}{l|l|r|l}\text { (d) } & \text { Tangent ruled at } x=2 & \text { B1 } & \\$\cline { 2 - 4 } 10 to 14 \& B2 \& $\left.\begin{array}{l}\text { Dep on correct tangent or close attempt at } \\ \text { tangent at } x=2\end{array} \\ \text { M1 for rise/run also dep on correct } \\ \text { tangent drawn or close attempt at tangent } \\ \text { Must see correct or implied calculation } \\ \text { from a drawn tangent }\end{array}\right]$

Question 48

| (a) | $3.5,15,3.9$ | 3 | B1 for each |
| :---: | :---: | :---: | :---: |
| (b) | Correct graph | 5 | B4 for correct curves but branches joined or touching $y$-axis <br> or B3FT 10 or 11 points <br> or B2FT for 8 or 9 points <br> or B1FT for 6 or 7 points <br> B1indep two separate branches not touching or crossing $y$-axis |
| (c) | 0.5 to 0.6 and 1.3 to 1.6 | 2 | B1 for each or both correct but in reverse order |
| (d) | 1 | 1 |  |
| (e)(i) | $y=3 x+1 \text { ruled }$ $\text { and } 0.3 \text { to } 0.49$ | 3 | B2 for correct ruled line that crosses their curve or $\mathbf{B 1}$ for $y=3 x+1$ soi or freehand line or ruled line with gradient 3 or with $y$-intercept at 1 (but not $y=1$ ) |
| (e)(ii) | $\begin{aligned} & {[a=]-6} \\ & {[b=]-2} \\ & {[c=]-4} \end{aligned}$ | 3 | M2 for $x^{4}+2-4 x=6 x^{3}+2 x^{2}$ or better seen or $\mathbf{B 1}$ for each correct value to a maximum of 2 marks <br> If 0 scored, $\mathbf{S C 1}$ for answer [ $a=] 6,[b=] 2$ and $[c=] 4$ or for $x^{5}+2 x-4 x^{2}=6 x^{4}+2 x^{3}$ or better |

Question 49

| I(i) | $(3,5.5)$ | $\mathbf{2}$ | B1 for either value correct |
| :--- | :--- | ---: | :--- |
| I(ii) | $\frac{5}{4} x+\frac{7}{4}$ final answer | $\mathbf{3}$ | $\mathbf{B 2}$ for answer $\frac{5}{4} x+c$ oe or for correct |
| equation in different form |  |  |  |
| or M1 for $\frac{8-3}{5-1}$ oe |  |  |  |
| and M1 for correct substitution shown of $(1$, |  |  |  |
| $3)$ or $(5,8)$ or their $(\mathrm{a})($ (i) into |  |  |  |
| $y=($ their $m) x+c$ oe |  |  |  |

Question 50

| 2(a)(i) | $\begin{array}{llll}3 & 2.25 & 1\end{array}$ | 3 | B1 for each |
| :---: | :---: | :---: | :---: |
| !(a)(ii) | Fully correct smooth curve | 4 | B3FT for 7 or 6 correct plots B2FT for 5 or 4 correct plots B1FT for 3 correct plots |
| (a)(iii) | $\begin{aligned} & -0.6 \text { to }-0.51,0.75 \text { to } 0.85, \\ & 1.7 \text { to } 1.85 \end{aligned}$ | 3 | B1 for each If 0 scored, SC1 for $y=1.5$ drawn |
| (a)(iv) | -3 or -2 or -1 or 0 | 1 |  |
| 2(b)(i) | Tangent ruled at $x=1$ | 1 |  |
| !(b)(ii) | 4.4 to 5.6 | 2 | Dep on tangent at $x=1$ or close attempt <br> M1 for rise/run for their line |
| (b)(iii) | $\begin{aligned} & y=(4.4 \text { to } 5.6) x-(1.8 \text { to } 2.2) \\ & \text { or } \\ & {[y=] \text { their }(\mathbf{b})(\mathbf{i i}) x+\text { their }(y \text {-intercept })} \end{aligned}$ | 2 | FT for any line but not horizontal or vertical line for 2 marks or B1 <br> B1FT for $[m=]$ their 5 <br> or for their $y$-intercept |

Question 51
(a)(i) $\left\lvert\,\left(-\frac{1}{2}, 4\right)\right.$ and $\left(\frac{1}{2}, 2\right)$
$5 \mid \mathbf{B 2}$ for $12 x^{2}-3[=0]$
or $\mathbf{B 1}$ for $12 x^{2}$ or -3
M1 for their derivative $=0$ or $\mathrm{d} y / \mathrm{d} x=0$
B1 for $[x=]-1 / 2$ and $1 / 2$ or one coordinate pair correct

(a)(ii) | $\left(-\frac{1}{2}, 4\right)$ Max with reason |  |
| :--- | :--- |
|  | $\left(\frac{1}{2}, 2\right)$ Min with reason |

(b) line $y=x+3$ ruled

|  |  |
| :--- | :--- |
| -0.7 to -0.8 |  |
| 2.7 to 2.8 |  |

Question 52

| (a)(i) | $x+5$ | $\mathbf{2}$ | B1 for linear equation with positive gradient <br> or intercept 5 |
| :--- | :--- | ---: | :--- |
| (a)(ii) $2 \sin x$ oe $\mathbf{2}$ <br> B1 for recognition of $\sin$ or $\cos (x-90)$   <br> (b) tangent ruled at $P$ B1 |  |  |  |
|  | 1.3 to 1.4 | B2 | dep on tangent drawn <br> M1 for rise/run |

Question 53

| (a)(i) | $\frac{3}{8}$ | $\mathbf{2}$ | M1 for $8 y=3 x+20$ or better |
| :--- | :--- | ---: | :--- |
| (a)(ii) | $(0,2.5)$ oe | $\mathbf{1}$ |  |
| (a)(ii) $(0,2.5)$ oe $\mathbf{1}$  <br> (b)(i) 15.6 or $15.62 \ldots$ $\mathbf{3}$ M2 for $\sqrt{(9--3)^{2}+(-2-8)^{2}}$ oe seen <br> or M1 for $(9--3)^{2}$ or $(-2-8)^{2}$ oe seen    |  |  |  |
| (b)(ii) | $y=-\frac{5}{6} x+4$ oe | $\mathbf{3}$ | M1 for gradient $\frac{-2-8}{9--3}$ oe <br> M1 for substituting $(6,-1)$ into a linear <br> equation oe |
| (b)(iii) | $y=\frac{6}{5} x-\frac{3}{5}$ oe | $\mathbf{4}$ | M1 for gradient $-1 /$ their $\left(-\frac{5}{6}\right)$ <br> B1 for midpoint at $(3,3)$ <br> M1 for their midpoint substituted into <br> $y=$ their $m \times x+c$ oe |

Question 54

| (a)(i) | 5 | $\mathbf{2}$ | $\mathbf{M 1}$ for $(-1)^{4}-4(-1)^{3}$ |
| :--- | :--- | :--- | :--- |
| (a)(ii) | $(0,0)$ and $(3,-27)$ | $\mathbf{6}$ | $\mathbf{B 2}$ for $4 x^{3}-12 x^{2}[=0]$ <br> or $\mathbf{B 1}$ for $4 x^{3}$ or $12 x^{2}$ |
| AND |  |  |  |
| $\mathbf{M 1}$ for derivative $=0$ or their derivative $=0$ |  |  |  |
| M1 for $4 x^{2}(x-3)[=0]$ |  |  |  |
| $\mathbf{B 1}$ for |  |  |  |
| $[x=] 0$ and $[x=] 3$ or $[y=] 0$ and $[y=]-27$ |  |  |  |
| or for one correct coordinate pair |  |  |  |

(b) $\left\lvert\, \begin{aligned} & {[p=] 11} \\ & {[q=] 5}\end{aligned}\right.$

2 B1 for each
or M1 for $\frac{d y}{d x}=p x^{p-1}+2 q x^{q-1}$

Question 55


2 B2 for correct quadratic curve with min touching $x$-axis
or B1 for parabola vertex downwards


Question 56

| )(a)(i) | 15.7 or $15.65 \ldots$ | 3 | M2 for $\sqrt{(4-10)^{2}+(4--3)^{2}}$ oe or M1 for $(-4-10)^{2}+(4--3)^{2}$ oe |
| :---: | :---: | :---: | :---: |
| (a)(ii) | $\frac{-10-4}{4--3}[=-2] \text { oe }$ | M1 |  |
|  | $\begin{aligned} & 10=-2(-3)+\mathrm{c} \\ & \text { Or }-4=-2(4)+\mathrm{c} \end{aligned}$ <br> and correct completion to $y=-2 x+4$ | A1 |  |
| (a)(iii) | $y=\frac{1}{2} x+\frac{11}{4}$ oe | $4$ | M1 for grad $=1 / 2$ soi <br> M1 for [midpoint $=$ ] $(1 / 2,3)$ <br> M1 for substitution of $(1 / 2,3)$ into their $y=m x+$ $c$ oe |
| )(b)(i) | $\left(-\frac{1}{3},-\frac{22}{27}\right)$ oe and $(-5,50)$ | 6 | B2 for $3 x^{2}+16 x+5$ <br> Or B1 for one correct <br> M1 for derivative $=0$ or their derivative $=0$ <br> M1 for $[x=]-\frac{1}{3}$ and $[x=]-5$ <br> B1 for $-\frac{22}{27}$ and 50 |
| (b)(ii) | $\left(-\frac{1}{3},-\frac{22}{27}\right)$ minimum $(-5,50)$ maximum with correct reasons | 3 | B2 for one correct with reason <br> or M1 for correct attempt e.g. $2^{\text {nd }}$ derivatives, gradients or sketching |

Question 57

| (a)(i) | Correct sketch | 2 | B1 for correct shape but inaccurate |
| :---: | :---: | :---: | :---: |
| (a)(ii) | Rotational [symmetry] order 2 [centre] $(180,0)$ | -0 | B1 for rotational [symmetry] |
| (b) | 48.6 or 48.59 to 48.60 and 131.4 or 131.40 to 131.41 | 3 | B2 for 48.6 or 48.59 to 48.60 or 131.4 or 131.40 to 131.41 <br> or M1 for $\sin x=0.75$ or better <br> If 0 scored, $\mathbf{S C 1}$ for two answers adding to 180 |
| (c)(i) | $(x+5)^{2}-11$ | 2 | M1 for $(x+5)^{2}+k$ <br> or $(x+\text { their } 5)^{2}+14-(\text { their } 5)^{2}$ or $a=5$ |
| (c)(ii) | Sketch of U-shaped parabola with a minimum indicated at $(-5,-11)$ with no part of graph in $4^{\text {th }}$ quadrant | 3 | FT their $(x+5)^{2}-11$ provided in that form B1 for $U$ shape curve B1FT for turning point at $(-5, k)$ or ( $k,-11$ ) |

## Question 58

(1.74, 7.21 to 7.24 )
and
$(-3.74,-9.20$ to -9.22$)$ сао

6 For the $y$ values accept any value rounded to 2 decimal places in the given range
$\mathbf{B 5}$ for (1.74, 7.21 to 7.24 )
or $(-3.74,-9.20$ to -9.22$)$
or $x=1.74$ and $x=-3.74$
OR
M2 for $2 x^{2}+4 x-13=0$
or $2 y^{2}+4 y-133=0$
or M1 for $2 x^{2}+7 x-11=3 x+2$
or $y=2\left(\frac{y-2}{3}\right)^{2}+7\left(\frac{y-2}{3}\right)-11$

AND
FT their quadratic expression (not $\left.2 x^{2}+7 x-11\right)$

M2FT for $\frac{-4 \pm \sqrt{4^{2}-4 \times 2 \times-13}}{2 \times 2}$
or $-1 \pm \sqrt{\frac{15}{2}}$ oe
or M1FT for $\sqrt{4^{2}-4 \times 2 \times-13}$ oe
or for $\frac{-4+\sqrt{k}}{2 \times 2}$ or $\frac{-4-\sqrt{k}}{2 \times 2}$
or $(x+1)^{2}[-13 / 2-1=0]$

Question 59
(a)(i) $\mid(8-x)(3+x)$
2 M1 for $8(3+x)-x(3+x)$ or $3(8-x)+x(8-x)$ or $(a-x)(b+x)$ where $a b=24$ or $a-b=5$

| (a)(ii) | $\begin{aligned} & {[a=]-3} \\ & {[b=] 8} \\ & {[c=] 24} \end{aligned}$ |  | 3 | FT their (a)(i) for $a$ and $b$ <br> B1FT for each of $a$ and $b$ or both correct <br> but reversed <br> B1 for [ $c=] 24$ |
| :---: | :---: | :---: | :---: | :---: |
| (a)(iii) | 8 |  | 3 | M2 for $5-2 x$ <br> or M1 for $-2 x$ or $5-k x, k \neq 0$ |
| '(b)(i) | Correct sketch: <br> positive cubic shape and max on the $y$-axis or to the right of $y$-axis <br> with one root at $(-1,0)$ <br> and <br> turning point at $(3,0)$ <br> and <br> $y$-intercept at $(0,9)$ all labelled |  | 4 | B1 for positive cubic shape with max on the $y$-axis or to the right of $y$-axis <br> B1 for root at $(-1,0)$ <br> B1 for turning point at $(3,0)$ <br> B1 for $y$-intercept $(0,9)$ <br> If 0 score $\mathbf{S C 1}$ for all three intercepts on axes identified |
| (b)(ii) | $x^{3}-5 x^{2}+3 x+9$ final answer |  | 3 | B2 for correct expansion of three brackets unsimplified <br> B1 for correct expansion of two brackets with at least 3 terms correct |
| Question 60 |  |  |  |  |
| '(a) | 2, 4.5 | 2 | B1 for each |  |
| '(b) | Correct graph |  | B3 FT for 6 or 7 correct points FT their table or B2 FT for 4 or 5 correct points FT their table <br> or B1 FT for 2 or 3 correct points FT their table |  |
| '(c)(i) | -0.5 to -0.4 | 1 |  |  |
| (c)(ii) | $\begin{aligned} & y=1-x \text { ruled } \\ & \text { and } \\ & -1.9 \text { to }-1.75 \end{aligned}$ | 2 | M1 for $[y=] 1-x$ or $\left[x^{2}+\frac{1}{x}=\right] 1-x$ soi or $\mathbf{B 1}$ for -1.9 to -1.75 |  |
| '(d) | Any integer $\geq 2$ | 1 |  |  |

Question 61


## Question 62

| (a) | 17 |  | M2 for $3 \times 2 x^{2}-7$ or better isw or M1 for $3 \times 2 x^{2}$ oe or $k x^{2}-7$ seen |
| :---: | :---: | :---: | :---: |
| (b)(i) | 13.4 or 13.41 to 13.42 | 3 | M2 for $\sqrt{(-5-7)^{2}+(8-2)^{2}}$ oe or M1 for $(-5-7)^{2}+(8-2)^{2}$ oe |
| (b)(ii) | [ $y=] 2 x+5$ final answer | 4 | M1 for [gradient of $A B=] \frac{8-2}{-5-7}$ oe <br> M1dep for gradient $p=-1 \div$ their $-\frac{1}{2}$ oe <br> M1dep on previous M1 for substituting $(-1,3)$ into $y=$ their $p x+c$ oe where their $p \neq 0$ |
| b)(iii) | $(5,0)$ |  | B3 for $\overrightarrow{A D}=\binom{-2}{-2}$ or $\overrightarrow{D A}=\binom{2}{2}$ <br> or coordinates of $C \quad(-7,6)$ and $[\overrightarrow{C D}=]\binom{12}{-6}$ oe seen <br> or $\mathbf{B} \mathbf{2}$ for $a=b=2$ soi or coordinates of $C \quad(-7,6)$ <br> or M1 for $a=b$ oe soi or for $a^{2}+b^{2}=(\sqrt{8})^{2}$ oe <br> or $\cos 45=\frac{a}{\sqrt{8}}$ oe <br> or for $[\overrightarrow{D C}=]\binom{-12}{6}$ or $[\overrightarrow{C D}=]\binom{12}{-6}$ seen or $\frac{y-8}{x--5}=1$ oe or $\frac{y-2}{x-7}=1$ |

## Question 63

| (a)(i) | 2.7 to 2.8 | 1 |  |
| :---: | :---: | :---: | :---: |
| (a)(ii) | tangent ruled at $x=-2$ | B1 |  |
|  | 6 to 10 | 2 | dep on B1 or a close attempt at tangent at $x=-2$ <br> or M1 for rise/run for their tangent, or close attempt, at any point Must see correct or implied calculation from a drawn tangent <br> After M0, SC1 for gradient of tangent (or close attempt) in range embedded in $y=m x+c$ |
| (a)(iii) | $\begin{aligned} & y=2 x-2 \text { ruled } \\ & \text { and } x=-2.9 \text { to }-2.8 \text { cao } \end{aligned}$ | 3 | B2 for correct ruled line <br> or $\mathbf{B 1}$ for short line or for freehand line or broken line or ruled line with gradient 2 or with $y$-intercept at -2 (but not $y=-2$ ) |
| (b) | $A(4,17) B(-1.5,0.5)$ | 5 | B4 for $(-1.5,0.5)$ and $(4,17)$, or for $x=4$ and $x=-1.5$ <br> OR <br> $\mathbf{B} 3$ for $\mathrm{A}(4,17)$ or $\mathrm{B}(-1.5,0.5)$ <br> OR <br> M1 for $2 x^{2}-2 x-7=3 x+5$ oe <br> AND <br> either <br> M2 for $(2 x+3)(x-4)$ <br> or M1 for $2 x(x-4)+3(x-4)$ <br> or $x(2 x+3)-4(2 x+3)$ <br> or $(2 x+c)(x+d)$ <br> where $c d=-12$ or $c+2 d=-5$ <br> [ $c$ and $d$ are integers] <br> OR <br> M2 for <br> $\frac{- \text { their } b \pm \sqrt{(\text { their } b)^{2}-4(\text { their } a)(\text { their } c)}}{2(\text { their } a)}$ <br> 2(their a) <br> or M1 for $\sqrt{(\text { their } b)^{2}-4(\text { their } a)(\text { their } c)}$ or for $p=-$ their $b, r=2($ their $a)$ if in the form $\frac{p+\sqrt{q}}{r}$ or $\frac{p-\sqrt{q}}{r}$ |

Question 64

| (a)(i) | $[a=] 4$ <br> $[b=]-3$ nfww | $\mathbf{2}$ | B1 for $[a=] 4$ <br> B1 for $[b=]-3$ nfww |
| :--- | :--- | ---: | :--- |
| (a)(ii) | $y=4$ oe | $\mathbf{1}$ |  |
| a)(iii) | $y=-6 x+7$ oe final answer | $\mathbf{2}$ | B1 for answer $-6 x+7$ <br> or answers $y=-6 x+c$ or $y=k x+7(k<0)$ |
| (b)(i) | 2.252 .673 .5 | $\mathbf{3}$ | B1 for each | | (b)(ii) |
| :--- |
| correct curve |

## Question 65

(a)(i) $\mid(2,7)$
(a)(ii) $\left\lvert\,-\frac{1}{2} x+8\right.$ oe

2 B1 for each coordinate
4 Correct equivalent in different form scores 3 marks.
M1 for gradient of $A B=\frac{9-5}{3-1}$ or $\frac{4}{2}$ or 2
M1 dep for gradient
$p=-\frac{1}{\text { their grad of } A B}$
M1 (dep on previous M1) for substitution of their midpoint into $y=($ their $p) x+c$ oe where their $p \neq 0$

Question 66

| (a)(i) | 2 | 2 | M1 for $x\left(x^{2}-4 x+4\right)$ or $x(x-2)^{2}$ or $\left(x^{2}-2 x\right)(x-2)$ or $x^{3}-2 a x^{2}+a^{2} x$ |
| :---: | :---: | :---: | :---: |
| (a)(ii) | Correct sketch with curve passing through $O$ and touching $(2,0)$ | 4 | B1 for any positive cubic <br> B1 for sketch through or touching $O$ B1 for sketch with min or max touching $x$-axis once only but not at $(0,0)$ <br> B1FT their (a)(i) for sketch with min or max touching $x$-axis at (their 2,0 ) and their 2 is labelled or clearly indicated |
| (b) | $y=20 x-64$ final answer nfww | 7 | B6 for equivalent correct equation OR <br> B2 for $3 x^{2}-8 x+4$ isw or B1 for $3 x^{2}$ or $-8 x$ seen <br> M2dep for $[\mathrm{grad}=] 20$ soi nfww or M1dep for substituting 4 into their derivative isw <br> B1 for $(4,16)$ soi <br> M1dep for $16=$ their $20 \times 4+c$ oe |

Question 67

| (a)(i) | $1,-0.5$ oe | 2 | B1 for each |
| :---: | :---: | :---: | :---: |
| (a)(ii) | Correct curve | 4 | B3FT for 6 or 7 correct plots or B2FT for 4 or 5 correct plots or B1FT for 2 or 3 correct plots |
| (b) | $y=2.5-2 x$ ruled | B2 | B1 for $y=k-2 x$ or $y=p x+2.5$ ruled ( $p \neq 0$ ) <br> or for $[y=] 2.5-2 x$ oe identified |
|  | 1.3 to 1.4 | B1 |  |
| (c) | -1 | B1 |  |
|  | $y=-1$ | B1 | FT their $k$ (must be negative) |

Question 68

| (a) | 0.75 | 3 | 7 | 3 | 0.75 |  | B2 for 4 or 3 correct <br> or $\mathbf{B 1}$ for 2 correct |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| (b) | correct curve |  | B3FT for 8 or 9 correct plots <br> B2FT for 6 or 7 correct plots <br> B1FT for 4or 5 correct plots |  |  |  |  |

Question 69

| (i) | $6 x^{2}-8 x$ final answer | 2 | B1 for each term in final answer or M1 for correct answer seen and spoilt |
| :---: | :---: | :---: | :---: |
| (ii) | 64 | 2 | FT their (c)(i) correctly evaluated provided at least 2 terms but not the original equation M1 for substituting $x=4$ into their (c)(i) |
| (iii) | $(0,6)$ <br> $\left(\frac{4}{3}, \frac{98}{27}\right)$ oe | 4 | M1 for their derivative $=0$ or $\frac{d y}{d x}=0$ soi B1 for $x=0$ and $x=\frac{4}{3}$ <br> M1dep for substituting one of their $x$ values into $y=2 x^{3}-4 x^{2}+6$ soi |

## Question 70



Question 71
\(\left.$$
\begin{array}{l|l|l|l}\text { (a) } & (2,-10) \text { and }(-2,22) & \mathbf{5} \left\lvert\, \begin{array}{l}\mathbf{B 2} \text { for } 3 x^{2}-12 \text { isw } \\
\text { or } \mathbf{B 1} \text { for } 3 x^{2}+k \text { or } p x^{2}-12(p \neq 0) \\
\text { or for } 3 x^{2}-12+6 \text { isw } \\
\text { M1 for setting their derivative }=0 \\
\text { or } \frac{d y}{d x}=0\end{array}\right. \\
\text { (b) } & \begin{array}{l}(2,-10) \text { minimum with correct reason } \\
\text { or sketch } \\
(-2,22) \text { maximum with correct reason } \\
\text { or sketch }\end{array} & \mathbf{3} \begin{array}{l}\text { Bair } x= \pm 2 \text { or for one correct coordinate } \\
\text { par }\end{array} \\
\hline \begin{array}{l}\text { B2 for 1 correct with correct reasoning } \\
\text { or B2FT for correct evaluation with correct } \\
\text { 2nd derivative for both of } t h e i r ~ d i f f e r e n t ~\end{array}
$$ <br>

values\end{array}\right\}\)| or M1 for showing [2nd derivative $=] 6 x$ |
| :--- |
| or gradients for one value on either side of |
| one correct stationary point |
| or for reasonable sketch of cubic |

Question 72

| (a) | $-1,-0.375,3$ | $\mathbf{3}$ | $\mathbf{B 1}$ for each |
| :--- | :--- | ---: | :--- |$⿻$| (b) | Correct graph | $\mathbf{4}$ |
| :--- | :--- | :--- |
| B3FT for 8 or 9 correct points <br> or B2FT for 6 or 7 correct points <br> or B1FT for 4 or 5 correct points |  |  |
| (c) | $y=2-x$ ruled correctly <br> AND <br> -0.45 to -0.35 <br> 1 <br> 2.35 to 2.45 | B2 for $y=2-x$ ruled <br> or $\mathbf{B} 1$ for $[y=] 2-x$ soi <br> or $y=k-x$ ruled <br> or $y=k x+2$ ruled, but not $y=2$ <br> B2 for all three values <br> or B1 for any two values |

Question73

| (i) | $20.25-(1.5+x)^{2}$ | 3 | Method 1 <br> B1 for $( \pm 1.5 \pm x)^{2}$ seen <br> B1 for $[\mathrm{b}=] 18+$ their $1.5^{2}$ <br> OR <br> Method 2 <br> B1 for $b-a^{2}-2 a x-x^{2}$ or for $\mathbf{b}=20.25$ B1 for $\mathrm{a}=1.5$ |
| :---: | :---: | :---: | :---: |
| (ii) | Correct sketch with max in correct quadrant at $(-1.5,20.25)$ | 3 | FT their $20.25-(\text { their } 1.5+x)^{2}$ provided in that form <br> B1 for $\cap$ shape or for $\cup$ shape if in form $c+(d+x)^{2}$ in part (b)(i) <br> B1 for TP at $(-1.5, k)$ or $(k, 20.25) \mathrm{FT}$ their $20.25 \pm(\text { their } 1.5+x)^{2}$ or for $(-1.5$, 20.25) seen |
| iii) | $[y=] 34-11 x$ | 6 | B2 for $-3-2 x$ <br> or B1 for either $\mathrm{k} x-3, k \neq 0$ or $-2 x+\mathrm{n}$ or for $18-3-2 x$ <br> M1dep for gradient $=$ their $(-3-2(4))$ <br> B1 for $y$-value at $x=4$, is -10 <br> M1dep for their $-10=($ their -11$) 4+c$ oe |

Question 74

| (a)(i) | 9.5, 4.8 and 8.5 | 3 | B1 for each |
| :---: | :---: | :---: | :---: |
| (a)(ii) | correct curve | 5 | B4 for correct curve, but branches joined or touching y axis <br> or B3FT for 9 or 10 correct plots or B2FT for 7 or 8 correct plots or B1FT for 5 or 6 correct plots <br> AND <br> B1 indep two separate branches not touching or cutting $y$-axis |
| (b) | $\begin{aligned} & y=\frac{24}{5}-2 x \text { ruled } \\ & \text { and } \\ & -0.4 \text { to }-0.2 \text { and } 1.45 \text { to } 1.7 \end{aligned}$ | $4$ | B2 for correct ruled line crossing curve twice <br> or B1 for correct freehand or for short ruled line or for line with negative gradient through ( $0,4.8$ ) or for line with gradient - 2 B1 for each value |
| (c) | $\begin{aligned} & {[a=] 10} \\ & {[b=] 20} \\ & {[c=]-48} \end{aligned}$ | 4 | B3 for $10 x^{3}-15=48 x-20 x^{2}$ oe or better or $\mathbf{B 2}$ for 2 correct values or B1 for 1 correct value or for $5 x^{2}-\frac{15}{2 x}=24-10 x$ or better or for $2 x^{3}-3=\frac{48}{5} x-4 x^{2}$ or better or for $x^{3}-\frac{3}{2}=\frac{24}{5} x-2 x^{2}$ <br> After 0 scored SC 1 for correct elimination of a denominator of $5, x$ or $2 x$ from a four term expression. |

## Question 75



B1 for any positive cubic shape
B1 for sketch with one max and one min and with 3 roots including zero

If 0 scored, SC1 for $x(x+2)(x-2)$ soi

Question 76

$$
\begin{aligned}
& a=-12 \\
& b=5 \\
& k=-11
\end{aligned}
$$

6 B5 for 2 correct
OR
B2 for $3 x^{2}+a$ or $\mathbf{B 1}$ for $3 x^{2}$ isw

M1dep on at least B1 for their $\frac{\mathrm{d} y}{\mathrm{~d} x}=0$
M1dep on at least B1M1 for $x=2$ or $x=-2$
substituted in their $\frac{\mathrm{d} y}{\mathrm{~d} x}=0$ equation
M1 for $k=2^{3}+2 \times$ their $a+b$ and
$10-k=(-2)^{3}+(-2) \times$ their $a+b$
Question 77

| (a) | $1[.0] \quad 0.9$ | $\mathbf{2}$ | B1 for each |
| :--- | :--- | ---: | :--- |
| (b) | correct curve | $\mathbf{4}$ | B3 FT for 6 or 7 points <br> B2 FT for 4 or 5 points <br> B1 FT for 2 or 3 points |
| (c) | ruled line at $y=-1$ | B1 |  |
|  | 0.3 to 0.32 | B1 |  |

Question 78


| '(b)(ii) | $9-2 x-3 x^{2}=0$ oe | B3 | B2 for $9-2 x-3 x^{2}$ <br> or B1 for two correct terms <br> M1 for their derivative $=0$ or stating $\frac{\mathrm{d} y}{\mathrm{~d} x}=0$ |
| :---: | :---: | :---: | :---: |
|  | $\frac{--2 \pm \sqrt{(-2)^{2}-4 \times-3 \times 9}}{2 \times-3}$ oe <br> OR <br> $-\frac{1}{3} \pm \sqrt{\frac{9}{3}+\left(\frac{1}{3}\right)^{2}}$ oe | B2 | FT their derivative <br> B1FT for $\sqrt{(-2)^{2}-4(-3)(9)}$ or better or for $\frac{-(-2)+\sqrt{q}}{2 \times-3}$ or $\frac{-(-2)-\sqrt{q}}{2 \times-3}$ OR <br> B1 for $\left(x+\frac{1}{3}\right)^{2}$ |
|  | -2.10 and 1.43 final answer | B2 | B1 for each or for answers -2.1 or $-2.097 \ldots$ and 1.4 or 1.430 to 1.431 or SC1 for -2.097 ... and 1.43 [0] to 1.431 seen in working or for -1.43 and 2.10 as final answer |
| (b)(iii) | $\begin{aligned} & {[a=]-6} \\ & {[b=] 17} \end{aligned}$ | 3 | B2 for either $a$ correct or $b$ correct or for [ $a=$ ] -5.04 or -5.049 to -5.05 and [ $b=$ ] 16.9... seen or M1 for substitution of one of their solutions into $9+9 x-x^{2}-x^{3}$ oe or SC1 for reversed answers, $a=17$, $b=-6$ |

Question 79

| 3(a)(i) | $(-0.5,1)$ | 2 | B1 for each |
| :---: | :---: | :---: | :---: |
| (a)(ii) | $\binom{7}{-3}$ | 2 | B1 for each |
| (a)(iii) | 7.62 or 7.615 to 7.616 | 2 | FT their (a)(ii) <br> M1 for $(\text { their } 7)^{2}+(\text { their }-3)^{2}$ oe |
| (a)(iv) | $[y=]-4 x-1$ final answer | 3 | B2 for answer $-4 x+c$ [oe] or for correct equation in different form or for $-4 x+-1$ or for $-4 m-1$ <br> OR <br> M1 for $\frac{-5-7}{1--2}$ oe <br> M1 for correct substitution shown of $(-2,7)$ or $(1,-5)$ or their $(-0.5,1)$ into $y=($ their $m) x+c$ oe OR <br> M1 for $7=-2 m+c$ and $-5=m+c$ <br> A1 for $m=-4$ and $c=-1$ |
| ;(a)(v) | $[y=] \frac{1}{4} x+\frac{11}{4}$ final answer | 3 | M1 for grad $=\frac{1}{4}$ oe nfww soi, FT negative reciprocal of their gradient from (iv) <br> M1 for correct substitution shown of $(5,4)$ into $y=($ their $m) x+c$ oe or, if no substitution shown, $(5,4)$ satisfies their final linear equation. |

(b) $\quad 2 x^{2}+11 x-21[=0]$

M2 or M1 for $8-5 x=2 x^{2}+6 x-13$ oe or better

M2 Allow correct method to solve their quadratic equation e.g. formula, complete the square but not for $2 x^{2}+6 x-13$

M1 FT their equation
for $2 x(x+7)-3(x+7)[=0]$
or $x(2 x-3)+7(2 x-3)[=0]$
or $(2 x+a)(x+b)[=0]$
where $a b=-21$ or $2 b+a=11$
OR
M1 for $\sqrt{11^{2}-4 \times 2 \times-21}$
or for $\frac{-11+\sqrt{k}}{2 \times 2}$ or $\frac{-11-\sqrt{k}}{2 \times 2}$
OR
M1 for $\left(x+\frac{11}{4}\right)^{2}$
B2 B1 for one correct pair or for 2 correct $x$-values or 2 correct $y$-values

Question 80

| (a) | $3 x^{2}-2 k x$ | M2 | M1 for $3 x^{2}$ or $-2 k x$ |
| :---: | :---: | :---: | :---: |
|  | $\text { their } \frac{\mathrm{d} y}{\mathrm{~d} x}=6$ | M1 | Dep on at least M1 for derivative |
|  | $x=2 \text { substituted in their } \frac{\mathrm{d} y}{\mathrm{~d} x}$ | M1 | Dep on at least M1 for derivative |
|  | Correct working leading to 1.5 oe | A1 | A0 if any errors in working leading to 1.5 |
| (b) | $(0,1)(1,0.5)$ | 4 | B3 for $x=0$ and $x=1$ or for $(1,0.5)$ OR <br> M1 for their $\frac{\mathrm{d} y}{\mathrm{~d} x}=0$ <br> B1 for $3 x^{2}-3 x$ oe or better |
| (c) | correct sketch | 2 | with max on positive $y$-axis and min in 1st quadrant <br> B1 for positive cubic or for graph with one max which is on pos $y$-axis and one min which is in 1st quadrant |

Question 81

| (a) | 8.25 or $8.246 \ldots$ | 3 | M2 for $(3--5)^{2}+(2-4)^{2}$ oe or better or M1 for $(3--5)$ and $(2-4)$ oe seen |
| :---: | :---: | :---: | :---: |
| (b) | $[y=] 4 x+7$ | 5 | B1 for [midpoint] $(-1,3)$ soi M1 for [gradient of $l=] \frac{4-2}{-5-3}$ oe M1 for gradient $-1 /$ their $\left(-\frac{1}{4}\right)$ <br> M1dep on at least M1 for their $(-1,3)$ substituted into $y=$ their $m \times x+c$ oe |

(c) $\quad(0,-8)$ and $(0,16)$

|  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |

4 B3 for $(0,-8)$ or $(0,16)$ or for -8 and 16 OR
B2 for distance $=[ \pm] 12$ soi or M1 for $13^{2}-(5[-0])^{2}$ oe
B1 for both answers $(0, k), k \neq 0$ or 4

## ALT METHOD

B3 for $(0,-8)$ or $(0,16)$ or for -8 and 16 OR
M2 for $y^{2}-8 y-128[=0]$ or for $(y-4)^{2}=144$ or better or M1 for $13^{2}=(-5-0)^{2}+(4-y)^{2}$ oe

B1 for both answers $(0, k), k \neq 0$ or 4

Question 82

| i(a)(i) | -3 | 1 |  |
| :---: | :---: | :---: | :---: |
| (a)(ii) | $\begin{aligned} & -1 \\ & 1.55 \text { to } 1.6 \\ & 4.4 \text { to } 4.45 \end{aligned}$ | 3 | B1 for each |
| (a)(iii) | -8 | 1 |  |
| (a)(iv) | Ruled line through origin intersecting curve once | 2 | B1 for ruled line through origin |
| i(b)(i) | 18 | 3 | B2 for $6 x-12$ <br> or $\mathbf{B 1}$ for $6 x$ or -12 |
| (b)(ii) | $(2,-5)$ | 2 | B1 for each. <br> If 0 scored, M1 for their $6 x-12=0$ or states $\frac{d y}{d x}=0$ |
| (c) | $\begin{aligned} & {[p=] 7} \\ & {[q=] 3} \end{aligned}$ | 2 | B1 for each |

## Question 82

| (a)(i) | Correct sketch of $3 x-4 y=12$ with <br> $y=-3$ and $x=4$ indicated on axes |  | $\mathbf{2}$ |
| :--- | :--- | :--- | :--- |

Question 83

| (a) | $(22,11)$ | 2 | B1 for each value |
| :---: | :---: | :---: | :---: |
| (b) | $\frac{\text { their } 11-3}{\text { their } 22-2}$ oe or better | M1 |  |
|  | $-\frac{1}{\text { their } m}$ | M1 |  |
|  | Substitution of $(12,7)$ into $y=($ their $m) x+c$ | M1 | Accept $y-7=$ their $m(x-12)$ oe |
|  | leading to $2 y+5 x=74$ final answer | A1 | Without error or omission |
| (c) | 32 | 1 |  |
| (d) | 145 | 2 | $\begin{aligned} & \text { M1 for } \frac{1}{2} \times(\text { their } 32-3) \times 10 \text { oe } \\ & \text { or } \\ & \frac{1}{2} \times \sqrt{(7-3)^{2}+(12-2)^{2}} \times \sqrt{(\text { their } 32-7)^{2}+(2-12)^{2}} \text { oe } \end{aligned}$ |

## Question 84

$6 x+4$

## Question 85



## Question 86

| (a)(i) | 28 | $\mathbf{1}$ |  |
| :--- | :--- | ---: | :--- |
| (a)(ii) | Correct curve | $\mathbf{4}$ | B3FT for 9 or 10 correct points <br> or B2FT for 7 or 8 correct points <br> or B1FT for 5 or 6 correct points |
| a)(iii) | 2.5 to 2.88 .2 to 8.5 | $\mathbf{2}$ | B1 for each value |
| (b)(i) | $2 x^{2}+4 x(9-x)$ oe | M1 | Accept the sum of individual areas if done in smaller parts |
|  | $2 x^{2}+36 x-4 x^{2}$ oe <br> Leading to $36 x-2 x^{2}$ | $\mathbf{A 1}$ | With intermediate step shown and brackets removed with no <br> errors or omissions |
| (b)(ii) | 144 | $\mathbf{3}$ | $\mathbf{B 1}$ for $x=6$ identified from graph or using calculus |

## Question 87

| )(a)(i) | 4 or 5 or 7 or 8 or 9 | 1 |  |
| :--- | :--- | ---: | :--- |
| (a)(ii)    <br> )(b) $[a=] 3,[b=] 10$ $\mathbf{2}$ B1 for each or for $a$ and $b$ transposed <br>  $6 x^{5}-30 x^{4}$ B2 B1 for $6 x^{5}$ or $-30 x^{4}$ <br>  their derivative $=0$. M1  <br>  $(0,0)$ and $(5,-3125)$ B2 B1 for $(5,-3125)$ or for $x=0$ and $x=5$ |  |  |  |

## Question 88



Question 89

| $5(\mathrm{a})$ | $y=4$ oe | 1 |  |
| :---: | :---: | :---: | :---: |
| j(b) | $[y=]-\frac{1}{2} x+4 \quad$ final answer | 2 | B1 for grad $=-\frac{4}{8}$ oe soi or $[y=] k x+4$ |
| 'c)(i) | $\text { Gradient }=\frac{-1}{\text { their gradient in }(b)}$ | M1 | Accept e.g. $2 \times-\frac{1}{2}=-1$ oe or states negative reciprocal of $-\frac{1}{2}=2$ |
|  | Substituting ( 2,3 ) in their equation. | M1 | $3=2 \times$ their $m+c$ |
|  | leading to $y=2 x-1$ | A1 | No errors or omissions |
| c)(ii) | 3.35 or 3.354... | 5 | B2 for $\left(\frac{1}{2}, 0\right)$ soi or $x$-coordinate of $D=\frac{1}{2}$ or M1 for $2 x-1=0$ <br> M2 for $\left(2-\text { their } \frac{1}{2}\right)^{2}+(3-\text { their } 0)^{2}$ oe or M1 for $\left(2-\right.$ their $\left.\frac{1}{2}\right)$ and (3-their 0$)$ oe |

Question 90

| (a) | $4 x^{3}-16 x$ cao | 2 | M1 for $4 x^{3}+k x$ or $k x^{3}-16 x$ or $4 x^{3}-16 x+k$ or $4 x^{3}-16$ as final answers |
| :---: | :---: | :---: | :---: |
| ! b | Their $\frac{\mathrm{d} y}{\mathrm{~d} x}=0$ or stating $\frac{\mathrm{d} y}{\mathrm{~d} x}=0$ | B1 |  |
|  | Correct method to solve their $4 x^{3}-16 x=0$ | M1 | e.g. $4 x\left(x^{2}-4\right)$ or $4 x(x-2)(x+2)$ oe |
|  | $[x=] 0,-2,2$ | A1 | Or B1 for $(-2,-11)$ and $(2,-11)$ |
|  | $(0,5)(-2,-11)(2,-11)$ | A1 |  |
| (c) | $(0,5)$ with correct reasoning | 2 | M1 for any of <br> - correct use of $2^{\text {nd }}$ derivative $12 x^{2}$ $-16$ <br> - evaluates correctly both values of $y$ on either side <br> - evaluates correctly the gradient on either side <br> - reasonable correct sketch |

## Question 91

| (a) | 10 | 3 | M2 for $(1--7)^{2}+(4--2)^{2}$ oe or M1 for ( $1--7$ ) or (4--2) oe |
| :---: | :---: | :---: | :---: |
| (b) | $\frac{4}{3} \text { or } \frac{8}{6}$ | 2 | M1 for $\frac{1--7}{4--2}$ oe |
| (c) | $y=-\frac{3}{4} x-\frac{9}{4}$ <br> or $4 y+3 x+9=0$ oe final answers | 4 | B3 for $-\frac{3}{4} x-\frac{9}{4}$ OR <br> B1 for midpoint $(1,-3)$ <br> M1 for gradient $-\frac{3}{4}$ or $-\frac{1}{\text { their (b) }}$ <br> M1 for substituting their $(1,-3)$ into $y=($ their $m) x+c$ or for their $m=\frac{y--3}{x-1}$ oe |

Question 92

| (a) | $-2.5-2 \quad-1$ | $\mathbf{3}$ | B1 for each |  |
| :--- | :--- | :--- | ---: | :--- |
| (b) | Correct curve | $\mathbf{4}$ | B3 FT for 8 or 7 correct plots <br> B2 FT for 6 or 5 correct plots <br> B1 FT for 4 or 3 correct plots |  |
|  |  |  |  |  |

Question 93
(a)(i)
\(\left.\left|\begin{array}{l|}Correct expansion of a pair of brackets <br>
x^{2}-4 x+[1] x-4 <br>
or x^{2}-4 x-2 x+8 <br>

or x^{2}+[1] x-2 x-2\end{array}\right|\)| $x^{3}-4 x^{2}+x^{2}-4 x-2 x^{2}+8 x-2 x+8$ |
| :--- |
| leading to and stating |
| $[y=] x^{3}-5 x^{2}+2 x+8$ | \right\rvert\,

M1 accept
$x^{2}-3 x-4$
or $x^{2}-6 x+8$
or $x^{2}-[1] x-2$
A1 Accept
$x^{3}-3 x^{2}-4 x-2 x^{2}+6 x+8$
or $x^{3}-6 x^{2}+[1] x^{2}+8 x-6 x+8$
or $x^{3}-[1] x^{2}-2 x-4 x^{2}+4 x+8$
leading to and stating
$[y=] x^{3}-5 x^{2}+2 x+8$
(a)(ii)

Correct labelled sketch positive cubic
Crossing $x$-axis at $-1,2$ and 4 only
Crossing $y$ - axis at 8 only


B1 for positive cubic
B2 for three intercepts only with $x$-axis labelled at -1,2 and 4
or B1 for 1 or 2 correctly labelled $x$ - intercepts
B1 for a single intercept on $y$-axis labelled at 8 but not if line $y=8$
(b)

|  |
| :--- |
| $3 x^{2}-10 x-8[=0]$ |
|  |
| $x=4$ and $x=-\frac{2}{3}$ |
| $(4,0)$ and $\left(-\frac{2}{3}, \frac{112}{27}\right)$ oe |
| $[y=] 10 x-40$ <br> and <br> $[y=] 10 x+\frac{292}{27}$ |

M3 $\mid$ B2 for derivative $=3 x^{2}-10 x+2$ isw OR
B1 for derivative with $3 x^{2}$ or $-10 x$ given in expression isw
M1dep on B1 for their first derivative $=$ 10

## B1 for each

or for two different equations of the form
$[y=] 10 x+c$ ( $c$ must be numeric)
or for $c=-40$ and $\frac{292}{27}$

Question 94

| i(a) | $\begin{aligned} & (5,2) \\ & (2,-2) \end{aligned}$ | 4 | B3 for 3 correct values or answers for $C$ and $D$ reversed or correct coordinates given on diagram wrongly labelled or B2 for one correct coordinate pair correctly labelled or M2 for $A, B, C$ and $D$ correctly plotted or M1 for $A$ and $B$ correctly plotted <br> If 0 or 1 scored instead award SC2 for answers $(-3,8)$ and $(-6,4)$ or answers $(1.5,1.5)$ and $(-2.5,4.5)$ |
| :---: | :---: | :---: | :---: |
| (b)(i) | $(2.5,3.5)$ oe | 2 | B1 for each |
| (b)(ii) | 7.07 or 7.071... | 3 | M2 for $(6--1)^{2}+(4-3)^{2}$ oe or M1 for $(6--1)$ or $(4-3)$ oe |
| b)(iii) | $\frac{1}{7}$ | 2 | M1 for $\frac{4-3}{6--1}$ oe |
| b)(iv) | $y=\frac{1}{7} x-\frac{2}{7}$ or $7 y=x-2$ oe final answer | 3 | $\mathbf{M 1}$ for gradient $=$ their (iii) <br> M1dep for substituting $(2,0)$ in a linear equation with their $m$ allow if $(2,0)$ satisfies $\mathrm{y}=($ their $(\mathbf{b})($ (iii) gradient) $x+c$ |

