Extended Mathematics

Topic :Graph

Year :May 2013 -May 2023

Paper - 4

Answers

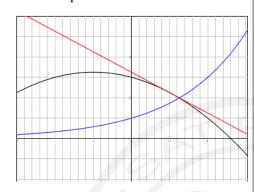
(a)	-5.04, 1.75, 0	3	B1 for each correct value
(b)	Fully correct curve	5	B3FT for 10 correct plots from their (a) B2FT for 8 or 9 correct plots or B1FT for 6 or 7 correct plots and SC1 for two branches not joined
(c)	-1.6 to -1.5 -0.4 to -0.3 1.8 to 1.9	1 1 1	RED
(d)	-2.6 to -2.5 www -0.4 to -0.3	1 1 1	After 0 scored, M1 for $y = 2x - 2$ drawn
(e)	3.25 to 4.25 with correct tangent	3	B1 for correct tangent
			B2 for answer in range dep on close attempt at tangent
	334.5	atp	M1dep for $[-]$ $\frac{\text{rise}}{\text{run}}$ used with values soi from tangent, dep on correct or close attempt at tangent

(a)	0, 2, 0, -3	3	B2 for 3 correct or B1 for 2 correct
(b)	Correct curve	B4	B3FT for 8 points B2FT for 7 or 6 points B1FT for 5 or 4 points
(c)	y = -1 indicated	B1	e.g. Could be mark[s] on curve
	x = 1.3 to 1.4 and 4.1 to 4.2	B1	isw other lines if not clearly used
(d) (i)	line drawn from (0, 2) to touch curve	M1	No daylight at point of contact If short, must cross at (0, 2) within ½ small square when extended
	(2.5 to 2.75, 3 to 3.4)	A1	REI
(ii)	rise/run e.g. (their y – 2)/their x	M1	dep on attempt at a tangent from (0, 2) in (d)(i) and uses scales correctly Can be implied from answer—check on tangent for their rise for a run of 1 (½ small square)
	0.4 to 0.48	A1	ww2 dep on attempt at a tangent from (0, 2) in (d)(i)

(a) 3, 0.33[3...], 1

(b) Correct quadratic curve

Correct exponential curve



(c) (i) Answer in range 1.2 < x < 1.4

(ii) Answer in range 1.2 < x < 1.35

(iii) Answer in range 0.55 < x < 0.7

(d) Correct tangent drawn And answer in range -2.5 < m < -1.5 3 B1 for each correct value

3 B2FT for 7 correct points or

B1FT for 5 or 6 correct points

3 B2FT for 7 correct points

B1FT for 5 or 6 correct points

Not from a line other than y = 4 (± 1 mm)

B1 for correct tangent at x = 0.5

B2 for answer in range dep on close attempt at tangent

M1 for $[-]\frac{rise}{run}$ used with values soi

from tangent, dep on close attempt at tangent or answer in range $1.5 \le m \le 2.5$

or

1

1

1

3

SC1 for close attempt at tangent to exponential curve and answer in the range 1.6 < m < 2.2

(i)
$$\frac{3}{2}$$
 or 1.5

(ii)
$$y = \frac{3}{2}x + 2$$
 oe

Question 5

- (a) 7, 11.5, 4.5
- **(b)** Correct curve cao

- (c) (i) 0.69 < x < 0.81
 - (ii) -2.3 < x < -2.2 -0.8 < x < -0.60.35 < x < 0.5
- (d) (i) y = 10 3x ruled correctly

$$-0.55 < x < -0.45$$

 $0.35 < x < 0.45$

(ii)
$$10 \quad 1 \quad -2$$
 or $-10 \quad -1 \quad 2$

2 M1 for
$$\frac{14-(-4)}{8-(-4)}$$
 oe

1,1,1

3

B2

B1dep

2 B1 for
$$y = their \frac{3}{2}x + c$$
 o.e.
or $y = mx + 2, m \neq 0$
SC1 for $\frac{3}{2}x + 2$

B3FT for 10 correct plots, on correct vertical grid line and within correct 2 mm square vertically

Or B2FT for 8 or 9 correct plots
Or B1FT for 6 or 7 correct plots
and B1 indep for two separate branches on either side of y-axis

B1 for each correct

After 0 scored, allow SC1 for drawing line y = 7.5 long enough to cross curve at least once

long enough to cross curve twice.

B1 for ruled line gradient -3 or y intercept at 10 but not y = 10

Or B1 for 'correct' but freehand

- B1dep Dependent on at least B1 scored for line
 - After 0 scored, **SC2** for -0.5 and 0.4 [from solving equation]
- 3 **B2** for $2 x 10x^2$ [= 0] oe
 - Or **B1** for $\frac{2}{x^2} \frac{1}{x} 10 = 0$ oe Correctly eliminating -3xOr **B1** for $2 - x - 3x^3 = 10x^2 - 3x^3$ oe Correctly clearing fractions

(i) 1.4 to 1.6

1

(ii) 1.15 to 1.25

1

(iii) -1

1

3

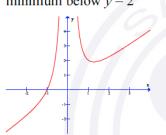
(iv) -2.25 to -2.1-0.9 to -0.752.2 to 2.35 **B2** for 2 correct or **B1** for one correct or **B1** for y = x drawn ruled to cut curve 3 times

Question 7

(a)

(b)

- 0, 4.5, 3.11[1...]
- Complete correct curve with minimum below y = 2



- 0.5 to 0.6 0.6 to 0.7 2.8 to 2.9
- Correct line or no line and -0.7 to -0.6 nfww

3 B1, B1, B1

5

1

1

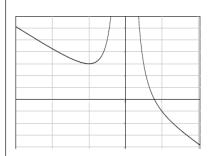
1

3

- B3 FT for 9 points correctly plotted
 B2 FT for 7 or 8 points correctly plotted
 or B1 FT 5 or 6 points correctly plotted
 - **and B1 indep** two separate branches not touching or cutting *y*-axis
 - if 0 SC1 for y = 3 indicated
 - Must check line not if wrong line **B2** for y = 1 x ruled correctly
 - or SC1 for ruled line with either gradient -1 or y-intercept 1 but not line y = 1 or correct freehand line

(a) 3, 3, -1

(b) Complete correct curve



(c) 0.5 to 0.6

(d) Correct line and 0.4 to 0.5 or no line and 0.4 to 0.5 nfww

(e) (i) Tangent at x = -1.5

(ii) -2 to -1

3 B1 B1 B1

2

5 B3FT 11 points or B2FT for 9 or 10 points or B1FT for 7 or 8 points

And Blindep two separate branches not touching or crossing *y*-axis

Must check line - not if wrong line **B2** for y = 2x + 3 ruled correctly **or SC1** for correct freehand line **or** ruled line with either gradient 2 or y-intercept 3 but not y = 3

No daylight at x = -1.5. Consider point of contact as midpoint between two vertices of daylight, the midpoint must be between x = -1.7 and -1.3

Dependent on tangent mark awarded
Allow integer/integer if in range
Or M1 for rise/run also dep on any tangent
drawn or close attempt at tangent at any point
Must see correct or implied calculation from a
drawn tangent

(a)	2.125 and 2.375	2	B1 for one correct value
(b)	Correct curve	B4	B3FT for 11 correct plots or B2FT for 9 or 10 correct plots or B1FT for 7 or 8 correct plots
(c)	Ruled tangent at $x = 2$	B1	No daylight at $x = 2$. Consider point of contact as midpoint between two vertices of daylight, this must be between $x = 1.8$ and 2.2
	Gradient from 7.8 to 10.2	PR	Dep on B1 awarded Allow integer/integer or a mixed number if within range or M1 dep for (change in y) ÷ (change in x) Dependent on any tangent drawn or close attempt at a tangent at any point Must see correct or implied calculation from a drawn tangent
(d)	0 and -1.75 to -1.65 and 1.65 to 1.75	2	B1 for two correct values
(e)	-1.2 to -0.8 < k < 2.8 to 3.2	2	B1 for each correct or SC1 for reversed answers

(i)
$$y = 3x - 1$$
 cao final answer

3 M1 for gradient = $\frac{8 - 4}{3 - 1}$ oe and M1 for substituting (3, 8) or (-1, -4) into their $y = 3x + c$ or for finding y-intercept is -1

(a)	-3, 7.375, 8.875	1, 1, 1	Accept 7.4 or 7.37 or 7.38 for 7.375 and 8.9 or 8.87 or 8.88 for 8.875
(b)	Correct curve	4	B3FT for 8 or 9 correct plots B2FT for 6 or 7 correct plots B1FT for 4 or 5 correct plots Point must touch line if exact or be in correct square if not exact (including boundaries)
(c) (i)	Any integer less than 7 or greater than 10	1	
(ii)	7, 8 or 9	1	
(d)	y = 15x + 2 ruled and fit for purpose	B2	B1 for short line but correct or freehand full length correct line or for ruled line through $(0, 2)$ (but not $y = 2$) or for ruled line with gradient 15 (acc ± 1 mm vertically for 1 horizontal unit)
	-1.45 to -1.35 and 0.4 to 0.5	B2	B1 for each
(e)	Tangent ruled at $x = 1.5$	B1	No daylight at point of contact. Consider point of contact as midpoint between two vertices of daylight, the midpoint must be between $x = 1.4$ and 1.6
	7 to 12	2	Dep on B1 or close attempt at tangent at $x = 1.5$ M1 for $y - \text{step}/x - \text{step}$ for their tangent

(a)	2 0 -2 2	3	B2 for 3 correct B1 for 2 correct
(b)	smooth correct curve through correct points	4	B3FT for 8 or 9 correct plots B2FT for 6 or 7 correct plots B1FT for 4 or 5 correct plots FT their table
(c)	line $y = \frac{1}{2}(x+1)$ ruled <u>and</u> -2.85 to -2.95		Line must be fit for purpose
	-1 0.85 to 0.95	4	B3 for correct line and 2 correct values or B2 for correct line and 1 correct value or B1 for correct line or SC2 for no/wrong line and 3 correct values or SC1 for no/wrong line and 2 correct values
(d)	tangent ruled	B1	No daylight between tangent and curve at point of contact. Consider point of contact as midpoint between two vertices of daylight, the midpoint must be between $x = -1.85$ and $x = -1.65$
	-1.1 to -1.5	tore	dep on B1 M1 for rise/run also dep on any tangent drawn or close attempt at tangent at any point Must see correct or implied calculation from a drawn tangent Accept M1 for answer in range 1.1 to 1.5 after B1

(a)	1 3 2.5	1 1 1	
(b)	Fully correct graph	5	B3FT for 11, 12 points correct or B2FT for 9, 10 correct points or B1FT for 7, 8 correct points B1 for branch each side of y-axis and not touching y-axis SC4 for correct graph but branches joined
(c)	-2.6 to -2.4	1	
(d)	Correct ruled line fit for purpose –1.0 to –1.3	2	SC1 for ruled line through $(0, 1)$ but not $y = 1$ or ruled line with gradient -1 or for correct line but freehand
(e)	Correct tangent and 0.9 ≤ grad ≤ 1.5	3 pre	Consider point of contact as midpoint between two vertices of daylight, the midpoint must be between $x = -3.4$ and -2.6 B2 if close attempt at correct tangent and answer in range (may be small amount of daylight) or B1 for ruled tangent at $x = -3$ within tolerance, no daylight at the point of contact and M1 (dep on B1 or close attempt at tangent) for a tangent at any point and $\frac{rise}{run}$ used

(a)	2 and 7	2	B1 for each value
(b)	Complete correct curve	5	B3 FT for <i>their</i> 9 or 10 points or B2 FT for <i>their</i> 7 or 8 points or B1 FT for <i>their</i> 5 or 6 points and B1 independent for one branch on each side of the <i>y</i> -axis and not touching the <i>y</i> -axis SC4 for correct curve with branches joined
(c)	Correct tangent and -13 ≤ grad ≤ -8	3 P	B2 for close attempt at tangent at $x = 1$ and answer in range OR B1 for ruled tangent at $x = 1$, no daylight at $x = 1$ Consider point of contact as midpoint between two vertices of daylight, the midpoint must be between $x = 0.8$ and 1.2 and M1 (dep on B1 or close attempt at tangent [at any point] for $\frac{rise}{run}$
(d) (i)	5 to 6	1	
(ii)	2 to 2.35 and -2.55 to -2.35	2FT	FT their k B1FT for each correct solution
(e)	[a =] -5 [b =] -1 [c =] 12	3 Itpr	B2 for two correct values or for $x^3 - 5x^2 - x + 12$ [= 0] oe or M1 for $x^2 - 2x + \frac{12}{x} = 3x + 1$

Question	14		
(a)	1.5 1.25 -0.75 0.5	4	B1 for each
(b)	Fully correct curve	5	B5 for correct curve over full domain or B3 FT for 11 or 12 points or B2 FT for 9 or 10 points or B1 FT for 7 or 8 points and B1 independent for one complete branch on each side of the y-axis and not touching or crossing the y-axis SC4 for correct curve with branches joined
(c)	-1.35 to -1.25	1	
	-0.27 to -0.251	1	
	1.51 to 1.55	1	
(d)	<i>k</i> < 1.2 or 1.15 to 1.25	2	SC1 for 1.15 to 1.25 seen or horizontal line drawn at min point
(e)	tangent ruled at $x = -1$	B1	No daylight at $x = -1$ Consider point of contact as midpoint between two vertices of daylight, the midpoint must be between $x = -1.1$ and -0.9
	-1.7 to -1.3	ore	dep on B1 or a close attempt at tangent at $x = -1$ or M1 for rise/run also dep on any tangent drawn or close attempt at tangent at any point. Must see correct or implied calculation from a drawn tangent

(a)	-1.5, 0.5	2	B1, B1
(b)	Correct curve	5	B3 FT for 10 or 11 points or B2FT for 8 or 9 points or B1FT for 6 or 7 points and B1 independent for two branches SC4 for correct curve but branches joined
(c)	1.25 to 1.35	1	
(d)	-1	1	
(e) (i)	2-x	1	D.
(ii)	Ruled line with gradient –1 through (0, 2) and fit for purpose	2FT	SC1 for ruled line, with gradient -1 or through $(0, 2)$, but not $y = 2$ FT their $y = mx + c$ from (e)(i), if $m \ne 0$ SC1FT for ruled line either with correct
	1.15 to 1.25 cao	1	gradient or through $(0, c)$, but not $y = c$
Question	16		
	0.605.0055		

(a)	0 4 0.625 0.875	1,1,1,1	
(b)	Fully correct smooth curve	4	B3 FT for 8 or 9 points or B2 FT for 6 or 7 points
(c)	line $y = x+1$ ruled and 0.2 to 0.3 and 1.8 to 1.95	1t3	or B1 FT for 4 or 5 points Line must be fit for purpose ie at least from $x = 0$ to $x = 2$ B2 for correct line and 1 correct value or B1 for correct line or SC1 for no/wrong line and 2 correct values

(d)	Tangent ruled at $x = -1.5$	B1	No daylight between tangent and curve at point of contact. Consider point of contact as midpoint between two vertices of daylight, the midpoint must be between $x = -1.6$ and $x = -1.4$
	2.2 to 5	2	dep on B1 M1 for $\frac{rise}{run}$ also dep on any tangent drawn or close attempt at tangent at any point Must see correct or implied calculation from a drawn tangent

(i)	0.25 oe and 1	2	B1 for each
(ii)	Correct curve	4	B3FT for 6 or 7 correct plots or B2FT for 4 or 5 correct plots or B1FT for 2 or 3 correct plots
(iii)	2.3	1FT	Correct or FT where $y = 5$ on <i>their</i> graph
(iv)	y = 3x - 1 oe 3 term equation	3	B2 for $3x - 1$ or $y = 3x$ [+ c]oe or for $m = 3$ and $c = -1$ or M1 for [gradient =] $\frac{8-2}{3-1}$ oe soi by $3x$ and M1 for substitution of $(1, 2)$ or $(3, 8)$ into their $y = mx + c$
(v)	-1.7 to -1.5 and 2	2 ore	B1 for either or M1 for $y = x + 2$ seen or drawn

(a)	3.5[0] 1.94 3.11	3	B1 for each
(b)	Fully correct curve	5	B3 FT for 10 or 11 points or B2 FT for 8 or 9 points or B1 FT for 6 or 7 points B1 indep two separate branches not touching or cutting y-axis
(c)	-0.7 to -0.6	1	SC4 for correct curve, but branches joined

(d) (i)	– 1	1	
	2.5	1	If 0,0, M1 for $y = 2.5 - x$ oe seen in working
(ii)	-0.6 to -0.5 with correct ruled line	3	B2FT for drawing <i>their</i> ruled line from (d)(i)
			or M1 for ruled line through (0, 2.5)FT or gradient –1 FT
(e)	Correct tangent and $0.5 \leq \text{grad} \leq 0.85$	3	B2 for close attempt at tangent at $x = 2$ and answer in range OR
			B1 for ruled tangent at $x = 2$, no daylight at $x = 2$
			Consider point of contact as midpoint between two vertices of daylight, the midpoint must be between $x = 1.8$ and 2.2
			and M1 (dep on B1 or close attempt at tangent
			[at any point] for $\frac{rise}{run}$
Question	19		
(a) (i)	10	1	

(a) (i)	10	1	
(ii)	-3.4 to -3.3 and -0.4 to -0.3 and 1.6 to 1.7	3	B1 for each
(iii)	y = -2.3 to -2.1 oe y = 10 to 10.1 oe	2	B1 for each
(b) (i)	2, -1, 4	3	B1 for each
(ii)	Fully correct curve drawn	4	SC3 for correct curves but branches joined or touching <i>y</i> -axis
	7. sat	ore'	or B2FT for 8 or 9 correct plots or B1FT for 6 or 7 correct plots
			and B1 indep for two separate branches not touching or crossing <i>y</i> - axis
(iii)	−3.4 to −3.2 and 1.8 to 1.9	2	B1 for each
(c)	3.2 oe	2FT	FT $2 \div their$ (a)(i) + 3 M1 for f(-2) = 10 or their (a)(i) used
(d)	1	1	

(a)	19[.0] or 18.97 nfww		M2 for $\sqrt{(4-2)^2 + (13-5)^2}$ oe or M1 for $(4-2)^2 + (13-5)^2$ oe
		l	l

(b)
$$[y =] 3x + 1$$

$$\mathbf{B2} \text{ for answer } [y =] 3x + c \text{ oe or answer } kx + 1 \ (k \neq 0)$$
or $\mathbf{M1} \text{ for } \frac{13 - 5}{4 - 2} \text{ oe or 3}$

$$\mathbf{and } \mathbf{M1} \text{ for correct substitution of } (-2, -5)$$
or $(4, 13) \text{ into } y = (their m)x + c \text{ oe}$

(c)
$$y = 3x - 5$$
 oe FT their gradient from (b) M1 for $y = mx - 5$ with other $m, m \ne 0$ or $y = \{their \text{ gradient from (b)}\}x + c$

(d)
$$y = -\frac{1}{3}x + \frac{13}{3}$$
 oe isw
5 B2FT for $-\frac{1}{3}x + c$ (c can be numeric or

algebraic)

FT –1/ their gradient from (b)

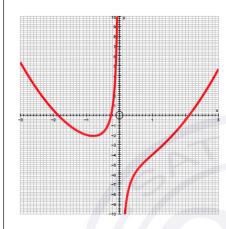
FT -1/ their gradient from (b) or M1 for -1/ their gradient from (b) soi

and B1 for [midpoint of AB = (1, 4)

and M1 for substitution of (1, k) or (k, 4) into a linear equation

(a) (i)
$$-2, -0.5 \text{ or } -\frac{1}{2}$$

(ii) Complete correct curve



(c) Any integer
$$k$$
 where $k \le -3$

(d) (i) Correct line y = -5x - 2 ruled and -0.4 to -0.2 0.55 to 0.75

(ii)
$$[a =] 5 \text{ and } [b =] - 2$$

2 B1 for each

5

SC4 for correct curves but branches joined or touching *y*-axis or B3FT 9 or 10 points or B2FT for 7 or 8 points or B1FT for 5 or 6 points

and **B1indep** two separate branches not touching or crossing *y*-axis

3 B1 for each

1

4

2

M2 for correct ruled line or M1 for correct line but freehand or for ruled line gradient – 5 or ruled line y-intercept – 2, but not y = -2and A1 for each correct solution dependent on at least M1

If 0 scored, SC1 for both correct with no line drawn

B1 for one correct value or **M1** for $x^3 + 5 x^2 - 2x - 1 = 0$ seen

(b) (i)

(a) (i)
$$-\frac{1}{2}x + 2$$
 oe

(ii)
$$\frac{16}{a^2} \left[+ \frac{0^{[2]}}{b^2} \right] = 1 \text{ or } \frac{4^2}{a^2} \left[+ \frac{0^{[2]}}{b^2} \right] = 1$$
and $a^{[2]} = 4^{[2]}$

$$\left[\frac{0^{[2]}}{a^2} \right] + \frac{4}{b^2} = 1 \text{ or } \left[\frac{0^{[2]}}{a^2} \right] + \frac{2^2}{b^2} = 1$$

SC2 for
$$y = -\frac{1}{2}x + c$$
 oe
or SC1 for $y = kx + 2$ oe, $k \ne 0$
or
M1 for [gradient =] $\frac{-2}{4}$
and M1 for substituting (4, 0) or (0, 2)
into $y = (their \ m)x + c$

$$\left[\frac{0^{[2]}}{a^2}\right] + \frac{4}{b^2} = 1 \text{ or } \left[\frac{0^{[2]}}{a^2}\right] + \frac{2^2}{b^2} = 1$$
and $b^{[2]} = 2^{[2]}$

1.73 or 1.732.. or $\sqrt{3}$

3 or M1 for
$$\frac{k^2}{4} = \frac{3}{4}$$
 or better or M1 for $\frac{2^2}{16} + \frac{k^2}{4} = 1$ oe

1

2FT

M2 for
$$2 \times \tan^{-1} \left(\frac{their\sqrt{3}}{2} \right)$$
 oe
or M1 for $\tan = \frac{their\sqrt{3}}{2}$ oe

(c) (i)
$$8\pi$$
 final answer

(ii) 72π final answer

FT their (c)(i) × 9 in terms of
$$\pi$$

M1 for area factor of 3^2 or 9
or [new a] = 12, [new b] = 6

(a)	9 10.5	1 1	
(b)	Fully correct curve	5	SC4 for correct curve, but branches joined
			B3 FT for 9 or 10 points plotted or B2 FT for 7 or 8 points plotted or B1 FT for 5 or 6 points plotted
			and B1 for two separate branches not touching or cutting <i>y</i> -axis
(c)	2.1 to 2.6	1	
	8.5 to 9	1	
(d)	2, 3, 5, 7	2	SC1 for correct 4 values and no more than one extra positive integer or ± 2 , ± 3 , ± 5 , ± 7 or 3 correct values and no extras
(e)	(-2, -12)	1	
(f) (i)	$20 + x^2 = x^3$	M1	Multiplication by <i>x</i>
	$x^3 - x^2 - 20 = 0$	A 1	No errors or omissions
(ii)	Fully correct curve $y = x^2$	2	SC1 for U – shaped parabola, vertex at origin
(iii)	2.5 to 3.5	1	
(iv)	3.[0] to 3.1 or FT their answer to (iii)	1FT	FT dep on (iii) > 0

(a)	-4.5 and 10.5	2	B1 for each value
(b)	Correct curve	5	B4 for correct curve with branches joined OR B3 FT for 9 or 10 points or B2 FT for 7 or 8 points or B1 FT for 5 or 6 points and B1 independent for one branch on each side of the y-axis and not touching or crossing the y-axis

(c)	5	1	
(d) (i)	Line $y = 15 - 3x$ ruled and -0.4 to $-0.310.35$ to $0.452.2$ to 2.3	4	B3 for correct line and 2 correct values or B2 for correct line or M1 for ruled line with gradient –3 or through (0, 15) or SC2 for no/wrong line and three correct values or SC1 for no/wrong line and two correct values or for correct freehand line
(ii)	[a=] 6 [b=] -14 [c=] 0	3	B2 for $6x^3 - 14x^2 + 2 = 0$ oe or M1 for correct removal of denominator or collection of terms on one side
Question	25		
(a)	1	1	

(a)	1	1 1	
(b)	Fully correct graph	4	B3FT for 6 or 7 points plotted or B2FT for 4 or 5 points plotted or B1FT for 2 or 3 points plotted
(c) (i)	-1 < ans < -0.8 1.25 < ans < 1.45 2.5 < ans < 2.6	1 1 1	
(ii)	-0.7 < ans < -0.5	2	M1 for evidence of $y = -x$ or $\frac{x^3}{3} - x^2 + 1 = -x$
(d) (i)	y = 1 to 1.1 oe	1FT	FT only if a clear maximum point
	y = -0.4 to -0.33 oe	1FT	FT only if a clear minimum point
(ii)	−0.4 to −0.33 oe	1FT	Correct or FT their graph

(-)						
~	Question 26					
(a)	0.92,, 0.5, -1,, -1, 0.5,, 0.92	3	B2 for 4 or 5 correct or B1 for 2 or 3 correct			
(b)	Fully correct graph	5	B4 for correct graph but branches joined OR B3FT for 11 or 12 correct points or B2FT for 9 or 10 correct points or B1FT for 7 or 8 correct points			

(c) (i)	Correct ruled line through $(-2, 1)$ and $(2, -3)$	2	Blindep for a branch on each side of the y-axis, without touching it Bl for straight line with gradient -1 or cutting y-axis at -1 or correct line but freehand or short correct ruled line
(ii)	0.7 to 0.95	1	
(iii)	[p =] 2 and $[q =] - 2$	3	B2 for $x^3 + 2x^2 - 2 = 0$ oe
			or B1 for $x^2 - 2 = -x^3 - x^2$ oe or better
			or $1+1-\frac{2}{x^2}+x = 0$ or better
(d) (i)	(1.3 to 1.6, 0)	1	RAIN
(ii)	Ruled line from $(0, -2)$ to intersection of <i>their</i> graph with positive <i>x</i> -axis	1FT	
(iii)	Tangent [to curve] A or (1.3 to 1.6, 0)	1	
Question	27		
(a)	0 0.5 oe 1.25 oe		1, 1, 1

l			
Question 27			
(a)	0 0.5 oe 1.25 oe	1, 1, 1	
(b)	Fully correct smooth curve	4	B3 FT for 7 or 8 points or B2 FT for 5 or 6 points or B1 FT for 3 or 4 points
(c)	3.6 to 3.8	2	M1 for $y = 3.5$ soi
(d)	line $y = x + 1$ ruled	M1	0.
	-1.55 to -1.40 4.55 to 4.8	A1 A1	If 0 scored SC1 for $y = x + 1$ stated or implied or for 2 correct values given
(e) (i)	Point plotted at (5, 5)	1	
(ii)	Tangent ruled from A	1	
(iii)	1.2 to 1.4	B2	B2 and M1 dep on reasonable attempt at tangent from (5, 5)
			M1 for change in y / change in x of their ruled line

(a)	0 2.25 2 1.25	4	B1 for each
(b)	Fully correct smooth curve	4	B3 FT for 7 or 8 points
			or B2 FT for 5 or 6 points
			or B1 FT for 3 or 4 points
3(c)	1	1	
3(d)(i)	[y=]x+1	1	
(d)(ii)	-2.2 to -2.1	1	
	-0.45 to -0.4	1	
	0.51 to 0.6	PR	If zero scored, SC1 for <i>their</i> line in (d)(i) drawn. It must be of the form $y = mx + c$ $(m \ne 0)$ and drawn 'fit for purpose'
3(e)	-1.33 < k < 0 to 0.1	2FT	FT Strict ft of <i>their</i> max point and min point dep on cubic graph or accept correct answer from calculus B1 for each If zero scored, SC1 for two correct values reversed
Questio	on 29		
4(a)	_1 75 to _1 7		

4(a)	−1.75 to −1.7	1	
	1.7 to 1.75	1	
(b)(i)	Correct ruled solid tangent at (-1.5, 3.5)	Sa	tpreP.co
(b)(ii)	−7 to −5	2 dep	dep on close attempt at ruled solid tangent at $x = -1.5$ in part (b)(i) M1 for rise/run dep on close attempt at ruled solid tangent at $x = -1.5$
(c)(i)	1	1	
(c)(ii)	Correct curve	3	B2 for 4 or 5 correct points or B1 for 2 or 3 correct points

(d)(i)	-0.95 to -0.8	1	
	1.1 to 1.45	1	
(d)(ii)	their (-0.95 to -0.8)< x < their (1.1 to 1.45) oe	1FT	correct or FT their (d)(i)
(e)(i)	0.125 oe and 0.03125 oe and 0.000976 to 0.000977 oe	1	
(e)(ii)	0	1	accept zero, nought, etc
	• •		

(a)	-1.6 to -1.4	1	
(b)	-0.5	1	
(c)	k > -4	2	B1 for identifying the -4 or for horizontal line drawn $y = -4$
(d)	y = x - 5 ruled and -2.3 to $-2.1-1.2$ to $-1.11.3$ to 1.4	3	B2 for correct line and 2 correct values or no line and 3 correct values or B1 for no line and 2 correct values or B1 for correct line
(e)	Tangent ruled at $x = 1$	B1	No daylight at point of contact. Consider point of contact as midpoint between two vertices of daylight, the midpoint must be between $x = 0.8$ and 1.2
	-6 to -4	atpre	Dep on B1 or close attempt at tangent at $x = 1$ M1 for rise/run for <i>their</i> tangent at $x = 1$

(a)(i)	5	1	
(a)(ii)	$-\frac{3}{2}$ oe	1	
8(b)	$\left(\frac{4}{5},\ 0\right)$ oe	2	M1 for $5x - 4 = 0$ soi

			(a) or $\mathbf{B1FT} \text{ for grad} = \frac{-1}{\text{their (a)(i)}} \text{ soi}$ and $\mathbf{M1}$ for substitution of (10, 9) into their equation
(d)	(2, 6)	3	M1 for elimination of one variable A1 for $x = 2$ or $y = 6$
(e)	13	3	M2 for (4 + 9) × their 2 ÷ 2 oe or B1 for 9 oe or 4 or -4 seen

(a)	9, -6, 9	3	B1 for each
(b)	Correct graph	4	B3FT for 6 or 7 correct points or B2FT for 4 or 5 correct points or B1FT for 2 or 3 correct points
(c)	−3.5 to −3.35 and 0.8 to 0.9	2FT	FT their graph B1FT for either
(d)	$a = \frac{5}{4} \text{ or } 1\frac{1}{4} \text{ or } 1.25$ $b = -\frac{49}{8} \text{ or } -6\frac{1}{8} \text{ or } -6.125$	3	B2 for either correct or M1 for $[2]\left(x+\frac{5}{4}\right)^2$ seen isw or for $2x^2 + 4ax + 2a^2 + b$

5(a)	3.2 or 3.15 or 3.152 to 3.153 5.2 or 5.19 or 5.20 or 5.196	2	B1 for each
5(b)	Correct graph for $0.5 \le x \le 3.5$	4	B3FT for 6 or 7 correct points or B2FT for 4 or 5 correct points or B1FT for 2 or 3 correct points
5(c)	1.7 to 1.8	1FT	FT their graph if one answer
(d)(i)	Any integer $k \geqslant -1$	1	
(d)(ii)	Any integer $k < -1$	1	
5(e)	Tangent ruled at $x = -3$	B1	
	2.5 to 4	B2	dep on tangent drawn at $x = -3$ or close attempt at tangent at $x = -3$ M1 for rise/run also dep on tangent at $x = -3$ or close attempt at tangent at $x = -3$

(f)(i)	y = 6 - x ruled accurately	M2	M1 for correct line but freehand or ruled line gradient -1.1 to -0.9 , or through $(0, 6)$ but not $y = 6$
	$2.85 \leqslant x \leqslant 3$	A1	
(f)(ii)	[a =] 8 [b =] -48 [c =] -16	4	B3 for 2 correct or $x^5 + 8x^3 - 48x^2 - 16 = 0$ seen or $-x^5 - 8x^3 + 48x^2 + 16 = 0$ seen or M2 for correct multiplication by $8x^2$ or B1 for answers $\pm 8, \pm 48, \pm 16$ or M1 for $\frac{x^2 \times x^3 - 8 \times 2}{x^2 \times 8} = 6 - x$ or M1 for correct multiplication by 8 or M1 for correct multiplication by x^2
Questio	on 34		
0(a)	10.8 or 10.81 to 10.82	3	M2 for $\sqrt{(6-3)^2 + (-2-4)^2}$ oe or M1 for $(6-3)^2 + (-2-4)^2$ oe
(b)(i)	(6, 4)	2	B1 for each
(b)(ii)	2	2	M1 for $\frac{12 - (-4)}{10 - 2}$ oe
(b)(iii)	$y = -\frac{1}{2}x + 4$ oe final answer	3	M1 for gradient = $-\frac{1}{2}$ or $-\frac{1}{their(\mathbf{b})(\mathbf{ii})}$ M1 for (2, 3) substituted into their $y = mx + c$ or $y - y_1 = m(x - x_1)$ oe
Questio	on 35	atn	re0.0
(a)	0 -0.17 2.4		B1 for each
(b)	Fully correct smooth curve	4	B3FT for 9 or 10 correct points or B2FT for 7 or 8 correct points or B1FT for 5 or 6 correct points
(c)	$x \le 0.17 \text{ to } 0.25$ and $x \ge 2.25 \text{ to } 2.3$	-	B2 for strict inequalities or one correct or B1 for 0.17 to 0.25 and 2.25 to 2.3 seen
(d)(i)	y = 4 - x oe final answer	2	B1 for $4-x$ or $y = k-x$ or $y = 4+kx$ oe
(d)(ii)	correct ruled line	1	FT if in form $y = mx + c$ oe $(m, c \neq 0)$
	0.125 to 0.2 and 2.15 to 2.2	2	B1 for each

?(a)(i)	1,, 16	2	B1 f	For each
(a)(ii)	14,, -2	2	B1 f	For each
2(b)	Fully correct smooth curves	6	or B or B or B	For correct curve of $y = 2^x$ 2FT for 4 or 5 correct points 1FT for 2 or 3 correct points For correct curve of $y = 14 - x^2$ 2FT for 4 or 5 correct points 1FT for 2 or 3 correct points
?(c)(i)	3.5 to 3.7	1		
(c)(ii)	2.65 to 2.8	1		
(d)(i)	Correct line	1	Rule	ed, through (4, 2) and gradient -4
(d)(ii)	Tangent (2, 10)	2	B1 f	For each
Questio	on 37			
(a)	-2[.0], -0.2, 2.5		3	B1 for each
(b)	Fully correct curve		5	or B3FT for 9 or 10 correct plots or B2FT for 7 or 8 correct plots or B1FT for 5 or 6 correct plots and B1 indep two separate branches not touching or cutting <i>y</i> -axis
i(c)(i)	Correct tangent and 3 ≤ grad ≤ 5	or@	3	B2 for close attempt at tangent to curve at $x = -2$ and answer in range OR B1 for ruled tangent at $x = -2$, no daylight at $x = -2$ and M1dep (dep on B1 or close attempt at tangent) [at $x = -2$] for $\frac{rise}{run}$
(c)(ii)	[$y =$] their(\mathbf{c})(\mathbf{i}) $x +$ their y -intercept final answer		2	Strict FT their y-intercept for their line M1 for $y = their(\mathbf{c})(\mathbf{i}) x + any value$ or 'c' oe seen or for $y = any value$ (non-zero) x or 'mx' + their y-intercept seen oe

(d)(i)	1.05 to 1.25	1	
(d)(ii)	-2.3 to -2.2 -0.4 to -0.3 0.3 to 0.4	3	B1 for each After 0 scored B1 for $y = -4$ ruled
i(e)	[a =] 2 [b =] 24 [n =] 5	3	B2 for 2 correct or for $2x^5 + 24x^2 [-3 = 0]$ or B1 for 1 correct or for $\frac{2x^5 - 3 + 4(6x^2)}{6x^2} [= 0]$ oe If 0 scored SC1 for $2x^5$ seen in final line of algebra
Questic	on 38	PR	
(a)	x = 0	1	
(b)	Tangent ruled at $x = 0.5$	B1	No daylight between tangent and curve at point of contact
	-9 to -6.5	2	dep on ruled tangent or close attempt a tangent at $x = 0.5$ M1 for rise/run also dep on tangent or close attempt at tangent at $x = 0.5$
(c)(i)	0 2.4 or better 4	3	B1 for each
(c)(ii)	Correct smooth curve	4 oreP	B3FT for 6 or 7 correct plots or B2 FT for 4 or 5 correct plots or B1 FT for 2 or 3 correct plots FT their table
(d)	$x^3 + 3x + 4 = 10 - 8x^2$ and correctly completed	1	
(e)	line $y = -2x + 2$ drawn and -0.45 to -0.35 nfww	3	B2 for ruled $y = -2x + 2$ or B1 for $-2x + 2$ seen or for line $y = -2x + c$ drawn or for $y = cx + 2$ $(c \ne 0)$ drawn and B1 for -0.45 to -0.35 nfww

(a)	-3, 17	2	B1 for each
(b)	Fully correct curve	4	B3 FT for 10 or 11 points or B2 FT for 8 or 9 points or B1 FT for 6 or 7 points
(c)(i)	Correct ruled tangent for <i>their</i> curve through $(0, -17)$	1	
(c)(ii)	(1.7 to 2.2, -1 to 2.5)	1	
(c)(iii)	[y =] 9x - 17 final answer	3	M2dep for answer $[y =] 9x[+] - c$ OR
	SPT	PF	M1dep for gradient = $\frac{rise}{run}$ for their tangent at any point B1 for answer $[y =]kx[+] - 17 (k \neq 0)$
(d)	y = 3x + 2 ruled correctly and -2.2 to -2.1 -0.6 to -0.4 2.6 to 2.8	4	B2 for $y = 3x + 2$ ruled or B1 for $[y =] 3x + 2$ soi or $y = 3x + k$ ruled or $y = kx + 2$ but not $y = 2$ B2 for all 3 values or B1 for 2 values
Questio	n 40		
(a)	(5, 6)		1- 1.5
(b)	$[y=]-\frac{4}{5}x+3 \text{ nfww}$	tpre	B2 for $[y =] -\frac{4}{5}x + c$ nfww or M1 for $\frac{rise}{run}$ using any two of (-5, 7) (0, 3) and (5, -1) and B1 for $[y =]mx + 3 \ (m \neq 0)$
(c)	$y = -\frac{4}{5}x - 2 \text{ oe}$		FT their gradient from 8(b) B1 for $y = (their \text{ gradient})x + c \text{ (}c \text{ not 0)}$ or for $y = mx - 2 \text{ (}m \neq 0 \text{)}$ or for $-\frac{4}{5}x - 2$ alone

(d)(i)	$y = \frac{5}{4}x + 4 \text{ oe}$	3	M1 for $-\frac{1}{their}$ gradient from 8(b) M1 for (8, 14) substituted into their $y = mx + c$ or $\frac{y-14}{x-8} = m$ or better
(d)(ii)	8.54 or 8.544	3	M2 for $(14-their 6)^2 + (8-their 5)^2$ or better or M1 for $14-their 6$ and $8-their 5$ seen
d)(iii)	(4, 6)	2	B1 for each

(a)	0 -2 0.9	3	B1 for each
(b)	Correct curve	4	B3 FT for 9 or 10 points or B2 FT for 7 or 8 points or B1 FT for 5 or 6 points
(c)	-0.45 to -0.35 1 2.35 to 2.45	3	FT their graph B1 for each in the correct position If zero scored, SC1FT for 3 correct values
(d)(i)	y=1-x oe	2	B1 for $y=1-kx$ oe, $k \neq 0$ or $y=k-x$ oe or $1-x$
(d)(ii)	Correct ruled line and 2.25 to 2.4	BP.3	B2FTdep for correct ruled line or B1 dep for line through $(0, 1)$ when extended but not $y = 1$ or with gradient -1.1 to -0.9 or correct line but freehand or SC2 for $y = x - 1$ ruled after answer [y =]x - 1 in (d)(i) and B1 for 2.25 to 2.4
(e)	Correct tangent and 1.7 to 3.7	3	No daylight between tangent and curve at $x = -0.25$. Point of contact is the midpoint between two vertices of daylight and this point of contact must be between -0.35 and -0.15 B2 for close attempt at tangent at $x = -0.25$ and answer in range

(, _			
(a)	-2.1, 1.6, -1.7, 2.1		3	B2 for 3 correct or B1 for 2 correct
(b)	Fully correct curve		4	B3FT for 8 or 9 correct plots or B2FT for 6 or 7 correct plots or B1FT for 4 or 5 correct plots
(c)	line $y = \frac{1}{2}(1-x)$ ruled		M2	M1 for line with gradient $-\frac{1}{2}$ M1 for line through $(0, \frac{1}{2})$ but not $y = \frac{1}{2}$
	-2.15 to -2.01 -0.45 to -0.2 2.25 to 2.45	PF	B2	B1 for two correct
(d)	number of intersections of <i>their</i> curve and the line $y = 1$	I	1	strict FT for their curve
Questic	on 43			
(a)(i)	-3	1		
(a)(ii)	6.2 to 6.4 oe	2	M	1 for 3 seen or used
(b)	y = 5 - 3x ruled	2	- 3 or B1	for $y = 5 - 3x$ soi or ruled line with gradient or with y – intercept at 5 (but not $y = 5$) FT for incorrect line equation/expression own in working and <i>their</i> line correctly drawn
	- 0.3 to - 0.2 1.65 to 1.8	2 nre	or	for each, dep on $y = 5 - 3x$ drawn FT their line provided equation/expression own in working, dep on B1FT for line
(c)	Tangent ruled at $x = -2$	1	B1	for correct tangent
	-4.5 to -2.5	2	M1 clos	p on B1 for tangent or close attempt at gent at $x = -2$ for rise/run also dep on tangent drawn or se attempt at correct tangent at see correct or implied calculation from a wn tangent
5(d)(i)	8, 4, 0.25 oe	3	B	1 for each
5(d)(ii)	Correct graph	3		2FT for 6 or 7 correct plots B1FT for 4 or 5 correct plots
(d)(iii)	1.8 to 1.9	1		

(a)	2.45, 0.25, -0.25	3	B1 for each
(b)	Fully correct smooth curve	4	B3FT for 6 or 7 points or B2 FT for 4 or 5 points or B1 FT for 2 or 3 points
(c)	0.7 to 0.8	1	FT their curve
(d)(i)	Correct ruled line	2	M1 for good freehand, or ruled line with gradient -1.05 to -0.95 or ruled line through $(0, 2)$ but not line $y = 2$
(d)(ii)	Both intersections of <i>their</i> (b) and <i>their</i> (d)(i)	2	Strict FT intersection of their (b) and their (d)(i)
	19 P		B1FT for one correct OR B2 for 0.27 to 0.28 and 2.38 to 2.39
(e)	Substitutes $x = \sqrt{2}$ into $\frac{1}{2x} - \frac{x}{4}$ OR Identifies $y = 0$ oe	M1	
	OR Correctly manipulates to a single fraction $2 - x^2$		
	e.g. $\frac{2-x^2}{4x}$ oe seen	ator	
	Concludes 'read the graph at $y = 0$ ' oe	A1	
	OR 1 r		
	Manipulates $0 = \frac{1}{2x} - \frac{x}{4}$ oe		
	leading to $x^2 = 2$		
	OR States $\frac{2-x^2}{4x}$ oe = 0 leading to		
	$x^2=2$		

(a)(i)	1.5 oe	1	
(a)(ii)	(0, 2)	1	
(b)(i)	y = -2x + 6 oe final answer	3	B2 for $y = -2x + c$ oe or $y = mx + 6$ oe $m \ne 0$ or for answer $-2x + 6$ or B1 for [gradient =] $-\frac{6}{3}$ oe or $c = +6$ soi
(b)(ii)	y = 0.5x - 1.5 oe final answer	3	B1 for [gradient =] – 1 divided by <i>their</i> gradient from (b)(i) evaluated soi M1 for substitution of (9, 3) into $y = (their \ m)x + c$ seen in working
(c)(i)	12.6 or 12.64 to 12.65	3	M2 for $\sqrt{(8-4)^2 + (5-1)^2}$ oe or M1 for $(8-4)^2 + (5-1)^2$ oe
(c)(ii)	(2, 3)	2	B1 for each

Question 46

(a)	2, 2, 6	3	B1 for each
(b)	Correct graph	4	B3FT for 10 or 11 correct plots or B2FT for 8 or 9 correct plots or B1FT for 6 or 7 correct plots
(c)	-3.3 to -3.1	1	FT their graph
(d)	y = -2x ruled	M1	or B1 for $y = -2x$ stated
	-2.6 to -2.45	A1	
(e)	3 or 4 or 5	1	FT their graph Allow more than one correct value

(a)	5, -3, 21	3	B1 for each
(b)	Fully correct curve	4	B3 FT for 9 or 10 points or B2 FT for 7 or 8 points or B1 FT for 5 or 6 points
(c)	-2.9 to -2.7 0 1.7 to 1.9	2	B1 for 2 correct values

(d)	Tangent ruled at $x = 2$	B1	
	10 to 14	B2	Dep on correct tangent or close attempt at tangent at $x = 2$
			M1 for rise/run also dep on correct tangent drawn or close attempt at tangent Must see correct or implied calculation from a drawn tangent
(e)	6	1	

(a)	3.5, 15, 3.9	3	B1 for each
(b)	Correct graph	5	B4 for correct curves but branches joined or touching y-axis or B3FT 10 or 11 points or B2FT for 8 or 9 points or B1FT for 6 or 7 points B1indep two separate branches not touching or crossing y-axis
(c)	0.5 to 0.6 and 1.3 to 1.6	2	B1 for each or both correct but in reverse order
(d)	1	1	
(e)(i)	y = 3x + 1 ruled and 0.3 to 0.49	Satpre	B2 for correct ruled line that crosses <i>their</i> curve or B1 for $y = 3x + 1$ soi or freehand line or ruled line with gradient 3 or with y – intercept at 1 (but not $y = 1$)
(e)(ii)	[a =] -6 [b =] -2 [c =] -4	3	M2 for $x^4 + 2 - 4x = 6x^3 + 2x^2$ or better seen or B1 for each correct value to a maximum of 2 marks If 0 scored, SC1 for answer $[a =] 6, [b =] 2$ and $[c =] 4$ or for $x^5 + 2x - 4x^2 = 6x^4 + 2x^3$ or better

)(i)	(3, 5.5)	2	B1 for either value correct
)(ii)	$\frac{5}{4}x + \frac{7}{4}$ final answer	3	B2 for answer $\frac{5}{4}x + c$ oe or for correct equation in different form or M1 for $\frac{8-3}{5-1}$ oe and M1 for correct substitution shown of (1, 3) or (5, 8) or <i>their</i> (a)(i) into $y = (their\ m)x + c$ oe

Question 50

2(a)(i)	3 2.25 1	3	B1 for each
!(a)(ii)	Fully correct smooth curve	4	B3FT for 7 or 6 correct plots B2FT for 5 or 4 correct plots B1FT for 3 correct plots
(a)(iii)	-0.6 to -0.51, 0.75 to 0.85, 1.7 to 1.85	3	B1 for each If 0 scored, SC1 for $y = 1.5$ drawn
(a)(iv)	−3 or −2 or −1 or 0	1	
2(b)(i)	Tangent ruled at $x = 1$	1	
!(b)(ii)	4.4 to 5.6	2	Dep on tangent at $x = 1$ or close attempt M1 for rise/run for <i>their</i> line
(b)(iii)	y = (4.4 to 5.6)x - (1.8 to 2.2) or [y =] their (b)(ii)x + their(y-intercept)	2 ore	FT for any line but not horizontal or vertical line for 2 marks or B1 B1FT for [m =] their 5 or for their y-intercept

(a)(i)
$$\left(-\frac{1}{2}, 4\right) \text{ and } \left(\frac{1}{2}, 2\right)$$

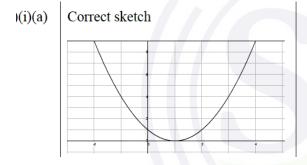
$$\mathbf{B2} \text{ for } 12x^2 - 3[=0]$$
or $\mathbf{B1} \text{ for } 12x^2 \text{ or } -3$

$$\mathbf{M1} \text{ for their derivative } = 0 \text{ or } \frac{dy}{dx} = 0$$

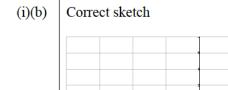
$$\mathbf{B1} \text{ for } [x =] -\frac{1}{2} \text{ and } \frac{1}{2} \text{ or one coordinate pair correct}$$

(a)(ii)	$\left(-\frac{1}{2}, 4\right)$ Max with reason $\left(\frac{1}{2}, 2\right)$ Min with reason	3	B2 for one correct with reason or M1 for correct attempt to find e.g. 2nd derivative/gradients/sketch
!(b)	line $y = x + 3$ ruled	M2	B1 for $[y=]x+3$ identified or rules $y=x+k$ or $y=px+3$
	-0.7 to -0.8 2.7 to 2.8	A1	
Questi	on 52		
(a)(i)	x + 5	2	B1 for linear equation with positive gradient or intercept 5
(a)(ii)	$2 \sin x$ oe	2	B1 for recognition of sin or $cos(x - 90)$
(b)	tangent ruled at P	B1	
•	1.3 to 1.4	B2	dep on tangent drawn M1 for rise/run
Questi (a)(i)	on 53 $\frac{3}{8}$	2	M1 for $8y = 3x + 20$ or better
(a)(ii)	(0, 2.5) oe	1	- /.5/
(a)(ii)	(0, 2.5) oe	1	60.
(b)(i)	15.6 or 15.62	tpi3	M2 for $\sqrt{(9-3)^2 + (-2-8)^2}$ oe seen or M1 for $(9-3)^2$ or $(-2-8)^2$ oe seen
(b)(ii)	$y = -\frac{5}{6}x + 4$ oe	3	M1 for gradient $\frac{-2-8}{93}$ oe M1 for substituting $(6, -1)$ into a linear
			equation oe
			T. Control of the con
(b)(iii)	$y = \frac{6}{5}x - \frac{3}{5}$ oe	4	M1 for gradient -1 / their $\left(-\frac{5}{6}\right)$

)(a)(i)	5	2	M1 for $(-1)^4 - 4(-1)^3$
(a)(ii)	(0, 0) and (3, -27)	6	B2 for $4x^3 - 12x^2$ [= 0] or B1 for $4x^3$ or $12x^2$
			AND
			M1 for derivative = 0 or <i>their</i> derivative = 0 M1 for $4x^2(x-3)[=0]$ B1 for $[x=] 0$ and $[x=] 3$ or $[y=] 0$ and $[y=] -27$ or for one correct coordinate pair
)(b)	[p =] 11 $[q =] 5$	2 P	B1 for each or M1 for $\frac{dy}{dx} = px^{p-1} + 2qx^{q-1}$



- 2 B2 for correct quadratic curve with min touching x-axis
 - or **B1** for parabola vertex downwards



- B2 for correct straight line intersecting curve on
 - or **B1** for straight line with positive gradient and positive y-intercept

2.8[0] or 2.795... :)(ii)

B3 for $x^2 - \frac{5}{2}x = 0$ oe

or **M1** for $(x-1)^2 = \frac{1}{2}x+1$ **B1** for $[(x-1)^2 =]x^2 - x - x + 1$

AND

B2 for (0, 1) and $\left(\frac{5}{2}, \frac{9}{4}\right)$ oe

or **B1** [x =] 0 and $\frac{5}{2}$ oe

AND

M1 for (difference in x)² + (difference in y)²

-			
)(a)(i)	15.7 or 15.65	3	M2 for $\sqrt{(4-10)^2 + (4-3)^2}$ oe or M1 for $(-4-10)^2 + (4-3)^2$ oe
)(a)(ii)	$\frac{-10-4}{43}$ [= -2] oe	M1	
	10 = -2(-3) + c Or $-4 = -2(4) + c$ and correct completion to $y = -2x + 4$	A1	
(a)(iii)	$y = \frac{1}{2} x + \frac{11}{4}$ oe	4	M1 for grad = $\frac{1}{2}$ soi M1 for [midpoint =] ($\frac{1}{2}$, 3) M1 for substitution of ($\frac{1}{2}$, 3) into their $y = mx + c$ oe
)(b)(i)	$\left(-\frac{1}{3}, -\frac{22}{27}\right)$ oe and (-5, 50)	6	B2 for $3x^2 + 16x + 5$ Or B1 for one correct M1 for derivative = 0 or <i>their</i> derivative = 0 M1 for $[x =] -\frac{1}{3}$ and $[x =] -5$ B1 for $-\frac{22}{27}$ and 50
(b)(ii)	$\left(-\frac{1}{3}, -\frac{22}{27}\right)$ minimum (-5, 50) maximum with correct reasons	3	B2 for one correct with reason or M1 for correct attempt e.g. 2 nd derivatives, gradients or sketching
Ouestio	n 57		

(a)(i)	Correct sketch	2	B1 for correct shape but inaccurate
(a)(ii)	Rotational [symmetry] order 2 [centre] (180, 0)	2	B1 for rotational [symmetry]
(b)	48.6 or 48.59 to 48.60 and 131.4 or 131.40 to 131.41	3	B2 for 48.6 or 48.59 to 48.60 or 131.4 or 131.40 to 131.41 or M1 for $\sin x = 0.75$ or better If 0 scored, SC1 for two answers adding to 180
(c)(i)	$(x+5)^2-11$	2	M1 for $(x + 5)^2 + k$ or $(x + their 5)^2 + 14 - (their 5)^2$ or $a = 5$
(c)(ii)	Sketch of U-shaped parabola with a minimum indicated at (-5, -11) with no part of graph in 4 th quadrant	3	FT their $(x + 5)^2 - 11$ provided in that form B1 for U shape curve B1FT for turning point at $(-5, k)$ or $(k, -11)$

6 For the y values accept any value rounded to 2 decimal places in the given range

B5 for
$$(1.74, 7.21 \text{ to } 7.24)$$
 or $(-3.74, -9.20 \text{ to } -9.22)$ or $x = 1.74$ and $x = -3.74$

OR

M2 for
$$2x^2 + 4x - 13 = 0$$

or $2y^2 + 4y - 133 = 0$

or M1 for
$$2x^2 + 7x - 11 = 3x + 2$$

or $y = 2\left(\frac{y-2}{3}\right)^2 + 7\left(\frac{y-2}{3}\right) - 11$

AND

FT their quadratic expression (not $2x^2 + 7x - 11$)

M2FT for
$$\frac{-4 \pm \sqrt{4^2 - 4 \times 2 \times -13}}{2 \times 2}$$

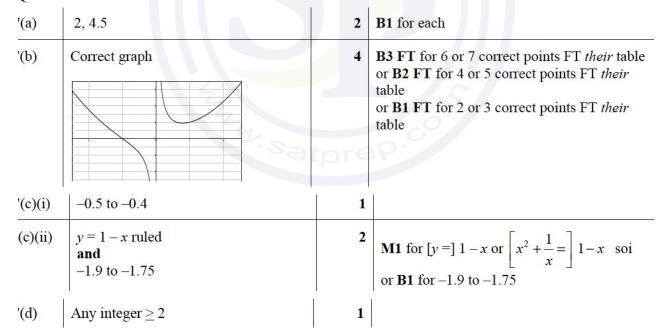
or $-1 \pm \sqrt{\frac{15}{2}}$ oe
or M1FT for $\sqrt{4^2 - 4 \times 2 \times -13}$ oe
or for $\frac{-4 + \sqrt{k}}{2 \times 2}$ or $\frac{-4 - \sqrt{k}}{2 \times 2}$
or $(x + 1)^2 [-13/2 - 1 = 0]$

(a)(i)
$$(8-x)(3+x)$$

2 M1 for
$$8(3+x) - x(3+x)$$

or $3(8-x) + x(8-x)$
or $(a-x)(b+x)$ where $ab = 24$ or $a-b=5$

(a)(ii)	[a =] -3 [b =] 8 [c =] 24	3	FT their (a)(i) for a and b B1FT for each of a and b or both correct but reversed B1 for $[c=]$ 24
(a)(iii)	8	3	M2 for $5-2x$ or M1 for $-2x$ or $5-kx$, $k \neq 0$
'(b)(i)	Correct sketch: positive cubic shape and max on the y-axis or to the right of y-axis with one root at (-1, 0) and turning point at (3, 0) and y-intercept at (0, 9) all labelled	4	B1 for positive cubic shape with max on the y-axis or to the right of y-axis B1 for root at (-1, 0) B1 for turning point at (3, 0) B1 for y-intercept (0, 9) If 0 score SC1 for all three intercepts on axes identified
(b)(ii)	$x^3 - 5x^2 + 3x + 9 \text{final answer}$	3	B2 for correct expansion of three brackets unsimplified B1 for correct expansion of two brackets with at least 3 terms correct



(a)(i)	A(-4, 0) B(1, 0) C(0, -4)	4	B3 for A and B correct Or B2 for B (-4, 0) and A (1, 0) Or B1 for $(x + 4)(x - 1)$ or for $\frac{-3 \pm \sqrt{3^2 - 4 \times 1 \times -4}}{2}$ oe and B1 for A or B correct B1 for C(0, -4) OR SC2 for -4, 1 and -4 in correct positions on the graph
(a)(ii)	$2x + 3 [\pm 0]$ final answer	2	B1 for answer $2x + c$ or for $ax + 3$, $a \ne 0$ or for correct answer seen
(a)(iii)	y = 7x - 8 oe	3	B2 for answer $7x - 8$ OR M1 for [gradient =] $2(2) + 3$ FT <i>their</i> part (a)(ii) of the form $ax + b$ M1dep for substitution of $(2, 6)$ into $y = their mx + c$ oe

(a)	17	3	M2 for $3 \times 2x^2 - 7$ or better isw or M1 for $3 \times 2x^2$ oe or $kx^2 - 7$ seen
4.50			1

(b)(i) 13.4 or 13.41 to 13.42
M2 for
$$\sqrt{(-5-7)^2 + (8-2)^2}$$
 oe or M1 for $(-5-7)^2 + (8-2)^2$ oe

(b)(ii)
$$[y =]2x + 5$$
 final answer

4 M1 for [gradient of $AB =]\frac{8-2}{-5-7}$ oe

M1dep for gradient $p = -1 \div their - \frac{1}{2}$ oe

M1dep on previous M1 for substituting $(-1, 3)$ into $y = their \ px + c$ oe where $their \ p \neq 0$

(5,0)

B3 for
$$\overrightarrow{AD} = \begin{pmatrix} -2 \\ -2 \end{pmatrix}$$
 or $\overrightarrow{DA} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$
or coordinates of C (-7,6) and $\begin{bmatrix} \overrightarrow{CD} = \end{bmatrix} \begin{pmatrix} 12 \\ -6 \end{pmatrix}$ oe seen or B2 for $a = b = 2$ soi or coordinates of C (-7,6) or M1 for $a = b$ oe soi or for $a^2 + b^2 = (\sqrt{8})^2$ oe

or
$$\cos 45 = \frac{a}{\sqrt{8}}$$
 oe
or for $\left[\overrightarrow{DC} = \right] \begin{pmatrix} -12 \\ 6 \end{pmatrix}$ or $\left[\overrightarrow{CD} = \right] \begin{pmatrix} 12 \\ -6 \end{pmatrix}$ seen
or $\frac{y-8}{x-5} = 1$ oe or $\frac{y-2}{x-7} = 1$

$x = -2$ or M1 for rise/run for their tangent, attempt, at any point Must see correct or implied calcula a drawn tangent After M0, SC1 for gradient of tang close attempt) in range embedded in $y = mx + c$ (a)(iii) $y = 2x - 2$ ruled and $x = -2.9$ to -2.8 cao B2 for correct ruled line or B1 for short line or fulled line with grading with y-intercept at -2 (but not $y = -2$). (b) $A(4, 17) B(-1.5, 0.5)$ B4 for $(-1.5, 0.5)$ and $(4, 17)$, or for $x = 4$ and $x = -1.5$ OR B3 for $A(4, 17)$ or $B(-1.5, 0.5)$ OR M1 for $2x^2 - 2x - 7 = 3x + 5$ or AND either M2 for $(2x + 3)(x - 4)$ or M1 for $2x(x - 4) + 3(x - 4)$	Question	11 03		
6 to 10 2 dep on B1 or a close attempt at tan $x = -2$ or M1 for rise/run for their tangent, attempt, at any point Must see correct or implied calcular a drawn tangent After M0, SC1 for gradient of tang close attempt) in range embedded in $y = mx + c$ 3 B2 for correct ruled line or B1 for short line or for freehand broken line or ruled line with gradie with y-intercept at -2 (but not $y = -2$). (a) (iii) $y = 2x - 2$ ruled and $x = -2.9$ to -2.8 cao 5 B4 for $(-1.5, 0.5)$ and $(4, 17)$, or for $x = 4$ and $x = -1.5$ OR B3 for A(4, 17) or B(-1.5, 0.5) OR M1 for $2x^2 - 2x - 7 = 3x + 5$ oe AND either M2 for $(2x + 3)(x - 4)$ or M1 for $2x(x - 4) + 3(x - 4)$	(a)(i)	2.7 to 2.8	1	
$x = -2$ or M1 for rise/run for their tangent, attempt, at any point Must see correct or implied calcula a drawn tangent After M0, SC1 for gradient of tange close attempt) in range embedded in $y = mx + c$ (a) (iii) $y = 2x - 2 \text{ ruled}$ $and x = -2.9 \text{ to } -2.8 \text{ cao}$ B2 for correct ruled line or B1 for short line or for freehand broken line or ruled line with gradie with y-intercept at -2 (but not $y = -2$) (b) $A (4, 17) B (-1.5, 0.5)$ 5 $B4 \text{ for } (-1.5, 0.5) \text{ and } (4, 17), \text{ or for } x = 4 \text{ and } x = -1.5$ OR B3 for A(4, 17) or B(-1.5, 0.5) OR M1 for $2x^2 - 2x - 7 = 3x + 5$ oe AND either M2 for $(2x + 3)(x - 4)$ or M1 for $2x(x - 4) + 3(x - 4)$	(a)(ii)	tangent ruled at $x = -2$	B1	
attempt, at any point Must see correct or implied calcular a drawn tangent After M0, SC1 for gradient of tang close attempt) in range embedded in $y = mx + c$ (a)(iii) $y = 2x - 2$ ruled and $x = -2.9$ to -2.8 cao B2 for correct ruled line or B1 for short line or for freehand broken line or ruled line with gradie with y -intercept at -2 (but not $y = -2.8$) by $y = 2x - 2$ for $y = -2.8$ cao B4 for $y = 2x - 2$ ruled and $y = -2.8$ cao B5 For short line or for freehand broken line or ruled line with gradie with y -intercept at $y = -2.8$ cao B6 For $y = 2x - 2$ for $y = -2.8$ cao B7 For $y = 2x - 2$ for $y = -2.8$ cao B8 For $y = 2x - 2$ for $y = -2.8$ cao B9 For $y = 2x - 2$ for $y = -2.8$ cao B9 For $y = 2x - 2$ for $y = -2.8$ cao B9 For $y = 2x - 2$ for $y = -2.8$ cao B9 For $y = 2x - 2$ for $y = -2.8$ cao B9 For $y = 2x - 2$ for $y = -2.8$ cao AND either B9 For $y = 2x - 2$ for $y =$		6 to 10	2	dep on B1 or a close attempt at tangent at $x = -2$
close attempt) in range embedded in $y = mx + c$ (a)(iii) $y = 2x - 2$ ruled and $x = -2.9$ to -2.8 cao B2 for correct ruled line or B1 for short line or for freehand broken line or ruled line with gradic with y-intercept at -2 (but not $y = -2.0$) by $y = -2.0$ for y				Must see correct or implied calculation from
and $x = -2.9$ to -2.8 cao or B1 for short line or for freehand broken line or ruled line with gradie with y -intercept at -2 (but not $y = -2.5$) B4 for $(-1.5, 0.5)$ and $(4, 17)$, or for $x = 4$ and $x = -1.5$ OR B3 for A(4, 17) or B(-1.5, 0.5) OR M1 for $2x^2 - 2x - 7 = 3x + 5$ oe AND either M2 for $(2x + 3)(x - 4)$ or M1 for $2x(x - 4) + 3(x - 4)$				After M0, SC1 for gradient of tangent (or close attempt) in range embedded in $y = mx + c$
or B1 for short line or for freehand broken line or ruled line with gradic with <i>y</i> -intercept at -2 (but not $y = -2$) (b) $A (4, 17) B (-1.5, 0.5)$ 5 $B4$ for $(-1.5, 0.5)$ and $(4, 17)$, or for $x = 4$ and $x = -1.5$ OR B3 for $A(4, 17)$ or $B(-1.5, 0.5)$ OR M1 for $2x^2 - 2x - 7 = 3x + 5$ oe AND either M2 for $(2x + 3)(x - 4)$ or M1 for $2x(x - 4) + 3(x - 4)$	(a)(iii)		3	B2 for correct ruled line
x = 4 and $x = -1.5OR B3 for A(4, 17) or B(-1.5, 0.5) OR M1 for 2x^2 - 2x - 7 = 3x + 5 oe AND either M2 for (2x + 3)(x - 4) or M1 for 2x(x - 4) + 3(x - 4)$				or B1 for short line or for freehand line or broken line or ruled line with gradient 2 or with y-intercept at -2 (but not $y = -2$)
OR M1 for $2x^2 - 2x - 7 = 3x + 5$ oe AND either M2 for $(2x + 3)(x - 4)$ or M1 for $2x(x - 4) + 3(x - 4)$	(b) 2	A (4, 17) B (-1.5, 0.5)	5	The state of the s
M1 for $2x^2-2x-7 = 3x + 5$ oe AND either M2 for $(2x + 3)(x - 4)$ or M1 for $2x(x - 4) + 3(x - 4)$				B3 for A(4, 17) or B(-1.5, 0.5)
AND either M2 for $(2x + 3)(x - 4)$ or M1 for $2x(x - 4) + 3(x - 4)$				OR
either M2 for $(2x+3)(x-4)$ or M1 for $2x(x-4)+3(x-4)$		2		M1 for $2x^2 - 2x - 7 = 3x + 5$ oe
or M1 for $2x(x-4) + 3(x-4)$		12		
or M1 for $2x(x-4) + 3(x-4)$.8.	ato	
			arp	
				or $x(2x+3)-4(2x+3)$
or $(2x+c)(x+d)$ where $cd = -12$ or $c + 2d = -5$				
[c and d are integers]				
OR				OR
M2 for				M2 for
				$-their b \pm \sqrt{(their b)^2 - 4(their a)(their c)}$
2(their a)				
				or M1 for $\sqrt{(their b)^2 - 4(their a)(their c)}$ or for $p = -their b$, $r = 2(their a)$ if in the
form $\frac{p + \sqrt{q}}{r}$ or $\frac{p - \sqrt{q}}{r}$				

(a)(i)	[a =] 4	2	B1 for $[a =]$ 4 B1 for $[b =]$ – 3 nfww
	[b=]-3 nfww		22.00 [0] 3 22.00
(a)(ii)	y = 4 oe	1	
(a)(iii)	y = -6x + 7 oe final answer	2	B1 for answer $-6x + 7$ or answers $y = -6x + c$ or $y = kx + 7$ ($k < 0$)
(b)(i)	2.25 2.67 3.5	3	B1 for each
(b)(ii)	correct curve	4	B3 FT for 7 or 8 points or B2 FT for 5 or 6 points or B1 FT for 3 or 4 points
(c)(i)	-0.78 to -0.72 and 0.55 to 0.59	2	B1 for each
(c)(ii)	$3x^3 - 9x^2 - 3x + 4$ [= 0] final answer	4 Itpr	B3FT for 3 out of 4 correct terms or for $bx^3 - 3bx^2 + (a - 1)x + 8 - 3a$ [= 0] oe or B2FT for 2 out of 4 correct terms or for 3 out of 4 terms from $bx^3 - 3bx^2 + (a - 1)x + 8 - 3a$ [= 0] or M1 for $1 + \frac{5}{3 - x} = their 4 + (their(-3))x^2$ oe

(a)(i)

(a)(ii) $-\frac{1}{2}x + 8$ oe

Correct equivalent in different form scores 3 marks. M1 for gradient of $AB = \frac{9-5}{3-1}$ or $\frac{4}{2}$ or 2

M1 dep for gradient

$$p = -\frac{1}{\text{their grad of } AB}$$

B1 for each coordinate

M1 (dep on previous M1) for substitution of their midpoint into y = (their p)x + c

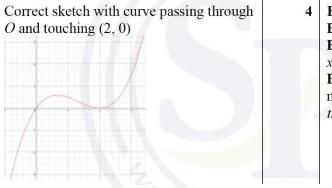
where their $p \neq 0$

Question 66

(a)(i)	2

M1 for $x(x^2 - 4x + 4)$ or $x(x - 2)^2$ or $(x^2 - 2x)(x - 2)$ or $x^3 - 2ax^2 + a^2x$

(a)(ii) O and touching (2, 0)



B1 for any positive cubic

B1 for sketch through or touching O

B1 for sketch with min or max touching x-axis once only but not at (0, 0)

B1FT their (a)(i) for sketch with min or max touching x-axis at (their 2, 0) and their 2 is labelled or clearly indicated

- y = 20x 64 final answer nfww (b)
- **B6** for equivalent correct equation

B2 for $3x^2 - 8x + 4$ isw

or **B1** for $3x^2$ or -8x seen

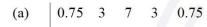
M2dep for [grad =] 20 soi nfww or M1dep for substituting 4 into their derivative isw

B1 for (4, 16) soi

M1dep for $16 = their 20 \times 4 + c$ oe

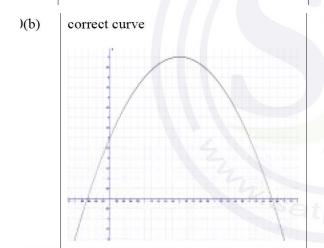
(a)(i)	1, -0.5 oe	2	B1 for each
(a)(ii)	Correct curve	4	B3FT for 6 or 7 correct plots or B2FT for 4 or 5 correct plots or B1FT for 2 or 3 correct plots
(b)	y = 2.5 - 2x ruled	B2	B1 for $y = k - 2x$ or $y = px + 2.5$ ruled $(p \neq 0)$ or for $[y =]$ 2.5 - 2x oe identified
	1.3 to 1.4	B1	
(c)	-1	B1	
	y = -1	B1	FT their k (must be negative)

Question 68



0.2 to 0.3

3 B2 for 4 or 3 correct or B1 for 2 correct



4.2 to 4.3

B3FT for 8 or 9 correct plots
B2FT for 6 or 7 correct plots
B1FT for 4 or 5 correct plots

)(c)	Accept any integer ≥ 8	1	
)(d)	line $y = 4 - \frac{1}{2}x$ ruled	В3	B2 for $[y=]4-\frac{1}{2}x$ identified
			or B1 for ruled line with gradient $-\frac{1}{2}$
			or B1 for ruled line through $(0, 4)$ but not $y = 4$

B1

(i)	$6x^2 - 8x$ final answer	2	B1 for each term in final answer or M1 for correct answer seen and spoilt
(ii)	64	2	FT their (c)(i) correctly evaluated provided at least 2 terms but not the original equation M1 for substituting $x = 4$ into their (c)(i)
(iii)	$(0, 6)$ $\left(\frac{4}{3}, \frac{98}{27}\right)$ oe	4	M1 for their derivative = 0 or $\frac{dy}{dx}$ = 0 soi B1 for $x = 0$ and $x = \frac{4}{3}$ M1dep for substituting one of their x values into $y = 2x^3 - 4x^2 + 6$ soi

(a)	(0, 0), (1, 0), (2, 0)	2	B1 for any two correct
			If 0 scored, SC1 for all three x values clearly identified
(b)	$x(x^2-x-2x+2)$ or $(x^2-x)(x-2)$	2	B1 for $x(x^2 - x - 2x + 2)$
	or $(x-1)(x^2-2x)$		or $(x^2-x)(x-2)$
	leading to $x^3 - 3x^2 + 2x$ with no errors or omissions		or $(x-1)(x^2-2x)$
(c)	$3x^2 - 6x + 2$	B2	B1 for 2 correct terms
	their $\frac{dy}{dx} = 0$	M1	
	their $\frac{-(-6) \pm \sqrt{(-6)^2 - 4(3)(2)}}{2(3)}$	M2	M1 for $\sqrt{(-6)^2 - 4(3)(2)}$ or for
	2(3)		$p = -(-6)$ and $r = 2(3)$ if in form $\frac{p \pm \sqrt{q}}{r}$
	(0.4, 0.4) (1.6, -0.4)	В3	B2 for 0.4 or 0.42 and 1.6 or 1.57 to 1.58 or for one correct pair of coordinates or B1 for 0.4 or 0.42 or 1.6 or 1.57 to 1.58 If 0 scored SC1 for $1 + \sqrt{\frac{1}{3}}$ and $1 - \sqrt{\frac{1}{3}}$ or better or for one correct pair of coordinates in any form
(d)	Correct sketch	2	FT their (c) but must be cubic i.e. correct shape cubic through origin and max and min in correct quadrants B1 for cubic shape sketch

(a)	(2, -10) and (-2, 22)	5	B2 for $3x^2 - 12$ isw or B1 for $3x^2 + k$ or $px^2 - 12$ ($p \ne 0$) or for $3x^2 - 12 + 6$ isw M1 for setting <i>their</i> derivative = 0 or $\frac{dy}{dx} = 0$ B1 for $x = \pm 2$ or for one correct coordinate pair
(b)	(2, -10) minimum with correct reason or sketch (-2, 22) maximum with correct reason or sketch	3 PA	B2 for 1 correct with correct reasoning or B2FT for correct evaluation with correct 2nd derivative for both of <i>their</i> different <i>x</i> values or M1 for showing [2nd derivative =] 6 <i>x</i> or gradients for one value on either side of one correct stationary point or for reasonable sketch of cubic
Questi	on 72		- 111

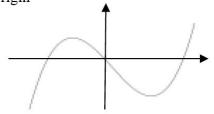
(a)	-1, -0.375, 3	3	B1 for each
(b)	Correct graph	4	B3FT for 8 or 9 correct points or B2FT for 6 or 7 correct points or B1FT for 4 or 5 correct points
(c)	y = 2 - x ruled correctly AND -0.45 to -0.35 1 2.35 to 2.45	tpre	B2 for $y = 2 - x$ ruled or B1 for $[y =]2 - x$ soi or $y = k - x$ ruled or $y = kx + 2$ ruled, but not $y = 2$ B2 for all three values or B1 for any two values

(i)	$20.25 - (1.5 + x)^2$	3	Method 1 B1 for $(\pm 1.5 \pm x)^2$ seen B1 for $[b =] 18 + their 1.5^2$ OR Method 2 B1 for $b - a^2 - 2ax - x^2$ or for $b = 20.25$ B1 for $a = 1.5$
(ii)	Correct sketch with max in correct quadrant at (-1.5, 20.25)	3	FT their $20.25 - (their 1.5 + x)^2$ provided in that form B1 for \cap shape or for \cup shape if in form $c + (d + x)^2$ in part (b)(i) B1 for TP at $(-1.5, k)$ or $(k, 20.25)$ FT their $20.25 \pm (their 1.5 + x)^2$ or for $(-1.5, 20.25)$ seen
iii)	[y =] 34 – 11x	ore!	B2 for $-3 - 2x$ or B1 for either $kx - 3$, $k \ne 0$ or $-2x + n$ or for $18 - 3 - 2x$ M1dep for gradient = their $(-3 - 2(4))$ B1 for y-value at $x = 4$, is -10 M1dep for their $-10 = (their -11)4 + c$ oe

(a)(i)	9.5, 4.8 and 8.5	3	B1 for each
(a)(ii)	correct curve	5	B4 for correct curve, but branches joined or touching y axis
			or B3FT for 9 or 10 correct plots or B2FT for 7 or 8 correct plots or B1FT for 5 or 6 correct plots
			AND
			B1 indep two separate branches not touching or cutting <i>y</i> -axis
(b)	$y = \frac{24}{5} - 2x \text{ ruled}$	4	B2 for correct ruled line crossing curve twice
	and - 0.4 to - 0.2 and 1.45 to 1.7		or B1 for correct freehand or for short ruled line or for line with negative gradient through (0, 4.8) or for line with gradient – 2 B1 for each value
(c)	[a =] 10 [b =] 20 [c =] -48	4	B3 for $10x^3 - 15 = 48x - 20x^2$ oe or better or B2 for 2 correct values or B1 for 1 correct value or for $5x^2 - \frac{15}{2x} = 24 - 10x$ or better
	Satur Satur		or for $2x^3 - 3 = \frac{48}{5}x - 4x^2$ or better or for $x^3 - \frac{3}{2} = \frac{24}{5}x - 2x^2$
			After 0 scored SC1 for correct elimination of a denominator of 5, x or 2x from a four term expression.

Question 75

correct shape and passes through origin



3 B1 for any positive cubic shape

B1 for sketch with one max and one min and with 3 roots including zero

If 0 scored, SC1 for x(x+2)(x-2) soi

$$a = -12$$

$$b = 5$$

$$k = -11$$

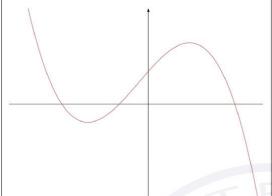
6 B5 for 2 correct
OR
B2 for
$$3x^2 + a$$

or B1 for $3x^2$ isw

M1dep on at least B1 for their $\frac{dy}{dx} = 0$ **M1dep** on at least B1M1 for x = 2 or x = -2substituted in their $\frac{dy}{dx} = 0$ equation **M1** for $k = 2^3 + 2 \times their \, a + b$ and $10 - k = (-2)^3 + (-2) \times their \, a + b$

(a)	1[.0] 0.9	2	B1 for each
(b)	correct curve	4	B3 FT for 6 or 7 points B2 FT for 4 or 5 points B1 FT for 2 or 3 points
(c)	ruled line at $y = -1$	B1	
	0.3 to 0.32	B1	.5

(a) Correct sketch of negative cubic crossing the x-axis at -3, -1 and 3 and crossing the y-axis at 9



4 B1 for any negative cubic shape with two turning points

B2 for three intercepts only with x-axis labelled at -3, -1 and 3 or **B1** for one or two correctly labelled x-intercepts

B1 for intercept with y-axis labelled at 9

If no graph drawn, SC1 for all four intercepts labelled on axes.

(b)(i) $3-x+3x-x^2$ or better or $3+x+3x+x^2$ or better or $9[-3x+3x]-x^2$

M1 At least 3 of the four terms correct

or for the correct expansion of all three brackets with all 8 terms correct

Correct completion to $[y =] 9 + 9x - x^2 - x^3$

A1 with no errors or omissions seen

'(b)(ii)	$9 - 2x - 3x^2 = 0$ oe	В3	B2 for $9 - 2x - 3x^2$ or B1 for two correct terms M1 for <i>their</i> derivative = 0 or stating $\frac{dy}{dx} = 0$
	$\frac{2\pm\sqrt{(-2)^2-4\times-3\times9}}{2\times-3} \text{ oe}$ OR	B2	FT their derivative B1FT for $\sqrt{(-2)^2 - 4(-3)(9)}$ or better or for $\frac{-(-2) + \sqrt{q}}{2 \times -3}$ or $\frac{-(-2) - \sqrt{q}}{2 \times -3}$
	$-\frac{1}{3} \pm \sqrt{\frac{9}{3} + \left(\frac{1}{3}\right)^2} \text{oe}$		OR B1 for $\left(x + \frac{1}{3}\right)^2$
	-2.10 and 1.43 final answer	B2	B1 for each or for answers –2.1 or –2.097 and 1.4 or 1.430 to 1.431 or SC1 for –2.097 and 1.43[0] to 1.431 seen in working or for –1.43 and 2.10 as final answer
(b)(iii)	[a =] - 6 [b =] 17	3	B2 for either <i>a</i> correct or <i>b</i> correct or for $[a =]$ –5.04 or –5.049 to –5.05 and $[b =]$ 16.9 seen or M1 for substitution of one of <i>their</i> solutions into $9 + 9x - x^2 - x^3$ oe or SC1 for reversed answers, $a = 17$, $b = -6$
	3h.satp		.00.

3(a)(i)	(-0.5, 1)	2	B1 for each
(a)(ii)	$\begin{pmatrix} 7 \\ -3 \end{pmatrix}$	2	B1 for each
(a)(iii)	7.62 or 7.615 to 7.616	2	FT their (a)(ii) M1 for $(their 7)^2 + (their -3)^2$ oe
(a)(iv)	[y=]-4x-1 final answer	3	B2 for answer $-4x + c$ [oe] or for correct equation in different form or for $-4x + -1$ or for $-4m - 1$ OR M1 for $\frac{-5 - 7}{12}$ oe M1 for correct substitution shown of $(-2, 7)$ or $(1, -5)$ or their $(-0.5, 1)$ into $y = (their \ m)x + c$ oe OR M1 for $7 = -2m + c$ and $-5 = m + c$ A1 for $m = -4$ and $c = -1$
3(a)(v)	$[y =] \frac{1}{4}x + \frac{11}{4} \text{ final answer}$	rey	M1 for grad = $\frac{1}{4}$ oe nfww soi, FT negative reciprocal of <i>their</i> gradient from (iv) M1 for correct substitution shown of (5, 4) into $y = (their \ m)x + c$ oe or, if no substitution shown, (5, 4) satisfies <i>their</i> final linear equation.

(b)	$2x^2 + 11x - 21 [= 0]$	M2	or M1 for $8 - 5x = 2x^2 + 6x - 13$ oe or better
	$(2x-3)(x+7) = 0 $ oe or $-11 \pm \sqrt{11^2 - 4 \times 2 \times (-21)}$ 2×2	M2	Allow correct method to solve <i>their</i> quadratic equation e.g. formula, complete the square but not for $2x^2 + 6x - 13$
	or		M1 FT <i>their</i> equation for $2x(x+7) - 3(x+7) = 0$
	$-\frac{11}{4} \pm \sqrt{\frac{21}{2} + \left(\frac{11}{4}\right)^2}$ oe		or $x(2x-3) + 7(2x-3) = 0$
			or $(2x + a)(x + b) = 0$ where $ab = -21$ or $2b + a = 11$
	TF		OR M1 for $\sqrt{11^2 - 4 \times 2 \times -21}$
	9		or for $\frac{-11+\sqrt{k}}{2\times 2}$ or $\frac{-11-\sqrt{k}}{2\times 2}$ OR
			M1 for $\left(x + \frac{11}{4}\right)^2$
	$\left(\frac{3}{2}, \frac{1}{2}\right)$ and (-7, 43)	B2	B1 for one correct pair or for 2 correct x-values or 2 correct y-values

(a)	$3x^2 - 2kx$	M2	M1 for $3x^2$ or $-2kx$
	their $\frac{dy}{dx} = 6$	M1	Dep on at least M1 for derivative
	$x = 2$ substituted in their $\frac{dy}{dx}$	M1	Dep on at least M1 for derivative
	Correct working leading to 1.5 oe	A1	A0 if any errors in working leading to 1.5
(b)	(0, 1) (1, 0.5)	4	B3 for $x = 0$ and $x = 1$ or for $(1, 0.5)$ OR
	GP	F	M1 for their $\frac{dy}{dx} = 0$ B1 for $3x^2 - 3x$ oe or better
(c)	correct sketch	2	with max on positive y-axis and min in 1st quadrant B1 for positive cubic or for graph with one max which is on pos y-axis and one min which is in 1st quadrant

(a)	8.25 or 8.246	atp ³	M2 for $(35)^2 + (2-4)^2$ oe or better or M1 for (35) and $(2-4)$ oe seen
(b)	[y=] 4x + 7	5	B1 for [midpoint] $(-1, 3)$ soi M1 for [gradient of $l = \frac{4-2}{-5-3}$ oe M1 for gradient -1 / their $\left(-\frac{1}{4}\right)$ M1dep on at least M1 for their $(-1, 3)$ substituted into $y = their \ m \times x + c$ oe

(c)	(0, -8) and $(0, 16)$

B3 for (0, −8) or (0, 16) or for −8 and 16 OR **B2** for distance = [±]12 soi

B2 for distance = $[\pm]12$ soi or **M1** for $13^2 - (5[-0])^2$ oe **B1** for both answers (0, k), $k \ne 0$ or 4

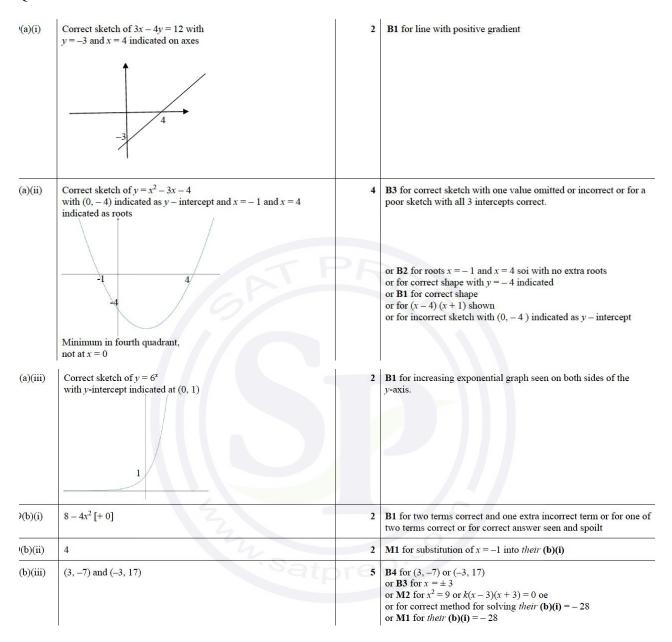
ALT METHOD

 $\boldsymbol{B3}$ for (0,-8) or (0 , 16) or for - 8 and 16 OR

M2 for $y^2 - 8y - 128$ [= 0] or for $(y - 4)^2 = 144$ or better or M1 for $13^2 = (-5 - 0)^2 + (4 - y)^2$ oe

B1 for both answers (0, k), $k \neq 0$ or 4

i(a)(i)	-3	1	PA
(a)(ii)	-1 1.55 to 1.6 4.4 to 4.45	3	B1 for each
(a)(iii)	-8	1	
(a)(iv)	Ruled line through origin intersecting curve once	2	B1 for ruled line through origin
(b)(i)	18	3	B2 for $6x - 12$ or B1 for $6x$ or -12
(b)(ii)	(2, -5)	2	B1 for each. If 0 scored, M1 for their $6x - 12 = 0$ or states $\frac{dy}{dx} = 0$
	[p=] 7 $[q=] 3$	2	B1 for each



(a)	(22, 11)	2	B1 for each value
(b)	$\frac{their11-3}{their22-2}$ oe or better	M1	
	$-\frac{1}{theirm}$	M1	
	Substitution of (12, 7) into $y = (their m)x + c$	M1	Accept $y - 7 = their m(x - 12)$ oe
	leading to $2y + 5x = 74$ final answer	A1	Without error or omission
(c)	32	1	
(d)	145	2	M1 for $\frac{1}{2} \times (their\ 32 - 3) \times 10$ oe or $\frac{1}{2} \times \sqrt{(7-3)^2 + (12-2)^2} \times \sqrt{(their\ 32 - 7)^2 + (2-12)^2}$ oe

Question 84

6x + 4

2

B1 for 6x or 4 or 6x + 4 with one extra term seen

Question 85

(a)(i)	$\frac{14}{18}$ oe	1	
(a)(ii)	17.5	4	M3 for $\frac{1}{2}(10+24)18+22\times24-134=40v$ oe
			or M2 for $\frac{1}{2}(10+24)18+22\times24$ oe
			or B2 for [distance covered by bus =] 700
			or M1 for correct method for any partial area for the car
1			or for 40v
(b)	92.8 or $92\frac{4}{5}$	3	M1 for $\frac{figs162[4]}{their10 \min 30 sec}$ oe
	34.80		M1 for correct conversion to km/h, e.g. $\times \frac{60}{1000}$

(a)(i)	28	1
(a)(ii)	Correct curve	4 B3FT for 9 or 10 correct points or B2FT for 7 or 8 correct points or B1FT for 5 or 6 correct points
a)(iii)	2.5 to 2.8 8.2 to 8.5	2 B1 for each value
(b)(i)	$2x^2 + 4x(9-x)$ oe	M1 Accept the sum of individual areas if done in smaller parts
	(iii) Correct curve 4 B3F or B or B or B (iii) 2.5 to 2.8 8.2 to 8.5 2 B1 f (iii) $2x^2 + 4x(9 - x)$ oe M1 According to $36x - 4x^2$ oe Leading to $36x - 2x^2$ A1 With Leading to $36x - 2x^2$ B1 f	A1 With intermediate step shown and brackets removed with n errors or omissions
(b)(ii)	144	3 B1 for $x = 6$ identified from graph or using calculus
		M1 for $36 \times their6 - 2 \times (their 6)^2$

)(a)(i)	4 or 5 or 7 or 8 or 9	1	
(a)(ii)	[a=] 3, [b=] 10	2	B1 for each or for <i>a</i> and <i>b</i> transposed
)(b)	$6x^5 - 30x^4$	B2	B1 for $6x^5$ or $-30x^4$
	their derivative = 0.	M1	
	(0, 0) and (5, -3125)	B2	B1 for $(5, -3125)$ or for $x = 0$ and $x = 5$

(a)	-3	3 B2 for $3x^2 - 6x$ or B1 for $3x^2 - kx$ or for $kx^2 - 6x$ or for $3x^2 - 6x + c$	
(b)	(0, -4) and $(2, -8)$	4 B3 for $x = 0$ and 2 or for $(2, -8)$ OR	
	RTF	M1 for <i>their</i> $3x^2 - 6x = 0$ or stating $\frac{dy}{dx} = 0$ oe	
		M1 for correct method to solve <i>their</i> $3x^2 - 6x = 0$	
(c)	Correct sketch	2 Max on negative y-axis and min in correct quadrant and extends into first quadrant	
		B1 for positive cubic graph and two turning points	
Que	stion 89		
1000			

5(a)	y = 4 oe	1	
5(b)	$[y =] -\frac{1}{2}x + 4$ final answer	2	B1 for grad = $-\frac{4}{8}$ oe soi or $[y =]kx + 4$
(e)(i)	Gradient = $\frac{-1}{their \text{ gradient in}(b)}$	M1	Accept e.g. $2 \times -\frac{1}{2} = -1$ oe or states negative reciprocal of $-\frac{1}{2} = 2$
	Substituting (2, 3) in their equation.	M1	$3 = 2 \times their \ m + c$
	leading to $y = 2x - 1$	A1	No errors or omissions
c)(ii)	3.35 or 3.354	5	B2 for $\left(\frac{1}{2}, 0\right)$ soi or <i>x</i> -coordinate of $D = \frac{1}{2}$ or M1 for $2x - 1 = 0$ M2 for $(2 - their \frac{1}{2})^2 + (3 - their 0)^2$ oe or M1 for $(2 - their \frac{1}{2})$ and $(3 - their 0)$ oe

(a)	$4x^3 - 16x \text{cao}$	2	M1 for $4x^3 + kx$ or $kx^3 - 16x$ or $4x^3 - 16x + k$ or $4x^3 - 16$ as final answers
!b	Their $\frac{dy}{dx} = 0$ or stating $\frac{dy}{dx} = 0$	B1	
	Correct method to solve <i>their</i> $4x^3 - 16x = 0$	M1	e.g. $4x(x^2 - 4)$ or $4x(x - 2)(x + 2)$ oe
	[x=]0,-2,2	A1	Or B1 for (-2, -11) and (2, -11)
	(0,5) $(-2,-11)$ $(2,-11)$	A1	
(c)	(0, 5) with correct reasoning	2	 M1 for any of correct use of 2nd derivative 12x² -16 evaluates correctly both values of y on either side evaluates correctly the gradient on either side reasonable correct sketch
Questi	ion 91		

(a)	10	3	M2 for $(17)^2 + (42)^2$ oe or M1 for (17) or (42) oe
(b)	$\frac{4}{3}$ or $\frac{8}{6}$	2	M1 for $\frac{17}{42}$ oe
(c)	$y = -\frac{3}{4}x - \frac{9}{4}$ or $4y + 3x + 9 = 0$ oe final answers	ref	B3 for $-\frac{3}{4}x - \frac{9}{4}$ OR B1 for midpoint $(1, -3)$ M1 for gradient $-\frac{3}{4}$ or $-\frac{1}{their}$ (b) M1 for substituting their $(1, -3)$ into $y = (their \ m)x + c$ or for their $m = \frac{y3}{x - 1}$ oe

(a)	-2.5 -2 -1	3	B1 for each
(b)	Correct curve	4	B3 FT for 8 or 7 correct plots B2 FT for 6 or 5 correct plots B1 FT for 4 or 3 correct plots
(c)	2.3 to 2.4	1	
(d)	ruled line $y = x - 1.5$	M2	M1 for $y = x - 1.5$ soi or for $2^x - 3 = x - 1.5$ seen. or $y = x + k$ or $y = kx - 1.5$ drawn Do not accept $y = -1.5$
	-1 and 1.55 to 1.7	A2	A1 for each

(a)(i)	Correct expansion of a pair of brackets
	7

$$x^{2}-4x+[1]x-4$$

or $x^{2}-4x-2x+8$
or $x^{2}+[1]x-2x-2$

M1 accept

$$x^2 - 3x - 4$$

or $x^2 - 6x + 8$
or $x^2 - [1]x - 2$

$$x^3 - 4x^2 + x^2 - 4x - 2x^2 + 8x - 2x + 8$$

leading to and stating $[y =]x^3 - 5x^2 + 2x + 8$

Accept

$$x^3 - 3x^2 - 4x - 2x^2 + 6x + 8$$

or $x^3 - 6x^2 + [1]x^2 + 8x - 6x + 8$
or $x^3 - [1]x^2 - 2x - 4x^2 + 4x + 8$
leading to and stating
 $[y =]x^3 - 5x^2 + 2x + 8$

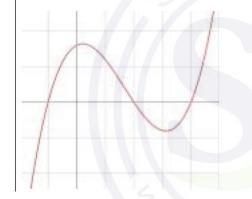
Correct labelled sketch positive cubic

Crossing x-axis at -1, 2 and 4 only Crossing y – axis at 8 only



B1 for positive cubic

B2 for three intercepts only with x -axis labelled at -1, 2 and 4



or **B1** for 1 or 2 correctly labelled x – intercepts

B1 for a single intercept on y-axis labelled at 8 but not if line y = 8

(b) $3x^2 - 10x - 8 = 0$

M3 B2 for derivative = $3x^2 - 10x + 2$ isw OR

B1 for derivative with $3x^2$ or -10x given in expression isw

M1dep on B1 for *their* first derivative = 10

$$x = 4$$
 and $x = -\frac{2}{3}$

B1

$$(4, 0)$$
 and $\left(-\frac{2}{3}, \frac{112}{27}\right)$ oe

B1

$$[y =] 10x - 40$$

and

$$[y=] \ 10x + \frac{292}{27}$$

B2 B1 for each

or for two different equations of the form [y =] 10x + c (c must be numeric)

or for
$$c = -40$$
 and $\frac{292}{27}$

i(a)	(5, 2) (2, -2)	4	B3 for 3 correct values or answers for C and D reversed or correct coordinates given on diagram wrongly labelled or B2 for one correct coordinate pair correctly labelled or M2 for A,B,C and D correctly plotted or M1 for A and B correctly plotted If 0 or 1 scored instead award SC2 for answers (-3, 8) and (-6, 4) or answers (1.5,1.5) and (-2.5, 4.5)
(b)(i)	(2.5, 3.5) oe	2	B1 for each
(b)(ii)	7.07 or 7.071	3	M2 for $(61)^2 + (4-3)^2$ oe or M1 for (61) or $(4-3)$ oe
(b)(iii)	1 7	2	M1 for $\frac{4-3}{6-1}$ oe
(b)(iv)	$y = \frac{1}{7}x - \frac{2}{7}$ or $7y = x - 2$ oe	3	M1 for gradient = their (iii)
	final answer		M1dep for substituting (2, 0) in a linear equation with their <i>m</i> allow if (2,0) satisfies y=(their(b)(iii) gradient)x+c