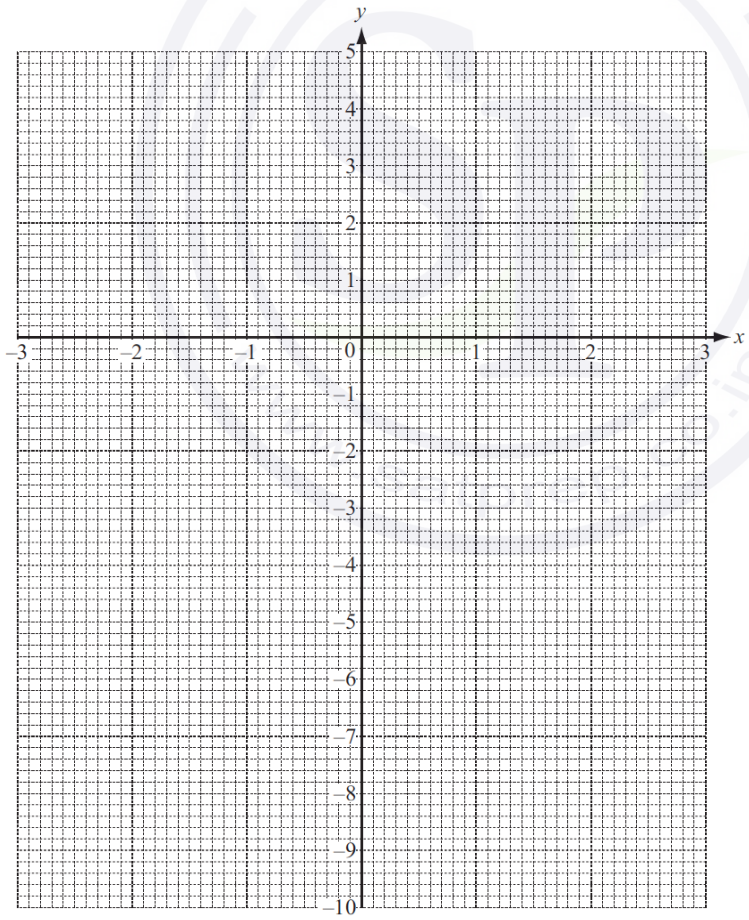
Extended Mathematics
Topic : Graph
Year :May 2013 -May 2023
Paper - 4
Questions Booklet

Question 1

(a) Complete this table of values for the function $f(x) = \frac{1}{x} - x^2$, $x \neq 0$

x	–3	–2	–1	–0.5	–0.2	0.2	0.5	1	2	3
f(x)	–9.33	–4.5	–2	–2.25		4.96			–3.5	–8.67

(b) Draw the graph of $f(x) = \frac{1}{x} - x^2$, $-3 \leq x \leq -0.2$ and $0.2 \leq x \leq 3$



[5]

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(c) Use your graph to solve $f(x) = -3$.

Answer(c) $x =$ ..... or $x =$ ..... or $x =$ ..... [3]

(d) By drawing a suitable line on your graph, solve the equation $f(x) = 2x - 2$.

Answer(d) $x =$ ..... or $x =$ ..... or $x =$ ..... [3]

(e) By drawing a suitable tangent, work out an estimate of the gradient of the curve at the point where $x = -2$.

You must show your working.

Answer(e) ..... [3]

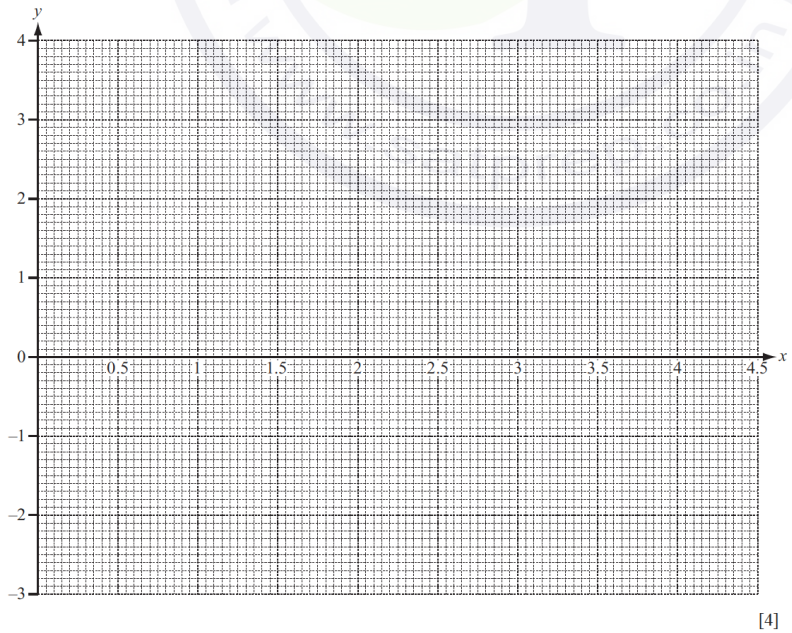
Question 2

The table shows some values for the function $y = 11x - 2x^2 - 12$ for $1 \leqslant x \leqslant 4.5$.

x	1	1.5	2	2.5	3	3.5	4	4.5
y	–3		2	3	3			

(a) Complete the table of values. [3]

(b) On the grid below, draw the graph of $y = 11x - 2x^2 - 12$ for $1 \leqslant x \leqslant 4.5$.



[4]

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(c) By drawing a suitable line, use your graph to solve the equation $11x - 2x^2 = 11$.

Answer(c) $x =$ ..... or $x =$ ..... [2]

(d) The line $y = mx + 2$ is a tangent to the curve $y = 11x - 2x^2 - 12$ at the point P.

By drawing this tangent,

(i) find the co-ordinates of the point P,

Answer(d)(i) (..... , ...) [2]

(ii) work out the value of m.

Answer(d)(ii) $m =$ ..... [2]

Question 3

$f(x) = 3 - x - x^2$ $\quad$ $g(x) = 3^x$

(a) Complete the tables of values for $f(x)$ and $g(x)$.

x	–1.5	–1	–0.5	0	0.5	1	1.5
$f(x)$	2.25	3	3.25		2.25	1	–0.75

x	–1.5	–1	–0.5	0	0.5	1	1.5
$g(x)$	0.19		0.58		1.73	3	5.20

[3]

(b) On the grid, draw the graphs of $y = f(x)$ and $y = g(x)$ for $-1.5 \leqslant x \leqslant 1.5$.

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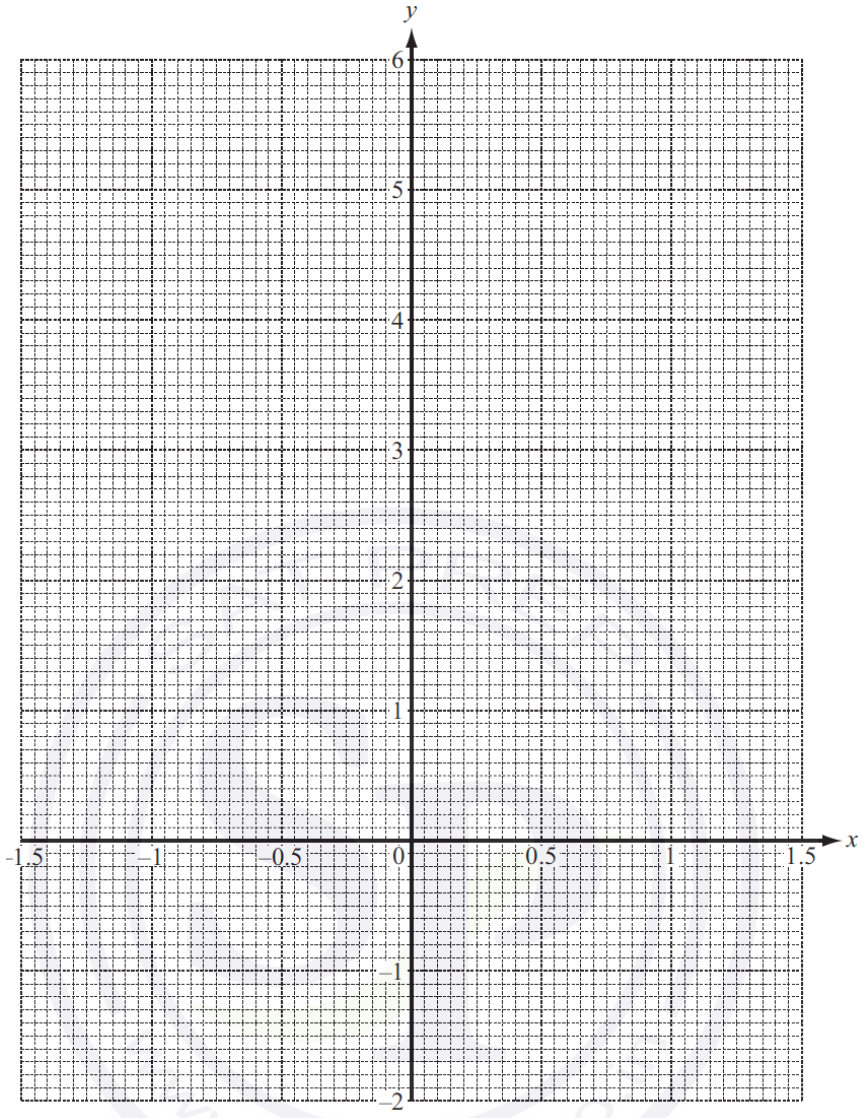

[6]

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(c) For $-1.5 \leqslant x \leqslant 1.5$, use your graphs to solve

(i) $f(x) = 0$,

Answer(c)(i) $x =$ ..... [1]

(ii) $g(x) = 4$,

Answer(c)(ii) $x =$ ..... [1]

(iii) $f(x) = g(x)$.

Answer(c)(iii) $x =$ ..... [1]

(d) By drawing a suitable tangent, find an estimate of the gradient of the graph of $y = f(x)$ when $x = 0.5$.

Answer(d) ..... [3]

Question 4

The co-ordinates of P are $(-4, -4)$ and the co-ordinates of Q are $(8, 14)$.

(i) Find the gradient of the line PQ.

Answer(a)(i) ..... [2]

(ii) Find the equation of the line PQ.

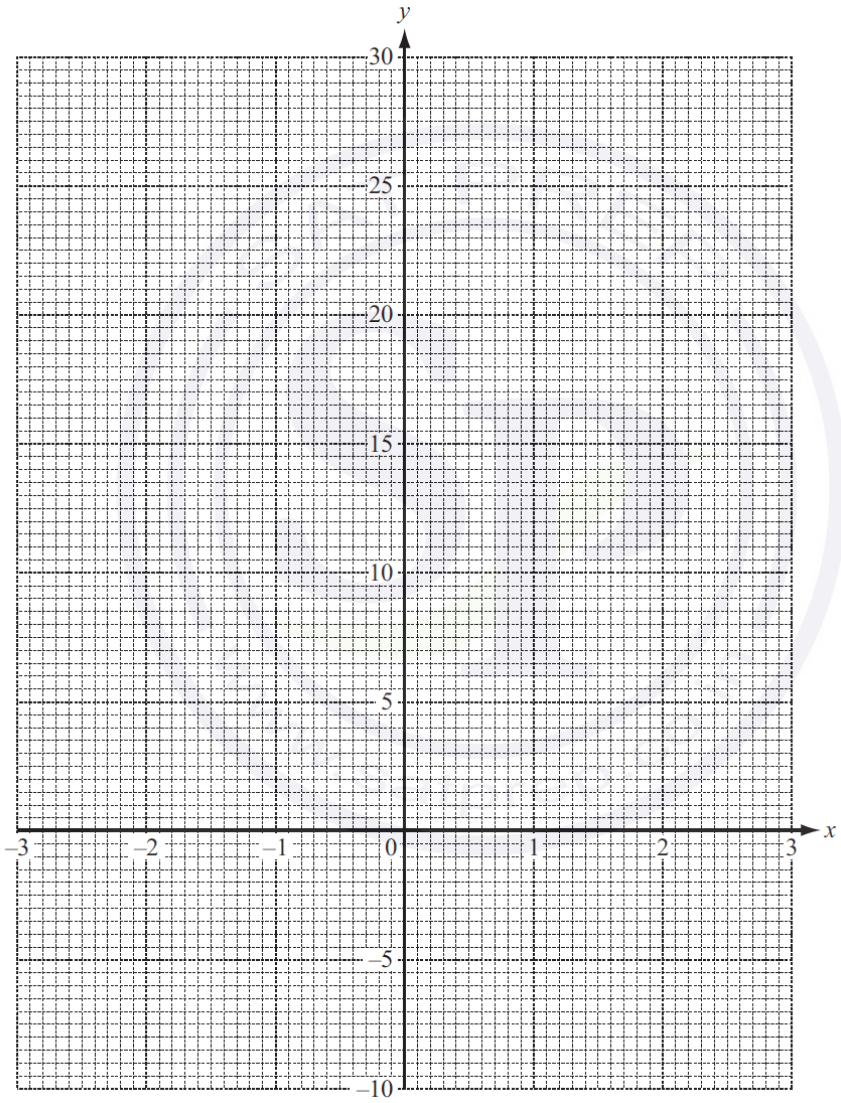
Answer(a)(ii) ..... [2]

Question 5

(a) Complete the table of values for $y = \frac{2}{x^2} - \frac{1}{x} - 3x,\ x \neq 0$

x	–3	–2	–1	–0.5	–0.3		0.3	0.5	1	2	3
y	9.6		6		26.5		18.0		–2	–6	–9.1

(b) Draw the graph of $y = \frac{2}{x^2} - \frac{1}{x} - 3x, -3 \leq x \leq -0.3$ and $0.3 \leq x \leq 3$



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(c) Use your graph to solve these equations.

(i) $\frac{2}{x^2} - \frac{1}{x} - 3x = 0$

Answer(c)(i) $x =$ ..... [1]

(ii) $\frac{2}{x^2} - \frac{1}{x} - 3x - 7.5 = 0$

Answer(c)(ii) $x =$ ..... or $x =$ ..... or $x =$ ..... [3]

(d) **(i)** By drawing a suitable straight line on the graph, solve the equation $\frac{2}{x^2} - \frac{1}{x} - 3x = 10 - 3x$.

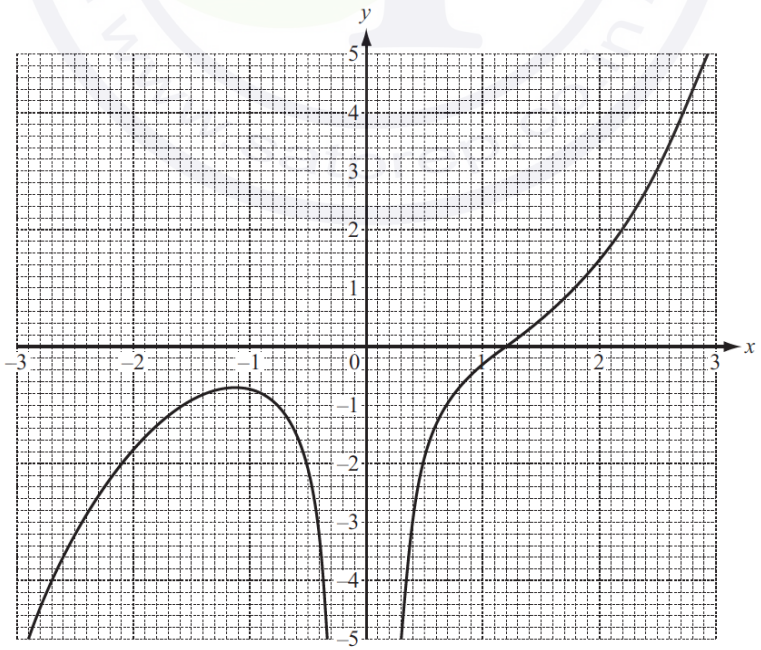
Answer(d)(i) $x =$ ..... or $x =$ ..... [4]

(ii) The equation $\frac{2}{x^2} - \frac{1}{x} - 3x = 10 - 3x$ can be written in the form $ax^2 + bx + c = 0$ where a, b and c are integers.

Find the values of a, b and c.

Answer(d)(ii) $a =$ ..... , $b =$ ..... , $c =$ ..... [3]

Question 6



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The diagram shows the graph of $y = f(x)$ for $-3 \leqslant x \leqslant 3$.

(i) Find f(2).

Answer(a)(i) ..... [1]

(ii) Solve the equation $f(x) = 0$.

Answer(a)(ii) $x =$ ..... [1]

(iii) Write down the value of the largest **integer**, k, for which the equation $f(x) = k$ has 3 solutions.

Answer(a)(iii) $k =$ ..... [1]

(iv) By drawing a suitable straight line, solve the equation $f(x) = x$.

Answer(a)(iv) $x =$ ..... or $x =$ ..... or $x =$ ..... [3]

Question 7

The table shows some values for the function $y = \frac{1}{x^2} + x$, $x \neq 0$.

x	–3	–2	–1	–0.5	0.5	1	2	3	4
y	–2.89	–1.75		3.5		2	2.25		4.06

(a) Complete the table of values. [3]

(b) On the grid, draw the graph of $y = \frac{1}{x^2} + x$ for $-3 \leqslant x \leqslant -0.5$ and $0.5 \leqslant x \leqslant 4$.

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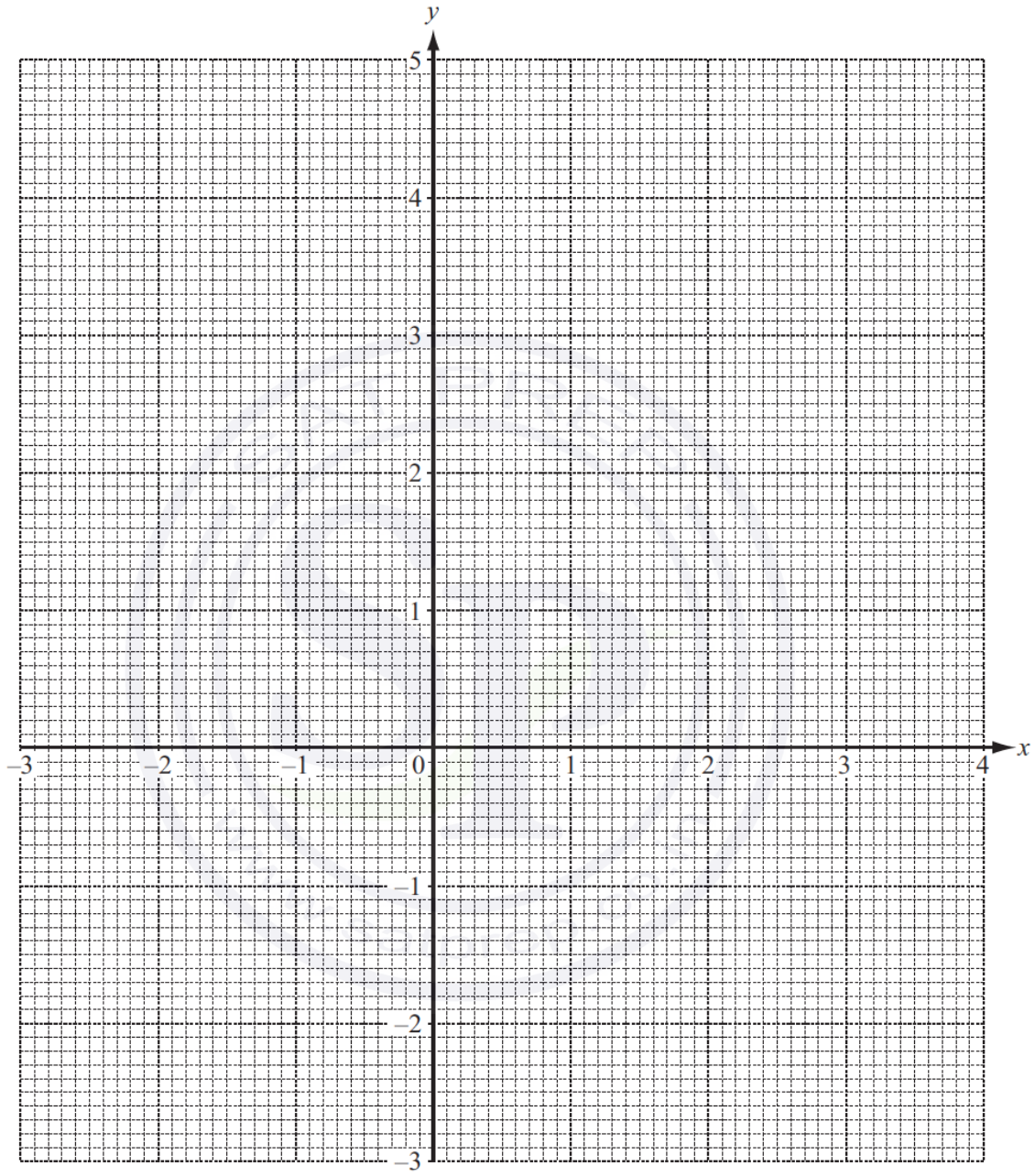

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(c) Use your graph to solve the equation $\frac{1}{x^2} + x - 3 = 0$.

Answer(c) $x =$ ..... or $x =$ ..... or $x =$ ..... [3]

(d) Use your graph to solve the equation $\frac{1}{x^2} + x = 1 - x$.

Answer(d) $x =$ ..... [3]

(e) By drawing a suitable tangent, find an estimate of the gradient of the curve at the point where $x = 2$.

Answer(e) ..... [3]

(f) Using algebra, show that you can use the graph at $y = 0$ to find $\sqrt[3]{-1}$.

Answer(f)

[3]

Question 7

$f(x) = \frac{1}{x^2} - 2x, \quad x \neq 0$

(a) Complete the table of values for $f(x)$.

x	–3	–2.5	–2	–1.5	–1	–0.5	0.4	0.5	1	1.5	2
$f(x)$	6.1	5.2	4.3	3.4		5	5.5			–2.6	–3.8

[3]

(b) On the grid, draw the graph of $y = f(x)$ for $-3 \leqslant x \leqslant -0.5$ and $0.4 \leqslant x \leqslant 2$.

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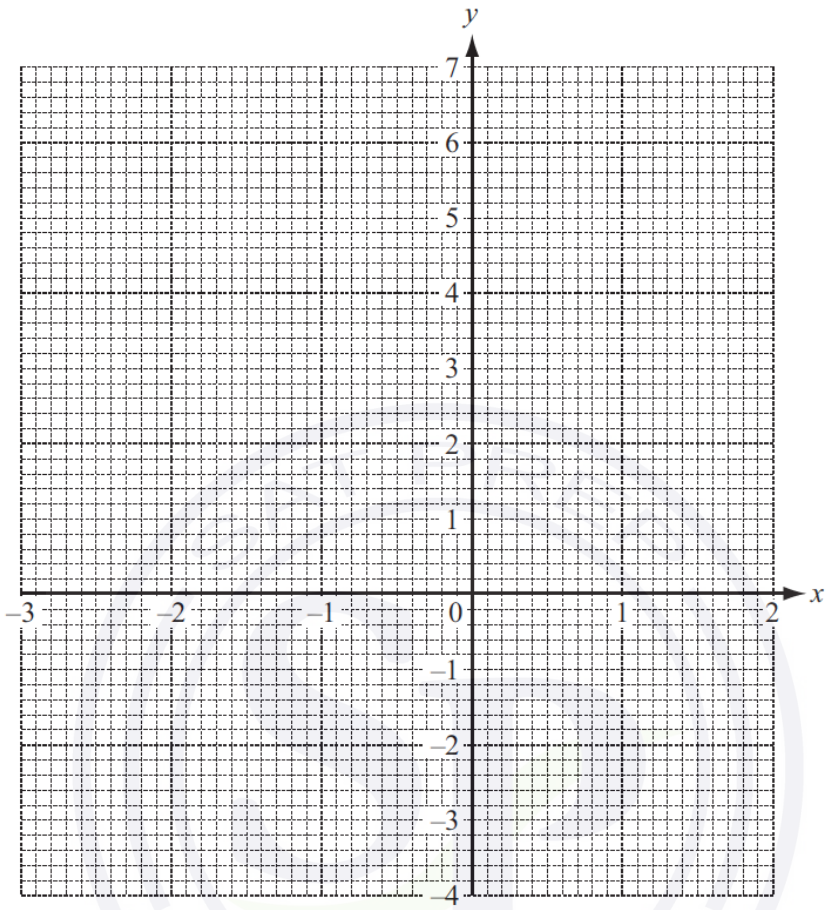

[5]

(c) Solve the equation $f(x) = 2$.

Answer(c) $x =$ ..... [1]

(d) Solve the equation $f(x) = 2x + 3$.

Answer(d) $x =$ ..... [3]

(e) **(i)** Draw the tangent to the graph of $y = f(x)$ at the point where $x = -1.5$. [1]

(ii) Use the tangent to estimate the gradient of the graph of $y = f(x)$ where $x = -1.5$.

Answer(e)(ii) ..... [2]

Question 8

(a) Complete the table of values for $y = x^3 - 3x + 1$.

x	–2.5	–2	–1.5	–1	–0.5	0	0.5	1	1.5	2	2.5
y	–7.125	–1		3		1	–0.375	–1	–0.125	3	9.125

[2]

(b) Draw the graph of $y = x^3 - 3x + 1$ for $-2.5 \leqslant x \leqslant 2.5$.

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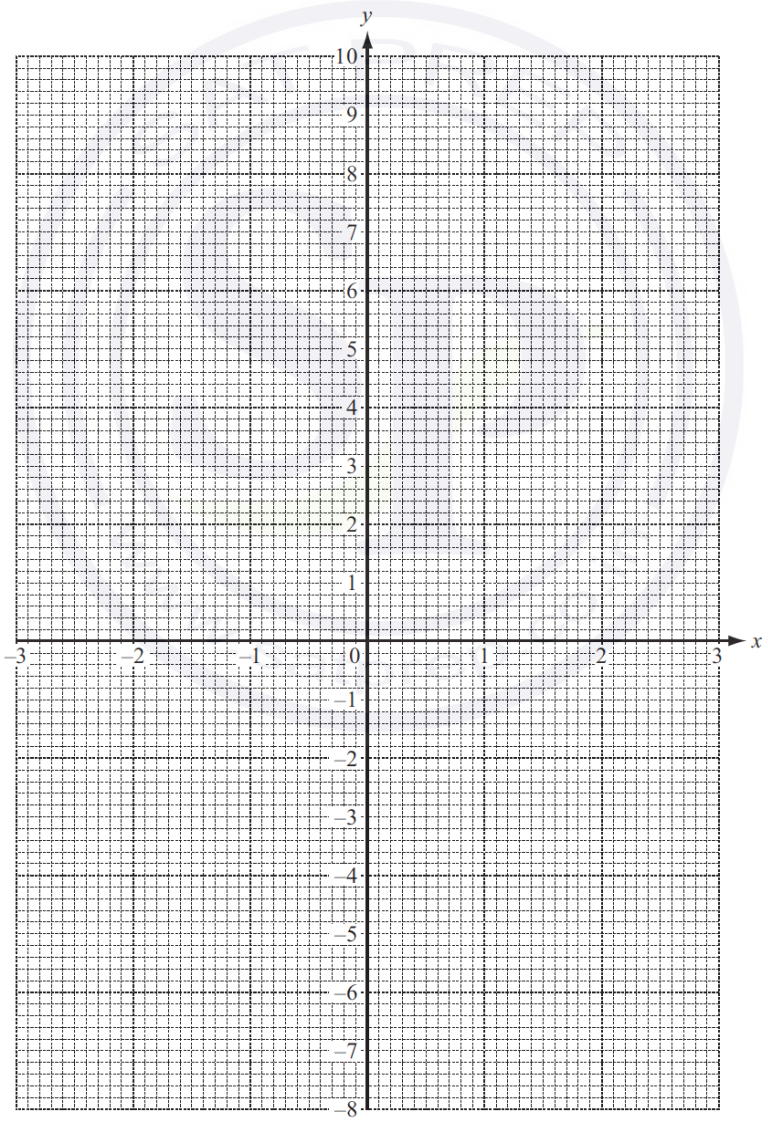

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(c) By drawing a suitable tangent, estimate the gradient of the curve at the point where $x = 2$.

Answer(c) ..... [3]

(d) Use your graph to solve the equation $x^3 - 3x + 1 = 1$.

Answer(d) $x =$ ..... or $x =$ ..... or $x =$ ..... [2]

(e) Use your graph to complete the inequality in k for which the equation

$x^3 - 3x + 1 = k$ has three different solutions.

Answer(e) ..... $< k <$ ..... [2]

Question 9

A straight line joins the points (–1, –4) and (3, 8).

(i) Find the midpoint of this line.

Answer(a)(i) (..... , ...) [2]

(ii) Find the equation of this line.
Give your answer in the form $y = mx + c$.

Answer(a)(ii) $y =$ ..... [3]

Question 10

$f(x) = 5x^3 - 8x^2 + 10$

(a) Complete the table of values.

x	–1.5	–1	–0.5	0	0.5	0.75	1	1.5	2
$f(x)$	–24.9			10	8.6	7.6	7		18

[3]

(b) Draw the graph of $y = f(x)$ for $-1.5 \leqslant x \leqslant 2$.

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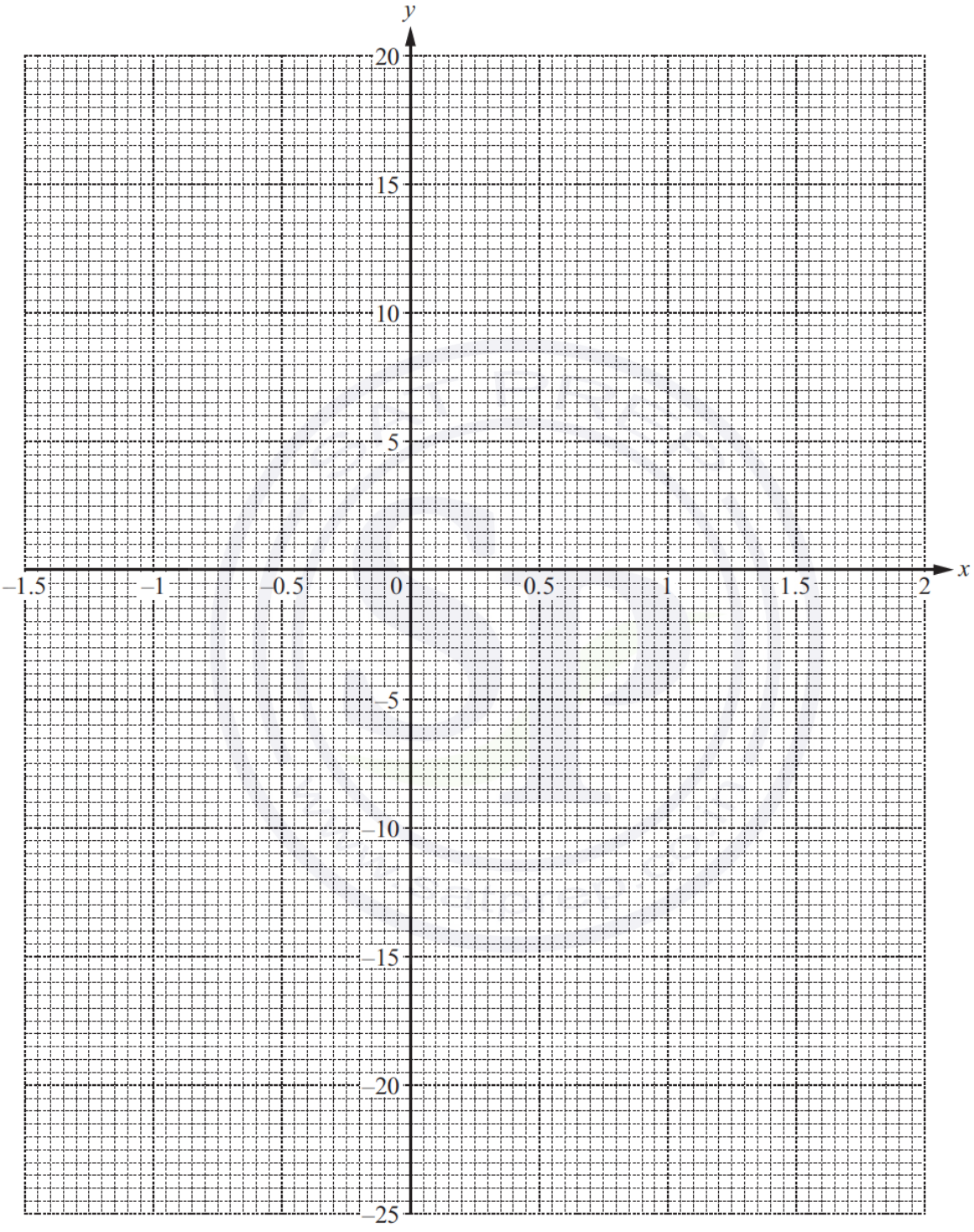

[4]

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(c) Use your graph to find an **integer** value of k so that $f(x) = k$ has

(i) exactly one solution,

Answer(c)(i) $k =$ ..... [1]

(ii) three solutions.

Answer(c)(ii) $k =$ ..... [1]

(d) By drawing a suitable straight line on the graph, solve the equation $f(x) = 15x + 2$ for $-1.5 \leqslant x \leqslant 2$.

Answer(d) $x =$ ..... or $x =$ ..... [4]

(e) Draw a tangent to the graph of $y = f(x)$ at the point where $x = 1.5$.

Use your tangent to estimate the gradient of $y = f(x)$ when $x = 1.5$.

Answer(e) ..... [3]

Question 11

The table shows some values of $y = x^3 + 3x^2 - 2$.

x	–3	–2.5	–2	–1.5	–1	–0.5	0	0.5	1
y	–2	1.13		1.38		–1.38		–1.13	

(a) Complete the table of values. [3]

(b) On the grid, draw the graph of $y = x^3 + 3x^2 - 2$ for $-3 \leqslant x \leqslant 1$.

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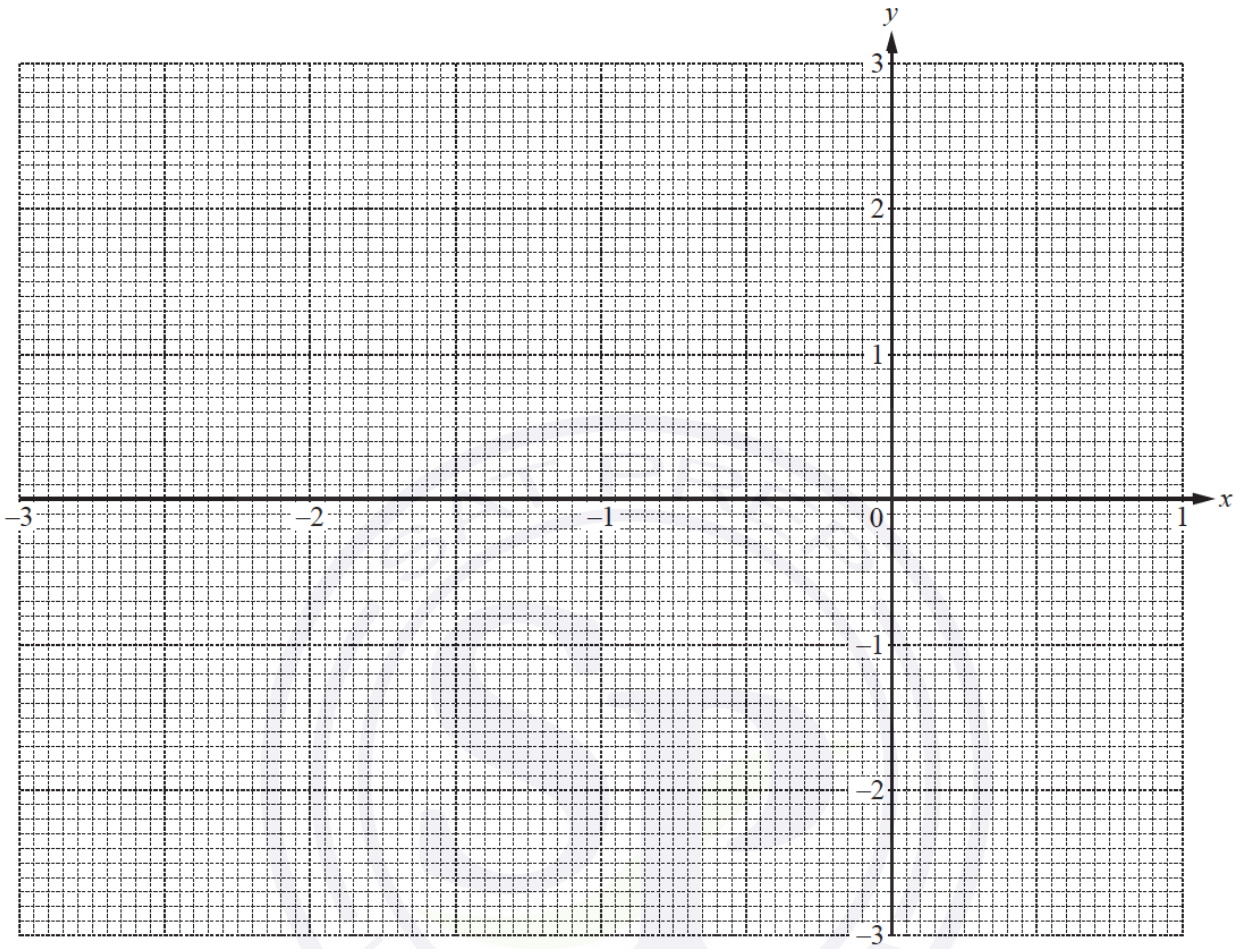


[4]

(c) By drawing a suitable line, solve the equation $x^3 + 3x^2 - 2 = \frac{1}{2}(x + 1)$.

Answer(c) $x =$ ..... or $x =$ ..... or $x =$ ..... [4]

(d) By drawing a suitable tangent, find an estimate of the gradient of the curve at the point where $x = -1.75$.

Answer(d) ..... [3]

Question 12

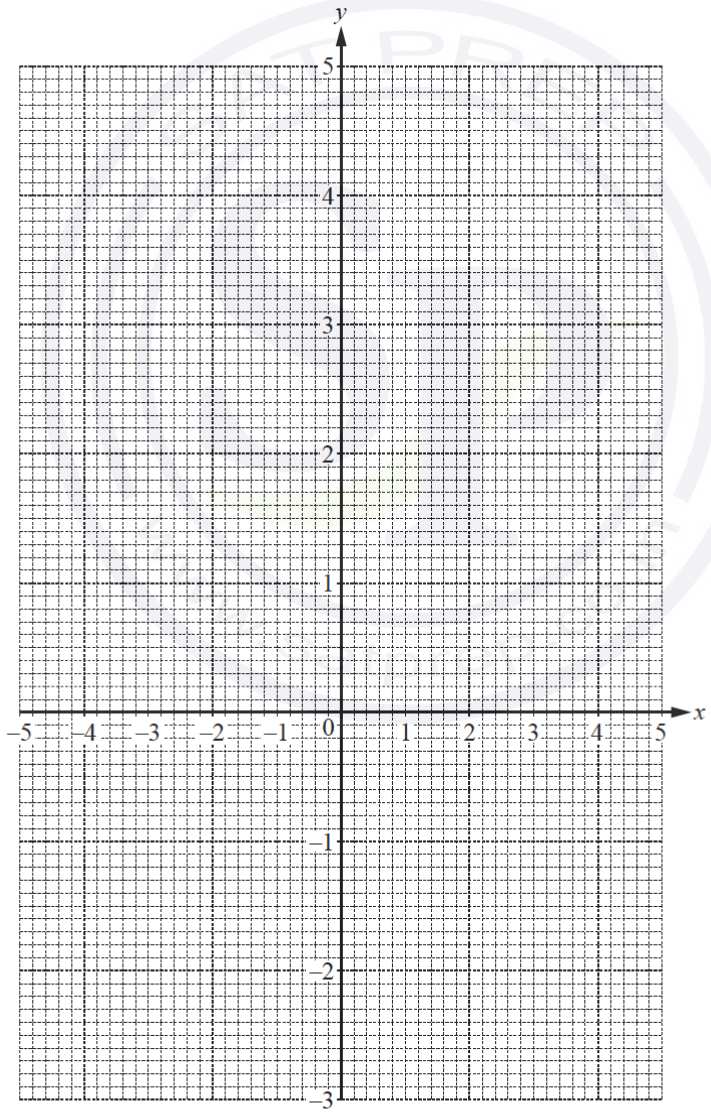
$$f(x) = \frac{8}{x^2} + \frac{x}{2}, \quad x \neq 0.$$

(a) Complete the table of values for f(x).

x	–5	–4	–3	–2	–1.5		1.5	2	2.5	3	3.5	4	5
f(x)	–2.2	–1.5	–0.6		2.8		4.3		2.5	2.4	2.4		2.8

[3]

(b) On the grid, draw the graph of $y = f(x)$ for $-5 \leqslant x \leqslant -1.5$ and $1.5 \leqslant x \leqslant 5$.



[5]

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(c) Solve $f(x) = 0$.

Answer(c) $x =$ ..... [1]

(d) By drawing a suitable line on the grid, solve the equation $f(x) = 1 - x$.

Answer(d) $x =$ ..... [3]

(e) By drawing a tangent at the point $(-3, -0.6)$, estimate the gradient of the graph of $y = f(x)$ when $x = -3$.

Answer(e) ..... [3]

Question 13

$$y = x^2 - 2x + \frac{12}{x}, \quad x \neq 0$$

(a) Complete the table of values.

x	–4	–3	–2	–1	–0.5		0.5	1	2	3	4
y	21	11		–9	–22.75		23.25	11	6		11

[2]

(b) On the grid, draw the graph of $y = x^2 - 2x + \frac{12}{x}$ for $-4 \leqslant x \leqslant -0.5$ and $0.5 \leqslant x \leqslant 4$.

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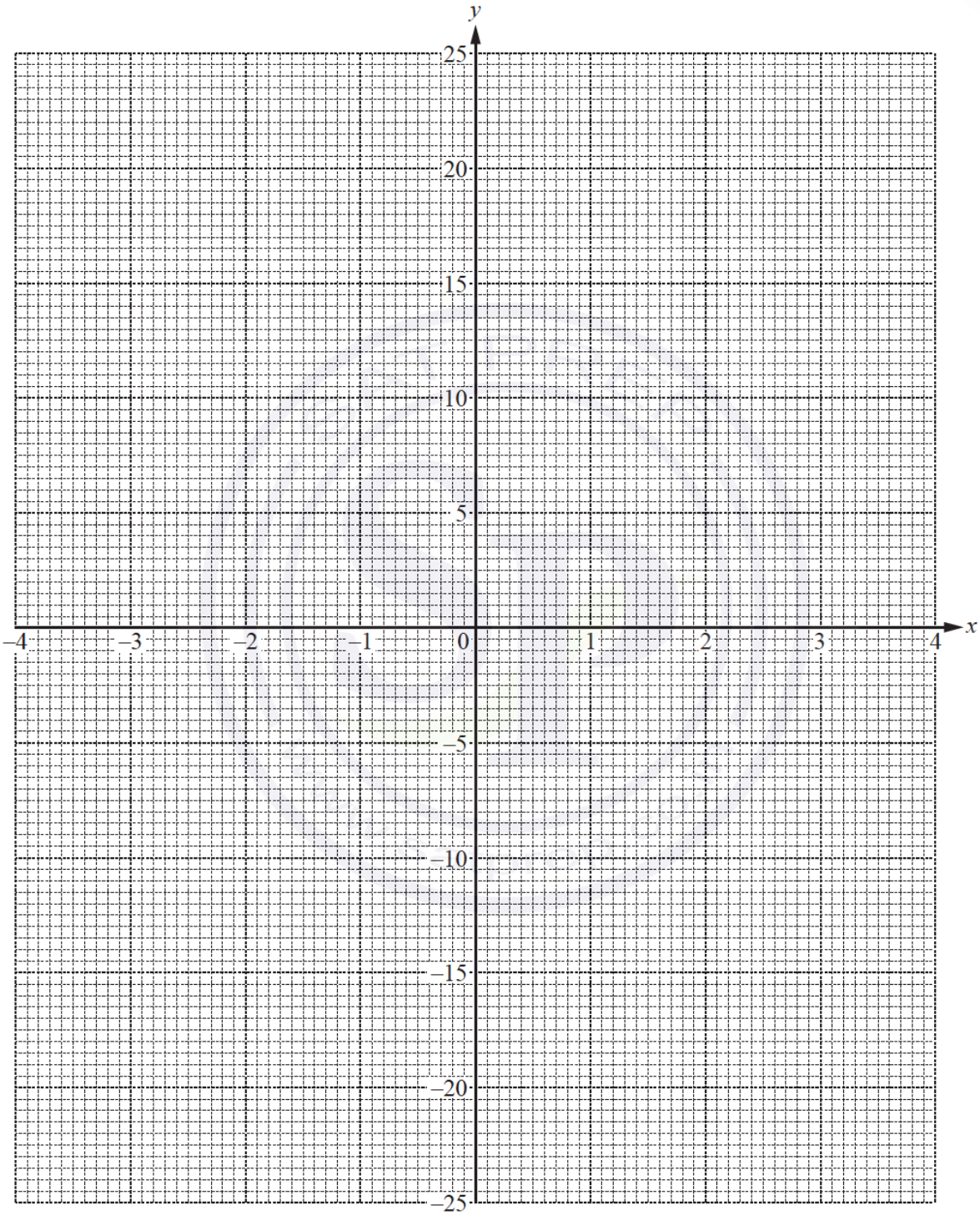

[5]

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(c) By drawing a suitable tangent, find an estimate of the gradient of the graph at the point (1, 11).

Answer(c) ..... [3]

(d) The equation $x^2 - 2x + \frac{12}{x} = k$ has exactly two distinct solutions.

Use the graph to find

(i) the value of k,

Answer(d)(i) k = ... [1]

(ii) the solutions of $x^2 - 2x + \frac{12}{x} = k$.

Answer(d)(ii) x = ... or x = ... [2]

(e) The equation $x^3 + ax^2 + bx + c = 0$ can be solved by drawing the line $y = 3x + 1$ on the grid.

Find the value of a, the value of b and the value of c.

Answer(e) a = ...

b = ...

c = ... [3]

Question 14

2 The table shows some values for $y = x^2 - \frac{1}{2x}$, $x \neq 0$.

x	–2	–1.5	–1	–0.5	–0.25	–0.2	0.2	0.25	0.5	1	1.5	2
y	4.25	2.58			2.06	2.54	–2.46	–1.94			1.92	3.75

(a) Complete the table of values. [4]

(b) On the grid, draw the graph of $y = x^2 - \frac{1}{2x}$ for $-2 \leqslant x \leqslant -0.2$ and $0.2 \leqslant x \leqslant 2$.

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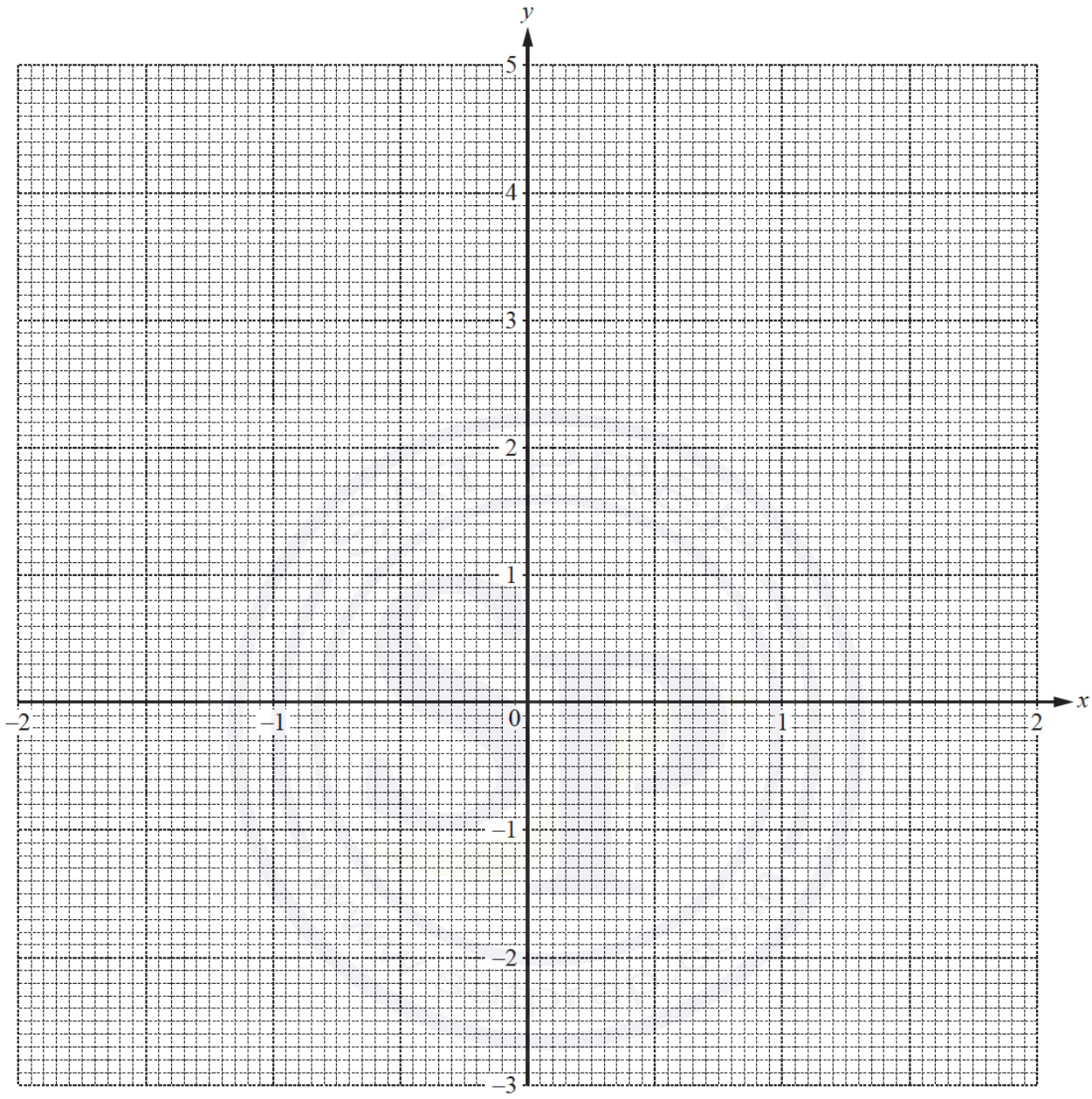


[5]

(c) By drawing a suitable line, use your graph to solve the equation $x^2 - \frac{1}{2x} = 2$.

Answer(c) $x =$ ..... or $x =$ ..... or $x =$ ..... [3]

Answer(e) ..... [3]

Question 15

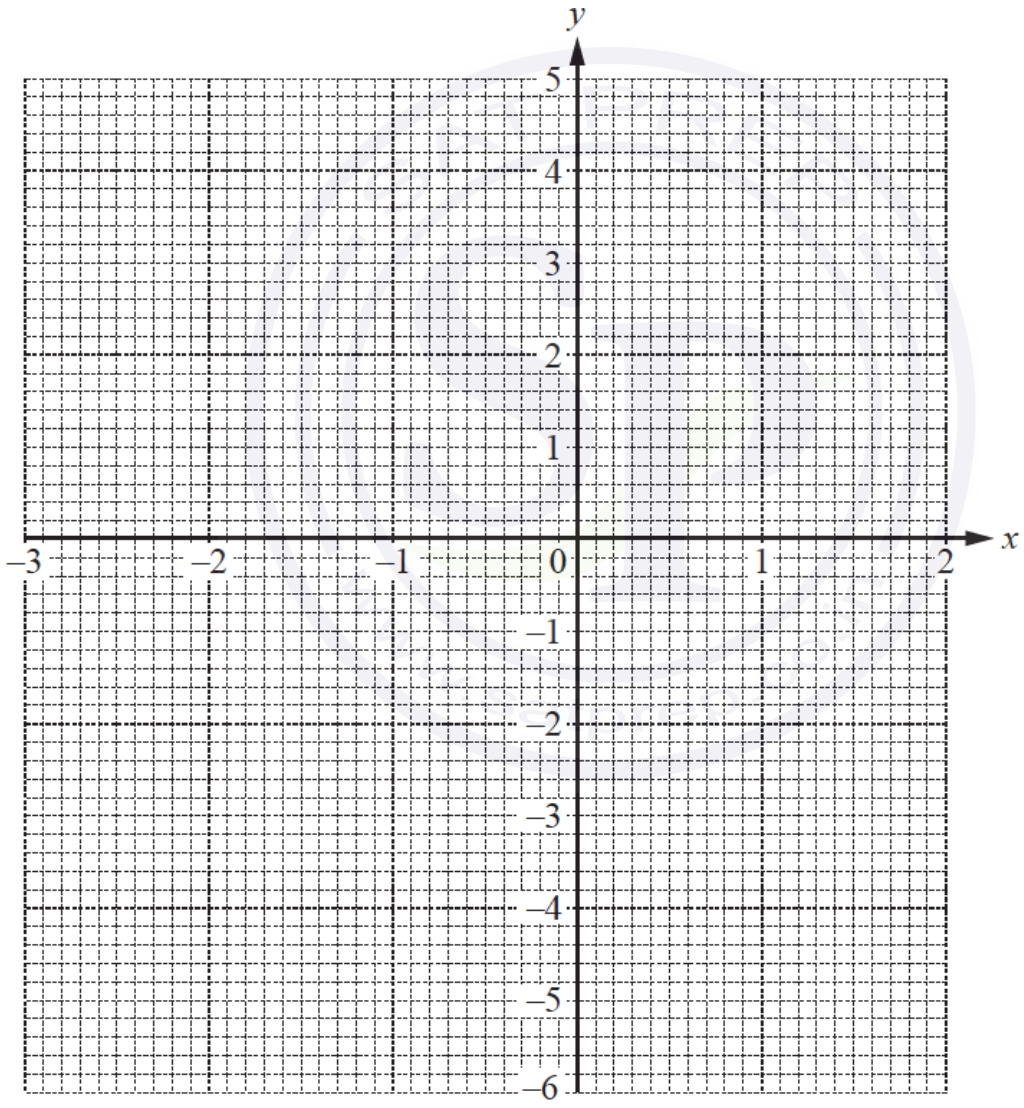
$$f(x) = x - \frac{1}{2x^2}, \quad x \neq 0$$

(a) Complete the table of values.

x	–3	–2	–1.5	–1	–0.5	–0.3		0.3	0.5	1	1.5	2
$f(x)$	–3.1	–2.1	–1.7		–2.5	–5.9		–5.3	–1.5		1.3	1.9

[2]

(b) On the grid, draw the graph of $y = f(x)$ for $-3 \leqslant x \leqslant -0.3$ and $0.3 \leqslant x \leqslant 2$.



[5]

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(c) Use your graph to solve the equation $f(x) = 1$.

Answer(c) $x =$ ..... [1]

(d) There is only one negative integer value, k, for which $f(x) = k$ has only one solution for all real x.

Write down this value of k.

Answer(d) $k =$ ..... [1]

(e) The equation $2x - \frac{1}{2x^2} - 2 = 0$ can be solved using the graph of $y = f(x)$ and a straight line graph.

(i) Find the equation of this straight line.

Answer(e)(i) $y =$ ..... [1]

(ii) On the grid, draw this straight line and solve the equation $2x - \frac{1}{2x^2} - 2 = 0$.

Answer(e)(ii) $x =$ ..... [3]

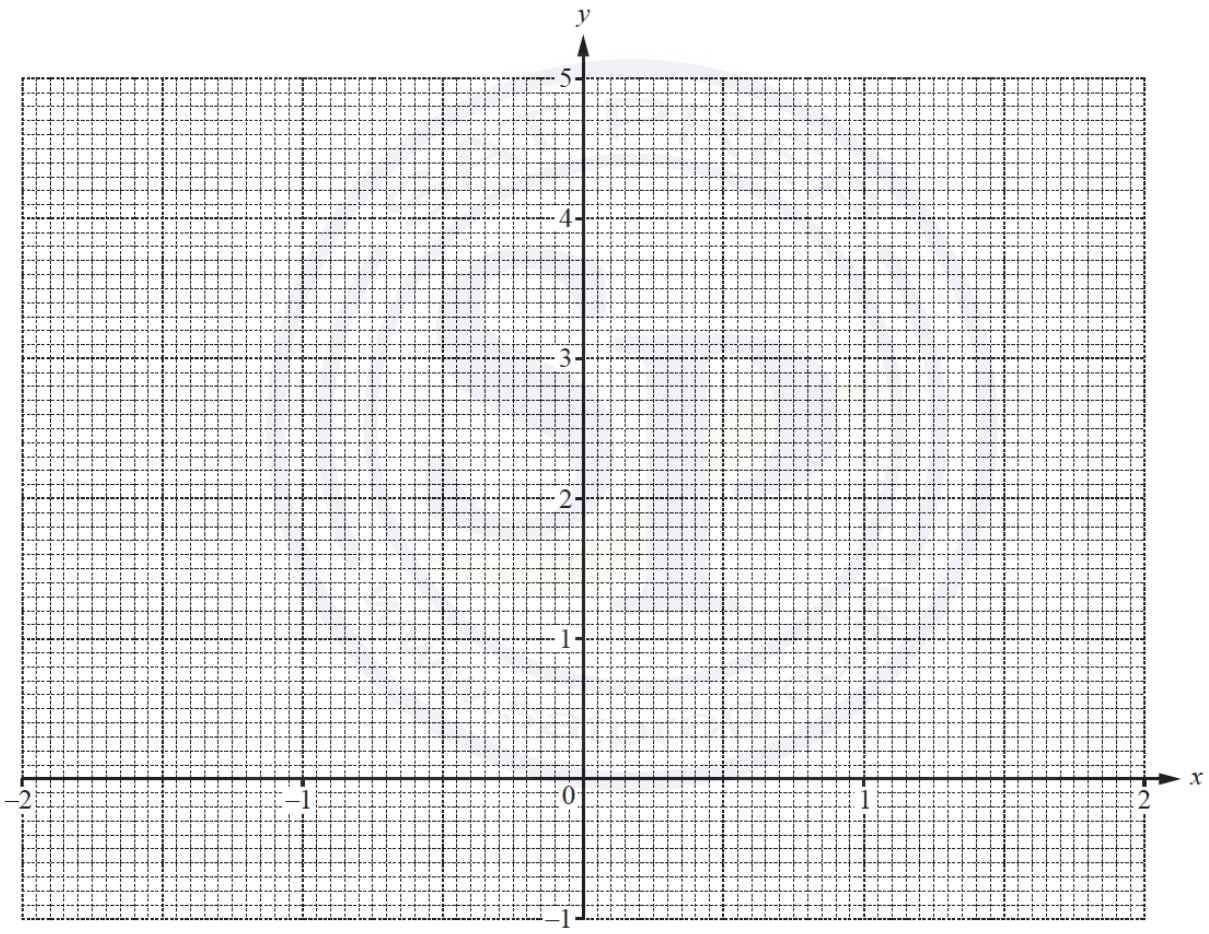
Question 16

The table shows some values for $y = x^3 - 3x + 2$.

x	–2	–1.5	–1	–0.5	0	0.5	1	1.5	2
y		3.125		3.375	2		0		4

(a) Complete the table of values. [4]

(b) On the grid, draw the graph of $y = x^3 - 3x + 2$ for $-2 \leqslant x \leqslant 2$.



[4]

(c) By drawing a suitable line, solve the equation $x^3 - 3x + 2 = x + 1$ for $-2 \leqslant x \leqslant 2$.

Answer(c) $x =$ ..... or $x =$ ..... [3]

(d) By drawing a suitable tangent, find an estimate of the gradient of the curve at the point where $x = -1.5$.

Answer(d) ..... [3]

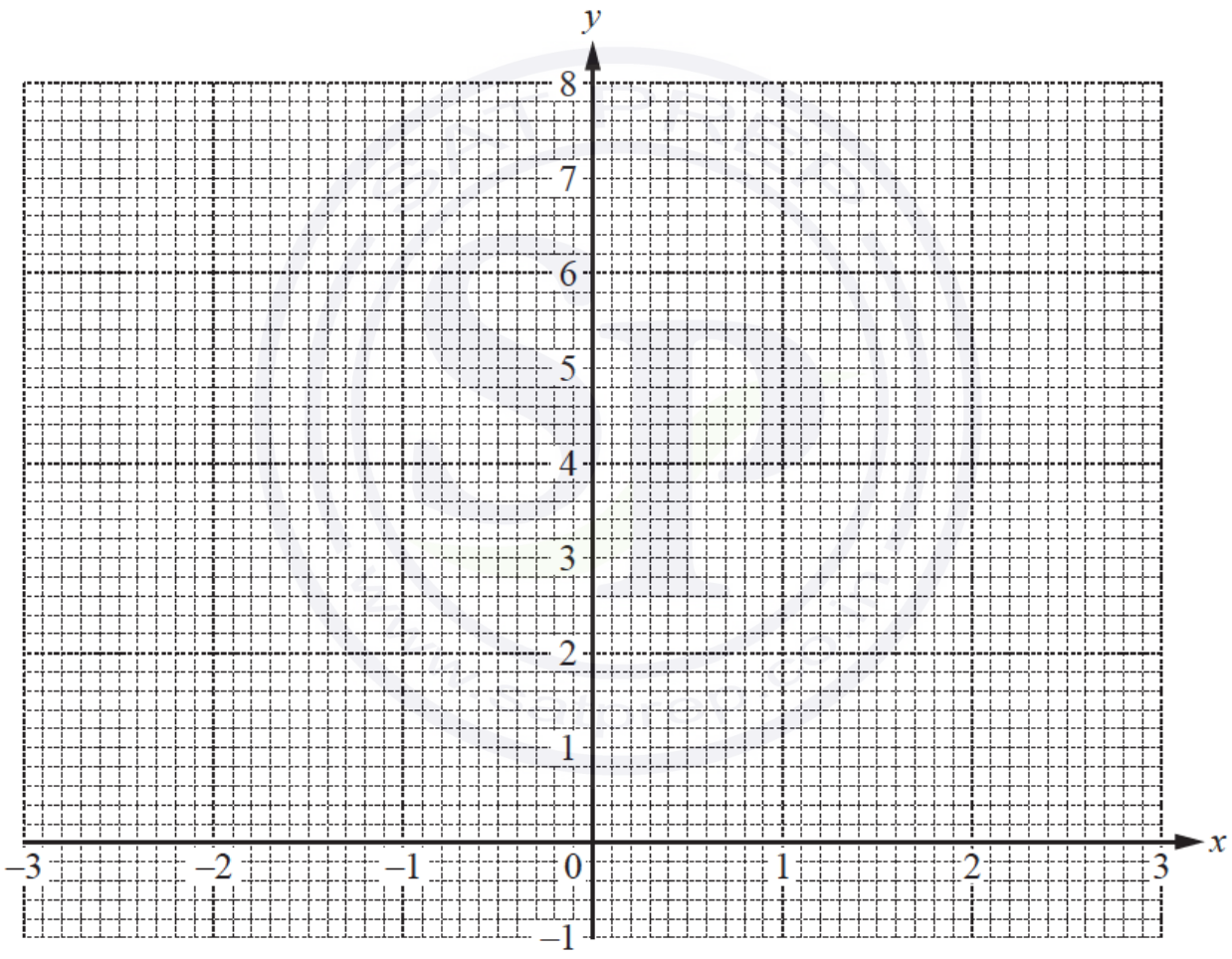
Question 17

(i) Complete this table of values for $y = 2^x$.

x	–3	–2	–1	0	1	2	3
y	0.125		0.5		2	4	8

[2]

(ii) On the grid, draw the graph of $y = 2^x$ for $-3 \leqslant x \leqslant 3$.



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[4]

(iii) Use your graph to solve $2^x = 5$.

Answer(c)(iii) $x =$ ..... [1]

(iv) Find the equation of the line joining the points (1, 2) and (3, 8).

Answer(c)(iv) ..... [3]

(v) By drawing a suitable line on your graph, solve $2^x - 2 - x = 0$.

Answer(c)(v) $x =$ ..... or $x =$ ..... [2]

Question 18

The table shows some values of $y = x + \frac{1}{x^2}$, $x \neq 0$.

x	–2	–1.5	–1	–0.75	–0.5		0.5	0.75	1	1.5	2	3
y	–1.75	–1.06	0	1.03			4.50	2.53	2		2.25	

(a) Complete the table of values. [3]

(b) On the grid, draw the graph of $y = x + \frac{1}{x^2}$ for $-2 \leqslant x \leqslant -0.5$ and $0.5 \leqslant x \leqslant 3$.

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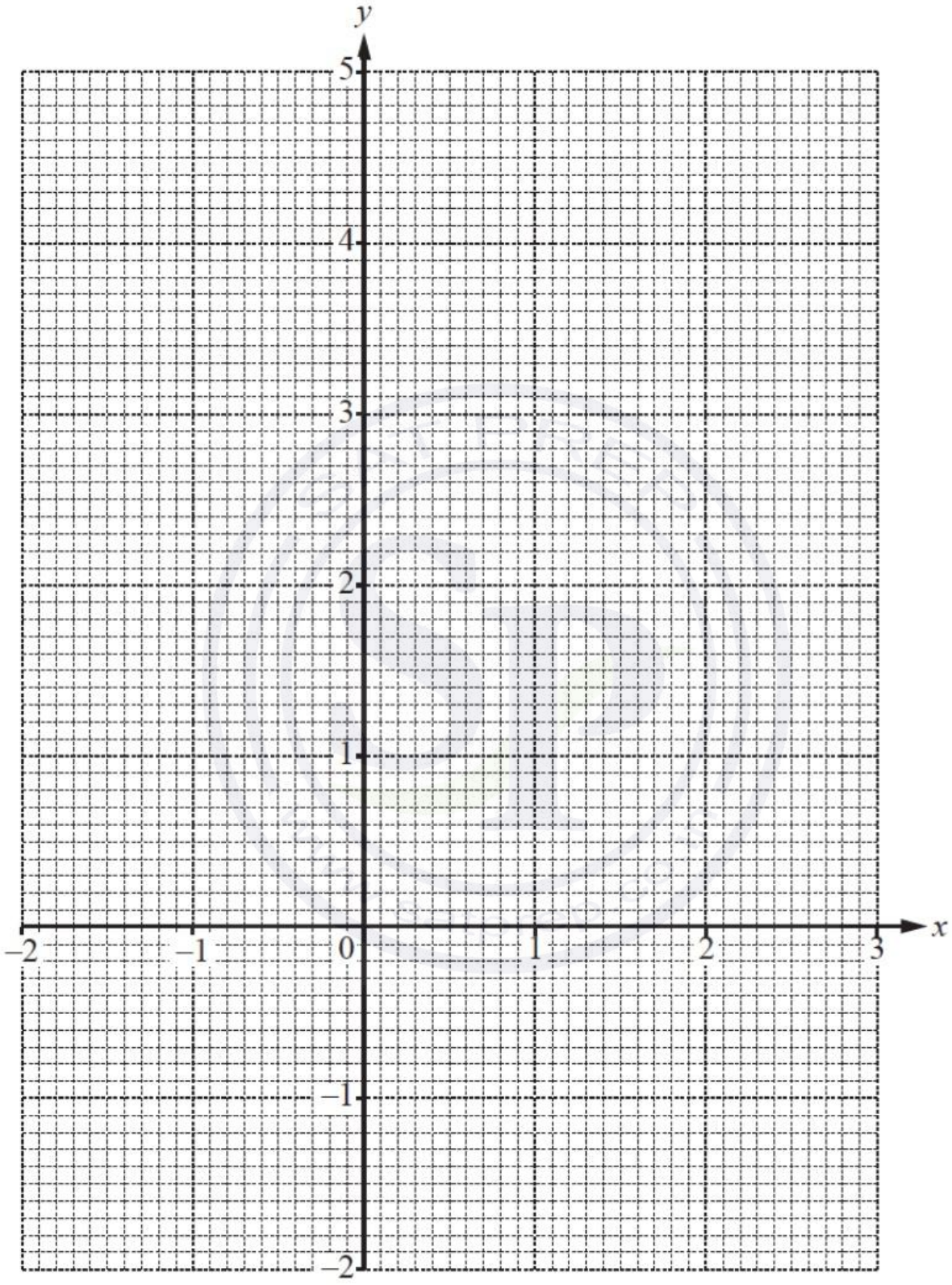

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(c) Use your graph to solve the equation $x + \frac{1}{x^2} = 1.5$.

$x =$ ..... [1]

(d) The line $y = ax + b$ can be drawn on the grid to solve the equation $\frac{1}{x^2} = 2.5 - 2x$.

(i) Find the value of a and the value of b.

$a =$ .....

$b =$ ..... [2]

(ii) Draw the line $y = ax + b$ to solve the equation $\frac{1}{x^2} = 2.5 - 2x$.

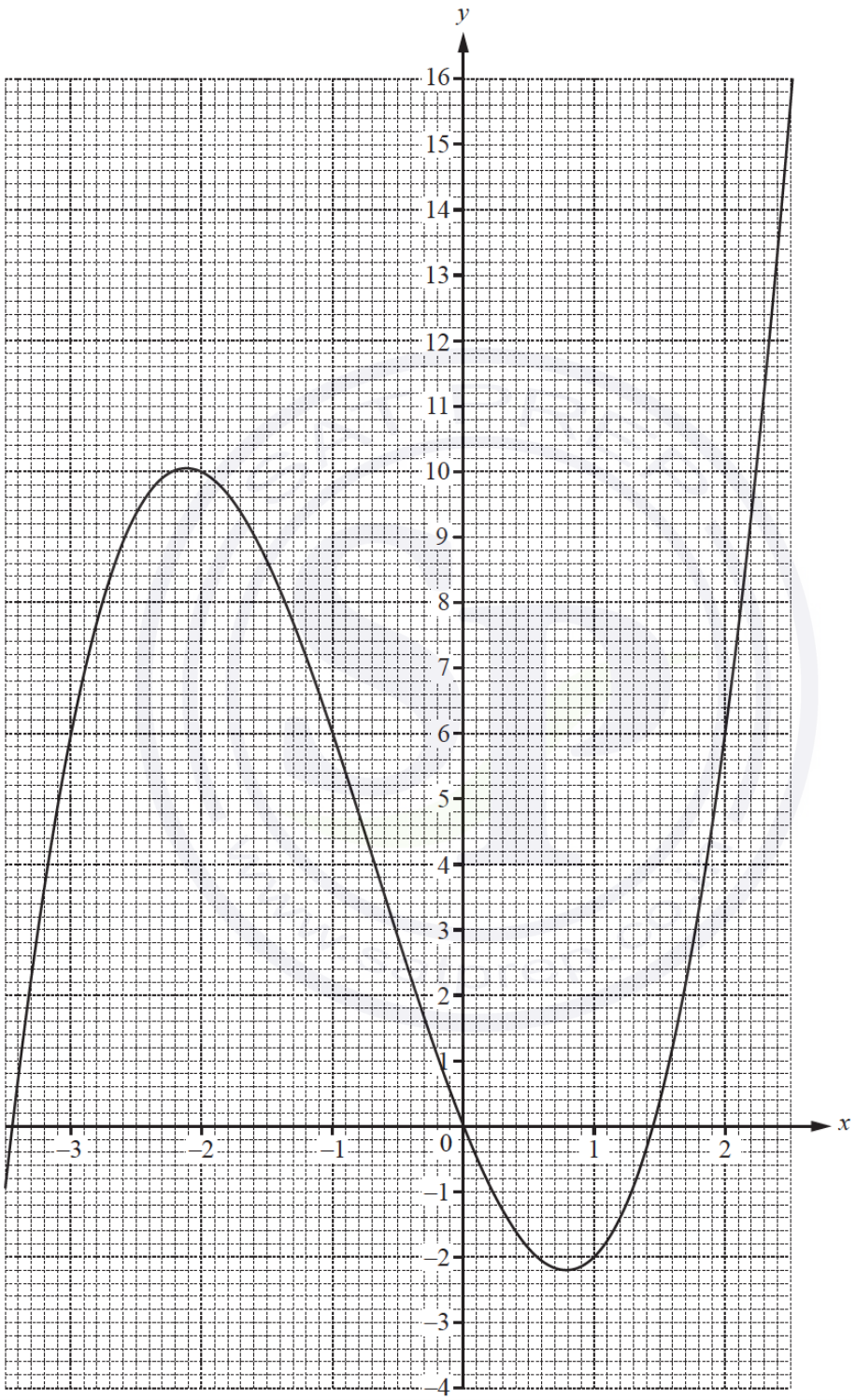
$x =$ ..... [3]

(e) By drawing a suitable tangent, find an estimate of the gradient of the curve at the point where $x = 2$.

..... [3]

Question 19

The diagram shows the graph of $y = f(x)$ for $-3.5 \leqslant x \leqslant 2.5$.



Continue on the next page..

(a) **(i)** Find f(– 2).

..... [1]

(ii) Solve the equation $f(x) = 2$.

x = ... or x = ... or x = ...[3]

(iii) Two tangents, each with gradient 0, can be drawn to the graph of $y = f(x)$.

Write down the equation of each tangent.

.....

..... [2]

(b) **(i)** Complete the table for $g(x) = \frac{2}{x} + 3$ for $-3.5 \leqslant x \leqslant -0.5$ and $0.5 \leqslant x \leqslant 2.5$.

x	–3.5	–3	–2	–1	–0.5		0.5	1	2	2.5
$g(x)$	2.4	2.3		1			7	5		3.8

[3]

(ii) On the grid opposite, draw the graph of $y = g(x)$. [4]

(iii) Use your graph to solve the equation $f(x) = g(x)$.

x = ... or x = ...[2]

(c) Find gf(–2).

..... [2]

(d) Find $g^{-1}(5)$.

..... [1]

Question 20

A line joins the points A (–2, –5) and B (4, 13).

(a) Calculate the length AB.

$AB =$ ..... [3]

(b) Find the equation of the line through A and B.
Give your answer in the form $y = mx + c$.

$y =$ ..... [3]

(c) Another line is parallel to AB and passes through the point (0, –5).

Write down the equation of this line.

..... [2]

(d) Find the equation of the perpendicular bisector of AB.

..... [5]

Question 21

$$f(x) = x^2 - \frac{1}{x} - 4\,, \quad x \neq 0$$

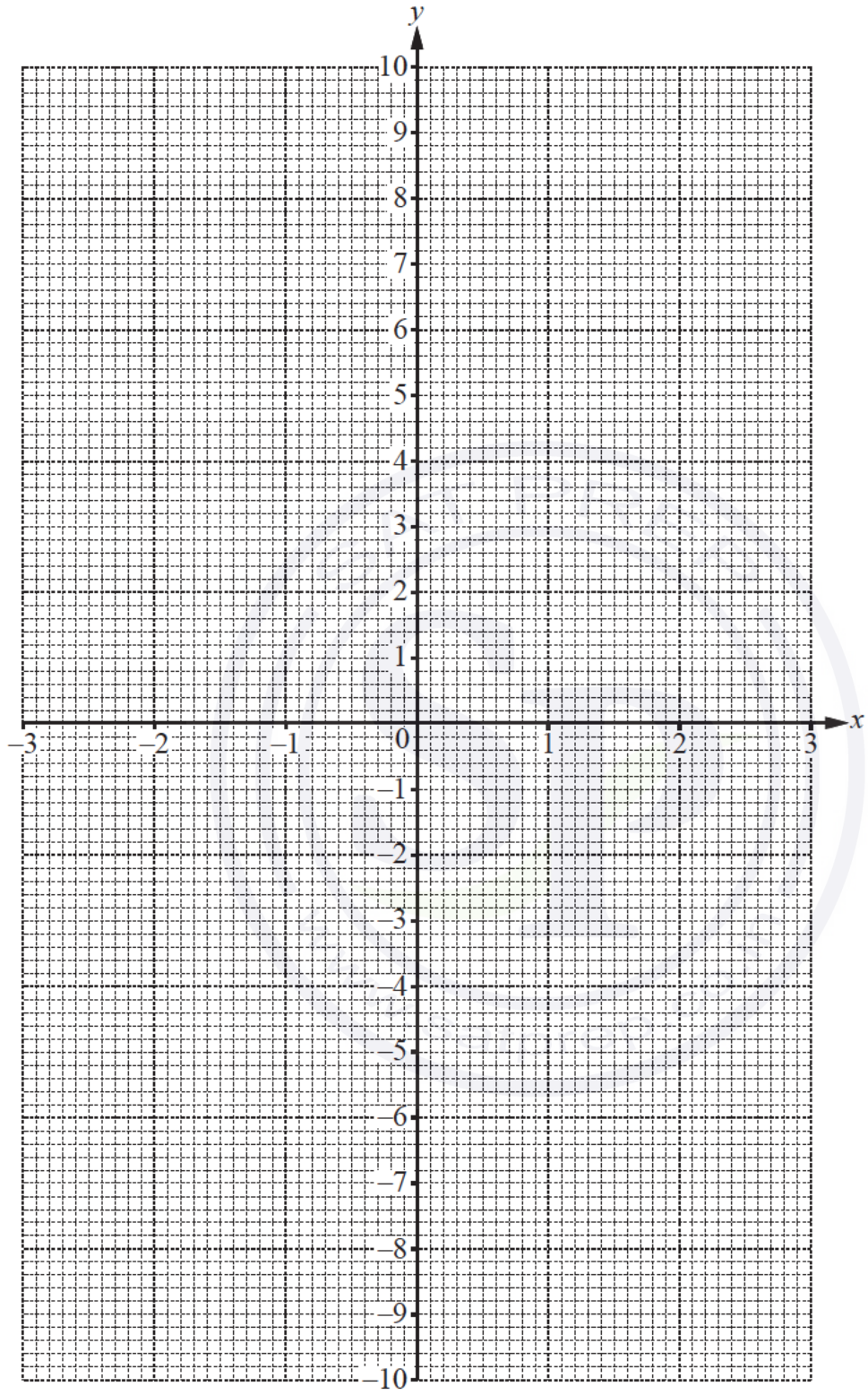
(a) **(i)** Complete the table.

x	–3	–2	–1	–0.5	–0.1	0.2	0.5	1	2	3
f(x)	5.3	0.5		–1.8	6.0	–9.0	–5.8	–4		4.7

[2]

(ii) On the grid, draw the graph of $y = f(x)$ for $-3 \leqslant x \leqslant -0.1$ and $0.2 \leqslant x \leqslant 3$.

Continue on the next page..



[5]

Continue on the next page..

(b) Use your graph to solve the equation $f(x) = 0$.

$x =$ ..... or $x =$ ..... or $x =$ ..... [3]

(c) Find an integer k, for which $f(x) = k$ has one solution.

$k =$ ..... [1]

(d) **(i)** By drawing a suitable straight line, solve the equation $f(x) + 2 = -5x$.

$x =$ ..... or $x =$ ..... [4]

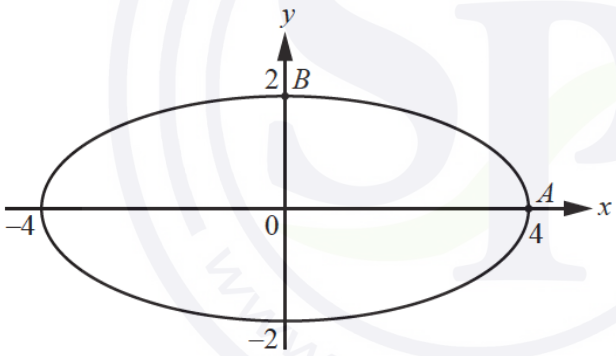
(ii) $f(x) + 2 = -5x$ can be written as $x^3 + ax^2 + bx - 1 = 0$.

Find the value of a and the value of b.

$a =$ .....

$b =$ ..... [2]

Question 22



NOT TO SCALE

The diagram shows a curve with equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

(a) A is the point (4, 0) and B is the point (0, 2).

(i) Find the equation of the straight line that passes through A and B.
Give your answer in the form $y = mx + c$.

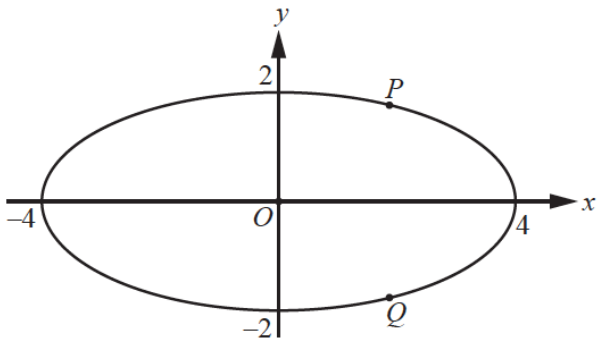
$y =$ ..... [3]

(ii) Show that $a^2 = 16$ and $b^2 = 4$.

[2]

Continue on the next page..

(b)



NOT TO SCALE

$P\,(2, k)$ and $Q\,(2, -k)$ are points on the curve $\frac{x^2}{16} + \frac{y^2}{4} = 1$.

(i) Find the value of k.

$k =$ ..... [3]

(ii) Calculate angle POQ.

Angle $POQ =$ ..... [3]

(c) The area enclosed by a curve with equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is πab.

(i) Find the area enclosed by the curve $\frac{x^2}{16} + \frac{y^2}{4} = 1$.

Give your answer as a multiple of π.

..... [1]

(ii) A curve, mathematically similar to the one in the diagrams, intersects the x-axis at $(12, 0)$ and $(-12, 0)$.

Work out the area enclosed by this curve, giving your answer as a multiple of π.

..... [2]

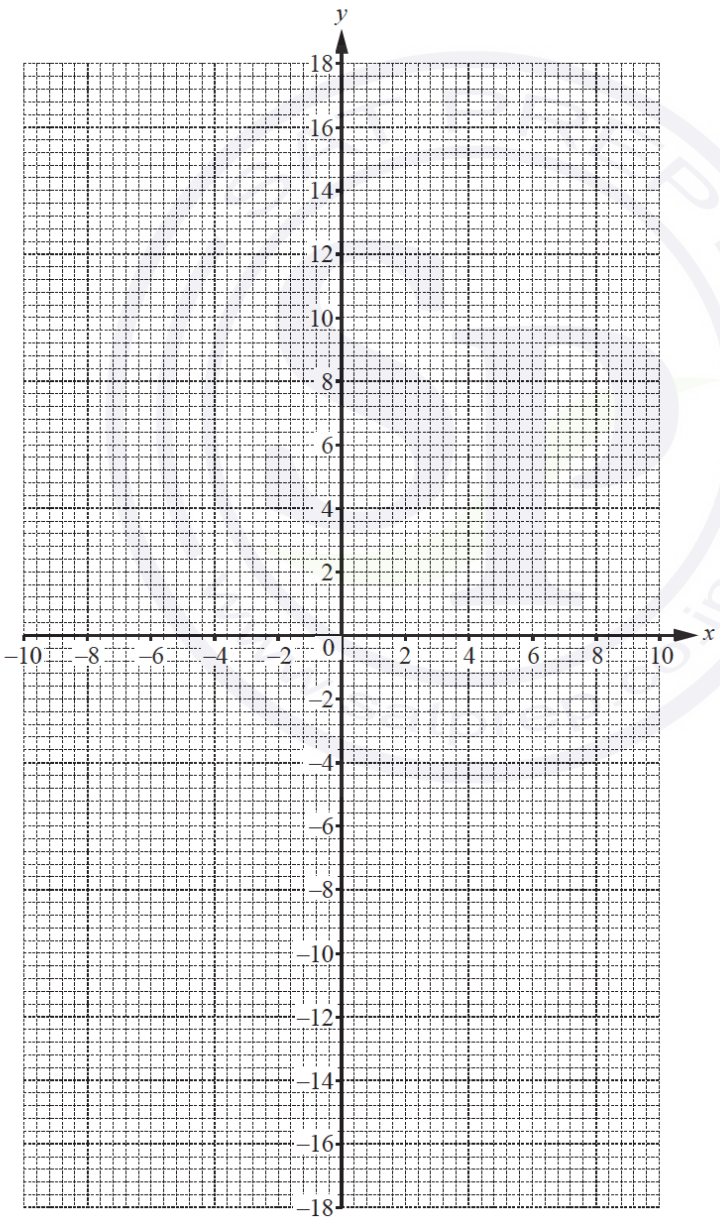
Question 23 [2]

$$f(x) = \frac{20}{x} + x, \quad x \neq 0$$

(a) Complete the table.

x	−10	−8	−5	−2	−1.6	1.6	2	5	8	10
f(x)	−12	−10.5	−9	−12	−14.1	14.1	12			12

(b) On the grid, draw the graph of $y = f(x)$ for $-10 \leqslant x \leqslant -1.6$ and $1.6 \leqslant x \leqslant 10$.



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(c) Using your graph, solve the equation $f(x) = 11$.

$x =$ ..... or $x =$ ..... [2]

(d) k is a prime number and $f(x) = k$ has no solutions.

Find the possible values of k.

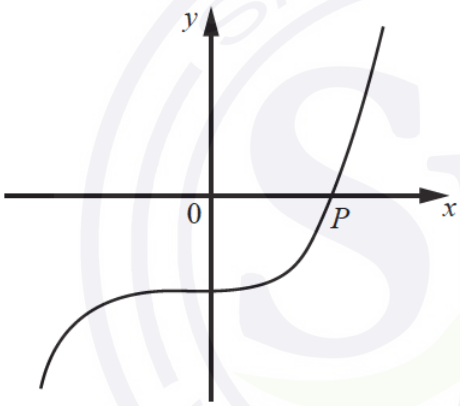
[2]

(ii) On the grid opposite, draw the graph of $y = x^2$ for $-4 \leqslant x \leqslant 4$. [2]

(iii) Using your graphs, solve the equation $x^3 - x^2 - 20 = 0$.

$x =$ ..... [1]

(iv)



NOT TO SCALE

The diagram shows a **sketch** of the graph of $y = x^3 - x^2 - 20$.
P is the point $(n, 0)$.

Write down the value of n.

$n =$ ..... [1]

Question 24

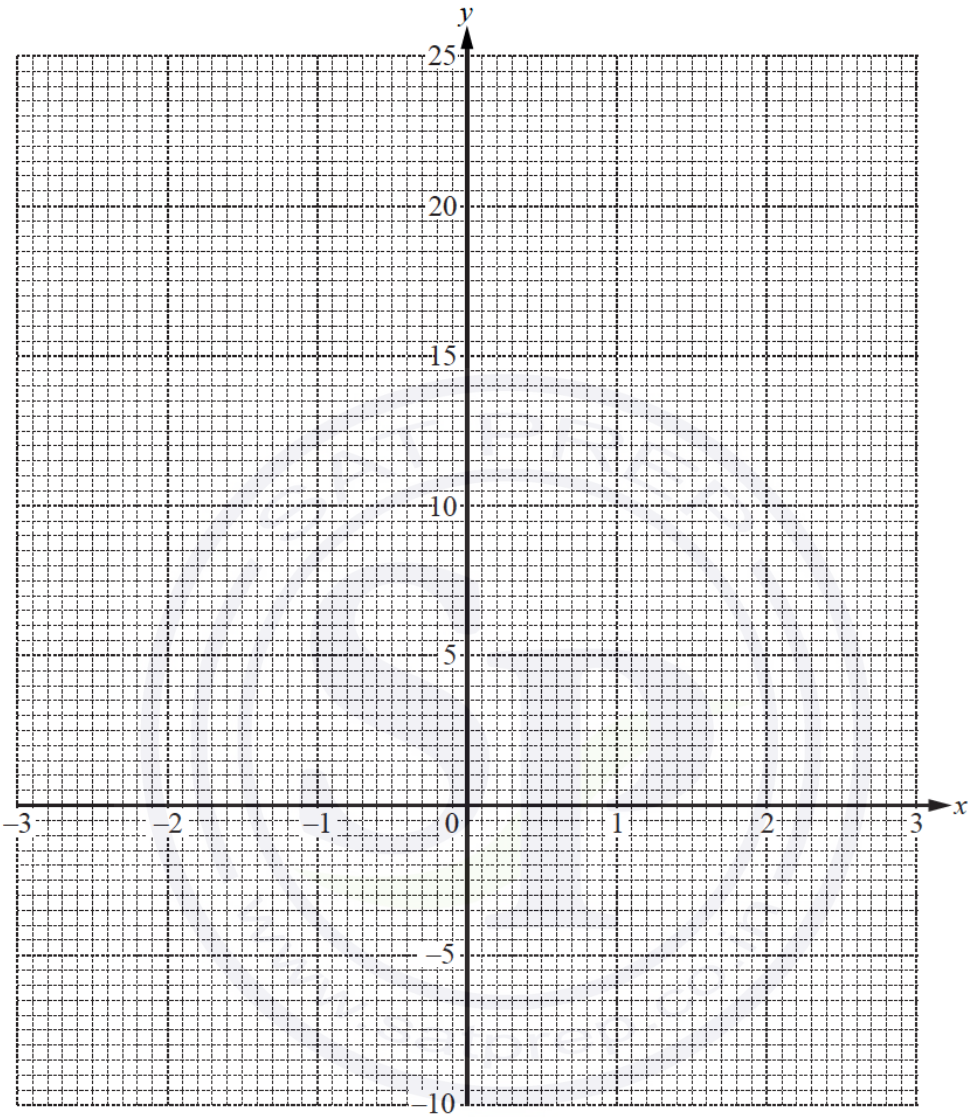
(a) Complete the table for $y = 3x + \frac{2}{x^2} + 1, x \neq 0$.

x	–3	–2	–1	–0.5	–0.3		0.3	0.5	1	2	3
y	–7.8		0	7.5	22.3		24.1		6	7.5	10.2

[2]

Continue on the next page..

(b) On the grid, draw the graph of $y = 3x + \frac{2}{x^2} + 1$ for $-3 \leqslant x \leqslant -0.3$ and $0.3 \leqslant x \leqslant 3$.



[5]

(c) Write down the value of the largest integer, k, so that the equation $3x + \frac{2}{x^2} + 1 = k$ has exactly one solution.

$k =$ ..... [1]

(d) **(i)** By drawing a suitable straight line on the grid, solve $3x + \frac{2}{x^2} + 1 = 15 - 3x$.

$x =$ ..... or $x =$ ..... or $x =$ ..... [4]

Continue on the next page..

(ii) The equation $3x+\frac{2}{x^2}+1=15-3x$ can be written in the form $ax^3+bx^2+cx+2=0$, where a, b and c are integers.

Find a, b and c.

$a =$ .....

$b =$ .....

$c =$ ..... [3]

Question 25

(a) Complete the table of values for $y=\frac{x^3}{3}-x^2+1$.

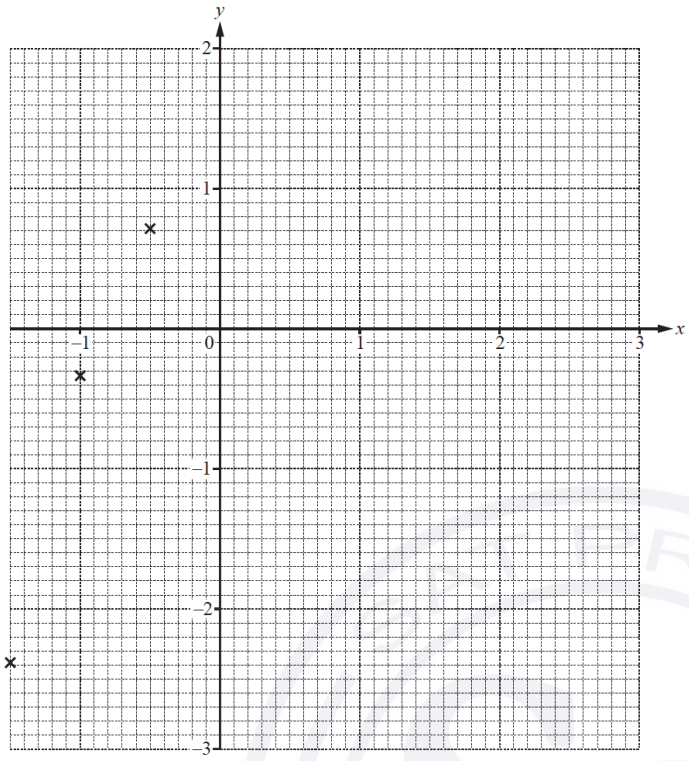
x	–1.5	–1	–0.5	0	0.5	1	1.5	2	2.5	3
y	–2.38	–0.33	0.71		0.79	0.33	–0.13	–0.33	–0.04	

[2]

(b) Draw the graph of $y=\frac{x^3}{3}-x^2+1$ for $-1.5 \leqslant x \leqslant 3$.

The first 3 points have been plotted for you.

Continue on the next page..



[4]

(c) Using your graph, solve the equations.

(i) $\frac{x^3}{3} - x^2 + 1 = 0$

$x =$ ..... or $x =$ ..... or $x =$ .....[3]

(ii) $\frac{x^3}{3} - x^2 + x + 1 = 0$

$x =$ ..... [2]

(d) Two tangents to the graph of $y = \frac{x^3}{3} - x^2 + 1$ can be drawn parallel to the x-axis.

(i) Write down the equation of each of these tangents.

.....

..... [2]

(ii) For $0 \leqslant x \leqslant 3$, write down the smallest possible value of y.

$y =$ ..... [1]

Question 26

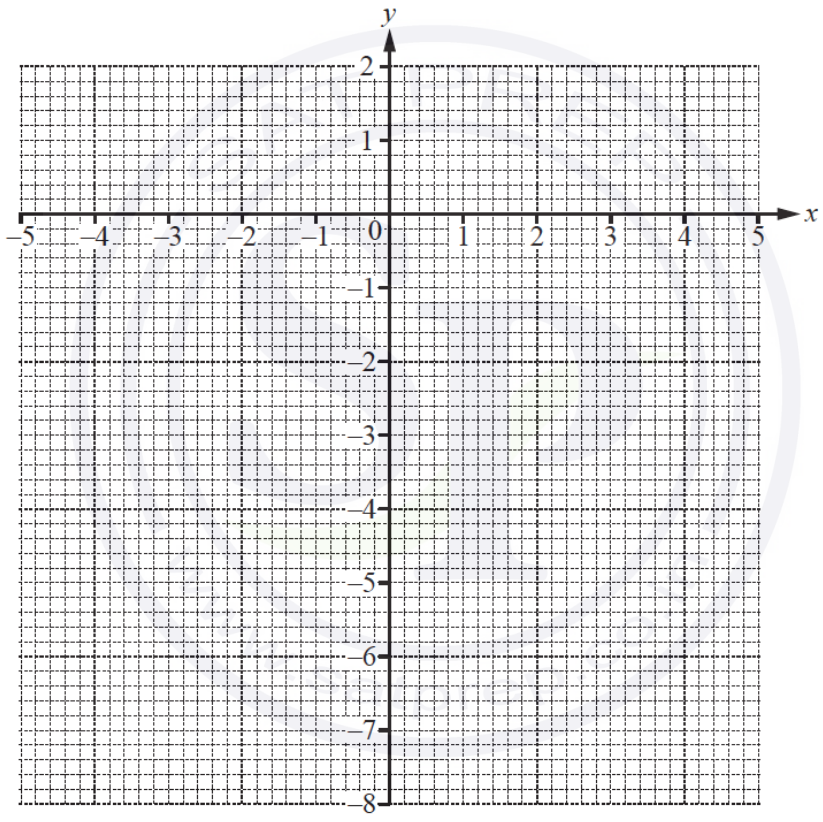
$$y = 1 - \frac{2}{x^2}, \quad x \neq 0$$

(a) Complete the table.

x	–5	–4	–3	–2	–1	–0.5		0.5	1	2	3	4	5
y		0.88	0.78			–7		–7			0.78	0.88	

[3]

(b) On the grid, draw the graph of $y = 1 - \frac{2}{x^2}$ for $-5 \leqslant x \leqslant -0.5$ and $0.5 \leqslant x \leqslant 5$.



[5]

(c) **(i)** On the grid, draw the graph of $y = -x - 1$ for $-3 \leqslant x \leqslant 5$. [2]

(ii) Solve the equation $1 - \frac{2}{x^2} = -x - 1$.

$x =$ ..... [1]

(iii) The equation $1 - \frac{2}{x^2} = -x - 1$ can be written in the form $x^3 + px^2 + q = 0$.

Find the value of p and the value of q.

$p =$ .....

$q =$ ..... [3]

(d) The graph of $y = 1 - \frac{2}{x^2}$ cuts the positive x-axis at A.

B is the point $(0, -2)$.

(i) Write down the co-ordinates of A.

(..... , ...) [1]

(ii) On the grid, draw the straight line that passes through A and B. [1]

(iii) Complete the statement.

The straight line that passes through A and B is a .....

at the point ..... [2]

Question 27

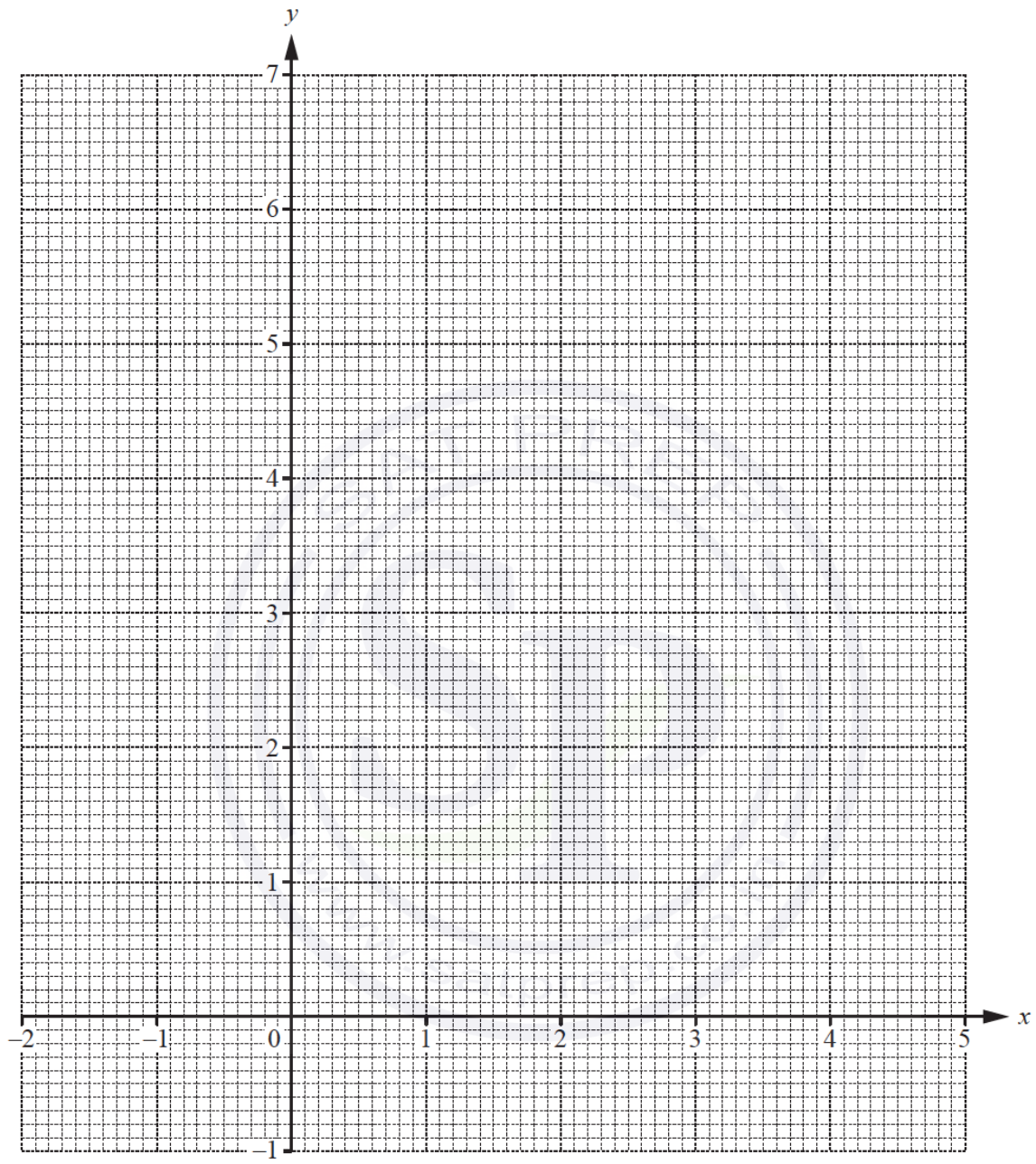
The table shows some values for $y = 1.5^x - 1$.

x	−2	−1	0	1	2	3	4	5
y	−0.56	−0.33				2.38	4.06	6.59

(a) Complete the table. [3]

Continue on the next page..

(b) Draw the graph of $y = 1.5^x - 1$ for $-2 \leqslant x \leqslant 5$.



[4]

Continue on the next page..

(c) Use your graph to solve the equation $1.5^x - 1 = 3.5$.

$x =$ ..... [2]

(d) By drawing a suitable straight line, solve the equation $1.5^x - x - 2 = 0$.

$x =$ ..... or $x =$ .....[3]

(e) **(i)** On the grid, plot the point A at (5, 5). [1]

(ii) Draw the tangent to the graph of $y = 1.5^x - 1$ that passes through the point A. [1]

(iii) Work out the gradient of this tangent.

..... [2]

Question 28

The table shows some values for $y = 2x^3 + 4x^2$.

x	–2.2	–2	–1.5	–1	–0.5	0	0.5	0.8
y	–1.94				0.75	0		3.58

(a) Complete the table. [4]

(b) Draw the graph of $y = 2x^3 + 4x^2$ for $-2.2 \leqslant x \leqslant 0.8$.

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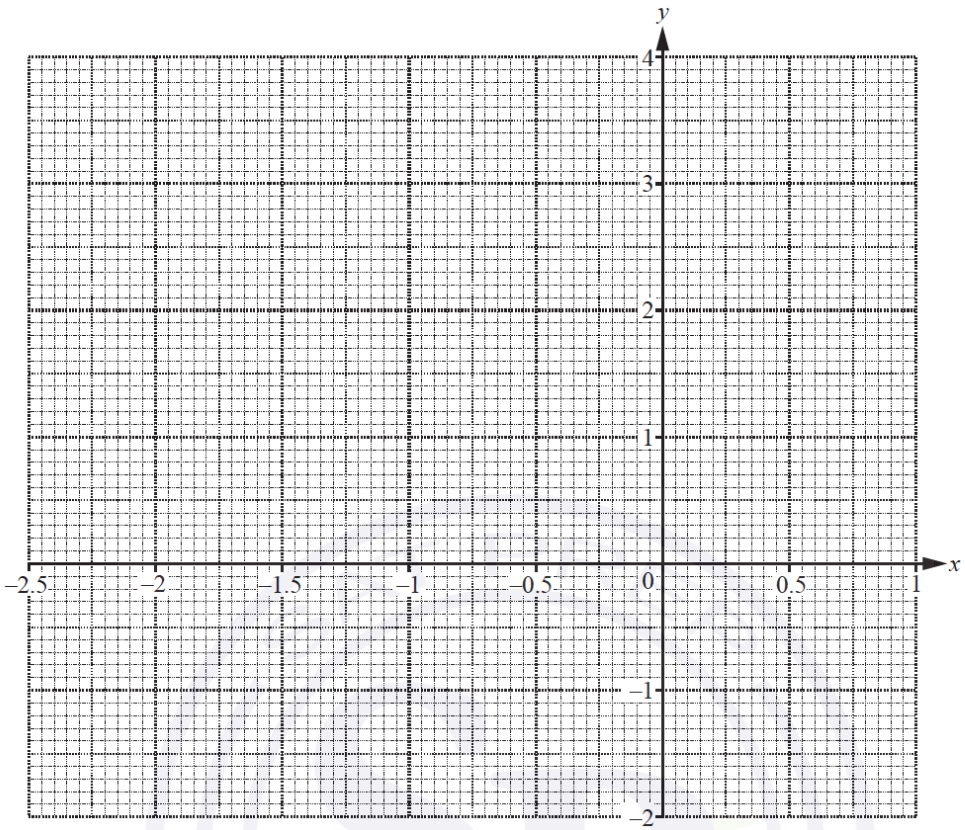


[4]

(c) Find the number of solutions to the equation $2x^3 + 4x^2 = 3$.

..... [1]

(d) **(i)** The equation $2x^3 + 4x^2 - x = 1$ can be solved by drawing a straight line on the grid.

Write down the equation of this straight line.

$y =$ ..... [1]

(ii) Use your graph to solve the equation $2x^3 + 4x^2 - x = 1$.

$x =$ ..... or $x =$ ..... or $x =$ .....[3]

(e) The tangent to the graph of $y = 2x^3 + 4x^2$ has a negative gradient when $x = k$.

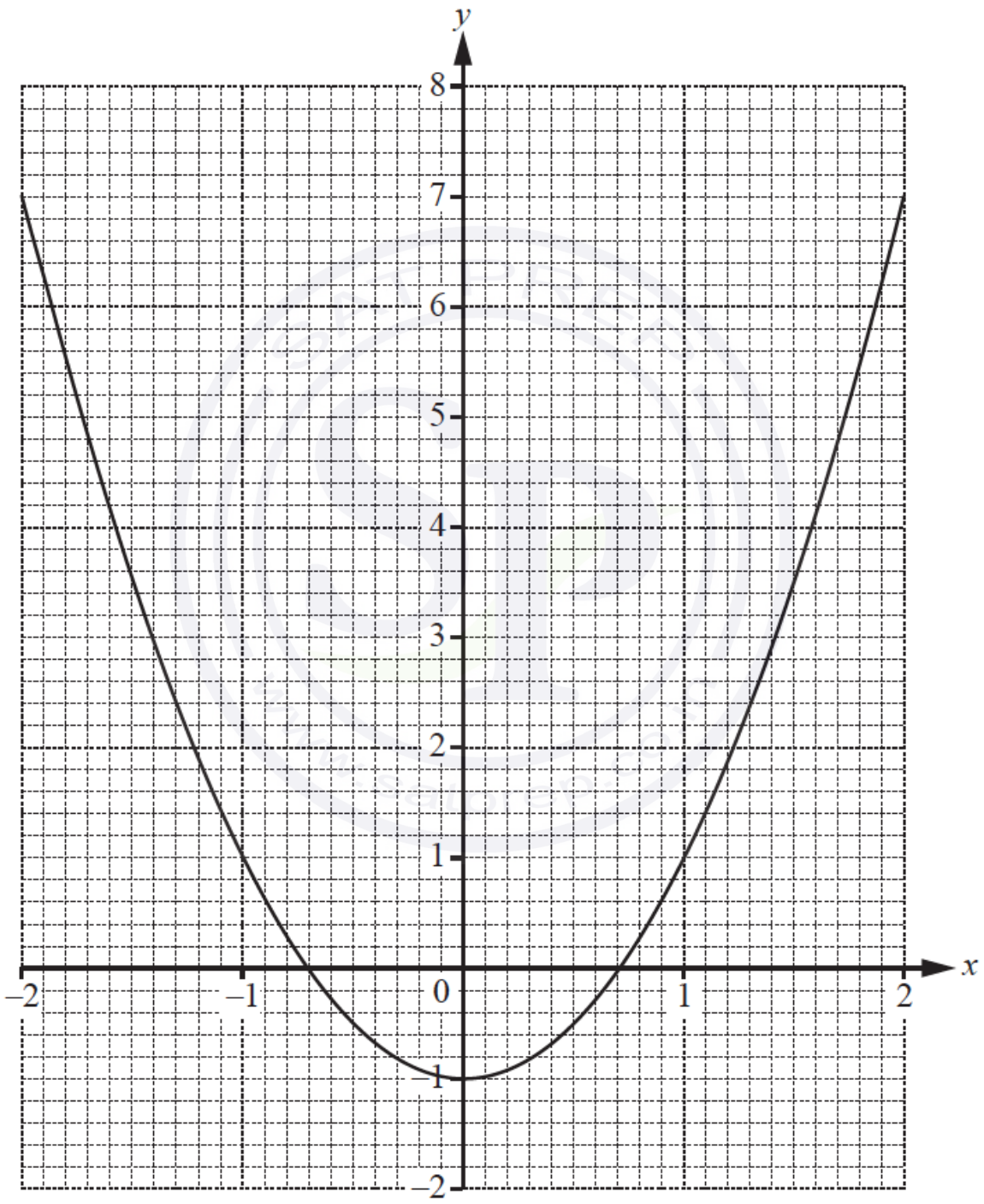
Complete the inequality for k.

..... $< k <$ .....[2]

Question 29

$f(x) = 2x^2 - 1$

The graph of $y = f(x)$, for $-2 \leqslant x \leqslant 2$, is drawn on the grid.



Continue on the next page..

(a) Use the graph to solve the equation $f(x) = 5$.

$x =$ ..... or $x =$ ..... [2]

(b) **(i)** Draw the tangent to the graph of $y = f(x)$ at the point $(-1.5, 3.5)$. [1]

(ii) Use your tangent to estimate the gradient of $y = f(x)$ when $x = -1.5$.

..... [2]

(c) $g(x) = 2^x$

(i) Complete the table for $y = g(x)$.

x	-2	-1	0	1	2
y	0.25	0.5		2	4

[1]

(ii) On the grid opposite, draw the graph of $y = g(x)$ for $-2 \leqslant x \leqslant 2$. [3]

(d) Use your graphs to solve

(i) the equation $f(x) = g(x)$,

$x =$ ..... or $x =$ ..... [2]

(ii) the inequality $f(x) < g(x)$.

..... [1]

(e) **(i)** Write down the three values.

$g(-3) =$ ..... $g(-5) =$ ..... $g(-10) =$ ..... [1]

(ii) Complete the statement.

As x decreases, $g(x)$ approaches the value ..... [1]

Question 30

(a) Find f(1).

..... [1]

(b) Solve $f(x) = 3$.

$x =$ ..... [1]

(c) The equation $f(x) = k$ has only one solution for $-2.5 \leqslant x \leqslant 2$.

Write down the range of values of k for which this is possible.

..... [2]

(d) By drawing a suitable straight line, solve the equation $f(x) = x - 5$.

$x =$ ..... or $x =$ ..... or $x =$ ..... [3]

(e) Draw a tangent to the graph of $y = f(x)$ at the point where $x = 1$.

Use your tangent to estimate the gradient of $y = f(x)$ when $x = 1$.

..... [3]

Question 31

Line A has equation $y = 5x - 4$.
Line B has equation $3x + 2y = 18$.

(a) Find the gradient of

(i) line A,

..... [1]

(ii) line B.

..... [1]

(b) Write down the co-ordinates of the point where line A crosses the x-axis.

(....., ...) [2]

(c) Find the equation of the line perpendicular to line A which passes through the point (10, 9).
Give your answer in the form $y = mx + c$.

$y =$ ..... [4]

(d) Work out the co-ordinates of the point of intersection of line A and line B.

(....., ...) [3]

(e) Work out the area enclosed by line A, line B and the y-axis.

..... [3]

Question 32

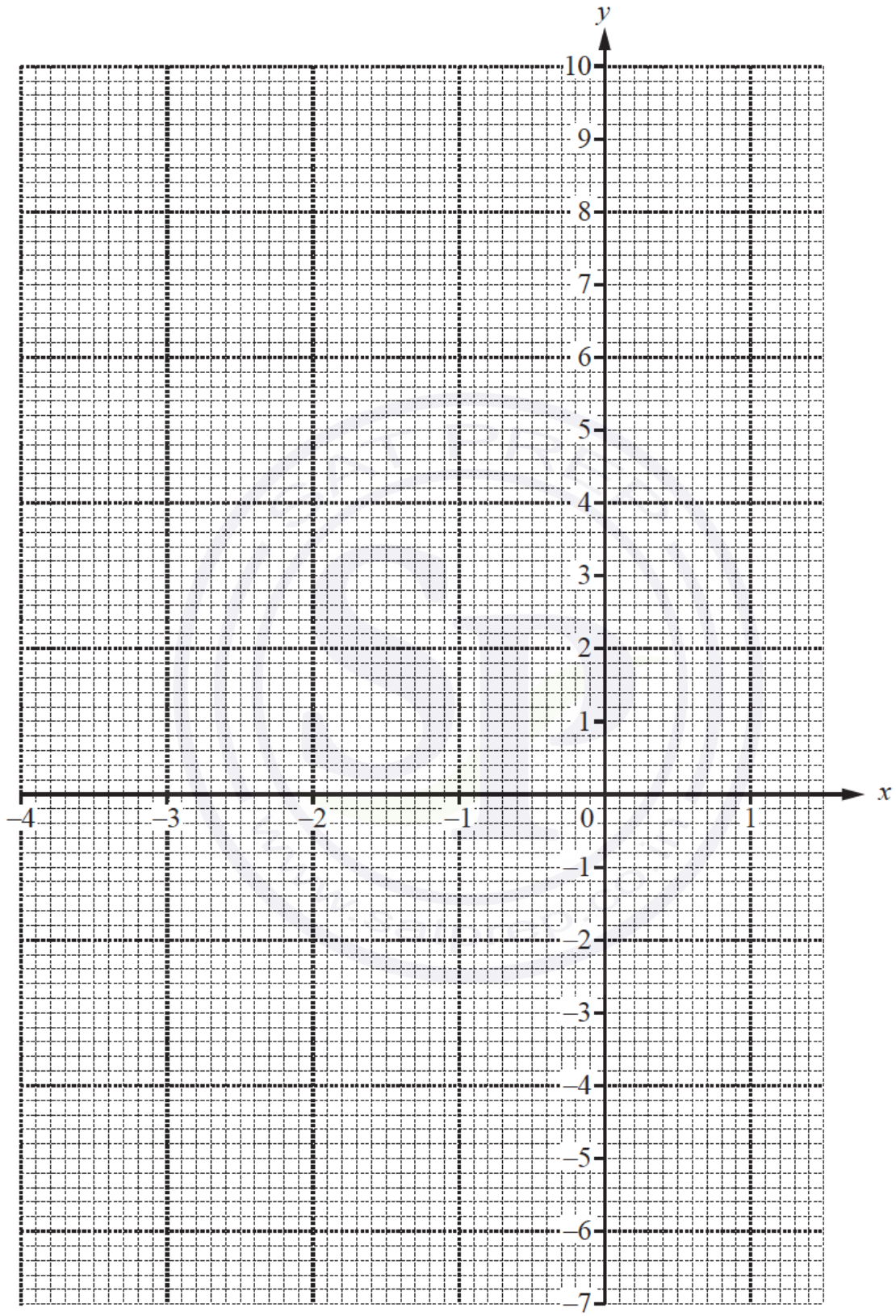
The table shows some values of $y = 2x^2 + 5x - 3$ for $-4 \leqslant x \leqslant 1.5$.

x	-4	-3	-2	-1	0	1	1.5
y		0	-5		-3	4	

(a) Complete the table. [3]

(b) On the grid, draw the graph of $y = 2x^2 + 5x - 3$ for $-4 \leqslant x \leqslant 1.5$.

Continue on the next page..



Continue on the next page..

[4]

(c) Use your graph to solve the equation $2x^2 + 5x - 3 = 3$.

$x =$ ..... or $x =$ ..... [2]

(d) $y = 2x^2 + 5x - 3$ can be written in the form $y = 2(x+a)^2 + b$.

Find the value of a and the value of b.

$a =$ .....

$b =$ ..... [3]

Question 33

$y = \frac{x^3}{8} - \frac{2}{x^2}$, $x \neq 0$

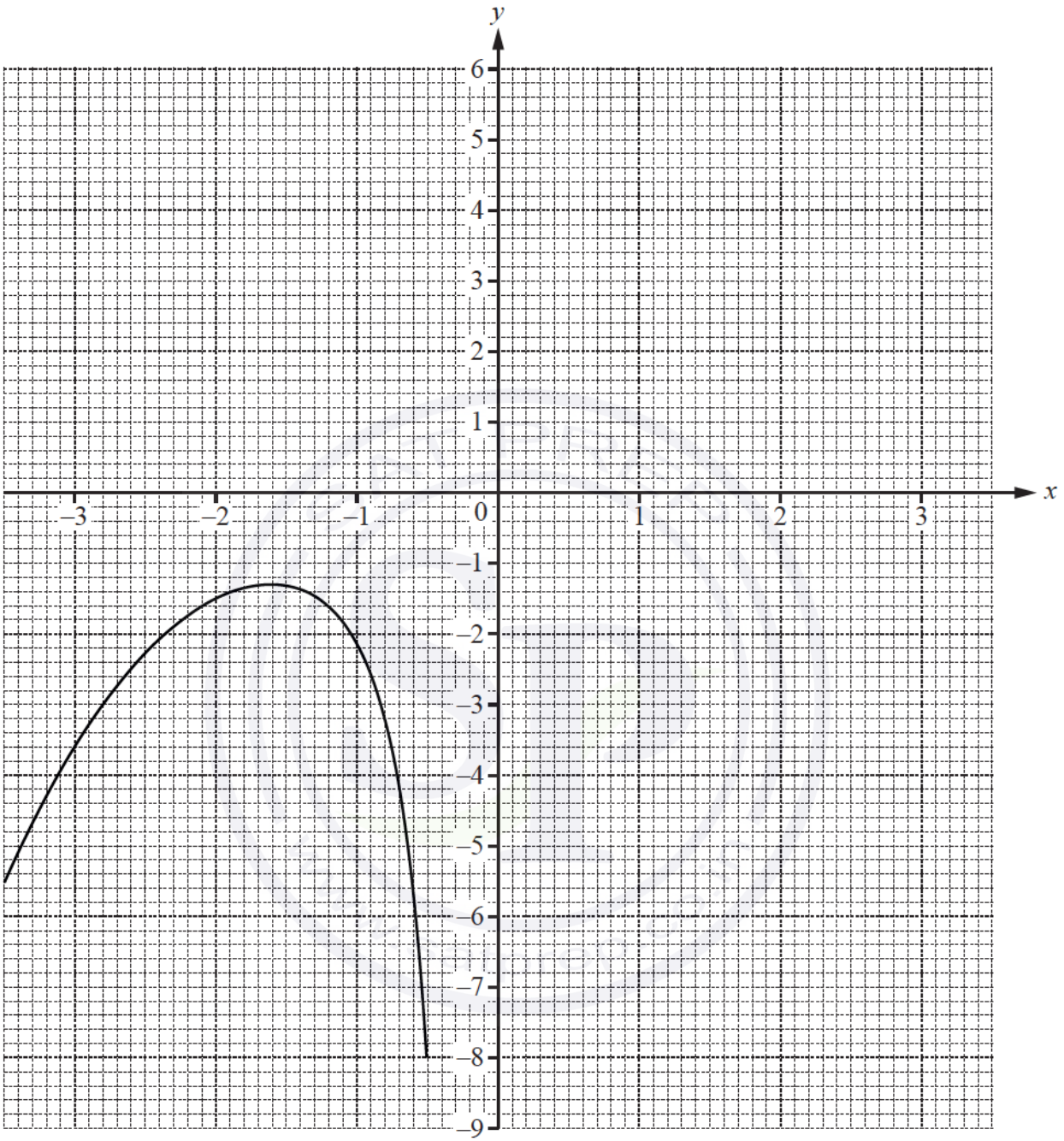
(a) Complete the table of values.

x	0.5	1	1.5	2	2.5	3	3.5
y	−8.0	−1.9	−0.5	0.5	1.6		

[2]

Continue on the next page..

[b]



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The graph of $y = \frac{x^3}{8} - \frac{2}{x^2}$ for $-3.5 \leqslant x \leqslant -0.5$ has already been drawn.

On the grid, draw the graph of $y = \frac{x^3}{8} - \frac{2}{x^2}$ for $0.5 \leqslant x \leqslant 3.5$. [4]

(c) Use your graph to solve the equation $\frac{x^3}{8} - \frac{2}{x^2} = 0$.

$x =$ ..... [1]

(d) $\frac{x^3}{8} - \frac{2}{x^2} = k$ and k is an integer.

Write down a value of k when the equation $\frac{x^3}{8} - \frac{2}{x^2} = k$ has

(i) one answer, $k =$ ..... [1]

(ii) three answers. $k =$ ..... [1]

(e) By drawing a suitable tangent, estimate the gradient of the curve where $x = -3$.

..... [3]

(f) **(i)** By drawing a suitable line on the grid, find x when $\frac{x^3}{8} - \frac{2}{x^2} = 6 - x$.

$x =$ ..... [3]

(ii) The equation $\frac{x^3}{8} - \frac{2}{x^2} = 6 - x$ can be written as $x^5 + ax^3 + bx^2 + c = 0$.

Find the values of a, b and c.

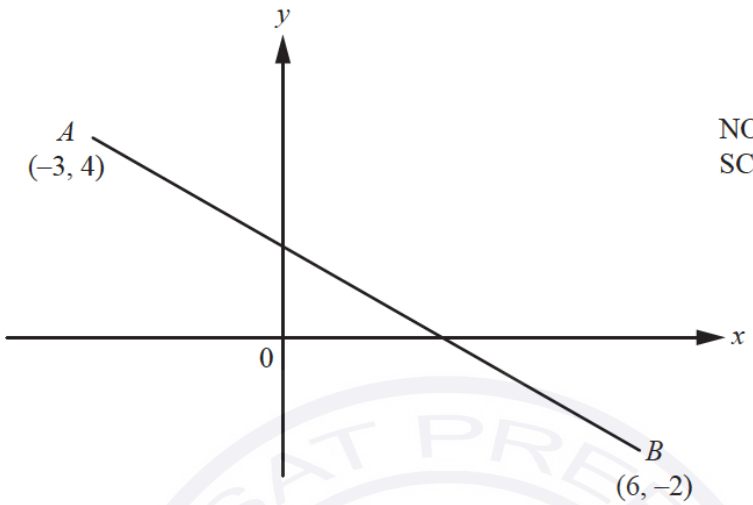
$a =$ .....

$b =$ .....

$c =$ ..... [4]

Question 34

(a)



Calculate the length of AB.

..... [3]

(b) The point P has co-ordinates $(10, 12)$ and the point Q has co-ordinates $(2, -4)$.

Find

(i) the co-ordinates of the mid-point of the line PQ,

(..... , ...) [2]

(ii) the gradient of the line PQ,

..... [2]

(iii) the equation of a line perpendicular to PQ that passes through the point $(2, 3)$.

..... [3]

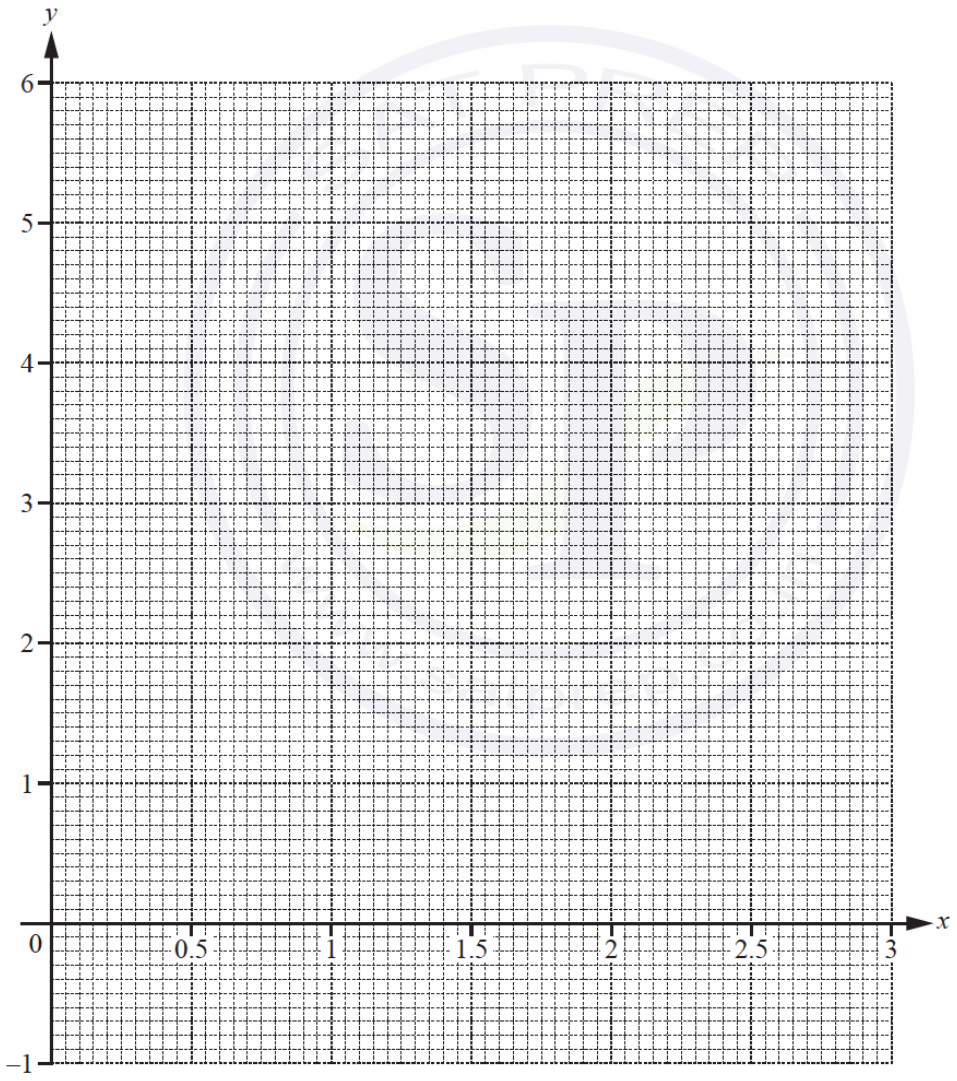
Question 35

The table shows some values for $y = 2x + \frac{1}{x} - 3$ for $0.125 \leqslant x \leqslant 3$.

x	0.125	0.25	0.375	0.5	0.75	1	1.5	2	2.5	3
y	5.25	1.5	0.42			0	0.67	1.5		3.33

(a) Complete the table. [3]

(b) On the grid, draw the graph of $y = 2x + \frac{1}{x} - 3$ for $0.125 \leqslant x \leqslant 3$.



[4]

Continue on the next page..

(c) Use your graph to solve $2x+\frac{1}{x}-3 \geqslant 2$.

.....

..... [3]

(d) The equation $\frac{1}{x}=7-3x$ can be solved using your graph in **part (b)** and a straight line.

(i) Write down the equation of this straight line.

..... [2]

(ii) Draw this straight line and solve the equation $\frac{1}{x}=7-3x$.

$x =$ ..... or $x =$ ..... [3]

Question 36

(a) (i) $y = 2^x$

Complete the table.

x	0	1	2	3	4
y		2	4	8	

[2]

(ii) $y = 14 - x^2$

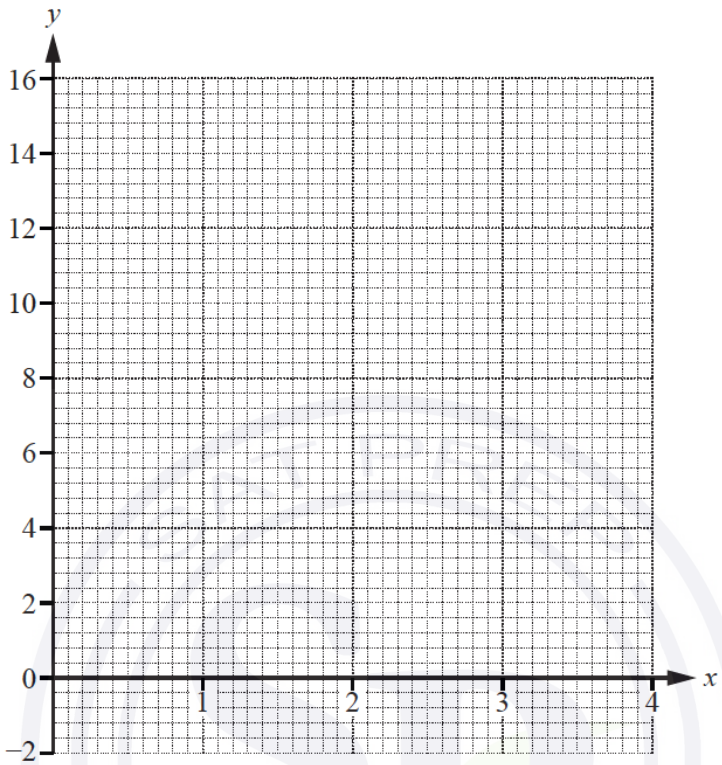
Complete the table.

x	0	1	2	3	4
y		13	10	5	

[2]

Continue on the next page..

(b) On the grid, draw the graphs of $y = 2^x$ and $y = 14 - x^2$ for $0 \leqslant x \leqslant 4$.



[6]

(c) Use your graphs to solve the equations.

(i) $2^x = 12$

$x =$ ..... [1]

(ii) $2^x = 14 - x^2$

$x =$ ..... [1]

(d) **(i)** On the grid, draw the line from the point (4, 2) that has a gradient of -4. [1]

(ii) Complete the statement.

This straight line is a ..... to the graph of $y = 14 - x^2$

at the point (..... , ...). [2]

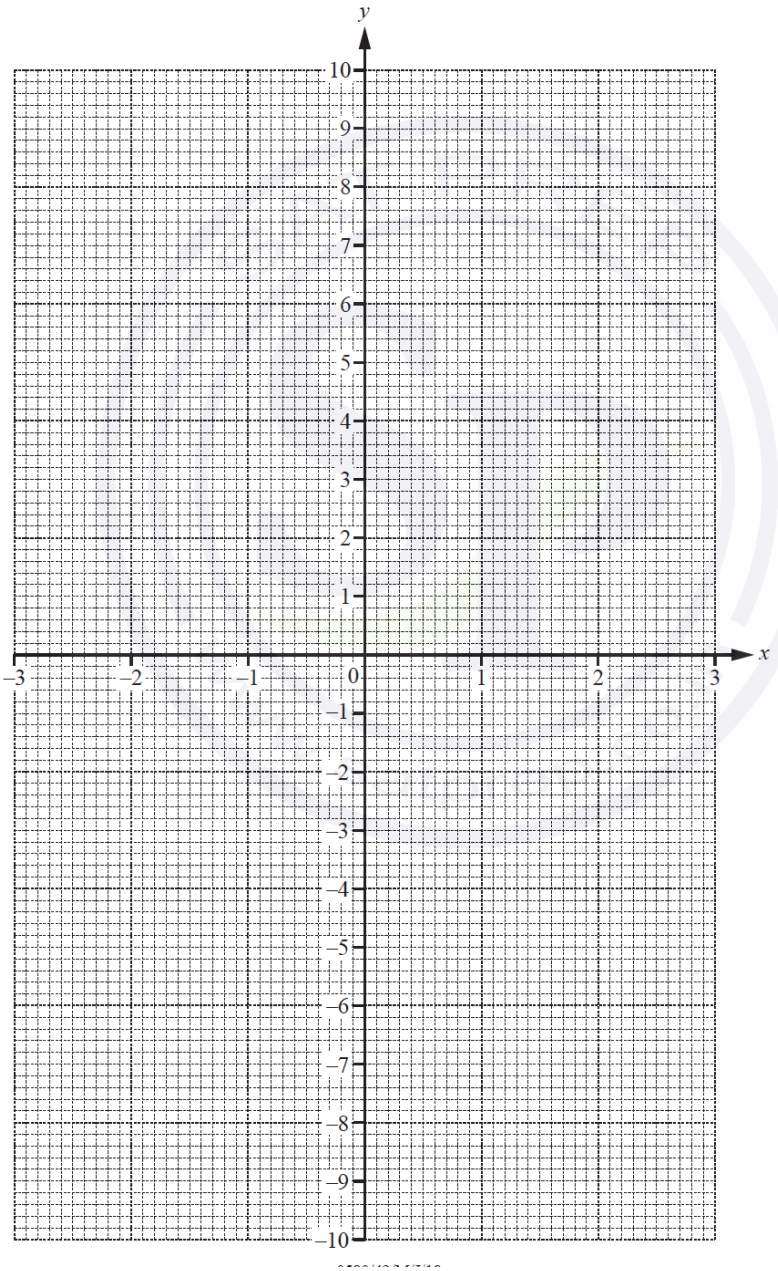
Question 37

(a) Complete the table of values for $y = \frac{x^3}{3} - \frac{1}{2x^2}$, $x \neq 0$.

x	-3	-2	-1	-0.5	-0.3		0.3	0.5	1	2	3
y	-9.1	-2.8	-0.8		-5.6		-5.5	-2.0			8.9

[3]

(b) On the grid, draw the graph of $y = \frac{x^3}{3} - \frac{1}{2x^2}$ for $-3 \leqslant x \leqslant -0.3$ and $0.3 \leqslant x \leqslant 3$.



[5]

Continue on the next page..

(c) **(i)** By drawing a suitable tangent, find an estimate of the gradient of the curve at $x = -2$.

..... [3]

(ii) Write down the equation of the tangent to the curve at $x = -2$.
Give your answer in the form $y = mx + c$.

$y =$ ..... [2]

(d) Use your graph to solve the equations.

(i) $\frac{x^3}{3} - \frac{1}{2x^2} = 0$

$x =$ ..... [1]

(ii) $\frac{x^3}{3} - \frac{1}{2x^2} + 4 = 0$

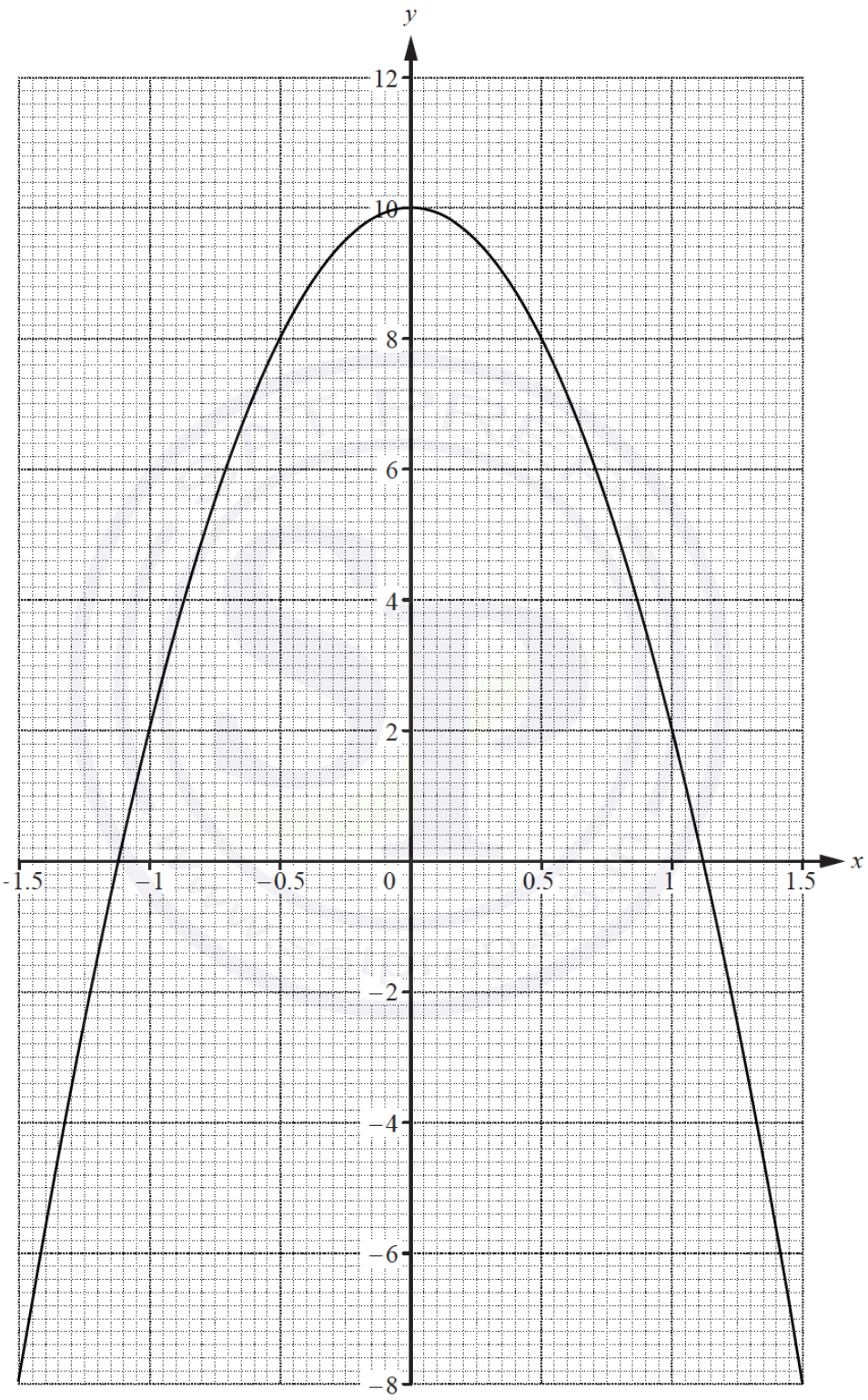
$x =$ ..... or $x =$ ..... or $x =$ ..... [3]

(e) The equation $\frac{x^3}{3} - \frac{1}{2x^2} + 4 = 0$ can be written in the form $ax^n + bx^{n-3} - 3 = 0$.

Find the value of a, the value of b and the value of n.

Question 38

The graph of $y = 10 - 8x^2$ for $-1.5 \leqslant x \leqslant 1.5$ is drawn on the grid.



Continue on the next page..

(a) Write down the equation of the line of symmetry of the graph.

..... [1]

(b) On the grid opposite, draw the tangent to the curve at the point where $x = 0.5$.
Find the gradient of this tangent.

..... [3]

(c) The table shows some values for $y = x^3 + 3x + 4$.

x	-1.5	-1	-0.5	0	0.5	1	1.5
y	-3.9				5.6	8	11.9

(i) Complete the table. [3]

(ii) On the grid opposite, draw the graph of $y = x^3 + 3x + 4$ for $-1.5 \leqslant x \leqslant 1.5$. [4]

(d) Show that the values of x where the two curves intersect are the solutions to the equation $x^3 + 8x^2 + 3x - 6 = 0$.

[1]

(e) By drawing a suitable straight line, solve the equation $x^3 + 5x + 2 = 0$ for $-1.5 \leqslant x \leqslant 1.5$.

$x =$ ..... [3]

Question 39

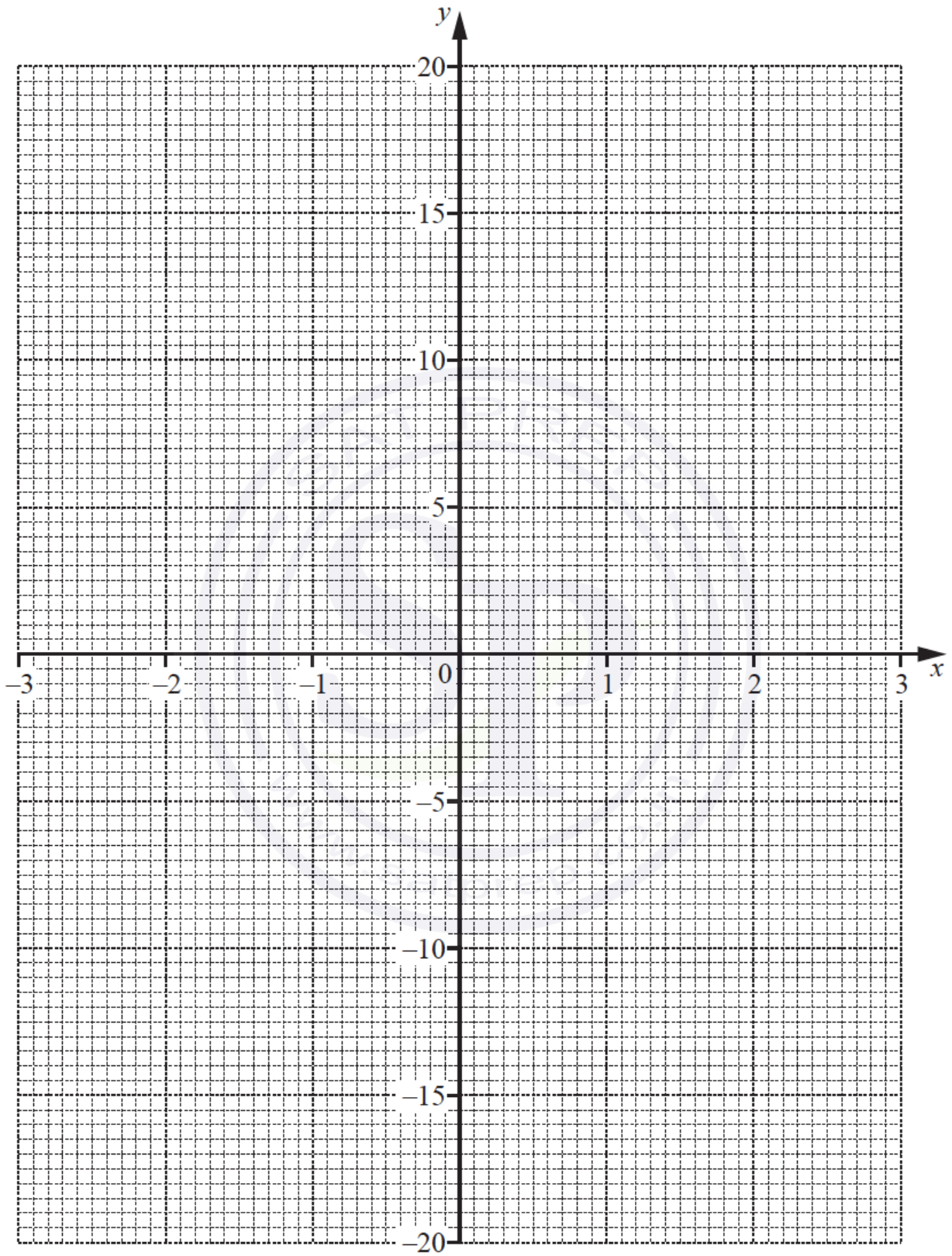
5 The table shows some values of $y = x^3 - 3x - 1$.

x	–3	–2.5	–2	–1.5	–1	0	1	1.5	2	2.5	3
y	–19	–9.1		0.1	1	–1	–3	–2.1	1	7.1	

(a) Complete the table of values. [2]

(b) Draw the graph of $y = x^3 - 3x - 1$ for $-3 \leqslant x \leqslant 3$.

Continue on the next page..



Continue on the next page..

[4]

(c) A straight line through (0, –17) is a tangent to the graph of $y = x^3 - 3x - 1$.

(i) On the grid, draw this tangent. [1]

(ii) Find the co-ordinates of the point where the tangent meets your graph.

(..... , ...) [1]

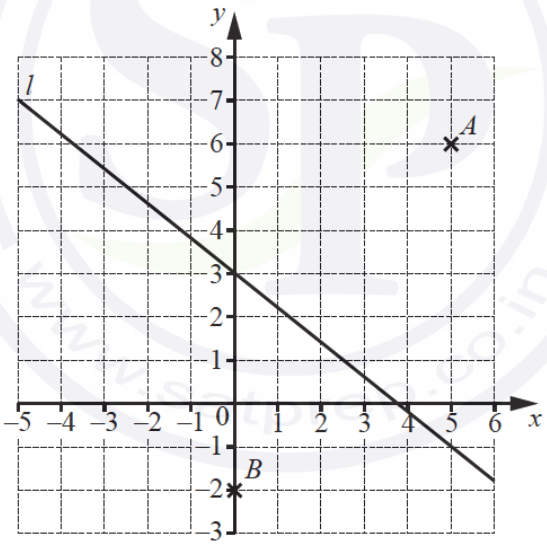
(iii) Find the equation of the tangent.
Give your answer in the form $y = mx + c$.

$y =$ ..... [3]

(d) By drawing a suitable straight line on the grid, solve the equation $x^3 - 6x - 3 = 0$.

$x =$ ..... or $x =$ ..... or $x =$ ..... [4]

Question 40



(a) Write down the co-ordinates of A.

(..... , ...) [1]

(b) Find the equation of line l in the form $y = mx + c$.

$y =$ ..... [3]

Continue on the next page..

(c) Write down the equation of the line parallel to line l that passes through the point B.

..... [2]

(d) C is the point (8, 14).

(i) Write down the equation of the line perpendicular to line l that passes through the point C.

..... [3]

(ii) Calculate the length of AC.

..... [3]

(iii) Find the co-ordinates of the mid-point of BC.

(..... , ...) [2]

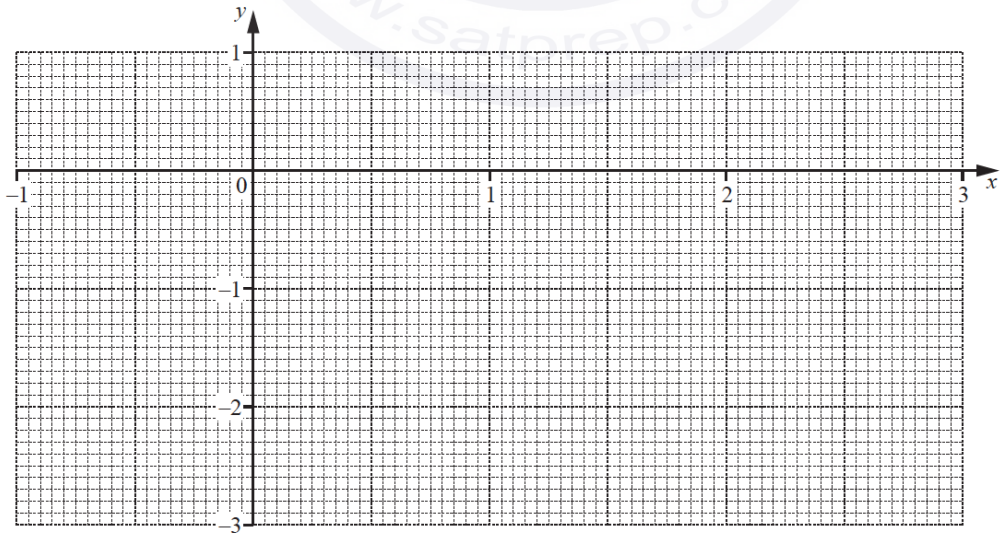
Question 41

3 The table shows some values of $y = x^3 - 3x^2 + x$.

x	−0.75	−0.5	−0.25	0	0.5	1	1.5	2	2.5	2.75
y	−2.9	−1.4	−0.5		−0.1	−1	−1.9		−0.6	

(a) Complete the table. [3]

(b) On the grid, draw the graph of $y = x^3 - 3x^2 + x$ for $-0.75 \leqslant x \leqslant 2.75$. [4]



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(c) Use your graph to complete the inequalities in x for which $y > -1$.

..... $< x <$ ..... and $x >$ ..... [3]

(d) The equation $x^3 - 3x^2 + 2x - 1 = 0$ can be solved by drawing a straight line on the grid.

(i) Write down the equation of this line.

..... [2]

(ii) On the grid, draw this line and use it to solve the equation $x^3 - 3x^2 + 2x - 1 = 0$.

$x =$ ..... [3]

(e) By drawing a suitable tangent, find an estimate for the gradient of the graph of $y = x^3 - 3x^2 + x$ at $x = -0.25$.

..... [3]

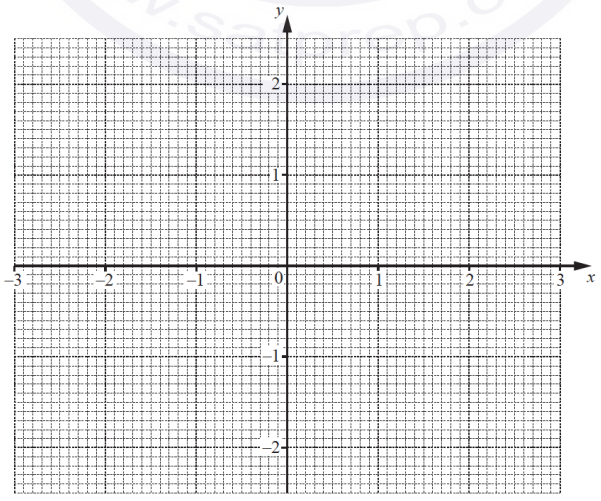
Question 42

The table shows some values for $y = \frac{3}{10}x^3 - 2x$ for $-3 \leqslant x \leqslant 3$.

x	−3	−2	−1.5	−1	0	1	1.5	2	3
y			2.0	1.7	0		−2.0	−1.6	

(a) Complete the table. [3]

(b) On the grid, draw the graph of $y = \frac{3}{10}x^3 - 2x$ for $-3 \leqslant x \leqslant 3$.



[4]

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(c) On the grid opposite, draw a suitable straight line to solve the equation $\frac{3}{10}x^3 - 2x = \frac{1}{2}(1-x)$ for $-3 \leqslant x \leqslant 3$.

$x =$ ..... or $x =$ ..... or $x =$ ..... [4]

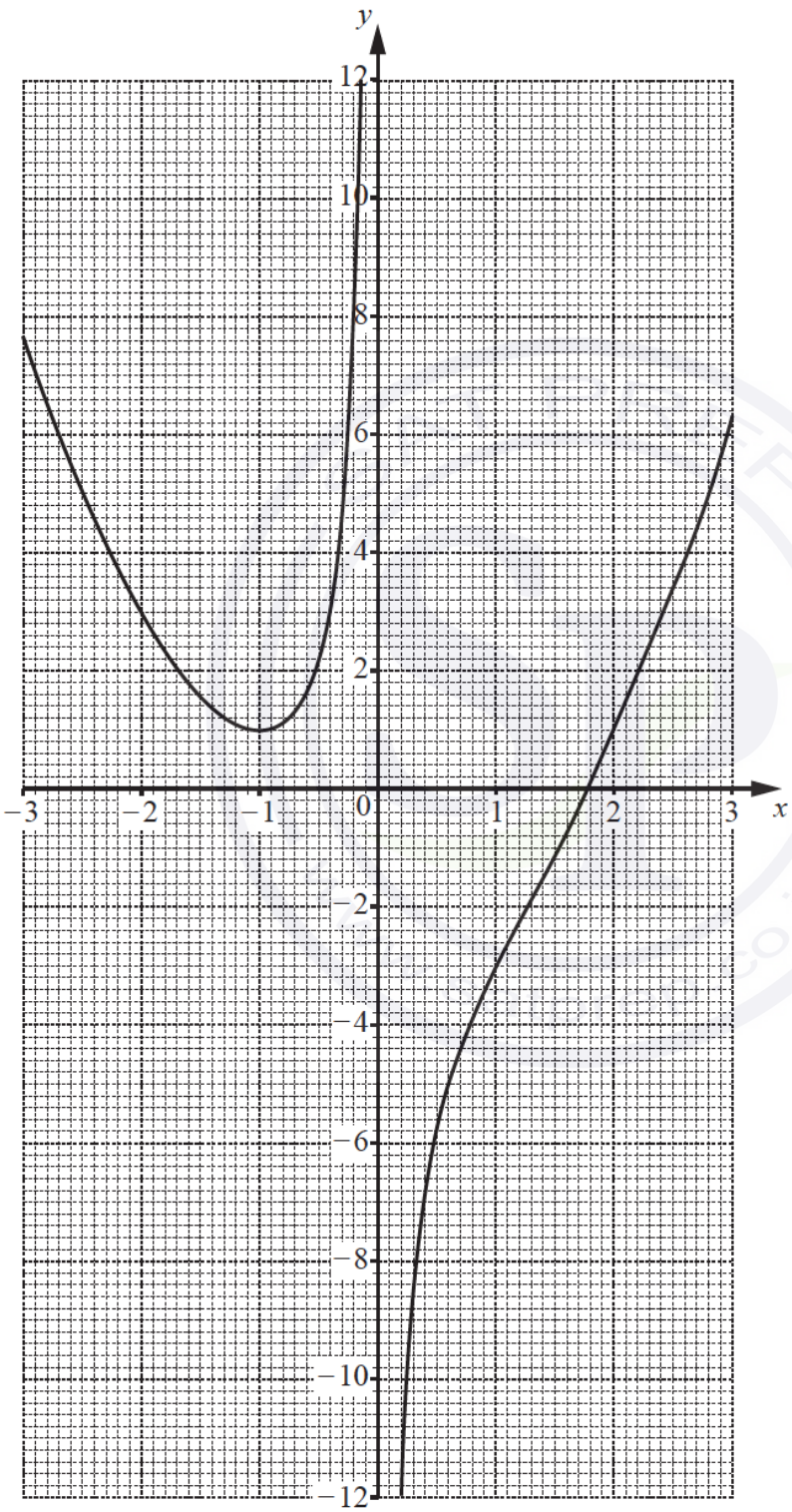
(d) For $-3 \leqslant x \leqslant 3$, the equation $\frac{3}{10}x^3 - 2x = 1$ has n solutions.

Write down the value of n.

$n =$ ..... [1]

Question 43

The diagram shows the graph of $y = f(x)$ where $f(x) = x^2 - \frac{2}{x} - 2, x \neq 0$.



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(a) Use the graph to find

(i) f(1),

..... [1]

(ii) ff(−2).

..... [2]

(b) On the grid opposite, draw a suitable straight line to solve the equation

$x^2 - \frac{2}{x} - 7 = -3x$ for $-3 \leqslant x \leqslant 3$.

$x =$ ..... or $x =$ ..... [4]

(c) By drawing a suitable tangent, find an estimate of the gradient of the curve at $x = -2$.

..... [3]

(d) **(i)** Complete the table for $y = g(x)$ where $g(x) = 2^{-x}$ for $-3 \leqslant x \leqslant 3$.

x	−3	−2	−1	0	1	2	3
y			2	1	0.5		0.125

[3]

(ii) On the grid opposite, draw the graph of $y = g(x)$. [3]

(iii) Use your graph to find the **positive** solution to the equation $f(x) = g(x)$.

$x =$ ..... [1]

Question 44

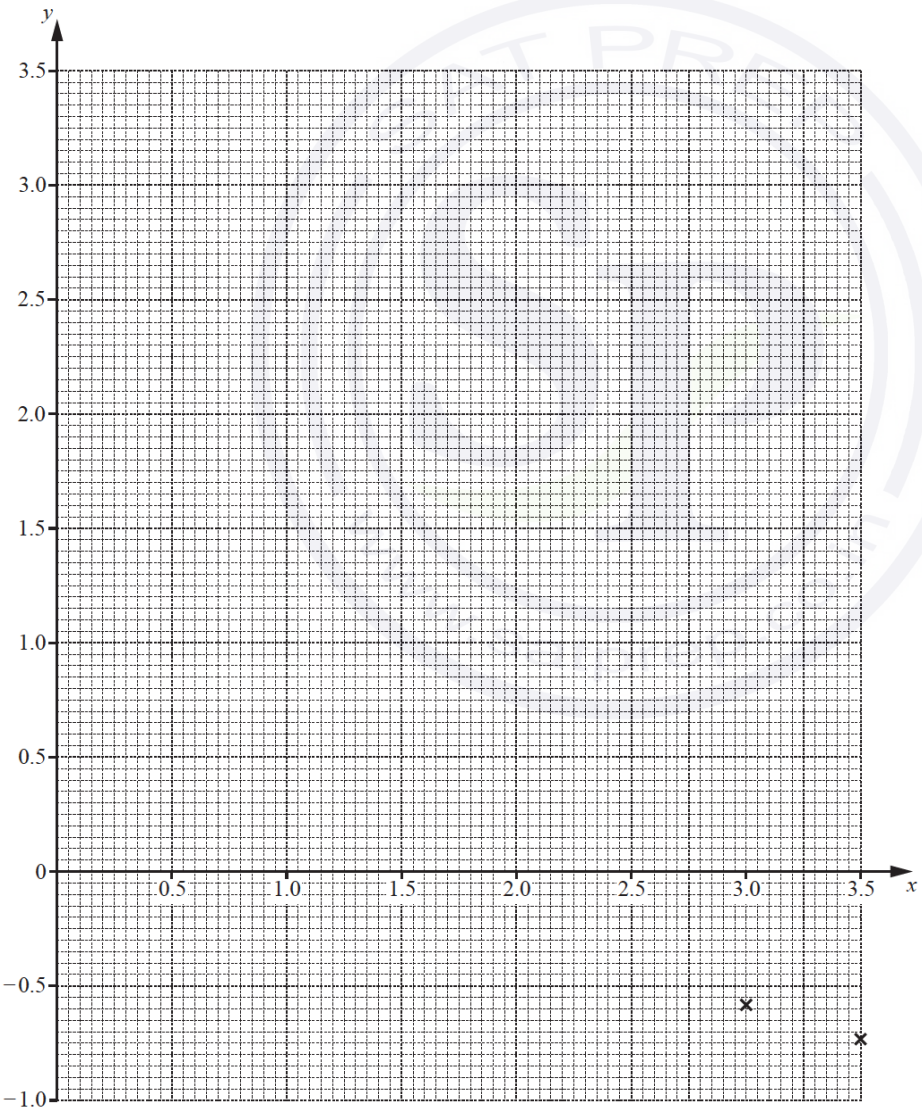
The table shows some values of $y = \frac{1}{2x} - \frac{x}{4}$ for $0.15 \leqslant x \leqslant 3.5$.

x	0.15	0.2	0.5	1	1.5	2	2.5	3	3.5
y	3.30		0.88		−0.04		−0.43	−0.58	−0.73

(a) Complete the table. [3]

(b) On the grid, draw the graph of $y = \frac{1}{2x} - \frac{x}{4}$ for $0.15 \leqslant x \leqslant 3.5$.

The last two points have been plotted for you.



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(c) Use your graph to solve the equation $\frac{1}{2x} - \frac{x}{4} = \frac{1}{2}$ for $0.15 \leqslant x \leqslant 3.5$.

$x =$ ..... [1]

(d) **(i)** On the grid, draw the line $y = 2 - x$. [2]

(ii) Write down the x co-ordinates of the points where the line $y = 2 - x$ crosses the graph of $y = \frac{1}{2x} - \frac{x}{4}$ for $0.15 \leqslant x \leqslant 3.5$.

$x =$ ..... and $x =$ ..... [2]

(e) Show that the graph of $y = \frac{1}{2x} - \frac{x}{4}$ can be used to find the value of $\sqrt{2}$ for $0.15 \leqslant x \leqslant 3.5$.

[2]

Question 45

(a) The equation of a straight line is $2y = 3x + 4$.

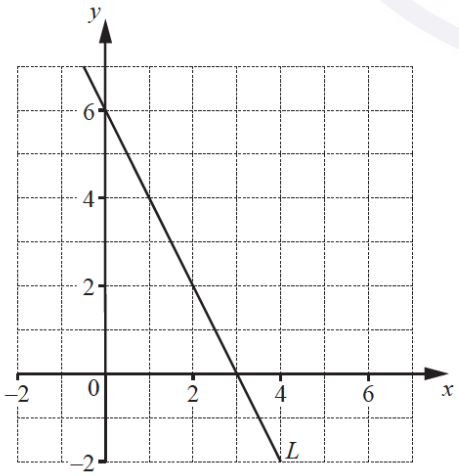
(i) Find the gradient of this line.

..... [1]

(ii) Find the co-ordinates of the point where the line crosses the y-axis.

(..... , ...) [1]

(b) The diagram shows a straight line L.



(i) Find the equation of line L.

..... [3]

(ii) Find the equation of the line perpendicular to line L that passes through (9, 3).

(c) A is the point (8, 5) and B is the point $(-4, 1)$.

(i) Calculate the length of AB.

..... [3]

(ii) Find the co-ordinates of the midpoint of AB.

(..... , ...) [2]

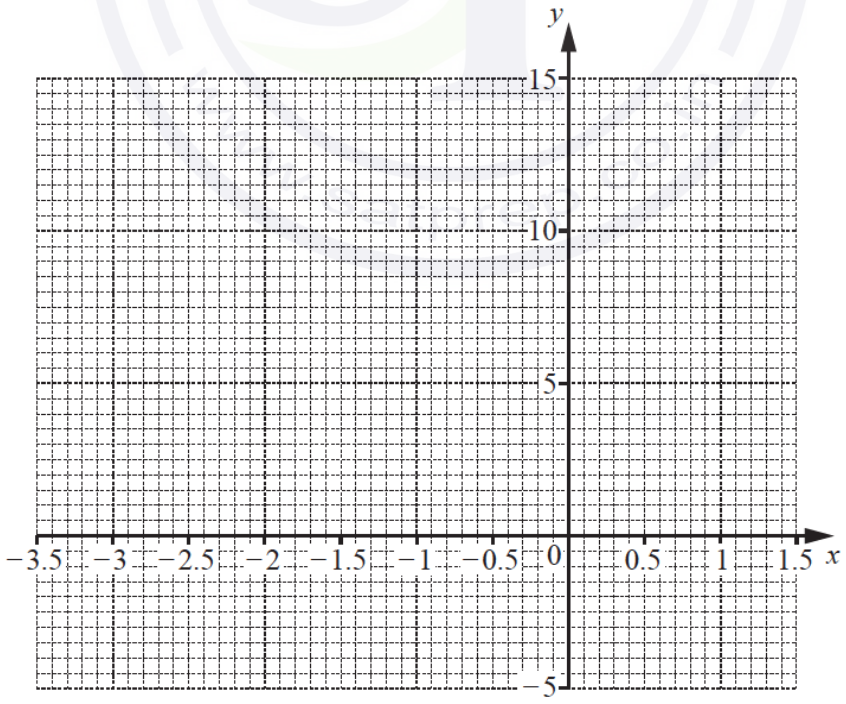
Question 46

The table shows some values for $y = x^3 + 3x^2 + 2$.

x	−3.5	−3	−2.5	−2	−1.5	−1	−0.5	0	0.5	1	
y	−4.1		5.1	6	5.4	4	2.6		2.9		

(a) Complete the table.

(b) On the grid, draw the graph of $y = x^3 + 3x^2 + 2$ for $-3.5 \leqslant x \leqslant 1.5$.



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[4]

(c) Use your graph to solve the equation $x^3 + 3x^2 + 2 = 0$ for $-3.5 \leqslant x \leqslant 1.5$.

$x =$ ..... [1]

(d) By drawing a suitable straight line, solve the equation $x^3 + 3x^2 + 2x + 2 = 0$ for $-3.5 \leqslant x \leqslant 1.5$.

$x =$ ..... [2]

(e) For $-3.5 \leqslant x \leqslant 1.5$, the equation $x^3 + 3x^2 + 2 = k$ has three solutions and k is an integer.

Write down a possible value of k.

$k =$ ..... [1]

Question 47

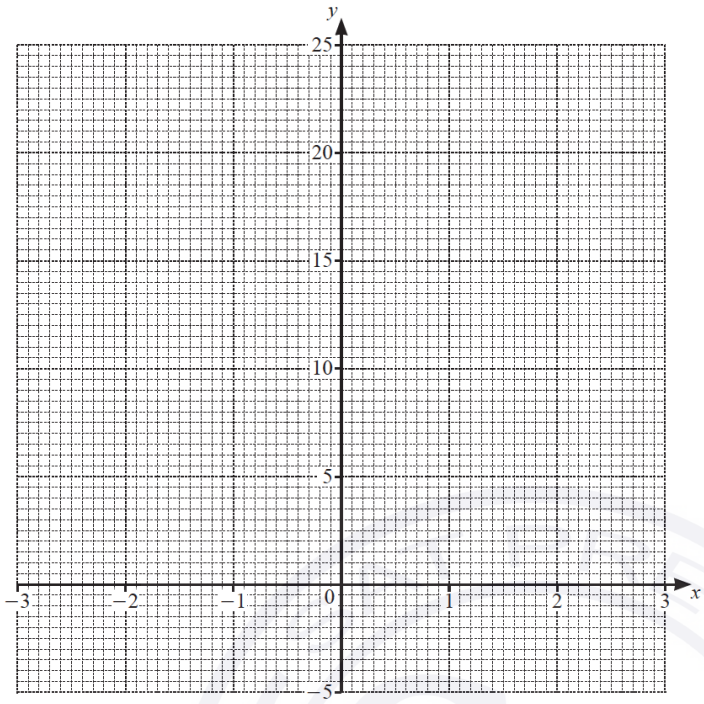
The table shows some values for $y = x^3 + x^2 - 5x$.

x	-3	-2	-1.5	-1	0	1	1.5	2	2.5	3
y	-3	6	6.4		0		-1.9	2	9.4	

(a) Complete the table. [3]

(b) On the grid, draw the graph of $y = x^3 + x^2 - 5x$ for $-3 \leqslant x \leqslant 3$.

Continue on the next page..



[4]

(c) Use your graph to solve the equation $x^3 + x^2 - 5x = 0$.

$x =$ ..... or $x =$ ..... or $x =$ ..... [2]

(d) By drawing a suitable tangent, find an estimate of the gradient of the curve at $x = 2$.

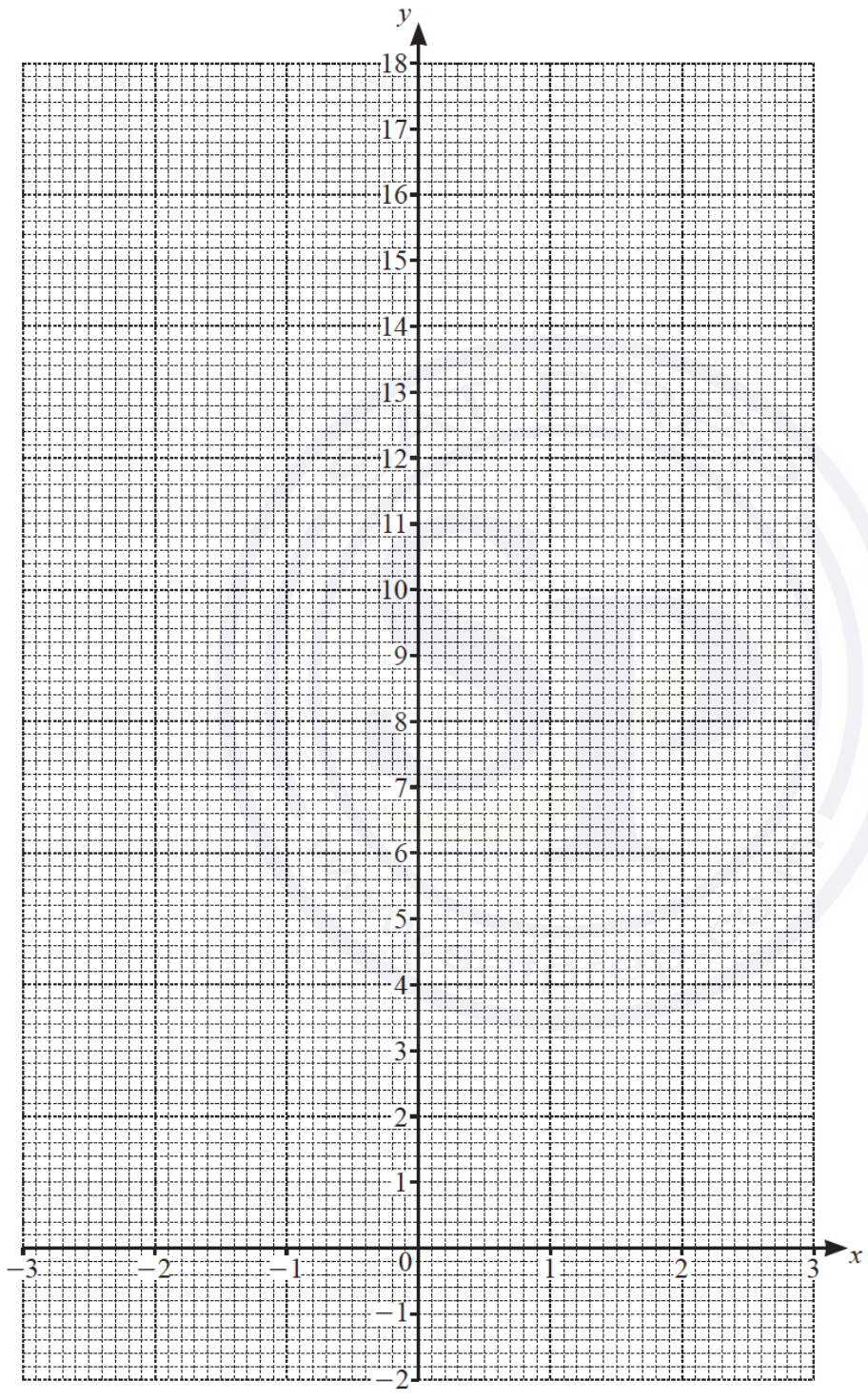
..... [3]

(e) Write down the largest value of the integer, k, so that the equation $x^3 + x^2 - 5x = k$ has three solutions for $-3 \leqslant x \leqslant 3$.

$k =$ ..... [1]

Question 48

The table shows some values of $y = \frac{x^2}{2} + \frac{1}{x^2} - \frac{2}{x}, \ x \neq 0.$



[5]

Continue on the next page..

(c) Use your graph to solve $\frac{x^2}{2}+\frac{1}{x^2}-\frac{2}{x}\leqslant 0$.

..... $\leqslant x \leqslant$ ..... [2]

(d) Find the smallest positive integer value of k for which $\frac{x^2}{2}+\frac{1}{x^2}-\frac{2}{x}=k$ has two solutions for $-3\leqslant x\leqslant -0.3$ and $0.2\leqslant x\leqslant 3$.

..... [1]

(e) **(i)** By drawing a suitable straight line, solve $\frac{x^2}{2}+\frac{1}{x^2}-\frac{2}{x}=3x+1$ for $-3\leqslant x\leqslant -0.3$ and $0.2\leqslant x\leqslant 3$.

$x =$ ..... [3]

(ii) The equation $\frac{x^2}{2}+\frac{1}{x^2}-\frac{2}{x}=3x+1$ can be written as $x^4+ax^3+bx^2+cx+2=0$.

Find the values of a, b and c.

$a =$ .....

$b =$ .....

$c =$ ..... [3]

Question 49

A line joins A (1, 3) to B (5, 8).

(i) Find the midpoint of AB.

(..... , ...) [2]

(ii) Find the equation of the line AB.
Give your answer in the form $y = mx + c$.

$y =$ ..... [3]

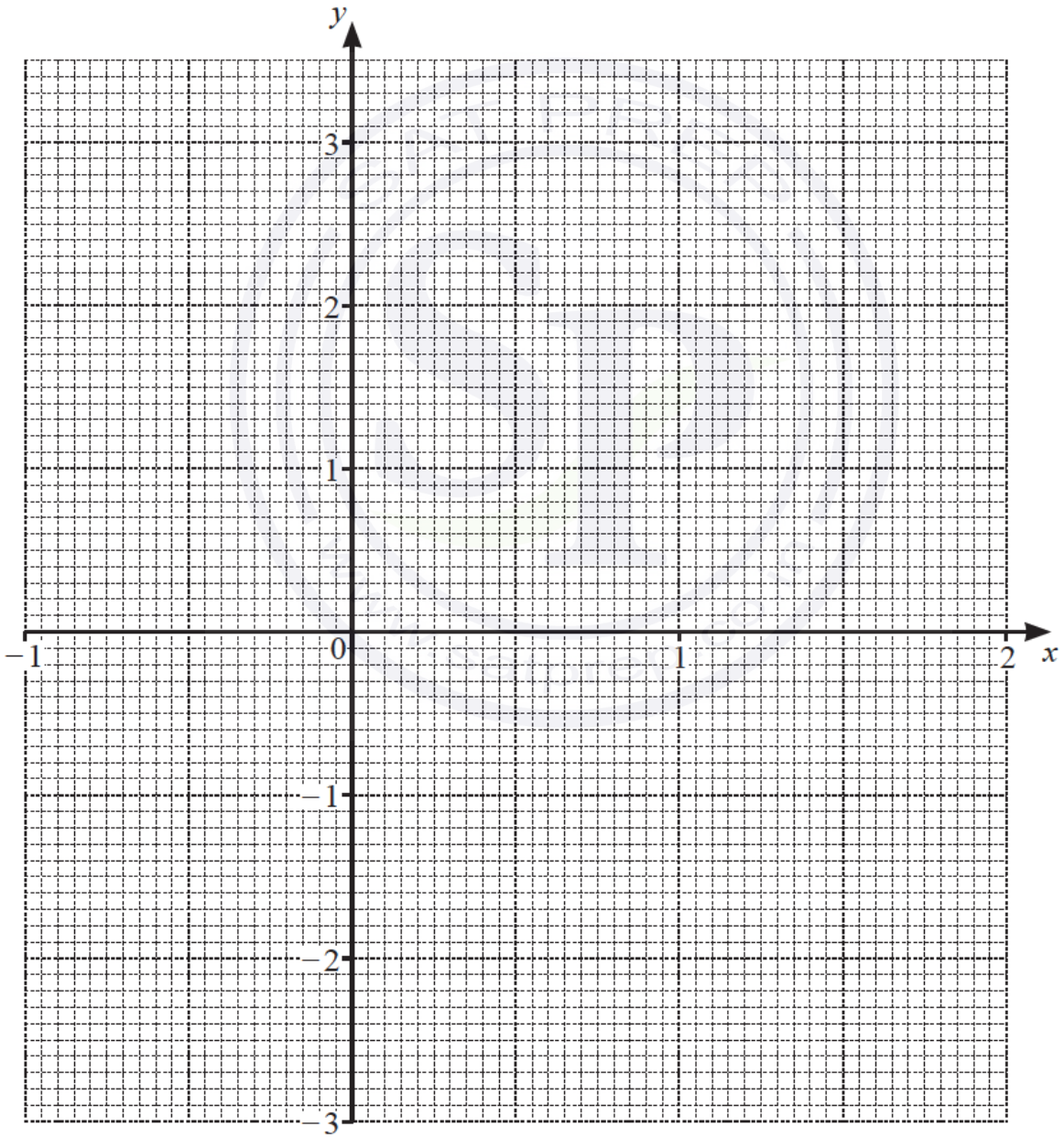
Question 50

(a) The table shows some values for $y = 2x^3 - 4x^2 + 3$.

x	-1	-0.5	0	0.5	1	1.5	2
y	-3	1.75				0.75	3

(i) Complete the table. [3]

(ii) On the grid, draw the graph of $y = 2x^3 - 4x^2 + 3$ for $-1 \leqslant x \leqslant 2$.



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[4]

(iii) Use your graph to solve the equation $2x^3 - 4x^2 + 3 = 1.5$.

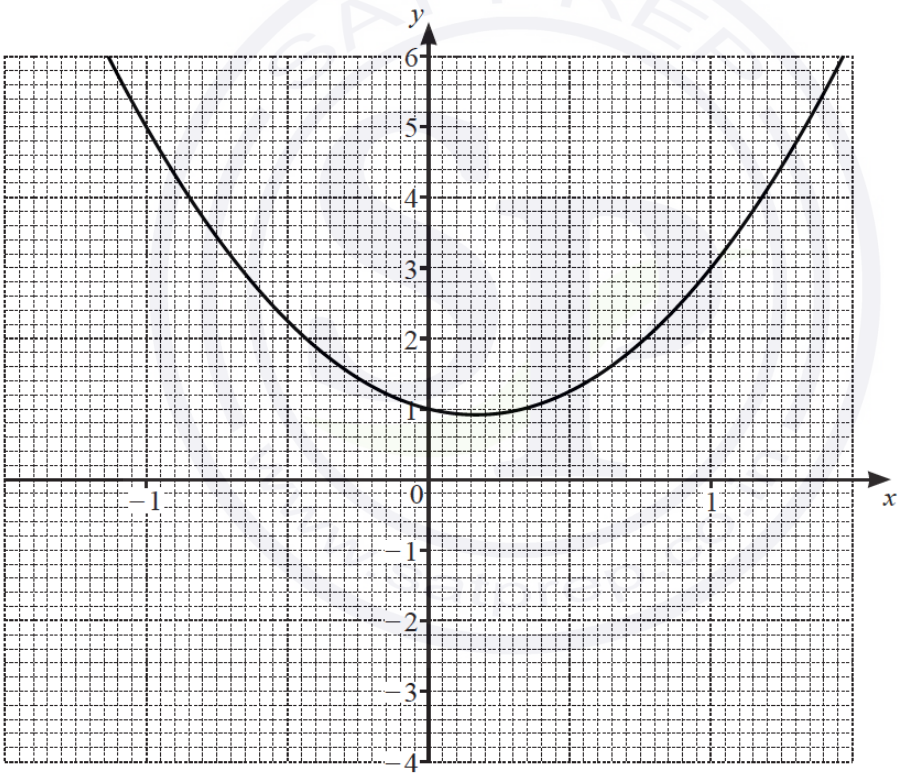
$x =$ ..... or $x =$ ..... or $x =$ ..... [3]

(iv) The equation $2x^3 - 4x^2 + 3 = k$ has only one solution for $-1 \leqslant x \leqslant 2$.

Write down a possible integer value of k.

..... [1]

(b)



(i) On the grid, draw the tangent to the curve at $x = 1$. [1]

(ii) Use your tangent to estimate the gradient of the curve at $x = 1$.

..... [2]

(iii) Write down the equation of your tangent in the form $y = mx + c$.

$y =$ ..... [2]

Question 51

(a) A curve has equation $y = 4x^3 - 3x + 3$.

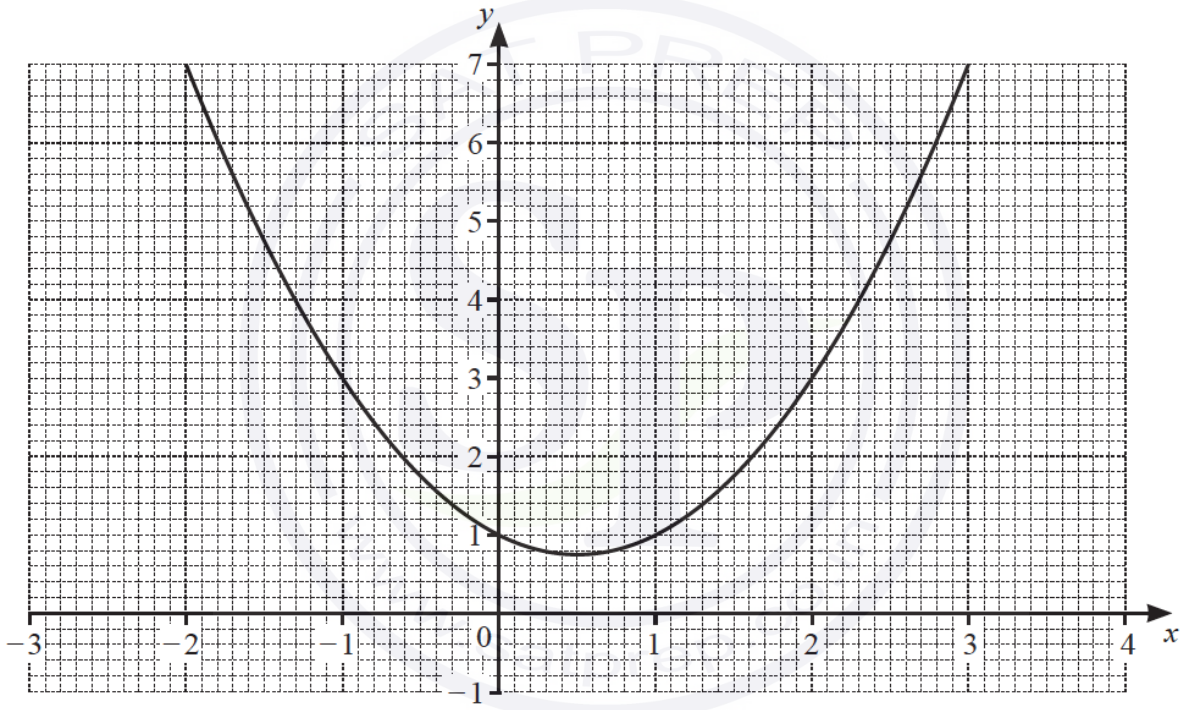
(i) Find the coordinates of the two stationary points.

(..... , ...) and (..... , ...) [5]

(ii) Determine whether each of the stationary points is a maximum or a minimum.
Give reasons for your answers.

[3]

(b) The graph of $y = x^2 - x + 1$ is shown on the grid.



By drawing a suitable line on the grid, solve the equation $x^2 - 2x - 2 = 0$.

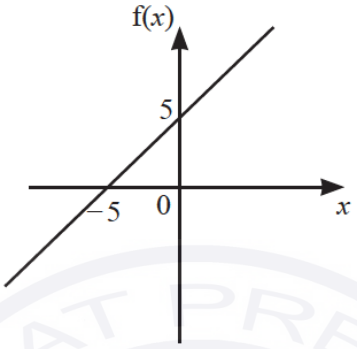
$x =$ ..... or $x =$ ..... [3]

Question 52

(a) The diagrams show the graphs of two functions.

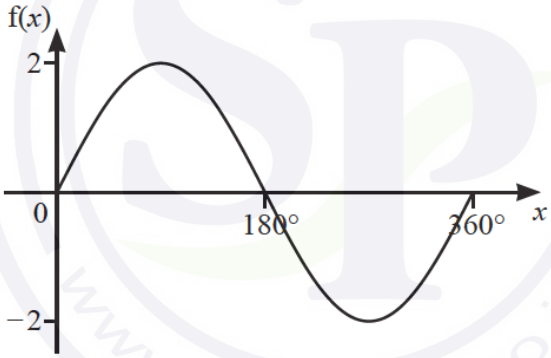
Write down each function.

(i)



f(x) = ... [2]

(ii)



f(x) = ... [2]

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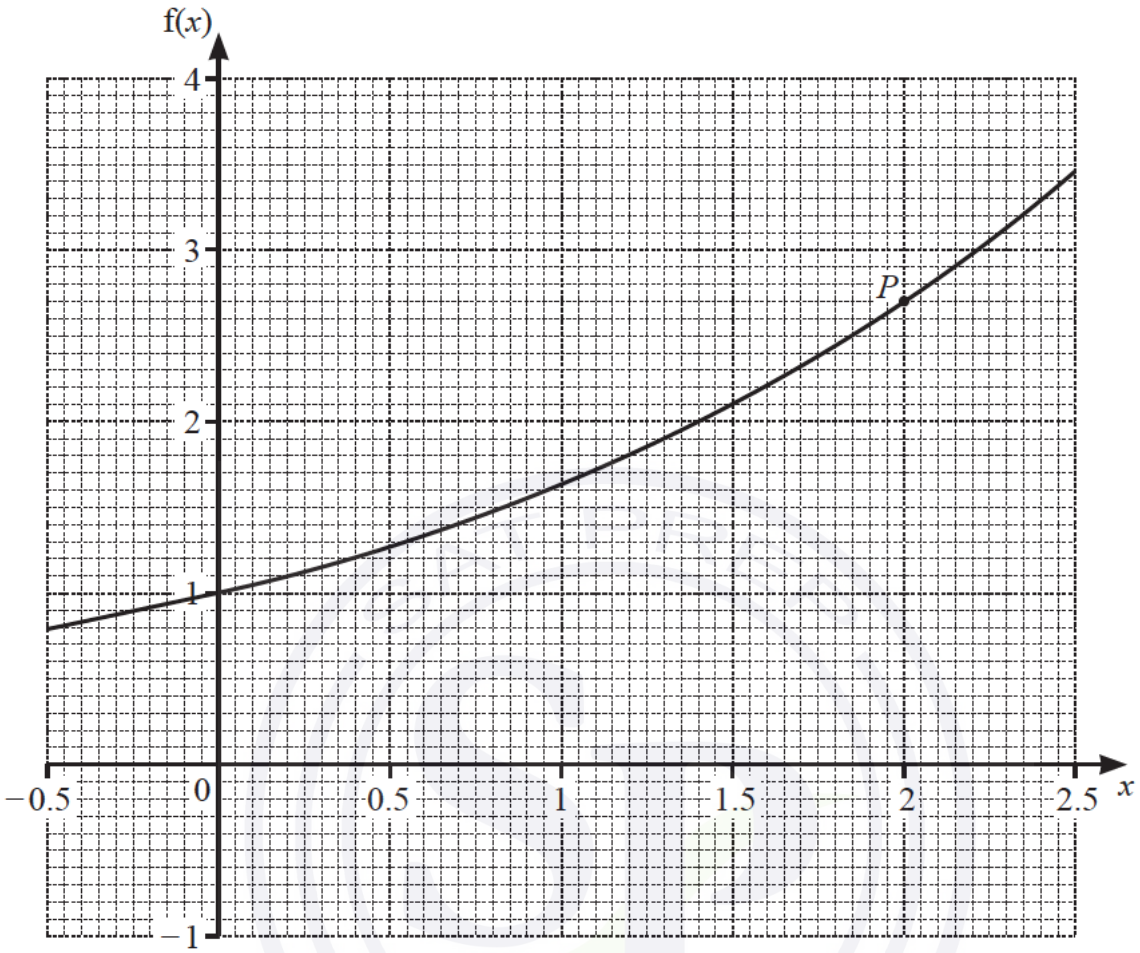


The diagram shows the graph of another function.

By drawing a suitable tangent, find an estimate for the gradient of the function at the point P.

..... [3]

Question 53

(a) The equation of line L is $3x-8y+20=0$.

(i) Find the gradient of line L.

..... [2]

(ii) Find the coordinates of the point where line L cuts the y-axis.

(..... , ...) [1]

Continue on the next page..

(b) The coordinates of P are $(-3, 8)$ and the coordinates of Q are $(9, -2)$.

(i) Calculate the length PQ.

..... [3]

(ii) Find the equation of the line parallel to PQ that passes through the point $(6, -1)$.

..... [3]

(iii) Find the equation of the perpendicular bisector of PQ.

..... [4]

Question 54

(a) $y = x^4 - 4x^3$

(i) Find the value of y when $x = -1$.

$y =$ ..... [2]

(ii) Find the two stationary points on the graph of $y = x^4 - 4x^3$.

(..... , ...)

(..... , ...) [6]

(b) $y = x^p + 2x^q$

$\frac{dy}{dx} = 11x^{10} + 10x^4$, where $\frac{dy}{dx}$ is the derived function.

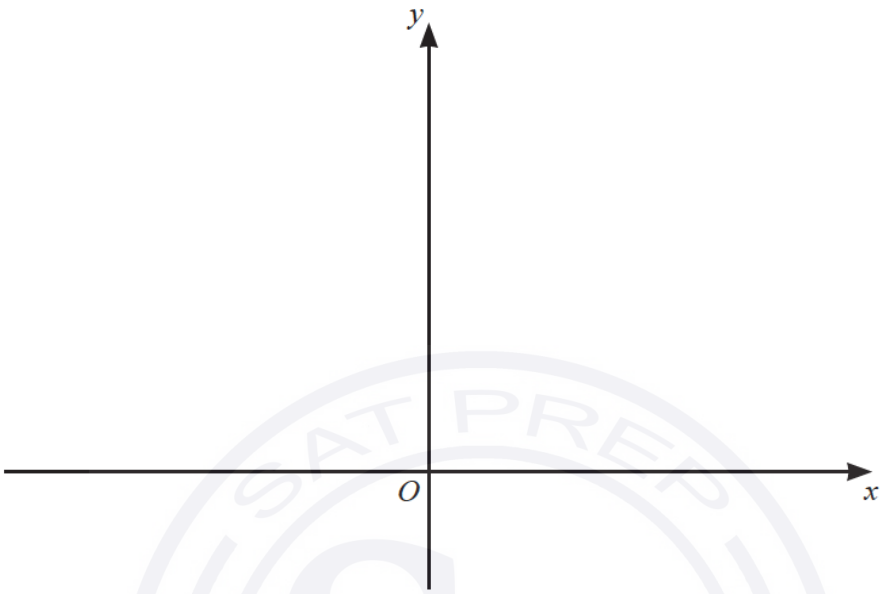
Find the value of p and the value of q.

$p =$ .....

$q =$ ..... [2]

Question 55

(i)



On the diagram,

(a) sketch the graph of $y = (x-1)^2$, [2]

(b) sketch the graph of $y = \frac{1}{2}x + 1$. [2]

(ii) The graphs of $y = (x-1)^2$ and $y = \frac{1}{2}x + 1$ intersect at A and B.

Find the length of AB.

$AB =$ ..... [7]

Question 56

(a) A rhombus $ABCD$ has a diagonal AC where A is the point $(-3, 10)$ and C is the point $(4, -4)$.

(i) Calculate the length AC.

..... [3]

(ii) Show that the equation of the line AC is $y = -2x + 4$.

Continue on the next page..

[2]

(iii) Find the equation of the line BD.

..... [4]

(b) A curve has the equation $y = x^3 + 8x^2 + 5x$.

(i) Work out the coordinates of the two turning points.

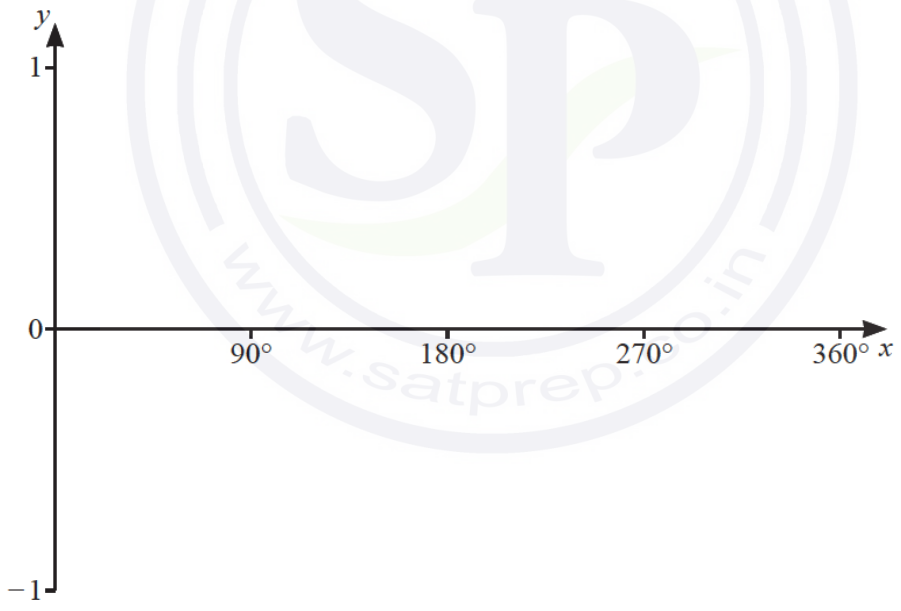
(..... , ...) and (..... , ...) [6]

(ii) Determine whether each of the turning points is a maximum or a minimum.
Give reasons for your answers.

[3]

Question 57

(a) **(i)** On the axes, sketch the graph of $y = \sin x$ for $0° \leqslant x \leqslant 360°$.



[2]

Continue on the next page..

(ii) Describe fully the symmetry of the graph of $y = \sin x$ for $0° \leqslant x \leqslant 360°$.

.....

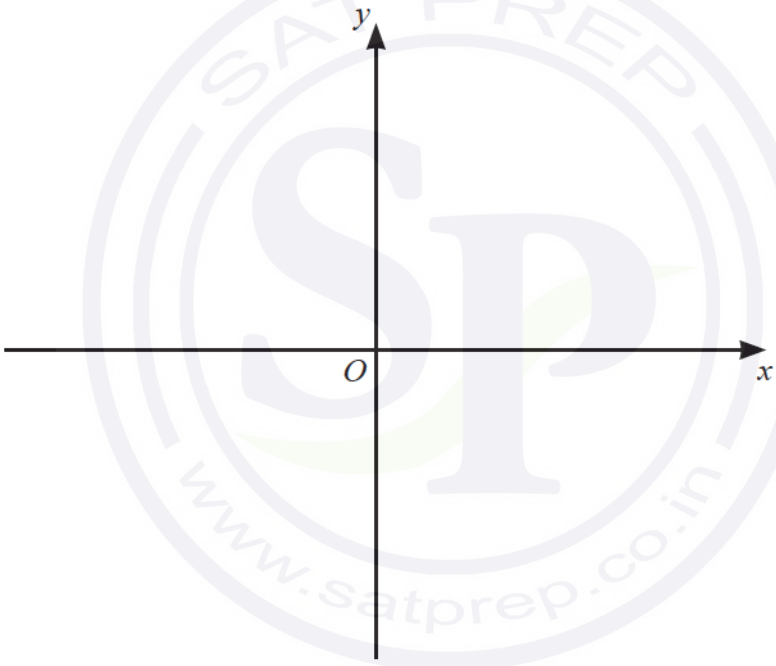
..... [2]

(b) Solve $4\sin x - 1 = 2$ for $0° \leqslant x \leqslant 360°$.

$x =$ ..... and $x =$ ..... [3]

..... [2]

(ii) On the axes, sketch the graph of $y = x^2 + 10x + 14$, indicating the coordinates of the turning point.



[3]

Question 58

The straight line $y = 3x + 2$ intersects the curve $y = 2x^2 + 7x - 11$ at two points.

Find the coordinates of these two points.
Give your answers correct to 2 decimal places.

(..... , ...)

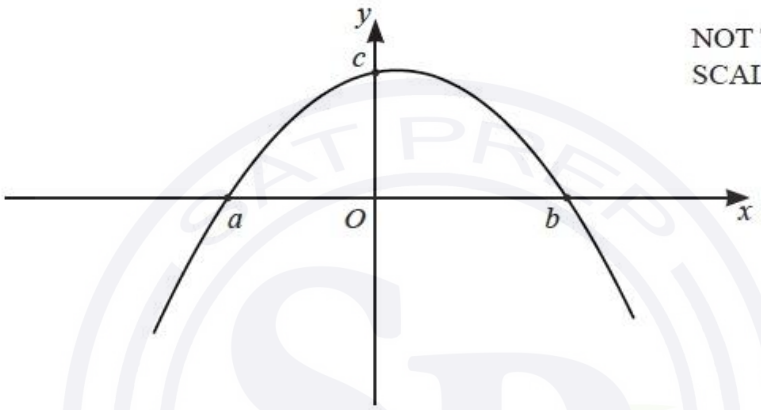
(..... , ...) [6]

Question 59

(a) (i) Factorise $24+5x-x^2$.

..... [2]

(ii) The diagram shows a sketch of $y=24+5x-x^2$.



NOT TO SCALE

Work out the values of a, b and c.

$a =$ .....

$b =$ .....

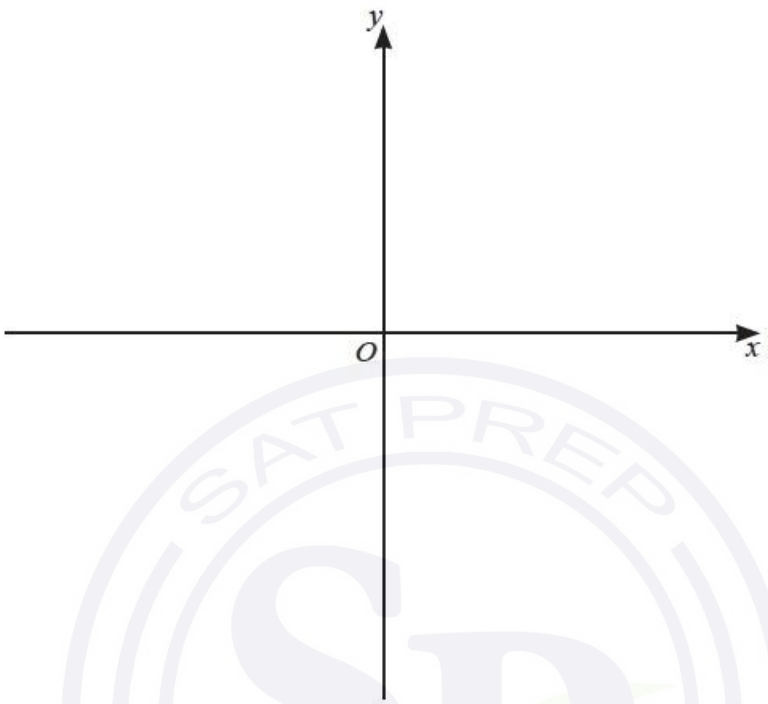
$c =$ ..... [3]

(iii) Calculate the gradient of $y=24+5x-x^2$ at $x=-1.5$.

..... [3]

Continue on the next page..

(b) **(i)** On the diagram, sketch the graph of $y = (x+1)(x-3)^2$.
Label the values where the graph meets the x-axis and the y-axis.



[4]

(ii) Write $(x+1)(x-3)^2$ in the form ax^3+bx^2+cx+d.

..... [3]

Question 60

$$y = x^2 + \frac{1}{x},\ x \neq 0$$

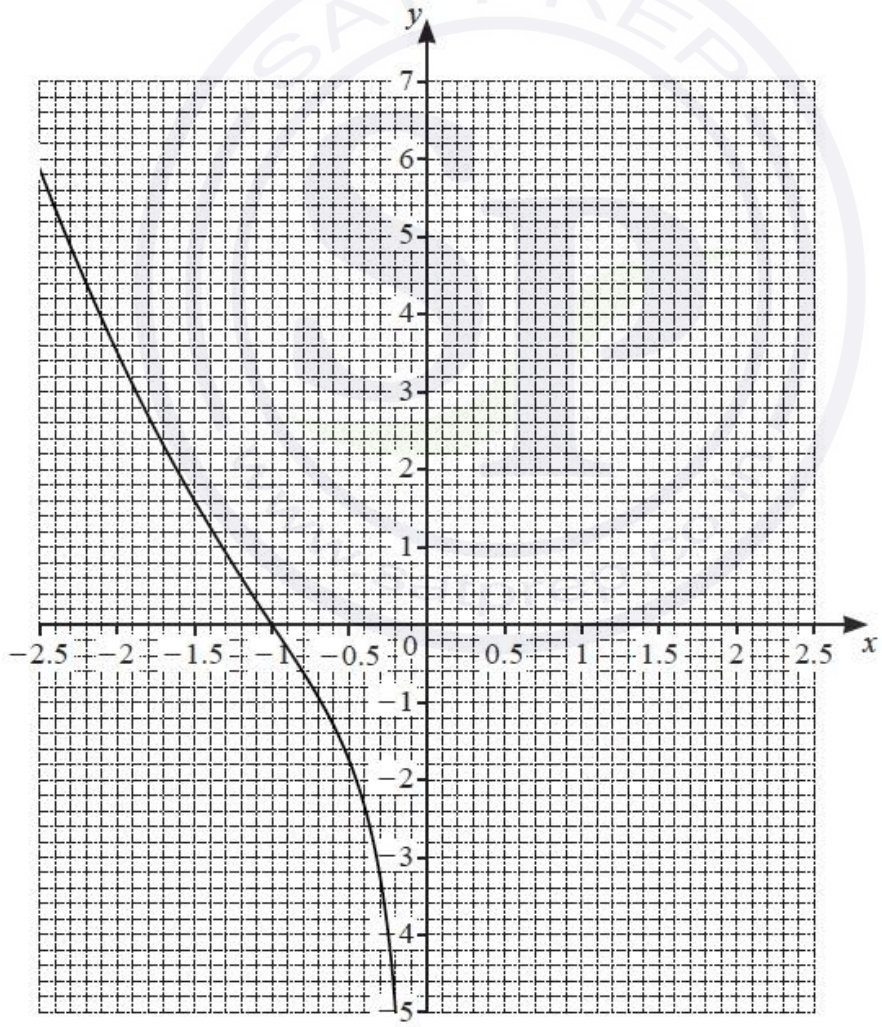
(a) Complete the table.

x	0.2	0.3	0.5	1	1.5	2	2.5
y	5.0	3.4	2.3		2.9		6.7

[2]

(b) On the grid, draw the graph of $y = x^2 + \frac{1}{x}$ for $0.2 \leqslant x \leqslant 2.5$.

The graph of $y = x^2 + \frac{1}{x}$ for $-2.5 \leqslant x \leqslant -0.2$ has been drawn for you.



[4]

Continue on next page…

(c) By drawing suitable straight lines on the grid, solve the following equations.

(i) $x^2 + \frac{1}{x} = -2$

$x =$ ..... [1]

(ii) $x^2 + \frac{1}{x} + x - 1 = 0$

$x =$ ..... [2]

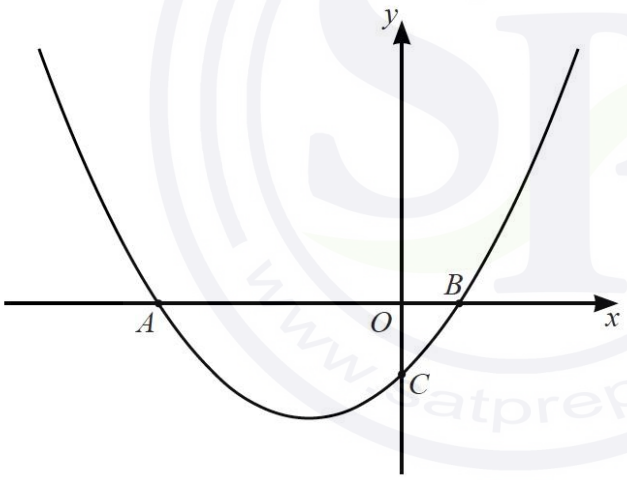
(d) k is an integer and the equation $x^2 + \frac{1}{x} = k$ has three solutions.

Write down a possible value of k.

$k =$ ..... [1]

Question 61

(a)



NOT TO SCALE

The diagram shows a sketch of the curve $y = x^2 + 3x - 4$.

(i) Find the coordinates of the points A, B and C.

A (..... , ...)

B (..... , ...)

C (..... , ...) [4]

Continue on the next page..

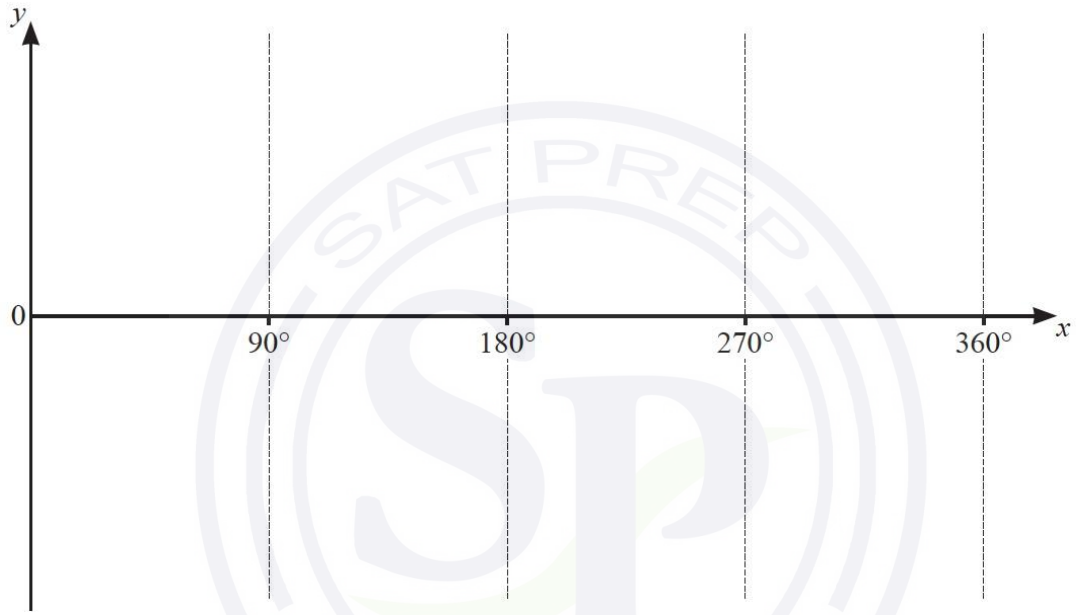
(ii) Differentiate $x^2 + 3x - 4$.

..... [2]

(iii) Find the equation of the tangent to the curve at the point (2, 6).

..... [3]

(b)



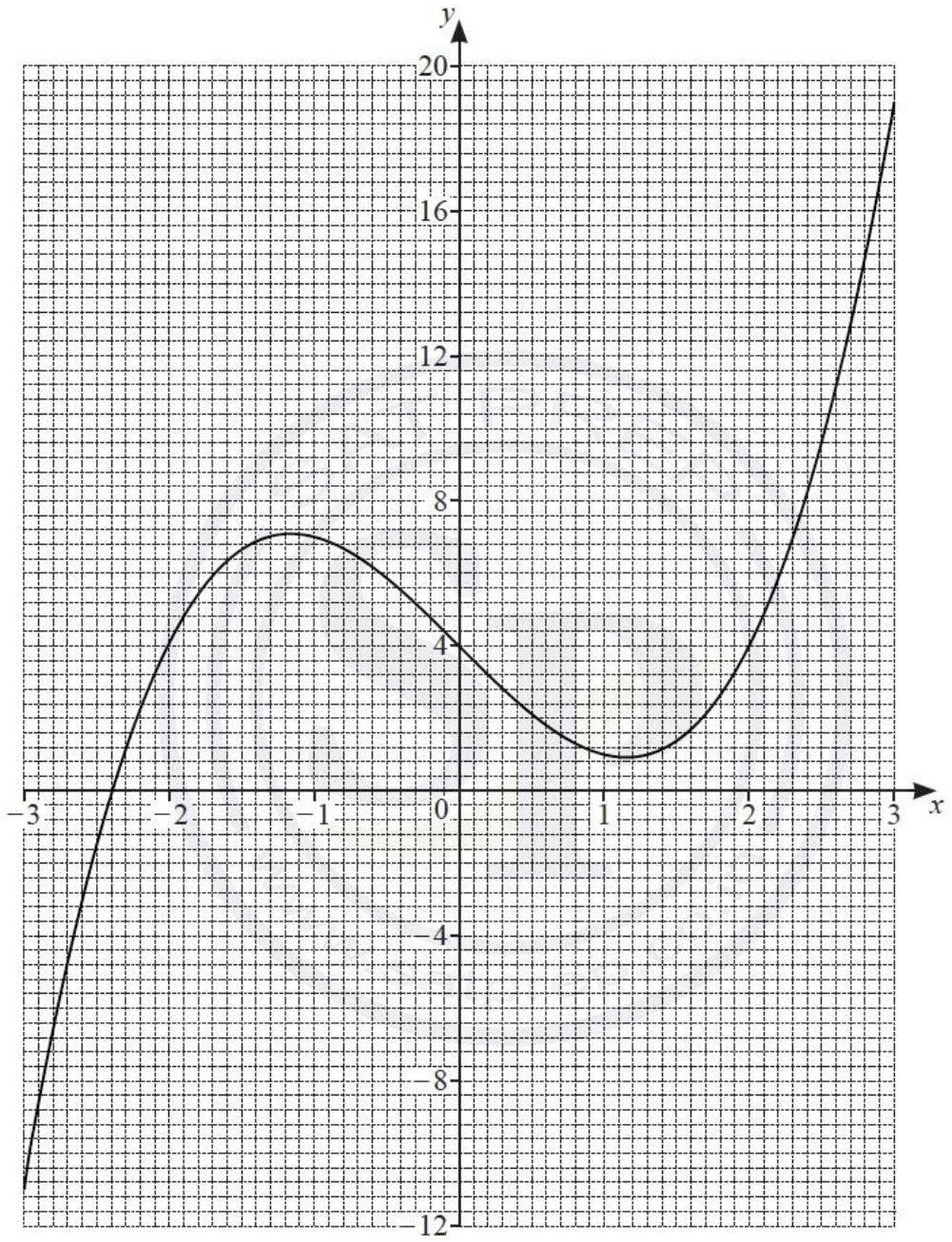
(i) On the diagram, sketch the graph of $y = \tan x$ for $0° \leqslant x \leqslant 360°$. [2]

(ii) Solve the equation $5 \tan x = -7$ for $0° \leqslant x \leqslant 360°$.

$x =$ ..... or $x =$ ..... [3]

Question 62

(a) The diagram shows the graph of $y = f(x)$ for $-3 \leqslant x \leqslant 3$.



(i) Solve $f(x) = 14$.

$x =$ ..... [1]

(ii) By drawing a suitable tangent, find an estimate of the gradient of the graph at the point $(-2, 4)$.

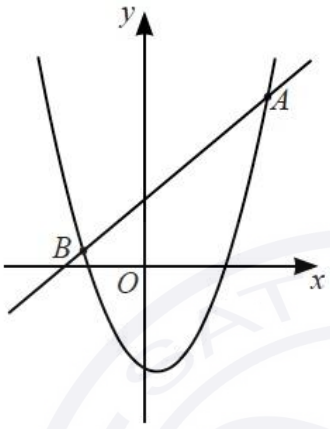
Continue on the next page..

..... [3]

(iii) By drawing a suitable straight line on the grid, solve $f(x) = 2x - 2$ for $-3 \leqslant x \leqslant 3$.

$x =$ ..... [3]

(b)



NOT TO SCALE

The diagram shows a curve with equation $y = 2x^2 - 2x - 7$.
The straight line with equation $y = 3x + 5$ intersects the curve at the points A and B.

Find the coordinates of the points A and B.

A (..... , ...)

B (..... , ...) [5]

Question 63

(a) Find the gradient of the curve $y = 2x^3 - 7x + 4$ when $x = -2$.

..... [3]

(b) A is the point (7, 2) and B is the point (−5, 8).

(i) Calculate the length of AB.

..... [3]

(ii) Find the equation of the line that is perpendicular to AB and that passes through the point (−1, 3).
Give your answer in the form $y = mx + c$.

$y =$ ..... [4]

Continue on the next page..

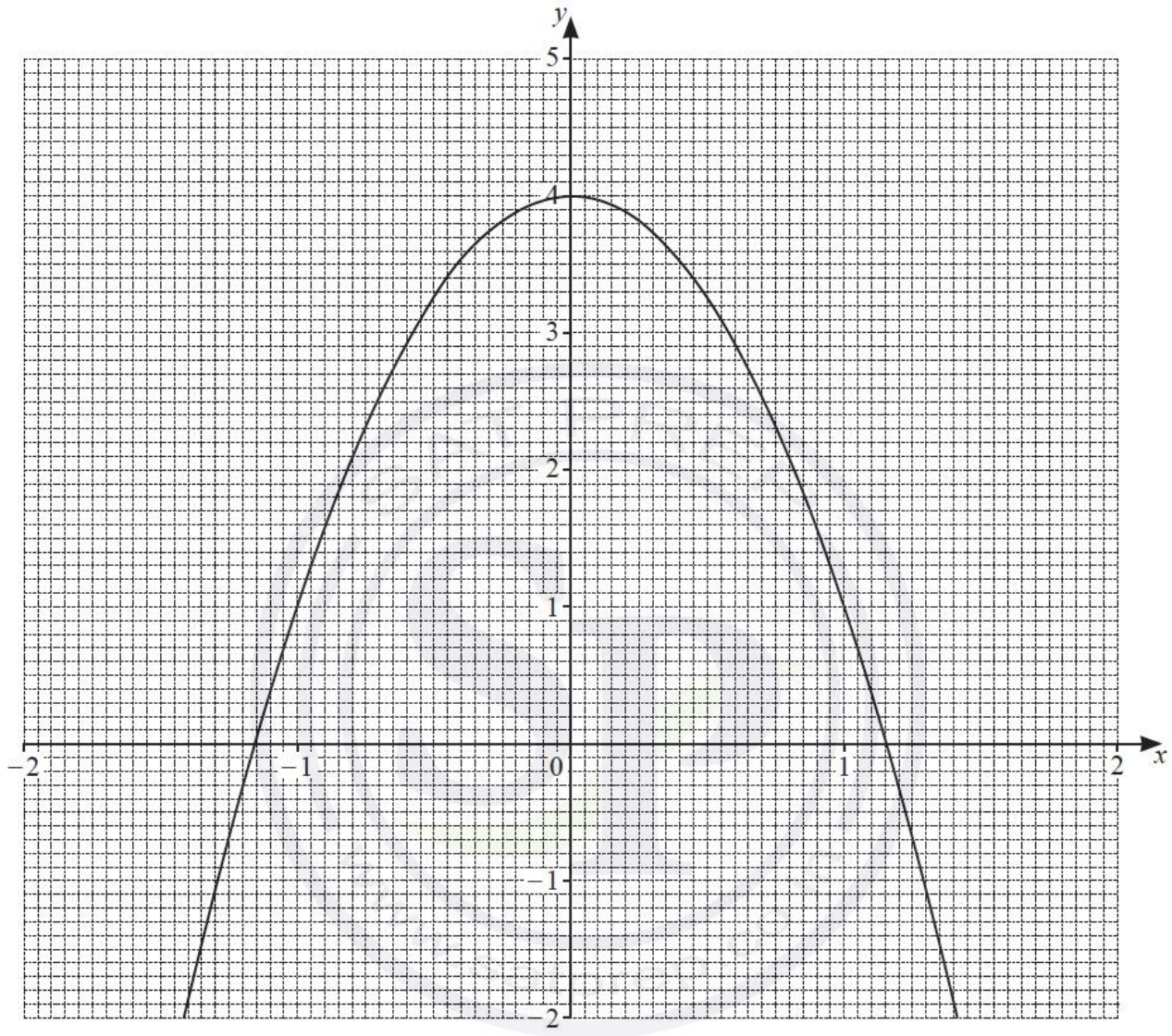
(iii) AB is one side of the parallelogram $ABCD$ and

- $\overrightarrow{BC} = \begin{pmatrix} -a \\ -b \end{pmatrix}$ where $a > 0$ and $b > 0$
- the gradient of BC is 1
- $\left|\overrightarrow{BC}\right| = \sqrt{8}$.

Find the coordinates of D.

(..... , ...) [4]

Question 64



(a) The grid shows the graph of $y = a + bx^2$.

The graph passes through the points with coordinates (0, 4) and (1, 1).

(i) Find the value of a and the value of b.

$a =$ .....

$b =$ ..... [2]

Continue on the next page..

(ii) Write down the equation of the tangent to the graph at (0, 4).

..... [1]

(iii) The equation of the tangent to the graph at $x = -1$ is $y = 6x + 7$.

Find the equation of the tangent to the graph at $x = 1$.

..... [2]

(b) The table shows some values for $y = 1 + \frac{5}{3-x}$ for $-2 \leqslant x \leqslant 1.5$.

x	-2	-1.5	-1	-0.5	0	0.5	1	1.5
y	2	2.11		2.43		3		4.33

(i) Complete the table. [3]

(ii) On the grid, draw the graph of $y = 1 + \frac{5}{3-x}$ for $-2 \leqslant x \leqslant 1.5$. [4]

(c) (i) Write down the values of x where the two graphs intersect.

$x =$ ..... or $x =$ ..... [2]

(ii) The answers to **part(c)(i)** are two solutions of a cubic equation in terms of x.

Find this equation in the form $ax^3 + bx^2 + cx + d = 0$, where a, b, c and d are integers.

..... [4]

Question 65

A is the point (1, 5) and B is the point (3, 9).
M is the midpoint of AB.

(i) Find the coordinates of M.

(..... , ...) [2]

(ii) Find the equation of the line that is perpendicular to AB and passes through M.
Give your answer in the form $y = mx + c$.

$y =$ ..... [4]

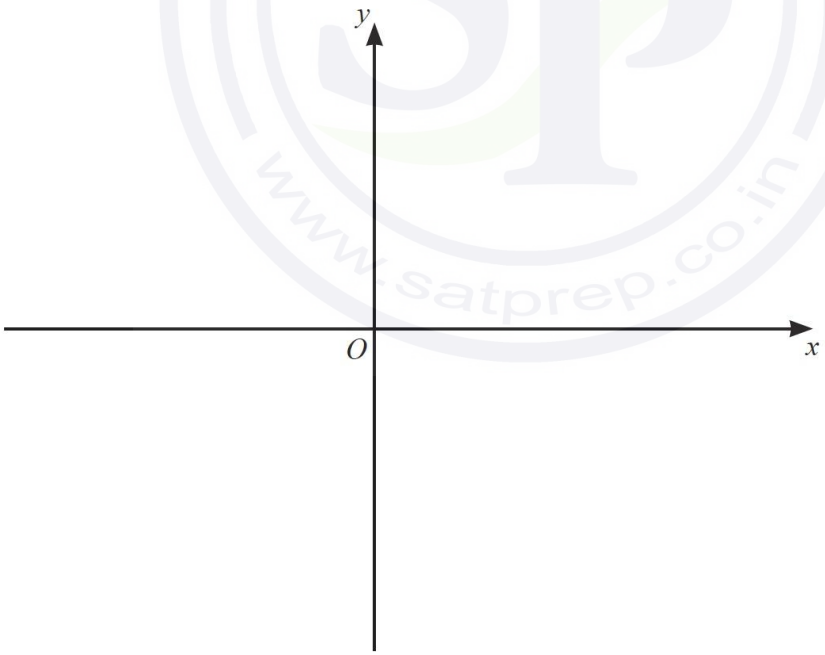
Question 66

(a) (i) The equation $y = x^3 - 4x^2 + 4x$ can be written as $y = x(x-a)^2$.

Find the value of a.

$a =$ ..... [2]

(ii) On the axes, sketch the graph of $y = x^3 - 4x^2 + 4x$, indicating the values where the graph meets the axes.



[4]

(b) Find the equation of the tangent to the graph of $y = x^3 - 4x^2 + 4x$ at $x = 4$.
Give your answer in the form $y = mx + c$.

$y =$ ..... [7]

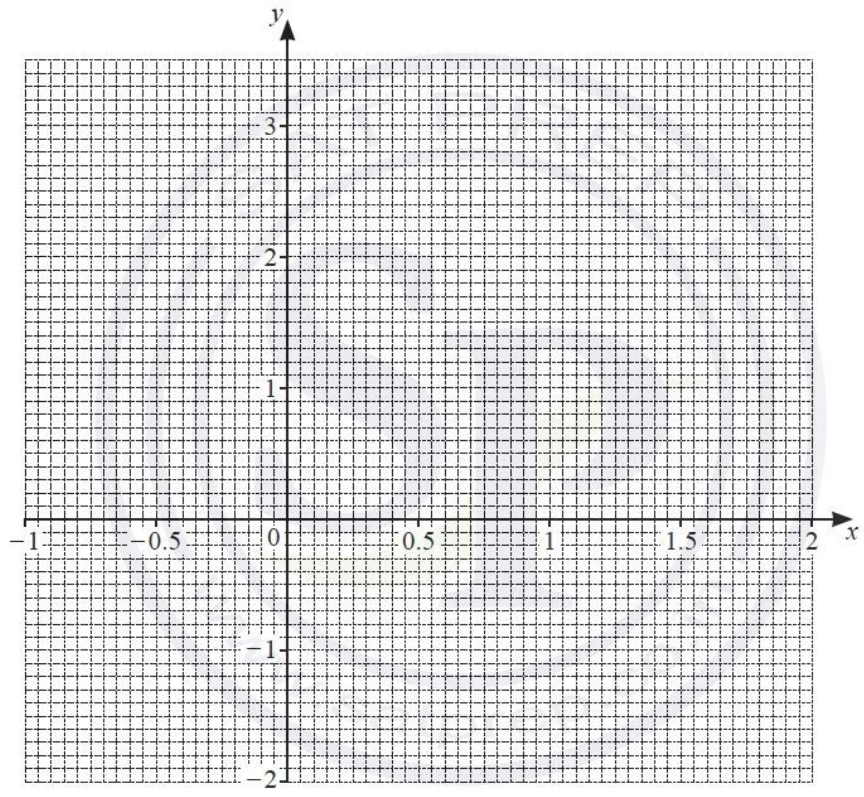
Question 67

The table shows some values for $y = 2 \times 0.5^x - 1$.

x	-1	-0.5	0	0.5	1	1.5	2
y	3	1.83		0.41	0	-0.29	

(a) (i) Complete the table. [2]

(ii) On the grid, draw the graph of $y = 2 \times 0.5^x - 1$ for $-1 \leqslant x \leqslant 2$.



[4]

(b) By drawing a suitable straight line, solve the equation $2 \times 0.5^x + 2x - 3.5 = 0$ for $-1 \leqslant x \leqslant 2$.

$x =$ ..... [3]

(c) There are no solutions to the equation $2 \times 0.5^x - 1 = k$ where k is an integer.

Complete the following statements.

The highest possible value of k is .....

The equation of the asymptote to the graph of $y = 2 \times 0.5^x - 1$ is ..... [2]

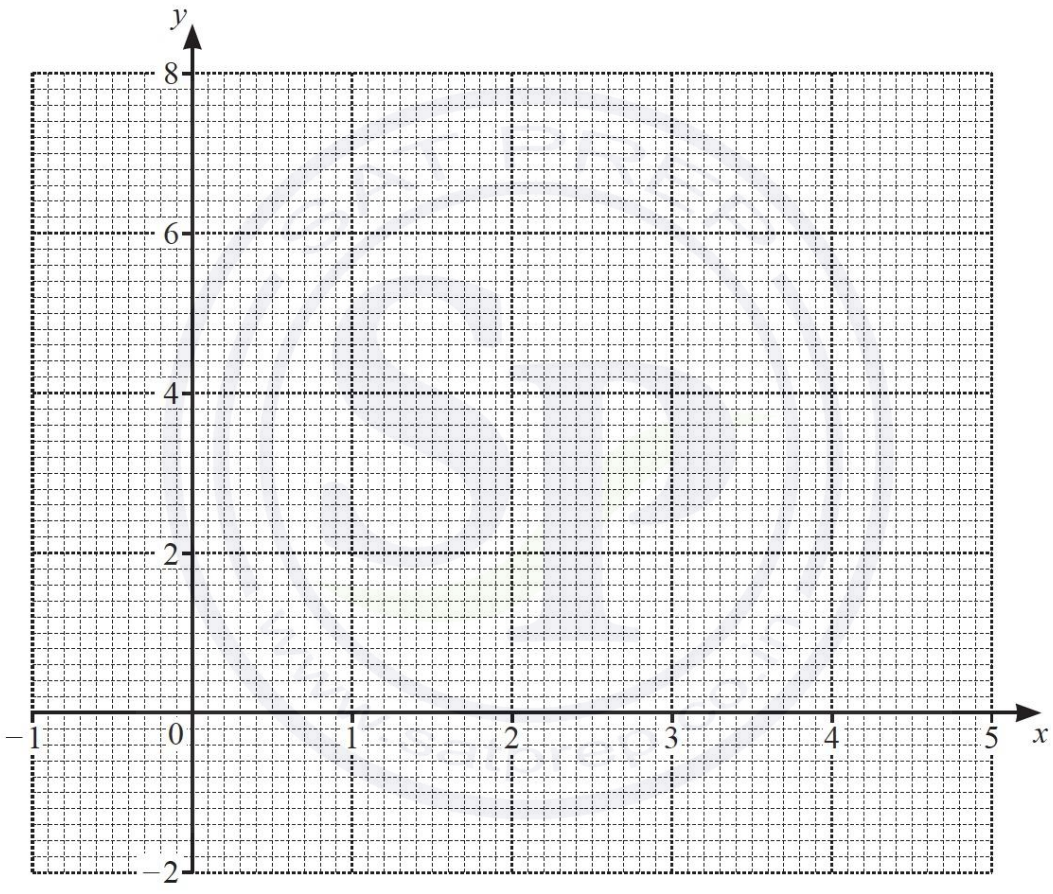
Question 68

The table shows some values of $y = 3 + 4x - x^2$ for $-1 \leqslant x \leqslant 5$.

x	-1	-0.5	0	1	2	3	4	4.5	5
y	-2			6		6			-2

(a) Complete the table. [3]

(b) On the grid, draw the graph of $y = 3 + 4x - x^2$ for $-1 \leqslant x \leqslant 5$.



[4]

(c) Write down an **integer** value of k for which the equation $3 + 4x - x^2 = k$ has no solutions.

..... [1]

(d) By drawing a suitable straight line on the grid, solve the equation $-1 + \frac{9}{2}x - x^2 = 0$.

$x =$ ..... or $x =$ ..... [4]

Question 68

A curve has equation $y = 2x^3 - 4x^2 + 6$.

(i) Find $\frac{dy}{dx}$, the derived function of y.

..... [2]

(ii) Calculate the gradient of the curve $y = 2x^3 - 4x^2 + 6$ at $x = 4$.

..... [2]

(iii) Find the coordinates of the two stationary points on the curve.

(..... , ...) and (..... , ...) [4]

Question 69

$f(x) = x(x-1)(x-2)$

(a) Find the coordinates of the points where the graph of $y = f(x)$ crosses the x-axis.

(..... , ...)

(..... , ...)

(..... , ...) [2]

(b) Show that $f(x) = x^3 - 3x^2 + 2x$.

[2]

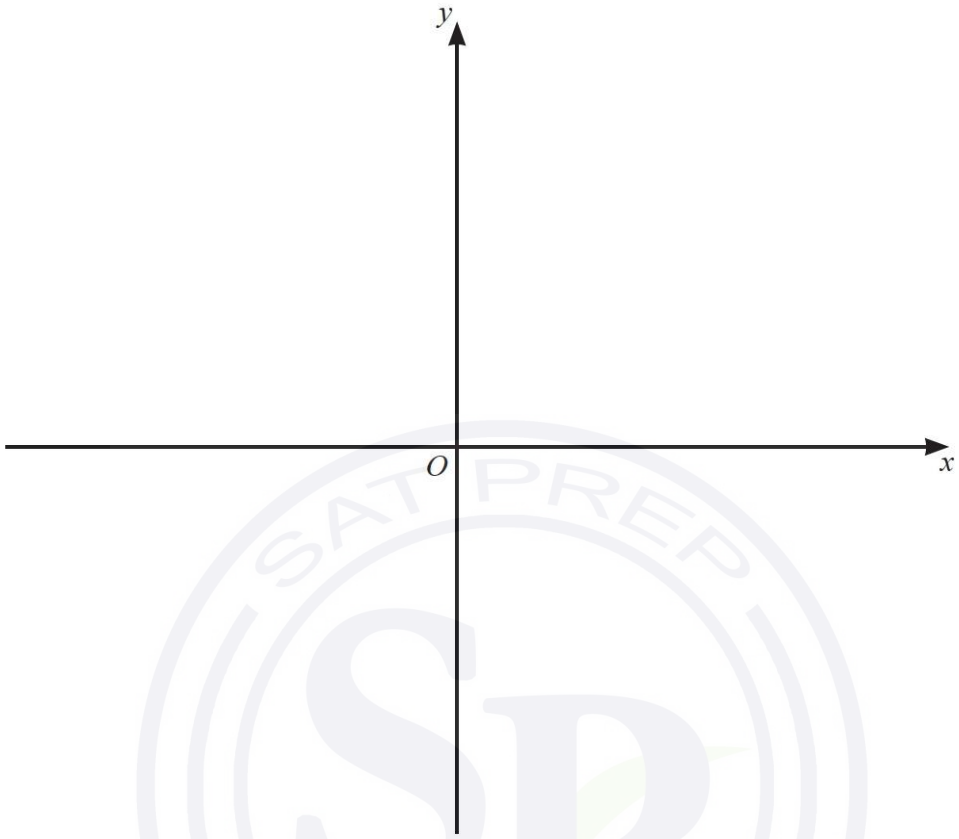
(c) Find the coordinates of the turning points of the graph of $y = f(x)$.
Show all your working and give your answers correct to 1 decimal place.

(..... , ...)

(..... , ...) [8]

Continue on the next page..

(d) Sketch the graph of $y = f(x)$.



[2]

Question 70

(a) Find the coordinates of the turning points of the graph of $y = x^3 - 12x + 6$.
You must show all your working.

(..... , ...) and (..... , ...) [5]

(b) Determine whether each turning point is a maximum or a minimum.
Show how you decide.

[3]

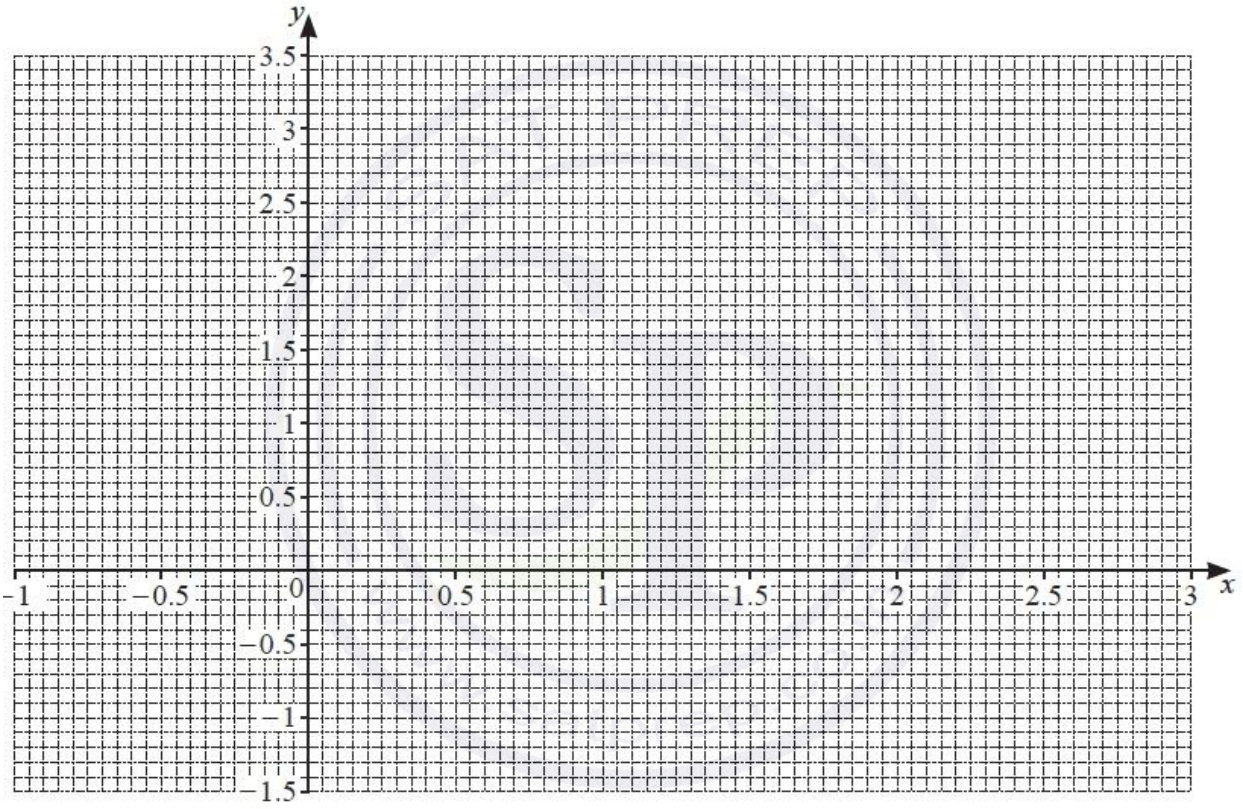
Question 71

The table shows some values for $y = x^3 - 3x^2 + 3$.

x	-1	-0.5	0	0.5	1	1.5	2	2.5	3
y		2.125	3	2.375	1		-1	-0.125	

(a) Complete the table. [3]

(b) On the grid, draw the graph of $y = x^3 - 3x^2 + 3$ for $-1 \leqslant x \leqslant 3$.



[4]

(c) By drawing a suitable straight line on the grid, solve the equation $x^3 - 3x^2 + x + 1 = 0$.

$x =$ ..... or $x =$ ..... or $x =$ ..... [4]

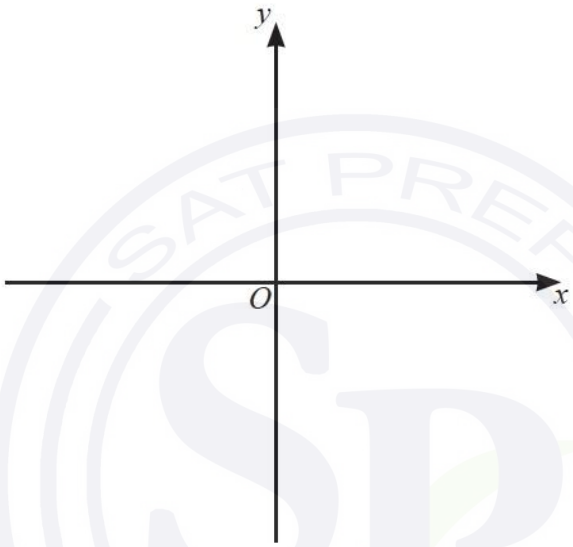
Question 72

$g(x) = 18 - 3x - x^2$

(i) Write $g(x)$ in the form $b - (a + x)^2$.

..... [3]

(ii) Sketch the graph of $y = g(x)$.
On your sketch, show the coordinates of the turning point.



[3]

(iii) Find the equation of the tangent to the graph of $y = 18 - 3x - x^2$ at $x = 4$.
Give your answer in the form $y = mx + c$.

$y =$ ..... [6]

Question 73

The table shows some values for $y = x^2 - \frac{3}{2x}$, $x \neq 0$, given correct to 1 decimal place.

x	−3	−2	−1	−0.5	−0.2		0.2	0.5	1	2	3
y			2.5	3.3	7.5		−7.5	−2.8	−0.5	3.3	

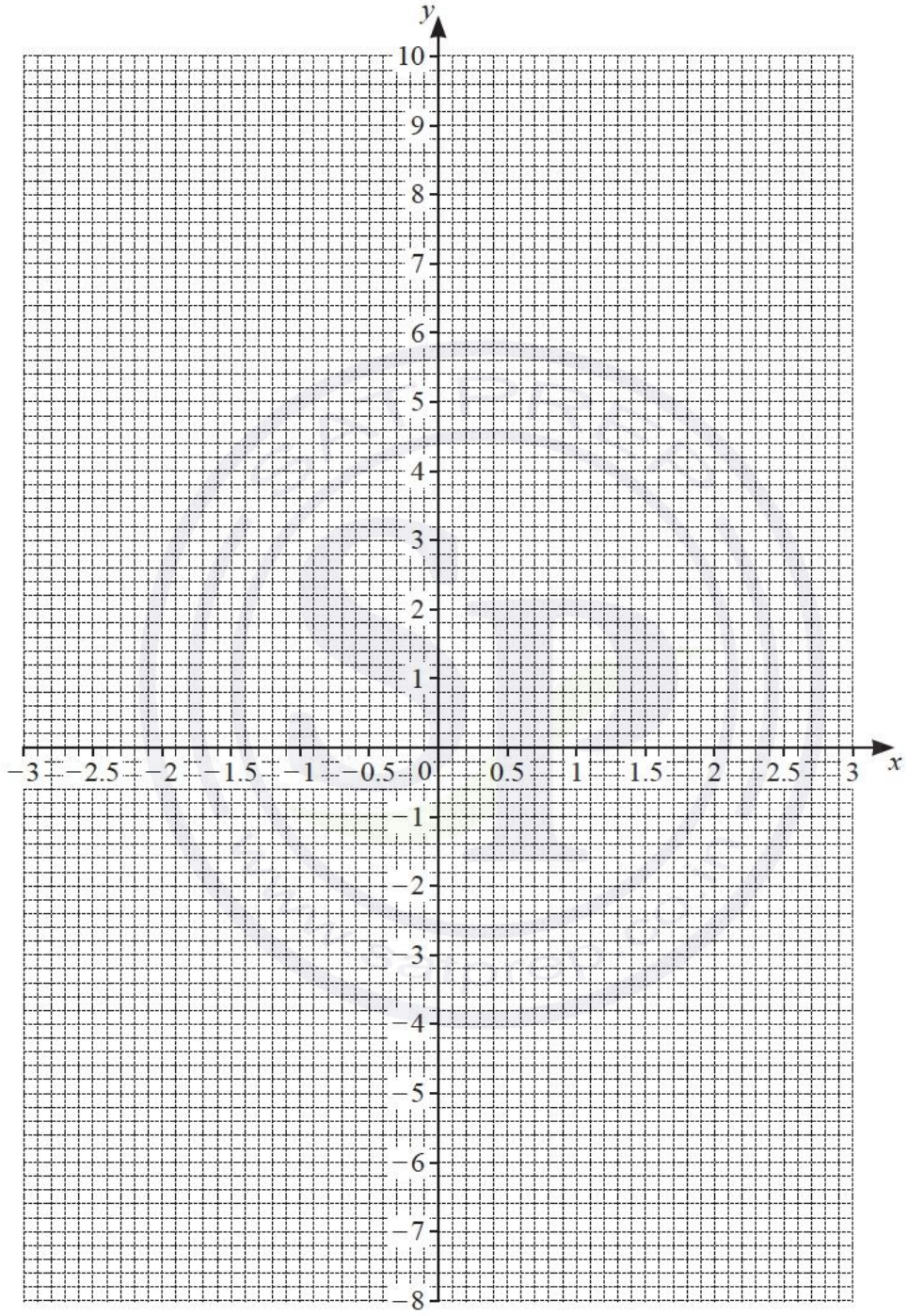
(a) (i) Complete the table. [3]

(ii) On the grid, draw the graph of $y = x^2 - \frac{3}{2x}$ for $-3 \leqslant x \leqslant -0.2$ and $0.2 \leqslant x \leqslant 3$.

Continue on the next page..

(a) **(i)** Complete the table. [3]

(ii) On the grid, draw the graph of $y = x^2 - \frac{3}{2x}$ for $-3 \leqslant x \leqslant -0.2$ and $0.2 \leqslant x \leqslant 3$.



[5]

(b) By drawing a suitable straight line on the grid, solve the equation $x^2 - \frac{3}{2x} = \frac{24}{5} - 2x$ for $-3 \leqslant x \leqslant -0.2$ and $0.2 \leqslant x \leqslant 3$.

$x =$ ..... or $x =$ ..... [4]

(c) The solutions to the equation $x^2 - \frac{3}{2x} = \frac{24}{5} - 2x$ are also the solutions to an equation of the form $ax^3 + bx^2 + cx - 15 = 0$ where a, b and c are integers.

Find the values of a, b and c.

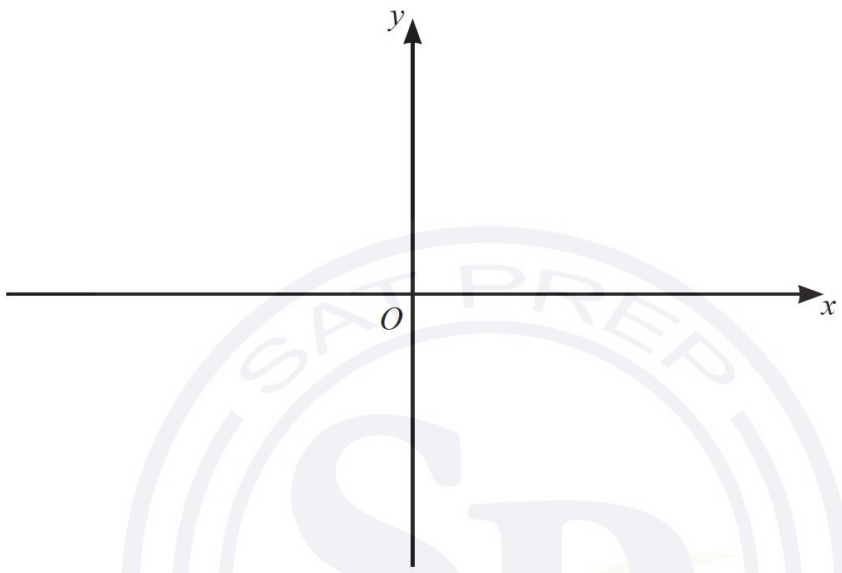
$a =$ .....

$b =$ .....

$c =$ ..... [4]

Question 74

Sketch the curve $y = x^3 - 4x$.



[3]

Question 75

A curve has equation $y = x^3 + ax + b$.
The stationary points of the curve have coordinates $(2, k)$ and $(-2, 10 - k)$.

Work out the value of a, the value of b and the value of k.

$a =$ ..... , $b =$ ..... , $k =$ ..... [6]

Question 76

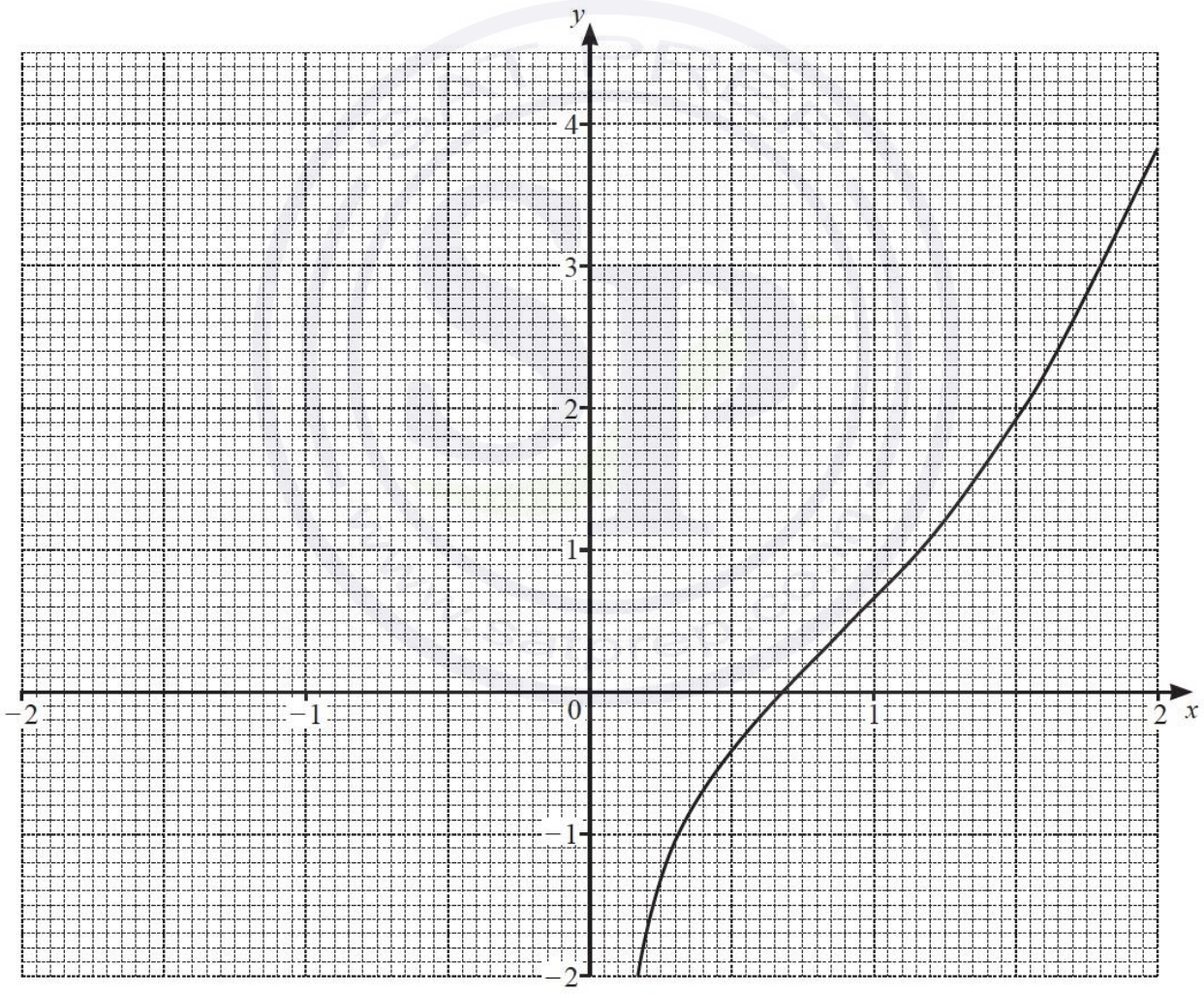
The table shows some values for $y = x^2 - \frac{1}{3x}$, $x \neq 0$.
The y-values are rounded to 1 decimal place.

x	-2	-1.5	-1	-0.75	-0.5	-0.25	-0.1
y	4.2	2.5	1.3			1.4	3.3

(a) Complete the table. [2]

(b) On the grid, draw the graph of $y = x^2 - \frac{1}{3x}$ for $-2 \leqslant x \leqslant -0.1$.

The graph of $y = x^2 - \frac{1}{3x}$ for $x > 0$ has been drawn for you.



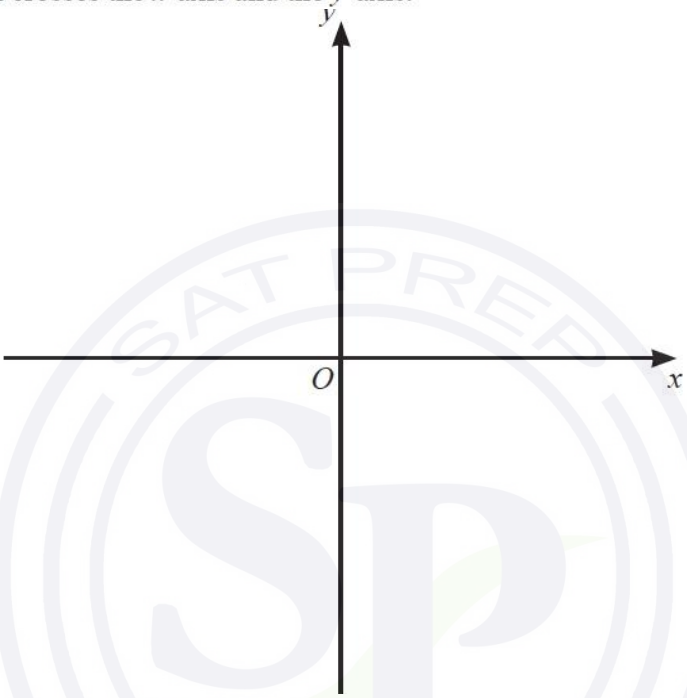
[4]

(c) By drawing a suitable line on the grid, solve the equation $x^2 - \frac{1}{3x} + 1 = 0$.

$x =$ ..... [2]

Question 77

(a) Sketch the graph of $y = (x+1)(3-x)(3+x)$, indicating the coordinates of the points where the graph crosses the x-axis and the y-axis.



[4]

(b) **(i)** Show that $y = (x+1)(3-x)(3+x)$ can be written as $y = 9+9x-x^2-x^3$.

[2]

(ii) Calculate the x–values of the turning points of $y = 9+9x-x^2-x^3$.
Show all your working and give your answers correct to 2 decimal places.

$x =$ ....., $x =$ ..... [7]

(iii) The equation $9+9x-x^2-x^3 = k$ has one solution only when $k < a$ and when $k > b$, where a and b are integers.

Find the maximum value of a and the minimum value of b.

$a =$ .....

$b =$ ..... [3]

Question 78

(a) A has coordinates $(-2,7)$, B has coordinates $(1,-5)$ and C has coordinates $(5,4)$.

(i) Find the coordinates of the midpoint of the line AB.

(..... , ...) [2]

(ii) Find $\overrightarrow{AC}$.

$\overrightarrow{AC} = \begin{pmatrix} \quad \\ \quad \end{pmatrix}$ [2]

(iii) Find $\left|\overrightarrow{AC}\right|$.

..... [2]

(iv) Find the equation of the line AB.
Give your answer in the form $y = mx + c$.

$y =$ ..... [3]

(v) Find the equation of the line perpendicular to AB that passes through C.
Give your answer in the form $y = mx + c$.

$y =$ ..... [3]

(b) The graphs of $y + 5x = 8$ and $y = 2x^2 + 6x - 13$ intersect at the points P and Q.

Find the coordinates of P and the coordinates of Q.
Show all your working.

P (..... , ...)

Q (..... , ...) [6]

Question 79

A curve has equation $y = x^3 - kx^2 + 1$.
When $x = 2$, the gradient of the curve is 6.

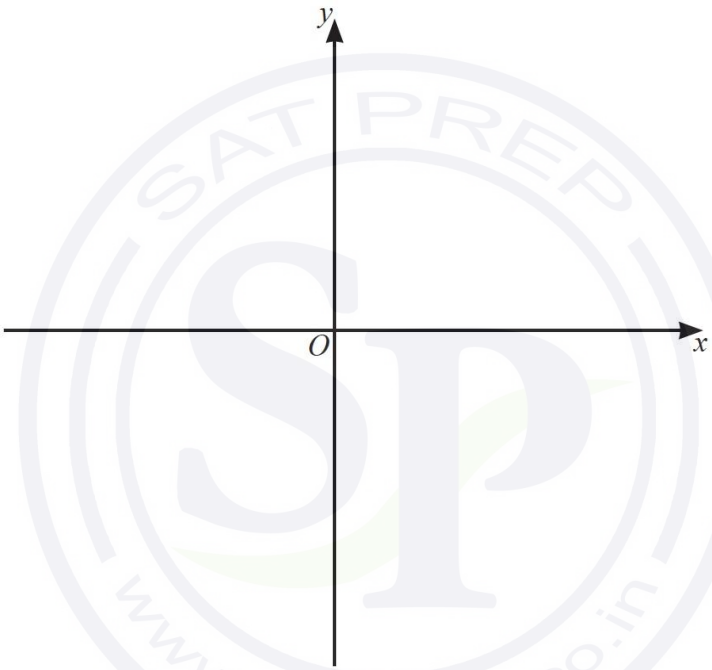
(a) Show that $k = 1.5$.

[5]

(b) Find the coordinates of the two stationary points of $y = x^3 - 1.5x^2 + 1$.
You must show all your working.

(....., ...) and (....., ...) [4]

(c) Sketch the curve $y = x^3 - 1.5x^2 + 1$.



[2]

Question 80

A line, l, joins point F (3, 2) and point G (− 5, 4).

(a) Calculate the length of line l.

..... [3]

(b) Find the equation of the perpendicular bisector of line l in the form $y = mx + c$.

$y =$ ..... [5]

(c) A point H lies on the y-axis such that the distance $GH = 13$ units.

Find the coordinates of the two possible positions of H.

(....., ...) and (....., ...) [4]

Question 81

The diagram shows the graph of $y = f(x)$ for $-1.5 \leqslant x \leqslant 5$.

(i) Find $f(2)$.

..... [1]

(ii) Solve the equation $f(x) = 0$ for $-1.5 \leqslant x \leqslant 5$.

$x =$ ..... or $x =$ ..... or $x =$ ..... [3]

(iii) $f(x) = k$ has three solutions for $-1.5 \leqslant x \leqslant 5$ where k is an integer.

Find the smallest possible value of k.

$k =$ ..... [1]

(iv) On the grid, draw a line $y = mx$ so that $f(x) = mx$ has exactly one solution for $-1.5 \leqslant x \leqslant 5$. [2]

(b) $y = 3x^2 - 12x + 7$

(i) Find the value of $\frac{dy}{dx}$ when $x = 5$.

..... [3]

(ii) Find the coordinates of the point on the graph of $y = 3x^2 - 12x + 7$ where the gradient is 0.

(c) When $y = 2x^p + qx^2$, $\frac{dy}{dx} = 14x^6 + 6x$.

Find the value of p and the value of q.

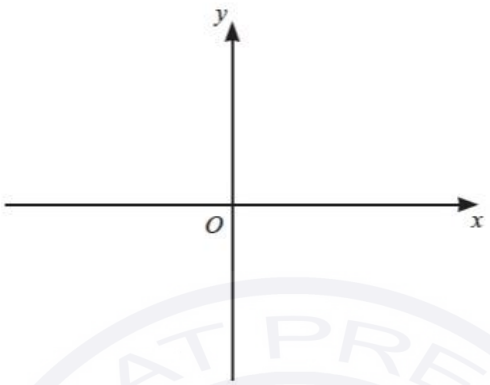
$p =$ .....

$q =$ ..... [2]

Question 82

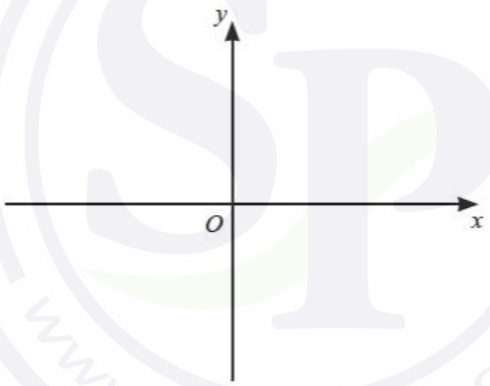
(a) Sketch the following graphs.
On each sketch, indicate any intercepts with the axes.

(i) $3x-4y=12$



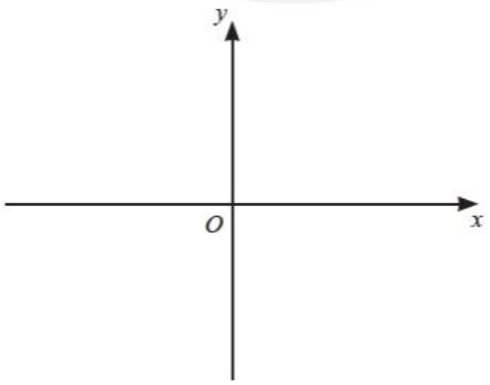
[2]

(ii) $y=x^2-3x-4$



[4]

(iii) $y=6^x$



[2]

Continue on the next page…

(b) **(i)** Find the derivative, $\frac{dy}{dx}$, of $y = 5 + 8x - \frac{4}{3}x^3$.

..... [2]

(ii) Find the gradient of $y = 5 + 8x - \frac{4}{3}x^3$ at $x = -1$.

..... [2]

(iii) A tangent is drawn to the graph of $y = 5 + 8x - \frac{4}{3}x^3$.
The gradient of the tangent is -28.

Find the coordinates of the two possible points where this tangent meets the graph.

(..... , ...)

(..... , ...) [5]

Question 83

AB is a line with midpoint M.
A is the point (2, 3) and M is the point (12, 7).

(a) Find the coordinates of B.

(..... , ...) [2]

(b) Show that the equation of the perpendicular bisector of AB is $2y + 5x = 74$.

[4]

(c) The perpendicular bisector of AB passes through the point N.
The point N has coordinates $(2, n)$.

Find the value of n.

$n =$ ..... [1]

(d) Points A, M and N form a triangle.

Find the area of the triangle.

..... [2]

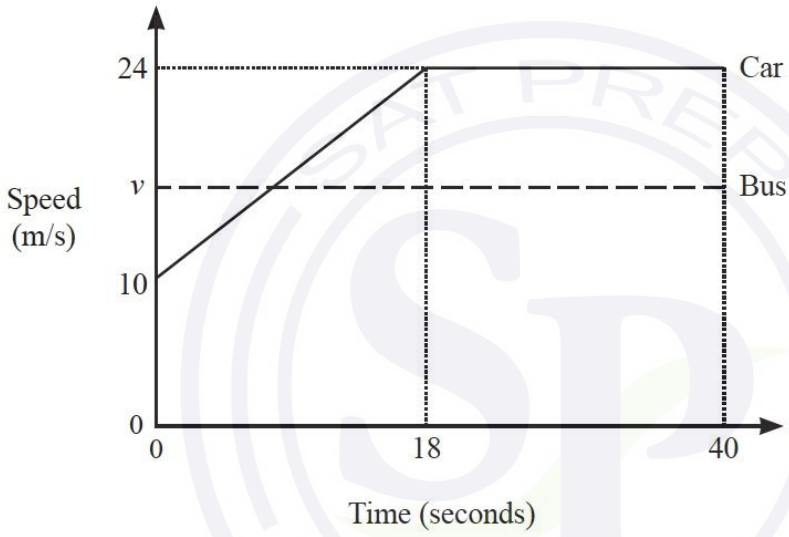
Question 84

Find the derivative, $\frac{dy}{dx}$, of $y = 3x^2 + 4x - 1$.

..... [2]

Question 85

(a) The diagram shows the speed–time graph for part of a journey for two vehicles, a car and a bus.



NOT TO SCALE

(i) Calculate the acceleration of the car during the first 18 seconds.

..... m/s^2 [1]

(ii) In the first 40 seconds the car travelled 134 m more than the bus.

Calculate the constant speed, v, of the bus.

v = ... m/s [4]

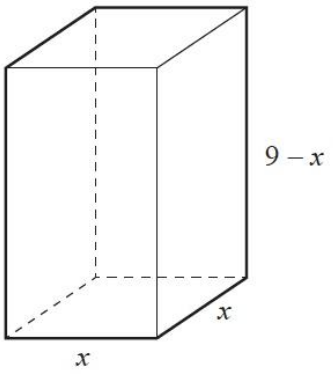
(b) A train takes 10 minutes 30 seconds to travel 16 240 m.

Calculate the average speed of the train.
Give your answer in kilometres per hour.

..... km/h [3]

Question 86

All the lengths in this question are measured in centimetres.



NOT TO SCALE

The diagram shows a solid cuboid with a square base.

(a) The volume, $V\,\text{cm}^3$, of the cuboid is $V = x^2(9-x)$.
The table shows some values of V for $0 \leqslant x \leqslant 9$.

x	0	1	2	3	4	5	6	7	8	9
V	0	8		54	80	100	108	98	64	0

(i) Complete the table.

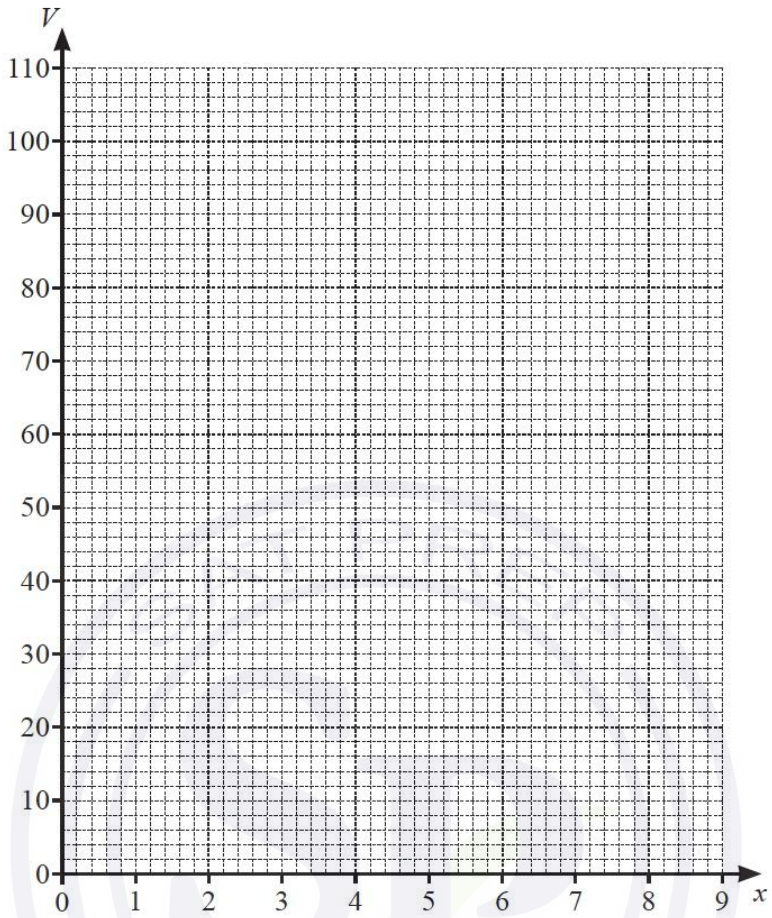
[1]

(ii) On the grid on the opposite page, draw the graph of $V = x^2(9-x)$ for $0 \leqslant x \leqslant 9$. [4]

(iii) Find the values of x when the volume of the cuboid is $44\,\text{cm}^3$.

$x =$ ..... or $x =$ ..... [2]

Continue on the next page…



(b) (i) Show that the total surface area of the cuboid is $(36x - 2x^2)\text{cm}^2$.

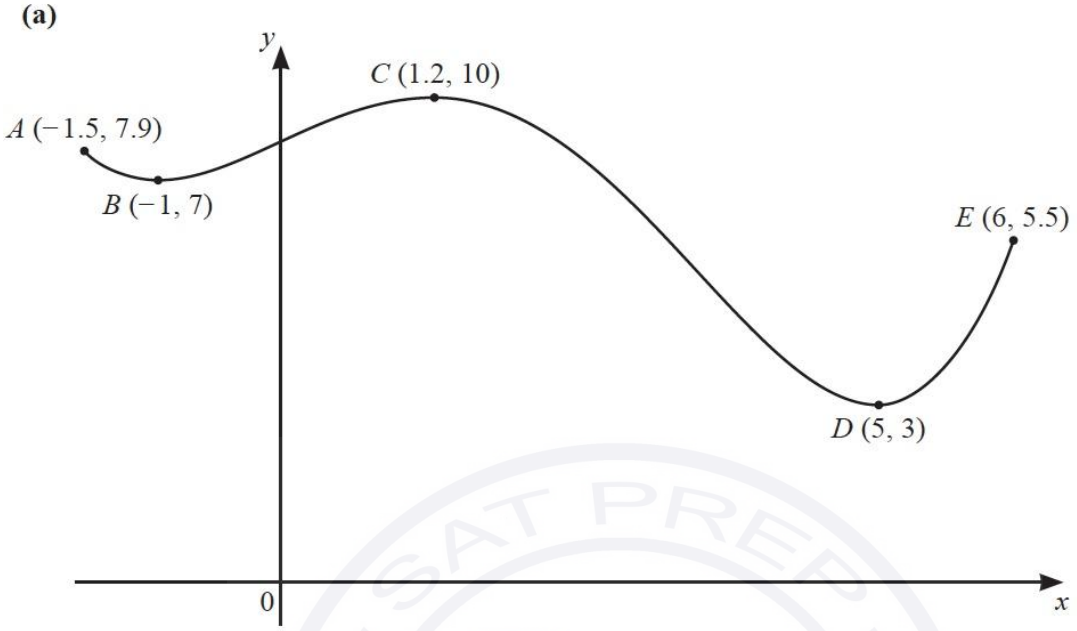
[2]

(ii) Find the surface area when the volume of the cuboid is a maximum.

..... cm^2 [3]

Question 87

(a)



NOT TO SCALE

The diagram shows a sketch of the graph of $y = f(x)$ for $-1.5 \leqslant x \leqslant 6$.
The coordinates of five points on the graph of $y = f(x)$ are shown on the diagram.

(i) $f(x) = k$ has two solutions in the interval $-1.5 \leqslant x \leqslant 6$.

Write down a possible integer value of k.

$k =$ ..... [1]

(ii) $f(x) = j$ has no solutions in the interval $-1.5 \leqslant x \leqslant 6$ when $j < a$ or $j > b$.

Find the maximum value of a and the minimum value of b.

$a =$ .....

$b =$ ..... [2]

(b) Find the coordinates of the two stationary points on the graph of $y = x^6 - 6x^5$.
You must show all your working.

(..... , ...)

(..... , ...) [5]

Question 88

$$f(x) = x^3 - 3x^2 - 4$$

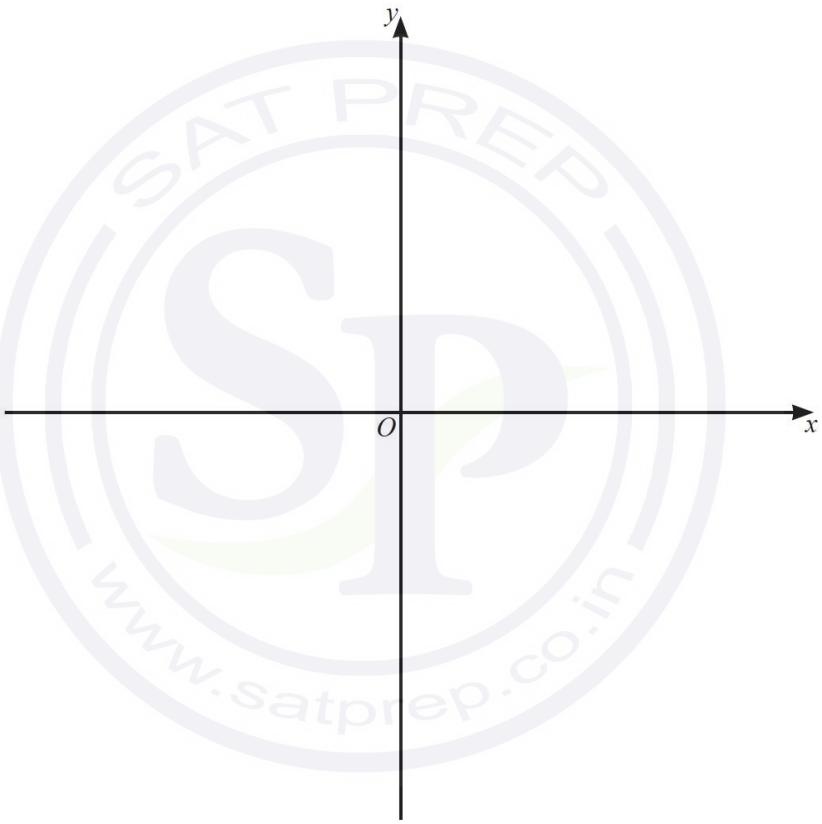
(a) Find the gradient of the graph of $y = f(x)$ where $x = 1$.

…….. [3]

(b) Find the coordinates of the turning points of the graph of y = f(x).

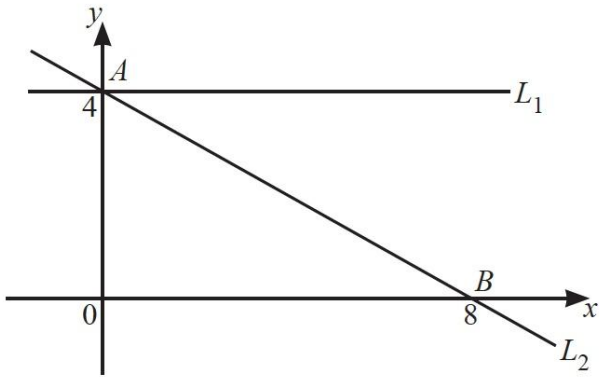
(..... , ...) , (..... , ...) [4]

(c) Sketch the graph of y = f(x).



[2]

Question 89



NOT TO SCALE

A is the point (0, 4) and B is the point (8, 0).
The line L_1 is parallel to the x-axis.
The line L_2 passes through A and B.

(a) Write down the equation of L_1.

..... [1]

(b) Find the equation of L_2.
Give your answer in the form $y = mx + c$.

$y =$ ..... [2]

(c) C is the point (2, 3).
The line L_3 passes through C and is perpendicular to L_2.

(i) Show that the equation of L_3 is $y = 2x - 1$.

[3]

(ii) L_3 crosses the x-axis at D.

Find the length of CD.

..... [5]

Question 90

The equation of a curve is $y = x^4 - 8x^2 + 5$.

(a) Find the derivative, $\left(\frac{dy}{dx}\right)$, of $y = x^4 - 8x^2 + 5$.

..... [2]

(b) Find the coordinates of the three turning points.
You must show all your working.

(..... , ...) and (..... , ...) and (..... , ...) [4]

(c) Determine which one of these turning points is a maximum.
Justify your answer.

[2]

Question 91

M has coordinates (4, 1) and N has coordinates (-2, -7).

(a) Find the length of MN.

..... [3]

(b) Find the gradient of MN.

..... [2]

(c) Find the equation of the perpendicular bisector of MN.

..... [4]

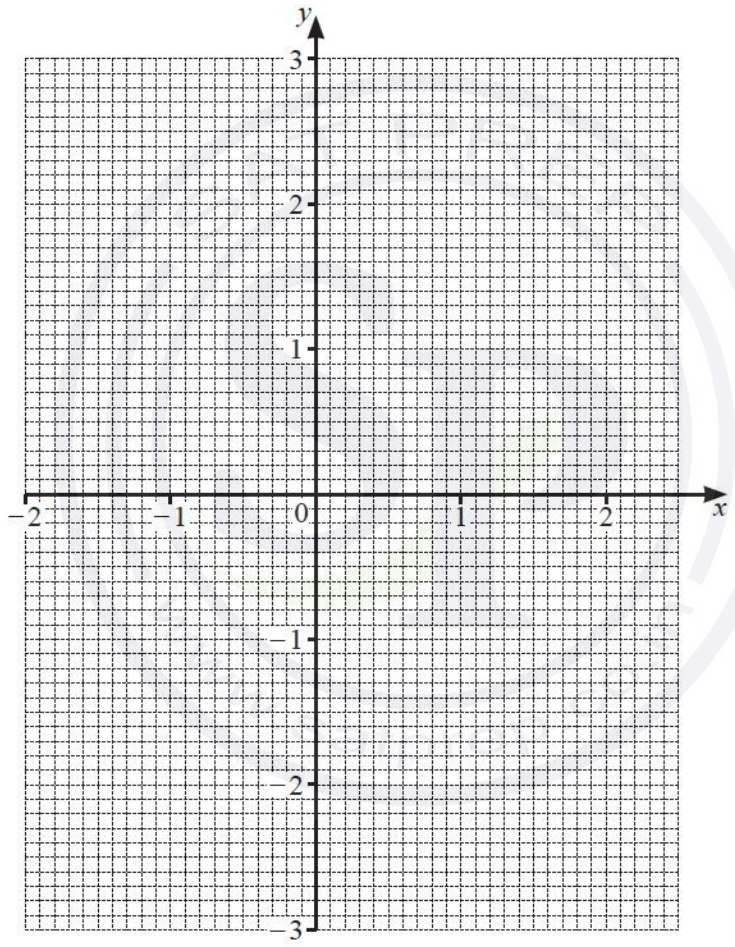
Question 92

The table shows some values for $y = 2^x - 3$.

x	-2	-1	0	0.5	1	1.5	2	2.5
y	-2.75			-1.58		-0.17	1	2.66

(a) Complete the table. [3]

(b) On the grid, draw the graph of $y = 2^x - 3$ for $-2 \leqslant x \leqslant 2.5$.



[2]

(c) Use your graph to solve the equation $2^x - 3 = 2$.

$x =$ ..... [1]

(d) By drawing a suitable straight line, solve the equation $2^x - x - 1.5 = 0$.

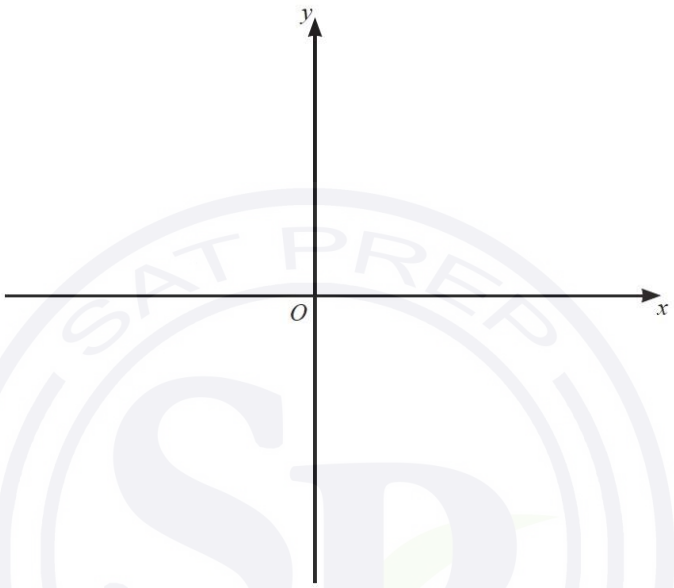
$x =$ ..... or $x =$ ..... [4]

Question 93

(a) **(i)** Show that the equation $y = (x-4)(x+1)(x-2)$ can be written as $y = x^3 - 5x^2 + 2x + 8$.

[2]

(ii) On the diagram, sketch the graph of $y = x^3 - 5x^2 + 2x + 8$, indicating the values where the graph crosses the axes.



[4]

(b) The graph of $y = x^3 - 5x^2 + 2x + 8$ has two tangents with a gradient of 10.

Find the equations of these two tangents.
You must show all your working and give your answers in the form $y = mx + c$.

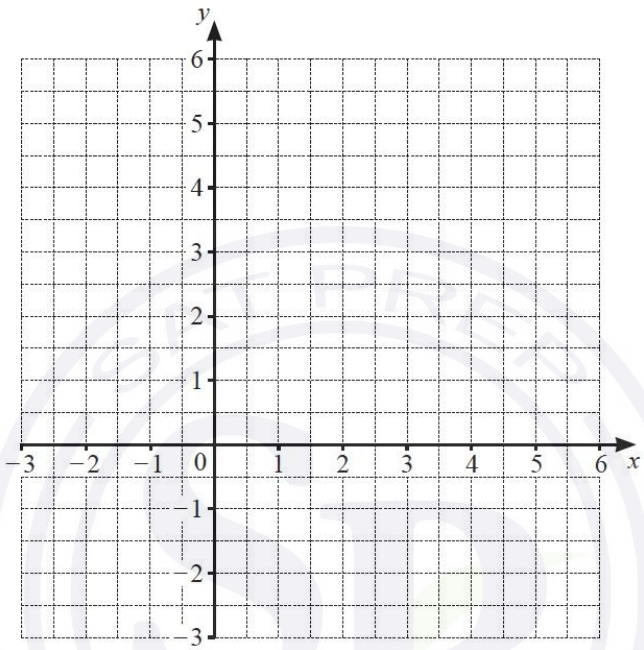
$y =$ .....

$y =$ ..... [7]

Question 94

(a) In the square $ABCD$, A has coordinates $(-2, 1)$ and B has coordinates $(1, 5)$.
C has coordinates (a, b), where a and b are both positive integers.

Find the coordinates of C and the coordinates of D.
You may use the grid to help you.



C (..... , ...)

D (..... , ...) [4]

(b) P has coordinates $(-1, 3)$ and Q has coordinates $(6, 4)$.

(i) Find the coordinates of the midpoint of PQ.

(..... , ...) [2]

(ii) Find the length PQ.

..... [3]

(iii) Find the gradient of PQ.

..... [2]

(iv) Find the equation of the line parallel to PQ that crosses the x-axis at $x = 2$.

..... [3]