

**A-level**  
**Topic : Binomial Theorem**  
**May 2013-May 2023**  
**Questions**

Question 1

Expand  $\frac{1+3x}{\sqrt{1+2x}}$  in ascending powers of  $x$  up to and including the term in  $x^2$ , simplifying the coefficients. [4]

Question 2

Expand  $(1+3x)^{\frac{1}{3}}$  in ascending powers of  $x$ , up to and including the term in  $x^3$ , simplifying the coefficients. [4]

Question 3

Show that, for small values of  $x^2$ ,

$$(1-2x^2)^{-2} - (1+6x^2)^{\frac{2}{3}} \approx kx^4,$$

where the value of the constant  $k$  is to be determined. [6]

Question 4

Given that  $\sqrt[3]{1+9x} \approx 1+3x+ax^2+bx^3$  for small values of  $x$ , find the values of the coefficients  $a$  and  $b$ . [3]

Question 5

Expand  $\frac{1}{\sqrt{1-2x}}$  in ascending powers of  $x$ , up to and including the term in  $x^3$ , simplifying the coefficients. [4]

Question 6

Expand  $(2-x)(1+2x)^{-\frac{3}{2}}$  in ascending powers of  $x$ , up to and including the term in  $x^2$ , simplifying the coefficients. [4]

Question 7

Expand  $\frac{1}{\sqrt[3]{1+6x}}$  in ascending powers of  $x$ , up to and including the term in  $x^3$ , simplifying the coefficients. [4]

Question 8

Expand  $(3+2x)^{-3}$  in ascending powers of  $x$  up to and including the term in  $x^2$ , simplifying the coefficients. [4]

Question 9

Expand  $\sqrt[4]{(1 - 4x)}$  in ascending powers of  $x$ , up to and including the term in  $x^3$ , simplifying the coefficients. [4]

Question 10

Expand  $\frac{4}{\sqrt{(4 - 3x)}}$  in ascending powers of  $x$ , up to and including the term in  $x^2$ , simplifying the coefficients. [4]

Question 11

Find the coefficient of  $x^3$  in the expansion of  $(3 - x)(1 + 3x)^{\frac{1}{3}}$  in ascending powers of  $x$ . [4]

Question 12

(a) Expand  $(2 - 3x)^{-2}$  in ascending powers of  $x$ , up to and including the term in  $x^2$ , simplifying the coefficients. [4]

(b) State the set of values of  $x$  for which the expansion is valid. [1]

Question 13

(a) Expand  $\sqrt[3]{1 + 6x}$  in ascending powers of  $x$ , up to and including the term in  $x^3$ , simplifying the coefficients. [4]

(b) State the set of values of  $x$  for which the expansion is valid. [1]

Question 14

Expand  $(1 + 3x)^{\frac{2}{3}}$  in ascending powers of  $x$ , up to and including the term in  $x^3$ , simplifying the coefficients. [4]

Question 15

When  $(a + bx)\sqrt{1 + 4x}$ , where  $a$  and  $b$  are constants, is expanded in ascending powers of  $x$ , the coefficients of  $x$  and  $x^2$  are 3 and  $-6$  respectively.

Find the values of  $a$  and  $b$ . [6]

Question 16

(a) Expand  $(2 - x^2)^{-2}$  in ascending powers of  $x$ , up to and including the term in  $x^4$ , simplifying the coefficients. [4]

(b) State the set of values of  $x$  for which the expansion is valid. [1]

Question 17

Expand  $\sqrt{\frac{1+2x}{1-2x}}$  in ascending powers of  $x$ , up to and including the term in  $x^2$ , simplifying the coefficients. [5]

Question 18

$$\text{Let } f(x) = \frac{2x^2 + 7x + 8}{(1+x)(2+x)^2}.$$

- (a) Express  $f(x)$  in partial fractions. [5]
- (b) Hence obtain the expansion of  $f(x)$  in ascending powers of  $x$ , up to and including the term in  $x^2$ . [5]

Question 19

$$\text{Let } f(x) = \frac{21 - 8x - 2x^2}{(1+2x)(3-x)^2}.$$

- (a) Express  $f(x)$  in partial fractions. [5]
- (b) Hence obtain the expansion of  $f(x)$  in ascending powers of  $x$ , up to and including the term in  $x^2$ . [5]

Question 20

Find the coefficient of  $x^3$  in the binomial expansion of  $(3+x)\sqrt{1+4x}$ . [4]