

**A-level**  
**Topic :Binomial Theorem**  
**May 2013-May 2025**  
**Answers**

Question 1

Obtain $1-x$ as first two terms of $(1+2x)^{-\frac{1}{2}}$	B1
Obtain $+\frac{3}{2}x^2$ or unsimplified equivalent as third term of $(1+2x)^{-\frac{1}{2}}$	B1
Multiply $1+3x$ by attempt at $(1+2x)^{-\frac{1}{2}}$ , obtaining sufficient terms	M1
Obtain final answer $1+2x-\frac{3}{2}x^2$	A1 [4]

Question 2

State a correct unsimplified version of the $x$ or $x^2$ or $x^3$ term	M1
State correct first two terms $1-x$	A1
Obtain the next two terms $2x^2 - \frac{14}{3}x^3$	A1 + A1 4
[Symbolic binomial coefficients, e.g. $\binom{-\frac{1}{3}}{3}$ are not sufficient for the M mark.]	

Question 3

Obtain correct (unsimplified) version of $x^2$ or $x^4$ term in $(1-2x^2)^{-2}$	M1
Obtain $1+4x^2$	A1
Obtain ... $+12x^4$	A1
Obtain correct (unsimplified) version of $x^2$ or $x^4$ term in $(1+6x^2)^{\frac{2}{3}}$	M1
Obtain $1+4x^2-4x^4$	A1
Combine expansions to obtain $k=16$ with no error seen	A1

Question 4

<u>Either</u> State correct unsimplified $x^2$ or $x^3$ term	M1
Obtain $a=-9$	A1
Obtain $b=45$	A1
 <u>Or</u> Use chain rule to differentiate twice to obtain form $k(1+9x)^{\frac{5}{3}}$	 M1
Obtain $f''(x) = -18(1+9x)^{-\frac{5}{3}}$ and hence $a=-9$	A1
Obtain $f'''(x) = 270(1+9x)^{-\frac{8}{3}}$ and hence $b=45$	A1 [3]

Question 5

State a correct un-simplified version of the  $x$  or  $x^2$  or  $x^3$  term

**M1**

State correct first two terms  $1 + x$

**A1**

Obtain the next two terms  $\frac{3}{2}x^2 + \frac{5}{2}x^3$

**A1 A1** [4]

[Symbolic binomial coefficients, e.g.  $\binom{-\frac{1}{2}}{3}$  are not sufficient for the M mark.]

Question 6

State correct unsimplified first two terms of the expansion of  $(1 + 2x)^{-\frac{3}{2}}$ , e.g.  $1 + (-\frac{3}{2})(2x)$

**B1**

State correct unsimplified term in  $x^2$ , e.g.  $(-\frac{3}{2})(-\frac{3}{2}-1)(2x)^2 / 2!$

**B1**

Obtain sufficient terms of the product of  $(2 - x)$  and the expansion up to the term in  $x^2$

**M1**

Obtain **final** answer  $2 - 7x + 18x^2$  Do not ISW

**A1** [4]

Question 7

State a correct unsimplified version of the  $x$  or  $x^2$  or  $x^3$  term in the expansion of  $(1 + 6x)^{-\frac{1}{3}}$

**(M1**

State correct first two terms  $1 - 2x$

**A1**

Obtain term  $8x^2$

**A1**

Obtain term  $-\frac{112}{3}x^3 \left(37\frac{1}{3}x^3\right)$  in final answer

**A1)**

Question 8

State a correct unsimplified version of the  $x$  or  $x^2$  term in the expansion of  $(1 + \frac{2}{3}x)^{-3}$  or  $(3 + 2x)^{-3}$

[Symbolic binomial coefficients, e.g.  $\binom{-3}{2}$ , are not sufficient for **M1**.]

State correct first term  $\frac{1}{27}$

**B1**

Obtain term  $-\frac{2}{27}x$

**A1**

Obtain term  $\frac{8}{81}x^2$

**A1)**

Question 9

State a correct unsimplified version of the $x$ or $x^2$ or $x^3$ term	<b>M1</b>
State correct first two terms $1 - x$	<b>A1</b>
Obtain the next two terms $-\frac{3}{2}x^2 - \frac{7}{2}x^3$	<b>A1 + A1</b>
	<b>4</b>

Question 10

Obtain a correct unsimplified version of the $x$ or $x^2$ term of the expansion of $(4 - 3x)^{\frac{1}{2}}$ or $\left(1 - \frac{3}{4}x\right)^{\frac{1}{2}}$	<b>M1</b>
State correct first term 2	<b>B1</b>
Obtain the next two terms $\frac{3}{4}x + \frac{27}{64}x^2$	<b>A1 + A1</b>
<b>Total:</b>	<b>4</b>

Question 11

State unsimplified term in $x^2$ , or its coefficient in the expansion of $(1 + 3x)^{\frac{1}{3}} \left( \frac{\frac{1}{3} \times \frac{-2}{3}}{2} (3x)^2 \right)$	<b>B1</b>
State unsimplified term in $x^3$ , or its coefficient in the expansion of $(1 + 3x)^{\frac{1}{3}} \left( \frac{\frac{1}{3} \times \frac{-2}{3} \times \frac{-5}{3}}{6} (3x)^3 \right)$	<b>B1</b>
Multiply by $(3 - x)$ to give 2 terms in $x^3$ , or their coefficients	<b>M1</b>
Obtain answer 6	<b>A1</b>
	<b>4</b>

### Question 12

(a)	State a correct unsimplified version of the $x$ or $x^2$ term of the expansion of $(2 - 3x)^{-2}$ or $\left(1 - \frac{3}{2}x\right)^{-2}$	<b>M1</b>
	State correct first term $\frac{1}{4}$	<b>B1</b>
	Obtain the next two terms $\frac{3}{4}x + \frac{27}{16}x^2$	<b>A1 + A1</b>
		<b>4</b>
(b)	State answer $ x  < \frac{2}{3}$ , or equivalent	<b>B1</b>
		<b>1</b>

### Question 13

(a)	State a correct unsimplified version of the $x$ or $x^2$ or $x^3$ term	<b>M1</b>	For the given expression
	State correct first two terms $1 + 2x$	<b>A1</b>	
	Obtain the next two terms $-4x^2 + \frac{40}{3}x^3$	<b>A1 + A1</b>	One mark for each correct term. ISW Accept $13\frac{1}{3}$ The question asks for simplified coefficients, so candidates should cancel fractions.
		<b>4</b>	
(b)	State answer $ x  < \frac{1}{6}$	<b>B1</b>	OE. Strict inequality
		<b>1</b>	

### Question 14

State correct first two terms $1 + 2x$	<b>B1</b>
State a correct unsimplified version of the $x^2$ or $x^3$ term	<b>M1</b>
Obtain the next term $-x^2$	<b>A1</b>
Obtain the final term $\frac{4}{3}x^3$	<b>A1</b>
	<b>4</b>

### Question 15

State or imply $1 + 2x$ as first terms of the expansion of $\sqrt{1+4x}$	<b>B1</b>	Allow for correct unsimplified expression.
State or imply $-2x^2$ as third term of the expansion of $\sqrt{1+4x}$	<b>B1</b>	Allow for correct unsimplified expression.
Form an expression for the coefficient of $x$ or coefficient of $x^2$ in the expansion of $(a+bx)\sqrt{1+4x}$ and equate to given coefficient	<b>M1</b>	All relevant terms considered.
Obtain $2a + b = 3$ , or equivalent	<b>A1</b>	One correct equation.
Obtain $-2a + 2b = -6$ or equivalent	<b>A1</b>	Second correct equation.
Obtain answer $a = 2$ and $b = -1$	<b>A1</b>	
	<b>6</b>	

### Question 16

(a)	State a correct unsimplified version of the $x^2$ or the $x^4$ term of the expansion of $(2-x^2)^{-2}$ or $(1-\frac{1}{2}x^2)^{-2}$	<b>M1</b>	$\frac{1}{4} \left( 1 + 2\frac{x^2}{2} + \frac{-2 \cdot -3}{2} \left( \frac{x^2}{2} \right)^2 \dots \right)$ Symbolic binomial coefficients are not sufficient for the M1.
	State correct first term $\frac{1}{4}$	<b>B1</b>	Accept $2^{-2}$ .
	Obtain the next two terms $\frac{1}{4}x^2 + \frac{3}{16}x^4$	<b>A1 A1</b>	A1 for each one correct ISW. Full marks for $\frac{1}{4}(1+x^2+\frac{3}{4}x^4)$ ISW.
			<b>SC</b> allow <b>M1 A1 A1</b> for $\frac{1}{4}$ and $1+x^2+\frac{3}{4}x^4$ SOI. <b>SC</b> allow <b>M1 A1</b> for $1+x^2+\frac{3}{4}x^4$
		<b>4</b>	
(b)	State answer $ x  < \sqrt{2}$	<b>B1</b>	Or $-\sqrt{2} < x < \sqrt{2}$ .
		<b>1</b>	

### Question 17

State a correct unsimplified term in $x$ or $x^2$ of the expansion of either $(1+2x)^{\frac{1}{2}}$ or $(1-2x)^{\frac{1}{2}}$	<b>B1</b>	
State correct unsimplified expansion of $(1+2x)^{\frac{1}{2}}$ up to the term in $x^2$	<b>B1</b>	
State correct unsimplified expansion of $(1-2x)^{\frac{1}{2}}$ up to the term in $x^2$	<b>B1</b>	
Obtain sufficient terms of the product of the expansions	<b>M1</b>	
Obtain final answer $1+2x+2x^2$	<b>A1</b>	

### Question 18

(a)	State or imply the form $\frac{A}{1+x} + \frac{B}{2+x} + \frac{C}{(2+x)^2}$	<b>B1</b>	
	Use a correct method to find a constant	<b>M1</b>	
	Obtain one of $A=3$ , $B=-1$ and $C=-2$	<b>A1</b>	SR after B0 can score M1A1 for one correct value
	Obtain a second value	<b>A1</b>	
	Obtain the third value	<b>A1</b>	$\frac{A}{1+x} + \frac{Dx+E}{(2+x)^2}$ , where $A=3$ , $D=-1$ and $E=-4$ , is awarded B1 M1 A1 A1 A1 as above.
		<b>5</b>	
(b)	Use a correct method to find the first two terms of the expansion of $(1+x)^{-1}$ , $(2+x)^{-1}$ , $\left(1+\frac{1}{2}x\right)^{-1}$ , $(2+x)^{-2}$ or $\left(1+\frac{1}{2}x\right)^{-2}$	<b>M1</b>	For the $A, D, E$ form of fractions, award M1 A1FT A1FT for the expanded partial fractions, then if $D \neq 0$ , M1 for multiplying out fully, and A1 for the final answer.
	Obtain correct unsimplified expansions up to the term in $x^2$ of each partial fraction	<b>A3 FT</b>	$3(1-x+x^2 \dots)$ $-\frac{1}{2}\left(1-\frac{x}{2}+\frac{x^2}{4} \dots\right)$ $-\frac{2}{4}\left(1-x+\frac{3}{4}x^2\right)$
	Obtain final answer $2-\frac{9}{4}x+\frac{5}{2}x^2$	<b>A1</b>	
		<b>5</b>	

### Question 19

(a)	State or imply the form $\frac{A}{1+2x} + \frac{B}{3-x} + \frac{C}{(3-x)^2}$	<b>B1</b>	Alternative form: $\frac{A}{1+2x} + \frac{Dx+E}{(3-x)^2}$ .
	Use a correct method to find a constant	<b>M1</b>	Incorrect format for partial fractions: Allow M1 and a possible A1 if obtain one of these correct values. Max 2/5 Allow M1 even if multiply up by $(1+2x)(3-x)^3$ .
	Obtain one of $A=2$ , $B=2$ and $C=-3$	<b>A1</b>	Alternative form: obtain one of $A=2$ , $D=-2$ and $E=3$ .
	Obtain a second value	<b>A1</b>	
	Obtain the third value	<b>A1</b>	Do not need to substitute values back into original form.
		<b>5</b>	If $\frac{A}{1+2x} + \frac{B}{3-x} + \frac{Cx+D}{(3-x)^2}$ B0 but M1 A1 for $A$ , A1 for $B$ and A1 for $C$ and $D$ . If $C=0$ then recovers B1 from above.

<p>(b) Use a correct method to obtain the first two terms of one of the unsimplified expansions</p> $(1+2x)^{-1}, \left(1-\frac{1}{3}x\right)^{-1}, \left(1-\frac{1}{3}x\right)^{-2}, (3-x)^{-1}, (3-x)^{-2}$	<p><b>M1</b></p> $(1+2x)^{-1} = 1 + (-1)(2x) + \dots$ $\left(1-\frac{1}{3}x\right)^{-1} = 1 + (-1)(-x/3) + \dots$ $\left(1-\frac{1}{3}x\right)^{-2} = 1 + (-2)(-x/3) + \dots$ $(3-x)^{-1} = 3^{-1} + (-1)3^{-2}(-x) \dots$ $(3-x)^{-2} = 3^{-2} + (-2)3^{-3}(-x) + \dots$
<p>Obtain the correct unsimplified expansions up to the term in <math>x^2</math> for each partial fraction</p> <p>If correct, should be working with</p> $\frac{2}{1+2x} + \frac{2}{3-x} - \frac{3}{(3-x)^2} \text{ or } \frac{2}{1+2x} + \frac{-2x+3}{(3-x)^2}$	<p><b>A1 FT</b> Follow through on <i>their</i> A, B, C</p> $A(1 + (-1)(2x) + ((-1)(-2)/2)(2x)^2 + \dots)$ <p><b>A1 FT</b> <math>\frac{B}{3} (1 + (-1)(-x/3) + ((-1)(-2)/2)(-x/3)^2 + \dots)</math></p> <p><b>A1 FT</b> <math>\frac{C}{3^2} (1 + (-2)(-x/3) + ((-2)(-3)/2)(-x/3)^2 + \dots)</math></p> <p>Must be <i>their</i> coefficients from (a) but may be unsimplified expansions for FT marks.</p> <p>If correct, expect to see</p> $2(1 - 2x + (2x)^2) \text{ or } 2 - 4x + 8x^2$ $\frac{2}{3} \left(1 + \frac{x}{3} + \left(\frac{x}{3}\right)^2\right) \text{ or } \frac{2}{3} + \frac{2}{9}x + \frac{2}{27}x^2$ $-\frac{1}{3} \left(1 + \frac{2x}{3} + (3)\left(\frac{x}{3}\right)^2\right) \text{ or } -\frac{1}{3} - \frac{2}{9}x - \frac{x^2}{9}$
<p>Obtain final answer <math>\frac{7}{3} - 4x + \frac{215}{27}x^2</math></p>	<p><b>A1</b> Accept <math>2\frac{1}{3} - 4x + 7\frac{26}{27}x^2</math>. No ISW.</p>

**Question 20**

<p>State unsimplified term in <math>x^3</math>, or its coefficient, in the expansion of <math>(1+4x)^{\frac{1}{2}}</math></p>	<p><b>B1</b></p> $\frac{1}{2} \times \frac{-1}{2} \times \frac{-3}{2} (4x)^3 (= 4) \text{ Must expand binomial coefficient.}$
<p>State unsimplified term in <math>x^2</math>, or its coefficient, in the expansion of <math>(1+4x)^{\frac{1}{2}}</math></p>	<p><b>B1</b></p> $\frac{1}{2} \times \frac{-1}{2} (4x)^2 (= -2) \text{ Must expand binomial coefficient.}$
<p>Multiply by <math>(3+x)</math> and combine terms in <math>x^3</math>, or their coefficients</p>	<p><b>M1</b></p> $(3 \times 4 - 1 \times 2)$ <p>Allow if they expanded with <math>x</math> rather than <math>4x</math>.</p>
<p>Obtain answer 10</p>	<p><b>A1</b> Accept <math>10x^3</math></p>

### Question 21

(a)	State unsimplified term in $x$ , or its coefficient, in the expansion of $(4-x)^{\frac{1}{2}}$	<b>B1</b>	$4^{\frac{1}{2}} \times \frac{1}{2} \times \left(\frac{-x}{4}\right) = \frac{-x}{4}$ .
	State unsimplified term in $x^2$ , or its coefficient, in the expansion of $(4-x)^{\frac{1}{2}}$	<b>B1</b>	$4^{\frac{1}{2}} \times \frac{\frac{1}{2} \times \frac{-1}{2}}{2} \times \left(\frac{-x}{4}\right)^2 = \frac{-x^2}{64}$ . Allow $\left(\frac{x}{4}\right)^2$ .
	Multiply by $(2x-5)$ and obtain 2 terms in $x^2$ , allow even if errors in $4^{\frac{1}{2}}$ , signs, etc.	<b>M1</b>	Allow unsimplified $2x$ . $4^{\frac{1}{2}} \times \frac{1}{2} \times \left(\frac{-x}{4}\right) - 5$ . $4^{\frac{1}{2}} \times \frac{\frac{1}{2} \times \frac{-1}{2}}{2} \times \left(\frac{-x}{4}\right)^2$ . Allow $\left(\frac{x}{4}\right)^2$ . $2x \times \left(\frac{-x}{4}\right)(-5) \times \frac{-x^2}{64}$ or $2 \times \left(\frac{-1}{4}\right)(-5) \times \left(\frac{-1}{64}\right)$ .
	Obtain $-\frac{27}{64}$ or $-0.421875$ or $-\frac{54}{128}$	<b>A1</b>	Allow in a full expansion up to $x^2$ , ignore extra terms even if they contain errors.
		<b>4</b>	
(b)	$ x  < 4$	<b>B1</b>	or $-4 < x < 4$ .
		<b>1</b>	

### Question 22

State correct unsimplified first two terms of the expansion of $(1-2x)^{\frac{1}{2}}$ , e.g. $1 + \frac{1}{2}(-2x)$	<b>B1</b>	Symbolic coefficients are not sufficient. $1-x$
State correct unsimplified term in $x^2$ , e.g. $\frac{\frac{1}{2}\left(\frac{1}{2}-1\right)(-2x)^2}{2!}$	<b>B1</b>	Symbolic coefficients are not sufficient. $-\frac{1}{2}x^2$
Obtain sufficient terms of the product of $(3+x)$ and the expansion up to the term in $x^2$	<b>M1</b>	
Obtain final answer $3 - 2x - \frac{5}{2}x^2$	<b>A1</b>	
	<b>4</b>	

### Question 23

Obtain a correct unsimplified version of the $x$ or $x^2$ term of the expansion of $(9-3x)^{\frac{1}{2}}$ or $\left(1-\frac{1}{3}x\right)^{\frac{1}{2}}$	<b>M1</b>	E.g. $-\frac{1}{2} \times \frac{1}{3}x$ or $-\frac{\frac{1}{2} \times \frac{1}{2}}{2} \times \frac{1}{9}x^2$ or $\frac{1}{2}9^{-\frac{1}{2}}(-3x)^1$ or $\frac{\frac{1}{2} \times \frac{-1}{2}}{2}9^{-\frac{3}{2}}(-3x)^2$ . Not for symbolic coefficients in the form ${}^nC_r$ .
State correct first term 3	<b>B1</b>	
Obtain the next two terms $-\frac{1}{2}x - \frac{1}{24}x^2$	<b>A1 A1</b>	A1 for each term correct. Do not ISW.
	<b>SC M1A1</b>	for $1 - \frac{1}{6}x - \frac{1}{72}x^2$ seen on its own or as a factor.
	<b>4</b>	

### Question 24

(a)	Find the first two terms of the expansion of $(1 - 2x)^{\frac{3}{2}}$	<b>B1</b>	
	Obtain correct third term $-\frac{3}{2}\left(-\frac{3}{2}-1\right)\frac{(-2x)^2}{2!}$ or $-\frac{3}{2}\left(-\frac{3}{2}-1\right)\frac{(2x)^2}{2!}$	<b>B1</b>	$\frac{15}{2}x^2$ Ignore extra terms.
	Multiply <i>their</i> 3 term expansion $a + bx + cx^2$ by $(6 - x)$ obtaining all necessary terms	<b>M1</b>	$6 + 18x + 45x^2 - x - 3x^2 \dots$ Ignore extra terms.
	$6 + 17x + 42x^2$	<b>A1</b>	Ignore extra terms. Allow with the terms in any order.
		<b>4</b>	
(b)	$ x  < \frac{1}{2}$ or $-\frac{1}{2} < x < \frac{1}{2}$ or $(-0.5, 0.5)$ or $] -0.5, 0.5[$	<b>B1</b>	OE B0 for an ambiguous statement. Must be strict inequality.
		<b>1</b>	

