**A-level**

**Topic : Differential Calculus**

**May 2013-May 2023**

**Answer**

Question 1

| | | | |
|---|---|---|---|
| **(i)** | Use correct quotient rule or equivalent | M1 | |
| | Obtain $\frac{(1+e^{2x})2x-(1+x^2)2e^{2x}}{(1+e^{2x})^2}$ or equivalent | A1 | |
| | Substitute $x=0$ and obtain $-\frac{1}{2}$ or equivalent | A1 | [3] |
| **(ii)** | Differentiate $y^3$ and obtain $3y^2\frac{dy}{dx}$ | B1 | |
| | Differentiate $5xy$ and obtain $5y+5x\frac{dy}{dx}$ | B1 | |
| | Obtain $6x^2+5y+5x\frac{dy}{dx}+3y^2\frac{dy}{dx}=0$ | B1 | |

Question 2

| | |
|---|---|
| State $2ay\frac{dy}{dx}$ as derivative of $ay^2$ | B1 |
| State $y^2+2xy\frac{dy}{dx}$ as derivative of $xy^2$ | B1 |
| Equate derivative of LHS to zero and set $\frac{dy}{dx}$ equal to zero | M1 |
| Obtain $3x^2+y^2-6ax=0$, or horizontal equivalent | A1 |
| Eliminate $y$ and obtain an equation in $x$ | M1 |
| Solve for $x$ and obtain answer $x=\sqrt{3}a$ | A1 |

Question 3

| | | | |
|---|---|---|---|
| **(i)** | Use correct quotient or chain rule to differentiate sec $x$ | M1 | |
| | Obtain given derivative, sec $x$ tan $x$, correctly | A1 | |
| | Use chain rule to differentiate $y$ | M1 | |
| | Obtain the given answer | A1 | [4] |
| **(ii)** | Using $\mathrm{d}x\sqrt{3}\sec^2\theta\ \mathrm{d}\theta$, or equivalent, express integral in terms of $\theta$ and d$\theta$ | M1 | |
| | Obtain $\int \sec\theta\ \mathrm{d}\theta$ | A1 | |
| | Use limits $\frac{1}{6}\pi$ and $\frac{1}{3}\pi$ correctly in an integral form of the form $k\ \ln(\sec\theta + \tan\theta)$ | M1 | |
| | Obtain a correct exact final answer in the given form, e.g. $\ln\left(\frac{2+\sqrt{3}}{\sqrt{3}}\right)$ | A1 | [4] |

Question 4

| | | | |
|---|---|---|---|
| | Use correct product or quotient rule at least once | M1* | |
| | Obtain $\frac{\mathrm{d}x}{\mathrm{d}t} = \mathrm{e}^{-t}\sin t - \mathrm{e}^{-t}\cos t$ or $\frac{\mathrm{d}y}{\mathrm{d}t} = \mathrm{e}^{-t}\cos t - \mathrm{e}^{-t}\sin t$, or equivalent | A1 | |
| | Use $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}t} \div \frac{\mathrm{d}x}{\mathrm{d}t}$ | M1 | |
| | Obtain $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\sin t - \cos t}{\sin t + \cos t}$, or equivalent | A1 | |
| *EITHER:* | Express $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of tan $t$ only | M1(dep*) | |
| | Show expression is identical to $\tan\left(t - \frac{1}{4}\pi\right)$ | A1 | |
| *OR:* | Express $\tan\left(t - \frac{1}{4}\pi\right)$ in terms of tan $t$ | M1 | |
| | Show expression is identical to $\frac{\mathrm{d}y}{\mathrm{d}x}$ | A1 | **[6]** |

Question 5

| | | |
|---|---|---|
| Differentiate $y^3$ to obtain $3y^2\frac{\mathrm{d}y}{\mathrm{d}x}$ | B1 | |
| Use correct product rule at least once | *M1 | |
| Obtain $6\mathrm{e}^{2x}y + 3\mathrm{e}^{2x}\frac{\mathrm{d}y}{\mathrm{d}x} + \mathrm{e}^{x}y^3 + 3\mathrm{e}^{x}y^2\frac{\mathrm{d}y}{\mathrm{d}x}$ as derivative of LHS | A1 | |
| Equate derivative of LHS to zero, substitute $x = 0$ and $y = 2$ and find value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ | M1(d*M) | |
| Obtain $-\frac{4}{3}$ or equivalent as **final answer** | A1 | [5] |

Question 6

| | | |
|---|---|---|
| Obtain $\frac{2}{2t+3}$ for derivative of $x$ | B1 | |
| Use quotient of product rule, or equivalent, for derivative of $y$ | M1 | |
| Obtain $\frac{5}{(2t+3)^2}$ or unsimplified equivalent | A1 | |
| Obtain $t=-1$ | B1 | |
| Use $\frac{dy}{dx}=\frac{dy}{dt}/\frac{dx}{dt}$ in algebraic or numerical form | M1 | |
| Obtain gradient $\frac{5}{2}$ | A1 | [6] |

Question 7

| | | | |
|---|---|---|---|
| **(i)** | State $\frac{dx}{dt}=1-\sec^2 t$, or equivalent | B1 | |
| | Use chain rule | M1 | |
| | Obtain $\frac{dy}{dt}=-\frac{\sin t}{\cos t}$, or equivalent | A1 | |
| | Use $\frac{dy}{dx}=\frac{dy}{dt}\div\frac{dx}{dt}$ | M1 | |
| | Obtain the given answer correctly. | A1 | **5** |
| **(ii)** | State or imply $t=\tan^{-1}(\frac{1}{2})$ | B1 | |
| | Obtain answer $x=-0.0364$ | B1 | **2** |

Question 8

| | | | |
|---|---|---|---|
| **(i)** | Use product rule | M1 | |
| | Obtain derivative in any correct form | A1 | |
| | Differentiate first derivative using the product rule | M1 | |
| | Obtain second derivative in any correct form, e.g. $-\frac{1}{2}\sin\frac{1}{2}x-\frac{1}{4}x\cos\frac{1}{2}x-\frac{1}{2}\sin\frac{1}{2}x$ | A1 | |
| | Verify the given statement | A1 | **5** |
| **(ii)** | Integrate and reach $kx\sin\frac{1}{2}x+l\int\sin\frac{1}{2}x\,dx$ | M1* | |
| | Obtain $2x\sin\frac{1}{2}x-2\int\sin\frac{1}{2}x\,dx$, or equivalent | A1 | |
| | Obtain indefinite integral $2x\sin\frac{1}{2}x+4\cos\frac{1}{2}x$ | A1 | |
| | Use correct limits $x=0, x=\pi$ correctly | M1(dep*) | |
| | Obtain answer $2\pi-4$, or exact equivalent | A1 | **5** |

Question 9

| | | |
|---|---|---|
| Obtain correct derivative of RHS in any form | B1 | |
| Obtain correct derivative of LHS in any form | B1 | |
| Set $\frac{dy}{dx}$ equal to zero and obtain a horizontal equation | M1 | |
| Obtain a correct equation, e.g. $x^2 + y^2 = 1$, from correct work | A1 | |
| By substitution in the curve equation, or otherwise, obtain an equation in $x^2$ or $y^2$ | M1 | |
| Obtain $x = \frac{1}{2}\sqrt{3}$ | A1 | |
| Obtain $y = \frac{1}{2}$ | A1 | **7** |

Question 10

| | | | |
|---|---|---|---|
| **(i)** | Substitute for $x$ and d$x$ throughout using $u = \sin x$ and d$u = \cos x$ d$x$, or equivalent | M1 | |
| | Obtain integrand $e^{2u}$ | A1 | |
| | Obtain indefinite integral $\frac{1}{2}e^{2u}$ | A1 | |
| | Use limits $u = 0$, $u = 1$ correctly, or equivalent | M1 | |
| | Obtain answer $\frac{1}{2}(e^2 - 1)$, or exact equivalent | A1 | **5** |
| **(ii)** | Use chain rule or product rule | M1 | |
| | Obtain correct terms of the derivative in any form, e.g. $2\cos x\, e^{2\sin x} \cos x - e^{2\sin x} \sin x$ | A1 + A1 | |
| | Equate derivative to zero and obtain a quadratic equation in $\sin x$ | M1 | |
| | Solve a 3-term quadratic and obtain a value of $x$ | M1 | |
| | Obtain answer 0.896 | A1 | **6** |

Question 11

| | | | |
|---|---|---|---|
| **(i)** | Use chain rule correctly at least once | M1 | |
| | Obtain either $\frac{dx}{dt} = \frac{3\sin t}{\cos^4 t}$ or $\frac{dy}{dt} = 3\tan^2 t \sec^2 t$, or equivalent | A1 | |
| | Use $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$ | M1 | |
| | Obtain the given answer | A1 | **[4]** |
| **(ii)** | State a correct equation for the tangent in any form | B1 | |
| | Use Pythagoras | M1 | |
| | Obtain the given answer | A1 | **[3]** |

Question 12

| | | |
|---|---|---|
| Use correct product rule or correct chain rule to differentiate $y$ | M1 | |
| Use $\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$ | M*1 | |
| Obtain $\frac{-4\cos\theta\sin^2\theta + 2\cos^3\theta}{\sec^2\theta}$ or equivalent | A1 | |
| Express $\frac{dy}{dx}$ in terms of $\cos\theta$ | DM*1 | |
| Confirm given answer $6\cos^5\theta - 4\cos^3\theta$ legitimately | A1 | [5] |

Question 13

| | | |
|---|---|---|
| Differentiate to obtain form $a\sin 2x + b\cos x$ | M1 | |
| Obtain correct $-6\sin 2x + 7\cos x$ | A1 | |
| Use identity $\sin 2x = 2\sin x\cos x$ | B1 | |
| Solve equation of form $c\sin x\cos x + d\cos x = 0$ to find at least one value of $x$ | M1 | |
| Obtain 0.623 | A1 | |
| Obtain 2.52 | A1 | |
| Obtain 1.57 or $\frac{1}{2}\pi$ from equation of form $c\sin x\cos x + d\cos x = 0$ | A1 | |
| Treat answers in degrees as MR – 1 situation | | [7] |

Question 14

| | | | |
|---|---|---|---|
| **(i)** | Use product rule to find first derivative | M1 | |
| | Obtain $2xe^{2-x} - x^2e^{2-x}$ | A1 | |
| | Confirm $x = 2$ at $M$ | A1 | [3] |
| **(ii)** | Attempt integration by parts and reach $\pm x^2e^{2-x} \pm \int 2xe^{2-x}\,dx$ | *M1 | |
| | Obtain $-x^2e^{2-x} + \int 2xe^{2-x}\,dx$ | A1 | |
| | Attempt integration by parts and reach $\pm x^2e^{2-x} \pm 2xe^{2-x} \pm 2e^{2-x}$ | *M1 | |
| | Obtain $-x^2e^{2-x} - 2xe^{2-x} - 2e^{2-x}$ | A1 | |
| | Use limits 0 and 2 having integrated twice | M1 dep *M | |
| | Obtain $2e^2 - 10$ | A1 | [6] |

Question 15

| | | |
|---|---|---|
| Obtain $\frac{dx}{dt} = \frac{2}{t+2}$ and $\frac{dy}{dt} = 3t^2 + 2$ | B1 | |
| Use $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$ | M1 | |
| Obtain $\frac{dy}{dx} = \frac{1}{2}(3t^2 + 2)(t+2)$ | A1 | |
| Identify value of $t$ at the origin as $-1$ | B1 | |
| Substitute to obtain $\frac{5}{2}$ as gradient at the origin | A1 | [5] |



Question 16

| | | | |
|---|---|---|---|
| EITHER: | Use correct product rule | M1 | |
| | Obtain correct derivative in any form, e.g. $-\sin x \cos 2x - 2\cos x \sin 2x$ | A1 | |
| | Use the correct double angle formulae to express derivative in $\cos x$ and $\sin x$, or $\cos 2x$ and $\sin x$ | M1 | |
| *OR*1: | Use correct double angle formula to express $y$ in terms of $\cos x$ and attempt differentiation | M1 | |
| | Use chain rule correctly | M1 | |
| | Obtain correct derivative in any form, e.g. $-6\cos^2 x \sin x + \sin x$ | A1 | |
| *OR*2: | Use correct factor formula and attempt differentiation | M1 | |
| | Obtain correct derivative in any form, e.g. $-\frac{3}{2}\sin 3x - \frac{1}{2}\sin x$ | A1 | |
| | Use correct trig formulae to express derivative in terms of $\cos x$ and $\sin x$, or $\sin x$ | M1 | |
| Equate derivative to zero and obtain an equation in one trig function | | M1 | |
| Obtain $6\cos^2 x = 1$, $6\sin^2 x = 5$, $\tan^2 x = 5$ or $3\cos 2x = -2$ | | A1 | |
| Obtain answer $x = 1.15$ (or 65.9°) and no other in the given interval | | A1 | **[6]** |

Question 17

| | | |
|---|---|---|
| Use correct quotient or product rule | M1 | |
| Obtain correct derivative in any form | A1 | |
| Equate derivative to zero and obtain a horizontal equation | M1 | |
| Carry out complete method for solving an equation of the form $ae^{3x} = b$, or $ae^{5x} = be^{2x}$ | M1 | |
| Obtain $x = \ln 2$, or exact equivalent | A1 | |
| Obtain $y = \frac{1}{3}$, or exact equivalent | A1 | **6** |

Question 18

| | | | |
|---|---|---|---|
| **(i)** | State $\frac{\mathrm{d}x}{\mathrm{d}t} = -4a\cos^3 t\sin t$, or $\frac{\mathrm{d}y}{\mathrm{d}t} = 4a\sin^3 t\cos t$ | B1 | |
| | Use $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}t} \div \frac{\mathrm{d}x}{\mathrm{d}t}$ | M1 | |
| | Obtain correct expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ in a simplified form | A1 | **3** |
| **(ii)** | Form the equation of the tangent | M1 | |
| | Obtain a correct equation in any form | A1 | |
| | Obtain the given answer | A1 | **3** |
| **iii)** | State the $x$-coordinate of $P$ or the $y$-coordinate of $Q$ in any form | B1 | |
| | Obtain the given result correctly | B1 | **2** |

Question 19

| | | | |
|---|---|---|---|
| **(i)** | State or imply that the derivative of $e^{-2x}$ is $-2e^{-2x}$ | **B1** | |
| | Use product or quotient rule | **M1** | |
| | Obtain correct derivative in any form | **A1** | |
| | Use Pythagoras | **M1** | |
| | Justify the given form | **A1** | [5] |
| **(ii)** | Fully justify the given statement | **B1** | [1] |
| **(iii)** | State answer $x = \frac{1}{4}\pi$ | **B1** | [1] |

Question 20

| | | | |
|---|---|---|---|
| **(i)** | Use the quotient rule | **M1** | |
| | Obtain correct derivative in any form | **A1** | |
| | Equate derivative to zero and solve for $x$ | **M1** | |
| | Obtain answer $x = \sqrt[3]{2}$, or exact equivalent | **A1** | [4] |
| **(ii)** | State or imply indefinite integral is of the form $k\ln(1+x^3)$ | **M1** | |
| | State indefinite integral $\frac{1}{3}\ln(1+x^3)$ | **A1** | |
| | Substitute limits correctly in an integral of the form $k\ln(1+x^3)$ | **M1** | |
| | State or imply that the area of $R$ is equal to $\frac{1}{3}\ln(1+p^3)-\frac{1}{3}\ln 2$, or equivalent | **A1** | |
| | Use a correct method for finding $p$ from an equation of the form $\ln(1+p^3)=a$ or $\ln((1+p^3)/2)=b$ | **M1** | |
| | Obtain answer $p = 3.40$ | **A1** | [2] |

Question 21

| | | |
|---|---|---|
| Use correct quotient rule or equivalent to find first derivative | **M1*** | |
| Obtain $\dfrac{-(1+\tan x)\sec^2 x - \sec^2 x(2-\tan x)}{(1+\tan x)^2}$ or equivalent | **A1** | |
| Substitute $x = \frac{1}{4}\pi$ to find gradient | **dep M1*** | |
| Obtain $-\frac{3}{2}$ | **A1** | |
| Form equation of tangent at $x = \frac{1}{4}\pi$ | **M1** | |
| Obtain $y = -\frac{3}{2}x + 1.68$ or equivalent | **A1** | [6] |

Question 22

| | | | |
|---|---|---|---|
| **(i)** | *EITHER*: State correct derivative of $\sin y$ with respect to $x$ | **B1** | |
| | Use product rule to differentiate the LHS | **M1** | |
| | Obtain correct derivative of the LHS | **A1** | |
| | Obtain a complete and correct derived equation in any form | **A1** | |
| | Obtain a correct expression for $\frac{dy}{dx}$ in any form | **A1** | |
| | *OR*: State correct derivative of $\sin y$ with respect to $x$ | **B1** | |
| | Rearrange the given equation as $\sin y = x/(\ln x + 2)$ and attempt to differentiate both sides | **B1** | |
| | Use quotient or product rule to differentiate the RHS | **M1** | |
| | Obtain correct derivative of the RHS | **A1** | |
| | Obtain a correct expression for $\frac{dy}{dx}$ in any form | **A1** | [5] |
| **(ii)** | Equate $\frac{dy}{dx}$ to zero and obtain a horizontal equation in $\ln x$ or $\sin y$ | **M1** | |
| | Solve for $\ln x$ | **M1** | |
| | Obtain final answer $x = 1/e$, or exact equivalent | **A1** | [3] |

Question 23

| | |
|---|---|
| Use product rule | **M1** |
| Obtain correct derivative in any form, e.g. $\cos x \cos 2x - 2\sin x \sin 2x$ | **A1** |
| Equate derivative to zero and use double angle formulae | **M1** |
| Remove factor of $\cos x$ and reduce equation to one in a single trig function | **M1** |
| Obtain $6\sin^2 x = 1$, $6\cos^2 x = 5$ or $5\tan^2 x = 1$ | **A1** |
| Solve and obtain $x = 0.421$ | **A1** |
| | [6] |

Question 24

| | | |
|---|---|---|
| **(i)** | State or imply $6xy + 3x^2\frac{dy}{dx}$ as derivative of $3x^2y$ | **B1** |
| | State $3y^2\frac{dy}{dx}$ as derivative of $y^3$ | **B1** |
| | Equate attempted derivative of the LHS to zero and solve for $\frac{dy}{dx}$ | **M1** |
| | Obtain the given answer | **A1** |
| | | [4] |
| **(ii)** | Equate numerator to zero | **M1*** |
| | Obtain $x = 2y$, or equivalent | **A1** |
| | Obtain an equation in $x$ or $y$ | **DM1*** |
| | Obtain the point $(-2, -1)$ | **A1** |
| | State the point $(0, 1.44)$ | **B1** |
| | | [5] |

Question 25

| | | |
|---|---|---|
| State or imply derivative of $(\ln x)^2$ is $\dfrac{2\ln x}{x}$ | B1 | |
| Use correct quotient or product rule | M1 | |
| Obtain correct derivative in any form, e.g. $\dfrac{2\ln x}{x^2}-\dfrac{(\ln x)^2}{x^2}$ | A1 | |
| Equate derivative (or its numerator) to zero and solve for $\ln x$ | M1 | |
| Obtain the point (1, 0) with no errors seen | A1 | |
| Obtain the point $(e^2,\ 4e^{-2})$ | A1 | [6] |

Question 26

| | | |
|---|---|---|
| **(i)** | State $\dfrac{dx}{dt}=1-\sin t$ | B1 |
| | Use chain rule to find the derivative of $y$ | M1 |
| | Obtain $\dfrac{dy}{dt}=\dfrac{\cos t}{1+\sin t}$, or equivalent | A1 |
| | Use $\dfrac{dy}{dx}=\dfrac{dy}{dt}\div\dfrac{dx}{dt}$ | M1 |
| | Obtain the given answer correctly | A1 [5] |
| **(ii)** | State or imply $t=\cos^{-1}(\frac{1}{3})$ | B1 |
| | Obtain answers $x = 1.56$ and $x = -0.898$ | B1 + B1 [3] |

Question 27

| | | |
|---|---|---|
| *EITHER*: | State $2xy+x^2\dfrac{dy}{dx}$, or equivalent, as derivative of $x^2y$ | B1 |
| | State $6y^2+12xy\dfrac{dy}{dx}$, or equivalent, as derivative of $6xy^2$ | B1 |
| *OR*: | Differentiating LHS using correct product rule, state term $xy(1-6\dfrac{dy}{dx})$, or equivalent | B1 |
| | State term $(y+x\dfrac{dy}{dx})(x-6y)$, or equivalent | B1 |
| | Equate attempted derivative of LHS to zero and set $\dfrac{dy}{dx}$ equal to zero | M1* |
| | Obtain a horizontal equation, e.g. $6y^2-2xy=0$ (from correct work only) | A1 |
| | Explicitly reject $y = 0$ as a possibility $py^2-qxy=0$ | A1 |
| | Obtain an equation in $x$ or $y$ | DM1 |
| | Obtain answer $(-3a, -a)$ | A1 |

Question 28

| | | | |
|---|---|---|---|
| **(i)** | Use the correct product rule | **M1** | |
| | Obtain correct derivative in any form, e.g. $(2-2x)e^{\frac{1}{2}x}+\frac{1}{2}(2x-x^2)e^{\frac{1}{2}x}$ | **A1** | |
| | Equate derivative to zero and solve for $x$ | **M1** | |
| | Obtain $x=\sqrt{5}-1$ only | **A1** | [4] |
| **(ii)** | Integrate by parts and reach $a(2x-x^2)e^{\frac{1}{2}x}+b\int(2-2x)e^{\frac{1}{2}x}\,dx$ | **M1*** | |
| | Obtain $2e^{\frac{1}{2}x}(2x-x^2)-2\int(2-2x)e^{\frac{1}{2}x}dx$, or equivalent | **A1** | |
| | Complete the integration correctly, obtaining $(12x-2x^2-24)e^{\frac{1}{2}x}$, or equivalent | **A1** | |
| | Use limits $x = 0$, $x = 2$ correctly having integrated by parts twice | **DM1** | |
| | Obtain answer 24 – 8e, or exact simplified equivalent | **A1** | [5] |

Question 29

| | | |
|---|---|---|
| Use correct quotient or product rule | **M1** | |
| Obtain correct derivative in any form | **A1** | |
| Use Pythagoras to simplify the derivative to $\frac{1}{1+\cos x}$, or equivalent | **A1** | |
| Justify the given statement, $-1 < \cos x < 1$ statement, or equivalent | **A1** | [4] |

Question 30

| | |
|---|---|
| Use product rule | **M1** |
| Obtain correct derivative in any form | **A1** |
| Equate derivative to zero, use Pythagoras and obtain a quadratic equation in $\tan x$ | **M1** |
| Obtain $\tan^2 x - a\tan x + 1 = 0$, or equivalent | **A1** |
| Use the condition for a quadratic to have only one root | **M1** |
| Obtain answer $a = 2$ | **A1** |
| Obtain answer $x = \frac{1}{4}\pi$ | **A1** |
| **Total:** | **7** |

Question 31

| | | |
|---|---|---|
| (i) | State or imply derivative is $2\frac{\ln x}{x}$ | B1 |
| | State or imply gradient of the normal at $x = \mathrm{e}$ is $-\frac{1}{2}\mathrm{e}$, or equivalent | B1 |
| | Carry out a complete method for finding the $x$-coordinate of $Q$ | M1 |
| | Obtain answer $x = \mathrm{e} + \frac{2}{\mathrm{e}}$, or exact equivalent | A1 |
| | **Total:** | **4** |
| (ii) | Justify the given statement by integration or by differentiation | B1 |
| | **Total:** | **1** |
| (iii) | Integrate by parts and reach $ax(\ln x)^2 + b\int x.\frac{\ln x}{x}\ \mathrm{d}x$ | M1* |
| | Complete the integration and obtain $x(\ln x)^2 - 2x\ln x + 2x$, or equivalent | A1 |
| | Use limits $x = 1$ and $x = \mathrm{e}$ correctly, having integrated twice | DM1 |
| | Obtain exact value $\mathrm{e} - 2$ | A1 |
| | Use $x$- coordinate of $Q$ found in part **(i)** and obtain final answer $\mathrm{e} - 2 + \frac{1}{\mathrm{e}}$ | B1√ |

Question 32

| | | |
|---|---|---|
| (i) | Use chain rule to differentiate $x$ $\left(\frac{dx}{d\theta} = -\frac{\sin\theta}{\cos\theta}\right)$ | **M1** |
| | State $\frac{dy}{d\theta} = 3 - \sec^2\theta$ | **B1** |
| | Use $\frac{dy}{dx} = \frac{dy}{d\theta} \div \frac{dx}{d\theta}$ | **M1** |
| | Obtain correct $\frac{dy}{dx}$ in any form e.g. $\frac{3-\sec^2\theta}{-\tan\theta}$ | **A1** |
| | Obtain $\frac{dy}{dx} = \frac{\tan^2\theta - 2}{\tan\theta}$, or equivalent | **A1** |
| | **Total:** | **5** |
| (ii) | Equate gradient to $-1$ and obtain an equation in $\tan\theta$ | **M1** |
| | Solve a 3 term quadratic $(\tan^2\theta + \tan\theta - 2 = 0)$ in $\tan\theta$ | **M1** |
| | Obtain $\theta = \frac{\pi}{4}$ and $y = \frac{3\pi}{4} - 1$ only | **A1** |
| | **Total:** | **3** |

Question 33

| | |
|---|---|
| State or imply $du = -\sin x \, dx$ | **B1** |
| Using correct double angle formula, express the integral in terms of $u$ and $du$ | **M1** |
| Obtain integrand $\pm(2u^2 - 1)^2$ | **A1** |
| Change limits and obtain correct integral $\int_{\frac{1}{\sqrt{2}}}^{1} (2u^2 - 1)^2 \, du$ with no errors seen | **A1** |
| Substitute limits in an integral of the form $au^5 + bu^3 + cu$ | **M1** |
| Obtain answer $\frac{1}{15}(7 - 4\sqrt{2})$, or exact simplified equivalent | **A1** |
| **Total:** | **6** |

| | | |
|---|---|---|
| (ii) | Use product rule and chain rule at least once | **M1** |
| | Obtain correct derivative in any form | **A1** |
| | Equate derivative to zero and use trig formulae to obtain an equation in $\cos x$ and $\sin x$ | **M1** |
| | Use correct methods to obtain an equation in $\cos x$ or $\sin x$ only | **M1** |
| | Obtain $10\cos^2 x = 9$ or $10\sin^2 x = 1$, or equivalent | **A1** |
| | Obtain answer 0.32 | **A1** |
| | **Total:** | **6** |

Question 34

| | | |
|---|---|---|
| (i) | State $\frac{dy}{dt} = 4 + \frac{2}{2t-1}$ | **B1** |
| | Use $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$ | **M1** |
| | Obtain answer $\frac{dy}{dx} = \frac{8t-2}{2t(2t-1)}$, or equivalent e.g. $\frac{2}{t} + \frac{2}{4t^2 - 2t}$ | **A1** |
| | **Total:** | **3** |
| (ii) | Use correct method to find the gradient of the normal at $t = 1$ | **M1** |
| | Use a correct method to form an equation for the normal at $t = 1$ | **M1** |
| | Obtain final answer $x + 3y - 14 = 0$, or horizontal equivalent | **A1** |
| | **Total:** | **3** |

Question 35

| | | |
|---|---|---|
| (i) | Use quotient or chain rule | **M1** |
| | Obtain given answer correctly | **A1** |
| | **Total:** | **2** |
| (ii) | *EITHER*:<br>Multiply numerator and denominator of LHS by $1+\sin\theta$ | **(M1** |
| | Use Pythagoras and express LHS in terms of sec $\theta$ and $\tan\theta$ | **M1** |
| | Complete the proof | **A1)** |
| | *OR1:*<br>Express RHS in terms of cos $\theta$ and sin $\theta$ | **(M1** |
| | Use Pythagoras and express RHS in terms of sin $\theta$ | **M1** |
| | Complete the proof | **A1)** |
| (iii) | Use the identity and obtain integral $2\tan\theta+2\sec\theta-\theta$ | **B2** |
| | Use correct limits correctly in an integral containing terms $a$ $\tan\theta$ and $b$ $\sec\theta$ | **M1** |
| | Obtain answer $2\sqrt{2}-\frac{1}{4}\pi$ | **A1** |
| | **Total:** | **4** |

Question 36

| | | |
|---|---|---|
| (i) | Use the chain rule | **M1** |
| | Obtain correct derivative in any form | **A1** |
| | Use correct trigonometry to express derivative in terms of $\tan x$ | **M1** |
| | Obtain $\frac{dy}{dx}=-\frac{4\tan x}{4+\tan^2 x}$, or equivalent | **A1** |
| | **Total:** | **4** |
| (ii) | Equate derivative to –1 and solve a 3–term quadratic for tan $x$ | **M1** |
| | Obtain answer $x$=1.11 and no other in the given interval | **A1** |
| | **Total:** | **2** |

Question 37

| | |
|---|---|
| Use correct quotient rule or product rule | **M1** |
| Obtain correct derivative in any form | **A1** |
| Equate derivative to zero and solve for $x$ | **M1** |
| Obtain $x = 2$ | **A1** |
| **Total:** | **4** |

Question 38

| | | |
|---|---|---|
| (i) | State or imply $y^3 + 3xy^2 \frac{dy}{dx}$ as derivative of $xy^3$ | **B1** |
| | State or imply $4y^3 \frac{dy}{dx}$ as derivative of $y^4$ | **B1** |
| | Equate derivative of the LHS to zero and solve for $\frac{dy}{dx}$ | **M1** |
| | Obtain the given answer | **A1** |
| | | **4** |
| (ii) | Equate numerator to zero | ***M1** |
| | Obtain $y = -2x$, or equivalent | **A1** |
| | Obtain an equation in $x$ or $y$ | **DM1** |
| | Obtain final answer $x = -1, y = 2$ and $x = 1, y = -2$ | **A1** |
| | | **4** |

Question 39

| | | |
|---|---|---|
| (i) | Use correct product or quotient rule | **M1** |
| | Obtain correct derivative in any form | **A1** |
| | Equate derivative to zero and obtain a 3 term quadratic equation in $x$ | **M1** |
| | Obtain answers $x = 2 \pm \sqrt{3}$ | **A1** |
| | | **4** |
| (ii) | Integrate by parts and reach $k(1+x^2)e^{-\frac{1}{2}x} + l\int xe^{-\frac{1}{2}x}\,dx$ | ***M1** |
| | Obtain $-2(1+x^2)e^{-\frac{1}{2}x} + 4\int xe^{-\frac{1}{2}x}\,dx$, or equivalent | **A1** |
| | Complete the integration and obtain $(-18-8x-2x^2)e^{-\frac{1}{2}x}$, or equivalent | **A1** |
| | Use limits $x = 0$ and $x = 2$ correctly, having fully integrated twice by parts | **DM1** |
| | Obtain the given answer | **A1** |
| | | **5** |

Question 40

| | | |
|---|---|---|
| (i) | Use correct product or quotient rule or rewrite as $2\sec x - \tan x$ and differentiate | **M1** |
| | Obtain correct derivative in any form | **A1** |
| | Equate the derivative to zero and solve for $x$ | **M1** |
| | Obtain $x = \frac{1}{6}\pi$ | **A1** |
| | Obtain $y = \sqrt{3}$ | **A1** |
| | | **5** |
| (ii) | Carry out an appropriate method for determining the nature of a stationary point | **M1** |
| | Show the point is a minimum point with no errors seen | **A1** |
| | | **2** |

Question 41

| | | |
|---|---|---|
| (i) | State or imply $3x^2y + x^3\frac{dy}{dx}$ as derivative of $x^3y$ | **B1** |
| | State or imply $9xy^2\frac{dy}{dx} + 3y^3$ as derivative of $3xy^3$ | **B1** |
| | Equate derivative of the LHS to zero and solve for $\frac{dy}{dx}$ | **M1** |
| | Obtain the given answer AG | **A1** |
| | | **4** |
| (ii) | Equate numerator to zero and use $x = -y$ to obtain an equation in $x$ or in $y$ | **M1** |
| | Obtain answer $x = a$ and $y = -a$ | **A1** |
| | Obtain answer $x = -a$ and $y = a$ | **A1** |
| | Consider and reject $y = 0$ and $x = y$ as possibilities | **B1** |
| | | **4** |

Question 42

| | | |
|---|---|---|
| (i) | State or imply $y^3 + 3xy^2\frac{dy}{dx}$ as derivative of $xy^3$ | **B1** |
| | State or imply $4y^3\frac{dy}{dx}$ as derivative of $y^4$ | **B1** |
| | Equate derivative of the LHS to zero and solve for $\frac{dy}{dx}$ | **M1** |
| | Obtain the given answer | **A1** |
| | | **4** |
| (ii) | Equate numerator to zero | ***M1** |
| | Obtain $y = -2x$, or equivalent | **A1** |
| | Obtain an equation in $x$ or $y$ | **DM1** |
| | Obtain final answer $x = -1$, $y = 2$ and $x = 1$, $y = -2$ | **A1** |
| | | **4** |

Question 43

| | | |
|---|---|---|
| (i) | Use correct product or quotient rule | **M1** |
| | Obtain correct derivative in any form | **A1** |
| | Equate derivative to zero and obtain a 3 term quadratic equation in $x$ | **M1** |
| | Obtain answers $x = 2 \pm \sqrt{3}$ | **A1** |
| | | **4** |
| (ii) | Integrate by parts and reach $k(1+x^2)e^{-\frac{1}{2}x} + l\int xe^{-\frac{1}{2}x}\,dx$ | ***M1** |
| | Obtain $-2(1+x^2)e^{-\frac{1}{2}x} + 4\int xe^{-\frac{1}{2}x}\,dx$, or equivalent | **A1** |
| | Complete the integration and obtain $(-18-8x-2x^2)e^{-\frac{1}{2}x}$, or equivalent | **A1** |
| | Use limits $x = 0$ and $x = 2$ correctly, having fully integrated twice by parts | **DM1** |
| | Obtain the given answer | **A1** |
| | | **5** |

Question 44

| | | | |
|---|---|---|---|
| (i) | State correct derivative of $x$ or $y$ with respect to $t$ | | **B1** |
| | Use $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$ | | **M1** |
| | Obtain $\frac{dy}{dx} = \frac{4\sin 2t}{2+2\cos 2t}$, or equivalent | | **A1** |
| | Use double angle formulae throughout | | **M1** |
| | Obtain the given answer correctly | **AG** | **A1** |
| | | | **5** |
| (ii) | State or imply $t = \tan^{-1}\left(-\frac{1}{4}\right)$ | | **B1** |
| | Obtain answer $x = -0.961$ | | **B1** |
| | | | **2** |

Question 45

| | |
|---|---|
| Use quotient or product rule | **M1** |
| Obtain correct derivative in any form | **A1** |
| Equate derivative to zero and obtain a quadratic in $\tan\frac{1}{2}x$ or an equation of the form $a\sin x = b$ | **M1 *** |
| Solve for $x$ | **M1(dep*)** |
| Obtain answer 0.340 | **A1** |
| Obtain second answer 2.802 and no other in the given interval | **A1** |
| | **6** |

Question 46

| | | |
|---|---|---|
| (i) | State or imply $3y^2\frac{dy}{dx}$ as derivative of $y^3$ | **B1** |
| | State or imply $6xy+3x^2\frac{dy}{dx}$ as derivative of $3x^2y$<br><br>*OR* State or imply $2x(x+3y)+x^2\left(1+3\frac{dy}{dx}\right)$ as derivative of $x^2(x+3y)$ | **B1** |
| | Equate derivative of the LHS to zero and solve for $\frac{dy}{dx}$ | **M1** |
| | Obtain the given answer | **A1** |
| | | **4** |

| | | |
|---|---|---|
| (ii) | Equate derivative to – 1 and solve for $y$ | M1* |
| | Use their $y = -2x$ or equivalent to obtain an equation in $x$ or $y$ | M1(dep*) |
| | Obtain answer (1, – 2) | A1 |
| | Obtain answer $(\sqrt[3]{3}\ , 0)$ | B1 |
| | | 4 |

Question 47

| | | | |
|---|---|---|---|
| (i) | Use correct product or quotient rule | M1 | $\frac{dy}{dx} = -\frac{1}{3}(x+1)e^{-\frac{1}{3}x} + e^{-\frac{1}{3}x}$ or $\frac{dy}{dx} = \frac{e^{\frac{1}{3}x} - (x+1)\frac{1}{3}e^{\frac{1}{3}x}}{e^{\frac{2}{3}x}}$ |
| | Obtain complete correct derivative in any form | A1 | |
| | Equate derivative to zero and solve for $x$ | M1 | |
| | Obtain answer $x = 2$ with no errors seen | A1 | |
| | | 4 | |
| (ii) | Integrate by parts and reach $a(x+1)e^{-\frac{1}{3}x} + b\int e^{-\frac{1}{3}x}\,dx$ | M1* | |
| | Obtain $-3(x+1)e^{-\frac{1}{3}x} + 3\int e^{-\frac{1}{3}x}\,dx$, or equivalent | A1 | $-3xe^{-\frac{1}{3}x} + \int 3e^{-\frac{1}{3}x}\,dx - 3e^{-\frac{1}{3}x}$ |
| | Complete integration and obtain $-3(x+1)e^{-\frac{1}{3}x} - 9e^{-\frac{1}{3}x}$, or equivalent | A1 | |
| | Use **correct** limits $x = -1$ and $x = 0$ **in the correct order**, having integrated twice | M1(dep*) | |
| | Obtain answer $9e^{\frac{1}{3}} - 12$, or equivalent | A1 | |
| | | 5 | |

Question 48

| | | |
|---|---|---|
| (i) | State or imply $3y^2 \dfrac{dy}{dx}$ as derivative of $y^3$ | B1 |
| | State or imply $3y^2 + 6xy \dfrac{dy}{dx}$ as derivative of $3xy^2$ | B1 |
| | Equate derivative of LHS to zero and solve for $\dfrac{dy}{dx}$ | M1 |
| | Obtain the given answer | A1 |
| | **Total:** | **4** |
| (ii) | Equate denominator to zero and solve for $y$ | M1* |
| | Obtain $y = 0$ and $x = a$ | A1 |
| | Obtain $y = \alpha x$ and substitute in curve equation to find $x$ or to find $y$ | M1(dep*) |
| | Obtain $x = -a$ | A1 |
| | Obtain $y = 2a$ | A1 |
| | **Total:** | **5** |

Question 49

| | | |
|---|---|---|
| (i) | Obtain $\dfrac{dx}{d\theta} = 2\cos\theta + 2\cos 2\theta$ or $\dfrac{dy}{d\theta} = -2\sin\theta - 2\sin 2\theta$ | B1 |
| | Use $dy/dx = dy/d\theta \div dx/d\theta$ | M1 |
| | Obtain correct $\dfrac{dy}{dx}$ in any form, e.g. $-\dfrac{2\sin\theta + 2\sin 2\theta}{2\cos\theta + 2\cos 2\theta}$ | A1 |
| | | 3 |
| (ii) | Equate denominator to zero and use any correct double angle formula | M1* |
| | Obtain correct 3-term quadratic in $\cos\theta$ in any form | A1 |
| | Solve for $\theta$ | depM1* |
| | Obtain $x = 3\sqrt{3}/2$ and $y = \dfrac{1}{2}$, or exact equivalents | A1 |
| | | 4 |

Question 50

| | | |
|---|---|---|
| '(i) | Use product rule | M1* |
| | Obtain correct derivative in any form | A1 |
| | Equate derivative to zero and obtain an equation in a single trig function | depM1* |
| | Obtain a correct equation, e.g. $3\tan^2 x = 2$ | A1 |
| | Obtain answer $x = 0.685$ | A1 |
| | | 5 |
| (ii) | Use the given substitution and reach $a\int\left(u^2 - u^4\right)\mathrm{d}u$ | M1 |
| | Obtain correct integral with $a = 5$ and limits 0 and 1 | A1 |
| | Use correct limits in an integral of the form $a\left(\frac{1}{3}u^3 - \frac{1}{5}u^5\right)$ | M1 |
| | Obtain answer $\frac{2}{3}$ | A1 |
| | | 4 |

Question 51

| | | |
|---|---|---|
| (i) | Use correct quotient or product rule | M1 |
| | Obtain correct derivative in any form | A1 |
| | Equate numerator to zero | M1 |
| | Use $\cos^2 x + \sin^2 x = 1$ and solve for $\sin x$ | M1 |
| | Obtain coordinates $x = -\pi/6$ and $y = \sqrt{3}$ ISW | A1 + A1 |
| | | |
| | | 6 |

| | | |
|---|---|---|
| (ii) | State indefinite integral of the form $k \ln(2 + \sin x)$ | M1* |
| | Substitute limits correctly, equate result to 1 and obtain $3 \ln(2 + \sin a) - 3 \ln 2 = 1$ | A1 |
| | Use correct method to solve for $a$ | M1(dep*) |
| | Obtain answer $a = 0.913$ or better | A1 |
| | | 4 |

Question 52

| | | |
|---|---|---|
| (i) | Obtain $\frac{dx}{d\theta} = 2\cos\theta + 2\cos 2\theta$ or $\frac{dy}{d\theta} = -2\sin\theta - 2\sin 2\theta$ | B1 |
| | Use $dy/dx = dy/d\theta \div dx/d\theta$ | M1 |
| | Obtain correct $\frac{dy}{dx}$ in any form, e.g. $-\frac{2\sin\theta + 2\sin 2\theta}{2\cos\theta + 2\cos 2\theta}$ | A1 |
| | | 3 |
| (ii) | Equate denominator to zero and use any correct double angle formula | M1* |
| | Obtain correct 3-term quadratic in $\cos\theta$ in any form | A1 |
| | Solve for $\theta$ | depM1* |
| | Obtain $x = 3\sqrt{3}/2$ and $y = \frac{1}{2}$, or exact equivalents | A1 |
| | | 4 |

Question 53

| | | |
|---|---|---|
| '(i) | Use product rule | **M1*** |
| | Obtain correct derivative in any form | **A1** |
| | Equate derivative to zero and obtain an equation in a single trig function | **depM1*** |
| | Obtain a correct equation, e.g. $3\tan^2 x = 2$ | **A1** |
| | Obtain answer $x = 0.685$ | **A1** |
| | | **5** |
| '(ii) | Use the given substitution and reach $a\int(u^2 - u^4)\,\mathrm{d}u$ | **M1** |
| | Obtain correct integral with $a = 5$ and limits 0 and 1 | **A1** |
| | Use correct limits in an integral of the form $a\left(\frac{1}{3}u^3 - \frac{1}{5}u^5\right)$ | **M1** |
| | Obtain answer $\frac{2}{3}$ | **A1** |
| | | **4** |

Question 54

| | |
|---|---|
| State $\cos y\frac{\mathrm{d}y}{\mathrm{d}x}$ as derivative of $\sin y$ | **B1** |
| State correct derivative in terms of $x$ and $y$, e.g. $\sec^2 x / \cos y$ | **B1** |
| State correct derivative in terms of $x$, e.g. $\frac{\sec^2 x}{\sqrt{1-\tan^2 x}}$ | **B1** |
| Use double angle formula | **M1** |
| Obtain the given answer correctly | **A1** |
| | **5** |

Question 55

| | | |
|---|---|---|
| (i) | State or imply $du = -\sin x \, dx$ | B1 |
| | Using Pythagoras express the integral in terms of $u$ | M1 |
| | Obtain integrand $\pm\sqrt{u}\left(1-u^2\right)$ | A1 |
| | Integrate and obtain $-\frac{2}{3}u^{\frac{3}{2}}+\frac{2}{7}u^{\frac{7}{2}}$, or equivalent | A1 |
| | Change limits correctly and substitute correctly in an integral of the form $au^{\frac{3}{2}}+bu^{\frac{7}{2}}$ | M1 |
| | Obtain answer $\frac{8}{21}$ | A1 |
| | | 6 |
| (ii) | Use product rule and chain rule at least once | M1 |
| | Obtain correct derivative in any form | A1 + A1 |
| | Equate derivative to zero and obtain a horizontal equation in integral powers of sin $x$ and cos $x$ | M1 |
| | Use correct methods to obtain an equation in one trig function | M1 |
| | Obtain $\tan^2 x = 6$, $7\cos^2 x = 1$ or $7\sin^2 x = 6$, or equivalent, and obtain answer 1.183 | A1 |
| | | 6 |

Question 56

| | |
|---|---|
| State or imply $3y^2 + 6xy\frac{dy}{dx}$ as derivative of $3xy^2$ | B1 |
| State or imply $3y^2\frac{dy}{dx}$ as derivative of $y^3$ | B1 |
| Equate derivative of LHS to zero, substitute (1, 3) and find the gradient | M1 |
| Obtain final answer $\frac{10}{3}$ or equivalent | A1 |
| | 4 |

Question 57

| | |
|---|---|
| Use chain rule | M1 |
| Obtain correct answer in any form | A1 |

Question 58

| | |
|---|---|
| Use correct quotient rule | M1 |
| Obtain correct derivative in any form | A1 |
| Equate derivative to $\frac{1}{4}$ and obtain a quadratic in $\ln x$ or $(1+\ln x)$ | M1 |
| Reduce to $(\ln x)^2 - 2\ln x + 1 = 0$ | A1 |
| Solve a 3-term quadratic in $\ln x$ for $x$ | M1 |
| Obtain answer $x = \mathrm{e}$ | A1 |
| Obtain answer $y = \frac{1}{2}\mathrm{e}$ | A1 |
| | 7 |

Question 59

| | | | |
|---|---|---|---|
| (i) | State correct expansion of $\sin(3x+x)$ or $\sin(3x-x)$ | B1 | B0 If their formula retains ± in the middle |
| | Substitute expansions in $\frac{1}{2}(\sin 4x+\sin 2x)$ | M1 | |
| | Obtain $\sin 3x\cos x=\frac{1}{2}(\sin 4x+\sin 2x)$ correctly | A1 | Must see the $\sin 4x$ and $\sin 2x$ or reference to LHS and RHS for A1<br>**AG** |
| | | 3 | |
| (ii) | Integrate and obtain $-\frac{1}{8}\cos 4x-\frac{1}{4}\cos 2x$ | B1 B1 | |
| | Substitute limits $x=0$ and $x=\frac{1}{3}\pi$ correctly | M1 | In their expression |
| | Obtain answer $\frac{9}{16}$ | A1 | From correct working seen. |
| | | 4 | |
| (iii) | State correct derivative $2\cos 4x+\cos 2x$ | B1 | |
| | Using correct double angle formula, express derivative in terms of $\cos 2x$ and equate the result to zero | M1 | |
| | Obtain $4\cos^2 2x+\cos 2x-2=0$ | A1 | |
| | Solve for $x$ or $2x$ (could be labelled $x$)<br>$\left(\cos 2x=\frac{-1\pm\sqrt{33}}{8}\right)$ | M1 | Must see working if solving an incorrect quadratic<br>The roots of the correct quadratic are -0.843 and 0.593<br>Need to get as far as $x=\ldots$<br>The wrong value of $x$ is 0.468 and can imply M1 if correct quadratic seen<br>Could be working from a quartic in $\cos x$:<br>$16\cos^4 x-14\cos^2 x+1=0$ |
| | Obtain answer $x=1.29$ only | A1 | |
| | | 5 | |

Question 60

| | | |
|---|---|---|
| (i) | Use the quotient or product rule | **M1** |
| | Obtain correct derivative in any form | **A1** |
| | Reduce to $-\dfrac{2e^{-x}}{\left(1-e^{-x}\right)^2}$ , or equivalent, and explain why this is always negative | **A1** |
| | | **3** |
| (ii) | Equate derivative to $-1$ and obtain the given equation | **B1** |
| | State or imply $u^2-4u+1=0$ , or equivalent in $e^a$ | **B1** |
| | Solve for $a$ | **M1** |
| | Obtain answer $a=\ln\left(2+\sqrt{3}\right)$ and no other | **A1** |
| | | **4** |

Question 61

| | | |
|---|---|---|
| (i) | Use product rule | **M1** |
| | Obtain correct derivative in any form | **A1** |
| | | **2** |
| (ii) | Equate derivative to zero and use correct $\cos(A+B)$ formula | **M1** |
| | Obtain the given equation | **A1** |
| | | **2** |
| (iii) | Use correct method to solve for $x$ | **M1** |
| | Obtain answer, e.g. $x=\dfrac{1}{12}\pi$ | **A1** |
| | Obtain second answer, e.g. $\dfrac{7}{12}\pi$ , and no other | **A1** |
| | | **3** |

Question 62

| | | |
|---|---|---|
| (i) | Use correct quotient rule | **M1** |
| | Obtain $\frac{dy}{dx} = -\text{cosec}^2 x$ correctly | **A1** |
| | | **2** |
| (ii) | Integrate by parts and reach $ax\cot x + b\int \cot x \, dx$ | ***M1** |
| | Obtain $-x\cot x + \int \cot x \, dx$ | **A1** |
| | State $\pm \ln \sin x$ as integral of $\cot x$ | **M1** |
| | Obtain complete integral $-x\cot x + \ln \sin x$ | **A1** |
| | Use correct limits correctly | **DM1** |
| | Obtain $\frac{1}{4}(\pi + \ln 4)$ following full and exact working | **A1** |
| | | **6** |

Question 63

| | | |
|---|---|---|
| (i) | Use $\cos(A + B)$ formula to express $\cos 3x$ in terms of trig functions of $2x$ and $x$ | **M1** |
| | Use double angle formulae and Pythagoras to obtain an expression in terms of $\cos x$ only | **M1** |
| | Obtain a correct expression in terms of $\cos x$ in any form | **A1** |
| | Obtain $\cos 3x \equiv 4\cos^3 x - 3\cos x$ | **A1** |
| | | **4** |
| (ii) | Use identity and solve cubic $4\cos^3 x = -1$ for $x$ | **M1** |
| | Obtain answer 2.25 and no other in the interval | **A1** |
| | | **2** |

Question 64

| | |
|---|---|
| State $4xy + 2x^2 \dfrac{dy}{dx}$, or equivalent, as derivative of $2x^2y$ | **B1** |
| State $y^2 + 2xy \dfrac{dy}{dx}$, or equivalent, as derivative of $xy^2$ | **B1** |
| Equate attempted derivative of LHS to zero and set $\dfrac{dy}{dx}$ equal to zero (or set numerator equal to zero) | ***M1** |
| Reject $y = 0$ | **B1** |
| Obtain $y = 4x$ | **A1** |
| Obtain an equation in $y$ (or in $x$) and solve for $y$ (or for $x$) in terms of $a$ | **DM1** |
| Obtain $y = -2a$ | **A1** |
| | **7** |

Question 65

| | | |
|---|---|---|
| (i) | State or imply ordinates 1, 1.2116…, 2.7597... | **B1** |
| | Use correct formula, or equivalent, with $h = 0.6$ | **M1** |
| | Obtain answer 1.85 | **A1** |
| | | **3** |
| (ii) | Explain why the rule gives an overestimate | **B1** |
| | | **1** |
| (iii) | Differentiate using quotient or chain rule | **M1** |
| | Obtain correct derivative in terms of $\sin x$ and $\cos x$ | **A1** |
| | Equate derivative to 2, use Pythagoras and obtain an equation in $\sin x$ | **M1** |
| | Obtain $2\sin^2 x + \sin x - 2 = 0$ | **A1** |
| | Solve a 3-term quadratic for $x$ | **M1** |
| | Obtain answer $x = 0.896$ only | **A1** |
| | | **6** |

Question 66

| | | |
|---|---|---|
| (i) | Use product rule and chain rule at least once | **M1** |
| | Obtain correct derivative in any form | **A1** |
| | Equate derivative to zero, use Pythagoras and obtain an equation in cos $x$ | **M1** |
| | Obtain $\cos^2 x + 3\cos x - 1 = 0$, or 3-term equivalent | **A1** |
| | Obtain answer $x = 1.26$ | **A1** |
| | | **5** |
| (ii) | Using $du = \pm \sin x \, dx$ express integrand in terms of $u$ and $du$ | **M1** |
| | Obtain integrand $e^u(u^2 - 1)$ | **A1** |
| | Commence integration by parts and reach $ae^u(u^2 - 1) + b\int ue^u \, du$ | ***M1** |
| | Obtain $e^u(u^2 - 1) - 2\int ue^u \, du$ | **A1** |
| | Complete integration, obtaining $e^u(u^2 - 2u + 1)$ | **A1** |
| | Substitute limits $u = 1$ and $u = -1$ (or $x = 0$ and $x = \pi$), having integrated completely | **DM1** |
| | Obtain answer $\frac{4}{e}$, or exact equivalent | **A1** |
| | | **7** |

Question 67

| | | |
|---|---|---|
| '(a) | State or imply $3y^2 + 6xy\dfrac{dy}{dx}$ as derivative of $3xy^2$ | **B1** |
| | State or imply $3y^2\dfrac{dy}{dx}$ as derivative of $y^3$ | **B1** |
| | Equate attempted derivative of LHS to zero and solve for $\dfrac{dy}{dx}$ | **M1** |
| | Obtain the given answer correctly | **A1** |
| | | **4** |
| '(b) | Equate denominator to zero | ***M1** |
| | Obtain $y = 2x$, or equivalent | **A1** |
| | Obtain an equation in $x$ or $y$ | **DM1** |
| | Obtain the point (1, 2) | **A1** |
| | State the point $\left(\sqrt[3]{5}, 0\right)$ | **B1** |
| | | **5** |

Question 68

| | | |
|---|---|---|
| (a) | Use product rule | **M1** |
| | Obtain derivative in any correct form e.g. $2e^{2x}(\sin x + 3\cos x) + e^{2x}(\cos x - 3\sin x)$ | **A1** |
| | Equate derivative to zero and obtain an equation in one trigonometric ratio | **M1** |
| | Obtain $x = 1.43$ only | **A1** |
| | | **4** |
| (b) | Use a correct method to determine the nature of the stationary point<br>e.g. $x = 1.42,\ y' = 0.06e^{2.84} > 0$<br>$x = 1.44,\ y' = -0.07e^{2.88} < 0$ | **M1** |
| | Show that it is a maximum point | **A1** |
| | | **2** |

Question 69

| | | |
|---|---|---|
| (a) | Use quotient or product rule | M1 |
| | Obtain derivative in any correct form e.g. $\dfrac{-\sin x(1+\sin x)-\cos x(\cos x)}{(1+\sin x)^2}$ | A1 |
| | Use Pythagoras to simplify the derivative | M1 |
| | Justify the given statement | A1 |
| | | 4 |
| (b) | State integral of the form $a\ln(1+\sin x)$ | *M1 |
| | State correct integral $\ln(1+\sin x)$ | A1 |
| | Use limits correctly | DM1 |
| | Obtain answer $\ln\dfrac{4}{3}$ | A1 |
| | | 4 |

Question 70

| | |
|---|---|
| Use correct product rule | M1 |
| Obtain correct derivative in any form, e.g. $-\sin x\sin 2x+2\cos x\cos 2x$ | A1 |
| Use double angle formula to express derivative in terms of $\sin x$ and $\cos x$ | M1 |
| Equate derivative to zero and obtain an equation in one trig function | M1 |
| Obtain $3\sin 2x=1$, or $3\cos 2x=2$ or $2\tan 2x=1$ | A1 |
| Solve and obtain $x=0.615$ | A1 |
| | 6 |

Question 71

| | | |
|---|---|---|
| (a) | Use quotient or product rule | M1 |
| | Obtain correct derivative in any form e.g. $\dfrac{(1+3x^4)-x\times 12x^3}{(1+3x^4)^2}$ | A1 |
| | Equate derivative to zero and solve for $x$ | M1 |
| | Obtain answer 0.577 | A1 |
| | | 4 |

| | | |
|---|---|---|
| (b) | State or imply $du = 2\sqrt{3x}\,dx$, or equivalent | B1 |
| | Substitute for $x$ and $dx$ | M1 |
| | Obtain integrand $\dfrac{1}{2\sqrt{3}(1+u^2)}$, or equivalent | A1 |
| | State integral of the form $a\tan^{-1}u$ and use limits $u=0$ and $u=\sqrt{3}$ (or $x=0$ and $x=1$) correctly | M1 |
| | Obtain answer $\dfrac{\sqrt{3}}{18}\pi$, or exact equivalent | A1 |
| | | 5 |

Question 72

| | | |
|---|---|---|
| (a) | Use the product rule | M1 |
| | State or imply derivative of $\tan^{-1}(\frac{1}{2}x)$ is of the form $k/(4+x^2)$, where $k = 2$ or 4, or equivalent | M1 |
| | Obtain correct derivative in any form, e.g. $\tan^{-1}\left(\dfrac{1}{2}x\right)+\dfrac{2x}{x^2+4}$, or equivalent | A1 |
| | | 3 |
| (b) | State or imply $y$-coordinate is $\dfrac{1}{2}\pi$ | B1 |
| | Carry out a complete method for finding $p$, e.g. by obtaining the equation of the tangent and setting $x=0$, or by equating the gradient at $x=2$ to $\dfrac{\frac{1}{2}\pi - p}{2}$ | M1 |
| | Obtain answer $p=-1$ | A1 |
| | | 3 |

Question 73

| | |
|---|---|
| State or imply $\dfrac{dx}{d\theta} = 2\sin 2\theta$ or $\dfrac{dy}{d\theta} = 2+2\cos 2\theta$ | B1 |
| Use $\dfrac{dy}{dx} = \dfrac{dy}{d\theta} \div \dfrac{dx}{d\theta}$ | M1 |
| Obtain correct answer $\dfrac{dy}{dx} = \dfrac{2+2\cos 2\theta}{2\sin 2\theta}$ | A1 |
| Use correct double angle formulae | M1 |
| Obtain the given answer correctly $\dfrac{dy}{dx} = \cot\theta$ | A1 |

Question 74

| | | |
|---|---|---|
| (a) | Use correct product or quotient rule | *M1 |
| | Obtain correct derivative in any form | A1 |
| | Equate derivative to zero and solve for $x$ | DM1 |
| | Obtain $x = 4$ | A1 |
| | Obtain $y = -2e^{-2}$, or exact equivalent | A1 |
| | | 5 |
| (b) | Commence integration and reach $a(2-x)e^{-\frac{1}{2}x} + b\int e^{-\frac{1}{2}x}\,dx$ | *M1 |
| | Obtain $-2(2-x)e^{-\frac{1}{2}x} - 2\int e^{-\frac{1}{2}x}\,dx$ | A1 |
| | Complete integration and obtain $2xe^{-\frac{1}{2}x}$ | A1 |
| | Use correct limits, $x = 0$ and $x = 2$, correctly, having integrated twice | DM1 |
| | Obtain answer $4e^{-1}$, or exact equivalent | A1 |

Question 75

| | | |
|---|---|---|
| 5(a) | State $\frac{dx}{d\theta} = \sec^2\theta$ or $\frac{dy}{d\theta} = -2\sin\theta\cos\theta$ | **B1** |
| | Use $\frac{dy}{dx} = \frac{dy}{d\theta} \div \frac{dx}{d\theta}$ | **M1** |
| | Obtain $\frac{dy}{dx} = -2\sin\theta\cos^3\theta$ from correct working | **A1** |
| | **Alternative method for question 5(a)** | |
| | Convert to Cartesian form and differentiate | **M1** |
| | $\frac{dy}{dx} = \frac{-2x}{\left(1+x^2\right)^2}$ | **A1** |
| | Obtain $\frac{dy}{dx} = -2\sin\theta\cos^3\theta$ from correct working | **A1** |
| | | **3** |

| | | |
|---|---|---|
| (b) | Use correct product rule to obtain $\frac{\mathrm{d}}{\mathrm{d}\theta}\left(\pm 2\cos^3\theta\sin\theta\right)$ | **M1** |
| | Obtain correct derivative in any form | **A1** |
| | Equate derivative to zero and obtain an equation in one trig ratio | **A1** |
| | Obtain answer $x = -\frac{1}{\sqrt{3}}$ | **A1** |
| | **Alternative method for question 5(b)** | |
| | Use correct quotient rule to obtain $\frac{\mathrm{d}^2 y}{\mathrm{d}x^2}$ | **M1** |
| | Obtain correct derivative in any form | **A1** |
| | Equate derivative to zero and obtain an equation in $x^2$ | **A1** |
| | Obtain answer $x = -\frac{1}{\sqrt{3}}$ | **A1** |
| | | **4** |

Question 76

| | | |
|---|---|---|
| (a) | State or imply $\mathrm{d}u = \cos x \, \mathrm{d}x$ | **B1** |
| | Using double angle formula for $\sin 2x$ and Pythagoras, express integral in terms of $u$ and $\mathrm{d}u$. | **M1** |
| | Obtain integral $\int 2\left(u - u^3\right)\mathrm{d}u$ | **A1** |
| | Use limits $u = 0$ and $u = 1$ in an integral of the form $au^2 + bu^4$, where $ab \neq 0$ | **M1** |
| | Obtain answer $\frac{1}{2}$ | **A1** |
| | | **5** |
| (b) | Use product rule | **M1** |
| | Obtain correct derivative in any form | **A1** |
| | Equate derivative to zero and use a double angle formula | ***M1** |
| | Obtain an equation in one trig variable | **DM1** |
| | Obtain $4\sin^2 x = 1$, $4\cos^2 x = 3$ or $3\tan^2 x = 1$ | **A1** |
| | Obtain answer $x = \frac{1}{6}\pi$ | **A1** |
| | | **6** |

Question 77

| | | |
|---|---|---|
| (a) | Calculate the values of a relevant expression or pair of expressions at $x = 1$ and $x = 1.5$ | **M1** |
| | Complete the argument correctly with correct calculated values | **A1** |
| | | **2** |
| (b) | Use the iterative formula $x_{n+1} = \frac{e^{2x_n}+1}{e^{2x_n}-1}$, or equivalent, correctly at least once | **M1** |
| | Obtain final answer 1.20 | **A1** |
| | Show sufficient iterations to 4 dp to justify 1.20 to 2 dp, or show there is a sign change in the interval (1.195,1.205) | **A1** |
| | | **3** |
| (c) | Use quotient rule | **M1** |
| | Obtain correct derivative in any form | **A1** |
| | Equate derivative to – 8 and obtain a quadratic in $e^{2x}$ | **M1** |
| | Obtain $2\left(e^{2x}\right)^2 - 5e^{2x} + 2 = 0$ | **A1** |
| | Solve a 3-term quadratic in $e^{2x}$ for $x$ | **M1** |
| | Obtain answer $x = \frac{1}{2}\ln 2$, or exact equivalent, only | **A1** |

Question 78

| | | |
|---|---|---|
| (a) | Use quotient or product rule | **M1** |
| | Obtain correct derivative in any form | **A1** |
| | Equate derivative to zero and solve for $x$ | **M1** |
| | Obtain $x = \sqrt[4]{e}$ and $y = \dfrac{1}{4e}$, or exact equivalents | **A1** |
| | | **4** |
| (b) | Commence integration and reach $ax^{-3}\ln x + b\int x^{-3}.\ \dfrac{1}{x}\ dx$ | ***M1** |
| | Obtain $-\dfrac{1}{3}x^{-3}\ln x + \dfrac{1}{3}\int x^{-3}.\ \dfrac{1}{x}\ dx$ | **A1** |
| | Complete integration and obtain $-\dfrac{1}{3}x^{-3}\ln x - \dfrac{1}{9}x^{-3}$ | **A1** |
| | Substitute limits correctly, having integrated twice | **DM1** |
| | Obtain answer $\dfrac{1}{9} - \dfrac{1}{3}a^{-3}\ln a - \dfrac{1}{9}a^{-3}$ | **A1** |
| | Justify the given statement | **A1** |
| | | **6** |

Question 79

| | | |
|---|---|---|
| (a) | State $\frac{dx}{dt} = 1 + \frac{1}{t+2}$ | **B1** |
| | Use product rule | **M1** |
| | Obtain $\frac{dy}{dt} = e^{-2t} - 2(t-1)e^{-2t}$ | **A1** |
| | Use $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$ | **M1** |
| | Obtain correct answer in any simplified form, e.g. $\frac{(3-2t)(t+2)}{t+3}\ e^{-2t}$ | **A1** |
| | | **5** |
| (b) | Equate derivative to zero and solve for $t$ | **M1** |
| | Obtain $t = \frac{3}{2}$ and obtain answer $y = \frac{1}{2}e^{-3}$, or exact equivalent | **A1** |
| | | **2** |

Question 80

| | |
|---|---|
| Use correct product (or quotient) rule | **M1** |
| Obtain $\frac{dy}{dx} = -5e^{-5x}\tan^2 x + 2e^{-5x}\tan x\sec^2 x$ | **A1** |
| Equate *their* derivative to zero and obtain an equation in $\sin x$ and $\cos x$ | **M1** |
| Obtain $5\cos x\sin x = 2$ | **A1** |
| State answer $x = 0$ | **B1** |
| Use double angle formula or square both sides and solve for $x$ | **M1** |
| Obtain answer, e.g. 0.464 | **A1** |
| Obtain second non-zero answer, e.g. 1.107 and no other in the given interval | **A1** |
| | **8** |

Question 81

| | | |
|---|---|---|
| (a) | Use correct product rule or correct quotient rule | **M1** |
| | Obtain correct derivative in any form | **A1** |
| | Equate 2 term derivative to zero and solve for $x$ | **M1** |
| | Obtain answer $x = e^{\frac{3}{2}}$ | **A1** |
| | Obtain answer $y = \frac{3}{2e}$ | **A1** |
| | | **5** |

| (b) | | |
|---|---|---|
| | Commence integration and reach $ax^{\frac{1}{3}}\ln x + b\int x^{\frac{1}{3}}.\frac{1}{x}\mathrm{d}x$ | ***M1** |
| | Obtain $3x^{\frac{1}{3}}\ln x - 3\int x^{\frac{1}{3}}.\frac{1}{x}\mathrm{d}x$ | **A1** |
| | Complete the integration and obtain $3x^{\frac{1}{3}}\ln x - 9x^{\frac{1}{3}}$ | **A1** |
| | Use limits correctly in an expression of the form $px^{\frac{1}{3}}\ln x + qx^{\frac{1}{3}}$ $(pq \neq 0)$ | **DM1** |
| | Obtain $18\ln 2 - 9$ from full and correct working | **A1** |
| | | **5** |

Question 82

| (a) | | |
|---|---|---|
| | Use correct chain rule **or** correct quotient rule to differentiate $x$ or $y$ | **M1** |
| | Obtain $\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{3}{2+3t}$ **or** $\frac{\mathrm{d}y}{\mathrm{d}t} = \frac{2}{(2+3t)^2}$ | **A1** |
| | Use $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}t} \div \frac{\mathrm{d}x}{\mathrm{d}t}$ | **M1** |
| | Obtain answer $\frac{2}{3(2+3t)}$ | **A1** |
| | Explain why this is always positive | **A1** |

| (b) | | |
|---|---|---|
| | Obtain $y = -\frac{1}{3}$ when $x = 0$ | **B1** |
| | Use a correct method to form the given tangent | **M1** |
| | Obtain answer $3y = 2x - 1$ | **A1** |
| | | **3** |

Question 83

| | | | |
|---|---|---|---|
| (a) | Use quotient or product rule | **M1** | |
| | Obtain correct derivative in any form | **A1** | |
| | Equate derivative to zero and solve for $x$ | **M1** | |
| | Obtain answer $x = 3$ | **A1** | |
| | | **4** | |
| (b) | State $\frac{du}{dx} = \frac{1}{2\sqrt{x}}$, or $dx = 2\sqrt{x}du$, or $2u\ du = dx$ | **B1** | |
| | Substitute and obtain integrand $\frac{2}{9-u^2}$ | **B1** | |
| | Use given formula for the integral or integrate relevant partial fractions | **M1** | |
| | Obtain integral $\frac{1}{3}\ln\left(\frac{3+u}{3-u}\right)$ | **A1** | OE |
| | Use limits $u = 0$ and $u = 2$ correctly | **M1** | |
| | Obtain the given answer correctly | **A1** | |
| | | **6** | |

Question 84

| | | | |
|---|---|---|---|
| (a) | Use chain rule to differentiate LHS | ***M1** | |
| | Obtain $\frac{1}{x+y}\left(1+\frac{dy}{dx}\right)$ | **A1** | |
| | Equate derivative of LHS to $1 - 2\frac{dy}{dx}$ and solve for $\frac{dy}{dx}$ | **DM1** | |
| | Obtain the given answer correctly | **A1** | |
| | | **4** | |
| (b) | State $x + y = 1$ | **B1** | |
| | Obtain or imply $x - 2y = 0$ | **B1** | |
| | Obtain coordinates $x = \frac{2}{3}$ and $y = \frac{1}{3}$ | **B1** | |
| | | **3** | |

Question 85

| | | |
|---|---|---|
| Use chain rule | **M1** | Allow if not starting with the correct index. |
| Obtain correct derivative in any form | **A1** | e.g. $\frac{dy}{dx} = \frac{\sec^2 x}{2\sqrt{\tan x}}$ |
| Use correct Pythagoras to obtain correct derivative in terms of tan $x$ | **A1** | e.g. $\frac{dy}{dx} = \frac{1+\tan^2 x}{2\sqrt{\tan x}}$ |
| Use a correct derivative to obtain $\frac{dy}{dx} = 1$ when $x = \frac{1}{4}\pi$ | **B1** | Confirm the **given statement** from correct work. Should see at least $\frac{2}{2} = 1$. |
| | **4** | |

## Question 86

| | | | |
|---|---|---|---|
| (a) | State correct derivative of $ye^{2x}$ with respect to $x$ | B1 | $2ye^{2x}+e^{2x}\frac{dy}{dx}$ |
| | State correct derivative of $y^2e^x$ with respect to $x$ | B1 | $2ye^x\frac{dy}{dx}+y^2e^x$ |
| | Equate attempted derivative of the LHS to zero and solve for $\frac{dy}{dx}$ | M1 | |
| | Obtain $\frac{dy}{dx}=\frac{2ye^x-y^2}{2y-e^x}$ | A1 | Obtain the **given answer** correctly. Condone multiplication by $\frac{-1}{-1}$ and cancelling of $e^x$ without comment. |
| (b) | Equate denominator to zero and substitute for $y$ or for $e^x$ in the equation of the curve | *M1 | |
| | Obtain equation of the form $ae^{3x}=b$ or $cy^3=d$ | DM1 | $(e^{3x}=8,\quad y^3=1)$ SOI |
| | Obtain $x=\ln 2$ | A1 | Accept $\frac{1}{3}\ln 8$ ISW |
| | Obtain $y=1$ | A1 | |
| | | 4 | |

## Question 87

| | | | |
|---|---|---|---|
| (a) | Use correct product rule | M1 | |
| | Obtain correct derivative in any form | A1 | $\frac{dy}{dx}=e^{1-2x}-2xe^{1-2x}$ |
| | Equate derivative to zero and solve for $x$ | M1 | |
| | Obtain $x=\frac{1}{2}$ and $y=\frac{1}{2}$ | A1 | |
| | | 4 | |
| (b) | Use a correct method for determining the nature of a stationary point | M1 | e.g. $\frac{d^2y}{dx^2}=-2e^{1-2x}-2(1-2x)e^{1-2x}$ |
| | Show that it is a maximum point | A1 | |
| | | 2 | |

## Question 88

| | | |
|---|---|---|
| State $\frac{dx}{d\theta} = \sin\theta$ or $\frac{dy}{d\theta} = -\sin\theta + \frac{1}{2}\sin 2\theta$ | **B1** | |
| Use $\frac{dy}{dx} = \frac{dy}{d\theta} \div \frac{dx}{d\theta}$ | **M1** | |
| Obtain correct answer in any form | **A1** | e.g. $\dfrac{-\sin\theta + \frac{1}{2}\sin 2\theta}{\sin\theta}$ |
| Use double angle correctly to obtain $\frac{dy}{dx}$ in terms of $\theta$ | **M1** | $\sin 2\theta = 2\sin\theta\cos\theta$ |
| Obtain the given answer with no errors seen $-2\sin^2\left(\frac{1}{2}\theta\right)$ | **A1** | AG.<br>Requires correct cancellation of ALL $\sin\theta$ terms and $\cos\theta = 1 - 2\sin^2\left(\frac{1}{2}\theta\right)$ seen<br>**SC** For incorrect signs, consistent throughout max. B0, M1, A0, M1, A1 |
| | **5** | |

## Question 89

| | | | |
|---|---|---|---|
| (a) | Use chain rule at least once | **M1** | Needs $\frac{dy}{dt} = \frac{1}{\tan t}\frac{d}{dt}(\tan t)$<br>or $\frac{dx}{dt} = (-1)(\cos^{-2}t)\frac{d}{dt}(\cos t)$.<br>BOD if + and (– 1) ( – 1) not seen.<br>$\frac{dx}{dt} = \sec t\tan t$ (from List of Formulae MF19) M1 A1.<br>If $\frac{dx}{dt} = -\sec t\tan t$ M1 A0. |
| | Obtain $\frac{dx}{dt} = \sec t\tan t$ | **A1** | OE e.g. $\sin t(\cos t)^{-2}$ .<br>If e.g. $\frac{dx}{dt} = \sec x\tan x$ or $\sec\theta\tan\theta$ or $\sec t\tan x$, condone recovery on next line. |
| | Obtain $\frac{dy}{dt} = \frac{\sec^2 t}{\tan t}$ | **A1** | OE e.g. $\frac{1}{\sin t\cos t}$.<br>If e.g. $\frac{dy}{dt} = \frac{\sec^2 x}{\tan x}$ or $\frac{\sec^2\theta}{\tan\theta}$, condone recovery on next line.<br>Only penalise notation errors **once** in $\frac{dx}{dt}$ and $\frac{dy}{dt}$ if no recovery. |
| | Use $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$ | **M1** | Allow even if previous M0 scored, but must be using derivatives. |
| | Obtain given answer $\frac{\cos t}{\sin^2 t}$ | **A1** | AG After $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$ used, any notation error A0.<br>Must cancel $\cos t$ correctly. |
| | | **5** | |

| | | | |
|---|---|---|---|
| (b) | State or imply $t = \frac{1}{4}\pi$ when $y = 0$ | B1 | |
| | Form the equation of the tangent at $y = 0$ or find $c$ | M1 | $x = \sqrt{2}$, $\frac{dy}{dx} = \sqrt{2}$ and $y = 0$, *their* coordinates and gradient used in $y = mx + c$. |
| | Obtain answer $y = \sqrt{2}x - 2$ | A1 | OE e.g. $y = \sqrt{2}(x - \sqrt{2})$ ISW. Allow $y = 1.41x - 2[.00]$ or $1.41(x - 1.41)$. |
| | | 3 | |

# Question 90

| | | | |
|---|---|---|---|
| (a) | Use correct product rule or quotient rule, and attempt at chain rule | M1 | $ke^{-4x}\tan x + e^{-4x}\sec^2 x$ or $\frac{e^{4x}\sec^2 x - \tan x(ke^{4x})}{(e^{4x})^2}$<br>Need to see $d(\tan x)/dx = \sec^2 x$ (formula sheet) and attempt at $ke^{-4x}$, where $k \neq 1$. |
| | Obtain correct derivative in any form | A1 | $-4e^{-4x}\tan x + e^{-4x}\sec^2 x$ or $\frac{e^{4x}\sec^2 x - \tan x(4e^{4x})}{(e^{4x})^2}$ |
| | Use trigonometric formulae to express derivative in the form<br>$ke^{-4x}\sin x\cos x\sec^2 x + ae^{-4x}\sec^2 x$ or $ke^{-4x}\frac{\sin x\cos x}{\cos x\cos x} + ae^{-4x}\sec^2 x$ or<br>$\sec^2 x(ke^{-4x}\sin x\cos x + ae^{-4x})$<br>Allow $\frac{1}{\cos^2 x}$ instead of $\sec^2 x$ | M1 | Need to use $\frac{\tan x}{\sec^2 x} = \sin x\cos x$ or $\tan x = \frac{\sin x}{\cos x}.\frac{\cos x}{\cos x}$ OE.<br>M1 is independent of previous M1, but expression must be of appropriate form. |
| | Obtain correct answer with $a = 1$ and $b = -2$ | A1 | At least one line of trigonometric working is required from $-4e^{-4x}\tan x + e^{-4x}\sec^2 x$ to given answer $\sec^2 x(1 - 2\sin 2x)e^{-4x}$ with elements in any order.<br>If only error: $4\sin x\cos x = 4\sin 2x$ M1 A1 M1 A0. |
| | | 4 | |
| (b) | Equate derivative to zero and use correct method to solve for $x$ | M1 | $\sin 2x = \frac{1}{2}$, hence $x = \frac{1}{2}\sin^{-1}\frac{1}{2}$ or $x = \tan^{-1}(2 \pm \sqrt{3})$<br>Allow M1 for correct method for non-exact value. |
| | Obtain answer, e.g. $x = \frac{1}{12}\pi$ | A1 | [0.262 M1 A0] |
| | Obtain second answer, e.g. $\frac{5}{12}\pi$ and no other in the given interval | A1 FT | FT $\frac{\pi - their\,2x}{2}$ if exact values; $x$ must be $< \frac{\pi}{2}$.<br>Ignore answers outside the given interval. Treat answers in degrees as a misread. 15°, 75°.<br>**SC** No values found for $a$ and $b$ in **4(a)** but chooses values in **4(b)**: max **M1** for $x$. |
| | | 3 | |

## Question 91

| | | | |
|---|---|---|---|
| (a) | State or imply $6xy+3x^2\frac{dy}{dx}$ as derivative of $3x^2y$ | **B1** | |
| | State or imply $3y^2\frac{dy}{dx}$ as derivative of $y^3$ | **B1** | Allow B1 B1 for $(3x^2dx+)6xydx+3x^2dy-3y^2dy[=0]$ |
| | Equate attempted derivative of left-hand side to zero and solve to obtain an equation with $\frac{dy}{dx}$ as subject | **M1** | Allow if zero implied by subsequent working. Allow if recover from an extra $\frac{dy}{dx}=\dots$ at the beginning of the left-hand side. |
| | Obtain $\frac{dy}{dx}=\frac{x^2+2xy}{y^2-x^2}$ correctly | **A1** | AG Accept $y'$ for $\frac{dy}{dx}$ . |
| | | **4** | |
| (b) | Equate numerator to zero | ***M1** | Must be using the given derivative. |
| | Obtain $x = -2y$, or equivalent | **A1** | An equation with $x$ or $y$ as the subject SOI. |
| | Use $x^3+3x^2y-y^3=3$ to obtain an equation in $x$ or $y$ | **DM1** | $-8y^3+12y^3-y^3=3$ or $x^3-\frac{3}{2}x^3+\frac{1}{8}x^3=3$ or any equivalent form (do not need to evaluate powers). |
| | Obtain the point (– 2, 1) and no others from solving their cubic equation | **A1** | Allow if each component stated separately. ISW. |
| | State the point $(0, -\sqrt[3]{3})$, or equivalent from correct work | **B1** | Accept $(0, \sqrt[3]{-3})$, or (0, – 1.44) (-1.44225). Allow if each component stated separately. ISW. |
| | | **5** | |

## Question 92

| | | |
|---|---|---|
| Use the correct product rule and then the chain rule to differentiate either $\cos^3 x$ or $\sqrt{\sin x}$ | **M1** | e.g. two terms with one part of $\frac{dy}{dx}=p\cos^2 x\sin x\sqrt{\sin x}+q\frac{\cos^3 x\cos x}{\sqrt{\sin x}}$. |
| Obtain correct derivative in any form e.g. $\frac{dy}{dx}=-3\cos^2 x\sin x\sqrt{\sin x}+\frac{\cos^3 x\cos x}{2\sqrt{\sin x}}$ | **A1 A1** | A1 for each correct term substituted in the complete derivative. |
| Equate their derivative to zero and obtain a horizontal equation with positive integer powers of sin $x$ and/or cos $x$ from an equation including $\sqrt{\sin x}$ or $\frac{1}{\sqrt{\sin x}}$ using sensible algebra. | **M1** | e.g. $-3\cos^2 x\sin^2 x+\frac{1}{2}\cos^4 x=0$ |
| Use correct formula(s) to express *their* equation/derivative in terms of one trigonometric function | **M1** | Can be awarded before the previous M1. May involve more than one trigonometric term. |
| Obtain $7\cos^2 x=6$, $7\sin^2 x=1$, or $6\tan^2 x=1$, or equivalent, and obtain answer $x = 0.388$ | **A1** | CAO. The question asks for 3 sf. Ignore additional answers outside $\left(0,\frac{\pi}{2}\right)$. $22.2°$ is A0. |
| | **6** | |

## Question 93

| | | | |
|---|---|---|---|
| (a) | State or imply $3y^2 \frac{dy}{dx}$ as derivative of $y^3$ | **B1** | |
| | State or imply $2y + 2x \frac{dy}{dx}$ as derivative of $2xy$ | **B1** | |
| | Complete differentiation and equate attempted derivative to zero and solve for $\frac{dy}{dx}$ | **M1** | |
| | Obtain answer $-\frac{3x^2+2y}{3y^2+2x}$ | **A1** | |
| | | **4** | |
| (b) | Find gradient at either (0, – 2) or (– 2, 0) | **M1** | |
| | Obtain answers $\frac{1}{3}$ and 3 | **A1 A1** | |
| | Use $\tan(A \pm B)$ formula to find $\tan\alpha$ | **M1** | |
| | Obtain answer $\tan\alpha = \frac{4}{3}$ | **A1** | |
| | | **5** | |

## Question 94

| | | |
|---|---|---|
| State or imply $\frac{dx}{dt} = 2 - \sec^2 t$ or $\frac{dy}{dt} = 2\frac{\cos 2t}{\sin 2t}$ | **B1** | |
| Use $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$ | **M1** | |
| Obtain correct answer in any form | **A1** | |
| Use double angle formula to express derivative in terms of cos $x$ and sin $x$ | **M1** | |
| Obtain the given answer correctly | **A1** | AG |
| | **5** | |

## Question 95

| | | |
|---|---|---|
| Use correct product rule on given expression | ***M1** | |
| Obtain correct derivative in any form | **A1** | e.g. $\cos x \sin 2x + 2\sin x \cos 2x$ |
| Use correct double angle formulae to express derivative in terms of $\sin x$ and $\cos x$ | ***M1** | |
| Equate derivative to zero and obtain an equation in one trig variable | **DM1** | dependent on the 2 previous M Marks. |
| Obtain $3\sin^2 x = 2$, $3\cos^2 x = 1$ or $\tan^2 x = 2$ | **A1** | OE |
| Solve and obtain $x$ = 0.955 | **A1** | 3 sf only.<br>Final answer in degrees is A0.<br>Ignore any attempt to find the corresponding value of $y$. |

## Question 96

| | | |
|---|---|---|
| Use correct product or quotient rule | ***M1** | |
| Obtain correct derivative in any form | **A1** | e.g. $\frac{dy}{dx} = -e^{-\frac{x}{3}} - \frac{1}{3}(3-x)e^{-\frac{x}{3}}$ |
| Equate their derivative to zero and solve for $x$ | **DM1** | |
| Obtain $x = 6$ | **A1** | |
| Obtain $y = -3e^{-2}$ | **A1** | Or exact equivalent. |
| | **5** | |

## Question 97

| | | | |
|---|---|---|---|
| (a) | Obtain $\frac{dx}{dt} = e^{2t} + 2te^{2t}$ | **B1** | OE |
| | Use $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$ | **M1** | $\frac{dy}{dx} = \frac{2t+1}{e^{2t}(1+2t)}$. |
| | Obtain the given answer $\frac{dy}{dx} = e^{-2t}$ | **A1** | AG Need to see $e^{2t}(1+2t)$ in denominator. |
| | | **3** | |
| (b) | Obtain $x = -e^{-2}$ or $-\frac{1}{e^2}$ and $y = 3$ at $t = -1$ | **B1** | |
| | Obtain gradient of normal $= -e^{-2}$ or $-\frac{1}{e^2}$ | **B1** | |
| | $x = 0$ substituted into equation of normal or use of gradients to give $y = 3 - \frac{1}{e^4}$ with no errors | **B1** | Equation of normal $y - 3 = -e^{-2}\left(x - -e^{-2}\right)$.<br>AG<br>**SC** Decimals **B0 B1 B0** − 0.135 . |
| | | **3** | |

## Question 98

| | | |
|---|---|---|
| Use the product rule correctly | ***M1** | $x^3\, d/dx(\ln x) + d/dx(x^3) \ln x$. |
| Obtain the correct derivative in any form | **A1** | e.g. $\frac{x^3}{x} + 3x^2 \ln x$ . |
| Equate derivative to zero and solve exactly for $x$ | **DM1** | Reaching $x = e^a$. |
| Obtain answer $\left(\frac{1}{\sqrt[3]{e}}, -\frac{1}{3e}\right)$ or exact equivalent | **A1** | ISW |
| | **4** | |

## Question 99

| | | |
|---|---|---|
| Use the product rule correctly to obtain $p(x+5)(3-2x)^n + q(3-2x)^{\frac{1}{2}}$ | *M1 | Allow with incorrect chain rule. BOD over sign errors unless an incorrect rule is quoted. |
| Obtain correct derivative in any form | A1 | e.g. $-(x+5)(3-2x)^{-\frac{1}{2}} + (3-2x)^{\frac{1}{2}}$. |
| Equate derivative to zero and obtain a linear equation | DM1 | Allow with surd factor e.g. $(3-2x)^{-\frac{1}{2}}\left(-(x+5)+(3-2x)\right)=0$. |
| Obtain a correct linear equation. | A1 | e.g. $-(x+5)+3-2x=0$. |
| Obtain answer $\left(-\frac{2}{3}, \frac{13\sqrt{39}}{9}\right)$. | A1 | Or exact equivalent e.g. $\left(-\frac{2}{3}, \frac{13\sqrt{13}}{3\sqrt{3}}\right)$ or $\left(-\frac{2}{3}, \frac{\sqrt{2197}}{\sqrt{27}}\right)$. Accept with $x$, $y$ stated separately. ISW |
| **Alternative Method for Question 10(a)** | | |
| Obtain $y^2$ and differentiate | *M1 | Ignore *their* left hand side i.e. *their* $\frac{d}{dx}y^2$. |
| Obtain correct derivative in any form | A1 | e.g. $-6x^2-34x-20$. |
| Equate derivative to zero and solve for $x$ | DM1 | |
| Obtain $-\frac{2}{3}$ | A1 | Ignore –5 if seen. |
| Obtain answer $\left(-\frac{2}{3}, \frac{13\sqrt{39}}{9}\right)$ only | A1 | Or exact equivalent e.g. $\left(-\frac{2}{3}, \frac{13\sqrt{13}}{3\sqrt{3}}\right)$ or $\left(-\frac{2}{3}, \frac{\sqrt{2197}}{\sqrt{27}}\right)$. ISW |
| | 5 | |

## Question 100

| | | |
|---|---|---|
| State $\frac{dy}{d\theta}=1-2\sin\theta$ | B1 | Ignore left side throughout $dx/dt$, $dy/dt$, $dx$, $dy$ but must see $\frac{dy}{dx}$ for final A1. |
| Use correct quotient rule, or product rule if rewrite $x$ as $\cos\theta(2-\sin\theta)^{-1}$ | M1 | Incorrect formula seen M0 A0 otherwise BOD. |
| Obtain $\frac{dx}{d\theta}=\frac{-(2-\sin\theta)\sin\theta+\cos^2\theta}{(2-\sin\theta)^2}$ o.e. | A1 | $-\sin\theta(2-\sin\theta)^{-1}-\cos\theta(2-\sin\theta)^{-2}(-\cos\theta)$ or equivalent. |
| Use $\frac{dy}{dx}=\frac{dy}{d\theta}\div\frac{dx}{d\theta}$ | M1 | $\left(\frac{dy}{dx}=(1-2\sin\theta)\div\frac{1-2\sin\theta}{(2-\sin\theta)^2}\right)$. Allow M1 even if errors in both derivatives. |
| Obtain $\frac{dy}{dx}=(2-\sin\theta)^2$. | A1 | AG – must see working in above cell to gain final A1. Allow $\cos^2\theta+\sin^2\theta=1$ to be implied. $x$ instead of $\theta$ or missing $\theta$ more than twice on right side then A0 final mark. |
| | 5 | |

## Question 101

| | Answer | Mark | Guidance |
|---|---|---|---|
| (a) | State or imply $6y\frac{dy}{dx}$ as the derivative of $3y^2$ | B1 | Allow $y'$ for $\frac{dy}{dx}$ throughout. Accept $\frac{\partial f}{\partial x} = 6x + 4y$. |
| | State or imply $4x\frac{dy}{dx} + 4y$ as the derivative of $4xy$ | B1 | Accept $\frac{\partial f}{\partial y} = 4x + 6y$. |
| | Equate derivative of LHS to zero and solve for $\frac{dy}{dx}$ | M1 | Allow an extra $\frac{dy}{dx}$ in front of their differentiated equation.<br>Allow if '= 0' is implied but not seen.<br>Allow $\frac{dy}{dx} = -\frac{\partial f}{\partial x} \Big/ \frac{\partial f}{\partial y}$ |
| | Obtain $\frac{dy}{dx} = -\frac{3x+2y}{2x+3y}$ | A1 | AG – must come from correct working.<br>The position of the negative must be clear. |
| | | 4 | |

| | Answer | Mark | Guidance |
|---|---|---|---|
| (b) | Equate $\frac{dy}{dx}$ to –2 and solve for $x$ in terms of $y$ or for $y$ in terms of $x$ | *M1 | Must be using the given derivative. |
| | Obtain $x = -4y$ or $y = -\frac{x}{4}$ | A1 | Seen or implied by correct later work. |
| | Substitute *their* $x = -4y$ or *their* $y = -\frac{x}{4}$ in curve equation | DM1 | Allow unsimplified. |
| | Obtain $y = \pm\frac{1}{\sqrt{7}}$ or $x = \pm\frac{4}{\sqrt{7}}$ | A1 | Or exact equivalent.<br>Or $x = \frac{4}{\sqrt{7}}$ and $y = -\frac{1}{\sqrt{7}}$ or exact equivalent. |
| | Obtain both pairs of values | A1 | Or $x = -\frac{4}{\sqrt{7}}$ and $y = \frac{1}{\sqrt{7}}$ or exact equivalent.<br>A1 A0 for incorrect final pairing. |
| | | 5 | |

## Question 102

| | Answer | Mark | Guidance |
|---|---|---|---|
| (a) | State or imply $2xy + x^2\frac{dy}{dx}$ as derivative of $x^2y$ | B1 | Accept partial: $\frac{\partial}{\partial x} \to 2xy$. |
| | State or imply $2ay\frac{dy}{dx}$ as derivative of $ay^2$ | B1 | Accept partial: $\frac{\partial}{\partial y} \to x^2 - 2ay$. |
| | Equate attempted derivative to zero and solve for $\frac{dy}{dx}$ | M1 | |
| | Obtain answer $\frac{dy}{dx} = \frac{2xy}{2ay - x^2}$ from correct working | A1 | AG |
| | | 4 | |

| | Answer | Mark | Guidance |
|---|---|---|---|
| (b) | State or imply $2ay - x^2 = 0$ | *M1 | |
| | Substitute into equation of curve to obtain equation in $x$ and $a$ or in $y$ and $a$ | DM1 | e.g. $2ay^2 - ay^2 = 4a^3$ or $\frac{x^4}{2a} - \frac{x^4}{4a} = 4a^3$. |
| | Obtain one correct point | A1 | e.g. $(2a, 2a)$. |
| | Obtain second correct point and no others | A1 | e.g. $(-2a, 2a)$. |
| | | 4 | **SC**: Allow A1 A0 for $x = \pm 2a$ or for $y = 2a$. |