A-level

Topic : Differential Calculus

May 2013-May 2025

Answer

Question 1

(i)	Use correct quotient rule or equivalent	M1	
	Obtain $\frac{(1+e^{2x})2x-(1+x^2)2e^{2x}}{(1+e^{2x})^2}$ or equivalent	A1	
	Substitute $x=0$ and obtain $-\frac{1}{2}$ or equivalent	A1	[3]
(ii)	Differentiate y^3 and obtain $3y^2\frac{dy}{dx}$	B1	
	Differentiate $5xy$ and obtain $5y+5x\frac{dy}{dx}$	B1	
	Obtain $6x^2+5y+5x\frac{dy}{dx}+3y^2\frac{dy}{dx}=0$	B1	

Question 2

State $2ay\frac{dy}{dx}$ as derivative of ay^2	B1
State $y^2+2xy\frac{dy}{dx}$ as derivative of xy^2	B1
Equate derivative of LHS to zero and set $\frac{dy}{dx}$ equal to zero	M1
Obtain $3x^2+y^2-6ax=0$, or horizontal equivalent	A1
Eliminate y and obtain an equation in x	M1
Solve for x and obtain answer $x=\sqrt{3}a$	A1

Question 3

(i)	Use correct quotient or chain rule to differentiate sec x	M1	
	Obtain given derivative, sec x tan x, correctly	A1	
	Use chain rule to differentiate y	M1	
	Obtain the given answer	A1	[4]
(ii)	Using $\mathrm{d}x\sqrt{3}\sec^2\theta\ \mathrm{d}\theta$, or equivalent, express integral in terms of θ and $\mathrm{d}\theta$	M1	
	Obtain $\int \sec\theta\ \mathrm{d}\theta$	A1	
	Use limits $\frac{1}{6}\pi$ and $\frac{1}{3}\pi$ correctly in an integral form of the form $k\ln(\sec\theta + \tan\theta)$	M1	
	Obtain a correct exact final answer in the given form, e.g. $\ln\left(\frac{2+\sqrt{3}}{\sqrt{3}}\right)$	A1	[4]

Question 4

	Use correct product or quotient rule at least once	M1*	
	Obtain $\frac{\mathrm{d}x}{\mathrm{d}t} = \mathrm{e}^{-t}\sin t - \mathrm{e}^{-t}\cos t$ or $\frac{\mathrm{d}y}{\mathrm{d}t} = \mathrm{e}^{-t}\cos t - \mathrm{e}^{-t}\sin t$, or equivalent	A1	
	Use $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}t} \div \frac{\mathrm{d}x}{\mathrm{d}t}$	M1	
	Obtain $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\sin t - \cos t}{\sin t + \cos t}$, or equivalent	A1	
EITHER:	Express $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of tan t only	M1(dep*)	
	Show expression is identical to $\tan\left(t - \frac{1}{4}\pi\right)$	A1	
OR:	Express $\tan\left(t - \frac{1}{4}\pi\right)$ in terms of tan t	M1	
	Show expression is identical to $\frac{\mathrm{d}y}{\mathrm{d}x}$	A1	**[6]**

Question 5

Differentiate y^3 to obtain $3y^2\frac{\mathrm{d}y}{\mathrm{d}x}$	B1	
Use correct product rule at least once	*M1	
Obtain $6\mathrm{e}^{2x}y + 3\mathrm{e}^{2x}\frac{\mathrm{d}y}{\mathrm{d}x} + \mathrm{e}^{x}y^3 + 3\mathrm{e}^{x}y^2\frac{\mathrm{d}y}{\mathrm{d}x}$ as derivative of LHS	A1	
Equate derivative of LHS to zero, substitute $x = 0$ and $y = 2$ and find value of $\frac{\mathrm{d}y}{\mathrm{d}x}$	M1(d*M)	
Obtain $-\frac{4}{3}$ or equivalent as **final answer**	A1	[5]

Question 6

Obtain $\frac{2}{2t+3}$ for derivative of x	B1	
Use quotient of product rule, or equivalent, for derivative of y	M1	
Obtain $\frac{5}{(2t+3)^2}$ or unsimplified equivalent	A1	
Obtain $t = -1$	B1	
Use $\frac{dy}{dx} = \frac{dy}{dt} / \frac{dx}{dt}$ in algebraic or numerical form	M1	
Obtain gradient $\frac{5}{2}$	A1	[6]

Question 7

(i)	State $\frac{dx}{dt} = 1 - \sec^2 t$, or equivalent	B1	
	Use chain rule	M1	
	Obtain $\frac{dy}{dt} = -\frac{\sin t}{\cos t}$, or equivalent	A1	
	Use $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$	M1	
	Obtain the given answer correctly.	A1	**5**
(ii)	State or imply $t = \tan^{-1}(\frac{1}{2})$	B1	
	Obtain answer $x = -0.0364$	B1	**2**

Question 8

(i)	Use product rule	M1	
	Obtain derivative in any correct form	A1	
	Differentiate first derivative using the product rule	M1	
	Obtain second derivative in any correct form, e.g. $-\frac{1}{2}\sin\frac{1}{2}x - \frac{1}{4}x\cos\frac{1}{2}x - \frac{1}{2}\sin\frac{1}{2}x$	A1	
	Verify the given statement	A1	**5**
(ii)	Integrate and reach $kx\sin\frac{1}{2}x + l\int\sin\frac{1}{2}x\,dx$	M1*	
	Obtain $2x\sin\frac{1}{2}x - 2\int\sin\frac{1}{2}x\,dx$, or equivalent	A1	
	Obtain indefinite integral $2x\sin\frac{1}{2}x + 4\cos\frac{1}{2}x$	A1	
	Use correct limits $x = 0, x = \pi$ correctly	M1(dep*)	
	Obtain answer $2\pi - 4$, or exact equivalent	A1	**5**

Question 9

Obtain correct derivative of RHS in any form	B1	
Obtain correct derivative of LHS in any form	B1	
Set $\frac{dy}{dx}$ equal to zero and obtain a horizontal equation	M1	
Obtain a correct equation, e.g. $x^2 + y^2 = 1$, from correct work	A1	
By substitution in the curve equation, or otherwise, obtain an equation in x^2 or y^2	M1	
Obtain $x = \frac{1}{2}\sqrt{3}$	A1	
Obtain $y = \frac{1}{2}$	A1	**7**

Question 10

(i)	Substitute for x and dx throughout using $u = \sin x$ and $du = \cos x\, dx$, or equivalent	M1	
	Obtain integrand e^{2u}	A1	
	Obtain indefinite integral $\frac{1}{2}e^{2u}$	A1	
	Use limits $u = 0$, $u = 1$ correctly, or equivalent	M1	
	Obtain answer $\frac{1}{2}(e^2 - 1)$, or exact equivalent	A1	**5**
(ii)	Use chain rule or product rule	M1	
	Obtain correct terms of the derivative in any form, e.g. $2\cos x\, e^{2\sin x}\cos x - e^{2\sin x}\sin x$	A1 + A1	
	Equate derivative to zero and obtain a quadratic equation in $\sin x$	M1	
	Solve a 3-term quadratic and obtain a value of x	M1	
	Obtain answer 0.896	A1	**6**

Question 11

(i)	Use chain rule correctly at least once	M1	
	Obtain either $\frac{dx}{dt} = \frac{3\sin t}{\cos^4 t}$ or $\frac{dy}{dt} = 3\tan^2 t\sec^2 t$, or equivalent	A1	
	Use $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$	M1	
	Obtain the given answer	A1	**[4]**
(ii)	State a correct equation for the tangent in any form	B1	
	Use Pythagoras	M1	
	Obtain the given answer	A1	**[3]**

Question 12

Use correct product rule or correct chain rule to differentiate y	M1	
Use $\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$	M*1	
Obtain $\frac{-4\cos\theta\sin^2\theta + 2\cos^3\theta}{\sec^2\theta}$ or equivalent	A1	
Express $\frac{dy}{dx}$ in terms of $\cos\theta$	DM*1	
Confirm given answer $6\cos^5\theta - 4\cos^3\theta$ legitimately	A1	[5]

Question 13

Differentiate to obtain form $a\sin 2x + b\cos x$	M1	
Obtain correct $-6\sin 2x + 7\cos x$	A1	
Use identity $\sin 2x = 2\sin x\cos x$	B1	
Solve equation of form $c\sin x\cos x + d\cos x = 0$ to find at least one value of x	M1	
Obtain 0.623	A1	
Obtain 2.52	A1	
Obtain 1.57 or $\frac{1}{2}\pi$ from equation of form $c\sin x\cos x + d\cos x = 0$	A1	
Treat answers in degrees as MR – 1 situation		[7]

Question 14

(i)	Use product rule to find first derivative	M1	
	Obtain $2xe^{2-x} - x^2e^{2-x}$	A1	
	Confirm $x = 2$ at M	A1	[3]
(ii)	Attempt integration by parts and reach $\pm x^2e^{2-x} \pm \int 2xe^{2-x}\,dx$	*M1	
	Obtain $-x^2e^{2-x} + \int 2xe^{2-x}\,dx$	A1	
	Attempt integration by parts and reach $\pm x^2e^{2-x} \pm 2xe^{2-x} \pm 2e^{2-x}$	*M1	
	Obtain $-x^2e^{2-x} - 2xe^{2-x} - 2e^{2-x}$	A1	
	Use limits 0 and 2 having integrated twice	M1 dep *M	
	Obtain $2e^2 - 10$	A1	[6]

Question 15

Obtain $\frac{dx}{dt} = \frac{2}{t+2}$ and $\frac{dy}{dt} = 3t^2 + 2$	B1	
Use $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$	M1	
Obtain $\frac{dy}{dx} = \frac{1}{2}(3t^2+2)(t+2)$	A1	
Identify value of t at the origin as -1	B1	
Substitute to obtain $\frac{5}{2}$ as gradient at the origin	A1	[5]

Question 16

EITHER:	Use correct product rule	M1	
	Obtain correct derivative in any form, e.g. $-\sin x \cos 2x - 2\cos x \sin 2x$	A1	
	Use the correct double angle formulae to express derivative in $\cos x$ and $\sin x$, or $\cos 2x$ and $\sin x$	M1	
*OR*1:	Use correct double angle formula to express y in terms of $\cos x$ and attempt differentiation	M1	
	Use chain rule correctly	M1	
	Obtain correct derivative in any form, e.g. $-6\cos^2 x \sin x + \sin x$	A1	
*OR*2:	Use correct factor formula and attempt differentiation	M1	
	Obtain correct derivative in any form, e.g. $-\frac{3}{2}\sin 3x - \frac{1}{2}\sin x$	A1	
	Use correct trig formulae to express derivative in terms of $\cos x$ and $\sin x$, or $\sin x$	M1	
Equate derivative to zero and obtain an equation in one trig function		M1	
Obtain $6\cos^2 x = 1$, $6\sin^2 x = 5$, $\tan^2 x = 5$ or $3\cos 2x = -2$		A1	
Obtain answer $x = 1.15$ (or 65.9°) and no other in the given interval		A1	**[6]**

Question 17

Use correct quotient or product rule	M1	
Obtain correct derivative in any form	A1	
Equate derivative to zero and obtain a horizontal equation	M1	
Carry out complete method for solving an equation of the form $ae^{3x} = b$, or $ae^{5x} = be^{2x}$	M1	
Obtain $x = \ln 2$, or exact equivalent	A1	
Obtain $y = \frac{1}{3}$, or exact equivalent	A1	**6**

Question 18

(i)	State $\frac{dx}{dt} = -4a\cos^3 t\sin t$, or $\frac{dy}{dt} = 4a\sin^3 t\cos t$	B1	
	Use $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$	M1	
	Obtain correct expression for $\frac{dy}{dx}$ in a simplified form	A1	**3**
(ii)	Form the equation of the tangent	M1	
	Obtain a correct equation in any form	A1	
	Obtain the given answer	A1	**3**
iii)	State the x-coordinate of P or the y-coordinate of Q in any form	B1	
	Obtain the given result correctly	B1	**2**

Question 19

(i)	State or imply that the derivative of e^{-2x} is $-2e^{-2x}$	**B1**	
	Use product or quotient rule	**M1**	
	Obtain correct derivative in any form	**A1**	
	Use Pythagoras	**M1**	
	Justify the given form	**A1**	[5]
(ii)	Fully justify the given statement	**B1**	[1]
(iii)	State answer $x = \frac{1}{4}\pi$	**B1**	[1]

Question 20

(i)	Use the quotient rule	**M1**	
	Obtain correct derivative in any form	**A1**	
	Equate derivative to zero and solve for x	**M1**	
	Obtain answer $x=\sqrt[3]{2}$, or exact equivalent	**A1**	[4]
(ii)	State or imply indefinite integral is of the form $k\ln(1+x^3)$	**M1**	
	State indefinite integral $\frac{1}{3}\ln(1+x^3)$	**A1**	
	Substitute limits correctly in an integral of the form $k\ln(1+x^3)$	**M1**	
	State or imply that the area of R is equal to $\frac{1}{3}\ln(1+p^3)-\frac{1}{3}\ln 2$, or equivalent	**A1**	
	Use a correct method for finding p from an equation of the form $\ln(1+p^3)=a$ or $\ln((1+p^3)/2)=b$	**M1**	
	Obtain answer $p=3.40$	**A1**	[2]

Question 21

Use correct quotient rule or equivalent to find first derivative	**M1***	
Obtain $\dfrac{-(1+\tan x)\sec^2 x-\sec^2 x(2-\tan x)}{(1+\tan x)^2}$ or equivalent	**A1**	
Substitute $x=\frac{1}{4}\pi$ to find gradient	**dep M1***	
Obtain $-\frac{3}{2}$	**A1**	
Form equation of tangent at $x=\frac{1}{4}\pi$	**M1**	
Obtain $y=-\frac{3}{2}x+1.68$ or equivalent	**A1**	[6]

Question 22

(i) *EITHER*: State correct derivative of $\sin y$ with respect to x **B1**
Use product rule to differentiate the LHS **M1**
Obtain correct derivative of the LHS **A1**
Obtain a complete and correct derived equation in any form **A1**
Obtain a correct expression for $\frac{dy}{dx}$ in any form **A1**

OR: State correct derivative of $\sin y$ with respect to x **B1**
Rearrange the given equation as $\sin y = x/(\ln x + 2)$ and attempt to differentiate both sides **B1**
Use quotient or product rule to differentiate the RHS **M1**
Obtain correct derivative of the RHS **A1**
Obtain a correct expression for $\frac{dy}{dx}$ in any form **A1** [5]

(ii) Equate $\frac{dy}{dx}$ to zero and obtain a horizontal equation in $\ln x$ or $\sin y$ **M1**
Solve for $\ln x$ **M1**
Obtain final answer $x = 1/\mathrm{e}$, or exact equivalent **A1** [3]

Question 23

Use product rule **M1**
Obtain correct derivative in any form, e.g. $\cos x \cos 2x - 2\sin x \sin 2x$ **A1**
Equate derivative to zero and use double angle formulae **M1**
Remove factor of $\cos x$ and reduce equation to one in a single trig function **M1**
Obtain $6\sin^2 x = 1$, $6\cos^2 x = 5$ or $5\tan^2 x = 1$ **A1**
Solve and obtain $x = 0.421$ **A1**
[6]

Question 24

(i) State or imply $6xy + 3x^2\frac{dy}{dx}$ as derivative of $3x^2y$ **B1**
State $3y^2\frac{dy}{dx}$ as derivative of y^3 **B1**
Equate attempted derivative of the LHS to zero and solve for $\frac{dy}{dx}$ **M1**
Obtain the given answer **A1**
[4]

(ii) Equate numerator to zero **M1***
Obtain $x = 2y$, or equivalent **A1**
Obtain an equation in x or y **DM1***
Obtain the point $(-2, -1)$ **A1**
State the point $(0, 1.44)$ **B1**
[5]

Question 25

State or imply derivative of $(\ln x)^2$ is $\dfrac{2\ln x}{x}$	**B1**
Use correct quotient or product rule	**M1**
Obtain correct derivative in any form, e.g. $\dfrac{2\ln x}{x^2} - \dfrac{(\ln x)^2}{x^2}$	**A1**
Equate derivative (or its numerator) to zero and solve for $\ln x$	**M1**
Obtain the point (1, 0) with no errors seen	**A1**
Obtain the point $(e^2, 4e^{-2})$	**A1** [6]

Question 26

(i)	State $\dfrac{dx}{dt} = 1 - \sin t$	**B1**
	Use chain rule to find the derivative of y	**M1**
	Obtain $\dfrac{dy}{dt} = \dfrac{\cos t}{1 + \sin t}$, or equivalent	**A1**
	Use $\dfrac{dy}{dx} = \dfrac{dy}{dt} \div \dfrac{dx}{dt}$	**M1**
	Obtain the given answer correctly	**A1** [5]
(ii)	State or imply $t = \cos^{-1}(\frac{1}{3})$	**B1**
	Obtain answers $x = 1.56$ and $x = -0.898$	**B1 + B1** [3]

Question 27

EITHER:	State $2xy + x^2\dfrac{dy}{dx}$, or equivalent, as derivative of x^2y	**B1**
	State $6y^2 + 12xy\dfrac{dy}{dx}$, or equivalent, as derivative of $6xy^2$	**B1**
OR:	Differentiating LHS using correct product rule, state term $xy(1 - 6\dfrac{dy}{dx})$, or equivalent	**B1**
	State term $(y + x\dfrac{dy}{dx})(x - 6y)$, or equivalent	**B1**
	Equate attempted derivative of LHS to zero and set $\dfrac{dy}{dx}$ equal to zero	**M1***
	Obtain a horizontal equation, e.g. $6y^2 - 2xy = 0$ (from correct work only)	**A1**
	Explicitly reject $y = 0$ as a possibility $py^2 - qxy = 0$	**A1**
	Obtain an equation in x or y	**DM1**
	Obtain answer $(-3a, -a)$	**A1**

Question 28

(i)	Use the correct product rule	**M1**	
	Obtain correct derivative in any form, e.g. $(2-2x)e^{\frac{1}{2}x}+\frac{1}{2}(2x-x^2)e^{\frac{1}{2}x}$	**A1**	
	Equate derivative to zero and solve for x	**M1**	
	Obtain $x=\sqrt{5}-1$ only	**A1**	[4]
(ii)	Integrate by parts and reach $a(2x-x^2)e^{\frac{1}{2}x}+b\int(2-2x)e^{\frac{1}{2}x}\,dx$	**M1***	
	Obtain $2e^{\frac{1}{2}x}(2x-x^2)-2\int(2-2x)e^{\frac{1}{2}x}dx$, or equivalent	**A1**	
	Complete the integration correctly, obtaining $(12x-2x^2-24)e^{\frac{1}{2}x}$, or equivalent	**A1**	
	Use limits $x = 0$, $x = 2$ correctly having integrated by parts twice	**DM1**	
	Obtain answer 24 – 8e, or <u>exact</u> simplified equivalent	**A1**	[5]

Question 29

Use correct quotient or product rule	**M1**	
Obtain correct derivative in any form	**A1**	
Use Pythagoras to simplify the derivative to $\dfrac{1}{1+\cos x}$, or equivalent	**A1**	
Justify the given statement, $-1 < \cos x < 1$ statement, or equivalent	**A1**	[4]

Question 30

Use product rule	**M1**
Obtain correct derivative in any form	**A1**
Equate derivative to zero, use Pythagoras and obtain a quadratic equation in $\tan x$	**M1**
Obtain $\tan^2 x - a\tan x + 1 = 0$, or equivalent	**A1**
Use the condition for a quadratic to have only one root	**M1**
Obtain answer $a = 2$	**A1**
Obtain answer $x=\frac{1}{4}\pi$	**A1**
Total:	**7**

Question 31

(i)	State or imply derivative is $2\frac{\ln x}{x}$	**B1**
	State or imply gradient of the normal at $x = \mathrm{e}$ is $-\frac{1}{2}\mathrm{e}$, or equivalent	**B1**
	Carry out a complete method for finding the x-coordinate of Q	**M1**
	Obtain answer $x = \mathrm{e} + \frac{2}{\mathrm{e}}$, or exact equivalent	**A1**
	Total:	**4**
(ii)	Justify the given statement by integration or by differentiation	**B1**
	Total:	**1**
(iii)	Integrate by parts and reach $ax(\ln x)^2 + b\int x.\frac{\ln x}{x}\,\mathrm{d}x$	**M1***
	Complete the integration and obtain $x(\ln x)^2 - 2x\ln x + 2x$, or equivalent	**A1**
	Use limits $x = 1$ and $x = \mathrm{e}$ correctly, having integrated twice	**DM1**
	Obtain exact value $\mathrm{e} - 2$	**A1**
	Use x- coordinate of Q found in part **(i)** and obtain final answer $\mathrm{e} - 2 + \frac{1}{\mathrm{e}}$	**B1**√

Question 32

(i)	Use chain rule to differentiate x $\left(\frac{dx}{d\theta} = -\frac{\sin\theta}{\cos\theta}\right)$	**M1**
	State $\frac{dy}{d\theta} = 3 - \sec^2\theta$	**B1**
	Use $\frac{dy}{dx} = \frac{dy}{d\theta} \div \frac{dx}{d\theta}$	**M1**
	Obtain correct $\frac{dy}{dx}$ in any form e.g. $\frac{3-\sec^2\theta}{-\tan\theta}$	**A1**
	Obtain $\frac{dy}{dx} = \frac{\tan^2\theta - 2}{\tan\theta}$, or equivalent	**A1**
	Total:	**5**
(ii)	Equate gradient to -1 and obtain an equation in $\tan\theta$	**M1**
	Solve a 3 term quadratic $\left(\tan^2\theta + \tan\theta - 2 = 0\right)$ in $\tan\theta$	**M1**
	Obtain $\theta = \frac{\pi}{4}$ and $y = \frac{3\pi}{4} - 1$ only	**A1**
	Total:	**3**

Question 33

State or imply $du = -\sin x \, dx$	**B1**
Using correct double angle formula, express the integral in terms of u and du	**M1**
Obtain integrand $\pm(2u^2 - 1)^2$	**A1**
Change limits and obtain correct integral $\int_{\frac{1}{\sqrt{2}}}^{1} (2u^2 - 1)^2 \, du$ with no errors seen	**A1**
Substitute limits in an integral of the form $au^5 + bu^3 + cu$	**M1**
Obtain answer $\frac{1}{15}(7 - 4\sqrt{2})$, or exact simplified equivalent	**A1**
Total:	**6**

(ii)	Use product rule and chain rule at least once	**M1**
	Obtain correct derivative in any form	**A1**
	Equate derivative to zero and use trig formulae to obtain an equation in $\cos x$ and $\sin x$	**M1**
	Use correct methods to obtain an equation in $\cos x$ or $\sin x$ only	**M1**
	Obtain $10\cos^2 x = 9$ or $10\sin^2 x = 1$, or equivalent	**A1**
	Obtain answer 0.32	**A1**
	Total:	**6**

Question 34

(i)	State $\frac{dy}{dt} = 4 + \frac{2}{2t-1}$	**B1**
	Use $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$	**M1**
	Obtain answer $\frac{dy}{dx} = \frac{8t-2}{2t(2t-1)}$, or equivalent e.g. $\frac{2}{t} + \frac{2}{4t^2 - 2t}$	**A1**
	Total:	**3**
(ii)	Use correct method to find the gradient of the normal at $t = 1$	**M1**
	Use a correct method to form an equation for the normal at $t = 1$	**M1**
	Obtain final answer $x + 3y - 14 = 0$, or horizontal equivalent	**A1**
	Total:	**3**

Question 35

(i)	Use quotient or chain rule	**M1**
	Obtain given answer correctly	**A1**
	Total:	**2**
(ii)	*EITHER*: Multiply numerator and denominator of LHS by $1+\sin\theta$	**(M1**
	Use Pythagoras and express LHS in terms of sec θ and $\tan\theta$	**M1**
	Complete the proof	**A1)**
	OR1: Express RHS in terms of cos θ and sin θ	**(M1**
	Use Pythagoras and express RHS in terms of sin θ	**M1**
	Complete the proof	**A1)**
(iii)	Use the identity and obtain integral $2\tan\theta + 2\sec\theta - \theta$	**B2**
	Use correct limits correctly in an integral containing terms $a\tan\theta$ and $b\sec\theta$	**M1**
	Obtain answer $2\sqrt{2} - \frac{1}{4}\pi$	**A1**
	Total:	**4**

Question 36

(i)	Use the chain rule	**M1**
	Obtain correct derivative in any form	**A1**
	Use correct trigonometry to express derivative in terms of $\tan x$	**M1**
	Obtain $\frac{dy}{dx} = -\frac{4\tan x}{4+\tan^2 x}$, or equivalent	**A1**
	Total:	**4**
(ii)	Equate derivative to –1 and solve a 3–term quadratic for tan x	**M1**
	Obtain answer x=1.11 and no other in the given interval	**A1**
	Total:	**2**

Question 37

Use correct quotient rule or product rule	**M1**
Obtain correct derivative in any form	**A1**
Equate derivative to zero and solve for x	**M1**
Obtain $x = 2$	**A1**
Total:	**4**

Question 38

(i)	State or imply $y^3 + 3xy^2\frac{dy}{dx}$ as derivative of xy^3	**B1**
	State or imply $4y^3\frac{dy}{dx}$ as derivative of y^4	**B1**
	Equate derivative of the LHS to zero and solve for $\frac{dy}{dx}$	**M1**
	Obtain the given answer	**A1**
		4
(ii)	Equate numerator to zero	***M1**
	Obtain $y = -2x$, or equivalent	**A1**
	Obtain an equation in x or y	**DM1**
	Obtain final answer $x = -1, y = 2$ and $x = 1, y = -2$	**A1**
		4

Question 39

(i)	Use correct product or quotient rule	**M1**
	Obtain correct derivative in any form	**A1**
	Equate derivative to zero and obtain a 3 term quadratic equation in x	**M1**
	Obtain answers $x = 2 \pm \sqrt{3}$	**A1**
		4
(ii)	Integrate by parts and reach $k(1+x^2)e^{-\frac{1}{2}x} + l\int xe^{-\frac{1}{2}x}\,dx$	***M1**
	Obtain $-2(1+x^2)e^{-\frac{1}{2}x} + 4\int xe^{-\frac{1}{2}x}\,dx$, or equivalent	**A1**
	Complete the integration and obtain $(-18-8x-2x^2)e^{-\frac{1}{2}x}$, or equivalent	**A1**
	Use limits $x = 0$ and $x = 2$ correctly, having fully integrated twice by parts	**DM1**
	Obtain the given answer	**A1**
		5

Question 40

(i)	Use correct product or quotient rule or rewrite as $2\sec x - \tan x$ and differentiate	**M1**
	Obtain correct derivative in any form	**A1**
	Equate the derivative to zero and solve for x	**M1**
	Obtain $x = \frac{1}{6}\pi$	**A1**
	Obtain $y = \sqrt{3}$	**A1**
		5
(ii)	Carry out an appropriate method for determining the nature of a stationary point	**M1**
	Show the point is a minimum point with no errors seen	**A1**
		2

Question 41

(i)	State or imply $3x^2y + x^3\frac{dy}{dx}$ as derivative of x^3y	B1
	State or imply $9xy^2\frac{dy}{dx} + 3y^3$ as derivative of $3xy^3$	B1
	Equate derivative of the LHS to zero and solve for $\frac{dy}{dx}$	M1
	Obtain the given answer AG	A1
		4
(ii)	Equate numerator to zero and use $x = -y$ to obtain an equation in x or in y	M1
	Obtain answer $x = a$ and $y = -a$	A1
	Obtain answer $x = -a$ and $y = a$	A1
	Consider and reject $y = 0$ and $x = y$ as possibilities	B1
		4

Question 42

(i)	State or imply $y^3 + 3xy^2\frac{dy}{dx}$ as derivative of xy^3	B1
	State or imply $4y^3\frac{dy}{dx}$ as derivative of y^4	B1
	Equate derivative of the LHS to zero and solve for $\frac{dy}{dx}$	M1
	Obtain the given answer	A1
		4
(ii)	Equate numerator to zero	*M1
	Obtain $y = -2x$, or equivalent	A1
	Obtain an equation in x or y	DM1
	Obtain final answer $x = -1$, $y = 2$ and $x = 1$, $y = -2$	A1
		4

Question 43

(i)	Use correct product or quotient rule	**M1**
	Obtain correct derivative in any form	**A1**
	Equate derivative to zero and obtain a 3 term quadratic equation in x	**M1**
	Obtain answers $x = 2 \pm \sqrt{3}$	**A1**
		4
(ii)	Integrate by parts and reach $k(1+x^2)e^{-\frac{1}{2}x} + l\int xe^{-\frac{1}{2}x}\,dx$	***M1**
	Obtain $-2(1+x^2)e^{-\frac{1}{2}x} + 4\int xe^{-\frac{1}{2}x}\,dx$, or equivalent	**A1**
	Complete the integration and obtain $(-18-8x-2x^2)e^{-\frac{1}{2}x}$, or equivalent	**A1**
	Use limits $x = 0$ and $x = 2$ correctly, having fully integrated twice by parts	**DM1**
	Obtain the given answer	**A1**
		5

Question 44

(i)	State correct derivative of x or y with respect to t	**B1**
	Use $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$	**M1**
	Obtain $\frac{dy}{dx} = \frac{4\sin 2t}{2 + 2\cos 2t}$, or equivalent	**A1**
	Use double angle formulae throughout	**M1**
	Obtain the given answer correctly **AG**	**A1**
		5
(ii)	State or imply $t = \tan^{-1}\left(-\frac{1}{4}\right)$	**B1**
	Obtain answer $x = -0.961$	**B1**
		2

Question 45

Use quotient or product rule	**M1**
Obtain correct derivative in any form	**A1**
Equate derivative to zero and obtain a quadratic in $\tan\frac{1}{2}x$ or an equation of the form $a \sin x = b$	**M1 ***
Solve for x	**M1(dep*)**
Obtain answer 0.340	**A1**
Obtain second answer 2.802 and no other in the given interval	**A1**
	6

Question 46

(i)	State or imply $3y^2\frac{dy}{dx}$ as derivative of y^3	**B1**
	State or imply $6xy+3x^2\frac{dy}{dx}$ as derivative of $3x^2y$ *OR* State or imply $2x(x+3y)+x^2\left(1+3\frac{dy}{dx}\right)$ as derivative of $x^2(x+3y)$	**B1**
	Equate derivative of the LHS to zero and solve for $\frac{dy}{dx}$	**M1**
	Obtain the given answer	**A1**
		4

(ii)	Equate derivative to – 1 and solve for y	M1*
	Use their $y = -2x$ or equivalent to obtain an equation in x or y	M1(dep*)
	Obtain answer (1, – 2)	A1
	Obtain answer $(\sqrt[3]{3}, 0)$	B1
		4

Question 47

(i)	Use correct product or quotient rule	M1	$\frac{dy}{dx} = -\frac{1}{3}(x+1)e^{-\frac{1}{3}x} + e^{-\frac{1}{3}x}$ or $\frac{dy}{dx} = \frac{e^{\frac{1}{3}x} - (x+1)\frac{1}{3}e^{\frac{1}{3}x}}{e^{\frac{2}{3}x}}$
	Obtain complete correct derivative in any form	A1	
	Equate derivative to zero and solve for x	M1	
	Obtain answer $x = 2$ with no errors seen	A1	
		4	
(ii)	Integrate by parts and reach $a(x+1)e^{-\frac{1}{3}x} + b\int e^{-\frac{1}{3}x}\,dx$	M1*	
	Obtain $-3(x+1)e^{-\frac{1}{3}x} + 3\int e^{-\frac{1}{3}x}\,dx$, or equivalent	A1	$-3xe^{-\frac{1}{3}x} + \int 3e^{-\frac{1}{3}x}\,dx - 3e^{-\frac{1}{3}x}$
	Complete integration and obtain $-3(x+1)e^{-\frac{1}{3}x} - 9e^{-\frac{1}{3}x}$, or equivalent	A1	
	Use correct limits $x = -1$ and $x = 0$ **in the correct order**, having integrated twice	M1(dep*)	
	Obtain answer $9e^{\frac{1}{3}} - 12$, or equivalent	A1	
		5	

Question 48

(i)	State or imply $3y^2 \dfrac{dy}{dx}$ as derivative of y^3	B1
	State or imply $3y^2 + 6xy \dfrac{dy}{dx}$ as derivative of $3xy^2$	B1
	Equate derivative of LHS to zero and solve for $\dfrac{dy}{dx}$	M1
	Obtain the given answer	A1
	Total:	**4**
(ii)	Equate denominator to zero and solve for y	M1*
	Obtain $y = 0$ and $x = a$	A1
	Obtain $y = \alpha x$ and substitute in curve equation to find x or to find y	M1(dep*)
	Obtain $x = -a$	A1
	Obtain $y = 2a$	A1
	Total:	**5**

Question 49

(i)	Obtain $\dfrac{dx}{d\theta} = 2\cos\theta + 2\cos 2\theta$ or $\dfrac{dy}{d\theta} = -2\sin\theta - 2\sin 2\theta$	B1
	Use $dy/dx = dy/d\theta \div dx/d\theta$	M1
	Obtain correct $\dfrac{dy}{dx}$ in any form, e.g. $-\dfrac{2\sin\theta + 2\sin 2\theta}{2\cos\theta + 2\cos 2\theta}$	A1
		3
(ii)	Equate denominator to zero and use any correct double angle formula	M1*
	Obtain correct 3-term quadratic in $\cos\theta$ in any form	A1
	Solve for θ	depM1*
	Obtain $x = 3\sqrt{3}/2$ and $y = \dfrac{1}{2}$, or exact equivalents	A1
		4

Question 50

'(i)	Use product rule	M1*
	Obtain correct derivative in any form	A1
	Equate derivative to zero and obtain an equation in a single trig function	depM1*
	Obtain a correct equation, e.g. $3\tan^2 x = 2$	A1
	Obtain answer $x = 0.685$	A1
		5
(ii)	Use the given substitution and reach $a\int(u^2 - u^4)\,du$	M1
	Obtain correct integral with $a = 5$ and limits 0 and 1	A1
	Use correct limits in an integral of the form $a\left(\frac{1}{3}u^3 - \frac{1}{5}u^5\right)$	M1
	Obtain answer $\frac{2}{3}$	A1
		4

Question 51

(i)	Use correct quotient or product rule	M1
	Obtain correct derivative in any form	A1
	Equate numerator to zero	M1
	Use $\cos^2 x + \sin^2 x = 1$ and solve for $\sin x$	M1
	Obtain coordinates $x = -\pi/6$ and $y = \sqrt{3}$ ISW	A1 + A1
		6

(ii)	State indefinite integral of the form $k \ln(2 + \sin x)$	M1*
	Substitute limits correctly, equate result to 1 and obtain $3 \ln(2 + \sin a) - 3 \ln 2 = 1$	A1
	Use correct method to solve for a	M1(dep*)
	Obtain answer $a = 0.913$ or better	A1
		4

Question 52

(i)	Obtain $\frac{dx}{d\theta} = 2\cos\theta + 2\cos 2\theta$ or $\frac{dy}{d\theta} = -2\sin\theta - 2\sin 2\theta$	B1
	Use $dy/dx = dy/d\theta \div dx/d\theta$	M1
	Obtain correct $\frac{dy}{dx}$ in any form, e.g. $-\frac{2\sin\theta + 2\sin 2\theta}{2\cos\theta + 2\cos 2\theta}$	A1
		3
(ii)	Equate denominator to zero and use any correct double angle formula	M1*
	Obtain correct 3-term quadratic in $\cos\theta$ in any form	A1
	Solve for θ	depM1*
	Obtain $x = 3\sqrt{3}/2$ and $y = \frac{1}{2}$, or exact equivalents	A1
		4

Question 53

'(i)	Use product rule	**M1***
	Obtain correct derivative in any form	**A1**
	Equate derivative to zero and obtain an equation in a single trig function	**depM1***
	Obtain a correct equation, e.g. $3\tan^2 x = 2$	**A1**
	Obtain answer $x = 0.685$	**A1**
		5
'(ii)	Use the given substitution and reach $a\int\left(u^2 - u^4\right)\mathrm{d}u$	**M1**
	Obtain correct integral with $a = 5$ and limits 0 and 1	**A1**
	Use correct limits in an integral of the form $a\left(\frac{1}{3}u^3 - \frac{1}{5}u^5\right)$	**M1**
	Obtain answer $\frac{2}{3}$	**A1**
		4

Question 54

State $\cos y\frac{\mathrm{d}y}{\mathrm{d}x}$ as derivative of $\sin y$	**B1**
State correct derivative in terms of x and y, e.g. $\sec^2 x / \cos y$	**B1**
State correct derivative in terms of x, e.g. $\frac{\sec^2 x}{\sqrt{1-\tan^2 x}}$	**B1**
Use double angle formula	**M1**
Obtain the given answer correctly	**A1**
	5

Question 55

(i)	State or imply $du = -\sin x\, dx$	B1
	Using Pythagoras express the integral in terms of u	M1
	Obtain integrand $\pm\sqrt{u}\left(1-u^2\right)$	A1
	Integrate and obtain $-\frac{2}{3}u^{\frac{3}{2}}+\frac{2}{7}u^{\frac{7}{2}}$, or equivalent	A1
	Change limits correctly and substitute correctly in an integral of the form $au^{\frac{3}{2}}+bu^{\frac{7}{2}}$	M1
	Obtain answer $\frac{8}{21}$	A1
		6
(ii)	Use product rule and chain rule at least once	M1
	Obtain correct derivative in any form	A1 + A1
	Equate derivative to zero and obtain a horizontal equation in integral powers of sin x and cos x	M1
	Use correct methods to obtain an equation in one trig function	M1
	Obtain $\tan^2 x = 6$, $7\cos^2 x = 1$ or $7\sin^2 x = 6$, or equivalent, and obtain answer 1.183	A1
		6

Question 56

State or imply $3y^2 + 6xy\frac{dy}{dx}$ as derivative of $3xy^2$	B1
State or imply $3y^2\frac{dy}{dx}$ as derivative of y^3	B1
Equate derivative of LHS to zero, substitute (1, 3) and find the gradient	M1
Obtain final answer $\frac{10}{3}$ or equivalent	A1
	4

Question 57

Use chain rule	M1
Obtain correct answer in any form	A1

Question 58

Use correct quotient rule	M1
Obtain correct derivative in any form	A1
Equate derivative to $\frac{1}{4}$ and obtain a quadratic in $\ln x$ or $(1+\ln x)$	M1
Reduce to $(\ln x)^2 - 2\ln x + 1 = 0$	A1
Solve a 3-term quadratic in $\ln x$ for x	M1
Obtain answer $x = \mathrm{e}$	A1
Obtain answer $y = \frac{1}{2}\mathrm{e}$	A1
	7

Question 59

(i)	State correct expansion of $\sin(3x+x)$ or $\sin(3x-x)$	B1	B0 If their formula retains $\pm$ in the middle
	Substitute expansions in $\frac{1}{2}(\sin 4x+\sin 2x)$	M1	
	Obtain $\sin 3x\cos x=\frac{1}{2}(\sin 4x+\sin 2x)$ correctly	A1	Must see the $\sin 4x$ and $\sin 2x$ or reference to LHS and RHS for A1 **AG**
		3	
(ii)	Integrate and obtain $-\frac{1}{8}\cos 4x-\frac{1}{4}\cos 2x$	B1 B1	
	Substitute limits $x=0$ and $x=\frac{1}{3}\pi$ correctly	M1	In their expression
	Obtain answer $\frac{9}{16}$	A1	From correct working seen.
		4	
(iii)	State correct derivative $2\cos 4x+\cos 2x$	B1	
	Using correct double angle formula, express derivative in terms of $\cos 2x$ and equate the result to zero	M1	
	Obtain $4\cos^2 2x+\cos 2x-2=0$	A1	
	Solve for x or $2x$ (could be labelled x) $\left(\cos 2x=\frac{-1\pm\sqrt{33}}{8}\right)$	M1	Must see working if solving an incorrect quadratic The roots of the correct quadratic are -0.843 and 0.593 Need to get as far as $x=\dots$ The wrong value of x is 0.468 and can imply M1 if correct quadratic seen Could be working from a quartic in $\cos x$: $16\cos^4 x-14\cos^2 x+1=0$
	Obtain answer $x=1.29$ only	A1	
		5	

Question 60

(i)	Use the quotient or product rule	**M1**
	Obtain correct derivative in any form	**A1**
	Reduce to $-\dfrac{2e^{-x}}{\left(1-e^{-x}\right)^2}$, or equivalent, and explain why this is always negative	**A1**
		3
(ii)	Equate derivative to -1 and obtain the given equation	**B1**
	State or imply $u^2 - 4u + 1 = 0$, or equivalent in e^a	**B1**
	Solve for a	**M1**
	Obtain answer $a = \ln\left(2+\sqrt{3}\right)$ and no other	**A1**
		4

Question 61

(i)	Use product rule	**M1**
	Obtain correct derivative in any form	**A1**
		2
(ii)	Equate derivative to zero and use correct $\cos(A + B)$ formula	**M1**
	Obtain the given equation	**A1**
		2
(iii)	Use correct method to solve for x	**M1**
	Obtain answer, e.g. $x = \dfrac{1}{12}\pi$	**A1**
	Obtain second answer, e.g. $\dfrac{7}{12}\pi$, and no other	**A1**
		3

Question 62

(i)	Use correct quotient rule	**M1**
	Obtain $\frac{dy}{dx} = -\text{cosec}^2 x$ correctly	**A1**
		2
(ii)	Integrate by parts and reach $ax\cot x + b\int \cot x \, dx$	***M1**
	Obtain $-x\cot x + \int \cot x \, dx$	**A1**
	State $\pm \ln \sin x$ as integral of $\cot x$	**M1**
	Obtain complete integral $-x\cot x + \ln \sin x$	**A1**
	Use correct limits correctly	**DM1**
	Obtain $\frac{1}{4}(\pi + \ln 4)$ following full and exact working	**A1**
		6

Question 63

(i)	Use $\cos(A + B)$ formula to express $\cos 3x$ in terms of trig functions of $2x$ and x	**M1**
	Use double angle formulae and Pythagoras to obtain an expression in terms of $\cos x$ only	**M1**
	Obtain a correct expression in terms of $\cos x$ in any form	**A1**
	Obtain $\cos 3x \equiv 4\cos^3 x - 3\cos x$	**A1**
		4
(ii)	Use identity and solve cubic $4\cos^3 x = -1$ for x	**M1**
	Obtain answer 2.25 and no other in the interval	**A1**
		2

Question 64

State $4xy + 2x^2 \frac{dy}{dx}$, or equivalent, as derivative of $2x^2y$	B1
State $y^2 + 2xy \frac{dy}{dx}$, or equivalent, as derivative of xy^2	B1
Equate attempted derivative of LHS to zero and set $\frac{dy}{dx}$ equal to zero (or set numerator equal to zero)	*M1
Reject $y = 0$	B1
Obtain $y = 4x$	A1
Obtain an equation in y (or in x) and solve for y (or for x) in terms of a	DM1
Obtain $y = -2a$	A1
	7

Question 65

(i)	State or imply ordinates 1, 1.2116…, 2.7597…	**B1**
	Use correct formula, or equivalent, with $h = 0.6$	**M1**
	Obtain answer 1.85	**A1**
		3
(ii)	Explain why the rule gives an overestimate	**B1**
		1
(iii)	Differentiate using quotient or chain rule	**M1**
	Obtain correct derivative in terms of $\sin x$ and $\cos x$	**A1**
	Equate derivative to 2, use Pythagoras and obtain an equation in $\sin x$	**M1**
	Obtain $2\sin^2 x + \sin x - 2 = 0$	**A1**
	Solve a 3-term quadratic for x	**M1**
	Obtain answer $x = 0.896$ only	**A1**
		6

Question 66

)(i)	Use product rule and chain rule at least once	**M1**
	Obtain correct derivative in any form	**A1**
	Equate derivative to zero, use Pythagoras and obtain an equation in cos x	**M1**
	Obtain $\cos^2 x + 3\cos x - 1 = 0$, or 3-term equivalent	**A1**
	Obtain answer $x = 1.26$	**A1**
		5
(ii)	Using du = ± sin x dx express integrand in terms of u and du	**M1**
	Obtain integrand $e^u(u^2 - 1)$	**A1**
	Commence integration by parts and reach $ae^u(u^2 - 1) + b\int ue^u \, du$	***M1**
	Obtain $e^u(u^2 - 1) - 2\int ue^u \, du$	**A1**
	Complete integration, obtaining $e^u(u^2 - 2u + 1)$	**A1**
	Substitute limits $u = 1$ and $u = -1$ (or $x = 0$ and $x = \pi$), having integrated completely	**DM1**
	Obtain answer $\frac{4}{e}$, or exact equivalent	**A1**
		7

Question 67

'(a)	State or imply $3y^2 + 6xy\dfrac{dy}{dx}$ as derivative of $3xy^2$	B1
	State or imply $3y^2\dfrac{dy}{dx}$ as derivative of y^3	B1
	Equate attempted derivative of LHS to zero and solve for $\dfrac{dy}{dx}$	M1
	Obtain the given answer correctly	A1
		4
'(b)	Equate denominator to zero	*M1
	Obtain $y = 2x$, or equivalent	A1
	Obtain an equation in x or y	DM1
	Obtain the point (1, 2)	A1
	State the point $\left(\sqrt[3]{5}, 0\right)$	B1
		5

Question 68

(a)	Use product rule	M1
	Obtain derivative in any correct form e.g. $2e^{2x}(\sin x + 3\cos x) + e^{2x}(\cos x - 3\sin x)$	A1
	Equate derivative to zero and obtain an equation in one trigonometric ratio	M1
	Obtain $x = 1.43$ only	A1
		4
(b)	Use a correct method to determine the nature of the stationary point e.g. $x = 1.42,\ y' = 0.06e^{2.84} > 0$ $x = 1.44,\ y' = -0.07e^{2.88} < 0$	M1
	Show that it is a maximum point	A1
		2

Question 69

(a)	Use quotient or product rule	M1
	Obtain derivative in any correct form e.g. $\dfrac{-\sin x(1+\sin x)-\cos x(\cos x)}{(1+\sin x)^2}$	A1
	Use Pythagoras to simplify the derivative	M1
	Justify the given statement	A1
		4
(b)	State integral of the form $a\ln(1+\sin x)$	*M1
	State correct integral $\ln(1+\sin x)$	A1
	Use limits correctly	DM1
	Obtain answer $\ln\dfrac{4}{3}$	A1
		4

Question 70

Use correct product rule	M1
Obtain correct derivative in any form, e.g. $-\sin x \sin 2x + 2\cos x \cos 2x$	A1
Use double angle formula to express derivative in terms of $\sin x$ and $\cos x$	M1
Equate derivative to zero and obtain an equation in one trig function	M1
Obtain $3\sin 2x = 1$, or $3\cos 2x = 2$ or $2\tan 2x = 1$	A1
Solve and obtain $x = 0.615$	A1
	6

Question 71

(a)	Use quotient or product rule	M1
	Obtain correct derivative in any form e.g. $\dfrac{(1+3x^4)-x\times 12x^3}{(1+3x^4)^2}$	A1
	Equate derivative to zero and solve for x	M1
	Obtain answer 0.577	A1
		4

(b)	State or imply $du = 2\sqrt{3x}\,dx$, or equivalent	B1
	Substitute for x and dx	M1
	Obtain integrand $\frac{1}{2\sqrt{3}(1+u^2)}$, or equivalent	A1
	State integral of the form $a\tan^{-1}u$ and use limits $u = 0$ and $u = \sqrt{3}$ (or $x = 0$ and $x = 1$) correctly	M1
	Obtain answer $\frac{\sqrt{3}}{18}\pi$, or exact equivalent	A1
		5

Question 72

(a)	Use the product rule	M1
	State or imply derivative of $\tan^{-1}(\frac{1}{2}x)$ is of the form $k/(4+x^2)$, where $k = 2$ or 4, or equivalent	M1
	Obtain correct derivative in any form, e.g. $\tan^{-1}\left(\frac{1}{2}x\right)+\frac{2x}{x^2+4}$, or equivalent	A1
		3
(b)	State or imply y-coordinate is $\frac{1}{2}\pi$	B1
	Carry out a complete method for finding p, e.g. by obtaining the equation of the tangent and setting $x = 0$, or by equating the gradient at $x = 2$ to $\frac{\frac{1}{2}\pi - p}{2}$	M1
	Obtain answer $p = -1$	A1
		3

Question 73

State or imply $\frac{dx}{d\theta} = 2\sin 2\theta$ or $\frac{dy}{d\theta} = 2 + 2\cos 2\theta$	B1
Use $\frac{dy}{dx} = \frac{dy}{d\theta} \div \frac{dx}{d\theta}$	M1
Obtain correct answer $\frac{dy}{dx} = \frac{2+2\cos 2\theta}{2\sin 2\theta}$	A1
Use correct double angle formulae	M1
Obtain the given answer correctly $\frac{dy}{dx} = \cot\theta$	A1

Question 74

(a)	Use correct product or quotient rule	*M1
	Obtain correct derivative in any form	A1
	Equate derivative to zero and solve for x	DM1
	Obtain $x = 4$	A1
	Obtain $y = -2e^{-2}$, or exact equivalent	A1
		5
(b)	Commence integration and reach $a(2-x)e^{-\frac{1}{2}x} + b\int e^{-\frac{1}{2}x}\,dx$	*M1
	Obtain $-2(2-x)e^{-\frac{1}{2}x} - 2\int e^{-\frac{1}{2}x}\,dx$	A1
	Complete integration and obtain $2xe^{-\frac{1}{2}x}$	A1
	Use correct limits, $x = 0$ and $x = 2$, correctly, having integrated twice	DM1
	Obtain answer $4e^{-1}$, or exact equivalent	A1

Question 75

5(a)	State $\frac{dx}{d\theta} = \sec^2\theta$ or $\frac{dy}{d\theta} = -2\sin\theta\cos\theta$	B1
	Use $\frac{dy}{dx} = \frac{dy}{d\theta} \div \frac{dx}{d\theta}$	M1
	Obtain $\frac{dy}{dx} = -2\sin\theta\cos^3\theta$ from correct working	A1
	Alternative method for question 5(a)	
	Convert to Cartesian form and differentiate	M1
	$\frac{dy}{dx} = \frac{-2x}{\left(1+x^2\right)^2}$	A1
	Obtain $\frac{dy}{dx} = -2\sin\theta\cos^3\theta$ from correct working	A1
		3

(b)	Use correct product rule to obtain $\frac{\mathrm{d}}{\mathrm{d}\theta}\left(\pm 2\cos^3\theta\sin\theta\right)$	**M1**
	Obtain correct derivative in any form	**A1**
	Equate derivative to zero and obtain an equation in one trig ratio	**A1**
	Obtain answer $x = -\frac{1}{\sqrt{3}}$	**A1**
	Alternative method for question 5(b)	
	Use correct quotient rule to obtain $\frac{\mathrm{d}^2 y}{\mathrm{d}x^2}$	**M1**
	Obtain correct derivative in any form	**A1**
	Equate derivative to zero and obtain an equation in x^2	**A1**
	Obtain answer $x = -\frac{1}{\sqrt{3}}$	**A1**
		4

Question 76

(a)	State or imply du = cos x dx	**B1**
	Using double angle formula for sin2x and Pythagoras, express integral in terms of u and du.	**M1**
	Obtain integral $\int 2\left(u-u^3\right)\mathrm{d}u$	**A1**
	Use limits $u = 0$ and $u = 1$ in an integral of the form $au^2 + bu^4$, where $ab \neq 0$	**M1**
	Obtain answer $\frac{1}{2}$	**A1**
		5
(b)	Use product rule	**M1**
	Obtain correct derivative in any form	**A1**
	Equate derivative to zero and use a double angle formula	***M1**
	Obtain an equation in one trig variable	**DM1**
	Obtain $4\sin^2 x = 1$, $4\cos^2 x = 3$ or $3\tan^2 x = 1$	**A1**
	Obtain answer $x = \frac{1}{6}\pi$	**A1**
		6

Question 77

(a)	Calculate the values of a relevant expression or pair of expressions at $x = 1$ and $x = 1.5$	**M1**
	Complete the argument correctly with correct calculated values	**A1**
		2
(b)	Use the iterative formula $x_{n+1} = \dfrac{e^{2x_n}+1}{e^{2x_n}-1}$, or equivalent, correctly at least once	**M1**
	Obtain final answer 1.20	**A1**
	Show sufficient iterations to 4 dp to justify 1.20 to 2 dp, or show there is a sign change in the interval (1.195,1.205)	**A1**
		3
(c)	Use quotient rule	**M1**
	Obtain correct derivative in any form	**A1**
	Equate derivative to – 8 and obtain a quadratic in e^{2x}	**M1**
	Obtain $2\left(e^{2x}\right)^2 - 5e^{2x} + 2 = 0$	**A1**
	Solve a 3-term quadratic in e^{2x} for x	**M1**
	Obtain answer $x = \dfrac{1}{2}\ln 2$, or exact equivalent, only	**A1**

Question 78

(a)	Use quotient or product rule	**M1**
	Obtain correct derivative in any form	**A1**
	Equate derivative to zero and solve for x	**M1**
	Obtain $x = \sqrt[4]{e}$ and $y = \frac{1}{4e}$, or exact equivalents	**A1**
		4
(b)	Commence integration and reach $ax^{-3}\ln x + b\int x^{-3}.\ \frac{1}{x}\ dx$	***M1**
	Obtain $-\frac{1}{3}x^{-3}\ln x + \frac{1}{3}\int x^{-3}.\ \frac{1}{x}\ dx$	**A1**
	Complete integration and obtain $-\frac{1}{3}x^{-3}\ln x - \frac{1}{9}x^{-3}$	**A1**
	Substitute limits correctly, having integrated twice	**DM1**
	Obtain answer $\frac{1}{9} - \frac{1}{3}a^{-3}\ln a - \frac{1}{9}a^{-3}$	**A1**
	Justify the given statement	**A1**
		6

Question 79

(a)	State $\dfrac{dx}{dt} = 1 + \dfrac{1}{t+2}$	**B1**
	Use product rule	**M1**
	Obtain $\dfrac{dy}{dt} = e^{-2t} - 2(t-1)e^{-2t}$	**A1**
	Use $\dfrac{dy}{dx} = \dfrac{dy}{dt} \div \dfrac{dx}{dt}$	**M1**
	Obtain correct answer in any simplified form, e.g. $\dfrac{(3-2t)(t+2)}{t+3}\ e^{-2t}$	**A1**
		5
(b)	Equate derivative to zero and solve for t	**M1**
	Obtain $t = \dfrac{3}{2}$ and obtain answer $y = \dfrac{1}{2}e^{-3}$, or exact equivalent	**A1**
		2

Question 80

Use correct product (or quotient) rule	**M1**
Obtain $\frac{dy}{dx} = -5e^{-5x}\tan^2 x + 2e^{-5x}\tan x\sec^2 x$	**A1**
Equate *their* derivative to zero and obtain an equation in $\sin x$ and $\cos x$	**M1**
Obtain $5\cos x\sin x = 2$	**A1**
State answer $x = 0$	**B1**
Use double angle formula or square both sides and solve for x	**M1**
Obtain answer, e.g. 0.464	**A1**
Obtain second non-zero answer, e.g. 1.107 and no other in the given interval	**A1**
	8

Question 81

(a)	Use correct product rule or correct quotient rule	**M1**
	Obtain correct derivative in any form	**A1**
	Equate 2 term derivative to zero and solve for x	**M1**
	Obtain answer $x = e^{\frac{3}{2}}$	**A1**
	Obtain answer $y = \frac{3}{2e}$	**A1**
		5

(b)	Commence integration and reach $ax^{\frac{1}{3}}\ln x + b\int x^{\frac{1}{3}}.\frac{1}{x}\,dx$	*M1
	Obtain $3x^{\frac{1}{3}}\ln x - 3\int x^{\frac{1}{3}}.\frac{1}{x}\,dx$	A1
	Complete the integration and obtain $3x^{\frac{1}{3}}\ln x - 9x^{\frac{1}{3}}$	A1
	Use limits correctly in an expression of the form $px^{\frac{1}{3}}\ln x + qx^{\frac{1}{3}}$ $(pq \neq 0)$	DM1
	Obtain $18\ln 2 - 9$ from full and correct working	A1
		5

Question 82

(a)	Use correct chain rule **or** correct quotient rule to differentiate x or y	M1
	Obtain $\frac{dx}{dt} = \frac{3}{2+3t}$ **or** $\frac{dy}{dt} = \frac{2}{(2+3t)^2}$	A1
	Use $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$	M1
	Obtain answer $\frac{2}{3(2+3t)}$	A1
	Explain why this is always positive	A1
(b)	Obtain $y = -\frac{1}{3}$ when $x = 0$	B1
	Use a correct method to form the given tangent	M1
	Obtain answer $3y = 2x - 1$	A1
		3

Question 83

(a)	Use quotient or product rule	**M1**	
	Obtain correct derivative in any form	**A1**	
	Equate derivative to zero and solve for x	**M1**	
	Obtain answer $x = 3$	**A1**	
		4	
(b)	State $\frac{du}{dx} = \frac{1}{2\sqrt{x}}$, or $dx = 2\sqrt{x}du$, or $2u\ du = dx$	**B1**	
	Substitute and obtain integrand $\frac{2}{9-u^2}$	**B1**	
	Use given formula for the integral or integrate relevant partial fractions	**M1**	
	Obtain integral $\frac{1}{3}\ln\left(\frac{3+u}{3-u}\right)$	**A1**	OE
	Use limits $u = 0$ and $u = 2$ correctly	**M1**	
	Obtain the given answer correctly	**A1**	
		6	

Question 84

(a)	Use chain rule to differentiate LHS	***M1**	
	Obtain $\frac{1}{x+y}\left(1+\frac{dy}{dx}\right)$	**A1**	
	Equate derivative of LHS to $1 - 2\frac{dy}{dx}$ and solve for $\frac{dy}{dx}$	**DM1**	
	Obtain the given answer correctly	**A1**	
		4	
(b)	State $x + y = 1$	**B1**	
	Obtain or imply $x - 2y = 0$	**B1**	
	Obtain coordinates $x = \frac{2}{3}$ and $y = \frac{1}{3}$	**B1**	
		3	

Question 85

Use chain rule	**M1**	Allow if not starting with the correct index.
Obtain correct derivative in any form	**A1**	e.g. $\frac{dy}{dx} = \frac{\sec^2 x}{2\sqrt{\tan x}}$
Use correct Pythagoras to obtain correct derivative in terms of $\tan x$	**A1**	e.g. $\frac{dy}{dx} = \frac{1+\tan^2 x}{2\sqrt{\tan x}}$
Use a correct derivative to obtain $\frac{dy}{dx} = 1$ when $x = \frac{1}{4}\pi$	**B1**	Confirm the **given statement** from correct work. Should see at least $\frac{2}{2} = 1$.
	4	

Question 86

(a)	State correct derivative of ye^{2x} with respect to x	B1	$2ye^{2x}+e^{2x}\frac{dy}{dx}$
	State correct derivative of y^2e^x with respect to x	B1	$2ye^x\frac{dy}{dx}+y^2e^x$
	Equate attempted derivative of the LHS to zero and solve for $\frac{dy}{dx}$	M1	
	Obtain $\frac{dy}{dx}=\frac{2ye^x-y^2}{2y-e^x}$	A1	Obtain the **given answer** correctly. Condone multiplication by $\frac{-1}{-1}$ and cancelling of e^x without comment.
(b)	Equate denominator to zero and substitute for y or for e^x in the equation of the curve	*M1	
	Obtain equation of the form $ae^{3x}=b$ or $cy^3=d$	DM1	$(e^{3x}=8,\quad y^3=1)$ SOI
	Obtain $x = \ln 2$	A1	Accept $\frac{1}{3}\ln 8$ ISW
	Obtain $y = 1$	A1	
		4	

Question 87

(a)	Use correct product rule	M1	
	Obtain correct derivative in any form	A1	$\frac{dy}{dx}=e^{1-2x}-2xe^{1-2x}$
	Equate derivative to zero and solve for x	M1	
	Obtain $x=\frac{1}{2}$ and $y=\frac{1}{2}$	A1	
		4	
(b)	Use a correct method for determining the nature of a stationary point	M1	e.g. $\frac{d^2y}{dx^2}=-2e^{1-2x}-2(1-2x)e^{1-2x}$
	Show that it is a maximum point	A1	
		2	

Question 88

State $\frac{dx}{d\theta} = \sin\theta$ or $\frac{dy}{d\theta} = -\sin\theta + \frac{1}{2}\sin 2\theta$	**B1**	
Use $\frac{dy}{dx} = \frac{dy}{d\theta} \div \frac{dx}{d\theta}$	**M1**	
Obtain correct answer in any form	**A1**	e.g. $\frac{-\sin\theta + \frac{1}{2}\sin 2\theta}{\sin\theta}$
Use double angle correctly to obtain $\frac{dy}{dx}$ in terms of θ	**M1**	$\sin 2\theta = 2\sin\theta\cos\theta$
Obtain the given answer with no errors seen $-2\sin^2\left(\frac{1}{2}\theta\right)$	**A1**	AG. Requires correct cancellation of ALL $\sin\theta$ terms and $\cos\theta = 1 - 2\sin^2\left(\frac{1}{2}\theta\right)$ seen **SC** For incorrect signs, consistent throughout max. B0, M1, A0, M1, A1
	5	

Question 89

(a)	Use chain rule at least once	**M1**	Needs $\frac{dy}{dt} = \frac{1}{\tan t}\frac{d}{dt}(\tan t)$ or $\frac{dx}{dt} = (-1)(\cos^{-2} t)\frac{d}{dt}(\cos t)$. BOD if + and (– 1) (– 1) not seen. $\frac{dx}{dt} = \sec t \tan t$ (from List of Formulae MF19) M1 A1. If $\frac{dx}{dt} = -\sec t \tan t$ M1 A0.
	Obtain $\frac{dx}{dt} = \sec t \tan t$	**A1**	OE e.g. $\sin t (\cos t)^{-2}$. If e.g. $\frac{dx}{dt} = \sec x \tan x$ or $\sec\theta\tan\theta$ or $\sec t \tan x$, condone recovery on next line.
	Obtain $\frac{dy}{dt} = \frac{\sec^2 t}{\tan t}$	**A1**	OE e.g. $\frac{1}{\sin t \cos t}$. If e.g. $\frac{dy}{dt} = \frac{\sec^2 x}{\tan x}$ or $\frac{\sec^2\theta}{\tan\theta}$, condone recovery on next line. Only penalise notation errors **once** in $\frac{dx}{dt}$ and $\frac{dy}{dt}$ if no recovery.
	Use $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$	**M1**	Allow even if previous M0 scored, but must be using derivatives.
	Obtain given answer $\frac{\cos t}{\sin^2 t}$	**A1**	AG After $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$ used, any notation error A0. Must cancel $\cos t$ correctly.
		5	

(b)	State or imply $t = \frac{1}{4}\pi$ when $y = 0$	B1	
	Form the equation of the tangent at $y = 0$ or find c	M1	$x = \sqrt{2}$, $\frac{dy}{dx} = \sqrt{2}$ and $y = 0$, *their* coordinates and gradient used in $y = mx + c$.
	Obtain answer $y = \sqrt{2}x - 2$	A1	OE e.g. $y = \sqrt{2}\,(x - \sqrt{2})$ ISW. Allow $y = 1.41x - 2[.00]$ or $1.41(x - 1.41)$.
		3	

Question 90

(a)	Use correct product rule or quotient rule, and attempt at chain rule	M1	$ke^{-4x}\tan x + e^{-4x}\sec^2 x$ or $\frac{e^{4x}\sec^2 x - \tan x(ke^{4x})}{(e^{4x})^2}$ Need to see $d(\tan x)/dx = \sec^2 x$ (formula sheet) and attempt at ke^{-4x}, where $k \neq 1$.
	Obtain correct derivative in any form	A1	$-4e^{-4x}\tan x + e^{-4x}\sec^2 x$ or $\frac{e^{4x}\sec^2 x - \tan x(4e^{4x})}{(e^{4x})^2}$
	Use trigonometric formulae to express derivative in the form $ke^{-4x}\sin x\cos x\sec^2 x + ae^{-4x}\sec^2 x$ or $ke^{-4x}\frac{\sin x\cos x}{\cos x\cos x} + ae^{-4x}\sec^2 x$ or $\sec^2 x(ke^{-4x}\sin x\cos x + ae^{-4x})$ Allow $\frac{1}{\cos^2 x}$ instead of $\sec^2 x$	M1	Need to use $\frac{\tan x}{\sec^2 x} = \sin x\cos x$ or $\tan x = \frac{\sin x}{\cos x}.\frac{\cos x}{\cos x}$ OE. M1 is independent of previous M1, but expression must be of appropriate form.
	Obtain correct answer with $a = 1$ and $b = -2$	A1	At least one line of trigonometric working is required from $-4e^{-4x}\tan x + e^{-4x}\sec^2 x$ to given answer $\sec^2 x(1 - 2\sin 2x)\,e^{-4x}$ with elements in any order. If only error: $4\sin x\cos x = 4\sin 2x$ M1 A1 M1 A0.
		4	
(b)	Equate derivative to zero and use correct method to solve for x	M1	$\sin 2x = \frac{1}{2}$, hence $x = \frac{1}{2}\sin^{-1}\frac{1}{2}$ or $x = \tan^{-1}(2 \pm \sqrt{3})$ Allow M1 for correct method for non-exact value.
	Obtain answer, e.g. $x = \frac{1}{12}\pi$	A1	[0.262 M1 A0]
	Obtain second answer, e.g. $\frac{5}{12}\pi$ and no other in the given interval	A1 FT	FT $\frac{\pi - \textit{their}\,2x}{2}$ if exact values; x must be $< \frac{\pi}{2}$. Ignore answers outside the given interval. Treat answers in degrees as a misread. 15°, 75°. **SC** No values found for a and b in **4(a)** but chooses values in **4(b)**: max **M1** for x.
		3	

Question 91

(a)	State or imply $6xy + 3x^2 \frac{dy}{dx}$ as derivative of $3x^2y$	B1	
	State or imply $3y^2 \frac{dy}{dx}$ as derivative of y^3	B1	Allow B1 B1 for $(3x^2dx+)6xydx + 3x^2dy - 3y^2dy[=0]$
	Equate attempted derivative of left-hand side to zero and solve to obtain an equation with $\frac{dy}{dx}$ as subject	M1	Allow if zero implied by subsequent working. Allow if recover from an extra $\frac{dy}{dx} = \ldots$ at the beginning of the left-hand side.
	Obtain $\frac{dy}{dx} = \frac{x^2 + 2xy}{y^2 - x^2}$ correctly	A1	AG Accept y' for $\frac{dy}{dx}$.
		4	
(b)	Equate numerator to zero	*M1	Must be using the given derivative.
	Obtain $x = -2y$, or equivalent	A1	An equation with x or y as the subject SOI.
	Use $x^3 + 3x^2y - y^3 = 3$ to obtain an equation in x or y	DM1	$-8y^3 + 12y^3 - y^3 = 3$ or $x^3 - \frac{3}{2}x^3 + \frac{1}{8}x^3 = 3$ or any equivalent form (do not need to evaluate powers).
	Obtain the point (– 2, 1) and no others from solving their cubic equation	A1	Allow if each component stated separately. ISW.
	State the point $(0, -\sqrt[3]{3})$, or equivalent from correct work	B1	Accept $(0, \sqrt[3]{-3})$, or (0, – 1.44) (-1.44225). Allow if each component stated separately. ISW.
		5	

Question 92

Use the correct product rule and then the chain rule to differentiate either $\cos^3 x$ or $\sqrt{\sin x}$	M1	e.g. two terms with one part of $\frac{dy}{dx} = p\cos^2 x \sin x\sqrt{\sin x} + q\frac{\cos^3 x \cos x}{\sqrt{\sin x}}$.
Obtain correct derivative in any form e.g. $\frac{dy}{dx} = -3\cos^2 x \sin x\sqrt{\sin x} + \frac{\cos^3 x \cos x}{2\sqrt{\sin x}}$	A1 A1	A1 for each correct term substituted in the complete derivative.
Equate their derivative to zero and obtain a horizontal equation with positive integer powers of sin x and/or cos x from an equation including $\sqrt{\sin x}$ or $\frac{1}{\sqrt{\sin x}}$ using sensible algebra.	M1	e.g. $-3\cos^2 x\sin^2 x + \frac{1}{2}\cos^4 x = 0$
Use correct formula(s) to express *their* equation/derivative in terms of one trigonometric function	M1	Can be awarded before the previous M1. May involve more than one trigonometric term.
Obtain $7\cos^2 x = 6$, $7\sin^2 x = 1$, or $6\tan^2 x = 1$, or equivalent, and obtain answer $x = 0.388$	A1	CAO. The question asks for 3 sf. Ignore additional answers outside $\left(0, \frac{\pi}{2}\right)$. 22.2° is A0.
	6	

Question 93

(a)	State or imply $3y^2 \dfrac{dy}{dx}$ as derivative of y^3	**B1**	
	State or imply $2y + 2x \dfrac{dy}{dx}$ as derivative of $2xy$	**B1**	
	Complete differentiation and equate attempted derivative to zero and solve for $\dfrac{dy}{dx}$	**M1**	
	Obtain answer $-\dfrac{3x^2+2y}{3y^2+2x}$	**A1**	
		4	
(b)	Find gradient at either (0, – 2) or (– 2, 0)	**M1**	
	Obtain answers $\dfrac{1}{3}$ and 3	**A1 A1**	
	Use $\tan(A \pm B)$ formula to find $\tan\alpha$	**M1**	
	Obtain answer $\tan\alpha = \dfrac{4}{3}$	**A1**	
		5	

Question 94

State or imply $\dfrac{dx}{dt} = 2 - \sec^2 t$ or $\dfrac{dy}{dt} = 2\dfrac{\cos 2t}{\sin 2t}$	**B1**	
Use $\dfrac{dy}{dx} = \dfrac{dy}{dt} \div \dfrac{dx}{dt}$	**M1**	
Obtain correct answer in any form	**A1**	
Use double angle formula to express derivative in terms of cos x and sin x	**M1**	
Obtain the given answer correctly	**A1**	AG
	5	

Question 95

Use correct product rule on given expression	***M1**	
Obtain correct derivative in any form	**A1**	e.g. $\cos x \sin 2x + 2\sin x \cos 2x$
Use correct double angle formulae to express derivative in terms of $\sin x$ and $\cos x$	***M1**	
Equate derivative to zero and obtain an equation in one trig variable	**DM1**	dependent on the 2 previous M Marks.
Obtain $3\sin^2 x = 2$, $3\cos^2 x = 1$ or $\tan^2 x = 2$	**A1**	OE
Solve and obtain $x = 0.955$	**A1**	3 sf only. Final answer in degrees is A0. Ignore any attempt to find the corresponding value of y.

Question 96

Use correct product or quotient rule	***M1**	
Obtain correct derivative in any form	**A1**	e.g. $\frac{dy}{dx} = -e^{-\frac{x}{3}} - \frac{1}{3}(3-x)e^{-\frac{x}{3}}$
Equate their derivative to zero and solve for x	**DM1**	
Obtain $x = 6$	**A1**	
Obtain $y = -3e^{-2}$	**A1**	Or exact equivalent.
	5	

Question 97

(a)	Obtain $\frac{dx}{dt} = e^{2t} + 2te^{2t}$	**B1**	OE
	Use $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$	**M1**	$\frac{dy}{dx} = \frac{2t+1}{e^{2t}(1+2t)}$.
	Obtain the given answer $\frac{dy}{dx} = e^{-2t}$	**A1**	AG Need to see $e^{2t}(1 + 2t)$ in denominator.
		3	
(b)	Obtain $x = -e^{-2}$ or $-\frac{1}{e^2}$ and $y = 3$ at $t = -1$	**B1**	
	Obtain gradient of normal = $-e^{-2}$ or $-\frac{1}{e^2}$	**B1**	
	$x = 0$ substituted into equation of normal or use of gradients to give $y = 3 - \frac{1}{e^4}$ with no errors	**B1**	Equation of normal $y - 3 = -e^{-2}\left(x - -e^{-2}\right)$. AG **SC** Decimals **B0 B1 B0** − 0.135 .
		3	

Question 98

Use the product rule correctly	***M1**	$x^3\, d/dx(\ln x) + d/dx(x^3) \ln x$.
Obtain the correct derivative in any form	**A1**	e.g. $\frac{x^3}{x} + 3x^2 \ln x$.
Equate derivative to zero and solve exactly for x	**DM1**	Reaching $x = e^a$.
Obtain answer $\left(\frac{1}{\sqrt[3]{e}}, -\frac{1}{3e}\right)$ or exact equivalent	**A1**	ISW
	4	

Question 99

Use the product rule correctly to obtain $p(x+5)(3-2x)^n + q(3-2x)^{\frac{1}{2}}$	*M1	Allow with incorrect chain rule. BOD over sign errors unless an incorrect rule is quoted.
Obtain correct derivative in any form	A1	e.g. $-(x+5)(3-2x)^{-\frac{1}{2}} + (3-2x)^{\frac{1}{2}}$.
Equate derivative to zero and obtain a linear equation	DM1	Allow with surd factor e.g. $(3-2x)^{-\frac{1}{2}}\left(-(x+5)+(3-2x)\right)=0$.
Obtain a correct linear equation.	A1	e.g. $-(x+5)+3-2x=0$.
Obtain answer $\left(-\frac{2}{3}, \frac{13\sqrt{39}}{9}\right)$.	A1	Or exact equivalent e.g. $\left(-\frac{2}{3}, \frac{13\sqrt{13}}{3\sqrt{3}}\right)$ or $\left(-\frac{2}{3}, \frac{\sqrt{2197}}{\sqrt{27}}\right)$. Accept with x, y stated separately. ISW
Alternative Method for Question 10(a)		
Obtain y^2 and differentiate	*M1	Ignore *their* left hand side i.e. *their* $\frac{d}{dx}y^2$.
Obtain correct derivative in any form	A1	e.g. $-6x^2-34x-20$.
Equate derivative to zero and solve for x	DM1	
Obtain $-\frac{2}{3}$	A1	Ignore -5 if seen.
Obtain answer $\left(-\frac{2}{3}, \frac{13\sqrt{39}}{9}\right)$ only	A1	Or exact equivalent e.g. $\left(-\frac{2}{3}, \frac{13\sqrt{13}}{3\sqrt{3}}\right)$ or $\left(-\frac{2}{3}, \frac{\sqrt{2197}}{\sqrt{27}}\right)$. ISW
	5	

Question 100

State $\frac{dy}{d\theta}=1-2\sin\theta$	B1	Ignore left side throughout dx/dt, dy/dt, dx, dy but must see $\frac{dy}{dx}$ for final A1.
Use correct quotient rule, or product rule if rewrite x as $\cos\theta(2-\sin\theta)^{-1}$	M1	Incorrect formula seen M0 A0 otherwise BOD.
Obtain $\frac{dx}{d\theta}=\frac{-(2-\sin\theta)\sin\theta+\cos^2\theta}{(2-\sin\theta)^2}$ o.e.	A1	$-\sin\theta(2-\sin\theta)^{-1}-\cos\theta(2-\sin\theta)^{-2}(-\cos\theta)$ or equivalent.
Use $\frac{dy}{dx}=\frac{dy}{d\theta}\div\frac{dx}{d\theta}$	M1	$\left(\frac{dy}{dx}=(1-2\sin\theta)\div\frac{1-2\sin\theta}{(2-\sin\theta)^2}\right)$. Allow M1 even if errors in both derivatives.
Obtain $\frac{dy}{dx}=(2-\sin\theta)^2$.	A1	AG – must see working in above cell to gain final A1. Allow $\cos^2\theta+\sin^2\theta=1$ to be implied. x instead of θ or missing θ more than twice on right side then A0 final mark.
	5	

Question 101

(a)	State or imply $6y\frac{\mathrm{d}y}{\mathrm{d}x}$ as the derivative of $3y^2$	**B1**	Allow y' for $\frac{\mathrm{d}y}{\mathrm{d}x}$ throughout. Accept $\frac{\partial f}{\partial x} = 6x + 4y$.
	State or imply $4x\frac{\mathrm{d}y}{\mathrm{d}x} + 4y$ as the derivative of $4xy$	**B1**	Accept $\frac{\partial f}{\partial y} = 4x + 6y$.
	Equate derivative of LHS to zero and solve for $\frac{\mathrm{d}y}{\mathrm{d}x}$	**M1**	Allow an extra $\frac{\mathrm{d}y}{\mathrm{d}x}$ in front of their differentiated equation. Allow if '= 0' is implied but not seen. Allow $\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{\partial f}{\partial x} \Big/ \frac{\partial f}{\partial y}$
	Obtain $\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{3x+2y}{2x+3y}$	**A1**	AG – must come from correct working. The position of the negative must be clear.
		4	
(b)	Equate $\frac{\mathrm{d}y}{\mathrm{d}x}$ to –2 and solve for x in terms of y or for y in terms of x	***M1**	Must be using the given derivative.
	Obtain $x = -4y$ or $y = -\frac{x}{4}$	**A1**	Seen or implied by correct later work.
	Substitute *their* $x = -4y$ or *their* $y = -\frac{x}{4}$ in curve equation	**DM1**	Allow unsimplified.
	Obtain $y = \pm\frac{1}{\sqrt{7}}$ or $x = \pm\frac{4}{\sqrt{7}}$	**A1**	Or exact equivalent. Or $x = \frac{4}{\sqrt{7}}$ and $y = -\frac{1}{\sqrt{7}}$ or exact equivalent.
	Obtain both pairs of values	**A1**	Or $x = -\frac{4}{\sqrt{7}}$ and $y = \frac{1}{\sqrt{7}}$ or exact equivalent. A1 A0 for incorrect final pairing.
		5	

Question 102

(a)	State or imply $2xy + x^2\frac{\mathrm{d}y}{\mathrm{d}x}$ as derivative of x^2y	**B1**	Accept partial: $\frac{\partial}{\partial x} \to 2xy$.
	State or imply $2ay\frac{\mathrm{d}y}{\mathrm{d}x}$ as derivative of ay^2	**B1**	Accept partial: $\frac{\partial}{\partial y} \to x^2 - 2ay$.
	Equate attempted derivative to zero and solve for $\frac{\mathrm{d}y}{\mathrm{d}x}$	**M1**	
	Obtain answer $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2xy}{2ay - x^2}$ from correct working	**A1**	AG
		4	
(b)	State or imply $2ay - x^2 = 0$	***M1**	
	Substitute into equation of curve to obtain equation in x and a or in y and a	**DM1**	e.g. $2ay^2 - ay^2 = 4a^3$ or $\frac{x^4}{2a} - \frac{x^4}{4a} = 4a^3$.
	Obtain one correct point	**A1**	e.g. $(2a, 2a)$.
	Obtain second correct point and no others	**A1**	e.g. $(-2a, 2a)$.
		4	**SC**: Allow A1 A0 for $x = \pm 2a$ or for $y = 2a$.

Question 103

Use the product rule correctly on $y = x \cos 2x$	M1	$\mathrm{d}x/\mathrm{d}x \cos 2x + x\, \mathrm{d}/\mathrm{d}x(\cos 2x)$ attempted.
Obtain the correct derivative in any form	A1	e.g. $\cos 2x - 2x \sin 2x$. If $\cos 2x + x{-}2\sin 2x$, not recovered, max M1A0A1FTA0 but can recover for full marks by seeing correct substitution.
Obtain $y = -\frac{\pi}{2}$ and $\frac{\mathrm{d}y}{\mathrm{d}x} = -1$ when $x = \frac{\pi}{2}$	A1FT	FT *their* $\frac{\mathrm{d}y}{\mathrm{d}x}$ with $x = \frac{\pi}{2}$ substituted.
Obtain answer $x + y = 0$	A1	OE CWO Need to see y and $\mathrm{d}y/\mathrm{d}x$ at $x = \frac{\pi}{2}$.
	4	

Question 104

(a)	State or imply $2y\frac{\mathrm{d}y}{\mathrm{d}x}$ as the derivative of y^2	B1	Allow for $3x^2\mathrm{d}x + 2y\mathrm{d}y$ or $F_x = 3x^2 + 6x$ and $F_y = 2y + 3$.
	Equate derivative of LHS to zero and solve for $\frac{\mathrm{d}y}{\mathrm{d}x}$	M1	$3x^2 + 2y\frac{\mathrm{d}y}{\mathrm{d}x} + 6x + 3\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ or $3x^2\mathrm{d}x + 2y\mathrm{d}y + 6x\mathrm{d}x + 3\mathrm{d}y = 0$ or $\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{F_x}{F_y}$ need evidence from B1 mark or formula must be seen. Allow errors.
	Obtain the given answer	A1	AG $\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{3x^2 + 6x}{2y + 3}$ not $\frac{-3x^2 - 6x}{2y + 3}$. Must factorise with $\frac{\mathrm{d}y}{\mathrm{d}x}$ e.g. $3x^2 + 6x + \frac{\mathrm{d}y}{\mathrm{d}x}(2y + 3) = 0$ or $3x^2\mathrm{d}x + 6x\mathrm{d}x + \mathrm{d}y(2y + 3) = 0$.
		3	
(b)	Equate numerator to zero and solve for x	*M1	Allow for just one x value.
	Obtain $x = 0$ and $x = -2$ only	A1	
	Substitute their x, [$x = 0$ or $x = -2$] in curve equation to obtain quadratic equation in y equal to 0	DM1	$y^2 + 3y - 4 = 0$ or $y^2 + 3y = 0$.
	Obtain $y = 1$ and $y = -4$ [when $x = 0$]	A1	
	Obtain $y = 0$ and $y = -3$ [when $x = -2$]	A1	ISW If forget $x = 0$ then max 3/5.
		5	

Question 105

Use correct product or quotient rule	M1	Need attempt at both derivatives condone errors in chain rule. In quotient rule allow BOD in formula if $\pm\ 2x$ seen unless clear that incorrect formula has been used. If omit denominator or forget to square or complete reversal of signs then M0 A0 M1 A1 A1 A1.
Obtain correct derivative in any form, e.g. $\dfrac{6x(1-x^2)e^{3x^2-1}+2xe^{3x^2-1}}{\left(1-x^2\right)^2}$	A1	If $6x\left(1-x^2\right)e^{3x^2-1}+2xe^{3x^2-1}=0$ from the start, with no wrong formula seen, award M1A1.
Equate derivative (or its numerator) to zero and solve for x	M1	$6x - 6x^3 + 2x = 0$ and solve. Allow for just one x value. Allow if from solution of 3 term quadratic equation, but if they get $x = 0$ the x must factorise out
Obtain the point $(0, e^{-1})$ or exact equivalent	A1	Or for all three x coordinates found 0, $\pm\dfrac{2\sqrt{3}}{3}$ oe and no extras but if this is the case then one pair of correct coordinates A1 and both other pairs of correct coordinates A1. Accept, e.g. $x = 0, y = e^{-1}$ ISW for last 3 marks.
Obtain the point $\left(\dfrac{2\sqrt{3}}{3}, -3e^3\right)$ or exact equivalent	A1	Allow $\sqrt{(4/3)}$.
Obtain the point $\left(-\dfrac{2\sqrt{3}}{3}, -3e^3\right)$ or exact equivalent	A1	
	6	

Question 106

Use correct product rule	*M1	As far as $p\cos x\cos 2x + q\sin x\sin 2x$ or full working (u, v, du/dx, dv/dx) shown.
Obtain $\dfrac{dy}{dx}=\cos x\cos 2x - 2\sin x\sin 2x$	A1	OE
Equate derivative to zero and use correct double angle formulae	DM1	Allow if only have one double angle in their derivative.
Obtain $\cos x\left(1-6\sin^2 x\right)=0$ or equivalent	A1	e.g. $\cos x\left(6\cos^2 x-5\right)=0$, $5\tan^2 x=1$. Simplified but not necessarily factorised - like terms must be collected.
Obtain $a=0.42$	A1	Only. Accept $x=0.42$.

Question 107

Obtain $\frac{\mathrm{d}x}{\mathrm{d}t}=\frac{2}{t}\ln t$	**B1**	Any equivalent form.
Obtain $\frac{\mathrm{d}y}{\mathrm{d}t}=-2t\mathrm{e}^{2-t^2}$	**B1**	Any equivalent form.
$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\mathrm{d}y}{\mathrm{d}t}\div\frac{\mathrm{d}x}{\mathrm{d}t}$ and substitute $t=\mathrm{e}$	**M1**	Correct use of chain rule for $\frac{\mathrm{d}y}{\mathrm{d}x}\left(\frac{-2\mathrm{e}^2\mathrm{e}^{2-\mathrm{e}^2}}{2\ln\mathrm{e}}\right)$. Condone an error between correct combination of the derivatives and attempt to substitute e.
Obtain $-\mathrm{e}^{4-\mathrm{e}^2}$	**A1**	ISW Accept $-0.0337(405..)$. Accept $-\mathrm{e}^4\mathrm{e}^{-\mathrm{e}^2}$, $\frac{-\mathrm{e}^4}{\mathrm{e}^{\mathrm{e}^2}}$ and $-\mathrm{e}^2\mathrm{e}^{2-\mathrm{e}^2}$. Allow M1A1 for a correct decimal answer following B1B1 seen.
	4	

Question 108

Use the correct product rule	***M1**	Condone error in chain rule.
Obtain correct derivative in any form	**A1**	e.g. $\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{x^2}{2}\mathrm{e}^{-\frac{x^2}{4}}+\mathrm{e}^{-\frac{x^2}{4}}$.
Equate derivative to zero and solve for x	**DM1**	
Obtain answer $\left(\sqrt{2},\sqrt{2}\mathrm{e}^{-\frac{1}{2}}\right)$	**A1**	Or exact equivalent. Can state the components separately.
	4	

Question 109

(a)	State correct derivative of x **or** y with respect to t	**B1**	$\frac{\mathrm{d}x}{\mathrm{d}t}=\frac{1}{2}t^{-\frac{1}{2}}$, $\frac{\mathrm{d}y}{\mathrm{d}t}=\frac{1}{t}$.
	Use $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\mathrm{d}y}{\mathrm{d}t}\times\frac{\mathrm{d}t}{\mathrm{d}x}$	**M1**	Use correct chain rule.
	Obtain answer $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{2}{\sqrt{t}}$	**A1**	Or simplified equivalent e.g $2t^{\frac{-1}{2}}$ or $\frac{2\sqrt{t}}{t}$.
		3	
(b)	State or imply *their* $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{2}$	**M1**	
	Obtain $\sqrt{t}=4$	**A1**	Or equivalent.
	Obtain answer (7, ln 16)	**A1**	Or exact equivalent. Can state the two components separately.
		3	

Question 110

Use correct quotient or product rule	***M1**	
Obtain correct derivative in any form	**A1**	e.g. $\frac{(1-3x)2x-x^2(-3)}{(1-3x)^2}\left(=\frac{2x-3x^2}{(1-3x)^2}\right)$ or $3x^2(1-3x)^{-2}+2x(1-3x)^{-1}$.
Equate derivative to 8 and solve for x	**DM1**	$75x^2-50x+8=(15x-4)(5x-2)$.
Obtain answers $x=\frac{2}{5}$ and $\frac{4}{15}$	**A1**	Exact values required.
Obtain answers $y=-\frac{4}{5}$ and $\frac{16}{45}$	**A1**	Allow A1 for one correct point.
	5	

Question 111

(a)	State or imply $4y\frac{dy}{dx}$ as the derivative of $2y^2$	**B1**	**SC** If $\frac{dy}{dx}$ introduced instead of $\frac{d}{dx}$ then allow **B1** for both, followed by correct method **M1** Max 2.
	State or imply $3y+3x\frac{dy}{dx}$ as the derivative of $3xy$	**B1**	Allow extra $\frac{dy}{dx}$ = correct expression to collect all marks if correct.
	Complete the differentiation, all 4 terms, isolate 2 $\frac{dy}{dx}$ terms on LHS or bracket $\frac{dy}{dx}$ terms and solve for $\frac{dy}{dx}$	**M1**	
	Obtain $\frac{dy}{dx}=\frac{2x-3y-1}{4y+3x}$	**A1**	Answer Given – need to have seen $4y\frac{dy}{dx}+3x\frac{dy}{dx}=2x-3y-1$ or $(4y+3x)\frac{dy}{dx}-2x+3y=-1$. Need to see = $2x$ or = 0 consistently throughout otherwise **M1 A0**. No recovery allowed. When all terms are included then must be an equation.
		4	Allow all marks if using dx and dy.
(b)	Equate numerator to zero, obtaining $2x = 3y + 1$ or $3y = 2x - 1$ and form equation in x only or y only from $2y^2 + 3xy + x = x^2$	**M1***	e.g. $\frac{2}{9}(2x-1)^2+x(2x-1)+x=x^2$ or $2y^2+\frac{3}{2}(1+3y)y+\frac{1}{2}(1+3y)=\frac{1}{4}(1+3y)^2$. Allow errors.
	Obtain $\frac{2}{9}(2x-1)^2=-x^2$ or a 3 term quadratic in one unknown and try to solve. If errors in quadratic formulation allow solution, applying usual rules for solution of quadratic equation, and allow **M1**	**DM1**	e.g. $17x^2-8x+2=0$ $(b^2-4ac=-72)$ or $17y^2+6y+1=0$ $(b^2-4ac=-32)$. $x = 4/17 \pm (3\sqrt{2}/17)i$, $y = -3//17 \pm (2\sqrt{2}/17)i$.
	Conclude that the equation has no [real] roots	**A1**	Given Answer. CWO
		3	

Question 112

Use correct product rule $\cos 2x$ may be $1 - 2\sin^2 x$ or …	**M1**	$ae^{2x}\sin 2x + e^{2x}b\cos 2x$. Need a or $b = 2$. Allow M1 if only error is e^x instead of e^{2x} in one of terms, then maximum 1/5.
Obtain correct derivative $2e^{2x}\sin 2x + 2e^{2x}\cos 2x$	**A1**	OE, e.g. $4e^{2x}\sin x\cos x + 2e^{2x}\left(\cos^2 x - \sin^2 x\right)$.
Equate derivative of the form $ae^{2x}\sin 2x + e^{2x}b\cos 2x$ to 0 and solve for $2x$ or x using a correct method Note may have substituted for $\sin 2x$ and/or $\cos 2x$	**M1**	Obtain $2x = \tan^{-1}(-$ *their b/their a*$)$ OE. Allow one slip in rearranging. Allow degrees. Variety of other methods available, such as solving quadratic equation in $\sin x$ or $\tan x$ e.g. $\tan^2 x - 2\tan x - 1 = 0$ leading to $x = \tan^{-1}(1+\sqrt{2})$.
Obtain $x = \frac{3}{8}\pi$ only or exact equivalent	**A1**	CWO 67.5° gets A0. Ignore any answers outside interval $0 \leqslant x \leqslant \frac{\pi}{2}$.
Obtain $y = \frac{1}{2}\sqrt{2}e^{\frac{3}{4}\pi}$ only or exact simplified equivalent	**A1**	CWO, ISW. Not $\sin\left(\frac{3}{4}\pi e^{\frac{3}{4}\pi}\right)$. Ignore any answers using x outside interval $0 \leqslant x \leqslant \frac{\pi}{2}$.
	5	

Ques113

Use correct product rule	***M1**	Or equivalent. Condone incorrect chain rule. M0 if a value is used for a (not equivalent work).
Obtain correct derivative	**A1**	E.g. $\frac{dy}{dx} = -axe^{-ax} + e^{-ax}$
Equate derivative to zero and solve for x	**DM1**	
Obtain $x = \frac{1}{a}$, $y = \frac{1}{ae}$	**A1**	ISW Or exact equivalent.
	4	

Question 114

Substitute $y=1$ and obtain $e^x = a$, where $a > 0$	**M1**	Must come from a quadratic in e^x. $\left(e^{2x}+e^x-6=0\right)$ Ignore any negative solution.
Obtain $e^x = 2$ only	**A1**	Or equivalent e.g. $x = \ln 2$. Condone $x = 0.693\ldots$..
State or imply $\frac{d}{dx}\left(ye^{2x}\right) = 2ye^{2x} + e^{2x}\frac{dy}{dx}$	**B1**	Accept y' for $\frac{dy}{dx}$.
State or imply $\frac{d}{dx}\left(y^2e^x\right) = y^2e^x + 2ye^x\frac{dy}{dx}$	**B1**	Accept y' for $\frac{dy}{dx}$.
Differentiate RHS of given equation to obtain zero (could be implied by subsequent work), substitute for *x* and *y* and obtain $\frac{dy}{dx} = \ldots$	**M1**	Independent. $\left[2\times4+4\frac{dy}{dx}+2+4\frac{dy}{dx}=0\right]$ NB: Could also rearrange to obtain $\frac{dy}{dx}$ then substitute. Both steps are needed for M1.
Obtain $-\frac{5}{4}$ or -1.25	**A1**	Correct answer from correct working only Accept $\frac{-10}{8}$. -1.2499 is A0.

Alternative method for Question 4: Dividing through by e^x

Substitute $y=1$ and obtain $e^x = a$, where $a > 0$	**(M1)**	Must come from a quadratic in e^x. $\left(e^{2x}+e^x-6=0\right)$ Ignore any negative solution.
Obtain $e^x = 2$ only	**(A1)**	Or equivalent e.g. $x = \ln 2$. Condone $x = 0.693\ldots$..
State or imply $\frac{d}{dx}\left(y^2\right) = 2y\frac{dy}{dx}$	**(B1)**	Accept y' for $\frac{dy}{dx}$.
State or imply $\frac{d}{dx}\left(ye^x\right) = ye^x + e^x\frac{dy}{dx}$	**(B1)**	Accept y' for $\frac{dy}{dx}$.
Differentiate RHS of given equation to obtain $-6e^{-x}$, substitute for *x* and *y* and obtain $\frac{dy}{dx} = \ldots..$	**(M1)**	Independent. $\left[1\times2+2\frac{dy}{dx}+2\frac{dy}{dx}=-\frac{6}{2}\right]$ NB: Could also rearrange to obtain $\frac{dy}{dx}$ then substitute. Both steps are needed for M1.
Obtain $-\frac{5}{4}$ or -1.25	**(A1)**	Correct answer from correct working only Accept $\frac{-10}{8}$. -1.2499 is A0.
	6	

Question 115

Obtain $2 = \sec^2 y\,\frac{dy}{dx}$ or equivalent	**B1**	E.g. $2\,\frac{dx}{dy} = \sec^2 y$ by differentiation with respect to *y*.
Use $\sec^2 y = 1 + \tan^2 y$	**M1**	
Replace $\tan y$ with $2x$ and rearrange to obtain given answer $\frac{dy}{dx} = \frac{2}{1+4x^2}$	**A1**	
	3	

Question116

Use correct quotient (or product) rule	*M1	
Obtain correct derivative $\dfrac{e^{\sin x}\cos^3 x-(-2e^{\sin x}\sin x\cos x)}{\left(\cos^2 x\right)^2}$ or equivalent	A1	
Equate numerator to zero	DM1	
Obtain equation in one unknown	DM1	E.g. $\sin^2 x - 2\sin x - 1 = 0$.
Solve a 3 term quadratic in $\sin x$ to find a value for x	M1	
Obtain a correct solution to the quadratic equation, e.g. 3.57°	A1	At least 3sf.
Obtain a further correct solution, e.g. x = 5.86° and no others in the interval	A1FT	At least 3sf. FT 3π – *their* 3.57.
Alternative Method for the first 3 marks:		
Take logarithms of both sides and simplify	(*M1)	$\ln y = \sin x - 2\ln\cos x$ or equivalent.
Obtain $\dfrac{1}{y}\dfrac{dy}{dx}=\cos x+2\dfrac{\sin x}{\cos x}$	(A1)	Or equivalent.
Equate $\dfrac{dy}{dx}$ to zero	(DM1)	
Continue as for the original		
	7	

Question 117

Use of correct product rule and correct chain rule	M1	$\dfrac{dy}{dx}=A\cos x\sqrt{2+\cos x}+\dfrac{B\sin x\sin x}{\sqrt{2+\cos x}}$
Obtain $\dfrac{dy}{dx}=2\cos x\sqrt{2+\cos x}-\dfrac{2\sin^2 x}{2\sqrt{2+\cos x}}$	A1	OE
Equate the derivative to zero and obtain a horizontal 3 term quadratic equation or 4 term quartic equation in $\cos a$ If M0 earlier then needs that expression to be such that arrive at 3 term quadratic or 4 term quartic equation in $\cos x$ without further trig errors. The only error in the form of the differential allowed is for $(2+\cos x)^{-\frac{1}{2}}$ to be $(2+\cos x)^{+\frac{1}{2}}$ or $(2+\cos x)^{-\frac{3}{2}}$	*M1	Accept in $\cos x$. E.g. $3\cos^2 x + 4\cos x - 1 = 0$. E.g. $3\cos^4 x + 16\cos^3 x + 18\cos^2 x - 1 = 0$.
Solve for $\cos a$	DM1	$\left(\cos a=\dfrac{-2+\sqrt{7}}{3}\text{ or }0.215\right)$ Allow presence of other solution(s).
Obtain $a = 4.93$	A1	Allow more accurate, e.g. 4.929… even though question states 2 dp. If x = 1.35 leads to x = 4.93 award A1 BOD. If x = 1.35 and x = 4.93 award A0.
	5	

Question 118

	Answer	Mark	Guidance
(a)	Obtain $\frac{dx}{dt} = 6\cos 2t$	B1	Allow $3.2\cos 2t$. $dx = 6\cos 2t$ is B0.
	Obtain $\frac{dy}{dt} = \sec^2 t - \operatorname{cosec}^2 t$	B1	Any equivalent form. $dy = \sec^2 t - \operatorname{cosec}^2 t$ is B0, but $\frac{dy}{dx} = \frac{\sec^2 t - \operatorname{cosec}^2 t}{6\cos 2t}$ can go on to gain M1M1A1, so 3/5 possible.
	Use $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$	*M1	$\frac{dy}{dx} = \frac{\sec^2 t - \operatorname{cosec}^2 t}{6\cos 2t}$
	Express as a single fraction With $\frac{dy}{dt}$ correctly simplified as a single fraction in terms of sin t and cos t Allow with $6\cos 2t$ expressed as $\frac{1}{6\cos 2t}$ outside bracket	DM1	Allow $\frac{\sin^2 t - \cos^2 t}{\cos^2 t \sin^2 t} \times \frac{1}{6\cos 2t}$ or $\frac{\sin^2 t - \cos^2 t}{\cos^2 t \sin^2 t} \div 6\cos 2t$ or $\frac{\frac{\sin^2 t - \cos^2 t}{\cos^2 t \sin^2 t}}{6\cos 2t}$
	Obtain $\left(\frac{-4\cos 2t}{6\cos 2t \times \sin^2 2t} = \right)\frac{-2}{3\sin^2 2t}$ from full and correct working Numerator and denominator must have identical terms before cancelling, both $\cos 2t$ or both + and – $(\sin^2 t - \cos^2 t)$	A1	AG Allow slips in θ and x for t to recover earlier marks, provided these are corrected before the final line. Do not allow serious errors in working for the final mark.

	Answer	Mark	Guidance
'(a)	**Alternative Method for Question 7(a):**		
	$y = \frac{6}{x}$	B2	Using $y = \frac{\tan^2 t + 1}{\tan t} = \frac{\sec^2 t}{\frac{\sin t}{\cos t}} = \frac{1}{\sin t \cos t}$
	$\frac{dy}{dx} = -6x^{-2}$	B1	
	$\frac{dy}{dx} = -\frac{6}{9\sin^2 2t}$	M1	
	Obtain $\frac{dy}{dx} = \frac{-2}{3\sin^2 2t}$ from full and correct working	A1	
		5	
'(b)	Gradient of normal $= \frac{3}{2}$	B1	
	Use correct method to find the equation of the normal	M1	E.g. $(y-2) = \frac{3}{2}(x-3)$ or find c in $y = \frac{3}{2}x + c$. Allow a wrong value for x or y but not both, with *their* normal gradient.
	Obtain $2y - 3x + 5 = 0$	A1	Or $k(2y - 3x + 5) = 0$, where k is an integer.
		3	

Question 119

(a)	Use correct product rule or chain rule to find derivative of x with respect to t	**M1**	Obtain $k\tan 2t\sec^2 2t$.
	Obtain $\frac{dx}{dt} = 4\tan 2t\sec^2 2t$ oe	**A1**	
	$\frac{dy}{dt} = -2\sin 2t$	**B1**	
	Use $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$ to obtain **given answer** $\frac{dy}{dx} = -\frac{1}{2}\cos^3 2t$	**B1**	Condone if $\frac{dy}{dx}$ missing.
		4	
(b)	Obtain $x = 1$ and $y = \frac{\sqrt{2}}{2}$	**B1**	Accept $y = 0.707\ldots$
	State or imply gradient of tangent is $\frac{-\sqrt{2}}{8}$ or gradient of normal is $4\sqrt{2}$	**B1**	Any equivalent form, e.g. $2^{\frac{5}{2}}$. Accept -0.177 or 5.66.
	Use correct method to find equation of **normal** using *their* values	**M1**	Need a fully substituted equation for the normal (in any form) or to get at least as far as finding value for m and expression for c.
	Obtain equation of normal is $y = 4\sqrt{2}x - \frac{7\sqrt{2}}{2}$ or equivalent 3 term equation	**A1**	E.g., $y = 5.66x - 4.95$. Must be $y = \ldots$
		4	

Question 120

State or imply $\frac{1+\frac{dy}{dx}}{x+y}$ as the derivative of $\ln(x+y)$	**B1**	
State or imply $6xy + 3x^2\frac{dy}{dx}$ as the derivative of $3x^2y$	**B1**	
Substitute $(1, 0)$ and solve for $\frac{dy}{dx}$	**M1**	Having the correct form for at least one of the above.
Obtain $\frac{dy}{dx} = \frac{1}{2}$ or 0.5	**A1**	
Alternative Method for Question 3:		
Rewrite as $x + y = e^{3x^2y}$ and state or imply $1+\frac{dy}{dx}$ as the derivative of the LHS	**B1**	
State or imply $\left(6xy + 3x^2\frac{dy}{dx}\right)e^{3x^2y}$ as the derivative of e^{3x^2y}	**B1**	
Substitute $(1, 0)$ and solve for $\frac{dy}{dx}$	**M1**	Having the correct form for at least one of the above.
Obtain $\frac{dy}{dx} = \frac{1}{2}$ or 0.5	**A1**	
	4	

Question 121

State or imply $\dfrac{1+2\dfrac{dy}{dx}}{x+2y}$ as the derivative of ln $(x + 2y)$	**B1**	
State derivative of xy^2 is $x2y\dfrac{dy}{dx}+y^2$	**B1**	$x^2y\dfrac{dy}{dx}+y^2$
Obtain $y^2+2xy\dfrac{dy}{dx}+\dfrac{1+2\dfrac{dy}{dx}}{x+2y}=0$	**B1**	OE May be implied by correct final answer.
Obtain $y=\dfrac{1}{2}e$ when $x = 0$	**B1**	OE Allow $\frac{1}{2}\times 2.718$ or 1.36 or better e.g. 1.359… May be implied by correct final answer.
Obtain $\dfrac{dy}{dx}=-\dfrac{1}{8}\left(e^3+4\right)$	**B1**	OE Accept AWRT –3.01. ISW.
	5	

Question 122

(a)	State or imply $y+x\dfrac{dy}{dx}$ as the derivative of xy	**B1**	
	State or imply $2ye^{-x}\dfrac{dy}{dx}-y^2e^{-x}$ as the derivative of y^2e^{-x}	**B1**	
	Equate attempted derivative to zero and solve for $\dfrac{dy}{dx}$	**M1**	Need implicit differentiation and attempt at a product.
	Obtain $\dfrac{dy}{dx}=\dfrac{y^2-ye^x}{xe^x+2y}$ from correct working	**A1**	AG Need to see sufficient correct detail.
	Alternative Method for Question 5(a)		
	State or imply $xe^x\dfrac{dy}{dx}+y\left(xe^x+e^x\right)$ as the derivative of xye^x	**B1**	Using $xye^x+y^2-4e^x=0$.
	State or imply $2y\dfrac{dy}{dx}-4e^x$ as the derivative of y^2-4e^x	**B1**	Using $xye^x+y^2-4e^x=0$.
	Equate attempted derivative to zero and solve for $\dfrac{dy}{dx}$	**M1**	Must make use of $4-xy=y^2e^{-x}$
	Obtain $\dfrac{dy}{dx}=\dfrac{y^2-ye^x}{xe^x+2y}$ from correct working	**A1**	AG Need to see sufficient correct detail.
		4	

(b)	Obtain one correct gradient	**B1**	E.g. $\dfrac{1}{2}$ at $(0, 2)$.
	Obtain second correct gradient	**B1**	E.g. $\dfrac{-3}{2}$ at $(0, -2)$.
		2	

Question 123

Differentiate $\cos^2 x$ to obtain $-2\sin x\cos x$	**B1**	OE Could be stated as $-\sin 2x$.
Use correct product rule	***M1**	With a '+' in the middle.
Obtain derivative $-10\sin 2x\sin x\cos x+10\cos 2x\cos^2 x$	**A1**	OE
Equate derivative to zero and obtain an equation in one trig function	**DM1**	Trigonometry formulas used need to be correct.
Obtain $3\tan^2 x = 1$, $4\sin^2 x = 1$ or $4\cos^2 x = 3$ or $\cos 3x = 0$	**A1**	OE
Obtain $x = \frac{1}{6}\pi$ only	**A1**	
Alternative Method for the Question 11(a)		
Use double angle formula to obtain $y = 10\sin x\cos^3 x$	**B1**	
Use correct product rule	***M1**	
Obtain derivative $10\cos^4 x - 30\sin^2 x\cos^2 x$	**A1**	OE
Equate derivative to zero and obtain an equation in one trig function	**DM1**	Trigonometry formulas used need to be correct.
Obtain $3\tan^2 x = 1$, $4\sin^2 x = 1$ or $4\cos^2 x = 3$ or $\cos 3x = 0$	**A1**	OE
Obtain $x = \frac{1}{6}\pi$ only	**A1**	

Question 124

$\frac{d}{dx}\sqrt{\sin 2x} = \frac{2\cos 2x}{2\sqrt{\sin 2x}}$ Accept $k\frac{\cos 2x}{\sqrt{\sin 2x}}$	**B1**	SOI
Use correct product rule	**M1***	
Obtain correct derivative in any form	**A1**	E.g. $\frac{dy}{dx} = -\sin x\sqrt{\sin 2x} + \cos x \times \frac{2\cos 2x}{2\sqrt{\sin 2x}}$.
Equate the derivative to zero and obtain a horizontal equation	**DM1**	E.g. $-\sin a\sin 2a + \cos a\cos 2a = 0$
Use correct trig formulae to obtain an equation in one trig function	**DM1**	E.g. $\cos 3a = 0$ or $\sin^2 a = \frac{1}{4}$.
Obtain $a = \frac{1}{6}\pi$	**A1**	Exact answer in radians only.
	6	

Question 125

At $(\mathrm{e}, 3)$, $\tan t = 1$ or $t = \frac{\pi}{4}$	**B1**	SOI
$\frac{\mathrm{d}x}{\mathrm{d}t} = \sec^2 t \times \mathrm{e}^{\tan t}$	**B1**	
$\frac{\mathrm{d}y}{\mathrm{d}t} = 6\tan t \times \sec^2 t$	**B1**	
$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{6\tan t \times \sec^2 t}{\sec^2 t \times \mathrm{e}^{\tan t}}$ $\left(= 6\tan t \times \mathrm{e}^{-\tan t}\right)$	***M1**	Correct use of *their* derivatives.
$y - 3 = \frac{6}{e}(x - \mathrm{e})$	**DM1**	Substitute for *t*, and use correct method for the equation of the line.
$y = \frac{6}{\mathrm{e}}x - 3$	**A1**	Or exact equivalent.

Alternative Method for Question 4

Correct cartesian form, e.g. $x = \mathrm{e}^{\sqrt{\frac{y}{3}}}$ or $\ln x = \sqrt{\frac{y}{3}}$ or $y = 3(\ln x)^2$	**B1**	SOI
Differentiate function of a function	***M1**	Complete method.
Obtain $\frac{\mathrm{d}x}{\mathrm{d}y} = k\sqrt{\frac{e}{y}}\mathrm{e}^{\sqrt{\frac{y}{3}}}$ or $\frac{\mathrm{d}y}{\mathrm{d}x} = k \times \frac{1}{x}\ln x$	**A1**	
Obtain $\frac{\mathrm{d}x}{\mathrm{d}y} = \frac{1}{6}\sqrt{\frac{3}{y}}\mathrm{e}^{\sqrt{\frac{y}{3}}}$ or $\frac{\mathrm{d}y}{\mathrm{d}x} = 6 \times \frac{1}{x}\ln x$	**A1**	
$y - 3 = \frac{6}{e}(x - \mathrm{e})$	**DM1**	Use correct method for the equation of the line.
$y = \frac{6}{\mathrm{e}}x - 3$	**A1**	Or exact equivalent.
	6	