A-level

Topic : Integral Calculus

May 2013-May 2025

Answers

Question 1

(a)	Carry out integration by parts and reach $ax^2\ln x + b\int \frac{1}{2}x^2\mathrm{d}x$	M1*	
	Obtain $2x^2\ln x - \int \frac{1}{x} . 2x^2 \, \mathrm{d}x$	A1	
	Obtain $2x^2\ln x - x^2$	A1	
	Use limits, having integrated twice	M1 (dep*)	
	Confirm given result $56\ln 2 - 12$	A1	[5]
(b)	State or imply $\frac{\mathrm{d}u}{\mathrm{d}x} = 4\cos 4x$	B1	
	Carry out complete substitution except limits	M1	
	Obtain $\int(\frac{1}{4} - \frac{1}{4}u^2)\,\mathrm{d}u$ or equivalent	A1	
	Integrate to obtain form $k_1u + k_2u^3$ with non-zero constants k_1, k_2	M1	
	Use appropriate limits to obtain $\frac{11}{96}$	A1	[5]

Question 2

(i)	Use correct quotient or chain rule to differentiate sec x	M1	
	Obtain given derivative, sec x tan x, correctly	A1	
	Use chain rule to differentiate y	M1	
	Obtain the given answer	A1	[4]
(ii)	Using $\mathrm{d}x\sqrt{3}\sec^2\theta\ \mathrm{d}\theta$, or equivalent, express integral in terms of θ and $\mathrm{d}\theta$	M1	
	Obtain $\int\sec\theta\ \mathrm{d}\theta$	A1	
	Use limits $\frac{1}{6}\pi$ and $\frac{1}{3}\pi$ correctly in an integral form of the form $k\ln(\sec\theta + \tan\theta)$	M1	
	Obtain a correct exact final answer in the given form, e.g. $\ln\left(\frac{2+\sqrt{3}}{\sqrt{3}}\right)$	A1	[4]

Question 3

(i)	State $R = 2$	B1	
	Use trig formula to find α	M1	
	Obtain $\alpha = \frac{1}{6}\pi$ with no errors seen	A1	[3]
(ii)	Substitute denominator of integrand and state integral $k\tan(x - \alpha)$	M1*	
	State correct indefinite integral $\frac{1}{4}\tan\left(x - \frac{1}{6}\pi\right)$	A1√	
	Substitute limits	M1 (dep*)	
	Obtain the given answer correctly	A1	[4]

Question 4

Integrate by parts and reach $kx^{\frac{1}{2}}\ln x - m\int x^{\frac{1}{2}}.\frac{1}{x}\,dx$ M1*

Obtain $2x^{\frac{1}{2}}\ln x - 2\int \frac{1}{x^{\frac{1}{2}}}\,dx$, or equivalent A1

Integrate again and obtain $2x^{\frac{1}{2}}\ln x - 4x^{\frac{1}{2}}$, or equivalent A1

Substitute limits $x = 1$ and $x = 4$, having integrated twice M1(dep*)

Obtain answer $4(\ln 4 - 1)$, or exact equivalent A1

Question 5

(i) Use Pythagoras M1

Use the sin2A formula M1

Obtain the given result A1 **[3]**

(ii) Integrate and obtain a $k\ln\sin\theta$ or $m\ln\cos\theta$ term, or obtain integral of the form $p\ln\tan\theta$ M1*

Obtain indefinite integral $\frac{1}{2}\ln\sin\theta - \frac{1}{2}\ln\cos\theta$, or equivalent, or $\frac{1}{2}\ln\tan\theta$ A1

Substitute limits correctly M1(dep)*

Obtain the given answer correctly having shown appropriate working A1 **[4]**

Question 6

Carry out complete substitution including the use of $\frac{du}{dx} = 3$ M1

Obtain $\int\left(\frac{1}{3} - \frac{1}{3u}\right)du$ A1

Integrate to obtain form $k_1u + k_2\ln u$ or $k_1u + k_2\ln 3u$ where $k_1k_2 \neq 0$ M1

Obtain $\frac{1}{3}(3x+1) - \frac{1}{3}\ln(3x+1)$ or equivalent, condoning absence of modulus signs and $+c$ A1 [4]

Question 7

State $\frac{du}{dx} = 3\sec^2 x$ or equivalent B1

Express integral in terms of u and du (accept unsimplified and without limits) M1

Obtain $\int \frac{1}{3}u^{\frac{1}{2}}\,du$ A1

Integrate $Cu^{\frac{1}{2}}$ to obtain $\frac{2C}{3}u^{\frac{3}{2}}$ M1

Obtain $\frac{14}{9}$ A1 [5]

Question 8

(i)	Use product rule	M1	
	Obtain derivative in any correct form	A1	
	Differentiate first derivative using the product rule	M1	
	Obtain second derivative in any correct form, e.g. $-\frac{1}{2}\sin\frac{1}{2}x - \frac{1}{4}x\cos\frac{1}{2}x - \frac{1}{2}\sin\frac{1}{2}x$	A1	
	Verify the given statement	A1	**5**
(ii)	Integrate and reach $kx\sin\frac{1}{2}x + l\int\sin\frac{1}{2}x\,dx$	M1*	
	Obtain $2x\sin\frac{1}{2}x - 2\int\sin\frac{1}{2}x\,dx$, or equivalent	A1	
	Obtain indefinite integral $2x\sin\frac{1}{2}x + 4\cos\frac{1}{2}x$	A1	
	Use correct limits $x = 0, x = \pi$ correctly	M1(dep*)	
	Obtain answer $2\pi - 4$, or exact equivalent	A1	**5**

Question 9

(i)	Use a correct method for finding a constant	M1	
	Obtain one of $A = 3, B = 3, C = 0$	A1	
	Obtain a second value	A1	
	Obtain a third value	A1	**4**
(ii)	Integrate and obtain term $-3\ln(2 - x)$	B1√	
	Integrate and obtain term of the form $k\ln(2 + x^2)$	M1	
	Obtain term $\frac{3}{2}\ln(2 + x^2)$	A1√	
	Substitute limits correctly in an integral of the form $a\ln(2 - x) + b\ln(2 + x^2)$, where $ab \neq 0$	M1	
	Obtain given answer after full and correct working	A1	**5**

Question 10

(i)	Substitute for x and dx throughout using $u = \sin x$ and $du = \cos x\,dx$, or equivalent	M1	
	Obtain integrand e^{2u}	A1	
	Obtain indefinite integral $\frac{1}{2}e^{2u}$	A1	
	Use limits $u = 0, u = 1$ correctly, or equivalent	M1	
	Obtain answer $\frac{1}{2}(e^2 - 1)$, or exact equivalent	A1	**5**
(ii)	Use chain rule or product rule	M1	
	Obtain correct terms of the derivative in any form, e.g. $2\cos x\, e^{2\sin x}\cos x - e^{2\sin x}\sin x$	A1 + A1	
	Equate derivative to zero and obtain a quadratic equation in $\sin x$	M1	
	Solve a 3-term quadratic and obtain a value of x	M1	
	Obtain answer 0.896	A1	**6**

Question 11

(i)	State or imply ordinates 2, 1.1547…, 1, 1.1547…	B1	
	Use correct formula, or equivalent, with $h = \frac{1}{6}\pi$ and four ordinates	M1	
	Obtain answer 1.95	A1	**[3]**
(ii)	Make recognisable sketch of $y = \operatorname{cosec} x$ for the given interval	B1	
	Justify a statement that the estimate will be an overestimate	B1	**[2]**

Question 12

(i)	State or imply correct ordinates 1, 0.94259…, 0.79719…, 0.62000…	B1	
	Use correct formula or equivalent with $h = 0.1$ and four y values	M1	
	Obtain 0.255 with no errors seen	A1	[3]
(ii)	Obtain or imply $a = -6$	B1	
	Obtain x^4 term including correct attempt at coefficient	M1	
	Obtain or imply $b = 27$	A1	
	<u>Either</u> Integrate to obtain $x - 2x^3 + \frac{27}{5}x^5$, following their values of a and b	B1√	
	Obtain 0.259	B1	
	<u>Or</u> Use correct trapezium rule with at least 3 ordinates	M1	
	Obtain 0.259 (from 4)	A1	[5]

Question 13

State or imply $\frac{du}{dx} = e^x$	B1	
Substitute throughout for x and dx	M1	
Obtain $\int \frac{u}{u^2 + 3u + 2}\,du$ or equivalent (ignoring limits so far)	A1	
State or imply partial fractions of form $\frac{A}{u+2} + \frac{B}{u+1}$, following their integrand	B1	
Carry out a correct process to find at least one constant for their integrand	M1	
Obtain correct $\frac{2}{u+2} - \frac{1}{u+1}$	A1	
Integrate to obtain $a\ln(u+2) + b\ln(u+1)$	M1	
Obtain $2\ln(u+2) - \ln(u+1)$ or equivalent, follow their A and B	A1√	
Apply appropriate limits and use at least one logarithm property correctly	M1	
Obtain given answer $\ln\frac{8}{5}$ legitimately	A1	[10]

Question 14

Attempt calculation of at least 3 ordinates	M1	
Obtain 9, 7, 1, 17	A1	
Use trapezium rule with $h=1$	M1	
Obtain $\frac{1}{2}(9+14+2+17)$ or equivalent and hence 21	A1	[4]

Question 15

(a)	Use identity $\tan^2 2x = \sec^2 2x - 1$	B1	
	Obtain integral of form $ax + b\tan 2x$	M1	
	Obtain correct $3x + \frac{1}{2}\tan 2x$, condoning absence of $+c$	A1	[3]
(b)	State $\sin x \cos \frac{1}{2}\pi + \cos x \ \sin \frac{1}{6}\pi$	B1	
	Simplify integrand to $\cos \frac{1}{6}\pi + \frac{\cos x \sin \frac{1}{6}\pi}{\sin x}$ or equivalent	B1	
	Integrate to obtain at least term of form a In(sin x)	*M1	
	Apply limits and simplify to obtain two terms	M1 dep *M	
	Obtain $\frac{1}{8}\pi\sqrt{3} - \frac{1}{2}\ln(\frac{1}{\sqrt{2}})$ or equivalent	A1	[5]

Question 16

(i)	Use product rule to find first derivative	M1	
	Obtain $2xe^{2-x} - x^2e^{2-x}$	A1	
	Confirm $x = 2$ at M	A1	[3]
(ii)	Attempt integration by parts and reach $\pm x^2e^{2-x} \pm \int 2xe^{2-x}\,dx$	*M1	
	Obtain $-x^2e^{2-x} + \int 2xe^{2-x}\,dx$	A1	
	Attempt integration by parts and reach $\pm x^2e^{2-x} \pm 2xe^{2-x} \pm 2e^{2-x}$	*M1	
	Obtain $-x^2e^{2-x} - 2xe^{2-x} - 2e^{2-x}$	A1	
	Use limits 0 and 2 having integrated twice	M1 dep *M	
	Obtain $2e^2 - 10$	A1	[6]

Question 17

State or imply ordinates 0, 0.405465…, 0.623810…, 0.693147…	B1	
Use correct formula, or equivalent, with $h = \frac{1}{6}\pi$ and four ordinates	M1	
Obtain answer 0.72	A1	**[3]**

Question 18

(i)	State or imply $\mathrm{d}u = -\dfrac{1}{2\sqrt{x}}\mathrm{d}x$, or equivalent	B1	
	Substitute for x and $\mathrm{d}x$ throughout	M1	
	Obtain integrand $\dfrac{\pm 2(2-u)^2}{u}$, or equivalent	A1	
	Show correct working to justify the change in limits and obtain the given answer with no errors seen	A1	**[4]**
(ii)	Integrate and obtain at least two terms of the form $a\ln u$, bu, and cu^2	M1*	
	Obtain indefinite integral $8\ln u - 8u + u^2$, or equivalent	A1	
	Substitute limits correctly	M1(dep*)	
	Obtain the given answer correctly having shown sufficient working	A1	**[4]**

Question 19

(i)	State or imply $\mathrm{f}(x) \equiv \dfrac{A}{2x-1} + \dfrac{B}{x+2} + \dfrac{C}{(x+2)^2}$	B1	
	Use a relevant method to determine a constant	M1	
	Obtain one of the values $A = 2$, $B = -1$, $C = 3$	A1	
	Obtain the remaining values A1 +	A1	**5**
	[Apply an analogous scheme to the form $\dfrac{A}{2x-1} + \dfrac{Dx+E}{(x+2)^2}$; the values being $A = 2$, $D = -1$, $E = 1$.]		
(ii)	Integrate and obtain terms $\dfrac{1}{2}\cdot 2\ln(2x-1) - \ln(x+2) - \dfrac{3}{x+2}$	B1√ + B1√ + B1√	
	Use limits correctly, namely substitution must be seen in at least two of the partial fractions to obtain M1 Integrate all 3 partial fractions and substitute in all three partial fractions for A1 since AG.	M1	
	Obtain the given answer following full and exact working	A1	**5**

Question 20

(i)	Use the quotient rule	**M1**	
	Obtain correct derivative in any form	**A1**	
	Equate derivative to zero and solve for x	**M1**	
	Obtain answer $x = \sqrt[3]{2}$, or exact equivalent	**A1**	[4]
(ii)	State or imply indefinite integral is of the form $k\ln(1+x^3)$	**M1**	
	State indefinite integral $\frac{1}{3}\ln(1+x^3)$	**A1**	
	Substitute limits correctly in an integral of the form $k\ln(1+x^3)$	**M1**	
	State or imply that the area of R is equal to $\frac{1}{3}\ln(1+p^3)-\frac{1}{3}\ln 2$, or equivalent	**A1**	
	Use a correct method for finding p from an equation of the form $\ln(1+p^3)=a$ or $\ln((1+p^3)/2)=b$	**M1**	
	Obtain answer $p = 3.40$	**A1**	[2]

Question 21

State $\mathrm{d}u = 3\sin x\,\mathrm{d}x$ or equivalent	**B1**	
Use identity $\sin 2x = 2\sin x\cos x$	**B1**	
Carry out complete substitution, for x and $\mathrm{d}x$	**M1**	
Obtain $\int \frac{8-2u}{\sqrt{u}}\,\mathrm{d}u$, or equivalent	**A1**	
Integrate to obtain expression of form $au^{\frac{1}{2}} + bu^{\frac{3}{2}}$, $ab \neq 0$	**M1***	
Obtain correct $16u^{\frac{1}{2}} - \frac{4}{3}u^{\frac{3}{2}}$	**A1**	
Apply correct limits correctly	**dep M1***	
Obtain $\frac{20}{3}$ or exact equivalent	**A1**	[8]

Question 22

(i)	Either	Substitute $x=-1$ and evaluate	**M1**	
		Obtain 0 and conclude $x+1$ is a factor	**A1**	
	Or	Divide by $x+1$ and obtain a constant remainder	**M1**	
		Obtain remainder = 0 and conclude $x+1$ is a factor	**A1**	[2]

(ii)	Attempt division, or equivalent, at least as far as quotient $4x^2+kx$	**M1**	
	Obtain complete quotient $4x^2-5x-6$	**A1**	
	State form $\frac{A}{x+1}+\frac{B}{x-2}+\frac{C}{4x+3}$	**A1**	
	Use relevant method for finding at least one constant	**M1**	
	Obtain one of $A=-2, B=1, C=8$	**A1**	
	Obtain all three values	**A1**	
	Integrate to obtain three terms each involving natural logarithm of linear form	**M1**	
	Obtain $-2\ln(x+1)+\ln(x-2)+2\ln(4x+3)$, condoning no use of modulus signs and absence of $\ldots+c$	**A1**	[8]

Question 23

(i)	State or imply $dx=\sqrt{3}\sec^2\theta\, d\theta$	**B1**	
	Substitute for x and dx throughout	**M1**	
	Obtain the given answer correctly	**A1**	[3]
(ii)	Replace integrand by $\frac{1}{2}\cos 2\theta+\frac{1}{2}$	**B1**	
	Obtain integral $\frac{1}{4}\sin 2\theta+\frac{1}{2}\theta$	**B1**√	
	Substitute limits correctly in an integral of the form $c\sin 2\theta+b\theta$, where $cb\neq 0$	**M1**	
	Obtain answer $\frac{1}{12}\sqrt{3}\pi+\frac{3}{8}$, or exact equivalent	**A1**	[4]

Question 24

(i)	State or obtain $A=3$	**B1**	
	Use a relevant method to find a constant	**M1**	
	Obtain one of $B=-4$, $C=4$ and $D=0$	**A1**	
	Obtain a second value	**A1**	
	Obtain the third value	**A1**	[5]
(ii)	Integrate and obtain $3x-4\ln x$	**B1**√	
	Integrate and obtain term of the form $k\ln(x^2+2)$	**M1**	
	Obtain term $2\ln(x^2+2)$	**A1**√	
	Substitute limits in an integral of the form $ax+b\ln x+c\ln(x^2+2)$, where $abc\neq 0$	**M1**	
	Obtain given answer $3-\ln 4$ after full and correct working	**A1**	[5]

Question 25

Integrate by parts and reach $axe^{-2x} + b\int e^{-2x}\,dx$	**M1**
Obtain $-\frac{1}{2}xe^{-2x} + \frac{1}{2}\int e^{-2x}\,dx$, or equivalent	**A1**
Complete the integration correctly, obtaining $-\frac{1}{2}xe^{-2x} - \frac{1}{4}e^{-2x}$, or equivalent	**A1**
Use limits $x = 0$ and $x = \frac{1}{2}$ correctly, having integrated twice	**M1**
Obtain answer $\frac{1}{4} - \frac{1}{2}e^{-1}$, or exact equivalent	**A1**
	[5]

Question 26

Integrate by parts and reach $ax^2\cos 2x + b\int x\cos 2x\,dx$	**M1***	
Obtain $-\frac{1}{2}x^2\cos 2x + \int x\cos 2x$, or equivalent	**A1**	
Complete the integration and obtain $-\frac{1}{2}x^2\cos 2x + \frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x$, or equivalent	**A1**	
Use limits correctly having integrated twice	**DM1***	
Obtain answer $\frac{1}{8}(\pi^2 - 4)$, or exact equivalent, with no errors seen	**A1**	[5]

Question 27

(i)	State or imply the form $A + \dfrac{B}{2x+1} + \dfrac{C}{x+2}$	**B1**	
	State or obtain $A = 2$	**B1**	
	Use a correct method for finding a constant	**M1**	
	Obtain one of $B = 1$, $C = -2$	**A1**	
	Obtain the other value	**A1**	[5]
(ii)	Integrate and obtain terms $2x + \frac{1}{2}\ln(2x+1) - 2\ln(x+2)$	**B3**√	
	Substitute correct limits correctly in an integral with terms $a\ln(2x+1)$ and $b\ln(x+2)$, where $ab \neq 0$	**M1**	
	Obtain the given answer after full and correct working	**A1**	[5]

Question 28

(i)	State or imply $du = 2x\ dx$, or equivalent	**B1**
	Substitute for x and dx throughout	**M1**
	Reduce to the given form and justify the change in limits	**A1**
		[3]
(ii)	Convert integrand to a sum of integrable terms and attempt integration	**M1**
	Obtain integral $\frac{1}{2}\ln u + \frac{1}{u} - \frac{1}{4u^2}$, or equivalent	**A1 + A1**
	(deduct A1 for each error or omission)	
	Substitute limits in an integral containing two terms of the form $a\ln u$ and bu^{-2}	**M1**
	Obtain answer $\frac{1}{2}\ln 2 - \frac{5}{16}$, exact simplified equivalent	**A1**
		[5]

Question 29

(i)	*EITHER*: Use tan $2A$ formula to express LHS in terms of $\tan\theta$	**M1**	
	Express as a single fraction in any correct form	**A1**	
	Use Pythagoras or cos $2A$ formula	**M1**	
	Obtain the given result correctly	**A1**	
	OR: Express LHS in terms of $\sin 2\theta$, $\cos 2\theta$, $\sin\theta$ and $\cos\theta$	**M1**	
	Express as a single fraction in any correct form	**A1**	
	Use Pythagoras or cos $2A$ formula or $\sin(A - B)$ formula	**M1**	
	Obtain the given result correctly	**A1**	[4]
(ii)	Integrate and obtain a term of the form $a\ln(\cos 2\theta)$ or $b\ln(\cos\theta)$ (or secant equivalents)	**M1***	
	Obtain integral $-\frac{1}{2}\ln(\cos 2\theta) + \ln(\cos\theta)$, or equivalent	**A1**	
	Substitute limits correctly (expect to see use of <u>both</u> limits)	**DM1**	
	Obtain the given answer following full and correct working	**A1**	[4]

Question 30

(i)	Use the correct product rule	**M1**	
	Obtain correct derivative in any form, e.g. $(2-2x)e^{\frac{1}{2}x} + \frac{1}{2}(2x - x^2)e^{\frac{1}{2}x}$	**A1**	
	Equate derivative to zero and solve for x	**M1**	
	Obtain $x = \sqrt{5} - 1$ only	**A1**	[4]
(ii)	Integrate by parts and reach $a(2x - x^2)e^{\frac{1}{2}x} + b\int(2-2x)e^{\frac{1}{2}x}\ dx$	**M1***	
	Obtain $2e^{\frac{1}{2}x}(2x - x^2) - 2\int(2-2x)e^{\frac{1}{2}x}dx$, or equivalent	**A1**	
	Complete the integration correctly, obtaining $(12x - 2x^2 - 24)e^{\frac{1}{2}x}$, or equivalent	**A1**	
	Use limits $x = 0$, $x = 2$ correctly having integrated by parts twice	**DM1**	
	Obtain answer $24 - 8e$, or <u>exact</u> simplified equivalent	**A1**	[5]

Question 31

(i)	State or imply $du = \frac{1}{2\sqrt{x}}dx$	**B1**	
	Substitute for x and dx throughout	**M1**	
	Justify the change in limits and obtain the given answer	**A1**	[3]
(ii)	Convert integrand into the form $A + \frac{B}{u+1}$	**M1***	
	Obtain integrand $A = 1$, $B = -2$	**A1**	
	Integrate and obtain $u - 2\ln(u+1)$	**A1√ + A1√**	
	Substitute limits correctly in an integral containing terms au and $b\ln(u+1)$, where $ab \neq 0$	**DM1**	
	Obtain the given answer following full and correct working	**A1**	[6]

Question 32

(i)	State or imply derivative is $2\frac{\ln x}{x}$	**B1**
	State or imply gradient of the normal at $x = e$ is $-\frac{1}{2}e$, or equivalent	**B1**
	Carry out a complete method for finding the x-coordinate of Q	**M1**
	Obtain answer $x = e + \frac{2}{e}$, or exact equivalent	**A1**
	Total:	**4**
(ii)	Justify the given statement by integration or by differentiation	**B1**
	Total:	**1**
(iii)	Integrate by parts and reach $ax(\ln x)^2 + b\int x.\frac{\ln x}{x}\,dx$	**M1***
	Complete the integration and obtain $x(\ln x)^2 - 2x\ln x + 2x$, or equivalent	**A1**
	Use limits $x = 1$ and $x = e$ correctly, having integrated twice	**DM1**
	Obtain exact value $e - 2$	**A1**
	Use x- coordinate of Q found in part **(i)** and obtain final answer $e - 2 + \frac{1}{e}$	**B1√**

Question 33

(i)	Remove logarithms correctly and obtain $e^x = \frac{1-y}{y}$	B1
	Obtain the **given answer** $y = \frac{e^{-x}}{1+e^{-x}}$ following full working	B1
	Total:	**2**
(ii)	State integral $k\ln(1+e^{-x})$ where $k = \pm 1$	*M1
	State correct integral $-\ln(1+e^{-x})$	A1
	Use limits correctly	DM1
	Obtain the **given answer** $\ln\left(\frac{2e}{e+1}\right)$ following full working	A1
	Total:	**4**

Question 34

State or imply $du = -\sin x \, dx$	**B1**
Using correct double angle formula, express the integral in terms of u and du	**M1**
Obtain integrand $\pm(2u^2 - 1)^2$	**A1**
Change limits and obtain correct integral $\int_{\frac{1}{\sqrt{2}}}^{1} (2u^2 - 1)^2 \, du$ with no errors seen	**A1**
Substitute limits in an integral of the form $au^5 + bu^3 + cu$	**M1**
Obtain answer $\frac{1}{15}(7 - 4\sqrt{2})$, or exact simplified equivalent	**A1**
Total:	**6**

(ii)	Use product rule and chain rule at least once	**M1**
	Obtain correct derivative in any form	**A1**
	Equate derivative to zero and use trig formulae to obtain an equation in $\cos x$ and $\sin x$	**M1**
	Use correct methods to obtain an equation in $\cos x$ or $\sin x$ only	**M1**
	Obtain $10\cos^2 x = 9$ or $10\sin^2 x = 1$, or equivalent	**A1**
	Obtain answer 0.32	**A1**
	Total:	**6**

Question 35

(i)	Use quotient or chain rule	**M1**
	Obtain given answer correctly	**A1**
	Total:	**2**
(ii)	*EITHER*: Multiply numerator and denominator of LHS by $1+\sin\theta$	**(M1**
	Use Pythagoras and express LHS in terms of sec θ and $\tan\theta$	**M1**
	Complete the proof	**A1)**
	OR1: Express RHS in terms of cos θ and sin θ	**(M1**
	Use Pythagoras and express RHS in terms of sin θ	**M1**
	Complete the proof	**A1)**
(iii)	Use the identity and obtain integral $2\tan\theta + 2\sec\theta - \theta$	**B2**
	Use correct limits correctly in an integral containing terms a $\tan\theta$ and b $\sec\theta$	**M1**
	Obtain answer $2\sqrt{2} - \frac{1}{4}\pi$	**A1**
	Total:	**4**

Question 36

Integrate by parts and reach $a\theta\cos\frac{1}{2}\theta + b\int\cos\frac{1}{2}\theta\ d\theta$	***M1**
Complete integration and obtain indefinite integral $-2\theta\cos\frac{1}{2}\theta + 4\sin\frac{1}{2}\theta$	**A1**
Substitute limits correctly, having integrated twice	**DM1**
Obtain final answer $(4-\pi)/\sqrt{2}$, or exact equivalent	**A1**
Total:	**4**

Question 37

State or imply ordinates 1.6487…, 1.3591…, 1.4938…	**B1**
Use correct formula, or equivalent, with $h = 1$ and three ordinates	**M1**
Obtain answer 2.93 only	**A1**
Total:	**3**

Explain why the estimate would be less than E	**B1**

Question 38

(i)	State or imply the form $\frac{A}{x} + \frac{B}{x^2} + \frac{C}{3x+2}$	**B1**
	Use a relevant method to determine a constant	**M1**
	Obtain one of the values $A = 3$, $B = -2$, $C = -6$	**A1**
	Obtain a second value	**A1**
	Obtain the third value [Mark the form $\frac{Ax+B}{x^2} + \frac{C}{3x+2}$ using same pattern of marks.]	**A1**
	Total:	**5**
(ii)	Integrate and obtain terms $3\ln x = \frac{2}{x} - 2\ln(3x+2)$ [The FT is on A, B and C] **Note:** Candidates who integrate the partial fraction $\frac{3x-2}{x^2}$ by parts should obtain $3\ln x + \frac{2}{x} - 3$ or equivalent	**B3 FT**
	Use limits correctly, having integrated all the partial fractions, in a solution containing terms $a\ln x + \frac{b}{x} + c\ln(3x+2)$	**M1**
	Obtain the given answer following full and exact working	**A1**
	Total:	**5**

Question 39

(i)	Use a relevant method to determine a constant	**M1**
	Obtain one of the values $A = 2$, $B = 2$, $C = -1$	**A1**
	Obtain a second value	**A1**
	Obtain the third value	**A1**
		4
(ii)	Integrate and obtain terms $2x + 2\ln(x+2) - \frac{1}{2}\ln(2x-1)$ (deduct **B1** for each error or omission) [The FT is on A, B and C]	**B2 FT**
	Substitute limits correctly in an integral containing terms $a\ln(x+2)$ and $b\ln(2x-1)$, where $ab \neq 0$	***M1**
	Use at least one law of logarithms correctly	**DM1**
	Obtain the given answer after full and correct working	**A1**
		5

Question 40

(i)	Use correct product or quotient rule	**M1**
	Obtain correct derivative in any form	**A1**
	Equate derivative to zero and obtain a 3 term quadratic equation in x	**M1**
	Obtain answers $x = 2 \pm \sqrt{3}$	**A1**
		4
(ii)	Integrate by parts and reach $k(1+x^2)e^{-\frac{1}{2}x} + l\int xe^{-\frac{1}{2}x}\,dx$	***M1**
	Obtain $-2(1+x^2)e^{-\frac{1}{2}x} + 4\int xe^{-\frac{1}{2}x}\,dx$, or equivalent	**A1**
	Complete the integration and obtain $(-18 - 8x - 2x^2)e^{-\frac{1}{2}x}$, or equivalent	**A1**
	Use limits $x = 0$ and $x = 2$ correctly, having fully integrated twice by parts	**DM1**
	Obtain the given answer	**A1**
		5

Question 41

(i)	State or imply ordinates 0.915929…, 1, 1.112485…	**B1**
	Use correct formula, or equivalent, with $h = 1.2$ and three ordinates	**M1**
	Obtain answer 2.42 only	**A1**
		3
(ii)	Justify the given statement	**B1**
		1

Question 42

(i)	Use a relevant method to determine a constant	**M1**
	Obtain one of the values $A = 2$, $B = 2$, $C = -1$	**A1**
	Obtain a second value	**A1**
	Obtain the third value	**A1**
		4
(ii)	Integrate and obtain terms $2x + 2\ln(x+2) - \frac{1}{2}\ln(2x-1)$ (deduct **B1** for each error or omission) [The FT is on *A*, *B* and *C*]	**B2 FT**
	Substitute limits correctly in an integral containing terms $a\ln(x+2)$ and $b\ln(2x-1)$, where $ab \neq 0$	***M1**
	Use at least one law of logarithms correctly	**DM1**
	Obtain the given answer after full and correct working	**A1**
		5

Question 43

(i)	Use correct product or quotient rule	**M1**
	Obtain correct derivative in any form	**A1**
	Equate derivative to zero and obtain a 3 term quadratic equation in x	**M1**
	Obtain answers $x = 2 \pm \sqrt{3}$	**A1**
		4
(ii)	Integrate by parts and reach $k(1+x^2)e^{-\frac{1}{2}x} + l\int xe^{-\frac{1}{2}x}\ dx$	***M1**
	Obtain $-2(1+x^2)e^{-\frac{1}{2}x} + 4\int xe^{-\frac{1}{2}x}\ dx$, or equivalent	**A1**
	Complete the integration and obtain $(-18-8x-2x^2)e^{-\frac{1}{2}x}$, or equivalent	**A1**
	Use limits $x = 0$ and $x = 2$ correctly, having fully integrated twice by parts	**DM1**
	Obtain the given answer	**A1**
		5

Question 44

State or imply ordinates 1, 0.8556…, 0.6501…, 0	**B1**
Use correct formula, or equivalent, with $h = \frac{1}{12}\pi$ and four ordinates	**M1**
Obtain answer 0.525	**A1**
	3

Question 45

(i)	State correct expansion of $\cos(3x+x)$ or $\cos(3x-x)$	**B1**
	Substitute in $\frac{1}{2}(\cos 4x + \cos 2x)$	**M1**
	Obtain the given identity correctly **AG**	**A1**
		3
(ii)	Obtain integral $\frac{1}{8}\sin 4x + \frac{1}{4}\sin 2x$	**B1**
	Substitute limits correctly	**M1**
	Obtain the given answer following full, correct and exact working **AG**	**A1**
		3

Question 46

(i)	State or imply the form $\frac{A}{2x+1} + \frac{Bx+C}{x^2+9}$	**B1**
	Use a correct method for finding a constant	**M1**
	Obtain one of $A = 3$, $B = 1$ and $C = 0$	**A1**
	Obtain a second value	**A1**
	Obtain the third value	**A1**
		5

(ii)	Integrate and obtain term $\frac{3}{2}\ln(2x+1)$ (FT on A value)	**B1 FT**
	Integrate and obtain term of the form $k\ln(x^2+9)$	**M1**
	Obtain term $\frac{1}{2}\ln(x^2+9)$ (FT on B value)	**A1 FT**
	Substitute limits correctly in an integral of the form $a\ln(2x+1)+b\ln(x^2+9)$, where $ab \neq 0$	**M1**
	Obtain answer ln 45 after full and correct working	**A1**
		5

Question 47

(i)	State or imply $dx = -2\cos\theta\sin\theta\,d\theta$, or equivalent	**B1**
	Substitute for x and dx, and use Pythagoras	**M1**
	Obtain integrand $\pm 2\cos^2\theta$	**A1**
	Justify change of limits and obtain given answer correctly	**A1**
		4
(ii)	Obtain indefinite integral of the form $a\theta + b\sin 2\theta$	**M1***
	Obtain $\theta + \frac{1}{2}\sin 2\theta$	**A1**
	Use correct limits correctly	**M1(dep*)**
	Obtain answer $\frac{1}{6}\pi$ with no errors seen	**A1**
		4

Question 48

(i)	Use correct double angle formulae and express LHS in terms of $\cos x$ and $\sin x$	M1	$\dfrac{2\sin x - 2\sin x\cos x}{1-\left(2\cos^2 x - 1\right)}$
	Obtain a correct expression	A1	
	Complete method to get correct denominator e.g. by factorising to remove a factor of $1-\cos x$	M1	
	Obtain the given RHS correctly *OR (working R to L)*:	A1	
	$\dfrac{\sin x}{1+\cos x}\times\dfrac{1-\cos x}{1-\cos x}=\dfrac{\sin x-\sin x\cos x}{1-\cos^2 x}$ $=\dfrac{2\sin x-2\sin x\cos x}{2-2\cos^2 x}$ M1A1		Given answer so check working carefully
	$=\dfrac{2\sin x-\sin 2x}{1-\cos 2x}$ M1A1		
		4	
(ii)	State integral of the form $a\ln(1+\cos x)$	M1*	If they use the substitution $u=1+\cos x$ allow M1A1 for $-\ln u$
	Obtain integral $-\ln(1+\cos x)$	A1	
	Substitute correct limits in correct order	M1(dep)*	
	Obtain answer $\ln\left(\frac{3}{2}\right)$, or equivalent	A1	
		4	

Question 49

(i)	Use correct product or quotient rule	M1	$\dfrac{dy}{dx}=-\frac{1}{3}(x+1)e^{-\frac{1}{3}x}+e^{-\frac{1}{3}x}$ or $\dfrac{dy}{dx}=\dfrac{e^{\frac{1}{3}x}-(x+1)\frac{1}{3}e^{\frac{1}{3}x}}{e^{\frac{2}{3}x}}$
	Obtain complete correct derivative in any form	A1	
	Equate derivative to zero and solve for x	M1	
	Obtain answer $x = 2$ with no errors seen	A1	
		4	
(ii)	Integrate by parts and reach $a(x+1)e^{-\frac{1}{3}x}+b\int e^{-\frac{1}{3}x}\,dx$	M1*	
	Obtain $-3(x+1)e^{-\frac{1}{3}x}+3\int e^{-\frac{1}{3}x}\,dx$, or equivalent	A1	$-3xe^{-\frac{1}{3}x}+\int 3e^{-\frac{1}{3}x}dx-3e^{-\frac{1}{3}x}$
	Complete integration and obtain $-3(x+1)e^{-\frac{1}{3}x}-9e^{-\frac{1}{3}x}$, or equivalent	A1	
	Use correct limits $x=-1$ and $x=0$ **in the correct order**, having integrated twice	M1(dep*)	
	Obtain answer $9e^{\frac{1}{3}}-12$, or equivalent	A1	
		5	

Question 50

Integrate by parts and reach $ax\sin 3x + b\int \sin 3x\,dx$	**M1***
Obtain $\frac{1}{3}x\sin 3x - \frac{1}{3}\int \sin 3x\,dx$, or equivalent	**A1**
Complete the integration and obtain $\frac{1}{3}x\sin 3x + \frac{1}{9}\cos 3x$, or equivalent	**A1**
Substitute limits correctly having integrated twice and obtained $ax \sin 3x + b \cos 3x$	**M1(dep*)**
Obtain answer $\frac{1}{18}(\pi - 2)$ OE	**A1**
Total:	**5**

Question 51

(i)	State answer $R = \sqrt{5}$	**B1**
	Use trig formulae to find $\tan \alpha$	**M1**
	Obtain $\tan\alpha = 2$	**A1**
	Total:	**3**
(ii)	State that the integrand is $3\sec^2(\theta - \alpha)$	**B1FT**
	State correct indefinite integral $3\tan(\theta - \alpha)$	**B1FT**
	Substitute limits correctly	**M1**
	Use $\tan(A \pm B)$ formula	**M1**
	Obtain the given exact answer correctly	**A1**
	Total:	**5**

Question 52

'(i)	Use product rule	M1*
	Obtain correct derivative in any form	A1
	Equate derivative to zero and obtain an equation in a single trig function	depM1*
	Obtain a correct equation, e.g. $3\tan^2 x = 2$	A1
	Obtain answer $x = 0.685$	A1
		5
(ii)	Use the given substitution and reach $a\int\left(u^2 - u^4\right)\mathrm{d}u$	M1
	Obtain correct integral with $a = 5$ and limits 0 and 1	A1
	Use correct limits in an integral of the form $a\left(\frac{1}{3}u^3 - \frac{1}{5}u^5\right)$	M1
	Obtain answer $\frac{2}{3}$	A1
		4

Question 53

(i)	State or imply the form $\dfrac{A}{2-x} + \dfrac{B}{3+2x} + \dfrac{C}{(3+2x)^2}$	B1
	Use a correct method to find a constant	M1
	Obtain one of $A = 1$, $B = -1$, $C = 3$	A1
	Obtain a second value	A1
	Obtain the third value [Mark the form $\dfrac{A}{2-x} + \dfrac{Dx+E}{(3+2x)^2}$, where $A = 1$, $D = -2$ and $E = 0$, B1M1A1A1A1 as above.]	A1
		5
(ii)	Integrate and obtain terms $-\ln(2-x) - \dfrac{1}{2}\ln(3+2x) - \dfrac{3}{2(3+2x)}$	B3ft
	Substitute correctly in an integral with terms $a\ln(2-x)$, $b\ln(3+2x)$ and $c/(3+2x)$ where $abc \neq 0$	M1
	Obtain the given answer after full and correct working [Correct integration of the A, D, E form gives an extra constant term if integration by parts is used for the second partial fraction.]	A1
		5

Question 54

(i)	Integrate by parts and reach $a\dfrac{\ln x}{x^2} + b\int \dfrac{1}{x}.\dfrac{1}{x^2}\,dx$	M1*
	Obtain $\pm\dfrac{1}{2}\dfrac{\ln x}{x^2} \pm \int \dfrac{1}{x}.\dfrac{1}{2x^2}\,dx$, or equivalent	A1
	Complete integration correctly and obtain $-\dfrac{\ln x}{2x^2} - \dfrac{1}{4x^2}$, or equivalent	A1
		3

(ii)	Substitute limits correctly in an expression of the form $a\frac{\ln x}{x^2}+\frac{b}{x^2}$ or equivalent	M1(dep*)
	Obtain the **given answer** following full and **exact** working	A1
		2

Question 55

(i)	Use correct quotient or product rule	M1
	Obtain correct derivative in any form	A1
	Equate numerator to zero	M1
	Use $\cos^2 x+\sin^2 x=1$ and solve for $\sin x$	M1
	Obtain coordinates $x=-\pi/6$ and $y=\sqrt{3}$ ISW	A1 + A1
		6

(ii)	State indefinite integral of the form $k \ln(2 + \sin x)$	M1*
	Substitute limits correctly, equate result to 1 and obtain $3 \ln(2 + \sin a) - 3 \ln 2 = 1$	A1
	Use correct method to solve for a	M1(dep*)
	Obtain answer $a = 0.913$ or better	A1
		4

Question 56

'(i)	Use product rule	M1*
	Obtain correct derivative in any form	A1
	Equate derivative to zero and obtain an equation in a single trig function	depM1*
	Obtain a correct equation, e.g. $3\tan^2 x = 2$	A1
	Obtain answer $x = 0.685$	A1
		5

'(ii)	Use the given substitution and reach $a\int(u^2 - u^4)\,du$	M1
	Obtain correct integral with $a = 5$ and limits 0 and 1	A1
	Use correct limits in an integral of the form $a\left(\frac{1}{3}u^3 - \frac{1}{5}u^5\right)$	M1
	Obtain answer $\frac{2}{3}$	A1
		4

Question 57

(i)	State or imply the form $\frac{A}{2-x}+\frac{B}{3+2x}+\frac{C}{(3+2x)^2}$	B1
	Use a correct method to find a constant	M1
	Obtain one of $A=1$, $B=-1$, $C=3$	A1
	Obtain a second value	A1
	Obtain the third value [Mark the form $\frac{A}{2-x}+\frac{Dx+E}{(3+2x)^2}$, where $A=1$, $D=-2$ and $E=0$, B1M1A1A1A1 as above.]	A1
		5
(ii)	Integrate and obtain terms $-\ln(2-x)-\frac{1}{2}\ln(3+2x)-\frac{3}{2(3+2x)}$	B3ft
	Substitute correctly in an integral with terms $a\ln(2-x)$, $b\ln(3+2x)$ and $c/(3+2x)$ where $abc \neq 0$	M1
	Obtain the given answer after full and correct working [Correct integration of the A, D, E form gives an extra constant term if integration by parts is used for the second partial fraction.]	A1
		5

Question 58

Integrate by parts and reach $ax^{-\frac{1}{2}}\ln x+b\int x^{-\frac{1}{2}}.\frac{1}{x}\mathrm{d}x$	M1*
Obtain $-2x^{-\frac{1}{2}}\ln x+2\int x^{-\frac{1}{2}}.\frac{1}{x}\mathrm{d}x$, or equivalent	A1
Complete the integration, obtaining $-2x^{-\frac{1}{2}}\ln x-4x^{-\frac{1}{2}}$, or equivalent	A1
Substitute limits correctly, having integrated twice	M1(dep*)
Obtain the given answer following full and correct working	A1
	5

Question 59

)(i)	State or imply $du = -\sin x\, dx$	B1
	Using Pythagoras express the integral in terms of u	M1
	Obtain integrand $\pm\sqrt{u}\left(1-u^2\right)$	A1
	Integrate and obtain $-\frac{2}{3}u^{\frac{3}{2}} + \frac{2}{7}u^{\frac{7}{2}}$, or equivalent	A1
	Change limits correctly and substitute correctly in an integral of the form $au^{\frac{3}{2}} + bu^{\frac{7}{2}}$	M1
	Obtain answer $\frac{8}{21}$	A1
		6
(ii)	Use product rule and chain rule at least once	M1
	Obtain correct derivative in any form	A1 + A1
	Equate derivative to zero and obtain a horizontal equation in integral powers of $\sin x$ and $\cos x$	M1
	Use correct methods to obtain an equation in one trig function	M1
	Obtain $\tan^2 x = 6$, $7\cos^2 x = 1$ or $7\sin^2 x = 6$, or equivalent, and obtain answer 1.183	A1
		6

Question 60

State or imply ordinates 3, 2, 0, 4	B1
Use correct formula, or equivalent, with $h = 1$ and four ordinates	M1
Obtain answer 5.5	A1
	3

Question 61

(i)	State correct expansion of $\sin(2x+x)$	B1
	Use trig formulae and Pythagoras to express $\sin 3x$ in terms of $\sin x$	M1
	Obtain a correct expression in any form	A1
	Obtain $\sin 3x \equiv 3\sin x - 4\sin^3 x$ correctly **AG**	A1
		4
(ii)	Use identity, integrate and obtain $-\frac{3}{4}\cos x + \frac{1}{12}\cos 3x$	B1 B1
	Use limits correctly in an integral of the form $a\cos x + b\cos 3x$, where $ab \neq 0$	M1
	Obtain answer $\frac{5}{24}$	A1
		4

Question 62

(i)	State or imply the form $\frac{A}{2x+1} + \frac{B}{2x+3} + \frac{C}{(2x+3)^2}$	B1
	Use a correct method to find a constant	M1
	Obtain the values $A = 1$, $B = -1$, $C = 3$	A1 A1 A1
	[Mark the form $\frac{A}{2x+1} + \frac{Dx+E}{(2x+3)^2}$, where $A = 1$, $D = -2$ and $E = 0$, B1M1A1A1A1 as above.]	
		5
(ii)	Integrate and obtain terms $\frac{1}{2}\ln(2x+1) - \frac{1}{2}\ln(2x+3) - \frac{3}{2(2x+3)}$ [Correct integration of the A, D, E form of fractions gives $\frac{1}{2}\ln(2x+1) + \frac{x}{2x+3} - \frac{1}{2}\ln(2x+3)$ if integration by parts is used for the second partial fraction.]	B1 B1 B1
	Substitute limits correctly in an integral with terms $a\ln(2x+1)$, $b\ln(2x+3)$ and $c/(2x+3)$, where $abc \neq 0$ If using alternative form: $cx/(2x+3)$	M1
	Obtain the given answer following full and correct working	A1
		5

Question 63

(i)	State correct expansion of $\sin(3x+x)$ or $\sin(3x-x)$	B1	B0 If their formula retains $\pm$ in the middle
	Substitute expansions in $\frac{1}{2}(\sin 4x+\sin 2x)$	M1	
	Obtain $\sin 3x\cos x=\frac{1}{2}(\sin 4x+\sin 2x)$ correctly	A1	Must see the $\sin 4x$ and $\sin 2x$ or reference to LHS and RHS for A1 **AG**
		3	
(ii)	Integrate and obtain $-\frac{1}{8}\cos 4x-\frac{1}{4}\cos 2x$	B1 B1	
	Substitute limits $x=0$ and $x=\frac{1}{3}\pi$ correctly	M1	In their expression
	Obtain answer $\frac{9}{16}$	A1	From correct working seen.
		4	
(iii)	State correct derivative $2\cos 4x+\cos 2x$	B1	
	Using correct double angle formula, express derivative in terms of $\cos 2x$ and equate the result to zero	M1	
	Obtain $4\cos^2 2x+\cos 2x-2=0$	A1	
	Solve for x or $2x$ (could be labelled x) $\left(\cos 2x=\frac{-1\pm\sqrt{33}}{8}\right)$	M1	Must see working if solving an incorrect quadratic The roots of the correct quadratic are -0.843 and 0.593 Need to get as far as $x=\ldots$ The wrong value of x is 0.468 and can imply M1 if correct quadratic seen Could be working from a quartic in $\cos x$: $16\cos^4 x-14\cos^2 x+1=0$
	Obtain answer $x=1.29$ only	A1	
		5	

Question 64

Commence integration and reach $ax^2 \sin 2x + b\int x \sin 2x \, dx$	M1*
Obtain $\frac{1}{2}x^2 \sin 2x - \int x \sin 2x \, dx$, or equivalent	A1
Complete the integration and obtain $\frac{1}{2}x^2 \sin 2x + \frac{1}{2}x \cos 2x - \frac{1}{4}\sin 2x$, or equivalent	A1
Use limits correctly, having integrated twice	DM1
Obtain given answer correctly	A1
	5

Question 65

(i)	Use double angle formulae and express entire fraction in terms of $\sin\theta$ and $\cos\theta$	M1
	Obtain a correct expression	A1
	Obtain the given answer	A1
		3
(ii)	State integral of the form $\pm \ln \cos\theta$	M1*
	Use correct limits correctly and insert exact values for the trig ratios	DM1
	Obtain a correct expression, e.g. $-\ln\frac{1}{\sqrt{2}} + \ln\frac{\sqrt{3}}{2}$	A1
	Obtain the given answer following full and exact working	A1
		4

Question 66

(i)	Use correct quotient rule	M1
	Obtain $\frac{dy}{dx} = -\text{cosec}^2 x$ correctly	A1
		2
(ii)	Integrate by parts and reach $ax\cot x + b\int \cot x\,dx$	*M1
	Obtain $-x\cot x + \int \cot x\,dx$	A1
	State $\pm\ln\sin x$ as integral of $\cot x$	M1
	Obtain complete integral $-x\cot x + \ln\sin x$	A1
	Use correct limits correctly	DM1
	Obtain $\frac{1}{4}(\pi + \ln 4)$ following full and exact working	A1
		6

Question 67

(i)	Use $\cos(A + B)$ formula to express $\cos 3x$ in terms of trig functions of $2x$ and x	M1
	Use double angle formulae and Pythagoras to obtain an expression in terms of $\cos x$ only	M1
	Obtain a correct expression in terms of $\cos x$ in any form	A1
	Obtain $\cos 3x \equiv 4\cos^3 x - 3\cos x$	A1
		4
(ii)	Use identity and solve cubic $4\cos^3 x = -1$ for x	M1
	Obtain answer 2.25 and no other in the interval	A1
		2

Question 68

(i)	State or imply the form $\frac{A}{2x-1}+\frac{Bx+C}{x^2+2}$	B1
	Use a correct method for finding a constant	M1
	Obtain one of $A=4$, $B=-1$, $C=0$	A1
	Obtain a second value	A1
	Obtain the third value	A1
		5
(ii)	Integrate and obtain term $2\ln(2x-1)$	B1FT
	Integrate and obtain term of the form $k\ln(x^2+2)$	*M1
	Obtain term $-\frac{1}{2}\ln(x^2+2)$	A1FT
	Substitute limits correctly in an integral of the form $a\ln(2x-1)+b\ln(x^2+2)$, where $ab \neq 0$	DM1
	Obtain answer ln 27 after full and correct exact working	A1
		5

Question 69

(i)	State or imply ordinates 1, 1.2116…, 2.7597...	**B1**
	Use correct formula, or equivalent, with $h = 0.6$	**M1**
	Obtain answer 1.85	**A1**
		3
(ii)	Explain why the rule gives an overestimate	**B1**
		1
(iii)	Differentiate using quotient or chain rule	**M1**
	Obtain correct derivative in terms of $\sin x$ and $\cos x$	**A1**
	Equate derivative to 2, use Pythagoras and obtain an equation in $\sin x$	**M1**
	Obtain $2\sin^2 x + \sin x - 2 = 0$	**A1**
	Solve a 3-term quadratic for x	**M1**
	Obtain answer $x = 0.896$ only	**A1**
		6

Question 70

(i)	Use product rule and chain rule at least once	**M1**
	Obtain correct derivative in any form	**A1**
	Equate derivative to zero, use Pythagoras and obtain an equation in cos x	**M1**
	Obtain $\cos^2 x + 3\cos x - 1 = 0$, or 3-term equivalent	**A1**
	Obtain answer $x = 1.26$	**A1**
		5
(ii)	Using $\mathrm{d}u = \pm \sin x \, \mathrm{d}x$ express integrand in terms of u and $\mathrm{d}u$	**M1**
	Obtain integrand $\mathrm{e}^u(u^2 - 1)$	**A1**
	Commence integration by parts and reach $a\mathrm{e}^u(u^2 - 1) + b\int u\mathrm{e}^u \, \mathrm{d}u$	***M1**
	Obtain $\mathrm{e}^u(u^2 - 1) - 2\int u\mathrm{e}^u \, \mathrm{d}u$	**A1**
	Complete integration, obtaining $\mathrm{e}^u(u^2 - 2u + 1)$	**A1**
	Substitute limits $u = 1$ and $u = -1$ (or $x = 0$ and $x = \pi$), having integrated completely	**DM1**
	Obtain answer $\frac{4}{\mathrm{e}}$, or exact equivalent	**A1**
		7

Question 71

Integrate by parts and reach $ax\tan x + b\int \tan x\,dx$	**M1***
Obtain $x\tan x - \int \tan x\,dx$	**A1**
Complete the integration, obtaining a term $\pm\ln\cos x$, or equivalent	**M1**
Obtain integral $x\tan x + \ln\cos x$, or equivalent	**A1**
Substitute limits correctly, having integrated twice	**DM1**
Use a law of logarithms	**M1**
Obtain answer $\frac{5}{18}\sqrt{3}\pi - \frac{1}{2}\ln 3$, or exact simplified equivalent	**A1**
	7

Question 72

(a)	Commence division and reach quotient of the form $2x + k$	**M1**
	Obtain quotient $2x - 1$	**A1**
	Obtain remainder 6	**A1**
		3
(b)	Obtain terms $x^2 - x$ (**FT** on quotient of the form $2x + k$)	**B1FT**
	Obtain term of the form $a\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)$	**M1**
	Obtain term $\frac{6}{\sqrt{3}}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)$ (**FT** on a constant remainder)	**A1FT**
	Use $x = 1$ and $x = 3$ as limits in a solution containing a term of the form $a\tan^{-1}(bx)$	**M1**
	Obtain final answer $\frac{1}{\sqrt{3}}\pi + 6$, or exact equivalent	**A1**
		5

Question 73

(a)	Use quotient or product rule	M1
	Obtain derivative in any correct form e.g. $\dfrac{-\sin x(1+\sin x)-\cos x(\cos x)}{(1+\sin x)^2}$	A1
	Use Pythagoras to simplify the derivative	M1
	Justify the given statement	A1
		4
(b)	State integral of the form $a\ln(1+\sin x)$	*M1
	State correct integral $\ln(1+\sin x)$	A1
	Use limits correctly	DM1
	Obtain answer $\ln\dfrac{4}{3}$	A1
		4

Question 74

Commence integration and reach $ax^{\frac{5}{2}}\ln x + b\int x^{\frac{5}{2}}.\frac{1}{x}\mathrm{d}x$	M1*
Obtain $\frac{2}{5}x^{\frac{5}{2}}\ln x - \frac{2}{5}\int x^{\frac{5}{2}}.\frac{1}{x}\mathrm{d}x$	A1
Complete the integration and obtain $\frac{2}{5}x^{\frac{5}{2}}\ln x - \frac{4}{25}x^{\frac{5}{2}}$, or equivalent	A1
Use limits correctly, having integrated twice e.g $\frac{2}{5}\times 32\ln 4 - \frac{4}{25}\times 32 - \left(\frac{2}{5}\times 0\right) + \frac{4}{25}$	DM1
Obtain answer $\frac{128}{5}\ln 2 - \frac{124}{25}$, or exact equivalent	A1
	5

Question 75

(a)	Use quotient or product rule	M1
	Obtain correct derivative in any form e.g. $\dfrac{(1+3x^4)-x\times 12x^3}{(1+3x^4)^2}$	A1
	Equate derivative to zero and solve for x	M1
	Obtain answer 0.577	A1
		4

(b)	State or imply $du = 2\sqrt{3x}\,dx$, or equivalent	B1
	Substitute for x and dx	M1
	Obtain integrand $\dfrac{1}{2\sqrt{3}(1+u^2)}$, or equivalent	A1
	State integral of the form $a\tan^{-1}u$ and use limits $u=0$ and $u=\sqrt{3}$ (or $x=0$ and $x=1$) correctly	M1
	Obtain answer $\dfrac{\sqrt{3}}{18}\pi$, or exact equivalent	A1
		5

Question 76

Commence integration and reach $a(2-x)e^{-2x} + b\int e^{-2x}\,dx$, or equivalent	M1*
Obtain $-\dfrac{1}{2}(2-x)e^{-2x} - \dfrac{1}{2}\int e^{-2x}\,dx$, or equivalent	A1
Complete integration and obtain $-\dfrac{1}{2}(2-x)e^{-2x} + \dfrac{1}{4}e^{-2x}$, or equivalent	A1
Use limits correctly, having integrated twice	DM1
Obtain answer $\dfrac{1}{4}(3-e^{-2})$, or exact equivalent	A1
	5

Question 77

(a)	State or imply the form $\dfrac{A}{2x-1} + \dfrac{B}{2x+1}$ and use a relevant method to find A or B	M1
	Obtain $A = 1$, $B = -1$	A1
		2
(b)	Square the result of part **(a)** and substitute the fractions of part **(a)**	M1
	Obtain the given answer correctly	A1
		2
(c)	Integrate and obtain $-\dfrac{1}{2(2x-1)} - \dfrac{1}{2}\ln(2x-1) + \dfrac{1}{2}\ln(2x+1) - \dfrac{1}{2(2x+1)}$, or equivalent	B3, 2, 1, 0
	Substitute limits correctly	M1
	Obtain the given answer correctly	A1
		5

Question 78

(a)	Use correct product or quotient rule	***M1**
	Obtain correct derivative in any form	**A1**
	Equate derivative to zero and solve for x	**DM1**
	Obtain $x = 4$	**A1**
	Obtain $y = -2e^{-2}$, or exact equivalent	**A1**
		5
(b)	Commence integration and reach $a(2-x)e^{-\frac{1}{2}x} + b\int e^{-\frac{1}{2}x}\,dx$	***M1**
	Obtain $-2(2-x)e^{-\frac{1}{2}x} - 2\int e^{-\frac{1}{2}x}\,dx$	**A1**
	Complete integration and obtain $2xe^{-\frac{1}{2}x}$	**A1**
	Use correct limits, $x = 0$ and $x = 2$, correctly, having integrated twice	**DM1**
	Obtain answer $4e^{-1}$, or exact equivalent	**A1**

Question 79

(i)	State or imply the form $\frac{A}{x}+\frac{B}{x^2}+\frac{C}{x+2}$	B1
	Use a correct method for finding a constant	M1
	Obtain one of $A=-1$, $B=3$, $C=2$	A1
	Obtain a second value	A1
	Obtain the third value	A1
		5
(ii)	Integrate and obtain terms $\ln x-\frac{3}{x}+2\ln(x+2)$	B1FT + B1FT + B1FT
	Substitute limits correctly in an integral with terms $a\ln x$, $\frac{b}{x}$ and $c\ln(x+2)$, where $abc\neq 0$	M1
	Obtain $\frac{9}{4}$ following full and exact working	A1
		5

Question 80

(a)	State or imply $du = \cos x \, dx$	**B1**
	Using double angle formula for $\sin 2x$ and Pythagoras, express integral in terms of u and du.	**M1**
	Obtain integral $\int 2\left(u - u^3\right) du$	**A1**
	Use limits $u = 0$ and $u = 1$ in an integral of the form $au^2 + bu^4$, where $ab \neq 0$	**M1**
	Obtain answer $\frac{1}{2}$	**A1**
		5
(b)	Use product rule	**M1**
	Obtain correct derivative in any form	**A1**
	Equate derivative to zero and use a double angle formula	***M1**
	Obtain an equation in one trig variable	**DM1**
	Obtain $4\sin^2 x = 1$, $4\cos^2 x = 3$ or $3\tan^2 x = 1$	**A1**
	Obtain answer $x = \frac{1}{6}\pi$	**A1**
		6

Question 81

(a)	Carry out a relevant method to determine constants A and B such that $\frac{5a}{(2x-a)(3a-x)} = \frac{A}{2x-a} + \frac{B}{3a-x}$	**M1**
	Obtain $A = 2$	**A1**
	Obtain $B = 1$	**A1**
		3
(b)	Integrate and obtain terms $\ln(2x-a) - \ln(3a-x)$	**B1 FT** **B1 FT**
	Substitute limits correctly in a solution containing terms of the form $b\ln(2x-a)$ and $c\ln(3a-x)$, where $bc \neq 0$	**M1**
	Obtain the given answer showing full and correct working	**A1**
		4

Question 82

(a)	Use quotient or product rule	**M1**
	Obtain correct derivative in any form	**A1**
	Equate derivative to zero and solve for x	**M1**
	Obtain $x = \sqrt[4]{e}$ and $y = \frac{1}{4e}$, or exact equivalents	**A1**
		4

(b)	Commence integration and reach $ax^{-3}\ln x + b\int x^{-3}.\ \frac{1}{x}\ dx$	*M1
	Obtain $-\frac{1}{3}x^{-3}\ln x + \frac{1}{3}\int x^{-3}.\ \frac{1}{x}\ dx$	A1
	Complete integration and obtain $-\frac{1}{3}x^{-3}\ln x - \frac{1}{9}x^{-3}$	A1
	Substitute limits correctly, having integrated twice	DM1
	Obtain answer $\frac{1}{9} - \frac{1}{3}a^{-3}\ln a - \frac{1}{9}a^{-3}$	A1
	Justify the given statement	A1
		6

Question 83

(a)	State or imply the form $\frac{A}{1+2x} + \frac{B}{4-x}$ and use a correct method to find a constant	M1
	Obtain one of $A = 4$ and $B = -1$	A1
	Obtain the second value	A1
		3
(b)	Integrate and obtain terms $2\ln(1+2x) + \ln(4-x)$	B1FT +B1FT
	Substitute limits correctly in an integral of the form $a\ln(1+2x) + b\ln(4-x)$, where $ab \neq 0$	M1
	Obtain final answer $\ln\left(\frac{50}{27}\right)$	A1
		4

Question 84

(a)	Express the LHS in terms of $\cos 2\theta$ and $\sin 2\theta$	**B1**
	Use correct double angle formulae to express the LHS in terms of $\cos\theta$ and $\sin\theta$	**M1**
	Obtain $\tan\theta$ from correct working	**A1**
(b)	State integral of the form $\mp\ln\cos\theta$ or $\pm\ln\sec\theta$	***M1**
	Use correct limits correctly and insert exact values for the trigonometric ratios	**DM1**
	Obtain a correct expression, e.g. $-\ln\frac{1}{2}+\ln\frac{1}{\sqrt{2}}$	**A1**
	Obtain $\frac{1}{2}\ln 2$ from correct working	**A1**
		4

Question 85

Commence integration and reach $ax\tan^{-1}\frac{1}{2}x+b\int x\frac{1}{c+x^2}\,\mathrm{d}x$	***M1**
Obtain $x\tan^{-1}\left(\frac{1}{2}x\right)-\int x.\ \frac{2}{4+x^2}\ \mathrm{d}x$	**A1**
Complete integration and obtain $x\tan^{-1}\left(\frac{1}{2}x\right)-\ln\left(4+x^2\right)$	**A1**
Substitute limits correctly in an expression of the form $px\tan^{-1}x+q\ln\left(c+x^2\right)$	**DM1**
Obtain final answer $\frac{1}{2}\pi-\ln 2$	**A1**

Use the substitution $\theta = \tan^{-1}\frac{x}{2}$ to obtain $\lambda\int 2\theta\sec^2\theta\,d\theta$ and reach $p\theta\tan\theta + q\int\tan\theta\,d\theta$	*M1
Obtain $2\theta\tan\theta - 2\int\tan\theta\,d\theta$	A1
Complete integration and obtain $2\theta\tan\theta + 2\ln(\cos\theta)$	A1
Substitute correct limits correctly in an expression of the form $r\theta\tan\theta + s\ln(\cos\theta)$	DM1
Obtain final answer $\frac{1}{2}\pi - \ln 2$	A1
	5

Question 86

(a)	Use correct product rule or correct quotient rule	M1
	Obtain correct derivative in any form	A1
	Equate 2 term derivative to zero and solve for x	M1
	Obtain answer $x = e^{\frac{3}{2}}$	A1
	Obtain answer $y = \frac{3}{2e}$	A1
		5

)(b)	Commence integration and reach $ax^{\frac{1}{3}}\ln x + b\int x^{\frac{1}{3}}.\frac{1}{x}\mathrm{d}x$	*M1
	Obtain $3x^{\frac{1}{3}}\ln x - 3\int x^{\frac{1}{3}}.\frac{1}{x}\mathrm{d}x$	A1
	Complete the integration and obtain $3x^{\frac{1}{3}}\ln x - 9x^{\frac{1}{3}}$	A1
	Use limits correctly in an expression of the form $px^{\frac{1}{3}}\ln x + qx^{\frac{1}{3}}$ $(pq \neq 0)$	DM1
	Obtain $18\ln 2 - 9$ from full and correct working	A1
		5

Question 87

l(a)	Use correct double angle formula or *t*-substitution twice	M1
	Obtain $\frac{1-\cos 2\theta}{1+\cos 2\theta} = \tan^2\theta$ from correct working	A1
		2

(b)	Express $\tan^2\theta$ in terms of $\sec^2\theta$	M1
	Integrate and obtain terms $\tan\theta - \theta$	A1
	Substitute limits correctly in an integral of the form $a\tan\theta + b\theta$, where $ab \neq 0$	M1
	Obtain answer $\frac{2}{3}\sqrt{3} - \frac{1}{6}\pi$	A1
		4

Question 88

(a)	Use quotient or product rule	M1	
	Obtain correct derivative in any form	A1	
	Equate derivative to zero and solve for x	M1	
	Obtain answer $x = 3$	A1	
		4	
(b)	State $\frac{\mathrm{d}u}{\mathrm{d}x} = \frac{1}{2\sqrt{x}}$, or $\mathrm{d}x = 2\sqrt{x}\mathrm{d}u$, or $2u\ \mathrm{d}u = \mathrm{d}x$	B1	
	Substitute and obtain integrand $\frac{2}{9-u^2}$	B1	
	Use given formula for the integral or integrate relevant partial fractions	M1	
	Obtain integral $\frac{1}{3}\ln\left(\frac{3+u}{3-u}\right)$	A1	OE
	Use limits $u = 0$ and $u = 2$ correctly	M1	
	Obtain the given answer correctly	A1	
		6	

Question 89

Commence integration and reach $ax\cos\frac{1}{2}x + b\int\cos\frac{1}{2}x\,\mathrm{d}x$	*M1	
Obtain $-2x\cos\frac{1}{2}x + 2\int\cos\frac{1}{2}x\,\mathrm{d}x$	A1	OE
Complete integration obtaining $-2x\cos\frac{1}{2}x + 4\sin\frac{1}{2}x$	A1	OE
Use limits correctly, having integrated twice	DM1	
Obtain answer $2 + \frac{\sqrt{3}}{3}\pi$, or exact equivalent	A1	
	5	

Question 90

(a)	State correct expansion of $\sin(3x+2x)$ or $\sin(3x-2x)$	**B1**	
	Substitute expansions in $\frac{1}{2}(\sin 5x+\sin x)$, or equivalent	**M1**	
	Simplify and obtain $\frac{1}{2}(\sin 5x+\sin x)=\sin 3x\cos 2x$	**A1**	Obtain the **given identity** correctly.
		3	
(b)	Obtain integral $-\frac{1}{10}\cos 5x-\frac{1}{2}\cos x$, or equivalent	**B1**	
	Substitute limits correctly in an expression of the form $p\cos 5x+q\cos x$	**M1**	Correct limits and subtracted the right way around. Do not need values of trig functions for M1. Maximum one slip.
	Obtain $\frac{1}{5}(3-\sqrt{2})$	**A1**	Substitute values and obtain the **given answer** following full, correct and exact working.
		3	

Question 91

State that $\frac{du}{dx}=\frac{1}{2\sqrt{x}}$ or $du=\frac{1}{2\sqrt{x}}\,dx$	**B1**	
Substitute throughout for x and dx	**M1**	
Obtain a correct integral with integrand $\frac{2}{u^2+1}$	**A1**	
Integrate and obtain term of the form $k\tan^{-1}u$	**M1**	$(2\tan^{-1}u)$
Use limits $\sqrt{3}$ and ∞ for u or equivalent and evaluate trig.	**A1**	e.g. $2\left(\frac{\pi}{2}-\frac{\pi}{3}\right)$ Must be working in radians.
Obtain answer $\frac{1}{3}\pi$	**A1**	Or equivalent single term.
	6	

Question 92

(a)	Use correct product rule or chain rule	M1	
	Obtain correct derivative in any form	A1	$\cos x.\cos 2x - \sin x.2\sin 2x$
	Equate derivative to zero and use a correct double angle formula	*M1	If chain rule used then derivative set to 0 gains M1 since correct double angle formula has already been used.
	Obtain an equation in one trigonometric variable	DM1	Allow following from coefficient errors in differentiation only
	Obtain $6\sin^2 x = 1$, $6\cos^2 x = 5$ or $5\tan^2 x = 1$	A1	One of these 3 expressions
	Obtain final answer $x = 0.421$	A1	Must be 3s.f.
		6	

(b)	State or imply $\mathrm{d}u = -\sin x \ \mathrm{d}x$	B1	
	Using double angle formula, express integral in terms of u and $\mathrm{d}u$	M1	Use $\cos 2x = 2\cos^2 x - 1$
	Integrate and obtain $\pm\left(u - \frac{2}{3}u^3\right)$	A1	
	Use limits $u = 1$, $u = \frac{1}{\sqrt{2}}$ in an integral of the form $au + bu^3$, where $ab \neq 0$	M1	Require both limits substituted twice in $au + bu^3$ for M1. Do not condone decimals.
	Obtain $\frac{1}{3}\left(\sqrt{2} - 1\right)$ or $\frac{1}{3}\sqrt{2}\ \frac{1}{3}$ or $\frac{2}{3}\left(\frac{1}{\sqrt{2}}\right)\frac{1}{3}$ or simplified equivalent	A1	ISW
		5	

Question 93

3(a)	Commence division and reach quotient of the form $2x \pm 1$	M1	Or by inspection $8x^3 + 4x^2 + 2x + 7 = (4x^2 + 1)(2x \pm 1) + \text{r}$
	Obtain (quotient) $2x + 1$	A1	
	Obtain (remainder) 6	A1	
		3	

(b)	Obtain terms $x^2 + x$	B1	OE
	Obtain term of the form $a\tan^{-1}2x$	M1	
	Obtain term $3\tan^{-1}2x$	A1	OE
	Use $x = 0$ and $x = \frac{1}{2}$ as limits in a solution containing a term of the form $a\tan^{-1}2x$	M1	$\left(\frac{1}{2}\right)^2 + \frac{1}{2} + a\frac{\pi}{4}$, need $\frac{\pi}{4}$ seen or implied
	Obtain final answer $\frac{3}{4}(1 + \pi)$, or exact equivalent	A1	ISW, Answers in degrees score A0.
		5	

Question 94

(a)	State or imply the form $\frac{A}{3x-1} + \frac{Bx+C}{x^2+3}$	**B1**	
	Use a correct method for finding a constant	**M1**	
	Obtain one of $A = 1$, $B = 0$ and $C = 3$ from correct working	**A1**	A maximum of M1 A1 is available after B0.
	Obtain a second value from correct working	**A1**	
	Obtain the third value from correct working	**A1**	
		5	
(b)	Integrate and obtain term $\frac{1}{3}\ln(3x-1)$	**B1 FT**	OE e.g. $\frac{1}{3}\ln(x-\frac{1}{3})$. The FT is on the value of A.
	Obtain term of the form $k\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)$	**M1**	
	Obtain term $\sqrt{3}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)$	**A1 FT**	OE. The FT is on the value of C.
	Substitute correct limits in an integral of the form $a\ln(3x-1)+k\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)$, where $ak \neq 0$, and evaluate trigonometry	**M1**	Must be subtracted the right way round. $\left(\frac{1}{3}\ln 8-\frac{1}{3}\ln 2+\sqrt{3}\times\frac{\pi}{3}-\sqrt{3}\times\frac{\pi}{6}\right)$ Angles should be in radians. Condone angles as decimals.
	Obtain answer $\frac{2}{3}\ln 2 + \frac{\sqrt{3}\pi}{6}$ from correct working in part **8(b)**	**A1**	Or exact 2-term equivalent e.g. $\frac{1}{3}\ln 4 + \frac{\pi}{2\sqrt{3}}$ ISW.
		5	

Question 95

(a)	State or imply $dx = 3\sec^2\theta\, d\theta$	**B1**	
	Substitute throughout for x and dx	**M1**	
	Obtain any correct form in terms of θ	**A1**	e.g. $\int \frac{81\sec^2\theta}{(9+9\tan^2\theta)^2} d\theta$
	Justify change of limits and obtain $\int_0^{\frac{\pi}{4}} \cos^2\theta\, d\theta$ correctly	**A1**	AG
		4	
(b)	Obtain indefinite integral of the form $\int a + b\cos 2\theta\, d\theta$, where $ab \neq 0$	***M1**	
	Obtain $\frac{1}{2}\theta + \frac{1}{4}\sin 2\theta$	**A1**	
	Use correct limits correctly in an expression containing $p\theta$ and $q\sin 2\theta$ where $pq \neq 0$	**DM1**	$\frac{\pi}{8}+\frac{1}{4}(-0)$
	Obtain answer $\frac{1}{8}(\pi+2)$	**A1**	Or exact equivalent e.g. $\frac{1}{8}\pi + \frac{1}{4}$.
		4	

Question 96

Commence integration by parts and reach $x\tan x \pm \int \tan x.1\,dx$	*M1	
Use a correct method to integrate $\tan x$	M1	
Obtain integral $x\tan x - \ln\sec x$, or equivalent	A1	
Use limits correctly, having integrated twice	DM1	
Obtain answer $\frac{1}{4}\pi - \frac{1}{2}\ln 2$, or exact equivalent	A1	
	5	

Question 97

(a)	State or imply the form $\frac{A}{3-x} + \frac{Bx+C}{1+3x^2}$	B1	
	Use a correct method to find a constant	M1	
	Obtain one of $A = 2$, $B = 0$ and $C = 1$	A1	
	Obtain a second value	A1	
	Obtain the third value	A1	
		5	
(b)	Integrate and obtain term $-2\ln(3-x)$	B1 FT	
	Obtain term of the form $b\tan^{-1}\left(\sqrt{3}x\right)$	M1	
	Obtain term $\frac{1}{\sqrt{3}}\tan^{-1}\left(\sqrt{3}x\right)$	A1 FT	
	Substitute limits correctly in an integral with terms $a\ln(3-x)$ and $b\tan^{-1}\left(\sqrt{3}x\right)$, where $ab \neq 0$	M1	
	Obtain answer $2\ln\frac{3}{2} + \frac{1}{3\sqrt{3}}\pi$, or equivalent	A1	
		5	

Question 98

(a)	State $(a =)\ \pi^2$	B1	Allow 32400, 180^2 . Accept $(x =)\pi^2$.
		1	
(b)	State or imply $dx = 2u\ du$ or equivalent	B1	e.g. $\frac{du}{dx} = \frac{1}{2\sqrt{x}}$ Incorrect statements e.g. $du = \frac{1}{2\sqrt{x}}$ is B0.
	Substitute for x and dx throughout the integral	M1	
	Obtain $\int 2u \sin u\, du$	A1	Allow with missing du.
	Commence integration of $\int ku \sin u\, du$ by parts and reach $\mp ku \cos u \pm \int k \cos u\, du$	*M1	
	Obtain integral $-ku\cos u + k\sin u$	A1	
	Substitute limits $u = 0$ and $u = \sqrt{their\ a}$, $a \neq 0$, a in radians or $x = 0$ and *their a* in the complete integral	DM1	$-2\pi\cos\pi + 2\sin\pi(+0 - 2\sin 0)$ Need limits stated but condone if zeros not shown in substitution.
	Obtain answer 2π	A1	
		7	

Question 99

(a)	State or imply the form $\frac{A}{1+x} + \frac{Bx+C}{2+x^2}$	B1	
	Use a correct method for finding a constant	M1	
	Obtain one of $A = 2$, $B = -1$ and $C = 0$	A1	**SC**: A maximum of M1A1 is available for obtaining $A = 2$ after scoring B0.
	Obtain a second value	A1	
	Obtain the third value	A1	
		5	
(b)	Integrate and obtain term $2\ln(1+x)$	B1 FT	$A\ln(1+x)$
	Integrate and obtain term of the form $k\ln(2+x^2)$ from an integral of the correct form	*M1	Ignore any separate working relating to $C \neq 0$.
	Obtain term $-\frac{1}{2}\ln(2+x^2)$	A1 FT	$\frac{B}{2}\ln(2+x^2)$
	Substitute limits in an integral containing terms of the form $a\ln(1+x) + b\ln(2+x^2)$, where $ab \neq 0$	DM1	Ignore working relating to $C \neq 0$. $\left(2\ln 5 - 2\ln 1 - \frac{1}{2}\ln 18 + \frac{1}{2}\ln 2\right)$ Dependent on the first M1. Must be subtracting the correct way round. Must have an exact substitution.
	Obtain answer $\ln\left(\frac{25}{3}\right)$	A1	ISW Any exact equivalent e.g. $\ln\frac{25\sqrt{2}}{\sqrt{18}}$ with no number in front of the logarithm.
		5	

Question 100

Commence integration and reach $a(3-x)e^{-\frac{1}{3}x} + b\int e^{-\frac{1}{3}x}\,dx$, where $ab \neq 0$	*M1	
Obtain $-3(3-x)e^{-\frac{1}{3}x} - 3\int e^{-\frac{1}{3}x}\,dx$, or equivalent	A1	
Complete integration and obtain $3xe^{-\frac{1}{3}x}$, or equivalent	A1	$-3e^{-\frac{x}{3}}(3-x)+9e^{-\frac{x}{3}}$
Substitute limits $x = 0$ and $x = 3$, having integrated twice	DM1	
Obtain answer $\frac{9}{e}$, or exact equivalent	A1	
	5	

Question 101

Integrate by parts and reach $ax^4 \ln x + b\int(x^4/x)\,dx$	*M1	
Obtain $\frac{x^4}{4}\ln x - \frac{1}{4}\int(x^4/x)\,dx$	A1	OE
Complete integration and obtain $\frac{x^4}{4}\ln x - \frac{x^4}{16}$	A1	OE
Use limits of $x = \frac{1}{2}$ and $x = 1$ in the correct order, having integrated twice	DM1	Correct substitution [(1/4)ln1 or 0 − 1/16] – [(1/64)ln(1/2) – $(1/16)^2$] or minus this value CWO. Allow omission of (1/4)ln1 or 0.
Obtain answer $\frac{15}{256} - \frac{1}{64}\ln 2$ or exact equivalent final answer	A1	
	5	

Question 102

(a)	State or imply the form $\frac{A}{1+x} + \frac{Bx+C}{2+x^2}$	**B1**	
	Use a correct method for finding a constant	**M1**	
	Obtain one of $A = 2$, $B = -1$ and $C = 0$	**A1**	**SC**: A maximum of M1A1 is available for obtaining $A = 2$ after scoring B0.
	Obtain a second value	**A1**	
	Obtain the third value	**A1**	
		5	
(b)	Integrate and obtain term $2\ln(1+x)$	**B1 FT**	$A\ln(1+x)$
	Integrate and obtain term of the form $k\ln(2+x^2)$ from an integral of the correct form	***M1**	Ignore any separate working relating to $C \neq 0$.
	Obtain term $-\frac{1}{2}\ln(2+x^2)$	**A1 FT**	$\frac{B}{2}\ln(2+x^2)$
	Substitute limits in an integral containing terms of the form $a\ln(1+x)+b\ln(2+x^2)$, where $ab \neq 0$	**DM1**	Ignore working relating to $C \neq 0$. $\left(2\ln 5 - 2\ln 1 - \frac{1}{2}\ln 18 + \frac{1}{2}\ln 2\right)$ Dependent on the first M1. Must be subtracting the correct way round. Must have an exact substitution.
	Obtain answer $\ln\left(\frac{25}{3}\right)$	**A1**	ISW Any exact equivalent e.g. $\ln\frac{25\sqrt{2}}{\sqrt{18}}$ with no number in front of the logarithm.
		5	

Question 103

(a)	$\frac{du}{dx} = -\sin x$	**B1**	SOI
	Use double angle formula and substitute for x and dx throughout the integral	**M1**	All x's must be removed, can be coefficient errors provided 2 seen in working.
	Obtain $\pm\int 2ue^{2u}\,du$	**A1**	Limits may be omitted, or left as 0 and π, during the change of variable stage.
	Justify new limits and obtain $\int_{-1}^{1} 2ue^{2u}\,du$ from correct working	**A1**	AG Must see $x = 0$, $u = 1$ and $x = \pi$, $u = -1$. Inequalities alone e.g. $0 \leqslant x \leqslant \pi$ and $1 \leqslant u \leqslant -1$ or $-1 \leqslant u \leqslant 1$ for limits are insufficient A0 If sign in expression and order of limits incorrect then A0. If negative sign is present in the integrand then this can be removed and limits introduced in correct order in a single step.
		4	

Question 104

Use the given substitution and reach $a\int\left(\frac{13}{2}-\frac{u}{2}\right)u^{\frac{1}{2}}\ \mathrm{d}u$	*M1	OE Need to see -2 or -½ used. Condone if du missing or the integral sign is missing. Allow M1A0 for complete substitution into $\int x\sqrt{3-2x}\,\mathrm{d}x$ to obtain first term of the line below.
Obtain correct integral $-\frac{1}{2}\int\left(\frac{13}{2}-\frac{u}{2}\right)u^{\frac{1}{2}}\ \mathrm{d}u$	A1	OE e.g. $-\frac{1}{2}\left[\int\frac{3-u}{2}\sqrt{u}\,\mathrm{d}u+5\int\sqrt{u}\,\mathrm{d}u\right]$. Ignore limits at this stage. Condone if du missing.
$x=-5$ and $\frac{3}{2}$	B1	SOI e.g. by $u = 13$ and 0. In any order and at any stage.
Use correct limits the right way round in an integral of the form $a\left(\frac{26}{3}u^{\frac{3}{2}}-\frac{2}{5}u^{\frac{5}{2}}\right)$	DM1	
Obtain answer $\frac{169}{15}\sqrt{13}$ or $a=\frac{169}{15}$	A1	or exact equivalents.
	5	

(b)

Commence integration and reach $aue^{2u}+b\int e^{2u}\mathrm{d}u$, where $ab\neq 0$, $b<0$	M1*	Condone dx.
Complete integration and obtain $ue^{2u}-\frac{1}{2}e^{2u}$	A1	OE Allow $(2u\frac{1}{2}e^{2u})-\frac{1}{2}e^{2u}$.
Use correct limits correctly in $cue^{2u}+d\,e^{2u}$ having integrated twice or in $c\cos x\,e^{2\cos x}+d\,e^{2\cos x}$	DM1	1 and −1 for u, 0 and π for x e.g. $ce^2+de^2-(-ce^{-2}+de^{-2})$. Not decimals. Allow one sign error at most in going from $cue^{2u}+d\,e^{2u}$ or $c\cos x\,e^{2\cos x}+d\,e^{2\cos x}$ to $ce^2+de^2-(-ce^{-2}+de^{-2})$. $[e^2-\frac{1}{2}e^2-(-e^{-2}-\frac{1}{2}e^{-2})]$ Complete reversal of sign by converting back to cos x and not making $x = 0$ upper limit is DM0 A0.
Obtain $\frac{1}{2}e^2+\frac{3}{2}e^{-2}$	A1	ISW Or equivalent 2-term expression e.g. $\frac{e^4+3}{2e^2}$ or $\frac{1}{2}\left(e^2+\frac{3}{e^2}\right)$.
	4	

Question 105

(a)

State or imply the form $\dfrac{A}{1+2x}+\dfrac{B}{2-x}+\dfrac{C}{(2-x)^2}$	**B1**	Alternative form: $\dfrac{A}{1+2x}+\dfrac{Dx+E}{(2-x)^2}$
Use a correct method for finding a coefficient	**M1**	e.g. $A(2-x)^2+B(1+2x)(2-x)+C(1+2x)$ $=2x^2+17x-17$ and compare coefficients or substitute for x. $A(2-x)^3+B(1+2x)(2-x)^2+C(1+2x)(2-x)$ $=2x^2+17x-17$ scores M0.
Obtain one of $A=-4$, $B=-3$ and $C=5$	**A1**	
Obtain a second value	**A1**	
Obtain the third value	**A1**	Extra term in partial fractions, then B0 unless recover at end. Allow the marks for any constants found correctly. Missing terms in partial fractions, B0 but M1A1 is available for a correct method that obtains at least one correct constant (e.g. cover-up rule) Max 2/5. Ignore any substitution back into their original expression. If alternative form used: $A=-4$, $D=3$ and $E=-1$.
	5	

(b)

Integrate and obtain terms $-2\ln(1+2x)+3\ln(2-x)+\dfrac{5}{2-x}$	**B1FT** **B1FT** **B1FT**	OE The FT is on correct use of *their* A, B and C; or on A, D and E. If using the A, D, E form then B1 for the A term, but no further marks until partial fractions are used to split the second term or they use integration by parts to obtain $\dfrac{Dx+E}{2-x}-\displaystyle\int\dfrac{D}{2-x}\,dx$ for the 2nd B1 and 3rd B1 for correct completion. B0FT, B0FT, B0FT if they place *their* A, B, C with incorrect denominators.
Substitute limits correctly in an integral with two terms (obtained correctly) of the form $a\ln(1+2x)+b\ln(2-x)+\dfrac{c}{2-x}$, where $abc\neq 0$	**M1**	Condone minor slips in substitution. Exact substitution required.
Obtain answer $\dfrac{5}{2}-\ln 72$ after full and correct working	**A1**	AG – evidence of some correct work to combine or simplify logs is required e.g. allow from $-\ln 9+\ln\dfrac{1}{8}$ or $-\ln 2^3-\ln 3^2$.
	5	

Question 106

(a)	State or imply the form $\frac{A}{2x+1}+\frac{B}{x+2}+\frac{C}{(x+2)^2}$	B1	Accept $\frac{A}{2x+1}+\frac{Dx+E}{(x+2)^2}$.
	Use a correct method for finding a constant	M1	
	Obtain one of $A=1, B=-2, C=3$	A1	For alternative form: $A=1, D=-2, E=-1$.
	Obtain a second value	A1	
	Obtain the third value	A1	
		5	
(b)	Integrate and obtain one of $\frac{1}{2}\ln(2x+1), -2\ln(x+2), \frac{-3}{x+2}$	B1 FT	The follow through is on *their* A, B, C.
	Obtain a second term	B1 FT	If the alternative form is used, then either need to use integration by parts or split the fraction further.
	Obtain the third term	B1 FT	
	Substitute limits correctly in an integral with at least two terms of the form $\frac{1}{2}\ln(2x+1), -2\ln(x+2)$ and $\frac{-3}{x+2}$ and subtract in correct order	M1	The terms used need to have been obtained correctly. Must be exact values, not decimals.
	Obtain $1-\ln 3$	A1	
		5	

Question 107

Integrate by parts and reach $ax\sin 2x + b\int \sin 2x\,dx$	*M1	
Obtain $\frac{1}{2}x\sin 2x - \frac{1}{2}\int \sin 2x\,dx$	A1	OE
Complete integration and obtain $\frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x$	A1	OE
Use limits of $x=0$ and $x=\frac{\pi}{4}$ in the correct order, having integrated twice to obtain $ax\sin 2x + c\cos 2x$	DM1	If correct, $\frac{1}{2}\left(\frac{\pi}{4}\right)\sin\frac{2\pi}{4}+\frac{1}{4}\cos\frac{2\pi}{4}-\frac{1}{4}\cos 0$ or $\frac{1}{2}\left(\frac{\pi}{4}\right)\sin\frac{2\pi}{4}-\frac{1}{4}\cos 0$. Max one substitution error.
Obtain answer $\frac{\pi}{8}-\frac{1}{4}$ or exact simplified two term equivalent	A1	ISW Accept $\frac{\pi-2}{8}$. Accept $\frac{1}{2}x\sin 2x+\frac{1}{4}\cos 2x$ then final answer.
	5	

Question 108

Use double angle formula and obtain $p\cos^3 x + q\cos x$ correctly	*M1	e.g. from $\int 2\cos^2 x\sin x - \sin x \,dx$.
Obtain $\pm\left(-\frac{2}{3}\cos^3 x + \cos x\right)$	A1	Correct for *their* integral.
Correct use of limits $\frac{1}{4}\pi$ and $\frac{3}{4}\pi$ (or use double the integral from $\frac{1}{4}\pi$ to $\frac{1}{2}\pi$)	DM1	OE $\pm\left(-\frac{2}{3}\left[\left(\frac{-1}{\sqrt{2}}\right)^3 - \left(\frac{1}{\sqrt{2}}\right)^3\right] - \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right)$.
Obtain $\frac{2\sqrt{2}}{3}$	A1	Or simplified exact equivalent. Final answer must be positive.
Alternative method 1 for question 9(b)		
Use integration by parts **twice** and obtain $r\cos x\cos 2x + s\sin x\sin 2x$	*M1	Seen, not just implied.
Obtain $\frac{1}{3}\cos x\cos 2x + \frac{2}{3}\sin x\sin 2x$	A1	Accept $\pm$ (correct for *their* integral).
Correct use of limits $\frac{1}{4}\pi$ and $\frac{3}{4}\pi$ (or use double the integral from $\frac{1}{4}\pi$ to $\frac{1}{2}\pi$)	DM1	OE $\pm\frac{1}{3}\left(0 + 2\times\frac{1}{\sqrt{2}}\times -1 - 0 - 2\times\frac{1}{\sqrt{2}}\times 1\right)$.
Obtain $\frac{2\sqrt{2}}{3}$	A1	Or simplified exact equivalent. Final answer must be positive.

Question 109

Split fraction to obtain $1 + \frac{x-4}{x^2+4}$	B1	
Attempt integration and obtain $p\ln\left(x^2+4\right)$ or $q\tan^{-1}\left(\frac{x}{2}\right)$ from correct working	M1	Allow for $p\ln\left(x^2+4\right)$ from $\int \frac{x}{x^2+4}dx$ but only if a correct method for splitting has been used.
Obtain $\frac{1}{2}\ln\left(x^2+4\right)$	A1 FT	Follow through is on their coefficients in the partial fraction. Allow from $\frac{x^2}{x^2+4} + \frac{x}{x^2+4}$ even if the split of the fraction is not complete. If $1 - \frac{4}{x^2+4} + \frac{x}{x^2+4}$ later seen or implied, award the B1. Only available from a correct split, not from an approach using parts that is incomplete.
Obtain $-2\tan^{-1}\left(\frac{x}{2}\right)$	A1 FT	Only available from a correct split, not from an approach using parts that is incomplete.
Correct use of correct limits 0 and 6 in an expression involving $p\ln\left(x^2+4\right)$, $q\tan^{-1}\left(\frac{x}{2}\right)$ and no incorrect terms.	M1	p and q should be constants. The x term is not required at this stage.
Obtain $6 + \frac{1}{2}\ln 10 - 2\tan^{-1} 3$	A1	ISW Or three term equivalent. (Must combine the ln terms.) Accept with $\frac{1}{2}\ln\lvert 10\rvert$.

Alternative method for question 5

Use the substitution $x = 2\tan\theta$ to obtain $\int 2\tan^2\theta + \tan\theta \, d\theta$	B1	
Attempt integration and obtain $p\tan\theta$ or $r\ln(\sec\theta)$ from correct working	M1	
Obtain $2\tan\theta\,(-2\theta)$ and	A1 FT	Follow through on *their* coefficients after the substitution.
Obtain $\ln\sec\theta$	A1 FT	Follow through on *their* coefficients after the substitution.
Use correct limits 0 and $\tan^{-1}3$ in an expression involving $u\tan\theta$, $v\ln\sec\theta$ and no incorrect terms	M1	u and v should be constants. The θ term is not required at this stage.
Obtain $6 + \ln\left\lvert\sec\left(\tan^{-1}3\right)\right\rvert - 2\tan^{-1}3$	A1	ISW Or three term equivalent. Not required to simplify $\ln\left\lvert\sec\left(\tan^{-1}3\right)\right\rvert$.
	6	

Question 110

State or imply $dx = \frac{1}{2}u^{-\frac{1}{2}}du$	B1	Or equivalent e.g. $du = 2x\,dx$. Alternative substitution: $u = -\frac{1}{4}x^2$.
Substitute for x and dx	M1	
Obtain correct integral $\frac{1}{2}\int e^{-\frac{1}{4}u}\,du$	A1	OE
Use correct limits in an integral of the form $ae^{-\frac{1}{4}u}$ or $ae^{-\frac{1}{4}x^2}$	M1	$u = 9$ and $u = 0$ or $x = 3$ and $x = 0$.
Obtain answer $2 - 2e^{-\frac{9}{4}}$	A1	Or exact equivalent.
Alternative Method for Question 9(b)		
$\int xe^{-\frac{1}{4}x^2}dx = ae^{-\frac{1}{4}x^2}$	M1	Recognition used.
a negative	A1	
$a = -2$	A1	
Use correct limits in an integral of the form $ae^{-\frac{1}{4}x^2}$	M1	$x = 3$ and $x = 0$.
Obtain answer $2 - 2e^{-\frac{9}{4}}$	A1	Or exact equivalent.
	5	

Question 111

(a)	State or imply the form $\frac{A}{2a+x}+\frac{B}{2a-x}+\frac{C}{5a-2x}$	**B1**	Allow if seen prior to assigning a value for a.
	Use a correct method for finding a coefficient	**M1**	
	Obtain one of $A=1, B=9, C=-16$	**A1**	
	Obtain a second value	**A1**	
	Obtain the third value	**A1**	
		5	**SC** $\frac{Dx+E}{4a\,^\wedge 2-x\,^\wedge 2}+\frac{C}{5a-2x}$ **B0 M1** and $C=-16$ **A1** Max 2/5. **SC** Allow **M1** only for other incorrect partial fraction.
(b)	Integrate and obtain one of the terms $\ln\lvert 2a+x\rvert-9\ln\lvert 2a-x\rvert+8\ln\lvert 5a-2x\rvert$	**B1 FT**	Condone missing modulus signs. Use *their* A, B and C.
	Obtain a second correct term	**B1 FT**	
	Obtain the third correct term	**B1 FT**	Max 3/5 if value is assigned for a (award M0 A0).
	Substitute limits correctly in an integral of the form $p\ln\lvert 2a+x\rvert+q\ln\lvert 2a-x\rvert++r\ln\lvert 5a-2x\rvert$ and remove all a's	**M1**	Either (i) collect terms with same coeeficient and remove all a's e.g. $p\ln 3a - p\ln a + q\ln a - q\ln 3a + r\ln 3a - r\ln 7a$ hence $p\ln 3 - q\ln 3 + r\ln 3 - r\ln 7$ or (ii) collect same ln terms and remove all a's e.g. $(p-q+r)\ln 3a - (p-q)\ln a - r\ln 7a$ and $-(p-q)\ln a = (-p+q-r)\ln a + r\ln a$ hence $p\ ln\ 3 - q\ ln 3 + r\ln 3 - r\ ln\ 7$.
	Obtain $18\ln 3-8\ln 7$ from correct working	**A1**	A0 if the solution involves logarithms of negative numbers.
		5	

Question 112

State or imply $2\mathrm{d}x = \sec^2\theta\,\mathrm{d}\theta$ or $(1+4x^2)\,\mathrm{d}\theta$ or equivalent e.g. $\frac{\mathrm{d}x}{\mathrm{d}\theta} = 0.5\sec^2\theta$ or $\frac{\mathrm{d}\theta}{\mathrm{d}x} = 2\cos^2\theta$	**B1**	$2\mathrm{d}x = \sec^2 x\,\mathrm{d}\theta$ seen **B0**. Allow BOD cancellation of $\sec^2 x$ and $\sec^2\theta$ and all remaining marks, so 8/9 possible.
Use $1+\tan^2\theta = \sec^2\theta$ in $\mathrm{d}x$ or in $\frac{1}{(1+4x^2)^2}$	**B1**	Or $1+\frac{\sin^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$. Must substitute for $\mathrm{d}x$ or no more marks possible.
Use $\cos^2\theta = \frac{1}{2}(1+\cos 2\theta)$	**B1**	$\mathrm{d}\theta$ seen once is sufficient. If never seen, this B1 mark is not available; max 8/9 possible.
Obtain integral of the form $\int p(1+\cos 2\theta)\,\mathrm{d}\theta$	**M1**	Any non-zero p.
Obtain $\int 3(1+\cos 2\theta)\,\mathrm{d}\theta$	**A1**	Allow $6\times\frac{1}{2}$.
Integrate to obtain $p\theta + q\sin 2\theta$	***M1**	Any non-zero q but same p as before.
Obtain correct $3\theta + \frac{3}{2}\sin 2\theta$	**A1**	
Use limits $\theta = \frac{\pi}{4}$ and 0 correctly in an expression of the form $p\theta + q\sin 2\theta$ or $x = \frac{1}{2}$ and 0 in an appropriate expression in terms of x	**DM1**	$p\frac{\pi}{4} + q\sin\frac{\pi}{2} - [p.0 + q.0]$ May be implied.
Obtain $\frac{3}{2} + \frac{3}{4}\pi$ or exact equivalent e.g. $\frac{3}{4}\pi + \frac{3}{2}$ but not $3\left(\frac{1}{2} + \frac{1}{4}\pi\right)$	**A1**	Must be in the form $a + b\pi$ where are a and b are rational numbers, but ISW after correct form seen (e.g. $1.5 + 0.75\pi$ scores A1).
	9	

Question 113

(a)	State $R = \sqrt{12}$ or exact equivalent	**B1**	ISW
	Use trig formula to find α	**M1**	Allow $\alpha = 30°$ or $\tan^{-1}\left(\frac{\pm\sqrt{3}}{3}\right)$ or $\cos^{-1}\left(\pm\frac{\sqrt{3}}{2}\right)$ or $\sin^{-1}\left(\pm\frac{1}{2}\right)$ Allow M1 if $-\tan^{-1}\left(\frac{\sqrt{3}}{3}\right)$ etc. NB: If $\cos\alpha = 3$ and $\sin\alpha = \sqrt{3}$ seen then M0 A0.
	Obtain $\alpha = \frac{1}{6}\pi$	**A1**	CWO, so A0 if from $\tan^{-1}\left(-\frac{\sqrt{3}}{3}\right)$.
		3	

(b)			
	Express integral in the form $A\int \sec^2(2x+...)\mathrm{d}x$ or $A\int \sec^2(2x-...)\mathrm{d}x$	B1FT	FT α from **(a)**.
	Integrate and reach $B\tan(2x+...)$ or $B\tan(2x-...)$	B1FT	FT α from **(a)**. Where $B = A$ or $2A$ or $0.5A$.
	Obtain $\frac{1}{8}\tan(2x+...)$	B1FT	OE FT α from **(a)**. Allow $\frac{1}{8}$ as $\frac{1}{4}\times\frac{1}{2}$. Coefficient must be correct.
	Use limits of $x=0$ and $x=\frac{1}{12}\pi$ in the correct order in expression of form $B\tan(2x\pm...)$ so $B\tan\left(\frac{\pi}{6}+...\right)-B\tan(...)$ or $B\tan\left(\frac{\pi}{6}-...\right)-B\tan(-...)$	M1	Allow with tan still present. FT α from **(a)**. **SC:** B1 $\frac{\sqrt{3}}{12}$ OE after $\frac{1}{8}\tan\left(2x+\frac{1}{6}\pi\right)$ with no working.
	Obtain answer $\frac{1}{12}\sqrt{3}$ or $\frac{1}{4\sqrt{3}}$ or $\frac{1}{\sqrt{48}}$ or single term exact equivalent	A1	$\frac{1}{8}\left(\sqrt{3}-\frac{1}{\sqrt{3}}\right)=\frac{1}{8}\left(\frac{3-1}{\sqrt{3}}\right)$ needs simplifying.
		5	Note: allow all marks in **(b)** even if $\alpha=\frac{1}{6}\pi$ found by an incorrect method in **(a)**.

Question 114

(a)			
	Factorise LHS using difference of 2 squares	*M1	$\left((\cos^2\theta-\sin^2\theta)(\cos^2\theta+\sin^2\theta)\right)$
	Simplify	DM1	$\cos^2\theta+\sin^2\theta=1$ must be seen or implied, e.g. $(\cos^2\theta-\sin^2\theta)(\cos^2\theta+\sin^2\theta)=(\cos^2\theta-\sin^2\theta)$.
	Obtain $\cos^4\theta-\sin^4\theta\equiv\cos 2\theta$ from correct working	A1	AG
	Alternative Method for Question 7(a)		
	Use of correct rearrangements of double angle formulae	(*M1)	E.g. $\left(\frac{1+\cos 2\theta}{2}\right)^2-\left(\frac{1-\cos 2\theta}{2}\right)^2$ Only condone $\left(\frac{1+\cos 2\theta}{2}\right)^2-\left(\frac{\cos 2\theta-1}{2}\right)^2$ if correct expression for $\sin^2\theta$ seen.
	Expand and simplify	(DM1)	Collect like terms. Condone recovery from missing brackets.
	Obtain $\cos^4\theta-\sin^4\theta\equiv\cos 2\theta$ from correct working	(A1)	AG
	Alternative Method 2 for Question 7(a)		
	Correct use of Pythagoras	(*M1)	E.g. $(1-\sin^2\theta)^2-\sin^4\theta$ or $\cos^2\theta(1-\sin^2\theta)-\sin^2\theta(1-\cos^2\theta)$
	Expand and simplify	(DM1)	Condone recovery from missing brackets.
	Obtain $\cos^4\theta-\sin^4\theta\equiv\cos 2\theta$ from correct working	(A1)	AG
		3	

'(b)	Use part (a) and correct double angle formula to obtain expression involving $\int \sin^2 2\theta \, d\theta$ or $\int \cos^2 2\theta \, d\theta$	M1	$\int \cos^4\theta - \sin^4\theta + 4\sin^2\theta\cos^2\theta \, d\theta = \int \cos 2\theta + \sin^2 2\theta \, d\theta$ Allow BOD for $2\sin^2 2\theta$ if $\sin 2\theta = 2\sin\theta\cos\theta$ seen.
	$\int \cos 2\theta d\theta = \frac{1}{2}\sin 2\theta$	B1	Seen or implied.
	Use of correct double angle formula on second part of the integral to obtain a form that can be integrated directly	M1	e.g. $\int \sin^2 2\theta \, d\theta = \int \frac{1-\cos 4\theta}{2} d\theta$
	Obtain $\frac{1}{2}\theta - \frac{1}{8}\sin 4\theta$	A1	Condone a mixture of x and θ.
	Correct use of limits $\pm\frac{\pi}{8}$ in an expression of the form $p\theta + q\sin 2\theta + r\sin 4\theta$ and evaluate the trig	M1	$\left(2\left(\frac{1}{2} \times \frac{1}{\sqrt{2}} + \frac{\pi}{16} - \frac{1}{8}\right)\right)$
	Obtain $\frac{1}{2}\sqrt{2} + \frac{1}{8}\pi - \frac{1}{4}$	A1	ISW Or exact equivalent from exact working.
		6	

Question 115

(a)	Use correct product rule	*M1	Or equivalent. Condone incorrect chain rule. M0 if a value is used for a (not equivalent work).
	Obtain correct derivative	A1	E.g. $\frac{dy}{dx} = -axe^{-ax} + e^{-ax}$
	Equate derivative to zero and solve for x	DM1	
	Obtain $x = \frac{1}{a}$, $y = \frac{1}{ae}$	A1	ISW Or exact equivalent.
		4	
(b)	Use integration by parts to obtain $pxe^{-ax} + q\int e^{-ax} dx$	*M1	Condone sign error in parts formula and omission of dx. M0 if a value is used for a (not equivalent work).
	Obtain $-\frac{1}{a}xe^{-ax} + \frac{1}{a}\int e^{-ax} dx$	A1	OE
	Complete integration to obtain $-\frac{1}{a}xe^{-ax} - \frac{1}{a^2}e^{-ax}$	A1	OE
	Correct use of limits 0 and $\frac{2}{a}$ in an expression of the form $rxe^{-ax} + se^{-ax}$	DM1	$\left(\frac{-2}{a^2}e^{-2} - \frac{1}{a^2}e^{-2} + 0 + \frac{1}{a^2}\right)$
	Obtain $\frac{1}{a^2}\left(1 - 3e^{-2}\right)$	A1	ISW Or simplified 2-term equivalent, e.g. $\frac{e^2 - 3}{a^2 e^2}$.
		5	

Question 115

(a)	Obtain $2 = \sec^2 y \dfrac{dy}{dx}$ or equivalent	B1	E.g. $2\dfrac{dx}{dy} = \sec^2 y$ by differentiation with respect to y.
	Use $\sec^2 y = 1 + \tan^2 y$	M1	
	Replace $\tan y$ with $2x$ and rearrange to obtain given answer $\dfrac{dy}{dx} = \dfrac{2}{1+4x^2}$	A1	
		3	
(b)	Integrate by parts and reach $ax^2 \tan^{-1} 2x + b\int \dfrac{x^2}{1+4x^2}\, dx$	*M1	
	Obtain $\frac{1}{2}x^2 \tan^{-1} 2x - \int \dfrac{x^2}{1+4x^2}\, dx$	A1	OE
	Reduce integral to expression of the form $\int m + \dfrac{n}{1+4x^2}\, dx$	M1	
	Complete integration and reach $px^2 \tan^{-1} 2x + qx + r\tan^{-1} 2x$	M1	
	Obtain $\frac{1}{2}x^2 \tan^{-1} 2x - \frac{1}{4}x + \frac{1}{8}\tan^{-1} 2x$	A1	OE
	Use limits of $x = \frac{1}{2}$ and $x = \frac{1}{2}\sqrt{3}$ in the correct order, having integrated twice	DM1	
	Obtain answer $\frac{5}{48}\pi - \frac{1}{8}\sqrt{3} + \frac{1}{8}$ or exact equivalent	A1	
		7	

Question 116

State or imply $du = -\cos x\, dx$	B1	
Use $\sin 2x = 2\sin x\cos x$ and write the integral in terms of u	*M1	
Obtain $\pm 2\int \dfrac{(1-u)}{\sqrt{u}}\, du$ or equivalent	A1	
Integrate correctly to obtain $au^{\frac{1}{2}} + bu^{\frac{3}{2}}$	DM1	
Obtain correct $-4u^{\frac{1}{2}} + \dfrac{4}{3}u^{\frac{3}{2}}$	A1	
Correctly use limits $u = 2$ and 0 in an expression of the form $au^{\frac{1}{2}} + bu^{\frac{3}{2}}$ OR limits $x = \frac{3}{2}\pi$ and $\frac{1}{2}\pi$ in an expression of the form $a(1-\sin x)^{\frac{1}{2}} + b(1-\sin x)^{\frac{3}{2}}$	DM1	
Obtain $\frac{8}{3} - \frac{4}{3}\sqrt{2}$	A1	
	7	

Question 117

(a)	Use of correct product rule and correct chain rule	M1	$\frac{dy}{dx} = A\cos x\sqrt{2+\cos x} + \frac{B\sin x\sin x}{\sqrt{2+\cos x}}$
	Obtain $\frac{dy}{dx} = 2\cos x\sqrt{2+\cos x} - \frac{2\sin^2 x}{2\sqrt{2+\cos x}}$	A1	OE
	Equate the derivative to zero and obtain a horizontal 3 term quadratic equation or 4 term quartic equation in $\cos a$ If M0 earlier then needs that expression to be such that arrive at 3 term quadratic or 4 term quartic equation in cos x without further trig errors. The only error in the form of the differential allowed is for $(2+\cos x)^{-\frac{1}{2}}$ to be $(2+\cos x)^{+\frac{1}{2}}$ or $(2+\cos x)^{-\frac{3}{2}}$	*M1	Accept in $\cos x$. E.g. $3\cos^2 x + 4\cos x - 1 = 0$. E.g. $3\cos^4 x + 16\cos^3 x + 18\cos^2 x - 1 = 0$.
	Solve for cos a	DM1	$\left(\cos a = \frac{-2+\sqrt{7}}{3} \text{ or } 0.215\right)$ Allow presence of other solution(s).
	Obtain $a = 4.93$	A1	Allow more accurate, e.g. 4.929… even though question states 2 dp. If $x = 1.35$ leads to $x = 4.93$ award A1 BOD. If $x = 1.35$ and $x = 4.93$ award A0.
		5	
(b)	State or imply $du = -\sin x\,dx$	B1	OE If B0, max M1M1M1.
	Substitute throughout for u and du	M1	
	Obtain $-\int 2\sqrt{u}du$	A1	OE. Ignore limits if $-\int 2\sqrt{u}du$, but if $+\int 2\sqrt{u}du$, then must have correct limits $\int_1^3 2\sqrt{u}du$. (See final M1)
	Integrate to obtain $ku^{\frac{3}{2}} (+C)$	M1	Constant of integration not required
	Use correct limits correctly in an expression of the form $ku^{\frac{3}{2}}$ or $k(2+\cos x)^{\frac{3}{2}}$	M1	1 and 3 for u, or 0 and π for x.
	Obtain $\frac{4}{3}(3\sqrt{3}-1)$ or $4\sqrt{3}-\frac{4}{3}$ or $\frac{4}{3}\sqrt{27}-\frac{4}{3}$	A1	OE. Allow, e.g., $\sqrt{3}^3$ for $\sqrt{27}$. ISW but don't ignore e.g. multiplying throughout by 3. If the answer is changed from negative to positive value at end, then A0. Last M1A1 can use modulus, providing no errors seen.
		6	

Question 118

(a)	Divide to obtain quotient $x^2 + k$	M1	k is a constant.
	Obtain quotient $x^2 - 4$	A1	If quotient stated separately, mark at this stage.
	Obtain remainder 32	A1	If remainder stated separately, mark at this stage. Need not state which is quotient and remainder, but if stated wrongly, max 2/3. After a correct division, still allow the marks if then written as $x^2 - 4 + \frac{32}{x^2+4}$.

(b)	$\frac{1}{3}x^3 - 4x$	B1 FT	Follow *their* quotient of form $Ax^2 + B$.
	Obtain $p\tan^{-1}qx$ where $q = 2$ or $q = \frac{1}{2}$	M1	
	Obtain $16\tan^{-1}\frac{1}{2}x$	A1 FT	Follow *their* constant remainder, i.e. $\left(\frac{\textit{their}\text{ constant remainder}}{2}\right)\tan^{-1}\frac{1}{2}x$.
	Use limits correctly in an expression containing $p\tan^{-1}qx$ where $q = 2$ or $q = \frac{1}{2}$ and $rx^3 + sx$	M1	Terms need not be evaluated, e.g. $\left[8\sqrt{3} - 8\sqrt{3}\right] + 16\tan^{-1}\sqrt{3} - \left(\frac{8}{3} - 8 + 16\tan^{-1}1\right)$ or $\frac{8}{3} - 8$ can be $-\frac{16}{3}$, $16\tan^{-1}\sqrt{3}$ can be $\frac{16\pi}{3}$, $16\tan^{-1}1$ can be 4π.
	Obtain $\frac{4}{3}(\pi + 4)$ from full and correct working	A1	AG
		5	

Question 120

(a)	Use correct quotient rule NB the question asks for $f'(x)$ so need complete form	M1	Or correct product rule.
	Obtain correct derivative in any form, e.g. $\frac{4e^{2x}(e^{2x} - 3e^x + 2) - 2e^{2x}(2e^{2x} - 3e^x)}{\left(e^{2x} - 3e^x + 2\right)^2}$	A1	
	Equate *their* derivative to zero	*M1	Can be implied by numerator equated to zero fo quotient rule. $\left(8 = 6e^x\right)$
	Solve for x to obtain $x = \ln a$	DM1	a positive.
	Obtain $x = \ln\frac{4}{3}$ and $y = -16$	A1	No errors seen. Accept equivalent exact forms, e.g. $x = \ln\frac{8}{6}$.

(a)	**Alternative Method for Question 11(a)**		
	Complete method to express $f(x)$ in partial fractions	M1	As far as $p + \frac{q}{e^x - 2} + \frac{r}{e^x - 1}$ with values for p, q and r $\left(2 + \frac{8}{e^x - 2} - \frac{2}{e^x - 1}\right)$. Allow in $u\left(u = e^x\right)$.
	Differentiate to obtain $f'(x) = \frac{se^x}{\left(e^x - 2\right)^2} + \frac{te^x}{\left(e^x - 1\right)^2}$	*M1	Note: the question requires f'(x) so if they have substituted for e^x, they will also need chain rule.
	Obtain $f'(x) = \frac{-8e^x}{\left(e^x - 2\right)^2} + \frac{2e^x}{\left(e^x - 1\right)^2}$	A1	From correct work.
	Equate derivative to zero and solve for x to obtain $x = \ln a$	DM1	Must follow correctly to give a positive value of a.
	Obtain $x = \ln\frac{4}{3}$ and $y = -16$	A1	No errors seen. Accept $x = \ln\frac{8}{6}$, or equivalent.
		5	

(b)	State or imply $\frac{du}{dx}=e^x$	B1	
	Obtain $\int \frac{2u}{u^2-3u+2}\,du$ or equivalent Or₁ $\int\left(\frac{2}{u}+\right)\frac{8}{u(u-2)}-\frac{2}{u(u-1)}du$	B1	Correct expression in u. Condone missing du or missing integral but not both. Allow FT if using their partial fractions from **(a).**
	State or imply partial fractions of the form $\frac{A}{u-1}+\frac{B}{u-2}$ Or₁ $\frac{C}{u}+\frac{D}{u-2}+\frac{E}{u-1}$ Or₂ $\frac{2u-3}{u^2-3u+2}+\frac{3}{u^2-3u+2}=\frac{2u-3}{u^2-3u+2}+\frac{F}{u-2}+\frac{G}{u-1}$	B1 FT	Complete reduction to partial fractions. Correct form for *their* integrand.
	Use a correct method for finding a constant	M1	Available if they have incorrect form.
	Obtain correct $\frac{-2}{u-1}+\frac{4}{u-2}$ Or₂ $\frac{2u-3}{u^2-3u+2}+\frac{3}{u-2}-\frac{3}{u-1}$	A1	
	Integrate to obtain $a\ln(u-1)+b\ln(u-2)$ or equivalent	*M1	M0 if they have additional terms that do not cancel out.
	Obtain correct $-2\ln(u-1)+4\ln(u-2)$ or equivalent	A1FT	FT values of *their* partial fraction coefficients.
	Correctly use limits $u=5$ and 3 in an expression of the form $a\ln(u-1)+b\ln(u-2)$ or $x=\ln 5$ and $\ln 3$ in an expression of the form $a\ln(e^x-1)+b\ln(e^x-2)$	DM1	
	Obtain $\ln\frac{81}{4}$	A1	Accept $\ln 20.25$.
		9	

Question 121

(a)	Correct use of product rule to differentiate	*M1	$2\cos 2x(1+\sin 2x)+\sin 2x\times 2\cos 2x$, or $2\cos^2 x-2\sin^2 x+8\sin x\cos^3 x-8\cos x\sin^3 x$. All terms needed but could have errors in the coefficients.
	Obtain $2\cos 2x+4\cos 2x\sin 2x$	A1	OE
	Equate derivative to zero and solve for $2x$ or x	DM1	$2\cos 2x(1+2\sin 2x)=0 \Rightarrow x=\frac{1}{2}\sin^{-1}\left(-\frac{1}{2}\right)$ Condone if they only consider $\cos 2x=0$.
	Obtain $x=\frac{7}{12}\pi,\ y=-\frac{1}{4}$	A1	Mark degrees as a misread. The Q asks for an exact answer.
		4	
(b)	Use correct double angle formula to integrate Or use integration by parts and correct double angle formula	M1	$\int \sin 2x+\frac{1-\cos 4x}{2}dx$ Or $-\frac{1}{2}\cos 2x(1+\sin 2x)+\int\frac{1+\cos 4x}{2}dx$.
	Obtain $-\frac{1}{2}\cos 2x+\frac{x}{2}-\frac{1}{8}\sin 4x\ (+C)$	A1	Or $-\frac{1}{2}\cos 2x-\frac{1}{2}\cos 2x\sin 2x+\frac{x}{2}+\frac{1}{8}\sin 4x\ (+C)$
	Use limits 0 and $\frac{1}{2}\pi$ correctly in a solution containing $p\cos 2x$ and $q\sin 4x$	M1	$\frac{1}{2}+\frac{1}{4}\pi-0+\frac{1}{2}-0+0$
	Obtain $\frac{1}{4}\pi+1$	A1	
		4	

Question 122

Integrate to obtain $px^3\ln 3x + q\int x^2\,dx$	M1*	
Obtain $\frac{1}{3}x^3\ln 3x - \frac{1}{3}\int x^2\,dx$	A1	Or unsimplified equivalent.
Complete integration to obtain $\frac{1}{3}x^3\ln 3x - \frac{1}{9}x^3$	A1	
Use correct limits correctly in an expression of the form $rx^3\ln 3x + sx^3$	DM1	$9\ln 9 - 3 - \frac{1}{3}\ln 3 + \frac{1}{9}$ An exact expression for *their* integral.
Obtain $\frac{53}{3}\ln 3 - \frac{26}{9}$	A1	Or 2-term equivalent.
	5	

Question 123

Commence integration by parts and reach $Ax^2\sin\frac{1}{3}x \pm \int Bx\sin\frac{1}{3}x\,dx$	*M1	OE BOD on ± otherwise scores 0/6.
Obtain $3x^2\sin\frac{1}{3}x - \int 6x\sin\frac{1}{3}x\,dx$	A1	OE Allow 3×2 for 6.
Complete integration by parts and reach $Ax^2\sin\frac{1}{3}x + Bx\cos\frac{1}{3}x + C\sin\frac{1}{3}x$	*DM1	OE
Obtain $3x^2\sin\frac{1}{3}x + 18x\cos\frac{1}{3}x - 54\sin\frac{1}{3}x$ oe	A1	Allow 6 × 3 for 18, 9 × 6 for 54 OE.
Substitute limits correctly in an expression of the form $Ax^2\sin\frac{1}{3}x + Bx\cos\frac{1}{3}x + C\sin\frac{1}{3}x$, where $ABC \neq 0$	DM1	Dependent on both previous M1 marks. Need to use $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$, and $\cos\frac{\pi}{3} = \frac{1}{2}$ to obtain $A\pi^2\frac{\sqrt{3}}{2} + \frac{B\pi}{2} + \frac{C\sqrt{3}}{2}$.
Obtain answer $\frac{3\sqrt{3}}{2}\pi^2 + 9\pi - 27\sqrt{3}$ or exact equivalent ISW	A1	Allow $\frac{\sqrt{27}}{2}$ for $\frac{3\sqrt{3}}{2}$, $\frac{18}{2}$ for 9, $\frac{54}{2}$ for 27, $\sqrt{2187}$ for $27\sqrt{3}$ etc.
Alternative Method for first 4 marks:		
Commence integration by parts and reach $Ax^2\sin\frac{1}{3}x + Bx\cos\frac{1}{3}x$	*M1	
Obtain $3x^2\sin\frac{1}{3}x + 18x\cos\frac{1}{3}x$	A1	OE Allow 6 × 3 for 18.
Complete integration by parts and reach $Ax^2\sin\frac{1}{3}x + Bx\cos\frac{1}{3}x + C\sin\frac{1}{3}x$	*DM1	
Obtain $3x^2\sin\frac{1}{3}x + 18x\cos\frac{1}{3}x - 54\sin\frac{1}{3}x$	A1	OE Allow 6 × 3 for 18, 9 × 6 for 54, OE.
	6	

Question 124

(a)	State or imply the form $\frac{A}{(1+x)}+\frac{Bx+C}{(4+x^2)}$	**B1**	
	Use a correct method for finding a constant Even with incorrect PF denominators	**M1**	$A(4+x^2)+(Bx+C)(1+x)+=-7x^2+2x-6$
	Obtain one of $A=-3$, $B=-4$ and $C=6$	**A1**	
	Obtain a second value	**A1**	
	Obtain a third value	**A1**	
			Special Case 1: $\frac{A}{(1+x)}+\frac{C}{(4+x^2)}+$ Find A, **M1 A1**. Max 2/5. **Special Case 2**: $\frac{A}{(1+x)}+\frac{Bx}{(4+x^2)}+$ Find A, **M1 A1**. Max 2/5.
		5	
(b)	Obtain term $-3\ln(1+x)$	**B1 FT**	OE FT $A\ln(1+x)$
	Obtain term $-2\ln(4+x^2)$	**B1 FT**	OE FT $\frac{B}{2}\ln(4+x^2)$
	Obtain integral of the form $c\tan^{-1}dx$ with $d\neq 1$ following separation into two expressions	**M1**	$d=\frac{1}{2}$ or 2 only.
	Obtain $3\tan^{-1}\frac{x}{2}$	**A1 FT**	FT $\frac{C}{2}\tan^{-1}\frac{x}{2}$
	Substitute correct limits correctly in an expression (obtained correctly) of the form $a\ln(1+x)$, $b\ln(4+x^2)$, and $c\tan^{-1}\left(\frac{1}{2}x\right)$, where $a, b, c\neq 0$.	**M1**	$a\ln(3)+b\ln(8)-b\ln(4)+c\left(\frac{1}{4}\pi\right)$, where $a, b, c\neq 0$. Do not allow slips, and must get to $c\left(\frac{1}{4}\pi\right)$.
	Obtain answer $\frac{3}{4}\pi-\ln 108$	**A1**	Must be in the form $a\pi-\ln b$.
		6	

Question 125

Use correct double angle formula	**M1***	$\frac{1}{2}(1+\cos 10x)$
Obtain $\frac{3}{20}\sin 10x+\frac{3}{2}x$	**A1**	OE
Use correct limits correctly	**DM1**	$\frac{3}{20}(-0)+\frac{15\pi}{40}-\frac{12\pi}{40}$
Obtain $\frac{3}{20}+\frac{3}{40}\pi$	**A1**	Or exact simplified equivalent.
	4	

Question 126

(a)	Differentiate $\cos^2 x$ to obtain $-2\sin x\cos x$	**B1**	OE Could be stated as $-\sin 2x$.
	Use correct product rule	***M1**	With a '+' in the middle.
	Obtain derivative $-10\sin 2x\sin x\cos x+10\cos 2x\cos^2 x$	**A1**	OE
	Equate derivative to zero and obtain an equation in one trig function	**DM1**	Trigonometry formulas used need to be correct.
	Obtain $3\tan^2 x = 1$, $4\sin^2 x = 1$ or $4\cos^2 x = 3$ or $\cos 3x = 0$	**A1**	OE
	Obtain $x = \frac{1}{6}\pi$ only	**A1**	
	Alternative Method for the Question 11(a)		
	Use double angle formula to obtain $y = 10\sin x\cos^3 x$	**B1**	
	Use correct product rule	***M1**	
	Obtain derivative $10\cos^4 x - 30\sin^2 x\cos^2 x$	**A1**	OE
	Equate derivative to zero and obtain an equation in one trig function	**DM1**	Trigonometry formulas used need to be correct.
	Obtain $3\tan^2 x = 1$, $4\sin^2 x = 1$ or $4\cos^2 x = 3$ or $\cos 3x = 0$	**A1**	OE
	Obtain $x = \frac{1}{6}\pi$ only	**A1**	

1(a)	**Alternative Method 2 for the Question 11(a)**		
	Use double angle formula to obtain $y = \frac{5}{2}\sin 2x(\cos 2x + 1)$	**B1**	
	Use double angle formula to obtain $y = \frac{5}{4}\sin 4x + \frac{5}{2}\sin 2x$	***M1**	
	Obtain derivative $5\cos 4x + 5\cos 2x$	**A1**	
	Equate derivative to zero and obtain an equation in $\cos 2x$	**DM1**	$2\cos^2 2x + \cos 2x - 1 = 0$
	Obtain $\cos 2x = \frac{1}{2}$ only	**A1**	
	Obtain $x = \frac{1}{6}\pi$ only	**A1**	
		6	

(b)	$\frac{du}{dx} = -\sin x$	**B1**	SOI
	Reach an integral of the form $\int Au^3\,du$	***M1**	OE The question requires use of the substitution method.
	Obtain $-10\int u^3 du$	**A1**	OE Ignore limits, but check order of limits if no minus sign.
	Substitute correct limits correctly in an expression of the form Cu^4 or $C\cos^4 x$	**DM1**	$u = 1$ and $u = \frac{\sqrt{2}}{2}$ $x = 0$ and $x = \frac{1}{4}\pi$ $-10\int_1^{\frac{1}{\sqrt{2}}} u^3 du$ or $10\int_{\frac{1}{\sqrt{2}}}^1 u^3 du$ Allow the correct answer from the correct integration and relevant limits to imply M1.
	Obtain answer $\frac{15}{8}$ or 1.875	**A1**	WWW ISW
		5	

Question 127

Commence integration by parts and reach $Ax^2\tan^{-1}2x \pm \int x^2 \frac{B}{C+Dx^2}dx$	***M1**	OE
Obtain $\frac{1}{2}x^2\tan^{-1}2x - \int \frac{x^2}{1+4x^2}dx$	**A1**	OE
Integrate $\int \frac{x^2}{1+4x^2}dx$ to obtain an expression of the form $p\tan^{-1}2x + qx$	**DM1**	
Complete integration and obtain $\frac{1}{2}x^2\tan^{-1}2x + \frac{1}{8}\tan^{-1}2x - \frac{1}{4}x$	**A1 FT**	OE FT *their* constant quotient and remainder from **(a),** and $\frac{k}{8}\tan^{-1}2x - \frac{k}{4}x$ from *their* $\frac{kx^2}{1+4x^2}$.
Substitute limits correctly in an expression of the form $Fx^2\tan^{-1}2x + G\tan^{-1}2x + Hx$	**DM1**	Need some evidence that they have considered the lower limit, e.g. sight of 0 in the working. No need to evaluate trigonometry. If in stages, then the 0 needs to be seen for each part.
Obtain answer $\frac{1}{16}\pi - \frac{1}{8}$ or exact one- or two-term equivalent with trigonometry evaluated	**A1**	
	6	

Question 128

(a)	$\frac{d}{dx}\sqrt{\sin 2x} = \frac{2\cos 2x}{2\sqrt{\sin 2x}}$ Accept $k\frac{\cos 2x}{\sqrt{\sin 2x}}$	**B1**	SOI
	Use correct product rule	**M1***	
	Obtain correct derivative in any form	**A1**	E.g. $\frac{dy}{dx} = -\sin x\sqrt{\sin 2x} + \cos x \times \frac{2\cos 2x}{2\sqrt{\sin 2x}}$.
	Equate the derivative to zero and obtain a horizontal equation	**DM1**	E.g. $-\sin a \sin 2a + \cos a \cos 2a = 0$
	Use correct trig formulae to obtain an equation in one trig function	**DM1**	E.g. $\cos 3a = 0$ or $\sin^2 a = \frac{1}{4}$.
	Obtain $a = \frac{1}{6}\pi$	**A1**	Exact answer in radians only.
		6	
(b)	Use $[\pi]\int y^2\,dx$	**M1***	
	Use double angle formula to obtain a form that can be integrated directly, e.g. $\int \cos^2 x \sin 2x\,dx = \int 2\cos^3 x \sin x\,dx$	**DM1**	Or $\int \frac{1}{4}\sin 4x + \frac{1}{2}\sin 2x\,dx$, or $\int 2(u - u^3)\,du$ by substituting $u = \sin x$.
	Obtain $-(\pi) \times \frac{1}{2}\cos^4 x$	**A1**	Or $-[\pi]\left(\frac{1}{16}\cos 4x + \frac{1}{4}\cos 2x\right)$, OE.
	Use correct limits correctly	**DM1**	$\left[-[\pi] \times \frac{1}{2}\cos^4 x\right]_0^{\frac{\pi}{2}} = -[\pi]\left(\frac{1}{2}\cos\left(\frac{1}{2}\pi\right) - \frac{1}{2}\right)$, or $-[\pi]\left(\frac{1}{16}\cos 2\pi + \frac{1}{4}\cos \pi - \frac{1}{16} - \frac{1}{4}\right)$ OE.
	Obtain final answer $\frac{1}{2}\pi$	**A1**	If the factor of π is missing throughout, allow the first 4 marks and A0 here.
		5	